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USE BOOLEAN NETWORK TO MODEL AND CONTROL WITHIN- AND BETWEEN-PERSON DYNAMICS

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ABSTRACT

This body of work introduces and forwards a Boolean network-based method for studying psychological dynamics, both within-person and between-persons. I outline the Boolean network method, provide a guide for implementation, and illustrate how the method is applied in two empirical settings – study of children's self-regulation, and study of group-therapy processes. The work highlights the utility of the method for obtaining intuitive descriptions of individual or group processes and deriving strategies for directing the individual or group towards desired outcomes.

Developmental science is making use of dynamical system methods to explain the mechanisms of change driving human development and to predict how and when individuals or groups will change. A natural next step is to understand how to intervene when problematic patterns or change arise. Although psychological researchers have proposed and explored use of network methods to design interventions, applications are sparse. My aim is to enrich the repertoire of methods researchers can use to learn about and direct individuals' and groups' psychological functioning, and in doing so to prompt further use of network methods for modeling behavior change.

In **Chapter 1**, I outline the motivation for introducing a Boolean network method that can be used to describe psychological systems and design interventions that may optimize how those systems function. Although a number of researchers have outlined the possibility of using dynamical system methods to guide psychological processes to desired levels, methods for deriving control strategies have remained theoretical. In this chapter, I identify a gap in the research on methods for analysis of developmental and psychological change processes – specifically, the sparsity of empirical applications of control system design despite its theoretical importance – and introduce how a Boolean network control method (Kauffman, 1969; 1993) can address this gap. Second, I briefly explain why network control is useful for guiding developmental processes, and how methods at the overlap between dynamical systems methods and network analysis can be used to develop that guidance. Third, I clarify how within- and between-person dynamics are conceptualized in this project, and how the definitions used here are analogous to other terms used in psychology. Fourth, I explain why the same dynamical system method can be used to describe both within- and between-person dynamics. I then briefly outline two empirical studies where I demonstrate how the Boolean network method can be applied to study and control of both within- and between-person dynamics.

In **Chapter 2**, I revisit how dynamical system methods are used to model the *nonlinear* dynamics of multivariate systems. Despite the interest and advancement of control theory to direct psychological dynamics toward desired goals, control has been less studied and rarely applied in *nonlinear* psychological systems.

We introduce the Boolean network method to address this gap. This method is useful because it can be used to model the *nonlinear* dynamics in multivariate systems and to develop network control strategies that might be used to manage the system toward a desired state. The Boolean network method is a discrete-time dynamical system method, and we introduce this method in three steps: (1) inference of the temporal relations between multiple binary variables as Boolean functions and construction of Boolean networks in which the binary variables are nodes and the Boolean functions are edges, (2) extraction of attractors based on the inferred dynamics and assignment of desirability for each attractor, and (3) design of network control to direct a psychological system toward a desired attractor by identifying how the Boolean network needs to be updated.

To demonstrate how the Boolean network can describe and prescribe control for emotion regulation dynamics, we applied this method to an observational dataset of children's regulation of anger using bidding and/or distraction behavior (N = 120, T = 480 seconds). Network control strategies were designed to move the child into attractors where anger is OFF. The sample shows heterogeneous emotion regulation dynamics across children in 22 distinct Boolean networks, and heterogeneous control strategies regarding which behavior to perturb and how to perturb it.

The presentation and illustration forward the Boolean network method as a novel method to describe nonlinear dynamics in multivariate psychological systems and a control method to guide nonlinear psychological systems toward desired goals.

In **Chapter 3**, I revisit theories suggesting group processes can induce desired or undesired behavior change in individuals in a group because they are under social influence. Empirical modeling of group processes often assumes the social influence is assimilative only, and network-based interventions that aim to manage group processes and promote desired behavior change does not apply when the social network is fully connected.

We introduce the Boolean network method to address these two gaps because it allows both assimilative and repulsive social influence to be modeled simultaneously, and prescribes network control strategies by changing a few group members' behavior regardless of network topology. The Boolean network method is a dynamical system method that models the group-specific temporal relations between group members' behavior as a Boolean network, and also allows for control theory to design group management strategies and direct the groups toward desired behavior.

The Boolean network method is applied to empirical data of individuals' self-disclosure behavior in multi-week therapy groups (N = 155, 18 groups, T = 10~16 weeks), to model and manage group-specific processes of self-disclosure. Results show the method can estimate each group member's self-disclosure with error rate of 0.14 (SD = 0.10). Both assimilative and repulsive social influence are found in 14 out of 18 groups. Group-specific network control strategies were designed to elicit the majority of the group self-disclose by encouraging a few group members' self-disclose behavior.

This example illustrates the Boolean network as a flexible method that allows for modeling of assimilative and repulsive social influences that simultaneously operate in a group process and design of strategies that can be used to direct the group process to desired states (without manipulating the social ties).

This dissertation introduces and forwards the Boolean network method as a method that can be used to describe and control a system's trajectory. The final chapter, **Chapter 4**, summarizes the contribution of this dissertation in terms of method innovation, theory, data, and potential applications, and begins to elaborate how the method might be extended further. To our knowledge, this is the first application of the Boolean network method in describing and controlling nonlinear psychological processes. The Boolean network method follows the long-standing tradition of using dynamical system methods to explain, model, and predict how complex psychological systems operate and change over time. This dissertation adds to that literature by providing the methodological steps and empirical examples that will enable control system design for nonlinear within- and between-person dynamics. Our demonstration emphasizes the appeal of this method for both theory and practice – providing simple descriptions and explanations of system dynamics and system control strategies. Altogether, this dissertation forwards and provides access to a useful tool that can help researchers discover, understand, and shape many different kinds of psychological dynamics.

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Chapter 1 INTRODUCTION

In this introduction chapter, I will first discuss a gap in the methodology used to study developmental and psychological change processes – sparsity of empirical applications of control system design despite its theoretical importance – and introduce how a Boolean network control method (Kauffman, 1969; 1993) can address this gap. Second, I will briefly explain why network control is useful for guiding developmental processes, and how methods at the overlap between dynamical systems methods and network analysis can be used to develop that guidance. Third, I will clarify how within- and between-person dynamics are conceptualized in this project, and how the definitions used here are analogous to other terms used in psychology. Fourth, I will explain why a single dynamical system method can be used to describe both within- and between-person dynamics. I then briefly outline the content of two empirical studies where I demonstrate how the Boolean network method can be applied to study and control of both within- and between-person dynamics.

Gap in the Methodology

Dynamical system methods have been instrumental in researchers' attempts to explain mechanisms of change and predict future trajectories and states of psychological systems (Boker & Graham, 1998; Chow, Ferrer, & Hsieh, 2010). A natural next step is to use these methods to design control strategies and guide system trajectories toward desired states and outcomes (Carver & Scheier, 1998; Molenaar & Nesselroade, 2015). So far, only a few empirical studies published in psychology journals have actually attempted to design control strategies using dynamical system methods (Wang et al., 2014).

In this dissertation, I will fill this gap by introducing a Boolean network method (Kauffman, 1969; 1993) into the psychology literature. Compared to other dynamical system methods, the Boolean network method provides an intuitive and parsimonious representation of system dynamics using only AND (multiplicative), OR (additive), and NOT (negation) rules. The Boolean network method is able to recover complex patterns (e.g., oscillatory) in the time-series simulated from the true model or the shape of the time-series using experiment data, despite the lack of detailed kinetics information (Berestovsky & Nakhleh, 2013). Benefits of having an intuitive representation of the dynamics include ease of interpretation and simplified design and communication of interventions that might optimize the system's progression toward desirable.

In the core of the dissertation (Chapters 2 and 3), I will apply this Boolean network method in studies of within- and between-person dynamics (clarified later in this chapter), and design network control in the form of edge perturbation (revising the dynamics) or node perturbation (manipulating the states of nodes). Since this dissertation is, to my knowledge, the first application of Boolean network in psychology, I will also introduce the Boolean network method in detail, as preparation and foundation for the introduction of network control design, including inference of the Boolean functions, extraction of attractors.

Overlap Between Dynamical System Method and Network

Human development – how a person changes over their life span – can be conceptualized and quantified as the product of a dynamical system. Individual development has been conceived as the product of multiple components (e.g., biology, emotions, thoughts, behaviors) that *interact* with each other at multiple time-scales and across multiple levels of analysis (Baltes, Lindenberger, & Staudinger, 2006; Ford & Lerner, 1992; Magnusson & Cairns, 1996).

Dynamical system methods use a variety of quantitative methods to model the temporal dependencies exhibited by one or multiple components, where the future value of these components are predicted by prior values (Thelen & Smith, 1994; Carver & Scheier, 2002; Boker & Bisconti, 2006). In principle and practice, the temporal relations captured by dynamical system methods can be used to describe the interactions among multiple components described by developmental theory.

Development is not the result of one psychological component or sums of multiple components, but rather is the result of ongoing, mutual influence among multiple processes or components of an interdependent network. Network science and methodology has been gradually adopted in the psychology literature in recent years to study psychological dynamics as networks (Borsboom & Cramer, 2013), including emotion network within a person (Bringmann, Vissers, Ceaulemans, Borsboom, Vanpaemel, Tuerlinckx, & Kuppens, 2016), and affect/behavior network between persons (Beltz, Beekman, Molenaar, & Buss, 2013). In all of the network representations of patterns embedded in longitudinal data, the nodes are the components/variables in the psychological system, and the edges are the temporal relations among those nodes – the dynamics. That is, the networks are constructed using multivariate dynamical system methods (Bringmann, et al., 2016; Yang, Ram, Gest, Lydon, Conroy, Pincus, & Molenaar, 2018) – meaning that when it comes to studying multivariate dynamical systems, network science and multivariate dynamical system methods have become synonymous.

Despite the overlaps in conception and application, the network literature in psychology and the dynamic systems literature in psychology have emerged separately. The primary advantage of viewing the dynamical system as a network is to operationalize, explore, and test, the view that different components of the system are connected and influence one another. When researchers adopt the idea that the phenomena of interest can be conceived as a network, an abundance of network theories (or graph theories) and network analysis methods (Barabasi, 2016) can be used to explore and test psychological theory about those phenomena. Indeed, psychologists are advancing understanding of a variety of phenomena by using the network metrics (e.g., density, centrality) to summarize and interpret observed behavior (Bringmann, et al., 2016). A lot more network theories and analytical methods have yet to be explored, and hold great potential of new discoveries. This dissertation, for example, adapts and uses network control methods from biology (Zanudo & Albert, 2015) to model and increase understanding of individual-level and group-level psychological systems.

Within- and Between-Person Dynamics

This dissertation project, which is built around two empirical examinations of dynamics, demonstrates the value of using the Boolean network method to examine different types of psychological processes. I use the terms *within-person dynamics* and *between-person dynamics* to distinguish the two substantive research questions that are explored using the Boolean network method. *Within-person dynamics* is used to refer to the research questions that mostly concern the dynamics that occur within a person, such as emotion regulation dynamics. Social behaviors, which happen between people, can be part of the within-person dynamics, but the research question is focused on analysis of the individual person, with the aim to describe, explain, and predict interindividual differences and intraindividual variability. *Between-person dynamics* is used to refer to the social dynamics that manifest as multiple persons interact in a shared environment. This kind of between-person or social network dynamics is the focus of study in, for example, studies of social influence and how that influences, at the micro-level, individuals’ behavioral development (e.g., aggression, substance use), and, at the macro-level, spread of behavior across persons.

Modeling Multivariate Dynamics

In this dissertation, I use the same Boolean network method to study both within- and between-person dynamics. Here, I explain why the same dynamical system method can be applied to the two types of dynamics by first comparing the research questions and data structure, and then explaining why the same analytical method can be used in both settings.

When studying within-person dynamics, our goal is to describe and explain the interactions between psychological components and use a network-based representation of the within-person dynamics to explain how an individual changes over time. When studying between-person dynamics, our goal is to describe and explain the spread of behavior from individuals to their peers and use a network-based representation of the dynamics to explain how a group of individuals changes over time. These two types of research questions use different types of data, and may or may not focus on the same mechanism or psychological components.

Methodological separation of within-person and between-person dynamics makes use of terminologies created by Cattell (Cattell, 1952) to refer to the data structures used in different applications. Cattell's data box consists of three dimensions, namely person (or participants), occasion, and variable. Answering different research questions requires slicing the databox in different ways, as shown in Figure 1.1a. For within-person dynamics, since we focus on the mechanistic interaction between psychological components (i.e., variables) within a person, we usually take the P-data – a slice of one person's data in the databox (N =1, p = multiple variables; and T = multiple ordered occasions), as shown in Figure 1.1b. P-data, sometimes called intensive longitudinal data (when there are many occasions), could be modeled by dynamical system methods. For between-person dynamics, since we focus on the social interaction and behavior influence between people, we usually take the T-data – a slice of one (behavior) variable's data in the databox (p = 1, N = multiple people, T = multiple ordered occasions), as shown in Figure 1.1c. This type of data is often seen in social network studies, where a variable of interest is collected from each member in the network longitudinally (e.g., prosocial and antisocial behavior, Osgood et al., 2013).

The data structures supporting examination of within-person and between-person dynamics provide for different kinds of substantive inferences, the same analytical methods can be used in both cases. This is because P-data and T-data slices both have an "occasions" dimension where T = many occasions are temporally ordered. The essence of dynamical system methods is to utilize the temporal dependencies in the data – information obtained from temporally ordered observations.

Dynamical system methods use the temporally ordered observations (occasion dimension) to set up models and infer how variables/persons are related to each other over time, relations between state at time t and state at time $t+1$. For within-person dynamics, the analytical method to analyze P-data generally need to estimate the temporal relations between variables – how variables predict each other from time t to time t+1. This is often written in equation from as,

$$v_{t+1} = \boldsymbol{\Phi} v_t + \zeta_t$$

where v_t and v_{t+1} are the vectors of *variables* at time t and time t+1, $\boldsymbol{\Phi}$ is a matrix of temporal relations between v_t and v_{t+1}, ζ_t are the innovations coming from external environment into the system. We can depict this model as a network, where the nodes are the variables in v, and the edges are the temporal relations between nodes in $\boldsymbol{\Phi}$. For example, we show in Figure 1.1d how a 3-node network look like, and the temporal relations could be expressed in the matrix form $\boldsymbol{\Phi} = \begin{bmatrix} 0 & 0.5 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix}$ or as a network of edges from the predictor to the outcome. For example, the edge from $v_{3,t}$ to $v_{1,t+1}$ indicates $v_{3,t}$ influenced $v_{1,t+1}$ and the magnitude of this temporal influence is 0.4. Inference from networks derived from a single slide of P-data apply to a single individual.

For between-person dynamics, the same analytical method could be used on T-data. We can apply the dynamical system method on the T-data to analyze behavior dynamics in a social network, so that we know which peers significantly influenced each individual. Here, we need to estimate the temporal relations between persons (of the same variable) – how one person's behavior predicts each other's behavior from time *t* to time *t*+1. This can be written as,

$$p_{t+1} = \boldsymbol{\Phi} p_t + \zeta_t$$

where p_{t+1} and p_t are the vectors of different *persons* at time t and time t+1, $\boldsymbol{\Phi}$ is a matrix of temporal relations between p_t and p_{t+1}, ζ_t are the innovations coming from external environment into the social network system. We can depict this model as a network, where the nodes are the persons in p, and the edges are the temporal relations between nodes in $\boldsymbol{\Phi}$. For example, we show in Figure 1.1d how a 3-node network look like, and the temporal relations could be expressed in the matrix form $\boldsymbol{\Phi} = \begin{bmatrix} 0 & 0.5 & 0.4 \\ 0.6 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix}$ or as a network of edges from the predictor to the outcome (e.g., the temporal relation from $p_{3,t}$ to $p_{1,t+1}$ is 0.4). Here the edges are the temporal relations between people's behavior, which is conceptualized as the social influence, as shown in Figure 1.1e. Inference from networks derived from a single slide of T-data apply to a group of individuals.

From the panels in Figure 1.1, we can see clearly the data structure (Figure 1.1b and 1.1c) share the occasions dimension and dynamical system applied on this data structure (Figure 1.1d and 1e) can be the same; the primary difference between the two the two slices of data are the content of the rows – variables or persons – each of which is mapped to a specific type of research question, either how variables influence each other in within-person dynamics (Figure 1.1b, 1.1d), or how people influence each other in between-person dynamics (Figure 1.1c, 1.1e). This is analogous to the comparison between P-technique and R-technique factor analysis. The analytical procedure is exactly the same – factor analysis (Ram, Brose, & Molenaar, 2013) – but in P-technique, the procedure is applied to P-data (single person, multiple variables, multiple occasions), while in R-technique, the procedure is applied to R-data (single occasion, multiple variables, multiple persons). The same technique can be used to answer different research questions, namely whether the observed differences, within-person across occasions or between-persons at one occasion, explained by a smaller number of latent constructs. There is however, a minor difference in how model constraints are used in the two settings. In between-person dynamics, it is usually assumed the social influence only exists when there are social ties. Thus, the constraints on how individuals can influence each other should be applied when fitting such a model. In my applications of the Boolean network method, I will highlight when such constraints are needed.

To summarize, understanding the connections of analytical procedure between the two research questions explains why the same method can be used to operationalize and answer different kinds of research questions with different kinds of data.

Content of Dissertation Studies

In this dissertation, I include two papers that demonstrate the value of using and how to use the Boolean network method to model the dynamics embedded in multivariate binary time-series.

In Chapter 2, I introduce and forward the Boolean network method as a method that can provide intuitive explanation of psychological dynamics that are useful for theorists and suggestions for network control that are useful for practitioners. A Boolean network $G(V,B)$ consists of a set of binary variables $V = \{x_1, \ldots, x_n\}$ as nodes and corresponding Boolean functions $B = \{f_1, \ldots, f_n\}$ as edges. The Boolean functions consist of predictors connected by Boolean operators "AND" and "OR" and are inferred to find the best prediction for the outcome variable by an algorithmic search. Attractors can be derived based on the inferred Boolean functions by permutation of all states as initial condition, and they can be classified as desirable or undesirable guided by theoretical or practical reasons. Based on the desirability of attractors, control strategies can be designed to direct the system out of undesired attractors by revising the dynamics.

The utility of the Boolean network method for study of within-person dynamics is illustrated using multivariate binary time-series data (*T*=480 second) obtained in a study of children's self-regulatory strategy (*N* = 120, age 36-month-old). Our example illustrates how the Boolean network method can identify the behavior that (dys)regulates each child's anger and provide personalized advice that would improve each child's emotion regulation skills. This study provides a novel method to analyze psychological dynamics and to designs control for multivariate psychological systems.

In Chapter 3, I introduce and forward the Boolean network method as a method that can be used to model social influence dynamics and to identify strategies that can be used to manage the spread of behavior. I propose Boolean network method to address the tension between social influence theory and previous methods because it allows both assimilative and repulsive social influence to be modeled. Then attractors can be extracted and desirability can be assigned to each attractor, regarding the spread of (un)desirable behavior. The Boolean network method can prescribe network control, by identifying the subset of nodes and associated states to perturb.

The utility of the Boolean network method for study of between-person dynamics is illustrated using empirical data obtained from individuals participating in multi-week therapy groups (*N* = 155, mean group size =6.6, *T* = 10~16 weeks). Results show evidence of both assimilative and repulsive social influence in each group. Some group processes have two attractors. One attractor, an undesirable one from a therapeutic perspective, is characterized by a pattern where only one or two participants self-disclose constantly. The other attractor, a more desirable one, is characterized by a pattern where most participants self-disclose in a rotating manner. Using these maps, network control strategies were designed that might prevent the multi-person systems from moving into the less desirable attractor. The analysis illustrates how the Boolean network method can be used to analyze group processes and to make suggestions on how to avoid the group getting stuck in undesirable scenarios.

The dissertation closes, Chapter 4, with a general discussion about how the work contributes in terms of method innovation, theory, data, outlines other potential applications, and begins to elaborate how the method might be extended further.

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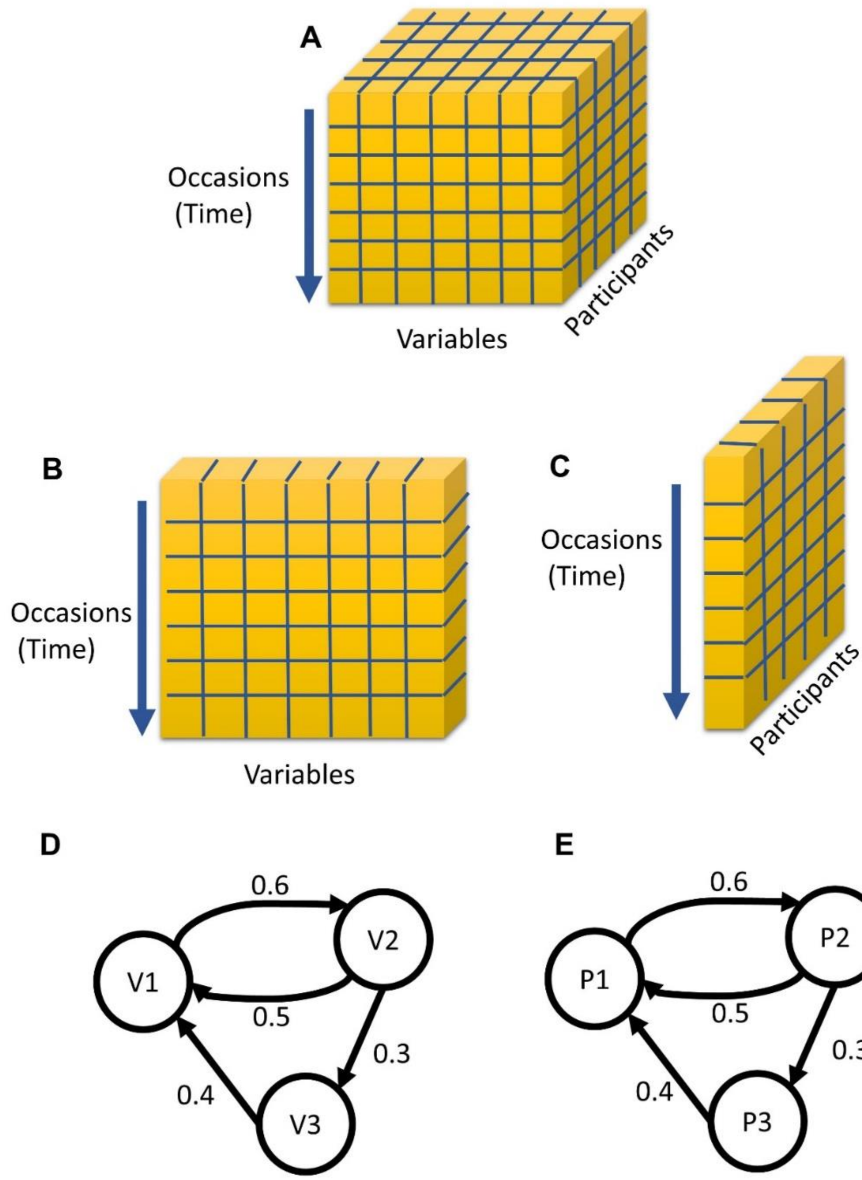

Figure 1.1 From Cattell's databox to the data structure and corresponding network illustration for within- and between-person dynamics.
Panel A: Cattell's data box with occasions, variables, and participants as 3 dimensions. Panel B: P-data as a slice of a single person's data. Panel C: T-data as a slice of data of a single variable. Panel D: Estimated dynamical system (3-variable example) depicted as a network, where nodes are variables, and directed edges correspond to the directional temporal relations. Panel E: Estimated dynamical system (3-person example) depicted as a network, where nodes are persons, and directed edges correspond to the directional temporal relations.

Chapter 2 DESCRIBING AND CONTROLLING MULTIVARAITE NONLINEAR DYNAMICS: A BOOLEAN NETWORK APPROACH

Introduction

In this paper, I introduce and forward the Boolean network method as a method that can model and describe nonlinear psychological dynamics from multivariate binary time-series and provide network control to ensure the dynamical system stays in desired state(s).

A Boolean network $G(\boldsymbol{X}, \boldsymbol{B})$ consists of a set of binary variables $\boldsymbol{X} = \{x_1, \ldots, x_n\}$ as nodes and corresponding Boolean functions $\boldsymbol{B} = \{f_1, \ldots, f_n\}$ as edges. The Boolean functions consist of predictors connected by Boolean operators "AND", "OR", and "NOT", and are inferred to find the best prediction for the outcome variable by an algorithmic search. Attractors are extracted based on the inferred Boolean functions by permutation of all states as initial condition, and they can be classified as desirable or undesirable guided by practical reasons. Based on the desirability of attractors, control strategies can be designed to direct the system out of undesired attractors by revising the dynamics.

The utility of the Boolean network method for study of within-person dynamics is illustrated using multivariate binary time-series data (*T*=480 second) obtained in a study of children's self-regulatory strategy (*N* = 120, age 36-month-old). Our example illustrates how the Boolean network method can identify the behavior that (dys)regulates each child's anger and provide personalized advice that would improve each child's emotion regulation skills. This study provides a novel method to analyze psychological dynamics and to design control for multivariate psychological systems.

Motivating Problem: Development of Emotion Regulation

Emotion regulation is conceptualized as a *dynamic, interactive process* between prepotent responses and executive process (Cole, Ram, & English, 2018). Prepotent response (PR) refers to "highly probable actions that take priority over other response options under specific conditions (Arnold, 1960)"; an example of PR is intensifying effort to overcome an obstacle (e.g., pounding on a vending machine for a jammed snack). Executive process (EP) refers to "cognitive processes aid in regulating whether and who a prepotent response is enacted – attention, memory, language, and reasoning"; an example of EP is shifting attention away from desired object (e.g., walking away from a vending machine that jammed a snack). Emotion regulation is defined as "the *influence* of strategies that recruit the executing capacities of human cognition to alter the intrinsic dynamics of prepotent response" (Cole, Martin, & Dennis, 2004; Kopp, 1982). In other words, emotion regulation is in the temporal dynamics between PR and EP.

From a developmental perspective, the emotion regulation dynamics also develop with age and are expected to change over time (Cole et al., 2018; Morales et al., 2018). By the time they enter school, children are expected to have sufficient regulatory skills to delay their own goals while focusing on teacher-defined activities (Cole, Tan, Hall, Zhang, Crnic, Blair, & Li, 2012). For example, children should be able to regulate their negative emotions (e.g., frustration and anger) when they are asked to wait for a desired object or fun activity. In the context of children's waiting tasks, the dynamic relation between negative emotion and attention focus could be summarized as this: when attention is shifted *away* from a desired object (e.g., a toy that they are told to wait to open), then anger is reduced; when attention is *focused* on the desired object, then anger is perpetuated. Consequently, children are expected to *learn to use* behaviors that shift attention *away* from the desired object (e.g., distraction) in order to reduce anger, and *learn to avoid* behaviors that shift attention *toward* the desired object (e.g., bidding about the object) and perpetuate anger. The *developmental goal* of emotion regulation is not to erase negative emotions, but to *effectively* engage executive control and enact *behaviors* that reduce negative emotions.

In sum, successful development of emotion regulation requires both a dynamic process wherein negative emotions are altered by regulatory strategies, and a control strategy wherein specific regulatory strategies are invoked when they are needed.

Binary Time-Series and Methods to Describe and Control Multivariate Dynamical System

Studies of children's emotion regulation often make use of laboratory based experimental and observation research paradigms (e.g., delayed gratification; Mischel, Shoda, Rodriguez, 1989). Binary coding is a common practice to identify the moment-to-moment presence or absence of psychological states and behaviors (e.g., emotions and regulatory behaviors; Cole et al., 2012). The experimental paradigm and behavioral coding produce abundant binary time-series data that is used to study a wide variety of phenomena, including children's emotion regulation.

To *describe* the emotion regulation dynamics in the binary behavioral codes, we need a dynamical system model that can model the temporal dynamics in binary time-series. There have been a few methods proposed to study binary time-series and applied in psychology, such as Ising model (Ising, 1925) and Markov chain method (Meyn & Tweedie, 2005). The Ising model focuses on describing the interactive dynamics in a system of binary nodes, and estimates node-specific activation thresholds and node-pairwise interactions. Psychologists have applied the Ising model on psychopathology symptom networks to model the (de)activation of symptoms, while accounting for the reinforcing relations between symptoms (van Borkulo et al., 2015; Cramer et al., 2016; Epskamp, Maris, Wardorp, Borsboom, 2018). The Markov chain method focuses on the transition probabilities between different states from t to $t+1$. This method can be applied to study the state transition in a binary system, where the states are vectors of the binary variables. Psychologists have applied more advanced variations of the Markov chain method to model state transition probabilities between observed and latent psychological states, e.g., reciprocal Markov modeling; Lu, Pan, Zhang, Dube, & Ip, 2015), mixed Markov modeling (de Haan-Rietdijk, Kuppens, Bergeman, Sheeber, Allen, Hamaker, 2017). In this paper we forward the Boolean network as an alternative to these methods to model temporal dynamics.

To *achieve* the developmental goal of emotion regulation, children often require at least some guidance on how to modify their behaviors so that they can effectively regulate emotions. Control theory (or control system design), a subfield of mathematics and engineering, focuses on moving dynamic systems toward desired goals. For example, engineers and mathematicians have developed control systems to guide planes so that they land successfully by adjusting engine thrust and steering in real time to accommodate on-going changes in the environment (e.g., wind). A variety of mathematical tools and algorithms have been developed to determine the specific actions (control input) that will guide the behavior of a dynamical system so that it follows a desired trajectory or settles into a desired state (Lewis, Vrabie, Syrmos, 2012; Liu & Barabasi, 2016).

Psychology researchers have, for decades, proposed to use control theory in psychological systems (Carver & Scheier, 1998; Molenaar & Ram, 2010; Molenaar & Nesselroade, 2015). There have been a few advanced applications of linear control theory into psychological systems. Prior work by Sinclair & Molenaar (2008) introduced an advanced optimal control technique, receding horizon feedback-feedforward control, to prescribe optimal amount of intervention (insulin input) for Type I diabetes patients. Each patient's bivariate dynamical system consists of concentration of insulin and glucose. This receding horizon feedback-feedforward control method combines an estimator of the dynamical system and a controller to direct the dynamical system. The estimator uses Extended Kalman filtering with Iteration and Smoothing (EKFIS) to estimate the state-space models with time-varying parameters in the bivariate system. The controller uses recursive optimal control techniques to control a single person's glucose at the desired level while minimizing the fluctuations in the glucose during dynamic control process. This receding horizon feedback-feedforward control method later was empirically applied to Type I diabetes patients' data and performed well in both estimation and control (Wang et al., 2014). Other recent empirical work creatively uses controllability as a network metric to inform therapists about how to optimally select symptom(s) as intervention targets, and uses linear quadratic regulator (LQR) to evaluate the efficacy of interventions (Henry, Robinaugh, & Fried, in press). Despite the interest and advancement of control theory methods, the empirical applications of control conducted in psychology remains scarce, and even fewer control methods introduced to guide *nonlinear* psychological systems.

In this paper, I aim to accomplish both describing and controlling nonlinear dynamics by the Boolean network method. The Boolean network method describes the temporal dynamics by Boolean operators, including AND, OR, and NOT; and the Boolean network method can also design network control to ensure the dynamical system stays in desired states.

Boolean Network

The Boolean networks (BN) method, originally introduced by Kauffman (1969, 1993) to study gene regulatory networks, has been extensively applied in systems biology to model and prescribe control for nonlinear dynamical systems (Cheng & Qi 2009; Zanudo & Albert, 2015).

A Boolean network $G(X(t), B)$ is defined by a set of nodes $\boldsymbol{X}(t) = \{x_1(t), x_2(t), \dots, x_n(t)\}$, where x_i is the i^{th} node, and a set of Boolean functions $B = \{f_1, f_2, \dots, f_n\}$, where each Boolean function $f_i(x_{i1}, x_{i2}, \dots, x_{ik})$ with *k* specific input nodes for node x_i is used to determine the state of x_i at time t+1. When $x_i = 1$, the node is in an activated state (i.e., behavior occurring), and when $x_i = 0$, the node is dormant (i.e., behavior not occurring). The vector formed by the current state of all $i = 1$ to n nodes, $\boldsymbol{X}(t)$, describes the current state of the system, and the set of Boolean functions, *B*, describe the dynamics of the system, how the states change from $\boldsymbol{X}(t)$ to $\boldsymbol{X}(t+1)$. The Boolean functions represent the temporal dynamics between nodes (i.e., how the nodes are influenced by other nodes), and consist of three Boolean operators (AND $\wedge$, OR $\vee$, NOT $\bar{x}$) that can be combined together to express complex relations.

Here we give a quick definition of each Boolean operator. The AND ($\wedge$) operator is defined as ***all*** input variables have to be ON to turn the outcome ON; the OR ($\vee$) operator is defined as ***any*** input variables being ON can turn the outcome ON; the NOT ($\bar{x}$) operator simply takes the opposite state of the input variable. Table 1 shows how these rules produce different outcome based on the input of two variables.

The nonlinear dynamics in a Boolean network are reflected by the Boolean operators. When a Boolean function contains two variables connected with an AND operator, it represents a multiplicative relation between the two input variables, e.g., z = x AND y is similar to z = x × y. This kind of multiplicative relation is a *nonlinear* relation between the input variables and the outcome variable. Given that the AND operator and OR operator can be transformed to each other, the OR operator can also capture nonlinear dynamics. And, when combined together with the NOT operator, lots of different kinds of nonlinear dynamics can be captured (Whitesitt, 2010).

To give an intuitive illustration of the Boolean dynamic network, we use a two-node network consisting of x_1 and x_2. The observed time-series of x_1 and x_2 are shown in Figure 2.1a, where the states of x_1 and $x_2$are ordered by time-steps *t1*, *t2*, *t3, t4*…. From this observed binary time-series, we can infer the Boolean functions

$$x_1(t+1) = x_1(t) \wedge \overline{x_2(t)}$$
$$x_2(t+1) = x_2(t)$$

(the details of how rules are inferred will be outlined later in the Boolean Network Implementation section). $x_1(t+1) = x_1(t) \wedge \overline{x_2(t)}$ indicates x_1 at *t+1* depends on both x_1 and x_2 at *t*. The state of $x_1(t+1)$ is predicted by both x_1 and x_2 at *t* after taking the AND operation aforementioned, e.g., if x_1 is ON and x_2 is also ON at *t*, x_1 will be turned OFF at *t+1*. $x_2(t+1) = x_1(t)$ indicates x_2 at *t+1* depends on x_2 itself at *t*. The state of $x_2(t+1)$ is predicted by x_2 at *t*, e.g., when x_2 is ON at *t*, x_2 will continue to be ON at *t+1*.

The network graph of these Boolean functions is shown in Figure 2.1b, where the first function "$x_1(t+1) = x_1(t) \wedge \overline{x_2(t)}$" is expressed using an expanded Boolean network diagram (Wang & Albert, 2011). In expanded Boolean network diagrams, AND operator will be expressed using a composite node; so $x_1(t) \wedge \overline{x_2(t)}$ is plotted as a composite node separately from x_1 and x_2 as a separate node, indicating it is an AND operator connecting x_1 and x_2. The composite node has a green edge from x_1 and a red edge from x_2, indicating equal and negative relation respectively. Then this composite node has a green edge pointing to x_1, indicating the equal relation between the composite node to x_1.

The second function "$x_2(t+1) = x_2(t)$" is indicated by a green edge from x_2 to x_2 representing the effect of $x_2(t)$ on $x_2(t+1)$.

We can use the above Boolean network to describe emotion regulation dynamics. Suppose x_1 and x_2 represent a child's anger and distraction. From the first Boolean function, we know that anger is regulated by distraction because whenever distraction is ON at *t*, anger will be turned OFF at *t+1*. From the second Boolean function, we know distraction is not activated by anger. Altogether, the Boolean network provides a dynamical system model that can be used to *describe* the emotion regulation dynamics embedded in the binary time-series data.

Attractors and Their Desirability

Once the temporal dynamics are described by Boolean functions, attractors can be extracted based on the dynamics. The purpose of extracting attractors is to articulate what persistent patterns individuals get stuck in and determines whether to intervene. Researchers can determine the desirability of each attractor based on domain knowledge and practical concerns. For example, we define an undesirable attractor as anger is persistently activated. If a child does not have this kind of attractor, then the child does not need external help to regulate anger; if a child does have this kind of attractor, maybe only as one of the child's many attractors, the child needs external help only when he gets stuck in this attractor.

Attractors of a dynamical system indicate the long-term behavior of the system, and a dynamical system can have various types of attractors and multiple attractors. A working definition of attractor is an area of phase space – the set of possible states of the system – the system occupies or approaches more frequently than others (Carver & Scheier, 2002). If a dynamical system is thought as a physical object whose movement is governed by the dynamical rules, then an attractor is where the dynamical system prefers to stay in this physical space (Thelen & Smith, 2005). It is worth noting there are many forms of attractors, such as fixed-point attractor, limit cycle, saddle, cusp, etc. In this paper, we will only need to use fixed-point attractor and limit cycle; the other forms of attractors are beyond the scope of this paper (for a review of various attractors, see Thelen & Smith, 2005). A fixed-point attractor means the attractor has a single fixed value, e.g., a pendulum *with* friction eventually returns to a stable point. A limit cycle attractor means the attractor has a set of states with different values and the system will cycle through these states, e.g., a pendulum *without* friction oscillates (Thelen & Smith, 2005). It is also worth noting nonlinear dynamical system can have multiple attractors (Boker & Graham, 1998). A dynamical system with multiple fixed-point attractors is also called *multi-stable*, as it has multiple stable points. Take smoking behavior as an example, smokers who are trying to quit might have multi-stable dynamical system with smoking and quitting as two attractors. On some days, smokers who try to quit can put in efforts to maintain non-smoking and stay in the quitting attractor; but on other days when cravings are strong, they can be pulled back to the smoking attractor.

The desirability of attractors depends on the domain knowledge of a dynamical system and the (un)desirability of attractors can inform when a dynamical system needs external help. For example, cancerous attractor is undesirable for a biological system (Campbell & Albert, 2019), behavior habits like smoking is an undesirable attractor of behavior for smokers (Boker & Graham, 1998), coactivation of many depressive symptoms is an undesirable attractor of mental activities for depressed patients (Cramer et al., 2016). Sometimes attractors change within the same developmental system and the desirability also changes according to the developmental context. For example, crawling is an attractor of locomotion for children before they develop abilities to walk (Thelen & Smith, 2005), which means crawling maybe desirable for a younger age but undesirable when children are expected to walk.

The Boolean network method can extract multi-stability of a dynamical system and various kinds of attractors. Continuing with the two-node Boolean network as an illustrative example, we can extract attractors from state transition graph using these Boolean functions (the details of how derivation works will be introduced in the Data Analysis section). If we represent the state of x_1 and x_2 as a tuple of the two variables (x_1, x_2), e.g., (0,0) means both x_1 and x_2 are OFF, we can express the state transition as a graph, shown in Figure 2.1c. For the given Boolean network in Figure 2.1b, the state of system will transition from (1,1) to (0,1), and then from (0,1) to (0,1), indicating (0,1) is an attractor as the system will not transition to any other states once it is in (0,1). Similarly, we can know from the state transition graph in Figure 2.1c that (0,0) and (1,0) are also fixed-point attractors. Using the above Boolean network to describe emotion regulation dynamics, where x_1 and x_2 represent a child's anger and distraction, then we can define (0,0) and (0,1) are both desirable attractors because anger (the first element in the tuple of two states) is OFF, and (1,0) is an undesirable attractor. Thus, using the Boolean network method, we discover the system has three attractors, and one of them is an undesirable attractor. The child will be emotionally neutral for a long time if the child goes into the desirable attractors; but when the child gets stuck in the undesirable attractor, the child will be angry for a long time and need external help to be regulated. We demonstrate in this example the Boolean network extracts three attractors, all of which are fixed-point attractors, and the Boolean network has been used to study limit cycle attractors from previous research (Kauffman, 1993; Aracena, Gomez, Salinas, 2013).

Boolean Network Control and Emotion Regulation

The Boolean network method utilizes the differential desirability of attractors *within the same dynamical system* and design network control strategies to move the dynamical system from an undesirable attractor to a desirable attractor. Because not all attractors are desirable with a dynamical system, we can assign desirability to each attractor based on practical concerns. For example, an attractor in which negative emotion is OFF is desirable; on the contrary, an attractor in which negative emotion is ON is undesirable.

Control strategy can be derived based on the distance from undesirable attractor to desirable attractor basin (the details of how derivation works will be introduced in the Data Analysis section). Continuing with the two-node Boolean network as an illustrative example, we can design network control based on the extracted attractors, and one network control strategy is to perturb x_2 when x_1 is ON as shown in Figure 2.1d, so that the system will transition from (1,0) to (1,1), and then goes to (0,1) because x_2 can turn x_1 OFF. This new state transition is also depicted in Figure 2.1d.

If we continue to use the above Boolean network to control emotion regulation dynamics, where x_1 and x_2 represent a child's anger and distraction. Based on the extracted attractors in Figure 2.1c, we already know attractor (1,0) is undesirable, and attractors (0,0) and (0,1) are desirable, based on the state of anger (the first element in the tuple). To move the system from attractor (1,0) to attractor (0,1), we can turn distraction ON when anger is ON, and anger will eventually be turned OFF. Therefore, the control strategy is to turn distraction ON when anger is ON.

Intermediate Summary

To circle back to the developmental goal of emotion regulation, which requires not only the dynamic process wherein negative emotions are altered by regulatory strategies, but also the effective use of behaviors as regulatory strategies to reduce negative emotions when they occur. This example illustrates exactly how network control design *fulfills* the developmental goal. The emotion regulation dynamics in our illustrative example can be summarized as anger is regulated by distraction but distraction is not activated by anger, and the network control strategy suggests to turn distraction ON when anger is ON. If parent or teacher can help children develop this control strategy, then the control strategy will help fulfill the developmental goal of "effective use of behaviors as regulatory strategies to reduce negative emotions when they occur".

Boolean Network Implementation

Because the Boolean network method is new to the psychology literature, we synthesized the Boolean network method introduced in system biology literature, to cover the essential concepts and methodological details to enable readers to have a solid understanding of the Boolean network method as a method for network modeling and network control.

The Boolean network method we introduce here has three major steps: (1) inference of the temporal relations between multiple binary variables as Boolean functions and construction of a Boolean network in which the binary variables are nodes and the Boolean functions are edges, (2) extraction of attractors based on the inferred dynamics and assignment of desirability for each attractor, and (3) design of network control to direct a psychological system toward a desired attractor by identifying how the Boolean network needs to be updated.

Inference of Boolean Functions and Construction of Boolean Networks

The Boolean functions can be inferred from the observed time-series of all the variables using a variety of algorithms (Lähdesmäki, Shmulevich, & Yli-Harja, 2003). Simulation studies provide good evidence that these algorithms can accurately recover the key features of continuous-scale dynamics (e.g., oscillation) in a variety of settings (Berestovsky & Nakhleh, 2013). To illustrate how the inference proceeds, we describe a Boolean computation algorithm introduced by Akutsu and colleagues (Akutsu, Miyano, & Kuhara, 2000). Input variable refers to the variables that produce the outcome variable, similar to predictors. Number of input variable used to infer Boolean functions is denoted by *K*, and the size of the network is denoted by *N*, so the Boolean network with size *N* and input *K* is sometimes called a *NK* Boolean network. When *K* is higher it increases the complexity of the system and decreases parsimony; thus a lower *K* is preferred when no improvement is found. The inference proceeds iteratively, starting with *K* = 1, then with *K* = 2, and so on to the pre-selected level of complexity.

Inference when *K* = 1. When *K* = 1, Boolean functions are inferred by comparing the time-series of the outcome variable and *one* input variable. In the case of *K* = 1, the Boolean functions can be the following categories:

CONSTANT: 0, 1

UNARY: $x, \bar{x}$

where expression $\bar{x}$ means the negative of x or NOT x. By simply comparing the target variable (or outcome variable) with the Boolean function (CONSTANT or UNARY, meaning there is one input variable) of the input variable. We identify the Boolean function that has the minimal error. Error is defined as the sum of false positive (type 1 error) and false negative (type 2 error) (Lähdesmäki et al., 2003). For example, for each x_j (*j* could be any variable in the system, including *i*), we test the following four functions, and select the function that has the minimal error rate as the inferred Boolean function.

$$x_i(t+1) = 0$$
$$x_i(t+1) = 1$$
$$x_i(t+1) = x_j(t)$$
$$x_i(t+1) = \overline{x_j}(t)$$

Inference when *K* = 2. When *K* = 2, Boolean functions are inferred by comparing the time-series of the outcome variable and *two* input variables, and these two input variables can be combined by any Boolean operators, including AND, OR, NOT. The inference uses matrix multiplication and become building blocks for Boolean function inference when *K* > 2. In the case of *K* = 2, the search for Boolean functions are conducted in the following order:

AND: $x_i \wedge x_j$, $x_i \wedge \overline{x_j}$, $\overline{x_i} \wedge x_j$, $\overline{x_i} \wedge \overline{x_j}$

OR: $x_i \vee x_j$, $x_i \vee \overline{x_j}$, $\overline{x_i} \vee x_j$, $\overline{x_i} \vee \overline{x_j}$

XOR: $x_i \oplus x_j$, $x_i \oplus \overline{x_j}$, $\overline{x_i \oplus x_j}$

where the XOR rule means only when the two input variables have opposite values, the outcome is 1. XOR is a succinct way to describe the rule $x_i \oplus x_j = \overline{x_i} \wedge x_j \vee (x_i \wedge \overline{x_j})$. We describe inference of each kind of function in turn.

First, we explain the inference of AND rules. The essence of the algorithm is to *compare* the multiplication product of each pair of two variables in the system at time *t* and the outcome variable at time *t+1*, then *select* the two variables with the product that is the closest match to the outcome variable.

The multiplication product of each two input variables in the system at time *t* is computed as a matrix $\boldsymbol{Z}$, which is a multiplication product of two matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$. The first matrix $\boldsymbol{X}$ is defined as $\boldsymbol{X}_{i,t} = x_i(t=t) \cdot 2^{t-1} mod\ p$, and the second matrix $\boldsymbol{Y}$ is the time-series $[x_1(t), x_2(t), \dots, x_n(t)]$. The multiplication of these two matrices $\boldsymbol{Z} = \boldsymbol{X^T Y}$ has the multiplication of two variables, in such a way that $\boldsymbol{Z}_{i,j} = \sum_{t=1}^{T} x_i \wedge x_j \cdot 2^{t-1}\ mod\ p$. The fingerprint function of outcome variable $x_k(t+1)$ is defined as $F_p(x_k(t+1)) = \sum_{t=1}^{T} x_k(t+1) \cdot 2^{t-1}\ mod\ p$. Note here the multiplier in $F_p(x_k(t+1))$ is still 2^{t-1} to align with the index of time in $\boldsymbol{X}$. Then we compare the elements in the product of these two matrices $\boldsymbol{Z}_{i,j}$

with the fingerprint function of the outcome variable $F_p\left(x_k(t+1)\right)$. If a particular element $\boldsymbol{Z}_{i,j}$ and the fingerprint function of outcome variable $x_k(t+1)$ has a minimal mismatch, then x_i and x_j will be selected as the best fit for the AND rule, or $x_k = x_i \wedge x_j$.

Here, we show this computation in a 3-node network as example, the observed times-series is $\boldsymbol{Y} = (x_1(t), x_2(t), x_3(t)) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, then $\boldsymbol{X} = \begin{pmatrix} 0 & 2^0 & 0 \\ 2^1 & 0 & 2^1 \\ 2^2 & 2^2 & 0 \\ 0 & 2^3 & 0 \end{pmatrix} \; mod\; p$. Suppose the outcome we want to predict is $x_1(t+1)$, and $x_1(t+1)$=$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$. $\boldsymbol{Z} = \boldsymbol{X^T Y} =$

$$\begin{pmatrix} 0 & 2^1 & 2^2 & 0 \\ 2^0 & 0 & 2^2 & 2^3 \\ 0 & 2^1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2^1 + 2^2 & 2^2 & 2^1 \\ 2^2 & 2^0 + 2^2 + 2^3 & 0 \\ 2^1 & 0 & 2^1 \end{pmatrix}$$, and the finger print function of $F_p\left(x_1(t+1)\right) = (2^0 \quad 2^1 \quad 2^2 \quad 2^3) \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$=$2^0 + 2^1$. This $F_p\left(x_1(t+1)\right)$ matches $Z_{1,3}$ most closely, with 1 mismatch – 2^0. Hence, $x_1 \wedge x_3$ is selected as the best fit for this rule. Note here, we did not consider the diagonal line of $\boldsymbol{Z}$ because they will collapse as a unary rule, as $x_i \wedge x_i = x_i$.

Second, we explain the inference of OR rules. Since any OR rules could be rewritten as AND rules, using $a \vee b = \overline{\bar{a} \wedge \bar{b}}$, so the inference of OR rules can utilize the same computation s AND rules with proper transformation, e.g., the search of $x_i \vee x_j$ is the same as search for $\overline{\overline{x_i} \wedge \overline{x_j}}$, which is the negation of $\overline{x_i} \wedge \overline{x_j}$. This could be achieved by the same algorithm introduced for AND rules, adding the negation to $\boldsymbol{Z}$.

Third, we explain the inference of $y = x_i \oplus x_j$ type of functions. First note that $x_i \oplus x_j = y$ if $x_i \oplus y = x_j$. So to find the x_j that satisfies $y = x_i \oplus x_j$, we simply compare the time-series of $x_i \oplus y$ with the time series of every x_j, and the best matching time-series is the x_j that satisfies $y = x_i \oplus x_j$. The inference of $x_i \oplus \overline{x_j}$ will follow a similar procedure with the input $\overline{x_j}$.

When the unary rule has equal error rate with *K* = 2, the unary rule would be chosen for parsimony reasons. The other two types of AND rules, namely $x_i \wedge \overline{x_j}$, $\overline{x_i} \wedge \overline{x_j}$, can be searched by replacing either $\boldsymbol{Y}$ or both $\boldsymbol{X}$ and $\boldsymbol{Y}$ matrix with the negation of the matrix itself.

Inference when *K* > 2. When *K* > 2, the inference can be done recursively with the input of two variables as the foundation. **In the case of *K* ≥ 2,** a recursive algorithm is used to build the inference upon the case of *K* = 2. Let us consider a case of three input variables. We can infer the Boolean functions $f\left(x_i, x_j, x_k\right)$ in the following way. First, infer the rules of two input variables, x_i, x_j, denoted as $f_1\left(x_i, x_j\right)$, that that matches the value of the outcome y when $x_k(t) = 1$, and then infer the rules of two input variables, denoted as $f_2\left(x_i, x_j\right)$, that matches the value of the outcome y when $x_k(t) = 0$.

The inferred Boolean function is as follows:

$$f\left(x_i, x_j, x_k\right) = \left(x_k \wedge f_1\left(x_i, x_j\right)\right) \vee \left(\overline{x_k} \wedge f_2\left(x_i, x_j\right)\right) \tag{1}$$

Here is the proof. When $x_k(t) = 1$, Equation 1 can be transformed in the following equation:

$$f(x_i, x_j, x_k) = \left(1 \wedge f_1(x_i, x_j)\right) \vee \left(0 \wedge f_2(x_i, x_j)\right)$$
$$= \left(1 \wedge f_1(x_i, x_j)\right) = f_1(x_i, x_j) \qquad (2)$$

Since we inferred $f_1(x_i, x_j)$ by matching to the outcome y, when $x_k(t) = 1$, thus $f(x_i, x_j, x_k) = y$. Similarly, we consider when $x_k(t) = 0$, Equation 1 can be transformed as the following equation:

$$f(x_i, x_j, x_k) = \left(0 \wedge f_1(x_i, x_j)\right) \vee \left(1 \wedge f_2(x_i, x_j)\right)$$
$$= 1 \wedge f_2(x_i, x_j) = f_2(x_i, x_j) \qquad (3)$$

Given that $y = f_2(x_i, x_j)$ when $x_k(t) = 0$, $f(x_i, x_j, x_k) = y$. Note here f_1 and f_2 have to have the same input variables x_i, x_j, so that the number of input variables is kept at 3 after adding x_k.

Any higher number of input variables could be inferred by the similar principle recursively. For example, when the number of input variable is 4, we can infer the Boolean functions $f(x_i, x_j, x_k, x_l)$ for fixed x_l in the following way. First, infer the rules of three input variables, denoted as $f_1(x_i, x_j, x_k)$, that matches the value of the outcome y when $x_l(t) = 1$, and then infer the rules of three input variables, denoted as $f_2(x_i, x_j, x_k)$, that matches the value of the outcome y when $x_l(t) = 0$. The new rules are the different combinations of f_1 and f_2 in the form of $\left(x_l \wedge f_1(x_i, x_j, x_k)\right) \vee \left(\overline{x_l} \wedge f_2(x_i, x_j, x_k)\right)$.

Choice of *K*. We inferred the Boolean functions and constructed the Boolean network using the R package BoolNet (Müssel, Hopfensitz, & Kestler, 2010). When inferring the Boolean functions, we specify the number of input variables (*K*) and search algorithm. In this paper, we specify *K* = 2, this is because when *K* > 2, the system will likely to be chaotic and not able to extract meaningful attractors to facilitate control design. We identify the Boolean function that has the minimum error, where error is defined as the sum of false positives (type 1 error) and false negatives (type 2 error) (Lähdesmäki et al., 2003).

The construction of Boolean network is putting all the inferred Boolean functions together. As a hypothetical example, Figure 2.1b shows the construction of a two-node Boolean network based on the inferred Boolean functions of the three nodes.

Extraction of Attractors

After the Boolean functions are inferred, a state transition graph can be constructed by exhaustively search all the state transition sequence from each permutation of initial conditions. The attractors are identified by constructing the state transition graph and finding the absorbing states, that is, states that will transition back to itself.

We demonstrate this procedure using the two-node network introduced above in Figure 2.1,

$$x_1(t+1) = x_1(t) \wedge \overline{x_2(t)}$$
$$x_2(t+1) = x_2(t) \qquad (4)$$

There are $2^N = 2^2 = 4$ possible initial conditions (x_1, x_2) for the 2-node network, (0,0), (0,1), (1,0), (1,1). From each initial condition, we can use the set of inferred Boolean functions, *B*, to compute the state of the system at all subsequent time steps. For example, when the system starts with initial condition (0,0), meaning x_1=0, x_2= 0 at *t* = 1, then $x_1(t=2)$=$x_1(t=1) \wedge \overline{x_2(t=1)}$= 0 ∧ $\overline{0}$= 0 ∧ 1 = 0, and $x_1(t=2)$ =$x_1(t=1)$ = 0. We then know (0,0) will transition to (0,0). Similarly, we can compute the subsequent state of the system for all the other initial conditions: (0,1), (1,0), and (1,1). The collection of state transitions are shown in Table 2.

The state transitions described by the Boolean functions can be drawn as a state transition graph as shown in Figure 2.1c. All the possible states of the system, (x_1, x_2), are represented as circles, and the arrows indicate what state will follow at time $t + 1$ when the system is in a particular state at time t. The arrows that point back to the same state indicate the absorbing states where once the system is in that state it will remain in that state. Here, (0,0), (0,1), and (1,0) are all transitioning back to themselves, and thus are the attractors of this dynamical system.

Design of Network Control

The purpose of network control is to direct the system from undesirable attractors to desirable attractors (Shmulevich & Dougherty, 2010; Campbell & Albert, 2019). In our substantive example, we use network control to suggest behavior modification that will help children develop regulatory skill and not get stuck in negative emotions.

Network control in this paper focuses on modifying the state space transition graph in a Boolean network. The output of control system design should include three pieces of information: (1) which node(s) or Boolean function(s) needs to be changed, (2) which states node(s) need to be changed to – the specific strategy of network control, and (3) what is the condition to change – the context of deploying network control. These three pieces of information will be useful to drive the system into more desirable states.

The general procedure for identifying control strategy is as follows:

1. *Formulate the goal of network control in terms of the desirable attractor(s) and undesirable attractor(s).* Then the goal of network control is to move the system out of the undesirable attractor, and into the desirable attractor. Assignment of desirability to each attractor could be done based on practical concerns.
2. *Compute the Hamming distance from an undesirable attractor to the states in a desirable attractor basin.* An *attractor basin* is defined as the set of states that will eventually go to the same attractor. We consider moving the system from an undesirable attractor to the basin of a desirable attractor because the system will eventually go to the desirable attractor once the system is in the desirable attractor basin. The distance from undesirable attractor to each state in the desirable basin then can the *number of nodes and their Boolean functions we need to manipulate*. Distance here is computed using Hamming distance (Hamming, 1950), where we compare two states digit by digit, and then sum the number of digits that differ from each other as the distance. For example, a state (0,0,1) has distance of 2 from another state (1,0,0) because the first and third digits differ from each other.
3. *Identify the control strategy.* Suppose there the undesirable attractor has a Hamming distance of 1 to a state in the desirable attractor basin, and the 1 difference is caused by node x_i. Suppose x_i is OFF in the undesirable attractor. Then "turning node x_i ON when the system is in the undesirable attractor" is our *control strategy*. There could be multiple states that have distance of 1 to the undesirable attractor, indicating multiple control strategies. It is also possible there is no state that has distance of 1 to the undesirable attractor, indicating more nodes need be to manipulated to control the system.
4. *Formulate the new Boolean function for node x_i.* After the node x_i needs to be perturbed is identified, the new form of this node's function should obey the following equations

$$f_{x_j} = \begin{cases} f_{x_j}, \ if \ (x_1, x_2, \dots) \neq undesirable \ attractor \ state \\ \overline{f_{x_j}}, \ if \ (x_1, x_2, \dots) = undesirable \ attractor \ state \end{cases} \tag{5}$$

So far, we have introduced how to identify which node to perturb, and under what circumstances we should perturb it. We now use an example to demonstrate the network control with Boolean functions and state transition graph before and after network control, shown in Figure 2.2. The original Boolean functions are in equation 4. We know the attractors based on the state transition graph are (0,0), (1,0), (0,1) from Figure 2.2b. Suppose we define x_1= 1 is undesirable, then attractor (1,0) is undesirable, and the other two are desirable. Therefore, the control strategy is to perturb x_2 when the system is in state (1,0), because the distance from (1,0) to (1,1) is 1, and (1,1) belongs to the desirable attractor basin. The new function of f_{x_2} is as follows:

$$f_{x_2} = \begin{cases} x_2, \ \ if \ (x_1, x_2) \neq (1{,}0) \\ \overline{x_2}, \ \ if \ (x_1, x_2) = (1{,}0) \end{cases} \tag{6}$$

The function under specific condition can be expressed as a product of the function and the condition with an AND operator, e.g., $x_2, \ \ if \ (x_1, x_2) \neq (1{,}0)$ can be expressed as $x_2 \wedge \overline{(x_1 \wedge \overline{x_2})}$. The two conditions can be combined by an OR operator, e.g., $f_{x_2} = (x_2 \wedge \overline{(x_1 \wedge \overline{x_2})}) \vee (\overline{x_2} \wedge (x_1 \wedge \overline{x_2}))$= $x_2(t) \vee (x_1(t) \wedge \overline{x_2(t)})$. Comparing this new Boolean function with the original inferred Boolean function of x_2, we see that $x_1 \wedge \overline{x_2}$ is added to the original function with an OR operator, and this function means to turn x_2 ON when x_1 is ON and x_2 is OFF – the undesirable attractor state.

To confirm the new Boolean function can direct the system from undesirable attractor to desirable attractor basin, we plot the state transition graph after perturbation in Figure 2.3d.

In sum, we have introduced the Boolean network method, focusing on how to model and control a multivariate dynamical system. Specifically, we have synthesized the Boolean network method into three steps, (1) inference of the Boolean functions and construction of the Boolean network; (2) extraction of attractors; and (3) design of network control that can direct the system into the researcher-defined desirable attractors. Following these three steps, we can describe the dynamics underlying multivariate time-series data as a Boolean network, and identify network control strategies that can move the system into desirable states. In principle, the network control strategy is the basis for designing behavior modification programs and interventions that can help individuals achieve desirable developmental goals.

Empirical Example

We illustrate how the Boolean network method can be used to examine emotion regulation dynamics and support development of effective emotion regulation using data from a study of how children regulate their emotions during a frustration-inducing wait task at age 36 months, an age when children are developing the ability to regulate emotions (Kopp, 1989). Comprehensive information about the Development of Toddlers Study can be found in Cole et al., 2012. Details relevant to the present study are given below.

Participants

The analysis sample consisted of 117 children (64 boys) described by their mother as White (92%) or biracial (8%) (see also Cole et al., 2011 for complete demographic information). All procedures were approved by the Pennsylvania State University's Institutional Review Board (IRB protocol #18993 and #45013).

Procedure

We examined the dynamics of children's emotion regulation in the context of a frustration-inducing wait task (Vaugh, Kopp, & Krakow, 1984). During the task, the child and mother were seated in an observation room. A research assistant provided the child with a boring toy (e.g., broken car) and a gift wrapped in shiny paper, and the mother with questionnaires to complete. The children were instructed by their mother to wait until she finishes her work to open the gift. Mothers were instructed in advance that the experiment was to see how children develop the ability to wait and that while she completed the questionnaires, she should do whatever she ordinarily does when her child needed to wait for something. After 8 minutes, the research assistant returned and the mother let the child open the gift.

Measures

The procedure was video recorded, and the videos were then coded for the child's expression of emotion and behavior. Here, we examine the dynamics of a specific negative emotion, *anger*, and two regulatory strategies, *bidding to mother* and *focused distraction*.

Anger expressions. Children's anger expressions were coded second by second using a system based on facial expressions and vocal quality (Cole, Zahn-Waxler, & Smith, 1994). Anger intensity was coded for each second on a 0-4 scale (0 = *not present*, 1 = *low intensity*, 2 = *moderate intensity*, 3 = *high intensity*). Interrater reliability was good (Cohen's κ= .86). For the purpose of demonstrating the Boolean network method, we simplified the codes as binary data 0 and 1 with all codes above 0 as 1.

Regulatory strategies. Two regulatory strategies were of interest: *bids* to mother about demands of the wait (e.g., asking how much longer the wait is) and *focused distraction* that was initiated by the child and not done in a disruptive manner (e.g., becoming absorbed in an alternate activity, such as playing with the boring toy). Interrater reliability for strategies were good (Cohen's κ= .84). Like anger, bids and focused distraction were coded as binary variables that indicated whether the behavior had not (=0) or had (=1) occurred in each second.

Results

I will introduce the results in two parts. The first part uses analysis of one person to illustrate how the three steps are applied and interpreted. The second part summarizes the analysis of all 117 individuals in the sample.

Person-Specific Analysis: Identifying a Behavior Modification Strategy

Following the procedures outlined above, we (1) inferred the Boolean functions and constructed the Boolean network, (2) extracted the attractors, and (3) designed network control strategy for each of the 117 children.

To further illustrate the model results, we also go through one child's example to explain the data analysis results in three steps.

Inference of Boolean functions and construction of Boolean networks. For network construction, the time-series of anger, distraction, and bidding were entered in the BoolNet R package to construct a person-specific Boolean network. The R package will find the best fit for each variable at time *t+1* in the form of a Boolean function that use the same three variables as input variables at time *t*. All the Boolean functions together consist the Boolean network of the given child.

As an example, we show a particular child's binary time-series data, and the inferred Boolean functions in Figure 2.3a and 2.3b. The inferred Boolean functions are as follows:

$$anger(t+1) = anger(t) \wedge \overline{distraction(t)} \quad (7)$$

$$bidding(t+1) = bidding(t) \quad (8)$$

$$distraction(t+1) = distraction(t) \qquad (9)$$

We can see distraction can regulate anger, because if distraction is ON (1), then $\overline{distraction(t)}$ is OFF (0), and anger will become OFF (0) because of the AND (∧) operator. It is also worth noting distraction is not affected by anger yet, and bidding is independent from anger and distraction.

Extraction of attractors. For each child, we then used the person-specific Boolean networks to make a state-transition graph and identify attractors – the states that transition back to themselves. Based on the general developmental goal that children will eventually be able to wait without expressing anger, we formally defined attractor states where anger = ON as undesirable attractors and attractor states where anger = OFF as desirable attractors.

The state transition for our example child is shown in Figure 2.3c, where each dot represents a state of the system, which is a 3-digit string ordered as anger, bidding, and distraction. Six attractors are identified and each attractor basin is highlighted by a different color in. The desirable attractors (0,0,0), (0,1,0), (0,0,1), and (0,1,1), where anger is OFF. The undesirable attractors are (1,0,0) and (1,1,0) where anger is ON.

Design of network control. For network control design, we search control strategies using the algorithm introduced earlier in the Boolean Network Implementation section.

In our example child, one behavior modification strategy is found to move the system from attractor (1,0,0) to attractor (0,0,1) and from attractor (1,1,0) to attractor (0,1,0), and the new Boolean function of distraction, shown in Figure 2.3d. The new Boolean function of distraction means to maintain the same state as distraction at *t-1*, which is expressed as $distraction(t)$), or take the opposite state of distraction when anger is ON, which is expressed as $anger(t) \wedge \overline{distraction(t)}$. According to this new Boolean function, when the system is in the state of anger = ON and distraction = OFF at *t*, distraction will become ON at *t+1*, as it is taking the opposite state of distraction; then, anger will be turned OFF at *t+2*, as we know distraction can regulate anger based on Equation 4. Figure 2.3e shows the updated the state transition graph and how the system will transition from its previously undesirable attractor to a desirable attractor basin, e.g., the undesirable attractor state (1,0,0), which used to be an undesirable attractor, now transitions to (1,0,1) (highlighted by a green arrow), and then transitions to (0,0,1), which is a desirable attractor.

Between-Person Differences: Different Network, Attractors, and Control Strategies

After conducting analysis for each of the 117 children separately, we can summarize the findings for the whole sample in Table 3 and Figure 2.4. Table 3 shows the result of the 3 analytical steps, namely inferred Boolean functions, state transition graph with attractors depicted, and control strategy (showing count of participants for each row).

We did not find guidelines for a cutoff of acceptable error rate from the literature introducing inference of Boolean functions (Akutsu, et al. 2000), so we report the empirical distribution of error rate in our sample. Error rate for Boolean function inference is on average 0.04 (*SD* = 0.03); error rate by variable are: error rate of inferring Boolean function of anger is on average 0.03 (*SD* = 0.04), error rate of bidding is on average 0.06(*SD* = 0.03, error rate of distraction is on average 0.03 (*SD* = 0.02).

Overall, there are 22 different Boolean networks found, *based on the Boolean function expressions*, as show in Table 3. Subsequently, each Boolean network will have their own unique state transition graph with associated attractors, and their own associated control strategy, shown in Table 3 and Figure 2.4.

There are totally 40 children whose anger is predicted to be 0/OFF with different functions for bidding and/or distraction (row 1 to row 4 in Table 3), and all the attractors has anger = 0/OFF (in Figure 2.4a-d), hence no control strategy is needed.

There are totally 8 children whose *bidding is regulating anger* with different functions for bidding and/or distraction (row 5 to row 9 in Table 3). The regulating effect of bidding is indicated by the Boolean function: anger(t+1) = anger(t) AND NOT bid(t). These networks have both desirable and undesirable attractors except one (in Figure 2.4e-i except 4g), and the corresponding control strategy is to *turn bidding ON when anger is ON*. There is one network (row 7 in Table 3, Figure 2.4g) does not have control strategy available. This is because bidding is 0/OFF in the Boolean function inference; therefore, there is no bidding = ON in the state transition graph. The search algorithm for control strategy cannot find a state transition to turn bidding ON.

There are totally 7 children whose *distraction is regulating anger* with different functions for bidding and distraction (row 10 to 12 in Table 3). The regulating effect of distraction is indicated by the Boolean function: anger(t+1) = anger(t) AND NOT distraction(t). These networks have both desirable and undesirable attractors (in Figure 2.4j-l), and the corresponding control strategy is to *turn distraction ON when anger is ON*.

There are totally 9 children whose *bidding is dysregulating anger* (row 13 in Table 3). The dysregulating effect of bidding is indicated by the Boolean function: anger(t+1) = anger(t) AND bidding(t). These networks have both desirable and undesirable attractors (in Figure 2.4m), and the corresponding control strategy is to *turn bidding OFF when anger is ON*.

There are totally 2 children whose *distraction is dysregulating anger* with different functions for bidding and distraction (row 14 to 15 in Table 3). The dysregulating effect of distraction is indicated by the Boolean function: anger(t+1) = anger(t) AND distraction(t). One network has both desirable and undesirable attractors (in Figure 2.4n), and the corresponding control strategy is to *turn distraction OFF when anger is ON*. The other network has only desirable attractor (in Figure 2.4o), so no control strategy is needed.

There are totally 53 children whose *anger is not affected by neither bidding nor distraction* with different functions for bidding and distraction (row 16 to 22 in Table 3). The no effect on anger is indicated by the Boolean function: anger(t+1) = anger(t). These networks have both desirable and undesirable attractors (in Figure 2.4p-v), but there is no control strategy available because anger is not affected by either of the behavior, so modifying either behavior will not turn anger OFF.

Substantive Interpretations. I want to *highlight several important findings* in the results. First, there is striking *heterogeneity* within the sample in terms of the Boolean networks, and their associated attractors and control design. One network can differ from another network in at least one Boolean function for one node (e.g., row 1 versus row 2), or they can differ by all three Boolean functions for all three nodes (e.g., row 4 versus row 5). Correspondingly, the state transition graph shows heterogeneous trajectories of each child’s emotion system in the binary state space. Depending on the initial state of a system, the system can end up in one attractor (Figure 2.4d), two attractors (Figure 2.4c), three attractors (Figure 2.4a, f), four attractors (Figure 2.4b, g, h, o, r, t), five attractors (Figure 2.4k, l, n), six attractors (Figure 2.4e, i, j, m, p, s, u, v), and eight attractors (Figure 2.4q).

The second important finding is control strategy depends on the regulatory dynamics between behavior (bidding or distraction) and anger. When a behavior, being it distraction or bidding, is regulating anger, the control strategy is turning that behavior ON when anger is ON.

when a behavior, being it distraction or bidding, is dysregulating anger, the control strategy is turning that behavior OFF when anger is ON.

The third important finding is not every network is controllable. The lack of controllability of the Boolean networks in this empirical example is due to anger is not affected by any behavior, and the state of anger determines desirability. We also constrained the control strategy could be only turning the behavior variable, because turning anger ON/OFF is not considered a feasible strategy.

From a substantive perspective, we can summarize the children into three types concerning their emotion regulation ability: *underdeveloped*, *developing*, and *well-developed*. The *underdeveloped* type is referring to the networks with no available control strategy (row 7 and row 16 to 22 in Table 3). The *developing* type is referring to the networks with available control strategy (row 5 to 6 and 8 to 14 in Table 3). The *well-developed* type is referring to the networks that do not need control strategy (row 1 to row 4 and row 15 in Table 3).

Discussion

This paper introduces the Boolean network method as a method to describe and prescribe control strategy for nonlinear, multivariate psychological dynamics. This method provides a novel method to model the nonlinear dynamics in multivariate psychological systems, and it also provides a control method to prescribe control strategies for nonlinear dynamical systems.

A Novel Method to Describe Nonlinear Dynamics

The Boolean network method provides a novel method to model and describe nonlinear dynamics for binary time-series, and I will compare the Boolean network method and the commonly used methods for binary nonlinear dynamics – Ising model and Markov Chain method – to discuss the unique contribution of the Boolean network method.

Comparison with Ising. The first and foremost difference is the mathematical expression of dynamics are different. A Boolean function that contains two input variables connected with an AND operator, it represents a multiplicative relation between the two input variables, e.g., z = x AND y is similar to z = x × y. This kind of multiplicative relation is a *nonlinear* relation between the input variables and the outcome variable. The Ising model uses a form of exponential function, similar to logistic regression, to link the input variables/nodes and the outcome/node x_j (van Borkulo et al., 2015).

$$P\left(x_{j,t+1} \middle| x_{\setminus j,t}\right) = \frac{\exp\left[\tau_j x_j + x_j \sum_{k \in V_{\setminus j}} \beta_{jk} x_k\right]}{1 + \exp\left[\tau_j x_j + x_j \sum_{k \in V_{\setminus j}} \beta_{jk} x_k\right]} \qquad (11)$$

where x_j represent the j^{th} node in a network, τ_j is the threshold of activation for node x_j, and β_{jk} is the *pairwise interaction* (also called weight) between x_j and another node x_k (Note: $\setminus j$ indicates all indices but j). In the Ising model, the nonlinear dynamics are expressed in the pairwise interactions. The Boolean network method and the Ising model both provide important information to describe the nonlinear dynamics, but with different mathematical expressions. Depending on the need for interpreting dynamics, researchers can choose which method to use.

Comparison with Markov Chain. The Boolean network method also has strong similarity with the Markov chain method, and the unique contribution of the Boolean network method is it *provides explicit information about the dynamics between variables*. Using the emotion regulation example, we can learn distraction is a regulatory behavior if distraction turns anger OFF, and we can also learn distraction has no effect on anger if anger does not depend on distraction. The information regulatory dynamics is modeled and identified in the Boolean functions. As a contrast, the Markov Chain method only focuses on estimating the state transition probability from *t* to *t+1* (Meyn & Tweedie, 2005). The state here is the state of a system, which is a vector of all the variables in the system. The state transition in Markov chain is similar to the state transition graph in the Boolean network method, except the Boolean network method assumes the probability is 1 when there is a state transition, and 0 when there is not state transition. Even though the dynamics between variables are embedded in the state transition probability in the Markov chain method, but the dynamics are not modeled or reported explicitly. Take a 3-node network as an example, this network/system has $2^3 = 8$ distinct states, e.g., 000, 001, …, 111, the Markov chain focuses on estimating the transition probability matrix between any two states, e.g., from state 000 to state 001, but does not provide an explicit interpretation of which variable led the third node to be turned ON. In sum, an advantage of the Boolean network method is it provides explicit explanation of the dynamics between variables and help researchers understand the psychological dynamics.

Boolean Network Control of Nonlinear Dynamics

In this paper, we introduced the Boolean network method as a method to control nonlinear dynamics. Specifically, we introduced a kind of Boolean network control method by finding the attractors first for a nonlinear system and moving the system from an undesirable attractor to a desirable attractor basin by designing new Boolean functions.

There is an important clarification about this network control method. The desired states are constrained to the dynamical systems' naturally occurring attractors in the network control method we introduced in this paper (Campbell & Albert, 2019). The reason is this kind of control method was proposed because control for biological, social, or technological systems tend to involve only naturally occurring attractors, so relaxing the definition of full control, which refers to the ability to control the system to any desired state in the state-space, is more realistic while still incorporating the nonlinear dynamics (Mochizuki, Fiedler, Kurosawa, Saito, 2013; Zanudo & Albert, 2015). Other Boolean network control methods are available to search for control strategies to find a sequence of 0-1 vectors such that the system is directed to any desired state (Akutsu, Hayashida, & Tamura, 2008). In other words, there are available Boolean network control methods that can direct the system to non-naturally occurring attractors; we can choose to use these methods if there is such a need.

The Boolean network control methods contributes a method that can tackle the complexity of nonlinear dynamical systems, compared with linear control theory methods. Linear control theory methods focuses on stabilizing the dynamics in the system using linear algebra, designing an input/control into the system to place the real part of the eigen values of the dynamics matrix to negative values, so that the discrepancy between the system state and the goal state will exponentially decrease to zero (Lewis et al., 2012; Brunton & Kutz, 2017). But linear control theory essentially assumes there is only one equilibrium point, which does not fully capture the complexity of the nonlinear dynamics, which are often marked by multi-stability, limit cycle, and even transitions between different regimes (Tang & Bassett, 2018). Hence, the Boolean network method introduced in this paper accommodates the nonlinear dynamics by accounting for multiple attractors and provides control strategies that can direct the system from an undesirable attractor to a desirable attractor.

Fuzzy logic control (FLC) is an alternative approach to control nonlinear dynamical systems (Yager & Zadeh, 1992) using if-then rules. The commonality between the Boolean network control and FLC is they both use logic rules to make decisions about control (e.g., if anger is ON, then distraction should be ON). FLC is traditionally model-free and requires humans' creating a set of rules as the controller. Take driving for example, the fuzzy logic controller could be a set of rules created from human's expert opinion to decide to hit brake at different degrees (e.g., hard, a little hard, not at all), according to the distance from another car is (e.g., very close, medium close, not close). Some later development began to extend fuzzy logic control into model-based controllers (Palm, Driankov, & Hallendoorn, 1997). Future work can explore in more detail the overlaps and various benefits of the Boolean network approach in relation to the FLC approach.

Reducing Burden of Applying Control in Psychological Systems

We have demonstrated the simplicity of the control strategy produced by the Boolean network, which requires switching a node ON or OFF in the system to move it out of an undesirable attractor. This kind of control strategy will reduce burden of applying control in psychological systems.

Now let us go through a hypothetical control implementation to explain why the Boolean network method reduces the burden of applying control in psychological systems. A control system requires the controller of a system to *at least know what input variables they should manipulate and how to manipulate them according to the state of the system* (Molenaar & Nesselroade, 2015). A univariate discrete-time control system can be expressed as:

$$u^*(t) = [y^* - \beta y(t)]/\phi \qquad (10)$$

where $y(t)$ is the observed univariate developmental process, and $u^*(t)$ is the univariate external input that needs to be manipulated in order to control the system.

For a psychological system, if a human is put in the control system to be in charge of enacting control, e.g., a parent is put in the position to help regulate a child's negative emotion, the human must know the input variable $u^*(t)$ and how to manipulate them when $y(t)$ changes over time, e.g., $y(t)$ could be the expression of anger of a child. For a continuous-scale system, the input variable $u^*(t)$ is a continuous-scale variable, and that means the control strategy will vary according to the states of the system $y(t)$. In other words, the human controller must *adapt* to the state of the system $y(t)$ and *change their control strategy* $u^*(t)$ accordingly. This is cognitively demanding and might be a major reason contributing to the sparsity of application of control in psychology.

The advantage of the Boolean network method is it reduced the variation of input variable to a binary switch-like strategy (e.g., ON/OFF) and the state of the system to a specific context (e.g., stuck in undesirable attractor). When applied to emotion regulation dynamics, the network control is in the form of behavior modification, if available. The switch-like control strategy reduces the burden from the controller (e.g., parent, teacher) to memorize and implement control in the child's emotion regulation process. This advantage of reducing cognitive burden on human controllers will remove barriers of applying control to psychological dynamical systems, and encourage more empirical research in application of control.

Using Network Control Strategy to Assess Person-Specific Developmental Goal

In our empirical example of 3-year-old toddlers, which is the developmental age when children exhibit tremendous individual differences in emotion regulation abilities (Kopp, 1989). We applied the three steps of the Boolean network method *for each child's data*, constructed *person-specific* Boolean network and designed *person-specific* network control. The person-specific network control also can assess person- specific developmental goals.

Based on the need and availability of control strategy, we defined 3 types of emotion regulation ability: under-developed, developing, and well-developed emotion regulation. The under-developed type has no control strategy, indicating the child can get stuck in anger and behavior modification cannot prevent the child from getting stuck; the developing type has control strategies available which can prevent the child from getting stuck in anger; the well-developed type does not need control strategies, indicating the child can regulate anger effectively during the waiting task.

The *developing* emotion regulation indicates children need to activate behavior after anger is activated, even when regulatory behavior already emerges. This highlights the person-specific developmental goal, which is *the effective use of appropriate regulatory behavior* when negative emotions occur. The *well-developed* type indicates the child might have already learned how to use regulatory behavior effectively, and this implies the formerly stated goal – effective use of appropriate regulatory behavior when negative emotions occur – is already fulfilled and the child *is ready for more advanced developmental goals*. On the other hand, the developmental goal for the *under-developed* type is *a behavior need to have an effect on anger*, and then network control will identify which behavior can affect anger (being it regulating or dysregulating) and the child can then develop skills to effectively use the particular behavior.

In sum, the *person-specific* network control can provide person-specific behavior modification advices that matches the developmental goal of emotion regulation, and it can also provide information to re-assess what is the appropriate developmental goal for a particular child.

Limitations and Outlooks

Identification of Boolean functions. The identification of Boolean functions uses the criteria of minimal error rate. The error rate is the sum of false positive and false negative. False positive means predicting the outcome to be 1 but the observed value is 0, and false negative means predicting the outcome to be 0 but the observed value is 1. When the observed data can have unbalanced classes, e.g., fewer observations of anger = 1 than that of anger = 0, the error rate will bias the identification by weighting the false positive and false negative equally. In other words, because there are fewer positive cases than negative cases (e.g., 15 seconds of angry compared to 465 seconds of not angry), the false negative (predicting the case to be negative when it is actually positive) cases might be subsequently fewer than false positive (predicting the case to be positive when it is actually negative). Therefore, the false negative should be more heavily weighted to balance with the false positive. The original biology literature, where Boolean network method has been applied, did not take this into account, possibly because unbalanced cases are not an issue in biology data. Further research can implement the weighted error rate to identify the Boolean functions.

The choice of number of input variables, denoted as K in this paper, is another issue. Preselecting K is based on the time complexity of computation if K is not a constant, which is a nondeterministic polynomial hard (NP-hard) problem (Akutsu, et al. 2000). The choice of $K = 2$ specifically is because when $K = 3$, the system will likely to be chaotic, meaning attractors would not be periodic like a limit cycle. This $K = 3$ case is also called the edge of chaos (Derrida & Pomeau, 1986). The $K = 2$ might limit the number of behaviors that can affect emotion by only identifying 1 or 2 behavior that can affect emotion, but it has the benefit of extracting attractors that can inform network control system design and intervention. Hence, it is a tolerable compromise of the accuracy of identifying system dynamics and practical purpose.

Probabilistic Boolean Network. An important limitation of the Boolean network method introduced in this paper is the assumption that the dynamics of the system are fully deterministic. This assumption is useful to simplify the modeling process, but is not always the most realistic or accurate assumption. Probabilistic Boolean network method (Shmulevich & Dougherty, 2010) has been developed which assigns probabilities for multiple Boolean functions with an equal error rate. This is one way to model a stochastic dynamical system. Another way to model the stochasticity in a dynamical system would be a method to incorporate and model a noise term in Boolean functions, e.g., a Boolean function like z = x AND y can have a noise with a probability. No such method has been developed to my knowledge.

Discrete-time Dynamical System Modeling. The Boolean network method is a discrete-time dynamical system modeling, so inherently it has the flaws of discrete-time modeling (Ryan, Kuiper, Hamaker, 2019). For example, when the true time-scale of psychological processes is unclear (e.g., how fast can children regulate anger), we usually assume the time-scale is equal with the time-scale measurement, 1 second in the empirical example in this paper, and this assumption dictates how the discrete-time modeling discover temporal dynamics between emotion and behavior. The problem of this assumption is if the true time-scale is slower (e.g., 5 second), then we will miss the temporal dynamics between behavior and emotion. This problem can be alleviated by allowing the state transition to be updated asynchronously, meaning not strictly from *t* to *t+1* (Albert, Robeva, 2015). This will allow for attractors of multiple time-scales to be discovered. It is worth noting we are not modeling the temporal dynamics with multi-scales though, but allowing the time-scale to be changed from 1s to 2s, 3s, …. by updating the states asynchronously.

Empirical Example. The empirical example in this paper is from an early childhood emotion regulation study. The participants in this sample are somewhat homogeneous in ethnicity. Before generalizing to larger population and to contribute to child emotional development theories, it is useful to engage with other population, including children with other ethnicity and cultural background. In terms of variables, we select anger, bidding, and distraction as the minimal set of variables to study emotion regulation dynamics, where bidding is expected to dysregulate anger and distraction is expected to regulate anger as emotion regulation development literature suggested. There are more emotion and behavior variables (e.g., sad, self-soothing) coded for the experiment, which could have affected anger or be affected by anger, but for demonstrative purposes, we did not include these variables. The three variables were manually coded second by second, and we have totally 480 seconds as 480 observations, and this is often considered a long psychological time-series. Dynamical system models, including the Boolean network method, usually requires long time-series to allow for a good fit; and we were fortunate to have a time-series dataset with 480 observations. For time-series data that are shorter, a comprehensive simulation study will be needed to provide guidelines of the minimal number of observations, with respect to complexity of the network model.

Conclusion

We introduced the Boolean network method as a method to model and control multivariate dynamical systems in psychology. We synthesized the Boolean network method from the system biology literature, to cover the essential concepts and three steps with methodological details, including inference of Boolean functions and construction of Boolean network, extraction of attractors, design of network control. This introduction will enable readers to have a solid understanding of Boolean network method. We also used an empirical dataset of children's emotion regulation to demonstrate the utility of the Boolean network method to prescribe regulatory strategies. This is the first application of Boolean network in psychology literature to our knowledge. Our hope is that the detailed introduction of the methodological steps and empirical demonstration will open up discussions and invite more empirical studies of network control system design for psychological systems.

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Table 2.1 Table of input variable(s) and the outcome of AND (∧), OR (∨), and NOT rule ($\bar{x}$)

$\boldsymbol{x(t)}$	$\boldsymbol{y(t)}$	$\boldsymbol{x(t) \wedge y\ (t)}$
0	0	0
0	1	0
1	0	0
1	1	1

$\boldsymbol{x(t)}$	$\boldsymbol{y(t)}$	$\boldsymbol{x(t) \vee y(t)}$
0	0	0
0	1	1
1	0	1
1	1	1

$\boldsymbol{x(t)}$	$\boldsymbol{\bar{x}(t)}$
0	1
1	0

Table 2.2 Table of state transition

(x_1, x_2)	
t	***t+1***
(0,0)	(0,0)
(0,1)	(0,1)
(1,0)	(1,0)
(1,1)	(0,1)

Table 2.3 Twenty-two types of Boolean networks, their associated state transition graph with attractor information, and network control strategies

Index	Boolean Functions			Attractor*	Control Strategy			
	anger	**bidding**	**distraction**		**Perturb bidding (Strategy, count)**	**Perturb distraction (Strategy, count)**	**Not Available (Count)**	**Not needed (Count)**
1	0	bid AND NOT distraction	distraction	A				4
2	0	bid	distraction	B				30
3	0	0	distraction	C				4
4	0	0	0	D				1
5	anger AND NOT bid	bid	distraction	E	Turn bidding **ON** when anger is ON, 4			
6	anger AND NOT bid	bid	0	F	Turn bidding **ON** when anger is ON, 1			
7	anger AND NOT bid	0	distraction	G			1	
8	anger AND NOT bid	anger AND bid	distraction	H	Turn bidding **ON** when anger is ON, 1			
9	anger AND NOT bid	NOT anger AND bid	distraction	I	Turn bidding **ON** when anger is ON, 1			
10	anger AND NOT distraction	bid	distraction	J		Turn distraction **ON** when anger is ON, 5		
11	anger AND NOT distraction	bid	NOT bid AND distraction	K		Turn distraction **ON** when		

						anger is ON,1		
12	anger AND NOT distraction	NOT anger AND bid	distraction	L		Turn distraction **ON** when anger is ON, 1		
13	anger AND bid	bid	distraction	M	Turn bidding **OFF** when anger is ON, 9			
14	anger AND distraction	bid AND distraction	distraction	N		Turn distraction **OFF** when anger is ON, 1		
15	anger AND distraction	bid OR anger	NOT anger AND distraction	O				1
16	anger	bid AND NOT distraction	distraction	P			2	
17	anger	bid	distraction	Q			39	
18	anger	0	distraction	R			2	
19	anger	NOT anger AND bid	distraction	S			4	
20	anger	bid	0	T			3	
21	anger	bid	NOT bid AND distraction	U			1	
22	anger	bid	NOT anger AND distraction	V			1	
Total (117 participants)					**16**	**8**	**53**	**40**

*Attractors will be represented in the form of state transition graph in Figure 2.4, labeled with character from A to V in Table 3, with the matching state transition graph in Figure 2.4 Panel A to Panel V. For example, if column “Attractor” is “C”, then the state transition graph is Figure 2.4 Panel C.

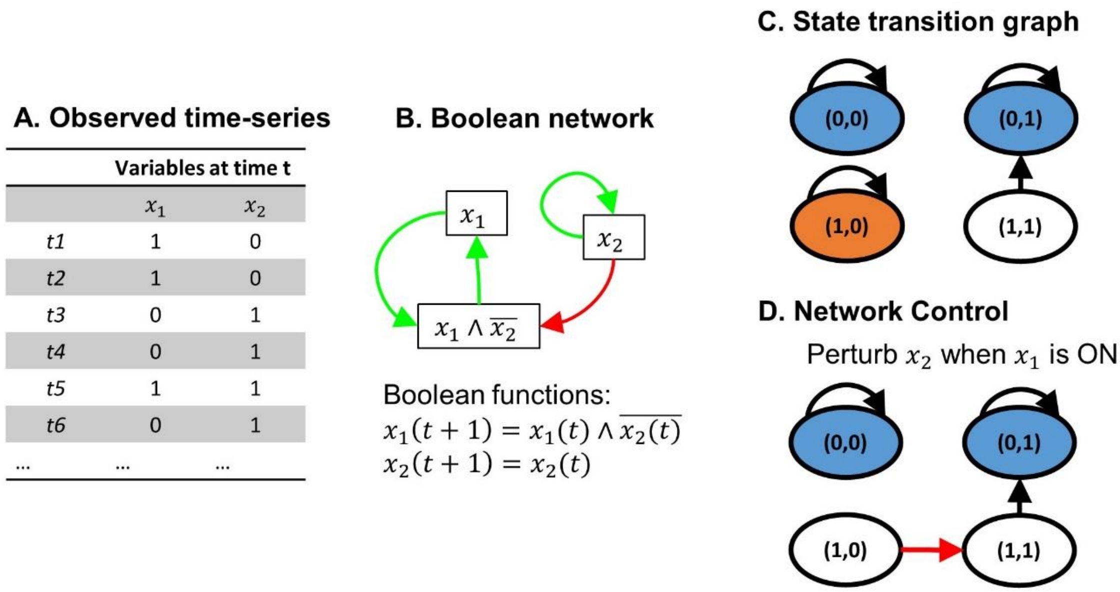

Figure 2.1 Example of a two-node network, consisting of x_1 and x_2.
Panel A is the observed binary time-series of x_1 and x_2 across time. Panel B is the Boolean network and Boolean functions inferred from the binary time-series in Panel A. Panel C is the state transition graph derived from the Boolean functions in Panel B. Panel D is the network control strategy, which is to perturb x_2 when x_1 is ON, and the updated state transition graph from Panel C, where the updated state transition is highlighted by a red arrow. Two-state tuple indicates the state of (x_1, x_2), e.g., (0,1) means (x_1=0, x_2=1).

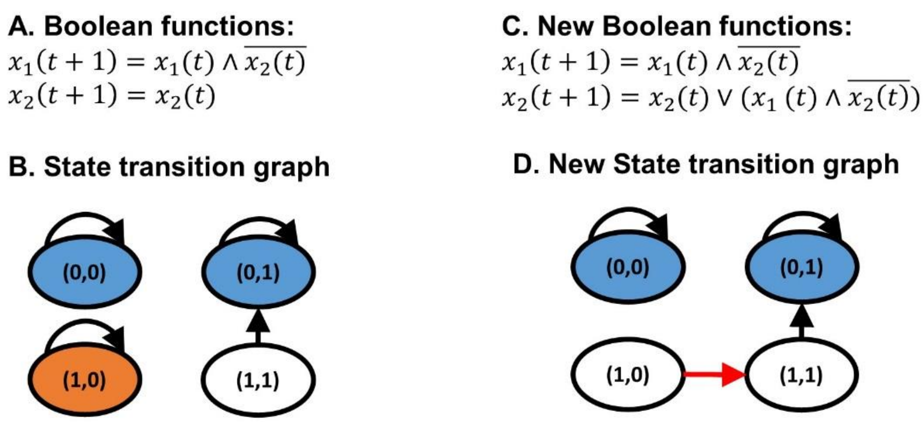

Figure 2.2 Illustration of design of network control

Panel A is the original Boolean network. Panel B is the associated state transition graph before network perturbation. Panel C is the updated Boolean network using the network control strategy. Panel D is the associated state transition graph if the network control is implemented.

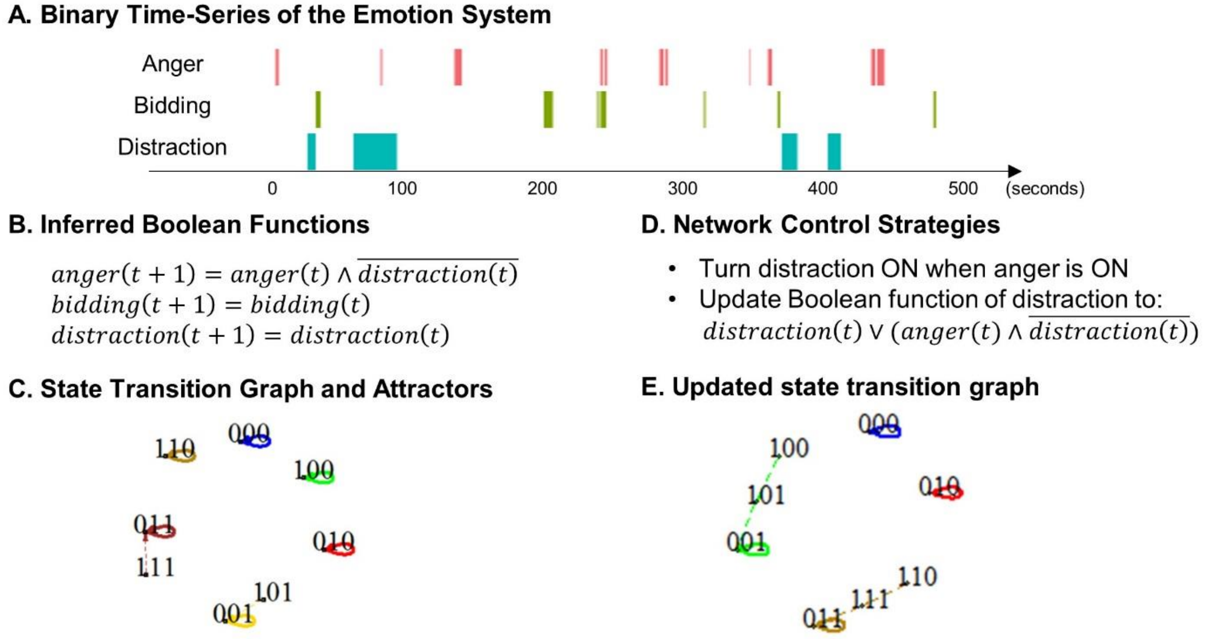

Figure 2.3 Illustration of a child's model results.
Panel A: Binary Time-Series of Emotion System. Binary time-series of anger, bidding, and distraction (colored bars = anger/bidding/distraction = ON, white space = anger/bidding/distraction = OFF). Panel B: Inferred Boolean functions. Inferred Boolean functions based on the time-series in Panel A. Panel C: State Transition Graph and Attractors. Each dot represents a state of the system (a 3-digit string ordered as anger, bidding, and distraction). Six attractors were extracted, which has a self-loop on the dot, e.g., 000 has a self-loop indicating it is an attractor. Different colors indicate different attractors and its basin (if applicable). Panel D: Network Control Strategies. One control strategy is found, to modify the behavior of distraction with a new Boolean function. Panel E: The updated state transition graph shows how undesirable attractor are eliminated and the state transition from an undesirable state to a desirable attractor basin, e.g., an undesirable attractor 100 (anger is ON, highlighted in green) will transition to the state 101 and eventually go to attractor 001 (anger is OFF).

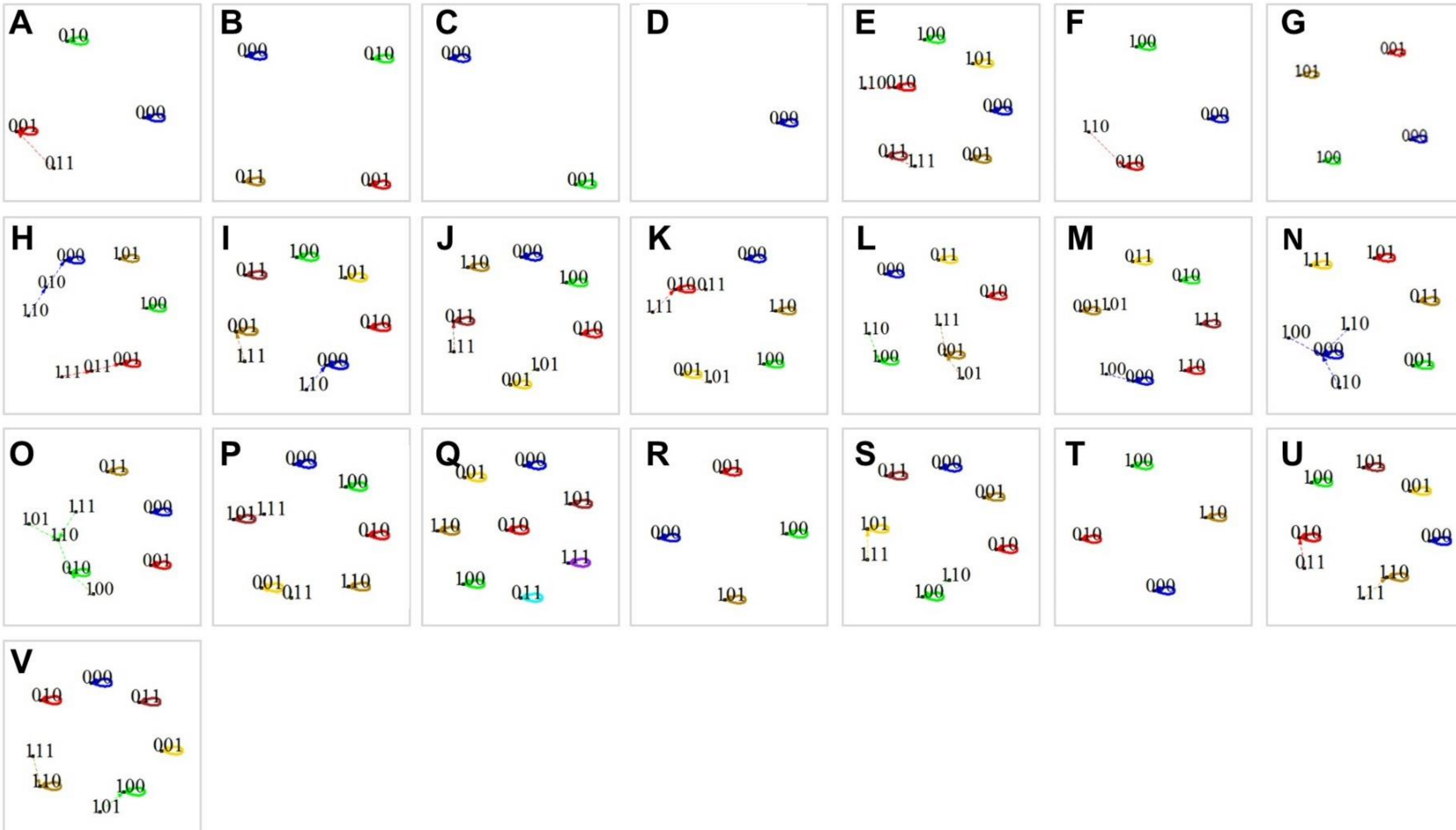

Figure 2.4 Illustration of twenty-two state-transition graphs.
In each panel, each color indicates a different attractor and its basin, and the self-loop indicates an attractor and dashed arrows indicate direction of state transition, e.g., Panel A show a state transition graph with three attractors (0,1,0) in green, (0,0,1) in red, (0,0,0) in blue, and (0,1,1) transitions to (0,0,1). The variables in the tuple/parenthesis are anger, bid, distraction.

Chapter 3 MODELING AND MANAGING BEHAVIOR CHANGE IN GROUPS: A BOOLEAN NETWORK METHOD

Introduction

Individuals often modify their behavior according to their observation of how other individuals in their social circle behave. Individuals may choose to conform to other's behavior because they feel safer to mimic their social peers' behaviors (e.g., conformity, Asch, 1956; group norm, Festinger, Schachter & Back, 1950); or they may choose to act differently from their peers because they disagree with their peers' behavior (Rosenbaum , 1986). Some individuals might be very consistent in their behavior and, although not changing their own behavior, might still influence their peers' behavior (Moscovici & Zavalloni, 1969). Because social influence facilitates individuals' behavior change in social group settings, the behavior changes among the individuals due to social influence can be conceptualized as a group process. The outcome of the group process of behavior change are often emergence of roles, group structures, group norms (Nowak, Szamrej, Latene, 1990; Arrow, 2010).

Theorists seeking to understand behavior change desire methods that can model social influence, and practitioners seeking to induce behavior change desire methods that can manage social influence. In this paper we introduce and forward a Boolean network method (Kauffman, 1969; 1993) that can estimate the presence of social influence and how it changes behavior within the same group, and provide strategies for network management that can promote desired behavior or prevent undesired behavior. Using empirical data from a longitudinal study of self-disclosing behavior in therapy groups on college campus, we demonstrate how the Boolean network method can be applied to longitudinal behavioral data to infer social influence and to manage the group toward a desired goal – the majority of group members will self-disclose.

Social Influence and Behavior Change in Social Groups

Theories of behavior change suggest individuals modify their behaviors according to the behaviors of their social peers. There are mainly two different types of social influence regarding behavior change, and each type of social influence has several corresponding mechanisms to explain why and how the social influences affects behavior change. Social influence can be broadly categorized as either assimilative or repulsive (Flache et al., 2017). When people are under *assimilative* social influence of their social peers, they tend to modify behavior to be more similar to peers' behavior. There are several possible mechanisms of assimilative social influence. The integrative theory of planned behavior (Yzer, 2012) suggests that perceived social norms (and social pressures to perform specific behaviors) motivate behavior change. Similarly, social learning theory suggests individuals imitate others' behaviors though observation learning, and can do so even without direct reinforcement (Bandura, 1963), and when there is uncertainty about the consequences (Bikhchandani, Hirshleifer, & Welch, 1992). In contrast, when people are under *repulsive* social influence of their social peers, they tend to modify behavior to be different from peers' behavior. There are several mechanisms for repulsive social influence. First, repulsive social influence could also be social learning when a specific behavior is associated with subsequent punishment (Bandura, 1963; 1977). Second, another mechanism of repulsive social influence is negative social ties, which refer to social ties that carry negative connotation between two persons, e.g., dislike (Veenstra, Dijkstra, Steglich, & Van Zalk, 2013; Harrigan, Labianca, & Agneessens, 2020). For example, if person A dislikes person B's behavior, then person A's behavior is more likely to move against person B's behavior. Third, dominance can also explain repulsive social influence, particularly in groups that have limited resources (Sade & Dow, 2013). Dominance explains why group members modify their behaviors – often related to occupation of resources – when in the presence of a more dominant peer. A group member may give up the resources voluntarily to a more dominant peer; the more dominant member can also actively fight and prevent the less dominant member from occupying resources, e.g., food (Senior, Lihoreau, Buhl, Raubenheimer, & Simpson, 2016), social status (Martin, 2009). Quantitatively, the behavior of less dominant member *moves in the opposite direction* from the behavior of the more dominant member, e.g., if the more dominant member occupies resources, then the less dominant member will *not* occupy resources.

Although many theories suggest that there are both assimilative and repulsive social influence, empirical studies of social influence based on behavior change dynamics often make assumption that social influence can only be assimilative. Prior empirical studies often studies social influence by examining the relation between the likelihood of adopting a behavior (behavior change) and the number of peers that already adopted that behavior (amount of peers that already have behavior change) (State & Adamic, 2015; Shameli, Althoff, Saberi, & Leskovec, 2017). The focus on the number of peers adopted a behavior is rooted in the threshold theory (Granovetter, 1978) which posits that the likelihood of adopting a novel behavior increases when the number of peers who adopted the novel behavior increase. This modeling approach assumes each dyad has assimilative social influence, so the effect of social influence on behavior from each peer *can be summed up to* measure the total social influence. Simulation Investigation of Empirical Network Analysis (SIENA; Snijders, 2017), another method to model behavior dynamics, also focus on only similarity effect (Veenstra et al., 2013), e.g., *average similarity effect* models a person's preference to be similar in behavior to their peers, *total similarity effect* models a person's preference to be similar in behavior to their peers in such a way that the total influence is proportional to total number of peers; *average alter effect* models when a person's have peers (called alter in SIENA) with higher values of a behavior, the person also has a higher value of the behavior.

There is no theoretical reason to assume only assimilative social influence exists in a social group; the lack of empirical studies focus on both assimilative and repulsive social influence might be due to a lack of available methods. Dynamical system method can fill this gap by estimating the dyad-level social influence based on the dyad's social time-series, without assuming the dyad-level social influence is uniform (Arrow, 2010). In a dynamical system, each group member's behavior is modeled as a set of temporal relations where the behavior at the current time is an outcome of their own and the other group members' behavior at a previous time point. The social influence can be inferred from the temporal relations between each dyad in the group, e.g., if person A's behavior at t has a temporal relation with person B's behavior at $t+1$, then person A's behavior influences person B's behavior. Depending on the direction of the temporal relation, we can infer whether it is assimilative or repulsive, e.g., if person A's behavior at t has a positive temporal relation with person B's behavior at $t+1$, then person A's behavior has assimilative influence on person B's behavior; otherwise, person A's behavior has repulsive influence on person B's behavior. Therefore, the dynamical system method can simultaneously accommodate both assimilative and repulsive social influence in one group.

Managing the Group Process of Behavior Change

Because individuals are under social influence and likely to modify their behavior due to other group members' behavior, the process of behavior change due to social influence in the social group context can be conceptualized as a *group process* of behavior change. Then there is potential to *manage the group process*, so that the majority of the group will adopt a desired behavior, or as many group members as possible will.

Previous literature suggests managing group process can utilize social influence from the central nodes to prevent undesirable behavior (e.g., aggression) from being spread via the social network (Osgood et al., 2013; Borek et al., 2019). The central nodes are the group members that receive the most friend nominations from others. The specific group management strategy is to intervene on the peer selection process, specifically to encourage group members to befriend those group members who show desired behavior (e.g., no substance use) and discourage group members to befriend those group members who show undesired behavior (e.g., substance use).

There are two important assumptions made for this group management strategy. The first important assumption is *the social influence is only assimilative among group members*. Given this assumption, a person with higher friend nominations (also more central nodes in the social network) is in the position to influence more people in the group, so positioning group members with desired behavior as the central nodes subsequently increase the number of group members that adopt desired behavior. On the other hand, positioning group members with undesired behavior as less central nodes will allow less people being influenced by them, and subsequently decrease the number of group members that adopt undesired behavior. The second important assumption of this group management strategy is the *network cannot be fully connected*. A fully connected network means every group member has a social tie with any other group member in the same network. Only when a group is *not* fully connected, it will be possible to change the friend nomination, to allow *some group members have more* friend nominations than others. This assumption is valid for large networks, because it is unrealistic to know everyone on large social networks (e.g., schools, corporations, the internet). But in small groups (e.g., therapy groups, sport teams, coauthor groups), this assumption is invalid because the small size of the group allows group members to share a social tie with any other group member.

When at least one of the two above assumptions are violated, we need a more flexible group management method. Once we can model the group process using dynamical system methods as previously introduced to relax the first assumption, we can also design network management strategies using control theory to relax the second assumption. Control theory (or control system design), a subfield of mathematics and engineering, focuses on moving dynamic systems toward desired goals. For example, engineers and mathematicians have developed automated systems to direct planes follow a designated trajectory and land at a designated lane by adjusting velocity and direction in real time to accommodate on-going changes in the environment (e.g., wind). A variety of mathematical tools and algorithms have been developed to determine the specific actions that will influence the behavior of a dynamical system so that it follows a desired trajectory or settles into a desired state (Lewis, et al., 2012; Molenaar & Nesselroade, 2015; Liu & Barabasi, 2016). In the context of group management, control theory methods focus on changing the behavior of specific group members instead of changing the social ties, which can relax the second assumption about social network topology. In this paper, we will apply the Boolean network method to model the group process, and then use the network control method based on the Boolean network method to identify specific ways to manage group processes.

Boolean Network

The Boolean network method provides a more realistic solution for both modeling and managing group process for two reasons. First, the Boolean network method does not impose social influence is only assimilative and can also model repulsive social influence or dominance.

Second, the Boolean network method can manage the group process, even for fully connected networks, by looking at the attractor states of a group.

Here, we introduce a Boolean network method that can: (1) allow both assimilative and repulsive social influence to be modeled for the same group, and (2) design network management strategy (also called network control; Shmulevich & Dougherty, 2010; Campbell & Albert, 2019) based on the dynamics of social influence and does not rely on manipulating the social ties.

Background. The Boolean networks (BN) model was originally introduced by Kauffman (Kauffman, 1969, 1993). In brief, a Boolean network $G(X(t), B)$ is defined by a set of nodes $X(t) = \{x_1(t), x_2(t), \dots, x_n(t)\}$, where x_i is the i^{th} node, and a set of Boolean functions $B = \{f_1, f_2, \dots, f_n\}$, where each Boolean function $f_i(x_{i1}, x_{i2}, \dots, x_{ik})$ with K specific input nodes for node x_i determines the value of x_i at time *t+1*. In this paper, the nodes represent a group member's behavior variable which is binary (1=ON, 0=OFF), e.g., self-disclosure behavior in a group setting. The Boolean functions represent the temporal dynamics between group members, i.e., how the group members' behavior (nodes) influence each other's behavior over time.

The Boolean functions are written using the Boolean operators: AND $\wedge$, OR $\vee$, NOT $\bar{x}$. The AND ($\wedge$) operator is defined as ***all*** input variables have to be ON to turn the outcome ON; the OR ($\vee$) operator is defined as ***any*** input variables being ON can turn the outcome ON; the NOT ($\bar{x}$) operator simply takes the opposite state of the input variable. Table 1 shows how these rules produce different outcome based on the input of two variables.

Modeling Assimilative and Repulsive Influence Simultaneously. To give an intuitive illustration of how the Boolean network method can be used to describe groups where assimilative and repulsive social influence operate simultaneously, we use a simple, fully connected three-person network. Member 1, member 2, and member 3's behaviors at multiple occasions are represented as x_1, x_2, and x_3, respectively. The observed time-series of each person's behavior x_1, x_2, and x_3 are shown in Figure 3.1a, where the states of x_1, x_2, and x_3 are ordered by time-steps t_1, t_2, t_3,…. From this observed binary time-series, we can infer the Boolean functions that have assimilative and repulsive social influence in the same group as follows:

$$x_1(t+1) = \overline{x_3(t)} \tag{1}$$
$$x_2(t+1) = x_1(t) \tag{2}$$
$$x_3(t+1) = x_3(t) \tag{3}$$

(the details of how rules are inferred will be introduced in the Data Analysis section).

1. $x_1(t+1) = \overline{x_3(t)}$ indicates *repulsive* social influence from person x_3 to person x_1: when x_3 is turned OFF at time *t*, x_1 will be turned ON at time *t+1*; when x_3 is turned ON at time *t*, x_1 will be turned OFF at time *t+1*. This can also be explained by *dominance*, member 3 dominates member 1: whenever member 3's self-disclosure is ON at time *t*, member 1's self-disclosure behavior is turned OFF at time *t*; only when member 3's self-disclosure is OFF, member 1's self-disclosure can be ON.
2. $x_2(t+1) = x_1(t)$ indicates *assimilative* social influence from member 1 to member 2: when x_1 is turned ON at time *t*, x_2 will be turned ON at time *t+1*; when x_1 is turned OFF at time *t*, x_2 can be turned OFF at time *t+1*.
3. $x_3(t+1) = x_3(t)$ indicates member 3's behavior depends on him/herself: when x_3 is turned ON at time *t*, x_3 will continue to be turned ON at time *t+1*; when x_3 is turned OFF at time *t*, x_3 will continue to be turned OFF at time *t+1*.

The Boolean functions in Equations 1 to 3 are used to construct a Boolean network, a graph of which is shown in Figure 3.1b. The first function "$x_1(t+1) = \overline{x_3(t)}$ " is expressed by a red edge from x_3 to x_1 representing the NOT effect of $x_3(t)$ on $x_1(t+1)$. The second function "$x_2(t+1) = x_1(t)$" is indicated by a green edge from x_1 to x_2 representing $x_2(t+1)$ depends on $x_1(t)$. The third function "$x_3(t+1) = x_3(t)$" is indicated a green edge pointing from x_3 back to itself – a self-loop.

In sum, this example illustrates that a Boolean network method can have *both assimilative and repulsive* social influence to be modeled for the same group. If we use another modeling framework rather than dynamical system method, such as multilevel modeling, then we will need to make assumptions that there is a prototypical value and direction (either positive or negative) of social influence. In other words, using multilevel modeling would only allow for either assimilative or repulsive social influence to be modeled. The Boolean network method, however, allows for cases where assimilative and repulsive social influences operate simultaneously in the same group.

Designing Network Control Without Manipulating Social Ties*.** Methodologically, network control focuses on modifying the state space transition graph that is derived from a Boolean network. Three pieces of information need to be identified: (1) which node needs to be perturbed, (2) which states to perturb the nodes to, and (3) what is the condition to perturb the node. These three pieces of information will be useful in diagnosing ***when and ***how*** to drive the system (social groups or social networks) into more desirable states.

We can extract attractors from state transition graph using these Boolean functions (the details of how derivation works will be introduced in the Data Analysis section). If we represent the state of x_1, x_2, and x_3 as a tuple of the three variables (x_1, x_2, x_3), e.g., (0,0,0) means all x_1, x_2, and x_3 are OFF, we can express the state transition as a graph, shown in Figure 3.1c. For the Boolean network in Figure 3.1c, where the arrows indicate the direction of transitions, the state of the system will transition from (0,0,0) to (1,0,0), and then from (1,0,0) to (1,1,0). Once the system enters (1,1,0), it will be absorbed in this state and it will not transition to other states. This kind of state, where the system stays, is an attractor.

The Boolean network method utilizes the differential desirability of attractors *within the same dynamical system* and design network control strategies to move the dynamical system from an undesirable attractor to a desirable attractor. Because not all attractors are desirable with a dynamical system, we can assign desirability to each attractor based on practical concerns. In our example, we define desirable as the majority of group members' self-disclosure are ON. For example, an attractor in which two group members' self-disclosure are ON, e.g., (1,1,0), is desirable; on the contrary, an attractor in which only one group members' self-disclosure is ON, e.g., (0,0,1), is undesirable.

Control strategy can be derived based on the distance from undesirable attractor to desirable attractor basin (the details of how derivation works will be introduced in the Data Analysis section). We can design network control based on the extracted attractors, and one network control strategy is to perturb x_3 when only x_3 is ON as shown in Figure 3.1d, so that the system will transition from (0,0,1) to (0,0,0) indicated by a red arrow, then goes to (1,0,0), and eventually (1,1,0), the desirable attractor. This new state transition is also depicted in Figure 3.1d. In sum, this example illustrates that a Boolean network method can design network management strategies based on the dynamics of social influence and does not rely on manipulating the social ties.

The Present Study

In the present study, we model social influence processes driving week-to-week disclosure behaviors of all members of a therapy groups. In doing so, we construct Boolean networks that simultaneously estimate assimilative and repulsive social influence from the Boolean functions. The assimilative social influence is shown in a function form like $x_j(t+1) = x_i(t)$, and a repulsive social influence is shown in a function form like $x_j(t+1) = \overline{x_i(t)}$. Then based on the Boolean network of each group, we can extract attractors for each group, and assign desirability to the attractors. When there are both desirable and undesirable attractors for the same group, control strategy will be searched and specific group members will be identified that can move the group into a desirable attractor. The strength and novelty of this Boolean network method includes (a) simultaneously estimating assimilative and repulsive social influence from dynamics of group members' behavior data, and (b) providing group management strategy by applying network control on the dynamics of group members' behavior data.

Method

Data for our empirical inquiry are drawn from a longitudinal study of how dynamics of on-campus counseling therapy groups that met weekly, and the health of individuals within those groups changed over 10 to 16 weeks. Comprehensive description of the larger study can be found in Molloy (2012). Details relevant to the present analysis are given below.

Participants and Procedure

Weekly data collection was done in a counseling setting where university students seek mental health services at the University's Center for Counseling and Psychological Services (CAPS) and get assigned to therapy groups that ranged in size from 5 to 8 persons, and met weekly for between 10 and 16 weeks. Participants were 119 individuals recruited from18 therapy groups (mean group size = 6.6, not including the therapists). Of the 18 groups, 17 were "general process" groups (e.g., no specific disorder or topic; 9 for undergraduate students, 8 for graduate students) and 1 was a substance abuse group (for both undergraduate and graduate students).

Measures

Measures for the present study were drawn from the American Group Psychotherapy Association's (AGPA) CORE-R Battery (Clinical Outcome Results Standardized Measures, Revised; see Burlingame et al., 2006; Strauss, Burlingame, & Bormann, 2008): a manual of evidence-based instruments that serve as a standardized "toolbox" for clinicians to systematically monitor and evaluate groups and their members. Since the original scale was often not administered on a weekly basis, many of the items were re-worded to be present-focused by adding, for example, the phrases "during today's session" or "today."

Participants' Weekly Self-Disclosure. To demonstrate the Boolean network modeling approach, we use a self-disclosure item from the Group Evaluation Scale (GES; Hess, 1966), a seven-item measure assessing the overall benefit that a client experienced during a given session. Traditional Likert-type response scales (e.g., 1 = 'strongly disagree' to 7 = 'strongly agree") were converted to "touch-point continuum" (slider-type) response scales (0 to 100) with end-point anchors. Examples of item stems include "Within the group today, I was … to self-disclose", with a response scale with end-point anchors "very uncomfortable" and "very comfortable". To prepare the variable for analysis using the Boolean network method, the interval-scale variable was binarized separately for each individual using their person-specific mean score across the repeated measures. Specifically, responses above the person-mean were coded as = 1, and responses equal to or below the person-mean were coded = 0. Thus, for each person in a group we obtained a binary time-series that is interpreted as indicating whether or not that individual did or did not engage in self-disclosure during each session.

Data Analysis

Because the Boolean network method is new to the psychology literature, we synthesized the Boolean network method introduced in system biology literature, to cover the essential concepts and methodological details to enable readers to have a solid understanding of the Boolean network method as a method for network modeling and network control.

The Boolean network method we introduce here has three major steps: (1) inference of Boolean functions and construction of a Boolean network, (2) extraction of attractors based on the inferred dynamics and assignment of desirability for each attractor, and (3) design of network control to direct a group toward a desired attractor and identify how the Boolean network needs to be updated.

Inference of Boolean Functions and Construction of Boolean Network

The Boolean functions can be inferred from the observed time-series of all the variables. Input variable refers to the variables that produce the outcome variable, similar to predictors. Number of input variable is usually denoted by k, and the size of the network is denoted by N, so the Boolean network with size N and input k is sometimes called a NK Boolean network.

Algorithms have been developed to infer Boolean functions (Lähdesmäki, Shmulevich, & Yli-Harja, 2003; Akutsu et al., 2000). The goals of inferring Boolean functions is to find the combination of input variables connected by the AND, OR, and NOT Boolean operators, that is the best fit of an outcome variable. The inference procedure utilized the matrix multiplication and finger print function to compare the time-series of outcome variable and the time-series of combinations of different input variables. The technical details of inference can be found in Chapter 2 "Data Analysis" section.

For network construction, each group member's self-disclosure measure within a group were entered in the BoolNet R package to construct a group-specific Boolean network. The R package will find the best fit for each variable – each member's self-disclosure – at time t in the form of a Boolean function that use the variables of the same group – group members' self-disclosure – as input variables at time $t-1$. All the Boolean functions together consist the Boolean network of the given therapy group.

Selection of number of input variable *k*. In this paper, we select $k = 1$, meaning each node is only predicted by one other node. The main reason for selecting $k = 1$ is due to the relatively short time-series (10~16 weeks), selecting $k = 1$ can avoid overfitting the data. Besides, selecting k=1 can also provide straightforward interpretation of the group process – assimilative or repulsive – that match more closely to the theory of social influence.

The Boolean network is the constructed by putting all the inferred Boolean functions together. As a hypothetical example, Figure 3.2a shows how the three-node Boolean network introduced earlier is constructed.

Extraction of Attractors

After the Boolean functions are inferred, the state transition graph can be constructed by exhaustive search of all possible state transition sequences from each permutation of initial conditions. The attractors are identified by constructing the state transition graph and finding the *absorbing states*, which are the *states that will transition to itself* due to the dynamics.

We will go through the same example of three-node network to demonstrate this procedure, using the Boolean network mentioned earlier in Figure 3.1 and Equation 1 to 3,

$$x_1(t+1) = \overline{x_3(t)} \tag{1}$$

$$x_2(t+1) = x_1(t) \quad (2)$$
$$x_3(t+1) = x_3(t) \quad (3)$$

For the 3-node network, there are $2^N = 2^3 = 8$ possible initial conditions: (0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0), (1,1,1). Using the Boolean functions, we can compute the state of the system for the next time point, t = 0 + 1. For example, when the system starts with initial condition at t = 1 of (0,0,0), meaning x_1=0, x_2= 0 , x_3= 0 at t = 1, then $x_1(t = 2)$=$\overline{x_3(t=1)}$= 1, $x_2(t=2)$ =$x_1(t=1) = 0$, $x_3(t=2)$ =$x_3(t) = 0$. We then know (0,0,0) will transition to (1,0,0). Similarly, we can compute the state when the system starts at the rest of the seven states. We have put the initial state and state at the next moment in Table 2.

The overall state transitions from Table 2 can be plotted as a state transition graph. Shown in Figure 3.2b, the states are represented in circles, and the arrows indicate the direction of change between different states. Attractors are identified as the states where the state at $t+1$ is identical to the state at t. One attractor (highlighted in blue) is (1,1,0), having two nodes x_1, x_2 turned ON. The other attractor (highlighted in orange) is (0,0,1), having one node x_3 turned ON.

It is worth mentioning attractors can take multiple forms. In this paper, we will have two forms, the fixed-point attractor when the system stays in one state, and a complex attractor when the system cycles through a finite set of states. The complex attractor is also called limit cycle. The limit cycle can be identified by visually checking the state transition graph, and if the system transitions/cycles through a set of states, there is a limit cycle with the set of states.

Design of Network Control

For this paper, we aim to suggest change to particular members' behavior so that a therapy group will not get stuck with only a few people self-disclosing all the time, but instead elicit more diverse participation in self-disclosure. Generally speaking, self-disclosure is a desirable behavior in therapy groups because it can strengthen the therapeutic process and group members' self-evaluation can enhance their own healing and change (Farber, 2006)

We introduce how to identify three pieces of information in this section: (1) which node needs to be perturbed, (2) which state to perturb them to, and (3) what is the condition to perturb the node. These three pieces of information will be useful in diagnosing ***when*** and ***how*** to drive the system into more desirable states for social groups. The procedure for identifying control strategy is as follows:

1. ***Formulate the goal of network control***. This is based on practical concern. For this paper, the goal is to promote the desired behavior – self-disclosure, and the desirability of attractors is determined by the number of participants who have self-disclosure ON at least once in the attractor. The desirability of an attractor is determined by the number of group members' self-disclosure ON. When there is only one attractor found, the attractor is desirable *if at least half of the group* have self-disclosure ON in the attractor; otherwise the attractor is undesirable. When there are multiple attractors found, the attractor is desirable on two conditions: (1) *if at least half of the group* have self-disclosure ON in the attractor, and (2) *if more group members'* have self-disclosure ON compared to the other attractors.
2. ***Compute the Hamming distance from an undesirable attractor to the states in a desirable attractor basin***. An attractor basin is defined as the set of states that will eventually go to a given attractor state. Because the system will eventually go to the desirable attractor once the system is in any state belonging to the desirable attractor basin, we can consider what actions are necessary to move the system from an

undesirable attractor state into a desirable attractor basin. Ideally, we would like to move the system from an undesirable attractor state to the closest state in the desirable attractor basin. The distance from an undesirable attractor state to each state in the desirable basin will be the number of nodes that need to be perturbed, thus shortest distance indicates fewest nodes to be perturbed. Formally, these distances are computed using Hamming distance (Hamming, 1950), which compares two binary strings of equal length and counts the number of bit positions in which the two bits are different. For example, the distance between state (0,0,1) and state (1,0,0) is 2 because two elements (x_1 and x_3) in the string are different, and need to be changed to move the system from one state to the other state.

3. ***Formulation of control strategy***. Once all of the Hamming distances are computed, we formulate control strategies from those that have the shortest distance. The Hamming distance indicate the number of nodes needs to be perturbed. So, the shortest Hamming distance indicate a control strategy with minimal number of nodes perturbed, which is what we choose as the control strategy. The *nodes to be perturbed* are the nodes that differ between the undesirable attractor state and the state in the desirable attractor basin state; the undesirable attractor state is the *condition to perturb the node*, the state in the desirable attractor basin state is *the state these nodes should be perturbed to*.

An example is shown in Figure 3.2. A three-node Boolean network and its state transition graph are given in Figure 3.2a and 3.2b. The state transition has one desirable attractor (1,1,0) in blue and one undesirable attractor (0,0,1) in orange, because the (1,1,0) has 2 group members' self-disclosure ON, compared with (0,0,1). The goal of network control is to move the system out of the undesirable attractor (0,0,1), and direct the system into the desirable attractor (1,1,0).

We compute the Hamming distances between the undesirable attractor (0,0,1) and every state in the attractor basin that includes the desirable attractor state (1,1,0). We then find the shortest distance, which in this case is from (0,0,1) to (0,0,0), and develop a control strategy that would facilitate that move.

Figure 3.2c shows the control strategy – turning node x_3 OFF – that when invoked can move the system towards the desirable attractor. The *node to perturb* is node x_3 because the undesirable attractor state (0,0,1) and the state in the desirable attractor basin that has the shortest Hamming distance (0,0,0) differ by the third node. *The condition to perturb* is when the system is stuck in the undesirable attractor (0,0,1), *the state to perturb x_3 to* is to turn x_3 to 0 (OFF), then the system goes to (0,0,0), and the state transition is highlighted as a red arrow in Figure 3.2c. The system will then transition to (1,0,0), and then to the desirable attractor (1,1,0), highlighted by blue, where two group members will have the desirable behavior = ON. It is worth noting there could be multiple states that have distance of 1 to the undesirable attractor, indicating multiple control strategies. It is also possible there is no state that has distance of 1 to the undesirable attractor, indicating no simple control strategy could be found.

Results

We used the Boolean network method to describe the group process of each of the 18 therapy groups self-disclosure, and to design a control strategy that would move each group towards a desirable outcome – where the majority of group members engage in self-disclosure. Results are reported in two parts. First, we go step-by-step through results obtained for an exemplar group. Then we summarize and identify patterns in the findings across all 18 groups.

Group-Specific Analysis: Identifying a Management Strategy

In this section we illustrate how the analyses proceeded for one exemplar group, Group #17 in Table 3. Group #17 consisted of 6 persons, x_1 to x_6, who participated in group therapy for 10 weeks. The self-disclosure behavior of each member is shown in Figure 3.3a, where the colored blocks indicate the weeks that each individual self-disclosed and the white spaces indicate the weeks they did not self-disclose.

Inference of Boolean functions and construction of Boolean networks. The Boolean functions inferred from this 6-dimensional binary time-series were, as also shown in Figure 3.3b,

$$x_1(t+1) = x_4(t) \qquad (4)$$

$$x_2(t+1) = x_6(t) \qquad (5)$$

$$x_3(t+1) = \overline{x_6(t)} \qquad (6)$$

$$x_4(t+1) = x_4(t) \qquad (7)$$

$$x_5(t+1) = 0 \qquad (8)$$

$$x_6(t+1) = \overline{x_2(t)} \qquad (9)$$

These functions indicate that the group process included both assimilative and repulsive social influences. Specifically, Equation 4 indicates an assimilative social influence from x_4 to x_1, in that x_1 always does what x_4 did the previous week. In contrast, Equation 6 indicates a repulsive social influence from x_6 to x_3, in that x_3 always does the opposite of what x_6 did the previous week. Collected together, the six Boolean functions form the Boolean network for Group #17 Altogether the group process demonstrates both assimilative and repulsive social influence co-exist in the same group.

Extraction of attractors. Based on the group-specific Boolean network, we then computed how the system evolved from *t* to *t+1* (e.g., as in Table 2), which is shown graphically in Figure 3.3c. The state transition graph in Figure 3.3c is obtained in the same way as introduced in the “Extraction of Attractors” section, but each state is depicted using dots in Figure 3.3c, instead of circles with states in Figure 3.2b, due to the lack of space.

Attractors – the states that transition back to itself – can be extracted. In our example group, two attractors are identified and each attractor basin is highlighted by a different color. Attractor basin 1 (highlighted in blue) has three nodes with self-disclosure = ON when it cycles through four different states. Attractor basin 2 (highlighted in green) has five nodes with self-disclosure ON when it cycles through four different states. As aforementioned, when the system cycles through a finite set of states, the attractor is a complex attractor or a limit cycle attractor. Based on the number of group members that have self-disclosure ON, Attractor 2 is desirable and Attractor 1 is undesirable.

Design of network control. Using the control strategy search algorithm described above we identified 4 strategies that might be used to move the system from Attractor 1 to Attractor 2. Hamming distance was calculated between the undesirable attractor to all states in desirable attractor basin. Figure 3.3d shows there are four states that have the minimal Hamming distance – one. The node that differ from the undesirable attractor to the desirable attractor basin is node 4, or x_4. These are shown in Figure 3.3d along with an indication of the fourth group member x_4 should be encourage to self-disclose. The first row in the table indicates that when person 4 self-discloses (1 = ON), the system will be moved from the state of (0,1,0,0,0,0) in Attractor 1 (3 members ON) to one state (0,1,0,1,0,0) in the desirable attractor basin, and eventually to Attractor 2 (5 members ON). The second row in the table indicates that when person 4 self-discloses (1 = ON), the system will be moved from the state of (0,0,1,0,0,0) in Attractor 1 (3 members ON) to one state (0,0,1,1,0,0) in the desirable attractor basin, and eventually to

Attractor 2 (5 members ON). The third row in the table indicates that when person 4 self-discloses (1 = ON), the system will be moved from the state of (0,1,0,0,0,1) in Attractor 1 (3 members ON) to one state (0,1,0,1,0,1) in the desirable attractor basin, and eventually to Attractor 2 (5 members ON). The fourth row in the table indicates that when person 4 self-discloses (1 = ON), the system will be moved from the state of (0,0,1,0,0,1) in Attractor 1 (3 members ON) to one state (0,0,1,1,0,1) in the desirable attractor basin, and eventually to Attractor 2 (5 members ON). In sum, all strategies indicate by turning the fourth node ON, the network can be moved from Attractor 1 to Attractor 2.

Between-Group Differences: Different Management Strategies

The above analysis was done for each of the 18 groups. Results are summarized in Table 3 (inferred Boolean functions, state transition graph, attractors, and control strategy) and Figure 3.4 (state transition graphs). As seen in the Boolean function expressions column of Table 3, each group had its own unique dynamics. All the Boolean networks were different. And thus, the state transition graphs derived from each Boolean network are unique. As seen in Figure 3.4, each group had a unique set of attractors, and as seen in Table 3, each group had a unique control strategy.

We did not find guidelines for a cutoff of acceptable error rate from the literature introducing inference of Boolean functions (Akutsu, et al. 2000), so we report the empirical distribution of error rate in our sample. Error rate of inference of Boolean function for individual group member's self-disclosure is on average 0.14 (*SD* = 0.10).

Assimilative and Repulsive Social Influences. Of the 18 groups, 14 included both assimilative and repulsive social influences. Specifically each of these groups' Boolean functions included a function with form of $x_j(t+1) = x_i(t)$ which indicates assimilative social influence where x_j moves toward x_i; *and* a function with form of $x_j(t+1) = \overline{x_i(t)}$ which indicates repulsive social influence with x_j is moving away from x_i. The co-existence of these Boolean functions in the same network indicates that social influence in therapy groups are not always assimilative. In all these groups it appears that some group members dominate the group discussion time and prevent other group members from participating. The other 4 groups had only assimilative social influence dynamics (Group 12) or only repulsive social influence dynamics (Groups 2, 8, and 16). When a Boolean function is fixed at 0, e.g., $x_1(t+1) = 0$ that indicates the behavior of the i^{th} group member is always OFF, and such members could be conceptualized as not affected by social influence (neither assimilative or repulsive), and never self-discloses in the group. When a Boolean function is fixed at 1, e.g., $x_i(t+1) = 1$ that indicates the behavior of the i^{th} group member is always ON and such members could be conceptualized as not affected by social influence either, and they are committed to self-disclosure every week.

Attractor States. The state transition graphs in Figure 3.4 show that most groups have only one attractor, and 4 groups have two attractors (Figure 3.4i, n, q, r). The state transition graph is obtained in the same way as introduced in the "Extraction of Attractors" section, and it is depicted using dots to represent states in Figure 3.4, instead of circles with state in Figure 3.2b, due to the lack of space. Attractors in the state transition graph are indicated by self-loops. It is worth noting when a group have a member whose behavior is fixed at 0 or 1, the number of states in the state transition graph will reduce by a half. This reduction is because only half of the initial states are permutated in the extraction of attractor step, with one node fixed at either 0 or 1. Thus, some state transition graphs have fewer states, e.g., Figure 3.4b only have two states, and the corresponding group 12 has 4 nodes being fixed at 1, and only one node x_4 is free to change its state. It is also worth noting some attractors are complex attractors (or limit cycle), indicating the system cycles through a set of states, e.g., Figure 3.4q shows two attractors and both are complex attractors.

Number and Desirability of Attractors. For groups that has only one attractor, we determine whether control strategy is needed based on the desirability of the attractor. If the attractor is a desirable attractor, meaning there are *at least half* of the group members' self-disclosure ON in the attractor, then we consider group members are doing well in terms of self-disclosure, so we labeled these groups as control strategy not needed, e.g., Group 1; otherwise, we label the groups as control strategy not available, since there is no more desirable attractor to move the system to, e.g., Group 10.

For groups that have two attractors with differential desirability, we search for the control strategy; otherwise, for groups that have two attractors without differential desirability, we determine whether the control strategy is needed based on the desirability to these attractors. Group 17 and 18 have two attractors with one desirable and the other undesirable, and we found control strategies for both groups. Group 7 and 14 both have two attractors with equal desirability, so whether the control strategy is needed is determined using the same principle as the one-attractor groups: the number of groups members = ON. As a result, group 7 does not need control strategy as the majority has self-disclosure ON and group 14 does not have available control strategy as the majority has self-disclosure OFF.

Control Strategies. Examination of the structure of groups' attractor basins and control strategies led us develop a typology of groups. Specifically, we identified three types of groups. These are described in Table 4. The first type is the *well-functioning group.* These groups (N = 9) had one or two attractors, and each attractor was desirable where the majority of the group members' self-disclosure = ON. Thus, these groups were already functioning well and no control strategy is needed. The second type is the *unmanageable groups*. These groups (N=7) had one or two attractors, but in each attractor the majority of the group members' self-disclosure = OFF, yet no control strategy is available because there is no alternative attractor to direct the system to. The third type is the *manageable groups*. These groups (N = 2) had two attractors which had differential desirability, and a control strategy was available to move the system into a more desirable attractor.

Discussion

In this paper, I have introduced the Boolean network as a method to model and manage group processes. The Boolean network method provides a more realistic solution for both modeling and managing group processes for two reasons. First, the Boolean network method can model both assimilative and repulsive social influence and does not impose a specific kind of social influence. Second, the Boolean network method can manage the group processes, even for social networks with both assimilative and repulsive social influence.

The utility of the Boolean network method is demonstrated through application of this method to empirical data about when individual's self-disclose in therapy groups (4 to 8 participants) over 10 to 16 consecutive weeks. In the model results, we found both assimilative and repulsive social influence recovered in the same group, and network control strategies can be designed for this group with both assimilative and repulsive social influence, to ensure more group members turning on the desirable behavior. The detailed introduction of the Boolean network method with accompanied tutorial and the empirical application together contribute to our knowledge and analytical repertoire about how to model and manage group processes realistically.

Assimilative and Repulsive Social Influence

The Boolean network method allows for estimation of assimilative and repulsive social influence, because social groups can have both positive and negative ties between group members (Harrigan, Labianca, & Agneessens, 2020). We found both assimilative and repulsive social influence in the same group, confirming our expectation of both types of social influence can co-exist. The empirical results demonstrate the Boolean network method can model group processes in a flexible and realistic way.

Uncovering repulsive social influence can facilitate therapist's understanding of group process. If only assimilative social influence is assumed and modeled, then even though there is repulsive social influence in the group, it remains undiscovered. As introduced earlier, the underlying mechanisms of repulsive social influence could be due to negative ties (Veenstra et al., 2013; Harrigan, Labianca, & Agneessens, 2020), social learning of negative consequences of self-disclosure (Bandura, 1977), or dominance (Martin, 2009; Sade & Dow, 2013). Identification of the dyad that has repulsive social influence can therefore inform therapist that some of the underlying mechanisms might exist in the group and allow intervention to target the repulsive social influence that the therapist considers harmful for the group process. For example, if the therapist considers the repulsive social influence from member A to member B is caused by dominance, the therapist can intervene by shortening the self-disclosure of member A and encouraging member B to take the opportunity to self-disclose.

An alternative explanation of what manifests as "repulsive" social influence in a can be obtained when the system is instead viewed as a manifestation of "cooperative" turn-taking. In this paper, we considered the NOT operator as an indicator of repulsive social influence, and used dominance, social learning, or negative ties to provide a potential explanation of why such group process would occur. An alternative interpretation is the NOT operator implies cooperative turn-taking behavior. For example, if group member 1's behavior at t+1 is predicted to be NOT group member 2's behavior at t, then that means when group member 2 speaks at t, then group member 1 will not self-disclose at t+1, which could be interpreted as group member 1 wants to give group member 2 the opportunity to continue self-disclosure. Further work can be done to identify the specific types of systems and settings where similar temporal dynamics indicate competitive and/or cooperative behavior patterns.

Network Control to Manage Group Processes

The Boolean network control method does not make assumptions about social influence and the network control design does not rely on manipulating social ties like previous network-based interventions. We demonstrated how the Boolean network methods allows for identification and design of group management strategies for groups with both assimilative and repulsive social influence. The dynamical system modeling framework enables extraction of attractors by deriving state transitions from t to $t + 1$ and identifying the state(s) the system moved toward and is absorbed in. As a result, the control system design is based on the extracted attractors and their differential desirability and the control design provides group management strategy, which focuses on changing a few group member's behavior. The network control design shows even if the network is fully connected, like the therapy groups in our empirical example, the control method can still find strategies to direct the majority of the group self-disclose.

This network control method can help group therapists manage group processes. In counseling practice, therapists can use the network control strategy to guide group therapy practice and to encourage specific group members to self-disclose when the group is stuck when only a few members self-disclose, which is an undesirable attractor, and then the group can be moved out of the undesirable attractor and have more members self-disclose. By managing the group and allowing more group members to self-disclosure, the therapy group is expected to have more effective counseling, as self-disclosure can strengthen the therapeutic process and the self-evaluation in self-disclosure can enhance healing and change (Farber, 2006).

The application of the Boolean network method is not limited to only therapy groups, nor the particular behavior (e.g., self-disclosure). The Boolean network method can be applied to a wide range of group processes that rely on social influence. Take prevention of substance use among adolescents as an example, the Boolean network control method can provide suggestions for school teachers or community stakeholders. In an adolescent social network within a school/community, the Boolean network control method will find strategies on how to reduce the number of adolescents that adopt substance use as a regular behavior, which corresponds to adolescents' substance use being ON in an attractor state. The strategies will require changing a few adolescents' substance use behavior, so that the whole social network will be moved to an attractor with less adolescents using substance.

Limitations and Outlooks

Controllability. The desirability of attractors, in this paper, was assigned in a relative sense. An attractor with more group members having desirable behavior is more desirable, compared to another attractor with less group members having desirable behavior. We found in our empirical dataset that not all groups have multiple attractors, nor do they always have differential desirability. This indicates some groups are not controllable with the current control system design method. Further methods to assess controllability of a Boolean network can provide quick evaluation of whether group is manageable (Cheng & Qi, 2009).

Efficiency of Attractor Extraction for Larger Networks. We presented the Boolean network method and an empirical application on modeling and controlling for dynamics of social influence. The empirical example here has the main goal to encourage a diverse participation of group dialogue. The size of network in the empirical example is relatively small (4 to 8), and some social networks are much larger, ranging from a classroom to the online social networks. The large size of the network will create challenge for attractor extraction, as it requires enumeration of all initial conditions, which is at the scale of 2^n, and n is the network size (Liu & Barabasi, 2016). Computational algorithms that can efficiently extract attractor are needed and will facilitate controlling the dynamics of large networks (Zanudo, Yang, Albert, 2017).

Complex Social Influence. In this paper, we preselect the number of input variables included in each Boolean function as $k = 1$ for the purpose of not overfitting short time-series and match the theory about assimilative and repulsive social influence. When we have more observations in the time-series, we can fit more complex social influence, e.g., $k = 2$. This has the advantage to model complex social influence in a multi-person group setting. For example, if two persons team up and dominate a third person, it can be modeled using $k = 2$, and the Boolean function will be in the form of $x_3(t+1) = \overline{x_1(t) \wedge x_2(t)}$, meaning only when both x_1 and x_2 are ON – occupying resources, x_3 will be turned OFF – not occupying resources. To understand complex social influence, we also need theories about social influence to pinpoint specific forms and mechanisms of social influence.

Peer Selection. In this study, we did not consider the process of peer selection also because we assumed no peer selection – befriending or stop befriending certain group members – occurs that would impact the effect of social influence on self-disclosure in the therapy group setting. For social influence problem that is intertwined with peer selection, more sophisticated model, e.g., stochastic actor-based models (Snijders, van de Bunt, & Steglich, 2010), is needed to properly model group processes. Ideally, a stochastic actor-based model that can model peer selection and peer/social influence and the social influence can be assimilative or repulsive would be an ideal model framework.

Empirical Example. The participants in this paper were young adults, and for privacy reasons, their demographic information and severity of psychopathology was not included or analyzed. Before generalizing to larger populations, it will be useful to examine the group process with other populations, including individuals with various degrees of psychopathology, and with both younger and older individuals. In terms of variables, we select self-disclosure over some other measures in the progress survey, such as self-efficacy, because one person's self-disclosure could be observed more easily by other group members, and generate social influence in the group process of therapy. A caveat with this measure is the exact questionnaire item was ease of self-disclosure, which may have some discrepancy from the actual self-disclosure behavior. For lack of a more precise measure of self-disclosure behavior, we use ease of self-disclosure as a proxy of self-disclosure behavior. The measurement in our dataset of self-disclosure behavior has 10 ~ 16 weeks/observations, and this is often considered a short time-series for dynamical system models. To avoid overfitting the data, we selected the number of input variable $k = 1$. For more complex social influence, such as higher number of input variables, more repeated measures of behavior data are required. Longer measurements of behavior data would be helpful to identify more complex social influence.

Conclusion

In this paper, we introduced the Boolean network method as a method to model and manage group processes. To demonstrate the utility of this method, we applied this method on an empirical dataset, focusing on managing self-disclosure behavior in group therapy settings. Our modeling approach addresses gaps in previous network-based intervention literature through construction of networks that simultaneously accommodate both assimilative and repulsive social influence, and that provision of design network control strategies for networks in which both types of social influence dynamics are operating. The Boolean network method is a more flexible, realistic, and precise method to design network-based intervention. We hope that this first application and demonstration of Boolean networks to repeated measurement of fully connected groups will open up discussions and invite more empirical studies of network control system design for social networks.

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Table 3.1 Table of input variable(s) and the outcome of AND, OR, and NOT rule

x	y	$x \wedge y$
0	0	0
0	1	0
1	0	0
1	1	1

x	y	$x \vee y$
0	0	0
0	1	1
1	0	1
1	1	1

x	$\overline{x}$
0	1
1	0

Table 3.2 Table of state transition

(x_1, x_2, x_3)	
t	$t+1$
(0,0,0)	(1,0,0)
(0,0,1)	**(0,0,1)**
(0,1,0)	(1,0,0)
(0,1,1)	(0,0,1)
(1,0,0)	(1,1,0)
(1,0,1)	(0,1,1)
(1,1,0)	**(1,1,0)**
(1,1,1)	(0,1,1)

Table 3.3 Each group's model result, including Boolean functions, state transition graph, attractors, and control strategy (totally 18 groups)

Index	**Boolean functions**	**State transition graph**	**Attractor***	**Control strategy**
1	$x_1(t+1) = 0$ $x_2(t+1) = \overline{x_6(t)}$ $x_3(t+1) = 0$ $x_4(t+1) = x_2(t)$ $x_5(t+1) = \overline{x_7(t)}$ $x_6(t+1) = 0$ $x_7(t+1) = \overline{x_7(t)}$	A	x_2, x_4, x_5, x_7	Not needed*
2	$x_1(t+1) = 1$ $x_2(t+1) = 1$ $x_3(t+1) = 1$ $x_4(t+1) = \overline{x_4(t)}$ $x_5(t+1) = 1$	B	x_1, x_2, x_3, x_4, x_5	Not needed
3	$x_1(t+1)$ $= \overline{x_3(t)}$ $x_2(t+1) = x_1(t)$ $x_3(t+1) = \overline{x_3(t)}$ $x_4(t+1) = \overline{x_2(t)}$ $x_5(t+1) = 0$ $x_6(t+1) =$ $\overline{x_8(t)}$ $x_7(t+1) = 0$ $x_8(t+1) = 0$	C	x_1, x_2, x_3, x_4, x_6	Not needed
4	$x_1(t+1)$ $= \overline{x_3(t)}$ $x_2(t+1) = 0$ $x_3(t+1) = 1$ $x_4(t+1) = \overline{x_7(t)}$ $x_5(t+1) = \overline{x_7(t)}$ $x_6(t+1) = 1$ $x_7(t+1) = 0$ $x_8(t+1) = \overline{x_7(t)}$	D	x_3, x_4, x_5, x_6, x_8	Not needed
5	$x_1(t+1)$ $= \overline{x_1(t)}$ $x_2(t+1) = 0$ $x_3(t+1) = 0$ $x_4(t+1) = 0$ $x_5(t+1) = \overline{x_1(t)}$ $x_6(t+1) =$ $\overline{x_1(t)}$ $x_7(t+1) = \overline{x_5(t)}$	E	x_1, x_5, x_6, x_7	Not needed
6	$x_1(t+1)$ $= x_6(t)$ $x_2(t+1) = \overline{x_5(t)}$ $x_3(t+1) = 1$	F	x_1, x_2, x_3, x_4, x_6	Not needed

Index	Boolean functions	State transition graph	Attractor*	Control strategy
	$x_4(t+1) = x_6(t)$ $x_5(t+1) = 0$ $x_6(t+1) = \overline{x_5(t)}$			
7	$x_1(t+1) = 1$ $x_2(t+1) = x_5(t)$ $x_3(t+1) = \overline{x_6(t)}$ $x_4(t+1) =$ $x_5(t)$ $x_5(t+1) = 0$ $x_6(t+1) =$ $\overline{x_6(t)}$	G	x_1, x_3, x_6	Not available
8	$x_1(t+1)$ $= \overline{x_6(t)}$ $x_2(t+1) = \overline{x_6(t)}$ $x_3(t+1) = x_6(t)$ $x_4(t+1) = x_6(t)$ $x_5(t+1) = 1$ $x_6(t+1) = 0$	H	x_1, x_2, x_5	Not available
9	$x_1(t+1) = 0$ $x_2(t+1) = x_5(t)$ $x_3(t+1)$ $= x_8(t)$ $x_4(t+1) = 0$ $x_5(t+1) = \overline{x_3(t)}$ $x_6(t+1) = 0$ $x_7(t+1) = 0$ $x_8(t+1) =$ $x_5(t)$	I	Attractor 1: x_2, x_3, x_5, x_8 Attractor 2*: x_2, x_3, x_5, x_8	Not available
10	$x_1(t+1) = 0$ $x_2(t+1) = \overline{x_4(t)}$ $x_3(t+1) = \overline{x_7(t)}$ $x_4(t+1) = \overline{x_5(t)}$ $x_5(t+1) = x_6(t)$ $x_6(t+1) = 0$ $x_7(t+1) = x_6(t)$	J	x_3, x_4	No available
11	$x_1(t+1)$ $= x_6(t)$ $x_2(t+1) = \overline{x_5(t)}$ $x_3(t+1) = \overline{x_7(t)}$ $x_4(t+1) = \overline{x_2(t)}$ $x_5(t+1) = \overline{x_7(t)}$ $x_6(t+1) = x_3(t)$ $x_7(t+1) = 0$	K	x_4, x_5	Not available
12	$x_1(t+1)$ $= x_3(t)$ $x_2(t+1) = x_5(t)$ $x_3(t+1) = 0$	L	No node is ON in the attractor	Not available

Index	Boolean functions	State transition graph	Attractor*	Control strategy
	$x_4(t+1) = x_8(t)$ $x_5(t+1) = 0$ $x_6(t+1) = x_1(t)$ $x_7(t+1) = x_1(t)$ $x_8(t+1) = x_7(t)$			
13	$x_1(t+1) = \overline{x_2(t)}$ $x_2(t+1) = 0$ $x_3(t+1) = \overline{x_6(t)}$ $x_4(t+1) = 0$ $x_5(t+1) = x_6(t)$ $x_6(t+1) = x_4(t)$	M	x_1, x_3	Not Available
14	$x_1(t+1) = 0$ $x_2(t+1) = x_1(t)$ $x_3(t+1) = \overline{x_7(t)}$ $x_4(t+1) = \overline{x_2(t)}$ $x_5(t+1) = 0$ $x_6(t+1) = 0$ $x_7(t+1) = x_7(t)$	N	Attractor 1: x_3, x_4 Attractor 2: x_4, x_7	Not available
15	$x_1(t+1) = \overline{x_6(t)}$ $x_2(t+1) = 0$ $x_3(t+1) = x_6(t)$ $x_4(t+1) = 0$ $x_5(t+1) = 0$ $x_6(t+1) = \overline{x_4(t)}$ $x_7(t+1) = \overline{x_7(t)}$	O	x_3, x_6, x_7	Not available
16	$x_1(t+1) = \overline{x_2(t)}$ $x_2(t+1) = \overline{x_5(t)}$ $x_3(t+1) = \overline{x_5(t)}$ $x_4(t+1) = \overline{x_3(t)}$ $x_5(t+1) = 0$	P	x_2, x_3	Not available
17	$x_1(t+1) = x_4(t)$ $x_2(t+1) = x_6(t)$ $x_3(t+1) = \overline{x_6(t)}$ $x_4(t+1) = \overline{x_4(t)}$ $x_5(t+1) = 0$ $x_6(t+1) = \overline{x_2(t)}$	Q	Attractor 1: x_2, x_3, x_6 Attractor 2: x_1, x_2, x_3, x_4, x_6	Turn x_4 ON
18	$x_1(t+1) = \overline{x_3(t)}$ $x_2(t+1) = \overline{x_7(t)}$ $x_3(t+1) = x_1(t)$	R	Attractor 1: x_1, x_2, x_3, x_5 Attractor 2: $x_1, x_2, x_3, x_4, x_5, x_6$	Turn x_6 ON

Index	Boolean functions	State transition graph	Attractor*	Control strategy
	$x_4(t+1) = x_6(t)$ $x_5(t+1) = \overline{x_7(t)}$ $x_6(t+1) = x_6(t)$ $x_7(t+1) = 0$			

Attractor*: Here we only list the nodes that have self-disclosure ON in the attractor.
Not needed*: Defined as at least half of the group members have self-disclosure ON.
Attractor 2*: Since only active nodes are listed here, we are not able to show attractor 1 and 2 have different states (Figure 3.3n shows one attractor 1 rotates between 2 states, while attractor 2 rotates between 6 states).

Table 3.4 Three scenarios of attractors and network control strategies (totally 18 groups)

Index	Type	Attractors and Network Control Strategy	Count (Groups)
1	Well-functioning groups	Has one or two attractors, no control strategy needed	6
2	Unmanageable groups	Has one or two attractors, no control strategy available	10
3	Manageable groups	Has two attractors with differential desirability, control strategy available	2

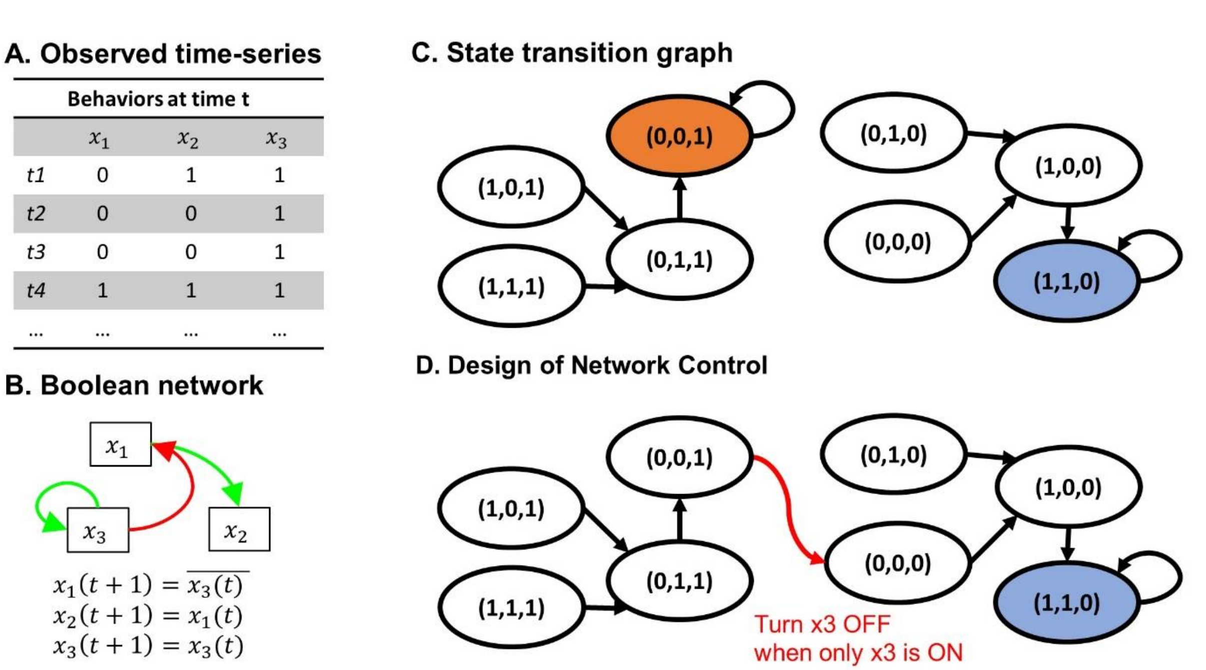


Figure 3.1 Example of a three-node network, consisting of x_1, x_2, and x_3.
Panel A is the observed binary time-series of x_1, x_2, and x_3 across time, Panel B is the Boolean network and Boolean functions inferred from the binary time-series in Panel A, and Panel C is the state transition graph derived from the Boolean functions in Panel B. Three-state tuple, e.g., (0,1,0) indicates the state of (x_1, x_2, x_3). Panel D. *Design of network management.* One strategy as an example is to turn x_3 OFF when only x_3 is ON, and this strategy will induce change in state transition graph highlighted by the red arrow.

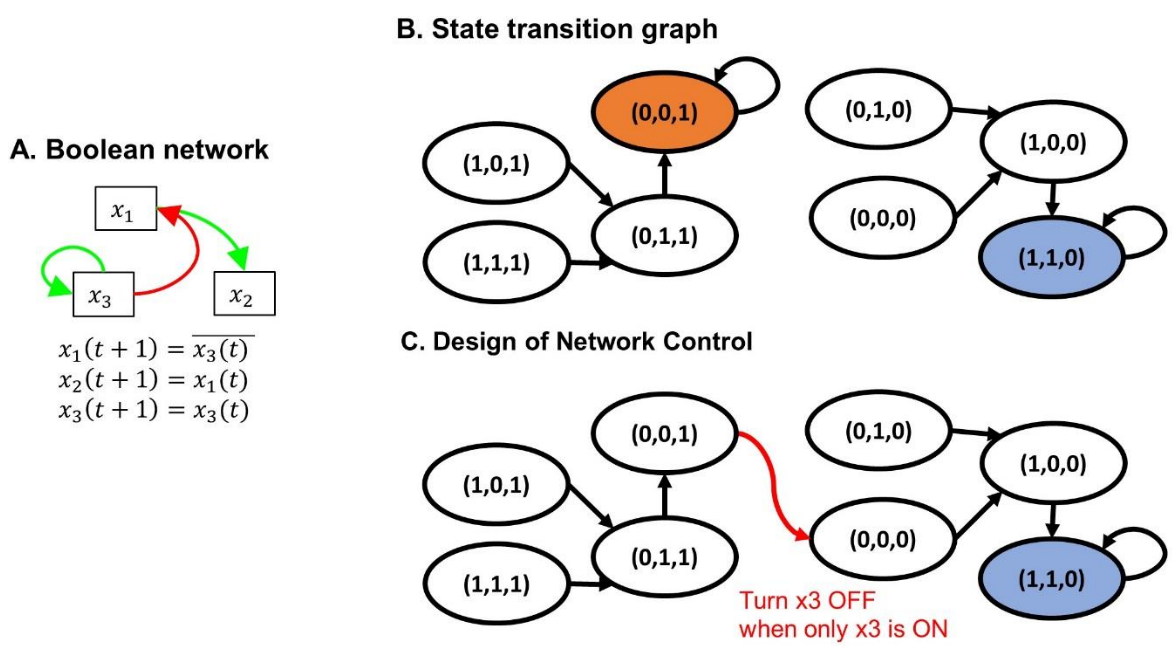
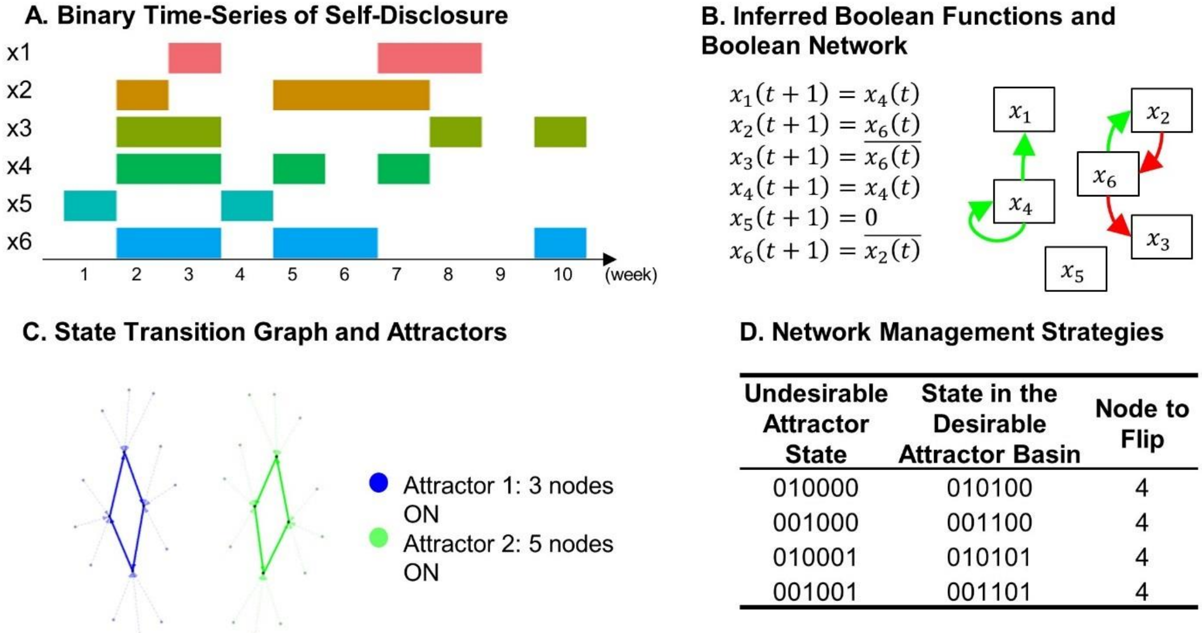


Figure 3.2 Illustration of design of network control
Panel A: Boolean network. The Boolean network and Boolean functions of a system. Panel B: State transition graph. The state transitions of all the states from t to t+1 are described as a state transition

graph, where each circle represents a state, and each arrow represents a state transition and its direction. Panel C: Design of network management. One strategy as an example is to turn x_3 OFF when only x_3 is ON, and this strategy will induce change in state transition graph highlighted by the red arrow.



Undesirable Attractor State	State in the Desirable Attractor Basin	Node to Flip
010000	010100	4
001000	001100	4
010001	010101	4
001001	001101	4

Figure 3.3 Illustration of a seven-member group's model results.
Panel A: Binary Time-Series of Self-Disclosure. Binary time-series of each group member's weekly self-disclosure behavior (colored bars indicate self-disclosure = ON, white space = self-disclosure = OFF). Panel B: Inferred Boolean functions and Boolean Network. Inferred Boolean functions based on the time-series in Panel A. The Boolean network represent the Boolean functions as edges and variables as nodes. Panel C: State Transition Graph and Attractors. Each dot represents a state, and two complex attractors were extracted (highlighted in blue and green). The Attractor 1 has 3 nodes ON and Attractor 2 has 5 nodes ON, both in a cyclic manner. Panel D: Network Management Strategies. Four strategies are found for this group. All strategies indicate by turning the fourth node ON, the network can be moved from Attractor 1 to Attractor 2.

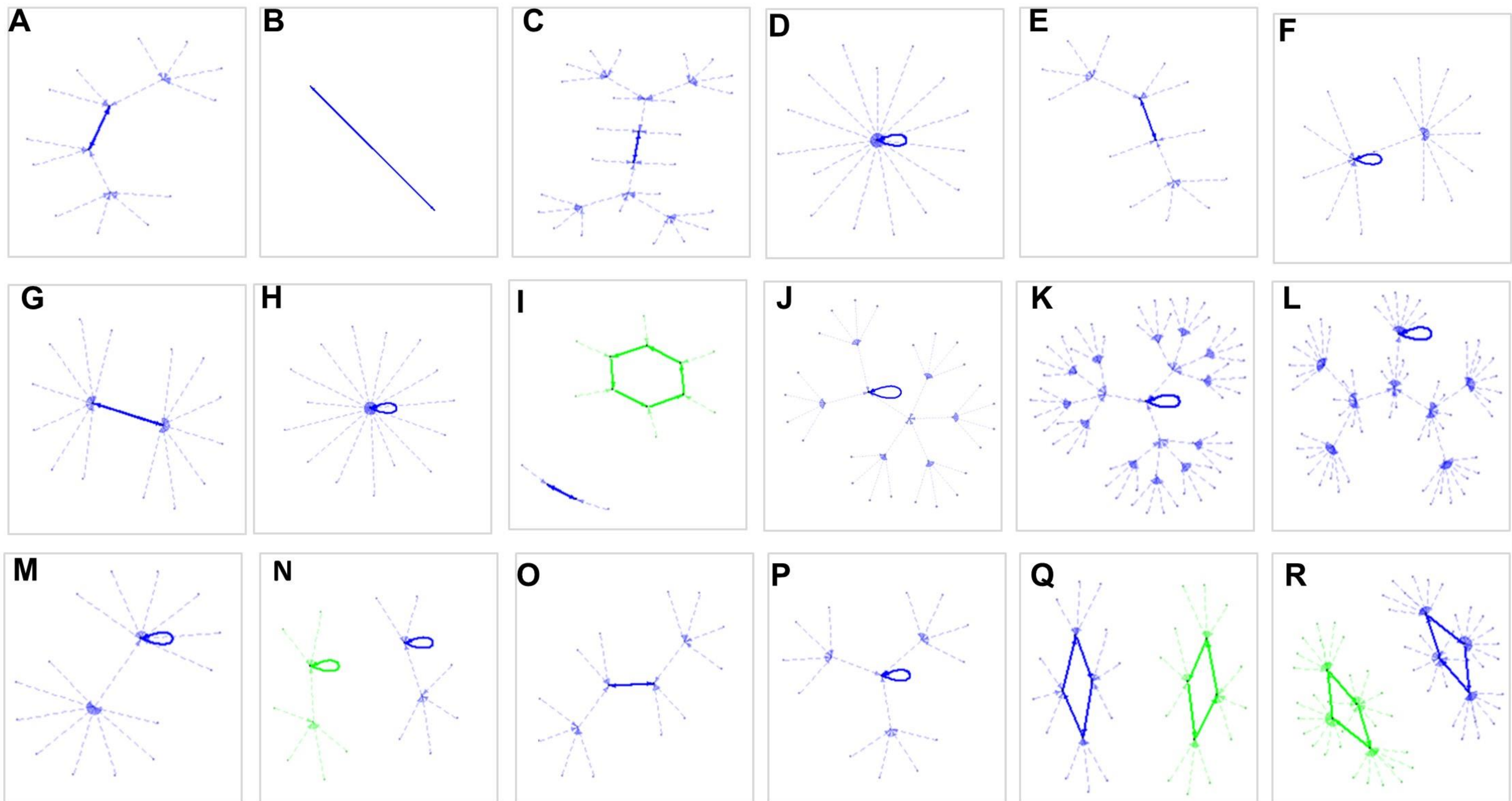


Figure 3.4 Illustration of eighteen state-transition graphs.

In each panel, each color indicates a different attractor and its basin, and the self-loop indicates an attractor and dashed arrows indicate direction of state transition, e.g., Panel H shows a state transition graph with one fixed point attractor, with a self-loop, Panel A shows a state transition graph with one limit cycle attractor where the system cycles through two states, Panel I shows a state transition graph with two limit cycle attractors labeled with different colors.

Chapter 4 DISCUSSION

Overview

The overarching goal of this dissertation is to introduce the Boolean network method as a method to model and develop control strategies for multivariate dynamical systems using binary time-series data. To my knowledge, this work is the first introduction and empirical application of the Boolean network method in psychological systems. Because the Boolean network method is new to the psychology literature, I synthesized the Boolean network method that was developed in system biology literature for this new audience, and reviewed the essential concepts and methodological details that enable psychologically-oriented readers to obtain a solid understanding of the method about network modeling and network control. To demonstrate the utility of the Boolean network method for study of psychological dynamical systems, I applied the method to two empirical data from two studies. The first example illustrated how the method can be used to study within-person dynamics of emotion regulation. The second example illustrated how the method can be used to study between-person dynamics of social influence.

Qualification about the Impetus for Studying Network Control

For clarification, my interest in exploring models that facilitate design of network control for psychological systems was to facilitate individual development in either within-person systems or between-person systems. The network control for a developmental system is designed according to the desirable goals for individual development in its specific context. For example, a desirable goal for early childhood emotion development is to be able to manage negative emotions when children are asked to wait, specifically to deploy a regulatory behavior to alter negative emotion when it occurs. In social groups, a desirable goal is to promote desirable behaviors to spread among group members to improve each individual's development.

From a methodology standpoint, we focus on *dynamical system-based control design* to achieve desirable goals. Control design is a subdiscipline of mathematics, and not to be confused with interventions about constructs related to "self-control" or "effortful control". The control design in this dissertation focuses on *identifying specific dynamics to manipulate,* so that a person's (or a group's) dynamical system can move from an undesirable attractor to a desirable attractor, which is also a desirable goal as aforementioned.

In this chapter, I summarize the contribution of this dissertation to our knowledge of modeling and controlling multivariate dynamical systems in psychology. In the following sections I address how the Boolean network method contributed to methods innovation and to theory, what the data requirements are, and point to other potential empirical applications. I finish with discussion of the limitations of the method and some exciting future directions.

Method Innovation

A Novel Method to Describe Nonlinear Multivariate Dynamics

The Boolean network method is a method for modeling dynamical systems in discrete-time with binary time-series data. In Chapter 2, I discussed the differences between the Boolean network method and other discrete-time dynamical system methods to model the dynamics driving changes in multiple variables with binary states, including the Ising model and Markov chain models.

In summary, the Boolean network method provides novelty that is complementary to these other methods. First, the Boolean network method provides a new form to express temporal relations. The Ising model expresses the temporal relations in the form of exponential functions, similar to logistic regression, that link the input variables/nodes and the outcome/node; and the Boolean network method expresses the temporal relations in Boolean operators, including AND, OR, and NOT. Both forms of expression can explain nonlinear dynamics and provide for interpretation.

Second, the Boolean network method provides explicit explanation of the dynamics. For example, in the empirical example in Chapter 2, regulatory dynamics where anger is *regulated* by distraction can be expressed as anger(t+1) = anger(t) AND NOT distraction(t). The expression indicates when distraction is ON, anger at the next time point will be OFF. That is how we identified anger is *regulated* by distraction. In Markov chain method, the state of the system is expressed as a vector of all variables. Take the anger-distraction relation as a two-node network example, there are four states of the system, which are (anger = 0, distraction = 0), (anger = 0, distraction = 1), (anger = 1, distraction = 0), and (anger = 1, distraction = 1). The Markov chain will focus on estimating the transition probability between any of the two states, e.g., the probability from (anger = 1, distraction = 1) at t to (anger = 0, distraction = 1) at $t+1$ has probability of 1. Even though this implies distraction can turn anger OFF, the Markov chain method does not formally verify this temporal relation or report it.

With the novelties coming with the Boolean network method, there are also some limitations. A major limitation is that the Boolean network method simplifies the dynamics into Boolean/logic dynamics, and also assumes the state transition is deterministic (either 0 or 1). In other words, the Boolean network does not provide detailed information about the dynamics like the Ising model, nor does it provide detailed estimation of the state transition probability like the Markov chain.

In sum, the Boolean network method provides a novel method to describe nonlinear multivariate dynamics. Choosing the Boolean network method should consider a few factors, including hypothesis about interactions between multiple input variables, direct test and interpretation of the dynamics, and the level of detail in the analysis.

Boolean Network Control of Nonlinear Dynamics

The Boolean network method provides a novel control method for nonlinear dynamics. In the discussion of Chapter 2, we clarified the rationales for choosing a function perturbation control method (Shmulevich & Dougherty, 2010; Campbell & Albert, 2019), and the rationale for only considering the naturally occurring attractors as desirable states. We also discussed why the Boolean network method provides a useful control method for nonlinear dynamics.

I will reiterate a clarification briefly as a background for the subsequent discussion. The desired states are constrained to the dynamical systems' naturally occurring attractors. We chose this method because it is more realistic to design control based on the naturally occurring attractors (Mochizuki, Fiedler, Kurosawa, Saito, 2013; Zanudo & Albert, 2015).

The Boolean network control methods contributes a method that can tackle the complexity of nonlinear dynamical systems, compared with linear control theory methods. Linear control theory methods focus on stabilizing the dynamics in the system using matrix algebra, designing an input/control into the system so that the discrepancy between the system state and the goal state will exponentially decrease to zero (Lewis et al., 2012). Linear control theory methods essentially assumes there is only one equilibrium point, which does not fully capture the complexity of the nonlinear dynamics, which are often marked by multiple equilibria, limit cycle, and even transitions between different regimes (Tang & Bassett, 2018). Hence, the Boolean network method provides control strategies that accommodates the nonlinear dynamics by accounting for multiple attractors.

Reducing Burden of Applying Control in Psychological Systems

Although advanced methods in control theory have been introduced by psychologists with empirical examples (receding horizon control, Sinclair & Molenaar, 2008; linear quadratic Gaussian control, Molenaar & Ram, 2010; linear quadratic regulator, Henry et al., in press) and obtained superb performance (Wang et al., 2014), the application of control methods in study of psychological systems remains quite sparse (Molenaar & Nesselroade, 2015). This dissertation pushes the possibility for applying control in psychological systems forward.

There are a few advantages of using the Boolean network method (as a network control method) to design control of psychological systems. The most prominent advantage is that the Boolean logic embedded in the network representation, and the design of control in Boolean logic terms, makes it easy to implement control in terms of simple (perhaps easy to remember) rules or heuristics (Gigerenzer, Goldstein, 1996). In Chapter 2, I discussed how control implementation requires an individual to at least know the *input* into a control system. A univariate discrete-time control system can be expressed as:

$$u^*(t) = [y^* - \beta y(t)]/\phi$$

where $y(t)$ is the state of the system, and $u^*(t)$ is the univariate external *input* that needs to be manipulated in order to control the system. As it is seen in the equation, the input $u^*(t)$ changes as the state of the system $y(t)$ changes. That means the individual or caretaker needs to be able to compute this equation or memorize the mapping between $y(t)$ and $u^*(t)$. It can be quite challenging when the individual needs to make a quick decision with time-pressing issues, or simply does not have the luxury to compute $u^*(t)$ in real-time. The Boolean network method circumvents this practical burden by using a simplified mapping. For example, we showed in our study of children's emotion regulation dynamics, that Boolean networks could be used to identify specific control strategies where turning specific behaviors ON or OFF would invoke regulation. These strategies were stated as simple if-then rules of the form, "*if* the child is stuck in anger, *then* turn [a specific behavior] [ON]/[OFF]". The simplicity of the control strategy can reduce the parent's burden to memorize complex mapping relations between the child's emotional states and the control strategy. In contrast, control designed using a continuous-scale dynamical system method would be much more complicated. Similarly, in Chapter 3, we suggest that in group settings, and the therapy group setting in particular, the Boolean network method can reduce the burden on group leaders' memorization of complex control strategies and make it easy for them to manage group dynamics.

It is worth highlighting that, like other methods, the Boolean network method relies on a stationary assumption. The dynamics of the system are assumed to be stationary, meaning the Boolean functions are not changing with time. Subsequently, the control strategy could be used at any time to guide the system into desired states. However, take the control strategy in Chapter 2 as an example, if the child develops new regulatory skills, it will change the emotion regulation dynamic. In that paper we found when anger is regulated by distraction, and distraction is only predicted by itself from previous time points, the control strategy indicated by the updated Boolean function is that distraction should be activated by anger. We assume the emotion regulation dynamics do not change for a period of time, so that the control strategy could be implemented and accomplish the desired goal. In the emotion regulation example, when the child learns to voluntarily use a regulatory strategy, meaning the control strategy is accomplished by the child without external/parental control input, the child's emotion regulation dynamics will be changed to a new Boolean function.

The Boolean network method provides a novel method to describe interactive dynamics among multiple predictors. The Boolean network method also provides a network control method that can identify how to direct nonlinear dynamics to a desired state and reduce cognitive burden when applying the control on psychological dynamics.

Theory Contribution: Heterogeneity in Psychological Dynamics

Based on the empirical example of children's emotion regulation in Chapter 2 and young adults' self-disclosure in therapy groups in Chapter 3, we found striking heterogeneity in psychological dynamics between children and between groups.

Heterogeneity of Emotion Regulation Dynamics and Control Strategies

In Chapter 2, with 117 children's emotion regulation dynamics, even when they were given the exactly same experimental task, we found 22 distinct Boolean networks consisting of anger, distraction, and bidding. And, the behavior that can regulate anger differs from child to child; there are children whose distraction can turn anger OFF, and there are also children whose bidding can turn anger OFF. This provides new insights to the emotion regulation development theory (Cole et al., 2012), which posits behaviors that shift attention away from desired object – here it is distraction rather than bidding – can regulate anger. A possible explanation is the empirical support for emotion regulation development theory has taken a nomothetic approach, aggregating a sample of children's data and assuming the data conform to the prototypical dynamic. In Chapter 2, I took the person-specific approach, and let each child's data be modeled separately, and observe whether the estimated dynamics do conform to the same dynamics. The results in Chapter 2 shows they do not. Thus, as pointed out by Molenaar (2004), caution is warranted when interpreting results derived from nomothetic analyses.

The heterogeneous networks correspond to heterogeneous control strategies. Some children need to turn distraction ON when they are angry, and some others need to turn bidding ON when they are angry. Which behavior is to be turned ON depends on whether it can regulate anger. Again, this result of control strategy also warrants caution when taking the nomothetic approach when making recommendations of how to improve children's emotion regulation skills, as the same advice, e.g., shifting attention away from the desired object, will only work for children whose distraction can regulate anger, but will not work for children whose distraction cannot regulate anger.

In sum, the Boolean network method provides person-specific/personalized advice, specifically which behavior is regulatory/dysregulatory and whether to turn this behavior ON/OFF, for parents and teachers to improve children's emotion regulation skills.

Heterogeneity of Group Dynamics and Group Management Strategies

In Chapter 3, the Boolean network that describes group dynamics and models whether group members' self-disclosure behavior will elicit or prevent others' self-disclosure behavior; if eliciting other's self-disclosure, it is considered an assimilative social influence, and if preventing other's self-disclosure, it is considered a repulsive social influence.

We observe heterogeneity in the Boolean networks when comparing across groups. The result shows 14 groups had both assimilative and repulsive social influence, while only 4 groups only had assimilative or repulsive social influence. No two groups had the same network configuration, which could be observed from the 18 heterogeneous state transition graphs.

Based on the information of attractors, we can see some group has zero group members' self-disclosure ON in the attractor, while some have all group members' self-disclosure ON in the attractor. That indicates in some groups where zero member had self-disclosure ON, group members had trouble self-disclosing, and maybe these groups have more punishing atmosphere in the group; in other groups where all members had self-disclosure ON, group members were motivated to self-disclose, and maybe these groups have more encouraging atmosphere in the group. Subsequently, the control strategy differs group by group. The Boolean network method provides group-specific suggestions for group therapists, specifically which group members can move the whole system into a desirable attractor.

Theories of group dynamics and social influence should take into account of both assimilative and repulsive social influence, which is a more accurate and realistic way to analyze the social influence and its impact on group-level behavior change. Omitting the repulsive social influence might be easier to conceptualize the system, but it will not be able to explain complex phenomenon, such as multi-stability or limit cycle, which we have found in our empirical data of self-disclosure in therapy groups. Furthermore, without accounting for both types of social influence, intervention would only focus on assimilative social influence and how to optimize the group process using assimilative social influence, while the Boolean network method and other nonlinear system control methods could contribute to the knowledge of how to manage group processes.

Data Requirement: Intensive Longitudinal Data

As with other dynamical system methods, use of the Boolean network method requires intensive longitudinal data. The minimal number of observations per Boolean network model depends on both the complexity of the Boolean functions (governed by selection of k = number of input variables) and network size N. When k is higher, more complex Boolean functions can be inferred. However, this complexity could lead to overfitting. Higher number of observations are needed to support the complexity and to avoid overfitting. Larger network size N indicates more potential input variables will be searched in the inference of Boolean functions, so it also requires higher number of observations (T) to avoid misidentification. Let us go through a hypothetical example to elaborate this point. Imagine there are 100 people in a social network (N=100), but only 3 measurements are taken (T=3), then the sequence of each individuals behavior has 3 digits of 0 or 1, which has $2^3 = 8$ possible combinations. There are at least 12, which is the round down number of 100 people divided by 8 possible combinations, sharing the same sequence for each possible combination. If we were to infer one target person's Boolean function (number of input variable k=1) as an outcome of other group member's behavior, then there will be at least 12 tied Boolean functions that can equally match the target person's behavior. More complex model (higher number of input variables k) and larger network size will require more intensive data to fit the Boolean network model, and a comprehensive simulation study to examine the minimal requirement of number of observations will provide useful guidance to analysis.

In both of the empirical examples, we applied the Boolean network method to multivariate binary time-series data. In Chapter 2, we applied the method to binary time-series data of a type that has been obtained in many observational studies that make used of behavioral coding. However, it is possible to apply the Boolean network method to continuous-scale time series data, albeit after these data have been binarized, As demonstrated in Chapter 3, we binarized the self-disclosure variable from a continuous-scale variable into a binary variable before inferring the Boolean functions. Previous research has studied the effect of such binarization on the model results and provided evidence that the key characteristics of time-series are in some cases maintained and can be discovered in the binarized data. For example, oscillation patterns present in continuous-scale time-series may be reflected by a sequence of ON and OFF in the binary time-series (Berestovsky & Nakhleh, 2013). Thus, there is some evidence that the Boolean network method may be useful in analysis of other types of intensive longitudinal data, including experience sampling data, and physiological data (e.g., wearable sensors, EEG, fMRI).

Potential Empirical Applications in Psychology

This dissertation provides the first demonstrations of how the Boolean network control method can be applied to study and control within-person and between-person psychological dynamics. Chapter 2 demonstrated how control design can facilitate children's development of emotion-regulation, and Chapter 3 demonstrated how control design can promote spread of desirable behaviors in a social group. Beyond these two demonstrations, the Boolean network method offers opportunities for many practical applications.

One important potential application is to use the Boolean network method to assist diagnosis or treatment of psychopathology. Evidence from a large body of work has shown that emotion regulation dynamics are associated with a variety of mental health outcomes (Cole, Ramsook, & Ram, 2019). There has been an increasing interest in using network models to understand emotion dynamics and to inform diagnosis and intervention from a clinical perspective (Bringmann et al. 2016; Cramer et al., 2016; Yang et al., 2018; Yang et al., 2019). The Boolean network method can be used to identify network control strategies that move an individual's emotion dynamics to a more desirable attractor and subsequently influence their long-term mental health. The person-specific approach also allows for designing personalized interventions. Additionally, the prescribed control strategy could be used as a data-driven approach to inform or confirm therapist's diagnosis about a patient's treatment plan. With the increasing use of smartphones and mental health improvement applications, the Boolean network method can be implemented on smartphones to model psychological dynamics and provide control strategies as a mobile health tool.

Another important future potential application is to model and control social dynamics. In Chapter 3, we focused on small therapy groups of 5 to 8 group members. The Boolean network method can also be applied on other social dynamics that are under the social influence, e.g., in a high-school classroom, the Boolean network control method can be used to identify strategies that will facilitate desired behavior such as academic performance (DeLay et al., 2016), or will help prevent undesired behavior such as substance use (Rulison, Gest, Osgood, 2015). Another possibility is the group dynamics that manifest on public social media platforms (e.g., twitter). Commonly studied topics related to individual development includes dissemination of health-related information (Guilbeault & Centola, 2019). Because the opinion dynamics on social media platforms are also susceptible to social influence, we can apply the Boolean network method to model the social influence based on the observed opinion, like how we modeled it in Chapter 3 based on self-disclosure behavior.

In sum, there are a wide range of empirical problems in individual development, from psychopathology treatment to adopting health-related behavior in the social group context, can use the Boolean network method. The dynamics in each kind of network can be uncovered by the Boolean functions, and control strategy to move the person toward a healthier state or adopting a healthier behavior can be prescribed.

Limitations and Future Directions

Modeling Uncertainty in the Boolean Network Method

An important limitation of the Boolean network method introduced in this dissertation is it assumes the dynamics in the system are deterministic. This means that the uncertainty in a dynamical system is not incorporated. I propose there are three ways uncertainty can be incorporated into Boolean networks through incorporation of the concepts of measurement error, process error (Deboeck & Boker, 2010), and regime-switching (Yang & Chow, 2010; Chow & Zhang, 2013).

For measurement error, we consider the uncertainty is caused by errors when measuring the outcome (e.g., anger is not coded correctly for a few seconds), and the error only affects the outcome while the dynamic process is not affected. Mathematically, the measurement error could be set up as such: there is a probability of error coming into the system, the probability conforms to a Bernoulli distribution at each time point, where the probability of occurrence of an error denoted as p. When the error occurs, the value of the outcome variable is taking the opposite value of the Boolean function expression of the outcome variable. A Boolean function equation is shown as follows to express measurement error:

$$x_{i,obs}(t+1) = x_i(t+1) \oplus e(t)$$

$$x_i(t+1) = f_i\left(x_j(t)\right)$$

$$e(t) \sim Bernouli(p)$$

where$x_{i,obs}$ is the observed value of x_i, x_i is the true value of the outcome variable, $f_i(x_j(t))$ is the Boolean function to predict x_i, $e(t)$ is the measurement error that follows a Bernoulli distribution with parameter p. $\oplus$ represents the XOR operator, meaning only when the two input variables have opposite values, the outcome is 1. So whenever e(t) is 1, meaning there is a measurement error, the value of $x_i(t+1)$ is taking the opposite value, e.g., if $x_i(t+1)$ =1, $x_{i,obs}(t+1) = 1 \oplus 1 = 0$; if $x_i(t+1)$ =0, $x_{i,obs}(t+1) = 0 \oplus 1 = 1$. whenever $r(t)$ is 0, meaning there is no measurement error, the value of $x_i(t+1)$ does not change its value, e.g., if $x_i(t+1)$ =1, $x_{i,obs}(t+1) = 1 \oplus 0 = 1$; if $x_i(t+1)$=0, $x_{i,obs}(t+1) = 0 \oplus 0 = 0$.

For process error, we consider the uncertainty is caused by noise going into the dynamic process (e.g., a child gets angry at one point because his mom ignored him for a little bit during the experiment). The difference between process error and measurement error is the process error can be added into any timestep with a given probability, and gets iterated into future time-series timestep by timestep. A Boolean function equation is shown as follows to express process error:

$$x_i(t+1) = f_i(x_j(t)) \oplus e(t)$$

$$e(t) \sim Bernouli(p)$$

where $f_i(x_j(t))$ is the Boolean function to predict x_i, $e(t)$ is the process error that follows a Bernoulli distribution with parameter p. Similarly, the noise can change the value of x_i using the XOR operator, and the noise gets iterated into all the following timesteps.

For regime-switching, we consider the uncertainty is caused by multiple regimes switching between each other probabilistically (e.g., a child gets angry when he focuses on the desired object, but he can also regulate himself if distracted). The probabilistic Boolean network (Shmulevich & Dougherty, 2010) estimates multiple Boolean functions when the error rate (here is estimation error, not measurement error) ties, and assign the probability equally across the Boolean functions. The multiple Boolean functions can be considered as multiple regimes. A Boolean function equation is shown as follows to express process error:

$$f_{x_i} = \begin{cases} f_{x_{i1}}, & p = \frac{1}{n} \\ f_{x_{i2}}, & p = \frac{1}{n} \\ & \dots \\ f_{x_{in}}, & p = \frac{1}{n} \end{cases}$$

where $f_{x_{i1}}, f_{x_{i2}}, \dots, f_{x_{in}}$ are n Boolean function that can predict x_i with equal probability p. The transition probability between regimes are not modeled based on the previous state like the regime-switch models (Yang & Chow, 2010; Chow & Zhang, 2013), instead it is evenly split by the number of regimes in the probabilistic Boolean network method.

The advantage of incorporating uncertainty into the Boolean network method is to allow for more realistic inference of the dynamics. Then, in the subsequent steps such as extraction of attractors, the uncertainty incorporated in the inference step can derive the probability distribution of the system going into different attractors and control strategies. Future work can investigate the estimation procedure associated with each kind of uncertainty mechanism. The probabilistic Boolean network method provides a solution when the uncertainty-generating mechanism is regime-switch. The estimation of the other two uncertainty-generating mechanisms is yet to be developed to my knowledge.

Scalability of the Boolean Network Method

Another important limitation of the Boolean network method introduced in this dissertation is time complexity of attractor extraction. In Chapter 2, I introduced the attractor extraction, which requires searching through all combinations of initial states. Thus, given a network of *N* nodes, the time complexity will be $O(2^N)$ and the computation time will be unscalable when the network size *N* increases, which is especially common for social network applications. Fortunately, network scientists began solving this problem and have suggested using structure-based search algorithm to reduce time complexity (Liu & Barabasi, 2016; Zanudo, Yang, Albert, 2017). The structure-based control design relies on the identification of specific network structures – motifs. Stable motifs are identified in the Boolean network structure, which are defined as structures in the network that are not affected by external inputs once they are activated, e.g., anger at *t+1* predicted by itself at *t* is a stable motif. This structure-based control design will be a valuable future direction to take to enhance the applicability of the Boolean network method on larger networks.

Timescale: Psychological Process, Measurement, and Models

We assumed the timescale of underlying process is the same with measurement timescale, e.g., Chapter 2 used second-by-second as the timescale, and Chapter 3 used week-by-week as the timescale. However, the time-scales embedded in the data (i.e., through measurement decisions) may not necessarily match the actual time-scale on which the underlying psychological and social processes actually proceed. To elaborate this point, two scenarios are discussed. The first scenario is when the underlying process has a faster time-scale than the measurement time-scale, e.g., the behavior change process in group settings happen within each week's session, rather than from one week to the next week, then the dynamics within each weekly session might be missed by the model. The second scenario is when the underlying process has a slower time-scale than the measurement time-scale, e.g., the emotion regulation dynamics happen every ten seconds, rather than every second, then the dynamics captured by the model might be overwhelmed by the auto-regression, e.g., anger(*t+1*)=anger(*t*). Both scenarios can lead to misrepresentations of the actual dynamics. A potential solution for the second scenario, when the underlying process has a slower timescale than the measurement timescale, is to test multiple timescales and compare estimation errors when from multiple timescales from a model comparison perspective. For the first scenario, when the measurement is slower than actual process, there is not yet a solution to test the true timescale using multiple timescales, because the data collected missed psychological process between measurements, so we cannot model the psychological process where we do not measure. Future work can include a systematic analysis of how collections of models might be used to identify the most likely time-scale and representation of the dynamic system.

Conclusion

This dissertation introduces and forwards a Boolean network-based method for studying psychological dynamics, both within-person and between-persons. I outline the Boolean network method, provide a guide for implementation, and illustrate how the method is applied in two empirical settings – study of children's self-regulation, and study of group-therapy processes. The work highlights the utility of the method for obtaining intuitive descriptions of individual or group dynamics and deriving strategies for directing the individual or group towards desired outcomes. To our knowledge, this is the first application of the Boolean network method in study of psychological processes. Our demonstration emphasizes the appeal of this method for both theory and practice – providing simple descriptions and explanations of system dynamics and system control strategies. Altogether, this dissertation forwards and provides access to a useful tool that can help researchers discover, understand, and shape many different kinds of psychological dynamics.

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