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ESSAYS ON LABOR, NON-HOMOTHETICITY AND WELFARE IN
DEVELOPING COUNTRIES

A Dissertation in
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by
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Abstract

This dissertation is composed of two, largely unrelated parts. In the first part (chapters 1 and 2) I explore the distributional effects of a negative terms-of-trade shock to an oil-exporting country. Specifically, I begin by developing a computable Roy model of the Nigerian economy with non-homothetic preferences, after-tax income effects, and government endogenous transfers through oil subsidies. I then use this model to simulate the impact of the global oil price shock that hits the Nigerian economy in 2016. In doing so, I focus on differential impacts across workers due to their different skills, locations, and comparative advantages. Finally, I use the model to assess whether Nigeria's policy of removing the oil subsidies was welfare improving, and to analyze complementary ways to improve the welfare outcome of low income workers in the wake of the subsidy removal.

The second part of this thesis (chapter 3) develops a dynamic model of child labor that challenges the view that child labor contributes to human capital accumulation (Dessy and Pallage, 2005; Sugawara, 2011). Therein, I argue that this view is valid but only in a static framework. Indeed, when accounting for the dynamic of human capital accumulation, child labor - including normal forms of labor - always negatively impacts children's future earning. I show through a theoretical argument that the apparent positive role played by child labor in a mincer-type regression merely reflects a disguised positive role played by children's ability in a sub-optimal equilibrium characterized by constrained credit and high incidence of child labor.

Table of Contents

List of Figures	vii
List of Tables	ix
Acknowledgments	x
Chapter 1	
Quantifying the incidence of a global oil price shock in Nigeria: a Roy model with non-homothetic preferences and endogenous transfers	1
1.1 Introduction	2
1.2 Related Literature	6
1.3 Motivating Facts	9
1.3.1 Data and Notations	9
1.3.2 Pre-Shock Descriptives	10
1.4 Model	14
1.4.1 Workers	16
1.4.1.1 Characteristics	16
1.4.1.2 Utility	17
1.4.1.3 Employment Choice Problem	18
1.4.1.4 Goods Allocation Sub-Problem	19
1.4.2 Government	21
1.4.3 Trade and Prices	22
1.4.4 General Equilibrium	24
1.4.4.1 Characterization	24
1.4.4.2 Comparative Statics	28

1.4.5	Welfare Changes Metric	33
1.5	Calibrating the 2016 Shocks	35
1.5.1	Pre-Shock Macroeconomic Descriptives	37
1.5.1.1	Sectoral Activities	37
1.5.1.2	Government Resources	38
1.5.2	Description of Scenario	40
1.5.3	Main Results	41
1.5.3.1	Fit of the Models	42
1.5.3.2	Main Outcomes	44
1.6	Conclusion	57

Chapter 2

Quantifying the incidence of a global oil price shock in Nigeria:		
	Policy counterfactual Exercises	59
2.1	Introduction	60
2.2	Should the government have removed the subsidies?	61
2.2.1	Uniform tax hike counterfactuals	61
2.2.2	Progressive tax hike counterfactuals	65
2.3	Complementary welfare improving mitigating counterfactuals	71
2.3.1	Increase in foreign borrowings	71
2.3.2	Progressive and neutral tax redistribution	75
2.4	Conclusion	79

Chapter 3

Child Labor and Future Human Capital: Understanding the		
	Role of Ability	81
3.1	Introduction	82
3.2	Models	86
3.2.1	Unconstrained Equilibrium	87
3.2.2	Constrained Equilibrium	88
3.3	Implications for Future Human Capital	91
3.3.1	Child Labor and Future Human Capital	91
3.4	Conclusion	94

Appendix A

On Chapters 1 - 2 (Part I)		95
A.1	Motivating Facts	95
A.2	Constructing Key Variables	101
A.3	Data, Parameters: Model Summary	102
A.4	Description of Solution Algorithm	103

A.4.1	Objects of the System of Equations	103
A.4.2	Description of the Algorithm	103
A.4.2.1	Primitives	103
A.4.2.2	Main Equations	103
A.4.2.3	Solving for Primitives (Steps)	103
A.4.2.4	Predicted outcomes	104
A.5	Computational Results	105
Appendix B		
On Chapter 3 (Part II)		110
B.1	Theoretical Predictions	110
B.1.1	A1: Proof to Lemma 1	110
B.1.2	A2: Proof to Lemma 2	113
B.1.3	A3: Proof to Proposition 1	113
B.1.4	A4: Proof to Proposition 2	115
Bibliography		118

List of Figures

1.1	<i>Baseline scenario: within-group sectoral labor reallocation: non homothetic model (in % units)</i>	43
1.2	<i>Baseline scenario: within-group sectoral labor reallocation: homothetic model (in % units)</i>	44
1.3	<i>Baseline scenario: changes in consumption prices (in %)</i>	46
1.4	<i>Baseline scenario: changes in wages (in %)</i>	47
1.5	<i>Baseline scenario: changes in net income (in %): non-homothetic vs homothetic models</i>	49
1.6	<i>Baseline scenario: changes in price index (in %): non-homothetic model</i>	50
1.7	<i>Baseline scenario: changes in total welfare (in %): non-homothetic model</i>	52
1.8	<i>Baseline scenario: changes in real consumption (in %): homothetic model</i>	52
1.9	<i>Baseline scenario: changes in social welfare at group and country levels (in %)</i>	54
1.10	<i>Baseline scenario: changes in real return to skill and location (in %)</i>	56
2.1	<i>Uniform tax hike counterfactual: welfare change relative to baseline scenario: non-homothetic model (in % units)</i>	62
2.2	<i>Uniform tax hike counterfactual: welfare change relative to baseline scenario: homothetic model (in % units)</i>	63
2.3	<i>Uniform tax hike counterfactual: social welfare changes relative to baseline scenario (in % units)</i>	64
2.4	<i>Uniform tax hike counterfactual: social welfare changes: non-homothetic vs homothetic models (in %)</i>	65
2.5	<i>Progressive tax hike counterfactual: welfare change relative to baseline scenario: non-homothetic model (in % units)</i>	67
2.6	<i>Progressive tax hike counterfactual: welfare change relative to baseline scenario: homothetic model (in % units)</i>	68

2.7	<i>Progressive tax hike counterfactual: social welfare changes relative to baseline scenario (in % units)</i>	69
2.8	<i>Progressive tax hike counterfactual: social welfare changes: non homothetic vs homothetic models (in %)</i>	70
2.9	<i>High foreign borrowing counterfactual: welfare change relative to baseline scenario: non-homothetic model (in % units)</i>	72
2.10	<i>High foreign borrowing counterfactual: welfare change relative to baseline scenario: homothetic model (in % units)</i>	73
2.11	<i>High foreign borrowing counterfactual: social welfare changes relative to baseline scenario (in % units)</i>	73
2.12	<i>High foreign borrowing counterfactual: social welfare changes: non homothetic vs homothetic models (in %)</i>	74
2.13	<i>Progressive tax redistribution counterfactual: welfare change relative to baseline scenario: non-homothetic model (in % units)</i>	76
2.14	<i>Progressive tax redistribution counterfactual: welfare change relative to baseline scenario: homothetic model (in % units)</i>	77
2.15	<i>Progressive tax redistribution counterfactual: social welfare changes relative to baseline scenario (in % units)</i>	77
2.16	<i>Progressive tax redistribution counterfactual: social welfare changes: non-homothetic vs homothetic models (in %)</i>	78

List of Tables

1.1	<i>Labor composition by skill and location (in %)</i>	10
1.2	<i>Within-group allocation of labor across sectors (π_{gi}, in %)</i>	11
1.3	<i>Average income across skills and locations</i>	12
1.4	<i>Parameters calibration: symbol and value</i>	35
1.5	<i>Sectoral components of GDP (in %)</i>	38
1.6	<i>Mix (share) of varieties in sectoral national absorption (in %)</i>	38
1.7	<i>Taxation policy profile: income brackets and tax rates</i>	39
1.8	<i>Contribution of tax revenue and foreign borrowing to government's resources (in %)</i>	39
1.9	<i>Baseline scenario: calibration of shocks and other exogenous changes</i>	40
1.10	<i>Baseline scenario: endogenous changes in key rates (in %)</i>	42
1.11	<i>Changes in output (in %)</i>	42
1.12	<i>Baseline scenario: changes in prices and wages (in %)</i>	45
2.1	<i>Uniform tax hike counterfactual: changes in key rates (in %)</i>	61
2.2	<i>Progressive tax hike counterfactual: changes in key rates (in %)</i>	66
2.3	<i>High foreign borrowing counterfactual: changes in key rates (in %)</i>	71
2.4	<i>Progressive tax redistribution counterfactual: changes in key rates (in %)</i>	75
3.1	<i>Human capital outcomes by type of equilibrium</i>	92

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Dedication

To my parents, Melanie Apedo and Marc Abalo

To my partner Janine Schuette

To my son Jonathan (Joni) Abalo

Chapter 1

Quantifying the incidence of a global oil price shock in Nigeria: a Roy model with non-homothetic preferences and endogenous transfers

1.1 Introduction

Context and motivation. Nigeria is a major crude oil producer, and as such, most (over 85%) of its export revenue comes from the oil sector. In the 1960s, the Nigerian government introduced petroleum subsidies with the objectives of strengthening the domestic refinery industry and improving product affordability. However, due to deficiencies in the country's refinery systems, today Nigeria continues to import over 80% of its petroleum. This reliance on imports makes the subsidy policy highly sensitive to shocks to the global price of oil, as well as potentially expensive in the case of international price spikes¹. Concretely, these subsidies take the form of a flexible discount (subsidy rate) on the consumption price, which is endogenously determined by both the international price of oil and the government's resources. The government's resources are derived primarily from tax revenues². In general, subsidy levels increase when global oil prices are high, both because tax revenues grow with oil exports and because the subsidies serve as a buffer between global price spikes and domestic prices. Although the subsidy policy is very expensive, Nigeria has maintained it because of a combination of strong economic growth and favorable current account balance, mostly driven by oil exports and foreign borrowings. In 2016, however, the price of oil dropped by about 40% in the international markets, leading to a steep decline in oil exports and a reduced access to foreign capital³, a depreciation of the national currency, and an economic recession. The subsequent fall in the tax revenues and foreign capital forced the government to entirely remove the subsidies.

In this chapter, I show that the oil shock had differentiated impacts on wages across income groups. Indeed, by leading to a depreciation of the Nigerian currency, the shock changed in each sector the mix of domestic demand for foreign and local varieties on the one hand, and the volume of foreign demand for Nigerian goods on the other hand. These changes led to differentiated changes in the sectoral equilibrium wages, as different sectors have different levels of exposure to trade. Ultimately,

¹For example, in the years before the 2016's price crash, the subsidies accounted on average for 40% of the government's budget. And that is in spite of global prices of oil having already engaged in a downward spiral.

²In 2015, 85% of public resources came from tax revenues. The other component includes net capital inflows.

³There was a strong push from the International Monetary Fund [IMF] at that time aimed at addressing the fiscal deficit.

this led to distributional income effects across different income groups, as workers within each respective group tend to specialize differently across sectors⁴. I also show that the change to the domestic prices of oil resulting from both the oil shock and the removal of the subsidies had significant welfare distributional consequences on the cost of living. The reason for this is that Nigerian consumers not only spend a significant amount of their income on petroleum, but they also spend an increasing share as their incomes rise, making petroleum a luxury good. This expenditure pattern can be traced to an unreliable electrical grid, which induces consumers to run in-house lamps or electrical generators, both of which are powered by petroleum. Therefore, while removing the subsidies can increase the cost of living, rich consumers may be more affected than poor consumers. The endogenous adjustments in other sectoral prices are also likely to affect the cost of living of rich consumers and poor consumers differently, as the two groups generally tend to enjoy different baskets of consumption.

Based on these two arguments, one could conclude that the oil shock and the subsequent removal of the subsidies had the potential to affect income and cost of living differentially, and that the magnitude of the net effects are ambiguous across different income groups. In this chapter, I develop a theoretical structural framework that sheds light on this ambiguity. Concretely, I build and quantify a general equilibrium structural Roy multi-sector model and use it to investigate the general equilibrium effects of the oil shock. In conducting welfare distributional analyses, I measure the ex-post after-tax (net) income effect and cost of living effect across different income groups. I define income groups using heterogeneous groups of workers who differ in skill levels, locations, and sectors of employment⁵. Concretely, using skill and location, I construct groups of workers and define within each group different subgroups characterized by each sector of employment. In order to conduct the welfare analyses, I construct for each subgroup-representative worker an otherwise non-observed real consumption metric whose change is driven by changes in after-tax income on the one hand and in the price index on the other hand. Hence, with the real consumption metric, I am able to explore both an *income channel* and a *cost of living channel* of the welfare distributional changes, a combination that is under-

⁴For instance, low-income groups tend to have heavy concentrations of agricultural workers.

⁵I distinguish among three skill levels (non, low, high); two types of location (rural, urban); and four sectors of employment (crude oil, agriculture, manufacturing, service).

explored in the existing literature⁶. The income channel explores the distributional changes in after-tax income across all groups and subgroups of workers. I enrich this latter channel with a taxation sub-channel, improving on recent literature that has generally ignored the adjustments in government taxes that accompany major terms of trade shocks in developing countries. The cost of living channel captures the heterogeneity in the changes to the cost of living. Capturing this latter heterogeneity represents an improvement to the literature on welfare changes, which has focused largely on the distributional effects through the income channel and considered the cost of living channel to be distributionally neutral.

Research questions. This chapter aims on the one hand at measuring the welfare distributional consequences of the oil shock and the subsequent removal of the subsidies that ensued. Concretely, I develop a methodology to quantify the unequal effects to workers' net income using their comparative advantage. I also quantify the unequal effects to their cost of living by allowing for non-homothetic preferences⁷. On the other hand, in the wake of the negative external shock, it is important that the government adopts the appropriate mitigating policies. In that regard, I investigate whether removing the oil subsidies was the best strategy regarding welfare outcomes, especially for low-income workers. Third, I explore different ways for the government to improve the welfare outcomes of low-income workers in the wake of the shock and subsidy response. Finally, I explore and quantify the role (if any) of non-homotheticity in preferences in determining the respective welfare outcomes.

Methodology. I use a three-step methodology to achieve the above-mentioned objectives. First, I build a multi-sector general equilibrium structural model that puts three forces in play: after-tax income effects, non-homothetic preferences, and endogenous subsidy levels. The model characterizes the behavior of workers of different skill levels and locations who, given their comparative advantage and idiosyncratic preferences, are able to choose the sector in which they work. Each worker

⁶Over the past 15 years, many researchers have used calibrated models to quantify the distributional effects of trade shocks on heterogeneous workers (Helpman et al., 2012; Galle et al., 2016; Cantore et al., 2012). With only a few exceptions (He and Zhang, 2017; Fajgelbaum and Khandelwal, 2016; Borusyak and Jaravel, 2018), this literature has focused exclusively on income effects.

⁷These preferences make it possible for consumers with different income levels not only to have different baskets of consumption, but also to differently adjust their expenditure behavior in response to price shocks.

earns an income determined by their labor endowment and their wage in the sector chosen. Then, she pays an income-proportional tax, and spends the resulting net income on the mix of local and foreign goods that maximizes her welfare, subject to the rate of oil subsidies. My treatment of labor markets builds on Rodriguez-Clare et al. (2018), who develop a Roy characterization of workers' employment choices. My characterization of consumption behavior is based on the recently introduced generalized non-homothetic constant elasticity of substitution (CES) preferences, which feature not only non-linear Engel curves but also income-specific price indices. In terms of my characterization of government behavior, I consider the government to play a passive role that consists of raising revenue mainly through taxation and setting a level of oil subsidies that balances its resource constraint, while keeping all other public expenditures constant. Second, using the hat-algebra methodology (Dekle et al., 2008), I derive the changes in sectoral wages and prices; and in labor allocation across sectors, including within each skill-location group; and in consumption prices. Ultimately, I obtain for each subgroup-representative consumer the expressions for the changes in net income, in sectoral expenditures, and in the price index. Ultimately, I obtain the changes in the subgroup-representative consumer's real consumption as a function of a net income effect and a cost of living effect. Third, I solve the model with respect to the 2016 oil shock (*baseline scenario*) and different public financing policy changes (*counterfactual scenarios*)⁸.

Summary of results. Solving the model for the 2016 oil shock (baseline scenario), I find that the oil shock led to negative GDP growth in real terms⁹, which when combined with reduced access to foreign capital, led to a phasing out of the subsidies. Furthermore, the shock made Nigerian agricultural and manufacturing goods relatively less expensive compared to foreign varieties. This led to significant rises in exports in these sectors, and more importantly, to a shift in domestic consumption from foreign to local varieties. As a result, the output, relative wages, and labor increases in these sectors, reflecting a reverse Dutch disease phenomenon. In terms of welfare, the shock led to a 4.2% decline in aggregate real consumption as a result of a decrease in cost of living and an even greater decline in net income. That impact was substantially different across workers. It was also progressive, ranging from -3.01% for the poorest group (non-skilled workers in urban areas) to -7.83% for

⁸See Chapter 2.

⁹I use the country-level consumer price index as an approximation of the GDP deflator.

the group of highly skilled workers in rural areas. Finally, I find that not accounting for non-homotheticity still leads to decreasing and regressive welfare changes but to a higher extent (100% higher on average).

Structure of the chapter. The rest of the chapter is structured as follows. Section 2 presents a short literature review. Section 3 provides some motivating facts. Section 4 presents the theoretical model. Section 5 discusses the quantitative results. Section 6 concludes.

1.2 Related Literature

This chapter contributes to different strands of literature.

First, on the theoretical side, this chapter contributes to the literature on Roy labor allocation models (Burststein et al., 2015; Lagakos and Waugh, 2013). Closer to my framework is Galle et al. (2016), who use a similar setting to study the distributional impact of trade on workers from different regions. I extend their model by generalizing their preferences setting in order to account for non-homotheticity. Concretely, I use the generalized non-homothetic CES preferences recently introduced by Comin et al. (2018). This chapter is the first to link non-homotheticity in preferences to a Roy-type labor allocation framework.

Second, as previously stated, most of the existing methodologies measuring aggregate and individual welfare changes and their distribution focus exclusively on the income channel (Helpman et al., 2012; Galle et al., 2016), overlooking the cost of living channel. A key reason for such a lack of focus on this latter channel is the tractability challenges that arise from the use of non-homothetic preferences¹⁰. In this chapter, I develop a framework that allows for within- and across-groups heterogeneity in income and cost of living effects. This thesis is the first to use the recently introduced generalized non-homothetic CES to perform such exercises. In doing so, I provide new evidence of the distributional welfare effects of external shocks through both the income and cost of living channels; I thereby contribute to the emerging literature on the joint analysis of these channels. But I also contribute to the literature linking cost of living channel and non-homotheticity in preferences. To the

¹⁰The use of non-homothetic preferences is not a necessary, but a sufficient condition in order to allow for heterogeneous changes in the cost of living.

best of my knowledge, there are only four papers describing such combined analyses. Porto (2006) uses a set of reduced-form estimations to compute trade-induced welfare changes through a set of price and wage elasticities; however, his approach is purely empirical and does not allow for flexible policy counterfactuals. He and Zhang (2017) extend a cost of living channel-based structural approach built on the non-homothetic almost-ideal demand systems (Fajgelbaum and Khandelwal, 2016) to the income channel. Finally, Borusyak and Jaravel (2018) use a non-homothetic nested CES preferences to evaluate through a structural framework the distributional effects of trade on education groups in the United States. I take a similar approach by developing a structural general equilibrium model based on the generalized non-homothetic CES, which I solve using readily available statistics obtained from micro-survey and macroeconomic data and calibrated parameters.

Third, this chapter lies in the existing literature on welfare variations at the household and individual levels that explores a partial equilibrium approach (Goldberg and Pavcnik, 2005; Edmonds and Pavcnik, 2005; Attanasio et al., 2004; Friedman and Levinsohn, 2002) or a general equilibrium approach (Porto, 2006; Fajgelbaum and Khandelwal, 2016). Furthermore, I provide a simple computational technique to compute post-shock changes in income and price indices for the class of quasi-separable utility, which includes the non-homothetic CES preferences. The empirical results of this method can be compared to those derived from a newly developed econometric estimation technique introduced by Atkin et al. (2018) and which is applicable to the aforementioned class of preferences.

Fourth, this chapter contributes to the literature on the distributional impact of oil shocks in oil-producing countries (Nkang, 2018; Bussolo et al., 2017; Cantore et al., 2012; Essama-Nssah et al., 2008). Closer to my framework is Nkang (2018), who uses a general equilibrium model to show that the 2016 oil shock led in Nigeria to an increase in the gross domestic output and supply of composites in the agricultural sector and caused agriculture prices to decline; This finding is corroborated in the present chapter. He concludes that lower international oil prices may boost output in other sectors, exhibiting as in the present chapter a “*reverse Dutch disease*” phenomenon (Essama-Nssah et al., 2008). Finally, unlike Nkang’s work, this chapter not only models the oil subsidy policy but also provides specific mitigating interventions against the negative welfare impact on households that may generate strong

progressive welfare implications.

Finally, the results and methodology presented in the chapter have three useful implications for public policy. First, the results show that the presence of oil subsidies not only induce inefficiency but also induce further welfare losses in the wake of oil crashes. Second, accounting for the cost of living channel under non-homothetic preferences is important for a more precise measure of the welfare changes deriving from the shock. Third, my methodology provides any policymaker with a simple and novel path to quantify and evaluate policy interventions while targeting all or only some groups within the population.

1.3 Motivating Facts

1.3.1 Data and Notations

This chapter uses data on expenditures, income, and employment from the Living Standards Measurement Study (LSMS). The LSMS combines an individual-level labor market survey with a household-level consumer expenditure survey, collected prior to the oil shock, i.e. in 2015 (pre-shock period). I link individuals (head of household) employment characteristics to their corresponding household-level expenditure. In particular, I observe for each head of household (unit of observation) indexed by $h \in \mathcal{L} = \{1, \dots, L\}$: (i) the total expenditure level denoted by e_h ¹¹; (ii) the sectoral expenditures $e_{h,k}$ for each sectoral good indexed by k ; (iii) the sectoral expenditure shares denoted by $\Omega_{h,k}$ ¹² for each sectoral good indexed by k ; (iv) the location (*rural areas* vs. *urban areas*); (v) the level of education, which I use to construct a skill-level variable (*non-skilled* vs. *low-skilled* vs. *high-skilled*)¹³; (vi) and finally, the sector of employment indexed by i . I distinguish four final goods and four sectors of employment, respectively indexed by k and i and regrouped in the sets \mathcal{F} and \mathcal{L} :

$$k \in \mathcal{F} \equiv \{\text{refined oil, agriculture, manufacturing, service}\}$$

$$i \in \mathcal{L} \equiv \{\text{crude oil, agriculture, manufacturing, service}\}$$

Therefore, each head of household is a *consumer-worker* agent who works in a particular sector and in a particular location; earns a labor income y_h ; pays an income-based tax t_h ; and allocates the after-tax income (i.e. expenditure or net income)

¹¹The total expenditure is equivalent to the net income, i.e. the after-tax income. I only observe the expenditure levels. Therefore, I determine the implied corresponding values of each consumer's tax t_h and income y_h by using the Nigerian tax policy (income-based and proportional), which distinguishes among six income brackets indexed by r . The corresponding income brackets and tax rates are as follows: $\tau_1 = 7\%$ for $y_h \leq \text{N}300,000$ ($br = 1$); $\tau_2 = 11\%$ for $\text{N}300,000 < y_h \leq \text{N}600,000$ ($br = 2$); $\tau_3 = 15\%$ for $\text{N}600,000 < y_h \leq \text{N}1,100,000$ ($br = 3$); $\tau_4 = 19\%$ for $\text{N}1,100,000 < y_h \leq \text{N}1,600,000$ ($br = 4$); $\tau_5 = 21\%$ for $\text{N}1,600,000 < y_h \leq \text{N}3,200,000$ ($br = 5$); and $\tau_6 = 24\%$ for $y_h > \text{N}3,200,000$ ($br = 6$). Hence, the expression of the tax of any consumer h is given by $t_h = \tau_r y_h \mathbb{1}_{h \in r}$. Given the observed expenditure level e_h , the income level can be derived for consumer h as $y_h = \frac{e_h}{1-\tau_r} \mathbb{1}_{h \in r}$.

¹²I use $\Omega_{h,k} = \frac{e_{h,k}}{e_h} = \frac{e_{h,k}}{\sum_k e_{h,k}}$.

¹³The non-skilled level corresponds to no or primary school; the low-skilled level corresponds to secondary school; and the high-skilled level corresponds to a college degree.

$e_h = y_h - t_h$ to the consumption of a bundle of S sectoral consumption goods. The heterogeneity in income level derives from the following comparative advantage-inducing characteristics of workers: skill levels and work locations. Using these two *productivity* factors, I construct six heterogeneous groups of workers indexed by $g \in \mathcal{G}$ and defined by

$$g \in \mathcal{G} \equiv \{non-skilled, low\ skill, high\ skill\} \times \{urban, rural\}$$

Throughout the chapter, I exploit this labor productivity heterogeneity and focus the analysis on characterizing the behavior of group- g and subgroup- (g, i) representative consumers-workers. Furthermore, within each group, I closely analyze the allocation of workers across all sectors of employment.

1.3.2 Pre-Shock Descriptives

Observation 1. *on the heterogeneity in labor productivity*

Table 1.1 gives the distribution of labor across skill levels and locations.

Table 1.1: *Labor composition by skill and location (in %)*

Groups	Urban	Rural	All
Non skilled	29.15 34.08	70.85 53.32	100.00 45.79
Low skilled	44.55 38.06	55.45 30.51	100.00 33.47
High skilled	52.58 27.85	47.42 16.17	100.00 20.74
All	39.20 100.00	60.80 100.00	100.00 100.00

Note. *This table presents the distribution of each skill type across locations in bolded text by row and the distribution of each location type across skills in non-bolded text by column. Source: LSMS*

The vast majority of the population is non-skilled or low-skilled and lives in rural

areas (*table 1.1*). Rural workers account for 60.8% of the entire population. Non-skilled workers account for 45.79%, followed by low-skilled workers who account for 33.47%). Highly skilled workers only account for 20.74%.

At group-level (*table 1.2*), this translates into the group of non-skilled workers living in rural areas making up the biggest share of the active population (32.4%), followed by the group of low-skilled workers living in rural areas (18.6%). The group of highly skilled workers living in rural areas has the lowest labor share (9.8% of the entire workforce).

Observation 2. *on the within-group (skill-location) sectoral allocation of labor*

In 2015 (i.e, before the oil shock), most workers were employed in the agriculture and service sectors (*table 1.2*). Indeed, the majority of workers is active in the agricultural sector (62.16%), followed by the service (21.76%) and manufacturing (16.79%) sectors. Less than 2% work in the crude oil sector.

I also compute for each group g the share π_{gs} of labor that is allocated to each sector of employment (for all i). Most agricultural workers live in rural areas. Furthermore, highly skilled workers are most likely to work in the crude oil and service sectors, while the least skilled workers are employed primarily in the manufacturing sector and even more so in the agriculture sector.

Table 1.2: *Within-group allocation of labor across sectors (π_{gi} , in %)*

Groups	Sectors of employment				Initial labor size by group (%)
	Crude oil	Agriculture	Manufacturing	Service	
<i>Urban, non skilled</i>	0.73	63.87	26.64	8.76	13.35
<i>Urban, low skilled</i>	1.63	45.10	33.01	20.26	14.91
<i>Urban, high skilled</i>	4.02	20.98	15.18	59.82	10.91
<i>Rural, non skilled</i>	0.15	88.59	5.71	5.56	32.44
<i>Rural, low skilled</i>	0.52	68.24	14.17	17.06	18.56
<i>Rural, high skilled</i>	2.48	32.67	2.97	61.88	9.84
<i>Labor size by sector (%)</i>	1.17	62.16	14.91	21.76	100.00

Source: LSMS

Observation 3. *on the heterogeneity in income across-groups*

In *table 1.3*, I compute the group-level expenditure averages. The average expenditure levels are relatively higher in urban areas compared to rural areas. However, average expenditure increases with skill level regardless of location. Skilled workers living in urban areas have the highest average expenditure, while non-skilled workers living in rural areas have the smallest average expenditure.

Table 1.3: *Average income across skills and locations*

<i>Groups</i>	<i>Labor shares (%)</i>	<i>Average income in Naira</i>	<i>Average income in USD</i>
<i>Urban, non skilled</i>	<i>16.30</i>	<i>574 126</i>	<i>2 577</i>
<i>Urban, low skilled</i>	<i>19.60</i>	<i>647 210</i>	<i>2 905</i>
<i>Urban, high skilled</i>	<i>12.91</i>	<i>1 213 869</i>	<i>5 449</i>
<i>Rural, non skilled</i>	<i>28.59</i>	<i>348 352</i>	<i>1 564</i>
<i>Rural, low skilled</i>	<i>14.77</i>	<i>480 408</i>	<i>2 156</i>
<i>Rural, high skilled</i>	<i>7.83</i>	<i>760 904</i>	<i>3 415</i>
<i>All groups</i>	<i>100.00</i>	<i>570 367</i>	<i>2 615</i>

Note. *This table presents the labor shares $\frac{L_g}{L}$ and the average income y_g for each group g . L_g and L are the group's labor size and the total labor size, respectively.*

Source: LSMS

Observation 1.4. *heterogeneity in expenditure behavior*

In order to investigate possible evidence of non-homotheticity in the preferences, I build polynomial representations of the Engel curves for each sectoral good (see *figure a1 to figure a8 in Appendix A.1*). These figures show that the income levels are significant in explaining the levels of expenditure shares on sectoral goods, and that these income-expenditure relationships are good-specific. This specificity suggests heterogeneity in sectoral expenditure behavior for workers across the income distribution. In general, richer workers generally have relatively higher expenditure shares in refined oil and service goods, while poorer workers have relatively higher expenditure shares in manufacturing and agricultural goods: this strongly hints to

the presence of non-homothetic preferences in the data.

Accounting for non-homotheticity in preferences, I can therefore construct sectoral expenditure shares for any income group-representative consumer. In particular, using the LSMS micro-data, I compute in *table a1* of *Appendix A.1* the sectoral expenditure shares of each subgroup (g, i) -representative consumer using

$$\Omega_{gi,k} = \sum_h \left(\frac{e_h^{emp.}}{\sum_{h'} e_{h'}^{emp.}} \right) \Omega_{h,k}^{emp.} \mathbb{1}_{h \in g, h \in i} \quad (1.3.1)$$

where $e_h^{emp.}$ and $\Omega_{h,k}^{emp.}$, respectively, are the empirical expenditure level of h and the empirical expenditure share of h in the sectoral good k . The heterogeneity in sectoral expenditures across all subgroups can be seen in *graph a1* of *Appendix A.1*. For each sectoral good, the graph shows the degree of heterogeneity across all 24 subgroups in the expenditure shares. For instance, there is more heterogeneity in service expenditures compared to refined oil expenditures. Furthermore, expenditures in refined oil are generally increasing as the income level increases, which makes refined oil a luxury good.

Conclusion. Since each group is characterized by different income levels, different expenditure shares, different contribution shares to overall tax revenue, and different degrees of concentration in the employment sectors¹⁴, accounting for income heterogeneity across and within these groups (i.e. across all subgroups) provides a simple path to capture most of the income heterogeneity while quantifying the unequal welfare gains or losses.

¹⁴Consequently, they are also differentially opened to trade.

1.4 Model

Production and trade. I distinguish five goods indexed by k : four traded goods (refined oil - oi_r , crude oil - oi_c , agriculture - ag , manufacturing - ma) and a non-traded good (service - se). The traded goods are regrouped in the set $\mathcal{T} = \{oi_r, oi_c, ag, ma\}$, and the non-traded good belongs to the single set $\mathcal{N} = \{se\}$.

For each traded good, I distinguish local and foreign varieties. The former is locally produced, whereas the latter is imported from the rest of the world.

Agricultural, manufacturing, service, and refined oil are the final goods regrouped into set \mathcal{F} . That is,

$$k \in \mathcal{F} \equiv \{oi_r, ag, ma, se\}$$

Moreover, part of the crude oil is exported, and the rest is used as input (intermediate) to produce refined oil locally. The locally produced refined oil is exclusively destined for local consumption whereas the crude oil is destined either for export or local intermediate consumption. In all but the refined oil sector k , the local variety is produced by constant returns to scale (CRS) firms operating under perfect competition, endowed with sector-specific and deterministic productivity Z_k , and using a Ricardian technology (labor) to produce. Labor is remunerated at a competitive per-efficiency unit rate w_k . In the refined oil sector, the local variety is produced by CRS firms operating under perfect competition, endowed with non-Ricardian technology, and using crude oil as an intermediate for production. That is,

$$\begin{cases} Q_k &= Z_k L_k^d \quad \forall \quad k \in \mathcal{T} \cup \mathcal{N} - \{oi_r\} \\ Q_{oi_r} &= \lambda I_{oi_c}^q \end{cases} \quad (1.4.1)$$

where L_k^d is the amount of labor used to produce good $k \in \mathcal{F}$, $I_{oi_c}^q$ is the quantity of crude oil used as an intermediate in producing refined oil, and λ is the transformation rate of crude oil to refined oil such that $0 < \lambda < 1$ ¹⁵.

¹⁵ λ is approximated by the ratio of the international prices of refined and crude oil, i.e. $\frac{P_{oi_r}^*}{P_{oi_c}^*}$. This ratio determines how many barrels of crude oil are needed to produce a barrel of refined oil.

Note that in the crude oil sector, since the only local use is for refined oil production, the total quantity produced Q_{oi_c} is such that

$$Q_{oi_c} = X_{oi_c}^q + I_{oi_c}^q \quad (1.4.2)$$

where $X_{oi_c}^q$ is the amount exported of crude oil.

Prices. In each sector k , each unit of the local type (if any) is produced at a production price $p_k^{p,L}$, while each unit of the foreign variety (if any) is imported at a production¹⁶ price $p_k^{p,F} = p_k^*$ where p_k^* is an exogenous international price expressed in world currency¹⁷. Because of perfect competition, production and consumption prices are equal. However, in the refined oil sector, the consumption price is subsidized by the government at a discount rate β with $0 \leq \beta \leq 1$. Indeed, producers or suppliers sell each unit at a competitive price $p_{oi_r}^p$, whereas consumers purchase at a discounted price $p_{oi_r}^c$ with $p_{oi_r}^c = (1 - \beta)p_{oi_r}^p$.

Employment. I distinguish four sectors of employment indexed by i : crude oil, agriculture, manufacturing, and service¹⁸.

$$i \in \mathcal{L} \equiv \{ oi_c, ag, ma, se \}$$

Furthermore, there are L workers indexed by h , and each worker belongs to one of the previously defined groups indexed by $g \in \mathcal{G} = \{1, \dots, G\}$. In accordance with their comparative advantage defined by their particular group of belonging, a worker inelastically supplies their labor to a particular sector and earns a corresponding labor income. For instance, a worker h from group g (type- g worker, hereafter) deciding to work in sector i belongs to the subgroup (g, i) and will earn a gross income y_{hgi} .

Consumption. On the one hand, workers use their labor income to consume final goods, including both local and foreign varieties, across sectors. The local and foreign varieties of each final good are aggregated using an homogeneous CES

¹⁶Throughout the chapter, I use the superscript p for production prices and the superscript c for consumption prices.

¹⁷Both domestic and foreign prices are expressed in terms of world currency.

¹⁸This list regroups all goods produced using labor. Note that the refined oil sector is not included since the technology of production does not involve labor.

Armington aggregator, whereas the sectoral final goods are aggregated using a non-homothetic CES aggregator. On the other hand, the government collects revenue by taxing labor income at a rate $\tau_r \in (0, 1)$ where r represents the subgroup (since there is no income heterogeneity within subgroups). For instance, a worker belonging to the subgroup (g, i) pays tax $\tau_{gi}y_{gi}$ and spends net income $e_{gi} = (1 - \tau_{gi})y_{gi}$ on a bundle of sectoral goods, both local and foreign. The government then uses the collected taxes and foreign borrowing to finance public spending, including the refined oil subsidies.

1.4.1 Workers

1.4.1.1 Characteristics

Each type- g worker h is endowed with a number of deterministic and time-invariant efficiency units of labor z_{gi} in each sector of employment i . Hence, z_{gi} reflects the labor productivity of any type- g worker in sector i ¹⁹. All workers within the same sub-group (g, i) get the same z draw. Henceforth, there is income heterogeneity within-groups across sectors of employment, but not within-subgroups.

Further, each type- g worker draws a non pecuniary idiosyncratic taste shock ε_{hgi} for each sector i . ε_{hgi} is expressed in terms of utility and reflects the worker's heterogeneous preference vis-à-vis working in different sectors (Caliendo et al., 2018). I consider that each draw ε_{hgi} follows a Fréchet distribution with a scale parameter $a_{gi} > 0$ and a shape parameter $\theta > 1$. The shape parameter is indicative of the within-group degree of dispersion in the tastes across workers. For instance, a smaller θ reflects more dispersion across individuals (Lagakos and Waugh, 2012).

Since the taste draws are non pecuniary, any type- g worker h who works in sector i earns a wage given by $y_{gi} = w_i z_{gi}$ with w_i , the per-efficiency unit wage rate that is earned in the sector of employment i ²⁰. She then pays a proportion τ_{gi} ²¹ of her wage to the government as an income tax. Finally, her net income is given by $e_{gi} = (1 - \tau_{gi})y_{gi}$. Hence, her real income and real consumption are respectively

¹⁹Equivalently, z_{gi} reflects any type- g worker's comparative advantage in sector i .

²⁰Since the sectoral labor endowments are time invariant, any change in income levels would not come from changes in labor productivity, but rather from changes in the sectoral wages. This is a reasonable assumption within the scope of the short-run analysis conducted in this chapter.

²¹ τ_{gi} is a micro-based exogenous tax rate which is computed using the actual income and tax of all workers belonging to subgroup (g, i) in the micro-data.

given by $r_{gi} = \frac{y_{gi}}{P_{gi}}$ and $c_{gi} = \frac{e_{gi}}{P_{gi}}$, where P_{gi} is her non-homothetic CES derived price index, itself a function of the real consumption, c_{gi} .

1.4.1.2 Utility

Following Caliendo et al. (2018), I define worker h 's utility level in the sector i as $u_{hgi} = c_{gi}\varepsilon_{hgi}$ ²².

Consider a worker h belonging to subgroup (g, i) . She has a net income e_{gi} which she allocates over a bundle of sectoral final goods $\{c_{gi,k}\}_{k \in \mathcal{F}}$. This bundle yields a level of real consumption C_{gi} defined according to the non-homothetic CES preferences, which are characterized by the following implicit function (Comin et al., 2015):

$$\sum_{k=1}^4 (c_{gi}^{\epsilon_k})^{\frac{1}{\sigma}} (c_{gi,k})^{\frac{\sigma-1}{\sigma}} = 1 \quad (1.4.3)$$

$$\sum_k \mathbf{p}_k^c c_{gi,k} = e_{hgi} \quad (1.4.4)$$

\mathbf{p}_k^c is the consumption price of good k ; σ is the elasticity of substitution; and ϵ_k is a non-zero positive or negative income elasticity identifying each sectoral good k and driving the income elasticity of demand for k ²³.

A special case to these generalized non-homothetic CES preferences yields the standard (homothetic) CES aggregator, which assumes away non-homotheticity. To this end, I posit

$$\epsilon_k = 1 - \sigma, \quad \forall k \in \mathcal{F}$$

Furthermore, the magnitudes of the income elasticities are indicative of the char-

²²Note that c_{gi} is not worker- h specific. Indeed, since the wage is the same for any subgroup (g, i) worker, the price index is uniform across the subgroup. And so is the real consumption c_{gi} . Hence, each member of a subgroup (g, i) enjoy the same real consumption, although the subgroup is characterized by heterogeneous utility levels (coming from the non-pecuniary taste draws).

²³A number of parametric restrictions are required in order to ensure a well defined real consumption and the uniqueness of the optimal bundle. In particular, two restrictions are needed to ensure that the aggregator (or real consumption index) c_{gi} is globally monotonically increasing and quasi-concave (Comin et al., 2018): (i) $\sigma > 0$; (ii) $\sigma < 1$ (resp. > 1) for gross complements (resp. substitutes).

acteristics (types) of the final goods. Indeed, the income elasticities are relatively higher for goods that are relatively more consumed by high income consumers. That is,

$$\epsilon_{ag} \leq \epsilon_{ma} \leq \epsilon_{oi_r} \leq \epsilon_{se}$$

The objective of any type- g worker h is to maximize her level of utility. That is, $\max_i \{u_{hgi}\}$. To achieve her objective, she recursively solves a problem of optimal choice of a sector of employment i , and a sub-problem of optimal allocation of the net income implied by the employment choice across all sectoral goods.

1.4.1.3 Employment Choice Problem

In all but refined oil sector, representative firm endowed with sector-specific productivity Z_i uses labor to produce the sectoral final good. The aggregated production of the final good sector i is defined as $Q_i = Z_i L_i^d$. Therefore, the domestic firms' profit maximization under perfectly competitive labor and good markets implies that the marginal revenue product of labor in the sector equals the production unit price of the local variety in the sector. This pins down the optimal competitive wages as²⁴

$$w_i = Z_i p_i^{p,L}, \quad \forall i \in \mathcal{L} \quad (1.4.5)$$

Any type- g worker h chooses the sector of employment that maximizes her utility conditional on her labor endowment vector $\{z_{gi}\}_i$, her employment tastes vector $\{\varepsilon_{hgi}\}_i$, and given the equilibrium wages $\{w_i\}_i$. She endogenously chooses to work in a particular sector s if and only if her vector of idiosyncratic tastes $\{\varepsilon_{hgi}\}_i$ belongs to the set

$$\vartheta_s^g \equiv \left\{ \{\varepsilon_{hgi}\}_{h,i} : \frac{z_{gs} w_s}{P_{gs}} \varepsilon_{hgs} \geq \frac{z_{gi} w_i}{P_{gi}} \varepsilon_{hgi} \quad \forall i \right\}$$

Assuming independence of workers' taste draws across sectors, I define a joint probability distribution F_g^ε of all workers belonging to group g i.e. over $\{\varepsilon_{hgi}\}_{h,i}$ (Galle et al., 2018). Using the properties of the Fréchet distribution, I then derive the share of workers from group g who work in sector s (expressed in worker counts - percentages)

²⁴Note that the local wages are function of both demand and supply of labor through general equilibrium.

as $\pi_{gs} = \int_{\vartheta_s^g} dF_g^\varepsilon$. That is,

$$\pi_{gs} = \frac{a_{gs} w_s^\theta z_{gs}^\theta P_{gs}^{-\theta}}{\Phi_g^\theta} \quad \text{with} \quad (1.4.6)$$

$$\Phi_g^\theta = \sum_i a_{gi} w_i^\theta z_{gi}^\theta P_{gi}^{-\theta} \quad (1.4.7)$$

with $\sum_i \pi_{gi} = 1$ and $\pi_{gs} L_g \equiv L_{gs}$. Φ_g is a measure of the real return to labor (or real income) in group g , i.e. the return to the skill and location characterizing group g .

I also derive the total supply of labor (in terms of efficiency units) which is provided by all workers from subgroup (g, s) as $S_{gs} = L_g z_{gs} \int_{\vartheta_s^g} dF_g^\varepsilon$. That is,

$$S_{gs} = z_{gs} \pi_{gs} L_g \quad (1.4.8)$$

I finally derive the aggregate labor income in each subgroup (g, i) as $Y_{gi} = w_i S_{gi}$, i.e.

$$Y_{gi} = y_{gi} \pi_{gi} L_g \quad \text{with} \quad y_{gi} = w_i z_{gi} \quad (1.4.9)$$

I can also use the within-group labor allocation across sectors to obtain the average income or expenditure (net income) in each group g respectively as $y_g = \sum_i \pi_{gi} Y_{gi}$ and $e_g = \sum_i \pi_{gi} e_{gi}$.

1.4.1.4 Goods Allocation Sub-Problem

The type- g worker h who decides to work in the utility maximizing sector i , also simultaneously allocates (optimally) her implied net income across all sectoral goods. In other words, conditional on her receiving the net income $e_{gi} = (1 - \tau_{gi}) w_i z_{gi}$, and given the consumption prices $\{\mathbf{p}_k^c\}_{k \in \mathcal{F}}$ ²⁵, she chooses an optimal level of consumption in all final goods, domestic and foreign.

The consumer optimization problem consists of maximizing the real consumption level $c_{gi} = \frac{e_{gi}}{P_{gi}} = \frac{1}{P_{gi}} (1 - \tau_{gi}) w_i z_{gi}$ defined in equation (1.4.3) under the budget constraint given in equation (1.4.4). Given the consumption price vector $\{\mathbf{p}_k^c\}_{k \in \mathcal{F}}$, the expenditure shares in each sectoral good k for a worker in subgroup (g, i) - or

²⁵ \mathbf{p}_k^c is an index that combines local and foreign variety prices (see section 1.4.3).

equivalently of the representative worker in subgroup (g, i) - is given by

$$\Omega_{gi,k} = \left(\frac{\mathbf{p}_k^c}{\mathbf{P}_{gi}} \right)^{1-\sigma} (c_{gi})^{\epsilon_k + \sigma - 1} \quad (1.4.10)$$

$$\mathbf{P}_{gi}^{1-\sigma} = \sum_{k=1}^4 c_{gi}^{\epsilon_k + \sigma - 1} (\mathbf{p}_k^c)^{1-\sigma} \quad (1.4.11)$$

The price index of each (g, i) -representative consumer is such that $\sum_k \mathbf{p}_k^c c_{gi,k} = e_{gi} = \mathbf{P}_{gi} c_{gi}$. Further, her price index is endogenously determined by her real consumption level, which is itself a function of her income. Furthermore, the income elasticity of demand for each sectoral final good $k \in \mathcal{F}$ is not only good-specific as expected in a non-homothetic framework, but also consumer (subgroup-level) specific. This latter specificity sources from the fact that the income elasticity of demand is also an endogenous function of the consumer's sectoral expenditure shares. Indeed, I have

$$\psi_{gi,k} \equiv \frac{\partial \log c_{gi,k}}{\partial \log e_{gi}} = \sigma + (1 - \sigma) \frac{\epsilon_k}{\sum_l \Omega_{gi,l} \epsilon_l} \quad (1.4.12)$$

Finally, for any consumer in subgroup (g, i) , the ratio of the expenditure shares between goods k and l is given as in Galle et al. (2018) by

$$\frac{\Omega_{gi,k}}{\Omega_{gi,l}} = \left(\frac{\mathbf{p}_k^c}{\mathbf{p}_l^c} \right)^{1-\sigma} (c_{gi})^{\epsilon_k - \epsilon_l} \quad (1.4.13)$$

Regardless of the level of real consumption, the relative expenditure share in good k with respect to good l is increasing (resp. decreasing) in the relative price of good k with respect to good l when $\sigma < 1$ (resp. $\sigma > 1$). Furthermore, the relation between the real consumption and the relative expenditure shares depends on the typology of the goods involved, i.e. on the magnitude of the respective income elasticities. For example, for a pair of goods (k, l) such that $\epsilon_k - \epsilon_l < 0$, the relative expenditure share in good k is decreasing in the real consumption levels. That is, a worker belonging to a subgroup endowed with a relatively larger real consumption will consume relatively less of the good k in comparison to good l . Furthermore, the relative expenditure share in good k is decreasing in the magnitude of the gap between ϵ_k and ϵ_l .

1.4.2 Government

The government's resources come primarily from the labor income taxes collected from workers, and secondarily from an exogenous positive or negative net capital inflows from the rest of the world, which runs through an exogenous trade imbalance²⁶ Λ (Dekle et al., 2008) where $\Lambda \equiv M - X$. M and X respectively represents endogenous aggregate imports and exports.

The government's resources are used to finance total public spending denoted G , of which there are two types: (i) the endogenous refined oil subsidy transfers $G_{oi_r} \equiv \beta p_{oi_r}^{p,L} C_{oi_r}^L + \beta p_{oi_r}^{p,F} C_{oi_r}^F$ where $C_{oi_r}^L$ and $C_{oi_r}^F$ are respectively the aggregate (national) consumption in the local and foreign varieties of refined oil; and (ii) a fixed and time-invariant residual $G - G_{oi_r}$ allocated to 3 other sectors $k \in \{ag, ma, se\}$. Concretely, each of these sectors receives an exogenous amount G_k with $\sum_{k \neq oi_r} G_k = G - G_{oi_r}$.

The government's is a passive planner who balances its resource constraint, given an endogenous level of subsidy rate β . Hence, the government's resource constraint reflects its endogenous transfers of refined oil subsidies to consumers through the discount rate applied to the consumption price (of refined oil). It is described as

$$G_{oi_r} + \sum_{k \neq oi_r, oi_c} G_k = \sum_{g,i} \tau_{gi} \pi_{gi} L_g y_{gi} + \Lambda \quad (1.4.14)$$

The government's resources can be either directly affected by the exogenous change in the net capital inflow, or indirectly affected through the general equilibrium effects on tax revenues. The external borrowing, or equivalently foreign lending are exogenously given and are affected by changes in net capital inflows. As for the tax revenues, they can be on the one hand endogenously affected by the changes in the incomes $\{y_{gi}\}_{g,i}$ and in the within-group labor allocations across sectors $\{\pi_{gi}\}_{g,i}$, and on the other hand exogenously affected by the change in the taxation policy through the tax rates $\{\tau_{gi}\}_{g,i}$. Hence, because all other public sectoral expenditures $\{G_k\}_{k \neq oi_r, oi_c}$ are fixed, any exogenous change to the foreign borrowing or to the taxation policy, by changing the government's resources, will also directly translate into an endogenous change in

²⁶I assume no saving and no private borrowing from the rest of the world. Hence, the current account balance is equivalent to the trade imbalance and represents a net capital inflows. Therefore, if Nigeria is rationed in global capital markets, net capital inflows can be viewed as driven by exogenous factors.

the subsidy rate. This will ultimately change the level of expenditures allocated to the refined oil subsidies $G_{oi,r}$. The counterfactual exercises in section 6 will explore different scenarios of exogenous changes to the tax policy or foreign borrowing and study their impact on the economy, including in terms of welfare across subgroups of workers.

1.4.3 Trade and Prices

For each good k , all consumers in the country derive the same utility from homogeneous bundles of foreign and domestic varieties. In other words, they aggregate domestic and foreign varieties according to the same sub-utility function defined by an Armington aggregator²⁷. Hence, for any subgroup- (g, i) representative consumer and each traded sectoral good ($k \in \mathcal{T}$), the consumption levels of the local variety C_k^L and the foreign variety (imports) C_k^F can be aggregated into C_k . That gives,

$$C_{gi,k} = \left((\delta_k^L)^{\frac{1}{\rho}} (C_k^L)^{\frac{\rho-1}{\rho}} + (\delta_k^F)^{\frac{1}{\rho}} (C_k^F)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \quad (1.4.15)$$

where δ_k^L and δ_k^F are uniform and deterministic preference weights that Nigerian consumers attaches to respectively local and foreign varieties of good k with $\delta_k^L + \delta_k^F = 1$ for all k ²⁸; and $\rho > 1$ is the Armington elasticity, i.e. the elasticity of substitution between the local and foreign variety of any traded good $k \in \mathcal{T}$ ²⁹.

Given any amount of expenditure on goods k , each consumer maximizes the flow of utility by combining the domestic and foreign varieties, respectively valued at a consumption price $p_k^{c,L} = (1 - \beta \mathbb{1}_{k=oi}) p_k^{c,L}$ for all $k \in \mathcal{T} \cup \mathcal{N}$ and $p_k^{c,F} = (1 - \beta \mathbb{1}_{k=oi}) p_k^{c,F}$ for all $k \in \mathcal{T}$. The optimal expenditure share in the local and foreign varieties are

²⁷That is, the preferences across sectoral good are non-homothetic CES, but the utility within each sector across varieties is defined by the Armington aggregator.

²⁸In the non traded sectors $k \in \mathcal{N}$, $\delta_k^L = 1$ and $\delta_k^F = 0$. Note that crude oil is used either by local firms for refined oil production, or directly exported. Hence, there is no import of crude oil from the rest of the world. Therefore, I have $\delta_{oi_c}^F = 0$.

²⁹The domestic and foreign varieties of good $k \in \mathcal{T}$ are perfect substitutes when $\rho \rightarrow \infty$.

given by

$$\begin{cases} \Omega_k^L &= \delta_k^F \left(\frac{p_k^{c,L}}{p_k^c} \right)^{1-\rho} \\ \Omega_k^f &= \delta_k^F \left(\frac{p_k^{c,F}}{p_k^c} \right)^{1-\rho} = 1 - \Omega_k^L \end{cases} \quad (1.4.16)$$

where p_k^c is the consumption price in sector k ³⁰ obtained as a weighted average of the consumption prices of the local and foreign varieties of good k . That is³¹,

$$p_k^c = \left(\delta_k^L [p_k^{c,L}]^{1-\rho} + \delta_k^F [p_k^{c,F}]^{1-\rho} \right)^{\frac{1}{1-\rho}} \quad (1.4.17)$$

In each sector, the consumption price p_k^c can also be written in terms of the production price p_k^p as

$$\begin{cases} p_k^c &= (1 - \beta \mathbb{1}_{k=\{oi\}}) p_k^p \quad \text{with} \\ p_k^p &= \left(\delta_k^L [p_k^{p,L}]^{1-\rho} + \delta_k^F [p_k^{p,F}]^{1-\rho} \right)^{\frac{1}{1-\rho}} \end{cases} \quad (1.4.18)$$

The domestic firms' profit maximization under perfectly competitive labor and good markets implies that the marginal revenue product of labor equates the production price of the locally produced good in each sector k . This pins down the optimal competitive wages in both traded and non-traded sectors, previously defined in equation (1.4.5).

Also, the first order conditions of the Armington problem provide useful ratios of equilibrium prices between local and foreign varieties of each traded good $k \in \mathcal{T}$. That is,

$$\frac{p_k^{c,L}}{p_k^{c,F}} = \left(\frac{\delta_k^L}{\delta_k^F} \right)^{\frac{1}{\rho-1}} \left(\frac{1}{\Omega_k^L} - 1 \right)^{\frac{1}{\rho-1}} \quad (1.4.19)$$

³⁰Note that Ω_k^L inversely depend on the ratio of foreign and local price as $\frac{\partial(p_k^{c,L}/p_k^{c,F})}{\partial \Omega_k^L} < 0$.

³¹In each non-traded sectors $k \in \mathcal{N}$, $\Omega_k^L = 1$ as $\delta_k^L = 1$, $\delta_k^F = 0$ and subsequently, $p_k^c = p_k^{c,L}$.

1.4.4 General Equilibrium

The general equilibrium can be defined as a set of production and consumption prices, wages, consumer-specific price indices, and labor allocations such that (i) domestic firms take the competitive domestic prices $\{p_m^{p,L}\}_{k \in \mathcal{T} \cup \mathcal{N}}$ as given and optimize their production under perfectly competitive markets, (ii) workers on the one hand take the competitive wage rates $\{w_i\}_{i \in \mathcal{F}}$ as given and choose their sector of employment; and on the other hand take the final consumption prices $\{p_k^{p,c}\}_{k \in \mathcal{F}}$ and the tax rates $\{\tau_{gi}\}_{g,i}$ as given and choose their basket of consumption, (iii) labor markets clear, and (iv) the government chooses a level of oil price subsidy rate in order to balance its resource constraint.

1.4.4.1 Characterization

Condition 1. Labor market equilibrium. *There is zero excess demand of labor efficiency units in each sector. That is, $EDL_k \equiv D_k - S_k = 0$ in each sector $k \in \mathcal{T} \cup \mathcal{N}$.*

Good market clearing conditions. First, I derive the market clearing condition for each good k as

$$Y_k + \Lambda_k \mathbb{1}_{k \in \mathcal{T}} = A_k \quad (1.4.20)$$

where Y_k , Λ_k and A_k are respectively the national income, net import and absorption in sector k .

Sectoral income. For all but the refined oil sector, labor is the only factor of production and firms operate under perfect competition (zero profit). Hence, the value of the production corresponds to the wage bill and is equivalent to the aggregate labor income across all workers active in sector $k \neq oi_r$. Using equations (1.4.8) or (1.4.9), that gives

$$Y_k = \sum_g y_{gk} \pi_{gk} L_g, \quad \forall k \neq oi_r \quad (1.4.21)$$

In the refined oil sector, firms still operate under perfect competition, but use crude oil and non-Ricardian technology for production. Further, I assume that an exoge-

nous share χ^{32} of the crude oil extracted is locally use to produced refined oil in Nigeria. Therefore, the value of the production is simply given by

$$Y_{oi_r} = \lambda I_{oi_c}^{val} \quad (1.4.22)$$

with

$$I_{oi_c}^{val} = \chi \times \sum_g y_{goi_c} \pi_{goi_c} L_g \quad (1.4.23)$$

$$\lambda = \frac{P_{oi_r}^*}{P_{oi_c}^*} \quad (1.4.24)$$

where λ is an exogenous transformation rate from crude to refined oil, approximated by the ratio of international prices of refined and crude oil.

Sectoral net imports. Λ_k is the trade imbalance in sector $k \in \mathcal{T}$, i.e. the difference between the imports M_k and the exports X_k .

$$\Lambda_k = M_k - X_k \quad (1.4.25)$$

National absorption. A_k is the sum of all consumption in sector k . It is obtained by aggregating all local and foreign varieties consumed in the country as final or intermediate goods. That is,

$$A_k = Y_k^l + Y_k^f \quad \forall k \quad (1.4.26)$$

where $Y_k^l \equiv p_k^{c,L} C_k^L$ is the national spending on the local variety of good k and $Y_k^f \equiv p_k^{c,F} C_k^F = M_k$ is the national spending on the foreign variety (i.e. the imports) of good k . Noting that $Y_k - X_k = Y_k^l$ and $Y_k^f = M_k$, equation (1.4.26) can be

³²For the sake of simplicity, I assume a constant share of crude output going into refining rather than assuming a fixed refining capacity. This is a realistic assumption as the deficient refinery production in Nigeria is not due to capacity issues, but rather to dysfunction in the industry. Refineries are mostly owned by crude oil exporting multinational firms. These firms have shown a strict preference for exporting crude oil (and produce refined oil overseas) rather than locally transform it into refined oil. But in order to comply with the Nigerian government's stated (political) objective of developing a local refinery industry, they are required to commit some quota (share) of their crude oil extracted for local transformation. Further, they tend to under-invest in their own local refineries, which makes most of them inoperable most of the year, and justify why most of the crude oil produced has to be shipped outside of Nigeria.

rewritten as an equivalent of the market clearing condition, i.e.

$$Y_k + M_k \mathbb{1}_{k \in \mathcal{T}} = A_k + X_k \mathbb{1}_{k \in \mathcal{T}}, \quad \forall k$$

The national absorption can also be equivalently defined as the aggregate expenditure in the country coming from all consumers, firms³³ and the government. In the refined oil sector, the national absorption also incorporates the government's subsidy spending. That gives $A_{oi_r} = \sum_{g,i,k=\{oi_r\}} \Omega_{gi,k} L_g \pi_{gi} e_{gi} + E_{sub}$. Since $Y_k - X_k \equiv Y_k^L$, I have that $E_{sub} = \beta(Y_{oi_c}^L + Y_{oi_r}^F) = \beta A_{oi_r}$. Hence, I have $(1 - \beta)A_{oi_r} = \sum_{g,i,k=\{oi\}} \Omega_{gi,k} L_g \pi_{gi} e_{gi}$, which implies that $A_{oi} = \frac{1}{1-\beta} \sum_{g,i,k=\{oi\}} \Omega_{gi,k} L_g \pi_{gi} e_{gi}$ ³⁴. Further, in the crude oil sector, the national absorption only includes firms' intermediate consumption use for refined oil production. Finally, in all but the refined and crude oil sectors, the national absorption also include the previously defined fixed public sectoral spending. I have

$$A_k = E_k^{cons} \mathbb{1}_{k \neq \{oi_c\}} + G_k \mathbb{1}_{k \neq \{oi_r, oi_c\}} + I_k^{val} \mathbb{1}_{k=\{oi_c\}} \quad \forall k \quad (1.4.27)$$

with

$$E_k^{cons} = (1 - \beta \mathbb{1}_{k=\{oi\}})^{-1} \sum_{g,i} \Omega_{gi,k} L_g \pi_{gi} e_{gi} \mathbb{1}_{k \neq \{oi_c\}} \quad (1.4.28)$$

Now using the expression the sectoral national absorption derived in equation (1.4.27), and the Armington framework³⁵, I obtain an expression for the sectoral import in the refined oil, agricultural and manufacturing sectors as $M_k = \Omega_k^f A_k$, i.e

$$M_k = \delta_k^F \left(\frac{P_k^{c,F}}{P_k^c} \right)^{1-\rho} A_k \quad \forall k \in \mathcal{T} - \{oi_c\} \quad (1.4.29)$$

As for the exports, I model differently crude oil on one hand, and the other traded goods on the other hand. For the crude oil exports, I use the identity (1.4.2) and

³³As intermediate goods in the refined oil sector.

³⁴Note that A_{oi} increases in β . Indeed, more subsidy encourages more imports and more domestic consumption of refined oil.

³⁵Via the homogeneous shares in foreign varieties derived in equation (1.4.16).

derive that

$$X_{oi_c} = \left(1 - \frac{p_{oi_r}^*}{p_{oi_c}^*}\right) \Gamma_{oi_c}^{val} \quad (1.4.30)$$

As for the exports of agricultural and manufacturing goods, I assume exogenous world price index p_k^W and foreign expenditures A^W , which is a reasonable assumption given the negligible size of the Nigerian economy relative to the world. Using the Armington framework, I then derive X_k in the agricultural and manufacturing sectors as a function of the domestic local price, and of an exogenous world price index p_k^W in sector k . I have

$$X_k = \left(\frac{p_k^{p,L}}{p_k^W}\right)^{1-\rho} A^W \quad \forall k \in \mathcal{T} - \{oi_r, oi_c\} \quad (1.4.31)$$

Labor Demand. With the good market clearing conditions now clearly defined, I now use the zero profit condition deriving from the perfect competition assumption. This latter assumption implies that the total sales for final consumption are entirely spent on the wage bill in each sector k that uses labor for production ($k \in \mathcal{L}$). The demand of effective units of labor in each sector k is therefore given by the ratio $\frac{Y_k}{w_k}$. That is,

$$D_k = \frac{1}{w_k} A_k + \frac{1}{w_k} X_k \mathbb{1}_{k \in \mathcal{T}} - \frac{1}{w_k} M_k \mathbb{1}_{k \in \mathcal{T}} \quad , \quad \forall k \in \mathcal{L} \quad (1.4.32)$$

Labor Supply. The total supply of efficiency units of labor to any sector $k \in \mathcal{L}$ can be derived by aggregating all subgroup-specific supply to sector k . Using equation (1.4.21), that gives

$$S_k = \frac{1}{w_k} \sum_g y_{gk} \pi_{gk} L_g \quad (1.4.33)$$

Equilibrium. Using equations (1.4.32) and (1.4.33), I define the zero excess demand of labor, i.e. $EDL_k = 0$ for each $k \in \mathcal{L}$ as

$$\frac{1}{w_k} \sum_g y_{gk} \pi_{gk} L_g + \frac{1}{w_k} M_k \mathbb{1}_{k \in \mathcal{T}} = \frac{1}{w_k} A_k + \frac{1}{w_k} X_k \mathbb{1}_{k \in \mathcal{T}} \quad (1.4.34)$$

In the refined oil sector where labor is not used, using equations (1.4.22) and (1.4.23),

the condition 1 is equivalent to the good market clearing condition, i.e.

$$\lambda \chi \sum_g y_{g oi_c} \pi_{g oi_c} L_g + M_{oi_c} = I_{oi_c}^{val} + X_{oi_c} \quad (1.4.35)$$

Condition 2. Equality between Nigeria's current account deficit and net capital inflows minus net international factor payments. *The sum of all sectoral net imports is equivalent to the net capital inflows minus net international factor payments*

There is no saving and no private borrowing from the rest of the world. Hence, the current account deficit is equivalent to the net capital inflows minus net international factor payments. Since foreign borrowing is exogenous, Λ is also exogenous. However, the sectoral components of the trade imbalance are endogenous. I have

$$\Lambda = \sum_{k \in \mathcal{T}} M_k - \sum_{k \in \mathcal{T}} X_k \quad (1.4.36)$$

Condition 3. Public budget constraint. *The government resource constraint holds*

Given fixed and exogenous public expenditures in the other sectors $\{G_k\}_{k \neq oi_r, oi_c}$, the government chooses an optimal level of subsidy that balances its resource constraint derived in equation (1.4.14). I have

$$\beta A_{oi} + \sum_{k \neq oi_r, oi_c} G_k = \sum_{g,i} \tau_{gi} \pi_{gi} L_g y_{gi} + \Lambda \quad (1.4.37)$$

1.4.4.2 Comparative Statics

The general equilibrium system of equations (1.4.34) – (1.4.37) can be expressed in terms of the wage vector $\{w_k\}_{k \in \mathcal{L}}$ and the subsidy rate β . I use the exact hat-algebra method introduced by Dekle et al. (2008) and solve the model in proportional changes. To do so, I rewrite the all objects in the system of equations (1.4.34) – (1.4.37) in terms of changes between the counterfactual (or ex-post) equilibrium and the pre-shock (or ex-ante) equilibrium. That is, I can rewrite any post-shock or variable x' such as $x' = x\hat{x}$ where x is the pre-shock value of the variable, and \hat{x} is the post-shock proportional change in the variable x .

Solution approach. Considering exogenous changes in the foreign prices of the traded goods $\hat{p}_{k \in \mathcal{T}}^*$, and in the net capital inflows $\hat{\Lambda}$, given macro and micro data on labor and income allocations, and given a set of calibrated parameters $\Theta \equiv \{\{\epsilon_k\}_{k \in \mathcal{F}}; \sigma; \rho; \theta\}$, I use the hat-algebra methodology to solve the general equilibrium system of equations (1.4.34) – (1.4.37). I then obtain a vector Υ of four endogenous primitives³⁶ on which all other endogenous variables in the model depend.

$$\Upsilon \equiv \left\{ \left\{ \hat{p}_k^{\text{p,L}} \right\}_{k \neq \{oi_r, oi_c\}} ; \hat{\beta} \right\} \quad (1.4.38)$$

Endogenous changes in prices. Using the exogenous international price shocks³⁷, I derive that

$$\hat{p}_k^{\text{p,F}} = \hat{p}_k^* , \quad \forall k \in \mathcal{T} \quad (1.4.39)$$

$$\hat{p}_k^{\text{c,F}} = \frac{1 - \hat{\beta} \mathbb{1}_{k=\{oi_r\}}}{1 - \beta \mathbb{1}_{k=\{oi_r\}}} \times \hat{p}_k^{\text{p,F}} , \quad \forall k \in \mathcal{T} \quad (1.4.40)$$

$$\hat{p}_k^{\text{p,L}} = \hat{p}_k^{\text{p,F}} , \quad \forall k = \{oi_r, oi_c\} \quad (1.4.41)$$

$$\hat{p}_k^{\text{c,L}} = \frac{1 - \hat{\beta} \mathbb{1}_{k=\{oi_r\}}}{1 - \beta \mathbb{1}_{k=\{oi_r\}}} \times \hat{p}_k^{\text{p,L}} , \quad \forall k \in \mathcal{T} \cup \mathcal{N} \quad (1.4.42)$$

$$\hat{p}_k^{\text{c}} = \left[\Omega_k^l (\hat{p}_k^{\text{c,L}})^{1-\rho} + (1 - \Omega_k^l) (\hat{p}_k^{\text{c,F}})^{1-\rho} \right]^{\frac{1}{1-\rho}} , \quad \forall k \in \mathcal{T} \cup \mathcal{N} \quad (1.4.43)$$

Equation (1.4.39) expresses the post-shock change in the production price of the foreign variety of each traded sectoral good. Equation (1.4.40) derives from equation (1.4.39) and expresses the post-shock change in the consumption price of the foreign variety. Equation (1.4.41) derives from equation (1.4.5) and expresses the post-shock change in the production price of the local variety of each good. Equation (1.4.42) derives from equation (1.4.41) and expresses the post-shock change in the consumption price of the local variety. Equation (1.4.43) derives from equation (1.4.18) and expresses the post-shock change in the consumption price (index) of each sectoral good k .

³⁶I approximate the sectoral local production prices for refined and crude oil by their international counterparts.

³⁷Given by $\{\hat{p}_k^*\}_{k \in \mathcal{T}}$.

Core equations. The post-shock general equilibrium can be formalized by a system of equations expressed in terms of (Υ, Θ) . This system reflects the post-shock zero excess demand of labor conditions deriving from equations (1.4.34) and (1.4.35), the post-shock current account balance equation deriving from equation (1.4.36), and the post-shock government's budget constraint deriving from equation (1.4.37). That gives respectively,

$$\hat{Y}_k Y_k + \hat{M}_k M_k \mathbb{1}_{k \in \mathcal{T}} = \hat{A}_k A_k + \hat{X}_k X_k, \quad \forall k \in \mathcal{T} \cup \mathcal{N} \quad (1.4.44)$$

$$\hat{\Lambda} \Lambda = \sum_{k \in \mathcal{T}} \hat{M}_k M_k - \sum_{k \in \mathcal{T}} \hat{X}_k X_k \quad (1.4.45)$$

$$\hat{T} T + \hat{\Lambda} \Lambda = \hat{G} G \quad (1.4.46)$$

with

$$\hat{Y}_k Y_k = \sum_g Y_{gk} \hat{Y}_{gk} \hat{\pi}_{gk}, \quad \forall k \in \mathcal{L} \quad (1.4.47)$$

$$\hat{Y}_{oi_r} Y_{oi_r} = \lambda \chi \sum_g Y_{goi_c} \hat{Y}_{goi_c} \hat{\pi}_{goi_c} \quad (1.4.48)$$

$$\hat{A}_k A_k = \hat{E}_k^{cons} E_k^{cons} \mathbb{1}_{k \in \mathcal{F}} + \hat{G}_k G_k \mathbb{1}_{k \neq \{oi_r, oi_c\}} + \hat{I}_k^{val} I_k^{val} \mathbb{1}_{k = \{oi_c\}} \quad (1.4.49)$$

$$\hat{E}_k^{cons} E_k^{cons} = \frac{1}{1 - \hat{\beta} \beta \mathbb{1}_{k = \{oi\}}} \sum_{g,i} \Omega_{gi,k} Y_{gi} (1 - \tau_{gi}) \hat{\Omega}_{gi,k} \hat{\pi}_{gi} \hat{e}_{gi}, \quad k \in \mathcal{F} \quad (1.4.50)$$

$$\hat{I}_{oi_c}^{val} I_{oi_c}^{val} = \chi \times \sum_g Y_{goi_c} \hat{Y}_{goi_c} \hat{\pi}_{goi_c} \quad (1.4.51)$$

$$\hat{M}_k M_k = \Omega_k^f \left(\frac{\hat{p}_k^{c,F}}{\hat{c}_k^p} \right)^{1-\rho} \hat{A}_k A_k, \quad \forall k \in \mathcal{T} \quad (1.4.52)$$

$$\hat{X}_k X_k = X_k \left(\left(\frac{\hat{p}_k^{p,L}}{\hat{p}_k^w} \right)^{1-\rho} \hat{A}^w \right), \quad \forall k \in \mathcal{T} - \{oi_c\} \quad (1.4.53)$$

$$\hat{X}_{oi_c} X_{oi_c} = \left(1 - \lambda \frac{p_{oi_r}^*}{p_{oi_c}^*} \right) \hat{I}_{oi_c}^{val} I_{oi_c}^{val}, \quad \forall k = \{oi_c\} \quad (1.4.54)$$

$$\hat{\mathbf{T}}\mathbf{T} = \sum_{g,i} \tau_{gi} Y_{gi} \hat{\tau}_{gi} \hat{\pi}_{gi} \hat{Y}_{gi} \quad (1.4.55)$$

$$\hat{\mathbf{G}}\mathbf{G} = \hat{\beta} \beta \hat{\mathbf{A}}_{oi} \mathbf{A}_{oi} + \mathbf{G}_k \mathbb{1}_{k \neq \{oi\}} \quad (1.4.56)$$

and

$$\hat{\mathbf{P}}_{gi}^{1-\sigma} = \sum_k \Omega_{gi,k} (\hat{Y}_{gi} \hat{\mathbf{P}}_{gi}^{-1})^{\epsilon_k + \sigma - 1} \left(\frac{1 - \hat{\tau}_{gi} \tau_{gi}}{1 - \tau_{gi}} \right)^{\epsilon_k + \sigma - 1} (\hat{\mathbf{p}}_k^c)^{1-\sigma} \quad (1.4.57)$$

$$\hat{\Phi}_g^\theta = \sum_i \pi_{gi} \hat{\mathbf{a}}_{gi} \hat{w}_i^\theta \hat{z}_{gi}^\theta \hat{\mathbf{P}}_{gi}^{-\theta} \quad (1.4.58)$$

$$\hat{\pi}_{gi} = \frac{\hat{\mathbf{a}}_{gi} \hat{w}_i^\theta \hat{z}_{gi}^\theta \hat{\mathbf{P}}_{gi}^{-\theta}}{\hat{\Phi}_g^\theta} \quad (1.4.59)$$

$$\hat{Y}_{gi} = \hat{w}_i \hat{z}_{gi} = \hat{\Phi}_g \hat{\mathbf{P}}_{gi} \hat{\pi}_{gi}^{\frac{1}{\theta}} \quad (1.4.60)$$

$$\hat{Y}_g = \sum_i \frac{Y_{gi}}{Y_g} \hat{\pi}_{gi} \hat{Y}_{gi} \quad (1.4.61)$$

$$\hat{Y} = \sum_g \frac{Y_g}{Y} \hat{Y}_g \quad (1.4.62)$$

$$\hat{Y}_{gi} = \hat{Y}_{gi} \hat{\pi}_{gi} \quad (1.4.63)$$

$$\hat{Y}_g = \sum_i \frac{Y_{gi}}{Y_g} \hat{Y}_{gi} \quad (1.4.64)$$

$$\hat{\mathbf{e}}_{gi} = \frac{1 - \hat{\tau}_{gi} \tau_{gi}}{1 - \tau_{gi}} \hat{Y}_{gi} \quad (1.4.65)$$

$$\hat{\mathbf{e}}_g = \sum_i \frac{Y_{gi}}{Y_g} \hat{\pi}_{gi} \hat{\mathbf{e}}_{gi} \quad (1.4.66)$$

$$\hat{\mathbf{e}} = \sum_g \frac{Y_g}{Y} \hat{\mathbf{e}}_g \quad (1.4.67)$$

$$\hat{\Omega}_{gi,k} = (\hat{\mathbf{p}}_k^c)^{1-\sigma} \hat{\mathbf{P}}_{gi}^{-\epsilon_k} \hat{\mathbf{e}}_{gi}^{\epsilon_k + \sigma - 1}, \quad \forall k \in \mathcal{F} \quad (1.4.68)$$

$$\hat{\Omega}_k^l = \hat{\delta}_k^L \left(\frac{\hat{p}_k^{p,L}}{\hat{p}_k^p} \right)^{1-\rho}, \quad \forall k \in \mathcal{T} \quad (1.4.69)$$

With the exact hat-algebra approach, I can classify all objects of the general equilibrium system of equations into four categories. First, the *fundamental parameters*: elasticity of labor supply θ , Armington elasticity ρ , elasticity of substitution σ , and income elasticities $\{\epsilon_k\}_{k \in \mathcal{F}}$. Second, the *other time-invariant exogenous parameters*: scale parameter of the taste draws $\{a_{gi}\}_{g,i}$, Armington-based preference weights δ_k^L and δ_k^F , and changes in the labor endowment parameters $\{z_{gi}\}_{g,i}$ ³⁸. Third, the *observed data*: subgroup-level tax rates $\{\tau_{gi}\}_{g,i}$, oil subsidy rate β , labor allocations within group and across sectors of employment $\{\pi_{gi}\}_{g,i}$, $\{L_g\}_g$, and subgroup-level aggregate income $\{Y_{gi}\}_{g,i}$. Finally, the *endogenous variables* such as the changes in the worker-specific price indices \hat{P}_{gs} , group-level real return to labor $\hat{\Phi}_g$, subgroup-level expenditure shares $\hat{\Omega}_{gs,k}$, etc.

Henceforth, assuming the pre-shock data as a proxy for the pre-shock equilibrium, I can identify the impact of the oil shock on all endogenous variables, by solving the general equilibrium system of equations for some choice of the parameters Θ ³⁹. In doing so, I obtain the values of the primitives Υ , and ultimately obtain the predictions of the model for the post-shock or post-counterfactual changes in sectoral consumption and production prices as defined in equations (1.4.39) – (1.4.43), and for other objects of interest as the price indices, labor allocations, incomes and expenditures shares, etc. as defined in equations (1.4.44) – (1.4.56) on the one hand, and equations (1.4.57) – (1.4.69) on the other hand. In solving the model, I use four main internal validation criterion. First, the sum of the post-shock expenditure shares for each subgroup representative consumer must sum up to 1 across goods. This reflects the fact that each consumer uses the totality of her post-shock net income for consumption; second, the post-shock within-group sectoral labor reallocations are such that the post-shock labor shares across sectors sum up to 1 within each group; third, the first order condition deriving from the optimal mix of foreign and local varieties as derived in equations (1.4.16) is respected in the post-shock equilibrium; fourth, the post-shock subsidy rate is bounded by 0 and 1. In summary, I have

³⁸That is, $\hat{\delta}_k^L = \hat{\delta}_k^F = 1$ for all k ; $\hat{a}_{gi} = \hat{z}_{gi} = 1$ for all g, i .

³⁹A summary of all objects in the system of equations is in *table a2* (*Appendix A.3*)

$$\begin{aligned}
\sum_k \hat{\Omega}_{gi,k} \Omega_{gi,k} &= 1 \quad , \quad \forall g \quad \text{and} \quad \forall i \\
\sum_i \hat{\pi}_{gi} \pi_{gi} &= 1 \quad , \quad \forall g \\
\left(\frac{1 - \hat{\Omega}_k^L \Omega_k^L}{\hat{\Omega}_k^L - \hat{\Omega}_k^L \Omega_k^L} \right)^{\frac{1}{\rho-1}} &= \frac{\hat{P}_k^{p,L}}{\hat{P}_k^{p,F}} \quad , \quad \forall k \in \mathcal{T} \\
0 &\leq \hat{\beta} < \frac{1}{\beta}
\end{aligned}$$

1.4.5 Welfare Changes Metric

I consider a money-metric welfare⁴⁰ defined by the real consumption, i.e. the ratio of the net income to the price index. Therefore, the post-shock change in the welfare is given for each subgroup representative worker by

$$\hat{\mathcal{W}}_{gi} = \underbrace{\hat{e}_{gi}}_{\text{net income effect}} \times \underbrace{\hat{P}_{gi}^{-1}}_{\text{price index effect}}$$

$\hat{\mathcal{W}}_{gi}$ is a measure of the total welfare change expressed as a combination of a net income effect and a price index effect. The net income effect is a combination of income and tax effects. The former reflects the growth rate in the income while the latter reflects the impact of the tax burden. As for the price index effect, it reflects an inflationary effects and measures the change in the cost of living. Henceforth, $\hat{\mathcal{W}}_{gi}$ captures both the income channel (*net income effect*) and the price or cost of living channel (*price index effect*) of the welfare changes. $\hat{\mathcal{W}}_{gi} > 1$ reflects a welfare gain whereas $\hat{\mathcal{W}}_{gi} < 1$ reflects a welfare loss. Using equations (1.4.57), (1.4.60) and (1.4.65), that gives

$$\hat{\mathcal{W}}_{gi} = \frac{1 - \hat{\tau}_{gi} \tau_{gi}}{1 - \tau_{gi}} \times \frac{\hat{w}_i \hat{z}_{gi}}{\hat{P}_{gi}} \quad (1.4.70)$$

⁴⁰As opposed to the utility metric which incorporate a non-metric component in the employment tastes.

The changes in the subgroup-level net income expressed by equation (1.4.60) are exclusively driven by the endogenous changes in the sectoral wages, i.e. by a wage structure effects. Indeed, the heterogeneity in labor productivity conferred by the skill-location differentiated comparative advantages does not play any active role in determining the changes in the income. This is because the labor endowment z_{gi} s are time-invariant in the short-run⁴¹. That is, $\hat{z}_{gi} = 1$ for all groups g and each sector of employment (i). Hence, the welfare distributional impacts coming from the income channel are driven by two potential factors: the wage structure effects $\{\hat{w}_i\}_i$, and the tax redistribution effects $\{\hat{\tau}_{g,i}\}_{g,i}$. The latter effects can potentially come from exogenous changes in the taxation rates (via tax policy reforms⁴²).

Second, the changes in the subgroup-level price indices expressed by equation (1.4.57) are functions not only of the endogenous net income changes, but also the ex-ante expenditure shares profile (through the income elasticities), which in itself captures the across-subgroups income heterogeneity. Therefore, the changes in the price indices accounts for the heterogeneity in the income distribution as reflected by the heterogeneity in the subgroup-level sectoral expenditure shares. Hence, the heterogeneity in the price indices reflects not only the sector-specific differences captured by the changes in the net income, but also for the differences in the expenditure shares driven by the skill-location differentiated comparative advantages. Therefore, the welfare distributional impacts coming from the cost of living channel are driven by the net income changes (i.e. the wage structure effects) and the tax redistribution effects, but also by the initial income heterogeneity patterns reflected in the differentiated ex-ante expenditure shares.

⁴¹This is the scope of analysis in this chapter.

⁴²The subgroup-level changes will then be exogenously determined using exogenous changes to the actual Nigerian tax policy (income bracket-specific tax rates) and the LSMS microdata - see equation (A.2.6).

1.5 Calibrating the 2016 Shocks

In this section, I solve the model and analyze the distribution of the welfare changes across skill levels, locations, and sectors of employment with respect to the actual 2016 oil shock (*baseline scenario*).

Solving the baseline scenario, I obtain predictions for all subsequent changes in prices, wages, incomes, expenditures, and real consumption levels. Furthermore, I perform counterfactual exercises that help me evaluate how an exogenous change in, say, the taxation rates $\{\hat{\tau}_{gi}\}_{g,i}$ might affect all or some of the aforementioned variables.

Using readily available micro-based (labor and income distribution, and taxation rates across subgroups) and macro-based data (trade imbalances) for the pre-shock equilibrium as well as a few parameters (*table 1.4*), including the elasticity of labor supply, the Armington elasticity, the elasticity of substitution, and the income elasticities, I am able to identify the impact of the oil shock or any exogenous tax or borrowing policy change on all endogenous variables. To do so, I use the exact hat-algebra methodology⁴³

Table 1.4: *Parameters calibration: symbol and value*

Parameters		
<i>Elasticity of substitution</i>	σ	0.43
<i>Armington elasticity</i>	ρ	6
<i>Fréchet shape parameter</i>	θ	6.99
<i>Income elasticities, oil</i>	ϵ_{oi}	1.2
<i>Income elasticities, agriculture</i>	ϵ_{ag}	0.05
<i>Income elasticities, manufacturing</i>	ϵ_{ma}	1
<i>Income elasticities, service</i>	ϵ_{se}	1.2

⁴³A summary of the shock and micro and macro data can be found in *table a2* in *Appendix A.3*.

I obtain the labor and income distribution across subgroups from the nationally representative 2015-2016 living standard measurement survey (LSMS); I obtain all other national macroeconomic data from the Nigerian Central Bank. The elasticity of substitution and the income elasticities are obtained from Comin et al. (2018)⁴⁴. Importantly, the relative values of the income elasticities reflect that refined oil, service, and to a lesser extent manufacturing goods are of the luxury type, and agricultural goods are of the necessity type. The Armington elasticity is obtained from Ruhl (2008). Finally, the labor supply elasticity is obtained from Morales (2018).

While these parameters are important, they already have been estimated in several papers across the literature and their estimation is not a key contribution of this chapter. Therefore, rather than estimating them, I use calibrated values and show how the resulting predictions derived from solving the model in the baseline scenario compare to the actual changes observed in the data between 2015 and 2016. In doing so, I am able to validate not only the machinery of the model, but also the values chosen for the parameters.

In the following subsections, I first present the results derived from solving the general equilibrium system of equations for the 2016 oil shock (namely, the *baseline scenario*). The general state of the economy is characterized by parameters ϵ_k for all k , θ , ρ and σ . Furthermore, before the *baseline scenario* occurred, each subgroup (g, i) -representative worker could be characterized in 2015 by an income level y_{gi} , an expenditure level e_{gi} , an average tax rate τ_{gi} , a price index P_{gi} and sectoral expenditure shares $\Omega_{gi,k}$ for all final goods k . The 2016 oil price shock affected the relative wages and prices, which respectively led to a reallocation of labor across sectors and changes in net incomes on the one hand, and changes in expenditure shares and price indices on the other hand. The changes in net incomes and price indices ultimately translated into changes in real consumption levels. Hence, when using real consumption as a metric for welfare, it appears that the welfare changes are driven on one hand by changes in net incomes (income channel) and on the other hand by changes in price indices i.e. a combination of changes in consumption prices,

⁴⁴Comin et al. (2008) consider three sectors: agriculture, manufacturing, and services. I approximate the income elasticity of refined oil by that of service, as both belong to the same class of luxury goods.

income, tax, and expenditure shares (cost of living channel)⁴⁵.

As an endogenous response to the shock, I obtain the adjusted oil subsidies rate as implied by the ex-post change in the government's resources. I also obtain the total welfare changes of all subgroup-representative consumers as a combination of net income effects and price index effects. Finally, I summarize these welfare changes at the group level, sector level, and country level by defining corresponding measures for a social welfare change metric, in addition to the directly computed welfare changes at the subgroup levels. Finally, I quantify in each exercise the role of non-homotheticity in the preferences. To do so, I simply assume away non-homotheticity by calibrating all income elasticities as

$$\epsilon_k = 1 - \sigma, \quad \forall k \in \mathcal{F}$$

1.5.1 Pre-Shock Macroeconomic Descriptives

Before presenting the results, the general macroeconomic picture of Nigeria in 2015 merits consideration.

1.5.1.1 Sectoral Activities

Table 1.5 gives an overview of Nigeria's GDP components in 2015 at the aggregate and sectoral levels. The agriculture and service sectors are the two main contributors to the GDP.

Crude oil exports represent 87% of the aggregate exports, whereas manufacturing imports represent a majority (67%) of aggregate imports. Furthermore, there is a trade deficit⁴⁶ equivalent to about 2% of the GDP in 2015. Both the agricultural and manufacturing sectors run trade deficits, while the oil sector has a trade surplus. The oil and manufacturing sectors are significantly more open to trade than the agricultural sector.

Furthermore, most refined oil consumed domestically is supplied by foreign firms. All crude oil used as an intermediary in locally producing refined oil is supplied

⁴⁵In the next chapter, I perform a series of government budget balancing tax counterfactuals in order to (i) assess whether removing the subsidy was the best policy option, and (ii) to explore further policies the government could take in order to improve the welfare outcomes of the poor.

⁴⁶The trade deficit is equivalent to the current account balance and matches the fiscal deficit.

domestically. Most of the agricultural goods consumed domestically are supplied by local firms (*table 1.6*). In the manufacturing sector, the mix of local and foreign varieties is more balanced.

Table 1.5: *Sectoral components of GDP (in %)*

<i>items</i>	<i>oi_r</i>	<i>oi_c</i>	<i>ag</i>	<i>ma</i>	<i>se</i>	(% of GDP)
GDP	0.67	11.94	23.11	13.57	50.71	100
<i>Consumer expenditures</i>	11.6	0	33.7	30.2	24.4	86
<i>National absorption</i>	14.0	1.4	20.1	18.2	46.7	102
<i>Total exports</i>	0	90.0	5.00	5.00	0	13
<i>Total imports</i>	18.0	0	15.0	67.0	0	15
<i>Openness to trade</i>	0.03	0.11	0.03	0.11	0	0

Source: *Central Bank*

Table 1.6: *Mix (share) of varieties in sectoral national absorption (in %)*

<i>items</i>	<i>oi_r</i>	<i>oi_c</i>	<i>ag</i>	<i>ma</i>	<i>se</i>
<i>From local production</i>	20.0	100	90.0	57.0	100
<i>From imports</i>	80.0	0	10.0	43.0	0.00

Source: *Central Bank*

1.5.1.2 Government Resources

The taxation policy in Nigeria is progressive. It can be summarized in *table 1.7*. The taxation rates range from 7% for the poorest workers to 24% for the richest workers, with a national average tax $\bar{\tau}$ rate of 14.15%. Furthermore, tax revenues

provide most (86%) of the resources that the government uses to finance all public expenditures, including the endogenous oil subsidies (*table 1.8*). In case of uniform tax rate changes, all income brackets are subject to the same percentage change. A progressive (res. regressive) tax reform can be obtain by redistributing, reducing, or increasing the taxation rate in a way that benefits (resp. hurts) or that is less (resp. more) penalizing to those in lower-income brackets.

Table 1.7: *Taxation policy profile: income brackets and tax rates*

<i>annual income in USD</i>	<i>tax rates (in %)</i>
<i>less than 1,345.6</i>	7.00
<i>from 1,345.6 to 2,693.1</i>	11.0
<i>from 2,693.1 to 4,937.4</i>	15.0
<i>from 4,937.4 to 7,181.7</i>	19.0
<i>from 7,181.7 to 14,363.4</i>	21.0
<i>more than 14,363.4</i>	24.0
<i>national average, $\bar{\tau}$</i>	14.15

Source: Nigerian IRS

Table 1.8: *Contribution of tax revenue and foreign borrowing to government's resources (in %)*

<i>Components</i>	<i>Share</i>	<i>in % of GDP</i>
<i>Tax revenue</i>	86.3	14.2
<i>Net capital inflows, Λ</i>	13.7	2.24

Source: *Federal Government*

1.5.2 Description of Scenario

The baseline scenario (*table 1.9*) is characterized by international price shocks in the traded sectors, including refined and crude oil. The oil shock resulted into a 40% decrease in the international price of both refined and crude oil: that is, $\hat{p}_{oi_r}^* = \hat{p}_{oi_c}^* = 0.6$ (40% decrease). Furthermore, using the small open economy assumption, I calibrate the changes in the international prices of the other traded goods. I obtain $\hat{p}_{ag}^* = 0.98$ (2% decrease) and $\hat{p}_{ma}^* = 0.99$ (1% decrease). The former is an approximation based on the World Food Index while the latter is obtained from the OECD⁴⁷. Finally, in order to identify the impact of the international price shocks $\{\hat{p}_k^*\}_{k \in \mathcal{T}}$ on the other macroeconomic variables, I use the exogenous changes in trade deficit, $\hat{\Lambda}$. I have $\hat{\Lambda} = 0.42$. Indeed, the trade deficit that represented 2.24% of the GDP in 2015 fell by 58% in 2016, reflecting reduced access to foreign borrowing.

Table 1.9: *Baseline scenario: calibration of shocks and other exogenous changes*

<i>International price shocks</i>		
<i>Refined oil</i>	$p_{oi_r}^*$	– 40%
<i>Crude oil</i>	$p_{oi_c}^*$	– 40%
<i>Agricultural good</i>	p_{ag}^*	– 2%
<i>Manufacturing good</i>	p_{ma}^*	– 1%
<i>Other shock</i>		
<i>Current account balance</i>	Λ	– 58%

Source: OECD

The main international price shocks occurred for oil products (-40% for both crude and refined oil). In the other traded sectors, the decline in the international prices is almost negligible. Furthermore, the trade deficit exogenously decreases by 58%, leading to a reduction in the current account balance and a reduction in the fiscal deficit.

⁴⁷I use the average production price indices for all OECD countries.

In summary, to solve the model, I consider Nigeria as a small open economy and calibrate all international price shocks observed in 2016. I also calibrate the rationing in foreign borrowings. Further, I use microdata from a consumer expenditure and labor survey to characterize the pre-shock equilibrium. I then use the shocks, the pre-shock data, and the calibrated parameters to solve the model for endogenous post-shock changes in all domestic sectoral consumption prices and wages. Ultimately, I obtain the endogenous changes in net income and consumer price index for all skill-location income groups.

1.5.3 Main Results

In the following, I present the main results associated with the baseline scenario. In order to evaluate the model, I use key statistics such as changes in the GDP growth rate and in the subsidy levels, and the sectoral labor reallocation patterns. I then use the predicted sectoral price and wage changes to evaluate the income effects and the price index effects across all subgroups, and ultimately across groups and at the national level.

The macroeconomic predictions of both the non-homothetic and homothetic models are fairly close to the actual changes observed in the data⁴⁸. The relatively small gap between the predictions of both models seems to indicate that they both perform well enough in terms of mirroring the macroeconomic changes that occurred in 2016. Furthermore, it indicates that the calibrated values used in *table* 1.4 for the parameters are reasonably close to the actual values. Both models predict changes in relative wages and prices that reflect a reverse Dutch disease, i.e. an expansion of output and employment in the agricultural and manufacturing sectors. Furthermore, sharp output decreases were observed in the crude oil and service sectors, leading to declining relative wages and employment losses in the sectors. As a result, significant differences appear when measuring the welfare changes across subgroups as they tend to specialize differently across sectors. Moreover, in general, not accounting for non-homotheticity leads to an overestimation⁴⁹ of welfare losses across subgroups. Two factors explain these differences: first, the differences in the net income changes predicted by the two models, and second, the differences in the changes to the cost of

⁴⁸However, the non-homothetic model's predictions are relatively closer to the data.

⁴⁹Overestimation occurs by a factor of almost 2.

living predicted by both models. The differences in the welfare change derived from both models suggest that although not accounting for non-homotheticity may not be a major issue when predicting macroeconomic changes (such as GDP growth or within-group labor reallocation across sectors), it can lead to biases and sometimes misleading results when measuring the welfare distributional implications emanating from the oil shock.

1.5.3.1 Fit of the Models

GDP and Subsidy Rates

Both non-homothetic and homothetic models predict a total removals of oil subsidies (*table 1.10*). Indeed, the oil shock led to a decline in crude oil exports, declining GDP⁵⁰ and ultimately declining tax revenue. This latter decline combined with a reduced access to foreign capital forced the government to phase out oil subsidies.

Table 1.10: *Baseline scenario: endogenous changes in key rates (in %)*

<i>item</i>	<i>non-homothetic</i>	<i>homothetic</i>	<i>actual data</i>
GDP <i>growth rate</i>	− 4.95	− 8.70	− 2.00
<i>Subsidy rate</i>	− 100	− 100	− 100

Table 1.11: *Changes in output (in %)*

<i>model</i>	<i>oi_r</i>	<i>oi_c</i>	<i>ag</i>	<i>ma</i>	<i>se</i>
<i>Homothetic</i>	− 40	− 40	+ 15.0	+ 17.0	− 10.0
<i>Non-homothetic</i>	− 40	− 40	+ 11.0	+ 14.0	− 11.0

⁵⁰The main driver of the growth difference between the homothetic and non-homothetic models is the output growth in the service sector, which contributes half (50.71%) of the GDP. The homothetic model predicts relatively higher decreases in service output (*table 1.11*).

Labor reallocation patterns

The changes in within-group labor allocation (in efficiency units) across sectors are given in *figure 1.1* and *figure 1.2* for the non-homothetic and homothetic models, respectively. Both models predict positive labor inflows in the agricultural and manufacturing sectors and negative labor inflows in the crude oil and service sectors. These reallocation patterns, especially the rise in agricultural and manufacturing labor, are consistent with the changes exhibited in the macrodata (Nkang, 2018).

Figure 1.1: *Baseline scenario: within-group sectoral labor reallocation: non homothetic model (in % units)*

Groups	Sectors of employment				Initial labor size by group (%)
	Crude oil	Agriculture	Manufacturing	Service	
Urban, non skilled	-0.01	0.00	0.05	-0.04	13.35
Urban, low skilled	-0.01	0.02	0.11	-0.12	14.91
Urban, high skilled	-0.02	0.20	0.09	-0.27	10.91
Rural, non skilled	0.00	0.03	0.01	-0.04	32.44
Rural, low skilled	0.00	0.02	0.09	-0.10	18.56
Rural, high skilled	-0.01	0.21	0.02	-0.22	9.84
Ex-ante labor size by sector (%)	1.17	62.16	14.91	21.76	100.00

Note. Figures 1.1 (non-homothetic model) and 1.2 (homothetic model) give the within-group sectoral reallocation of labor $\{\hat{\pi}_{gi}\}_{g,i}$, i.e. the changes in the labor size of dyads $\{(g,i)\}_i$ within each group g . Orange (resp. green) indicates an increase (resp. decrease). The length of each bar is indicative of the magnitude of the change.

Figure 1.2: *Baseline scenario: within-group sectoral labor reallocation: homothetic model (in % units)*

Groups	Sectors of employment				Initial labor size by group (%)
	Crude oil	Agriculture	Manufacturing	Service	
Urban, non skilled	-0.01	0.01	0.05	-0.05	13.35
Urban, low skilled	-0.01	0.03	0.09	-0.11	14.91
Urban, high skilled	-0.02	0.11	0.12	-0.21	10.91
Rural, non skilled	0.00	0.02	0.01	-0.03	32.44
Rural, low skilled	0.00	0.06	0.04	-0.09	18.56
Rural, high skilled	-0.01	0.19	0.03	-0.21	9.84
Labor size by sector (%)	1.17	62.16	14.91	21.76	100.00

1.5.3.2 Main Outcomes

Consumption Prices and Wages

First, note that the endogenous changes in the domestic consumption prices (*figure 1.3* and *table 1.12*) source from three factors⁵¹: first, the relative weights of local varieties in the national sectoral spending, expressed by the share Ω_k^L ; second, the change in the relative prices of foreign and local varieties; and third, the endogenous change in the oil subsidies levels as dictated by the endogenous government transfers⁵².

Crude oil sector. In the crude oil sector, the 40% negative global price shock translated into an equivalent decrease in the domestic (producer) price and ultimately equivalent declines in the values of production and exports. As a consequence, the output, relative wages, and employment (in terms of efficiency units of labor and number of workers) fell across the sector.

Refined oil sector. In the refined oil sector, the 40% negative global price shock translated into a similar decrease in the domestic production price. However, because of the removal of the pre-shock 40% petroleum subsidies, the consumption price

⁵¹Using equation (1.4.43).

⁵²This third potential source only directly affects the refined oil price.

of refined oil remained unchanged. Ultimately, the lower-than-expected⁵³ decrease in the refined oil's consumption price led, on the one hand, to an increase in the relative consumption prices of refined oil with respect to other (domestic) sectoral goods, and on the other hand, to an increase of its relative price with respect to foreign varieties⁵⁴. The consequences are twofold. On one hand, the national output fell significantly in the sector as the country moved in favor of the significantly less expensive foreign varieties. On the other hand, there was a decrease in the relative demand addressed for refined oil by Nigerian consumers.

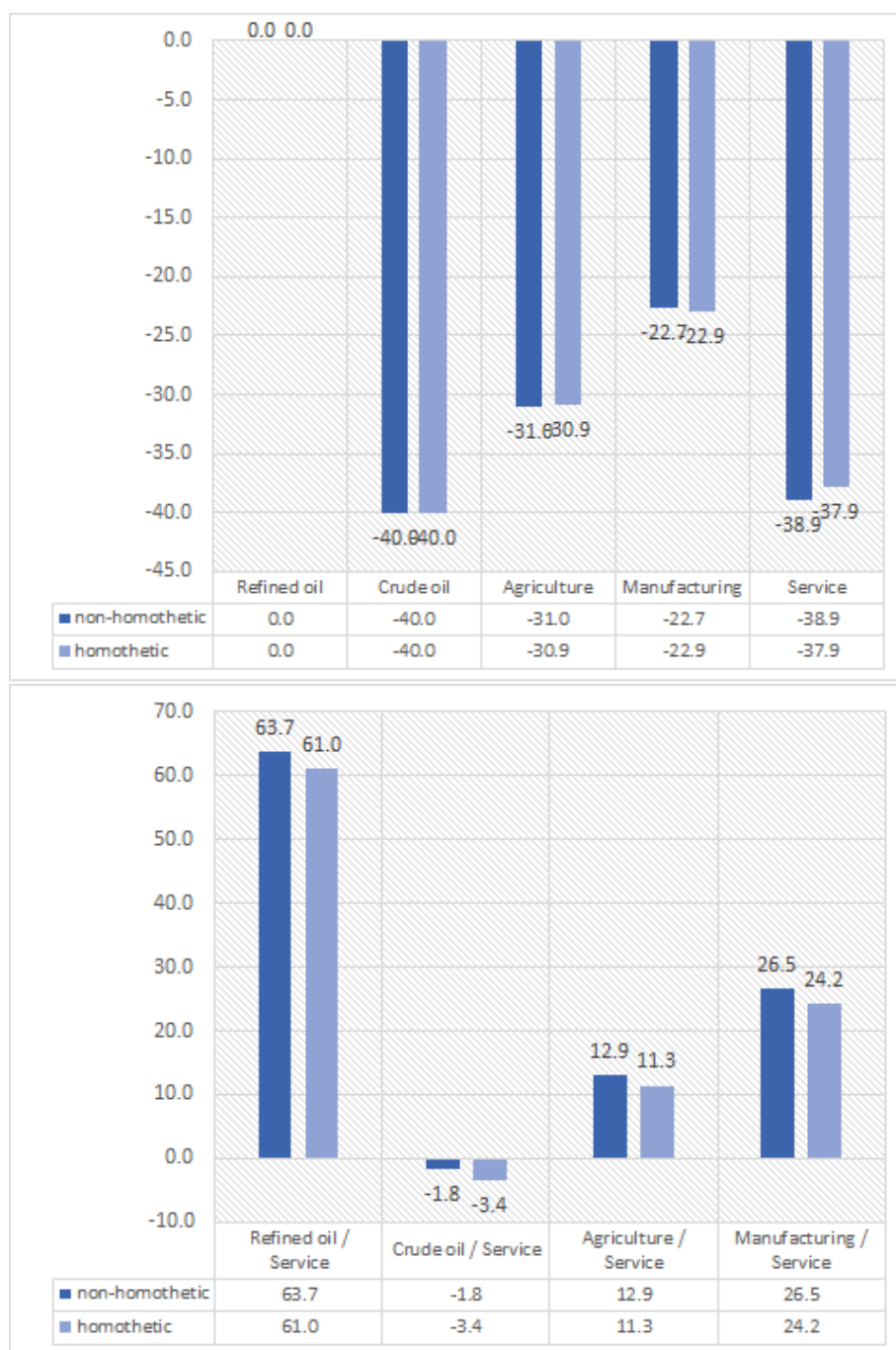
Table 1.12: *Baseline scenario: changes in prices and wages (in %)*

<i>model</i>	<i>oi_r</i>	<i>oi_c</i>	<i>ag.</i>	<i>ma.</i>	<i>se.</i>
Non-homothetic					
<i>Prices, foreign variety</i>	– 40.0	– 40.0	– 2.0	– 1.0	<i>na</i>
<i>Prices, local variety</i>	– 40.0	– 40.0	– 34.7	– 28.7	– 38.0
<i>Production prices</i>	– 40.0	– 40.0	– 33.3	– 20.7	– 38.0
<i>Consumption prices*</i>	– 0.0	– 40.00	– 33.3	– 20.7	– 38.0
<i>Wages</i>	– 40.0	– 40.00	– 34.7	– 28.7	– 38.0
Homothetic					
<i>Prices, foreign variety</i>	– 40.0	– 40.0	– 2.0	– 1.0	<i>na</i>
<i>Prices, local variety</i>	– 40.0	– 40.0	– 34.6	– 28.1	– 37.0
<i>Production prices</i>	– 40.0	– 40.0	– 33.2	– 20.2	– 37.0
<i>Consumption prices*</i>	– 0.0	– 40.00	– 33.2	– 20.2	– 37.0
<i>Wages</i>	– 40.0	– 40.00	– 34.2	– 28.1	– 37.0

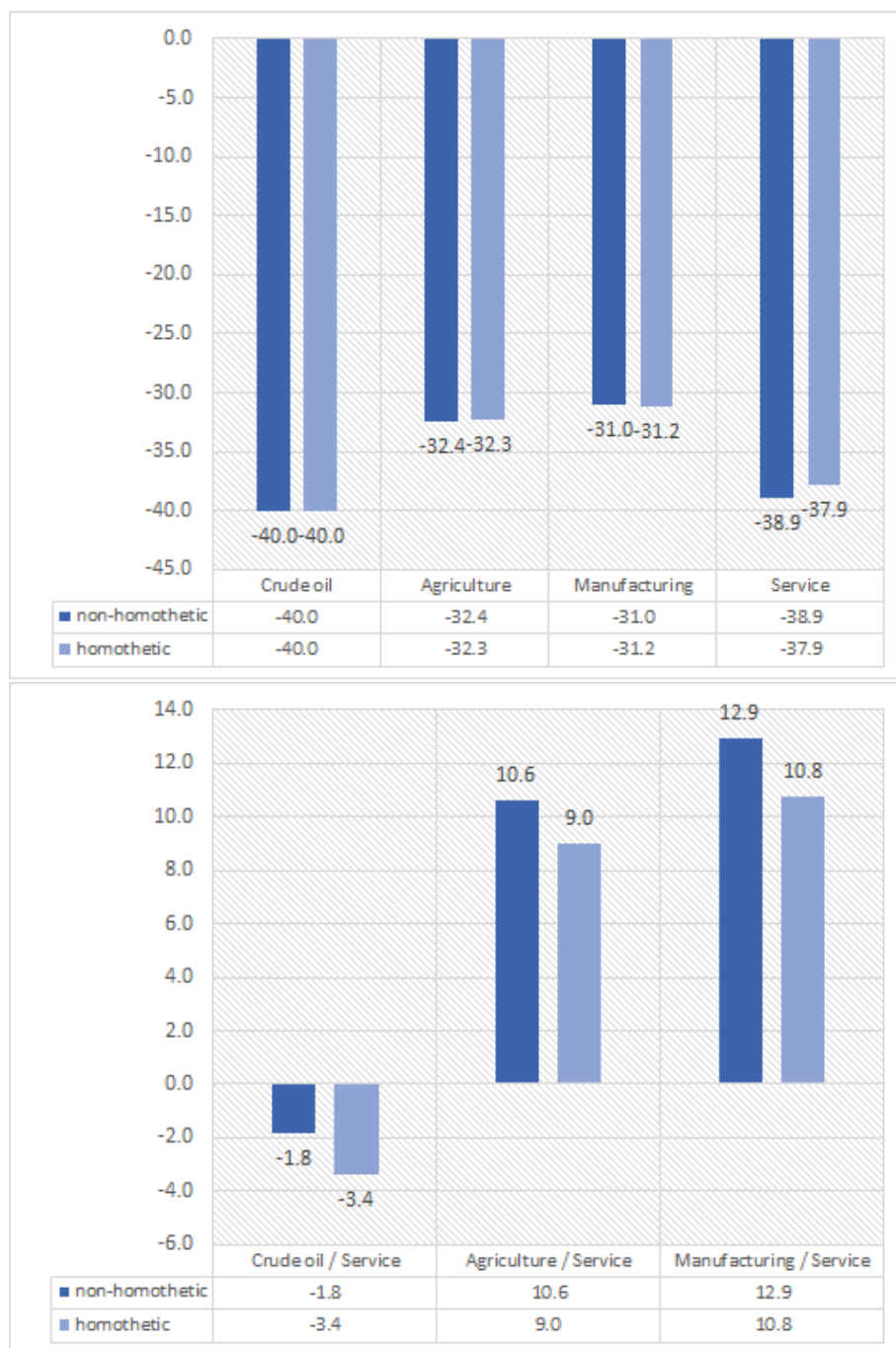
*intermediary for crude oil

⁵³Relative to the much larger 40% decline on the international markets.

⁵⁴The exceptions are for refined and crude oil whose domestic and foreign varieties are perfect substitutes and therefore have same price.

Figure 1.3: *Baseline scenario: changes in consumption prices (in %)*

The upper panel illustrates the changes in sectoral prices whereas the lower panel illustrates the changes to relative prices with respect to the non-traded (service) sector

Figure 1.4: *Baseline scenario: changes in wages (in %)*

The left panel illustrates the changes in sectoral wages whereas the right panel illustrates the changes to relative wages with respect to the non-traded (service) sector

Manufacturing sector. The manufacturing sector experienced a significant boost in output (17% with the non-homothetic model) because of significant rises in exports and local demand. First, the increasing price competitiveness of the Nigerian manufacturing goods relative to foreign varieties led to a significant increase in foreign demand. Second, there was the significant shift in value of domestic demand from foreign to local varieties. The total manufacturing national demand (including both local and foreign varieties) declined. However, the significant rise in foreign demand, combined with the sufficiently significant rise in local demand for local varieties⁵⁵ have led to an increase in output. Therefore, in the sector, there was an increase in both the relative prices and wages, which ultimately translated into an increase in aggregate efficiency units and total number of workers⁵⁶. Ultimately, in the manufacturing sector, the shock led to an expansion of output, an increase in relative prices and wages, and increased employment.

Agricultural sector. The agricultural sector experienced a significant (but moderate in comparison to the manufacturing sector) boost in output (15% with the non-homothetic model) sourcing because of a slight rise in both exports and a significant increase in local demand. The rise in exports is due to the added price competitiveness of Nigerian varieties. It is moderate compared to the manufacturing export rise because the post-shock relative foreign vs. local prices change was lower for agricultural goods (1.39), compared to manufacturing goods (1.5). Furthermore, as with manufacturing goods, there was a rise in local demand that was driven by two factors. First, there was a shift in national consumption from foreign to local varieties as local varieties became relatively less expensive⁵⁷. Second, there was rising output and demand for necessity goods, a typical feature observed especially for low-income workers in the wake of negative income shocks. The output in the sector rose,

⁵⁵This shift was significant because the pre-shock share of foreign varieties (57%, representing 67% of the aggregate imports) in the manufacturing absorption was significantly high (*table 1.5*). Consequently, this allows for much more room in terms of shifting from foreign to local varieties, boosting output in spite of the negative shock.

⁵⁶The rise in terms of efficiency units is slightly lower than the rise in terms of numbers. This is because the shock led to some service and crude oil workers moving into the manufacturing sector. However, because of their comparative advantage, such workers are likely to have lower productivity for manufacturing labor.

⁵⁷However, because the pre-shock share of foreign varieties was much smaller (10% vs 57% in the manufacturing sector), this shifting preference toward locally produced goods was not sufficiently high, partly explaining (along with the more moderate foreign demand) the more moderate increase in production.

and the rise in production went along with moderate increases in both foreign and local demand. Ultimately, a combination of a rising output and a more moderate rise in demand led to a decline in relative (consumption) prices and a moderate rise in relative wages in the sector. Ultimately, both the number of workers and the number of aggregate efficiency units rose in the sector⁵⁸.

Service sector. In the service sector, there is a decrease in the output by 10%. This decrease is indeed to be expected from a negative shock, since consumers tend to consume fewer services (considered a luxury) when they experience negative income shocks. Ultimately, relative prices and wages decreased and the sector experienced labor losses, both in terms of efficiency units and number of workers.

Net Income Effects: Income Channel of Welfare Changes

The changes in the subgroup-level net income $\{e_{gi}\}_{g,i}$ are given in *figure 1.5* for both models and reflect the changes in the sectoral wages⁵⁹. Hence, the changes in the net income are uniform across groups and only dependent upon the choice of sector of employment⁶⁰.

Figure 1.5: *Baseline scenario: changes in net income (in %): non-homothetic vs homothetic models*

Groups	Sectors of employment			
	Crude oil	Agriculture	Manufacturing	Service
non-homothetic model	-40.00	-32.40	-30.99	-38.85
homothetic model	-40.00	-32.29	-31.19	-37.95

Note. This table gives the net income effects $\{\hat{e}_{gi}\}_{g,i}$ for both non-homothetic and homothetic models regardless of the subgroups $\{(g,i)\}_{g,i}$. The orange color is indicative of declining income levels

⁵⁸The rise in terms of efficiency units is significantly higher than the rise in terms of numbers. The shock led to some service and crude oil workers moving into the agricultural sector. However, such workers tend to have significantly lower productivity for agricultural labor.

⁵⁹There is no change to the taxation policy in the baseline scenario, and workers' labor productivities are unchanged in the short term.

⁶⁰Indirectly, they also reflect workers' comparative advantages, which along with salary changes drive their respective employment choices.

In line with the changes observed in the relative wages (*figure 1.4*)⁶¹, workers employed in the crude oil and service sectors experienced the biggest decreases in net income, whereas workers active in agricultural and manufacturing sectors experienced the smallest decreases in net income (*figure 1.5*). Furthermore, the non-homothetic model predicts a slightly larger decrease in net income (compared to the homothetic model) for agricultural and service workers, and a slightly smaller decrease for manufacturing workers.

Price Index Effects: Cost of Living Channel of Welfare Changes

The changes in the subgroup-level price indices are given in *figure 1.6*. There is no heterogeneity in the changes to the price indices in the homothetic case, as income level does not play any role. The homothetic model predicts a uniform decrease in the price index by 26.92%.

Figure 1.6: *Baseline scenario: changes in price index (in %): non-homothetic model*

Groups	Sectors of employment			
	Crude oil	Agriculture	Manufacturing	Service
Urban, non skilled	-29.65	-30.12	-29.94	-32.36
Urban, low skilled	-27.86	-30.24	-30.35	-30.79
Urban, high skilled	-30.19	-32.51	-29.49	-30.70
Rural, non skilled	-30.46	-29.46	-28.99	-29.08
Rural, low skilled	-32.34	-29.37	-31.09	-30.10
Rural, high skilled	-30.52	-29.36	-28.00	-29.91

Note. This table gives the price index effects $\{\hat{P}_{gi}\}_{g,i}$ for the non-homothetic model in all subgroups, i.e. all dyads $\{(g,i)\}_{g,i}$. The orange color is indicative of declining cost of living. Also note that the homothetic model predicts a uniform **26.92%** decline in the price index across all subgroups

However, in the non-homothetic case, the changes are heterogeneous across con-

⁶¹The relative wages increase in the agricultural and manufacturing sectors, while they decrease in the crude oil and service sectors.

sumers and reflect not only the changes in the consumption prices, but also the net income effects combined with the role of income elasticities. The non-homothetic model consistently predicts larger decreases in the cost of living compared to the homothetic model. Indeed, it predicts much larger increases in the relative prices of refined oil and manufacturing goods, and a slightly larger increase in the relative price of agricultural goods (*figure 1.3*). Hence, the subsequent decreases in relative demand⁶² for refined oil, manufacturing goods, and agricultural goods are much larger in the non-homothetic model. As a consequence, the non-homothetic model induces higher declines in the cost of living across all consumers.

Further, in general, rich workers experienced lower declines in the cost of living as they benefited more from the oil subsidies, which were subsequently phased out. They also consumed relatively more manufacturing goods, which also experienced rises in relative prices. As a result, changes in the price indices are more favorable to low-income workers.

Total Welfare Effects

The total welfare effect combines net income and cost of living effects. That is,

$$\underbrace{\hat{\mathcal{W}}_{gi}}_{\text{total welfare effect}} = \underbrace{\hat{e}_{gi}}_{\text{net income effect}} \times \underbrace{\hat{P}_{gi}^{-1}}_{\text{price index effect}}$$

In order to understand the change in real consumption levels, one needs to consider three factors: the change in the sectoral wage, the change in consumption prices, and the change in the subgroup-representative consumer's expenditure shares. For a particular subgroup (g, i) , the change in the sectoral wage w_i determines the income effect. Meanwhile, the changes in the consumption prices vector $\{p_k^c\}_{k \in \mathcal{T} \cup \mathcal{N}}$, in net income y_{gi} , and in expenditure shares $\{\Omega_{gi,k}\}_{k \in \mathcal{T} \cup \mathcal{N}}$ determine the cost of living effect, i.e. the change in the price index P_{gi} .

First, I summarize for each subgroup the net income and price index effects in *figure a9* and *figure a10* in the *Appendix A.5*.

⁶²Due to the negative income shock.

Figure 1.7: *Baseline scenario: changes in total welfare (in %): non-homothetic model*

Groups	Sectors of employment			
	Crude oil	Agriculture	Manufacturing	Service
Urban, non skilled	-14.72	-3.27	-1.50	-9.59
Urban, low skilled	-16.82	-3.10	-0.92	-11.65
Urban, high skilled	-14.05	0.16	-2.12	-11.76
Rural, non skilled	-13.71	-4.17	-2.82	-13.77
Rural, low skilled	-11.32	-4.29	0.15	-12.52
Rural, high skilled	-13.64	-4.31	-4.15	-12.76

Note. The orange (resp. green) color is indicative of declining (increasing) welfare

Figure 1.8: *Baseline scenario: changes in real consumption (in %): homothetic model*

Groups	Sectors of employment			
	Crude oil	Agriculture	Manufacturing	Service
Urban, non skilled	-17.90	-7.35	-5.84	-15.09
Urban, low skilled	-17.90	-7.35	-5.84	-15.09
Urban, high skilled	-17.90	-7.35	-5.84	-15.09
Rural, non skilled	-17.90	-7.35	-5.84	-15.09
Rural, low skilled	-17.90	-7.35	-5.84	-15.09
Rural, high skilled	-17.90	-7.35	-5.84	-15.09

The combined effects, i.e. the total welfare effects, are presented in *figure 1.7*

for the non-homothetic model and in *figure 1.8* for the homothetic model. In both cases, both net income and price indices decrease. The cost of living decreases less than the income does. In other words, the fall in the cost of living is not sufficient to offset the fall in the net income, resulting in decreasing real consumption.

Second, the homothetic model predicts relatively larger welfare losses across all subgroups. This gap stems primarily from the cost of living channel, as the homothetic model predicts significantly smaller declines in price indices than the non-homothetic model (see the footnote to *figure 1.6*). Secondly, the gap comes from the income channel, although there is not much difference in the changes to net income between the two models (see *figure 1.5*).

Finally, regardless of the model, workers in the crude oil and service sectors experienced the biggest welfare losses. Indeed, the gaps between income and price index effects are significantly more important for these workers.

Summary of Welfare Changes

I now define a social welfare metric at both the group and national levels. First, I derive a group-level social welfare change $\hat{\mathcal{W}}_{soc}^g$ as a weighted average of the relevant subgroup-level changes in real income, using the post-shock within-group sectoral allocations of labor as weights. This allows me to account for the the ex-post relative size of each skill group and each location group⁶³. I proceed similarly in defining a sector-level social welfare change $\hat{\mathcal{W}}_{soc}^i$ as a weighted average of the relevant subgroup-level changes in real income, using the post-shock within-sector allocations of labor across groups (namely, $\pi_{ig} \equiv \frac{L_{gi}}{L_i}$ with $L_i = \sum_g L_{gi}$) as weights. In doing so, I am able to account for the the ex-post relative size of each sector of employment.

$$\hat{\mathcal{W}}_{soc}^g = \sum_i \pi_{gi} \hat{\pi}_{gi} \hat{\mathcal{W}}_{gi} \quad (1.5.1)$$

$$\hat{\mathcal{W}}_{soc}^i = \sum_g \pi_{ig} \hat{\pi}_{ig} \hat{\mathcal{W}}_{gi} \quad (1.5.2)$$

⁶³The across-groups heterogeneity in $\hat{\mathcal{W}}_{soc}^g$ in the homothetic case (where the changes in real consumption are uniform across subgroups within each group) comes from the group specificity in the sectoral composition of labor. Indeed, each group has a different specialization profile dictated by the differentiated comparative advantages of its members.

$\pi_{ig}\hat{\pi}_{ig}$ is the ex-post within-sector labor shares across groups. Finally, I derive a country-level social welfare change $\hat{\mathcal{W}}_{soc}$ as a weighted average of the group-level or sector-level social welfare changes (reported in the last row of *figure 1.9*).

$$\hat{\mathcal{W}}_{soc} = \frac{1}{L} \sum_g L_g \hat{\mathcal{W}}_{soc}^g = \frac{1}{L} \sum_i L_i \hat{\mathcal{W}}_{soc}^i \quad (1.5.3)$$

Both the non-homothetic and the homothetic models predict relative welfare losses across all six groups (*figure 1.9*). As expected from *figure 1.7* (non-homothetic case) and *figure 1.8* (homothetic case), at the sectoral level, crude oil and service workers are the most affected by the negative welfare changes.

Figure 1.9: *Baseline scenario: changes in social welfare at group and country levels (in %)*

Groups	Non-Homothetic		Homothetic	
Urban, non skilled		-3.01		-7.18
Urban, low skilled		-2.93		-7.48
Urban, high skilled		-4.58		-10.11
Rural, non skilled		-4.27		-7.44
Rural, low skilled		-3.88		-7.70
Rural, high skilled		-7.83		-10.58
All groups		-4.21		-8.06

Note. This table gives the social welfare changes $\{\hat{\mathcal{W}}_{soc}^g\}_g$ and $\hat{\mathcal{W}}_{soc}$ (all groups) for both non-homothetic and homothetic models. The orange (resp. green) color is indicative of declining (increasing) social welfare

To understand these results, recall from the income channel of the welfare changes that crude oil and service workers were hit the hardest by the income losses, as they experience the biggest relative wage declines. Hence, the welfare losses generally increasing in the income levels and are much larger for highly skilled workers. Indeed, skilled workers tend to specialize in the crude oil or service sector because of their

comparative advantage. Therefore, they suffer more income losses. Furthermore, as seen with the price index effects (cost of living channel), high-income consumers are likely to experience lower declines in the cost of living. Ultimately, skilled workers are likely to experience higher welfare losses.

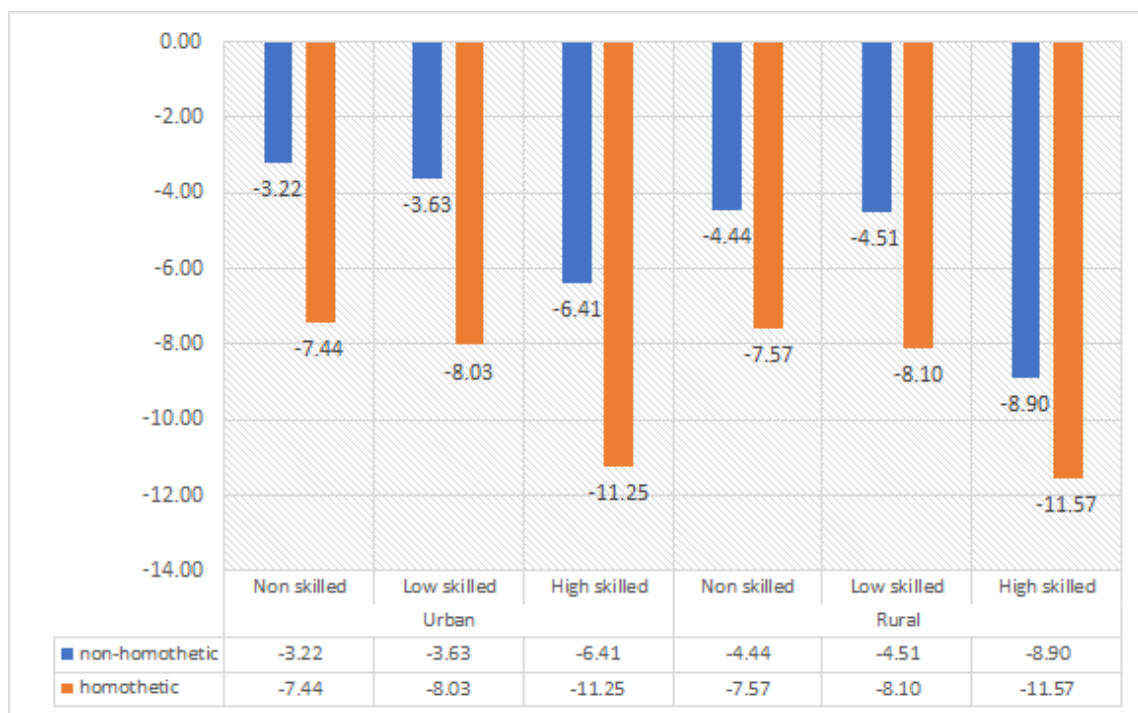
Regardless of the skill level, the welfare losses are relatively more important in rural areas. This is particularly noticeable for highly skilled workers. This is because these workers have fewer second-best employment choices in rural areas when leaving crude oil or service jobs. Indeed, because of the differences in the profiles of the alternative jobs available, highly skilled workers living in urban areas are more likely to switch to manufacturing jobs (adequately paid), whereas those living in rural areas are more likely to switch to agricultural jobs (less adequately paid).

To further illustrate these progressive welfare changes across groups, I can alternatively analyze the changes to average real income across groups. Indeed, the reallocation of labor across sectors⁶⁴ within each group will result in a change in the actual efficiency units supplied by the group, and ultimately in a change of the average real income of the group, given by $\hat{\Phi}_g$. Using equation (1.4.7), I have for each skill-location group g

$$\hat{\Phi}_g = \left[\sum_i \left(\pi_{gi} \frac{\hat{w}_i \hat{z}_{gi}}{\hat{P}_{gi}} \right)^{-\theta} \right]^{\frac{1}{\theta}} \quad \text{with} \quad \hat{z}_{gi} = 1 \quad \forall i \quad ; \quad \frac{\hat{w}_i \hat{z}_{gi}}{\hat{P}_{gi}} = \hat{c}_{gi}$$

As expected, the average returns to labor are decreasing across all skills and locations (*figure 1.10*). Furthermore, the losses (declining returns) are increasing in the skill level, and slightly larger in rural areas, especially for highly skilled workers. Finally, as expected, the losses predicted by the non-homothetic model are relatively smaller.

⁶⁴Recall that workers within each groups get different draws of labor productivity for different sectors of employment.

Figure 1.10: *Baseline scenario: changes in real return to skill and location (in %)*

1.6 Conclusion

This chapter introduced a approach to obtain the post-shock or post-counterfactual changes in the after-tax incomes and price indices of different income groups, and therefore to measure the unequal welfare changes through the income and cost of living channels. The approach is built on readily available micro and macro data and a few calibrated parameters. The model generates macroeconomic predictions that are staggeringly close to the actual data in spite of the use of calibrated rather than directly estimated parameters.

Solving the model for the 2016 oil shock, I found that the oil shock led to a reverse Dutch disease phenomenon characterized by the expansion of relative wages and employment in the agricultural and manufacturing sectors and a contraction in the crude oil sector. In terms of welfare, the shock led to a 4.21% decline in the aggregate real consumption stemming from decreasing price indices (cost of living channel) and even higher decreases in net income (income channel).

When comparing different income groups, the losses appear regressive in nature, ranging from -3.01% for the group of non-skilled workers living in urban areas to as much as -7.83% for the group of highly skilled workers living in rural areas. Low-income workers suffered smaller income losses and a slightly larger decline in costs of living, suggesting the progressive nature of the welfare losses. Although the oil shock and the subsequent removal of the subsidies did hurt all workers, poor workers were hurt less because of their comparative advantage and the composition of their basket of consumption.

By exploring a special case of the model that reduces to a homothetic CES, I find that overlooking non-homotheticity in preferences still leads to negative and regressive welfare losses but to a much larger extent (100% more on average). The gap can be mostly explained by the homothetic model predicting significantly smaller reductions in the cost of living.

Finally, it is worth noting three key qualitative implications deriving from the chapter. First, both the non-homothetic and the homothetic models fairly accurately match changes in some key macroeconomic data (e.g., real GDP growth, subsidy rate, within-group sectoral labor reallocation patterns). Second, in addition to being

inefficient, the oil subsidies induce further welfare losses in the aftermath of oil crashes if maintained. Finally, failing to account for heterogeneity in expenditure behavior can introduce biases in the magnitude of the welfare changes and therefore mislead policymakers as they develop mitigating response strategies, especially in the case of low-income workers.

Chapter 2

Quantifying the incidence of a global oil price shock in Nigeria: Policy counterfactual Exercises

2.1 Introduction

As a passive planner, the government can aim at reducing the welfare losses observed in the baseline scenario, especially for low-income workers. To this end, two public policy tools are available: tax policy and foreign borrowing. Indeed, in order to finance the oil subsidy policy and other public expenditures, the government relies primarily on tax revenue(internal funding)¹ and secondarily on foreign borrowing (external funding).

In this chapter, I explore different policy reform scenarios in order to assess two questions: (1) Was removing the subsidies the best scenario for welfare outcomes? In other words, which is the better welfare-improving scenario (i) maintaining the status quo prevailing in the baseline scenario, i.e., a total suppression of the oil subsidies, or (ii) collecting more revenue in order to allow for a lower post-shock decline in the subsidy rate compared to the baseline scenario?²; (2) With the subsidies removed, how can the government improve the welfare outcomes observed in the baseline scenario, especially for poor workers?

To answer the first question, I allow for the government to prevent a total suppression of the subsidies by raising more revenue via uniform (*Count 1*) or progressive (*Count 2*) tax hikes. To answer the second question, I allow for the government, while maintaining the zero subsidy level, to borrow more from the rest of the world (*Count 3*) or to redistribute the burden of tax in a fiscally neutral and progressive manner (*Count 4*)³.

¹Tax revenue is itself a function of income levels and tax rates.

²In this latter case, the magnitude of the tax rate changes or the amount of additional foreign borrowing would affect public resources and spending. Ultimately, they would impact the level of subsidies; the relative prices and wages; and the net income, prices, and real consumption across all income groups.

³I do not explore a tax cut counterfactual for two reasons: First, it trivially leads to improving welfare for all; second, it implies that the government reduces other essential public expenditures $\{G_k\}_{k \neq oi}$ (in order to balance its resource constraint).

2.2 Should the government have removed the subsidies?

2.2.1 Uniform tax hike counterfactuals

Count 1 (*uniform tax hike*): *baseline* + uniform tax increase (from 14.15% to 26%) preserving the pre-shock subsidy level⁴

In this counterfactual, there is a further decrease in the GDP in comparison to the baseline scenario (*table 2.1*). The government's resources are negatively impacted by the declining GDP but positively affected by the significant rise in the taxation rates.

Table 2.1: *Uniform tax hike counterfactual: changes in key rates (in %)*

<i>item</i>	<i>baseline</i>	<i>count 1</i>	<i>gap</i>
<i>non-homothetic</i>			
<i>GDP growth rate</i>	– 4.95	– 6.67	– 1.71
<i>Subsidy rate</i>	– 100	– 0.00	+ 100
<i>homothetic</i>			
<i>GDP growth rate</i>	– 8.70	– 10.9	– 2.19
<i>Subsidy rate</i>	– 100	– 0.00	+ 100
<i>National average tax, $\bar{\tau}$</i>	14.15%	26%	

The net effect on government revenue is positive, i.e. the additional revenue stemming from increased national average tax is sufficiently high to more than counter the losses deriving from the further decline in the GDP, comparing to the baseline scenario. Overall, the national average tax goes up from $\bar{\tau}_{baseline} = 14.15\%$ to $\bar{\tau}_{count\ 1} = 26\%$. As a result, there is a more moderate decline in government's re-

⁴This is such that $\hat{\beta} = 1$. An 85% increase in the tax rate delivers such an objective. That is, the national average tax rate increases from 14.15% to 26%.

sources relative to the baseline scenario. That combined with cheaper refined oil in the international markets made possible the keeping of the pre-shock subsidy level, as opposed to the total removal enacted in the baseline scenario.

Total Welfare Effects

In order to understand the change in subgroup-level real consumption, one needs to consider four factors. In addition to the three factors considered in the baseline scenario, i.e., (i) the change in the sectoral wage rates, (ii) the change in the sectoral price, and (iii) the change in the subgroup-representative consumer's expenditure shares, one also needs to account for the (iv) the change in the taxation policy.

Figure 2.1: *Uniform tax hike counterfactual: welfare change relative to baseline scenario: non-homothetic model (in % units)*

Groups	Sectors of employment			
	Crude oil	Agriculture	Manufacturing	Service
Urban, non skilled	-6.30	-6.19	-9.62	-8.06
Urban, low skilled	-6.51	-6.52	-8.86	-5.81
Urban, high skilled	-7.37	-11.81	-11.54	-7.88
Rural, non skilled	-1.28	-5.17	-8.45	-5.00
Rural, low skilled	-3.61	-6.09	-10.19	-6.23
Rural, high skilled	-7.27	-6.76	-8.17	-6.93

Note. This table gives the relative welfare changes with respect to the baseline scenario for the non-homothetic model in all subgroups, i.e. all dyads. The orange (resp. green) color is indicative of a relative welfare gain (loss) with respect to the baseline scenario

For a particular subgroup (g, i) , the changes in the sectoral wage w_i and in the taxation policy measured ex-post by $\{\tau_{gi}\}_{g,i}$ determine the net income effects. Furthermore, the changes in the consumption price $\{p_k^c\}_{k \in \mathcal{T} \cup \mathcal{N}}$, in the subgroup-level

H

Figure 2.2: *Uniform tax hike counterfactual: welfare change relative to baseline scenario: homothetic model (in % units)*

Groups	Sectors of employment			
	Crude oil	Agriculture	Manufacturing	Service
Urban, non skilled	-6.37	-7.21	-9.15	-9.30
Urban, low skilled	-7.28	-8.51	-8.77	-7.13
Urban, high skilled	-10.25	-13.86	-15.49	-11.36
Rural, non skilled	-2.34	-4.92	-7.85	-4.35
Rural, low skilled	-2.34	-5.98	-9.82	-6.53
Rural, high skilled	-7.75	-9.39	-11.73	-8.94

Note. This table gives the relative welfare changes with respect to the baseline scenario for the homothetic model in all subgroups, i.e. all dyads. The orange (resp. green) color is indicative of a relative welfare gain (loss) with respect to the baseline scenario

taxation rates τ_{gi} and in the expenditure shares $\{\Omega_{gi,k}\}_{k \in T \cup N}$ determine the price index effects.

I compute for each subgroup the relative changes (with respect to the baseline scenario) in the income and price indices (*figure a11* and *figure a12*). For each subgroup, the summary of the combined net income and cost of living effects, i.e. the changes in the real consumption levels, are given in *figure 2.1* (non-homothetic model) and *figure 2.2* (homothetic model).

All net income effects and price index effects are negative, reflecting not only relative decreases in expenditures stemming primarily from the taxation hikes, but also relative decreases in the cost of living. However, in every subgroup, the net income decreases more than the cost of living does, which explains the further losses in terms of welfare. Furthermore, rich consumers (higher skills, urban) experience higher relative decreases in the cost of living rise (as they tend to consume more refined oil that is now partially subsidized). However, they also experience higher (and even

larger compared to other consumers) relative income losses. Hence, although they benefit more from the further decline in cost of living, they also experience much larger additional income losses, and these income losses are so large that they end up experiencing larger additional welfare losses.

Using both models, this counterfactual leads to relative (with respect to the baseline scenario) welfare losses for all other workers, including most notably those employed in the agricultural and manufacturing sectors (*figure 2.1* and *figure 2.2*).

Social Welfare Changes

Compared to the baseline scenario, all groups are worse off in this counterfactual scenario (*figure 2.3*). This further decline in welfare is relatively homogeneous across all groups and similar in both models. In absolute terms, the welfare losses shown in *figure 2.4* are still distributed in a progressive manner, as they tend to increase in the skill levels.

Figure 2.3: *Uniform tax hike counterfactual: social welfare changes relative to baseline scenario (in % units)*

Groups	Non-Homothetic		Homothetic	
<i>Urban, non skilled</i>		-7.38		-7.93
<i>Urban, low skilled</i>		-7.54		-8.54
<i>Urban, high skilled</i>		-10.67		-13.38
<i>Rural, non skilled</i>		-5.38		-5.12
<i>Rural, low skilled</i>		-7.09		-6.75
<i>Rural, high skilled</i>		-6.93		-9.45
<i>All groups</i>		-7.02		-7.63

Note: *Orange indicates a relative decrease in the social welfare with respect to the baseline scenario*

Hence, raising tax in order to maintain a relatively high level of subsidy is welfare-deteriorating, even for high-income workers. Rich workers benefit more from the

decline in the cost of living, but they continue to be the most hit by income losses. Poor consumers also continue to experience negative welfare losses. These losses are relatively larger compared to the baseline scenario, although they are less large than those experienced by rich workers. In conclusion, keeping all or parts of the subsidies is more welfare-deteriorating than not keeping them at all, as in the baseline scenario.

Figure 2.4: *Uniform tax hike counterfactual: social welfare changes: non-homothetic vs homothetic models (in %)*

Groups	Non-Homothetic		Homothetic	
	Count 1	Baseline	Count 1	Baseline
Urban, non skilled	-10.40	-3.01	-15.11	-7.18
Urban, low skilled	-10.47	-2.93	-16.02	-7.48
Urban, high skilled	-15.25	-4.58	-23.49	-10.11
Rural, non skilled	-9.65	-4.27	-12.55	-7.44
Rural, low skilled	-10.97	-3.88	-14.45	-7.70
Rural, high skilled	-14.76	-7.83	-20.03	-10.58
All groups	-11.23	-4.21	-15.69	-8.06

Note. This table gives the social welfare changes $\{\hat{W}_{soc}^g\}_g$ and \hat{W}_{soc} for both non-homothetic and homothetic models. The orange (resp. green) color is indicative of declining (increasing) welfare

2.2.2 Progressive tax hike counterfactuals

Count 2 (progressive tax hike): Increasing tax for all income brackets in a progressive manner⁵ with an increase in the national tax average from 14.15% to

⁵The tax rates increase are as follows: +67% for the first bracket; +75% for the second bracket; +80% for the third bracket; +82% for the fourth bracket; +87% for the fifth bracket; and +90% for the sixth bracket. See *table 1.7* for references to income brackets and ex-ante taxation rates. As a consequence, the tax hikes affect highly skilled workers more than they do low-skilled workers since the former are mostly concentrated in the top three income brackets.

26% that preserves the pre-shock subsidy level.⁶

In this counterfactual, there is a further decrease in the GDP in comparison to the baseline scenario (*table 2.2*). The government's resources are negatively impacted by the declining GDP but positively affected by the significant rise in the taxation rates (40% at the national level).

Table 2.2: *Progressive tax hike counterfactual: changes in key rates (in %)*

<i>item</i>	<i>baseline</i>	<i>count 2</i>	<i>gap</i>
<i>non-homothetic</i>			
<i>GDP growth rate</i>	– 4.95	– 6.67	– 1.71
<i>Subsidy rate</i>	– 100	– 0.00	+ 100
<i>homothetic</i>			
<i>GDP growth rate</i>	– 8.70	– 10.8	– 2.05
<i>Subsidy rate</i>	– 100	– 0.00	+ 100
<i>National average tax, $\bar{\tau}$</i>	14.15%	26.2%	

The net effect on government revenue is positive. Indeed, the additional revenue stemming from the increased national average tax is sufficiently high to more than counter the losses deriving from the further decline in the GDP, compared to the baseline scenario. Overall, the national average tax increases from $\bar{\tau}_{baseline} = 14.15\%$ to $\bar{\tau}_{count\ 1} = 26\%$. As a result, there is a more moderate decline in the government's resources relative to the baseline scenario. That in addition to cheaper international refined oil made possible the keeping of the pre-shock subsidy level, contrary to the baseline scenario in which it was totally phased out.

⁶The pre-shock subsidy level is preserved for the sake of comparison with *count 1* (and to understand the role of progressiveness in taxation policy). Hence, the increases although non-uniform are made in order to reflect a 85% increase in the national tax average (i.e. from 14.15% to 26%).

Total Welfare Effects

I now compute for each subgroup the relative changes with respect to the baseline scenario in the income and price index effects (*figure a13* and *figure a14*). The summary for each subgroup of the combined net income and cost of living effects, i.e. the changes in the real consumption levels, are given in *figure 2.5* (non-homothetic model) and *figure 2.6* (homothetic model).

Figure 2.5: *Progressive tax hike counterfactual: welfare change relative to baseline scenario: non-homothetic model (in % units)*

Groups	Sectors of employment			
	Crude oil	Agriculture	Manufacturing	Service
Urban, non skilled	-6.06	-5.81	-9.36	-8.08
Urban, low skilled	-6.34	-6.38	-8.59	-5.57
Urban, high skilled	-7.83	-12.37	-12.43	-8.23
Rural, non skilled	-0.91	-4.60	-8.24	-4.33
Rural, low skilled	-3.06	-5.56	-10.21	-5.75
Rural, high skilled	-7.12	-6.74	-8.16	-6.89

Note. This table gives the relative welfare changes with respect to the baseline scenario for the non-homothetic model in all subgroups, i.e. all dyads. The orange (resp. green) color is indicative of a relative welfare gain (loss) with respect to the baseline scenario

All net income and price indices further decrease compared to the baseline scenario. This reflects further decreases in expenditures because of the tax rate hikes, as well as further decreases in the cost of living. Furthermore, both models predict that the cost of living will decrease less than the income does for workers across all sectors. Moreover, as with the uniform tax hike scenario, rich consumers (higher skills, urban) experience higher decreases in the cost of living rise (benefiting more from oil subsidies), but also even higher decreases in income and ultimately higher

additional welfare losses. Further, these losses are exacerbated by the fact that rich consumers are subject to the highest tax rate hikes. Hence, in all sectors, the welfare losses are increasing in the income levels (i.e. larger at higher skill levels and higher in urban areas, as expected from a progressive tax hike reform).

In both models, this counterfactual leads to relative welfare losses for all workers, including those employed in the agricultural sector (*figure 2.5* and *figure 2.6*). However, the relative welfare losses are more progressively distributed, i.e. impact rich workers relatively more.

Figure 2.6: *Progressive tax hike counterfactual: welfare change relative to baseline scenario: homothetic model (in % units)*

Groups	Sectors of employment			
	Crude oil	Agriculture	Manufacturing	Service
Urban, non skilled	-6.19	-6.74	-8.87	-9.30
Urban, low skilled	-7.08	-8.22	-8.45	-6.84
Urban, high skilled	-10.56	-14.37	-16.18	-11.55
Rural, non skilled	-1.90	-4.39	-7.63	-3.78
Rural, low skilled	-1.90	-5.47	-9.82	-6.05
Rural, high skilled	-7.59	-9.19	-11.38	-8.78

Note. This table gives the relative welfare changes with respect to the baseline scenario for the homothetic model in all subgroups, i.e. all dyads. The orange (resp. green) color is indicative of a relative welfare gain (loss) with respect to the baseline scenario

Social Welfare Changes

Compared to the baseline scenario, all groups are worse off in this counterfactual scenario (*figure 2.7*). This further decline in welfare is relatively greater for highly skilled workers. It is also slightly overstated with the homothetic model, which is primarily due to the larger further declines in price indices in the non-homothetic

case, as suggested in *figure a13* and *figure a14* of *Appendix A.5*.

Hence, raising tax in order to maintain a relatively high level of subsidy is welfare-deteriorating for all workers, including poor workers, even in case of progressive tax hikes. Further, more subsidy is welfare-deteriorating compared to no subsidy at all. Finally, note that in comparison to the uniform 85% tax hike explored in *count 1*, switching to a progressive tax hike does improve the welfare outcome of low-income workers (non- and low-skilled workers living in both urban and rural areas). However, it does not prevent a further decrease in low-skilled workers' welfare in comparison to the baseline scenario.

Figure 2.7: *Progressive tax hike counterfactual: social welfare changes relative to baseline scenario (in % units)*

Groups	Non-Homothetic		Homothetic	
Urban, non skilled		-7.07		-7.54
Urban, low skilled		-7.35		-8.24
Urban, high skilled		-11.21		-13.81
Rural, non skilled		-4.84		-4.61
Rural, low skilled		-6.70		-6.34
Rural, high skilled		-6.93		-9.27
All groups		-6.76		-7.33

Note: Orange indicates a relative decrease in the social welfare with respect to the baseline scenario

In absolute terms, the welfare losses shown in *figure 2.8* are still distributed in a progressive manner, as they are increasing in skill levels. Furthermore, they are now clearly relatively larger in urban areas.

Figure 2.8: *Progressive tax hike counterfactual: social welfare changes: non homothetic vs homothetic models (in %)*

Groups	Non-Homothetic		Homothetic	
	Count 2	Baseline	Count 2	Baseline
Urban, non skilled	-10.08	-3.01	-14.72	-7.18
Urban, low skilled	-10.28	-2.93	-15.72	-7.48
Urban, high skilled	-15.79	-4.58	-23.92	-10.11
Rural, non skilled	-9.11	-4.27	-12.05	-7.44
Rural, low skilled	-10.58	-3.88	-14.04	-7.70
Rural, high skilled	-14.75	-7.83	-19.85	-10.58
All groups	-10.97	-4.21	-15.39	-8.06

Note. This table gives the social welfare changes $\{\hat{W}_{soc}^g\}_g$ and \hat{W}_{soc} for both non-homothetic and homothetic models. The orange (resp. green) color is indicative of declining (increasing) welfare

2.3 Complementary welfare improving mitigating counterfactuals

2.3.1 Increase in foreign borrowings

Count 3 (*high foreign borrowing*): offset the 58% decrease in foreign borrowing and borrow even more: tripling the level of the pre-shock borrowing

Table 2.3: *High foreign borrowing counterfactual: changes in key rates (in %)*

<i>item</i>	<i>baseline</i>	<i>count 3</i>	<i>gap</i>
<i>non-homothetic</i>			
<i>GDP growth rate</i>	− 4.95	+ 1.00	+ 5.57
<i>Subsidy rate</i>	− 100	− 53.2	+ 46.8
<i>homothetic</i>			
<i>GDP growth rate</i>	− 8.70	− 3.40	+ 5.34
<i>Subsidy rate</i>	− 100	− 50.6	+ 49.4
<i>Foreign borrowing, $\bar{\Lambda}$</i>	$0.41x\Lambda_{baseline}$	$3x\Lambda_{baseline}$	

There is positive growth in the real GDP, a clear improvement compared to the negative growth observed in the baseline scenario (*table 2.3*). The government's revenue is positively impacted by increases in both taxation revenue (via GDP growth) and foreign borrowing. As a result, there is a relative increase in the government's resources, leading to a more moderate decline in the subsidy rate relative to the baseline scenario. More precisely, the non-homothetic and homothetic models predict a 53.2% decrease ($\uparrow \hat{\beta} : 0 \rightarrow 0.468$) and a 50.6% decrease ($\uparrow \hat{\beta} : 0 \rightarrow 0.494$), respectively, in the oil subsidy compared to the 100% decrease observed in the baseline scenario.

Total Welfare Effects

First, I compute for each subgroup the relative changes with respect to the baseline scenario in the income and price index effects (*figure a15* and *figure a16*). I also summarize for each subgroup the combined net income and cost of living effects, i.e. the changes in the real consumption levels in *figure 2.9* (non-homothetic model) and *figure 2.10* (homothetic model).

Figure 2.9: *High foreign borrowing counterfactual: welfare change relative to baseline scenario: non-homothetic model (in % units)*

Groups	Sectors of employment			
	Crude oil	Agriculture	Manufacturing	Service
Urban, non skilled	-2.78	2.19	1.62	4.17
Urban, low skilled	-1.49	2.64	1.63	4.89
Urban, high skilled	-1.80	0.94	3.17	5.62
Rural, non skilled	-2.11	2.23	2.34	4.78
Rural, low skilled	-4.07	2.32	1.37	4.97
Rural, high skilled	-2.85	2.97	4.28	5.58

With both models (*figure 2.9* and *figure 2.10*), this counterfactual leads to relative (with respect to the baseline scenario) welfare losses for all crude oil workers and relative welfare gains for all other workers. Oil workers experience relative increases in the cost of living and an unchanged net income effect, compared to the baseline scenario. The other workers experience relative increases in cost of living and even more significant subsequent increases in the net income.

Figure 2.10: *High foreign borrowing counterfactual: welfare change relative to baseline scenario: homothetic model (in % units)*

Groups	Sectors of employment			
	Crude oil	Agriculture	Manufacturing	Service
Urban, non skilled	-1.57	3.25	3.24	5.28
Urban, low skilled	-1.57	3.25	3.24	5.28
Urban, high skilled	-1.57	3.25	3.24	5.28
Rural, non skilled	-1.57	3.25	3.24	5.28
Rural, low skilled	-1.57	3.25	3.24	5.28
Rural, high skilled	-1.57	3.25	3.24	5.28

Figure 2.11: *High foreign borrowing counterfactual: social welfare changes relative to baseline scenario (in % units)*

Groups	Non-Homothetic	Homothetic
Urban, non skilled	2.03	3.26
Urban, low skilled	2.18	3.29
Urban, high skilled	2.34	3.65
Rural, non skilled	2.24	3.26
Rural, low skilled	2.07	3.29
Rural, high skilled	3.67	3.73
All groups	2.32	3.36

Social Welfare Changes

Increasing the fiscal and current account deficits in order to maintain a relatively high level of subsidy (with respect to the baseline scenario) improves current welfare (*figure 2.11*). These improvements are relatively larger for highly skilled workers. However, these welfare gains must be weighed against the reductions in future welfare due to debts incurred.

In absolute terms, the policy change still leads to welfare losses (*figure 2.12*) that continue to be distributed in a progressive manner.

Figure 2.12: *High foreign borrowing counterfactual: social welfare changes: non homothetic vs homothetic models (in %)*

Groups	Non-Homothetic		Homothetic	
	Count 3	Baseline	Count 3	Baseline
Urban, non skilled	-0.98	-3.01	-3.92	-7.18
Urban, low skilled	-0.75	-2.93	-4.19	-7.48
Urban, high skilled	-2.24	-4.58	-6.45	-10.11
Rural, non skilled	-2.03	-4.27	-4.18	-7.44
Rural, low skilled	-1.81	-3.88	-4.41	-7.70
Rural, high skilled	-4.16	-7.83	-6.85	-10.58
All groups	-1.89	-4.21	-4.70	-8.06

Note. This table gives the social welfare changes $\{\hat{\mathcal{W}}_{\text{soc}}^g\}_g$ and $\hat{\mathcal{W}}_{\text{soc}}$ for both non-homothetic and homothetic models. The orange (resp. green) color is indicative of declining (increasing) welfare

2.3.2 Progressive and neutral tax redistribution

Count 4 (*fiscally neutral progressive tax redistribution*): cutting tax for the 3 lowest tax income brackets and raising tax for the 3 highest tax income bracket⁷ while maintaining the ex-ante 14.15% national tax average

There is a lower decline in the real GDP in comparison to the baseline scenario (*table 2.4*). The government's revenue is positively impacted by the slight increase in the GDP. But with the fiscally neutral tax reform that maintained the 14.15% national tax average, the additional tax revenue is negligible. As a result, the subsidies' removal remains in place.

Table 2.4: *Progressive tax redistribution counterfactual: changes in key rates (in %)*

<i>item</i>	<i>baseline</i>	<i>count 4</i>	<i>gap</i>
<i>non-homothetic</i>			
<i>GDP growth rate</i>	− 4.95	− 4.95	+ 0.00
<i>Subsidy rate</i>	− 100	− 100	− 0.00
<i>homothetic</i>			
<i>GDP growth rate</i>	− 8.70	− 8.70	− 0.00
<i>Subsidy rate</i>	− 100	− 100	+ 0.00
<i>National average tax, $\bar{\tau}$</i>	14.15%	14.15%	

Total Welfare Effects

First, I compute for each subgroup the relative changes with respect to the baseline scenario in the income and price index effects (*figure a17* and *figure a18*). I also

⁷That is, -11% for first bracket; -9% for second bracket; -5% for third bracket; +5% for fourth bracket; +10% for fifth bracket; and +20% for sixth bracket - see *table 1.7* for references on income brackets and ex-ante taxation rates. As a consequence, the tax hikes mostly affect highly skilled workers since they are mostly concentrated in the top 3 income brackets; similarly, the tax cuts benefit non- and low-skilled workers as they are mostly concentrated in the bottom 3 income brackets.

summarize for each subgroup the combined net income and cost of living effects, i.e. the changes in the real consumption levels in *figure 2.13* (non-homothetic model) and *figure 2.14* (homothetic model).

Figure 2.13: *Progressive tax redistribution counterfactual: welfare change relative to baseline scenario: non-homothetic model (in % units)*

Groups	Sectors of employment			
	Crude oil	Agriculture	Manufacturing	Service
Urban, non skilled	0.60	0.50	0.13	-0.09
Urban, low skilled	0.13	0.29	0.15	0.41
Urban, high skilled	-0.73	-1.37	-1.68	-0.69
Rural, non skilled	0.74	0.64	0.16	0.82
Rural, low skilled	0.74	0.69	-0.29	0.59
Rural, high skilled	0.10	0.08	-0.31	0.04

In the non-homothetic model (*figure 2.13*), this counterfactual leads to relative (with respect to the baseline scenario) welfare losses for all highly skilled workers living in urban areas, all workers employed in the manufacturing sector, and rural low- and highly skilled workers in the manufacturing sector. However, the counterfactual leads to relative welfare gains for all other workers, including those employed in the agricultural sector. The welfare losses are all explained by further decreases in the cost of living and even further decreases in the income, compared to the baseline scenario (*figure a17*). As for the welfare gains, they can be explained by the relative increase in the cost of living and even larger relative increase in the net income, compared to the baseline scenario. In the homothetic case (*figure 2.14*), the picture is similar to the non-homothetic model (with details in *figure a18*).

Figure 2.14: *Progressive tax redistribution counterfactual: welfare change relative to baseline scenario: homothetic model (in % units)*

Groups	Sectors of employment			
	Crude oil	Agriculture	Manufacturing	Service
Urban, non skilled	0.73	0.59	0.22	-0.17
Urban, low skilled	0.14	0.34	0.25	0.43
Urban, high skilled	-0.88	-1.74	-2.10	-0.97
Rural, non skilled	0.92	0.69	0.23	0.83
Rural, low skilled	0.92	0.74	-0.26	0.60
Rural, high skilled	0.14	0.09	-0.32	-0.02

Figure 2.15: *Progressive tax redistribution counterfactual: social welfare changes relative to baseline scenario (in % units)*

Groups	Non-Homothetic	Homothetic
Urban, non skilled	0.35	0.44
Urban, low skilled	0.23	0.31
Urban, high skilled	-1.17	-1.53
Rural, non skilled	0.61	0.66
Rural, low skilled	0.46	0.55
Rural, high skilled	0.03	0.02
All groups	0.24	0.26

Social Welfare Changes

This progressive and fiscally neutral tax redistributive counterfactual is welfare-improving for all groups except for highly skilled workers in urban areas, as shown in both models (*figure 2.15*). The relative welfare gains are larger in rural areas. Hence, this counterfactual delivers a slightly progressive welfare improvement while maintaining being fiscally responsible.

In absolute terms, these welfare changes still reflect losses (*figure 2.16*) and continue to be distributed in a progressive manner, as they are increasing in skill levels and relatively larger in rural areas. Finally, the homothetic model predicts higher welfare losses in the group of highly skilled urban workers, but also higher welfare gains across all other groups.

Figure 2.16: *Progressive tax redistribution counterfactual: social welfare changes: non-homothetic vs homothetic models (in %)*

Groups	Non-Homothetic		Homothetic	
	Count 4	Baseline	Count 4	Baseline
Urban, non skilled	-2.66	-3.01	-6.74	-7.18
Urban, low skilled	-2.70	-2.93	-7.17	-7.48
Urban, high skilled	-5.75	-4.58	-11.63	-10.11
Rural, non skilled	-3.66	-4.27	-6.78	-7.44
Rural, low skilled	-3.42	-3.88	-7.15	-7.70
Rural, high skilled	-7.80	-7.83	-10.56	-10.58
All groups	-3.98	-4.21	-7.80	-8.06

Note. This table gives the social welfare changes $\{\hat{W}_{soc}^g\}_g$ and \hat{W}_{soc} for both non-homothetic and homothetic models. The orange (resp. green) color is indicative of declining (increasing) welfare

2.4 Conclusion

Understanding the macroeconomic and welfare implications of domestic oil price subsidies' removal in the wake of an external oil shock is an important empirical question that has not been studied very much in developing countries, especially through the lens of structural models that would make it possible to examine the welfare implications of public mitigating policies that are complementary or alternative to the subsidy removal. In this chapter, I studied the welfare implications of different policy options, including some leading up to a partial reinstatement of the subsidies.

First, in the wake of a negative external shock, it is important for the government to adopt the appropriate mitigating policies. The first question I investigated was as follows: Was removing the oil subsidies the best strategy? Concretely, should the government have sought more internal resources in order to maintain relatively high levels of subsidy, or was the subsidy removal the best strategy? To answer this question, I performed two counterfactuals. The first two explored different tax hike schemes: I allowed for the government to raise more revenue through either uniform or progressive tax hikes. I found that although tax hike scenarios can lead to non-zero subsidy levels, they also lead to further welfare losses with respect to the zero subsidy-induced baseline scenario. In other words, maintaining the subsidies would have led to further welfare losses across all income groups. This makes the removal an imperfect but necessary measure, as maintaining the subsidies would have had led to worse welfare outcomes.

I also studied other policy options that the government could explore to facilitate welfare-improving outcomes for the poor while maintaining the zero-subsidy policy. First, I showed that an increase in foreign borrowing leads to relative welfare gains. However, this counterfactual change should be considered carefully for two reasons: first, the welfare gains are regressive as they benefit high-income workers more than they do low-income workers; second, since the model does not incorporate debt reimbursement⁸, there may be a missing welfare-loss inducing component coming from future (and non accounted for in this model) negative impacts of debt reimbursement. Second, I showed that fiscally neutral progressive tax redistribution schemes a la robin-des-bois⁹ can deliver relative welfare gains with respect to the

⁸There is no dynamic in the model.

⁹Tax cut scenarios could also be explored. However, they lead to obvious welfare-improving

2016 welfare outcomes, as well as a weakly progressive redistribution of these gains that primarily benefits poor workers. In other words, this type of tax scheme leads not only to relative welfare gains, but also to a transfer of welfare to some poor workers. Additionally, I showed that not accounting for non-homotheticity leads to biases when evaluating the magnitude of the subsequent (relative) welfare losses or gains.

Finally, it is worth noting that the results and methodology presented in this chapter have useful implications for public policy in two regards. First, the results show that the oil subsidy is not only inefficient, but also leads to welfare losses, especially for poor consumers. Second, the methodology developed provides a path for targeted policy changes, allowing the government to use readily available micro and macro data to measure the changes in income and cost of living across groups of a population that can be defined in various ways. This makes it possible for the government to conduct policy interventions while targeting all or some groups or subgroups of populations¹⁰.

outcomes. Furthermore, cutting tax would affect the other sectoral expenditures that I am keeping constant. For these reasons, I do not report the results associated with tax cut scenarios.

¹⁰For example, the government could target groups defined by race, gender, region, or state, among other demographics

Chapter 3

Child Labor and Future Human Capital: Understanding the Role of Ability

3.1 Introduction

The question as to how child labor impacts human capital has long been uncontroversial. Indeed, most studies (Becker et al., 1991; Basu & Van, 1998; Baland & Robinson, 2000; Das & Deb, 2006) have embraced the view that child labor undermines human capital because it interferes with school performance thanks to work-related high stress levels or other health hazards (ILO¹, 1999). As a consequence, two modeling assumptions have been used extensively in the literature: (i) there is a trade-off between child labor and school (Glomm, 1997; Baland & Robinson, 2000); and (ii) human capital solely depends - in a positive way - on education and thus depends - in a negative way - on child labor. Despite this widespread consensus, in the 2000s, a set of empirical (descriptive) studies (Anker, 2000; ILO, 2011) concluded that some but not all forms of child labor are harmful to children. This new line of studies² pointed to the fact that the normal forms of child labor, unlike the worst forms of child labor³, contribute to building up children's human capital through learning-by-doing (Boyden et al., 1998). As a consequence, the newly emerging view on the role of child labor, led by Dessy and Pallage, suggests that child labor can actually make both positive and negative contributions to human capital, depending upon its gravity (Dessy & Pallage, 2005; Sugawara, 2011)⁴.

In this chapter, I show that the aforementioned empirical findings are relevant only from a contemporaneous perspective. Indeed, child labor can be seen as creating a learning-by-doing type of experience for children. Therefore, it can have a positive return to earnings if measured in young ages (childhood). However, once I introduce dynamics⁵ and turn to future or adulthood earnings, I show that child labor is always harmful to children. Therefore, I argue that the approach introduced by Dessy and Pallage in modeling child labor as a two-way contributor to human capital can be

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²It is worth noting that these studies have no theoretical foundation, and the derived conclusions arise only from empirical (descriptive) analyses.

³"Normal forms of child labor" refers to any labor done by a child aged 5 to 14 (Convention 138, ILO). "Worst forms of child labor," in contrast, refers to any type of labor that jeopardizes the physical, mental, or moral well-being of a child, either because of the nature of the labor or because of the conditions under which it is carried out (Convention 182, ILO).

⁴A key modeling assumption now embraced in the literature is that both child labor and education contribute to human capital and that child labor's contribution can be positive.

⁵This is in the sense of looking at childhood + adulthood and thereby analyzing lifetime human capital accumulation patterns.

misleading⁶. I make a theoretical argument that the apparent positive role played by child labor in a mincer-type of regression or in descriptive analyses reflects either a disguised positive role of the children's ability or a simple positive correlation between child labor and human capital for which there is no causal interpretation⁷. A logical consequence of my argument is that there is no such distinction to be made between the normal and worst forms of child labor when considering their (opposite) contributions to human capital, on contrary to Dessy and Pallage (2005). Furthermore, I show that although child labor is always harmful in the long run, it can give rise to higher future wages in some instances⁸. However, these instances are characterized by high levels of child labor (greater than the optimal levels), low levels of education, and relatively small returns to education. On a macroeconomic scale, this describes a bad equilibrium with low-skilled workers and poorly paid informal jobs, which is the scenario observed in many developing countries⁹.

Concretely, I build a simple two-period-lived individuals model of human capital,¹⁰ in which both inter-generational transfers (financial assets from parents) and inter-temporal transfers (borrowings or savings) can potentially take place. In order to better capture the human capital accumulation process, I divert from the standard literature on child labor by considering a critical question raised in the education literature: the role of ability in shaping human capital. Indeed, I assume that human capital is a not only a function of education, as is commonly assumed in the child labor literature but is also a function of *ability*¹¹ (Lochner et al., 2011). Moreover, for the sake of this modeling exercise, I assume as is standard in the literature on child labor, that labor and schooling are the only competing claims on children's time (Glomm, 1997; Baland & Robinson, 2000). Furthermore, in order to better fit the context of a developing economy, which is the context of this chapter, I use a set of assumptions that makes it possible to guarantee an environment that complies with the reality observed in these economies. First, I consider the cost of schooling and therefore account for the fact that education can be a financial burden on individu-

⁶The empirical evidence supporting child labor as a potential builder of long-term human capital can be misleading.

⁷This positive correlation turns negative once I introduce ability into the estimation (see **Section 3**).

⁸I show that a positive correlation is observed between child labor and future wages.

⁹This is simply an intuition worth testing; it is not the focus of this chapter.

¹⁰Individuals work and/or go to school in the first period, and they work in the second period.

¹¹Every child is endowed with a certain amount of ability.

als. Second, although I allow for the existence of a credit market and assume that individuals can borrow from the credit market in order to finance their consumption or school cost, I distinguish unconstrained credit (perfect capital market) from constrained credit (imperfect capital market) cases. I specifically focus on a setting with credit constraints, as this setting is more likely to occur in these economies (see Banerjee & Duflo, 2004, 2005), and because this setting is likely to make it difficult and occasionally impossible for some individuals to finance their education.

I therefore use the possibility of frictions on the credit market to derive various types of equilibrium¹² by examining both unconstrained and constrained credit cases. I subsequently link the conditions derived in each of these equilibria to the interactions among ability, child labor, and future human capital or earnings. Ultimately, I demonstrate that child labor always contributes negatively to human capital, but in some specific instances, it can also be positively correlated to human capital¹³. I show that this latter correlation is in fact the rationale behind the the aforementioned studies' empirical findings. One should not jump to conclusion that child labor possibly contributes positively to human capital, however, as this positive correlation has no causal interpretation.

More broadly, I contribute to the theoretical and empirical literature on child labor, education, and human capital. First, I show that there exists a selection mechanism that operates in the decision-making process of parents in the sense that the level of ability will determine the time allocation decisions. I show that this mechanism can go in both directions. Indeed, as is standard in the literature, I find that children with high ability are likely to work less, or in other words, to allocate more time to school because of their capacity to perform well in school. I call this phenomenon “negative selection” to indicate that child labor is negatively selected with respect to ability. However, I also prove that in particular instances¹⁴, children

¹²I derive an unconstrained credit equilibrium (optimal) and a constrained credit equilibrium (non-optimal).

¹³I show that child labor can be either negatively correlated to human capital, in which case it is harmful, or positively correlated to human capital. In this latter case, the correlation reflects a disguised role of ability under a constrained credit equilibrium and does not carry any causal implications.

¹⁴Intuitively, one can think of very poor individuals (also hit the hardest by the credit constraint), given that on the one hand, education for these individuals is a luxury good, and on the other hand, they are likely to put high-ability children to work as they would perform as child laborers and therefore earn higher wages

with high ability are likely to work more, i.e. allocate less time to school because of their capacity to perform well in manual work. I call this phenomenon “positive selection” to indicate that child labor is positively selected with respect to ability. To the best of my knowledge, this thesis is the first to highlight the existence of a positive selection phenomenon¹⁵. Second, using the selection mechanism, I show that ability actually has both direct and indirect effects on human capital, although the indirect effect has been overlooked in the literature. I derive an actual return to ability as a sum of these two effects and show that the indirect component is always different from zero, which means that a simple return to ability featuring only its direct effect, as derived in most of the literature, can be misleading. I show that this actual return to ability can be smaller than the direct return to ability. I call this phenomenon the “curse of ability”: Relatively high-ability children work relatively more, hence accumulate relatively lower levels of human capital, and subsequently earn lower wages in adulthood.

On the basis of these results, I am able to challenge the position taken by Dessy and Pallage (2005) in modeling child labor as a potential contributor (positive return) to human capital on the sole basis of the empirically tested positive relationship between child labor and human capital or earnings. My findings show that regardless of the type of equilibrium, child labor always negatively contributes to human capital, although there are certain instances in which it can be positively or negatively correlated to human capital. The positive correlation case explains how these empirical findings were obtained. This positive correlation should not be confused with a causality relationship¹⁶. These findings suggest the importance of reconsidering the relevance of the framework promoted by Dessy and Pallage (2005), especially in terms of its aforementioned new modeling assumptions.

The rest of the chapter is organized as follows. First, I present the theoretical model. Second, I derive a set of theoretical predictions linking a selection mechanism to future human capital. Third, I conclude.

¹⁵Negative selection, which is a quite intuitive result, is derived for example in Lochner et al. (2011) as a positive relationship between education and ability, i.e. a negative relationship between child labor and ability.

¹⁶One can also think of it as indicative of the disguised positive role of ability through the positive selection that occurs in that case

3.2 Models

I consider a model of human capital investment in which two-period-lived individuals invest in both schooling and work in the first period, and invest only in work in the second period (Lochner et al., 2011). Their preferences are

$$U(c_1, c_2) = U(c_1) + \beta U(c_2) \quad (3.2.1)$$

where c_t is consumption in periods $t \in \{1, 2\}$ and $\beta \in (0, 1)$ is a discount factor. In addition, $U(\cdot)$ is strictly increasing and strictly concave, and it satisfies the standard Inada conditions.

Each individual is endowed with a unit time in each of both periods. She is also endowed with financial asset $A \geq 0$ and ability $a > 0$ but only in the first period. Initial assets capture all familial transfers, while ability reflects innate factors, early parental investments, and other characteristics that shape the returns to education. I take (A, a) as given.

At the beginning of the first period, individuals (children) allocate their unit time between labor l and education $\tau = 1 - l$ ¹⁷. On the one hand, children can work for wage w_1 and therefore earn revenue $y_1 = w_1 l$. On the other hand, they can make human capital investments in the form of labor allocated to education at a cost θ (for tuition and fees), using their financial endowment and eventually supplementing it with an amount borrowed from a credit market. Young individuals can borrow d (or save, in which case $d < 0$) at gross interest rate $R > 1$. I consider the possibility of a credit constraint by imposing a fixed and exogenous borrowing constraint \bar{d} on the amount of borrowing, i.e. by assuming $d \leq \bar{d}$ where $0 \leq \bar{d} < \infty$ is uniform across agents.

In the second period, grown children, i.e. adults, work with an accumulated level of human capital that increases their post-school labor earnings to $y_2 = w_2 h(\tau, a)$ where $w_2 \geq w_1$ is the wage earned in adulthood and h is the level of human capital that is a positive, strictly increasing, and strictly concave function in its two terms

¹⁷As is standard in the literature on child labor, work and schooling are considered the only competing claims on a child's time (Glomm, 1997; Baland & Robinson, 2000).

with $h_{\tau a}, h_{a\tau} > 0$ ¹⁸. h_{τ} is the return to education and h_a is the return to ability¹⁹. The budget constraints in both periods are as follows:

$$c_1 = A + w_1 l + d - \theta(1 - l) \quad (3.2.2)$$

$$c_2 = w_2 h(1 - l, a) - dR \quad (3.2.3)$$

The optimization problem consists of maximizing (3.2.1) under both periods' budget constraints, (3.2.2) and (3.2.3). For the sake of simplicity, I assume that there is an interior optimum level of child labor and assume away corner solutions.

3.2.1 Unconstrained Equilibrium

In case of an unconstrained credit market, i.e. when $d < \bar{d}$, the optimal level of child labor l^u is such that the first-order conditions with respect to child labor and borrowing are respected. I have

$$\begin{cases} (w_1 + \theta) U'(c_1) = \beta w_2 h_{\tau}(1 - l, a) U'(c_2) \\ U'(c_1) = \beta R U'(c_2) \end{cases} \quad (3.2.4)$$

The optimality condition is therefore

$$\frac{w_2}{w_1 + \theta} h_{\tau}(1 - l^u, a) = R \quad (3.2.5)$$

I show in **Appendix B.1.1** that in case of an unconstrained equilibrium, child labor is strictly decreasing in children's ability a and independent of initial wealth A . High-ability children work less and study more since they are likely to perform well at school because of the strategic complementarity between ability and school. Therefore, they can expect a high return to education, which combined with their expected high actual return to ability, will yield a high level of human capital and ultimately high earnings.

I also show that borrowing is strictly decreasing in wealth and increasing in

¹⁸This reflects a strategic complementarity relationship.

¹⁹I later show that $h \equiv h(1 - l(a), a)$ and derive $\hat{h}_a = -\frac{\partial l}{\partial a} h_{\tau} + h_a$, which I call the "actual return to ability." The return to ability h_a accounts for the direct effect of ability; however, there is also an indirect effect of ability that should be added to the direct effect. I call the two combined effects the "actual return to ability," or \hat{h}_a .

ability. On the one hand, individuals who inherit higher initial wealth are less likely to borrow. On the other hand, ability increases borrowing because more capable individuals are likely to take more credit to finance their schooling or to increase their level of consumption in the first period, since they expect a higher net lifetime income.

I now describe in *lemma 1* (proof in **Appendix B.1.1**) the negative selection phenomenon, a corollary to the unrestricted optima result derived by Lochner & Monge-Naranjo (2011).

Lemma 1. - *unconstrained equilibrium and negative selection:* *When the credit market is perfect (unconstrained credit), child labor is a strictly decreasing function of ability; this implies a negative selection phenomenon.*

I now study the implications of an imperfect credit market, particularly the case in which constraints exist on the credit market.

3.2.2 Constrained Equilibrium

I now impose a constraint on the credit market, reflected by $d(a, A) = \bar{d}$.

In the following lemma, I show that the corresponding level of child labor, l^c , is higher than that in the unconstrained equilibrium. I also show that this level of child labor is decreasing in the borrowing limit (proofs in **Appendix B.1.2**).

Lemma 2. - *constrained level of child labor:* *When $d = \bar{d}$, $\frac{w_2}{w_1 + \theta} h_\tau(1 - l, a) > R$ and the corresponding level of child labor, l^{c20} , is strictly greater than l^u . Furthermore, l^c is decreasing in \bar{d} and strictly decreasing in A .*

For each level of a , a constraint in the credit defines a wealth threshold A_{min} below which an individual is constrained. Indeed, with $d(a, A) = \bar{d}$, and knowing that d is increasing in a and decreasing in A , I can define a minimal threshold level for initial wealth, A_{min} as an increasing function of a . I can also determine who is constrained ($A < A_{min}(a)$) and who is unconstrained ($A \geq A_{min}(a)$).

I show in the **Appendix B.1.2** that in case of credit constraint, child labor depends upon both ability and wealth. It is strictly decreasing in initial wealth

²⁰ l^c is the constrained equilibrium level of child labor.

A. However, the way individuals determine child labor allocation in response to children's ability differs and depends upon their degree of aversion to risk. I show that child labor is increasing in ability for more risk-averse individuals and decreasing in ability for less risk-averse individuals. I now define in **Proposition 1** this double selection phenomenon that occurs in the case of an imperfect credit market (proof in **Appendix B.1.3**).

Proposition 1. - unconstrained level of child labor: *Assume $A < A_{min}$ so that the credit constraint binds. There then exists an endogenous threshold level $\gamma > 0$ for Absolute Risk Aversion (ARA)²¹ with $\gamma = h_{\tau a}(\tau, a)/(w_2 h_{\tau}(\tau, a) h_a(\tau, a))$.²², such that when $ARA \leq \gamma$, child labor strictly decreases with the level of ability (negative selection) and when $ARA > \gamma$, child labor strictly increases with the level of ability (positive selection).*

It appears that risk aversion plays an important role in determining the selection mechanism that is in place. To better elucidate this argument, I first note that individuals who are more affected by the credit constraint are likely to be facing financing issues since borrowing is limited. Therefore, their level of aversion to risk, which determines their preference for immediate versus future consumption²³, will have an impact on their time endowment allocation decisions in young childhood.

Thus, when $ARA \leq \gamma$, i.e. in cases where individuals are moderately averse to risk, the individuals prefer to consume more in the future. However, their limited financing capacity (due to the credit constraint) will force them to work more than they would have liked to, and this explains why they end up with a level of child labor that is greater than in the unconstrained cases (see *lemma 1*). Furthermore, in this subgroup of the population, higher-ability individuals will work less in childhood in order to earn a higher salary in the future and subsequently to consume more; this explains the negative selection result.

As for the more risk-averse individuals ($ARA > \gamma$), their strong preference for immediate consumption along with constrained credit will prompt them to always

²¹Absolute Risk Aversion is defined as $-\frac{U''(c_2)}{U'(c_2)}$

²²In the proof, I show that the threshold for the ARA depends upon the return to school h_{τ} , the return to ability h_a , the degree of complementarity between school and ability $h_{\tau a}$, and the wage rate in old age w_2 .

²³The more averse one is to risk, the more likely it is that she will prefer immediate consumption to future consumption.

prefer to work more in childhood - even more than the moderately averse individuals - leading to a very high level of child labor ($l_{ARA>\gamma}^c > l_{ARA<\gamma}^c > l^u$). Furthermore, in this subgroup of the population, higher-ability individuals will work more in childhood so that they can increase their labor income and subsequently their current consumption; this explains the positive selection result.

3.3 Implications for Future Human Capital

In this section, I use the results derived from the model to make connections among ability, child labor, and human capital. I then link this exercise to the empirical results obtained in the 2000s regarding the role of child labor in human capital formation. Finally, I discuss the implications of these results and use them to shed light on some theoretical ambiguity in the framework developed by Dessy and Pallage (2005).

3.3.1 Child Labor and Future Human Capital

I use the link between type of equilibrium and selection mechanism in order to determine a structural relationship among ability, child labor, and future human capital. From the previously obtained results, I know that ability and child labor time allocation are related in the sense that children's labor decisions are a function of their ability. I also know that a particular selection mechanism corresponds to each type of equilibrium²⁴. Hence, I can derive for each type of equilibrium key outcomes such as the returns²⁵ to school, child labor, and ability, as well as the correlation between human capital and child labor.

First, I make use of the selection mechanism and compute the actual return to ability, \tilde{h}_a , as the sum of an indirect effect of ability and a direct effect of ability. Using $l \equiv l(a)$, I find

$$h(\tau, a) = h(1 - l, a) \equiv h(1 - l(a), a)$$

Therefore, I derive

$$\tilde{h}_a \equiv \frac{\partial h(1 - l(a), a)}{\partial a} = \underbrace{-\frac{\partial l}{\partial a} \times h_\tau(\tau, a)}_{\text{Indirect Effect}} + \underbrace{h_a(\tau, a)}_{\text{Direct Effect}}$$

Therefore, the actual return to ability is always greater (resp. smaller) than the return to ability in **Case 1** and **Case 2** (resp. **Case 3**). This means that not

²⁴Indeed, I derive that an unconstrained equilibrium corresponds to negative selection, while a constrained equilibrium corresponds to either positive or negative selection, depending upon whether individuals are highly averse to risk,

²⁵(The returns to school are considered with respect to human capital.

accounting for the selection phenomenon could lead to an undervaluation or overvaluation of the return to ability or future human capital in general.

In the following proposition, I show the link between the selection mechanism and the human capital outcomes.

Proposition 2. : *In the unconstrained equilibrium, or in the constrained equilibrium with moderately low-risk agents, the actual return to ability \tilde{h}_a is greater than the direct return to ability h_a . However, in the constrained equilibrium with highly risk-averse individuals, the actual return to ability \tilde{h}_a is smaller than the direct return to ability h_a . In all cases, child labor is negatively correlated to future human capital or earnings; that is, child labor is always harmful to children's future earnings.*

The positive selection phenomenon leads to a relatively smaller actual return to ability and future earnings. Indeed, a high level of ability might help increase income in childhood by shifting choices from school to work. However, it also leads to a relative decrease in the return to ability on future income. In the case of negative selection, there is no credit constraint, or else there is a preference for future consumption (low-risk agents). Therefore, high-ability children tend to increase the amount of time they participate in schooling and end up with a relatively higher return to ability and higher earnings in adulthood.

Table 3.1: *Human capital outcomes by type of equilibrium*

Types of Equilibrium	Selection	$\tilde{h}_a > h_a$	$\text{corr}(h, l)$
Case 1 - Unconstrained	—	<i>yes</i>	—
Case 2 - Constrained with $\text{ARA} \leq \gamma$	—	<i>yes</i>	—
Case 3 - Constrained with $\text{ARA} > \gamma$	+	<i>no</i>	—

In *table 3.1*, I show a summary of the results, including the signs of the future return to school h_τ and return to child labor h_l , as well as how the direct return to ability h_a compares to the actual return to ability \tilde{h}_a . The table also shows the implications in terms of the sign of the correlation between child labor and future human capital, $\text{corr}(h, l)$ ²⁶.

²⁶I note that the return to school is always positive and the return to child labor is always

The proof to this proposition (see **Appendix B.1.4**) clearly shows that a strong preference for immediate consumption will prompt individuals to always want to work more in childhood. They can therefore choose to use up a very large amount of their time available for working and allocate only minimal time to education²⁷.

negative. Indeed, $h_\tau > 0$, and using $l = 1 - \tau$, I derive $h_l = -h_\tau < 0$. I also note that the return to ability h_a is always positive.

²⁷ < 1 since I have imposed an interior solution for child labor.

3.4 Conclusion

In this chapter, I showed that child labor can play an important role in shaping and determining future skill. I find that regardless of case, the return to child labor is always negative. However, child labor and future human capital can be influenced in different ways by the level of children's ability. The selection mechanism and the level of risk aversion seem to play key roles in determining the different outcomes.

I improve upon the framework developed by Dessy and Pallage (2005) by showing clearly the distinction that needs to be made between correlation and causality in terms of the relationship between child labor and future human capital. I also suggest more structural and general conditions yielding positive selection, instead of focusing on the opposite roles played by the worst forms and normal forms of child labor²⁸. Positive selection suggests that performing more labor in childhood might be beneficial when children reach adulthood. However, it actually leads to a poverty trap phenomenon.

²⁸The results clearly indicate that the form of child labor does not matter. Its return is always negative. However, the return in childhood can vary depending on the type of labor in the sense that the worst forms of child labor, which are dangerous and often illegal, are usually paid better than the normal forms of child labor. This is not the focus of this chapter, but this exemplifies the argument of Dessy and Pallage (2005), who consider normal (resp. worst) forms of child labor to yield positive (resp. negative) returns.

Appendix A

On Chapters 1 - 2 (Part I)

A.1 Motivating Facts

Graph a1. Sectoral expenditures across all subgroups (source: LSMS)



Table a1. *Subgroup-level sectoral expenditure shares (in %)*

<i>subgroups (g,i)</i>		<i>expenditure shares</i>			
<i>group</i>	<i>employment</i>	<i>refined oil</i>	<i>agriculture</i>	<i>manufacturing</i>	<i>service</i>
<i>Urban Non Skilled</i>	<i>oil</i>	0.10	0.33	0.36	0.21
	<i>agriculture</i>	0.11	0.33	0.32	0.24
	<i>manufacturing</i>	0.10	0.34	0.34	0.23
	<i>service</i>	0.09	0.30	0.24	0.37
<i>Urban Low Skilled</i>	<i>oil</i>	0.13	0.37	0.36	0.13
	<i>agriculture</i>	0.13	0.33	0.28	0.26
	<i>manufacturing</i>	0.10	0.32	0.32	0.26
	<i>service</i>	0.11	0.31	0.30	0.28
<i>Urban High Skilled</i>	<i>oil</i>	0.16	0.31	0.25	0.27
	<i>agriculture</i>	0.10	0.27	0.24	0.39
	<i>manufacturing</i>	0.17	0.28	0.31	0.24
	<i>service</i>	0.15	0.29	0.26	0.30
<i>Rural Non Skilled</i>	<i>oil</i>	0.15	0.34	0.24	0.27
	<i>agriculture</i>	0.10	0.38	0.32	0.19
	<i>manufacturing</i>	0.11	0.38	0.33	0.18
	<i>service</i>	0.08	0.38	0.37	0.17
<i>Rural Low Skilled</i>	<i>oil</i>	0.08	0.32	0.25	0.35
	<i>agriculture</i>	0.10	0.39	0.32	0.19
	<i>manufacturing</i>	0.10	0.34	0.27	0.29
	<i>service</i>	0.10	0.37	0.30	0.23
<i>Rural High Skilled</i>	<i>oil</i>	0.11	0.33	0.30	0.26
	<i>agriculture</i>	0.14	0.31	0.33	0.22
	<i>manufacturing</i>	0.19	0.33	0.31	0.16
	<i>service</i>	0.13	0.34	0.30	0.24

Source: LSMS

Expenditure Patterns: Engel Curves (Linear vs Polynomial)

Figure a1. *Pre-shock Engel Curves for Agricultural Goods*

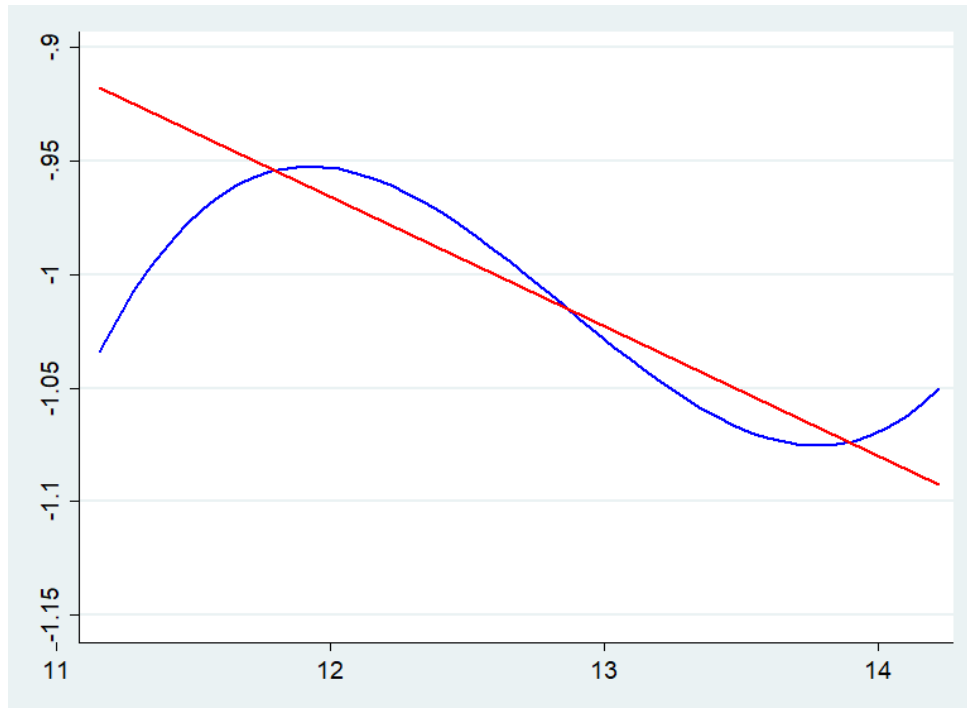


Figure a2. *Post-shock Engel Curves for Agricultural Goods*

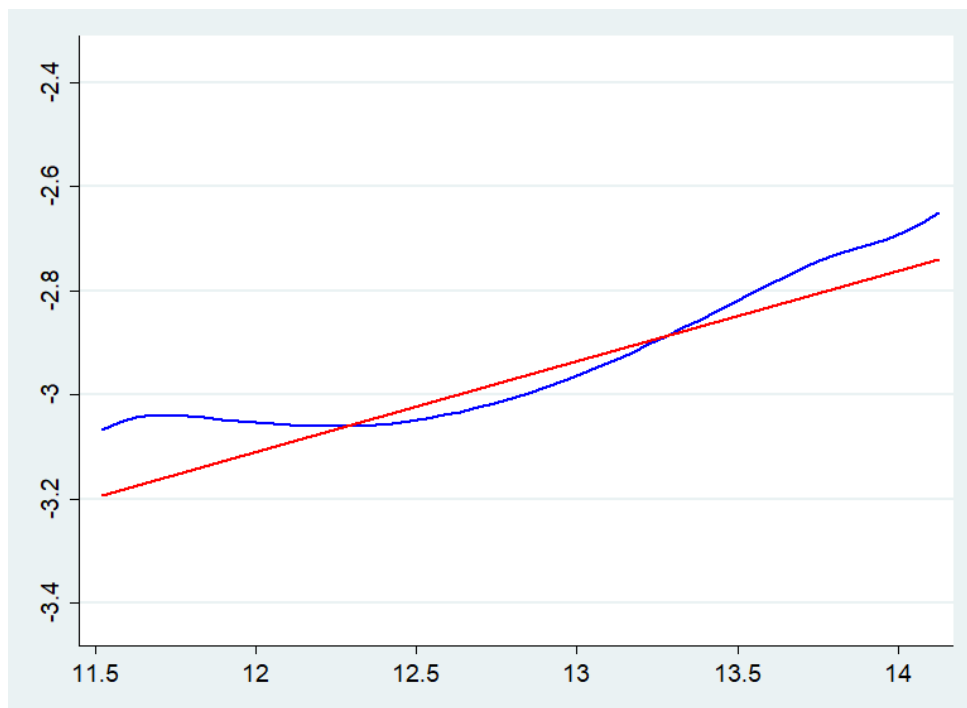


Figure a3. *Pre-shock Engel Curves for Manufacturing Goods*

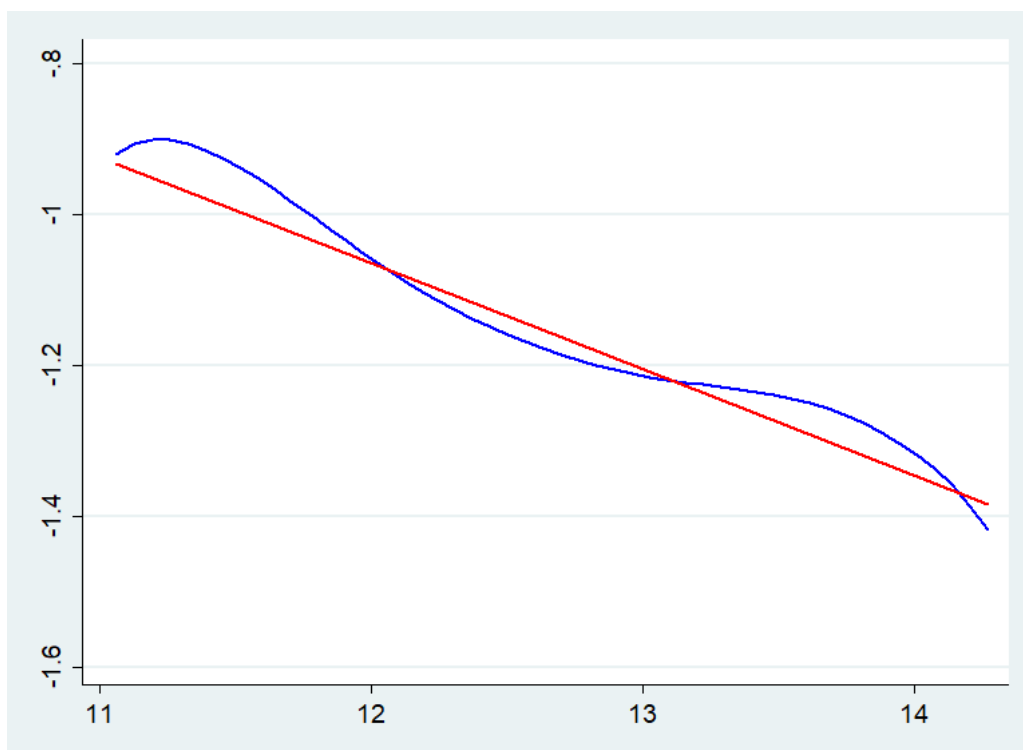


Figure a4. *Post-shock Engel Curves for Manufacturing Goods*

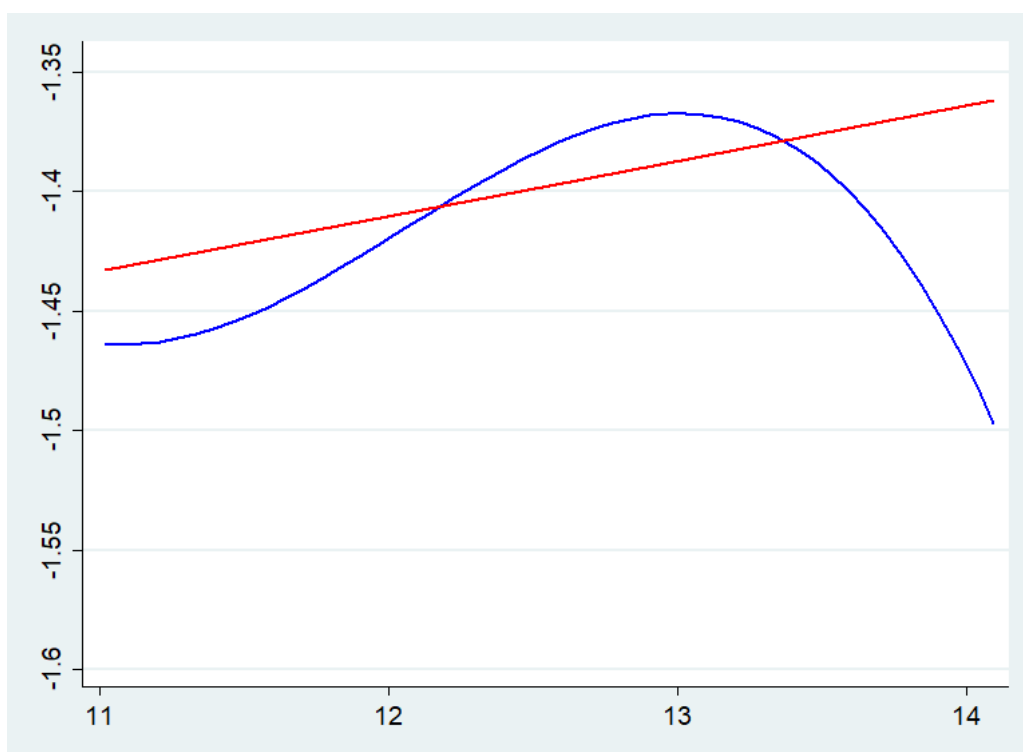


Figure a5. *Pre-shock Engel Curves for Service Goods*

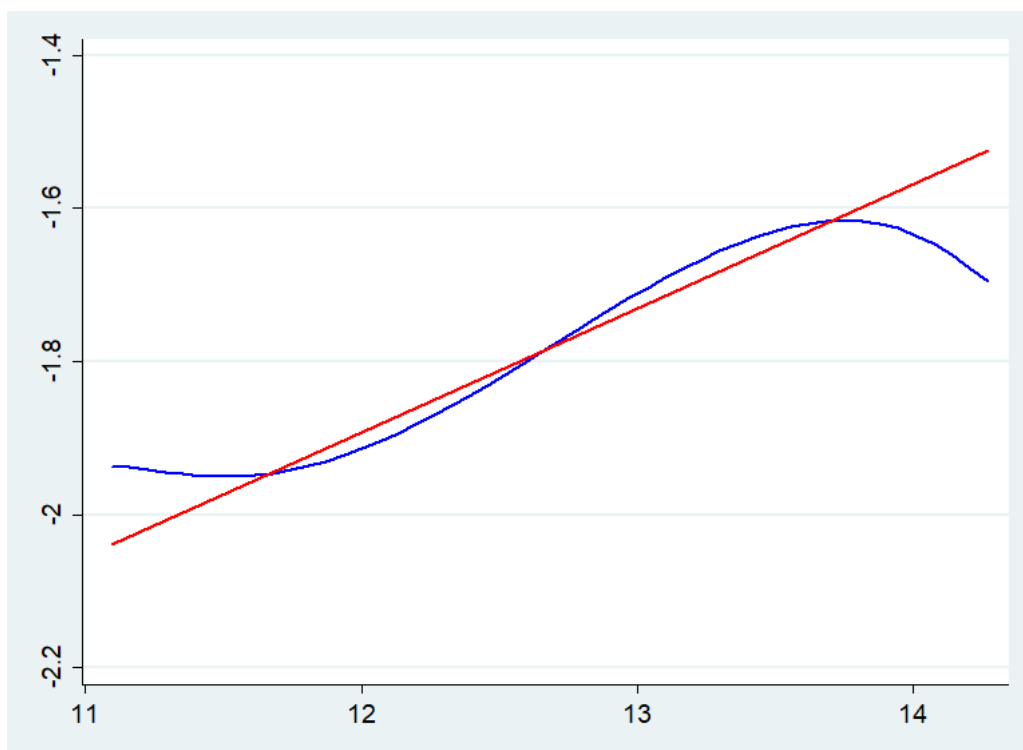


Figure a6. *Post-shock Engel Curves for Service Goods*

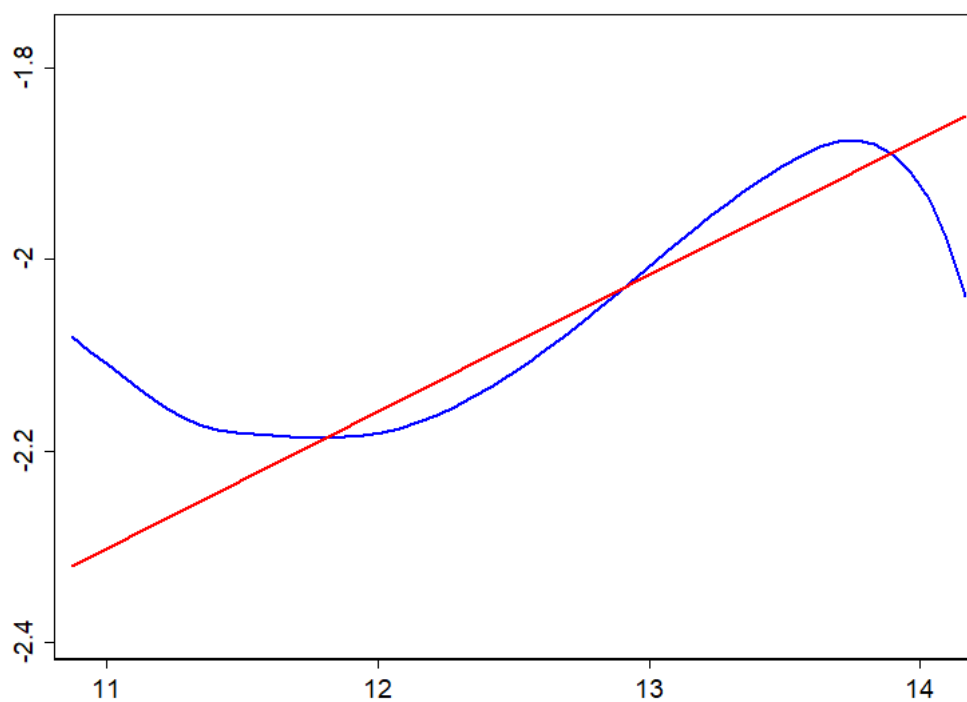
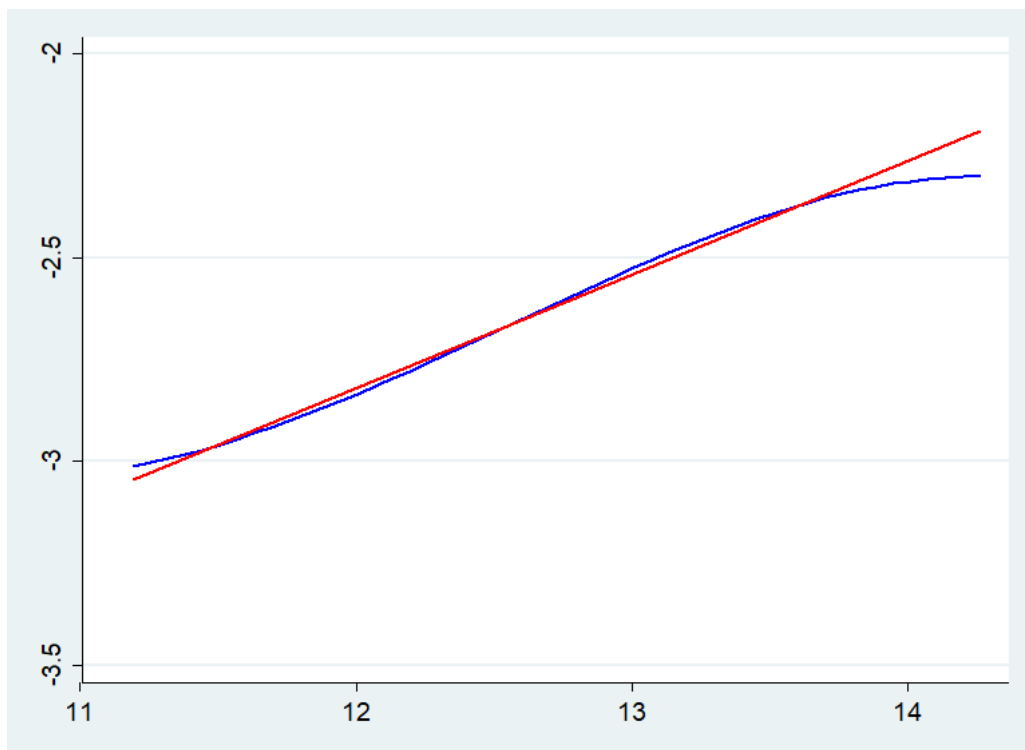
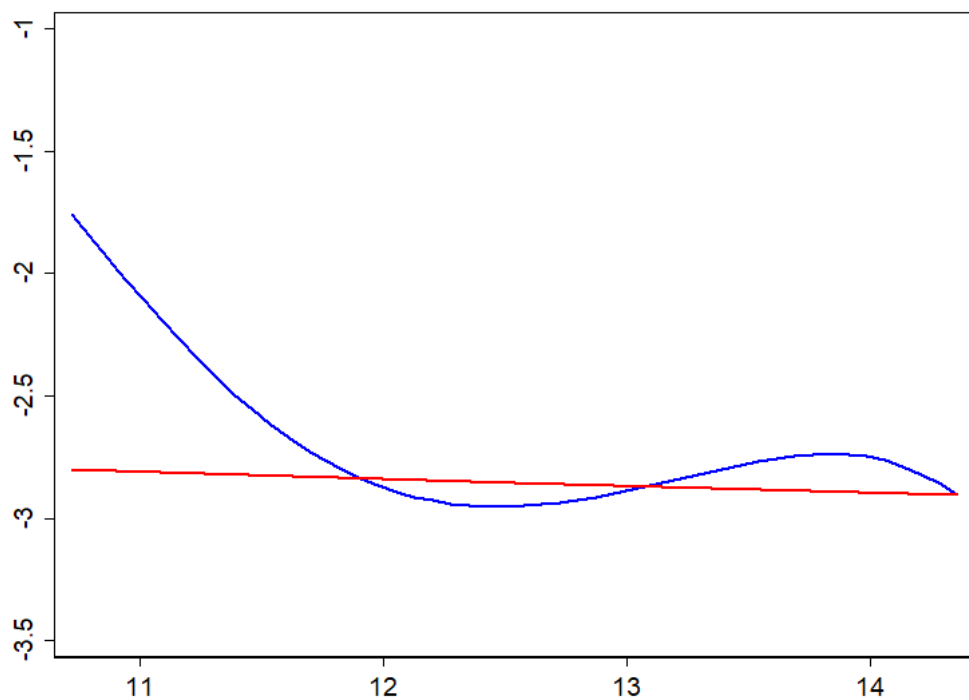


Figure a7. *Pre-shock Engel Curves for Oil Goods***Figure a8.** *Post-shock Engel Curves for Oil Goods*

A.2 Constructing Key Variables

Using micro-data on sectoral consumption prices, $\{p_k^{c,emp.}\}_k$; total number of consumers $L^{emp.}$; consumers' expenditure level, $\{e_h^{emp.}\}_h$; consumers' sectoral expenditure levels, $\{e_{h,k}^{emp.}\}_{h,k}$, I construct individual, subgroup and national levels expenditure shares, incomes, tax and price indices as

$$\Omega_{h,k} = \frac{e_{h,k}^{emp.}}{e_h^{emp.}} \quad (A.2.1)$$

$$y_h^{emp.} = \frac{e_h^{emp.}}{1 - \tau_{br}} \mathbb{1}_{h \in br} \quad (A.2.2)$$

$$t_h^{emp.} = y_h^{emp.} - e_h^{emp.} \quad (A.2.3)$$

$$P_h^{1-\sigma} = \sum_{k=1}^S \left(\zeta_k e_h^{\epsilon_k + \sigma - 1} (p_k^{c,emp.})^{1-\sigma} \right)^{\frac{1-\sigma}{\epsilon_k}} (\Omega_{h,k})^{\frac{\epsilon_k + \sigma - 1}{\epsilon_k}} \quad (A.2.4)$$

$$y_{gi} = \sum_h \frac{y_h^{emp.}}{L_g^{emp.}} \mathbb{1}_{h \in g, h \in i} \quad (A.2.5)$$

$$\tau_{gi} = \frac{T_{gi}}{Y_{gi}} = \frac{\sum_h t_h^{emp.}}{\sum_h y_h^{emp.}} \mathbb{1}_{h \in g, h \in i} \quad (A.2.6)$$

$$t_{gi} = y_{gi} \tau_{gi} \quad (A.2.7)$$

$$e_{gi} = y_{gi} (1 - \tau_{gi}) \quad (A.2.8)$$

$$\Omega_{gi,k} = \sum_h \left(\frac{e_h^{emp.}}{\sum_{h'} e_{h'}^{emp.}} \right) \Omega_{h,k} \mathbb{1}_{h \in g, h \in i} \quad (A.2.9)$$

$$P_{gi}^{1-\sigma} = \sum_{k=1}^S \left(\zeta_k e_g^{\epsilon_k + \sigma - 1} (p_k^{c,emp.})^{1-\sigma} \right)^{\frac{1-\sigma}{\epsilon_k}} (\Omega_{gi,k})^{\frac{\epsilon_k + \sigma - 1}{\epsilon_k}} \quad (A.2.10)$$

$$y = \sum_h \frac{y_h^{emp.}}{L^{emp.}} \quad and \quad (A.2.11)$$

$$\tau = \frac{T}{Y} = \frac{\sum_h t_h^{emp.}}{\sum_h y_h^{emp.}} \quad (A.2.12)$$

$$t = y \bar{\tau} \quad (A.2.13)$$

$$e = y (1 - \bar{\tau}) \quad (A.2.14)$$

$$\Omega_k = \sum_h \left(\frac{e_h^{emp.}}{\sum_{h'} e_{h'}^{emp.}} \right) \Omega_{h,k} \quad (A.2.15)$$

$$P^{1-\sigma} = \sum_{k=1}^S \left(\zeta_k e^{\epsilon_k + \sigma - 1} (p_k^{c,emp.})^{1-\sigma} \right)^{\frac{1-\sigma}{\epsilon_k}} \Omega_k^{\frac{\epsilon_k + \sigma - 1}{\epsilon_k}} \quad (\text{A.2.16})$$

A.3 Data, Parameters: Model Summary

Table a2. *Micro and macro-data, and parameters: symbol and sources*

Micro-based data		
<i>Income</i>	y_h, Y_g, Y_{gi}	LSMS
<i>Expenditure</i>	e_h, E_g, E_{gi}	LSMS
<i>Tax</i>	τ_h, t_h, T_g, T_{gi}	LSMS
<i>Consumer sectoral expenditure shares</i>	$\Omega_{h,k}, \Omega_{gi,k}$	LSMS
<i>Labor sizes and shares</i>	$L_g, L_{gi}, \pi_{gi}, \pi_{ig}$	LSMS
Macro-based data		
<i>Trade imbalance</i>	Λ	CBN
Active shocks, i.e. $\neq 1$		
<i>International Price shocks</i>	$\hat{p}_{oi_c}^*, \hat{p}_{oi_r}^*, \hat{p}_{ag}^*, \hat{p}_{ma}^*, \hat{p}_{se}^*$	OECD, WFO
<i>Trade imbalance</i>	$\hat{\Lambda}$	CBN
Potentially active shocks, i.e. $= 1$		
<i>Tax shocks</i>	$\hat{\tau}_h, \hat{\tau}_{gi}$	macro data
Inactive shocks		
<i>Sectoral productivity shocks</i>	\hat{Z}_k	na
<i>Armington weights</i>	$\hat{\delta}_k$	na
<i>Scale parameter of preference tastes</i>	\hat{a}_{gi}	na
Parameters		
<i>Income elasticities</i>	ϵ_k	Comin et al., 2018
<i>Elasticity of substitution</i>	σ	Comin et al., 2018
<i>Armington elasticity</i>	ρ	Ruhl, 2008
<i>Fréchet shape parameter</i>	θ	Morales, 2018

Note. CBN: *Central Bank of Nigeria*

A.4 Description of Solution Algorithm

A.4.1 Objects of the System of Equations

Recall that the general equilibrium system of equations is described by equations (1.4.38) – (1.4.56), with its main components given by equations (1.4.44) – (1.4.46). Furthermore, the active shocks, potentially active shocks, other data and parameters are described in *table a3* of *appendix A.3*. Finally, the endogenous objects are described by equations (1.4.39) – (1.4.70).

A.4.2 Description of the Algorithm

A.4.2.1 Primitives

The system of equation is written as a function of six primitives, including the numeraire:

$$\Upsilon \equiv \left\{ \left\{ \hat{w}_k \right\}_{k \in \mathcal{T} \cup \mathcal{N}} ; \hat{\beta} \right\}$$

A.4.2.2 Main Equations

The core system is made of five equations: four deriving from the post-shock equilibrium on the labor market, including an equation deriving from the current account balance identity, and a fifth equation deriving from the post-shock government's budget constraint. That is,

$$\hat{Y}_k Y_k + \hat{M}_k M_k \mathbb{1}_{k \in \mathcal{T}} = \hat{A}_k A_k + \hat{X}_k X_k, \quad \forall k \in \mathcal{T} \cup \mathcal{N}$$

$$\hat{\Lambda} \Lambda = \sum_{k \in \mathcal{T}} \hat{M}_k M_k - \sum_{k \in \mathcal{T}} \hat{X}_k X_k$$

$$\hat{T} T + \hat{\Lambda} \Lambda = \hat{G} G$$

A.4.2.3 Solving for Primitives (Steps)

1. Given the shock \hat{p}_{oi}^* , the exogenous changes \hat{p}_{ag}^* , \hat{p}_{ma}^* , $\hat{\Lambda}$, the data and the parameters Θ , solve the system of equations (1.4.39) – (1.4.46)

- (a) Set first initial values: $\{\hat{w}_k^o\}_{k \in \mathcal{T} \cup \mathcal{N}}$ and $\hat{\beta}^o$
- Solve for $\{\hat{P}_{gi}^o\}_{g,i}$ using $y_{gi} = z_{gi}w_i$ for each subgroup (g, i) , and equation (1.4.57)
 - Plug the values of $\{\hat{P}_{gi}^o\}_{g,i}$ back into equations (1.4.44) – (1.4.46)
 - Solve the system of equations (1.4.39) – (1.4.70) for new values of primitives, $\{\hat{w}_k^1\}_{k \in \mathcal{T} \cup \mathcal{N}}$, $\hat{\beta}^1$
- (b) Set new initial values as $\{\hat{w}_k^1\}_{k \in \mathcal{T} \cup \mathcal{N}}$ and $\hat{\beta}^1$
- Repeat the sub-step (a) for these new initial values and obtain $\{\hat{w}_k^2\}_{k \in \mathcal{T} \cup \mathcal{N}}$, $\hat{\beta}^2$ in the current iteration
- (c) Set new initial values as $\{\hat{w}_k^2\}_{k \in \mathcal{T} \cup \mathcal{N}}$ and $\hat{\beta}^2$
- Repeat the sub-step (a) for these initial values and obtain $\{\hat{w}_k^3\}_{k \in \mathcal{T} \cup \mathcal{N}}$, $\hat{\beta}^3$ in the current iteration
- ...
- (d) Set next initial values (at iteration $t - 1$) as $\{\hat{w}_k^{t-1}\}_{k \in \mathcal{T} \cup \mathcal{N}}$ and $\hat{\beta}^{t-1}$
- Repeat the sub-step (a) for these initial values and obtain $\{\hat{w}_k^t\}_{k \in \mathcal{T} \cup \mathcal{N}}$, $\hat{\beta}^t$ in the current iteration

2. Repeat these iterations until convergence at iteration T , i.e. until, say,

$$\|\hat{e}_{gi}^T - \hat{e}_{gi}^{T-1}\| < \text{tolerance}$$

- Obtain final solutions $\{\hat{w}_k^{sol}\}_{k \in \mathcal{T} \cup \mathcal{N}}$ and $\hat{\beta}^{sol}$

A.4.2.4 Predicted outcomes

Using the values of the primitives $\hat{w}_{oi}^{(sol)}$, $\hat{w}_{ag}^{(sol)}$, $\hat{w}_{ma}^{(sol)}$, $\hat{w}_{se}^{(sol)}$ and $\hat{\beta}^{(sol)}$, I obtain all endogenous variables defined in the system (1.4.39) – (1.4.70).

A.5 Computational Results

Figure a9. *Baseline scenario: net income and price index effects: non-homothetic model*
(in %)

Groups	Sectors of employment				Effects
	Crude oil	Agriculture	Manufacturing	Service	
Urban, non skilled	-40.00	-32.40	-30.99	-38.85	net income
	-29.65	-30.12	-29.94	-32.36	price index
Urban, low skilled	-40.00	-32.40	-30.99	-38.85	net income
	-27.86	-30.24	-30.35	-30.79	price index
Urban, high skilled	-40.00	-32.40	-30.99	-38.85	net income
	-30.19	-32.51	-29.49	-30.70	price index
Rural, non skilled	-40.00	-32.40	-30.99	-38.85	net income
	-30.46	-29.46	-28.99	-29.08	price index
Rural, low skilled	-40.00	-32.40	-30.99	-38.85	net income
	-32.34	-29.37	-31.09	-30.10	price index
Rural, high skilled	-40.00	-32.40	-30.99	-38.85	net income
	-30.52	-29.36	-28.00	-29.91	price index

Figure a10. *Baseline scenario: net income and price index effects: homothetic model*
(in %)

Groups	Sectors of employment				Effects
	Crude oil	Agriculture	Manufacturing	Service	
Urban, non skilled	-40.00	-32.29	-31.19	-37.95	net income
	-26.92	-26.92	-26.92	-26.92	price index
Urban, low skilled	-40.00	-32.29	-31.19	-37.95	net income
	-26.92	-26.92	-26.92	-26.92	price index
Urban, high skilled	-40.00	-32.29	-31.19	-37.95	net income
	-26.92	-26.92	-26.92	-26.92	price index
Rural, non skilled	-40.00	-32.29	-31.19	-37.95	net income
	-26.92	-26.92	-26.92	-26.92	price index
Rural, low skilled	-40.00	-32.29	-31.19	-37.95	net income
	-26.92	-26.92	-26.92	-26.92	price index
Rural, high skilled	-40.00	-32.29	-31.19	-37.95	net income
	-26.92	-26.92	-26.92	-26.92	price index

Figure a11. *Uniform tax hike counterfactual: changes in net income and price index effects relative to baseline scenario (in % unit): non-homothetic model*

Groups\ Sectors of employment	Sectors of employment				Effects
	Crude oil	Agriculture	Manufacturing	Service	
Urban, non skilled	-8.89	-9.50	-11.61	-10.30	net income
	-5.65	-5.71	-5.47	-5.89	price index
Urban, low skilled	-9.49	-10.37	-11.36	-8.87	net income
	-6.25	-6.44	-5.75	-5.88	price index
Urban, high skilled	-11.45	-13.93	-15.80	-11.65	net income
	-8.03	-6.74	-8.88	-7.70	price index
Rural, non skilled	-6.23	-7.97	-10.75	-7.04	net income
	-6.28	-4.78	-5.35	-4.30	price index
Rural, low skilled	-6.23	-8.68	-12.05	-8.47	net income
	-4.45	-4.89	-5.59	-5.07	price index
Rural, high skilled	-9.81	-10.95	-13.32	-10.06	net income
	-6.01	-6.94	-8.48	-6.48	price index

Figure a12. *Uniform tax hike counterfactual: changes in net income and price index effects relative to baseline scenario (in % unit): homothetic model*

Groups\ Sectors of employment	Sectors of employment				Effects
	Crude oil	Agriculture	Manufacturing	Service	
Urban, non skilled	-9.05	-10.22	-11.62	-11.19	net income
	-5.80	-5.80	-5.80	-5.80	price index
Urban, low skilled	-9.66	-11.10	-11.36	-9.73	net income
	-5.80	-5.80	-5.80	-5.80	price index
Urban, high skilled	-11.66	-14.70	-15.88	-12.57	net income
	-5.80	-5.80	-5.80	-5.80	price index
Rural, non skilled	-6.34	-8.68	-10.75	-7.86	net income
	-5.80	-5.80	-5.80	-5.80	price index
Rural, low skilled	-6.34	-9.40	-12.07	-9.32	net income
	-5.80	-5.80	-5.80	-5.80	price index
Rural, high skilled	-9.98	-11.69	-13.36	-10.94	net income
	-5.80	-5.80	-5.80	-5.80	price index

Figure a13. *Progressive tax hike counterfactual: changes in net income and price index effects relative to baseline scenario (in % unit): non-homothetic model*

Groups\ Sectors of employment	Sectors of employment				Effects
	Crude oil	Agriculture	Manufacturing	Service	
Urban, non skilled	-8.47	-8.89	-11.15	-10.08	net income
	-5.31	-5.32	-5.14	-5.60	price index
Urban, low skilled	-9.08	-9.91	-10.86	-8.41	net income
	-5.86	-6.03	-5.39	-5.51	price index
Urban, high skilled	-11.44	-14.11	-16.10	-11.60	net income
	-7.65	-6.56	-8.59	-7.37	price index
Rural, non skilled	-5.56	-7.29	-10.31	-6.35	net income
	-5.78	-4.43	-5.01	-4.01	price index
Rural, low skilled	-5.56	-8.03	-11.79	-7.88	net income
	-4.08	-4.55	-5.28	-4.73	price index
Rural, high skilled	-9.42	-10.57	-12.85	-9.73	net income
	-5.65	-6.53	-7.95	-6.10	price index

Figure a14. *Progressive tax hike counterfactual: changes in net income and price index effects relative to baseline scenario (in % unit): homothetic model*

Groups\ Sectors of employment	Sectors of employment				Effects
	Crude oil	Agriculture	Manufacturing	Service	
Urban, non skilled	-8.47	-9.39	-10.92	-10.73	net income
	-5.20	-5.20	-5.20	-5.20	price index
Urban, low skilled	-9.08	-10.40	-10.63	-9.06	net income
	-5.20	-5.20	-5.20	-5.20	price index
Urban, high skilled	-11.44	-14.58	-15.88	-12.25	net income
	-5.20	-5.20	-5.20	-5.20	price index
Rural, non skilled	-5.56	-7.80	-10.08	-6.98	net income
	-5.20	-5.20	-5.20	-5.20	price index
Rural, low skilled	-5.56	-8.54	-11.56	-8.52	net income
	-5.20	-5.20	-5.20	-5.20	price index
Rural, high skilled	-9.42	-11.06	-12.62	-10.38	net income
	-5.20	-5.20	-5.20	-5.20	price index

Figure a15. Increasing foreign borrowing counterfactual: changes in net income and price index effects relative to baseline scenario (in % unit): non-homothetic model

Groups\ Sectors of employment	Sectors of employment				Effects
	Crude oil	Agriculture	Manufacturing	Service	
Urban, non skilled		0.00	3.76	3.56	5.68
		2.37	2.25	2.42	3.02
Urban, low skilled		0.00	3.76	3.56	5.68
		1.31	1.93	2.41	2.46
Urban, high skilled		0.00	3.76	3.56	5.68
		1.49	3.09	1.31	1.90
Rural, non skilled		0.00	3.76	3.56	5.68
		1.74	2.23	1.91	2.51
Rural, low skilled		0.00	3.76	3.56	5.68
		3.26	2.16	2.58	2.39
Rural, high skilled		0.00	3.76	3.56	5.68
		2.37	1.68	0.48	1.90

Figure a16. Increasing foreign borrowing counterfactual: changes in net income and price index effects relative to baseline scenario (in % unit): homothetic model

Groups\ Sectors of employment	Sectors of employment				Effects
	Crude oil	Agriculture	Manufacturing	Service	
Urban, non skilled		0.00	3.75	3.76	5.15
		1.43	1.43	1.43	1.43
Urban, low skilled		0.00	3.75	3.76	5.15
		1.43	1.43	1.43	1.43
Urban, high skilled		0.00	3.75	3.76	5.15
		1.43	1.43	1.43	1.43
Rural, non skilled		0.00	3.75	3.76	5.15
		1.43	1.43	1.43	1.43
Rural, low skilled		0.00	3.75	3.76	5.15
		1.43	1.43	1.43	1.43
Rural, high skilled		0.00	3.75	3.76	5.15
		1.43	1.43	1.43	1.43

Figure a17. *Progressive tax redistribution counterfactual: changes in net income and price index effects relative to baseline scenario (in % unit): non-homothetic model*

Groups\ Sectors of employment	Sectors of employment				Effects
	Crude oil	Agriculture	Manufacturing	Service	
Urban, non skilled	0.53	0.44	0.13	-0.07	net income
	0.12	0.10	0.03	-0.02	price index
Urban, low skilled	0.10	0.26	0.15	0.36	net income
	0.01	0.06	0.05	0.09	price index
Urban, high skilled	-0.65	-1.26	-1.57	-0.65	net income
	-0.16	-0.33	-0.41	-0.19	price index
Rural, non skilled	0.67	0.51	0.13	0.65	net income
	0.17	0.07	0.02	0.08	price index
Rural, low skilled	0.67	0.55	-0.22	0.48	net income
	0.18	0.07	-0.02	0.08	price index
Rural, high skilled	0.10	0.08	-0.27	0.03	net income
	0.03	0.02	-0.05	0.00	price index

Figure a18. *Progressive tax redistribution counterfactual: changes in net income and price index effects relative to baseline scenario (in % unit): homothetic model*

Groups\ Sectors of employment	Sectors of employment				Effects
	Crude oil	Agriculture	Manufacturing	Service	
Urban, non skilled	0.53	0.43	0.16	-0.13	net income
	0.00	0.00	0.00	0.00	price index
Urban, low skilled	0.10	0.25	0.18	0.31	net income
	0.00	0.00	0.00	0.00	price index
Urban, high skilled	-0.65	-1.27	-1.54	-0.71	net income
	0.00	0.00	0.00	0.00	price index
Rural, non skilled	0.67	0.50	0.17	0.61	net income
	0.00	0.00	0.00	0.00	price index
Rural, low skilled	0.67	0.54	-0.19	0.43	net income
	0.00	0.00	0.00	0.00	price index
Rural, high skilled	0.10	0.06	-0.24	-0.02	net income
	0.00	0.00	0.00	0.00	price index

Appendix B

On Chapter 3 (Part II)

B.1 Theoretical Predictions

B.1.1 A1: Proof to Lemma 1

Proof: In the optimum, I have

$$\frac{w_2}{w_1 + \theta} h_\tau(1 - l^u, a) = R$$

- **Negative Selection**

As $a \uparrow$, $h_\tau(1 - l^u, a) \uparrow$ since $h_{12} > 0$. Therefore, I need $1 - l^u$ to increase, i.e. for l^u to decrease, in order to maintain the equality. Therefore, in the optimum, the children's time allocated to labor is strictly decreasing in the ability level.

More formally, I can define $S = \frac{w_2}{w_1 + \theta} h_\tau(1 - l, a) - R$ and use an implicit differentiation to compute $\frac{\partial l}{\partial a}$. I get

$$\begin{aligned} \frac{\partial S}{\partial a} &= \frac{w_2}{w_1 + \theta} h_{\tau a} \\ \frac{\partial S}{\partial A} &= 0 \\ \frac{\partial S}{\partial l} &= -\frac{w_2}{w_1 + \theta} h_{\tau l} \end{aligned}$$

Thus, I have

$$\frac{\partial l}{\partial a} = -\frac{\frac{\partial S}{\partial a}}{\frac{\partial S}{\partial l}} = \frac{h_{\tau a}}{h_{\tau\tau}} < 0$$

Indeed, $h_{\tau\tau} < 0$ and $h_{\tau a} > 0$, meaning that their ratio is negative. Also,

$$\frac{\partial l}{\partial A} = -\frac{\frac{\partial S}{\partial A}}{\frac{\partial S}{\partial l}} = \frac{0}{h_{\tau\tau}} = 0$$

Therefore, I have

$$\frac{\partial l}{\partial a} < 0 \quad , \quad \frac{\partial l}{\partial A} = 0$$

Hence, l is decreasing in a and independent of A .

• Determinants of borrowings

From the F.O.Cs, I know that borrowing, d , satisfies $U'(c_1) = \beta R U'(c_2)$.

Therefore, I define

$$H \equiv U' \left(A + (w_1 + \theta) l^u(a) + d - \theta \right) - \beta R U' \left(w_2 h(1 - l^u(a), a) - R d \right) = 0$$

I use the implicit function theorem and derive

$$\begin{aligned} \frac{\partial H}{\partial a} &= \frac{\partial l}{\partial a} (w_1 + \theta) U''(c_1) + \beta R w_2 h_\tau \frac{\partial l}{\partial a} U''(c_2) - \beta R w_2 h_a U''(c_2) \\ \frac{\partial H}{\partial A} &= U''(c_1) \\ \frac{\partial H}{\partial d} &= U''(c_1) + \beta R^2 U''(c_2) \end{aligned}$$

Therefore, I have¹

$$\begin{aligned} \frac{\partial d}{\partial a} = -\frac{\frac{\partial H}{\partial a}}{\frac{\partial H}{\partial d}} &= -\frac{\frac{\partial l}{\partial a} (w_1 + \theta) U''(c_1) + \beta R w_2 h_\tau \frac{\partial l}{\partial a} U''(c_2) - \beta R w_2 h_a U''(c_2)}{U''(c_1) + \beta R^2 U''(c_2)} \\ &= -\frac{\partial l}{\partial a} (w_1 + \theta) \frac{U''(c_1) + \beta R \frac{w_2 h_\tau}{(w_1 + \theta)} U''(c_2) - \beta R \frac{w_2 h_a}{\frac{\partial l}{\partial a} (w_1 + \theta)} U''(c_2)}{U''(c_1) + \beta R^2 U''(c_2)} \end{aligned}$$

¹I will use the optimality condition, i.e. $\frac{w_2}{w_1 + \theta} h_\tau (1 - l, a) = R$

$$\begin{aligned}
&= -\frac{\partial l}{\partial a}(w_1 + \theta) \frac{U''(c_1) + \beta R^2 U''(c_2) - \beta R \frac{w_2 h_a}{\frac{\partial l}{\partial a}(w_1 + \theta)} U''(c_2)}{U''(c_1) + \beta R^2 U''(c_2)} \\
&= -\frac{\partial l}{\partial a}(w_1 + \theta) + \frac{\beta R w_2 h_a U''(c_2)}{U''(c_1) + \beta R^2 U''(c_2)} > 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial d}{\partial A} &= -\frac{\frac{\partial H}{\partial A}}{\frac{\partial H}{\partial d}} = -\frac{U''(c_1)}{U''(c_1) + \beta R^2 U''(c_2)} \\
&= \frac{-1}{1 + \beta R^2 \frac{U''(c_2)}{U''(c_1)}} < 0
\end{aligned}$$

Indeed, since, $\frac{\partial l}{\partial a} < 0$, the first term of $\frac{\partial d}{\partial A}$ is positive. Since $h_a > 0$ and $U'' < 0$, its second term is positive as well². Furthermore, $\frac{\partial d}{\partial A}$ is negative since its denominator is positive. I therefore have

$$\frac{\partial d}{\partial a} > 0 \quad , \quad \frac{\partial d}{\partial A} < 0$$

Hence, d is increasing in a and decreasing in A .

A different method for proving these results is to consider the initial equality

$$U' \left(A + (w_1 + \theta) l^u(a) + d - \theta \right) = \beta R U' \left(w_2 h (1 - l^u(a), a) - R d \right)$$

By earlier argument (negative selection proof), as $a \uparrow$, $l \downarrow$ and this increasing h since $h_\tau > 0$, $h_a > 0$. Therefore $c_2 \uparrow$ and RHS goes down. At the same time, the decrease in l provokes a decrease in c_1 and ultimately an increase of the LHS. In order to restore the equality, I need for the only endogenous variable, i.e. d to increase. Indeed, $d \uparrow$ will decrease c_2 and ultimately increase the RHS, and at the same time, increase c_1 so that the LHS goes back down. Hence, $a \uparrow \implies d \uparrow$.

Second, as $A \uparrow$, c_1 increases, and this decreasing the LHS. Therefore, in order to maintain the equality, I need for the only endogenous variable, i.e. d to decrease. Indeed, $d \downarrow$ will increase c_2 and ultimately decrease the RHS. Hence, $A \uparrow \implies d \downarrow$.

²Both the numerator and the denominator are negative.

B.1.2 A2: Proof to Lemma 2

Proof: I know that if $d = \bar{d}$, $U'(c_1) > RU'(c_2)$. I have

$$U'(c_1) > RU'(c_2) = R\beta \frac{w_1 + \theta}{\beta w_2 h_\tau} U'(c_1)$$

This implies that

$$\frac{w_2}{w_1 + \theta} h_\tau (1 - l^c, a) > R \quad (B.1.1)$$

The optimality condition given by (3.2.5) is no more respected and the corresponding level of child labor, l^c , is therefore non optimal.

Also, (B.1.1) which represents a deviation from (3.2.5), implies an increase in the level of child labor, comparing to the unconstrained level l^u . Indeed, in order to get back to equality, i.e. (3.2.5), one would need to decrease l^c since

$$\bullet \quad l^c \downarrow \implies 1 - l^c \uparrow \implies \text{LHS of (B.1.1)} \downarrow \text{ as } h_{\tau\tau} < 0.$$

Therefore, the level of child labor, l^c , corresponding to (B.1.1) is higher than l^u ³ and I have

$$\frac{\partial l}{\partial a} < 0$$

B.1.3 A3: Proof to Proposition 1

I know from the F.O.C that child labor l satisfies $(w_c + \theta) U'(c_1) = \beta w_2 h_\tau (1 - l, a) U'(c_2)$.

I define

$$G \equiv (w_1 + \theta) U' \left(A + (w_1 + \theta) l + \bar{d} - \theta \right) - \beta w_2 h_\tau (1 - l, a) U' \left(w_2 h (1 - l, a) - R \bar{d} \right) = 0$$

I then use the implicit function theorem and derive

$$\begin{aligned} \frac{\partial G}{\partial a} &= -\beta w_2 h_{12} U'(c_2) - \beta w_2^2 h_\tau h_a U''(c_2) \\ \frac{\partial G}{\partial A} &= (w_1 + \theta) U''(c_1) \end{aligned}$$

³This result holds as long as $\bar{d} \geq 0$

$$\begin{aligned}\frac{\partial G}{\partial l} &= (w_1 + \theta)^2 U''(c_1) + \beta w_2 h_{\tau\tau} U'(c_2) + \beta w_2^2 h_\tau^2 U''(c_2) \\ \frac{\partial G}{\partial d} &= (w_1 + \theta) U''(c_1) + \beta R w_2 h_\tau U''(c_2)\end{aligned}$$

Therefore, I have

- **With respect to ability, a**

$$\frac{\partial l}{\partial a} = -\frac{\frac{\partial G}{\partial a}}{\frac{\partial G}{\partial l}} = -\frac{-\beta w_2 h_{\tau a} U'(c_2) - \beta w_2^2 h_\tau h_a U''(c_2)}{(w_1 + \theta)^2 U''(c_1) + \beta w_2 h_{\tau\tau} U'(c_2) + \beta w_2^2 h_\tau^2 U''(c_2)}$$

I first note that the denominator is negative. Indeed, its first term is negative as $U'' < 0$; its second term is also negative since $h_{\tau\tau} < 0$ and $U' > 0$; and its third term is negative as well since $U'' < 0$. Therefore, the sign of $\frac{\partial l}{\partial a}$ is the same as the sign of $\frac{\partial G}{\partial a}$.

I note that the first term of $\frac{\partial G}{\partial a}$ is negative while its second term is positive. The sign will then depend on which term dominates the other. I have

$$\begin{aligned}-\beta w_2 h_{\tau a} U'(c_2) - \beta w_2^2 h_\tau h_a U''(c_2) \geq 0 &\implies \beta w_2 h_{\tau a} U'(c_2) + \beta w_2^2 h_\tau h_a U''(c_2) \leq 0 \\ &\implies h_{\tau a} U'(c_2) \leq -w_2 h_\tau h_a U''(c_2) \\ &\implies h_{\tau a} \leq -w_2 h_\tau h_a \frac{U''(c_2)}{U'(c_2)} \\ &\implies \frac{h_{\tau a}}{w_2 h_\tau h_a} \leq -\frac{U''(c_2)}{U'(c_2)} \equiv \Gamma \\ &\implies \Gamma \geq \frac{h_{\tau a}}{w_2 h_\tau h_a} \equiv \gamma > 0\end{aligned}$$

where Γ is the Arrow-Pratt measure of absolute risk-aversion (ARA). Also note that the threshold γ depends upon the wage rate in adulthood w_2 , and the following endogenous measures: the return to education $h_\tau(\tau, a)$, the return to ability $h_a(\tau, a)$, the degree of complementarity between ability and school $h_{\tau a}(\tau, a)$.

In conclusion to the proof, I note that there exists a positive ARA threshold $\gamma \equiv \gamma(h_\tau(\tau, a), h_a(\tau, a), h_{\tau a}(\tau, a), w_2)$ such that

$$\begin{cases} \frac{\partial l^c}{\partial a} > 0 & \text{if } \Gamma \geq \gamma \\ \frac{\partial l^c}{\partial a} < 0 & \text{if } \Gamma \leq \gamma \end{cases} \quad (\text{B.1.2})$$

Hence, l is decreasing in a for the less risk averse persons and increasing in a for the more risk averse persons.

- **With respect to initial wealth, A**

I have

$$\frac{\partial l}{\partial A} = -\frac{\frac{\partial G}{\partial A}}{\frac{\partial G}{\partial l}} = -\frac{(w_1 + \theta) U''(c_1)}{(w_c + \theta)^2 U''(c_1) + \beta w_2 h_{\tau\tau} U'(c_2) + \beta w_2^2 h_\tau^2 U''(c_2)}$$

Since the sign of the denominator is negative, the sign of that derivative is the same as $\frac{\partial G}{\partial A}$, i.e negative as $U'' < 0$. I have

$$\frac{\partial l^c}{\partial A} < 0 \quad (\text{B.1.3})$$

Hence, l is decreasing in A .

- **With respect to borrowing limit, \bar{d}**

I have

$$\frac{\partial l}{\partial \bar{d}} = -\frac{\frac{\partial G}{\partial \bar{d}}}{\frac{\partial G}{\partial l}} = -\frac{(w_1 + \theta) U''(c_1) + \beta R w_2 h_\tau U''(c_2)}{(w_c + \theta)^2 U''(c_1) + \beta w_2 h_{\tau\tau} U'(c_2) + \beta w_2^2 h_\tau^2 U''(c_2)}$$

Since the sign of the denominator is negative, the sign of that derivative is the same as $\frac{\partial G}{\partial \bar{d}}$, i.e negative as $U'' < 0$ and $h_\tau > 0$. I have

$$\frac{\partial l^c}{\partial \bar{d}} < 0$$

Hence, l is decreasing in A .

B.1.4 A4: Proof to Proposition 2

The actual return to ability can be expressed as a sum of indirect and direct return to ability as

$$\tilde{h}_a \equiv \frac{\partial h(1 - l(a), a)}{\partial a} = -\frac{\partial l}{\partial a} \times h_\tau + h_a \quad (\text{B.1.4})$$

Hence, I distinguish two cases:

- When $\frac{\partial l}{\partial a} < 0$ (negative selection), $\tilde{h}_a > h_a > 0$
- When $\frac{\partial l}{\partial a} > 0$ (positive selection), $\tilde{h}_a < h_a > 0$

Also, the return to education is bound by 0 and Δ . Indeed, since $\tilde{h}_a > 0$, I have that

$$h_a > \frac{\partial l}{\partial a} \times h_\tau \Leftrightarrow \Delta > h_\tau > 0 \quad \text{with} \quad \Delta \equiv \frac{h_a}{\frac{\partial l}{\partial a}}$$

Furthermore, in comparing the actual return to ability and the return to education, I have that

$$\begin{aligned} \tilde{h}_a &= -\frac{\partial l}{\partial a} h_\tau + h_a > h_\tau \Leftrightarrow h_\tau < \frac{h_a}{1 + \partial l / \partial a} < \frac{h_a}{\partial l / \partial a} = \Delta \\ \tilde{h}_a &= -\frac{\partial l}{\partial a} h_\tau + h_a < h_\tau \Leftrightarrow h_\tau > \frac{h_a}{1 + \partial l / \partial a} = \Delta' \quad \text{with} \quad \Delta' < \Delta \end{aligned}$$

Hence,

$$\begin{aligned} \tilde{h}_a &> h_\tau \Leftrightarrow h_\tau < \Delta \\ \tilde{h}_a &< h_\tau \Leftrightarrow h_\tau > \Delta' \end{aligned}$$

I now make use of these facts when needed in proceeding with our proofs. I distinguish different cases:

Unconstrained equilibrium case or Constrained equilibrium case with $\text{ARA} \leq \gamma$

- I know from **Proposition 1** and **Proposition 2** that in these cases, $\frac{\partial l}{\partial a} < 0$ (negative selection). Therefore,
 - using (B.1.4), I determine that $\tilde{h}_a > 0$ since $h_\tau, h_a > 0$. Indeed, negative selection and $h(1 - l(a), a)$ imply the following: as $a \uparrow, l(a) \downarrow$, i.e. $1 - l(a) \uparrow$ and this implies that $h(1 - l(a), a) \uparrow$ since $h_\tau > 0$ and $h_a > 0$: hence, $\tilde{h}_a > 0$.
 - From the negative selection mechanism, I know that high ability children are subject to less child labor. Consequently, in this setting, a decrease in the level of child labor is associated with an increase in the level of ability and therefore an increase in future earnings (from $\tilde{h}_a > 0$): child labor is therefore negatively correlated to future earnings, i.e. $\text{corr}(h, l) < 0$.

- Conclusion: in these **case 1** (unconstrained equilibrium) and **case 2** (constrained equilibrium with moderate ARA), ability has a positive contribution to future level of human capital and therefore to future earnings⁴. Also, higher levels of child labor is associated with lower levels of future earnings, meaning that child labor is harmful to children's future earnings.

Constrained equilibrium case with $\text{ARA} \geq \gamma$

In here, since $\frac{\partial l}{\partial a} > 0$ (positive selection). Therefore,

- Positive selection and $h(1 - l(a), a)$ imply the following: as $a \uparrow$, $l(a) \uparrow$, i.e. $1 - l(a) \downarrow$, which implies that $h(1 - l(a), a) \downarrow$ since $\frac{\partial l}{\partial a} h_\tau > h_a$, i.e. $\tilde{h}_a < h_\tau$: hence, $\tilde{h}_a < 0$
- From the positive selection mechanism, I know that high ability children are subject to more child labor. Consequently, in this setting, an increase in the level of child labor is associated with an increase in the level of ability, and therefore a decrease in future earnings (from $\tilde{h}_a < 0$): child labor is negatively correlated to future earnings, i.e. $\text{corr}(h, l) < 0$.
- Conclusion: in this **case 3**, ability has a negative contribution to future level of human capital and therefore to future earnings: this is the curse of ability phenomenon. Also, high level of child labor is associated with low level of future earnings, meaning that child labor is harmful to children's future earnings⁵.

⁴Earning is simply the stock of human capital multiplied by the constant wage rate, $w_2 > 0$.

⁵Note that child labor is harmful whenever the curse of ability phenomenon holds.

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- **Ph.D in Economics.** The Pennsylvania State University, 2014 – 2020
- **Master of Science in Statistics and Applied Economics,** Graduate School of Statistics and Applied Economics (ENSEA in *French*), 2011
- **Master of Science in Applied Statistics,** Graduate School of Statistics and Applied Economics, 2008
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Research Interest

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Work Experience

- **August 2014 to February 2020:** *Teaching Assistant, Summer Instructor* at Penn State, College of the Liberal Arts
- **December 2013 to June 2014:** *Research Fellow* at United Nations Economic Commission for Africa (Ethiopia)
- **November 2012 to November 2013:** *Project Analyst* at Presidency State Office (Togo)
- **October 2011 to October 2012:** *Program Manager* at Planning and Development Ministry (Togo)

Skills

- Language: English (*fluent*), French (*native*), Ewe (*native*), Ana-Ife (*native*)
- Software: SPSS, Stata, Spad, Eviews, CSpro, Sphinx, Epi-Info, ArcGIS, MS Excel, MS Access, Adobe Creative Suite
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