The Pennsylvania State University
The Graduate School

TRIANGULATING MULTINATIONALS AND TRADE

A Dissertation in
Economics
by
Yuta Watabe

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

August 2021

The dissertation of Yuta Watabe was reviewed and approved by the following:

Jonathan Eaton
Distinguished Professor of Economics
Thesis Advisor
Chair of Committee

Stephen Yeaple
Professor of Economics

Marc Henry
Professor of Economics
Graduate Program Director

Aydin Alptekinoglu
Associate Professor of Supply Chain Management

Abstract

Multinationals play a dominant, if sometimes contentious, role in the global economy. Quantifying their welfare implication requires not only knowing where firms from different countries locate production, but the destinations of what they produce. Data on bilateral trade and multinational production do not fully identify multinational activities without additional assumptions. I develop a model that allows a complete range of multinational activities, including selling only in the host country (horizontal FDI), selling only in the country of origin (vertical FDI), and selling to third countries (export platform FDI). Using the model and the available data, I bound outcomes from various counterfactual scenarios without imposing specific assumptions about multinationals. As a theoretical matter, I show that, for any country, gains from openness (from both trade and multinational activity) are at a maximum with horizontal FDI and a minimum with vertical FDI. Empirical results show that a wide range of outcomes are consistent with the data; for example, the welfare gains from openness in Germany range between 4.5% and 14.5%. While the incompleteness of the data leaves open a wide range of outcomes, what data are available still bring clear judgment on policy evaluation. Penalizing U.S. multinationals for offshoring never benefits the U.S.

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1.1 Introduction

Foreign direct investment (FDI) or multinational production (MP) occurs for various reasons; multinational firms may produce abroad to access the market of the host country (horizontal FDI), may produce abroad to reduce the cost of production and serve the home country (vertical FDI) or may produce abroad to access the broader markets (export platform FDI).

If there is enough data, the distinction is unimportant to develop a quantitative model of trade and multinational production. The complete data on multinational production disaggregated by the firm origin, the production location, and the sales destination — which I define as an *allocation*[1] — are sufficient to perform counterfactual experiments in the quantitative model. The model does not require any assumptions on the motive of multinational production. However, the complete data on allocation does no exist. The data on multinational production and its sales destinations are recorded only in a few countries and are insufficient to calibrate the quantitative model.[2]

Previous studies on trade and multinational production impose a particular assumption on the allocation based on the distinction; some studies assume that multinational production is purely horizontal (i.e., the output of the affiliates are all sold in the host country), purely vertical (i.e., the output of the affiliates are all sold in the headquarters country), or an export platform (i.e., the output of the affiliates are export all over the world). Imposing these assumptions allows them to calibrate the model, perform coun-

[1]Formally, the allocation is a vector where each component is output by the different firm origin, production location, and final destination. The mathematical definition is discussed later.

[2]The noticeable exception is the automobile industry. Head and Mayer (2019) use the automobile data compiled by IHS Markit, whose sales data includes the country of headquarters, production, and sales for the majority of countries. As they acknowledge, data limitation is still a major challenge for the economy-wide model of multinationals.

terfactual experiments and provide an outcome such as counterfactual welfare. But the approach faces serious limitations. Different assumptions provide different counterfactual outcomes, and there is no guidance on the right assumption. Furthermore, these assumptions mask the incompleteness of the data and derive overly precise counterfactual outcomes.

I develop an alternative approach. Instead of providing a particular counterfactual outcome with strong assumptions, I derive an interval of outcomes using available data without assumptions on the allocation. This approach integrates previous studies; the interval includes the values derived from the assumptions used in the literature. The inclusion of these values allows my approach to evaluate and compare the results from past studies in a unified manner. Furthermore, the interval quantifies how the incomplete data results in a wide range of possible counterfactual outcomes. The approach proceeds in three steps: (i) Restrict possible allocations consistent with the observed data, (ii) create a quantitative model for the counterfactual experiments, (iii) characterize a set of possible counterfactual outcomes that is consistent with the data.

The first step – which I call *triangulation* – is particularly important. The triangulation defines the set of allocations consistent with the data on bilateral trade and multinational production. The allocation is defined by the two accounting identities. The data on multinational production records the output in country l by firms from country i. If the allocation is summed over the final destination, the summation must coincide with the bilateral multinational production. The data on bilateral trade records the export from country l to country m. If the allocation is summed over the headquarters location, the summation must coincide with the bilateral trade. These accounting identities provide a set of allocations that is consistent with the data, which I define as *triangulated set*.

Following the triangulation, I develop a quantitative model that augments the Armington model of trade with multinational production. It augments the Armington model by allowing firms to produce the goods abroad and export from their affiliates. In the model, the allocation and two elasticity parameters are sufficient to perform various counterfactual experiments. The model is compatible with the triangulation; it accommodates any allocation in the triangulated set as an equilibrium outcome of the model. In addition, the model encompasses previous studies (which pick particular allocation from the triangulated set) as a special case. The model generalizes the models of quantitative export platform FDI such as Ramondo and Rodríguez-Clare (2013), Arkolakis et al. (2018) and Wang (2020), and further incorporates pure horizontal FDI and pure vertical FDI as testable special cases.

I combine the model with the triangulation to derive a *set* of possible counterfactual outcomes consistent with the data. The set consists of counterfactual outcomes from the model calibrated to some allocation in the triangulated set. The set provides counterfactual results without relying on particular assumptions on the allocation. The set is easy to characterize; if the outcome is scalar and is a continuous function of the allocation, the set of counterfactual outcomes forms an *interval.* The interval can be found by searching the allocation in the triangulated set that provide the maximum and the minimum of the counterfactual outcome.

This interval includes various models in the literature as points in the interval. This feature allows me to evaluate the quantitative implications of different assumptions imposed in the literature. I theoretically show that the intervals of gains from openness — the counterfactual welfare gain from trade and multinational production — are characterized by the pure horizontal FDI assumption and the pure vertical FDI assumption. Formally, the maximum of the gains from openness is achieved by the allocation of pure horizontal FDI, and the minimum of that is achieved by the allocation of pure vertical FDI. This analysis shows that previous studies assuming these two types of multinational activities are taking an extreme stand on gains from openness.

I perform counterfactual experiments for 15 countries using the World Input-Output Database and OECD Analytical AMNE Database. For each country, I provide an interval constructed from a triangulated set and values derived from the special cases. The first three counterfactual outcomes are gains from openness, gains from trade, and gains from multinationals. All of them, especially for gains from openness and gains from multinationals, indicate that the wide range of outcomes is consistent with the data: For example, in Germany, the gains from openness range from 4.5% to 14.5%, which is economically sizable. The wide range of the interval raises a concern about the robustness of previous studies, which assign a particular value within the interval based on the assumption on the allocation.

If the interval is so wide, what can we learn from the counterfactual experiments? I show that the interval is still useful for policy evaluation. Inspired by an actual policy proposed by Joe Biden, I evaluate a policy that penalizes offshoring — the multinational production that is brought back to headquarters country — by U.S. multinationals. Through the lens of this model, the welfare consequence of the policy is conclusive for the U.S.; the policy never raises the real wage of the U.S. people. I augment the data for U.S. foreign affiliates' sales to the U.S. (taken from the Bureau of Economic Analysis) and show that the U.S. may lose from 0.0005% to 0.2% in terms of the real wage. The approach is also useful to recommend a more desirable policy for the U.S. The policy penalizing U.S. multinational production (not only offshoring) may benefit the U.S. by raising their real wage.

The triangulation can be extended to include intermediate goods using global input-output tables and the multinational production data. Combining the triangulation and the model with intermediate goods provides an interval of possible counterfactual outcomes. I show how the intervals change for gains from openness when the intermediate goods are included. The inclusion of intermediate goods further enlarges the range of intervals, suggesting there is a serious lack of information when intermediate goods are considered together with multinational production.

This paper contributes to a wide array of studies involving multinationals. The first contribution is to the literature on horizontal FDI and vertical FDI. The early theories of horizontal FDI and vertical FDI can be found in Markusen (1984) and Helpman (1984), respectively, and surveyed in Antràs and Yeaple (2014), Helpman (2006), and Yeaple (2013). There are numerous empirical studies testing these two theories. These tests look at the determinant of multinational production using aggregate data. The early empirical test on horizontal FDI can be found in Brainard (1997). Subsequent studies are extended so that the test includes vertical FDI and the hybrid of these two (Markusen and Maskus 2002, Carr et al. 2001, and Blonigen et al. 2003), mainly supporting horizontal FDI and the hybrid model. Some studies, such as Braconier et al. (2005) and Davies (2008), find empirical support for vertical FDI. This paper provides a quantitative model for pure horizontal FDI and pure vertical FDI and an alternative testing strategy using the triangulated set, which complements the argument in the literature.

There is a growing literatuer on the quantitative model of trade and multinational production. There are handful of papers on horizontal FDI —see, for example, Irarrazabal et al. (2013), Ramondo (2014), Gumpert et al. (2020), and McGrattan and Waddle (2020)—, and on vertical FDI, such as Garetto (2013) and Boehm et al. (2019)). Ramondo and Rodríguez-Clare (2013), Tintelnot (2016), Arkolakis et al. (2018), Alviarez (2019), Head and Mayer (2019), Garetto et al. (2019), Fan (2020), Wang (2020) and Li (2021) formulate affiliates as export platform FDI. While different in the details, my model aims to connect these different studies to compare the welfare implications in a unified manner. Specifically, the model of this paper generalizes Ramondo and Rodríguez-Clare (2013) which Arkolakis et al. (2018), Wang (2020), Fan (2020) are based on, and integrates the features of Ramondo (2014) and Garetto (2013).

This paper is in line with the studies that acknowledge the limitation of the data and provide a bound (not a point) on counterfactual outcomes. de Gortari (2020) develop an approach to bound a counterfactual outcome in the model of the global value chain. He shows that multiple global value chains are consistent with the global input-output tables we observe, and we cannot calibrate the model uniquely. Instead of imposing a particular assumption on the structure of the global value chain, he constructs a bound on the global value chain and counterfactual outcomes. The current paper is closely related to Wang (2020). Using data on affiliates located in China, Wang (2020) rejects the assumption in Ramondo and Rodríguez-Clare (2013) which attains the allocation from the triangulated set. He extends the model of Ramondo and Rodríguez-Clare (2013) and performs counterfactual experiments providing both the point (with some additional assumptions) and the bound. My paper is independently developed and has several differences over Wang (2020). The set constructed in his paper excludes some allocations from the triangulated set. Specifically, the constructed set precludes pure horizontal FDI and pure vertical FDI, which are crucial components of this paper. The exclusion of these allocations hinders Wang (2020) from analyzing the interval in a theoretical manner. The exclusion of certain allocations results in a narrower range of possible counterfactual outcomes; Wang's presumption is restrictive in terms of counterfactual experiments.

The remainder of the dissertation is structured as follows. The remainder of this chapter explains the concept of triangulation. Chapter 2 introduces the model of trade and multinational production and illustrates special cases, including both models from past studies and canonical theories. Chapter 3 establishes a framework to characterize counterfactual outcomes based on the triangulated set and a theorem to characterize the set of gains from openness. Chapter 4 displays quantitative results for various counterfactual experiments. Chapter 5 extends the triangulation and the model with intermediate goods, and chapter 6 concludes the dissertation.

1.2 Triangulation

I consider an economy with N countries indexed by $i, l, m \in \{1, ..., N\}$ and define a variable X_{ilm} as a gross value of goods made by country i's firm, produced in country l and consumed by consumers in country m. From now on, I use these subscripts i, l and m as a general notion for firm origin, production location, and consumption location, respectively. I denote T_{lm} the gross trade flow from country l to country m, and M_{il} as an output in country l by firms from country i (e.g, output in China by Japanese firms is denoted $M_{JPN,CHN}$). Denote the vector $\{X_{ilm}\}_{i=1,\ldots,N,l=1,\ldots,N,m=1,\ldots N}$, $\boldsymbol{X}$ and call this an *allocation*. In my setting, the allocation $\boldsymbol{X}$ is not observed. What I observe are vectors $\boldsymbol{T} \equiv \{T_{lm}\}_{l=1,\ldots,N,m=1,\ldots,N}$ and $\boldsymbol{M} \equiv \{M_{il}\}_{i=1,\ldots,N,l=1,\ldots,N}$, which are the data on bilateral trade and multinational production.

While the allocation is not directly observed, I can use the data on bilateral trade and multinational production to restrict the possible allocation; I call this *triangulation*. To triangulate the allocation with the observed data, I use the following accounting identities:

$$M_{il} = \sum_{m=1}^{N} X_{ilm}$$
$$T_{lm} = \sum_{i=1}^{N} X_{ilm},$$

The first identity indicates that the total production of goods in country l by firms from country i will be sold in some countries. The second identity implies that all the goods delivered from country l to country m must be produced by firms from some countries. In addition, elements of the allocation also must be non-negative since elements are gross values. These restrictions do no pin down the allocation; multiple allocations satisfy these restrictions [3]. However, they are still useful to define a set of allocations consistent with the data. A triangulated set $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M})$ is a set of allocations that satisfy the accounting identities and non-negativity. Specifically, any allocation $\boldsymbol{X}$ in $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M})$ satisfies

$$M_{il} = \sum_{m=1}^{N} X_{ilm} \;\forall\; i, l$$
$$T_{lm} = \sum_{i=1}^{N} X_{ilm} \;\forall\; l, m$$
$$0 \le X_{ilm} \;\forall\; i, l, m.$$

These equations restrict the set of allocations and hence partially identify the state of the economy.

[3] There are N^3 elements in the allocation while there are only $2N^2$ constraints.

Chapter 2 Quantitative Model

In this chapter, I construct a model that incorporates multinationals into the Armington model (Armington 1969). The concept of multinational production is translated into the model as a firm being able to produce goods in foreign countries. This model is isomorphic (provides the same aggregate implication) to the multinational Eaton-Kortum model as in Ramondo and Rodríguez-Clare (2013)[1], and can naturally be extended to the multinational Melitz model as in Arkolakis et al. (2018).

2.1 Model

There are N countries in the economy with representative firms and consumers. A consumer earn wages from her labor (where a representative consumer in country l provide L_i amount of labor inelastically) and purchase goods.

Goods are differentiated across the origin of the firm and the country of production. I denote C_{ilm} as the consumption of goods produced by a firm from country i, produced in country l and consumed in country m; I denote p_{ilm} as the price of such goods. The utility function of a representative consumer in country m is:

$$U_m = \left(\sum_{i=1}^{N} \left(\sum_{l=1}^{N} C_{ilm}^{\frac{\epsilon}{\epsilon+1}} \right)^{\frac{\epsilon+1}{\epsilon} \frac{\theta}{\theta+1}} \right)^{\frac{\theta+1}{\theta}}. \tag{2.1}$$

I assume $\theta < \varepsilon$. This assumption implies that if the firm origin is the same, the goods are less differentiated compared to goods with a different firm origin. Define $\rho \equiv \frac{\epsilon-\theta}{\epsilon}$, where $0 \leq \rho < 1$ for $\epsilon > \theta$. The parameter θ is a usual trade elasticity, and the

[1]For the time being, this paper abstracts intra-firm trade in Ramondo and Rodríguez-Clare (2013) since this model does not incorporate intermediate goods. The triangulation and the model with intermediate goods are shown later.

parameter ρ is a relative elasticity of substitution between the goods from the same firm origin and the goods from a different firm origin. If $\rho = 0$, the goods from the same firm origin (but from different production locations) are as differentiated as goods from different firm origins. When $\rho \approx 1$, the goods from the same firm origin are close to perfect substitutes. The parameter ρ can be viewed as a cannibalization parameter as in Ramondo and Rodríguez-Clare (2013).

The expenditure of goods X_{ilm} is

$$X_{ilm} = \frac{P_{im}^{-\theta}}{\sum_{j=1}^{N} P_{jm}{}^{-\theta}} \frac{p_{ilm}^{-\theta/(1-\rho)}}{\sum_{k=1}^{N} p_{ikm}^{-\theta/(1-\rho)}} X_m \tag{2.2}$$

where X_m is total absorption in country m and $P_{im} \equiv \left(\sum_{k=1}^{N}(p_{ikm}^{-\theta/(1-\rho)})\right)^{-(1-\rho)/\theta}$ is the price index in country m for goods produced by a firm from country i. The price index of country m is

$$P_m = \left[\sum_{i=1}^{N} P_{im}^{-\theta}\right]^{-1/\theta}. \tag{2.3}$$

I assume perfect competition; the price of the goods is the marginal cost of the goods. Labor is the only factor of production. Multinationals employ labor in country l to produce in country l. Denote the wage in country l w_l. A firm from country i requires an amount τ_{ilm} of labor to produce a unit of goods in country l and deliver to country m. Therefore, the price of the goods p_{ilm} is

$$p_{ilm} = w_l \tau_{ilm}. \tag{2.4}$$

Here $\tau_{ilm} \in (0, \infty)$ is a composite of productivity and various frictions (e.g., trade cost, knowledge transfer cost and marketing cost) associated with multinationals and trade. When $\tau_{ilm} = \infty$, there is no possible technology to produce the goods for this purpose, hence $X_{ilm} = 0$.

In the model, there is a trade deficit, whch is an exogenous transfer between countries. Denote D_m as the trade deficit of country m. The absorption of country m is a summation of labor income (total production) and trade deficit:

$$X_m = w_m L_m + D_m. \tag{2.5}$$

The market clearing condition is

$$X_l = \sum_{k=1}^{N}\sum_{m=1}^{N} X_{klm} + D_l. \tag{2.6}$$

Denote the vectors of labor endowment, wage, labor requirement, τ, price and trade deficit as $\boldsymbol{L}$, $\boldsymbol{w}$,$\boldsymbol{\tau}$, $\boldsymbol{p}$ and $\boldsymbol{D}$. Given $\boldsymbol{L}, \boldsymbol{\tau}$ and $\boldsymbol{D}$, an equilibrium is a wage vector $\boldsymbol{w}$, a price vector $\boldsymbol{p}$, and an allocation $\boldsymbol{X}$ that satisfy the consumer optimization, the producer optimization and the market clearing condition.

2.2 Implication of the model

The model rationalizes any allocation as an equilibrium outcome.

Proposition 2.2.1. *For any θ, ρ, and $\boldsymbol{X}$, there is a set of variables $(\boldsymbol{L}, \boldsymbol{D}, \boldsymbol{\tau})$ such that $\boldsymbol{X}$ is an outcome of the equilibrium of the model. If $\boldsymbol{L}$ and $\boldsymbol{D}$ are observed in addition, for any θ, ρ $\boldsymbol{L}$, $\boldsymbol{D}$, and $\boldsymbol{X}$, there exists $\boldsymbol{\tau}$ such that $\boldsymbol{X}$ is an outcome of the equilibrium of the model.*

Proof. See appendix A. □

This feature of the model is neccesary to combine the model with the triangulation. The triangulation restricts the allocation only from the data, and the model will not further restrict the possible allocation. In previous models in the literature, some allocations are precluded. Previous studies impose a structure on $\boldsymbol{\tau}$ to reduce the dimension of $\boldsymbol{\tau}$. Ramondo and Rodríguez-Clare (2013), Arkolakis et al. (2018), and Fan (2020) assume $\boldsymbol{\tau}$ is the product of two bilateral costs: the cost of delivering knowledge from country i to country of l, and the cost of delivering goods from country l to country m. This implies $\tau_{ilm} = \gamma_{il}\xi_{lm}$, where γ_{il} is a knowledge cost, and ξ_{lm} is a trade cost. This specification identifies the allocation in the triangulated set, given a parameter ρ. For further usage, I denote the allocation from this specification $\boldsymbol{X}^{RRC}(\boldsymbol{T}, \boldsymbol{M}; \rho)$. This allocation assumes proportionality; there exists a^1_{il} and a^2_{lm} such that $X_{ilm} = a^1_{il}a^2_{lm}$ for any i, l and m.

Wang (2020) assumes that $\boldsymbol{\tau}$ is a multiplication of three bilateral costs and is written as $\tau_{ilm} = \gamma_{il}\xi_{lm}\zeta_{im}$ $\forall i, l, m$. In addition to the two costs specified in Ramondo and Rodríguez-Clare (2013), Wang adds ζ_{im}, a cost of marketing goods from country i to country m. Given a parameter ρ, this specification provides the set of allocations

$\mathbb{X}^{Wang}(\boldsymbol{T}, \boldsymbol{M}; \rho)$; the allocation must be in the triangulated set and must also be rationalized as an equilibrium outcome of his model. Wang's set also requires proportionality. In Wang's model, there are set of variables a^1_{il}, a^2_{lm}, and a^3_{im} such that $X_{ilm} = a^1_{il} a^2_{lm} a^3_{im}$ for any i, l and m.

In addition to the explanation of proportionality restriction, I exemplify the restrictiveness of Wang's set. In Wang's model, the export of affiliates is restricted by the export of headquarters. Specifically, if the headquarters export to a country, their affiliates must also export to the country. [2] This is why I call his model an export platform model. In contrast, my model allows arbitrary patterns of sales for both headquarters and affiliates, such as pure horizontal FDI and pure vertical FDI.[3]

This separability also excludes some plausible cost structures of multinational production. For example, people in the headquarters, the affiliate, and the sales destination may conduct a Zoom meeting. The Zoom meeting must be conducted when all the participants are awake. The possible time window depends on the trilateral time differences, not on two bilateral time differences; the cost of such Zoom meeting is not multiplicatively separable.

2.3 Canonical theories of foreign direct investment

The model encompasses canonical theories of multinationals: pure horizontal foreign direct investment (pure horizontal FDI), pure vertical foreign direct investment (pure vertical FDI) and proportional export platform foreign direct investment (proportional export platform FDI). In pure horizontal FDI, all the output of the affiliates is sold to the host country (production location). In pure vertical FDI, all the output of the affiliates is sold to the firm origin (headquarters location).[4] In proportional export platform FDI, affiliates export to multiple countries, and the export of the affiliates (and headquarters) is proportional to the total export of the host country. These three theories pin down a

[2]Think of three countries i, l and m. Suppose we consider an allocation with $X_{ilm} = 0$ and $X_{iim} > 0$. If the data indicates $M_{il} > 0$ and $T_{lm} > 0$, such allocation is not in the Wang's set. Specifically, if $X_{ilm} = 0$, γ_{il}, ξ_{lm} or ζ_{im} must be infinitely high. However, if $\zeta_{im} = \infty$, then it contradicts the fact that X_{iim} being positive. If $\gamma_{il} = \infty$, then it contradicts the data indicating M_{il} being positive. Similarly, if $\xi_{lm} = \infty$, then it contradicts the data indicating T_{lm} being positive.

[3]I show in appendix A that with a similar data structure, pure horizontal FDI and pure vertical FDI are excluded from Wang's set.

[4]There are no intermediate goods in this setting. Vertical FDI indicates that the affiliates' production is sold to the headquarters location. The formulation with intermediate goods from affiliates to headquarters is discussed later.

specific allocation in the triangulated set.[5]

Denote the allocation of pure horizontal FDI $\boldsymbol{X}^{HFDI}(\boldsymbol{T}, \boldsymbol{M})$. This allocation assumes all the output of affiliates is sold in the host country. Specifically, this allocation can be characterized by

$$X_{ilm}^{HFDI} \equiv \begin{cases} M_{mm} - \sum_{k \neq m}^{N} T_{mk} & if \ i = m, l = m \\ T_{im} & if \ i = l, l \neq m \\ M_{im} & if \ i \neq l, l = m \\ 0 & otherwise. \end{cases}$$

Here, the output of affiliates from country i in country m is all delivered to country m ($M_{im} = X_{imm}^{HFDI}$), and all the export from country i to country m is attributed to the export of firms from country i ($T_{im} = X_{iim}^{HFDI}$). The output of goods where the firm origin, production, and consumption are all in country m is the total output of headquarters in the country subtracting the export of the country ($X_{mmm}^{HFDI} = M_{mm} - \sum_{k \neq m}^{N} T_{mk}$). This allocation satisfies the accounting identities by construction. The allocation may contain negative value, hence may not be in the triangulated set. For the allocation of pure horizontal FDI to be in the triangulated set, for all the countries, the output of the headquarters must be larger than the total export of the country:

$$0 \leq M_{mm} - \sum_{k \neq m}^{N} T_{mk} \ \forall \ m.$$

Denote the allocation of pure vertical FDI $\boldsymbol{X}^{VFDI}(\boldsymbol{T}, \boldsymbol{M})$. This allocation assumes all the production of affiliates is sold to the headquarters location. I call such flow of goods offshoring. Specifically, this allocation can be characterized by

$$X_{ilm}^{VFDI} = \begin{cases} T_{mm} & if \ i = m, l = m \\ T_{im} - M_{mi} & if \ i = l, i \neq m \\ M_{mi} & if \ i \neq l, i = m \\ 0 & if \ otherwise. \end{cases}$$

Here, the output of affiliates from country i in country m is all delivered to the consumer

[5] Given the proposition of the model, there is always a $\boldsymbol{\tau}$ which rationalizes the allocation as an equilibrium outcome. Therefore, I do not discuss the assumption on $\boldsymbol{\tau}$, but I directly discuss the assumption on $\boldsymbol{X}$.

in country i ($M_{im} = X^{VFDI}_{imi}$). The export from country l to country m is a summation of the export of affiliates to the headquarters location and the export of headquarters ($T_{im} = X^{VFDI}_{iim} + X^{VFDI}_{mim}$). The output of goods where the firm origin, production, and consumption are all in country m is equal to the total output of goods produced and consumed in country m ($T_{mm} = X^{VFDI}_{mmm}$). This allocation satisfies the accounting identities by definition. The allocation is in the triangulated set if all the elements of the allocation are non-negative. For the allocation of the vertical FDI to be in the triangulated set, there must be more export from country i to country m than the output of firms from country m in country i:

$$0 \leq T_{im} - M_{mi} \ \forall \ i, m.$$

Denote the allocation of proportional export platform FDI $\boldsymbol{X}^{PFDI}(\boldsymbol{T}, \boldsymbol{M})$. This allocation assumes that affiliates and headquarters have the same export intensity if they are located in the same country. Specifically, this allocation is described as

$$X^{PFDI}_{ilm} = \frac{M_{il}}{\sum_{j=1} M_{jl}} T_{lm} \ \forall \ i, l, m.$$

This proportionality assumption is closely related to the separability assumption proposed by Ramondo and Rodríguez-Clare (2013). Specifically, when $\rho = 0$, these two assumptions provide the same allocation (See appendix A). By definition, this allocation is always in the triangulated set.

2.4 Three measures of gains from multinationals and trade

For the counterfactual experiments, I define a notion of real wage of country q, namely $W_q = \frac{w_q}{P_q}$ which is a utility measure of country q.

I introduce three values from the counterfactual experiments in the literature: gains from openness, gains from trade, and gains from multinationals.[6] Gains from openness of country q are changes in a real wage of country q, W_q, by moving from the counterfactual equilibrium without trade or multinational production to the current observed equilibrium. Gains from openness summarize the dependency of the country on trade

[6] Derivation of gains from openness follow Arkolakis et al. (2018). Derivation of gains from trade and gains from multinationals are shown in appendix A.

and multinational production. Following Arkolakis et al. (2018), gains from openness for country q GO_q are expressed as: [7]

$$GO_q(\boldsymbol{X}) = \left(\frac{\sum_{l=1}^{N} X_{qlq}}{X_q}\right)^{-1/\theta} \left(\frac{X_{qqq}}{\sum_{l=1}^{N} X_{qlq}}\right)^{-(1-\rho)/\theta}. \tag{2.7}$$

Gains from openness can be decomposed into two parts. The first term captures the gains from foreign technology. With multinationals and trade, a consumer in country q consumes goods produced by foreign firms, including the goods produced in country q. The second term captures gains from offshoring. With multinationals and trade, firms from country q can use foreign labor to offshore production and serve the consumer in country q.

The gains from trade of country q are changes in the real wage of country q, W_q, by moving from the counterfactual equilibrium without trade to the current observed equilibrium. I denote it GT_q:

$$GT_q(\boldsymbol{X}) = \left(\frac{\sum_{i=1}^{N} \left(\sum_{l=1}^{N} X_{ilm}\right)^{\rho} X_{imm}^{(1-\rho)}}{X_m}\right)^{-1/\theta}. \tag{2.8}$$

Gains from multinationals of country q is a change in the real wage q, W_q, by moving from the counterfactual equilibrium without multinational production to the current observed equilibrium. I denote it GM_q:

$$GM_q(\boldsymbol{X}) = \left(\frac{\sum_{i=1}^{N} \left(\sum_{l=1}^{N} X_{ilm}\right)^{\rho} X_{iim}^{(1-\rho)}}{X_m}\right)^{-1/\theta}. \tag{2.9}$$

For gains from multinationals, the counterfactual equilibrium is different from that of gains from openness and gains from trade. In the counterfactual equilibrium without multinational production, the wage is fixed to the level of the observed equilibrium.[8] This assumption is required for the allocation to be the sufficient statistic of gains from multinationals.[9]

[7] Arkolakis et al. (2018) have profit margins. The implication of gains from openness does not change moving from perfect competition to monopolistic competition. Specifically gains from openness with monopolistic competitions is $GO_q(\boldsymbol{X}) = \left(\frac{\sum_{l=1} X_{qkq}}{X_q}\right)^{-1/\theta} \left(\frac{X_{qqq}}{\sum_{k=1}^{N} X_{qkq}}\right)^{-(1-\rho)/\theta} \frac{\sum_{l=1}^{N} M_{ql}}{M_{qq}}$. The third term only depends on the data, which is due to the fact that the profit of firms are proportional to their sales.

[8] This can be rationalized by assuming some numeraire sector with free trade.

[9] Trade remains in the counterfactual equilibrium for gains from multinationals. This implies there is

still an exchange of labor through trade; hence the change in relative wage must be taken into account (which is not the case for the other two, since there is no trade). Tracking the change in relative wage requires solving the counterfactual equilibrium (which is not possible only with $\boldsymbol{X}$).

Chapter 3
Theoretical Implications

In this chapter, I show that the possible outcomes from counterfactual experiments can be represented as an interval and show that the canonical theories characterize the interval of gains from openness.

3.1 Characterizing Counterfactuals

I focus on counterfactual experiments where the outcome can be calculated with $\boldsymbol{X}$, θ, and ρ. I assume θ and ρ are fixed and known.[1] I denote a scalar outcome of a counterfactual experiment $F : \boldsymbol{X} \to \mathbb{R}$ (e.g., gains from openness for country q, GO_q). Assuming a specific allocation provides a unique counterfactual outcome $F(\boldsymbol{X})$. Instead of assuming a specific allocation, I consider the set of allocations in the triangulated set and construct a set of outcomes:

$$\mathbb{F}(\boldsymbol{T}, \boldsymbol{M}) \equiv \{V | V = F(\boldsymbol{X}) \mid \boldsymbol{X} \in \mathbb{X}(\boldsymbol{T}, \boldsymbol{M})\}.$$

I show that if the counterfactual function F is a continuous function of $\boldsymbol{X}$, the set can be characterized in a convenient manner.

Proposition 3.1.1. *If F is a continuous function of $\boldsymbol{X}$ and satisfies some regularity conditions, then $\mathbb{F}(\boldsymbol{T}, \boldsymbol{M})$ is an interval. The interval may not be bounded.*

The regularity conditions and the proof is shown in appendix B.[2] All the counterfactual function F (i.e., Gains from openness, gains from trade, gains from multinationals,

[1] The consequence of not knowing θ and ρ is discussed later.

[2] This proposition is not established in Wang (2020). In his paper, Wang claims that $\mathbb{X}^{Wang}$ is a convex set, but he disregards the fact that his model precludes some allocation even though it is consistent with the data. This implies that the set of possible allocations in the Wang model may not be connected, which is necessary for the proof. He did not provide the proof that his set $\mathbb{X}^{Wang}$ is connected.

and outcomes from exact hat-algebra) in this paper is a continuous function of $\boldsymbol{X}$ and satisfies the regularity conditions.

If the interval is bounded, this proposition implies that calculating the maximum and minimum of the set $\mathbb{F}$ is sufficient to characterize the set of counterfactual outcomes. When the interval is bounded[3] , I calculate the maximum of $\mathbb{F}$ by solving the following problem:

$$
\begin{aligned}
F^U &= maximize\ F(\boldsymbol{X}) \text{ over } \boldsymbol{X} \\
s.t.\ M_{il} &= \sum_{m=1}^{N} X_{ilm}\ \forall\ i, l \\
T_{lm} &= \sum_{i=1}^{N} X_{ilm}\ \forall\ l, m \\
X_{ilm} &\geq 0\ \forall\ i, l, m.
\end{aligned}
$$

The minimum of $\mathbb{F}$ can be similarly calculated.

3.2 Quantitative implication of the canonical theory

I define $\mathbb{GO}_q(\boldsymbol{T}, \boldsymbol{M})$ as the set of gains from openness for country q that is consistent with the data. Since gains from openness are a continuous function of the allocation $\boldsymbol{X}$, the set is an interval:

$$\mathbb{GO}_q(\boldsymbol{T}, \boldsymbol{M}) = [GO_q^L(\boldsymbol{T}, \boldsymbol{M}), GO_q^U(\boldsymbol{T}, \boldsymbol{M})].$$

I show that the allocations of pure horizontal FDI and pure vertical FDI are the limiting cases.

Theorem 3.2.1. *For any country q, if the pure horizontal FDI allocation is in the triangulated set, then $GO_q^U(\boldsymbol{T}, \boldsymbol{M}) = GO_q\left(\boldsymbol{X}^{HFDI}(\boldsymbol{T}, \boldsymbol{M})\right)$. If the pure vertical FDI allocation is in the triangulated set, $GO_q^L(\boldsymbol{T}, \boldsymbol{M}) = GO_q\left(\boldsymbol{X}^{VFDI}(\boldsymbol{T}, \boldsymbol{M})\right)$.*

The intuition is explained as follows. Notice that $GO_q(\boldsymbol{X})$ is a decreasing function of X_{qqq} and $\{X_{qlq}\}_{l=1,..N,l\neq q}$; higher X_{qqq} implies higher demand for the domestic goods that remain available in autarky, and higher $\{X_{qlq}\}_{l=1,..N,l\neq q}$ implies higher demand for

[3] In practice, the interval may not be bounded. I show in appendix B how we can address the unboundedness of the interval for some cases.

offshored goods, which are more substitutable than the goods made by foreign firms. In the proof, I show that these two allocations are the extremes in terms of these variables. In the pure horizontal FDI allocation, there is no offshoring (minimize $\{X_{qlq}\}_{l=1,..N,l\neq q}$) and everything that foreign firms produce in country q is consumed in country q (minimize X_{qqq}). In the pure vertical FDI allocation, everything that firms from country q produce abroad is sent back to country q (maximize $\{X_{qlq}\}_{l=1,..N,l\neq q}$) and everything that foreign firms produce in country q is exported back, hence not consumed in country q (maximize X_{qqq}). Since $GO_q(\boldsymbol{X})$ is a decreasing function of both elements, these allocations characterize the maximum and the minimum of the interval. The formal proof is stated below.

Proof. I first show the bounds for X_{ilm}. Formally, the bounds are inequalities that any X_{ilm} must satisfy if the allocation is in the triangulated set. While these bounds only use a subset of constraints, they are useful to calculate bounds for the gains from openness:

$$0 \leq X_{ilm} \tag{3.1}$$
$$X_{ilm} \leq M_{il} \tag{3.2}$$
$$X_{ilm} \leq T_{lm} \tag{3.3}$$
$$M_{il} - \sum_{n\neq m}^{N} T_{ln} \leq X_{ilm}. \tag{3.4}$$

I use the subset of these bounds to construct bounds for X_{qqq} and $\{X_{qlq}\}_{l=1,..N,l\neq q}$:

$$0 \leq X_{qlq} \leq M_{ql}\ q,\ l \neq q. \tag{3.5}$$
$$M_{qq} - \sum_{n\neq q} T_{qn} \leq X_{qqq} \leq T_{qq}\ q. \tag{3.6}$$

I first show the case of GO_q^U. Notice that maximizing gains from openness corresponds to minimizing X_{qqq} and $\{X_{qlq}\}_{l=1,..N,l\neq q}$. I take the lower bound for each variable, which are

$$0 \leq X_{qlq}\ \forall\ l \neq q \tag{3.7}$$
$$M_{qq} - \sum_{n\neq q} T_{qn} \leq X_{qqq}. \tag{3.8}$$

Any allocation where X_{qqq} and $\{X_{qlq}\}_{l=1,..N,l\neq q}$ achieve these lower bounds (if in the triangulated set) achieves maximum gains from openness. The pure horizontal FDI allocation achieves the lower bound for each variable. By assumption, the allocation is in the triangulated set. Therefore, the allocation achieves the maximum gains from openness.

A similar argument can be made for GO_q^L. Notice that minimizing the gains from openness corresponds to maximizing X_{qqq} and $\{X_{qlq}\}_{l=1,..N,l\neq q}$. I take the upper bounds for each variable which are

$$X_{qlq} \leq M_{ql} \ \forall \ l \neq m \tag{3.9}$$

$$X_{qqq} \leq T_{qq}. \tag{3.10}$$

Any allocation where X_{qqq} and $\{X_{qlq}\}_{l=1,..N,l\neq q}$ achieve these upper bounds, if in the triangulated set, achieves the minimum for the gains from openness. The pure vertical FDI allocation achieves the upper bound for each variable. By assumption, the allocation is in the triangulated set. Therefore, the allocation achieves the minimum for gains from openness.

□

Hereafter, for notational simplicity, I will omit $(\boldsymbol{T}, \boldsymbol{M})$ from the notion of each specific allocation. If both the allocations of horizontal FDI and the vertical FDI are in $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M})$, then $\mathbb{GO}_q = \left[GO_q\left(\boldsymbol{X}^{VFDI}\right), GO_q\left(\boldsymbol{X}^{HFDI}\right)\right]$.

There are several remarks to this theorem. Firstly, this theory only uses the monotonicity of gains from openness; hence the result does not depend on the specific parameter value of θ and ρ. The proof holds if $\{X_{qlq}\}_{l=1,..N}$ fully characterizes gains from openness for country q. Second, these two allocations achieve the maximum or the minimum of gains from openness for any country simultaneously. If one assumes the allocation of pure horizontal FDI or the allocation of pure vertical FDI, the model achieves the maximum or the minimum of the gains from openness for all the countries. Third, this result crucially depends on the assumption that the allocation is in the triangulated set. The specific method to obtain the maximum and the minimum if these allocations are not in the triangulated set is explained in the empirical section.

Chapter 4
Quantitative Experiments

In this chapter, I quantify various counterfactual experiments discussed in the literature and demonstrate the usefulness of this approach. For the benchmark result, I set $\theta = 4.5$ and $\rho = 0.55$ following Arkolakis et al. (2018).[1] Computational details are described appendix C.

I use two sets of data for bilateral trade and multinational production. The data on bilateral trade is taken from the World Input-Output Database, which is collected by 12 research institutes headed by the University of Groningen, The Netherlands, and covers trade in both goods and services (Timmer et al., 2015). Bilateral trade flow is constructed by aggregating the input-output table over the purchase dimension. For the data on bilateral multinational production, I use the analytical AMNE (Activity of Multinational Enterprises) Database constructed by OECD.[2] I use the data for 2013. I aggregate the industry dimension, which includes primary, manufacturing and services, and aggregate the data to 15 countries that are common in these two data. The countries are Australia, Brazil, Canada, China, France, Germany, Indonesia, Italy, Japan, Mexico, Russia, Spain, the United Kingdom, the United States and the rest of the world.

4.1 Gains from trade and multinational production

I firstly verify if the allocations of horizontal FDI and vertical FDI are in the triangulated set. The allocation of vertical FDI is not in the triangulated set, while the allocation

[1] Without additional data on $\boldsymbol{X}$ and cost shifter / demand shifter, it is not possible to identify the parameter θ and ρ. Since $\boldsymbol{X}$ can be rationalized as an equilibrium outcome for any parameter θ and ρ, the allocation $\boldsymbol{X}$, in isolation, cannot identify θ and ρ; hence the data $(\boldsymbol{M}, \boldsymbol{T})$ cannot identify the parameter. The possible consequence of varying these parameters are discussed in appendix B.

[2] The final product of analytical AMNE calculates the trade of multinationals, but it relies on some assumptions. I use the intermediate calculation of an analytical AMNE database, which calculates the bilateral flow of multinational production.

of horizontal FDI is. This implies that the data refute vertical FDI and does not refute horizontal FDI.

4.1.1 Gains from openness

I utilize the theorem to calculate the interval of gains from openness. The maximum gains from openness are calculated as $GO_q(\boldsymbol{X}^{HFDI})$. For the minimum gains from openness, the theory does not apply since the vertical FDI allocation is not in the triangulated set. I modify the vertical FDI allocation and verify that it achieves the minimum if it is in the triangulated set. Denote this modified allocation $\boldsymbol{X}^{MVFDI,q}$. For each country q, I guess a subset of the allocation $\{X_{qlq}^{MVFDI,q}\}_{l=1,\ldots,N}$,

$$X_{qlq}^{MVFDI,q} = min\left(M_{ql}, T_{lq}\right),$$

which is a simple upper bound for the relevant variables. If there is an allocation that satisfies this restriction (and in the triangulated set), gains from openness from this allocation are the minimum of gains from openness for country q. The existence of such allocation can be verified using linear programming techniques, and the minimum is:

$$GO_q^L\left(\boldsymbol{X}^{MVFDI,q}\right) = \left(\frac{X_{qqq}^{MVFDI,q}}{X_q}\right)^{-(1-\rho)/\theta}\left(\frac{\sum_{k=1}^{N} X_{qkq}^{MVFDI,q}}{X_q}\right)^{-\rho/\theta}.$$

I verify that in the data there is an allocation in the triangulated set that achieves this value and I use the value as the minimum of gains from openness.[3]

In addition to the interval of gains from openness, I calculate values from four allocations: (i) proportional export platform FDI allocation, (ii) allocation from Ramondo Rodríguez-Clare's model, (iii) allocation from Wang's model that maximizes gains from openness (I denote this Wang-max allocation), and (iv), allocation from Wang's model that minimizes gains from openness (I denote this Wang-min allocation). The result is shown in Figure 4.1.

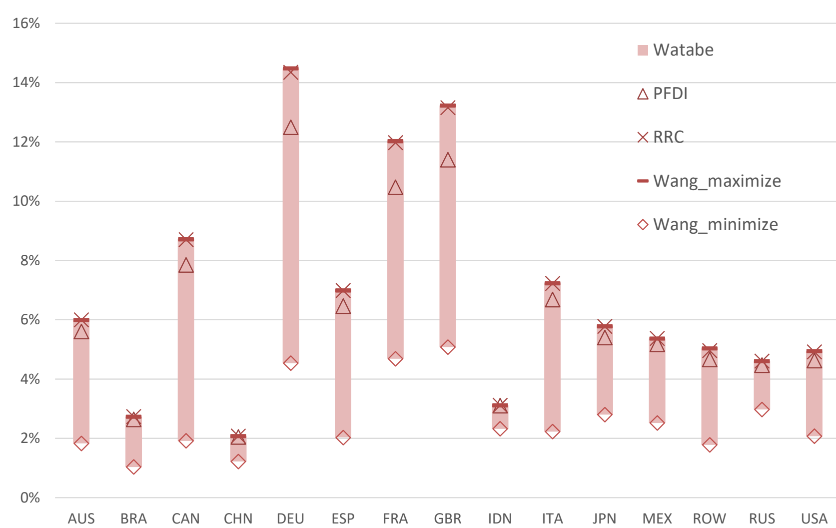
There is a wide range of gains from openness consistent with the data. For example, the gains from openness in Germany range from 4.5% to 14.5%. In general, European countries, which are open both in trade and multinational production, tend to have wider intervals compared to other countries.

[3]There is a similar approach when horizontal FDI is rejected. Construct a subset of the modified horizontal FDI allocation, which is $X_{qlq}^{MHFDI,q} = max\left(0, M_{il} - \sum_{n\neq m} T_{ln}\right)$. If the allocation is feasible, such aalocation achieves the maximum.

In most countries, the allocation from Ramondo and Rodríguez-Clare's model achieves a value close to the maximum. For example, the gains from openness of Germany with Ramondo and Rodríguez-Clare's model are 14.3%, which is almost the maximum of the interval. This suggests that, in this data, the assumption by Ramondo and Rodríguez-Clare (2013) may overestimate gains from openness. The value from the proportional export platform FDI allocation is similar. For example, gains from openness of the U.S. with the proportional export platform FDI allocation are 4.6% while that of the U.S. with Ramondo and Rodríguez-Clare's model is 4.9%.

Wang's model seems to achieve the maximum and the minimum of the interval. The minimum and the maximum in Wang's model for Germany are 4.5% and 14.5%, respectively, equal to the minimum and the maximum achieved from triangulation. Since gains from openness only depend on a small subset of the variables in the allocation, it is not surprising that Wang's model has enough flexibility to achieve the minimum and the maximum.

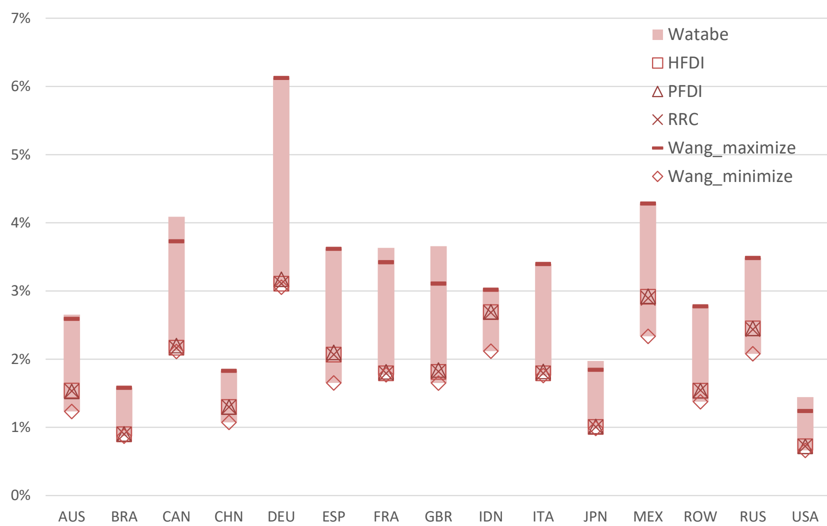
Figure 4.1: Gains from openness



4.1.2 Gains from trade

For gains from trade, I show the interval and the five values from the following allocations: (i) proportional export platform FDI allocation, (ii) allocation from Ramondo and Rodríguez-Clare's model, (iii) allocation of horizontal FDI, (iv) allocation from Wang's model that maximizes gains from trade and (v) allocation from Wang's model that minimizes gains from trade. The result is shown in Figure 4.2.

Figure 4.2: Gains from trade



The range of the interval for gains from trade is much narrower than that of gains from openness. For example, while gains from openness in Germany range from 4.5% to 14.5%, gains from trade in Germany range only from 3.1% to 6.1%. The low maximum of the interval is because gains from trade are bounded above by gains from trade without multinationals.[4]

While the allocation of horizontal FDI achieves the maximum for gains from openness, the same is not the case for the gains from trade; the allocation of horizontal FDI attains low value in the interval. For example, gains from trade in Germany for the horizontal FDI allocation is 3.1%, which is almost the lower end of the interval (the minimum value is 3.05%). In the allocation of horizontal FDI, trade and multinational production are two distinct and substitutable ways to serve the foreign country. Given that horizontal FDI exhibits substitution between trade and multinational production, the gains from trade of that should be small. Wang's model achieves a narrower range compared to the interval from the triangulation. In the U.K., while the interval from the triangulated set

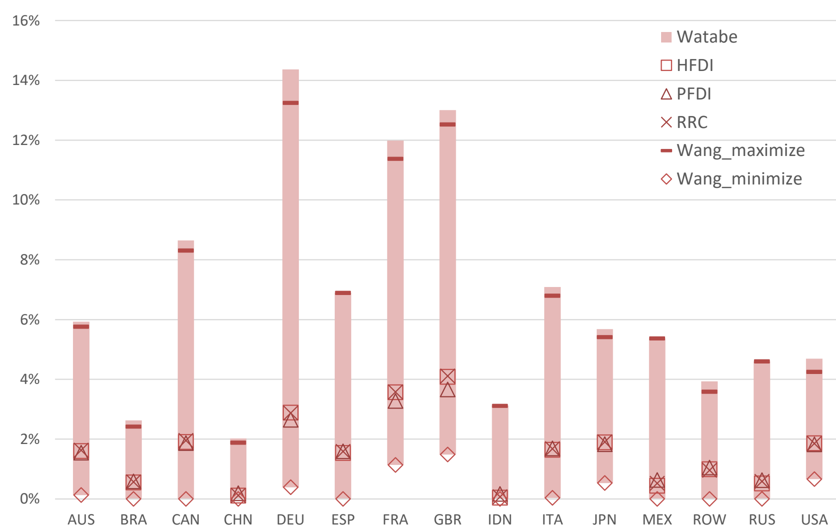
[4]Since multinational production works as a substitute for trade, gains from trade with multinationals are always smaller than those without multinationals. When $\rho = 0$, the gains from trade are uniquely identified and follow the ACR formula without multinationals.

ranges from 1.7% to 3.7%, the value in Wang's model only ranges from 1.7% to 3.1%. This result highlights the restrictiveness of Wang's model in terms of gains from trade.

4.1.3 Gains from multinationals

For gains from multinationals, I show the interval and the values from five allocations: (i) proportional export platform FDI allocation, (ii) allocation from Ramondo-Rodríguez Clare's model, (iii) allocation of horizontal FDI, (iv) allocation from Wang's model that maximizes gains from the multinationals and (v) allocation from Wang's model that minimizes gains from multinationals. The result is shown in 4.3.

Figure 4.3: Gains from multinationals



The range of the interval for gains from multinationals is wide as (or even wider than) that of gains from openness. For example, for Germany, gains from multinationals range from 0.4% to 14.4% (the gains from openness range from 4.5% to 14.5%) and in France, gains from multinationals range from 1.1% to 12.0% (the gains from openness range from 4.6% to 12.0%).

In some countries—notably Brazil, Canada, China, Indonesia, Italy, Mexico, Russia and Spain—the lower end of gains from multinationals is zero. This is when the consumer in the country does not consume goods produced by affiliates.[5] In this case, there is no

[5]This is also due to fixed wage. Since the wages are fixed, only the price change in the consumption basket matters.

loss of consumption due to this counterfactual experiment.

In contrast, in most countries, the maximum of gains from multinationals is equal to the maximum gains from openness. If all of the country's import is indeed made by affiliates from some country (including that country), abandoning multinational production also abandons trade.[6] Thus, the range of the interval for gains from multinationals is wide as that of gains from openness, and is much wider than that of gains from trade.

Similar to gains from trade, gains from multinationals from the horizontal FDI allocation are in the lower end of the interval. In the U.K., the gains from multinationals for the allocation of horizontal FDI are 4.0%, where the interval ranges from 1.4% to 13.0%. The explanation is similar to that of gains from trade. For horizontal FDI, trade and multinational production are two distinct ways to serve the market; hence trade is mitigating the loss from abandoning multinationals. Both gains from multinationals of proportional export platform FDI allocation and that of Ramondo and Rodríguez-Clare's model are at the lower side of the interval. Similar to gains from trade, Wang's model has a smaller range of gains from multinationals. One example is Germany, in which the interval ranges from 0.4% to 14.4% while Wang's model's value ranges from 0.4% to 13.2%.

Overall, gains from openness, gains from trade, and gains from multinationals exhibit economically sizable indeterminacy (range of the interval). Numerical results indicate that the results from the previous studies, which impose various assumptions to the allocation, must be treated with caution.

4.1.4 Why are intervals for gains from trade and gains from multinationals so different?

As shown in this section, the range of gains from trade is significantly narrower than that of gains from multinationals. There are two factors that determine the gains: the expenditure on the goods that are not available in the counterfactual equilibrium and how substitutable these goods are. Variation in these two factors within the triangulated set produces the interval. In gains from trade, the first source is point-identified by looking at the bilateral trade data; imported goods are unavailable by closing trade, and the value of this is observed. The only source of variation is the second factor, which is from offshored goods; these goods are more substitutable than the goods made by foreign firms, thus reducing the loss from closing trade. The interval for gains from

[6] The upper end is not the consequence of partial equilibrium. When trade is abandoned, a general equilibrium effect does not exist.

trade is constructed by varying the amount of offshored goods. In gains from multinationals, the data do not identify the expenditure on goods that are unavailable in the counterfactual equilibrium. A good example is the goods produced by affiliates. In the horizontal FDI allocation, the goods made by foreign affiliates are solely consumed in the host country, while in the vertical FDI allocation, they are solely consumed in the headquarters location. This variation provides an additional range in the intervals.

4.1.5 Bounding gains from trade and gains from multinationals

In the case of gains from openness, intervals are derived by calculating gains from openness for horizontal FDI and vertical FDI. In the case of gains from trade and gains from multinationals, intervals are calculated numerically. To calculate the intervals, I derive the maximum and the minimum of the intervals by searching over possible allocations within the triangulated set (which is how de Gortari (2020) and Wang (2020) calculate the intervals). There is no guarantee that these values attained are indeed maximum or minimum for gains from trade or gains from multinationals. In appendix C, in addition to these values, I provide the upper bounds and the lower bounds on gains from trade and gains from multinationals. The bounds are the values that may not be attainable from the triangulated set (hence may not be maximum nor minimum), but it is guaranteed that any outcomes are between the upper bound and the lower bound. The bounds of gains from trade and gains from multinationals are similar (up to a 2% point difference) to this section's benchmark value.[7]

4.2 Policy evaluation

In this section, I show how this framework can be used for policy evaluations. Inspired by the proposal of the 2020 U.S. democratic presidential nominee, Joe Biden, I evaluate the policy which penalizes offshoring by U.S. multinationals. For this purpose, I develop an exact hat algebra to calculate a counterfactual equilibrium. The evaluation is done in four steps: First, I show the interval of real wage change due to this policy. Secondly, to analyze where the interval stems from, I compare two allocations that are the best and the worst for the U.S. In the third step, I incorporate the data from the BEA (U.S. Bureau of Economic Analysis) to show how the additional data tighten the intervals. In the fourth step, I propose an alternative policy and compare that to the original policy.

[7] Unfortunately, the method I use to calculate the bounds is not practical for the policy evaluation below. Details are in appendix C

4.2.1 Counterfactual using exact hat algebra

I start by developing the exact hat algebra for the model. Exact hat-algebra is a method to compute counterfactual equilibrium in terms of proportional change. I denote $\hat{x}$ a proportional change in a variable x. Formally, $\hat{x} = x'/x$ where x is a value in the observed equilibrium, and x' is the corresponding value in the counterfactual equilibrium. A particular policy experiment is $\hat{\boldsymbol{\tau}}$, a proportional change in the vector $\boldsymbol{\tau}$. For example, reducing the cost of trade from country o to country q by $(1-z)\%$ is formulated as:

$$\hat{\tau}_{ilm} = \begin{cases} z \text{ if } l = o,\ m = q \\ 1 \text{ otherwise }. \end{cases}$$

Given $\hat{\boldsymbol{\tau}}$, exact hat algebra solves a system of nonlinear equations:

$$\hat{p}_{ilm} = \hat{w}_l \hat{\tau}_{ilm} \tag{4.1}$$

$$\hat{\pi}_{ilm} = \frac{\hat{P}_{im}^{-\theta} \hat{p}_{ilm}^{-\theta/(1-\rho)}}{\sum_{j=1}^{N} \pi_{jm} \hat{P}_{jm}^{-\theta} \sum_{k=1}^{N} \pi_{km|i} \hat{p}_{ikm}^{-\theta/(1-\rho)}} \tag{4.2}$$

$$\hat{P}_{im} = \left(\sum_{k=1}^{N} \pi_{km|i} \hat{p}_{ikm}^{-\theta/(1-\rho)} \right)^{-(1-\rho)/\theta} \tag{4.3}$$

$$\hat{P}_m = \left(\sum_{i=1}^{N} \pi_{im} \hat{P}_{im}^{-\theta} \right)^{-1/\theta} \tag{4.4}$$

$$X'_m = \hat{w}_m Y_m + D_m \tag{4.5}$$

$$\hat{w}_m Y_m = \sum_{j=1}^{N} \sum_{k=1}^{N} \hat{\pi}_{kmj} \pi_{kmj} X'_j \tag{4.6}$$

where

$$\pi_{ilm} = \frac{X_{ilm}}{X_m} \tag{4.7}$$

$$\pi_{im} = \frac{\sum_{k=1}^{N} X_{ikm}}{X_m} \tag{4.8}$$

$$\pi_{im|l} = \frac{X_{ilm}}{\sum_{k=1}^{N} X_{ikm}}. \tag{4.9}$$

I denote the variables in the counterfactual equilibrium $\hat{\boldsymbol{x}}$ and the system of nonlinear equations $H(X, \hat{\boldsymbol{\tau}}, \hat{\boldsymbol{x}}) = 0$. I focus on the interval of changes in the real wage ($\hat{W}_q = \hat{w}_q / \hat{P}_q$), where I denote $\hat{\mathbb{W}}_q \equiv [\hat{W}_q^L(\boldsymbol{T}, \boldsymbol{M}, \hat{\boldsymbol{\tau}}), \hat{W}_q^U(\boldsymbol{T}, \boldsymbol{M}, \hat{\boldsymbol{\tau}})]$. The maximum

$\hat{W}_q^U(\boldsymbol{T}, \boldsymbol{M}, \hat{\boldsymbol{\tau}})$ can be calculated by solving a following problem:

$$\begin{aligned} \hat{W}_q^U(\boldsymbol{T}, \boldsymbol{M}, \hat{\boldsymbol{\tau}}) &= maximize_{\boldsymbol{X}, \hat{\boldsymbol{x}}} \ \hat{w}_q / \hat{P}_q \\ s.t. \ M_{il} &= \sum_{m=1}^{N} X_{ilm} \ \forall \ i, l \\ T_{lm} &= \sum_{i=1}^{N} X_{ilm} \ \forall \ l, m \\ X_{ilm} &\geq 0 \ \forall \ i, l, m. \\ H(X, \hat{\tau}, \hat{x}) &= 0, \end{aligned}$$

and the minimum can be similarly calculated.I solve these problems for each country to obtain the interval.

4.2.2 Policy experiment

In this paper, I focus on a protectionist policy that involves both trade and multinational production. On September 9th, 2020, Joe Biden proposed a 10% surtax to goods produced by U.S. affiliates abroad that are sold to U.S. consumers (Wilkie (2020)). This policy aims to discourage offshoring by U.S. multinationals and bring jobs back to the United States. In my model, the policy is modeled as a 10% increase in the cost of production when firms from the U.S. produce goods in foreign countries and is sold to the U.S. consumers:

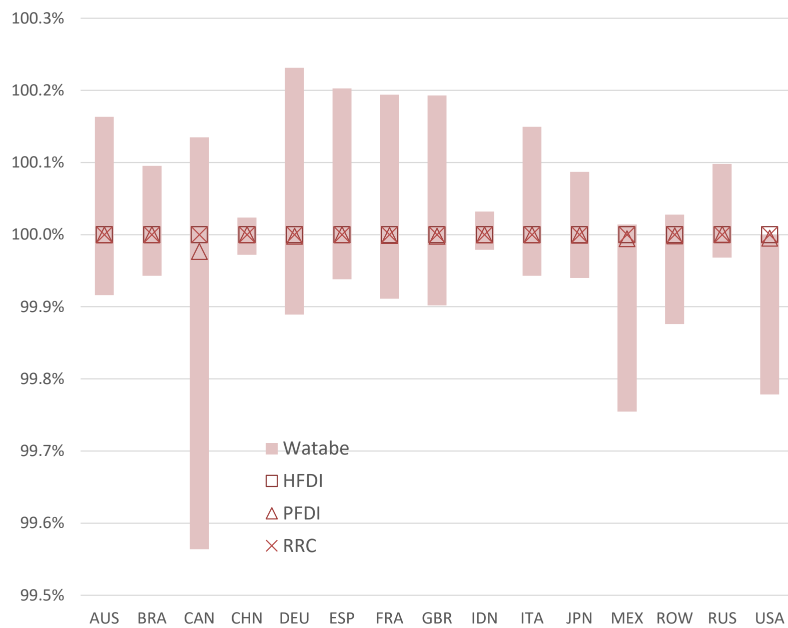
$$\hat{\tau}_{ilm} = \begin{cases} 1.1 & \text{if } i = U.S., l \neq U.S., m = U.S. \\ 1 & \text{otherwise.} \end{cases}$$

For U.S. consumers, the effect of this policy is ambiguous. On the one hand, the policy increases the price of the offshored goods, which will hurt the U.S. consumer. On the other hand, by suppressing the demand for foreign labor and bringing the production back to the U.S., this policy can improve terms of trade for the U.S. The net effect of the policy should be calculated by solving the counterfactual equilibrium. A similar argument can be made for the consumers in other countries.

The welfare prediction is shown in terms of intervals. For each country, I calculate the interval of welfare change (change in the real wage). I also calculate the outcome for three allocations: (i) proportional export platform FDI allocation, (ii) allocation from

Ramondo and Rodríguez-Clare's model, (iii) horizontal FDI allocation.[8]

Figure 4.4: Biden's policy : Benchmark



Note: 100.0% corresponds to the status quo.

The result is shown in Figure 4.4. There are three noticeable features in the result. Firstly, the policy has no effect in the horizontal FDI allocation. This is because in the horizontal FDI allocation, there is no offshoring of multinationals. Since the other two allocations are similar to the allocation of horizontal FDI, they have similar predictions. Secondly, for countries other than the U.S., the effect of the policy could either be positive or negative. This suggests that the data on bilateral trade and multinational production are insufficient to judge if the policy is beneficial for other countries. Third, the prediction on the U.S. is clear in this model: The interval ranges from 0% to -0.2%, which implies that the policy does not benefit the U.S. consumer. The direct effect on the consumer price outweighs the terms of trade improvement.

To anatomize the variation in the outcome, I examine two allocations: the allocation that maximizes the welfare gain of the U.S. consumer and the allocation that minimizes the welfare gain of the U.S. consumer. These two allocations take two extremes in terms of offshoring of U.S. multinationals. In the allocation for the maximum gain (no change

[8] I did not include Wang's model in this exercise. This is because performing exact hat algebra with Wang's assumption is challenging in my computational resources. In many cases, solvers cannot provide a solution within the allotted time.

in the U.S. real wage), there is no offshoring in the allocation. In the allocation for the minimum gain (0.2% loss in the U.S. real wage), 82% of U.S. import is offshored goods. This suggests that the amount of offshoring by U.S. multinationals is crucial to identify the consequence of the policy.

4.2.3 Narrowing the interval using additional data

To further narrow the interval, I add information on U.S. multinationals' offshoring activity. BEA provides the data on sales of U.S. affiliates abroad to the U.S. Specifically, I collect the data for the affiliates in Canada, China, Germany and Japan ($\{X_{USA,l,USA}\}_{l=CAN,CHN,DEU,JPN}$ and include them as additional constraints on the allocation. The result is shown in Figure 4.5. I show two sets of intervals: Intervals without additional data on the allocation (which are the intervals on Figure 4.4), and intervals with additional information on the allocation.

There are three points to be noticed. First, the intervals with additional data on the allocation have a narrower range. Second, the reduction of the range does happen not only in the countries with additional data but also in other countries. The general equilibrium feature of the model propagates the information to narrow the policy effect for countries without additional data. The additional data are most informative for Canada; without additional data, the change in welfare ranges from -0.44% to 0.13%, while with the additional data, it ranges from -0.26% to 0.019%. This is natural given that I added the data for Canada, and the U.S. and Canada have a large volume of trade and multinational production. Third, the additional data further shows that U.S. consumers lose from this policy in terms of welfare; at best U.S. consumers lose around 0.005% and at worst, they lose around 0.2%.

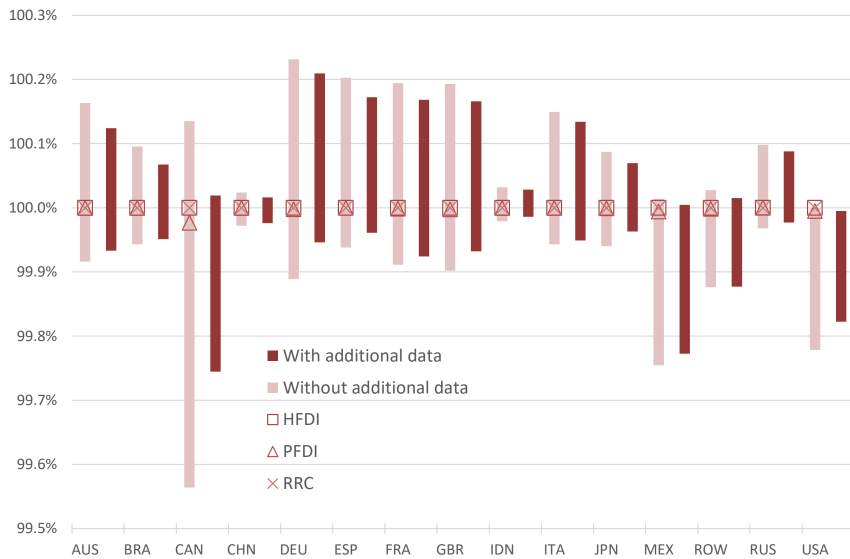
4.2.4 Is there a better policy?

I further explore whether there is a possible policy that is more beneficial to the U.S. consumer. Simple alternative to this policy is to penalize U.S. multinationals' production, regardless of the sales destination. The alternative policy can be described as follows:

$$\hat{\tau}_{ilm} = \begin{cases} 1.1 & \text{if } i = U.S., l \neq U.S. \\ 1 & \text{otherwise.} \end{cases}$$

I use the augmented data and derive the welfare intervals for each country. Figure

Figure 4.5: Biden's policy : Augmented data

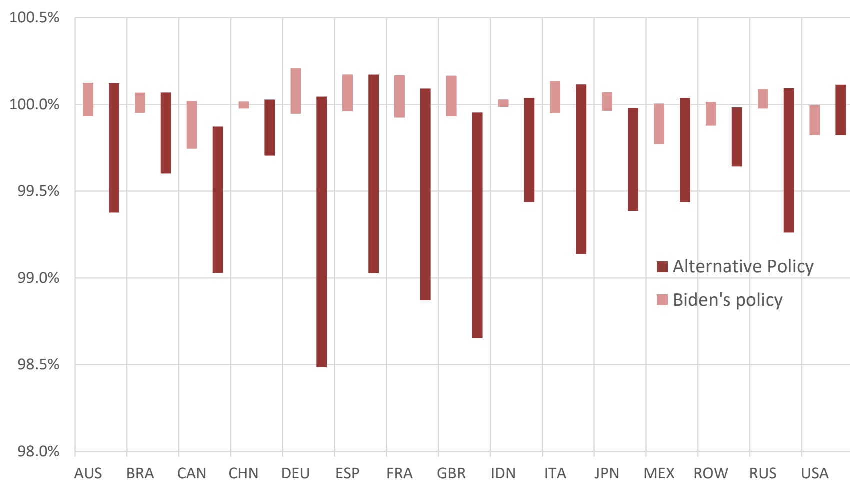


Note: 100.0% corresponds to the status quo.

4.6 compares the intervals of the original policy and the alternative policy. Most of the countries, other than the U.S., are likely to be worse off when the alternative policy is implemented.[9] The welfare change from the alternative policy tends to have a lower maximum and a lower minimum than the original policy. The interval is also wider for the alternative policy; the consequence of the alternative policy is much more indeterminate than the original one. This is because this alternative policy is agnostic on who bears the direct cost of the policy (the direct cost of the original policy is concentrated on the U.S. consumer). The policy is likely to be better for the United States. Both policies produce identical welfare loss for the worst case, while the alternative policy has a brighter possibility; at best, the U.S. may gain about 0.1% from this policy. The interval suggests that the alternative policy may be better for the U.S. while potentially harmful for other countries.

[9]Notice that this is not an allocation-wise comparison. There may be an allocation that the original policy is more harmful than the alternative policy.

Figure 4.6: Bident's policy : Comparison



Note: 100.0% corresponds to the status quo.

Chapter 5
Incorporating intermediate goods

In this chapter, I discuss the triangulation with intermediate goods and the model associated with it. Now the goods can be used as an intermediate input and consumption. In addition to the origin country i of the exporting firm, for intermediate goods, there is country n, the origin of the importing firm. Denote the value of goods used as intermediate goods produced by firms from country i, produced in country l, delivered to country m and used by the firms from country n as $X_{ilnm,int}$. Similarly, denote the value of goods used as final consumption produced by firms from country i, produced in country l and delivered to country m as $X_{ilm,f}$.

5.1 Triangulation

I use global input-output tables and the multinational production data for the triangulation. Accounting identities for multinational production are:

$$M_{il} = \sum_{n=1}^{N} \sum_{m=1}^{N} X_{ilnm,int} + \sum_{m=1}^{N} X_{ilm,f}$$

where now M_{il} is a summation of the production of intermediate goods and the production of final goods. The global input-output table records $T_{lm,int}$, a flow of intermediate goods from country l to country m; $T_{lm,f}$ a flow of final goods from country l to country m. The accounting identities of the global input-output talbes are:

$$T_{lm,int} = \sum_{i=1}^{N} \sum_{n=1}^{N} X_{ilnm,int}$$
$$T_{lm,f} = \sum_{i=1}^{N} X_{ilm,f},$$

where the trade flow of intermediate goods is a summation of $X_{ilnm,int}$ over the origin i of the exporting firm and the origin n of the importing firm. Similarly the trade flow of final goods is a summation of $X_{ilm,f}$ of over the origin of the exporting firm i. I denote $\boldsymbol{X}_{int}$ and $\boldsymbol{X}_f$ as the vector notations of the intermediate goods flow and the final goods flow. In addition to the non-negativity constraint for $\boldsymbol{X}_{int}$ and $\boldsymbol{X}_f$, I restrict the value added to be non-negative:

$$\sum_{n}^{N}\sum_{m}^{N} X_{nmil,int} \leq \sum_{n}^{N}\sum_{m}^{N} X_{ilnm,int} + \sum_{m=1}^{N} X_{ilm,f}$$

The triangulated set $\mathbb{X}$ is a set of $\boldsymbol{X}_{int}$ and $\boldsymbol{X}_f$ that satisfies the conditions above.

5.2 Special case

I propose a special case that identifies the allocation from the data. The allocation of proportional export platform FDI with intermediate goods is expressed as follows:

$$\begin{aligned} X_{ilnm,int}^{PFDI} &= T_{lm,int}\frac{M_{il}}{\sum_{j=1}^{N} M_{jl}}\frac{M_{nm}}{\sum_{j=1}^{N} M_{jm}} \\ X_{ilm,f}^{PFDI} &= T_{lm,f}\frac{M_{il}}{\sum_{j=1}^{N} M_{jl}}. \end{aligned}$$

The first equation implies that the composition of intermediate goods are common across the importing firms regardless of their origin if they are located in the same production location. This allocation is always in the triangulated set. The accounting identities on multinational production are satisfied as:

$$\begin{aligned} \sum_{n=1}^{N}\sum_{m=1}^{N} X_{ilnm,int} + \sum_{m=1}^{N} X_{ilm,f} &= \sum_{m=1}^{N} \left(T_{lm,int} + T_{lm,f}\right)\frac{M_{il}}{\sum_{j=1}^{N} M_{jl}} \\ &= M_{il}. \end{aligned}$$

The accounting identities on the global input-output table are trivially satisfied. The non-negativity of the allocation and the value-added can be verified through a simple calculation.

5.3 Model

I develop a model that incorporates multinationals into the Armington model (Armington 1969) with input-output linkage. This model generalizes the model developed by Li (2021)[1]. There are N countries in the economy with representative firms and consumers. A consumer earn wages from her labor (where a representative consumer in country l provide L_i amount of labor inelastically) and purchase goods. There is a trade deficit in the model, which is an exogenous transfer between countries.

As in the original model, the utility function of a representative consumer in country m is:

$$U_m = \left(\sum_{i=1}^{N} \left(\sum_{l=1}^{N} C_{ilm,f}^{\frac{\epsilon}{\epsilon+1}} \right)^{\frac{\epsilon+1}{\epsilon}\frac{\theta}{\theta+1}} \right)^{\frac{\theta+1}{\theta}}. \tag{5.1}$$

Denote I_m as a total income for the consumer in country m. The expenditure of the final goods $X_{ilm,f}$ is

$$X_{ilm,f} = \frac{P_{im,f}^{-\theta}}{\sum_{j=1}^{N} P_{jm,f}{}^{-\theta}} \frac{p_{ilm,f}^{-\theta/(1-\rho)}}{\sum_{k=1}^{N} p_{ikm,f}^{-\theta/(1-\rho)}} I_m \tag{5.2}$$

where $P_{im,f} \equiv \left(\sum_{k=1}^{N} (p_{ikm,f}^{-\theta/(1-\rho)})\right)^{-(1-\rho)/\theta}$ is the price index in country m for final goods produced by the firms from country i. The price index of final goods in country m is

$$P_{m,f} = \left[\sum_{i=1}^{N} P_{im,f}^{-\theta} \right]^{-1/\theta}. \tag{5.3}$$

Firms combine labor and intermediate goods in a Cobb-Douglas manner to produce both the final goods and the intermediate goods. The cost share of the labor input is β_{il} (the cost share of the intermediate goods is $1 - \beta_{il}$). The labor share β_{il} depends on firm origin i and production location l. Intermediate goods from different firm origins and production locations are aggregated into composite intermediate goods in a nested CES manner (similar to the utility function for the representative consumer). Denote τ_{ilnm}^{int} as the quantity of (Cobb-Douglas composite of) labor and composite intermediate goods required by firms from country i, produced in country l, used in the production of firms from country n and delivered to country m. Denote τ_{ilm}^{f} as the quantity of

[1] Li 2021 assumes separability on $\boldsymbol{\tau}$ similar to Wang (2020)

(Cobb-Douglas composite of) goods required for final consumption produced by firms from country i, produced in country l, delivered to country m.

Perfect competition implies the price is set to the marginal cost of production. The price of intermediate goods $p_{ilmn,int}$ (which is the price corresponding to $X_{ilnm,int}$) is

$$p_{ilnm,int} = \tau_{ilnm} w_l^{\beta_{il}} P_{il,int}^{1-\beta_{il}}, \tag{5.4}$$

where

$$P_{inm,int} = \left(\sum_{k=1}^{N} p_{iknm,int}^{-\theta/(1-\rho)} \right)^{-(1-\rho)/\theta} \tag{5.5}$$

$$P_{nm,int} = \left(\sum_{j=1}^{N} P_{jnm,int}^{-\theta} \right)^{-1/\theta} . \tag{5.6}$$

and the expenditure on such intermediate goods is

$$X_{ilnm,int} = \frac{P_{inm,int}^{-\theta}}{\sum_{j=1}^{N} P_{jnm,int}{}^{-\theta}} \frac{p_{ilnm,int}^{-\theta/(1-\rho)}}{\sum_{k=1}^{N} p_{iknm,int}^{-\theta/(1-\rho)}} (1-\beta_{nm}) \left(\sum_{j=1}^{N} \sum_{k=1}^{N} X_{nmjk,int} + \sum_{k=1}^{N} X_{nmk,f} \right). \tag{5.7}$$

Similarly, the price of final goods $p_{ilm,f}$ delivered to country m produce in country l by firms from country i is

$$p_{ilmn,int} = \tau_{ilm} w_l^{\beta_{il}} P_{il,int}^{1-\beta_{il}}. \tag{5.8}$$

Total income of the representative consumer in country l is the labor income from the production and the transfer D_l:

$$I_l = w_l L_l + D_l = (1-\beta_{il}) \left(\sum_{i=1}^{N} \sum_{m=1}^{N} \sum_{n=1}^{N} X_{ilnm,int} + \sum_{i=1}^{N} \sum_{m=1}^{N} X_{ilm,f} \right) + D_l \tag{5.9}$$

Denote the vector of variables in the bold font. Given $\boldsymbol{D}$, $\boldsymbol{L}$, $\boldsymbol{\beta}$, $\boldsymbol{\tau}_{int}, \boldsymbol{\tau}_f$, an equilibrium is a wage vector $\boldsymbol{w}$, a price vector $\boldsymbol{p}_{int}$, $\boldsymbol{p}_f$, a final goods allocation $\boldsymbol{X}_{int}$ and a intermediate goods allocation $\boldsymbol{X}_f$ that satisfy the consumer optimization, the producer optimization and the market clearing condition.

5.4 Gains from Openness

Gains from openness with intermediate goods is written as follows:

$$GO_q(\boldsymbol{X}_{int}, \boldsymbol{X}_f) = \left(\frac{\sum_{k=1}^N X_{qkq,f}}{I_q}\right)^{-1/\theta} \left(\frac{X_{qqq,f}}{\sum_{k=1}^N X_{qkq,f}}\right)^{-(1-\rho)/\theta}$$
$$\left(\frac{\sum_{k=1}^N X_{qkqq,int}}{\sum_{i=1}^N \sum_{k=1}^N X_{ikqq,int}}\right)^{-(1-\beta_{qq})/\beta_{qq}\theta} \left(\frac{X_{qqqq,int}}{\sum_{k=1}^N X_{qkqq,int}}\right)^{-(1-\beta_{qq})(1-\rho)/\beta_{qq}\theta} \tag{5.10}$$

where β_{qq} is derived from the cost share:

$$\beta_{qq} = \frac{\sum_n^N \sum_m^N X_{nmil,int}}{\sum_n^N \sum_m^N X_{ilnm,int} + \sum_{m=1}^N X_{ilm,f}}. \tag{5.11}$$

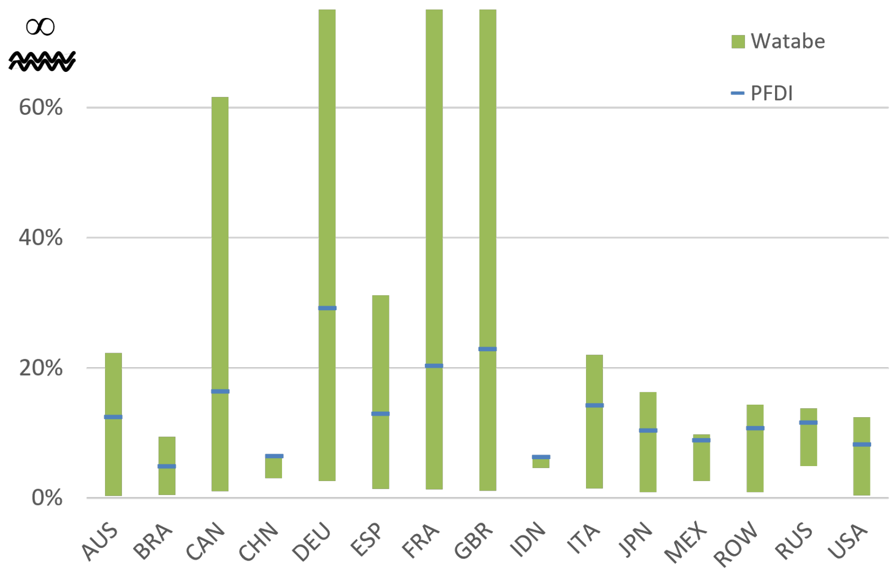
5.5 Quantification

I use the exact same data for the multinational production data while I disaggregate the bilateral trade flow into the intermediate goods flow and the final goods flow. I use the same parameter for θ and ρ. I calculate the intervals of gains from openness for 15 countries.[2] I include proportional export platform FDI as a special case and combine that with the interval. The result is shown in figure 5.1:

In countries like Germany, France and the U.K., the gains from openness are unbounded above. A simple example for the economy with infinite gains from openness is the case where the production of the domestic firm solely relies on intermediate inputs produced by foreign firms or imported. The autarky counterfactual makes the domestic firms impossible to use these intermediate goods; there is no domestic good produced to consume. Even for other countries with bounded intervals, the intervals tend to be wider in range. For example, with intermediate goods, the U.S. has gains from openness ranging from 0.04% to 12.4%, while without intermediate goods, the gains from openness for the U.S. ranges from 2.0% to 4.9%. The wider range suggests assuming particular allocation maybe be problematic. For many countries, gains from openness calculated from proportional export platform FDI is neither close to the maximum or the minimum, which implies the assumption is not informative to predict gains from

[2]While it is theoretically possible to perform policy experiments using exact hat-algebra for this model, obtaining an interval is too computationally costly (in terms of memory) to perform.

Figure 5.1: Gains from openness with intermediate goods



Note: Any interval ranging over the broken axis indicates that the interval is unbounded from above.

openness.

Chapter 6
Conclusion

6.1 Conclusion

Multinational production occurs various motives: multinational firms may produce abroad to access the foreign country they invested in, reduce production costs, or access the regional markets. Previous studies specify the particular motive and develop a quantitative model to perform counterfactual experiments. Different specifications provide different results, and there is no consensus about the correct specifications. I develop an empirical approach — *triangulation* and a quantitative model that is compatible with the triangulation — that allows multinational production to occur for various motives and synthesize models in the literature as special cases. The model and the triangulation provide an interval of counterfactual outcomes, including outcomes from the past studies. The interval shows possible outcomes without imposing any assumption on the motives and also summarizes the indeterminacy of the counterfactual outcomes due to the incomplete data. I use the interval to theoretically analyze the role of assumptions in the literature on quantification. I show that, for any country, gains from openness (from both trade and multinational activity) are at a maximum with pure horizontal FDI and a minimum with pure vertical FDI. Empirical results show that a wide range of outcomes is consistent with the data; for example, the welfare gains from openness in Germany range between 4.5% and 14.5%. While the incompleteness of the data leaves open a wide range of outcomes, what data are available still brings clear judgment on policy evaluation; for example, penalizing U.S. multinationals for offshoring never benefits the United States.

Appendix A
Quantitative model

A.1 Constructing a model with arbitrary allocation

Here I show that given $\theta, \rho, \boldsymbol{L}$ and $\boldsymbol{D}$, for any $\boldsymbol{X}$, there exists $\boldsymbol{\tau}$ such that $\boldsymbol{X}$ is an equilibrium outcome. Firstly, I set $\boldsymbol{w}$ as

$$w_l L_l = \sum_m \sum_k X_{klm} + D_l,$$

so that the economy satisfies goods market equilibrium. Now for the elements of $\boldsymbol{X}$ where the value is zero, I set $\tau_{ilm} = \infty$. $p_{ilm} = \infty$, eliminating the expenditure for such goods. For the non-zero elements of $\boldsymbol{X}$, I set τ_{ilm} so that the demand of goods to be what is observed in $\boldsymbol{X}$. Specifically, I set $\tau_{ilm} = p_{ilm}/w_l$ so that

$$X_{ilm} = \frac{P_{im}^{-\theta}}{\sum_j P_{jm}{}^{-\theta}} \frac{p_{ilm}^{-\theta/(1-\rho)}}{\sum_k p_{ikm}^{-\theta/(1-\rho)}} X_m.$$

For each market m, there are N^2 equations (ignoring the zeros) and N^2 parameters τ_{ilm}. The $\boldsymbol{\tau}$ satisfying the equation can be found following the steps in the appendix of Berry 1994. The goods are gross substitute and the demand function is differentiable, which is the sufficient condition for the steps to work.

A.2 The equivalence between proportionality assumption and the separability assumption

With parameter $\rho = 0$, the allocation $\boldsymbol{X}^{PFDI}$ is equivalent to $\boldsymbol{X}^{RRC}$. Formally stated, this implies

Proposition A.2.1. *If* $\rho = 0$ *and* $\boldsymbol{X}^{RRC} \in \mathbb{X}$, $\boldsymbol{X}^{RRC} = \boldsymbol{X}^{PFDI}$.

Proof. Notice that Ramondo and Rodríguez-Clare (2013) assumes $\tau_{ilm} = \gamma_{il}\zeta_{lm}$. Combined with $\rho = 0$, this implies

$$X_{jlm} = \left(\frac{\gamma_{il}}{\gamma_{jl}}\right)^{\theta} X_{ilm}$$

Now I show that this implies

$$X_{ilm} = \frac{M_{il}}{\sum_{j}^{N} M_{jl}} T_{lm}$$

This is satisfied by the following reformulation:

$$\begin{aligned}
\frac{M_{il}}{\sum_{j=1}^{N} M_{jl}} T_{lm} &= \frac{\sum_{n=1}^{N} X_{iln}}{\sum_{j=1}^{N}\sum_{n=1}^{N} X_{jln}} \sum_{j=1}^{N} X_{jlm} \\
&= \frac{\sum_{n=1}^{N} X_{iln}}{\sum_{j=1}^{N}\sum_{n=1}^{N} \left(\frac{\gamma_{il}}{\gamma_{jl}}\right)^{\theta} X_{iln}} \left(\sum_{j=1}^{N}\left(\frac{\gamma_{il}}{\gamma_{jl}}\right)^{\theta} X_{ilm}\right) \\
&= X_{ilm} \frac{\sum_{n=1}^{N} X_{iln}}{\sum_{j=1}^{N}\sum_{n=1}^{N} \left(\frac{\gamma_{il}}{\gamma_{jl}}\right)^{\theta} X_{iln}} \sum_{j=1}^{N}\left(\frac{\gamma_{il}}{\gamma_{jl}}\right)^{\theta} \\
&= X_{ilm} \frac{\sum_{j=1}^{N}\left(\frac{\gamma_{il}}{\gamma_{jl}}\right)^{\theta} \sum_{n=1}^{N} X_{iln}}{\sum_{j=1}^{N}\left(\frac{\gamma_{il}}{\gamma_{jl}}\right)^{\theta} \sum_{n=1}^{N} X_{iln}} \\
&= X_{ilm}
\end{aligned}$$

which implies for any i, l, m, $X_{ilm}^{EXP} = X_{ilm}^{RRC}$. □

A.3 Wang (2020) and canonical theories

Here I show a simple example that $\boldsymbol{X}^{HFDI}$ and $\boldsymbol{X}^{VFDI}$ are not in $\mathbb{X}^{Wang}$ while they are in the triangulated set. Consider three symmetric countries a, country b and country c. The flow of multinational production and trade are $M_{il} = M_f > 0, T_{lm} = T_f > 0$ for all $i \neq l$ and $l \neq m$. The domestic production and trade flow are same across countries, and are noted as $M_{ii} = M_d$, $T_{ii} = T_d$ $\forall i$. I assume:

$$M_d > 2T_f$$

$$T_f > M_f$$

which implies that both horizontal FDI and vertical FDI are in the triangulated set. Furthermore, this implies that there must be a positive export from headquarters to other countries for both allocations for horizontal FDI and vertical FDI. Both horizontal FDI assumption and vertical FDI assumption implies $X_{abc} = 0$. Then the separability assumption implies:

$$\gamma_{ab}\zeta_{bc}\xi_{ac} = \infty.$$

This means at least one of the separable components must be arbitrarily large. Think of the case with $\zeta_{ab} = \infty$. This indicates $M_{ab} = M_f = 0$, which contradicts the observed multinational production. Similarly, $\zeta_{bc} = \infty$ indicates $T_{bc} = T_f = 0$, which contradicts the observed trade flow. The only possibility left is $\xi_{ac} = \infty$, but this requires $X_{aac} = 0$. Given $M_d > 2T_f$, the horizontal FDI allocation implies $X_{aac} > 0$ which contradicts $\xi_{ac} = \infty$. Similarly, given $T_f > M_f$, the vertical FDI assumption implies $X_{aac} > 0$, which contradicts with $\xi_{ac} = \infty$. Therefore, both horizontal FDI allocation and vertical FDI allocation are not in $\mathbb{X}^{Wang}$ while they are in $\mathbb{X}$.

A.4 Deriving gains from trade and gains from multinationals

In this section I show how to derive gains from trade and gains from multinationals. I denote x' a variable x in a counterfactual equilibrium. For gains from trade, this is for the counterfactual equilibrium without trade, and for gains from multinationals, this is for the counterfactula equilibrium without multinationals. I reiterate the additional notion used for covenience:

$$\pi_{ilm} = \frac{P_{im}^{-\theta}}{\sum_j P_{jm}{}^{-\theta}} \frac{p_{ilm}^{-\theta/(1-\rho)}}{\sum_k p_{ikm}^{-\theta/(1-\rho)}}$$
$$\pi_{il} = \frac{P_{im}^{-\theta}}{\sum_j P_{jm}{}^{-\theta}}$$
$$\pi_{lm|i} = \frac{p_{ilm}^{-\theta/(1-\rho)}}{\sum_k p_{ikm}^{-\theta/(1-\rho)}}.$$

A.4.1 Gains from trade

If there is no trade, there is no exchange of labor embodied in trade; I can disregard the change in the wages. I only need to track the changes in the price index. The price index of the country q is

$$P_q = \left[\sum_{i=1}^{N} P_{iq}^{-\theta}\right]^{-1/\theta}.$$

The counterfactual price index in trade autarky is

$$P'_q = \left[\sum_{i=1}^{N} P_{iq}'^{-\theta}\right]^{-1/\theta}.$$
$$P'_{iq} = p'_{iqq} = p_{iqq}.$$

Without trade, the only goods available from firms from country i are the goods produced in country q. Hence $P'_{iq} = p'_{iqq}$. Since the wage does not change, the price of such goods does not change. Notice that

$$\begin{aligned} P_{iq} &= \left(\sum_{l=1}^{N} p_{ilq}^{-\theta/(1-\rho)}\right)^{-(1-\rho)/\theta} \\ &= p_{iqq}\left(\sum_{l=1}\left(\frac{p_{ilq}}{p_{iqq}}\right)^{-\theta/(1-\rho)}\right)^{-(1-\rho)/\theta} \end{aligned}$$

and since $\frac{X_{ilq}}{X_{iqq}} = \left(\frac{p_{ilq}}{p_{iqq}}\right)^{-\theta/(1-\rho)}$, I have

$$\begin{aligned} P_{iq} &= p_{iqq}\left(\sum_{l=1}^{N}\frac{X_{ilq}}{X_{iqq}}\right)^{-(1-\rho)/\theta} \\ &= p_{iqq}\pi_{qq|i}^{(1-\rho)/\theta}. \end{aligned}$$

I derive an expression for $\frac{p_{iqq}}{p_{qqq}}$:

$$\begin{aligned} \frac{\sum_{l=1}^{N} X_{qlq}}{\sum_{l=1}^{N} X_{ilq}} = \frac{\pi_{qq}}{\pi_{iq}} &= \left(\frac{P_{qq}}{P_{iq}}\right)^{-\theta} \\ &= \left(\frac{p_{qqq}}{p_{iqq}}\right)^{-\theta}\left(\frac{\pi_{qq|q}}{\pi_{qq|i}}\right)^{-(1-\rho)} \end{aligned}$$

$$\left(\frac{p_{iqq}}{p_{qqq}}\right)^{-\theta} = \left(\frac{\pi_{iq}}{\pi_{qq}}\right)\left(\frac{\pi_{qq|q}}{\pi_{qq|i}}\right)^{-(1-\rho)}.$$

Using these equations, the price index of country q is

$$\begin{aligned}
P_q &= \left[\sum_{i=1}^{N} p_{iqq}^{-\theta}\pi_{qq|i}^{-(1-\rho)}\right]^{-1/\theta}. \\
&= p_{qqq}\left[\sum_{i=1}^{N}\left(\frac{p_{iqq}}{p_{qqq}}\right)^{-\theta}\pi_{qq|i}^{-(1-\rho)}\right]^{-1/\theta} \\
&= p_{qqq}\left[\sum_{i=1}^{N}\frac{\pi_{iq}}{\pi_{qq}}\left(\frac{\pi_{qq|q}}{\pi_{qq|i}}\right)^{-(1-\rho)}\pi_{qq|i}^{-(1-\rho)}\right]^{-1/\theta} \\
&= p_{qqq}\left[\sum_{i=1}^{N}\frac{\pi_{im}}{\pi_{qq}}\pi_{qq|q}^{-(1-\rho)}\right]^{-1/\theta} \\
&= p_{qqq}\pi_{qq}^{1/\theta}\pi_{qq|q}^{(1-\rho)/\theta}
\end{aligned}$$

and the price index in trade autarky is

$$\begin{aligned}
P'_q &= \left[\sum_{i=1}^{N} p_{iqq}^{-\theta}\right]^{-1/\theta} \\
&= p_{qqq}\left[\sum_{i=1}^{N}\left(\frac{p_{iqq}}{p_{qqq}}\right)^{-\theta}\right]^{-1/\theta} \\
&= p_{qqq}\left[\sum_{i=1}^{N}\frac{\pi_{iq}}{\pi_{qq}}\left(\frac{\pi_{qq|q}}{\pi_{qq|i}}\right)^{-(1-\rho)}\right]^{-1/\theta} \\
&= p_{qqq}\pi_{qq}^{1/\theta}\pi_{qq|q}^{(1-\rho)/\theta}\left[\sum_{i=1}^{N}\pi_{iq}\pi_{qq|i}^{(1-\rho)}\right]^{-1/\theta}.
\end{aligned}$$

Then the gains from trade is

$$\begin{aligned}
GT_q = \frac{P'_q}{P_q} &= \left[\sum_{i=1}^{N}\pi_{iq}\pi_{qq|i}^{(1-\rho)}\right]^{-1/\theta} \\
&= \left[\sum_{i=1}^{N}\pi_{iq}^{\rho}\pi_{iqq}^{(1-\rho)}\right]^{-1/\theta} \\
&= \left[\sum_{i=1}^{N}\left(\sum_{l=1}^{N}X_{ilq}\right)^{\rho}X_{iqq}^{(1-\rho)}\right]^{-1/\theta} X_q^{1/\theta}.
\end{aligned}$$

A.4.2 Gains from multinationals

In the case of gains from multinationals, I cannot ignore the change in the wages, since there is an exchange of labor through trade. I assume that there is no change in the wages, which can be justified by assuming a freely tradable numeraire sector, which fixes the wage. The price index of the country q is

$$P_q = \gamma \left[\sum_i P_{iq}^{-\theta} \right]^{-1/\theta}.$$

Now I state the counterfactual price index in multinational autarky. This is

$$P'_q = \gamma \left[\sum_{i=1}^{N} P_{iq}'^{-\theta} \right]^{-1/\theta}.$$
$$P'_{iq} = p_{iiq}.$$

Here $P'_{iq} = p'_{iiq}$ since without trade, the only goods available from a firm from country i is the goods produced in country i. The price of such goods does not change since the wage is fixed. Notice that

$$\begin{aligned} P_{iq} &= \left(\sum_{l=1}^{N} p_{ilq}^{-\theta/(1-\rho)} \right)^{-(1-\rho)/\theta} \\ &= p_{iiq} \left(\sum_{l=1}^{N} \left(\frac{p_{ilq}}{p_{iiq}} \right)^{-\theta/(1-\rho)} \right)^{-(1-\rho)/\theta}, \end{aligned}$$

and since $\frac{X_{ilq}}{X_{iiq}} = \left(\frac{p_{ilq}}{p_{iiq}} \right)^{-\theta/(1-\rho)}$, I have

$$\begin{aligned} P_{iq} &= p_{iiq} \left(\sum_{l=1}^{N} \frac{X_{ilq}}{X_{iiq}} \right)^{-(1-\rho)/\theta} \\ &= p_{iiq} \pi_{iq|i}^{(1-\rho)/\theta} \end{aligned}$$

I calculate $\frac{p_{iiq}}{p_{qqq}}$:

$$\begin{aligned} \frac{\pi_{qq}}{\pi_{iq}} &= \left(\frac{P_{qq}}{P_{iq}} \right)^{-\theta} \\ &= \left(\frac{p_{qqq}}{p_{iiq}} \right)^{-\theta} \left(\frac{\pi_{qq|q}}{\pi_{iq|i}} \right)^{-(1-\rho)} \end{aligned}$$

$$\left(\frac{p_{iiq}}{p_{qqq}}\right)^{-\theta} = \left(\frac{\pi_{iq}}{\pi_{qq}}\right)\left(\frac{\pi_{qq|q}}{\pi_{iq|i}}\right)^{-(1-\rho)}.$$

The price index is already derived in the previous section, which is

$$P_q = \gamma p_{qqq} \pi_{qq}^{1/\theta} \pi_{qq|q}^{(1-\rho)/\theta}.$$

The price index for multinational autarky is

$$\begin{aligned}
P'_q &= \gamma \left[\sum_{i=1}^{N} p_{iiq}^{-\theta}\right]^{-1/\theta} \\
&= p_{qqq} \left[\sum_{i=1}^{N} \left(\frac{p_{iiq}}{p_{qqq}}\right)^{-\theta}\right]^{-1/\theta} \\
&= p_{qqq} \left[\sum_{i=1}^{N} \frac{\pi_{iq}}{\pi_{qq}} \left(\frac{\pi_{qq|q}}{\pi_{iq|i}}\right)^{-(1-\rho)}\right]^{-1/\theta} \\
&= p_{qqq} \pi_{qq}^{1/\theta} \pi_{iq|q}^{(1-\rho)/\theta} \left[\sum_{i=1}^{N} \pi_{iq} \pi_{iq|i}^{(1-\rho)}\right]^{-1/\theta}.
\end{aligned}$$

So the gains from multinational are

$$\begin{aligned}
\frac{P'_q}{P_q} &= \left[\sum_{i=1}^{N} \pi_{iq} \pi_{iq|i}^{(1-\rho)}\right]^{-1/\theta} \\
&= \left[\sum_{i=1}^{N} \pi_{iq}^{\rho} \pi_{iiq}^{(1-\rho)}\right]^{-1/\theta} \\
&= \left[\sum_{i=1}^{N} \left(\sum_{l=1}^{N} X_{ilq}\right)^{\rho} X_{iiq}^{(1-\rho)}\right]^{-1/\theta} X_q^{-1/\theta}.
\end{aligned}$$

A.5 Gains from Openness with intermediate goods

In this section I drive gains from openness with the model with intermediate goods. I denote x' a variable x in a counterfactual equilibrium. The price of intermediate goods $p_{qqqq,int}$ in autarky is:

$$p'_{qqqq,int} = w'_q (\tau'_{qqqq,int})^{1/\beta_{qq}}$$

so $P'_{qq,int} = w'_q(\tau'_{qqqq,int})^{1/\beta_{qq}}$. Now I look the price index in current observed equilibrium. The price index of the intermediate goods is

$$\begin{aligned}
P_{qqq,int} &= \left(\sum_{k=1}^{N} p_{qkqq,int}^{-\theta/(1-\rho)}\right)^{-(1-\rho)/\theta} \\
&= p_{qqqq,int}\left(\sum_{k=1}^{N} \frac{p_{qkqq,int}^{-\theta/(1-\rho)}}{p_{qqqq,int}^{-\theta/(1-\rho)}}\right)^{-(1-\rho)/\theta} \\
&= p_{qqqq,int}\left(\sum_{k=1}^{N} \frac{\pi_{qkqq,int}}{\pi_{qqqq,int}}\right)^{-(1-\rho)/\theta} \\
&= p_{qqqq,int}\pi_{qkq|q,int}^{(1-\rho)/\theta} \\
P_{qq,int} &= \left(\sum_{i=1}^{N} P_{iqq,int}^{-\theta}\right)^{-1/\theta} \\
&= P_{qqq,int}\left(\sum_{i=1}^{N} \left(\frac{P_{iqq,int}}{P_{qqq,int}}\right)^{-\theta}\right)^{-1/\theta} \\
&= P_{qqq,int}\left(\sum_{i=1}^{N} \frac{\pi_{iqq,int}}{\pi_{qqq,int}}\right)^{-1/\theta} \\
&= p_{qqq,int}\pi_{qqq|q,int}^{(1-\rho)/\theta}\pi_{qqq,int}^{1/\theta} \\
p_{qqqq,int} &= \tau_{qqqq,int}w_l^{\beta_{qq}}P_{qq,int}^{1-\beta_{qq}} \\
&= \tau_{qqqq,int}w_l^{\beta_{qq}}p_{qqq,int}^{1-\beta_{qq}}\pi_{qqq|q,int}^{(1-\rho)(1-\beta_{qq})/\theta}\pi_{qqq,int}^{(1-\beta_{qq})/\theta}
\end{aligned}$$

Therefore,

$$\begin{aligned}
P_{qq,int} &= \tau_{qqqq,int}^{1/\beta_{qq}}w_l\pi_{qqq|q,int}^{(1-\rho)(1-\beta_{qq})/\theta\beta_{qq}}\pi_{qqq,int}^{(1-\beta_{qq})/\theta\beta_{qq}}\pi_{qqq|q,int}^{-(1-\rho)/\theta}\pi_{qqq,int}^{1/\theta} \\
&= \tau_{qqqq,int}^{1/\beta_{qq}}w_l\pi_{qqq|q,int}^{(1-\rho)/\theta\beta_{qq}}\pi_{qqq,int}^{1/\theta\beta_{qq}}
\end{aligned}$$

and

$$\frac{P_{qq,int}}{P'_{qq,int}} = \pi_{qqq|q,int}^{(1-\rho)/\theta\beta_{qq}}\pi_{qqq,int}^{1/\theta\beta_{qq}}.$$

Using this equation to the final goods price index implies gains from openness is:

$$GO_q = \frac{P_{q,f}}{P'_{q,f}} = \frac{p_{qqq,f}\pi_{qq|q,f}^{(1-\rho)/\theta}\pi_{qq,f}^{\theta}}{p'_{qqq,f}}$$

$$
\begin{aligned}
&= \frac{\tau_{qqq,f} w_q^{\beta_{qq}} P_{qq,int}^{1-\beta_{qq}} \pi_{qq|q,f}^{(1-\rho)/\theta} \pi_{qq,f}^{\theta}}{\tau_{qqq,f} w_q^{\beta_{qq}} (P'_{qq,int})^{1-\beta_{qq}}} \\
&= \pi_{qq|q,f}^{(1-\rho)/\theta} \pi_{qq,f}^{\theta} \pi_{qqq|q,int}^{(1-\rho)(1-\beta_{qq})/\theta\beta_{qq}} \pi_{qqq,int}^{(1-\beta_{qq})/\theta\beta_{qq}} \\
&= \left(\frac{\sum_{k=1}^{N} X_{qkq,f}}{I_q} \right)^{-1/\theta} \left(\frac{X_{qqq,f}}{\sum_{k=1}^{N} X_{qkq,f}} \right)^{-(1-\rho)/\theta} \\
&\left(\frac{\sum_{k=1}^{N} X_{qkqq,int}}{\sum_{i=1}^{N} \sum_{k=1}^{N} X_{ikqq,int}} \right)^{-(1-\beta_{qq})/\beta_{qq}\theta} \left(\frac{X_{qqqq,int}}{\sum_{k=1}^{N} X_{qkqq,int}} \right)^{-(1-\beta_{qq})(1-\rho)/\beta_{qq}\theta}
\end{aligned}
$$

where β_{qq} is

$$
\beta_{qq} = \frac{\sum_{n}^{N} \sum_{m}^{N} X_{nmil,int}}{\sum_{n}^{N} \sum_{m}^{N} X_{ilnm,int} + \sum_{m=1}^{N} X_{ilm,f}}.
$$

Appendix B Characeterizing counterfactuals

B.1 Varying parameters

I discuss the consequence of varying the parameter θ and ρ. First, for any allocation, $GO(\boldsymbol{X})$, $GT(\boldsymbol{X})$ and $GM(\boldsymbol{X})$ are non-increasing in θ and ρ. Since they are both elasticities of substitution between goods, the higher the value is, the lower the gains are. The maximum and the minimum of the intervals of these three gains will be lower when θ or ρ is higher. Changing θ does not change the allocations that maximize or minimize the gains; hence the intervals for different θ can be easily calculated. Changing parameter ρ may or may not change the allocations.

There is a better insight for the intervals for gains from openness. I first discuss the role of ρ on the intervals. Notice that the allocations $\boldsymbol{X}^{HFDI}$, $\boldsymbol{X}^{VFDI}$ and $\boldsymbol{X}^{MVFDI,q}$ do not depend on the parameters θ and ρ. In horizontal FDI allocation, gains from openness do not depend on ρ:

$$GO_q\left(\boldsymbol{X}^{HFDI}(\boldsymbol{T},\boldsymbol{M})\right) = \left(\frac{X_{qqq}^{HFDI}}{X_q}\right)^{-1/\theta}.$$

In horizontal FDI allocations, there is no offshoring. Countries do not substitute offshored goods (X_{qlq}) for domestic goods (X_{qqq}); hence ρ does not affect gains from openness in $\boldsymbol{X}^{HFDI}(\boldsymbol{T},\boldsymbol{M})$. This is not true for $\boldsymbol{X}^{VFDI}$ and $\boldsymbol{X}^{MVFDI,q}$. Since there are positive amounts of offshoring in these allocations, gains from openness calculated from these allocations are decreasing in ρ. Therefore, higher ρ implies the same maximum but the lower minimum of gains from openness.

Unfortunately for exact hat-algebra, there is no clear prediction on the consequences of varying parameters. For different parameters, the allocation that maximizes (minimizes) the real wage must be calculated.

B.2 Conditions for $\mathbb{F}$ to be an interval

Here are the regularity conditions for the proposition. I redefine $\mathbb{F}$ in a rigorous manner:

$$\mathbb{F}\boldsymbol{(T, M)} \equiv \{V | V = F\left(\boldsymbol{X}\right) \ | \ \boldsymbol{X} \in \mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(F)\}.$$

Where $Dom(F)$ is a domain of the function F. The most general version of the statement is:

Proposition B.2.1. *If F is continuous on $\boldsymbol{X}$ and $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(F)$ is connected, then $\mathbb{F}\boldsymbol{(T, M)}$ is an interval (may or may not have finite endpoints).*

Proof. Since $Dom(F) \cap \mathbb{X}(\boldsymbol{T}, \boldsymbol{M})$ is connected and F is a continuous function, $\mathbb{F}(\boldsymbol{T}, \boldsymbol{M})$ must be connected (this is a multivariate extension of intermediate value theorem). This implies that $\mathbb{F}(\boldsymbol{T}, \boldsymbol{M})$ is an interval. □

If $Dom(F)$ is larger than $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M})$, then I can further show that $\mathbb{F}$ is an closed interval.

Corollary B.2.1.1. *If F is a continuous on $\boldsymbol{X}(\boldsymbol{T}, \boldsymbol{M})$ and $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \subset Dom(F)$, then $\mathbb{F}\boldsymbol{(T, M)}$ is a closed interval.*

Proof. Notice that $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(F) = \mathbb{X}(\boldsymbol{T}, \boldsymbol{M})$. The triangulated set $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M})$ is a set defined by a system of linear equations (without strict inequality). This indicates that the set is a closed convex set which is connected. Since F is continuous, by the Weierstrass Extreme Value Theorem, $\mathbb{F}$ has both a maximum F^U and a minimum F^L. Since the set is connected and F is a continuous function, $\mathbb{F}(\boldsymbol{T}, \boldsymbol{M})$ must be connected. This implies that $\mathbb{F}(\boldsymbol{T}, \boldsymbol{M})$ is a closed interval $[F^L, F^U]$. □

Now I move on to the specific cases:

Corollary B.2.1.2. $\mathbb{GO}_q(\boldsymbol{T}, \boldsymbol{M}) =\equiv \{GO_q\left(\boldsymbol{X}\right) \ | \ \boldsymbol{X} \in \mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(GO_q)\}$ *is an interval.*

Proof. GO_q is not defined in some allocations. Specifically, GO_q takes finite value (only) when $X_{qqq} > 0$ (If $X_{qqq} = 0$ then $GO_q = \infty$). Then $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(GO_q)$ is defined by these equations:

$$
\begin{aligned}
M_{il} &= \sum_{m=1}^{N} X_{ilm} \;\forall\; i, l \\
T_{lm} &= \sum_{i=1}^{N} X_{ilm} \;\forall\; l, m \\
X_{ilm} &\geq 0 \;\forall\; i, l, m \\
X_{qqq} &> 0.
\end{aligned}
$$

The set $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(GO_q)$ is a set defined by a system of linear equations (which includes both inequalities and strict inequalities). The set is a convex set which is connected. By proposition B.2.1, $\mathbb{GO}(\boldsymbol{T}, \boldsymbol{M})$ is an interval. $\square$

In addition, it is straightforward to verify whether $\mathbb{GO}_q(\boldsymbol{T}, \boldsymbol{M})$ is bounded or not.

Corollary B.2.1.3. *If there is an allocation $\boldsymbol{X}$ such that $X_{qqq} = 0$, then $\mathbb{GO}_q(\boldsymbol{T}, \boldsymbol{M})$ is unbounded from above. If thre is no such allocation in the triangulated set, then $\mathbb{GO}_q(\boldsymbol{T}, \boldsymbol{M})$ is bounded.*

Proof. Recall that gains from openness is a decreasing function of X_{qqq} and can be arbitrary large by choosing a small value of X_{qqq}. As long as $X_{qqq} > 0$, gains from openness take a finite value (The second part of the corollary is verified). Suppose there exists an allocation such that $X_{qqq} = 0$. Think of another allocation $\boldsymbol{X'}$ in $(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(GO_q)$. Since $(\boldsymbol{T}, \boldsymbol{M}) \cap Dom(GO_q)$ is convex, any convex combination of $\boldsymbol{X}$ and $\boldsymbol{X'}$ is in the set. By choosing the allocation close enough to $\boldsymbol{X}$ (by changing the convex weight), one can attain arbitrary small X_{qqq}. and arbitrary large value of $GO_q(\boldsymbol{X})$. This implies that $\mathbb{GO}_q$ is unbounded from above. $\square$

For the exact hat-algebra, $\mathbb{X}(\boldsymbol{T}, \boldsymbol{M}) \subset Dom(F)$, so the counterfactual outcomes consistent with the data constitutes a closed interval.

Appendix C Computation

C.1 Computational details and the bounds

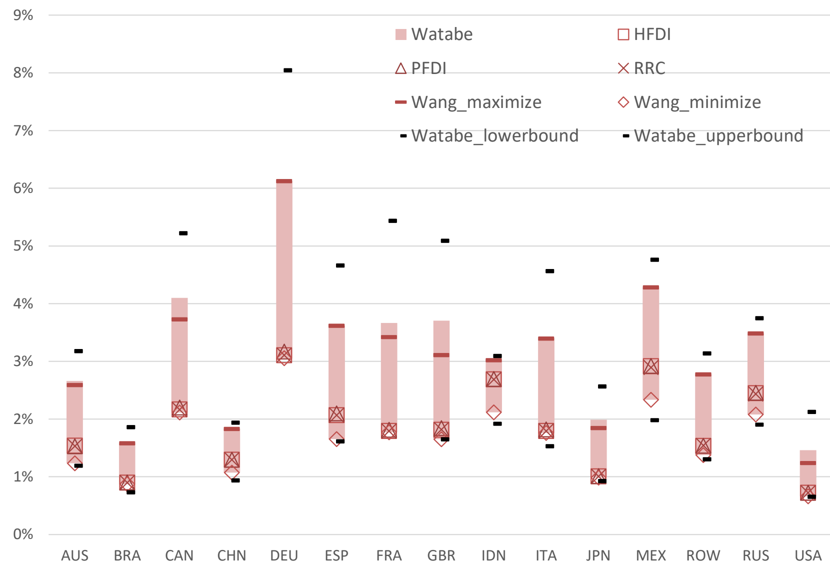
I calculate intervals from gains from openness by deriving gains from openness in the horizontal FDI allocation and the modified vertical FDI allocation. I calculate gains from trade and gains from multinationals by numerically solving a constrained optimization problem. For this purpose, I use the NEOS server, which is an internet-based client-sever application providing various solvers for optimization problems. I use two solvers provided in the NEOS server.

The first solver is Knitro, which combines interior-point methods and active-set methods for nonlinear programming. An advantage of Knitro is that it provides a solution within the time allotted (an eight hours limit for the NEOS server). While useful, this does not guarantee that the solution will be (globally) optimal. The second solver I use is BARON, which uses various techniques of the branch-and-reduce method to solve global optimization problems. The advantage of this solver is that it provides the upper bound and the lower bound for each problem. The disadvantage of this solver is that it may require a massive amount of time and memory. In some problems BARON does not provide a solution since there is not enough memory for BARON to solve the problem.

Given the constraint on the computational resource, I supplement the solution of BARON by the solution of Knitro. Specifically, if BARON solves the problem, I use the value provided by BARON. If BARON cannot solve the problem within the limit of the computational resource, I use the solution provided by Knitro. The results with the upper bound and the lower bound are shown Figure C.1 and Figure C.2 . The lower bound of gains from trade for Canada, Germany and France are not provided because BARON reached the memory limit for these problems.

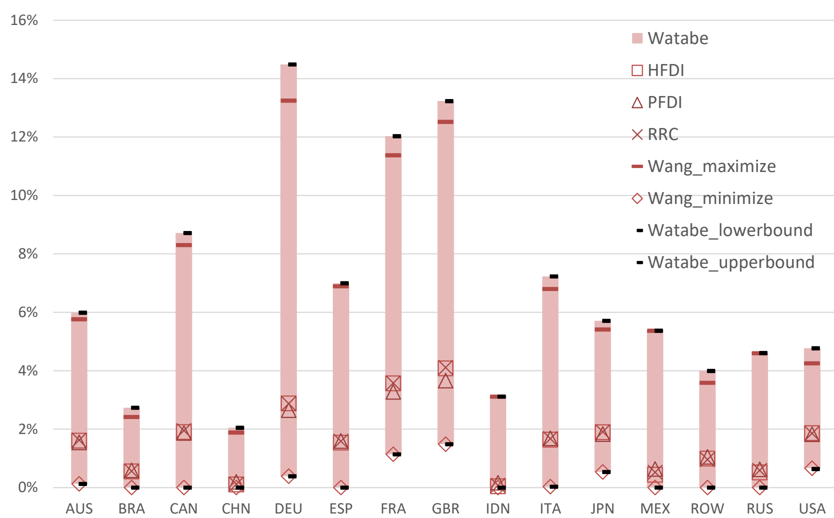
For exact hat-algebra, I only use the solution by Knitro. The problem seems to be

Figure C.1: Gains from trade: Bounds



too complicated for BARON; most of the time BARON does not provide a solution within the limitation of time and memory.

Figure C.2: Gains from multinationals: Bounds



Bibliography

Alviarez, V. (2019). Multinational production and comparative advantage. *Journal of International Economics*, *119*, 1–54. https://doi.org/https://doi.org/10.1016/j.jinteco.2019.03.004

Antràs, P., & Yeaple, S. (2014). Multinational Firms and the Structure of International Trade. *Handbook of international economics* (pp. 55–130).

Arkolakis, C., Ramondo, N., Rodríguez-Clare, A., & Yeaple, S. (2018). Innovation and Production in the Global Economy. *American Economic Review*, *108*(8), 2128–2173. https://doi.org/10.1257/aer.20141743

Armington, P. S. (1969). A Theory of Demand for Products Distinguished by Place of Production (Une théorie de la demande de produits différenciés d'après leur origine) (Una teoría de la demanda de productos distinguiéndolos según el lugar de producción). *Staff Papers (International Monetary Fund)*, *16*(1), 159–178. http://www.jstor.org/stable/3866403

Berry, S. T. (1994). Estimating Discrete-Choice Models of Product Differentiation. *The RAND Journal of Economics*, *25*(2), 242–262. http://www.jstor.org/stable/2555829

Blonigen, B. A., Davies, R. B., & Head, K. (2003). Estimating the Knowledge-Capital Model of the Multinational Enterprise: Comment. *American Economic Review*, *93*(3), 980–994. https://doi.org/10.1257/000282803322157214

Boehm, C., Flaaen, A., & Pandalai-Nayar, N. (2019). *Multinationals, Offshoring and the Decline of U.S. Manufacturing* (Working Paper No. 25824). National Bureau of Economic Research. https://doi.org/10.3386/w25824

Braconier, H., Norbäck, P.-J., & Urban, D. (2005). Multinational enterprises and wage costs: Vertical FDI revisited. *Journal of International Economics*, *67*(2), 446–470. https://doi.org/https://doi.org/10.1016/j.jinteco.2004.08.011

Brainard, S. L. (1997). An Empirical Assessment of the Proximity-Concentration Trade-off Between Multinational Sales and Trade. *American Economic Review*, *87*(4), 520–544. http://www.jstor.org/stable/2951362

Carr, D. L., Markusen, J. R., & Maskus, K. E. (2001). Estimating the Knowledge-Capital Model of the Multinational Enterprise. *American Economic Review*, *91*(3), 693–708. http://www.jstor.org/stable/2677888

Davies, R. B. (2008). Hunting High and Low for Vertical FDI. *Review of International Economics*, *16*(2), 250–267. https://doi.org/10.1111/j.1467-9396.2008.00735.x

de Gortari, A. (2020). Disentangling Global Value Chains. *mimeo*.

Fan, J. (2020). Talent, Geography, and Offshore R&D. *mimeo*.

Garetto, S. (2013). Input Sourcing and Multinational Production. *American Economic Journal: Macroeconomics*, *5*(2), 118–51. https://doi.org/10.1257/mac.5.2.118

Garetto, S., Oldenski, L., & Ramondo, N. (2019). *Multinational Expansion in Time and Space* (Working Paper No. 25804). National Bureau of Economic Research. https://doi.org/10.3386/w25804

Gumpert, A., Li, H., Moxnes, A., Ramondo, N., & Tintelnot, F. (2020). The Life-Cycle Dynamics of Exporters and Multinational Firms. *mimeo*.

Head, K., & Mayer, T. (2019). Brands in Motion: How Frictions Shape Multinational Production. *American Economic Review*, *109*(9), 3073–3124. https://doi.org/10.1257/aer.20161345

Helpman, E. (1984). A Simple Theory of International Trade with Multinational Corporations. *Journal of Political Economy*, *92*(3), 451–71. https://EconPapers.repec.org/RePEc:ucp:jpolec:v:92:y:1984:i:3:p:451-71

Helpman, E. (2006). Trade, FDI, and the Organization of Firms. *Journal of Economic Literature*, *44*(3), 589–630. https://doi.org/10.1257/jel.44.3.589

Irarrazabal, A., Moxnes, A., & Opromolla, L. D. (2013). The Margins of Multinational Production and the Role of Intrafirm Trade. *Journal of Political Economy*, *121*(1), 74–126. https://EconPapers.repec.org/RePEc:ucp:jpolec:doi:10.1086/669877

Li, H. (2021). Multinational Production and Global Shock Propagation during the Great Recession. *mimeo*.

Markusen, J. R. (1984). Multinationals, multi-plant economies, and the gains from trade. *Journal of International Economics*, *16*(3), 205–226. https://doi.org/https://doi.org/10.1016/S0022-1996(84)80001-X

Markusen, J. R., & Maskus, K. E. (2002). Discriminating Among Alternative Theories of the Multinational Enterprise. *Review of International Economics*, *10*(4), 694–707. https://doi.org/10.1111/1467-9396.00359

McGrattan, E. R., & Waddle, A. (2020). The Impact of Brexit on Foreign Investment and Production. *American Economic Journal: Macroeconomics*, *12*(1), 76–103. https://doi.org/10.1257/mac.20170399

Ramondo, N. (2014). A quantitative approach to multinational production. *Journal of International Economics*, *93*(1), 108–122. https://doi.org/10.1016/j.jinteco.2014.01.004

Ramondo, N., & Rodríguez-Clare, A. (2013). Trade, Multinational Production, and the Gains from Openness. *Journal of Political Economy*, *121*(2), 273–322. https://EconPapers.repec.org/RePEc:ucp:jpolec:doi:10.1086/670136

Tintelnot, F. (2016). Global Production with Export Platforms. *The Quarterly Journal of Economics*, *132*(1), 157–209. https://doi.org/10.1093/qje/qjw037

Wang, Z. (2020). Headquarters Gravity: How Multinationals Shape International Trade. *mimeo*.

Wilkie, C. (2020). Biden proposes a tax penalty for offshoring and new credits for manufacturing investments. *CNBC*. https://www.cnbc.com/2020/09/09/biden-proposes-tax-penalty-for-offshoring-tax-credit-for-us-investment.html

Yeaple, S. (2013). The Multinational Firm. *Annual Review of Economics*, *5*(1), 193–217. https://doi.org/10.1146/annurev-economics-081612-071350

Vita

Yuta Watabe

Yuta Watabe was born in Tokyo, Japan. He spent his childhood in Japan, Sri Lanka, Laos, and Thailand. He studied economics at Keio University, where he took his bachelor's degree in 2012 and his master's degree in 2014. Yuta Watabe began graduate school in August 2015 at the Penn State University and began working on international economics under the supervision of Prof. Jonathan Eaton.