VIBRATIONAL ANALYSIS OF ASH AND COMPOSITE HURLEYS

A Thesis in
Acoustics
by
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Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

May 2020
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Abstract

The primary piece of equipment for the Irish sport of hurling is the hurley, an ash, composite, or plastic stick with a flat end (bás) for hitting a ball (sliotar) during gameplay. The size, shape and material of hurleys vary greatly due to nonrestrictive regulations from the Gaelic Athletic Association and the Camogie Association. This leads to hurleys of similar length having different vibrational behaviors, which could have implications for gameplay use. Measuring the mode shapes, frequencies, and damping characteristics of hurleys, as well as their moment of inertia and center of percussion, provides insight into hurley behavior and how it might relate to players’ perception. Ten 35-inch (89 cm) hurleys of various materials (ash, composite, plastic) from different manufacturers were compared in this study. The moment of inertia and center of percussion relative to a 7-inch (18 cm) pivot point are reported for each hurley. The chosen pivot was based on previous work for the center of percussion for hurleys and baseball bats, which may not be the best representation of the pivot of a hurley swing. Experimental modal analysis was used to compare the frequencies and patterns of mode shapes within the frequency range of human sensation. Both bending and torsional modes appear within this range and affect the perception of “feel” of the hurley. The bending mode frequencies of each hurley were curve fit to give a simple parameter for comparison across hurleys. The modal damping rates of the lowest five modes of each hurley was also measured. This allowed comparison of damping rates both between modes of a hurley and between different hurleys. This thesis discusses hurley behavior compared by material, which could inform next steps for creating sustainable replacements for ash hurleys. Additional work involves testing different sized hurleys to establish scaling for their properties and biomechanical analysis of hurley swings to determine the actual pivot point of each swing.
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Chapter 1
History of Hurling

1.1 Introduction

The field sports of hurling and camogie originated in Ireland and are strongly associated with Irish identity and heritage. Hurling and camogie are nearly identical stick-and-ball team sports played with a hurley (the stick) and sliotar (the ball). Evidence for hurling in Ireland can be found going back centuries, which helped establish a historical basis when the game was codified for men in the late nineteenth century. Camogie was created by, and for, women a few decades after hurling was codified in order to involve women in a sport that had by then become a large part of Irish culture. Both hurling and camogie were added to UNESCO’s Representative List of the Intangible Cultural Heritage of Humanity in 2018. The listing states that “Hurling is considered as an intrinsic part of Irish culture and plays a central role in promoting health and wellbeing, inclusiveness and team spirit” [19]. While they remain closely tied to an Irish identity, the sports have gained traction abroad, including in the United States.

1.2 The Game

The games of hurling and camogie as they are played now are nearly identical. Two opposing teams of 15 men (hurling) or 15 women (camogie) compete on a rectangular
Figure 1.1. The official camogie pitch. For hurling, the line at 45 meters shifts to 65 meters. Source: [1]

field to score points and goals at each end. Games are played in two thirty-minute halves [1, 20]. Whichever team has the highest score at the end wins the match. Ties are determined during additional time added at the end of the match.

The field for these sports is very large. An acceptable pitch falls within the range of 130 to 145 meters long by 80 to 90 meters wide [1, 20]. This is more than one and a half times the area of an international regulation soccer pitch. Figure 1.1 shows a typical pitch, including lines that are marked on the field to assist with player positioning, penalties and refereeing. At either end of the pitch is an H-shaped goal, which resembles a soccer goal with net, but with the side posts extending up above the crossbar. The goalposts and net are visible in the background of Fig. 1.2. A point is scored when the sliotar is passed between the goalposts above the crossbar. A goal is scored when the sliotar is passed between the goalposts below the crossbar into the net and is worth three points. Games are scored as goals-points for each team, with the winner being determined by who has the highest score using the following equation: \[ \text{score} = 3 \times (\# \text{ of goals}) + (\# \text{ of points}). \]

The sliotar may not be picked up from the ground, carried or thrown with the hands.
It may, however, be caught in the air with one hand, or lifted off the ground using the toe of the hurley. If a player wishes to move on the field while maintaining possession of the sliotar, it is balanced or hopped on the bás of the hurley as the player runs, called the solo run or soloing [3, 20]. The sliotar may be held in the hand for no more than four steps, after which it must be struck with the hand (called handpassing), tossed and struck with the hurley (striking from the hand), or soloed [1, 3]. The sliotar may also be struck on the ground by the hurley, called pucking, which is the move that starts the game and restarts play after penalties and scores.

Player to player contact is allowed in both hurling and camogie, although the accepted types of contact are much stricter for camogie [1, 20]. Hooking is allowed in both sports, where a player uses their hurley to “hook” the hurley of an opposing player who has possession of the sliotar. Incidental contact is unavoidable in both hurling and camogie,
but only hurling explicitly allows, and even encourages physical contact. In fact, one of the skills taught to children under 8 is called the shoulder clash, shown in Fig. 1.3, where opposing players who are attempting to gain possession of a sliotar contact each other from shoulder to hip [3].

### 1.2.1 Safety

Because of the physical nature of the games, and presence of wooden or plastic sticks being swung with great force, there is a strong emphasis on safety and learning safe technique. This is especially evident in youth programs. For example, as part of learning to catch the sliotar in the air, children are taught to guard their catching hand with their own hurley in case another player is going for the sliotar with a hurley instead of a hand [21]. Players are often reminded to position themselves closer to their opponent because “the most dangerous place to be is a hurley’s length away from an opponent” [3]. Players who start learning the skills of hurling and camogie at a later age can have more trouble learning to play safely [22].

Helmets with faceguards are now required by both the Gaelic Athletic Association (GAA) and the Camogie Association. An analysis of sports injuries in 1984, before helmets...
were integrated into the sports, found that facial injuries were the second most frequent injury sustained during hurling and camogie, primarily broken noses and cheekbones [23]. Helmets were introduced in lower levels of hurling and camogie to reduce these injuries and were finally made mandatory at the senior level of hurling in 2010 [24]. Further safety equipment such as a mouthguard is allowed, provided the referee deems it to not interfere with gameplay.

1.3 Historical Origins

Hurling and camogie, as they are played today, are traceable directly to Gaelic cultural nationalist organizations around the turn of the twentieth century. In a country that had been struggling under English occupation for hundreds of years, hurling, and later camogie, was intentionally reintroduced as a way for Irish people to connect to a national identity, styled as Gaelic culture. The existence of early accounts of hurling in Ireland helped tie the sport to the land. A narrative of a continuous linear history is entrenched in the collective consciousness surrounding Gaelic games.

It is important to note that this section deals solely with historic mentions of hurling, only because camogie was created in 1904 and thus did not exist yet. There is no evidence precluding women’s participation in stick-and-ball sports before that, although it was primarily men who made the cut for written mention. Oral tradition suggests that women may have been more involved in hurling and hurling activities than written evidence suggests [25].

1.3.1 Hurling from the beginning to 1884

While tracing the history of hurling through the last millennium, it is tempting to present a picture of a cohesive, continuously played sport that survived multiple periods of occupation and hardship without lapsing, which is unique and specific to Ireland.
However, remarkable similarities between early hurling, early field hockey in England, and shinty in Scotland make these declarations more perilous. A leading Scottish expert on shinty, Hugh Dan MacLennan, points out that “not only were all these games, which were played with a ‘clubbe or hurl batte’ indigenous, but they all derive from one common ancestor, to wit from the game to which such frequent reference is made in the Celtic story” [26]. Michael Cronin extends this argument to hurling, asking if we can “simply believe that hurling existed in Ireland in splendid isolation, when the origins of so many other games with club and ball are difficult to locate?” [26]. It is with this caveat in mind that the early history of hurling in Ireland is presented here.

The idea of hurling as a sport or contest of prowess has been present in Ireland for centuries. Hurling is mentioned in ancient Brehon Laws from the seventh and eighth centuries, which detail compensation for various injuries [5,27]. Some of the first mentions of hurling appear in the Book of Leinster, annals compiled in the twelfth century, a page of which is shown in Fig. 1.4. In the Book of Leinster, hurling is mentioned in the record of a battle that happened in 1272 BCE [27]. The Book of Leinster also includes the myth of Cú Chulainn, which is more popular as the mythic origin of hurling. In this story, a hero, Cú Chulainn defeats a dog by hitting the sliotar down the dog’s throat [28].

Hurling’s status as a widely played pastime was such that its popularity constituted a threat to English occupiers in the Middle Ages, who sought to curb playing of the game. The Statutes of Kilkenny of 1367 are often cited as concrete evidence of hurling’s prevalence in early Ireland. These statutes banned many expressions of Gaelic culture with the hope of preventing English colonizers from assimilating too far into the culture of the land they were occupying [26]. The Statutes of Kilkenny framed hurling as a frivolous distraction and therefore a threat to the safety of the land and its people, urging them to “…use not henceforth the games which men call hurlings, with great clubs at ball upon the ground, from which great evils and maims have arisen, to the weakening of the defence of the said land…” but instead adapt to using bows and lances; noncompliant
Figure 1.4. An illuminated page from the Book of Leinster, in which the legend of the heroic warrior/hurley Cú Chulainn is first recorded. Source: [4]

people were to be imprisoned and fined [29]. Despite England’s best efforts, the statute was only partially successful. England kept control of Ireland, and English colonizers kept their English identity, but hurling was resilient. A century and a half after the Statutes of Kilkenny, the city of Galway found it necessary to ban hurling again [27]. Hurling had survived and was played to the extent that it was a safety concern in the city.

During the middle ages, two distinct types of hurling coalesced based on both season and geographical location. Winter hurling was played on the ground with a wood ball and a thinner stick. The game resembled the Scottish stick-and-ball game shinty. Summer hurling was played on a large field or cross-country with very large teams. The hurleys used for summer hurling were broader and were more like those used today and the balls were made of hair [5]. Modern hurling more closely resembles historical summer hurling. Summer hurling balls have been recovered in Irish bogs dating as far back as the twelfth century [30]. The discovery of these balls appears to corroborate contemporaneous written accounts of hurling as well as the statutes of Kilkenny and Galway.
Written evidence of hurling from the seventeenth century is not readily available. King asserts that this paucity of written accounts can be attributed to hurling’s status as a sport for commoners [27]. He maintains that the seventeenth and eighteenth centuries were the “Golden Age of Hurling,” whereas other historians attribute the scarcity of record as evidence of the sport’s decline [26, 27, 31]. The existence and popularity of hurling is more certain in the eighteenth century, when landlords would organize teams and host matches. There is some written evidence of hurling in this time period. In their private journals, travelers mentioned the games while reverends disparaged them [27]. Newspapers published announcements of games and accounts of recent victories. King attributes the decline of this era to economic hardship, famine, and religious efforts to minimize sporting of all kinds [27]. Hurling would not be revived on a grand scale until the concerted efforts of Irish nationalists at the end of the nineteenth century.

1.4 Hurling and Camogie are Codified After 1884

Hurling’s strong association with Irish identity is no accident. Gaelic cultural nationalist groups fostered the game’s resurgence for many of the same reasons that English occupiers had tried to ban it hundreds of years before.

1.4.1 The GAA brings back hurling

Michael Cusack is commonly credited with leading the vanguard that brought back Gaelic sports. His 1884 article “A word about Irish athletics,” in the publication *United Ireland*, called for replacement of English sports and pastimes with those which were uniquely Irish. Cusack also called for the creation of an Irish association to regulate Irish sports [31]. In response, the Gaelic Athletic Association was formed. The GAA was closely linked with political nationalist movements, some quite radical, and was “dominated and driven by the demands of nationalist politics and identity” [26]. The
activities of these organizations fueled the push for widespread adoption of Gaelic games. The two most successful introductions of the GAA were standardized rules for hurling and for Gaelic football. The first set of rules for hurling was drawn up by Maurice Davin in 1885 [31]. Both hurling and Gaelic football are still played today and are regulated in Ireland by the GAA. Both of these Gaelic games involve large amounts of physical contact and require strength and endurance. They were “fashioned to display male prowess, and in doing so, to show that the Irish were not weak.” [31]. The GAA’s focus on a masculine image and the organization’s patriarchal structure, joined with the prejudices of contemporary society towards women, led to the complete exclusion of women in formal iterations of the original revival of hurling. This did not go on for long.

1.4.2 Women create camogie

At the turn of the twentieth century, the GAA was only one of many Irish organizations continuing to promote Gaelic culture. Another organization was the Gaelic League, a cultural organization that accepted male and female members more equally than others at the time [25]. In the very early years of the twentieth century, women from the Gaelic League established a set of rules for camogie. This feminized version of hurling required long skirts, visible in Fig. 1.5, disallowed physical contact, and was played on a smaller field, in addition to a few other adaptations [31]. Around this time, more women were entering the workforce and universities. These women had more free time to fill, and it was their participation that helped camogie gain traction in urban areas [5]. By 1905, the Camogie Association (An Cumann Camógaíochta) had been formed, and still governs camogie in Ireland and abroad to this day. The spread of camogie followed networks already established by the Gaelic League, and later camogie was strongly promoted at the university level so that those graduates would go on to establish teams at the secondary school level and continue to grow the sport [25]. Since its inception, camogie has been a vehicle for challenging gender inequality. One of the early teams in Dublin
was named the Gra’inne Mhaol Club, after “the sixteenth-century Irish pirate queen who transgressed colonially imposed gender roles and challenged authority” [25]. Camogie, and its lack of adherence to certain gender norms, was often condemned by clergy and others who had certain ideas of propriety [25]. In fact, some early camogie players felt it necessary to conceal their hurleys while traveling to their playing grounds [5].

Over time, the rules and play of camogie have inched closer to hurling, although most rough play will still result in a foul [1]. In 1999, the Camogie Association approved the switch to a larger field, measuring the same as a hurling pitch [5]. Due to its frequent treatment as the women’s counterpart to a men’s sport, camogie sometimes still struggles to be recognized in the greater picture of Gaelic games, despite having existed for more than a century. Notably, *A History of Hurling* by Seamus King, a former GAA official, does not mention camogie even once in its 400-page account of the “standard modern history” of hurling [27]. That the book was published in 1996 and re-released in 2005 without addressing camogie underscores the continued exclusion of camogie, and therefore women, from analysis and history of Gaelic games. This dearth of information is being addressed by both women’s sports historians and camogie players such as Mary Moran,
who published *A Game of Our Own: Camogie’s Story* in 2011 [25, 31]. Unfortunately the book is not widely available, a fact that reinforces the lack of representation camogie players and fans may already feel.

### 1.4.3 Connection to Irish identity

As the GAA pushed for the adoption of Gaelic games as replacement for English traditional sports such as cricket, it relied on the legacy of hurling in Ireland. Overlooking hurling’s near-century-long absence in the 1800s, the GAA romanticized the sport as a tenacious survivor of English rule. This mimicked the real perseverance of Irish people and Gaelic culture under English rule and oppression. However, the practice of hurling had been effectively extinct for a century, so the assumption of continuity is an invention of the GAA. This is still the prevalent narrative in the collective consciousness of the sport, which “...allows the “script” of the sport to become a millennia-old story instead of a conscious adaptation...” [31]. Measures were enforced to further solidify the adaptation and authenticity of Gaelic games. For example, members of the GAA were banned from attending or participating in games that were deemed foreign [31]. This ban was mirrored in a similar ruling by the Camogie Association in 1930 to prevent camogie players from participating in field hockey or even attending events hosted by field hockey associations [25]. It seems that these early efforts were very effective, and to this day hurling and camogie are strongly associated with an Irish identity. These sports are now so ingrained in the culture of Ireland that recent immigrants to Ireland often take up the sport as a way to feel more integrated into Irish society and culture [22].

### 1.4.4 Spread to other countries

Gaelic games’ connection to an Irish identity is also manifested in communities with Irish heritage abroad. Hurling and camogie both spread with the emigrant population of Ireland. The United States and Canada both have large organizations that govern Gaelic
games, due to a large amount of Irish immigration in the twentieth century. The United States GAA boasts more than 130 adult and youth clubs for hurling, camogie and Gaelic football [32]. In the United States, participation in Gaelic games is a meaningful way for both recent Irish emigrants and people of Irish heritage to connect to Irish culture [22].

1.5 Equipment

The two main pieces of equipment used to play hurling and camogie are the hurley (the stick) and the sliotar (the ball). These have developed over time, both for performance and material variability.

1.5.1 Hurley

The same kind of hurleys are used in both hurling and camogie. A hurley has a handle and a flat surface at the other end called the bás. The sides of the bás are referred to as the toe and the heel because of the hurley’s vague resemblance to a foot. Figure 1.6 is an illustration of an ash hurley. Traditionally, hurleys are made of ash wood, chosen for its elasticity and its “exceptional strength perpendicular to the grain” [6]. The part of the tree closest to the root is used to take advantaged of the curved grain [6,33]. Most ash hurleys are still handmade or utilize minimal automated assistance [33]. As such, every ash hurley is slightly different, depending on the specific piece of ash with which it is made. Many ash hurleys are “banded” with a piece of metal to help prevent the hurley from splitting along the grain. Players whose banded ash hurleys have split occasionally continue to play with the hurley, visible in Fig. 1.7.

In the second half of the twentieth century, manufacturers began to develop hurleys made of alternate materials in response to, first, the desire for increased durability and consistency in hurley behavior and, second, a concern for the environmental impact of overharvesting ash trees. Hurleys made of plastic and composite materials are widely
available and used by players at all levels. Plastic and composite hurleys do not have the metal band that ash hurleys do, and are designed to fracture or break in a safe way [34,35].

The official gameplay rules of both hurling and camogie only state one rule regarding hurleys: the hurley must not exceed 13 cm in width at its widest point. The exact shape, mass, and toe-heel balance are not standardized, but manufacturers seem to have agreed on general shape for modern hurleys. Figure 1.8 shows the similarities in shape of the hurleys used in this study.

Hurleys are sized by their length, which ranges from 18–36 inches (46–91 cm). Hurley lengths are chosen based on the height of the players, so any given team might have many different sized hurleys on the field at the same time [36]. The hurleys used in this study are 35-inch hurleys, which corresponds roughly to a 5’10” player. This is coincidentally the average height of men of playing age in both Ireland and the USA [37]. While there are certainly tall women who could use these large hurleys effectively, a 35-inch hurley is not likely a good representation of “average” use in camogie. For example, I am the mean height for women in the USA and Ireland and I use a 32-inch hurley.

**Figure 1.6.** An ash hurley. The contour of the hurley follows the curve of the wood grain. Source: [6, Figure 1]
This camogie player’s ash hurley has split, but the metal band around the bás keeps the hurley functional. In camogie, the metal bands must be covered with tape, whereas in hurling metal bands may be kept bare [1]. Source: [7]

Each hurley has a different outline. The bottom set of outlines are the 35-inch hurleys used in this thesis. Notice how some of the hurleys are shaped to have more of the bás toward the toe and some have the bás fuller in the heel.

1.5.2 Sliotar

The sliotar is a ball composed of a leather wrapping around a core. It resembles a baseball with ridges instead of stitching. Hurling uses a standard size 5 sliotar, while camogie uses a size 4 sliotar. The following are the specifications for a size 5 sliotar from
the GAA official rulebook:

The diameter of the Sliotar - not including the rim (rib) - shall be between 69mm. and 72mm. The mass of the Sliotar shall be between 110 and 120 grams. The rim (rib) height shall be between 2.0mm. and 2.8mm. The rim (rib) width shall be between 3.6 mm. and 5.4mm. The thickness of the leather cover shall be between 1.8mm. and 2.7mm. and shall not be laminated with a coating greater than 0.15mm [20].

A size 4 sliotar weighs 90 to 110 grams and is 21 centimeters in circumference [1]. Sliotars are tested by the GAA and the Camogie association, and only sliotars from accepted manufacturers are allowed.

Sliotars are being developed with different materials for both core and cover. These new, sometimes synthetic materials can behave quite differently than the traditional cork-and-leather sliotars. Collins, et al. investigated impact characteristics of four different ball types in 2010 in order to establish a standard test method [38]. The stiffness of traditional ball types was found to be significantly greater than those with polymer cores, which affects the impact time [38].

1.5.3 Development

Early hurleys were essentially crooked sticks, likely with a flattened bás. Early Irish law called for bronze bands on the hurleys of royalty [9]. This practice is still in use to increase the longevity of ash hurleys. Figure 1.9 shows some hurleys from around 1900, which are some of the oldest remaining. Earlier hurleys may have been unearthed in excavations, but were not recognized as such and were therefore discarded [9]. Examples of early sliotars have been found in bogs throughout Ireland. These were made of cow hair and were likely used for summer hurling [9].
1.6 This Thesis

This thesis is primarily concerned with how different hurley materials and constructions affect their modal characteristics: mode shapes, frequencies, and damping rates. Measurements of moment of inertia and center of percussion for each hurley are also included. The modal characteristics are interpreted by how they might affect what a player feels when using the hurley. Hurleys made of alternate materials are compared to ash hurleys to determine what qualities differ and where they are similar.

Twelve hurleys are included in the study: four ash hurleys, three plastic hurleys with a solid bás, and five composite hurleys with a hollow bás. There are two sets of composite hurleys that are the same model, so the doubles are omitted from full modal analysis.

Chapter 2 evaluates the hurleys using measurements of the moment of inertia and
center of percussion based on standards developed for other sports. These affect and are affected by the way a hurley is swung. Chapter 3 establishes the procedure by which modal data is collected. It also introduces concepts of human vibration sensation and the simple example of a uniform beam. The key results of modal analysis are presented in Chapter 4, including mode shapes and frequencies. Full mode shapes for all hurleys are contained in Appendix A. Chapter 4 also identifies regions that are likely the sweet spot of each hurley. Chapter 5 reports measured modal damping rates for the first several modes of hurleys. This provides insight on how long identified modes might continue to vibrate after impact. Chapter 6 summarizes the key findings of this thesis and identifies areas that further research might clarify or extend.
Chapter 2  
The Center of Percussion and Moment of Inertia

2.1 Introduction

In gameplay, hurleys are held in the hands and swung to hit the sliotar either in the air or on the ground. As a player swings a hurley, it rotates around an axis. This axis or pivot has not been defined in the current literature for hurling. Quantities that relate to rotational motion can still be used to compare between hurleys.

As with any object that rotates, the moment of inertia is used to quantify the ease with which that rotation can be changed. The moment of inertia about a pivot point on the hurley is reported as a means of comparing hurleys of varying materials and mass distributions. The moment of inertia is a factor in determining how a hurley feels when the player is swinging it.

For any physical pendulum there is a location on the pendulum that, when struck with an impulsive force, produces no reaction at the pivot point. This location is referred to as the “center of percussion” (COP). When the COP is struck, the translational and rotational displacements at the pivot cancel each other perfectly and the pivot experiences no acceleration or force. Every pivot point on a physical pendulum has a different corresponding center of percussion. For a hurley, every place that a player’s
hand might act as a pivot has one corresponding location on the bàs that causes no reaction in the hands when struck by the sliotar. The COP is often reported, along with the moment of inertia, as a quantity for comparing baseball bats and other equipment.

2.2 The Moment of Inertia, The Center of Percussion, and The Hurley Swing

Historically, the moment of inertia and the center of percussion have both been used to quantify characteristics of handheld sports equipment and allow them to be compared to each other. In 1944, Sears used the example of a swinging baseball bat to demonstrate the concept of the center of percussion [39]. In 2012, Schorah, et al. conducted a meta-analysis which focused on the moment of inertia (therein called “swing weight”) and swing acceleration of equipment from many different sports [40]. The swing acceleration (tip velocity divided by swing time) that a player can achieve with a piece of equipment was found to be inversely proportional to its moment of inertia.

2.2.1 Moment of inertia of a swinging bat

For baseball bats, Koenig, et al. found that “bat speed approximately correlates with the moment of inertia of the bat about a vertical axis of rotation through the batter’s body, the speed generally decreasing as this moment of inertia increases” [41]. Because of conservation of momentum, the speed at which the ball comes off the bat depends strongly on the speed of the swing. Therefore, the moment of inertia is used to characterize baseball bats, softball bats, cricket bats, and other similar equipment [41,42].
2.2.2 The COP as a sweet spot

The center of percussion is often chosen as a likely candidate for the sweet spot of baseball bats [39, 43, 44]. If the motion of the bat or other equipment can be described with motion around a pivot located at the hands, a player might judge that hitting at the center of percussion is “sweeter” than hitting at a location that does not cause a force in the hands [45]. The existence of the center of percussion is contingent entirely on the bat’s motion being constrained to a pivot at the time of impact. It has been shown that the motion involved in swinging a baseball bat, illustrated in Fig. 2.1, has an effective pivot just off of the bat past the knob [10]. Since the pivot is not located on the bat, any center of percussion calculated relative to a point on the handle would have no consequence at the player’s hands. This renders measurement of the center of percussion for baseball bats inconsequential to the player. Regardless of this new development, an ASTM standard exists for measuring the center of percussion of a bat which is still in use by most baseball and softball governing bodies [15].
2.2.3 Past studies of hurleys

Fahey, Hassett and Ó Brádaigh determine the moment of inertia and center of percussion of six hurleys of various manufacture and undisclosed length using a pivot located at 18 cm from the top of the handle; the pivot point lines up with the distal hand in the hurley grip (left hand for a right-handed player). The moment of inertia and center of percussion were determined using a compound pendulum approach, although only the center of percussion is reported in the article. The centers of percussion reported in the article are slightly further from the pivot point than those found the present study; therefore, Fahey, et al. likely used 36-inch hurleys, although this is uncorroborated. Fahey, et al. also note variation of up to 20% in the measured moment of inertia between the different hurleys studied [6]. Their study was conducted in 1998, before biomechanical studies called into question the assumption of a bat as a pivoted object at the moment of impact [10].

An undergraduate research group at the University of Hartford has also done some experimental measurements to determine the center of percussion of three different hurley models [46]. Instead of deriving the center of percussion from period measurements and the moment of inertia, they impacted the bás at multiple locations and reported the response from accelerometers distributed along the handle. This then defined a range along the bás that, when impacted, causes minimal acceleration in most of the accelerometers simultaneously [46]. This does does not quite track with the standard definition of the center of percussion, where each chosen pivot point has just one specific conjugate point. The Hartford study does, however imply the idea of a zone of minimal excitation, which is a promising direction.
2.2.4 A note about the hurley swing

From personal experience in fourth grade softball, this author can say definitively that swinging a hurley does not feel like swinging a baseball bat. The most immediate difference is that the dominant hand is placed closer to the top of the hurley, opposite to the standard grip used for baseball. On visual inspection, the motion used to swing a hurley more closely resembles the motion of a person swinging a golf club [33]. This is especially relevant when the player is hitting the sliotar from the ground (pucking). The arms are held straighter than a baseball swing, and the torso rotates in a similar way. Golf club swings are often modeled as double pendulums, see Fig. 2.2 [11,47]. There is no apparent single pivot point on the club anywhere in the swing. It is unclear whether the cross-handed grip on a hurley might add a pivot point somewhere, but if the hurley is not being swung as a pivot at the time of contact, there is no center of percussion. Players also occasionally “choke up” on the hurley to make shorter passes. This alternate grip likely has a completely different pivot point than the one for a full swing. There are occasions when players have their hands further apart, such as when they are blocking
the sliotar overhead. The movement exhibited by the hurley when the sliotar hits it in this hand position might even be rotational and also have some sort of pivot point. The three different hand positions are shown in Fig. 2.3.

The specific model chosen for a hurley swing is unlikely to affect trends in the moment of inertia data for hurleys as it does not depend on the existence of a pivot. Players of differing swing strength may find hurleys of differing moment of inertia easier or more difficult to swing. Later investigation into the biomechanics of a hurley being swung may reveal the existence of a pivot point which would certainly bring the center of percussion back into favor as a parameter defining the hurley. On this chance, center of percussion measurements are reported. The methods used in this chapter to collect moment of inertia and center of percussion information from hurleys are adapted from the ASTM standard and other previous studies [15].
2.3 Theory for the Moment of Inertia and Center of Percussion

2.3.1 Moment of inertia

The moment of inertia is a “quantity that determines the torque needed for a desired angular acceleration about a rotational axis” [48]. The moment of inertia of an object is entirely dependent on its mass distribution about the chosen pivot. For a continuous object, if each differential mass $dm$ is located at distance $r$ from the axis of rotation, the moment of inertia can be calculated with the integral

$$ I = \int r^2 dm. \quad (2.1) $$

The moment of inertia can also be calculated experimentally from measuring the period of oscillation of the object as a pendulum. Gravity provides the restoring force for the oscillation of any pendulum. For small angles, the period of a simple pendulum is given by

$$ T = 2\pi \sqrt{\frac{l}{g}}, \quad (2.2) $$

where $l$ is the length of the pendulum and $g$ is acceleration due to gravity. For a physical pendulum, Eq. 2.2 changes to

$$ T = 2\pi \sqrt{\frac{I}{mgh}}, \quad (2.3) $$

where $I$ is the moment of inertia of the object, $m$ is its mass, and $h$ is the distance between the center of mass (also called balance point $BP$) and the pivot point. The distance $h$ and other relevant quantities are marked in Fig. 2.4. Equation 2.3 can be rearranged to show that
\[ I = \frac{T^2}{4\pi^2} mgh. \]  \hspace{1cm} (2.4)

Since \( h = BP - a \), all that is required for determination of the moment of inertia is a measurement of the balance point and the period of oscillation. It is shown in Section 2.3.2 that \( \frac{T^2}{4\pi^2} = COP - a \), so Eq. 2.4 can be rewritten as

\[ I = m(BP - a)(COP - a), \]  \hspace{1cm} (2.5)

which is essentially the form that appears in the 2015 ASTM standards without accounting for the apparatus moment of inertia [15].

### 2.3.2 Center of percussion for a physical pendulum

The center of percussion describes the concept of two related points on an object where an impact at one of them will produce no acceleration or force at the other, so long as the motion at the second point is constrained as a pivot. This is because the translational
and rotational momentum produced by an impact at that point are equal and opposite at the other point. This relationship only exists if the point that is not being impacted is constrained so that the object must pivot around that point. This relation goes both ways, the pivot and impact point can be switched, to the same effect. For clarity in this section, the point $COP$ is set as the point of impact and $a$ is the corresponding pivot, separated by distance $l$. This derivation follows the derivation given in *University Physics* by Sears [39].

An impulse $J$ changes both translational and rotational momentum. The change in translational momentum, $\Delta p$, of the object after impact is given by

$$\Delta p = J = mv, \quad (2.6)$$

where $v$ is the velocity of the center of mass. The change in rotational momentum, $\Delta L$, around the pivot point $a$ is

$$\Delta L = Jl = I\omega. \quad (2.7)$$

The linear velocity $v$ at point $a$ is related to the angular velocity $\omega$ by $v = h\omega$, where $h$ is the distance from the center of mass to the pivot point. The distance $l$ between center of percussion location $COP$ and pivot point $a$ can be found by solving Eqs. 2.6 and 2.7 to obtain [39]

$$l = \frac{I}{mh}. \quad (2.8)$$

Equation 2.4 eliminates $I$ and gives

$$l = \frac{T^2g}{4\pi^2} = COP - a \quad (2.9)$$

in meters, which is then multiplied by 100 to give a value in centimeters. Because the $COP = l + a$, the only measurement required to determine the center of percussion is the period of the object’s oscillation and knowledge of the pivot point location $a$. 

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2.4 Measuring COP and I

2.4.1 Pivot apparatus

In keeping with previous work by Fahey, et al., the pivot point for 35-inch (88.9 cm) hurleys was set at 18 cm [6]. A 18 cm pivot aligns with the center of this author’s lower hand in a standard grip.

The apparatus used to swing the hurleys as a physical pendulum is made of two main parts: supports and a pivot. The pivot, shown in Fig. 2.5, clamps around the handle of the hurley and the triangular bars that extend to either side rest on the supports. In this way, the hurley was free to oscillate from the chosen pivot point through a photogate below. The photogate is visible as a grey apparatus in Fig. 2.5b. The photogate was connected to a CPO Science Timer IIe, which produced period measurements accurate to ±0.0001 seconds. The moment of inertia of the pivot was determined using a uniform aluminum calibration rod. All subsequent calculations account for the moment of inertia of the pivot.

2.4.2 Pivot calibration

The uniform aluminum rod used to calibrate the pivot apparatus was 0.91 meters long, with a diameter of ~ 13 mm and mass of 0.8893 kilograms. The calibration pivot location was chosen to be the same as used for 35-inch hurleys, 18 cm. The parallel axis theorem is used to calculate the theoretical moment of inertia of the rod pivoting about the chosen point. The moment of inertia about the center of mass, $I_{cm}$ of a uniform rod of mass $m_r$ and length $L_r$ is known to be

$$I_{cm} = \frac{m_r L_r^2}{12}. \quad (2.10)$$

The moment of inertia of the rod at a different pivot, our $a$, is found by relating the new pivot to the center of mass, which for a uniform rod lies at the point $\frac{L_r}{2}$. The distance
Figure 2.5. The pivot apparatus used to measure the period of each hurley. (a) shows how the hurley was held in the apparatus. (b) shows how the pivot balanced on the bars and swings the hurley through the photogate.

from the center of mass to the pivot, \(a\), is then \(d = \frac{L_r}{2} - a\). The parallel axis theorem states that

\[
I_a = I_{cm} + m_r d^2, \tag{2.11}
\]

which in this case means

\[
I_a = \frac{m_r L_r^2}{12} + m_r \left( \frac{L_r}{2} - a \right)^2. \tag{2.12}
\]

For our 0.91 meter, 0.8893 kg rod pivoted about point \(a\) at 18 cm, Eq. 2.12 gives a theoretical value \(I_a = 0.1297 \text{ kg m}^2\).

Obtaining the moment of inertia of the pivot apparatus requires measurement of the moment of inertia of the apparatus and the calibration rod together. The moment of inertia is calculated from measurement of the period \(T_{rod} = 1.4590 \text{ s}\) and center of mass using Eq. 2.4. The theoretical moment of inertia of the rod is then subtracted off of the measured value, leaving a measure of the moment of inertia of the pivot apparatus, which here is \(9.2411 \times 10^{-4} \text{ kg m}^2 (0.0092 \text{ g cm}^2)\).
2.4.3 Period and Balance Point Measurements

Each hurley was placed in the pivot apparatus at the chosen pivot point 18 cm from the end of the handle. It was displaced a small angle from vertical and then allowed to swing freely. After a few initial oscillations, the value of the period was recorded from the CPO Science Timer IIe, which was accurate to ±0.0001 seconds. Waiting until several oscillations had occurred reduced the chances of recording a measurement influenced by transient effects of pulling the hurley back by hand.

The balance point was measured using two scales and an apparatus developed to measure bats for ASTM standards, shown in Fig. 2.6. The two supports on the apparatus make it simple to obtain simultaneous measurements of the mass of the hurley at 15.24 and 60.96 cm (6 and 24 in). The AND ED-1200i compact balances (scales) measure mass to ±0.1g. The balance point was determined by a simple weighted average,

\[ BP = \frac{15.24 m_6 + 60.96 m_{24}}{m_6 + m_{24}}, \]

(2.13)

which is given in the ASTM standard [15].

Once these two measurements were obtained, the center of percussion and moment of inertia of the hurley were calculated using Eqs. 2.9 and 2.4.
2.4.4 Error

As with any measurements, uncertainty introduced at any point propagates through calculations and adds error to the final numbers. An error propagation analysis is presented using the calibration rod measurements to give an idea of the uncertainty in reported values for the hurleys. The following analysis follows the partial derivative method to obtain the uncertainty in a calculation \( \delta q \) [49]. The uncertainty \( \delta q \) of a calculated quantity is given by

\[
\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + \left(\frac{\partial q}{\partial y}\delta y\right)^2 + \left(\frac{\partial q}{\partial z}\delta z\right)^2},
\]

(2.14)

where \( x, y \) and \( z \) are measured variables [49]. The balance point equation only involves mass measurements (which measured to \( \pm 0.1 \) g), so \( \delta m_6 = \delta m_{24} = 0.1 \). Using Eq. 2.14 applied to Eq. 2.13, we get

\[
\delta BP_{rod} = \sqrt{\left(-45.72 \frac{m_{24}}{(m_6 + m_{24})^2}\delta m_6\right)^2 + \left(45.72 \frac{m_6}{(m_6 + m_{24})^2}\delta m_{24}\right)^2} = 3.862 \times 10^{-5} \text{ cm},
\]

(2.15)

which is smaller than we can even measure without using electronic means because the scales are so accurate. This error would grow if we accounted for the small deviation (< 1 mm) of the supports from their theoretical location values. For period measurements, the error in the timer was \( \delta T = \pm 0.0001 \) s, and the uncertainty in precise pivot location is \( \delta a = 0.001 \) m. Equation 2.9 gives an error in the center of percussion of

\[
\delta COP_{rod} = \sqrt{\left(\frac{Tg}{2\pi^2}\delta T\right)^2 + (1 \delta a)^2} = 0.1235 \text{ cm}.
\]

(2.16)

It is easiest to use Eq. 2.5 in analysis of the moment of inertia error, because \( \delta BP \) and \( \delta COP \) have both already been obtained, and \( \delta = \pm 0.1 \) g comes from the uncertainty in
the scales. The uncertainty in moment of inertia measurement becomes

\[
\delta I_{rod} = \sqrt{\left(\frac{Tmg(BP-a)}{2\pi^2}\delta T\right)^2 + \left(\frac{T^2g(BP-a)}{4\pi^2}\delta m\right)^2 + \left(\frac{T^2mg}{4\pi^2}\delta BP\right)^2 + \left(\frac{T^2mg}{4\pi^2}\delta a\right)^2} = 0.005 \text{ g cm}^2.
\]  

(2.17)

Thus for hurleys we can expect error in measurement to be a little over a millimeter for the center of percussion and likely for the balance point too. Because the moment of inertia of hurleys are a little above 1 g cm\(^2\) the error in this calculation ends up out in insignificant figures.

The moment of inertia of the pivot was measured to be 0.0092 g cm\(^2\), which is on the same order of magnitude as the error in the measurement. The moment of inertia of the pivot is accounted for in the reported values of moment of inertia, but barely makes a difference in the final value.

### 2.5 Discussion

Many of these calculations are made to quite fine resolution, but it is important to keep in mind that an impact with the sliotar is not a point impulse. The sliotar deforms on impact with the bás and can cover an area with a diameter of several centimeters [38,46]. Part of the sliotar might touch the center of percussion even when the strike is not centered on the center of percussion.

It should be noted that the balance point and center of percussion were both calculated as if the hurleys were radially symmetric like baseball bats. Hurleys are both flat and asymmetric on the flat part (bás). Depending on the shape and mass distribution of each individual bás, the actual center of mass may stray from the center line of the handle and may even appear off of the physical hurley. Because the center of percussion lies on a line that extends from the pivot and through the center of mass, the center of percussion would shift from the center line of the hurley in the same direction. This shift is unlikely
In-plane (▽) and out-of-plane (△) measurements of period $T$ gave an average difference in MOI of only $\sim 1.2\%$.

to be large enough to make a difference.

As a physical pendulum, the two most logical swinging directions for the hurley are in the flat plane of the bás and perpendicular to that plane. In gameplay the hurley is much more likely to be swung perpendicular to the plane of the bás (out of plane). Swinging the hurley consistently out of plane was difficult to do in the lab, perhaps due to air resistance. Figure 2.7 shows the difference between calculated moment of inertia using period measurements in- and out-of-plane. Out-of-plane measurements of $T$ increased the moment of inertia by an average of $\sim 1.2\%$. The in-plane period measurements were more consistent and repeatable, so they were chosen for use in further calculations. The summary table 2.1 uses only the in-plane period measurement.

Some of the hurleys were received with cushioned tape grips on the handles. Measurement of the COP and moment of inertia were made both with the tape and after the tape was removed to facilitate modal analysis. The tapes were weighed after removal and were found to have the following masses: hurley 5 - 25.5 g, hurley 6 - 20.7 g, hurley 7 - 38.6 g, and hurley 10 - 9.2 g. The main effect of removal of the tape was that the balance point moved further from the handle of the hurley. The differences were not negligible, but small. Because only about half of the hurleys were measured with and without tape,
Figure 2.8. Values with tape (○) and without tape (●) for (a) Center of Percussion, (b) Balance Point, (c) Moment of Inertia for hurleys 5, 6, 7, 10.

Table 2.1 shows only tapeless values. Figure 2.8 shows the minimal difference in values for taped versus untaped hurleys. Note that hurley number 7 had tape both at the handle and near the balance point.

2.6 Results

The values measured and calculated in this chapter are summarized in Table 2.1. It is useful to visualize the calculated values in terms of the mass of each hurley. Figures 2.9, 2.10, and 2.11 show the balance point, moment of inertia, and the center of percussion measurements of all of the hurleys, organized by each hurley’s mass. Since all of the hurleys are built so differently, there are not many identifiable trends between hurleys. Hurleys with balance points and centers of percussion further from the handle also have a higher moment of inertia, but this comes from Eq. 2.4 on both of these values.

2.6.1 Ash hurleys (1 & 2, 3, 4)

Ash hurleys are still predominantly handmade with emphasis on using the natural curve of the ash grain to inform the exact outline of the final hurley [33]. Because ash is a natural material, this means that every ash hurley is unique, even when they are...
Table 2.1. Summary table of the values measured and calculated in Chapter 2

<table>
<thead>
<tr>
<th>Hurley</th>
<th>Material</th>
<th>Mass (grams) ±0.1 g</th>
<th>Balance Point (cm) ±1 mm</th>
<th>Center of Percussion ±1.2 mm</th>
<th>Moment of Inertia (g cm²) ±0.005 g cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ash</td>
<td>572.2</td>
<td>58.8</td>
<td>77.2</td>
<td>1.384</td>
</tr>
<tr>
<td>2</td>
<td>Ash</td>
<td>573.7</td>
<td>60.9</td>
<td>77.3</td>
<td>1.465</td>
</tr>
<tr>
<td>3</td>
<td>Ash</td>
<td>447.6</td>
<td>59.5</td>
<td>77.0</td>
<td>1.097</td>
</tr>
<tr>
<td>4</td>
<td>Ash</td>
<td>510.2</td>
<td>55.4</td>
<td>76.5</td>
<td>1.118</td>
</tr>
<tr>
<td>5</td>
<td>Plastic</td>
<td>620.7</td>
<td>57.5</td>
<td>77.2</td>
<td>1.454</td>
</tr>
<tr>
<td>6</td>
<td>Composite</td>
<td>585.3</td>
<td>55.6</td>
<td>73.0</td>
<td>1.216</td>
</tr>
<tr>
<td>7</td>
<td>Composite</td>
<td>521.3</td>
<td>60.0</td>
<td>76.1</td>
<td>1.273</td>
</tr>
<tr>
<td>8</td>
<td>Composite</td>
<td>480.1</td>
<td>61.0</td>
<td>74.9</td>
<td>1.176</td>
</tr>
<tr>
<td>9</td>
<td>Composite</td>
<td>499.4</td>
<td>60.6</td>
<td>74.8</td>
<td>1.211</td>
</tr>
<tr>
<td>10</td>
<td>Composite</td>
<td>507.5</td>
<td>59.8</td>
<td>75.6</td>
<td>1.224</td>
</tr>
<tr>
<td>11</td>
<td>Composite</td>
<td>497.8</td>
<td>60.5</td>
<td>75.8</td>
<td>1.226</td>
</tr>
<tr>
<td>12</td>
<td>Plastic</td>
<td>606.1</td>
<td>50.8</td>
<td>71.1</td>
<td>1.059</td>
</tr>
</tbody>
</table>

supposed to be nearly identical. Hurleys 1 and 2 were sold as the same size and model, and would be expected to have consistent characteristics. While they are approximately the same mass, hurleys 1 and 2 have different balance points (see Fig. 2.9) because, while hurley 1 has a thicker base, it is slightly narrower going from the handle into the toe.

Figure 2.9. The distance from the end of the handle to the balance point of each hurley, organized by mass. (△) Wood; (☆) Composite; (★) Plastic.
The wood density might also vary along each hurley, though this is difficult to measure. The difference in mass distribution also affects the moment of inertia so hurleys 1 and 2 would also feel different to swing (see Fig. 2.10).

Ash hurley 3 has much less mass than hurleys 1 and 2, but it has a similar center of percussion and balance point. The difference in mass makes hurley 3’s moment of inertia much lower than that of hurley 1 and 2 because of Eq. 2.1’s dependence on mass. Hurley four’s balance point is lower than the other ash hurleys, but it has more mass than hurley 3, which causes the two to have similar moment of inertia. The center of percussion of all four of the ash hurleys falls within a one centimeter range. If the center of percussion is indeed relevant to hurling, a player would not have to change much in terms of positioning their swing to make the sliotar impact at the center of percussion.

2.6.2 Plastic solid bás hurleys (5, 6, 12)

Hurley 5, hurley 6, and hurley 12 are the three heaviest hurleys, but have stark differences in characteristics. Their mass distributions are very different, evident in Fig. 2.9. Hurley 12 (the outlier) has a completely different outline than all of the other hurleys. The

![Figure 2.10.](image)

**Figure 2.10.** The moment of inertia of each hurley, organized by mass. (△) Wood; (☆) Composite; (●) Plastic.
Figure 2.11. The distance from the end of the handle to the center of percussion of each hurley, organized by mass. (△) Wood; (☆) Composite; (★) Plastic.

stick widens constantly from the handle, so the balance point is further from the báis and it is shorter than all of the other hurleys by approximately 5 cm. Hurley 5 has a more standard 35-inch hurley outline, and its balance point is similar to that of the other hurleys. This also affects their moments of inertia (1.059 g cm\(^2\) for hurley 12 and 1.454 g cm\(^2\) for hurley 5). The contrast in moment of inertia between these two hurleys make it so that swinging each hurley would feel noticeably different to a player. While hurley 5 is heavier than any ash hurley measured, swinging the hurley is likely to feel more familiar to a player accustomed to ash. Hurley 12 might feel too “easy” to swing.

Hurley 6 is classified as a 35-inch hurley, but is actually approximately 2 centimeters shorter than the other hurleys. This difference, combined with its mass distribution, brings down all three calculated values.

### 2.6.3 Composite hollow báis hurleys (7, 8 & 9, 10 & 11)

Hurleys 8, 9, 10 and 11 all come from the same manufacturer; hurleys 8 and 9 are the same model, and hurleys 10 and 11 are a pair of a different model. All four of these hurleys have exactly the same outline and are hollow, but hurleys 8 & 9 are prototypes
with material added in the handle meant to improve the feel of the hurley [34,50]. The difference between the two model pairs is readily distinguishable only in the center of percussion (Fig. 2.11), where 8 and 9 have a center of percussion closer to the handle. It is interesting to note the variation in mass between hurleys of the same model, perhaps due to allowed margins in manufacture.

Hurley 7 actually shares the same outline with hurleys 8, 9, 10 and 11, although it is made by a different manufacturer. While its balance point is similar to the group of four, hurley 7 has more mass and therefore a higher moment of inertia.

### 2.7 Summary

Moment of inertia and center of percussion are both related to the mass and mass distribution of the hurley. Ash hurleys differ greatly in their moment of inertia, but their centers of percussion relative to a 18 cm pivot point are quite consistent. The moment of inertia of composite hurleys fall within the wide range defined by ash hurleys, which could lead to no definitive swing “feel” for ash hurleys. The difference in location of the center of percussion (relative to a 18 cm pivot) for plastic and composite hurleys might lead a player to change their swing in order to attempt to hit on the center of percussion, but this would have the consequence of changing the pivot point of a standard swing and therefore the location of the center of percussion that they are changing their swing to hit! Whether or not the idea of the center of percussion is relevant to hurley swings is dependent on further research to investigate the actual pivot point of the different types of swing.
Chapter 3  Modal analysis

3.1 Introduction

In acoustics and vibration, we are particularly interested in the ways that systems vibrate in response to inputs. In the case of hurleys, this is the vibration response of the hurley after it impacts the sliotar. The modes of vibration intrinsic to the hurley are excited during an impact, but it is difficult to measure this in play conditions. Information about these modes is instead accessed by conducting modal analysis. The frequency responses of many points on the hurley are taken to be the ratio of the response of an accelerometer to the input force from a modal hammer. These frequency responses are curve-fitted using STAR Modal software [51], and mode shape data is extracted. These mode shapes and the frequencies at which they occur can then be analyzed and compared.

3.2 Setup

Roving hammer excitation was chosen as the most efficient method of excitation for this application. Using the hammer eliminates possible effects of mass loading that can arise when a shaker is used. A shaker would also require longer measurements of noise or sine sweeps whereas the hammer excites all of the frequencies in its range simultaneously. With a roving hammer and fixed accelerometer, the experimentalist can
move from excitation point to excitation point without losing time to reset. The force hammer excitation and accelerometer response signals are collected using a Stanford Research Systems SR785 2 Channel Dynamic Signal Analyzer. Further data processing is conducted on a computer using STAR Modal to extract modal parameters.

The hurley is assumed to exhibit linear behaviour for the range of input forces used for modal analysis. This means that the amplitude of the accelerometer response should scale with input force, which in turn means that the frequency response function, which is fundamentally a ratio, should not change based on the input force. The hammer used to excite the hurleys is a PCB model 086E80 miniature instrumented impulse hammer, which weighs less than 5 grams but is still sufficient to excite the structure [52]. The hammer is shown next to a larger impulse hammer in Fig. 3.1. The small hammer was chosen to minimize instances of double tap impulses and to combat researcher fatigue.

The PCB 086E80 hammer delivers an impulse with a very flat frequency spectrum with less than 10 dB rolloff up to about 3000 Hz when using the metal tip. This mild rolloff is visible in the impulse spectrum in Fig. 3.2. Frequencies above this range are not relevant to this investigation; therefore the rolloff at higher frequencies is not an issue.

The accelerometer is a PCB Model 352C22 ceramic shear accelerometer. It is very small and lightweight compared to the hurleys, ~5 grams, with a published frequency range of 1-10,000 Hz [53]. This accelerometer is unlikely to mass load the hurley and is functional in the frequency range that is of interest. Because this investigation is primarily concerned with comparing mode shapes and node locations, not exact amplitudes, calibration was not necessary.

Based on the size of the hurleys and the expected shapes of the first several modes, a one-inch grid was deemed to provide sufficient spatial resolution to capture modal information up to and beyond 1000 Hz on hurleys. Because each hurley has a different shape, a new grid was created for each hurley, with measurement points positioned at intersections of the grid and around the perimeter. As previously shown in Fig. 1.8, most
Figure 3.1. The PCB Model 086E80 (top) and the PCB Model 086C01 (bottom) impulse hammers. The smaller hammer is used for modal analysis and the larger hammer is used for boundary condition determination and damping measurements.

Figure 3.2. The input spectrum for the small modal hammer on 4 different hurley surfaces. The spectrum is attenuated by less than 10 dB for frequencies under 1000 Hz.

Figure 3.3 shows the STAR Modal grid and analysis points for hurley 3. Having two rows of points going down the handle of the hurley facilitates viewing of modes that include torsion on the handle, resolution that was not achieved in previous modal analysis of hurleys, which only used one line of measurement points down the handle [46].

On every hurley, the accelerometer was positioned at point 1, the toe side of the end of the handle. This location was chosen because it is unlikely to be a node for the lower order modes on which this investigation focused. The accelerometer was placed
Figure 3.3. Measurement points with grid for hurley 3. All hurleys were numbered in the same pattern, but the grid locations differed slightly based on the shape of the hurley. Point 1 (○) is nominally the same for all hurleys.

on the underside of the hurley to allow measurement of the frequency response at the accelerometer location. The direction of the accelerometer response was accounted for in the STAR Modal post processing. The edges of hurleys are often rounded, especially on the handle. This made impacts at the outline points difficult. Care was taken to ensure that impacts were exactly vertical.

### 3.3 Frequency Range Determination

There are a number of factors to consider in determination of the frequency range. One must keep in mind human sensitivity, interaction of sliotar and hurley, and damping of higher modes. When holding a vibrating object in the hand, humans do not perceive every frequency equally. Reynolds, et al. identified threshold curves of the amplitude necessary for humans to perceive the motion of a handle [16]. Subjects in this study held a vibrating handle and indicated whether or not they could sense vibration at various frequencies and amplitudes [16]. This established a curve for the threshold of minimum amplitude vibration required of a handheld object in order to be sensed by a human. From this curve, a frequency range at which humans are most sensitive can be extrapolated, as shown in Fig. 3.4. The extreme sensitivity range is ∼150 Hz to ∼400 Hz. It is reasonable to assume that humans can feel vibrations above the 1000 Hz cutoff in Reynolds, et al.’s collected data, but the next question to ask is whether a hurley would end up vibrating with a large enough amplitude for perception at those higher
frequencies. Chapter 5 shows that damping for higher order modes is much higher than
damping for the fundamental and first few modes. A study on tennis rackets measured
up to 1500 Hz based on the concentration of energy under 1500 Hz [54].

The final question in the range determination is the span of frequencies actually
excited by impact with a sliotar. A study on cricket bats chose its frequency range based
the idea that the length of a typical ball/bat impact corresponds to the period of the
highest frequency excited. A 0.8 ms cricket ball/bat impact led the authors to choose
1200 Hz as the maximum frequency of interest [42]. It might make more physical sense to
theorize the impact duration as the half-period or three-quarter-period of the oscillation.
This depends on where in the oscillation of the hurley the sliotar leaves the surface, which
is beyond the scope of this investigation. Regardless, applying this logic to hurley/sliotar
impacts gives an approximate range of excitation to expect. A typical hurley/sliotar
impact duration is about 2.6 ms, but impacts can last longer than 3 ms [55]. If the
duration of the typical impact is taken as half of the period $T$ of the highest frequency
oscillation, then

$$f_{max} = \frac{1}{T} = \frac{1}{1.3 \times 10^{-3}} \approx 1080 \text{ Hz.}$$

(3.1)

If the impact time is instead taken as a whole period of the upper frequency, the period
equation gives

$$f_{max} = \frac{1}{T} = \frac{1}{2.6 \times 10^{-3}} \approx 540 \text{ Hz,}$$

(3.2)

which is quite low. It is reasonable to assume that an impact by a sliotar would excite
frequencies at least to 540 Hz, and possibly up to 1080 Hz. This potential sliotar
excitation range fully encompasses the sensitive range of human perception as defined by
Reynolds. This justifies omitting analysis of modes that greatly exceed 1000 Hz.

Based on the input spectrum in Fig. 3.2, it is certain that the hammer is exciting
all of the modes that a sliotar would excite and the frequencies of vibration at which
humans are sensitive.

### 3.4 Uniform Beams

Before delving into boundary conditions and mode shapes, it is useful to first look at
the uniform beam in free and clamped conditions. The frequencies of bending modes of
mode number $m$ for a free-free beam approximately follow the pattern

$$f_{b(\text{free})} = A\sqrt{\frac{Y}{\rho}} [3.011^2, s^2, (2m + 1)^2],$$

(3.3)
where $Y$ is the elastic modulus and $\rho$ is density \([56]\). The quantity $A$ is a scalar quantity that contains information about the geometry of the system including the radius of gyration $\kappa$. Torsional modes of mode number $m$ follow approximately

$$f_{t(\text{free})} = B \sqrt{\frac{GK}{\rho}} m,$$  \hspace{1cm} (3.4)

where $GK$ is the effective torsional stiffness factor and $B$ is a scalar that contains information about the geometry of the system [56]. From these equations, it is clear that $f_B \propto (2m + 1)^2$ and $f_T \propto m$. These general patterns for flexural bending and torsional modes are useful when it comes time to compare different beams, or even hurleys. Figure 3.5 shows a contrived plot for a free-free ash beam with torsional and bending modes. Notice that the bending modes follow a parabola and the torsional modes follow a straight line because of each kind of vibration’s dependence on mode order.

**Figure 3.5.** Ash beam (.02x.03x.889 m) using ash properties from Bucur’s *Acoustics of Wood* [17] and Eqs. 3.3 and 3.4. A curve fit for the bending modes gives $f_m = 120.25 m^{1.612}$.
Because hurleys are essentially beams with an uneven distribution of mass, it stands to reason that patterns like those that appear for beams will show up. Beams with one clamped end and one free end have bending modes that show up at much lower frequencies. Assuming the same beam properties as in Eq. 3.3, the bending modes for the beam clamped at one end are given by approximately

\[ f_{b(\text{clamped-free})} = A \sqrt{\frac{Y}{\rho}} [1.194^2, 2.988^2, (2n - 1)^2], \]  

(3.5)

where \( A \) is the same coefficient used in Eq. 3.3. This allows direct calculation of the proportionality of the frequency at which the first bending mode appears for free-free versus clamped-free. The ratio of free-free first bending to clamped-free first bending is \( \frac{3.011^2}{1.194^2} = 6.359 \). This ratio will prove useful for determining boundary conditions.

### 3.5 Boundary Conditions

For baseball bats, Brody, et al. found that a free-free boundary condition was considered sufficient to use because it more closely resembled a bat held in the hands than a clamped condition did [57]. They determined this by examining oscilloscope traces of bats in the three conditions. For tennis rackets, Banwell, et al. found that holding the racket with the hands resembled a free boundary, albeit with added damping and shift in frequency [54]. Brooks, et al. measured the mode shapes and frequencies of the first four bending modes of a cricket bat in the free, handheld, and clamped condition [42]. They also found the handheld condition to be closest to a free boundary with added damping.

As shown in section 3.4, the first bending mode of a beam clamped at one end is quite low, more than 6 times less than the first bending mode of a beam that is free at all boundaries. It is simple to measure the frequency response function of a hurley in both a clamped-free and free-free condition and then compare to the behavior of a hurley held in hands. The accelerometer was placed on the bás of ash hurley 1, and the
hurley was struck on the bás with the mid-size modal hammer that is used to measure damping in Chapter 5, pictured in Fig. 3.1 with the tiny hammer. For the free boundary condition, the hurley was suspended by the handle from rubber bands, the configuration also used for damping measurements in Chapter 5. The clamped boundary condition was achieved by securing it tightly to an optical table using the hardware shown in Fig. 3.6.

It was found that gripping the hurley with two hands, as in the first tile of Fig. 3.7, adds so much damping that modes are near-unidentifiable. The black trace in Fig. 3.8 has a small hump around 50 Hz, but it is difficult to pull out definite modal information for comparison. This suggests that gripping tightly both damps vibration and mass loads the hurley enough to shift the bending frequencies. Holding the hurley flat with one hand (center photo of Fig. 3.7) shows a peak at 61 Hz, but it proved difficult to obtain repeatable results in that position. To keep the hurley held flat, it needed to be
Figure 3.7. The 3 different positions for handheld hurley measurement. Two-handed provided too much damping for helpful analysis. Flat one-handed hold had strange arm/elbow bouncing mode from compensating for the impact. Vertical one-handed did not have the arm/elbow mode and did not require as tight a grip as the other two configurations.

gripped tightly at the handle because the mass is concentrated at the other end. There was also a small peak at 12.5 Hz which was determined to be rigid body motion due to compensation from the arm at the elbow after impact. Stabilizing the elbow against a pole while still gripping with one hand (right photo of Fig. 3.7) eliminated the reaction from the arm and resulted in a higher modal peak (less damping) because the heavy hurley did not need to be gripped as tightly upright.

Figure 3.8 shows that the clamped handle boundary exhibits a clear modal peak at 8.5 Hz. This corresponds to the first bending mode (diving board mode). The second peak around 50 Hz was confirmed to be a torsional mode that was excited to a large amplitude because of accelerometer placement on one side of center line of the hurley and the impact occurring on the other. Neither the free boundary trace nor the one-hand hang trace exhibit the very low bending mode present in the clamped boundary condition. In fact, the frequency of the first bending mode of the free hurley is 62 Hz and the first modal peak that appears in the one-hand hang frequency response function is at 62.5 Hz. The ratio between the first bending modes of the free-free condition and the clamped-free
condition is $\frac{62}{8.5} = 7.294$. This is not exactly the ratio that is obtained between a beam that is free or clamped, but it does demonstrate that the hurley is behaving nominally like a beam, whether clamped or free. Both the near perfect frequency match between the handheld and free condition and the absence of a very low diving board mode in the handheld condition confirm that a free condition is more similar to the handheld condition. Thus the modal analysis experiments in this thesis were conducted using a free condition.

### 3.5.1 Rubber bands

Dangling the hurley from rubber bands as in Fig. 3.6a is not a very secure or consistent environment in which to conduct modal analysis. In order to secure the hurleys but not
Rubber band scheme for creating a free boundary for the hurleys. The setup holds the hurleys still enough for modal analysis without adding damping to the system. The natural frequency for rigid body motion of a hurley on the rubber bands ranged from \( \sim 20 \) to \( 30 \) Hz.

introduce too much damping, the hurleys were suspended above the optical table using multiple rubber bands at various points. The location of the rubber bands in the bás varied slightly as the shape of the bás of the various hurleys also changed. The general configuration of the modal analysis setup is shown in Fig. 3.9. The rubber bands provide a boundary condition that is sufficiently free at all locations on the hurley. The frequency of rigid body oscillation of the hurleys in the rubber bands was clearly identifiable in the frequency response functions and ranged between \( 20 \) and \( 30 \) Hz. This frequency range is lower than the first bending mode of all of the hurleys except for hurley 12. Consequently, the first bending mode of hurley 12 is best represented as a video because the modal fit combines the bending mode with rocking on the rubber bands.

### 3.6 Frequency Response Function Formation

The frequency response function \( H_{ij} \) created for every impulse point, \( j \) on a hurley is

\[
H_{ij}(\omega) = \frac{Y_i(\omega)}{X_j(\omega)},
\]

the frequency domain ratio of the response of the accelerometer at point \( i \), \( Y_i(\omega) \) to the impact force from the hammer at point \( j \), \( X_j(\omega) \). In this case \( i \) is fixed at \( i = 1 \) because there is only one accelerometer, positioned at point 1.
To transform the input time signals from excitation $j$ and response $i$ into the frequency domain, the SR785 uses a Fast Fourier Transform (FFT) [58]. By definition, the FFT requires periodicity so both time signals are windowed in order to fulfill this requirement. The force/exponential filter pair is commonly used during impact excitation testing to do so [59]. The force window keeps only the part of the impact signal that contains the impulse, and the exponential window decays the time signal to ensure that the response signal is sufficiently reduced by the end of the time record to prevent leakage. The exponential window also adds damping so damping is addressed in a separate measurement in Chapter 5. Using the force/exponential filter pair along with averaging drives down the signal-to-noise ratio in the final frequency response functions. The signal-to-noise ratio is driven down further by taking the average of three separate impacts for each measurement point.

### 3.6.1 Coherence

Some of the points on the hurleys were very sensitive to small changes in impact location. This means that the frequency response function could change drastically if the impact location varies. The coherence function is used as a check to ensure that impacts are consistent in location, direction and force. The coherence function falls between 0 and 1 at every frequency, and is “a measure of how linearly related the response of the structure is to the excitation force” [60]. The coherence for each frequency is given by

$$\gamma^2(f) = \frac{|G_{XY}(f)|^2}{G_{XX}(f)G_{YY}(f)}, \quad (3.7)$$

where $G_{xx}$ and $G_{yy}$ are the averaged power spectra of the response and excitation signals and $G_{xy}$ is the averaged cross power spectrum [60].

If, for example, the researcher were to impact the structure once exactly on the marked point, and then again a centimeter away, the change in the averaged frequency
response function might be difficult to notice, but the coherence function would show that there is some difference in the two measurements taken. The further the coherence is from 1, the more likely the impacts were in different positions. Nonlinearities due to variations in hammer impact force are unlikely due to the very small mass of the PCB 086E80.

### 3.7 Processing & Analysis of Test Result

Once the frequency response functions of all the points on a hurley were collected, the files were transferred from the SR785, converted into a universal file format, and processed with STAR Modal. STAR Modal allows for easy identification of modes and quasi-automatic curve fitting. In a typical frequency response function (see Fig. 3.10), modes are identifiable by visual inspection as peaks in the frequency response function magnitude. STAR Modal has an additional feature, called mode indicators to automatically find candidates for structural modes [61]. This obviates manually checking about 135 frequency response functions per hurley, although a quick perusal ensures nothing glaring is missed. Once modes are identified, the peak in the frequency response function for each selected mode is curve fitted as a single degree-of-freedom oscillator (also in Fig. 3.10).

The curve fit for the $k^{th}$ mode, identifies modal parameters and can be modeled mathematically by the function

$$H_{ij}^{(k)}(\omega) = \left| \frac{R_{ij}^{(k)}}{(j\omega - p_k)} - \frac{R_{ij}^*}{(j\omega - p_k^*)} \right|.$$

(3.8)

where $p_k = -\sigma_k + j\omega_d$ is the pole location of mode $k$ and $R_{ij}^{(k)}$ is the residue for mode $k$ [62]. The pole location for each mode contains the modal damping $\sigma$ and damped natural frequency $\omega_d$, for the single-degree-of-freedom oscillator model of the mode. The residue is "a mathematical concept that has no direct interpretation in physical terms," [63] but is the dot product of the mode shape component for the mode at the excitation point.
and the mode shape component at the response point [64]. The acceleration frequency response function output by the SR785 has both real and imaginary values at every frequency. In STAR Modal, “the FRF value at the maximum magnitude peak on the FRF data in the band is taken as the residue, and the frequency of the peak is taken as the modal frequency” [61]. For a selected mode, STAR Modal computes the modal parameters for each measurement point, and then connects the residue information to create the mode shape.

STAR Modal has the capacity to curve fit identified modes with five different curve fit methods. All hurleys were fit with either quadrature or coincident curve fitting, whichever gave the smoothest mode shapes. The difference between the two curve fits is whether the result is placed in the real or imaginary part of the residue. For a quadrature fit, the result is placed in the real part of the residue; for a coincident fit, the result is placed in the imaginary part [61]. Curve fit results from STAR Modal were then transferred into MATLAB to facilitate interpretation of mode shapes.

![Figure 3.10. Curve fitting on STAR Modal. The top graph (blue) shows mode indicators, the bottom graph (pink) is the frequency response function for one excitation point on the hurley. The vertical green lines are the bounds that STAR Modal will use to curve fit the frequency response functions. Source: Screenshot from STAR7](image)
Chapter 4
Modal analysis results

4.1 Introduction

This chapter contains discussion of results of the modal analyses of the hurleys. Mode shapes are quantitatively analyzed and modal frequencies are compared between hurleys. For most hurleys the first 8 observed bending and torsional modes fall within the frequency range of interest, below 1000 Hz. These are likely to both be excited by impact with a sliotar and sensed by a human holding the hurley. A selection of mode shapes are given here to illustrate trends and conclusions. The full set of mode shapes collected for each hurley are given in Appendix A. Mode shapes are described in inches, as that was the basis for the measurement grid.

4.2 Bending and Torsional Modes for Hurley 6

Hurley 6 is a solid bás hurley. It is presented as a representative hurley because the bending and torsional modes are clearly identifiable from visual inspection of the mode shape data. The modes were also clearly defined in the frequency response functions of the measurement points and STAR Modal’s “mode indicators” function. Looking at the mode animation videos on STAR Modal helps to classify modes as bending or torsional, although the actual locations of node lines are not always clear. Looking at
individual frames of these videos, as in Fig. 4.1, is even less helpful for torsional modes. Instead, the STAR Modal mode shapes are exported into MATLAB [65] and presented using colormaps. The colormaps are pink and blue at antinodes, with nodes marked by white lines. The first five bending modes and first four torsional modes of hurley 6 are presented in Fig. 4.2. The full set of modes for hurley 6 is in Appendix A.

The first bending mode is characterized by two node lines, one about 10 inches from the top of the handle, the other near where the bás joins the handle (∼28 inches from the top of the handle). These node lines do not cross the hurley perpendicularly, as they are both slanted. This first bending mode vibrates at 68 Hz, below the very sensitive region of human hand vibration sensation (Fig. 3.4). The second bending mode has three nodes, one near the top of the handle, one at the approximate midpoint of the hurley (∼18 inches), and one through the bás. The node in the bás is quite slanted and points to the toe of the hurley. This second bending mode occurs at 184 Hz, which is right at the bottom of Reynolds et al.’s vibration sensation curve (Fig. 3.4), the middle of the

\[ \text{Figure 4.1.} \]\]  

STAR Modal animation frames for (a) bending modes and (b) torsional modes of hurley 6. Nodes are more clearly identified in the bending modes than in the torsional modes.
Figure 4.2. The first five bending modes and four torsional modes of composite hurley 6.

sensitive region. The third bending mode occurs at 372 Hz, as humans are starting to become less sensitive to vibration. It has three nodes in the handle and one in the bás, all of which are slanted. The fourth bending mode finally sees an antinode placed in the bás, which for the first three bending modes had a node running through the middle. The fourth bending mode vibrates at 588 Hz, past the upper limit of the sensitive region, but still more easily felt than the fifth bending mode at 864 Hz. The fifth bending mode has four nodes in the handle and two in the bás. The bás node lines are starting to curve.
Curved node lines become more common with even higher order bending modes, which can be seen in the full mode shapes in Appendix A.

For symmetric objects, torsional modes are characterized by a node line down the center, about which the object twists. Because hurleys are asymmetric, the node lines of their torsional modes are not as distinct. The first four torsional modes of hurley 6 are also presented in colormap form in Fig. 4.2. The twisting quality of these modes was more easy to identify in the animations in STAR Modal, but once one knows what is going on, the torsion is visible in the format in Fig. 4.2. The first torsional mode is much weaker than any of the other modes presented here. The mode is only visible in the mode indicators function if it is presented in log magnitude format, and might be discounted as an anomaly if one were going through all the frequency response functions themselves. This has the side effect of a sub-optimal curve fit, but there is a clear node at approximately 9 inches, where the direction of twisting in the handle switches. 102 Hz is within the sensitive range of human hands, but the mode was excited at such a weak amplitude for modal analysis that the likelihood of the mode being excited when the hurley is held is quite low.

The second, third, and fourth torsional modes were much stronger than the first, and are clearly defined in the frequency response functions of hurley 6. The plus-sign shaped node intersection in the bás in the third bending mode indicates that even the bás twists during some torsional modes. The fourth torsional mode starts to pick up some bending properties in the bás. It could be coupled to a bending mode, facilitated by the asymmetry of the hurley. Figure 4.3 displays the frequencies of hurley 6’s bending and torsional modes. The bending modes were fit with a power curve fit to obtain

$$f_{B_m} = 65.473m^{1.584},$$

(4.1)

where \( m \) is the mode order. Remember that the mode number relation for free beam bending modes from Section 3.4 is \( f_m \propto (2m+1)^2 \), and the power curve fit is \( f_m \propto m^{1.612} \).
Figure 4.3. Curve fits for first five bending modes and first four torsional modes of hurley 6. (green) $f_{Tm} = 326.37m - 275.24$; (pink) $f_{Bm} = 65.473m^{1.584}$

so the power growth of bending mode frequencies for hurley 6 ($f_m \propto m^{1.584}$) is quite close to that of beams. Because beams are a useful comparison, the torsional modes were fit with a linear fit to give

$$f_{Tm} = 326.37m - 275.24. \quad (4.2)$$

The $R^2$ value of this linear fit is $R^2 = .9755$, whereas the power fit has a value of $R^2 = .9984$, which suggests that, while a power fit is a good indicator of bending mode frequency relation, a linear fit might not be the most appropriate for hurley torsion. The $R^2$ value also reflects scatter, so anomalies might affect it too.
4.3 Bending Mode Frequencies

A common way to analyze the bending modes of similar objects is to compare mode order frequency relationships. Section 3.4 shows that the frequencies of the torsional modes of a free-free beam are proportional to \( m \) and bending mode frequencies of the beam are proportional to \((2m + 1)^2\). Fitting a power curve to the bending mode frequencies of the beam gives a curve fit of \( m^{1.612} \). The bending mode curve fit for hurleys is similar, and it can be used to compare different hurleys. The first five bending modes for the hurleys were identified using visual inspection of their mode shape plots. Bending mode curves were fit to this data using Microsoft Excel’s built-in “power trendline” fit. This fits the hurley bending with an equation of the form

\[ f_m = C m^b, \quad (4.3) \]

where \( m \) is the order of the bending mode. Specific hurleys have been chosen for comparison in this section, but all bending mode curve fits are reported in Appendix A. These fits had an \( R^2 \) minimum value of \( R_{avg}^2 = 0.9922 \), with an average value of \( R_{avg}^2 = 0.9969 \). This indicates that the power fits are quite a good estimation of the pattern of bending mode frequencies.

Because they are made of natural material, and are often handmade, there is much variation between ash hurleys. Even the two ash hurleys sold as the same model (1 & 2) vary in mass distribution, even though their total mass and shape are the same. Figure 4.4 shows the frequencies at which the bending modes of the ash hurleys appeared and the curve fit equations for each hurley. It is interesting to note that hurley 2 is more similar to hurley 3 than it is to hurley 1, its supposed twin. Hurley 3 weighs approximately 60 g less than hurley 4, which contributes to its lower curve. Using the average of the exponents of the equations for the four curve fits for ash bending modes, a general pattern of \( f_m \propto m^{1.727} \) is determined. This is a slightly higher value of exponent.
Figure 4.4. Curve fits for the first 5 bending modes for the four ash hurleys. Hurley 1: $f_m = 61.325 m^{1.763}$; Hurley 2: $f_m = 56.636 m^{1.748}$; Hurley 3: $f_m = 57.914 m^{1.7203}$; Hurley 4: $f_m = 69.014 m^{1.706}$

than for a uniform beam (1.6127). Because the four curves in Fig. 4.4 are similar, it can be expected that the ash hurleys will feel similar when evaluated for vibration. Ash hurley 1 is selected for comparison to hurleys of other materials because its bending mode curve falls approximately in the middle of the other ash hurleys.

The frequencies at which bending modes appear for solid plastic hurleys are much lower than those of ash hurleys. Figure 4.5a compares the two plastic hurleys (5 & 12) with ash hurley 1. Both plastic hurley bending mode curves are much lower than ash hurley 1. The exponents of the two fit equations differ slightly from the ash fit: hurley 5’s exponent is higher, hurley 12’s exponent is lower. This suggests that the noticeable difference between curves comes from the value of $C$, which is dependent on the material properties and physical dimensions of the hurleys. The lower curves pack more bending modes into the frequency range where humans are most sensitive to vibration in the
Figure 4.5. (a) Curve fits for hurley 1, 5, and 12 (ash versus solid bás plastic).
Hurley 1: $f_m = 61.325 \, m^{1.763}$; Hurley 5: $f_m = 42.118 \, m^{1.773}$; Hurley 12: $f_m = 30.996 \, m^{1.688}$.
(b) Curve fits for hurley 1 and 10 (ash versus hollow bás composite).
Hurley 1: $f_m = 61.325 \, m^{1.763}$; Hurley 10: $f_m = 68.6 \, m^{1.655}$.

hands. This means that plastic hurleys can be expected to feel noticeably different than ash.

The bending frequency curves for composite hurleys more closely resembled the curves for ash hurleys. Figure 4.5b shows the curves of hurley 1 and hurley 10, which overlap by quite a lot. This means that, in terms of frequencies and feel, the composite hurley (10) should feel very much like an ash hurley.

4.4 A Closer Look at Torsional Modes

Torsional modes are often omitted in discussion of symmetric equipment like baseball bats and cricket bats. For example, Brooks, et al. ignore the first torsional mode because a typical cricket shot on the center of the bat would not excite it [42]. Torsional modes
are important to tennis racket behavior, as they show up at frequencies in the sensitive range [54].

Because of the asymmetrical shape of hurleys, torsional modes appear at frequencies that are likely to be felt by players. These torsional modes appear in both the bás and the handle. It is possible that torsional modes might feel different in the hands than bending modes, but perceptive testing is beyond the scope of this investigation. Torsional modes are often ignored in studies of baseball, softball and cricket bats because good hits don’t excite them. The round cross-section of a baseball bat might also reduce the effect of torsional modes versus the flatter handle of a hurley. The first torsional mode of hurleys, however, appeared as a very weak mode, possibly due to the location of the accelerometer in relation to the node lines. If it isn’t completely subsumed into a bending mode as it is for some hurleys, it still might be missed if one is not specifically looking for it. A first torsional mode was identified for hurleys 1, 2, 6, 7, 9, and 10, shown in Fig. 4.6. These six colormaps show that the first torsional mode is not a clean mode shape.

The second torsional mode is much easier to identify, as for most of the hurleys it is a strong mode which shows up at a frequency that is not too near those of other modes. When it is not detectable, the second torsional mode likely occurs at a frequency very close to that of the third bending mode and is therefore indistinguishable from it. On many of the hurleys that showed a second torsional mode, the nodes of the mode lined up with the nodes of the third bending mode (hurleys 1, 2, 6, 7, and 10). Figure 4.7 shows the alignment of node lines for hurley 6 and hurley 10. On hurleys that don’t display a distinct second torsional mode, it could be that both the frequency of the second torsional and third bending modes were close and their node lines aligned well enough to superpose the modes together into one bending/torsional mode. It is interesting to note that this second torsional mode showed up in a different order between bending mode frequencies for each hurley.
Figure 4.6. First torsional mode for hurleys (1,2,6,7,9,10). There is a node in the handle, and the hurley twists in opposite directions on either side of the node. It is also possible that there is a node on the bás, where the twisting again changes direction. Note that the shape fitting is messier than for stronger modes because of the relative weakness of the first torsional mode.

Figure 4.7. The third bending mode (top) and second torsional mode (bottom) for hurleys 6 and 10. For hurley 6, the node lines at approximately 20 inches and 30 inches line up quite well. For hurley 10, the node lines at approximately 14 inches nominally align. If either were to shift much, a coupled mode might arise.
4.5 Ash as an Orthotropic Material

Ash is natural material, and is therefore subject to organic phenomena that do not necessarily show up in materials like composites, which do not necessarily have an organized grain structure. Wood is an orthotropic material with “three mutually perpendicular mirror planes of symmetry ... related to the direction of growth” [17]. Ash can usually be considered as transversely orthotropic, with properties along the grain differing from those that cross it [6]. Ash hurleys 1, 2, and 3 have relatively evenly spaced grain, and can be described as such. Hurley 4 has a section of its grain that is very closely spaced, as if the tree had a colder or droughty decade, marked in Fig. 4.5. The dense section is basically the entire handle, and extends through the middle of the bás. This is significant because at certain frequencies (fourth bending mode) the dense section appears to behave completely out of phase with the rest of the hurley. This behavior is visible in both the STAR Modal animation and the colormap in Fig. 4.5. It is possible that at 744 Hz, the difference in behavior between the dense section and the other parts of the hurley is so significant that it causes local abnormalities. A repeat modal analysis of the bás of hurley 4 was much less resistant to STAR Modal curve fitting. This could mean that the problems of the first analysis are only due to measurement error, but it is worth noting that no errors of this sort happened for the non-wood hurleys.

Figure 4.8. The fourth bending mode of Ash hurley (4). The handle and points on the bás that line up with it appeared to be 180 degrees out of phase with the rest of the bás (top). A second modal analysis of the bás (bottom), was less resistant to STAR Modal curve fitting.
4.6 Trampoline Effect in Composite Hurleys

In some of the composite hurleys (7, 9, 10), some of the higher modes appeared to show evidence of a possible trampoline effect. In hollow baseball bats, the trampoline effect occurs when the bat is less stiff than the ball and supplies the energy to make the ball leave the bat, instead of dissipating energy into dissipation of the ball [18]. Figure 4.10 shows the difference between a non-trampoline impact and an impact that exhibits the trampoline effect. The presence of a trampoline effect depends on the restitutive properties of both the hurley and the sliotar [38]. This suggests that sliotars with different stiffness might experience more or less trampoline effect on the same hurley. In this application, the restitutive property of the hurley is in part determined by the frequency of its vibration.

As shown in Fig. 4.11, the perimeter of the bás has joined into a ring of the same phase, with the encircled area of the bás out of phase. It was previously conjectured that these might be breathing modes where the top and bottom of the hollow bás were oscillating out of phase with each other, similar to a hoop mode in a baseball bat [66].
To test this theory, the hurley was turned over and impulse data was collected for a few measurement points on the bás and into the middle of the hurley. Processing in STAR Modal showed that the top and bottom of the bás are moving in phase at the trampoline frequency, as shown in Fig. 4.12. The edges of the bás, which are not hollow, move with much less amplitude than the more compliant center. There was no apparent trampoline mode for any of the solid bás hurleys. The presence of trampoline modes in hollow or filled core composite hurleys opens up the possibility of being able to tune these to the duration of a sliotar impact to optimize performance [67]. In the hurleys in this study, the modes that might exhibit a trampoline effect show up between 900 and 1000 Hz, which corresponds to a period of $\sim 1 \text{ ms}$. A typical sliotar impact lasts about 2.6 ms [55], so the trampoline mode is likely to be damped out during impact and have a diminished effect.

**Figure 4.10.** Schematic of the trampoline effect for a hard wall and a flexible bar. Source: Modified from *Physics of Baseball and Softball* [18, 13.1]
Figure 4.11. The trampoline modes of hurleys 7, 9, and 10. These are composite hurleys that are either hollow or filled with materials that are less rigid than the edge of the hurley. The phase of hurley 10 was flipped for uniformity with the other two plots.

Figure 4.12. Confirming the phase relationship of the top and bottom of the bás of Hurley 9 by adding measurement points on the bottom of the bás to the modal analysis. The points on the bottom of the bás are in phase with the points on the top of the bás. This eliminates the possibility of a breathing mode at this frequency.
4.7 Identifying Sweet Spot Candidates

The “sweet spot” for baseball bats has been determined to be either the barrel node of either the first or second bending modes of the bat, or some combination of the two [44–46]. Students at University of Hartford identified a preliminary “sweet zone” for hurleys that included the two bás-end nodes of the first two bending modes and the center of percussion as they measured it [46]. Because the location of the center of percussion varies by pivot location (which depends on hand position and swing biomechanics), its relevance to a “universal” sweet spot that works for all hand positions is unlikely.

For all hurleys in this study (except 12), the frequency of the first bending mode is between 40 and 75 Hz. This is not in the extremely sensitive region of human sensation, but can be sensed at approximately an order of magnitude smaller displacement than the lowest frequencies in Reynolds, et al.’s sensation curve, Fig. 3.4. The frequencies of the second mode of vibration are generally at the extreme of the sensation curve, approximately 150 to 200 Hz. If the second mode is excited while a player is gripping an antinode of that mode, a player is likely to feel it, even at low displacement amplitude. The third bending mode occurs as the vibration sensitivity curve in Fig. 3.4 is climbing back out of the sensitive region, around 400 Hz. The contribution of each of these modes to vibration in the hands depends on how strongly they are excited by an impact, which in turn depends on whether the impact occurs at a node, antinode, or somewhere in between. It is possible to visualize candidates for the sweet spot defined by these nodes by combining the mode shapes of the first and second or second and third bending modes of each hurley.

Because only node lines are of interest in this analysis, the absolute value of each mode shape is taken and the values are normalized. Small numbers and zeroes stay small so that when both mode shapes are multiplied together point by point, the node lines of both modes are visible as white areas. When both mode shapes are added together, only
locations that are zero or small for both mode shapes show up as white areas. Areas that are vibrating at a high amplitude for both modes are darker colors.

Figure 4.13 shows the combined first and second bending modes of the hurleys in this study. Hurley 12 was omitted because the curve fit for the first bending mode was dependent on rigid body motion in the rubber bands. Multiplying the shapes together (purple) shows all of the areas that are a node of either the first or second mode. Adding the mode shapes together (green) only shows locations where both mode shapes are zero (have nodes). The nodes in the handle seem to line up with typical hand positions. The wide region of low excitation near the top of the handle covers a great portion of the region in which a player would hold the hurley if puckering or hitting a long ball. The node in the middle of the handle might inform where a player holds the hurley for a shorter pass or to block. The chart of added modes (green) shows where a sliotar impact would excite neither the first nor the second bending modes. Each hurley has a defined area where the first and second modes both have a node. Hurley 4 has one, it is just not as bright as the others which means that the modes do not overlap as precisely. The light spot on each hurley in Fig. 4.13b is a candidate for the sweet spot of that hurley.
Figure 4.13. Left chart identifies regions that are nodes for either the first or second bending mode. Right chart identifies regions that are only nodes for BOTH the first and second bending mode.
The frequency of the first bending mode is so low for all hurleys that it may be of diminished importance when it comes to vibration sensation in the hands. It may be more useful to look at the combined effects of the second and third bending modes, both of which fall into the approximate region of sensitivity in human hands. Figure 4.14 shows the combined node lines of the second and third bending modes of the hurleys in this study. The multiplied chart, Fig 4.14a, shows all node lines of both modes. The lines located in the handle do not appear to have as strong of a correspondence with typical hand positions. This could be an indicator that the sweet spot identified in Fig 4.14b is even more importance to ensure that the second and third modes are not excited in the handle. The size of the sweet spot region is larger and better defined for the combined second and third modes than it was for the combined first and second modes. This is because the bás nodes for the second and third modes are more closely aligned. Anecdotal corroboration of the sweet spot for hurleys can be found in Appendix B.
Figure 4.14. Left chart identifies regions that are nodes for either the second or third bending mode. Right chart identifies regions that are only nodes for BOTH the second and third bending mode.
4.8 The Sweet Spot and Center of Percussion

Now that potential sweet spots for hurleys have been identified based on their bending modes, they are compared to the location of the center of percussion calculated in Chapter 2. Figure 4.15 shows the center of percussion (relative to 7 inches) on top of several of the sweet spot diagrams from Fig. 4.13 and Fig. 4.14. The center of percussion relative to a 7-inch (18 cm) pivot overlaps the sweet spot better for the first and second bending modes than for the second and third. This would be significant if hurley swings were determined to have a pivot point at 7 inches (18 cm) from the top of the handle. It is possible that a center of percussion relative to other pivot points might also align with the sweet spot of the hurley as determined by either pair of bending modes. If a better measure of the actual pivot point and center of percussion for different swing styles were developed, a perceptual study on players could better establish if the center of percussion is a relevant quantity for hurleys.
Figure 4.15. Sweet spot indicator with center of percussion (7-inch pivot) marking for (a) first and second and (b) second and third bending modes. The center of percussion aligns better with the sweet spot as determined by the first and second modes.
5.1 Introduction

An impulse applied to any object excites vibrations in the object at specific modes of vibration. Which modes of vibration are excited depends on whether the object was impacted at a node or elsewhere. Impacts at or near a node of a mode lessen or eliminate the contribution of that mode to the total response of the object. If the impact is at an antinode, that mode contributes more strongly to the total response. Over time, these vibrations decrease in amplitude until the system is again at rest. The rate at which these oscillations decrease is determined by the damping rate of each mode. The same is true for hurleys. After striking the sliotar a hurley continues to vibrate due to the impact. These vibrations eventually abate, although they may cause the player discomfort if they are excited at high amplitudes and take a long time to die out.

Vibration in the handles of baseball bats and field hockey sticks has been connected to the feeling of “sting” [44,68]. For baseball bats, it has been determined that adding damping mechanisms tuned to damp vibration from the second bending mode reduces the feeling of “sting” [69]. It is not quite clear if this is solely due to the location of the hands or if the frequency of the mode has an effect. Russell emphasizes the relationship between hand position, node location, and “sting” sensation [68]. For most applications, baseball bats are gripped the same way every time; the variability in the way hurleys are
held might require different analysis.

Characterizing each hurley by the damping rates of each of its modes is a method that allows hurleys to be compared to each other. If future research determines that a particular mode contributes most to player discomfort, the trends identified in this section might be used to predict preference.

5.2 Measuring Damping

5.2.1 Damping rate

The total frequency response of a hurley can be modeled as a superposition of single degree of freedom systems, each being a damped harmonic oscillator with its own (damped) natural frequency and damping. These single degree of freedom systems correspond to the modes of the hurley. The $k^{th}$ mode/1-DOF model can be characterized by its impulse response function and its frequency response function, both of which can be used to describe the damping of the system. The damping rate for the $k^{th}$ mode $\sigma_k$ is a frequency independent quantity that allows comparison of damping across modes and hurleys. The damping rate $\sigma_k$ describes the shape of exponential decay in the time signal of the damped oscillation. An alternate parameter $\zeta_k$ is the damping ratio (the ratio of damping to critical damping), which is frequency dependent and not as useful in this application. Both $\sigma_k$ and $\zeta_k$ appear in this section because of the way that the SR785 measures damping, but keep in mind that the final goal is to obtain values of $\sigma_k$ for hurley modes.

The damping rate, $\sigma$ is most apparent in the time domain as the rate of exponential decay of the mode’s impulse response,

$$h_{ij}^{(k)}(t) = \left|R_{ij}^{(k)}\right| e^{-\sigma_k t} \sin(\omega_k t + \phi_{ij}^{(k)})$$  \hspace{1cm} (5.1)
Figure 5.1. Simple single degree of freedom system at 10 Hz with two different damping rates. (a) Impulse Response, (b) Frequency Response

where $\sigma_k$ is the rate of exponential decay of the sinusoid oscillating at $\omega_k$ [62]. The residue, $\left|R_{ij}^{(k)}\right|$ gives information on the strength of the mode. This time-domain model is useful for conceptualizing damping; Figure 5.1 shows impulse responses with two different values of $\sigma$. However, it is nearly impossible to isolate modes in a time signal so finding the damping is typically done in the frequency domain.

The same mode $k$ can be represented in the frequency domain in pole-residue form by

$$H_{ij}^{(k)}(\omega) = \frac{R_{ij}^{(k)}}{(j\omega - p_k)} - \frac{R_{ij}^{*(k)}}{(j\omega - p_k^*)}$$

(5.2)

where the pole $p_k = -\sigma_k + j\omega_k$ [62]. The pole of the curve fit directly gives a value for the damping rate of the curve. Unfortunately, obtaining values of $\sigma_k$ directly from a frequency response function on the SR785 Analyzer is not so straightforward.

The SR785 Dynamic Signal Analyzer fits each modal peak with Eq. 5.2 when it is commanded to calculate modal parameters [58]. It does not report the value of $\sigma_k$ that it just calculated. The analyzer instead displays the value of the damping ratio, $\zeta_k$, calculated from the pole values determined in the curve fit using

$$\zeta_k = \frac{\sigma_k}{\sqrt{\sigma_k^2 + \omega_0^2}} = \frac{\sigma_k}{\omega_{0k}},$$

(5.3)

where $\omega_{0k} = \sqrt{\sigma_k^2 + \omega_k^2}$ is the undamped natural frequency, calculated from the magnitude
of the pole. This means that $\zeta_k$ is a frequency dependent quantity [58, 62, 67]. The damping ratio is so named because $\zeta_k = 1$ when $\omega_k = \omega_{0k}$. When this condition is met, the system is “critically damped,” and returns to equilibrium as quickly as possible without overshooting [56]. This is useful for describing individual systems, but $\zeta_k$’s frequency dependence makes it difficult to compare different modes, not to mention different hurleys, all of which have different modal frequencies [62, 67].

The frequency-independent quantity $\sigma$ is a direct measure of the decay rate of each mode, and allows comparison across modes and hurleys [62]. It must be backed out of reported quantities. Equation 5.3 can be rearranged to give

$$\sigma_k = \sqrt{\frac{\omega_k^2 \zeta_k^2}{1 - \zeta_k^2}},$$

which can be calculated directly from the values of $\omega_k$ and $\zeta_k$ reported by the SR785.

### 5.2.2 Setup

The damping rates of the modes of each hurley are found from the frequency response between an impact in the bás of the hurley and an accelerometer at point 1. The location of point 1 is the same on every hurley and can be seen in Fig. 3.3. Figure 5.2 shows how the hurley is suspended from the handle by rubber bands and impacted on the bás. The accelerometer is the same as used for modal analysis, the PCB Model 352C22 ceramic shear accelerometer. It is very small and unlikely to mass load the hurley, and is functional in the desired frequency range. The hammer used to excite the hurleys for the damping measurement is about twenty times heavier than the one used for full modal analysis, and is shown in Fig. 3.1. The PCB 086C01 impulse hammer is fitted with a plastic tip in an attempt to better replicate the frequency content that a sliotar impact might deliver. Each hurley is impacted at a spot on the bás that is not a node for any of the lowest modes of the hurley.
Figure 5.2. Measurement layout for Damping Rate measurement. Hurley is suspended by rubber bands (not illustrated), accelerometer is at point 1. Hurley is hit by modal hammer on the bás at a point that is not a node for low modes.

The frequency response function for each measurement is the average of three impacts. Figure 5.3 shows the frequency response function hurley 4 after impact. The height and width of the peak are indicators of the amount of damping for each mode. The SR785 gives the damping ratio $\zeta_k$ and frequency $\omega_k$ of each modal peak $k$ identified, from which $\sigma_k$ is calculated. Notice in Fig. 5.3 how the third peak is much shorter and wider than the others. This mode would have a much higher value of $\sigma$ than the others.

Because the hurley is impacted with considerable force for this measurement, there is more variability due to human inconsistency. There is also a higher chance of encountering nonlinearities at higher force amplitude. To offset these effect, 10 frequency response functions were taken for each hurley. The reported values for $\sigma_k$ are averages of all 10 measured values of $\sigma_k$ for each mode $k$. The error for each measurement of $\sigma_k$ is the
standard deviation of the 10 measured values. This changed with the size of $\sigma_k$, with larger $\sigma_k$ having a larger standard deviation. For all hurleys except hurley 8 (which was itself an outlier), the error of each measurement was within 10% of the measured value of $\sigma_k$.

The frequencies of some of the modal peaks differed slightly from the frequencies of the modes calculated during modal analysis in Chapter 3. This is because the frequency response function for damping was only collected for one measurement point, whereas modal analysis takes the whole hurley into account. There may be local effects at play but it is not of great significance to the measurement of damping.

5.2.3 Human vibration sensation

Human vibration sensation was explained in depth in Section 3.3. As identified in Fig. 3.4, the most sensitive region of hand vibration sensation is approximately between 150 and 400 Hz. For most hurleys, only the first three or four bending modes and the first torsional mode are below 500 Hz. Because of this, only the damping rates for the first five modal peaks in each hurley’s frequency response function were calculated.
5.3 Measured damping values for hurleys

The results of the damping test are organized first by mode. The modes were determined by matching the frequency of the modal peak in the damping frequency response function to the mode shape with a close frequency as determined by modal analysis in Chapter 3. Table 5.1 shows the damping rate and frequency of the first four bending modes and the second torsional mode of each hurley. The first torsional mode is such a weak mode it does not show up on the frequency response function. The second torsional mode shows more strongly, and therefore was identifiable. On hurley 5 it was difficult to identify a second torsional mode. It is possible that the second torsional mode is so close in frequency to the fourth bending mode that it does not appear as a separate mode. The second torsional mode does not show up consistently between the same two bending modes for every hurley. Some times it falls between the second and third bending modes, others between the third and fourth. For clarity, the second torsional mode is listed in the table even though the frequencies might belong between two bending modes.

Figure 5.4 breaks the data up into the different bending modes so they may be compared one by one. Again, plastic hurley 12 is a visible outlier with both low modal frequencies and low values of damping rate. The low damping rate means that it will take a long time for vibrations to damp out. Hurley 7 is noticeable in that it always has a high modal frequency, but damping rates in the lower range of values. In Chapter 2, hurley 7 usually showed up in a cluster with hurleys 8, 9, 10, and 11, and it has a similar mass and outline, but in terms of damping it appears to have diverged from the pack. Hurley 7 is from a different manufacturer, so this may come down to different material or manufacturing technique.

While none of the hurleys follow the same exact damping rate pattern, some interesting trends do arise. It would be expected for most materials that the damping rate would increase with the frequency of the mode. Ash hurley 4, and all of the plastic and
Table 5.1. Frequencies and Damping Rate $\sigma$ for first 4 bending modes of hurleys and the second torsional mode.

<table>
<thead>
<tr>
<th>Hurley</th>
<th>First Bending $f_{B1}$ [Hz]</th>
<th>$\sigma_{B1}$ [s$^{-1}$]</th>
<th>Second Bending $f_{B2}$</th>
<th>$\sigma_{B2}$</th>
<th>Third Bending $f_{B3}$</th>
<th>$\sigma_{B3}$</th>
<th>Fourth Bending $f_{B4}$</th>
<th>$\sigma_{B4}$</th>
<th>Second Torsional $f_{T2}$</th>
<th>$\sigma_{T2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ash</td>
<td>62</td>
<td>4.7</td>
<td>439</td>
<td>21.5</td>
<td>709</td>
<td>24.2</td>
<td>297</td>
<td>34.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 ash</td>
<td>58</td>
<td>5.3</td>
<td>397</td>
<td>28.1</td>
<td>654</td>
<td>20.9</td>
<td>286</td>
<td>29.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 ash</td>
<td>60</td>
<td>2.3</td>
<td>400</td>
<td>35.8</td>
<td>639</td>
<td>23.7</td>
<td>335</td>
<td>27.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 ash</td>
<td>70</td>
<td>2.8</td>
<td>448</td>
<td>16.2</td>
<td>745</td>
<td>23.4</td>
<td>327</td>
<td>23.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 plastic</td>
<td>41</td>
<td>1.7</td>
<td>131</td>
<td>10.3</td>
<td>300</td>
<td>30.5</td>
<td>709</td>
<td>71.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 composite</td>
<td>60</td>
<td>3.9</td>
<td>177</td>
<td>9.7</td>
<td>375</td>
<td>45.5</td>
<td>595</td>
<td>51.2</td>
<td>282</td>
<td>66.7</td>
</tr>
<tr>
<td>7 composite</td>
<td>72</td>
<td>1.4</td>
<td>227</td>
<td>4.5</td>
<td>439</td>
<td>13.0</td>
<td>796</td>
<td>25.3</td>
<td>538</td>
<td>36.2</td>
</tr>
<tr>
<td>8 composite</td>
<td>73</td>
<td>4.2</td>
<td>206</td>
<td>26.8</td>
<td>432</td>
<td>33.1</td>
<td>761</td>
<td>166.8</td>
<td>562</td>
<td>106.5</td>
</tr>
<tr>
<td>9 composite</td>
<td>70</td>
<td>4.6</td>
<td>199</td>
<td>30.4</td>
<td>426</td>
<td>41.4</td>
<td>748</td>
<td>51.7</td>
<td>556</td>
<td>88.3</td>
</tr>
<tr>
<td>10 composite</td>
<td>67</td>
<td>2.3</td>
<td>213</td>
<td>5.5</td>
<td>426</td>
<td>21.9</td>
<td>726</td>
<td>32.8</td>
<td>528</td>
<td>30.4</td>
</tr>
<tr>
<td>11 composite</td>
<td>68</td>
<td>2.4</td>
<td>212</td>
<td>14.3</td>
<td>425</td>
<td>25.0</td>
<td>699</td>
<td>59.8</td>
<td>541</td>
<td>63.4</td>
</tr>
<tr>
<td>12 plastic</td>
<td>32</td>
<td>1.3</td>
<td>96</td>
<td>3.8</td>
<td>184</td>
<td>6.5</td>
<td>327</td>
<td>11.0</td>
<td>275</td>
<td>19.8</td>
</tr>
</tbody>
</table>

Composite hurleys follow this rule of thumb, albeit at vastly differing rates. Ash hurleys 1, 2, and 3 break the trend of increasing damping rate between the third and fourth bending modes. The damping rate for their fourth bending mode is less than that of their third. It may be that the grain structure and orthotropy of the sticks adds more damping at the third bending mode than at the fourth.

Figure 5.5 shows the damping ratio for the second torsional mode. This figure clearly shows the divide in where the second torsional mode shows up. Ash and solid-bás hurleys have their second torsional mode at lower frequencies than the hollow-bás composite hurleys. The damping rate for ash hurleys are clustered in the range $(20 < \sigma_{T2} < 35)$, which is a much smaller spread than that of the composite hurleys, whose values of $\sigma_{T2}$ have a spread of $\Delta \sigma_{T2} \approx 70$, which is more than quadruple the size.
Figure 5.4. A closer look at the damping rate $\sigma$ versus frequency for the first four bending modes. Note that the y-axis scale for $\sigma$ changes for each mode. (△) Wood; (☆) Composite; (★) Plastic.
5.3.1 Hurleys 8 and 9

It is interesting that the paired composite hurleys 8 & 9 and 10 & 11 do not exhibit consistent behavior. This could be due to any number of factors, including manufacturing discrepancies or wear. In Fig. 5.4, it can be seen that hurley 8 deviates quite spectacularly at the fourth bending mode with $\sigma_{B4} = 166\,s^{-1}$. Because only hurley 9 from the pair was analyzed in Chapter 3, there are no mode shapes to compare. This particular aspect of hurley 8 remains a mystery for now.

The hurley pair 8 & 9 are prototype composites with a modification in the handle intended to reduce vibration and improve feel [50]. This modification added a great deal of damping into the system. This is quite apparent in the second mode, where the values of $\sigma$ for hurleys 8 and 9 are anywhere from double to ten times the value of $\sigma$ for the other hurleys. They exhibit more damping than ash hurleys at all bending modes, except for ash hurley 3’s third bending mode. Figure 5.6 shows how the second bending mode for hurley 9 would decay over time compared to hurley 7, if they started oscillating at the same amplitude. Hurley 7 was chosen because it has a very similar mass and outline.
Figure 5.6. Schematic of ring-down time for the second bending mode of hurleys 7 and 9, \( h(t) = e^{-\sigma^2} \sin(2\pi f_2 t) \), if both were excited at the same initial displacement amplitude. Notice that hurley 7 vibrates almost 10 times longer than hurley 9.

In the schematic, the amplitude of vibration of hurley 9 is approximately an order of magnitude smaller than hurley 7 within the first 100 ms. If the amplitude of vibration were to start in the range in which humans can sense it, hurley 7 would tend to oscillate in that amplitude range for much longer. This is likely to affect the feel of the hurley, and might even influence player preference.
Chapter 6
Conclusions and Future work

6.1 Scope

Hurling and camogie are an important part of Irish culture and heritage. The sports and their equipment have developed over time. The performance of the hurley and sliotar affect the way the game is played and therefore it is important to understand their characteristics. This thesis presents measurements of 12 hurleys of varying materials, from different manufacturers. The main results in this thesis come primarily from modal analysis of hurleys, including their mode shapes, modal frequencies, and modal damping. Other reported quantities relevant to hurley behavior are the moment of inertia and center of percussion of each hurley. Measuring the mode shapes, frequencies, and damping characteristics of hurleys, as well as their moment of inertia and center of percussion, provides insight into hurley behavior and how it might relate to players’ perception.

6.2 Moment of Inertia and Center of Percussion

Measuring the moment of inertia allows us to compare the way that hurleys swing. Two hurleys of similar mass but different moment of inertia (hurley 5 and 12, for example) might feel similar to pick up, but quite different to actually swing. There was not an identifiable trend in the moment of inertia based on hurley material. Moment of inertia
is instead dependent on hurley mass distribution, which is governed by its shape and whether it is hollow or solid. The center of percussion relative to a 7-inch (18 cm) pivot was found to be \( \sim 77 \) cm from the top of the handle for ash hurleys, with less than a centimeter spread in measured values. The center of percussion relative to a 7-inch pivot for composite hurleys was anywhere from 1 cm to 4 cm closer to the top of the handle.

### 6.3 Modal Analysis and Damping

The vibrational pattern of a handheld hurley was determined to be more similar to a free-free boundary than a clamped-free boundary. Gripping the hurley tightly does add both damping and mass loading. The peak of the first bending mode is much broader, and it shifts down \( \sim 20\% \) in frequency for the tightly held condition. This frequency, however, is nowhere near the 8.5 Hz first bending mode of the hurley clamped at the handle. A free-free boundary was determined to be sufficient for modal analysis. It would be useful to establish a scale for the damping added by different grip strengths. Further research could also establish the approximate mass-loading added by the hand, as Banwell, et al. did for tennis rackets [54].

Using modal analysis, bending and torsional mode shapes were identified. From these classifications, modal frequency curves were created, which facilitated direct comparison of different hurleys. Curve fitting of the bending frequencies for all hurleys resulted in a fit of \( f = Am^b \) with values of \( b \) between 1.5 and 1.8 and the value of \( A \) depending on the material properties.

Wood hurleys exhibited very similar curve fits for the frequencies of their bending modes, \( f_m = Am^b \), with \( 56.636 < A < 69.014 \) and \( 1.703 < b < 1.763 \), giving an average curve for ash hurleys of \( f_m \approx 61.222 m^{1.727} \). The bending mode frequency curves of composite hurleys are closely matched to the curves of ash hurleys. This could lead to even wider acceptance of composite hurleys as a suitable alternative to ash.

Hurley 12 was a plastic hurley introduced in the 1970’s [50]. It had the lowest bending
frequency curve, and therefore has many bending and torsional modes packed into the sensitive range of human hands. The other heavy plastic hurley (5) also had modal frequencies lower than the others, but not as drastically low as hurley 12.

6.4 The Sweet Spot

Section 4.7 identified potential sweet spots based on either the first and second bending modes or the second and third bending modes. Clear zones of minimal excitation were revealed when the mode shapes were layered. Adding together the first and second bending mode revealed a clear candidate for the sweet spot. The white spots in Fig. 4.13 are the areas where an impact would excite neither the first nor second bending mode. These fall in the approximate center of the bás. However, the first bending mode of all of the hurleys falls below of the frequency range in which hands are the most sensitive. It is possible that sensation of hurleys is governed by the second and third bending modes instead, which both fall in the most sensitive range. Analysis of the layered mode shapes in Fig. 4.14 showed a sweet spot that is slightly closer to the handle than the sweet spot determined from the first and second bending modes. A sliotar impact, however, is not a point impulse. In fact, during impact with the hurley, a sliotar can deform and spread to be in contact with the hurley in a circle with diameter of approximately 5 cm [38, 46]. This means that impacts could cover both sweet spot candidates and render the difference between the two unimportant.

6.5 Damping

The modal damping rates measured for ash hurleys exhibited less spread than that of the composite hurleys. This suggests that, although the ash differs between hurleys, damping properties of ash itself may be more consistent than the various materials used for composite hurleys.
Hurley 12 was found to have more bending and torsional frequencies within and close to the sensitive range as defined by Fig. 3.4. This characteristic, combined with the low damping rates of these modes, likely contributed to the hurley’s discontinuation shortly after its introduction in the 1970s [50,66]. The modal damping rates for Hurley 5 are on par with the damping of the ash and composite hurleys. Hurley 5 is less likely to feel different from the other hurleys than Hurley 12.

Composite materials also have more opportunity to increase the damping rate versus natural materials. Intentional manipulation of damping in composite hurleys could lead to a change in the feel, even when their modes are at similar frequencies.

### 6.6 Future Work

The hurleys in this study were all 35-inch hurleys, the second-largest size offered. Hurley sizing does not work on a linear scale, as the bás needs to have a certain surface area for the sliotar to contact. An in-depth study on hurleys of the same model but different sizes could establish a general idea of the way mode shapes and frequencies scale with hurley size. This would help inform hurley design for people shorter than 5’10”. This is important because camogie is the most popular women’s sport in Ireland [70]. Irish women average 5’5” tall and likely use a 32- or 33-inch hurl [36,37].

Investigation into the biomechanics of the different types of hurley swing would establish a pivot point for each swing. This would inform further investigation into centers of percussion relative to the measured pivot points and determine their relevance to hurling.

Many of the conclusions in this thesis would benefit from corroboration with field tests using players. Exciting the hurley at each of the first three bending modes while it was being held could confirm which modes are more important to feel. This would indicate which sweet spot candidate is more correct. Impact testing at the center of percussion that corresponds to the pivot of each type of swing could also clarify the
relative importance of vibrational properties versus impact.

Modal analysis of hurleys can inform hurley design that emphasizes or diminishes the effects of different modes. For example, composite hurleys 8 & 9, which were designed to increase damping. Manipulating the shape or mass distribution of the hurley could change the frequency of the second torsional mode, or force it to the same frequency as the third bending mode so they show up superposed. Consolidating torsional modes into bending modes in the handle could also affect the feel or perception of the hurley by reducing the number of distinct modal frequencies present in the hurley. The hollow bás in composite hurleys offers the opportunity to exploit a trampoline effect, if the mode were tuned to the impact time of the sliotar.
Appendix A

Full modal characteristics of hurleys

A.1 Introduction

This appendix contains details of the data collected for each hurley. The material and measured properties from Chapter 2 are reported, as well as mode shapes for the first five bending modes and four torsional modes of each hurley. These are the modes that occur at frequencies that are likely to be excited by impact with the sliotar and felt by the player holding the hurley. If a mode is not present in the mode shape figures, it either is not a strong enough mode in the hurley to show up in this modal analysis, or it overlaps with another mode. Modes that overlap are marked as such. Modal analysis was not conducted for hurleys 8 and 11 because they were part of identical pairs.
A.2 Hurley 1

<table>
<thead>
<tr>
<th>Material</th>
<th>Ash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair</td>
<td>Hurley 2</td>
</tr>
<tr>
<td>Mass</td>
<td>572.2 g</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>1.3854 g cm$^2$</td>
</tr>
<tr>
<td>Center of percussion relative to 180 mm pivot</td>
<td>77.2 cm</td>
</tr>
<tr>
<td>Bending mode curve</td>
<td>$f_m = 61.325 m^{1.736}$</td>
</tr>
</tbody>
</table>

![Figure A.1](image.png)

**Figure A.1.** First five bending modes and four torsional modes, with bending mode curve fit.
Figure A.2. First 5 bending modes for hurley 1.

Figure A.3. First 4 torsional modes for hurley 1.
### A.3 Hurley 2

<table>
<thead>
<tr>
<th>Material</th>
<th>Ash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair</td>
<td>Hurley 1</td>
</tr>
<tr>
<td>Mass</td>
<td>573.7 g</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>1.465 g cm$^2$</td>
</tr>
<tr>
<td>Center of percussion relative to 180 mm pivot</td>
<td>77.3 cm</td>
</tr>
<tr>
<td>Bending mode curve</td>
<td>$f_m = 56.636 m^{1.749}$</td>
</tr>
</tbody>
</table>

![Figure A.4. First five bending modes and four torsional modes, with bending mode curve fit.](image-url)

Figure A.4. First five bending modes and four torsional modes, with bending mode curve fit.
Figure A.5. First 5 bending modes for hurley 2.

Figure A.6. First 4 torsional modes for hurley 2.
A.4 Hurley 3

<table>
<thead>
<tr>
<th>Material</th>
<th>Ash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair</td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>447.5 g</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>1.097 g cm²</td>
</tr>
<tr>
<td>Center of percussion relative to 180 mm pivot</td>
<td>77.0 cm</td>
</tr>
<tr>
<td>Bending mode curve</td>
<td>$f_m = 57.914 m^{1.720}$</td>
</tr>
</tbody>
</table>

Figure A.7. First five bending modes and torsional modes 1, 2, and 4, with bending mode curve fit.
Figure A.8. First 5 bending modes for hurley 3.

Figure A.9. Torsional modes 1, 2, and 4 for hurley 3.
A.5 Hurley 4

<table>
<thead>
<tr>
<th>Material</th>
<th>Ash</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair</td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>510.2 g</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>1.118 g cm$^2$</td>
</tr>
<tr>
<td>Center of percussion relative to 180 mm pivot</td>
<td>76.5 cm</td>
</tr>
<tr>
<td>Bending mode curve</td>
<td>$f_m = 69.014 m^{1.706}$</td>
</tr>
</tbody>
</table>

![Figure A.10. First five bending modes torsional modes 1 and 3, with bending mode curve fit.](image)

Figure A.10. First five bending modes torsional modes 1 and 3, with bending mode curve fit.
Figure A.11. First 5 bending modes for hurley 4.

Figure A.12. Torsional modes 1 and 3 for hurley 4.
A.6 Hurley 5

Material | Plastic solid bás
Pair |
Mass | 620.7 g
Moment of Inertia | 1.454 g cm²
Center of percussion relative to 180 mm pivot | 77.2 cm
Bending mode curve | \( f_m = 42.118 m^{1.773} \)

Figure A.13. First five bending modes and four torsional modes, with bending mode curve fit.
Figure A.14. First 5 bending modes for hurley 5.

Figure A.15. First 4 torsional modes for hurley 5. Hurley 5 has a very rounded handle, and repeatable measurements were difficult to obtain near the edge. These points were omitted from curve-fitting.
A.7 Hurley 6

Material                  Plastic solid bás
Pair                      
Mass                       585.3 g
Moment of Inertia        1.216 g cm$^2$
Center of percussion relative to 180 mm pivot  73.0 cm
Bending mode curve       $f_m = 65.473 \, m^{1.584}$

Figure A.16. First five bending modes and four torsional modes, with bending mode curve fit.
Figure A.17. First 5 bending modes for hurley 6.

Figure A.18. First 4 torsional modes for hurley 6.
A.8 Hurley 7

<table>
<thead>
<tr>
<th>Material</th>
<th>Composite hollow bás</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair</td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>521.3 g</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>1.273 g cm²</td>
</tr>
<tr>
<td>Center of percussion</td>
<td>76.1 cm</td>
</tr>
<tr>
<td>to 180 mm pivot</td>
<td></td>
</tr>
<tr>
<td>Bending mode curve</td>
<td></td>
</tr>
<tr>
<td>$f_m = 75.249 m^{1.673}$</td>
<td></td>
</tr>
</tbody>
</table>

Figure A.19. First five bending modes and four torsional modes, with bending mode curve fit.
Figure A.20. First 5 bending modes for hurley 7.

Figure A.21. First 4 torsional modes for hurley 7.
### A.9 Hurley 8

<table>
<thead>
<tr>
<th>Material</th>
<th>Composite hollow bás</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair</td>
<td>Hurley 9</td>
</tr>
<tr>
<td>Mass</td>
<td>480.1 g</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>1.176 g cm²</td>
</tr>
<tr>
<td>Center of percussion relative to 180 mm pivot</td>
<td>74.9 cm</td>
</tr>
</tbody>
</table>
A.10 Hurley 9

<table>
<thead>
<tr>
<th>Material</th>
<th>Composite hollow bās</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair</td>
<td>Hurley 8</td>
</tr>
<tr>
<td>Mass</td>
<td>499.4 g</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>1.211 g cm$^2$</td>
</tr>
<tr>
<td>Center of percussion relative to 180 mm pivot</td>
<td>74.8 cm</td>
</tr>
<tr>
<td>Bending mode curve</td>
<td>$f_m = 70.037 m^{1.626}$</td>
</tr>
</tbody>
</table>

Figure A.22. First five bending modes and three torsional modes, with bending mode curve fit.
Figure A.23. First 5 bending modes for hurley 9.

Figure A.24. First 3 torsional modes for hurley 9.
A.11 Hurley 10

<table>
<thead>
<tr>
<th>Material</th>
<th>Composite hollow bās</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair</td>
<td>Hurley 11</td>
</tr>
<tr>
<td>Mass</td>
<td>507.5 g</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>1.224 g cm$^2$</td>
</tr>
<tr>
<td>Center of percussion relative to 180 mm pivot</td>
<td>75.6 cm</td>
</tr>
<tr>
<td>Bending mode curve</td>
<td>$f_m = 68.600 m^{1.656}$</td>
</tr>
</tbody>
</table>

**Figure A.25.** First five bending modes and two torsional modes, with bending mode curve fit.
Figure A.26. First 5 bending modes for hurley 10.

Figure A.27. First 2 torsional modes for hurley 10.
### A.12 Hurley 11

<table>
<thead>
<tr>
<th>Material</th>
<th>Composite hollow bát</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair</td>
<td>Hurley 10</td>
</tr>
<tr>
<td>Mass</td>
<td>497.8 g</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>1.226 g cm$^2$</td>
</tr>
<tr>
<td>Center of percussion relative to 180 mm pivot</td>
<td>75.8 cm</td>
</tr>
</tbody>
</table>
A.13 Hurley 12

Material  Plastic solid bás
Pair
Mass  606.1 g
Moment of Inertia  1.059 g cm$^2$
Center of percussion relative to 180 mm pivot  71.1 cm
Bending mode curve  $f_m = 30.996 m^{1.689}$

Figure A.28. First five bending modes and four torsional modes, with bending mode curve fit.
Recall that hurley 12’s first bending mode occurs at 33 Hz, which is very close to the frequency of rigid body motion of the hurley in the rubber bands. The mode had to be fit in a way that accounted for the rubber bands, which did not translate to a colormap because of the rigid body motion.

**Figure A.29.** Bending modes 2-6 for hurley 12.
Figure A.30. Torsional modes 2-4 for hurley 12.
Appendix B

Anecdotal Sweet Spot Determination

Insofar as it is not a rigorous investigation, Fig. B.1 shows the bás of my own hurley, with the region that I usually hit the sliotar identified. As an amateur with no training except for YouTube and trial and error, my experience is limited to gripping near the top of the handle for looping swings that go far. The region might shift if I were to make short passes holding closer to the middle of the hurley. The region identified in Fig. B.1 aligns nominally with the double-node region identified for the first and second bending modes in Fig. 4.13, as it is further from the end of the hurley, closer to where the bás joins the handle.

Figure B.1. The general region that most swings hit, based on the scuffing relative to the rest of the hurley. Dark scuffs around the edge of the bás are from bad hits that equate to a pop fly in baseball, related to a timing issue.
Appendix C
Higher Order Modes

C.1 Introduction

While the body of this thesis deals with vibrational modes that are both in the range of human sensation and likely to be excited with a typical sliotar impact, mode shapes for higher order modes were recorded for some hurleys. These tended to be some combination of bending and torsional modes. The following are some of the higher order modes recorded for the hurleys.

Figure C.1. Higher order modes for hurley 1.
Figure C.2. Higher order modes for hurley 2.

Figure C.3. Higher order modes for hurley 3.

Figure C.4. Higher order modes for hurley 4.
Figure C.5. Higher order modes for hurley 5.

Figure C.6. Higher order modes for hurley 6.

Figure C.7. Higher order modes for hurley 7.
Figure C.8. Higher order modes for hurley 9.

Figure C.9. Higher order modes for hurley 10.
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