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CONSIDERING THICKNESS IN CYLINDRICAL DEVELOPABLE MECHANISMS

A Thesis in

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by

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ABSTRACT

The field of Developable Mechanisms (DMs) holds potential to support the creation of compact mechanisms and to reduce general mechanism footprint, both of which are important considerations for many application areas. However, currently available literature in the field predominantly discusses the design, motion, and construction of DMs when they are modeled as zero-thickness geometries that conform to or lie within the developable surfaces from which the DMs take their shape. Considering zero-thickness introduces potential difficulties when fabricating DMs as fabrication requires thickness which can lead to unexpected behavior. Moreover, this work posits that this approach might lead to the omission of potentially valuable DM variants which are made possible solely by the virtue of body thickness. The current work addresses this gap by exploring the changes that are necessary to support the modeling and consideration of shape-conforming mechanisms with inherent thickness similar to DMs. This study analyzes the utility of a reference surface construct as a modeling guide when it works under the zero-thickness assumption and explores its limitations. The study then proposes an alternative novel approach of modeling DMs on thick bodies using boundary surfaces as the guiding construct.

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Chapter 1

Introduction

Mechanism design is an ever-expanding field of research which takes inspiration and improvements from a variety of fields spanning from core scientific disciplines like biology [1,2] to art and fiction [3]. This field has profound effects on almost every modern system ranging from home appliances to living environments to space exploration. As such, research efforts that make mechanisms more efficient, spatially compact, and generally sustainable are of key interest and importance. To aid such endeavors, design engineers and researchers often seek inspiration from seemingly unrelated fields to conceptualize novel mechanism design methods. For example, adoption of the traditional art of paper folding known as ‘origami’ has propelled innovation in compact folding mechanisms better known as ‘deployable mechanisms’ (for example [4–6]). Additionally, origami has found its application as an enabler of technology by enabling the creation of compact systems in robotics [7], architecture [8], and even biomedical sciences [9]. This practice of amalgamating traditional engineering with art not only helps in the creation of new mechanisms but also informs our scientific understanding of the application-based constraints that the problem case exhibits. In the case of origami-based designs, one of the key concerns while designing and constructing the mechanism is addressing thickness while replicating paper-like motion. Therefore, a plethora of research has been done to adapt origami techniques into modern mechanisms by translating and redefining constraints presented by structurally viable materials to deliver the same mobility as a folded piece of paper [10,11].

This attitude towards adaptation of origami-based mechanisms in mechanism design has led to the development of new and innovative families of mechanisms. Developable mechanisms are one such family, taking inspiration from the concept of developable surfaces. Developable surfaces are resulting contours that a planar sheet can take when bent without tearing and stretching [12]. Developable surfaces exist in many forms but can be narrowed down as constituents of four fundamental shapes: planes, generalized cylinders, generalized cones, and tangent-developable surfaces [14]. Mechanisms that conform to such surfaces in any particular configuration without affecting their own mobility are known as developable mechanisms [13]. There are four separate classes of developable mechanisms that follow the general shape of their respective developable surfaces. These different categories of DMs present different motion, construction opportunities and challenges. Extensive work has been done in understanding the construction and motion of cylindrical and conical developable mechanisms [15,16].

Developable mechanisms are typically characterized by their ability to conform to or within the façade of the body on which they are mounted, therefore providing space-saving and material-saving potential. DMs hold potential to aid such design requirements [21], addressing rising interests in the creation of compact mechanisms and reduction in general mechanism footprint especially in surgical devices [17,18] and space exploration-based applications [19,20]. However, existing work on DMs solely discusses their construction and motion when they are conceptualized as having no thickness (known as the zero-thickness assumption). This method of modeling a DM, though useful, stands to omit modeling possibilities and particular constraints that only become apparent when dealing with body thickness. For instance, using the body's thickness can enable the creation of unique linkage shapes, or introduce flexibility in the placement of hinges. Moreover, modeling with zero-thickness and then fabricating with thickness can lead to unexpected issues pertaining to motion and interference.

This study shows how making gradual relaxations to the rules to model a DM (and the implicit conditions that come in action when the zero-thickness assumption is used) can help expand the design space and result in a wider array of general shape conforming mechanisms. The introduction of thickness during the modeling process of DMs leads to identification of a conceptual ambiguity, as mechanisms formed using thickness start shifting into the larger domain of shape conforming mechanisms from developable mechanisms. This study investigates how such mechanisms are constructed and also formulates the kinematics of the mechanisms formed using thickness. Different cases discussed in this study are shown to serve different utilities in terms of design freedom and complexity. The study then explores the implicit ambiguity that occurs when thickness is introduced to the process of modeling DMs and presents modeling constructs that can help model DMs in thickness while navigating through the ambiguity.

Chapter 2

Background

This chapter provides an overview of the current framework and concepts used in the definition of developable surfaces and in designing of developable mechanisms. This chapter also presents the current work being done in developable mechanisms and its dependency on the current framework.

2.1 Developable Surfaces

Mathematically speaking, developable surfaces are surfaces that have a Gaussian curvature of zero and can be mapped onto a plane isometrically, meaning that the mapping can take place without any distortion of the surface curve [14,22]. In simpler terms, a developable surface is any contour which a planar sheet can take without tearing or stretching when bent in a certain way. Owing to their unique construction, developable surfaces find their application in a wide variety of fields like architecture [23], garment design [24] and creation of innovative compliant mechanisms [25]. Moreover, developable surfaces are typically divided into four fundamental shapes: planes, cylinders, cones, and tangent-developable surfaces. Building on this, the aforementioned four classifications are determined by ruling lines, which are straight lines that govern the contour of a developable surface [13]. Developable surfaces are categorized according to the patterns followed by the ruling lines of the surface. For planar surfaces, ruling lines can exist in any orientation within the plane. For generalized cylindrical surfaces, all ruling lines should be parallel to each other. For generalized cones, the ruling lines converge or extend to intersect at a point. And lastly, in the case of tangent-developed surface, the ruling lines are tangent to a spatial curve.

2.2 Developable Mechanisms and Zero-Thickness Assumption

The concept of using developable surfaces as a guiding construct to make mechanisms was first presented by Nelson et al. [13] in an attempt to conceptualize space-effective mechanisms. In that work, the authors define mechanisms as DMs if they satisfy three criteria:

- they are contained within or conform to a developable surface when both are modeled with zero-thickness
- do not require the developable surface to deform to enable the mechanism's movement
- have mobility

The last two criteria of the definition address the requirement of mobility and non-deformation, both of which are intuitively understandable as they pertain to the standard functionality of mechanisms and definition of developable surfaces. Both criteria have important implications. For example, Hyatt et al. [26] in their study present a modeling paradigm that uses flat patterns to create developable mechanism. Similarly, work that couples developable mechanisms and Lamina Emergent Mechanisms [27–29] remains an active area of study. The interpretations presented in these studies are possible due to the criteria that discuss mobility and non-deformation.

The first criterion of the definition, however, relies upon the zero-thickness assumption and demands a deeper examination for the current work, which seeks to permit thickness in mechanism design. This criterion effectively dictates that a mechanism in its conformed state, when modeled with zero-thickness, should resemble the developable surface through which it was modeled. The modeling paradigm of hypothetically using zero-thickness to make the mechanism and the surface coincident to each other is the zero-thickness assumption. This is an elegant approach that bridges the gap between concept and construction and aides in modeling a developable mechanism.

While the zero-thickness assumption has utility in conceptualizing and modeling the kinematics of DMs, it may lead to omission of some practical considerations that are inherent to bodies with thickness. Primarily, the formalization of the zero-thickness assumption within the modeling criteria itself requires some reframing. Conceptually, a zero-thickness DM may conform to a zero-thickness surface, but it is counter-intuitive to consider a zero-thickness DM to be embedded within a zero-thickness surface. Furthermore, investigation is required to understand how the thickness of a body affects the mechanism's functionality, capabilities, construction, and the design space available as these are potential study areas that escape exploration when DMs are modeled under the zero-thickness assumption. Additionally, capturing a new perspective that steps away from the current usage of the zero-thickness assumption holds potential to extend the elegant solutions that the zero-thickness assumption provides. This in turn, opens room for creation of even more robust and exhaustive set of mechanisms.

2.3 Reference Surface

In Nelson's introductory work [13] on developable mechanisms, two conditions were introduced to assist and guide the design of DMs. The first of these conditions, known as the hinge axis ruling condition, states that the joint axes of a DM must be coincident with the ruling lines on the developable surface. The hinge axis ruling condition governs the mobility of a developable mechanism as different alignments of ruling lines and therefore hinge axes map to different mechanisms [13]. The second condition, referred to as the link shaping condition, states that the links must conform to the developable surface. These conditions help distinguish DMs from other mechanisms, but their interpretation has traditionally been dependent upon the zero-thickness assumption.

This dependency of interpretation also brings us to introduce ‘reference surface’, a construct used to currently model developable mechanisms. A reference surface (as interpreted in the context of the zero-thickness assumption) refers to the surface to which the DM conforms; it also defines the location of a mechanism’s joints. It may or may not be represented by a physical surface [15]. In essence, a reference surface, when interpreted in the current framework of the zero-thickness assumption, is a non-physical conceptualization of the parent developable surface. A reference surface is an important tool because of the ease in modeling and analysis it provides when making DMs. For that reason, reference surfaces are ubiquitous in the subsequent work done in the field. For example, work done by Greenwood et al. [15] defines concepts like intramobility, extramobility and transmobility as a developable mechanism’s ability to move inside, outside, and both inside and outside of the reference surface, respectively. Work done by Butler et al. [30,31] uses these concepts to study the behavior of cylindrical developable mechanisms, while Woodland et al. [32] incorporate reference surface in their analysis of conical DMs.

Interestingly, working under the zero-thickness assumption, the reference surface exhibits peculiar behavior. Owing to the zero-thickness interpretation of the ‘hinge axis ruling condition’ and the ‘link shaping condition’, the reference surface currently assumes the function of providing both the shape of the linkages in the mechanism and the limiting positions where the hinges of the mechanism can be placed. Having multiple functions associated with a singular construct can be useful but also unnecessarily constraining. For example, with zero-thickness link shaping can only happen according to the reference surface.

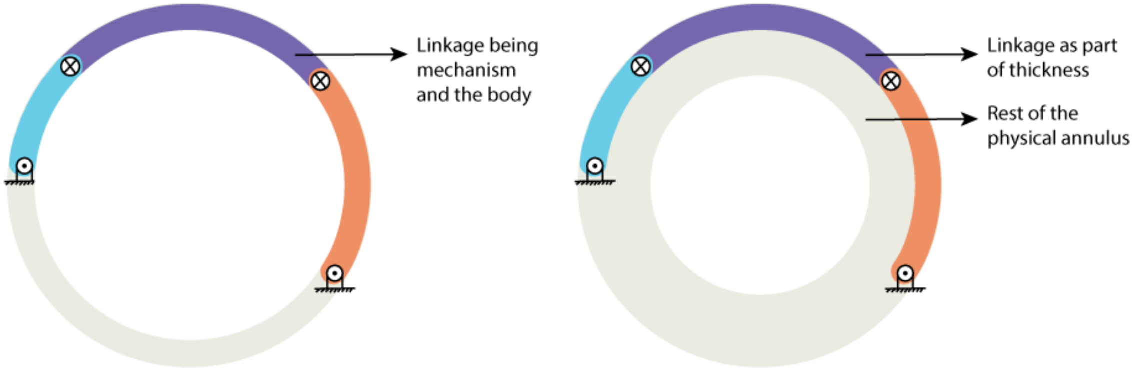
This occurrence of interdependencies shows that the entire framework of modeling DMs works under the zero-thickness assumption. The introduction of thickness therefore establishes the need to evaluate the exhaustiveness of the current mode of modeling developable mechanisms. Though the usage of a zero-thickness construct to conceptualize mechanisms is elegant, additional perspectives may be gained by relaxing the zero-thickness assumption.

Chapter 3

Developable Mechanisms for Thick Bodies

Traditionally, developable mechanisms are defined through their ability to conform to the parent developable surface from which they are modeled. This approach works well within the zero-thickness framework as it is impossible to have a linkage within the thickness of a construct that has no thickness. This chapter builds up on this paradigm to understand the implicit functioning of the zero-thickness assumption and discusses the kinematics of a developable mechanism thus created using it. Following this, gradual changes are made to the traditional way of modeling developable mechanisms by induction of thickness during the process of modeling to explore how it can open room for more design variations.

Developable mechanisms are currently made using the zero-thickness assumption. In the zero-thickness approach a DM is fabricated such that the linkages themselves correspond to the physical form of the body i.e. the modeled mechanism and the physical form are one and the same. However, introducing thickness in the process of modeling acts as a new parameter. It allows both the above mentioned roles (i.e. being the mechanism and the body) to be separated as the linkages of mechanism can be created as parts of the thick annulus’s cross-section rather than being the entire annulus itself (as done in the traditional method). This becomes clearer from Fig. 3-1 where Fig. 3-1 (a) represents a mechanism obtained through traditional modeling whereas Fig. 3-1 (b) represents a mechanism that can be formed when thickness is included while modeling. Inclusion of thickness increases the link shaping options available to the designer when making a DM. However, this can also potentially lead to creation of mechanisms that do not conform to a reference surface (which is central for a mechanism to be a DM).



(a) (b)

Figure **3-1**: DMs where (a) Linkage is the entire thickness and (b) Linkage is part of the thickness

This chapter identifies the possibility of such a deviation as a grey area as there has been little discussion pertaining to thickness of body of a DM in the literature. Seymour et al. [17] discussed variations of DMs that utilize thickness and cross-sectional kinematics which do not resemble traditional DMs, but a study on exact frameworks that discuss how thickness can be used to create DMs is lacking. We call these mechanisms that use thickness of the body, 'thick DMs' and they are primarily identified by their utility that they share with traditional DMs which is 'their ability to conform to the façade of the body on which they are mounted'. Based on the relaxations made, the chapter discusses two thick DM constructs and their kinematics. This shift fundamentally expands the design space and empowers us to consider mechanisms that might be omitted from the traditional framework.

3.1 Reference Surface Thickening Approach

Presently, to create a physical developable mechanism, designers generally model a developable mechanism using the tools and following the conditions mentioned in the previous chapter and then thickening the model symmetrically around the reference surface. We refer to this approach as ‘reference surface thickening approach’ from this point (shown in Fig. 3-2).

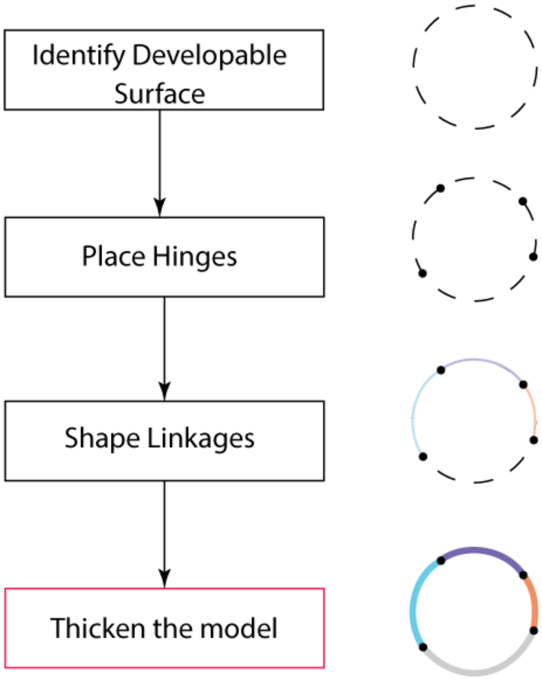


Figure **3-2**: Design process flow of the reference surface thickening approach

When constructing DMs using the reference surface thickening approach, designers intend to create thick mechanisms that conform to the reference surface of their model. However, the act of thickening the model inadvertently leads to creation of a mechanism that not only conforms to the reference surface of the skeletal model but also the boundary surfaces of the annulus of the contraption thus formed. This becomes evident from Fig. 3-3 where the basic zero-thickness model in Fig. 3-3 (a) thickens to become the mechanism in Fig. 3-3 (b) with Surface A and Surface B as the boundary surfaces to which the mechanism conforms.

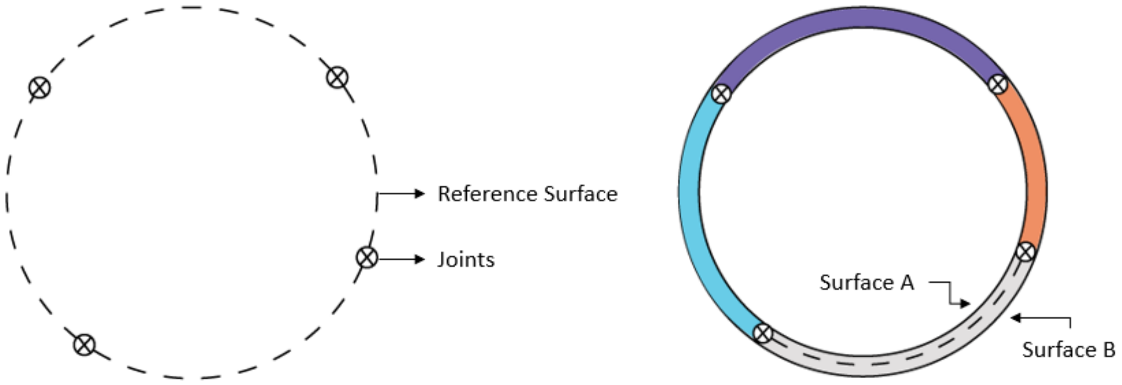


Figure **3-3**: (a) Reference Skeleton and (b) Final Thickened mechanism in reference surface thickening approach

Currently, the reference surface acts as the construct that controls the hinge placement and the linkage shape. Among these two, the linkage shaping role exists solely because the reference surface is the only surface to which the developable mechanism conforms. The presence of two other surfaces to which the mechanism conforms means the reference surface does not need to perform both the roles and therefore its functions can be decoupled. This possibility of decoupling brings us to an important decision which is, given the roles that the reference surface can take, which one should it be associated with. Traditionally, the reference surface is identified with its ability to conform to the DM, but that practice can also be interpreted as an implication of the reference surface working under the zero-thickness assumption.

3.1.1 Kinematics

This section discusses the kinematics of a four bar regular cylindrical DM created using the reference surface thickening approach. Formulating and understanding the kinematics of a mechanism helps us understand its construction in a well laid out analytical method and is a step towards mechanism synthesis. For purposes of this study, interference is not considered while discussing these frameworks.

In Fig. 3-4, if R is the radius of the reference surface and O the origin with Φ_P the angle that hinge point P takes from the ground and Φ_1, Φ_2, Φ_3 and Φ_4 the angles between the hinges then,

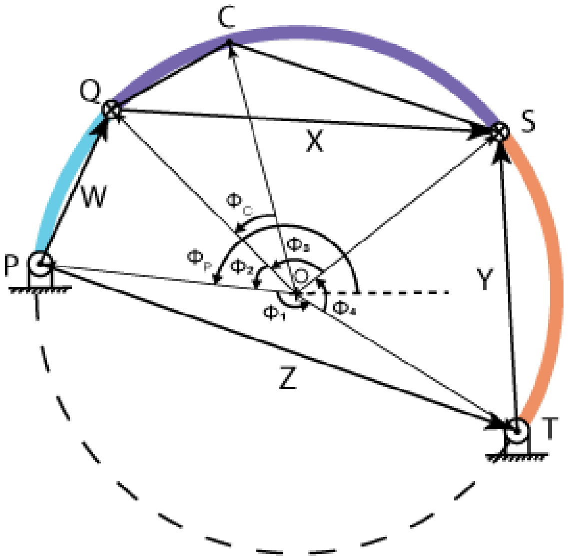


Figure **3-4**: Kinematic breakdown of the zero-thickness model used in the reference surface thickening approach

Position equations for the four hinge positions

$$\mathbf{P} = R\, e^{\mathbf{j}\, \Phi_P}$$

$$\mathbf{Q} = R\, e^{\mathbf{j}\, (\Phi_P - \Phi_2)}$$

$$\mathbf{S} = R\, e^{\mathbf{j}\, (\Phi_P - \Phi_2 - \Phi_3)} \qquad (1)$$

$$\mathbf{T} = R\, e^{\mathbf{j}\, (\Phi_P + \Phi_1)}$$

Link equations

$$\mathbf{W} = \mathbf{Q} - \mathbf{P} \Rightarrow \mathbf{W} = R\,(e^{\mathbf{j}(\Phi_P - \Phi_2)} - e^{\mathbf{j}\Phi_P})$$
$$\mathbf{X} = \mathbf{S} - \mathbf{Q} \Rightarrow \mathbf{X} = R\,(e^{\mathbf{j}(\Phi_P - \Phi_2 - \Phi_3)} - e^{\mathbf{j}(\Phi_P - \Phi_2)}) \qquad (2)$$
$$\mathbf{Y} = \mathbf{S} - \mathbf{T} \Rightarrow \mathbf{Y} = R\,(e^{\mathbf{j}(\Phi_P - \Phi_2 - \Phi_3)} - e^{\mathbf{j}(\Phi_P + \Phi_1)})$$
$$\mathbf{Z} = \mathbf{T} - \mathbf{P} \Rightarrow \mathbf{Z} = R\,(e^{\mathbf{j}(\Phi_P + \Phi_1)} - e^{\mathbf{j}\Phi_P})$$

Linkage Equations

The elegance and the simplicity of the reference surface thickening approach becomes apparent when we formulate the linkage equations of a mechanism made using this construct. Given that all the linkages in this construct are supposed to be on the reference surface, for any point C on the linkage making an angle Φ_C with hinge Q can be represented by,

$$\mathbf{C} = R\, e^{\mathbf{j}(\Phi_P - \Phi_2 - \Phi_C)} \qquad (3)$$

In equation sets (1), (2) and (3) the distance remains constant (i.e. R) as all the hinge points and the points on the linkages lie on the reference surface.

3.2 Reference surface acting only as hinge anchor

The zero-thickness assumption, as it is used in the reference surface thickening approach, is elegant because it aggregates five surfaces into one, thereby simplifying the design process. These surfaces are 1) the outer boundary surface of the annulus, 2) the outer surface of the linkage, 3) the surface on which the hinges are positioned, 4) the inner surface of the linkage and 5) the inner boundary surface of the annulus. Here,

- Boundary Surface is the imaginary boundary of the annulus on which the mechanism is being modeled. These surfaces demarcate the extent of the to which the thickness of the body can extend.
- Linkage Surface is the physical peripheral surface of the linkage.

In the reference surface thickening approach, these surfaces are merged into the reference surface and only materialize at the end stage when the linkages are thickened (see Fig. 3-5). Moreover, owing to the way the reference surface thickening approach is utilized, the boundary surfaces of the annulus are still merged with their respective linkage counterparts thereby finally resulting in two surfaces (as seen in Fig. 3-3 with surface A and B).

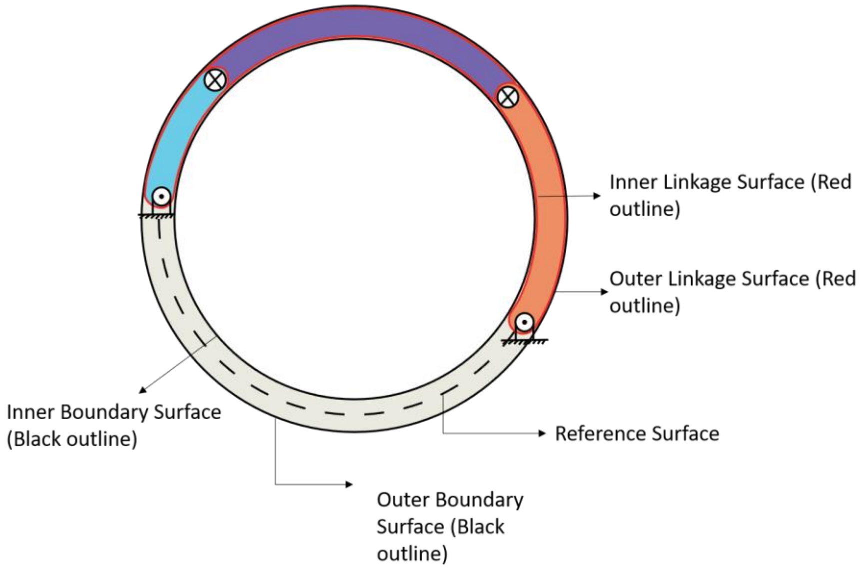


Fig **3-5**: 5 surfaces in a developable mechanism

Keeping this in consideration, if the roles performed by the reference surface are decoupled, the number of surfaces under consideration can increase to provide more design control. This act of decoupling also hints at stepping away from what is traditionally understood as a developable mechanism and opens room for more flexibility during design.

A logical step would be to follow a modeling construct where the reference surface still acts as the hinge placement guide while the linkage shape is set free and is only constrained by the boundary surfaces of the thick cylinder as shown in Fig. 3-6. This approach, unlike traditional practice, associates the role of hinge placement to the reference surface and decouples it from link shaping. Therefore, in a construct like this the reference surface would be a regular cylindrical surface suspended within the thickness of a regular thick cylinder and locations on its cross-sectional periphery would be the only positions where the hinges of the mechanisms can be placed. Meanwhile, instead of being governed by the reference surface, the linkages can take any shape as long as they are accommodated by the thickness of the body and therefore contained within the boundary surfaces. Fig. 3-7 shows the design process of such a modeling construct.

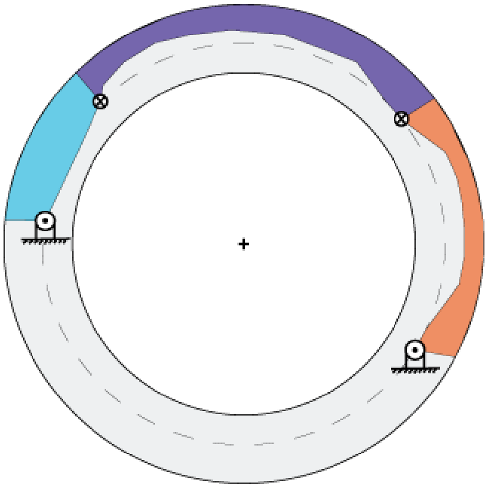


Figure **3-6**: Reference Surface as the hinge anchor but not the link shaping guide

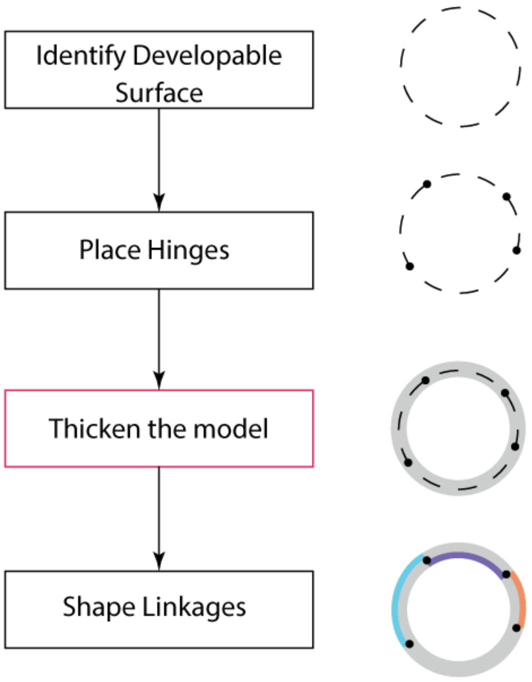


Figure **3-7**: Design process flow of the reference surface as the hinge anchor construct

But deviation from the traditional way of conceptualizing linkages opens up room for discussion on linkage thickness as reviewed earlier. For any mechanism to be shape conforming, it is required that all its surfaces in at least one configuration subsume within the entire façade of the body on which the mechanism is mounted. That effectively implies that in a particular configuration at least one surface of all the linkages should coincide with their respective boundary surface i.e. inner surface of a linkage should coincide with the inner boundary surface of the annulus and the outer surface of the linkage should coincide with the outer boundary surface of the annulus. For simplicity, we will call the boundary surface that a linkage's surface needs to coincide with the 'active surface' for that linkage. This is aptly represented in Fig. 3-8. The figure shows 3 linkages (A, B and C) of a 4-bar mechanism. For linkages A and C, their outer linkage surface coincides with the outer boundary surface of the annulus and therefore the outer boundary surface becomes the active surface for these linkages. However, for linkage B, its inner linkage surface coincides with the inner boundary surface which therefore becomes linkage B’s active surface. Linkage surfaces can also coincide with both the boundary surfaces.

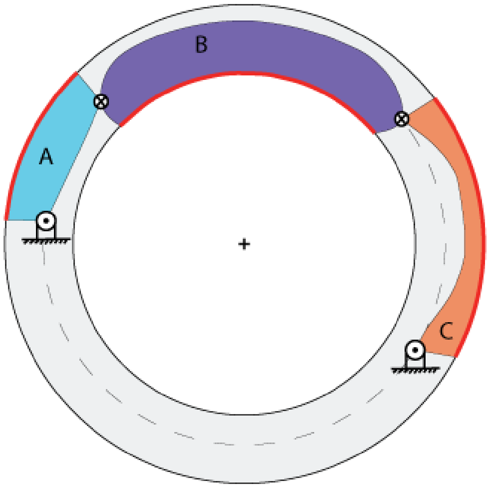


Figure **3-8**: Linkages with their respective active surfaces

This method of modeling a thick developable mechanism exhibits two distinctive merits. Firstly, it stands to incorporate more variations of mechanisms that were otherwise being omitted in the earlier modeling iterations and traditional developable mechanisms. This construct uses the full thickness of the thick body and leverages that thickness to attain unique shapes for its linkages (See Fig. 3-9). Secondly, because the hinge positions are still anchored to a regular cylindrical reference surface suspended in the thick body, all the work done so far except linkage shaping to understand and interpret the kinematic behavior of developable mechanisms in the literature remains applicable to this construct.

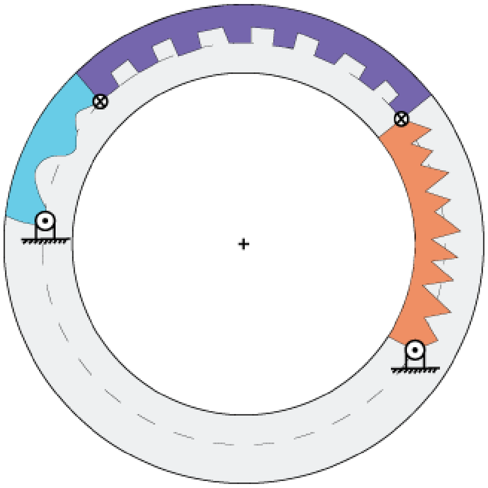


Figure **3-9**: Allowance of arbitrary link shapes within the thick annulus

This construct is a useful alternative for modeling thick developable mechanisms because it permits a wide variety of mechanisms while ensuring that prior work on kinematic behavior is still applicable. This is especially true when the designer is working under conditions where the hinge placement is constrained to a circular periphery.

3.2.1 Kinematics

This section discusses and formulates the kinematics of a four-bar regular cylindrical thick DM, modeled using the reference surface as just the hinge anchor construct. The position and link equations of such a mechanism are discussed. We also discuss and formulate the boundary conditions that can contain and represent flexibly shaped linkages made in this construct. The methods presented in this section can be used to fabricate mechanisms encapsulated by this construct. Interference is not considered.

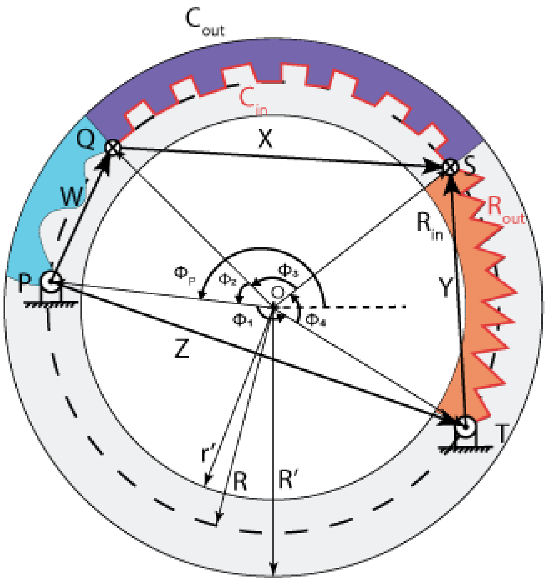


Figure **3-10**: Kinematic breakdown of the construct that uses reference surface just for hinge anchoring (Surface-based Linkage equation formulation)

Position and Link equations

As mentioned in section 3.2, in the construct where the reference surface acts just as the hinge anchoring surface, the kinematics related to position and link equations do not change when compared with the reference surface thickening approach. This is again observable from Fig. 3-10 where the reference surface, mechanism’s hinges and the links are same as Fig. 3-4. This means that equation set (1) and (2) sufficiently represent the position and link equations of this construct.

Linkage equations

Surface-based linkage equations,

Linkages exhibit interesting behavior in this construct. Given that this construct provides us control over the linkage shaping of the mechanism, it subsequently implies that we also gain control over the thickness of the linkage. This implies that while making a linkage in this construct, we endorse the responsibility of defining both the linkage surfaces. And the linkage is represented by the area enclosed by these surfaces.

In Fig. 3-10 let, r' is the radius inner boundary surface of the annulus and R' is the radius of the outer boundary surface of the annulus. If the inner linkage of the surface of the coupler (say C_{in}) can be represented as a function of angle and its distance from the origin such that,

$$C_{in} = f(r_c, \Phi_c), \text{ where } r' \leq r_c \leq R' \text{ and, } \Phi_c \text{ lies within the angle that the coupler linkage subtends on the center} \quad (4)$$

Then due to conformance requirements of the construct, the outer linkage surface (say C_{out}) will be,

$$C_{out} = R' \, \Phi_{CF}, \text{ where } \Phi_{CF} \text{ is angle that the coupler linkage subtends on the center} \quad (5)$$

Similarly, if the outer linkage surface of the rocker (say R_{out}) is represented as a function of angle and it of distance from the origin such that,

$$R_{out} = g(r_r, \Phi_r), \text{ where } r' \leq r_r \leq R' \text{ and, } \Phi_r \text{ lies within the angle that the rocker linkage subtends on the center} \quad (6)$$

Then due to conformance requirements of the construct, the outer linkage surface (say R_{in}) will be,

$$R_{in} = r' \, \Phi_{RF}, \text{ where } \Phi_{RF} \text{ is angle that the rocker linkage subtends on the center} \quad (7)$$

The points lying between C_{in} and C_{out} and R_{in} and R_{out} represent all the points on the linkages and making any change to r_c and r_r stands to reflect the changes in the shape of the coupler and the rocker respectively as required by the designer. Here Φ_{CF} and Φ_{RF} are not the angles between hinges as linkages can have appendages that extend beyond hinge points.

Based on the equation (4), (5), (6) and (7), we can present generalized linkage surface equations. A generalized linkage surface L can be represented as,

$$L_{in} = \begin{cases} f(r, \Phi);\ \mathit{if\ the\ outer\ linkage\ surface\ is\ the\ active\ surface} \\ r'\ \Phi_O;\ \mathit{if\ the\ inner\ linkage\ surface\ is\ the\ active\ surface} \end{cases}$$

$$L_{out} = \begin{cases} R'\ \Phi_O;\ \mathit{if\ the\ outer\ linkage\ surface\ is\ the\ active\ surface} \\ f(r, \Phi);\ \mathit{if\ inner\ linkage\ surface\ is\ the\ active\ surface} \end{cases} \qquad (8)$$

where, $r' \leq r \leq R'$,

Φ lies within the angle that the coupler linkage subtends on the center, and

Φ_O is angle that the coupler linkage subtends on the center

A designer can use equation set (8) to model a thick DM based on this construct by fitting any curve as a function of radius and/or angle.

3.3 Bounding surface as the only modeling tool

The construct discussed in section 3.2, though more permissive, exhibits one major constraint. The construct, by the virtue of its dependence on the reference surface, requires that the hinges of the thick developable mechanism being formed must be placed along a circular periphery. This situation does not achieve full generality as it omits mechanisms that can have hinges which do not follow a circular spread and are instead subsided within a thick body. Fig. 3-12 provides an example of this case. The figure shows a cross-sectional annulus of a thick cylindrical body with six arbitrarily chosen non-coaxial points representing hinge placements. In cases when a reference surface is used as the guiding construct for a layout like in Fig. 3-12 with any reference circle, out of the 6 points shown, only a single point would emerge as legitimate positions where a hinge can be placed (in this case point C). However, if an area-based approach is used, more points can be captured. In this example the grey area bands all the 6 points together. This demonstrates that using an area-based approach can help capture larger potential hinge placement positions. In lieu of this fact, we identify that the area encapsulated by the boundary surfaces is the maximum area the annulus can have and therefore captures all the potential hinge placement positions.

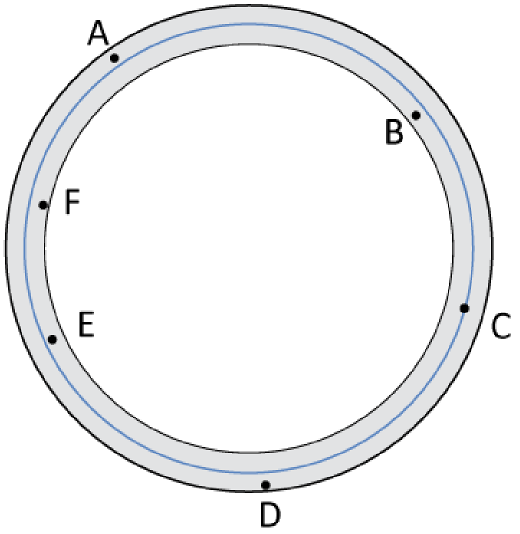


Figure **3-11**: Inability of reference surface to capture all potential hinge placement

Following the above finding, this section introduces a construct where we stop using the reference surface altogether when modeling a thick developable mechanism and instead use the pair of boundary surfaces as the modeling guide. By moving away from the reference surface, we now permit the hinge placement positions to be less bounded as compared to the previous constructs. In a construct like this the hinge positioning is constrained just by the bounding surfaces of the annulus and any position or location on the annulus qualifies as a legitimate option for hinge placement. The design flow of such a construct is shown in Fig. 3-13. Though, the hinge placement is freed up in comparison to the construct discussed in section 3.2, the linkage shaping paradigm still remains similar to it i.e. the mechanism in its conformed state should have its linkages conform to the active surface of the linkage.

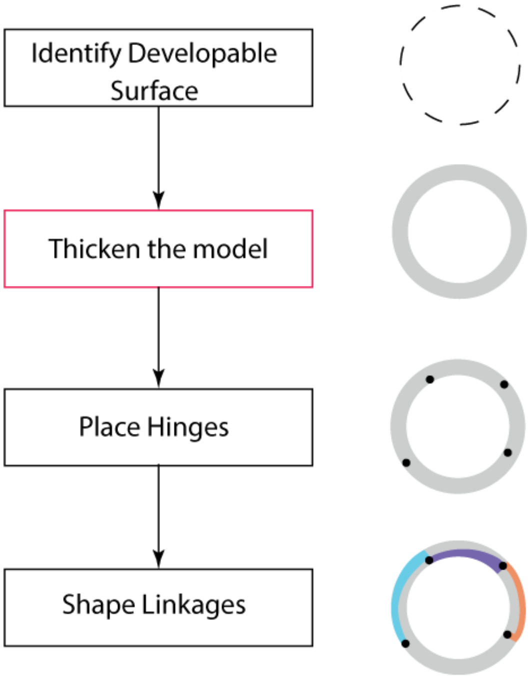


Figure **3-12**: Design process flow of the boundary surfaces as the guiding construct

3.3.1 Kinematics

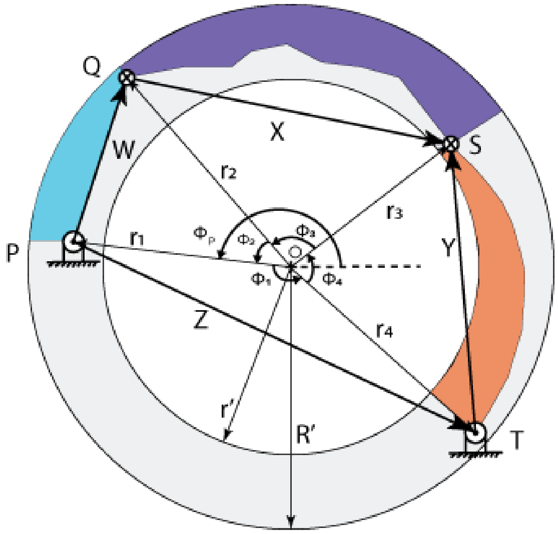


Figure **3-13**: Kinematic breakdown of the construct that uses boundary surfaces as the guiding construct

The kinematic behavior exhibited by this construct takes inspiration from both the above constructs and takes a step further for increased generality. As shown in Fig. 3-14, all the hinges in this construct are placed at differing radii within the annulus such that $r' \leq r_1, r_2, r_3, r_4 \leq R'$.

Position equations for the four hinge positions

$$\begin{aligned} \mathbf{P} &= r_1\, e^{\mathbf{j}\,\Phi_P} \\ \mathbf{Q} &= r_2\, e^{\mathbf{j}\,(\Phi_P - \Phi_2)} \\ \mathbf{S} &= r_3\, e^{\mathbf{j}\,(\Phi_P - \Phi_2 - \Phi_3)} \\ \mathbf{T} &= r_4\, e^{\mathbf{j}\,(\Phi_P + \Phi_1)} \end{aligned} \tag{9}$$

Link equations

$$\begin{aligned} \mathbf{W} &= \mathbf{Q} - \mathbf{P} \Rightarrow \mathbf{W} = r_2\, e^{\mathbf{j}\,(\Phi_P - \Phi_2)} - r_1\, e^{\mathbf{j}\,\Phi_P} \\ \mathbf{X} &= \mathbf{S} - \mathbf{Q} \Rightarrow \mathbf{X} = r_3\, e^{\mathbf{j}\,(\Phi_P - \Phi_2 - \Phi_3)} - r_2\, e^{\mathbf{j}\,(\Phi_P - \Phi_2)} \\ \mathbf{Y} &= \mathbf{S} - \mathbf{T} \Rightarrow \mathbf{Y} = r_3\, e^{\mathbf{j}\,(\Phi_P - \Phi_2 - \Phi_3)} - r_4\, e^{\mathbf{j}\,(\Phi_P + \Phi_1)} \\ \mathbf{Z} &= \mathbf{T} - \mathbf{P} \Rightarrow \mathbf{Z} = r_4\, e^{\mathbf{j}\,(\Phi_P + \Phi_1)} - r_1\, e^{\mathbf{j}\,\Phi_P} \end{aligned} \tag{10}$$

Linkage equations

As mentioned in section 3.3, the linkage equations in this construct follow the same treatment as linkages of the construct in section 3.2 and therefore equation set (8) aptly represents the linkages for this construct.

Chapter 4

Thick Developable Mechanism or Shape Conforming Mechanism

At the beginning of Chapter 3 we mention that introduction of thickness in designing of DMs leads to an ambiguous area where we find mechanisms that in a generalized and utilitarian sense resemble traditional developable mechanisms but do not exactly adhere to the defining qualities of traditional DMs. These include issues like looser or no conformance to the reference surface, arbitrary linkage shapes, usage and dependence on surfaces apart from the reference surface while modeling and general difference in design process flow. Owing to their working and common functionality with traditional DMs, they can be considered ‘thick DMs’ but it is important to note that this ambiguity makes it difficult to determine if these mechanisms are DMs in the traditional sense. This becomes apparent when thick DM modeling constructs like the ones discussed in section 3.3 allow construction of mechanisms that are perceivably neither developable nor shape-conforming, as mentioned in case 4.1.

4.1 Case: Embedded Mechanism Formation

Section 3.3 provides us with a generalized construct that helps cover any mechanism or contraption that can be imagined to be working under the confines of a thick cylindrical body. However, it is this same generality that also makes it less definitive as it risks capturing mechanisms that are not generally associated with conventional understanding of a developable mechanism. For example, following the proposed construct, if any set of 4 points can qualify to be legitimate positions for hinge placement, then contraptions that are embedded within the thickness of the thick cylinder with no appendage or extension coming out of the body on movement become possible (as shown in Fig. 4-1 (a)). This case would then closely resemble to embedding a micro-mechanism within a certain thickness and would not exactly represent a DM or a shape conforming mechanism. At the same time, mechanisms exhibiting the same kinematics but also consisting of appendages that conform to the boundary surface (as shown in Fig. 4-1 (b)) see more DM-like. But given the ambiguity over what qualifies as a DM with thickness in play, forming conditions where mechanisms can be strictly differentiated on their type (like in this case), is difficult.

(a) (b)

Figure **4-1**: Mechanism with small back and forth movement, (a) shows a mechanism that resembles embedded micro-mechanism and (b) shows a mechanism with same kinematics but more DM-like behavior

4.2 Modeling Conditions for Thick DMs

Based on the case discussed in section 4.1, it is clear that while modeling thick DMs we require some guiding conditions (like the hinge axis ruling condition and the link shaping condition in reference surface thickening approach) that help us model mechanisms that exhibit developability. For instance, in the modeling paradigms discussed in section 3.2 and 3.3, the constructs mandate some conformity to the boundary surfaces. This can be viewed as a modified form of the link shaping condition that governs the shape and curvature of the linkages in a zero-thickness situation and can be postulated as,

> Modified Link Shaping Condition: At least one linkage surface should conform to the shape and curvature of the active boundary surface in the mechanism's conformed state.

This condition expands the parameters of the previous condition to make the mechanism conform itself within the boundary surfaces in at least one configuration. This condition is a recognition of the ability of a thick DM to conform to the façade of its parent body. Though the thickness allows attaining arbitrary linkage shapes (withing the annulus), this condition identifies that in at least one configuration, the linkages should conform to the boundary surfaces to manifest the parent body's façade.

Possibility of creation of mechanisms that do not exhibit developability requires and leads to generation of counter conditions that ensure that the mechanisms that are generated using novel modeling paradigms (likes the ones discussed in this study) are always developable. The modified linkage shaping condition is an example of such counter conditions. The conditions that we formulate are also highly dependent on the way we define thick DMs. In this study, define thick DMs as mechanisms that can subside within the façade of the body on which they are mounted. Different modes of defining a DM when thickness is considered during the process of modeling can lead to creation of different conditions and there needs to be a discussion on a commonly accepted definition of thick DMs.

Chapter 5

Conclusion and Future Scope

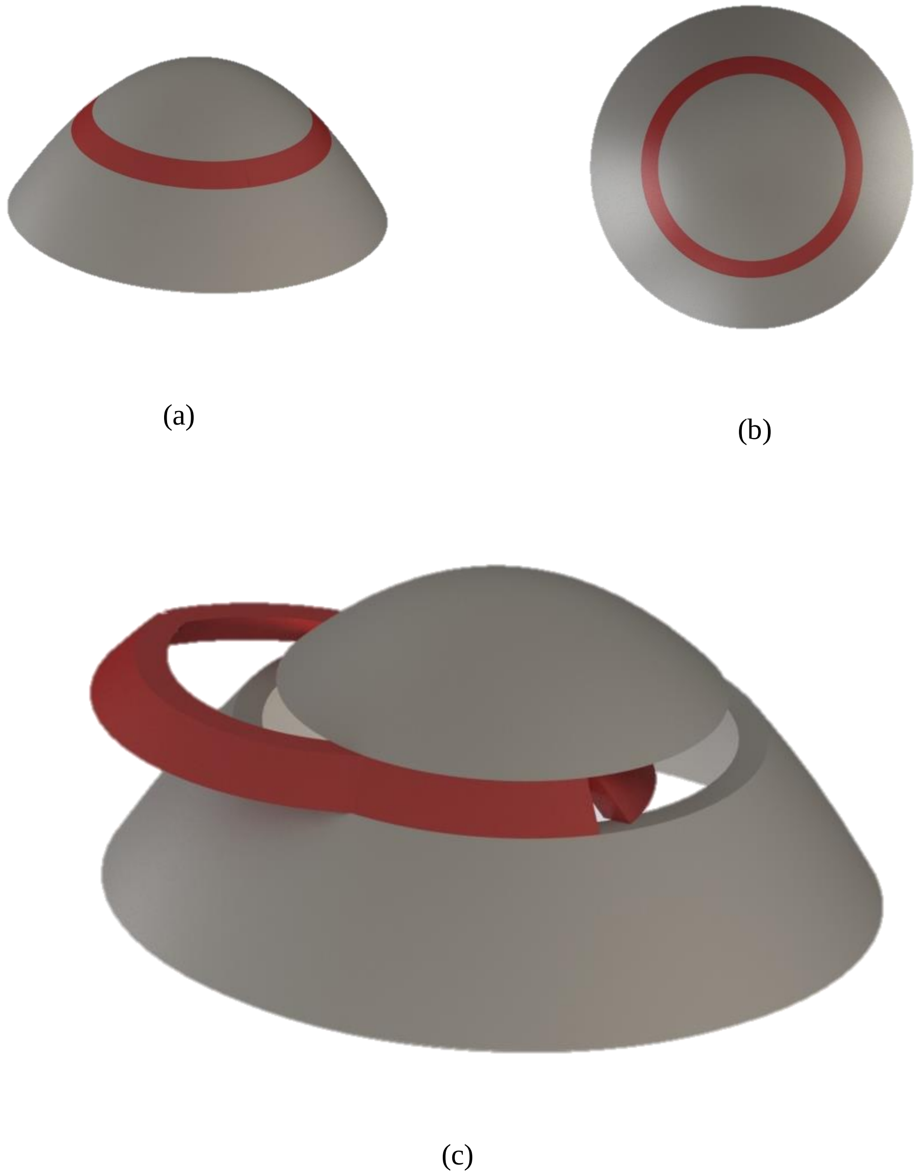
Developable mechanisms hold tremendous potential in creation of compact mechanisms that provide both material and space conservation. However, the current paradigm of modeling developable mechanisms is highly dependent on the zero-thickness assumption which is central to one of the definitive criteria of developable mechanisms. The current study identifies the benefits and the limitations imposed by the zero-thickness assumption while modeling DMs and explores a novel approach for designing and modeling of developable mechanisms on thick bodies. It presents that the freedom of hinge placement coupled with the freedom of shaping the linkages arbitrarily as required by designer encapsulates a wider design space provided by a thick body and therefore proposes the need for a modeling paradigm that fully uses the body thickness. This in turn, underlines the requirement of defining what qualifies as a DM in thickness as ambiguity in literature holds back proper identification and definition. In this study we propose that the usage of two bounding surfaces instead of a singular reference surface as the modeling guide is one method to capture the thickness.

We show the kinematics that encapsulate the positional analysis of the constructs presented. We use these equations to show how thick DMs can be modeled by presenting boundary conditions for linkage shape generation. We present two distinctive methods for linkage shape formulation which can be used depending on the intended complexity of the linkage shape. We can also use these equations to formulate movement based distinctions for thick DMs as intramobility, extramobility and transmobility does for traditional DMs. We also present a modified modeling condition that better captures the thick DMs that can be modeled using the constructs presented in the study. We highlight that the condition presented is dependent on the way we define thick DMs and can change with any change in definition of thick DMs.

The methods presented in the study with certain modifications, can be further employed to model developable mechanisms on conical and tangent-developable surface inspired bodies as well. The current work could be further informed by a kinematic study that undertakes mechanism synthesis and therefore would help solidify the proposed system by providing a robust analytical and graphical analysis. Self-intersection and interference were another aspect that was not considered during the modeling of constructs discussed in section 3.2 and 3.3 and models that utilize or consider these effects can make these constructs even more robust.

We limited the current work by focusing extensively on regular cylindrical thick bodies paired with co-axial regular cylindrical developable surface. Cases where the developable surface from which the mechanism takes inspiration and the shape of the thick body on which the mechanism is mounted are different from each other are possible. Coupling differently shaped thick bodies with generalized developable surfaces (even non-coaxial ones) can lead to creation of even more generalized solutions. This includes creation of thick DMs on conventionally non-developable shaped bodies (like a thick paraboloid) because in the given context of thick DMs developability comes because of the thick annulus as shown in Figures 5-1, 5-2 and 5-3.

Apart from the above future work avenues, the current work can also progress with a detailed exploration of mobility-based grading of thick DMs. Currently, traditional DMs are defined in 3 categories based on their mobility, the categories being: intramobility, extramobility and transmobility. Given that proper identifying criteria for thick DMs does not exist yet, defining its mobility is a task yet to be done.

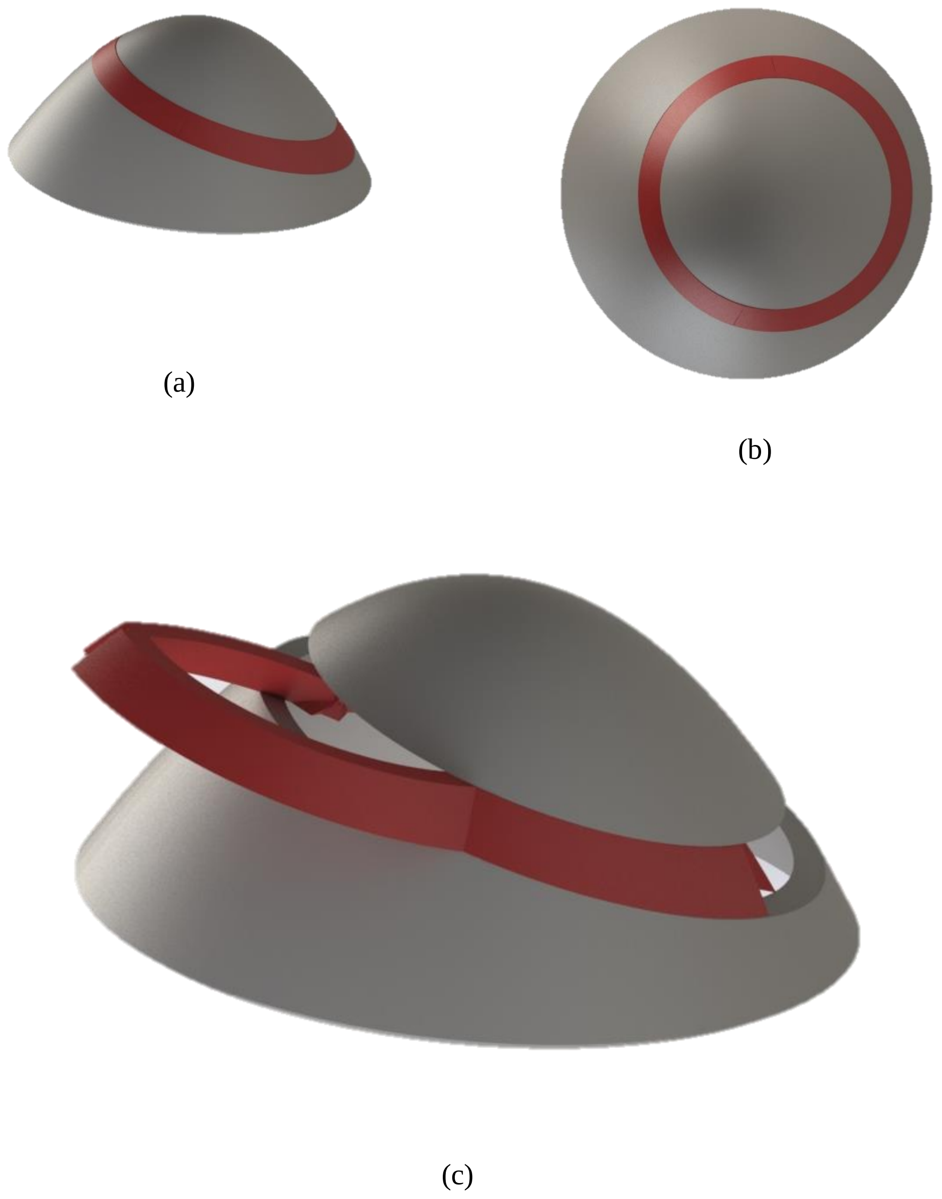


(a)

(b)

(c)

Figure **5-1**: Shows a vertical co-axial cylindrical thick DM made on a paraboloid. (a) Isometric view when the mechanism is conformed, (b) Top view of the conformed system and (c) Isometric view of the mechanism in deployed state.

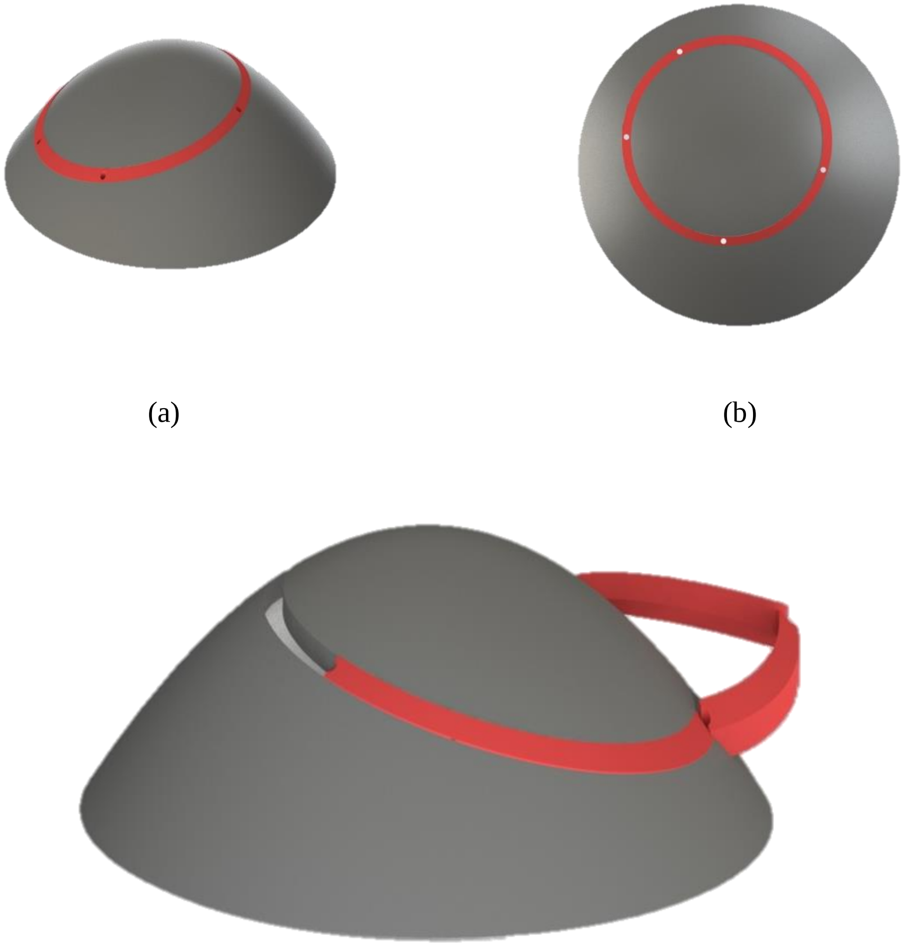


(a)

(b)

(c)

Figure **5-2**: Shows a cylindrical thick DM with slanted axis made on a paraboloid. (a) Isometric view when the mechanism is conformed, (b) Top view of the conformed system and (c) Isometric view of the mechanism in deployed state.



(a) (b)

(c)

Figure **5-3**: Shows a vertical cylindrical thick DM with shifted axis made on a paraboloid. (a) Isometric view when the mechanism is conformed, (b) Top view of the conformed system and (c) Isometric view of the mechanism in deployed state.

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