INVESTIGATION OF BEAMED-MICROWAVE PLASMA
GENERATION IN SUPersonic FLOW

A Dissertation in
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by
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ABSTRACT

Beamed-energy propulsion provides a possible advanced propulsion method for Earth-launch systems with high-specific-impulse electric thrust and high payload mass fraction. The concept behind beamed-energy propulsion is the generation of thrust by heating up propellant with off-vehicle beamed energy, either by laser or microwaves, instead of with the inner (chemical) energy of propellants in traditional propulsion systems. This method can provide higher theoretical specific impulse than traditional chemical propellants. Furthermore, the separation of the energy source from the vehicle can increase the payload mass fraction.

In this research, the proposed method to transform beamed energy of continuous-wave (CW) microwaves to thrust is by the focusing of beamed microwaves onto a supersonic nitrogen flow to generate a plasma that can absorb the microwave energy and heat the supersonic flow. In this dissertation, work in three aspects were reported: theoretical investigation, generating Mach 5 supersonic flow in a parabolic supersonic nozzle, and simulation of the electric field distribution in the parabolic nozzle.

The mechanisms about how energy transfers from a microwave to gas were investigated. After a microwave breakdown, the energy in the microwaves is transferred first to the kinetic energy of electrons. Due to the large mass difference between an electron and a gas particle, the energy of electrons is transferred to gas particles mainly through inelastic collisions. Energy absorption and release via three types of inelastic collisions and excitations of rotational states, vibrational states, and electronic states are discussed in this
dissertation. With the understanding of how the energy is transferred from microwaves to a gas, the coupling coefficients between microwaves and supersonic flow were estimated for two situations. One situation is simulating the environment at sea level. At sea level, a launch vehicle applies highest thrust to lift the payload and all the fuel. In this situation, it is the coupling between a high-power microwave and a supersonic flow with high flow rate. The other situation is simulating the low-pressure environment at high altitude. A launch vehicle needs lower thrust compared with the thrust at sea level due to most of the propellant having been consumed by the time the launch vehicle has reached high altitude. In this situation, coupling occurs between moderate-power microwaves and supersonic flow with low flow rate. Quasi-one-dimensional simulations were conducted to understand the thrust augmentations in different heating conditions in a supersonic flow. The quasi-one-dimensional simulation can be used to analyze the experimental data in future and increase our understanding about the coupling between a microwave and a supersonic flow.

To generate a supersonic flow in our converging–diverging (supersonic) nozzle, a gas feed system capable of supplying higher flow rate is needed according to preliminary experiments. The earlier gas feed system failed to supply enough mass flow rate and generated a shock wave in the nozzle. An improved gas feed system was designed to supply a mass flow rate of 2.53 kg/s and stagnation pressure higher than 2000 psig. The pressure distribution in the improved gas feed system and the maximum flow rate of the system were estimated. At the same time, flow tests were executed to understand the capability of the improved gas feed system.
Computational electromagnetics was utilized to simulate the electric field distribution in our parabolic nozzle. The simulation results obtained with COMSOL Multiphysics showed that there was resonance in the stagnation chamber of our first nozzle design. To eliminate the resonance in the stagnation chamber, a new nozzle was designed with the aid of the computational simulation. At the same time, the electric field distributions in the new nozzle under different incident microwaves were simulated to further understand the electric field distribution in real condition.
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Chapter 1

Introduction

Nowadays, sending an object to space still costs a great deal. The combustion efficiency of a chemical rocket engine, the mainstream propulsion system used in a launch vehicle, achieves 94–99 %.\(^1\) The combustion efficiency is defined as the ratio of the actual energy released to the ideal energy released. This high efficiency implies that current chemical propulsion technology is so efficient that almost all the energy in the chemical reaction can be used by the rocket engine. This also implies that it is difficult to achieve a breakthrough based on current chemical rocket engine technologies. Several methods of advanced propulsion have been proposed for improving the launch technologies, including the method researched in this project: beamed-microwave propulsion. In this chapter, the technology of current rocket engines, advanced propulsion methods, and beamed-energy propulsion are introduced.

1-1 Space Access

To send an object from the ground to low Earth orbit (LEO), one must apply a velocity change of 9.5 km/s to the object. The orbital speed at LEO is about 7.7 km/s. 1.1–1.5 km/s is used to overcome the gravitational potential of Earth. 0.1–0.5 km/s is consumed due to the atmospheric drag and steering the object to orbit. At the same time, roughly 0.4 km/s is obtained from the rotation of Earth, depending on launch site. Thus, a launch vehicle
needs to provide a velocity change of 9.1 km/s to send a payload to LEO. The current source of thrust to send an object to space are chemical rocket engines. This section introduces the basic theory of a rocket engine, contemporary rocket engines, and recent developments of advanced technologies to generate thrust.

1-1-1 Basic rocket theory

A rocket engine produces thrust by ejecting mass with exhaust velocity $v_e$. The behavior of the whole system satisfies Newton’s second law. The governing equation can be derived as

$$F = m \frac{dv}{dt} = - \frac{dm}{dt} v_e. \quad (1-1)$$

In the case of a constant exhaust velocity, the final velocity $v_f$ can be solved as

$$v_f = v_e \ln \frac{m_0}{m_f}, \quad (1-2)$$

where $m_0$ is the initial mass of the entire system and the $m_f$ is the final mass. This is the famous Tsiolkovski Equation. Assuming the initial mass $m_0$ of a launch vehicle is the sum of the final mass $m_f$ and the mass of propellant $m_p$, then the Tsiolkovski Equation can be rearranged to yield

$$v_f = -v_e \ln \left(1 - \frac{m_p}{m_0}\right). \quad (1-3)$$

As shown by Equation 1-3, higher exhaust velocities can achieve a fixed final velocity with a smaller mass ratio of propellant.

In practice, the effect of the ambient pressure should also be considered. The thrust produced by a rocket engine is given by
\[ \tau = \dot{m} v_e + (P_e - P_a)A_e, \]  

(1-4)

where \( \dot{m} \) is the mass flow rate, \( P_e \) is the pressure of the flow at the exit, \( P_a \) is the ambient pressure and \( A_e \) is the area of the exhaust. Considering the effect of the pressure, the equivalent exhaust velocity \( v_{eq} \) is used to estimate the needed amount of propellant to achieve a final velocity change. The equivalent exhaust velocity is defined as:

\[ v_{eq} = v_e + \frac{(P_e - P_a)A_e}{\dot{m}}. \]  

(1-5)

Another similar parameter often used for a propulsion system is the specific impulse \( I_{sp} \), which is defined as:

\[ I_{sp} = \frac{\tau}{\dot{m}g_0} \frac{v_{eq}}{g_0}, \]  

(1-6)

where \( g_0 \) is the gravitational acceleration at sea level and the unit of specific impulse is seconds.

For an ideal rocket, the stagnation chamber is filled with a perfect gas with molar mass \( MW \) at temperature of \( T_c \), and pressure of \( P_c \). The flow in the nozzle is assumed to be isentropic. The exhaust velocity of an ideal rocket can be derived as:

\[ v_e = \sqrt{\frac{2\gamma R T_c}{(\gamma - 1)MW} \left[ 1 - \left( \frac{P_e}{P_c} \right)^{(\gamma - 1)/\gamma} \right]}, \]  

(1-7)

where \( \gamma \) is the ratio of heat capacities of the gas and \( R \) is the ideal gas constant. From Equation 1-7, a gas with low molar mass at high temperature will achieve a high exhaust velocity.

1-1-2 Contemporary rocket engines

Contemporary rocket engines use the chemical energy stored in the propellant to
generate thrust. Current launch vehicles mainly use two kinds of rocket engines: the liquid-propellant rocket engine and the solid-propellant rocket engine. A liquid-propellant rocket engine uses liquid hydrogen (LH₂), liquid methane, or kerosene as the fuel combined with liquid oxygen (LOX) as the oxidizer. A contemporary liquid-propellant rocket engine can achieve a specific impulse between 250 and 450 seconds. A liquid-propellant rocket engine is comprised of two tanks, one for the fuel and one for the oxidizer, two sets of liquid feed systems, the combustion chamber, and the nozzle. It is straightforward to adjust the output thrust and to restart a liquid-propellant rocket engine after engine shut down. However, the complexity of the feed systems and the devices to ensure high efficiency combustion are relatively expensive. The advantages of this type of rocket are ease of adjusting the output thrust, high operational flexibility, and high specific impulse. The drawbacks of liquid-propellant rockets are the complex fuel–oxidizer feed system, large volume due to the low density of the fuel–oxidizer, and the high cost of the entire system.

A solid-propellant rocket engine typically uses a hydrocarbon-based polymer as the fuel and ammonium perchlorate (AP) as the oxidizer. The fuel and the oxidizer are cast into a solid bulk. Current solid-propellant rocket engines can achieve specific impulses between 200–300 seconds. The thrust profile of a solid-propellant rocket engine during operation is determined by the shape of the interior flow cross section. It is almost impossible to reignite a solid-propellant rocket engine after the engine stops. The advantages of the solid-propellant rocket engine are its low volume due to the high density of the fuel and the oxidizer, ease of storage, and low cost. The drawbacks are low specific
impulse compared with liquid rockets and the lack of operational flexibility.

For a single-stage launch vehicle, the practical maximum value of the ratio of the initial mass to the final masses is approximately 8. When using a rocket engine with a specific impulse of 450 seconds—the maximum value that can be achieved by a chemical rocket engine—the final velocity is only 9.17 km/s. This ideal velocity change is just enough to send an object to the LEO. In reality, the specific impulse is lower in the atmosphere so it is even harder to send an object to space with a single-stage launch vehicle using a contemporary rocket engine. To overcome this restriction, multi-stage launch vehicles and strap-on boosters were developed. The multi-stage launch vehicle stacks together several rocket sections. The lower stage is dropped after it burns out to achieve a higher final velocity. Strap-on boosters are boosters installed on a launch vehicle to increase the initial thrust of the launch vehicle. These boosters are also dropped after they burn out to increase the final velocity of the payload. These two designs are typical for a contemporary launch vehicle. For example, the Atlas V is composed of three stages combined with 0-to-5 solid rocket boosters. The Delta IV Medium is a 3-stage vehicle with 0, 2, or 4 solid rocket boosters. The use of multi-stage and strap-on boosters overcomes the insufficient velocity change of a single-stage launch vehicle, but these configurations also increase the structural complexity and the cost of a launch vehicle.

1-1-3 Advanced concepts for space access

As discussed in the previous section, the reason why a launch vehicle costs so much is the exhaust velocity of the propellant is not high enough to send an object to space with
a practical mass ratio. Several advanced propulsion concepts have been proposed in order to overcome this difficulty. These concepts are well compiled in the excellent review written by Ketsdever et al. Some concepts are described in this dissertation. These concepts can be categorized into two types: employing propellant and propellantless. The concepts employing propellant include nuclear propulsion, advanced chemical, air-breathing engines, and beamed-energy propulsion.

Nuclear propulsion is a propulsion system that employs a nuclear reactor as the energy source. The propellant absorbs the heat produced by the reactor and reaches a temperature higher than the temperature that can be achieved using a chemical rocket engine. The propellant uses hydrogen, ammonia, or methane. The architecture of a nuclear propulsion rocket engine is simpler than that of a chemical liquid rocket engine. Nuclear propulsion needs only one propellant feed system to flow the propellant through the nuclear reactor. The only constraint to the chamber temperature is the temperature limit of the material. At the same time, nuclear propulsion use propellants with small molar weight. The specific impulses of fission devices are around 800–1000 seconds. This technology is the one with the highest possibility to operate in the near future. However, a radiation shield is needed for the payload on a nuclear launch system to protect humans and electric devices onboard, which increases mass and decreases its mass ratio. At the same time, nuclear fuels require increased system safety concerns, and access to nuclear fuels is extremely limited and costly.

One of the most important factors determining the performance of a chemical rocket
engine is the energy released in the chemical reaction of the propellants. Among the propellants used in current rockets, the combustion of liquid hydrogen and liquid oxygen can release the highest energy, roughly 10 MJ/kg. Looking for a new high-energy-density propellant is a direction to improve the specific impulse of a rocket engine. One direction focuses on hydrocarbon fuels with novel molecular arrangements that contain more energy. Another direction is polynitrogen. It is expected that the decomposition reaction of N$_4$ or N$_8$ into N$_2$ can release a large amount of energy. But polynitrogen has yet to be produced. A conceptual design using metallic hydrogen as the propellant is proposed by Cole et al. Metallic hydrogen is a solid with metal properties and composed of atomic hydrogen. This material is predicted to be metastable and can release 216 MJ/kg during recombination into molecular hydrogen, much higher than that released in the combustion of hydrogen and oxygen. The initial calculation shows metallic hydrogen can achieve a temperature of 5600 °C and a specific impulse of 1700 seconds. Methods for producing metallic hydrogen are still in the research phase. The transition to metallic hydrogen is observed by Dias et al. at the pressure of 495 GPa. The stability and other properties of metallic hydrogen are not well studied.

Beamed-energy propulsion is the concept that uses an off-vehicle energy source to generate or increase the thrust. The energy is transmitted to the engine via a laser beam or a microwave beam. There are several concepts proposed to transform the exterior energy into thrust. These concepts are explained in detail in the next section.

As shown in Figure 1-1, the specific impulses of various air-breathing systems are
higher than a rocket engine in different Mach number ranges. It is reasonable to consider taking advantage of this for space access. Various concepts of the launch sequences with combined systems are proposed. Two main categories of concepts are the rocket-based combined cycle (RBCC) and the turbine-based combined cycle (TBCC). Traditionally, these two cycles are common in the last three cycles (ram, scram, and rocket) and the only difference is the first cycle. A rocket is used for the RBCC and the turbine engine for the TBCC. The possibility of using different combinations in the middle cycles are also discussed. Both cycles are assumed to cost less than a rocket vehicle. However, recent analysis shows that the costs of the combined cycles might be the same or even higher.\textsuperscript{9,10}

![Figure 1-1: The performance of air-breathing engines and rockets for different Mach number ranges.\textsuperscript{3}](image-url)
Propellantless advanced propulsion concepts includes gun launch systems, and a space elevator and tower. The gun launch system accelerates the payload to the required velocity in a short distance, i.e., literally a gun. These systems can be divided according to the source of acceleration: the electromagnetic rail system (railgun), and the gas gun system.

The projectile of a railgun system is accelerated by the Lorenz force generated by the interaction between the current in the projectile and an exterior magnetic field. The current in the projectile is supplied by two rails around the projectile. The acceleration of the projectile is determined by the strength of the current and the magnetic field. The structure of a railgun system can be simpler compared with a traditional gas gun system because there is no gas with extremely high temperature and pressure. The railgun also can achieve a higher muzzle velocity if sufficient power can be supplied. A railgun system has been demonstrated the ability to accelerate a 4-kg projectile to a velocity of 2 km/s.\textsuperscript{11}

The gas gun system accelerates a projectile via a high-pressure gas. The high-pressure gas is generated either by the combustion of propellants or by a piston driven by the explosion of powder. The High Altitude Research Project (HARP) gun system adapted from a surplus naval gun tube can fire a 180-kg projectile to a muzzle velocity of 3.6 km/s.\textsuperscript{12} The Super High Altitude Research Project (SHARP) demonstrated the ability to fire a 5-kg projectile to a velocity of 3 km/s.\textsuperscript{12} Both railgun and gas gun systems need to improve their performance to achieve the velocity needed for space access. At the same time, the issues of the aerothermal effect experienced during flight and the high acceleration at the beginning of the launch are waiting to be solved. Besides directly using these gun systems...
to launch an object to space, using the gun systems as the launch assist systems is also being considered.

The space elevator, platforms, and towers are similar ideas. All of them propose building a tower to orbital altitude. The height of the platform or tower is around 100 km. Launching from such height can reduce the cost of space access. However, the cost of space access is mainly on achieving the orbital velocity. Launch from such a height does not save much. The space elevator is a building extending to geosynchronous equatorial orbit (GEO). An object there can easily stay in space without any further acceleration. Space elevator designers envision a cable extending from the ground to GEO and a climber to deliver the payloads from ground to space. The main challenge of the space elevator is that a material with a very high tensile strength (65–120 GPa) is needed. The carbon nanotube (CNT) is a possible solution to this challenge. There are other issues needed to be solved, such as high construction expense and the energy source for the climber.

1-2 Beamed-Energy Propulsion

The basic concept of beamed-energy propulsion is generating or increasing thrust by adding energy from an exterior source to the propulsion system through the beamed energy instead of only using the energy in the propellant. Energy can be transmitted by either a laser beam or a microwave beam. The concept of laser propulsion was first introduced by Arthur Kantrowitz in 1972. The concept of the microwave rocket was first proposed by Shad and Moriarty in 1965. Because of the beam divergence of microwaves, research on
beamed-energy propulsion has primarily focused on transmitting the energy by laser beam. In recent years, however, high-power microwave sources at higher frequencies have been developed, which mitigated the disadvantage of beam divergence. Hence, beamed-microwave propulsion has recently received renewed research interest. This section introduces past research on the beamed-laser propulsion and beamed-microwave propulsion.

1-2-1 Beamed-laser-energy propulsion

There are four basic approaches to using a laser beam to generate thrust: the laser sail, laser ablation, laser detonation, and via a heat exchanger.

The laser sail uses a sail craft to intercept the laser beam and utilize the photon pressure of the high-power laser beam. It is essentially a photon propulsion utilizing an incident laser beam. The momentum-coupling coefficient $C_m$ is defined as the ratio of the thrust generated to the input power of the laser beam. The $C_m$ of a sail craft (only 6.7 nN/W\textsuperscript{16}) is so low that it is not considered a pragmatic thrust source for a launch vehicle.

Laser-ablation propulsion is a method that focuses a laser beam onto the surface of a solid or liquid propellant to produce a jet of vapor or plasma. The resulting reaction force on the surface generates thrust. Phipps et al. provides a good review of the development of this technology.\textsuperscript{16} The system based on this concept can be as simple as a sheet of propellant waiting to be illuminated by a laser beam. The thrust can be controlled by the intensity of the incident laser beam. Specific impulses in the range of 200–3600 seconds have been demonstrated.\textsuperscript{17} One issue with this concept is the choice of the propellant to
generate high $C_m$. Several materials were investigated and found to have $C_m$ in the range of 100–10000 N/MW.$^{16}$ A specially designed propellant sheet composed of multi-layers of different materials can achieve a $C_m$ of 3 kN/MW and operate for hours.$^{18}$ Despite these breakthroughs, the laser-ablation propulsion is still uneconomical for ground launch compared with contemporary rocket engines.$^{19}$

Another method to add the energy of a laser beam to a propellant is by generating a laser breakdown in the propellant. The propagation process of the energy in the plasma into ambient air after a breakdown is generated is described in detail by Root.$^{20}$ The main mechanism of the laser energy absorption in the plasma after the breakdown is inverse bremsstrahlung. When the temperature is less than 10,000 K, electron–neutral inverse bremsstrahlung is the main absorption process. When the ionization rate of the air reaches 1%, the main absorption process switches to electron–ion inverse bremsstrahlung. This plasma cloud tends to move in the direction toward the laser and forms an absorption wave. At the same time, there is a shock wave generated due to the quick temperature rise. The ambient air gains energy via the absorption wave and the shock wave. There are three plasma wave modes observed: the laser-supported-combustion (LSC) wave, the laser-supported-detonation (LSD) wave, and the laser-supported-radiation (LSR) wave. These three waves move at different velocities and result in different pressure, temperature, and density changes at the transition zone between the plasma and ambient air as shown in Figure 1-2. The LSD wave moves at the fastest velocity and generates the highest impulse to the ambient air. Pulsed-detonation-laser propulsion uses this impulse to provide thrust.
Pulsed-laser-detonation propulsion is an air breathing system and carries no propellant on board. The operating cycle of pulsed-laser-detonation propulsion uses a pulsed laser to induce a detonation in the nozzle and then the air flows back from ambient to the nozzle until the next laser pulse occurs. A vehicle flying with a pulsed detonation laser engine developed by Myrabo was first demonstrated in 1998. That vehicle demonstrated the ability to fly to the height of 71 m with the mass up to 50 g using a 10-kW CO\textsubscript{2} pulsed laser. The maximum $C_m$ demonstrated in the experiment was 460 N/MW. Sasoh proposed the concept of the Laser-driven In-Tube Accelerator (LITA) as shown in Figure 1-3. The LITA concept places the vehicle in a tube, and projects a laser beam on the front of the vehicle. The laser is scattered by the fore body of the vehicle first, and then refocused to the rear of the vehicle by mirrors built around the tube. The vehicle is accelerated by the detonation induced by the refocused laser beam. Ageichik et al. proposed another system, called the AeroSpace Laser Propulsion Engine (ASLPE) as shown in Figure 1-4. The concept of the ASLPE shares the feature of refocusing the laser from the front to the rear of the vehicle. However, the mirror to refocus the laser is built around the vehicle in the ASLPE. The experiments with ASLPE gave a $C_m$ in the range of 250–400 $\mu$N/W.
The laser heat exchanger (HX) thruster is a design that uses a heat exchanger array to absorb the energy of the laser beam and the flow passing through the heat exchanger array to gain temperature. Similar to the nuclear thruster, the flow system on board a HX thruster is essentially a cold flow system. A vehicle design capable of bringing 122 kg of payload to orbit was described by Kare in a report in 2003. That vehicle planned to use a 100-MW laser and was estimated to achieve a specific impulse of ~600 seconds.

Figure 1-2: The structure of the transition regions between the plasma and the air for the LSC, LSD, and LSR waves.20
Figure 1-3: Illustration of the LITA concept, upstream beam operation.\textsuperscript{22}

Figure 1-4: The ASLPE vehicle.\textsuperscript{23}
1-2-2 Beamed-microwave-energy propulsion

In the last 20 years, microwave sources with megawatt-level power at 100 GHz have been developed.\textsuperscript{25} Compared with the era from 1960–1975, when available high average power sources were all below 10 GHz,\textsuperscript{25} the disadvantage of beam divergence is mitigated. At the same time, high conversion efficiency from electricity to microwave and a lower cost of development of the microwave source make microwave energy propulsion an attractive option.\textsuperscript{26}

The development of microwave propulsion is well described in a review by Komurasaki et al.\textsuperscript{27} Two methods were developed to convert the microwave energy into thrust: the microwave-supported-detonation (MSD) and the heat exchange engine.

An MSD engine was first demonstrated by Nakagawa et al.\textsuperscript{28} An air-breathing vehicle with a parabolic nozzle was built to focus the microwave beam. This 10-g-mass vehicle was lifted to the altitude of 2 m. A gyrotron with output power of 500 kW to 1 GW at a frequency of 110 GHz was employed in this experiment, which demonstrated a maximum $C_m$ of 395 N/MW. In 2009, a 126-g-mass vehicle built by Oda et al. was lifted to an altitude of 1.2 m.\textsuperscript{29} A time-averaged thrust of 2.3 N was measured. The $C_m$ in this experiment was 100 N/MW. In 2011, a thruster with higher thrust and $C_m$ was achieved by the Komatsu et al.\textsuperscript{30} The improvement was mainly due to an improved switch in the gyrotron that increased the duration and the repetitive rate of the microwave pulses. This thruster achieved a thrust of 30 N and a $C_m$ of 360 N/MW.

One difficulty of an MSD thruster is the air-breathing mechanism. The influence of
the efficiency of air-breathing was investigated by Shiraishi et al.\textsuperscript{31} The air-breathing mechanism of an MSD thruster should be able to refill the whole cylinder back to pressure before the next detonation in the time between pulses or the $C_m$ will decrease. A reed valve system is proposed by Fukunari et al. to refill the thruster efficiently.\textsuperscript{32} The switch in the reed valve system is controlled by the difference between the pressure in the thruster and pressure outside of it. Another difficulty is the size of the microwave beam increasing as the altitude increases. A thruster with a microwave concentrator was designed and tested by Fukunari et al.\textsuperscript{33} A cone-shaped microwave concentrator with a 250-mm diameter at one end and 50.4-mm diameter at the other end was attached to an MSD thruster. This design achieved a $C_m$ of 204 N/MW. At the same time, abnormal air breakdown was observed in a multi-pulse test.

The concept of a microwave thermal thruster was proposed by Pakin.\textsuperscript{34} A vehicle with a heat exchanger on the side and the hydrogen flowing through the heat exchanger was described in the report. A specific impulse of 700 seconds and the temperature at the exit of the heat exchanger of 1800–2200 K were estimated. In 2014, a 1.8-kg MTLS vehicle was built and lifted to a height of 10 m.\textsuperscript{35} The rocket could produce a peak thrust of 88 N when a microwave beam with an average intensity of 26 W/cm$^2$ illuminated the heat exchanger. Compared with the 72-N peak thrust without the heat exchanger operating, this represents a 22% increase.
1-3 Project Description

Our idea for utilizing beamed energy, as shown in Figure 1-5, is to intercept the microwave beam with the wall of the diverging section of a converging–diverging, or supersonic, nozzle, which focuses the energy of the beam along the axis of nozzle, and generates a gaseous plasma. Plasma absorbs the electromagnetic energy from the beam and heats the supersonic flow. The concept of absorbing beam energy from laser to generate a plasma has also been proposed. Focusing a microwave beam transmitted from several meters away and creating a gaseous plasma was initially demonstrated by Moriarty and Brown in 1968. Heating a flow with a plasma can achieve a higher temperature than the heat exchanger concept. Higher temperature leads to higher exhaust velocity and pressure in the exhaust and thus higher thrust and higher specific impulse can be obtained. Another advantage of heating using a plasma is the temperature increase in the stagnation chamber of the nozzle can be avoided because the process of capturing energy from beam is mainly in the diverging section of the nozzle where the temperature is falling.
Preliminary tests of igniting and sustaining a gaseous plasma in a supersonic flow of nitrogen were conducted at Penn State in 2008. In these experiments a converging–diverging nozzle was formed from a 6-mm-ID quartz tube. Nitrogen was metered through the tube at a rate of 106 mg/s and the supersonic region downstream of the nozzle throat was positioned to traverse through a half-height rectangular microwave waveguide. Plasma was able to be ignited due to 2.45-GHz microwave surface waves at an input power of 650 W and sustained down to a power of 200 W as shown in Figure 1-6.

Figure 1-5: Schematic of beamed energy concept wherein microwaves are directed into the diverging (supersonic) portion of a nozzle and focused to heat a propellant to plasma temperatures.
In this research, the concepts of igniting a gaseous plasma in a supersonic flow with a beam of continuous microwaves and capturing the energy of the beam to generate thrust were investigated. This time, besides igniting a plasma in a supersonic flow, the thrust produced by the nozzle and pressure in the nozzle is also to be measured. A converging–diverging nozzle, which can accelerate the gas flow to Mach 5, was designed and manufactured. At the same time, the diverging section must be able to focus the microwave beam. Ports along the diverging section of the nozzle were included for the installation of pressure transducers. A thrust measurement instrument was also designed. Thrust and pressure measurement instruments were all integrated and tested with a nitrogen flow.

In addition to an experiment with 94-GHz microwaves, a plasma ignition test in a

Figure 1-6: Microwave nitrogen plasma in supersonic \((M = 2.82)\) portion of converging–diverging nozzle with 650 W of applied power.

1-4 Contributions of this Research

In this research, the concepts of igniting a gaseous plasma in a supersonic flow with
vacuum chamber is also planned. The vacuum chamber simulates the environment at high altitude. A TWTA that can generate microwaves at 30 GHz and maximum output power of 175 W will be used.

A quasi-one-dimensional numerical simulation was executed. With a quasi-one-dimensional assumption, the results of different heating power and regions were simulated. By comparing experimental and simulation results, properties of the coupling between the supersonic flow and microwaves can be better understood. Simulations with COMSOL Multiphysics were conducted to calculate the focusing of the microwaves at different frequencies in the divergent portion of the nozzle.

1-5 Dissertation Overview

In this dissertation, this first chapter introduces the advanced concepts for space access and beamed-energy propulsion. The second chapter introduces the basic theories associated with this project, including the properties of microwave, the plasma theories associated with how microwave energy is transferred to a gas, and the flow theories describing how a heated gas generates thrust. Quasi-one-dimensional simulations predicting the thrust and pressure in the nozzle under different coupling conditions between the microwave and a supersonic flow are also shown in this chapter. The third chapter reports the effort to achieve a supersonic flow with a stagnation pressure of 2000 psig in a supersonic nozzle. A gas feed system was built and tested. The gas feed system can supply a maximum flow rate of 2.0 kg/s, which is not enough to achieve a supersonic flow with a
stagnation pressure of 2000 psig in previous nozzle. However, it is enough for a new nozzle with a throat diameter of 8 mm. The fourth chapter shows the simulation results of the electric field distribution in the supersonic nozzle. This chapter reports the process of designing a new nozzle with the aid of simulations to eliminate the high strength spots of electric field in the stagnation chamber. In addition, the electric field distributions in the new nozzle under different conditions are also shown in Chapter 4. The final chapter provides conclusions and proposes future research.
Chapter 2

Basic Theory

2-1 Microwaves

2-1-1 Microwaves in free space

The term “microwave” is used for electromagnetic waves with wavelengths between 1 mm and 10 cm, corresponding to the frequency between 3 GHz to 300 GHz. To describe the behavior of electromagnetic waves in space involves solving Maxwell’s Equations. The frequency of microwaves is too high to use standard circuit theory, which assumes the electric potential at any point along a line is the same. This assumption is not satisfied for microwave propagation. At the same time, the wavelength of microwaves is too long to apply geometric optics in many situations. Geometric optics is applicable to describe the behavior of microwaves in some circumstances, which is referred as quasi-optical.

The behavior of a microwave propagating in free space can be described by the solution of Maxwell’s Equations. Maxwell’s Equations in a source free and homogeneous region filled with media with permittivity as \( \varepsilon \) and permeability as \( \mu \) are:

\[
\nabla \cdot \vec{E} = 0 , \\
\n\nabla \cdot \vec{B} = 0 , \\
\n\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} , \text{ and} \\
\n\n\nabla \times \vec{B} = \mu \varepsilon \frac{\partial \vec{E}}{\partial t} ,
\]

\( (2-1-1) \) \( \quad (2-1-2) \) \( \quad (2-1-3) \) \( \quad (2-1-4) \)
where \( \mathbf{E} \) is the electric field and \( \mathbf{B} \) is the magnetic field.

Assuming \( e^{i \omega t} \) dependence for the oscillating electric and magnetic field in Maxwell’s Equations, we obtain\(^{39}\)

\[
\nabla \times \mathbf{E} = -i \omega \mathbf{B}, \quad (2-1-5)
\]
\[
\nabla \times \mathbf{B} = i \omega \mu \varepsilon \mathbf{E}. \quad (2-1-6)
\]

Taking the curl of Equation 2-1-5 and substituting in Equation 2-1-6, we obtain

\[
\nabla \times \nabla \times \mathbf{E} = -i \omega \nabla \times \mathbf{B} = \omega^2 \mu \varepsilon \mathbf{E}. \quad (2-1-7)
\]

The left side can be rewritten as

\[
\nabla \times \nabla \times \mathbf{E} = \nabla \left( \nabla \cdot \mathbf{E} \right) - \nabla^2 \mathbf{E}. \quad (2-1-8)
\]

Finally, we obtain\(^{39}\)

\[
\nabla^2 \mathbf{E} + \omega^2 \mu \varepsilon \mathbf{E} = 0. \quad (2-1-9)
\]

This equation is the Helmholtz Equation for \( \mathbf{E} \). We can treat the magnetic field in the same manner to obtain\(^{39}\)

\[
\nabla^2 \mathbf{B} + \omega^2 \mu \varepsilon \mathbf{B} = 0. \quad (2-1-10)
\]

The wave number can be defined as

\[
k = \omega \sqrt{\mu \varepsilon}. \quad (2-1-11)
\]

The general solution of the electric field is\(^{39}\)

\[
\mathbf{E}(\mathbf{r}, t) = \mathbf{E}^+ \cos \left( \omega t - \mathbf{k} \cdot \mathbf{r} \right) + \mathbf{E}^- \cos \left( \omega t + \mathbf{k} \cdot \mathbf{r} \right). \quad (2-1-12)
\]

The direction of the wave number points in the direction of travel of the wave. \( \mathbf{E}^+ \) and \( \mathbf{E}^- \) are the constant amplitudes for the component of waves traveling in the \( +\mathbf{k} \) direction and the \( -\mathbf{k} \) direction, respectively. The phase velocity is
Because this velocity does not involve the transmission of material or energy, the phase velocity can exceed the speed of light. The group velocity is defined as

\[ v_g = \left( \frac{d\omega}{dk} \right)^{-1} \bigg|_{\omega = \omega_0}, \]

where \( \omega_0 \) is the frequency. This is the velocity that the energy of the wave travels at, so this velocity must be smaller than the speed of light.

Rearranging Equation 2-1-5 we obtain the relation between electric field and the magnetic field,

\[ \vec{B} = \frac{i}{\omega} \vec{\nabla} \times \vec{E} = \frac{i}{\omega} \vec{k} \times \vec{E} = i \sqrt{\mu \epsilon} \frac{\vec{k} \times \vec{E}}{k} = \frac{1}{v_p} \frac{\vec{k} \times \vec{E}}{k} e^{i\pi}. \]

We can see that the magnetic field is perpendicular to the electric field with a 90° phase shift. The magnitude of the magnetic field is the magnitude of the electric field divided by the phase velocity in that media.

The energy radiated by an EM wave is given by the Poynting vector defined as

\[ \vec{S} = \frac{1}{\mu} \vec{E} \times \vec{B} \]

The time-averaged energy intensity is given by

\[ I = \frac{1}{2} ceE^2 \]

For a lossy media, an electromagnetic wave attenuates in media because of the conductivity or damping in the media. The dielectric and magnetic damping can be described by introducing the complex electric susceptibility and permittivity. Combining with the conductivity \( \sigma \), the propagation equation becomes\(^{39} \)
We define the complex propagation constant $\gamma$ as

$$\gamma = i\omega \sqrt{\mu\epsilon \left(1 - i \frac{\sigma}{\omega\epsilon}\right)} = \alpha + i\beta,$$  \hspace{1cm} (2-1-19)

where $\alpha$ is the attenuation constant, and $\beta$ is the phase constant. The temporal and spatial dependence of the solved fields becomes

$$e^{-\alpha r} \cos(\omega t - \vec{\beta} \cdot \vec{r}),$$  \hspace{1cm} (2-1-20)

the phase velocity becomes

$$v_p = \frac{\omega}{\beta},$$  \hspace{1cm} (2-1-21)

and the group velocity becomes

$$v_g = \left(\frac{d\beta}{d\omega}\right)^{-1} \bigg|_{\omega = \omega_0}. \hspace{1cm} (2-1-22)$$

If we expand the complex electric susceptibility $\epsilon = \epsilon' - i\epsilon''$, the complex propagation constant $\gamma$ can be written as

$$\gamma = i\omega \sqrt{\mu\epsilon \left(1 - i \frac{\sigma}{\omega\epsilon}\right)} = i\omega \sqrt{\mu\epsilon' \left(1 - \tan\delta\right)},$$  \hspace{1cm} (2-1-23)

where $\tan\delta = \frac{\omega\epsilon'' + \sigma}{\omega\epsilon'}$ is called loss tangent.

In the atmosphere, the attenuation is mainly due to the absorption by water and oxygen molecules. There are several frequencies of the absorption spectrum of water and oxygen molecules in the microwave region. The attenuation coefficient will peak around those frequencies. For water molecules, the absorption frequencies in the microwave range are 22.2 GHz and 183.3 GHz. For oxygen molecules, 60 GHz and 120 GHz are in its absorption
spectrum. The resultant atmospheric attenuation coefficient versus frequency is shown in Figure 2-1. There is little atmospheric attenuation for the microwaves below 10 GHz. For higher frequencies, the frequency regions near 35 GHz, 94 GHz, and 135 GHz can transmit with minimized attenuation.

![Atmospheric attenuation versus frequency at sea level and at 9150-m altitude.

Figure 2-1: Atmospheric attenuation versus frequency at sea level and at 9150-m altitude.]

2-1-2 Microwaves in waveguides

To transmit microwaves with low power loss, waveguides or transmission lines are used. For propagation within waveguides or transmission lines, there are several transverse modes that can exist. These transverse modes can be divided into four categories by the electric field and magnetic field component in the transverse direction (the $z$-direction
The transverse electromagnetic modes, or TEM modes, are the modes with $E_z = 0$ and $B_z = 0$. The transverse electric modes, or TE modes, are the modes with $E_z = 0$, but $B_z \neq 0$. The transverse magnetic modes, or TM modes, are the modes with $B_z = 0$, but $E_z \neq 0$. And the hybrid modes, or HE and EH modes are the modes with $E_z \neq 0$ and $B_z \neq 0$. The TEM mode can only exist where there are two or more separated conductors, such as in transmission lines. In rectangular and circular waveguides, the TE and TM modes are the modes used. In corrugated waveguides, the hybrid modes, HE and EH modes, are mainly considered.

To understand the characteristics of the waveguides and transmission lines, we must start by solving for the electric and magnetic fields inside. Other properties of a waveguide such as suitable frequency, impedance, and conductor loss can be derived from the distribution of electric and magnetic fields inside.

For a rectangular waveguide, the width of the waveguide is $a$ along the $x$-direction and $b$ along $y$-direction. By convention, the longer side is along the $x$-direction, namely $a \geq b$. The boundary conditions are

$$E_x = 0, \text{ at } y = 0, b \quad \text{and}$$

$$E_y = 0, \text{ at } x = 0, a . \quad (2-1-24)$$

For TE modes, $E_z = 0$. The other components of electric and magnetic fields can be solved as

$$H_z(x, y, z) = A_{mn} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-i\beta z} , \quad (2-1-26)$$

$$E_x(x, y, z) = \frac{i\omega \mu \pi}{bk} A_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{-i\beta z} , \quad (2-1-27)$$
\[ E_y(x, y, z) = \frac{-im\omega \mu \pi}{ak_r^2} A_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-i\beta z}, \quad (2-1-28) \]
\[ H_x(x, y, z) = \frac{im\beta \pi}{ak_r^2} A_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-i\beta z}, \quad (2-1-29) \]
\[ H_y(x, y, z) = \frac{in\beta \pi}{bk_r^2} A_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{-i\beta z}, \quad (2-1-30) \]

where \( \vec{H} = \vec{B}/\mu \), \( k_c^2 = \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \), \( A_{mn} \) is the amplitude of this mode, namely the \( \text{TE}_{mn} \) mode, and the propagation constant \( \beta \) is
\[ \beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left( \frac{m\pi}{a} \right)^2 - \left( \frac{n\pi}{b} \right)^2}. \quad (2-1-31) \]

The propagation constant \( \beta \) should be real, or the wave will decay as it travels in the waveguide. Therefore, the wave number \( k \) should satisfy
\[ k > k_c = \sqrt{\left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2}. \quad (2-1-32) \]

The term \( k_c \) is called the cutoff wave number. The cutoff frequency of each mode (each combination of \( m \) and \( n \)) is given as
\[ f_{cnm} = \frac{1}{2\pi \sqrt{\mu\varepsilon}} \sqrt{\left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2}. \quad (2-1-33) \]

The lowest cutoff frequency among all the TE modes is the frequency of the \( \text{TE}_{10} \) mode, which is
\[ f_{c10} = \frac{1}{2a\sqrt{\mu\varepsilon}}. \quad (2-1-34) \]

For TM modes, \( B_z = 0 \). The other components of electric and magnetic fields are
\[ E_x(x, y, z) = B_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{-i\beta z}. \quad (2-1-35) \]
\[ H_x(x, y, z) = \frac{im\omega \pi}{bk_r^2} B_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-i\beta z}. \quad (2-1-36) \]
The cutoff frequency of each mode is also

\[ f_{cnm} = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}. \]  

(2-1-40)

According to the solution of \( E_z \), \( E_z \) will disappear when either \( m \) or \( n \) is zero. The lowest cutoff frequency among TM modes is the frequency of the TM\(_{11}\) mode, which is

\[ f_{c11} = \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}. \]  

(2-1-41)

The energy loss in a waveguide is contributed by the dielectric loss and the conductor loss. The dielectric loss is the loss due to the dielectric damping of the media filling the waveguide. The conductor loss is mainly due to the induced current in the walls of the waveguide.

The dielectric loss in the media filling the waveguide can be derived by generalizing the propagation constant in a waveguide into a complex number. At the same time, we assume the conductivity of the media is almost zero and the loss tangent of the media is small. Combining Equation 2-1-23 and Equation 2-1-31, we can generalize the propagation constant into

\[ \sqrt{k_c^2 - \omega^2\mu\varepsilon(1 - i\tan\delta)} \approx \sqrt{k_c^2 - k^2} + \frac{i k^2 \tan\delta}{2 \sqrt{k_c^2 - k^2}} \]  

(2-1-42)

where \( k = \sqrt{\omega^2\mu\varepsilon} \). We obtain the attenuation constant as
\[ \alpha = \frac{k^2 \tan \delta}{2 \beta}. \]

The conductor loss in a waveguide can be derived by calculating the power loss per unit length at the wall of the waveguide and then obtaining the attenuation constant. From Equation 2-1-20, we know that the electric and magnetic fields attenuate as \( e^{-ar} \) after traveling a distance \( r \) in a lossy media. Combining with Equation 2-1-16, we find that the power intensity decays \( e^{-2ar} \) after traveling a distance \( r \). The relation between power loss rate, the power loss per unit length, \( P_1 \) and incident power \( P_0 \) can be derived

\[
P_1 = - \frac{\partial(P_0 e^{-2ar})}{\partial r} \bigg|_{r=0} = 2\alpha P_0 \tag{2-1-44}
\]

Therefore,

\[
\alpha = \frac{P_1}{2P_0} \tag{2-1-45}
\]

The power loss at the wall of a waveguide can be calculated by integrating the outward Poynting vectors all over the inner space of a waveguide. For the \( TE_{mn} \) modes, the attenuation constant due to conductor loss \( \alpha_c \) is

\[
\alpha_c = \frac{2}{\sigma \delta_s b n} \sqrt{1 - \frac{f_{enm}^2}{f^2}} \left\{ \left(1 + \frac{b}{a}\right) \frac{f_{enm}^2}{f^2} + \left(1 - \frac{f_{enm}^2}{f^2}\right) \left[\frac{b^2_{enm} a^2 m^2 + n^2}{a^2 b^2 m^2 + n^2}\right]\right\}, \tag{2-1-46}
\]

where \( \sigma \) and \( \delta_s \) are the conductivity and skin depth of the wall material, and \( \eta \) is the intrinsic impedance of the material in the waveguide. The attenuation constant \( \alpha_c \) of the \( TM_{mn} \) modes is

\[
\alpha_c = \frac{2}{\sigma \delta_s b n} \sqrt{1 - \frac{f_{enm}^2}{f^2}} \left[\frac{b^3_{enm} a^2 m^2 + n^2}{a^2 b^2 m^2 + n^2}\right]. \tag{2-1-47}
\]

For a circular waveguide with inner radius \( a \), we obtain the cylindrical Bessel
function for the $\rho$-dependent component of electric or magnetic fields in the $z$-direction.

The solutions are the combinations of the Bessel functions of the first and the second kind. However, the second kind goes to infinity when $\rho = 0$, so it must be eliminated.

For the TE mode, the electric and magnetic fields are solved as

$$H_z(\rho, \phi, z) = (A \sin \phi + B \cos \phi)J_n(k_c \rho) e^{-i\beta z}, \quad (2-1-48)$$

$$E_\rho(\rho, \phi, z) = \frac{-i n \omega \mu}{\rho k_c^2} (A \cos \phi - B \sin \phi)J_n(k_c \rho) e^{-i\beta z}, \quad (2-1-49)$$

$$E_\phi(\rho, \phi, z) = \frac{i \omega \mu}{k_c} (A \sin \phi + B \cos \phi)J'_n(k_c \rho) e^{-i\beta z}, \quad (2-1-50)$$

$$H_\rho(\rho, \phi, z) = \frac{-i \beta}{k_c} (A \sin \phi + B \cos \phi)J'_n(k_c \rho) e^{-i\beta z}, \quad (2-1-51)$$

$$H_\phi(\rho, \phi, z) = \frac{-in \beta}{\rho k_c^2} (A \cos \phi - B \sin \phi)J_n(k_c \rho) e^{-i\beta z}, \quad (2-1-52)$$

where $J_n(k_c \rho)$ is the Bessel function of the first kind

$$J_n(k_c \rho) = \sum_{m=0}^{\infty} (-1)^m \frac{(\frac{1}{2}k_c \rho)^{2m+n}}{m!(m+n)!}. \quad (2-1-53)$$

The cutoff wavenumber $k_c$ is

$$k_c = \frac{p'_{nm}}{a}, \quad (2-1-54)$$

where $p'_{nm}$ is the $m$th solution of the derivative of the Bessel function of first kind $J'_n(k_c \alpha) = 0$. The cutoff frequency of a TE$_{nm}$ mode is

$$f_{c_{nm}} = \frac{p'_{nm}}{2\pi a \sqrt{\mu \epsilon}}. \quad (2-1-55)$$

The attenuation constant due to conductor wall loss is

$$\alpha_c = \frac{1}{\sigma \delta_s b m \sqrt{1-f_{c_{nm}}^2/f^2}} \left( f^2 - \frac{n^2}{p'_{nm}^2 - n^2} \right). \quad (2-1-56)$$

For the TM modes, the electric and magnetic fields are solved as
The cutoff wavenumber $k_c$ is

$$k_c = \frac{p_{nm}}{a},$$  \hspace{1cm} (2-1-62)

where $p_{nm}$ is the mth solution of the Bessel function of first kind $J_n(k_c a) = 0$. The cutoff frequency of a $TM_{nm}$ mode is

$$f_{c_{nm}} = \frac{p_{nm}}{2\pi a \sqrt{\mu \epsilon}}.$$  \hspace{1cm} (2-1-63)

The attenuation constant due to conductor wall loss is

$$\alpha_c = \frac{1}{\sigma \delta_{s b n} \epsilon_n \sqrt{1 - f_{c_{nm}}^2 / f^2}}.$$  \hspace{1cm} (2-1-64)

Typically, a waveguide is operated with a single transmission mode; the mode with the lowest frequency is usually chosen. For the rectangular waveguide, the $TE_{10}$ mode is the lowest mode and the $TE_{11}$ mode is for the circular waveguide. While using the lowest mode, other modes are dissipated by operating the microwave with frequency higher than the cutoff frequency of the lowest mode but lower than the cutoff frequency of other modes. However, the lowest mode of a waveguide does not match well with the focused field in a reflector antenna. This leads to the development of circular corrugated waveguides that
can operate in hybrid modes as well as their basic modes. A circular corrugated waveguide is a circular waveguide with ring slots on its inner wall as shown in Figure 2-2. The fundamental transmission modes inside are the hybrid modes, HE and EH modes. In the HE modes, the TE and TM components are in phase, while the TE and TM modes are out of phase in the EH modes. The lowest mode of those hybrid transmission modes, the HE_{11} mode, matches well with the focused field in a reflector antenna. This results in a better efficiency when emitting or receiving microwaves via a reflector antenna. Furthermore, the loss of the HE_{11} mode is lower than the loss of the modes in smooth circular waveguides.

In our case, high frequency (higher than 90 GHz) and high power (100-kW level) microwaves are planned to be used in future experiments. The method of transmitting 100-kW, 94-GHz, continuous-wave (CW) microwaves to an experimental area at AFRL/RDHPS at Kirtland AFB is shown in Figure 2-3. In this case, convertors transform
a Gaussian beam to the HE_{11} mode and the circular corrugated waveguide is used to transmit the microwaves with high power. The reason why we are using corrugated waveguide are its lower loss and good match with the Gaussian beam. The HE_{11} mode can be converted from a Gaussian beam with 98% conversion efficiency.\textsuperscript{42}

Figure 2-3: Schematic illustration of how microwave power is directed from gyrotron to laboratory.\textsuperscript{43}

2-1-3 Antenna

An antenna is a device that converts electromagnetic energy of a plane wave in space to the transmission modes in the waveguides or transmission lines. Antennas can be divided into two categories by the source of radiation. One type is the conduction current type antenna. The source of radiation is the conduction current on the metallic surface. The oscillating current in the antenna produces electromagnetic waves, and the shape of the antenna determines the strength of the radiated fields in each direction. The linear, loop,
helix, and spiral antennas are in this category. The other type is the displacement current type. Unlike the conduction current type, this kind of antenna adjusts existing electromagnetic waves into another form or mode. The best example is the reflector antenna. The reflector antenna reflects electromagnetic waves and reshapes waves. The slot antennas and horn antennas are of this type.

Radiation pattern, beam width, antenna gain, and directivity are used to describe the characteristics of an antenna. The radiation pattern is the distribution of the power intensity radiating in all directions. Beam width is the angle size of a radiation lobe, generally the lobe that contains the most radiated power. There are several definitions of beam width to describe the shape of a radiation pattern. The half power beam width (HPBW) is defined as the angle between the points where the power intensity is −3 dB compared with the maximum intensity in the lobe. The first null beam width (FNBW) is the angle between the first null points in the pattern.

The antenna gain $G$ is the angular power intensity $U(\theta, \phi)$ radiating in the direction $(\theta, \phi)$ by an antenna compared to the mean input angular power intensity $P_{in}/4\pi$, namely

$$G(\theta, \phi) = 4\pi \frac{U(\theta, \phi)}{P_{in}}.$$  \hspace{1cm} (2-1-65)

If the direction is not stated, it is usually the maximum gain the antenna can achieve. The directivity is a similar parameter to the gain. Its definition is

$$D(\theta, \phi) = 4\pi \frac{U(\theta, \phi)}{P_T},$$  \hspace{1cm} (2-1-66)

where $P_T$ is the total output power of the antenna. The directivity describes how the power can be directed to a certain direction and the gain describes the ratio between the radiated
power and the input power. The relation between the gain and directivity is

$$G(\theta, \phi) = e_{\text{antenna}} D(\theta, \phi), \quad (2-1-67)$$

where $e_{\text{antenna}}$ is the efficiency of the antenna and is defined as $e_{\text{antenna}} = P_T / P_{in}$.

In this research, a parabolic reflector is used to focus the energy of a microwave beam. A parabolic antenna can focus a plane wave with a uniform phase wave front at the aperture to focal point in phase, which means all the microwaves reach the focal point with the same phase. To achieve this, the curvature of the reflector must obey

$$x^2 + y^2 = 4fz, \quad (2-1-68)$$

where $f$ is the focal length and $x$, $y$, and $z$ are the positions in the $x$, $y$, and $z$ directions. The central axis of the paraboloid is set on the $z$-axis. The qualitative analysis above about a paraboloidal reflector antenna is based on geometric optics. Further analysis needs to solve the electric and magnetic field distributions in the reflector. The power gain $G$ achieved by a parabolic reflector is given by

$$G = 0.75 \frac{\pi^2 D^2}{A^2}, \quad (2-1-69)$$

where $D$ is the diameter of the reflector and an efficiency of 75% is assumed. In our case, the diameter of the reflector is set by the beam width, 50 mm. The wavelength corresponding to 94 GHz is 3.19 mm. Therefore, the power gain achieved by our reflector will be 1819, or 32.6 dB.

2-1-4 Microwave source

The devices generating microwaves can be grouped into one of two categories: solid-state or vacuum tube devices. Solid-state devices are semiconductor-based. Solid-state
electronic components include oscillators and amplifiers. This type of microwave source is primarily used in low output power and lower frequency applications, although the output power and frequency range of solid-state sources continues to increase as technology advances. Vacuum tube–type microwave sources are generally lower cost and have better performance in high power and high frequency applications, as shown in Figure 2-4. In our experiment, a microwave source that can output CW with high power is required. A vacuum tube–type microwave source is a better, or perhaps, the only choice.

The vacuum tube–type microwave source, also called a microwave tube, uses the interaction between an electron beam and electromagnetic field to generate microwaves. To prevent electron loss due to collisions with molecules of air, the electron beams must be sealed in a vacuum. This is the reason why microwave tubes are usually bulkier than solid-state microwave sources. At the same time, there must be “window” for the microwaves to get out or in (if that tube can be used as amplifier). This makes microwave tubes complex to produce.
Microwave tubes can be divided into three categories according to the chamber geometry and the microwave generation mechanism: linear-beam tubes, cross-field tubes, and fast-wave tubes. The linear-beam tube features an electron beam traveling linearly to a collector. There is an RF circuit surrounding the path of the electron beam. The input signal first modulates the electron beam. The modulated electron beam then induces the RF circuit and causes resonance. This process goes back and forth several times in the tube. Signals with resonant frequency will be amplified in these processes and output at the end of the RF circuit. The klystron and traveling wave tube (TWT) are in this category. The difference between them is the klystron uses discrete cavities as the RF circuit and the TWT uses a continuous RF circuit.

Figure 2-4: The power versus frequency of microwave tube and solid state.
The cross-field tube features a cylindrical cathode at the center and a magnetic field applied along the axis of the cathode. The RF circuit in a cross-field tube circles around the cathode. The electron beam travels radially outward to the RF circuit. At the same time, the electron beam also rotates around the cathodes because of the influence of the magnetic field. The interaction between the electron beams and surrounding RF circuit amplifies signals at the resonant frequency. Magnetron and crossed-field amplifiers are in this category. They are mainly different in the RF circuit: the magnetron has discrete cavities, whereas the crossed-field amplifier is continuous as in a TWT.

The configuration of a fast-wave tube is similar to a linear-beam tube, i.e., an electron gun at one side of a linear cavity with a collector at the other side. However, the basic principle is different from other kinds of microwave tubes. There is no RF circuit in fast-wave tubes. The resonance is achieved by the gyro-motion of electron beams caused by a magnetic field in the fast-wave tubes instead of an RF circuit in other kinds of microwave tubes. This kind of tube is also called a gyro-device. The electron travels with transverse speed roughly equal to 1.5–2 times the speed drifting toward the collector. The resonant frequency of a gyro-device is determined by the strength of the applied magnetic field instead of the RF circuit dimensions in other types. This makes it easier for a gyro-device to achieve high power and high frequency.

In our case, a microwave source that can output high power microwaves at high frequency is needed. As shown in Figure 2-5, only a gyrotron can fulfill the 100-kW level output at a frequency around 100 GHz. We plan to eventually use the existing 100-kW, 94-
GHz, CW microwave source located at AFRL/RDHPS at Kirtland AFB. Figure 2-6 shows a photo of the facility. The microwaves there are generated by a gyrotron, converted from a Gaussian beam to HE$_{11}$ mode, and transmitted with a corrugated waveguide through HE$_{11}$ mode from gyrotron to laboratory area as mentioned in Section 2-1-2. At the same time, we are going to use a Ka band TWT Xicom XTD-175Ka. This TWT can output up to 175 W of CW microwave power. This power level should be enough for the low flow rate in the vacuum chamber.

Figure 2-5: Power versus frequency performance of microwave tubes.$^{39}$
In a region filled with air and illuminated by a high-intensity microwave beam, the following phenomena will occur. First the high-strength electric field of the incident microwave induces breakdown. Then, the plasma generated by the breakdown absorbs the energy of the incident microwave and forms a region of high temperature. Finally, the heat in the high-temperature region conducts into the ambient air and increases the surrounding gas temperature. In the first two subsections of this section, the basic mechanism of the microwave breakdown and the basic properties of the plasma are introduced. In the last subsection, research results about the transmitting mechanisms of energy from the microwaves to gas are introduced.

2-2-1 Microwave breakdown

Electric breakdown is the phenomenon in which an insulator becomes conductive under an electric field exceeding its breakdown strength. Microscopically, this

Figure 2-6: Enclosure of 100-kW, 94-GHz gyrotron at Kirtland AFB.
phenomenon happens when the electric field is so strong that sufficient electrons are removed from neutral particles and a current composed of moving electrons is generated. In a gas, these electrons and ions form a plasma.

Consider the breakdown in a plane gap between an anode and a cathode. The gap with a width $d$ is filled with gas. When a DC voltage, $V$, is applied between the anode and the cathode, an electron current, $I_0$, from the cathode can ionize the gas particles in the gap and result in an electron current, $I_0 e^{\alpha d}$, at the anode. The term $\alpha$ is the Townsend ionization coefficient, which describes the rate of ionization of gas particles per distance in an electric field. At the same time, the ions close to the cathode can drift back to the cathode and generate secondary electrons. The total current between the gap is given by the Townsend formula

$$I = \frac{I_0 e^{\alpha d}}{1 - \gamma (e^{\alpha d} - 1)},$$

where $\gamma$ is the secondary ion–electron emission coefficient. When there is a breakdown, the current becomes self-sustaining. The condition to achieve a breakdown in the Townsend formula is

$$1 - \gamma (e^{\alpha d} - 1) = 0, \quad \alpha d = \ln \left(1 + \frac{1}{\gamma}\right).$$

This condition is called Townsend’s breakdown criteria. Combining this condition and a semi-empirical relation between $\alpha$ and the pressure $p$, the breakdown voltage can be calculated by

$$V = \frac{B(pd)}{C + \ln (pd)},$$
where $B$ and $C$ are constants to be determined by experiments for different gases. The curve that plots the breakdown voltage $V$ against the product of pressure $p$ and the gap distance $d$ is called the Paschen curve.

When microwaves are applied to gas, the situation becomes more complex. The microwaves can be considered as an AC electric field. In a DC electric field, the charged particles are only accelerated in one direction. However, charged particles in an AC electric field will be accelerated in one direction for a short time and the quickly be decelerated because the electric field reverses to the opposite direction. The mean power transmitted by the electric field to charged particles will be zero. Nevertheless, the energy gained from the electrostatic force can be transferred to other particles, such as gas molecules, or the directions perpendicular to the AC electric field through collisions. The mean mechanism of energy exchange between an electric field and gas becomes collisions in an AC field instead of an electrostatic force in a DC field. Electron–neutral collisions are considered the primary process, because other charged particles are too heavy to be accelerated to a high velocity in a short time.

The electron–neutral collisions can generate electrons. The main mechanisms of the electron generation are direct ionization and stepwise ionization. Direct ionization is the ionization of a particle at the electronic ground state and stepwise ionization is the ionization of a particle at an electronic excited state. Compared with direct ionization, stepwise ionization requires less energy. In a normal gas, the amount of stepwise ionization is small because of the lack of particles at electronic excited states. However, the
breakdown provides a huge number of electrons and generates a huge number of particles at excited electronic states. The main mechanism of electron generation switches from direct ionization to stepwise ionization after a breakdown. This is why sustaining a breakdown requires less power than initiating one.

Besides generating electrons, the electron–neutral collisions can also lead to the loss of the electrons through attachment and recombination. In addition to these mechanisms, diffusion can also result in the loss of electrons.

To initiate a self-sustaining discharge, the rate of electron generation must be higher than the rate of electron loss. Townsend’s breakdown criteria in the microwave discharge becomes the condition that the rate of electron generation and the rate of electron loss are equal. To calculate the breakdown strength of an electric field in a gas, one must consider how much energy can be transferred to electrons via electron–neutral collisions, the rates of the ionization, attachment, recombination; and diffusion at a given pressure and microwave strength.

Experimental data in Figure 2-7, also called the Paschen curve, show that the field strength for breakdown will decrease initially when the gas pressure decreases and then increase when the gas pressure is further decreased. This situation can be explained by the processes above, the loss rate of electrons decreases faster than the production rate when the pressure decreases. It is easier to induce breakdown in this situation. However, when the pressure is further decreased, it becomes hard to produce electrons because the particle density is too low for electrons to hit neutral gas particles to produce new electrons.
The analysis of experimental data showed that a relation $\Lambda E_e = f(\Lambda p)$ was satisfied at all the frequencies, where $\Lambda$ is the diffusion length, $E_e$ is the effective breakdown electric field, $E_e = \frac{E}{1+\omega^2/\nu_c^2}$, $\nu_c$ is the frequency of collisions, $\omega$ is the frequency of the microwaves, and $E$ is the electric field of the microwaves. This implies that a higher electric field is required to achieve breakdown when the microwave frequency increases at the same pressure and diffusion length.

Figure 2-7: Continuous-wave breakdown in air, oxygen, and nitrogen at 9.4 GHz, characteristic diffusion length 0.40 cm.\textsuperscript{47}
2-2-2 Plasma

After breakdown occurs at the focal point of the nozzle, the interaction between the generated plasma and microwaves plays an important role in the coupling between microwaves and supersonic flow. In this subsection, basic plasma characteristics are introduced.

The definition of a plasma is a mixture of neutral and charged particles that exhibit collective behavior. To describe the state of a plasma, it is necessary to describe the state of the neutrals, electrons, and ions. Though they exhibit collective behavior, their states are not always related to each other. For example, the density of ions and electrons is constrained by charge neutrality. The density of ions equals the density of electrons, so there is no net charge. However, there is no specific relation between the density of neutrals and the density of charged particles. Another example is temperature. The temperature of each species in a plasma is defined by the average energy of each species. In fluid dynamics, there is usually one temperature at each position. But for a plasma, it is usual that the temperature of the ions and electrons are different at one location.

One method to describe the behavior of a plasma is using the fluid theory of plasma. The fluid theory of plasma is basically a fluid theory considering multiple interpenetrating fluids—electrons and one kind of ion in the simplest case—combined with electromagnetics, namely Maxwell’s Equations. The only differences in Maxwell’s Equations in the fluid theory of plasma are the charge density, $\rho$, and the current density, $j$. In fluid theory of plasma, they are rewritten as
where \( q_i \) is the charge of an ion, \( n_i \) is the density of ions, \( e \) is the charge of an electron, \( n_e \) is the density of electrons, \( v_i \) is the average velocity of the ion fluid and \( v_e \) is the average velocity of the electron fluid. Besides Maxwell’s Equations, the fluid theory of plasma also contains the continuity equation, momentum equation, and the state equation for each fluid in the plasma. The continuity equation and state equation in a plasma are

\[
\frac{\partial n_j}{\partial t} = \nabla \cdot \left( n_j \vec{v}_j \right), \quad j = i, e, \quad \text{and} \quad \frac{\partial p}{\partial t} = C_j n_j^{\gamma_j}, \quad j = i, e. \tag{2-2-7}
\]

where \( p_j \) is the pressure of each fluid, \( C_j \) is the constants for each fluid, and \( \gamma_j \) is the ratio of specific heats of each fluid. The forms of these two equations are similar to the corresponding equations in usual fluid theory. For the momentum equation, it is essentially the usual momentum equation combined with the effect of electric fields, magnetic fields, and collisions, i.e.,

\[
m_j n_j \left[ \frac{\partial \vec{v}_j}{\partial t} + (\vec{v}_j \cdot \nabla) \vec{v}_j \right] = -\nabla p_j + q_j n_j (\vec{E} + \vec{v}_j \times \vec{B}) + \vec{P}_j, \quad j = i, e, \tag{2-2-8}
\]

where \( m_j \) is the mass of each fluid particle and \( P_j \) is the momentum change due to collisions. For collisions between charged particles and neutral particles,

\[
\vec{P}_j = m_j n_j (\vec{v}_j - \vec{v}_0) \nu_{j0}, \tag{2-2-9}
\]

where \( \nu_0 \) is the average velocity of neutral particles and \( \nu_{j0} \) is the frequency of collisions with neutral particles for each fluid.
One fundamental characteristic of a plasma is that it shields electric potentials within it. For an object with electric potential $\varphi_0$ in a plasma, the potential distribution around the object decays faster than in free space because the potential is shielded by charged particles surrounding the object. In free space, the strength of an applied potential is inversely proportional to the distance to the object if the distance is much larger than the size of object. In a plasma, the strength of the potential is given by

$$\varphi = \varphi_0 e^{-\frac{x}{\lambda_D}},$$

where $x$ is the distance from the object and $\lambda_D$ is the Debye length. The definition of the Debye length is

$$\lambda_D \equiv \sqrt{\frac{\varepsilon_0 k T_e}{n e^2}},$$

where $\varepsilon_0$ is the vacuum permittivity, $k$ is Boltzmann’s constant, $T_e$ is the temperature of the electrons, and $n$ is the plasma density. To achieve the capability of shielding a potential, a plasma should fulfill two criteria

$$\lambda_D \ll L; \quad N_D \gg 1,$$

where $L$ is the dimension of system and $N_D$ is the total number of plasma particles inside a “Debye sphere.” $N_D$ equals the volume of a sphere with radius $\lambda_D$ times the density, i.e.,

$$N_D = n \frac{4}{3} \pi \lambda_D^3.$$

Another criterion a plasma must fulfill is that the dominant behavior should be determined by electromagnetic forces rather than collisions. If the frequency of plasma
oscillations is $f_p$ and the mean frequency of collisions is $f_c$, the plasma should fulfill:

$$f_p > f_c .$$  \hspace{1cm} (2-2-14)

For a plasma in an exterior magnetic field, the charged particles with velocity perpendicular to the magnetic field will gyrate around the direction of the magnetic field because of the magnetic force. The radius of this gyration is called the Larmor radius $r_L$, which is

$$r_L = \frac{mV_\perp}{|q|B},$$  \hspace{1cm} (2-2-15)

where $m$ is the mass of the particle, $V_\perp$ is the velocity perpendicular to the magnetic field, $q$ is the charge of the particle, and $B$ is the magnetic field. The gyration frequency is $\omega_g$ given by

$$\omega_g = \frac{eB}{m}.$$  \hspace{1cm} (2-2-16)

This gyration results in a charged particle in a magnetic field only being able to travel freely along the direction of the magnetic field. Motion perpendicular to a magnetic field will be hindered by the magnetic force. This leads to several different modes for wave propagation with or without a magnetic field.

For an electromagnetic wave in a plasma, despite the relation between propagation direction and magnetic field, the propagation mode is also affected by the propagation media, either electrons or ions. Basic governing equations of electromagnetic waves in a plasma are

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \text{ and}$$  \hspace{1cm} (2-2-17)
These two equations can be combined into\(^{48}\)
\[
\vec{\nabla} \times \vec{B} = \mu_0 \left( \vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right). \tag{2-2-18}
\]

To connect current with electric and magnetic fields, two other equations are needed, i.e.,\(^{48}\)
\[
\vec{\nabla} (\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \left( \frac{\partial \vec{j}}{\partial t} + \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \right) . \tag{2-2-19}
\]

where \( q \) and \( m \) are the charge and mass of each particle (electron or ion). Equation 2-2-21 describes the motion of particles assuming the temperature is 0 K, namely no random thermal motion. The propagation mode of electromagnetic waves changes in different media and different relations between the propagation direction and magnetic field. In this dissertation, only the mode propagated by electrons with null magnetic field is derived.

For electromagnetic waves propagated by electrons without an exterior magnetic field, the dispersion relation is\(^{48}\)
\[
\omega^2 = c^2 k^2 + \omega_p^2, \tag{2-2-22}
\]
where \( \omega_p^2 = \frac{n e^2}{\epsilon_0 m_e} \), which is the frequency of plasma oscillations for electrons. The wave number of electromagnetic waves in a plasma is\(^{48}\)
\[
k = \frac{1}{c} \sqrt{\omega^2 - \omega_p^2}. \tag{2-2-23}
\]

The electromagnetic wave can propagate in a plasma only if the frequency is higher than the plasma oscillation frequency. Otherwise the wave number becomes a purely imaginary number. This means the amplitude of waves decays exponentially as it
propagates in the plasma and results in reflection or absorption if the frequency is lower than the plasma oscillation frequency. The skin depth, \( \delta \), is the parameter used to indicate how deep microwaves can penetrate, and is given by\(^{48} \)

\[
\delta = \frac{1}{|k|} = \frac{c}{\sqrt{\omega^2 - \omega_p^2}}.
\] (2-24)

For electromagnetic waves with a frequency larger than the plasma oscillation frequency, the phase velocity is\(^{48} \)

\[
v_{\phi}^2 = c^2 + \frac{\omega_p^2}{k^2}.
\] (2-25)

And the group velocity is\(^{48} \)

\[
v_g = \frac{d\omega}{dk} = \frac{c^2}{v_{\phi}}.
\] (2-26)

For microwaves with a frequency \( f_0 \), the minimum density of the plasma to block it is called the cutoff density. The cutoff density is the density with the frequency of plasma oscillations for electrons equal to the given frequency. The cutoff density \( n_c \) for a frequency \( f_0 \) can be derived as

\[
n_c = \frac{4\pi^2 e m_e}{e^2} f_0^2.
\] (2-27)

2-2-3 Gas heating by microwaves

In this project, the energy coupling efficiency between the microwaves and a supersonic flow is a key factor affecting the final thrust. The energy coupling efficiency is the ratio of the energy transferred into the supersonic flow to the input microwave energy. Unfortunately, there are few direct results about the energy coupling efficiency from past experiments or theoretical models. An energy coupling efficiency of 20–50% was reported.
by Moriarty and Brown in 1968. In that experiment, a 3-GHz pulsed microwave was focused to a flow of helium with a flow rate of 200–400 l/min. The energy coupling efficiency increased when the flow rate increased. Bollen et al. described an experiment focusing a microwave into a 1.7 cm² spot in 1983. In this experiment, a pulsed microwave with a frequency of 35 GHz is focused by a dielectric lens in nitrogen with a pressure of 25 Torr. An electron temperature of 3.8 eV was measured in this experiment. Simulations in the same paper predicted the gas temperature to be 0.027 eV, equivalent to 313 K, or lower. Another experiment executed by Grachev et al. in 1999 focused a pulsed microwave with a wavelength of 8.5 cm to ignite a plasma in a supersonic flow of air with different gas pressures. This report mainly showed the patterns of the discharge in the flows with different pressures. No energy coupling efficiency was reported. An experiment reported by Vikharev et al. in 2015 focused a 30-GHz microwave with a maximum CW power of 12 kW to generate a discharge in an argon and hydrogen gas mixture. A temperature of 3000 K was observed around the area of the discharge. The temperature was independent of the input power of the microwave. No further calculation revealed the energy coupling efficiency in that experiment.

The energy coupling efficiency equals the absorption efficiency multiplied by the ratio of the energy converted into gas heating to the total energy absorbed by a plasma. Though the energy coupling coefficient itself is not obtained, there are some previous studies about these two coefficients.

Concerning the absorption efficiency, Moriarty and Brown reported experimental
results in 1968. In their experiment, the microwave was transmitted to a resonant chamber filled with helium or argon gas via waveguides. For helium gas, the absorption efficiency is around 60% when the pressure is 0 psig. Then, the absorption efficiency increased to 80% when the pressure increased to 10 psig, and the efficiency decreased to 60% when the pressure increased to 20 psig. For argon gas, the absorption efficiency decreased from the highest value exceeding 95% at the pressure of 0 psig to the lowest value around 80% at the pressure of 50 psig. Bollen et al. also reported an absorption efficiency of 72–79% in nitrogen at the pressure of 25 Torr in their paper in 1983.

Woo and Degroot presented a simulation based on the hydrodynamics around the microwave discharge in air. The process of a microwave discharge in air is described in detail. The plasma density will grow quickly to the cutoff density of the incident microwave at the beginning region of the discharge. Then the plasma region will start to expand to the neighboring region. In the case of the discharge induced by a high-intensity microwave beam without focusing, the discharge region will move toward the source of the microwave beam. The plasma at the initial region will gradually disappear since the microwave is cutoff by the newly generated discharge region. In the case of a focused microwave, the plasma region will expand and the electric field at the center region will decrease to the breakdown value of the steady state. In a strongly collisional atmosphere, the absorption efficiency increases when the microwave intensity increases and decreases when the pressure decreases. In a weakly collisional atmosphere, the plasma density will easily grow to a value higher than the cutoff density and result in high reflection. The reflection depends
only on the collisionality, the ratio of the collision frequency to the incident microwave frequency. The higher collisionality results in the lower reflection.

The absorbed microwave energy is mainly transferred into the kinetic energy of electrons first, and then transferred to neutral gas particles. However, due to inefficiency of the energy transferring mechanisms, the temperature of electrons and neutral particles are usually different in a microwave plasma. Namely, the microwave plasma does not reach a thermal equilibrium. This kind of plasma is called a non-thermal plasma or non-equilibrium plasma.

The electron temperature in such plasma can be derived from averaging the electron drift velocity in an electric field

\[
\frac{3}{2} \delta T_e = \frac{e^2 E^2}{m_e v_{en}^2},
\]

where \( \delta \) is the fraction of electron energy lost in one collision, and \( v_{en} \) is the frequency of collisions between electrons and neutral particles. In a non-thermal plasma, \( v_{en} \) can be approximated as

\[
v_{en} = n_0 (\sigma_{en}) \left( \frac{8T_e}{\pi m_e} \right)^{1/2},
\]

where \( n_0 \) is the density of neutral particles, \( \sigma_{en} \) is the cross section of electron–neutral collision. The temperature of electrons can be rearranged into

\[
T_e = \frac{e}{(\sigma_{en})} \sqrt{\frac{\pi}{12 \delta}} \left( \frac{E}{n_0} \right),
\]

where \( \frac{E}{n_0} \) is the reduced electric field. The unit of the reduced electric field is V·cm\(^2\) or Townsend (Td). 1 Td equals 10\(^{-17}\) V·cm\(^2\).
In a non-thermal plasma, the rates of mechanisms of energy absorption and release are dependent on the electron temperature, and the electron temperature is proportional to the reduced electric field. Therefore, the reduced electric field can be used as an index for determining the absorption modes for a non-thermal plasma.

Electrons at a high temperature can heat gas via elastic collisions between electrons and neutral particles. However, elastic collisions are inefficient to transfer the kinetic energy from electrons to neutral gas particles due to the huge mass difference between an electron and a gas particle. The energy of electrons is mainly transferred to neutral gases through inelastic collisions, such as excitation, dissociation, ionization, and attachment.

The inelastic collision between an electron and a neutral gas particle includes rotational excitation, vibrational excitation, electronic excitation, attachment, dissociation, and ionization. In a non-thermal plasma composed of polyatomic gas, the rotational excitation absorbs most of the kinetic energy of electrons at a low reduced electric field. When the reduced electric field increases, the vibrational excitation takes over and absorbs most of the electron energy. If the reduced electric energy increases further, the electronic excitation, or even the ionization and dissociation, will dominate the energy absorption.

In the case of nitrogen, roughly 40% of the energy of electrons transfers to the rotational energy and almost the same fraction of energy goes to the vibrational energy when the reduced electric field is at 1 Td, as shown in Figure 2-8. When the reduced electric field is smaller than 100 Td, the vibrational degrees of freedom dominate the absorption of electron energy. When the reduced electric field is in the range between 100–600 Td, most
of the electron energy transfers to the electronic excitation. When reduced electric field goes higher than 600 Td, the dominating energy absorption mechanism switches to the dissociation.

![Graph showing energy loss fraction of electron against the reduced electric field and electron energy.](image)

Figure 2-8: The energy loss fraction of electron against the reduced electric field and electron energy.\textsuperscript{53}

The energy in a rotational excited state can be transferred to the rotational state of another particle through rotational–rotational (RR) relaxation or it can be transferred to the translational degrees of freedom through rotational–translational (RT) relaxation. These two processes are non-adiabatic and fast. Therefore, the rotational degrees of freedom are usually considered in quasi-equilibrium with the translational degrees of freedom.

For the energy stored in a vibrational excited state, it can be transferred to the
vibrational energy of another particle through vibrational–vibrational (VV) relaxation or to the translational energy through vibrational–translational (VT) relaxation. Comparing with the VT relaxation, the VV relaxation is usually a much faster process. Therefore, the vibrational energy tends to be constant for a long time.

VT relaxation processes can be categorized into adiabatic and non-adiabatic processes. The adiabatic one is the elementary mechanism for the vibrational excited state to transfer energy to the translational degrees of freedom. With the adiabatic VT relaxation process only, release of vibrational energy would be a very slow process. For example, in an experiment conducted by Alphen,

\[ ^53 \] it took more than 500 \( \mu s \) to achieve equilibrium between the vibrational temperature and the gas temperature, as shown in Figure 2-9. The non-adiabatic VT relaxation processes are faster than the adiabatic ones. However, the non-adiabatic VT relaxation processes exist only in some specific combinations of particles. For example, a collision between a nitrogen molecule and an atomic oxygen can lead to the vibronic transfer and the vibrational energy in a nitrogen molecule can be transferred into the translational degrees of freedom. Another kind of VT relaxation process involves the intermediate formation of complexes such as the collisions of \( H_2O^+ - H_2O \) and \( CO_2 - H_2O \).
The energy stored in an electronic excited state can be transferred to translational, rotational and vibrational degrees of freedom. The elementary process of the energy transfer from an electronic excited state to the translational degrees of freedom is also a slow and adiabatic process. The transfer processes from electronic excited states to vibrational or rotational states are much faster than the elementary transfer process to the translational degrees of freedom. These fast energy transfer processes to the rotational and vibrational degrees of freedom are non-adiabatic and release some energy into the translational degrees of freedom. These processes are considered to be the main processes to achieve the fast heating, in a time scale no longer than $3 \times 10^{-7}$ s, in a plasma ignited in Figure 2-9: The temperature of vibration and gas against time when the gas was heated by a microwave pulse with a pulse width of 233 µs.\textsuperscript{53}
air at a reduced electric field higher than 100 Td.\textsuperscript{54}

There are some experimental results on the energy fraction of gas heating—the ratio of the energy converted into gas heating to the total absorbed energy by a plasma—in air on a timescale much shorter than the time needed for the adiabatic vibrational relaxation. When the reduced electric field was around 40 Td, the energy fraction of gas heating did not exceed 3.5\%.\textsuperscript{54} For the experiments in the reduced electric field in the range between 80–120 Td, the energy fractions of gas heating increased to 10–15\%.\textsuperscript{54} For the experiments in a reduced electric field in the range of 200–1200 Td, the energy fraction of gas heating varied with the gas pressure. Znamenskaya et al. conducted an experiment using surface-barrier discharges with a voltage of 25–30 kV to excite a gas.\textsuperscript{55} The energy fraction of gas heating of 20–30\% was measured at the pressure of 25–60 Torr. The energy fraction of gas heating increased to 60\% when the pressure increased to 175–250 Torr. Another experiment executed by Alexsandrov et al. used surface discharges.\textsuperscript{56} An energy fraction of gas heating of 60–70\% was measured at a pressure of 1 atm. Another experiment conducted in the reduced electric field varied between 200–400 Td and pressure of 3–9 mbar, and an energy fraction of gas heating of about 20\% was reported.\textsuperscript{57}

In this project, a microwave beam will be focused into a supersonic flow composed of pure nitrogen. One experiment will focus a microwave beam with a power of 100 kW into a supersonic flow with a stagnation pressure of 2000 psig. The reduced electric field at the focal point in this experiment is about 10 Td. The other experiment will focus a microwave beam with a power of 100 W into a supersonic flow with a stagnation pressure
about 1 psia in a vacuum chamber. The reduced electric field at the focal point in this experiment is around several hundred Td. According to the experimental results discussed above, the coupling efficiencies and the heating pattern can be expected to be highly different in these two experiments because of the huge difference on the reduced electric field and pressure.

In the high-pressure flow experiment, the absorption rate of the microwaves in the plasma should be higher than 80% due to the high pressure. After the absorption of the microwave energy by electrons, the energy primarily transfers to the vibrational degrees of freedom and transfers to the gas through the adiabatic VT relation when pure nitrogen is used. The gas heating pattern in the nozzle in this experiment would be a smooth curve. The total energy fraction of gas heating might be around 10–20% because the residence time in the nozzle is about 50 µs. The total coupling efficiency might be around 8–16%. If air is used, the coupling efficiency can increase because of the non-adiabatic VT relaxation process caused by the collisions between nitrogen molecules and oxygen atoms.

For the experiment in vacuum, the absorption rate of the microwave energy in the plasma should be around 75% according to the experiment conducted by Bollen et al. Because of the high reduced electric field, the electron energy mainly transfers into electronic excitation. In the case of pure nitrogen, the main energy-releasing mechanism will be still the adiabatic VT relaxation because it lacks efficient processes like fast heating or non-adiabatic VT relaxation in pure nitrogen. The gas heating pattern in the nozzle would be also a smooth curve. The total energy fraction of gas heating might be around
10–20% because of the residence time. If air is used, the experiments conducted by others at a similar pressure and reduced electric field showed that fast heating processes transfer 20% of electron energy into gas heating. Therefore, the gas heating pattern in the nozzle in the vacuum experiment should be a high spike that transfers 20% of the total absorbed energy around the focal point caused by the fast heating and then a low and smooth curve that contains about 15–25% of total absorbed energy caused by the VT relaxation. The total energy fraction of gas heating might be 35–45%, and the total coupling efficiency might be 25–33%.

2-3 Compressible Flows

To generate a supersonic flow, one needs a converging–diverging nozzle and a gas feed system that can provide the high flow rate needed to sustain the supersonic flow in the nozzle. The performance and flow properties of the gas feed system and supersonic nozzle can be described by compressible flow with the quasi-one-dimensional assumption. Fluid behavior obeys three basic principles: continuity, momentum conservation, and energy conservation. In this section, these principles for compressible flow are introduced. Combining these principles and the quasi-one-dimensional assumption, the governing equations describing flow properties in an area-changing nozzle are derived. At the same time, the quasi-one-dimensional compressible flow with friction is also introduced. This is suitable for describing the flow properties in the gas feed system. Furthermore, several numerical simulations based on quasi-one-dimensional flow with heat addition in a nozzle
are demonstrated in this section. These simulations provide some insights about the change of flow properties due to the coupling between microwaves and a supersonic flow.

2-3-1 Basic governing equations and quasi-one-dimensional assumption

Continuity is the mass conservation principle, which states that “mass can be neither created nor destroyed.” When we consider fluid in a controlled region \( \mathcal{V} \), the rate of mass change in region \( \mathcal{V} \) equals the net mass flow rate in or out across a boundary \( \mathcal{S} \) around region \( \mathcal{V} \). This principle can be written as

\[
\frac{\partial}{\partial t} \iiint_\mathcal{V} \rho \, d\mathcal{V} = -\iint_\mathcal{S} \rho \vec{V} \cdot d\vec{S},
\]

where \( \rho \) is the density of the fluid and \( \vec{V} \) is the fluid velocity. The density \( \rho \) is variable here because we are dealing with compressible flow. The negative sign is because the direction of the area vector of boundary \( d\vec{S} \) is set as outward.

Momentum conservation implies that the rate of momentum change equals the net force. When we consider fluid in a controlled region \( \mathcal{V} \), there are two sources of momentum change: momentum change caused by net incoming or outgoing flow and momentum change caused by the change of density or velocity in that region. These can be summed up as

\[
\frac{d\vec{p}}{dt} = \iint_\mathcal{S} (\rho \vec{V} \cdot d\vec{S}) \vec{V} + \iiint_\mathcal{V} \frac{\partial (\rho \vec{V})}{\partial t} \, d\mathcal{V}.
\]

There are two sources for the force: a body force and a surface force. Body force is the force acting from a distance, such as gravitational or electric force. Surface force is the force due to the interaction between fluid particles and can be separated into pressure and shear stress. Since we assume the flow is inviscid, the surface force is only due to pressure
on the control volume surface. Summing all the above, the net force can be written as \(^5^8^\)

\[
\mathbf{F} = \iiint_V \rho \mathbf{f} dV - \oint_S p d\mathbf{S}.
\]  

(2-3-3)

The negative sign at the second term in Equation 2-3-3 also comes from the direction of the area vector of boundary \(d\mathbf{S}\). Combining the momentum change and force, we obtain \(^5^8^\)

\[
\iiint_V \left( \rho \ddot{\mathbf{V}} \cdot d\mathbf{S} \right) \mathbf{V} + \iiint_V \frac{\partial (\rho \mathbf{V})}{\partial t} dV = \iiint_V \rho \mathbf{f} dV - \oint_S p d\mathbf{S}.
\]  

(2-3-4)

The meaning of energy conservation is “energy can be neither created nor destroyed,” just as for mass conservation. Again, when we consider fluid in a controlled region \(\mathcal{V}\), the rate of energy change in region \(\mathcal{V}\) equals the net energy flows in or out of region \(\mathcal{V}\). The energy of fluid in region \(\mathcal{V}\) is the summation of internal energy and kinetic energy, i.e., \(^5^8^\)

\[
\iiint_V \rho \left( e + \frac{v^2}{2} \right) dV,
\]  

(2-3-5)

where \(e\) is the internal energy of the fluid. The change of the energy can be divided into two parts: the change due to the internal energy or the fluid velocity change and the change caused by the net incoming or outgoing flow. They can be written as \(^5^8^\)

\[
\iiint_V \frac{\partial}{\partial t} \left[ \rho \left( e + \frac{v^2}{2} \right) \right] dV + \iiint_S \rho \left( e + \frac{v^2}{2} \right) \mathbf{V} \cdot d\mathbf{S}.
\]  

(2-3-6)

Energy can flow in (or out) through two forms: work done by force or heat. The rate of work done by surface and body forces is \(^5^8^\)

\[
- \iiint_S (p d\mathbf{S} \cdot \mathbf{V}) + \iiint_V (\rho \mathbf{f} dV \cdot \mathbf{V}).
\]  

(2-3-7)

The rate of heat addition is \(^5^8^\)

\[
\iiint_V \rho \mathbf{q} dV.
\]  

(2-3-8)

Summing them we obtain \(^5^8^\)
The governing equations from above can be rearranged into differential form:

\[
\frac{\partial}{\partial t} \left[ \rho \left( e + \frac{v^2}{2} \right) \right] dV + \oint_S \rho \left( e + \frac{v^2}{2} \right) \vec{V} \cdot d\vec{S} = - \oint_S (p d\vec{S}) \cdot \vec{V} + \oint_V (\rho \vec{f} dV) \cdot \vec{V} + \oint_V \rho q dV.
\]

(2-3-9)

The governing equations from above can be rearranged into differential form:

\[
\frac{\partial \rho}{\partial t} = - \vec{V} \cdot (\rho \vec{V}),
\]

(2-3-10)

\[
[\vec{V} \cdot (\rho \vec{V})] \vec{V} + \rho \vec{V}(\vec{V} \cdot \vec{V}) + \frac{\partial (\rho \vec{V})}{\partial t} = \rho \vec{f} - \vec{V}P, \text{ and}
\]

(2-3-11)

\[
\frac{\partial}{\partial t} \left[ \rho \left( e + \frac{v^2}{2} \right) \right] + \vec{V} \cdot \left[ \rho \left( e + \frac{v^2}{2} \right) \vec{V} \right] = \rho \vec{f} \cdot \vec{V} - \vec{V} \cdot (P \vec{V}) + \rho \dot{q}.
\]

(2-3-12)

For steady flow, all the time differential terms are zero. At the same time, we can ignore the body force and assume no heat flow. The governing equations can be rearranged into

\[
\vec{V} \cdot (\rho \vec{V}) = 0,
\]

(2-3-13)

\[
\rho \vec{V}(\vec{V} \cdot \vec{V}) = - \vec{V}P, \text{ and}
\]

(2-3-14)

\[
\vec{V} \left( e + \frac{p}{\rho} + \frac{v^2}{2} \right) = 0.
\]

(2-3-15)

The quasi-one-dimensional assumption assumes that the flow properties through an area-varying duct depend only on $x$, the axial position in the duct. This assumes the flow properties passing any cross section of the duct are uniform and have no dependence in other directions. Under this assumption, the governing equations become:

\[
\frac{d}{dx} (\rho AU) = 0,
\]

(2-3-16)

\[
\rho U \left( \frac{dU}{dx} \right) = - \frac{dp}{dx}, \text{ and}
\]

(2-3-17)
where \( \mathcal{U} \) is the velocity in the \( x \)-direction and \( h \) is the enthalpy, \( h = e + \frac{p}{\rho} \).

### 2-3-2 Flow and shock waves in a supersonic nozzle

To understand the properties of a flow at different Mach numbers, i.e., the ratio of flow velocity to the sound speed, we first rewrite the continuity into an equation including Mach number. The differential form of the continuity equations can be rewritten as\(^{58}\)

\[
\frac{d\rho}{\rho} + \frac{d\mathcal{U}}{\mathcal{U}} + \frac{dA}{A} = 0. \tag{2-3-19}
\]

Meanwhile, momentum equation under quasi-one-dimensional assumption can be rewritten as\(^{58}\)

\[
\frac{d\rho}{\rho} \frac{dp}{d\rho} = -\mathcal{U}d\mathcal{U}. \tag{2-3-20}
\]

The \( \frac{dp}{d\rho} \) term is the square of the velocity of sound \( a^2 \), i.e.,

\[
\frac{dp}{d\rho} = \left( \frac{\partial p}{\partial \rho} \right)_s = a^2. \tag{2-3-21}
\]

After rearranging, we obtain

\[
\frac{dp}{\rho} = -\frac{\mathcal{U}d\mathcal{U}}{a^2} = -\frac{u^2}{a^2} \frac{d\mathcal{U}}{\mathcal{U}} = -M^2 \frac{d\mathcal{U}}{\mathcal{U}}, \tag{2-3-22}
\]

where \( M \) is the Mach number. Finally we obtain\(^{58}\)

\[
\frac{dA}{A} = (M^2 - 1) \frac{d\mathcal{U}}{\mathcal{U}}. \tag{2-3-23}
\]

According to Equation 2-3-23, to accelerate flow speed while \( M < 1 \), a converging \((dA < 0)\) nozzle is needed. When \( M > 1 \), a diverging nozzle \((dA < 0)\) is needed to accelerate the flow speed. Thus in order to obtain a supersonic flow from a stagnation state,
the nozzle shape should be converging–diverging. In addition, the sonic flow ($M = 1$) appears at the minimum area position or throat, where $dA = 0$.

If we consider the working fluid as an ideal gas, enthalpy, $h$, can be written as $C_p T$, where $C_p$ is the specific heat at constant pressure. The energy equation can be rewritten as

\[
\frac{d}{dx} \left( C_p T + \frac{u^2}{2} \right) = 0. \tag{2-3-24}
\]

Analytic solutions of each variable can be derived as

\[
\left( \frac{A}{A^*} \right)^2 = \frac{1}{M^2} \left[ \frac{2}{y+1} \left( 1 + \frac{y-1}{2} M^2 \right) \right]^{(y+1)/(y-1)}, \tag{2-3-25}
\]

\[
\frac{P}{P_0} = \left( 1 + \frac{y-1}{2} M^2 \right)^{-\gamma/(y-1)}, \tag{2-3-26}
\]

\[
\frac{T}{T_0} = \left( 1 + \frac{y-1}{2} M^2 \right)^{-1}, \tag{2-3-27}
\]

where $A^*$ is the area at the throat, $P_0$ and $T_0$ are the stagnation pressure and temperature, and $\gamma$ is defined as

\[
\gamma \equiv \frac{C_p}{C_v}. \tag{2-3-28}
\]

The thrust produced by the nozzle can be calculated by

\[
T = \dot{m}_e V_e - \dot{m}_i V_i + (P_e - P_\infty)A_e, \tag{2-3-29}
\]

where the subscript $e$ means at the exit, $i$ means at the entrance, $\infty$ means the property of the environment, and $\dot{m}$ is the mass flow rate. Mass flow rate can be calculated by

\[
\dot{m} = \frac{P_0 A^*}{\sqrt{T_0}} \sqrt{\gamma \left( \frac{2}{y+1} \right)^{(y+1)/(y-1)}}, \tag{2-3-30}
\]

where $R$ is the ideal gas constant.
In our case, we need a nozzle with $M_e = 5$ and the diameter at exit as 50 mm. Exit diameter is fixed to match the size of the microwave beam. According to the theory above, a converging–diverging nozzle with throat diameter 10 mm is needed. While the stagnation pressure is set as 2000 psig, environmental pressure 83,319 Pa, and temperature is 300 K, $P_e$ is calculated as 26,083 Pa, $T_e$ is 50 K, flow rate of mass $\dot{m}$ is 2.53 kg/s, and the thrust is 1680 N.

The flow is supersonic at the exit of a nozzle only when the pressure in the stagnation chamber is high compared with the ambient pressure. Otherwise there will be a shock wave inside the nozzle and the flow becomes subsonic after the shock wave. A normal shock wave is assumed to be a sudden discontinuity in a flow. The flow properties across the shock immediately change. The flow properties across the shock wave are assumed to satisfy mass, momentum, and energy conservations. At the same time, the gas is assumed to be a calorically perfect gas. Therefore, we have\textsuperscript{58}

\begin{align}
\rho_1 u_1 &= \rho_2 u_2, \\
p_1 + \rho_1 u_1^2 &= p_2 + \rho_2 u_2^2, \\
h_1 + \frac{u_1^2}{2} &= h_2 + \frac{u_2^2}{2}, \\
p_1 &= \rho_1 RT_1, \text{ and} \\
p_2 &= \rho_2 RT_2,
\end{align}

where the variables with subscript 1 are the flow properties of the upstream and the ones with subscript 2 are the properties of the downstream of the shock. The relations between upstream and downstream are\textsuperscript{58}
After the shock wave, the flow continues the isentropic expansion in the divergent section of the nozzle. For subsonic flow, the pressure at the exit must match the ambient pressure. The position of the shock wave can be calculated by matching the exit pressure at different shock wave positions to the ambient pressure. As the pressure in the stagnation chamber increases, the shock wave will be blown toward the exit. The critical pressure to blow the shock wave out of the nozzle is the chamber pressure that can generate a shock wave at the exit. In our nozzle, the critical pressure is 268 psia at 1 atm ambient pressure.

### 2-3-3 Compressible flow with friction

A key component is the gas feed system that can provide high flow rate and sustain the flow pressure. In our nozzle, we wish to achieve a 2.53 kg/s flow rate at 2000 psig pressure in the stagnation chamber. The gas feed system is constrained by two main factors: choking and friction. The highest flow rate in a tube with a fixed radius is the flow rate when \( M = 1 \) at the exit of the tube. If the radius in one section is too small, the flow rate cannot be sustained. Meanwhile, the pressure of the flow decreases due to friction. The quasi-one-dimensional compressible flow with friction is suitable to describe this process in our experiment. Under the quasi-one-dimensional assumption, only the shear stress...
between the flow and the wall of tubes is considered. At the same time, the shear stress does not result in the energy change of the flow.

With the friction, the differential form of the momentum equation becomes

\[ dp + d(\rho U^2) = -\pi D \tau w dx, \]  

where \( A \) is the cross area of the tube, \( D \) is the diameter of the tube, and the \( \tau_w \) is the shear stress between the wall and the flow. Combining with \( A = \frac{\pi D^2}{4} \) and \( \tau_w = \frac{1}{2} \rho U^2 f \), where \( f \) is the friction coefficient, we get

\[ dp + d(\rho U^2) = -\frac{1}{2} \rho U^2 \frac{4fx}{D}. \]

After introducing the Mach number and integrating, we obtain the relation of the Mach number at 2 different points \( x_1 \) and \( x_2 \)

\[ \int_{x_1}^{x_2} \frac{4fx}{D} dx = \left[ -\frac{1}{\gamma M^2} - \frac{\gamma+1}{2\gamma} \ln \left( \frac{M^2}{1+\frac{\gamma-1}{2} M^2} \right) \right]_{M_1}^{M_2}. \]

The relations of temperature, pressure and density are

\[ \frac{T_2}{T_1} = \frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}, \]

\[ \frac{p_2}{p_1} = \frac{M_1}{M_2} \left[ \frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2} \right]^{1/2}, \text{ and} \]

\[ \frac{\rho_2}{\rho_1} = \frac{M_1}{M_2} \left[ \frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2} \right]^{-1/2}. \]

2-3-4 Heat addition to the flow in a supersonic nozzle

To understand the effect of coupling between the supersonic flow and the microwaves under different coupling conditions (transmitted power, coupling region, etc.), quasi-one-dimensional numerical simulations of the compressible nozzle flow with area change and
heat addition were conducted. In this simulation, the coupling between the supersonic flow and the microwaves is considered as a quasi-one-dimensional heat source. Flow properties and thrust under different coupling conditions are simulated to compare with experiment data.

Governing equations in these simulations are the basic governing equations of quasi-one-dimension compressible flow derived above plus the ideal gas equation:

\[
\frac{d}{dx}(\rho AU) = 0, \tag{2-3-45}
\]

\[
\rho U \frac{dU}{dx} = -\frac{dP}{dx}, \tag{2-3-46}
\]

\[
\frac{d}{dx} \left( C_p T + \frac{u^2}{2} \right) = 0, \text{ and} \tag{2-3-47}
\]

\[
\frac{1}{P} \frac{dP}{dx} = \frac{1}{\rho} \frac{d\rho}{dx} + \frac{1}{T} \frac{dT}{dx}. \tag{2-3-48}
\]

To describe the energy addition caused by the coupling between the supersonic flow and the microwaves, a heat flow term, \( \frac{dQ_{EM}}{dx} \), is added to the energy equation. The energy equation becomes

\[
\frac{d}{dx} \left( C_p T + \frac{u^2}{2} \right) = \frac{dQ_{EM}}{dx}. \tag{2-3-49}
\]

The value of \( \frac{dQ_{EM}}{dx} \) represents the total heat added per distance at position \( x \). Under the quasi-one-dimensional assumption, this heat is applied uniformly across the cross-sectional area at that \( x \) position. The \( \frac{dQ_{EM}}{dx} \) term in this simulation is set as a step function, the value is set to 0 in the region out of the coupling region, and as \( \frac{Q_{total}}{m_{L, C}} \), where \( Q_{total} \) is the total
microwave power transmitted into the supersonic flow, \( \dot{m} \) is the mass flow rate, and \( L_c \) is the length of the coupling region. Simulations were executed with Wolfram Mathematica. The “NDSolve” instruction was used to numerically solve the set of differential equations including Equations 2-3-45, 2-3-46, 2-3-48, and 2-3-49. One can change the value of \( Q_{\text{total}} \) and the coupling region to simulate the flow properties and performance of different coupling conditions, and compare to experimental data to understand how microwaves and supersonic flow couple to each other.

Two sets of simulations were conducted to estimate the flow properties in the high-pressure flow experiment and the experiment in vacuum. In the first set, the situation using a supersonic flow with a stagnation pressure of 2000 psig in the 40-mm nozzle illuminated by a microwave beam with a power of 100 kW at an ambient pressure of 1 atm was simulated. Different values of \( Q_{\text{total}} \) were used to simulate different energy coupling efficiencies. The energy coupling efficiency was set as 15% in one simulation and 30% in the other. The coupling patterns were set as straight lines from the focal point to the exhaust in order to simulate the heating caused by the vibrational relaxation. Simulation results showed that a 100-kW microwave beam with an energy coupling efficiency of 15% can result in a thrust gain of 0.5% and an energy coupling efficiency of 30% can result in a thrust gain of 1%. The simulated pressure distributions in the nozzle with different energy coupling efficiencies are shown in Figure 2-10.

In the second set of simulations, the situation using a supersonic flow with a stagnation pressure of 0.25 psia in the same nozzle illuminated by a microwave beam with a power of
100 W at an ambient pressure of 0.01 Torr was simulated. The coupling patterns used in this simulation were composed of two parts. One part was a line section with a length of 3 mm around the focal point with an energy coupling efficiency of 20%, which simulated the fast heating. The other part was a line section from the focal point to the exhaust with an energy coupling efficiency of 20%, which simulated the heating due to the vibrational relaxation. Two simulations were conducted. One simulated that the supersonic flow was heated only by the vibrational relaxation with a total energy coupling efficiency of 20%. The other simulated that the flow was heated by the vibrational relaxation and fast heating with a total energy coupling efficiency of 40%. Simulation results showed that the heating caused only by the vibrational relaxation can produce a thrust gain of 4.7% and the heating caused by the fast heating and the vibrational relaxation can result in a thrust gain of 14.2%. The simulated pressure distributions in the nozzle with these two heating conditions were showed in Figure 2-11.
Figure 2-10: Pressure distributions in the nozzle for supersonic flows with a stagnation pressure of 2000 psig and different heating conditions. The blue line is calculated from the analytical solution (Equations 2-3-25 and 2-3-26) to simulate the flow without heat addition. Red and green lines are numerical simulation results with different energy coupling efficiencies.
Figure 2-11: Pressure distributions in the nozzle for supersonic flows with a stagnation pressure of 0.25 psia and different heating conditions. The blue line is calculated from the analytical solution (Equations 2-3-25 and 2-3-26) to simulate the flow without heat addition. Red and green lines are numerical simulation results with different energy coupling efficiencies and coupling patterns.
Chapter 3

Supersonic Flow in the Nozzle

Preliminary flow tests of the nozzle show that the pressure in the stagnation chamber cannot exceed 150 psia. At the same time, the flow at the exit of the nozzle is not supersonic due to a shock wave in the nozzle. This situation is because the prior gas feed system could not provide sufficient mass flow rate to sustain a supersonic flow in the nozzle. This chapter describes the improved experiment setup, estimation of the pressure drop in the gas feed system, and cold flow tests to achieve a supersonic flow with Mach 5 at the exhaust and 2000 psig chamber pressure.

3-1 Preliminary Experiment Results

The plot of the measured pressure in the stagnation chamber against the regulator pressure—the pressure measured at the outlet of the regulator when the flow is not applied—is shown in Figure 3-1. The stagnation pressure increases with the regulator pressure at the beginning, but decreases when the regulator pressure exceeds 2000 psig. At the same time, the measured stagnation pressures are less than 1/10th of the regulator pressures. This result implies that the previous gas feed system could not sustain the flow rate needed for a flow in the nozzle with stagnation pressure higher than 150 psia.

The pressure distribution in the divergent section of the nozzle is shown in Figure 3-2. The measured pressures in the nozzle are all close to 14.7 psia, 1 standard atmosphere.
However, minimum pressures appearing at the beginning of measurement show a pressure drop at the position close to the nozzle throat. The pressure distribution implies that the system fails to generate a supersonic flow in the nozzle. The stagnation pressure is so low that there is a shock wave in the nozzle.

Figure 3-1: The chamber pressure against the regulator pressure.
The purpose of this project is to demonstrate the gaseous plasma generation in supersonic flow by CW beamed microwaves and observing the coupling between the microwaves and the supersonic flow. The requirements of the nozzle are the ability to generate supersonic flow and focus the microwaves. At the same time, ports for pressure measurements are also needed.

As shown in Section 2-3, a converging–diverging nozzle with a throat diameter of 5 mm and an exit diameter of 50 mm is needed to produce supersonic flow with a Mach number equal to 5 at the exit. To focus the microwaves, the diverging section is designed

3-2 Experimental Setup

3-2-1 Nozzle design

The measured pressure distribution in the nozzle comparing with theoretical pressure. The stagnation pressure in this case is 125 psia.
as a parabola with the focal point at 3.0 mm from the bottom of the parabola, or 49.1 mm from the nozzle exit. Mach number at the focal point is 1.8 for a cold flow.

The nozzle assembly is separated into two parts: nozzle and chamber. An o-ring is used on the surface connecting these two parts to prevent gas leakage. The outer profile of the parts is constrained by other connecting parts. At the chamber part, two 3/4” NRT ports are used for the gas input and chamber pressure measurement. For the nozzle part, six ports are used for pressure measurement as a function of axial location in the nozzle.

The nozzle material is 303 stainless steel because of its good machinability, high temperature resistance, and ease of procurement. Machinability is important to reduce imperfections of the parabolic section. The high temperature resistance is due to the need to resist possible high temperatures during the experiment.

The tolerance of the parabolic section is assigned as RMS 0.001 inch (0.025 mm) and other dimensions are assigned as 0.01 inch (0.25 mm). The 0.01-inch tolerance for most parts is due to the need of precise assembly and alignment. To avoid the shift of focal point due to an imperfect shape of the parabolic section and high loss caused by a rough surface, a high tolerance and smooth finish on the parabolic section is needed. However, achieving high tolerance and a perfect mirror finish such as micron- or nanometer-level tolerance costs more and is not necessary at microwave frequencies. Fortunately, losses under 0.1 dB can be achieved with tolerance smaller than 0.012λ.\(^59\) The wavelength of microwaves at a frequency of 90 GHz is 0.3189 cm or 0.1256 inch. The appropriate tolerance to achieve 0.1 dB for our project is 0.0038 cm, or 0.0014 inch. RMS 0.001” tolerance is a good
compromise between cost and low reflection loss.

3-2-2 Gas feed system

The goal of the gas feed system is to supply a mass flow rate of 2.53 kg/s and stagnation pressure higher than 2000 psig to the nozzle. There are two main factors constraining the flow rate and pressure: the choking and the friction between the tube wall and the flow. For a flow in a tube, the highest flow rate is the flow rate when Mach 1 is reached at the exit of a tube. Therefore, the highest flow rate is proportional to the inner area of a tube. The friction decreases the total pressure of the flow. The friction is proportional to the square of the flow speed. For a flow with fixed mass flow rate, the tube with larger diameter results in less pressure reduction due to friction. Both factors prefer a tube with a larger diameter, so the selection of tubing in the gas feed system should keep the tube diameter large. Another requirement is that the parts in the gas feed system before the nozzle should be capable of bearing the pressure higher than 2000 psig.

Another important component of the gas feed system is the pressure regulator, a device that can keep the outlet flow at a constant pressure. The requirements on the pressure regulator are to keep the outlet flow with a stagnation pressure higher than 2000 psig and mass flow rate higher than 2.53 kg/s. The Swagelok RSH 10 regulator fulfills this requirement.

The last part of the tubing system is the source of gas. The 6K cylinders filled with nitrogen are pressurized to 6000 psig. This type of cylinder can meet the requirement for pressure, but fails to satisfy the flow rate. The orifice diameter of the 6K cylinder is only
0.156”. The highest flow rate achieved by one cylinder is only 1.19 kg/s. Three cylinders are used in our system to achieve the 2.53 kg/s mass flow rate. At the same time, the parts in the high-pressure section of gas feed system should be able to contain 6000 psig pressure.

The setup of the improved gas feed system is shown in Figure 3-3. An RSH 10 regulator is used to control the pressure and separate the high-pressure and low-pressure sections. The main line of the high-pressure section consists of 1” tubing. The branch line from the main line to each cylinder is composed of two 1/2” tubes due to the lack of strength to endure the pressure of 6000 psig for a tube with larger size. At the same time, the small diameter of the 1/2” tubes may cause the huge reduction in pressure, so two 1/2” tubes are used. In addition, a pressure-sensing relief valve and a manual relief valve are included for safety. The pressure-sensing relief valve is set at 5800 psia, the maximum pressure at which the regulator can work. To protect the regulator, the cylinders are vented to a pressure lower than 5500 psig before an experiment. The low-pressure section consists of a section of 1” tube, a 1” flexible tube, and a ball valve. The flexible tube is set at the final section connecting to the nozzle to minimize the disturbance to thrust measurements due to the stiffness of the tubes. A solenoid valve and a pneumatic actuator are used to control the ball valve remotely for safety.
A highly automated DAQ system is used to execute the experiment. The DAQ system controls the flow, measures the pressure at different positions in the nozzle, and measures the thrust of the nozzle. The DAQ system is based on the NI USB 6211 and LabVIEW software. The instruments connected, input signal range, and output value of each channel are listed in Table 3-1.

The thrust of the nozzle is measured by a load cell, a device that can output a signal proportional to the force exerted on it. The load cell can measure force in the range of 0–
1000 lbf, or 0–4450 N. The range of the output signal of the load cell is 0–22 mV. It needs a 10-V excitation voltage. At the same time, the nozzle is set on a slide stage composed of air bearings to minimize friction. The load cell is set in front of the slide stage to measure the thrust. With a flow applied, there are four forces exerted on the slide stage along the direction of the flow: thrust generated by the flow, friction, the force from the load cell, and the force due to the stiffness of the 1” flexible tube. To measure thrust accurately, the friction and the force caused by the flexible tube must be minimized. The friction is minimized by the air bearings. To minimize the force from the flexible tube, the neutral position of the slide stage is set at the position where the stage touches the load cell. Therefore, the position of the slide table will move with the minimum distance while the gas is flowing and minimize the force produced by the bending of the flexible tube.

To measure the pressure distribution in the nozzle, seven pressure transducers are located at different positions in the nozzle. One is in the stagnation chamber to monitor the stagnation pressure and six are at ports in the diverging section of the nozzle. The transducer measuring the pressure in the stagnation chamber can measure up to 2500 psia pressure and the range of the output signal is 0–10 V. The exciting voltage for this transducer is 24 V. The transducers in the nozzle can measure the pressure up to 100 psia and the range of the output signal is 0–100 mV. These six transducers require a 10-V excitation voltage.

In the flow control part, the ball valve is controlled by a spring-return pneumatic actuator. A solenoid valve is set to control the motion of the actuator. When the solenoid
valve is opened, the pneumatic actuator is pressurized and opens the ball valve. When the solenoid valve is closed, the pneumatic actuator is depressurized and the spring inside pulls the ball valve back to the close position. The solenoid valve needs 12 V at 0.5 A to open. A Crydom DC60S7 relay circuit is used to relay the signal from the DAQ to the solenoid valve. The DC60S7 will connect the output ports when the input voltage is higher than 3.5 V and break the connection between the output ports when the input voltage is lower than 1 V. The input signal for the DC60S7 from the NI USB 6211 is set at 5 V. A panic bottom is set at the input side to immediately cut the signal at the input of the relay circuit and stop the flow.

Table 3-1: List of channels in the DAQ system.

<table>
<thead>
<tr>
<th>CHANNEL</th>
<th>DESCRIPTION</th>
<th>WORKING VOLTAGE</th>
<th>SIGNAL RANGE</th>
<th>SET VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>INPUT 0</td>
<td>Load cell</td>
<td>10–12 V</td>
<td>0–22 mV</td>
<td>N/A</td>
</tr>
<tr>
<td>INPUT 1–6</td>
<td>Transducers at the divergent section</td>
<td>10–12 V</td>
<td>0–100 mV</td>
<td>N/A</td>
</tr>
<tr>
<td>INPUT 7</td>
<td>Transducer at stagnation chamber</td>
<td>24 V</td>
<td>0–10 V</td>
<td>N/A</td>
</tr>
<tr>
<td>OUTPUT 0</td>
<td>Valve control</td>
<td>12 V</td>
<td>N/A</td>
<td>5 V</td>
</tr>
</tbody>
</table>

3-3 Theoretical Pressure Drop in the Gas Feed System

A gas feed system to deliver a flow with 2.53 kg/s mass flow rate and 2000 psig stagnation pressure to the nozzle is designed as described in Section 3-2-3. An estimation
of the pressure drop in the gas feed system is made. In this estimation, the friction between the wall of the gas feed system and the flow is considered. The flow in a tube is considered to be a compressible flow experiencing friction. The friction factor is set as 0.0172. At the same time, the flow in an adaptor between different sizes of tubes is considered to be an isentropic flow to simplify the calculation.

The gas feed system is divided into two sections: high-pressure and low-pressure sections. The borderline is the pressure regulator. There are some devices in the pressure regulator to control the pressure of the outlet flow. The effect of those devices on the properties of a flow is beyond the scope of this dissertation. Fortunately, Swagelok provides the chart describing the flow rate versus the pressure of outlet flow. It is assumed in this estimation that the flow in the pressure regulator is an isothermal flow. This assumption also applies to the ball valve. Swagelok also provides the plot of flow rate versus pressure drop of the ball valve.

The high-pressure section is composed of tubes with three sizes. After the orifice with diameter of 0.156", there is a 9-mm-long 1/4" tube with inner diameter of 0.18"; 20" long parallel tubing composed of two 1/2" tubes with inner diameter of 0.334", and finally 6 sets of 1/2" tubes converging into a 15"-long 1" tube with inner diameter of 0.69" connecting to the inlet port of the regulator. In the 1/2" parallel tubes, the flow is assumed to perform as in a tube with a diameter of $\sqrt{2} \times 0.334"$. At the converging point between 6 sets of 1/2" tubes and the 1" tube, it is assumed to be an isentropic process for a tube with a diameter of $\sqrt{6} \times 0.334"$ into a tube with a diameter of 0.69".
The low-pressure section is composed of 1" tubes. There is a 2.6"-long 1" tube with 0.80" inner diameter connecting to the outlet port of the RSH10 regulator, then the ball valve, and finally an 18"-long 1" flexible tube with 1" inner diameter connecting to the inlet port of the stagnation chamber of the nozzle.

In this estimation, the maximum flow rate that can be achieved by the improved gas feed system is the flow rate when the flow at the end of the 1/4" tube reaches Mach 1. In this situation, the Mach number at the orifice of the cylinder is 0.76. At 5500-psig pressure in the cylinders, the maximum mass flow rate supplied by the improved gas feed system is 3.0597 kg/s. The static and stagnation pressure drops in the high-pressure section are shown in Figure 3-4 and Figure 3-5. Figure 3-4 shows the static pressure of the flow against the distance from the cylinders. The discontinuities in the figure are due to the adaptors. The theoretical static pressure of the flow at the end of the high-pressure section is 3756 psia. Figure 3-5 shows the stagnation pressure of the flow against the distance from the cylinders. The theoretical stagnation pressure of the flow at the end of the high-pressure section is 3796 psia. Most of the pressure reduction is in the section of the 1/4" tube. The flow velocity there is high so the friction is also high.
Figure 3-4: Static pressure versus the distance from the cylinder. The discontinuities are due to the adaptors.
In the low-pressure section, the static pressure of the outlet flow from the regulator is set as 2400 psia, and the flow rate as 3.06 kg/s as the inlet flow. The resultant pressure drop is shown in Figures 3-6 and 3-7. Figure 3-6 shows the static pressure and Figure 3-7 shows the stagnation pressure. The discontinuities in both figures are due to the ball valve. The ball valve results in a 150-psi pressure drop. The theoretical stagnation pressure at the inlet of the nozzle is 2238 psia.

Figure 3-5: Stagnation pressure versus the distance from the cylinder. Due to the conservation of momentum, the stagnation pressure is continuous in the gas feed system.
Figure 3-6: Static pressure versus the distance from the regulator. The discontinuity comes from the pressure drop due to the ball valve.

Figure 3-7: Stagnation pressure versus the distance from the regulator. The discontinuity comes from the pressure drop due to the ball valve.
3-4 Experimental Results

In the experiments, the highest pressure in the stagnation chamber that could be achieved by the improved gas feed system is 1593 psia, equivalent to the mass flow rate of 2.01 kg/s. In the estimation provided in Section 3-3, the highest mass flow rate is 3.06 kg/s. The discharge coefficient of this gas feed system is 0.66.

Figure 3-8 shows the measured thrust against the measured pressure in the stagnation chamber. The theoretical curve is also shown in the same figure. The measured thrust is higher than the theoretical thrust when the pressure in the stagnation chamber is lower than 600 psia. However, when the stagnation pressure is higher than 600 psia, the measured thrust becomes lower than the theoretical thrust.

![Figure 3-8: Measured thrust versus the stagnation pressure.](image)

The Figure 3-9 shows the pressure distribution in the nozzle at the stagnation pressure.
of 1593 psia, the highest stagnation pressure this system can achieve. The measured values match well with the theoretical curve. This measured distribution shows the flow in the nozzle is totally supersonic, i.e., no shock waves.

Figure 3-10 shows the pressure distribution in the nozzle at the stagnation pressure of 246 psia. This is a typical measured pressure distribution when there is a shock wave in the nozzle. The last four ports close to the exhaust have pressure values close to the ambient pressure. There should be a shock wave between the fourth and fifth ports from the nozzle exit. The position of the shock wave is much deeper than the position predicted by the theory.

![Graph showing pressure distribution](image)

Figure 3-9: Pressure distribution in the nozzle at the stagnation chamber pressure of 1596 psia.
The improved gas feed system can only achieve 65% of the mass flow rate predicted by theory. The possible reasons might be: pressure drop in the adaptors, pressure drop due to turns in the gas lines, and the effects due to the regulator.

When a flow in a tube experiences a change of the diameter of tube, there will some permanent pressure loss due to friction, eddies, and noise produced inside. Except friction, these effects cannot be described by the quasi-one-dimensional theory used in the estimation. In this system, there are several positions changing the diameter other than the

**3-5 Discussion**

**3-5-1 Difference between real mass flow rate and theoretical value**

The improved gas feed system can only achieve 65% of the mass flow rate predicted by theory. The possible reasons might be: pressure drop in the adaptors, pressure drop due to turns in the gas lines, and the effects due to the regulator.

Figure 3-10: Pressure distribution in the nozzle at the stagnation chamber pressure of 246 psia. Unlike the pressure distribution with higher pressure, the measured pressure decreases at the beginning and then increases after flow passes the second port counting from the throat.
adaptors. The 1” tube section in the system is, in fact, composed of several parts with different diameters due to the lack of strength for the 1” tube. The inner diameter of this section varies between 0.69” and 1.17”. The 1” section is, in fact, equivalent to a series of adaptors. This situation also happens in the ball valve. The orifice of the valve is only 0.472”, but the diameter of inlet and outlet are 0.80”. The bending of tubes also leads to similar permanent pressure loss. There are two 90° turns in each of the 1/2” tubes and a 90° turn in the 1” section.

The basic operating principle of the pressure regulator is that there is a device in the regulator that can compare the difference between the pressure of the flow in the regulator and the set pressure. At the same time, this device can control the size of the orifice inside the regulator. If the flow pressure is higher than the set pressure, the device will shrink the orifice to decrease the flow rate and the pressure. If the flow pressure is lower than the set pressure, the device will enlarge the orifice to increase the flow rate and the pressure. The flow through the regulator experiences several expanding and compressing processes to achieve a flow with constant pressure. It is difficult to describe the effect by the quasi-one-dimensional theory.

The effects on the flow due to these three factors are complex and highly dependent on the geometry inside. It is difficult to understand the detailed effects caused by these factors without a detailed understanding of the structure and further simulations. They are ignored in the estimation for convenience.
3-5-2 Critical pressure for the shock wave

As mentioned in Section 2-3-2, the stagnation pressure of a supersonic flow to generate a shock wave at the exhaust of the nozzle is 268 psia at 1-atm ambient pressure. In theory, there is no shock wave in the nozzle when a higher pressure is applied in the stagnation chamber. However, in the experiment with a flow with a pressure of 246 psia in the stagnation chamber, the shock wave position is much deeper than the position predicted by theory as shown in Figure 3-10. In those experiments with the stagnation pressure higher than the 268 psia, there is still a pressure jump at the position close to the exhaust most. The pressure jump disappears when the stagnation pressure is higher than 600 psia. The reason why the measured critical pressure is higher than the value predicted by theory might be the nozzle shape cannot keep the flow inside as stable as the quasi-one-dimensional theory described. Simulation with more realistic assumptions, such as 2D or 3D, is needed to understand the shock wave inside the parabolic converging–diverging nozzle.

3-5-3 Difference between measured thrust and theoretical value

As mentioned in Section 3-4 and shown in Figure 3-8, the measured thrust is lower than the theoretical value while the stagnation pressure is higher than 600 psia, but the measured thrust becomes higher than the theoretical value while the stagnation pressure is lower than 600 psia.

In Section 3-2-3, the possible factors of inaccurate thrust measurement are listed: friction, force due to the flexible tube, and viscosity in the flow in the nozzle. The friction
is minimized by the air bearing. If the friction does not become zero, the measured thrust should be smaller than the real value. For the force produced by the flexible tube, if the position of the nozzle moves a lot from the neutral position of the flexible tube when measurement occurs, there will be a force needed to bend and stretch the flexible tube. This will also result in a reduction of the measured thrust. The viscosity decreases the flow pressure in the nozzle so it also reduces the thrust. All factors result in a lower measured value, but values of thrust higher than the theoretical value are measured when the pressure in the stagnation chamber is lower than 600 psia. The reason why a higher value of thrust measured at low stagnation pressure could be the shock wave. As shown in Figure 3-8, there is a section of curve with a different curvature in the low-pressure region in the thrust curve predicted by the quasi-one-dimensional theory. It is the region with a shock wave. The measured critical pressure of the shock wave in this nozzle is around 600 psia, similar to the pressure that is the difference between the measured thrust and the theoretical value inverted from positive to negative. Therefore, this phenomenon could be relative to the shock wave. Further understanding also requires simulations with more realistic assumptions.

For the measured thrust in the region with the stagnation pressure higher than 600 psia, the difference between the thrust predicted by quasi-one-dimensional theory and the linear regression of measured thrust is a line with a slope of 0.054 N/psi and an intercept of −3.7 N. Theoretically, the slope is due to the effects proportional to the pressure, viscosity in this case, and the intercept is due to the effects independent of the pressure, the force
produced by the flexible tube in this case. The reason why the intercept is negative might be that the effect of viscosity is not linear to the stagnation pressure.

3-5-4 Freezing of the DAQ system

During flow tests, the DAQ program sometimes froze in the middle of the test, which left the ball valve open. Furthermore, when the pressure in the stagnation chamber rises to a value higher than 800 psia, the DAQ system definitely froze soon after the flow was applied. After a series of tests, two reasons were surmised: a voltage spike from the transducer measuring the stagnation pressure and the mechanical shock due to the thrust.

Figure 3-11: Measured pressure in stagnation chamber versus time. The output of the transducer is connected to the DAQ directly. A spike is shown at the beginning of the flow.
As shown in Figure 3-11, the measured stagnation pressure after the ball valve opened jumped sharply to a high value, then decreased a little bit and became stable or decreased slowly. This spike was due to air stored in the tube between the ball valve and the cylinders. The pressure of the air there is high but the amount is small. Therefore, after the valve opened, the air in this region can produce a flow with a pressure higher than the flow that could be supplied steadily from the cylinders. After the air in the region had run out, the pressure in the stagnation chamber decreased to the pressure that could be steadily supplied by the gas feed system. This voltage spike resulted in the freezing of the DAQ program.

To eliminate this voltage spike, a bypass capacitor was installed. The bypass capacitor can filter out signals with high frequency. The measured pressure with a bypass capacitor installed is shown in Figure 3-12. The spike at the beginning of the flow has disappeared. At the same time, there is no distortion for the rest of the measurement.

The other reason resulting in the freezing of DAQ program is the shock due to the beginning of the flow. The LabVIEW program uses a large amount of virtual memory, namely, it uses a part of the hard disk as its memory. If there is a shock while the hard disk is operating, it results in a failure of operation and causes the program to freeze. To solve this problem, the DAQ PC was moved away from the bench where the nozzle was set on.
Figure 3-12: Measured pressure in stagnation chamber versus time. There is a bypass capacitor installed across the output of the transducer. The spike shown in Figure 3-11 is disappeared.
Chapter 4

Computational Electromagnetics

According to geometrical optics, a parabola can focus a beam to a small point at the focal point. However, a parabolic nozzle is not a perfect parabola since a nozzle throat is needed at the apex to feed the flow. To understand the electric field distribution in a parabolic nozzle in detail, numerical simulations are needed. In this chapter, the simulation of the electric field distribution in the thruster’s nozzle is presented. The simulation was performed with COMSOL Multiphysics software. The result implies that some energy of the incident microwave is transmitted into the stagnation chamber and forms regions with high electric field strength. For the previous nozzle design, as described in Section 3-2-1, the maximum magnitude of the electric field in the stagnation chamber is higher than the electric field at the focal point of the parabolic nozzle. Another parabolic nozzle was designed to eliminate the high strength of electric field in the stagnation chamber. The first section of this chapter describes the principles of the simulation, and the second section shows the simulation result for the previous nozzle. The third section describes the process of designing a new nozzle utilizing the simulation. The last section shows the simulation results with different conditions of incident microwave beams to match the conditions in real experiments.
4-1 Basic Principles of Simulation

In the radio frequency (RF) module of the COMSOL Multiphysics software, the finite element method (FEM) is applied to solve Maxwell’s Equations for the electric and magnetic fields generated by an electromagnetic wave. FEM is widely used in numerical simulations since it can solve variable problems with high accuracy in a short time. At the same time, the axisymmetric 2D method was applied in the simulations of the electric field in nozzles to save time and computational resources. The following two subsections introduce the FEM and the polarization in the axisymmetric 2D method.

4-1-1 Finite element method

The concept behind FEM is to use basic functions to simulate the parameter of interest in a small region, utilizing these basic functions in the governing function to find the relations among the basic functions in neighbor regions, and solving the new relations to get the entire distribution of the parameter of interest in the whole region of interest. For example, the electric field in a one-dimensional question can be simulated by $a_n x^2 + b_n x + c_n$ in a small region when a quadratic method is used in an FEM simulation. This quadratic function is centered at the grid point $#n$ and only works in the region around the grid point $#n$. The value of the electric field at the grid point $#n$ equals $c_n$ and the values of the electric field in the region between two grid points equal the superposition of the two functions centered at the grid points at the tips of the region. The parameter of interest must obey the governing equation so the relations among the coefficients in each regional basic function can be derived from the governing equation. The collection of these relations in
the whole region of interest can be written into a matrix form

\[ [K] \{C\} = \{b\}, \]  

(4-1)

where \([K]\) is called the conductivity matrix, \(\{C\}\) is the collection of the coefficients in regional basic functions, called the weight vector, and \(\{b\}\) is called the load vector. The conductivity matrix and the load vector can be calculated from known parameters and boundary conditions. After the conductivity matrix and the load vector are calculated, the weight vector can be solved by inverting the conductivity matrix and calculating the product of the inverted conductivity matrix and the load vector.

Because FEM solves problems by inverting a matrix, a large number of iterations in other numerical methods like the finite difference method can be avoided. At the same time, the calculation of the components of the inverted matrix and weight vector are independent to each other. It is ideally suited for parallel computing. However, FEM consumes a huge amount of memory because the total amount of components is proportional to the square of the number of grid points.

4-1-2 Polarization in an axisymmetric 2D simulation

In this project, microwaves with a frequency of 94 GHz are used. The size of the nozzle is large compared with the wavelength at 94 GHz, which is 3.18 mm. Using the 3D method to simulate the electric field distribution in the whole nozzle would consume a large amount of computational resources. Fortunately, simulations can be conducted with the axisymmetric 2D method because of the axial symmetry of the nozzle.

The electric field in an axisymmetric 2D method can be reduced to
\( \vec{E}(r, \phi, z) = \vec{E}(r, z)e^{-im\phi} \), \hspace{1cm} (4-2)

where \( m \) is the azimuthal mode number. One inherent property of the axisymmetric 2D method is that the basic modes, i.e., the basic solutions of the governing equation, of the electric field are axisymmetric. However, microwaves in free space are usually linearly or circularly polarized. Combination of basic modes is necessary to simulate linear and circular polarizations.

According to the coordinate transformation, the relations between the unit vectors in Cartesian and cylindrical coordinates are

\[
\hat{x} = \cos \phi \hat{r} - \sin \phi \hat{\phi},
\]
\[
\hat{y} = \sin \phi \hat{r} + \cos \phi \hat{\phi}.
\]

(4-3)
(4-4)

For a microwave beam with the electric field polarized in the \( x \)-direction and \( y \)-direction, the electric field can be expressed as

\[
E_0 \hat{x} = E_0 \left[ (e^{-i\phi} + e^{i\phi}) \hat{r} - i(e^{-i\phi} - e^{i\phi}) \hat{\phi} \right] \quad \text{and}
\]
\[
E_0 \hat{y} = E_0 \left[ i(e^{-i\phi} - e^{i\phi}) \hat{r} + (e^{-i\phi} + e^{i\phi}) \hat{\phi} \right].
\]

(4-5)
(4-6)

Therefore, the linear polarizations can be simulated with the superposition of the modes \( m = -1 \) and \( m = 1 \) in cylindrical coordinates. For circular polarization, the electric fields are

\[
\frac{E_0}{\sqrt{2}} (\hat{x} + i\hat{y}) = \frac{E_0}{\sqrt{2}} [\hat{r} + i\hat{\phi}] e^{-i\phi} \quad \text{and}
\]
\[
\frac{E_0}{\sqrt{2}} (\hat{x} - i\hat{y}) = \frac{E_0}{\sqrt{2}} [\hat{r} - i\hat{\phi}] e^{i\phi}.
\]

(4-7)
(4-8)

Thus, circular polarization can be simulated with one azimuthal mode.
4-2 Simulation Results of the Previous Nozzle

The simulation results of the electric field distributions in the previous nozzle when illuminated by uniform, parallel, and circular polarized microwave beams at 94 GHz and 30 GHz are shown in Figures 4-1 and 4-2. The electric field of the incident microwave beam is 1 V/m.

At 94 GHz, the entire nozzle assembly can achieve a gain of 31 dB. However, the position of the maximum electric field strength is in the stagnation chamber. In the diverging section, the highest gain is only around 23 dB. When a microwave beam is applied, the breakdown might occur in the stagnation chamber instead of the diverging section of present nozzle. The energy of the microwave beam will heat the stagnation chamber instead of coupling with the supersonic flow. Observing the coupling between a microwave and a supersonic flow, the target of this research, is not able to be conducted with this nozzle. Designing a new nozzle was necessary.
At 30 GHz, the situation is similar to the 94 GHz. The position with the maximum gain of 25.7 dB is located in the converging section of present nozzle. The maximum gain in the diverging section is only around 18 dB.

Figure 4-1: The electric field distribution with a 94 GHz microwave beam illuminated from the exit of the nozzle. The incident electric field is set as 1 V/m. The unit of the electric field in the figure is V/m. The values marked above and below the color bar are the maximum and minimum values.
As shown in the previous section, the maximum gains of the previous nozzle with microwave beams at 94 and 30 GHz are not in the diverging section, where the supersonic flow exists. A new nozzle had to be designed to prevent the breakdown in the stagnation.

Figure 4-2: The electric field distribution with a 30 GHz microwave beam radiating from the exit of the nozzle. The incident electric field is set as 1 V/m. The unit of the electric field in the figure is V/m. The values marked above and below the color bar are the maximum and minimum values.

4-3 Designing a New Nozzle

As shown in the previous section, the maximum gains of the previous nozzle with microwave beams at 94 and 30 GHz are not in the diverging section, where the supersonic flow exists. A new nozzle had to be designed to prevent the breakdown in the stagnation.
chamber. At the same time, the gas feed system can only supply the gas flow rate up to 2.0 kg/s. A new nozzle with a smaller nozzle throat is needed to achieve a supersonic flow with a stagnation pressure of 2000 psig.

Considering the stagnation chamber as a resonant chamber, the converging section of the nozzle works as a horn antenna transmitting the microwave beam at the throat to the stagnation chamber. To eliminate the resonance in the chamber with a 94-GHz microwave beam, there are several methods available, such as changing the geometry of the stagnation chamber to change the resonant frequencies and modes, or fine-tuning the converging–diverging nozzle to decouple the incident microwave beam from the resonant modes of the stagnation chamber. Because a nozzle with a smaller throat diameter is needed, the method of fine-tuning the nozzle was chosen.

For a converging–diverging nozzle with a parabolic diverging section and a simple conically shaped converging section, the shape is controlled by five parameters: focal length of the parabolic section, converging angle, inlet radius, exit radius, and throat radius. In this case, the inlet radius was fixed by the stagnation chamber. The exit radius is related to the throat radius by a fixed area ratio to achieve Mach 5 supersonic flow. Hence, only three parameters were free variables: focal length of the parabolic section, converging angle, and throat radius.

The electric field at the throat is the superposition of the electric field due to the microwaves directly illuminating the throat and the electric field of the microwaves reflected from the remainder of the nozzle. Since the focal point is close to the throat, the
The electric field at the nozzle is strong. Increasing the focal length is chosen as the first step of the fine-tuning to decouple the incident microwave beam from the resonance in the chamber. Three values of focal length were first simulated: 3 mm (the focal length in present nozzle), 4 mm, and 5 mm. As shown in Figure 4-3, the region with the highest electric field strength is in the stagnation chamber for all three cases. The value of maximum gain in the chamber and the length of the parabolic diverging section are listed in Table 4-1. The maximum gain in the chamber decreases as the focal length increases. The design with a focal length of 4 mm was selected for further development since the length of the diverging section of the design with a 5-mm focal length is short. A short length for the diverging section leads to difficulty in installing the pressure transducers.

The second parameter for fine-tuning the converging–diverging nozzle is the converging angle. Simulations of the electric field distribution in nozzles with the converging angle from 25° to 45° with 5° step size were conducted. The maximum gains in the stagnation chamber are plotted in Figure 4-4. The maximum gain in the diverging section in all cases is around 23 dB. The combination of 4-mm focal length and 30° converging angle is the only combination able to maintain the electric field in the entire stagnation chamber lower than the electric field at the focal point of the parabolic nozzle.
Table 4-1: Maximum gain and length of the parabolic diverging section for nozzles with different focal lengths of the parabolic section with the other parameters the same as the present nozzle.

<table>
<thead>
<tr>
<th>Focal length</th>
<th>Max gain</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 mm</td>
<td>31.0 dB</td>
<td>50.0 mm</td>
</tr>
<tr>
<td>4 mm</td>
<td>30.2 dB</td>
<td>37.5 mm</td>
</tr>
<tr>
<td>5 mm</td>
<td>25.0 dB</td>
<td>30.0 mm</td>
</tr>
</tbody>
</table>

Figure 4-3: The electric field distributions in the nozzles with different focal lengths.

A) 3 mm, B) 4 mm and C) 5 mm.
The last step of fine-tuning is shrinking the throat radius. Electric field distributions in the designs with throat radii of 3.5–5.0 mm were simulated. The values of the gain at the focal point, the mass flow rate at the stagnation pressure of 2000 psig, and the length of the parabolic section are listed in Table 4-2. The 4.0-mm throat radius was chosen since the mass flow rate is at a level that can be supplied by the current gas feed system. At the same time, the length of the parabolic nozzle is sufficiently long such that the desired number of pressure transducers can be installed.

The simulated electric distributions of the final design at 94 GHz and 30 GHz are shown in Figures 4-5 and 4-6. The value of maximum electric field in the chamber at 94 GHz is less than 70% of the value at the focal point. In the case of 30 GHz, the maximum
strength of electric field is in the parabolic section, but a small region with high strength of electric field in the converging section is also observed.

Table 4-2: The list of the simulated gain at the focal point, mass flow rate for a stagnation pressure of 2000 psig, and the length of the parabolic nozzle for designs with different throat radii.

<table>
<thead>
<tr>
<th>Throat radius</th>
<th>Gain at focal point</th>
<th>Mass flow rate</th>
<th>Nozzle length</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0 mm</td>
<td>23.4 dB</td>
<td>2.54 kg/s</td>
<td>37.5 mm</td>
</tr>
<tr>
<td>4.5 mm</td>
<td>24.3 dB</td>
<td>2.06 kg/s</td>
<td>30.4 mm</td>
</tr>
<tr>
<td>4.0 mm</td>
<td>22.7 dB</td>
<td>1.62 kg/s</td>
<td>24.0 mm</td>
</tr>
<tr>
<td>3.5 mm</td>
<td>20.9 dB</td>
<td>1.24 kg/s</td>
<td>18.4 mm</td>
</tr>
</tbody>
</table>
Figure 4-5: The simulated electric field distribution in the new design with a 94-GHz microwave beam feeding from the exit of the nozzle. The strength of the electric field of the microwave beam is set at 1 V/m and the unit of the color bar is V/m.
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Different Incident Conditions

In the process of designing the new nozzle, the incident microwave power was assumed to be a uniform, parallel, and circular polarized beam propagating along the central axis of nozzle. This incident condition may not be the real case under experimental conditions. Further simulations were conducted to realize the effect under different incident conditions.

Figure 4-6: The simulated electric field distribution in the new design with a 30-GHz microwave beam feeding from the exit of the nozzle. The strength of the electric field of the incident microwave beam is set as 1 V/m and the unit of the color bar is V/m.
4-4-1 Linear polarization

Compared with a circularly polarized microwave source, a linearly polarized microwave source is much easier to obtain. Simulations were conducted to understand the focusing pattern of the new nozzle illuminated by uniformly, parallel, and linearly polarized microwave beams at 30 and 94 GHz.

The simulated focus pattern caused by a linear polarized microwave beam at 94 GHz in the 40-mm nozzle is shown in Figure 4-7. The shape of the region with the strength of electric field higher than 70% of the maximum strength is shown in Figure 4-8. Compared with the focal pattern caused by a circularly polarized microwave beam, the gain increased to 25.6 dB and the envelope of the zone with 70% of the maximum strength changed into two small balls from a ring shape in the circular polarization.
Figure 4-7: The simulated electric field distribution in the new nozzle radiated by a linear polarized microwave beam at 94 GHz. The upper one is the cut view in the $x$-$z$ plane and the lower one is the cut view in the $y$-$z$ plane. The strength of the electric field of the incident microwave beam is set as 1 V/m and the unit of the color bar is V/m.
The simulated focal pattern caused by a linear polarized microwave beam at 30 GHz in the 40-mm nozzle is shown in Figure 4-9. The shape of the region with the strength of electric field higher than 70% of the maximum strength is shown in Figure 4-10. Compared with the focus pattern caused by a circular polarized microwave beam, the gain increased to 18.1 dB and the shape of the zone with 70% maximum strength became two balls larger than the ones at 94 GHz and a small spot in the converging section of the nozzle.
Figure 4-9: The simulated electric field distribution in the new nozzle radiated by a linear polarized microwave beam at 30 GHz. The upper one is the cut view in the $x$-$z$ plane and the lower one is the cut view in the $y$-$z$ plane. The strength of the electric field of the incident microwave beam is set as 1 V/m and the unit of the color bar is V/m.
In experiments (as well in an actual launch system), perfect alignment between a nozzle and a microwave beam is difficult to achieve. Simulations were conducted to understand the effect on the maximum gain in the nozzle caused by a misaligned microwave beam. The microwave beams in these simulations were set to uniform, parallel, and linear polarization. At the same time, the incident microwave was tilted in two directions: along the direction of polarization and perpendicular to polarization.

Figure 4-10: The shape of the region with the strength of electric field higher than 70% of the maximum strength produced by a linear polarized microwave beam at 30 GHz in the 40-mm nozzle.

4-4-2 Misaligned nozzle and microwave beam
In the case of 94 GHz, the maximum gain against the incident angle is shown in Figure 4-11. The gain dropped about 1 dB over the first 2 degrees of misalignment in both directions. With further tilt, the maximum gain became higher in the case of tilting the microwave along the direction of polarization. In contrast, the maximum gain became lower in the case of tilting the microwave perpendicular to the polarization. With incident angle less than 5 degrees, the loss in gain is less than 1.5 dB. Therefore, a misalignment less than 5 degrees can be tolerated in the experiment using a microwave beam at 94 GHz.

Figure 4-11: The maximum gain in the 40-mm nozzle versus the incident angle of the microwave beam at 94 GHz.

For the case of 30 GHz, the maximum gain in the 40-mm nozzle against the incident angle is shown in Figure 4-12. In the case of tilting the microwave along the direction of
polarization, the maximum gain increased a little at the first two degrees and dropped when the microwave was further tilted. In the case of tilting the microwave perpendicular to the polarization, the maximum gain decreased when the incident angle increased. The change of the maximum gain was less than 0.5 dB when the incident angle was smaller than 5 degrees. Therefore, a misalignment less than 5 degrees can also be tolerated in the experiment with a microwave beam at 30 GHz.

Figure 4-12: The maximum gain in the 40-mm nozzle versus the incident angle of the microwave beam at 30 GHz.
4-4-3 Gaussian beam

In the high-pressure experiment, a dielectric lens with a diameter of 33 cm and a focal length of 130 cm will be used to focus the microwave. Around the focal point, the microwave becomes a Gaussian beam with a size similar to the 40-mm nozzle. Simulations were conducted to estimate the gain of the 40-mm nozzle illuminated by a Gaussian beam.

The shape of a Gaussian beam is determined by two parameters: the minimum beam radius, \( \omega_0 \), and the Rayleigh range, \( z_0 \). The minimum beam radius is given by

\[
2\omega_0 = \frac{4\lambda F}{\pi D}.
\]  

(4-9)

where \( \lambda \) is the microwave wavelength, \( F \) is the focal length of the lens, and \( D \) is the diameter of the lens. The Rayleigh range is given by

\[
z_0 = \frac{\pi \omega_0^2}{\lambda}.
\]  

(4-10)

The beam radius, \( \omega_z \), at a distance of \( z \) from the focal point in the direction of beam propagating is given by

\[
\omega_z^2 = \omega_0^2 \left[ 1 + \left( \frac{z}{z_0} \right)^2 \right].
\]  

(4-11)

The minimum beam diameter in the high-pressure experiment is 16 mm, and the beam diameter becomes 40 mm at a distance of 144 mm from the focal point. The exhaust of the nozzle was placed at these two positions in the simulations. The beam profile around the focal point and the positions of the exhaust in the simulations are shown in Figure 4-13.
Two simulations were conducted, one that simulates the situation in which the nozzle is placed between the lens and the focal point to intercept the converging section of the Gaussian beam and the result is shown in Figure 4-14. In this situation, the ratio of the maximum strength of the electric field in the nozzle to the maximum strength of the electric field at the focal point is 5.11. The other simulates the situation that the nozzle intercepts the diverging section of the Gaussian beam after the focal point and the result is shown in Figure 4-15. In this situation, the ratio of the maximum strength of the electric field in the nozzle to the maximum strength of the electric field at the focal point is 3.9.

Figure 4-13: The profile of the Gaussian beam generated by the dielectric lens around the focal point. The red dot line marks the focal point and the black dot lines mark the positions with a beam diameter of 40 mm.
Figure 4-14: The simulated electric field distribution in the new nozzle intercepting the converging section of the Gaussian beam. The upper one is the cut view in the $x$-$z$ plane and the lower one is the cut view in the $y$-$z$ plane. The maximum electric field of the Gaussian beam at the focal point was set as 1 V/m.
Figure 4-15: The simulated electric field distribution in the new nozzle intercepting the diverging section of the Gaussian beam. The upper one is the cut view in the x-z plane and the lower one is the cut view in the y-z plane. The maximum electric field of the Gaussian beam at the focal point was set as 1 V/m.
Though intercepting the beam at the same distance from the focal point, the gains and the focal patterns are different when intercepting the converging and diverging beams. When intercepting a converging beam, the gain is higher and the focal region is closer to the nozzle throat. The converging beam turns closer to a parallel one when the beam propagating in the parabolic nozzle because the converging beam is in the process of focusing. Oppositely, the diverging beam turns farther away from a parallel one in the nozzle. That is the reason why the gain is higher when intercepting the converging beam.

Concerning the focal pattern, the incident angle of a diverging beam is smaller than the one of a parallel beam and the incident angle of a converging beam is larger as shown in Figure 4-16. According to the law of reflection, the reflection angle of a diverging beam is also smaller than that of a parallel beam and the reflection angle of a converging beam is also larger. Therefore, the focal region of a diverging beam will be closer to the aperture of a parabolic reflector than the focal point. Oppositely, the focal region of a converging beam will be further from the aperture of the parabolic reflector than the focal point. These features are also shown in Figure 4-14 and 4-15.

In a Gaussian beam with a total power of \( P_\infty \), the maximum electric field, \( E_0 \), at the focal point is given by

\[
E_0 = \sqrt{\frac{4P_\infty}{c\varepsilon_0\omega_0^2}},
\]

where \( c \) is the speed of light and \( \varepsilon_0 \) is the vacuum permittivity. Assuming the total power at the focal point is 100 kW, it corresponds to 887 kV/m at the center of the Gaussian beam at the focal point. The maximum strengths of the electric field in the nozzle are 4.53 MV/m.
when intercepting the converging section of the Gaussian beam and 3.45 MV/m when intercepting the diverging section.

Figure 4-16: The illustration of the incident angles of a diverging beam, a parallel beam and a converging beam on a surface.
Chapter 5

Conclusions and Proposed Future Work

5-1 Conclusions

In this study, directly coupling the energy of a CW microwave beam to a supersonic flow in the diverging section of a converging–diverging nozzle was proposed as an alternative method to generate thrust for a launch vehicle. Because the energy source is off-vehicle, this method is expected to achieve a higher specific impulse and a higher payload mass ratio compared with contemporary launch vehicles using chemical rocket engines.

In the proposed method, a CW microwave beam is focused with a parabolic nozzle into a supersonic flow. The high-strength electric field at the focal point induces breakdown in the supersonic flow. The density of the plasma generated by the breakdown quickly rises to a density capable of absorbing the incoming microwave energy. The energy absorbed by the plasma then transfers the rotational, vibrational, or electronic degrees of freedom via inelastic collisions. Gas can be heated via the relaxation processes of excited states or the non-adiabatic transferring process from the electronic excited states to the vibrational degrees of freedom. Finally, the heated gas augments thrust by increasing the pressure in the nozzle. Some experiments and theoretical models are surveyed to understand the tendency of the energy coupling coefficient in different situations. Numerical simulations
based on the quasi-one-dimensional assumption were conducted to understand the thrust augmentation due to different heating conditions.

Besides the theoretical investigation, instruments for thrust and pressure measurements were set up and cold flow tests of the present nozzle were also conducted. In the preliminary tests, the issue of the insufficient mass flow rate in the prior gas feed system was identified. A new gas feed system was designed and tested. The performance of the new gas feed system was estimated. The new gas feed system is capable of supplying a mass flow rate of 2.01 kg/s and generate a flow with a stagnation pressure of 1600 psia in the present nozzle. The difference between the test results and the estimation were discussed.

To understand the electric field distribution in the present nozzle in detail, simulations with COMSOL Multiphysics software were conducted. An issue was found with the present nozzle is that is can result in a stronger electric field in the stagnation chamber than the electric field at the focal point of the parabolic section. To avoid this electric field distribution coupling the energy of a microwave beam with the stagnation chamber instead of the supersonic flow, a new nozzle was designed. The COMSOL Multiphysics software was utilized to find a new nozzle design without generating a resonance at 30 and 94 GHz in the stagnation chamber. At the same time, electric field distributions in the new nozzle due to different incident microwave conditions, such as linear polarization, off axis, and Gaussian beam, were also simulated.
5-2 Proposed Future Work

There are two experiments planned in the future with the new nozzle design. One is generating a supersonic flow with a stagnation pressure of 2000 psig in atmosphere and illuminated by a 94-GHz CW microwave beam with a power of 100 kW. The experimental results will be compared with the result of the quasi-one-dimensional simulation to understand the coupling between a supersonic flow and a microwave beam in atmosphere in detail. According to the investigation about the microwave-plasma-flow interaction, the energy coupling coefficient might be 8–16% when nitrogen is used. At the same time, quasi-one-dimensional simulations for this experiment with energy coupling coefficients of 15 and 30% were conducted. Thrust augmentations of 1% and 1.5% were predicted.

The other planned experiment will be conducted in a vacuum chamber to simulate the environment at high altitude. A supersonic flow with a stagnation pressure of a few psia will be generated by the nozzle in a vacuum chamber and a 30-GHz CW microwave source with a maximum power of 175 W will be used. The experimental results will be analyzed to further understand the coupling between microwaves and a supersonic flow with a low stagnation pressure into a low-pressure environment. Theoretical investigation led to the conclusion that the energy coupling coefficient might be in the 25–33% when air is used. Quasi-one-dimensional simulations were conducted to simulate a flow with a stagnation pressure of 0.25 psia coupling with a microwave with a heating power of 100 W with two different heating patterns. A thrust augmentation of 4.7% was predicted when a smooth heating pattern with a total energy coupling coefficient of 20% was used. At the same time,
a thrust augmentation of 14.2% was predicted when a smooth pattern with a 20% energy coupling coefficient and an abrupt heating pattern with a 20% energy coupling coefficient were both used.
References


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