ADDITIVELY MANUFACTURED METALS: EFFECT OF MICROSTRUCTURE AND DEFECTS ON MULTIAXIAL PLASTICITY AND FRACTURE BEHAVIOR

A Dissertation in
Materials Science and Engineering
by
Alexander E. Wilson-Heid

© 2021 Alexander E. Wilson-Heid

Submitted in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy

May 2021
The dissertation of Alexander E. Wilson-Heid was reviewed and approved by the following:

Allison M. Beese
Associate Professor of Materials Science and Engineering and Mechanical Engineering
Dissertation Advisor
Chair of Committee

Todd A. Palmer
Professor of Engineering Science and Mechanics and Materials Science and Engineering
Director, Center for Innovative Sintered Products

Jingjing Li
Associate Professor of Industrial and Manufacturing Engineering

Guha Manogharan
Assistant Professor of Mechanical Engineering and Industrial and Manufacturing Engineering

John C. Mauro
Professor of Materials Science and Engineering
Chair of the Intercollege Graduate Degree Program in Materials Science and Engineering
Abstract

Metal additive manufacturing (AM) processes build 3-dimensional (3D) components in a layer-by-layer fashion, which allows for the manufacturing of geometrically complex components that cannot be produced via traditional manufacturing methods, making it an attractive option in many industries. Laser powder bed fusion (L-PBF) AM is a process category of AM that involves using a focused laser to selectively melt and fuse metallic powder to make a component. A common type of defect in AM is lack-of-fusion porosity, where the newly melted powder fails to fully-fuse to the material adjacent or below due to imperfect selection of processing parameters. Understanding the mechanical behavior of material used in AM is important for the safe, reliable, and repeatable application of the technology in industry. This thesis work provides a new understanding of the static uniaxial and multiaxial plasticity and fracture properties of L-PBF Ti-6Al-4V and stainless steel 316L in two material orientations. In particular, a series of experiments over a wide range of stress states that included uniaxial tension, equibiaxial tension, plane strain tension, pure shear, and combined tension/shear were probed to characterize both the plasticity and subsequent failure behavior. Finite element method simulations were used in combination with the experiments to calibrate and validate the stress state dependent, anisotropic plasticity behavior. Fracture behavior of both alloys was found to be stress state dependent in both orientations, and the equivalent plastic strain to failure under the wide range of stress states studied was accurately captured using calibrated ductile fracture criteria, namely the modified Mohr-Coulomb and Hosford-Coulomb models. Hypothesized microstructural driven shear softening in 316L shear dominated experiments was captured in simulations by adopting shear damage criterion in
conjunction with the anisotropic plasticity model. Furthermore, this thesis for the first time characterized the effect of penny-shaped defects of varying size on the deformation and failure response of L-PBF 316L under uniaxial tension and three high stress triaxiality stress states. Pores were intentionally introduced using the unique capability of AM to embed enclosed defects at the center of samples. As a result, the 316L material was found to be defect tolerant under uniaxial tension; where the pore did not impact material ductility until the pore was 9% of the sample cross-sectional area. Strain to failure was stress state dependent until a large pore size occupying 4% of the sample cross-sectional area was introduced and then failure became pore size dependent and stress state independent.
# Table of Contents

List of Figures ........................................................................................................................ xi

List of Tables ........................................................................................................................... xxiii

Acknowledgements ............................................................................................................... xxv

Chapter 1 Introduction .......................................................................................................... 1

1.1. Laser powder bed fusion ......................................................................................... 2

1.2. Materials and properties ....................................................................................... 4

1.2.1. Ti-6Al-4V .................................................................................................. 4

1.2.2. Stainless steel 316L ............................................................................... 5

1.3. Defects ............................................................................................................... 6

1.3.1. Characterization of defects ....................................................................... 9

1.4. Stress state dependent plasticity and fracture .................................................. 10

1.5. Thesis outline ..................................................................................................... 14

Chapter 2 Quantitative relationship between anisotropic strain to failure and grain morphology in additively manufactured Ti-6Al-4V ................................................................. 17

2.1. Introduction ......................................................................................................... 17

2.2. Experimental methods ....................................................................................... 18

2.2.1. Fabrication ............................................................................................... 18

2.2.2. Mechanical Testing .................................................................................. 20

2.2.3. Sample Characterization ......................................................................... 21
2.3. Results and discussion ................................................................. 23

2.3.1. Overview: continuous-wave versus pulsed laser .................. 23

2.3.2. Anisotropy ................................................................................. 27

2.3.3. Effect of processing on microstructure ................................. 28

2.3.4. Quantitative relationship between microstructure and anisotropic ductility ................................................................. 32

2.3.5. Effect of surface roughness on properties .......................... 37

2.4. Summary and Conclusions ......................................................... 39

Chapter 3 Anisotropic multiaxial plasticity model for laser powder bed fusion additively manufactured Ti-6Al-4V ......................................................... 41

3.1. Introduction .................................................................................. 41

3.2. Experimental methods ............................................................... 42

3.2.1. Sample fabrication ................................................................. 42

3.2.2. Mechanical testing ................................................................. 45

3.3. Results and discussion ............................................................... 47

3.3.1. Uniaxial tension results ......................................................... 47

3.3.2. Hardening behavior ............................................................... 48

3.3.3. Multiaxial loading results .................................................... 50

3.4. Modeling ..................................................................................... 52

3.4.1. Plasticity modeling ............................................................... 52
Chapter 4 Fracture of laser powder bed fusion additively manufactured Ti–6Al–4V under multiaxial loading: Calibration and comparison of fracture models .................................................. 59

4.1. Introduction ................................................................................................ 59

4.2. Experimental methods ............................................................................... 60

4.3. Finite element simulations ......................................................................... 66

4.4. Calibration of fracture models ................................................................... 70

4.4.1. Constant equivalent strain to fracture criterion ................................... 72

4.4.2. Johnson-Cook fracture criterion ............................................................. 72

4.4.3. Two-branch empirical fit fracture criterion ......................................... 73

4.4.4. Maximum shear stress fracture criterion .............................................. 74

4.4.5. Modified Mohr-Coulomb fracture criterion ......................................... 75

4.4.6. Hosford-Coulomb fracture criterion ................................................... 76

4.5. Results and discussion ............................................................................... 78

4.5.1. Comparison of models in 2D space of $\varepsilon_f$ versus $\eta$ ..................... 79

4.5.2. Comparison of models in 3D space of $\varepsilon_f$ versus $\eta$ versus $\theta$ .......... 85

4.6. Conclusions ................................................................................................ 91
Chapter 5 Multiaxial plasticity and fracture behavior of stainless steel 316L by laser powder bed fusion: Experiments and computational modeling ........................................ 93

5.1. Introduction ................................................................................................ 93

5.2. Experimental methods ........................................................................... 94

5.2.1. Material fabrication and characterization ........................................... 94

5.2.2. Plasticity tests .................................................................................... 96

5.2.3. Fracture tests .................................................................................... 99

5.3. Plasticity behavior: experimental results and model ......................... 102

5.3.1. Experimental results ........................................................................ 102

5.3.2. Plasticity modeling ........................................................................ 105

5.4. Fracture behavior: experimental results and model ......................... 109

5.4.1. Experimental results ........................................................................ 110

5.4.2. Fracture modeling .......................................................................... 112

5.5. Summary and conclusions .................................................................... 126

Chapter 6 Characterization of the effects of internal pores on tensile properties of additively manufactured austenitic stainless steel 316L .................................... 128

6.1. Introduction .......................................................................................... 128

6.2. Experimental methods ........................................................................ 129

6.2.1. Fabrication ..................................................................................... 129

6.2.2. Archimedes method ...................................................................... 132
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.2.3. 2D radiograph method</td>
<td>133</td>
</tr>
<tr>
<td>6.2.4. X-ray CT method</td>
<td>134</td>
</tr>
<tr>
<td>6.2.5. 2D cross-section method</td>
<td>135</td>
</tr>
<tr>
<td>6.2.6. Evaluation of uncertainty</td>
<td>136</td>
</tr>
<tr>
<td>6.2.7. Mechanical testing</td>
<td>139</td>
</tr>
<tr>
<td>6.3. Results and discussion</td>
<td>140</td>
</tr>
<tr>
<td>6.3.1. Bulk porosity analysis</td>
<td>140</td>
</tr>
<tr>
<td>6.3.2. Characterization of intentional pores</td>
<td>141</td>
</tr>
<tr>
<td>6.3.3. Tensile testing</td>
<td>145</td>
</tr>
<tr>
<td>6.4. Conclusion</td>
<td>149</td>
</tr>
</tbody>
</table>

Chapter 7 Combined effects of porosity and stress state on the failure behavior of laser powder bed fusion stainless steel 316L

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1. Introduction</td>
<td>151</td>
</tr>
<tr>
<td>7.2. Experimental methods</td>
<td>152</td>
</tr>
<tr>
<td>7.2.1. Sample fabrication</td>
<td>152</td>
</tr>
<tr>
<td>7.2.2. Fracture test sample geometries</td>
<td>153</td>
</tr>
<tr>
<td>7.2.3. Mechanical testing</td>
<td>155</td>
</tr>
<tr>
<td>7.2.4. X-ray computed tomography</td>
<td>156</td>
</tr>
<tr>
<td>7.3. Results and discussion</td>
<td>156</td>
</tr>
<tr>
<td>7.3.1. Fracture behavior: experimental results</td>
<td>156</td>
</tr>
</tbody>
</table>
7.3.2. Characterization of pores via XCT ................................................... 159
7.3.3. Fracture behavior: simulations.......................................................... 162
7.4. Summary and conclusions ....................................................................... 168

Chapter 8 Summary and conclusions.............................................................. 170
Appendix........................................................................................................ 174
Published papers during Ph.D. study .............................................................. 174
Bibliography .................................................................................................. 176
List of Figures

Figure 1.1. (a) Schematic of the laser powder bed fusion process. Schematic plot of power as a function of time for the laser beam in (b) continuous wave and (c) pulsed laser powder bed fusion processes. .......................................................................................................... 3

Figure 2.1. Geometries of tensile samples (a) extracted from walls made using CW-LPBF (EOSINT M280), and (b) directly built using P-LPBF (Renishaw AM250). Dimensions are in mm. ......................................................................................................................... 18

Figure 2.2. Representative engineering stress-strain curves for samples fabricated using CW-LPBF and P-LPBF in two orientations, showing little anisotropy in strength in the CW-LPBF samples, and a higher strength in the \( \perp BD \) than the BD in P-LPBF samples. With both processing conditions, samples in the \( \perp BD \) had lower ductility than those in the BD; however, this difference is more dramatic in the P-LPBF samples. ......................... 25

Figure 2.3. Hardness as a function of position for the (a) CW-LPBF samples and (b) P-LPBF samples. Both plots show little variation in microhardness as a function of distance from the substrate, indicating that a steady-state hardness regime has been reached throughout the samples. .................................................................................................... 26

Figure 2.4. Optical micrographs of a CW-LPBF Ti-6Al-4V sample, in which build direction is vertical, showing (a) the equiaxed prior-\( \beta \) grain morphology; (b) an enlarged image of the inset in (a) showing prior-\( \beta \) grains in more detail and \( \alpha \)-laths within the prior-\( \beta \) grains; and (c) an enlarged image of the inset in (b) showing the acicular \( \alpha \)-lath morphology. ...................................................................................................................... 29

Figure 2.5 Optical micrographs of a P-LPBF Ti-6Al-4V sample, in which build direction is vertical, showing (a) the elongated prior-\( \beta \) grain morphology; (b) an enlarged image of
the inset in (a) showing prior-β grains in more detail and α-laths within the prior-β grains; and (c) an enlarged image of the inset in (b) showing the acicular α-lath morphology. 

Figure 2.6. Elongation ratio (BD sample elongation, t/BD sample elongation, l) versus grain aspect ratio (grain height/grain width) for tests performed in the present study as well as data from literature [106–109,111,112,122]. These data show that as the grain aspect ratio increases above 6, the elongation ratio, and therefore anisotropy in ductility, increases substantially.

Figure 3.1. Geometry of tensile samples extracted from as-built walls. Dimensions are in mm.

Figure 3.2. Geometry of multiaxial loading samples extracted from as-built walls. Dimensions are in mm. Adapted from [130].

Figure 3.3. Schematic of sample orientation nomenclature with respect to build direction (BD) where the outlines correspond to sample geometries in Figures 3.1 and 3.2. All of the specimens were extracted from the x₁-x₂ plane, where x₂ is the vertical build direction.

Figure 3.4. Micrographs of L-PBF Ti-6Al-4V in the (a) x₁-x₂ plane and (b) x₁-x₃ plane. The melt pool is shown to have influence on the morphology of the prior-β grains, which are filled with acicular α or α’ laths. The scale bar is equivalent to the thickness of the gauge regions of mechanical test specimens shown in Figures 3.1 and 3.2.

Figure 3.5. Schematic of the dual-actuator load frame used to perform multiaxial tests (pure shear, plane strain tension, β = 30°, and β = 60°) using specimens shown in Figure 3.2.
Figure 3.6. Engineering stress-strain curves for uniaxial tension in the build direction and the perpendicular build direction. ................................................................. 48

Figure 3.7. Lower bound von Mises equivalent stress-plastic strain curves under uniaxial tension in the (a) build direction and (b) perpendicular build direction for L-PBF Ti-6Al-4V along with a Swift law fit prior to necking, and a linear extrapolation after necking. (c) Engineering stress-strain curves for uniaxial tension from experiments (symbols) and simulations based on the inputs in (a) and (b) (lines). ...................................................... 49

Figure 3.8. Von Mises equivalent stress-plastic strain curves for (a) pure shear, (b) combined loading with $\beta = 30^\circ$, (c) uniaxial tension, (d) combined loading with $\beta = 60^\circ$, and (e) plane strain tension, in two directions. In all cases except for combined loading with $\beta = 60^\circ$, the mechanical behavior showed notable anisotropy............................................. 51

Figure 3.9. Von Mises equivalent stress versus equivalent plastic strain for all stress states evaluated in both the (a) build direction and (b) perpendicular BD directions, indicating that the yield and strain hardening behavior of the material is stress state dependent. .... 52

Figure 3.10. Anisotropic Hill48 2D yield surface (for plane stress) at initial yield (0.2% strain) and subsequent strain (1.0%) of L-PBF Ti-6Al-4V. The fitted yield surfaces are shown as lines, while experimental data are shown as symbols. Solid symbols represent perpendicular build direction samples, while open symbols represent build direction samples................................................................................................................................. 55

Figure 3.11. (a) Normal and (b) shear engineering stress-strain curves for the four multiaxial loading conditions, showing the comparison between experimental data (symbols) and the simulation results (lines). ................................................................. 57
Figure 4.1. Schematic of the butterfly sample used for pure shear and combined loading stress states (dimensions in mm). Geometry from [137].

Figure 4.2. Schematics of the flat fracture specimens tested. (a) Equibiaxial tension punch, (b) 1.5 mm thick central hole, and 1.5 mm thick notched tension specimens with cutout radii of (c) 2.67 mm, (d) 4 mm, and (e) 8 mm. All dimensions in mm.

Figure 4.3. Stress triaxiality versus Lode angle parameter for the condition of plane stress (dashed curve) along with stress state locations of all samples tested (symbols). Square data points represent samples that did not perfectly meet the plane stress condition (as shown by their distance from the ideal plane stress dashed line).

Figure 4.4. Comparison of experimentally measured and simulated force vs. displacement results for the (a) central hole tension; (b) R = 2.67 mm, (c) R = 4 mm, and (d) R = 8 mm notched tension tests; (e, f) $\beta = 30^\circ$ combined loading; (g) pure shear loading; and (h) equibiaxial tension punch tests.

Figure 4.5. Evolution of the (a, b) stress triaxiality and (c, d) Lode angle parameter (solid lines) up to failure (symbols) for each sample geometry in both the BD (a, c) and $\perp$BD (b, d) with the average value of each stress state parameter used in fracture model calibration represented by a vertical dashed line.

Figure 4.6. Constant equivalent strain fracture locus (solid black line) for L-PBF Ti-6Al-4V, based only on strain to failure under uniaxial tension (central hole tests) whose tensile axis is in the (a) vertical build direction and (b) perpendicular to the vertical build direction compared to all experimental data (symbols). Shaded area bounded with dashed lines represents $\pm 15\%$ margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.
Figure 4.7. Johnson-Cook fracture locus (solid black line) for L-PBF Ti-6Al-4V whose tensile axis is in the (a) vertical build direction and (b) perpendicular to the vertical build direction compared to all experimental data (symbols). Shaded area bounded with dashed lines represents ±15% margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.

Figure 4.8. Two-branch empirical fit fracture locus (solid black line) for L-PBF Ti-6Al-4V whose tensile axis is in the (a) vertical build direction and (b) perpendicular to the vertical build direction compared to all experimental data (symbols). Shaded area bounded with dashed lines represents ±15% margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.

Figure 4.9. Maximum shear stress criterion fracture locus (solid black line) for L-PBF Ti-6Al-4V whose tensile axis is in the (a) vertical build direction and (b) perpendicular to the vertical build direction compared to all experimental data (symbols). Shaded area bounded with dashed lines represents ±15% margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.

Figure 4.10. Modified Mohr-Coulomb fracture locus (solid black line) for L-PBF Ti-6Al-4V whose tensile axis is in the (a) vertical build direction and (b) perpendicular to the vertical build direction compared to all experimental data (symbols). Shaded area bounded with dashed lines represents ±15% margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.

Figure 4.11. Hosford-Coulomb fracture locus (solid black line) for L-PBF Ti-6Al-4V whose tensile axis is in the (a) vertical build direction and (b) perpendicular to the vertical build direction compared to all experimental data (symbols). Shaded area bounded with
dashed lines represents ±15% margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.

**Figure 4.12.** Calibrated three-dimensional MMC fracture locus (surface) for L-PBF Ti-6Al-4V in the (a) build direction and (b) perpendicular build direction compared to all experimental data (symbols). Dashed line on the surface represents the plane stress relationship between the two stress state parameters. Arrows, if present, indicate the distance and direction of the experimental data point to the calibrated fracture surface. Filled data points were used for calibration of the model.

**Figure 4.13.** Calibrated three-dimensional HC fracture locus (surface) for L-PBF Ti-6Al-4V in the (a) build direction and (b) perpendicular build direction compared to all experimental data (symbols). Dashed line on the surface represents the plane stress relationship between the two stress state parameters. Arrows, if present, indicate the distance and direction of the experimental data point to the calibrated surface. Filled data points were used for calibration of the model.

**Figure 4.14.** Calibrated MMC fracture locus (solid black line) in the space of Lode angle parameter versus strain to failure, highlighting the importance of Lode angle parameter, for L-PBF Ti-6Al-4V in the (a) build direction and (b) perpendicular build direction compared to all experimental data (symbols). Shaded area bounded with dashed lines represents ±15% margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.

**Figure 4.15.** Calibrated HC fracture locus (solid black line) in the space of Lode angle parameter versus strain to failure, highlighting the importance of the Lode angle parameter, for L-PBF Ti-6Al-4V in the (a) build direction and (b) perpendicular build direction.
compared to all experimental data (symbols). Shaded area bounded with dashed lines represents ±15% margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.

**Figure 5.1.** (a, b) Optical micrographs showing the melt pool and grain structure in the three planes of the build. (c) EBSD images of all three planes of the build showing the epitaxial grain growth and chevron morphology of the grains in the XZ and YZ planes. For all three planes, the color shown corresponds to the hkl direction parallel to the vertical build direction (Z). (d) SEM image of the subgrain cellular structure in the XZ plane.

**Figure 5.2.** (a) Uniaxial tension and (b) multiaxial plasticity specimen geometries used to evaluate the elastoplastic behavior of the L-PBF 316L material in two orientations – BD (tension along the z-axis) and ⊥BD (tension along the y-axis). Units in mm. (c) Schematic of the sample orientations and nomenclature with respect to the baseplate, not to scale.

**Figure 5.3.** (a) Central hole tension and (b) butterfly specimens used to evaluate the fracture behavior of the L-PBF 316L material in two orientations – BD (tension along the z-axis) and ⊥BD (tension along the y-axis). (c) Equibiaxial tension specimens that were fabricated in the x-y plane and loaded out of plane (along the build axis). Red circles indicate the locations at which fracture is assumed to initiate, where the location is at the through-thickness center in (a) and (b) and on the top surface in (c). Units in mm.

**Figure 5.4.** (a) Engineering stress-strain data for uniaxial tension in both orientations. (b) Flow versus plastic strain for a representative ⊥BD uniaxial tension test along with a Swift law fit prior to necking, and a linear extrapolation after necking.

**Figure 5.5.** Comparison of experimental (symbols) and simulated (lines) force versus displacement curves for (a) pure shear, (b) plane strain tension (c) combined loading with
β=30°, and (d) combined loading with β=60° in both orientations. The mechanical response in the tension dominated tests show notable anisotropy .................................................. 104

**Figure 5.6.** Anisotropic Hill48 2D yield surface (for plane stress) at initial yield (0.5% Hill48 equivalent plastic strain strain) and subsequent increments of strain up to 15% for L-PBF 316L. The yield surfaces are shown as lines, while experimental data are shown as symbols. ........................................................................................................................................... 108

**Figure 5.7.** Comparison of fracture geometry experimental and simulated force versus displacement curves for (a) pure shear, (b) plane strain tension (c) combined loading with β=30°, (d) combined loading with β=60°, (e) equibiaxial tension, and (f) central hole tension in two orientations. .................................................................................................................. 111

**Figure 5.8.** Comparison of the shear fracture geometry force vs. displacement experimental behavior of rolled austenitic stainless steel 316 sheet and L-PBF 316L material. The force of the calibrated Abaqus/Standard simulation without damage was increased by 15% to match the rolled experimental behavior and the comparison shows that the rolled material does not undergo shear softening. ................................................................. 116

**Figure 5.9.** Evolution of the (a, b) stress triaxiality and (c, d) Lode angle parameter (solid lines) up to failure (symbols) for each sample geometry in both the BD (a, c) and ⊥BD (b, d) directions with the average value of each stress state parameter used in fracture model calibration represented by a vertical dashed line. ........................................................................ 118

**Figure 5.10.** (a,b) Damage accumulation at fracture and (c,d) relative displacement to failure comparisons of the four different fracture models: isotropic HC, anisotropic HC, isotropic MMC, and anisotropic MMC. In the legends, the value in parenthesis indicates the mean absolute percentage error between the specified model predictions for each stress
state studied in both orientations and a value of 1. The boxed x-axes labels indicate tests that were not used in calibration of the models. ............................................................. 124

**Figure 5.11.** Schematic comparisons of the anisotropic HC and MMC fracture surfaces in three-dimensional space of stress triaxiality vs. Lode angle parameter vs. equivalent plastic strain to failure for the (a) build direction and (b) perpendicular build direction samples. Symbols indicate the locations of each stress state directly on the anisotropic HC surface at the average location of the stress triaxiality and Lode angle parameter for each stress state under assumed proportional loading. Damage accumulates slower at locations where the surface is higher for each model. .............................................................................. 126

**Figure 6.1.** Cross-section geometry of (a) as-built cylinders and (b) uniaxial tension samples in compliance with ASTM E8 [129], where dashed horizontal line indicates the intentionally introduced pore (here, showing a 3 mm diameter pore) at the center of the specimen. Dimensions in mm. (c) 3D CAD rendering of a tensile sample with an internal penny-shaped pore. ......................................................................................................... 131

**Figure 6.2.** Witness sample with pores vertically positioned every 2 mm, starting with the 150 μm closest to the baseplate and the 4800 μm farthest from the baseplate. The pore diameter/volume fractions in the witness sample were characterized with each porosity analysis technique. Dimensions are given in mm. ....................................................................................... 132

**Figure 6.3.** 2D radiograph image centered at the 1800 μm diameter pore, as indicated by the arrow. Each pore was centered with respect to the X-ray source for dimensional analysis to ensure elimination of skewed features, as seen in the larger pores in this image. ........................................................................................................................................ 134
Figure 6.4. Polished cross-section of the 2400 μm internal pore in the witness sample, which was used to measure dimensions for 2D cross-section analysis in ImageJ........ 136

Figure 6.5. Comparison of the pore diameter measurement and uncertainty using different viewing directions in 2D radiography and the effect of using both planes for a more accurate value of the pore diameter. ................................................................. 138

Figure 6.6. 3D X-ray CT reconstruction of the eleven pores in the witness sample within the 6 mm diameter cylinder. ................................................................................................. 142

Figure 6.7. X-ray CT images of the reconstructed pores in different planes for the (a) 4800 μm, (b) 1800 μm, and (c) 300 μm pores. ................................................................. 142

Figure 6.8. Comparison of the 2D analysis techniques as related to their similarity with the pore diameter measured in the 3D X-ray CT analysis using percent difference. A positive percentage indicates the 2D measurement was larger than the 3D X-ray CT measurement and a negative percentage indicates the 2D measurement was smaller than 3D X-ray CT measurement. Both methods are shown to more likely underestimate the pore diameter that is measured in 3D X-ray CT. ........................................................................ 144

Figure 6.9. Representative engineering stress-strain curves for samples with each initial pore diameter. A is representative of the dense samples, while B-L correspond to samples with increasing pore diameters. See Table 6.3 for nomenclature of B-L pore diameters. .................................................................................................................. 145

Figure 6.10. Engineering stress-strain plots for each uniaxial tension test............. 147

Figure 6.11. Box plots displaying the minimum, maximum, first and third quartiles, exclusive median (central line), and mean (x) of (a) tensile elongation to failure and (b) ultimate tensile strength as a function of pore diameter with the pore diameter at which
each of these properties begins to be negatively impacted shown. Inset of (a) is given in (c).

**Figure 7.1** Geometries of cylindrical notched tension specimens with three different notch radii, which result in different stress triaxiality values at the center of the specimen, with Bridgman approximated triaxiality values of: (a) $R = 12$ mm giving $\eta cB = 0.5$, (b) $R = 5$ mm giving $\eta cB = 0.7$, and (c) $R = 3$ mm giving $\eta cB = 0.9$. All dimensions in mm.

**Figure 7.3.** Representative force versus displacement curves, as a function of the intentional pore diameter, for each notched tension geometry: (a) $R = 12$ mm, (b) $R = 5$ mm, and (c) $R = 3$ mm.

**Figure 7.4.** Trends for the (a) maximum force and (b) displacement to failure averaged across three tests for each test condition as a function of the intentional pore diameter.

**Figure 7.5.** (a) Comparison of the force versus displacement behavior for the $R = 5$ mm geometry, with an 1800 µm pore (9% of sample cross-sectional area), pulled to failure versus one unloaded after 86% displacement to failure and subsequently loaded to failure. (b) X-ray computed tomography reconstruction of the intentional pore prior to testing and (c) reconstruction of the pore after 86% displacement to failure. (d) Comparison of the force versus displacement behavior for the $R = 12$ mm geometry, with a 600 µm pore (1% of sample cross-sectional area), pulled to failure versus one unloaded after 79% displacement to failure and subsequently loaded to failure. (e) X-ray computed tomography reconstruction of the intentional pore prior to testing and (f) reconstruction of the pore after 79% displacement to failure.
Figure 7.6. Effect of stress triaxiality on pore growth behavior as measured by change in volume for all interrupted tests for which the displacement at unloading was on average 70% of the eventual displacement to failure. ................................................................. 162

Figure 7.7. Comparison of the force versus displacement behavior of representative dense experimental data and simulated data for all three notched tension geometries. Error bars for displacement at each maximum force represent the range of displacement that fall within 0.5% of the maximum force. All maximum force values in the simulations was 5% lower than that in experiments................................................................. 164

Figure 7.8. (a) Evolution of the Hill48 equivalent plastic strain versus stress triaxiality, for all three geometries, up to the displacement to failure in the dense samples (solid lines). Symbols along the solid lines indicate the equivalent plastic strain (from dense simulations) at the average displacement to failure of each set of samples with intentional pores. The dashed lines connecting these points each correspond to the same intentional pore diameter, showing how increasing pore size reduces the strain to failure. Dashed vertical lines represent the average stress triaxiality, throughout loading, for the dense samples. (b) Equivalent plastic strain to failure as a function of percent solid cross-sectional area at the sample’s minimum diameter, accounting for the designed penny-shaped pore geometry. ................................................................................................. 166
List of Tables

Table 2.1 Processing parameters used for manufacturing samples via CW-LPBF and P-LPBF. ................................................................................................................................ 19

Table 2.2. Comparison of tensile mechanical properties for samples made with CW-LPBF and P-LPBF lasers, where n indicates the number of samples tested in each condition. Values reported are average ± standard deviation. ................................................................................. 24

Table 2.3. Grain aspect ratio (grain height/grain width) and elongation ratio (BD elongation/⊥BD elongation) for tests performed in the present study as well as data from literature for both powder bed fusion and directed energy deposition AM. Values reported are average ± standard deviation. ...................................................................................... 33

Table 2.4. Surface roughness (Ra) measurements of the pulsed laser samples in the as-built and machined conditions. Measurements taken in the gauge region. Values reported are average ± standard deviation. ........................................................................................... 38

Table 3.1. As-built wall sample dimensions and number of samples tested in each orientation. ................................................................................................................................ 43

Table 3.2. Strain hardening parameters determined through experimental fitting of lower bound uniaxial tension tests in each orientation. ............................................................................................. 50

Table 3.3. Calibrated Hill48 yield surface model parameters. ................................................... 55

Table 4.1. Average values of the strain to failure, stress triaxiality, and Lode angle parameter for each test performed. ........................................................................................................... 66

Table 4.2. Calibrated fracture model parameters for L-PBF Ti-6Al-4V. ........................ 78

Table 5.1. Calibrated plasticity model parameters: Swift law hardening with linear extrapolation and Hill48 yield function. ........................................................................................................... 105
Table 5.2. Average values of the strain to failure, stress triaxiality, and Lode angle parameter for each test performed. These values were used for MMC model calibration and validation. ................................................................. 118

Table 5.3. Calibrated anisotropic and isotropic HC and MMC model parameters. .... 123

Table 6.1. Processing parameters used for L-PBF manufacturing of 316L samples in the present study. ................................................................. 130

Table 6.2. Chemical composition (wt.%) of the as-built stainless steel 316L in study. 130

Table 6.3. Diameters of 180 µm tall penny-shaped pores designed into the cylindrical tensile samples. ................................................................. 130

Table 6.4. Parameters used in X-ray 2D radiographs and 3D CT scans. ................. 133

Table 7.1. Comparison of designed penny-shaped pore diameters to average X-ray CT measured values and measured pore diameters as a fraction of sample diameter (6 mm) and sample cross-sectional area. ................................................................. 154

Table 7.2. Effect of pore size on fracture as a function of sample geometry, presented as the percent of equivalent plastic strain to failure reached in a sample with a pore compared to that of the corresponding dense notched tension sample. ..................... 167
Acknowledgements

This thesis and my growth as a scientist and person would not have been possible without the initial belief and continuous support and guidance of my advisor, Professor Allison Beese, and for that I am thankful beyond measure. Specifically, the generosity with her time and commitment to our work was more than I could have ever asked for in a mentor.

I am very thankful to the other members of my committee, Professors Jingjing Li, Guha Manogharan, and Todd Palmer for their suggestions, which improved this thesis.

I would like to thank the members of the Beese Research Group, Dr. Shipin Qin, Dr. Zhuqing Wang, Lourdes Bobbio, John Shimanek, Cole Britt, John O’Brien, Erik Furton, Nancy Huang, and Qixiang Luo. During my time at Penn State, I could not imagine having a better group of people to work with on a daily basis who provide helpful feedback, a true collaborative environment, and most importantly levity. I also want to thank the many friends made along the way, most notably those within my MatSE cohort.

I gratefully acknowledge the financial support of my research from National Science Foundation through award numbers CMMI-1402978 and CMMI-1652575. I would also like to thank Penn State’s Center for Innovative Materials Processing through Direct Digital Deposition (CIMP-3D), and the staff at Penn State’s Center for Quantitative Imaging (CQI) and Materials Characterization Lab (MCL).

Finally, the biggest and most important thank you is to my entire family for their love, support, and constant motivation. This is most true of my parents, Eric and Wendy, and my best friend Gabby.
Chapter 1 Introduction

Additive manufacturing (AM) is a term used to describe a layer-by-layer manufacturing process used to create 3-dimensional (3D) components [1]. These processes allow for the production of custom, complex, and near-net-shape components [2]. Reliable and repeatable material properties in components manufactured via AM is one of the necessary steps for AM to become a mature technology that is as accepted as traditional subtractive manufacturing methods. The number of options for manufacturing methods (e.g., laser powder bed fusion (L-PBF), powder-based directed energy deposition (DED), wire-based deposition), parameters within each method (e.g., laser power, laser spot size, scanning speed, layer height, hatch spacing, scan pattern), and metal feedstock (feedstock composition, powder particle size distribution, virgin or recycled material, etc.), are all factors that contribute to the reproducibility and reliability of additively manufactured parts [1,3].

Parts reproduced from:
As the field of metal additive manufacturing grows from a useful tool for fast prototyping and novelty items into a technology that transforms important industries such as aerospace, biomedical sciences, consumer products, defense, energy production, tool and die making, and transportation it becomes necessary to develop a thorough understanding of the mechanical response of additively manufactured materials. Enabling load bearing applications of additively manufactured parts is in an important step for widespread adoption of the technology. Safe implementation and design against failure of additively manufactured components necessitates a description of how additively manufactured deform and fail under load.

1.1. Laser powder bed fusion

Laser powder bed fusion can be used to manufacture solid metallic components starting with raw powder metal feedstock and a laser heat source. In L-PBF, a laser is used to selectively melt a 2D pattern in a thin layer of powder, typically between 30 – 100 μm tall, fusing the current layer to either the baseplate or a previously solidified layer below. Layers are continuously added by lowering the baseplate by the desired layer height, spreading a new layer of powder, melting the new 2D layer pattern into the new powder layer using the laser heat source, and repeating the process until the 3D component is fabricated, as schematically shown in Figure 1.1. The final part(s) and baseplate are then extracted from the surrounding un-melted powder, and the part(s) removed from the baseplate.
Figure 1.1. (a) Schematic of the laser powder bed fusion process. Schematic plot of power as a function of time for the laser beam in (b) continuous wave and (c) pulsed laser powder bed fusion processes.

The L-PBF process requires rapid melting, solidification, and repeated heating and cooling cycles with the addition of layers, resulting in unique process-structure-property relationships in parts made by L-PBF. Material in L-PBF can undergo cooling rates of up to $10^6$ K/s at the solidification front [4], and after solidification undergoes thermal cycles as material is added above or adjacent to the previously solidified material. The most commonly adjusted processing parameters in L-PBF include laser power, scanning speed, hatch spacing, layer height, and scan pattern. Altering these parameters results in varying morphology and size of both grains and defects in a fabricated component [5].

Laser based PBF systems use either continuous-wave (CW) laser emission [6–10] or power modulated “pulsed” fiber laser emission [10–12] as the heat source for selectively melting the powder metal feedstock. A CW laser produces a continuous, uninterrupted beam of light with a constant output power, as shown in Figure 1.1b, while pulsed fiber
lasers in PBF are power modulated to generate pulsed (e.g., a square wave) patterns as a function of time, as shown in Figure 1.1c [13].

1.2. Materials and properties

1.2.1. Ti-6Al-4V

Titanium-6 wt.% aluminum-4 wt.% vanadium (Ti-6Al-4V), the most commonly used titanium alloy globally, is an α/β phase alloy that is used in applications where high strength, stiffness, and corrosion resistance are desired [14]. This alloy has been widely studied in the AM field, both because of its suitability for building complex part geometries for use in the aerospace and biomedical industries [15,16] and because of the high cost associated with traditional subtractive machining of this material [17]. The AM community has performed significant research aimed at understanding the behavior of additively manufactured Ti-6Al-4V in areas such as: strength, ductility, anisotropy, fatigue, and corrosion resistance [18–21]. However, there is limited published data on the elastoplastic and fracture behavior of additively manufactured Ti-6Al-4V over a broad range of stress states [22,23], as most research focuses only on mechanical properties under uniaxial tension (UT).

In UT testing of Ti-6Al-4V, both the yield and ultimate tensile strengths have been reported as greater than those values seen in traditionally manufactured material [24–26]. This result can be explained by the rapid cooling in AM leading to the formation of fine acicular α and α’ laths in the L-PBF material, which are stronger than the lamellar α+β structure seen in as-cast and annealed versions of this alloy [27]. However, when compared to traditionally manufactured counterparts, Ti-6Al-4V that has been fabricated via L-PBF
has lower ductility, which can also be explained by microstructural differences [26]. In the L-PBF condition, lack-of-fusion (LoF) defects between laser passes can result from non-optimal processing parameters, laser power or beam size fluctuations, or recoating errors [28,29]. Furthermore, \( \alpha \) and \( \alpha' \) phases, which are the primary phases in as-built Ti-6Al-4V made by L-PBF and contribute to the high strength, have limited plastic deformability compared to the \( \beta \) phase typically present in the conventionally processed material [30]. Together, these features result in a decrease in the macroscopic ductility of additively manufactured Ti-6Al-4V compared to conventionally processed Ti-6Al-4V.

1.2.2. Stainless steel 316L

While many steels have been studied for application with the layer-by-layer process of AM [31–33], austenitic AISI Type 316L stainless steel (316L) is one of the most widely studied alloys in the AM industry [34]. The broad interest in the alloy stems from its current wide use in industry as a conventionally processed alloy, the ease of weldability and therefore manufacturability with AM, high corrosion and oxidation resistance, and combination of strength and elongation to failure [1,35]. The inherent rapid thermal cycling, repeated melting and solidification, of material in the AM process has been found to result in key hierarchical microstructural features including small 10-500 \( \mu \)m grains [36–39], fine intergranular cellular structures [38–42], and high dislocation density [36–38,41,43] that lead to an increase of yield strength relative to conventionally processed 316L. However, through computational analysis, stainless steel 316L has been found to be an alloy that is susceptible to lack-of-fusion defects making it an ideal candidate in which to study the effects of internal pores on mechanical properties [44].
Anisotropic mechanical behavior of L-PBF 316L has been shown in the literature for L-PBF 316L. The yield and ultimate tensile strength (UTS) is lower along the vertical build direction (BD) than the horizontal direction, referred to here as perpendicular to the build direction ($\perp$BD) [36,37,39,45,46]. A potential explanation proposed for the anisotropic strength is the solidification direction of the sub-granular cells and columnar grains [36,38,47], both of which grow along the highest thermal gradient during processing. The columnar grains can grow epitaxially through multiple layers, providing statistically more grain and cell boundaries that act as dislocation barriers under UT along the $\perp$BD compared to the BD, resulting in increased yield strength along the $\perp$BD. Like Ti-6Al-4V, the anisotropic behavior of 316L has primarily been investigated under uniaxial tension, which does not provide a description of material response under other stress states.

1.3. Defects

In AM, internal defects and heterogeneous microstructures, caused by rapid solidification and thermal cycling, are frequently present [1]. Understanding how the microstructure and defects affect the mechanical behavior of components is required in order to design against failure in additively manufactured components. This includes understanding which techniques are best for characterizing defects and determining tolerable limits of defects within AM components that still result in structurally sound parts.

Additive manufacturing introduces the risk of porosity in components – especially the laser powder bed fusion technique, where the powder packing density and the optimization of other parameters such as hatch spacing for a given layer height, power, and
scan speed combination are important to produce fully dense parts [48]. Porosity in additively manufactured 316L is generally from non-optimized laser parameters that result in: spherical gas entrapped pores formed during solidification [49], vapor entrapment pores from operating close to or in the keyholing mode [50,51], or lack-of-fusion type pores with irregular morphologies at the boundaries between layers or adjacent meltpools [36,42,47,52,53]. LoF pores, which can range from 15 - 600 µm [54–56] and are irregular in morphology, have been shown to be detrimental to mechanical properties of materials because their sharp features act as stress concentration sites [18,21,55]. The orientation dependence of the pores, where LoF pores form at the layer boundaries and their major axes are normal to the vertical build direction, contributes to the anisotropy in ductility behavior of 316L material. Ronneberg et al., found that the presence and orientation of LoF pores were correlated with a reduction of elongation to failure [36]. Carlton et al. evaluated the role of LoF pores on tensile behavior of L-PBF stainless steel 316L using in situ X-ray computed tomography (CT) testing and found that samples with high porosity (>2.2%) displayed flaw-driven failure where cracks initiated at pre-existing defects [55]. Stef et al. found that the formation of these pores are closely related to the laser scan pattern, where LoF porosity is more common in areas where the overlapping of the laser scan pattern exists between sequential layers [57]. Spierings et al. showed that increased scanning speed from 300 to 850 mm/s at a constant power of 104 W resulted in an increase of LoF porosity formation and overall porosity of 0.03% to 9.02% for stainless steel 316L samples built with L-PBF [58], highlighting the importance of using optimized process parameters.

In addition to studying the impact of unwanted defects, in AM the layer-by-layer processing can be used to intentionally manufacture internal pores into samples, which is
not possible with conventional manufacturing techniques, in order to directly assess the impact of void size on ductile fracture. Fadida et al. embedded a single spherical pore at the center of 4 mm diameter Ti-6Al-4V dynamic tensile samples manufactured via L-PBF and found that samples with a 600 μm or greater diameter pore had significantly lower tensile ductility compared to dense samples. They also showed that at this critical pore diameter (with respect to sample gauge diameter), the failure in samples always occurred at the pore location. Kim et al. introduced pores into L-PBF 17-4 PH stainless steel cylindrical uniaxial tension samples (3.98 mm² cross-sectional area) in the form of a single intentional pore (1 mm² cross-sectional area) and randomly generated LoF type pores (in a 1.77 mm² cross-sectional area) via changes in process parameters. Using in situ XCT to capture pore sizes and locations, and finite element analysis to study the corresponding elastic stress fields, the authors found that locally high values of stress triaxiality (0.5-0.75, further explained in Section 1.4) correlated well with the locations of failure [59]. With AM, the effect of pores can be isolated to determine their impact on mechanical properties giving insight to void growth in both AM and conventional materials. Although it is known that LoF pores negatively impact the mechanical behavior of AM components, quantifying the impact of internal pores is important for defining defect tolerances in AM.

The role of internal pores on the mechanical properties of ductile metals is of interest to not only the AM community, but to the entire fracture community. There has been much classical research on the role of pores in ductile plasticity [60,61] and fracture, including theoretical work on the growth of spherical voids [62] and cylindrical voids [63].
1.3.1. Characterization of defects

The presence of internal defects can be assessed and quantified using different methods. The classic Archimedes method has been used to quantify bulk porosity in AM components [64,65]. This method is non-destructive, but the results can vary based on part surface finish and surface-breaking porosity [58]. The Archimedes method provides bulk porosity information, but does not provide information about pore size distribution, pore locations, or pore morphology, all which have been found to be important factors when assessing mechanical properties [55]. X-ray radiography (2D) and X-ray computed tomography (XCT, 3D) are also non-destructive methods for investigating internal porosity. In 2D radiography, a shadowgraph of a stationary 3D sample is produced by directing radiation energy through a sample and measuring attenuation with a detector on the opposite side. This method can be used to assess variations in 2D grayscale projections of the sample that can be generated quickly to provide for assessment of pore cross-sections.

In X-ray CT, the sample is rotated in angular increments ranging from 0° to 360° and is exposed to the source generated X-rays. The attenuated X-rays are captured by a detector that uses a mathematical algorithm to convert the 2D data into a 3D reconstruction of the entire sample or region of interest [66]. In X-ray CT the size of quantifiable pores in a sample is directly related to the voxel size that is used in the analysis, where pores smaller than the voxel size cannot be measured [66], and typically, the convention is to assume that any pores that can be reliably detected are 3 times the voxel size or larger. Voxel size is dictated by the desired magnification or field of view [67]. This is an important consideration when determining the desired scan resolution and a compromise
must be made between desired porosity resolution, volume of area analyzed, time, and economics [67]. The biggest advantage to using X-ray CT is that it can be used to non-destructively characterize the 3D morphology of pores, the spatial distribution and location of pores, and the distribution of pore sizes in a metallic sample.

For destructive evaluation of pores, serial sectioning and optical microscopy may be used. With this method only one 2D cross-section of a component can be analyzed at a time, providing limited information on pore size, morphology, and distribution. This method is prone to selection bias and the appropriate magnification must be selected to achieve consistent pore resolution [58].

1.4. Stress state dependent plasticity and fracture

One method for generating a more complete understanding of the elastoplastic and fracture behavior of ductile metals like Ti-6Al-4V and 316L is to evaluate the materials under a wide range of complex stress states. The stress state dependent, multiaxial mechanical data and accompanying models that describe the behavior can then be used to design against failure in components, especially those with complex geometries allowed through the application of AM in design. The stress state can be characterized by a combination of two dimensionless parameters: stress triaxiality ($\eta$) and Lode angle parameter ($\tilde{\theta}$). The stress triaxiality is the ratio of the mean stress ($\sigma_m$) and von Mises equivalent stress ($\bar{\sigma}_{VM}$),

$$\eta = \frac{\sigma_m}{\bar{\sigma}_{VM}} \text{ with } \sigma_m = \frac{1}{3} I_1 \text{ and } \bar{\sigma}_{VM} = \sqrt{3J_2}$$

(1.1)
where $I_1 = \sigma_{kk}$ is the first invariant of the stress tensor, $\sigma$, and $J_2 = \frac{1}{2} \sum_{i,j} s_{ij} s_{ij}$ is the second invariant of the deviatoric stress tensor, $s$. The normalized Lode angle parameter is a function of the third invariant of the deviatoric stress tensor, $J_3 = \text{det}(s_{ij})$, and is defined as:

$$\bar{\theta} = 1 - \frac{2}{\pi} \arccos \left[ \frac{27}{2} \frac{J_3}{\bar{\sigma}^3_{VM}} \right] \quad (1.2)$$

Under plane stress conditions ($\sigma_3 = 0$), the relationship between the normalized Lode angle parameter and stress triaxiality is given as:

$$\sin \left( \frac{\pi}{2} \bar{\theta} \right) = -\frac{27}{2} \eta \left( \eta^2 - \frac{1}{3} \right) \quad (1.3)$$

It is widely understood that the strain to failure in ductile metals is stress state dependent [62,63,68–71]. Many experimental studies have focused on the effect of stress state on failure behavior, showing that, for a constant Lode angle parameter, material ductility decreases with increasing stress triaxiality [70,72–75] as the ductile fracture processes of void nucleation, growth, and coalescence are aided by high stress triaxiality. There are many models that define the ductile fracture strain of a material as a function of stress state. Some models are physically informed and consider void nucleation, growth, and coalescence in high stress triaxiality fracture [62,63,76,77], or shear band formation in shear dominated fracture [78]. Other models are empirical and strictly based on experimental studies that assess the role of stress triaxiality on the fracture behavior [69,70,79]; this includes the well-known Johnson-Cook model, which captures the decreasing strain to failure with increasing stress triaxiality [79]. More recent models have pointed out the need for considering the impact of the third invariant, or the incorporation
of Lode angle dependence, to accurately capture fracture behavior of ductile metals [71,75].

Wierzbicki et al. compared the ability of seven different fracture models to capture the multiaxial ductile failure behavior of a conventionally processed metal (2024-T351 aluminum alloy) in the equivalent plastic strain to failure versus stress triaxiality space, and found that the maximum shear stress criterion [80] worked well to predict failure over a wide range of stress triaxialities while only requiring one test for calibration [74]. However, they found that the Xue-Wierzbicki model, which incorporates the effect of stress triaxiality and a parameter dependent on the third invariant of the stress deviator, more accurately captured the failure behavior of the aluminum alloy studied [74]. Studies on conventionally processed Ti-6Al-4V have found that the mechanical properties of this material are often anisotropic and stress state dependent.

Using both stress state parameters ($\eta$ and $\bar{\theta}$), Bai et al. proposed an asymmetric fracture model that describes the equivalent strain to failure ($\bar{\varepsilon}_f$) as function of both stress triaxiality and Lode angle parameter, referred to as the modified Mohr-Coulomb (MMC) fracture model [71]. The model, which takes into account the contribution of the Lode angle parameter on the failure strain of a material, has been successfully used to capture the fracture behavior of many ductile metals [81–83]. In addition to the MMC model, the Hosford-Coulomb (HC) fracture initiation model developed by Mohr and Marcadet [84] also is in the $[\bar{\varepsilon}_f, \eta, \bar{\theta}]$ 3D space. A primary difference between the MMC and the HC model is that the dependence on shear stress in the HC model is incorporated through the Hosford equivalent stress, which takes into account the second principal stress [85], while the MC model, on which the MMC model is based, uses the Tresca equivalent stress, or
maximum shear stress [86]. The HC model has been used to effectively characterize the failure behavior of high strength steels and an aluminum alloy [84,87].

Evaluation of materials under different combinations of stress triaxiality and Lode angle parameter is required for an accurate description of the initial yield behavior, subsequent hardening, and eventual fracture under a range of different stress states. This procedure has been used for additively manufactured materials such as Ti-6Al-4V by directed energy deposition [22,23] and stainless steel 304L by DED [88,89]. An anisotropic plasticity and fracture model was developed for L-PBF 316L by Tancogne-Dejean et al. [90]. However, due to limited data available, the model was calibrated using only uniaxial tension and build direction double notched tension data; therefore, the authors assumed that fracture behavior was stress state independent and adopted a constant equivalent strain to fracture model.

In addition to the effect of stress state, some models have been developed to also consider the effect of internal pores. Models by McClintock [63], Rice and Tracey [76], Hancock and Mackenzie [72], and Gurson [62] consider the effect of stress triaxiality and pore growth on ductile fracture. Weck et al. experimentally evaluated void growth and coalescence using XCT analysis of copper-based tension samples with an array of laser-drilled pores resulting in a localized void volume fraction of 6.5% [91]. The authors found that experimental void growth as a function of true strain was well predicted with the Rice and Tracey model when considering the increase in stress triaxiality due to macroscale necking.

The stress state in an AM part under applied load can vary spatially, for example in parts designed using topology optimization, which result in geometrically complex parts.
Therefore, it is important to understand the effect of stress state on fracture behavior over a wide range of stress states to predict and prevent failure as well as to help develop acceptance criteria for AM parts with and without defects.

1.5. Thesis outline

The objective of this thesis research was to experimentally and computationally quantify process-structure-property relationships for Ti-6Al-4V and stainless steel 316L manufactured via laser powder bed fusion additive manufacturing. Plasticity and fracture properties of the two alloys were evaluated using uniaxial tensile and multiaxial testing to probe properties over a wide range of stress states. Two orientations were evaluated in both alloys to investigate the effect of microstructural features on mechanical properties. Experimental data was used to calibrate material models using computational methods. Intentional defects were used to probe the effect of structure on properties in L-PBF 316L. The thesis is arranged with the following chapters:

In Chapter 2, the mechanical properties of Ti-6Al-4V manufactured via two different L-PBF methods, continuous wave and pulsed laser beam, were compared. Existing data from the literature were integrated with the collected data to identify a general quantitative relationship between anisotropic ductility and grain morphology in additively manufactured Ti-6Al-4V.

In Chapter 3, the multiaxial yield and plastic flow behavior of Ti-6Al-4V manufactured in two orientations via L-PBF additive manufacturing was investigated. The stress state dependent mechanical properties of the material were examined through multiaxial loading under five stress states. A plasticity model was calibrated and validated
using experiments and finite element simulations to capture the anisotropic, stress state
dependent properties.

In Chapter 4, the fracture behavior of L-PBF additively manufactured Ti-6Al-4V
alloy manufactured in two orientations was examined using a combined experimental and
computational simulation approach. The results from mechanical tests subjecting the
material to seven distinct stress states were evaluated in both orientations. Six existing
fracture criteria, with varying complexities, were calibrated and their ability to capture
and/or predict the fracture behavior over a wide range of stress states was assessed.

In Chapter 5, the multiaxial large deformation and ductile fracture behavior of L-
PBF additively manufactured austenitic 316L stainless steel was experimentally measured.
Data from tests in two orientations, under five dissimilar stress states was used to calibrate
and validate anisotropic plasticity and fracture models. The implementation of a damage
sensitive material model was used to capture the experimental behavior driven by
microstructural features. Two fracture models that consider the effect of both stress
triaxiality and Lode angle parameter were calibrated and compared.

In Chapter 6, the effect of internal pores on the tensile behavior of austenitic
stainless steel 316L manufactured with L-PBF additive manufacturing was experimentally
determined. Single, penny-shaped internal pores of varying size were deliberately
fabricated in the center of cylindrical tensile samples during AM processing. To link the
pore size and morphology to the mechanical properties, the sizes of the initial pores were
evaluated using non-destructive Archimedes measurements, 2D X-ray radiography, 3D X-
ray computed tomography, and destructive 2D optical microscopy. The effect of the pore
as function of cross-sectional area on the elongation to failure and ultimate tensile strength in uniaxial tension were discussed.

In Chapter 7, the effects of both internal pores and stress state on the ductile failure behavior of laser powder bed fusion additively manufactured 316L stainless steel were explored with experiments and simulations. The results of tests whose geometries result in different macroscopic stress states with intentional penny-shaped pores of varying size at the center of samples were discussed. Strain to failure from finite element simulations using the calibrated model in Chapter 5 were compared for each test condition and an engineering approach to account for the effect of pore size and stress state is shown.

In Chapter 8, a summary and conclusions of the thesis are presented.
Chapter 2 Quantitative relationship between anisotropic strain to failure and grain morphology in additively manufactured Ti-6Al-4V

2.1. Introduction

This chapter focuses on identifying quantitative links between the anisotropy in ductility and the microstructure in additively manufactured Ti-6Al-4V. The microstructure, namely the β grain morphology, and tensile mechanical properties of samples fabricated using CW-LPBF and P-LPBF were investigated. Additionally, studies from literature were analyzed to quantify grain morphology and its impact on anisotropic ductility. From these data, a quantitative correlation describing anisotropic tensile ductility based on grain morphology is developed. Additionally, this study presents the effect of processing conditions on the microstructure and tensile mechanical properties of Ti-6Al-4V fabricated via CW-LPBF and P-LPBF. The effect of microstructure (CW versus pulsed) on properties, direction (anisotropic microstructure in CW versus pulsed) on properties, and surface roughness (macroscopic structure in P-LPBF) was investigated.

2.2. Experimental methods

2.2.1. Fabrication

To identify the difference of the impact of CW laser versus a pulsed laser processing in PBF on microstructure and properties, Ti-6Al-4V samples were fabricated using both of these laser heat sources. For CW-LPBF (EOSINT M280), 32 mm x 30 mm x 4 mm walls were fabricated on a 280 mm x 280 mm annealed Ti-6Al-4V substrate in an argon filled chamber to minimize oxygen contamination. Samples were fabricated using the CW laser parameters in Table 2.1. Prior to sample removal from the substrate, the entire plate was subjected to a standard PBF stress relief in Ar at 650 °C for 3 hours. Uniaxial tension samples were extracted from the deposited walls in the vertical BD and \( \perp \)BD using wire electrical discharge machining (EDM). The tensile sample geometry used was in accordance with ASTM E8 [98], with a gauge length of 6 mm and cross sectional area of 1.5 mm\(^2\) as shown in Figure 2.1a.

![Figure 2.1](image)

**Figure 2.1.** Geometries of tensile samples (a) extracted from walls made using CW-LPBF (EOSINT M280), and (b) directly built using P-LPBF (Renishaw AM250). Dimensions are in mm.
Table 2.1 Processing parameters used for manufacturing samples via CW-LPBF and P-LPBF.

<table>
<thead>
<tr>
<th></th>
<th>Continuous-wave laser</th>
<th>Pulsed laser</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Substrate temperature (°C)</strong></td>
<td>25</td>
<td>170</td>
</tr>
<tr>
<td><strong>Laser power (W)</strong></td>
<td>340</td>
<td>188</td>
</tr>
<tr>
<td><strong>Scanning speed (mm/s)</strong></td>
<td>1250</td>
<td>1000</td>
</tr>
<tr>
<td><strong>Hatch spacing (μm)</strong></td>
<td>140</td>
<td>100</td>
</tr>
<tr>
<td><strong>Layer height (μm)</strong></td>
<td>60</td>
<td>30</td>
</tr>
<tr>
<td><strong>Laser spot size (μm)</strong></td>
<td>70-80</td>
<td>70</td>
</tr>
</tbody>
</table>

Conversely, samples made using P-LPBF (Renishaw AM 250) were fabricated directly in the uniaxial tension sample geometry shown in Figure 2.1b, which was also in accordance with ASTM E8, with a gauge length of 21.3 mm and a cross sectional area of 8 mm². Samples were fabricated in two orientations, vertical and horizontal, on a 250 mm x 250 mm substrate in an Ar filled chamber with the baseplate heated to 170°C throughout the build process. The fabrication parameters used for the near-net-shaped P-LPBF samples are specified in Table 2.1. Although there was a smaller gauge region in the CW-LPBF samples, the sample dimensions incorporated the same order of magnitude of grains for mechanical testing to be representative of bulk properties. For P-LPBF samples, ten samples were fabricated such that their tensile axes were parallel to the vertical build direction, and ten were fabricated such that their tensile axes were perpendicular to the vertical build direction. The grips of the \( \perp \)BD samples were fabricated directly on the substrate, while support structure was used in the gauge region to maintain sample
geometry during fabrication. Prior to sample removal from the substrate, the entire plate was subjected to the same stress relief heat treatment as the CW-LPBF samples. Samples were removed from the substrate using wire-EDM, and subjected to a grit blasting to remove un-melted powder. Half of the samples in each orientation were left in this as-built condition for tensile testing, while the gauge regions of the other half of the samples were milled to determine how the surface finish impacted the measured properties.

2.2.2. Mechanical testing

To quantify the mechanical properties that result from CW-LPBF versus P-LPBF processing, samples from each build were subjected to tensile testing. Uniaxial tensile testing of the P-LPBF samples was performed on an electromechanical testing frame (Instron 4202) with a 10 kN load cell (Instron model 2518-804). Uniaxial tension testing of the smaller CW-LPBF samples was performed on a servo-hydraulic load frame (810 MTS) with a 25 kN load cell (MTS model 661 20E-01). All tensile tests were performed under quasi-static conditions with a strain rate on the order of $10^{-4}$ s$^{-1}$. Digital image correlation (DIC), a non-contact method for measuring surface deformations, was used to compute surface strains using correlation software (Vic2D, Correlated Solutions). Sample gauge regions were painted with a white basecoat followed by a random black speckled pattern on top. A digital camera (Point Grey GRAS-50S5M-C) was used to take images of the deforming gauge region of the sample at 1 Hz during each test. The surface deformations in the gauge region of each sample were computed from the digital images using a cubic B-spline interpolation algorithm with a subset size of 21 pixels and a step size of 5 pixels. The axial strain in the gauge section of each sample was measured using
a vertical virtual extensometer measuring 5.5 mm for the CW-LPBF samples and 20 mm for the P-LPBF samples.

Microhardness was measured using a Vickers indenter (Leco MHT Series 200) with a load of 300 g and a dwell time of 15 s. At least nine locations at varying heights from the substrate were analyzed for microhardness, with five indentations at each height, in both P-LPBF and CW-LPBF samples.

### 2.2.3. Sample characterization

X-ray computed tomography (CT), a non-destructive measurement technique, was used to visualize internal porosity. For this study, X-ray CT (General Electric phoenix v|tome|x m) was used to identify any lack-of-fusion porosity in two samples, one from each orientation, made by P-LPBF and a wall from which the CW-LPBF samples were extracted. Scans were performed using a 300 kV micro-focus X-ray source with a GE DXR250 flat panel detector with a 200 mm pitch. An accelerating voltage of 200 kV, a tube current of 125 μA, and a voxel size of 20 μm were used with 950 projections per scan. This voxel size allows for the identification of pores 40-60 μm in diameter or larger [66,99]. The scans were analyzed using VGStudio Max 2.2 visualization and analysis software to identify pore morphology.

The Archimedes method for determining density was used to quantify porosity. The sample density was computed as:

\[
\rho = \frac{m_{\text{dry}} \times \rho_{\text{theor}}}{m_{\text{soak}} - m_{\text{sub}}} \tag{2.1}
\]
where $m_{\text{dry}}$ is the measured mass of the sample in air, $m_{\text{sub}}$ is the measured mass of the sample in water, $m_{\text{soak}}$ is the sample weighed in the air after patting the samples dry to remove water on the surface, and $\rho_{\text{theor}}$ is the density of water, which was assumed to be 1.0 g/cm$^3$. Calculated densities were compared with the theoretical density of Ti-6Al-4V, 4.43 g/cm$^3$ [100], to determine the percent density in the analyzed samples.

For microstructural analysis, samples were prepared using standard metallurgical procedures with a final polish using 0.05 µm colloidal silica, and etched using Kroll’s reagent (2 vol% hydrofluoric acid and 3 vol.% nitric acid in distilled water). Images of the microstructures were taken using a digital optical microscope (OM, Keyence VHX-2000). The digital OM was also used for 3D surface reconstruction of the samples for surface roughness analysis.

To quantify the grain dimensions in the present study and from reported data, selected micrographs were overlaid with a five by five grid pattern aligning with the build and perpendicular build directions. Grain dimensions were computed based on how many grain boundaries each line intersected. Due to the elongated morphology of grains, the line lengths were dictated by grain boundary locations, so that a line started on a grain boundary and ended on another. Measurements were made for each individual line three times for a total of fifteen measurements in each orientation. To ensure proper statistics of measurements, this procedure was repeated for each micrograph by five individuals. The twenty-five grain dimensions per orientation were used to determine the average width/height of the grains for each micrograph.
2.3. Results and discussion

2.3.1. Overview: continuous-wave versus pulsed laser

The measured mechanical properties for all samples are given in Table 2.2, and representative engineering stress-strain curves of CW-LPBF, as-deposited P-LPBF, and machined P-LPBF samples under uniaxial tension are shown in Figure 2.2. The variations in both ultimate tensile strength (UTS) and elongation to failure indicate that differences in processing conditions and surface finish influenced the bulk material properties. In general, the CW-LPBF samples had higher yield strength (0.2% offset), UTS, and elongation to failure in both orientations tested compared to the corresponding machined P-LPBF samples. In the same direction, CW-LPBF samples had higher elongations than the as-built and machined P-LPBF samples. The average yield strength of CW-LPBF samples was 8.3% higher, and the average UTS 6.7% higher, than that of the machined P-LPBF samples.
Table 2.2. Comparison of tensile mechanical properties for samples made with CW-LPBF and P-LPBF lasers, where n indicates the number of samples tested in each condition. Values reported are average ± standard deviation.

<table>
<thead>
<tr>
<th>Laser Type</th>
<th>Condition</th>
<th>n</th>
<th>Modulus (GPa)</th>
<th>Yield strength (MPa)</th>
<th>Ultimate tensile strength (MPa)</th>
<th>Elongation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous-wave laser</td>
<td>∠BD</td>
<td>16</td>
<td>103 ± 2.0</td>
<td>1137 ± 10.3</td>
<td>1204 ± 13.0</td>
<td>8.5 ± 1.1</td>
</tr>
<tr>
<td></td>
<td>BD</td>
<td>3</td>
<td>103 ± 2.4</td>
<td>1141 ± 1.5</td>
<td>1207 ± 6.7</td>
<td>12.7 ± 1.1</td>
</tr>
<tr>
<td>Pulsed laser</td>
<td>∠BD, as-built</td>
<td>3</td>
<td>93 ± 0.9</td>
<td>893 ± 5.8</td>
<td>912 ± 5.9</td>
<td>1.4 ± 0.2</td>
</tr>
<tr>
<td></td>
<td>∠BD, as-built, area corrected for E = 110 GPa</td>
<td>3</td>
<td>110</td>
<td>1060 ± 10.0</td>
<td>1082 ± 15.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>∠BD, machined</td>
<td>3</td>
<td>110 ± 5.6</td>
<td>1078 ± 2.9</td>
<td>1131 ± 17.4</td>
<td>2.8 ± 0.8</td>
</tr>
<tr>
<td></td>
<td>BD, as-built</td>
<td>4</td>
<td>95 ± 1.7</td>
<td>916 ± 4.8</td>
<td>992 ± 5.9</td>
<td>6.5 ± 1.0</td>
</tr>
<tr>
<td></td>
<td>BD, as-built, area corrected for E = 110 GPa</td>
<td>4</td>
<td>110</td>
<td>1060 ± 18.3</td>
<td>1149 ± 15.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BD, machined</td>
<td>3</td>
<td>110 ± 4.6</td>
<td>1010 ± 0.0</td>
<td>1117 ± 1.2</td>
<td>7.8 ± 1.1</td>
</tr>
</tbody>
</table>
Figure 2.2. Representative engineering stress-strain curves for samples fabricated using CW-LPBF and P-LPBF in two orientations, showing little anisotropy in strength in the CW-LPBF samples, and a higher strength in the $\perp$BD than the BD in P-LPBF samples. With both processing conditions, samples in the $\perp$BD had lower ductility than those in the BD; however, this difference is more dramatic in the P-LPBF samples.

Microhardness as a function of vertical position from the substrate was measured to determine if the mechanical properties, and indirectly, microstructure, had a location dependence that would impact the properties within the gauge regions of the samples. In the CW-LPBF samples, the centers of the gauge region for the $\perp$BD and BD samples were approximately 5 mm and 13 mm from the baseplate, respectively, and microhardness measurements were taken at heights from 3 mm to 24 mm from the baseplate. In P-LPBF
samples, the center of the gauge region for the \( \perp \)BD and BD samples was approximately 5 mm and 32 mm away from the baseplate, respectively, and microhardness measurements were taken at heights from 1 mm to 9 mm away from the baseplate. As shown in Figure 2.3, the microhardness as a function of distance from the substrate was found to be constant by the time the gauge region was reached in all samples. Thus, the gauge regions are sufficiently far from the baseplate such that the microstructure has reached a steady state in all of the tested samples.

![Figure 2.3](image)

**Figure 2.3.** Hardness as a function of position for the (a) CW-LPBF samples and (b) P-LPBF samples. Both plots show little variation in microhardness as a function of distance from the substrate, indicating that a steady-state hardness regime has been reached throughout the samples.

The average hardness in the CW-LPBF samples was 403 ± 8 HV compared to 375 ± 7 HV in the P-LPBF samples. These values agree with existing literature, for which the hardness of Ti-6Al-4V fabricated by PBF has been reported to be greater than traditionally processed Ti-6Al-4V (341-369 HV) [2,101–103].
X-ray CT scans revealed that neither CW-LPBF nor P-LPBF samples contained lack-of-fusion porosity. The representative as-built ⊥BD and BD P-LPBF samples examined with the Archimedes method showed that the samples were 99.7 ± 1.3% and 98.7 ± 2.2% dense, respectively. The CW-LPBF deposited wall had a density of 98.2 ± 1.9% measured via Archimedes method. These analyses indicate samples were near fully dense with only gaseous, spherical porosity present.

2.3.2. Anisotropy

The tensile mechanical properties of samples were measured in the ⊥BD and BD orientations with respect to the build layers to determine the degree of anisotropy of components made by PBF. As shown in Table 2.2, samples fabricated via CW-LPBF exhibited isotropic yield strength and UTS, but anisotropic strain to failure, with a higher elongation in the BD samples. Near isotropic yield strength and UTS behavior was observed in the milled P-LPBF samples, and similar to CW-LPBF samples, the elongation in the milled P-LPBF samples was higher in the BD samples than in the ⊥BD samples.

The isotropic strength behavior in both sets of samples can be explained by the low strain hardening behavior in Ti-6Al-4V [104]. The tensile stress versus plastic strain curve can be fit with a power law (e.g., [105]) equation, given as:

\[ \sigma = k \varepsilon_p^n \]  

(2.2)

where \( \sigma \) is the true stress, \( \varepsilon_p \) is the plastic strain, \( k \) is the strength coefficient, and \( n \) is the strain hardening exponent. The strain hardening exponents for CW-LPBF and P-LPBF samples were found to be 0.06 and 0.08, respectively. The very low strain hardening rate in both sets of samples, in which the yield strength was isotropic, resulted in isotropic UTS
Despite anisotropy in elongation to failure. We note that although the prior-\(\beta\) grain morphologies differ, which impacts elongation, as discussed in Section 2.3.3, the plasticity behavior in Ti-6Al-4V is dictated by the \(\alpha\) lath morphology and preferred orientation [106], neither of which varied within each sample set, as shown in Figures 2.4 and 2.5.

2.3.3. Effect of processing on microstructure

To explain the differences in the mechanical properties of Ti-6Al-4V samples manufactured using CW or pulsed lasers, the microstructures were analyzed, and the prior-\(\beta\) grain dimensions were quantified. Prior-\(\beta\) grains that grow epitaxially across several build layers in AM have been widely reported in literature [106–113]. The microstructures in the CW-LPBF and P-LPBF samples are shown in Figures 2.4 and 2.5, respectively. There is a clear difference in prior-\(\beta\) grain size and morphology between the two processing methods. The microstructure of the sample made via CW-LPBF (Figure 2.4) contains small, nearly equiaxed prior-\(\beta\) grains, with average measured widths of 96.3 ± 18.0 \(\mu\)m and lengths of 125.3 ± 14.4 \(\mu\)m, and the prior-\(\beta\) grains contain acicular \(\alpha\) laths. The samples made with P-LPBF (Figure 2.5) also have prior-\(\beta\) grains containing acicular \(\alpha\)-laths; however, the prior-\(\beta\) grains are elongated and extend across multiple build layers, with the measured dimensions of 150.3 ± 22.7 \(\mu\)m wide by 1201.8 ± 190.2 \(\mu\)m long (in build direction).
Figure 2.4. Optical micrographs of a CW-LPBF Ti-6Al-4V sample, in which build direction is vertical, showing (a) the equiaxed prior-β grain morphology; (b) an enlarged image of the inset in (a) showing prior-β grains in more detail and α-laths within the prior-β grains; and (c) an enlarged image of the inset in (b) showing the acicular α-lath morphology.
Figure 2.5 Optical micrographs of a P-LPBF Ti-6Al-4V sample, in which build direction is vertical, showing (a) the elongated prior-β grain morphology; (b) an enlarged image of the inset in (a) showing prior-β grains in more detail and α-laths within the prior-β grains; and (c) an enlarged image of the inset in (b) showing the acicular α-lath morphology.
In welding research, it has been shown that the morphology and size of grains are controlled by the relationships between thermal gradient (G) and the solidification growth rate (R) of as-deposited material [114]. The ratio G/R influences grain morphology, where a low value results in the formation of equiaxed grains, which transition to columnar grains with increasing G/R values. The product GR, or the solidification cooling rate, controls the size of the grain structure, with higher values resulting in smaller grains. These relationships have also been noted in AM simulations and experiments [115–117]. The microstructural variations between the two processing methods in the current study can be explained by relating the input parameters and the knowledge of thermal behavior in AM to G and R.

In CW-LPBF, with increasing scanning speed, the melt pool becomes elongated behind the laser spot, resulting in a decreasing thermal gradient, increasing solidification growth velocity at the melt pool boundary, and decreasing G/R ratio [115,118]. This results in fine, equiaxed grains, which are seen in Figure 2.4 for the CW-LPBF build.

The laser in P-LPBF build does not simultaneously supply energy to the build and vary position, but rather supplies bursts of power at discrete locations. This results in a higher thermal gradient at the boundary of the melt pool compared to a continuously scanning laser, and is similar to spot welding [119]. Therefore, the solidification growth velocity in P-LPBF is not influenced by the scanning speed of the laser, rather it is driven by the cooling rate. In a prior study of pulsed versus continuous-wave laser processing, for an equivalent thermal gradient, the cooling rate in P-LPBF of AlSi10Mg was found to be orders of magnitude higher (1.1x10^5 K/s) than that in CW-LPBF (4.0x10^3 – 1.0x10^3 K/s) [120]. The P-LPBF parameters used in the present study resulted in a fine, columnar
structure (Figure 2.5), which has also been observed in other studies for builds with large G/R ratios [117,121].

2.3.4. Quantitative relationship between microstructure and anisotropic ductility

To quantitatively compare the two microstructures, the grain aspect ratio, or grain height divided by grain width, was computed for CW-LPBF and P-LPBF samples, as well as microstructures in prior literature, as tabulated in Table 2.3. In the CW-LPBF samples, this aspect ratio was 1.3, indicating nearly equiaxed grains, while grains in the P-LPBF samples had a much higher aspect ratio of 8.0. This significant difference is attributed to the disparate thermal histories of these samples, which were dictated by different energy inputs, different scanning approaches, and different thermal conditions during fabrication, as described in Section 2.3.3.
Table 2.3. Grain aspect ratio (grain height/grain width) and elongation ratio (BD elongation/⊥BD elongation) for tests performed in the present study as well as data from literature for both powder bed fusion and directed energy deposition AM. Values reported are average ± standard deviation.

<table>
<thead>
<tr>
<th>Grain aspect ratio</th>
<th>Elongation ratio</th>
<th>AM system</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.20 ± 0.42</td>
<td>1.00 ± 0.21</td>
<td>Realizer SLM100</td>
<td>[112]</td>
</tr>
<tr>
<td>1.30 ± 0.29</td>
<td>1.49 ± 0.23</td>
<td>EOSINT M280</td>
<td></td>
</tr>
<tr>
<td>2.38 ± 0.65</td>
<td>1.06 ± 0.34</td>
<td>Custom DED</td>
<td>[122]</td>
</tr>
<tr>
<td>4.77 ± 0.86</td>
<td>1.05 ± 0.00</td>
<td>EOS M270</td>
<td>[109]</td>
</tr>
<tr>
<td>5.22 ± 1.18</td>
<td>0.98 ± 0.17</td>
<td>EOS M270</td>
<td>[108]</td>
</tr>
<tr>
<td>5.30 ± 1.82</td>
<td>0.98 ± 0.15</td>
<td>EOS M270</td>
<td>[111]</td>
</tr>
<tr>
<td>5.79 ± 0.90</td>
<td>1.32 ± 0.19</td>
<td>Custom DED</td>
<td>[106]</td>
</tr>
<tr>
<td>6.71 ± 1.60</td>
<td>1.55 ± 0.39</td>
<td>Concept LaserM2 Cusing SLM</td>
<td>[25]</td>
</tr>
<tr>
<td>8.00 ± 1.75</td>
<td>2.79 ± 0.89</td>
<td>Renishaw AM250</td>
<td>Present study, P-LPBF</td>
</tr>
</tbody>
</table>
Carroll et al. found that prior-β grain boundaries, which contain semi-continuous α phase, act as damage accumulation sites under load [106]. The α phase is more brittle than the β phase [14]. When tension is applied to separate prior-β grains, therefore subjecting the grain boundary α to tension, these grain boundaries act as damage accumulation pathways. Therefore, the presence of high aspect ratio prior-β grains, which present long, continuous grain boundaries in the build direction, results in a lower strain to failure in \( \perp \text{BD} \) samples, than BD samples as described in more detail in [106].

We propose using the grain morphology to describe and potentially predict anisotropic elongation behavior in Ti-6Al-4V. To elucidate any quantitative links between the anisotropic microstructure and the anisotropic ductility in Ti-6Al-4V made by AM, the elongation to failure ratio, defined as the strain to failure in the BD samples divided by that in the \( \perp \text{BD} \) samples, versus the grain aspect ratio, is plotted in Figure 2.6 and tabulated in Table 2.3, for data reported in the literature and the present study. Each data point included comes from a study that reported elongation in the BD and \( \perp \text{BD} \) as well as a micrograph showing complete prior-β grains. Studies in which the grain lengths were not fully encompassed in the micrographs were not included, as the grain dimensions could not be fully quantified. Additionally, in order to isolate the effect of grain morphology only, data for which the elongation in the BD samples was less than that in the \( \perp \text{BD} \) samples, resulting in an elongation ratio less than 1, were excluded due to presumed or identified internal defects. In a fully-dense Ti-6Al-4V component, where grains grow vertically across build layers, the elongation in the build direction should be greater than that in the perpendicular build direction due to the preferential damage accumulation along prior-beta grain
boundaries [106]. However, sharp lack-of-fusion porosity, which is commonly present in additively manufactured components, can result in samples having an elongation ratio of less than 1 [123]. Lack-of-fusion pores have long axes perpendicular to the build direction, and act as stress risers when tension is applied in the build direction; thus, these internal defects limit ductility in the build direction, resulting in an elongation ratio of less than 1. Since the focus of the present study is on linking the grain morphology to ductility, these data were excluded.

Figure 2.6. Elongation ratio (BD sample elongation, t/BD sample elongation, l) versus grain aspect ratio (grain height/grain width) for tests performed in the present study as well as data from literature [106–109,111,112,122]. These data show that as the grain aspect ratio increases above 6, the elongation ratio, and therefore anisotropy in ductility, increases substantially.
The data of anisotropic elongation versus grain aspect ratio in Figure 2.6 were fitted with an exponential curve passing through the point (1.0, 1.0), corresponding to an isotropic sample. This resulted in the following empirical description of the elongation ratio (y) as a function of grain aspect ratio (x):

\[ y = 0.00125e^{0.91x} + 0.98 \]  

(2.3)

This expression can be used to describe and predict the strain to failure ratio between two directions in additively manufactured Ti-6Al-4V based only on the prior-\( \beta \) grain aspect ratio. These data show that grain aspect ratios below about 6 do not lead to significant anisotropy in elongation, as there is enough of a distribution in grain sizes in those samples to dilute the impact of anisotropic grains. However, once the grain aspect ratio exceeds 6, the anisotropy in elongation becomes significant as the elongated grains dominate the mechanics by furnishing significant damage accumulation paths when tension is applied in the perpendicular to the vertical build direction.

With respect to the present samples, when the P-LPBF samples are loaded in tension perpendicular to the vertical build direction, the grains are perpendicular to the loading axis, and are separated by grain boundaries, composed of grain-boundary \( \alpha \). Therefore, the grains are pulled apart in Mode I fracture opening fashion [106]. However, when applying tension along the build direction, the tension acts parallel to the long, continuous prior-\( \beta \) grain boundaries. This results in higher damage accumulation and reduced elongation to failure in the \( \perp \)BD samples compared to BD samples. Conversely, in the CW-LPBF samples, the nearly equiaxed prior-\( \beta \) grains do not provide for higher damage accumulation in one direction compared to another, resulting in nearly isotropic elongation behavior.
2.3.5. Effect of surface roughness on properties

As noted in prior literature, surface roughness of as-built samples, due to both the layered fabrication process as well as partially melted particles adhered to sample surfaces, can result in an overall decrease in the elongation to failure of the material [20,21,124]. The rough surface of as-built samples provides stress concentration sites under loading, which can lead to crack nucleation and early failure. This is particularly apparent when tension is applied in the build direction, as the sharp surface features, corresponding to subsequent layers, are subjected to Mode I opening loading. Conversely, the build layers in samples built perpendicular to the vertical build direction align with the tensile axis of these samples; thus, the ductility in these samples is not as significantly impacted by these features.

Here, the effect of surface finish was examined for the P-LPBF samples. The layer stepping effect was noticeable on the surface of these as-built samples, and the impact of surface finish on properties is seen when comparing the elongation of the as-built and machined P-LPBF samples in Table 2.4. The BD as-built samples had an elongation of 6.5 ± 1.0 %, which increased to 7.8 ± 1.1 % upon milling. Surface roughness was also apparent in the ⊥BD P-LPBF samples due to necessity of incorporating support structures for their fabrication, and as such, the elongation to failure increased in these samples from 1.4 ± 0.2 % to 2.8 ± 0.8% upon surface machining.

A quantitative assessment of the surface roughness was made by analyzing surface line profiles generated from OM 3D reconstruction images taken on the thin edge of the gauge region of the P-LPBF tensile samples, as shown in Table 2.4. The data show that
milling reduced surface roughness (Ra) from 33.90 ± 5.51 μm to 20.71 ± 3.50 μm for the BD samples. Milling was most effective for the ⊥BD samples that were built with support structures, reducing the Ra value from 144.31 ± 13.54 μm to 20.71 ± 3.50 μm. As milling removed the stress concentrations due to surface roughness, the elongation increased.

Table 2.4. Surface roughness (Ra) measurements of the pulsed laser samples in the as-built and machined conditions. Measurements taken in the gauge region. Values reported are average ± standard deviation.

<table>
<thead>
<tr>
<th>Surface roughness, Ra (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>As-built BD</td>
</tr>
<tr>
<td>As-built ⊥BD top surface</td>
</tr>
<tr>
<td>As-built ⊥BD bottom surface (support structure edge)</td>
</tr>
<tr>
<td>Machined (milled) ⊥BD</td>
</tr>
<tr>
<td>Typical range for milled condition [125]</td>
</tr>
<tr>
<td>Typical range for wire EDM condition [125]</td>
</tr>
</tbody>
</table>

Additionally, undulations in the cross-sectional area due to the layer-by-layer fabrication process, as well as partially melted particles on the surface, of as-built samples result in the overestimation of continuous load bearing cross-sectional area, and therefore, the underestimation of the sample strength [126]. To compare properties of samples with as-built surfaces to those reported with machined surfaces, both of which are presented in disparate studies in the literature, a correction technique is needed. In the present study, the cross-sectional area for each rough as-built P-LPBF sample was originally measured
using calipers to measure width and thickness of the gauge region. Due to the nearly fully dense samples verified by X-ray CT, the elastic modulus of as-built Ti-6Al-4V samples should have been equivalent to the milled samples, which was measured to be 110 GPa. To correct for the surface roughness and estimate the load-bearing yield and ultimate tensile strengths of the as-built samples, load-bearing cross-sectional areas were computed to be those that reproduced the expected elastic modulus of 110 GPa for each of the as-built samples. With these load-bearing cross-sectional areas, the yield and ultimate tensile strengths were recalculated. This correction resulted in strengths comparable to the milled samples for both orientations as shown in Table 2.4. This correction technique may be used for computing representative strength values for as-built samples, or for comparing strength data among studies in the literature with disparate surface finishes.

2.4. Summary and conclusions

In the present paper, the anisotropic microstructure in additively manufactured Ti-6Al-4V was quantitatively linked to the anisotropic ductility using samples manufactured with a continuous-wave laser PBF system, a pulsed laser PBF system, and data from literature. Additionally, a method for comparing strengths measured for samples with different surface roughness conditions was presented. The following primary conclusions can be drawn from this study:

- For fully dense, near-defect free Ti-6Al-4V fabricated using AM, the elongation to failure was determined to be anisotropic with prior-β grain aspect ratios greater than 6. In these samples, the elongation was higher in the vertical build direction than perpendicular to the build direction. The quantitative relationship proposed herein
to link microstructure to anisotropic elongation can be used to predict the anisotropic elongation to failure behavior in additively manufactured Ti-6Al-4V with knowledge of only prior-\(\beta\) grain morphology.

- The variations in AM processing conditions studied here resulted in variations in grain morphology, and consequently mechanical properties. The P-LPBF samples contained prior-\(\beta\) grains with a high aspect ratio induced by a high G/R ratio. This led to anisotropic tensile elongation. The CW-LPBF samples had nearly equiaxed grains due to a low G/R ratio, resulting in nearly isotropic elongation behavior.

- The finer grains in the CW-LPBF samples compared to the P-LPBF samples resulted in higher ultimate tensile strength, yield strength, and elongation in the CW-LPBF samples.

- Correcting for surface roughness in as-built P-LPBF samples through calculation of a representative load-bearing cross-sectional area can be used to compare strengths of as-built samples rough surfaces to those of samples with machined surfaces reported in the literature.
Chapter 3 Anisotropic multiaxial plasticity model for laser powder bed fusion additively manufactured Ti-6Al-4V

3.1. Introduction

In this chapter, we experimentally characterized and computationally modeled the plasticity behavior of L-PBF fabricated Ti-6Al-4V, considering both uniaxial and multiaxial stress states. An accurate plasticity model that predicts the mechanical response of complex-shaped additively manufactured Ti-6Al-4V components under loading is required for the adoption of these components in structural applications. Here, L-PBF Ti-6Al-4V was studied under uniaxial tension, pure shear (PS), plane strain tension (PST), and combined tension/shear loading conditions, and the initial yield and strain hardening behavior were measured. A plasticity model that captures the anisotropic and stress state dependent mechanical behavior of this material under multiaxial loading was calibrated based on experimental results and validated using finite element simulations.

3.2. Experimental methods

3.2.1. Sample fabrication

Ti-6Al-4V walls were fabricated using L-PBF AM (EOSINT M280). The pre-alloyed Ti-6Al-4V powder was manufactured by EOS Gmbh via argon gas atomization and had a chemical composition in accordance with ASTM F1472 and F2924 [127,128]. Standard EOS processing parameters for Ti-6Al-4V with a 60 µm layer thickness were used, resulting in a linear heat input of 0.27 J/mm and a volumetric heat input of 32.4 J/mm³ [1]. All specimens used for mechanical testing were extracted from the thin walls, which had the dimensions given in Table 3.1. Prior to machining the walls off of the build plate, or machining test specimens out of the walls, the entire build plate was subjected to a standard stress relief heat treatment of 650°C for 3 hours in an argon environment. Uniaxial tension, in accordance with ASTM E8 [129], (Figure 3.1) and multiaxial loading (Figure 3.2) specimens were extracted from the walls in the vertical build direction and perpendicular to the vertical build direction using wire electrical discharge machining. The gauge thickness of both uniaxial tension and multiaxial loading specimens was 0.5 mm to remove any potential size effects in the mechanical property measurements. A schematic of the orientation nomenclature is given in Figure 3.3. Note that all the specimens were extracted from the x₁-x₂ plane. While x₂ is the vertical build direction, all specimens possess a plane stress condition along the x₃ direction due to the thin thickness along this direction. In addition, the width to height aspect ratio of the gauge region in the multiaxial loading specimen geometry also allows for plane strain conditions along the horizontal
direction of the specimen (x_1 direction for BD samples and x_2 direction for \(\perp\)BD samples) [130].

Table 3.1. As-built wall sample dimensions and number of samples tested in each orientation.

<table>
<thead>
<tr>
<th>Sample Description</th>
<th>As-built wall dimensions: width x thickness x height (mm)</th>
<th>Number of Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>UT – BD</td>
<td>12 x 4 x 65.5</td>
<td>2</td>
</tr>
<tr>
<td>UT (\perp) BD</td>
<td>65.5 x 4 x 12</td>
<td>2</td>
</tr>
<tr>
<td>Multiaxial Loading – BD</td>
<td>70 x 3.5 x 27.4</td>
<td>8</td>
</tr>
<tr>
<td>Multiaxial Loading (\perp) BD</td>
<td>27.4 x 3.5 x 70</td>
<td>8</td>
</tr>
</tbody>
</table>

Figure 3.1. Geometry of tensile samples extracted from as-built walls. Dimensions are in mm.

Figure 3.2. Geometry of multiaxial loading samples extracted from as-built walls. Dimensions are in mm. Adapted from [130].
Figure 3.3. Schematic of sample orientation nomenclature with respect to build direction (BD) where the outlines correspond to sample geometries in Figures 3.1 and 3.2. All of the specimens were extracted from the $x_1$-$x_2$ plane, where $x_2$ is the vertical build direction.

Representative micrographs, in two perpendicular planes, of the L-PBF Ti-6Al-4V in this study are given in Figure 3.4. For microstructural analysis, the samples were prepared using standard metallurgical procedures with a final polish using 0.05 µm colloidal silica and etched using Kroll's reagent (2 vol% hydrofluoric acid and 3 vol% nitric acid in distilled water). The images of the microstructure were taken using a digital microscope (Keyence VHX- 2000).
Figure 3.4. Micrographs of L-PBF Ti-6Al-4V in the (a) $x_1$-$x_2$ plane and (b) $x_1$-$x_3$ plane. The melt pool is shown to have influence on the morphology of the prior-$\beta$ grains, which are filled with acicular $\alpha$ or $\alpha'$ laths. The scale bar is equivalent to the thickness of the gauge regions of mechanical test specimens shown in Figures 3.1 and 3.2.

3.2.2. Mechanical testing

An electromechanical load frame (MTS Criterion Model 43) with a 10 kN load cell (MTS model LPS-104A) was used to perform uniaxial tension tests. The tests were performed under displacement control with an applied strain rate on the order of $10^{-4}$ s$^{-1}$. Multiaxial loading tests were performed on a custom-built dual-actuator hydraulic loading machine (MTS Inc., see Figure 3.5). The dual-actuator test frame is equipped with two 100 kN load cells in the vertical direction ($y$) and one 50 kN load cell in the horizontal direction ($x$). To access plane strain tension, the samples were loaded at 0.1 mm/min in the vertical direction, with the horizontal actuator fixed. To access pure shear, the samples were loaded at 0.8 mm/min in the horizontal direction, with the vertical force set to zero.
For combined tension/shear tests, the ratio of the applied vertical force, $F_y$, to horizontal force, $F_x$, is described by the loading angle, $\beta$, where:

$$\beta = \tan \left( \frac{F_y}{F_x} \right)$$  \hspace{1cm} (3.1)

Combined tension/shear loading tests were performed under force control. For tests where $\beta=30^\circ$, values of $F_y = 0.866 \text{ kN/min}$ and $F_x = 1.5 \text{ kN/min}$ were used, and the opposite loading rates were used to access the $\beta=60^\circ$ condition.

**Figure 3.5.** Schematic of the dual-actuator load frame used to perform multiaxial tests (pure shear, plane strain tension, $\beta = 30^\circ$, and $\beta = 60^\circ$) using specimens shown in Figure 3.2.

For all tests, the surface deformation fields were measured using digital image correlation, a non-contact strain measurement technique (Vic2D software, Correlated Solutions). For the analysis, gauge regions of the samples were painted with a flat white basecoat, and a random black speckled pattern was painted on top of the basecoat. A digital camera (Point Grey GRAS-50S5M-C) was used to take images of the samples at a rate of
1 Hz during loading until fracture. The 2D surface deformation fields in the gauge region of each sample were computed from the digital images using a cubic B-spline interpolation algorithm. DIC parameters used were a subset size of 21 pixels, a step size of 5 pixels, and a strain window of 15 pixels for an overall virtual strain gauge of 71 pixels and spatial resolution of 91 pixels [131]. The axial strain in the uniaxial tension samples was computed using an 18 mm-long vertical virtual extensometer. For the multiaxial loading tests, the surface strains were evaluated at the center of the gage region, and axial and shear strains were computed with a 3 mm vertical virtual extensometer.

3.3. Results and discussion

3.3.1. Uniaxial tension results

Figure 3.6 shows the engineering stress-strain curves for the uniaxial tension tests. The elastic modulus of the material in both orientations was 103.2 ± 1.3 GPa. The yield strength (0.2% offset) and ultimate tensile strength of the material were similar in the BD and \( \perp \)BD directions. Specifically, the average yield strengths were measured to be 1060 MPa in the BD and 1031 MPa in the \( \perp \)BD, while the average ultimate tensile strengths were measured to be 1113 MPa in the BD and 1105 MPa in the \( \perp \)BD. However, the elongation to failure was orientation dependent, as the average ductility in the BD direction was 9.3%, compared to 6.7% in the \( \perp \)BD.
Figure 3.6. Engineering stress-strain curves for uniaxial tension in the build direction and the perpendicular build direction.

In order to propose a conservative plasticity model for this material, considering the fact that scatter of mechanical properties is often observed in materials made by AM [109], the lower bound of the uniaxial tension results were adopted for model development.

3.3.2. Hardening behavior

In the present study, the evolution of the yield strength can be described by the Swift law before necking and a linear extrapolation after necking as:

\[
\Delta \sigma_y = \begin{cases} 
  nA(\varepsilon_0 + \bar{\varepsilon}^p)^{n-1} \Delta \bar{\varepsilon}^p & \bar{\varepsilon}^p \leq 0.038 \\
  K\Delta \bar{\varepsilon}^p & \bar{\varepsilon}^p > 0.038 
\end{cases}
\]  

(3.2)
where $\sigma_y$ is the flow stress at an equivalent plastic strain of $\bar{\varepsilon}_p$, and $A$, $\varepsilon_0$, $n$, and $K$ are material constants. To determine the slope of the linear extrapolation portion of the hardening curve, an inverse method was used in which the criterion for acceptance was to reproduce the experimentally measured engineering stress-strain behavior under uniaxial tension. The experimental data used to fit the hardening behavior, compared with calibrated eq. 3.2, for both directions, are given in Figures 3.7a and 3.7b, while the calibrated parameters are given in Table 3.2. The simulated engineering stress-strain curves used for validation of these parameters are compared with experimentally measured values in Figure 3.7c.

Figure 3.7. Lower bound von Mises equivalent stress-plastic strain curves under uniaxial tension in the (a) build direction and (b) perpendicular build direction for L-PBF Ti-6Al-4V along with a Swift law fit prior to necking, and a linear extrapolation after necking. (c) Engineering stress-strain curves for uniaxial tension from experiments (symbols) and simulations based on the inputs in (a) and (b) (lines).
Table 3.2. Strain hardening parameters determined through experimental fitting of lower bound uniaxial tension tests in each orientation.

<table>
<thead>
<tr>
<th></th>
<th>A (MPa)</th>
<th>n</th>
<th>ε0</th>
<th>K (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BD</td>
<td>1349.4</td>
<td>0.042</td>
<td>0.002</td>
<td>950</td>
</tr>
<tr>
<td>⊥BD</td>
<td>1303</td>
<td>0.042</td>
<td>0.002</td>
<td>950</td>
</tr>
</tbody>
</table>

3.3.3. Multiaxial loading results

Multiaxial loading tests were performed to study the mechanical behavior of the material as a function of stress state and material orientation. Figure 3.8 shows the anisotropy of the plasticity behavior under five loading conditions, while Figure 3.9 highlights the stress state-dependency of the plasticity behavior. As shown in Figure 3.8, in all stress states studied, BD samples had higher yield strength (1017 ± 27 MPa) than their ⊥BD counterparts (961 ± 52 MPa). Under multiaxial loading, the L-PBF Ti-6Al-4V exhibited very low ductility, with none of the samples exceeding 5.5% equivalent plastic strain before failure. The plane strain tension and β = 60° samples all failed at or below 1.0% equivalent plastic strain.
**Figure 3.8.** Von Mises equivalent stress-plastic strain curves for (a) pure shear, (b) combined loading with $\beta = 30^\circ$, (c) uniaxial tension, (d) combined loading with $\beta = 60^\circ$, and (e) plane strain tension, in two directions. In all cases except for combined loading with $\beta = 60^\circ$, the mechanical behavior showed notable anisotropy.

Figure 3.9 shows that in each direction, the von Mises equivalent stress-strain curves depend on stress state. Therefore, the material behavior is anisotropic and stress state dependent, and the initial yield behavior and subsequent flow behavior cannot be captured using an isotropic von Mises, or J2 plasticity, framework.
Figure 3.9. Von Mises equivalent stress versus equivalent plastic strain for all stress states evaluated in both the (a) build direction and (b) perpendicular BD directions, indicating that the yield and strain hardening behavior of the material is stress state dependent.

3.4. Modeling

3.4.1. Plasticity modeling

A plasticity framework using the Hill 1948 anisotropic yield function (Hill48) was adopted to describe the multiaxial plasticity behavior of the current material [132]. The yield function is given as:

\[ f = \sigma_{\text{Hill48}} - \sigma_y = 0 \]  \hspace{1cm} (3.3)

where \( \sigma_y \) is the yield stress, and the equivalent Hill48 stress, \( \sigma_{\text{Hill48}} \), under the plane stress condition is defined as:

\[
\sigma_{\text{Hill48}} = \sqrt{(G + H)\sigma_{11}^2 - 2H\sigma_{11}\sigma_{22} + (F + H)\sigma_{22}^2 + 2N\tau^2}
\]  \hspace{1cm} (3.4)
where G, H, F, and N are constants that describe the material’s anisotropy, \( \sigma_{11} \) and \( \sigma_{22} \) are the normal stresses along \( x_1 \) and \( x_2 \) directions, respectively, as shown in Figure 3.3, and \( \tau \) is the shear stress component. Assuming an associated flow rule, the plastic strain increment can be calculated as:

\[
\varepsilon_{ij}^p = \dot{\gamma} \frac{\partial f(\sigma_{ij})}{\partial \sigma_{ij}}
\]

(3.5)

where \( \dot{\gamma} \) is the plastic multiplier.

In the current work, the specimens were built in two orientations, BD (y-loading direction parallel to the build direction) and \( \perp \)BD (y-loading direction perpendicular to the build direction). The multiaxial loading specimens were under plane strain, with zero strain along the x-direction in machine coordinates (see Figure 3.5), along the majority of the gauge region. The x-direction was parallel to the \( x_1 \) direction for BD specimens and parallel to the \( x_2 \) direction for \( \perp \)BD specimens, as shown in Figure 3.3. As a result, \( \varepsilon_{11} \approx 0 \) for BD and \( \varepsilon_{22} \approx 0 \) for \( \perp \)BD specimens. For plastic deformation, if \( \varepsilon_{22}^p \approx 0 \), the associated flow rule is given as:

\[
\varepsilon_{22}^p = \dot{\gamma} \frac{\partial f(\sigma_{22})}{\partial \sigma_{22}} = \dot{\gamma} \frac{2(F+H)\sigma_{22}-2H\sigma_{11}}{2\sigma_{Hill48}} = 0,
\]

(3.6)

which gives:

\[
\sigma_{22} = \frac{H}{F+H} \sigma_{11}
\]

(3.7)

Correspondingly, the plane strain condition for the BD specimens gives:

\[
\sigma_{11} = \frac{H}{G+H} \sigma_{22}
\]

(3.8)

However, if the plastic strain is very small, as was the case in this study, then \( \varepsilon_{22}^p \approx \varepsilon_{22} \approx 0 \) for \( \perp \)BD specimens, giving:
\[ \sigma_{22} = \nu \sigma_{11}, \quad (3.9) \]

where \( \nu \) is the Poisson ratio (\( \sigma_{33} = 0 \) due to out-of-plane plane stress condition). Similarly, for BD specimens, \( \varepsilon_{11}^e \approx \varepsilon_{11} = 0 \) and:

\[ \sigma_{11} = \nu \sigma_{22}. \quad (3.10) \]

Given the low level of plastic strain in the present study, eq. (3.9) and eq. (3.10) were used to compute the stress along the x-direction in the multiaxial loading tests.

### 3.4.2. Model calibration

The Hill48 yield surface and subsequent hardening law were calibrated using the lower bound uniaxial tension data. The sum \( G+H \) was assumed to be 1, and \( F+H \) was determined based on the yield strength ratio under uniaxial tension along the two orientations, which can be derived from eq. (3.4) as:

\[ \frac{G+H}{F+H} = \left( \frac{\sigma_{y,22}}{\sigma_{y,11}} \right)^2 \quad (3.11) \]

The values of \( H \) and \( N \) were determined using the experimental data from pure shear and \( \beta = 60^\circ \) tests along both orientations.

The calibrated model parameters are given in Table 3.3 and the calibrated Hill48 anisotropic yield surface for the L-PBF Ti-6Al-4V in this study is given in Figure 3.10. The calibrated yield surface captures the multiaxial experimental data well. It is noted that the adopted Hill48 parameters are close to the values for the isotropic von Mises criterion \((G + H = 1, F + H = 1, N = 1.5, \text{ and } H = 0.5)\). Therefore, the material exhibits a clear, but limited amount of anisotropy.
Table 3.3. Calibrated Hill48 yield surface model parameters.

<table>
<thead>
<tr>
<th>F</th>
<th>G</th>
<th>H</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.47</td>
<td>0.53</td>
<td>1.45</td>
</tr>
</tbody>
</table>

Figure 3.10. Anisotropic Hill48 2D yield surface (for plane stress) at initial yield (0.2% strain) and subsequent strain (1.0%) of L-PBF Ti-6Al-4V. The fitted yield surfaces are shown as lines, while experimental data are shown as symbols. Solid symbols represent perpendicular build direction samples, while open symbols represent build direction samples.

3.4.3. Finite element simulations

To calibrate and validate the continuum plasticity model, the model was implemented in commercial finite element analysis (FEA) software ABAQUS [133]. Comparing FEA simulated mechanical behavior with experimental results allows for
calibration of model parameters and subsequent validation of the calibrated model in all stress states studied.

Simulations of the uniaxial tension experiments were performed using a model of 1/8\textsuperscript{th} of the tensile specimen geometry, taking advantage of symmetry, and the geometry was discretized using 18,592 C3D8 elements. Symmetry boundary conditions were adopted along all three cut planes. A uniform vertical displacement was prescribed to the grip region.

Additionally, finite element models were created to simulate the multiaxial mechanical tests studied. To simulate the multiaxial loading tests, 2D single element models measuring 1 mm by 1 mm were used. Due to the through thickness plane stress condition of the multiaxial loading specimens, a four node 2D plane stress element (CPS4) was used. For all multiaxial loading cases, the lower boundary was fixed in the vertical direction, and the lengths of the bottom and top surfaces of the element were fixed. To evaluate plane strain tension, a uniform vertical displacement was applied to the two upper nodes, with the left and right edges constrained in the horizontal direction. For pure shear, a horizontal displacement was prescribed to the top nodes. For combined loading, concentrated forces in both the horizontal and vertical directions were applied to the top nodes such that the two desired loading angles were fulfilled.

3.4.4. Model validation

As model calibration was performed using data from uniaxial tension, pure shear, and combined tension/shear loading tests with $\beta=60^\circ$, the model was validated using data from plane strain tension and combined tension/shear loading tests with $\beta=30^\circ$. In both the BD and LD orientations, the experimental results lie on or very close to the calibrated
yield surface. The developed model is able to predict the evolving flow stress behavior of
the material in both build orientations and in all stress states evaluated as shown in Figure
3.11. The maximum percent difference in normal engineering stress for a given strain
between those predicted using the plasticity model and those experimentally measured is
4.1% for BD and 3.1% for \( \perp \text{BD} \) specimens across all stress states evaluated. The
maximum percent difference in engineering shear stress between plasticity simulations and
experiments at a given strain is 4.6% for BD and 6.0% for \( \perp \text{BD} \) specimens.

**Figure 3.11.** (a) Normal and (b) shear engineering stress-strain curves for the four
multiaxial loading conditions, showing the comparison between experimental data
(symbols) and the simulation results (lines).

Therefore, the proposed plasticity model captures and predicts the anisotropy and
stress state dependence of the plasticity behavior of L-PBF Ti-6Al-4V. Note that the
calibrated material parameters hold for this particular L-PBF Ti-6Al-4V material, with the
parameters used. This means that the approach for gathering the experimental data, and calibrating and validating the model can be applied to other additively manufactured materials, but the parameters need to be calibrated for the particular material being studied. For example, the model presented here applies to the nearly equiaxed grain structure in this L-PBF Ti-6Al-4V; however, it is anticipated that the anisotropy would be increased in a Ti-6Al-4V material with columnar grains such as in [134].

3.5. Summary and conclusions

The current work studied the multiaxial yield and plastic flow behavior of L-PBF Ti-6Al-4V. Using experimental results that are novel to the AM community, a calibrated and validated anisotropic and stress state dependent plasticity model was proposed. The main takeaways from this study include:

- L-PBF Ti-6Al-4V was found to be anisotropic under all stress states evaluated (uniaxial tension, plane strain tension, pure shear, and combined tension/shear loading).
- The yield and flow behavior of L-PBF Ti-6AL-4V was found to be stress state dependent.
- The calibrated plasticity model, consisting of an anisotropic Hill48 yield criterion, an associated flow rule, and an isotropic strain hardening law, captures and predicts the anisotropic and stress state dependent initial yield and subsequent flow behavior of L-PBF Ti-6Al-4V. The ability to describe and predict the multiaxial deformation behavior of additively manufactured Ti-6Al-4V is required for its safe adoption in structural applications.
Chapter 4 Fracture of laser powder bed fusion additively manufactured Ti–6Al–4V under multiaxial loading: Calibration and comparison of fracture models

4.1. Introduction

As demonstrated in conventionally processed ductile metals, ductile failure behavior depends on stress state; thus, engineers should not rely on UT data alone for predicting fracture of components. It is therefore critical to characterize materials under a range of stress states and to develop and calibrate models that capture stress state-dependent failure behavior to ensure the safety of structural components. This is particularly critical for the adoption of AM, where the design freedom that allows for complex geometries and custom components is a hallmark of the technology. To demonstrate the importance of understanding and describing the stress state-dependence of fracture in L-PBF Ti-6Al-4V, the present chapter evaluates the fracture behavior of this material under a range of stress states, and compares the ability of several fracture models, with varying complexity, to capture this behavior. This work highlights the importance of considering the effects of both stress triaxiality and Lode angle parameter in fracture models and provides insight on models that best capture this material’s fracture behavior.

---

4.2. Experimental methods

All Ti-6Al-4V samples were manufactured on an EOS M280 L-PBF system using the EOS standard processing parameters for Ti-6Al-4V with a 60 μm layer height, such that the volumetric heat input was 32.4 J/mm³. For all samples, blocks or walls were fabricated using the L-PBF process, then the entire build plate was subjected to a post-processing stress relief heat treatment of 650°C for 3 hours. The samples were then removed from the build plate by wire electrical discharge machining for final machining. Samples for the evaluation of fracture behavior of additively manufactured Ti-6Al-4V in this work were machined from the same L-PBF build as those used in the development of the plasticity model for this material [135], with the exception of the equibiaxial tension (EBT) punch samples that were manufactured separately. The microstructure of the Ti-6Al-4V material used in the current study was evaluated previously and is presented in reference [135].

Six different sample geometries were machined, from the additively manufactured walls, and used to evaluate the fracture behavior in seven different stress states and in two primary orientations: one in which the properties parallel to the vertical build direction were probed and one in which the properties perpendicular to the vertical build direction were probed. The equibiaxial tension punch samples, as detailed in reference [136], were machined from the plane orthogonal to the other samples.

Butterfly specimens with geometry optimized by Dunand and Mohr [137], as shown in Figure 4.1, were used to evaluate pure shear and combined tension/shear loading under plane stress. Wire EDM was used to machine the outer profile of the butterfly samples and CNC milling was used to machine the reduced thickness gauge region of these
samples. Central hole tension (CH) and notched tension (NT) samples, also designed to maintain a plane stress state at fracture with the geometries as shown in Figure 4.2, were machined using wire EDM. The central hole tension sample is designed such that it maintains a uniaxial tension stress state up to fracture at the perimeter of the circle in the center of the gauge region. The stress states in the notched tension samples have a smaller Lode angle parameter and slightly larger stress triaxiality, which increases with decreasing notch radii, than uniaxial tension, at the center of the gauge region. The equibiaxial tension punch samples have the highest stress triaxiality of all sample types studied and are the only samples with a negative Lode angle parameter. The set of test samples used was chosen to probe the failure behavior of L-PBF Ti-6Al-4V over a wide range of stress states, as well as to evaluate the ability of multiple fracture models to capture the resulting fracture behavior.

**Figure 4.1.** Schematic of the butterfly sample used for pure shear and combined loading stress states (dimensions in mm). Geometry from [137].
The relationship between the two stress state parameters, $\eta$ and $\bar{\theta}$, under plane stress, as well as the stress state of each sample type probed in this study at fracture are shown in Figure 4.3. It is noted that computational modeling of the experiments indicated that the notched tension samples were not under perfect plane stress for the duration of the experiments, but were very close.
Figure 4.3. Stress triaxiality versus Lode angle parameter for the condition of plane stress (dashed curve) along with stress state locations of all samples tested (symbols). Square data points represent samples that did not perfectly meet the plane stress condition (as shown by their distance from the ideal plane stress dashed line).

Mechanical tests of the butterfly samples were performed using a custom-built dual-actuator hydraulic loading machine (MTS, Inc.) schematically shown in reference [135]. The load frame is equipped with two 100 kN load cells to measure force in the vertical direction and one 50 kN load cell to measure force in the horizontal direction. Pure shear tests were completed for two samples in each orientation by loading each sample under displacement control at 0.4 mm/min in the horizontal direction, with the vertical actuator kept at zero force throughout the test. For the combined tension/shear loading tests, both actuators were run under force control, and the angle $\beta$ can be used to describe the ratio of vertical to horizontal forces during a test as:
\[
\beta = \tan^{-1}\left(\frac{\dot{F}_V}{\dot{F}_H}\right) \tag{4.1}
\]

Two butterfly samples in each orientation were subjected to \(\beta = 30^\circ\) loading, with a horizontal loading rate, \(\dot{F}_H\), of 0.75 kN/min and a vertical loading rate, \(\dot{F}_V\), of 0.433 kN/min.

Three tests in each orientation were performed on central hole samples and each of the notched tension geometries using an electromechanical load frame (MTS Criterion 43) with a 10 kN load cell. These tests were performed under displacement control loading with an applied strain rate on the order of \(10^{-4} \text{ s}^{-1}\). The equibiaxial tension, or punch tests, were performed on three samples. These were tested using a punch fixture consisting of a 25.4 mm diameter die and a 12.7 mm diameter hemispherical punch, modeled after the fixture in reference [136], using an electromechanical load frame (Instron model 4206) with a 150 kN load cell and a punch loading rate of 0.4 mm/min. Teflon sheets were used between the sample and the punch to minimize the effect of friction during loading.

Surface deformation fields were measured using digital image correlation, a non-contact deformation measurement technique. Three-dimensional DIC (3D DIC) was used for equibiaxial punch tests, while two-dimensional DIC (2D DIC) was used for all other tests. The gauge regions of all samples were painted with a flat white basecoat with a black random speckle pattern on top of the basecoat. Images of the samples were taken at a rate of 1 Hz using one camera for 2D DIC and two cameras for 3D DIC (Point Grey GRAS-50S5M-C) with data capture software (VicSnap, Correlated Solutions) until fracture. For the 3D DIC a calibration target with a 9 x 9 dot pattern and 2.5 mm spacing was used to calibrate the relative positions of the cameras with respect to each other. Vic2D and Vic3D software (Correlated Solutions) were used to compute strains from the images acquired.
during tests. The image analysis parameters for 2D DIC were set as a subset size of 21 pixels and a step size of 5 pixels, while for the 3D DIC the subset size was 29 pixels and the step size was 7 pixels. Displacement for each 2D DIC test was measured using a virtual vertical extensometer centered in the gauge region with a length of 10 mm for the central hole test, 18 mm for the notched tension samples, and 8 mm for the butterfly samples. Displacement for the punch test samples was calculated comparing the relative displacement of the pixel at the center of the sample, directly above the apex of the punch, and a pixel 11 mm away from the apex, close to the edge of the die. Force data for each test were directly exported from the respective load frames during testing.

Testing was stopped for each test after catastrophic failure of the sample, where failure was defined as separation of the material in the gauge region for all 2D DIC tests, or the appearance of a visible crack on the sample surface in the digital images for the punch tests. For the purposes of model calibration, failure in the current study was defined as the time (or displacement) corresponding to a drop in force by over 90% in the 2D DIC tests and exceeding 10% in the punch tests. This criteria for failure was adopted because of the low ductility and low strain hardening behavior of L-PBF Ti-6Al-4V. With this adopted fracture criteria, the average critical equivalent strain to fracture, stress triaxiality, and Lode angle parameter at fracture, computed using finite element analysis as described in Section 4.3, are given in Table 1. These values were used in the calibration of the fracture models described in Section 4.4.
Table 4.1. Average values of the strain to failure, stress triaxiality, and Lode angle parameter for each test performed.

<table>
<thead>
<tr>
<th>Sample Type</th>
<th>Orientation</th>
<th>$\bar{\varepsilon}_f$</th>
<th>$\eta$</th>
<th>$\bar{\theta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure Shear</td>
<td>BD</td>
<td>0.091</td>
<td>0.006</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>$\perp$BD</td>
<td>0.062</td>
<td>0.003</td>
<td>0.010</td>
</tr>
<tr>
<td>$\beta_{30}$</td>
<td>BD</td>
<td>0.115</td>
<td>0.170</td>
<td>0.490</td>
</tr>
<tr>
<td></td>
<td>$\perp$BD</td>
<td>0.109</td>
<td>0.171</td>
<td>0.491</td>
</tr>
<tr>
<td>CH</td>
<td>BD</td>
<td>0.324</td>
<td>0.361</td>
<td>0.905</td>
</tr>
<tr>
<td></td>
<td>$\perp$BD</td>
<td>0.230</td>
<td>0.376</td>
<td>0.856</td>
</tr>
<tr>
<td>R8</td>
<td>BD</td>
<td>0.164</td>
<td>0.489</td>
<td>0.626</td>
</tr>
<tr>
<td></td>
<td>$\perp$BD</td>
<td>0.148</td>
<td>0.494</td>
<td>0.611</td>
</tr>
<tr>
<td>R4</td>
<td>BD</td>
<td>0.125</td>
<td>0.540</td>
<td>0.440</td>
</tr>
<tr>
<td></td>
<td>$\perp$BD</td>
<td>0.135</td>
<td>0.550</td>
<td>0.413</td>
</tr>
<tr>
<td>R2.67</td>
<td>BD</td>
<td>0.112</td>
<td>0.555</td>
<td>0.315</td>
</tr>
<tr>
<td></td>
<td>$\perp$BD</td>
<td>0.110</td>
<td>0.564</td>
<td>0.285</td>
</tr>
<tr>
<td>Punch</td>
<td>-</td>
<td>0.120</td>
<td>0.667</td>
<td>-0.931</td>
</tr>
</tbody>
</table>

4.3. Finite element simulations

The previously developed, orientation dependent plasticity model was implemented in the finite element analysis (FEA) commercial software ABAQUS [133] and used for simulations of all fracture experiments in the present study. FEA simulations were used to determine the equivalent plastic strain and stress state history at the location of fracture for each sample type.

Simulations of the butterfly samples under pure shear and $\beta = 30^\circ$ were performed using a FEA model with half the thickness of the experimental geometry, taking advantage of sample symmetry to reduce computational time. The grip regions were modeled as rigid bodies, while the gauge region and transition region between the grips and the gauge region were modeled using the elasto-plastic model of L-PBF Ti-6Al-4V presented in reference [135]. The half butterfly geometry was discretized with 126,176 C3D8 elements. For both
stress states, two reference nodes were defined for the bottom and top rigid grip sections. For the pure shear condition, a horizontal displacement of 0.6 mm was applied to the top reference node, and all remaining degrees of freedom were fixed for the top and bottom grips. In the simulations of the $\beta = 30^\circ$ test, force boundary conditions were applied to the top grip reference node in the vertical and horizontal directions at a ratio to meet the $\beta = 30^\circ$ condition, and all other degrees of freedom for the top and bottom grips were fixed. The simulated displacements for both stress states were extracted from the equivalent top and bottom locations of the 8 mm tall experimental virtual extensometer at the center of the gauge region.

Simulations of the central hole and notched tension tests were performed using a 1/8th model geometry with symmetry boundary conditions applied on the three cut planes of the samples. The model geometries were discretized with the following number of C3D8 elements: 5,824 in the central hole geometry, 1,176 in the $R = 2.67$ mm NT geometry, 1,232 in the $R = 4$ mm NT geometry, and 1,456 in the $R = 8$ mm NT geometry. A 1 mm vertical displacement was applied to a reference node that dictated the vertical displacement of all nodes in the grip region of each sample. Displacement was calculated from nodes at equivalent locations to those of the experimental virtual extensometers.

For all 2D DIC samples, failure was assumed to occur in the center of the gauge region, except for the CH geometry in which failure was assumed to occur at the leftmost or rightmost point on the perimeter of the hole. These locations are where time histories of stress triaxiality, Lode angle parameter, and equivalent plastic strain were extracted for each simulation.
For the punch simulation, the full sample thickness was modeled, along with ¼ of the circle, and this geometry was discretized with 121,440 C3D8 elements. Symmetry boundary conditions along the two cut planes were applied to the ¼ geometry. In addition to the sample, the punch and clamping fixture were modeled as rigid bodies with frictionless contact between these entities and the sample. This is consistent with the approximation made in reference [136] for similar tests, and is justified due to the use of Teflon in the experiments. All degrees of freedom were constrained for nodes along the circumference of the punch specimen and the die/clamping fixture. A 1 mm vertical displacement was applied to the rigid punch during the simulation, while all other degrees of freedom of the punch were constrained. The punch displacement was calculated as the relative displacement between the apex node on the top surface of the specimen and a node 11 mm away from the apex (toward the die edge) on the top surface. The stress triaxiality, Lode angle parameter, and equivalent strain histories were extracted from the apex of the punch specimen, where failure was assumed to initiate.

The plasticity model proposed for L-PBF Ti-6Al-4V in reference [135] incorporates a conservative flow curve, which was the lowest stress-strain curve measured in each direction for this material. For the current study, in order to accurately capture the deformation behavior of all fracture tests performed, the flow stress in the plasticity model was taken to be that of the average material behavior in each orientation, implemented as a 4% increase in flow stress over that adopted in reference [135]. Comparisons of the simulated and experimentally measured force vs. displacement behavior for each of the samples tested are given in Figure 4.4. The model predicts well the elastic and plastic behavior of the material for each geometry, with the maximum error being an
overprediction of the maximum force by 2% in the build direction notched tension $R = 2.67$ mm sample.

**Figure 4.4.** Comparison of experimentally measured and simulated force vs. displacement results for the (a) central hole tension; (b) $R = 2.67$ mm, (c) $R = 4$ mm, and (d) $R = 8$ mm notched tension tests; (e, f) $\beta = 30^\circ$ combined loading; (g) pure shear loading; and (h) equibiaxial tension punch tests.
4.4. Calibration of fracture models

In this study, six different ductile fracture models, dependent on neither, one, or both of the stress state parameters of stress triaxiality and Lode angle parameter, were compared to determine their suitability for describing the multiaxial fracture behavior of L-PBF Ti-6Al-4V. While the stress state parameters evolve over the course of elasto-plastic deformation, as shown in Figure 4.5, for the description of a fracture condition, an average definition of stress state is needed. Thus, as monotonic loading is used in all tests presented, the average stress triaxiality is approximated as:

$$\eta_{av} = \frac{1}{\bar{\varepsilon}_f} \int_0^{\bar{\varepsilon}_f} \eta \, d\bar{\varepsilon}$$

(4.2)

and the average Lode angle parameter is approximated as:

$$\bar{\theta}_{av} = \frac{1}{\bar{\varepsilon}_f} \int_0^{\bar{\varepsilon}_f} \bar{\theta} \, d\bar{\varepsilon}$$

(4.3)

Four of the models described here consider the equivalent strain to failure only as a function of the average stress triaxiality, while two of the models consider the strain to failure as a function of both the average stress triaxiality and Lode angle parameter.
Figure 4.5. Evolution of the (a, b) stress triaxiality and (c, d) Lode angle parameter (solid lines) up to failure (symbols) for each sample geometry in both the BD (a, c) and \( \perp \text{BD} \) (b, d) with the average value of each stress state parameter used in fracture model calibration represented by a vertical dashed line.
4.4.1. Constant equivalent strain to fracture criterion

One of the simplest criteria to define the failure of ductile metals is the constant equivalent strain to fracture criterion, which states that fracture occurs in a material when

\[ \bar{\varepsilon} = \varepsilon_f \]  

(4.4)

where \( \varepsilon_f \) is the critical strain to failure under a given stress state used to define failure (here taken to be uniaxial tension), and \( \bar{\varepsilon} \) is the equivalent plastic strain in any stress state of interest. For the L-PBF Ti-6Al-4V material, the Hill 1948 description of the equivalent plastic strain under plane stress was used, which is given as [88,132]:

\[
d\bar{\varepsilon}^p = \left\{ \frac{1}{\alpha} \left[ (F + H)(d\varepsilon_{11}^p)^2 + (H + G)(d\varepsilon_{22}^p)^2 + 2H(d\varepsilon_{11}^p)(d\varepsilon_{22}^p) \right] \right. \\
+ \left. \frac{1}{2N} (2d\varepsilon_{12}^p)^2 \right\}^{0.5}
\]  

(4.5)

where \( G, H, F, \) and \( N \) are constants that describe the material’s anisotropy, \( \alpha = FG + GH + HF \), \( \varepsilon_{11}^p \) and \( \varepsilon_{22}^p \) are the normal plastic strains, and \( \varepsilon_{12}^p \) is the shear strain. In the present study, the strain to failure under uniaxial tension was obtained using the central hole tension test, which maintains a uniaxial tension state all the way to fracture. The average equivalent strain to failure, \( \bar{\varepsilon}_f \), for the central hole tests in each orientation is given in Table 4.1.

4.4.2. Johnson-Cook fracture criterion

The empirical Johnson-Cook ductile fracture model defines the equivalent strain to failure as monotonic decreasing function of stress triaxiality [81]:

\[ \bar{\varepsilon}_f = C_1 + C_2 \exp (C_3 \eta) \]  

(4.6)
where $C_1$, $C_2$, and $C_3$ are all calibration constants. This model captures well the experimentally observed decreasing ductility with increasing stress triaxiality in the high stress triaxiality regime. However, this model was not designed to capture the fracture behavior under low stress triaxialities. The Johnson-Cook model parameters for both material orientations in the present study were calibrated using nonlinear least-squares fitting of the Johnson-Cook equation to the experimental data from the central hole tension, all three notched tension tests, and the punch samples. The calibrated parameters are given in Table 4.2.

4.4.3. Two-branch empirical fit fracture criterion

Another empirical method for constructing a fracture locus for a ductile material in the $[\eta, \varepsilon]$ space, as described by Bao et al. [74], is to simply fit the experimental test data, in two distinct stress triaxiality ranges, with polynomials. While this criterion does not incorporate any physics, breaking the plane stress fracture criterion into two regimes does have a physical basis. Namely, Bao et al. hypothesized that in the first, low stress triaxiality range, $0 \leq \eta \leq 0.33$, fracture occurs due to a combination of shear deformation and void growth, while in the second, high stress triaxiality range, $\eta > 0.33$, fracture is assumed to occur only due to void formation and growth mechanisms. The calibrated empirical curve fit fracture loci for the L-PBF Ti-6Al-4V material are given as:

$$BD, \bar{\varepsilon}_f = \begin{cases} 2.67\eta^2 - 0.324\eta + 0.093 & 0 \leq \eta \leq 0.361 \\ 0.037\eta^{-2.12} & 0.361 < \eta \end{cases}$$

(4.7)

$$\perp BD, \bar{\varepsilon}_f = \begin{cases} 0.832\eta^2 - 0.135\eta + 0.062 & 0 \leq \eta \leq 0.376 \\ 0.059\eta^{-1.37} & 0.376 < \eta \end{cases}$$

(4.8)
The ranges of the low and high stress triaxiality regimes were adjusted for each orientation to capture the central hole test data in the current study. These equations and corresponding fracture locus are only valid for this particular data set for L-PBF Ti-6Al-4V, but the curve fitting does indicate the presence of two distinct regimes of failure for the additively manufactured Ti-6Al-4V material under plane stress.

4.4.4. Maximum shear stress fracture criterion

Plastic deformation occurs by dislocation motion, which is driven by shear stress. Additionally, experimental findings for ductile metals in which localized shear bands and eventual material separation occur at the angle of maximum shear stress/strain (45° from the tensile axis for isotropic materials) can be found in the literature [79]. Based on these observations, the maximum shear failure criterion, which hypothesizes that ductile fracture occurs on the plane of maximum shear stress/strain, defines failure to occur when the maximum shear stress in the stress state of interest is equal to the maximum shear stress at failure under uniaxial tension, and is given as [72]:

\[ \tau_{max} = (\tau_{max})_f \quad (4.9) \]

where

\[ \tau_{max} = \max \left\{ \frac{\sigma_1 - \sigma_2}{2}, \frac{\sigma_2 - \sigma_3}{2}, \frac{\sigma_3 - \sigma_1}{2} \right\} \quad (4.10) \]

To convert this failure criterion to strain space, a flow rule must be adopted. Approximating the flow behavior under uniaxial tension as a power law relationship between strain and stress as \( \sigma = Ke^n \), the fracture locus in the \([\eta,\varepsilon_f]\) space is given as [72]:

74
\[ \varepsilon_f = C \left\{ \frac{\sqrt{1 + \alpha + \alpha^2}}{2 + \alpha} \right\}^{\frac{1}{n}} \quad \text{for} \quad -\frac{1}{2} < \alpha < 1 \quad \text{or} \quad \frac{1}{3} < \eta < \frac{2}{3} \quad (4.11) \]

\[ \varepsilon_f = C \left\{ \frac{\sqrt{1 + \alpha + \alpha^2}}{1 - \alpha} \right\}^{\frac{1}{n}} \quad \text{for} \quad -2 < \alpha < -\frac{1}{2} \quad \text{or} \quad -\frac{1}{3} < \eta < \frac{1}{3} \quad (4.12) \]

where the relationship between \( \eta \) and \( \alpha \), the strain ratio parameter, is given as

\[ \eta = \left( \frac{1}{\sqrt{3}} \right) \left( \frac{1 + \alpha}{\sqrt{1 + \alpha + \alpha^2}} \right) \quad (4.13) \]

For the L-PBF Ti-6Al-4V material studied here, the strain hardening parameter, \( n = 0.113 \) for both build orientations was adopted. The calibrated \( C \) parameter for each orientation is given in Table 4.2 for the maximum shear stress fracture criterion.

**4.4.5. Modified Mohr-Coulomb fracture criterion**

To more accurately define fracture in ductile metals, Bai and Wierzbicki developed the Modified Mohr-Coulomb fracture model [77], which includes the effects of both stress triaxiality and Lode angle parameter (third invariant of deviatoric stress) in the classical stress-based MC failure criterion [92]. Physically, the MMC fracture criterion takes into account the critical combination of normal stress and shear stress that will cause failure on a given plane in a material. The MMC fracture locus, which is transformed into \([\varepsilon_f, \eta, \bar{\theta}]\) space as described in detail in reference [87], is given as:

\[ \varepsilon_f[\eta, \bar{\theta}] = \left\{ \frac{A}{c_2} \left[ c_{\bar{\theta}}^s + \frac{\sqrt{3}}{2 - \sqrt{3}} (c_{\bar{\theta}}^{\alpha \bar{\theta}} - c_{\bar{\theta}}^s) \left( \sec \left( \frac{\theta}{6} \right) - 1 \right) \right] \left[ \sqrt{\frac{1 + c_1^2}{3}} \cos \left( \frac{\theta}{6} \right) \left( \frac{\theta}{6} \right) \right] + \right. \]

\[ \left. c_1 \left( \eta + \frac{1}{3} \sin \left( \frac{\theta}{6} \right) \right) \right\} \left( \frac{1}{n} \right)^{\frac{1}{n}} \quad (4.14) \]

where
\( \tau_{\theta}^{ax} = \begin{cases} 1 & \bar{\theta} \geq 0 \\ c_{\theta} & \bar{\theta} < 0 \end{cases} \quad (4.15) \)

\( A \) and \( n \) are Swift hardening law parameters \( \sigma_y = A(\varepsilon_0 + \varepsilon^p)^n \), [22], and \( c_1, c_2, c_{\theta}, \) and \( c_{\theta}^s \) are model parameters, where \( c_1 \) is referred to as a “friction” coefficient, which describes the stress triaxiality dependence and asymmetry of the fracture surface with respect to the Lode angle parameter, \( c_2 \) influences the magnitude of the failure strains and represents the shear resistance to failure, \( c_{\theta}^s \) controls the Lode angle parameter dependence of the fracture surface, and \( c_{\theta}^g \) controls the asymmetry of the fracture locus with respect to the Lode angle parameter but does not impact the stress triaxiality dependence of the fracture locus [87].

In this study, the MMC model was calibrated using a Matlab function that finds the minimum of a constrained non-linear multivariable function and determines the model parameters for the best surface fitting of the experimental data points. The calibrated parameters for an MMC fracture model of L-PBF Ti-6Al-4V in two orientations are given in Table 4.2.

**4.4.6. Hosford-Coulomb fracture criterion**

The phenomenological Hosford-Coulomb model was developed by Mohr and Marcadet as a fracture initiation model based on the hypothesis that the formation of a primary or secondary band of localization corresponds to the start of fracture in a ductile metal [90]. The MC stress-based criterion was found to not fully capture experimental results where localization occurs before void coalescence. To remedy this issue, the HC model takes into account the intermediate principal stress contribution to failure by substituting the Hosford equivalent stress [91] for the maximum shear stress contribution in the MC criterion. The stress-based fracture criterion is then transformed from the
principal stress space to equivalent plastic strain space by assuming a flow rule, which in reference [90] was taken to be an isotropic hardening law. The Hosford-Coulomb fracture criterion in \([\bar{\varepsilon}_f, \eta, \bar{\theta}]\) space is given as [90]:

\[
\bar{\varepsilon}_f[\eta, \bar{\theta}] = b(1 + c)^{1/n} \left\{ \frac{1}{2} \left( (f_1 - f_2)^a + (f_2 - f_3)^a + (f_1 - f_3)^a \right)^{1/a} + c(2\eta + f_1 + f_3) \right\}^{\frac{1}{n}}
\]

(4.16)

where \(a\) is the model parameter the controls the dependence on the Lode angle parameter, \(b\) controls the overall magnitude of strain to failure and is defined as the average equivalent strain to failure under uniaxial tension in each orientation, and \(c\) controls the dependence on the stress triaxiality [22]. The Lode angle parameter is incorporated in the model through the following functions [90]:

\[
f_1[\bar{\theta}] = \frac{2}{3} \cos \left[ \frac{\pi}{6} (1 - \bar{\theta}) \right],
\]

\[
f_2[\bar{\theta}] = \frac{2}{3} \cos \left[ \frac{\pi}{6} (3 + \bar{\theta}) \right],
\]

\[
f_3[\bar{\theta}] = \frac{2}{3} \cos \left[ \frac{\pi}{6} (1 + \bar{\theta}) \right].
\]

(4.17)

Here, the model parameters \(a, b,\) and \(c\) for L-PBF Ti-6Al-4V were calibrated through the same methodology as that used to calibrate the MMC parameters as described in Section 4.5.
Table 4.2. Calibrated fracture model parameters for L-PBF Ti-6Al-4V.

<table>
<thead>
<tr>
<th>Maximum Shear Stress</th>
<th>Orientation</th>
<th>C</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BD</td>
<td>42.0</td>
<td>0.113</td>
</tr>
<tr>
<td></td>
<td>⊥BD</td>
<td>28.4</td>
<td>0.113</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Johnson-Cook</th>
<th>Orientation</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BD</td>
<td>0.105</td>
<td>16.7</td>
<td>-12.0</td>
</tr>
<tr>
<td></td>
<td>⊥BD</td>
<td>0.111</td>
<td>6.94</td>
<td>-10.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hosford-Coulomb</th>
<th>Orientation</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BD</td>
<td>1.37</td>
<td>0.324</td>
<td>0</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>⊥BD</td>
<td>1.44</td>
<td>0.230</td>
<td>0</td>
<td>0.042</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Modified Mohr-Coulomb</th>
<th>Orientation</th>
<th>$c_1$ (MPa)</th>
<th>$c_2$ (MPa)</th>
<th>$c_3\hat{s}$</th>
<th>$c_3\hat{c}$</th>
<th>A (MPa)</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BD</td>
<td>0</td>
<td>651.2</td>
<td>0.927</td>
<td>1.05</td>
<td>1349</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>⊥BD</td>
<td>0</td>
<td>627.2</td>
<td>0.933</td>
<td>1.05</td>
<td>1303</td>
<td>0.042</td>
</tr>
</tbody>
</table>

### 4.5. Results and discussion

Here, the ability of each of the six failure criteria described above to capture the stress state-dependent fracture behavior of L-PBF Ti-6Al-4V is assessed. The benefits and drawbacks of each model primarily lie in their ability to capture the stress-state dependent fracture behavior with the fewest number of experiments as well as the ease of model calibration. An overarching finding of the study was that similarly to the plasticity behavior, the fracture behavior of L-PBF Ti-6Al-4V was found to be stress state-dependent and anisotropic, which could be captured, at least to some extent, in all but one of the failure criteria studied here.
4.5.1. Comparison of models in 2D space of $\bar{\varepsilon}_f$ versus $\eta$

As noted in Wierzbicki et al. [72], it is difficult to directly compare stress based, strain based, or mixed stress-strain based fracture criteria; however, they can be directly compared for the plane stress condition (Eq. 1.3) for which there is a one-to-one mapping from one space to the other. Therefore, all fracture criteria examined here are plotted in the 2D space of strain to failure as a function of stress triaxiality for the case of plane stress. Only fracture criteria that explicitly incorporate a functional dependence on the Lode angle are plotted in the 3D space of strain to failure as a function of stress triaxiality and Lode angle parameter. For each fracture model considered, it was generally found that a $\pm 15\%$ margin was needed to capture the experimentally observed variation in strain to failure at a given stress state for L-PBF Ti-6Al-4V. This margin is shown on each of the 2D fracture locus plots.

The constant equivalent strain to failure fracture criterion, which requires a single experiment, assumes that the equivalent strain to failure is the same regardless of stress state. This criterion can vary widely depending on which stress state is used to define the failure equivalent strain. For L-PBF Ti-6Al-4V, if using a uniaxial tension state of stress to define the limit on the equivalent strain to failure, as shown in Figure 4.6, the strain to failure in all other stress states is overestimated. This presents two issues: first, this method cannot be used to predict and prevent fracture in stress states other than what it was calibrated based on, as an overestimation of the ductility of a material in engineering design will result in failure; second, if an engineer instead uses a different stress state to define failure strain, providing a lower bound constant strain to fracture criterion, the component may be overdesigned against failure if locations within the component access stress states
other than the one used to define failure. Thus, inability of the constant strain to failure fracture criterion to capture the stress state-dependent failure behavior makes this approach inappropriate for the design of components against failure.

**Figure 4.6.** Constant equivalent strain fracture locus (solid black line) for L-PBF Ti-6Al-4V, based only on strain to failure under uniaxial tension (central hole tests) whose tensile axis is in the (a) vertical build direction and (b) perpendicular to the vertical build direction compared to all experimental data (symbols). Shaded area bounded with dashed lines represents ±15% margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.

The calibrated Johnson-Cook model for L-PBF Ti-6Al-4V captures the equivalent strain to failure behavior for stress triaxialities that are equivalent to uniaxial tension or greater; however, as this model is not intended to predict failure for lower triaxialities, it significantly overestimates the failure behavior under low stress triaxialities as shown in Figure 4.7. This model requires tests be performed under at least three stress states for calibration, and model calibration is performed with a simple curve fit to the data. For
tension-dominated loading situations, in which void growth mechanisms are assumed to dominate the failure process, the Johnson-Cook model may suffice in describing and predicting fracture behavior.

Figure 4.7. Johnson-Cook fracture locus (solid black line) for L-PBF Ti-6Al-4V whose tensile axis is in the (a) vertical build direction and (b) perpendicular to the vertical build direction compared to all experimental data (symbols). Shaded area bounded with dashed lines represents ±15% margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.

The two-branch empirical fit fracture criterion is able to capture the data for all seven tests for which it was calibrated, as shown in Figure 4.8. However, this is due to the fact that it is composed simply of two empirical fits of the data, and there is no physical meaning incorporated in this model. Therefore, the calibrated equations are only valid for this specific material, and cannot be used to generally describe other material behavior. Given that this fracture criterion is based on simple empirical fits, the model accuracy will increase with the number of tests performed.
Figure 4.8. Two-branch empirical fit fracture locus (solid black line) for L-PBF Ti-6Al-4V whose tensile axis is in the (a) vertical build direction and (b) perpendicular to the vertical build direction compared to all experimental data (symbols). Shaded area bounded with dashed lines represents ±15% margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.

The maximum shear stress fracture criterion captures the stress triaxiality dependent failure behavior of the material very well, while only requiring one test condition - pure shear - for calibration. As shown in Figure 4.9, this fracture model predicts the equivalent strain to failure conservatively for both build orientations, in all stress states tested, which is preferred to overestimating the ductility. As described above, this model does have physical significance in that it assumes fracture occurs at a critical value of the maximum shear stress present in an arbitrary stress state. However, while this criterion requires only one test, pure shear tests are challenging to carry out due to either limitations in lab equipment (e.g., the access to only uniaxial test frames) or challenges with achieving
pure shear loading up to failure [138,139]. Therefore, the experimental definition of failure
strain under pure shear may be assumed to be a lower bound for failure [140].

**Figure 4.9.** Maximum shear stress criterion fracture locus (solid black line) for L-PBF
Ti-6Al-4V whose tensile axis is in the (a) vertical build direction and (b) perpendicular to
the vertical build direction compared to all experimental data (symbols). Shaded area
bounded with dashed lines represents ±15% margin in the average equivalent strain to
fracture represented by the solid line. Filled data points were used for calibration of the
model.

Figures 4.10 and 4.11 show the 3D MMC and HC models plotted in the 2D stress
triaxiality versus strain to failure space for the case of plane stress. The MMC and HC
models are very similar to each other and to the empirical curve fit and maximum shear
stress criteria in this space. Comparing just the MMC and HC models, the MMC model
predicts a higher equivalent strain to failure under perfect plane stress uniaxial tension for
both the BD and ⊥BD samples, by 21% or 43% change, respectively.
Figure 4.10. Modified Mohr-Coulomb fracture locus (solid black line) for L-PBF Ti-6Al-4V whose tensile axis is in the (a) vertical build direction and (b) perpendicular to the vertical build direction compared to all experimental data (symbols). Shaded area bounded with dashed lines represents $\pm 15\%$ margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.

Figure 4.11. Hosford-Coulomb fracture locus (solid black line) for L-PBF Ti-6Al-4V whose tensile axis is in the (a) vertical build direction and (b) perpendicular to the vertical build direction compared to all experimental data (symbols). Shaded area
bounded with dashed lines represents ±15% margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.

The MMC model, HC model, empirical curve fit criterion, and maximum shear stress criterion all accurately capture the equivalent strain to failure as a function of stress triaxiality, over a wide range of stress triaxiality values, and under plane stress, for L-PBF Ti-6Al-4V. However, for the fracture criterion considered in this study, the influence of the Lode angle parameter is only fully considered using the MMC and HC models.

4.5.2. Comparison of models in 3D space of $\bar{\varepsilon}_f$ versus $\eta$ versus $\bar{\theta}$

Both the MMC and HC fracture criteria explicitly incorporate the effect of Lode angle parameter on failure behavior, which was found to be important in fully understanding the stress state-dependence of the failure behavior of L-PBF Ti-6Al-4V. The 3D fracture surfaces of the MMC and HC models, using the calibrated parameters in Table 4.2, are shown in Figures 4.12 and 4.13. When plotting these 3D fracture loci on the 2D plane of equivalent strain to failure versus Lode angle parameter, the dependence of fracture strain on Lode angle parameter of the material is made clear, as shown in Figures 4.14 and 4.15.
Figure 4.12. Calibrated three-dimensional MMC fracture locus (surface) for L-PBF Ti-6Al-4V in the (a) build direction and (b) perpendicular build direction compared to all experimental data (symbols). Dashed line on the surface represents the plane stress relationship between the two stress state parameters. Arrows, if present, indicate the distance and direction of the experimental data point to the calibrated fracture surface. Filled data points were used for calibration of the model.
Figure 4.13. Calibrated three-dimensional HC fracture locus (surface) for L-PBF Ti-6Al-4V in the (a) build direction and (b) perpendicular build direction compared to all experimental data (symbols). Dashed line on the surface represents the plane stress relationship between the two stress state parameters. Arrows, if present, indicate the distance and direction of the experimental data point to the calibrated surface. Filled data points were used for calibration of the model.
Figure 4.14. Calibrated MMC fracture locus (solid black line) in the space of Lode angle parameter versus strain to failure, highlighting the importance of Lode angle parameter, for L-PBF Ti-6Al-4V in the (a) build direction and (b) perpendicular build direction compared to all experimental data (symbols). Shaded area bounded with dashed lines represents ±15% margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.
Figure 4.15. Calibrated HC fracture locus (solid black line) in the space of Lode angle parameter versus strain to failure, highlighting the importance of the Lode angle parameter, for L-PBF Ti-6Al-4V in the (a) build direction and (b) perpendicular build direction compared to all experimental data (symbols). Shaded area bounded with dashed lines represents ±15% margin in the average equivalent strain to fracture represented by the solid line. Filled data points were used for calibration of the model.

The MMC fracture criterion for L-PBF Ti-6Al-4V, calibrated using five test geometries, results in $c_1 = 0$ for both material orientations, as shown in Table 4.2. In this case, the MMC model has, for a given Lode angle, no dependence on stress triaxiality. The parameter $\eta_\theta^c$, calibrated to be 1.05 and 1.04 for the BD and $\perp$BD material, respectively, controls the asymmetry of the fracture locus with respect the Lode angle parameter. This asymmetry allows for the behavior of the negative Lode angle parameter equibiaxial tension samples to be captured by the calibrated fracture surface.

The HC fracture criterion, calibrated using five test geometries, results in a calibrated value of 0 for the parameter $c$ in both material orientations, meaning that, for a
constant Lode angle parameter, the calibrated criterion is independent of stress triaxiality, and the fracture locus is symmetric with respect to the Lode angle parameter \cite{90}. The symmetry, with respect to Lode angle parameter, in the HC model captures the experimental equivalent strain data points for all of the positive Lode angle parameter tests well; however, it fails to capture the equibiaxial tension failure. There is an 85\% and 57\% difference, for the BD and $\perp$BD material respectively, between the average experimental equibiaxial tension equivalent strain to failure and the fracture surface predicted strain to failure at the same ($\hat{\eta}, \hat{\theta}$) coordinates. One advantage of the calibrated HC model for the L-PBF Ti-6Al-4V material is that it captures stress state-dependent failure, while also more conservatively predicting the failure strain under uniaxial tension, compared to the calibrated MMC model, in both orientations.

However, the clear advantage of the MMC model over the HC model for L-PBF Ti-6Al-4V is the MMC model’s ability to capture any asymmetry in Lode angle parameter dependence through its $c_\theta^c$ model parameter. The HC model, with $c = 0$, does not have another parameter to capture asymmetry of equivalent strain to failure with respect to the Lode angle parameter and simultaneously remain independent of the stress triaxiality, as observed in the calibrated MMC model.

The stress state-dependent ductile failure of L-PBF Ti-6Al-4V is best captured with the maximum shear stress and MMC fracture criterions, whereas each of these models has benefits and drawbacks. The primary benefit of the maximum shear stress criterion is that it captures the trend of the equivalent failure strain over a wide range of stress states while only using a single test for calibration. The key drawbacks of the maximum shear stress failure criterion are that it generally underestimates the equivalent failure strain, which
could lead to overdesign of components, and that achieving the pure shear stress state in an experimental setting can be difficult. The primary benefits of the MMC fracture criterion are that it explicitly incorporates a dependence on both $\eta$ and $\bar{\theta}$ and it is able to capture the asymmetry of the failure behavior with respect to Lode angle parameter that was observed in the L-PBF Ti-6Al-4V material. A drawback of the MMC model is the number of tests required to accurately calibrate the model over a range of stress states.

### 4.6. Summary and conclusions

For the first time, the fracture behavior of L-PBF Ti-6Al-4V was measured over a wide range of stress states in two build orientations, and a range of ductile failure models were calibrated and compared. Using a combined experimental and computational approach, the equivalent strain to failure, as a function of stress state parameters stress triaxiality and Lode angle parameter, was calculated for seven different stress states. Six different failure criteria were compared and the primary conclusions are:

- The ductile failure of L-PBF Ti-6Al-4V was found to depend significantly on stress state, where both stress triaxiality and Lode angle parameter are needed to fully define the stress state and its influence on failure strain. This highlights the importance of understanding fracture properties of additively manufactured materials under a range of stress states in order to prevent failure of structural components designed with the freedom of AM.

- The MMC model, calibrated with five tests and able to capture the asymmetry of the equivalent strain to failure with respect to the Lode angle parameter, was
found to be most appropriate for capturing and predicting the effects of both of the stress state parameters on fracture.

- The maximum shear stress fracture criterion, which was calibrated with only one test, albeit one that can be challenging to conduct experimentally, captured conservatively the trend of equivalent strain to failure with respect to the stress triaxiality.

- The equivalent strain to failure for all stress states was higher for BD samples compared to \( \perp \)BD samples, which every model evaluated was able to capture.

- A margin of \( \pm 15\% \) surrounding the two-branch empirical fit, maximum shear stress, MMC, and HC fracture criterions was appropriate for describing the experimental variation of the equivalent strain to failure in L-PBF Ti-6Al-4V.
Chapter 5 Multiaxial plasticity and fracture behavior of stainless steel 316L by laser powder bed fusion: Experiments and computational modeling

5.1. Introduction

This chapter aims to address existing gaps in data with experiments and validated models that capture the effect of stress state and orientation on the plasticity and fracture behavior of L-PBF 316L. The stress state dependent plasticity and fracture behavior of L-PBF 316L was probed through experiments under shear, shear/tension combined loading, plane strain tension, uniaxial tension, and equibiaxial tension. The flow behavior was observed to be anisotropic in tension dominated tests, while the fracture behavior was found to be anisotropic and stress state dependent. The newly acquired experimental data, coupled with computational modeling and simulations, were used to calibrate and validate anisotropic plasticity and fracture models. The resultant models are able to capture the anisotropic flow behavior in tension dominated stress states, and the anisotropic and stress state dependent fracture behavior.

---

5.2. Experimental methods

The fabrication of the material, characterization of microstructure, and the specimens for mechanical testing used in the study are briefly summarized.

5.2.1. Material fabrication and characterization

The austenitic stainless steel 316L material in this study was manufactured via L-PBF AM on a 3DSystems ProX DMP 320 machine. Recycled pre-alloyed powder was used after having been sieved with a 60 μm screen. The standard 3DSystems 316L fabrication parameters for a 60 μm layer height were used: scanning speed of 900 mm/s, laser power of 300 W, and 100 μm hatch spacing. A scan strategy with outer contours and a full-length hatch fill (i.e., not checkerboard) was used for each individual component on the build plate. A 245° rotation of the hatch direction was implemented for each successive layer during the build. All microstructure characterization and mechanical testing was performed on the material in the as-built condition.

Representative samples were used for characterization of microstructure and sample density. One of the samples was sectioned into four sub-samples - one for Archimedes density analysis and three for microstructural analysis of three orthogonal planes of the material. The densities of three additional samples were analyzed with X-ray computed tomography (XCT, General Electric v|tome|x L300 nano/microCTsystem), using a voxel size of 14 μm. Following post processing in Avizo 9.3.0 software (Thermo Fisher Scientific), the samples were all found to be 99.99% dense. Archimedes density analysis, following the procedure in [141], found that samples in the study were 99.8 ± 0.1 % dense. Standard metallographic procedures were used to grind and polish each microstructural sub-sample with a final polishing step using 0.05 μm Al₂O₃. After sample
preparation, a scanning electron microscope (SEM, FEI Apreo) with an electron backscatter diffraction (EBSD, Oxford Symmetry) detector was used for grain morphology and sub-grain cell imaging, as shown in Figure 5.1c and 5.1d, respectively. For further analysis, samples were electrolytically etched in a 10% Oxalic acid and 90% water solution at 4 V and then imaged with a digital microscope (Keyence VHX-2000), as shown in Figure 5.1a-b.

Figure 5.1. (a, b) Optical micrographs showing the melt pool and grain structure in the three planes of the build. (c) EBSD images of all three planes of the build showing the epitaxial grain growth and chevron morphology of the grains in the XZ and YZ planes. For all three planes, the color shown corresponds to the hkl direction parallel to the
vertical build direction (Z). (d) SEM image of the subgrain cellular structure in the XZ plane.

The average equivalent diameter, referred to as the grain size after this point, in the XZ and YZ planes were found to be 33.6 μm and 30.0 μm, respectively, using the EBSD results. The average grain size in the XY plane was 28.2 μm.

5.2.2. Plasticity tests

The multiaxial plasticity behavior of the 316L material was assessed by performing mechanical tests on specimens with two different geometries, as shown in Figure 5.2. The two specimen geometries allowed for the evaluation of elastoplastic properties under five different stress states. Both specimen geometries were tested in two orientations, referred to as BD and ⊥BD and schematically shown in Figure 5.2. Three uniaxial tension specimens in both directions were profiled and sliced to thickness in accordance to ASTM E8 [129], from an 10 x 70 x 80 mm (thickness x width x height) wall using wire electrical discharge machining (wEDM). BD multiaxial plasticity specimens were machined from the same wall as the uniaxial tension specimens, while ⊥BD multiaxial plasticity specimens were machined from individual 3.5 x 27 x 70 mm (thickness x width x height) walls using wEDM for the outer profile and CNC milling to thickness in the gauge section and grip regions.
Figure 5.2. (a) Uniaxial tension and (b) multiaxial plasticity specimen geometries used to evaluate the elastoplastic behavior of the L-PBF 316L material in two orientations – BD (tension along the z-axis) and ⊥BD (tension along the y-axis). Units in mm. (c) Schematic of the sample orientations and nomenclature with respect to the baseplate, not to scale.

Before testing, the gauge sections for both sample types were spray painted with a white basecoat and black random speckle pattern for digital image correlation, a non-contact surface strain field measurement technique (Vic2D software, Correlated Solutions). A digital camera (Point Grey GRAS-50S5M-C) was used to take an image of the gauge sections during loading every one second of testing until sample failure. In multiaxial tests two identical cameras were used; one that had the entire gauge region in
the field of view and the other focused at the center of the gauge region on the reverse side of the sample.

The uniaxial tension samples were pulled under displacement control at an applied strain rate on the order of $10^{-4}$ s$^{-1}$ using an electromechanical load frame (MTS Criterion Model 43) with a 10 kN load cell. Vertical displacement was extracted from the DIC analysis with an 18 mm long virtual extensometer centered in the gauge section of the uniaxial tension samples, which was used to compute strain in the vertical direction.

The geometry of the multiaxial plasticity specimens, with a thin, high aspect ratio gauge section, allowed for plane stress through the thickness and plane strain along the long axis of the gauge section [130]. Samples were loaded in a custom dual-actuator hydraulic test machine (MTS System Corp.), schematically shown in ref. [135]. The vertical and horizontal actuators can independently apply any combination of normal ($F_V$) and shear ($F_H$) forces to the sample. An angle $\beta$, eq. (4.1), can be used to characterize the combined loading ratio.

In the current study four different stress states were evaluated with multiaxial plasticity specimens: shear ($\beta=0^\circ$), shear dominated combined loading ($\beta=30^\circ$), tension dominated combined loading ($\beta=70^\circ$), and plane strain tension (PST, $\beta=90^\circ$). The shear and plane strain tension tests were run in displacement control with a horizontal loading rate of 0.4 mm/min and a vertical loading rate of 0.1 mm/min, respectively. Combined loading tests were run in force control mode; for $\beta=30^\circ$, $F_H=1.5$ kN/min and $F_V=0.866$ kN/min, and for $\beta=70^\circ$, $F_H=0.546$ kN/min and $F_V=1.5$ kN/min. A vertical 3 mm long virtual extensometer at the center of the gauge section of the plasticity sample was used to measure
vertical and horizontal displacements, which were converted to vertical and in-plane shear strains [130].

This combination of experimental tests allowed for: (1) the orientation and stress state dependent elastoplastic behavior of the material to be measured, (2) the calibration and validation of a plasticity model that captured and predicted the observed behavior, and (3) the use of the plasticity model in a hybrid computational/experimental approach to quantify the multiaxial fracture behavior of L-PBF 316L, as discussed in upcoming sections.

5.2.3. Fracture tests

The onset of fracture of L-PBF 316L for a wide range of stress states was studied through experiments on samples extracted in two orientations. The three different geometries used to study fracture were selected because under applied macroscopic deformation, the strain in these geometries localizes at predictable locations, which is where failure is assumed to initiate. These locations for each sample geometry are shown in Figure 5.3: the surface of the hole in central hole tension samples, the inside center of the butterfly samples, and the center top surface of the punch samples. Note that under pure shear, failure may initiate at the edges of the butterfly sample before the sample fails at the center. However, as the central point in the butterfly sample during this loading remains under a constant state of shear throughout loading, using the strain at the center of the specimen up to the moment of specimen failure provides a lower bound for damage to failure under shear. The applied displacement to onset of failure was recorded in each experiment.
Figure 5.3. (a) Central hole tension and (b) butterfly specimens used to evaluate the fracture behavior of the L-PBF 316L material in two orientations – BD (tension along the z-axis) and \( \perp \)BD (tension along the y-axis). (c) Equibiaxial tension specimens that were fabricated in the x-y plane and loaded out of plane (along the build axis). Red circles indicate the locations at which fracture is assumed to initiate, where the location is at the through-thickness center in (a) and (b) and on the top surface in (c). Units in mm.

For sample fabrication, central hole (CH) tension specimens in both directions were profiled and sliced to thickness with wEDM from a 10 x 70 x 80 mm wall (thickness x width x height), with the central hole drilled and then cut to size with wEDM. Multiaxial fracture specimens, referred to as butterfly specimens [137], were machined with the same process as that for the plasticity specimens from individual 3.5 x 34 x 80 mm (thickness x
width x height) walls in both orientations. An extruded punch sample geometry was
directly manufactured and sliced to thickness with wEDM for the punch tests.

In central hole tension, a uniaxial tension stress state is maintained throughout
nearly the entire test at the edge of the central hole along the centerline of the specimen
normal the loading direction. Three central hole tension samples in each build orientation
were loaded on the same load frame as the uniaxial tension samples in Section 5.2.2 with
a loading rate of 0.0162 mm/s. A 10 mm virtual extensometer was used at the center of the
gauge section to measure displacement to onset of failure in the tests, defined by the first
visual identification of a crack.

The butterfly geometry featured a reduced thickness gauge region similar to the
plasticity specimens, as well as a non-uniform gauge height such that fracture is assumed
to start at the center of the shortest region of the gauge section at half through-thickness
[137]. Using DIC, with the camera focused on the center of the gauge section, the
displacement at the onset of failure was recorded for two samples of each stress state and
build orientation. The butterfly samples were tested using the same custom load frame as
the multiaxial plasticity specimens and using the same loading angles, β.

The fracture behavior of L-PBF 316L was also observed under equibiaxial tension
with a punch experiment. The punch test is widely used [90,136,142,143], and the
experimental set-up is identical to that presented in ref. [144] in which a disc specimen is
fixed in a die, and impacted with a hemispherical punch during quasistatic loading. For the
punch experiments, three-dimensional DIC was used to measure the surface deformation
field during loading up to the onset of failure (Vic3D, Correlated Solutions). Displacement
up to failure was measured using the crosshead displacement with an assumed compliance of 19 kN/mm.

The displacements to onset of fracture from the experiments presented in this section were used in the calibration and validation of a ductile fracture model for L-PBF 316L. The wide range of tests performed provide important information for understandings the orientation and stress state dependent ductile fracture behavior of L-PBF 316L.

5.3. Plasticity behavior: experimental results and model

The experimental findings of tests probing plasticity behavior, and their use in the calibration and validation of a plasticity model are presented in this section. Differences between experimental results and the finite element analysis (FEA) simulations of these using the plasticity model are discussed with respect to the microstructure and defects present in the L-PBF 316L material.

5.3.1. Experimental results

5.3.1.1. Uniaxial tension results

The engineering stress-strain behavior for the six uniaxial tension tests, three in each orientation, agreed well with the direction-dependent trends observed in the literature, as shown in Figure 5.4. The yield strength (⊥BD: 558 ± 3 MPa and BD: 497 ± 8 MPa) and UTS (⊥BD: 670 ± 1 MPa and BD: 600 ± 1 MPa) were greater in the ⊥BD orientations samples than BD samples. The ductility of the material was also anisotropic, BD samples had higher engineering strain to failure than ⊥BD samples, 62 ± 3% and 53 ± 1%, respectively. The anisotropic strength in the L-PBF 316L is hypothesized to have originated from the microstructural features, most notably the columnar grain morphology
that creates more barriers for dislocation motion in the \( \perp \text{BD} \) loaded samples, that were observed in our material and the literature [37,38].

**Figure 5.4.** (a) Engineering stress-strain data for uniaxial tension in both orientations. (b) Flow versus plastic strain for a representative \( \perp \text{BD} \) uniaxial tension test along with a Swift law fit prior to necking, and a linear extrapolation after necking.

### 5.3.1.2. Multiaxial plasticity results

The multiaxial plasticity tests revealed mixed isotropic and anisotropic behavior dependent on stress state, as shown in Figure 5.5. Tension dominated loading conditions, plane strain tension and \( \beta=70^\circ \), had similar behavior as seen under uniaxial tension in which larger vertical force was applied to the \( \perp \text{BD} \) samples compared to BD samples to achieve the same displacement. The force versus displacement behavior in shear dominated tests, shear and \( \beta=30^\circ \), was isotropic. The anisotropic, stress state dependent behavior of the material was similar to the behavior observed in L-PBF Ti-6Al-4V [135], except that the L-PBF 316L was significantly more ductile. All stress states except plane strain tension
and $\beta=70^\circ$ in the $\perp$BD reached an equivalent plastic strain of $\geq 20\%$. Based on similar anisotropic experimental behavior observed in the two studies a plasticity model using the same framework as that in ref. [135] was initially implemented with calibration of new model parameters for L-PBF 316L, as discussed in the next section.

**Figure 5.5.** Comparison of experimental (symbols) and simulated (lines) force versus displacement curves for (a) pure shear, (b) plane strain tension (c) combined loading with $\beta=30^\circ$, and (d) combined loading with $\beta=60^\circ$ in both orientations. The mechanical response in the tension dominated tests show notable anisotropy.
5.3.2. Plasticity modeling

5.3.2.1. Hardening model

To describe the hardening behavior of the material, a Swift law with linear extrapolation after necking was used. The hardening behavior is represented in the form of a piecewise function as:

\[
\text{∆σ}_y = \begin{cases} 
A(nA_0 + \bar{ε}^p)^{n-1} \Delta \bar{ε}^p & \bar{ε}^p \leq 0.032 \\
K\Delta \bar{ε}^p & \bar{ε}^p > 0.032
\end{cases}
\] (5.1)

where \(σ_y\) is the flow stress, \(\bar{ε}^p\) is the equivalent plastic strain, and \(A, ε_0, n,\) and \(K\) are material constants. The constants were calibrated for a representative \(⊥\)BD uniaxial tension test, and are given in Table 5.1. The \(⊥\)BD was chosen to be the reference orientation for the model as it would be minimally influenced by any potential LoF defects. A comparison between experimental data and the calibrated model is given in Figure 5.4.

Table 5.1. Calibrated plasticity model parameters: Swift law hardening with linear extrapolation and Hill48 yield function.

<table>
<thead>
<tr>
<th>Swift law parameters</th>
<th>A (MPa)</th>
<th>(ε_0)</th>
<th>(n)</th>
<th>(K) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1289</td>
<td>0.206</td>
<td>0.516</td>
<td>1000</td>
</tr>
<tr>
<td>Hill48 parameters</td>
<td>(F)</td>
<td>(G)</td>
<td>(H)</td>
<td>(N)</td>
</tr>
<tr>
<td></td>
<td>0.63</td>
<td>0.42</td>
<td>0.58</td>
<td>1.58</td>
</tr>
</tbody>
</table>
5.3.2.2. Yield function and flow rule

The Hill 1948 anisotropic yield function (Hill48) [132] was implemented to capture the multiaxial plasticity behavior of L-PBF 316L. The yield function is given as:

\[ f = \sigma_{\text{Hill48}} - \sigma_y = 0 \]  

(5.2)

where \( \sigma_y \) is the current yield stress, and the equivalent Hill48 stress, \( \sigma_{\text{Hill48}} \), under the plane stress condition is defined as:

\[ \sigma_{\text{Hill48}} = \sqrt{F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2} \]  

(5.3)

where \( F, G, H, L, M, \) and \( N \) are constants that describe the material’s anisotropy, \( \sigma_{11} \) and \( \sigma_{22} \) are the normal stresses along the \( \perp \)BD and BD directions, respectively, and \( \sigma_{23}, \sigma_{31}, \) and \( \sigma_{12} \) are the shear stress components. Assuming an associated flow rule, the plastic strain increment can be calculated using eq. (3.5). The equivalent plastic strain, \( \varepsilon_{\text{Hill48}}^p \), is defined as the accumulation of \( \dot{\gamma} \), i.e., \( \varepsilon_{\text{Hill48}}^p = \int_0^t \dot{\gamma} \, dt \), and its relationship with the plastic strain components is given through the following incremental form:

\[ d\varepsilon_{\text{Hill48}}^p = \sqrt{\frac{F(d\varepsilon_{11}^p)^2 + G(d\varepsilon_{22}^p)^2 + H(d\varepsilon_{33}^p)^2}{FG+FH+GH} + \frac{2(d\varepsilon_{23}^p)^2}{L} + \frac{2(d\varepsilon_{31}^p)^2}{M} + \frac{2(d\varepsilon_{12}^p)^2}{N}} \]  

(5.4)

5.3.2.3. Model calibration

The model in eq. (5.3) was calibrated with experimental data from uniaxial tension, shear, and \( \beta=70^\circ \) plasticity tests. Additionally, \( G+H \) was assumed to be equal to 1 because the strain hardening parameters were calibrated along the \( \sigma_{11} \) or \( \perp \)BD orientation. Using
the uniaxial tension initial yield data along the two orientations F+H was calculated to be 1.21, and L and M were both calculated to be 1.6. To calibrate N, $\sigma_{\text{Hill48}}$ was computed for the initial yield under uniaxial tension in the $\perp$BD direction and set equal to $\sigma_{\text{Hill48}} = \sqrt{2N(\sigma_{12}^\gamma)^2}$, where $\sigma_{12}^\gamma$ was the average yield stress in shear tests, resulting in $N=1.58$.

With all other model parameters calibrated except H, eq. (5.3) was used with the average $\beta=70^\circ$ initial yield stress components ($\sigma_{11}$, $\sigma_{22}$, and $\sigma_{12}$) from both orientations, which resulted in a calibrated value of $H=0.58$.

### 5.3.2.4. Model validation

The Hill48 yield surface and comparison with experiments for each stress state in both orientations is shown in Figure 5.6. The initial yield surface (0.5% equivalent plastic strain) agrees well with all the experimental data in both orientations, including those tests not used in calibration of the model parameters, $\beta=30^\circ$ and plane strain tension. Subsequent surfaces that represent the evolution of the Hill48 equivalent plastic strain are also shown in Figure 5.6. The model does well to capture the entire set of experimental data in for the $\perp$BD tests; however, the model does not capture the BD plane strain tension experimental data as well after initial yield. The model slightly overpredicts the experimental $\sigma_{\text{Hill48}}$ (average difference of 7%) with increased plastic strain for BD plane strain tension, as shown by the mismatch between the 15% evolved Hill48 equivalent plastic strain surface and the last experimental data point in Figure 5.6. This result can be explained when considering the effect of voids in the high stress triaxiality regime (for plane strain tension, $\eta=0.58$) where the voids are being opened in two directions – along the vertical applied tension direction and the horizontal direction due to the stress resulting from zero strain.
along the gauge width – having a larger reduction of strain to failure compared to other tests with a shear stress component and reduced $\sigma_{22}$.

**Figure 5.6.** Anisotropic Hill48 2D yield surface (for plane stress) at initial yield (0.5\% Hill48 equivalent plastic strain strain) and subsequent increments of strain up to 15\% for L-PBF 316L. The yield surfaces are shown as lines, while experimental data are shown as symbols.

The model was implemented into the commercial finite element analysis software Abaqus using the implicit solver [133]. To check the ability of the model to capture and predict the experimentally measured data, half-thickness multiaxial plasticity specimens were modeled and discretized into 14,336 C3D8R type elements, with a symmetry boundary condition applied along the sample thickness plane. The top and bottom grips
were defined as rigid bodies, and each grip was tied to a reference node. All degrees of freedom of the bottom reference node were constrained. For the shear and plane strain tension tests, a displacement was applied to the top grip reference node in the horizontal and vertical directions, respectively. The combined loading tests were simulated using force vectors applied to the top node corresponding to the representative $\beta$ angle. Force data were extracted from the top reference node in simulations. Horizontal and vertical displacement data were extracted from two nodes, one 3 mm above the other, in the center of the gauge section on the modeled sample surface, identical to the experimental DIC virtual extensometers. Figure 5.5 compares the force vs. displacement response in each of the simulations with their experimental counterparts. The calibrated and validated plasticity model captured the orientation and stress state dependent plasticity tests in the current study, even for those tests that were not used in the calibration of the model. The largest discrepancy was observed in the $\perp$BD shear test, where the maximum force was 13% larger in the simulation than in the experiments.

5.4. Fracture behavior: experimental results and model

The quantification of ductile strain to failure requires a well calibrated plasticity model, implemented in an FEA framework, for simulating stress state and plastic strain evolution to failure in fracture experiments. The procedure for quantifying the stress state dependent strain to failure and calibrating a suitable ductile fracture model is discussed in this section.
5.4.1. Experimental results

5.4.1.1. Central hole tension results

The force vs. displacement results for the four central hole tension tests, two in each orientation, are shown in Figure 5.7. The maximum force in the \( \perp \text{BD} \) sample was 11\% greater than measured in the BD tests. The displacement to failure was similar in three of the four tests, while one of the BD tests had higher strain to failure. These tests showed the same anisotropy as observed in the tension dominated plasticity tests, namely that the \( \perp \text{BD} \) samples required higher forces for the same deformation.

5.4.1.2. Butterfly results

A total of sixteen butterfly tests were completed, two replications for each combination of orientation and stress state. An anisotropic force vs. displacement response was observed in the tension dominated butterfly tests, while the shear dominated tests were isotropic, as shown in Figure 5.7. The differences in the maximum force at fracture for the vertical force component in the plane strain tension and \( \beta=70^\circ \) tests were 15\% and 14\%, respectively.

5.4.1.3. Punch results

The experimentally measured force vs. displacement curves for five tests are shown in Figure 5.7. The largest percent difference between any two tests for maximum force and displacement to failure was 7\% and 2\%, respectively. The principal stresses in the punch tests were out of plane relative to the BD and \( \perp \text{BD} \) orientations of all other samples; therefore, these tests were used in calibration of the fracture surfaces for both orientations.
Figure 5.7. Comparison of fracture geometry experimental and simulated force versus displacement curves for (a) pure shear, (b) plane strain tension (c) combined loading with $\beta=30^\circ$, (d) combined loading with $\beta=60^\circ$, (e) equibiaxial tension, and (f) central hole tension in two orientations.
5.4.2. Fracture modeling

5.4.2.1. Finite element modeling

To probe the evolution of stress triaxiality, Lode angle parameter, stress and plastic strain components, and equivalent plastic strain to failure in each fracture test in this study, a simulation of each test, using Abaqus/implicit, was performed using the calibrated plasticity model presented in Section 5.3. The evolution of stress state parameters and equivalent plastic strain were extracted from the locations denoted in Figure 5.3, which were assumed to be the locations of failure initiation in each geometry. Mesh size dependence studies were performed for all three geometries such that the mesh size was decreased by half until the computed strain to failure changed by less than 3% between subsequent simulations.

For the central hole tension tests, a 1/8th model geometry, with 136,448 C3D8R elements, was used. Symmetry boundary conditions were applied on each of the three cut planes, and a vertical displacement boundary was applied to a reference node that controlled the vertical displacement of all nodes. A half-thickness butterfly geometry was modeled with 738,241 C3D8R elements. Similar to the simulations of the plasticity specimens in Section 5.3.2.4, the grips were modeled as rigid, and the same displacement or force boundary conditions were used up to the experimentally observed displacement to failure. Punch tests were simulated with a full thickness, ¼ disc specimen geometry using 627,900 C3D8R and 3,588 C3D6R type elements for the disc specimen with all degrees of

---

6 The work in this section includes contributions from S. Qin
freedom constrained along the circumference of the disc and symmetry boundary conditions on the cut planes. The hemispherical punch and die were modeled as a rigid body analytical surfaces with a coefficient of friction of zero between the surfaces and the disc specimen. A vertical displacement boundary condition, using the same displacement needed for experimental failure, was applied to the punch, while the top die was fixed in all degrees of freedom.

Experimentally measured and computationally simulated force vs. displacement behavior were compared for each test. In the central hole and butterfly simulations, force was extracted from the top reference node and the displacements were extracted from nodes corresponding to the ends of the experimentally used virtual extensometers. The force and displacement in the punch simulations were extracted from the node at the apex of the rigid hemispherical punch, where the punch first contacted the disc specimen.

In the shear dominated tests (shear and $\beta=30^\circ$), the strain hardening behavior in the simulations exceeds that observed experimentally, resulting in a large divergence from experimental results at high strain. At the same time, there was very good agreement between simulations and experiments in the tension dominated stress states (plane strain tension and $\beta=70^\circ$). To better capture the behavior of the shear dominated butterfly tests, a damage initiation model was implemented. Damage models have been used to describe the influence of cracks and texture in L-PBF 316L in the literature [145], although ref. [145] used elements of crystal plasticity and only studied how the microscale features imparted damage in $\perp BD$ uniaxial tension tests. Here, the built-in shear damage initiation model in Abaqus was used, in order to capture in shear dominated tests without impacting the tension dominated simulations.
The shear damage initiation criterion in Abaqus is a phenomenological model based on damage initiating from shear band localization [133]. The equivalent plastic strain at the onset of damage in the model is dependent on the equivalent plastic strain rate ($\dot{\varepsilon}_p$) and shear stress ratio $\chi_s$:

$$
\chi_s = \frac{(\bar{\sigma}_V + k_s \sigma_m)}{\tau_{max}}
$$

(5.5)

where $k_s$ is a material parameter and $\tau_{max}$ is the maximum shear stress. Calibrated values of $k_s = 0.3$ and $\chi_s = 1.73$ at an equivalent plastic strain of 10% for damage initiation were determined through a parametric study. The criterion for damage initiation is met when:

$$
w_s = \int \frac{d\varepsilon_p}{\varepsilon_p(\chi_s, \dot{\varepsilon}_p)} = 1
$$

(5.6)

where the state variable $w_s$ increases monotonically with plastic deformation [133]. Once the damage initiation criterion was met, a linear damage evolution plus linear softening was adopted to implement a reduction in flow stress (compared to damage-free material) with accumulated plastic strain. The standard linear softening model in Abaqus was calibrated to capture the observed force vs. displacement experimental behavior under shear.

For simulations involving damage, Abaqus/explicit was used. For all models, a mass scaling factor was used to reduce simulation time while ensuring that the kinetic energy of the model was no greater than 5% of the total internal energy. The force vs. displacement results for each of the fracture test simulations with damage is shown in Figure 5.7. With the implementation of damage, the $\beta=30^\circ$ simulation agreed very well with the experiments and the pure shear simulation agreed better, but still has a force...
difference at failure of 9.8% and 19% in the BD and \( \perp \)BD orientations, respectively. Using the standard model with no consideration of damage the force difference at failure was 18% and 26% for the BD and \( \perp \)BD orientations, respectively. The incorporation of the damage did not influence the tension dominated fracture simulations and was also reincorporated to the plasticity type simulations with no effect, as shown in Figure 5.5.

In annealed stainless steel 316L, thermal softening due to the formation of adiabatic shear bands has been observed experimentally under shear loading at high strain rates [146,147]. Xue and Gray showed that a high defect density resulting in subgrain structures was a prerequisite for shear localization, and once deformation localized in shear bands, the local temperature in those shear bands was calculated to increase by 470 °C. Together the formation of the shear band and temperature increase resulted in softening. Softening from shear bands formed under quasi-static loading for iron has also been reported, but only for grain sizes below 300 nm [148]. For the present shear fracture tests of L-PBF 316L, a maximum temperature increase of only 9.6 °C was measured using an infrared camera (Optris PI300 and Optris PIX Connect software) during testing, thus, adiabatic shear banding was ruled out in these tests. In as-built L-PBF 316L the dislocation density has been found to be orders of magnitude larger than that observed in annealed rolled material [44]. Additionally, the sub granular cells, measuring 0.5 to 1 \( \mu \)m in smallest dimension [41,43], are partially responsible for increased yield strength in this material. Therefore, it is hypothesized that the initial large defect density fulfills the prerequisite of high defect density, and the sub granular cells provide significant boundaries for dislocation motion, similar to grain refinement, such that even at quasi-static strain rates the shear band localization occurs in L-PBF 316L.
A comparison of the experimental force versus displacement behavior of rolled austenitic stainless steel 316 sheet and the L-PBF material under pure shear loading using the butterfly geometry is shown in Figure 5.8. Adjusting the standard simulation (no damage) plasticity model to account for the initial offset in flow stress in the rolled sheet versus the L-PBF material shows that unlike the L-PBF material, the rolled material does not exhibit shear softening. The microstructural features therefore justify the application of the phenomenological shear damage initiation criterion in the computational material model.

![Graph showing force vs. displacement comparison](image)

**Figure 5.8.** Comparison of the shear fracture geometry force vs. displacement experimental behavior of rolled austenitic stainless steel 316 sheet and L-PBF 316L material. The force of the calibrated Abaqus/Standard simulation without damage was increased by 15% to match the rolled experimental behavior and the comparison shows that the rolled material does not undergo shear softening.
5.4.2.2. Calibration of fracture models

The evolution of the equivalent plastic strain up to failure as a function of the two stress state parameters is shown in Figure 5.9 and a summary of the average value of each stress state parameter up to fracture is given in Table 5.2. The fracture behavior of the material exhibited an orientation dependence, and the strains to fracture along BD were larger than their counterparts along PBD for most of the loading types, with the only exception being $\beta=70^\circ$ and PST. Therefore, an anisotropic fracture model was needed to describe the fracture behavior. In the current study, the anisotropic Hosford-Coulomb (HC) model [149] and the anisotropic modified Mohr-Coulomb (MMC) fracture model [150] were adopted. In what follows, the definition of both models will be recalled briefly, while references [149,150] provide further details.
Figure 5.9. Evolution of the (a, b) stress triaxiality and (c, d) Lode angle parameter (solid lines) up to failure (symbols) for each sample geometry in both the BD (a, c) and ⊥BD (b, d) directions with the average value of each stress state parameter used in fracture model calibration represented by a vertical dashed line.

Table 5.2. Average values of the strain to failure, stress triaxiality, and Lode angle parameter for each test performed. These values were used for MMC model calibration and validation.

<table>
<thead>
<tr>
<th>Sample Type</th>
<th>Orientation</th>
<th># of tests</th>
<th>Avg. $\bar{\varepsilon}_f$</th>
<th>Avg. $\eta$</th>
<th>Avg. $\bar{\theta}$</th>
</tr>
</thead>
</table>

118
For both models, a damage indicator, \( D \), is calculated as:

\[
D = \int_{0}^{\varepsilon_{frac}} \frac{1}{\bar{\varepsilon}_{frac}^{pr}} \, d\bar{\varepsilon}_{frac}^{pr}
\]

(5.7)

where \( \varepsilon_{frac}^{pr} \) is a model-dependent weighting function, and \( \varepsilon_{frac}^{p} \) is a model-dependent equivalent plastic strain. \( D = 0 \) for the virgin material, and it gradually increases with plastic deformation. The material is regarded to be failed when \( D = 1 \).

For the anisotropic HC model, \( \varepsilon_{frac}^{p} \) is the equivalent plastic strain defined in the plasticity model, which is the Hill48 equivalent plastic strain in the current case, eq. (5.4), and the weighting function \( \varepsilon_{frac}^{pr} \) is defined as:

\[
\varepsilon_{frac}^{pr} (M\sigma / \bar{\sigma}) = b \left( \frac{1 + c}{\sigma_{HC} (M\sigma / \bar{\sigma})} \right)^{\frac{1}{q}}
\]

(5.8)
where $b$, $c$ and $d$ are model parameters, $\bar{\sigma}$ is the von Mises equivalent stress, and $\sigma$ is the Cauchy stress tensor. In the current study, $\sigma$ is expressed in Voigt notation as $[\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31}]$. $M$ is a linear transformation matrix defined as:

$$
M = \begin{bmatrix}
1 & M_{12} & 0 & 0 & 0 & 0 \\
0 & M_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & M_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

(5.9)

where $M_{12}$, $M_{22}$, and $M_{44}$ are transformation coefficients that must be calibrated, and $g_{HC}$ is a function of the normalized stress tensor $\sigma/\bar{\sigma}$, which is in turn a function of $\eta$ and $\bar{\theta}$:

$$
g_{HC} \left( \frac{\sigma}{\bar{\sigma}} \right) = \left( \frac{1}{2} \left[ (f_1 - f_2)^a + (f_2 - f_3)^a + (f_1 - f_3)^a \right] \right)^{\frac{1}{a}} + c(2\eta + f_1 + f_3)
$$

(5.10)

where $a$ is a model parameter, and $f_1$, $f_2$, and $f_3$ are defined as:

$$
f_1 = \frac{2}{3} \cos \left[ \frac{\pi}{6} (1 - \bar{\theta}) \right]
$$

$$
f_2 = \frac{2}{3} \cos \left[ \frac{\pi}{6} (3 + \bar{\theta}) \right]
$$

(5.11)

$$
f_3 = -\frac{2}{3} \cos \left[ \frac{\pi}{6} (1 + \bar{\theta}) \right]
$$

By applying the transformation matrix $M$ to the stress tensor $\sigma$, $g_{HC}$ is changed from an isotropic function of $\sigma$ to an anisotropic function of $\sigma$, thus making the fracture model anisotropic.

For the anisotropic MMC model, the weighting function $\bar{\varepsilon}_f^{pr}$ is defined as:
\[ \varepsilon_f^{pr} = \left( \frac{A}{c_2} \left[ c_0^s + \frac{\sqrt{3}}{2 - \sqrt{3}} (1 - c_0^s) \left( \sec \left( \frac{\vartheta \pi}{6} \right) - 1 \right) \right] \right) \left[ \frac{1 + c_1^2}{3} \cos \left( \frac{\vartheta \pi}{6} \right) \right]^{-\frac{1}{n}} + c_1 \left( \eta + \frac{1}{3} \sin \left( \frac{\vartheta \pi}{6} \right) \right) \right]^{\frac{1}{n}} \]

(5.12)

where \( A \) and \( n \) are hardening parameters (the same variables presented in eq. (5.1)), and \( c_1, c_2, \) and \( c_0^s \) are model parameters [87,144].

The anisotropic MMC model uses an equivalent plastic strain definition that is independent of the plasticity model. Luo et al. adopted two equivalent plastic strain definitions for the MMC model: one that was equivalent to that in the plasticity model and one that was independent of the plasticity model [150]. They showed that the model accuracy was higher when adopting an equivalent plastic strain definition that was independent of the plasticity model. Following the framework in ref. [150], the equivalent plastic strain for the anisotropic MMC model is defined as:

\[ \varepsilon_f^{p} = \sqrt{\frac{2}{3} \left[ \mathbf{\beta} \cdot (d\varepsilon_p) \right] \cdot \left[ \mathbf{\beta} \cdot (d\varepsilon_p) \right]} \]

(5.13)

where \( \varepsilon_p \) is the plastic strain tensor expressed in Voigt notation as \( [\varepsilon_{11}^p, \varepsilon_{22}^p, \varepsilon_{33}^p, \sqrt{2} \varepsilon_{12}^p, \sqrt{2} \varepsilon_{23}^p, \sqrt{2} \varepsilon_{31}^p] \), and \( \mathbf{\beta} \) is a linear transformation matrix defined as:

\[
\mathbf{\beta} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & \beta_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & \beta_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(5.14)

where \( \beta_{22}, \beta_{33}, \) and \( \beta_{44} \) are transformation coefficients that must be calibrated.
To summarize, the parameters to be calibrated for the anisotropic HC model are: $a$, $b$, $c$, $d$, $M_{12}$, $M_{22}$, and $M_{44}$, and those for the anisotropic MMC model are $c_1$, $c_2$, $c_0^S$, $\beta_{22}$, $\beta_{33}$, and $\beta_{44}$. In the current study, a Matlab code was developed to determine the parameter values. The code optimized the parameter values using the interior-point algorithm so that the resultant $D$ values in the experiments used for calibration were as close to 1 as possible at the moment of fracture. For the anisotropic HC and MMC models, the tests used for calibration were: pure shear, $\beta=70^\circ$, plane strain tension, and central hole tension, leaving punch and $\beta=30^\circ$ for model validation. The calibrated parameter values are summarized in Table 5.3.

Further, the isotropic variations of the HC and MMC models were calibrated in each orientation independently [77,90]. The isotropic HC model is obtained by setting the $M$ matrix in the anisotropic HC model to the identity matrix. The isotropic MMC model is obtained by replacing the equivalent plastic strain definition in the anisotropic MMC model, eq. (5.13), with that in the plasticity model (i.e., $\bar{\varepsilon}_{frac}^p = \varepsilon_{\text{Hill}48}^p$). Using the same calibration tests and procedure, but only for the build direction tests, a direct comparison was made with the anisotropic models. The calibrated parameters for each model are given in Table 5.3.
Table 5.3. Calibrated anisotropic and isotropic HC and MMC model parameters.

<table>
<thead>
<tr>
<th>Hosford-Coulomb</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$M_{12}$</th>
<th>$M_{22}$</th>
<th>$M_{44}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anisotropic</td>
<td>0.970</td>
<td>1.184</td>
<td>0.127</td>
<td>0.091</td>
<td>-0.032</td>
<td>0.961</td>
<td>1.347</td>
</tr>
<tr>
<td>Isotropic - BD</td>
<td>0.694</td>
<td>1.300</td>
<td>0.244</td>
<td>0.199</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Modified Mohr-Coulomb</th>
<th>$c_1$</th>
<th>$c_2$ (MPa)</th>
<th>$c_\theta$</th>
<th>$\beta_{22}$</th>
<th>$\beta_{33}$</th>
<th>$\beta_{44}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anisotropic</td>
<td>0.860</td>
<td>1372</td>
<td>2.102</td>
<td>0.618</td>
<td>-0.461</td>
<td>-0.084</td>
</tr>
<tr>
<td>Isotropic - BD</td>
<td>1.150</td>
<td>1923</td>
<td>1.871</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.4.2.3. Results and discussion of fracture models

Using the calibrated parameter values, the accumulated damage at the moment of fracture (D) for each of the tests are shown in Figure 5.10. The isotropic versions of the HC and MMC fracture models, calibrated with the build direction tests, resulted in a mean absolute percentage error (MAPE) for damage prediction across all tests in both orientations of 11.3% and 13.7% for the HC and MMC models, respectively. The MAPE values for the HC and MMC models were reduced when adopting the anisotropic variations of each model, although the anisotropic HC model resulted in the lowest amount of error of all the models calibrated. This finding indicates that the anisotropic HC model best captures the multiaxial failure behavior of L-PBF 316L.
Figure 5.10. (a,b) Damage accumulation at fracture and (c,d) relative displacement to failure comparisons of the four different fracture models: isotropic HC, anisotropic HC, isotropic MMC, and anisotropic MMC. In the legends, the value in parenthesis indicates the mean absolute percentage error between the specified model predictions for each stress state studied in both orientations and a value of 1. The boxed x-axes labels indicate tests that were not used in calibration of the models.
The relative displacement to failure values, defined as the predicted displacement to failure divided by the measured displacement to failure, for each model are also shown in Figure 5.10. Note that in almost all cases, the relative displacement to failure is below 1, indicating that the calibrated models are largely conservative when predicting failure under the stress states considered. For example, the least agreement between the anisotropic HC model and experimental data was for $\beta=30^\circ$, where the average displacement to failure for samples in the BD was found to be 4.69 mm in experiments, while the model predicted failure at a displacement of 3.41 mm, 73% of the experimental value.

To visualize the differences between the anisotropic HC model and the anisotropic MMC model, the fracture surfaces along BD and $\perp$BD for both models are shown in Figure 5.11. In the plots, the maximum principal stress direction was assumed to be aligned with the BD or $\perp$BD, and the minimum principal stress direction was assumed to be aligned with the thickness direction. Note that the fracture surfaces in the figure can be interpreted as the fracture strain limit for the condition of proportional loading. In actual tests, in which the loading paths are typically nonproportional, the fracture surfaces should be interpreted as the reciprocal of the damage accumulation rate (i.e., the higher the fracture surface, the slower damage accumulates).

The fracture surfaces of both models have the same general shape, but the anisotropic HC model shows higher sensitivity to both $\eta$ and $\theta$, which explains why the anisotropic HC model was able to better capture the experimental data compared to the anisotropic MMC model. The higher stress state sensitivity of the anisotropic HC model
comes from its dependence on all three principal stresses, while the anisotropic MMC model depended on the first and third principal stresses only [90].

**Figure 5.11.** Schematic comparisons of the anisotropic HC and MMC fracture surfaces in three-dimensional space of stress triaxiality vs. Lode angle parameter vs. equivalent plastic strain to failure for the (a) build direction and (b) perpendicular build direction samples. Symbols indicate the locations of each stress state directly on the anisotropic HC surface at the average location of the stress triaxiality and Lode angle parameter for each stress state under assumed proportional loading. Damage accumulates slower at locations where the surface is higher for each model.

### 5.5. Summary and conclusions

The isotropic and anisotropic HC and MMC ductile fracture models were calibrated for L-PBF 316L in two orientations. The anisotropic HC model was able to accurately capture and predict the experimental data in the present study. In general, the L-PBF 316L
exhibited more anisotropy in strength than ductility, but only for tension dominated loading conditions. The major findings of the study are as follows:

- Anisotropic initial yield strength behavior, hypothesized to derive from grain morphology effects, was only observed in the tension dominated tests, which most directly probed the difference in grain dimensions along loading different orientations.

- The plasticity behavior of L-PBF 316L was found to be stress state dependent and somewhat anisotropic. The proposed plasticity model consisting of an anisotropic Hill48 initial yield criterion, associated flow rule, and isotropic hardening equation captured the large deformation experimental results for a wide range of stress states.

- Shear softening, hypothesized to be due to shear band formation in shear dominated experiments, was captured through the implementation of a shear damage criterion in the plasticity model. With the calibrated damage initiation and linear softening, agreement between experiments and simulations was improved for shear dominated stress states and had no effect on the tension dominated stress states.

- The ductile fracture behavior of L-PBF 316L was found to depend strongly on stress state and slightly on build direction. The experimental data were used to calibrate the anisotropic Hosford-Coulomb fracture model, which captured the accumulated damage at the moment of fracture and relative displacement to failure within 7.9% and 7.1% mean absolute percentage error, respectively, for L-PBF 316L.
Chapter 6 Characterization of the effects of internal pores on tensile properties of additively manufactured austenitic stainless steel 316L

6.1. Introduction

The aim of this chapter was to isolate the role of internal pores on the tensile properties of austenitic stainless steel samples made using L-PBF and provide a direct understanding of the impact of initial void size on the tensile properties of conventional materials. A secondary aim was to compare different methods used to quantify porosity in manufactured components. Leveraging the unique capabilities of AM, samples were manufactured with a penny-shaped internal pore, at the center of cylindrical specimens, to mimic the presence of a LoF pore or large pores formed during void growth. Four different methods were used to measure the embedded pore size and morphology. By changing the size of the pores within samples, the role of defect size on tensile mechanical properties was investigated.

---

6.2. Experimental methods

6.2.1. Fabrication

In this study, a ProX DMP 320 laser powder bed fusion machine by 3D Systems [151] was used to manufacture AISI Type 316L stainless steel round cylindrical samples. The processing parameters used to fabricate the 316L L-PBF cylinders are shown in Table 6.1. The powder was sieved using a 60 μm screen. The composition of the fabricated 316L material, shown in Table 6.2, was measured using combustion infrared detection for carbon and sulfur content (ASTM E1019-18, [152]), inert gas fusion for nitrogen content (ASTM E1019-18, [152]), and direct current plasma emission spectroscopy for all other elements (ASTM E 1097-12, [153]). Cylindrical samples with an external diameter of 10 mm were built in the vertical orientation, such that when machined to tensile specimen geometry the loading axis was parallel to the vertical build direction. Each cylinder, except for the fully dense samples, had a single, centrally located pore, designed to be 180 μm tall, to closely mimic the sharp features in LoF pores. Specimens with eleven different pore diameters, whose dimensions are in Table 6.3, and fully dense specimens were fabricated. The cylinders were removed from the baseplate using wire electrical discharge machining in the as-built condition, as stress-relief has been shown to have no significant effect on mechanical properties of L-PBF 316L [154]. Samples were then machined to the cylindrical tensile geometry in Figure 6.1 using a CNC lathe to remove the effect of surface finish, which has been shown to impact the mechanical behavior of AM components [155].
Table 6.1. Processing parameters used for L-PBF manufacturing of 316L samples in the present study.

<table>
<thead>
<tr>
<th>Laser Power (W)</th>
<th>Layer thickness (µm)</th>
<th>Hatch spacing (µm)</th>
<th>Scanning speed (mm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>60</td>
<td>100</td>
<td>900</td>
</tr>
</tbody>
</table>

Table 6.2. Chemical composition (wt.%) of the as-built stainless steel 316L in study.

<table>
<thead>
<tr>
<th>Element</th>
<th>wt.%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon, C</td>
<td>0.02</td>
</tr>
<tr>
<td>Manganese, Mn</td>
<td>0.79</td>
</tr>
<tr>
<td>Phosphorous, P</td>
<td>0.008</td>
</tr>
<tr>
<td>Sulfur, S</td>
<td>0.006</td>
</tr>
<tr>
<td>Silicon, Si</td>
<td>0.60</td>
</tr>
<tr>
<td>Chromium, Cr</td>
<td>17.80</td>
</tr>
<tr>
<td>Nickel, Ni</td>
<td>12.48</td>
</tr>
<tr>
<td>Molybdenum, Mo</td>
<td>2.41</td>
</tr>
<tr>
<td>Nitrogen, N</td>
<td>0.068</td>
</tr>
<tr>
<td>Iron, Fe</td>
<td>Bal.</td>
</tr>
</tbody>
</table>

Table 6.3. Diameters of 180 µm tall penny-shaped pores designed into the cylindrical tensile samples.

<table>
<thead>
<tr>
<th>Sample Name</th>
<th>Pore diameter (µm)</th>
<th>% of cross-sectional area of tensile sample</th>
<th>Pore diameter as measured by X-ray CT (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>150</td>
<td>0.06%</td>
<td>-</td>
</tr>
<tr>
<td>C</td>
<td>300</td>
<td>0.25%</td>
<td>282 ± 40</td>
</tr>
<tr>
<td>D</td>
<td>450</td>
<td>0.56%</td>
<td>455 ± 45</td>
</tr>
<tr>
<td>E</td>
<td>600</td>
<td>1%</td>
<td>538 ± 41</td>
</tr>
<tr>
<td>F</td>
<td>1200</td>
<td>4%</td>
<td>1135 ± 54</td>
</tr>
<tr>
<td>G</td>
<td>1800</td>
<td>9%</td>
<td>1781 ± 48</td>
</tr>
<tr>
<td>H</td>
<td>2400</td>
<td>16%</td>
<td>2373 ± 54</td>
</tr>
<tr>
<td>I</td>
<td>3000</td>
<td>25%</td>
<td>2961 ± 53</td>
</tr>
<tr>
<td>J</td>
<td>3600</td>
<td>36%</td>
<td>3595 ± 52</td>
</tr>
<tr>
<td>K</td>
<td>4200</td>
<td>49%</td>
<td>4132 ± 76</td>
</tr>
<tr>
<td>L</td>
<td>4800</td>
<td>64%</td>
<td>4789 ± 52</td>
</tr>
</tbody>
</table>
Figure 6.1. Cross-section geometry of (a) as-built cylinders and (b) uniaxial tension samples in compliance with ASTM E8 [129], where dashed horizontal line indicates the intentionally introduced pore (here, showing a 3 mm diameter pore) at the center of the specimen. Dimensions in mm. (c) 3D CAD rendering of a tensile sample with an internal penny-shaped pore.

In addition to the cylindrical tensile test samples, a cylindrical witness sample that contained all eleven penny-shaped pore geometries was fabricated as shown in Figure 6.2. This sample was used to characterize pore size and morphology using X-ray CT and cross-section analysis after machining to a 6 mm diameter, which is equivalent to the tensile sample gauge diameter. Characterization of the pores in the witness sample were used as a direct representation of the pore sizes and morphologies that was expected in the tensile samples.
Figure 6.2. Witness sample with pores vertically positioned every 2 mm, starting with the 150 μm closest to the baseplate and the 4800 μm farthest from the baseplate. The pore diameter/volume fractions in the witness sample were characterized with each porosity analysis technique. Dimensions are given in mm.

6.2.2. Archimedes method

The Archimedes method was used to determine sample density and porosity of all machined tensile samples. The Archimedes process involved weighing the samples in the dry state to obtain their dry mass, $m_{\text{dry}}$, on an analytical balance (Ohaus Adventurer, Model AX324) that had a precision of 0.1 mg. Samples were then submerged in reverse osmosis (RO) water ($\rho_{\text{theor}}$ assumed to be 1.0 g/cm$^3$) and put under vacuum for 24 hours, during which the air bubbles on the surface of the samples were removed after 12 hours to allow the water to continue to permeate the surface. After removal from the vacuum, samples
were weighed submerged in RO water to obtain $m_{\text{sub}}$, and then weighed in air after patting the samples dry to remove water on the surface, obtaining $m_{\text{soak}}$. All mass measurements were done at room temperature and each sample was measured five times in each state letting the scale reset to zero between measurements. Experimental density was calculated using eq. (2.1). A theoretical density for 316L of 7.99 g/cm$^3$ was assumed for all calculations of experimentally determined porosity [156].

6.2.3. 2D radiograph method

The cylindrical witness sample (Figure 6.2), which contained all pore geometries studied, was analyzed with a General Electric v|tome|x L300 nano/microCT system. Two 2D radiograph images were taken for each pore in two perpendicular directions with the parameters in Table 6.4. The X-ray source was adjusted to be normal to each of the eleven pores when the grayscale image was taken to avoid skew in the data and in the measurement of the pore in post-process analysis, as shown in Figure 6.3. ImageJ analysis software was used to measure the diameter and height of each pore in each 2D radiograph [157].

**Table 6.4. Parameters used in X-ray 2D radiographs and 3D CT scans.**

<table>
<thead>
<tr>
<th>Voxel size (µm)</th>
<th>Voltage (kV)</th>
<th>Current (mA)</th>
<th>Power (W)</th>
<th>Cu detector filter thickness (mm)</th>
<th>X-ray source to detector distance (mm)</th>
<th>Exposure time (ms)</th>
<th>Number of projections</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.45</td>
<td>150</td>
<td>90</td>
<td>13.5</td>
<td>0.5</td>
<td>500</td>
<td>500</td>
<td>1000</td>
</tr>
</tbody>
</table>

133
Figure 6.3. 2D radiograph image centered at the 1800 μm diameter pore, as indicated by the arrow. Each pore was centered with respect to the X-ray source for dimensional analysis to ensure elimination of skewed features, as seen in the larger pores in this image.

6.2.4. X-ray CT method

The same system and scan parameters that were used for 2D radiographs were used for the 3D X-ray computed tomography scan. A total of 1000 images, through rotations between 0° and 360°, were taken during the scan with an exposure time of 500 ms for each image and an averaging of three images per rotation. A voxel size of 13.45 μm was used,
which results in a minimum resolvable pore size of 40.35 μm [158]. The voxel size was calibrated prior to each scan using a calibrated ball bar with a known distance between the center of two ruby spheres; a correction was applied to the system after this calibration. These 2D images were reconstructed into a 3D dataset using the cone-beam algorithm developed by Feldkamp et al. [159]. The 32-bit reconstructed projections were imported to Avizo 9.3.0 software (Thermo Fisher Scientific) for 3D processing and analysis. The diameter for each intentionally introduced pore was measured from the scans. As X-ray CT provides full information on pore morphology and size, the diameter of each pore measured with this technique were assumed to be the pore diameter in the corresponding tensile specimens, for the respective pore geometry, meaning that the other pore measurement methods were compared against this measurement technique.

6.2.5. 2D cross-section method

After non-destructive evaluations of pore geometries, the witness sample was sectioned in half, parallel to the vertical build direction, to expose the pores for optical microscopy (OM) analysis. Half of the sample was mounted in epoxy, ground and polished using SiC abrasive papers, with a final polishing step using 0.05 μm colloidal silica. Images of the pores in the sample were taken using a digital optical microscope (Keyence VHX-2000). ImageJ software was used to analyze the cross-section images by first converting the images to a 16-bit greyscale image, as shown in Figure 6.4, and then a threshold value was used to distinguish between pores and solid material. The pore diameter and height were then measured using the modified image.
Figure 6.4. Polished cross-section of the 2400 μm internal pore in the witness sample, which was used to measure dimensions for 2D cross-section analysis in ImageJ.

6.2.6. Evaluation of uncertainty

In the measurement of the internal pore diameters with the different techniques, the uncertainty in the measurements was quantified to account for the different origins of uncertainty in each of the techniques evaluated. For the measurement of the diameters of the pores using X-ray CT, the pores in the witness sample were reconstructed with Avizo software [160]. The volumetric center of each pore was calculated and its z-slice number (height in the vertical build direction) was determined. The diameter of the pore was measured with a best-fit a circle on the 2D cross-section where the volumetric center was calculated to be. The best-fit diameter was adjusted and measured at eight equally spaced distances around the pore using the best-fit circle. For each of the eight measurements an individual percent systematic error, $\delta_{i,sys}$, was calculated using:

$$
\delta_{i,sys} = \frac{3 \times \text{voxel size}}{d_{mean}}
$$

(6.1)

where $d_{mean}$ is the mean diameter of the pore of interest as measured by the best-fit circle and the general rule for detectable features in a CT scan of three times the voxel size was
used in the numerator [158]. The overall systematic error, $\delta_{o,sys}$, for a given pore was defined as:

$$
\delta_{o,sys} = d_{mean} \times \frac{\sum \delta_{i,sys}}{n}
$$

(6.2)

where $n$ is the number of measurements. Random uncertainty, $\delta_{ran}$, was defined as the standard deviation of the mean, using the standard deviation with Bessel’s correction. Total uncertainty, $\delta_{tot}$, which was used in the calculation of uncertainty in the percent difference as described later, was defined as:

$$
\delta_{tot} = \sqrt{(\delta_{ran})^2 + (\delta_{o,sys})^2}
$$

(6.3)

For the 2D analysis techniques, measurements of the pore diameters were made in ImageJ software with the line tool and length was recorded in pixels. For the 2D radiograph images, three measurements of the pore diameter were made at different heights on the pore on the two perpendicular planes for a total of six measurements. Evaluating two perpendicular planes in 2D radiography analysis provides both more measurements and a more accurate assessment of the uncertainty in the measurements as shown in Figure 6.5.

For the 2D cross section method, five measurements of the pore diameter were made at different locations along the height of the pore. In both methods, at the edge of the pore there was a gradient of pixel colors or grayscale that could be considered the edge of the pore. The number of pixels in the gradient on one edge was recorded and multiplied by two to account for the uncertainty in the defined edge location at both sides of the measurement line. The ratio of total pixel gradient, $d_{grad}$, to the line length in pixels, $d_l$, was calculated for each line, and was defined as the individual systematic error $\delta_{i,sys}$.
In 2D cross-section analysis, this gradient was assumed to be the same for each of the five measurements of pore diameter. For the 2D radiography measurements, the gradient and therefore pixel ratio was redefined for each of the six measurements. The overall systematic error for a single pore was calculated using eq. (6.2). The random uncertainty for the 2D methods was calculated with the same methodology used for the X-ray CT. The total uncertainty was taken to be the systematic error and random uncertainty added in quadrature, as given in eq. (6.3).

\[
\delta_{\text{sys}} = \frac{d_{\text{grad}}}{d_l}
\]  

(6.4)

**Figure 6.5.** Comparison of the pore diameter measurement and uncertainty using different viewing directions in 2D radiography and the effect of using both planes for a more accurate value of the pore diameter.
The equation for percent difference, \( q \), was used to compare the 2D measurements against the X-ray CT measurements, and was defined as:

\[
q = \left| \frac{x_{2D} - y_{XCT}}{\frac{x_{2D} + y_{XCT}}{2}} \right| \times 100
\]  

(6.5)

where \( x_{2D} \) was the diameter of the pore as measured by the cross-section or radiography method and \( y_{XCT} \) is the diameter of the pore as measured by X-ray CT method. The percent difference equation is an equation of two variables where each variable has an associated total uncertainty. To propagate the uncertainty throughout the equation and into the uncertainty in the percent difference, \( \delta q \), the general formula for error propagation was used:

\[
\delta q = \sqrt{\left( \frac{\partial q}{\partial x} \delta x \right)^2 + \left( \frac{\partial q}{\partial y} \delta y \right)^2}
\]  

(6.6)

6.2.7. Mechanical testing

Uniaxial tension tests were performed on six to nine samples for each of the pore geometries and dense samples using an electromechanical load frame (MTS Criterion Model 45) with a 150 kN load cell. Quasi-static tension tests were performed under displacement control using an applied strain rate on the order of \( 10^{-4} \, \text{s}^{-1} \). The surface deformation fields were measured using 3D digital image correlation, a non-contact surface strain measurement technique (Vic3D software, Correlated Solutions). The gauge regions of the machined samples were painted with flat white basecoat with a random black speckle on top. Images of the sample during loading were taken at a rate of 1 Hz using two digital cameras (Point Grey GRAS-50S5M-C) and data capture software (VicSnap, Correlated Solutions). A calibration target with a 14 x 9 dot pattern and 3 mm spacing between dots.
was used to calibrate relative position of the cameras with respect to each other. A subset size of 29 pixels and a step size of 7 pixels was used for the surface deformation analysis of the images. To compute the evolution of vertical strain in the gauge region during post-processing a vertical virtual extensometer of length 24 mm was used. The engineering stress was calculated as a function of applied force and the cross-sectional area of each sample using the measured outer diameter of the gauge region (i.e., not considering a reduced cross-sectional area due to the introduction of a pore).

6.3. Results and discussion

6.3.1. Bulk porosity analysis

Using the Archimedes density technique, the average bulk porosity of the dense samples was found to be 0.60 ± 0.2 %. The bulk measure of porosity remains relatively constant up to the maximum designed 4800 μm pore samples, which had an average bulk porosity of 0.88 ± 0.2 %. The 4200 μm pore samples had the largest measured bulk porosity of 0.95 ± 0.1 %. Archimedes can only provide information on bulk porosity and does not provide any information on the size of pores, distribution of pores, or morphology of pores. Additionally, the Archimedes method does not account for the true volume of the intentional internal pore space as these pores also include trapped powder that remains during the fabrication, which does not aid mechanical properties, but adds uncertainty in this measurement technique. However, the low bulk porosity measured in the dense samples provides confidence that any changes in the mechanical properties of the samples with the intentional pores included in this study can be attributed to the intentional pore present at the center of each sample (i.e., not random porosity throughout, as this was not found in any of the measurement methods).
6.3.2. Characterization of intentional pores

The X-ray CT measurements give the most accurate measurement of the morphology of the pores when comparing the three local analysis techniques used because of the ability to resolve 3D features, as shown in Figure 6.6. The CT analysis also allowed for the measurement of the diameter of the pores in the XY plane, shown in Figure 6.7, as opposed to the vertical build direction planes evaluated in the 2D analysis. It should be noted that in all the measurement techniques the 150 μm designed pore was not resolvable, likely due to melt pool spreading during fabrication closing the pore, and is excluded from the analysis. Taking the X-ray CT reconstructed data as the measurement of the actual pore diameter and comparing this to the expected designed pore geometry in the CAD drawings, the average percent error of all resolvable pore diameters was -4.9%, and -2.4% when the smallest resolvable pore (300 μm) is excluded. This indicates that smaller pores were more challenging to make per the design than larger pores, which is likely due to the flow of the meltpool in the build plane, and dross formation in the vertical build direction. However, these measurements indicate that in this study, pores whose designed diameter were larger than 300 μm, can be assumed to have been accurately fabricated within the tensile sample, and any size effects on mechanical properties can be interpreted in terms of the designed pore diameter. This is opposed to the 2D cross-section and radiograph methods, which gave average percent errors of -11.6% and -13.1% compared to the designed pore diameter, respectively, but have other measurement artifacts that could impact measurements discussed below. Therefore, in all discussion and analysis below, the designed pore diameter is referred to, since X-ray CT found good agreement between the designed and actual pore diameters.
Figure 6.6. 3D X-ray CT reconstruction of the eleven pores in the witness sample within the 6 mm diameter cylinder.

Figure 6.7. X-ray CT images of the reconstructed pores in different planes for the (a) 4800 μm, (b) 1800 μm, and (c) 300 μm pores.
Taking the diameter measurements from 3D X-ray CT as the baseline with which to compare the two 2D analysis methods, it was found that for pores designed with a diameter of 450 to 1200 μm, the OM cross-section analysis more accurately resolved the pore diameters than 2D radiography. However, for pores 1800 μm or larger, the 2D radiography method provided more accurate measurement of the pores compared to the 2D cross-section method. These trends are shown in Figure 6.8, where the cross-section with OM method tended to overestimate the pore diameter for the smaller pores, while the 2D radiography method always underestimated the pore diameter, especially the 300 and 450 μm pores. The consistent underestimate of diameter in the 2D radiography method is a result of two main factors: the difficulty in differentiating the un-melted trapped powder at the edges of the pores, which have the same density, and therefore grayscale values as the solid material, resulting in challenges in defining the full geometrical extents of the pore; and the fact that the 2D images are a projection of 3D samples where the pores are surrounded by dense material resulting in gradient of grayscale values along the 2D cross-sectional projection that can also be difficult to threshold, as shown in Figure 6.3. For pores larger than 1800 μm, where the pore edges are close to the 6 mm outer diameter of the witness sample, the radiograph analysis was very close to the X-ray CT method. Using percent difference to compare measurements of the pores larger than 1800 μm, the 2D radiograph measured average diameter was only 4% smaller than the X-ray CT-measured diameters, and the diameter measured using the OM method was 19% smaller than the X-ray CT diameter. Given that smaller pores are more prevalent in AM builds, the cross-section with OM analysis technique provided more relevant information in the analysis of the intentionally embedded pore diameters, although there is no consistent trend of over or
under estimating the pore size. However, capturing the pore geometry with OM can be a challenge, and can result in skewed data for two primary reasons: first, if the pore is not sliced and viewed exactly in the center plane, this will result in an underestimate of diameter; second, if the sample is not sectioned and polished exactly parallel to the build axis, the pore will be viewed at an angle, resulting in a possible overestimate of the pore dimensions at the location viewed.

Figure 6.8. Comparison of the 2D analysis techniques as related to their similarity with the pore diameter measured in the 3D X-ray CT analysis using percent difference. A positive percentage indicates the 2D measurement was larger than the 3D X-ray CT measurement and a negative percentage indicates the 2D measurement was smaller than 3D X-ray CT measurement. Both methods are shown to more likely underestimate the pore diameter that is measured in 3D X-ray CT.
6.3.3. Tensile testing

Uniaxial tension testing of the dense samples and those with internal pores revealed clear trends with respect to mechanical properties as a function of pore diameter in the 6 mm diameter sample, as shown in Figure 6.9. The change in mechanical behavior was a function of the pore size within the constant outer diameter of the gauge region. The increasing pore size also corresponds to a decreasing number of grains in the remaining cross-sectional area. The average grain area in the XY plane of the 316L material in this study was measured to be 427 µm² or 15.7 µm in diameter. In the fully dense samples, this results in more than 66,000 grains within the cross-sectional area of the 6 mm gauge region of the tensile samples, which reduces to around 2,700 grains in samples with a 4800 µm pore, the largest intentional pore evaluated in the study. Thus, this work studies the impact of pore size with respect to the component size (e.g., thick and thin components), while still evaluating a bulk number of grains within a cross-section.

![Figure 6.9](image)

**Figure 6.9.** Representative engineering stress-strain curves for samples with each initial pore diameter. A is representative of the dense samples, while B-L correspond to samples with increasing pore diameters. See Table 6.3 for nomenclature of B-L pore diameters.
The presence of the pre-existing pore did not affect the material strength or ductility of the samples until the pore was greater than or equal to 600 \( \mu m \) in diameter, as shown in Figure 6.10. At a pore diameter of 600 \( \mu m \), corresponding to 10\% of the cross-sectional diameter, the inclusion of the pore resulted in significant spread in the data compared to samples with smaller pores or no pores. At a pore diameter of 1200 \( \mu m \), 20\% of the cross-sectional diameter, seven of the nine samples failed at the pore. It was evident when a sample did fail at a pore, because the pore, surrounded by shear lips, was clearly visible on the fracture surfaces in these samples. Once the initial pore was 1800 \( \mu m \) in diameter, 30\% of the cross-sectional diameter, there was a repeatable reduction in the elongation to failure of the samples to 26.5 \( \pm 0.5 \)% compared to 64 \( \pm 0.5 \)% in the dense samples. Increasing pore diameters resulted in decreased elongation to failure for all pore diameters exceeding 1800 \( \mu m \), as shown in Figure 6.11a. The samples with the largest pore, 4800 \( \mu m \) in diameter, had tensile elongations to failure of under 1.5\%. 
Deterioration of material strength occurred for pores greater than or equal to 2400 μm in diameter within the 6 mm diameter gauge region. The UTS of the material dropped from 619 ± 3 MPa in the dense samples to 580 ± 5 MPa for samples with an internal pore 2400 μm in diameter. The strength of the material continued to decrease with increasing pore diameter above 2400 μm all the way to an UTS of 297 ± 5 MPa in the sample with the maximum pore diameter of 4800 μm as shown in Figure 6.11b.

These findings indicate that L-PBF 316L is defect insensitive up to large pore sizes, with respect to sample dimensions. Samples, with a gauge diameter of 6 mm, and an internal pore diameter up to 2400 μm maintain the experimentally determined UTS of wrought 316L [161], while samples with a pore diameter of up to 3600 μm maintain the
experimentally determined UTS of as-cast 316L [162]. Additionally, the uniform elongation of the 6 mm gauge diameter sample exceeds that required of wrought material even with a 1200 μm intentional pore.

**Figure 6.11.** Box plots displaying the minimum, maximum, first and third quartiles, exclusive median (central line), and mean (x) of (a) tensile elongation to failure and (b) ultimate tensile strength as a function of pore diameter with the pore diameter at which each of these properties begins to be negatively impacted shown. Inset of (a) is given in (c).
6.4. Summary and conclusions

The current work evaluated the effect of internal pores on the tensile properties of L-PBF 316L and compared common techniques used to evaluate internal pores in metallic components. This study, which is unique in both the material evaluated and the fact that pores of varying sizes were directly fabricated inside of samples to link pore geometry to mechanical properties provides the following primary conclusions:

- The tensile strength of L-PBF 316L was shown to be insensitive of significant defects. Penny-shaped intentionally introduced pores with a height on the order of 180 μm, within tensile samples with a gauge diameter of 6 mm, only began to impact ultimate tensile strength when the pore diameter was 2400 μm, or 16% of the cross-sectional sample area.

- The tensile elongation to failure was slightly more sensitive to the internal defects as the elongation decreased substantially when the pore diameter was 1800 μm in diameter or 9% of the cross-sectional sample area.

- Samples with a pore diameter of 450 μm and smaller behaved very similarly to dense samples, with the mechanical properties showing very good repeatability, no drop-off in strength, and only 9% reduction in elongation to failure. The 600 μm and 1200 μm pore diameters led to significant scatter in the elongation to failure behavior.

- Internal pores were most accurately characterized using the 3D X-ray CT technique, which could capture pore morphology, pore size, pore location, and if multiple pores were present, the spatial distributions of pores and pore sizes; however, if this method is not an option, 2D cross-section analysis with OM of
pores 1200 μm and smaller provides data that captures the geometry of the pores within 20% on average. For larger pore or internal features, 2D radiographs provide fast and accurate shape information.
Chapter 7 Combined effects of porosity and stress state on the failure behavior of laser powder bed fusion stainless steel 316L

7.1. Introduction

This chapter presents an experimental investigation on the combined effects of porosity and stress state on fracture behavior of L-PBF 316L. Intentional, penny-shaped pores of varying diameter were included in notched tension specimens. The notch radii of these specimens were designed such that, while the initial Lode angle parameter at the center of the sample was the same in all samples, the stress triaxiality value when the minimum cross-sectional area became fully plastic was different for each notch radius. The intentional pores were characterized via XCT before and after elongation, to evaluate size, morphology, and pore growth characteristics as a function of stress state. An integrated experimental/computational approach was used to quantify the strain to failure in samples with and without pores. Finally, the effect of pores on the strain to failure is described in the space of equivalent plastic strain and stress triaxiality. Harnessing AM’s unique capability of being able to manufacture intentional internal pores, this study represents the first-time experimental data with tailored pore size has been used to determine the combined effect of pore size and stress state on ductile strain to failure.

---

7.2. Experimental methods

7.2.1. Sample fabrication

All parts in this study were built using a 3D Systems, Inc. ProX DMP 320 L-PBF AM system with gas atomized, pre-alloyed austenitic stainless steel 316L powder (LaserForm®316L(A) stainless steel powder from 3D Systems, Inc. [42]) meeting the compositional requirements of DIN X2CrNiMo17-12-2. The powder was sieved before use to only include powder particles 60 μm or smaller in diameter. A laser power of 300 W, scanning speed of 900 mm/s, hatch spacing of 100 μm, and layer height of 60 μm were used. Outer contours and a full-length hatch fill scan strategy were used for all samples. For each sequential layer of the build a 245° rotation of the hatch direction was used. Process parameters in the current study were chosen to achieve fully dense parts.

A total of 63 12.5 mm diameter and 68 mm tall cylinders were built on the baseplate. To study the influence of defects in the material, intentional penny-shaped pores of varying diameters were built at the center of 54 of the cylinders. The designed intentional pore diameters studied were: 300, 600, 900, 1200, 1800, and 2400 μm. Nine cylinders were built with each pore size, and nine cylinders were built to be dense. All intentional pores were designed to be 180 μm, or three build layers, tall. Wire electrical discharge machining was used to remove parts from the build plate in the as-built condition. All mechanical tests were completed on the material in the as-built condition; a stress-relief heat treatment was not used for the L-PBF 316L because it has been shown to have no effect on either the monotonic [43] or fatigue properties (in a 6 mm sample diameter, equivalent to that used in the current study) for this material [44]. A representative image of the microstructure for this build is given in ref. [39].
### 7.2.2. Fracture test sample geometries

The three notched tension geometries were chosen to probe a range of high stress triaxiality stress states with a nearly constant Lode angle parameter of 1, as shown in Figure 7.1. The approximate stress triaxiality when the minimum cross-section of each geometry has yielded through the diameter was calculated using the Bridgman formulation [45] with the $\sqrt{2}$ correction factor from Bao [46], given as:

$$
\eta_{CB} = \frac{1}{3} + \sqrt{2} \ln \left(1 + \frac{a}{2R}\right)
$$  \hspace{1cm} (7.1)

where $\eta_{CB}$ is the corrected Bridgman approximate stress triaxiality, $R$ is the radius of the neck in the round bar specimen, and $a$ is the radius of the smallest cross-section. In the three geometries studied, $a$ was held constant at 3 mm and $R$ values were 12 mm (R12), 5 mm (R5), and 3 mm (R3), which resulted in $\eta_{CB}$ of 0.5, 0.7, and 0.9, respectively. The relationship between designed and experimentally measured pore size, as well as the relationship of actual pore size to the minimum sample diameter and cross-sectional area, for all samples are shown in Table 7.1. The sample gauge regions were machined via a computer numeric control (CNC) lathe. A total of three samples for each sample geometry and dense/intentional pore size combination were machined.
Figure 7.1 Geometries of cylindrical notched tension specimens with three different notch radii, which result in different stress triaxiality values at the center of the specimen, with Bridgman approximated triaxiality values of: (a) $R = 12$ mm giving $\eta_{cB} = 0.5$, (b) $R = 5$ mm giving $\eta_{cB} = 0.7$, and (c) $R = 3$ mm giving $\eta_{cB} = 0.9$. All dimensions in mm.

Table 7.1. Comparison of designed penny-shaped pore diameters to average X-ray CT measured values and measured pore diameters as a fraction of sample diameter (6 mm) and sample cross-sectional area.

<table>
<thead>
<tr>
<th>Designed pore diameter (µm)</th>
<th>Average XCT measured diameter ± standard deviation (µm)</th>
<th>XCT measured pore diameter as % of sample diameter</th>
<th>XCT measured pore cross-sectional area as % of sample cross-sectional area</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>322 ± 24</td>
<td>5.4</td>
<td>0.3</td>
</tr>
<tr>
<td>600</td>
<td>603 ± 49</td>
<td>10.0</td>
<td>1.0</td>
</tr>
<tr>
<td>900</td>
<td>899 ± 35</td>
<td>15.0</td>
<td>2.2</td>
</tr>
<tr>
<td>1200</td>
<td>1240 ± 35</td>
<td>20.7</td>
<td>4.3</td>
</tr>
<tr>
<td>1800</td>
<td>1840 ± 35</td>
<td>30.7</td>
<td>9.4</td>
</tr>
<tr>
<td>2400</td>
<td>2413 ± 38</td>
<td>40.2</td>
<td>16.2</td>
</tr>
</tbody>
</table>
7.2.3. Mechanical testing

All notched tension samples were deformed in tension using an electromechanical load frame (MTS Criterion Model 45) with a 150 kN load cell. Tests were completed under displacement control using a loading rate of 0.005 mm/s. Prior to testing, the entire gauge region and 5 mm of the grip section on the top and bottom were painted with flat white basecoat and random black speckle pattern. Two digital cameras (Point Grey GRAS-50S5M-C) were used to take images, at a rate of 1 Hz, of the sample during loading; images were saved in the data capture software VicSnap (Correlated Solutions), and post-processing was performed using 3D digital image correlation (DIC). DIC is a non-contact surface strain measurement technique and 3D surface deformation fields were measured using Vic3D software by Correlated Solutions. To compute the displacement data from tests, a virtual extensometer was used with either end placed in the grip region, 1 mm past the transition from the notched region for all three geometries.

The first sample in each three-sample set of notch radius and pore size (21 samples in total) was deformed all the way to failure. The second group of 21 samples were: (1) scanned via XCT to characterize the initial size, location, and morphology of the internal pores, (2) pulled in tension until the displacement in the test reached a value of ~70% of the displacement to failure of the first test and then unloaded, (3) re-scanned via XCT, and (4) tested in tension until failure. The last group of 21 samples were (1) loaded until ~70% of the displacement to failure of the corresponding first test and then unloaded, (2) the internal pore in the elongated sample was characterized via XCT, and (3) the tension tests were completed to failure. The first group of samples revealed a baseline force versus displacement behavior for each test type (notch radius and pore diameter combination),
while the second and third groups provided insight to pore growth behavior as a function of stress state and pore size.

### 7.2.4. X-ray computed tomography

A General Electric v|tome|x L300 nano/microCT system was used for all XCT scans performed in this study. A voltage of 150 kV and current of 90 mA, for a power of 13.5 W, were used for each scan. A 0.5 mm thick copper sheet was placed on the detector, which was 500 mm away from the X-ray source. The pixel pitch for the flat panel detector was 200 µm. The notched tension samples were scanned individually and were located 35 mm from the X-ray source. A voxel size of 14 µm was used. For each sample, 1800 images were collected, through rotations between 0° and 360°, using an exposure time of 500 ms for each image with no averaging. A ball-bar calibration and correction was completed prior to each group of scans. Initial projection reconstruction was completed using phoenix datos|x 2.0 CT software and then segmentation and quantitative data analysis, including volume measurements, were completed in Avizo 9.7.0 software (Thermo Fisher Scientific).

### 7.3. Results and discussion

#### 7.3.1. Fracture behavior: experimental results

The effect of the six different sized intentional penny-shaped pores in samples with three different stress states was examined. Note that what was experimentally measured was the effect of increasing pore diameter within a sample on the displacement to failure
of that sample. In this way, we did not account for the potential change of stress state at the center of the sample due to the introduction of the pores. Instead, using a practical engineering approach, we considered the impact of increasing pore size on each sample’s displacement to failure to describe the effect of pores on the stress triaxiality-dependent failure behavior calculated via subsequent finite element analysis of the dense samples.

The expected trend in the force versus displacement behavior as a function of stress triaxiality was observed for the dense samples. The dense R12 geometry ($\eta_{cB} = 0.5$) had the highest average displacement to failure and lowest average maximum force value relative to the other two geometries (where average refers to the mean force or displacement of all similarly tested samples), as shown in Figure 7.2. Both the dense R5 ($\eta_{cB} = 0.7$) and R3 ($\eta_{cB} = 0.9$) samples had subsequently higher maximum force and reduced displacement to failure compared to the dense R12 samples. For the R12 samples, the drop in average maximum force was less precipitous than loss in ductility with the inclusion of the intentional pores. The R12 samples maintained >95% of the maximum force of the dense samples even with the 1800 µm pores (9% of sample cross-sectional area), unlike the other two geometries tested (R5 and R3), as shown in Figure 7.3. These findings support the known effect of stress triaxiality on ductile material failure, where, for a constant Lode angle parameter, increasing stress triaxiality results in decreasing ductility.
**Figure 7.2.** Representative force versus displacement curves, as a function of the intentional pore diameter, for each notched tension geometry: (a) $R = 12 \, \text{mm}$, (b) $R = 5 \, \text{mm}$, and (c) $R = 3 \, \text{mm}$.

**Figure 7.3.** Trends for the (a) maximum force and (b) displacement to failure averaged across three tests for each test condition as a function of the intentional pore diameter.

The 300 $\mu$m pore in the R12 samples resulted in an average loss of 11% in displacement to failure relative to the dense samples, while the 300 $\mu$m pore (0.3% of sample cross-
sectional area) in the R5 samples had a larger effect on the reduction of the displacement to failure, resulting in a 15% loss, compared to the R12 tests. In the highest stress triaxiality samples (R3), the reduction in displacement to failure as a percentage of the dense sample was always the lowest compared to the other geometries with the introduction of pores from 300 µm to 1200 µm in diameter (0.3% to 4% of sample cross-sectional area). For the 1800 µm and 2400 µm pores (9% and 16% of sample cross-sectional area), no trend between sample geometry and pore size was observed and the average reduction in displacement to failure compared to dense counterparts was relatively constant across all three geometries, 39 ± 1% and 32 ± 2%, respectively.

The observed experimental trend revealed that stress triaxiality is the dominating factor in loss of displacement to failure up until a pore diameter of 1200 µm (4% of sample cross-sectional area), while the pore size itself dominated the displacement to failure when initial pores were equal to or larger than 1800 µm in diameter (9% of sample cross-sectional area).

7.3.2. Characterization of pores via XCT

The pore volume was segmented from the reconstructed XCT scans for each of the scans completed on the as-built and elongated samples. Segmentation of the pore from the dense material was done using a consistent, manually determined thresholding of the grayscale values for each scanned sample, along with additional manual selection of pore space in volumes that were occupied by unmelted, trapped powder where grayscale intensities along the pore edges were more similar to the surrounding solidified material. The segmented as-built pores were measured to determine the difference between the dimensions prescribed in the CAD software for the build and the as-built geometries. To
determine the average maximum diameter of all pore sizes studied, or the maximum reduction of solid cross-sectional material, 3D axis-aligned bounding boxes were fit around each segmented pore. For the pores in the R3 samples, the bounding box was rotated 9° around the axis parallel to the vertical build direction from 0° to 72°, for a total of nine bounding boxes and 18 unique measurements. For the pores in the R5 and R12 samples, a single bounding box for each pore size was used, providing an additional four measurements per designed pore size. The two dimensions of each bounding box perpendicular to the height of the pore were averaged across all 22 measurements to determine an average pore size. The diameters of the as-built pores were on average 2.3% different than the designed diameters; for example, the average diameter of the designed 300 µm pore was measured as 322 ± 24 µm, and the average diameter of the 2400 µm pore was measured as 2413 ± 38 µm, as shown in Table 7.1. These measured values confirmed that the designed pores were accurately included in the fabricated samples.

After partial loading and unloading of the group 2 and 3 samples, as described in Section 2.3, the XCT data of the elongated pores of varying size in different sample geometries revealed that pore growth behavior was influenced by stress triaxiality. Examples of the pore growth analyzed via XCT are shown in Figure 7.4, while the overall trends of increased pore growth with increased stress triaxiality are shown in Figure 7.5. The intentional pores in the R12 and R5 geometries experienced on average 742% and 755% pore volume growth for interrupted tests in which samples were elongated to an average of 70% of the eventual displacement to failure prior to unloading and XCT evaluation. The highest stress triaxiality tests, R3 samples, had an average of 922% pore volume growth using the same criteria for elongation as the R12 and R5 samples. Pore
growth was mainly due to expansion of the pores in the loading direction, where the average pore growth along the loading direction for all pores in all three notched tension geometries was 211%, while the average growth in two orthogonal directions to the build direction was only 48%.

Figure 7.4. (a) Comparison of the force versus displacement behavior for the \( R = 5 \) mm geometry, with an 1800 µm pore (9% of sample cross-sectional area), pulled to failure versus one unloaded after 86% displacement to failure and subsequently loaded to failure. (b) X-ray computed tomography reconstruction of the intentional pore prior to testing and (c) reconstruction of the pore after 86% displacement to failure. (d) Comparison of the force versus displacement behavior for the \( R = 12 \) mm geometry, with a 600 µm pore (1% of sample cross-sectional area), pulled to failure versus one unloaded after 79% displacement to failure and subsequently loaded to failure. (e) X-ray computed tomography reconstruction of the intentional pore prior to testing and (f) reconstruction of the pore after 79% displacement to failure.
Figure 7.5. Effect of stress triaxiality on pore growth behavior as measured by change in volume for all interrupted tests for which the displacement at unloading was on average 70% of the eventual displacement to failure.

7.3.3. Fracture behavior: simulations

7.3.3.1. Finite element modeling of dense geometries

The three different geometries were simulated via finite element analysis using the commercial software Abaqus [47]. The proposed plasticity model consisting of an anisotropic Hill 1948 initial yield criterion [48], associated flow rule, isotropic hardening equation, and shear damage criterion from ref. [39] was used as the material model. The fully dense 3D geometry of each notched tension sample was modeled, including 5 mm of grip region above and below the notches, using C3D8R type elements. The R12, R5, and
R3 models had 1,149,808 elements, 951,648 elements, and 847,704 elements, respectively. All degrees of freedom for each node on the bottom face of the bottom grip section of each model were constrained, and a vertical displacement boundary condition was applied to a reference node on the top face of the top grip section of the geometry to replicate the experimental displacement of the dense samples. Simulated force data were extracted from the top reference node, while displacement data was taken from two nodes 1 mm from the edge of the transition between grip and notched region, replicating the process used with the virtual extensometer in DIC. The Hill48 equivalent plastic strain and stress components were extracted from the center node of each geometry, at the vertical location of minimum cross-sectional area, corresponding to the location of highest strain during deformation and presumed fracture initiation. The simulated stress state parameters were calculated using the extracted data from the center node. The Bridgman approximated stress triaxiality values of 0.5, 0.7, and 0.9 were found to be the same as those calculated in simulations once the minimum cross-section of each geometry had yielded through the diameter. However, due to the stress state evolving with plastic deformation in these geometries, the values of average stress triaxiality during deformation, $\eta_{avg}$, were 0.67 (R12), 0.80 (R5), and 0.97 (R3), calculated using eq. (4.2).

7.3.3.2. Comparison with experimental force versus displacement behavior

Good agreement between experimentally measured and computationally simulated force versus displacement behavior was achieved for all three geometries, as shown in Figure 7.6. The maximum force of the simulated tests was lower than that observed in
experiments, however the difference is small – no greater than 5% error between the simulated and experimentally measured maximum force for any of the three geometries. These comparisons confirmed that calibrated material model provided an accurate representation of experimental behavior.

**Figure 7.6.** Comparison of the force versus displacement behavior of representative dense experimental data and simulated data for all three notched tension geometries. Error bars for displacement at each maximum force represent the range of displacement that fall within 0.5% of the maximum force. All maximum force values in the simulations was 5% lower than that in experiments.
7.3.3.3. Effect of pores and stress triaxiality on equivalent plastic strain to failure

To understand the effect of the pore size on the fracture behavior of the 316L material, the simulations of the dense samples, in tandem with the experimental displacement to failure behavior of each pore size were used. By simulating dense samples up to the measured displacements to failure of samples with pores, we are presenting a method for understanding the impact pores have on reducing the ductility compared to fully dense material, while also presenting an engineering approach that could be used to limit the loading of a sample in service with an assumed pore size.

The evolution of the equivalent plastic strain to failure as function of stress triaxiality for the dense simulations is shown in Figure 7.7. For the engineering approach described above, the same evolution of these parameters was assumed for all specimens with pores, and the failure location along the evolution of equivalent plastic strain versus stress triaxiality for samples with each pore size studied is indicated in Figure 7.7 and given in Table 7.2.
Figure 7.7. (a) Evolution of the Hill48 equivalent plastic strain versus stress triaxiality, for all three geometries, up to the displacement to failure in the dense samples (solid lines). Symbols along the solid lines indicate the equivalent plastic strain (from dense simulations) at the average displacement to failure of each set of samples with intentional pores. The dashed lines connecting these points each correspond to the same intentional pore diameter, showing how increasing pore size reduces the strain to failure. Dashed vertical lines represent the average stress triaxiality, throughout loading, for the dense samples. (b) Equivalent plastic strain to failure as a function of percent solid cross-sectional area at the sample’s minimum diameter, accounting for the designed penny-shaped pore geometry.
Table 7.2. Effect of pore size on fracture as a function of sample geometry, presented as the percent of equivalent plastic strain to failure reached in a sample with a pore compared to that of the corresponding dense notched tension sample.

<table>
<thead>
<tr>
<th>Pore diameter (µm)</th>
<th>Designed pore as % sample cross-sectional area</th>
<th>R12</th>
<th>R5</th>
<th>R3</th>
<th>Average</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dense</td>
<td>-</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>N/A</td>
</tr>
<tr>
<td>300</td>
<td>0.3</td>
<td>84</td>
<td>84</td>
<td>71</td>
<td>80</td>
<td>6</td>
</tr>
<tr>
<td>600</td>
<td>1</td>
<td>68</td>
<td>67</td>
<td>60</td>
<td>65</td>
<td>4</td>
</tr>
<tr>
<td>900</td>
<td>2</td>
<td>52</td>
<td>62</td>
<td>52</td>
<td>56</td>
<td>5</td>
</tr>
<tr>
<td>1200</td>
<td>4</td>
<td>44</td>
<td>52</td>
<td>49</td>
<td>48</td>
<td>3</td>
</tr>
<tr>
<td>1800</td>
<td>9</td>
<td>31</td>
<td>37</td>
<td>40</td>
<td>36</td>
<td>4</td>
</tr>
<tr>
<td>2400</td>
<td>16</td>
<td>24</td>
<td>33</td>
<td>31</td>
<td>29</td>
<td>4</td>
</tr>
</tbody>
</table>

For the dense samples, the equivalent plastic strain to failure followed an expected trend where equivalent plastic strain to failure decreased with increasing stress triaxiality due to high triaxiality aiding the ductile fracture mechanisms of void nucleation, growth, and coalescence.

The effect of each pore size across all three stress triaxiality values evaluated is highlighted in Figure 7.7 with dashed horizontal lines connecting each pore size. With pores up to 1200 µm (4% of the cross-sectional area), the strain to failure decreased with increased stress triaxiality for a given pore size, and decreased with increasing pore size. This highlights that for the smaller pore sizes studied, stress triaxiality dominated the pore size dependent fracture behavior. For the 1800 µm (9% of the cross-sectional area) and 2400 µm (16% of the cross-sectional area) pores, the strain to failure was reduced compared to that of smaller pores, but more notably, the strain to failure was the same for
all three stress triaxiality values studied. This indicates that with this pore size, the failure strain is only dependent on pore size, and not stress triaxiality, indicating a critical pore size for stress state dependent failure. The presented results provide data and understanding on the effect of internal pores, as a fraction of a given feature size, under a range of high stress triaxiality stress states, which are at high risk of low ductility failure, especially in the presence of pre-existing porosity.

7.4. Summary and conclusions

The stress state and porosity dependent fracture behavior of L-PBF 316L was studied using the unique capability of AM to manufacture single, penny-shaped internal pores, of prescribed sizes in samples. The primary findings of this study are:

- In dense cylindrical notched tension samples increasing stress triaxiality resulted in lower equivalent plastic strain at the same Lode angle parameter value confirming the known effect of stress triaxiality on ductility of ductile metals.

- For all stress states studied the smallest pore (300 μm diameter within a 6 mm minimum cross-section, 0.3% cross-sectional area) reduced the displacement to failure compared to that of the corresponding dense samples. However, the higher the stress triaxiality, the more impact this pore had. Specifically, for the 300 μm pore, the displacement to failure dropped by 28%, 15%, and 11% for the R3 ($\eta_{avg} = 0.97$), R5 ($\eta_{avg} = 0.80$), and R12 ($\eta_{avg} = 0.67$) notched tension samples, respectively.

- The maximum force under all three stress triaxialities evaluated was defect tolerant. The maximum force dropped to $\leq90\%$ of the maximum force observed
for the dense samples when the pores were 1800 μm in diameter (9% of the cross-sectional area) for the R3 and R5 samples, and the same criteria was met for the 2400 μm diameter pore (16% of the cross-sectional area) in the R12 samples. Stress triaxiality had a limited effect on the maximum force response until the two largest diameter pore sizes, 1800 μm and 2400 μm, were introduced into samples.

• Up until the inclusion of a 1200 μm diameter, penny-shaped pore (4% cross-sectional area of the sample gauge sections), strain to failure was primarily dependent on stress triaxiality, while for larger pores (9% cross-sectional area or larger), strain to failure was independent of stress triaxiality, and only a function of pore diameter.
Chapter 8 Summary and conclusions

The uniaxial and multiaxial plasticity and fracture behavior, over a wide range of stress states, of Ti-6Al-4V and stainless steel 316L manufactured via laser powder bed fusion additive manufacturing were experimentally examined, computationally simulated, and modeled. The effects of microstructure were evaluated in the materials by testing samples in two orientations with respect to the vertical build direction, parallel and perpendicular, and by incorporating intentional defects in 316L. Using experimental test data, plasticity and fracture material models were calibrated to capture the anisotropic and stress state dependent properties in both alloys, and validated indicating the predictability of these appropriately calibrated models.

The effect of microstructural features on the mechanical anisotropy was evaluated for Ti-6Al-4V manufactured via AM. Uniaxial tension experiments on as-built continuous wave L-PBF and pulsed L-PBF material revealed that the nearly equiaxed grains in the continuous wave L-PBF led to greater yield strength, ultimate tensile strength, and ductility compared the pulsed L-PBF samples with elongated, columnar grains. Through an analysis of results in the literature, an exponential relationship was found to exist between the anisotropic grain morphology and anisotropic elongation to failure in additively manufactured Ti-6Al-4V.

The mechanical properties of L-PBF Ti-6Al-4V in two orientations were evaluated under uniaxial tension, plane strain tension, pure shear, and combined tension/shear loading. The mechanical behavior was found to be stress state dependent and slightly anisotropic. A plasticity model, consisting of a Hill 1948 anisotropic yield criterion,
associated flow rule, and an isotropic hardening law was calibrated and validated to describe the yield and plasticity behavior of L-PBF Ti-6Al-4V.

The stress state-dependent fracture behavior of L-PBF Ti-6Al-4V was quantified with tests in seven different stress states. Six different fracture criteria were calibrated and it was determined that fracture models that explicitly consider the effects of both the stress triaxiality and Lode angle parameter were able to more accurately capture the multiaxial failure behavior of L-PBF Ti-6Al-4V compared to models that do not consider stress state-dependence or incorporate only a dependence on stress triaxiality. Additionally, samples built in the vertical build direction had a higher ductility than corresponding samples built perpendicular to the vertical build direction.

The deformation and ductile fracture behavior of L-PBF 316L stainless steel was probed over a wide range of stress states. Shear softening, hypothesized to be due to shear band formation in the material due to high initial dislocation density and sub-micron cellular structures, was observed in shear dominated tests, and modeled through the adoption of a shear damage criterion in an anisotropic plasticity model. Using a combined experimental and computational approach, isotropic and anisotropic Hosford-Coulomb and modified Mohr-Coulomb ductile fracture models were calibrated for both sample orientations. The calibrated anisotropic Hosford-Coulomb fracture model best captures the stress state dependent and anisotropic failure behavior of L-PBF 316L.

316L stainless steel manufactured by L-PBF additive manufacturing was found to be defect-tolerant under uniaxial tension loading. Cylindrical, fully dense samples and samples with an intentional, penny-shaped internal pore of varying diameter were fabricated using the unique capabilities of AM. The intentional pore began to impact
ultimate tensile strength when the pore occupied 16% of the cross-sectional sample area. Elongation to failure was significantly affected when the pore diameter occupied 9% of the cross-sectional sample area.

Using both experiments and finite element simulations, the effect of both internal pores and stress state on the ductile failure behavior of L-PBF 316L stainless steel was measured. Using intentional pores at the center of sample geometries with varying macroscopic stress states, it was found that strain to failure depended strongly on stress triaxiality until a large pore size with a diameter occupying 4% of the cross-sectional area of the sample gauge sections, while strain to failure was independent of stress triaxiality, and only a function of pore diameter, with larger pores (9% cross-sectional area or larger).

The primary findings of this research can be summarized as follows:

- Showed that the plasticity and fracture behavior of L-PBF 316L was strongly dependent on stress state, in terms of both stress triaxiality and Lode angle parameter, and presented calibrated and validated plasticity and fracture models to capture this behavior.

- Showed that the plasticity and fracture behavior of L-PBF 316L was anisotropic with respect to the vertical build direction in loading conditions that were tension dominated, i.e., uniaxial tension, plane strain tension, and tension dominated combined tension/shear loading, in which samples were stronger when the applied tension was perpendicular to the build direction than the perpendicular build direction.

- Showed that the initial yield strength and strain to failure of L-PBF Ti-6Al-4V were slightly anisotropic and strongly dependent on stress state. Material loaded
with tension parallel to the vertical build direction was found to be stronger than when loaded perpendicular to the build direction, which is the opposite trend of the L-PBF 316L material. The fracture behavior was strongly dependent on the Lode angle parameter, where stress states with small or negative values had limited ductility.

- Experimentally measured defect tolerance of L-PBF 316L under four stress states, providing an understanding of when failure became pore size and/or stress state dependent.

- This research as a whole provides information needed to design against deformation and failure in additively manufactured metals, specifically addressing stress state, anisotropy, and the effect of internal defects.
Appendix

Published papers during Ph.D. study


Bibliography


https://doi.org/10.1016/j.jmbbm.2008.05.004.


https://doi.org/10.1179/1743290114Y.0000000092.

https://doi.org/10.1016/j.msea.2015.10.073.


https://doi.org/10.1016/j.matdes.2018.06.049.


https://doi.org/10.1016/j.ijmecsci.2005.03.003.


https://doi.org/10.1007/BF01139164.

https://doi.org/10.1016/j.ijplas.2007.09.004.


https://doi.org/10.1016/j.ijsolstr.2015.05.006.


[115] Y. Li, D. Gu, Parametric analysis of thermal behavior during selective laser melting


[122] J.S. Keist, T.A. Palmer, Role of geometry on properties of additively manufactured


[130] D. Mohr, M. Oswald, A New Experimental Technique for the Multi-axial Testing


[138] A. Ghahremaninezhad, K. Ravi-Chandar, Ductile failure behavior of polycrystalline


https://doi.org/10.1016/j.msea.2015.10.068.


https://www.3dsystems.com/materials/laserform-stainless-316l.


https://doi.org/10.1016/j.actamat.2020.08.066.


VITA

Alexander E. Wilson-Heid

Alexander E. Wilson-Heid was born in 1993 in Fort Lewis, WA. He obtained his A.A. from St. Petersburg College in 2013 and a B.S. degree in Materials Science and Engineering from University of Florida in 2016. He started his Ph.D. studies in Materials Science and Engineering at the Pennsylvania State University in August 2016. His research involves investigating the defect tolerance, plasticity, and fracture behavior of additively manufactured metals.