FLOWBACK RATE TRANSIENT ANALYSIS OF MULTI-FRACTURED HORIZONTAL WELLS IN TIGHT AND ULTRATIGHT RESERVOIRS

A Dissertation in
Energy and Mineral Engineering
by
Fengyuan Zhang

© 2020 Fengyuan Zhang

Submitted in Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy

December 2020
The dissertation of Fengyuan Zhang was reviewed and approved by the following:

Hamid Emami-Meybodi  
Assistant Professor of Petroleum and Natural Gas Engineering  
Dissertation Advisor  
Chair of Committee  

Luis F. Ayala H.  
William A. Fustos Family Professor  
Professor of Petroleum and Natural Gas Engineering  

Gregory King  
Professor of Practice, Petroleum and Natural Gas Engineering  

Ming Xiao  
Associate Professor of Civil and Environmental Engineering  

Mort Webster  
Professor of Energy Engineering  
Associate Department Head for Graduate Education
ABSTRACT

This dissertation covers theoretical analysis and mathematical modeling of flowback period for multi-fractured horizontal wells in tight and ultratight (shale) hydrocarbon reservoirs. The analysis of flowback data obtained right after hydraulic fracturing is of critical importance in the characterization of hydraulic fracture (HF), evaluation of stimulation job, and the simulation of hydraulically fractured reservoirs. Analysis of flowback data provides an early-time estimation of fracture characteristics and stimulation effects guiding the efficient development of unconventional reservoirs. Accordingly, the main goal of this research is to provide rigorous yet straightforward approaches for HF characterization by analyzing two-phase (i.e., water and hydrocarbon) flowback data. To achieve this goal, this dissertation follows four progressive steps.

First, a one-dimensional (1-D) two-phase (i.e., water and gas) flowback model is developed for the early-time flowback period when fluid influx from matrix remains insignificant and production is mainly from HF. A new method of calculating average pressure in the fracture is proposed for the 1-D system based on the solution of water-phase diffusivity equation. Three iterative workflows are proposed using the successive substitution method to calculate fracture attributes under different production conditions.

Second, a set of two-dimensional (2-D) two-phase (i.e., water and gas) semianalytical models is developed for the late-time flowback period when fluid influx from matrix to HF becomes significant. A modified material balance equation (MBE) is derived for the 2-D system to obtain average pressure in HF by calculating hydrocarbon influx from matrix under variable bottomhole pressure (BHP) condition based on Duhamel’s principle. Two workflows are proposed to quantify the fracture dynamics through the loss of both fracture volume and permeability by reconciling flowback and long-term production data.
Third, multiple water and gas storage and transport mechanisms in nanopores are incorporated into the 2-D flowback models to predict HF attributes and dynamics through initial pore-volume, initial permeability, compressibility, and HF permeability modulus. To represent the fluid transport in ultratight gas reservoirs, the matrix model considers desorption, slip flow, diffusion, stress dependence, and water film on pore walls. A modified MBE is proposed to account for the sorption phenomenon in the organic nanopores.

Finally, the 2-D flowback models are extended to tight and ultratight oil reservoirs by considering multiple water and oil transport mechanisms in nanopores. A new effective matrix permeability is defined for each phase to consider multiple transport mechanisms including two-phase flow, slippage effect, stress dependence, the variation of fluid properties in the bulk and near-wall regions, as well as the differences in slip length, pore size, and wettability of organic and inorganic nanopores.

Validation of the developed flowback models and the analysis workflows against numerical simulation shows the accuracy of proposed approaches and their capability in the quick interpretation of flowback data, estimation of HF properties, and evaluation of HF closure. Field applications in Horn River shale gas, Marcellus shale gas, and Eagle Ford shale oil reservoirs confirm the practicality and robustness of the proposed models. The findings of this dissertation provide insight into the HF characterization and the quantitative understanding of HF closure dynamics not only during flowback but also long-term production period.
# TABLE OF CONTENTS

LIST OF FIGURES ......................................................................................................................... viii

LIST OF TABLES ............................................................................................................................... xiii

ACKNOWLEDGEMENTS ..................................................................................................................... xv

DEDICATION ..................................................................................................................................... xvi

Chapter 1  Overview ........................................................................................................................... 1

1.1 Motivations and Objectives ......................................................................................................... 1
1.2 Dissertation Outline ....................................................................................................................... 3

Chapter 2  Multiphase Flowback Rate-Transient Analysis of Shale Gas Reservoirs ............ 5

2.1 Abstract ....................................................................................................................................... 5
2.2 Introduction ................................................................................................................................. 6
2.3 Theory and Methods .................................................................................................................... 9
  2.3.1 Conceptual Model .................................................................................................................. 9
  2.3.2 Flow Regime Analysis ......................................................................................................... 11
  2.3.3 Two-phase BDF Model ....................................................................................................... 13
    2.3.3.1 Constant BHP .................................................................................................................. 14
    2.3.3.2 Constant Rate ............................................................................................................... 17
    2.3.3.3 Variable Rate ............................................................................................................... 18
  2.3.4 Calculation of Average Pressure ......................................................................................... 19
    2.3.4.1 Diffusivity Equation Approach .................................................................................. 20
    2.3.4.2 Material Balance Approach .................................................................................... 23
  2.3.5 Analysis Technique ............................................................................................................. 24
    2.3.5.1 Calculation of Pseudotime and Pseudopressure ...................................................... 24
    2.3.5.2 Identification of Two-phase BDF regime .............................................................. 25
    2.3.5.3 Successive Substitution Method for $x_f, k_f, \text{ and } V_f$ .................................. 27
  2.4 Model Validation and Results ................................................................................................. 30
    2.4.1 Validation of Average Pressure Calculation ............................................................. 31
    2.4.2 Validation of Flowback Model ...................................................................................... 32
    2.4.3 Validation of Analysis Workflow .................................................................................. 38
  2.5 Field Application ...................................................................................................................... 41
  2.6 Discussion ............................................................................................................................... 45
  2.7 Summary and Conclusions ...................................................................................................... 46
  2.8 Nomenclatures ........................................................................................................................ 48
  2.9 Reference .................................................................................................................................. 51

Chapter 3  Flowback Fracture Closure of Multi-fractured Horizontal Wells in Shale Gas
Reservoirs ........................................................................................................................................ 54

3.1 Abstract ....................................................................................................................................... 54
3.2 Introduction ............................................................................................................................... 55
3.3 Theory and Methods .................................................................58
  3.3.1 Conceptual Model .................................................................59
  3.3.2 Flow Regime Analysis .............................................................61
  3.3.3 Mathematical Models ..............................................................63
    3.3.3.1 Two-phase Water Flowback Model ............................................63
    3.3.3.2 Two-phase Gas Flowback Model ...............................................65
  3.3.4 Analysis Technique ...............................................................70
    3.3.4.1 Calculation of Average Pressure in the Fracture .........................70
    3.3.4.2 Workflow for $x_f$, $k$, and $\gamma$ Calculation .............................72
  3.4 Model Validation and Results .....................................................74
    3.4.1 Model Validation .................................................................75
    3.4.1 Field Validation .................................................................81
  3.5 Summary and Conclusions .........................................................86
  3.6 Nomenclatures ..................................................................87
  3.7 Reference ..............................................................................92

Chapter 4 A Semianalytical Method for Two-Phase Flowback Rate Transient Analysis in Shale Gas Reservoirs .................................96

  4.1 Abstract ............................................................................96
  4.2 Introduction .......................................................................97
  4.3 Theory and Methods ..............................................................101
    4.3.1 Conceptual Model .................................................................101
    4.3.2 Flow Regime Analysis .............................................................103
    4.3.3 Mathematical Formulations .......................................................105
      4.3.3.1 Two-phase Fracture Model ......................................................106
      4.3.3.2 Two-phase Matrix Model .......................................................108
      4.3.3.3 Solution Procedure .................................................................110
    4.3.4 Analysis Technique ...............................................................111
      4.3.4.1 Two-phase Diagnostic Plot ......................................................111
      4.3.4.2 Workflow for $V_f$, $k$, $\gamma_f$, and $C_f$ Calculation ...................112
  4.4 Results ............................................................................114
    4.4.1 Model Validation .................................................................115
    4.4.2 Field Application .................................................................125
  4.5 Discussion and Recommendation ...............................................131
  4.6 Conclusions .......................................................................133
  4.7 Nomenclatures ....................................................................134
  4.8 Appendix A – Two-phase Fracture Model ......................................139
  4.9 Appendix B – Two-phase Matrix Model .........................................143
  4.10 Reference .......................................................................145

Chapter 5 Analysis of Early-time Production Data from Multi-fractured Shale Gas Wells by Considering Multiple Transport Mechanisms Through Nanopores ........................................149

  5.1 Abstract .............................................................................149
  5.2 Introduction .......................................................................150
  5.3 Theory and Model Development ...............................................154
  5.4 Results .............................................................................161
    5.4.1 Model Validation .................................................................161
Chapter 6 Water Production Analysis and Fracture Characterization of Hydraulically Fractured Reservoirs ................................................................. 199

6.1 Abstract ............................................................................................... 199
6.2 Introduction ......................................................................................... 200
6.3 Theory and Mathematical Development ........................................... 205
  6.3.1 Conceptual Model ........................................................................ 205
  6.3.2 Fracture Model ........................................................................... 209
  6.3.3 Matrix Model .............................................................................. 212
  6.3.4 Multiple Liquid Transport Mechanisms in Matrix ..................... 214
  6.3.5 Analysis Technique .................................................................... 219
6.4 Results .................................................................................................. 221
  6.4.1 Model Validation ........................................................................ 221
  6.4.2 Field Application ........................................................................ 232
6.5 Discussion and Recommendations ................................................... 239
6.6 Summary and Conclusions ............................................................... 241
6.7 Nomenclatures .................................................................................. 243
6.8 Reference ............................................................................................ 247

Chapter 7 Conclusion and Recommendations ........................................ 251

7.1 Conclusion ........................................................................................ 251
  7.1.1 Early-Time Flowback Analysis ................................................ 251
  7.1.2 Late-Time Flowback Analysis with Matrix Influx ...................... 252
  7.1.3 Flowback Analysis for Ultratight Gas Reservoirs ...................... 252
  7.1.4 Flowback Analysis for Tight and Ultratight Oil Reservoirs ....... 253
7.2 Recommendations for Future Research .......................................... 254
  7.2.1 Type Curve Analysis .................................................................. 254
  7.2.2 Three-Phase Flow ..................................................................... 254
  7.2.3 Complex Fracture Network ....................................................... 255
LIST OF FIGURES

Figure 2-1: A schematic of the conceptual model for flowback depicting flow through hydraulic fracture only under fracture depletion mechanism .................................................. 10

Figure 2-2: (a) Schematic of the three flow regimes and (b) Two-phase diagnostic plot modified from Zhang and Emami-Meybodi (2018) during early flowback period. ........ 13

Figure 2-3: Three workflows of the successive substitution method to evaluate $V_{fi}$, $x_f$, and $k_{fi}$. Workflow I applies the material balance approach to calculate average pressure in the fracture, whereas Workflow II and III use the diffusivity equation approach. Workflow III is an alternative method for constant BHP condition. ................................. 28

Figure 2-4: (a) Relative permeability curves for the fracture and (b) gas viscosity and formation volume factor vs. pressure ........................................................................................................ 31

Figure 2-5: Comparison of diffusivity equation approach and material balance approach for (a) Case 1, (b) Case 2, and (c) Case 3. ................................................................. 32

Figure 2-6: Two-phase (a) diagnostic plot and (b) specialty plot for Case 1-3 using constant BHP flowback model ...................................................................................................... 33

Figure 2-7: Two-phase (a) diagnostic plot and (b) specialty plot for Case 4-6 using constant rate flowback model ...................................................................................................... 34

Figure 2-8: Two-phase (a) diagnostic plot and (b) specialty plot for Case 3 using variable rate flowback model ...................................................................................................... 35

Figure 2-9: Pressure profile in 1D fracture under (a) constant BHP condition and (b) constant rate condition. ................................................................. 36

Figure 2-10: (a) Calculated average pressure and (b) generated specialty plot during iterations of Workflow I .......................................................................................... 39

Figure 2-11: (a) Calculated average pressure and (b) generated specialty plot during iterations of Workflow III .......................................................................................... 40

Figure 2-12: Production history of gas flowrate, water flowrate, and bottomhole pressure for the field case during (a) flowback and (b) long-term production .............. 42

Figure 2-13: Plots during flowback analysis after the convergence of $V_{fi}$: (a) Average pressure and permeability in the fracture during flowback period. (b) The path of average fracture water saturation vs. average fracture pressure. (c) Two-phase diagnostic plot of water where the pseudo-variables are calculated based on the average pressure from plot (a). (d) Specialty plot of two-phase water flowback model. ............................................................................. 43

Figure 2-14: Plots during long-term production data analysis: (a) Single-phase diagnostic plot of gas where the pseudo-variables are calculated based on the average pressure
in the distance of investigation in the matrix. (b) Specialty plot of single-phase gas formation transient linear flow. (c) History match of gas flowrate and (d) history match of bottomhole pressure during long-term production using the Horizontal-Multifractured-SRV model in IHS harmony software.

Figure 3-1: Schematic of the conceptual model for flowback depicting water flow through hydraulic fracture and gas flow through fracture and matrix.

Figure 3-2: (a) Different flow regimes during flowback and (b) two-phase diagnostic plot of water phase (Zhang and Emami-Meybodi, 2020).

Figure 3-3: (a) Different flow regimes during flowback and (b) gas phase diagnostic plot (Zhang and Emami-Meybodi, 2018).

Figure 3-4: Schematic of the variation of flowing pressure and intermediate parameters in the calculation.

Figure 3-5: Workflow to evaluate $x_f$, $k_{fi}$, and $\gamma$ using both water-phase and gas-phase flowback model.

Figure 3-6: Comparison of average fracture pressure calculated from material balance equation and numerical simulation for (a) Case 1 under constant gas rate and (b) Case 2 under constant water rate.

Figure 3-7: Two-phase diagnostic plot of water phase for (a) Case 1 with a constant gas rate and (b) Case 2 with a constant water rate.

Figure 3-8: Water-phase specialty plot for (a) Case 1 under constant gas rate and (b) Case 2 under constant water rate.

Figure 3-9: Two-phase diagnostic plot of gas phase for (a) Case 1 with a constant gas rate and (b) Case 2 with a constant water rate.

Figure 3-10: Gas-phase FMB plot for (a) Case 1 with a constant gas rate and (b) Case 2 with a constant water rate.

Figure 3-11: Calculated average pressure in the fracture during iterations of workflow.

Figure 3-12: Production history of gas flowrate, water flowrate, and bottomhole pressure for the field case during (a) flowback and (b) long-term production.

Figure 3-13: Plots during flowback analysis after the convergence of $x_f$ and $k_{fi}$: (a) Average pressure and permeability in the fracture during flowback period. (b) Two-phase diagnostic plot of water where the pseudo variables are calculated based on the average pressure from plot (a). (c) Specialty plot of two-phase water flowback model. (d) Specialty plot of two-phase gas flowback model.

Figure 3-14: History match of (a) gas flowrate and (b) bottomhole pressure during long-term production using the Horizontal-Multifractured-SRV model in IHS harmony software.
Figure 4-1: Schematic of the conceptual model for flowback depicting water and gas flow through fracture and matrix. ........................................................................................................... 102

Figure 4-2: (a) Schematic of FR1. (b) Two-phase water diagnostic plot. (c) Schematic of FR2. (d) Two-phase gas diagnostic plot. ................................................................. 105

Figure 4-3: Workflow to evaluate $V_{fi}$, $k_{fi}$, $y_{fi}$, and $C_f$ using flowback models with/without reconciliation of long-term RTA. .............................................................. 114

Figure 4-4: Pressure maps for a quarter element of numerical simulation model at 10 days and 20 days of production for (a-b) Case 1 and (c-d) Case 2. ................................. 116

Figure 4-5: Comparison of cumulative water and gas influx calculated from analytical solution and numerical simulation for (a-b) Case 1 and (c-d) Case 2 .......................... 118

Figure 4-6: Comparison of average pressure and average water saturation in the fracture calculated from analytical solution and numerical simulation for (a-b) Case 1 and (c-d) Case 2. .............................................................................................................. 118

Figure 4-7: Two-phase water and gas diagnostic plot for (a-b) Case 1 with constant gas rate and (c-d) Case 2 with constant water rate. ......................................................... 119

Figure 4-8: Water-phase (a) BDF specialty plot and (b) FMB plot, and gas-phase (c) BDF specialty plot and (d) FMB plot for Case 1 with a constant gas rate. ......................... 119

Figure 4-9: Water-phase (a) BDF specialty plot and (b) FMB plot, and gas-phase (c) BDF specialty plot and (d) FMB plot for Case 2 with a constant water rate .......................... 120

Figure 4-10: Calculated average pressure and water saturation in the fracture during iterations of the workflow. .................................................................................................. 124

Figure 4-11: Relative permeability curves for (a) HF and (b) shale matrix (Daigle et al., 2015). ............................................................................................................................. 126

Figure 4-12: Production history of gas flowrate, water flowrate, and bottomhole pressure for the field case during (a) flowback and (b) long-term production (Zhang and Emami-Meybodi, 2020a). ................................................................. 128

Figure 4-13: Flowback analysis after the convergence of $V_{fi}$ and $k_{fi}$: (a) Average pressure and permeability in the fracture during flowback period. (b) Water and gas influx rate from matrix. (c) Two-phase water diagnostic plot. (d) Two-phase gas diagnostic plot. (e) Two-phase water BDF specialty plot. (f) Two-phase gas BDF specialty plot. .......... 128

Figure 5-1: Schematic of the conceptual model for early-production depicting water and gas flow through fracture and matrix (SEM image from Naraghi and Javadpour (2015)). ......................................................................................................................... 156

Figure 5-2: Two-phase water and gas diagnostic plot and the schematic of FR1 and FR2 (modified from Zhang and Emami-Meybodi (2020d)). ................................................... 157
Figure 5-3: (a) Comparison of matrix apparent permeability with and without considering desorption, diffusion, and slip flow. (b) Comparison of gas flowrate and cumulative gas production with and without considering desorption, diffusion, and slip flow. 

Figure 5-4: Comparison of average fracture pressure, average pressure in the matrix DOI, and average fracture water saturation, and average water saturation in the matrix DOI calculated from analytical solution and numerical simulation for (a-b) Case 1 and (c-d) Case 2.

Figure 5-5: Comparison of cumulative gas influx calculated from analytical solution and numerical simulation for (a) Case 1 and (b) Case 2.

Figure 5-6: Two-phase water and gas diagnostic plot for (a-b) Case 1 with constant water rate and (c-d) Case 2 with constant gas rate.

Figure 5-7: Water-phase and gas-phase IALF specialty plot for (a-b) Case 1 with constant water rate and (c-d) Case 2 with constant gas rate.

Figure 5-8: Water-phase (a) BDF specialty plot and (b) FMB plot, and gas-phase (c) BDF specialty plot and (d) FMB plot for Case 1 with constant water rate.

Figure 5-9: Water-phase (a) BDF specialty plot and (b) FMB plot, and gas-phase (c) BDF specialty plot and (d) FMB plot for Case 2 with constant gas rate.

Figure 5-10: Flowback analysis after the convergence of $V_f$ and $k_f$: (a) Average pressure in the fracture and matrix DOI during flowback period. (b) Average permeability and water saturation in the fracture during flowback. (c) Gas influx rate and cumulative gas influx from matrix. (d) Water-phase IALF specialty plot. (e) Two-phase water diagnostic plot. (f) Two-phase gas diagnostic plot. (g) Two-phase water BDF specialty plot. (h) Two-phase gas BDF specialty plot.

Figure 6-1: Production history of a hydraulically fractured well in oil reservoir.

Figure 6-2: Schematic of the conceptual model depicting water and oil transport through fracture and matrix (SEM image from Naraghi and Javadpour (2015)).

Figure 6-3: Two-phase water and oil diagnostic plot and the schematic of FR1 and FR2 (modified from Zhang and Emami-Meybodi (2020d)).

Figure 6-4: (a) Comparison of apparent permeability for oil and water as well as absolute permeability without considering slip boundary and adsorbed/near-wall region. (b) Permeability multiplier for absolute permeability, apparent oil permeability, and apparent water permeability.

Figure 6-5: (a) HF relative permeability curves used in the numerical validation. (b) Original and adjusted matrix relative permeability curves used in the numerical validation.

Figure 6-6: Comparison of average fracture pressure, average pressure in the matrix DOI, and average fracture water saturation, and average water saturation in the matrix DOI.
calculated from analytical solution and numerical simulation for (a-b) Case 1 and (c-d) Case 2.................................................................228

Figure 6-7: Comparison of cumulative water and oil influx calculated from analytical solution and numerical simulation for (a-b) Case 1 and (c-d) Case 2...............................229

Figure 6-8: Two-phase water and oil diagnostic plot for (a-b) Case 1 with constant water rate and (c-d) Case 2 with constant oil rate.................................................................230

Figure 6-9: Water-phase and oil-phase IALF specialty plot for (a-b) Case 1 with constant water rate and (c-d) Case 2 with constant oil rate..........................................................231

Figure 6-10: Water-phase and oil-phase BDF specialty plot for (a-b) Case 1 with constant water rate and (c-d) Case 2 with constant oil rate.................................................................231

Figure 6-11: Relative permeability curves used in the semianalytical method in field example for (a) HF and (b) shale matrix (adopted from Clarkson and Williams-Kovacs (2013))..................................................................................................................233

Figure 6-12: Production history of oil-flow rate, water-flow rate, and bottomhole pressure for the field case during (a) flowback and (b) long-term production (Hossain et al., 2020). .................................................................................................................................234

Figure 6-13: Flowback analysis after the convergence of Vfi and kfi: (a) Average pressure in the fracture and matrix DOI during flowback. (b) Comparison of cumulative production from the well and cumulative influx from matrix for oil and water. (c) Two-phase water diagnostic plot. (d) Two-phase oil diagnostic plot. (e) Two-phase water BDF specialty plot. (f) Two-phase oil BDF specialty plot..............................................237

Figure 6-14: Long-term production history match for (a) oil-flow rate and (d) bottomhole pressure. ..........................................................................................................................238
LIST OF TABLES

Table 2-1: Input parameters used in numerical simulation..................................................30

Table 2-2: The numerical cases used for validation. ............................................................31

Table 2-3: Set and calculated values of fracture properties and relative error using the new
flowback model for the six cases and the comparison with the results analyzed by the
two-phase flowing material balance method of Xu et al. (2016) and Behmanesh et al.
(2018b). ............................................................................................................................37

Table 2-4: Comparison of the proposed model with two RTA models. .............................38

Table 2-5: Validation results of Workflow I.................................................................39

Table 2-6: Validation results of Workflow III ...............................................................40

Table 2-7: Real Field data from an MFHW in Horn River Shale........................................42

Table 2-8: Comparison of flowback analysis and long-term production data analysis
results. ..................................................................................................................................45

Table 3-1: Summary of input data used in numerical simulation........................................75

Table 3-2: A summary of input data used in numerical simulation and calculated
properties using flowback models.........................................................................................79

Table 3-3: Validation results of the workflow with the known γ. ........................................80

Table 3-4: Validation results of the workflow with the known $x_f$.....................................81

Table 3-5: Real Field data from a Marcellus MFHW........................................................82

Table 3-6: Analysis results using the flowback model and history matching.......................86

Table 4-1: Comparison of different diagnostic plots for multiphase flow.........................103

Table 4-2: Summary of the formulations for $V_\theta$ and $k_\theta$, as well as the slope and intercept
of the specialty plots. .........................................................................................................108

Table 4-3: Summary of input data used in numerical simulation........................................117

Table 4-4: Summary of input data used in numerical simulation and calculated properties
using flowback models........................................................................................................120

Table 4-5: Effect of initial fracture permeability on the calculated fracture properties. The
expected value of $V_f$ = 150 Mcf and the values of other input parameters are the same as
Case 1..................................................................................................................................122
Table 4-6: Effect of fracture half-length on the calculated fracture properties. The expected value of $k_f=3000$ md and the values of other input parameters are the same as Case 1. ................................................................................................. 122

Table 4-7: Effect of fracture permeability modulus on the calculated fracture properties. The expected value of $V_f=150$ Mcf, $k_f=3000$ md, and the values of other input parameters are the same as Case 1. ........................................................................................................ 123

Table 4-8: Effect of initial water saturation in the matrix on the calculated fracture properties. The expected value of $V_f=150$ Mcf, $k_f=3000$ md, and the values of other input parameters are the same as Case 1. ........................................................................................................ 123

Table 4-9: Validation results of the workflow by iterating $V_f$ with the known $\gamma_f$. ................. 124

Table 4-10: Validation results of the workflow by iterating $\gamma_f$ with the known $V_f$. ............... 125

Table 4-11: Field data from a Marcellus MFHW. ........................................................................ 126

Table 4-12: Interpretation results from flowback analysis and long-term production data analysis. ........................................................................................................................................... 130

Table 5-1: Summary of input data used in numerical simulation. .............................................. 162

Table 5-2: Summary of input data used in numerical simulation and calculated properties using the proposed method. ........................................................................................................ 168

Table 5-3: Field input data from a Marcellus MFHW. ................................................................... 170

Table 5-4: Interpreted results from flowback analysis and long-term production data analysis. ........................................................................................................................................... 173

Table 6-1: Summary of the formulations for $V_f$ and $k_f$, as well as the slope and intercept of the specialty plots. ............................................................................................................. 212

Table 6-2: Summary of input data used in numerical simulation. .............................................. 222

Table 6-3: Summary of input data used in numerical simulation and calculated properties using the proposed method. ........................................................................................................... 232

Table 6-4: Summary of input data used in field example. ........................................................... 234

Table 6-5: Results from flowback analysis and long-term production data analysis. ............... 239
ACKNOWLEDGEMENTS

First and foremost, I would like to express my sincere appreciation to my advisor Dr. Hamid Emami-Meybodi for his support, mentorship, and encouragement through the research and studies at The Pennsylvania State University (PSU). Without his help and detailed guidance, this dissertation would not have happened and I would not have these research accomplishments. For this, I am eternally thankful.

My sincere thanks also go to the members of my Ph.D. examining committee Dr. Luis Ayala, Dr. Gregory King, and Dr. Ming Xiao for their valuable feedback and comments, which significantly improved this dissertation. I would also like to thank all the faculty and staff of the Department of Energy and Mineral Engineering at PSU, especially Bob Byers, Jaime Harter, and Sue Hyde.

Furthermore, I thank my advisor Dr. Xinwei Liao and my friend Xiaojin Zheng during my MSc at the China University of Petroleum – Beijing. Without their support and help, I would not start my Ph.D. career abroad. I am thankful to my friends and colleagues at PSU, especially Wei Zhang, Zizhong Liu, Prakash Purswani, Daulet Magzymov, and Lu Lee, for many fruitful discussions about research and life.

I acknowledge the financial support from the Institutes of Energy and the Environment at PSU for the Seed Grant, Earth and Mineral Science College through Wilson Research Initiation Grant, SPE Foundation through the Nico van Wingen Memorial Graduate Fellowship, and Society for Mining, Metallurgy & Exploration through the WAAIME Scholarship.

Most importantly, I warmly thank my parents and my sister for their material and spiritual supports in all aspects of my life.
DEDICATION

I dedicate this dissertation to my parents and my sister in recognition of their tireless love, support, and encouragement. My time at Penn State has been a thoroughly enriching one.
Chapter 1

Overview

1.1 Motivations and Objectives

Unconventional hydrocarbon resources, such as tight and ultratight (shale) reservoirs, have continuously been playing a critical role in meeting the energy demand. Commercial production from the unconventional reservoirs has been enabled by multi-fractured horizontal wells (MFHWs) technology. The economics of these reservoirs is tied closely to the MFHWs performance, which is the most direct indicator of stimulation effectiveness. Thus, understanding and analysis of the factors affecting the performance of MFHW in these reservoirs are a critical need for efficient exploitation.

Production rates and flowing pressures gathered during all phases of fluid production from MFHW can be interpreted to obtain reservoir and stimulation attributes based on rate transient analysis (RTA) and pressure transient analysis (PTA). Although traditional PTA and RTA methods have been widely used to characterize hydraulic fracture and reservoir, their focus has been on long-term production data, which requires well to produce for several months and even years.

Water production data after hydraulic fracturing (i.e., flowback data) could provide information about hydraulic fracture (HF) and reservoir properties in the very early period of production. By analyzing flowback data, a quantitative understanding of HF properties and dynamics can be obtained through initial fracture pore volume, initial fracture permeability, fracture compressibility, and permeability modulus. The early estimation of fracture characteristics and stimulation effects not only provide an important constraint for long-term production data analysis and history matching, but also guides the efficient development of unconventional reservoirs in the future.
The accurate estimation of HF properties and stimulation performance in the currently available flowback models is challenged by the complexity in two-phase flow, fracture closure during production, as well as multiple storage and transport mechanisms in nanopores of tight and ultratight reservoirs. The traditional diagnostic plots designed for single-phase flow will exhibit significant error in identifying flow regimes during two-phase flowback and production. In addition, the current flowback models developed for two-phase flow systems are usually so complicated that they have to be solved numerically. The complexity in solving procedures restricts their practicality limiting them only to the direct modeling or history matching of production data.

Given the gaps in existing research in the field of flowback analysis, this dissertation aims to provide simple yet rigorous approaches for HF characterization by quantitatively analyzing two-phase (i.e., water and hydrocarbon) flowback data. The presented flowback data analysis therefore follows three interrelated tasks. The first task is to investigate the possible flow regimes for each phase during both early and late flowback period. By accomplishing the first task, several two-phase diagnostic plots are proposed to identify the different flow regimes and to understand the flow behaviors during flowback (Chapters 2 – 4). The second task is to develop mathematical solutions and workflows for inverse flowback analysis (Chapters 2 – 4). Developed models provide a convenient and precise way to evaluate production performance using flowback data by characterizing HF attributes, such as fracture pore-volume and permeability, as well as HF dynamics, which are controlled by HF compressibility and permeability modulus. The third task concerns the incorporation of multiple fluid storage and transport mechanisms in nanopores into the inverse analysis so that the developed flowback models, which are applicable for tight and ultratight gas and oil reservoirs (Chapters 5 and 6).
1.2 Dissertation Outline

This dissertation is comprised of seven chapters, in which five of them cover the core contents (Chapters 2 – 6) with an overview chapter (Chapter 1) at the beginning and a summary chapter (Chapter 7) at the end. Chapters 2 to 5 present flowback models for unconventional gas reservoirs and Chapter 6 presents a flowback model for tight and ultratight oil reservoirs.

Chapter 2 provides a water-phase diagnostic plot for 1-D two-phase flow inside fracture and a flowback model for shale gas reservoir under constant bottomhole pressure (BHP), constant rate, and variable rate/BHP conditions. This chapter also proposes a new diffusivity equation approach to calculate average pressure in the fracture, and three iterative workflows to estimate fracture attributes. This chapter is a modified version of the submitted peer-reviewed paper “Multiphase Flowback Rate-Transient Analysis of Shale Gas Reservoirs” published in the International Journal of Coal Geology 1.

Chapter 3 presents a gas-phase diagnostic plot and a 2-D two-phase flowback model for water-phase and gas-phase flow in the fracture, which considers gas influx from the matrix. A modified material balance equation (MBE) is developed to obtain average pressure in the fracture by calculating gas influx from matrix under variable BHP condition. An iterative workflow is proposed to analyze evaluate fracture closure and fracture properties. This chapter is a modified version of the article titled “Flowback Fracture Closure of Multi-Fractured Horizontal Wells in Shale Gas Reservoirs” published in the Journal of Petroleum Science and Engineering 2.

Chapter 4 presents a two-phase diagnostic plot for the coupled two-phase flow in the fracture and matrix system, and a flowback model for shale gas reservoir, which considers both

---

water and gas influx from the matrix. A workflow is proposed to quantify the fracture dynamics through the loss of both fracture volume and permeability by reconciling flowback and long-term production data. This chapter is a modified version of the article titled “A Semianalytical Method for Two-Phase Flowback Rate-Transient Analysis in Shale Gas Reservoirs” published in the SPE Journal 3.

Chapter 5 examines the multiple storage and transport mechanisms of gas in shale nanopores and presents a flowback model that incorporates desorption, slip flow, diffusion, stress dependence, and the effect of water film for the shale gas reservoir. This chapter is a modified version of the article titled “Analysis of Early-time Production Data from Multi-fractured Shale Gas Wells by Considering Multiple Transport Mechanisms Through Nanopores” published in the Journal of Petroleum Science and Engineering 4.

Chapter 6 examines the multiple liquid transport mechanisms for tight and ultratight oil reservoir and presents a flowback model that considers two-phase flow, slippage effect, stress dependence, the variation of fluid properties in the bulk and near-wall regions, as well as the differences in slip length, pore size, and wettability of organic and inorganic nanopores. This chapter is a modified version of the article titled “Water Production Analysis and Fracture Characterization of Hydraulically Fractured Reservoirs”, which is under review by a peer-review journal 5.

---

Chapter 2

Multiphase Flowback Rate-Transient Analysis of Shale Gas Reservoirs ††

2.1 Abstract

Multi-fractured horizontal well (MFHW) is commonly used in the development of unconventional reservoirs. Flowback data after hydraulic fracturing is of critical importance in the characterization of hydraulic fracture, stimulation evaluation, and reservoir simulation. This study presents a two-phase diagnostic plot and a semi-analytical flowback model for the early-time flowback period when fluid influx from matrix remains insignificant and production is mainly from the fracture network. The developed model considers the changes of water saturation in hydraulic fracture (HF) under fracture depletion mechanism and pressure-dependent permeability and porosity, and is able to predict HF attributes, such as fracture half-length, initial fracture permeability, and initial pore volume of fracture network. A new method of calculating average pressure in the fracture is developed based on the solution of water-phase diffusivity equation and is compared with the traditional material balance approach based on the tank model. Three iterative workflows are proposed using the successive substitution method to calculate fracture attributes under different production conditions. Validation of the developed flowback model, average pressure calculation method, and the analysis workflow, as well as their application in an MFHW drilled in Horn River Shale show the applicability of this study to interpret flowback data quickly and estimate HF properties accurately.

2.2 Introduction

Multi-fractured horizontal wells (MFHWs) have been widely used to economically produce from unconventional resources, such as tight and shale gas. Rate transient analysis (RTA) is a commonly used technique to conduct quantitative analysis on the well performance of MFHWs. Many mathematical tools have been investigated to characterize reservoir and HF properties using RTA (Clarkson, 2013). However, traditional RTA methods usually focus on long-term production data that require the well producing several months or even several years in ultratight reservoirs. In recent years, more and more studies focus on flowback data right after stimulation to obtain an early estimation of fracture characteristics and stimulation effects, which guides efficient exploitation in the future (Clarkson and Williams-Kovacs, 2013).

Early studies of flowback analysis had focused on qualitative insights and flow regime analysis. Crafton (1998) used the reciprocal productivity index method to analyze post-stimulation flowback data after hydraulic fracturing to evaluate productive capacity, and later studied the effect of flowback rate on well performance and recovery (Crafton, 2008). Ilk et al. (2010) gathered detailed time-rate-pressure flowback data from five wells in the same shale gas reservoir and proposed a procedure to perform qualitative analysis using numerous plots. Although this approach is not a model-based workflow that leads to the characterization of reservoir properties, it provides an important empirical diagnostic plot: gas water ratio (GWR) vs. cumulative gas production ($G_p$) plot in log-log scale, which is widely used by many authors. Clarkson and Williams-Kovacs (2013) applied the half-slope straight line of this plot to identify multiphase depletion flow in the fracture to conduct flowback analysis. Abbasi et al. (2014) pointed out that GWR vs. $G_p$ plot for shale gas wells shows two distinct regions of first decrease and then increase with $G_p$. Adefidipe et al. (2014) introduced early gas production and late gas production concepts to divide flowback period based on the “V” shape of GWR vs. $G_p$ plot.
In recent works, many authors investigated the mathematical development of flowback models to conduct quantitative analysis. During the early flowback period, the fluids (water or water and gas) inside the fracture network flow along the fractures to the wellbore. Meanwhile, formation gas may enter the fracture network creating a coupled fracture-matrix flow. Early attempts of flowback modeling focused on fluid flow in the fracture. Abbasi et al. (2014) developed a single-phase water model to analyze flowback data of MFHWs in shale gas or oil reservoir. But this model is only valid for the single-phase water dominated flow regime, which usually lasts a very short time. Alkouh et al. (2014) considered gas matrix influx into the fracture and proposed a governing equation that combines water and gas flow in the fracture. To obtain an analytical solution, simplification was applied to the definition of total compressibility and material balance equation. Uzun et al. (2016) applied a similar simplification on total compressibility in their work. However, this method has a limitation that it requires the diffusivity equations to be linear. Xu et al. (2016) developed a flowing material balance equation for flowback analysis based on gas-phase model, which might cause errors when is applied to early-time flowback data dominated by water-phase flow. Yang (2017) proposed two semi-analytical models, one for early gas production ignoring fluid influx into the fracture, and the other for late gas production assuming the same pressure drop in fracture and matrix. More recently, Jia et al. (2018) developed a one-dimensional (1D) two-phase flowback model based on constant rate solution using water flowing material balance equation in an iterative procedure. Their model is applicable to cases with small gradients of saturation and pressure, such as with small fracture half-length and high fracture permeability. However, the model may not be adequate for systems experiencing significant saturation and pressure gradients.

To consider fluid influx in flowback modeling, advanced analysis methods continue to evolve by coupling fracture flow with single-phase or two-phase influx from the matrix. Ezulike and Dehghanpour (2014) developed a two-phase bilinear flow model to analyze flowback data with
gas influx. They introduced Dynamic Relative Permeability (DRP) concept and applied Laplace transform to the coupled partial differential equations. Although this model is robust, the main drawback is that the proposed DRP function was obtained by curve fitting, which requires known matching parameters. Clarkson and Williams-Kovacs (2013) applied multiphase RTA model in CBM to determine fracture properties in multiphase fracture depletion flow during the boundary-dominated flow period. In an extended study, Williams-Kovacs and Clarkson (2016) divided the flowback period into three different regimes and modeled after-breakthrough flow period using a modified material balance equation. Later, Clarkson and Qanbari (2016a) adopted the Dynamic Drainage Area (DDA) concept from well testing to conduct flowback analysis based on boundary dominated flow (BDF) solution with material balance equation within the distance of investigation in the matrix. The DDA method for flowback modeling was later improved and applied to MFHWs in CBM, tight oil, and shale reservoirs for history matching, production forecasting, and RTA (Clarkson and Qanbari, 2016b, Clarkson et al., 2016, Qanbari and Clarkson, 2016). More recent studies pointed out the importance of incorporating complex fracture geometries to estimate fracture attributes using two-phase production data (AlTwaijri et al., 2018, Chen et al., 2018, Jia et al., 2017, Yang et al., 2017). However, such complex models demand expensive computational time due to the existence of local grid refinement or unstructured grids making their field application impractical. Apart from the flow-regime analysis, Jones and Blasingame (2019) applied a decline curve analysis to empirically estimate initial fracture volume and forecast total liquid productivity index.

The present study focuses on analyzing the flowback data during primary fracture depletion. We developed a 1D model to estimate HF attributes and forecast the flowback behavior during early gas production from the fracture network, and proposed a two-phase diagnostic plot to identify flow regimes. The developed model provides a quick estimation of fracture properties and can be used for further development of more complex scenarios. A new average pressure
calculation approach is proposed based on the solution of diffusivity equation, which releases the need for gas-phase flowback data as input. The proposed flowback model, average pressure method, and the analysis workflow are validated against several numerical simulations and applied to an MFHW drilled in Horn River Shale in British Columbia, Canada.

2.3 Theory and Methods

We first describe the conceptual model and the assumptions made in this study. RTA flow regime analysis on flowback data is introduced as a basis for further mathematical development and model application. Then, a 1D two-phase flowback model for boundary dominated flow (BDF) is developed. We also proposed a new analytical method to calculate average pressure in the fracture which does not rely on gas production data as the input, and compared the new method with the material balance approach. At the end of this section, the method of successive substitution is applied to iteratively obtain key fracture properties, such as fracture half-length, initial fracture permeability, and initial pore volume of the fracture.

2.3.1 Conceptual Model

As shown in Figure 2-1, we considered an element of symmetry from a planar fracture network for the modeling of 1D flow during the early gas production. Here, HF is treated as a porous medium initially saturated with fracturing liquid (water) and hydrocarbon (gas). Even though fracturing fluid will leak off during the stimulation job, most of the water in the matrix cannot be recovered and will be trapped due to imbibition and capillary pressure (Alkouh et al., 2014). Gas production only sources from initial free gas stored in fracture and gas influx from the matrix are negligible.
Based on these explanations, the main assumptions include:

- Water only sources from the hydraulic fracture and matrix contribution to water production is negligible due to low water mobility in the matrix.

- Gas only sources from the initial gas in fracture, and matrix contribution is negligible.

- Water is assumed to be slightly compressible, and water compressibility is constant.

- Water formation volume factor, water viscosity, fracture permeability, and fracture porosity are pressure-dependent variables, which are evaluated at average fracture pressure.

- While linear, elliptical, and radial flow may be observed for different fracture geometries, which are controlled by $x_f/h$ (Zhang and Emami-Meybodi, 2020), we only considered linear flow in the fracture.

- The gravity segregated relative permeability curves (Clarkson, 2012) are valid for the fracture.

Figure 2-1: A schematic of the conceptual model for flowback depicting flow through hydraulic fracture only under fracture depletion mechanism.
2.3.2 Flow Regime Analysis

Flow regime analysis is a useful tool to identify what production stages the well has experienced and to extract reservoir and fracture properties from specific flow regimes. The analysis is usually based on the long-term production data by generating the diagnostic plot, which is rate normalized pressure ($RNP$) and its derivative ($DRNP$) vs. superposition (material balance) time in log-log scale (Song and Ehlig-Economides, 2011). By considering the nonlinearity caused by pressure-dependent permeability, Zhang and Emami-Meybodi (2018) replaced $RNP$ and superposition time with rate normalized pseudopressure and superposition pseudotime, and generated diagnostic plots for long-term production to analyze flowback data. They proposed three possible water-phase flow regimes that may occur for the conceptual model depicted in Figure 2-1: (1) single-phase water fracture transient linear flow (FR1), (2) two-phase fracture transient linear flow (FR2), and (3) two-phase boundary dominated flow in fracture (FR3). In this work, we proposed a two-phase diagnostic plot (see Figure 2-2b) based on multiphase pseudopressure and pseudotime. The definitions of pseudo-variables used in the diagnostic plot are introduced in Section 2.3. Compared to the traditional single-phase diagnostic plots, the proposed diagnostic plot is applicable to both single- and two-phase flow systems. This understanding is consistent with Clarkson et al. (2019) that using single-phase diagnostic plots to identify flow regimes in multiphase flow systems may lead to erroneous results. Figure 2-2a and Figure 2-2b show the schematic of these three flow regimes and the modified two-phase diagnostic plot of water, respectively.

**FR1:** Single-phase water transient flow (infinite acting linear flow) is the first flow regime that may be observed during the flowback period. After well stimulation, the hydraulic fracture is mostly saturated with fracturing liquid. Therefore, single-phase water production dominates the fracture system at the beginning, which shows the half-slope straight line in the diagnostic plot.
Clarkson and Williams-Kovacs (2013) modeled a similar single-phase water transient flow in a radial system. Although the analysis on FR1 could calculate fracture permeability and has been validated by synthetic data, it may not be applicable to field data because the duration of FR1 is often too short.

**FR2:** During the two-phase transient flow, the diagnostic plot shows a half-slope straight line as well. The produced gas is the initial trapped gas inside the fracture network. Although half-slope straight line can also be observed, the single-phase transient flow model cannot be used for FR2 because gas production has a significant impact on total compressibility so that assuming only water expansion dominates flow will cause an enormous error (Zhang and Emami-Meybodi, 2018). Furthermore, FR2 is usually short making it difficult to conduct any meaningful analysis. Nonetheless, the end of the half-slope straight line could be used to identify the beginning of FR3.

**FR3:** Two-phase boundary dominated flow (BDF) in the fracture is the most dominant regime for water flow during the early flowback period exhibiting a unit-slope line in the diagnostic plot. Alkouh et al. (2014) proposed a model for FR3 using a simplified mobility and compressibility definition. Williams-Kovacs and Clarkson (2016) developed an “after-breakthrough” model for two-phase fracture depletion in a radial system. Jia et al. (2018) used the flowing material balance equation (constant rate solution of water-phase diffusivity equation) to analyze flowback data during FR3. In the present paper, we focus on this flow regime and present a new RTA model for both constant bottomhole pressure (BHP) and constant/variable-rate condition.
The behavior of gas production during flowback period is different from water production. As pointed out by Zhang and Emami-Meybodi (2018), the hydraulic fracture is initially saturated with water and very less trapped gas after stimulation job. During the early time of flowback, the produced gas mainly comes from the fracture depletion, and gas influx from matrix can be overlooked especially for shale gas reservoirs. Because the permeability of shale matrix is much smaller than fracture permeability, which makes the duration of fracture depletion flow longer than other types of reservoirs. But in the late flowback period, the fracture pressure drops significantly and most of the gas production comes from matrix influx. In this work, we only focus on the multiphase water and gas flow during the early flowback period. In the next work (Zhang and Emami-Meybodi, 2020), we elaborate on the gas-phase flow regimes in the fracture and matrix during late flowback period and discuss the coupling of fracture flow with matrix flow in details.

2.3.3 Two-phase BDF Model

Due to the high conductivity of hydraulic fracture, we assumed 1D BDF for the fracture. This work divides basic production conditions during flowback period into constant BHP and
constant rate, which can be extended to a more realistic variable BHP/rate condition using the superposition principle.

### 2.3.3.1 Constant BHP

We first developed a 1D two-phase flowback model under constant BHP conditions. The water-phase material balance equation for an infinitesimal fracture element is given by:

\[
q \rho_w \bigg|_{x+dx} - q \rho_w \bigg|_x = \frac{d}{dt} \left( dx w_j h \phi_f \rho_w s_w \right),
\]

where \(x\) is the location in the fracture, \(dx \ w_j h\) is the control volume, \(q\) is the water-phase downhole flowrate at a specific location \(x\), \(s_w\) is water saturation in the fracture, \(\rho_w\) is the water-phase density, \(w_j\) is the fracture width, \(h\) is the fracture height, \(\phi_f\) is the fracture porosity, and \(t\) is the time.

Dividing both sides of eq (2-1) by \(dx\), the following partial differential equation (PDE) can be obtained:

\[
\frac{\partial (q \rho_w)}{\partial x} = \frac{\partial}{\partial t} \left( w_j h \phi_f \rho_w s_w \right).
\]

Applying chain rule to the accumulative term (right-hand side) and substituting \(\rho_{wf} = \frac{\rho_{wsc}}{B_w}\), eq (2-2) can be expanded as:

\[
\frac{\partial}{\partial x} \left( \frac{q \rho_{wsc}}{B_w} \right) = w_j h \left[ \phi_f s_w \rho_{wsc} \left( \frac{1}{\rho_w} \frac{d \rho_w}{dp} \right) + \phi_f s_w \rho_{wsc} \left( \frac{1}{\phi_f} \frac{d \phi_f}{dp} \right) + \phi_f s_w \rho_{wsc} \left( \frac{1}{s_w} \frac{d s_w}{dp} \right) \right] \frac{\partial p}{\partial t},
\]

where \(\rho_{wsc}\) is the water density at the standard condition and \(B_w\) is the water formation factor.

Using the definitions of water and fracture compressibility (Jia et al., 2018) and Darcy’s water rate \(q = w_j h \frac{k_{rw} k_f}{\mu_w} \frac{\partial p}{\partial x}\), eq (2-3) can be written as:
\[ \frac{\partial}{\partial x} \left( \frac{k_w k_f}{B_w \mu_w} \frac{\partial p}{\partial x} \right) = \phi_f s_w \left( c_w + c_f + \frac{1}{s_w} \frac{ds_w}{dp} \right) \frac{\partial p}{\partial t}, \]

where \( c_w \) is the water compressibility, \( c_f \) is the fracture compressibility, \( p \) is the pressure in the fracture, and \( \mu_w \) is the water viscosity.

Equation (2-4) is subject to the following initial and boundary conditions:

\[ p(x,0) = p_i, \quad p(0,t) = p_{wf}, \quad \frac{\partial p(x_f,t)}{\partial x} = 0, \]

where \( p_i \) is the initial fracture pressure and \( p_{wf} \) is the bottomhole pressure.

The exponential relationship of pressure-dependent fracture permeability and porosity (Pedrosa Jr, 1986) were considered to include permeability and porosity changes with respect to pressure drop in the fracture:

\[ k_f = k_{fi} f(p) = k_{fi} \exp \left[ -\gamma (p_i - p) \right], \]

\[ \phi_f = \phi_{fi} g(p) = \phi_{fi} \exp \left[ -c_f (p_i - p) \right], \]

where \( \gamma \) is the permeability modulus, \( k_{fi} \) is the initial fracture permeability, and \( \phi_{fi} \) is initial fracture porosity.

We used the pseudopressure \( (p_p) \) and pseudotime \( (t_p) \) concepts to linearize the partial differential equation by evaluating pressure-dependent properties at average pressure \( (\bar{p}_f) \) and average water saturation \( (\bar{s}_w) \) in the fracture:

\[ p_p = \frac{\mu_w B_w}{k_{fi}} \int_{p_0}^p k_f(p) s_w(p) dp, \]

\[ t_p = \frac{\phi_{fi}}{k_{fi}} \int_0^{t'} \frac{k_{rw}(\bar{s}_w) k_f(\bar{p}_f)}{\bar{s}_w \mu_w \left( c_w + c_f + \frac{1}{s_w} \frac{ds_w}{d\bar{p}_f} \right) \phi_f (\bar{p}_f)} d\tau. \]

The governing equation (2-4), initial condition, and boundary condition in eq (2-5) can be linearized to be:
\[
\frac{\partial^2 p_p}{\partial x^2} = \frac{\phi_{fi}}{k_{fi}} \frac{\partial p_p}{\partial t_p},
\]

(2-10)

\[
p_p(x,0) = p_{pi}, \quad p_p(0,t_p) = p_{pwf}, \quad \frac{\partial p_p(x,f,t_p)}{\partial x} = 0.
\]

(2-11)

Equation (2-10) subject to eq (2-11) can be solved using the separation of variables method (Yang, 2017):

\[
p_p(x,t_p) = (p_{pi} - p_{pwf}) \sum_{n=1}^{\infty} \frac{2}{\beta_n} \sin \left( \beta_n \frac{x}{x_f} \right) \exp \left( -\beta_n^2 \frac{k_{fi}}{\phi_{fi} x_f^2} t_p \right),
\]

(2-12)

where \( \beta_n = (2n-1)\pi/2 \).

Taking the derivative of eq (2-12) with respect to \( x \) and evaluating it at \( x = 0 \) gives:

\[
\frac{\mu_w B_{wi} k_{fi} k_{rw}}{k_{fi} \mu_u B_w} \frac{\partial p}{\partial x} \bigg|_{x=0} = (p_{pi} - p_{pwf}) \sum_{n=1}^{\infty} \frac{2}{\beta_n} \exp \left( -\beta_n^2 \frac{k_{fi}}{\phi_{fi} x_f^2} t_p \right) \frac{\partial \sin \left( \beta_n \frac{x}{x_f} \right)}{\partial x} \bigg|_{x=0}.
\]

(2-13)

Applying Darcy’s water surface flowrate for the whole fracture \( q_w = 2w_f h \frac{k_{rw} k_f}{\mu_u B_w} \frac{\partial p}{\partial x} \bigg|_{x=0} \) to eq (2-13):

\[
q_w = (p_{pi} - p_{pwf}) \sum_{n=1}^{\infty} \frac{4w_f h k_{fi}}{x_f B_{wi} \mu_{wi}} \exp \left( -\beta_n^2 \frac{k_{fi}}{\phi_{fi} x_f^2} t_p \right),
\]

(2-14)

Equation (2-14) can be rearranged by defining rate normalized pressure (RNP):

\[
RNP = \frac{x_f B_{wi} \mu_{wi}}{4w_f h k_{fi}} \sum_{n=1}^{\infty} \frac{1}{\exp \left( -\beta_n^2 \frac{k_{fi}}{\phi_{fi} x_f^2} t_p \right)},
\]

(2-15)

where \( q_w \) is the water-phase flow rate from the whole fracture at surface condition, \( RNP = (p_{pi} - p_{pwf})/q_w \) is the rate normalized pseudopressure, and \( B_{wi} \) is the water formation volume factor evaluated at initial pressure.
Applying the long-term approximation described by Wattenbarger et al. (1998) to eq (2-15), a linear relationship between \( \ln(\text{RNP}) \) and \( t_p \) (i.e., constant BHP specialty plot) can be obtained:

\[
\ln(\text{RNP}) = m_p t_p + b_p ,
\]

(2-16)

\[
m_p = \frac{0.0156 k_f}{\phi_f x_f^2} ,
\]

(2-17)

\[
b_p = \ln \left( \frac{221.73 x_f B_{wi} \mu_{wi}}{w_f k_f h} \right) ,
\]

(2-18)

where \( m_p \) and \( b_p \) are the slope and intercept of \( \ln(\text{RNP}) \) vs. \( t_p \) specialty plot, respectively, and all the parameters have the field units.

### 2.3.3.2 Constant Rate

Constant rate case in flowback modeling is as important as constant BHP case, not only because it is the basic condition for well testing, but also it can be easily extended to variable rate/BHP case with superposition principle. With the same definition of pseudotime and pseudopressure in eqs (2-8) and (2-9), we used eq (2-10) in 1D fracture subject to the following initial and boundary conditions:

\[
p_p(x,0) = p_{pi}, \quad \frac{\partial p_p(0,t_p)}{\partial x} = - \frac{q_w B_{wi} \mu_{wi}}{2 k_f w_f h}, \quad \frac{\partial p_p(x_f,t_p)}{\partial x} = 0 .
\]

(2-19)

The analytical solution of eq (2-10) subject to eq (2-19) is given by Miller (1962):

\[
p_p(x,t_p) = p_{pi} - \frac{q_w x_f \mu_{wi} B_{wi}}{2 k_f w_f h} \left[ \frac{1}{3} + \frac{x^2}{2 x_f^2} - \frac{x}{x_f} + \frac{k_f t_p}{\phi_f x_f^2} - \sum_{n=1}^{\infty} \frac{2 \cos \left( \frac{n n x}{x_f} \right) \exp \left( - \frac{(n n)^2 k_f t_p}{\phi_f x_f^2} \right)}{(n n)^2} \right].
\]

(2-20)
We evaluate eq (2-20) at $x=0$ and substitute $p_p(0, t_p) = p_{p wf}$:

$$p_{pi} - p_{pwf} = \frac{q_w x_f \mu_{wi} B_{wi}}{2 k_f w_f h} \left[ 1 + \frac{k_f t_p}{\phi_i x_f^2} \sum_{n=1}^{\infty} 2 \exp \left( -\frac{(n\pi)^2 k_p t_p}{\phi_i x_f^2} \right) \right].$$  (2-21)

Taking the long-term approximation (Wattenbarger et al., 1998) and introducing $RNP$ definition from the constant BHP model, a linear relationship between $RNP$ and $t_p$ (i.e., constant rate specialty plot) can be obtained from eq (2-21) in field unit:

$$RNP = m_q t_p + b_q,$$  (2-22)

$$m_q = \frac{2.807 B_{wi} \mu_{wi}}{w_f h x_f \phi_i},$$  (2-23)

$$b_q = \frac{147.82 x_f B_{wi} \mu_{wi}}{w_f h k_i},$$  (2-24)

where $m_q$ and $b_q$ are the slope and intercept of $RNP$ vs. $t_p$ specialty plot, respectively.

### 2.3.3.3 Variable Rate

The constant rate solution of pseudopressure can be extended to variable rate condition by applying the superposition principle (Dake, 1983). By defining superposition pseudotime $t_{sppt}$ in eq (2-25), the solution of pseudopressure under variable rate condition is given in eq (2-26).

$$t_{sppt} = \sum_{j=1}^{N} \left[ \frac{(q_{w,j} - q_{w,j-1})(t_{p,N} - t_{p,j-1})}{q_{w,N}} \right],$$  (2-25)
\[ p_p(x,t_p) = p_{pi} - \frac{q_{w,N} x_f \mu_{wi} B_{wi}}{2 k_i w_i h} \left[ \frac{1}{3} + \frac{x^3}{2 x_f^3} - \frac{x}{x_f} + \frac{k_i t_{spt} x_f^2}{\phi_i x_f^2} \right] \sum_{n=1}^{\infty} \frac{q_{w,j} - q_{w,j-1}}{q_{w,N}} 2 \cos \left( \frac{n \pi x}{x_f} \right) \exp \left( -\frac{(n \pi)^2}{(n \pi)^2} \frac{k_i}{\phi_i x_f^2} \left( t_{p,N} - t_{p,j-1} - t_{p,j} \right) \right) \],

(2-26)

where \( q_{w,j} \) is the water-phase flow rate at the surface condition in the \( j \)th flow rate interval, \( t_{p,j} \) is the pseudotime at the \( j \)th flow rate interval.

Similarly, we evaluate eq (2-20) at \( x=0 \) and substitute \( p_p(0,t_p)=p_{p,wf} \):

\[ p_{pi} - p_{p,wf} = \frac{q_{w,N} x_f \mu_{wi} B_{wi}}{2 k_i w_i h} \left[ \frac{1}{3} + \frac{k_i t_{spt} x_f^2}{\phi_i x_f^2} \right] \sum_{n=1}^{\infty} \frac{2 (q_{w,j} - q_{w,j-1})}{q_{w,N} (n \pi)^2} \exp \left( -\frac{(n \pi)^2}{(n \pi)^2} \frac{k_i}{\phi_i x_f^2} \left( t_{p,N} - t_{p,j-1} - t_{p,j} \right) \right) \],

(2-27)

Introducing the definition of \( RNP \) and applying the same long-term approximation to variable-rate solution, we can obtain the linear relationship but between \( RNP \) and \( t_{spt} \) (i.e., variable-rate specialty plot), where the slope and intercept of specialty plot are the same as constant rate case (eqs (2-22) to (2-24)).

\[ RNP = m q_{spt} + b_{eq}. \]

(2-28)

According to eqs (2-16), (2-22) and (2-28), fracture properties, such as fracture half-length \( (x_f) \) and initial fracture permeability \( (k_f) \), are part of the slope and intercept of specialty plot. The procedure to obtain these attributes is discussed in Section 2.3.5 Analysis Technique.

### 2.3.4 Calculation of Average Pressure

Average pressure in the fracture is a critical parameter used to evaluate pressure dependent properties and calculate pseudopressure and pseudotime in the flowback model. Material balance equation is a simple and popular way to obtain average pressure, which requires both water and gas
production data as the input (Moghadam et al., 2011). Behmanesh et al. (2018a) derived an explicit expression to calculate average pseudopressure within the distance of investigation, but they didn’t provide the solution of average pressure. This section proposes a diffusivity equation approach to calculate average pressure in the fracture under constant BHP and constant/variable rate conditions and also derived a two-phase material balance equation in a tank system.

### 2.3.4.1 Diffusivity Equation Approach

The new method is based on the solution of pseudopressure from the diffusivity equation.

The volumetric average pseudopressure under constant BHP, constant rate, and variable rate conditions are calculated by taking the integral of eqs (2-12), (2-20), and (2-26) with respect to $x$:

\[
\overline{p_{pf}}(t_p)_{\text{constant BHP}} = \left[ \int_0^{x_f} \frac{p_p(x, t_p)}{x_f} \right]_{\text{constant BHP}} \, dx = \left( p_{pi} - p_{p_{pf}} \right) \sum_{n=1}^{\infty} \frac{2}{\beta_n^2} \exp \left\{ -\beta_n^2 \frac{k_f}{\phi_t x_f^2} t_p \right\},
\]

\[
\overline{p_{pf}}(t_p)_{\text{constant q}} = \left[ \int_0^{x_f} \frac{p_p(x, t_p)}{x_f} \right]_{\text{constant q}} \, dx = p_{pi} \left( 1 - \frac{B_{wi} q_w t_p}{p_{pi} V_f} \right),
\]

\[
\overline{p_{pf}}(t_p)_{\text{variable q}} = \left[ \int_0^{x_f} \frac{p_p(x, t_p)}{x_f} \right]_{\text{variable q}} \, dx = p_{pi} \left( 1 - \frac{B_{wi} \sum_{j=1}^{N} (q_{w,j} - q_{w,j-1}) \cdot (t_{p,N} - t_{p,j-1})}{p_{pi} V_f} \right),
\]

where $\overline{p_{pf}}$ is the average pseudopressure in the fracture.

According to the definition of pseudopressure in eq (2-8), real pressure can be converted from pseudopressure using the following relation:

\[
\frac{dp_p}{dt} = \mu_w B_w \frac{k_f(p) k_{rw} s_w}{k_f} \frac{dp}{dt},
\]
\[
\frac{dp}{dt} = \frac{k_{fi}}{\mu_{wi}B_{wi}} \frac{\mu_w(p)B_w(p)}{k_j(p)k_{rw}(s_w)} \frac{dp}{dt}
\]

(2-33)

\[
p = p_i + \frac{k_{fi}}{\mu_{wi}B_{wi}} \int_0^t \frac{\mu_w(p)B_w(p)}{k_j(p)k_{rw}(s_w)} \frac{dp}{dt} d\tau
\]

(2-34)

Assuming that pseudo average pressure can be approximated with average pseudopressure, eq (2-34) can be rewritten to calculate average pressure based on average pseudopressure:

\[
\bar{p}_f = p_i + \frac{k_{fi}}{\mu_{wi}B_{wi}} \int_0^t \frac{\mu_w(p)B_w(p)}{k_j(p)k_{rw}(s_w)} \frac{dp}{dt} d\tau \approx p_i + \frac{k_{fi}}{\mu_{wi}B_{wi}} \int_0^t \frac{\mu_w(p)B_w(p)}{k_j(p)k_{rw}(s_w)} \frac{d\bar{p}_{pf}}{dt} d\tau,
\]

(2-35)

where \(\bar{p}_{pf}\) is the pseudo average pressure in the fracture.

With the relation between pressure and pseudopressure in eq (2-35), we take the derivative of eqs (2-29), (2-30), and (2-31) with respect to \(t\):

\[
\left. \frac{d\bar{p}_{pf}(t_p)}{dt} \right|_{\text{constant } q} = -\frac{B_{wi}q_{wi} \phi_{fi}}{V_{fi} \kappa_{fi}} \frac{k_{rw}(\bar{s}_w)}{\bar{s}_w \mu_w} \frac{k_j(\bar{p}_f)}{k_j(\bar{p}_f)} \frac{ds_w}{dp_{pf}} \phi_j(\bar{p}_f)
\]

(2-36)

\[
\left. \frac{d\bar{p}_{pf}(t_p)}{dt} \right|_{\text{variable } q} = -\frac{B_{wi}q_{wi,N} \phi_{fi}}{V_{fi} \kappa_{fi}} \frac{k_{rw}(\bar{s}_w)}{\bar{s}_w \mu_w} \frac{k_j(\bar{p}_f)}{k_j(\bar{p}_f)} \frac{ds_w}{dp_{pf}} \phi_j(\bar{p}_f)
\]

(2-37)

(2-38)

where \(\zeta = k_{pf}/k_{wf}^2\) is the fracture coefficient.
Substituting eqs (2-36), (2-37), and (2-38) into eq. (2-35), we arrive at the expression of average pressure in the fracture under constant BHP, constant rate, and variable rate conditions in field unit:

\[
\bar{p}_f(t_p) \big|_{\text{constant BHP}} = p_i - \int_0^t \frac{0.0127 \zeta (p_{pf} - p_{w}) B_w (\bar{p}_f)}{\mu_{wi} B_m (\bar{w}_w c_w + \bar{w}_w c_f + \frac{d\bar{w}_w}{d\bar{p}_f})} \phi_f (\bar{p}_f)
\]

\[
\sum_{n=1}^{\infty} \exp \left[ -\int_{\tau_2}^{\tau_1} \frac{0.0156 \zeta (2n-1)^2 k_f (\bar{p}_f)}{k_n \mu_w (c_w + c_f + \frac{1}{\bar{w}_w} \frac{d\bar{w}_w}{d\bar{p}_f})} d\tau \right]
\]

\[
\bar{p}_f(t_p) \big|_{\text{constant q}} = p_i - \int_0^t \frac{5.615 q_w B_w (\bar{p}_f) \phi_{fi}}{V_{fi} \left(\bar{w}_w c_w + \bar{w}_w c_f + \frac{d\bar{w}_w}{d\bar{p}_f}\right) \phi_f (\bar{p}_f)} d\tau ,
\]

\[
\bar{p}_f(t_p) \big|_{\text{variable q}} = p_i - \int_0^t \frac{5.615 q_w A B_w (\bar{p}_f) \phi_{fi}}{V_{fi} \left(\bar{w}_w c_w + \bar{w}_w c_f + \frac{d\bar{w}_w}{d\bar{p}_f}\right) \phi_f (\bar{p}_f)} d\tau .
\]

We use the simplified material balance equation proposed by King (1993) to calculate average fracture water saturation in eqs (2-39) – (2-41).

\[
\bar{w}_w = \frac{(W_i - 5.615 W_p) B_w}{V_{fi} \left[1 - c_f (p_i - \bar{p}_f)\right]},
\]

\[
B_w = B_{wi} \exp \left[c_w (p_i - \bar{p}_f)\right] \approx B_{wi} \left[1 + c_w (p_i - \bar{p}_f)\right],
\]

where \(W_i\) is the initial water storage in the fracture at the surface condition, \(W_p\) is the cumulative water production at the surface condition, and \(V_{fi} = 2\phi_{fi} x_f w_f h\) is the initial pore volume of the fracture.
With the average water saturation, $\frac{d\tilde{\delta}_w}{d\tilde{\phi}_f}$ can be approximated using backward difference so that we can calculate average pressure in the fracture iteratively.

### 2.3.4.2 Material Balance Approach

Here, the fracture network is treated as a tank system in the early flowback without gas matrix influx acting on fracture flow. The material balance equation based on gas fracture storage can be written as (King, 1993):

$$G_p = G_i - G_r,$$

where $G_i$ is the initial gas storage in the fracture and $G_r$ is the remaining gas in the fracture, all at the surface condition. The definition of $G_i$ and $G_r$ are given by:

$$G_i = s_g V_{fi}, \quad G_r = \frac{V_f - V_{wr}}{B_g},$$

$$V_f = V_{fi} \left(1 - c_f \left(p_i - \bar{p}_f\right)\right), \quad V_{wr} = \left(W_{fi} - W_p\right) B_w, \quad W_{fi} = \frac{V_{fi} s_{wi}}{B_{wi}},$$

where $V_{wr}$ is the remaining water volume in the fracture. Substituting eqs (2-45) and (2-46) into eq (2-44), we obtain an implicit expression for average fracture pressure which can be solved by the secant method:

$$\bar{p}_f = p_i - \frac{V_{fi} - \left((G_i - G_p) B_g + \left(W_{fi} - 5.615 W_p\right) B_w\right)}{V_{fi} c_f},$$

$$B_g = \frac{p_{sc} ZT}{T_{sc} p},$$

where $p_{sc}$ is the pressure at standard condition and $T_{sc}$ is the temperature at standard condition.
2.3.5 Analysis Technique

Based on the proposed two-phase flowback model and the average fracture pressure calculation methods, we developed an analysis technique to interpret early-time flowback data and estimate fracture properties such as $x_f$, $k_f$, and $V_f$. The analysis technique consists of three steps: (1) calculation of pseudo-variables, (2) identification of two-phase BDF regime, and (3) calculation of fracture properties using the successive substitution method.

2.3.5.1 Calculation of Pseudotime and Pseudopressure

The specialty plots described in Sections 2.3.3 Two-phase BDF Model require the calculations of pseudopressure and pseudotime. Pseudotime $t_p$ is a function of average pressure in the fracture and can be calculated by substituting either of the eqs (2-39), (2-41), and (2-47) into eq (2-9). Choosing the appropriate average pressure formula depends on the well production condition and the availability of two-phase flowback data, which will be discussed the following section. The $\frac{1}{s_w} \frac{d\bar{s}_w}{d\bar{p}_f}$ term in $t_p$ formulation can be evaluated at the average pressure and water saturation in the fracture because pressure changes in distance can be ignored due to the high conductivity of the fracture (Zhang et al., 2013).

According to the definition of pseudopressure in eq (2-8), the initial pseudopressure drawdown $(p_{wi}-p_{pwf})$ changes with the lower limit of the integral $p_{wf}$ in constant/variable-rate flowback models. To calculate $(p_{wi}-p_{pwf})$, we have to obtain $\bar{p}_f$ first from eqs (2-41) or (2-47). Then $\bar{s}_w$ can be estimated corresponding to each $\bar{p}_f$ from eq (2-42). By interpolating the relation between $\bar{s}_w$ and $\bar{p}_f$, we can evaluate $(p_{wi}-p_{pwf})$ at each $p_{wf}$ point.

Unlike constant/variable-rate case, $(p_{wi}-p_{pwf})$ is a constant for constant BHP condition, which is independent of average fracture pressure. To obtain this constant, we have to know how
\( \bar{s}_w \) changes from \( p_{wf} \) to \( p_i \). This paper uses a linear function to denote the relation between \( \bar{s}_w \) and \( p_f \) as a simplification:

\[
\bar{s}_w = A \bar{p}_f + B, \tag{2-49}
\]

where \( A \) and \( B \) are the coefficients of linear correlation, \( s_{wf} \) is the average water saturation inside fracture when \( \bar{p}_f \) reaches \( p_{wf} \) and it depends on the estimated ultimate water recovery at the end of flowback. Here we used \( W_p \) at the end of flowback to approximate the ultimate water cumulative production so that \( s_{wf} \) can be approximated as:

\[
s_{wf} = \left( \frac{W_f - 5.615 W_{p_{wf}}}{V_p} \right) B_{wf} - c_f \left( p_i - p_{wf} \right). \tag{2-51}
\]

By substituting eqs (2-49), (2-43), and (2-6) into eq (2-8), we arrive at the initial pseudopressure drawdown \( (p_{pi} - p_{p_{wf}}) \) for constant BHP condition:

\[
p_{pi} - p_{p_{wf}} = \frac{\mu_{wi} B_{wi}}{k_{fi}} \int_{p_{wf}}^{p_i} \frac{k_f \exp[-\gamma(p_i - p)] \cdot (Ap + B) \cdot \mu_w(p) B_{wf} \exp[c_w(p_i - p)]}{\mu_w(p) B_{wf} \exp[c_w(p_i - p)]} dp. \tag{2-52}
\]

If we assume constant water viscosity and apply integration by parts, we can obtain the analytical expression for eq (2-52):

\[
p_{pi} - p_{p_{wf}} = \frac{1}{\gamma + c_w} \left\{ (Ap_i + B) - \frac{A p_{wf} + B}{e^{(\gamma + c_w)(p_i - p_{wf})}} \cdot \frac{1 - e^{-\gamma c_w(p_i - p_{wf})}}{\gamma + c_w} \right\}. \tag{2-53}
\]

### 2.3.5.2 Identification of Two-phase BDF regime

To perform analysis with the proposed flowback model, the data interval in BDF should be identified first. According to flow regime analysis presented in Section 2.3.2 Flow Regime...
Analysis, two-phase BDF flow (FR3) starts right after the end of two-phase transient flow in the fracture (FR2). Therefore, a diagnostic plot should be used to identify the end of FR2, which is the beginning of FR3. The new initial and boundary conditions for FR2 under constant BHP condition and constant rate condition are given by:

\[ p_p(x, 0) = p_{pi}, \quad p_p(0, t) = p_{pwf}, \quad p_p(x \rightarrow \infty, t) = p_{pi}, \]  
\[ p_p(x, 0) = p_i, \quad \frac{\partial p_p(0, t)}{\partial x} = -\frac{q_{wi} B_{wi} \mu_{wi}}{2 k_f w_j h}, \quad p_p(x \rightarrow \infty, t) = p_{pi}. \]  

Wattenbarger et al. (1998) gave the solutions of eq (2-10) together with the new outer boundary conditions. Rearranging the solutions in terms of \( RNP \) in the field unit gives:

\[ RNP |_{\text{constant BHP}} = 62.536 B_{wi} \mu_{wi} \sqrt{\frac{f}{p}}, \]  
\[ RNP |_{\text{constant q}} = 39.825 B_{wi} \mu_{wi} \sqrt{\frac{f}{p}}. \]  

Equation (2-57) can be easily extended to variable rate condition with superposition principle by replacing \( t_p \) with \( t_{qspt} \):

\[ RNP |_{\text{variable q}} = 39.825 B_{wi} \mu_{wi} \sqrt{\frac{f}{p}}. \]  

We defined the derivative of \( RNP \) (\( DRNP = dRNP/dt_{qspt} \)) with the new definitions of pseudopressure and pseudotime in this work. The two-phase transient flow solution under general production condition in eq (2-58) reveals that \( \log-log \; DRNP \) vs. \( t_{qspt} \) plot shows a half-slope straight line denoting FR1+FR2. The two-phase boundary dominated flow solution under variable rate condition in eq (2-28) indicates a unit-slope straight line in the same plot denoting FR3. Hence, we used log-log \( DRNP \) vs. \( t_{qspt} \) plot as the new two-phase diagnostic plot to identify the flow regimes.
2.3.5.3 Successive Substitution Method for $x_f$, $k_{fi}$, and $V_{fi}$

This section presents three workflows using a successive substitution method to extract $x_f$, $k_{fi}$, and $V_{fi}$ from flowback data. For the constant BHP flowback model in eq (2-16), one can use the slope $m_p$ and intercept $b_p$ of $\ln(RNP)$ vs. $t_p$ plot to calculate $x_{f,p}$, $k_{fi,p}$, $V_{fi,p}$, and $\zeta_p$ using the following expressions:

$x_{f,p} = \frac{3.459 B_{wi} \mu_{wi}}{m_p \phi_{fi} \exp(b_p) w_j h}, \quad (2-59)$

$k_{fi,p} = \frac{767.59 B_{wi}^2 \mu_{wi}^2}{m_p \phi_{fi} \left( \exp(b_p) w_f h \right)^2}, \quad (2-60)$

$V_{fi,p} = 2\phi_{fi} x_j w_j h = \frac{6.918 B_{wi} \mu_{wi}}{m_p \exp(b_p)}, \quad (2-61)$

$\zeta_p = 64.155 m_p \phi_{fi} \cdot \quad (2-62)$

For the constant rate or variable rate flowback model in eqs (2-22) and (2-28), the slope $m_q$ and intercept $b_q$ of $RNP$ vs. $t_p$ and $RNP$ vs. $t_{sp}$ plot can be used to calculate $x_{f,q}$, $k_{fi,q}$, and $V_{fi,q}$ using the following expressions:

$x_{f,q} = \frac{2.807 B_{wi} \mu_{wi}}{m_q w_j h \phi_{fi}}, \quad (2-63)$

$k_{fi,q} = \frac{147.82 x_j B_{wi} \mu_{wi}}{b_q w_j h}, \quad (2-64)$

$V_{fi,q} = 2\phi_{fi} x_j w_j h = \frac{5.614 B_{wi} \mu_{wi}}{m_q}. \quad (2-65)$

However, the calculation of average pressure in eqs (2-39), (2-41), and (2-47) requires $V_{fi}$ as a given input, which is usually unknown in field application. Figure 2-3 shows three workflows using successive substitution method to estimate $V_{fi}$ iteratively, and then calculate $x_j$ and $k_{fi}$. 

Workflow I and II have the same calculation procedures. The only difference is the method used to calculate average pressure in the fracture. They both follow seven steps:

1. Guess an initial value for \( V_{fi} \).
2. Obtain average fracture pressure \( \bar{p}_f \) from eqs (2-47) for Workflow I or from eq (2-41) for Workflow II. Calculate \( (p_{pi} - p_{pwr}) \) and evaluate \( t_p \) at \( \bar{p}_f \).
3. Generate two-phase diagnostic plot, \( DRNP vs. t_{esp} \) in log-log scale, to pick the time when half-slope straight line and unit-slope straight line intersects, which is the start of BDF regime (FR3).
4. Generate specialty plot \( \ln(RNP) vs. t_p \) if using constant BHP flowback model or \( RNP vs. t_{esp} \) for constant/variable-rate flowback model. Then, extract slope and intercept of the straight line after the end of half-slope interval in step 3.
5. Calculate a new value of \( V_{fi} \) using eq (2-61) or (2-65).
6. Repeat step 2 to 4 until the relative error of \( V_{fi} \) meets the tolerance we set.
7. Determine \( x_f \) and \( k_f \) using eq (2-59) to (2-60), or (2-63) to (2-64).

The two workflows cover both constant BHP and constant/variable-rate flowback model which are capable of analyzing complex flowback data in reality. Workflow I uses material balance approach to calculate average pressure in the fracture, which requires both water and gas-phase production data as the inputs. Whereas, Workflow II only requires water-phase production data and releases the need for gas data for flowback analysis by applying the new average pressure calculation method. For the field application, it depends on the availability and quality of gas-phase production data to choose the most appropriate workflow.

If the well is producing under constant BHP condition, Workflow III is an alternative way to analyze flowback data which uses the diffusivity equation approach to calculate average pressure in the fracture. It follows nine steps:

1. Guess an initial value for \( V_{fi} \) and \( \zeta \),
2. Obtain average fracture pressure \( \bar{p}_f \) from eq (2-39), calculate \( (p_{wi} - p_{pwi}) \) and evaluate \( t_p \) at each \( \bar{p}_f \),
3. Generate two-phase diagnostic plot, \( DRNP vs. t_{opt} \) in log-log scale, to pick the time when half-slope straight line and unit-slope straight line intersects, which is the start of BDF regime (FR3),
4. Generate specialty plot \( \ln(RNP) vs. t_p \) and extract slope and intercept of the straight line after the end of half-slope interval in step 3,
5. Calculate a new value of \( \zeta \) using eq (2-62),
6. Repeat step 2 to 5 until the relative error of \( \zeta \) meets the tolerance we set,
7. Calculate a new value of \( V_{fi} \) using eq (2-61),
8. Repeat step 2 to 7 until the relative error of \( V_{fi} \) meets the tolerance we set,
9. Determine \( x_f \) and \( k_f \) using eq (2-59) to (2-60).
Workflow III only requires water-phase production data as the input, like Workflow II, but it is only valid for constant BHP case. Therefore for flowback analysis under constant BHP condition, Workflow I, II and III all work.

2.4 Model Validation and Results

To validate the developed flowback model, average pressure calculation methods, and the proposed workflow, we conducted several numerical simulations for one fracture stage as shown in Figure 2-1 using IMEX—CMG to generate production data. A 1D flow system with 1000 uniform grids was considered as the physical model. To model fracture flow with high conductivity, we set the grid block width to 0.02 ft and permeability to 800 md. Considering that the early gas production period only lasts for a short time (Adefidipe et al., 2014), we conducted the simulation for 10 days, which are long enough to show boundary dominated flow during the flowback period. The permeability modulus values were obtained from the laboratory measurements from Ozkan et al. (2010). The input parameters used in the numerical simulations are shown in Table 2-1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial pressure, ( p_i ) (psi)</td>
<td>5000</td>
</tr>
<tr>
<td>Reservoir temperature, ( T ) (°F)</td>
<td>160</td>
</tr>
<tr>
<td>Specific gravity, ( S_G )</td>
<td>0.65</td>
</tr>
<tr>
<td>Water compressibility, ( c_w ) (psi(^{-1}))</td>
<td>8×10(^{-6})</td>
</tr>
<tr>
<td>Initial fracture porosity, ( \phi_f ) (%)</td>
<td>50</td>
</tr>
<tr>
<td>Fracture compressibility, ( c_f ) (psi(^{-1}))</td>
<td>5×10(^{-6})</td>
</tr>
<tr>
<td>Reservoir thickness, ( h ) (ft)</td>
<td>100</td>
</tr>
<tr>
<td>Fracture width, ( w_f ) (ft)</td>
<td>0.02</td>
</tr>
<tr>
<td>Initial water formation volume factor, ( B_{wi} ) (bbl/STB)</td>
<td>1.03</td>
</tr>
<tr>
<td>Water viscosity @( p_i ), ( \mu_w ) (cp)</td>
<td>0.6</td>
</tr>
<tr>
<td>Water density @( p_i ), ( \rho_w ) (lb/ft(^3))</td>
<td>62</td>
</tr>
<tr>
<td>Fracture half-length, ( x_f ) (ft)</td>
<td>1000</td>
</tr>
<tr>
<td>Initial fracture permeability, ( k_f ) (mD)</td>
<td>800</td>
</tr>
</tbody>
</table>
A list of simulation runs with different degrees of nonlinearity and production conditions are given in Table 2-2. We used the gravity segregated relative permeability curve (Clarkson, 2012) for the fracture, as shown in Figure 2-4a. Gas viscosity values vs. pressure is presented in Figure 2-4b.

Table 2-2: The numerical cases used for validation.

<table>
<thead>
<tr>
<th>Case #</th>
<th>Initial water saturation, $s_{wi}$</th>
<th>Fracture permeability modulus, $\gamma$(psi$^{-1}$)</th>
<th>Production condition</th>
<th>Bottom-hole pressure, $p_{wf}$(psi)</th>
<th>Water-phase surface flowrate, $q_w$(bbl/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.95</td>
<td>$1 \times 10^{-5}$</td>
<td>Constant BHP</td>
<td>700</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.9</td>
<td>$1 \times 10^{-5}$</td>
<td>Constant BHP</td>
<td>700</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>0.7</td>
<td>$5 \times 10^{-4}$</td>
<td>Constant BHP</td>
<td>700</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>0.95</td>
<td>$1 \times 10^{-5}$</td>
<td>Constant rate</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>0.9</td>
<td>$1 \times 10^{-5}$</td>
<td>Constant rate</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>0.7</td>
<td>$5 \times 10^{-4}$</td>
<td>Constant rate</td>
<td>-</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 2-4: (a) Relative permeability curves for the fracture and (b) gas viscosity and formation volume factor vs. pressure

2.4.1 Validation of Average Pressure Calculation

First, we tested the accuracy of the average fracture pressure calculation methods against the numerical simulation for Case 1, Case 2, and Case 3. These cases are all under constant BHP condition so that the diffusivity equation approach for constant BHP and variable rate case, and
material balance approach can be compared against each other and the numerical simulation results by considering $V_f$ as a known parameter. Figure 2-5 shows the comparison for the three cases.

Figure 2-5: Comparison of diffusivity equation approach and material balance approach for (a) Case 1, (b) Case 2, and (c) Case 3.

The results show that there is a good agreement between numerical simulation and analytical methods. Although the nonlinearity caused by two-phase flow and pressure dependent fracture permeability is increasing from Case 1 to Case 3, the good match of average pressure for all the cases indicates that the nonlinearity does not affect the accuracy of average pressure calculation. The average pressure values obtained from the variable-rate method are sensitive to the quality and trend of production data. Further investigation is required to improve the calculation of average pressure using variable-rate method.

2.4.2 Validation of Flowback Model

After validation of average pressure calculation methods, we tested the accuracy of the two-phase flowback model against the production data obtained from the numerical simulation, by considering $V_f$ as a known parameter. Material balance approach was used to calculate average pressure in this section. We analyzed Case 1, 2, and 3 with constant BHP flowback model and Case
4, 5, and 6 with constant rate model. The two-phase diagnostic plot and specialty plot for all the six cases are shown in Figure 2-6 and Figure 2-7. We also analyzed flowback data in Case 3 with the variable rate flowback model. The diagnostic plot and specialty plot are shown in Figure 2-8.

Figure 2-6: Two-phase (a) diagnostic plot and (b) specialty plot for Case 1-3 using constant BHP flowback model.
Figure 2-7: Two-phase (a) diagnostic plot and (b) specialty plot for Case 4-6 using constant rate flowback model.
The two-phase diagnostic plot for each case shows a clear half-slope straight line indicating the transient flow, which is expected based on eqs (2-56) – (2-58). We picked the intersection point between the half-slope and unit-slope straight line as the start of two-phase boundary dominated flow. After identifying the flow regimes, the corresponding data interval is selected to conduct flowback RTA in specialty plot. Based on the analytical two-phase BDF solution given by eq (2-16) for constant BHP condition and eq (2-22) for constant rate condition, the specialty plot is expected to show a straight line during the BDF. But the results show that only the specialty plots in Case 4 – 6 have the perfect straight line, whereas Case 1 – 3 are curved. That is because, applying pseudotime to linearize the governing equation has such an assumption that all the pressure dependent properties, such as \( s_w \), \( k_f \), and \( \phi_f \), are evaluated at average fracture pressure (\( \bar{p}_f \)) and average water saturation (\( \bar{s}_w \)). Although these properties vary along the fracture, an average value for each of these properties is selected to represent all the values in the entire domain. The pressure profile under constant BHP condition is very sharp at the beginning and drops unevenly in Figure 2-9a. Whereas the pressure profile under constant rate condition is gentler and drops with the same rate everywhere during BDF in Figure 2-9b. Therefore, the less error of interpretation results and the less curved the specialty plot are expected in constant rate flowback model.
With the slope and intercept of the specialty plot, we used eqs (2-59) to (2-65) to calculate $x_f$ and $k_{fi}$ for the six cases. For constant BHP cases, $(p_{pi}-p_{pwf})$ is estimated with eq (2-53). The calculated results and relative errors are compared with the inputs in the simulator in Table 2-3. The results show that the obtained fracture properties using constant rate model are very close to the set values in the numerical simulation and the relative errors are all less than 10%, which proves the accuracy. Although the $(p_{pi}-p_{pwf})$ calculation is proved to be accurate enough, the constant BHP model has much more error than constant rate model, which is expected. The error of both constant BHP model and constant rate model increases with the extent of nonlinearity. Jia et al. (2018) observed a similar behavior where fracture properties calculation errors increased with the nonlinearity caused by the strong water saturation and pressure gradients. Thus, the constant BHP flowback model is only valid when the nonlinearity is not significant, whereas the constant rate model always works. For Case 3 with the greatest extent of nonlinearity under constant BHP condition, the variable rate model provides more accurate results compared to constant BHP model.

Figure 2-9: Pressure profile in 1D fracture under (a) constant BHP condition and (b) constant rate condition.
Table 2-3: Set and calculated values of fracture properties and relative error using the new flowback model for the six cases and the comparison with the results analyzed by the two-phase flowing material balance method of Xu et al. (2016) and Behmanesh et al. (2018b).

<table>
<thead>
<tr>
<th>Case #</th>
<th>Set values</th>
<th>This work</th>
<th>Xu et al. (2016)</th>
<th>Behmanesh et al. (2018b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_f$ (ft)</td>
<td>$k_{fi}$ (md)</td>
<td>$p_{pi}-p_{pwf}$ (psi/cp)</td>
<td>$x_f$ (ft)</td>
</tr>
<tr>
<td>1</td>
<td>1000</td>
<td>800</td>
<td>6542</td>
<td>941</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
<td>800</td>
<td>6216</td>
<td>845</td>
</tr>
<tr>
<td>3</td>
<td>1000</td>
<td>800</td>
<td>2025</td>
<td>702</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>800</td>
<td>1008</td>
<td>1008</td>
</tr>
<tr>
<td>5</td>
<td>1000</td>
<td>800</td>
<td>1010</td>
<td>1010</td>
</tr>
<tr>
<td>6</td>
<td>1000</td>
<td>800</td>
<td>1052</td>
<td>1052</td>
</tr>
<tr>
<td>3(variable rate)</td>
<td>1000</td>
<td>800</td>
<td>1019</td>
<td>1019</td>
</tr>
</tbody>
</table>

To further validate our model, we compared the analysis results with the two-phase flowing material balance model proposed by Xu et al. (2016) and Behmanesh et al. (2018b) in Table 2-3 and listed the differences of the three models in Table 2-4. We extended the original model of Xu et al. (2016) from constant rate conditions to include variable-rate conditions by introducing superposition pseudotime. The estimated $x_f$ from Xu et al. (2016) shows to be accurate for all the numerical cases, but the calculated $k_{fi}$ has much more error than our flowback model in this work, especially for Case 1 – 3. It is because their model assumes constant $k_f$ and $\phi_f$, whereas in the numerical cases $k_f$ and $\phi_f$ are pressure-dependent properties. What’s more, the pressure and saturation gradient in Case 1 – 3 are greater than Case 4 – 6 as shown in Figure 2-9, which causes more error using the approach of Xu et al. (2016). We also extended the two-phase RTA model of Behmanesh et al. (2018b) from radial system to linear system by adopting the $b_{pfi}$ term in Zhang and Emami-Meybodi (2020), and to consider pressure-dependent permeability. The analysis results from Behmanesh et al. (2018b) in Table 2-3 show good accuracy in calculating both $k_f$ and $x_f$ for all the numerical cases. Compared to these two flowback models, our proposed flowback model only requires water production data and does not require gas production data during flowback analysis.
Table 2-4: Comparison of the proposed model with two RTA models.

<table>
<thead>
<tr>
<th></th>
<th>This work</th>
<th>Xu et al. (2016)</th>
<th>Behmanesh et al. (2018b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pseudopressure $p_p$</td>
<td>$\frac{\mu_w B_w}{k_f} \int_0^p \frac{k_f f(p)k_{rw}(s_w)}{p_b \mu_w(p)B_w(p)} dp$</td>
<td>$\int_0^p \frac{2p}{p_b \mu_g Z} dp$</td>
<td>$\frac{1}{k_f} \int_0^p \frac{p_{gsc} k_f k_{fg}}{\rho_{ref} \mu_g B_g} + \frac{p_{wsc} k_f k_{rw}}{\rho_{ref} \mu_w B_w} dp$</td>
</tr>
<tr>
<td>Pseudotime $t_p$</td>
<td>$\int_0^t \frac{k_{rw}(s_w) f(\beta_f)}{s_w \mu_w(c_w + c_f + s_w dp_f) / \beta_f} dt$</td>
<td>$\int_0^t k_{rg} \frac{dp}{p_b \mu_g c_t}$</td>
<td>–</td>
</tr>
<tr>
<td>Superposition</td>
<td>$\sum_{i=1}^N \left( \frac{q_{w,i} - q_{w,i-1}}{q_{w,N}} \right)(t_{p,N} - t_{p,i-1})$</td>
<td>$\sum_{i=1}^N \left( \frac{q_{g,i} - q_{g,i-1}}{q_{g,N}} \right)(t_{p,N} - t_{p,i-1})$</td>
<td>$\frac{\mu_d c_t}{\rho_g} \int_0^t \frac{q_{e}}{\rho_c} dt$</td>
</tr>
<tr>
<td>Flowrate</td>
<td>$q_w$</td>
<td>$q_g = \frac{q_w B_{gi} q_{w,b}}{k_{fg}}$</td>
<td>$q_e = \frac{p_{gsc}}{\rho_{ref}} q_g + \frac{p_{wsc}}{\rho_{wsc}} q_w$</td>
</tr>
<tr>
<td>RNP/PNR</td>
<td>$RNP = \frac{p_{wci} p_{pwf}}{q_w}$</td>
<td>$RNP = \frac{p_{wci} p_{pwf}}{q_g}$</td>
<td>$PNR = \frac{q_e}{p_{pwf}}$</td>
</tr>
<tr>
<td>Comments</td>
<td>• Considers pressure dependent $k_f$ and $\phi_f$</td>
<td>• Assumes constant $k_f$ and $\phi_f$</td>
<td>• Assumes constant $k_f$ and $\phi_f$</td>
</tr>
<tr>
<td></td>
<td>• Requires only water production data</td>
<td>• Requires both water and gas production data</td>
<td>• Requires both water and gas production data</td>
</tr>
</tbody>
</table>

1 Extended from original formulation to include pressure-dependant permeability.

2 $C_{ci} = \left( 1 - \frac{G_{ci}}{G_{fi}} \right) B_{gi} S_{ci} C_G + \left( 1 - \frac{W_{ci}}{W_{fi}} \right) S_{wi} C_W + \frac{1}{\nu_{ci}} \frac{\partial v_{ci}}{\partial P_f}.$

3 Extended from original formulation to include variable-rate conditions.

4 $\mu_e = \frac{1}{p_{gsc} k_f k_{fg} p_{wsc} k_f k_{rw} \rho_{ref} \mu_g B_g \rho_{ref} \mu_w B_w}.$

2.4.3 Validation of Analysis Workflow

In this section, we validated the proposed workflows of the successive substitution method in Figure 2-3. First, we validated Workflow I with Case 6. According to Table 2-1, $V_{fi}$ used in the numerical simulation is 2000 ft$^3$. However, here $V_{fi}$ is an unknown parameter and an initial guess (i.e., 1500 ft$^3$) is needed. Average fracture pressure is obtained first to generate the diagnostic plot and specialty plot for constant rate flowback model. The slope and intercept of specialty plot were used in eq (2-65) to update $V_{fi} = 1635$ ft$^3$. The relative error between the initial guess $V_{fi} = 1500$ ft$^3$ and output $V_{fi} = 1635$ ft$^3$ is 8.3%, which is greater than the acceptable 5% tolerance. Then, $V_{fi}$ is updated with the new calculated value 1635 ft$^3$. After five iterations, the relative error of $V_{fi}$ between
two consecutive iterations was less than the acceptable tolerance with the final $V_f=2125$ ft$^3$. Using the estimated $V_f$, $x_f$ and $k_f$ were calculated to be 1063 ft and 728 md, respectively, with the relative error of <10%. These calculations are tabulated in Table 2-5. The calculated average pressure and specialty plot gradually converge to the numerical simulation during the five iterations as shown in Figure 2-10.

<table>
<thead>
<tr>
<th>Iteration #</th>
<th>Input $V_f$, ft$^3$</th>
<th>Output $x_f$, ft</th>
<th>Output $k_f$, md</th>
<th>Output $V_f$, ft$^3$</th>
<th>Error of $V_f$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1500</td>
<td>817</td>
<td>1850</td>
<td>1635</td>
<td>8.3</td>
</tr>
<tr>
<td>2</td>
<td>1635</td>
<td>887</td>
<td>1149</td>
<td>1774</td>
<td>7.8</td>
</tr>
<tr>
<td>3</td>
<td>1774</td>
<td>953</td>
<td>905</td>
<td>1906</td>
<td>6.9</td>
</tr>
<tr>
<td>4</td>
<td>1906</td>
<td>1012</td>
<td>790</td>
<td>2025</td>
<td>5.9</td>
</tr>
<tr>
<td>5</td>
<td>2025</td>
<td>1063</td>
<td>728</td>
<td>2125</td>
<td>4.7</td>
</tr>
</tbody>
</table>

Figure 2-10: (a) Calculated average pressure and (b) generated specialty plot during iterations of Workflow I.

Workflow II follows a similar procedure as Workflow I, with the difference in average pressure calculation. Therefore, the above-mentioned validation is valid for Workflow II. We validated Workflow III with the flowback data generated from Case 1. With the set parameters $V_f = 2000$ ft$^3$ and $\zeta=0.0008$ md/ft$^2$ in the numerical simulation, we started the workflow with the initial value of 1000 ft$^3$ for $V_f$ and $4\times10^{-4}$ md/ft$^2$ for $\zeta$. The average pressure in the fracture is obtained at first to generate the diagnostic plot and specialty plot for constant BHP flowback model. With the
slope and intercept of specialty plot, we used eqs (2-61) and (2-62) to calculate $V_f$ to be 1127 ft³ and $\zeta$ to be $4.27 \times 10^{-4}$ md/ft². The relative error of $V_f$ and $\zeta$ between input and output are 11.3% and 6.4%, respectively. A smaller tolerance of 0.3 % is required in Workflow III to obtain a good match of average pressure and accurate interpretation results. Then, $V_f$ and $\zeta$ are updated. After 69 iterations, both relative errors meet the acceptable tolerance. Using the estimated $V_f$, $x_f$ and $k_{fi}$ were calculated to be 1137 ft and 875 md, respectively, with the relative error of <10%. Five of the 49 calculations are tabulated in Table 2-6 and shown in Figure 2-11.

Table 2-6: Validation results of Workflow III.

<table>
<thead>
<tr>
<th>Iteration #</th>
<th>Input $V_f$, ft³</th>
<th>Input $\zeta$, $10^{-4}$ md/ft²</th>
<th>Output $x_f$, ft</th>
<th>Output $k_{fi}$, md</th>
<th>Output $V_f$, ft³</th>
<th>Output $\zeta$, $10^{-4}$ md/ft²</th>
<th>Error of $V_f$, %</th>
<th>Error of $\zeta$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000</td>
<td>4.00</td>
<td>563</td>
<td>135</td>
<td>1127</td>
<td>4.27</td>
<td>11.2</td>
<td>6.4</td>
</tr>
<tr>
<td>6</td>
<td>1434</td>
<td>4.85</td>
<td>742</td>
<td>272</td>
<td>1484</td>
<td>4.94</td>
<td>3.4</td>
<td>1.8</td>
</tr>
<tr>
<td>18</td>
<td>1795</td>
<td>5.50</td>
<td>906</td>
<td>454</td>
<td>1813</td>
<td>5.53</td>
<td>1.0</td>
<td>0.6</td>
</tr>
<tr>
<td>49</td>
<td>2135</td>
<td>6.32</td>
<td>1071</td>
<td>727</td>
<td>2143</td>
<td>6.34</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>69</td>
<td>2270</td>
<td>6.74</td>
<td>1137</td>
<td>875</td>
<td>2276</td>
<td>6.76</td>
<td>0.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Figure 2-11: (a) Calculated average pressure and (b) generated specialty plot during iterations of Workflow III.
2.5 Field Application

We applied the flowback model to an MFHW drilled and completed in the Muskwa and Otter Park members of the Horn River Shale in part of British Columbia, Canada. This field example was analyzed by Xu et al. (2016) to estimate the initial effective fracture volume. The purpose of analyzing this field case is to verify the practical use of the new two-phase diagnostic plot and flowback model, which could respectively identify the water flow regimes in Figure 2-2 and quantitatively evaluate the fracture half-length and permeability.

Table 2-7 shows the real field data from the Horn River MFHW with 15 hydraulic fractures. The input parameters are consistent with those used by Xu et al. (2016) and Anderson et al. (2013) for the same field case. The fracture width used in this study adopts the primary fracture network width in the field case analyzed by Jia et al. (2018). The fracture permeability modulus refers to the empirical range of $1 \times 10^{-4} - 1 \times 10^{-3}$ psi$^{-1}$ from laboratory measurements (Ozkan et al., 2010) and is set to be $5 \times 10^{-4}$ psi$^{-1}$ for this case.

Figure 2-12 shows production history of water flowrate, gas flowrate and bottomhole pressure for this MFHW during flowback and long-term production period. After hydraulic fracturing, this well was flowed back for more than 24 days and then produced for 78 days. In the flowback period, water is dominantly produced with a high flowrate and then quickly decreases due to the fracture depletion. When it comes to long-term production, water flowrate is negligible. Whereas gas is mainly produced and matrix contribution of gas influx becomes more and more significant.
Table 2-7: Real Field data from an MFHW in Horn River Shale.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial pressure, $p_i$ (psi)</td>
<td>5900</td>
</tr>
<tr>
<td>Initial reservoir pressure, $p_{ir}$ (psi)</td>
<td>4900</td>
</tr>
<tr>
<td>Reservoir temperature, T (°F)</td>
<td>278</td>
</tr>
<tr>
<td>Specific gravity, $SG_g$</td>
<td>0.66</td>
</tr>
<tr>
<td>Water compressibility, $c_w$ (psi⁻¹)</td>
<td>3.33×10⁻⁶</td>
</tr>
<tr>
<td>Initial fracture porosity, $\phi_{fi}$ (%)</td>
<td>60</td>
</tr>
<tr>
<td>Matrix porosity, $\phi_m$ (%)</td>
<td>5</td>
</tr>
<tr>
<td>Matrix permeability, $k_m$ (md)</td>
<td>1×10⁻⁴</td>
</tr>
<tr>
<td>Fracture compressibility, $c_f$ (psi⁻¹)</td>
<td>8.14×10⁻⁵</td>
</tr>
<tr>
<td>Matrix compressibility, $c_m$ (psi⁻¹)</td>
<td>6.48×10⁻⁶</td>
</tr>
<tr>
<td>Reservoir thickness, $h$ (ft)</td>
<td>394</td>
</tr>
<tr>
<td>Horizontal wellbore length, $L_w$ (ft)</td>
<td>4593</td>
</tr>
<tr>
<td>Number of fractures</td>
<td>15</td>
</tr>
<tr>
<td>Fracture spacing, $L_f$ (ft)</td>
<td>306</td>
</tr>
<tr>
<td>Fracture width, $w_f$ (ft)</td>
<td>0.08</td>
</tr>
<tr>
<td>Initial water formation volume factor, $B_{wi}$ (bbl/STB)</td>
<td>1.03</td>
</tr>
<tr>
<td>Initial fracture water saturation, $s_{wi}$ (%)</td>
<td>85</td>
</tr>
<tr>
<td>Matrix water saturation, $s_{wm}$ (%)</td>
<td>10</td>
</tr>
<tr>
<td>Initial water viscosity, $\mu_w$ (cp)</td>
<td>0.33</td>
</tr>
<tr>
<td>Initial water density, $\rho_w$ (lb/ft³)</td>
<td>62</td>
</tr>
<tr>
<td>Fracture permeability modulus, $\gamma$ (psi⁻¹)</td>
<td>5×10⁻⁴</td>
</tr>
</tbody>
</table>

We applied the proposed two-phase water flowback model to analyze flowback data and used the diffusivity approach under variable rate condition to calculate the average pressure in the fracture. We started the Workflow II with an initial guess of fracture volume $V_{fi}=8\times10^5$ ft³ and generated the average pressure in the fracture, two-phase diagnostic plot, and specialty plot for water during each iteration. The tolerance of the relative error between $V_{fi}$ and $V_{fi,new}$ is set to be
After convergence, the essential plots of flowback analysis is shown in Figure 2-13. Figure 2-13a shows the average pressure in the fracture determined from the diffusivity approach and the average fracture permeability calculated using eq (2-6) during flowback. Figure 2-13b shows the saturation vs. pressure path obtained from eq (2-42), which is later extrapolated using exponential function to calculate pseudopressure. The two-phase diagnostic plot of water is plotted in Figure 2-13c to identify FR2 and FR3, which shows a half-slope and unit-slope straight line respectively. The flow regimes and sequence for this field case are consistent with those in Figure 2-2. With identified FR3, the specialty plot is generated in Figure 2-13d to calculate $x_f$ and $k_{fi}$ based on the slope and intercept of straight line. The flowback analysis results are listed in Table 2-8.

Figure 2-13: Plots during flowback analysis after the convergence of $V_f$: (a) Average pressure and permeability in the fracture during flowback period. (b) The path of average fracture water saturation vs. average fracture pressure. (c) Two-phase diagnostic plot of water where the pseudo-variables are calculated based on the average pressure from plot (a). (d) Specialty plot of two-phase water flowback model.
We also analyzed the long-term production data to extract the fracture attributes in the late time using the RTA technique for transient linear flow and history matching approach. Considering that gas is dominantly produced in the long-term period with water production negligible, we first use the single-phase gas formation linear flow model (Clarkson, 2013) to analyze the long-term production data. The gas-phase diagnostic plot in Figure 2-14a shows a clear half-slope straight line which illustrates the transient linear flow in the shale matrix. The matrix permeability for this MFHW is in a range of $5 \times 10^{-6} - 1 \times 10^{-4}$ md (Anderson et al., 2013), which leads to the interpreted $x_f$ in the range of 103 – 462 ft based on the slope of specialty plot in Figure 2-14b. Then, we use history matching approach to analyze the long-term production data with the single-phase gas Horizontal-Multifractured-SRV model in IHS Harmony software. Figure 2-14c and d show a good match of gas flowrate and bottomhole pressure history during long-term production. The estimated $x_f$ and $k_f$ are 430 ft and 212 md, respectively.

Figure 2-14: Plots during long-term production data analysis: (a) Single-phase diagnostic plot of gas where the pseudo-variables are calculated based on the average pressure in the distance of...
Table 2-8 summarized the flowback analysis results using the new flowback model and long-term production data analysis results using RTA technique and history matching approach for this field case. A significant decrease in $x_f$ from 750 ft to 430 ft and for $k_f$ from 883 md to 212 md is observed during production. It is consistent with the observation in Figure 2-13a that the $k_f$ drops from 883 md to 394 md after 24 days flowback. The average $k_f=212$ md from long-term production data analysis is slightly below the estimated ultimate $k_f=394$ md from flowback analysis, which is reasonable according to eq (2-6). Because the ultimate average pressure in the fracture during flowback period is lower than that in long-term production. The decrease in $x_f$ and $k_f$ is mainly caused by the partial closure of hydraulic fractures.

### Table 2-8: Comparison of flowback analysis and long-term production data analysis results.

<table>
<thead>
<tr>
<th>Fracture properties</th>
<th>Flowback analysis</th>
<th>Long-term production data analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Linear flow RTA technique</td>
</tr>
<tr>
<td>$x_f$, ft</td>
<td>750</td>
<td>103 – 462</td>
</tr>
<tr>
<td>$k_f$, md</td>
<td>883</td>
<td>-</td>
</tr>
<tr>
<td>Average $k_f$, md</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

### 2.6 Discussion

In this paper, we proposed a 1D two-phase model to analyze water-phase flowback data of MFHWs in shale reservoirs and a two-phase diagnostic plot to identify flow regimes during flowback period. Pressure-dependent fracture permeability and porosity are accounted for in this model. Compared to other flowback models, the proposed models consider water-phase relative permeability into pseudopressure to linearize the governing equation. Furthermore, this paper first developed flowback model under constant BHP condition although it is only valid for cases with low extent of nonlinearity, whereas other researchers derived flowback models under constant rate
condition. With the flowback model, we proposed a comprehensive analysis workflow to iteratively calculate $V_{fi}$, rather than considering it as a known parameter. The validation compares the analysis result with the two-phase flowing material balance models proposed by Xu et al. (2016) and Behmanesh et al. (2018b). The results show the superiority and practical applicability of the proposed model to estimate fracture properties.

In the derivation of the new average pressure calculation method, it is assumed that pseudo average pressure can be approximated by average pseudopressure. Therefore, the proposed method is not applicable to the systems with significant nonlinearity in two-phase flow and pressure dependent permeability. As the nonlinearity increases, the approximation is expected to be worse. Also, the variable rate method of calculating average pressure is very sensitive to the data quality, which needs careful attention to smoothing the production data. More work needs to improve this method to make it robust.

The flowback model in this paper is developed for early gas production period, which assumes water and gas sourced from the matrix are negligible. This could be invalid for reservoirs with relatively high matrix permeability or secondary fractures. When the fluid influx from the matrix cannot be overlooked, our model could be inapplicable. The modeling of 2D two-phase flow during the late flowback period needs further investigations.

2.7 Summary and Conclusions

This work presents a semi-analytical two-phase model to analyze early-time flowback data from MFHWs and evaluate fracture properties, such as fracture half-length, initial fracture permeability, and initial pore volume of fracture, under constant BHP, constant rate, and variable rate/BHP condition. The following important conclusions can be drawn:
1. The two-phase semi-analytical model is based on water diffusivity equation and considers pressure-dependent fracture permeability and porosity. By introducing new definitions of pseudopressure and pseudotime, the proposed two-phase diagnostic plot is able to identify flow regimes more accurately during flowback period.

2. A diffusivity equation approach is developed to calculate average pressure under constant BHP, constant rate, and variable rate/BHP condition. The method releases the demand for gas-phase production data as input and is not affected by the extent of nonlinearity.

3. Comparing against input fracture parameters in numerical simulation, the developed constant BHP flowback model has more error and is only applicable to cases with slight nonlinearity. The constant/variable-rate model is more accurate with relative error less than 10%.

4. The proposed three analysis workflows can closely predict fracture properties. The iterative procedure is robust and practical for the field applications. Workflow I is valid for cases with gas and water-phase flowback data both available, whereas Workflow II and III are more applicable and only need water-phase production data as input. Workflow III provides an alternative analysis method to calculate fracture properties under constant BHP condition.

5. A field example in Horn River Shale is analyzed using the proposed flowback model to demonstrate the practical application of the mathematical developments presented in this study. The field application shows that the fracture half-length and permeability are reduced to half and quarter, respectively, after three months of production.
2.8 Nomenclatures

\( b_p \): the intercept of \( \ln (RNP) \) vs. \( t_p \) plot

\( b_q \): the intercept of \( RNP \) vs. \( t_p \) plot

\( B_g \): gas formation factor, \( ft^3/scf \)

\( B_w \): water formation factor, \( bbl/STB \)

\( B_{wf} \): bottomhole water formation factor, \( bbl/STB \)

\( B_{wi} \): initial water formation factor, \( bbl/STB \)

\( c_f \): fracture compressibility, \( psi^{-1} \)

\( c_t \): total compressibility, \( psi^{-1} \)

\( c_w \): water compressibility, \( psi^{-1} \)

DDA: dynamic drainage area

\( DRNP \): derivative of rate normalized pseudopressure, \( psia \times d/STB \)

\( G_i \): initial gas storage in the fracture, \( scf \)

\( G_r \): remaining gas in fracture, \( scf \)

\( G_{ip} \): cumulative gas production, \( scf \)

\( h \): fracture height, \( ft \)

HF: hydraulic fracture

\( k_f \): fracture permeability, \( md \)

\( k_{f_i} \): initial fracture permeability, \( md \)

\( k_{rw} \): water relative permeability

\( m_p \): the slope of \( \ln (RNP) \) vs. \( t_p \) plot

\( m_q \): the slope of \( RNP \) vs. \( t_p \) plot

MFHWs: multi-fractured horizontal wells
\( p \): pressure, \( psia \)

\( p_b \): base pressure, \( psia \)

\( \bar{p}_f \): average pressure in the fracture, \( psia \)

\( p_i \): initial fracture pressure, \( psia \)

\( p_p \): pseudopressure, \( psia \)

\( \bar{p}_{pf} \): pseudo average pressure in the fracture, \( psia \)

\( \bar{p}_{pf} \): average pseudopressure in the fracture, \( psia \)

\( p_{pi} \): initial pseudopressure, \( psia \)

\( p_{pwf} \): bottomhole pseudopressure, \( psia \)

\( p_{sc} \): pressure at standard condition, \( psia \)

\( p_{w} \): bottom-hole pressure, \( psia \)

\( q_{D_{w}} \): dimensionless water flow rate

\( q \): the water-phase downhole flowrate, \( bbl/d \)

\( q_{w} \): the water-phase surface flowrate from the whole fracture, \( STB/d \)

\( RNP \): rate normalized pseudopressure, \( psia \times d/STB \)

\( RTA \): rate transient analysis

\( s_{g} \): gas saturation

\( s_{gi} \): initial gas saturation

\( s_{w} \): water saturation

\( \bar{s}_{w} \): average water saturation

\( s_{wf} \): average water saturation in fracture at the end of flowback

\( s_{wi} \): initial water saturation

\( V_{fi} \): initial pore volume of the fracture, \( ft^3 \)

\( V_{wr} \): remaining water volume in the fracture, \( ft^3 \)
$w_f$: hydraulic fracture width, $ft$

$W_{fi}$: initial water storage in fracture, $ft^3$

$W_p$: cumulative water production, $bbl$

$x$: location in the fracture, $ft$

$x_f$: fracture half-length, $ft$

$t$: time, $day$

$t_p$: pseudotime, $psia \times day/\text{cp}$

$t_{spt}$: superposition pseudotime, $psia \times day/\text{cp}$

$T_{sc}$: temperature at standard condition, °F

$\phi_f$: fracture porosity

$\phi_{fi}$: initial fracture porosity

$\rho_{gsc}$: gas density at standard condition, $lb/ft^3$

$\rho_{ref}$: reference density, $lb/ft^3$

$\rho_w$: water density, $lb/ft^3$

$\rho_{wsc}$: water density at standard condition, $lb/ft^3$

$\mu_w$: water viscosity, $\text{cp}$

$\mu_g$: gas viscosity, $\text{cp}$

$\gamma$: permeability modulus, $psi^{-1}$

$\zeta$: fracture coefficient, $md/ft^2$

Subscripts

$i$: initial

$j$: the $j^{th}$ flow rate interval

$fi$: initial fracture
$f$: fracture

$g$: gas

$sc$: standard condition

$w$: water

$D$: dimensionless

$p$: pseudo or constant BHP model

$q$: pseudo or constant rate mode

2.9 Reference


CLARKSON, C. 2012. Modeling 2-Phase Flowback of Multi-Fractured Horizontal Wells Completed in Shale. Paper SPE 162593 presented at SPE Canadian Unconventional


Chapter 3

Flowback Fracture Closure of Multi-fractured Horizontal Wells in Shale Gas Reservoirs‡‡

3.1 Abstract

In multi-fractured horizontal wells (MFHW), fracture properties such as permeability and fracture half-length significantly deteriorate during production, which negatively affects gas production from shale reservoirs. Therefore, it is crucial to evaluate the temporal changes in fracture properties using production data. This paper presents two multiphase flowback models for gas and water phase under boundary dominated flow (BDF) condition by considering gas influx from matrix. In addition, a workflow is proposed to quantitatively evaluate hydraulic fracture closure and fracture properties using both water-phase and gas-phase flowback data. A material balance equation (MBE) is derived to obtain average pressure in the fracture by calculating gas influx from matrix under variable bottomhole pressure (BHP) condition based on Duhamel’s principle. The flowback models, average pressure calculation method, and workflow are validated against numerical simulations and applied to an MFHW drilled in Marcellus Shale. The results show that the developed models are capable of predicting fracture properties, and MBE could accurately calculate average pressure in the fracture. Furthermore, the proposed workflow provides quantitative insights into the performance of stimulation jobs by accurately predicting fracture half-length, fracture permeability, and permeability modulus using flowback data. By reconciling

flowback analysis with long-term production data analysis, a significant decrease in fracture properties due to fracture closure is observed in the Marcellus MFHW.

3.2 Introduction

Multi-fractured horizontal wells (MFHWs) are widely used to acquire economic production from unconventional resources such as shale reservoirs. Evaluating hydraulic fracturing performance is very critical for MFHWs. One way to assess stimulation performance is rate transient analysis (RTA), which provides important matrix and fracture properties. In unconventional reservoirs such as shales, RTA usually focuses on analyzing long-term production data obtained over several months to even several years where various flow regimes can be identified (Clarkson, 2013). Apart from long-term RTA, fracture and reservoir properties could also be estimated from short-term data obtained during the flowback period. Flowback is the process of allowing injected fluids to flow from well to the surface after hydraulic fracturing. Although fracturing fluids leaks off to the formation, the flow in the matrix is still dominated by single-phase gas flow due to the low water relative permeability (Clarkson et al., 2017). Thus, two-phase flow mainly occurs in hydraulic fractures. Compared to long-term RTA, flowback RTA provides insights on fracture information at the early times and hence could be used to evaluate fracture closure during production.

Over the past decade, numerous studies have qualitatively analyzed flowback data. In 2008, Crafton studied the effects of flowback rate, fracture complexity, and shut-ins on well performance and recovery (Crafton, 2008). Ilk et al. (2010) presented a diagnostic log-log plot showing that gas water ratio (GWR) vs. cumulative gas production ($G_p$) plot has a half slope, which was later used by others (Ghanbari et al., 2013, Abbasi et al., 2014, Zhang and Ehlig-Economides, 2014). However, there is no mathematical proof for this empirical diagnostic plot. Clarkson and colleagues
used GWR vs. $G_p$ to identify multiphase fracture depletion flow regime and the linear flow regime (Williams-Kovacs and Clarkson, 2016, Clarkson and Williams-Kovacs, 2013b). Later, Clarkson and Williams-Kovacs (2013a) divided the flowback process into before-breakthrough (BBT) and after-breakthrough (ABT) according to the time when gas enters the fracture from matrix. During BBT, only single-phase water flows in fracture. When it comes to ABT, formation gas starts entering fracture and the two-phase flow occurs. Adefidipe et al. (2014) extended BBT period to two-phase flow with flowing fluids in-situ water and gas in the hydraulic fracture. They divided flowback period to early gas production (EGP) and late gas production (LGP) using GWR vs. time plot.

Numerous studies have addressed the modeling of single-phase flowback (Abbasi et al., 2012, Wattenbarger and Alkouh, 2013, Kurtoglu et al., 2015). However, single-phase model is only valid for the very early period because the flowback is multiphase flow most of the time. To model two-phase water and gas flow in the fracture, Ezulike et al. (2013) developed a two-phase flowback model for MFHWs which considers gas influx. To solve coupled equations, they proposed the Dynamic Relative Permeability (DRP) concept using Mellin transform. In an extended study, they improved their approach by simplifying DRP with an empirical function and used Laplace transforms to get the semi-analytical solution (Ezulike and Dehghanpour, 2014). Yang (2017) developed two-phase flowback models for both EGP and LGP, where the LGP model assumes the equal average pressure dropping rate inside fracture and matrix, which is far from the real case. Clarkson and Williams-Kovacs (2013a) extended RTA method designed for multi-phase flow in CBM to analyze early two-phase flowback data in cylindrical-source MFHWs. They coupled pseudo-steady state boundary dominated equation with MBE to develop a two-phase model for fracture flow (Williams-Kovacs et al., 2015). In order to model two-phase formation linear flow towards fracture, Clarkson and Qanbari (2016a) developed a semi-analytical model for the multi-phase flow of MFHWs in tight reservoirs using Dynamic Drainage Area (DDA) concept adopted
from welltest analysis (Shahamat et al., 2014). This concept was first applied to coal-bed methane (CBM) wells (Clarkson and Qanbari, 2016b) and extended to analyze multi-phase flowback period for tight oil reservoirs (Clarkson et al., 2016). Later, the DDA flowback model was modified by including fracture propagation before flowback and fracturing liquid leak-off during stimulation (Clarkson et al., 2017). However, fluid influx from the matrix is only reflected in material balance.

Recent studies have included complex fracture networks and fracture closure in flowback modeling. Jia et al. (2017) modeled flow through complex fracture networks in shale gas reservoirs for two-phase flowback using a hybrid numerical and analytical model. Yang et al. (2017) developed a semi-analytical LGP model for MFHWs in shale gas reservoir, which considered both gas macroscopic transport mechanism and complex fracture network geometries. However, this model can only be applied to forward simulation not fracture properties inversion. Fu et al. (2018) applied decline curve analysis to determine initial fracture volume and used fracture compressibility to evaluate fracture volume loss during flowback. Hossain et al. (2019) developed a post-flowback water production model to estimate the average fracture volume and observed a significant decrease in fracture volume during flowback by analyzing the field case. Jones and Blasingame (2019) proposed hyperbolic and modified-hyperbolic models to empirically history match and forecast total liquid productivity index and bottomhole pressure during flowback.

Flowing material balance method (FMB) has been widely used in flowback modeling (Abbasi et al., 2014, Clarkson and Williams-Kovacs, 2013a, Clarkson and Williams-Kovacs, 2013b, Jia et al., 2018). The FMB method could be traced back to the work by Palacio and Blasingame (1993), in which they developed a single-phase gas BDF model in a radial system and proposed material balance pseudo time to model variable rate production based on a constant rate (pseudo-steady state) solution. Agarwal et al. (1998) rearranged the BDF solution to the pressure normalized rate form. Clarkson et al. (2008) extended the single-phase FMB method to two-phase gas flow in CBM reservoirs by modifying the definition of pseudopressure and total compressibility
in pseudotime. However, their approach has many assumptions; for example, the production of CBM well is dominated by gas and total compressibility is approximated by gas compressibility to apply the Blasingme’s material balance pseudotime. Later, Clarkson et al. (2012) generated a two-phase FMB model and a workflow for horizontal CBM wells. Sun et al. (2018) considered pressure-dependent permeability into the two-phase FMB model. Recently, Shahamat and Clarkson (2018) developed a general FMB model for multiphase (oil, gas, and water) flow by redefining multiphase pseudopressures. However, FMB still approximates total compressibility with gas compressibility. Behmanesh et al. (2018b) relaxed the total compressibility assumption and derived a new generalized FMB equation based on the governing equation for total flowrate.

This paper presents semi-analytical models for gas and water flow through fractures by incorporating a pressure-dependent permeability, gas influx from matrix, and an MBE to obtain average pressure in fractures. A workflow is proposed to calculate fracture properties and evaluate fracture closure by combining water flowback model and gas flowback model. The flowback models, MBE, and the workflow are validated against numerical simulations and applied to an MFHW drilled in Marcellus Shale in Pennsylvania, USA. Accordingly, the contributions of this work are in: (1) development of a two-phase gas FMB equation for the fracture network in which the assumption of approximating total compressibility with gas compressibility is relaxed, (2) development of an MBE by considering the variable inflow pressure in the fracture, and (3) proposing a workflow for calculation of fracture half-length, fracture permeability, and permeability modulus, as well as evaluation of fracture closure using flowback data.

3.3 Theory and Methods

This section describes two-phase semi-analytical models to analyze water-phase and gas-phase flowback data. First, the conceptual model and flow regime analysis are introduced. Then,
mathematical formulations are presented. Last, a workflow is proposed that reconciles water-phase and gas-phase flowback data to evaluate hydraulic fracture properties and its closure.

3.3.1 Conceptual Model

Recently, we developed a 1-D two-phase model for the flow of gas and water in fracture toward a wellbore to analyze early-time flowback data (Zhang and Emami-Meybodi, 2020). We extend our previous work by considering two-phase water and gas flow in the fracture and single-phase gas influx from the matrix. The conceptual model is shown in Figure 3-1 and is comparable to that assumed by Jia et al. (2018).

![Figure 3-1: Schematic of the conceptual model for flowback depicting water flow through hydraulic fracture and gas flow through fracture and matrix.](image)

The main assumptions are as follows:

- Gas sources from the initial gas in fracture and matrix contribution due to the influx. Water only sources from the fracture (Clarkson and Williams-Kovacs, 2013a) and water
production from matrix is negligible due to the low mobility. Water saturation in the matrix is considered to be constant, which is a reasonable assumption during the flowback period (Williams-Kovacs and Clarkson, 2016).

- Water and gas flow in both fracture and matrix follow Darcy’s law.
- Fracture is an elastic porous medium and is assumed more compressible than matrix. Fracture permeability and porosity are pressure-dependent variables, whereas matrix permeability and porosity are constants. All the hydraulic fractures have the same attributes, i.e. length, width, and permeability.
- While linear, elliptical, and radial flow may be observed for different fracture geometries, which are controlled by $x/h$ (Al-Kobaisi et al., 2004), we only considered linear flow in the fracture.
- Fracture width and half-length are assumed to be constant during flowback period. However, the effect of fracture closure on production is considered through variation of fracture permeability quantified by permeability modulus.
- Water formation volume factor, water viscosity, gas compressibility, and gas viscosity are pressure-dependent variables. Water compressibility is constant.
- The variables used in the fracture model are evaluated at average fracture pressure. The variables used in the matrix model are evaluated at average pressure within the distance of investigation in the matrix.
- The gravity segregated relative permeability curves (Clarkson, 2012, Kanfar et al., 2014) are valid for the fracture.
3.3.2 Flow Regime Analysis

As mentioned above, water is the dominant flowing phase during the early flowback period and only flows inside fracture. As shown in Figure 3-2a, three flow regimes of water are expected to occur in the fracture: FR1_W single-phase water infinite acting linear flow (IALF), FR2_W two-phase water IALF, and FR3_W two-phase water boundary dominated flow. FR1_W appears in the very early time of flowback when the fracture is saturated with mostly water, i.e., high water saturation. In water diagnostic plot, FR1_W can be detected by a half-slope. Theoretically, the corresponding data for FR1_W could be used to estimate fracture permeability, but the short duration of this regime sometimes restricts the feasibility of using a single-phase model. If there is any gas left inside fractures after the stimulation, FR2_W would be the first flow regime to occur. Although a half-slope trend can be observed in the two-phase water diagnostic plot (Figure 3-2b), analyzing of FR2_W is impractical due to few numbers of data. When the effect of pressure drop reaches the fractures boundaries, FR3_W is observed with a unit-slope straight line in the two-phase water diagnostic plot (Figure 3-2b). Therefore, water flow in our conceptual model is always 1-D flow in spite of the complex gas flow, which is the coupling of matrix influx with the flow in the fracture. Here we used the two-phase diagnostic proposed by Zhang and Emami-Meybodi (2020), which is based on multiphase pseudopressure and pseudotime definitions. As pointed out by Clarkson et al. (2019), using single-phase diagnostic plots to identify flow regimes in multiphase flow systems may lead to erroneous results.
The behavior of gas production over time is different as compared with that of water. We generated the gas-phase diagnostic plot in Figure 3-3b, where the definitions of pseudo variables are listed in Zhang and Emami-Meybodi (2018) but are evaluated at average pressure in the distance of investigation in the matrix. The three flow regimes are expected to occur: gas bilinear flow (FR1_G), gas formation IALF flow (FR2_G), and gas formation boundary dominated flow (FR3_G). FR1_G corresponds to FR2_W and occurs at the beginning of production during which both water (in fractures) and gas (in fractures and influx from the matrix) flow toward wellbore. As water is produced from fracture, gas flow becomes dominant and can be identified by one-fourth slope in the gas-phase diagnostic plot. However, single-phase gas bilinear flow model is not accurate enough to extract fracture permeability from FR1_G data due to the influence of water flow in the fracture. When the effect of pressure reaches the boundary of fractures, FR2_G will appear with a half-slope (see Figure 3-3b), which usually takes a long time in shales due to the extremely low matrix permeability. As pressure further propagates in the matrix and reaches the symmetric line between two fractures, FR3_G appears and will show a unit-slope straight line in the diagnostic plot. Numerous studies have investigated gas flow in the matrix during FR2_G and FR3_G to develop formation linear flow and boundary dominated flow models (Nobakht and

3.3.3 Mathematical Models

Based on the flow regime analysis, we adopted the two-phase water BDF model developed by Zhang and Emami-Meybodi (2020) to analyze water-phase flowback data corresponding to FR3_W. For gas flow, we focused on FR2_G and developed a two-phase gas FMB to analyze gas-phase flowback data.

3.3.3.1 Two-phase Water Flowback Model

As discussed by Zhang and Emami-Meybodi (2020), constant BHP model has more error than the constant rate model, because the constant BHP case has stronger pressure and saturation
gradient in the fracture (i.e., the governing equation is more nonlinear). The stronger nonlinearity causes the assumption of applying pseudotime invalid for constant BHP model. Hence, we used constant rate model to analyze water-phase flowback data during FR3_W:

$$RNP_w = m_w t_{p,w} + b_w,$$  \hfill (3-1)

$$m_w = \frac{2.807 B_{wi} \mu_{wi}}{w_f h x_f \phi_{fi}},$$  \hfill (3-2)

$$b_w = \frac{147.82 x_f B_{wi} \mu_{wi}}{w_f h k_{fi}},$$  \hfill (3-3)

where, $RNP_w = (p_{pw} - p_{pwf})/q_w$ is the rate normalized pseudopressure for water, $q_w$ is the water-phase flowrate from the whole fracture at surface condition, $B_{wi}$ is the water formation volume factor evaluated at initial pressure, $\mu_{wi}$ is the water viscosity at initial pressure, $w_f$ is the fracture width, $h$ is the fracture height, $\phi_{fi}$ is the initial fracture porosity, $x_f$ is the fracture half-length, $k_{fi}$ is the initial fracture permeability, and $m_w$ and $b_w$ are the slope and intercept of $RNP_w$ vs. $t_{p,w}$ plot, respectively.

The definition of water-phase pseudopressure $p_p$ and water-phase pseudotime $t_{p,w}$ are given by:

$$p_p = \frac{\mu_{wi} B_{wi}}{k_{fi}} \int_{p_b}^{p} \frac{k_f(p) k_{rw}(s_w)}{\mu_w(p) B_w(p)} dp,$$  \hfill (3-4)

$$t_{p,w} = \frac{\phi_{fi}}{k_{fi}} \int_0^T \frac{k_{rw}(s_w) k_f(\bar{p}_f)}{s_w \mu_w(\bar{p}_f) \phi_f(\bar{p}_f) \left( c_w + c_f \frac{1}{s_w} \frac{d s_w}{d \bar{p}_f} \right)} d \tau,$$  \hfill (3-5)

where $k_{rw}$ is the water-phase relative permeability, $\bar{s}_w$ is the average water saturation in the fracture, $\bar{p}_f$ is the average fracture pressure, $p_b$ is the base pressure for integral, $c_w$ is the water compressibility, $c_f$ is the fracture compressibility.

Fracture permeability $k_f$ and porosity $\phi_f$ are pressure-dependent variables (Pedrosa Jr, 1986):

$$k_f = k_{fi} f(p) = k_{fi} \exp[-\gamma (p_i - p)],$$  \hfill (3-6)

$$\phi_f = \phi_{fi} g(p) = \phi_{fi} \exp[-C_f (p_i - p)],$$  \hfill (3-7)

where $\gamma$ is the permeability modulus.
Applying superposition principle (Dake, 1983), the constant rate flowback model can be extended to variable rate case by replacing pseudotime in eq (3-1) with superposition pseudotime:

\[ t_{\text{sppt},w} = \sum_{j=1}^{N} \left( \frac{(q_{w,j}-q_{w,j-1})(t_{p,w,N}-t_{p,w,j-1})}{q_{w,N}} \right), \]  

(3-8)

where \( q_{w,j} \) is the water-phase flow rate at the surface condition in the \( j^{th} \) flow rate interval, \( t_{p,w,j} \) is the water-phase pseudotime at the \( j^{th} \) flow rate interval, and \( t_{\text{sppt},w} \) is the water-phase superposition pseudotime.

With the two-phase water flowback model, we can generate \( RNP_w \) and the derivative of \( RNP_w \) (\( RNP_w = dRNP_w / dt_{\text{sppt},w} \)) vs. \( t_{\text{sppt},w} \) in log-log scale as a diagnostic plot to identify FR3_W regime (Zhang and Emami-Meybodi, 2020). Although the heterogeneity in fracture half-length and permeability will affect the transition from FR2_W to FR3_W, it will not impact the identification of FR3_W, which always shows a unit-slope straight line when flow in the fracture reaches the outer boundary. \( RNP_w \) vs. \( t_{\text{sppt},w} \) in normal scale can be generated as the water-phase specialty plot to calculate \( x_f \) and \( k_{fi} \) with the slope \( m_w \) and intercept \( b_w \) of the straight line part for general production conditions:

\[ x_{f,w} = \frac{2.807B_{wi}\mu_{wi}}{m_{w}w_{f}h\phi_{fi}}, \]  

(3-9)

\[ k_{fi,w} = \frac{147.82x_{f,w}B_{wi}\mu_{wi}}{b_{w}w_{f}h}. \]  

(3-10)

### 3.3.3.2 Two-phase Gas Flowback Model

Gas flowback model is different from water due to the pressure-dependent gas properties and gas influx from matrix. Based on the conceptual model in Figure 3-1, the gas-phase material balance equation for an infinitesimal element in the fracture is given by:

\[ q_{gf}\rho_{gf}\left|_{x+dx} - q_{gf}\rho_{gf}\right|_{x} + q_{gm}\rho_{gm}\left|_{y=0} = \frac{d}{dt}(\Delta x w_f h \phi_f S_g \rho_{gf}), \right. \]  

(3-11)
where $x$ is the location in the fracture, $y$ is the location in the matrix, $\Delta x \, w_f h$ is the control volume, $q_{gf}$ is the gas-phase downhole flowrate at a specific location $x$ in the fracture, $q_{gm}$ is the gas-phase downhole inflow rate at a specific location $y$ from matrix, $s_g$ is the gas saturation in the fracture, $\rho_{gf}$ is the gas-phase density in the fracture, and $\rho_{gm}$ is the gas-phase density in the matrix.

We neglected the effect of fracture pressure gradient on gas influx and assumed that gas flows uniformly into the fracture. Hence the gas influx term in eq (3-11) can be included in accumulation term on the right-hand side to treat as a function of time. Dividing both sides of eq (3-11) by $\Delta x$, the following equation can be obtained:

$$\frac{\partial (q_{gf} \rho_{gf})}{\partial x} = \frac{\partial}{\partial t} \left( w_f h \, \phi_f \, S_g \rho_{gf} - q_{gm} \rho_{gm} \bigg|_{y=0} \right). \tag{3-12}$$

Applying gas Equation of State $\rho_{gf} = \frac{p_f M}{Z R T}$, Darcy’s gas flowrate along the fracture $q_{gf} = w_f h \frac{k_{rg} k_f}{\mu_{gf}} \frac{\partial p_f}{\partial x}$, and inflow rate from matrix $q_{gm} = 2 h \Delta x \frac{k_m}{\mu_{gm}} \frac{\partial p_m}{\partial y}$, eq (3-12) can be expanded with chain rule by introducing effective gas-phase compressibility in the fracture:

$$\frac{\partial}{\partial x} \left( \frac{p_f k_{rg} k_f}{\mu_{gf}} \frac{\partial p_f}{\partial x} \right) = \frac{p_f}{Z} \phi_f c_{eff,g} \frac{\partial p_f}{\partial t}, \tag{3-13}$$

$$c_{eff,g} = \left[ s_g c_{gf} + s_g c_f + \frac{\partial s_g}{\partial p_f} \frac{1}{\phi_f c_{eff,g}} \frac{\partial p_f}{\partial t} \left( \int_0^t \frac{2}{w_f} \frac{k_m}{\mu_{gm}} \frac{\partial p_m}{\partial y} \rho_{gm} \, dt \right) \right], \tag{3-14}$$

where $\mu_{gf}$ is the gas viscosity in the fracture, $\mu_{gm}$ is the gas viscosity in the matrix, $p_f$ is the fracture pressure, $p_m$ is the matrix pressure, $k_m$ is the matrix permeability, $Z$ is gas deviation factor, $T$ is the temperature, $R$ is the ideal gas constant, $M$ is the mass of gas, $c_{eff,g}$ is the effective gas-phase compressibility, and $c_{gf}$ is the gas compressibility.

First, we derived gas flowback model under constant rate condition and then extended it to the variable rate condition. Equation (3-13) is subject to the following initial and boundary conditions:

$$p_f(x, 0) = p_i, \frac{\partial}{\partial x} \left( \frac{p_f M}{Z R T} \frac{k_{rg} k_f}{\mu_{gf}} \frac{\partial p_f}{\partial x} \right) \bigg|_{x=0} = \frac{p_{sc} M}{R T s_c} q_g, \frac{\partial p_f}{\partial x} \bigg|_{x=x_f} = 0, \tag{3-15}$$
where \(q_g\) is the gas-phase flow rate from the whole fracture at surface standard condition, \(Z_{sc}, p_{sc},\) and \(T_{sc}\) are gas deviation factor, pressure, and temperature at surface standard condition.

Considering pressure-dependent fracture permeability and porosity in eqs (3-6) and (3-7), we defined gas-phase pseudopressure \(m(p)\) and pseudotime \(t_{p,g}\) to linearize the governing equation by evaluating pressure-dependent parameters at the average fracture pressure and saturation:

\[
m(p) = \frac{\mu g_i Z_i}{k_f} \int^p_{p_b} \frac{p_f k_f f(p) k_{rg}(p)}{\mu_{gf}(p) Z(p)} dp,
\]

\[
t_{p,g} = \left(\mu g c_{eff,g}\right)_i \int^t_0 \frac{k_{rg}(\bar{p}) f(\bar{p})}{\mu_{g}(\bar{p}) c_{eff,g}(\bar{p})} d\tau.
\]

By applying pseudopressure and pseudotime, the linearized governing equation with initial and boundary conditions are:

\[
\frac{\partial^2 m(p_f)}{\partial x^2} = \frac{1}{\eta_g} \frac{\partial m(p_f)}{\partial t_{p,g}},
\]

\[
m(p_f(x, 0)) = m(p_i), \quad \frac{\partial m(p_f)}{\partial x} \bigg|_{x=0} = \frac{q_g \mu g_i p_{sc} Z_i T_{sc}}{2 \mu h w_f k_f T_{sc}} \frac{\partial m(p_f)}{\partial x} \bigg|_{x=x_f} = 0,
\]

where \(\eta_g = \frac{k_{fi}}{(\phi_f c_{eff,g} \mu_g)_i}\).

The analytical solution of eq (3-18) subject to eq (3-19) is given by Miller (1962):

\[
m(p_f) = m(p_i) - \frac{q_g \mu g_i p_{sc} x_f Z_i T_{sc}}{2 \mu h w_f k_f T_{sc}} \left[ \frac{1}{3} + \frac{x^2}{2 x_f^2} - \frac{x}{x_f} + \frac{\eta w t_{p,g}}{x_f^2} \right] \sum_{n=1}^{\infty} \exp \left( \frac{(n\pi)^2 \eta w t_{p,g}}{x_f^2} \right) \cos \left( \frac{n\pi x}{x_f} \right).
\]

Considering the long-term approximation for boundary dominated flow (Wattenbarger et al., 1998), the volumetric average pseudopressure can be obtained by taking an integration over eq (3-20) with respect to \(x\):
\[ \bar{m}(p_f) = \frac{1}{x_f} \int_{0}^{x_f} m(p_f) \, dx \approx m(p_i) - \frac{z_i T}{T_{sc}} \frac{500.2 \, p_{sc} q_g}{h w_f x_f (\phi_f c_{eff, g})} \, t_{p,g}, \] (3-21)

where \( \bar{m}(p_f) \) is the average pseudopressure in the fracture.

Assuming that the average pseudopressure can be approximated by pseudo-average-pressure \( m(\bar{p}_f) \), eq (3-21) can be rearranged to relate gas-phase pseudotime with pseudopressure drop:

\[ t_{p,g} = \frac{T_{sc} h w_f x_f (\phi_f c_{eff, g})}{z_i T} \left[ m(p_i) - m(\bar{p}_f) \right]. \] (3-22)

Substituting \( x=0 \) and \( p=p_{wf} \) into eq (3-20) and taking the long-term approximation for boundary dominated flow (Wattenbarger et al., 1998), we can obtain the \( RNP \) form solution in field unit:

\[ RNP_g = m_g t_{p,g} + b_{pss}, \] (3-23)

\[ m_g = \frac{500.2 \, p_{sc} z_i T}{V_f T_{sc} (c_{eff, g})}, \] (3-24)

\[ b_{pss} = \frac{26340.7 \mu_g p_{sc} x_f}{h w_f k_f} \frac{z_i T}{Z_{sc T_{sc}}}, \] (3-25)

where, \( RNP_g = (m(p_i)-m(p_{wf}))/q_g \) is the rate normalized pseudopressure for gas, \( V_f = x_f h \phi_f \) is the initial pore volume of half fracture.

Equation (3-23) can be rearranged to pressure normalized rate form:

\[ \frac{q_g}{m(p_i)-m(p_{wf})} = -\frac{1}{V_f b_{pss}} \frac{500.2 \, p_{sc} z_i T}{T_{sc} c_{eff, g}} t_{p,g} q_g + \frac{1}{b_{pss}}. \] (3-26)

Substituting eq (3-22) into (3-26), we arrive at the FMB equation for two-phase gas flow in the fracture:

\[ \frac{q_g}{m(p_i)-m(p_{wf})} = -\frac{1}{V_f b_{pss}} \frac{[m(p_i)-m(\bar{p}_f)]V_f}{[m(p_i)-m(p_{wf})]} + \frac{1}{b_{pss}}. \] (3-27)

Similar to the two-phase water model, the superposition principle can be applied to obtain the analytical solution of eq (3-18) and eq (3-19) for the variable gas-phase flowrate \( q_g \):
\[ m(p) = m(p_i) - \frac{q_{g,N} \mu_{gi} p_{sc} x_f Z_i T}{2h_w k_{fi} T_{sc}} \left[ \frac{1}{3} + \frac{x^2}{2x_f^2} - \frac{x}{x_f} + \frac{\eta_{w t_{sppt,g}}}{x_f^2} \right] - LT, \quad (3-28) \]

\[ t_{sppt,g} = \sum_{j=1}^{N} \left[ \frac{(q_{g,j} - q_{g,j-1})(t_{p,g,N} - t_{p,g,j-1})}{q_{g,N}} \right], \quad (3-29) \]

\[ LT = \frac{\mu_{gi} p_{sc} x_f Z_i T}{h_w k_{fi} T_{sc}} \sum_{j=1}^{N} \left[ \left( q_{g,j} - q_{g,j-1} \right) \sum_{n=1}^{\infty} \frac{\exp \left( \frac{-(n\pi)^2 \eta_w}{x_f^2} (t_{p,g,N} - t_{p,g,j-1}) \right)}{n \pi x_f^2} \cos \left( \frac{n \pi x_f}{x_f} \right) \right], \quad (3-30) \]

where \( q_{g,j} \) is the gas-phase flow rate at the surface condition in the \( j^{th} \) flow rate interval, \( t_{p,g,j} \) is the gas-phase pseudotime at the \( j^{th} \) flow rate interval, and \( t_{pppt,g} \) is the gas-phase superposition pseudotime.

Equation (3-28) extends the two-phase gas flowback model under constant rate condition to variable rate by replacing \( t_p \) and \( q_g \) in equation (3-21), (3-23), and (3-26) with \( t_{pppt,g} \) and \( q_{g,N} \). Accordingly, using the normalized cumulative production \( G_{pN} = \frac{[m(p)-m(p_j)] V_{fi}}{[m(p_i)-m(p_{wf})]} \) vs. pressure normalized rate \( PNR = \frac{q_{g,N}}{m(p_i)-m(p_{wf})} \) plot (i.e., gas-phase specialty plot or FMB plot), \( x_f \) and \( k_{fi} \) can be obtained from the slope \( m_{FMB} \) and intercept \( b_{FMB} \) of the straight line, which passes through the later portion of the FMB plot:

\[ x_{f,g} = -\frac{b_{FMB}}{m_{FMB} h \phi_{fi}}, \quad (3-31) \]

\[ k_{fi,g} = 26340.7 \frac{b_{FMB} \mu_{gi} p_{sc} x_f z_{f} Z_i T}{h_w z_{sc} T_{sc}}. \quad (3-32) \]

To determine \( x_f \) and \( k_{fi} \), the FMB plot \( PNR vs. G_{pN} \) needs average pressure in the fracture, whose calculation requires \( x_f \) itself. Therefore, an iterative procedure is required as discussed in the next section. We adopted the systematic procedure proposed by Shahamat and Clarkson (2018) to conduct the FMB analysis.
3.3.4 Analysis Technique

An analysis technique is proposed that includes the calculation of average pressure in the fracture and a workflow to calculate $x_f$, $k_{fi}$, and $\gamma$. In this study, fracture closure is quantified by $k_{fi}$ and $\gamma$ while length and width are kept fixed. In other words, changes in fracture dimensions during fracture closure is considered in $\gamma$. This is reasonable because fracture closure is usually characterized by fracture conductivity, which is the product of fracture width and fracture permeability. Note that the fracture closure is directly dependent on average fracture pressure, and the average pressure is affected by production rates.

3.3.4.1 Calculation of Average Pressure in the Fracture

In order to generate the specialty plot, we calculated pseudotime based on the average fracture pressure $\bar{p}_f$ at each time. Williams-Kovacs and Clarkson (2016) modified the MBE by King (1993) to calculate $\bar{p}_f$ and considered gas influx from matrix using the formation linear flow solution by Wattenbarger et al. (1998). They set the flowing pressure of inflow model to the average pressure in the fracture as an approximation, although the inflow model by Wattenbarger is derived for constant BHP condition. In this study, we modified their MBE by using the rigorous solution of gas influx under variable BHP condition.

Wattenbarger et al. (1998) derived the solution of gas influx during the formation IALF under constant BHP condition in field unit:

$$q_{g,inf} = \frac{[m_m(p_i)-m_m(p_{wf})]x_f\sqrt{k_m}h}{f_{cp}^315.4\sqrt{T}} \sqrt{\frac{(\phi_m\mu g \xi_m)}{i}}. \tag{3-33}$$
where \( q_{g,\text{inf}} \) is the flowrate of gas influx at surface condition, \( m_m(p) = 2 \int_0^p \frac{p}{\mu_g} dp \) is the real gas pseudopressure, \( c_m = c_{gm} + c_m \) is the matrix total compressibility, \( c_{gm} \) is the gas compressibility in the matrix, and \( c_m \) is the formation compressibility in the matrix.

\( f_{cp} \) is the correction factor proposed by Ibrahim and Wattenbarger (2006) and it is a function of dimensionless drawdown parameter \( D_D \):

\[
f_{cp} = 1 - 0.0852D_D - 0.0857D_D^2,
\]

\[
D_D = \frac{[m_m(p_i) - m_m(p_w)]}{m_m(p_i)}.
\]

In our conceptual model, the flowing pressure of the formation IALF model is the average pressure in the fracture, which is not a constant but changing with time. The variation of flowing pressure can be treated by a stepwise function as shown in Figure 3-4a. With the constant matrix water saturation \( s_{wm} \), flowrate \( q_{g,\text{inf}} \) and cumulative production \( G_{g,\text{inf}} \) of gas influx at surface condition can be obtained using Duhamel’s principle (Özisik et al., 1993):

\[
q_{g,\text{inf}} = \frac{[m_m(p_i) - m_m(p_b)] x_f \sqrt{k_m h} \left(1-s_{wm}\phi_m \mu_g c_{tm}\right)}{315.4 \tau} \frac{\Delta \zeta(\tau)_j}{f_{cp,j} \sqrt{(t-\tau_j)}},
\]

\[
G_{g,\text{inf}} = \frac{[m_m(p_i) - m_m(p_b)] x_f \sqrt{k_m h} \left(1-s_{wm}\phi_m \mu_g c_{tm}\right)}{0.3154 \tau} \frac{\Delta \zeta(\tau)_j}{f_{cp,j} \sqrt{(t-\tau_j)}} d\tau,
\]

\[
\zeta(\tau) = \frac{m_m(p_i) - m_m(p_f(\tau))}{m_m(p_i) - m_m(p_b)},
\]

\[
\Delta \zeta(\tau)_j = \zeta(\tau)_{j+} - \zeta(\tau)_{j-}.
\]

where the variations of \( \zeta(\tau) \) and \( \Delta \zeta(\tau)_j \) are treated by stepwise functions as shown in Figure 3-4b and 4c, respectively.

With the calculated gas influx, the average pressure in the fracture can be solved iteratively from the modified MBE:

\[
G_p = \frac{2V_{fi} s_{gi}}{B_{gi}} - \frac{2V_{fi} [1-c_f(p_i-p_f)] s_g}{B_g} + G_{g,\text{inf}},
\]
\[ 
\bar{B}_g = \frac{p_{sc} Z_T}{T_{sc} \bar{p}_f}, 
\]

where \( G_p \) is the cumulative gas production at the surface condition, \( B_g \) is the gas formation volume factor, and \( \bar{S}_g \) is the average gas saturation in the fracture which is calculated using the simplified MBE proposed by King (1993):

\[ 
\bar{S}_g = 1 - \bar{s}_w = 1 - \frac{(W_{fi} - 5.615W_p)\bar{B}_w}{2V_f[1 - \varepsilon_f(p_i - \bar{p}_f)]}, 
\]

\[ 
\bar{B}_w = B_{wl}[1 + c_w(p_i - \bar{p}_f)], 
\]

where \( W_{fi} \) is the initial water storage in the fracture at the surface condition, and \( W_p \) is the cumulative water production at the surface condition.

---

**3.3.4.2 Workflow for \( x_f, k_f, \) and \( \gamma \) Calculation**

To obtain \( x_f \) and \( k_f \) using the developed flowback models, the average pressure in the fracture is calculated based on MBE, which requires \( x_f \) to be known. Thus, an iterative procedure is needed. Here, we present a workflow that calculates fracture properties and evaluates fracture closure using water-phase and gas-phase flowback data. Figure 3-5 shows the workflow to estimate fracture attributes iteratively. The workflow follows nine steps:
1. Guess an initial value for \( x_f \) based on the field experience and assume \( \gamma = 10^{-3} \) psi\(^{-1} \), which is an upper bound based on experimental data (Ozkan et al., 2010),

2. Calculate average pressure in the fracture \( \bar{p}_f \) using eq (3-40) and the Newton-Raphson technique,

3. Evaluate \( t_{p,w} \) and \( p_p \) for water-phase model and \( m(p) \) for gas-phase flow model at each \( \bar{p}_f \),

4. Create two-phase diagnostic plot of water, \( t_{p,w} \) vs. \( DRNP_w \) in log-log scale, to identify water-phase BDF regime (FR3_W), as well as a specialty plot, \( t_{p,w} \) vs. \( RNP_w \) in normal scale, to calculate \( x_{f,w} \) and \( k_{fi,w} \) from eqs (3-9) and (3-10),

5. Generate FMB plot \( G_{pN} \) vs. \( PNR \) in normal scale and pass a straight line through the later portion to calculate \( x_{f,g} \) and \( k_{fi,g} \) from eqs (3-31) and (3-32). The gas-phase diagnostic plot shown in Figure 3b can assist to identify gas-phase fracture depletion flow regime (FR2_G and FR3_G) with the half-slope straight line,

6. Update \( x_{f,new} = (x_{f,w} + x_{f,g})/2 \),

7. Repeat step 2 to 6 until the relative error between \( x_{f,new} \) and \( x_{f,old} \) becomes less than an acceptable tolerance \( \epsilon_{xf} \),

8. Update \( \gamma \) with another guess in the range of \( 10^{-4} \) to \( 10^{-3} \) psi\(^{-1} \), and repeat step 2 to 7 until the relative error between \( k_{fi,w} \) and \( k_{fi,g} \) becomes less than an acceptable tolerance \( \epsilon_{k_f} \),

9. Report \( x_f \), \( k_f \), and \( \gamma \).
3.4 Model Validation and Results

In this section, first, a numerical simulation case is generated based on the reservoir and fracture properties in Marcellus Shale to test the accuracy of the proposed flowback models, average pressure calculation method, and the workflow. Next, the proposed models and workflow are applied to an MFHW in Marcellus Shale to characterize hydraulic fractures. The analysis results are compared with long-term production data analysis by history matching with the IHS Harmony (IHS, 2019) to have a quantitative understanding of temporal changes in fracture properties during the production life of a well.
3.4.1 Model Validation

We constructed a 2-D two-phase numerical model using IMEX—CMG to generate synthetic data. The relative permeability curve to water and gas in the fracture was assumed to be straight lines (Wei and Zhang, 2010). We chose a logarithmic grid size distribution along the fracture and matrix to accurately simulate flow near fracture and wellbore. Considering that gas influx usually becomes significant during late flowback period, we conducted the simulation for 200 days, which is long enough to observe the flow regimes depicted in Figure 3-2 and Figure 3-3. The permeability modulus in the fracture is set to be $3\times10^{-4}$ psi$^{-1}$ according to the laboratory measurements (Ozkan et al., 2010). The input data used in the numerical simulation are given in Table 3-1.

Table 3-1: Summary of input data used in numerical simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial pressure, $p_i$ (psi)</td>
<td>5000</td>
</tr>
<tr>
<td>Reservoir temperature, $T$ ($^\circ$F)</td>
<td>160</td>
</tr>
<tr>
<td>Specific gravity, $SG_a$</td>
<td>0.65</td>
</tr>
<tr>
<td>Water compressibility, $c_w$ (psi$^{-1}$)</td>
<td>$3\times10^{-6}$</td>
</tr>
<tr>
<td>Initial fracture porosity, $\phi_{f_i}$ (%)</td>
<td>50</td>
</tr>
<tr>
<td>Matrix porosity, $\phi_m$ (%)</td>
<td>5</td>
</tr>
<tr>
<td>Fracture initial permeability, $k_{f_i}$ (md)</td>
<td>300</td>
</tr>
<tr>
<td>Matrix permeability, $k_m$ (md)</td>
<td>$1\times10^{-5}$</td>
</tr>
<tr>
<td>Fracture compressibility, $c_f$ (psi$^{-1}$)</td>
<td>$2\times10^{-5}$</td>
</tr>
<tr>
<td>Matrix compressibility, $c_m$ (psi$^{-1}$)</td>
<td>$1\times10^{-6}$</td>
</tr>
<tr>
<td>Reservoir thickness, $h$ (ft)</td>
<td>300</td>
</tr>
<tr>
<td>Horizontal wellbore length, $L_w$ (ft)</td>
<td>10000</td>
</tr>
<tr>
<td>Number of fractures</td>
<td>125</td>
</tr>
<tr>
<td>Fracture spacing, $L_f$ (ft)</td>
<td>80</td>
</tr>
<tr>
<td>Fracture width, $w_f$ (ft)</td>
<td>0.02</td>
</tr>
<tr>
<td>Initial water formation volume factor, $B_{wt}$ (bbl/STB)</td>
<td>1.03</td>
</tr>
<tr>
<td>Initial fracture water saturation, $s_{wt}$ (%)</td>
<td>70</td>
</tr>
<tr>
<td>Matrix water saturation, $s_{wm}$ (%)</td>
<td>0</td>
</tr>
<tr>
<td>Initial water viscosity, $\mu_w$ (cp)</td>
<td>0.3</td>
</tr>
<tr>
<td>Initial water density, $\rho_w$ (lb/ft$^3$)</td>
<td>62</td>
</tr>
<tr>
<td>Fracture half-length, $x_f$ (ft)</td>
<td>500</td>
</tr>
<tr>
<td>Fracture permeability modulus, $\gamma$ (psi$^{-1}$)</td>
<td>$3\times10^{-4}$</td>
</tr>
</tbody>
</table>
We simulated two cases with different production conditions so that both constant rate and variable rate models can be validated. In Case 1, gas is produced at a constant flowrate of 625 Mscf/day while in Case 2 water is produced at a constant flowrate of 125 bbl/day.

First, we validated the modified MBE to calculate average pressure in the fracture with the known \( x_f \) as an input parameter. A comparison between the analytical approach and numerical simulation is shown in Figure 3-6. The results show a perfect match for Case 1 and Case 2, which validate the accuracy of the proposed MBE method.

![Figure 3-6](image)

Figure 3-6: Comparison of average fracture pressure calculated from material balance equation and numerical simulation for (a) Case 1 under constant gas rate and (b) Case 2 under constant water rate.

Here, the accuracy of the two-phase flowback model is tested against the production data obtained from the numerical simulation by considering \( \gamma \) as a known parameter. Two-phase water flowback model is validated first. Figure 3-7 and Figure 3-8 show the two-phase diagnostic plot for water and specialty plot for Case 1 and Case 2, respectively. Superposition pseudotime was used in Case 1 due to the variable flowrate of water. The diagnostic plots for both numerical cases show a half-slope straight line and a unit-slope straight line, which indicate the IALF regime (FR2_W) and the fracture BDF regime (FR3_W), respectively. By selecting the deviation point of the unit-slope straight lines, the start of water-phase BDF regime (FR3_W) was identified to be about 3.41 and 3.5 days for Case 1 and 2, respectively. Within the selected data interval, the slope
and intercept of specialty plot are extracted to calculate $x_{f,w}$ and $k_{f,w}$. The calculated results and relative error in Table 3-2 show that the estimated fracture attributes are very close to the set values and the relative errors are all less than 10%, which validate the accuracy of water-phase flowback model.

Figure 3-7: Two-phase diagnostic plot of water phase for (a) Case 1 with a constant gas rate and (b) Case 2 with a constant water rate.

Figure 3-8: Water-phase specialty plot for (a) Case 1 under constant gas rate and (b) Case 2 under constant water rate.

Here, the gas-phase flowback model is tested against numerical simulation results. We generated the gas-phase diagnostic plot presented by Zhang and Emami-Meybodi (2018) to study
the gas-phase flow regimes during flowback in Figure 3-9, in which the pseudo variables are evaluated at the average pressure in the distance of investigation of matrix obtained from the numerical simulation. The diagnostic plots for both numerical cases show a clear quarter-slope straight line and a half-slope straight line, which indicate the bilinear flow regime (FR1_G) and the formation IALF (FR2_G), respectively. By selecting the deviation point, the start of gas-phase fracture depletion regime (FR2_G and FR3_G) was identified to be about 1.8 day for Case 1 and 1.9 day for Case 2. Then, we generated the gas-phase specialty plot, as shown in Figure 3-10, to conduct FMB analysis. The slope and intercept of the straight line that passes through the late portion of the specialty plot are extracted to calculate $x_{f,g}$ and $k_{f,g}$. The calculated results and relative error given in Table 3-2 show that the evaluated fracture properties are very close to the set values and the relative errors are all less than 10%.

![Figure 3-9](image)

Figure 3-9: Two-phase diagnostic plot of gas phase for (a) Case 1 with a constant gas rate and (b) Case 2 with a constant water rate.
Table 3-2: A summary of input data used in numerical simulation and calculated properties using flowback models.

<table>
<thead>
<tr>
<th>Case #</th>
<th>Set values</th>
<th>Water-phase flowback model</th>
<th>Gas-phase flowback model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Calculated values</td>
<td>Relative error</td>
</tr>
<tr>
<td></td>
<td>$x_f$ (ft)</td>
<td>$k_f$ (md)</td>
<td>$x_f$ (ft)</td>
</tr>
<tr>
<td>1</td>
<td>500</td>
<td>300</td>
<td>499.6</td>
</tr>
<tr>
<td>2</td>
<td>500</td>
<td>300</td>
<td>499.8</td>
</tr>
</tbody>
</table>

The applicability of workflow depicted in Figure 3-5 is tested by calculating $x_f$, $k_f$, and $\gamma$ iteratively and comparing them against the set values in Case 1. The proposed workflow updates $x_f$ and $\gamma$ separately by checking the convergence of calculated $x_f$ and $k_f$ from the water-phase and gas-phase flowback data. Here the workflow is tested by setting $\gamma$ as a known parameter to see if the correct $x_f$ can be determined. An initial value of $x_f = 700$ ft is chosen to calculate average pressure and then generate specialty plots for water-phase and gas-phase flowback model. The fracture attributes are calculated from the slope and intercept of specialty plots. We compared the average of $x_f$ calculated from water and gas flowback data with $x_{f,\text{guess}}$ to check the convergence. After five iterations, the relative error meets the acceptable tolerance of 0.2%. The calculated average pressure for each iteration are shown in Figure 3-11. As given in Table 3-3, the estimated $x_f$ is very close to the set value after five iterations.
Table 3-3: Validation results of the workflow with the known $\gamma$.

<table>
<thead>
<tr>
<th>Iteration #</th>
<th>Input $x_f$, ft</th>
<th>Output $x_{fw}$, ft</th>
<th>Output $x_{fg}$, ft</th>
<th>$(x_{fw} + x_{fg})/2$, ft</th>
<th>Error of $x_{fw}$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>700</td>
<td>516.5</td>
<td>461.3</td>
<td>488.9</td>
<td>30.16</td>
</tr>
<tr>
<td>2</td>
<td>488.9</td>
<td>575.4</td>
<td>504.7</td>
<td>540.0</td>
<td>10.46</td>
</tr>
<tr>
<td>3</td>
<td>540.0</td>
<td>506.1</td>
<td>486.3</td>
<td>496.2</td>
<td>8.12</td>
</tr>
<tr>
<td>4</td>
<td>496.2</td>
<td>495.7</td>
<td>501.9</td>
<td>498.8</td>
<td>0.52</td>
</tr>
<tr>
<td>5</td>
<td>498.8</td>
<td>497.7</td>
<td>500.7</td>
<td>499.2</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Now, we test the proposed workflow by setting the known $x_f$ as input to iterate $\gamma$. The initial guess of $\gamma$ starts from $1 \times 10^{-3}$ psi$^{-1}$, which is the upper limit of the possible range ($10^{-4}$–$10^{-3}$ psi$^{-1}$). Following the workflow depicted in Figure 3-5, the calculated $k_{fw}$ and $k_{fg}$ are compared to check the tolerance. In the first iteration, the relative error of $k_{fi}$ doesn’t meet the acceptable tolerance 2.7%, thus another $\gamma$ value is picked as the new input for the second iteration. Similarly, iteration continues and for $\gamma = 3 \times 10^{-4}$ psi$^{-1}$ the tolerance is met. Table 3-4 shows that the calculated fracture properties are very close to the set values. With the calculated $k_{fi}$ and $\gamma$, we can predict the fracture permeability deterioration with the change of average fracture pressure due to fracture closure for
the long-term production. Accordingly, the proposed workflow is able to predict fracture properties and evaluate fracture closure.

Table 3-4: Validation results of the workflow with the known $x_f$.

<table>
<thead>
<tr>
<th>Iteration #</th>
<th>Input $\gamma$, psi $^1$</th>
<th>Output $x_{f,w}$, ft</th>
<th>Output $x_{f,g}$, ft</th>
<th>Error of $x_f$, %</th>
<th>Output $k_{f,w}$, md</th>
<th>Output $k_{f,g}$, md</th>
<th>Error of $k_f$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1x10^-3</td>
<td>501.2</td>
<td>503.1</td>
<td>0.39</td>
<td>336.0</td>
<td>326.2</td>
<td>2.92</td>
</tr>
<tr>
<td>2</td>
<td>7x10^-4</td>
<td>500.3</td>
<td>501.9</td>
<td>0.30</td>
<td>333.2</td>
<td>323.5</td>
<td>2.92</td>
</tr>
<tr>
<td>3</td>
<td>5x10^-4</td>
<td>499.8</td>
<td>501.0</td>
<td>0.25</td>
<td>331.4</td>
<td>321.7</td>
<td>2.93</td>
</tr>
<tr>
<td>4</td>
<td>3x10^-4</td>
<td>499.6</td>
<td>500.3</td>
<td>0.14</td>
<td>328.0</td>
<td>319.3</td>
<td>2.64</td>
</tr>
</tbody>
</table>

3.4.1 Field Validation

We applied the flowback models and workflow to a MFHW drilled in Marcellus Shale in Pennsylvania, USA, to estimate fracture properties and compared the results with long-term production data analysis to characterize the fracture closure during production. The purpose of analyzing this field case is to verify the practical use of the flowback model and to quantitatively evaluate the temporal changes of fracture half-length and permeability.

Table 3-5 shows the real field data from the Marcellus MFHW with 110 hydraulic fractures. Considering that only 40% - 60% of the fracture stages are found to contribute the production in practical stimulation job, we set the efficiency of hydraulic fracturing to be 60% for this case. Figure 3-12 shows the production history of gas flowrate, water flowrate, and bottomhole pressure for this well during flowback and long-term production period. After stimulation, the well was flowed back for 24 days and then continuously produced for 249 days without shut-in. In the first 4 days of early time flowback period, water flowrate is dominant and remains almost constant, and gas flowrate gradually increases. During the late time flowback, gas production is significant because of the gas influx from matrix, whereas water production quickly declines due to the lack
of water contribution. When it comes to long-term production, gas is dominantly produced with a negligible water flowrate.

**Table 3-5:** Real Field data from a Marcellus MFHW.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial pressure, $p_i$ (psi)</td>
<td>6580</td>
</tr>
<tr>
<td>Initial reservoir pressure, $p_{ir}$ (psi)</td>
<td>4480</td>
</tr>
<tr>
<td>Reservoir temperature, $T$ (°F)</td>
<td>140</td>
</tr>
<tr>
<td>Specific gravity, $SG_g$</td>
<td>0.6</td>
</tr>
<tr>
<td>Water compressibility, $c_w$ (psi⁻¹)</td>
<td>2.4×10⁻⁶</td>
</tr>
<tr>
<td>Initial fracture porosity, $\phi_{fi}$ (%)</td>
<td>60</td>
</tr>
<tr>
<td>Matrix porosity, $\phi_m$ (%)</td>
<td>7.75</td>
</tr>
<tr>
<td>Matrix permeability, $k_m$ (md)</td>
<td>1×10⁻³</td>
</tr>
<tr>
<td>Fracture compressibility, $c_f$ (psi⁻¹)</td>
<td>7×10⁻⁵</td>
</tr>
<tr>
<td>Matrix compressibility, $c_m$ (psi⁻¹)</td>
<td>1×10⁻⁶</td>
</tr>
<tr>
<td>Reservoir thickness, $h$ (ft)</td>
<td>80</td>
</tr>
<tr>
<td>Horizontal wellbore length, $L_w$ (ft)</td>
<td>4359</td>
</tr>
<tr>
<td>Number of fractures</td>
<td>110</td>
</tr>
<tr>
<td>Fracture spacing, $L_f$ (ft)</td>
<td>66</td>
</tr>
<tr>
<td>Fracture width, $w_f$ (ft)</td>
<td>0.04</td>
</tr>
<tr>
<td>Initial water formation volume factor, $B_{wi}$ (bbl/STB)</td>
<td>1.03</td>
</tr>
<tr>
<td>Initial fracture water saturation, $s_{wi}$ (%)</td>
<td>98</td>
</tr>
<tr>
<td>Matrix water saturation, $s_{wm}$ (%)</td>
<td>28</td>
</tr>
<tr>
<td>Initial water viscosity, $\mu_w$ (cp)</td>
<td>0.54</td>
</tr>
<tr>
<td>Initial water density, $\rho_w$ (lb/ft³)</td>
<td>62</td>
</tr>
</tbody>
</table>

**Figure 3-12:** Production history of gas flowrate, water flowrate, and bottomhole pressure for the field case during (a) flowback and (b) long-term production.
We applied the proposed workflow in Figure 3-5 to analyze flowback data. By starting the workflow with an initial guess of fracture half-length $x_f = 700$ ft and permeability modulus $\gamma = 1 \times 10^{-3}$ psi$^{-1}$, the average pressure in the fracture, two-phase diagnostic plot of water, and specialty plots for water and gas are generated during each iteration. The tolerances of the relative error between $x_{f,new}$ and $x_{f,guess}$, as well as $k_{fi,w}$ and $k_{fi,g}$ are both set to be 10%. Figure 3-13 presents the essential plots of flowback analysis after the convergence of both $x_f$ and $k_f$. Figure 3-13a shows the average fracture pressure obtained from the modified material balance equation and the average fracture permeability history during flowback which is determined based on the iterated $\gamma$ and $k_f$. Figure 3-13b provides the log-log plot of $DRNP_w$ vs. $t_{ppt,w}$ to identify FR3_W regime (Zhang and Emami-Meybodi, 2020). The half slope straight line in the diagnostic plot shows the FR2_W regime in the early time flowback period. A dominant unit-slope straight line is identified after 2.75 days flowback on the diagnostic plot, which shows the boundary dominated flow in the fracture. It can be seen that the flow regimes sequence for this well is consistent with our conceptual model in Figure 3-2, which verifies the practical use of the developed models and proposed workflow. With the identified flow regime, water and gas specialty plots are generated, Figure 3-13c and d, to calculate fracture properties based on the slope and intercept of straight line. The flowback analysis results are listed in Table 3-6.
We also analyzed the long-term production data to obtain the fracture properties in the late time. We use history matching approach to analyze the long-term production data with the single-phase gas Horizontal-Multifractured-SRV model in IHS Harmony software. Figure 3-14 shows a good match of gas flowrate and bottomhole pressure history. Although the history match result is not unique, it is still deemed good enough to yield accurate results as long as the model parameters are in acceptable ranges (Clarkson et al., 2019).
Table 3-6 summarizes the analysis results for the field case using the new flowback model and history matching approach. By comparing the flowback analysis with long-term production data analysis results, we observed a significant decrease in $x_f$ from 873 ft to 489 ft and for $k_f$ from 1227.5 md to 46.4 md during the production, which is mainly due to the fracture closure. Figure 3-14b shows that the bottomhole pressure declines slowly after 3 months of production, which is around 1200 psia. Applying the obtained $k_f$ and $\gamma$ from flowback analysis, we estimated the ultimate $k_f$ is 16.6 md when average fracture pressure reaches 1200 psia. The ultimate $k_f$ 16.6 md is slightly below the average $k_f$ 46.4 md obtained from history matching of long-term production data, which is reasonable according to eq (3-6) because the estimated ultimate fracture pressure 1200 psia is lower than the average fracture pressure during long-term production. Furthermore, the predicted ultimate $k_f$ 16.6 md falls into the permeability range between the fracture without proppant and with one layer of glass beads, which is obtained from laboratory experiments by Tan et al. (2018). Accordingly, the decrease in fracture permeability and fracture half-length is mainly caused by the removal of proppants and fracture closure.
Table 3-6: Analysis results using the flowback model and history matching.

<table>
<thead>
<tr>
<th>Fracture Properties</th>
<th>Flowback analysis</th>
<th>Long-term production data analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_f$, ft</td>
<td>873</td>
<td>489</td>
</tr>
<tr>
<td>$k_f$, md</td>
<td>1227.5</td>
<td>-</td>
</tr>
<tr>
<td>γ, psi-l</td>
<td>8x10^-4</td>
<td>-</td>
</tr>
<tr>
<td>Average $k_f$, md</td>
<td>-</td>
<td>46.4</td>
</tr>
</tbody>
</table>

3.5 Summary and Conclusions

We developed multiphase flowback models for both gas and water boundary dominated flow (BDF) by considering gas influx from matrix under both constant and variable-rate conditions. The flowback models consider pressure-dependent fracture permeability. We also developed a two-phase flowing material balance (FMB) equation to calculate average pressure in the fracture by using the variable BHP solution of gas inflow equation in the matrix. We proposed a workflow to reconcile water-phase and gas-phase flowback models and evaluate fracture closure through estimation of initial fracture permeability, fracture half-length, and fracture permeability modulus.

We validated the flowback models and workflow against the results obtained from numerical simulations. The validation results reveal that the two-phase flowback models can closely predict fracture properties. We applied the workflow to flowback data from a multi-fractured horizontal gas well in Marcellus Shale and utilized IHS Harmony software to analyze long-term production data. With the estimated fracture properties from flowback and long-term production data, the fracture closure was quantitatively evaluated. The Marcellus field application shows that the fracture half-length is reduced by half and the fracture permeability decreased by over an order of magnitude after nine months of production.

We assumed a uniform gas influx along the fracture in the gas flowback model, which is valid when the pressure gradient along the fracture is small. If the system has a substantial pressure gradient in the fracture, we suggest considering non-equivalent gas influx in the flowback model.
Another assumption made in this study is negligible water influx from matrix, which is valid when the water saturation is small, or the capillary pressure impedes water flow in the matrix. However, when the water leakoff during hydraulic fracturing is significant, the absorbed water in the shale matrix has a significant impact on flowback, so the two-phase flow is likely to appear in the near fracture region. Hence further study is needed to include the effect of water influx from matrix into the flowback models, as well as gas desorption and diffusion for shale gas reservoir (Xu et al., 2016, Zhang et al., 2018, Yang et al., 2017, Clarkson et al., 2017, Clarkson et al., 2016, Hamdi et al., 2018, Behmanesh et al., 2018a). Further, the modeling of two-phase bilinear flow in FR1_G needs further investigations, which is outside of the scope of this study.

3.6 Nomenclatures

ABT: after-breakthrough
BBT: before-break-through
BDF: boundary dominated flow
CBM: coalbed methane

\( b_{FMB} \): the intercept of \( G_{P_{norm}} \) vs. \( PNR \) plot

\( b_{pss} \): the intercept of \( t_{p,g} \) vs. \( RNP_g \) plot

\( b_w \): the intercept of \( t_{p,w} \) vs. \( RNP_w \) plot

\( B_g \): gas formation factor, \( ft^3/scf \)

\( B_w \): water formation factor, \( bbl/STB \)

\( c_{eff,g} \): effective gas compressibility, \( psi^{-1} \)

\( c_f \): fracture compressibility, \( psi^{-1} \)

\( c_{gf} \): gas compressibility in the fracture, \( psi^{-1} \)
\( c_{gm} \): gas compressibility in the matrix, \( psi^{-1} \)

\( c_{m} \): matrix compressibility, \( psi^{-1} \)

\( c_{tm} \): matrix total compressibility, \( psi^{-1} \)

\( c_{w} \): water compressibility, \( psi^{-1} \)

\( D_P \): dimensionless drawdown parameter

DDA: dynamic drainage area

\( DRNP_w \): derivative of water-phase rate normalized pseudopressure, \( psi/STB \)

DRP: dynamic relative permeability

EGP: early gas production

\( f_{cp} \): correction factor

FMB: flowing material balance

\( G_{g,inf} \): cumulative gas production of gas influx at surface condition, \( scf \)

\( G_p \): cumulative gas production, \( scf \)

\( G_{nN} \): normalized cumulative gas production, \( scf \)

\( GWR \): gas water ratio

\( h \): fracture height, \( ft \)

IALF: infinite acting linear flow

\( k_f \): fracture permeability, \( md \)

\( k_{fi} \): initial fracture permeability, \( md \)

\( k_m \): matrix permeability, \( md \)

\( k_{rg} \): gas relative permeability

\( k_{rw} \): water relative permeability

\( l_f \): fracture spacing, \( ft \)

LGP: late gas production
\( m(p) \): gas-phase pseudopressure, psia

\( m(\bar{p}_f) \): gas-phase pseudo average pressure in the fracture, psia

\( \bar{m}(p_f) \): average gas-phase pseudo pressure in the fracture, psia

\( m_{FMB} \): the slope of \( G_{P\text{norm}} \) vs. \( PNR \) plot

\( m_g \): the slope of \( t_{p,g} \) vs. \( RNP_g \) plot

\( m_m(p) \): real gas pseudopressure, \( \text{psia}^2/\text{cp} \)

\( m_w \): the slope of \( t_{p,w} \) vs. \( RNP_w \) plot

\( M \): mass of gas

MBE: material balance equation

MFHWs: multi-fractured horizontal wells

\( p \): pressure, psia

\( p_b \): base pressure, psia

\( p_f \): fracture pressure, psia

\( \bar{p}_f \): average pressure in the fracture, psia

\( p_i \): initial fracture pressure, psia

\( p_p \): water-phase pseudopressure, psia

\( p_{pi} \): initial water pseudopressure, psia

\( p_{pwf} \): bottomhole water pseudopressure, psia

\( p_{sc} \): pressure at standard condition, psia

\( p_{wf} \): bottom-hole pressure, psia

\( PNR \): pressure normalized rate, \( \text{psia} \cdot \text{d/Mscf} \)

\( q_{g} \): the gas-phase surface flowrate, \( \text{Mscf/d} \)

\( q_{gf} \): the gas-phase downhole flowrate, \( \text{Mscf/d} \)

\( q_{g,inf} \): the flowrate of gas influx at surface condition, \( \text{Mscf/d} \)
\( q_w \): the water-phase surface flowrate from the whole fracture, \( STB/d \)

\( R \): ideal gas constant

\( RNP_g \): gas-phase rate normalized pseudopressure, \( psia \cdot d/Mscf \)

\( RNP_w \): water-phase rate normalized pseudopressure, \( psia \cdot d/STB \)

RTA: rate transient analysis

\( s_g \): gas saturation in the fracture

\( \bar{s}_g \): average gas saturation in the fracture

\( s_w \): water saturation in the fracture

\( s_{wm} \): water saturation in the matrix

\( \bar{s}_w \): average water saturation in the fracture

\( t \): time, \( day \)

\( t_{p,g} \): gas-phase pseudotime, \( day \)

\( t_{p,w} \): water-phase pseudotime, \( psia \cdot day/cp \)

\( t_{sppt,g} \): gas superposition pseudotime, \( day \)

\( t_{sppt,w} \): water superposition pseudotime, \( psia \cdot day/cp \)

\( T \): temperature, \( R \)

\( T_{sc} \): temperature at standard condition, \( R \)

\( V_h \): initial pore volume of half fracture, \( f t^3 \)

\( w_f \): hydraulic fracture width, \( ft \)

\( W_{fi} \): initial water storage in fracture, \( Scf \)

\( W_p \): cumulative water production, \( STB \)

\( x \): location in the fracture, \( ft \)

\( x_f \): fracture half-length, \( ft \)

\( y \): location in the matrix, \( ft \)
$Z$: gas deviation factor

$\epsilon$: tolerance of iteration

$\phi_f$: fracture porosity

$\phi_m$: matrix porosity

$\rho_w$: water density

$\rho_{gf}$: gas-phase density in the fracture

$\rho_{gm}$: gas-phase density in the matrix

$\mu_w$: water viscosity, cp

$\mu_{gf}$: gas viscosity in the fracture, cp

$\mu_{gm}$: gas viscosity in the matrix, cp

$\gamma$: fracture permeability modulus, $psi^{-1}$

\textbf{Subscripts}

$i$: initial

$j$: the $j^{th}$ flowrate interval

$f$: fracture

$m$: matrix

$g$: gas

$w$: water

$D$: dimensionless
3.7 Reference


CLARKSON, C., QANBARI, F., WILLIAMS-KOVACS, J. & ZANGANENH, B. Fracture propagation, leakoff and flowback modeling for tight oil wells using the dynamic drainage area concept. SPE Western Regional Meeting, 2017. Society of Petroleum Engineers.


Chapter 4

A Semianalytical Method for Two-Phase Flowback Rate Transient Analysis in Shale Gas Reservoirs

4.1 Abstract

We propose a new semianalytical method for analyzing flowback water and gas production data to estimate hydraulic fracture (HF) properties and to quantify HF dynamics. The method includes a semianalytical flowback model, a set of two-phase diagnostic plots, and a workflow to evaluate initial fracture volume and permeability, as well as fracture compressibility and permeability modulus. The flowback model incorporates two-phase water and gas flow in both HF and matrix domains and considers variations of fluid and rock properties with pressure. The HF domain is modeled by boundary dominated flow, whereas an infinite acting linear flow is assumed for the matrix domain. The flowback model is developed by assigning the variable average pressure in the fracture as the inner boundary condition for matrix based on Duhamel’s principle. The average pressure in the fracture and distance of investigation (DOI) in the matrix are calculated from a modified material balance equation by updating the matrix DOI as well as phase saturation and relative permeability in both the fracture and matrix domains. A modified DOI equation is used for two-phase flow in the matrix, which considers the pressure-dependent fluid and rock properties in pseudotime. The diagnostic plots shed light on the identification of flow regimes during the coupled two-phase flow in both fracture and matrix. The proposed workflow quantifies the HF dynamics through the loss of both fracture volume and fracture permeability by reconciling flowback and long-term production data. The accuracy of the new method is tested against

---

numerical simulations conducted by a commercial numerical simulator. The validation results confirm that the proposed method accurately predicts initial fracture volume, permeability, and permeability modulus. Further, we use production data from a multi-fractured horizontal well (MFHW) drilled in Marcellus Shale to test the practicality of the proposed method. The results show a significant reduction in fracture volume and permeability during production due to the HF closure.

4.2 Introduction

The importance of flowback data analysis to evaluate the stimulation performance for multi-fractured horizontal wells (MFHWs) in shale reservoirs has been demonstrated by characterizing hydraulic fracture (HF) properties (Clarkson et al., 2016). Although the duration of flowback is short (usually several days to months) compared to long-term production, it has great potential to understand the performance of stimulation jobs and MFHWs; hence, acting as a critical constraint for long-term production data analysis (Yang et al., 2017).

The early work of flowback analysis mainly focused on qualitative studies. Crafton (1998) used four field examples to evaluate maximum productive capacity from flowback data right after stimulation and encouraged to evaluate these data to improve flowback management. Then, the impact of flowback rates and shut-ins on gas recovery was studied through laboratory experiments and numerical simulation (Crafton, 2008). Ilk et al. (2010) provided a comprehensive workflow to integrate flowback data by generating different types of visualization curves, such as rate vs. time and gas water ratio (GWR) vs. cumulative production (Gp). Although two distinct periods of flowback are observed in Ilk et al.’s curves, they didn’t provide the physical explanations behind the behaviors. Later, Clarkson and Williams-Kovacs (2013a) built a conceptual model for MFHW and categorized the flow regimes of flowback period into before-breakthrough (BBT) and after-
breakthrough (ABT) according to the time when matrix fluid fluxes into HF. Adefidipe et al. (2014) further elaborated on the physical meanings of two-phase flow behaviors and divided the flowback history into two periods based on the V-shaped GWR vs. Gp plot. They categorized the negative trend region of GWR vs. Gp plot to be early gas production (EGP) and the positive trend region to be late gas production (LGP). Different terminologies are used to name similar production behaviors during flowback in the literature, we refer to BBT and EGP as the “early-time” flowback period and to ABT and LGP as the “late-time” flowback period.

Numerous authors have proposed mathematical models to extract HF properties from flowback data. The first attempt of quantitative analysis is treating the early flowback period as the water dominated production by Abbasi et al. (2012). They developed a single-phase water model for fracture boundary dominated flow (BDF) to extract total storage coefficient and fracture permeability. Although the single-phase water model shows satisfactory behavior in some field cases with enough early-time flowback data for analysis (Clarkson and Williams-Kovacs, 2013a), the duration of single-phase water production period is usually too short for gas wells, especially in shale reservoir, to accurately evaluate the fracture properties. Alkouh et al. (2014) presented a two-phase model to determine effective fracture volume from the combined flowback and production water data. But Alkouh et al.’s model may not be valid when the water saturation in HF is a large value because of the approximation made in the definitions of total mobility and compressibility. To avoid such an issue, Clarkson and Williams-Kovacs (2013b) adopted a two-phase model, which was originally designed for the coalbed methane (CBM) reservoirs (Clarkson et al., 2012), to analyze the early-time flowback data. However, their model treats relative permeability independent of pressure in the pseudopressure definition. Xu et al. (2016) modified the general single-phase BDF model to two-phase BDF in HF and presented a flowing material balance equation to extract fracture permeability and storage coefficient. But their model is developed under constant rate condition and assumes constant fracture permeability and porosity.
Zhang and Emami-Meybodi (2018) developed a two-phase flowback model for early-time flowback period under constant bottomhole pressure (BHP) condition by including pressure-dependent fracture permeability, porosity, and relative permeability in pseudopressure and pseudotime definitions. Later, they extended their model by considering the variable rate and BHP conditions (Zhang and Emami-Meybodi, 2020b).

While all the above-mentioned models consider one dimensional (1D) flow in HF, many semianalytical models have been proposed to analyze late-time flowback data by including gas influx from matrix. Ezulike and Dehghanpour (2014b) derived a two-phase model with a gas influx from matrix to history match late-time flowback data. Their model is based on the linear dual-porosity model for single-phase flow (Ezulike and Dehghanpour, 2014a) and accounts for two-phase transient linear flow in the fracture by introducing a dynamic relative permeability concept. In an extended study, they proposed a complementary approach to reduce the non-uniqueness problem of production data analysis by updating the input fracture parameters (Ezulike and Dehghanpour, 2015). Williams-Kovacs and Clarkson (2016) considered matrix gas influx to conduct rate transient analysis (RTA) and history matching. Xu et al. (2017) extended their 1D model (Xu et al., 2016) to an open-tank model to estimate the fracture-matrix interface area using late-time flowback data. However, they used the constant BHP condition to derive the gas flow model in matrix. Jia et al. (2018) pointed out that the appearance of 1D water-phase flow in the fracture is independent of gas influx from the matrix. They proposed a two-phase flowback model by ignoring the change of water relative permeability in pseudopressure. Zhang and Emami-Meybodi (2020a) developed two multiphase flowback models for both water and gas flow in HF by considering gas influx under variable BHP condition and including pressure-dependent permeability and relative permeability in pseudopressure and pseudotime.

The assumption of single-phase gas influx from matrix may hold in formations with either low water saturation or high capillary pressure (Xu et al., 2017). However, two-phase water and
gas influx from the matrix becomes significant when the water saturation is high, which may affect flowback data. Clarkson and Qanbari (2016b) introduced the dynamic drainage area (DDA) concept to multiphase CBM modeling and proposed a coupled two-phase model for matrix transient linear flow. However, the saturation vs. pressure path in the pseudopressure is obtained from numerical simulations. They improved the DDA model by using an empirical function for saturation vs. pressure to calculate pseudopressure in an extended work (Clarkson and Qanbari, 2016a) and also extended the DDA approach to conducting RTA for transient linear flow in matrix (Qanbari and Clarkson, 2016). They later applied the DDA model to flowback analysis in tight oil wells by considering the propagation of pressure drop along and perpendicular to HF (Clarkson et al., 2017). Although the DDA approach is capable of modeling coupled two-phase flow in fracture and matrix, the pseudopressure definition still needs to be improved to consider pressure-dependent relative permeability. Furthermore, the assumption of constant fluid and rock properties needs to be relaxed when calculating the distance of investigation (DOI) in their DDA approach.

Apart from the above analytical and semianalytical models, many numerical models have been proposed to analyze flowback data by considering complex fracture network and other phenomena such as diffusion and adsorption (Cao et al., 2017, Yang et al., 2017, Jia et al., 2017, Chen et al., 2018, Wang et al., 2018). More recently, empirical models, such as decline curve analysis, are applied to forecast multiphase flowback data and estimate the effective fracture volume (Hossain et al., 2019, Jones and Blasingame, 2019, Fu et al., 2019). However, they are mostly developed for history matching and require the fracture properties as input parameters.

In this study, we extend our previous work (Zhang and Emami-Meybodi, 2020a) by developing a semianalytical model for two-phase flow in a coupled fracture/matrix system that incorporates matrix water/gas influx in pseudotime definition. The contribution of this paper is threefold. First, we propose a new set of two-phase diagnostic plots to identify different flow regimes for the coupled fracture/matrix flow system. Second, unlike other models for the coupled
two-phase system, our model can be used not only for history matching but also for straight-line analysis to estimate initial fracture volume and permeability. Although the nature of the system is a dynamic two-phase flow system, this study analyzes the water-phase and gas-phase flowback data separately. The analysis results from the two phases provide an important constraint to determine fracture permeability modulus and fracture compressibility iteratively. Third, the proposed workflow quantifies HF dynamics through the loss of both fracture volume and fracture permeability by reconciling flowback and long-term production data. We validate the new method by comparing the results of the semianalytical model against the numerical simulations and then apply it to a field example in Marcellus Shale.

4.3 Theory and Methods

In this section, first, we describe the two-phase conceptual model for a coupled fracture/matrix system and discuss different flow regimes that may appear during flowback. Then, we present the semianalytical method for analyzing water-phase and gas-phase flowback data, separately.

4.3.1 Conceptual Model

As shown in Figure 4-1, the conceptual model consists of HF and matrix domains. While HF could be saturated with single-phase water or two-phase water and gas, the matrix contains both gas and water. The model is expanded from the 1D water flow and 2D gas flow model by Zhang and Emami-Meybodi (2020a). Different from our previous work, this 2D two-phase model also considers the water influx from the matrix. The water and gas influx from the matrix begins when the average fracture pressure drops below the initial matrix pressure at the onset of flowback, which
is usually larger than the initial reservoir pressure due to water injection during stimulation. In this study, the instantaneous fluid influx is adopted (Ozkan et al., 2010) by assuming the initial matrix pressure at the onset of flowback equal to initial fracture pressure. Other main assumptions of the conceptual model are as follows:

- Water and gas flow obeys Darcy’s law in both fracture and matrix. Gravity and capillary pressure are negligible. Condensate is negligible during production.
- The fracture and matrix are homogeneous and isotropic with uniform geometry and attributes. The fracture height equals the formation thickness.
- HF applies the straight-line relative permeability curve (Clarkson and Williams-Kovacs, 2013b). The relative permeability in the matrix follows Corey formulation with the initial water saturation larger than the irreducible water saturation.
- The fracture and matrix are slightly compressible with constant rock compressibility. Permeability and porosity change exponentially with pressure (Yilmaz et al., 1994).
- Water is slightly compressible fluid with constant compressibility and pressure-dependent viscosity and formation volume factor (FVF). Gas is compressible fluid with pressure-dependent compressibility, FVF, and viscosity.

Figure 4-1: Schematic of the conceptual model for flowback depicting water and gas flow through fracture and matrix.
4.3.2 Flow Regime Analysis

To understand the gas and water production behavior, many authors attempt to use diagnostic plots to identify the different flow regimes for multiphase flow in the shale reservoir.

The comparison of different multiphase diagnostic plots is shown in Table 4-1.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Ilk et al. (2010)</th>
<th>Clarkson et al. (2020)</th>
<th>Zhang and Emami-Meybodi (2020a)</th>
<th>This work</th>
</tr>
</thead>
<tbody>
<tr>
<td>RNP</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Pseudo-pressure Material balance pseudotime</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Psuedotime</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Comments**
- Show a half-slope straight line during flowback.
- Identify multiphase depletion flow regime empirically.
- Can be used to identify three-phase flow regimes.
- Require numerical simulation to obtain the relation of pressure and saturation.
- Identify water-phase flow regime for 1D two-phase flow in the fracture.
- Identify gas-phase flow regime for 2D system.
- Two-phase flow in fractures, single-phase flow in matrix.
- Identify water-phase and gas-phase flow regimes for 2D system.
- Two-phase flow in both fracture and matrix.

\[ P_{inj}(p_i) - P_{inj}(p_{ur}) = \frac{\Delta P_{inj}}{k_{eff}} \left( \int_{p_{inj}}^{p_{ur}} \frac{\rho_i}{\mu_i} \frac{\partial \phi}{\partial p} dp + \int_{p_{inj}}^{p_{ur}} \frac{\rho_i}{\mu_i} \mu_i \frac{\partial \phi}{\partial p} dp + \int_{p_{inj}}^{p_{ur}} a_{gas} dp + \int_{p_{inj}}^{p_{ur}} a_{oil} dp \right) \]
\[ C_t = C_w + \frac{1}{\frac{d}{a_{oil}}}; \]
\[ C_{eff} = S_{ij} (C_{ij} + C_i) + \frac{d_{ij}}{\frac{d}{a_{oil}}}; \]
\[ \frac{d}{a_{oil}} \frac{\partial \phi}{\partial p} dp + \frac{d}{a_{oil}} \frac{\partial \phi}{\partial p} dp + \int_{p_{inj}}^{p_{ur}} a_{gas} dp + \int_{p_{inj}}^{p_{ur}} a_{oil} dp \]

In this study, we analyzed flow regimes during flowback by considering both water and gas influx from the matrix based on a new set of diagnostic plots. Due to the significant difference in permeability between HF and shale matrix, we assumed a uniform fluid influx from matrix by ignoring the gradient of influx rate along the fracture. We focus on the flow in HF and categorize the multiphase flow in this coupled fracture/matrix system into two regimes according to the time when the effect of pressure drop in the fracture reaches the tip boundary. This time is different for water and gas because of the different fluid attributes in hydraulic diffusivity coefficient. The
schematic of the two flow regimes in the conceptual model is shown in Figure 4-2a and Figure 4-2c. The first flow regime (FR1) denotes the fracture infinite acting linear flow (IALF) with an insignificant fluid influx from matrix. Although FR1 shows the behavior of two-phase bilinear flow based on the simultaneous IALF in both fracture and matrix, we decoupled the two linear flows by incorporating the effect of matrix linear influx into the pseudotime definition in the fracture model. Hence, we used the derivative of rate normalized pressure (RNP) vs. the corrected pseudotime in log-log scale to generate the new diagnostic plot based on the fracture model, in which FR1 shows a half-slope straight line representing the fracture IALF (Figure 4-2b and Figure 4-2d). During FR1, the matrix influx domain gradually propagates from near-wellbore to the fracture tip. When the distance of investigation (DOI) in the fracture equals the fracture half-length, the second flow regime (FR2) appears, which denotes the fracture BDF with a significant matrix influx. In other words, the matrix linear flow is considered into fracture BDF through our newly defined fracture effective compressibility term in pseudotime. Using the same diagnostic plot, FR2 will show a unit-slope straight line in the later portion with more datasets than FR1 to conduct analysis. A detailed derivation of the diagnostic plot is given in the following sections.
4.3.3 Mathematical Formulations

We developed a semianalytical model for FR2 since FR2 dominates most of the life of the well in shale gas reservoirs. The fracture model was first developed under a constant rate condition and extended to the variable rate condition using the Duhamel’s principle (i.e., superposition principle), whereas the matrix model was derived under constant BHP and later extended to variable BHP condition for general application.

Figure 4-2: (a) Schematic of FR1. (b) Two-phase water diagnostic plot. (c) Schematic of FR2. (d) Two-phase gas diagnostic plot.
4.3.3.1 Two-phase Fracture Model

The diffusivity equation for water and gas phase flow in the fracture can be represented as:

\[
\frac{\partial}{\partial x} \left( k_{rj} f \frac{\partial p_f}{\partial x} \right) = \frac{\phi_f c_{j,\text{eff}}}{B_{jf}} \frac{\partial p_f}{\partial t},
\]

(4-1)

where subscript \(j\) can be \(w\) for water or \(g\) for gas, \(x\) is the location in the fracture, \(k_f = k_0 \exp(-\gamma_f (p_f \cdot p_j))\) is the pressure-dependent fracture permeability, \(\gamma_f\) is the fracture permeability modulus, \(k_{rj,f}\) is the relative permeability in the fracture, \(\mu_{j,f}\) is the fluid viscosity in the fracture, \(B_{jf}\) is the FVF in the fracture, \(\phi_f = \phi_0 \exp(-C_f (p_f \cdot p_j))\) is the pressure-dependent fracture porosity, \(C_f\) is the fracture compressibility, \(p_f\) is the fracture pressure, \(C_{j,\text{eff}}\) is the effective compressibility in the fracture defined in Appendix A, and \(t\) is the time.

The pressure-dependent fluid and rock properties make the governing equation nonlinear. To linearize Eq. (4-1), the pseudopressure \(m_{j,f}\) and pseudotime \(t_{p,j,f}\) in the fracture are introduced by:

\[
m_{j,f}(p) = \left( \frac{\mu_{j,f} B_{jf}}{k_f} \right)_i \int_{p_b}^p \frac{k_f(p) k_{rj,f}(S_{jf})}{\mu_{j,f}(p) B_{jf}(p)} dp,
\]

(4-2)

\[
t_{p,j,f} = \left( \frac{\phi_f \mu_{j,f} c_{j,\text{eff}}}{k_f} \right)_i \int_0^t \frac{k_f(\bar{p}_f) k_{rj,f}(\bar{S}_{jf})}{\phi_f(\bar{p}_f) \mu_{j,f}(\bar{p}_f) c_{j,\text{eff}}(\bar{p}_f)} d\tau,
\]

(4-3)

where \(p_b\) is the base pressure, \(S_{jf}\) is the phase saturation in the fracture, and the subscript \(i\) means that the fluid or rock properties are evaluated at the initial condition.

After applying Eqs. (4-2) and (4-3) to Eq. (4-1), the linearized governing equation under constant rate condition can be solved using the method of Separation of Variables (Miller, 1962). After performing some algebraic manipulations as described in Appendix A, straight line form solutions in field unit can be obtained in terms of RNP \(\{\text{RNP}_j = (m_{j,f}(p_f) - m_{j,f}(p_{o_j}))q_j\}\) and pseudopressure normalized rate (PNR) \(\text{PNR}_j = q_j(m_{j,f}(p_f) - m_{j,f}(p_{o_j}))\):
where \( q_j \) is the flowrate from the whole fracture at surface condition, \( Q_{j,pN} = V_{fi} \frac{m_{j,f}(p_{fi})-m_{j,f}(p_f)}{m_{j,f}(p_{fi})-m_{j,f}(p_{wf})} \)

is the normalized cumulative production, \( V_f = 2x_f w_f h \phi_f \) is the initial fracture volume, \( x_f \) is the fracture half-length, \( w_f \) is the fracture width, and \( h \) is the fracture height.

For simplicity, we denote the RNP form solution in Eq. (4-4) as “boundary dominated flow (BDF) model” and the PNR form solution in Eq. (4-5) as “flowing material balance (FMB) model”. The FMB model can be directly extended to the variable rate condition without any manipulation. We applied the Duhamel’s principle to extend the BDF model from the constant rate condition to variable rate by replacing pseudotime with superposition pseudotime \( t_{sp,j} \):

\[
t_{sp,j} = \sum_{n=1}^{N} \frac{(q_{j,n}-q_{j,n-1})(t_{p|f,n}-t_{p|f,n-1})}{q_{j,n}},
\]

(4-6)

where \( t_{p|f,n} \) and \( q_{j,n} \) are the pseudotime and flowrate at the \( n \)th flowrate interval.

By generating RNP\(_j\) vs. \( t_{sp,j} \) (i.e., BDF specialty plot) or PNR\(_j\) vs. \( Q_{j,pN} \) (i.e., FMB specialty plot), we can calculate \( V_f \) and \( k_f \) based on the slope and intercept of the straight line, which falls into the FR2 identified by the diagnostic plot. The slope and intercept of Eqs. (4-4) - (4-5), as well as the formulations for \( V_f \) and \( k_f \), are listed in Table 4-2. To evaluate the pseudo-variables, we calculated the average fracture pressure and saturation using the water and gas material balance equations in the fracture:

\[
Q_w = V_{fi} \left( \frac{S_{w,f,i}}{B_{w,f}} - \frac{S_{w,f}}{B_{wf}} \left[ 1 - C_f \left( p_{fi} - \bar{p}_f \right) \right] \right) + Q_{w,inf},
\]

(4-7)

\[
Q_g = V_{fi} \left( \frac{S_{g,f,i}}{B_{g,f,i}} - \frac{S_{g,f}}{B_{g,f}} \left[ 1 - C_f \left( p_{fi} - \bar{p}_f \right) \right] \right) + 1000Q_{g,inf},
\]

(4-8)

where \( Q_w \) and \( Q_g \) are the cumulative water and gas production at the surface condition, and \( Q_{w,inf} \) and \( Q_{g,inf} \) are the cumulative water and gas matrix influx at the surface condition.
Table 4-2: Summary of the formulations for $V_{fi}$ and $k_{fi}$, as well as the slope and intercept of the specialty plots.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>BDF model ($RNP_i$ vs. $t_{pN}$)</th>
<th>FMB model ($FNR_i$ vs. $Q_{pN}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_j$</td>
<td>$a_{j,BDF} = a_j \left( \frac{B_{j,f}}{V_{f,ij,BDF}} \right)_i$</td>
<td>$a_{j,FMB} = -\frac{h^2 w_f^2}{\beta_j} \phi_f k_f$</td>
</tr>
<tr>
<td>$b_j$</td>
<td>$b_{j,BDF} = \beta_j \frac{1}{h^2 w_f^2} \left( \frac{V_{f,ij,BDF}}{\phi_f k_f} \right)_i$</td>
<td>$b_{j,FMB} = \frac{h^2 w_f^2}{\beta_j} \phi_f k_f$</td>
</tr>
<tr>
<td>$V_{f_2}$</td>
<td>$V_{f_2,BDF} = a_j \frac{1}{a_{j,BDF}} \left( \frac{B_{f,ij}}{V_{f,ij,BDF}} \right)_i$</td>
<td>$V_{f_2,FMB} = -b_{j,FMB}$</td>
</tr>
<tr>
<td>$k_{f_2}$</td>
<td>$k_{f_2,BDF} = \beta_j \frac{V_{f_2,BDF}(\mu_{2,ij}B_{f,ij}))}{h^2 w_f^2 \phi_f k_{f_2,BDF}}$</td>
<td>$k_{f_2,FMB} = b_{j,FMB} \frac{V_{f_2,FMB}(\mu_{2,ij}B_{f,ij}))}{h^2 w_f^2 \phi_f k_f}$</td>
</tr>
</tbody>
</table>

For $j=w$: $a_j = 1$ and $\beta_j = 13.16$ (Zhang and Emami-Meybodi, 2020b)
For $j=g$: $a_j = 1000.4$ and $\beta_j = 13170.35$ (Zhang and Emami-Meybodi, 2020a)

4.3.3.2 Two-phase Matrix Model

The diffusivity equation for water and gas phase flow in the matrix can be represented as:

$$\frac{\partial}{\partial y} \left( \frac{k_{f,lm}k_m}{\mu_{f,m}B_{f,m}} \frac{\partial p_m}{\partial y} \right) = \frac{\phi_m C_{ef,m} \partial p_m}{B_{f,m}} \frac{\partial p_m}{\partial t},$$

(4-9)

where $y$ is the location in the matrix towards the fracture, $k_m = k_{mi} \exp(-\gamma_m(p_{mi}, p_m))$ is the pressure-dependent matrix permeability, $\gamma_m$ is the matrix permeability modulus, $k_{f,lm}$ is the phase relative permeability in the matrix, $\mu_{f,m}$ is the fluid viscosity in the matrix, $B_{f,m}$ is the FVF in the matrix, $\phi_m = \phi_{mi} \exp(-C_m(p_{mi} - p_m))$ is the pressure-dependent matrix porosity, $C_m$ is the matrix compressibility, $p_m$ is the matrix pressure, $C_{f,ef,m}$ is the matrix effective compressibility.

To linearize the diffusivity equation, the pseudopressure and pseudotime are introduced by:

$$m_{j,m}(p) = \frac{\mu_{f,j,m}}{k} \int_{p_b}^{p} k_m(p) k_{f,j,m}(S_{f,m}) \frac{dp}{\mu_{f,j,m}(p) B_{f,m}(p)} d\tau,$$

(4-10)

$$t_{p,j,m} = \frac{\phi_{j,m} C_{ef,f}}{k} \int_{p_b}^{p} k_m(p) k_{f,j,m}(S_{f,m}) \frac{dp}{\mu_{f,j,m}(p) B_{f,m}(p) C_{ef,m}(p)} d\tau,$$

(4-11)

where $S_{f,m}$ is saturation in the matrix, $m_{j,m}$ and $t_{p,j,m}$ are the matrix pseudopressure and pseudotime.
After performing algebraic manipulations, which are described in Appendix B, the cumulative water and gas influx from the matrix are calculated using Duhamel’s principle (Özisik et al., 1993):

\[
Q_{w,inf} = \frac{[m_{w,m}(p_{mi}) - m_{w,m}(p_b)]}{\delta_w(\mu_Bw)} \int_0^t \Delta \zeta_w(\tau)_n \text{ d}\tau, 
\]

\[
(4-12)
\]

\[
Q_{g,inf} = \frac{[m_{g,m}(p_{mi}) - m_{g,m}(p_b)]}{\delta_g(\mu_bg)} \int_0^t \Delta \zeta_g(\tau)_n \text{ d}\tau, 
\]

\[
(4-13)
\]

\[
\Delta \zeta_j(\tau)_n = \zeta_j(\tau)_{n+} - \zeta_j(\tau)_{n-}, 
\]

\[
(4-14)
\]

\[
\zeta_j(\tau) = \frac{m_{j,m}(p_{mi}) - m_{j,m}(p_f(\tau))}{m_{j,m}(p_{mi}) - m_{j,m}(p_b)}, 
\]

\[
(4-15)
\]

where \( \delta_j \) is the unit conversion factor \( \delta_w = 5.575 \) and \( \delta_g = 5578.5 \), \( \tau_{p,j,m,n} \) is the pseudotime evaluated at the \( n \)th \( \bar{p}_f \) interval, \( \zeta_j(\tau) \) and \( \Delta \zeta_j(\tau)_n \) are stepwise functions at each \( \bar{p}_f \) interval, the subscript + or – means that \( \zeta_j(\tau) \) is evaluated at \( \bar{p}_f(\tau) \) at the current or former time step.

Similar to the fracture model, we evaluated the pseudo-variables in Eqs. (4-10) - (4-11) by calculating the average pressure and saturation in matrix but within the distance of investigation. We modified the matrix DOI definition used by Clarkson and Qanbari (2016a) and Behmanesh et al. (2018a) by incorporating the pressure-dependent fluid and rock properties into pseudotime. The water and gas material balance equations within the modified matrix DOI are presented as follows:

\[
Q_{w,inf} = 4V_{w,mi} \left( \frac{s_{w,mi}}{B_{w,mi}} - \frac{\bar{s}_{w,m}}{B_{w,m}} \left[ 1 - C_m(p_{mi} - \bar{p}_m) \right] \right), 
\]

\[
(4-16)
\]

\[
1000Q_{g,inf} = 4V_{g,mi} \left( \frac{s_{g,mi}}{B_{g,mi}} - \frac{s_{g,m}}{B_{g,m}} \left[ 1 - C_m(p_{mi} - \bar{p}_m) \right] \right), 
\]

\[
(4-17)
\]

\[
V_{j,mi} = y_{j,inv} x_f h \phi_{mi}, 
\]

\[
(4-18)
\]

\[
y_{j,inv} = 0.194 \sqrt{\frac{k_{mi}(\tau_{p,j,m})}{(\phi_m \mu_m C_{j,efm})}}, 
\]

\[
(4-19)
\]

where \( V_{j,mi} \) is the initial pore volume of the quarter fracture-matrix element, \( y_{j,inv} \) is the matrix DOI.
As mentioned earlier, $p_{mi}$ is set to equal $p_{fi}$ in this study, which leads to instantaneous water and gas influx from matrix. However, there may be some shale gas cases where $p_{mi}$ is below $p_{fi}$ due to the short stimulation duration or extreme low permeability of shale matrix. Although the assumption of instantaneous matrix influx is violated in these cases, the proposed method is still valid as long as $Q_{j,inf}$ is set to be zero when $\bar{p}_f$ stays above $p_{mi}$. In this period, there is only water and gas depletion in the fracture without any influx from matrix. The fluid leakoff into the shale reservoir could be negligible due to the fast depletion of hydraulic fracture. Once $\bar{p}_f$ drops below $p_{mi}$, gas and water flow from matrix and Eqs. (4-16) - (4-17) need to be used to calculate $Q_{j,inf}$.

### 4.3.3.3 Solution Procedure

At each time step, Eqs. (4-7) - (4-8), (4-12) - (4-13), and (4-16) - (4-17) are solved simultaneously using the fixed-point iteration scheme (Kreyszig, 2009) to calculate average pressure ($\bar{p}_f$) in fracture, average water saturation in fracture ($\bar{S}_{w,f}$), cumulative water influx ($Q_{w,inf}$), cumulative gas influx ($Q_{g,inf}$), average pressure in matrix ($\bar{p}_m$), and average water saturation in matrix ($\bar{S}_{w,m}$) within DOI. To ensure the robustness of the solving procedure, we made two approximations to improve the iteration scheme. Firstly in the calculation of pseudopressure in matrix, a saturation and pressure relation is required. We assumed the relative permeability inside matrix pseudopressure can be evaluated at the initial condition. Thus, the variation of $k_{rj,m}$ can be overlooked in the pseudopressure calculation as well. On the other hand, after magnitude analysis of different terms in $C_{j,efm}$, we found that the $dS_{j,m}/dp_m$ term is at least one order of magnitude smaller than the $S_{j,m}(C_{j,m}+C_m)$ term, and it does not affect the final calculations. Therefore, we neglected $dS_{j,m}/dp_m$ term to simplify the iterative procedure so that the convergence problem caused by this derivative term could be eliminated and the iteration stays robust.
After solving the system of six equations simultaneously, we substituted $\tilde{p}_f, S_{w,f}, Q_{w,inf}$ and $Q_{g,inf}$ into the fracture model to evaluate pseudo-variable in the HF. Accordingly, we generated specialty plots to calculate the fracture properties (i.e., $V_{fi}$ and $k_{fi}$) from Table 4-2.

4.3.4 Analysis Technique

This section presents an analysis technique to identify FR1 and FR2 as well as a workflow to calculate fracture properties and characterize HF dynamics. In our previous study, we only focused on the reduction of fracture permeability during production to evaluate HF closure (Zhang and Emami-Meybodi, 2020a). In this work, the HF dynamics are quantified through the loss of fracture volume and fracture permeability, which are controlled by $C_f$ and $\gamma_f$, respectively.

4.3.4.1 Two-phase Diagnostic Plot

Based on the flow regime analysis above, FR2 is the flow regime to perform BDF or FMB analysis. Therefore, a diagnostic plot is needed to identify the start of FR2. The initial and boundary conditions of HF for FR1 under constant rate condition are given:

$$p_f(x, 0) = p_{fi}, \quad p_f(x \to \infty, 0) = p_{fi}, \quad 2 w_f h \left( \frac{k_{ref} k_f}{\mu_{j,f} B_{j,f}} \frac{\partial p_f}{\partial x} \right) \bigg|_{x=0} = q_j, \quad p_f(x \to \infty, 0) = p_{fi}. \quad (4-20)$$

The same pseudopressure definition for BDF in Eq. (4-2) can be applied to linearize the boundary conditions in Eq. (4-20). The analytical solution of Eq. (4-1) subject to Eq. (4-20) is given in terms of RNP$_j$ in field unit by Wattenbarger et al. (1998):

$$\text{RNP}_j = \frac{\epsilon_f}{w_f h} \sqrt{\frac{\mu_{j,f} B_{j,f}^2}{\phi C_{j,f} k_f}} t_{pj,f}, \quad (4-21)$$

where $\epsilon_f$ is the unit conversion factor, $\epsilon_w = 7.093$ and $\epsilon_g = 7103.13$. 
Eq. (4-21) is the analytical solution for FR1 and can be extended to the variable rate condition by replacing pseudotime with superposition pseudotime defined in Eq. (4-6). By defining the derivative of RNP, as \( \text{DRNP}_j = \frac{d\text{RNP}_j}{d\ln t_{sp,j}} \), a half-slope straight line will be shown in the DRNP vs. \( t_{sp,j} \) plot in log-log scale representing FR1. Furthermore, Eq. (4-4) reveals a unit-slope straight line in the latter portion of the same plot for FR2. Therefore, we treated DRNP vs. \( t_{sp,j} \) in the log-log scale as a new diagnostic plot to identify FR1 and FR2 as depicted in Figure 4-2.

4.3.4.2 Workflow for \( V_{fi} \), \( k_{fi} \), \( \gamma_{fi} \), and \( C_f \) Calculation

To calculate \( \bar{p}_{f} \), \( \bar{S}_{w,f} \), \( Q_{w,inf} \), \( Q_{g,inf} \), \( \bar{p}_{m} \), and \( \bar{S}_{w,m} \) based on the system of six equations, we need \( V_{fi} \) as an input. Whereas \( V_{fi} \), \( k_{fi} \), and \( \gamma_{fi} \) are to be determined from the flowback model. Therefore, an iterative procedure is needed. We proposed a workflow in Figure 4-3 to iteratively calculate \( V_{fi} \), \( k_{fi} \), and \( \gamma_{fi} \) based on water and gas flowback data. Without the reconciliation of long-term RTA, the workflow of flowback analysis follows nine steps:

1. Guess an initial value for \( V_{fi} \) and \( \gamma_{fi} \) to start the workflow. The initial guess of \( \gamma_{fi} \) should fall into the empirical range of \( 10^{-4} \) to \( 10^{-3} \) psi\(^{-1} \) based on laboratory data (Ozkan et al., 2010).
2. Solve the Eqs. (4-7) - (4-8), (4-12) - (4-13), and (4-16) - (4-17) simultaneously using the fixed-point iteration scheme for the six unknowns: \( \bar{p}_{f} \), \( \bar{S}_{w,f} \), \( Q_{w,inf} \), \( Q_{g,inf} \), \( \bar{p}_{m} \), and \( \bar{S}_{w,m} \).
3. Evaluate the pseudo-variables \( m_{j,f} \) and \( t_{pj,f} \) using Eqs. (4-2) - (4-3).
4. Create the two-phase diagnostic plots DRNP vs. \( t_{sp,j} \) for water and gas to identify FR2 from the unit-slope straight line.
5. Generate RNP vs. \( t_{sp,j} \) or PNR vs. \( Q_{i,pV} \) to calculate \( V_{fi} \) and \( k_{fi} \) using water-phase and gas-phase flowback data, respectively.
6. Update \( V_{fi,new} = (V_{fi,w} + V_{fi,g})/2 \).
113

7. Repeat steps 2 to 6 until the relative error between $V_{fi,old}$ and $V_{fi,new}$ reaches the tolerance $\epsilon_{V_f}$. The relative error between $V_{fi,w}$ and $V_{fi,g}$ should be in an acceptable range as well.

8. Update $\gamma_f$ with another guess in the range of $10^{-4}$ to $10^{-3}$ psi$^{-1}$ until the relative error between $k_{fi,w}$ and $k_{fi,g}$ reaches the tolerance $\epsilon_{\gamma_f}$.

9. Report $V_{fi}$, $k_{fi}$, and $\gamma_f$.

If flowback data are reconciled with long-term production data in a field application, we can treat the long-term RTA as a constraint for fracture volume reduction and determine $C_f$ to fully quantify the HF dynamics. The workflow follows seven steps:

1. Guess an initial value for $C_f$ to start the flowback analysis. The initial guess of $C_f$ should fall into the empirical range of $7 \times 10^{-5}$ to $7 \times 10^{-4}$ psi according to recently published shale cases collected by Tan et al. (2019).

2. Follow the nine steps above to obtain $V_{fi}$, $k_{fi}$, and $\gamma_f$ from flowback data.

3. Estimate the ultimate $p_{fi}$ at the end of long-term production based on the BHP history.

4. With the ultimate $p_{fi}$ and $V_{fi}$, calculate final fracture volume $V_{ff} = V_{fi} \exp[-C_f(p_{fi} - \bar{p}_f)]$.

5. Use the long-term RTA technique to obtain $x_{f,LT} \sqrt{k_m}$ (Nobakht and Clarkson, 2012) and estimate final fracture volume based on long-term production data $V_{f,LT}=2 x_{f,LT} w_f h \phi_{fi}$.

6. Update $C_f$ with another guess in the range of $7 \times 10^{-5}$ to $7 \times 10^{-4}$ psi$^{-1}$ until the relative error between $V_{ff}$ and $V_{f,LT}$ reaches the tolerance $\epsilon_{C_f}$.

7. Report $V_{fi}$, $V_{ff}$, $k_{fi}$, $\gamma_f$, and $C_f$.

The entire workflow accounts for the loss of fracture volume from two angles. On one hand, we predict the reduction of fracture volume based on the $V_{fi}$ obtained from flowback analysis.

On the other hand, long-term production data provide us the cross-sectional area of formation transient linear flow towards fracture, which is applied to estimate the final fracture volume $V_{f,LT}$ from the perspective of matrix. Although the calculation of $V_{f,LT}$ converted the reduction of fracture
volume to the decrease in $x_f$ by ignoring the loss of $w_f$ and $\phi_f$, it still provides us an important constraint to reach unique interpretation results.

4.4 Results

To verify the accuracy of the proposed semianalytical method, we generated several numerical simulation cases based on the fracture and reservoir properties in Marcellus Shale and conducted a sensitivity analysis to demonstrate the model’s flexibility. After numerical validation, the proposed semianalytical method was applied to an MFHW drilled in Marcellus Shale to test the robustness and practicality.
4.4.1 Model Validation

We constructed two synthetic cases of MFHW using a commercial numerical simulator (CMG, 2019) having two different operating conditions: Case 1 is under a constant gas rate of 3000 Mscf/day and Case 2 is under constant water rate of 1000 STB/day. These two cases can provide us the validation of the semianalytical method for both variable water-rate and gas-rate conditions.

In the chosen quarter element of symmetry, fracture half-length, fracture half-spacing, and reservoir thickness are 500, 100, 300 ft, which are discretized by 500 × 100 × 1 grid blocks, respectively (Figure 4-4). Local-grid-refinement has been applied to both directions along and perpendicular to HF to accurately simulate the significant pressure drawdown near wellbore and HF. We conducted the production for 20 days to mimic the real flowback period in Marcellus Shale.

Table 4-3 lists reservoir and fracture properties for both cases. We applied the straight-line relative permeability curve (Clarkson et al., 2012) for HF and the Corey formulation ($N_w=N_g=3$) for matrix (Clarkson et al., 2016) to model the relative permeability of water and gas. We also assumed the HF and matrix systems have the same initial pressure during flowback period.

We first validated the solving procedure for $\bar{p}_f$, $\bar{S}_{w,f}$, $Q_{w,inf}$, and $Q_{g,inf}$ with the known $V_i$ as an input. Figure 4-4 illustrates the pressure profiles for the quarter element of numerical simulation model at 10 days and 20 days for Case 1 and Case 2, respectively. The even propagation of pressure drop front in the matrix confirms the validity of the uniform influx assumption. Figure 4-5 shows the comparison of $Q_{w,inf}$ and $Q_{g,inf}$ calculated from the analytical solution and numerical simulation. For both Case 1 and Case 2, the calculated $Q_{g,inf}$ reveals a perfect match with numerical simulation and $Q_{w,inf}$ generally agrees with simulation with an error of <10%. The error comes from the two approximations we made on the matrix pseudopressure and pseudotime calculations in the solution procedure. Although the analytical solution slightly overestimated the water influx, the error is still
acceptable for engineering purposes. The $\bar{p}_f$ and $\bar{S}_{w,f}$ between analytical solution and numerical simulation are compared in Figure 4-6, which shows a good agreement.

![Pressure maps for a quarter element of numerical simulation model at 10 days and 20 days of production for (a-b) Case 1 and (c-d) Case 2.](image)

Figure 4-6: Pressure maps for a quarter element of numerical simulation model at 10 days and 20 days of production for (a-b) Case 1 and (c-d) Case 2.

With the calculated $\bar{p}_f$, $\bar{S}_{w,f}$, $Q_{w,inf}$, and $Q_{g,inf}$, we used the known $\gamma_f$ as input and generated two-phase diagnostic plots for water and gas in Figure 4-7. A clear half-slope and unit-slope straight line are shown for both cases representing the FR1 and FR2 during flowback. Some data points in the diagnostic plot are slightly noisy due to the error cumulated from the calculation of the average pressure, saturation, and the derivative term during the solving procedure. We selected the starting point of unit-slope straight lines as the onset of FR2, after which the flowback data are used for straight line analysis. This time is different for water and gas due to the different fluid attributes in hydraulic diffusivity coefficient. For case 1, the starting time of FR2 is 18 hrs for water and 19 hrs for gas. For case 2, the starting time of FR2 is 21 hrs for water and 19 hrs for gas. Figure 4-8 and Figure 4-9 show the BDF and FMB specialty plots for Case 1 and Case 2, respectively. We extracted the slope and intercept of the specialty plots to calculate $V_{fi}$ and $k_{fi}$ based on the...
formulations given in Table 4-2. The analysis results in Table 4-4 show a good accuracy compared to the set values in the numerical simulation with relative errors <10%.

Table 4-3: Summary of input data used in numerical simulation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial pressure for fracture and matrix system, $p_i$ (psi)</td>
<td>5000</td>
</tr>
<tr>
<td>Temperature, $T$ (°F)</td>
<td>160</td>
</tr>
<tr>
<td>Specific gravity, $SG_g$</td>
<td>0.65</td>
</tr>
<tr>
<td>Water compressibility, $C_w$ (psi$^{-1}$)</td>
<td>$3 \times 10^{-6}$</td>
</tr>
<tr>
<td>Initial fracture porosity, $\phi_{fi}$ (%)</td>
<td>50</td>
</tr>
<tr>
<td>Initial matrix porosity, $\phi_{mi}$ (%)</td>
<td>5</td>
</tr>
<tr>
<td>Initial fracture permeability, $k_{fi}$ (md)</td>
<td>3000</td>
</tr>
<tr>
<td>Initial matrix permeability, $k_{mi}$ (md)</td>
<td>$1 \times 10^4$</td>
</tr>
<tr>
<td>Fracture compressibility, $C_f$ (psi$^{-1}$)</td>
<td>$2 \times 10^{-5}$</td>
</tr>
<tr>
<td>Matrix compressibility, $C_m$ (psi$^{-1}$)</td>
<td>$1 \times 10^{-6}$</td>
</tr>
<tr>
<td>Reservoir thickness, $h$ (ft)</td>
<td>300</td>
</tr>
<tr>
<td>Number of active fractures</td>
<td>50</td>
</tr>
<tr>
<td>Fracture width, $w_f$ (ft)</td>
<td>0.02</td>
</tr>
<tr>
<td>Initial water FVF, $B_{wi}$ (ft$^3$/STB)</td>
<td>5.78</td>
</tr>
<tr>
<td>Initial fracture water saturation, $S_{w,fi}$ (%)</td>
<td>70</td>
</tr>
<tr>
<td>Initial matrix water saturation, $S_{w,mi}$ (%)</td>
<td>30</td>
</tr>
<tr>
<td>Water viscosity, $\mu_w$ (cp)</td>
<td>0.3</td>
</tr>
<tr>
<td>Water density, $\rho_w$ (lbm/ft$^3$)</td>
<td>62</td>
</tr>
<tr>
<td>Fracture half-length, $x_f$ (ft)</td>
<td>500</td>
</tr>
<tr>
<td>Fracture permeability modulus, $\gamma_f$ (psi$^{-1}$)</td>
<td>$3 \times 10^{-4}$</td>
</tr>
<tr>
<td>Matrix permeability modulus, $\gamma_m$ (psi$^{-1}$)</td>
<td>$3 \times 10^{-5}$</td>
</tr>
</tbody>
</table>
Figure 4-5: Comparison of cumulative water and gas influx calculated from analytical solution and numerical simulation for (a-b) Case 1 and (c-d) Case 2.

Figure 4-6: Comparison of average pressure and average water saturation in the fracture calculated from analytical solution and numerical simulation for (a-b) Case 1 and (c-d) Case 2.
Figure 4-7: Two-phase water and gas diagnostic plot for (a-b) Case 1 with constant gas rate and (c-d) Case 2 with constant water rate.

Figure 4-8: Water-phase (a) BDF specialty plot and (b) FMB plot, and gas-phase (c) BDF specialty plot and (d) FMB plot for Case 1 with a constant gas rate.
Table 4-4: Summary of input data used in numerical simulation and calculated properties using flowback models.

<table>
<thead>
<tr>
<th>Case #</th>
<th>Set values</th>
<th>Water-phase flowback model</th>
<th>Gas-phase flowback model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>BDF analysis</td>
<td>FMB analysis</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_{fI}$ (Mcf) $k_{fI}$ (md)</td>
<td>$V_{fI}$ (Mcf) $k_{fI}$ (md)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BDF analysis</td>
<td>FMB analysis</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_{gI}$ (Mcf) $k_{gI}$ (md)</td>
<td>$V_{gI}$ (Mcf) $k_{gI}$ (md)</td>
</tr>
<tr>
<td>1</td>
<td>150</td>
<td>3000</td>
<td>150.75</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>3000</td>
<td>150.78</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>3000</td>
<td>152.19</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>3000</td>
<td>155.16</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>3000</td>
<td>153.42</td>
</tr>
</tbody>
</table>

As shown in the mathematical development, the derivation of our semianalytical method is based on three critical assumptions: (1) uniform matrix influx by ignoring the pressure and saturation gradient in the fracture; (2) calculating $\bar{p}_f$ in the entire fracture domain due to the short duration of FR1; (3) evaluating the relative permeability inside matrix pseudopressure at the initial condition. Accordingly, there are four possible parameters that affect the validity of these assumptions: initial fracture permeability, fracture half-length, fracture permeability modulus, and initial water saturation in the matrix. We conducted a sensitivity analysis on these four parameters.
based on Case 1 and the other 12 newly generated numerical cases by following the same analysis procedure above. The new cases adopted the same input parameters with Case 1 but different values for the analyzed parameters to see their effects on the interpreted results.

The initial fracture permeability is a key parameter that affects the accuracy of inverse analysis results. We generated four numerical cases with \( k_{fr} \) ranging from 800 md to 3000 md. The calculated \( k_{fr}, V_{fr} \), and the relative error using different methods are listed in Table 4-5. The results demonstrate that with the increasing \( k_{fr} \) in the simulation cases, the estimated \( V_{fr} \) and \( k_{fr} \) become closer to the input values. This is because the larger \( k_{fr} \) means the lower flow resistance for water and gas, leading to a smaller pressure and saturation gradient in the fracture. The assumption of uniform matrix influx and pore-volume averaged pressure and saturation will be more valid in our flowback model. On the other hand, the larger \( k_{fr} \) results in a shorter duration of FR1, which makes the calculation of \( \bar{p}_f \) and \( \bar{S}_{w,f} \) work well within the entire fracture domain.

The fracture half-length has an opposite effect to initial fracture permeability on the analysis results shown in Table 4-6. As the \( x_f \) increases from 500 ft to 1100 ft, the interpreted results using the water-phase and gas-phase flowback data deviate from the input values. The reason is that with the increasing \( x_f \), the pressure drop in the fracture needs more time to reach the outer boundary. The \( \bar{p}_f \) and \( \bar{S}_{w,f} \) should be calculated within the distance of investigation in the fracture, not the entire fracture domain during FR1. Furthermore, longer fracture half-length leads to more pressure and saturation gradient in the fracture, which causes more error associated with pseudotime definition and the assumption of uniform matrix influx.

Table 4-7 shows the analysis results of four cases with different fracture permeability modulus varying from \( 1 \times 10^{-4} \) psi\(^{-1} \) to \( 1 \times 10^{-3} \) psi\(^{-1} \). It can be seen that \( \gamma_f \) has little effect on the accuracy of the flowback model with the relative error of \( V_{fr} \) and \( k_{fr} \) both <10\% for most of the cases. The reason may be that a larger \( \gamma_f \) increases the nonlinearity of the fracture system by introducing the pressure dependent fracture permeability. More pressure and saturation gradient
are expected in the fracture, which causes more error in flowback analysis. However, the existence of $\gamma_f$ decreases fracture permeability during production, thus less flux from matrix due to the extra flow resistance in the fracture. It results in fewer errors of analysis results related to the two-phase matrix flow model. These errors offset with each other and make the effect of $\gamma_f$ not clear.

The effect of water influx on the calculated fracture properties is shown in Table 4-8. With the increasing initial water saturation in the matrix from 20% to 50%, the analysis results deviate more from the input values in the numerical simulation. This is because a larger $S_{w,m}$ allows more water influx from matrix. The assumption of evaluating $k_{rw,m}$ inside matrix pseudopressure at the initial condition will cause more error in the matrix flow model. Furthermore, the larger $S_{w,m}$ impedes gas feed from matrix to fracture and increases the fracture pressure gradient (Jia et al., 2018). Therefore, the estimated error will increase with larger $S_{w,m}$. Similarly, we can also observe more error by decreasing the exponent $N_w$ of Corey’s water-phase relative permeability curve for the matrix, since the less $N_w$ leads to higher $k_{rw,m}$.

<table>
<thead>
<tr>
<th>Set $k_{fi}$ (md)</th>
<th>Calculated values</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water-phase model</td>
<td>Gas-phase model</td>
</tr>
<tr>
<td></td>
<td>BDF</td>
<td>FMB</td>
</tr>
<tr>
<td>$V_f$ (Mcf)</td>
<td>$k_{fi}$ (md)</td>
<td>$V_f$ (Mcf)</td>
</tr>
<tr>
<td>800</td>
<td>146.3 917</td>
<td>145.0 901</td>
</tr>
<tr>
<td>1000</td>
<td>146.8 1130</td>
<td>146.6 1113</td>
</tr>
<tr>
<td>2000</td>
<td>150.3 2207</td>
<td>149.9 2194</td>
</tr>
<tr>
<td>3000</td>
<td>150.8 3288</td>
<td>150.8 3303</td>
</tr>
</tbody>
</table>

Table 4-5: Effect of initial fracture permeability on the calculated fracture properties. The expected value of $V_{fi}$ = 150 Mcf and the values of other input parameters are the same as Case 1.

<table>
<thead>
<tr>
<th>Set $x_f$ (ft)</th>
<th>Calculated values</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water-phase model</td>
<td>Gas-phase model</td>
</tr>
<tr>
<td></td>
<td>BDF</td>
<td>FMB</td>
</tr>
<tr>
<td>$V_f$ (Mcf)</td>
<td>$k_{fi}$ (md)</td>
<td>$V_f$ (Mcf)</td>
</tr>
<tr>
<td>500</td>
<td>150.8 3288</td>
<td>150.8 3303</td>
</tr>
<tr>
<td>700</td>
<td>209.4 3347</td>
<td>208.7 3290</td>
</tr>
<tr>
<td>900</td>
<td>264.3 3353</td>
<td>263.9 3330</td>
</tr>
<tr>
<td>1100</td>
<td>310.0 3491</td>
<td>308.1 3450</td>
</tr>
</tbody>
</table>

Table 4-6: Effect of fracture half-length on the calculated fracture properties. The expected value of $k_{fi}$ = 3000 md and the values of other input parameters are the same as Case 1.
The sensitivity analysis shows that the accuracy of the proposed semianalytical method depends on the validity of the three key assumptions in the mathematical development, which are mainly controlled by \( k_f, x_f, \) and \( S_w,m \). The effect of \( \gamma_f \) on the analysis results is not clear since the errors are offsetting with each other, and needs further investigation in a future study. Although there are some deviations of the calculated fracture properties from the set values, most of the relative errors are less than 10\%, which is acceptable for engineering purposes.

The validation mentioned above assumes the \( V_f \) and \( \gamma_f \) are the known parameters. Here, we used Case 1 to validate the workflow without reconciliation of long-term RTA in Figure 4-3 by calculating \( V_{fr}, k_f, \) and \( \gamma_f \) iteratively. \( C_f \) is treated as a known parameter in the workflow validation since the given numerical simulator cannot model the reduction of fracture half-length. Based on the analysis results in Table 4-4, the BDF model shows more accuracy than FMB model for synthetic cases. Therefore, we used the BDF model to conduct the straight-line analysis in the following validation. We first set \( \gamma_f \) as a given value to check the convergence of \( V_{fr} \). An initial

---

### Table 4-7: Effect of fracture permeability modulus on the calculated fracture properties. The expected value of \( V_f = 150 \text{ Mcf}, k_f = 3000 \text{ md}, \) and the values of other input parameters are the same as Case 1.

<table>
<thead>
<tr>
<th>Set ( \gamma_f ) (10^-4 psi^-1)</th>
<th>Calculated values</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water-phase model</td>
<td>Gas-phase model</td>
</tr>
<tr>
<td></td>
<td>BDF</td>
<td>FMB</td>
</tr>
<tr>
<td>( V_f ) (Mcf) &amp; ( k_f ) (md)</td>
<td>( V_f ) (Mcf) &amp; ( k_f ) (md)</td>
<td>( V_f ) (Mcf) &amp; ( k_f ) (md)</td>
</tr>
<tr>
<td>1</td>
<td>150.9 &amp; 3301</td>
<td>150.8 &amp; 3297</td>
</tr>
<tr>
<td>3</td>
<td>150.8 &amp; 3288</td>
<td>150.8 &amp; 3303</td>
</tr>
<tr>
<td>7</td>
<td>151.1 &amp; 3366</td>
<td>150.7 &amp; 3304</td>
</tr>
<tr>
<td>10</td>
<td>150.9 &amp; 3356</td>
<td>150.7 &amp; 3297</td>
</tr>
</tbody>
</table>

### Table 4-8: Effect of initial water saturation in the matrix on the calculated fracture properties. The expected value of \( V_f = 150 \text{ Mcf}, k_f = 3000 \text{ md}, \) and the values of other input parameters are the same as Case 1.

<table>
<thead>
<tr>
<th>Set ( S_{w,m} ) (%)</th>
<th>Calculated values</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water-phase model</td>
<td>Gas-phase model</td>
</tr>
<tr>
<td></td>
<td>BDF</td>
<td>FMB</td>
</tr>
<tr>
<td>( V_f ) (Mcf) &amp; ( k_f ) (md)</td>
<td>( V_f ) (Mcf) &amp; ( k_f ) (md)</td>
<td>( V_f ) (Mcf) &amp; ( k_f ) (md)</td>
</tr>
<tr>
<td>20</td>
<td>150.8 &amp; 3236</td>
<td>150.8 &amp; 3239</td>
</tr>
<tr>
<td>30</td>
<td>150.8 &amp; 3288</td>
<td>150.8 &amp; 3303</td>
</tr>
<tr>
<td>40</td>
<td>151.5 &amp; 3370</td>
<td>151.3 &amp; 3409</td>
</tr>
<tr>
<td>50</td>
<td>153.7 &amp; 3632</td>
<td>153.5 &amp; 3635</td>
</tr>
</tbody>
</table>
guess of \( V_f = 300 \) Mcf is set to start the workflow. Figure 4-10 shows the converging procedure of \( \bar{p}_f \) and \( \bar{S}_{w,f} \) as an example. After four iterations, the relative error between \( V_{f,old} \) and \( V_{f,new} \) reaches the tolerance \( \epsilon_{V_f} = 0.1\% \) and the relative error between \( V_{f,w} \) and \( V_{f,g} \) falls into the acceptable range 5%. The calculated \( V_f = 150.9 \) Mcf is very close to the set value 150 Mcf. Table 4-9 lists the validation results for \( V_f \) iteration. Furthermore, we tested the workflow by setting \( V_f \) to be known and updating \( \gamma_f \). An initial guess of \( 1 \times 10^{-3} \) psi\(^{-1}\) was set for \( \gamma_f \) to start the workflow and \( \gamma_f \) was updated by deducting \( 1 \times 10^{-4} \) psi\(^{-1}\) during each iteration. Once \( \gamma_f \) was set to \( 3 \times 10^{-4} \) psi\(^{-1}\), the relative error of \( k_f \) reached the tolerance value of \( \epsilon_{\gamma_f} = 5\% \). The calculated \( V_f = 150.9 \) Mcf and \( k_f = 3212.5 \) md are both very close to the set values 150 Mcf and 3000 md. Table 4-10 lists the validation results for \( \gamma_f \) iteration. Accordingly, the workflow is verified to estimate fracture properties accurately.

![Figure 4-10: Calculated average pressure and water saturation in the fracture during iterations of the workflow.](image)

**Table 4-9**: Validation results of the workflow by iterating \( V_f \) with the known \( \gamma_f \)．

<table>
<thead>
<tr>
<th>Iteration #</th>
<th>Input ( V_f ) (Mcf)</th>
<th>Output ( V_{f,w} ) (Mcf)</th>
<th>Output ( V_{f,g} ) (Mcf)</th>
<th>( V_{f,new} ) (Mcf)</th>
<th>Relative error between ( V_f ) and ( V_{f,new} ), %</th>
<th>Relative error between ( V_{f,w} ) and ( V_{f,g} ), %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300.0</td>
<td>157.1</td>
<td>129.4</td>
<td>143.3</td>
<td>52.24</td>
<td>17.61</td>
</tr>
<tr>
<td>2</td>
<td>143.3</td>
<td>149.1</td>
<td>154.4</td>
<td>151.8</td>
<td>-5.93</td>
<td>-3.52</td>
</tr>
<tr>
<td>3</td>
<td>151.8</td>
<td>151.5</td>
<td>150.1</td>
<td>150.8</td>
<td>0.63</td>
<td>0.90</td>
</tr>
<tr>
<td>4</td>
<td>150.8</td>
<td>151.2</td>
<td>150.6</td>
<td>150.9</td>
<td>-0.06</td>
<td>0.39</td>
</tr>
</tbody>
</table>
Table 4-10: Validation results of the workflow by iterating $\gamma_f$ with the known $V_{fi}$.

<table>
<thead>
<tr>
<th>Iteration #</th>
<th>Input $\gamma_f$ (psi$^{-1}$)</th>
<th>Output $V_{fi,w}$ (Mcf)</th>
<th>Output $V_{fi,g}$ (Mcf)</th>
<th>Relative error between $V_{fi}$ and $V_{fi,new}$, %</th>
<th>Output $k_{fi,w}$ (md)</th>
<th>Output $k_{fi,g}$ (md)</th>
<th>Relative error between $k_{fi,w}$ and $k_{fi,g}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1\times10^{-3}$</td>
<td>151.2</td>
<td>151.9</td>
<td>1.04</td>
<td>3378.4</td>
<td>3197.7</td>
<td>5.65</td>
</tr>
<tr>
<td>2</td>
<td>$7\times10^{-4}$</td>
<td>151.0</td>
<td>151.5</td>
<td>0.85</td>
<td>3339.2</td>
<td>3171.7</td>
<td>5.28</td>
</tr>
<tr>
<td>3</td>
<td>$5\times10^{-4}$</td>
<td>150.9</td>
<td>151.3</td>
<td>0.72</td>
<td>3313.3</td>
<td>3154.3</td>
<td>5.04</td>
</tr>
<tr>
<td>4</td>
<td>$3\times10^{-4}$</td>
<td>150.8</td>
<td>151.0</td>
<td>0.59</td>
<td>3288.0</td>
<td>3136.9</td>
<td>4.81</td>
</tr>
</tbody>
</table>

4.4.2 Field Application

In this section, we tested the applicability of the semianalytical method by analyzing flowback and production data from an MFHW in Marcellus Shale, Appalachian Basin, Pennsylvania, USA. This field case was analyzed by our previous work (Zhang and Emami-Meybodi, 2020a) without the consideration of water influx from matrix. In this study, we revisited this example using the new method to see how the water influx impacts the final results.

Table 4-11 shows the input reservoir and fracture properties for this field case. Due to the lack of experimental data for the relative permeability curve, we applied the straight-line relative permeability for HF and the Corey formulation ($N_w=2.8, N_g=1$) for matrix based on the shale sample experiment conducted by Daigle et al. (2015), which are shown in Figure 4-11. This well was hydraulically fractured into 22 stages with 5 clusters per stage. Considering that usually 40% - 60% of HF are active and contribute to production after stimulation, we set 60% as the fracture efficiency in calculation. After stimulation, this well was flowed back for 24 days and then produced for more than eight months without shut-ins. The fracture and matrix systems are assumed to have the same initial pressure during flowback period. When it comes to the long-term production period, we used the initial reservoir pressure to perform long-term production data analysis. Recently, Clarkson et al. (2017) proposed a method to estimate the initial conditions of fluid pressure and saturations at the start of flowback by coupling DDA model with frac model.
Later, Zhang et al. (2018) expanded the enhanced fracture region in Clarkson’s method to a dual-porosity system and fully coupled the frac model with DDA model to provide a more rigorous estimation of initial conditions for flowback. These techniques can be applied to the proposed RTA method to provide an estimation of initial conditions and improve the interpretation results.

Table 4-11: Field data from a Marcellus MFHW.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial fracture pressure, $p_{fi}$ (psi)</td>
<td>6580</td>
</tr>
<tr>
<td>Initial reservoir pressure, $p_{ri}$ (psi)</td>
<td>4480</td>
</tr>
<tr>
<td>Reservoir temperature, $T$ (°F)</td>
<td>140</td>
</tr>
<tr>
<td>Specific gravity, $SG_0$</td>
<td>0.6</td>
</tr>
<tr>
<td>Water compressibility, $C_w$ (psi⁻¹)</td>
<td>2.4×10⁻⁶</td>
</tr>
<tr>
<td>Initial fracture porosity, $\phi_{fi}$ (%)</td>
<td>60</td>
</tr>
<tr>
<td>Initial matrix porosity, $\phi_{mi}$ (%)</td>
<td>7.75</td>
</tr>
<tr>
<td>Matrix permeability, $k_m$ (md)</td>
<td>1×10⁻⁵</td>
</tr>
<tr>
<td>Matrix compressibility, $C_m$ (psi⁻¹)</td>
<td>1×10⁻⁶</td>
</tr>
<tr>
<td>Reservoir thickness, $h$ (ft)</td>
<td>80</td>
</tr>
<tr>
<td>Number of active fractures</td>
<td>66</td>
</tr>
<tr>
<td>Fracture width, $w_f$ (ft)</td>
<td>0.04</td>
</tr>
<tr>
<td>Initial water FVF, $B_{wi}$ (ft³/STB)</td>
<td>5.78</td>
</tr>
<tr>
<td>Initial fracture-water saturation, $S_{w,fi}$ (%)</td>
<td>98</td>
</tr>
<tr>
<td>Initial matrix-water saturation, $S_{w,mi}$ (%)</td>
<td>28</td>
</tr>
<tr>
<td>Irreducible water saturation in the matrix, $S_{wir}$ (%)</td>
<td>20</td>
</tr>
<tr>
<td>Residual gas saturation in matrix, $S_{grw}$ (%)</td>
<td>15</td>
</tr>
<tr>
<td>Water viscosity, $\mu_w$ (cp)</td>
<td>0.54</td>
</tr>
<tr>
<td>Water density, $\rho_w$ (lbm/ft³)</td>
<td>62</td>
</tr>
</tbody>
</table>

Figure 4-11: Relative permeability curves for (a) HF and (b) shale matrix (Daigle et al., 2015).
We applied the proposed semianalytical method and the entire workflow to analyze flowback data and long-term production data shown in Figure 4-12 by setting the initial guess of $V_{fi}, \gamma_f,$ and $C_f$ to be 300 Mcf, $1 \times 10^{-3}$ psi$^{-1}$, and $7 \times 10^{-5}$ psi$^{-1}$. The error tolerance for $V_{fi}, \gamma_f,$ and $C_f$ are all set to be 2%. We chose the BDF model to calculate fracture properties based on the comparison analysis with FMB model in Table 4-4. After convergence, Figure 4-13a shows the average fracture pressure and permeability predicted by the obtained $k_f$ and $\gamma_f$. By comparing the matrix influx rate in Figure 4-13b with the flowback history in Figure 4-12a, we find that most of the water production comes from the fracture water depletion and the contribution of water influx from matrix is very small. Whereas, gas production mainly comes from matrix influx, which can be verified by the same behavior of production rate and influx rate history in Figure 4-13a and Figure 4-13b, respectively. A unit-slope straight line in the proposed two-phase diagnostic plot (Figure 4-13c-d) identifies the FR2 for water-phase after 59 hrs and for gas-phase after 47 hrs. The noise in Figure 4-13c-d is due to the error when calculating the derivative of rate normalized pressure. Whereas, the noise in the early portion of Figure 4-13e-f sources from the fluctuation of flowrates and the lack of enough data during early flowback period. By extracting the slope and intercept of the straight lines as shown in Figure 4-13e-f during FR2, we calculated the initial fracture volume $V_{fi,w,BDF} = 183.8$ Mcf, $V_{fi,g,BDF} = 175.2$ Mcf, and initial fracture permeability $k_{fi,w,BDF} = 901.5$ md, $k_{fi,g,BDF} = 883.7$ md, using the formula in Table 4-2. The results obtained using water-phase and gas-phase data confirm the accuracy of our method.
Figure 4-12: Production history of gas flowrate, water flowrate, and bottomhole pressure for the field case during (a) flowback and (b) long-term production (Zhang and Emami-Meybodi, 2020a).

Figure 4-13: Flowback analysis after the convergence of $V_{fi}$ and $k_{fi}$: (a) Average pressure and permeability in the fracture during flowback period. (b) Water and gas influx rate from matrix. (c) Two-phase water diagnostic plot. (d) Two-phase gas diagnostic plot. (e) Two-phase water BDF specialty plot. (f) Two-phase gas BDF specialty plot.
Table 4-12 shows the calculated HF properties for the field example using the new semianalytical method and our previous flowback model (Zhang and Emami-Meybod, 2020a). The relative errors of interpreted $V_{fi}$, $k_{fi}$, and $\gamma_f$ between the newly developed model and our previous model are about 19%, 27%, and 6%, respectively. The analysis results reveal that ignoring water influx from matrix and the HF dynamics will overestimate the calculated values of $V_{fi}$, $k_{fi}$, and $\gamma_f$, with the relative errors up to 27%. The observed difference mainly comes from the constraint we set for the fracture volume reduction from flowback to long-term production period. However, the water influx from matrix has less impact on the error. This is because for the field case, $S_{w,mi}=28\%$ and is close to critical saturation based on the chosen relative permeability curves (Daigle et al., 2015), as shown in Figure 4-11b. Therefore, the contribution of water influx from matrix remains relatively small. It is expected the relative error increases in the shales with higher initial water saturation and higher water relative permeability because the water influx will have more impact on the results of flowback analysis, and hence the proposed flowback model will show more superiority.

As a part of workflow, we also performed long-term production data analysis to evaluate fracture properties during the production of hydrocarbon. Considering that gas is the dominant phase in long-term production, we used the single-phase gas RTA model for transient linear flow to extract $x_f$ (Nobakht and Clarkson, 2012). To evaluate pseudo-variables, we calculated average pressure in matrix DOI using the material balance equation. Following the procedures shown in Figure 4-3, the calculated final fracture volume $V_{f,LT}=118.6$ Mcf. We also used the single-phase gas MFHW model in commercial software (IHS, 2019) to obtain the fracture properties by history matching the flow rate and BHP. Table 4-12 shows the calculated $V_{f,LT}=123.9$ Mcf from history matching, which shows good consistency and accuracy with RTA technique. To obtain $V_f$ from flowback data, we selected 1200 psi as the ultimate $p_f$ at the end of long-term production, since the BHP drops to around 1200 psi after three months of production and remains almost steady for
the following five months. The comparison of $V_{f,LT}$ calculated from long-term production data against $V_f = 123.1$ Mcf predicted from the flowback model provides an important constraint of $C_f$ to be $7 \times 10^{-5}$ psi$^{-1}$, which is used for characterizing HF dynamics.

We demonstrated the calculated permeability modulus from the field case in the following two aspects. Firstly, with the formulation of pressure-dependent fracture permeability and the interpreted $k_f$ and $\gamma_f$, we estimated the final $k_f$ to be 15.8 md when $\bar{p}_f$ drops to 1200 psi, (i.e., the final BHP). The estimated final $k_f$ falls into the range of permeability for the fractures without proppant and with one layer of glass beads, according to the laboratory experiments on shale fracture by Tan et al. (2018). This observation is consistent with the industry experience that the proppants usually crush after long-term production of hydrocarbon, which leads to the partial fracture closure. Furthermore, we also noticed that the estimated average $k_f$, 46.4 md, obtained from history matching is on the same order of magnitude with the ultimate $k_f$, 15.8 md, predicted from flowback analysis. The difference may because of the higher fracture pressure averaged throughout the long-term production period than the ultimate BHP. By reconciling short-term flowback data and long-term production data analysis results, we can see a significant decrease in $V_f$ from 179.5 Mcf to 123.1 Mcf and in $k_f$ from 892.6 md to 15.8 md due to HF closure.

Table 4-12: Interpretation results from flowback analysis and long-term production data analysis.

<table>
<thead>
<tr>
<th>Fracture properties</th>
<th>Flowback analysis</th>
<th>Long-term production data analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Zhang and Emami-Meybodi (2020a)</td>
<td>Semianalytical Method</td>
</tr>
<tr>
<td>$V_f$, Mcf</td>
<td>221.3</td>
<td>179.5</td>
</tr>
<tr>
<td>$k_f$, md</td>
<td>1227.5</td>
<td>892.6</td>
</tr>
<tr>
<td>$\gamma_f$, psi$^{-1}$</td>
<td>8$\times$10$^{-4}$</td>
<td>7.5$\times$10$^{-4}$</td>
</tr>
<tr>
<td>$C_f$, psi$^{-1}$</td>
<td>-</td>
<td>7$\times$10$^{-5}$</td>
</tr>
<tr>
<td>$V_f$ or $V_{f,LT}$, Mcf</td>
<td>-</td>
<td>123.1</td>
</tr>
<tr>
<td>Final $k_f$, md</td>
<td>-</td>
<td>15.8</td>
</tr>
<tr>
<td>Average $k_f$, md</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
4.5 Discussion and Recommendation

The mathematical procedure and assumption made to develop the semianalytical method created some inherent limitations. First, the error of interpreted $k_{fi}$ is usually larger than that of $V_{fi}$ and can be even >10% in some cases. This is because the $k_{fi}$ value comes from the intercept of the straight line, whereas $V_{fi}$ is calculated from the slope of the straight line. The error in the straight line has much more impact on the value of intercept than that of slope. To reduce the error of flowback analysis for large $S_{w,mi}$, we need to apply a more rigorous pressure-saturation relationship in the two-phase matrix model. Recently, an empirical function (Clarkson and Qanbari, 2016a) and an approximation relation at limiting cases (Behmanesh, 2016) were proposed to provide the saturation and pressure relation during two-phase transient linear flow. Material balance method could help to obtain the saturation and pressure relation for boundary dominated flow in matrix. These approaches will be considered in future work to improve the flowback model and extend the matrix flow model from IALF to BDF due to the inter-fracture depletion in our short-term research plan. Another assumption made in the derivation of two-phase matrix model is neglecting the gradient of matrix influx rate by considering a uniform influx from matrix along HF. In other words, the semianalytical model is applicable to the system with a small pressure gradient in the fracture. This limitation emanates from the introduction of pseudotime to linearize the governing equation. To relax the assumption, one can introduce the new transformations of pseudo variables (Yuan et al., 2019) to linearize the diffusivity equation.

Second, desorption was not included in the analysis, which is reasonable during early-time flowback but may impact the accuracy of analysis results when the pressure drop in the shale matrix becomes significant. A following work shall include gas desorption in the proposed method based on current shale gas models (Cao et al., 2017, Wang et al., 2018, Wang et al., 2019). Apart from Darcy flow, other transport mechanisms are usually observed in the shale gas reservoir, such as slip
flow, kerogen diffusion, surface diffusion, and Knudsen diffusion. In an extended study, one can apply the concept of apparent permeability, which is a form of converted permeability to account for the total gas influx due to these transport mechanisms, to consider all these flow regimes (Shi et al., 2013, Feng et al., 2019). Skin and wellbore storage effect shall be included in future studies, as well.

Third, as pressure drop below dew-point, condensate will dropout in the wellbore, fracture, or even formation near wellbore. The appearance of condensate will lead to the loss of gas relative permeability, gas flowrate, and well productivity (Safari-Beidokhti and Hashemi, 2016), further affecting the flow regime identification and the accuracy of straight-line analysis using the proposed method. To consider condensate in flowback, Kanfar and Clarkson (2016) applied a triple porosity simulation model to history match three-phase flowback and production data. Behmanesh et al. (2018b) proposed a generalized FMB model for three-phase flow. Future study shall also incorporate three-phase flow during flowback when condensate appears in shale reservoir or wellbore (the current work only accounts for two-phase flow of water and gas).

Finally, although the proposed method could provide an estimation of HF petrophysical parameters, such as fracture compressibility and permeability modulus, the uncertainties in other reservoir and fracture inputs still limit the accuracy of HF characterization using this method. For example, the estimation of matrix permeability can vary by orders of magnitude depending on techniques used to determine it. It is highly recommended to provide the laboratory-derived pressure-dependent fracture permeability and porosity, matrix permeability and porosity, as well as the relative permeability data for propped and unpropped fracture and reservoir to assist the flowback analysis in practical field application. Since these laboratory geomechanical data are highly uncertain and of great value in constraining interpretation results (Zhang et al., 2018).
4.6 Conclusions

We developed a semianalytical method that can be used for straight-line analysis to characterize fracture properties and HF dynamics for MFHWs completed in shale gas reservoir. The method incorporates the two-phase matrix influx into the pseudotime definition in the fracture and considers the pressure-dependent relative permeability, fluid, and rock properties. With the corrected pseudo-variables, we proposed a two-phase flowback diagnostic plot to identify different flow regimes for the coupled fracture/matrix system. We also proposed a workflow to iteratively calculate fracture properties, such as initial fracture volume, initial fracture permeability, fracture compressibility, and permeability modulus, and to quantify HF dynamics by reconciling flowback and long-term production data. The interpretation result from the proposed method could provide important pre-estimation of fracture attributes before history matching with numerical simulation. We demonstrated the accuracy of the flowback model with numerical simulations and verified the practical application of the semianalytical method through a field example of MFHW in Marcellus Shale. The key findings of this work are:

- The accuracy of the new method is influenced by initial fracture permeability, fracture half-length, and the initial water saturation in matrix. With increasing initial fracture permeability, the analysis results will be more accurate. Whereas calculation error increases with increasing fracture half-length and initial water saturation in matrix.
- Ignoring water influx from matrix and the constraint for the fracture volume loss during flowback and long-term production will cause up to 27% error in calculations of fracture properties for the analyzed MFHW. It is expected that the error increases in the shales with higher initial water saturation and water relative permeability as water influx has more impact on the results of flowback analysis.
The field application results reveal that fracture closure causes the reduction of the fracture volume by 30% and the fracture permeability by over an order of magnitude after 24 days of flowback and eight months of production.

### 4.7 Nomenclatures

- \(a_{j,BDF}\) = slope of \(RNP_j\) vs. \(t_{sp,j}\) plot
- \(a_{j,FMB}\) = slope of \(PNR_j\) vs. \(Q_{j,PN}\) plot
- \(b_{j,BDF}\) = intercept of \(RNP_j\) vs. \(t_{sp,j}\) plot
- \(b_{j,FMB}\) = intercept of \(PNR_j\) vs. \(Q_{j,PN}\) plot
- \(B_{g,f}\) = gas FVF in the fracture, \(ft^3/scf\)
- \(B_{g,m}\) = gas FVF in the matrix, \(ft^3/scf\)
- \(B_{w,f}\) = water FVF in the fracture, \(ft^3/STB\)
- \(B_{w,m}\) = water FVF in the matrix, \(ft^3/STB\)
- \(C_f\) = fracture compressibility, \(psi^{-1}\)
- \(C_m\) = matrix compressibility, \(psi^{-1}\)
- \(C_{j,\text{eff}}\) = effective compressibility in the fracture, \(psi^{-1}\)
- \(C_{j,\text{efm}}\) = effective compressibility in the matrix, \(psi^{-1}\)
- \(C_{j,f}\) = fluid compressibility in the fracture, \(psi^{-1}\)
- \(C_{j,m}\) = fluid compressibility in the matrix, \(psi^{-1}\)
- \(C_t\) = total compressibility, \(psi^{-1}\)
- \(\text{DRNP}_g\) = derivative of gas-phase rate normalized pseudopressure, \(psi \cdot \text{day}/(Mscf \cdot \ln(\text{day}))\)
\[ DRNP_w = \text{derivative of water-phase rate normalized pseudopressure, } psi \cdot \text{day}/(STB \cdot \ln(\text{day})) \]

\[ h = \text{fracture height, ft} \]
\[ k_f = \text{fracture permeability, md} \]
\[ k_{fi,j,BDF} = \text{initial fracture permeability obtained from the BDF specialty plot, md} \]
\[ k_{fi,j,FMB} = \text{initial fracture permeability obtained from the FMB specialty plot, md} \]
\[ k_m = \text{matrix permeability, md} \]
\[ k_{roi} = \text{initial oil relative permeability, -} \]
\[ k_{rj,f} = \text{relative permeability in the fracture, -} \]
\[ k_{rj,m} = \text{relative permeability in the matrix, -} \]
\[ L_f = \text{fracture spacing, ft} \]
\[ m_{j,f}(p) = \text{pseudopressure in the fracture, psi} \]
\[ m_{j,m}(p) = \text{pseudopressure in the matrix, psi} \]
\[ N_g = \text{Corey gas exponent} \]
\[ N_w = \text{Corey water exponent} \]
\[ p_b = \text{base pressure, psi} \]
\[ p_f = \text{fracture pressure, psi} \]
\[ p_{gc} = \text{pressure for which critical gas saturation is achieved, psi} \]
\[ p_m = \text{matrix pressure, psi} \]
\[ p_p = \text{pseudopressure for water or gas, psi} \]
\[ p_{po} = \text{oil pseudopressure, psi} \]
\[ p_{sc} = \text{pressure at standard condition, psi} \]
\[ p_{wf} = \text{bottomhole pressure, psi} \]
PNR\(_g\) = gas-phase pseudopressure normalized rate, \(Mscf/(psi \cdot day)\)

PNR\(_w\) = water-phase pseudopressure normalized rate, \(STB/(psi \cdot day)\)

\(q_g\) = gas flowrate from the whole fracture at surface condition, \(Mscf/day\)

\(q_o\) = oil production rate, \(STB/day\)

\(q_w\) = water flowrate from the whole fracture at surface condition, \(STB/day\)

\(q_{g,f}\) = gas downhole flowrate at a specific location \(x\) in the fracture, \(Mcf/day\)

\(q_{g,m}\) = gas downhole flowrate at a specific location \(y\) in the matrix, \(Mcf/day\)

\(q_{w,f}\) = water downhole flowrate at a specific location \(x\) in the fracture, \(bbl/day\)

\(q_{w,m}\) = water downhole flowrate at a specific location \(y\) in the matrix, \(bbl/day\)

\(q_{g,inf}\) = gas influx rate from matrix at surface condition, \(Mscf/day\)

\(q_{w,inf}\) = water influx rate from matrix at surface condition, \(STB/day\)

\(Q_g\) = cumulative gas production from the whole fracture at surface condition, \(scf\)

\(Q_{g,inf}\) = cumulative gas influx from matrix at surface condition, \(Mscf\)

\(Q_w\) = cumulative water production from the whole fracture at surface condition, \(STB\)

\(Q_{w,inf}\) = cumulative water influx from matrix at surface condition, \(STB\)

\(Q_{j,p,N}\) = normalized cumulative production, \(ft^3\)

RNP\(_g\) = gas-phase rate normalized pseudopressure, \(psi \cdot day/Mscf\)

RNP\(_w\) = water-phase rate normalized pseudopressure, \(psi \cdot day/STB\)

\(S_{oi}\) = initial oil saturation, -

\(S_{j,f}\) = phase saturation in the fracture, -

\(S_{j,m}\) = phase saturation in the matrix, -

\(t\) = time, \(day\)

\(T\) = reservoir temperature, \(R\)

\(T_{sc}\) = temperature at standard condition, \(R\)
\( t_{ao} \) = oil pseudotime, day

\( t_{cao} \) = oil material balance pseudotime, day

\( t_{MBp} \) = gas-phase material balance pseudotime, day

\( t_p \) = gas-phase pseudotime, day

\( t_{p,j,f} \) = pseudotime in the fracture, day

\( t_{p,j,m} \) = pseudotime in the matrix, day

\( t_{p,w} \) = water-phase pseudotime, \( psi \cdot day/cp \)

\( t_{sp,j} \) = superposition pseudotime in the fracture, day

\( V_{fi} \) = initial pore volume of half fracture, \( ft^3 \)

\( V_{j,mi} \) = initial pore volume of quarter fracture-matrix element, \( ft^3 \)

\( w_f \) = fracture width, \( ft \)

\( x \) = location in the fracture, \( ft \)

\( x_f \) = fracture half-length, \( ft \)

\( V_{fi,j,BDF} \) = initial fracture volume obtained from the BDF specialty plot, \( ft \)

\( V_{fi,j,FMB} \) = initial fracture volume obtained from the FMB specialty plot, \( ft \)

\( y \) = location in the matrix, \( ft \)

\( y_{j,inv} \) = distance of investigation in the matrix, \( ft \)

\( Z_f \) = gas deviation factor in the fracture, -

\( Z_m \) = gas deviation factor in the matrix, -

condition, -

\( \alpha \) = oil flowing efficiency parameter

\( \alpha_{OGW} \) = oil flowing efficiency parameter for oil, gas, and water

\( \alpha_{OW} \) = oil flowing efficiency parameter for oil and water

\( \alpha_j \) = unit conversion factor in \( a_{j,BDF} \)
\( \beta \) = oil storage parameter

\( \beta_j \) = unit conversion factor in \( b_{j,BDF} \)

\( \gamma_f \) = fracture permeability modulus, \( psi^{-1} \)

\( \gamma_m \) = matrix permeability modulus, \( psi^{-1} \)

\( \delta_j \) = unit conversion factor in \( q_{j,inf} \)

\( \epsilon_j \) = unit conversion factor in diagnostic plot solution

\( \epsilon_{cf} \) = error tolerance of \( C_f \)

\( \epsilon_{\gamma f} \) = error tolerance of \( \gamma_f \)

\( \epsilon_{vf} \) = error tolerance of \( V_{fi} \)

\( \phi_f \) = fracture porosity

\( \phi_m \) = matrix porosity

\( \mu_{j,f} \) = fluid viscosity in the fracture, \( cp \)

\( \mu_{j,m} \) = fluid viscosity in the matrix, \( cp \)

\( \mu_{oi} \) = initial oil viscosity, \( cp \)

\( \rho_{j,f} \) = fluid density in the fracture at downhole condition, \( lbm/ft^3 \)

\( \rho_{j,m} \) = fluid density in the matrix at downhole condition, \( lbm/ft^3 \)

\( \rho_{j,sc} \) = fluid density at standard condition, \( lbm/ft^3 \)

**Subscripts**

BDF = boundary dominated flow

FMB = flowing material balance

\( f \) = fracture

\( ff \) = final fracture

\( g \) = gas
\( i = \text{initial} \)

\( inf = \text{influx} \)

\( j = w \text{ or } g \)

\( LT = \text{long-term} \)

\( n = \text{the nth flowrate interval} \)

\( m = \text{matrix} \)

\( p = \text{pseudo} \)

\( sp = \text{superposition pseudo} \)

\( sc = \text{standard condition} \)

\( w = \text{water} \)

**Superscripts**

\( \overline{\text{--}} = \text{average} \)

### 4.8 Appendix A – Two-phase Fracture Model

The water or gas phase material balance equation for an infinitesimal fracture element is:

\[
q_{j,f} \rho_{j,f} \bigg|_{x+\Delta x} - q_{j,f} \rho_{j,f} \bigg|_x + 2h\Delta x \left( \frac{k_{j,m}}{\mu_{j,m}} \frac{\partial p_m}{\partial y} \right) \bigg|_{y=0} \cdot \rho_{j,m} \bigg|_{y=0} = \frac{\partial}{\partial t} \left( \Delta x w_f h \phi_f S_{j,f} \rho_{j,f} \right),
\]

(4-22)

where subscript \( j \) can be \( w \) for water phase or \( g \) for gas phase, \( \Delta x w_f h \) is the control volume of the infinitesimal fracture element, \( q_{j,f} \) is the downhole flowrate at a specific location \( x \) in the fracture, \( \rho_{j,f} \) is the fluid density in the fracture at the downhole condition, \( \rho_{j,m} \) is the fluid density in the matrix at the downhole condition.
Rearranging Eq. (4.22) by including the matrix influx term into the accumulation term on the right-hand side and dividing $\Delta x$ on both sides, the governing equation for each phase is given by:

$$
\frac{\partial}{\partial x} \left( q_{j,f} \rho_{j,f} \right) = \frac{\partial}{\partial t} \left( w_f h \phi_f S_{j,f} \rho_{j,f} - 2 h \int_0^t \left( \frac{k_{j,m}}{\mu_{j,m}} \frac{\partial p_m}{\partial y} \bigg|_{y=0} \right) \cdot \rho_{j,m} \bigg|_{y=0} \right) dt. \tag{4-23}
$$

Applying Darcy’s flowrate along the fracture and matrix and introducing the FVF $B_{j,f} = \rho_{j,sc} / \rho_{j,f}$:

$$
q_{j,f} \rho_{j,f} = \rho_{j,sc} w_f h \frac{k_{r,j,f} k_f}{\mu_{j,f} B_{j,f}} \frac{\partial p_f}{\partial x}, \tag{4-24}
$$

$$
q_{j,inf} B_{j,f} \rho_{j,f} = 4 h \int_0^x \left( \frac{k_{j,m}}{\mu_{j,m}} \frac{\partial p_m}{\partial y} \bigg|_{y=0} \right) \cdot \rho_{j,m} \bigg|_{y=0} \right) dx, \tag{4-25}
$$

where $\rho_{j,sc}$ is the fluid density at standard condition, $q_{j,inf}$ is the influx rate from the entire matrix domain at the surface condition. We assumed the uniform fluid influx from matrix, which means $\left. \frac{k_{j,m}}{\mu_{j,m}} \frac{\partial p_m}{\partial y} \right|_{y=0}$ is independent of $x$. By substituting Eqs. (4-24) and (4-25) into Eq. (4-23) and applying some cancelations, we have:

$$
\frac{\partial}{\partial x} \left( \rho_{j,sc} \frac{k_{r,j,f} k_f}{\mu_{j,f} B_{j,f}} \frac{\partial p_f}{\partial x} \right) = \frac{\partial}{\partial t} \left( \phi_f S_{j,f} \rho_{j,f} - \int_0^t \frac{q_{j,inf} B_{j,f} \rho_{j,f}}{2 h x_f w_f} dt \right). \tag{4-26}
$$

The right-hand side of Eq. (4-26) can be expanded with chain rule by introducing the definition of $C_{j,eff}$ the effective compressibility in the fracture, and $C_f$ the fluid compressibility in the fracture. Then, the simplified governing equation for the two-phase flow in the fracture is given by:

$$
\frac{\partial}{\partial x} \left( \frac{k_{r,j,f} k_f}{\mu_{j,f} B_{j,f}} \frac{\partial p_f}{\partial x} \right) = \frac{\phi_f C_{j,eff}}{B_{j,f}} \frac{\partial p_f}{\partial t}, \tag{4-27}
$$

$$
C_{j,eff} = S_{j,f} \left( C_{j,f} + C_f \right) + \frac{dS_{j,f}}{dp_f} - \frac{q_{j,inf} B_{j,f} \phi_f}{2 h x_f w_f} \frac{\partial t}{\partial p_f}. \tag{4-28}
$$
We first developed the fracture model under constant rate condition and later expanded to variable rate. The initial and boundary conditions for fracture BDF regime are given by:

\[
p_f(x, 0) = p_{fi}, \quad 2w_f h \left( \frac{\kappa_{rj,f} \kappa_f \partial p_f}{\mu_j,f B_{j,f}} \right) \bigg|_{x=0} = q_j, \quad \frac{\partial p_f}{\partial x} \bigg|_{x=x_f} = 0. \tag{4-29}
\]

Considering that all the fluid and rock properties in Eq. (4-27) are pressure dependent, we introduced the definitions of pseudopressure and pseudotime to linearize the governing equation:

\[
m_{j,f}(p) = \left( \frac{\mu_{j,f} B_{j,f} \kappa_f}{k_f} \right)_l \int_{p_b}^{p} \frac{k_f(p) \kappa_{rj,f}(\tilde{S}_{j,f})}{\mu_{j,f}(p) B_{j,f}(p)} \, dp,
\]

\[
t_{p,j,f} = \left( \frac{\phi_f \mu_{j,f} C_{j,eff}}{k_f} \right)_l \int_{p_b}^{p} \frac{k_f(\tilde{p}) k_{rj,f}(\tilde{S}_{j,f})}{\phi_f(\tilde{p}) \mu_{j,f}(\tilde{p}) C_{j,eff}(\tilde{p})} \, d\tilde{p}.
\]

The linearized governing equation together with initial and boundary conditions become:

\[
\frac{\partial^2 m_{j,f}(p_f)}{\partial x^2} = \frac{1}{\eta_{j,f}} \frac{\partial m_{j,f}(p_f)}{\partial t_{p,j,f}},
\]

\[
m_{j,f} \left( p_f(x, 0) \right) = m_{j,f} \left( p_{fi} \right), \quad \frac{\partial m_{j,f}(p_f)}{\partial x} \bigg|_{x=0} = q_j \frac{\mu_{j,f} B_{j,fi}}{2w_f h k_f}, \quad \frac{\partial m_{j,f}(p_f)}{\partial x} \bigg|_{x=x_f} = 0, \tag{4-33}
\]

where \( \eta_{j,f} = \left( \frac{k_f}{\phi_f \mu_{j,f} C_{j,eff}} \right)_l \).

The analytical solution of Eq. (4-32) subject to Eq. (4-33) is given by Miller (1962). By defining \( \text{RNP}_j = \left( m_{j,f}(p_0) - m_{j,f}(p_{w0}) \right)/q_j \) and following the same procedure in Zhang and Emami-Meybodi (2020a), we can reach the straight-line form analytical solution in the field unit:

\[
\text{RNP}_j = a_{j,BDF} t_{p,j,f} + b_{j,BDF}, \tag{4-34}
\]

\[
a_{j,BDF} = \alpha_j \left( \frac{B_{j,f}}{\nu_f C_{j,eff}} \right)_l, \tag{4-35}
\]

\[
b_{j,BDF} = \beta_j \frac{1}{h^2 w_f} \left( \frac{\nu_f \mu_{j,f} B_{j,f}}{\phi_f k_f} \right)_l, \tag{4-36}
\]

where,

\[
\nu_f = 2x_f w_f h \phi_{fi}, \tag{4-37}
\]

\[
\bar{B}_{w,f} = B_{w,fi} \left[ 1 + C_{w,f} \left( p_{fi} - \bar{p}_f \right) \right], \tag{4-38}
\]
\[ \bar{B}_{g,f} = \frac{p_{sc} T \bar{Z}_f}{T_{sc} \bar{p}_f}, \]  

where \( Z_f \) is the gas deviation factor in the fracture, \( T \) is the reservoir temperature, \( p_{sc} \) and \( T_{sc} \) are the pressure and temperature at standard condition.

If we plot RNP, vs. \( t_{sp,j} \) (i.e., BDF specialty plot), there will be a straight line shown during the boundary dominated flow. With the slope and intercept of the straight line, we can determine \( V_{fi} \) and \( k_{fi} \) from the BDF specialty plot:

\[ V_{fi,j,BDF} = \alpha_j \frac{1}{a_{j,BDF}} \left( \frac{B_{jf}}{c_{j,eff}} \right), \]  

\[ k_{fi,j,BDF} = \beta_j \frac{V_{fi,j,BDF} (\mu_{j,f} B_{jf})}{h^2 w_f^2 \phi_f (b_{j,BDF})}. \]

Equation (4-34) can also be rearranged in terms of pseudopressure normalized rate \( PNR_j = q_i / (m_{j,f} (p_i) - m_{j,f} (p_{wf})) \) using the approach introduced in Zhang and Emami-Meybodi (2020a):

\[ PNR_j = a_{j,FMB} Q_{j,pN} + b_{j,FMB}, \]  

\[ a_{j,FMB} = -\frac{1}{V_f b_{j,BDF}}, \]  

\[ b_{j,FMB} = \frac{1}{b_{j,BDF}}. \]

where \( Q_{j,pN} = V_f i \frac{m_{j,f} (p_i) - m_{j,f} (p_{wf})}{m_{j,f} (p_i) - m_{j,f} (p_{wf})} \) is the normalized cumulative production.

If we plot PNR, vs. \( Q_{j,pN} \) (i.e., FMB specialty plot), we can determine \( V_{fi} \) and \( k_{fi} \) based on the slope and intercept of the straight line of FMB specialty plot during the boundary dominated flow:

\[ V_{fi,j,FMB} = \frac{b_{j,FMB}}{a_{j,FMB}}, \]  

\[ k_{fi,j,FMB} = \beta_j \frac{b_{j,FMB} V_{fi,j,FMB} (\mu_{j,f} B_{jf})}{h^2 w_f^2 \phi_f (b_{j,BDF})}. \]

The derivations mentioned above are all under the constant rate condition (either constant water rate or constant gas rate). We can apply the superposition principle (Dake, 1983) to extend
the constant rate model to variable rate condition by replacing $q_j$ and $t_{pf,j}$ in Eq. (4-34) with $q_{j,N}$ and $t_{sp,j}$, where $t_{sp,j}$ is the superposition pseudotime defined by:

$$t_{sp,j} = \sum_{n=1}^{N} \frac{(q_{j,n} - q_{j,n-1})(t_{pf,j,N} - t_{pf,j,n-1})}{q_{j,N}}. \quad (4-47)$$

### 4.9 Appendix B – Two-phase Matrix Model

The water or gas material balance equation for an infinitesimal element in the matrix is expressed as:

$$q_{j,m} \rho_{j,m} \bigg|_{y+\Delta y} - q_{j,m} \rho_{j,m} \bigg|_{y} = \frac{\partial}{\partial t} \left( \Delta y x_f h \phi_m S_{j,m} \rho_{j,m} \right). \quad (4-48)$$

where $\Delta y x_f h$ is the control volume of the infinitesimal matrix element, $q_{j,m}$ is the downhole flowrate at a specific location $y$ in the matrix.

Rearranging Eq. (4-48) by applying Darcy’s flowrate along the matrix $q_{j,m} \rho_{j,m} = \rho_{j,sc} x_f h k_{rf,m} \mu_{j,m} B_{j,m} \phi_m S_{j,m} \rho_{j,m}$ and dividing $\Delta y$ on both sides, the governing equation is given by:

$$\frac{\partial}{\partial y} \left( \rho_{j,sc} k_{rf,m} \mu_{j,m} B_{j,m} \phi_m S_{j,m} \rho_{j,m} \right) = \frac{\partial}{\partial t} \left( \phi_m S_{j,m} \rho_{j,m} \right). \quad (4-49)$$

The right-hand side of Eq. (4-49) can be expanded with chain rule by introducing the definitions of $B_{j,m} = \rho_{j,sc} \phi_m$ (the FVF), $C_{j,m}$ (the fluid compressibility in the matrix), and $C_{j,efm}$ (the effective compressibility in the matrix). Then, the simplified governing equation for the two-phase flow in the matrix is given by:

$$\frac{\partial}{\partial y} \left( k_{rf,m} \mu_{j,m} B_{j,m} \phi_m S_{j,m} \rho_{j,m} \right) = \frac{\partial}{\partial t} \left( \phi_m C_{j,efm} \rho_{j,m} \right). \quad (4-50)$$

$$C_{j,efm} = S_{j,m} \left( C_{j,m} + C_m \right) + \frac{dS_{j,m}}{dp_m}. \quad (4-51)$$

The matrix model is coupled with the fracture model by assigning the average pressure in the fracture as the inner boundary condition. The initial and boundary conditions for matrix IALF are given by:
\[ p_m(y, 0) = p_{mi}, p_m(0, t) = \bar{p}_f, p_m(\infty, t) = p_{mi}. \]  

(4-52)

Considering that all the fluid and rock properties in Eq. (4-50) are pressure dependent, we introduced the definitions of pseudopressure and pseudotime to linearize the governing equation:

\[ m_{j,m}(p) = \left( \frac{\mu_{j,m} B_{j,m}}{k_m} \right)_i \int p \frac{k_m(p) k_{r,j,m}(s_{j,m})}{\mu_{j,m}(p) B_{j,m}(p)} \, dp, \]  

(4-53)

\[ t_{p,j,m} = \left( \frac{\phi_m \mu_{j,m} c_{j,efm}}{k_m} \right)_i \int_0^t \frac{k_m(p_m) k_{r,j,m}(s_{j,m})}{\phi_m(p_m) \mu_{j,m}(p_m) c_{j,efm}(p_m)} \, dr. \]  

(4-54)

The linearized governing equation together with the initial and boundary conditions become:

\[ \frac{\partial^2 m_{j,m}(p_m)}{\partial y^2} = \frac{1}{\eta_{j,m}} \frac{\partial m_{j,m}(p_m)}{\partial t_{p,j,m}}, \]  

(4-55)

\[ m_{j,m}(y, 0) = m_{j,m}(p_{mi}), m_{j,m}(0, t_{p,j,m}) = m_{j,m}(\bar{p}_f), m_{j,m}(\infty, t_{p,j,m}) = m_{j,m}(p_{mi}), \]  

(4-56)

where \( \eta_{j,m} = \left( \frac{k_m}{\phi_m \mu_{j,m} c_{j,efm}} \right)_i. \)

We first developed the matrix model under constant BHP condition and later expanded to variable BHP. The analytical solution of Eq. (4-55) subject to the homogeneous inner boundary condition of Eq. (4-56) is given by Wattenbarger et al. (1998):

\[ m_{j,m}(p_m) = m_{j,m}(\bar{p}_f) + \left[ m_{j,m}(p_{mi}) - m_{j,m}(\bar{p}_f) \right] \text{erf} \left( \frac{y}{2 \sqrt{\eta_{j,m} t_{p,j,m}}} \right). \]  

(4-57)

Applying Duhamel’s principle in the discontinuous form (Özisik et al., 1993), we can extend the Eq. (4-57) to the solution under the nonhomogeneous inner boundary condition of Eq. (4-56), which is under variable BHP condition:

\[ m_{j,m}(p_m) = m_{j,m}(p_{mi}) - \left[ m_{j,m}(p_{mi}) - m_{j,m}(p_b) \right]. \]

\[ \sum_{n=0}^{N-1} \text{erfc} \left( \frac{y}{2 \sqrt{\eta_{j,m} (t_{p,j,m} - t_{p,j,m,n})}} \right) \Delta \zeta_j(\tau)_n, \]  

(4-58)

\[ \Delta \zeta_j(\tau)_n = \zeta_j(\tau)_{n+} - \zeta_j(\tau)_{n-}, \]  

(4-59)
\[ \zeta_j(\tau) = \frac{m_{jm}(p_m) - m_{jm}(\bar{p}_f(\tau))}{m_{jm}(p_m) - m_{jm}(p_b)}. \] (4-60)

By substituting Darcy’s flowrate in Eq. (4-25) into Eq. (4-58), we obtain matrix influx rate in field unit:

\[ q_{j,\text{inf}} = \frac{[m_{jm}(p_m) - m_{jm}(p_b)] \chi_f \sqrt{K_{mi}} h \sqrt{(\phi_m \mu_{jm} C_{j,efm})_i}}{\delta_{jm,ml} B_{jm,mi} \sum_{n=0}^{N-1} \frac{\Delta \zeta_j(\tau)_n}{(t_{p,j,m} - t_{p,j,m,n})}}, \] (4-61)

\[ Q_{j,\text{inf}} = \int_0^t q_{j,\text{inf}} \, dt, \] (4-62)

\[ \bar{B}_{w,m} = B_{w,m} [1 + C_{w,m}(p_m - \bar{p}_m)], \] (4-63)

\[ \bar{B}_{g,m} = \frac{p_{sc} T Z_m}{T_{sc}} \] (4-64)

where \( Z_m \) is the gas deviation factor in the matrix.

### 4.10 Reference


CLARKSON, C., QANBARI, F., WILLIAMS-KOVACS, J. & ZANGANEH, B. Fracture propagation, leakoff and flowback modeling for tight oil wells using the dynamic drainage area concept. SPE Western Regional Meeting, 2017. Society of Petroleum Engineers.


Chapter 5

Analysis of Early-time Production Data from Multi-fractured Shale Gas Wells by Considering Multiple Transport Mechanisms Through Nanopores

5.1 Abstract

Analysis of early-time production and flowback data from multi-fractured shale gas wells is of critical importance in the characterization of hydraulic fractures (HF). However, the accurate estimation of HF properties in the current models is challenged by the complexity in multiple transport mechanisms and two-phase flow of shale gas reservoir. This study presents a new semi-analytical method to estimate HF attributes for shale gas wells exhibiting two-phase flow based on straight line analysis. The proposed method considers two-phase infinite acting linear flow (IALF) and boundary dominated flow (BDF) for both HF and matrix domains. In addition, matrix flow considers desorption, diffusion, slip flow, continuum flow, stress dependence rock, and adsorbed water film in the nanopores. A modified material balance equation is proposed to calculate average pressure in the fracture and distance of investigation (DOI) in the matrix by considering gas desorption and diffusion in the matrix domain. The accuracy of the proposed method is tested against numerical results obtained from commercial software (IMEX-CMG). The validation results confirm that the developed method can closely estimate initial fracture volume, permeability, and permeability modulus from early-time production data exhibiting two-phase IALF and BDF regimes. Furthermore, the results indicate that the consideration of desorption, diffusion, and slip

---

flow plays an important role in the modeling of gas transport in shale matrix. The impact of these transport mechanisms on production modeling as well as characterization of HF properties becomes significant when the matrix experiences substantial pressure drops. For the purpose of field application, the proposed method is used to analyze flowback data from a multi-fractured horizontal well. The field results reveal that the calculated fracture properties are consistent between the proposed method and the long-term production data analysis method.

5.2 Introduction

Understanding reservoir and fracture properties is one of the most important tasks the reservoir engineers are trying to work on all the time. Rate transient analysis (RTA), as an established reservoir engineering technique, is widely used in unconventional reservoirs to provide quantitative estimation of reservoir and fracture attributes due to its rigorousness in theory and convenience in application. Early RTA techniques focus on interpreting long-term production data, which exhibit single-phase oil or gas production behavior (Clarkson, 2013). In recent years, early-time production and flowback data demonstrate more and more potential in the characterization of hydraulic fractures (HF); hence, playing an important role in constraining long-term production data analysis results (Kanfar and Clarkson, 2016).

Numerous researchers have proposed mathematical models to analyze the early-time production data and extract HF properties. The main complexity in quantitative study is the feature of multiphase flow in the HF and matrix domains. The early work of flowback models focused on single-phase or two-phase HF depletion right after the well stimulation and ignored the fluid influx from matrix (Abbasi et al., 2012, Alkouh et al., 2014, Xu et al., 2016, Zhang and Emami-Meybodi, 2018, Zhang and Emami-Meybodi, 2020c). Further studies assume single-phase flow in the shale matrix and two-phase flow in the HF. The representative models can be categorized into the
analytical and numerical simulation models (Clarkson and Williams-Kovacs, 2013, Williams-Kovacs and Clarkson, 2013, Williams-Kovacs and Clarkson, 2016, Jia et al., 2019), the dynamic relative permeability models (Ezulike and Dehghanpour, 2014, Ezulike and Dehghanpour, 2015) and the flowing material balance (FMB) models (Xu et al., 2017, Jia et al., 2018, Zhang and Emami-Meybodi, 2020a). To consider two-phase flow in both HF and matrix domains, many methods are proposed to analyze the late-time flowback data, such as the semianalytical simulation model (Yang et al., 2017, Chen et al., 2018), the dynamic drainage area model (Clarkson and Qanbari, 2016b, Qanbari and Clarkson, 2016) and the improved FMB model (Zhang and Emami-Meybodi, 2020d, Zhang and Emami-Meybodi, 2020b).

In addition to the complexity in two-phase flow, the accurate estimation of HF properties using RTA models is challenged by the specific storage mechanism of adsorption and desorption in shale gas and coalbed methane (CBM) formations. The preliminary work incorporating mechanisms of sorption mainly focused on modeling single-phase flow. For example, Bumb and McKee (1988) proposed an analytical model for single-phase gas flow in CBM and shale formation by defining desorption compressibility, which considers the contribution of desorption to the effective compressibility. Their work shows that the effect of desorption on total production is substantial and could be reflected in the increased compressibility. Gerami et al. (2007) adopted the desorption compressibility proposed by Bumb and McKee (1988) and extended the traditional RTA type curve of flowing material balance to the analysis of production data in dry CBM reservoir. Their model assumes instantaneous gas desorption in the formation, i.e., equilibrium sorption prevails.

More recently, numerous studies published by Clarkson consider the sorption mechanism in modeling two-phase flow inside HF. In the early work by Clarkson et al. (2008), an RTA model is proposed to analyze two-phase production data from CBM wells by considering relative permeability in gas pseudopressure and desorption in total compressibility. Later, Clarkson and
Williams-Kovacs (2013) provided a method to interpret HF properties from two-phase flowback data in the saturated shale reservoir. The method includes desorption effects in both total compressibility and MBE but didn’t consider pressure-dependent permeability and relative permeability into pseudopressure definition. In an extended study, Williams-Kovacs and Clarkson (2016) improved the MBE to include fracture closure and incorporated pressure-dependent porosity and permeability into the modified pseudotime.

Different from the works above considering sorption mechanism from the macroscopic view, recent studies focused on the microscopic scale by incorporating desorption into the reduction of organic nanopore size (Wu et al., 2016). Similarly, Sun et al. (2018) coupled the positive effect of matrix shrinkage due to desorption and negative effect of stress dependence into the absolute permeability, and proposed an FMB model for the undersaturated CBM by defining a critical desorption pressure.

Apart from storage mechanism in shale, the specific gas transport mechanisms also play an important role in modeling. Considering that the transport mechanisms could be different for pore scales from nano to micro meter, it is inappropriate to simply use Darcy’s law to describe gas flow in the shale matrix (Shi et al., 2014, Mi et al., 2019, Yang et al., 2020). The most popular approach to incorporate the gas transport mechanisms is modifying the matrix permeability with apparent permeability in the Darcy equation. As presented by Javadpour (2009), the apparent permeability considers Knudsen diffusion and slip flow in shale rock and the results show that as pore size reduces, the contribution of Knudsen diffusion and slip flow on gas transport increases. Instead of directly summing up slip flow and Knudsen diffusion by Javadpour (2009), Shi et al. (2013) combined these two mechanisms using an empirical weight factor from curve fitting. Later, Wu et al. (2016) improved the apparent permeability model by coupling slip flow and Knudsen diffusion with a more rigorous weight factor based on the mathematical development and
incorporating the contribution of surface diffusion (Wu et al., 2015), desorption, and stress dependence into gas transport through nanopores.

The apparent permeability above mainly focused on the organic nanopores. To model gas transport in the inorganic nanopores, Sun et al. (2017b) presented an apparent permeability by considering the presence of immobile water film on the inorganic pore surface. To further relax the assumption of immobile adsorbed water, Zhang et al. (2018) proposed a two-phase flow model in nanotube using the fractal theory. Wang et al. (2018b) coupled the gas transport in the organic and inorganic shale matrix and proposed a total apparent permeability.

A few researchers have attempted to incorporate the multiple gas storage and transport mechanisms in the analysis of flowback and early-time production data. Yang et al. (2017) developed a semi-analytical model to predict two-phase flowback data of shale gas wells, which considers desorption, molecular diffusion, and slip flow. Cao et al. (2017) derived a dual-porosity dual-permeability model for the coupled two-phase flow in HF and matrix. Gas desorption, Knudsen diffusion, and surface diffusion are taken into account in the shale matrix. Similarly, Cui et al. (2020) used a dual-porosity model to develop a more complete flowback model that considers the non-Darcy effect. Although these studies incorporated the multiple mechanisms for shale reservoir into the two-phase flow model, the coupled governing equations are usually so complicated that they have to be solved numerically with either commercial software or discretized numerical solver. The complexity in solving procedure restricts their practicality, which can only be applied to the direct modeling or history matching of production data. Therefore, it is imperative to combine the specific mechanisms for shale reservoir into an inverse RTA method that can be simple yet rigorously applied to characterize HF properties.

In this study, we extend our previous work (Zhang and Emami-Meybodi, 2020d) by overcoming the three limitations that were recognized by the authors: 1) the early-time flowback data during two-phase fracture infinite acting linear flow (IALF) were not utilized, which may
provide valuable information about HF attributes, such as fracture permeability; 2) the assumption of transient linear flow in the shale matrix restricts the model’s application since the fracture interference appears more common with the modern completion styles; 3) ignoring the specific storage and transport mechanisms of shale reservoir may lead to some error in the analysis results.

To address these limitations, a new semi-analytical approach is developed with a more extensive application scope in analyzing early-time production data. The contribution of this paper is fourfold. First, a two-phase fracture IALF model is proposed to extract initial fracture permeability from the earlier flowback period than the previous work, which can be treated as an important constraint in HF characterization. Second, a solution for both IALF and BDF is developed for matrix model that considers the inter-fracture depletion for the later production period. Third, the new approach considers the specific mechanisms for shale gas reservoirs and, to our best knowledge, is the first RTA approach incorporating two-phase flow, desorption, diffusion, slip flow, stress dependence, and the effect of water film into the inverse analysis of HF properties using straight line approach. Fourth, we modified the MBE, matrix effective compressibility, and matrix pseudotime to account for desorption as an important mechanism during production. The new method is validated against the numerical simulations and then applied to the same Marcellus field example that is previously analyzed for comparison.

5.3 Theory and Model Development

The illustration of the conceptual model depicting water and gas flow through fracture and matrix is shown in Figure 5-1. The model is expanded from the 2D two-phase flow model developed by Zhang and Emami-Meybodi (2020d) and hence has the same assumptions made on HF as before. However, this model considers single-phase gas influx with immobile water in the shale matrix, since the previous study shows that water influx only has a slight influence on the
analysis results during early production (Ezulike and Dehghanpour, 2014, Xu et al., 2017). Furthermore, this model considers multiple storage and transport mechanisms for shale reservoir and divides the shale matrix into organic matter and inorganic matter with nanopores. The matrix model agrees with the scanning electron microscope (SEM) image of shale sample (Naraghi and Javadpour, 2015, Riewchotisakul and Akkutlu, 2016).

Other main assumptions of the conceptual model are as follows:

- The matrix has the same stress dependence on organic matter and inorganic matter with the same pore pressure. Matrix permeability and porosity change exponentially with pressure (Yilmaz et al., 1994).
- Gas only adsorbs on the surface of organic nanopores following Langmuir monolayer isothermal adsorption model (Langmuir, 1916). Desorption starts when the pore pressure drops below the critical desorption pressure.
- Water in the matrix only exists in the inorganic nanopores and remains immobile during the production (Sun et al., 2017b). The surface diffusion in the inorganic nanopores is negligible due to existence of water film in the inorganic matter.
- The transport mechanisms in the organic nanopores include continuum flow, slip flow, Knudsen diffusion, and surface diffusion. Whereas in the inorganic nanopores, continuum flow, slip flow, and Knudsen diffusion are the driving mechanisms (Wang et al., 2018b).
- The organic and inorganic nanopores are arranged in parallel without fluid transport between each other (Sun et al., 2017b, Wang et al., 2018b, Li et al., 2020).
This study adopted the two-phase diagnostic plot (Figure 5-1) proposed by Zhang and Emami-Meybodi (2020d) to identify the different flow regimes during production, i.e., FR1 fracture IALF and FR2 fracture BDF. Although the diagnostic plot was originally proposed by assuming two-phase matrix transient linear flow, it is still applicable to the present system with single-phase gas flow in the matrix. To consider the fracture interference, we corrected the influx calculation with the full solution for matrix and modified the MBE, which will be introduced in the mathematical development below.
The RTA methods usually ignore the early-time production data during fracture IALF due to the complexity in two-phase flow and the short duration of flow regime compared to the fracture BDF. However, according to our latest field case study from a multi-fractured horizontal well (MFHW) in Marcellus Shale, the FR1 could last for around 69 hours, which is long enough to conduct effective analysis based on the hourly recorded flowback data. Therefore, different from our previous studies that focused on FR2, we extended the application scope of the new semi-analytical method to the earlier production period during FR1 and developed a two-phase RTA model to extract initial fracture permeability.

The solutions for straight line analysis are given in Eq. (5-1) for FR1 (the detailed derivation is shown in Appendix A), and Eqs. (5-2) – (5-3) for FR2 (Zhang and Emami-Meybodi, 2020d):

\[
RNP_j = a_{j,IALF} \sqrt{t_{sp,j,IALF}},
\]

\[
RNP_j = a_{j,BDF} t_{sp,j,BDF} + b_{j,BDF},
\]

\[
PNR_j = a_{j,FMB} Q_j,pN + b_{j,FMB},
\]

where subscript \( j \) can be \( w \) for water or \( g \) for gas, \( RNP_j = (m_{j,f} (p_h) - m_{j,f} (p_w))/q_j \) is the rate normalized pseudopressure, \( PNR_j = q_j/(m_{j,f} (p_h) - m_{j,f} (p_w)) \) is the pseudopressure normalized rate,
$q_j$ is the flowrate from fracture at surface condition, $Q_{j,pN} = V_{fi} \frac{m_{j,f}(p_f) - m_{j,f}(p_f)}{m_{j,f}(p_f)}$ is the normalized cumulative production, $V_f = 2x_f w_f h \phi_f$ is the initial fracture volume, $t_{sp,j}$ is the superposition pseudotime, $x_f$ is the fracture half-length, $w_f$ is the fracture width, $h$ is the fracture height, $\phi_f = \phi_f \exp(-C_f(p_f - p_i))$ is the pressure-dependent fracture porosity, $p_f$ is the fracture pressure, $C_f$ is the fracture compressibility, $a_{j,IALF}$ is the slope of IALF specialty plot (i.e., $RNP_j$ vs. $\sqrt{t_{sp,j,IALF}}$), $a_{j,BDF}$ and $b_{j,BDF}$ are the slope and intercept of BDF specialty plot (i.e., $RNP_j$ vs. $t_{sp,j,BDF}$), $a_{j,FMB}$ and $b_{j,FMB}$ are the slope and intercept of FMB specialty plot (i.e., $PNR_j$ vs. $Q_{j,pN}$), $t_{sp,j,IALF}$ is the superposition time for FR1, which is defined in Appendix A, and $t_{sp,j,BDF}$ is the superposition time for FR2, which is defined as follows (Zhang and Emami-Meybodi, 2020d):

$$t_{sp,j,BDF} = \sum_{n=1}^{N} \frac{(q_{j,n} - q_{j,n-1})(t_{p,f,n} - t_{p,f,n-1})}{q_{j,n}},$$

(5-4)

By plotting IALF specialty plot, one can use Eq. (5-5) to calculate the initial fracture permeability $k_{fi,j}$ based on the slope of the straight line, which falls into the FR1 identified by the diagnostic plot:

$$k_{fi,j,IALF} = \left( \frac{\mu_j}{\phi_{C,j,eff}} \right) f_i \left( \frac{\varepsilon_j B_j}{w_f h a_{j,IALF}} \right)^2,$$

where $\mu_j$ is the fluid viscosity in the fracture, $C_{j,eff}$ is the effective compressibility in the fracture, $\varepsilon_j$ is the unit conversion factor, $\varepsilon_w = 7.093$ and $\varepsilon_g = 7103.13$.

By plotting BDF specialty plot or FMB specialty plot, one can obtain the $V_f$ and $k_f$ from the straight line during FR2 (Zhang and Emami-Meybodi, 2020d). To estimate the pseudo variables, the modified MBE is applied to calculate average pressure ($\bar{p}_f$) and water saturation ($\bar{S}_w,f$) in the fracture, average pressure ($\bar{p}_m$) and water saturation ($\bar{S}_w,m$) in the matrix DOI:

$$Q_w = V_{fi} \left( \frac{s_{w,f}}{B_{w,f}} - \frac{s_{w,f}}{B_{w,f}} [1 - C_f(p_{fi} - \bar{p}_f)] \right),$$

(5-6)

$$Q_g = V_{fi} \left( \frac{s_{g,f}}{B_{g,f}} - \frac{s_{g,f}}{B_{g,f}} [1 - C_f(p_{fi} - \bar{p}_f)] \right) + 1000Q_{g,inf},$$

(5-7)
\[ 0 = 4V_{mi} \left( \frac{s_{w,mi}}{B_{w,mi}} - \frac{s_{wm}}{B_{w,m}} [1 - C_m(p_{mi} - \bar{p}_m)] \right), \]  
(5-8)

\[ 1000Q_{g,inf} = 4V_{mi} \left( \frac{s_{g,mi}}{B_{g,mi}} - \frac{s_{gm}}{B_{g,m}} [1 - C_m(p_{mi} - \bar{p}_m)] \right), \]  
(5-9)

where \( Q_w \) and \( Q_g \) are the cumulative water and gas production at the surface condition, \( p_m \) is the matrix pressure, \( C_m \) is the matrix compressibility, \( B_{j,f} \) and \( B_{j,m} \) are the fluid formation volume factor in the fracture and matrix, and \( Q_{g,inf} \) is the cumulative gas influx from matrix.

To account for desorption and diffusion during production, we calculated the initial pore volume of the quarter fracture-matrix element, \( V_{mi} \), using the modified matrix pseudotime, \( t_{pg,m} \), and matrix effective compressibility, \( C_{g,efm} \): (the detailed derivation is shown in Appendix B)

\[ V_{mi} = y_{inv} x_f h \phi_{mi}, \]  
(5-10)

\[ y_{inv} = 0.194 \sqrt{\frac{k_{app}(t_{pg,m})}{(\phi_m \mu_{g,m} C_{g,efm})}}, \]  
(5-11)

\[ t_{pg} = \frac{\phi_m \mu_{g,m} C_{g,efm}}{k_{app}} \int_0^t \frac{k_{app}(\bar{p}_m)}{\phi_m(\bar{p}_m) \mu_{g,m}(\bar{p}_m) C_{g,efm}(\bar{p}_m)} dt, \]  
(5-12)

\[ C_{g,efm} = S_{g,m} (C_{g,m} + C_m) + \frac{dS_{g,m}}{dp_m} + \frac{H(p_d - p_m) B_{g,m} p_L V_L}{\phi_m(p_m + p_L)^2}, \]  
(5-13)

where \( y_{inv} \) is the matrix DOI, \( \phi_m = \phi_{mi} \exp(-C_m(p_{mi} - p_m)) \) is the pressure-dependent matrix porosity, \( C_{g,m} \) and \( \mu_{g,m} \) are the gas compressibility and viscosity in the matrix, \( H(p_d - p_m) \) is the Heaviside function, \( p_d \) is the critical desorption pressure, \( p_L \) is the Langmuir pressure, and \( V_L \) is the Langmuir volume at standard condition.

The cumulative gas influx \( Q_{g,inf} \) from matrix for both IALF and BDF flow are given as follows to consider the inter-fracture depletion during the later production period, which is described in Appendix B:

\[ Q_{g,inf,IALF} = \frac{m_{g,m}(p_{mi}) - m_{g,m}(p_0)}{\delta_{g,IALF} (\mu_g B_g)_{mi} \eta_{g,m}} \int_0^t \left[ \frac{\Delta \zeta(t)}{\sqrt{(t_{pg,m} - t_{pg,m,n})}} \right] dt, \]  
(5-14)
\[
\frac{m_{g,m}(p_m) - m_{g,m}(p_b)}{\delta_{g,full}(\mu_g B_g) n_m Y_m} \int_0^t \sum_{n=0}^{N-1} \exp \left( -\beta_i^2 \eta_i m (t_{pg,m} - t_{pg,m,n}) \right) \Delta \xi(t_n) dt, \quad (5-15)
\]
where \(\beta_i = (2l - 1)\pi/2\), \(\delta_i\) is the unit conversion factor, \(\delta_{g,IAF} = 5578.5\) and \(\delta_{g,full} = 19752.5\).

Different from our previous study, the \(Q_{g,inf}\) not only includes the continuum gas influx driven by Darcy’s law but also considers the contribution of gas desorption, diffusion, and slip flow, the effects of adsorbed water film on the pore wall, as well as the stress dependence. These storage and transport mechanisms in organic and inorganic shale nanopores are incorporated into the apparent permeability \(k_{app}\), which is derived in Appendix C:

\[
k_{app} = \psi k_{app,om} + (1 - \psi) k_{app,im}, \quad (5-16)
\]
where \(k_{app,om}\) and \(k_{app,im}\) are the apparent permeability for organic and inorganic matter, respectively, and \(\psi\) is the organic matter (kerogen) volume fraction representing the ratio of organic matter volume to rock volume.

After simultaneously solving the modified MBE in Eqs. (5-6) – (5-9) with the fixed-point iteration scheme (Kreyszig, 2009) for each time step, \(\bar{p}_f, \bar{S}_{w,f}, \bar{p}_m, \bar{S}_{w,m}\) can be obtained to estimate the pseudo variables and generate the specialty plots. Although water is assumed immobile in the matrix, there is still a slight change in \(\bar{S}_{w,m}\) due to the gas influx from matrix into HF. By applying the workflow proposed by Zhang and Emami-Meybodi (2020d), the early-time production data can be analyzed to extract HF properties \((V_f, k_f, C_f,\) and fracture permeability modulus \(\gamma_f)\) and quantify HF closure with the reconciliation of long-term production data. The interpreted \(k_f\) from FR1 provides an earlier understanding of HF conductivity and constrains the calculated \(k_f\) from FR2 to help reach the unique analysis results.
5.4 Results

To test the accuracy of the new semi-analytical approach in estimating fracture properties, several numerical simulation cases were generated based on the reservoir and fracture attributes in Marcellus Shale. After synthetic case validation, this study revisited the field case that was analyzed in our previous studies, and compared the interpretation results to test the robustness and practicality of the proposed semi-analytical method.

5.4.1 Model Validation

We constructed two numerically-simulated cases of MFHW using commercial software (CMG, 2019) having different operation conditions: Case 1 is produced under constant water rate of 1100 STB/day and Case 2 is produced under constant gas rate of 20,000 Mcf/day. So that the proposed method can be verified under variable rate condition of gas and water, respectively, for general application. The input reservoir and fracture properties, as well as the parameters related to the desorption and diffusion, are the same for the two cases and listed in Table 5-1. The input reservoir and fracture properties are adopted from Zhang and Emami-Meybodi (2020d), which represent the shale reservoir. The source of desorption and diffusion parameters are listed in Table 5-1. The synthetic cases were run for 14 days to mimic the real early-time production period in shale reservoirs. The HF domain adopts the relative permeability curve of straight-line (Clarkson et al., 2012), whereas the matrix domain is set to be single-phase gas flow by assigning matrix water saturation below the irreducible water saturation all the time. The shale matrix is initialized at the undersaturated condition by setting the critical desorption pressure 30 psi lower than the initial reservoir pressure.
Table 5-1: Summary of input data used in numerical simulation.

<table>
<thead>
<tr>
<th>Reservoir and fracture parameters</th>
<th>Value</th>
<th>Desorption and diffusion parameters</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial pressure for fracture and matrix system, $p_f$ (psi)</td>
<td>5000</td>
<td>Langmuir maximum adsorbed gas volume, $V_L$ (scf/ft$^3$)</td>
<td>7.646</td>
<td>Williams-Kovacs and Clarkson (2016)</td>
</tr>
<tr>
<td>Temperature, $T$ (°F)</td>
<td>160</td>
<td>Langmuir critical adsorption pressure, $p_d$ (psi)</td>
<td>4970</td>
<td>Assumption</td>
</tr>
<tr>
<td>Specific gravity, $SG_b$ (-)</td>
<td>0.65</td>
<td>Langmuir pressure, $p_L$ (psi)</td>
<td>500</td>
<td>Williams-Kovacs and Clarkson (2016)</td>
</tr>
<tr>
<td>Water compressibility, $C_w$ (psi$^{-1}$)</td>
<td>$3 \times 10^6$</td>
<td>Bulk rock density, $\rho_b$ (lb/ft$^3$)</td>
<td>168.6</td>
<td>Yu et al. (2018)</td>
</tr>
<tr>
<td>Initial fracture porosity, $\phi_{fi}$ (%)</td>
<td>50</td>
<td>Density of kerogen, $\rho_{ker}$ (lb/ft$^3$)</td>
<td>103.63</td>
<td>Ward (2010)</td>
</tr>
<tr>
<td>Initial matrix porosity, $\phi_{mi}$ (%)</td>
<td>5</td>
<td>Ratio of organic / inorganic pore size, $r_{om}/r_{im}$ (-)</td>
<td>5</td>
<td>Li et al. (2020)</td>
</tr>
<tr>
<td>Initial fracture permeability, $k_{fi}$ (mD)</td>
<td>3000</td>
<td>Initial porosity of organic matter, $\phi_{om,i}$ (%)</td>
<td>8.65</td>
<td>Yu et al. (2018)</td>
</tr>
<tr>
<td>Initial apparent permeability, $k_{app}$ (mD)</td>
<td>$1 \times 10^3$</td>
<td>Gas molar mass (methane), $M$ (kg/mol)</td>
<td>0.016</td>
<td>Given</td>
</tr>
<tr>
<td>Fracture compressibility, $C_f$ (psi$^{-1}$)</td>
<td>$2 \times 10^5$</td>
<td>Diameter of methane molecule, $d_m$ (m)</td>
<td>$3.8 \times 10^{-10}$</td>
<td>Given</td>
</tr>
<tr>
<td>Matrix compressibility, $C_m$ (psi$^{-1}$)</td>
<td>$1 \times 10^6$</td>
<td>Gas universal constant, $R$ (J/mol/K)</td>
<td>8.314</td>
<td>Given</td>
</tr>
<tr>
<td>Fracture permeability modulus, $\gamma_f$ (psi$^{-1}$)</td>
<td>$3 \times 10^4$</td>
<td>Rarefaction coefficient fitting constant, $\alpha_4$ (-)</td>
<td>4</td>
<td>Beskok and Karniadakis (1999)</td>
</tr>
<tr>
<td>Matrix permeability modulus, $\gamma_m$ (psi$^{-1}$)</td>
<td>$3 \times 10^5$</td>
<td>Rarefaction coefficient fitting constant, $\beta$ (-)</td>
<td>0.4</td>
<td>Beskok and Karniadakis (1999)</td>
</tr>
<tr>
<td>Initial water formation volume factor, $R_w$ (ft$^3$/STB)</td>
<td>5.78</td>
<td>Slippage constant, $b$ (-)</td>
<td>-1</td>
<td>Beskok and Karniadakis (1999)</td>
</tr>
<tr>
<td>Initial fracture water saturation, $S_{w,fi}$ (%)</td>
<td>70</td>
<td>Fractal dimension of the organic pore surface roughness, $D_{f,om}$ (-)</td>
<td>3</td>
<td>Coppens and Dammers (2006)</td>
</tr>
<tr>
<td>Initial matrix water saturation, $S_{w,mi}$ (%)</td>
<td>30</td>
<td>Fractal dimension of the inorganic pore surface roughness, $D_{f,im}$ (-)</td>
<td>2</td>
<td>Coppens and Dammers (2006)</td>
</tr>
<tr>
<td>Water viscosity, $\mu_w$ (cp)</td>
<td>0.3</td>
<td>Isosteric adsorption heat when $\theta=0$, $\Delta H$ (J/mol)</td>
<td>16000</td>
<td>Li et al. (2020)</td>
</tr>
<tr>
<td>Water density, $\rho_w$ (lb/ft$^3$)</td>
<td>62</td>
<td>Ratio of rate constant for blockage and migration, $k_B/k_m$ (-)</td>
<td>0.5</td>
<td>Li et al. (2020)</td>
</tr>
<tr>
<td>Fracture half-length, $x_f$ (ft)</td>
<td>500</td>
<td>Avogadro’s constant, $N_A$ (mol-1)</td>
<td>$6.02 \times 10^{23}$</td>
<td>Given</td>
</tr>
<tr>
<td>Reservoir thickness, $h$ (ft)</td>
<td>300</td>
<td>Total organic carbon, TOC (%)</td>
<td>5</td>
<td>Milliken et al. (2012)</td>
</tr>
<tr>
<td>Number of active fractures</td>
<td>50</td>
<td>Kerogen correction factor, $\kappa_{TOC}$ (-)</td>
<td>0.8</td>
<td>Crain (2015)</td>
</tr>
<tr>
<td>Fracture half spacing, $y_m$ (ft)</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fracture width, $w_f$ (ft)</td>
<td>0.02</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To represent the multiple gas transport mechanisms in the semi-analytical approach, we generated $k_{app}$ vs. $p_m$ using the formula listed in Appendix C and input this correlation into the numerical simulator through the permeability multiplier table. The red dots in Figure 5-3a show the $k_{app}$ correlation for the synthetic cases, which agrees with the $k_{app}$ trend from experimental data and analytical models (Sun et al., 2017b, Wang et al., 2018b). When ignoring the gas storage and transport mechanisms in the matrix model and only considering Darcy’s law as the previous study
(Zhang and Emami-Meybodi, 2020d), the variation of $k_{app}$ with pressure is shown as the blue curve in Figure 5-3a. The initial $k_{app}$ is reduced by 17% due to the exclusion of desorption, diffusion, and slip flow, and $k_{app}$ decreases as pressure drops, which is solely controlled by the stress dependence.

To have a quantitative understanding of how much the gas storage and transport mechanisms affect the early-time production data, we compared the gas flowrate and cumulative gas production under constant BHP condition using the two different $k_{app}$ correlation shown in Figure 5-3a. The results in Figure 5-3b indicate that ignoring the desorption, diffusion, and slip flow for shale reservoir will underestimate the gas flowrate and cumulative gas production, and the difference is expected to grow with the increase of pressure drop in matrix.

First, we validated the modified MBE of the semi-analytical approach for the Case 1 and Case 2. By iteratively solving the Eqs. (5-6) – (5-9), the obtained $\bar{p}_f$, $\bar{S}_{w,f}$, $\bar{p}_m$, and $\bar{S}_{w,m}$ are shown in Figure 5-4. The calculated $\bar{p}_f$ and $\bar{S}_{w,f}$ from MBE show a perfect match with the numerical simulation. As shown in the plot, the average pressure drop in the matrix DOI is less than that in the fracture due to the much smaller matrix permeability. The $\bar{S}_{w,m}$ doesn’t change much since water remains immobile in the matrix. The slight increase in $\bar{S}_{w,m}$ solely results from the slight

Figure 5-3: (a) Comparison of matrix apparent permeability with and without considering desorption, diffusion, and slip flow. (b) Comparison of gas flowrate and cumulative gas production with and without considering desorption, diffusion, and slip flow.
expansion of water as gas is drained from the matrix. With the average properties, the pseudopressure and the modified pseudotime in the matrix are calculated and the cumulative gas influx is determined using Eqs. (5-47) – (5-49) with the consideration of desorption and diffusion. The results in Figure 5-5 also show a perfect match of gas influx with numerical simulation for both cases.

Figure 5-4: Comparison of average fracture pressure, average pressure in the matrix DOI, and average fracture water saturation, and average water saturation in the matrix DOI calculated from analytical solution and numerical simulation for (a-b) Case 1 and (c-d) Case 2.
Then, we estimated the pseudopressure and pseudotime in the fracture with the obtained average properties and gas influx so that the two-phase diagnostic plots can be generated in Figure 5-6. A clear half-slope and unit-slope straight lines are shown on the diagnostic plots for both cases, denoting FR1 and FR2, respectively. Besides, there is a transitional period connecting the two flow regimes, which lasts for 1 – 2 days. For case 1, the FR1 ending time and FR2 starting time are 5 and 57 hrs for water-phase, as well as 15 and 42 hrs for gas-phase. For case 2, the FR1 ending time and FR2 starting time are 3 and 37 hrs for water-phase, as well as 11 and 30 hrs for gas-phase. With the identified flow regimes, we conducted straight line analysis for FR1 by generating the IALF specialty plot in Figure 5-7. By applying Eq. (5-5), the calculated \(k_{fi,w} = 3035\) mD and \(k_{fi,g} = 3055\) mD for Case 1, \(k_{fi,w} = 3236\) mD and \(k_{fi,g} = 2855\) mD for Case 2, which are very close to the set \(k_{fi} = 3000\) mD in the simulation with the relative error <8%. Figure 5-8 and Figure 5-9 show the the BDF and FMB specialty plots during FR2 for Case 1 and Case 2, respectively. The calculated \(V_{fi}\) and \(k_{fi}\) are presented in Table 5-2, with the relative error all <10%. Therefore, the new semi-analytical method is successfully validated for FR1 and FR2, which considered the contribution of desorption and diffusion from shale matrix.
Figure 5-6: Two-phase water and gas diagnostic plot for (a-b) Case 1 with constant water rate and (c-d) Case 2 with constant gas rate.

Figure 5-7: Water-phase and gas-phase IALF specialty plot for (a-b) Case 1 with constant water rate and (c-d) Case 2 with constant gas rate.
Figure 5-8: Water-phase (a) BDF specialty plot and (b) FMB plot, and gas-phase (c) BDF specialty plot and (d) FMB plot for Case 1 with constant water rate.
Table 5-2: Summary of input data used in numerical simulation and calculated properties using the proposed method.

<table>
<thead>
<tr>
<th>Case #</th>
<th>Set values</th>
<th>Water-phase flowback model</th>
<th>Gas-phase flowback model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>IALF analysis</td>
<td>BDF analysis</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_f$ (Mcf)</td>
<td>$k_f$ (mD)</td>
</tr>
<tr>
<td>1</td>
<td>150 3000</td>
<td>3035</td>
<td>150.4</td>
</tr>
<tr>
<td>2</td>
<td>150 3000</td>
<td>3236</td>
<td>147.4</td>
</tr>
</tbody>
</table>

5.4.2 Field Application

The field example presented in this study is an MFHW drilled in Marcellus Shale. The late-time flowback data during FR2 and long-term production data were analyzed in Zhang and Emami-Meybodi (2020a) and Zhang and Emami-Meybodi (2020d) to estimate $V_f$, $k_f$, $C_f$, and $\gamma_f$ via straight
line analysis. However, the earlier-time flowback data during FR1 were not utilized, and the gas storage and transport mechanisms for shale reservoir were not incorporated. In this study, we revisited this field case to interpret the entire flowback data and considered the contribution of desorption and diffusion into matrix model using the proposed approach. Table 5-3 lists the input parameters for reservoir and apparent permeability model. The reservoir parameters used in this study are consistent with our previous works, and the parameters associated with gas desorption and diffusion come from either the current field case or the field examples of shale gas reservoir analyzed by other researchers. In the previous works, the matrix permeability was set to be a constant value of $10^{-5}$ mD based on the history matching of long-term production data. However in this study, gas storage and transport mechanisms in the shale matrix are considered in the newly defined apparent permeability $k_{app}$. To be consistent, the constant matrix permeability from the previous works should represent the initial $k_{app}$ in this work. Accordingly, the initial radius of organic nanopores is obtained to be $r_{om,i} = 2$ nm, which is consistent with the scale of pore size in shale reservoir (Wang et al., 2018b).

With $r_{om,i}$ and other input parameters listed in Table 5-3, the new semi-analytical method is applied together with the workflow proposed by Zhang and Emami-Meybodi (2020d) to analyze the entire flowback and long-term production data. After the convergence of workflow, the calculated $\bar{p}_f$ and $\bar{p}_m$ during flowback are shown in Figure 5-10a. The average pressure drop in the fracture is much larger than that in the matrix DOI, which is consistent with the observation from numerical simulation due to the extremely low permeability in the matrix. The reduction of average fracture permeability in Figure 5-10b indicates the significant fracture closure during flowback. The rapid drop of $\bar{S}_{w,f}$ in Figure 5-10b is due to the displacement of gas influx from matrix, which is shown in Figure 5-10c. By generating the two-phase diagnostic plots for water and gas phase data, a clear unit-slope straight line is shown in Figure 5-10e and Figure 5-10f denoting FR2. To be noted, Figure 5-10e also indicates a half-slope straight line in the early flowback period showing
the FR1. In contrast, the FR1 cannot be clearly identified from the gas-phase diagnostic plot due to the bad quality of fluctuated gas production data. The FR1 for water-phase data lasts for 69 hours, which is long enough to conduct an effective straight line analysis. By generating the IALF specialty plot in Figure 5-10d, we calculated $k_{f,IALF} = 899$ mD based on the extracted slope of the straight line during FR1. Figure 5-10g and Figure 5-10h show the BDF specialty plots for water and gas phase data. With the slope and intercept of the straight lines during FR2, we determined the $V_g = 144.6$ Mcf and $k_{f,BDF} = 903$ mD. The $k_f$ estimated from the IALF model and BDF model shows an excellent agreement, which provides an important constraint to reach the unique interpretation results for RTA inverse analysis.

Table 5-3: Field input data from a Marcellus MFHW.
Figure 5-10: Flowback analysis after the convergence of $V_f$ and $k_f$: (a) Average pressure in the fracture and matrix DOI during flowback period. (b) Average permeability and water saturation in the fracture during flowback. (c) Gas influx rate and cumulative gas influx from matrix. (d) Water-phase IALF specialty plot. (e) Two-phase water diagnostic plot. (f) Two-phase gas diagnostic plot. (g) Two-phase water BDF specialty plot. (h) Two-phase gas BDF specialty plot.
The workflow reconciled the flowback data with long-term production data and yielded $\gamma_f = 9.5 \times 10^{-4}$ psi$^{-1}$ and $C_f = 7 \times 10^{-5}$ psi$^{-1}$, which fall into the empirical range of $10^{-4}$ to $10^{-3}$ psi$^{-1}$ (Ozkan et al., 2010) and $7 \times 10^{-5}$ to $7 \times 10^{-4}$ psi$^{-1}$ (Tan et al., 2019), respectively. With the obtained fracture properties, the final $V_f$ and $k_f$ are predicted to be 99.2 Mcf and 5.5 mD when $p_f$ drops to 1200 psi (i.e., the ultimate BHP). The ultimate $k_f$ falls into the permeability range of fracture without proppant (0.4 – 8.0 mD) obtained from the laboratory experiments (Tan et al., 2018, Tan et al., 2019), which confirms the industrial experience of proppants crushing and HF closure after nine months of production. The ultimate $V_f$ is 16% smaller than the $V_f$ from long-term RTA. This could be reasonable and caused by the assumption of ignoring the loss of $w_f$ during long-term production. The interpreted results for the field case are listed in Table 5-4 and referred to as “Analysis A”.

To have a quantitative understanding of the impact of changes in apparent permeability caused by desorption, diffusion, and slip flow on the flowback analysis, we repeated the above-mentioned analysis (Analysis A) by neglecting the pressure-dependency of $k_{app}$. In this repeated analysis (Analysis B), a constant apparent permeability of $10^{-5}$ mD is set to exclude diffusion and slip flow and sets zero $V_L$ to exclude desorption. The comparison of interpreted results between analysis A and B in Table 5-4 indicates that neglecting the gas storage and transport mechanisms in the chosen field case causes about 2% error in the calculation of HF attributes from early-time production data. The influence of gas storage and transport mechanisms in this field case is small due to the small pressure drop within matrix DOI (700 psi) and is expected to increase with larger matrix pressure drop, i.e., long flowback periods.

A comparison of interpreted results between the analysis B of this study and Zhang and Emami-Meybodi (2020d) shows that considering two-phase flow in the shale matrix causes 23% and 3% difference in $V_{fi}$ and $k_{fi}$ estimation. The calculated $k_{fi}$ doesn’t show as much difference as $V_{fi}$ since the update of $\gamma_f$ during iteration transfers the error of $k_{fi}$ to the 21% error in $\gamma_f$ estimation. Although the model in Zhang and Emami-Meybodi (2020a) has the same assumption of single-
phase gas flow in the matrix as this study, the calculated \( V_{fi} \) and \( k_{fi} \) are 52% and 33% larger than that estimated in analysis B. These differences are due to the fact that this study adopts the workflow proposed by Zhang and Emami-Meybodi (2020d), which quantifies HF dynamics through the loss of both fracture volume and permeability based on the early-time and long-term production data.

Whereas, the workflow in Zhang and Emami-Meybodi (2020a) only used a single constraint, i.e., fracture permeability, during the iteration of HF properties. Therefore, the number of constraints during workflow iteration has the greatest impact on characterizing HF attributes, then two-phase flow, and finally gas storage and transport mechanisms during early-time production.

Table 5-4: Interpreted results from flowback analysis and long-term production data analysis.

<table>
<thead>
<tr>
<th>Fracture properties</th>
<th>Flowback analysis</th>
<th>Long-term production data analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Zhang and Emami-Meybodi (2020a)</td>
<td>Zhang and Emami-Meybodi (2020d)</td>
</tr>
<tr>
<td>( V_{fi} ), Mcf</td>
<td>221.3</td>
<td>179.5</td>
</tr>
<tr>
<td>( k_{fi,BDF} ), mD</td>
<td>1227.5</td>
<td>892.6</td>
</tr>
<tr>
<td>( k_{fi,IALF} ), mD</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \gamma_f ), psi(^{-1} )</td>
<td>8\times10(^{-4} )</td>
<td>7.5\times10(^{-4} )</td>
</tr>
<tr>
<td>( C_f ), psi(^{-1} )</td>
<td>-</td>
<td>7\times10(^{-5} )</td>
</tr>
<tr>
<td>Final ( V_{fi} ), Mcf</td>
<td>-</td>
<td>123.1</td>
</tr>
<tr>
<td>Final ( k_{fi} ), mD</td>
<td>-</td>
<td>15.8</td>
</tr>
<tr>
<td>Average ( k_{fi} ), mD</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Comments:
- Matrix: single-phase IALF for gas, constant km
- Fracture: two-phase BDF
- Workflow: Iterate \( V_{fi} \), \( k_{fi} \), \( \gamma_f \), and \( C_f \)
- Fracture: two-phase IALF
- Workflow: Iterate \( V_{fi} \), \( k_{fi} \), \( \gamma_f \), and \( C_f \)
- Fracture: single-phase gas IALF/BDF, kapp includes desorption & diffusion
- Fracture: two-phase IALF / BDF
- Workflow: Iterate \( V_{fi} \), \( k_{fi} \), \( \gamma_f \), and \( C_f \)
- Fracture: two-phase IALF / BDF
- Workflow: Iterate \( V_{fi} \), \( k_{fi} \), \( \gamma_f \), and \( C_f \)
- Fracture: single-phase gas IALF/BDF, constant kappi
- Fracture: two-phase IALF / BDF
- Workflow: Iterate \( V_{fi} \), \( k_{fi} \), \( \gamma_f \), and \( C_f \)

NA
5.5 Discussion and Recommendation

The new semi-analytical approach presented in this work is believed to provide a reasonable and accurate estimate of fracture properties by addressing several limitations of our previous work (Zhang and Emami-Meybodi, 2020d). In Zhang and Emami-Meybodi (2020d), the fracture IALF regime in the early period was only identified by the diagnostic plot but not analyzed to extract fracture attributes. Also, the fracture interference was neglected by assuming transient linear flow in the matrix. In the present work, we not only proposed a two-phase fracture IALF model to extract $k_{fi}$ from the earlier flowback period than the previous work, but also developed a solution for both IALF and BDF flow in the matrix model that considers the inter-fracture depletion for the later production period. The interpreted $k_{fi}$ from FR1 provides a quantitative understanding of fracture conductivity in the much earlier period and is an important constraint to help FR2 analysis reach the unique interpretation results. One of the key benefits of the new method is incorporating the gas storage and transport mechanisms for shale reservoir into the early-time production data analysis, as opposed to just using Darcy’s law to model continuum flow of the matrix in the previous study. Furthermore, the new model is, to our best knowledge, the first RTA method to incorporate two-phase flow in fractures as well as desorption, diffusion, slip flow, stress dependence, and the effect of water film in matrix into the inverse characterization of HF properties using straight line analysis. Different from history matching and direct reservoir modeling, the straight line analysis approach is rigorous yet much simpler and faster to provide the key information of HF attributes and flow regimes.

The mathematical assumptions made in developing the semi-analytical approach inevitably create some limitations. First, the matrix model assumes the organic and inorganic nanopores are arranged in parallel without fluid transport between each other. This could be valid when the permeability is in the same order of magnitude in organic and inorganic matter. Recently, many
researchers applied dual-porosity models to simulate gas transportation between the two types of nanopores and also considered the natural fracture network in the shale matrix (Cao et al., 2017, Wang et al., 2018a, Cui et al., 2020). Hence, one can use the dual-porosity dual-permeability model to extend this work. Second, the adsorbed water on the surface of inorganic nanopores was assumed immobile in the analysis, which is reasonable for the reservoir with low initial water saturation but may impact the accuracy of interpretation results when water saturation becomes larger than irreducible saturation. The following work shall consider mobile water in the inorganic nanopores (Zhang et al., 2018). The saturation vs. pressure relation is the critical point to consider two-phase flow in the shale matrix. As future research, one may introduce the empirical correlation by Clarkson and Qanbari (2016a) or the approximation relation at limiting cases by Behmanesh (2016) to modify the pseudopressure and pseudotime in the matrix model. The three-phase transient linear flow model proposed by Hamdi et al. (2018) may also be incorporated to consider the gas condensate dropped out in the wellbore, or near fracture region. Finally, the uncertainties of input parameters used for the reservoir, fracture, desorption, and diffusion limit the accuracy of the interpreted results. It is recommended to apply this method together with the laboratory experiments conducted on the shale core samples to obtain more information about desorption and diffusion input parameters. Type curve solutions shall be generated in future research to forecast the production and analyze the critical parameters affecting production performance for different periods.

5.6 Summary and Conclusions

Analysis of early-time production data using RTA techniques has been proposed to estimate HF and reservoir properties, right after well stimulation. However, the complexity in two-phase flow and gas storage and transport mechanisms for shale reservoir challenges the accurate
characterization of HF properties and its closure. In this study, a new semi-analytical method is developed to evaluate HF attributes and its closure for fractured shale wells exhibiting two-phase flow. The new method provides four important improvements in analyzing early-time production data during FR1 with a new two-phase fracture IALF model; considering inter-fracture depletion in FR2 using the full solution of matrix flow; incorporating desorption, slip flow, diffusion, stress dependence, and the effect of water film into the matrix model; as well as modifying the MBE, matrix effective compressibility, and matrix pseudotime to account for desorption in the undersaturated organic shale matrix. The accuracy of the proposed method is verified with numerical simulation and demonstrated the applicability and superiority than our previous work by revisiting the same field example of MFHW in Marcellus Shale. The key conclusions of this work are as follows:

- The new semi-analytical method is significantly improved over the previous one to characterize HF properties and its closure during early-time production period. This is mainly due to the inclusion of both IALF and BDF in fracture and matrix models, as well as the storage and flow mechanisms within the shale matrix.

- The proposed fracture IALF model is capable of estimating initial fracture permeability from the production data during FR1 and provides an important constraint for the interpreted HF properties.

- Numerical validation shows that ignoring the gas storage and transport mechanisms for shale reservoir will underestimate the matrix permeability as well as the gas flowrate and cumulative production. The difference in the modeling of gas transport in shale matrix is expected to grow as pressure drops in the matrix.

- The field application indicates that the number of constraints during workflow iteration has the greatest impact on characterizing HF attributes during early-time production period; the two-phase flow in matrix takes second place, and the multiple mechanisms within shale
matrix at last. It is expected that the impact of desorption, slip flow, and diffusion on interpreting HF properties increases with a longer production period and a larger pressure drop in the matrix.

- While the changes of apparent permeability due to storage and transport mechanisms are minimal in short flowback cases, these mechanisms have a significant impact on the initial value of apparent permeability for the matrix, which should be considered to avoid underestimation of gas flux from the matrix to fractures.

5.7 Nomenclatures

\[ a_{j,BDF} = \text{the slope of } RNP_j \text{ vs. } t_{sp,j,BDF} \text{ plot} \]

\[ a_{j,FMB} = \text{the slope of } PNR_j \text{ vs. } Q_{j,pN} \text{ plot} \]

\[ a_{j,IALF} = \text{the slope of } RNP_j \text{ vs. } \sqrt{t_{sp,j,IALF}} \text{ plot} \]

\[ b = \text{slippage constant, } - \]

\[ b_{j,BDF} = \text{the intercept of } RNP_j \text{ vs. } t_{sp,j,BDF} \text{ plot} \]

\[ b_{j,FMB} = \text{the intercept of } PNR_j \text{ vs. } Q_{j,pN} \text{ plot} \]

\[ B_{g,f} = \text{gas formation volume factor in the fracture, } ft^3/scf \]

\[ B_{g,m} = \text{gas formation volume factor in the matrix, } ft^3/scf \]

\[ B_{w,f} = \text{water formation volume factor in the fracture, } ft^3/STB \]

\[ B_{w,m} = \text{water formation volume factor in the matrix, } ft^3/STB \]

\[ C_f = \text{fracture compressibility, psi}^{-1} \]

\[ C_m = \text{matrix compressibility, psi}^{-1} \]

\[ C_{j,eff} = \text{effective compressibility in the fracture, psi}^{-1} \]

\[ C_{g,efm} = \text{effective compressibility in the matrix, psi}^{-1} \]
\( C_{j,f} \) = fluid compressibility in the fracture, \( psi^{-1} \)

\( C_{g,m} \) = gas compressibility in the matrix, \( psi^{-1} \)

\( d_m \) = diameter of gas molecule, \( m \)

\( D_f \) = fractal dimension of the pore surface roughness, -

\( DRNP_g \) = derivative of gas-phase rate normalized pseudopressure, \( psi \cdot day/(Mscf \cdot ln(day)) \)

\( DRNP_w \) = derivative of water-phase rate normalized pseudopressure, \( psi \cdot day/(STB \cdot ln(day)) \)

\( D_{s0} \) = surface diffusion coefficient when \( \theta \) is zero, \( m^2/s \)

\( h \) = fracture height, \( ft \)

\( H(\_\_\_\_) \) = Heaviside function, -

\( \Delta H \) = isosteric adsorption heat when \( \theta \) is zero, \( J/mol \)

\( J_{b,om} \) and \( J_{b,im} \) = mass flux of bulk gas transport for organic and inorganic matter, \( kg/(m^2 \cdot s) \)

\( J_{k,om} \) and \( J_{k,im} \) = mass flux of Knudsen diffusion for organic and inorganic matter, \( kg/(m^2 \cdot s) \)

\( J_{sf,om} \) = mass flux of surface diffusion for organic matter, \( kg/(m^2 \cdot s) \)

\( J_{st,om} \) and \( J_{st,im} \) = mass flux of slip flow for organic and inorganic matter, \( kg/(m^2 \cdot s) \)

\( J_{t,om} \) and \( J_{t,im} \) = total mass flux for organic and inorganic matter, \( kg/(m^2 \cdot s) \)

\( k_{app} \) = apparent permeability, \( md \)

\( k_{app,om} \) and \( k_{app,im} \) = apparent permeability for organic and inorganic matter, \( md \)

\( k_f \) = fracture permeability, \( md \)

\( k_m \) = matrix permeability, \( md \)

\( k_{om} \) = permeability of organic matter only under Darcy’s law, \( md \)
\(k_{r,j,f} = \) relative permeability in the fracture, 

\(K_{n,om} \) and \(K_{n,im} = \) Knudsen number in organic and inorganic matter, 

\(m_{j,f}(p) = \) pseudopressure in the fracture, \(psi\)

\(m_{g,m}(p) = \) gas matrix pseudopressure, \(psi\)

\(M = \) gas molar mass, \(kg/mol\)

\(N_A = \) Avogadro’s constant, \(mol^{-1}\)

\(p_b = \) base pressure, \(psi\)

\(p_d = \) critical desorption pressure, \(psi\)

\(p_f = \) fracture pressure, \(psi\)

\(p_L = \) Langmuir pressure, \(psi\)

\(p_m = \) matrix pressure, \(psi\)

\(p_{sc} = \) pressure at standard condition, \(psi\)

\(p_{wf} = \) bottomhole pressure, \(psi\)

\(PNR_g = \) gas-phase pseudopressure normalized rate, \(Mscf/(psi \cdot day)\)

\(PNR_w = \) water-phase pseudopressure normalized rate, \(STB/(psi \cdot day)\)

\(q_g = \) gas flowrate from the whole fracture at surface condition, \(Mscf/day\)

\(q_w = \) water flowrate from the whole fracture at surface condition, \(STB/day\)

\(q_{g,m} = \) gas downhole flowrate at a specific location \(y\) in the matrix, \(Mcf/day\)

\(q_{g,inf} = \) gas influx rate from matrix at surface condition, \(Mcf/day\)

\(Q_g = \) cumulative gas production from the whole fracture at surface condition, \(scf\)

\(Q_{g,inf} = \) cumulative gas influx from matrix at surface condition, \(Mscf\)

\(Q_w = \) cumulative water production from the whole fracture at surface condition, \(STB\)

\(Q_{j,p,N} = \) normalized cumulative production, \(ft^3\)

\(r_{om} \) and \(r_{im} = \) radius of organic and inorganic nanopores at certain condition, \(m\)
\( r_{om,s} \) and \( r_{im,s} \) = radius of organic and inorganic nanopores under effective stress condition, \( m \)

\( r_{om,i} \) and \( r_{im,i} \) = initial radius of organic and inorganic nanopores, \( m \)

\( R \) = gas universal constant, \( J/(mol \cdot K) \)

\( RNP_g \) = gas-phase rate normalized pseudopressure, \( psi \cdot day/Mscf \)

\( RNP_w \) = water-phase rate normalized pseudopressure, \( psi \cdot day/STB \)

\( S_{j,f} \) = phase saturation in the fracture, -

\( S_{j,m} \) and \( S_{j,mi} \) = phase saturation in the entire shale matrix at certain and initial condition,

\( S_{w,im} \) = water saturation in the inorganic nanopore, -

\( t \) = time, day

\( t_{p,j,f} \) = pseudotime in the fracture, day

\( t_{p,g,m} \) = gas pseudotime in the matrix, day

\( t_{sp,j} \) = superposition pseudotime in the fracture, day

\( T \) = reservoir temperature, \( R \)

\( T_{sc} \) = temperature at standard condition, \( R \)

\( TOC \) = total organic carbon, -

\( V_a \) = volume of adsorbed gas per unit volume of bulk matrix at standard condition, scf/scf

\( V_{fl} \) = initial pore volume of half fracture, \( ft^3 \)

\( V_L \) = Langmuir volume, scf/scf

\( V_{mi} \) = initial pore volume of quarter fracture-matrix element, \( ft^3 \)

\( w_f \) = fracture width, \( ft \)

\( x \) = location in the fracture, \( ft \)
\( x_f = \text{fracture half-length, } ft \)
\( y = \text{location in the matrix, } ft \)
\( y_{inv} = \text{distance of investigation in the matrix with the DOI of matrix, } ft \)
\( y_m = \text{fracture half-spacing, } ft \)
\( Z_m = \text{gas deviation factor in the matrix, } - \)
\( \alpha = \text{rarefraction coefficient, } - \)
\( \alpha_1 = \text{fitting constant for rarefaction coefficient, } - \)
\( \beta = \text{fitting constant for rarefaction coefficient, } - \)
\( \gamma_f = \text{fracture permeability modulus, } psi^{-1} \)
\( \gamma_m = \text{matrix permeability modulus, } psi^{-1} \)
\( \delta_y = \text{unit conversion factor in } q_{g,inf} \)
\( \varepsilon_j = \text{unit conversion factor in IALF solution} \)
\( \lambda = \text{mean free path of gas molecules, } m \)
\( \kappa_b = \text{rate constant for blockage, } m/s \)
\( \kappa_m = \text{rate constant for migration, } m/s \)
\( \kappa_{TOC} = \text{kerogen correction factor, } - \)
\( \phi_f = \text{fracture porosity, } - \)
\( \phi_m \text{ and } \phi_{mi} = \text{entire shale matrix porosity at certain and initial condition, } - \)
\( \phi_{om} \text{ and } \phi_{im} = \text{porosity of organic and inorganic matter at certain condition, } - \)
\( \phi_{om,i} \text{ and } \phi_{im,i} = \text{initial porosity of organic and inorganic matter, } - \)
\( \mu_{j,f} = \text{fluid viscosity in the fracture, } cp \)
\( \mu_{g,m} = \text{gas viscosity in the matrix, } cp \)
\( \rho_{g,m} = \text{gas density in the matrix at downhole condition, } lb/ft^3 \)
\( \rho_{g,sc} = \text{gas density at standard condition, } lb/ft^3 \)
\( \rho_{kr} \) and \( \rho_r \) = density of kerogen and shale rock, \( lb/ft^3 \)

\( \psi \) = organic matter volume fraction, -

\( \tau \) = tortuosity of shale matrix, -

\( \theta \) = gas coverage for organic nanopores, -

**Subscripts**

IALF = infinite acting linear flow

BDF = boundary dominated flow

FMB = flowing material balance

\( f \) = fracture

\( full \) = full solution for matrix

\( g \) = gas

\( i \) = initial

\( im \) = inorganic matter

\( inf \) = influx

\( j = w \) or \( g \)

\( m \) = matrix

\( n \) = the nth flowrate interval

\( om \) = organic matter

\( p \) = pseudo

\( sp \) = superposition pseudo

\( sc \) = standard condition

\( w \) = water
Superscripts

- $= \text{average}$

5.8 Appendix A - Fracture IALF Model

The governing equation for water or gas flow in the hydraulic fracture is given in Eq. (5-17). Different from the BDF and FMB model developed for FR2 in Zhang and Emami-Meybodi (2020d), this model focuses on analyzing the earlier flowback data during FR1. Therefore, the adjusted initial and boundary conditions for fracture IALF regime are given in (5-18):

\[
\frac{\partial}{\partial x} \left( k_{rjf} \frac{\partial p_f}{\partial x} \right) = \frac{\phi_f c_{j,eff} \frac{\partial p_f}{\partial t}}{B_{jf}}, \quad (5-17)
\]

\[
p_f(x,0) = p_{fi}, \quad 2w_fh \left( k_{rjf} \frac{\partial p_f}{\partial x} \right)_{x=0} = q_j, \quad p_f(\infty,0) = p_{fi}, \quad (5-18)
\]

\[
C_{w,eff} = S_{w,f} \left( C_{w,f} + C_f \right) + \frac{dS_{w,f}}{dp_f}, \quad (5-19)
\]

\[
C_{g,eff} = S_{g,f} \left( C_{g,f} + C_f \right) + \frac{dS_{g,f}}{dp_f} - \frac{q_g \eta_f B_{g,f}}{2h x_f w_f \phi_f \frac{\partial p_f}{\partial t}}, \quad (5-20)
\]

where $k_{rjf}$ is the relative permeability in the fracture, $C_{j,f}$ is the fluid compressibility in the fracture, $q_{g,inf}$ is the gas influx rate from the entire matrix domain at the surface condition and includes the contribution of desorption and diffusion from matrix.

By introducing the pseudopressure $m_{j,f}$ and pseudotime $t_{p,f}$ for fracture, the governing equation is linearized as follows:

\[
\frac{\partial^2 m_{j,f}(p_f)}{\partial x^2} = \frac{1}{\eta_{j,f}} \frac{\partial m_{j,f}(p_f)}{\partial t_{p,j,f}}, \quad (5-21)
\]

\[
m_{j,f}(x,0) = m_{j,f}(p_{fi}), \quad \left. \frac{\partial m_{j,f}(p_f)}{\partial x} \right|_{x=0} = \frac{q \mu_{j,f} B_{j,f} l}{2w_fh k_{fi}}, \quad m_{j,f}(\infty, t_{p,j,f}) = m_{j,f}(p_{fi}), \quad (5-22)
\]

\[
m_{j,f}(p) = \left( \frac{\mu_{j,f} B_{j,f}}{k_f} \right) \int_{p_b}^{p} \frac{k_f(p) k_{r,j,f}(S_{j,f})}{\mu_{j,f}(p) B_{j,f}(p)} dp, \quad (5-23)
\]
\[ t_{pj,f} = \left( \frac{\phi_f \mu_j f C_{j,eff}}{k_f} \right) \int_0^t \frac{k_f(\varphi_f)\mu_{r,j,f}(S_{j,f})}{\phi_f(\varphi_f)\mu_{r,j,f}(\varphi_f)C_{j,eff}(\varphi_f)} \, dt, \quad (5-24) \]

where \( \eta_{j,f} = \left( \frac{k_f}{\phi_f \mu_j f C_{j,eff}} \right) \).

Due to the time-dependent inner boundary condition in Eqs. (5-18) and (5-22), we solved the auxiliary problem first with homogeneous inner boundary condition, i.e., under constant rate condition. The analytical solution of Eq. (5-21) in terms of \( RNP_j \) subject to the constant rate condition is given by Wattenbarger et al. (1998) in field unit:

\[ RNP_j = \varepsilon_j \frac{\varepsilon_j}{w_{rf h}} \sqrt{\left( \frac{\mu_j B_j^2}{\varepsilon_j C_{j,eff} k} \right) t_{pj,f}}, \quad (5-25) \]

where \( \varepsilon_j \) is the unit conversion factor, \( \varepsilon_w = 7.093 \) and \( \varepsilon_g = 7103.13 \).

Applying the discontinuous Duhamel’s principle (Özisik et al., 1993), we can extend the solutions in Eq. (5-25) to the problem with variable rate inner boundary condition by replacing \( t_{pj,f} \) with \( t_{sp,j,IALF} \):

\[ t_{sp,j,IALF} = \sum_{n=1}^{N} \frac{(q_{j,n} - q_{j,n-1})[t_{pj,f,N} - t_{pj,f,n-1}]}{q_{j,n}^2} \quad (5-26) \]

By generating \( RNP_j \) vs. \( \sqrt{t_{sp,j,IALF}} \), we can use the following equation to calculate \( k_{fi,j} \) based on the slope of the straight line, which falls into the FR1 identified by the diagnostic plot:

\[ k_{fi,j,IALF} = \left( \frac{\mu_j}{\phi C_{j,eff}} \right) \left( \frac{\varepsilon_j B_{j,fi}}{w_{rf h} a_{j,IALF}} \right)^2. \quad (5-27) \]

According the Wattenbarger et al. (1998), the end time of fracture IALF, (i.e., FR1) for each phase can be obtained as follows:

\[ t_{end,j} = \frac{x_f^2}{0.113^2} \left( \frac{\phi \mu_j C_{j,eff}}{k} \right) f_i. \quad (5-28) \]

According to \( t_{end,j} \), the duration of FR1 not only depends on the fracture properties but also is controlled by the matrix attributes. From the aspect of fracture properties, the FR1 will exhibit a longer duration for the reservoirs with larger \( x_f, \phi_f, \mu_f, \) and \( C_{j,eff} \), or smaller \( k_f \). From the aspect of
matrix attributes, the gas influx from matrix maintains the average pressure in the HF and extends the time for pressure drop reaching the boundary of fracture tip. Therefore, the significant gas influx, especially for shale reservoirs with the contribution of desorption and diffusion, will lead to the larger $C_{j,\text{eff}}$ and accordingly the longer FR1.

### 5.9 Appendix B – Matrix Model with Multiple Mechanisms

The material balance equation for gas flow in an infinitesimal element in the shale matrix is expressed as:

$$q_{g,m}\rho_{g,m}\bigg|_{y+\Delta y} - q_{g,m}\rho_{g,m}\bigg|_y = \frac{\partial}{\partial t}(\Delta y x_f h \phi_m S_{g,m}\rho_{g,m} + \Delta y x_f h V_a \rho_{g,sc}), \quad (5-29)$$

where $\Delta y x_f h$ is the control volume of the infinitesimal matrix element, $q_{g,m}$ is the downhole gas flowrate at a specific location $y$ in the matrix, $\rho_{g,m}$ is the gas density in the matrix at downhole condition, $\rho_{g,sc}$ is the gas density at standard condition, $V_a$ is the volume of adsorbed gas per unit volume of bulk matrix at standard condition and can be expressed by the Langmuir monolayer isothermal adsorption model (Langmuir, 1916):

$$V_a = \frac{V_L p_m}{p_m + p_L}. \quad (5-30)$$

To consider gas transport mechanisms apart from continuum flow, we introduced the apparent permeability $k_{app}$, which is described in Appendix C. By applying the corrected Darcy’s flowrate along the matrix $q_{g,m}\rho_{g,m} = \rho_{g,sc} x_f h \frac{k_{app}}{\mu_{g,m} B_{g,m}} \frac{\partial p_m}{\partial y}$, we rearranged Eq. (5-29) to the governing equation:

$$\frac{\partial}{\partial y} \left( \rho_{g,sc} \frac{k_{app}}{\mu_{g,m} B_{g,m}} \frac{\partial p_m}{\partial y} \right) = \frac{\partial}{\partial t} \left( \phi_m S_{g,m}\rho_{g,m} + V_a \rho_{g,sc} \right). \quad (5-31)$$
The right-hand side of Eq. (5-31) can be expanded with chain rule by introducing the definitions of \( B_{g,m} = \rho_{g,sc} / \rho_{g,m} \) (the gas formation volume factor) and \( C_{g,efm} \) (the effective compressibility in the matrix). Then, the simplified governing equation for matrix gas flow is:

\[
\frac{\partial}{\partial y} \left( \frac{k_{app} \mu_g, m B_{g,m}}{\partial p_m} \frac{\partial p_m}{\partial y} \right) = \frac{\phi_m C_{g,efm}}{B_{g,m}} \frac{\partial p_m}{\partial t}. \tag{5-32}
\]

\[
C_{g,efm} = S_{g,m} (C_{g,m} + C_m) + \frac{dS_{g,m}}{dp_m} + \frac{H(p_d-p_m)B_{g,m}p_LV_L}{\phi_m(p_m+p_L)^2}. \tag{5-33}
\]

The last term in the effective compressibility is the desorption compressibility (Gerami et al., 2007), which can be applied to both saturated and undersaturated shale reservoir by introducing the Heaviside function (Sun et al., 2017a). The desorption compressibility term could also be replaced with other expression according to the chosen adsorption model. The matrix model is coupled with the fracture model by assigning the average fracture pressure as the inner boundary condition of Eq. (5-32). We derived matrix flow model for the infinite and sealed outer boundary, respectively. The IALF solution can significantly save time for modeling before the matrix pressure drop reaches the boundary. The initial and boundary conditions for the infinite and sealed outer boundary are respectively given by:

\[
p_m(y, 0) = p_{mi}, \quad p_m(0, t) = \tilde{p}_f, \quad p_m(\infty, t) = p_{mi}, \tag{5-34}
\]

\[
p_m(y, 0) = p_{mi}, \quad p_m(0, t) = \tilde{p}_f, \quad \frac{\partial p_m}{\partial y} \bigg|_{y=y_m} = 0, \tag{5-35}
\]

where \( y_m \) is the fracture half-spacing.

Gas pseudopressure \( m_{g,m} \) and pseudotime \( t_{p,g,m} \) in the matrix are introduced to linearize the governing equation:

\[
m_{g,m}(p) = \left( \frac{\mu_{g,m}B_{g,m}}{k_{app}} \right) \int_{p_b}^{p} \frac{k_{app}(p)}{\mu_{g,m}(p)B_{g,m}(p)} dp, \tag{5-36}
\]

\[
t_{p,g,m} = \left( \frac{\phi_m \mu_{g,m} C_{g,efm}}{k_{app}} \right) \int_0^t \frac{k_{app}(\tilde{p}_m)}{\phi_m(\tilde{p}_m)\mu_{g,m}(\tilde{p}_m)C_{g,efm}(\tilde{p}_m)} d\tilde{t}. \tag{5-37}
\]

The linearized governing equation for Eq. (5-32) together with the initial and boundary conditions in Eqs. (5-34) and (5-35) are as follows:
\[ \frac{\partial^{2}m_{g,m}(p_{m})}{\partial y^{2}} = \frac{1}{\eta_{g,m}} \frac{\partial m_{g,m}(p_{m})}{\partial t_{p,g,m}}, \]  
(5-38)

\[ m_{g,m}(y,0) = m_{g,m}(p_{mi}), \quad m_{g,m}(0,t_{p,g,m}) = m_{g,m}(\bar{p}_{f}), \quad m_{g,m}(\infty,t_{p,g,m}) = m_{g,m}(p_{mi}), \]  
(5-39)

\[ m_{g,m}(y,0) = m_{g,m}(p_{mi}), \quad m_{g,m}(0,t_{p,g,m}) = m_{g,m}(\bar{p}_{f}), \quad \left. \frac{\partial m_{g,m}(p_{m})}{\partial y} \right|_{y=y_{m}} = 0, \]  
(5-40)

where \( \eta_{g,m} = \left( \frac{k_{app}}{\phi_{m} \mu_{g,m} C_{g,cf,m} \gamma_{i}} \right) \).

Due to the non-homogeneous inner boundary condition in Eqs. (5-39) and (5-40), we solved the auxiliary problem with homogeneous inner boundary condition first, i.e., constant bottomhole pressure (BHP) condition. The analytical solutions of Eq. (5-38) subject to the homogeneous inner boundary condition of Eqs. (5-39) and (5-40) are respectively given by Wattenbarger et al. (1998):

\[ m_{g,m}(p_{m}) = m_{g,m}(\bar{p}_{f}) + \left[ m_{g,m}(p_{mi}) - m_{g,m}(\bar{p}_{f}) \right] \text{erf} \left( \frac{y}{2 \sqrt{\eta_{g,m} t_{p,g,m}}} \right), \]  
(5-41)

\[ m_{g,m}(p_{m}) = m_{g,m}(\bar{p}_{f}) + \left[ m_{g,m}(p_{mi}) - m_{g,m}(\bar{p}_{f}) \right] \sum_{i=1}^{\infty} \frac{2}{\beta_{i}} \sin \left( \frac{\beta_{i} y_{m}}{y_{m}} \right) \exp \left( -\frac{\beta_{i}^{2} \eta_{g,m} t_{p,g,m} y_{m}^2}{2} \right), \]  
(5-42)

where \( \beta_{i} = (2l-1)\pi/2 \).

Applying the discontinuous Duhamel’s principle (Özisik et al., 1993), we can extend the solutions in Eqs. (5-41) and (5-42) to the problem with non-homogeneous inner boundary condition, i.e., variable BHP condition:

\[ m_{g,m}(p_{m}) = m_{g,m}(p_{mi}) - \left[ m_{g,m}(p_{mi}) - m_{g,m}(p_{b}) \right] \sum_{n=0}^{N-1} \text{erf} \left( \frac{y}{2 \sqrt{\eta_{g,m} (t_{p,g,m}-t_{p,g,m,n})}} \right) \Delta \zeta(t)_{n}, \]  
(5-43)

\[ \sum_{i=1}^{\infty} \frac{2}{\beta_{i}} \sin \left( \frac{\beta_{i} y_{m}}{y_{m}} \right) \exp \left( -\beta_{i}^{2} \eta_{g,m} (t_{p,g,m} - t_{p,g,m,n}) \right) \Delta \zeta(t)_{n}. \]  
(5-44)
\[
\Delta \zeta(t)_n = \zeta(t)_{n+} - \zeta(t)_{n-}, \quad (5-45)
\]
\[
\zeta(t) = \frac{m_{g.m}(p_{mi}) - m_{g.m}(p_b)}{m_{g.m}(p_{mi}) - m_{g.m}(p_b)} \frac{h x f k_{appi}}{\mu g B_g m |\bar{g}_m|} \sum_{n=0}^{N-1} \frac{\Delta \zeta(t)_n}{\sqrt{(t_{pg.m} - t_{pg.m,n})}}, \quad (5-46)
\]

By substituting Darcy’s flowrate into Eqs. (5-43) and (5-44) (Zhang and Emami-Meybodi, 2020d), we obtain the IALF solution and full solution of gas influx rate \(q_{g,inf}\) and cumulative influx \(Q_{g,inf}\) from matrix in field unit:

\[
q_{g,inf,IALF} = \left[ \frac{m_{g.m}(p_{mi}) - m_{g.m}(p_b)}{\delta g_{IALF} (\mu g B_g)_m |\bar{g}_m|} \right] h x f k_{appi} \sum_{n=0}^{N-1} \frac{\Delta \zeta(t)_n}{\sqrt{(t_{pg.m} - t_{pg.m,n})}},
\]

\[
q_{g,inf,full} = \left[ \frac{m_{g.m}(p_{mi}) - m_{g.m}(p_b)}{\delta g_{full} (\mu g B_g)_m |\bar{g}_m|} \right] h x f k_{appi} \sum_{n=0}^{N-1} \left[ \sum_{l=1}^{\infty} \exp \left( -\beta l^2 \frac{n_{g.m}(t_{pg.m} - t_{pg.m,n})}{y_m^2} \right) \right] \Delta \zeta(t)_n, \quad (5-48)
\]

\[
Q_{g,inf} = \int_0^t q_{g,inf} \, dt, \quad (5-49)
\]

where \(\delta_g\) is the unit conversion factor, \(\delta_{g,IALF} = 5578.5\) and \(\delta_{g,full} = 19752.5\).

### 5.10 Appendix C –Apparent Permeability for Matrix Model

As elaborated in the assumptions of conceptual model, the gas transport and storage in organic nanopores consider bulk gas mechanisms (continuum flow, slip flow, and Knudsen diffusion), surface diffusion, and gas desorption. Whereas, the gas flow in the inorganic nanopores only considers bulk gas mechanisms and the effect of adsorbed water film. Both the organic and inorganic nanopores take into account the stress dependence and real gas effect.

#### 5.10.1 Gas Transport in Organic Nanopores

We assume the porosity and permeability of organic matter follow the exponential function due to the stress dependence (Pedrosa Jr, 1986):
\[ \phi_{om} = \phi_{om,i} \exp[-c_m(p_{mi} - p_m)], \]  
(5-50)

\[ k_{om} = k_{om,i} \exp[-\gamma_m(p_{mi} - p_m)], \]  
(5-51)

where \( \phi_{om,i} \) and \( \phi_{om} \) are the porosity of organic matter before and after the consideration of stress dependence, \( k_{om,i} \) and \( k_{om} \) are the permeability of organic matter before and after the consideration of stress dependence, \( \gamma_m \) is the matrix permeability modulus. It is noteworthy that \( \phi_{om,i} \) is the ratio of initial organic nanopore volume to organic matter bulk volume and can be estimated using the kerogen porosity equation (Yu et al., 2018).

By applying Darcy’s law to nanotube model, we can obtain the Hagen-Poiseuille equation (Darabi et al., 2012):

\[ k_{om,i} = 10^{15} \frac{\phi_{om,i} r_{om,i}^2}{8 \tau}; k_{om} = 10^{15} \frac{\phi_{om} r_{om,s}^2}{8 \tau}, \]  
(5-52)

where \( r_{om,i} \) and \( r_{om,s} \) are the radius of organic nanopores before and after consideration of stress dependence, \( \tau \) is the tortuosity that can be estimated by numerous methods (Ghanbarian et al., 2013). The numerical validation in this paper adopted the tortuosity model proposed by Bo-Ming and Jian-Hua (2004) for porous media due to the relatively high matrix permeability. Whereas the field example shows an extremely low permeability, hence we used the tortuosity model by Alharthy et al. (2018): \( \tau = \phi^{1-m} \), where \( m = 2.25 \) for shale and tight formations.

Substituting Eqs. (5-50) and (5-51) into (5-52), the pore radius considering stress dependence is:

\[ r_{om,s} = r_{om,i} \exp\left[ \frac{1}{2} (c_m - \gamma_m)(p_{mi} - p_m) \right]. \]  
(5-53)

Equations (5-50) – (5-52) indicate that nanopore is compressed with the increasing effective stress during the pressure drawdown of the reservoir, which results in a reduction of the matrix permeability. At the same time, the pressure drawdown leads to gas desorption from organic nanopores, which is accompanied by the enlargement of pore size due to matrix shrinkage. We
employ the Langmuir isothermal adsorption model to quantify the volume occupied by adsorbed gas (Langmuir, 1916):

\[
\theta = \frac{p_m/Z_m}{p_m/Z_m + p_L},
\]

(5-54)

where \( \theta \) is the gas coverage under a certain pressure, \( Z_m \) is the gas deviation factor in the matrix, \( p_m \) remains the Langmuir critical adsorption pressure \( p_d \) until the pore pressure drops below \( p_d \). When pore pressure is lower than \( p_d \), \( p_m \) becomes the in-situ pore pressure.

As the pressure drops in the organic nanopores, the adsorbed gas gradually desorbs from the surface of nanopores and the adsorption layer thickness shrinks. Thus, the organic nanopore size will increase, leading to the permeability enhancement. The radius of organic nanopores accounting for stress dependence and gas desorption, \( r_{om} \), is presented by:

\[
r_{om} = r_{om, s} - d_m \theta,
\]

(5-55)

where \( d_m \) is the diameter of gas molecule.

In shale organic nanopores, the pore size may range from nanometer to micrometer. Accordingly, the bulk gas transport mechanism could be different in the various dimensions of organic pore size. Knudsen number is commonly used to classify the different bulk gas flow mechanisms:

\[
K_{n, om} = \frac{\lambda}{2r_{om}},
\]

(5-56)

\[
\lambda = 1.081 \times 10^{-7} \frac{\mu_{g,m}}{p_m} \sqrt{\frac{\pi Z_m RT}{2M}},
\]

(5-57)

where \( K_{n, om} \) is the Knudsen number for real gas in organic nanopores, \( \lambda \) is the mean free path of gas molecules, \( M \) is the gas molar mass, \( T \) is the reservoir temperature, and \( R \) is the gas universal constant.

When \( K_{n, om} < 0.001 \), the continuum flow dominates the shale matrix and is traditionally modeled by Darcy’s law (Darabi et al., 2012). When \( 0.001 < K_{n, om} < 0.1 \), the Darcy’s law has to be
modified to consider the slip velocity jump near the pore walls. The mass flux of slip flow for organic matter, \( J_{sl,om} \), can be written as (Wang et al., 2018b):

\[
J_{sl,om} = -2.809 \times 10^{11} \frac{\phi_{om} r_{om}^2 M p_m}{8 \pi \mu_{g,m} z_m R T} \left( 1 + \alpha K_{n,om} \right) \left( 1 + \frac{4 K_{n,om}}{1 - b K_{n,om}} \right) \frac{\partial p_m}{\partial y},
\]

\[\alpha = \frac{128}{15 \pi^2} \tan^{-1} \left( K_{n,om}^\beta \right),\]

where \( \alpha \) is the rarefaction coefficient evaluated at certain \( K_n \), \( \alpha_1 \) and \( \beta \) are the fitting constants determined from experiments or molecular simulation, \( b \) is the slippage constant.

As \( K_{n,om} \) increases and falls into the range of 0.1 to 10, the gas transport mechanism will exhibit transition flow between slip flow and the free-molecule flow. Wu et al. (2016) used a weighted averaged model of slip flow and Knudsen diffusion to characterize transition flow, which will be introduced later in this section. When \( K_{n,om} > 10 \), the collision between gas molecules and pore wall plays an important role in gas transport, which is categorized to be Knudsen diffusion.

The mass flux of Knudsen diffusion for organic matter, \( J_{k,om} \), can be described by (Wu et al., 2016):

\[
J_{k,om} = -3.0357 \times 10^3 \frac{2 \phi_{om} r_{om} M p_m C_{g,m}}{3 r_{om}} \left( \frac{d_m}{2 r_{om}} \right)^{D_f - 2} \left( \frac{8 Z_m M}{\pi R T} \right)^{0.5} \frac{\partial p_m}{\partial y},
\]

where \( D_f \) is the fractal dimension of the pore surface roughness varying between 2 for smooth and 3 for space-filling (Coppens and Dammers, 2006).

The continuum flow, slip flow, and Knudsen diffusion comprise the bulk gas flow mechanism, which transports gas molecules through the bulk space of the nanopores. In addition to the bulk gas transport, there is still a certain amount of gas transport through the surface of organic nanopores, which shouldn’t be neglected. The mass flux of surface diffusion in the organic matter, \( J_{sf,om} \), can be expressed as follows (Sun et al., 2017b, Wu et al., 2015):

\[
J_{sf,om} = -3.281 D_{s0} \frac{(1 - \theta) + \frac{K}{2} (2 - \theta) + [H(1 - \kappa)] (1 - \kappa) \frac{\theta^2}{2}}{(1 - \theta + \frac{K}{2} \theta)^2} \frac{\phi_{om}}{\tau} \frac{4 \theta M}{\pi d_m^3 N_A p_m} \frac{\partial p_m}{\partial y},
\]

\[D_{s0} = 8.29 \times 10^{-7} \times (0.555556 T)^{0.5} \exp \left( -\frac{\Delta H^{0.5}}{0.555556 R T} \right),\]
where \(D_{s0}\) is the surface diffusion coefficient when \(\theta\) is zero, \(\Delta H\) is the isosteric adsorption heat when \(\theta\) is zero, \(H(1-\kappa)\) is the Heaviside function, \(\kappa = \kappa_b / \kappa_m\) is the ratio of the rate constant for blockage \(\kappa_b\) to the ratio of the rate constant for migration \(\kappa_m\), and \(N_A\) is the Avogadro’s constant.

The total mass flux in the organic matter, \(J_{t,om}\), is the combination of bulk gas transport and surface diffusion. As for the mass flux of bulk gas transport, the contribution weight of different mechanisms depends on \(K_{n,om}\), which could vary from 0.01 to 6 in typical shale reservoirs (Wu et al., 2015). By introducing the weighting coefficients proposed by Wu et al. (2016), we can obtain a weighted averaged mass flux for bulk gas transport, \(J_{b,om}\), and accordingly the total mass flux in organic matter:

\[
J_{t,om} = J_{b,om} + J_{sf,om} = \left(\frac{1}{1+K_{n,om}}\right)J_{sl,om} + \left(\frac{1}{1+1/K_{n,om}}\right)J_{k,om} + J_{sf,om}. \tag{5-63}
\]

For the convenience of reservoir modeling, we applied Darcy’s law to convert the total gas flux into the apparent permeability for organic matter:

\[
k_{app,om} = -2.759 \times 10^6 J_{t,om} \frac{\mu_{g,m} \partial y}{\rho_{g,m} \partial p_m}, \tag{5-64}
\]

\[
\rho_{g,m} = 774.97 \frac{p_m M}{Z_m RT}. \tag{5-65}
\]

### 5.10.2 Gas Transport in Inorganic Nanopores

To derive the apparent permeability for inorganic matter, we introduced the organic matter volume fraction (Yu et al., 2018), \(\psi\), and the initial porosity of inorganic matter, \(\phi_{im,i}\):

\[
\psi = \frac{\text{volume of organic matter (kerogen)}}{\text{volume of shale rock}} = \frac{\text{TOC}}{\kappa_{TOC}}, \tag{5-66}
\]

\[
\phi_{im,i} = \frac{\text{inorganic pore volume}}{\text{volume of inorganic matter}} = \frac{\phi_{mi} - \psi \phi_{om,i}}{1 - \psi}, \tag{5-67}
\]

where \(TOC\) (total organic carbon) is the amount of carbon bound in organic compounds of a rock that is reported in weight percentage, \(\kappa_{TOC}\) is the kerogen correction factor and denotes the weight fraction of organic carbon in kerogen. \(\kappa_{TOC}\) usually ranges between 0.68 and 0.9, and defaults 0.8
\( \rho_r \) and \( \rho_{kr} \) are the density of shale rock and kerogen, \( \phi_{mi} \) is the initial porosity of the entire shale matrix (including organic and inorganic matter).

We assume the inorganic nanopores follow the same stress dependence of porosity and permeability with organic nanopores, which are controlled by \( C_m \) and \( \gamma_m \). The pore pressure of inorganic nanopores is assumed to be the same with organic pores. The pore radius considering the stress dependence will be:

\[
r_{im,s} = r_{im,i} \exp \left[ \frac{1}{2} \frac{C_m - \gamma_m}{\phi_m - p_m} \right],
\]

where \( r_{im,i} \) and \( r_{im,s} \) are the radius of inorganic nanopores before and after consideration of stress dependence.

The clay-rich inorganic nanopores are very apt to adsorb water on the pore surface to form water film due to the hydrophilic feature. The adsorbed water film will reduce the effective flowing pore space for gas and decrease the porosity and radius of organic nanopores. To consider the effect of water film, we define the water saturation in the inorganic nanopore, \( S_{w,im} \):

\[
S_{w,im} = \frac{\text{water volume in the inorganic nanopore}}{\text{inorganic pore volume}} = \frac{S_{w,m}\phi_m}{\phi_{im,s}(1-\psi)},
\]

where \( \phi_m = \phi_{mi} \exp(-C_m(p_{mi} - p_m)) \) and \( \phi_{im,s} = \phi_{im,i} \exp(-C_m(p_{mi} - p_m)) \) are the porosity of entire matrix and inorganic matter after the consideration of stress dependence, \( S_{w,m} \) is the water saturation in the entire shale matrix (including organic and inorganic matter).

During flowback and early production period, it is reasonable to consider the immobile water film (Sun et al., 2017b, Wang et al., 2018b, Li et al., 2020). As the pressure drops in the inorganic nanopores, \( S_{w,m} \) and \( S_{w,im} \) will slightly increase due to the production of gas. The radius \( r_{im} \) and porosity \( \phi_{im} \) of inorganic nanopores accounting for the effects of stress dependence and water film, are presented as follows (Shi et al., 2013):

\[
r_{im} = r_{im,s} \sqrt{1 - S_{w,im}},
\]
\[ \phi_{im} = (1 - S_{w,im})\phi_{im,s}. \]  
(5-71)

Similar to the organic nanopores, we can have Knudsen number for real gas in inorganic nanopores, \( K_{n,im} \), to classify the different bulk gas flow mechanisms:

\[ K_{n,im} = \frac{\lambda}{2r_{im}}. \]  
(5-72)

The surface diffusion doesn’t exist in the inorganic nanopores due to the adsorbed water film on the pore surface. Therefore the total mass flux in the inorganic matter, \( J_{t,im} \), solely comes from the mass flux of bulk transport, \( J_{b,om} \), which includes continuum flow, slip flow, and Knudsen diffusion:

\[ J_{t,im} = J_{b,im} = \left( \frac{1}{1 + K_{n,im}} \right) J_{sl,im} + \left( \frac{1}{1 + 1/K_{n,im}} \right) J_{k,im}, \]  
(5-73)

\[ J_{sl,im} = -2.809 \times 10^{11} \frac{\phi_{im} r_{im}^2 M_p}{8 r_{m} \mu_{g,m} Z_m R T} \left( 1 + \alpha K_{n,im} \right) \left( 1 + \frac{4K_{n,im}}{1 - bK_{n,im}} \right) \frac{\partial p_m}{\partial y}, \]  
(5-74)

\[ J_{k,im} = -3.0357 \times 10^{3} \frac{2 \phi_{im} r_{im}^2 M_p c_{g,m}}{3 r_{m} Z_m} \left( \frac{d_{m}}{2 r_{im}} \right)^{D_f - 2} \left( \frac{8 Z_m M_p}{\pi R T} \right)^{0.5} \frac{\partial p_m}{\partial y}, \]  
(5-75)

where \( J_{sl,im} \) is the mass flux of slip flow in the inorganic matter, \( J_{k,im} \) is the mass flux of Knudsen diffusion in the inorganic matter, and \( \alpha \) in Eq. (5-74) is evaluated at \( K_{n,im} \).

By applying Darcy’s law, the total gas flux can be converted into the apparent permeability for inorganic matter:

\[ k_{app,im} = -2.759 \times 10^{6} J_{t,im} \frac{\mu_{g,m} \partial y}{\rho_{g,m} \partial p_m}. \]  
(5-76)

Based on the assumption of parallel arrangement for organic and inorganic matter, the total apparent permeability for gas flow in the entire shale matrix can be obtained as the sum of \( k_{app,om} \) and \( k_{app,im} \) weighted by \( \psi \) (Wang et al., 2018b, Li et al., 2020):

\[ k_{app} = \psi k_{app,om} + (1 - \psi) k_{app,im}. \]  
(5-77)
5.11 Reference


Chapter 6

Water Production Analysis and Fracture Characterization of Hydraulically Fractured Reservoirs

6.1 Abstract

Analyzing water production data after hydraulic fracturing (i.e., flowback analysis) is a practical tool for characterizing hydraulic fracture (HF) properties. However, the primary complications in the current flowback analysis are the single-phase assumption in fracture-matrix flow and exclusion of liquid transport behaviors through nanopores. Accordingly, we present a new semianalytical approach to characterize HF attributes and dynamics using multi-phase flowback data from hydraulically fractured oil wells. The proposed approach includes a two-phase diagnostic plot, a fracture model for HF properties calculation, and a matrix model capable of characterizing water and oil transport in nanopores. The approach is based on fracture infinite acting linear flow as well as boundary dominated flow solutions, which treats HF as an open tank with a variable production rate at the well and the liquid contribution from matrix within the distance of investigation (DOI). We introduced a newly defined effective matrix permeability for each phase to consider multiple liquid transport mechanisms in nanopores including two-phase flow, slippage effect, stress dependence, the variation of fluid properties in the bulk and near-wall regions, as well as the differences in slip length, pore size, and wettability of organic and inorganic nanopores. We tested the accuracy of the proposed method against numerical results obtained from commercial software and verified its applicability by analyzing the flowback and long-term production data from a field example in Eagle Ford shale. The numerical validation confirms that our method can

closely calculate water and oil influx from matrix as well as the average pressure and saturation in the HF and matrix DOI. The accurate estimation of fracture properties, including the initial fracture permeability, pore volume, compressibility, and permeability modulus, demonstrates the accuracy of the proposed method in characterizing HF attributes and closure dynamics. Furthermore, the analysis results from numerical simulation and a field example both indicate a clear flow enhancement and deficit for oil and water transport, respectively, due to the slippage effect and the variation of fluid properties inside nanopores. The difference between apparent water permeability and apparent oil permeability becomes significant as the pore size decreases.

6.2 Introduction

Hydraulic fracturing has been widely used to enhance the hydrocarbon production from unconventional reservoirs, such as tight/ultratight (shale) oil reservoirs (Jia et al., 2017, Zeng et al., 2019). After well stimulation, the hydraulic fracturing liquid (mainly water) is flowed back from the reservoir before the production of hydrocarbon, which is called flowback process. As shown in Figure 6-1, flowback usually lasts for several days. The produced water mainly comes from the fracturing liquid inside hydraulic fracture and may also contain the leak-off water from formation matrix. Analyzing water production data after hydraulic fracturing (i.e., flowback analysis) is demonstrating more and more value in characterizing hydraulic fracture (HF) properties and evaluating the performance of well stimulation (Zhang and Emami-Meybodi, 2018).
The preliminary work of quantitative flowback analysis in tight and ultratight oil reservoirs focuses on the early period of single-phase water production. Abbasi et al. (2012) proposed a single-phase model to analyze water production data from the HF. Their attempt to extract HF properties during flowback encourages the more careful measurement of pressure and flowrate data in the industry and the development of mathematical models in academia. Clarkson and Williams-Kovacs (2013) classified the flowback stages of the tight oil wells into before breakthrough and after breakthrough, and developed analytical models to extract fracture half-length and permeability from both of the stages. Later, they improved their models to consider different fracture geometries and used stochastic simulation to better understand the uncertainty of unknown parameters during history matching (Williams-Kovacs and Clarkson, 2013).

To consider multi-phase flow during flowback, Kurtoglu et al. (2015) developed an analytical model by incorporating the water, oil, and gas flow into the total flowrate and total mobility. However, this treatment requires the linearized diffusivity equations for each phase, which may restrict the accuracy in application. Ezulike et al. (2016) proposed a two-phase tank model to estimate the fracture pore volume and fracture closure by defining total effective

Figure 6-1: Production history of a hydraulically fractured well in oil reservoir.
compressibility. Similarly, Fu et al. (2017) applied the single-phase tank model developed by Abbasi et al. (2012) to extract the fracture pore volume from the tight oil wells and proposed a workflow to determine the fracture compressibility using Diagnostic Fracture Injection Test data. Hossain et al. (2020) assumed a negligible fluid influx from the unstimulated matrix and presented a two-phase tank model to describe water and oil flow in the stimulated reservoir volume.

To consider fluid influx for the late-flowback stage, Clarkson et al. (2016) introduced the dynamic-drainage-area (DDA) model, which is adopted from well test (Shahamat et al., 2015), to analyze the two-phase flowback data in tight oil reservoirs. Water and oil influx from matrix is incorporated into the DDA model to extract fracture properties through history matching. In an extended study, the pre-flowback processes of fracture propagation and leakoff during hydraulic fracturing treatment are included in the DDA modeling to estimate the initial pressure and saturation of reservoir and fracture at the start of flowback (Clarkson et al., 2017). Later, they improved the DDA model by changing the enhanced fracture region to a dual-porosity system and reducing the uncertainties of geomechanical parameters with laboratory experiment data (Zhang et al., 2018b). Instead of using bi-wing fracture geometry, Chen et al. (2018) presented a flowback model for shale oil reservoirs with complex fracture networks. The numerical solving procedure and inversion are required to consider two-phase water and oil flow in both fracture and matrix systems in their model. Jia et al. (2019) coupled the numerical fracture model with an analytical transient linear model in the matrix to consider water and oil influx during flowback of tight oil wells.

In recent years, many researchers have used empirical decline curves to analyze multi-phase flowback data. Jones and Blasingame (2019) applied the hyperbolic and modified-hyperbolic models to forecast the flowrates of multi-phase during flowback. Fu et al. (2019) and Hossain et al. (2020) observed a harmonic decline behavior of water flowback and estimated the initial fracture volume with the ultimate water production through decline curve analysis.
In addition to the complexity in multi-phase flow, the accurate characterization of HF attributes and dynamics in shale oil reservoir is challenged by the multiple mechanisms of liquid transport in organic and inorganic nanopores. Early study of the liquid transport mechanisms in shale focuses on water flow in shale matrix during leakoff of hydraulic fracturing. Javadpour et al. (2015) corrected the traditional flow model with liquid slip length to consider the slippage effect at the pore walls and used atomic force microscope metrology to measure the slip coefficient for shale samples. Afsharpoor and Javadpour (2016) developed a generalized liquid flow equation for various geometric and slip conditions in a nano-duct. Li et al. (2016) conducted laboratory experiments to investigate the water sorption and distribution characteristics inside clay and shale samples. To consider the spatial variation of water viscosity in the bulk and near-wall region inside the nanopores, Wu et al. (2017) proposed a model to calculate water influx based on the slip boundary condition. They incorporated the changing water viscosity into the slippage effect by defining an effective slip length, which is the sum of true slip and apparent slip. Wu’s method shows robustness in avoiding tedious calculations for the modeling of confined water. Zhang et al. (2018a) developed a stochastic apparent permeability model to characterize water transport in shale matrix with dual-wettability systems, which requires the pore size distribution from scanning electron microscopy (SEM) images for implementation. Zeng et al. (2020) adopted the concept of effective slip length proposed by Wu et al. (2017) to model water transport in a single nanopore and developed an apparent permeability model for the shale matrix using the fractal theory.

More emphasis has been recently put on the oil storage and transport mechanisms in shale nanopores. Wang et al. (2015) and (2016) studied the oil adsorption behavior in organic shale nanopores and the transport behavior in the inorganic nanoslits using molecular dynamics simulation (MDS). Cui et al. (2017) presented a flow enhancement model to account for the slip flow and oil physical adsorption in organic nanopores. Zhang et al. (2017) proposed an apparent liquid permeability for shale oil reservoir, which is the weighted sum of apparent permeabilities in
organic and inorganic pores by total organic carbon (TOC). Zhang’s model considers the differences in slip length, pore size and wettability of organic and inorganic nanopores, and included the different oil properties, such as viscosity and density, in the bulk and near-wall regions. Different from the assumption of circular geometry of nanopores, Wang et al. (2019) developed an apparent permeability model for nanopores with elliptical cross-sections in shale reservoirs. Cui (2019) incorporated the mixed wettability and effects of wettability alteration, pore size shrinkage, and viscosity increase caused by asphaltenes into the apparent oil permeability. To characterize the oil transport in complex pore structures, Zhang et al. (2019) established a model for single-nanopore considering slip boundary and changing oil viscosity then scaled it up to the porous media using lattice Boltzmann method.

Given the gaps in existing research in the field of water flowback analysis, we developed a new semianalytical approach to analyze two-phase flowback data in tight and ultratight oil reservoirs. The contribution of this study is threefold. First, the proposed method incorporates the flow enhancement or deficit due to multiple mechanisms of liquid transport in organic and inorganic nanopores into flowback modeling. Second, the semianalytical model is, to our best knowledge, the first approach incorporating two-phase water and oil influx from matrix into the inverse analysis of fracture properties and dynamics using straight-line analysis, instead of history matching. Third, we proposed a new effective permeability model, which not only considers two-phase flow and stress dependence but also accounts for the variation of fluid properties in the bulk and near-wall regions, the differences in slip length, pore size, and wettability of organic and inorganic nanopores. Compared to numerical simulation, the proposed method not only covers more physics of two-phase liquid transport in nanopores and quantitative understanding of flow regimes, but also is much simpler and faster to implement because of excluding grids discretization, integral transformation/inversion, and numerical solver. Compared to empirical decline curve analysis, our approach not only shows more rigorousness and robustness but also provides a more
accurate long-term production forecast with the interpreted HF properties and dynamics. The new method is successfully validated against numerical simulations using commercial software and applied to an Eagle Ford oil well.

6.3 Theory and Mathematical Development

In this section, we first describe the conceptual model and the assumptions made in this study. Flow regime analysis is briefly introduced as a basis for further model development. Then, mathematical models for two-phase flow in fracture and matrix are respectively developed for all the possible flow regimes. We also proposed a new apparent permeability model to consider multiple fluid transport mechanisms in nanopores. At the end of this section, the analysis technique is introduced to iteratively obtain fracture properties and dynamics.

6.3.1 Conceptual Model

The conceptual flowback model describing water and oil flow through fracture and matrix is depicted in Figure 6-2. This model is expanded from the 2D two-phase model developed for shale gas reservoir by Zhang and Emami-Meybodi (2020d) and thus has the same assumptions of HF as before. To capture the complexity of multiple transport mechanisms in shale matrix, we divided the matrix system into the organic and inorganic matter with nanopores, as illustrated in the SEM image of shale sample (Naraghi and Javadpour, 2015). This model can also be used for the tight oil reservoirs by ignoring organic matter. As introduced by Wang et al. (2015), the organic matter usually behaves hydrophobic/oil-wet since it consists of the carbonaceous materials. Whereas the inorganic matter is mainly composed of water-wet minerals, such as clay, silica, and quartz, hence behaves hydrophilic (Zhang et al., 2017).
To consider the complexity of two-phase flow in the matrix, we generated two basic scenarios of fluid transport in the nanopores: Scenario 1 is a special case of water-saturated reservoir without oil in the nanopores, which may appear during the leakoff of hydraulic fracturing (Javadvpour et al., 2015). Water, as a strongly polar molecule, will adsorb on the surface of inorganic nanopore to form an adsorbed layer. Water molecules in the adsorbed layer would hardly move due to the strong water-wall molecular interactions, and the water viscosity ($\mu_{w,im}$) and density ($\rho_{w,im}$) in this region are larger than that of bulk water. Such an extra flow resistance in the adsorbed layer

Figure 6-2: Schematic of the conceptual model depicting water and oil transport through fracture and matrix (SEM image from Naraghi and Javadpour (2015)).
usually leads to a small value of slip length ($\lambda_{w,im}$) or even no-slip boundary condition (Wu et al., 2017). Whereas in organic nanopores, the solid hydrophobic substrate shows more repulsion to the water molecules in the near-wall region than bulk water. Hence, the water viscosity ($\mu_{w,om}$) and density ($\rho_{w,om}$) in the near-wall region usually go down, which results in a significant value of slip length ($\lambda_{w,om}$) at the flow boundary (Zeng et al., 2020).

Scenario 2 is a special case of the oil-saturated reservoir without water in the nanopores, which could be valid for the shales with Type I or Type II kerogen and small oxygen index (Curiale and Curtis, 2016). This scenario is widely used in long-term production data analysis. In contrast to water, liquid hydrocarbons are usually nonpolar molecules and show a different trend of transport behavior from water. Oil molecules are apt to form an adsorbed layer on the surface of organic nanopores with larger viscosity ($\mu_{o,om}$) and density ($\rho_{o,om}$) than bulk oil, as well as smaller slip length ($\lambda_{o,om}$). For the inorganic nanopores, the viscosity ($\mu_{o,im}$) and density ($\rho_{o,im}$) of near-wall oil become smaller, and the slip length ($\lambda_{o,im}$) goes up due to the weak wall liquid-wall interactions (Zhang et al., 2017). To model two-phase flow in the matrix, we first derived apparent permeability expressions for Scenarios 1 and 2, and then multiply them by a relative permeability to characterize the fraction of water and oil flow in nanopores.

Other main assumptions of the conceptual model are as follows:

- The matrix has the same stress dependence on organic matter and inorganic matter with the same pore pressure. The exponential function of stress dependence is adopted in fracture and matrix (Yilmaz et al., 1994).
- The organic and inorganic nanopores are arranged in parallel without fluid transport from each other (Sun et al., 2017, Wang et al., 2018, Li et al., 2020).
- Fluid transports in the nanopores with a slip boundary condition and negligible effect of capillary pressure.
• The fluid properties, such as viscosity, density, and formation volume factor, are pressure-dependent variables in the bulk region of nanopores.

• The circular cross-sectional shape of the organic and inorganic nanopore is adopted.

• The reservoir remains under-saturated condition throughout flowback period.

A preliminary step before conducting flowback analysis is the flow regime identification, which helps to choose the appropriate model for straight-line analysis accordingly. A set of two-phase diagnostic plots have been proposed to identify flow regimes during flowback of shale gas reservoir (Zhang and Emami-Meybodi, 2020c, Zhang and Emami-Meybodi, 2020b, Zhang and Emami-Meybodi, 2020d). In this paper, we expanded the diagnostic plot proposed by Zhang and Emami-Meybodi (2020d) to oil reservoirs to identify the fracture infinite acting linear flow (IALF), i.e., FR1, and fracture boundary dominated flow (BDF), i.e., FR2. As shown in Figure 6-3, FR1 and FR2 exhibit a half-slope and unit-slope straight line in the diagnostic plot, respectively. Although the current flowback models usually ignore the FR1 due to its short duration, it can still provide important information about HF property in the very early period if the matrix influx is significant or the flowback data can be recorded more intensively.
6.3.2 Fracture Model

According to Zhang and Emami-Meybodi (2020d), the governing equation for water or oil flow in the hydraulic fracture is given in Eq. (6-1). The initial and boundary conditions for FR1 and FR2 are described in Eqs. (6-2) and (6-3), respectively. The effect of fluid influx from matrix on fracture flow is included in the fracture effective compressibility in Eq. (6-4), which is derived in the work by Zhang and Emami-Meybodi (2020d).

\[
\frac{\partial}{\partial x} \left( \frac{k_{r,j,f} k_f \partial p_f}{\mu_{j,f} B_{j,f} \partial x} \right) = \frac{\phi_f C_{ej,f}}{B_{j,f}} \frac{\partial p_f}{\partial t},
\]

(6-1)

\[
p_f(x, 0) = p_{fl}, 2w_f h \left( \frac{k_{r,j,f} k_f \partial p_f}{\mu_{j,f} B_{j,f} \partial x} \right) \bigg|_{x=0} = q_j, p_f(\infty, 0) = p_{fl},
\]

(6-2)

\[
p_f(x, 0) = p_{fl}, 2w_f h \left( \frac{k_{r,j,f} k_f \partial p_f}{\mu_{j,f} B_{j,f} \partial x} \right) \bigg|_{x=0} = q_j, \frac{\partial p_f}{\partial x} \bigg|_{x=x_f} = 0,
\]

(6-3)

\[
C_{ej,f} = S_{j,f} (C_{j,f} + C_f) + \frac{dS_{j,f}}{dp_f} - \frac{q_{sj,B_{j,f}}}{2h x_f w_f \phi_f} \frac{\partial t}{\partial p_f},
\]

(6-4)
where subscript \( j \) can be \( w \) for water or \( o \) for oil, \( x \) is the location in the fracture, \( k_{j,f} \) is the relative permeability in the fracture, \( k_f = k_i \exp(-\gamma_f(p_f \cdot p_i)) \) is the pressure-dependent fracture permeability, \( \gamma_f \) is the fracture permeability modulus, \( p_i \) is the initial fracture pressure, \( \mu_{j,f} \) is the fluid viscosity in the fracture, \( B_{j,f} \) is the fluid formation volume factor in the fracture, \( \phi_f = \phi_i \exp(-C_f(p_i \cdot p_f)) \) is the pressure-dependent fracture porosity, \( C_f \) is the compressibility of fracture, \( t \) is the time, \( C_{j,f} \) is the effective compressibility in the fracture, \( q_f \) is the flowrate from fracture at surface condition, \( q_{ij} \) is the fluid influx rate from the entire matrix domain at the surface condition, \( x_f \) is the fracture half-length, \( w_f \) is the fracture width, \( h \) is the fracture height, \( S_{j,f} \) is the phase saturation in the fracture, and \( C_{j,f} \) is the fluid compressibility in the fracture.

The Eqs. (6-1) - (6-3) can be linearized into Eqs. (6-5) - (6-7) by defining fracture pseudopressure \( m_{j,f}(p) \) in Eq. and fracture pseudotime \( t_{p,j,f} \) in Eq. (6-9):

\[
\frac{\partial^2 m_{j,f}(p_f)}{\partial x^2} = \frac{1}{\eta_{j,f}} \frac{\partial m_{j,f}(p_f)}{\partial t_{p,j,f}}, \tag{6-5}
\]

\[
m_{j,f}[p_f(x, 0)] = m_{j,f}(p_{f_i}), \quad \frac{\partial m_{j,f}(p_f)}{\partial x} \bigg|_{x=0} = \frac{q_f \mu_{j,f} B_{j,f} C_{j,f}}{2w_f h k_{f_i}}, \quad m_{j,f}[p_f(\infty, t)] = m_{j,f}(p_{f_i}), \tag{6-6}
\]

\[
m_{j,f}[p_f(x, 0)] = m_{j,f}(p_{f_i}), \quad \frac{\partial m_{j,f}(p_f)}{\partial x} \bigg|_{x=0} = \frac{q_f \mu_{j,f} B_{j,f} C_{j,f}}{2w_f h k_{f_i}}, \quad \frac{\partial m_{j,f}(p_f)}{\partial x} \bigg|_{x=x_f} = 0, \tag{6-7}
\]

\[
m_{j,f}(p) = \left(\frac{\mu_{j,f} B_{j,f}}{k_f}\right) \int_{p_f}^{p} \frac{k_f(p) k_{j,f} C_{j,f}(S_{j,f})}{\mu_{j,f}(p) B_{j,f}(p)} \, dp, \tag{6-8}
\]

\[
t_{p,j,f} = \left(\frac{\phi_f \mu_{j,f} C_{j,f}}{k_f}\right) \int_0^t \frac{k_f(\bar{p}_f) k_{j,f}(S_{j,f})}{\phi_f(\bar{p}_f) \mu_{j,f}(\bar{p}_f) C_{j,f}(\bar{p}_f)} \, dt, \tag{6-9}
\]

where \( \eta_{j,f} = \left(\frac{k_f}{\phi_f \mu_{j,f} C_{j,f}}\right) \), \( \bar{p}_f \) is the average pressure in the fracture, and \( \bar{S}_{j,f} \) is the average saturation in the fracture.

We first achieve the solutions under constant rate condition and later extend to variable rate condition with Duhamel’s principle. The analytical solution of Eq. (6-5) subject to the initial
and boundary conditions of Eqs. (6-6) and (6-7) are presented in Eqs. (6-10) and (6-11), respectively, in straight-line form (Zhang and Emami-Meybodi, 2020d):

\[ RNP_j = a_{j,IALF} \sqrt{t_{pj,f}}, \quad (6-10) \]
\[ RNP_j = a_{j,BDF} t_{pj,f} + b_{j,BDF}, \quad (6-11) \]

where \( RNP_j = \frac{(m_{j,f} (p_i) - m_{j,f} (p_{wf}))}{q_j} \) is the rate normalized pseudopressure, \( p_{wf} \) is the bottomhole pressure, \( a_{j,IALF} \) is the slope of straight line for the plot of \( RNP_j \) vs. \( \sqrt{t_{pj,f}} \), \( a_{j,BDF} \) and \( b_{j,BDF} \) are the slope and intercept of straight line for the plot of \( RNP_j \) vs. \( t_{pj,f} \).

Eq. (6-10) is the solution for the fracture IALF (FR1), whereas Eq. (6-11) is the long-term approximation of the sealed-boundary solution for fracture BDF (FR2) (Zhang and Emami-Meybodi, 2020d). The formulations for \( a_{j,IALF}, a_{j,BDF}, \) and \( b_{j,BDF} \) are shown in Table 6-1. The constant rate solutions can be extended to variable rate conditions by replacing \( \sqrt{t_{pj,f}} \) with \( \sqrt{t_{spj,FR1}} \) and \( t_{pj,f} \) with \( t_{spj,FR2} \):

\[ \sqrt{t_{spj,FR1}} = \sum_{n=1}^{N} \frac{(q_{j,n} - q_{j,n-1})}{q_{j,N}} \sqrt{t_{pj,f,N} - t_{pj,f,n-1}}, \quad (6-12) \]
\[ t_{spj,FR2} = \sum_{n=1}^{N} \frac{(q_{j,n} - q_{j,n-1})}{q_{j,N}} (t_{pj,f,N} - t_{pj,f,n-1}), \quad (6-13) \]

where \( t_{spj,FR1} \) and \( t_{spj,FR2} \) are the superposition pseudotime for FR1 and FR2, respectively.

By plotting \( RNP_j \) vs. \( \sqrt{t_{spj,FR1}} \) (i.e., IALF specialty plot), one can obtain the initial fracture permeability \( k_{fi} \) based on the slope of the straight line during FR1. By plotting \( RNP_j \) vs. \( t_{spj,FR2} \) (i.e., BDF specialty plot), can also obtain \( k_{fi} \) and initial fracture pore volume \( V_{fi} \) based on the slope and intercept of the straight line during FR2. The formulations for \( k_{fi} \) and \( V_{fi} \) calculations are shown in Table 6-1 and are used to analyze the water-phase and oil-phase flowback data separately, whose results constrain with each other to help reach the unique interpreted fracture attributes.
Table 6-1: Summary of the formulations for \(V_{fi}\) and \(k_{fi}\), as well as the slope and intercept of the specialty plots.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>IALF model (RNP(<em>j) vs. (\sqrt{t</em>{e\text{p}j,FR1}}))</th>
<th>BDF model (RNP(<em>j) vs. (t</em>{e\text{p}j,FR2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_j)</td>
<td>(a_{j,IALF} = \frac{e}{w_f h} \sqrt{\frac{\mu_j \beta^2_j}{\phi c_{ej,f_i}}})</td>
<td>(a_{j,BDF} = \alpha \left(\frac{B_{ij,f}}{V_{f_i} c_{ej,f_i}}\right))</td>
</tr>
<tr>
<td>(b_j)</td>
<td>-</td>
<td>(b_{j,BDF} = \beta \frac{1}{h^2 w_f^2} \left(\frac{V_{f_i} \mu_j \beta^2_j}{\phi f k_f}\right))</td>
</tr>
<tr>
<td>(V_{fi})</td>
<td>-</td>
<td>(V_{f_i,j} = \alpha \frac{1}{a_{j,BDF}} \left(\frac{B_{ij,f}}{c_{ej,f_i}}\right))</td>
</tr>
<tr>
<td>(k_{fi})</td>
<td>(k_{fi,j,IALF} = \left(\frac{a_j}{\phi c_{ej,f_i}}\right) \left(\frac{e B_{ij,f_i}}{w_f h a_{j,IALF}}\right)^2)</td>
<td>(k_{fi,j,BDF} = \beta \frac{V_{f_i,j}(\mu_j \beta^2_j)}{h^2 w_f^2 \phi f_i b_{j,BDF}})</td>
</tr>
</tbody>
</table>

\(\alpha = 1, \beta = 13.16, \text{ and } \varepsilon = 7.093\) (Zhang and Emami-Meybodi, 2020d).

6.3.3 Matrix Model

To evaluate effective compressibility in Eq. (6-4), matrix model are developed to obtain the fluid influx rate \(q_{sj}\) from matrix. The governing equation for water or oil flow in the matrix system is given in Eq. (6-14). Different from the model which is only developed for the matrix infinite acting linear flow (Zhang and Emami-Meybodi, 2020d), this study derived the matrix model in a new flow regime, i.e., matrix boundary dominated flow. The new model in this work is able to account for the inter-fracture depletion when the pressure drop in the matrix reaches the symmetrical line between the two fractures. The initial and boundary conditions for the infinite and sealed outer boundaries are described in Eqs. (6-15) and (6-16), respectively. The inner boundary condition is set as the average fracture pressure to couple the fracture and matrix system together.

\[
\frac{\partial}{\partial y} \left( k_{ej} \frac{\partial p_m}{\partial y} \right) = \frac{\phi_m c_{ej,m}}{B_{j,m}} \frac{\partial p_m}{\partial t}, \quad (6-14)
\]

\[
p_m(y, 0) = p_{mi}, \quad p_m(0, t) = \bar{p}_f, \quad p_m(\infty, t) = p_{mi}, \quad (6-15)
\]

\[
p_m(y, 0) = p_{mi}, \quad p_m(0, t) = \bar{p}_f, \quad \left. \frac{\partial p_m}{\partial y} \right|_{y = y_m} = 0, \quad (6-16)
\]

\[
C_{ej,m} = S_{j,m}(C_{j,m} + C_m) + \frac{dS_{j,m}}{dp_m}, \quad (6-17)
\]
where \( y \) is the location in the matrix (perpendicular to the fracture), \( k_{ej} \) is the effective fluid permeability in the matrix, \( \mu_{j,m} \) is the fluid viscosity in the matrix, \( B_{j,m} \) is the fluid formation volume factor in the matrix, \( p_{mi} \) and \( p_m \) are the matrix pressure at the initial and a given condition, \( C_{ej,m} \) is the effective compressibility in the matrix, \( \phi_m = \phi_m \exp(-C_m (p_{mi} - p_m)) \) is the pressure-dependent matrix porosity, \( C_m \) is the matrix compressibility, \( \bar{p}_f \) is the average pressure in the fracture, \( y_m \) is the fracture half-spacing, \( S_{j,m} \) is the phase saturation in the matrix, and \( C_{j,m} \) is the fluid compressibility in the matrix.

The Eqs. (6-14) - (6-16) can be linearized into Eqs. (6-18) - (6-20) by defining matrix pseudopressure \( m_{j,m}(p) \) in Eq. (6-21) and matrix pseudotime \( t_{pj,m} \) in Eq. (6-22):

\[
\frac{\partial^2 m_{j,m}(p_m)}{\partial y^2} = \frac{1}{\eta_{j,m}} \frac{\partial m_{j,m}(p_m)}{\partial t_{pj,m}}, \tag{6-18}
\]

\[
m_{j,m}[p_m(y,0)] = m_{j,m}(p_{mi}), \quad m_{j,m}[p_m(0,t)] = m_{j,m}(\bar{p}_f), \quad m_{j,m}[p_m(\infty,t)] = m_{j,m}(p_{mi}), \tag{6-19}
\]

\[
m_{j,m}[p_m(y,0)] = m_{j,m}(p_{mi}), \quad m_{j,m}[p_m(0,t)] = m_{j,m}(\bar{p}_f), \quad \left. \frac{\partial m_{j,m}(p_m)}{\partial y} \right|_{y=y_m} = 0, \tag{6-20}
\]

\[
m_{j,m}(p) = \left( \frac{\mu_{j,m} B_{j,m}}{k_{ej}} \right) \int_{p_{b}}^{p} \frac{k_{ej}(p,S_{j,m})}{\phi_m(p) B_{j,m}(p)} dp, \tag{6-21}
\]

\[
t_{pj,m} = \left( \frac{\phi_m \mu_{j,m} C_{ej,m}}{k_{ej}} \right) \int_{0}^{t} \frac{k_{ej}(\bar{p}_m,\bar{S}_{j,m})}{\phi_m(\bar{p}_m) \mu_{j,m}(\bar{p}_m) C_{ej,m}(\bar{p}_m)} dt, \tag{6-22}
\]

where \( \eta_{j,m} = \left( \frac{k_{ej}}{\phi_m \mu_{j,m} C_{ej,m}} \right) \), \( \bar{p}_m \) is the average pressure in the matrix DOI, and \( \bar{S}_{j,m} \) is the average saturation in the matrix DOI.

Applying Duhamel’s principle and Darcy’s law, the analytical solution of Eq. (6-18) subject to the initial and boundary conditions of Eqs. (6-19) and (6-20) are presented in Eqs. (6-23) and (6-24), respectively, in terms of \( q_{ji} \):

\[
q_{s_{j,IALF}} = \left[ \frac{m_{j,m}(p_{mi})-m_{j,m}(p_{b})}{\delta_{IALF}(\mu B_j)} \right] m(\sqrt{\eta_{j,m}}) \sum_{n=0}^{N-1} \Delta \zeta_{j}(t_n) \sqrt{t_{p,j,m} - t_{p,j,m,n}}, \tag{6-23}
\]
\[ q_{sj,\text{full}} = \frac{[m_{j,m}(p_{mi})-m_{j,m}(p_b)] x f h k_{ej}}{\delta_{\text{full}}(\mu B_j)_{mi} y_m} \sum_{n=0}^{N-1} \left[ \sum_{t=1}^{\infty} \exp \left( -\beta t \frac{\eta_{j,m}(t_{p_{j,m}}-t_{p_{j,m,n}})}{y_m^2} \right) \right] \Delta \zeta_j(t)_n \].

(6-24)

\[ Q_{sj} = \int_0^t q_{sj} \, dt, \]

(6-25)

\[ \Delta \zeta_j(t)_n = \zeta_j(t)_{n+} - \zeta_j(t)_{n-}, \]

(6-26)

\[ \zeta_j(t) = \frac{m_{j,m}(p_{mi})-m_{j,m}(p_f(t))}{m_{j,m}(p_{mi})-m_{j,m}(p_b)}, \]

(6-27)

where \( \beta t = (2l - 1)\pi/2 \), \( p_b \) is the base pressure, \( Q_{sj} \) is the cumulative water or oil influx from matrix, \( \delta_j \) is the unit conversion factor: \( \delta_{\text{full}} = 19.757 \) and \( \delta_{\text{IALF}} = 5.575 \).

Equation (6-24) is the new full-solution of a sealed-boundary that considers both matrix IALF regime and BDF regime, whereas Eq. (6-23) is the short-term approximation of the full solution for matrix IALF. The calculation of influx can be optimized to reduce the simulation time by using the IALF solution before the matrix DOI reaches the fracture half-spacing and applying the full solution after that.

### 6.3.4 Multiple Liquid Transport Mechanisms in Matrix

Different from conventional matrix models, this study proposed a new apparent permeability model to consider multiple fluid transport mechanisms in nanopores. Specifically, the fluid influx in Eq. (6-23) - (6-24) considers the slippage effect during fluid transport in shale matrix, the variation of fluid properties in the bulk and near-wall region, as well as the differences in slip length, pore size, and wettability of organic and inorganic nanopores. These effects are incorporated into the effective fluid permeability in the matrix \( k_{ej} \), which accounts for stress dependence and two-phase fluid transport in the shale matrix.
To reach the formulation of \( k_{oj} \), we assume the same pore pressure of organic and inorganic nanopores and consider the same stress dependence in the porosity and permeability for organic and inorganic matter using the exponential function (Pedrosa Jr, 1986):

\[
\phi_{om} = \phi_{om,i} \exp[-C_m(p_{mi} - p_m)]; \quad \phi_{im} = \phi_{im,i} \exp[-C_m(p_{mi} - p_m)],
\]

\[ (6-28) \]

\[
k_{om} = k_{om,i} \exp[-\gamma_m(p_{mi} - p_m)]; \quad k_{im} = k_{im,i} \exp[-\gamma_m(p_{mi} - p_m)],
\]

\[ (6-29) \]

where \( \phi_{om,i} \) and \( \phi_{om} \) are the porosity of organic matter before and after the consideration of stress dependence, \( \phi_{im,i} \) and \( \phi_{im} \) are the porosity of inorganic matter before and after the consideration of stress dependence, \( k_{om,i} \) and \( k_{om} \) are the permeability of organic matter before and after the consideration of stress dependence, \( k_{im,i} \) and \( k_{im} \) are the permeability of inorganic matter before and after the consideration of stress dependence, and \( \gamma_m \) is the matrix permeability modulus. It is noteworthy that \( \phi_{im,i} \) is the ratio of initial inorganic nanopore volume to inorganic matter bulk volume.

The permeability can also be expressed as a function of nanopore radius using the Hagen-Poiseuille (HP) equation (Darabi et al., 2012):

\[
k_{om,i} = 10^{15} \frac{\phi_{om,i}R_{om,i}^2}{8\tau_{om,i}}; \quad k_{om} = 10^{15} \frac{\phi_{om}R_{om}^2}{8\tau_{om}}.
\]

\[ (6-30) \]

\[
k_{im,i} = 10^{15} \frac{\phi_{im,i}R_{im,i}^2}{8\tau_{im,i}}; \quad k_{im} = 10^{15} \frac{\phi_{im}R_{im}^2}{8\tau_{im}}.
\]

\[ (6-31) \]

where \( R_{om,i} \) and \( R_{om} \) are the radius of organic nanopores before and after consideration of stress dependence, \( R_{im,i} \) and \( R_{im} \) are the radius of inorganic nanopores before and after consideration of stress dependence, \( \tau_{om} \) and \( \tau_{im} \) are the tortuosity for organic and inorganic matter and can be calculated by \( \tau = \phi^{1-m} \), where \( m \) varies from 2.2 to 2.3 for shale and tight formations (Alharthy et al., 2018).

By combining Eqs. (6-28) - (6-29) with (6-30) - (6-31), we can obtain the radius of organic and inorganic nanopores considering the stress dependence:
\[ R_{om} = R_{om,t} \exp \left[ \frac{1}{2} (mc_m - \gamma_m) (p_{mi} - p_m) \right]; R_{im} = R_{im,t} \exp \left[ \frac{1}{2} (mc_m - \gamma_m) (p_{mi} - p_m) \right]. \] (6-32)

As illustrated in Figure 6-2, we adopted a radial-composite model of bulk and near-wall region to characterize the fluid transport in shale nanopores. Based on the analysis of the conceptual model, the flow equations are the same for single-phase oil or water transport in organic and inorganic nanopores. The only difference in transport behavior between water and oil is the variation trend of fluid properties, i.e., viscosity, density, and slip length, in the nanopores, as described in the section of Results. Hence, we develop a general apparent permeability model for single-phase oil or water transport in Scenarios 1 and 2 and later extend to the two-phase case. The velocity profile of slip-corrected HP flow in the inorganic nanopores can be described as (Zhang et al., 2017):

\[ v_{bj}(r) = \frac{\partial p_m}{\partial y} \frac{R_{im}^2 - r^2}{4\mu_{j,m}} + c_1, 0 \leq r \leq R_{im} - \delta_{j,im}, \] (6-33)

\[ v_{nwj}(r) = \frac{\partial p_m}{\partial y} \frac{R_{im}^2 - r^2}{4\mu_{j,im}} + c_2, R_{im} - \delta_{j,im} \leq r \leq R_{im}, \] (6-34)

where \( \delta_{j,im} \) is the thickness of near-wall region for oil or adsorbed region for water (i.e., adsorbed/near-wall region) in the inorganic nanopores, \( v_{bj} \) and \( v_{nwj} \) are the flow velocity at \( r \) in the bulk and adsorbed/near-wall region of inorganic nanopores, \( \mu_{j,m} \) and \( \mu_{j,im} \) are the fluid viscosity in the bulk and adsorbed/near-wall region, \( c_1 \) and \( c_2 \) are the unknowns to be determined related to slip velocity.

Based on the continuous velocity and shear stress at the interface as well as the slip-corrected boundary, the inner and outer boundary conditions of Eqs. (6-33) and (6-34) are presented:

\[ \frac{\partial v_{bj}}{\partial r} \bigg|_{r=0} = 0, \] (6-35)

\[ v_{bj} \bigg|_{r=R_{im}-\delta_{j,im}} = v_{nwj} \bigg|_{r=R_{im}-\delta_{j,im}}, \] (6-36)
where $\lambda_{j,im}$ is the slip length in the inorganic nanopores.

By solving the flow equations subject to the boundary conditions, we can get the velocity profiles in bulk and adsorbed/near-wall regions of the inorganic nanopore:

$$v_{bj} = 2.26 \times 10^7 \left[ \frac{R_{im}^2 - \mu_{j,im}^2}{4\mu_{j,im}} + \frac{\mu_{j,im} - \mu_{j,im}}{4\mu_{j,im}} (R_{im} - \delta_{j,im})^2 + \frac{\lambda_{j,im} r_{im}}{2\mu_{j,im}} \right] \frac{\partial p_m}{\partial y}, \quad 0 \leq r \leq R_{im} - \delta_{j,im},$$

$$v_{nwj} = 2.263 \times 10^7 \left( \frac{R_{im}^2 - r^2}{4\mu_{j,im}} + \frac{\lambda_{j,im} r_{im}}{2\mu_{j,im}} \right) \frac{\partial p_m}{\partial y}, \quad R_{im} - \delta_{j,im} \leq r \leq R_{im}.$$
\[ J_{b,j,om} = 3.62 \times 10^8 \frac{\rho_j \pi (R_{om} - \delta_{j,om})^2}{8} \left[ \frac{4R_{om}\delta_{j,om} - 2\delta_{j,om}^2 + 4\lambda_{j,om}R_{om}}{\mu_{j,om}} + \frac{(R_{om} - \delta_{j,om})^2}{\mu_{j,m}} \right] \frac{\partial p_m}{\partial y}, \]  

(6-44)

\[ J_{nw,j,om} = 3.62 \times 10^8 \frac{\rho_{nw,j,om}(2R_{om}\delta_{j,om} - \delta_{j,om}^2)}{8\mu_{j,om}} \left( 2R_{om}\delta_{j,om} - \delta_{j,om}^2 + 4\lambda_{j,om}R_{om} \right) \frac{\partial p_m}{\partial y}, \]  

(6-45)

\[ J_{j,om} = J_{b,j,om} + J_{nw,j,om}, \]  

(6-46)

where \( J_{b,j,om} \) and \( J_{nw,j,om} \) are the mass flux from the bulk and near-wall regions of the organic nanopores, and \( J_{j,om} \) is the total fluid mass flux from the organic nanopores.

Zhang et al. (2017) assumed no-slip boundary condition for oil transport in organic nanopores due to the adsorption. Similarly, the no-slip boundary is assumed in modeling water transport in the inorganic nanopores due to the water adsorbed on the hydrophilic pore wall (Wu et al., 2017). Our solutions of mass flux can be simplified to represent these two scenarios of oil and water transport by setting \( \lambda_{o,om}=0 \) and \( \lambda_{w,im}=0 \), respectively. The adsorbed layer can be further assumed immobile in the cases with significant liquid-wall molecular interaction by setting \( \mu_{o,om}=\infty \) and \( \mu_{w,im}=\infty \).

For the convenience of reservoir modeling, we applied Darcy’s law to convert the fluid mass flux into the apparent liquid permeability of the organic and inorganic matter, \( k_{j,om} \) and \( k_{j,im} \). The structural parameters of porosity and tortuosity are used to extend the oil flux from nanopore to porous media:

\[ k_{j,om} = 2.76 \times 10^6 \frac{\phi_{om}}{\tau_{om}} \frac{J_{j,om}}{\pi R_{om}^2} \frac{\mu_{j,m}}{\rho_j} \frac{\partial y}{\partial p_m}, \]  

(6-47)

\[ k_{j,im} = 2.76 \times 10^6 \frac{\phi_{im}}{\tau_{im}} \frac{J_{j,im}}{\pi R_{im}^2} \frac{\mu_{j,m}}{\rho_j} \frac{\partial y}{\partial p_m}. \]  

(6-48)

According to the assumption of parallel arrangement for organic and inorganic matter, we introduce the organic matter volume fraction (Yu et al., 2018), \( \psi \), and obtain the total apparent liquid permeability, \( k_{aj} \), as the weighted sum of \( k_{j,om} \) and \( k_{j,im} \) by \( \psi \) (Zhang and Emami-Meybodi, 2020a).
\[ k_{aj} = \psi k_{j,om} + (1 - \psi)k_{j,im}. \]  
\[ (6-49) \]

The apparent liquid permeability above only works for Scenarios 1 and 2 when single-phase water or single-phase oil exists and transports in the shale nanopores based on the assumptions we made in the derivation. Whereas, the flowback right after stimulation or early-time production of multi-fractured shale wells always exhibits the behavior of two-phase water and oil flow in the matrix. The traditional methods to model two-phase water and oil transport in porous media are multiplying one constant apparent permeability by relative permeability to consider the flow fraction for each phase (Zhang et al., 2018b, Jia et al., 2019). To incorporate the two limiting cases of single-phase water and oil transport into two-phase transport in shale nanopores, we propose a new definition of effective fluid permeability \( k_{ej} \) for the shale matrix:

\[ k_{ej} = k_{rj,m}k_{aj}, \]  
\[ (6-50) \]

where \( k_{rj,m} \) is the relative permeability in the matrix.

The new effective permeability is applied to calculate fluid influx from matrix in Eqs. (6-23) and (6-24), so that the multiple mechanisms of fluid transport in shale nanopores can be taken into account.

**6.3.5 Analysis Technique**

To evaluate pseudopressure and pseudotime in Eqs. (6-8) - (6-9) and (6-21) - (6-22), we used the following material balance equations to estimate the average pressure and saturation in the fracture and the matrix DOI:

\[ Q_w = V_{fi}\left(\frac{S_{w,f_i}}{B_{w,f_i}} - \frac{\bar{S}_w}{B_{w,f}} [1 - C_f(p_{f_i} - \bar{p}_f)]\right) + Q_{sw}, \]  
\[ (6-51) \]

\[ Q_o = V_{fi}\left(\frac{S_{o,f_i}}{B_{o,f_i}} - \frac{\bar{S}_o}{B_{o,f}} [1 - C_f(p_{f_i} - \bar{p}_f)]\right) + Q_{so}, \]  
\[ (6-52) \]
\[ Q_{sw} = 4V_{w,mi} \left( \frac{S_{w,mi}}{B_{w,mi}} - \frac{S_{w,m}}{B_{w,m}} \left[ 1 - C_m \left( p_{mi} - \bar{p}_m \right) \right] \right), \]  

(6-53)

\[ Q_{so} = 4V_{o,mi} \left( \frac{S_{o,mi}}{B_{o,mi}} - \frac{\bar{S}_o,m}{B_{o,m}} \left[ 1 - C_m \left( p_{mi} - \bar{p}_m \right) \right] \right), \]  

(6-54)

where \( Q_w \) and \( Q_o \) are the cumulative water and oil production at the surface condition, \( V_{fi} \) and \( V_{j,mi} \) are the initial pore volume of fracture and quarter element of matrix DOI, whose formulations are described as:

\[ V_{fi} = 2x_{wj} h \phi_{fi}, \]  

(6-55)

\[ V_{j,mi} = y_j x_{f} h \phi_{mi}, \]  

(6-56)

\[ y_j = 0.194 \sqrt{\frac{k_{ej,l_p,j,m}}{(\phi_{m}\mu_{j,m}\sigma_{e,j,m})}}, \]  

(6-57)

where \( y_j \) is the matrix DOI for each phase.

The material balance equations in Eqs. (6-51) - (6-54) are simultaneously solved using the fixed-point iteration (Kreyszig, 2009) to obtain \( \bar{p}_f, \bar{S}_{w,f}, \bar{p}_m, \) and \( \bar{S}_{w,m} \) at each time step. The fluid influx from matrix used in the material balance equation are obtained from Eq. (6-23) or (6-24). Then, the pseudo-variables can be estimated to generate the IALF and BDF specialty plots and calculate \( V_{fi} \) and \( k_f \). Considering that \( V_{fi} \) is not only a parameter to be determined but also an input in the material balance equations, we adopted the workflow proposed by Zhang and Emami-Meybodi (2020d) to estimate the fracture properties iteratively and quantify the HF dynamics by reconciling flowback and long-term production data. The water-phase and oil-phase flowback data were separately analyzed to constrain the extracted HF properties (\( V_{fi}, k_f, C_f \), and fracture permeability modulus \( \gamma_f \)).
6.4 Results

In this section, the accuracy of the proposed semianalytical method in extracting fracture properties is tested with the numerical simulation populated with the inputs for a shale oil well. After numerical validation, we applied the proposed method to analyze the flowback and long-term production data from a multi-fractured horizontal well (MFHW) in Eagle Ford shale oil reservoir to demonstrate the practicality and robustness.

6.4.1 Model Validation

We established two fine-grid numerical models of MFHW to generate synthetic flowback data using commercial software (CMG, 2019). Case 1 is produced with a constant water flowrate of 1,000 STB/day, and Case 2 is produced with a constant oil flowrate of 1,000 STB/day. The numerical models are simulated for 14 days to stand for the actual field operation. The pressure in fracture and matrix is kept above bubble point pressure during simulation so that oil remains undersaturated throughout the flowback period. The input fluid, fracture, and reservoir properties are the same for the two cases as listed in Table 6-2.
The source of input parameters for oil transport in nanopores is discussed below:

- **Viscosity ratio**: We adopted the oil viscosity ratio $\mu_{o,im}/\mu_{o,m} = 0.9$ for inorganic nanopores used by Zhang et al. (2017) and Wang et al. (2019), and $\mu_{o,om}/\mu_{o,m} = 1.1$ for organic nanopores obtained from matching the MDS data by Zhang et al. (2019).

- **Density ratio**: According to the density profiles of n-octane generated by MDS (Wang et al., 2016), the oil density does not change much in the inorganic nanopores, which was set constant by Wang et al. (2019), and oil density ratio $\rho_{o,om}/\rho_{o,m} = 1.1$ is widely used for the organic nanopores (Wang et al., 2015; Cui et al., 2017; Zhang et al., 2017).
- **Thickness of adsorbed/near-wall region**: The MDS results show that there are four layers of n-octane molecules adsorbed on the carbonaceous slit (Wang et al., 2015). However, the third and fourth layers can be ignored due to the slight density deviation from bulk oil (Wang et al., 2016). Therefore, the thickness of adsorbed oil in the organic nanopores is 0.96 nm (single layer 0.48 nm). As for the inorganic nanopores, this paper adopts the assumption of the same thickness of near-wall region with that of adsorbed layer by Zhang et al. (2017).

- **Slip length**: Wang et al. (2016) used an empirical exponential function to fit the correlation between slip length and pore size from MDS results. Zhang et al. (2019) modified the coefficient of exponential function for small external force condition and obtained the expressions to calculate the slip length of oil transport in organic and inorganic nanopores:
  \[
  \lambda_o,om = 10^{-9} \times [311.367 \exp(-2 \times 10^9 R_{om} / 1.0236) + 94.801] \quad \text{and} \quad \lambda_o,im = 10^{-9} \times [0.387 \exp(-2 \times 10^9 R_{im} / 32.47) + 0.74],
  \]
  respectively.

The source of input parameters for water transport in nanopores is discussed as follows:

- **Viscosity ratio**: The water viscosity in the nanopores is a function of wall wettability. Based on the laboratory experiment and MDS results, Wu et al. (2017) proposed a linear relationship between the viscosity ratio and contact angle: \(\mu_{W,om}/\mu_{W,m} = -0.018\theta + 3.25\), which is valid for both organic and inorganic nanopores when the pore radius is larger than 0.7 nm. The contact angle and viscosity of bulk water can be estimated with the formula proposed by Wu et al. (2017).

- **Density ratio**: According to the water density profiles generated by MDS (Sendner et al., 2009), the density of water confined between the hydrophobic substrates doesn’t change much and thus is set as a constant in organic nanopores in this paper. The density profiles of water confined between hydrophilic substrates indicate that \(\rho_{W,im}\) in inorganic nanopore is about 1.1 times larger than \(\rho_{W}\).
• Thickness of adsorbed/near-wall region: The water sorption isotherms with different wettabilities indicate monolayer adsorption on the surface of hydrophobic organic nanopores, i.e., $\delta_{w,om} = 0.4$ nm, (Li et al., 2016). The thickness of adsorbed water in the inorganic nanopores can be estimated with the apparent slip length proposed by Wu et al. (2017). For simplicity, this paper adopts $\delta_{w,im} = 1$ nm according to the water sorption isotherm for $\theta_{im} = 20^\circ$ (Li et al., 2016).

• Slip length: Huang et al. (2008) fitted the MDS results to obtain an empirical correlation between slip length and contact angle: $\lambda_{w,om} = C / (\cos \theta_{om} + 1)^2$, where $C=0.41 \times 10^{-9}$ for water flow in nanopores fitted by the MDS results (Wu et al., 2017). For water confined in the inorganic nanopores, the assumption of no-slip boundary can be valid due to the water adsorption on the hydrophilic substrate (Wu et al., 2017). Thus, we assume $\lambda_{w,im} = 0$ in this study.

• Water contact angle: Li et al. (2016) indicated that the graphite surface could represent the strongly hydrophobic organic pores, on which the water contact angle is around $100^\circ$. They also estimated the water contact angle on the surface of inorganic nanopore to be about $20^\circ$, which agrees with the experiment result conducted on a clay-rich shale sample.

The source of other input parameters is explained below:

• The organic matter volume fraction $\psi$ is estimated with the correlation introduced by Zhang and Emami-Meybodi (2020a): $\psi = \frac{TOC}{\kappa_{TOC} \rho_r} \rho_{kr}$, where $TOC$ is the amount of total organic carbon bound in a rock, $\kappa_{TOC}$ is the kerogen correction factor and defaults 0.8 (Crain, 2015, Tissot and Welte, 1984), $\rho_r$ and $\rho_{kr}$ are the density of shale rock and kerogen.

• The initial porosity of inorganic matter $\phi_{im,i}$ is calculated from the equation proposed by Zhang and Emami-Meybodi (2020a): $\phi_{im,i} = \frac{\phi_{mi} - \psi \phi_{om,i}}{1 - \psi}$. The initial porosity of organic matter $\phi_{om,i}$ is estimated using the kerogen porosity equation (Yu et al., 2018).
• We set the average radius of organic nanopores at the initial condition to be 10 nm in the numerical simulation, according to the pore size data from Eagle Ford Shale sample (Zhang et al., 2019). The ratio of $R_{om}/R_{im}$ is set to be 10 based on the pore size distribution in organic and inorganic pores (Javadpour et al., 2015).

• The relative permeability in the matrix is modeled with Corey’s formulation by assigning Corey’s exponent to 2 as well as the endpoint water and oil relative permeability to 0.85 and 0.75. This correlation is obtained from history-matching the flowback data in a tight oil reservoir by Zhang et al. (2018b). The HF relative permeability adopts the straight-line correlation used by Clarkson et al. (2012).

The semianalytical approach requires two apparent permeabilities for water and oil, i.e., $k_{ao}$ and $k_{aw}$, whose pressure dependence can be different and is solely controlled by the matrix stress dependence, i.e., $C_m$ and $\gamma_m$. Whereas, the numerical simulator can only input one permeability value as the initial matrix permeability and one permeability multiplier table for stress dependence. To make the numerical simulation consistent with our semianalytical approach, we first set the same stress dependence correlation for $k_{ao}$ and $k_{aw}$ by setting $C_m = 4.5 \times 10^{-6}$ psi$^{-1}$ and $\gamma_m = 1 \times 10^{-5}$ psi$^{-1}$, so that we can use one set of permeability multiplier, i.e., $k_{aj}/k_{aji}$, for both water and oil apparent permeabilities in the commercial software.

As elaborated in the previous section, the multiple fluid mechanisms are all included in the new effective permeability for each phase in the semianalytical approach. To implement the proposed effective permeability in the numerical simulator, we transformed the expression of effective permeability in Eq. (6-50) into Eqs. (6-58) - (6-59). By applying the transformations in Eq. (6-58) - (6-59), $k_{rj}^*$, $k_{mi}^*$, and $k_{aj}/k_{aji}$ are the adjusted inputs for commercial software so that the numerical simulation can fully represent the proposed semianalytical model of matrix flow.

$$k_{ej} = k_{rj}^* k_{mi}^* \frac{k_{aj}}{k_{aji}},$$

(6-58)
\[ k^*_{rj} = k_{rj,m} \frac{k_{a/ji}}{k^*_{mi}}, \quad (6-59) \]

\[ k^*_{mi} = \max(k_{aoi}, k_{awi}), \quad (6-60) \]

where \( k^*_{rj} \) is the modified relative permeability, \( k^*_{mi} \) is the modified initial matrix permeability that is used to normalize \( k^*_{rj} \) between 0 and 1, and \( k_{a/ji} \) is the apparent permeability multiplier.

Using the inputs listed in Table 6-2, we generated the pressure-dependent apparent permeability for water and oil as well as the absolute permeability without considering slip boundary and adsorbed/near-wall regions for comparison in Figure 6-4a. As shown in the figure, a clear flow enhancement can be observed in the apparent oil permeability curve due to the slippage effect. Whereas, the apparent water permeability is slightly lower than the absolute permeability due to the elevated water viscosity in the adsorbed/near-wall layer than the bulk region. The results in Figure 6-4a indicate that ignoring the multiple mechanisms of fluid transport in nanopores will lead to a 24\% difference in the estimation of apparent permeability, and the difference is expected to grow with the decrease of pore size. Figure 6-4b shows the same permeability multiplier for \( k_{ao}, k_{aw}, \) and absolute permeability, as described above. If the slip length, thickness of adsorbed/near-wall region, as well as the ratio of fluid properties between bulk and adsorbed/near-wall region are not constant but a function of pressure, they will show the impacts on the pressure dependence of \( k_{ao} \) and \( k_{aw} \). Figure 6-5a shows the HF relative permeability curves for numerical simulation, which are the straight-line correlation used by Clarkson et al. (2012). After applying the transformations in Eq. (6-58) - (6-59), the matrix relative permeability curves for numerical simulation are adjusted from the blue-color dots to the blue dash curve in Figure 6-5b. In other words, the flow enhancement of oil phase in the matrix in Figure 6-4a is converted to the drop of water relative permeability in Figure 6-5b.
We verified the accuracy of $\bar{p}_f$, $\bar{S}_{w,f}$, $\bar{p}_m$, and $\bar{S}_{w,m}$ calculation using material balance equations with a known $V_\beta$ as input. By simultaneously solving Eqs. (6-51) - (6-54), the average pressure and water saturation in the fracture and matrix DOI are obtained in Figure 6-6 for the two cases. The results show a good match of $\bar{p}_f$ and $\bar{S}_{w,f}$ from semianalytical approach with that from numerical simulation. The pressure drop and saturation variation in the matrix DOI are much less than that in HF due to the much smaller matrix permeability. Figure 6-7 illustrates the results of the calculated fluid influx from matrix using Eqs. (6-23) - (6-24) by considering the multiple mechanisms of two-phase transport. The calculated $Q_{sw}$ from analytical solution shows a perfect
match with numerical simulation for both cases, whereas $Q_{sw}$ exhibits a 6% difference at the late time. The slight overestimation of $Q_{sw}$ is caused by the approximation in calculating matrix pseudopressure and pseudotime (Zhang and Emami-Meybodi, 2020d), which is still acceptable for engineering purpose.

Figure 6-6: Comparison of average fracture pressure, average pressure in the matrix DOI, and average fracture water saturation, and average water saturation in the matrix DOI calculated from analytical solution and numerical simulation for (a-b) Case 1 and (c-d) Case 2.
With the calculated fluid influx and average pressure and saturation, we estimated the pseudo-variables for HF and generated the two-phase water and oil diagnostic plots in Figure 6-8. A clear half-slope and unit-slope straight line are shown in the plot to identify FR1 and FR2. Due to the short transition time between two flow regimes, we select the intersection point of two straight lines as the starting time of FR2, i.e., the ending time of FR1. For Case 1, the FR2 starts at 1.5 hrs and 1.2 hrs for water-phase and oil-phase. For Case 2, the FR2 starts at 1 hrs and 1.1 hrs for water-phase and oil-phase. After flow regime identification, we performed a straight-line analysis to estimate the fracture properties. Figure 6-9 presents the water and oil IALF specialty plots for the two cases. With the extracted the slope of the straight line during FR1, the \( k_{f,w} \) and \( k_{f,o} \) are calculated to be 3,201 mD and 3,268 mD for Case 1, and 3,255 mD and 3,248 mD for Case 2.

Figure 6-7: Comparison of cumulative water and oil influx calculated from analytical solution and numerical simulation for (a-b) Case 1 and (c-d) Case 2.
estimated initial fracture permeabilities are all close to the set value of 3,000 mD in the numerical simulation with the relative error < 10%. The water and oil BDF specialty plots are generated in Figure 6-10 to analyze flowback data during FR2. By extracting the slope and intercept of the straight lines, we obtain the calculated $V_{fi,w}$ and $V_{fi,o}$ to be 122.86 Mcf and 125.47 Mcf for Case 1, and 122.60 Mcf and 127.69 Mcf for Case 2, which are all close to the set value of 120 Mcf in the numerical simulation with the relative error < 5%. The estimated $k_{fi,w}$ and $k_{fi,o}$ from BDF specialty plots are listed in Table 6-3 with the relative error < 10%. Note that the calculated initial fracture pore volume shows more accuracy than the initial fracture permeability, which is consistent with the observation by Zhang and Emami-Meybodi (2020d). The estimated $V_f$ shows more robustness because it is solely calculated from the slope of the straight line, whereas $k_f$ is calculated from the not only slope but also intercept during straight-line analysis. Therefore, the accuracy of the semianalytical approach is successfully verified.

Figure 6-8: Two-phase water and oil diagnostic plot for (a-b) Case 1 with constant water rate and (c-d) Case 2 with constant oil rate.
Figure 6-9: Water-phase and oil-phase IALF specialty plot for (a-b) Case 1 with constant water rate and (c-d) Case 2 with constant oil rate.

Figure 6-10: Water-phase and oil-phase BDF specialty plot for (a-b) Case 1 with constant water rate and (c-d) Case 2 with constant oil rate.
Table 6-3: Summary of input data used in numerical simulation and calculated properties using the proposed method.

<table>
<thead>
<tr>
<th>Case #</th>
<th>Set values</th>
<th>Water-phase flowback model</th>
<th>Oil-phase flowback model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>IALF analysis</td>
<td>BDF analysis</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_{fi}$ (Mcf)</td>
<td>$k_{fi}$ (mD)</td>
</tr>
<tr>
<td>1</td>
<td>120</td>
<td>3,000</td>
<td>3.201</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
<td>3,000</td>
<td>3.255</td>
</tr>
</tbody>
</table>

6.4.2 Field Application

To test the applicability of the proposed semianalytical method, we analyzed the flowback and long-term production data from an MFHW drilled in Eagle Ford shale formation, Texas, USA. This field example was one of the six wells (i.e., “Well C”) presented in the study of Hossain et al. (2020). Hossain et al provided the quantitative analysis for all the wells, except Well C, using the proposed multiphase flowing material balance model. However, Well C was not analyzed in their study since it doesn’t show the behavior of multiphase depletion from a closed tank (i.e., boundary dominated flow). In this study, we provided a quantitative understanding of this well by using the proposed approach, which not only considers the oil and water inflow from matrix but also incorporates the multiple liquid transport mechanisms in the shale nanopores.

This well was hydraulically fractured into 44 stages with seven clusters per stage. Fracturing efficiency is set to be 60% in this study according to the field experience (Zhang and Emami-Meybodi, 2020d). The core sample analysis from the laboratory experiment provides the possible range of porosity (average 11.7 ± 0.55%), permeability (average 4.7 ± 2.7 nD), and water saturation (average 17.2 ± 2.7%) in the shale matrix. To account for the water leakoff and imbibition into shale matrix during shut-in period right after well stimulation, we set initial water saturation in the fracture to be 40% to better represent the initial reservoir condition before flowback. Due to the lack of experimental data for the relative permeability, we adopted the relative permeability curves for HF and matrix shown in Figure 6-11 from history matching by Clarkson.
and Williams-Kovacs (2013). The oil properties are based on the 14-component PVT data from Eagle Ford oil using direct flash calculation (Cronin et al., 2020). Other input parameters are listed in Table 6-4. The well was first conservatively flowed back for five days, after which it was shut-in for nine days to install production facilities, then was continuously produced for 203 days. The production history of oil-flow rate, water-flow rate, and bottomhole pressure during flowback and long-term production is shown in Figure 6-12. The bottomhole pressure is above the initial reservoir pressure of 8,000 psi for most of the flowback period, whereas stays below 8,000 psi during long-term production. Accordingly, the initial pressure of fracture and matrix system is set to be 9,600 psi and 8,000 psi for flowback and long-term production, respectively, to account for the supercharge effect and stabilized condition during shut-in. Only two-phase flow of oil and water is observed throughout the production since the bottomhole pressure always stays above the bubble point pressure of 2,410 psi. The flowrates during flowback significantly fluctuate due to the frequent change of choke size. Figure 6-12a shows a short-time (nine hours) single-phase water production at the beginning of flowback due to fracture depletion, after which oil influx from matrix dominates the two-phase flow in the late flowback. Figure 6-12b illustrates the dominant oil production and negligible water production during the long-term production period.

Figure 6-11: Relative permeability curves used in the semianalytical method in field example for (a) HF and (b) shale matrix (adopted from Clarkson and Williams-Kovacs (2013)).
Table 6-4: Summary of input data used in field example

<table>
<thead>
<tr>
<th>Fluid, reservoir and fracture parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial pressure of fracture and matrix for flowback, $p_{fi}$ and $p_{mi}$ (psi)</td>
<td>9,600</td>
</tr>
<tr>
<td>Initial pressure of fracture and matrix for long-term production, $p_{rl}$ (psi)</td>
<td>8,000</td>
</tr>
<tr>
<td>Bubble point pressure, $p_{bb}$ (psi)</td>
<td>2,410</td>
</tr>
<tr>
<td>Temperature, $T$ (°F)</td>
<td>278</td>
</tr>
<tr>
<td>Undersaturated oil compressibility, $C_o$ (psi⁻¹)</td>
<td>$1.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>Water compressibility, $C_w$ (psi⁻¹)</td>
<td>$3.3 \times 10^{-6}$</td>
</tr>
<tr>
<td>Initial fracture porosity, $\phi_{fi}$ (%)</td>
<td>50</td>
</tr>
<tr>
<td>Initial matrix porosity, $\phi_{mi}$ (%)</td>
<td>11.7</td>
</tr>
<tr>
<td>Matrix compressibility, $C_m$ (psi⁻¹)</td>
<td>$4 \times 10^{-6}$</td>
</tr>
<tr>
<td>Matrix permeability modulus, $\gamma_m$ (psi⁻¹)</td>
<td>$9 \times 10^{-5}$</td>
</tr>
<tr>
<td>Oil formation volume factor at initial pressure, $B_{oil}$ (ft³/STB)</td>
<td>6.22</td>
</tr>
<tr>
<td>Water formation volume factor at initial pressure, $B_{wli}$ (ft³/STB)</td>
<td>5.78</td>
</tr>
<tr>
<td>Initial fracture water saturation, $S_{w,fi}$ (%)</td>
<td>70</td>
</tr>
<tr>
<td>Initial matrix water saturation, $S_{w,mi}$ (%)</td>
<td>40</td>
</tr>
<tr>
<td>Bulk oil viscosity, $\mu_o$ (cp)</td>
<td>0.35</td>
</tr>
<tr>
<td>Bulk water viscosity, $\mu_w$ (cp)</td>
<td>0.38</td>
</tr>
<tr>
<td>Bulk oil density, $\rho_o$ (lb/ft³)</td>
<td>44</td>
</tr>
<tr>
<td>Bulk water density, $\rho_w$ (lb/ft³)</td>
<td>62</td>
</tr>
<tr>
<td>Reservoir thickness, $h$ (ft)</td>
<td>45</td>
</tr>
<tr>
<td>Number of active fractures</td>
<td>185</td>
</tr>
<tr>
<td>Fracture half spacing, $y_m$ (ft)</td>
<td>23</td>
</tr>
<tr>
<td>Fracture width, $w_f$ (ft)</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Figure 6-12: Production history of oil-flow rate, water-flow rate, and bottomhole pressure for the field case during (a) flowback and (b) long-term production (Hossain et al., 2020).

To consider the multiple transport mechanisms of water and oil in the shale nanopores, we calculated the pressure-dependent apparent oil permeability and water permeability using the approach in Section 6.3.4. The matrix permeability from core sample analysis (average $4.7 \pm 2.7$ nD) represents the absolute permeability without the effects of slip boundary and adsorbed/near-
wall regions. With the input parameters related to apparent permeability listed in Table 6-2, we calculated the initial radius of organic nanopores $R_{om,i} = 1$ nm. The ratio of $R_{om}/R_{im} = 3$ is adopted from the pore-scale distribution data derived from Eagle Ford Shale samples (Zhang et al., 2019). Accordingly, the initial apparent oil permeability and water permeability are calculated to be $k_{omi} = 98.7$ nD and $k_{awi} = 3.6$ nD. The significant difference between $k_{omi}$ and $k_{awi}$ indicates the flow enhancement and deficit for oil and water in shale nanopores, respectively, which agrees with the observation in the section of numerical validation.

In this study, the two-phase flowback data were interpreted with the developed semianalytical method, whereas the oil-phase data during long-term production were analyzed by commercial software (IHS, 2019) via history matching. The interpreted results from flowback and long-term production data analysis are reconciled using the workflow presented by Zhang and Emami-Meybodi (2020d) to provide a quantitative characterization of HF properties and dynamics. After convergence of the workflow, the calculated average pressure in the fracture and matrix DOI during flowback are shown in Figure 6-13a. The average pressure drop in the fracture is much larger than that in the matrix DOI due to the low permeability in shale matrix. We can observe a clear build-up of average fracture pressure in Figure 6-13a, which agrees with the build-up of bottomhole pressure in Figure 6-12a caused by a shut-in during flowback. Figure 6-13b illustrates the comparison of cumulative production and matrix influx for water and oil. The close match of oil production and influx reveals that the produced oil from the well mainly comes from oil influx from the matrix, which is contributed by oil flow enhancement through nanopores. Whereas, the significant difference between water production and influx indicates the negligible water contribution from matrix, which results from the flow deficit in apparent water permeability. The comparison in Figure 6-13b quantitatively proves the observation by Hossain et al. (2020) that oil is mainly produced from highly stimulated matrix while the produced water mainly comes from fractures for the given field example. Figure 6-13c and Figure 6-13d show the two-phase diagnostic
plot of water and oil. Although fluctuation exists due to the frequent change of choke size, we can still observe a trend of unit-slope straight line indicating the FR2, i.e., fracture BDF regime. The water and oil BDF specialty plots are shown in Figure 6-13e and Figure 6-13f to perform straight-line analysis. With the extracted slope and intercept from straight lines, we calculated the initial fracture pore volume $V_{fi,w} = 179.5$ Mcf and $V_{fi,o} = 170.4$ Mcf, as well as initial fracture permeability $k_{fi,w} = 1,634.4$ mD and $k_{fi,o} = 1,766.6$ mD, using the formula listed in Table 6-1. The relative errors for the interpreted $V_f$ and $k_f$ from water and oil phase data are 5.3% and 7.5%, which are acceptable for engineering purposes.
We also analyzed long-term production data of this well to constrain the fracture compressibility and quantify HF dynamics. Considering that oil is dominantly produced during long-term production with negligible water, we used commercial software (IHS, 2019) to perform history matching with the Horizontal-Multifractured-SRV model for single-phase oil flow. The

Figure 6-13: Flowback analysis after the convergence of Vfi and kfi: (a) Average pressure in the fracture and matrix DOI during flowback. (b) Comparison of cumulative production from the well and cumulative influx from matrix for oil and water. (c) Two-phase water diagnostic plot. (d) Two-phase oil diagnostic plot. (e) Two-phase water BDF specialty plot. (f) Two-phase oil BDF specialty plot.
fluid and reservoir properties in history matching are adopted from Table 6-4 for consistency. Figure 6-14 shows a good match of oil flowrate and bottomhole pressure. The slight deviation in the early portion of the plot is caused by the fluctuation of flowrates and the two-phase production behavior. The estimated fracture pore volume and fracture permeability from long-term production history matching are $V_{f,LT} = 133.1 \text{ Mcf}$ and $k_{f,LT} = 978.9 \text{ mD}$. These two parameters can also be predicted based on the interpreted $V_{fi}$ and $k_{fi}$ from flowback data using the exponential correlation:

$$V_{f,FB} = V_{fi} \exp(-C_f(p_{fi} - p_{LT}))$$
$$k_{f,FB} = k_{fi} \exp(-\gamma_f(p_{fi} - p_{LT}))$$

The ultimate pressure during long-term production, $p_{LT}$, can be evaluated to be 4000 psi, which is the stabilized bottomhole pressure in the late-production period. The comparison between the predicted values from flowback data (i.e., $V_{f,FB} = 147.9 \text{ Mcf}$ and $k_{f,FB} = 971.3 \text{ mD}$) and history matched values from long-term production data (i.e., $V_{f,LT} = 133.1 \text{ Mcf}$ and $k_{f,LT} = 978.9 \text{ mD}$) provides an important constraint of $C_f$ and $\gamma_f$ to be $3 \times 10^{-4} \text{ psi}^{-1}$ and $1 \times 10^{-4} \text{ psi}^{-1}$.

![Figure 6-14: Long-term production history match for (a) oil-flow rate and (d) bottomhole pressure.](image)

By reconciling the interpreted results from flowback and long-term production data, we can observe a significant decrease in fracture permeability from 1,700 to 979 mD and in fracture pore volume from 175 to 133 Mcf due to fracture closure. Although the fracture permeability has reduced by almost half, it still remains large enough after eight months production due to the conservative flowback operation (i.e., slowback). The fracturing fluid used in the well stimulation
(i.e., cross-linked gel) confirms the slowback behavior because of its low mobility. The reduction of fracture pore volume and permeability is expected to be more significant for more aggressive flowback operations with less viscous fracturing fluid, such as slickwater. Therefore, the HF dynamics due to partial fracture closure during production are successfully characterized with the interpreted HF properties from flowback and long-term production data, as shown in Table 6-5.

Table 6-5: Results from flowback analysis and long-term production data analysis.

<table>
<thead>
<tr>
<th>Fracture properties</th>
<th>Flowback</th>
<th>Long-term production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial fracture pore volume ( V_{f_i} ), Mcf</td>
<td>175.0</td>
<td>-</td>
</tr>
<tr>
<td>Initial fracture permeability ( k_{f_i} ), md</td>
<td>1,700.5</td>
<td>-</td>
</tr>
<tr>
<td>Fracture permeability modulus ( \gamma_f ), psi-1</td>
<td>( 1 \times 10^{-4} )</td>
<td>-</td>
</tr>
<tr>
<td>Fracture compressibility ( C_f ), psi-1</td>
<td>( 3 \times 10^{-5} )</td>
<td>-</td>
</tr>
<tr>
<td>( V_{f,FB} ) predicted from flowback for long-term, Mcf</td>
<td>147.9</td>
<td>-</td>
</tr>
<tr>
<td>( V_{f,LT} ) obtained from long-term analysis, Mcf</td>
<td>-</td>
<td>133.1</td>
</tr>
<tr>
<td>( k_{f,FB} ) predicted from flowback for long-term, md</td>
<td>971.3</td>
<td>-</td>
</tr>
<tr>
<td>( k_{f,LT} ) obtained from long-term analysis, md</td>
<td>-</td>
<td>978.9</td>
</tr>
</tbody>
</table>

6.5 Discussion and Recommendations

The new semianalytical method is demonstrated to provide an accurate estimation of HF properties and dynamics, such as initial fracture pore volume and permeability, as well as fracture compressibility and permeability modulus, based on flowback and long-term production data from oil reservoirs. The method consists of a two-phase diagnostic plot, a fracture model for HF properties calculation, and a new matrix model capable of characterizing water and oil transport in shale nanopores. The fracture model not only considers the inter-fracture depletion in the late-time flowback period through the full solution of matrix model, but also includes a fracture IALF solution to provide the estimation of HF permeability during the early-time flowback period. The interpreted fracture properties from water-phase and oil-phase flowback data are constrained with each other to help reach the unique analysis results.
One of the key contributions of this work is incorporating multiple fluid transport mechanisms into a newly defined effective permeability for each phase, as opposed to using just one matrix permeability under Darcy’s law. The multiple complexities of fluid transport considered in the matrix model include the two-phase flow, slippage effect, stress dependence, variation of fluid properties in the bulk and near-wall regions, as well as the differences in slip length, pore size, and wettability of organic and inorganic nanopores. More importantly, to our best knowledge, the proposed method is the first semianalytical approach for oil reservoirs that is capable of including the aforementioned complexities into the straight-line analysis instead of history matching. While the presented field example is from shale oil formation, the proposed method can be applied to tight oil reservoirs as well.

Compared to numerical simulation, the new method not only covers more physics of two-phase liquid transport in nanopores and quantitative understanding of flow regimes, but also is much simpler and faster to implement because of excluding grids discretization, integral transformation/inversion, and numerical solver. Compared to empirical decline curve analysis, our approach not only shows more rigorousness and robustness but also can provide a more accurate long-term production forecast with the interpreted HF properties and dynamics.

The assumptions made on the mathematical models inevitably create some limitations. First, a pressure-saturation relation is required to estimate the pseudopressure and pseudotime in the matrix accurately. In this study, we evaluated the relative permeability in matrix pseudopressure at the initial condition and ignored $dP_m/dS_{jm}$ in matrix pseudotime after magnitude analysis of different terms in $C_{j,efm}$. Such an approximation is valid with small pressure drop and saturation variation in the matrix DOI, but could lead to some error when the matrix pressure drop becomes significant. Second, the proposed method assumes undersaturated oil in the matrix and fracture throughout the flowback period. This may cause an error if the reservoir is initialized with a pressure slightly above or even below the bubble point pressure, in which the dissolved gas may
appear in the wellbore, HF, or reservoir. Third, it is recommended to reconcile the proposed approach with the laboratory experiments in the application so that more constraints can be provided to reach the unique interpretation results. Finally, modern methods involving liquid transport in shale nanopores can be categorized into digital cores simulation, experimental or molecular dynamic simulation, and formula derivation (Feng et al., 2019). However, most of these studies are performed for single-phase water or oil flow in organic and inorganic nanopores, and scarce research has been done on the two-phase transport with the multiple mechanisms considered. This study, as a preliminary attempt, is based on the formula derivation of apparent two-phase permeability. The following work shall focus on using laboratory experiments, digital cores, or molecular dynamic simulation to improve the apparent permeability model and quantitatively characterize two-phase transport in microscale.

### 6.6 Summary and Conclusions

Analysis of two-phase water and oil flowback data right after hydraulic fracturing has been proposed to characterize hydraulic fractures (HF) and reservoir attributes. However, the accuracy of HF properties estimation is challenged by the complexities in two-phase flow and multiple liquid transport mechanisms in tight/shale reservoirs. In this paper, we present a new semianalytical approach to perform flowback analysis for hydraulically fractured wells with two-phase flow. The proposed approach provides a quantitative understanding of HF properties and dynamics through initial fracture pore volume, initial fracture permeability, fracture compressibility and permeability modulus. The newly defined effective permeability in matrix model accounts for multiple liquid transport mechanisms, including two-phase flow, slippage effect, stress dependence, variation of fluid properties in the bulk and near-wall regions, as well as the differences in slip length, pore size, and wettability of organic and inorganic nanopores. The nature of water and oil transport with
multiple mechanisms in shale nanopore is examined in numerical simulation using a modified relative permeability, a modified initial matrix permeability, and an apparent permeability multiplier as inputs for the simulator. We successfully applied the proposed method to a field case to analyze the flowback and long-term production data from an MFHW in the Eagle Ford shale formation. The key conclusions of this work are as follows:

- The proposed infinite acting linear flow model (IALF) is capable of calculating initial fracture permeability, which provides an important constraint for the interpreted the initial fracture pore volume and permeability from the boundary dominated flow (BDF) model.
- The accuracy of the proposed method in estimating HF properties are demonstrated using the numerically simulated data. The estimated initial fracture pore volume shows more accuracy than initial fracture permeability due to the different calculation sources in straight-line analysis, i.e., the former one is from the slope, and the latter one is from both the slope and intercept. Both relative errors are less than 10%, which confirms the applicability of the model for engineering purposes.
- The change of apparent permeability with pressure is mainly caused by stress dependence, i.e., matrix compressibility and permeability modulus. The behaviors of pressure dependence for apparent water permeability and apparent oil permeability can be different.
- The field application analyzed by the proposed method shows that oil is sequentially produced from fractures and highly stimulated matrix, while the produced water mainly comes from fractures.
- The interpreted results from the field example reveal that HF closure leads to the reduction of fracture permeability and fracture pore volume by 42% and 24% after eight months of production. The deterioration of HF properties is expected to grow with more aggressive flowback operations and less viscous fracturing fluid.
A clear flow enhancement and a slight flow deficit for oil and water transport in shale nanopores, respectively, are observed in synthetic cases and confirmed in a field example. Ignoring the multiple fluid transport mechanisms leads to a 24% difference in the estimated apparent permeability in the synthetic case. The difference is expected to grow with the decrease of pore size and can be as large as 20 times, as shown in the field example.

6.7 Nomenclatures

- \( a_{j,BDF} \) = slope of \( RNP_j \) vs. \( t_{spj} \) plot
- \( a_{j,IALF} \) = slope of \( RNP_j \) vs. \( \sqrt{t_{spj}} \) plot
- \( b_{j,BDF} \) = intercept of \( RNP_j \) vs. \( t_{spj} \) plot
- \( B_{j,f} \) and \( B_{j,m} \) = fluid formation volume factor in the fracture and matrix, \( ft^3/STB \)
- \( B_{ob} \) and \( B_{oi} \) = oil formation volume factor evaluated at bubble point pressure or initial pressure, \( ft^3/STB \)
- \( B_{wi} \) = water formation volume factor evaluated at initial pressure, \( ft^3/STB \)
- \( c_1 \) and \( c_2 \) = unknowns to be determined related to slip velocity, \( m/s \)
- \( C_{ej,f} \) and \( C_{ej,m} \) = fluid effective compressibility in the fracture and matrix, \( psi^{-1} \)
- \( C_f \) and \( C_m \) = compressibility of fracture and matrix, \( psi^{-1} \)
- \( C_{j,f} \) and \( C_{j,m} \) = fluid compressibility in the fracture and matrix, \( psi^{-1} \)
- \( DRNP_j \) = derivative of rate normalized pseudopressure, \( psi \cdot day/(STB \cdot ln(day)) \)
- \( h = \) fracture height, \( ft \)
- \( J_{b,j,im} \) and \( J_{nw,j,im} \) = mass flux from the bulk and adsorbed/near-wall regions of the inorganic nanopores, \( kg/s \)
$J_{b,j,om}$ and $J_{nw,j,om}$ = mass flux from the bulk and adsorbed/near-wall regions of the organic nanopores, $kg/s$

$J_{j,om}$ and $J_{j,im}$ = total fluid mass flux from organic and inorganic nanopores, $kg/s$

$k_{aj}$ = total apparent liquid permeability for single-phase oil or water transport, $md$

$k_{a,j,om}$ and $k_{a,j,im}$ = apparent liquid permeability in the organic and inorganic matter, $md$

$k_{ej}$ = effective fluid permeability in the matrix during two-phase flow, $md$

$k_{f}$ = fracture permeability, $md$

$k_{f,FB}$ and $k_{f,LT}$ = fracture permeability predicted from flowback for long-term and obtained from long-term analysis, $md$

$k_{om}$ and $k_{im}$ = permeability of organic and inorganic matter at certain condition, $md$

$k_{om,i}$ and $k_{im,i}$ = permeability of organic and inorganic matter at initial condition, $md$

$k_{r,j,f}$ and $k_{r,j,m}$ = relative permeability in the fracture and matrix, -

$k_{r,j}^*$ = modified relative permeability input in commercial software, -

$k_{m}^*$ = modified initial matrix permeability input in commercial software, $md$

$m_{j,f}(p)$ and $m_{j,m}(p)$ = pseudopressure in the fracture and matrix, psi

$p_{b}$ = base pressure, psi

$p_{bb}$ = bubble point pressure, psi

$p_{f}$ = fracture pressure, psi

$p_{m}$ = matrix pressure, psi

$p_{wf}$ = bottomhole pressure, psi

$q_{j}$ = fluid flowrate from the whole fracture at surface condition, $STB/day$

$q_{sj}$ = fluid influx rate from matrix at surface condition, $STB/day$

$Q_{j}$ = cumulative fluid production from the whole fracture at surface condition, $STB$
\( Q_{sj} = \text{cumulative fluid influx from matrix at surface condition, STB} \)

\( R_{om} \) and \( R_{im} = \text{radius of organic and inorganic nanopores at certain condition, m} \)

\( RNP_j = \text{rate normalized pseudopressure, psi} \cdot \text{day/STB} \)

\( S_{j,f} \) and \( S_{j,m} = \text{phase saturation in the fracture and matrix, -} \)

\( t = \text{time, day} \)

\( t_{p,f} \) and \( t_{p,m} = \text{pseudotime in the fracture and matrix, day} \)

\( t_{sp,j} = \text{superposition pseudotime in the fracture, day} \)

\( v_{b,j} \) and \( v_{nw,j} = \text{flow velocity at r in the bulk and adsorbed/near-wall region of inorganic nanopores, m/s} \)

\( V_{fi} = \text{initial pore volume of fracture, ft}^3 \)

\( V_{f,FB} \) and \( V_{f,LT} = \text{fracture pore volume predicted from flowback for long-term and obtained from long-term analysis, ft}^3 \)

\( V_{j,mi} = \text{initial pore volume of quarter fracture-matrix element, ft}^3 \)

\( w_f = \text{fracture width, ft} \)

\( x = \text{location in the fracture, ft} \)

\( x_f = \text{fracture half-length, ft} \)

\( y = \text{location in the matrix, ft} \)

\( y_j = \text{distance of investigation in the matrix for each phase, ft} \)

\( y_m = \text{fracture half-spacing, ft} \)

\( \alpha, \beta, \) and \( \varepsilon = \text{unit conversion factor in specialty plots} \)

\( \delta_{full} \) and \( \delta_{IALF} = \text{unit conversion factor in influx calculation, -} \)

\( \delta_{j,om} \) and \( \delta_{j,im} = \text{thickness of adsorbed/near-wall region in organic and inorganic nanopores, m} \)
\( \lambda_{j,om} \) and \( \lambda_{j,im} \) = slip length in the organic and inorganic nanopores, \( m \)

\( \mu_{j,f} \) and \( \mu_{j,m} \) = fluid viscosity in the fracture and matrix, \( cp \)

\( \mu_{j,om} \) and \( \mu_{j,im} \) = the fluid viscosity in adsorbed/near-wall region of organic and inorganic nanopores, \( cp \)

\( \rho_{j} \) = bulk fluid density in the matrix, \( lb/ft^{3} \)

\( \rho_{j,om} \) and \( \rho_{j,im} \) = fluid density in the adsorbed/near-wall region of organic and inorganic nanopores, \( lb/ft^{3} \)

\( \phi_{f} \) and \( \phi_{m} \) = porosity of fracture and matrix, -

\( \phi_{om} \) and \( \phi_{im} \) = porosity of organic and inorganic matter at certain condition, -

\( \phi_{om,i} \) and \( \phi_{im,i} \) = porosity of organic and inorganic matter at initial condition, -

\( \psi \) = organic matter volume fraction, -

\( \tau_{om} \) and \( \tau_{im} \) = tortuosity for organic and inorganic matter, -

\( \theta_{om} \) and \( \theta_{im} \) = contact angle on the surface of organic and inorganic substrate, °

\( \gamma_{f} \) and \( \gamma_{m} \) = permeability modulus of fracture and matrix, \( psi^{-1} \)

**Subscripts**

IALF = infinite acting linear flow

BDF = boundary dominated flow

\( b \) = base or bulk

\( e \) = effective

\( f \) = fracture

\( full \) = full solution for matrix

\( i \) = initial

\( im \) = inorganic matter
\[ s = \text{influx/source} \]

\[ j = w \text{ or } o \]

\[ m = \text{matrix} \]

\[ n = \text{the nth flowrate interval} \]

\[ nw = \text{adsorbed/near-wall region} \]

\[ o = \text{oil} \]

\[ om = \text{organic matter} \]

\[ p = \text{pseudo} \]

\[ sp = \text{superposition pseudo} \]

\[ w = \text{water} \]

\textbf{Superscripts}

\[ \bar{ } = \text{average} \]

\subsection*{6.8 Reference}


Chapter 7

Conclusion and Recommendations

The following chapter is a summary of important conclusions from this dissertation along with a set of recommendations for future research based on these findings.

7.1 Conclusion

7.1.1 Early-Time Flowback Analysis

During the early-time flowback period, the hydraulic fracture can be treated as a closed tank with a negligible fluid influx from matrix. I have developed a semi-analytical two-phase model to analyze early-time flowback data from MFHWs and evaluate fracture properties, such as initial fracture permeability and initial pore volume of fracture, under constant bottomhole pressure (BHP), constant rate, and variable rate/BHP condition. Comparing to input fracture parameters in numerical simulation, the developed constant BHP flowback model has more error and is only applicable to cases with slight nonlinearity. The constant/variable-rate model is more accurate, with a relative error of less than 10%.

A diffusivity equation approach has been developed to calculate average pressure under constant BHP, constant rate, and variable rate/BHP condition. The method releases the demand for gas-phase production data as input and is not affected by the extent of nonlinearity.
7.1.2 Late-Time Flowback Analysis with Matrix Influx

During the late-time flowback period, the water and hydrocarbon production is mainly due to the fluid influx from matrix. I have developed a semi-analytical method that can be used for straight-line analysis to characterize fracture properties (initial fracture pore-volume and permeability) and HF closure dynamics (fracture compressibility and permeability modulus) for MFHWs completed in shale gas reservoir. The method incorporates the two-phase matrix influx into the pseudotime definition in the fracture and considers the pressure-dependent relative permeability, fluid, and rock properties.

Ignoring water influx from matrix and the constraint for the fracture volume loss during flowback and long-term production will cause up to 27% error in calculations of fracture properties for the analyzed MFHW. The field application results reveal that fracture closure causes the reduction of the fracture volume by 30% and the fracture permeability by over an order of magnitude after 24 days of flowback and eight months of production.

7.1.3 Flowback Analysis for Ultratight Gas Reservoirs

The complexity in gas storage and transport mechanisms for shale gas reservoir challenges the accurate characterization of HF properties and dynamics in flowback analysis. To make flowback models applicable to shale gas reservoirs, I have developed a semi-analytical method for HF characterization that incorporates desorption, slip flow, diffusion, stress dependence, and the effect of water film.

Numerical validation shows that ignoring the gas storage and transport mechanisms for shale reservoir will underestimate the matrix permeability as well as the gas flowrate and cumulative production. The field application indicates that the number of constraints during
workflow iteration has the greatest impact on characterizing HF attributes during early-time production period; the two-phase flow in matrix takes second place, and the multiple mechanisms within shale matrix at last.

7.1.4 Flowback Analysis for Tight and Ultratight Oil Reservoirs

To make flowback models applicable to tight and ultratight oil reservoirs, I have developed a semi-analytical approach to analyze two-phase flowback data by incorporating multiple fluid transport mechanisms into a newly defined effective permeability for each phase, as opposed to using just a matrix permeability under Darcy’s law. The newly defined effective permeability in matrix model accounts for multiple liquid transport mechanisms, including two-phase flow, slippage effect, stress dependence, variation of fluid properties in the bulk and near-wall regions, as well as the differences in slip length, pore size, and wettability of organic and inorganic nanopores. Ignoring the multiple fluid transport mechanisms leads to a 24% difference in the estimated apparent permeability in the synthetic case.
7.2 Recommendations for Future Research

Three important future research directions are type curve analysis, three-phase flow, and complex fracture network that are briefly described in the following sections.

7.2.1 Type Curve Analysis

Type-curve methods, as a powerful technique for quantitative production data analysis, are playing an essential role in estimating reservoir and fracture properties, such as fracture half-length, fracture and reservoir permeability, as well as reservoir drainage area. However, the appearance of two-phase flow after hydraulic fracturing, e.g., water/gas flow for shale gas reservoir and water/oil flow for shale oil reservoir, introduces the nonlinearities into governing equations and makes the use of single-phase type-curve analysis erroneous. It is recommended to apply the semi-analytical methods developed in this dissertation to obtain the two-phase type curve solutions for flowback analysis.

7.2.2 Three-Phase Flow

For the gas condensate reservoirs, as pressure drops below the dewpoint pressure, condensate will drop out in the wellbore, fracture, or even the formation near the wellbore. Similarly for the volatile oil reservoirs, solution gas will become free gas as pressure drops to the bubble point. The appearance of condensate in gas reservoirs or solution gas in oil reservoirs will lead to the variation of relative permeability, saturation, and flowrate for each phase, further affecting the flow-regime identification and the accuracy of straight-line analysis using the proposed methods in this dissertation. Future research shall incorporate three-phase flow during flowback when condensate or solution gas appears in the reservoir or wellbore.
7.2.3 Complex Fracture Network

The methods presented in this dissertation could not describe complex fracture geometries. In other words, the flowback models in this study are based on the assumption that only planar fractures are created after stimulation. However, available micro-seismic data show that hydraulic fracturing in unconventional plays could generate very complex fracture networks, instead of simple planar fractures. Therefore, future study is recommended to consider the complex fracture geometry in fracture characterization using flowback data.
Copyright Letters of Permission

Chapter 2

Copyright

Describes the rights related to the publication and distribution of research. It governs how authors (as well as their employers or funders), publishers and the wider general public can use, publish and distribute articles or books.

Journal author rights

In order for Elsevier to publish and disseminate research articles, we need publishing rights. This is determined by a publishing agreement between the author and Elsevier. This agreement deals with the transfer or license of the copyright to Elsevier and authors retain significant rights to use and share their own published articles. Elsevier supports the need for authors to share, disseminate and maximize the impact of their research and these rights, in Elsevier proprietary journals* are defined below:

<table>
<thead>
<tr>
<th>For subscription articles</th>
<th>For open access articles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Authors transfer copyright to the publisher as part of a journal publishing agreement, but have the right to:</td>
<td>Authors sign an exclusive license agreement, where authors have copyright but license exclusive rights in their article to the publisher**. In this case authors have the right to:</td>
</tr>
<tr>
<td>• Share their article for <a href="https://www.sciencedirect.com/terms/personal-use">Personal Use, Internal Institutional Use and Scholarly Sharing purposes</a>, with a DOI link to the version of record on ScienceDirect (and with the Creative Commons CC-BY-NC-ND license for author manuscript versions)</td>
<td>• Share their article in the same ways permitted to third parties under the relevant user license (together with Personal Use rights) so long as it contains a CrossMark logo, the end user license, and a DOI link property rights (including research data).</td>
</tr>
<tr>
<td>• Retain own trademark and other intellectual ....</td>
<td>• Proper attribution and credit for the published work.</td>
</tr>
</tbody>
</table>

*Please note that society or third party owned journals may have different publishing agreements. Please see the journal’s guide for authors for journal specific copyright information.

**This includes the right for the publisher to make and authorize commercial use, please see “Rights granted to Elsevier” for more details.

Help and Support

• Download a sample publishing agreement for subscription articles in [English](https://www.sciencedirect.com/terms/personal-use) and [French](https://www.sciencedirect.com/terms/personal-use).
• Download a sample publishing agreement for open access articles for authors choosing a [commercial user license](https://www.sciencedirect.com/terms/personal-use) and [non-commercial user license](https://www.sciencedirect.com/terms/personal-use).
• For authors who wish to self-archive see our [sharing guidelines](https://www.sciencedirect.com/terms/self-archiving).
• See our [author pages](https://www.sciencedirect.com/terms/author) for further details about how to promote your article.
• For use of Elsevier material not defined below please see our [permissions page](https://www.sciencedirect.com/terms/permissions) or visit the [Permissions Support Center](https://www.sciencedirect.com/terms/permissions-support-center).
Government employees

Elsevier has specific publishing agreements with certain government and inter-governmental organizations for their employee authors. These agreements enable authors to retain substantially the same rights as detailed in the "Author Rights section" but are specifically tailored for employees from the relevant organizations, including:

- World Bank
- World Health Organization
- For US government employees, works created within the scope of their employment are considered to be public domain and Elsevier's publishing agreements do not require a transfer or license of rights for such works.
- In the UK and certain commonwealth countries, a work created by a government employee is copyrightable but the government may own the copyright (Crown copyright). Click here for information about UK government employees publishing open access

Rights granted to Elsevier

For both subscription and open access articles, published in proprietary titles, Elsevier is granted the following rights:

- The exclusive right to publish and distribute an article, and to grant rights to others, including for commercial purposes.
- For open access articles, Elsevier will apply the relevant third party user license where Elsevier publishes the article on its online platforms.
- The right to provide the article in all forms and media so the article can be used on the latest technology even after publication.
- The authority to enforce the rights in the article, on behalf of an author, against third parties, for example in the case of plagiarism or copyright infringement.

Protecting author rights

Copyright aims to protect the specific way the article has been written to describe an experiment and the results. Elsevier is committed to its authors to protect and defend their work and their reputation and takes allegations of infringement, plagiarism, ethic disputes and fraud very seriously.

If an author becomes aware of a possible plagiarism, fraud or infringement we recommend contacting their Elsevier publishing contact who can then liaise with our in-house legal department. Note that certain open access user licenses may permit quite broad re-use that might otherwise be counted as copyright infringement. For details about how to seek permission to use an article see our permission page.

Open access

How copyright works with open access licenses

For Elsevier proprietary journals the following steps apply:

1. Authors sign a publishing agreement where they will have copyright but grant broad publishing and distribution rights to the publisher, including the right to publish the article on Elsevier's online platforms.
2. The author chooses an end user license under which readers can use and share the article.
3. The publisher makes the article available online with the author's choice of end user license.
Quick definitions

Personal use
Authors can use their articles, in full or in part, for a wide range of scholarly, non-commercial purposes as outlined below:

- Use by an author in the author's classroom teaching (including distribution of copies, paper or electronic)
- Distribution of copies (including through e-mail) to known research colleagues for their personal use (but not for Commercial Use)
- Inclusion in a thesis or dissertation (provided that this is not to be published commercially)
- Use in a subsequent compilation of the author's works
- Extending the Article to book-length form
- Preparation of other derivative works (but not for Commercial Use)
- Otherwise using or re-using portions or excerpts in other works

These rights apply for all Elsevier authors who publish their article as either a subscription article or an open access article. In all cases we require that all Elsevier authors always include a full acknowledgement and, if appropriate, a link to the final published version hosted on Science Direct.

Commercial use
This is defined as the use or posting of articles:

- For commercial gain without a formal agreement with the publisher.
  - For example by associating advertising with the full-text of the article, by providing hosting services to other repositories or to other organizations (including where an otherwise non-commercial site or repository provides a service to other organizations or agencies), or charging fees for document delivery or access
- To substitute for the services provided directly by the journal.
  - For example article aggregation, systematic distribution of articles via e-mail lists or share buttons, posting, indexing, or linking for promotional/marketing activities, by commercial companies for use by customers and intended target audiences of such companies (e.g. pharmaceutical companies and healthcare professionals/physician-prescribers).

If you would like information on how to obtain permission for such uses click here or if you would like to make commercial use of the article please visit the Permissions Support Center.

Internal institutional use
- Use by the author's institution for classroom teaching at the institution and for internal training purposes (including distribution of copies, paper or electronic, and use in coursepacks and courseware programs, but not in Massive Open Online Courses)
- Inclusion of the Article in applications for grant funding
- For authors employed by companies, the use by that company for internal training purposes
Chapter 3

Copyright

Describes the rights related to the publication and distribution of research. It governs how authors (as well as their employers or funders), publishers and the wider general public can use, publish and distribute articles or books.

Journal author rights

In order for Elsevier to publish and disseminate research articles, we need publishing rights. This is determined by a publishing agreement between the author and Elsevier. This agreement deals with the transfer or license of the copyright to Elsevier and authors retain significant rights to use and share their own published articles. Elsevier supports the need for authors to share, disseminate and maximize the impact of their research and these rights, in Elsevier proprietary journals* are defined below:

<table>
<thead>
<tr>
<th>For subscription articles</th>
<th>For open access articles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Authors transfer copyright to the publisher as part of a</td>
<td>Authors sign an exclusive license agreement, where authors have copyright but license</td>
</tr>
<tr>
<td>journal publishing agreement, but have the right to:</td>
<td>exclusive rights in their article to the publisher**. In this case authors have the</td>
</tr>
<tr>
<td>• Share their article for Personal Use, Internal</td>
<td>right to:</td>
</tr>
<tr>
<td>Institutional Use and Scholarly Sharing purposes,</td>
<td>• Share their article in the same ways permitted to third</td>
</tr>
<tr>
<td>with a DOI link to the version of record on ScienceDirect (and with the Creative Commons CC-</td>
<td>parties under the relevant user license (together with Personal Use rights) so long as it</td>
</tr>
<tr>
<td>BY-NC-ND license for author manuscript versions)</td>
<td>contains a CrossRef logo, the end user license, and a DOI link</td>
</tr>
<tr>
<td>• Preprint on preprint.zebu.org and other preprint website</td>
<td>property rights (including research data).</td>
</tr>
<tr>
<td>• Proper attribution and credit for the published work.</td>
<td>• Proper attribution and credit for the published work.</td>
</tr>
</tbody>
</table>

*Please note that society or third party owned journals may have different publishing agreements. Please see the journal’s guide for authors for journal specific copyright information.

**This includes the right for the publisher to make and authorize commercial use, please see “Rights granted to Elsevier” for more details.

Help and Support

• Download a sample publishing agreement for subscription articles in English and French.
• Download a sample publishing agreement for open access articles for authors choosing a commercial user license and non-commercial user license.
• For authors who wish to self-archive see our sharing guidelines
• See our author pages for further details about how to promote your article.
• For use of Elsevier material not defined below please see our permissions page or visit the Permissions Support Center.
Government employees

Elsevier has specific publishing agreements with certain government and inter-governmental organizations for their employee authors. These agreements enable authors to retain substantially the same rights as detailed in the "Author Rights section" but are specifically tailored for employees from the relevant organizations, including:

- World Bank
- World Health Organization
- For US government employees, works created within the scope of their employment are considered to be public domain and Elsevier's publishing agreements do not require a transfer or license of rights for such works.
- In the UK and certain Commonwealth countries, a work created by a government employee is copyrightable but the government may own the copyright (Crown copyright). Click here for information about UK government employees publishing open access.

Rights granted to Elsevier

For both subscription and open access articles, published in proprietary titles, Elsevier is granted the following rights:

- The exclusive right to publish and distribute an article, and to grant rights to others, including for commercial purposes.
- For open access articles, Elsevier will apply the relevant third party user license where Elsevier publishes the article on its online platforms.
- The right to provide the article in all forms and media so the article can be used on the latest technology even after publication.
- The authority to enforce the rights in the article, on behalf of an author, against third parties, for example in the case of plagiarism or copyright infringement.

Protecting author rights

Copyright aims to protect the specific way the article has been written to describe an experiment and the results. Elsevier is committed to its authors to protect and defend their work and their reputation and takes allegations of infringement, plagiarism, ethic disputes and fraud very seriously.

If an author becomes aware of a possible plagiarism, fraud or infringement we recommend contacting their Elsevier publishing contact who can then liaise with our in-house legal department. Note that certain open access user licenses may permit quite broad re-use that might otherwise be counted as copyright infringement. For details about how to seek permission to use an article see our permission page.

Open access

How copyright works with open access licenses

For Elsevier proprietary journals the following steps apply:

1. Authors sign a publishing agreement where they will have copyright but grant broad publishing and distribution rights to the publisher, including the right to publish the article on Elsevier's online platforms.
2. The author chooses an end user license under which readers can use and share the article.
3. The publisher makes the article available online with the author's choice of end user license.
Quick definitions

Personal use
Authors can use their articles, in full or in part, for a wide range of scholarly, non-commercial purposes as outlined below:

- Use by an author in the author’s classroom teaching (including distribution of copies, paper or electronic)
- Distribution of copies (including through e-mail) to known research colleagues for their personal use (but not for Commercial Use)
- Inclusion in a thesis or dissertation (provided that this is not to be published commercially)
- Use in a subsequent compilation of the author’s works
- Extending the Article to book-length form
- Preparation of other derivative works (but not for Commercial Use)
- Otherwise using or re-using portions or excerpts in other works

These rights apply for all Elsevier authors who publish their article as either a subscription article or an open access article. In all cases we require that all Elsevier authors always include a full acknowledgement and, if appropriate, a link to the final published version hosted on ScienceDirect.

Commercial use
This is defined as the use or posting of articles:

- For commercial gain without a formal agreement with the publisher.
  - For example by associating advertising with the full-text of the article, by providing hosting services to other repositories or to other organizations (including where an otherwise non-commercial site or repository provides a service to other organizations or agencies), or charging fees for document delivery or access
- To substitute for the services provided directly by the journal.
  - For example article aggregation, systematic distribution of articles via e-mail lists or share buttons, posting, indexing, or linking for promotional/marketing activities, by commercial companies for use by customers and intended target audiences of such companies (e.g. pharmaceutical companies and healthcare professionals/physician-prescribers).

If you would like information on how to obtain permission for such uses click here or if you would like to make commercial use of the article please visit the Permissions Support Center.

Internal institutional use
- Use by the author's institution for classroom teaching at the institution and for internal training purposes (including distribution of copies, paper or electronic, and use in coursepacks and courseware programs, but not in Massive Open Online Courses)
- Inclusion of the Article in applications for grant funding
- For authors employed by companies, the use by that company for internal training purposes
Chapter 4

Society of Petroleum Engineers (SPE) - License Terms and Conditions

This is a License Agreement between Fengyuan Zhang (Penn State University) (“You”) and Society of Petroleum Engineers (SPE) (“Publisher”) provided by Copyright Clearance Center (“CCC”). The license consists of your order details, the terms and conditions provided by Society of Petroleum Engineers (SPE), and the CCC terms and conditions.

All payments must be made in full to CCC.

<table>
<thead>
<tr>
<th>Order Date</th>
<th>20-Oct-2020</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order license ID</td>
<td>1071571-1</td>
</tr>
<tr>
<td>ISSN</td>
<td>1060-3680</td>
</tr>
<tr>
<td>Type of Use</td>
<td>Publisher</td>
</tr>
<tr>
<td>Portion</td>
<td>Chapter/article</td>
</tr>
<tr>
<td>Republish in a thesis/dissertation</td>
<td>The SOCIETY, Chapter/article</td>
</tr>
</tbody>
</table>

**LICENSED CONTENT**

<table>
<thead>
<tr>
<th>Publication Title</th>
<th>SPE JOURNAL -RICHARDSON -SOCIETY OF PETROLEUM ENGINEERS (U.S.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author/Editor</td>
<td>01/01/1996</td>
</tr>
<tr>
<td>Date</td>
<td>English</td>
</tr>
<tr>
<td>Language</td>
<td>United States of America</td>
</tr>
<tr>
<td>Country</td>
<td>Society of Petroleum Engineers (SPE)</td>
</tr>
<tr>
<td>Rightsholder</td>
<td>Journal</td>
</tr>
<tr>
<td>Publication Type</td>
<td></td>
</tr>
</tbody>
</table>

**REQUEST DETAILS**

<table>
<thead>
<tr>
<th>Portion Type</th>
<th>Chapter/article</th>
</tr>
</thead>
<tbody>
<tr>
<td>Page range(s)</td>
<td>1599-1622</td>
</tr>
<tr>
<td>Total number of pages</td>
<td>24</td>
</tr>
<tr>
<td>Format (select all that apply)</td>
<td>Electronic</td>
</tr>
<tr>
<td>Academic institution</td>
<td>Life of current and all future editions</td>
</tr>
<tr>
<td>Up to 14,999</td>
<td></td>
</tr>
<tr>
<td>Life time Unit Quantity</td>
<td></td>
</tr>
<tr>
<td>Rights Requested</td>
<td>No</td>
</tr>
<tr>
<td>Main product</td>
<td>Worldwide</td>
</tr>
<tr>
<td>Distribution</td>
<td>No</td>
</tr>
<tr>
<td>Translation</td>
<td>Yes</td>
</tr>
<tr>
<td>Copies for the disabled?</td>
<td>No</td>
</tr>
<tr>
<td>Minor editing privileges?</td>
<td>No</td>
</tr>
<tr>
<td>Incidental promotional use?</td>
<td>No</td>
</tr>
<tr>
<td>Currency</td>
<td>USD</td>
</tr>
</tbody>
</table>

**NEW WORK DETAILS**

<table>
<thead>
<tr>
<th>Title</th>
<th>Flowback Rate Transient Analysis of Multi-Fractured Horizontal Wells in Tight and Ultra Tight Reservoirs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructor name</td>
<td>Fengyuan Zhang</td>
</tr>
<tr>
<td>Institution name</td>
<td>The Pennsylvania State University</td>
</tr>
<tr>
<td>Expected presentation date</td>
<td>2020-10-23</td>
</tr>
</tbody>
</table>

**ADDITIONAL DETAILS**

<table>
<thead>
<tr>
<th>Order reference number</th>
<th>N/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>The requesting person / organization to appear on the license</td>
<td>Fengyuan Zhang (Penn State University)</td>
</tr>
</tbody>
</table>

**REUSE CONTENT DETAILS**

<table>
<thead>
<tr>
<th>Title, description or numeric reference of the portion(s)</th>
<th>The entire contents of SPE-201225-PA “A Semianalytical Method for Two-Phase Flowback Rate-Transient Analysis in Shale Gas Reservoirs”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Editor of portion(s)</td>
<td>N/A</td>
</tr>
<tr>
<td>Volume of serial or monograph</td>
<td>25</td>
</tr>
<tr>
<td>Page or page range of portion</td>
<td>1599-1622</td>
</tr>
<tr>
<td>Title of the article/chapter the portion is from</td>
<td>SPE-201225-PA “A Semianalytical Method for Two-Phase Flowback Rate-Transient Analysis in Shale Gas Reservoirs”</td>
</tr>
<tr>
<td>Author of portion(s)</td>
<td>N/A</td>
</tr>
<tr>
<td>Issue, if republishing an article from a serial</td>
<td>Society of Petroleum Engineers (U.S.)</td>
</tr>
<tr>
<td>Publication date of portion</td>
<td>04</td>
</tr>
<tr>
<td>Date</td>
<td>2020-08-01</td>
</tr>
</tbody>
</table>

**PUBLISHER SPECIAL TERMS AND CONDITIONS**

The person requesting republication permission is an author of SPE-201225-PA.
Chapter 5

Copyright

Describes the rights related to the publication and distribution of research. It governs how authors (as well as their employers or funders), publishers and the wider general public can use, publish and distribute articles or books.

Journal author rights

In order for Elsevier to publish and disseminate research articles, we need publishing rights. This is determined by a publishing agreement between the author and Elsevier. This agreement deals with the transfer or license of the copyright to Elsevier and authors retain significant rights to use and share their own published articles. Elsevier supports the need for authors to share, disseminate and maximize the impact of their research and these rights, in Elsevier proprietary journals*, are defined below:

<table>
<thead>
<tr>
<th>For subscription articles</th>
<th>For open access articles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Authors transfer copyright to the publisher as part of a journal publishing agreement, but have the right to:</td>
<td>Authors sign an exclusive license agreement, where authors have copyright but license exclusive rights in their article to the publisher**. In this case authors have the right to:</td>
</tr>
<tr>
<td>• Share their article for Personal Use, Internal Institutional Use and Scholarly Sharing purposes, with a DOI link to the version of record on ScienceDirect (and with the Creative Commons CC-BY-NC-ND license for author manuscript versions)</td>
<td>• Share their article in the same ways permitted to third parties under the relevant user license (together with Personal Use rights) so long as it contains a CrossMark logo, the end user license, and a DOI link property rights (including research data).</td>
</tr>
<tr>
<td>• Print, copy, tweet, and other intellectual property rights</td>
<td>• Proper attribution and credit for the published work.</td>
</tr>
</tbody>
</table>

*Please note that society or third party owned journals may have different publishing agreements. Please see the journal's guide for authors for journal specific copyright information.

**This includes the right for the publisher to make and authorize commercial use, please see "Rights granted to Elsevier" for more details.

Help and Support

• Download a sample publishing agreement for subscription articles in English and French.
• Download a sample publishing agreement for open access articles for authors choosing a commercial user license and non-commercial user license.
• For authors who wish to self-archive see our sharing guidelines
• See our author pages for further details about how to promote your article.
• For use of Elsevier material not defined below please see our permissions page or visit the Permissions Support Center.
Government employees

Elsevier has specific publishing agreements with certain government and inter-governmental organizations for their employee authors. These agreements enable authors to retain substantially the same rights as detailed in the "Author Rights section" but are specifically tailored for employees from the relevant organizations, including:

- World Bank
- World Health Organization
- For US government employees, works created within the scope of their employment are considered to be public domain and Elsevier's publishing agreements do not require a transfer or license of rights for such works.
- In the UK and certain commonwealth countries, a work created by a government employee is copyrightable but the government may own the copyright (Crown copyright). Click here for information about UK government employees publishing open access

Rights granted to Elsevier

For both subscription and open access articles, published in proprietary titles, Elsevier is granted the following rights:

- The exclusive right to publish and distribute an article, and to grant rights to others, including for commercial purposes.
- For open access articles, Elsevier will apply the relevant third party user license where Elsevier publishes the article on its online platforms.
- The right to provide the article in all forms and media so the article can be used on the latest technology even after publication.
- The authority to enforce the rights in the article, on behalf of an author, against third parties, for example in the case of plagiarism or copyright infringement.

Protecting author rights

Copyright aims to protect the specific way the article has been written to describe an experiment and the results. Elsevier is committed to its authors to protect and defend their work and their reputation and takes allegations of infringement, plagiarism, ethic disputes and fraud very seriously.

If an author becomes aware of a possible plagiarism, fraud or Infringement we recommend contacting their Elsevier publishing contact who can then liaise with our in-house legal department. Note that certain open access user licenses may permit quite broad re-use that might otherwise be counted as copyright infringement. For details about how to seek permission to use an article see our permission page.

Open access

How copyright works with open access licenses

For Elsevier proprietary journals the following steps apply:

1. Authors sign a publishing agreement where they will have copyright but grant broad publishing and distribution rights to the publisher, including the right to publish the article on Elsevier's online platforms.
2. The author chooses an end user license under which readers can use and share the article.
3. The publisher makes the article available online with the author's choice of end user license.
Quick definitions

Personal use
Authors can use their articles, in full or in part, for a wide range of scholarly, non-commercial purposes as outlined below:

• Use by an author in the author’s classroom teaching (including distribution of copies, paper or electronic)
• Distribution of copies (including through e-mail) to known research colleagues for their personal use (but not for Commercial Use)
• Inclusion in a thesis or dissertation (provided that this is not to be published commercially)
• Use in a subsequent compilation of the author’s works
• Extending the Article to book-length form
• Preparation of other derivative works (but not for Commercial Use)
• Otherwise using or re-using portions or excerpts in other works

These rights apply for all Elsevier authors who publish their article as either a subscription article or an open access article. In all cases we require that all Elsevier authors always include a full acknowledgement and, if appropriate, a link to the final published version hosted on Science Direct.

Commercial use
This is defined as the use or posting of articles:

• For commercial gain without a formal agreement with the publisher.
  • For example by associating advertising with the full-text of the article, by providing hosting services to other repositories or to other organizations (including where an otherwise non-commercial site or repository provides a service to other organizations or agencies), or charging fees for document delivery or access
• To substitute for the services provided directly by the journal.
  • For example article aggregation, systematic distribution of articles via e-mail lists or share buttons, posting, indexing, or linking for promotional/marketing activities, by commercial companies for use by customers and intended target audiences of such companies (e.g. pharmaceutical companies and healthcare professionals/physician-prescribers).

If you would like information on how to obtain permission for such uses click here or if you would like to make commercial use of the article please visit the Permissions Support Center.

Internal institutional use
• Use by the author’s institution for classroom teaching at the institution and for internal training purposes (including distribution of copies, paper or electronic, and use in coursepacks and courseware programs, but not in Massive Open Online Courses)
• Inclusion of the Article in applications for grant funding
• For authors employed by companies, the use by that company for internal training purposes
Vita

Fengyuan holds a master’s degree in Oil and Gas Field Development Engineering and a bachelor’s degree in Petroleum Engineering from China University of Petroleum, Beijing. He specializes in pressure and rate transient analysis of flowback and long-term production data in tight and ultratight reservoir. His current research interests include analytical modeling for rate transient analysis and production data analysis, multiphase flow and transport in porous media, hydraulic fracture characterization, and carbon dioxide sequestration in geological formations. During his Ph.D., Fengyuan is the recipient of the 2019 Nico van Wingen Memorial Graduate Fellowship from the Society of Petroleum Engineers and the 2019 WAAIME Scholarship from the Society for Mining, Metallurgy & Exploration. He also received the 2020 Graduate Merit Award from the department of Energy and Mineral Engineering at the Pennsylvania State University.

This dissertation was typed by Fengyuan Zhang

© 2020 Fengyuan Zhang