PASSIVE BALANCER DEVELOPMENT FOR ROTOR SYSTEMS IN ICING CONDITIONS

A Thesis in
Aerospace Engineering
by
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ABSTRACT

For a rotary system, the minimization of detrimental fixed-frame forces and rotor vibration is critically important to ensuring the safety of the system and its operator. For modern VTOL aircraft, rotors are precisely fabricated to certify that their mass is evenly distributed about their axis of rotation. However, significant mass eccentricity can arise in a rotor system, operating in adverse icing conditions, that experiences an asymmetric ice shed event, as ice accretes on the blades and is subsequently removed by centrifugal force. This mass eccentricity creates large dynamic loading and vibration, leading to shortened fatigue life, component failure, or catastrophic failure. As VTOL flight continues to evolve, there is a need for a low-power, low-complexity solution to rotor mass eccentricity that arises during supercritical rotor operation.

A solution for mass eccentric vibration is presented in the form of a passive balancing device. A passive balancing device consist of several masses that are free to move circumferentially along a track concentric to the axis of rotation. Beyond the shaft first flexural natural frequency speed, referred to as supercritical operation, the phase of the system response is oriented 180 degrees relative to the asymmetric loading caused by the imbalance, which causes the masses to assume positions opposed to the direction of the imbalance and reduces vibration. A comprehensive model of this nonlinear phenomenon was developed in previous studies and it was experimentally validated for a partitioned ball-type balancer on small scale applications.

In the study outlined in this thesis, a partitioned ball-type passive balancing device was investigated as a possible solution to address the fixed-frame loading and rotor vibration created by asymmetric ice shed events. A mathematical model of a flexible, passively balanced rotor system was employed for the prediction of system fixed frame loading in response to a given mass imbalance condition. Three rotor configurations were considered for icing experiments in the Adverse Environment Rotor Test Stand (AERTS), consisting of a representative rotor mounted to
a cantilevered shaft inside an icing chamber. Icing experiments consisted of spinning up the rotor to its supercritical operating region and introducing realistic icing conditions in the continuous icing envelope to observe the system vibration response by measuring the fixed frame loading following each ice shed event. An analytical model for the ice mass of each shed event was developed from commercially available techniques to facilitate the modeling of each observed mass imbalance condition. An unbalanced rotor configuration was tested to provide the baseline for the vibration response without a passive balancing device. An initial device was designed from design trends observed in literature that dictate maximizing balancing authority, minimizing distance between the balancer and imbalance planes, and implementing multiple track partitions. The initial device was fabricated and tested to determine its vibration reduction performance and identify key design parameters limiting its effectiveness. The final design was formulated from an iterative design process, which varied key design parameters to improve predicted performance, provided by the model, until convergence. The final design was fabricated and tested to determine its vibration reduction performance. Experimental results from all configurations were compared to model predictions to validate the mathematical model of the system. The performance of the final passive balancer was compared to the initial to determine the effectiveness of implementing an iterative design process based on parametric analysis.

Experiments conducted on the unbalanced rotor configuration revealed a closely linear correlation between mass imbalance magnitude and fixed frame loading, with some mass imbalance conditions approaching the design loading limit of the system. The model prediction of the unbalanced rotor configuration loading was 18.7% different on average, underpredicting the load in most cases. Experimental results for the initial passive balancer configuration exhibited a close correlation to unbalanced rotor results in trend and magnitude. The model prediction of the initial passive balancer configuration loading was 22.5% different on average, overpredicting values for the higher imbalance cases. It was calculated that the implementation of the initial
passive balancer increased fixed frame loading by 8.63% on average, illustrating that a design formulated without numerical analysis to precisely tune parameters can be detrimental to the system. Examining loading time history and balancing mass positions, it was determined that excess resistance force between the ball and track outer wall were fixing the masses in place prior to reaching supercritical speeds where passive balancing behavior occurs. In addition, the balancer was mounted too close to the fixed boundary to allow for significant displacement in response to asymmetric loading, which is necessary for passive balancing. Key design parameters were identified as the passive balancer track radius, axial position along the shaft, frictional dynamics, and balancing authority. Parametric analysis was conducted on each parameter successively in an iterative design loop until performance predicted by the model converged. The final design was fabricated to conform to the results of the parametric analysis for the most ideal model prediction. Experimental results for the final passive balancer illustrated the effectiveness of the device with a significant reduction in fixed-frame loading in all observed mass imbalance conditions, as compared to prior configurations. The model prediction of the final passive balancer configuration loading was 64.8% different on average; however, the practical significance of this value is diminished when considering the magnitude of the observed loading. The model prediction was within 20 lbf. on average, which is well within the loading tolerance limits for most rotor systems, illustrating that the model remains practically accurate. It was calculated that the implementation of the final passive balancing device reduced fixed frame loading by 78.8% on average. The significant performance improvement between the initial and final passive balancing devices illustrates that a passive balancing device, when carefully designed using numerical analysis, can effectively reduce mass eccentric rotor vibration in response to asymmetric ice shed events.
TABLE OF CONTENTS

LIST OF FIGURES ................................................. x
LIST OF TABLES ..................................................... xvi
LIST OF SYMBOLS ................................................... xviii
ACKNOWLEDGEMENTS ............................................... xxiv

Chapter 1
Introduction ............................................................. 1
1.1 Background and Motivation ........................................... 1
1.2 Mass Eccentric Rotor Vibration ....................................... 3
1.3 Passive Balancing Dynamics ......................................... 7
1.4 Review of Passive Balancing System Development and Modeling ... 10
1.5 Design Performance Parameters ..................................... 17
1.6 Refinement of Passive Balancer Designs ................................ 21
1.7 Rotorcraft Icing ...................................................... 27
1.8 Practical Application to eVTOL Aircraft ............................... 35
1.9 Theis Objectives ....................................................... 37

Chapter 2
Mathematical Model .................................................... 43
2.1 Coordinate System ................................................... 44
2.2 Model Components .................................................. 49
2.2.1 Shaft ............................................................ 49
2.2.2 Hub ............................................................ 51
2.2.3 Imbalance ....................................................... 52
2.2.4 External Damper ................................................ 53
2.3 Passive Balancer ...................................................... 54
2.3.1 Energy Equations ............................................... 55
2.3.2 Friction ........................................................ 56
2.3.3 Drag ........................................................... 60
2.3.4 Inelastic Collisions ............................................. 60
2.3.5 Centrifugal Clamps ................................................................. 61
2.4 System Discretization ................................................................. 63
2.5 System Total Energy ................................................................. 64
2.6 System Degrees of Freedom ....................................................... 64
2.7 Equations of Motion ................................................................. 65

Chapter 3
Testing Facility and Experimental Design 66
3.1 Facility Overview ........................................................................... 66
3.2 System Components ..................................................................... 71
  3.2.1 Shaft ......................................................................................... 71
  3.2.2 Rotor Hub ................................................................................. 73
  3.2.3 Rotor Blades ............................................................................. 73
  3.2.4 Initial Passive Balancer .............................................................. 77
  3.2.5 Tapered Roller Bearing .............................................................. 87
3.3 Instrumentation ............................................................................ 95
  3.3.1 Load Cell ................................................................................. 95
  3.3.2 Torque Sensor .......................................................................... 97
  3.3.3 RPM Sensor ........................................................................... 99
  3.3.4 Optical Displacement Sensor ................................................... 99
  3.3.5 Hall Effect Sensor and Strobe Light System ............................. 100
  3.3.6 IP Cameras ............................................................................. 102
  3.3.7 High-Speed Camera ............................................................... 106
3.4 Ice Accretion Experimental Methods ............................................. 108
  3.4.1 Temperature ........................................................................... 108
  3.4.2 Liquid Water Content ............................................................. 110
  3.4.3 Ice Shape Cross Section ....................................................... 116
  3.4.4 Ice Shed Mass ....................................................................... 119
  3.4.5 The Continuous Multi-Shed Icing Test .................................... 121
3.5 Test Matrix .................................................................................. 124

Chapter 4
Initial Passive Balancer Results and Discussion 126
4.1 Analytical Ice Shed Mass Validation ............................................. 126
Chapter 4
Unbalanced Rotor Configuration Results .......................................................... 137
  4.2.1 Fixed Frame Loading Amplitude ............................................................. 138
  4.2.2 Fixed Frame Loading Time History .......................................................... 141
4.3 Initial Passive Balancer Configuration Results .............................................. 142
  4.3.1 Fixed Frame Loading Amplitude ............................................................. 143
  4.3.2 Fixed Frame Loading Time History .......................................................... 147
  4.3.3 Balancing Mass Positions ................................................................. 148
4.4 Summary ........................................................................................................ 153

Chapter 5
Parametric Analysis and Experimental Validation of Final Design 155
5.1 Parametric analysis ......................................................................................... 155
  5.1.1 Passive Balancer Track Radius ................................................................. 159
  5.1.2 Passive Balancer Axial Position ............................................................... 160
  5.1.3 Ball-Track Frictional Model ................................................................. 162
  5.1.4 Balancing Authority ............................................................................. 165
  5.1.5 Summary .............................................................................................. 166
5.2 Final Passive Balancer Configuration Components ......................................... 168
  5.2.1 Shaft Extension ..................................................................................... 168
  5.2.2 Final Passive Balancer ........................................................................... 170
5.3 Test Matrix ..................................................................................................... 174
5.4 Final Passive Balancer Configuration Results ................................................ 175
  5.4.1 Fixed Frame Loading Amplitude ............................................................. 175
  5.4.2 Fixed Frame Loading Time History .......................................................... 184
  5.4.3 Balancing Mass Positions ..................................................................... 187
5.5 Summary ........................................................................................................ 193

Chapter 6
Conclusions 195
6.1 Passive Balancer Performance ................................................................... 196
6.2 Model Accuracy ............................................................................................. 199
6.3 Passive Balancer Design .......................................................................... 200
6.4 Recommendations for Future Work ............................................................ 202
  6.4.1 Numerical Analysis ............................................................................... 202
6.4.2 Modeling Capabilities ................................................................. 203
6.4.3 Experimental Work ................................................................. 204

Bibliography 205
LIST OF FIGURES

1-1. Pegasus PAV Prototype ........................................................................................................... 2
1-2. Mass Eccentric Rotor ................................................................................................................ 4
1-5. Dynamic Response of a Typical Rotor ....................................................................................... 8
1-6. Subcritical response of a PBS .................................................................................................. 9
1-7. Critical Response of a PBS ....................................................................................................... 10
1-8. Supercritical Response of a PBS .............................................................................................. 10
1-9. Illustrations of Early Passive Balancer Designs ........................................................................ 11
1-11. Flexible Shaft-Passive Balancing System with Explicit Rotor Axial Offset [40] ................. 15
1-12. Penn State Univ. Tail rotor Driveshaft Test Rig [48] ............................................................... 16
1-13. CAD Model of Test Configuration [48] .................................................................................. 17
1-14. Partitioned PBS Design ......................................................................................................... 22
1-15. Ball-Rod-Spring Balancer Design .......................................................................................... 23
1-16. Multi-Track PBS Design ....................................................................................................... 24
1-17. Spatial Spring PBS Design ..................................................................................................... 25
1-18. FAA Regulation Icing Envelopes [71] .................................................................................. 31
1-19. Ice Regime Examples [73] .................................................................................................... 32
1-20. Icing Regime Conditions Summary [73] .............................................................................. 33
1-22. Passive Balancer Development Methodology Flow Chart ................................................... 40

2-1. System Model Coordinate Systems [48] ................................................................................. 43
2-2. Shaft rotation angles from $x_g$ to $n$ reference frames ..................................................45
2-3. Body rotation angles from $a$ to $b$ reference frames ..........................................................46
2-4. Hub Component ......................................................................................................................52
2-5. Schematic of External Damper Component .............................................................................53
2-6. Passive Balancer Schematic ..................................................................................................54
2-7. Ball Motion Resistance in Model ............................................................................................56
2-8. Friction Dynamics Algorithm with Ball-Track Interaction ....................................................58
2-9. Schematic of Passive Balancer with Friction .........................................................................60
2-10. Passive Balancer with Centrifugal Clamping Mechanism ....................................................62

3-1. AERTS Original Rotor stand Configuration [89]....................................................................67
3-2. AERTS Rotor stand Modified Configuration ..........................................................................68
3-3. AERTS Freezer Diagram [89] ..................................................................................................69
3-4. AERTS Atomizing Nozzle Spray System [90]. ....................................................................69
3-5. AERTS Rotor stand Control Front Panel ................................................................................70
3-6. NASA Standard Atomizing Nozzle Calibration Chart [91]. ..................................................70
3-7. Shaft Schematic ......................................................................................................................72
3-8. Rotor Hub 3-View Drawing (Top, Front, Right) .....................................................................73
3-9. Blade Radial Center of Gravity Estimation ............................................................................74
3-10. Blade Length Scale Pattern ..................................................................................................75
3-11. Iced Rotor Blade ....................................................................................................................76
3-12. Imbalance Mass Position from Shed Event Imagery ............................................................77
3-13. AERTS Aluminum Plate Balancer .......................................................................................78
3-14. Aluminum Plate Balancer Configuration Top View .............................................................79
3-15. Factor of Safety Plot (Top View). ..........................................................................................80
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-16</td>
<td>Sectional Factor of Safety Plot at Balancer Mid-thickness Plane</td>
</tr>
<tr>
<td>3-17</td>
<td>MATLAB Ball Position Region of Interest</td>
</tr>
<tr>
<td>3-18</td>
<td>HDPE-Al COR Test 2</td>
</tr>
<tr>
<td>3-19</td>
<td>HPDE-Aluminum COR Results Summary</td>
</tr>
<tr>
<td>3-20</td>
<td>Steel-Aluminum COR Results Summary</td>
</tr>
<tr>
<td>3-21</td>
<td>Relation of Normal Load to HDPE-Al Static Coefficient of Friction [94]</td>
</tr>
<tr>
<td>3-22</td>
<td>Relation of Non-dimensional Normal Load to Steel-Al Coefficient of Static Friction [96]</td>
</tr>
<tr>
<td>3-23</td>
<td>Drag Coefficient and Reynold’s Number Relation in Different Flow Types</td>
</tr>
<tr>
<td>3-24</td>
<td>Tapered Roller Bearing Unit</td>
</tr>
<tr>
<td>3-25</td>
<td>Bearing-Shaft Collar Contact</td>
</tr>
<tr>
<td>3-26</td>
<td>System Damping Test Configuration</td>
</tr>
<tr>
<td>3-27</td>
<td>Full System Damping Test Displacement Data</td>
</tr>
<tr>
<td>3-28</td>
<td>Unfiltered and Filtered Displacements Data Comparison</td>
</tr>
<tr>
<td>3-29</td>
<td>Simulated and Experimental Response for the 11th Applied Impulse</td>
</tr>
<tr>
<td>3-30</td>
<td>Damping Coefficient Results</td>
</tr>
<tr>
<td>3-31</td>
<td>Damped Natural Frequency Results</td>
</tr>
<tr>
<td>3-32</td>
<td>Undamped Natural Frequency Results</td>
</tr>
<tr>
<td>3-33</td>
<td>ATI Omega160 Load Cell Technical Drawing [102]</td>
</tr>
<tr>
<td>3-34</td>
<td>LabView Load Cell Front Panel</td>
</tr>
<tr>
<td>3-35</td>
<td>Instrumentation Configuration Below the Rotor stand</td>
</tr>
<tr>
<td>3-36</td>
<td>Optical Displacement Sensor Calibration [48]</td>
</tr>
<tr>
<td>3-37</td>
<td>Shimpo DT315A Stroboscope</td>
</tr>
<tr>
<td>3-38</td>
<td>Strobe Light Mounting Configuration</td>
</tr>
<tr>
<td>3-39</td>
<td>Frame Capture from Icing Characterization Test 15 at 1200RPM</td>
</tr>
<tr>
<td>3-40</td>
<td>Ceiling Mounted IP Cameras</td>
</tr>
</tbody>
</table>
3-41. Wall Mounted IP Cameras ............................................................................................................. 103
3-42. Reolink Camera Interface ............................................................................................................. 104
3-43. Frame Capture from Continuous Icing Test 12 Prior to Shed Event ........................................ 105
3-44. Frame Capture from Continuous Icing Test 12 Immediately Following Shed Event ... 105
3-45. High-Speed Camera Field of View ................................................................................................. 106
3-46. FLIR Blade Surface Temperature Image ......................................................................................... 110
3-47. Collection Efficiency as a Function of the Modified Inertia Parameter for an NACA0012 Airfoil [10]. ................................................................................................................................. 111
3-48. Effective LWC Iterative Algorithm Flowchart [104] ................................................................. 113
3-49. Stagnation Thickness Measurements ........................................................................................... 115
3-50. Effective LWC Results ................................................................................................................. 115
3-51. FAA Appendix C Continuous Icing Envelope [71] ...................................................................... 116
3-52. Ice Shape Digitization .................................................................................................................. 117
3-53. Experimental Ice Shape and LEWICE Prediction Comparison ....................................................... 118
3-54. Experimental Ice Shed Length ...................................................................................................... 121
3-55. Continuous Multi-Shed Icing Test Flow Chart ............................................................................. 123

4-1. Ice Shed Characterization Test 1 Cross Section Comparison (-12 °C) ........................................ 127
4-2. Ice Shed Characterization Test 2 Cross Section Comparison (-12 °C) .......................................... 128
4-3. Ice Shed Characterization Test 3 Cross Section Comparison (-12 °C) .......................................... 128
4-4. Ice Shed Characterization -12 °C Experimental Cross Section Image ...................................... 129
4-5. Ice Shed Characterization -12 °C Experimental Shed Length Image ........................................ 130
4-6. Ice Shed Characterization Test 7 Cross Section Comparison (-8 °C) ........................................... 133
4-7. Ice Shed Characterization Test 8 Cross Section Comparison (-8 °C) ........................................... 133
4-8. Ice Shed Characterization -8 °C Experimental Shed Length Image ........................................... 134
4-9. Ice Shed Characterization -12 °C Experimental Cross Section Image ...................................... 135
4-10. Unbalanced Configuration Loading Profile During Spin-Up (Test 10) .................. 137
4-11. Simulated Unbalanced Configuration Frequency Response .............................. 138
4-12. Unbalanced Configuration Experimental and Modeled Loading Results .............. 139
4-13. X-Direction Loading Time History (Test 13) .................................................. 142
4-14. Initial Passive Balancer Configuration Experimental and Modeled Loading Results .... 143
4-15. Initial Passive Balancer and Unbalanced Rotor Configuration Loading Comparison .. 146
4-16. Y-direction Loading Time History (Test 22) .................................................. 147
4-17. Initial Passive Balancer Subcritical Ball Positions (500RPM) ......................... 148
4-18. Initial Passive Balancer Critical Speed Transition Ball Positions (800RPM) ............ 149
4-19. Initial Passive Balancer Supercritical Ball Positions (1200RPM) ...................... 149
4-20. Ice Shed Response (Test 25) ................................................................. 151
4-21. Ball Position and Rotor Speed Time History for Simulated Spin Up ($Imb_{sys} = 16000$ g-mm) ................................................................. 152

5-1. Passive Balancer Parametric Design Process ...................................................... 157
5-2. Rotor stand Mass Imbalance Normal Distribution ............................................. 158
5-3. Passive Balancer Track Radius Initial Design Loop Results .............................. 160
5-4. Passive Balancer Axial Position Initial Design Loop Results ............................ 161
5-5. Teflon and Nylon Static Coefficient of Friction Under Normal Load [105] ............. 162
5-6. Passive Balancer Frictional Model Initial Design Loop Results .......................... 164
5-7. Ball Position and Rotor Speed Time History for Nylon Model Simulation ($Imb_{sys} = 16000$ g-mm) ................................................................. 164
5-8. Ball Position and Rotor Speed Time History for Teflon Model Simulation ($Imb_{sys} = 16000$ g-mm) ................................................................. 165
5-9. Passive Balancer Balancing Authority Initial Design Loop Results ...................... 166
5-10. Predicted Configuration Fixed Frame Loading Comparison ............................ 167
5-11. Final Passive Balancer Configuration Experimental Assembly ........................ 168
5-12. Shaft Extension CAD Front and Top View .......................................................... 169
5-13. Final Passive Balancer CAD Top and Right View ............................................. 170
5-14. Final Passive Balancer Mid-Thickness Plane Sectional Factor of Safety Plot ...... 172
5-15. Final Passive Balancer Factor of Safety Distribution (Side View) ...................... 172
5-16. Nylon-Steel Coefficient of Restitution Results Summary ..................................... 173
5-17. Final Passive Balancer Configuration Experimental and Modeled Loading Results .... 176
5-18. Fixed Frame Loading Amplitude Configuration Comparison .............................. 181
5-19. Final Passive Balancer Configuration Performance Results .............................. 184
5-20. X-direction Loading Time History (Test 32) .................................................... 185
5-21. Y-direction Loading Time History (Test 29) .................................................... 187
5-22. Final Passive Balancer Subcritical Ball Positions (500RPM) ............................ 188
5-23. Final Passive Balancer Critical Speed Transition Ball Positions (~800RPM) ...... 189
5-24. Final Passive Balancer Supercritical Ball Positions (1200RPM) ....................... 190
5-25. Final Passive Balancer Supercritical Ball Positions, Pre-Shed (Test 36) ............ 191
5-26. Final Passive Balancer Supercritical Ball Positions, Stable Post-Shed (Test 36) .... 192

6-1. Measured Fixed Frame Loading Configuration Comparison ............................. 196
6-2. Passive Balancer Performance Comparison ...................................................... 197
6-3. Model Accuracy Summary .................................................................................. 199
LIST OF TABLES

1-1. Published Values for Shear Adhesion Strength of Ice on Al. (T_a = -11°C)..........................34

3-1. Shaft Properties...................................................................................................................71
3-2. Blade Surface Roughness Characterization.................................................................75
3-3. AERTS Aluminum Plate Balancer Physical Properties .............................................78
3-4. AERTS Aluminum Plate Balancer Dynamic Properties .............................................81
3-5. Aluminum Plate Balancer COR Results........................................................................85
3-6. Bearing Unit and Adapter Block Physical Properties....................................................89
3-7. System Damping Characterization Results.................................................................95
3-8. Effective LWC Characterization Results.......................................................................114
3-9. Ice Shed Characterization Test 3 Parameters .............................................................117
3-10. Initial Passive Balancer Configuration Test Matrix ...................................................125

4-1. -12 °C Cross Sectional Results Summary......................................................................131
4-2. -12 °C Ice Shed Mass Results Summary.........................................................................131
4-3. -8 °C Cross Sectional Results Summary.........................................................................136
4-4. -8 °C Ice Shed Mass Results Summary..........................................................................136
4-5. Unbalanced Configuration Ice Shed Characterization Results Summary ....................140
4-6. Unbalanced Configuration Continuous Icing Results Summary..................................140
4-7. Initial Passive Balancer Configuration Ice Shed Characterization Results Summary......144
4-8. Initial Passive Balancer Configuration Continuous Icing Results Summary ...............145
4-9. Initial Passive Balancer Configuration Performance Summary ....................................146
4-10. Initial Configuration Balancing Mass Ice Shed Response Ice Shed Events...............151
5-1. Frictional Model Coefficients................................................................. 163
5-2. Final Passive Balancer Key Design Parameters................................. 167
5-3. Shaft Extension Properties.................................................................... 169
5-4. Passive Balancer Physical Properties Comparison............................. 171
5-5. Final Passive Balancer Dynamic Properties......................................... 173
5-6. Final Passive Balancer Configuration Test Matrix............................... 174
5-7. Final Passive Balancer Configuration Continuous Icing Results Summary
     (Ta = -12 °C).......................................................................................... 178
5-8. Final Passive Balancer Configuration Continuous Icing Results Summary
     (Ta = -8 °C).......................................................................................... 179
5-9. Final Passive Balancer Configuration Performance Summary (Ta = -12°C)........... 182
5-10. Final Passive Balancer Configuration Performance Summary (Ta = -8°C)......... 183
5-11. Final Configuration Balancing Mass Ice shed Response Ice Shed Events............. 191

6-1. Model Accuracy Summary..................................................................... 199
LIST OF SYMBOLS

\( a \)  Shaft-fixed rigid body rotating frame

\( b \)  Shaft-fixed deformed body rotating frame

\( C \)  System damping matrix

\( F \)  System load vector

\( F_g \)  Gravitational body force, N

\( Imb_{sys} \)  System mass imbalance vector, g mm

\( K \)  System stiffness matrix

\( M \)  System mass matrix

\( n \)  Inertial coordinate frame

\( Q \)  Generalized force vector, varies

\( q \)  Generalized coordinate vector, m

\( q_{PB} \)  Passive balancer generalized coordinate vector, rad

\( q_{shaft} \)  Shaft generalized coordinate vector, m

\( R_{OB_i} \)  Position vector locating the \( i \)th balancing mass, m

\( R_{OC} \)  Position vector locating hub center, m

\( R_{OL_i} \)  Position vector locating the \( i \)th centrifugal clamp arm, m

\( R_{OM} \)  Position vector locating imbalance point mass, m

\( T_{ab} \)  Transformation matrix from \( a \) to \( b \)

\( T_{na} \)  Transformation matrix from \( n \) to \( a \)

\( T_{nb} \)  Transformation matrix from \( n \) to \( b \)

\( T_{xn} \)  Transformation matrix from \( x_g \) to \( n \)

\( x_g \)  Earth fixed inertial coordinate frame
\( A_b \)  Balancing mass cross-sectional area, \( \text{m}^2 \)

\( B \)  Blade number

\( B_A \)  Balancing authority, \( \text{g mm} \)

\( C_D \)  Balancing mass drag coefficient

\( C_{R_p} \)  Coefficient of restitution for ball-partition collision

\( c \)  External damping coefficient, \( \text{kg s}^{-1} \)

\( c_h \)  Characteristic length, \( \text{m} \)

\( c_{PB} \)  Balancer track viscous damping coefficient, \( \text{kg s}^{-1} \)

\( D \)  Total system Rayleigh damping energy, \( \text{N m s}^{-1} \)

\( D_{damper} \)  External damper Rayleigh dissipation function, \( \text{N m s}^{-1} \)

\( D_{PB} \)  Passive balancer Rayleigh dissipation function, \( \text{N m s}^{-1} \)

\( D_{shaft} \)  Shaft Rayleigh dissipation function, \( \text{N m s}^{-1} \)

\( d_i \)  Displacement of the \( i \)th centrifugal clamp arm, \( \text{m} \)

\( E_{shaft} \)  Shaft modulus of elasticity, \( \text{N m}^2 \)

\( E^* \)  Modified material modulus of elasticity, \( \text{N m}^2 \)

\( F_{ball-track} \)  Kinetic friction force due to ball-track slipping, \( \text{N} \)

\( F_{sball-track} \)  Static friction at ball track contact surface, \( \text{N} \)

\( F_f \)  Total frictional force resisting balancing mass motion, \( \text{N} \)

\( F_N \)  Ball-track surface normal force, \( \text{N} \)

\( F_r \)  Rolling resistance force, \( \text{N} \)

\( f \)  Frequency of vibration in the fixed frame, \( \text{Hz} \)

\( f_R \)  Rotor rotational frequency, \( \text{Hz} \)

\( g \)  Earth gravitational constant, \( \text{N} \)

\( h \)  Angle rotated about the \( \mathbf{n}_1 \) axis with respect to the \( \mathbf{x}_g \) frame, \( \text{rad} \)
$i$ Integer index

$I_{bmass_i}$ Mass moment of inertia for the $i$th balancing mass, kg m$^2$

$I_{mhub}$ Hub transverse moment of inertia, kg m$^2$

$I_{shaft}$ Shaft second area moment of inertia, m$^4$

$I_{mhub}$ Hub polar moment of inertia, kg m$^2$

$I_{mshaft}$ Shaft polar moment of inertia, kg m$^2$

$K_\theta$ Modified droplet inertia parameter

$K$ Droplet inertia parameter

$k_a$ Molecular conductivity of air, J cm$^{-1}$ s$^{-1}$ K$^{-1}$

$L_{10}$ Bearing fatigue lifetime, hrs.

$L_{bmass}$ Balancer location along shaft length, m

$L_{damper}$ External damper location along the shaft length, m

$L_f$ Latent heat of fusion, J kg$^{-1}$

$L_{hub}$ Hub location along the shaft length, m

$L_L$ Centrifugal clamp arm location along shaft length, m

$L_{shaft}$ Shaft overall length, m

$m_{bmass_i}$ Mass of the $i$th balancing mass, kg

$m_{hub}$ Hub mass, kg

$m_{imb}$ Mass magnitude of imbalance point mass, kg

$m_{shaft}$ Mass of shaft, kg

$N$ Total number of degrees of freedom considered

$N_{bal}$ Number of balancing masses in a passive balancing device

$N_m$ Number of shaft flexural modes considered

$P$ Applied normal load, N
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{Dyn}$</td>
<td>Applied radial dynamic load, N</td>
</tr>
<tr>
<td>$P_B$</td>
<td>Bearing critical radial dynamic load, N</td>
</tr>
<tr>
<td>$P_C$</td>
<td>Critical normal load, N</td>
</tr>
<tr>
<td>$p$</td>
<td>Angle rotated about the $n_2$ axis with respect to the $x_g$ frame, rad</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynold’s number</td>
</tr>
<tr>
<td>$R_b$</td>
<td>Balancer track radius, m</td>
</tr>
<tr>
<td>$R_{hub}$</td>
<td>Hub radius, m</td>
</tr>
<tr>
<td>$R_{imb}$</td>
<td>Radial location of imbalance point mass, m</td>
</tr>
<tr>
<td>$R_{L_i}$</td>
<td>Initial radial location of the $i$th centrifugal clamp arm, m</td>
</tr>
<tr>
<td>$R_{shaft}$</td>
<td>Shaft radius, m</td>
</tr>
<tr>
<td>$r$</td>
<td>Angle rotated about the $n_3$ with respect to the $x_g$ frame, rad</td>
</tr>
<tr>
<td>$r_b$</td>
<td>Balancing mass radius, m</td>
</tr>
<tr>
<td>$r_{LE}$</td>
<td>Airfoil leading edge radius, m</td>
</tr>
<tr>
<td>$T$</td>
<td>Total system kinetic energy, J</td>
</tr>
<tr>
<td>$T_a$</td>
<td>Ambient temperature, K</td>
</tr>
<tr>
<td>$T_{hub}$</td>
<td>Hub kinetic energy, J</td>
</tr>
<tr>
<td>$T_{imb}$</td>
<td>Imbalance mass kinetic energy, J</td>
</tr>
<tr>
<td>$T_L$</td>
<td>Centrifugal clamp kinetic energy, J</td>
</tr>
<tr>
<td>$T_{PB}$</td>
<td>Passive balancer collective kinetic energy, J</td>
</tr>
<tr>
<td>$T_{shaft}$</td>
<td>Shaft kinetic energy, J</td>
</tr>
<tr>
<td>$t$</td>
<td>Time, s</td>
</tr>
<tr>
<td>$t_{hub}$</td>
<td>Hub thickness, m</td>
</tr>
<tr>
<td>$t_p$</td>
<td>Track partition wall thickness, m</td>
</tr>
<tr>
<td>$U$</td>
<td>Total system potential energy, J</td>
</tr>
</tbody>
</table>
$U_\infty$ Freestream velocity, m s$^{-1}$

$U_{damper}$ External damper potential energy, J

$U_{hub}$ Hub potential energy, J

$U_{imb}$ Imbalance mass potential energy, J

$U_L$ Centrifugal clamp total potential energy, J

$U_{PB}$ Passive balancer total potential energy, J

$U_{shaft}$ Shaft potential energy, J

$u_{1,i}$ Velocity of the $i$th balancing mass prior to collision, m/s

$u_{2,i}$ Velocity of the $i$th balancing mass after collision, m/s

$v$ Shaft displacement along the $a_2$ axis, m

$w$ Shaft displacement along the $a_3$ axis, m

$\beta$ Angle rotated about the $b_2$ axis with respect to the $a$ frame, rad

$\gamma$ Angle rotated about the $b_3$ axis with respect to the $a$ frame, rad

$\delta$ Droplet diameter, m

$\delta W_L$ Centrifugal clamp virtual work, N m

$\delta W_{PB}$ Passive balancer virtual work, N m

$\zeta$ System damping ratio

$\Theta_i$ Fixed-free Euler Bernoulli beam $i$th bending mode shape

$\mu_{air}$ Absolute viscosity of air, kg m$^{-1}$ s$^{-1}$

$\mu_{fluid}$ Dynamic viscosity of balancer track fluid, kg m$^{-1}$ s$^{-1}$

$\mu_{kball-track}$ Kinetic coefficient of friction between ball-track contact surface

$\mu_r$ Rolling friction coefficient

$\mu_{r,l}$ Rolling friction coefficient with dimension length, m

$\mu_{sball-track}$ Static coefficient of friction between ball-track contact surface
\[ \xi_{shaft} \]  Shaft structural damping coefficient

\[ \phi \]  Shaft total angular displacement, rad

\[ \phi_{bi} \]  Angular location of the \( i \)th balancing mass, rad

\[ \phi_{ci} \]  Angular location of the \( i \)th centrifugal clamp arm, rad

\[ \phi_m \]  Angular location of the imbalance mass, rad

\[ \rho_{fluid} \]  Density of fluid occupying balancer track, kg m\(^{-3}\)

\[ \rho_{ice} \]  Ice accretion density, kg m\(^{-3}\)

\[ \sigma \]  Droplet density, kg m\(^{-3}\)

\[ \sigma_Y \]  Material yield strength, N m\(^{-2}\)

\[ \pi_c \]  Ice density heat transfer parameter

\[ \pi_K \]  Ice density droplet inertia parameter

\[ \pi_\phi \]  Ice density equivalent cylinder Reynold’s number

\[ \psi_p \]  Track partition azimuth angle, rad

\[ \Omega \]  Shaft speed, rad s\(^{-1}\)

\[ \omega_B \]  Rotation rate of the \( b \) frame with respect to the \( n \), rad s\(^{-1}\)

\[ \omega_d \]  System damped natural frequency, Hz

\[ \omega_n \]  System undamped natural frequency, Hz

\[ \omega_{bmass_i} \]  Rotational velocity of the \( i \)th balancing mass, rad s\(^{-1}\)
ACKNOWLEDGEMENTS

I would like to express my sincerest gratitude to my thesis advisor and mentor, Dr. Jose Palacios. Your commitment and passion you have towards your work is inspiring and, especially now, as the world is facing such great adversity, gives me hope for the future. Your support and guidance was instrumental in my development as a research assistant, an engineer, and a person.

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Finally, to my incredible girlfriend, you know who you are, through all the ups and downs, shut downs and breakthroughs, late nights into early mornings, you have always been there, right beside me. Your constant love and encouragement gave me the strength to do what I thought I couldn’t. I owe you so much and I hope I have all the time in the world to make it up to you. I’m looking forward to writing my next chapter about the two of us.
Chapter 1 | 

Introduction 

1.1 Background and Motivation 

Rotary machines function as an integral component in many areas of technology. From everyday applications such as hand drills, to the helicopter rotors and turbine engines that propel modern flight, it is of the utmost importance that these systems are carefully designed with operator safety in mind. Due to the oscillatory nature of these machines, they are susceptible to transmitting unwanted vibrations throughout the system, the consequences of which vary based on the application. For example, it has been documented that working with intensely vibrating handheld machines leads to musculoskeletal disorders [1]. In gas turbine engines, blade vibrations can give rise to catastrophic failures and a reduction of the blade life due to fatigue related phenomena [2]. Therefore, the presence of undesired vibration leads to additional maintenance, and, in the case of blade failure, has the potential for loss of human life. In general, vibration mitigation is crucial to improving the safety and performance of rotary systems. In order to create safer, more reliable, rotary systems at minimal cost to functionality, passive methods of rotary vibration mitigation warrant further study. Therefore, the purpose of this study is to investigate the application of a Passive Balancing device for the vibration mitigation of a mass eccentric helicopter rotor. 

Unwanted vibrations arise in rotorcraft, resulting in increased fatigue of mechanical parts that leads to maintenance problems that affect the vehicle’s availability, higher probability of malfunctions in the avionics, a high level of vibration can limit the operational envelope and increased discomfort for passengers in civil applications. It is common to divide the rotorcraft vibrations into three categories: high frequency vibrations, where the frequency of vibration (f) is
very high compared to the rotor rotational frequency \( (f_R) \), \((f \geq 20f_R)\) these vibrations are mainly caused by the engine or gear-box, moderate frequency vibrations \((5f_R \leq f \leq 20f_R)\), where the tail rotor, and to a lesser extent the main rotor, are the sources of these vibrations, and finally, low frequency vibrations \((f \leq 5f_R)\), which can be caused by track and balance issues on the main rotor [3]. It should be noted that the low frequency vibrations have the most severe effect on human tolerance and fatigue of mechanical parts.

The potential for destruction of aircraft due to vibrations caused by track and balance issues was recently displayed in a June 2019 flight test of an Aurora Flight Sciences Pegasus Passenger Air Vehicle (PAV) unmanned aircraft system, shown in Figure 1-1 [4]. According to the National Transportation Safety Board (NTSB) accident report, the vehicle impacted the runway at a high vertical descent rate after the aircraft motors went idle several seconds into an automatic landing maneuver input by the remote operator [5]. On impact, the aft booms and horizontal stabilizer failed in addition to damage being done to a significant portion of the airframe. The normal landing maneuver for the PAV was an autoland, which establishes a vertical descent and transitions to on-ground mode using a combination of squat switches and a time derivative of acceleration known as "jerk" logic. When the aircraft contacts the ground and either the squat switches close or an acceleration spike is detected, the logic switches to ground mode and commands the vertical takeoff/landing (VTOL) motors to idle.
A review of the recorded data was provided by the operator/manufacturer revealed that airframe vibration occurred in a resonant mode and was transmitted through the structure into the flight controller. The accelerations resulting from the vibrations briefly exceeded the jerk logic threshold and the aircraft entered the ground mode, subsequently commanding the motors to shut down. Therefore, excessive airframe vibrations have the ability to cause catastrophic damage to rotorcraft, especially those that are heavily reliant on avionics for maneuvering. As the commercial and military rotorcraft industry work towards creating reliable unmanned aircraft, vibration mitigation remains a key design objective to ensure the safety of the vehicle and any passengers onboard.

Over the decades, a great deal of effort has gone into the research and modeling of rotary machine vibrations as well as techniques to mitigate them. This topic of research remains relevant today as advances in modeling approaches and computational power have allowed the exploration of low-power, high performance vibration reduction systems. This chapter provides an overview of the Passive Balancing method and its relevant applications to facilitating rotary machine research.

1.2 Mass Eccentric Rotor Vibration

An eccentric rotor is one that is displaced from its rotating axis. These eccentric rotors undergo a periodic oscillation, known as whirl, that is a function of their angular velocity and the extent of eccentricity [6]. The vibrations due to whirl extend to all the components in the system. There are two causes for a rotor to be eccentric. First, external forces acting off the rotational axis that are asymmetrically distributed in the rotor plane. Second, an eccentric mass imbalance in the rotor system. This leads to the system center of gravity not being coincident with the axis of rotation, and therefore, the eccentricity is caused by the system itself.
An uneven mass distribution in the rotating plane leads to an imbalance in the centrifugal loading about the axis of rotation. The imbalance in these forces is the source of the vibration transmitted to the rest of the system. Mass imbalance, illustrated in Figure 1-2, occurs when there is excess mass at a radial location in the rotating component, making the component asymmetric about the rotating axis. Mass imbalance can be introduced into the system during the manufacturing process or due to conditions that arise during system operation.

![Figure 1-2. Mass Eccentric Rotor](image)

Imbalance due to manufacturing defects is the most prominent source of mass eccentricity in rotary systems. Despite improvements in fabrication tools and processes, manufacturing tolerances are still necessary and even a small amount of mass eccentricity present in the system will cause vibrations. Depending on the process, tolerances can range from micrometers to tens of millimeters [7]. Figure 1-3 illustrates that tolerances are also dependent on feature size; therefore, mass imbalance due to manufacturing tolerances will scale with the size of the system.
It has been documented that even small manufacturing inaccuracies lead to vibrations in rotary systems. Chao et al. [8] notes the presence of unavoidable manufacturing tolerance in optical storage disks which are unique to each disk. There, it was shown that the imbalance results in detrimental vibrations of the system assembly when subject to high rotational velocities.

Manufacturing defects affect rotorcraft as well. Due to the complex geometry and composite construction of helicopter blades, it is very difficult to fabricate blades that are exactly identical. To address this, considerable effort is spent in the design, manufacture, and quality assurance testing of rotors to mitigate vibration during operation. The quality assurance process, dubbed “Rotor Track and Balance,” consists of two stages. First, while the rotorcraft is on the ground, blades are adjusted to make sure all blades tips are on the same horizontal plane during rotation. Second, rotor’s vibration magnitude and phase measurement during flight and detected
imbalance are counterbalanced using external weights on the rotor. Measurement and counterbalancing are performed several times to minimize vibrations caused by mechanical and aerodynamic imbalance to acceptable levels [9]. However, these alterations cannot account for mass eccentricity that can occur due to operating conditions.

A common example of imbalance due to operating environment is the asymmetric ice shed of a rotor operating in adverse environmental conditions. This is a unique case of post-production mass eccentricity that is relevant to helicopters. Soltis et al. studied ice adhesion on helicopter blades [10]. In their experiments, natural ice shedding occurred due to centrifugal forces resulting from the accumulated ice mass. An example case of an ice shedding event is shown in Figure 1-4. Ice was allowed to accumulate on the blades in an icing chamber until the ice shed on one blade leading to increased forces on the hub and shaft vibration.

The icing and shedding behaviour of a blade is dependent upon the environmental conditions, ambient temperature, Median Volumetric Diameter (MVD) of water droplets, Liquid Water Content (LWC), accretion time, Reynolds Number and Weber Number, as well as the properties of the blade, blade geometry, blade material, blade surface temperature, and blade
surface roughness[12-14]. Due to the presence of all these variables, it is difficult to precisely predict the shedding behaviour of any given blade during operation, and there will be variation from blade to blade. Therefore, rotors without active ice protection systems exposed to adverse conditions can expect asymmetric ice shedding events to occur, introducing mass eccentricity into the system. Rotors protected against ice accretion seldomly shed symmetrically, also having short intervals of high vibration. Looking to future development towards electric VTOL (eVTOL) vehicles, the power requirements for an electro-thermal ice protection system, such as the one used on the UH-60, are restrictively high, approximately 25 W/m² [83]. This kind of system is not feasible for smaller or electric rotorcraft as it comes at the cost of range which is already an issue for such aircraft. Therefore, there is a need for ice protective coatings to passively shed ice. These shedding events will introduce asymmetric ice shedding that will need to be addressed.

External loading on the rotor can also contribute to eccentric vibrations. Rotors operating in icing conditions will experience aerodynamic performance degradation due to the ice accumulating on the leading edge altering the blade geometry [15]. As discussed earlier, there is variation in icing and shedding behaviour between blades leading to asymmetric aerodynamic loading. During a shedding event, a portion of the blade is cleared from ice, restoring its sectional aerodynamic performance. In an asymmetric ice shed event, this leads to significant asymmetry in aerodynamic loading which translates into off-axis loads on the shaft, contributing to eccentric shaft vibrations.

1.3 Passive Balancing Dynamics

To compensate for these dynamic imbalances during operation, a self-compensating Passive Balancing System (PBS) can be mounted to the rotor system. A PBS consists of an annular section containing a track, or race, upon which several balancing masses are free to move angularly
at a fixed distance from the rotating axis. The principal concept behind PBS mechanisms is that there is a driving centrifugal force that acts on the balancing masses due to the shift in the apparent axis of rotation due to the mass eccentricity. The effect of this driving force on the balancing masses can be categorized into three regions of rotor operation, determined by the phase of the system response in the shaft bending mode. Consider a simple rotary machine with a dynamic response described by Figure 1-5. Once the system reaches its critical operation speed at 800RPM, the rotor shaft will undergo its first flexural resonance, and the phase of the system response will shift by 180 degrees in the operating region with angular speed greater than 800RPM, known as the supercritical region.

![Figure 1-5. Dynamic Response of a Typical Rotor](image)

In Figures 1-6, 1-7, and 1-8, C is the geometric center of the shaft and disk, and O is the axis about which the system rotates. "Imb" represents an imbalance mass, and the effect it has on the center of gravity of the system is shown. Geometrically, the phase angle, R, is the angle between the O-C axis and the O-CG axis, where CG is the center of gravity of the system. During subcritical
operation. The phase of the system is 0 degrees and the shaft eccentricity grows towards the imbalance, which introduces a centrifugal load on the masses that exacerbates the imbalance, illustrated in Figure 1-6.

At resonance, the system response is oriented 90 degrees with respect to the loading which does not contribute to the angular motion of the balancing masses, shown in Figure 1-7. Finally, a rotary system operating in its supercritical region exhibits a response oriented 180 degrees from the loading. Therefore, in the supercritical region the shaft eccentricity grows away from the imbalance and the resultant centrifugal force causes the masses to move away from the imbalance, illustrated in Figure 1-8. This motion mitigates the effects of the imbalance on the system and allows the masses to eventually reach asymptotically stable positions, correcting the imbalance. This describes the automatic balancing behaviour of freely rotating masses. Automatic balancing is the principle mechanism upon which Passive Balancing Systems take advantage to correct vibrations in mass eccentric rotors.
1.4 Review of Passive Balancing System Development and Modeling

The first observations of passive balancing systems date back to the early 20th century. In 1916, Leblanc invented a passive balancing device that consisted of an annular channel filled with Mercury to counter imbalances [16]. He observed that at certain speeds, the fluid would automatically redistribute itself, reducing shaft vibrations. Thearle built off of Leblanc’s concept, and in 1932, invented the ball balancer, replacing the Mercury with free-rolling masses, and
observed that the automatic balancing behavior only occurred at supercritical speeds [17]. Thearle proposed one of the first applications for the technology, which was to use the balancer to identify the angular positions that need mass added to them to permanently balance shafts. Thearle continued to study the technology. In 1950, he experimentally determined the performance of the Leblanc balancer, his ball balancer, and a balancer utilizing pendulum masses when integrated into automatic washing machines [18,19]. Illustrations of the three balancing systems are displayed in Figure 1-9. Examining them, it can be noted that all three consist of a concentric track with freely moving masses. The major differences between the three being the dynamics by which their respective masses traverse the track. Despite experimentally studying passive balancing systems, neither Leblanc or Thearle attempted to model their behavior mathematically.

![Figure 1-9. Illustrations of Early Passive Balancer Designs](image)

Since its inception, a vital step to progressing passive balancing technology to the point of practical application has been understanding and characterizing the dynamics of the system. In
1967, Inoue et al. studied the dynamics of the Thearle ball balancer, developing equations of motion for an ideal system consisting of a balancer with in-plane spring/dashpot supports [21]. It was determined that the ideal steady state positions of the balancing masses could be predicted analytically. In 1986, Kubo et al. resolved equations of motion for balancer consisting of pendulums attached to a solid rotating disk [20]. The limitations of the pendulum balancer were identified, achieving, at best, 70% vibration amplitude reduction within a narrow range of supercritical operation speeds. Outside of these frequencies, the pendulum balancer was only able to reduce vibration amplitude by a mere 10% and introduced instability in a broad range of supercritical operation speeds. Bovik and Hogsford employed perturbation theory to numerically study the spin-up and steady state response of planar passive balancing systems [22]. It was shown that complete rotor balancing could be achieved with a passive balancing system with a minimum of 2 freely moving balancer masses.

In 1988, Majewski constructed planar equations of motion and examined the effects of ball-ball rolling resistance and track eccentricity on the rotor-PBS system at steady state [23]. In 1993, Jinnouchi et al. illustrated that a PBS provides excellent balancing when the rotor is operating at supercritical speeds, but leads to an increase in vibrations at subcritical speeds [24]. Lindell experimentally studied the application of a PBS to reduce the mass eccentric vibrations of handheld grinding machine [25]. Under certain conditions, he was able to observe the balancer balls moving automatically to their steady-state positions, reducing the vibrations by at least half their amplitude. In 1998, Rajalingham et al. conducted parametric studies on the stability of an undamped Jeffcott rotor and a single-ball passive balancer system [26, 27]. The studies focused on the effects of balancer to rotor mass ratio and support damping, and it was concluded that large support damping and large mass ratios tended to decrease the stability of the system. Chung and Ro conducted a similar study in 1999, where nonlinear equations were generated for a rotating rigid track balancer with two spherical balancing masses [28]. These equations were used along with the Routh-Hurwitz
criteria to analyze the stability of the system with respect to the assumed viscous damping due to ball/track interaction. Numerical simulations were conducted to test the predicted system stability profile and steady state solutions matched the simulations.

In 2001, Kang et al. and Huang et al. formulated equations of motion for a planar PBS using Newtonian mechanics rather than Langrangian mechanics [29, 30]. Lu et al. examined the stability of single ball [31] and dual ball balancers [32] and validated predictions with experiments. It was determined that there is at most one solution for the stable equilibrium position of a one ball balancer and two solutions for a two-ball balancer. Green et al. conducted bifurcation analysis on a two ball PBS [33]. The nonlinear stability analysis yielded many solutions unique to a nonlinear system. Green et al. then analyzed a linear system and related the sensitivity of the eigenvalue perturbation to the transient response of the balancing system [34].

The Thearle ball balancer has been the focus of most numerical and experimental studies; however, recent studies have explored the potential other balancing configurations. In 2007, Sohn et al. demonstrated that the pendulum balancer was capable of balancing at subcritical speeds [35]. In 2011, Urbiola-Soto et al. examined the stability of the Leblanc balancer numerically and experimentally via particle image velocimetry [36].

The modeling efforts presented up to this point have only considered a planar PBS connected to a rigid shaft. More recent efforts have been made to model three-dimensional dynamic behavior of passive balancing systems including in plane twisting, out of plane tilting, and gyroscopic effects. Specifically, Chung and Jang [37] and Rajalingham and Bhat [38] explored single-plane passive balancing rotor disks mounted to a massless, flexible cantilevered shaft. Similar to the purely planar studies, balanced equilibrium was only stable during supercritical speeds. In addition, other coexisting stable limit cycles were identified via direct numerical time-domain simulation. However, only the effects of shaft flexibility at the location of the balancer only
the shaft was considered. Another limitation of these studies was the shaft was modeled as massless and therefore, higher vibration modes were not considered.

In 2007, Chao et al. modeled vertical optical disk drive spindle with a mounted PBS as a rigid system with a flexible support, shown in Figure 1-10, that allowed for teetering [8]. Chao et al. demonstrated analytically and experimentally that both radial displacement and teetering could be reduced simultaneously with proper positioning of a PBS below the imbalance plane of the rotor, confirming balancing is still possible in teetering conditions [8, 39]. In 2009, DeSmidt generated a theoretical model that predicts and numerically simulates vibrations across the length of a flexible shaft with a PBS mounted at a defined position along the shaft length [40]. The study yielded insight on the effects of bearing support stiffness, axial mounting offset between the rotor plane and PBS plane, and ball/track viscous damping on the performance of the configuration shown in Figure 1-11. Kim et al. concluded that the stability of a shaft/passive balancing system could be adequately predicted with a reduced model that which neglects the out of plane tilting of the system and only considered in plane deflections and rotations of the passive balancer/rotor system [41].

![Figure 1-10. Model of a Teetering Rigid Shaft with Rotor and Balancer [8]](image-url)
Recent improvements in available computational power and the robustness of numerical models has facilitated the study of more complex shaft-PBS configurations. In 2008, Green et al. employed numerical continuation analysis methods to analyze in plane rotors fitted with multi-ball balancers with two or more balls [42, 43]. The balanced equilibrium attraction basins for a two-ball balancer were characterized as a function of ball initial positions. When the analysis was extended to a balancer with more than two balancing masses, it was found that increasing the number of masses enlarged the basin of attraction with respect to the mass initial position. However, this also tended to reduce the range of design parameters for which balancing was possible, limiting applications for multi-ball balancers. In 2008, as opposed to increasing the number of masses in a single balancer, Rodrigues studied multiple balancing devices mounted on a single rigid shaft [44]. Ehyaei and Moghaddam expanded on this concept by considering the balancers mounted to a flexible shaft instead [45]. It was demonstrated that the axial offset between the rotor and the balancers affect the vibration reduction performance, with the greatest performance achieved with both devices in close proximity with the imbalanced rotor. In 2015, Majewski et al. conducted a similar study modeling 2 passive balancing systems mounted to a
flexible shaft that are coupled via loads only, ignoring shaft teetering [46]. In the same year, Haidar provided a comprehensive model of a single passive balancer mounted to a flexible shaft [47]. Haidar expanded on the model in 2018, considering ball friction dynamics, multi-body impacts, track partitions, and centrifugal clamps [48]. The models were experimentally validated at steady supercritical speeds on the Penn State Tailrotor Driveshaft Test Rig configuration shown in Figure 1-12 and 1-13, consisting of a simply supported shaft with external damper, a partitioned and non-partitioned PBS, and a rotor hub with adjustable imbalance. The recorded metrics used for validation were in plane shaft displacement amplitude recorded by high-speed camera and optical displacement sensor and then compared to numerical predictions. The validated model was then used to conduct parametric studies to determine the key parameters that govern passive balancer performance. The model is also the basis for the dynamic modeling efforts contained within this document.

Figure 1-12. Penn State Univ. Tailrotor Driveshaft Test Rig [48]
1.5 Design Performance Parameters

Passive balancing systems have demonstrated, both analytically and experimentally, the ability to suppress mass eccentric vibrations under ideal conditions. However, there exists several parameters that require attention in the modeling, application, and design of passive balancing devices. This section outlines the possible adverse behaviors of passive balancers, their cause, and studies to optimize the performance of the system. With careful design, most of these adverse effects can be minimized or mitigated entirely.

Most of the studies of passive balancing dynamics have been focused on the steady state supercritical operation, as this is the ideal operating region. However, careful attention must be paid to the transient response of the system as it transitions up through critical speed and back down as
these regions of operation will be accompanied by dramatic increases in shaft vibration and system loading. The additional vibration in the transient regions due to the passive balancer has a detrimental side effect on shaft support bearings to which heavy loads result in early failure due to fatigue [49]. Mitigating the increased vibration common to passive balancers through critical speed transitions is desired for practical application.

In 1988, Tadeusz examined sources of position errors of balancing masses [50]. It was determined that the source of the position error, and residual imbalance, was the eccentricity of the passive balancing track itself to the shaft. If the device track is not concentric to the shaft it is mounted to, the balancing masses will not assume their ideal positions with respect to the imbalance, limiting the system from achieving a completely balanced configuration. Therefore, Tadeusz concluded that pendulums attached eccentrically to a rotor will not be capable of suppressing vibration completely. Huang et al. reached the same conclusion after performing a similar theoretical parametric study on ball balancer system [30]. It is suggested by these conclusions that a PBS with minimized track eccentricity will exhibit optimal performance.

As previously discussed, DeSmidt explored the effect of balancer axial offset from the rotor on vibration reduction performance [40]. In 2018, Haidar applied the comprehensive single balancer/flexible shaft model to conduct a parametric study on axial balancer placement relative to the imbalance [48]. It was determined that vibration reduction improved significantly as the balancer approached the source of the imbalance. In addition, Haidar concluded that a perfectly balanced configuration was only possible if the balancer was positioned in the plane of the imbalance. Therefore, in practical applications, the balancer should be positioned as close to the rotor as possible, or ideally, built into the planar structure of the rotor.

Due to the nonlinear behavior of passive balancing devices, system response is acutely sensitive to the initial positions of the masses during the transition stage of operation. Green et al. discovered that the stability and performance of a passive balancing device is sensitive to the initial
positions of the balancing masses [33,42]. It was found that for a two-ball balancer, masses initially in diametrically opposed positions result in a stable equilibrium, assuming masses hold fixed locations at steady state speed. Lu and Hang illustrated numerically that varying the initial positions of balancing masses at transition can vary the length of the transient response [51].

Rodrigues et al. determined, via numerical analysis, that stable equilibrium of passive balancing systems is more dependent on the initial positions of masses than their respective initial velocities [44]. The dependence was illustrated through a simulated scenario with one initial condition resulting in adequate balancing of the system while another resulted in the masses engaging in periodic motion, severely increasing the vibration amplitude of the system. In 2010, DeSmidt investigated the sensitivity of the system to varying initial positions and confirmed the phenomena through numerical simulation [52]. Presently, no experimental study has been conducted to test the effect initial conditions have balancer/rotor system stability.

In addition to sensitivity to initial conditions, the nonlinear behavior of passive balancing devices allows for the existence of several unstable states the system could undergo that a linearized system could not. In 1960, Caughey conducted a study involving the motion of a Hula-Hoop, and recognized that a pendulum excited by oscillatory motion at its base could enter a limit cycle state where the pendulum mass rotates periodically [53]. In a limit cycle state, the mass freely rotates at subsynchronous speeds. In 1967, Inoue et al. described similar behavior observed in a two-dimensional analytical model of a PBS [21]. In 2006, Green et al. performed bifurcation analysis to study the dynamic response of passive balancers [33]. It was shown that stable limit cycle states can coexist with stable equilibrium states. In addition, it was found that there is narrow region of frequencies immediately following the first resonant frequency where a limit cycle state is always reached. DeSmidt predicted limit cycle frequency ranges and demonstrated limit cycle behavior via numerical simulation [52].
In 2012, Lu and Hung extensively studied the limit cycle motion of a two-ball passive balancer analytically [51]. It was found that the rotational speed history of the system affects the resulting state of the system and whether stable equilibrium and limit cycles states can coexist. It was observed that if rotation speed was gradually increased, the system will enter a limit cycle state above the natural frequency and below the critical speed. However, after passing the critical speed, balancing resumes again and limit cycle state vanishes. Finally, the analysis showed that the critical speed can be reduced by increasing support damping hence narrowing the range where limit cycle states occur. In 2016, Jung et al. confirmed limit cycle states can coexist with synchronous states [54]. It was also demonstrated that for certain operating speeds and system parameters, limit cycle states can be made unstable guaranteeing a balanced state instead.

Due to the physical contact between the balancing masses and the track that defines their motion, frictional forces play a significant role in balancer design and performance. Tadeusz recognized that balancer mass resistive forces including friction could result in residual imbalance and therefore residual vibrations in the system as opposed to perfect balancing [50]. In 2001, Kang et al. studied the steady state ball locations with introduced rolling friction [29]. In the year following, Huang et al. used the same approach for friction as Kang et al. to study the steady state effect of friction on achieving complete vibration suppression [30]. Both studies confirmed residual vibrations at steady state due to the inclusion of friction.

In 2005, Yang et al. formulated a two-dimensional model of a rigid rotor and ball balancing device [55]. They performed simulations for multiple shaft speeds and did experiments with low imbalance. It was shown experimentally that friction compromises the balancing potential at low imbalance since the frictional loads are greater than the inertial loads driving the balancing masses. In the model used in this study, however, the friction could induce motion of the masses when the friction force was larger than the balancing force which is not true physically. In 2005, Van de Wouw et al. analytically and experimentally found steady state ball positions for a two-dimensional
rigid rotor system which included Coulomb friction similar to the approach used by Kang et al. and Huanh et al [56]. The results here also confirmed the deteriorating performance of the ball-type balancer at low imbalances and the existence of residual vibrations at any rotor speed. In 2012, Ishida et al. studied the effects that friction has on balancing performance when operating at critical speeds [57,58]. Here, experiments demonstrated that friction on the tracks made solutions probabilistic. In addition, more tracks improved the probability of greater vibration reduction at resonance. However, no model or analytical study was provided with the experimental results.

In 2016, Haidar et al. experimentally demonstrated the detrimental performance of a ball-type passive balancer at steady state that is due to friction [59]. A model was provided which was able to mirror the experimental results. The model employed an experiment-based parametric study to determine viscous damping and friction.

1.6 Refinement of Passive Balancer Designs

The designs of passive balancing devices have changed little since their inception by Leblanc and Thearle. In addition, much of the modeling efforts have been performed on the ideal Thearle and Leblanc balancers with little modifications. However, recent improvements in the numerical analysis methods used to predict the behavior of such devices have identified where enhancements can be made to improve performance. The new designs presented in this section address some undesired behaviors present in passive balancing devices.

In 1993, Jinnouchi et al. made the first improvement on the conventional balancer by proposing a passive balancer with a partitioned track [24]. It was found to reduce the self-excited vibrations associated with critical speed transitions. A schematic of a partitioned multi-ball balancer is provided in Figure 1-14. In 2008, Green et al. suggested that a partitioned track reduced the need for optimal initial conditions during rotor spin up as the partitions kept the masses separated [43].
However, this design reduced the magnitude of balancing the system could correct for, also known as the system balancing authority, reducing the effectiveness at supercritical speeds.

In 2011, Urbiola-Soto analyzed the use of partitions in a Leblanc balancer [60]. The partitions provided more robustness to the stability of the system due to the fluid dynamics interacting with them. Ishida et al. experimentally demonstrated that partitions do not negatively impact the balancing stability of a ball-type passive balancer [57,58]. In fact, they demonstrated that partitions narrowed the range at which the balancing balls were subject to limit cycle behavior. Haidar conducted experiments comparing a partitioned PBS to a conventional device [48]. On average, it was found that the partitioned device effectively reduced vibration amplitude in all 3 operating regions, while the conventional device contributed to dramatic increase in vibration during critical transition up and down. In addition, the conventional device was observed to slightly increase vibration even during steady state supercritical operation compared to a system with no such device.

Figure 1-14. Partitioned PBS Design
In 2011, Lu and Wang proposed a unique passive balancing device design consisting of a system of balls rods and springs [61]. A schematic of their design is displayed in Figure 1-15, illustrating how the balancing masses are able to move both radially and circumferentially, along the rods. Equations of motion unique to this design were formulated and through the analysis it was determined that this design possesses a larger stable region than the conventional Thearle balancer. The range of stable frequencies was demonstrated to be similar to pendulum balancers. However, the analysis also illustrated that the ball-rod-spring balancer performs as effectively as the Thearle balancer in the supercritical region, where the pendulum balancer fails to be as effective.

![Figure 1-15. Ball-Rod-Spring Balancer Design](image)

Based on their findings examining the contact friction between balancing masses and track walls, Ishida et al. developed a novel design for a multi-track ball type balancer [57, 58]. The design, displayed in Figure 1-16, consists of multiple concentric channels with balancing masses contained in each. It was found experimentally that a ball-type passive balancer with multiple channels where balancing balls can rotate freely reduced the negative influence of friction to ideal levels at resonance.
Finally, Kim et al. suggested a novel ball-type balancer design meant to mitigate the transient response of the system through critical speed transition [62]. The design, shown in Figure 1-17 consisted of a single channel containing multiple balancing masses separated by springs. The purpose of the springs is to ensure the masses are held at ideal initial positions for stable equilibrium. This mechanism allowed for a controlled transient response where vibrations do not exceed the vibrations that the system would experience without a passive balancing device. However, the balancing performance was slightly degraded with the introduction of springs and with the increased stiffness of the springs. Overall, this design effectively addressed the increase in vibrations associated with passive balancing device transient response.
In 2018, Haidar formulated a comprehensive model for balancer performance and conducted a series of parametric studies on partitioned, non-partitioned and multi-track balancer configurations [48]. Key design parameters, described below, were identified as having potential influence over passive balancer vibration suppression performance at steady state supercritical operation and during critical speed transitions. Comparisons were drawn between the 3 designs of PBS being considered.

The parametric study on balancer configuration demonstrated that ball mass does not influence performance effects when rolling resistance is considered, while an increase track radius, associated with a multi-track balancer, degraded performance. Partitions improved performance during critical speed transitions: more partitions further improve performance as all collisions were avoided and “hoola-hoop” or limit cycle modes were prevented.

The passive balancer location along the shaft proved critical in optimizing balancer performance. The balancer performance improved significantly as its location approached the source of imbalance on the shaft.
The effect of shaft damping on balancer performance was determined. The damping parameters had little impact on supercritical and speed down performance while had a larger influence on speed up performance. Increased damping allowed the partitioned damper to match non-balanced vibration levels during the first critical speed transition.

Rolling resistance and drag were nonlinearly related to balancer performance for the partitioned and conventional cases. Differences between the balancer configurations were, first, the ability of the partitioned passive balancer to synchronize with the shaft speed at low drag damping unlike the non-partitioned balancer. Second, the non-partitioned balancer had a higher tolerance for increased rolling resistance while maintaining performance at steady state supercritical operation. Overall, a low rolling resistance coefficient is desired.

The effect of shaft acceleration rate was determined for two of the balancer types. It was shown that with the partitioned balancer, and at sufficiently high shaft acceleration rates, the balls adhere to the partition walls during shaft speed up and speed down matching the performance of the non-balanced configuration in those regions. This is an improvement over conventional ball-type balancers which lack partitions and cause an increase of vibration in those regions.

As design parameters are studied and the design process for passive balancing devices is refined, practical applications for the technology need to be considered. There have been many studies dedicated to examining the effectiveness of passive balancing devices for various applications. In 1994, Conrad studied the use of passive balancing devices in washing machines [63]. Rajalingham and Rakheja first proposed using passive balancers in hand-held power tools for whirl suppression [26]. In 2001, several studies were conducted to characterize ball-type passive balancing devices in high-speed optical disk drivers [8, 30]. In 2011, Uribolo-Soto et al. revisited the dynamics of Leblanc balancers in washing machines [60], while Majewski et al. studied the application of passive balancers in centrifuges [64].
Recently a unique application of passive balancing devices was presented: to suppress vibrations associated with mass eccentricity that is introduced when the system is already operating in the supercritical region. In 2010, DeSmidt investigated the effectiveness of passive balancers when a bladed-disk on a rotor was subjected to a blade loss event [52]. Most relevant to this study, Haidar and Palacios suggested the use of a passive balancing device to suppress vibration due to ice shedding events on bladed-disks [65].

Passive balancing technology is at a crucial stage in its development. Two dimensional rigid models are documented and well understood. However, with the increase in computing capabilities, it has become feasible to formulate more complex models. Comprehensive models have been experimentally validated and applied to identify design parameters to optimize the performance of passive balancing devices. Passive balancing devices are now poised to be designed for practical applications. Therefore, investigations into the feasibility of implementing passive balancing devices is warranted to determine the limitations of the technology. In this work, passive balancing to counter asymmetric ice shedding events on supercritical rotor systems is investigated.

1.7 Rotorcraft Icing

The practical application of passive balancing devices being considered in this study is vibration suppression of rotors operating in adverse environmental conditions that have undergone an asymmetric ice shed event. Therefore, a review of rotorcraft icing research and relevant icing physics is provided to facilitate better understanding of the test set up and application.

At the inception of powered flight, aviators were not concerned with aircraft icing and the problems it would pose. These early pilots lacked instruments which were crucial for flying without having visual references and therefore, did their best to steer clear of clouds. This began to change in the mid-1920s with the arrival of flight instrumentation such as gyroscopic turn indicators, ball-
bank indicators, and airspeed indicators. Using these new instruments, pilots began to adopt Instrument Flight Rules (IFR) and fly in conditions where there were not any visual references. The pilots of the U.S. Air Mail Service’s New York and Chicago route became the first group of aviators to face icing challenges on a regular basis. These pilots believed that icing was the greatest hazard of their flights [66].

Beginning in the 1970s, there was a significant amount of interest in rotorcraft icing research. The interest was driven by new design requirements and the opportunities new technology presented. At the time, both the military and commercial companies had a similar need for helicopters that could operate in icing environments. The military needed helicopters that were capable of operating throughout Europe during the winter, and commercial companies needed helicopters that were capable of servicing oil rigs off the Alaskan coast and in the North Sea [66].

To address these design requirements, researchers began to apply deicing systems for fixed wing aircraft to rotorcraft blades. Under an army contract in the 1970’s, Lockheed Aircraft began exploring an improved version of the Goodrich de-icer boot, a simple pneumatic device. The boot was originally designed using neoprene rubber. This material was unable to withstand heavy rain and centrifugal forces and the concept was rejected. After investigating many materials, a polyurethane boot design was evaluated in 1979 and determined to successful, providing the first ice protective system for rotorcraft [66].

By the mid-1980s, research efforts shifted towards a more fundamental understanding of the physics behind icing and ways to model ice accretion on airfoils. NASA developed a standard model for icing predictions known as LEWICE [66]. The LEWICE software contains an analytical ice accretion model that evaluates the thermodynamics of the freezing process that occurs when super-cooled water droplets impinge on a body [67]. To determine the shape of the ice, the atmospheric and meteorological conditions must be known as inputs. Atmospheric conditions
consist of temperature, pressure and velocity, while meteorological conditions are composed of liquid water content, droplet diameter and relative humidity.

The software has four major modules: flow field calculation (panel method), particle trajectory and impingement evaluation based on Messinger’s model [68], thermodynamic and ice growth calculation, and modification of current geometry by adding ice growth [69]. Initially, the flow field and droplet impingement characteristics are determined for the clean, user-provided geometry. The thermodynamic model determines the ice accretion growth rate on each segment of the surface. LEWICE applies a time-stepping procedure to modify the ice accretion growth. When a time increment is specified in the main input file, this growth rate can be interpreted as an ice thickness and the body coordinates are adjusted to account for the accreted ice. This procedure is repeated until the predefined icing time in the main input has been reached.

LEWICE is capable of predicting two-dimensional ice accretion shapes for cylinders and multi-element airfoils with high fidelity. In 1998, Wright and Rutkowski, compared LEWICE predictions on seven unique airfoils over a range meteorological conditions to those experimentally obtained in the Icing Research Tunnel (IRT) at NASA Glenn Research Center [70]. The results of shape comparison were analyzed and it was found that, on average, the variation between the experimental data and the LEWICE predictions was 7.2%, while the variation in the experimental data itself was 2.5%, illustrating the ability for LEWICE to predict two-dimensional ice shapes accreted on geometries in predefined conditions.

Ice accretion is the process by which supercooled water droplets, droplets whose temperature is below 0 ℃, impinge on a surface and deposit ice. The rate of accretion and the resulting ice shape is dependent upon the local airspeed and several atmospheric conditions in an icing cloud. These parameters that characterize an icing cloud are air temperature, water droplet size, amount of water in the cloud for a given unit of volume, and ice accretion time.
The ambient temperature of the air determines if the supercooled droplets will freeze on impact or initially form a water layer that will freeze after sufficient heat transfer to initiate a phase change. This is largely the determining factor for the type of accreted ice shape. Because an icing cloud will contain a distribution of droplet sizes, the median volumetric diameter (MVD) is used as the representative value to characterize the droplet size of the cloud. The amount of water present per unit volume in a cloud is defined as the liquid water content (LWC). Higher LWC will lead to an increase in the ice accretion rate.

The three atmospheric parameters described above categorize the icing envelope as described by the Federal Aviation Administration (FAA). The FAA defines two icing envelopes, intermittent and continuous icing conditions, in Federal Aviation Regulations Part 25 and 29 Appendix C for aircraft and rotorcraft respectively [71]. The intermittent icing envelope, displayed in Figure 1-18, represents extreme icing conditions. In this envelope, LWC is large, typically ranging from 0.3 to 2.9 g/m$^3$. As the label suggests, these icing conditions are normally experienced sporadically, but can be more dangerous for aircraft due to the higher ice accretion rates. The continuous icing envelope, highlighted in Figure 1-18, represents less severe icing conditions that are more commonly seen by aircraft. In this envelope, LWC is low, ranging from 0.06 to 0.8 g/m$^3$, and therefore, these conditions are less dangerous for aircraft. When evaluating component performance in icing conditions, intermittent icing conditions must be considered to be conservative, despite being less commonly experienced than continuous icing.
Accreted ice can be described by three regimes, glaze, rime, and mixed. Each regime can be identified based upon the characteristic shape of the accreted ice. However, this is fairly subjective and there are a broad range of factors that affect ice shape. Generally, the ice regime is dependent upon the freezing fraction, defined as the volumetric ratio of frozen layer to water layer for the accreted ice, the ambient temperature, LWC and MVD. While it is much simpler to determine the icing regime after examining accreted ice, there are computationally based prediction models available to predict ice shapes, such as LEWICE [69]. For all regimes, the ice shape can be discretized into a main ice shape accompanied by feathers, delicate single columns of ice crystals which appear behind the main ice shape.

The rime icing regime is typically encountered at low ambient temperatures, low LWC and the MVD of the cloud is relatively small. Fully rime conditions correspond to a freezing fraction of 1. In this regime water droplets freeze fully on impact to the rotor surface and conform tightly to the leading edge of the rotor cross section. Due to this immediate freezing, air becomes trapped in the formation giving the ice its characteristic opaque white appearance. The surface of the
accreted ice is typically rough leading to an increase in the drag force experienced by the rotor [72].

An example of a typical rime ice formation is displayed in Figure 1-19c.

![Figure 1-19: Ice regime examples: (a) Glaze; (b) Mixed; (c) Rime [73]](image)

Glaze ice formations are observed when the ambient temperature is close to the freezing temperature of water, high LWC and the MVD of the cloud is relatively high. Glaze conditions have a freezing fraction less than one, which means there is a water layer and an ice layer present in the formation. Because the water droplets do not freeze on impact and a water layer is present on the surface, the water will tend to runback and freeze further away from the leading edge than it initially impinged. This phenomenon, known as “runback,” is what contributes to the large horns of ice above and below the main ice shape in glaze formations. Because of this horn shape, glaze ice typically exhibits the largest aerodynamic penalty of the three regimes [72]. In addition, because the water does not freeze on impact, less air is trapped in the formation, making glaze ice clear in appearance. An example of typical glaze ice formation is shown in Figure 1-19a.

Finally, the mixed ice regime displays characteristics of both glaze and rime ice. As such, there is no explicit set of conditions in which the transition from one regime to another takes place. Therefore, the transition region between rime and glaze is simply defined as mixed. Mixed ice formations typically exhibit a clear main ice shape, associated with glaze ice, surrounded by
feathers similar to those formed in rime ice. An example of a mixed regime ice formation is shown in Figure 1-19b.

Figure 1-20 summarizes the three icing regimes in terms of the three relevant atmospheric parameters. However, it should be noted that this figure exists only as a loose guideline.

![Figure 1-20. Icing Regime Conditions Summary [73]](image)

To properly model ice accretion over the leading edge of a rotor blade and characterize its ice shed behavior, the material properties of ice must be known. Ice is complex material by nature, with its properties being largely dependent upon the conditions in which it was formed. In spite of its complex nature, efforts have been made to develop predictions of ice behavior. These can be split into two types of models, physical and empirical. Physical models require specific knowledge of the test conditions. These types of models typically require large amount of computational power. However, as computing power has increased so has their accuracy. Empirical models rely on data collected experimentally and statistical trends to predict behavior. Given the amount of computing power available today, physical models are more widely used but empirical models can be better suited to specific applications [74].

The major property that governs ice shed behavior is the ice adhesion strength between the main ice shape and the blade surface. Like many other ice material properties, adhesion strength is
sensitive to a variety of test conditions. Brouwers et al. summarized the inconsistency present in published values for the ice adhesion strength of an Aluminum plate at -11 °C between various researchers [75]. Examining the values provided in Table 1-1, it is evident that differences in ice accretion method as well as testing procedure produces significant variability in the data. One of the limitations of the ice adhesion strength testing by these facilities involves transporting accreted ice from an icing chamber, such as an icing wind tunnel, to a centrifuge to observe its shedding behavior under centrifugal load. During the transfer process, thermal and mechanical shocks are introduced to the ice formation that introduce uncertainty in the measurement of ice adhesion strength. In addition, Knuth showed that the surface roughness of the blades affects shedding behavior, with rougher surfaces leading to an increase in adhesion strength.

<table>
<thead>
<tr>
<th>Researcher Date [Ref.]</th>
<th>Test Type</th>
<th>Type of Ice</th>
<th>Shear Adhesion Strength (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LoughBorough 1946 [76]</td>
<td>Pull</td>
<td>Freezer Ice</td>
<td>81</td>
</tr>
<tr>
<td>Stallabrass and Price 1962 [77]</td>
<td>Rotating Instrumented Beam</td>
<td>Impact Ice</td>
<td>14</td>
</tr>
<tr>
<td>Itagaki 1983 [78]</td>
<td>Rotating Rotor</td>
<td>Impact Ice</td>
<td>4-23</td>
</tr>
<tr>
<td>Scavuzzo and Chu 1987 [79]</td>
<td>Shear Window</td>
<td>Impact Ice</td>
<td>13-42</td>
</tr>
<tr>
<td>Reich 1994 [80]</td>
<td>Pull</td>
<td>Freezer Ice</td>
<td>130</td>
</tr>
</tbody>
</table>

The method of ice removal must also be considered, as a system that assumes pure shear failure that places additional moments on the ice will likely result in an uncharacteristically low value. To standardize the ice adhesion strength testing procedure, Soltis et al. developed methodology for a novel hands-off approach meant to mimic in flight ice shedding of rotorcraft as closely as possible by accreting and shedding the ice in one facility [10]. The methodology was applied to construct the Adverse Environment Rotor Test Stand (AERTS) facility in 2009 [81]. The
ice is shed naturally via centrifugal force, ensuring the ice is loaded under pure shear. In addition, the AERTS facility is capable of precise control over relevant test conditions, namely LWC, MVD, ambient temperature, and rotor speed. The AERTS facility makes use of instrumented blades to monitor the amount ice accreted and eventually shed over the outer span of the blade, the procedure for which is summarized in Ref. 10. However, for tests that involve high rotational velocities instrumented blades may not be feasible due to limitations with transferring signals from the rotational frame to the fixed frame.

In cases where instrumentation is not available, knowledge of other ice material properties can be used determine the shedding behavior. Jones developed a physical thermodynamic model, based on Meissinger’s model, to determine the density of ice accreted on airfoils as a function of nondimensional conditions [82]. This information is invaluable for determining the mass of ice that was shed off of a length of blade. If the ice accretion conditions are known, the ice shape can be determined from LEWICE simulation and, assuming uniform ice shape along the span of the shed, the volume of ice shed can be determined. Coupled with Jones’ model of ice density, this can be used to provide a close estimate to the mass of a shed event which is a critical parameter in modeling balancer performance during asymmetric ice shed.

1.8 Practical Application to eVTOL Aircraft

As described in Section 1.3, passive balancing behavior occurs only in rotary systems operating in their supercritical region. This limits the rotorcraft systems that passive balancing technology can be applied to, as the main rotors on most modern helicopters do not operate at supercritical speeds under any flight conditions, and only some tail rotors reach supercritical speed. However, the interest in eVTOL aircraft to function as passenger air vehicles for improved urban
mobility, dubbed “air-taxis,” presents a uniquely suited application for passive balancing technology.

First, the emergence of Distributed Electric Propulsion (DEP) systems in eVTOL configurations, such as the Joby Passenger Air Vehicle shown in Figure 1-21, illustrates that the future of vertical flight vehicles will feature multiple smaller rotors as opposed to one large rotor. DEP systems are implemented as a means to improve hover and propulsive efficiency, enable new capabilities in vehicle control and reduce rotor noise, which is especially important in urban environments [107]. Based on basic momentum theory, to produce the equivalent thrust of a single main rotor to maintain hover, the smaller diameter rotors will have to rotate at much higher speeds. Therefore, it is likely that these rotor systems will operate in their supercritical region, making the practical application of passive balancing possible.

Secondly, in the interest of preserving the vehicle range, eVTOL aircraft prefer to implement passive systems rather than active systems. As previously mention in Section 1.2, eVTOL aircraft cannot afford the power requirements of an active electrothermal deicing system, making the rotor susceptible to asymmetric ice shed. Active balancing systems implemented into rotor systems would increase weight, complexity, and power requirements making the system
inefficient. In contrast, passive balancing devices are simple in their construction, lightweight and occupy a small amount space on the rotor shaft, and, most significant, require no powered components.

Finally, eVTOL aircraft have lower tolerance for airframe vibration and rotor noise, suggesting a system to mitigate both phenomena must be implemented. Due to limitations in the power density of state-of-the-art batteries and electric motors, eVTOL airframes must be lightweight and therefore are not as robust as most other vehicle airframes to fixed frame loading and fatigue cycles. Examining the proposed function of eVTOL aircraft as passenger air vehicles, the minimization of airframe vibration in any operating condition is paramount to passenger safety. The Aurora Flight Sciences PAV crash, summarized in Section 1.1, illustrated the destructive capabilities of resonant airframe vibration with avionics, which is becoming especially relevant as flight controls are becoming more automated. In addition to safety, passenger experience must be considered as well, as these aircraft are presented as a luxury method to travel around urban environments, which will be degraded by the presence of any perceptible airframe vibration. The potential noise pollution introduced in an urban environment where eVTOL vehicles are ubiquitous dictates that efforts to minimize rotor noise must be taken. Therefore, mitigating the contribution of rotor vibration to rotor noise improves the acoustic compliance of eVTOL aircraft. The multiple benefits a passive balancing device presents to an eVTOL rotor system warrants the study and development of the technology for this specific application.

1.9 Thesis Objectives

The objective of this study was to investigate the application of a passive balancing device to full-scale rotor systems subject to impulsive mass eccentricity during operation. More specifically, the investigation focuses on the device’s ability to suppress eccentric vibrations that
are caused by a sudden asymmetric ice shed event on a supercritical rotor configuration. The rotor configuration in the AERTS facility for ice accretion testing will be subjected to asymmetric ice shed events in its supercritical operating region and the fixed frame loading response will be recorded with and without a passive balancing device in the system. By comparing the measurements, the ability for a passive balancing device to effectively reduce mass eccentric vibration in supercritical rotors experiencing ice shed events will be quantified. Presently, no experimental studies have been conducted on passive balancing performance in a full-scale rotorcraft configuration subjected to impulsive mass eccentricity.

In addition to the experimental comparison, a passive balancing device design process will be formulated, incorporating in-house modeling capabilities and performance prediction tools established by Haidar, to create a refined design for the AERTS rotor stand [48]. The methodology is summarized in Figure 1-22. The procedure for developing and experimentally validating the design process is as follows:

• Characterize the physical parameters AERTS rotor stand configuration and its components
• Adapt the passive balancer system mathematical model, developed by Haidar, to conduct numerical simulations of the rotor stand
• Conduct ice shed characterization and continuous icing experiments on the rotor stand configuration without a passive balancing device, referred to as the unbalanced rotor configuration, and record the fixed frame loading response for each observed mass eccentric condition
• Compare experimental results to predictions from numerical simulations to experimentally validate the model of the unbalanced rotor configuration
• Design and fabricate an initial passive balancer from design principles outlined in literature and expected mass imbalance conditions for the rotor stand
• Incorporate the initial passive balancer into the rotor stand configuration, referred to as the initial passive balancer configuration, and conduct icing experiments and record the fixed frame loading response for each observed mass eccentric condition, as well as capture the balancing mass positions in the various system operating regions and in response to ice shed events

• Compare experimental results to model predictions from numerical simulations to experimentally validate the model of the initial passive balancer configuration and quantify the initial passive balancer performance

• Analyze the experimental data and observations of the initial passive balancer configuration to identify key design parameters limiting performance for parametric analysis

• Conduct parametric analysis to improve predicted design performance across the range of mass imbalance conditions experienced by the rotor stand

• Design and fabricate a final passive balancer from the results of the parametric analysis

• Incorporate the final passive balancer into the rotor stand configuration, referred to as the final passive balancer configuration, and conduct icing experiments and record the fixed frame loading response for each observed mass eccentric condition, as well as capture the balancing mass positions in the various system operating regions and in response to ice shed events

• Compare experimental results to model predictions from numerical simulations to experimentally validate the model of the final passive balancer configuration and quantify the final passive balancer performance

• Compare the experimental results across the three configurations considered in the study to determine the effectiveness of a passive balancing device in reducing mass eccentric vibration and to validate the passive balancer design process
Unbalanced Rotor Configuration:
- Characterize system
- Adapt mathematical model
- Conduct icing experiments
- Compare to model predictions

Initial Passive Balancer Configuration:
- Design from literature trends
- Fabricate and install onto rotor
- Conduct icing experiments
- Compare to model predictions
- Quantify performance
- Determine parameters degrading performance

Parametric Analysis:
- Start with initial passive balancer design
- Vary key design parameters to improve predicted performance
- Iterate passive balancer design until performance prediction converges

Final Passive Balancer Configuration:
- Design from parametric analysis
- Fabricate and install onto rotor
- Conduct icing experiments
- Compare to model predictions
- Quantify performance

Passive Balancer Effective?

Design Process = Performance Improvement?

Figure 1-22. Passive Balancer Development Methodology Flow Chart
The performance of each passive balancing device is calculated by comparing the experimentally measured fixed frame loading to the model prediction for the unbalanced rotor configuration for a given mass imbalance condition, arising from an ice shed event. By comparing the measured performance of the initial passive balancer, which did not utilize any numerical analysis in its design, to that of the final passive balancer, which was designed from parametric analysis results, the presented design process will be experimentally validated if a significant improvement is observed.

This thesis is distributed into the following chapters:

Chapter 2: Mathematical Model

A comprehensive, physics-based mathematical model of a passive balancing system is described. The equations of motion for a multi-mass partitioned passive balancing device, including gravitational effects, coulomb friction, viscous damping, drag and ball slip is presented.

Chapter 3: Testing Facility and Experimental Design

An overview of the AERTS rotor stand facility and its relevant systems is provided. Experimental approaches to quantify balancer performance and rotor operating conditions are presented. Experiments conducted to characterize the physical properties of the facilities and model system mass eccentricity are summarized.

Chapter 4: Initial Balancer Results and Discussion

The results of experiments carried out on the unbalanced rotor configuration and the initial balancer configuration are summarized and the major findings are discussed. Experimental values are compared to model predictions and balancer performance is quantified.
Chapter 5: Parametric Analysis and Experimental Validation of Final Design

Parametric studies are conducted on key balancing design parameters to improve predicted performance. The enhanced design is fabricated and experimentally compared to the initial design to validate the use of the model as a comprehensive design tool for practical applications.

Chapter 6: Conclusions

The final chapter draws conclusions based upon the findings of the study. Recommendations for future analytical and experimental work are presented.
Chapter 2 | Mathematical Model

To analytically predict the behavior of a passive balancing device, equations of motion must be derived. Multiple cartesian coordinate frames are employed to analyze the system in the rotating-frame and fixed frame. This chapter presents the derivation of equations of motion for each component in the modeled system, adapted from the model originally developed by Haidar [48]. Haidar’s work concerned passive balancing on fixed-pinned shafts to reflect a tail rotor configuration. The model presented utilizes the same derivation, however it considers a fixed-free shaft and model parameters were altered to reflect a single small main rotor, as part of a multirotor configuration. The model is composed of a shaft, hubs, imbalance masses, a passive balancer and a bearing, modeled as an external damping device.

![System Model Coordinate Systems](image.png)

Figure 2-1. System Model Coordinate Systems [48]
2.1 Coordinate Systems

The model reference frame definitions are defined in this section. First, the Earth fixed inertial frame is denoted by $x_g$, where gravity acts in the $-x_{g1}$ direction. The $n$ inertial frame, or Newtonian frame, describes the orientation of the system with respect to the $x_g$ frame. Two rotating frames are employed. First, the $a$ frame denotes the shaft-fixed frame that rotates with the shaft. $n_1$ and $a_1$ both run parallel to the longitudinal direction of the shaft. Second, the $b$ frame is the body-fixed frame that rotates and follows the motion of the shaft. These coordinate reference frames are illustrated in figure 2-1.

A set of transformation matrices is employed to facilitate the use of local coordinate frames for energy calculations. Equation 2.1 describes the relationship between the $x_g$ and $n$ coordinate frames with the transformation matrix $[T_{xn}]$. $[T_{xn}]$ is defined in its expanded form in Equation 2.2, where $h$ represents the angle rotated about $x_{g1}$, $r$ represents the angle rotated about $x_{g2}$, and $p$ represents the angle rotated about $x_{g3}$, shown in Figure 2-2.

\[
\begin{align*}
\begin{bmatrix}
  n_1 \\
  n_2 \\
  n_3
\end{bmatrix} &= [T_{xn}]egin{bmatrix}
  x_{g1} \\
  x_{g2} \\
  x_{g3}
\end{bmatrix} \\
&T_{xn} = 
\begin{bmatrix}
  \cos (p) & \sin (p) & 0 \\
  -\sin (p) & \cos (p) & 0 \\
  0 & 0 & 1
\end{bmatrix} \\
&\quad \begin{bmatrix}
  \cos (r) & 0 & -\sin (r) \\
  0 & 1 & 0 \\
  \sin (r) & 0 & \cos (r)
\end{bmatrix} \\
&\quad \begin{bmatrix}
  1 & 0 & 0 \\
  0 & \cos (h) & \sin (h) \\
  0 & -\sin (h) & \cos (h)
\end{bmatrix}
\end{align*}
\]
The relationship between the \( \mathbf{n} \) and \( \mathbf{a} \) coordinate frames is defined by Equation 2.3, where \( [T_{na}] \) is the corresponding transformation, shown in expanded form in Equation 2.4. \( [T_{na}] \) is a function of \( \phi \), which is the total angular displacement of the shaft. Angular displacement is determined by the shaft speed (\( \Omega \)) time history, illustrated by Equation 2.5, where \( t \) denotes time.

\[
\begin{bmatrix}
a_1 \\
a_2 \\
a_3 \\
\end{bmatrix} = [T_{na}] 
\begin{bmatrix}
n_1 \\
n_2 \\
n_3 \\
\end{bmatrix} \tag{2.3}
\]

\[
T_{na} = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(\phi) & \sin(\phi) \\
0 & -\sin(\phi) & \cos(\phi)
\end{bmatrix} \tag{2.4}
\]
\[ \phi = \int_0^t \Omega(t) \, dt \quad (2.5) \]

The relationship between the shaft-fixed rotating frame, \( a \), and the body-fixed rotating frame, \( b \) is defined by Equation 2.6. The transformation matrix from the \( a \) to \( b \) frame is represented by \([T_{ab}]\), shown in expanded form in Equation 2.7. \([T_{ab}]\) is a function of \( \gamma \) and \( \beta \), the angles rotated about \( a_3 \) and \( a_2 \) respectively. The definition of \( \gamma \) and \( \beta \) is demonstrated in Figure 2-3.

\[
\begin{bmatrix}
    b_1 \\
    b_2 \\
    b_3
\end{bmatrix} = [T_{ab}]
\begin{bmatrix}
    a_1 \\
    a_2 \\
    a_3
\end{bmatrix} \quad (2.6)
\]

\[
T_{ab} = \begin{bmatrix}
    \cos(-\gamma) & \sin(-\gamma) & 0 \\
    -\sin(\gamma) & \cos(\gamma) & 0 \\
    0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
    \cos(\beta) & 0 & -\sin(\beta) \\
    0 & 1 & 0 \\
    \sin(\beta) & 0 & \cos(\beta)
\end{bmatrix} \quad (2.7)
\]

\[\text{(a) \ b frame rotation about } a_3\]

\[\text{(b) \ b frame rotation about } a_2\]

Figure 2-3. Body rotation angles from \( a \) to \( b \) reference frames
In this study, due to the stiffness characteristics of the shaft, it is safe to assume small deflections which allows the angles \( \gamma \) and \( \beta \) to be approximated as:

\[
\gamma \approx v'(x,t) \quad \beta \approx w'(x,t)
\]  

(2.8)

In Equation 2.8, \( v \) denotes the shaft displacement along the \( a_2 \) axis and \( w \) denotes the shaft displacement along the \( a_3 \) axis as a function of position along the shaft, \( x \), and time, \( t \). In addition, by the small angle approximation, the elements of \( [T_{ab}] \) can be simplified using the following:

\[
\sin(\gamma) \approx v'(x,t) \quad \cos(\gamma) = 1 - \frac{v'(x,t)^2}{2}
\]

\[
\sin(\beta) \approx w'(x,t) \quad \cos(\beta) = 1 - \frac{w'(x,t)^2}{2}
\]  

(2.9)

The small angle approximations are derived from the first term of the Maclaurin series expansion of the sine function and the first two terms of the Maclaurin series expansion of the cosine function, given in Equations 2.10 and 2.11 respectively. Applying these relations, \( [T_{ab}] \) is rewritten in terms of \( v \) and \( w \) according to Equation 2.12.

\[
\sin(\gamma) = \sum_{i=0}^{\infty} \frac{(-1)^i \gamma^{2i+1}}{(2i+1)!} = \gamma - \frac{\gamma^3}{3!} + \frac{\gamma^5}{5!} - \frac{\gamma^7}{7!} + \ldots \]  

(2.10)

\[
\cos(\gamma) = \sum_{i=0}^{\infty} \frac{(-1)^i \gamma^{2i}}{(2i)!} = 1 - \frac{\gamma^2}{2!} + \frac{\gamma^4}{4!} - \frac{\gamma^6}{6!} + \ldots \]  

(2.11)

\[
T_{ab} = \begin{bmatrix}
1 - \frac{v'(x,t)^2}{2} & -v'(x,t) & 0 \\
v'(x,t) & 1 - \frac{w'(x,t)^2}{2} & 0 \\
0 & 0 & 1 - \frac{w'(x,t)^2}{2}
\end{bmatrix}
\]  

(2.12)

Finally, the \( b \) is related to the \( n \) frame by Equation 2.13, where \( [T_{nb}] \) denotes the transformation matrix from the \( n \) frame to the \( b \) frame. \( [T_{nb}] \) is obtained from the product of the \( [T_{ab}] \) and \( [T_{na}] \) transformation matrices, shown in Equation 2.14. Applying the inverse of the
\([T_{nb}]\) matrix to rotating body-fixed frame values yields their resultant inertial frame values in the orientation of the system.

\[
\begin{bmatrix}
\{b_1 \\
\{b_2 \\
\{b_3
\end{bmatrix} = [T_{nb}]
\begin{bmatrix}
\{n_1 \\
\{n_2 \\
\{n_3
\end{bmatrix}
\]
(2.13)

\[
[T_{nb}] = [T_{ab}][T_{na}]
\]
(2.14)

To account for the rotational inertia of system components, the angular velocity is calculated in the \(b\) frame. Differentiating the \(b\) frame unit vector with respect to time and applying the chain rule, \(\dot{b}\), the time rate of change of the \(b\) frame, is determined, illustrated in Equations 2.15-2.16.

\[
\dot{b} = \frac{\partial}{\partial t} \begin{bmatrix}
\{b_1 \\
\{b_2 \\
\{b_3
\end{bmatrix} = \frac{\partial}{\partial t} \left([T_{nb}] \begin{bmatrix}
\{n_1 \\
\{n_2 \\
\{n_3
\end{bmatrix}\right) = \frac{\partial}{\partial t} ([T_{nb}]) \begin{bmatrix}
\{n_1 \\
\{n_2 \\
\{n_3
\end{bmatrix} + [T_{nb}] \frac{\partial}{\partial t} \begin{bmatrix}
\{n_1 \\
\{n_2 \\
\{n_3
\end{bmatrix}
\]
(2.15)

Applying Equation 2.14, \(\dot{b}\) can be written in terms of \([T_{nb}]\) and \(b\):

\[
\dot{b} = \frac{\partial}{\partial t} ([T_{nb}])^{-1} \begin{bmatrix}
\{b_1 \\
\{b_2 \\
\{b_3
\end{bmatrix}
\]
(2.16)

Because \(b\) is a basis, the rate of change of \(b\) with respect to time is defined as the angular velocity in the body frame with respect to the inertial frame crossed with the body frame unit vector as described by Equation 2.17. \(N_{\omega b}\) represents the vector of the angular rates of the \(b\) frame with respect to the \(n\) frame. Equation 2.18 defines a skew-symmetric angular velocity tensor which is equivalent to the cross-product result of Equation 2.17. The derivation of the skew-symmetric tensor can be found in literature [84].

\[
\dot{b} = N_{\omega b} \times \dot{b}
\]
(2.17)
\[ \frac{\partial}{\partial t} ([T_{nb}]([T_{nb}])^{-1} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \] (2.18)

\[ \frac{\partial}{\partial t} ([T_{nb}]([T_{nb}])^{-1} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \] (2.19)

Simplifying these expressions yields the relationship between the \( b \) frame angular velocities and the transformation matrix, \([T_{nb}]\), described by Equation 2.19. Solving for the angular velocities, produces an expression for the \( b \) frame angular velocities, \( \omega_B \), in terms of shaft displacements and shaft speed, shown by Equation 2.20.

\[
\begin{bmatrix} \omega_{B1} \\ \omega_{B2} \\ \omega_{B3} \end{bmatrix} = -\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} \Omega - v'\dot{w}' - \frac{1}{2}\Omega(v'^2 + w'^2) \\ -\Omega v' - \dot{w}' \\ \dot{v}' - \Omega \dot{w}' \end{bmatrix} \] (2.20)

### 2.2 Model Components

In order to model the response of each component in the system, energy equations are derived and compiled to construct corresponding equations of motion by applying LaGrange’s equations.

#### 2.2.1 Shaft

To examine the effect of non-planar motion on passive balancing performance, the system driveshaft is assumed to be flexible. The shaft is modeled as a cantilevered Euler-Bernoulli beam, which is valid assuming small deflections. Applying Euler-Bernoulli beam theory to the shaft, facilitates simplified shaft deflection prediction from equations of motion.
The shaft position vector \( \mathbf{R}_{OC} \), located on its inertial axis in the \( \mathbf{a} \) frame, is defined by the deflections, \( v \) and \( w \), shown in Equation 2.21. The kinetic energy of the shaft (\( T_{shaft} \)) is given in Equation 2.22, where \( L_{shaft} \) represents the overall shaft length and \( m_{shaft} \) is the mass per unit length. \( J_{m_{shaft}} \) represents the shaft polar moment of inertia, for a cylinder of radius \( R_{shaft} \), and is defined by Equation 2.23 [85].

\[
\mathbf{R}_{OC} = v(x, t)a_2 + w(x, t)a_3 \tag{2.21}
\]

\[
T_{shaft} = \frac{1}{2} \int_0^{L_{shaft}} \left\{ m_{shaft} \left( \dot{\mathbf{R}}_{OC} \cdot \mathbf{R}_{OC} \right) + J_{m_{shaft}} \omega_B^2 \right\} dx \tag{2.22}
\]

\[
J_{m_{shaft}} = \frac{1}{2} m_{shaft} R_{shaft}^2 \tag{2.23}
\]

The potential energy of the shaft is described by Equation 2.24, where \( E_{shaft} \) is the shaft modulus of elasticity, \( I_{shaft} \) is the shaft second area moment of inertia, and \( \mathbf{F}_g \) is the gravitational body force. The dot product of the second derivative of the position vector can be simplified according to Equation 2.25. \( I_{shaft} \) is a geometric property defined for a solid uniform rod of circular cross section by Equation 2.26, [85]. The gravitational body force, \( \mathbf{F}_g \), acts in the Earth inertial frame in the \(-x_{g1}\) direction, and its magnitude is determined by the gravitational constant \( g \), as shown in Equation 2.27.

\[
U_{shaft} = \frac{1}{2} \int_0^{L_{shaft}} \left\{ E_{shaft} l_{shaft} \left( \mathbf{R}'_{OC} \cdot \mathbf{R}'_{OC} \right) + 2 m_{shaft} \left( -\mathbf{F}_g \cdot \mathbf{R}_{OC} \right) \right\} dx \tag{2.24}
\]

\[
\mathbf{R}'_{OC} \cdot \mathbf{R}'_{OC} = v'^2 + w'^2 \tag{2.25}
\]

\[
l_{shaft} = \frac{\pi}{4} R_{shaft}^4 \tag{2.26}
\]
\[ F_g = -gx_g3 \] (2.27)

The Rayleigh dissipation energy of the shaft, \( D_{shaft} \), is described by Equation 2.28, where the shaft structural damping is represented by \( \xi_{shaft} \). In 1976, Zorzi et al. demonstrated that internal viscous damping in the time domain for beam bending can be modeled as a dissipation function [86].

\[
D_{shaft} = \frac{1}{2} \xi_{shaft} \int_0^{L_{shaft}} \{E_{shaft} I_{shaft} (\dot{R}_{OC}' \cdot \ddot{R}_{OC}')\} dx \quad (2.28)
\]

### 2.2.2 Hubs

A hub component in this model is defined as a rigid cylinder concentrically attached to the shaft at \( L_{hub} \). The two system components modeled as hubs in this model are the passive balancer frame and the rotor blades. By modeling the rotor as a hub, aerodynamic and inertial forces due to blade dynamics are ignored to simplify the analysis. Because the rotor hub is modeled as uniform and concentric to the shaft, the center of gravity will be coincident to the \( b_1 \) axis along the length of the shaft, illustrated by Figure 2-4, and any mass eccentricity of the rotor will be modeled as a separate point mass. The kinetic energy of a hub component is given by Equation 2.29, and includes the hub rotary inertia term, where \( J_{m_{hub}} \) is the hub polar moment of inertia and \( I_{m_{hub}} \) is the transverse moment of inertia. \( J_{m_{hub}} \) and \( I_{m_{hub}} \) are defined for a cylindrical hub in Equations 2.30 and 2.31 respectively, where \( m_{hub} \) is the hub mass, \( R_{hub} \) is the hub radius, and \( t_{hub} \) is the hub thickness.

\[
T_{hub} = \left[ \frac{1}{2} m_{hub} (\dot{R}_{OC} \cdot \ddot{R}_{OC}) + \frac{1}{2} \left( J_{m_{hub}} \omega_{B_1}^2 + I_{m_{hub}} \omega_{B_2}^2 + I_{m_{hub}} \omega_{B_3}^2 \right) \right]_{x=L_{hub}} \quad (2.29)
\]
\[ J_{\text{hub}} = \frac{m_{\text{hub}} R_{\text{hub}}^2}{2} \] (2.30)

\[ I_{m_{\text{hub}}} = \frac{1}{12} m_{\text{hub}} (3 R_{\text{hub}}^2 + t_{\text{hub}}^2) \] (2.31)

The hub potential energy is defined by Equation 2.32. The hub is assumed to be rigid and therefore does not have a strain energy term, leaving only the contribution from the gravitational force, \( F_g \).

\[ U_{\text{hub}} = [m_{\text{hub}} (-F_g \cdot R_{OC})]_{x=L_{\text{hub}}} \] (2.32)

Figure 2- 4. Hub Component

2.2.3 Imbalance Mass

As previously discussed, the imbalance mass is modeled as a point mass in the hub plane at \( L_{\text{hub}} \). The position vector of the point mass, \( R_{OM} \), is described by equation 2.33, and is a function of the radial location of the point mass, \( R_{imb} \), as well as its azimuth angle, \( \phi_m \). The kinetic and
potential energy of the imbalance mass are defined by Equations 2.34 and 2.35 respectively, where \(m_{imb}\) is the magnitude of the mass imbalance.

\[
R_{OM} = R_{OC}|_{x=l_{hub}} + R_{imb} \cos(\phi_m) b_2 + R_{imb} \sin(\phi_m) b_3
\]  

(2.33)

\[
T_{imb} = \frac{1}{2} m_{imb} (R_{OM} \cdot \dot{R}_{OM})
\]  

(2.34)

\[
U_{imb} = m_{imb} (-F_g \cdot R_{OM})
\]  

(2.35)

### 2.2.4 External Damper

The bearing mounted to the rotor stand frame to support the driveshaft is modeled as a planar external damping device, displayed in Figure 2-5, attached to the shaft at \(L_{damper}\). The bearing position is assumed to be fixed and therefore has no kinetic energy.

![Figure 2-5. Schematic of External Damper Component](image)

However, the bearing does provide stiffness to the system, resulting in potential energy described by Equation 2.36, where \(k\) is the bearing stiffness coefficient and the dot product of the shaft position vector can be simplified according to Equation 2.37. The Rayleigh dissipation energy
of the damping mechanism is a function of the damping coefficient, $c$, and the in-plane velocity of the shaft, illustrated by Equation 2.38.

$$U_{\text{damper}} = \frac{k}{2} [\mathbf{R}_{OC} \cdot \mathbf{R}_{OC}]_{x=L_{\text{damper}}}$$  \hspace{1cm} (2.36)$$

$$\mathbf{R}_{OC} \cdot \mathbf{R}_{OC} = v^2 + w^2$$  \hspace{1cm} (2.37)$$

$$D_{\text{damper}} = \frac{c}{2} [\dot{\mathbf{R}}_{OC} \cdot \dot{\mathbf{R}}_{OC}]_{x=L_{\text{damper}}}$$  \hspace{1cm} (2.38)$$

### 2.3 Passive Balancer

In this section, the equations that govern balancing behavior are derived for a single balancing mass of index, $i$. Each balancing mass is located at $L_{\text{mass}}$ along the shaft and the angular location of each balancing mass, $\phi_{bi}$, is measured from the positive $b_2$ axis, as illustrated by Figure 2-6. The radial location of each mass from the hub center of gravity is represented by $R_b$ and the $i$th ball position vector, $\mathbf{R}_{OBi}$, is defined by Equation 2.39.

![Figure 2-6. Passive Balancer Schematic](image-url)
\[ R_{OB_i} = (R_{OC} + R_{CB_i})_{x=L_{bmass}} \]

\[ = [v(x,t)a_2 + w(x,t)a_3 + R_b(\cos[\phi_{bi}(t)]b_2 + \sin[\phi_{bi}(t)]b_3)]_{x=L_{bmass}} \]

### 2.3.1 Energy Equations

The collective kinetic energy for all balancing masses is described by Equation 2.40, where \( N_{bal} \) is the total number of balancing masses in a passive balancing device and \( m_{bmass_i} \) is the mass of the \( i \)th balancing mass. \( I_{bmass_i} \) denotes the mass moment of inertia, defined for a spherical mass of radius \( r_b \) by Equation 2.41 [85]. The rotational velocity of the \( i \)th mass, \( \omega_{bmass_i} \), is related to its translational velocity by Equation 2.42.

\[ T_{PB} = \frac{1}{2} \sum_{i=1}^{N_{bal}} \{m_{bmass_i}(\dot{R}_{OB_i} \cdot \dot{R}_{OB_i}) + I_{bmass_i}\omega_{bmass_i}^2\} \quad (2.40) \]

\[ I_{bmass_i} = \frac{2}{5} m_{bmass_i} r_b^2 \quad (2.41) \]

\[ \omega_{bmass_i} = \frac{U_{bmass_i}}{r_b} = \frac{\dot{\phi}_b R_b}{r_b} \quad (2.42) \]

The total potential energy of the balancing masses is given by Equation 2.43 and consists solely of the summation of the gravitational body force terms. The Rayleigh dissipation energy of the balancing masses is calculated assuming the presence of viscous damping due to interactions between the balancer race and the balancing masses. Equation 2.44 defines the Rayleigh dissipation energy with the viscous damping coefficient, \( c_{PB} \), is determined from the material properties of the balancing masses and track.

\[ U_{PB} = \sum_{i=1}^{N_{bal}} m_{bmass_i}(-F_g \cdot R_{OB_i}) \quad (2.43) \]

\[ D_{PB} = \frac{1}{2} \sum_{i=1}^{N_{bal}} c_{PB} R_b^2 \dot{\phi}_b^2 \quad (2.44) \]
2.3.2 Friction

The model considers several mechanisms that oppose balancing mass motion. Namely, ball/track slipping, ball rolling, dynamic and static friction effects are accounted for. Figure 2-7 illustrates the slipping and rolling resistive forces. Frictional forces are modeled according to the Coulomb friction definition. The virtual work done by the total friction force, \( F_f \), is defined by Equation 2.45, where the function \( sgn(\dot{\phi}_{b_i}) \) is described by Equation 2.46.

\[
\delta W_{PB} = -sgn(\dot{\phi}_{b_i}) F_f R_b \delta \phi_{b_i} \tag{2.45}
\]

\[
sgn(\dot{\phi}_{b_i}) = \begin{cases} 
-1 & \text{if } \dot{\phi}_{b_i} < 0 \\
0 & \text{if } \dot{\phi}_{b_i} = 0 \\
1 & \text{if } \dot{\phi}_{b_i} > 0
\end{cases} \tag{2.46}
\]

The rolling resistance force, \( F_r \), is given by Equation 2.47, where \( \mu_{r,l} \) denotes the rolling friction coefficient with a unit of length and \( F_N \) is the normal force with respect to the ball-track

\[
F_r = \mu_{r,l} F_N \tag{2.47}
\]
contact surface. The rolling resistance force is due to the asymmetric contact pressure distribution of a rolling element, which causes the reacting force of the supporting surface to be offset from the surface normal, illustrated in Figure 2-7a \[87\].

\[ F_r = \frac{\mu_{rA}}{r_b} F_N \]  \hspace{1cm} (2.47)

The second friction resistance force, \( F_{\text{ball-track}} \), corresponds to the ball-track slipping case, as shown in Figure 2-7b. This occurs when the ball stops rolling but motion continues and the resistance force is defined by Equation 2.48, where \( \mu_{k\text{ball-track}} \) is the kinetic coefficient of friction between the ball contact surface and the track. The ball is only determined to be rolling when Equation 2.49 is satisfied, where the force on the ball is greater than the static friction between the ball and the track, \( F_{s\text{ball-track}} \). The magnitude of the static friction force is determined by the static coefficient of friction between the ball and the track, \( \mu_{s\text{ball-track}} \), as described by Equation 2.50.

\[ F_{\text{ball-track}} = \mu_{k\text{ball-track}} F_N \]  \hspace{1cm} (2.48)

\[ |F_b| > F_{s\text{ball-track}} \]  \hspace{1cm} (2.49)

\[ F_{s\text{ball-track}} = \mu_{s\text{ball-track}} F_N \]  \hspace{1cm} (2.50)

Criteria were established to ensure that the physics of the ball motion were realistic. For example, Equation 2.51 describes a criterion that ensures that friction forces are never the driving force when ball velocity is zero. The algorithm that accounts for the frictional dynamics of the balancing masses is displayed in Figure 2-8.

\[ \text{if } \text{sgn}(\dot{\phi}_b) \approx 0, \text{ then } \text{sgn}(F_f) = -\text{sgn}(F_b); \ |F_f| \leq |F_b| \]  \hspace{1cm} (2.51)
System State: $F_b, F_N, \phi_b, \dot{\phi}_b$

- yes, rolling
  - $|F_b| \leq F_{sb}^{ball-track}$?
    - yes, $F_f = F_r$, rotational inertia
    - no, $F_f = F_{kball-track}$, no rotational inertia

- no, slipping
  - $\phi_b = 0$?
    - yes
      - $|F_f| \geq |F_b|$?
        - yes, $F_f = -F_b$
        - no
          - $F_f = -\text{sgn}(F_b)|F_f|$

- no
  - $|F_f| \geq |F_b|$, and $\text{sgn}(\phi_{b,\text{old}}) \neq \text{sgn}(\dot{\phi}_b)$?
    - yes
      - Ball is stuck ($\dot{\phi}_b = 0$)
    - no
      - $F_b = F_b + F_f$

Figure 2-8. Friction Dynamics Algorithm with Ball-Track Interaction
The total normal force on a balancing mass, $F_{Ni}$, present in this model is described by Equation 2.52, where $\hat{R}_{CB}$ is the unit vector in the $R_{CB}$ direction, and $R_{OB}$ is the vector from the axis of rotation to the mass location. The shaft displacement is assumed to be small, therefore, the balancing mass motion, $\dot{\phi}_{bi}$, is about the origin, $O$, illustrated in Figure 2-9. The total normal force accounts for both gravitational effects, which are dominant at low shaft speeds, and centrifugal loading, which is dominant at higher shaft speeds. The model accounts for the misaligned centrifugal loading when the balancer is displaced from the axis of rotation, as shown in Figure 2-9.

$$F_{Ni} = m_{bmass}[\left(F_g \cdot \hat{R}_{CB}\right) + \left((\Omega + \dot{\phi}_{bi})^2 R_{OB} \cdot \hat{R}_{CB}\right)]_{x=L_{bmass}} \quad (2.52)$$

![Figure 2-9. Schematic of Passive Balancer with Friction](image)

The model also considers the transfer of friction resistance loads to the shaft, as resistance forces act tangent to the circumference of the balancer track. The sum of these loads in the balancer fixed frame is given by Equation 2.53. The dot product of the resulting load vector is then taken...
with the shaft horizontal and vertical displacement vectors and multiplied by the shaft bending
mode shape function to obtain the shaft-fixed frame force on the shaft due to resistance loads, as
shown in Equation 2.54.

\[ F_{\text{resistance}} = \sum_{i=1}^{N_{\text{bal}}} \left\{ -F_{f_i} \sin(\phi_{b_i}) \mathbf{b}_2 + F_{f_i} \cos(\phi_{b_i}) \mathbf{b}_3 \right\} \]  

(2.53)

\[ F_{\text{shaft}} = F_{\text{shaft}} + \left\{ \mathbf{\Theta} \cdot (F_{\text{resistance}} \cdot \mathbf{a}_2) \right\} \]

(2.54)

### 2.3.3 Drag

Drag force opposing the motion of the balancing masses is considered in the model. The
nonlinear relationship between the drag force and the difference in the rotating speed of the track
to the balancing mass motion is based on a linear aerodynamic model, defined as a function of
model parameters in Equation 2.55 [88]. \( \rho_{\text{fluid}} \) denotes the density of the fluid that occupies the
track, for this study that fluid is assumed to be air in ambient Standard Temperature and Pressure
(STP) conditions. \( A_b \) is the cross-sectional area of the balancing mass seen by the fluid flow. \( C_D \) is the
balancing mass drag coefficient, which is a function of the balancing mass Reynold’s number,
\( Re \), given by Equation 2.56, where \( \mu_{\text{fluid}} \) represents the dynamic viscosity of the track fluid.

\[ F_d = \frac{1}{2} \rho_{\text{fluid}} A_b C_D (Re) \phi_{b_i}^2 R_b^2 \text{sgn}(\dot{\phi}_{b_i}) \]  

(2.55)

\[ Re = \frac{\rho_{\text{fluid}} |R_b \phi_{b_i}|^2 R_b}{\mu_{\text{fluid}}} \]  

(2.56)

### 2.3.4 Inelastic Collisions

Track partitions are included as barriers in the passive balancer and limit the range of
anglular positions that can be occupied by each balancing mass. The parameters that define each
partition within a balancer track are wall thickness, $t_p$, azimuth angle, $\psi_p$, and coefficient of restitution, $C_{R_p}$. Inelastic collisions between the balancing masses and partitions are considered in the model, and the new trajectory and speed of the mass is calculated after a collision according to the coefficient of restitution between the mass and the partition wall. As a fixed barrier, the velocity of the partition wall is zero in the $b$ frame before and after the collision, therefore only the velocity vector of the balancing mass is considered in the conservation of momentum equation. Equation 2.57 describes the resultant velocity, $u_{2,i}$, of an inelastic collision between a balancing mass with velocity $u_{1,i}$ prior to collision and a partition wall with coefficient of restitution $C_{R_p}$. In this study, only partitioned track passive balancers with a single mass per partition are considered. Therefore, collision dynamics and contacts between balancing masses are not relevant.

$$u_{2,i} = C_{R_p} u_{1,i}$$ (2.57)

2.3.5 Centrifugal Clamps

Centrifugal clamps are used to fix balancing mass initial positions to avoid vibratory performance degradation due to the balancer in the subcritical and transition regions of operation. Centrifugal clamps are included in the model to determine the magnitude of their effect on the subcritical and transition performance of current and future balancer designs.
An example of a balancing configuration with centrifugal clamps is illustrated in Figure 2-10. The clamping mechanism consists of a spring-loaded clamp arm. The precompressed springs drive the clamp arm towards the balancing device’s center, initially locking the balancing masses in place. As shaft speed increases, the centrifugal effects increase until sufficient centrifugal force acts on the balancing mass and the clamp arm mass, $m_{L_i}$, to compress the springs according to the spring stiffness, $k_L$, moving the arm radially outwards, and freeing the masses. In order to optimize balancer performance, the mass of the balancing masses and clamp arm must be tuned to ensure the masses are released after transition occurs.

Equations of motion are derived for each centrifugal clamp arm, using only one degree of freedom per arm as it is assumed to only move radially. The $i$th clamp arm position vector, $R_{DL_i}$, is defined by Equation 2.58, where $L_{Li}$ is the clamp arm location along the shaft length, $R_{Li}$ is the initial radial location of the arm, $d_i$ is the arm radial displacement, and $\phi_{Li}$ is its azimuth angle.

$$R_{DL_i} = [R_{OC} + R_{CL_i}]_{X = L_{Li}} =$$

$$[v(x, t)a_2 + w(x, t)a_3 + \left( R_{Li} + d_i(t) \right) ( \cos[\phi_{Li}(t)] b_2 + \sin[\phi_{Li}(t)] b_3 )]_{X = L_{Li}}$$
The total kinetic and potential energy of all the clamp arms is given by Equations 2.59 and 2.60 respectively, where $N_L$ is the total number of clamp arms, $k_{Li}$ is the effective stiffness of the $i$th clamp, and $d_{Oi}$ is the difference in the initial spring compressed length relative to the uncompressed length. In addition, virtual work is done on the by the centrifugal force on the balancing masses described by Equation 2.61, where $F_N$ is equal to that defined by Equation 2.52.

$$T_L = \frac{1}{2} \sum_{i=1}^{N_L} m_{Li} (\dot{R}_{OLi} \cdot \dot{R}_{OLi})$$ (2.59)

$$U_L = \sum_{i=1}^{N_L} \left[ \frac{1}{2} k_{Li} (d_i(t) + d_{Oi})^2 + m_{Li} (-F_g \cdot R_{OBi}) \right]$$ (2.60)

$$\delta W_L = \sum F_N \delta d_i$$ (2.61)

2.4 System Discretization

Shaft deflections are approximated by an assumed modal expansion. The transverse displacements $v$ and $w$ are approximated by Equations 2.62 and 2.63 respectively, where $\Theta_i(x)$ represents the shape function for the $i$th elastic beam bending mode, with the model considering the first $N_m$ modes. $\eta_{v_i}$ and $\eta_{w_i}$ are the time-dependent modal coordinates that correspond to their respective transverse shaft displacement. By representing transverse displacements as modal expansions, the equations of motion for the system can be obtained using Lagrangian mechanics.

$$v(x,t) = \sum_{i=1}^{N_m} \Theta_i(x) \eta_{v_i}(t)$$ (2.62)

$$w(x,t) = \sum_{i=1}^{N_m} \Theta_i(x) \eta_{w_i}(t)$$ (2.63)
2.5 System Total Energy

Compiling the energy equations of all components considered by the model, the total kinetic energy is given by:

\[ T = T_{shaft} + T_{hub} + T_{PB} + T_{imb} + T_L \]  

(2.64)

The total potential energy of the system is:

\[ U = U_{shaft} + U_{hub} + U_{PB} + U_{damper} + U_{imb} + U_L \]  

(2.65)

The total system Rayleigh dissipation energy is:

\[ D = D_{shaft} + D_{damper} + D_{PB} \]  

(2.66)

Finally, the total virtual work done on the system is:

\[ \delta W = \delta W_{PB} = Q^T \delta q \]  

(2.67)

where \( Q \) is the generalized force vector and \( q \) is the generalized coordinate vector.

2.6 System Degrees of Freedom

The system degrees of freedom in the model are dependent upon the system composition. A system composed of a shaft with \( N_m \) flexural modes and \( N_{PB} \) passive balancing devices, each with \( N_{bal} \) balancing masses, will have degrees of freedom, \( N \), defined by Equation 2.68.

\[ N = 2N_m + N_{PB}N_{bal} \]  

(2.68)

The \( N \times 1 \) generalized coordinate vector, \( q \), is defined as:

\[ q = \begin{bmatrix} q_{shaft}^T & q_{PB}^T \end{bmatrix}^T \]  

(2.69)

where \( q_{shaft} \) is the \( 2N_m \times 1 \) generalized coordinate vector defined by equation 2.70 and \( q_{PB} \) is the \( N_{PB}N_{bal} \times 1 \) generalized coordinate vector defined by equation 2.71. Here, \( \eta_v \) and \( \eta_w \) are the shaft bending modal coordinate vectors in the \( a_2 \) and \( a_3 \) directions respectively.
\[ q_{\text{shaft}} = \{ \eta_v^T \eta_w^T \}^T \] (2.70)

\[ q_{PB} = \{ \phi_{b_1} + \phi_{b_2} + \phi_{b_3} + \cdots + \phi_{b_{N-2N_m}} \}^T \] (2.71)

### 2.7 System Equations of Motion

The system equations of motion are derived from LaGrange’s equation, Equation 2.72, where \( T \) is the total kinetic energy in the system, \( U \) is the total potential energy, \( D \) is the total Rayleigh dissipation energy and \( Q \) is the generalized force vector. \( q \) is the system generalized coordinate vector and \( \dot{q} \) is its derivative with respect to time.

\[
\frac{d}{dt} \left[ \frac{\partial T}{\partial \dot{q}} \right] - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} + \frac{\partial D}{\partial \dot{q}} = Q \tag{2.72}
\]

Evaluating LaGrange’s equation, the resultant system of nonlinear equations is put into standard matrix form (Equation 2.73), where \( M \) is the system mass matrix, \( C \) is the system damping matrix, \( K \) is the system stiffness matrix, and \( F \) is the system load vector. A 4\(^{th}\) order Runge-Kutta algorithm solves the system numerically in a MATLAB® environment to obtain the system response over a predefined range of time and operating conditions.

\[
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + K(q, \dot{q})q = F \tag{2.73}
\]
Chapter 3 | Testing Facility and Experimental Design

Experiments were conducted in the Penn State Adverse Environment Rotor Test Stand (AERTS) Laboratory. An overview of the test facility capabilities, experimental configuration components and instrumentation is provided. Procedure for obtaining relevant physical properties of the system and categorizing experimental conditions is described. Finally, a test matrix of the experiments to assess passive balancer performance for a rotor system in icing conditions is presented.

3.1 Facility Overview

The AERTS facility was designed as a state-of-the-art facility for the testing of truncated helicopter blades, propellers, and wind turbines operating in icing conditions. The original facility configuration, shown in Figure 3-1, consisted of a QH-50D rotor hub mounted to a splined stainless-steel shaft. The shaft was connected to a 125 HP motor with a universal joint to correct any misalignment, and mounted to a steel support structure with the QH-50D bell housing. The bell housing swashplate was attached to two linear actuators to control rotor collective and lateral pitch [89]. Fixed frame measurements are taken by an ATI Omega160, 6-axis load cell, is mounted to the bell housing to monitor the loads and moments experienced by the rotor system and a Cooper LXT971 Torque Sensor, mounted below the steel support structure, monitors the torque supplied by the motor to the shaft. The rotor configuration can be connected to a slip ring to transit power and electrical signals from the rotating frame to the fixed frame through 48 signal channels and 24
power channels. The inclusion of the slip ring facilitated the testing of instrumented rotor blades for precise ice adhesion strength testing as well as blades with powered de-icing systems.

Figure 3-1. AERTS Original Rotor stand Configuration [89]

Due to the limitations of the original configuration components, modifications to the rotor stand had to be made to allow for the testing of mass eccentric rotor blades in their supercritical region. The slip ring is only rated for low speed operating regions (<600RPM). Therefore, the slip ring is disconnected and all instrumentation utilized in this study takes measurements in the fixed frame. Because this study does not consider aerodynamic effects, symmetric airfoil rotor blades are mounted to the hub and set at a fixed 0-degree angle of attack. This eliminates the need for pitch control and the linear actuators were removed. With the anticipation of high loading due to asymmetric ice shed at supercritical speed, the rotor plane height needed to be reduced to minimize the moments experienced at the fixed shaft root. In addition, a system had to be implemented to fix the balancer directly to the shaft. Therefore, the bell housing was removed in favor of a simpler design to fix the shaft to the steel support structure while occupying minimal shaft length. The shaft was shortened, moving the rotor plane closer to the shaft root, and a keyed slot was machined into
the shaft to affix a bushing to mount the balancer below the rotor plane. The bell housing support structure is replaced with a Dodge FCE112R piloted, tapered roller bearing unit. The bearing is rigidly fixed to the ATI load cell with a stainless-steel adapter block that restrains the shaft axially through a thrust bearing contact between the piloted bearing and a shaft collar. The modified configuration without the rotor hub is shown in Figure 3-3 and, with the rotor hub, is capable of achieving speeds of 1600RPM during rotorcraft icing experiments.

The rotor stand is enclosed by a ballistic wall, for absorbing shed ice impacts, and contained within a 6 m x 6 m x 3.5 m industrial freezer, displayed in Figure 3-3. The freezer is instrumented with thermocouples distributed throughout the chamber to provide precise temperature control from ambient temperature to -25 ℃. Above the rotor stand, 2 concentric rings of NASA standard atomizing nozzles are mounted in the ceiling to provide the icing cloud, as shown in Figure 3-4. The inner ring contains 5 nozzles and the outer contains 10 nozzles for a total of 15 nozzles that can be used in any spraying combination depending on desired test conditions. All nozzles are connected to a central manifold where air and water pressure are controlled through pressure
transducers connected to the calibration nozzle in the outer ring. Constantly controlling the differential pressure between the air and water lines provides the operator control over the MVD of the icing cloud, with capabilities from 10 μm to 50 μm. The LWC is varied by changing the number of spraying nozzles as well as the air-line pressure, allowing the chamber LWC range from 0.2 g/m³ to 5.0 g/m³. LWC must be calibrated experimentally for each spray configuration. Combining these capabilities, the AERTS spray system is able to produce a precise, repeatable icing cloud in the intermittent and continuous icing envelopes.

Figure 3- 3. AERTS Freezer Diagram [89]

Figure 3- 4. AERTS Atomizing Nozzle Spray System [90]
Test conditions are controlled with a central LabView code run by the operator in the control room, located just outside the freezer. The front panel for the control code GUI is shown in Figure 3-5. The code takes inputs for air pressure, MVD, and ambient chamber temperature, and communicates with an external compressor unit to supply cooling through the freezer evaporator system and controls the airline pressure, to maintain desired conditions. Water pressure is controlled by a pump on a separate system from the compressor so water and air pressure can be independently controlled. The resultant MVD is then calculated from NASA’s standard atomizing nozzle calibration chart, shown in Figure 3-6 [91].

Figure 3- 5. AERTS Rotor stand Control Front Panel

Figure 3- 6. NASA Standard Atomizing Nozzle Calibration Chart [91]
3.2 System Components

3.2.1 Shaft

The rotor shaft is composed of 321 stainless-steel to be able to withstand adverse operating conditions without oxidizing. The shaft properties are given in Table 3-1, with the shaft modulus of elasticity and structural damping obtained from typical values provided in literature. A schematic of the shaft is provided in Figure 3-7, with relevant measurements. In this study, the shaft is considered to be a cantilever Euler-Bernoulli beam with the root (L=0) set at the bottom surface of the collar as this is the point where the shaft’s in-plane and axial motion is fixed to the load cell. This yields a modeled shaft length of 17.48 inches and a modeled shaft mass of 5.0517 kg, which assumes a uniform shaft cross-section and calculates mass based on the shaft length fraction.

<table>
<thead>
<tr>
<th>Material</th>
<th>321 Stainless Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>8027 kg/m³ [92]</td>
</tr>
<tr>
<td>Length</td>
<td>24.81 in. (630.174 mm)</td>
</tr>
<tr>
<td>Radius</td>
<td>0.87 in. (22.098 mm) (below spline)</td>
</tr>
<tr>
<td></td>
<td>0.815 in. (20.701 mm) (above spline)</td>
</tr>
<tr>
<td>Mass</td>
<td>15.81 lbs. (7.17 kg)</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>193 GPa [92]</td>
</tr>
<tr>
<td>Structural Damping Coefficient</td>
<td>0.0002 s [92]</td>
</tr>
</tbody>
</table>
Figure 3- 7. Shaft Schematic
3.2.2 Rotor Hub

The rotor hub, shown in Figure 3-8, mounted to the shaft is from the original QH-50D configuration. The hub is splined to the shaft with the center of gravity offset from the root plane by 11.575 inches. The component has a mass of 16.575kg. The rotor blades are bolted to the hub grips with adapters to allow a variety of rotor blades to be compatible with the rotor hub. The hub is normally free to both teeter and pitch the blades during operation, however, static pitch links and teeter locks ensure that the blades are level and set at 0 degrees angle of attack. Prior to test, the rotor hub was dynamically mass balanced to within 0.5 g.

![Figure 3-8. Rotor Hub 3-View Drawing (Top, Front, Right)](image)

3.2.3 Rotor Blades

Two main rotor blades from a Schweizer S300 helicopter were donated to the AERTS facility in order to study rotorcraft icing on a representative helicopter rotor. Due to the presence of the ballistic wall, the blades were cut to a length of 40 inches to comply with the 9-foot diameter rotor limit in the AERTS facility. The blade tip radius \( R_{tip} \) is located 4.5 feet from the rotating
axis. The blades have a uniform cross-section throughout the span, comprised of an NACA0015 airfoil with a chord of 7 inches. To ensure that the blades would not introduce any imbalance to the system, the radial center of gravity was determined for each blade. The blade is placed on a set of two scales on angle bracket to minimize contact area, as shown in Figure 3-9, and based on the readings the radial center of gravity relative to the blade root is determined. The estimated radial center of gravity for each blade was 18.9 in., which corresponds to a distance of 30.2 in. from the axis of rotation. The mass of each blade was also determined to be identical, at 5.985 kg, therefore, the blade mass distribution is assumed to be identical.

![Figure 3-9. Blade Radial Center of Gravity Estimation](image)

Due to conditions encountered in their original service life, the blades leading edge displayed uneven scratching and surface roughness down the span. To ensure that the blades produced consistent shedding behavior, the blade spanwise leading edge characteristics needed to be made uniform between the two blades. To address this the blade painted coating was removed with a handheld grinder to expose the bare aluminum layer, which was then sanded down to a uniform 400 grit spec, corresponding to a mean surface roughness of 0.23 μm Ra. This estimate was confirmed at the 90%, 75%, and 50% span locations with an optical profilometer. The measurements are summarized in Table 3-2.

To estimate the length of ice shed from the blades, an in-plane length scale was applied to the blades down the outer 50% span. Alternating strips of black were painted on the blades,
excluding the leading-edge surface, measuring 2.5 inches in width with gaps of 2.5 inches in between. The blade scale pattern is displayed in Figure 3-10.

Table 3-2. Blade Surface Roughness Characterization

<table>
<thead>
<tr>
<th>Span %</th>
<th>Average Surface Roughness (μm Ra)</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.31</td>
<td>7.432%</td>
</tr>
<tr>
<td>75</td>
<td>0.26</td>
<td>11.648%</td>
</tr>
<tr>
<td>90</td>
<td>0.28</td>
<td>9.891%</td>
</tr>
</tbody>
</table>

The blades are mounted to the rotor hub such that the plane of their center of gravity is assumed to be coincident to the hub center of gravity plane relative to the shaft root location. Therefore, the blades center of gravity is offset from the shaft root by 11.575 inches. An image of an iced rotor blade mounted to the rotor hub is provided in Figure 3-11.
It is assumed that the icing cloud is evenly distributed throughout the chamber during testing. Therefore, the ice accretion shapes are assumed to be identical between rotor blades and the ice mass accreted during testing is not accounted for in the model, as the mass distribution between blades should be similar until an asymmetric ice shed occurs. After an ice shed on a blade occurs, the imbalance mass is modeled as a point mass with a location and magnitude that are a function of the ice shed mass and geometry, as there is now an excess mass on the opposite blade equal to the mass that was just shed. The ice shape cross-section is assumed to uniform down the span of the blade, therefore the location of the point mass ($R_{imb}$) will be half the distance of the shed length from the blade tip, as shown in Figure 3-12. The shed length is obtained from imagery making use of the blade length scale pattern. The experimental procedure for determining the ice mass of the shed event is covered later in this section. With these two parameters, a mass imbalance due to asymmetric shed can be adequately represented in the model.
3.2.4 Initial Passive Balancer

The initial balancer investigated experimentally is the AERTS Aluminum Plate Balancer, shown in Figure 3-13, with the physical properties described by Table 3-3. This balancer was not designed using the balancer performance prediction model, but formulated based upon previously studied design trends for passive balancer performance. The design concept for this balancer was to maximize the system balancing authority, while implementing 4 track partitions to improve subcritical performance. The balancing authority is a measure of the maximum imbalance the passive balancing device can account for and is a function of the balancer partition thickness, $t_p$, the mass of a balancing mass, $m_{bmass}$, the mass radius, $r_p$, and the distance of the mass CG to the axis of rotation, $R_b$, described by Equation 3.1. Two choices for balancing masses were considered for this study: 3-inch diameter chrome steel ball bearings, and 2.8-inch diameter high-density polyethylene (HDPE) balls. The steel masses and HDPE masses represent the high and low balancing authority cases respectively.
\[ B_A = 2m_{bmass}R_b \left( \cos \left( \sin^{-1} \left( \frac{r_p}{R_b} \right) + \sin^{-1} \left( \frac{r_k}{R_b} \right) \right) + \cos \left( \frac{\pi}{2} + \sin^{-1} \left( \frac{r_p}{R_b} \right) + \sin^{-1} \left( \frac{r_k}{R_b} \right) \right) \right) \] (3.1)

Figure 3-13. AERTS Aluminum Plate Balancer

Table 3-3. AERTS Aluminum Plate Balancer Physical Properties

<table>
<thead>
<tr>
<th>Ball Material</th>
<th>Chrome Steel, High-Density Polyethylene</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ball Radius</td>
<td>1.5 in. (38.1 mm), 1.4 in. (35.56 mm)</td>
</tr>
<tr>
<td>Ball Mass</td>
<td>1 lb. (0.4536 kg), 0.3448 lbs. (0.1564 kg)</td>
</tr>
<tr>
<td>Distance to Axis of Rotation</td>
<td>12.25 in. (311.5 mm), 12.35 in. (313.69 mm)</td>
</tr>
<tr>
<td>Partition Thickness</td>
<td>2.15 in. (54.61 mm)</td>
</tr>
<tr>
<td>Balancing Authority</td>
<td>2.1735 x 10^5 g-mm, 7.6389 x 10^4 g-mm</td>
</tr>
<tr>
<td>Balancer Frame Material</td>
<td>6061-T6 Aluminum</td>
</tr>
<tr>
<td>Balancer Frame Mass</td>
<td>36.390 lbs. (16.506 kg)</td>
</tr>
<tr>
<td>Balancer Frame Outer Diameter</td>
<td>30 in. (762 mm)</td>
</tr>
<tr>
<td>Balancer Frame Thickness</td>
<td>3.27 in. (83.06 mm)</td>
</tr>
<tr>
<td>Balancer Track Radius</td>
<td>13.75 in. (349.25 mm)</td>
</tr>
<tr>
<td>Balancer Shaft Location (from L = 0)</td>
<td>6.45 in. (163.83 mm)</td>
</tr>
</tbody>
</table>
The balancer frame is composed of three Aluminum plates bolted together with Aluminum spacers in between in an effort to reduce weight and fabrication cost. The frame is enclosed on the top and bottom by plexiglass plates to allow observation of the balancing mass positions during operation, as shown in Figure 3-14. The outer Aluminum plates have spokes that run to the balancer bushing, which fixes the frame to the shaft. The bushing is made of Carbon Steel and has a mass of 5.96 kg. The bushing is keyed directly into the shaft and secured in place with 4 set screws.

Prior to testing, static stress analysis simulations were conducted on the Aluminum Plate Balancer design to ensure that it could withstand loading under its critical operating conditions within a reasonable factor of safety. The analysis was conducted with SOLIDWORKS Finite Element Analysis via a Direct Sparse Solver. The critical condition studied was an angular velocity of 1600RPM and chrome steel balancing masses aligned in the most mass eccentric track locations. The results of the study are displayed in Figures 3-15 and 3-16, illustrating the system has a minimum factor of safety of 1.54 located at the stress concentrations around the bolts in the balancer.
mid-thickness plane. Factor of safety calculations were done according to Von Mises stress criterion. Examining Figure 3-16, it can be seen that the centrifugal loading on the balancer frame is highly dependent upon the balancing mass location. Therefore, a minimum factor of safety of 1.54 is conservative as the heaviest masses are positioned at their most extreme positions.

Figure 3-15. Factor of Safety Plot (Top View)

Figure 3-16. Sectional Factor of Safety Plot at Balancer Mid-thickness Plane
Table 3-4. AERTS Aluminum Plate Balancer Dynamic Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ball-Wall Coefficient of Restitution (HDPE-Aluminum)</td>
<td>0.6144</td>
</tr>
<tr>
<td>Ball-Wall Coefficient of Restitution (Steel-Aluminum)</td>
<td>0.7167</td>
</tr>
<tr>
<td>Rolling Friction Resistance (HDPE-Aluminum)</td>
<td>7.620e-4 m [93]</td>
</tr>
<tr>
<td>Rolling Friction Resistance (Steel-Aluminum)</td>
<td>4.826e-4 m [93]</td>
</tr>
<tr>
<td>Ball-Track Static Coefficient of Friction (HDPE-Aluminum)</td>
<td>varies [94]</td>
</tr>
<tr>
<td>Ball-Track Kinetic Coefficient of Friction (HDPE-Aluminum)</td>
<td>0.2 [95]</td>
</tr>
<tr>
<td>Ball-Track Static Coefficient of Friction (Steel-Aluminum)</td>
<td>varies [96]</td>
</tr>
<tr>
<td>Ball-Track Kinetic Coefficient of Friction (Steel-Aluminum)</td>
<td>0.47 [95]</td>
</tr>
<tr>
<td>Coefficient of Drag</td>
<td>varies [97]</td>
</tr>
</tbody>
</table>

The coefficient of restitution for balancing mass-partition inelastic collisions was determined experimentally for each ball material. The experimental set up is, a Grayscale Hi-Speed camera was mounted above the ceiling of the AERTS chamber with the slip-ring opening providing clearance for the camera lens and LED light arrays providing high-lumen exposure of the subject matter. The shaft was removed and the Aluminum Plate balancer was fixed to the steel support structure. An impulse was applied to a single balancing mass by hand and its motion within its track was recorded by the Hi-Speed video, taken at 1000 Frames Per Second (FPS). The video frames were processed with the MATLAB Image Processing Toolbox. To record the position of the ball in each frame, a circular region of interest was identified and overlaid over the ball, illustrated by Figure 3-17, and its centroid was recorded in terms of image pixels.
The position data in the x and y was plotted separately for each test case approximately 150 milliseconds before and after collision with the partition. A linear regression was calculated to model the position change prior to the collision and after the collision, displayed in Figure 3-18. The slope of each regression yields the velocity vector in pixels s\(^{-1}\) and the magnitude of each velocity vector \((U_{11}, U_{12})\) is calculated. The coefficient of restitution is then calculated according to Equation 3.2. Three test cases were conducted for each balancing mass material. The results are summarized for HDPE and Steel masses in Figures 3-19 and 3-20, as well as Table 3-5.

\[
C_{Rp} = \frac{U_{12}}{U_{11}} \tag{3.2}
\]
Figure 3-18. HDPE-Al COR Test 2: (Top) X Position Plot, (Bottom) Y Position Plot
Figure 3-19. HPDE-Aluminum COR Results Summary

Figure 3-20. Steel-Aluminum COR Results Summary
<table>
<thead>
<tr>
<th>Material</th>
<th>Mean COR</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>High-Density Polyethylene</td>
<td>0.6144</td>
<td>7.943%</td>
</tr>
<tr>
<td>Chrome Steel</td>
<td>0.7167</td>
<td>2.827%</td>
</tr>
</tbody>
</table>

Constants for rolling friction resistance and kinetic coefficient of friction for both balancing mass materials were obtained in literature [93,95]. The coefficients of static friction for the HDPE and Chrome-Steel masses and the Aluminum track is based on experimental data collected by Benabdallah [94] and Chen and Wang [96] respectively. The data collected by Benabdallah, shown in Figure 3-21, demonstrates the dependence of static friction on the explicit magnitude of normal loading.

![Graph showing the relation of normal load to HDPE-Aluminum coefficient of static friction](image)

*Figure 3-21. Relation of Normal Load to HDPE-Al Static Coefficient of Friction [94]*

The data collected by Chen and Wang, represented in Figure 3-22, demonstrates the relationship between the coefficient of static friction and a dimensionless normal loading value
$(P/P_c)$, where $P$ is the applied normal load and $P_c$ is the critical normal load described by Equation 3.3 for a spherical friction contact. Here, $E^*$ is a function of both material’s Young’s Modulus, $(E_1, E_2)$, and Poisson’s Ratio, $(v_1, v_2)$, described by Equation 3.4, $\sigma_Y$ is the yield strength of the softer material, and $k$ is a function of the Poisson’s Ratio of the softer material defined by Equation 3.5. The critical normal load is defined as the transition point from an elastic contact to an elastoplastic contact. This definition of coefficient of static friction, based on non-dimensional normal load, was implemented into the model to provide robust approximation of static friction for a variety of balancing mass and track materials for future designs.

$$P_c = \frac{4}{3} \left( \frac{r_b}{E^*} \right)^2 \left( \frac{k}{\pi \sigma_Y} \right)^3$$

$$E^* = \left[ \frac{(1-v_1)^2}{E_1} + \frac{(1-v_2)^2}{E_2} \right]^{-1}$$

$$k = 1.295 e^{0.736v}$$

Figure 3-22. Relation of Non-dimensional Normal Load to Steel-Al Coefficient of Static Friction [96]
The coefficient of drag force on the balancing masses within the track fluid is a function of the Reynold’s number \((R_e)\) associated with the flow. However, due to the track width being close to the ball diameter, the effects of tube flow are considered. In addition, the ball coming into contact with the walls effects the drag coefficient. The drag coefficient on a sphere is shown in free flow [98], tube flow [97], and wall flow [99] as a function of \(R_e\) in Figure 3-23. For the model, tube flow is assumed and the fluid in the tracks is assumed to be uncompressed air with properties at Standard Temperature and Pressure. However, the dynamic pressure achieved by the balls in their limited range of motion is assumed to be small. Therefore, drag has a minimal impact on balancing mass motion.

![Figure 3-23. Drag Coefficient and Reynold’s Number Relation in Different Flow Types](image)

3.2.5 Tapered Roller Bearing

To provide the shaft support closer to the rotor plane, the shaft is coupled to a Dodge FCE112R tapered roller bearing unit, shown in Figure 3-24, which is modeled as a source of external damping and stiffness. The external damping provided by the bearing is used in the experiment to reduce transient vibration amplitudes and avoid whirl instability. The bearing
external damping coefficient is a critical parameter to modeling the vibration response. The bearing unit is bolted to an adapter block, which in turn is bolted to the load cell top plate. Therefore, it is assumed that any loads experienced by the bearing unit are transferred to the load cell. The configuration is more clearly visualized if the adapter block is made transparent, displayed in Figure 3-25, illustrating how the fixed bearing sits directly on top of the shaft collar, restraining any axial shaft displacement due to the rotor generating thrust. Annular thrust bearings are placed above and below the shaft collar to reduce friction between the shaft collar and the non-rotating components. The bearing unit and adapter block have the physical properties outlined in Table 3-6.

Figure 3-24. Tapered Roller Bearing Unit

Figure 3-25. Bearing-Shaft Collar Contact
<table>
<thead>
<tr>
<th><strong>Bearing Unit and Adapter Block Physical Properties</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bearing Unit Mass</strong></td>
</tr>
<tr>
<td><strong>Bearing Shaft Location (From L = 0)</strong></td>
</tr>
<tr>
<td><strong>Bearing Housing Material</strong></td>
</tr>
<tr>
<td><strong>Bearing Unit Bore Diameter</strong></td>
</tr>
<tr>
<td><strong>Radial Dynamic Load Capacity</strong> ($P_{Dyn}$)</td>
</tr>
<tr>
<td><strong>Radial Static Load Capacity</strong></td>
</tr>
<tr>
<td><strong>Bearing Unit Maximum RPM</strong></td>
</tr>
<tr>
<td><strong>L10 Fatigue Life Hours</strong></td>
</tr>
<tr>
<td><strong>External Damping Coefficient</strong></td>
</tr>
<tr>
<td><strong>Adapter Block Mass</strong></td>
</tr>
<tr>
<td><strong>Adapter Block Material</strong></td>
</tr>
</tbody>
</table>

The $L_{10}$ fatigue life hours parameter is defined as the number of hours under fatigue loading that 10% of manufactured bearings are expected to fail, and is the preferred bearing life estimate used by The American Bearing Manufacturers Association [101]. The $L_{10}$ life hours for the bearing unit were calculated according to Equation 3.6, assuming a critical dynamic loading ($P_B$) of ±600 lbf at 1600RPM, which are the load cell dynamic load limit and the rotor stand speed limit respectively. Here, $\kappa$ is an empirical constant that is a function of the bearing type, defined as 3.0 for spherical bearings and 10/3 for roller bearings. This analysis was conducted to ensure the bearing unit could operate under the large dynamic loading of a mass eccentric rotor in its supercritical region.

$$L_{10} = \left(\frac{P_{Dyn}}{P_B}\right)^{\kappa} \frac{60\Omega}{10^6}$$  (3.6)
The external damping introduced into the system for the first flexural mode was determined experimentally by recording the shaft impulse response while coupled to the bearing and calculating the damping ratio and damped natural frequency by log-decrement method and spectral analysis respectively. The experimental set up, displayed in Figure 3-26, consists of an optical displacement sensor connected to a LabView Data Acquisition Module. The impulse was applied with a rubber mallet in the direction of the sensor at the tip of the shaft 12 times with a gap of several seconds between impulses to allow the response to fully decay. The raw data from the full test is plotted in Figure 3-27, with the impulse locations highlighted.
The displacement data was divided into twelve 250 millisecond segments, with the applied impulse occurring at the beginning of each segment. Only the damping of first flexural mode is considered in the model. To isolate the first mode impulse response, a low pass filter is applied to each of the displacement data segments using the MATLAB Signal Processing Toolbox. A cutoff frequency of 50 Hz for the low pass filter was specified in order to filter out both the contributions of higher order shaft modes as well as electrical noise, which typically has a frequency of 60 Hz. The effectiveness of the signal filtration is illustrated in Figure 3-28.

Figure 3- 27. Full System Damping Test Displacement Data

Figure 3- 28. Unfiltered and Filtered Displacements Data Comparison
The first several peaks of the filtered signal for each segment are identified and the log decrement method is applied to obtain the system damping ratio. The log decrement method is described by Equation 3.7, where $\zeta$ is the system damping ratio, $A_0$ is the amplitude of the first peak and $A_1$ is the amplitude of the second peak. The damped natural frequency ($\omega_d$) is calculated from the time period observed between the first two peaks ($T_{01}$), as shown in Equation 3.8. The undamped natural frequency ($\omega_n$) is calculated according to Equation 3.9. Finally, the external damping coefficient ($c$) is determined from Equation 3.10, $m_{sys}$ is the assumed system mass equal to the combined mass of the shaft and bearing unit. The simulated system impulse response is compared to the experimental data, illustrated in Figure 3-29.

$$\zeta = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\ln\left(\frac{A_0}{A_1}\right)}\right)^2}} \quad (3.7)$$

$$\omega_d = \frac{2\pi}{T_{01}} \quad (3.8)$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} \quad (3.9)$$

$$c = 2\zeta m_{sys} \omega_n \ast (2\pi) \quad (3.10)$$

A summary of the system damping characterization results is illustrated in Figures 3-30, 3-31, and 3-32, and the mean values for system damping ratio, damped natural frequency and undamped natural frequency are provided in Table 3-7.
Figure 3-29. Simulated and Experimental Response for the 11th Applied Impulse

Figure 3-30. Damping Ratio Results
Figure 3-31. Damped Natural Frequency Results

Figure 3-32. Undamped Natural Frequency Results
Table 3-7. System Damping Characterization Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping Ratio</td>
<td>0.3694</td>
<td>9.762%</td>
</tr>
<tr>
<td>Damped Natural Frequency</td>
<td>22.910 Hz</td>
<td>5.294%</td>
</tr>
<tr>
<td>Undamped Natural Frequency</td>
<td>24.678 Hz</td>
<td>5.577%</td>
</tr>
</tbody>
</table>

3.3 Instrumentation

In preparation for this study, the AERTS facility was instrumented with a 6-axis load cell, a torque sensor, and an RPM sensor. In addition, a hall effect sensor, synchronized strobe lights, and several camera systems were implemented to facilitate the capture of high-quality imagery during test.

3.3.1 Load Cell

An ATI Omega160 load cell, shown in Figure 3-33, recorded the shaft fixed-end reaction forces and moments to hub loading due to imbalance. The load cell was secured to the rotor stand steel support structure and the shaft runs through the load cell bore. The load cell data was recorded for the entire duration of each test by a LabView Data Acquisition System at a sample rate of 1000Hz. The LabView load cell virtual interface, displayed in Figure 3-34, continuously plots the force and moment data, allowing the operator to monitor the state of the rotor system, during transition and steady state operation. This provides an indication of an ice shed event as the amplitude of the in-plane force data will increase due to the imbalance introduced by the shed. Prior to spin-up,
bias vector is calculated and applied to the load cell readings to ensure all data is centered around zero. The force calibration limits for the in-plane and axial forces are ±600 lbf and ±1500 lbf respectively. The moment calibration limit is ±3600 lbf-in. Force or moment measurements that approach these values are criteria for immediate system shutdown.

Figure 3-33. ATI Omega160 Load Cell Technical Drawing [102]

Figure 3-34. LabView Load Cell Front Panel

Real-time Force Data
Real-time Moment Data
3.3.2 Torque Sensor

A Cooper Systems LXT 971 Shaft-to-Shaft torque sensor is mounted between motor shaft and the universal joint that connect to the bottom of the rotor shaft, pictured in Figure 3-35, and is coupled to each. The torque sensor monitors the torque supplied to the rotor shaft by the motor. The motor is set to a constant speed control, through an ABB motor control system, and will provide an increase in torque if forces are introduced that impede rotor motion to preserve rotor angular velocity. During steady-state operation, the measured torque is assumed to be equivalent to the drag on the rotor. Therefore, examining the torque measurements over a period of ice accretion provides valuable data on blade performance degradation. However, during an ice shed event the blade section performance is restored leading to a decrease in the rotor drag and therefore, the measured torque. This provides the operator a quantifiable indication of an ice shed event as well as an estimate of the magnitude of the shed event based on the decrease in measured torque. The torque sensor is connected to a LabView Data Acquisition System and a LabView virtual interface displays the measured torque data. The measured torque is set to zero while the rotor is at rest prior to each test to ensure accurate readings.
Figure 3-35. Instrumentation Configuration Below the Rotor stand
3.3.3 RPM Sensor

An inductive proximity sensor is mounted below the rotor stand close to the motor shaft, highlighted in Figure 3-35, to monitor the rotor RPM during test. A steel gear, with eight equally distributed teeth, is mounted to the shaft in the same plane as the inductive sensing element such that the sensor produces a square wave signal as each gear tooth passes the element. This signal is recorded by a LabView Data Acquisition System and spectral analysis is conducted to determine its time-average frequency. The signal frequency is divided by 8 to provide the rotor angular velocity in revolutions per second, which is then converted to RPM. Because only 8 waves are recorded per revolution, the variability in the time-averaged angular speed is high at lower RPM and becomes more accurate as RPM increases. RPM is recorded alongside the force and torque data at a rate of 1000 Hz and examining how force data changes with RPM allows the transition speed of the system to be determined.

3.3.4 Optical Displacement Sensor

An optical displacement sensor was mounted close to the shaft end, as shown in Figure 3-26, in order to record the damped impulse response of the shaft-bearing unit system. Figure 3-36 provides the sensor calibration curve. The displacement sensor sensitivity in the linear region is 1.196 μm mV⁻¹. The optical displacement sensor was considered to record shaft vibration during rotor stand icing tests. However, the vibrations introduced by the rotor aerodynamics and the icing cloud in the chamber pollutes the shaft vibration signal beyond a point where data obtained from the sensor was considered unreliable.
3.3.5 Hall Effect Sensor and Strobe Light System

A synchronized strobe light system was implemented into the AERTS chamber in order to provide real-time imagery of the blades and accreted ice shapes during testing. The system consists of two Shimpo DT-315A Stroboscopes, pictured in Figure 3-37. The strobes were placed in temperature-controlled boxes, show in Figure 3-38, to facilitate their use in icing conditions without damaging their internal electronics, and attached to adjustable mounts on the ballistic wall.
The strobe illumination is controlled by an input signal on a rising voltage trigger. The input signal was produced by a Hall Effect sensor mounted close to the universal joint below the rotor stand, highlighted in Figure 3-35. A high-strength Neodymium magnet is attached to the universal joint in the same plane as the Hall sensor and produces a 1/rev frequency square wave signal as it passes the sensor. This forced synchronization between rotor speed and the strobe illumination produces a one still image of the rotor per revolution, allowing the observation of the rotor regardless of angular speed. The angular position of the magnet was tuned such that each
strobe light illuminates the blade mid-span, producing an image that captures the rotor scale and the accreted ice shape, illustrated by Figure 3-39.

Figure 3-39. Frame Capture from Icing Characterization Test 15 at 1200RPM

3.3.6 IP Cameras

In order to record the static rotor image produced by the synchronized strobe light system, four Reolink 5 Mega-Pixel Internet Protocol (IP) cameras were mounted inside the AERTS chamber. Two dome-type cameras were mounted to the chamber ceiling, pictured in Figure 3-40, to provide a top view of each rotor blade. Two bullet-type cameras were mounted to the ballistic wall, pictured in Figure 3-41, to provide a right-isometric view of each rotor blade. All four cameras transmit real-time video through ethernet cables to a central gigabit ethernet switch and simultaneous video playback is provided through the Reolink Camera control interface, displayed in Figure 3-42.
Figure 3-40. Ceiling Mounted IP Cameras

Figure 3-41. Wall Mounted IP Cameras
These weatherproof cameras are uniquely suited to providing imagery of the rotor during continuous icing tests. In many cases, the icing cloud is too thick for a standard visible light camera to provide adequate picture. However, each IP camera is equipped with an infrared LED illumination system as well as infrared sensors to provide a clear image of the rotor through the icing cloud. Video recordings of the 1/rev rotor image were collected by all cameras at 30FPS for each test, providing a clear time history of each test and a visual recording of all shed events. For example, Figures 3-43 and 3-44 depict consecutive frames before and after an ice shed event. The individual video frame captures of a shed event can be analyzed to determine ice shed length and the associated timestamps provide a precise measure of the icing time that a shed event occurred.
Figure 3- 43. Frame Capture from Continuous Icing Test 12 Prior to Shed Event

Figure 3- 44. Frame Capture from Continuous Icing Test 12 Immediately Following Shed Event
3.3.7 High-Speed Camera

A Phantom MIRO 310 series grayscale high-speed camera was mounted on the top outside surface of the freezer, with the attached 35 mm lens running through an insulated channel into the chamber that normally contains the slip-ring cables. The camera was mounted outside of the chamber to avoid exposing it to the adverse conditions in the freezer and to provide sufficient field of view to capture the entire top view of the Aluminum Plate Balancer, displayed in Figure 3-45. The camera is able to record at a maximum resolution of 1280x800 pixels and a maximum frame rate of 4100FPS. All high-speed recordings were captured at a frame rate of 3600FPS, providing exactly 180 frames per revolution for a rotor operating at 1200RPM.

![Figure 3-45. High-Speed Camera Field of View](image)

The high-speed camera continuously records footage and stores it within its dynamic memory. This dynamic memory has a predefined time length of footage that is continuously
overwritten, and the length of dynamic footage is determined by the frame rate and allotted memory for the recording. The camera is instructed to retroactively save the footage stored in its dynamic memory to static memory through a trigger system. To trigger the camera, a bayonet Neill-Concelman (BNC) connector is attached to the cameras trigger pin and an electrical impulse must be sent through the BNC. For this configuration, the BNC connector was a part of an open 10 Volt DC circuit that was closed by pressing a button in the AERTS control room. This allows the operator to quickly trigger the camera in response to an event and retroactively record the event.

The purpose of the high-speed camera in this study, is to monitor the position of the balancing masses during moments of interest during testing. These are generally either as the rotor goes through transition or after an ice shed event has occurred. Visualizing the response of the balancing masses to an impulsively applied imbalance is an important indication of balancing device performance. The camera can also be triggered at specific angular speeds to track how the balancing mass positions change with the phase of the system.

In order to record at such high frame rates, the camera exposure time is very small, on the order of microseconds, and without proper lighting the recording will be too dark. To compensate for the lack of exposure, increased illumination of the balancing device was provided by 4 ceiling mounted 45 Watt Bi-Color LED light arrays. However, during icing tests, the illumination does not prevent the icing cloud from obscuring the high-speed camera’s view. Therefore, high-speed video was only recorded for the rotor system during spin-up icing tests, where ice is accreted at a lower RPM for a predefined period of time and then the cloud is shut off and allowed to dissipate before the RPM is increased until either an ice shed event occurs or the RPM limit of the rotor stand is reached.
3.4 Ice Accretion Experimental Methods

To appropriately model the balancer performance in this study, the mass eccentricity of the rotor must be precisely determined. The ice shed mass is a function of the icing conditions the rotor is subjected to and therefore must be precisely characterized and maintained. The method to experimentally measure ice shed mass after a shed event involves stopping the rotor and removing an equivalent ice shape length to mass on a scale. This method is not only time consuming, but introduces multiple sources of variability, namely the removal method and human error, and limits the icing tests to one shed per test. If the ice shed mass could be determined analytically, these issues would be alleviated. This is accomplished by monitoring the icing cloud in the chamber and the rotor ice shedding behavior to implement a predictive analytical model to characterize ice accretion mass on the blade. The purpose of the icing model is to analytically determine the mass imbalance of a rotor after a shed event from the shed length and ice accretion time. This allows the mass eccentricity to be modeled for a rotor that has undergone multiple ice shed events over time, which is more representative of an in-flight icing condition, without the need to top the test and remove ice from the rotor to measure mass after each shed. This section details the experimental methods applied to identify the key icing parameters to develop the analytical ice mass model and its application to the experiments in this study.

3.4.1 Temperature

The ambient temperature in the chamber is a critical parameter for icing as it determines the temperature of the water droplets as they impact the rotor. Droplet temperature is a parameter that affects the type of impact ice regime and the ice adhesion strength, which ultimately determines shedding behavior. It is assumed that there is sufficient resonance time between the droplets exiting
the atomizing nozzle and impacting the rotor that the droplets have an average temperature equal to that of the chamber. The ambient air temperature is monitored by 4 Type J thermocouples distributed around the chamber to ensure the uniformity of chamber temperature. The temperature and is automatically maintained by the freezer-evaporator system that is controlled by the facility’s LabView software, which takes the thermocouple readings as the control input. For this study, ambient temperatures of -12 °C and -8 °C were considered in order to categorize glaze and mixed phase icing regimes in the continuous icing enveloped.

In addition to ambient air temperature, the blade surface temperature is a critical parameter in ice shedding behavior. For this study, the blade surfaces were not assumed to be constantly equal to the ambient temperature due to them having higher thermal inertia and the method for removing ice from the leading edge involving heating the surface with a heat gun. In order to monitor the blade temperature, a TG165 FLIR handheld thermal imager was used to image the blade leading edge prior to rotor spin-up. A thermal image of the blade leading edge for a -12 °C test case is shown in Figure 3-46. A surface temperature tolerance of ±0.5°C is used in this study to categorize a wider range of the continuous icing envelope.
3.4.2 Liquid Water Content

To determine the icing envelope and facilitate the modeling of the ice shape cross section, the effective Liquid Water Content (LWC) was experimentally determined. LWC is the characteristic water-to-air concentration in a two-phase flow and is typically given in units of g/m³. LWC is an important parameter to categorize icing conditions as it directly affects ice accretion rate. As previously mentioned, the chamber LWC is controlled by the number of spray nozzles, airline pressure, and MVD of the droplets in the cloud. One spray condition was considered for this study, utilizing 4 of the ten nozzles in the outer ring of the AERTS spray configuration, illustrated by Figure 3-47, with an airline pressure of 15 psi and a requested MVD of 20μm. Nozzles 1, 3, 5, and 7 were selected to provide relatively even distribution of icing cloud in the chamber. The airline pressure and MVD spray conditions were specified in the facility LabView software with the MVD calculated according to the atomizing nozzle calibration curve, Figure 3-6.
The three parameters described above determine the icing cloud conditions and therefore the chamber LWC. However, the effective LWC, which is the LWC that is “seen” by the rotor leading edge, is dependent on the blade geometry and local aerodynamics. These factors determine the collection efficiency of the blade cross section in the mixed phase flow conditions. Collection efficiency is defined as the proportion of the water droplets in rotor plane freestream flow that will impact the blade surface. Generally, thinner blade cross sections exhibit higher collection efficiencies. Bragg conducted similarity analysis on droplet trajectories and was able to categorize collection efficiency of symmetric blades as a function of their modified inertia parameter, $K_\theta$, in the flow, illustrated in Figure 3-47 [103]. $K_\theta$ is described by Bragg in Equation 3.11 as function of the Reynold’s Number ($R_e$) and droplet inertia parameter ($K$), where $\beta$ and $H$ are empirically determined constants based on the expected range of Reynold’s Numbers. $K$ is defined by Equation 3.12, where $\sigma$ is the droplet density, $\delta$ is the droplet diameter, $U_\infty$ is the freestream velocity, $c_h$ is the characteristic length, usually specified as the airfoil chord, and $\mu_\text{air}$ is the absolute air viscosity.
An iterative algorithm to experimentally determine effective LWC for symmetric airfoils was developed by Han and Palacios [104]. The algorithm takes inputs of ambient temperature, MVD, ice accretion time, freestream velocity, leading edge thickness and the measured ice thickness at the flow stagnation point. The model attempts to match the predicted ice thickness determined from the input conditions, Bragg’s droplet trajectory analysis, and varying the ice shape freezing fraction. The freezing fraction is defined as the fraction of water flux entering a control volume, in this case the blade section stagnation point, that freezes within it. The algorithm’s iterative process is outlined in Figure 3-48. First the collection efficiency is determined from input conditions using Bragg’s similarity analysis, to determine the proportion of water droplets impacting the surface. From here, the freezing fraction and LWC is iterated, assuming linear ice accretion with time, until the predicted thickness converges with the measured thickness. The assumption of constant ice accretion rate limits the accuracy of the model over for longer icing time. The algorithm assumes that the symmetric airfoil leading edge interacts with the flow according to its equivalent cylinder, the radius of which, \( r_{LE} \), is calculated from Equation 3.13, where \( c \) is the airfoil chord and \( t_{AF} \), the airfoil max thickness. However, as ice accretes on the airfoil the geometry of the cross-section changes, which changes the collection efficiency, and by extension, the ice accretion rate. Therefore, the icing time for all LWC experiments is set below 2 minutes to preserve the airfoil geometry throughout the ice accretion period. The algorithm also assumes a constant icing cloud presence experienced by the airfoil. However, the icing cloud in the
AERTS chamber takes approximately 10 seconds to stabilize, during which MVD can vary from 10 to 120μm.

\[ r_{LE} = 1.1019 \frac{c_{\max}}{c} \] (3.13)

**Figure 3-48. Effective LWC Iterative Algorithm Flowchart [104]**

The optimal effective LWC characterization test conditions for this study were determined to be an accretion time of 1 minute at an ambient temperature of -20°C. These conditions ensure a fully rime regime ice shape, which conforms closer to the original geometry, and a freezing fraction of 1. Following the accretion period at 1200RPM, three sectional thickness measurements were taken on each blade stagnation point at the 50%, 75%, and 90% span locations. This provides insight into the spanwise effective LWC distribution and the extent of the error associated with assuming uniform ice shape cross section along the blade span. The stagnation thicknesses were measured by cutting a thin channel in the ice with a hotplate at each location and obtaining the
thickness with a set of calipers cooled to chamber ambient temperature. A total of three LWC tests were conducted providing a total of 18 thickness measurements. A summary of the thickness measurements is provided in Table 3-8 and Figure 3-49 illustrates that the thickness is larger at outboard span locations. The measured thicknesses and icing conditions were input into the LWC algorithm and the associated LWC values were obtained, given in Table 3-8. Effective LWC varied greatly between the midspan and outboard sections, which can be accounted for by the spanwise variation in flow conditions. Figure 3-50 illustrates that the outboard sections have similar effective LWC values, while the midspan is much higher. Therefore, the assumption of constant ice shape cross section is valid for the outer portion of the blade but degrades towards the inboard section. Examining the FAA Appendix C icing envelope, shown in Figure 3-51, the obtained LWC values at the -8 °C and -12°C cases place the icing conditions in this study in the continuous icing envelope. Therefore, the observed ice shapes in this study are representative of those observed in flight by a helicopter in continuous icing conditions.

<table>
<thead>
<tr>
<th>Test #</th>
<th>Span Location</th>
<th>Avg. Stagnation Thickness (mm)</th>
<th>LWC (g/m³)</th>
<th>Average LWC (g/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50%</td>
<td>2.0828</td>
<td>0.436</td>
<td>0.3317</td>
</tr>
<tr>
<td></td>
<td>75%</td>
<td>2.2098</td>
<td>0.297</td>
<td></td>
</tr>
<tr>
<td></td>
<td>90%</td>
<td>2.3749</td>
<td>0.262</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>50%</td>
<td>2.1336</td>
<td>0.447</td>
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<td>2.1844</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>90%</td>
<td>2.3749</td>
<td>0.262</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>50%</td>
<td>2.0193</td>
<td>0.423</td>
<td>0.3317</td>
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<td></td>
<td>75%</td>
<td>2.2225</td>
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</tr>
<tr>
<td></td>
<td>90%</td>
<td>2.4765</td>
<td>0.273</td>
<td></td>
</tr>
</tbody>
</table>
Figure 3-49. Stagnation Thickness Measurements

Figure 3-50. Effective LWC Results
3.4.3 Ice Shape Cross Section

Characterization of the icing conditions and airfoil geometry facilitates the use of the LEWICE 3.0 software to model the ice accretion shape over time. This provides a powerful analytical tool for determining the volume of ice that was shed from the blade assuming the ice shape cross section is uniform for the length of the shed mass. The conditions provided in Table 3-9 were input into LEWICE 3.0 and compared to an experimental ice shape obtained under the same conditions. The conditions correspond to the ice shape cross section at the blade tip with a rotor angular velocity of 1200RPM. The experimental ice shape was digitized from a photo taken of the ice shape, shown in Figure 3-52, with the blade D-block thickness used as the in-plane scale. The image is processed using the MATLAB Image Processing Toolbox to identify the ice shape region by using the sharp contours between the ice and the background. The ice shapes were plotted on
top of each other in non-dimensionalized coordinates with the origin at the leading-edge stagnation point, displayed in Figure 3-53.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient Temperature</td>
<td>-12.0 °C</td>
</tr>
<tr>
<td>Blade Surface Temperature</td>
<td>-12.2 °C</td>
</tr>
<tr>
<td>LWC</td>
<td>0.2657 g/m³</td>
</tr>
<tr>
<td>MVD</td>
<td>20 μm</td>
</tr>
<tr>
<td>Span Location Radius</td>
<td>54 in. (1.3716 m)</td>
</tr>
<tr>
<td>Rotor Angular Velocity</td>
<td>1200 RPM</td>
</tr>
<tr>
<td>Freestream Tip Velocity</td>
<td>159.593 m/s</td>
</tr>
<tr>
<td>Accretion Time</td>
<td>280 sec</td>
</tr>
</tbody>
</table>

Figure 3-52. Ice Shape Digitization: (Left) Original Image, (Right) Close-up of Digitized Ice shape Region
Figure 3-53. Experimental Ice Shape and LEWICE Prediction Comparison

The major differences between the LEWICE prediction and the experimentally obtained ice shape can be explained by the assumptions made by the LEWICE 3.0 ice accretion algorithm. To determine the collection efficiency, LEWICE solves the flow field around the body by employing the Douglas Hess-Smith 2-D panel method [67]. This method assumes potential flow around the airfoil, which is both irrotational and incompressible. However, due to the blunt body geometry of the ice shape introducing trailing vortices to the flow and the rotor freestream velocity operating in the compressible flow region at the blade tip ($M=0.4923$). Therefore, the potential flow model assumptions breakdown quickly as the ice shape degrades the leading edge causing LEWICE to calculate ice impingement limits that are much more extreme than those observed in the experimental ice shape. However, the ice thickness at the stagnation point is similar between the two ice shapes, indicating that the ice accretion rate prediction at the stagnation point is accurate. The higher degree of runback observed in the LEWICE prediction could be explained by differences in the assumed heat transfer coefficients for an Aluminum substrate in the LEWICE model and those that correspond to the actual blade surface.
Despite differences in the ice shape cross section, the critical parameter for determining ice shed volume is the cross-sectional area which is a function of the blade collection efficiency over time. The predicted ice shape cross sectional area was determined from numerical integration of the LEWICE output and scaled into cm² using the chord length. The area of the digitized ice shape, given in pixels was provided by the region properties and scaled into cm² using the image reference length. Comparison of the obtained cross-sectional areas determines if LEWICE is a valid tool for determining ice shed volume in this study.

3.4.4 Ice Shed Mass

Provided LEWICE reliably determines the ice shed volume for an arbitrary ice shed event, accurate prediction of the ice density will yield an analytically obtained value for the ice shed mass, which is needed to model the system mass eccentricity. The ice density ($\rho_{\text{ice}}$) model, given in g/cm³ by Equation 3.14, was empirically derived by Jones in 1990 as a function of nondimensional icing parameters proportional to the convective heating and heat flux due to droplet freezing ($\pi_C$), the droplet inertia parameter ($\pi_K$), and the equivalent cylinder Reynold’s number ($\pi_\phi$). The nondimensional heat transfer parameter is described by Equation 3.15, where $k_a$ is the molecular conductivity of air, which is a function of ambient temperature ($T_a$) given by Equation 3.16, and $L_f$ is the water latent heat of fusion. The nondimensional flow parameter is given by Equation 3.17. The nondimensional droplet inertia parameter is defined by equation 3.18.

$$\rho_{\text{ice}} = 0.249 - 0.084 \ln(\pi_C) - 0.00624 \left(\ln(\pi_\phi)\right)^2 + 0.135 \ln(\pi_K) + 0.0185 \ln(\pi_K)\ln(\pi_\phi) - 0.0339(\ln(\pi_K))^2 \ [g/cm^3] \tag{3.14}$$

$$\pi_C = \frac{k_a(T_a)}{2\tau L E LWC U_\infty L_f} \tag{3.15}$$
\[
k_a(T_a) = 4.186 \times 10^{-7}(573 + 1.8T_a) \left[ \text{J cm}^{-1} \text{s}^{-1} \text{C}^{-1} \right]
\]  

(3.16)

\[
\pi_\phi = \frac{18\rho_{air}^2 r_{LE} U_\infty}{\sigma \mu_{air}}
\]

(3.17)

\[
\pi_K = \frac{\sigma \delta^2 U_\infty}{18 r_{LE} \mu_{air}}
\]

(3.18)

The Jones model also has expanded forms that consider ice accretion surface heat transfer, water evaporative diffusivity, and water vapor momentum diffusivity, but they were deemed to either have too little an effect on predicted density or could not be appropriately categorized for this study. The simplified model presented above accounts for 90% of the variability in ice density, and therefore, should provide a sufficient approximation of accreted ice density for this study.

Ice shed mass characterization tests were conducted for the 4-nozzle spray condition at \(-8^\circ\text{C}\) and \(-12^\circ\text{C}\). The ice mass of the first observed shedding event was obtained by immediately shutting down the rotor and icing cloud once a shed event was observed to preserve the ice shape to be analyzed once the rotor is static. The ice shed length was measured and an equivalent ice shape was sectioned from the intact ice shape on the opposite blade by applying a cut with a hot plate, illustrated in Figure 3-54. Because the cloud is assumed to be uniformly distributed throughout the chamber, the mass of the section ice shape is assumed to be equivalent to the ice shed mass. The ice section was removed from the blade surface by gently applying heat to the ice shape until it debonded from the leading edge and placed on a 200g scale to be massed. Any water droplets left behind were removed with a pipette to ensure they were included in the mass measurement.
With the precise control over the icing conditions in the AERTS chamber, the ice shed volume and density can be obtained analytically with the methods described above to yield the ice shed mass. The analytical ice masses were compared to those obtained experimentally to validate the LEWICE-Jones model method of determining the ice shed mass for an arbitrary ice shed event in known icing conditions.

3.4.5 The Continuous Multi-Shed Icing Test

With an experimentally validated model for ice shed mass, the rotor mass eccentricity can be modeled provided the icing conditions and ice shed length are known. The synchronized strobe-camera system is capable of determining the ice shed length on either blade from digitizing the ice shed from a recorded video frame using the in-plane scale painted on the blades. The total ice accretion time at each shed event can be determined quantitatively from the torque data or visually by examining the strobe light video frame timestamps. The necessary icing conditions are known from the chamber instrumentation or the experimental methods described in this section. Therefore, an experiment with a rotor in the AERTS chamber subjected to multiple shed events over time can be reliably modeled. The purpose of such a test is to evaluate the performance of a passive balancing device to mitigate shaft vibrations in a more realistic in-flight icing scenario.
The procedure for implementing the ice shed mass analytical model during a continuous icing tests is illustrated by Figure 3-55. Prior to testing, the MVD, Temperature, LWC, and Freestream Velocity are all known. The rotor is spun to the steady state super critical angular velocity and the icing cloud is dispersed into the chamber. The rotor RPM and icing cloud are maintained for 5 minutes. For each observed ice shed event within those 5 minutes, the corresponding ice accretion time is determined and entered into LEWICE to yield the ice shape cross section at shed. The portion of the blade that shed ice is identified from the strobe light video, yielding the shed length and distance from the rotational axis. The ice density is approximated by the Jones Model and multiplied by the cross-sectional area and length of the ice shed to yield the mass. The mass imbalance is approximated as a point mass at the mid span of the blade section that shed its ice shape. The calculated imbalance is assumed to be constant until another ice shed event occurs. The two blade leading edges are assumed to be diametrically opposed. Therefore, if the subsequent ice shed occurs on the opposite blade, which is typically the case. The corresponding mass eccentricity is subtracted from the calculated system imbalance due to the prior shed. If the subsequent shed occurs on the same blade, the resulting mass eccentricity is added to the system mass eccentricity. It should be noted that the same section of blade can shed multiple times in a single test. This is modeled by assuming that the ice accretion time resets to zero for the corresponding blade section at time of shed.

The system mass imbalance is recalculated following each ice shed event according to equation 3.19 for the \( i \)-th ice shed of the test. The system imbalance is described as the summation of all mass imbalance contributions of the previous ice sheds and the most immediate shed where \( sgn(B) \), defined by equation 3.20, is a function of the blade on which the ice was removed and determines the directionality of the \( R_{OM_i} \) vector.

\[
(Imb_{sys})_i = \sum_{k=1}^{i-1} (Imb)_k + sgn(B)m_iR_{OM_i}; \quad sgn(B) = \begin{cases} 1 & \text{for Blade 1} \\ -1 & \text{for Blade 2} \end{cases}
\]

(3.19, 3.20)
Test Conditions Specified:
MVD, Nozzle Configuration, Ambient Temperature, Steady State RPM, Blade Geometry

LWC:
Experimentally determined from specified conditions prior to test and measured ice thickness

Blade Surface Temperature:
Measured by FLIR prior to test

Test Start

Cloud On

LEWICE:
Inputs: MVD, LWC, Temperature, Freestream Velocity, Accretion Time, Airfoil Shape
Output: Cross Sectional Area

Jones Model:
Inputs: MVD, LWC, Temperature, Freestream Velocity, Airfoil Shape
Output: Ice Density

Ice Shed Event Strobe Data

Accretion Time

Shed Midspan
Radial Distance

CS Area

Shed Length

Ice Density

Ice Volume

Ice Shed Mass

Update System Imbalance:
\[ (Imb_{sys})_l = \sum_{k=1}^{l-1} (Imb)_k + sgn(B)\cdot m_ir_{OM}\]

Mass Imbalance

Accretion Time Limit Reached

End Test

Figure 3-55. Continuous Multi-Shed Icing Test Flow Chart
3.5 Test Matrix

Experiments were carried out without a passive balancing device to characterize the ice shed behavior of the blades in this study to determine the critical imbalance condition and to validate the LEWICE-Jones analytical method of obtaining ice shed mass. The system loads in the non-balanced configuration were recorded during spin-up to 1200RPM for 3 tests to determine the critical RPM where transition to the supercritical region occurs and compare to the mathematical model prediction. Two continuous icing tests, with an accretion time of 5 minutes, were conducted to observe the fixed frame loading of the unbalanced system after undergoing multiple shed events. Following the results of the unbalanced configuration experiments, the Aluminum Plate Balancer was installed and four continuous icing tests were conducted at -12 °C and -8 °C, stopping after the first shed event. The goal of these tests is to determine the balancer performance in response to a single asymmetric ice shed events in the continuous icing envelope. Two continuous icing tests were conducted, in order to evaluate balancer performance in a more realistic in-flight icing condition, where multiple asymmetric ice shed events will occur consecutively at varying mass imbalance magnitude. The experimental fixed frame loading results were compared to model predictions to validate the mathematical model of the AERTS rotor stand in this configuration. To capture the response of the balancing masses to asymmetric ice shed, ice was accreted at low speed for 5 minutes, the cloud was turned off, and then the rpm was increased until a shed event occurred. High-speed footage was captured of the balancing masses for two ice shed events that occurred unobstructed by the icing cloud. A relatively low number of tests were planned for this balancer design as its results were used to inform the design process for subsequent design iterations. A summary of the intended test matrix for this test entry is provided in Table 3-10.
Table 3-10. Initial Passive Balancer Configuration Test Matrix

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Test #</th>
<th>Rotor RPM</th>
<th>Icing Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice Shed Characterization</td>
<td>1:8</td>
<td>1200 RPM</td>
<td>Temperature: -8, -12 °C LWC: 0.3326 g/m³ MVD: 20 μm</td>
</tr>
<tr>
<td>(Unbalanced Configuration)</td>
<td>(1:4 @ -12 °C)</td>
<td></td>
<td>Accretion Time: Continuous until first shed</td>
</tr>
<tr>
<td></td>
<td>(5:8 @ -8 °C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transition Region Determination</td>
<td>9:11</td>
<td>1200 RPM</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Continuous Icing</td>
<td>12:13</td>
<td>1200 RPM</td>
<td>Temperature: -8, -12 °C LWC: 0.3326 g/m³ MVD: 20 μm</td>
</tr>
<tr>
<td>(Unbalanced Configuration)</td>
<td>(12 @ -12 °C)</td>
<td></td>
<td>Accretion Time: Multiple Shed Events</td>
</tr>
<tr>
<td></td>
<td>(13 @ -8 °C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ice Shed Characterization</td>
<td>14:21</td>
<td>1200 RPM</td>
<td>Temperature: -8, -12 °C LWC: 0.3326 g/m³ MVD: 20 μm</td>
</tr>
<tr>
<td>(Balancer Configuration)</td>
<td>(14:17 @ -12 °C)</td>
<td></td>
<td>Accretion Time: Continuous until first shed</td>
</tr>
<tr>
<td></td>
<td>(18:21 @ -8 °C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Continuous Icing</td>
<td>22:23</td>
<td>1200 RPM</td>
<td>Temperature: -8, -12 °C LWC: 0.3326 g/m³ MVD: 20 μm</td>
</tr>
<tr>
<td>(Balancer Configuration)</td>
<td>(22 @ -12 °C)</td>
<td></td>
<td>Accretion Time: Multiple Shed Events</td>
</tr>
<tr>
<td></td>
<td>(23 @ -8 °C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Balancing Mass Ice Shed Response</td>
<td>24:25</td>
<td>500 RPM,</td>
<td>Temperature: -8, -12 °C LWC: 0.3326 g/m³ MVD: 20 μm</td>
</tr>
<tr>
<td></td>
<td>(24 @ -12 °C)</td>
<td>shed</td>
<td>Accretion Time: 5 min. Ice Shed on Spin Up</td>
</tr>
<tr>
<td></td>
<td>(25 @ -8 °C)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 4 | Initial Passive Balancer Results and Discussion

Experimental results and model predictions obtained for the Unbalanced Rotor Configuration and the Initial Passive Balancer Configuration. The balancer performance is evaluated based on the steady state fixed frame loading following an asymmetric ice shed event. The model prediction for steady state loading is compared to experimental results for a given imbalance to evaluate the model accuracy.

4.1 Analytical Ice Shed Mass Validation

First, to validate the analytical method of determining ice shed mass, 4 tests were conducted at the nominal spray condition described in Table 3-10 for each of the two temperatures. Each test case begins with a clean rotor blade with the leading edge entirely free of ice and the blade surface temperature is obtained from FLIR imagery. The rotor is accelerated to 1200 RPM and the icing cloud is introduced into the chamber until an ice shed event is recorded. The accretion time between the cloud introduction and the first ice shed is recorded and the mass of the ice shed event is obtained from an equivalent length of ice from the intact ice shape of the opposite rotor blade. In cases where both blades experience ice shed events simultaneously, the ice shed mass for each side is obtained by removing an equivalent length of ice from the inboard section of the corresponding rotor. The experimental accretion time and specified icing parameters are input into the LEWICE 3.0 software to obtain an analytical ice shape cross section at the blade tip to be compared to the experimental ice shape digitized from a cross sectional image. The area of each cross section is determined and compared to assess the accuracy of the LEWICE software. The analytically obtained cross section area is multiplied by the experimentally measure ice shed length.
to obtain the ice shed volume and the ice density is determined from the ice accretion conditions using the Jones model (Equation 3.14). The analytical ice shed mass is compared to the experimental ice shed mass to determine whether this is an appropriate methodology to approximate the ice shed mass for the remainder of the study.

The cross-sectional results for the four -12 °C test cases are displayed in Figure 4-1. In the event of a simultaneously shed on both rotor blades, no experimental ice shape cross section was obtained, however, two experimental ice shed mass values were obtained. The experimental ice shape cross sections and shed lengths were determined from photos of the ice shape for each case, pictured in Figures 4-2 and 4-3 respectively. The results of the analytical and experimental ice shed characterization are summarized in Tables 4-1 and 4-2.

![Figure 4-1. Ice Shed Characterization Test 1 Cross Section Comparison (-12 °C)](image-url)
Figure 4-2. Ice Shed Characterization Test 2 Cross Section Comparison (-12 °C)

Figure 4-3. Ice Shed Characterization Test 3 Cross Section Comparison (-12 °C)
Figure 4-4. Ice Shed Characterization -12 °C Experimental Cross Section Image: (a) Test 1, (b) Test 2, (c) Test 3
Figure 4-5. Ice Shed Characterization -12 °C Experimental Shed Length Image: (a) Test 1, (b) Test 2, (c) Test 3, (d) Test 4 Blade 1, (e) Test 4 Blade 2
Table 4-1. -12 °C Cross Sectional Results Summary

<table>
<thead>
<tr>
<th>Test #</th>
<th>Shed Time (s)</th>
<th>Shed Length (cm)</th>
<th>Exp. CS Area (cm²)</th>
<th>LEWICE CS Area (cm²)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>246.96</td>
<td>17.780</td>
<td>0.953</td>
<td>1.019</td>
<td>6.97%</td>
</tr>
<tr>
<td>2</td>
<td>331.2</td>
<td>17.267</td>
<td>1.290</td>
<td>1.341</td>
<td>3.95%</td>
</tr>
<tr>
<td>3</td>
<td>280</td>
<td>14.046</td>
<td>1.212</td>
<td>1.163</td>
<td>3.99%</td>
</tr>
<tr>
<td>4a</td>
<td>241.02</td>
<td>32.652</td>
<td>-</td>
<td>1.017</td>
<td>-</td>
</tr>
<tr>
<td>4b</td>
<td>241.02</td>
<td>35.037</td>
<td>-</td>
<td>1.017</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Average</td>
<td>4.97%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Standard Deviation</td>
<td>1.73%</td>
</tr>
</tbody>
</table>

Table 4-2. -12 °C Ice Shed Mass Results Summary

<table>
<thead>
<tr>
<th>Test #</th>
<th>Temperature (°C)</th>
<th>Analytical Density (g/cm³)</th>
<th>Exp. Ice Mass (g)</th>
<th>Analytical Ice Mass (g)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-12.4</td>
<td>0.7420</td>
<td>11.7</td>
<td>13.448</td>
<td>14.94%</td>
</tr>
<tr>
<td>2</td>
<td>-11.8</td>
<td>0.7415</td>
<td>15.3</td>
<td>17.165</td>
<td>12.19%</td>
</tr>
<tr>
<td>3</td>
<td>-12.2</td>
<td>0.7418</td>
<td>12.5</td>
<td>12.120</td>
<td>3.04%</td>
</tr>
<tr>
<td>4a</td>
<td>-12.4</td>
<td>0.7420</td>
<td>19.4</td>
<td>24.650</td>
<td>27.06%</td>
</tr>
<tr>
<td>4b</td>
<td>-12.4</td>
<td>0.7420</td>
<td>21.2</td>
<td>26.450</td>
<td>24.7645%</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td>16.40%</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td></td>
<td></td>
<td></td>
<td>9.77%</td>
</tr>
</tbody>
</table>

Examining the cross-section comparisons in Figures 4-1 through 4-3, the impingement limits of the LEWICE ice shape prediction has much more extreme blade surface impingement limits than those observed in the experiment due to the assumptions of the LEWICE flow solver. This generally leads to an overestimation of the cross-sectional area. In addition, the cross section of interest is the ice core of the ice shape as the feathers above and below the core are not shed from the blade, as shown in Figure 4-5, feathers are still present on the upper surface of the blade. Despite
the consistent overestimation, the magnitude of the difference between the experimental and analytical area values is within an acceptable range considering the error which may be introduced in the image scaling for obtaining the experimental area.

The analytical density values are consistent between the tests and is within the range of reasonable values observed for mixed regime ice in literature. In 2010, Palacios et al. observed an ice density of 0.743 g/cm³ [81]. Generally, the analytical method continues to overestimate the ice shed mass. However, with a final average percent difference value of 16.40% the analytical method of determining ice mass is fairly accurate. There are also multiple sources of error introduced in the experimental mass measurement as some water that melts from the ice shape will be lost to evaporation or dripping during the debonding process. In addition, the assumption of constant spanwise ice shape cross section begins to breakdown towards the inboard portion of the blade where the experimental ice shape is obtained for a simultaneous shed event. This accounts for the increase in the percent difference between the first three tests and the fourth, where both blades shed. Therefore, the error associated an overprediction is conservative as some ice shed mass was likely lost in the experimental mass measurement, or the inboard sectioned ice sample likely had a lesser ice thickness than that of the blade tip.

The ice shed cross section comparison results for the -8 °C test cases are displayed in Figures 4-6 and 4-7. Test cases 5 and 6 both experienced simultaneous ice shed events on both blades, therefore, only 2 experimental ice shed cross sections were obtained. The ice shapes are rougher and adhere less to the blade geometry, indicating that ice accreted at this temperature is characteristic of glaze ice. The differences in the heat transfer model employed by LEWICE and the heat transfer experienced by the physical rotor are more apparent when comparing the ice shapes. The LEWICE predicted shapes have much further runback of water along the body, leading to an underprediction of the ice stagnation thickness.
Figure 4-6. Ice Shed Characterization Test 7 Cross Section Comparison (-8 °C)

Figure 4-7. Ice Shed Characterization Test 8 Cross Section Comparison (-8 °C)
Figure 4-8. Ice Shed Characterization -8°C Experimental Shed Length Image: (a) Test 5 Blade 1, (b) Test 5 Blade 2, (c) Test 6 Blade 2, (d) Test 6 Blade 1, (e) Test 7, (f) Test 8
The results for the glaze regime ice shape characterization test cases is summarized in Tables 4-3 and 4-4. Photographs of the ice shape cross sections at the rotor tip and the shed lengths are displayed in Figures 4-9 and 4-8 respectively. The observed shed times and shed masses are much lower than those seen in the mixed regime, which follows observed trends in literature where ice adhesion strength, which is proportional to ice shed mass, decreases with temperature [14]. The predicted areas are less accurate than those in the mixed regime, which is to be expected with differences in shape and therefore collection efficiency. However, the average percent difference is within a reasonable range at 13.95%. Similar to the mixed regime tests, the analytical model tends to overpredict the ice shed mass. The average percent difference was slightly higher than the mixed regime at 16.98%, but is still a conservative estimate.

The results validate that the analytical ice accretion model provides a more conservative and reliable estimate of ice shed mass than stopping the test after each shed event and physically removing ice from the blade. By applying this model, the ice shed mass can be easily obtained from the ice shed length and accretion time, facilitating the modeling of tests with multiple ice sheds without stopping the rotor to evaluate the balancer performance in conditions more representative of an in-flight icing condition.
Table 4-3. -8 ℃ Cross Sectional Results Summary

<table>
<thead>
<tr>
<th>Test #</th>
<th>Shed Time (s)</th>
<th>Shed Length (cm)</th>
<th>Exp. CS Area (cm²)</th>
<th>LEWICE CS Area (cm²)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>5a</td>
<td>132.3</td>
<td>14.859</td>
<td>-</td>
<td>0.6381</td>
<td>-</td>
</tr>
<tr>
<td>5b</td>
<td>132.3</td>
<td>14.199</td>
<td>-</td>
<td>0.6381</td>
<td>-</td>
</tr>
<tr>
<td>6a</td>
<td>99.4</td>
<td>18.514</td>
<td>-</td>
<td>0.3839</td>
<td>-</td>
</tr>
<tr>
<td>6b</td>
<td>99.4</td>
<td>22.946</td>
<td>-</td>
<td>0.3839</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>134.4</td>
<td>14.465</td>
<td>0.7258</td>
<td>0.6503</td>
<td>10.4%</td>
</tr>
<tr>
<td>8</td>
<td>140.6</td>
<td>15.547</td>
<td>0.8039</td>
<td>0.6632</td>
<td>17.5%</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>13.95%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.02%</td>
</tr>
</tbody>
</table>

Table 4-4. -8 ℃ Ice Shed Mass Results Summary

<table>
<thead>
<tr>
<th>Test #</th>
<th>Temperature (℃)</th>
<th>Analytical Density (g/cm³)</th>
<th>Exp. Ice Mass (g)</th>
<th>Analytical Ice Mass (g)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>5a</td>
<td>-8.6</td>
<td>0.8713</td>
<td>6.4</td>
<td>8.261</td>
<td>29.07%</td>
</tr>
<tr>
<td>5b</td>
<td>-8.6</td>
<td>0.8713</td>
<td>6.1</td>
<td>7.893</td>
<td>29.40%</td>
</tr>
<tr>
<td>6a</td>
<td>-7.9</td>
<td>0.8707</td>
<td>5.9</td>
<td>6.188</td>
<td>4.88%</td>
</tr>
<tr>
<td>6b</td>
<td>-7.9</td>
<td>0.8707</td>
<td>6.9</td>
<td>7.669</td>
<td>11.15%</td>
</tr>
<tr>
<td>7</td>
<td>-8.3</td>
<td>0.871</td>
<td>7.3</td>
<td>8.194</td>
<td>12.24%</td>
</tr>
<tr>
<td>8</td>
<td>-8</td>
<td>0.8708</td>
<td>7.8</td>
<td>8.979</td>
<td>15.12%</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16.98%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10.07%</td>
</tr>
</tbody>
</table>
4.2 Unbalanced Rotor Configuration Results

The loads obtained from the ice shed characterization tests were analyzed in order to examine the system response when mass eccentricity is introduced without a passive balancing system. Continuous icing tests were conducted in this configuration to determine the system fixed frame loading response to multiple ice shed events. These results serve as a baseline to compare the effect of passive balancing devices on shaft vibrations in the other tested configurations.

First, the system critical speed transition region was characterized to ensure the system is operating in its supercritical region at the shaft speed used for steady state icing tests. The first flexural natural frequency of the system in the non-balanced configuration was measured experimentally from the system loading profile compared to the measured RPM, given in Figure 4-10. Three measurements were taken yielding a mean critical frequency of 14.817 Hz, corresponding to a rotor speed of 889 RPM. The frequency response of the simulated system is provided in Figure 4-11. The first flexural resonance frequency of the simulated system was calculated to be 15.133 Hz, corresponding to a rotor speed of 908 RPM and a 2.1% difference between the experimentally measured value. At the speed selected for steady state supercritical operation, 1200 RPM, the phase of the simulated system is -149.3 degrees.

Figure 4-10. Unbalanced Configuration Loading Profile During Spin-Up (Test 10)
4.2.1 Fixed Frame Loading Amplitude

The measured, time averaged fixed frame loading, in the X and Y-direction defined by the load cell, following the 14 observed ice shed events is displayed in Figure 4-12, accompanied by the corresponding model predictions. As predicted, the fixed frame loading in both the X and Y directions measured by the load cell increases with the mass imbalance of the system. The mass imbalance due to asymmetric ice shed ranged in magnitude from 2580 to 16789 g-mm, with highest levels of imbalance producing fixed frame loading near the 600 lbf. limit of the system.

The experimental values and comparison to model predictions for the ice shed characterization tests and continuous icing tests are summarized in Tables 4-5 and 4-6 respectively. No values were considered for the simultaneous ice shed events on both blades in tests 4 through 6, as the resulting imbalance resulted in insignificant changes to system loading. The measured
loading in the X and Y direction was averaged to a single value and compared to the model prediction for fixed frame loading under the measured mass eccentricity and rotor speed of the experiment. A positive imbalance indicates ice removed from blade 1 while a negative imbalance corresponds to an ice shed on blade 2. A positive percent difference indicates that the predicted value was larger than the corresponding experimental value, while a negative indicates a predicted value lower than the experimental value. The model was most inaccurate for lower levels of imbalance, overpredicting the value in most of these cases. This could be explained by unaccounted for imbalance in the system on the same magnitude as the relatively small ice shed mass. Overall, the average magnitude of the percent difference between model predictions and experimental observations for the fixed frame loading response to each ice shed was 18.7%, with the model underpredicting the measured loading.

Figure 4-12. Unbalanced Configuration Experimental and Modeled Loading Results
Table 4-5. Unbalanced Configuration Ice Shed Characterization Results Summary

<table>
<thead>
<tr>
<th>Test #</th>
<th>Ice Shed Mass (g)</th>
<th>ROM (mm)</th>
<th>System Imbalance (g-mm)</th>
<th>Avg. Fixed Frame Load (lbf.)</th>
<th>Model Prediction % Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>1092.2</td>
<td>13819</td>
<td>485.66</td>
<td>-8.503</td>
</tr>
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<td>2</td>
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<td>-16789</td>
<td>496.93</td>
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<td>3</td>
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<td>4</td>
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<td>-</td>
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<td>-</td>
</tr>
<tr>
<td>5</td>
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<tr>
<td>6</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
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<td>-8215</td>
<td>328.14</td>
<td>-21.356</td>
</tr>
<tr>
<td>8</td>
<td>7.8</td>
<td>1114.5</td>
<td>8693</td>
<td>310.17</td>
<td>-10.115</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Average</td>
<td>11.119%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Std. Deviation</td>
<td>9.213%</td>
</tr>
</tbody>
</table>

Table 4-6. Unbalanced Configuration Continuous Icing Results Summary

<table>
<thead>
<tr>
<th>Test #</th>
<th>Ice Shed #</th>
<th>Ice Shed Mass (g)</th>
<th>ROM (mm)</th>
<th>System Imbalance (g-mm)</th>
<th>Avg. Fixed Frame Load (lbf.)</th>
<th>Model Prediction % Difference</th>
</tr>
</thead>
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<td>10.012</td>
<td>1144.4</td>
<td>11458</td>
<td>405.42</td>
<td>-9.539</td>
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<td></td>
<td>2</td>
<td>16.752</td>
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<td>3</td>
<td>8.877</td>
<td>943.8</td>
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<td>4</td>
<td>12.887</td>
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<td>-7575</td>
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<td>2</td>
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<td>3</td>
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<td></td>
<td>4</td>
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<td>969.8</td>
<td>3195</td>
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<td>5</td>
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<td>-931.8</td>
<td>-8194</td>
<td>293.59</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Average</td>
<td>22.555%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Std. Deviation</td>
<td>28.105%</td>
</tr>
</tbody>
</table>
4.2.2 Fixed Frame Loading Time History

A representative time history of the measured fixed frame loading for a continuous icing test in this configuration is displayed in Figure 4-13. The X-direction force amplitude in pounds is plotted on the left y-axis and the rotor speed in RPM is plotted on the right y-axis. The two values are included on the same plot to provide a reference for the system operating region for the measured loads. There is a slight increase in measured load around the system critical transition speed but the load remains under 50 pounds for the duration of spin up, illustrating that the system is well-balanced prior to the first ice shed event. The point in time each shed event occurred is represented by the red stars and the measured loading is calculated from the time average between shed events, where the mass imbalance is assumed constant. Examining the changes in loading between shed events, there is little change following the first shed as the magnitude of the imbalance it introduces to the system is relatively small. However, the second shed is of larger magnitude and dynamic fixed frame loading between 200 and 400 lbs. is measured for the next 75 seconds until the fourth ice shed event reduces the system imbalance. This dynamic loading behavior, with high magnitude and frequency, is that which a passive balancing system is intended to mitigate.
4.3 Initial Passive Balancer Configuration Results

Ice shed characterization tests and continuous icing tests were conducted at both temperatures for the rotor configuration with the initial Aluminum Plate Balancer design mounted to the shaft in the test configuration. Based on the magnitude of ice shed events observed for the unbalanced configuration, the HDPE balancing masses were selected for the initial passive balancer configuration icing tests. Experimental results for the measured fixed frame loading in response to ice shed events was compared to model predictions for the simulated unbalanced configuration under the calculated mass eccentricity. The comparison quantified the ability of this passive balancing device to mitigate system vibration following an ice shed event, referred to as the balancer performance. Hi-speed video footage was collected of the passive balancing device in subcritical, critical speed transition, and supercritical operation to monitor the balancing mass position response in reference to each operating region. In addition, Hi-speed footage of the
balancing mass positions was collected in response to an ice shed event to observe the ability of the balancing masses to compensate for an impulsive mass eccentricity during steady state supercritical operation.

4.3.1 Fixed Frame Loading Amplitude

The measured X and Y-direction fixed frame loading in response to 9 unique ice shed events and the model loading predictions for the simulated initial passive balancer configuration are displayed in Figure 4-14. Examining the plot, the measured fixed frame loading increases, close to linearly, with imbalance and is higher than expected for a system with a functioning passive balancing device. The system loading is reflected by model predictions, indicating that the potential issue with the balancing device may be captured in the numerical simulations of the system.

Figure 4-14. Initial Passive Balancer Configuration Experimental and Modeled Loading Results
The experimental values and comparison to model predictions for the ice shed characterization tests and continuous icing tests are summarized in Tables 4-7 and 4-8 respectively. All tests planned in the test matrix for this section of the study were not able to be conducted for this configuration due to limitations on facility access imposed in response to the COVID-19 global pandemic. However, sufficient data was collected to describe the model accuracy and system loading mitigation performance across a range of mass imbalance conditions from 276 to 17491 g-mm. Generally, the model underpredicts the fixed frame loading values for small imbalance and overpredicts for larger imbalance. Model predictions captured the trends and force magnitude observed in the experimental data. Overall, the model predictions had an average percent difference magnitude of 22.48%.

Table 4-7. Initial Passive Balancer Configuration Ice Shed Characterization Results Summary

<table>
<thead>
<tr>
<th>Test #</th>
<th>Ice Shed Mass (g)</th>
<th>$R_{OM}$ (mm)</th>
<th>System Imbalance (g-mm)</th>
<th>Avg. Fixed Frame Load (lbf.)</th>
<th>Model Prediction % Difference</th>
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<td>14</td>
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<tr>
<td>15</td>
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<tr>
<td>16</td>
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<td>1195.38</td>
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<td>297.085</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Average: 16.737%
Std. Deviation: 12.670%
Experimental loading values and model predictions are compared to that of the unbalanced configuration in Figure 4-15, illustrating that the introduction of the initial passive balancing device did not significantly improve the system loading in response to an ice shed event. In cases with lower magnitude imbalance, the measured loading was much larger than that predicted for an unbalanced configuration, which suggests that the source of the increased loading is the passive balancing device itself rather than the ice shed event. The passive balancing device could be introducing imbalance into the system through manufacturing inaccuracies in the frame and mounting structure or through the balancing masses assuming positions that exacerbate the system imbalance. A summary of the performance parameter for each test case, calculated relative to the simulated loading for the unbalanced configuration with identical mass eccentricity, is provided in Table 4-9. A positive performance value indicates an increase in the loading relative to the simulated unbalanced configuration, while a negative value denotes a decrease. The performance value for tests case 22.2 was excluded in the average and standard deviation calculation as the calculated mass imbalance present in the system was insignificant leading to an outlier in the
performance as the model prediction for this case was close to zero. Overall, the initial passive balancing device increased loading by 8.63% on average.

Figure 4- 15. Initial Passive Balancer and Unbalanced Rotor Configuration Loading Comparison

<table>
<thead>
<tr>
<th>Test-Shed #</th>
<th>System Imbalance (g-mm)</th>
<th>Avg. Load (lbf.)</th>
<th>Balancer Performance (%)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-14671</td>
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<td>+5.901</td>
</tr>
<tr>
<td>15-1</td>
<td>-7745</td>
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<td>-8.898</td>
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<td>11215</td>
<td>297.085</td>
<td>-14.569</td>
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<td>-17491</td>
<td>453.675</td>
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<td>22-4</td>
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<td>193.825</td>
<td>+101.631</td>
</tr>
</tbody>
</table>

| Average     | +8.630%                  |
| Std. Deviation | 38.110%               |
4.3.2 Fixed Frame Loading Time History

The loading time history over the 5-minute continuous icing test for this configuration is provided in Figure 4-16. Examining the loading prior to the first ice shed event, the cyclic loading between 200 to 250 pounds indicates that the system is not in a balanced mass distribution prior to the first ice shed event. This provides further data to suggest that the source of the system loading is the passive balancing device rather than the mass eccentricity introduced by the ice shed. It is expected that if the ice shed event is the source of the system fixed frame loads that the magnitude would change substantially following each shed event as observed for the unbalanced rotor configuration in Figure 4-13. The magnitude of the loading fluctuates following each shed event, however, there is little change in the loading behavior of the system.

Figure 4-16. Y-direction Loading Time History (Test 22)
4.3.3 Balancing Mass Positions

Analyzing the positions of the balancing masses within the balancer track in response to rotor spin-up and ice shed events provides insight into the cause of the increased loading due to the implementation of the initial passive balancer design.

Segments of Hi-Speed footage was captured at 500 RPM, 800 RPM and 1200 RPM, corresponding to the subcritical, critical speed transition, and supercritical operating region. Footage was rotationally stabilized based upon the measured rotor RPM and the footage frame rate such that the balancer appears fixed in space to provide clearer observation of the balancing mass positions and how they change over time. Representative frames collected from the rotor spin-up footage is displayed in Figures 4-17, 4-18, and 4-19, for the subcritical, critical and supercritical operating regions respectively.

![Image: Initial Passive Balancer Subcritical Ball Positions (500RPM)](image)

Figure 4-17. Initial Passive Balancer Subcritical Ball Positions (500RPM)
Figure 4-18. Initial Passive Balancer Critical Speed Transition Ball Positions (800RPM)

Figure 4-19. Initial Passive Balancer Supercritical Ball Positions (1200RPM)
Comparing the positions of the balancing masses observed in Figures 4-17, 4-18, and 4-19, the masses adhere to the partition walls due to their inertia as the system rotates counterclockwise. This behavior is expected in subcritical operation, as it has been observed experimentally as a major advantage to a partitioned track in limiting self-excited vibrations in this operating region. However, as the system reaches critical speed transition, the masses should be perturbed from their subcritical positions and rapidly oscillate within their range of motion as the balancer response resonates with the rotational speed of the system. Finally, upon reaching steady state supersonic operation, the masses should reach asymptotically stable positions oriented in diametrically opposed positions to any imbalance present in the system. The Hi-speed footage illustrates that the masses do not change positions as the system reaches critical speed and supersonic operation. This observation suggests that the ball-track resistance force reaches a sufficiently large magnitude between the subcritical and critical operating speeds to fix the masses in place for the remainder of system spin up.

This is further illustrated by footage of the balancing mass response to asymmetric ice shed events. Two tests were conducted to visualize the ice shed response of the balancing mass positions, by accreting 5 minutes of ice at 500RPM and increasing speed to 1200RPM, triggering the Hi-speed camera system immediately following the ice shed event to retroactively record the balancing mass positions prior to and following the ice shed. The mass imbalance introduced by each of the two ice shed events is provided in Table 4-10. The passive balancer performance was not calculated for these tests and the ice shed events were not considered to characterize the system shedding behavior, as the conditions for these tests were not representative of a realistic icing condition. Due to accreting ice at a lower RPM, a larger mass of ice was removed in each case as the centrifugal force during ice accretion is much lower at 500RPM. Therefore, ice shed events did not occur until ice accretion was stopped and the rotor RPM was increased to the supersonic operating region, where a larger portion of the blade was shed at once.
Table 4-10. Initial Configuration Balancing Mass Ice Shed Response Ice Shed Events

<table>
<thead>
<tr>
<th>Test #</th>
<th>Ice Shed Mass (g)</th>
<th>ROM (mm)</th>
<th>System Imbalance (g-mm)</th>
<th>Rotor Speed at Shed (RPM)</th>
<th>Avg. Fixed Frame Load (lbf.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>64.5</td>
<td>1092.2</td>
<td>70447</td>
<td>1181</td>
<td>1253</td>
</tr>
<tr>
<td>25</td>
<td>80.1</td>
<td>1047.75</td>
<td>83925</td>
<td>945</td>
<td>1232.5</td>
</tr>
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</table>

A comparison of the balancing mass positions immediately before and several tenths of a second following an ice shed event is displayed in Figure 4-20. Despite the large magnitude of the ice shed event and the shaft displacement observed in the footage, the balancing mass positions do not change. Therefore, it was hypothesized that the ball-track resistance force at 1200RPM is greater than the centrifugal force driving the passive balancing behavior as the passive balancing device is displaced from the rotational axis. The inability of this configuration to maintain the free circumferential motion of its balancing masses at supercritical speeds makes it impossible for the passive balancing device to benefit the system loading in response to mass eccentricity. Passive balancing device design parameters must be altered to tune ball-track resistance forces to allow for passive balancing at 1200RPM.

Figure 4-20. Ice Shed Response (Test 25): (Left) Ball Positions Prior to Shed, (Right) Ball Positions after Shed
The ball position time history obtained from numerical simulations of the initial passive balancer configuration supports the hypothesis that large ball-track resistance force, namely static friction, is fixing the balancing masses at low operating speeds, making the system unable to exhibit passive balancing behavior. The ball position and rotor speed time history from a numerical simulation of system spin up to 1200RPM with a mass imbalance of 16000 g-mm is displayed in Figure 4-21. In the figure, the balancing mass positions are plotted in degrees relative to the rotational frame axes and points in time where the masses are fixed in position by the ball-track static friction are highlighted. Comparing the highlighted regions for each mass position to the rotor speed time history, static friction begins to fix the ball positions at approximately 400RPM, well below the critical transition speed, and masses are unable to overcome static friction until after rotor speed drops below 400 RPM despite the force being put on the system by the 16000 g-mm imbalance. Therefore, there is sufficient evidence to determine that ball-track resistance force is a major contributor to the poor balancer performance, which the model effectively captures.

![Figure 4-21. Ball Position and Rotor Speed Time History for Simulated Spin Up (Imb_{sys} = 16000 g-mm)](image-url)
The ability of the comprehensive model to accurately simulate the frictional dynamics between the balancing masses and outer track wall presents an opportunity to conduct parametric studies in order to improve the predicted performance of the passive balancing device in this application. The experimental and model data indicate that there is insufficient force on the blancing masses due to the imbalance to overcome static friction and allow passive balancing at supercritical operating speed. To improve performance, balancer design parameters must be changed to reduce the ball-track static friction force and increase the magnitude of the force on the balancing masses due to mass imbalance.

4.4 Summary

Experiments were conducted to successfully measure and model the fixed frame loading of the system in response to asymmetric ice shed in representative icing conditions. The system mass eccentricity due to an arbitrary ice shed was modeled by applying the analytical ice mass model. The model was determined to be fairly accurate with an average model accuracy of 16.69% compared to experimentally measured ice mass values from ice shed characterization tests.

The unbalanced rotor configuration operating region critical speed was determined experimentally and numerically, with a difference of 2.1%. The unbalanced rotor configuration experienced adverse icing conditions and the fixed frame loading was measured in response to 14 unique ice shed events. Fixed frame loading increased close to linearly with mass imbalance, with the highest observed values reaching loading close to the facility limits. Analyzing the loading time history for continuous icing tests, the effect of large asymmetric ice shed events on fixed frame loading was illustrated. The system mass eccentricity for each shed event was calculated from the analytical ice mass model and experimental data and the system loading response was simulated.
using the comprehensive model. On average, model predictions were 18.7% different from experimental measurements.

The initial passive balancer configuration was subjected to the same icing conditions as the unbalanced rotor configuration and the fixed frame loading was measured in response to 9 unique ice shed events. Similar to the unbalanced configuration, fixed frame loading increased nearly linearly with mass imbalance, and with similar the magnitude. The performance of the initial passive balancing device was quantified by comparing experimental values to model predictions for the unbalanced configuration. Overall, the inclusion of the passive balancing device increased fixed frame loading 8.63%, illustrating that the balancing device is not improving the system loading as intended. The system response to each shed was modeled and compared experimental loading measurements. On average, model predictions were 22.48% different compared to experimental measurements, illustrating that the comprehensive model is capturing the poor performance of this passive balancing configuration. Analyzing the loading time history for continuous icing tests in this configuration, the high degree of loading prior to any ice shed event suggests that a large portion of the loading in this configuration is due to the system configuration itself, which accounts for loading much higher than model predictions at low levels of mass imbalance. The source of the vibration was hypothesized to be due to balancing masses being unable to reach positions to mitigate mass eccentricity in the system. Hi-speed footage of the balancing masses during system spin up and ice shed response supported this hypothesis, as masses were unable to be perturbed from positions observed during subcritical operation. It was determined from numerical simulations that the potential cause of the poor performance of the initial passive balancing device was the ball-track resistance force fixing balancing masses in place at subcritical speeds and restricting any motion at steadystate supercritical operation. It was hypothesized that altering passive balancer design parameters to reduce the ball-track resistance force and increase the passive balancing forces at supercritical operating speeds would improve the performance of the next design iteration.
Chapter 5 | Parametric Analysis and Experimental Validation of Final Design

Parametric analysis was conducted to identify key design parameters limiting the performance of the initial design and to formulate a final design with improved predicted vibration mitigation performance. The final passive balancer design was fabricated, and its physical quantities were characterized. Continuous icing experiments were conducted on the final passive balancer design configuration to characterize the final design performance and compare to model predictions. Hi-speed footage was collected to monitor the balancing mass positions in each operating region and in response to an ice shed event. Experimental and predicted model performance was compared across all configurations to experimentally validate the model as a design tool for practical applications of passive balancing devices.

5.1 Parametric Analysis

The numerical simulation capabilities provided by the comprehensive mathematical model were applied to the passive balancer design process through several parametric analyses with the goal of formulating a design with improved performance across the range of mass imbalance experienced by the rotor stand. The key design parameters selected for analysis:
1. **Passive Balancer Track Radius:** The radial distance the balancing masses are set from the rotational axis. Varying the track radius affects the normal force on the balancing masses for a given speed, which is proportional to the ball-track resistance force.

2. **Passive Balancer Axial Position:** The location of the balancer plane along the shaft axis. Due to the shaft fixed boundary condition, the balancer displacement from the rotational axis for a given mass imbalance is determined by the distance of the passive balancing device from the fixed boundary. The balancer displacement is proportional to the driving passive balancing force acting on the balancing mass, as it determines the centrifugal force vector on the balancing mass.

3. **Ball-Track Frictional Model:** The frictional dynamics between the balancing masses and the balancer track outer wall are critical to the ball-track resistance force. Varying frictional models, obtained from literature, are implemented into numerical simulations assuming various materials for the balancing masses and outer track wall contact point. Selecting material interactions with properly tuned frictional dynamics will ensure the balancing masses are free to move during steady state supercritical operation while allowing stable mass positions to be reached.

4. **Balancing Authority:** The balancing authority of the passive balancing device is varied for a set track radius by changing the collective mass of the balancing masses. Only sets of 4 identical balancing masses are considered in this study. Excess balancing mass is expected to generally improve balancer performance. However, considering the ball-track resistance force, excess balancing authority can also degrade performance due to increased normal loading.

The design approach was iterative, illustrated by Figure 5-1, beginning with the design parameters for the initial passive balancer configuration and assuming a critical mass imbalance of
16400 g-mm for the track radius and axial position variation. The critical mass imbalance condition was calculated based upon the previously observed system imbalance values from the icing experiments run on the previous two configurations. Assuming a normal distribution of system mass imbalance magnitude, shown in Figure 5-2, 16386 g-mm is the 93rd percentile of mass imbalance conditions. Therefore, a passive balancing device designed to account for this mass imbalance with a high level of performance will be conservative for that experienced by the rotor stand.

Figure 5-1. Passive Balancer Parametric Design Process
For each design iteration, the steady state loading response was calculated for the critical imbalance condition with the initial parameters. Each numerical simulation case, consists of an imbalance present in the system from rest, directed at 90 degrees in the rotational frame, spin up to 1200RPM, 3 seconds of steady state supercritical operation, and spin down to rest. The parameter used to quantify balancer performance was the time-average loading during steady state operation. Simulations were constructed in this way to maintain reasonable computational time for the large number of cases modeled for the purpose of the passive balancer design process. The track radius was varied and the critical imbalance condition loading was compared to select the track radius with the best balancing performance, which was carried over into the subsequent parametric analyses. The position of the balancer plane relative to the rotor plane was varied and the predicted loading for the critical imbalance predicted was compared to select the balancer axial position with
the most ideal performance. For the frictional model analyses, a set of frictional models for various material combinations was implemented into the model, assuming the balancing masses and track wall are composed of the corresponding materials. The balancer performance was predicted across a range of imbalance conditions from 0 to 20000 g-mm to compare the effects of the frictional models across the range of imbalance conditions experienced by the rotor stand. Finally, the collective mass of the balancing masses was varied to observe the effect of changing balancing authority, for a fixed track radius, on fixed frame loading for the critical imbalance condition. The balancing authority corresponding to the most ideal critical imbalance loading performance was selected and the predicted fixed frame loading was compared to that of the initial prediction from the previous design iteration. If greater than a 10 lbf. reduction was observed between the predicted critical imbalance loading at the beginning and the end of the parametric analysis design loop, the updated parameters are input back into the loop starting with the track radius parametric analysis. Otherwise, the design was considered to have converged and the key design parameters for the final passive balancer design were determined. Results for the initial design loop parametric analyses are shared in the following sections and trends observed in performance in response to the variation of each parameter are discussed.

5.1.1 Passive Balancer Track Radius

The predicted critical imbalance performance for varying balancer track radius from the first design iteration loop is displayed in Figure 5-3. The trends in model predictions illustrate the effect of balancer track radius on balancer performance as there was an approximately 250 lbf. difference in fixed frame loading solely by reducing the track radius from 12 inches to 6 inches. The improvement in performance is attributed to the reduction in ball-track resistance force associated with lesser balancer track radius. However, balancing authority decreases with track radius as well and performance degrades past 6 inches as there was insufficient balancing authority
to account for the critical mass imbalance condition. This result illustrates that the optimal track radius value is highly dependent on the balancing authority required for the critical imbalance case. The balancer track radius parameter was coarsely varied in the initial design iteration loop to reduce computational time, but is the range of values considered for the track radius is refined with each iteration.

![Graph](image)

**Figure 5- 3. Passive Balancer Track Radius Initial Design Loop Results**

### 5.1.2 Passive Balancer Axial Position

The passive balancer plane position along the shaft axis was defined relative to the rotor plane, which was set at 11.575 inches from the shaft fixed boundary. A negative value for balancer axial position indicates that the balancer plane was set beneath the rotor plane, while a positive value indicates a balancer plane set above the rotor plane. In cases where the balancer was set sufficiently high along the shaft axis, length was added to the shaft in the numerical simulation to accommodate the passive balancing device and keep the mass in the system representative of an
actual experimental configuration. The predicted critical imbalance loading in response to varied passive balancer axial position, above and below the rotor plane is displayed in Figure 5-4. The balancer critical loading performance improves as it is mounted above the rotor plane compared to below the rotor plane. The improvement approaches an asymptotic limit as position above the rotor plane increases. The improvement is attributed to larger passive balancer displacement from the rotational axis for the critical imbalance for balancer planes farther from the shaft fixed boundary. Therefore, the asymptotic performance improvement indicates that for that axial position, and those further from the fixed boundary, the balancing masses experience sufficient passive balancing force due to the imbalance to overcome static friction and reach asymptotically stable positions that correct for the critical imbalance. These findings contradict those of DeSmidt and Haidar, which illustrate that ideal passive balancer performance occurs for a balancer plane coincident with the mass imbalance [40,48]. However, neither study considered a cantilevered shaft with stiffness and length on the same scale as this study, suggesting that shaft boundary conditions and mechanical properties must be considered in passive balancing design for practical applications.

Figure 5-4. Passive Balancer Axial Position Initial Design Loop Results
5.1.3 Ball-Track Frictional Model

Three frictional models obtained from literature were considered for parametric analysis. The Aluminum-HDPE frictional model formulated by Benabdallah [94], used in numerical simulations of the initial passive balancer configuration, Polytetrafluoroethylene (PTFE)-316 Stainless Steel, referred to as the Teflon model, and Nylon-316 Stainless Steel, referred to as the Nylon model. The Teflon and Nylon models were formulated by Nuruzzaman et al. and are described by Table 5-1 and Figure 5-5, which illustrates the relationship between static friction coefficient and normal load [105]. It was noted that, while Teflon normally exhibits lower kinetic and static friction than Nylon, under normal loading, the Teflon static frictional coefficient increases while the Nylon static friction coefficient decreases.

![Figure 5-5. Teflon and Nylon Static Coefficient of Friction Under Normal Load [105]]
### Table 5-1. Frictional Model Coefficients

<table>
<thead>
<tr>
<th>Frictional Model</th>
<th>Kinetic Coefficient of Friction</th>
<th>Static Coefficient of Friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-HDPE</td>
<td>0.2</td>
<td>varies [95]</td>
</tr>
<tr>
<td>PTFE-316 SS</td>
<td>0.04</td>
<td>varies [105]</td>
</tr>
<tr>
<td>Nylon-316 SS</td>
<td>0.1</td>
<td>varies [105]</td>
</tr>
</tbody>
</table>

A comparison of the predicted fixed frame loading for passive balancing devices with different assumed frictional models is provided in Figure 5-6. Despite the Teflon model exhibiting the lowest kinetic friction coefficient, the frictional dynamics provided by the Nylon model produced the most ideal predicted performance across the majority of the range of mass imbalance considered. The improvement in performance over the Al-HDPE frictional dynamics is attributed to lower kinetic and static friction coefficients between the balancing masses and the track outer wall allowing balancing masses to reach more optimal positions in response to mass imbalance, as less ball-track resistance force in present in the system. However, there is sufficient ball-track resistance force acting on the balancing masses to dampen ball motion, allowing asymptotically stable positions to be reached during steady state operation. The balancing mass position time histories for the Nylon model and Teflon model numerical simulations at 16000 g-mm are displayed in Figure 5-7 and 5-8 respectively. The balancing masses experiencing Nylon model frictional dynamics were able to react to the mass imbalance following critical speed transition, and reach stable positions shortly after reaching steady state. In comparison, the masses with the Teflon model did not experience sufficient friction and continuously oscillated between partitions during steady state, preventing a balanced configuration from being reached and degrading the passive balancer performance. This behavior was exhibited for nearly all Teflon model numerical simulation cases, accounting for the degradation in performance compared to the Al-HDPE model cases. The results of implementing various frictional models into numerical simulations illustrated
that simply minimizing ball-track resistance force is not sufficient in the design of passive balancing devices. Finely tuned frictional dynamics must be present for the balancing masses to both react to mass imbalance in the system and reach asymptotically stable positions during steady state operation.

**Figure 5-6. Passive Balancer Frictional Model Initial Design Loop Results**

**Figure 5-7. Ball Position and Rotor Speed Time History for Nylon Model Simulation (\(I_m b_{sys} = 16000 \, \text{g-mm}\))**
5.1.4 Balancing Authority

The balancing authority parameter was varied by changing the mass of the balancer balls for a fixed track radius. The critical imbalance numerical simulation results for varying balancing authority for two balancers with fixed track radii of 2.5 and 4 inches is displayed in Figure 5-9. Balancer performance increased with balancing authority for both balancers up to a point, with the most ideal performance observed for the 2.5 in. track radius balancer with a balancing authority of 18890 g-mm. For the 2.5 in. balancer, excess balancing authority past 1.5 times the critical imbalance magnitude degraded performance due to larger inertia and ball-track resistance introduced by the increased ball mass. Similar trends are observed in the 4 in. balancer results, with performance degradation occurring at higher levels of balancing authority, illustrating that the degradation was a function of the ball mass rather than the balancing authority.
5.1.5 Summary

Parametric analyses were conducted on key balancer design parameters as a part of an iterative design process. At the end of the initial design loop, the lowest predicted critical imbalance loading was 27.638 lbf., representing a 93.22% reduction from that predicted for the initial passive balancer configuration and a 95.03% reduction from that predicted for the unbalanced rotor configuration. The design parameters were iterated through several successive loops with refined variation in parameters until the critical imbalance loading predictions converged. The final passive balancer design parameters obtained from the parametric analysis driven design process are summarized in Table 5-2. A comparison of the predicted fixed frame loading for the final passive balancer design to model predictions for the initial passive balancer configuration and the unbalanced rotor configuration is shown in Figure 5-10, illustrating the significant improvement in the performance of a passive balancer design formulated using a numerical design tool.
Table 5-2. Final Passive Balancer Key Design Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Value</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Track Radius</td>
<td>12.35 in. (313.69 mm)</td>
<td>2.55 in. (64.77 mm)</td>
</tr>
<tr>
<td>Axial Position Rel. to Rotor Plane</td>
<td>-5.125 in. (-130.175 mm)</td>
<td>+11.9 in. (+302.3 mm)</td>
</tr>
<tr>
<td>Frictional Model</td>
<td>Al-HDPE Model</td>
<td>Nylon-316 SS Model</td>
</tr>
<tr>
<td>Ball Mass</td>
<td>0.1564 kg</td>
<td>0.225 kg</td>
</tr>
<tr>
<td>Balancing Authority</td>
<td>76389 g-mm</td>
<td>17137 g-mm</td>
</tr>
<tr>
<td>Critical Imbalance Loading</td>
<td>525.39 lbf.</td>
<td>19.645 lbf.</td>
</tr>
</tbody>
</table>

Figure 5-10. Predicted Configuration Fixed Frame Loading Comparison
5.2 Final Passive Balancer Configuration Components

Physical quantities are provided for the final passive balancer configuration and changes made to system components are detailed. The updated configuration assembly is displayed in Figure 5-11.

![Figure 5-11. Final Passive Balancer Configuration Experimental Assembly](image)

5.2.1 Shaft Extension

The shaft was extended to provide a mounting position for the passive balancer above the rotor plane. The shaft extension component, displayed in Figure 5-12, was composed of 321 stainless steel and was slotted onto the upper 2 inches of the original shaft, with an outer bore flush to the original shaft. The extension was secured with a compression sleeve component that straddles the interface between the extension and the original shaft and was tightened to clamp the components together. A keyed slot was machined into the shaft extension outer bore to mount the passive balancing device. Relevant physical quantities are provided in Table 5-3. The component is
modeled by adding length to the shaft in numerical simulations, assuming the shaft functions as one continuous entity.

![Figure 5-12. Shaft Extension CAD Front and Top View](image)

### Table 5-3 Shaft Extension Properties

<table>
<thead>
<tr>
<th>Material</th>
<th>321 Stainless Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>8027 kg/m³ [92]</td>
</tr>
<tr>
<td>Length</td>
<td>10.88 in. (276.35 mm)</td>
</tr>
<tr>
<td>Radius</td>
<td>0.815 in. (20.701 mm)</td>
</tr>
<tr>
<td>Mass</td>
<td>2.195 kg</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>193 GPa [92]</td>
</tr>
</tbody>
</table>
5.2.2 Final Passive Balancer

The final passive balancing device, shown in Figure 5-13, was fabricated from alternating plates of 6061-T6 Aluminum and cast Nylon, designed to conform to the design parameters obtained from the parametric analyses. The Nylon plates provide the desired frictional dynamics, while the Aluminum plates provide lightweight structurally integrity to the system. The balancer track bottom plate was composed of cast Nylon and a 0.05 in. offset between outer wall radius of the Aluminum and Nylon plates making up the balancer track was specified to ensure the balancing masses were only in contact with Nylon parts. A clear, polycarbonate top plate was bolted to the frame to seal the balancer track from moisture, while facilitating the observation of balancing mass positions from above. Because this passive balancer design was much smaller than the previous iteration, the plates were bolted directly together as the weight of the frame was less significant. G25 precision Chrome Steel ball bearings served as the balancing masses with a diameter of 1.5 inches. The final passive balancer was mounted directly to the shaft extension with a matching key channel and tight tolerancing at the shaft-balancer interface such that the balancer was press-fit onto the shaft extension. Relevant physical properties for the final passive balancer are provided and compared to the initial design parameters in Table 5-4.

Figure 5-13. Final Passive Balancer CAD Top and Right View
Prior to fabrication, static stress analysis was conducted on the final passive balancer design, using SOLIDWORKS finite element analysis, to ensure the frame could withstand the centrifugal loading in the supercritical operating region within a reasonable factor of safety. The critical loading condition imposed corresponds to an angular speed of 1600 RPM with the balancing masses in the most bias circumferential positions. The factor of safety was calculated for each element based on the Von Mises yield stress criterion. The factor of safety output field in the balancer mid-thickness plane is displayed in Figure 5-14. The highest concentrations of stress were located at the point of contact between the Nylon outer wall and the balancing masses, illustrating that the magnitude of local stress concentrations was highly dependent on the position of the balancing masses. A minimum factor of safety of 1.817 located in the mid-thickness Nylon plate was observed under the conservative loading condition. A side view of the factor of safety output field, normal to the highest stress partition outer wall, is displayed in Figure 5-15 to illustrate the differences in factor of safety between the Nylon and Aluminum plates. The Aluminum plates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial Design</th>
<th>Final Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ball Material</td>
<td>HDPE</td>
<td>Chrome Steel</td>
</tr>
<tr>
<td>Ball Radius</td>
<td>1.4 in. (38.1 mm)</td>
<td>0.75 in. (19.05 mm)</td>
</tr>
<tr>
<td>Ball Mass</td>
<td>0.1564 kg</td>
<td>0.2268 kg</td>
</tr>
<tr>
<td>Distance to Axis of Rotation</td>
<td>12.35 in. (313.69 mm)</td>
<td>2.55 in. (64.77 mm)</td>
</tr>
<tr>
<td>Partition Thickness</td>
<td>2.15 in. (54.61 mm)</td>
<td>0.23228 in. (5.90 mm)</td>
</tr>
<tr>
<td>Balancing Authority</td>
<td>7.6389 x 10^4 g-mm</td>
<td>17746 g-mm</td>
</tr>
<tr>
<td>Balancer Frame Material</td>
<td>6061-T6 Aluminum</td>
<td>6061-T6 Aluminum, Nylon</td>
</tr>
<tr>
<td>Balancer Frame Mass</td>
<td>16.506 kg</td>
<td>2.125 kg</td>
</tr>
<tr>
<td>Balancer Frame Outer Diameter</td>
<td>30 in. (762 mm)</td>
<td>8 in. (203.2 mm)</td>
</tr>
<tr>
<td>Balancer Frame Thickness</td>
<td>3.27 in. (83.06 mm)</td>
<td>2.38 in. (60.45 mm)</td>
</tr>
<tr>
<td>Balancer Track Radius</td>
<td>13.75 in. (349.25 mm)</td>
<td>3.3 in. (83.82 mm)</td>
</tr>
<tr>
<td>Balancer Shaft Location (from L = 0)</td>
<td>6.45 in. (163.83 mm)</td>
<td>23.475 in. (596.27 mm)</td>
</tr>
</tbody>
</table>
exhibited a minimum factor of safety well over 50, indicating that the structural support provided by the Aluminum ensured the balancer frame was robust to yielding under centrifugal loading.

Figure 5-14. Final Passive Balancer Mid-Thickness Plane Sectional Factor of Safety Plot

Figure 5-15. Final Passive Balancer Factor of Safety Distribution (Side View)
The final passive balancer dynamic properties are summarized in Table 5-5. The friction coefficients between the track wall and balancing masses were determined from the Nylon-316 Stainless Steel model formulated by Noruzzaman et al., outlined in section 5.1.3 [105]. The coefficient of restitution for inelastic collisions between Steel balancing masses and Nylon track partitions was determined experimentally from processed Hi-speed footage of collisions, the same method applied to the initial passive balancer configuration. The results of the coefficient of restitution experiments are displayed in Figure 5-16. The average coefficient of restitution was 0.2019 with a standard deviation of 4.5%.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Table 5-5. Final Passive Balancer Dynamic Properties</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Ball-Wall Coefficient of Restitution</strong></td>
<td>0.2019</td>
</tr>
<tr>
<td><strong>Rolling Friction Resistance</strong></td>
<td>$4.7625 \times 10^{-5}$ m [93]</td>
</tr>
<tr>
<td><strong>Ball-Track Static Coefficient of Friction</strong></td>
<td><em>varies</em> [105]</td>
</tr>
<tr>
<td><strong>Ball-Track Kinetic Coefficient of Friction</strong></td>
<td>0.1 [105]</td>
</tr>
<tr>
<td><strong>Coefficient of Drag</strong></td>
<td><em>varies</em> [97]</td>
</tr>
</tbody>
</table>

![Figure 5-16. Nylon-Steel Coefficient of Restitution Results Summary](image-url)
5.3 Test Matrix

With sufficient data collected to characterize the rotor blade ice shedding behavior and experimentally validate the analytical ice mass model from the prior test campaign, the test matrix constructed to characterize performance of the final passive balancer configuration was composed solely of continuous icing and balancing mass ice shed response tests. Five continuous icing tests were conducted at each temperature to observe and model the system fixed frame loading response to a larger number of ice shed events in a realistic icing condition. Two balancing mass ice shed response tests were conducted, one at each temperature, to collect Hi-speed footage of balancing masses before and immediately following an asymmetric ice shed event. The test matrix for the final passive balancer configuration icing tests is summarized in Table 5-6.

Table 5-6. Final Passive Balancer Configuration Test Matrix

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Test #</th>
<th>Rotor RPM</th>
<th>Icing Conditions</th>
</tr>
</thead>
</table>
| Continuous Icing              | 26:35       | 1200 RPM  | Temperature: -8, -12 °C
LWC: 0.3326 g/m³
MVD: 20 μm
Accretion Time: 5 min. |
|                               | (26:30 @ -12 °C) |           | Multiple Shed Events                                                             |
|                               | (31:35 @ -8 °C)  |           |                                                                                  |
|                               | 36:37       | 500 RPM, shed | Temperature: -8, -12 °C
LWC: 0.3326 g/m³
MVD: 20 μm
Accretion Time: 5 min. |
| Balancing Mass Ice Shed       | 36 @ -12 °C |           | Ice Shed on Spin Up                                                             |
| Response                      | 37 @ -8 °C  |           |                                                                                  |
5.4 Final Passive Balancer Configuration Results

Fixed frame loading data was collected for continuous icing tests in the final passive balancer configuration. For each observed ice shed event, the system mass eccentricity was calculated from the analytical ice mass model and the measured time-average loading following the shed was compared to model predictions. The performance of the final passive balancer configuration was quantified for a given mass imbalance condition by comparing measured fixed frame loading to model predictions for the unbalanced rotor configuration. The loading time history recorded for continuous icing tests was analyzed to observe the system loading response to successive ice shed events and impulsive changes to system mass imbalance magnitude and direction. Hi-speed footage of the balancing mass position response was captured during system spin up to observe the changes in position between the subcritical, critical speed transition, and supercritical operating regions. The balancing mass ice shed response was observed from rotationally stabilized hi-speed footage of the balancing masses prior to ice shed, immediately following ice shed, and several seconds after ice shed, once masses had assumed asymptotically stable positions.

5.4.1 Fixed Frame Loading Amplitude

The X and Y-direction fixed frame loading was measured in response to 40 unique ice shed events, ranging from 736 to 21599 g-mm, observed over the 10 continuous icing tests conducted in the final passive balancer configuration. The mass eccentricity was calculated and numerical simulations were run for each observed imbalance condition. Experimental data and model predictions are plotted in Figure 5-17. Examining the experimental loading results, the force amplitude for the majority of cases fell below 50 lbf. and there was no clear correlation between the mass imbalance and force amplitude. The departure from trends and magnitude in measured
loading observed for the prior configurations illustrates that the final passive balancer had a significant effect on the loading behavior of the system. There was consistent scatter in the experimental results across the range of observed mass imbalance positions. These differences in loading for similar mass imbalance conditions could be explained by the varied mass distributions for each imbalance case, as the ice shed mass and radial position can vary greatly for nearly identical system imbalance due to the presence of multiple ice sheds. In addition, the probabilistic of the asymptotically stable positions assumed by the balancing masses leads to variation in the passive balancer performance for repeatable mass imbalance conditions.

Figure 5-17. Final Passive Balancer Configuration Experimental and Modeled Loading Results
Comparing experimental results to model predictions, the measured loading was larger than the model predictions for nearly all except the largest mass imbalance conditions, indicating that some model parameters affecting the predicted balancer performance may be inaccurate to the experimental configuration. One example of a potential inaccuracy was the static frictional model assumed for the contact between the balancing masses and the outer track wall was defined in literature for a normal load range up to 10 N. However, the normal load at the contact point at 1200RPM is approximately 231 N. Therefore, the frictional dynamics implemented in the model may not have been representative of those present in the experimental configuration.

Experimental values for system mass imbalance, fixed frame loading and comparison to model predictions are summarized in Tables 5-7 and 5-8, for the -12 °C and -8 °C conditions respectively. A positive percent difference denotes an overestimation and a negative percent difference denotes an underestimation of the measured loading for each mass imbalance condition. Simultaneous shed events, where ice sections were removed from both blades simultaneously, the shed event number is denoted as with an “a” and “b” to provide mass data for each section of ice. Examining how the fixed frame loading changes over the duration of each test, it was generally observed that the changes in mass imbalance conditions had an insignificant effect on system loading illustrating the ability of the final passive balancer to effectively respond to impulsive changes in mass distribution. However, there are several measured loading cases that exhibit much higher magnitude loads than those observed for the rest of the test. Namely those observed in tests 28, 29, 30, and 34, which exhibit outliers in fixed frame loading amplitude. For these cases, it is unlikely that the increased loading was due to mass imbalance outside the balancing authority of the configuration, as several of the cases had calculated mass imbalance close to zero. A possible explanation for the increase in loading was that the balancing masses entered an unstable mode in response to the impulse applied to them by the ice shed event, preventing asymptotically stable positions opposed to the system mass imbalance to be reached. The conditions that bring about
unstable modes in the balancing masses in the experimental configuration and methods to prevent them prompts further study.

Table 5-7. Final Passive Balancer Configuration Continuous Icing Results Summary ($T_a = -12 \, ^\circ C$)

<table>
<thead>
<tr>
<th>Test #</th>
<th>Ice Shed #</th>
<th>Ice Shed Mass (g)</th>
<th>$R_{OM}$ (mm)</th>
<th>System Imbalance (g-mm)</th>
<th>Avg. Fixed Frame Load (lbf.)</th>
<th>Model Prediction % Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>1</td>
<td>8.246</td>
<td>1199.1</td>
<td>9888</td>
<td>31.7185</td>
<td>-68.471</td>
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<tr>
<td></td>
<td>2</td>
<td>11.205</td>
<td>1045.2</td>
<td>21599</td>
<td>37.676</td>
<td>+132.129</td>
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<td></td>
<td>3</td>
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<td><strong>Std. Deviation</strong></td>
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Average: 68.120%
Std. Deviation: 23.884%
Table 5-8. Final Passive Balancer Configuration Continuous Icing Results Summary ($T_a = -8 \, ^{\circ}C$)

<table>
<thead>
<tr>
<th>Test #</th>
<th>Ice Shed #</th>
<th>Ice Shed Mass (g)</th>
<th>ROM (mm)</th>
<th>System Imbalance (g-mm)</th>
<th>Avg. Fixed Frame Load (lbf.)</th>
<th>Model Prediction % Difference</th>
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<td>-55.775</td>
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</table>

Average 61.569%
Std. Deviation 33.437%
Overall, the model predicted loading accuracy for this configuration was 64.844%, given in average percent difference magnitude. This value suggests large inaccuracies present in the model of the final passive balancer configuration. However, the practical significance of the average percent difference was less in this configuration due to the lesser magnitude of both the measured and predicted loading compared to the prior two configurations tested. A more representative parameter of model in this case would be the average difference between the measured load and predicted load given in pounds force. On average, the difference between experimental and predicted values for fixed frame loading was 19.729 lbf., which is well within the loading tolerances for most rotorcraft components. Therefore, the model retained a level of practical accuracy, as the measured and predicted loads for all observed mass imbalance cases fell within the acceptable range for rotorcraft operation, demonstrating the ability for the mathematical model to function as a predictive performance model to aid in the design of passive balancing devices for practical applications.

A comparison of the fixed frame loading results for the three configurations considered in this study is plotted in Figure 5-18. The plot illustrates the significant reduction in fixed frame loading in the final passive balancer configuration for all mass imbalance conditions as compared to the prior configurations. The reduction in fixed frame loading was quantified at each observed mass imbalance condition by calculating the performance of the final passive balancer configuration relative to the predicted loading of the unbalanced rotor configuration. The final passive balancer configuration performance is summarized in Tables 5-9 and 5-10, as in prior sections, a negative value denotes a reduction in fixed frame loading relative to the prediction for the unbalanced rotor configuration, while a positive value denotes an increase. The final passive balancer performance is displayed graphically in Figure 5-19. Examining the figure, passive balancing performance improved with increasing mass imbalance, suggesting that small imbalance conditions may not provide sufficient impulse to the balancing masses to overcome the static
friction between the masses and the track wall. There were two outliers located at approximately 2000 g-mm imbalance where performance was worse than the unbalanced case. The decrease in performance for the cases can be attributed to the balancing masses becoming fixed in positions that exacerbated the imbalance, while the loading magnitude was not sufficient to perturb them. Overall, the implementation of the final passive balancing device into the rotor stand system reduced fixed frame loading in response to asymmetric ice shed events by 78.821% on average, demonstrating the effectiveness of a passive balancing device finely tuned to a practical application using numerical analysis.

![Figure 5-18. Fixed Frame Loading Amplitude Configuration Comparison](image-url)
Table 5-9. Final Passive Balancer Configuration Performance Summary ($T_a = -12^\circ C$)

<table>
<thead>
<tr>
<th>Test-Shed #</th>
<th>System Imbalance (g-mm)</th>
<th>Avg. Load (lbf.)</th>
<th>Balancer Performance (%)</th>
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<tbody>
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Average: **-74.434%**

Std. Deviation: **19.022%**
Table 5-10. Final Passive Balancer Configuration Performance Summary ($T_a = -8^\circ C$)

<table>
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<th>Test-Shed #</th>
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<th>Avg. Load (lbf.)</th>
<th>Balancer Performance (%)</th>
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<td>40.73735</td>
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| Average     | -83.208%                |
| Std. Deviation | 19.174%                |
5.4.2 Fixed Frame Loading Time History

A representative time history of the measured system fixed frame loading and angular speed for a final passive balancer configuration continuous icing test is displayed in Figure 5-20.
As the rotor was spun up, the point at which critical speed transition was encountered is represented in the time history as the dramatic increase in fixed frame loading amplitude to approximately 475 lbf. The spike in loading was caused by transients experienced by the system as the critical speed rotor RPM approaches the damped natural frequency of the system. The transient loading led to shaft displacement and high frequency oscillations in the balancing mass positions. The transient behavior exhibited by the system was consistent with that observed in previous experimental studies of passive balancing devices found in literature. As transients loading subsided, the balancing masses assumed asymptotically stable positions in a balanced configuration, illustrated by the measured force amplitude less than 10 lbf. in the steady state region prior to the first ice shed event. The observed fixed frame loading response to each ice shed event was nearly identical. Following each ice shed, the balancing masses were perturbed from their stable position by the change in the system mass distribution, leading to the initial spike in measured load. The balancing mass positions oscillated for short period of time, on the scale of tenths of a second, before reaching stable positions opposed to the rotor mass eccentricity and returning the measured load amplitude to level similar to that observed prior to the ice shed event. The short ice shed response periods and low fixed frame loading magnitude across successive ice shed events exhibited in the time history, illustrated the responsiveness and robustness of the final passive balancer configuration.
As discussed in Section 5.4.1, there were several observed mass imbalance conditions which exhibit exceptionally high fixed frame loading despite the general effectiveness of the final passive balancing device. The loading time history for a continuous icing tests in which a possible unstable mode was reached is displayed in Figure 5-21. Examining the steady state supercritical region prior to the first ice shed event, the measured loading amplitude oscillated with increasing frequency and amplitude over time. This behavior suggested that, while the rotor is in a relatively balanced configuration, the balancing masses were unable to reach asymptotically stable positions, oscillating with greater frequency and range of motion over time. The measured loading amplitude following an ice shed event continues to oscillate, with a larger amplitude of nearly 25 lbf., contrasting with the steady measured values following ice shed events observed in Figure 5-20. The stability of the measured loading amplitude decreased with each ice shed event leading up to the dramatic spike in loading on the third shed, indicating that the balancing mass positions became increasingly unstable with successive shed events until a sufficiently large ice shed event created
an unstable mode, where the balancing masses exacerbate asymmetric system loading. Examining the measured loading following the third ice shed event, the loading amplitude decreases over a period of approximately ten seconds prior to spin down as the system damping and ball-track resistance forces dampen the balancing mass oscillations, returning the system to a more stable state. The behavior exhibited in Figure 5-21 was common across all continuous icing time histories for tests with outlying measured load data, illustrating that the cause of these outlier was likely increasing balancing mass instability with successive ice shed events.

Figure 5-21. Y-direction Loading Time History (Test 29)

5.4.3 Balancing Mass Positions

Hi-speed footage of the final passive balancer was captured during rotor spin up and rotationally stabilized to observe the changes in balancing mass positions in the system subcritical,
critical speed transition, and supercritical operating regions. The subcritical balancing mass positions are pictured in Figure 5-22. Balancer partitions were numbered to establish angular reference points in the rotating frame. In the subcritical region footage, all balancing masses were observed in stable positions. Three of the balancing masses are adhering to their respective partitions as the system rotates counterclockwise, with the fourth in biased position towards partition 2, indicating that the system has excess mass in the direction of partition 2.

![Figure 5-22. Final Passive Balancer Subcritical Ball Positions (500RPM)](image)

The footage captured in the critical speed transition operating region exhibited the transient balancing mass position oscillations that produce the sharp increase in loading through transition
during spin up observed in the loading time history. A frame of the critical speed transition footage is displayed in Figure 5-23. At this moment in the oscillation, the balancing masses are positioned at the extremes in opposition to the excess mass in the partition 2 direction, illustrating that passive balancing driving forces were acting on the masses to perturb them away from their subcritical positions and the masses were able to overcome the ball-track static friction at transition speed.

The footage captured at steady state supercritical operation depicted the balancing masses in asymptotically stable positions, displayed in Figure 5-24. The balancing masses assumed positions that were relatively evenly distributed about the rotational axis with a slight bias towards partition 4. Therefore, the excess mass that was present in the partition 2 direction, suggested by
Figure 5-22, was determined to be of small magnitude. In addition, the bias in the balancing mass positions in the direction opposite to those observed during subcritical operation is consistent with expected passive balancing behavior, as positions change due to the phase shift in system response at supercritical operation. The changes in balancing mass positions with operating region provide concrete examples of passive balancing behavior observed in the final passive balancer configuration that was not present in previous configurations.

Figure 5-24. Final Passive Balancer Supercritical Ball Positions (1200RPM)

Hi-speed footage was captured for two balancing mass ice shed response tests. The mass imbalance introduced by the two ice shed events is summarized in Table 5-11. Measured load data was not included in the final passive balancer performance characterization calculation, as the test
conditions specified for this test were not representative of a realistic icing condition. The footage was analyzed to observe the change in balancing mass positions in response to an asymmetric ice shed. Footage captured several milliseconds prior to an ice shed event depicted the balancing masses in stable positions shown in Figure 5-25. The observed positions show a slight bias towards partition 1, illustrating that the iced rotor was close to mass balanced prior to the ice shed event.

<table>
<thead>
<tr>
<th>Test #</th>
<th>Ice Shed Mass (g)</th>
<th>$R_{OM}$ (mm)</th>
<th>System Imbalance (g-mm)</th>
<th>Rotor Speed at Shed (RPM)</th>
<th>Avg. Fixed Frame Load (lbf.)</th>
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</thead>
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<td>961</td>
<td>217.717</td>
</tr>
</tbody>
</table>

Figure 5-25. Final Passive Balancer Supercritical Ball Positions, Pre-Shed (Test 36)
Hi-speed footage was captured as an asymmetric ice shed occurred on the blade span in the direction of partition 1 at 1126RPM. The balancing masses were immediately perturbed from their stable supercritical positions entering high frequency oscillations between their respective partitions and significant shaft displacement was observed. After approximately 300 milliseconds, the masses reached asymptotically stable positions, pictured in Figure 5-26, close to the most extreme bias towards partition 1 to account for the mass removed by the ice shed event. The footage of the balancing mass position response to asymmetric ice shed confirms that the reduction in fixed frame loading observed in this configuration was due to passive balancing behavior and a passive balancing device can effectively respond to the impulsive mass eccentricity introduced by an asymmetric ice shed event.

Figure 5-26. Final Passive Balancer Supercritical Ball Positions, Stable Post-Shed (Test 36)
5.5 Summary

The shortcomings of the initial passive balancer configuration were used to construct an iterative passive balancer design process based on parametric analyses of passive balancer track radius, axial position, frictional model, and balancing authority. Parametric analysis results identified two major issues with the initial design related to ball-track resistance force (friction) and limited shaft displacement in the balancer plane. Design parameters were tuned individually in each design loop. Numerical simulations evaluated at a critical imbalance condition, determined from experimental data, quantified the predicted performance of each design iteration until a final passive balancer design converged.

The final passive balancer and components crucial to its installation onto the AERTS rotor stand were fabricated and their relevant physical quantities were determined. A test matrix was formulated for the final passive balancer configuration, consisting of 10 continuous icing and 2 balancing mass ice shed response tests, to provide a comprehensive data set to characterize fixed frame loading mitigation performance.

Over the continuous icing tests, system response to 40 ice shed events was experimentally measured and numerically simulated using the mathematical model. The overall model accuracy was 64.844%, which was larger than that for the prior configuration models. The average difference in predicted and measured loading was 19.729 lbf., illustrating the model was accurate in a practical sense. Several outliers in fixed frame loading were observed and attributed to unstable modes arising due to impulsive changes in rotor mass distribution. The performance was calculated by comparing fixed frame loading measurements to predictions for the unbalanced rotor configuration. On average, fixed frame loading in response to asymmetric ice shed was reduced by 78.821%, bringing loading into an acceptable range for operation (< 50 lbf.) for the majority of observed mass imbalance cases. The dramatic improvement in performance from the initial passive balancer,
which did not consider numerical analysis in its design process, to the final passive balancer, whose key design parameters were determined from iterative parametric analyses, illustrates the need for a numerical tool to tune passive balancing design parameters to a specific practical application.

The loading time history for continuous tests was analyzed to observe the loading response for successive ice shed events and observe the time scale for passive balancer response. In the majority of cases, the increase in fixed frame loading introduced by the ice shed event was corrected within one second, returning the loading magnitude to pre-shed conditions. However, the time history for tests containing fixed frame loading outliers exhibited behavior that indicated that the balancing masses were unable to reach asymptotically stable positions and each successive ice led to increasing system instability. The conditions that led to system instability and methods to prevent it warrant further study.

Hi-speed footage recorded during system spin-up and ice shed events was analyzed to observe the response of the balancing masses to different operating regions and mass eccentricity. Behavior consistent with that described by passive balancing dynamics was observed, confirming that the reduction in fixed frame loading in this configuration was due to the presence of passive balancing behavior.
Chapter 6 | Conclusions

This study explored the practical application of passive balancing devices to mitigate vibration in rotor systems operating in adverse icing conditions subject to asymmetric ice shed. A comprehensive passive balancer system model, developed by Haidar, was modified to conduct numerical simulations of the vibration response of full-scale rotor systems. Parameters were adjusted to model a passive balancing device mounted to the AERTS rotor stand following an asymmetric ice shed event. The physical quantities of the rotor stand and its individual components were characterized for modeling purposes. An experimentally validated analytical ice mass model was developed to quantify the mass removed due to an arbitrary ice shed event for the purposes of calculating the system mass eccentricity without needing to stop the rotor and take a physical sample of ice.

Two passive balancing devices were designed and fabricated. An initial device was designed from general design trends observed in literature. A final passive balancer device design was formulated from parametric performance analysis provided by numerical simulations. Icing experiments were conducted on three rotor configurations in the continuous icing envelope at -8 °C and -12 °C to record the vibration response following ice shed events at steady state supercritical RPM. Fixed frame loading measurements following each observed ice shed event were recorded to compare to model predictions and determine passive balancer vibration mitigation performance. The practical application of a passive balancing device to reduce rotor vibration following an asymmetric ice shed event was evaluated.
6.1 Passive Balancer Performance

Rotor stand fixed frame loading was measured following ice shed events at 1200RPM. Continuous icing tests were conducted where multiple ice shed events occurred in quick succession over a 5-minute ice accretion period. Fixed frame loading results for the unbalanced rotor configuration, initial passive balance configuration and final passive balancer configuration are summarized in Figure 6-1. Comparing the configurations, the unbalanced and initial passive balancer configuration exhibited a similar positive trend and loading amplitude across the range of observed mass imbalance conditions. However, the final passive balancer configuration results exhibited no correlation between mass imbalance and fixed frame loading less than 50 lbf. across the majority of observed conditions.

![Figure 6-1. Measured Fixed Frame Loading Configuration Comparison](image-url)
The vibration mitigation performance of each balancing device was quantified by comparing measured fixed frame loading for a given mass imbalance to the model prediction for the unbalanced configuration. A comparison of the calculated balancer performance for the initial and final passive balancing devices is provided in Figure 6-2.

Overall, the initial passive balancing device increased fixed frame loading in response to ice shed events by 8.63%. The degradation in system loading due to the introduction of the initial passive balancer was attributed to large magnitude ball-track resistance forces preventing the free circumferential motion of the balancing masses. In addition, the proximity of the balancer frame to the shaft fixed position prevented sufficient shaft displacement for passive balancing behavior to
occur. These inferences were confirmed upon analysis of the loading time history and balancing mass position data collected for the initial passive balancer configuration.

The final passive balancer reduced fixed frame loading by 78.821% on average, with the largest reductions occurring for the largest observed mass imbalance conditions. The significant reduction in fixed frame loading observed in this study demonstrates the effectiveness of a properly tuned passive balancing device to applications for rotor systems encountering asymmetric ice shed events. Analyzing the loading time history and observed balancing mass position ice shed response, it was confirmed that the significant increase in performance was due to passive balancing dynamics. The loading time history illustrated the robustness of the system as it was able to correct for mass imbalance within several tenths of a second of an ice shed event and maintained low levels of loading throughout multiple ice shed events. The Hi-speed footage captured of the balancing mass positions during system spin up revealed that the behavior of the balancing mass dynamics through the various operating regions was consistent with that outlined by passive balancing physics. Hi-speed footage of the balancing mass response to ice shed event captured the change in position to account for the mass of ice removed from the blade, confirming the ability of a passive balancing device to respond to the impulsive imbalance introduced by the ice shed event. The degraded performance observed at low levels of imbalance was likely due to insufficient asymmetric loading present in the system to allow the balancing masses to overcome the static friction at the track wall and assume beneficial positions. However, performance at low magnitudes of mass imbalance is of little practical concern as, under those conditions, fixed frame loading will remain within system loading tolerances with or without a passive balancing device. Several outliers in fixed frame loading magnitude were identified in the measurements for the final passive balancer configuration. Analysis of the loading time history for the tests containing outliers revealed balancing mass position instability at steady state operation that degraded with each
successive ice shed event. The conditions that led to the system instability for those few cases were unclear and warrant further study.

6.2 Model Accuracy

Experimental fixed frame loading values for all observed mass imbalance conditions were compared to model predictions to validate the comprehensive model for full-scale mass eccentric rotors. The model comparisons for all three configurations tested in this study are summarized in Table 6-1 and Figure 6-3.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Average % Difference</th>
<th>Average Difference in Predicted Load (lbf.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unbalanced Rotor</td>
<td>18.700</td>
<td>38.329</td>
</tr>
<tr>
<td>Initial Passive Balancer</td>
<td>22.480</td>
<td>62.876</td>
</tr>
<tr>
<td>Final Passive Balancer</td>
<td>64.844</td>
<td>19.729</td>
</tr>
</tbody>
</table>

Figure 6-3. Model Accuracy Summary: (Left) Average Difference in Loading given as a Percentage (Right) Average Difference in Loading given as Pounds Force
Based on the average percent difference between model predictions and experimental measurements of fixed frame loading, the model retained accurate predictions for the unbalanced rotor and initial passive balancer configurations. The strong correlation in predictions for the unbalanced rotor configuration validated the method of quantifying balancer performance, and the model consistently underpredicted loading for this configuration, which ensured balancer performance calculations are conservative. The retention of accuracy for the initial passive balancer configuration illustrated the ability for the model to capture the passive balancer dynamics that were limiting the initial passive balancing device performance. Comparing the prior two configuration model accuracy to that of the final passive balancer configuration, there was a large difference in model predictions and experimental based on percentage. However, the practical significance of this value was diminished when considering the large difference in loading magnitude between the three configurations as the majority of loading measurements for the final passive balancer were below 50 lbf. A more practical measure of accuracy for this case was the average difference between the model predictions and experimental measurements in pounds, for which the final passive balancer model was the most accurate. With an average difference in loading of less than 20 lbf., which is well within the dynamic loading tolerance for most rotorcraft components, it was concluded that the model was able to accurately predict the final passive balancer loading for practical application.

6.3 Passive Balancer Design

The significant increase in passive balancer performance between the initial and final designs was precipitated by the design process for the final device. The initial design process sought to implement design trends observed in literature to formulate a robust passive balancing device
for ice shed events. The initial balancer was designed to implement a partitioned track, maximize balancing authority, minimize balancer frame weight and minimize distance between the balancer and the imbalance plane. The goal of the initial passive balancer test campaign was to observed the ability for an unrefined passive balancing device to reduce vibration and identify issues with the initial device. As shown in Figure 6-2, the initial passive balancing device was detrimental to the system fixed frame loading illustrating the sensitivity of performance to key design parameters and the need for an analytical tool to tune such design parameters for specific practical applications. The experimental measurements were compared to model predictions, which were able to effectively capture the issues in the passive balancer performance observed in the experiments.

Key design parameters were identified as balancer track radius, axial position, frictional model, and balancing authority. The experimentally validated model was used to conduct parametric analysis to improve predicted balancer performance in an iterative process that formed the basis for the final passive balancer. Major findings from the parametric analysis were as follows:

- The two major factors inhibiting the performance of the initial passive balancer were ball-track resistance force and limited shaft displacement in the plane of the passive balancer.
- Decreasing track radius improves performance until balancing authority is insufficient to correct for the system mass eccentricity.
- For the cantilevered shaft considered in this study, positioning the balancer plane further from the fixed shaft boundary improved the performance. It was demonstrated that this was true even past the plane of the imbalance.
- Frictional contacts between the balancing masses and outer track wall must be finely tuned to allow free circumferential motion at steady state supercritical operation, while providing enough resistance to allow asymptotically stable balancing mass positions to be reached.
- Excess balancing authority improves performance asymptotically before degrading slightly as resistance force increases with the magnitude of the ball mass.
The significant improvement in performance exhibited by the final passive balancer illustrates the ability for the comprehensive model to function as a design tool to aid in the design process of future applications of passive balancing technology.

6.4 Recommendations for Future Work

The following sections discuss recommendations for work to expand upon the work presented in this study to further the development of passive balancing technology and modeling techniques.

6.4.1 Numerical Analysis

For future work to be conducted on passive balancing devices utilizing numerical simulations, it is recommended that a more complex model of the system shaft be considered. The shaft in this study was assumed to behave as a cantilevered Euler-Bernoulli beam discretized with the corresponding mode shapes for the first three bending and twisting modes. It was observed that the parametric analysis results for position balancer axial position were a departure from those in literature. Constructing an explicit, dynamic finite element model of the shaft for numerical simulations would incorporate more complex shaft dynamics and allow for simpler variation in shaft parameters and boundary conditions to observe their effect on passive balancing performance.

In addition, it is recommended that numerical simulations of more complex rotor configurations be conducted for practical application to modern flight vehicles. For example, coaxial rotors would introduce multiple sources of imbalance at different shaft axial planes. Parametric studies on passive balancer configurations to address this type rotor should be conducted to determine the effect of passive balancer position and whether the presence of multiple passive
balancing devices improves the vibrational response of the system. While the rotor configuration in the AERTS facility was meant to be representative, numerical simulations of a canonical rotor system with a passive balancing device implemented on an actual flight vehicle across various flight conditions should be conducted to confirm the effectiveness of passive balancing technology to the practical application presented in this study.

6.4.2 Modeling Capabilities

For this study, the parameters used to model system components were derived from a mix of experimentally determined values and those obtained from literature. The frictional models implemented into the model were observed to have a significant effect on the predicted passive balancer performance, but were all obtained from studies that did not consider the range of normal force present in the system. Therefore, it is recommended that frictional models are developed based on experimental data of balancing mass response to known system loading conditions, in order to account for the ranges of normal loading experienced by the masses. Experimental approaches to improve the characterization of the system should be developed where possible.

The comprehensive model in this study did not consider any aerodynamic loading or blade dynamics of the rotor system. Computation Fluid Dynamics analysis should be conducted to characterize aerodynamic forces for a configuration with a passive balancing device and the development of equations of motion for blade dynamics in the rotating frame should be implemented into the model to consider a rotor in hover and forward flight. The dynamic loading in these conditions should be modeled to determine the effect of a passive balancing device on asymmetric loading due to unsteady aerodynamics and blade dynamics.
6.4.3 Experimental Work

Further experimental testing is recommended to determined other practical applications for passive balancing technology. The lack of power requirements and simplicity of passive balancing devices make them suitable for application in turbomachinery. Specifically, a passive balancing device implemented into the shaft of a turbine jet engine could provide vibration control in the extreme engine environment. A representative test rig should be constructed to provide experimental data to support this application.

While centrifugal clamp mechanisms and balancing mass drag were considered in the comprehensive model for this study, no experimental data was collected on the effect of either on passive balancer performance. It is recommended that designs for a centrifugal clamping mechanism be formulated and tested to observe the effect of such a mechanism on the and to determine the critical parameters for centrifugal clamp design. In addition, conducting experiments on passive balancing devices with fluids of various viscosity in the tracks would demonstrate the effect of drag on passive balancer performance, in an effort to identify a parameter to further improve the design of passive balancing devices.
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