EFFECTS OF DIMENSIONALITY, SAMPLE TOPOLOGY, AND DISORDER ON SUPERCONDUCTIVITY

A Thesis in

Physics

by

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Abstract

The primary goal of this thesis work is to explore various phenomena related to the effects of dimensionality, sample topology, and disorder on superconductivity, motivated by theoretical questions and potential applications of superconductivity. First, doubly connected ultrathin superconducting cylinders of Al with a diameter down to 100 nm were studied. Following the previous discovery of the destructive regime, the suppression of superconductivity at zero temperature around half-integer flux quanta, in these cylinders, we have carried out detailed resistance vs. temperature measurements, revealing the possible existence of phase separation near the quantum phase transition (QPT) at the onset of the destructive regime. Resistance vs. magnetic field measurements suggested that the QPT might be continuous, and the normal state inside the destructive regime was possibly characterized by quantum fluctuations of superconductivity. Other interesting phenomena, such as a new phase diagram not considered in the original theory of de Gennes and the thermally activated phase slips were also observed in these ultrathin cylinders.

Destructive regime physics of the Al cylinders relies on the doubly connected geometry. To gain further insight into the effect of sample topology on superconductivity in reduced geometry, we also studied the singly connected nanowires grown by electrochemical deposition in a porous host medium as a comparison to the doubly connected cylinders. Ru, with its oxide conducting, was chosen in this study for its potential to circumvent the oxidation problem encountered in measurements on individual nanowires.
released from their host medium. Electrical transport measurements on arrays of Ru nanowires as well as on individual nanowires were attempted down to 0.3 K in a $^{3}$He refrigerator, with a particular sample measured down to 50 mK in a dilution refrigerator. These Ru nanowires had a diameter $\geq 50$ nm depending on the pore size of the host medium. However, no superconductivity was found. The loss of superconductivity was attributed to the ultrasmall grain size ($\approx 2$ nm) in these nanowires as revealed by high-resolution transmission electron microscopy.

Finally, analysis of our previous measurements on mesoscopic superconducting Au$_{0.7}$In$_{0.3}$ rings is included in this thesis as an example of effects of strong disorder on mesoscopic superconductivity.
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Chapter 1

Superconductivity in Reduced Dimensions
and the Effects of Sample Topology and Disorder:
Introduction and Theoretical Backgrounds

In this chapter I will give a brief overview of superconductivity and related phenomena in disordered, low-dimensional systems. The purpose of this is to provide a context in which the experimental results are discussed in this thesis. The first section introduces some basic concepts in superconductivity, while the later sections are devoted to discussions of suppression of superconductivity due to low dimensionality, strong disorder, or sample topology.

1.1 Superconductivity and Off-Diagonal Long Range Order

The origin of superconductivity is the pairing of two electrons to form a Cooper pair. In 1956, Cooper showed that the Fermi sea of electrons is unstable against the formation of bound pairs as long as there is an attractive interaction between electrons around the Fermi surface [13]. For conventional superconductivity, the attractive interaction comes from the electron-phonon interaction, as suggested originally by the isotope effect [57, 70]. Consider now two electrons interacting via an attractive potential $-V$ above the Fermi sea ($V > 0$), which is itself not affected by $-V$. Cooper showed that
two electrons with zero total momentum have a bound state with an eigen energy

\[ E \approx 2E_F - 2\hbar \omega_D e^{-2/N(0)V} \quad (1.1) \]

in the weak-coupling limit, \( N(0)V \ll 1 \), where \( \omega_D \) is Debye frequency, \( N(0) \) is the density of states at the Fermi level, and \( E_F \) is the Fermi energy.

In the BCS theory [7], the ground state of a superconductor can be viewed as a linear superposition of paired states in which \( \vec{k} \uparrow \) and \( -\vec{k} \downarrow \) are either both occupied or both empty, in the form

\[ |\psi_G\rangle = \prod_{\vec{k}} (u_{\vec{k}} + v_{\vec{k}} c_{\vec{k}\uparrow}^+ c_{-\vec{k}\downarrow}^+) |0\rangle, \quad (1.2) \]

where \( |u_{\vec{k}}|^2 + |v_{\vec{k}}|^2 = 1 \), \( c_{\vec{k}\uparrow}^+ \) and \( c_{-\vec{k}\downarrow}^+ \) are creation operators that create an electron of momentum \( \vec{k} \) and spin up and another electron of momentum \( -\vec{k} \) and spin down, and \( |0\rangle \) is the vacuum state with no particle present. The formation of Cooper pairs makes the scatterings involving the electronic states \( (\vec{k} \uparrow, -\vec{k} \downarrow) \) correlated, which reflects the microscopic coherence in superconductivity. Essentially, any scattering of an electron from \( \vec{k} \uparrow \) to \( \vec{l} \uparrow \) due to an external perturbation must be accompanied by another scattering from \( -\vec{k} \downarrow \) to \( -\vec{l} \downarrow \) in the superconducting state. Therefore, the Hamiltonian of a superconducting state can be approximated by the pairing Hamiltonian,

\[ H = \sum_{\vec{k}\sigma} \epsilon_{\vec{k}\sigma} n_{\vec{k}\sigma} + \sum_{\vec{k}\vec{l}} V_{\vec{k}\vec{l}} c_{\vec{k}\uparrow}^+ c_{\vec{l}\downarrow} c_{-\vec{k}\downarrow} c_{\vec{l}\uparrow}, \quad (1.3) \]
where the first term gives the energy of the quasiparticles forming the pairs, and the second term characterizes the pairing interaction. $V_{\vec{k}\vec{l}}$ is the scattering matrix element between the pair states $\vec{k}$ and $\vec{l}$. If $V_{\vec{k}\vec{l}}$ can be simplified as $-V$ with the pair states within an energy $\hbar \omega_D$ near the Fermi surface and 0 otherwise, an energy gap $\Delta$ can be defined as

$$\Delta = \Delta_{\vec{k}} = -\sum_{\vec{l}} V_{\vec{k}\vec{l}} u_\vec{k} v_\vec{l} \approx 2 \hbar \omega_D e^{-1/N(0)} V$$ (1.4)

in the weak-coupling limit. The meaning of $\Delta$ is that for a quasiparticle excitation of momentum $\vec{k}$ in the superconducting state, the excitation energy relative to the Fermi surface is

$$E_{\vec{k}} = \sqrt{\Delta^2 + \varepsilon_{\vec{k}}^2},$$ (1.5)

where $\varepsilon_{\vec{k}}$ is the corresponding excitation energy in the normal state. Therefore near the Fermi surface, there is an energy gap of $2\Delta$ in the superconducting state.

A phenomenological theory for superconductivity, the Ginzburg-Landau (G-L) theory [35], can be more convenient to use in some cases, especially for describing spatial inhomogeneity in a superconductor. In the G-L theory, a complex function $\psi(\vec{r}) = \sqrt{n_s(\vec{r})} e^{-i\phi}$ was introduced as the superconducting order parameter, where $n_s(\vec{r})$ is the local superfluid density and $\phi$ is phase of the order parameter. The free energy density $f$ can be expanded in a series of the form

$$f = f_{n0} + \alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 + \frac{1}{2m^*} |(\hbar i \nabla - \frac{e^*}{c} \vec{A})\psi|^2 + \frac{\hbar^2}{8\pi},$$ (1.6)
where \( f_{n0} \) is the free energy density in normal state, \( m^* \) the effective mass of a Cooper pair, \( e^* \) the effective charge, \( \vec{A} \) the vector potential, \( h \) the local magnetic field and \( \alpha \) and \( \beta \) are temperature-dependent coefficients related to the parameters of the superconductor. Minimization of \( f \) yields the well-known G-L equations

\[
\alpha \psi + \beta |\psi|^2 \psi + \frac{1}{2m^*} \left( \frac{\hbar}{i} \nabla - \frac{e^*}{c} \vec{A} \right)^2 \psi = 0, \tag{1.7}
\]

and

\[
\vec{J} = \frac{e^*}{m^*} |\psi|^2 \left( \hbar \nabla \phi - \frac{e^*}{c} \vec{A} \right) = e^* |\psi|^2 \vec{v}_s, \tag{1.8}
\]

where \( \vec{J} \) is the supercurrent density and \( \vec{v}_s \) the supercurrent velocity.

Superconducting state is a quantum state with a long range order, which is manifested through the macroscopic coherence of the superconducting order parameter \( \psi(\vec{r}) \). For a long range order, the correlation function of the order parameter at two points separated by a distance \( \vec{r} \) is

\[
\langle \psi(0) \psi(\vec{r}) \rangle \sim C, \tag{1.9}
\]

where the angle brackets indicate thermodynamic averaging and \( C \) is a nonzero constant even as \( |\vec{r}| \to \infty \). The macroscopic coherence is also referred to as the off-diagonal long range order (ODLRO), a name suggested by Yang [88]. ODLRO can be defined mathematically through reduced density matrix of the form,

\[
\rho_N(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N; \vec{r}_1', \vec{r}_2', ..., \vec{r}_N') = \psi^*(\vec{r}_1, \vec{r}_2, ..., \vec{r}_N) \psi(\vec{r}_1', \vec{r}_2', ..., \vec{r}_N'), \tag{1.10}
\]
where \( \psi \) is the wave function and \( \vec{r}_1 \) and \( \vec{r}_1' \) are coordinates for individual particles. The reduced density matrix \( \rho_1(\vec{r}_1, \vec{r}_1') \) can be obtained from \( \rho_N \) by setting all but one set of coordinates to be equal and integrating them out,

\[
\rho_1(\vec{r}_1, \vec{r}_1') = \int d^3\vec{r}_2 d^3\vec{r}_3 \ldots d^3\vec{r}_N \rho_N(\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots, \vec{r}_N; \vec{r}_1', \vec{r}_2, \vec{r}_3, \ldots, \vec{r}_N).
\]  (1.11)

In the presence of translational symmetry

\[
R(r) = R(|\vec{r}_1 - \vec{r}_1'|) = \nu \rho_1(\vec{r}_1, \vec{r}_1'),
\]  (1.12)

where \( \nu \) is the volume of the system. The quantity \( \rho_1(\vec{r}_1, \vec{r}_1') \) is an alternative definition of the quantity \( \langle \psi^+(\vec{r})\psi(\vec{r}) \rangle \) [54]. In a superconducting state, \( R(r) \) goes to a constant as \( r \to \infty \), representing the ordering in the off-diagonal term of the reduced density matrix.

### 1.2 Fluxoid Quantization and Little-Parks Resistance Oscillation

Fluxoid quantization is a result of the global phase coherence, which leads to the Little-Parks (L-P) resistance oscillation [51]. Let us consider a doubly connected cylinder with magnetic field \( H \) applied along the cylinder axis. If the whole cylinder is superconducting such that it can be described by a single wave function \( \psi(\phi) \),

\[
\psi(\phi) = \psi(\phi + 2l\pi),
\]  (1.13)
where $\phi$ is the phase and $l$ is an integer. The superfluid velocity, $\vec{v}_s$, can be defined as

$$m^* \vec{v}_s = \hbar \nabla \phi - \frac{e^*}{c} \vec{A},$$

(1.14)

where $\vec{A}$ is the vector potential, $m^*$ is the effective mass of a Cooper pair and $e^*$ the effective charge.

Consider a contour circling around the cylinder. If the cylinder thickness $t \gg \lambda$, where $\lambda$ is the penetration depth, we can choose the contour deep inside the superconductor so that $\vec{v}_s = 0$. Therefore along the contour,

$$\hbar \oint \nabla \phi \cdot d\vec{l} = \frac{e^*}{c} \oint \vec{A} \cdot d\vec{l}.$$  

(1.15)

Integrating along the contour back to the starting point, we have

$$\Phi = l\Phi_0,$$

(1.16)

where $\Phi_0 = \frac{hc}{2e}$ in Gaussian units is the flux quantum $2.07 \times 10^{-7} \text{ Gcm}^2$. Therefore flux quantization is a consequence of global phase coherence. Experimentally, it was discovered in 1961 by Deaver and Fairbank [6] and Doll and N"abauer [17]. The flux quantization experiments provided direct evidence that the carriers in a superconductor are pairs of electrons.
Now consider the case $t < \lambda$. $\vec{v}_s \neq 0$ along the contour. Integrating Eq. 1.14 along the circumference gives the formula of fluxoid quantization

$$\Phi' = \Phi + \frac{m^* c}{e^*} \oint \vec{v}_s \cdot d\vec{s} \equiv l\Phi_0.$$  

(1.17)

Assuming $|\vec{v}_s| = v_s$ is independent of the radius which is valid for a film with thickness $t < \lambda$, we have

$$v_s = \frac{2\hbar}{m^* d}(l - \frac{\Phi}{\Phi_0}),$$  

(1.18)

where $l$ is the integer that minimizes $v_s$. It is seen that $v_s$ is modulated by the applied flux $\Phi$, so is the kinetic energy $m^* v_s^2/2$. The $\Phi$-dependences of $v_s$ and $m^* v_s^2/2$ are plotted in Fig. 1.1. The kinetic energy is maximal at half-integer flux quanta.

Fig. 1.1. a) $v_s$ as a function of $\Phi/\Phi_0$; b) $m^* v_s^2/2$ as a function of $\Phi/\Phi_0$. 
The modulation of the kinetic energy $m^*v_s^2/2$ leads to the modulation of $\Phi - T$ phase boundary and the Little-Parks resistance oscillation [51], as the resistance varies with the free energy of a superconductor in the transition region. At fixed $T$, free energy is maximal at half-integer flux quanta, so is the resistance. In the resistance vs. magnetic field measurement of a doubly connected superconductor at fixed $T$, resistance oscillates with a period of $hc/2e$ in Gaussian units.

1.3 Loss of Long Range Order in Low Dimensions

There has long been an understanding that the long range order rarely exists in low-dimensional systems. The simplest case is the 1D Ising model, where the entropy gain due to the formation of random domain walls in a chain of spins, with two states available for each spin and nearest neighbor exchange interaction, could destroy an ordered state at any finite temperature. Van Hove proved that an ordered state does not exist in a 1D classical gas with hard-core and finite-range interactions [40]. Hohenberg [39] showed that an inequality due to Bogoliubov [10] can be used to exclude superfluid or superconductivity defined in the conventional way in one or two dimensions.

The superconducting order parameter, $|\psi|e^{-i\phi}$, has two independent variables, the amplitude $|\psi|$ and the phase $\phi$. Therefore superconductivity can be described by the XY model. Here we consider the XY model in $D$ dimensions [9] following the original work of Mermin and Wagner [59]. We will show that there is no long range order in $D \leq 2$ dimensions at any finite temperature, $T \neq 0$. 
The effective Landau-Ginzburg Hamiltonian density is given by

\[ H_{LG} \equiv \frac{1}{2} \alpha^2 |\nabla \psi|^2 + \frac{1}{2} \mu^2 |\psi|^2 + \frac{1}{4!} \lambda (|\psi|^2)^2. \]  

(1.19)

In an ordered state, \( H_{LG} \) is minimized by a uniform order parameter \( \psi(x) = \psi_0 \) of constant magnitude \( |\psi_0|^2 = -6\mu^2/\lambda \). Consider the possible energy excitations of the ordered system with a configuration \( \psi(x) = \psi_0 + \delta \psi \), where \( \delta \psi \) represents the thermal fluctuations of the order parameter, which can be decomposed into \( \delta \psi_\parallel \) and \( \delta \psi_\perp \), corresponding to the longitudinal and transverse components, respectively. The components in \( \delta \psi \) that contribute to energy changes in \( H_{LG} \) are, neglecting the higher order terms,

\[ |\nabla \delta \psi|^2 = |\nabla \delta \psi_\parallel|^2 + |\nabla \delta \psi_\perp|^2, \]  

(1.20)

for the first term (gradient) on the right side of Eq. 1.19, and

\[ (\delta |\psi|)^2 = (\delta \psi_\parallel)^2 + O(\delta \psi_\parallel^3), \]  

(1.21)

for the second (quadratic) and third (quartic) terms on the right side of Eq. 1.19. Putting this all together, we have an effective Hamiltonian which is similar to a Gaussian model,

\[ \Delta H = \int d^Dx \left[ \frac{1}{2} \alpha^2 (|\nabla \delta \psi_\parallel|^2 + |\nabla \delta \psi_\perp|^2) + \mu^2 (|\delta \psi_\parallel|^2 + O(|\delta \psi|^3)) \right]. \]  

(1.22)
The mean-squared value of the fluctuations can be obtained by Fourier transform of the partition function in the Gaussian model with

\[ \delta \psi(x) = \int \frac{d^D k}{(2\pi)^D} e^{i k \cdot x} \tilde{\psi}(k). \]  

(1.23)

For \( \delta \psi_\parallel \), we have

\[ \langle \delta \psi_\parallel(x)\delta \psi_\parallel(x) \rangle = \int_0^\Lambda \frac{d^D k/(2\pi)^D}{\alpha^2 k^2 + 2|\mu|^2}, \]  

(1.24)

where \( \Lambda \) is the cutoff parameter, which excludes any fast variation of \( \psi(x) \) on a length scale \( \leq 1/\Lambda \). For \( \delta \psi_\perp \), on the other hand,

\[ \langle \delta \psi_\perp(x)\delta \psi_\perp(x) \rangle = \int_0^\Lambda \frac{d^D k/(2\pi)^D}{\alpha^2 k^2}. \]  

(1.25)

The integrand in Eq. 1.25 contains a singularity at \( k = 0 \). For \( D \geq 3 \), this singularity is integrable and the transverse fluctuations in the order parameter \( \delta \psi_\perp(x) \) are finite. However, for \( D \leq 2 \), this singularity is not integrable and the mean squared value of fluctuations of the assumed ordered state are infinite. Therefore the assumption of an ordered state is incorrect.

In conclusion, a long range order for a system of a multi-component order parameter cannot exist in dimensions of \( D \leq 2 \) at nonzero temperatures. This is known as the Mermin-Wagner theorem [59]. Real superconductivity with ODLRO, described by a two-component (amplitude and phase) order parameter, does not exist in 2D films and 1D wires at \( T \neq 0 \). However, superconductivity can exist in 2D films and is characterized by a quasi long range order at finite \( T \). The superconducting transition associated
with this quasi long range order is described by the Kosterlitz-Thouless (K-T) transition, discussed in next section.

1.4 Vortex-antivortex Unbinding and Kosterlitz-Thouless Transition in 2D

As discussed in the previous section, a true long range order is impossible at nonzero temperatures in 2D. Therefore, superconductivity in 2D at nonzero temperatures can only be characterized by the so-called quasi long range order, which requires that the correlation of the order parameter, \( \langle \psi(0)\psi(r) \rangle \), decrease as \( 1/r^{\eta} \), where \( r \) is the spatial separation, different from the invariant correlation for long range order and the \( e^{-r/\xi_l} \) correlation for non-ordered state, where \( \xi_l \) is the localization length.

The lack of long range order in a 2D superconducting film means that the superconducting order parameter tends to vary spatially even in zero magnetic field. There may be patches in the film where the order parameter tends to fluctuate to zero from time to time. These patches can be viewed as a collection of vortices and antivortices, which feature a normal core with a diameter \( \sim 2\xi \) surrounded by circulating supercurrents. Vortices and antivortices are topological excitations in 2D superconductors, which always appear in pairs in zero magnetic field. Figure 1.2 is an illustration of a vortex-antivortex pair in the presence of an applied supercurrent \( I \).

A 2D superconducting film will exhibit superconducting properties at low enough temperatures even though vortex-antivortex pairs exist. This is because vortices and antivortices are bound into pairs. The binding energy is \( \Delta E = 2\pi E_j \ln (r/\xi) \), where \( r \) is the separation between the two vortices, and \( E_j \sim \Phi_0^2/\pi \lambda_\perp \) and \( \lambda_\perp = \lambda^2/t \) where
Fig. 1.2. Schematic of a vortex-antivortex pair. The normal core has a diameter of $2\xi$, where $\xi$ is the superconducting coherence length. The vortex and antivortex experience forces $F$ and $-F$, respectively, exerted by the supercurrent $I$. This drawing was produced by N. Kurz.

$\lambda$ is the penetration depth and $t$ is the film thickness. As long as the vortices and antivortices are bound in pairs, they do not move freely in the film due to equal but opposite Lorentz forces they experience. The Lorentz force is in the form $\vec{f} = \vec{J} \times \Phi_0/c$ in Gaussian units, where $\vec{J}$ is the total supercurrent density at the location of the core of the vortex in consideration, $c$ the speed of light, and $\Phi_0$ the total flux threading a vortex [82]. Immobile vortices and antivortices give rise to no resistance since a resistive voltage is only generated across the film due to flux flow [82], with the voltage proportional to the number of free vortices and antivortices. However, as temperature increases to a critical value, $T_{KT}$, vortex-antivortex pairs start to unbind. The transition temperature can be estimated from the free energy change $\Delta F = \Delta E - T\Delta S$. Here $\Delta S = k_B \ln [(r/\xi)^4] - k_B \ln [(r/\xi)^2]$, where the first term on the right side is the entropy for a free vortex and a free antivortex, and the second term is for a vortex-antivortex pair. At $T_{KT}$, $\Delta F = 0$. Above $T_{KT}$, superconductivity is destroyed due to the freely
moving vortices and antivortices which give rise to a finite resistance. The transition is known as Kosterlitz-Thouless (K-T) transition [46, 82].

The experimental signatures of the K-T transition are a nonlinear $I - V$ characteristic and a particular temperature dependence of the sample resistance. The key point is that in order to see flux-flow dissipation, there must be free vortices and antivortices. Above $T_{KT}$, they exist due to thermal unbinding. Below $T_{KT}$, in a $I - V$ measurement, they are generated by current-induced unbinding. With a nonzero supercurrent density of $J$ in the film, vortices and antivortices experience a Lorentz force with a magnitude of $F = J\Phi_0/c$, which modifies the binding energy of a vortex-antivortex pair to $\Delta E' = 2\pi E_j \ln(r/\xi) - F \cdot r$. As a result, the number of current-induced free vortices and antivortices is given by $N_F \propto J^{\pi E_j/k_B T}$. Since the film resistance $R \sim N_F$,

$$V = RI \sim I^{1+(\pi E_j/k_B T)}.$$  \hfill (1.26)

$T_{KT}$ is defined by

$$k_B T_{KT} = (\pi/2)E_j(T_{KT}).$$  \hfill (1.27)

A renormalization group calculation taking into account the screening effects of other vortices, antivortices, and vortex-antivortex pairs resulted in only small corrections to the results presented above [46]. Therefore, near $T_{KT}$, $V \sim I^\alpha(T)$, where $\alpha > 3$ below $T_{KT}$, $\alpha = 3$ at $T_{KT}$, and $\alpha = 1$ above $T_{KT}$. The resistance right above $T_{KT}$ varies as

$$\exp \left[ -\sqrt{(T_{co} - T_{KT})/(T - T_{KT})} \right],$$

where $T_{co}$ is the mean-field transition temperature.
1.5 Quantum Phase Transitions in 2D Superconductors: Effects of Dimensionality and Disorder

1.5.1 2D Superconductor-to-Insulator Transition and Scaling

The interplay between superconductivity and localization is an important topic in condensed matter physics. Localization enhances the effective Coulomb repulsion by reducing the screening [50], which acts against electron pairing. Superconductivity in 2D as defined in the picture of K-T transition is suppressed with strong enough disorder, resulting in a superconductor-to-insulator transition (SIT) at zero temperature. It is generally believed that the 2D SIT is a continuous quantum phase transition (QPT), as suggested by a scaling analysis of the experimental data obtained in quench condensed Bi films [31]. Therefore we can map it to a classical phase transition in 3D. In the original study on Bi films [38], it was found that the film switched from insulating to superconducting as the film thickness was increased. A temperature-independent resistance separatix $R_0$ appeared to be present in the low-temperature limit [52].

An influential theory for 2D SIT is the so-called dirty Boson model. Here the description of the $T = 0$ SIT starts from the hypothesis that the transition is described by a model of charge-2e bosons moving in a 2D random potential in the superconducting phase [33]. In the insulating phase, the electron pairs are localized by the disorder. As argued by Fisher et al., the critical behavior is dominated by bosonic degree of freedom [33]. Alternatively, near SIT, the homogeneously disordered superconducting film can be considered as a network of Josephson-coupled superconducting “droplets”, because superconductivity is nearly suppressed [95]. Because of the relation, $\Delta N \Delta \phi > 1$, 

where \( N \) is the number of Cooper pairs and \( \phi \) is the phase of the superconducting order parameter of a droplet, the suppression in \( \Delta N \) due to the charging energy of a small droplet enhances \( \Delta \phi \), leading to the phase fluctuation, and loss of superconductivity at \( T = 0 \).

A continuous SIT can be characterized by a divergent correlation length and correlation time, \( i.e. \), the order parameter fluctuates coherently over increasing distance and time while approaching the quantum criticality. The correlation length, \( \xi \), diverges as \( \xi \sim \delta^{-\nu} \), where \( \delta \) measures the “distance” to the critical point, which can be, \( e.g., \), \( |d - d_c| \), where \( d \) is the thickness of the film and \( d_c \) is the critical thickness. The correlation-length exponent \( \nu \geq 2/D \) [34]. The time direction might involve a different set of interactions than the spatial directions, which leads to a distinct correlation time \( \xi_\tau \) [79]. \( \xi_\tau \sim \xi^z \sim \delta^{-\upsilon z} \), where \( z \) is the dynamical-scaling exponent.

Near a continuous QPT, the physics of the system is controlled by the diverging \( \xi \) and \( \xi_\tau \). As a consequence, all physical quantities scale with \( \xi \) and \( \xi_\tau \), which diverge as a function of \( \delta \), the “distance” from the quantum criticality. For an example, the energy density, \( f_s \), satisfies

\[
 f_s(\delta) \sim b^{-(D+z)} f_s(b^{1/\upsilon} \delta) \sim \delta^{\upsilon(D+z)} \sim \xi^{-(D+z)}, \tag{1.28}
\]

where \( b \) is a rescaling parameter. The second equality above follows from the choice \( b = \delta^{-\upsilon} \).

The superfluid density, \( \rho_s \), a measure of the spatial stiffness of the phase, \( \phi \), of the order parameter, \( \psi \), does not follow immediately from Eq. 1.28. The scaling behavior
was given by Fisher et al. [34] as

\[ \rho_s \sim \xi^{-(D+z-2)}. \] (1.29)

Scaling of the frequency-dependent conductivity near 2D SIT can be obtained from the relation

\[ \sigma(\omega) = (2e)^2 \rho_s(-i\omega)/(-im\omega), \] (1.30)

where \( \rho_s(\omega) = \langle |\psi|^2 \rangle \) is a generalized frequency-dependent superfluid density and \( m \) is the mass of a boson. Near the transition, \( \rho_s \) should scale with \( \xi \) and \( \xi_{\tau} \) in the form

\[ \rho_s(\omega) = \xi^{-D}(\xi/a)^{2-z} \tilde{\rho}_s(\omega\xi_{\tau}), \] (1.31)

where \( \tilde{\rho}_s \) is an appropriate dimensionless scaling function, and

\[ \xi_{\tau} = (ma^2/\hbar)(\xi/a)^z \] (1.32)

with \( a \) a short-distance cutoff. Introducing the short-distance cutoff \( a \) also matches the units in Eqs. 1.31 and 1.32. Equation 1.31 recovers Eq. 1.29 with \( \tilde{\rho}_s(x \to 0) \) approaches a constant. As both \( \xi \) and \( \xi_{\tau} \to \infty \) near the transition, \( \rho_s(\omega) \) is finite. Therefore \( \tilde{\rho}_s(\omega) = c_D x^{(D+z-2)/z} \), with \( x \equiv \omega\xi_{\tau} \) and \( c_D \) is a dimensionless constant. Combining this with Eq. 1.30 we have at criticality

\[ \sigma(\omega, \xi = \infty) = c_D(e^2/\hbar)a^{2-D}(-i\hbar\omega/ma^2)^{(D-2)/z}. \] (1.33)
The scaling theory therefore predicts that at \( T = 0 \), conductivity at the criticality is a finite constant, \( c_D e^2 / h \), in the dc limit. Therefore at the SIT criticality the system exhibits true metallic conduction even at \( T = 0 \), which was thought not possible in a 2D fermionic system [50].

### 1.5.2 2D Superconductor-to-Normal Metal Transition

The behavior of the amplitude of the superconducting order parameter near the SIT is of particular importance in deciding whether the dirty boson model is relevant. Tunneling studies of quench-condensed Pb and Sn films grown on \( \alpha \)-Ge substrates [42] and Bi films on \( \alpha \)-Sb substrates [43] appeared to imply that the amplitude of the superconducting order parameter vanishes as the SIT is approached. Mason and Kapitulnik also argued that there is a finite range of the control parameter in which metallic state exists, e.g., there is a superconductor-to-normal metal transition (SNT) tuned by a magnetic field applied perpendicular to the film. Their argument is based on their experimental data of MoGe films [56].

On the theoretical side, Feigelman and Larkin considered a model of superconducting islands of size \( u \) coupled via a disordered two-dimensional conductor with the dimensionless conductance \( g = h / (e^2 R_\square) \gg 1 \) [32]. They found that at \( T = 0 \) the macroscopic superconductivity is destroyed if the lattice spacing \( w \gg u \) and \( g < g_c \approx 0.1 \ln(w/u)^2 \), because of the fluctuation in the amplitude of the order parameter. However, the critical concentration of grains \( (\approx u/w) \) with \( w \gg u \) is very small, suggesting that realistically a SNT only happens in high sheet resistance films.
with $g$ close to 1. Experimentally, field-tuned SNT was observed in very low sheet resistance ($g \gg 1$) films of Au$_{0.7}$In$_{0.3}$ [72], where the critical concentration appears to be much larger than the theoretical value, and in other systems [56].

### 1.5.3 Superconducting Glass Phase and Droplet State

In a thin-enough film with a magnetic field applied parallel to the film, the suppression of superconductivity by the magnetic field is due to Zeeman splitting of electron energy levels. The value of the critical magnetic field $H^0_c$ is determined by the competition between the condensation energy density and the polarization energy density of the electron gas in the magnetic field. Zhou et al. calculated the total energy of a single domain of size $L_D \sim g\xi_0$ and found that its energy have random signs in a magnetic field $H$ close to $H^0_c$ such that $H \in [H^0_c - (H^0_c/g^2), H^0_c]$. Therefore, the global superconductivity is unstable with respect to the creation of normal regions, referred to as the superconducting glass phase [96].

In a subsequent paper [95], Zhou et al. considered the case with magnetic field approaching from well above the critical field $H^0_c$. They found that even in magnetic fields substantially higher than $H^0_c$, instead of forming a spin-polarized disordered Fermi liquids, a homogeneous film of superconducting metal is characterized by the nucleation of superconducting pairing states of mesoscopic scales at $T = 0$, referred to as the droplet state. The statistics of these pairing states can be studied with the generalized G-L equations. At $H - H^0_c \gg H^0_c/g^2$, the droplet has a size of $L_f = \xi_0 \sqrt{H^0_c/(H - H^0_c)}$, with the typical $\Delta$ on the order $\Delta_0/g$. The separation between these droplet, $L_d$, is
of order of \( L f \text{erfc}^{-1/2}\left[Bg\sqrt{(H - H_c^0)/H_c^0}\right] \) where \( \text{erfc}(z) = (2/\sqrt{\pi})\int_z^\infty e^{-t^2} dt \) is the complementary error function.

1.6 Phase Slips in 1D Superconductors

ODLRO is impossible in a strictly 1D system in the thermodynamic limit even at \( T = 0 \). However, in superconducting wires with a diameter reasonably large, superconductivity is still possible. The pioneering work on superconducting wires was carried out in early 1970s. Newbower, Beasley and Tinkham measured the superconducting transition \( R(T) \) of tin whiskers with a diameter of 0.5 \( \mu \)m [65]. It was found that the transition was broadened as compared to that of bulk or film, which the authors attributed to phase slip processes. The exact theory was formulized by Langer and Ambegaokar [48], and reexamined by McCumber and Halperin [58] (LAMH). During a phase slip, the order parameter fluctuates to zero at some point along the wire in the time scale of \( \tau_{GL} \), the Ginzburg-Landau time, allowing the relative phase across the point to slip by \( 2\pi \), resulting in a voltage pulse. The sum of these pulses gives the resistive voltage. In the LAMH theory, phase slips occur via thermal activation as system passes over a free-energy barrier proportional to a volume of about \( A \cdot \xi \), where \( A \) is the cross-sectional area of the wire. The time averaged effect of the thermally-activated phase slips (TAPS) gives a broadened resistive transition \( R(T) \).

Giordano measured In wires with a diameter as small as 40 nm in late 1980s [36] and found an unusual resistance tail at lower temperatures in \( R(T) \). He attributed this resistance tail to the occurrence of phase slips via macroscopic quantum tunneling (MQT) through the free-energy barrier, referred to as quantum phase slips (QPS). Tinkham’s
group observed similar features in Mo$_{0.79}$Ge$_{0.21}$ wires by carbon nanotube template and claimed that a combination of the TAPS, QPS and a parallel normal conduction channel inherent to the wire could explain their experimental results reasonably well [49]. Later experiments on single-crystal Sn nanowires by electrochemical deposition [27] and on Al nanowires by e-beam lithography [94] also confirmed that the QPS model could explain the results reasonably well.

However, the QPS is yet to be accepted. One opinion is that the QPS phenomenon reported at least in the earlier papers was related to structural defects or weak links in the sample, and was not an intrinsic feature of the 1D systems, as argued by Duan [18]. More recently, Rogachev et al. reported that no significant contribution of QPS was detected in their Nb and Mo$_{0.79}$Ge$_{0.21}$ nanowires measured in magnetic fields up to 11 T [71].

Below we will write down the explicit formulas describing TAPS and QPS, following Tinkham and co-workers [49]. The contribution due to TAPS according LAMH is

$$R_{LAMH} = \frac{\pi \hbar^2 \Omega}{2e^2 k_B T} e^{-\Delta F/k_B T}, \quad (1.34)$$

where $\Omega = (L/\xi)(\Delta F/k_B T)^{1/2}(1/\tau_{GL})$ is the attempt frequency, and

$\Delta F = (8\sqrt{2}/3)(H_c^2/8\pi)A\xi$ is the energy barrier. The contribution due to QPS is

$$R_{MQT} = B \frac{\pi \hbar^2 \Omega_{MQT}}{2e^2 (\hbar/\tau_{GL})} e^{-a\Delta F \tau_{GL}/\hbar}, \quad (1.35)$$
where \( \Omega_{MQT} = \frac{(L/\xi)|\Delta F/(h/\tau_{GL})|^{1/2}}{(1/\tau_{GL})} \), and \( a \) and \( B \) are possible numerical factors of order unity. The total resistance in the superconducting channel should be \( R_{LAMH} + R_{MQT} \). Combining with the parallel normal channel gives the total resistance

\[
R = \left[ R_N^{-1} + (R_{LAMH} + R_{MQT})^{-1} \right]^{-1}. \tag{1.36}
\]

Substituting the G-L parameters \( H_C, \xi, \) and \( \tau_{GL} \) with their explicit temperature-dependent forms, a further simplification of Eq. 1.34 gives

\[
R_{LAMH} = 8 \frac{R_N}{0.83\pi} \frac{1}{(1-t)^{9/4}} \frac{e^{-c(1-t)^{3/2}/t}}{t^{3/2}}, \tag{1.37}
\]

where \( c = 0.83(R_q/R_N)(L/\xi(0)) \), \( R_q = h/4e^2 \), and \( t \) is the normalized temperature \( T/T_c \). Similar simplification of Eq. 1.35 gives

\[
R_{MQT} = B \frac{\sqrt{\pi}R_N}{1.66\sqrt{2}} \frac{1}{(1-t)^{3/4}} e^{-a\pi c(1-t)^{1/2}/8}. \tag{1.38}
\]

1.7 **Destructive Regime in Doubly Connected Superconductors: Effect of Sample Topology**

In a 1968 paper by Groff and Park [37], the effect of pair-breaking due to flux-induced supercurrent (Eq. 1.18) was discussed. They argued that the pair-breaking effect could be so strong that the superconductivity would be lost in small enough cylinders. Separately, Tinkham obtained the equation describing the phase diagram of a doubly
connected geometry from linearized G-L equations [81]:

$$T_c(H) = T_c(0) - \frac{T_c(0) R^2}{8 \lambda_e^2(0) H c_B(0)} [(H \cos \theta - \frac{l \Phi_0}{\pi R^2})^2 + \frac{l^2}{3 R^2} H^2 \cos^2 \theta + 4 H^2 \sin^2 \theta], \quad (1.39)$$

where $R$ is the radius, $\lambda_e$ is the effective penetration depth, and $\theta$ is the angle between the cylinder axis and the magnetic field $H$. A slight revision of Eq. 1.39 with the superconducting coherence length $\xi$ replacing the penetration depth $\lambda_e$ gives

$$T_c(H) = T_c(0) - 4 T_c(0) \frac{\xi(0)}{d} (l - \frac{\Phi}{\Phi_0})^2 - T_c(0) \frac{16 l^2 \xi(0)^2}{3 d^4} \frac{\Phi}{\Phi_0}^2, \quad (1.40)$$

where $d$ is the diameter and the terms related to $\theta$ are omitted for simplicity. However, Tinkham did not consider the case for a cylinder with $d < \xi(0)$.

![Fig. 1.3. Schematic of the model in de Gennes’ theory. The coordinates along the ring circumference is $s$ and that along the side arm is $x$. The magnetic field is applied perpendicular to the ring.](image)

de Gennes considered in 1981 the magnetic response of granular superconductors using simplified model of a ring with a side arm as shown in Fig. 1.3. This particular
geometry was designed to capture the structural feature of granular superconductors. Using the gauge $A_{s,\parallel} = H d/4$ and $A_{x,\parallel} = 0$, de Gennes obtained the linearized Ginzburg-Landau equation along the ring as

$$[i \frac{\partial}{\partial s} - \chi]^2 \psi(s) = \frac{1}{\xi(T)^2} \psi(s),$$

(1.41)

where $\chi = (2/d)(\Phi/\Phi_0 - l)$ due to fluxoid quantization. Along the arm, however,

$$\frac{\partial^2}{\partial x^2} \psi(x) = -\frac{1}{\xi(T)^2} \psi(x).$$

(1.42)

Equation 1.41 and 1.42 have the solution of the forms

$$\psi(s) = e^{-i\chi s} (\alpha e^{is/\xi} + \beta e^{-is/\xi}),$$

(1.43)

and

$$\psi(x) = \psi_0 \cos (x/\xi) \sin (x/\xi),$$

(1.44)

respectively. The solution in Eq. 1.44 already takes into account the boundary condition, $\partial \psi(x)/\partial x = 0$ at $x = 0$ (no superfluid current at the end of the side arm). Matching the order parameter $\psi$ at the node where $x = L$, $s = 0$, and $s = \pi d$,

$$\psi_0 = \alpha + \beta,$$

(1.45)

and

$$\psi_0 = e^{-i\chi \pi d} (\alpha e^{i\pi d/\xi} + \beta e^{-i\pi d/\xi}).$$

(1.46)
Noting that at $x = L, s = 0$, and $s = \pi d$, the superfluid current is zero. Therefore

$$i(\alpha - \beta) - ie^{-i\chi\pi d}(\alpha e^{i\pi d/\xi} - \beta e^{-i\pi d/\xi}) + \psi_0 \tan \left( \frac{L}{\xi} \right) = 0. \quad (1.47)$$

The phase boundary of this superconducting structure can be obtained by solving Eqs. 1.45, 1.46 and 1.47 for three unknowns $\alpha$, $\beta$, and $\psi_0$, which have solutions only if the relevant determinant is zero. The equation for the phase boundary so obtained has the form,

$$\cos \left( 2\pi \frac{\Phi}{\Phi_0} \right) = \cos \left( \frac{\pi d}{\xi} \right) - \frac{1}{2} \sin \left( \frac{\pi d}{\xi} \right) \tan \left( \frac{L}{\xi} \right). \quad (1.48)$$

If $L = 0$, the sample structure reduces to a simple ring without the side arm. In this case, Eq. 1.48 becomes

$$\cos \left( 2\pi \frac{\Phi}{\Phi_0} \right) = \cos \left( \frac{\pi d}{\xi} \right), \quad (1.49)$$

where $\xi = \xi(T) \geq \xi(0)$. When $d < \xi(0)$, Eq. 1.49 has no valid solution at half-integer flux quanta, which means a superconducting state is impossible in rings with $d < \xi(0)$ around half-integer flux quanta, even in the zero temperature limit. de Gennes referred to the case of $d < \xi(0)$ as the destructive regime.
Chapter 2

Experimental Techniques

2.1 Preparation and Characterization of Ultrathin Superconducting Cylinders of Al

Figure 2.1 shows the process of fabricating a cylindrical sample of Al. To start with, a fine filament is pulled from quartz melt, which is laid down across the gap of a home-made U-shape glass slide with a thickness \( \approx 0.15 \) mm. Both ends of the filament are tacked down with a dab of G.E. vanish. The gap width of the U-shape glass slide is \( \leq 2 \) mm. The distance between the filament and the bottom of the gap is typically \( \geq 5 \) mm. The diameter of the filament is typically between 100 nm to 200 nm.

Filaments are screened first by eyes with strong light (Fiber-lite 190) shedding perpendicularly from the side and then by a 10 X optical lens together with a 1/2” CCD (Sony XC-999, digital magnification on our display \( \approx 42 \) X). Filaments with a diameter around 200 nm or less determined by the optical microscope are picked out.

Aluminum is thermally evaporated onto the rotating quartz filament with a vacuum in the range of \( 10^{-6} \) torr. The rotation speed of the filament is controlled at around 1 rev/s. The deposition rate ranges from below 20 Å to above 100 Å depending on the desired film property (Appendix A). During evaporation, a part of the filament’s circumference is partially shadowed by the bottom part of the U-shape glass slide and by a holder clamping the glass slide. However, the resulted variation of film thickness is
Fig. 2.1. Illustration of fabrication of a cylinder sample in the sequence from a to d.
estimated to be less than 3%. The film thickness \( t \) is nominally the value measured by the quartz crystal thickness monitor divided by \( \pi \), including the oxidized surface layer. Al is chosen for its long coherence length and homogeneity in its thin-film form.

After deposition, the U-shape glass slide with the Al cylinder is mounted on an alumina or quartz substrate. Meanwhile, another small piece of 0.15-mm-thick glass slide is used to support the suspended cylinder from underneath. At least four 25-\( \mu \)m-diam Au wires are attached to the cylinder using Ag epoxy or paste. Au wires are also attached to both ends of the cylinder on the glass slide, which can be used for current leads. The Ag epoxy or paste dots also served to thermally and structurally anchor the cylinder. Copper wires of 50 \( \mu \)m in diameter are then used to connect the Au wires to connectors on the sample stage for measurement. We tried to evaporate Au film onto the cylinder at several spots before the Au wires were attached, intended for better contacts, which did not yield a better result (Appendix A).

Samples are measured in a dilution or a \( ^3 \)He refrigerator. The cylinders are manually aligned to be parallel to the magnetic field, estimated to be within \( \pm 5^\circ \). The diameter of the cylinder is inferred from the resistance vs. magnetic field, \( R(H) \), measurement using

\[
d = \sqrt{\frac{4\Phi_0}{\pi \Delta H}},
\]

where \( \Phi_0 \) is the flux quantum \( 2.07 \times 10^{-7} \) Gcm\(^2\) and \( \Delta H \) is the magnetic field period of the resistance oscillation in \( R(H) \). The \( d \) inferred this way should correspond to \( d_f + t/2 \), where \( d_f \) is the filament diameter and \( t \) is the film thickness. The percentage error of \( d \)
due to a small uncertainty of $\Delta H$ is given by

$$\frac{\Delta d}{d} = -\frac{\pi}{8\Phi_0} d^2 \delta(\Delta H). \tag{2.2}$$

For a cylinder around 150 nm, a $\delta(\Delta H)$ of $\approx 20$ G, usually resulted from either trapped flux or large step size in $R(H)$ scan, only gives less than 1% error in $d$. The percentage error of $d$ due to a small misalignment is given by

$$\frac{\Delta d}{d} = \frac{1}{2} \cos(\theta)^{-3/2} \sin(\theta) \Delta \theta, \tag{2.3}$$

where $\theta$ is the misaligned angle between the cylinder axis and the magnetic field. A $\pm 5^\circ$ would give an error of $\Delta d/d$ less than 0.4%. Therefore the $d$ estimated from the magnetic field period $\Delta H$ in $R(H)$ oscillation is very accurate, agreeing very well with that from the scanning electron microscope (SEM) image of the cylinder.

### 2.2 Preparation and Characterization of Ru Nanowires

Nanowires of Ru were grown in commercial, track-etched polycarbonate membranes with various nominal pore diameters by our collaborators [24]. Membrane thickness is 6 $\mu$m and the pore density is $6 \times 10^8$ cm$^{-2}$. The pores in the membrane are separated straight channels and are perpendicular to the surface of the membrane within $\pm 17^\circ$. The electrolyte used for electrochemical deposition is commercially available Ru plating solution (purity 99.9%, Technic Inc.). Prior to the electrochemical deposition, a 200 nm layer of Ag or Au was evaporated onto one side of the membrane as the cathode.
A 0.8-mm-diam Pt wire was used as the anode. The electrochemical deposition was done under a constant voltage of 2.2 V. After depositing Ru, a layer of Ag or Au, used as the counter-electrode, was prepared on the finishing side of the membrane either by electrochemical deposition or thermal evaporation.

The diameters of Ru nanowires are usually different from the nominal pore diameters of the membranes specified by the vendor. For example, wires of 50 and 100 nm in diameters were obtained from membranes with nominal pore diameters 30 and 50 nm, respectively, as determined by transmission electron microscope (TEM) imaging. The Ru nanowires resulted from nominally 70-nm-diam-pore membranes were found to be 70 nm in diameter, inferred from imaging the Ru nanowires in measurement devices prepared by photolithography using atomic force microscope (AFM). The height of these Ru nanowires was close to be 70 nm while the width was found to range from 120 nm to 200 nm.Photoresist residue might have accumulated close to the wire, or alternatively the uncertainty originated from the shape of the AFM tip in determining the wire width.

Samples of arrays of Ru nanowires in the membrane were made by first cutting the membrane containing Ru nanowires into small pieces, and then attaching two Cu wires on the top and two on the bottom of a membrane piece using Ag epoxy, forming essentially a two-terminal probe. To avoid electrical shorts, the Au or Ag films on the edge of the membrane surfaces were carefully removed using cotton swabs either dry or with ethanol before attaching the Cu wires.

To measure individual Ru nanowires, the polycarbonate membrane was dissolved by dichloromethane. Drops of dichloromethane solution containing Ru nanowires were placed on Si wafers (with a thermally grown 200 nm SiO₂ top layer) and blown dry.
Fig. 2.2. Mask used in a two-stage photolithography process. a) The overall image of the mask used to put electrical leads onto Ru nanowires. The small leads in the center square (enlarged in c) directly connect to Ru nanowires; b) The mask used to generate large contact leads that connect to usable leads from a. The small end of this mask connects to the outside of the mask in a. The big end connects to Cu wires for measurements; c) Enlarged view of the center square of a.
Au_{0.9}In_{0.1} leads with good adhesion to SiO_2 substrate were prepared by a standard lift-off process using photolithography. We implemented a two-stage photolithography process to enhance the chance of covering individual Ru nanowires effectively. The design of the masks used for this purpose is shown in Fig. 2.2. The resulted structure was inspected using either a high-magnification optical microscope up to 1000 X (Fig. 2.3a) or an AFM (Fig. 2.3b). Electrical connections were made to pairs of leads with individual Ru nanowires attached.

![Fig. 2.3](image)

Fig. 2.3. a) Optical image of a Ru nanowire with electrodes on as indicated by arrow; b) AFM image of the same sample.
2.3 Low-Temperature Electrical Transport Measurements

2.3.1 \(^3\)He Evaporation and Dilution Refrigerators

Low temperature measurements were carried out in a \(^3\)He evaporation refrigerator down to 0.3 K or a dilution refrigerator down to 20 mK. Both refrigerators are equipped with a superconducting magnet up to 8 T.

The \(^3\)He cryostat consists of a continuous fill \(^4\)He pot and a closed \(^3\)He pot filled with 3 STP liters of \(^3\)He. The cryostat utilized an internal charcoal sorption pump to pump the condensed liquid \(^3\)He to gain temperatures as low as < 0.3 K, and a hold time can be > 24 hrs in normal operation.

The dilution refrigerator was originally an Oxford Kelvinox 100, consisting of a continuous fill \(^4\)He pot and a Kelvinox\(^{\text{IGH}}\) gas handling system, with an add-on Roots blower that boosts the cooling power to 180 \(\mu\)W at 100 mK. The Kelvinox\(^{\text{IGH}}\) is a closed circulation loop of \(^3\)He/\(^4\)He mixture, including an impedance line, a mixing chamber, a still, and two rotary pumps. It obtains cooling by “evaporating” \(^3\)He from the concentrated phase of \(^3\)He to the dilute phase since the enthalpy of \(^3\)He in these two phases is different. It has 42 STP liters of mixture, including 7 STP liters of \(^3\)He. The base temperature for this refrigerator is < 20 mK.

Further details of these refrigerators can be found in Refs. [90] and [72].

2.3.2 Measurement Instrument

The dilution and the \(^3\)He refrigerators both are equipped with room-temperature RF filters (very likely \(\approx 5\) nF capacitor-only low-pass filters) with an attenuation of 10
dB at 10 MHz, 30 dB at 100 MHz, and 50 dB at 300 MHz. Another 4K dip probe, without any filters, was used for preliminary measurements of Ru nanowires.

Either Keithley 220 or 2400 was used as the current source, with a nominal current resolution as small as 50 pA for 2400; Keithley 182 or 2182 was used as the voltmeter. The noise level of these digital voltmeters is usually around 10 nanovolts at low temperatures. In the 4K dip probe setup, Keithley 2002 was also used as the voltmeter. The Keithley instrument can generate unwanted noises. Keithley 220, an older model than 2400, was particularly bad. The noise spectrum for Keithley 2400 sourcemeter was found to range from slightly above d.c. all the way to RF frequency as revealed by spectrum analyzers (Stanford Research Systems SR760 FFT 0 Hz - 100 KHz and Agilent E4405B 9 KHz - 13.2 GHz), though instrument specification only states a peak-peak noise of 5 pA in the frequency range 0.1 Hz - 10 Hz using 50 pA programming resolution. Therefore, we used a battery current source to verify the measurement results obtained using Keithley instrument. It was found that two 1 MΩ resistors work well to damp the noise. Most measurements were done with two 1 MΩ resistors connected to the output of a digital current source.

We have made several two-stage LC filters with a cutoff frequency at either 6 KHz or 1 MHz to replace the 1 MΩ resistors. These LC filters were designed to incorporate very different values of L and C, and feedthrough capacitors to cover a wide range of frequency in case that some component fails to work above its resonant frequency. Analysis of their attenuation characteristic using a spectrum analyzer indicates a noise peak around 10 - 20 MHz for the 6 KHz filters (the 1 MHz filters have not been tested). It seems that their filtering capability do not improve very much compared to the 1 MΩ
resistors. However, they can be used to filter the voltage wires in a d.c. measurement setup while the 1 MΩ resistors cannot. They can also be used to replace the 1 MΩ resistors in the current wires when a high bias current is needed. In a test run, these filters burnt several cylinders due to static charges accumulated in the integrated capacitors with a value $\approx 1 \mu F$. Discharging the capacitors beforehand is necessary to avoid possible damage to mesoscopic samples.

### 2.3.3 Shielding and Grounding

Shielding and grounding are important issues for electrical transport measurements. Our measurement setup is configured in a way that the signal (current and voltage) loop is shielded all the way from the instrument to the sample, and is floating. There are two grounding lines for our equipment (see below). One grounding line is through the main pipe of the building’s water supply, referred to as direct ground; the other one is through the electrical power ground. We have found that measurements using Keithley instrument that were grounded to power ground usually resulted in noises.

Shielded BNC cables are connected to the RF filters on top of the cryostat. The shielding of the BNC cables is connected to the frame and the ground of the instruments (except for 2400 which uses banana connectors for output), and is also connected to the shielding of the GPIB cables. The sample itself is not grounded. Keithley instruments as well as other electronics used in the measurement (referred to as instrument side) are connected through the shielding of the GPIB cables and are grounded at a single point to the direct ground. This single point is the cryostat for the dilution system and is the
instrument rack for the $^3$He system. These instruments use a 500 W isolation power transformer so that their ground does not short to the electrical power ground directly. The other group of equipment including the magnetic power supply and the computer (referred to as computer side) are grounded through the power ground. A GPIB-120A optical bus isolator (National Instruments) is used to isolate the instrument side from the computer side electrically while maintaining the regular GPIB communications.

An exception to the above arrangement is that the Kelvinox$^{\text{IGH}}$ gas handling system (IGH) of the dilution refrigerator is connected directly to power ground as the IGH has a safety protection that requires the power ground to be connected. Therefore even though the IGH are directly connected to the dilution refrigerator, it is not electrically isolated from the power ground. It may be possible to connect the ground pole of the IGH power cord to the direct ground without compromising the safety. But this was not attempted. The IGH operates three rotary pumps, but it is electrically isolated from the pumps using relay switches and plastic connectors for pump lines. It connects electrically to some thermometers and heaters inside the internal vacuum can (IVC) of the dilution refrigerator, with the cable shielding connecting to the cryostat. Therefore it is desirable for IGH to be isolated from power ground. However, the large power consumption prevents us from using the 500W isolation power transformer that we have.

The temperature control system for the dilution refrigerator includes an AVS47 resistance bridge, an AVS47-IB computer interface, and a TS530 temperature controller. The AVS47-IB converts the RS232 interfaces of AVS47 and TS530 to GPIB so that they can talk to the computer directly. AVS47, AVS47-IB, and TS530 are shielded
Fig. 2.4. Desired connection scheme for the dilution refrigerator system in terms of grounding and shielding. The question marks indicate the places that need improvement. Note that there are some grounding/shielding loops in this scheme which can potentially cause noises.
and grounded all together. They are connected through shielded BNC cables to inside IVC with the shielding connecting to the cryostat. They were not isolated by power transformer while the measurements were done.

We have not achieved an ideal isolated measurement system for the dilution refrigerator due to limited power of our isolation transformer. A desired connection scheme is shown in Fig. 2.4, with question marks indicating the places where further work is needed.

2.3.4 Softwares and Measurement Techniques

In our measurements data is collected by a computer using Labview programs written by several generations of graduate students who worked in the lab. Electronic instrument is controlled using GPIB protocol. Resistance vs. temperature $R(T)$, resistance vs. magnetic field $R(H)$, and voltage vs. current $V(I)$ are the main types of measurements. All measurements are done in d.c. mode. First the bias current is sent to the sample. Consecutive voltage readings are taken and averaged after an appropriate delay. In the case that heating or current-induced sample change is a concern, pulse measurements are performed. The trigger-pulse mode features direct communication between the Keithley sourcemeter and nanovoltmeter. One measurement cycle finishes within around 30 ms and the time interval between adjacent two measurement cycles can be adjusted. The trigger-pulse mode is realized in various combinations of Keithley instrument in an effort undertaken previously by Karl Nelson. Some combinations using Keithley 2400 sourcemeter as the current source needs a manual trigger to start the measurement. Tunneling measurement on $dI/dV$ in a.c. mode using a lockin amplifier
(Stanford Research Systems SR830) was attempted and did not yield results better than those obtained from numerically differentiating voltage vs. current data taken in d.c. mode.

In some $R(H)$ measurements, the steps in ramping the field have to be small. Our power supply Oxford IPS-120 only provides a current resolution of $\approx 1 \text{ mA}$, corresponding to a field resolution of $1 \text{ G}$ \(^1\) (The Lakeshore power supply 622 is noisy, which also has a field resolution of $1 \text{ G}$). We use Keithley 2400 as the current source for magnet which improves the field resolution to $50 \text{ mG}$ in the field range from $100 \text{ G}$ to $1000 \text{ G}$. Smaller field range gives better field resolution. Since we did not implement a magnetic field shielding for the cylinder measurement. The earth background field is present and assumed to be constant during the measurement.

### 2.4 Low-Temperature Microwave Filters

Reducing noise is of particular importance in the electrical transport measurements on samples such as 2D ultrathin superconducting films, 1D nanowires, carbon nanotubes, and single-electron devices. Even though an experimental setup is good enough for measurements on bulk samples, it may not be good enough for measurements on small samples. Electrical and magnetic (EM) waves propagating through the measurement wires can couple into a small sample, leading to a heating problem. Due to the extremely weak electron-phonon coupling at ultra low temperatures ($< 0.1 \text{ K}$), electrons may not be at the same temperature as the lattice temperature. Experimentally,

\(^1\)The unit of magnetic field strength in Gaussian units is Oersted [44]. I use Gauss for unit of magnetic field throughout this thesis by noting that $H = B$ in vacuum in Gaussian units.
electron heating due to microwave noise were observed previously [86, 21]. Dephasing
due to microwave noise without heating of the electrons was also reported [86].

An impedance $Z_e(\omega)$ of the electrical circuitry at temperature $T_e$ creates photons
with an energy spectrum that follows the Planck radiation law [14]:

$$S_e(\omega) = \frac{2}{\pi} \frac{\Re[Z_e(\omega)] h \omega}{\exp \left( \frac{h \omega}{k_B T_e} \right) - 1}.$$  \hspace{1cm} (2.4)

At low frequency, $S_e(\omega)$ is the “white-noise” spectrum with constant value

$$S_e(\omega) = \frac{2}{\pi} k_B T_e \Re[Z_e(\omega)].$$  \hspace{1cm} (2.5)

Above the thermal cutoff frequency $k_B T_e/h$, the radiation $S_e(\omega)$ approaches zero ex-
ponentially. The thermal cutoff frequency of the input noise is equal to 20 GHz for an
impedance temperature $T_e$ of 1 K. RF filters (either RC or LC type) are incapable of
attenuating the noise at so high frequency–microwave noise [25].

Microwave noise can be attenuated effectively by a Cu (or Ag) powder filter as
invented by Martinis et al. [55], with the original design shown in Fig. 2.5. A powder
filter consists of measurement wires inside a copper tube filled with copper or silver
powder with a grain size of about 30 $\mu$m. Since each grain appears to be insulated from
its neighbors by a naturally grown oxide layer, the effective surface area is enormous,
and the skin-effect damping produces a substantial attenuation. The attenuation of such
a filter 0.1 m long was greater than 50 dB at 0.5 - 12 GHz in the original design.
Fig. 2.5. Schematic of a powder microwave filter due to Martinis et al. [55]. The $I-V$ connector connects to current and voltage wires. The microwave connector couples microwave power capacitively into the junction.

We plan to install LC filters (RF filters) and microwave powder filters on the sample stage of our dilution refrigerator to suppress the possible Nyquist noise ("white noise") coming from wires at temperatures higher than the sample stage temperature. We will adapt the powder filter design by Martinis et al. [55]. The LC filters will be made from surface-mount components for compact size. The powder filter needs to be $\geq 10$ cm for a reasonable attenuation. We will design a new sample stage to accommodate the filter assembly. A $2 \times 2 \times 2$ cm sample block will locate at the top of the new sample stage, as shown in Fig. 2.6. This sample stage will be connected to mixing chamber in an appropriated way to ensure that the samples are near the center of the magnetic coil, so as to obtain the most uniform magnetic field.
Fig. 2.6. Schematic of a new sample stage: a) a 3D view; b) front view; c) side view; d) top view. The height of the stage is less than 30 cm and the diameter of the stage is less than 5 cm, as limited by the space in IVC. Figures generated from drawings in TurboCad that are to scale.
Chapter 3

Doubly Connected Ultrathin Superconducting Cylinders

3.1 Introduction

The study of doubly connected superconducting cylinders started in 1960s [51, 37, 60], following the discovery of the Little-Parks (L-P) resistance oscillation [51], resulted from fluxoid quantization. The oscillation observed in the resistance vs. magnetic field, $R(H)$, measurement reflects the periodically modulated phase diagram in $\Phi - T$ space, with a period corresponding to $h/2e$ in flux. The phase diagram can be explained by Ginzburg-Landau theory [81, 60].

Moshchalkov et al. in 1995 [30] measured a filled and an open Al square with an area of $1 \mu m \times 1 \mu m$, and found that the phase boundaries of the two types of the samples, the singly and doubly connected squares, were different. Their interpretation was based on the numerical solution of the linearized G-L equation with appropriate boundary conditions, which are imposed by the sample geometry. Moshchalkov’s work demonstrated the importance of the sample topology on the behavior of mesoscopic superconductors.

de Gennes’ theory on destructive regime, the loss of superconductivity around half-integer flux quanta in the zero temperature limit in a ring with a diameter $d$ less than the zero-temperature superconducting coherence length $\xi(0)$, is another important
result on mesoscopic superconductors [16]. de Gennes’ theory can be applied to a doubly connected superconducting cylinder.

Experiments to confirm de Gennes’ theory on destructive regime was first carried out on mesoscopic superconducting rings fabricated using e-beam lithography, which failed due to a relatively short superconducting coherence length $\xi(0)$ in comparison with the ring diameter [75]. The unavoidable contamination by organic solvent during the lithography process and the large surface scattering in a ring of mesoscopic size probably are the main reasons for the $\xi(0)$ to be relatively short. Doubly connected superconducting cylinders prepared on ultrathin quartz filament turns out to a better system to achieve the destructive regime [22].

In this chapter, I will first describe experiments on Kosterlitz-Thouless transition in large cylinders. Then I will discuss work on the destructive regime, including quantum phase transition (QPT) occurring at the onset of the destructive regime, a possible phase separation near the QPT, and quantum fluctuations in the destructive regime.

### 3.2 Kosterlitz-Thouless Transition in Large Cylinders–Cylindrical Films

Kosterlitz-Thouless (K-T) transition has been studied extensively in various 2D systems such as disordered thin films [20] and 2D Josephson junction arrays [29]. Understanding the subtle effect on K-T transition offers insight into the nature of superconductivity and the existence of quasi long range order in 2D. In this section, we consider cylinders with a diameter $d > 4\xi(0)/\pi$. The criterion is chosen such that the circumference ($\pi d$) can accommodate at least two vortices [overall size $\geq 4\xi(0)$]. We consider a
doubly connected cylinder as a cylindrical film rolled up from 2D. The interesting question is how the K-T transition is affected. If the rolled-up film—a cylindrical film is big in diameter, no change is expected. However, what will happen if the diameter is so small that the circumference can barely accommodate a vortex-antivortex pair \( \pi d > 4\xi(0) \) as shown in Fig. 3.1? In this case, the vortex cores of a pair should start to touch and may even form a single normal band (Fig. 3.1b), a phenomenon that we will discuss in Section 3.3.4.

Doubly connected cylinders with a diameter smaller than 300 nm have been measured either in zero or a magnetic field applied along the axis of the cylinder. The field is applied to suppress \( T_c \), so that the transition is placed in a temperature range in which the temperature can be easily stabilized in our dilution refrigerator. At zero field, the \( T_c \) of the Al is 1.3 K, around which it is very difficult to control the temperature in our dilution refrigerator. Because of the misalignment, which should be within \( \pm 5^\circ \), a small field component perpendicular to the cylinder axis is present, which leads to the presence of certain field-induced vortices.

Figure 3.2a shows the \( I-V \) isotherms measured at \( H = 170 \) G for cylinder Al-6. \( d = 263 \) nm, \( \xi(0) = 150 \) m, and \( R_\square = 1.02 \) Ω. No destructive regime is expected in this cylinder because \( d > \xi(0) \). A crossover for \( \alpha \) from 3 to 1 was observed as the temperature is raised from 1.030 K to 1.035 K, where \( \alpha \) is the exponent in the power law \( V \sim I^\alpha \). The corresponding \( R(T) \) is shown in Fig. 3.2b. A good fit (solid line in Fig. 3.2b) according to K-T theory was obtained using \( T_{KT} \) estimated from the \( I-V \) isotherms.

It is important to understand the effect of the small perpendicular field on K-T transition. For the results of Al-6 shown in Fig. 3.2, a perpendicular field due to
Fig. 3.1. Schematic showing a) a film containing a vortex-antivortex pair that can be rolled up to form a cylinder; b) a cylindrical film with a very small diameter. The magnetic flux of a thermally induced vortex entering the cylinder will have to emerge from the cylinder through a vortex as well. As a result, vortices and antivortices always form in pairs. Drawings were produced by N. Kurz.

Fig. 3.2. a) $I - V$ isotherms taken at $H = 170$ G for cylinder Al-6. The temperatures from right to left are 1.010, 1.020, 1.025, 1.030, 1.035, 1.040, and 1.050 K. Solid line indicates a slope corresponding to $\alpha = 3$; b) $R(T)$ at $H = 170$ G for Al-6 (circles). Solid line is a fit with $T_{KT} = 1.03$ K estimated from Fig. 3.2a. Inset is the $R(T)$ at $H = 0$ showing a sharp transition.
misalignment can be up to 15 G. Abrikosov vortices can be induced by this field since the cylinder is in the type II region as a result of disorder (Table B.1). The field-induced vortices can renormalize the logarithmic interaction between a vortex and an antivortex in a pair, therefore affecting the K-T transition. The exact effect remains to be further explored theoretically. The effect of a parallel field is to reduce the meanfield $T_c$. However, at $H = 0$ (the $I - V$ isotherms of Al-6 was not measured because of the $T$ stabilization issue), we find that the $R(T)$ data does not fit to K-T theory—the $R(T)$ appears to be too sharp to be described by a K-T dependence.

The circumference of Al-6 is such that $\pi d \approx 1.37 \cdot 4\xi(0)$. Since the normal core of a vortex is around $2\xi(0)$, two vortices can be barely fitted into the circumference of the cylinder. For a small cylinder, the flux lines have to come in from one side and come out directly from the other side as shown in Fig. 3.1. Therefore it was a surprise that we actually observed two experimental signatures for K-T transition in such a small cylinder. More theoretical calculation is needed to understand why K-T transition was apparently observed.

Figure 3.3a shows the $I - V$ isotherms measured at $H = 230$ G for cylinder Al-5 with $d = 212$ nm, $\xi(0) = 146$ nm, and $R_\square = 1.68 \Omega$. A crossover for $\alpha$ from 3 to 1 is difficult to identify in this figure. The corresponding $R(T)$ is shown in Fig. 3.2b. Only low resistance part is plotted for fitting purpose due to the steplike features (refer to section 3.3.4). The steplike features in $R(T)$ indicate the formation of normal bands in the cylinder. Before the appearance of any steplike feature the resistance of the cylinder is determined by the unbinding process of thermally activated vortex-antivortex pairs. Therefore only the tail part of the $R(T)$ is fitted to K-T theory. Even over such a
Fig. 3.3. a) $I - V$ isotherms taken at $H = 230$ G for cylinder Al-5. The temperatures from right to left are 0.930, 0.940, 0.945, 0.950, 0.955, 0.960, 0.965, and 0.970 K. Solid line indicates a slope corresponding to $\alpha = 3$; b) Low resistance part of $R(T)$ at $H = 230$ G for Al-5 (circles). Solid line is a fit with $T_{KT} = 0.955$ K approximated from Fig. 3.3a. Inset is the full range of $R(T)$ at $H = 230$ G.

Fig. 3.4. a) $I - V$ isotherms taken at $H = 0$ for cylinder Al-21. The temperatures from right to left are 1.206, 1.207, 1.208, 1.209, 1.210, 1.211, 1.213, and 1.214 K. Solid line indicates a slope corresponding to $\alpha = 3$; b) $R(T)$ at $H = 0$ for Al-21 (circles). Solid line is a fit with $T_{KT} = 1.208$ K roughly approximated from Fig. 3.4a. Inset is the full range of $R(T)$ at $H = 0$. 
small range of resistance the K-T fit did not yield very satisfying result. It should be noted that for cylinder Al-5, $\pi d \approx 1.14 \cdot 4\xi(0)$. The space was even tighter than Al-6 to accommodate vortex pairs.

We also attempted to probe the $I - V$ characteristic in zero field despite of the difficulty in controlling the temperature. Figure 3.4 shows the results on cylinder Al-21 with $d = 206$ nm, $\xi(0) = 177$ nm, and $R_{\square} = 1.48$ Ω [$\pi d \approx 0.91 \cdot 4\xi(0)$]. The zero-field $R(T)$ shows a step, indicating that this cylinder is nonuniform with two different $T_c$. A crossover similar to K-T behavior in $I - V$ isotherms and a reasonable K-T fit to the tail part of the $R(T)$ were found.

In conclusion, our measurements suggest that K-T transition occurs in superconducting cylinders that are so small that vortex-antivortex unbinding picture of the 2D films appears to be not applicable. The effect of sample topology on superconductivity should be studied more systematically to fully understand this phenomenon in these ultrathin cylinders with varying $\pi d/4\xi(0)$ ratio.

3.3 Destructive Regime Physics in Quasi 1D Cylinders

3.3.1 Observation of Destructive Regime

de Gennes’ theory on a superconducting ring discussed in section 1.7 can be extended to a doubly connected cylinder, for which a destructive regime is also expected if $d < \xi(0)$. In the original experiment, Au$_{0.7}$In$_{0.3}$ and Al cylinders were used to confirm the existence of the destructive regime. The relative long superconducting coherence length and small diameter of the cylinder make observation of the destructive regime
possible. Even though a similar coherence length may be achieved in rings made by e-beam lithography [75], the ability of this technique to reach small diameter of the rings is usually limited.

Figures 3.5a and c show the resistance as a function of temperature for a Au$_{0.7}$In$_{0.3}$ cylinder AuIn-1 and an Al cylinder Al-1 at different applied flux values, respectively. At $\Phi = 0$, the superconducting transition was sharp. At $\Phi = \Phi_0/2$, no superconducting state was found—a resistance plateau was seen instead down to the lowest temperature measured, 20 mK. At $\Phi = \Phi_0$ for Al-1, a superconducting transition was seen again except for shifting to a lower temperature. The nonzero resistance around $\Phi = \Phi_0/2$ in the $T = 0$ limit could be better seen in resistance vs. normalized flux $R(\Phi/\Phi_0)$ as shown in Figs. 3.5b and d.

The zero temperature superconducting coherence length $\xi(0)$ estimated from the parallel upper critical field $H_{c2}$

$$H_{c2} = \frac{\sqrt{3} \Phi_0}{\pi t \xi(0)} \quad (3.1)$$

was 160 nm and the diameter $d$ estimated from the oscillation period in $R(H)$ was 154 nm for AuIn-1. $\xi(0) = 161$ nm and $d = 150$ nm for Al-1. Both satisfied de Gennes’ criterion. The estimated diameter $d$ agreed well with that from SEM image of a control sample. Similar measurements on cylinders with $d > \xi(0)$ only showed the conventional L-P oscillations.

Figure 3.6 shows the $\Phi - T$ phase diagram for Al-1. Two disconnected superconducting regions are separated by a normal state phase even down to zero temperature. A superconductor-to-normal metal quantum phase transition tuned by flux $\Phi$ is clearly
Fig. 3.5. a) Resistance as a function of temperature $R(T)$ at different applied flux values for cylinder AuIn-1 with $d = 154$ nm and $\xi(0) = 160$ nm; b) Resistance as a function of applied flux $R(\Phi)$ for AuIn-1; c) $R(T)$ for Al-1 with $d = 150$ nm and $\xi(0) = 161$ nm; d) $R(\Phi)$ for Al-1. Figures are from Ref. [22].
seen in this phase diagram. A fitting of the data according to Eq. 1.49 or Eq. 1.40 (essentially the same formulas) is shown as the solid line–de Gennes’ theory can explain the phase diagram quite well.

Fig. 3.6. $\Phi - T$ phase diagram determined using $R(T_c) = 0.05R_N$ for Al-1. The solid lines are fits to theory.

### 3.3.2 Phase Diagram Beyond de Gennes’ Theory

Figure 3.7 shows the $\Phi - T$ phase diagram for Al-14 with $\xi(0) = 137$ nm and $d = 174$ nm. No destructive regime is expected according to de Gennes’ theory. As shown in the figure, at $\Phi = \Phi_0/2$, it was superconducting as usual. However, at $\Phi = 3\Phi_0/2$, surprisingly, a normal-metal behavior in the zero-temperature limit was observed, suggesting that a destructive regime is present near $3\Phi_0/2$. 
Careful examination of the phase diagram shown in Fig. 3.7 reveals subtlety of the destructive regime physics. For the three domes in the phase diagram corresponding to angular momentum number \( l = 0, 1, \) and 2 in Eq. 1.18, respectively, we can identify a consistent curvature change of these domes, through fitting of the data according to Eq. 1.40, as indicated by the solid line in Fig. 3.7. The similar curvature change can also be found in phase diagrams of other cylinders.

![Phase diagram](image)

**Fig. 3.7.** \( \Phi - T \) phase diagram determined using \( R(T_c) = 0.01 R_N \) for Al-14 with \( \xi(0) = 137 \text{ nm} \) and \( d = 174 \text{ nm} \). The solid lines are fits to theory.

A possible way to understand the phase diagram shown in Fig. 3.7 is to assume a field-dependent zero-temperature superconducting coherence length \( \xi(0, H) \). The curvature of the domes is given by \( 4T_c(0)[\xi(0, H)/d]^2 \). The inferred \( \xi(0, H)/d \) vs. \( H/H_{c2}^\parallel \) data for Al-14 are shown in Fig. 3.8. Although a small set of data points is obtained due
Fig. 3.8. a) $\xi(0, H)/d$ vs. $H/H_{c2}^{\parallel}$ inferred from the phase diagram in Fig. 3.7 (see text). Solid line represent a fitting attempt (see text). The crosses of the dashed lines indicate that $\xi(0, H)/d$ is about larger than 1 at the field corresponding to $\Phi = 3\Phi_0/2$, leading to a destructive regime.

To limited number of domes (determined by the low $H_{c2}^{\parallel}$ and the relative large period in magnetic field corresponding to $\Phi_0$), it is seen that $\xi(0, H)$ increases with field and tends to diverge at the parallel upper critical field $H_{c2}^{\parallel}$. The solid curve is a fit based on $\xi(0) = \hbar v_F/\pi\Delta(0, H)$, where $\Delta(0, H) = \Delta(0, 0)\sqrt{1 - (H/H_{c2}^{\parallel})}$ (However, this formula for $\Delta(0, H)$ is flawed due to the subtle effect of magnetic field on the energy gap [82]).

If a field-dependent $\xi(0, H)$ is possible, we are able to explain the new phase diagram in Fig. 3.7 within de Gennes` criterion of $d < \xi(0, H)$. Since $\xi(0, H)$ increased with field $H$ and also with flux $\Phi$ assuming that the cross-sectional area remained constant, $\xi(0, H)$ at the field corresponding to $3\Phi_0/2$ became larger than diameter $d$ so that a destructive regime was observed. As a consistency check, we can see that the crosses of the dashed lines in Fig. 3.7 which locate the $3\Phi_0/2$ positions indeed indicate a $\xi(0, H)/d$ value just about larger than 1.
However, fundamental inconsistency is present in the above picture. A field-dependent $\xi(0, H)$ is not expected within G-L theory, upon which de Gennes’ theory was based. In Tinkham original deduction of Eq. 1.39, he used the penetration depth $\lambda_e$ as the fundamental parameter, which was generally considered as field-independent. Furthermore, the estimate of the $\xi(0)$ in G-L theory using Eq. 3.1 is based on the parallel upper critical field $H_{c2}$, again showing paradox if $\xi(0) = \xi(0, H)$. A field-dependent $\xi(0, H)$ was also reported in 1D wires [71, 1]. In that study a series of $R(T)$ curves at different magnetic fields was fitted using the model for thermally activated phase slips and a field-dependent $\xi(0, H)$ was assumed. However, no justification of the assumption was provided.

An alternative way to understand the new phase diagram is to return to the point on the competition between the kinetic energy density of the supercurrent, $n_s m^* v_s^2/2$ (where $n_s$ is the number density of Cooper pairs), and the superconducting condensation energy density in an applied field, $H_c^2/8\pi - H_a^2/8\pi$ (where $H_a$ is the applied field). The kinetic energy density at $\Phi_0/2$ as well as that at $3\Phi_0/2$ is maximized as $n_s h^2/2m^*d^2$ (Eq. 1.18). However, the superconducting condensation energy density at $\Phi_0/2$ is larger than that at $3\Phi_0/2$ due to the pair-breaking of magnetic field. As a result, a superconducting state could exist at $\Phi_0/2$ but not at $3\Phi_0/2$ if the value of $n_s h^2/2m^*d^2$ is in between the superconducting condensation energy density at $3\Phi_0/2$ and that at $\Phi_0/2$. A destructive regime emerges only at $3\Phi_0/2$ in this case as observed in Al-14. To fully understand this new phase diagram, a microscopic theory of the destructive regime may be needed.
3.3.3 Phase Boundaries Defined by Different Criteria

The phase boundary of the cylinder was determined by tracing the onset of finite resistance \( R(T_c) = 0.05R_N \) for Al-1 and \( R(T_c) = 0.01R_N \) for Al-14. A value of \( R(T_c) \) larger than \( 0.05R_N \) can also be used to define the boundary. Although no qualitative difference was found in previous research [60], we did see differences by choosing different \( R(T_c) \) values to define the phase boundary. Fig. 3.9 shows the phase boundaries defined by \( R(T_c) = 0.01R_N \) (dots with lines) and \( R(T_c) = 0.5R_N \) (hollow squares), respectively, for Al-14. As can be seen, an extra superconducting dome corresponding to \( l = 3 \) was found with \( R(T_c) = 0.5R_N \).

![Phase Boundaries Diagram](image_url)

Fig. 3.9. \( \Phi - T \) phase boundaries determined using \( R(T_c) = 0.01R_N \) (dots with lines) and \( R(T_c) = 0.5R_N \) (open squares) for Al-14.
Fig. 3.10. $\xi(0, H)/d$ vs. $H/H_{c2}^\parallel$ inferred from the phase boundaries in Fig. 3.9 defined by two different $R(T_c)$ values at $0.01R_N$ (dots) and $0.5R_N$ (open squares), respectively. Solid lines are fitting attempts (see Section 3.3.2).

Different phase boundaries defined by different $R(T_c)$ will also lead to difference in $\xi(0, H)$, within the scheme discussed in the previous section. Figure 3.10 shows two sets of $\xi(0, H)$ inferred from $R(T_c) = 0.01R_N$ (circles) and $= 0.5R_N$ (open squares), respectively.

### 3.3.4 Phase Separation near the Quantum Phase Transition

The existence of an extra degree of freedom in an ultrathin cylinder rather than a ring considered by de Gennes provides opportunity to observe phenomena not predicted in de Gennes’ theory. For example, the superconducting order parameter may vary along the cylinder axis. The kinetic energy cost due to the modulation may be sufficiently small to be compensated by the interface energy, as in the case for vortices in type II superconductors.
Cylinders possessing a destructive regime may be considered as a quasi 1D system as their circumferences, \(\pi d\), is only slightly larger than or comparable to twice of the zero temperature superconducting coherence length, \(2\xi(0)\), over which the order parameter can change from and recover to a constant value. Since the destructive regime exists at \(T = 0\), the onset of the destructive regime represents a superconductor-to-normal metal quantum phase transition (QPT), which can be precisely tuned by the magnetic field. The question is whether any interesting phenomena can be found near the QPT.

Figures 3.11a and c show the resistance as a function of temperature, \(R(T)\), at different applied flux values from 0 to \(\Phi_0/2\) for Cylinders Al-1 and Al-3, respectively. Parameters for these and other cylinders used in this study are summarized in Table B.1. Cylinder Al-1 was one of the samples used in the original experiment on the destructive regime [22]. As the system approaches \(\Phi_0/2\), regular steplike features, identified alternatively as minima in \(dR/dT\), are seen at fixed resistance values. At low fields, these features become less distinct, and disappear when the field is sufficiently small. Even though a single step is seen at zero fields in Al-1, Al-3, and several other samples we measured, it always disappears at slightly higher fields. Although the precise physical origin for this step is not known, we believe that it is a sample-specific feature unrelated to those seen in higher fields. The regular steplike features near the half-flux quantum are clearly induced by magnetic field. Similar steplike features are also observed in the resistance vs. magnetic field measurements in the intermediate temperature range (Fig. 3.12).

Even though both samples shown in Fig. 3.11 were measured in a 2-wire configuration, the steplike features are not due to 2-wire measurements because of the following
Fig. 3.11. a) Resistance as function of temperature, $R(T)$, at different applied flux values for Cylinder Al-1, steps in $R(T)$ are marked by arrows. $n$ denotes the order for the step to appear as the temperature is increased. Between 490 G and 580 G, the field values are 500, 505, 510, 515, 520, 524, 526, 527, 528, 530, 535, 540, 550, and 560 G. $\Phi_0/2$ corresponds to 585.5 G; b) $dR/dT$ for data shown in 1a. Steps in $R(T)$ shown in a correspond to minima in $dR/dT$; c) $R(T)$ curves for Al-3. Between 400 G and 450 G, the field values are 405, 410, 415, 420, 425, 430, 435, 440, 441.4, and 445 G. $\Phi_0/2$ corresponds to 580 G; d) $dR/dT$ for data shown in c. The bias currents were 25 nA for Al-1 and 100 nA for Al-3.
Fig. 3.12. a) Resistances as functions of magnetic field, $R(H)$, at different temperatures for Cylinder Al-3. The steps in resistance, which are most pronounced at 0.5 K, are at the same resistance values as those observed in $R(T)$ (Fig. 3.11). The steps become less obvious or even disappearing at low temperatures; b) $dR/dH$ at $T = 0.5$ K. The minima correspond to the steps in $R(H)$. 
Fig. 3.13. Comparison of resistances as functions of temperature obtained from 2-wire (the contact resistance is subtracted) and 4-wire measurements, respectively, at zero field and 350 G for a short section of Cylinder Al-7. The 2-wire data (dots) and 4-wire data (lines) share essentially the same features.
reasons. First, these features were seen in both 2- and 4-wire samples (see below). Furthermore, in a control experiment, 2- and 4-wire measurements were carried out on the same cylinder with multiple leads \(d \approx 0.2 \mu m\). Essentially identical steplike features in \(R(T)\) were found in both cases as shown in Fig. 3.13. Sample inhomogeneities featuring variation in local \(T_c\) can also be excluded from being the cause of the regular steplike features. To begin with, SEM studies have shown that our cylinders are quite uniform. For those on which sharp SEM images were obtained, grains of Al with rather uniform size were seen. In addition, if the sample were inhomogeneous with separate regions of different \(T_c\) values, the superconducting transition in zero fields would not be as sharp as what was observed experimentally. The regular steplike features would also have persisted down to low and zero fields, inconsistent with experimental result.

Can the steplike features be due to phase slip centers (PSCs) formed at sections of the sample with a small local critical current \(I_c\) [78]? We believe that they are not based on the following observations. First, the detailed evolution of the steplike features as a function of magnetic field suggest that the steplike features were not caused by PSCs; Second, we measured a cylinder (Al-3) with different bias currents. It appears that the steplike features are present even at 10 nA. More importantly, at high bias currents, some steplike features presented at lower currents actually disappeared, again inconsistent with the PSC picture (Fig. 3.14a); Finally, we measured \(R(T)\) at fixed magnetic field with a battery rather than a digital current source that may have introduced electrical noises to the sample despite the damping of the RF filters. Since electrical noises are known to generate PSCs, stronger steplike features are expected for digital current source
Fig. 3.14. a) $R(T)$ for Al-3 measured (using a digital current source) at 410 G with different bias currents as indicated. Curves are shifted along $T$ axis for clarity; b) $R(T)$ measured using a battery and digital current source (Keithley 2400) at 100nA and 10nA, respectively, in the same field. Curves measured by battery are shifted towards higher $T$ for clarity.
measurements if these features were due to PSCs. Experimentally, the opposite is true (Fig. 3.14b).

Figure 3.15 shows $R(T)$ traces at fixed flux values up to half-flux quantum for cylinders with a diameter slightly larger than that required for destructive regime (Table B.1). Multiple, but relatively irregular steplike features were observed in Al-4, which is only slightly larger than Al-1 and Al-3. The irregularity in the steplike features appears to be related to the slight variation in diameter (Table B.1) as revealed by the $R(H)$ measurements. The overall trend of the steplike features is very similar to those found in Al-1 and Al-3. In Al-5, which is larger in diameter but uniform, only a couple of steplike features were found. For Al-6, with the largest diameter (Fig. 3.15d), no steplike features were observed. This systematic behavior was observed for all cylinders (20 in total) in which the presence of such steplike features was examined closely, suggesting that the steplike features can be induced by magnetic field so long as the diameter of the cylinder is sufficiently close to that required for possessing a destructive regime.

For the two samples shown in Figs. 3.11a and b, the empirical $\Phi - T$ phase diagrams are constructed (Fig. 3.16a and b). The phase diagrams suggest that for cylinders with sufficiently small diameters to host a destructive regime, the steplike features were found near the QPT between the superconducting and normal ground states at $T = 0$.

If we consider the diameter of the cylinder as the third axis in the parameter space, the above results seem to suggest that the steplike features emerge in a quantum critical regime. It is interesting to note that the appearance of the steplike features shown in Figs. 3.11 and 3.15 is accompanied by a broadening of $R(T)$, a feature very similar to that observed near a superconductor-insulator transition (SIT) in 2D [53] homogeneous
Fig. 3.15. a-c) $R(T)$ curves at various flux values for cylinders with a larger diameter (Table B.1). The magnetic field corresponding to $\Phi_0/2$ is 460-533 G for Al-4 which is slightly non-uniform in diameter, 295 G for Al-5, and 190 G for Al-6. The measurement current was 100 nA for all three cylinders; d) An SEM picture for Al-6. The Al grains are seen to be quite uniform with an average size of 69±16 nm. The diameter measured in the SEM image is 268±10 nm, as compared to 263±7 nm obtained from the $R(H)$ measurements.
Fig. 3.16. Empirical phase diagram for Al-1 a) and Al-3 b). The upper curve marks the highest $T$ at which a step was identified at fixed flux while the lower curve shows the onset of finite resistance.

Fig. 3.17. a) Step resistance as function of the index $n$ (Fig. 3.11) for Al-1 and Al-3. The lines indicate exponential dependence; b) Proposed bifurcation process for the normal band formation in ultrathin cylinders. c) Illustration of a single normal band. Drawing is prepared by N. Kurz.
systems. Even though whether the superconductor-normal metal transition at onset of the destructive regime is a continuous QPT featuring a critical regime is yet to be established, it is likely that whatever drives this QPT is also responsible for the emergence of the regular steplike features.

In 2D SIT, the QPT is widely believed to be driven by phase fluctuation due to the suppression of fluctuation in the number of Cooper pairs, \( N \). Because of the relation, \( \Delta N \Delta \phi > 1 \), where \( \phi \) is the phase of the superconducting order parameter, the suppression in \( \Delta N \) enhances \( \Delta \phi \). However, \( \Delta N \) is not suppressed near the destructive regime. Therefore it seems that even though the phase fluctuation may be present because of the reduced dimension, it can not dominate the QPT. A different path must be explored in order to understand this QPT and the accompanied steplike features. In this regard, the remarkable regularity of the latter as revealed in Fig. 3.11 may provide useful hints. To quantify this regularity, in Fig. 3.17a, we plot \( R_{\text{step}} \), at which the steplike features were found, as a function of \( n \), which denote the order for the emergence of the steplike feature as \( T \) increases (Fig. 3.11). We see clearly that \( \log_{10} R_{\text{step}} \propto n \), namely, \( R_{\text{step}} \) grows exponentially.

To explain such exponential growth in \( R_{\text{step}} \), and the emergence of the steplike features, we propose the following phenomenological model. As the system approaches the destructive regime, normal regions will nucleate in a homogeneous superconducting state. Each normal region will encircle the entire cylinder, forming a ring, or a band, referred here as a normal band. The first normal band will form near the center of a uniformly superconducting cylinder as \( T \) is raised. With increasing \( T \), each of the two superconducting sections will break into two sections, followed by breaking the 4
superconducting sections, and so on (Fig. 3.17b). Such a bifurcation process can lead to an exponential growth in the number of normal bands, \( N \), given by \( N = 2^n - 1 \). If all normal bands have the same length, we have \( R_{\text{step}} = N \cdot R_1 \), where \( R_1 \) is the resistance associated with a single normal band (Fig. 3.17c), leading to evenly spaced steps on a logarithmic scale as seen experimentally.

Further analysis shows good self consistency in this normal-band model. The slope in Fig. 3.17a is only slightly smaller than expected \( \log_{10} 2 \). The value of \( R_1 \) is also consistent with that expected for a single normal band. The length of a normal band, \( L_b \), should be \( 2\xi(0) \) based on energetic considerations (see below). However, \( R_1 \) should correspond to the resistance of a length twice of \( \Lambda_Q \), the charge imbalance length [12]. Typically \( \xi(0) \ll \Lambda_Q \). Therefore,

\[
R_1 = 2\Lambda_Q \cdot \rho_N / A, \tag{3.2}
\]

where \( \Lambda_Q = \sqrt{D\tau_Q} \), \( A \) is the cross-section area, and \( \tau_Q \) is the branch imbalance relaxation time, similar to that for PSCs [78]. \( \tau_Q \) is determined by the inelastic electron-phonon collision time \( \tau_E \) together with the pair-breaking time due to parallel magnetic field \( H \), \( \tau_s \). In the limit \( \tau_s \ll \tau_E \), which is the case for our cylinders, \( \tau_Q \) is given by [45],

\[
\tau_Q(T) \approx \frac{2\sqrt{2}k_B T}{\pi \Delta} \sqrt{\tau_E / \tau_s}. \tag{3.3}
\]

Based on the experimental value of \( R_1 \), \( \Lambda_Q \approx 2 \mu m \) for Al-1 and 3, a very reasonable number for Al films [23, 45]. In principle, \( \Lambda_Q \) is a function of the applied field and
temperature [45, 61]. Therefore $R_1$ at different $\Phi$ values should be different. However, our calculations according to Eq. 3.2 and 3.3 show that $\Lambda_{\Omega}$ varies within 10% for all curves with steplike features in Fig. 3.11. Such a variation in $R_1$ is invisible in a logarithmic plot.

Even though this picture of normal-band bifurcation seems to provide a consistent account of our data, it is surprising that a regular spatial variation of the order parameter should be allowed as this would in general cost energy. On the other hand, as the destructive regime is approached, the free energy of the normal state is only slightly higher than that of the superconducting state because of the large $v_s$. To minimize the free energy cost, a normal band should only be long enough to support two superconducting-normal (S-N) interfaces. As a result, $L_b = 2\xi(0)$. Once a normal band is formed, two S-N interfaces should bring about an interface energy. The applied field in this case is perpendicular rather than parallel to the interface, different from the typical situation considered in bulk Type I or Type II superconductors. The energy associated with such an interface has not been calculated. Two adjacent superconducting sections may also be coupled by Josephson coupling, likely to lead to a gain (lowering) of free energy. All these factors have to be considered to provide an energetic underpinning for the normal-band formation.

It is interesting to note that steps in $R(T)$ were reported long ago in Al cylinders with a diameter larger than or equal to 1.4 $\mu$m in a narrow temperature range [60]. These cylinders were too large to possess a destructive regime. Most importantly, all steps seen at finite fields were also found at zero fields. Therefore the physical origin of those steps, not identified in the original work, cannot be the same as what we have
observed in ultrathin superconducting cylinders. It was proposed previously that a heterogeneous mixed state featuring isolated superconducting spots would be formed at the high-temperature part of the superconducting transition due to impurity and strain effects [68]. It was further predicted that the presence of these superconducting spots will lead to steps in $R(T)$. No steps were actually observed in their $R(T)$ curves taken on cylinders with a diameter larger than or equal to 1.2 µm [68], which were again too large to exhibit a destructive regime.

In summary, we have observed steplike features near a QPT at the onset of the destructive regime in ultrathin, doubly connected superconducting cylinders. A tentative model based on phase separation is proposed to explain the emergence of these steplike features. More theoretical input and further experimental studies will be needed to fully understand the physical origin of these steplike features. For example, reducing the wall thickness of the cylinder will increase the amount of disorder and decrease the orbital effects. Varying the magnetic field angle with respect to the axis of the cylinder, which were found to give rise to Abrikosov vortices in a large cylinder, may shed light on the microscopic origin of the proposed phase separation near the QPT in our ultrathin cylinders.

3.3.5 Continuous Quantum Phase Transition

The quantum phase transition (QPT) associated with the destructive regime is interesting in its own right. In QPTs studied in other low-dimensional superconductors, superconductivity is destroyed by strong disorder, a magnetic field, or a Coulomb repulsion. In this case, however, superconductivity at zero temperature in a doubly connected
cylinder is destroyed by the rise of kinetic energy of the supercurrent circulating the cylinder. Different from the destruction of superconductivity at applied critical current, here the superconductor is in equilibrium state. An advantage of studying this QPT is that the transition can be precisely tuned by applied magnetic field (a field resolution better than 50 mG has been achieved in our setup). Meanwhile, since the material properties of the sample, such as the film morphology and the amount of disorder, remain unchanged across the transition, the analysis of the QPT is simplified.

As the resistance vs. temperature measurement revealed the existence of phase separation near the QPT. A more direct resistance vs. magnetic field measurement at the base temperature (approaching the zero temperature limit) would provide insight into the nature of the QPT.

Fig. 3.18. Resistance vs. magnetic field measured at base temperature for cylinder Al-3 a) and Al-16 b). Fields were ramped up and down showing no hysteresis. The ramping rates are labeled.
We have done measurements near the QPT on several cylinders that possess a
destructive regime. Figures 3.18a and b show the results on cylinders Al-3 and Al-16,
respectively, with significantly different field ramping rates as indicated. No hysteresis
was found even with a fast ramping rate at 10G/s for Al-16. The rate at 10G/s is
comparable to that used in the $R(H)$ hysteresis measurement on ultrathin Al films
revealing a first order transition [87]. The absence of hysteresis suggest that the QPT is
likely to be continuous.

Groff and Parks [37] pointed out that the pair-breaking due to magnetic-field-
induced orbital supercurrent indeed causes a continuous transition. They argued that
experiments in which the current is externally supplied can only give a first-order tran-
sition to the normal state in a thin film, since a maximum (critical) supercurrent is
reached when $|\psi|^2/\psi^2_\infty = 2/3$, where $\psi_\infty$ is the full value order parameter without any
pair-breaking [82]. $\psi$ is suppressed all the way to zero as $v_s$ reaches maximum, not
the supercurrent $J_s$. However, if an external agent like the axial magnetic field is used
instead of the current, it is possible to obtain a second-order transition if the condition

$$\frac{\lambda^2(T)}{(d/2)t} > 1 \quad (3.4)$$

is satisfied, where $d$ is diameter and $t$ is the film thickness. From G-L theory, for a film
with thickness

$$t < \sqrt{5}\lambda(T), \quad (3.5)$$
the transition from the normal to superconducting state due to a parallel magnetic field is second order \[82\]. However, Eq. 3.4 is a more stringent condition which must replace Eq. 3.5 as the criterion for a second-order phase transition \[37\].

Establishing a continuous phase transition is important because only a continuous QPT features quantum critical fluctuations. Near the transition, the behavior of the system is dominated by a characteristic length (correlation length) and a characteristic time, which enables a scaling analysis, as we have discussed in Section 1.5.1.

Fig. 3.19. Fine resistance \(v.s\). magnetic field scan at the bottom of the QPT for cylinder Al-3 a) and Al-12 b). The transition is very sharp, less than 1 G.

Figures 3.19a and b shows a fine hysteresis scan at the bottom of QPT for cylinders Al-3 and Al-12, respectively. As can be seen, the transition width is less than 1 G. The
transition from a superconducting state to a finite-resistance state [with its resistance value at least higher than that of the first steplike feature in $R(T)$] can be very sharp.

### 3.3.6 Resistance in the Destructive Regime

The nature of the normal state in the destructive regime is important to consider. In Fig. 3.5c, a significant resistance drop in $R(T)$, around 60% of the sample’s full normal-state resistance $R_N$, was found at $\Phi = \Phi_0/2$. Such a resistance drop was not discussed in de Gennes’ theory. Further analysis based on the Aslamazov-Larkin (A-L) theory [5] predicted that the Gaussian fluctuation conductivity in the neighborhood of the QPT is 4 orders of magnitude smaller than observed in Fig. 3.5c [83].

A possibility for the unusually large conductance in the destructive regime is that either $d$ or $\xi(0)$ varies along the cylinder. As a result, some part of the cylinder may still be in the superconducting state in the destructive regime.

Examination of the data on Al-1 excludes the possibility of nonuniformity of the cylinder diameter $d$ being responsible for the observed 60% resistance drop, which would require that more than 60% of the sample (in terms of the length) is not in the destructive regime. A monotonic change of the diameter $d$, in pulling quartz melt, is expected. On the other hand, the optical and scanning electron microscopy (SEM) images taken on a typical sample revealed no significant change of $d$.

Variation in $\xi(0)$ along the cylinder axis caused by sample inhomogeneity is more difficult to determine. The SEM images show that the coated Al film had a uniform granular structure (Fig. 3.15d); The resistance transition at zero field is sharp; The magnetic field corresponding to $\Phi_0/2$ is much smaller than $H_{c2}$ so that the effect of the
time reversal symmetry breaking should be weak; The relatively long coherence length $\xi(0)$ for our samples (160 nm for Al-1 and 190 nm for Al-3) suggests that our cylinders are quite homogeneous. As a result, it is difficult to see how variation in $\xi(0)$ can cause 60% drop in the sample resistance at $\Phi = \Phi_0/2$ for Al-1.

Oreg [66] considered the resistance drop observed in our experiments in the Aslamasov-Larkin fluctuation picture [5]. In this picture, the resistance drop would be related to fluctuations of the amplitude of the superconducting order parameter in the zero-temperature limit since $T = 0$ in the destructive regime. Phenomenologically, we found that the strength of the fluctuations seems to be determined by the ratio of $d/\xi(0)$. Figure 3.20 shows the normalized resistance vs. normalized flux values at base temperature for several cylinders possessing a destructive regime, $d/\xi(0) < 1$. The resistance peaks around half-integer flux quanta, as indicated by arrows, were seen varying in height. In Fig. 3.21 we plot the $R(\Phi = \Phi_0/2)/R_N$ as a function of $d/\xi(0)$ for cylinders shown in Fig. 3.20. Below a value of $\approx 0.77$ for $d/\xi(0)$ (indicated by the dashed line) the resistance of the sample in the destructive regime seems to recover its normal-state value, $R_N$.

Previously, the presence of localized Cooper pairs [33] in the possible metallic phase near the 2D SIT [38] was considered. It is possible that the existence of localized Cooper pairs are responsible for the low resistance value in the destructive regime, a resistance drop from $R_N$ [83].
Fig. 3.20. Normalized resistance $vs.$ normalized flux at base temperature for several small cylinders with $d/\xi(0) < 1$. Two arrows indicate the positions of destructive regime along the axis of flux, $\Phi = \pm \Phi_0/2$. 
3.4 Thermally Activated Phase Slips

As the diameter $d$ becomes smaller than $\xi(0)$, thermally activated phase slips become relatively easy to observe. We analyze the series of $R(T)$ curves taken at different flux values shown in Fig. 3.11a for cylinder Al-1 below the resistance corresponding to the first steplike feature. Other quasi 1D cylinders share similar $R(T)$ characteristic, and yield similar results as Al-1.

Before the entire cylinder ceases to be fully superconducting (Fig. 3.22b), the family of $R(T)$ at different flux values can be fitted using the phase slip model according to Eqs. 1.36, 1.37, and 1.38 as shown in Figure 3.23. The adjustable parameters $T_c$ and $\xi(0)$ are assumed to be flux-dependent. At flux values $\Phi \leq 0.447\Phi_0$ where the $R(T)$ transitions are relatively sharp, the Langer-Ambegaobar and McCumber-Halperin (LAMH) theory (Eq. 1.37) appears to be adequate to describe the data; Right at the
Fig. 3.22.  a) Redrawing of the phase separation model in Fig. 3.17b; b) Illustration of a phase slip process which occurs at the $n = 0$ stage in Fig. 3.22a.

Fig. 3.23.  Numerical fittings according the TAPS and QPS model (solid lines) to the segment of the $R(T)$ data of Al-1 at the $n = 0$ stage in the phase separation model (dots).
onset of the destructive regime ($\Phi = 0.449\Phi_0$), deviation from LAMH behavior becomes noticeable.

A more serious problem in the above fit can be found in the value of the parameter used in the fit. In the fitting of the $R(T)$ data in Fig. 3.23, we have assumed a flux-dependent zero-temperature superconducting coherence length, denoted as $\xi'(0, \Phi)$. The flux-dependent $\xi'(0, \Phi)$ is plotted in Fig. 3.24. It is seen that $\xi'(0, \Phi)$ diverges very quickly with increasing $\Phi/\Phi_0$. It is very difficult to believe that $\xi'(0, \Phi)$ can be the true zero-temperature superconducting coherence length. Therefore whether our fit of data to LAMH is meaningful needs to be further examined.
It is interesting to note that a field-dependent superconducting coherence length was proposed for singly connected Mo$_{0.79}$Ge$_{0.21}$ and Nb wires [71]. However, the coherence length increases much slower as the field increases in their work, very different from that encountered in the present work.

Another question is whether quantum phase slips may have played a role in the behavior of $R(T)$ near the destructive regime. This question seems to be reasonable given that $R \sim \ln T$, which may be considered to be a signature of QPS in previous studies. However, given the uncertainty in $\xi'(0, \Phi)$, this question is difficult to address with the data that we have.

### 3.5 Effect of a Reduced Wall Thickness

So far all the results that we reported are from cylinders with a film thickness $t$ around 30 nm. The penetration depth $\lambda$ ranges from 100 nm to 200 nm in our cylinders (Table B.1), the effect due to a finite film thickness can be observed in Fig. 3.5c. In the $R(T)$ transitions for cylinder Al-1, curves at $\Phi_0$ is about as sharp as the $\Phi = 0$ one except for a shift in $T_c$. The shift was related to the parallel field on thin films, similar as the screening effects if the cylinder had a break in it and was singly connected. In G-L theory, the suppression of order parameter in thin films is given as [82]

$$|\psi|^2 = \psi_\infty^2 (1 - \frac{t^2 H^2}{24 \lambda^2 H_c^2}). \quad (3.6)$$

In the limit $t = 0$, there is no suppression due to $H$. The similar conclusion can be drawn by taking $t = 0$ in Eq. 1.40, therefore eliminating the $H^2$ term.
Several new features are expected from reducing the cylinder wall thickness $t$. In the phase diagram of cylinder Al-14 (Fig. 3.7), the low upper critical field $H_{c2}$ leads to a low highest quantum number $l$, corresponding to few superconducting domes in the phase diagram. So far we have only observed domes up to $l = 2$ choosing $R(T_c) = 0.01R_N$. So far, disorder effect on the destructive regime physics has not been explored. The disorder shorten the coherence length so that a type II superconducting cylinder is expected even though we start from a type I superconductor of Al. As a result, vortices are expected in addition to normal bands. We have reduced the film thickness down to 15nm to increase disorder, and observed a high parallel upper critical field $H_{c2}^{||}$, a therefore short coherence length (Fig. 3.25).

Reducing the film thickness can allow us to study the effect of geometry constraints on some interesting phenomena already observed in 2D ultrathin films. An example is the superconducting glass phase in ultrathin granular Al film with typical grain diameter 25 nm and thickness near 5 nm [87]. An experimental signature of this glass phase is the hysteresis region nearby the $H_{c2}^{||}$, which may be found in ultrathin cylinders with a wall thickness less than those studied so far.
Fig. 3.25. a) Resistance vs. magnetic field at different temperatures as indicated showing well-defined resistance oscillations for cylinder Al-15 with a film thickness of 15 nm. No hysteresis was found at 60 mK. $H_{c2}$ was 8500 G, a value nearly 4 times larger than that for 30 nm cylinders, resulting a significantly smaller $\xi(0)$; b) Resistance vs. temperature in zero field. $T_c$ was raised by around 0.15 K compared to 30 nm cylinders with $T_c$ around 1.2 K.
Chapter 4

Ru Nanowires

Superconductivity in quasi-1D with diameter approaching nanometer scale is a subject of recent interest. Experimentally, In wires made by step-edge lithography [36], Pb wires prepared by combining e-beam lithography and quench deposition [76], and MoGe wires grown on carbon nanotube substrates [8] have been explored previously. However, all these superconducting nanowires were polycrystalline, or amorphous, with potential of having structural defects forming weak links that may have dominated the physics of the system [18]. Recently superconducting single-crystal Pb and Sn wires were prepared by electrochemical deposition in porous media [26], raising the possibility that intrinsic properties of superconducting nanowires can be measured without the complications of structural defects.

However, electrical transport measurements on all superconducting nanowires prepared by electrochemical deposition was in most cases limited to arrays of nanowires embedded in the host medium in a two-point configuration [26, 89, 28, 19]. If the wires are taken out of the host medium, an insulating oxide layer forms quickly, making it difficult to prepare electrical contacts. Ru, an elemental superconductor with a transition temperature \( T_c \) of 0.5 K, may provide an opportunity to circumvent this problem because Ru oxide is itself conducting.
Motivated by this consideration, we have studied nanowires of Ru grown in commercial, track-etched polycarbonate membranes by our collaborators [24]. Electrical transport measurements on arrays of Ru wires in the membrane and individual wires were carried out in a $^4\text{He}$, $^3\text{He}$, or dilution refrigerator using a d.c. current source and a nanovoltmeter. The details of the sample fabrication and measurement procedure have been described in Chapter 2.

Structural characterization of the Ru nanowires was carried out by high-resolution transmission electron microscopy (TEM), electron diffraction (ED), and energy-dispersive x-ray (EDX) spectrum in a JEOL 2010F field-emission transmission electron microscope. Figures 4.1a and b show the TEM images of freestanding Ru nanowires obtained from membranes with quoted nominal pore diameters of 30 nm. The actual diameter is around 50 nm. The wires are uniform in long segment. The inset of Fig. 4.1b shows a typical ED pattern of a selected spot on a 50-nm-diam Ru nanowire, showing that the wire is polycrystalline. Figures 4.1c and d gives HRTEM images of the edges of the nanowires with diameters 50 nm and 100 nm, respectively. The dark clusters in the image of the wire were identified as ultra small grains with a diameter around 2 nm. Amorphous regions surrounding Ru grains (corresponding to RuO$_2$) could also be identified. Ru wires grown in membranes with quoted nominal pore diameter of 50 nm were found to have an actual diameter around 100 nm and an essentially identical structure. Ru wires from membranes with a nominal pore diameter of 70 nm (membranes obtained separately from the 30 nm and 50 nm ones) were used to make measurement devices by photolithography and were imaged only by AFM. Their diameters by AFM were believed
Fig. 4.1. a) and b) Bright field TEM images of freestanding 50-nm-diam Ru nanowires at two different scales. Inset: Typical electron diffraction pattern on a selected spot of the wire; c) HRTEM image of a 50-nm-diam Ru nanowire; d) HRTEM image of a 100-nm-diam Ru nanowire.
to be around 70 nm as discussed in Chapter 2. They were expected to have a similar structure as those from 30 nm and 50 nm membranes.

Figure 4.2a shows the normalized resistances as functions of temperature $T$ for arrays of Ru nanowires of 50 and 100 nm diameters. Similar results on two individual Ru nanowires of a 70 nm diameter connected in series are shown in Fig. 4.2c. Although a small negative $dR/dT$ is seen at low temperatures, these Ru nanowires are clearly metallic. In this low-$T$ regime, the temperature dependence of the conductance change, $R_{\text{min}}^{-1} - R^{-1}(T)$, was found to follow $\ln T$ behavior (Figs. 4.2b and d). No superconductivity was found down to 0.3 K, well below the bulk $T_c$ of Ru, $T_c = 0.5$ K.

Figure 4.3 shows the resistance as a function of temperature for two Ru nanowires in parallel measured down to 50 mK in the dilution fridge. The diameter of wires was about 70 nm. A good metallic behavior with low resistance values was seen in the high temperature range (4.3a) followed by a weak insulating behavior measured below 1.1 K (4.3b). No feature related to superconductivity was observed even down to 50 mK.

Are individual Ru grains superconducting? If they were but the nanowire was not, the magnetoresistance (MR) of the wire should be large. We carried out MR measurements up to 8 T on the array samples (Fig. 4.4), with the field aligned both parallel and perpendicular to the Ru nanowires. For both field orientations, the MR was found to be small, a few tenths of percents at maximum, comparable with that of a typical normal elemental metal. The MR data therefore suggests that superconductivity is suppressed even in individual Ru grains. The linear MR and the $\ln T$ behavior shown in Figs. 4.2b and d are not expected in the 1D or 3D weak localization, or interaction, theories [50]. The physical origins of these observations are yet to be determined.
Fig. 4.2. a) Normalized resistances as functions of temperature for two Ru arrays. Insets show a schematic of the sample configuration; b) \( \ln T \) behavior of the conductance at low temperatures for data in a; c) Resistance as a function of temperature for two individual Ru wires in series. The diameter of the wires was 70 nm. Insets show an AFM image of the sample; d) \( \ln T \) behavior of the conductance at low temperatures for data in c.
Fig. 4.3. a) Resistance as a function of temperature $R(T)$ for two Ru nanowires in parallel; b) $R(T)$ for the same sample measured down to 50 mK. Insets show an AFM image of the sample. The nanowires look thicker than their nominal diameter $d = 70$ nm possibly due to some photoresist residues.
Fig. 4.4. Magnetoresistance (MR) at $T = 0.3$ K measured on two arrays of 50- (a) and 100-nm-diam (b) Ru nanowires. The MR were obtained with the magnetic field perpendicular to or parallel with the wires as indicated.
The suppression of superconductivity in these Ru wires up to 100 nm in diameter is surprising. Polycrystalline Pb wires of small diameters (40 and 44 nm) were found to be non-superconducting down to 4.2 K but 100-nm-diam ones were superconducting with a $T_c$ close to that in the bulk [28]. The impurity level of the nanowires is low (around or less than 0.1%). Therefore superconductivity in Ru nanowires could not have been suppressed by impurities. For thin Pb nanowires, insulating oxide layer covering Pb grains tends to confine electrons. The discreteness of energy levels of confined electrons is known to lead to the suppression of superconductivity when the average level spacing becomes comparable or larger than the bulk superconducting energy gap [3]. For typical metal, this corresponds to a grain diameter around 10 nm [11]. Alternatively, the spatial confinement of electrons will lead to fluctuation in the phase of the order parameter, again suppressing superconductivity [82]. Neither mechanism seems to be applicable to Ru nanowires in which electrons are not confined.

We proposed that superconductivity is suppressed in the Ru nanowires because of the change of electron-phonon interaction in ultrasmall Ru grains embedded in amorphous RuO$_2$. It is known that RuO$_2$ in single-crystal form is a good metal, but non-superconducting. Amorphous RuO$_2$ formed in our Ru nanowires appears to retain its metallic behavior. However, given the small size of the Ru grain, the electronic states, and therefore, the electron-phonon interaction, can be strongly modified. Alternatively, even if the modified electron-phonon interaction still leads to the formation of Cooper pairs, the strong fluctuation in the amplitude of superconducting order parameter on ultrasmall grains embedded in a normal-metal matrix may suppress superconductivity [80].
In conclusion, even though superconductivity was not observed in these Ru nanowires, metallic contacts and two-point electrical transport measurements on individual nanowires of an elemental superconductor prepared by electrochemical deposition were accomplished. No similar work had been reported previously. It will be of interest if the superconducting Ru nanowires can be prepared so that electrical transport measurements on individual superconducting nanowires could be made.
Chapter 5

\textbf{Au}_{0.7}\textbf{In}_{0.3} \textbf{Rings}

5.1 Introduction

Superconductivity in the mesoscopic superconductors, which may be defined in general as superconductors with at least two of their dimensions comparable to or smaller than the characteristic lengths of the superconductor, the superconducting coherence length or penetration depth in the zero-temperature limit, has been a very active area of research in recent years. The advances in nano-fabrication technologies have made it possible to fabricate devices with various geometries and material characteristics in which novel phenomena were discovered. For example, a highly unusual resistance anomaly, a resistance peak with the peak resistance larger than the normal-state resistance at the onset of the superconductivity, was found in mesoscopic 0D or 1D \cite{74, 63, 23, 4}, as well as 2D (2D) \cite{67} samples, and explained either in the picture of charge imbalance process near the superconductor-normal metal (S-N) interface \cite{63, 67, 23, 64} or that of a non-uniform distribution of the current within the locus of the S-N interface \cite{4, 47}. In both pictures, the presence of an S-N interface in the sample is necessary for the resistance anomaly to be observed.

Recent structural and electrical transport studies of \textbf{Au}_{0.7}\textbf{In}_{0.3} films revealed an interesting phase separation in which In-rich grains, most likely intermetallic compound
of AuIn, precipitate in a uniform In-dilute matrix, most likely Au$_{0.9}$In$_{0.1}$. The superconducting transition temperature of the In-rich grains is substantially higher than that of the In-dilute matrix, forming an array of superconductor-normal metal-superconductor (SNS) Josephson junctions [93] and a network of S-N interfaces. Interesting phenomena were found in planar and cylindrical films of Au$_{0.7}$In$_{0.3}$ [91, 92]. In particular, an $h/4e$, rather than $h/2e$, resistance oscillation was found in doubly connected cylinders of Au$_{0.7}$In$_{0.3}$. The $h/4e$ oscillation was attributed tentatively to the existence of $\pi$-junctions along the circumference of the cylinder [92]. In single mesoscopic rings of Au$_{0.7}$In$_{0.3}$ the presence of such $\pi$-junctions will lead to a phase shift of $\pi$ in the $h/2e$ oscillation if the total number of $\pi$-junctions along the ring is odd.

In this chapter, we report our experimental studies of mesoscopic superconducting Au$_{0.7}$In$_{0.3}$ rings. Conventional $h/2e$ L-P resistance oscillation was observed in one of the samples. However, whether there was a phase shift of $\pi$ was not determined primarily for technical reasons (see below). Surprisingly, in the samples showing an $h/2e$ resistance oscillation with an oscillation amplitude much smaller than that expected for L-P oscillation, double resistance anomalies and a magnetic-field-induced metallic state with excessive resistance were found. We attribute these observations to the separation of In-rich and In-dilute phases in this type of rings, which was absent in the sample that exhibited conventional L-P effect.

### 5.2 Sample Preparation and Characterization

Conventional e-beam lithography was used to prepare the Au$_{0.7}$In$_{0.3}$ rings. The ring pattern was generated using double-layer PMMA/MMA resist on polished $1 \times 1$
Fig. 5.1.  a) AFM image of a 1-µm-diam Au$_{0.7}$In$_{0.3}$ ring (6-100, see Table 5.1); b) Schematic corresponding to the AFM image in Fig. 5.1a. The linewidth of all rings is nominally 100 nm, though in AFM it is found to be slightly larger, e.g., 130 nm for Ring 6-100. However, whether the increase is Au$_{0.7}$In$_{0.3}$ or just organic residues from the lift-off process is not clear; c) Height profile along the ring arms showing surface roughness.
Sequential thermal evaporation of alternating 99.9999% pure Au and In layers, with the layer thickness determined by the appropriate atomic ratio of Au to In, was carried out at ambient temperature in a conventional evaporator with a vacuum of $1 \times 10^{-6}$ torr or slightly better. The ring pattern was placed with respect to the Au and In sources so as to minimize the shadow effect during evaporation. AFM was used to image the resulted Au$_{0.7}$In$_{0.3}$ rings before and/or after the measurements.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$d$ ($\mu$m)</th>
<th>$R_{N,\Box}$ ($\Omega$)</th>
<th>$\rho_N$ ($\mu\Omega$ cm)</th>
<th>$T_c$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-075</td>
<td>0.75</td>
<td>13.5</td>
<td>40.6</td>
<td>0.450</td>
</tr>
<tr>
<td>6-100</td>
<td>1.00</td>
<td>13.5</td>
<td>40.4</td>
<td>0.380</td>
</tr>
<tr>
<td>7-100</td>
<td>1.00</td>
<td>12.5</td>
<td>37.4</td>
<td>0.370</td>
</tr>
<tr>
<td>8-075</td>
<td>0.75</td>
<td>11.3</td>
<td>33.8</td>
<td>0.359</td>
</tr>
<tr>
<td>8-100</td>
<td>1.00</td>
<td>12.4</td>
<td>37.2</td>
<td>0.355</td>
</tr>
</tbody>
</table>

Five rings of Au$_{0.7}$In$_{0.3}$ were measured in this study. Their structural and electrical parameters are listed in Table 5.1. Two rings have a diameter of 0.75 $\mu$m and the rest have a diameter of 1 $\mu$m. The thickness of all rings is nominally 30 nm. The linewidth of all rings is nominally 100 nm, though in AFM it is found to be slightly larger. For example, the linewidth is 130 nm for Ring 6-100 as shown in Fig. 5.1. However, whether the increase is Au$_{0.7}$In$_{0.3}$ or just organic residues from the lift-off process is
not clear. The AFM image of Ring 6-100, a 1-µm-diam ring, is shown in Fig. 5.1a, with a schematic shown in Fig. 5.1b to detail the various dimensions of the sample. Rings with a 0.75 µm diameter have the same layout with a 4 µm voltage-probe separation. Two features of the samples should be noted. First, as seen in the schematic of the sample (Fig. 5.1b), the linewidth increases slightly at the nodes of the voltage leads going towards the large contact pads. Such a variation of the linewidth, observed in all samples, was probably due to an overexposure while writing the large contact pads; Second, the surface roughness as seen by AFM increases at the narrow part of the sample (within the voltage leads, see Fig. 5.1c). This roughness might be due to the contamination by the organic residues such as PMMA and/or developer during the lift-off process, which may explain the increase of the normal-state sample resistivity, $\rho_N$ (Table 5.1), compared to that of the Au$_{0.7}$In$_{0.3}$ films with the same thickness prepared under nominally the same conditions [93].

5.3 Results on Double Resistance Anomalies

In Fig. 5.2a, normalized resistance as functions of temperature $R(T)$ in zero magnetic field are shown for all five rings. It is clear that there exist two types of behaviors among these rings. For Ring 6-075, a smooth resistive transition was seen. For other four rings, however, two resistance peaks were found near the onset of the superconducting transition. It is seen that the low-temperature resistance peak (LTRP) is relatively sharp, with a resistance about 30-70% higher than the normal-state resistance, $R_N$. The high-temperature resistance peak (HTRP) is broader and smaller in height (10-20% higher than $R_N$) than LTRP.
Fig. 5.2.  a) Normalized resistances as functions of temperature $R(T)$ for five Au$_{0.7}$In$_{0.3}$ rings. Rings are labeled as indicated. Parameters for these rings are shown in Table 5.1; b) A LAMH theory fit (solid line) to $R(T)$ data of Ring 6-100 (see text).
The parameters of our Au$_{0.7}$In$_{0.3}$ rings are shown in Table 5.1. Another important parameter, the superconducting coherence length in the zero-temperature limit, which can be used to judge whether our samples are in the mesoscopic limit, can also be estimated. Within the Ginzburg-Landau theory, the superconducting coherence length can be estimated from the upper critical field, $\xi(0) = \sqrt{3}\Phi_0/\pi w H_{c2}$, where $\Phi_0 = h/2e$ is the flux quantum and $w$ is the linewidth, which yields a $\xi(0)$ value of 88 nm for Ring 6-100 based on experimental value of $H_{c2}(20 \text{ mK}) = 1300 \text{ G}$. Alternatively, we can also estimate $\xi(0)$ using the microscopic theory for dirty superconductors, $\xi_{0,\text{dirty}} \approx 0.86(\xi_0 l_{\text{el}})^{1/2}$, where $\xi_0 = h v_F/\pi \Delta$ is the coherence length in the clean limit and $l_{\text{el}}$ is the mean-free path [82]. Using a gap value of Au$_{0.7}$In$_{0.3}$, $\Delta \approx 0.088 \text{ meV}$, obtained from planar tunneling measurements on Au$_{0.7}$In$_{0.3}$ films, and a mean-free path of 1.68 nm using $l_{\text{el}} = m v_F/\rho n e^2$, where $m$ is the mass of electron, $v_F$ is the Fermi velocity, $\rho$ is the resistivity, and $n$ is the free electron density, we obtain a value of $\xi_{0,\text{dirty}}$ of 68 nm. Therefore it is likely that $\xi(T)$ is longer than the linewidth of the ring (nominally 100 nm) at temperatures sufficiently close to the superconducting transition temperature.

Furthermore, for all samples shown in Fig. 5.2a, the tail of the resistive transition in $R(T)$ can well be described by the LAMH theory [48, 58] of thermally activated phase slips in 1D limit (An example of fitting is shown in Fig. 5.2b), providing additional support that our samples were in the 1D limit.

Resistance anomaly featuring a single resistance peak at the onset of superconductivity has been widely observed in mesoscopic superconductors [74, 63, 23, 4] as well as in 2D superconducting films [67]. It seems reasonable that the double resistance anomalies observed in the present work and the previously observed resistance anomaly
Fig. 5.3.  

Fig. 5.3.  a) $R(T)$ for Ring 6-100 in zero field. The temperatures at which $dV/dI$ were taken (shown in b) are indicated by arrows; b) Dynamical resistance $dV/dI$ (calculated from d.c. current biased $I-V$ curves using numerical derivatives) at various temperatures as indicated. Two critical currents are indicated by the dashed line and by the arrow, respectively. All curves except the one for $T = 0.26$ K are shifted for clarity.
share a similar physical origin. Since the resistance anomaly was found to occur at the onset of superconductivity, the occurrence of the double resistance anomalies appears to indicate the existence of two $T_c$'s. In Fig. 5.3b, we plot the dynamical resistance $dV/dI$ as functions of the bias current $I$ for Ring 6-100 at different temperatures within the resistive transition as marked by arrows in Fig. 5.3a. At $T = 0.26$ K, two sharp rises in $dV/dI$ were seen (shown by the dash lines and by arrows at 0.26 K), which appear to correspond to two critical currents, $I_c$'s. Indeed the dynamical resistance was vanishingly small below $\approx 0.05$ $\mu$A, the smaller $I_c$, evolving into a central peak near zero bias current as temperature was raised to 0.30 K. At this temperature, the dynamic resistance showed a sharp rise at $\approx 0.2$ $\mu$A, the larger $I_c$. While the smaller $I_c$ was found to become zero around 0.30 K, the larger $I_c$ survived at least up to 0.34 K, roughly where the HTRP was located. Above this temperature, the entire sample turned normal. The presence of two $T_c$'s and two $I_c$'s suggests strongly the presence of two superconducting phases in the sample.

Similar to planar Au$_{0.7}$In$_{0.3}$ films [93], these two phases should be the In-rich grains, most likely intermetallic compound of AuIn, and the Josephson coupled In-rich grains embedded in the uniform In-dilute matrix, most likely Au$_{0.9}$In$_{0.1}$. The superconducting transition temperatures of the In-rich grains are higher than that of the In-dilute matrix. As a result, the sample can be viewed as a random array of SNS Josephson junctions. Such underlying structure will not only give rise to many S-N interfaces, but also lead to the non-uniform distribution of current and the creation of phase slip centers in the sample, favoring the occurrence of resistance anomaly. Despite of the randomness, there exist two typical $T_c$'s. Therefore, HTRP should correspond to the resistance
Fig. 5.4. $R(T)$ at several magnetic fields as indicated for Ring 6-100. The fields were applied perpendicular to the plane of the ring. A resistance plateau higher than $R_N$ is seen down to the lowest temperature available ($<20$ mK) for $H = 300$ and 900 G.
anomaly of the individual In-rich grains (with higher $T_c$ and larger $I_c$) while LTRP should correspond to that of the junction array formed by the In-rich grains and the In-dilute matrix (with lower $T_c$ and smaller $I_c$). Interestingly, features found in the dynamical resistance in currents below the larger $I_c$ ($\approx 0.2 \, \mu\text{A}$) and their temperature dependence, i.e., the sharp rise above the smaller $I_c$ ($\approx 0.05 \, \mu\text{A}$) and the emergence of a central peak near zero bias current, are similar to those observed in mesoscopic Al wires showing resistance anomaly [74], which supports our assigning the lower $T_c$ and smaller $I_c$ to the overall sample.

Double resistance anomalies were observed previously [73, 62]. In one experiment [62], two resistance peaks were observed in a filled but not in an open Al square of mesoscopic size $1 \times 1 \, \mu\text{m}^2$ (our Au$_{0.7}$In$_{0.3}$ rings are open) below the onset of superconductivity, not at the onset of superconductivity as seen in our Au$_{0.7}$In$_{0.3}$ rings. Furthermore, the resistance peaks were only seen in a finite field and their resistance values were below the normal-state resistance. Therefore we believe that those resistance peaks were unrelated to the physics that is responsible for the double resistance anomalies observed in our Au$_{0.7}$In$_{0.3}$ rings. However, the double resistance anomalies and a common feature seen in the Au$_{0.7}$In$_{0.3}$ rings, namely, the LTRP is always higher and sharper than the HTRP, were found in one of the granular films prepared from pre-formed In clusters embedded in a Kr matrix [73] at large excitation currents. Obviously, many differences exist between the two very different systems, which may be responsible for the presence of triple resistance anomalies found in In-Kr films in small excitation currents that were not observed in our Au$_{0.7}$In$_{0.3}$ rings. It should be noted that the current dependence of these resistance anomalies, in which the higher temperature peak
Fig. 5.5. a) Dynamical resistance $dV/dI$ at various temperatures, as indicated, for Ring 6-100 at $H = 300$ G; b) $dV/dI$ at $H = 900$ G. $dV/dI$ curves are based on numerical derivatives of d.c. current biased $I-V$ measurements. All curves except the one at $T = 0.10$ K are shifted for clarity in both panels.
was suppressed first by increasing excitation current, is very different from that in the 
Au$_{0.7}$In$_{0.3}$ rings. Similar to our Au$_{0.7}$In$_{0.3}$ samples, this film can be modeled as an array 
of superconductor-insulator-superconductor Josephson junctions [73], with a $T_c$ smaller 
than that of the individual In clusters.

The measurements on the magnetic field dependence of the double resistance 
anomalies, shown in Fig. 5.4, provide further support to the picture that the presence 
of two superconducting phases is responsible for the double resistance anomalies. It is 
seen that LTRP was affected significantly at a field as low as 100 G, becoming barely 
visible at 300 G, which appears to be a critical field, $H_c$. On the other hand, in a field 
as high as 900 G, even though HTRP was broadened and shifted to lower temperatures, 
it was clearly visible. In fact, even at field up to 1300 G, HTRP could still be identified 
close to the lowest temperature, 20 mK. It is therefore evident that the HTRP was 
associated with In-rich grains with a critical field of 1300 G while LTRP belongs to the 
SNS Josephson junction array with a critical field of 300 G.

The dynamical resistance curves taken at 300 G (Fig. 5.5a) show that, at 0.1 K, 
the minimum near the zero bias current was replaced by a peak, indicating that the 
critical current of the SNS Josephson junction array was essentially zero at 300 G. This 
is consistent with results of Fig. 5.4, which in turn supports the assessment that LTRP 
was associated with the SNS Josephson junction array. The sharp rises in $dV/dI$ at high 
bias current, on the other hand, was clearly visible at $H = 900$ G (Fig. 5.5b), suggesting 
that HTRP was associated with the In-rich grains.
5.4 Results on the Low-Temperature Metallic State

An interesting feature emerging from Fig. 5.4 is that, in the intermediate field range, the broadened HTRP extended to the lowest temperature we measured. At $H = 900$ G, in particular, the resistance became independent of temperature down to 20 mK, with a resistance value larger than that of the normal-state resistance, $R_N$. The existence of this low-temperature resistance plateau, referred to here as excessive resistance, appears to suggest the existence of a metallic state in which the In-rich grains were superconducting, but not Josephson coupled. This metallic state, with its onset temperature $> 0.2$ K, should not be due to heating as it is generally believed that only below 0.1 K electron dephasing and heating from the coupling to the environment start to be a serious concern. The observation of a metallic state with its resistance larger than $R_N$ and its onset temperature as high as 0.2 K is a strong evidence that this metallic state is intrinsic. The physics of such a novel metallic state is yet to be explored.

5.5 Results and Discussions of $h/2e$ Resistance Oscillations

For Ring 6-075 with a smooth $R(T)$ showing no double resistance anomalies, its onset $T_c$ was around 0.45 K, close to that observed in thinnest planar films (thickness $\leq 15$ nm) of Au$_{0.7}$In$_{0.3}$ [93]. In those thinnest films, the interdiffusion of Au and In was suppressed by the close proximity to the substrate, resulting in films that were uniform rather than phase separated. Ring 6-075 might be of similarly uniform structure. This assessment is consistent with the observation of conventional $h/2e$ L-P resistance oscillation in this sample as shown in Fig. 5.6a. Unfortunately, whether there was a
Fig. 5.6.  a) Little-Parks resistance oscillation of Ring 6-075 at various temperatures as indicated. The period of the oscillation corresponds to $h/2e$; b) Altshuler-Aronov-Spivak (AAS) resistance oscillations enhanced by superconductivity (see text) of Ring 6-100 at various temperatures as indicated. The oscillation period corresponds to $h/2e$. The resistance variation corresponding to $20e^2/h$ is shown.
phase shift of $\pi$ in the $h/2e$ oscillation in Ring 6-075, the question that motivated this work originally, could not be determined because it is difficult to determine the flux sufficiently precisely because of the possibility of trapping flux in the superconducting magnet.

For the four rings showing double resistance anomalies, a weak $h/2e$ resistance oscillation was found as shown in Figs. 5.6b and 5.7 for Ring 6-100. The oscillation was most pronounced in the intermediate field range, weaker or even absent in the low field range. For example, for the $R(H)$ curve at $T = 0.3$ K, the oscillation was pronounced between 350 G and 600 G, less obvious between 100 G and 300 G, and indiscernible below 100 G ($H < 0$ data not shown for clarity). A Fourier analysis of the data with different field ranges generated similar findings. The oscillation was completely quenched above $H_c$ or $T_c$.

It is of interest to ask whether the $h/2e$ resistance oscillation observed in these rings showing double resistance anomalies is L-P resistance oscillation. Anomalous L-P oscillation was observed in Al loops in a previous study in low field range, where the oscillation was distorted due to the complication of resistance anomaly and restored to the conventional L-P with increasing field [84]. However, we believe that the oscillation in the current work is unlikely to be related to the L-P effect. First, the resistance oscillation was absent in low fields, inconsistent with the L-P behavior; Second, the amplitude of the resistance oscillation is very small in comparison with the conventional L-P resistance oscillation shown in Fig. 5.6a—The oscillation amplitude is less than 5 $\Omega$ for Ring 6-100 but as large as 50 $\Omega$ in Ring 6-075. The superconducting coherence lengths $\xi$ estimated from their upper critical field values or using the formula for dirty
Fig. 5.7. Resistance as a function of applied magnetic field $R(H)$ at several temperatures, as indicated. The period of the resistance oscillations is $h/2e$. The negative magnetoresistance in the intermediate fields is related to the suppression of the resistance anomalies by the field. All curves except the one at $T = 20$ mK are shifted for clarity.
superconductors are not very different for the two samples. The difference in diameter along cannot explain the large difference in amplitude since the L-P amplitude is proportional to \( [\xi(0)/d]^2 (\Delta R/\Delta T) \), where \( d \) is the ring diameter and \( \Delta R/\Delta T \) is the measured slope in the transition region [82]; Our Au\(_{0.7}\)In\(_{0.3}\) rings consist of a random SNS junction array because of the phase separation, which will give rise to three temperature regimes at given applied flux values. At the lowest temperatures the whole ring will be superconducting while at sufficiently high temperatures the ring will be normal. In the intermediate temperatures, however, only the In-rich grains will be superconducting, with no global phase coherence. L-P resistance oscillation results from the modulation of the phase boundary of the fully superconducting phase (with global phase coherence) by the applied flux. For our Au\(_{0.7}\)In\(_{0.3}\) rings featuring double resistance anomalies, the resistive transition as a function of magnetic flux at this phase boundary is extremely sharp as shown in Fig. 5.7, making it difficult to observe the L-P resistance oscillation. In this regard, it will be interesting to prepare rings of a lithographically defined, regular SNS junction array with less sharp resistive transition to compare with our Au\(_{0.7}\)In\(_{0.3}\) rings.

For our Au\(_{0.7}\)In\(_{0.3}\) samples, superconductivity in individual In-rich grains is suppressed over a rather wide range of magnetic field, in which the \( h/2e \) resistance oscillation was observed away from the boundary of the fully superconducting phase.

We believe that the \( h/2e \) resistance oscillation in rings showing double resistance anomalies is the Altshuler-Aronov-Spivak (AAS) resistance oscillation [2] enhanced by superconductivity. The AAS effect is a result of the coherent backscattering of normal electrons in disordered systems, as first experimentally seen by Sharvin and Sharvin [77]. AAS oscillation was shown to be enhanced in normal metal samples in contact with one
or more superconducting islands in several experiments [15, 69]. The amplitude of AAS resistance oscillation is determined by both $L$, the length of the circumference of the time-reversal path, and $L_{\varphi}$, the electron dephasing length, in the form $\propto \exp \left( -L/L_{\varphi} \right)$. In our experiment, when In-rich grains became superconducting, $L$, which included only the normal part of the ring, decreased. Meanwhile, $L_{\varphi}$ increased because of the superconducting fluctuations. As a result, the amplitude of the resistance (or conductance) oscillation was enhanced as $\Delta G \approx 11e^2/h$ for Ring 6-100 (Here an effective normal-state resistance of the ring itself, 105.8 $\Omega$, estimated from the sample geometry, rather than the total resistance of the sample, 510 $\Omega$, is used). Such an enhanced amplitude is comparable to previous observations [15, 69]. The AAS effect enhanced by superconductivity also explains the fact that the oscillation was quenched above $T_c$ or $H_c$ (Figs. 5.6b and 5.7).

5.6 Possible Explanation for Double Resistance Anomalies

In closing, we would like to discuss briefly the possible physical origin of the double resistance anomalies. As mentioned above, the double resistance anomalies should be due to a physical process similar to the resistance anomaly observed previously in mesoscopic superconducting samples. The physical origin of the resistance anomaly is still subject to an intensive debate [74, 63, 67, 23, 4, 47, 64] and is not settled down. One school of thoughts has argued that the charge imbalance near the S-N interface contributes to the resistance anomaly [63, 67, 23, 64]. Charge imbalance is a nonequilibrium process
Fig. 5.8. Schematic of a charge imbalance process explaining resistance anomaly. $\mu_p$, $\mu_q$, and $\mu_n$ are the chemical potentials of Cooper pairs and quasiparticles in the superconductor (S), and normal electrons in the normal metal (N), respectively. $\xi$ is the superconducting coherence length. $\Lambda_Q$ is the charge imbalance length. $x$ is the distance between the voltage leads ($V_-$) and the S-N interface where $\Delta = 0$. Since a superconducting probe ($V_-$) detects $\mu_p$ instead of $\mu_q$, an extra voltage drop of $\Delta V$ is picked up, which gives resistance anomaly.
related to the injection of normal electrons through the S-N interface. Consider now an 1D S-N interface at which the energy gap $\Delta = 0$ (Fig. 5.8). The gap $\Delta$ reaches the bulk value (full value) over the coherence length $\xi$. Close to $T_c$, the injection of normal current from the normal side (N) causes nonequilibrium distribution of quasiparticles on the superconducting side (S), which relaxes back to its equilibrium value over the charge imbalance length $\Lambda_Q$. The electroneutrality requires a corresponding decrease in the charge of the condensate to counter the increase of quasiparticle charge, thus leading to a spatial variation of chemical potential of quasi-particles, $\mu_q$, and that of Cooper pairs, $\mu_p$. In the normal side of the S-N interface, $\mu_n$ varies linearly spatially and matches with $\mu_q$ and $\mu_p$ at the interface [85]. Inside the superconducting region, in the absence of any supercurrent (near $T_c$), $\mu_q$ can be approximated as [41]

$$\mu_q(x) = -\rho I \Lambda_Q I \tanh(x/\Lambda_Q),$$  \hspace{1cm} (5.1)

where $\rho I$ is the normal-state resistance per unit length, $I$ is the current, and $x$ is the distance from the S-N interface. Because the superconducting voltage leads probe $\mu_p$, a potential difference between the extrapolated $\mu_n$ and the $\mu_p$, $\Delta V$ (Fig. 5.8), gives rise to a resistance peak when the voltage leads are placed at $x < \Lambda_Q$ from the interface. The magnitude of this resistance rise is roughly given by

$$\Delta R(x) = \rho_I (\Lambda_Q - x).$$  \hspace{1cm} (5.2)
The S-N interface can be a result of either phase slip centers at weak links or sample inhomogeneity. Superconducting voltage probes are required to observe the anomaly [67]; The other school has pursued an alternative model based on the non-uniform distribution of the current across the sample within the locus of S-N interface [4, 47]. For example, compare the effect of a tilted S-N interface and that of a non-tilted S-N interface as shown in Figs. 5.9a and b, respectively. Solving the Laplace equation for scaler electrical potential in the configurations in Fig. 5.9, it is found that the local current density nearby the voltage probe when S-N is tilted, $j_t$, will be larger than that when S-N is non-tilted, $j_n$ [4]. Therefore, the voltage due to such a current density change picked up by the voltage probe nearby the interface is roughly $\rho \Delta L j_t$ for tilted interface, as compare to $\rho \Delta L j_n$ for non-tilted, where $\rho$ is resistivity and $\Delta L$ is the length of the segment nearby the voltage probe where a change of the current density due to a tilted S-N interface is effective. Clearly, an enhancement of the voltage signal, therefore the resistance anomaly, can be detected due to a tilted S-N interface as compared to a non-tilted one. The S-N interface originates from sample inhomogeneity and non-superconducting voltage probes can also lead to the resistance anomaly. In both scenarios, an S-N interface is necessary in order to observe the resistance anomaly, which indicates that the sample consists of superconducting and normal patches at the time of observation.

$\text{Au}_{0.7}\text{In}_{0.3}$ rings are composed of superconducting grains (In-rich phase) embedded in normal metal matrix (In-dilute phase). Therefore there exist preformed S-N interfaces inside the sample as the temperature is first below the onset $T_c$ of the grains; Further decreasing of temperature helps establishing the Josephson coupling among neighboring
Fig. 5.9. Schematic of a tilted S-N interface formed nearby the voltage probe a) and that of a non-tilted S-N interface b). The local current density nearby the voltage probe in a is larger than that in b due to a tilted interface.
grains so that large superconducting patches can form, resulting in large S-N interface inside the sample.

Can the double-peak anomaly be explained in terms of the formation of two kinds of superconducting patches (grains and Josephson coupled grains) inside the sample? In our picture, the formation of superconducting grains with small S-N interfaces gives the HTRP which is small in height and occurs at high temperature; The formation of superconducting patches from Josephson coupled grains gives LTRP which is large in height and occurs at low temperature. It should be noted that the two debating strategies to explain the resistance anomaly are both qualitatively compatible with our picture. For example, superconducting grains and their associated small S-N interfaces can be viewed to give a reduced charge imbalance effect since the real charge imbalance process happens in superconducting region with size larger than the charge imbalance length \( \Lambda_Q \) (up to a few microns in Au\(_{0.7}\)In\(_{0.3}\), Fig. 5.8). A reduced charge imbalance effect, therefore, gives small resistance peak as HTRP observed in experiment; In the other thought, small S-N interfaces have less modification on the current flow, resulting in less non-uniform distribution of current and therefore a small peak as HTRP. However, a serious quantitative comparison between the two strategies applying to our picture on the double-peak resistance anomaly in these phase separated Au\(_{0.7}\)In\(_{0.3}\) rings is yet to be explored to further clarify the origin of resistance anomaly.
Chapter 6

Conclusion

In this thesis we have examined the effects of dimensionality, sample topology, and disorder on superconductivity in superconducting systems, with particular focus on doubly connected ultrathin superconducting cylinders of Al. In the cylinders, the possible phase separation region near the quantum phase transition that features the formation of normal bands in an otherwise homogeneous superconducting cylinder represents a spatial modulation of the superconducting order parameter, similar to the superconducting droplet state in 2D films. We show that the quantum phase transition associated with the destructive regime may be continuous, associated with quantum critical behavior. We investigated the normal state in the destructive regime and found that the peak resistance inside the destructive regime depended on the $d/\xi(0)$ ratio, saturating at $d/\xi(0) \approx 0.77$. This result suggests that $d/\xi(0)$ is a measure of the superconducting fluctuation strength.

We also found, surprisingly, a K-T behavior in cylinders with very limited size of circumference for vortex pairs to appear. We observed an interesting phase diagram beyond de Gennes’ original theory. Thermally activated phase slips were found to possibly account for the $R(T)$ data at low temperatures away from the destructive regime. Experiments on systems other than the cylinders revealed the suppression of superconductivity in Ru nanowires with ultrasmall grains, and double resistance anomalies in phase separated Au$_{0.7}$In$_{0.3}$ rings.
Most of these observations are new and cannot be explained by mean-field theories. No microscopic theory has been proposed to explain the phase separation near the QPT in the ultrathin cylinders. The superconducting fluctuation in the destructive regime remains to be further explored. A scaling analysis of the QPT associated with the destructive regime, which requires carrying out the experiments on cylinders with systematically varying diameter and wall thickness, has not been attempted.
Appendix A

Further Information on Sample Fabrication

The fabrication of doubly connected cylinders has been introduced in chapter 2 and also in some earlier work [90, 72]. An optical image of a real sample was shown in thesis [90]. Here we will summarize the fabrication process.

A.1 Pulling and Seeing Quartz Filaments

Pulling a quartz filament down to almost 100 nm in diameter was done by hand, though a micropipette puller (Sutter instrument co. model P-2000) was tried. It turns out that the puller yielded poor results. It is possible that the pulling force for a micropipette puller is too different from what is used by hand, or the laser used by the puller could not melt the quartz sufficiently before pulling. It is even difficult for us to use the puller to get a submicron filament with a usable length (> 1 mm).

Quartz filaments down to 100 nm in diameter were seeable by eyes with a strong light (Fiber-lite 190) shedding perpendicularly from the side, possibly due to diffraction. One can even estimate whether it is small enough (< 180 nm) by eyes. We also used a 10 X optical lens together with a 1/2” CCD (Sony XC-999) to image the filament on screen (CCD digital magnification on our display screen ≈ 42 X). Suspended filaments thinner than 180 nm were impossible to see on screen under normal conditions. A strong light from the side and a dark background could help locate the filament easily.
Most filament on screen (diffraction pattern) looked shiny, indicating that the filaments were smooth without bumps on the surface. This is expected as resulted from pulling of quartz melt. However, the optical image could not be used to estimate the diameter since the diffraction pattern instead of the cylinder was being imaged.

### A.2 Coating an Al Film

Conventional thermal evaporation of Al was used while rotating the quartz filament. Rotation speed was controlled at around 1 rev/sec. Higher rotation speed tended to yield samples with a much higher critical field, therefore more disordered. Significantly higher rotation speed yielded samples 100% not conducting. A hypothesis is that fast motion of substrate resulted in a less efficient deposition of Al atoms. Therefore the real thickness of Al film on the filament was far less than what the thickness monitor indicated for a very fast rotation. The film thickness that we recorded was a nominal value.

The deposition rate of Al film ranged from below 20 Å to more than 100 Å (real thickness monitor readings). Our experience shows that higher rate tended to give higher critical field and shorter coherence length, therefore more disordered. The deposition rate between 20 Å to 30 Å tended to give a critical field less than 3000 G, desirable for destructive regime physics. However, the relation between the deposition rate and the film quality were not always the same since we had a few samples showing the reversed trend. Other parameters, for example, the humidity could also be a factor to degrade the film quality after its being made.
Some of the good samples with a low critical field were made with deposition rate around 50 Å or slightly higher. An advantage of higher deposition rate is that it gave less trouble for electrical contacts. This is highly important because not all the contacts could survive the thermal cycling. We quite often ended up with a 3-wire or 2-wire sample. A bad contact was characterized by an insulating behavior with a steep slope $> 20 \, \Omega/K$ in $R(T)$ below the transition of Al, which was often associated with a negative magnetoresistance in low fields. Bad contacts made the subtraction of the contact resistance from the $R(T)$ data very troublesome. Our hypothesis for a bad contact is that a superconductor-normal metal interface formed near the contact point, possibly leading to a charge imbalance process and a large slope in $R(T)$. This is similar to the resistance anomaly in Au$_{0.7}$In$_{0.3}$ rings. However, how the slow deposition rate led to a bad contact is not clear.

### A.3 Making Electrical Contact

Ag epoxy or paste were regularly used to make electrical contact. They made contact to cylinders but the contact usually degraded with time significantly. Evaporating Au to cover the cylinder for better contact was attempted. For the thin film cylinder it worked where Au is in between the cylinder and the Ag epoxy. However we did not achieve significantly higher yield rate, and the contact degraded with time and finally died even with Au covering the Al film. Since our cylinders were more likely to be floating above the small glass slide support in the gap of the U-shape glass slide (Fig. 2.1), a thick film of Au could not by itself cover the cylinders well. We demonstrated attracting the cylinder to the substrate by incorporating the cylinder into a capacitor.
Humidity is bad for preparing working contacts. We noticed big difference in the surviving rates of our samples under different humidities.

We found that by adding some small Ag particles with around 150 nm in diameter (Inframat advanced materials) into the Ag epoxy, we could slightly raise the yield of our working samples. However, same procedure with Ag paste totally ruined the contact. We believe that small Ag particles could fill into the gap between the large Ag particles in epoxy (typically more than microns in size), making better contacts onto the curled surface of a tiny cylinder with less than 200 nm in diameter.

During the cooling down from 77 K to 4 K, a few contacts died each time, which was likely resulted from different thermal contractions of Ag particles and Al films.

The contact between the Ag particles and cylindrical Al films can be modeled as a touch-ball contact. It is usually very weak, as we confirmed that most time a non-conducting sample was due to the bad contact instead of the sample itself being non-conducting.

Samples are fragile. It was confirmed that the discharge of a $1 \mu$F capacitor (static charges had built up for a few days) integrated in a home-made low-pass filter (cut-off frequency $\approx 6$ KHz) burnt a couple of samples.
Appendix B

Summary of Cylinder Properties

B.1 Cylinder Morphology

The cylinders were long and straight, as shown by an optical image of a typical sample in Fig. B.1. The scanning electron microscopy (SEM) image of a sample was shown in Fig. 3.15d, revealing uniform granular morphology of the coated Al film.

![Optical microscope image of a cylinder between two Ag epoxy islands that connect the measurement leads (500 X).](image)

Fig. B.1. Optical microscope image of a cylinder between two Ag epoxy islands that connect the measurement leads (500 X).

B.2 List of Selected Parameters of All Cylinders
Table B.1.
Parameters of cylinders reported in this work. $\xi(0)$ and $\lambda(0)$ are estimated from the parallel upper critical field, $H_{c2}^\parallel$; the length of the cylinder, $L$, is defined by the distance between the edges of the adjacent Ag epoxy or paste dots (in the case Au was used, $L$ is defined by adjacent edges of the Au films); the mean-free path, $l_{el}$, is estimated from the normal-state resistivity, $\rho_N$, using free-electron gas model.

<table>
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<th>$d$ (nm)</th>
<th>$t$ (nm)</th>
<th>$L$ (nm)</th>
<th>$\xi(0)$ (nm)</th>
<th>$l_{el}$ (nm)</th>
<th>$\lambda(0)$ (nm)</th>
<th>deposit. rate (Å/s)</th>
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References


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Vita

Haohua Wang was born in Yichang, Hubei, P. R. China in 1977. He is the younger son of Yonggui Wang and Yongying Han. In 1999 he received the B.S. degree in Physics, from Nankai University, P. R. China. In 2001 he enrolled in the Ph. D. program in Physics at the Pennsylvania State University. Since 2001 he has been employed in the Physics Department of the Pennsylvania State University as a teaching assistant for five years and as a research assistant for one year.

Haohua Wang is a member of the American Physical Society.

Selected Publications


