NONLINEAR YAW CONTROL OF A COMPOUND HELICOPTER

A Thesis in
Electrical Engineering
by
Venkatakrishnan V. Iyer

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The thesis of Venkatakrishnan V. Iyer was reviewed and approved by the following:

Minghui Zhu
Associate Professor of Electrical Engineering
Thesis Co-Adviser

Constantino Lagoa
Professor of Electrical Engineering

Eric N. Johnson
Professor of Aerospace Engineering
Thesis Co-Adviser

Kultegin Aydin
Professor of Electrical Engineering
Head of the Department, Electrical Engineering
Abstract

In this thesis, a Nonlinear Dynamic Inversion (NDI) based controller is designed and implemented to control the forward acceleration and yaw rate control of a compound helicopter with a Vectored Thrust Ducted Propeller (VTDP) tail rotor. The VTDP configuration facilitates yaw control and an increase in forward velocities of the helicopter with the tail rotor acting as a pusher propeller. The wing in the compound helicopter provides additional lift thus allowing reduction of main rotor speed which in turn prevents issues associated with high rotor speeds and retreating blade stall. The compound helicopter considered is a scaled model of the Piasecki X-49 helicopter with certain changes in the configuration such as tail rotor speed control instead of tail rotor collective pitch control. Helicopter yaw control is achieved through a combination of rudder and sector deflection and variation of tail rotor speed. A detailed model of the helicopter is developed with specific emphasis on the modeling of the VTDP tail rotor configurations. Simulation results of the model are compared with the static test stand data to validate the model.

Neural network-based feedback linearization is used to compensate for the approximation in the dynamic inversion along with a Proportional + Derivative (PD) compensator to achieve the desired performance. Pseudo Control Hedge (PCH) signals are used in the reference model to cater to factors such as actuator position limits, rate limits, etc. A Model Reference Adaptive Control (MRAC) architecture is used to ensure that the reference command tracking error is zero. Different scenarios for the approximate dynamic inversion are analyzed with a suitable control algorithm to address them. The efficacy of the model and control law is tested in a simulation model involving basic helicopter maneuvers such as takeoff, yaw turns and forward flight.
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List of Symbols

\( a \)  
Lift curve slope

\( a_0 \)  
Blade coning angle

\( a_1 \)  
Blade longitudinal flapping coefficient

\( A \)  
Rotor or duct area

\( a, A \)  
Scalar and Matrix State transition element in state space representation

\( A_b \)  
Blade Area

\( \alpha \)  
Angle of Attack (AoA)

\( b \)  
Number of blades

\( b_1 \)  
Blade lateral flapping coefficient

\( b_v, b_w \)  
Bias terms in neural networks

\( \beta \)  
Side-slip angle

\( \beta \)  
Blade flapping angle

\( b, B \)  
Scalar and Vector plant input term in state space representation

\( c \)  
Chord Length

\( C \)  
Plant Output matrix in state space representation

\( C_L, C_D, C_T \)  
Lift, drag and thrust coefficients

\( C_{D0} \)  
Drag Coefficient

\( \delta_{beep} \)  
Beep input to tail rotor

\( \delta \)  
Actual actuator position

\( \hat{\delta} \)  
Estimate of actuator position

\( \delta_{cmd} \)  
Commanded input to the plant
δ_r  Rudder deflection  
δ_{r_{cmd}}  Commanded rudder deflection  
D  Diameter  
D  Drag force  
Δ  Approximate dynamic inversion model error  
\frac{\partial L}{\partial b_1}, \frac{\partial M}{\partial a_1}  Main rotor primary flapping stiffness  
e  Rotor hinge offset  
e  Error signal  
e_{c}  Command tracking error  
e_{crm}  Reference model command tracking error  
e_{rm}  Reference Model tracking error  
\hat{f}  Nominal plant model  
F  Force vector consisting of forces in the x, y, z directions  
g  Acceleration due to gravity  
γ  Lock Number  
γ  Scalar adaptation rate  
Γ  Circulation  
Γ  Adaptation Rate Matrix  
i  Incidence of horizontal stabilizer or vertical fin  
I  Inertia Matrix  
I  Identity matrix  
I_b  Blade Inertia  
i_s  Longitudinal shaft tilt of main rotor  
J  Advance ratio  
J  Rotor Rotational Inertia  
k, K  Scalar and Matrix Feedback gain  
κ  E-modification factor  
k_c  Lateral and longitudinal cross coupling  
L, M, N  rolling, pitching and yawing moments in body axes system  
L  Lift force
\( l_s \)  
Lateral shaft tilt of main rotor

\( m \)  
Mass

\( \mathcal{M} \)  
Moment vector consisting of moments in the \( x, y, z \) axes

\( M_r \)  
Rotor Moment

\( \hat{\mathbf{n}} \)  
Unit normal to a cross section

\( \omega \)  
Angular rates of the helicopter

\( \Omega \)  
rotor rpm (rad/sec)

\( \Omega_{cmd} \)  
Commanded tail rotor RPM

\( P \)  
Rotor Power

\( p, q, r \)  
\( x, y, z \) components of angular rates in body axes system

\( P, Q \)  
Matrices of standard Lyapunov function

\( \phi \)  
Vector of known basis functions

\( \phi, \theta, \psi \)  
Roll, Pitch and Yaw angles of the helicopter

\( \psi \)  
Blade azimuth angle

\( q \)  
Quaternion

\( q_0, q_1, q_2, q_3 \)  
Elements of Quaternion

\( \dot{Q} \)  
Quaternion error matrix

\( r \)  
Desired reference signal

\( R \)  
Rotor radius

\( R \)  
Modified yaw rate pseudo-control signal

\( \rho \)  
ar density

\( Q_E \)  
Engine Torque

\( Q_R \)  
Rotor Torque

\( \mathbb{R} \)  
Set of Real numbers

\( S \)  
Reference area

\( \sigma \)  
Rotor Solidity

\( \sigma \)  
Sigmoid activation function

\( T \)  
Thrust

\( \theta \)  
Duct angle

\( \theta_{1c} \)  
Lateral Cyclic input

xi
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{1s}$</td>
<td>Longitudinal Cyclic input</td>
</tr>
<tr>
<td>$\theta_{coll}$</td>
<td>Main rotor collective input</td>
</tr>
<tr>
<td>$\theta_{twist}$</td>
<td>Blade linear twist angle</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Vector of unknown elements of matched uncertainty</td>
</tr>
<tr>
<td>$u$</td>
<td>Plant input in a state space representation</td>
</tr>
<tr>
<td>$u, v, w$</td>
<td>$x, y, z$ components of velocity in body axes system</td>
</tr>
<tr>
<td>$u_b, w_b$</td>
<td>Velocity at the blade</td>
</tr>
<tr>
<td>$u_r, w_r$</td>
<td>Velocity at the rotor</td>
</tr>
<tr>
<td>$U$</td>
<td>Modified beep requirement</td>
</tr>
<tr>
<td>$v$</td>
<td>Pseudo-control signal</td>
</tr>
<tr>
<td>$V$</td>
<td>Helicopter forward speed</td>
</tr>
<tr>
<td>$V$</td>
<td>Lyapunov Function</td>
</tr>
<tr>
<td>$V$</td>
<td>Velocity in control volume</td>
</tr>
<tr>
<td>$V, W$</td>
<td>Neural Network weight matrix</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Volume of a control volume</td>
</tr>
<tr>
<td>$v_i$</td>
<td>Induced velocity</td>
</tr>
<tr>
<td>$w_0$</td>
<td>Axial velocity along propeller</td>
</tr>
<tr>
<td>$w_a$</td>
<td>Axial Induced velocity of propeller</td>
</tr>
<tr>
<td>$w_i$</td>
<td>Main Rotor downwash effect</td>
</tr>
<tr>
<td>$x$</td>
<td>State vector</td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>Components of $x, y, z$ displacement in NED frame</td>
</tr>
<tr>
<td>$X, Y, Z$</td>
<td>$x, y, z$ components of force in body axes system</td>
</tr>
<tr>
<td>$y$</td>
<td>Plant output $y$ in state space representation</td>
</tr>
<tr>
<td>$z$</td>
<td>Axial distance from propeller</td>
</tr>
</tbody>
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### Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>ADAPT</td>
<td>Adaptive Digital Automated Pilotage Technology</td>
</tr>
<tr>
<td>AGL</td>
<td>Above Ground Level</td>
</tr>
<tr>
<td>AoA</td>
<td>Angle of Attack</td>
</tr>
<tr>
<td>BL</td>
<td>Butt Line</td>
</tr>
<tr>
<td>CG</td>
<td>Center of Gravity</td>
</tr>
<tr>
<td>CV</td>
<td>Control Volume</td>
</tr>
<tr>
<td>CV</td>
<td>Controlled Variables</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>FARA</td>
<td>Future Attack Reconnaissance Aircraft</td>
</tr>
<tr>
<td>FRD</td>
<td>Forward Right Down</td>
</tr>
<tr>
<td>GUST</td>
<td>Georgia Tech UAV Simulation Tool</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multi Input Multi Output</td>
</tr>
<tr>
<td>MRAC</td>
<td>Model reference Adaptive Control</td>
</tr>
<tr>
<td>NACA</td>
<td>National Advisory Committee for Aeronautics</td>
</tr>
<tr>
<td>NDI</td>
<td>Nonlinear Dynamic Inversion</td>
</tr>
<tr>
<td>NED</td>
<td>North East Down</td>
</tr>
<tr>
<td>NN</td>
<td>Neural Network</td>
</tr>
<tr>
<td>PCH</td>
<td>Pseudo Control Hedge</td>
</tr>
<tr>
<td>PD</td>
<td>Proportional + Derivative</td>
</tr>
<tr>
<td>PURL</td>
<td>Pennsylvania State University UAV Research Laboratory</td>
</tr>
<tr>
<td>RC</td>
<td>Remote Control</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Full Form</td>
</tr>
<tr>
<td>--------------</td>
<td>-----------</td>
</tr>
<tr>
<td>ReLU</td>
<td>Rectified Linear Unit</td>
</tr>
<tr>
<td>RPM</td>
<td>Revolutions Per Minute</td>
</tr>
<tr>
<td>SHL</td>
<td>Single Hidden Layer</td>
</tr>
<tr>
<td>STA</td>
<td>Station Line</td>
</tr>
<tr>
<td>TED</td>
<td>Trailing Edge Down</td>
</tr>
<tr>
<td>TER</td>
<td>Trailing Edge Right</td>
</tr>
<tr>
<td>TEU</td>
<td>Trailing Edge Up</td>
</tr>
<tr>
<td>TPP</td>
<td>Tip Path Plane</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
</tr>
<tr>
<td>US</td>
<td>United States</td>
</tr>
<tr>
<td>VTDP</td>
<td>Vectored Thrust Ducted Propeller</td>
</tr>
<tr>
<td>VTOL</td>
<td>Vertical Take Off and Landing</td>
</tr>
<tr>
<td>WL</td>
<td>Water Line</td>
</tr>
</tbody>
</table>
Acknowledgments

The work presented in this thesis is a culmination of the efforts over the last two years. Despite my personal efforts, the success of this project is attributed to many people who have played a direct or indirect role. I would like to thank Dr. Eric Johnson for his constant support. He has been a great source of knowledge and motivation. The discussions with him have always been enlightening. He has the knack of making the most difficult approaches seem relatively easy. I would also like to thank Dr. Minghui Zhu for always being available and for giving me a free hand in the direction that I wanted to pursue my research. My sincere gratitude to Dr. Constantino Lagoa who agreed to be a part of this committee and was glad to provide me with any help that I needed. I am also thankful to my lab mates at PURL with whom I have had a very good time. The bonhomie and helpful nature that they have portrayed would be hard to come across with any other team. A special mention is due for Jeffrey Lewis with whom I have had many discussions about my thesis. Our discussions would always result in better clarity of the task at hand. I would also like to thank my friends in State College, namely Amit, Amrat, Mitansh, Snehal and Sujay who have made my stay at State College one that I will never forget. I am ever so grateful to my parents Venkatashalam Iyer and Hema Iyer for always being there for me and believing in me. They have shaped me the way I am today. They have been able to look beyond my flaws and discover the goodness in me. A big thanks to my brother Srinivasan who has allowed me to pursue my dreams while ensuring that I need not worry about the difficulties back home. He has been a pillar to my parents; an ideal son that I so strive to be. Lastly, my greatest gratitude and devotion to Lord Krishna, my friend and savior who has held my hand at every moment in my life. I bow in salutation to Thee and hope that you always lead me on the path of righteousness.
1.1 Compound Helicopters

Compound Helicopters have long existed in the aviation industry. A compound helicopter is a configuration that consists of additional lifting surfaces or propulsive methods to increase the speed of the helicopter. Typical configurations include the use of a lifting wing apart from the main rotor or some additional propulsion source such as a pusher propeller. Co-axial rotors are another configuration used in the development of compound helicopters. The need for a compound helicopter arises from the desire to fly at higher speeds and improve performance metrics such as lift-to-drag ratio, propulsive efficiency, etc. [1]. The edgewise operation of the rotor blades entails aerodynamic limitations owing to which the forward speed of helicopters is limited [2]. The purpose of compound helicopters is to overcome these shortcomings to facilitate higher speeds of operation.

Although the concept of compound helicopters has been explored for a considerable period, very few configurations have ever made it to full-scale production. Helicopters such as the McDonnel XV1 and the Fairey Rotodyne were developed in the 1950s wherein the main rotor operation was based on the tip jet concept. Although these designs allowed higher helicopter speeds, they suffered from tip-jet noise [3]. The AH-56 Cheyenne developed by Lockheed in the 1960s had a conventional tail rotor along with a pusher propeller and wing to provide additional forward speeds. While the use of compound helicopters is beneficial in terms of higher forward speeds, drawbacks such as an increase in empty weight, loss of payload capability, increased penalties in hover due to wing interaction with rotor
downwash, reduced vertical rate of climb, etc. [1] have hindered the development of compound helicopters.

Despite the perceived limitation with compound helicopters, there has been a renewed interest in the development of new configurations. With the US Army’s Future Attack Reconnaissance Aircraft (FARA) program launched in 2018, many companies have utilized compound helicopter configurations to meet the program requirements. Further, with the advancement in research and novel configurations, compound helicopters such as the Airbus X3 and the Sikorsky Technology Demonstrator X2 have been capable of breaching the speed thresholds of 250 knots [4].

While other VTOL configurations such as tilt-rotor aircraft (for example the Bell Boeing V-22 Osprey) do exist that allow for higher forward speeds in comparison to compound helicopters, their utility in terms of low-speed operations including hovering is limited. With an increase in forward velocities of 80 to 100 knots and additional benefits such as a higher lift to drag ratio in higher forward speeds [5], the use of compound helicopters is bound to continue with advanced research being conducted in new configurations and materials.

Piasecki Corporation has been involved in the development of compound helicopters for a considerable period. The 16H-1 compound helicopter, developed in 1965 had a wing and empennage along with a ducted fan at the tail [3]. The Piasecki X-49 helicopter which is an adaptation of the Blackhawk helicopter is a similar configuration with both wings and a ducted propeller in the tail rotor. It also includes an elevator in the horizontal stabilizer. The tail rotor has an additional modification of sectors and a rudder that can be deflected to achieve the desired yaw control [6]. This configuration is known as Vectored Thrust Ducted Propeller (VTDP). The wing consists of flaperons to generate additional lift/drag and rolling moments. Further, the tail rotor consists of a thrust vectoring sectors that can be deflected along with a rudder to generate the desired yaw moment.

The company is currently involved in the research of a flight control software package development program called Adaptive Digital Automated Pilotage Technology (ADAPT) that aims at the development of a digital framework that can be used to increase the safety, performance, survivability, etc. of US military aircraft. The objective is to develop an adaptive flight control that is capable of tasks such as reconfigurable controls in case of failure of any effectors, flying in a manner to optimize the aircraft performance, etc.
The work in this thesis is motivated based on the development of such a suitable control for a scaled demonstrator of the Piasecki X-49 helicopter. The scaled version used in the simulation is a large RC helicopter. The tail rotor is a ducted propeller with fixed pitch, variable RPM control instead of a variable pitch propeller. As the tail rotor can produce forward thrust, the yaw control problem has dual objectives of achieving the desired yaw and forward thrust from the tail rotor.

1.2 Tail Rotor Modeling

One of the primary objectives of this thesis is to arrive at a suitable model for the VTDP tail rotor. This analysis includes considering the effects of the thrust due to a propeller, the effects of a duct on the thrust of the propeller and the thrust vectoring aspect of the rudders and sectors. Ducted propellers are known to increase the efficiency of the propellers thus producing a higher thrust for the same power, in other words, generating the same thrust at a lower power [7]. The increase in the efficiency of ducted propellers is governed by the design of the ducts. Ducted propellers also offer an additional safety feature owing to the rotors being enclosed and thus making them suitable for various unmanned operations [8]. The improvement in thrust due to ducts is well presented by Martin and Tung [9] who state that while the thrust coefficient of a propeller remains relatively constant with the RPM, there is an increase in the thrust coefficient with RPM for a ducted propeller.

McCormick [7] presents a vortex ring approach to estimate the thrust generated by the duct. This analysis is based on replacing a duct by a vortex ring placed at the quarter chord location of the duct and analyzing the boundary conditions at the three-quarter position of the duct. The thrust estimation of the duct carried out in this thesis is based on this approach. A similar approach is presented by Mendenhall and Spangler [10] wherein the performance of a ducted fan in uniform flow is analyzed. The report explores the performance at very low advance ratios as well as computing the performance at various angles of attack. The performance of a ducted propeller for angles of attack ranging from 0° to 90° is presented by Parlett [11] wherein tests have been performed on a small shrouded propeller to determine the effects of airspeed and angle of attack on the lift, drag and pitching moment of the ducted propeller. The report also analyzes the effect of the inlet-lip
cross-sectional radius on the static thrust. The report suggests that the efficiency of ducted propellers increases with an increase in the lip cross-sectional diameter.

Mendenhall and Spangler [10] developed a flow model which considers the interaction between the fan, the duct, and the center body. For the blade, computations are performed at several radial locations by considering vortex cylinders to be shed from each radial location. Thus, the fan wake is assumed to consist of concentric vortex cylinders and the wake vortex strength is determined. The radial and axial velocity distributions are then determined using Fourier analysis. The ring vortex distribution is computed in an iterative process until a converged solution is obtained. Using the velocities and vorticities, the forces and moments of the duct are then determined.

As stated above, the analysis of the effect of ducts on the propeller thrust is performed using the method stated by McCormick [7]. The thrust of the propeller is determined based on an iterative process suggested by Heffley and Mnich [12] and Johnson and Turbe [13]. The effect of the sectors is analyzed considering standard flow analysis techniques of linear momentum analysis in a control volume. Details of the analysis along with underlying assumptions are covered in chapter 3.

1.3 Control Allocation

With an increase in the number of lifting surfaces in a compound helicopter, there is an increase in the number of redundant controls to achieve the same desired forces and moments on the helicopter which leads to the requirement of Control Allocation. Control allocation is required in cases of over-actuated systems and deals with distributing the total control among the individual actuators [14]. Thus, for an aircraft, this means that the desired force or moments can be generated using one or more of the control surfaces and the control allocation problem is to determine the best possible control to achieve the desired force/momentum.

The problem of control allocation deals with the task of determining the most suitable control distribution for an over-actuated system. The suitability may be determined based on various objectives depending on the application. Typical examples include optimization requirements in terms of the desired forces and moments or in terms of the least possible power utilization to achieve the desired forces and moments using the actuators. The use of linear control allocation techniques has
been well studied by Wayne Durham. Methods such as dynamic control allocation were also introduced by Durham in 1993. While various control allocation schemes exist, the choice of any particular method is application dependent. The following paragraphs provide an overview of different control allocation techniques.

Page and Steinberg [15] compare the performance of multiple control allocation schemes such as linear programming, direct allocation, quadratic programming and variations of pseudo-inverse based techniques. A performance metric used to analyze these schemes is the percentage volume of the attainable moments that these schemes can produce. The authors compare the open-loop and closed-loop performance of different control allocation schemes and state that the performance in open and closed-loop configuration are not the same. Further, the authors also conclude that pseudo-inverse methods perform the worst based on open-loop performance measures. The performance metric stated above is the difference between the desired moment and the moment that the control allocation scheme can generate. It should be noted that as the control allocation method generates an optimum solution based on a specific cost function, the moments generated by different control allocation methods will be different.

Durham explores the constrained control allocation problem in [16]. Constrained control allocation deals with achieving specific objectives through effective utilization and blending of control effectors based on their constraints. Details of the constrained control allocation problem are explored by Durham in [17] and [18]. Durham states the process to determine the attainable moment set for a two-moment problem in [17] and extends the analysis to the three-moment problem in [18]. However, the analysis presented in [18] is complicated, the difficulty of which is resolved in [16]. The algorithm presented in [16] possesses the advantage of fast computation as it is capable of identifying the entire moment set without any search or iterations which is usually the case in determining the attainable moment set. Additionally, closed-form solutions of the constrained control allocation problem using generalized inverses are presented by Bordignon and Durham [19]. While the above-referenced sources present the problem of control allocation and the method to determine the attainable moments, Durham [20] provides details of a computationally efficient method to determine the near-optimal solutions to linear control allocation problems. The author introduces the concept of bisecting edge searching instead of the facet searching techniques that were followed earlier. The
The process of solving the control allocation solution can be computationally intensive based on the number of control effectors and the number of desired moments.

The above-cited sources detail the process to determine the attainable moment set. Many times, it is possible that the desired moment cannot be achieved. In such a scenario, the near-optimal solution can be selected based on the desired objective i.e. we can either choose to maximize the attainable force/moment in the desired direction or we may choose to achieve the maximum possible force/moment in a particular direction. One such approach is presented by Enns [21] who introduces the method of axis priority weighting i.e. maximize the force/moment in a particular direction.

Davidson et al. [22] have proposed a method based on the frequency response of the control surfaces. The control allocation scheme consists of frequency-apportioned control wherein the frequency response of the actuators is used to determine the most appropriate control to be used to ensure a quick response. The method is particularly useful in scenarios where the control requirements are changing rapidly. In such cases, control surfaces that have a high-frequency response can be utilized to achieve the immediate control requirement. Thus, high-frequency commands are issued to actuators with higher rate limits and low-frequency commands to highly effective controls. The paper uses a weighted pseudo-inverse control allocation method wherein the weights are determined based on the frequency response of the actuators. Another frequency-based approach is presented by Härkegård [23] wherein a quadratic programming approach is utilized with penalization on the actuator rates apart from their limits. This penalty on the rates results in a frequency-based approach that accounts for the actuator bandwidth.

While most of the control allocation techniques presented establish a linearity assumption, most of the systems are non-linear. Thus, limitations may be encountered with the linearity assumption. To overcome this problem, dynamic inversion based control allocation techniques have been proposed in various works [24,25]. Reiner et al. [25] include robustness analysis along with the control allocation problem while Doman and Oppenheimer [24] aim at minimizing the error norm and at the same time control the deviation from the desired value.

Apart from the techniques presented above, additional control allocation techniques exists. Methods such as re-configurable control allocation [22], control allocation to minimize drag [26], etc. are available based on the desired application.
While multiple sources exist for a review of control allocation techniques, the thesis by Härkegård [14] provides an excellent overview of different control allocation techniques along with a detailed analysis of $l_2$ optimal control allocation methods using linear control allocation. Examples with applications to flight controls are also provided. The works of Durham and others are presented in the form of a textbook [27] which contains a detailed analysis of various control allocation techniques based on their research and publications.

In the case of the Piasecki X-49, the helicopter consists of multiple effectors such as the main rotor, flaperons, the rudder and sectors, variable pitch/RPM tail rotor and elevators. Thus, a suitable control allocation scheme is required to distribute the controls to achieve the desired forces and three primary moments. The rotor cyclic and rudder pedal forms the three primary controls while the flaperons, elevator and tail rotor pitch/RPM form the secondary control [6]. With additional redundant controls, there are numerous solutions possible to achieve a desired force/moment combination. This leads to the need for a suitable control allocation algorithm that distributes the control to achieve a desired objective. In this thesis, it is assumed that a suitable control allocation scheme exists, and each control effector is commanded to generate a part of the force/moment that is desired.

### 1.4 Dynamic Inversion

Some of the early research in control allocation was based on model following and dynamic inversion based control laws which involve determining the desired control effector deflections to achieve a desired force or moment [27]. Dynamic inversion is a technique of inverting the plant non-linearities such that a linear relation exists between the output and a pseudo-control input [28]. This helps to eliminate the need for gain scheduling which is typically used in aircraft control design. In Gain Scheduling, the aircraft is trimmed around an operating point and a linear controller is used to achieve the desired performance around the operating point. As the aircraft operates in different flight envelopes, this leads to the requirement of scheduling gains for each phase of flight corresponding to the appropriate operating point.

The concept of Feedback Linearization was first introduced by Roger Brockett
who is considered as the father of feedback linearization [29]. The initial concept was introduced in 1978 [30] followed by which additional work was carried out by Jakubczyk-Respondek [31] and Hunt et al. [32]. While the concept of Dynamic Inversion or Feedback Linearization has existed for a considerable period, it has found limited use in the design of flight controllers for aircraft. However, with changes in current trends, dynamic inversion based controllers are being used in multiple aerospace applications.

Dynamic inversion provides the advantage of having a universal control law that can be used for all phases of flight without the need for gain scheduling as is the typical norm. This, however, is based on the assumption that the actual plant model is accurately known thus allowing inversion of the plant facilitating a linear controller between the pseudo-control signal and the plant output.

While using dynamic inversion, care needs to be ensured that the plant does not have any non-minimum phase zeros as an inversion of the plant model can lead to unstable poles in the system. While it is preferable that the plant model is accurately represented for dynamic inversion, methods exist to cater to cases where a nominal simplified plant model is used instead of the complex actual plant model. Methods such as feedback linearization using Neural Networks form an effective solution to approximations in the plant models.

As Neural Networks (NN) are good approximators of non-linear functions [33], an SHL-NN with the correct number of neurons can approximate any non-linear function [34]. The primary use of neural networks in controls application is that of a function approximator [35]. Some of the typical advantages of using Neural Networks include their ability to approximate non-linear functions, ease of implementation due to their parallel structure, their capability to adapt and learn and the possibility to cater to multivariate systems [35].

Dynamic inversion based control architectures has been applied to aerospace applications. The use of dynamic inversion for aerospace applications was explored in the late 1980s and early 1990s [36,37]. Leitner et al. [38] apply the dynamic inversion control design to a nonlinear model of an attack helicopter. Rysdyk and Calise [39] extend the use of dynamic inversion based control to a tilt-rotor aircraft. The concept has also been applied to aircraft and space vehicles. The use of a similar concept for reusable launch vehicles is presented by Johnson and Calise [40]. While most of the above implementations were performed in simulation, the use of
dynamic inversion based control was applied to hardware in the loop simulation of a fighter aircraft by Miller [41]. For the application of dynamic inversion based control to rotorcrafts, the reader is referred to the article by Horn [42].

In this work, a model following dynamic inversion-based controller with feedback linearization using neural networks is used to control the helicopter. Separate inner loop and outer loop controls are used to control the moments and forces. A detailed analysis of feedback linearization is presented in chapter 3 along with the analysis of the helicopter control architecture in chapter 4.

1.5 Thesis Scope

The thesis is motivated by current research in the development of fault-tolerant and optimal control methods for use in a compound helicopter. In this thesis, a scaled model of the Piasecki X-49 helicopter has been considered. The scaled model considered is a large Remote Controller (RC) autonomous helicopter with a modified tail rotor. The objective of this thesis is to develop a suitable control law to control the yaw rate of the Vectored Thrust Ducted Propeller (VTDP) tail rotor.

Unlike conventional helicopters, the VTDP employs pusher ducted propellers along with sectors to direct the tail rotor thrust in the desired direction to achieve the desired yaw rate and provide a forward thrust, thus allowing higher forward speeds to the helicopter. Owing to the change in tail rotor configuration, a detailed model of the tail needs to be developed to implement the control algorithm.

The scope of this thesis is thus limited to the modeling of the VTDP tail rotor configuration along with the design of a Nonlinear Dynamic Inversion (NDI) based control law to control the yaw rate and forward acceleration requirements. The yaw rate controller is designed to cater primarily to the yaw rate requirement, in case the combination of yaw rate and forward velocity cannot be achieved. Thus, the controller can prioritize the axis based on the requirement.

As stated earlier, the research is based on the underlying assumption that a control allocation scheme exists to determine the force and moment requirements from each control surface. Further, the research utilizes existing helicopter simulation models along with suitable control architectures in all axes. The scope of this thesis is limited to modeling the new tail rotor configuration and developing a new yaw rate controller with the controller for all other axes already existing.
1.6 Thesis Outline

The thesis is organized as follows - Chapter 2 details the mathematical model of the helicopter with specific emphasis on the modeling of the Vectored Thrust Ducted Propeller (VTDP) tail rotor configuration. A model of the helicopter is developed including the first order flapping dynamics of the main rotor. A separate governor-based speed control model for the main rotor and tail rotor is considered.

Chapter 3 provides a foundation of various control techniques including dynamic inversion and Model Reference Adaptive Control (MRAC). The basic concept of dynamic inversion for both linear and nonlinear systems is explored. The chapter also provides a brief overview of the MRAC technique for a first-order scalar system. Finally, the concept of Pseudo Control Hedging (PCH) is introduced which helps in addressing issues with adaptive control systems that may tend to learn incorrectly due to restrictions in the plant.

In chapter 4, the control system architecture of the helicopter is presented. The architecture is divided into outer and inner loops wherein the outer loop addresses the force requirements while the inner loop satisfies the moment requirements. Separate dynamic inversion based MRAC architecture with PCH is considered for the inner and outer loop. As the primary objective of the thesis is yaw rate control, the inner loop control architecture is discussed in detail along with the development of the yaw rate controller.

Simulation results based on the control laws developed in chapter 4 are presented in chapter 5. The simulations results include basic helicopter operations such as takeoff and hover and includes details of forward flight. The test cases for the simulation are developed to evaluate the performance of various scenarios encountered by the controller.

Finally, chapter 6 lists the conclusions derived from the analysis along with future work that can be continued based on the work presented in this thesis. While the current scope was limited to the yaw rate control, future work such as the development of control allocation algorithms and outer loop integration with the inner loop are discussed.
In this chapter, a mathematical model of the helicopter is developed with specific emphasis on the tail rotor model. The nonlinear dynamic model includes the equations of motion of the helicopter along with the forces and moments generated by each aerodynamic surface of the helicopter. The model is developed in a simulation platform called GUST (Georgia Tech UAV Simulation Tool) which allows multiple functionalities such as full-scale simulation, manned/unmanned flight, hardware in the loop simulation and many more features. The chapter details the mathematical model used in the simulation software along with the additional modifications to cater to specific helicopter requirements.

The mathematical model of the helicopter is derived based on the works of Heffley and Mnich [12] and Ren et al. [43]. Typical rigid body assumptions and negligible effects of Earth’s curvature and rotation are used in the equations of motion. Additional assumptions are stated in the appropriate sections. The degrees of freedom include the velocity components, the angular rates, first-order flapping dynamics and RPM degree of freedom of the main rotor and tail rotor.

2.1 Assumptions

The primary objective of the thesis is the evaluation of the control law for the yaw control of the helicopter. To achieve the same, a simulation model of the helicopter is considered with sufficient fidelity such that the controllability and handling qualities of the helicopter can be suitably evaluated. Certain assumptions have been made while modeling different dynamics of the helicopter. The major assumptions in arriving at the model are listed below:
• The inflow through the main rotor and tail rotor is uniform. Inflow dynamics have not been considered in the simulation model.

• The main rotor Tip Path Plane (TPP) flapping dynamics have been modeled as a first-order model instead of the second-order flapping dynamics.

• While it is possible to model the lift, drag and pitching moment of the annular duct, studies have revealed that the effect of an annular ring can be approximated by the use of an elliptical loaded wing with suitable dimensions [7,44]. Thus, the effect of the duct of the propeller has been factored in the area of the horizontal stabilizer.

• Although the helicopter is not symmetric about the XZ plane, this is a reasonable assumption for basic helicopter control analysis and performance analysis. Thus, the assumption results in the helicopter inertia $I_{xy}$ and $I_{yz}$ being 0.

2.2 Equations of Motion

The following section details the equations of motion of the helicopter. The equations of motion define the basis for the evolution of the helicopter states. All equations of motion are presented in the body axis of the helicopter. The local frame considered is the standard North East Down (NED) frame and the helicopter body axes frame is the Forward Right Down (FRD) frame. The equations can be classified into four sets i.e. force equations, moment equations, kinematic equations, and navigation equations. Apart from the four sets of equations, additional equations may be included based on the number of states considered. The four sets of equations are detailed below.

2.2.1 Force Equations

The force equations of the helicopter in the body axes frame including the Coriolis acceleration due to body rotation can be represented as:
\[
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{w}
\end{bmatrix} = -
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix} \times
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix} +
\begin{bmatrix}
-g \sin \theta \\
g \sin \phi \cos \theta \\
g \cos \phi \cos \theta
\end{bmatrix} + \frac{1}{m}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
\] (2.1)

where, \(u, v, w\) are the body axes velocities, \(p, q, r\) are the angular rates, \(g\) is the acceleration due to gravity and \(m\) is the mass of the helicopter. \(X, Y, Z\) represent the total aerodynamic forces on the helicopter in the corresponding axis excluding the force due to gravity.

### 2.2.2 Moment Equations

The Moment equations of the helicopter are given by:

\[
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = -I^{-1}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix} \times I(
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}) + I^{-1}
\begin{bmatrix}
L \\
M \\
N
\end{bmatrix}
\] (2.2)

where, \(L, M, N\) are the total moments exerted at the helicopter CG due to the forces acting on various aerodynamic surfaces of the helicopter including the torque generated by the main and tail rotor. \(I\) represents the rotation inertia matrix and is given by:

\[
I =
\begin{bmatrix}
I_{xx} & 0 & -I_{xz} \\
0 & I_{yy} & 0 \\
-I_{xz} & 0 & I_{zz}
\end{bmatrix}
\] (2.3)

Most of the off-diagonal terms in equation (2.3) are zero owing to the symmetry of the helicopter about the XZ plane (ideally this is not the case owing to the presence of the tail rotor but assumption holds for basic simulations).

### 2.2.3 Kinematic Equations

The kinematic equation of the helicopter for the three Euler angles \(\phi, \theta, \psi\) that define the roll, pitch and yaw angles of the helicopter respectively can be represented as:

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
1 \tan \theta \sin \phi & \tan \theta \cos \phi \\
0 & \cos \phi & -\sin \phi \\
0 & \frac{\sin \phi}{\cos \theta} & \frac{\cos \phi}{\cos \theta}
\end{bmatrix}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\] (2.4)
Although the helicopter may be fully represented using the Euler angle representation, the same is not a practical choice for state computation owing to the singularity of the pitch angle at 90°. To avoid this problem, the quaternion representation for the helicopter attitudes may be used which eliminates the singularity issues. While the quaternions are mathematically more convenient, it is difficult to comprehend the orientation of the helicopter in terms of quaternions. Thus, periodic conversion between Euler angles and quaternions may be used. The kinematic equation (2.4) can be expressed in terms of quaternions as [28]:

\[
\begin{bmatrix}
\dot{q}_0 \\
\dot{q}_1 \\
\dot{q}_2 \\
\dot{q}_3
\end{bmatrix}
= \frac{1}{2}
\begin{bmatrix}
0 & -p & -q & -r \\
p & 0 & r & -q \\
q & -r & 0 & p \\
r & q & -p & 0
\end{bmatrix}
\begin{bmatrix}
q_0 \\
q_1 \\
q_2 \\
q_3
\end{bmatrix}
\] (2.5)

### 2.2.4 Navigation Equations

Finally, the navigation equations in the local frame can be expressed as:

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix}
= \begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix}
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix}
\] (2.6)

Equations (2.1), (2.2), (2.5), (2.6) results in 13 equations that form the basic non-linear equations of motion for the 13 states of the helicopter. The state vector considered includes the 13 states and additional states such as the main and tail rotor angular speed and the longitudinal and lateral flapping dynamics. Thus, the total state vector comprises of 17 states given by:

\[
x = [u, v, w, q_0, q_1, q_2, q_3, p, r, x, y, z, \Omega_{mr}, \Omega_{tr}, a_1, b_1]^T
\] (2.7)

\(\Omega_{mr}, \Omega_{tr}\) are the main rotor and tail rotor angular speeds respectively and \(a_1, b_1\) are the longitudinal and lateral flapping coefficients of the main rotor which are further explained in section 2.4.2.

The forces \(X\), \(Y\) and \(Z\) and moments \(L\), \(M\) and \(N\) in equations (2.1) and (2.2) respectively represent the total aerodynamic and propulsive forces and moments.
acting on the helicopter determined in the body axes of the helicopter. These forces and moments are a result of the various components of the helicopter such as the main rotor, tail rotor, wing, fuselage, horizontal stabilizer, vertical fin, etc. The forces and moments generated by each component of the helicopter are detailed in the subsequent sections. The total forces and moments acting on the helicopter is a sum of all the forces and moments and is detailed in section 2.3.

2.3 Total Forces and Moments on the Helicopter

The total forces and moments acting on the helicopter is a result of the individual forces and moments acting on each component of the helicopter. Thus, the total forces and moments on the helicopter can be represented as:

\[
F = F_{MR} + F_{FUS} + F_{HS} + F_{W} + F_{VF} + F_{TR} \tag{2.8}
\]

\[
\mathcal{M} = \mathcal{M}_{MR} + \mathcal{M}_{FUS} + \mathcal{M}_{HS} + \mathcal{M}_{W} + \mathcal{M}_{VF} + \mathcal{M}_{TR} \tag{2.9}
\]

where,

\[
F = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \text{ and } \mathcal{M} = \begin{bmatrix} L \\ M \\ N \end{bmatrix} \tag{2.10}
\]

The following sections detail the aerodynamic analysis of each component of the helicopter and thus determine the forces and moments generated by the respective component.

2.4 Main Rotor

2.4.1 Main Rotor Thrust

The thrust generated by the rotor is determined based on the method suggested by Heffley and Mnich [12] and Johnson and Turbe [13]. The velocity at the rotor is computed as:

\[
w_r = w + (a_1 + i_s)u - b_1 v \tag{2.11}
\]
where, $u, v$ and $w$ are the velocities of the helicopter in the body frame, $a_1, b_1$ are the longitudinal and lateral flapping coefficients respectively and $i_s$ is the longitudinal incidence angle of the main rotor shaft.

The performance of a rotor having a blade with a linear twist is equivalent to a rotor with the blade having a constant pitch corresponding to the pitch of the linearly twisted blade at 75% of its radius \cite{1}. Thus, for a blade with a linear twist ($\theta_{\text{twist}}$), the velocity at the blades can be expressed as \cite{12,13}:

$$w_b = w_r + \frac{2}{3} \Omega R \left( \theta_{\text{coll}} + \frac{3}{4} \theta_{\text{twist}} \right)$$

(2.12)

Here, $\theta_{\text{coll}}$ is the collective input to the main rotor. In the current case, the rotor blade is of uniform chord and no twist. Thus, $\theta_{\text{twist}} = 0$.

Due to the inter-dependency of the rotor thrust and induced velocity, they are determined through an iterative process. The method typically yields convergence in a few iterations. Assuming an initial non-zero induced velocity, the thrust generated is computed using equation (2.13) \cite{12,13}.

$$T = \frac{1}{4} (w_b - v_i) \Omega R^2 \rho abc$$

(2.13)

where $v_i$ is the induced velocity, $a$ is the lift slope curve of the main rotor, $b$ is the number of blades and $c$ is the average chord of the blade. The intermediate velocity $V'$ is then determined as \cite{13,45}:

$$V' = \sqrt{u^2 + v^2 + (w_r - v_i)^2}$$

(2.14)

Finally, the induced velocity is determined using equation (2.15) as indicated below \cite{13,45}:

$$v_i = T/2\rho \pi R^2 V'$$

(2.15)

Equations (2.13) to (2.15) are solved in an iterative process until the values converge.

The aerodynamic moment exerted on the rotor can be determined based on the power consumed by the main rotor. The induced power and profile power of the main rotor is \cite{12,13}:

$$P_{\text{ind}} = T (v_i - w_r)$$

(2.16)

$$P_{\text{prof}} = \frac{1}{8} \rho f_i R \Omega \left[ (R \Omega)^2 + 4.6 \left( u^2 + v^2 \right) \right]$$

(2.17)
where,

\[ f_r = C_{D_0} Rbc \]  \hspace{1cm} (2.18)

Thus, the moment on the rotor exerted by the air can be stated as:

\[ M_r = (P_{\text{ind}} + P_{\text{prof}}) / \Omega \]  \hspace{1cm} (2.19)

The longitudinal \((a_1)\) and lateral \((b_1)\) flapping coefficients are determined considering first-order flapping dynamics as indicated in the subsequent section. The horizontal force generated by the rotor has been considered negligible and is ignored in the simulation model. Thus, for a longitudinal shaft tilt \(i_s\) and lateral shaft tilt \(l_s\), the total forces generated by the main rotor is given by:

\[ X_{mr} = -T(a_1 - i_s) \]  \hspace{1cm} (2.20)

\[ Y_{mr} = T(b_1 + l_s) \]  \hspace{1cm} (2.21)

\[ Z_{mr} = -T \]  \hspace{1cm} (2.22)

The moments generated by the main rotor is given by [12]:

\[ L_{mr} = -Ty - T(b_1 + l_s)z + \frac{\partial L}{\partial b_1}(b_1 + \zeta \dot{b}_1) \]  \hspace{1cm} (2.23)

\[ M_{mr} = Tx - T(a_1 - i_s)z + \frac{\partial M}{\partial a_1}(a_1 + \zeta \dot{a}_1) \]  \hspace{1cm} (2.24)

\[ N_{mr} = Q_E G_R \]  \hspace{1cm} (2.25)

where \(Q_E\) is the engine torque and \(G_R\) is the gear reduction ratio. Here, \(\zeta\) is the main rotor damping factor. \(\frac{\partial L}{\partial b_1}\) and \(\frac{\partial M}{\partial a_1}\) are the rotor primary flapping stiffness given by [46]:

\[ \frac{\partial L}{\partial b_1} = \frac{\partial M}{\partial a_1} = \frac{3}{4} \frac{e A_b \rho R a (\Omega R)^2}{\gamma} \]  \hspace{1cm} (2.26)

where \(A_b\) is the blade area, \(e\) is the main rotor hinge offset and \(\gamma\) is the Lock number, given by [1]:

\[ \gamma = \frac{\rho a c R^4}{I_b} \]  \hspace{1cm} (2.27)

and \(I_b\) is the blade inertia.
2.4.2 Main Rotor Flapping Dynamics

The flapping motion of the blade at the Tip Path Plane (TPP) can be approximated considering the first-order harmonics as [45]:

$$\beta = a_0 - a_1 \cos(\psi) - b_1 \sin(\psi)$$  \hspace{1cm} (2.28)

where $a_0$ is the blade coning angle, $a_1$ and $b_1$ are the longitudinal and lateral flapping coefficients respectively.

The flapping coefficients are governed by a second-order differential equation for the TPP dynamics which can be approximated by the first-order differential equations given by [12]:

$$\dot{a}_1 = \frac{\gamma \Omega}{16} (-a_1 + \theta_{1s} + k_c b_1 - \frac{\partial a_1}{\partial u} u - \frac{p}{\Omega}) - q$$  \hspace{1cm} (2.29)

$$\dot{b}_1 = \frac{\gamma \Omega}{16} (-b_1 + \theta_{1c} - k_c a_1 - \frac{\partial b_1}{\partial v} v + \frac{q}{\Omega}) - p$$  \hspace{1cm} (2.30)

where $\theta_{1s}$ and $\theta_{1c}$ and the longitudinal and lateral cyclic inputs. The terms, $\frac{\partial a_1}{\partial u}$ and $\frac{\partial b_1}{\partial v}$ represent the dihedral effect due to a change in the rotor sideslip.

Dihedral effects are a result of the rotor longitudinal flapping aligning itself to a change in the flight path angle due to a change in the sideslip of the helicopter. This re-alignment results in a rolling moment similar to that generated by a dihedral wing of a fixed-wing aircraft [46]. The longitudinal and lateral dihedral effects of the main rotor can be computed as [12]:

$$\frac{\partial a_1}{\partial u} = -\frac{\partial b_1}{\partial v} = \frac{2}{\Omega R} \sqrt{\frac{8 C_T}{a \sigma} + \sqrt{\frac{C_T}{2}}}$$  \hspace{1cm} (2.31)

where $C_T$ is the main rotor thrust coefficient given by [1]:

$$C_T = \frac{T}{\rho A \Omega^2 R^2}$$  \hspace{1cm} (2.32)

and $\sigma$ is the rotor solidity given by [1]:

$$\sigma = \frac{b c}{\pi R}$$  \hspace{1cm} (2.33)
In equations (2.29) and (2.30), $k_c$ is the cross-coupling term given by [46]:

$$k_c = -\left(\frac{3}{4}\right)\Omega\left(\frac{e}{R}\right)\left(\frac{16}{\gamma\Omega}\right)$$  \hspace{1cm} (2.34)

Apart from the flapping dynamics, another additional parameter to be considered is the main rotor RPM degree of freedom. The main rotor RPM is modeled as an RPM governor. The same principle is also applied to the tail rotor. The analysis of the RPM Governor model is included in the tail rotor section 2.9.2 and a similar analysis follows for the main rotor as well.

## 2.5 Fuselage

Due to the variations in the shapes and sizes of the fuselage, it is difficult to obtain an aerodynamic database to model the forces on the fuselage. Methods such as wind tunnel testing may need to be adopted to estimate the fuselage aerodynamic characteristics. However, this can be an expensive and time-consuming process. An alternate approximate method is used which estimates the flat plate area of the fuselage in the three directions. The aerodynamic forces are then determined based on this flat plate area. The total velocities on the fuselage can be expressed as [43]:

$$\begin{bmatrix} u_{FU} \\ v_{FU} \\ w_{FU} \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ w_{iFU} \end{bmatrix}$$  \hspace{1cm} (2.35)

where $w_{iFU}$ is the effect of the rotor induced velocity on the fuselage. Thus, the total forces and moments generated due to the fuselage is expressed as:

$$\begin{bmatrix} X_{FU} \\ Y_{FU} \\ Z_{FU} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\rho u_{FU}^2 S_{front} \\ \frac{1}{2}\rho v_{FU}^2 S_{side} \\ \frac{1}{2}\rho w_{FU}^2 S_{top} \end{bmatrix}$$  \hspace{1cm} (2.36)

$$\begin{bmatrix} L_{FU} \\ M_{FU} \\ N_{FU} \end{bmatrix} = -\begin{bmatrix} \text{STA}_{FU} - \text{STA}_{CG} \\ \text{BL}_{FU} - \text{STA}_{CG} \\ \text{WL}_{FU} - \text{WL}_{CG} \end{bmatrix} \times \begin{bmatrix} X_{FU} \\ Y_{FU} \\ Z_{FU} \end{bmatrix}$$  \hspace{1cm} (2.37)

where $S$ represents the front plate area.
### 2.6 Horizontal Stabilizer

The horizontal stabilizer is mainly used to generate a trim load that can compensate for the blade flapping. The aerodynamic forces generated by the horizontal stabilizer provide a stabilizing pitching moment to the helicopter. The velocity components acting on the horizontal stabilizer or tail is given as \[43\]:

\[
\begin{bmatrix}
    u_{HS} \\
    v_{HS} \\
    w_{HS}
\end{bmatrix} = \begin{bmatrix}
    u \\
    v \\
    w
\end{bmatrix} + \begin{bmatrix}
    STA_{HS} - STA_{CG} \\
    BL_{HS} - BL_{CG} \\
    WL_{HS} - WL_{CG}
\end{bmatrix} \times \begin{bmatrix}
    p \\
    q \\
    r
\end{bmatrix} + \begin{bmatrix}
    0 \\
    0 \\
    w_{ihs}
\end{bmatrix}
\]

The \(w_{ihs}\) term is the effect of the rotor induced velocity on the horizontal stabilizer.

The angle of attack of the stabilizer is given as \[43\]:

\[
\alpha_{HS} = \tan^{-1}\left(\frac{u_{HS}}{V_{HS}}\right)
\]

In case the horizontal stabilizer is installed at an angle to the XY plane of the helicopter, the effective angle of attack of the horizontal stabilizer is then given by \[43\]:

\[
\alpha_{eff} = \alpha_{HS} + i_{HS}
\]

where \(i_{HS}\) is the angle of incidence of the horizontal stabilizer.

In the case of the current helicopter, the entire tail is similar to a tilt-wing and can be set to any desired orientation. Thus, the angle of incidence considered is the elevator input \(\delta_{el}\) that is provided. However, as control allocation schemes have not been incorporated, the \(\delta_{el}\) input has not been considered in the system simulation. Thus, the incidence angle of the horizontal stabilizer is zero.

The sideslip can be expressed as \[43\]:

\[
\beta_{HS} = \sin^{-1}\left(\frac{u_{HS}}{V_{HS}}\right)
\]

where,

\[
V_{HS} = \sqrt{u_{HS}^2 + v_{HS}^2 + w_{HS}^2}
\]
Thus, the Lift and Drag of the horizontal stabilizer are:

\[ L_{HS} = \frac{1}{2} \rho V_{HS}^2 S_{HS} C_{L_{HS}} \]  

(2.43)

\[ D_{HS} = \frac{1}{2} \rho V_{HS}^2 S_{HS} C_{D_{HT}} \]  

(2.44)

where,

\[ C_{L_{HS}} = a \alpha_{eff} \]  

(2.45)

and,

\[ C_{D_{HS}} = 0.0087 - 0.0216 \alpha_{eff} + 0.4 \alpha_{eff}^2 \]  

(2.46)

The expression for the drag coefficient assumes that the stabilizer has NACA 23012 profile [5].

As the lift and drag are the only two forces acting on the stabilizer, the total forces and moments in the body frame due to the horizontal stabilizer are [43]:

\[
\begin{bmatrix}
X_{HS} \\
Y_{HS} \\
Z_{HS}
\end{bmatrix} =
\begin{bmatrix}
\cos \alpha_{HS} & 0 & -\sin \alpha_{HS} \\
0 & 1 & 0 \\
\sin \alpha_{HS} & 0 & \cos \alpha_{HS}
\end{bmatrix}
\begin{bmatrix}
\cos \beta_{HS} & -\sin \beta_{HS} & 0 \\
\sin \beta_{HTS} & \cos \alpha_{HS} & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
-D_{HS} \\
0 \\
-L_{HS}
\end{bmatrix}
\]

(2.47)

\[
\begin{bmatrix}
L_{HS} \\
M_{HS} \\
N_{HS}
\end{bmatrix} = -
\begin{bmatrix}
STA_{HS} - STA_{CG} \\
BL_{HS} - BLCG \\
WL_{HS} - WL_{CG}
\end{bmatrix}
\times
\begin{bmatrix}
X_{HS} \\
Y_{HS} \\
Z_{HS}
\end{bmatrix}
\]

(2.48)

**2.7 Wing**

The analysis of the wing is similar to that of the horizontal stabilizer with the forces and moments determined similarly. The primary difference will be the downwash velocity component acting on the wing. As the wing is directly under the main rotor, the effect of the downwash will be greater as compared to the horizontal stabilizer. As the effect of flaperons is not considered in the current analysis, modeling of the flaperons can be ignored and the wing can be modeled as a single entity.
2.8 Vertical Fin

The aerodynamic forces on the vertical fin are again similar to that of the horizontal tail except for the change in orientation. The velocities on the vertical fin can be expressed as [43]:

\[
\begin{bmatrix}
  u_{VF} \\
  v_{VF} \\
  w_{VF}
\end{bmatrix} =
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix} +
\begin{bmatrix}
  STA_{VF} - STA_{CG} \\
  BL_{VF} - BL_{CG} \\
  WL_{VF} - WL_{CG}
\end{bmatrix} \times
\begin{bmatrix}
  p \\
  q \\
  r
\end{bmatrix} +
\begin{bmatrix}
  0 \\
  v_{it} \\
  w_{iVF}
\end{bmatrix}
\] (2.49)

Note here that apart from the main rotor induced velocity term \( w_{iVF} \), there is an additional term \( v_{it} \) which is the effect of the tail rotor downwash on the vertical fin. The angle of attack and sideslip of the vertical fin is given by [43]:

\[
\alpha_{VF} = \tan^{-1}\left( \frac{v_{VF}}{u_{VF}} \right)
\] (2.50)

\[
\beta_{VF} = \sin^{-1}\left( \frac{w_{VF}}{V_{VF}} \right)
\] (2.51)

where,

\[
V_{VF} = \sqrt{u_{VF}^2 + v_{VF}^2 + w_{VF}^2}
\] (2.52)

Similar to the horizontal stabilizer, if the vertical fin is mounted at an angle to the XZ plane, the effective angle of attack of the vertical fin can be expressed as:

\[
\alpha_{eff} = \alpha_{VF} + i_{VF}
\] (2.53)

Here, \( i_{VF} \) is the angle of incidence of the vertical fin.

With the velocity at the vertical fin known, along with the angle of attack and sideslip angle, the total forces and moments due to the vertical fin can be computed like that of the horizontal stabilizer.

2.9 Tail Rotor

This section details the modeling of the Vectored Thrust Ducted Propeller (VTDP) configuration of the tail rotor. The aerodynamic analysis and modeling of the tail rotor form one of the primary objectives of this thesis.
As stated in chapter 1, the helicopter considered is a scaled version of the Piasecki X-49 aircraft. Figure 2.1 obtained from the paper by Geiger et al. [6] details the architecture of the tail rotor. As stated earlier, the tail rotor consists of a ducted pusher propeller along with a rudder and sectors to direct the flow.

![Figure 2.1. Tail Rotor configuration of the Piasecki X-49 Helicopter](image)

In the original X-49 configuration, the variable parameters in the tail rotor are the rudder and sectors deflection and the pitch of the tail rotor blades. The tail rotor RPM is linked to the main rotor similar to a conventional helicopter through a drive-train system. The tail rotor performs dual roles of providing additional forward thrust by varying the tail rotor pitch and anti-torque and yaw control through deflection of the rudder and sectors. The sectors help to deflect the flow thus allowing vectoring of the thrust generated by the tail rotor. As explained in the paper by Geiger et al. [6], the rudder deflection and tail rotor pitch are controlled through the 'beep' setting. In low-speed conditions, the rudder and sector deflection is primarily controlled to provide the desired yaw control and
anti-torque moment. The motion of the propeller pitch is limited during low-speed flights. During high-speed requirements, larger emphasis is provided to the blade pitch to generate a higher thrust. The deflection of the rudder and sectors are limited in this case. Thus, the 'beep' setting helps determine the dominant control in a particular phase of flight.

The tail rotor analyzed in this thesis has a variation as compared to the X-49 configuration. The rudder and sectors in the scaled model perform a similar function as that of the X-49. However, the tail rotor/propeller pitch is fixed. Instead of varying the tail rotor pitch, the RPM of the propeller is varied to generate the desired thrust. Further, as the main rotor direction of rotation is clockwise when viewed from the top, the sectors are placed on the left side of the duct with rudder and sector deflection towards the right to provide the anti-torque moment in the right direction.

The aerodynamic modeling and analysis of the tail rotor has been performed considering each component of the tail rotor. From figure 2.1, the forces and moments contributed by the tail rotor can be attributed to four main components i.e. the propeller, effect of the duct on the propeller thrust as well as the lift and drag contributed by the duct as an annular ring, the effect of the rudder and finally, the contribution of the sectors which help in orienting the tail rotor thrust. In the following sections, the forces generated due to each of these components is considered to determine the total forces and moments generated by the tail rotor.

### 2.9.1 Propeller Thrust

The thrust generated by the propeller is computed similar to the main rotor. As the propeller is mounted perpendicular to the X axis of the helicopter, the velocity at the rotor is determined as:

\[
  u_r = -u + yr - zq
\]

where,

- \(u\) is the velocity of the helicopter in the body frame,
- \(q, r\) are the pitching and yawing rates of the helicopter respectively and
- \(y, z\) are the distance of the propeller hub from the CG of the helicopter along the \(y\) and \(z\) axis respectively.

It has been stated earlier that the propeller is a fixed-pitch propeller, a linear
distribution of pitch is assumed. Thus, for a blade with a linear twist ($\theta_{\text{twist}}$), the velocity at the blades can be expressed as [12, 13]:

$$u_b = u_r + \frac{2}{3} \Omega R \left( \frac{3}{4} \theta_{\text{twist}} \right)$$  

(2.55)

As in the case of the main rotor, the thrust of the propeller is determined as [12, 13]:

$$T = \frac{1}{4} (u_b - v_i) \Omega R^2 \rho abc$$  

(2.56)

where $v_i$ is the induced velocity, $a$ is the lift slope curve of the propeller, $b$ is the number of blades and $c$ is the average chord of the blade.

The intermediate velocity $V'$ is determined as [13, 45]:

$$V' = \sqrt{v^2 + w^2 + (u_r - v_i)^2}$$  

(2.57)

Finally, the induced velocity is determined using equation (2.58) as indicated below [13, 45]:

$$v_i = \frac{T}{2 \rho \pi R^2 V'}$$  

(2.58)

Equations (2.56) to (2.58) are solved in an iterative process until the values converge. Equation (2.56) is the total force generated by the propeller. This force acts in the x-direction of the aircraft. No forces are acting in the y and z direction due to the thrust of the propeller.

### 2.9.2 Tail Rotor RPM Governor

As stated earlier, the tail rotor in the configuration considered has two degrees of freedom; one in terms of the rudder and sector deflections and the second in terms of the speed control of the rotor RPM. Due to the RPM degree of freedom, the tail rotor RPM control is modeled as an RPM governor model assuming simplified engine dynamics. The modeling is based on the methodology adopted by Johnson and Turbe [13] and Talbot et al. [47]. The tail rotor speed is controlled based on the torque requirement and engine power available. For any change in yaw, the torque requirement from the tail rotor will vary which, in turn, will result in a
change in the tail rotor RPM. The RPM speed control law can be written as [47]:

\[ Q_E - Q_R = J \dot{\Omega} \]  

(2.59)

where \( Q_E \) is the engine torque, \( Q_R \) is the required torque, \( J \) is the rotor rotational inertia and \( \dot{\Omega} \) is the rotor speed rate of change. As the tail rotor hinge offset is negligible, the tail rotor rotational inertia \( J \) can be approximately written as:

\[ J = b I_b \]  

(2.60)

where \( b \) is the number of blades and \( I_b \) is the flapping inertia of a single blade.

While the torque generated by the tail rotor depends on various factors, the torque can be determined based on the power consumed by the tail rotor. The power consumed by the tail rotor depends on the induced power and profile power. The induced power and profile power of the tail rotor is computed similar to the main rotor as [12,13]:

\[ P_{\text{ind}} = T (v_i - u_r) \]  

(2.61)

\[ P_{\text{prof}} = \frac{1}{8} \rho f_r R \Omega \left[ (R \Omega)^2 + 4.6 \left( v^2 + w^2 \right) \right] \]  

(2.62)

where,

\[ f_r = C_{D_0} R b c \]  

(2.63)

Thus, the torque on the rotor exerted by the air can be stated as:

\[ Q_R = \frac{(P_{\text{ind}} + P_{\text{prof}})}{\Omega} \]  

(2.64)

The moment generated by the tail rotor is a result of the distance from the helicopter CG as well as the torque generated by the tail rotor. As indicated above, this moment generated by the tail rotor is used in determining the update law for the tail rotor RPM. The "beep" input of the tail rotor is mapped to the tail rotor RPM. Changes in the beep and desired yaw acceleration determine the change required in the tail rotor RPM to generate the desired force and yaw moment. Details of the same are further elaborated in the next chapter.

Equation (2.64) determines the torque generated by the tail rotor. The additional torque available depends on the maximum torque of the motor as well as the amount of beep control available. For a given beep control, the engine torque is determined
using equation (2.65) as:

\[ Q_E = Q_{E_{max}} \delta_{beep} \]  

(2.65)

Thus, the update law for the tail rotor RPM is determined as [13]:

\[ \dot{\Omega} = \frac{Q_E - Q_R}{b I_b} \]  

(2.66)

where \( i_b \) is the inertia of a single blade.

In the above analysis, the force generated by the propeller is determined assuming an open propeller. As the propeller is enclosed in a duct, the performance of the propeller will be different as compared to an open propeller. The duct can result in an increase or decrease in thrust depending on the configuration of the duct. The subsequent section explains the effect of the duct on the thrust generated by the propeller.

### 2.9.3 Effect of Duct on Propeller Thrust

While the thrust generated by a propeller depends on various factors such as blade design and rotation speed, further enhancement in efficiency i.e. higher thrust for the same power can be achieved with the use of ducts around propellers [7]. A proper design of the shroud can result in a higher thrust being produced. If a duct is so shaped that it accelerates the flow at the rotor, the ducted propeller will produce a higher thrust as compared to an open propeller. The following analysis of the additional thrust generated by a shroud is based on the theory described by McCormick [7].

Ignoring the effects of the rotor, the flow in a duct is influenced by two aspects viz. the camber of the duct and the convergence angle. Consider figure 2.2 obtained from the textbook by McCormick [7].

![Figure 2.2. Types of ducts](image)

The figure illustrates three duct shapes with positive, zero and negative convergence respectively. A duct with a positive convergence will diffuse the flow. A
similar effect is observed if the duct is cambered. Positive camber will result in positive convergence.

It can be seen that due to the duct, the airflow near the leading edge of the duct is oriented radially either inward or outward based on the duct shape. This effect can be analyzed by neglecting the duct and considering a sequence of continuously distributed vortex rings. While this method leads to an accurate estimation of the thrust generated by the duct, the process is complicated and does not lead to a closed-form solution. An approximation to this process, known as Weissinger’s approximation is to consider a vortex at the quarter chord distance of the duct and analyze the boundary conditions at the three-quarter distance. The analysis can be performed with the aid of figure 2.3 obtained from the textbook by McCormick [7].

![Figure 2.3. Duct geometry for thrust estimation](image)

The following analysis holds with an assumption that the camber of the duct is reasonably small such that the straight-line horizontal distance between the quarter-chord point and the three-quarters point can be approximated as half the chord distance. The radially induced velocity due to the duct at the three-quarter point can be expressed as [7]:

\[
v_{ir} = \frac{\Gamma}{\pi D_{1/4}} f \left( \frac{c}{D_{1/4}}, \frac{D_{3/4}}{D_{1/4}} \right)
\]

(2.67)

where \( \Gamma \) is the circulation, \( D_{1/4} \) and \( D_{3/4} \) are the duct diameters at one-quarter and three-quarter distance respectively and the function \( f \) can be obtained from the curve indicated in figure 2.4 obtained from the textbook by [7] based on the geometry of the shroud. A combination of the propeller and shroud is illustrated
Using the above relations and continuity analysis, the radially induced velocity due to the rotor is given by [7]:

\[ v_{iR} = -\frac{1}{2} \frac{RR_R^2 w_0}{(z^2 + R_R^2)^{3/2}} \]  \hspace{1cm} (2.68)

where \( R_R \) is the radius of the open propeller, \( R \) is the radius of the duct, \( z \) is the
axial distance from the propeller and $w_0$ is the axial induced velocity given by [7]:

$$w_0 = \frac{1}{2} \left\{ -u_r + \left[ u_r^2 + \frac{T_R}{(\rho/2)\pi R^2_{R}} \right]^{1/2} \right\} \quad (2.69)$$

Thus, due to the vortex ring at the quarter chord point, the total circulation is given by equation (2.70) [7]:

$$\Gamma = \frac{\pi D_{1/4}}{f \left( c/D_{1/4}, D_{3/4}/D_{1/4} \right)} \left[ -v_{iR_{3/4}} - \theta \left( u_r + w_{a_{3/4}} \right) \right] \quad (2.70)$$

and the resulting thrust by the shroud is expressed as [7]:

$$T_s = -\rho \left( \pi D_{1/4} \right)^2 \frac{v_{iR_{3/4}} v_{iR_{1/4}}}{f \left( c/D_{1/4}, D_{3/4}/D_{1/4} \right)} \quad (2.71)$$

where $\theta$ is the duct angle as indicated in figure 2.5. Equation (2.71) is used to compute the additional thrust generated by the duct.

Assuming a straight duct i.e. $\theta = 0$, the thrust equation (2.71) is simplified to equation (2.72) [7]:

$$T_s = \frac{\rho \left( \pi D_{1/4} \right)^2 v_{iR_{1/4}} v_{iR_{3/4}}}{f \left( c/D_{1/4}, D_{3/4}/D_{1/4} \right)} \quad (2.72)$$

In the above equation, the product $v_{iR_{1/4}} v_{iR_{3/4}}$ depends on the location of the rotor in the duct. It has a maximum value of $0.13w_0^2$ for the rotor at the center of the duct and a minimum value of $0.09w_0^2$ if the rotor is at the quarter chord or three quarter chord point in the duct [7].

Thus, assuming the rotor to be located around the one-third chord location, considering a value of 0.1 for the product $v_{iR_{1/4}} v_{iR_{3/4}}/w_0^2$, the additional thrust due to the duct can be written as:

$$T_s = 0.1 \ w_0^2 \ \frac{\rho \left( \pi D_{1/4} \right)^2}{f \left( c/D_{1/4}, D_{3/4}/D_{1/4} \right)} \quad (2.73)$$

It should be noted that in the above analysis, the duct has been assumed to be straight. Thus, even a straight duct can increase the overall efficiency of the
Another perspective to understand the effect of the duct on the propeller efficiency is from the viewpoint of the tip vortex generated by the propeller. It is known that tip vortices lead to a loss in power i.e. efficiency of the rotor/propeller. The use of a duct helps in considerably reducing the tip vortices which in turn helps in improving the efficiency of the rotor. Thus, the total forces by the ducted propeller is given by:

\[ F_{\text{prop}} = T + T_s \]  

(2.74)

The above analysis has dealt with the effect of the duct on the propeller thrust ignoring the effects of the drag caused by the duct and the additional lift that would be generated by the duct owing to the annular ring shape. The following section analyzes the effect of the duct lift and drag by considering it as an annular ring without considering the propeller. This is a reasonable assumption because a propeller in the duct typically does not increase the lift of the duct [7].

2.9.4 Forces due to Duct

The analysis of the lift and drag forces generated by the duct is based on the estimated lift, drag and moment coefficients of an annular ring as determined by Fletcher [48] who details the experimental results of five annular rings with different aspect ratios. The analysis presented below utilizes the coefficients determined from Fletcher [48] and computes the effective lift, drag and self-moment of the duct. As stated earlier, the effect of the propeller is not considered in the duct analysis as the same does not affect the lift generated by the annular duct [7].

The lift and drag acting on the duct depend on the velocity at the duct. The velocity is influenced by the motion of the helicopter as well as the downwash of the main rotor on the duct. The velocity at the duct is determined as:

\[
\begin{bmatrix}
  u_d \\
v_d \\
w_d
\end{bmatrix} = \begin{bmatrix}
u \\
v \\
w
\end{bmatrix} + \begin{bmatrix}
\text{STA}_d - \text{STA}_{CG} \\
\text{BL}_d - \text{BL}_{CG} \\
\text{WL}_d - \text{WL}_{CG}
\end{bmatrix} \times \begin{bmatrix}
p \\
q \\
r
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
w_{i\text{mr}}
\end{bmatrix}
\]  

(2.75)

where \(w_{i\text{mr}}\) is the effect of the main rotor downwash at the duct. The magnitude of the velocity at the duct is given by:

\[ V_d = \sqrt{u_d^2 + v_d^2 + w_d^2} \]  

(2.76)
The duct angle of attack and sideslip can then be computed as:

$$\alpha_d = \tan^{-1}\left(\frac{w_d}{u_d}\right)$$ (2.77)

$$\beta_d = \sin^{-1}\left(\frac{v_d}{V_d}\right)$$ (2.78)

The lift, drag, and moment coefficients at different angles of attack are determined from a lookup table based on the graphs obtained from the technical note by Fletcher [48]. Based on these values, the lift, drag, and self-moment of the annular duct can be determined using equations (2.79) to (2.81).

$$L = \frac{1}{2} \{V_d\} S_d C_L$$ (2.79)

$$D = \frac{1}{2} \{V_d\} S_d C_D$$ (2.80)

$$M = \frac{1}{2} \{V_d\} S_d c_{duct} C_M$$ (2.81)

where $S_d$ is the reference area given by:

$$S_d = D_{duct} c_{duct}$$ (2.82)

and $D_{duct}$ and $c_{duct}$ represent the duct diameter and chord.

As the self-pitching moment of the duct is relatively small, the same can be ignored. The pitching moment of the duct would be due to the forces on the duct and the relative distance between the duct and the helicopter CG. Finally, the resultant forces of the duct as expressed in the helicopter body axes is given by:

$$X_{duct} = -D \cos \alpha_d \cos \beta_d + L \cos \alpha_d \sin \beta_d$$ (2.83)

$$Y_{duct} = -D \sin \beta_d - L \cos \beta_d$$ (2.84)

$$Z_{duct} = -D \sin \alpha_d \cos \beta_d + L \sin \alpha_d \sin \beta_d$$ (2.85)

The above forces due to the duct would have a major contribution only in forward/rearward flight and during climb. Although the rotor downwash would be considerable during hover, the effective forces generated would be minimal and the same can be ignored.
With the forces of the propeller and duct being considered, the remaining components that contribute to the helicopter tail rotor forces are the rudder and the sectors. The forces generated by these two components are considered in the following sections. The effect of a rudder would be considerable even in hover due to the downwash of the ducted propeller. The analysis of the rudder would be identical to that of the vertical fin and is elaborated in the next section.

### 2.9.5 Forces due to Rudder

Apart from the forces generated by the propeller, forces and moments are also generated due to the aerodynamic forces acting on the rudder. The forces and moments generated by the rudder have been determined by modeling the rudder similar to a vertical fin [43]. The velocity components on the rudder, expressed in the body axes can be represented as:

\[
\begin{bmatrix}
    u_{\text{rud}} \\
    v_{\text{rud}} \\
    w_{\text{rud}}
\end{bmatrix} = \begin{bmatrix}
    u \\
    v \\
    w
\end{bmatrix} + \begin{bmatrix}
    \text{STA}_{\text{rud}} - \text{STA}_{\text{CG}} \\
    \text{BL}_{\text{rud}} - \text{BL}_{\text{CG}} \\
    \text{WL}_{\text{rud}} - \text{WL}_{\text{CG}}
\end{bmatrix} \times \begin{bmatrix}
    p \\
    q \\
    r
\end{bmatrix} + \begin{bmatrix}
    w_{a} \\
    0 \\
    0
\end{bmatrix}
\]

(2.86)

where \(w_{a}\) is the axial velocity of the propeller at distance \(z\) (distance between rudder and propeller) from the propeller given by [7]:

\[
\frac{w_{a}(z)}{v_{i}} = 1 + \frac{z/R}{\sqrt{1 + (z/R)^2}}
\]

(2.87)

where \(v_{i}\) is the induced velocity of the tail rotor.

The angle of attack and sideslip of the rudder is computed as:

\[
\alpha_{\text{rud}} = \tan^{-1}\left(\frac{v_{\text{rud}}}{u_{\text{rud}}}\right)
\]

(2.88)

\[
\beta_{\text{rud}} = \sin^{-1}\left(\frac{w_{\text{rud}}}{V_{\text{rud}}}\right)
\]

(2.89)

\[
V_{\text{rud}} = \sqrt{u_{\text{rud}}^2 + v_{\text{rud}}^2 + w_{\text{rud}}^2}
\]

(2.90)

The effective angle of attack of the rudder considering the rudder deflection can be
expressed as:

\[ \alpha_{\text{eff}} = \alpha_{\text{rud}} + \delta_r \]  

(2.91)

where \( \delta_r \) is the rudder deflection.

The lift and drag of the rudder in the local wind axis can be expressed as:

\[ L_{\text{rud}} = \frac{1}{2} \rho V_{\text{rud}}^2 S_{\text{rud}} C_{L_{\text{rud}}} \]  

(2.92)

\[ D_{\text{rud}} = \frac{1}{2} \rho V_{\text{rud}}^2 S_{\text{rud}} C_{D_{\text{rud}}} \]  

(2.93)

where, \( S_{\text{rud}} \) is the rudder reference area. The lift and drag coefficient are computed as [43]:

\[ C_{L_{\text{rud}}} = a \alpha_{\text{eff}} \]  

(2.94)

and,

\[ C_{D_{\text{rud}}} = 0.0087 - 0.0216 \alpha_{\text{eff}} + 0.4 \alpha_{\text{eff}}^2 \]  

(2.95)

The expression for the drag coefficient assumes that the rudder is NACA 23012 profile. Thus, the total forces in the body frame due to the rudder are:

\[
\begin{bmatrix}
X_{\text{rud}} \\
Y_{\text{rud}} \\
Z_{\text{rud}}
\end{bmatrix} =
\begin{bmatrix}
\cos \beta_{\text{rud}} & 0 & -\sin \beta_{\text{rud}} \\
0 & 1 & 0 \\
\sin \beta_{\text{rud}} & 0 & \cos \beta_{\text{rud}}
\end{bmatrix}
\begin{bmatrix}
\cos \alpha_{\text{rud}} & -\sin \alpha_{\text{rud}} & 0 \\
\sin \alpha_{\text{rud}} & \cos \alpha_{\text{rud}} & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
-D_{\text{rud}} \\
-L_{\text{rud}} \\
0
\end{bmatrix}
\]

(2.96)

2.9.6 Forces due to Sectors

The sector is ganged with the rudder such that the deflection of one surface results in the deflection of the other. The rudder and the sectors deflection (\( \theta \)) is between 0 to 90°. For simplicity of analysis, the rudder and sectors are considered as separate entities with the assumption that the aerodynamic effects of both surfaces are independent without any cross-coupling. The forces and moments generated by the rudder have been analyzed in the previous section. The analysis of the sectors is detailed below. The sectors deflect the flow in a particular direction such that the force generated by the sectors is in a direction opposite to that of the flow. A Control Volume (CV) analysis is performed on the sectors to determine the total forces acting on the sector. The CV analysis is performed assuming the flow to be
2-dimensional i.e. in the XY plane with no resultant flow in the Z direction.

Figure 2.6 illustrates the CV defined for the sectors. As stated above, the sectors are analyzed independently without considering the effects of the rudder. The downwash from the propeller enters section 1 of area $A_1$ with velocity $V_1$ and exits section 2 of area $A_2$ with velocity $V_2$. It is assumed that areas $A_1$ and $A_2$ are the same. Further, it is also assumed that the inlet velocity $V_1$ and outlet velocity $V_2$ are identical. In the previous section, the velocity at an axial distance from the rotor was computed using equation (2.87). As the rudder and the sector are at the same axial distance from the propeller, $V_1 = u_{rud}$.

Linear Momentum analysis is used on the control volume to determine the total forces acting on the sectors. To apply the linear momentum equation, it is assumed that the control volume is fixed and non-deforming. The linear momentum equation for the control volume can then be expressed as [49]:

$$\frac{\partial}{\partial t} \int_{cv} \rho V \, dV + \int_{cs} V \rho V \cdot \hat{n} \, dA = \sum F_{CV}$$  \hspace{1cm} (2.97)

The x and y components of equation (2.97) can be written as:

$$\frac{\partial}{\partial t} \int_{CV} V_x \rho dV + \int_{cs} u \rho V \cdot \hat{n} \, dA = \sum F_x$$  \hspace{1cm} (2.98)

$$\frac{\partial}{\partial t} \int_{CV} V_y \rho dV + \int_{cs} v \rho V \cdot \hat{n} \, dA = \sum F_y$$  \hspace{1cm} (2.99)

where $\sum F_x$ and $\sum F_y$ are the net forces acting in the x and y direction and $V_x$ and
$V_y$ are the velocity components in the respective directions.

It should be noted that the flow in the duct is considered to be uniform and steady. Thus, the first terms in equations (2.98) and (2.99) will be 0. Further, as the air enters and leaves the control volume in atmospheric pressure, no pressure forces are assumed to act on the body.

In figure 2.6, $\mathbf{n}_1$ and $\mathbf{n}_2$ represent the unit normal vectors at the input and output of the control volume. Thus, $V_1 \cdot \mathbf{n}_1 = -V_1$ and $V_2 \cdot \mathbf{n}_2 = -V_2$. Using the above information, equations (2.98) and (2.99) are written as:

$$V_1\rho (- V_1) A_1 + V_1 \cos \theta \rho (V_1) A_2 = X_{sect}$$

(2.100)

$$(0)\rho (- V_1) A_1 + V_1 \sin \theta \rho (V_1) A_2 = Y_{sect}$$

(2.101)

It has been previously stated that the areas in both sections of the control volume are assumed to be same with the velocity also being constant across both the sections. Therefore, we have $V_1 A_1 = V_2 A_2$. The rudder and the sectors are approximately at the same STA location in the tail rotor. Thus, the velocity entering the sector would be the same as the velocity at the rudder. Further, as the sectors immediately follow the duct, it can be assumed that the only component of velocity acting on the sectors is the $u$ component of the rudder velocity. Thus, in the above equations, we have $V_1 = u_{rud}$ and $\theta = \delta_r$. With the above considerations, equations (2.100) and (2.101) result in the following forces in the x and y directions:

$$X_{sect} = -\rho A_1 u_{rud}^2 + \rho A_1 u_{rud}^2 \cos \delta_r = -\rho A_1 u_{rud}^2 (1 - \cos \delta_r)$$

(2.102)

$$Y_{sect} = \rho A_1 u_{rud}^2 \sin \delta_r$$

(2.103)

Equations (2.102) and (2.103) represent the total forces acting on the sector. As stated earlier, forces are assumed to act only in the x and y-direction. Thus, the force exerted by the sector in the z-direction is zero.

### 2.9.7 Total Forces and Moments due to Tail Rotor

The total forces generated by the tail rotor in the body axes are the sum of the forces contributed by each element of the tail rotor i.e. the propeller, duct, rudder and sectors. The sum of all these forces is the resultant force generated by the tail
rotor on the helicopter. These forces result in the corresponding moments that act at the CG of the helicopter. The total forces generated by the tail rotor can thus be represented as:

\[ F_{TR} = F_{prop} + F_{duct} + F_{rud} + F_{sect} \]  \hspace{1cm} (2.104)

where,

\[ F_{TR} = \begin{bmatrix} X_{TR} \\ Y_{TR} \\ Z_{TR} \end{bmatrix} \]  \hspace{1cm} (2.105)

Thus, the total moments generated by the tail rotor is:

\[ \begin{bmatrix} L_{TR} \\ M_{TR} \\ N_{TR} \end{bmatrix} = - \begin{bmatrix} \text{STA}_{TR} - \text{STACG} \\ \text{BL}_{TR} - \text{BLCG} \\ \text{WL}_{TR} - \text{WL}_{CG} \end{bmatrix} \times \begin{bmatrix} X_{TR} \\ Y_{TR} \\ Z_{TR} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ Q_{E} \end{bmatrix} \]  \hspace{1cm} (2.106)

### 2.10 Comparison of Tail Rotor Model with Test Data

The above-stated tail rotor model was compared with the test data obtained from the actual helicopter. The test data is a static data of the tail rotor which was generated by controlling the RPM of the tail rotor and the rudder and sector deflection. The forces generated in the X and Y directions were then measured. A comparison of the model and test data was performed, and it was observed that the model is a reasonable fit to the data. The test stand data is not included in the thesis owing to the proprietary nature of the data.

Although the model is a good fit to the data, variations exist between the model and test data. These variations may exist due to various factors such as flutter in the rudder, additional drag effects of the sectors, assumption of uniform inflow through the duct, assumption of uniform and steady flow through the sectors, possible interference between the rudder and sectors, etc. A higher accuracy model can be achieved by addressing these factors. However, the model is a good fit for the analysis of basic helicopter control and handling qualities. Thus, the tail rotor model explained in the previous section is used for the design of the controller.
Dynamic Inversion and Adaptive Control

3.1 Dynamic Inversion

Dynamic Inversion, also known as feedback linearization, is a method that facilitates the conversion of a non-linear relation between input and output to a linear relationship between the output and a pseudo-control input, thus allowing the use of linear control techniques for analysis and design. Linear control techniques of non-linear systems typically involve linearizing the system around an operating point and designing a controller in a close regime to the linearization point. For typical flight control applications, the flight envelope involves multiple operating points. Suitable trim points are identified for each phase of flight and linear controllers are designed around these operating points. This method, known as Gain Scheduling, is suitable if the regime of operation is close to the linearization point. While using Gain Scheduling, controllers have multiple look-up tables with scheduled gains that are used in various phases of flight.

An alternative to this methodology of design is the use of dynamic inversion based controllers. In dynamic inversion, the non-linearity in the plant is typically inverted such that a linear relation exists between a pseudo-control and the output, thus facilitating the use of linear control techniques. The method can help eliminate the need for gain scheduling in a manner such that the same controller can be used in all phases of flight. Thus, dynamic inversion is an alternative technique that can deal directly with the known non-linearities of the aircraft dynamics and use
these non-linearities in the controller to improve the system performance [28]. The following sections provide a brief overview of the use of dynamic inversion for linear and non-linear systems.

3.1.1 Dynamic Inversion in Linear Systems

While dynamic inversion is more suited for non-linear systems, the analogy is better understood when dealt with using a linear system. The following sections regarding linear and nonlinear dynamic inversion are based on the theory stated in Stevens et al. [28]. Consider a plant expressed by the state space representation:

\[ \dot{x} = Ax + Bu \]  
\[ y = Cx \]

where \( x(t) \in \mathbb{R}^n \), \( u(t) \in \mathbb{R}^m \) and \( y(t) \in \mathbb{R}^p \).

Typically, the assumption is that the system is square i.e. the number of inputs and outputs are equal. To obtain a relation between the input and the output, the output equation (3.2) is continuously differentiated until a relation exists between the output and input. Thus, differentiating equation (3.2), we get:

\[ \dot{y} = C\dot{x} = CAx + CBu \]

For dynamic inversion to work, the \( CB \) term in equation (3.3) should be non-zero. We will continue to differentiate equation (3.2) if the term \( CB \) in equation (3.3) is zero. We will stop differentiating once the multiplication factor for the \( u \) term is non-zero. If we define an auxiliary input \( v(t) \) such that,

\[ v = CBu + CAx - \dot{r} \]  

where \( r \) is the desired reference signal. Then, the input \( u \) to the plant can be represented as:

\[ u = (CB)^{-1}(\dot{r} - CAx + v) \]

Thus, equation (3.3) becomes:
\[ \dot{y} = CAx + CB \left[ (CB)^{-1}(\dot{r} - CAx + v) \right] \]
\[ = CAx + \dot{r} - CAx + v \]
\[ = \dot{r} + v \]  
(3.6)

The objective of dynamic inversion is to ensure that the output follows a reference signal. Thus, the error signal can be defined as:

\[ e(t) = r(t) - y(t) \]  
(3.7)

Using equations (3.6) and (3.7), we have the error dynamics which is given by:

\[ \dot{e} = -v \]  
(3.8)

Thus, suitably selecting the pseudo-control signal \( v \) will ensure that the error dynamics are stable. A simple choice for the pseudo-control signal is \( v = Ke \) with \( K \) being positive definite so that the error dynamics is stable and becomes:

\[ \dot{e} = -Ke \]  
(3.9)

Equation (3.5) is the desired dynamic inversion control input to the plant. This input ensures that a linear relation exists between the auxiliary input \( v \) and the plant output \( y \). Thus, the system is modified to a set of \( p \) integrators for the \( p \) inputs and outputs. While selecting the auxiliary input \( v \), it can be ensured that the error dynamics are stable.

With the use of dynamic inversion, it is also necessary to observe the zero-dynamics of the system. Dynamic inversion ensures that the \( p \) poles of the system are stable. However, as the system is of order \( n \), the remaining \( p - n \) poles of the system are not observable through the output \( y \) and hence cannot be controlled [28]. These poles are given by the zero dynamics of the system and should be analyzed. In the case of any non-minimum phase zeros in the system, they will manifest as unstable poles due to the dynamic inversion process and result in an unstable closed-loop system. A proper selection of the Controlled Variables (CV) i.e. the plant output \( y(t) \) can help ensure that the zero dynamics of the system are stable.
3.1.2 Non-linear Dynamic Inversion (NDI)

Owing to the inherent non-linearity in real-world systems, dynamic inversion is a useful tool for the control of non-linear systems. Consider a non-linear system represented in the form:

\[ \dot{x} = f(x) + g(x)u \]  
\[ y = h(x) \]  
(3.10)  
(3.11)

The input, output, and state dimensions are assumed to be the same as their linear counterpart and the assumption that the system is square also holds. It should also be noted that the system is assumed to be affine in control. Now, differentiating the output equation (3.11), we get,

\[ \dot{y} = \frac{\partial h}{\partial x} \dot{x} = \frac{\partial h}{\partial x} f(x) + \frac{\partial h}{\partial x} g(x)u \]
\[ \equiv F(x) + G(x)u \]  
(3.12)

Using the analogy in the previous section, the control input \( u \) to the plant can be represented as:

\[ u = G^{-1}(x)[-F(x) + \dot{r} + v] \]  
(3.13)

Equations (3.7), (3.8) and (3.9) which define the error dynamics for linear systems also hold for the non-linear system. Care needs to be ensured that \( G(x) \) is non-singular. If it is singular, the output equation should be repeatedly differentiated as in the case of linear systems. The use of the dynamic inversion controller thus results in a linear relation from the pseudo-control input \( v \) to the plant output \( y \).

It should be noted that with the use of a dynamic inversion controller, the plant dynamics are utilized in the controller to cancel the non-linearities in the system. Thus, while using the dynamic inversion controller, the implied assumption holds that the plant dynamics are accurately known. However, errors or limitations in the plant model may manifest as undesirable characteristics while designing a controller. A typical solution employed to mitigate this limitation is the use of neural networks in the feedback linearization loop. Details of the same are
elaborated in the subsequent section.

A typical implementation of dynamic inversion in flight control application obtained from the article by Horn [42] is illustrated in figure 3.1. The dynamic inversion/feedback linearization typically forms the inner loop of the feedback control design with the outer loop forming the linear controller.

![Figure 3.1. Dynamic Inversion based Control Architecture](image)

3.2 Feedback Linearization using Neural Networks

As explained in the previous section, to completely invert the plant, the dynamics of the plant should be known accurately. Often, the dynamics of the plant can be complicated, and it may not be possible to obtain a suitable inverse. Dynamic inversion requires that the non-linearities in the plant are accurately known. In cases where the plant is not accurately known, feedback linearization can be performed using Single Hidden Layer (SHL) Neural Networks [50]. As Neural Networks (NN) are good approximators of non-linear functions [33], an SHL-NN with the correct number of neurons can approximate any non-linear function [34]. The primary use of neural networks in controls application is that of a function approximator [35]. Some of the typical advantages of using Neural Networks include their ability to approximate non-linear functions, ease of implementation due to their parallel structure, their capability to adapt and learn and the possibility to cater to multivariate systems [35]. The following analysis of feedback linearization using neural networks is based on the results presented by Johnson and Calise [34, 40]. An SHL-NN architecture obtained from the thesis by Johnson [34] is indicated in
Here, $x_{in}$ represents the $n_1$ inputs to the system and $v_{ad_n}$ represents the $n$ outputs of the system. $v_{ad}$ represents the correction to the feedback linearization that is not addressed by the nominal dynamic inversion model used. The SHL consists of $n_2$ neurons. $b_v > 0$ and $b_w > 0$ represent the bias terms and $V$ and $W$ are the weight matrices for the inputs to the SHL and the output layer. The weight matrices are represented as indicated in equations (3.14) and (3.15).

$$V = \begin{bmatrix} \theta_{v,1} & \cdots & \theta_{v,n_2} \\ v_{1,1} & \cdots & v_{1,n_2} \\ \vdots & \ddots & \vdots \\ v_{n_1,1} & \cdots & v_{n_1,n_2} \end{bmatrix} \tag{3.14}$$

$$W = \begin{bmatrix} \theta_{w,1} & \cdots & \theta_{w,n} \\ w_{1,1} & \cdots & w_{1,n} \\ \vdots & \ddots & \vdots \\ w_{n_2,1} & \cdots & w_{n_2,n} \end{bmatrix} \tag{3.15}$$

In figure 3.2, $\sigma$ represents the activation function that is used to activate or "fire" the neuron. The output of a neuron in the NN is based on a nonlinear mathematical function. This function results in a non-linear behavior of the NN that allows it to learn and adapt. Lack of an activation function will result in the neuron mapping a linear function that limits the scope of the neuron.

Neural networks can employ different types of activation functions such as the
sigmoid function, hyperbolic tangent, Rectified Linear Unit (ReLU), etc. While
the sigmoid function is limited to values between 0 and 1, the hyperbolic tangent
function allows values between -1 and 1. As the adaptation law for the neuron
weights depends on the derivative of the activation function, the activation function
should be chosen such that its first derivative exists [35]. For feedback linearization,
a sigmoid function is used which is represented as:

\[
\sigma_j(z) = \frac{1}{1 + e^{-a_jz}}
\]  

(3.16)

where \(a\) is the slope of the sigmoid function. Thus, the activation function for the
SHL can be represented in vector form as:

\[
\sigma^T(z) = \begin{bmatrix} b_w & \sigma(z_1) & \sigma(z_2) & \ldots & \sigma(z_n_2) \end{bmatrix}
\]  

(3.17)

and the vectors \(z\) and \(\bar{x}\) are defined as:

\[
z = V^T \bar{x}
\]  

(3.18)

\[
\bar{x}^T = \begin{bmatrix} b_v & \bar{x}_{in}^T \end{bmatrix}
\]  

(3.19)

Finally, the output of the NN which represents the input-output mapping is
expressed as:

\[
v_{ad,k} = b_w\theta_{w,k} + \sum_{j=1}^{n_2} w_{j,k}\sigma_j \left( b_v\theta_{v,j} + \sum_{i=1}^{n_1} v_i,j \bar{x}_{in_i} \right)
\]  

(3.20)

The output can also be expressed in matrix form as:

\[
v_{ad}(W, V, \bar{x}) = W^T \sigma \left( V^T \bar{x} \right)
\]  

(3.21)

The NN needs to learn to adapt to the requirements of the system. The output
\(v_{ad}\) of the neural network may not be equal to the modeling error in the approximate
dynamic inversion model. The usual methodology adopted to train a NN is by
using the error signal between the desired output and the actual output to update
the weights matrices. This update mechanism is known as backpropagation. The
limitation of using backpropagation for closed-loop control is the dependency of
the gradients on the Jacobian of the unknown plant that is being controlled [35].
Thus, the update laws for the weights need to be modified to suit applications
involving closed-loop feedback control. A sample update law that can be used for the weight matrices $W$ and $V$ is indicated in equations (3.22) and (3.23). This law is representative and suitable laws can be developed based on the desired application.

\[
\dot{W} = - \left[ \sigma \left( V^T \bar{x} \right) - \sigma' \left( V^T \bar{x} \right) V^T \bar{x} \right] e^T P B + \kappa \| e \| W \Gamma_w \tag{3.22}
\]

\[
\dot{V} = - \Gamma_v \left[ \bar{x} e^T P B W^T \sigma' \left( V^T \bar{x} \right) + \kappa \| e \| V \right] \tag{3.23}
\]

Here, $\Gamma_w$ and $\Gamma_v$ are positive definite matrices. $\kappa > 0$ and $P$ is a positive definite matrix which is a solution to the Lyapunov equation $A^T P + PA = -Q$ with $Q > 0$. $A$ and $B$ are specific matrices that are defined in the next chapter. $\sigma'$ is the derivative matrix of the sigmoid function which is given by:

\[
\sigma'(z) = \begin{bmatrix}
0 & \ldots & 0 \\
\frac{\partial \sigma(z_1)}{\partial z_1} & \ldots & 0 \\
0 & \ldots & \frac{\partial \sigma(z_{n_2})}{\partial z_{n_2}}
\end{bmatrix} \tag{3.24}
\]

As stated earlier, the purpose of the NN is to approximate any nonlinear function. Consider a function $f(x) = [f_1(x), f_2(x), ... f_n(x)]$ which is a function from $\mathbb{R}^{n_1}$ to $\mathbb{R}^n$. For $x$ belonging to a compact set in $\mathbb{R}^{n_1}$, if there are a sufficient number of neurons in the hidden layer, it can be shown that [35]:

\[
\left| f_k(x) - \left[ b_u \theta_{w,k} + \sum_{j=1}^{n_2} w_{j,k} \sigma_j \left( b_v \theta_{v,j} + \sum_{i=1}^{n_1} v_{i,j} x_{i} \right) \right] \right| \leq \epsilon_k(x) \tag{3.25}
\]

Equation (3.25) shows that a NN with a sufficient number of neurons in the hidden layer is capable of approximating any continuous function in a compact set [35]. The neural network function approximation error $\epsilon_k(x)$ can be made as small as desired using the correct number of neurons in the hidden layer.

### 3.3 Model Reference Adaptive Control (MRAC)

The following section provides a brief overview of Model Reference Adaptive Control (MRAC) based on the literature presented by Nguyen [51]. MRAC, as the name
suggests is a direct adaptive control technique wherein the controller is adapted in a manner so that the plant follows a reference model. The adaption law for the controller is based on the error between the actual plant output and the reference model output. The basic architecture of an MRAC obtained from the textbook by Nguyen [51] is illustrated in figure 3.3.

Figure 3.3. Model Reference Adaptive Control Architecture

As can be seen from the figure, the update for the adaptive law is based on the error i.e. the difference between the reference model states and the actual plant states. The reference model is chosen in a manner such that the reference model state follows the desired trajectory. The error signal $e$ is known as the tracking error which represents the deviation of the plant output from the reference model output [52]. In case the plant output matches the reference model output, the error signal is zero and the gains of the controller are no longer updated. Most of the adaptive control architectures are either based on direct or indirect adaptive implementations. MRAC is a form of direct adaptive control wherein the controller parameters are directly updated instead of updating the plant parameters which is typically performed in indirect adaptive control.

The following subsection presents a brief overview of MRAC for a scalar system based on the theory presented by Nguyen [51]. Further a detailed study on MRAC, the reader may refer to various adaptive control textbooks currently available.
3.3.1 MRAC for scalar systems

In this section, MRAC for a scalar system is developed. The scalar system is represented in the form:

\[
\dot{x} = ax + b[u + f(x)]
\]  
(3.26)

where \( f(x) \) is a nonlinear structured matched uncertainty that is parametrized as:

\[
f(x) = \sum_{i=1}^{p} \theta_i^* \phi_i(x) = \Theta^* \Phi(x)
\]  
(3.27)

Here, \( \Theta^* = [\theta_1 \theta_2 \ldots \theta_p]^T \) is an unknown constant vector and, \( \Phi(x) = [\phi_1(x) \phi_2(x) \ldots \phi_p(x)]^T \) is a vector of known bound basis functions. For the following analysis, it is assumed that the parameters \( a \) and \( b \) of the plant are unknown, but the sign of \( b \) is known.

Consider a stable reference model \( (a_m < 0) \) of the form:

\[
\dot{x}_m = a_m x_m + b_m r
\]  
(3.28)

Assuming that an ideal controller can be defined which cancels out the matched uncertainty and ensures that the plant follows the reference model, the controller can be defined as:

\[
u^* = k_x^* x + k_r^* r(t) - \Theta^* \Phi(x)
\]  
(3.29)

Thus, the closed-loop plant model becomes:

\[
\dot{x} = (a + bk_x^*) x + bk_r^* r
\]  
(3.30)

Comparing equation (3.30) to equation (3.28), the ideal controller gains can be determined using the matching conditions as:

\[
k_x^* = \frac{a_m - a}{b}
\]

\[
k_r^* = \frac{b_m}{b}
\]  
(3.31)

Thus, with the model matching conditions providing two equations, a solution to the ideal controller gains can always be found. The above approach to determining the controller gains works if the plant parameters \( a \) and \( b \) are accurately known.
However, as the parameters may not be known accurately, an adaptive controller is required to ensure that the tracking error approaches 0. Let the actual controller be represented as:

$$u = k_x(t)x + k_r(t)r - \Theta^T(t)\Phi(x)$$ \hspace{1cm} (3.32)

where \(k_x(t), k_r(t),\) and \(\Theta^T\) are the estimates of \(k_x^*, k_r^*,\) and \(\Theta^*_x\) respectively. The estimation errors can then be expressed as:

$$\tilde{k}_x(t) = k_x(t) - k_x^*$$ \hspace{1cm} (3.33)

$$\tilde{k}_r(t) = k_r(t) - k_r^*$$ \hspace{1cm} (3.34)

$$\tilde{\Theta}(t) = \Theta(t) - \Theta^*$$ \hspace{1cm} (3.35)

Using the above equations, the closed-loop plant model becomes:

$$\dot{x} = \left( a + b\tilde{k}_x^* + b\tilde{k}_x \right)x + \left( b\tilde{k}_r^* + b\tilde{k}_r \right)r - b\tilde{\Theta}^T\Phi(x)$$ \hspace{1cm} (3.36)

With the tracking error defined as \(e(t) = x_m - x\), the closed loop tracking error of the system becomes:

$$\dot{e} = \dot{x}_m - \dot{x}$$

$$= a_m e - b\tilde{k}_x x - b\tilde{k}_r r + b\tilde{\Theta}^T \Phi(x)$$ \hspace{1cm} (3.37)

The purpose of the adaptive controller is to ensure that the tracking error \(e\) becomes zero. Hence, it is necessary to ensure that the tracking error dynamics defined in equation (3.37) is stable. Thus, the controller parameters \(k_x(t), k_r(t)\) and \(\Theta^T\) need to be adjusted such that the tracking error goes to 0. The adaptive laws for the controller parameters can be determined by using Lyapunov theory with a suitable Lyapunov function. Consider the positive scalar Lyapunov function \(V\) given by:

$$V = e^2 + |b| \left( \frac{\tilde{k}_x^2}{\gamma_x} + \frac{\tilde{k}_r^2}{\gamma_r} + \tilde{\Theta}^T \Gamma^{-1} \tilde{\Theta} \right)$$ \hspace{1cm} (3.38)

where \(\gamma_x > 0\) and \(\gamma_r > 0\) are the adaptation or learning rates and \(\Gamma\) is a positive definite symmetric adaptation matrix for \(\Theta\). For the system to be stable, as per Lyapunov’s theory, the derivative of the Lyapunov function \(V\) should be negative.
semidefinite. The derivative of the Lyapunov function is:

\[
\dot{V} = 2\epsilon \dot{\epsilon} + |b| \left( \frac{2\dot{k}_x \dot{k}_x}{\gamma_x} + \frac{2\dot{k}_r \dot{k}_r}{\gamma_r} + 2\dot{\Theta}^\top \Gamma^{-1} \dot{\Theta} \right)
\]

\[
= 2a_m e^2 + 2\ddot{k}_x \left( -ebx + |b| \frac{\dot{k}_x}{\gamma_x} \right) + 2\ddot{k}_r \left( -ebr + |b| \frac{\dot{k}_r}{\gamma_r} \right)
\]

\[
+ 2\dot{\Theta}^\top \left[ eb\Phi(x) + |b|\Gamma^{-1} \dot{\Theta} \right]
\]

Now, \(a_m < 0\) and the sign of \(b\) is known, thus, \(b = |b|sgn(b)\). For equation (3.39) to be negative semi-definite, the following conditions must hold:

\[
-ex \text{sgn}(b) + \frac{\dot{k}_x}{\gamma_x} = 0 \tag{3.40}
\]

\[
-e \text{r} \text{sgn}(b) + \frac{\dot{k}_r}{\gamma_r} = 0 \tag{3.41}
\]

\[
e\Phi(x) \text{sgn}(b) + \Gamma^{-1} \dot{\Theta} = 0 \tag{3.42}
\]

As \(k_x^*, k_r^*\) and \(\Theta_x^*\) are constant, \(\dot{k}_x(t) = \dddot{k}_x(t), \dot{k}_r(t) = \dddot{k}_r(t)\) and \(\dot{\Theta} = \ddot{\Theta}\). Thus, equations (3.40), (3.41) and (3.42) result in the following adaptive laws for the controller parameters:

\[
\dot{k}_x = \gamma_x xe \text{sgn}(b) \tag{3.43}
\]

\[
\dot{k}_r = \gamma_r xe \text{sgn}(b) \tag{3.44}
\]

\[
\dot{\Theta}_x = \Gamma \Phi(x) e \text{sgn}(b) \tag{3.45}
\]

The adaptive laws in equations (3.43) to (3.45) ensure that \(\dot{V} = 2a_m e^2 \leq 0\). It can be further proved that \(\dot{V} \to 0\) and that the tracking error is asymptotically stable. However, it can only be shown that the controller parameters are bounded \([51]\). Thus, the above analysis demonstrates that the adaptive controller can ensure that the plant follows the reference model and at the same time also ensuring that the controller parameters are bounded. The convergence rate of the controller can be varied by varying the adaptation or learning rate. A trade-off should be made regarding the convergence rate because a very high convergence rate would result in faster convergence but, in turn, degrades the robustness of the system \([51]\). The
adaptive laws presented in the previous section are based on a similar approach using a suitable Lyapunov function. For details of the proof, the reader is referred to the work by Johnson and Calise [40]. MRAC can also be applied to Multi-Input Multi-Output (MIMO) systems and further details of the same can be found in any standard adaptive control textbook.

### 3.4 Pseudo Control Hedging (PCH)

Pseudo Control Hedging (PCH) is a technique used to prevent the adaptive element from adapting to specific plant characteristics such as saturation, rate limits, etc. [34]. Using PCH, the adaptive law is made oblivious to reference model tracking errors due to certain plant characteristics [40]. The effect of the plant characteristics such as saturation is addressed in the reference model tracking error using PCH so that the limitations of the system do not manifest as a reference model error thus causing the adaptive law to learn incorrectly. As all physical systems exhibit such characteristics, PCH helps in addressing these aspects.

In PCH, the reference model is corrected in a manner such that the adaptive law does not learn any particular characteristic that is not desired. The correction is performed so that the undesirable characteristics are subtracted from the reference model in the tracking error loop and not in the feed-forward path [34]. An implementation of PCH with MRAC obtained from the thesis by Johnson [34] is illustrated in figure 3.4.

As can be seen from the figure, the inclusion of PCH results in an additional signal to the reference model. Thus, the reference model output with PCH is given by:

$$\ddot{x}_{rm} = v_{crm}(x_{rm}, \dot{x}_{rm}, x_c) - v_h$$  \hspace{1cm} (3.46)

where $v_h$ is the PCH signal which is determined as:

$$v_h = \hat{f}(x, \dot{x}, \delta_{cmd}) - \hat{f}(x, \dot{x}, \hat{\delta})$$ \hspace{1cm} (3.47)

Here, $\hat{f}$ is the approximate plant model used in the dynamic inversion. From equation (3.47), the PCH signal consists of the difference between the expected plant output and the actual plant output. If the expected and actual plant output is the same, then the PCH signal is zero and there is no correction to the reference
Figure 3.4. Model Reference Adaptive Control with PCH

model dynamics. In case of any difference between the expected plant output and the actual plant output, the PCH signal is non-zero and the signal is subtracted from the reference model so that the same is not reflected in the tracking error dynamics.
Chapter 4  |  Yaw Rate Controller

This chapter provides an overview of the flight control architecture used on the aircraft along with an analysis of the inner loop yaw rate controller development. Different scenarios that can result from the pseudo-control requirements are also discussed along with the implementation of axis-prioritization in case the control authority is insufficient to meet the yaw and beep requirements of the helicopter.

4.1 Helicopter Flight Control Architecture

A Model Reference Adaptive Control (MRAC) architecture with Nonlinear Dynamic Inversion (NDI) and Pseudo Control Hedging (PCH) is used for the flight control system of the helicopter. The control architecture used in the thesis is based on the implementations by Johnson and Kannan [34, 53]. The flight control problem is managed in terms of inner loop control and outer loop control. The outer loop is used to control the forces on the helicopter to achieve the desired position and velocities. This is achieved by varying the magnitude of the rotor thrust by the use of the collective. The inner loop controls the control surfaces responsible to achieve the desired moments and angular rates of the helicopter [34]. A separated Navigation and Guidance loop determines the desired/commanded inputs to the outer and inner loop.

Approximate dynamic inversion based control along with PCH is utilized separately for the inner and outer loops. A NN based feedback linearization mechanism is used to cater to the uncertainties in the approximate dynamic inversion model. The PCH signal ensures that the adaptive element does not adapt to unwanted system characteristics. The inner loop PCH is used to prevent adaptation to
characteristics such as actuator position and rate limits while the outer loop PCH signal is used to prevent the outer loop adaptation to inner loop dynamics [53]. A macro-level architecture of the control scheme for the helicopter obtained from the paper by Johnson and Kannan [53] is illustrated in figure 4.1.

\[ \delta_f \text{ and } \delta_m \text{ represent the force and moment controls of the helicopter.} \]

The controls are classified based on the primary action that results due to the action of these controls. For example, the moment controls are primarily responsible for varying the moments of the helicopter although changes in the moments also result in changes in the forces. However, they are classified as moment controls because of the primary action that they intend to cause. A similar analogy applies to the force controls. \( p_c, v_c, q_c, w_c \) are the commanded position, velocity, attitudes and attitude rates that are determined by the navigation and guidance loop. \( \alpha_d \) and \( \alpha_d \) represent the pseudo-control adaptive elements corresponding to the linear and angular accelerations.

A detailed view of the control architecture obtained from the thesis by Johnson [34] is illustrated in figure 4.2. As stated earlier, the outer loop controls the forces by varying the thrust of the main rotor through the helicopter collective. The inner loop controls the helicopter moments through the lateral and longitudinal sticks and the helicopter rudder pedal. As the tail rotor can provide forward thrust, the inner loop is also responsible for meeting the "beep" requirements as may be

\[ \text{Figure 4.1. Macro Control System Architecture of the Helicopter} \]
required by the outer loop.

The objective of this thesis is the design of the yaw rate control of the helicopter. As the yaw rate control is addressed by the inner loop, the control architecture of the inner loop specific to yaw rate control is addressed in this work. The following section explains the details of the inner loop followed by the design of the yaw rate controller.

### 4.2 Inner Loop Control

The existing simulation model of the helicopter addresses the outer loop control and inner loop control. This existing design is based on the works of Johnson and Kannan [34, 53, 54]. It is assumed that the outer loop caters to the position and velocity requirements and the inner loop control of the pitch and roll rates are addressed by the existing controller. This thesis primarily deals with the design of the yaw rate controller for the VTDP tail rotor configuration.

The yaw rate controller is responsible for achieving and maintaining the desired yaw rate and the forward velocity as needed based on the beep command. The
desired yaw rate and forward velocity is determined by the outer loop and provided to the inner loop. The inner loop is responsible to achieve the desired input. The detailed architecture of the inner loop obtained from the thesis by Johnson [34] is illustrated in figure 4.3. The following subsections provide an overview of each component of the inner loop.

![Figure 4.3. Control Architecture of Inner Loop](image)

4.2.1 Nominal Plant Model

For the inner loop control, the plant model can be considered of the form [34]:

\[
\dot{q} = Q(q, \omega) \tag{4.1}
\]

\[
\dot{\omega} = f(q, \omega, \delta) \tag{4.2}
\]

\[
\dot{\delta} = g(q, \omega, \delta, \delta_{cmd}) \tag{4.3}
\]

where \( q \in \mathbb{R}^4 \) is the quaternion of the helicopter attitudes with respect to the inertial frame, \( \omega \in \mathbb{R}^3 \) is the vector of angular rates of the helicopter and \( \delta \) and \( \delta_{cmd} \) are the actuator positions and commands respectively.
The plant model expressed in equations (4.1) to (4.3) represents the helicopter model derived in chapter 2. Equation (4.1) represents the kinematic equations relating the helicopter attitudes to the angular rates, equation (4.2) is the moment equation of the helicopter and equation (4.3) is the actuator model which includes actuator dynamics such as position and rate limits. It is assumed that the actuator dynamics are unknown.

From the helicopter model derived in chapter 2, it is evident that the relation between the angular rates and the helicopter controls is not affine. Thus, it is difficult to arrive at the dynamic inversion of the plant model. An approximate simplified dynamic inversion model is thus assumed. The approximation error between the actual inversion and the approximate inversion is addressed by the adaptive element.

### 4.2.2 Linear Compensator

A Proportional + Derivative (PD) control is used that acts on the reference model tracking error. The gains of the PD compensator are suitably chosen to meet the required reference model performance and to ensure that the reference model tracking error is zero. The pseudo-control signal of the PD compensator is given by:

\[
v_{pd} = K_p(x_{rm} - x) + K_d(\dot{x}_{rm} - \dot{x})
\]

(4.4)

Combining the tracking error and it’s derivative in a single vector, the compensator signal \( V_{pd} \) can be expressed as:

\[
v_{pd} = \begin{bmatrix} K_p & K_d \end{bmatrix} e_{rm}
\]

(4.5)

where the error signal \( e_{rm} \) is given by:

\[
e_{rm}^T = \begin{bmatrix} (x_{rm} - x)^T & (\dot{x}_{rm} - \dot{x})^T \end{bmatrix}^T
\]

(4.6)

As the inner loop represents the control of the pitch, roll and yaw axis, \( K_p \) and \( K_d \) are positive definite 3 × 3 diagonal matrices.
4.2.3 Approximate Dynamic Inversion

Section 4.2.1 described the nominal plant model of the helicopter for the inner loop control. The approximate plant model considering unknown plant elements can be expressed as:

\[ \dot{v} = \hat{f}(q, \omega, \delta) \]  \hspace{1cm} (4.7)

\[ \hat{\delta} = \hat{g}(q, \omega, \delta_{cmd}) \]  \hspace{1cm} (4.8)

If the actuator positions are accurately known, equation (4.8) would be the same as equation (4.3). Thus, the approximate dynamic inversion is given by:

\[ \delta_{cmd} = \hat{f}^{-1}(q, \omega, v) \]  \hspace{1cm} (4.9)

where \( v \) is the pseudo-control signal given by equation (4.11). It represents the desired angular acceleration which the commanded input \( \delta_{cmd} \) is expected to achieve.

4.2.4 PCH Signal

The PCH signal is computed based on the estimate of the actuator position. The estimate of the actuator position is used to compute the difference between the desired pseudo-control and the actual pseudo-control. Thus, the PCH signal is given by:

\[ v_h = \hat{f}(q, \omega, \delta_{cmd}) - \hat{f}(q, \omega, \hat{\delta}) = v - \dot{v} \]  \hspace{1cm} (4.10)

Note that the PCH signal is only used in the reference model for tracking error dynamics and does not feature in the pseudo-control signal input to the actual plant.

4.2.5 Reference Model Tracking Error Dynamics

The pseudo-control signal to the approximate dynamic inversion model is given by:

\[ v = v_{crm} + v_{pd} - v_{ad} \]  \hspace{1cm} (4.11)

where, \( v_{pd} \) is the output of the linear compensator described in section 4.2.2 and \( v_{ad} \) is the output of the neural network-based adaptive element.
The tracking error of the inner loop was defined in equation (4.6). As the attitude of the helicopter is expressed in terms of quaternions, the tracking error can thus be expressed as:

\[ e_{rm} = \begin{bmatrix} \tilde{Q}(q_{rm}, q) \\ \omega_{rm} - \omega \end{bmatrix} \] (4.12)

where the matrix \( \tilde{Q} \) is the attitude error given by [34]:

\[
\tilde{Q}(q, p) = 2 \text{sign} \left( q^T p \right) \begin{bmatrix} -q_2 + q_4 - q_3 \\ -q_3 + q_4 + q_1 + q_2 \\ -q_4 + q_3 - q_2 + q_1 \\ +p_2 - p_1 - p_4 + p_3 \\ +p_3 + p_4 - p_1 - p_2 \\ +p_4 - p_3 + p_2 - p_1 \end{bmatrix}
\] (4.13)

\[ p = 2 \text{sign} \left( q^T p \right) \Omega_q^{(-1)} p \]

where \( \Omega \) is the matrix defined in equation (4.14) below. Equation (2.5) in chapter 2 expressed the kinematic relationship between the helicopter angular rates and attitudes expressed in quaternions. The equation can be modified to the form expressed in equation (4.14) [34].

\[
\dot{q} = Q(q, \omega) = \frac{1}{2} \begin{bmatrix} 0 & -\omega_1 & -\omega_2 & -\omega_3 \\ \omega_1 & 0 & \omega_3 & -\omega_2 \\ \omega_2 & -\omega_3 & 0 & \omega_1 \\ \omega_3 & \omega_2 & -\omega_1 & 0 \end{bmatrix} q = \frac{1}{2} \begin{bmatrix} -q_2 & -q_3 & -q_4 \\ q_1 & -q_4 & q_3 \\ q_4 & q_1 & -q_2 \\ -q_3 & q_2 & q_1 \end{bmatrix} \omega = \frac{1}{2} \Omega_q \omega
\] (4.14)

The reference model tracking error is obtained by differentiating equation (4.12) and is given by:

\[ \dot{e} = Ae + B \left[ v_{ad}(x, \dot{x}, \hat{\delta}) - f(q, \omega, g(q, \omega, \delta_{cmd})) + \dot{f}(q, \omega, \hat{g}(q, \omega, \delta_{cmd})) \right] \] (4.15)

where, \( A \) and \( B \) are matrices given by:

\[
A = \begin{bmatrix} 0 & I \\ -K_p & -K_d \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I \end{bmatrix}
\] (4.16)

Here, \( K_p \) and \( K_d \) are the same gains used in the linear compensator. Thus, the
tracking error dynamics can be expressed as:

\[
\dot{e} = Ae + B \left[ v_{ad}(x, \dot{x}, \delta) - \Delta(x, \dot{x}, \delta) \right]
\]  

(4.17)

where, \( \Delta(x, \dot{x}, \delta) \) is the model error given by:

\[
\Delta(q, \omega, \delta_{cmd}) = f(q, \omega, g(q, \omega, \delta_{cmd})) - \hat{f}(q, \omega, \hat{g}(q, \omega, \delta_{cmd}))
\]  

(4.18)

The model error expressed in equation (4.18) is used to train the NN adaptive element.

### 4.2.6 Reference Model

The reference model governs the type of response that is desired from the system. The parameters for the reference model are selected based on desired handling qualities. The reference model dynamics \( v_{crm} \) in equation (4.11) represents the desired response to command tracking errors. The criteria for the selection of the reference model are detailed in the works of Johnson [34]. The method used considers an integrator backstepping approach to determine the appropriate Lyapunov function. The method aims to allow the analysis of the entire system with some assumptions removed.

The system is analyzed as a non-adaptive system wherein the reference model tracking error is assumed to be zero so that the NN weights correspond to their ideal weights. This ensures that the states of the plant are the same as the reference model i.e. \( q = q_{rm} \) and \( \omega = \omega_{rm} \). Thus, the non-adaptive system is represented as:

\[
\dot{q} = Q(q, \omega)
\]  

(4.19)

\[
\dot{\omega} = f \left( q, \omega, g(q, \omega, \hat{f}^{-1}(q, \omega, v_{crm} - v_{ad}^*)) \right)
\]  

(4.20)

where \( v_{ad}^* \) is the adaptive controller output for the ideal weights. Thus, the reference model can be represented as:

\[
v_{cm} = [K_{pc} \ K_{dc}] e_{cm}
\]  

(4.21)

where \( K_{pc} \) and \( K_{dc} \) are diagonal matrices with gains similar to that of the linear

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compensator which is chosen such that the matrix $A_c$ corresponding to the Lyapunov function $A_c^TP_c + P_cA_c = -Q_c$ with $Q > 0$ is Hurwitz and is given by:

$$A_c = \begin{bmatrix} 0 & I \\ -K_{pc} & -K_{dc} \end{bmatrix}$$

(4.22)

Thus, the reference model command tracking error is given by:

$$e_{crm} = \begin{bmatrix} \bar{Q}(q_c, q_{rm}) \\ \omega_c - \omega_{rm} \end{bmatrix}$$

(4.23)

The above analysis is based on the assumption that the system is non-adaptive. For the non-adaptive system, the reference model tracking error is zero so that $e_{crm} = e_c$. Thus, the command tracking error can be expressed as:

$$e_c = \begin{bmatrix} \bar{Q}(q_c, q) \\ \omega_c - \omega \end{bmatrix}$$

(4.24)

### 4.2.7 Neural Network Adaptive Element

The adaptive laws used for updating the neural network weights were detailed in section 3.2. The section explained the inputs and outputs of the neural network along with the weights of the individual layers. While using the neural network-based feedback linearization, there exist certain assumptions regarding the neural networks. The following assumptions hold in ensuring that the neural network output and the plant states remain bounded.

1. A static mapping $\delta = \delta(x, \dot{x}, \hat{\delta})$ exists between the actual actuator position $\delta$ and the estimated actuator position $\hat{\delta}$ [40].

2. It is assumed that the norm of the ideal weights of the neural network are bounded. Here, ideal weights $(V^*, W^*)$ refer to those values that ensure that the output of the neural network is within bounds of the function approximation error [53]. Thus, the assumption can be stated as:

$$0 < \|Z^*\|_F \leq \bar{Z}$$

(4.25)

where, $\|\cdot\|_F$ is the Frobenius norm.
3. The external commanded signal \( (x_c) \) to the system is bounded.

\[ \|x_c\| \leq \bar{x}_c \] (4.26)

4. The states and signals of the reference model are bounded for permissible plant and actuator dynamics [40].

Based on these assumptions, the adaptive law for the inner loop control as expressed in section 3.2 is restated below:

\[ \dot{W} = - \left[ (\sigma \left( V^T \ddot{x} \right) - \sigma' \left( V^T \ddot{x} \right) V^T \ddot{x}) e_{rm}^T PB + \kappa \|e_{rm}\| W \right] \Gamma_w \] (4.27)

\[ \dot{V} = - \Gamma_v \left[ \ddot{x} e_{rm}^T P B \sigma' \left( V^T \ddot{x} \right) + \kappa \|e_{rm}\| V \right] \] (4.28)

where, the matrix \( P \) is the positive definite solution to the Lyapunov equation \( A^T P + PA = -Q \) with \( Q > 0 \). The matrix \( Q \) is chosen as [40]:

\[ Q = \begin{bmatrix} K_d K_p & 0 \\ 0 & K_d K_p^2 \end{bmatrix} \frac{1}{\frac{1}{4} n_2 + b_w^2} \] (4.29)

For details of the derivation of the adaptive laws and stability and boundedness proofs of the same, the reader is referred to the work by Johnson and Calise [40]. Thus, the output of the adaptive element is given as:

\[ v_{ad} = W^T \sigma \left( V^T \ddot{x} \right) \] (4.30)

Equation (4.18) listed the tracking error dynamics that is used to train the neural network. As the error signal depends on parameters such as \( q_c, \omega, e_c, e_{rm}, v_{ad} \) and the neural network weights [34], the input vector to the neural network is given by [34]:

\[ \ddot{\tilde{x}} = \begin{bmatrix} b_v & q_c^T & \omega_c^T & e_c^T & e_{rm}^T & v_{ad}^T & \|Z\| \end{bmatrix} \] (4.31)

### 4.3 Yaw Rate Controller

The above sections provided an overview of the helicopter control system architecture including a detailed analysis of the inner loop control. The inner loop control
previously discussed consists of the control design for all three moments of the helicopter which is a generalized philosophy that can be adopted for a conventional helicopter. In the current VTDP configuration, owing to the change in the tail rotor controls, the architecture was suitably modified to include the tail rotor speed control as well as the rudder and sectors deflection. The forces and moments generated by the tail rotor are governed by the yaw rate requirement as well as the "beep" requirement. The beep requirement governs the desired forward acceleration needed from the tail rotor. This input is determined by the outer loop based on a suitable control allocation scheme. Thus, the dynamic inversion based controller was modified to cater to this requirement. This section details the updated yaw rate controller for the helicopter.

The approximate plant model for the yaw rate and beep command in the inner loop is given as:

\[
\hat{f}_r = \dot{v}_r = \dot{r}_{\text{des}} = a\ddot{r} + b_r \Omega_{\text{cmd}}^2 \sin \delta_{\text{cmd}} \\
\hat{f}_u = \dot{v}_u = \dot{u}_{\text{des}} = b_u \Omega_{\text{cmd}}^2 \cos \delta_{\text{cmd}}
\] (4.32)

Here, \(v_r\) and \(v_u\) are the pseudo-control signals corresponding to the yaw angular acceleration and the forward acceleration and \(\Omega_{\text{cmd}}\) and \(\delta_{\text{cmd}}\) are the commanded control inputs to achieve the required pseudo-controls. \(b_r\) and \(b_u\) are the gains that determine the total forward acceleration and yaw acceleration that can be satisfied by the tail rotor. \(\dot{r}_{\text{des}}\) and \(\dot{u}_{\text{des}}\) are the desired yaw angular acceleration and forward linear accelerations which are determined by the outer loop and navigation and guidance loop. It is assumed that the forward acceleration requirement from the inner loop is determined by the outer loop control which has a suitable control allocation scheme to distribute the control between the helicopter longitudinal cyclic and the tail rotor beep control.

In section 2.9.2, it was stated that the torque exerted by the engine is determined based on the maximum engine torque and the beep command. The beep command is mapped to the RPM of the tail rotor. In typical aircraft applications, the general practice involves mapping the maximum aircraft surface deflection to a range of -1 to 1 or 0 to 1. Thus, the commanded inputs computed by both the inner and outer loop is a value that varies between these limits. The selection i.e -1 to 1 or 0 to 1 may depend on the control surface. In the case of the tail rotor, the RPM command and the rudder/sector deflection are mapped from 0 to 1. Thus, the
beep command used in equation (2.65) in section 2.9.2 can be mapped to the RPM command \( \Omega_{cmd} \) by the following relation:

\[
\delta_{\text{beep}} \propto \Omega_{cmd}^2 \quad (4.34)
\]

The solution to equations (4.32) and (4.33) will result in the desired commanded inputs to the plant to achieve the corresponding yaw rate and beep requirements. This is the approximate dynamic inversion for the yaw and beep control. Equations (4.32) and (4.33) represent the approximate plant model used for the dynamic inversion and may be re-written in the form:

\[
R = \Omega_{cmd}^2 \sin \delta_{r_cmd} \quad (4.35)
\]

\[
U = \Omega_{cmd}^2 \cos \delta_{r_cmd} \quad (4.36)
\]

where,

\[
R = \frac{v_r - ar}{b_r} \quad (4.37)
\]

and,

\[
U = \frac{v_u}{b_u} \quad (4.38)
\]

The solution to equations (4.32) and (4.33) results in the commanded inputs to the plant. However, the existence of a solution depends on certain conditions. A quick analysis of equation (4.33) will reveal that the beep command \( v_u \) (and hence \( U \)) should always be greater than or equal to zero. No solution exists if this criterion is not met. With this underlying restriction, the existence of a solution to (4.32) and (4.33) is governed by five possible scenarios:

1. The desired beep \( v_u \) (or \( U \)) is greater than or equal to the minimum beep command \( v_{u_{\text{min}}} \) (or \( U_{\text{min}} \)) and the value of \( R \) is greater than or equal to 0.

2. The desired beep \( v_u \) (or \( U \)) is lesser than the minimum beep command \( v_{u_{\text{min}}} \) (or \( U_{\text{min}} \)) and the value of \( R \) is greater than or equal to 0.

3. The combined requirement of \( R \) and \( U \) is greater than the total control authority available and \( R \) is lesser than or equal to \( R_{\text{max}} \).

4. The desired value of \( R \) is greater than the maximum control authority available.
5. The value of $R$ is less than 0.

Each of the above five cases can be represented graphically as indicated in figure 4.4. The figure illustrates the relation of $U$ versus $R$ in the first two quadrants. Due to the restriction on $U$, the third and fourth quadrants are not applicable. Each of the five cases is indicated as different zones in the figure.

![Figure 4.4. Inner Loop forward pseudo-control versus yaw pseudo-control](image)

To understand the figure, the relation between $U$ and $R$ needs to be analyzed. This can be achieved using equations (4.35) and (4.36). The relation between $R$ and $U$ can be expressed as:

$$U = \frac{R}{\tan \delta_{cmd}}$$

(4.39)

Further, from equations (4.35) and (4.36), it can be seen that:

$$U^2 + R^2 = \Omega^4$$

(4.40)

The minimum value of $U$ will correspond to a maximum value of $\delta_{cmd}$. Thus, assuming that $\delta_{r_{max}} < 90^\circ$, when $\delta_{cmd} = \delta_{r_{max}}$, a linear relation exists between $U$ and $R$. Now, as the requirement for $R$ increases, the same is met with an increase in the tail rotor RPM. This results in a corresponding increase in $U$. Thus, the
minimum value of $U$ increases with an increase in $R$. The line separating the boundaries of case 1 and case 2 in figure 4.4 represents the minimum value of $U$ as $R$ increases. In case the rudder/sector deflection attains the maximum value of 90°, the minimum possible $U$ will be 0 and the line will coincide with the $R$ axis.

From equation (4.40), it can be seen that the maximum value of $R$ and $U$ is governed by $\Omega_{cmd}$. Thus, $R$ and $U$ are bounded by an upper value of 1. The achievable values of $U$ and $R$ will depend on the available control authority. As the priority of control is assigned to $R$, the maximum value of $U$ that can be achieved depends on the corresponding value of $R$. For a particular value of $R$, the maximum value $U$ that can be achieved lies on a circle of radius 1. This is the boundary separating cases 1 and 3.

The maximum value of $R$ is governed by the maximum rudder deflection and maximum tail rotor RPM. In case of full rudder deflection, $R_{\text{max}} = 1$, else $R_{\text{max}} < 1$. At maximum RPM, the value of $R$ is limited by the rudder deflection. This value is indicated by the line separating the zones corresponding to case 3 and case 4. However, the maximum value of $U$ is not affected by rudder deflection.

Each of the five cases is indicated by shaded regions in the figure. The shaded region between the $U_{\text{min}}$ line and $U_{\text{max}}$ curve is the region of 'Achievable Control' and corresponds to case 1. In this region, both the desired $U$ and $R$ can be achieved by the tail rotor. The region below the $U_{\text{min}}$ line is case 2 where the desired $U$ is lower than the minimum possible $U$. In this region, the desired value of $R$ can be achieved but the value of $U$ achieved would be greater than the desired value. Here, the outer loop will need to compensate for the additional forward acceleration generated by the tail rotor.

The region marked case 3 corresponds to a region where the combined requirement of $U$ and $R$ is greater than what can be achieved by the tail rotor. However, it may be possible to satisfy the requirement of $R$. There are two possible approaches to obtain the solution in this case - direction preservation or axis prioritization [27]. The concept is illustrated in figure 4.5.

Here, point A represents the total pseudo-control requirement which is outside the zone marked case 1. Thus, it is not possible to achieve both $R$ and $U$. In such a case, it may be possible to achieve maximum control in the direction of OA i.e. OB or prioritize one axis over the other i.e. OC. It can be seen that selecting OC results in the yaw requirement being satisfied while compromising on the beep
requirement. OB would have resulted in an optimal solution that tries to maximize both the yaw and beep requirements. However, as the primary purpose of the tail rotor is to meet the yaw requirement, axis prioritization is adopted over direction preservation. Thus, the control algorithm will determine the control along OC instead of OB.

The shaded region marked case 4 represents the region where control authority is insufficient to meet the requirements of both $R$ and $U$. In this region, the maximum possible yaw command will be generated while compromising the beep requirements. As the total yaw acceleration generated will be insufficient to meet the requirement, the response of the helicopter will be slower than desired. This might typically be one of the inner loop dynamics that need to be addressed by the outer loop PCH signal.

Case 5 corresponds to the case of $R < 1$ and hence encompasses the entire second quadrant. Here, as $R$ is negative, only the desired beep requirement can be met. The line corresponding to $U = 1$ represents the maximum $U$ that can be achieved. If the desired $U$ is lower than the maximum value, the same can be achieved by varying the RPM of the tail rotor with zero rudder deflection. The maximum possible negative yaw acceleration that can be achieved in this
case is governed by the torque generated by the main rotor. As the tail rotor is not generating any anti-torque due to zero rudder deflection, a minimum possible negative yaw acceleration can be achieved by the main rotor engine torque.

The inner loop control is so designed to address all five cases based on their occurrence. For cases 1 to 3, it is assumed that the desired value of $R$ is lesser than the maximum control authority of the helicopter. The solutions to equations (4.32) and (4.33) along with the appropriate control strategy for each of the five cases is explained below.

4.3.1 Case 1: $R$ is greater than or equal to 0 and $U$ is greater than or equal to minimum beep

From equation (4.32), it can be seen that a solution exists if the value of $R$ is greater than or equal to 0. For the case when $R$ is 0, as the tail rotor RPM cannot be 0, the only possible solution is to set the rudder deflection to 0. The desired tail rotor RPM is then purely governed by the beep required. In case $R$ is not 0, the tail rotor RPM and rudder/sectors deflection are so adjusted to meet both requirements. Thus, for case 1, the solution to equations (4.32) and (4.33) is given by:

$$\Omega_{cmd}^2 = \sqrt{R^2 + U^2} \quad (4.41)$$

$$\delta_{r_{cmd}} = \tan^{-1} \frac{R}{U} \quad (4.42)$$

Note that if $U = 0$, $\delta_{r_{cmd}} = 90^\circ$. Thus, as the beep requirement is 0, the rudder will be at full deflection so that the tail rotor only generates the desired yaw moment without any additional forward thrust. The tail rotor RPM, in this case, is governed by the yaw requirement. Thus, as long as the desired yaw rate is lower than the maximum possible yaw rate, the tail rotor will be able to satisfy the requirement through variation of the RPM.

4.3.2 Case 2: $R$ is greater than or equal to 0 and $U$ is lesser than minimum beep

Restrictions may exist in the helicopter design that limits the maximum motion of the control surfaces. In such cases, the control authority is governed by the
maximum surface deflection possible. A case may exist wherein the rudder/sector deflection may be limited to some value lower than 90° i.e. $\delta_{r\text{max}} < 90^\circ$. This may be owing to a limitation in the maximum surface deflection or due to effects such as fluttering of the rudder/sector due to high velocities, etc. Under such scenarios, there might be a minimum forward thrust that is generated by the tail rotor. The solution detailed in case 1 would no longer be applicable in case the desired beep is lower than the minimum beep that is generated by the tail rotor. In such a case, the most optimum solution is to ensure that the rudder and sectors deflection is maximum to the permissible extent. Thus, the desired rudder deflection will be given by:

$$\delta_{r\text{cmd}} = \delta_{r\text{max}}$$ \hfill (4.43)

For the maximum rudder deflection, the desired tail rotor RPM command can be obtained using the following expression:

$$\Omega_{\text{cmd}}^2 = \frac{R}{\sin\delta_{r\text{max}}}$$ \hfill (4.44)

The solution to case 2 results in the tail rotor producing a forward acceleration that is greater than that demanded by the outer loop. Thus, the additional forward acceleration generated by the tail rotor will need to be addressed by the main rotor. This will typically be achieved by a pitch-up motion by the helicopter to reduce the forward velocity. The effect of such a case can be more pronounced in hover that might result in the helicopter having an upward orientation to negate the forward thrust generated by the tail rotor.

### 4.3.3 Case 3: U and R combined are greater than maximum control authority but R is achievable

At times, the combined requirement of $U$ and $R$ may be greater than the total control authority possible. However, it may be a case wherein the beep requirement is too high, and the yaw requirement can still be achieved. In the case of lack of control authority, the case leads to the need for either axis prioritization or direction preservation. In the case of direction preservation, the control will be so managed to meet partial requirements of $U$ and $R$ such that the overall direction of the moments, although lower, is in the direction of the desired moment. In the case
of axis prioritization, it would be attempted to achieve one of the two requirements while compromising the other.

Thus, the rudder/sectors will be deflected in such a manner along with suitable RPM control so that the desired yaw rate will be achieved. However, it will not be possible to meet the beep requirements. As indicated in figure 4.4, there exists a maximum value of $U$ corresponding to an achievable value of $R$. Thus, the solution to equations (4.32) and (4.33) will result in the desired value of $R$ being achieved along with a limit on the beep command. This solution is given by:

$$\Omega_{cmd}^2 = \Omega_{max}^2 \quad (4.45)$$

$$\delta_{r cmd} = \tan^{-1} \frac{R}{U_{max}} \quad (4.46)$$

### 4.3.4 Case 4: R is greater than maximum control authority

Case 4 is an extension of case 3 wherein the control authority is now insufficient to meet both the yaw as well as the beep requirement. While case 3 resulted in the desired yaw rate being achieved, case 4 will result in insufficient yaw and beep value. Thus, the achievable yaw and forward acceleration will be lower than desired leading to a slower response from the helicopter.

This case typically arises when the available control is lower than the desired control i.e. there exists insufficient control authority. A high yaw requirement may typically be in cases of maneuvers. In terms of yaw control, the lack of control authority could be due to rudder/sector deflection limitation or tail rotor RPM limits. Regarding rudder deflection, if the maximum possible rudder deflection is $90^\circ$, then the lack of control authority is due to a limitation of the maximum tail rotor speed.

As we consider axis prioritization, the solution to this case would typically result in maximum rudder deflection with maximum tail rotor RPM so that the desired yaw rate requirement can be met to the maximum extent possible. The solution to equations (4.32) and (4.33) for case 4 is:

$$\Omega_{cmd}^2 = \Omega_{max}^2 \quad (4.47)$$

$$\delta_{r cmd} = \delta_{r max} \quad (4.48)$$
4.3.5 Case 5: R is lesser than 0

The above four cases have dealt with the scenario where $R \geq 0$. At times it may be possible that the desired $R$ is less than 0. A look at equation (4.35) will reveal that a solution to the same does not exist owing to the limitation of the rudder deflection from $0^\circ$ to $90^\circ$. Thus, in case $R$ is negative, the requirement cannot be satisfied. In this case, the control strategy followed is to assume $R = 0$ and satisfy the beep requirement. Thus, the solution to the system will be of the form:

$$\Omega_{cmd}^2 = U$$

(4.49)

$$\delta r_{cmd} = 0$$

(4.50)

4.4 Yaw Controller Implementation Logic

The previous section detailed the requirements that the yaw rate controller needs to meet to achieve the desired yaw rate and forward velocity control. As explained, there are five possible scenarios based on the values of the intermediate variable $R$ and $U$. The implementation of the controller is explained through the flowchart indicated in figure 4.6 along with a description.

1. The first step for the controller is to compute the intermediate signal $R$. The resulting action is based on the sign of $R$. If $R$ is greater than or equal to zero, proceed to step 2, else proceed to step 8.

2. If $R$ is positive, the next step is to check the total pseudo-control requirement. There are four possible scenarios. If the total pseudo-control can be achieved, cases 1 and 2 results. In this case, proceed to step 3. Cases 3 and 4 result when the total pseudo-control is greater than the tail rotor control authority. In this case, proceed to step 5.

3. If the total pseudo-control requirement can be met, compare the value of the desired $U$ to the minimum value of $U$. If $U \geq U_{min}$ the condition satisfies the requirements of case 1, the desired yaw rate and beep can be achieved using equations (4.41) and (4.42) and the desired dynamic inversion based control is achieved.
4. If $U < U_{\text{min}}$ case 2 results. Here, as the desired beep command is lower than that physically possible, the rudder deflection is set to maximum and the yaw rate requirement is be satisfied using equation (4.44). As the beep requirement is not met, the forward velocity provided by the tail rotor would be greater than the desired forward velocity. In this case, the increase in forward velocity can be negated by pitching up the helicopter.

5. If the desired pseudo-control is greater than the control authority, it needs to be determined whether the requirement for $R$ can be met. If $R$ can be satisfied, proceed to step 6, else proceed to step 7.

6. If $R$ can be achieved, case 3 results. Here, the tail rotor RPM is set to the maximum value and the rudder deflection is determined based on the maximum $U$ that can be achieved for the corresponding $R$. Thus, the tail rotor RPM command and rudder/sector deflection can be computed using equations (4.45) and (4.46).

---

Figure 4.6. Flowchart of Yaw rate controller implementation
7. If $R$ cannot be achieved, then the rudder deflection is set to maximum and the tail rotor RPM is also set to maximum speed. In this scenario, neither yaw nor forward acceleration can be met, and the controller tries to achieve the maximum yaw control possible. Thus, equations (4.47) and (4.48) can be used to determine the control commands.

8. If the event that $R$ is negative, as the current helicopter configuration will not be able to achieve this requirement, the best action is to meet the beep requirement while ignoring the yaw rate requirement. This is achieved by setting the control inputs using equations (4.49) and (4.50) as determined in case 5.
Chapter 5  
Simulation Results

The previous chapter described the helicopter control architecture and provided a detailed analysis of the development of the yaw rate controller. The implementation of the yaw rate controller was tested in the simulation environment GUST (Georgia Tech UAV Simulation Tool) which was explained in chapter 2. The tail rotor in the existing helicopter model was modified in accordance with the VTDP configuration and the new yaw controller was implemented. Simulation results of some basic helicopter operations are provided below. The simulations are performed in a manner to simulate all possible cases explained above.

As the maximum rudder deflection was considered as \(90^\circ\), the minimum value of \(U\) for all values of \(R\) was 0. The basic restriction for the analysis presented above is that \(U \geq 0\). Thus, as \(U_{\min} = 0\), test case 2 was not applicable as it was encompassed by test case 1. To simulate case 2, the physical characteristics of the model need to be modified so that the maximum rudder/sector deflection is limited to a value less than \(90^\circ\).

### 5.1 Takeoff and Hover

The initial simulation was performed with the tail rotor beep set to 0. Thus, the only function of the tail rotor is to generate the desired yaw moment. A simulation of the helicopter taking off and hovering at a stipulated altitude (30 ft AGL) is illustrated in figure 5.1. The plot includes the desired \(R\) to achieve the maneuver along with the commanded controls to achieve the desired \(R\).

From figure 5.1, it can be seen that during the initial phase, the rudder deflection is fluctuating between the maximum and minimum value. This is due to \(R\)
fluctuating on either side of 0. The magnitude of $R$ on the ground is very small. As this value is changing between a positive and negative value, the rudder deflection is fluctuating between the maximum and minimum value in accordance with case 5. Case 5 results when $R$ becomes negative. In such a case, as the requirement cannot be satisfied, the tail rotor tries to meet the beep requirement and hence the rudder deflection is set to 0. The controller designed is mainly applicable after takeoff. Thus, the value of $R$ indicated on the ground is very small due to which the commanded RPM ($\Omega_{cmd}^2$) is closer to 0.

Once the helicopter takes off, there is a high requirement of $R$ to meet the anti-torque requirement. As the beep requirement is 0 and $R$ is less than 1, there is sufficient control authority to meet the requirement and case 1 results. The tail rotor RPM and rudder deflection are computed using equations (4.41) and (4.42). As the beep command is 0, this results in maximum rudder deflection as can be seen from the figure. Also, from equation (4.41), it can be seen that $\Omega_{cmd}^2 = R$. This is reflected in the figure where $\Omega_{cmd}^2$ follows $R$. 

Figure 5.1. Plot of $R$ and commanded controls for take-off to hover
5.2 Right and Left Yaw Turns

Following stabilization in hover, the helicopter was commanded to yaw to the right by 90°. A plot for the same is illustrated in figure 5.2.

![Plot of R versus time for right turn yaw with U = 0](image1)

![Plot of \( \dot{\gamma} \) versus time for right turn yaw with U = 0](image2)

![Plot of \( \delta \) versus time for right turn yaw with U = 0](image3)

**Figure 5.2.** Plot of R and commanded controls for right side yaw with 0 beep

It can be seen that at the instance of commanding the right turn yaw, there is a very high requirement of \( R \). This is because the direction of rotation of the main rotor is clockwise and hence, higher torque is needed to yaw to the right. The value of \( R \) crosses the maximum of 1. This results in case 4 where the helicopter control authority is insufficient. Thus, the RPM control is capped at its maximum value in an attempt to meet the maximum possible yaw requirement. The oscillatory nature of \( R \) is due to the system being under-damped. There is an overshoot from the desired position that is corrected by decreasing the value of \( R \) as can be seen from the figure. The helicopter subsequently stabilizes at the desired position.

A 90° left yaw turn was commanded next and the behavior of the helicopter was analyzed. A plot of the control requirements is illustrated in figure 5.3. It can be seen that the plot is similar to the case of right turn yaw except that the
requirement of $R$ is inverted. This is because, for a left side turn, sufficient torque is provided by the main rotor which reduces the $R$ requirement from the tail rotor. As explained in the case of the right turn, the oscillatory nature of the response is due to the system being under-damped.

![Graphs showing R versus time for left turn yaw with U = 0](image)

**Figure 5.3.** Plot of $R$ and commanded controls for left side yaw with 0 beep

### 5.3 Hover with non-zero beep

The above simulation results demonstrated the inner loop control for cases 1, 4 and 5 with the beep requirement set to zero. The subsequent simulation was performed with a beep requirement of 0.4 with the beep gain ($b_u$) set to 10. Commanded yaw turns in the right and left direction were performed. The plots for $U$, $R$ and the commanded controls is presented in figure 5.4.

With an increase in the commanded beep signal, there is a reduction in $R$. At the instance of the change in beep input, there is a corresponding increase in the RPM command to meet the beep requirement. Also, the rudder/sector deflection is reduced to meet the beep requirement. As the total requirement in less than the
maximum control authority, the instance of change in beep command corresponds to case 1.

A right yaw turn, and left yaw turn were initiated at approximately 250 sec and 280 sec respectively. As in the case with 0 beep, the plots for $R$ are the reciprocal of each other for the right and left turn. The initial part of the turn corresponds to case 1 as the total control requirement is satisfied. However, after the overshoot in position, $R$ becomes negative in the case of a right turn. At this instance, it can be seen that the RPM command follows the beep requirement and the rudder/sector deflection command is set to 0, in accordance with case 5. In case of the left turn, such a phenomenon is not observed although the control requirement approaches close to 1.

In the previous section, it was stated that the tail rotor provides the desired beep and the position and velocity requirements are regulated by the outer loop. As a beep command of 0.4 was provided to the tail rotor, the outer loop is required to negate this effect to maintain the position. Thus, the outer loop is expected to correct for the increased forward velocity by a pitch-up action. The same is
illustrated in figure 5.5. It can be seen in the figure that the beep command has resulted in the outer loop commanding an increase in the helicopter pitch angle to ensure that the desired position and velocity are maintained.

![Plot of U versus time](image)

![Plot of Pitch angle (θ) versus time](image)

**Figure 5.5.** Plot of beep versus helicopter pitch in hover

To simulate case 3, the beep requirement from the tail rotor was increased to 0.6. In the case of \( U = 0.4 \), it was observed that the control authority was close to saturation. The beep command was thus increased to cause saturation to simulate case 3. The results of the same are illustrated in figure 5.6.

It can be seen from figure 5.6 that an increase in the beep command has resulted in control saturation during the turns. The turns were performed similarly to the previous cases i.e. a right turn followed by a left turn. In the case of the right turn, when the requirement for \( R \) increases, as the total requirement exceeds the control authority, the beep requirement is compromised to meet the yaw requirement. Thus, it can be seen that the achieved beep dips below 0.6 at the instance of the right turn. This is also associated with the RPM command hitting the maximum and the rudder deflection increasing to meet the yaw and partial beep requirement. In the case of the left turn, there is a higher decrease in the achievable beep command due to a higher requirement of \( R \).
5.4 Forward Flight

The final simulation that was performed was to test the efficacy of the tail rotor model and controller in forward flight. The helicopter was provided with a navigation command to fly in a loop at 150 ft AGL at a velocity of 50 ft/s. The beep command on the ground was set to 0.6. After completion of one round along the loop, the beep command was set to 0. The helicopter flight was continued for a short duration without beep followed by a beep command of 0.2. The rationale of the particular test case is explained below. The path flown by the helicopter is indicated in figure 5.7.

As can be seen from figure 5.7, the performance of the helicopter and controller is satisfactory in forward flight. The helicopter is able to fly along the stipulated path. It can be seen from the figure that there is a change in altitude at the points marked A and B. These are the points at which the beep command to the helicopter was changed.

The helicopter flight was initiated with the beep command set to 0.6. After
Figure 5.7. Helicopter flight trajectory in forward flight

the completion of a lap, the beep command was set to 0 at point A. As the initial beep was set to 0.6, the helicopter flies with a pitch up orientation. When the beep command is removed at point A, the helicopter pitches down instantaneously to negate the change in velocity. This pitch down motion results in a decrease in altitude which the helicopter subsequently recovers. After a brief duration, the helicopter beep command was set to 0.2 at point B. This results in the helicopter pitching up to maintain the desired velocity. The pitch up motion increases the altitude as reflected in the figure.

The above exercise was performed to observe the behavior of the helicopter at different beep levels and evaluate the performance of the yaw rate controller. The beep command was later removed to determine the recovery performance of the helicopter. The behavior is analogous to a case of a sudden gust that the helicopter must be capable of recovering from. Finally, the beep command of 0.2 at point B is provided to illustrate the possibility of better handling qualities of the helicopter with a VTDP configuration. The additional thrust generated by the tail rotor allows for a lower forward tilt to achieve the desired forward velocity. This can help in a better overall orientation of the helicopter, thus increasing the level of comfort in case of manned flight. The change in helicopter pitch with changes in beep is illustrated in figure 5.8.

As can be seen from the figure, the helicopter pitch angle varies with changes in the beep command. The pitch angle becomes negative at around 120 seconds
Figure 5.8. Plot of beep versus helicopter pitch in forward flight

corresponding to point A when the beep command is set to 0. The pitch angle increases at around 150 seconds when the beep is set to 0.2. It can be seen that a small beep command can help in ensuring a suitable pitch angle such that the helicopter is nearly horizontal. This orientation may be more comfortable for the people on board. Thus, the desired beep command from the tail rotor can be based on the forward velocity requirements or from a perspective of handling qualities. Finally, the plot of $U$, $R$ and the desired controls is illustrated in figure 5.9.

It can be seen that the achievable beep becomes 0 at approximately 10 seconds during the takeoff phase. At this point, the yaw requirement from the tail rotor is very high owing to the desired right turn towards the desired flight path. Thus, as $U$ becomes 0, this corresponds to case 4. The controller tries to satisfy the maximum yaw requirement as possible. Subsequently, the desired yaw and beep commands are within the achievable control region and correspond to case 1. Changes in the commanded RPM and rudder deflection are appropriately achieved based on the pseudo-control requirements.

The above results demonstrated the capability of the control algorithm to satisfy all the possible scenarios. While case overlapped with case 1 owing to the current definition of the tail rotor configuration, appropriate logic has been incorporated in the algorithm to cater to any restrictions/limitations in the rudder/sector deflection. Apart from hover, forward flight in a predefined trajectory was also simulated and
Figure 5.9. Plot of pseudo and commanded controls for forward flight

the performance of the helicopter with the VTDP configuration of the tail rotor was found satisfactory.
Chapter 6  |  Conclusion and Future Work

6.1 Conclusion

A model of the tail rotor for the Vectored Thrust Ducted Propeller (VTDP) has been developed which has been validated with static test stand data. A yaw controller to control the yaw rate and forward acceleration requirement from the tail rotor has also been developed. The efficacy of the controller has been tested for helicopter operations such as takeoff, hover and forward flight.

The model of the tail rotor was separated into its associate components and the aerodynamic analysis of each component was conducted separately. The analysis was based on some underlying assumptions such as uniform inflow through the propeller, ignoring compressibility and cross-coupling effects, etc. These assumptions have enabled in simplifying the development of the model. As the model is mainly used for analyzing the controller performance and handling qualities requirements, the fidelity of the model is sufficient. The forces generated by the tail rotor were compared to static test stand data available and the model was found to be a reasonable fit to the data. Variations in the model and data can be attributed to the simplifications in the design analysis as well as some of the assumptions that were used in the aerodynamic modeling.

The VTDP configuration of the helicopter can help in improving the handling qualities of the helicopter. With the tail rotor generating additional forward thrust, a control allocation scheme can be used to distribute the propulsive force requirements between the main rotor and the tail rotor. In a conventional helicopter, as the propulsive force is generated only by the main rotor, a forward tilt of the
helicopter is needed to generate the forward force. Thus, with increasing forward speeds, the helicopter tilt tends to increase. This can be a source of discomfort for the pilots. By distributing the requirements between the main rotor and tail rotor, the forward tilt needed to meet the propulsive requirement reduces and leads to a possible reduction of discomfort to the pilots. This aspect was verified through the helicopter pitch angle plot that was compared to the change in beep command.

The controller design was based on an existing nonlinear dynamic inversion based controller on the helicopter. The existing architecture consisted of a model following dynamic inversion based controller for the inner loop and the outer loop. The yaw controller designed in this work was used to modify the existing architecture to meet the requirements of the VTDP configuration and has shown to be capable of addressing both the forward acceleration and yaw requirements from the tail rotor.

A simplified dynamic inversion model was used to linearize the relation between the pseudo-control and output owing to the complexity of the actual plant. The dynamic inversion model has a valid solution despite the system not being affine in control. The simplified dynamic inversion model resulted in the need for an adaptive neural network feedback system. The neural network-based feedback linearization helped to approximate the nonlinearities in the plant. The proof of stability and boundedness for the adaptive neural network architecture exists and is available in the works of Johnson [34].

As the tail rotor configuration is not capable of satisfying negative yaw requirements, forward velocity requirement is prioritized over the yaw control. The desired negative yaw is satisfied by the natural torque moment generated by the main rotor rotation. Thus, the maximum possible negative yaw rate is limited by the torque generated by the main rotor. The VTDP architecture thus limits the yaw rate in the direction opposite to the rotation of the main rotor.

Different scenarios were determined to compute the solution of the dynamic inversion model. These scenarios were based on the possible control requirement from the plant. Cases that resulted in the lack of sufficient control have also been addressed. For cases where the total control requirement is greater than the control authority available, the possibility of satisfying the yaw rate requirement was analyzed. In cases where the yaw rate requirement is lower than the maximum yaw rate possible, axis prioritization was adopted over direction preserving control. This
philosophy is followed owing to the higher priority of the yaw requirement over the forward acceleration requirement. For cases where the yaw rate requirement exceeds the maximum possible yaw rate, the controller addresses the yaw requirement to the maximum extent possible ignoring the beep requirements. Thus, all possible cases have been identified and the controller is designed to address these scenarios.

The performance of the controller was evaluated for different helicopter operations in the simulation environment. Test cases were so designed to test the functionality of the tail rotor for most of the possible scenarios. The performance of the controller was found to be satisfactory in all the test cases. The helicopter performance was also evaluated in forward flight and the performance was satisfactory. The following sections detail additional research that can be undertaken based on the design presented in this work.

6.2 Future Work

The analysis presented in this work deals with the development of the VTDP model and the yaw rate controller design. Further research is possible in continuation with the analysis presented here and can be broadly classified into the following categories:

1. Improvements in helicopter modeling fidelity including effects of additional aerodynamic surfaces.

2. Integration of the outer loop controller with inner loop controller for beep management.

3. Development of Control Allocation schemes for redundant controls of the helicopter.


6.2.1 Improvement in Modeling Fidelity

The helicopter model developed in this work has utilized many basic assumptions to simplify the development of the model. The rationale behind the same is that the primary purpose of the model development was to analyze the performance of
the controller. However, in the case of additional analysis, a higher fidelity model can be developed utilizing the existing model. The controller developed in this work has been tested with a simulation model of the helicopter. A higher fidelity model will help to evaluate the controller performance before testing on an actual helicopter. Some of the areas identified for improvement of the model are listed below:

- The horizontal forces of the main rotor have not been considered in the force computations. The same can be included to develop an accurate estimation of the helicopter forces and moments.

- The inflow dynamics of the main and tail rotor have been ignored assuming uniform inflow. Additional inflow dynamics can be modeled, and appropriate states included in the state vector.

- The forces on the tail rotor have been developed assuming no interaction between the rudders and sectors. However, there is bound to exist inter-coupling between these surfaces. Thus, additional analysis of the same can help achieve a better match between the static test stand data and the helicopter model.

- Performance of the helicopter during hover will be affected due to the wing being directly in the path of the main rotor downwash. Additional analysis of the same will help in better control of the helicopter.

- The wings and horizontal stabilizer have been modeled without considering the effects of the flaperons and the elevator. Modeling of these elements is needed to develop the control allocation scheme for the redundant controls.

The above items list some of the possible improvements that can be implemented to develop a high-fidelity helicopter model. However, the scope of improvements will be governed by the objective of the proposed application and the performance metrics that need to be evaluated. With an increase in fidelity, the complexity of the model will increase which may not be desired for a particular application. Thus, the need for improvement will be governed by the appropriate design choice.
6.2.2 Integration of Outer loop and Inner loop

The current beep configuration is designed such that the tail rotor balances the yaw and beep requirement. The work in this thesis was based on the assumption that a control allocation scheme exists to distribute the control between the main rotor and tail rotor. Thus, in continuation with the present work, a suitable control allocation scheme can be developed.

The control architecture was so developed that the outer loop addresses the force requirements and the inner loop satisfies the moment requirement. This architecture is suitable for a conventional helicopter wherein there are no redundant controls to satisfy a force or moment. With the current VTDP configuration, as the tail rotor can generate forward thrust, the loops would have to suitable integrated using a control allocation scheme to distribute the forward acceleration requirements.

The integration between the outer loop and inner loop will have to address the five scenarios that can result based on the yaw and beep requirements. In case the tail rotor is unable to satisfy the beep requirement, the same should be communicated to the outer loop so that it can address any deficiencies. Further, it is also possible that the tail rotor generates a forward thrust higher than that demanded from it. The outer loop should be capable of addressing this as well. A proper integration scheme can thus be developed to integrate the outer and inner loop for effective control of the helicopter.

6.2.3 Control Allocation Scheme for Redundant Controls

An extension of the discussion of the previous section is the development of a control allocation scheme for all redundant surfaces on the helicopter. Redundant surfaces are those surfaces apart from the primary surfaces that can generate additional forces and moments to augment the primary surfaces. The redundant surfaces on the helicopter include the flaperons and elevators.

The wing helps in increasing the lift of the helicopter which can help in the reduction of the main rotor RPM. Effective use of the flaperons can help in increasing the lift generated by the wing. If both flaperons are deflected together, the lift of the wing can be increased or decreased. Also, differential deflection of the surface can help in operation as an aileron, thus generating a rolling moment. Thus, there
is reasonable scope to control the RPM governor of the main rotor as well as distribute the controls between the helicopter lateral cyclic and ailerons to satisfy the roll requirements.

Another redundant surface that exists is the elevator in the horizontal stabilizer. While the horizontal stabilizer is used to trim the helicopter, the use of elevators can help in offsetting the helicopter longitudinal cyclic requirements. Thus, as additional pitching moment can be generated by the elevator, the main rotor can be better utilized to meet all collective requirements.

A suitable control allocation scheme can be developed to distribute the controls among the control surfaces available. Different optimization criteria can be used to develop the control allocation scheme. Objectives such as optimum control, best possible maneuver, drag optimization, etc. can be used to determine the control allocation scheme. Methods such as frequency response based control can also be developed. In the current helicopter configuration, possible control of yaw includes tail rotor RPM control or rudder/sectors deflection. As the rudder/sectors deflection may have a higher frequency response in comparison to RPM variation, the initial yaw requirement can be satisfied through rudder/sectors deflection while the RPM builds up.

The selection of the control allocation scheme will depend on the effectiveness of each control surface. The effectiveness may be a function of factors such as the forward speed of the helicopter. As the control effectiveness matrix may vary with the phase of flight, the control allocation scheme will have to be accordingly adapted. Thus, it is important to determine the control effectiveness of all surfaces before finalizing the allocation scheme.

### 6.2.4 Development of Fault-Tolerant Design

Another area of research that can be built upon the current work is the development of a fault-tolerant design for the helicopter. Suitable fault cases can be identified, and suitable mechanisms can be developed to isolate these faults so that maximum/partial control of the helicopter can still be maintained. Thus, along with the development of a control allocation scheme, the architecture should also be capable of re-configuring certain controls in case of failure of any surfaces.

A typical example of a fault might be due to the rudder/sectors being jammed
resulting in insufficient yaw being generated. Thus, in such a case, the helicopter control architecture should be able to identify such faults so that they can accordingly be addressed. In this particular case, additional yaw may be generated by increasing the tail rotor RPM. Another example of a fault case may be due to one of the sectors being damaged or lost. The current model of the tail is not designed to address such a scenario. Thus, a suitable analysis can be performed to address such cases so that the accurate control requirement can be determined.

The above sections listed some of the possible tasks that can be undertaken in continuation of the existing research. Additional scope can exist based on the application. All these aspects can be addressed, some with minimal adaptation to the existing work while others requiring considerable modifications based on the final objective of the research. The work presented in this thesis provides a fundamental building block to further the research in the VTDP compound helicopter configuration.
Appendix
Coordinate Transformations & Conventions

A.1 Transformation Matrix

The Euler angle transformation $T_{2,1}$ from coordinate frame $F_1$ to coordinate frame $F_2$ can be obtained by rotations along the three axes in the sequence 3-2-1. This is represented as [55]:

$$
T_{2,1} = \begin{bmatrix}
\cos \theta_z & \cos \theta_y \sin \theta_z & -\sin \theta_y \\
\sin \theta_x \sin \theta_y \cos \theta_z - \cos \theta_x \sin \theta_z & \sin \theta_x \sin \theta_y \sin \theta_z + \cos \theta_z \cos \theta_x & \sin \theta_x \sin \theta_y \cos \theta_z \\
\cos \theta_x \sin \theta_z \cos \theta_y + \sin \theta_x \sin \theta_z & \cos \theta_x \sin \theta_z \sin \theta_y - \sin \theta_x \cos \theta_z & \cos \theta_x \sin \theta_y \cos \theta_z \\
\end{bmatrix}
$$

(1)

Here, $\theta_x$, $\theta_y$ and $\theta_z$ represent the rotations along the three orthogonal axes.

A.2 Euler Angle Names

The Euler angles used in the transformation from one coordinate system to another are given in the table below [55]:

<table>
<thead>
<tr>
<th>$T_{F_2,F_1}$</th>
<th>$\theta_x$</th>
<th>$\theta_y$</th>
<th>$\theta_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{B,H}$</td>
<td>$\phi$</td>
<td>$\theta$</td>
<td>$\psi$</td>
</tr>
<tr>
<td>$T_{W,H}$</td>
<td>$\mu$</td>
<td>$\gamma$</td>
<td>$\chi$</td>
</tr>
<tr>
<td>$T_{B,W}$</td>
<td>0</td>
<td>$\alpha$</td>
<td>$-\beta$</td>
</tr>
</tbody>
</table>

Here, W, B, and H are the wind, body and horizontal axes respectively. $\phi, \theta, \psi$ are
the three body orientation angles, $\mu, \gamma, \chi$ are the wind axis bank angle, flight path angle and tracking angle respectively and $\alpha, \beta$ are the aerodynamic angles.

A.3 Euler Angle Sign Conventions

The sign conventions for various Euler angles are [55]:

- **Yaw** - Right-handed rotation about the z-axis, or positive $\psi$.
- **Pitch** - Right-handed rotation about the y-axis, or positive $\theta$.
- **Roll** - Right-handed rotation about the x-axis, or positive $\phi$.
- **AoA** - Angle of Attack ($\alpha$) is positive when the relative wind is from below the X axis.
- **Sideslip** - The sideslip angle ($\beta$) is positive when the relative wind is from the right of the plane of symmetry.

A.4 Aircraft Control Effectors Angles

The sign convention for control effector angles is determined by the right-hand rule where the thumb points along the axis of the primary moment being generated, and the curled fingers point in the direction of positive deflection. The sign conventions are [28,55].

- $\delta_a$ is positive when the right aileron is Trailing Edge Down (TED) and the left is up (TEU).
- $\delta_e$ is positive when the surface is TED.
- $\delta_r$ is positive when the surface is Trailing Edge Right (TER).

The same can be summarized in a table as indicated below [28]:

<table>
<thead>
<tr>
<th>Surface</th>
<th>Deflection</th>
<th>Sense</th>
<th>Primary Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevator</td>
<td>TED</td>
<td>Positive</td>
<td>Negative pitching moment</td>
</tr>
<tr>
<td>Rudder</td>
<td>TEL</td>
<td>Positive</td>
<td>Negative yawing moment</td>
</tr>
<tr>
<td>Ailerons</td>
<td>Right Wing TED</td>
<td>Positive</td>
<td>Negative rolling moment</td>
</tr>
</tbody>
</table>
References


