FINITE-VOLUME ANALYSIS OF TWO-PHASE SPLIT AT T-JUNCTIONS

A Thesis in
Petroleum & Mineral Engineering
by
Gurpreet Singh

© 2009 Gurpreet Singh

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

May 2009
The thesis of Gurpreet Singh was reviewed and approved* by the following:

Luis Ayala
Assistant Professor of Petroleum & Natural Gas Engineering
Thesis Advisor

Vladislav Kecojevic
Associate Professor of Mining Engineering
Centennial Career Development Professorship in Mining Engineering

Yaw D. Yeboah
Professor of Energy & Mineral Engineering
Head of the Department of Energy & Mineral Engineering

*Signatures are on file in the Graduate School
ABSTRACT

A two-fluid finite-volume model for predicting liquid route selection for two-phase gas-liquid flow at diverging T-junctions is presented. Stratified smooth, wavy stratified and mist flow patterns for low liquid loading systems were investigated. A steady state mass and momentum conservation with appropriate closure correlations applied to finite volume discretized tee is used to simulate pressure, velocity and liquid holdup conditions at and in near proximity of T-junctions. A simultaneous solution approach (Generalized Newton Raphson) was used to solve model governing equations. Previous models rely upon specification of branch gas mass fraction, information which is not readily available for large pipeline networks. Additionally, a pressure difference between run and branch arms was considered as a derivative for two phase split undermining the significance of absolute pressures. The proposed model formulation is capable of handling available field data to affect two phase flow split at a Tee. Pressure, liquid holdup and phase velocity profiles predictions can be generated instead of a single-point split at T-junctions, as suggested in earlier studies. A simultaneous solver employed allows study of interconnectivity between system elements and renders the model an inherent dependency on existing flow parameters. Several parametric studies and comparisons are presented to demonstrate model capabilities.
# TABLE OF CONTENTS

LIST OF FIGURES ........................................................................................................ vii

LIST OF TABLES ............................................................................................................ xii

ACKNOWLEDGEMENTS ................................................................................................. xv

NOMENCLATURE .......................................................................................................... xviii

Chapter 1 Introduction ................................................................................................. 1

Chapter 2 Literature Survey ....................................................................................... 3

Chapter 3 Problem Statement ..................................................................................... 12

Chapter 4 Model Description ...................................................................................... 14

  4.1 Flow at T-Junctions ............................................................................................... 16
  4.2 Two Fluid Finite Volume Derivation .................................................................. 17
    4.2.1 Mass Conservation ....................................................................................... 17
    4.2.2 Momentum Conservation ............................................................................. 19
  4.3 Donor Cell Scheme .............................................................................................. 20
  4.4 Primary Unknowns ............................................................................................... 22
  4.6 Finite Volume Discretization – Staggered Gridding ........................................... 23
  4.7 Boundary Conditions ........................................................................................... 25
  4.8 Staggered Approach - Pipe Flow ........................................................................ 27
    4.8.1 Forward Staggered Formulation ................................................................ 28
    4.8.2 Backward Staggered Formulation ............................................................... 29
  4.9 Staggered Approach - Tee Junction .................................................................... 30
    4.9.1 Forward Staggered Formulation ................................................................ 31
    4.9.2 Backward Staggered Formulation ............................................................... 33
  4.10 Information Translation Approach: An Alternative to Staggering .................... 34
  4.11 Closure Relationships ......................................................................................... 37
    4.11.1 Wall Shear Force ($F_{\text{wall}}$) .................................................................... 37
    4.11.2 Interfacial Shear Force ($F_{\text{int}}$) ................................................................. 39
      4.11.3 Tee Junction Losses ................................................................................ 39
      4.11.3.1 Energy Dissipation Approach ............................................................. 39
      4.11.3.2 Interfacial Drag Force Approach ......................................................... 42
    4.11.4 Stratified Smooth Flow Pattern ................................................................... 44
      4.11.4.1 Wall Shear Force Coefficients ............................................................ 45
      4.11.4.2 Interfacial Shear Force Coefficients ..................................................... 45
    4.11.5 Wavy Stratified Flow Pattern ..................................................................... 47
      4.11.5.1 Wetted Shear Coefficients ................................................................. 47
      4.11.5.2 Interfacial Shear Force Coefficients ..................................................... 49
4.10.6 Mist Flow Pattern ................................................................. 50
4.11.6.1 Wall Shear Force Coefficients ......................................... 51
4.11.6.2 Interfacial Shear Force Coefficients ................................... 51

Chapter 5  Numerical Treatment.......................................................... 53

5.1  Newton-Raphson Method ............................................................... 53
5.1.1 Geometrical Interpretation ......................................................... 54
5.1.2 Generalized Newton Raphson ...................................................... 56
5.1.3 Convergence Issues .................................................................. 58
5.2  Initialization of Variables ............................................................... 61
5.2.1 Single Phase Flow-Variable Initialization ................................. 61
5.2.2 Two Phase Flow-Variable Initialization ...................................... 62

Chapter 6  Results and Discussion ........................................................ 64

6.1  System Description ........................................................................ 64
6.2  Single Phase Steady State Analysis ............................................... 65
6.2.1 Single Phase Pipe Flow ............................................................... 65
6.2.1.1 Variation with Pipeline Roughness ....................................... 68
6.2.1.2 Effect of Grid Resolution (Number of Grid Blocks) .............. 70
6.2.2 Single Phase Flow at Tee ............................................................ 71
6.2.2.1 Effect of Grid Resolution (Number of Grid Blocks) .......... 73
6.2.2.2 Variation with Branch Arm Angle (β) ............................... 74
6.2.2.3 Variation with Branch Arm Pressure .................................. 76
6.3  Comparison between Single and Two Phase Flow in Pipelines ......... 77
6.4  Two Phase Steady State Analysis .................................................. 79
6.4.1 Two Phase Pipe Flow - Smooth Stratified Flow Pattern .......... 79
6.4.1.1 Variation with Inlet Liquid Holdup ..................................... 82
6.4.2 Two Phase Tee Split - Smooth Stratified Flow ....................... 85
6.4.2.1 Variation with Inlet Liquid Mass Flow-rate ....................... 88
6.4.2.2 Variation with Branch Arm Pressure ................................ 91
6.4.3 Two Phase Pipe Flow - Mist Flow Pattern .............................. 94
6.4.3.1 Variation with Inlet Liquid Holdup ..................................... 94
6.4.4 Two Phase Tee Split – Mist Flow ............................................... 96
6.4.4.1 Variation with Branch Arm Pressure .................................. 99
6.4.4.2 Variation with Branch Arm Angle ................................. 100
6.4.4.3 Variation with Inlet Liquid Flow-rate .............................. 102
6.4.5 Wavy Stratified Flow ............................................................... 104
6.5  Additional Model Capabilities: Converging Tees ......................... 105

Chapter 7  Summary and Conclusions .................................................. 108

7.1  Future Work and Recommendations .......................................... 110

Bibliography ................................................................................. 111
Appendix A Three Block Problem ................................................................. 115
  A.1 Forward Staggered Approach: Two Phase Pipe Flow ...................... 115
  A.2 Backward Staggered Approach: Two Phase Pipe Flow .................... 117
  A.3 Inventory of Unknowns and Specifications: Two Phase Pipe Flow ...... 118
  A.4 Two Phase Flow at T-junction: Forward Staggered Approach .......... 119
  A.5 Two Phase Flow at T-junction: Backward Staggered Approach ........ 121
    A.5.1 Inventory of Unknowns and Specifications: Two Phase Tee Split .... 122
  A.6 Nomenclature .................................................................................. 123

Appendix B Programs and Subroutines ..................................................... 124
  B.1 Two Phase Pipe Flow ........................................................................ 124
  B.2 Two Phase Flow at T-junction .......................................................... 138
LIST OF FIGURES

Figure 2.1: Tee Split (Saba and Lahey, 1984) ................................................................. 9

Figure 3.1: Tee Junction ................................................................................................... 12

Figure 4.1: Discretization Schemes (a) Finite Difference Grid(left) (b) Finite
Volume Grid(right) ........................................................................................................... 15

Figure 4.2: Classification of Tee Splits.............................................................................. 16

Figure 4.3: Mass Conservation ......................................................................................... 17

Figure 4.4: Momentum Conservation ................................................................................ 19

Figure 4.5: Nomenclature ................................................................................................ 22

Figure 4.6: Checkerboard Problem – Patankar (1980) .................................................. 23

Figure 4.7: Zig-zag Pressure Path ..................................................................................... 24

Figure 4.8: Staggered v/s Coincidental Grids ................................................................. 25

Figure 4.9: Outflow boundary condition – Versteeg and Malalasekera (1995) .......... 26

Figure 4.10: Zero-Curvature boundary condition ......................................................... 27

Figure 4.11: Pipe Flow Grid Block Nomenclature ............................................................. 27

Figure 4.12: Forward Staggered Grid Structure ............................................................... 28

Figure 4.13: Backward Staggered Grid Structure ............................................................. 29

Figure 4.14: Tee Split Grid Block Nomenclature ............................................................... 31

Figure 4.15: Forward Staggered Momentum Grid for Tee Split ............................... 31

Figure 4.16: Backward Staggered Momentum Grid for Tee Split ............................... 33

Figure 4.17: Coincidental gridding Scheme ...................................................................... 35

Figure 4.18: Geometrical representation of Stratified Smooth Flow (Ayala, 2001).. 44

Figure 4.19: Geometrical representation of Wavy Stratified Flow (Chen et al, 1997) ..................................................................................................................... 47

Figure 4.20: Mist Flow Pattern ....................................................................................... 50
Figure 5.1: Geometrical Representation – Newton Raphson ........................................... 54

Figure 5.2: Convergence Issues – Newton Raphson ....................................................... 59

Figure 5.3: Errors in numerical differentiation ................................................................. 60

Figure 6.1: Single phase pipe flow pressure profile - Backward Staggering ............... 66

Figure 6.2: Single phase pipe flow velocity profile - Backward Staggering ............... 66

Figure 6.3: Single phase pipe flow wall shear force profile - Backward Staggering ................................................................. 67

Figure 6.4: Single phase pipe flow pressure profile - Forward Staggering ............... 67

Figure 6.5: Single phase pipe flow velocity profile - Forward Staggering ............... 67

Figure 6.6: Single phase pipe flow wall shear force profile - Forward Staggering .... 68

Figure 6.7: Single phase pipe flow pressure profile variation with roughness parameter ............................................................................................................. 69

Figure 6.8: Single phase pipe flow velocity profile variation with roughness parameter ............................................................................................................. 69

Figure 6.9: Single phase pipe flow wall shear force profile variation with roughness ............................................................................................................. 70

Figure 6.10: Grid block resolution - Single phase pipe flow ........................................ 71

Figure 6.11: Hart’s single phase pressure profile (Hart, 1990) ..................................... 71

Figure 6.12: Model generated pressure profile ................................................................. 72

Figure 6.13: Model generated velocity profile ................................................................. 73

Figure 6.14: Grid block resolution - Single phase tee flow ......................................... 74

Figure 6.15: Effect of branch arm angle on pressure spiking - Single Phase .......... 75

Figure 6.16: Effect of branch arm angle on pressure spiking - Single Phase (Magnified Section) ......................................................................................... 75

Figure 6.17: Effect of branch arm angle on branch gas mass fraction - Single Phase ............................................................................................................. 76

Figure 6.18: Effect of branch arm pressure on branch gas mass fraction - Single Phase ............................................................................................................. 77
Figure 6.19: Comparison between single and two phase flow in pipelines

Figure 6.20: Determination of initial estimate for implicit liquid holdup calculation

Figure 6.21: Comparison between forward and backward staggered pressure profile - Smooth stratified pipe flow

Figure 6.22: Comparison between forward and backward gas velocity profile - Smooth stratified pipe flow

Figure 6.23: Comparison between forward and backward liquid velocity profile - Smooth stratified pipe flow

Figure 6.24: Comparison between forward and backward liquid holdup profiles - Smooth stratified pipe flow

Figure 6.25: Two phase pipe flow pressure profile variation with inlet liquid holdup - Smooth stratified flow pattern

Figure 6.26: Two phase pipe flow gas velocity profile variation with inlet liquid holdup - Smooth stratified flow pattern

Figure 6.27: Two phase pipe flow liquid velocity profile variation with inlet liquid holdup – Smooth stratified flow pattern

Figure 6.28: Two phase pipe flow liquid holdup profile variation with inlet liquid holdup – Smooth stratified flow pattern

Figure 6.29: Two phase pipe flow liquid wall shear variation with inlet liquid holdup – Smooth stratified flow pattern

Figure 6.30: Two phase pipe flow gas wall shear variation with inlet liquid holdup – Smooth stratified flow pattern

Figure 6.31: Branch arm gas and liquid mass fractions

Figure 6.32: Pressure profile for two phase tee split for stratified flow - Effect of junction losses

Figure 6.33: Liquid phase velocity profile for two phase tee split for stratified flow - Effect of junction losses

Figure 6.34: Gas phase velocity profile for two phase tee split for stratified flow - Effect of junction losses
Figure 6.35: Liquid holdup profile for two phase tee split for stratified flow - Effect of junction losses........................................................................................................ 88

Figure 6.36: Inlet liquid holdup v/s inlet liquid mass flow rate - Smooth stratified flow ...................................................................................................................................................... 89

Figure 6.37: Branch gas mass fraction v/s inlet liquid mass flow rate - Smooth stratified flow ...................................................................................................................................................... 90

Figure 6.38: Branch liquid mass fraction v/s inlet liquid mass flow rate - Smooth stratified flow ...................................................................................................................................................... 90

Figure 6.39: Branch liquid mass fraction v/s branch arm pressure - Smooth stratified flow ...................................................................................................................................................... 91

Figure 6.40: Branch gas mass fraction v/s branch arm pressure - Smooth stratified flow ...................................................................................................................................................... 92

Figure 6.41: Hart’s branch liquid mass fraction v/s gas mass fraction for $U_{sg} = 5.1\text{m/s}$ – Stratified Flow (Hart, 1990). ...................................................................................................................................................... 92

Figure 6.42: Branch gas mass fraction v/s branch arm pressure - Smooth stratified flow ...................................................................................................................................................... 94

Figure 6.43: Two phase pipe flow pressure profile variation with inlet liquid holdup-Mist flow pattern ...................................................................................................................................................... 95

Figure 6.44: Two phase pipe flow gas velocity profile variation with inlet liquid holdup-Mist flow pattern ...................................................................................................................................................... 95

Figure 6.45: Two phase pipe flow liquid velocity profile variation with inlet liquid holdup-Mist flow pattern ...................................................................................................................................................... 96

Figure 6.46: Two phase pipe flow liquid holdup profile variation with inlet liquid holdup-Mist flow pattern ...................................................................................................................................................... 96

Figure 6.47: Pressure profile for two phase tee split – Mist flow pattern.............. 97

Figure 6.48: Gas phase velocity profile for two phase tee split – Mist flow pattern. 98

Figure 6.49: Liquid phase velocity profile for two phase tee split – Mist flow pattern ...................................................................................................................................................... 98

Figure 6.50: Liquid holdup profile for two phase tee split – Mist flow pattern ...... 98

Figure 6.51: Branch liquid and gas mass fractions v/s branch arm pressure – Mist flow pattern ...................................................................................................................................................... 100
Figure 6.52: Branch gas mass fraction v/s branch liquid mass fraction – Mist flow pattern ................................................................. 100

Figure 6.53: Branch gas mass fraction v/s branch arm angle – Mist flow pattern ..... 101

Figure 6.54: Branch liquid mass fraction v/s branch arm angle – Mist flow pattern ........................................................................ 102

Figure 6.55: Branch gas mass fraction v/s inlet liquid mass flow rate – Mist flow pattern ..................................................................... 103

Figure 6.56: Branch liquid mass fraction v/s inlet liquid mass flow rate – Mist flow pattern ................................................................. 103

Figure 6.57: Inlet liquid holdup v/s inlet liquid mass flow rate – Mist flow pattern .. 104

Figure 6.58: Objective function v/s (θi) – red (θ = 15), green (θ = 30), blue (θ = 60) .................................................................................................. 105

Figure 6.59: Converging T-junction Example .......................................................................................................................... 106

Figure 6.60: Converging tee – pressure profile ......................................................... 106

Figure 6.61: Converging tee – gas phase velocity profile ........................................ 107

Figure 6.62: Converging tee – liquid phase velocity profile ................................. 107

Figure 6.63: Converging tee – liquid holdup profile .............................................. 107

Figure A1.1: Three block problem: Grid block nomenclature .............................. 115

Figure A1.2: Three block problem: Two phase pipe flow (Forward Staggered Approach) ........................................................................ 115

Figure A1.3: Three block problem: Two phase pipe flow (Backward Staggered Approach) ...................................................................... 117

Figure A1.4: Three block problem :Two phase tee split (Forward Staggered Approach) ........................................................................ 119

Figure A1.5: Three block problem :Two phase tee split (Backward Staggered Approach) ...................................................................... 121
LIST OF TABLES

Table 4.1: Inventory of Unknowns (Two Phase Flow) ............................................. 22
Table 4.2: Inventory of Unknowns (Single Phase Flow) ............................................. 22
Table 4.3: Specifications - Forward Staggered Pipe Flow ........................................... 28
Table 4.4: Specifications - Backward Staggered Pipe Flow ......................................... 30
Table 4.5: Specifications – Forward Staggered Tee Junction ....................................... 32
Table 4.6: Specifications – Backward Staggered Tee Junction ..................................... 34
Table 4.7: Pipe Specifications: Information translation approach ................................... 36
Table 4.8: Tee Specifications: Information translation approach ..................................... 36
Table 4.9: Drag Coefficients, \( w = \log_{10} \text{Re} \), (Cliff et al., 1978), (Ayala, 2001) ....... 52
Table 5.1: Single Phase Gas Flow Unknowns .............................................................. 61
Table 5.2: Two Phase Gas-Liquid Flow Unknowns ....................................................... 62
Table 6.1: Gas-liquid and pipeline property data .......................................................... 65
Table 6.2: Single phase forward and backward staggering input parameters ............... 66
Table 6.3: Input parameters for single phase pipe flow with varying roughness
(Forward Staggered Approach) .................................................................................. 68
Table 6.4: Input parameters for single phase pipe flow - Grid block resolution .......... 70
Table 6.5: Input parameters - Model generated single phase pressure profile .............. 72
Table 6.6: Input parameters for single phase tee split - Grid resolution ................. 74
Table 6.7: Input parameters for single phase tee split with varying branch arm angle .......................................................... 75
Table 6.8: Input parameters for single phase tee split with varying branch arm outlet pressure ............................................................................. 76
Table 6.9: Input parameters for two-phase mist flow: Comparison between single and two phase pressure losses in pipeline ................................................... 77
Table 6.10: Two phase forward and backward staggering input parameters ........ 80

Table 6.11: Input parameter for two-phase pipe flow with varying inlet liquid holdup: Smooth stratified flow (Backward Staggered Approach) .................. 82

Table 6.12: Input and output parameters for two phase tee split for smooth stratified flow: Effect of junction losses (Forward Staggered Approach) .......... 86

Table 6.13: Input parameters for two-phase tee split for smooth stratified flow: Variation with inlet liquid mass flow-rate (Forward Staggered Approach) ........ 89

Table 6.14: Output parameters for two-phase tee split for smooth stratified flow: Variation with inlet liquid mass flow-rate (Forward Staggered Approach) ........ 89

Table 6.15: Input parameters for two phase tee split for smooth stratified flow: Variation with branch arm pressure for $m_{l,inlet} = 0.05$ kg/sec (Forward Staggered Approach) ................................................................. 91

Table 6.16: Output parameters for two phase tee split for smooth stratified flow: Variation with branch arm pressure for $m_{l,inlet} = 0.05$ kg/sec (Forward Staggered Approach) ................................................................. 91

Table 6.17: Input parameters for two phase tee split for smooth stratified flow: Variation with branch arm pressure for $m_{l,inlet} = 0.005$ kg/sec (Forward Staggered Approach) ................................................................. 93

Table 6.18: Output parameters for two phase tee split for smooth stratified flow: Variation with branch arm pressure for $m_{l,inlet} = 0.005$ kg/sec (Forward Staggered Approach) ................................................................. 93

Table 6.19: Input parameters for two phase pipe flow with varying inlet liquid holdup: Mist flow (Backward Staggered Approach) ......................... 94

Table 6.20: Input parameters for two phase tee split for mist flow: Generic profiles (Forward Staggered Approach) .................................................. 97

Table 6.21: Output parameters for two phase tee split for mist flow: Generic profiles (Forward Staggered Approach) .................................................. 97

Table 6.22: Input parameters for two phase tee split for mist flow: Variation with branch arm outlet pressure (Forward Staggered Approach) ................. 99

Table 6.23: Output parameters for two phase tee split for mist flow: Variation with branch arm outlet pressure (Forward Staggered Approach) ................. 99

Table 6.24: Input parameters for two phase tee split for mist flow: Variation with branch arm inclination (Forward Staggered Approach) ...................... 101
Table 6.25: Output parameters for two phase tee split for mist flow: Variation with branch arm inclination (Forward Staggered Approach) ........................................ 101

Table 6.26: Input parameters for two phase tee split for mist flow: Variation with inlet liquid mass flow-rate (Forward Staggered Approach) ........................................ 102

Table 6.27: Output parameters for two phase tee split for mist flow: Variation with inlet liquid mass flow-rate (Forward Staggered Approach) ........................................ 103

Table 6.28: Converging T-junction: Mist Flow Pattern (Forward Staggered Approach) ........................................................................................................................................... 106

Table A1.1: Inventory of unknowns: Two phase pipe flow ........................................ 118

Table A1.2: Specifications: Two phase pipe flow .................................................. 118

Table A1.3: Inventory of unknowns: Two phase tee split ..................................... 122

Table A1.4: Specifications: Two phase tee split .................................................. 123
NOMENCLATURE

\( m_{g/l} = \) Mass flow rate of gas/liquid phase

\( M_{g/l} = \) Mass of gas/liquid inside control volume

\( A_{g/l} = \) Cross sectional areas occupied by gas/liquid phase

\( A_{k,wall} = \) Wall-phase ‘k’ contact area (m\(^2\))

\( A_{int,pipe} = \) Interfacial area for pipe flow

\( A_{int} = \) Gas/liquid interfacial contact area (m\(^2\))

\( A_{k,flow} = \) Cross-sectional flow area for phase ‘k’

\( A_{int,eqv} = \) Equivalent interfacial area

\( A_{int,tee} = \) Equivalent interfacial area for tee flow

\( \Delta x = \) Length of finite volume block

\( P = \) Pressure exerted

\( F_{g/l,wall} = \) Gas/liquid wall shear force

\( F_{int} = \) Gas-liquid interfacial shear force

\( F_{int,eqv} = \) Equivalent interfacial drag force

\( D = \) diameter of pipe or annulus

\( D_b = \) Liquid droplet diameter.

\( d_{hg/l} = \) Gas (or liquid) hydraulic diameters (m)

\( f_{k,wall} = \) Fanning friction factor for phase ‘k’ (dimensionless)

\( f_{int} = \) Interfacial fanning friction factor (dimensionless)

\( f_{k,wall} = \) Fanning friction factor for phase ‘k’ (dimensionless)

\( \rho_{g/l} = \) Density of gas/liquid phase
\( r = \text{Radius of radiasation} \)

\( \beta_{g/l,\text{inlet}} = \text{Inlet gas/liquid velocity profile correction factor (dimensionless)} \)

\( \rho_{g/l,\text{inlet}} = \text{Inlet gas/liquid density (kg/m}^3\text{)} \)

\( U_{g/l,\text{inlet}} = \text{Inlet gas/liquid velocities (m/sec}^2\text{)} \)

\( U_k = \text{Velocity of phase ‘k’ in x-direction} \)

\( V_k = \text{Velocity of phase ‘k’ in y-direction} \)

\( F_{\text{tee-loss(x)}} = \text{T-junction losses in x-direction} \)

\( F_{\text{tee-loss(y)}} = \text{T-junction losses in y-direction} \)

\( C_D = \text{Interfacial drag force coefficient} \)

\( C_d = \text{Drag coefficient} \)

\( h_l = \text{Liquid phase depth for stratified flow} \)

\( \rho_{gst} = \text{Gas phase density at standard temperature and pressure conditions} \)

\( \alpha_{g/l} = \text{Gas (or liquid) holdup (dimensionless)} \)

\( \alpha_k = \text{Holdup value for phase ‘k’} \)

\( \theta = \text{Liquid holdup parameter} \)

\( \beta = \text{Angle between run and branch arms} \)
\( \mu_k = \) Viscosity of phase ‘k’ where, \( k = \text{gas (g) / liquid (l)} \)

\( W_f = \) Wetted wall fraction (dimensionless)

\( g = \) Acceleration due to gravity (m/sec\(^2\))

\( \sigma = \) Interfacial surface tension (N/m)
ACKNOWLEDGEMENTS

I am grateful to God who bestowed me with the tools and strength to confront every obstacle and to come across all challenges in life.

I express my gratitude to Dr. Luis Ayala for allowing me to complete my thesis. I appreciate his guidance during the course of this study, which I will remember for my entire life. I extend my sincerest gratitude to Dr Yaw D. Yeboah and Dr. Vladislav Kecojevic for honoring me in serving as members of my committee. I greatly appreciate their presence for my thesis defense at such an odd time of the year.

My utmost respect for Dr. Abrahm Sageev Grader who has been a teacher, a mentor and a friend in times of need and for whom my loyalty will forever hold. I am greatly obliged for his advice and continuous help during inclement circumstances.

My special thanks to Dr. Alan Scaroni, Dr. Yaw D. Yeboah and Dr. Sarma V. Pisupati for providing me with the much needed financial support without which this thesis would not have been possible. I am also thankful to Dr. Sarma V. Pisupati and Dr. Yaw D. Yeboah for making my experience as a teaching assistant enjoyable.

I am thankful to my parents for their teachings to stand ground in every battle and to persevere. I am grateful to my father, Kabal Singh, who taught me sacrifice, endurance, responsibility and whom I epitomize as my ideal. To my mother, Shivraj Kaur, for her religious guidance and her caring, loving and forgiving nature.

I would also like to acknowledge my friends Rituraj Nandan, Amit Arora, Rohit Rai for their cherished friendship and unabated support.
Chapter 1

Introduction

The issue of liquid phase route selectivity for two-phase, gas-liquid flow in complex pipeline networks via model formulations for predicting pressure and velocity profiles in pipelines has been extensively studied in the last few decades. However, most of these existing models either rely upon empirical correlations making them specific to a particular network under investigation, or are based upon phenomenological considerations along with conservation principles. This greatly limits their range of applicability and scope of future developments in a similar direction.

A more comprehensive and generic model formulation can be developed following a mechanistic approach which entails implementation of conservation principles (mass and momentum) on each fluidic phase. This approach was coined as ‘two-fluid’ by nuclear industry for predicting air water flow in pipeline network entities. An appropriate selection of empirical correlations to attain closure on mass and momentum balance yields governing equations representative of the system being studied.

Two phase flow studies play a vital role in natural gas transportation through complex pipeline networks in midstream operations. Natural gas recently gained interest in Pennsylvania due to developments in Marcellus Shale, tight-gas reservoirs. The production of natural gas from well heads follows transportation through complex pipeline network before reaching an end user. The role of compression in this process is instrumental in determining economic feasibility of pipe-line transportation. A large pressure head maintained at production end, using compressors, and a low pressure at consumption end due to frictional head losses, drives the transmission process. The friction between flowing fluids and pipeline walls thus becomes a necessary evil during gas transmission.
However, appearance of an additional liquid-phase either due to retrograde condensation or precipitation of moisture content greatly increases frictional losses thereby increasing compression costs. Retrograde condensation is a frequently encountered phenomenon during midstream operations due to separation of condensate at high pressures. The liquid phase thus formed acts as an additional frictional component drastically increasing head losses during transmission. Experimental investigations ordered to characterize behavior of this additional phase revealed preferential pathways followed by liquid at T-joints in the pipeline network. Models must therefore be developed to predict flow conditions at T-junctions to isolate and suggest liquid removal scenarios.

In this study, a two-fluid approach in conjunction with finite-volume discretization was undertaken to propose a new model formulation predicting flow conditions at a tee junction. A steady state, isothermal assumption along with one dimensional area and time averaging of flow parameter was used to reduce system intricacies. Model formulation was developed for low liquid loading systems, predominant in midstream operations, for stratified wavy, stratified smooth and mist flow patterns. A constant density liquid and constant super-compressibility gas was taken as additional simplifications. The resulting two-fluid finite-volume constitutive equations were solved simultaneously using generalized Newton-Raphson technique.
Chapter 2

Literature Survey

Oranje (1973) pioneered two phase split studies at T junctions. The empirical model proposed was amongst the first few who put forth a thermodynamic view to the problem by presenting a section of phase envelope (dew point curve) that encompasses the region of pipeline operations in general. Thus an investigation was ordered serving two principal objectives:

1. Occurrence of condensation in the pipeline network.
2. Path selectivity of the liquid phase in the network.

An experimental study on splitting phenomenon at T junction was carried out on two experimental installations. The first one was a transparent Tee designed for pressures up to 21.8 psig. Both straight and reduced Tees were studied with branch arm diameters of 60 and 40 mm respectively for a run arm diameter of 60 mm with known amount of liquid (180 cc/min) being injected into the inlet stream. A distance of 10 m was maintained between point of injection and T joint to ensure fully developed flow pattern upstream of the Tee. Flow meters were installed in the branch and run arms to measure gas flow rates. A second setup was developed for higher pressure operations (up to 435 psig) for reduced and straight branch arms of 50 and 75 mm at inlet arm diameter of 75 mm. Liquid injection rate was fixed at 50 cc/min. The model was based upon a simple formulation maintaining same gas velocity heads as were encountered during normal pipeline operations given by Eqn. (2.1).

\[
\frac{1}{2} \rho v^2 = velocity\ -\ head = const. \tag{2.1}
\]

Where,

\[\rho_{\text{gas}} = \text{density of gas at flowing conditions (kg/m}^3)\]

\[V_{\text{gas}} = \text{velocity of gas (m/sec)}\]
A visual analysis of the transparent tee developed especially for the purpose of this study by Oranje (1973) established three types of flow pattern configurations—annular, stratified smooth and wavy flow. Branch liquid take off (fraction of liquid entering branch arm) was plotted as a function of branch gas take off (fraction of gas phase entering branch arm) for upstream gas velocities of 3, 5 and 7 m/s for reduced and straight Tee case. These plots suggested that a critical or threshold value of branch gas take off existed below which no liquid enters branch arm. Oranje (1973) also confirmed the existence of an upper limit to branch gas mass fraction beyond which all liquid enters the branch arm. This behavior was usually encountered over small branch gas mass fraction referred to as ‘flip-flop’ behavior. It was also interesting to note that the former phenomenon occurred at lower gas flow rates and tends to decrease as upstream gas flow rates increase. In his study, Oranje (1973) suggested several mechanisms to account for this characteristic ‘flip-flop’ behavior:

1. Under-pressure on the inside-bend of the T joint at which gas enters into branch arm. This was attributed to the centrifugal forces acting on gas phase creating a vacuum at the bend.
2. Liquid phase inertia while neglecting gas phase inertia.
3. Upstream flow pattern dependency.
4. Geometry of Tee joint—straight, reduced, sharp, radiused.

In his study, Oranje (1973) suggested the above factors can be related to gas velocity head by introducing dimensionless parameters or empirical correlations. The results and observations made were extended and applied to similar pipeline configurations. However, range of applicability of the model was restricted due to its inherent dependency on experimental characteristic for a specific pipeline configuration.

The issue of phase splitting at T-junctions applies to a large number of chemical industries e.g., petrochemical refineries, pharmaceutical plants etc which thrive upon continuous supply of feed material to various sub-units through pipeline transportation. Fluid transport over long distances is mainly achieved by transmitting them through pipelines or annuli. In natural gas industry, gas movement is achieved by creating a pressure gradient. Thus, compression costs are instrumental while developing and
constructing gas pipe line networks. Appearance of liquid phase drastically increases pressure losses and thus the compression costs. An extensive investigation on to optimize and predict flow behavior accurately is therefore abundant in literature. A detailed study of existing articles suggests that two phase flow models can be broadly classified into three categories:

a) **Empirical Models**: These were developed for a particular set of conditions based upon formulation of dimensional as well as dimensionless independent variables which characterize the problem at hand. Its inherent dependency on experimental data for generation of correlations greatly narrows down the validity and applicability of predicted data for conditions outside experimentally verified range.

b) **Phenomenological Models**: Models which require a physical definition of the system before conservation principals are applied fall under this category. Geometrical models, which will be discussed shortly, are symbolic of this approach, and require a definition of flow pattern upstream. A formulation based upon geometrical considerations, constraints accounting a phenomenon and conservation principles result in a constitutive equation set representative of the system being studied.

c) **Mechanistic Models**: Mechanistic models rely upon formulation of mathematical equations based upon principles of conservation when applied to the system being studies. In general, continuity, momentum and energy balance employed along with appropriate closure relationships (empirical or otherwise) form a governing equation set.

A geometrical model developed by Shoham et al (1984) was based upon momentum balance and definition of fixed geometry of gas liquid split at the T junction. Shoham (1984) presented one of the first models to consider streamline approach for predicting branch liquid take off. A separating streamline was defined each for gas and liquid phase dividing each fluid stream into inlet-run and inlet branch flow streams at the T-junction. It was proposed that liquid phase splitting occurred due to the centripetal forces acting on liquid stream at the inside edge. A centripetal force expression was
defined as a function of gas and liquid phase velocities and tee geometry. This centripetal force incorporated in the gas and liquid phase momentum balances affected the split.

The upstream flow pattern dependency, as predicted by Oranje (1973) were confirmed from independent experiments conducted by Shoham (1984) for smooth and wavy stratified flow patterns. The experimental studies were later extended to annular flow pattern as well by Shoham (1987). The liquid and gas take-off fractions in the branch and run arm were plotted to distinguish between annular flow, wavy stratified and smooth stratified flow patterns at different inlet liquid superficial velocities and constant inlet gas superficial velocities.

The experimental findings suggested that split at T-junction is affected by upstream conditions such as flow pattern, gas and liquid superficial velocities. The results also show that for wavy stratified flow (high gas flow rates) most of the liquid tends to enter branch arm at lower liquid superficial velocities and decreases as the liquid velocity is increased. This is evident because as the liquid inertial force increases more liquid will by-pass the branch arm and at low liquid velocities opposite will occur due to lower inertial force and film stopping phenomenon coming into play due to low liquid momentum. However, contrary to the above case for the smooth stratified flow case (Vsg=2.5 m/s) significant liquid flow into the run arm occurs for same range of liquid superficial velocities.

In addition to this, observations also show that a larger threshold value of gas take-off in the branch arm is required for liquid take-off in the branch arm to occur. This feature might be attributed to the centrifugal forces at T junction, since for the wavy stratified case at higher inlet gas superficial velocity (Vsg=6.1 m/s), a higher gas flow rate in the branch arm at the same branch gas fraction will be encountered thereby increasing the under. A similar rationale can be provided for lower branch liquid fraction for the smooth stratified case (Vsg= 2.5 m/s). Thus, a balance between, centrifugal and inertial forces acting on the liquid phase at the Tee may be defined as momentum balance component charactering the transition from wavy to smooth stratified flow. An extensive literature was found (James P. Brill, 1999; Shoham, 2005) where the above transition is characterized using liquid holdup and upstream gas and liquid superficial velocities, for
two phase flow in straight pipelines. Further on, for annular flow case ($V_{sg} = 26$ m/s), most of the experimentally collected data points lie below the line of equal phase split depicting that most of the liquid enters branch arm and decreases as liquid superficial velocity is increased. Thus, one can infer that centrifugal forces dominate momentum balance for high gas flow rates at low liquid holdup values.

A ‘unified model’ was later presented by Shoham et al (1996) for two phase flow splitting at reduced T junctions accounting for a wide range of branch arms inclinations with respect to inlet and run arms. The choice of centripetal force expression in the former geometrical model (Shoham, 1984) was unclear and hence the split dependency on centripetal force was removed in this formulation. It was suggested that splitting of liquid at T junction was affected by the dominant forces controlling physical phenomenon:

a) *Gravity forces:* This is divided in two parts, one part accounts for the inclination of branch arm whereas the other accounts for the elevation difference between branch and inlet arms of a reduced Tee. For straight Tees the latter becomes zero.

b) *Inertial Force:* This represents the flow momentum of the two phases, for the model under consideration gas phase inertial momentum is assumed to be zero. Due to smaller diameter of reduced Tees, higher liquid velocities will lead to lower liquid runoffs in the branch arm, since most of the liquid will by-pass branch arm.

c) *Pressure-Drop Driving force:* Pressure drop has a significant impact on liquid run off in branch arms. Thus a component accounting for it must be included in the momentum balance.

A recent experimental investigation carried out by Mak and Azzopardi (2006) on two phase split at a vertical T-junction presents comparison with one of its predecessors (Stacey et al, 2000). Small pipe diameters ranging from 5 mm to 10 mm were used with inlet gas superficial velocities ranging from 3.01 m/s to 14.63 m/s, wherein liquid holdup due to entrainment can be neglected. Observations suggested the phase split is independent of inlet gas velocity for the above mentioned range. However, considerable
difference at higher inlet gas velocities is indicative of existence of a threshold gas superficial velocity. More liquid take off in branch arm occurs at higher inlet gas velocities which can be attributed to higher gas flow rate entering the branch arm at lower branch gas takeoff, leading to large under-pressures. The split is independent of liquid superficial velocity at low gas superficial velocities, but is a strong function of liquid velocity at high gas velocities. Thus, it can be inferred that split can be affected by using a certain ratio of gas to liquid superficial velocity at high gas velocities.

It was suggested that for the above mentioned diameter range phase split is a weak function of T joint configuration or inclination due to the absence of flooding phenomenon. The interfacial and wall shear is much more pronounced, as compared to our case wherein these forces have a weak influence. One of the major issues with application of this experimental data is scalability to large diameter pipelines.

Inferences were also drawn from air-water split models used in nuclear power industry. Cooling of reactor core is the single most critical factors in safe operation of a nuclear reactor. Water (light or heavy) is amongst the most commonly used coolants, because of its high latent heat of vaporization and low capital requirements. T-joints are often encountered in all industrial material transport applications and it is evident from the study conducted by Oranje (1973) that an uneven split is frequently observed. This mal-distribution of water and steam may send steam, which has a low sensible heat, into the reactor core thereby drastically reducing cooling efficiency and possible blow down of core termed as Loss of Coolant Accident (LOCA).

Saba and Lahey (1984) put forth a phenomenological model predicting air water split at T junctions for a hypothetical loss of coolant accident (LOCA). An experimental setup was prepared to isolate parameters characterizing the split. The setup contained a T joint with pressure taps located along the length of inlet, branch and run arms to obtain the pressure distribution located at section 1, 2 and 3 shown in Figure 2.1. Appropriate lengths of arms were used to ensure fully developed flow upstream of Tee and to reduce any possible downstream static pressure interference at the Tee. Compressed air and water was passed simultaneously through a mixing Tee to achieve two phase flow in the inlet arm. A combination of separator and air water orifice meters was located at each of
the outlet arms which provided air and water flow rates, measure as mass fluxes (kg/m$^2$/hr).

Saba and Lahey (1984) identified three different qualitative volume elements, shown in Figure 2.1, for a T-junction. A model formulation was then developed based upon momentum conservation on inlet-run and inlet-branch flow streams. The momentum conservation equation was expressed in terms of irreversible pressure losses at T-junction later calculated from empirical correlations. Azzopardi (1999) presented a brief overview of control volume representing the Tee junction. One can qualitatively deduce the parameters characterizing control volume as:

1) **Developed Flow**: The inlet, branch and run arm lengths selected must be such that a fully developed flow enters and leaves Tee control volume. This also ensures any recirculation of liquid phase back into the due to Tee configuration (topology, branch arm inclination etc.) or turbulence, are accounted for.

2) **Spacing between network entities**: The models developed so far do not take into account interaction of Tee with neighboring network entities which are common in any gas transmission network. In other words, Tee control volume is free from external influences such as pressure heads upstream or downstream. Thus, a bend or orifice, etc. in the vicinity of a Tee is a completely different problem and must be addressed separately.
It is worthwhile to note that, the two factors discussed above are counter intuitive, since developed flow criteria suggests inclusion of infinite arm lengths to ensure fully developed flows whereas the spacing between already fixed network entities rules out infinite (large) arm lengths. A balance between these factors will enable us to identify the optimal control volume in future developments.

Saba and Lahey (1986) later presented a comparative review on phenomenological and empirical models present at that time with his mechanistic model. It was suggested that none of the models until then were able to holistically address the issue of two phase split at T junction. Further on, contemporary models at that time (Azzopardi and Whalley, 1982) were accurate only for low branch liquid take off. An outline of an interim model was proposed by Saba and Lahey (1986), in an attempt towards a more generic model. Thus the resulting model can thus be considered as a compendium which encompassed nearly all models proposed in the near time frame.

In 1990, Lahey came up with a comprehensive 3-Dimensional model for phase split at Tee junctions containing separate conservation equations for each phase or the so called “Two Fluid Model”. One peculiar feature of this model is the level of detail gained by a thorough description of interfacial and wall force balance and transfer laws which is lost in 1-D modeling due to space and time averaging. Even more intriguing is the consideration of energy transfer, and dissipation due to turbulence (eddies).

Although, experimental investigations were based on bubbly flow in a vertical pipeline, a brief description for modeling stratified flow was provided along with modifications in the correlations suggested for bubbly flow. Before we delve any deeper into modeling approach, it is advisable to for the reader to get acquainted with description of shear stresses associated with turbulent flow. Turbulent flow is characterized by the formation of vortices or eddies which contribute to dissipation of energy when an eddy is destroyed by viscous forces. An additional term is thereby used in energy conservation equations to account for this dissipation which is otherwise absent in laminar flow. A stochastic analysis of turbulence though can provide us with useful insights into the mechanisms of turbulence. Lahey (1990) suggested that the effect of turbulence can be neglected for stratified flow in inlet, run and branch arms. However, same cannot be said
for flow splitting at T-junction. It is interesting to note that a modified form of drag force was used to account for junction momentum dissipation assuming bubbly flow indirectly accounting for turbulence at the tee.

Hart, Fortuin, & Hamersma (1990) proposed a ‘double stream’ model based upon separate energy balance equations for gas and liquid phase following inlet-branch and inlet-run arms. The double stream model (DSM) explained static pressure loss and gain in the run and branch arm respectively observed by Hart (1990) during his phase split experiments. Gas and liquid phase friction loss coefficients were defined to account for junction energy losses. Gardel’s correlations (1957), developed for single-phase gas-flow at T-junctions, were used to calculate these friction factors. Ottens et al (2001) later improved upon this with an ‘advanced double stream’ model (ADSM) based upon separate friction loss coefficients for liquid and gas phase. A liquid velocity profile correction was also incorporated to account for the parabolic liquid velocity profile which is lost during area averaging in 1D modeling. A mechanistic approach based upon was later presented by Margaris (2007) wherein momentum and mass conservation principles were applied to obtain governing equations representative of flow system. It was suggested that appropriate closure relations are crucial for closely predicting flow conditions existing at T-junctions.
Chapter 3

Problem Statement

Complex pipeline networks used to transport natural gas often encounter condensate separation. From a thermodynamic aspect, this separation of liquid phase from natural gas mixture occurs at high pressures and is referred to as retrograde condensation. The appearance of a liquid phase exacerbates the already complex situation of modeling single phase flow to two phase flow in gas pipeline networks. Although the liquid content is rather small relative to gas volumes, liquid loading as small as 5% (Ottens, 2001) drastically increases pressure loss during transmission. This results in increased compression costs required to maintain a pressure derivative for gas transmission. Thus, isolation and removal of condensate phase becomes imperative for economical operation. Another source of liquids is formation water, thus pipeline networks rarely operate in single phase conditions.

Maldistribution of liquid phase at T-junctions is one of the major issues while developing models for gas pipeline networks. Any complex pipeline network can be reconstructed by using variants of two network entities: (a) pipeline and (b) tee junction. A T-junction is characterized by joining of three flow lines shown as equal diameter inlet, run and branch arms in Figure 3.1 for a horizontal, regular, straight Tee.

![Figure 3.1: Tee Junction](image-url)
Extensive experimental data is present in literature on ‘flip-flop’ phenomena (Oranje, 1973) observed in liquid phase tee split. Depending upon inlet and outlet flow conditions, liquid could flow completely either into run or branch arms. An actual phase split was observed only for a narrow range of experimental specifications (Oranje, 1973; Hart, 1990). Thus, liquid phase follows a specific path in gas transmission network depending upon flow conditions. Prediction of preferential pathways followed by liquid phase will enable us to isolate specific locations in the network and suggest possible liquid removal scenarios.

A number of two phase flow models are available in literature (discussed in Chapter 2) addressing the issue of liquid phase split at tee junctions. However, such an independent treatment of tee junctions will not be able to capture the influence of interconnectivity on phase split while predicting flow conditions in a complex pipeline networks. This study endeavors to develop a steady-state, two-fluid finite-volume (TFFV) model in order to evaluate and predict flow conditions for the two network entities (pipelines and tees) on a common basis of mass and momentum conservation. The gas and liquid phase temperatures were considered to be equal assuming isothermal conditions and similar flow pattern for inlet, branch and run arms during model formulation.

Numerical solution of governing TFFV equations was accomplished using Generalized Newton-Raphson (GNR) technique. GNR allows simultaneous solution of linear algebraic equations thereby preserving interconnectivity between network entities. This study focuses mainly on small network entities i.e., tee junctions, thus gas-liquid inter-phase mass transfer is neglected. The length and diameter of inlet run and branch arms were kept equal however, this model is not restricted by preceding assumption. An incompressible liquid phase was considered along with the assumption of constant super-compressibility ($Z = \text{constant}$) gas phase. The model applicability is restricted to low liquid loading systems generally encountered in gas pipeline networks.
Chapter 4

Model Description

Analytical solution of PDEs (Partial Differential Equations) is not always possible and alternative solution techniques are thus imperative. A numerical solution, on the other hand, can be accomplished with relative ease. Two widely used numerical methods for solving PDEs have been discussed along with their pros and cons. The aptness of FVM in contrast to FDM is also inspected for the purpose of two phase split study at Tee junctions. Finite Difference Method (FDM) relies upon converting PDEs into corresponding algebraic equations wherein differential operators are replaced by difference operators using Taylor series expansion after suitable truncation. This replaces otherwise continuous system represented by PDEs into discrete nodes representing finite elements. The validity of conservation principles is requisite if a physically realizable solution is to be reached. These algebraic equations solved simultaneously provide desired system parameters at each node for the desired numerical precision (restricted by processor bit length).

The Finite Volume (FV) approach begins by discretizing a system into predefined volume elements known as control volumes (CV). The corresponding extensive system-parameters are assumed to be constant over each control volume. Conservation principles (mass and momentum balance) are then applied over this fixed-volume, open-system to yield representative algebraic equations. Another way of developing linear algebraic constitutive representing the system is to integrate the corresponding PDEs over a control volume. A derivation of FV constitutive equations from PDEs (for which conservation principles are valid) integrated over control volumes was presented by Alp (2007). Thus, conservation principles are equally valid on the resulting.

The two approaches discussed differ solely in the way algebraic equations are formulated. FVM renders a physical sense allowing an independent model formulation representative of the system. Governing equations were obtained by directly applying
conservation principles on a constant-volume (control volume) open-system. Fig. 4.1 (b) demonstrates inherent flexibility of FV discretization wherein a FV approach allows ease of formulation of governing equations on a non rectangular grid block. Another advantage of implementing finite volume approach is discussed later in section 4.10. Wherein, mathematical connectivity of flow parameters at finite volume grid blocks is emphasized. Effective translation of information from grid block where specifications are provided to generic blocks where unknowns are calculated serves as the rationale behind this approach. This is of prime importance while studying role of interconnectivity between system elements.

![Discretization Schemes](image)

**Figure 4.1:** Discretization Schemes (a)Finite Difference Grid(left) (b) Finite Volume Grid(right)

However, an inappropriate choice of difference operators replacing differential operators in PDE might inadvertently introduce instability in both FD (Finite Difference) and FV (Finite Volume) formulations. Both FDM and FVM are prone to precision errors often introduced in FD equation set when a Taylor series is truncated to obtain a difference expression corresponding to a differential. These error expressions are similar to mathematical forms representative of physical phenomena such as dispersion and diffusion. An iterative solution might therefore leads to loss of precision due to growth of error at each step.
4.1 Flow at T-Junctions

The split is affected mainly by pressure conditions downstream in the branch and run arms, flow parameters upstream and additional losses due to Tee geometry. Ideally, flow at a tee junction can be classified into three major categories (a) diverging, (b) converging, and (c) impacting tee based upon flow directions in each of the inlet, run and branch arms. Previous work in this field presents models which are specific for each case based upon highly empirical approach. Hence, their applicability is limited due to subjective nature of model predictions.

An intuitive insight suggests that the aforementioned three T-split cases differ in losses encountered at the junction due to varying flow patterns. This study presents a novel method capable of addressing the three cases, as shown in Fig. 4.2 at a generic level. The model allows ease of modification of junction losses for each of the above case so that a slight degree of subjectivity can be incorporated with an increased range of applicability. This investigation mainly considered diverging split cases for straight and reduced regular, horizontal tee junctions. However, a parametric study for converging tee case is presented later in section 6.5 demonstrating applicability of the proposed model to cases other than diverging T-junction.

In actual practice, edges at T-junction are smoothed during manufacturing either by casting or machining with varying angles between run and branch arms. Additional parameters were incorporated in Tee junction losses to account for the effect of radiasation and angle between run and branch arms. A different expression accounting
for Tee losses is required for each flow pattern will be discussed in following sections while considering each case separately.

4.2 Two Fluid Finite Volume Derivation

Governing equations were formulated by applying conservation principles on mass and momentum (Newton’s second law) on a constant volume open system (control volume). A detailed derivation was also presented by Alp (2007) wherein conservation principles were applied followed by Reynolds transport theorem to convert from a control mass system to control volume system. In this section, straightforward mass (Fig. 4.3) and momentum (Fig. 4.4) balances on a bounded, constant volume, cylindrical element yields constitutive equations representative of single or two phase flow are presented.

4.2.1 Mass Conservation

Fig. 4.3 below shows a FV block of length ‘Δx’ depicting two phase gas/liquid flow. The shaded region represents volume occupied by liquid phase inside the cylindrical element whereas gas phase occupies the rest. Mass balance for any bounded system can be written as:

\[
\text{(Rate of mass flow in)} - \text{(Rate of mass flow out)} = \text{(Rate of mass accumulation)}
\]  

(4.1)
Applying mass conservation on gas and liquid phase for the system depicted by Fig. 4.3 yields:

\[
(m_g)_{i\frac{1}{2}} - (m_g)_{i+\frac{1}{2}} = \frac{dM_g}{dt}
\]

(4.2)

\[
(m_l)_{i\frac{1}{2}} - (m_l)_{i+\frac{1}{2}} = \frac{dM_l}{dt}
\]

(4.3)

Assuming constant gas/liquid phase cross-sectional areas \(A_{g/l}\) and densities \(\rho_{g/l}\) over a control volume mass conservation equations for gas and liquid phases can be expressed as:

\[
(\rho_g A_g U_g)_{i\frac{1}{2}} - (\rho_g A_g U_g)_{i+\frac{1}{2}} = \left[\frac{d(\rho_g A_g \Delta x)}{dt}\right]_{i}
\]

(4.4)

\[
(\rho_l A_l U_l)_{i\frac{1}{2}} - (\rho_l A_l U_l)_{i+\frac{1}{2}} = \left[\frac{d(\rho_l A_l \Delta x)}{dt}\right]_{i}
\]

(4.5)

Where,
- \(m_{g/l}\) = Mass flow rate of gas/liquid phase
- \(M_{g/l}\) = Mass of gas/liquid inside control volume
- \(A_{g/l}\) = Cross sectional areas occupied by gas/liquid phase
- \(\Delta x\) = Length of finite volume block
- \(\rho_{g/l}\) = Density of gas/liquid phase
- \(U_{g/l}\) = Gas/liquid phase velocities
4.2.2 Momentum Conservation

Fig. 4.4 below shows a free body diagram of a FV block of length ‘Δx’ depicting forces acting on gas and liquid phases. Pressure being an isotropic property was assumed to be equal for gas and liquid phases at cell block edges and hence phase subscripts were dropped. Momentum balance can be written similar to mass balance as:

\[
\text{(Rate of momentum flow in)} - \text{(Rate of momentum flow out)} + \text{(Net forces acting in the flow direction)} = \text{(Rate of momentum accumulation)} \tag{4.6}
\]

Since momentum is a vector quantity a positive x and y axes were chosen as positive directions conventionally during model formulation. Assuming constant gas/liquid cross-sectional areas (\(A_{g/l}\)) and densities (\(\rho_{g/l}\)) over a control volume momentum conservation equations for each phase can be expressed as:

**Gas Phase Momentum Conservation**

\[
(\rho g_A g |U_g|)_{i-\frac{1}{2}} - (\rho g_A g |U_g|)_{i+\frac{1}{2}} + \left[ (PA_g)_{i-\frac{1}{2}} - (PA_g)_{i+\frac{1}{2}} - (F_{g,wall})_i - (F_{int})_i \right] = \left[ \frac{d(\rho g_A g \Delta x U_g)}{dt} \right]_i
\]  \tag{4.7}
Liquid Phase Momentum Conservation

\[
\begin{align*}
(\rho A_i U_i | U_i |)_{i-\frac{1}{2}} - (\rho A_i U_i | U_i |)_{i+\frac{1}{2}} + \\
\left[ (PA_i)_{i-\frac{1}{2}} - (PA_i)_{i+\frac{1}{2}} - (F_{i,\text{wall}}) + (F_{\text{int}}) \right]_i = \left[ \frac{d(\rho A_i \Delta x U_i)}{dt} \right]_i
\end{align*}
\]

(4.8)

Where,

- \( P \) = Pressure exerted
- \( F_{g/l, \text{wall}} \) = Gas/liquid wall shear force
- \( F_{\text{int}} \) = Gas-liquid interfacial shear force

Steady state flow conditions were assumed during model development thus time varying terms were neglected. The values of parameter at cell edges are contingent upon choice of staggered or coincidental gridding scheme discussed in later sections. Thus, Eq. (4.4) thru Eq. (4.8) form the basis of two-fluid finite-volume (TFFV) formulation. Constitutive equations for single-phase gas flow can be inferred from two-phase flow equations by neglecting liquid phase mass and momentum conservation i.e, Eq. (4.5) and Eq. (4.8). Since only gas phase is being considered interfacial force has no significance and additionally flow area is same as pipe cross-sectional area. These features can be realized by implementing following conditions in Eq. (4.9):

\[
F_{\text{int}} = 0 \\
A_g = A_{\text{cross-section}} = \pi D^2
\]

(4.9)

Where, \( D \) = diameter of pipe or annulus

4.3 Donor Cell Scheme

An ‘upwind donor cell’ or ‘advective property’ scheme was adopted for mass conservation during equation formulation step. Patankar (1980) suggested that for flow
conditions with high Peclet numbers (ratio of convective to diffusive flow), flow parameters are solely influenced by upstream conditions. Thus, mass transfer due to diffusion in principal flow direction of fluids is negligible as compared to convective or advective flow. For pipeline networks, gas/liquid flow rates are relatively large therefore diffusive flow due to concentration difference (compositional model) can be neglected. Thus, this study assumes a constant gas and liquid composition while formulating governing equations. Therefore, diffusive flow mechanisms were not considered during model formulation providing a justification for implementing an upwind scheme.

An arbitrary selection of forward, backward or central difference schemes while replacing differential operators in a PDE with a difference operator to obtain a linear algebraic equation during FDM might lead to instability. An upwind scheme incorporated in FD formulation avoids instability by embedding a dependency on upstream conditions. Thus, gas and liquid phase velocities at cell block edges are expressed as functions of adjacent upstream block parameters. Whereas, in FV formulation advective property scheme determines density and fluid flow areas at cell block edges as functions of upstream block density and flow area respectively. Thus, while donor cell scheme is a mere justification to circumvent instability in FD formulation, for FV formulation it has a physical significance.

An advective property or donor cell scheme assumes flow parameters at block edges are strongly influenced by upstream conditions. Hence, cross-sectional flow areas, densities at FV block cell edges were held equal to respective adjacent upstream block values. Eq. (4.4) and Eq. (4.5) can then be written as:

Gas Phase Mass Conservation

\[
\left( \rho_g A_g U_g \right)_{i-1} - \left( \rho_g A_g U_g \right)_i = \left[ \frac{d \left( \rho_g A_g \Delta x \right)}{dt} \right]_i
\]  

(4.10)

Liquid Phase Mass Conservation

\[
\left( \rho_l A_l U_l \right)_{i-1} - \left( \rho_l A_l U_l \right)_i = \left[ \frac{d \left( \rho_l A_l \Delta x \right)}{dt} \right]_i
\]  

(4.11)
4.4 Primary Unknowns

The selection of primary unknowns or independent variables is based upon flow parameter to be determined, choice of convenient specifications and effective translation of information during numerical solution. For e.g., pressure information can be propagated using density by selecting an equation of state. However, a weaker interconnectivity is observed as compared to the case wherein pressure is the independent variable. Fig. 4.20 presents the nomenclature of variables used along each flow direction.

Table 4.1: Inventory of Unknowns (Two Phase Flow)

<table>
<thead>
<tr>
<th>Non Tee Cell</th>
<th></th>
<th>Tee Cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>P</td>
<td>Pressure</td>
</tr>
<tr>
<td>Liquid Holdup</td>
<td>( \alpha_l )</td>
<td>Liquid Holdup</td>
</tr>
<tr>
<td>Gas Velocity</td>
<td>( U_g )</td>
<td>Gas Velocity</td>
</tr>
<tr>
<td>Liquid Velocity</td>
<td>( U_l )</td>
<td>Liquid Velocity</td>
</tr>
<tr>
<td><strong>Total Unknowns</strong></td>
<td>4</td>
<td><strong>Total Unknowns</strong></td>
</tr>
</tbody>
</table>

Table 4.2: Inventory of Unknowns (Single Phase Flow)

<table>
<thead>
<tr>
<th>Non Tee Cell</th>
<th></th>
<th>Tee Cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>P</td>
<td>Pressure</td>
</tr>
<tr>
<td>Gas Velocity</td>
<td>( U_g )</td>
<td>Gas Velocity</td>
</tr>
<tr>
<td><strong>Total Unknowns</strong></td>
<td>2</td>
<td><strong>Total Unknowns</strong></td>
</tr>
</tbody>
</table>

Figure 4.5: Nomenclature
Pressure (P), gas/liquid phase velocities (U_{g/l}) and liquid holdup (\alpha_l) were considered as independent variables. Liquid holdup (\alpha_l) defined as fraction of volume occupied by liquid was used later on to replace gas and liquid flow areas (A_{g/l}) depending upon flow geometry. For single phase flow, holdup value is constant, thus pressure and gas or liquid phase velocity (U_g or U_l) are the only primary unknowns. Similarly, for two-phase flow a total of four independent variables exist at each grid block. Isothermal conditions were assumed at all times during model development, thus thermal energy balances over control volumes was neglected. Table 4.1 and Table 4.2 below provide a description of unknowns for single and two phase flow in pipe and T-junction cell blocks.

4.6 Finite Volume Discretization – Staggered Gridding

FV discretization of pipe and tee junction was accomplished by defining constant-volume open-systems called control volumes to form governing equation. Conventionally, mass and momentum conservation principles were applied on overlapping control volumes with flow parameters representing a volumetric average, known as coincidental gridding. Patankar (1980) proposed a staggered grid approach, discussed by Versteeg & Malalasekera (2007), used a separate grid for momentum conservation. A staggered grid was defined for momentum conservation relative to mass balance grid.

![Checkerboard Problem – Patankar (1980)](image)

Figure 4.6: Checkerboard Problem – Patankar (1980)

The rationale suggested behind this approach is threefold. Firstly, staggering avoids checkerboard problem discussed by Patankar (1980), resulting in an unrealistic solution to a set of algebraic equations for coincidental grids. Patankar considered an
example wherein, pressure drop \( \frac{dP}{dx} \) integrated over a control volume shown (shaded region) in Fig. 4.6 is given by the following expression:

\[
\left( \frac{dP}{dx} \right)_b = P_{b-\frac{1}{2}} - P_{b+\frac{1}{2}}
\] (4.12)

Although edge values can be expressed as complex functions of grid point values, Patankar suggested an oversimplified case for the ease of understanding assuming piecewise linear profile over the control volume. In order to express edge values in terms of grid point values, a midpoint averaging was applied resulting in:

\[
P_{b-\frac{1}{2}} - P_{b+\frac{1}{2}} = \frac{P_a + P_b}{2} - \frac{P_c + P_b}{2}
\] (4.13)

A zig-zag pressure profile, shown in Fig. 4.6, forms a possible solution for the system represented by Eq. (4.13) referred to as ‘checkerboard problem’. Similar arguments were put forth for zig-zag velocity profile.

Figure 4.7: Zig-zag Pressure Path

Secondly, Patankar (1980) also suggested that coincidental grid defines all primary unknowns at grid points. While expressing edge parameters in terms of adjacent grid point parameters, as discussed before, a coarser grid is generated resulting in reduced solution accuracy. This can be more clearly understood from Fig. 4.8(b) below. Flow-parameters at cell edges marked by ‘A’ and ‘B’ are expressed as functions of adjacent grid block parameters thereby making the actual control volume larger.

Lastly, staggering of momentum blocks allows velocities to be defined at the cell block edges providing equation formulation with a physical relevance. Fig. 4.8(a) shows a generic FV (Finite Volume) block ‘i’ for a forward staggered momentum grid wherein pressures \( P_i \) and \( P_{i+1} \) act as a driving force for block velocity \( U_j \). Substitution of zig-zag velocity and pressure profiles in Eq. (4.4) and Eq. (4.7), assuming single phase gas flow, satisfy governing equations if and only if a constant density, zero shear force system is
considered. Thus, the example presented by Patankar (1980) assumes mass and momentum conservation equations to be decoupled. However, in this study constitutive equations have a high degree of correlation.

For single phase gas flow, pressure dependency of density, and velocity dependency of shear force at each grid block. For two-phase flow, liquid holdup serves as an additional synergistic parameter, coupling gas and liquid phase equations. Thus, ‘checker board’ problem does not pose any threat for the proposed model predictions. The shear forces in Eq. (4.7) and Eq. (4.8) are functions of central grid block phase velocities. Hence, a coarser grid is not encountered even if edge values are expressed in terms of adjacent block parameters during momentum conservation. Moreover, mass conservation is based upon the assumption of advective property i.e. edge values expressed as functions of adjacent upstream grid block values.

4.7 Boundary Conditions

So far, we have only discussed development of governing equations based upon conservation principles. In order to solve a system of equations, the number of unknowns (n) must be equal to the number of equations (m). However, this is seldom the case during formulation of constitutive equations representing a system. Thus remaining (n-m) unknowns must either be specified or additional equations, also known boundary conditions, must be developed to account for unknowns.
Boundary conditions represent either a physical boundary such as a no flow boundary or a constraint on parameters at the inlet or outlet. Patankar (1980) proposed ‘outflow’ or zero curvature (zero-gradient) boundary condition constraining flow parameters at the inlet or outlet. The applicability of ZC (zero curvature) is based upon assumption of fully developed flow with no recirculation suggested by Versteeg and Malalasekera (1995) as shown in Fig. 4.9. The boundary condition is depicted by the blue line where fully developed flow conditions exist. This preserves assumptions of unidirectional flow and constant cross-sectional area average velocity value at a FV grid block. The zero curvature boundary condition written for an independent variable (P) is:

\[
\frac{\partial^2 P}{\partial x^2} = 0 \Rightarrow \frac{dP}{dx} = \text{const.} \quad (4.14)
\]

Integrating, Eq. (4.14) over a control volume domain, wherein discretization is such that each block has same length \((\int dx = \text{const.})\), results in:

\[
\int dP = c \int dx \Rightarrow \Delta P = c \cdot \Delta x
\]

For the case shown in Fig. 4.10, the zero curvature boundary condition is given by Eq. (4.15) where \(\Delta x_1\) and \(\Delta x_2\) distance between centres of the grid blocks.

\[
\frac{P_i - P_{i-1}}{\Delta x_i} = \frac{P_{i+1} - P_i}{\Delta x_{i+1}} \quad (4.15)
\]
If equal volume cell blocks are considered i.e. $\Delta x_i = \Delta x_{i+1}$ this reduces to,

$$\Rightarrow P_i - P_{i-1} = P_{i+1} - P_i = \text{const}$$

$$\Rightarrow P_{i+1} = 2P_i - P_{i-1}$$  \hspace{1cm} (4.16)

Mathematically, this is equivalent to linear extrapolation of unknown $P$ from two adjacent values. Outflow boundary conditions was used to reduce velocity and liquid holdup specifications either at the outlet or inlet while using forward, backward and information translation grid approaches discussed in sections 4.8 thru 4.10. However, it can also be used to switch from one set of specifications to other both at inlet and outlet.

4.8 Staggered Approach - Pipe Flow

Two gridding schemes were implemented wherein momentum grid was either staggered in forward or backward direction. Mass and momentum conservations principles applied on control volumes then yield governing equations representative of the system. In both cases, an upwind donor cell scheme, as discussed in section 4.2., was adopted for an advective flow formulation.
4.8.1 Forward Staggered Formulation

A non overlapping BC (Body Centered) momentum grid shifted in the flow direction relative to BC mass grid was considered as shown in Fig. 4.12. The block centers represent average (time and area) values of gas/liquid flow areas \( A_{g/l} \), densities \( \rho_{g/l} \) and pressure \( P \) for a mass block and phase velocities \( U_{g/l} \) for a momentum block. The centers of momentum block representative of block velocity were defined on right cell edges of the corresponding mass block.

Figure 4.12: Forward Staggered Grid Structure

Rewriting Eq. (4.4) thru Eq. (4.7) for two phase gas-liquid flow for forward staggered momentum grid, where ‘\( A_{fk} \)’ is cross sectional flow area for phase ‘\( k \)’, we obtain:

**Mass Conservation**

\[
\rho_{k,i-1} A_{fk,i-1/2} U_{k,i-1} - \rho_{k,i} A_{fk,i+1/2} U_{k,i} = 0
\]

**Momentum Conservation**

\[
\rho_{k,i} A_{fk,i} U_{k,i} |U_{k,i}| - \rho_{k,i+1} A_{fk,i+1} U_{k,i+1} |U_{k,i+1}| + P_i A_{fk,i} - P_{i+1} A_{fk,i+1} \\
\pm F_{int,i} - F_{wall,i} = 0
\]

Table 4.3: Specifications - Forward Staggered Pipe Flow

<table>
<thead>
<tr>
<th>Set</th>
<th>Inlet</th>
<th>Outlet</th>
<th>Boundary Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mass flow rate for each phase</td>
<td>( A_k ) (or ( \alpha_k )), ( P )</td>
<td>Outlet ( U_k )</td>
</tr>
<tr>
<td>2</td>
<td>( A_k ) (or ( \alpha_k )), ( U_k )</td>
<td>( P )</td>
<td>Inlet ( P ), and outlet ( A_k ) (or ( \alpha_k )) and ( U_k )</td>
</tr>
</tbody>
</table>
Zero curvature boundary condition (discussed in section 4.7) was applied to reduce number of specified unknowns for the above two specification sets as follows:

\[
U_{k,j+1} = 2U_{k,j} - U_{k,j-1}
\]
\[
\alpha_{k,i+1} = 2\alpha_{k,i} - \alpha_{k,i-1}
\]
\[
P_{k,i+1} = 2P_{k,i} - P_{k,i-1}
\]  

Eq. (4.17) and Eq. (4.18) suggest that phase velocities, densities and liquid holdup values at the inlet must be specified (alternatively mass flow rates were specified at inlet) in addition to pressure and liquid holdup specification at outlet. The number of unknowns twice the number of grid blocks for single phase flow and four times number of grid blocks for two phase flow as shown previously in section.

### 4.8.2 Backward Staggered Formulation

Pressure values are generally available upstream of the block where flow parameters are required. Thus, requirement of pressure specification downstream of the block poses certain degree of difficulty for field deployment. A momentum grid staggered in backward direction relative to mass grid resolves this issue. However, this formulation hardly differs from the former in terms of final solution.

[Figure 4.13: Backward Staggered Grid Structure]

Rewriting Eq. (4.4) thru Eq. (4.7) for two phase gas-liquid flow using forward staggered momentum grid, where ‘\(A_k\)’ is cross sectional flow area for phase ‘k’, we obtain a constitutive equation set different from Eq. (4.17) and Eq. (4.18):
Mass Conservation

\[ \rho_{k,i-1}A_{j,k,i-1/2}U_{k,j-1} - \rho_{k,i}A_{j,k,i+1/2}U_{k,j} = 0 \]  \hspace{1cm} (4.20)

Momentum Conservation

\[ \rho_{k,i-1}A_{j,k,i-1}U_{k,j-1} |U_{k,j-1}| - \rho_{k,i}A_{j,k,i}U_{k,j} |U_{k,j}| + P_{i-1}A_{j,k,i-1} - P_iA_{j,k,i} \pm F_{kint,i} - F_{kwall,i} = 0 \]  \hspace{1cm} (4.21)

Table 4.4: Specifications - Backward Staggered Pipe Flow

<table>
<thead>
<tr>
<th>Set</th>
<th>Inlet</th>
<th>Outlet</th>
<th>Boundary Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(A_k) (or (\alpha_k)), (U_k) and (P)</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>2</td>
<td>Mass flow rates for each phase, (A_k) (or (\alpha_k)) and (P)</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

Table 4.4 provides inlet and outlet specifications required for a backward staggered momentum grid formulation. Since mass and momentum conservation equations are independent of upstream block parameters an outlet specification for single or two phase pipe flow is not required as for forward staggered. A similar analysis of Eq. (4.20) and Eq. (4.21) as done before suggests gas/liquid mass flow rates and pressure be specified at the outlet. Since pressure specification is required at pipe inlet backward staggering gains higher grounds over forward staggering for pipe flow study. The number of unknowns is twice the number of grid blocks for single phase flow and four times the number of grid blocks for two phase flow as shown previously in section.

4.9 Staggered Approach - Tee Junction

Governing equations for inlet, run and branch arms are identical to pipe flow equations discussed above. However, a separate set of equations is required for the tee cell FVM block. A modified form of steady state pipe flow mass and momentum conservation is used to account for the flow in two principal directions i.e. inlet (or run)
and branch arms. Fig. 4.14 presents the grid block nomenclature which will be followed in this section for tee split considerations. Three non-overlapping grids were considered, one for mass conservation and two for momentum conservation in two directions of flow.

![Tee Split Grid Block Nomenclature](image)

**Figure 4.14:** Tee Split Grid Block Nomenclature

### 4.9.1 Forward Staggered Formulation

Fig. 4.15 shows forward staggered scheme wherein block ‘4’ is representative of Tee split. A similar approach, as discussed in section 4.4, based upon mass and momentum conservation was used to obtain governing equations on the tee cell block. The two momentum balances given by Eq. (4.25) and Eq. (4.26) are representative of flow in two principal directions assuming inlet and branch arm directions coincide.

![Forward Staggered Momentum Grid for Tee Split](image)

**Figure 4.15:** Forward Staggered Momentum Grid for Tee Split
Mass Conservation (Block 3)

\[ \rho_{k2} A_{jk2} U_{k2} - \rho_{k3} A_{jk3} U_{k3} = 0 \]  \hspace{1cm} (4.22)

Momentum Conservation (Block 3)

\[ \rho_{k,3} A_{jk,3} U_{k,3} \left| U_{k,3} \right| - \rho_{k,4} A_{jk,4} \left| U_{k,4} \right| + P_{3} A_{jk,3} - P_{4} A_{jk,4} \]
\[ \pm F_{int,3} - F_{wall,3} - \rho_{k,4} A_{jk,4} V_{k,4} \left| V_{k,4} \right| \cos \beta = 0 \]  \hspace{1cm} (4.23)

Mass Conservation (Block 4)

\[ \rho_{k,3} A_{jk,3} U_{k,3} - \rho_{k,4} A_{jk,4} U_{k,4} - \rho_{k,4} A_{jk,4} V_{k,4} = 0 \]  \hspace{1cm} (4.24)

X-Direction Momentum Conservation (Block 4)

\[ \rho_{k,4} A_{jk,4} U_{k,4} \left| U_{k,4} \right| - \rho_{k,5} A_{jk,5} U_{k,5} \left| U_{k,5} \right| + P_{4} A_{jk,4} - P_{5} A_{jk,5} \]
\[ - F_{\text{tee-loss}}(x) = 0 \]  \hspace{1cm} (4.25)

Y-Direction Momentum Conservation (Block 4)

\[ \rho_{k,4} A_{jk,4} V_{k,4} \left| V_{k,4} \right| - \rho_{k,8} A_{jk,8} V_{k,8} \left| V_{k,8} \right| + P_{4} A_{jk,4} - P_{8} A_{jk,8} \]
\[ - F_{\text{tee-loss}}(y) = 0 \]  \hspace{1cm} (4.26)

Additional losses, encountered at tee due to formation of vortices and boundary layer separation, were accounted for by incorporating a loss component in momentum balances. The calculation of tee momentum losses is discussed in detail in section 4.11.3

Table 4.5: Specifications – Forward Staggered Tee Junction

<table>
<thead>
<tr>
<th>Set</th>
<th>Inlet</th>
<th>Run</th>
<th>Branch</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \rho_{k},A_{jk}/\alpha_{k}, U_{k} )</td>
<td>( P_{out} )</td>
<td>( P_{out} )</td>
</tr>
<tr>
<td>2</td>
<td>Inlet mass flow rates</td>
<td>( P_{out} )</td>
<td>( P_{out} )</td>
</tr>
</tbody>
</table>

A forward staggered approach (Table 4.5) requires liquid and gas flow rates at inlet. An additional pressure specification both for branch or run arm outlet must also be provided. The ready availability of pressure information as compared to flow rate information for a gas pipe-line network and the split dependency on downstream pressures in each arm renders forward staggering an inherent advantage. Thus, a forward
staggered formulation was considered more appropriate to simulate two phase flow at a tee junction.

4.9.2 Backward Staggered Formulation

Fig. 4.16 shows backward staggered scheme wherein block ‘4’ is representative of Tee split. A modified mass balance and consideration of momentum conservation in two principal directions yield the governing FV equations. However, since momentum grids are staggered against direction of flow, the tee cell momentum block in Y-direction represents pipe wall (or no flow boundary). Since no momentum flow can occur through pipe wall, a momentum balance in the branch flow direction was neglected.

Figure 4.16: Backward Staggered Momentum Grid for Tee Split

Mass Conservation (Block 3)

\[ \rho_{k2} A_{jk2} U_{k2} - \rho_{k3} A_{jk3} U_{jk3} = 0 \]  \hspace{1cm} (4.27)

Momentum Conservation (Block 3)

\[ \rho_{k2} A_{jk2} U_{k2} \mid U_{k2} \mid - \rho_{k3} A_{jk3} U_{k3} \mid U_{k3} \mid \pm F_{kint,3} - F_{kwall,3} = 0 \]  \hspace{1cm} (4.28)
**Mass Conservation (Block 4)**

\[ \rho_{k3} A_{f3} U_{k3} - \rho_{k4} A_{f4} U_{k4} - \rho_{k4} A_{f4} V_{k8} = 0 \]  \hspace{1cm} (4.29)

**X--Direction Momentum Conservation (at Block 4)**

\[ \rho_{k3} A_{f3} U_{k3} | U_{k3} | - \rho_{k4} A_{f4} U_{k4} | U_{k4} | - \rho_{k4} A_{f4} V_{k8} | V_{k8} | \cos \beta \\
+ P_3 A_{f3} - P_4 A_{f4} - F_{tee-loss(x)} = 0 \]  \hspace{1cm} (4.30)

**Table 4.6: Specifications – Backward Staggered Tee Junction**

<table>
<thead>
<tr>
<th>Set</th>
<th>Inlet</th>
<th>Run</th>
<th>Branch</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A/ak, U_k, P_in</td>
<td>U_k, out</td>
<td>NA</td>
</tr>
<tr>
<td>2</td>
<td>A/ak, U_k, P_in</td>
<td>NA</td>
<td>V_k, out</td>
</tr>
<tr>
<td>3</td>
<td>P_in, inlet mass flow rates</td>
<td>U_k, out</td>
<td>NA</td>
</tr>
<tr>
<td>4</td>
<td>P_in, inlet mass flow rates</td>
<td>NA</td>
<td>V_k, out</td>
</tr>
</tbody>
</table>

It can be inferred that the formulation requires inlet pressure and gas/liquid mass flow rate specifications. A slight modification during numerical solution of the set of equation formulated requires a consistent combination of one pressure and gas/liquid flow rate at inlet and run arm outlets. Since outlet gas/liquid flow rates are generally unknowns in gas transmission networks, backward staggering approach is not pursued extensively.

It is often necessary to determine liquid holdup values in run and branch arms for predicting liquid preferential pathways. Flow parameters such as gas/liquid mass flow rates and pressure are readily available for gas transmission networks however, measurements for liquid holdup is seldom available. The liquid holdup was obtained by assuming a ‘no slip’ condition at the inlet as discussed in section 4.7.

**4.10 Information Translation Approach: An Alternative to Staggering**

Another insight into the formulation of governing equations can be attained by considering interconnectivity of primary of independent unknowns at each grid block. For a generic finite volume block (Fig. 4.19) four set of governing equations can be
obtained from downwind and upwind mass balance and forward and backward staggering for momentum balance. Mathematically, it is equivalent to block flow parameter dependency on either of the adjacent downstream or upstream parameters. A selection can later be made based upon the system being studied, available specifications and choice of convenient boundary conditions specific to each set.

Conservation principles applied to the control volume edges (i-1/2 and i+1/2) of a generic grid block ‘i’ in a coincidental gridding scheme, as shown in figure Fig. 4.17, yield following four governing equations. Fig. 4.17 illustrates that a definition of unknown parameters at cell block edges is not required as for staggered grid approach. The naming below is solely for the purpose of distinguishing one from another and to provide a physical sense.

1. **Downwind Mass Balance (at cell block edge i+1/2)**

\[
\rho_{k,i} A_{jk,i} U_{k,j} - \rho_{k,i+1} A_{jk,i+1} U_{k,j+1} = 0
\]  

(4.31)

2. **Upwind Mass Balance (at cell block edge i-1/2)**

\[
\rho_{k,i-1} A_{jk,i-1} U_{k,i-1} - \rho_{k,i} A_{jk,i} U_{k,i} = 0
\]

(4.32)

3. **Forward Staggered Momentum Balance (at cell block edge i+1/2)**

\[
\rho_{k,j-1} A_{jk,j} U_{k,j} |U_{k,j}| - \rho_{k,j} A_{jk,j+1} U_{k,j+1} |U_{k,j+1}| + P_{k} A_{jk,j} - P_{i+1} A_{jk,j+1} \\
\pm F_{kint,j} - F_{kwall,i} = 0
\]

(4.33)

4. **Backward Staggered Momentum Balance (at cell block edge i-1/2)**

\[
\rho_{k,j-1} A_{jk,j} U_{k,j-1} |U_{k,j-1}| - \rho_{k,j} A_{jk,j} U_{k,j} |U_{k,j}| + P_{i-1} A_{jk,j-1} - P_{k} A_{jk,j} \\
\pm F_{kint,i} - F_{kwall,j} = 0
\]

(4.34)
Table 4.7 demonstrates the variation in specifications (inlet or outlet) corresponding to different combinations of mass and momentum conservation equations. A translation from one set of model inputs to another set can be achieved with relative ease as compared to staggered grid approach.

Table 4.7: Pipe Specifications: Information translation approach

<table>
<thead>
<tr>
<th>Set</th>
<th>Inlet</th>
<th>Outlet</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&amp;3</td>
<td>NA</td>
<td>Phase mass flow-rates, Pressure</td>
</tr>
<tr>
<td>2&amp;3</td>
<td>Phase mass flow-rates</td>
<td>Pressure</td>
</tr>
<tr>
<td>1&amp;4</td>
<td>Pressure</td>
<td>Phase mass flow-rates</td>
</tr>
<tr>
<td>2&amp;4</td>
<td>Phase mass flow-rates,</td>
<td>NA</td>
</tr>
<tr>
<td></td>
<td>Pressure</td>
<td></td>
</tr>
</tbody>
</table>

A constitutive equation set can be formed by combining either of two mass balances with one of momentum balance equation. As discussed before, the selection is solely based upon desired parameters at inlet/outlet and convenience of specifications. A similar approach, as for pipe flow yields four sets of specifications for T-junctions, shown in Table 4.8. Other factors influencing the selection are system being depicted, validity of formulation for that particular system and physical sense as discussed before for finite volume approach. This approach differs from staggering only in the sense that it stresses on mathematical interconnectivity between flow parameters in addition to preserving physical sense of the system.

Table 4.8: Tee Specifications: Information translation approach

<table>
<thead>
<tr>
<th>Set</th>
<th>Inlet</th>
<th>Run</th>
<th>Branch</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&amp;3</td>
<td>NA</td>
<td>Pout, Phase mass flow rates</td>
<td>Pout</td>
</tr>
<tr>
<td>2&amp;3</td>
<td>Phase mass flow rates</td>
<td>Pout</td>
<td>Pout</td>
</tr>
<tr>
<td>1&amp;4</td>
<td>Pin</td>
<td>Phase mass flow rates, Pout (when Pout for branch is not specified)</td>
<td>Pout (when Pout for run is not specified)</td>
</tr>
<tr>
<td>2&amp;4</td>
<td>Pin, Phase mass flow rates</td>
<td>Pout (when Pout for branch is not specified)</td>
<td>Pout (when Pout for run is not specified)</td>
</tr>
</tbody>
</table>


4.11 Closure Relationships

In gas transmission networks we seldom encounter systems with high liquid loading. Therefore, study, only low liquid holdup systems were studied with conditions restricted to stratified, annular and mist flow patterns. A comprehensive study was presented by Ayala (2001) for different flow patterns with low liquid holding in horizontal pipe flow. Closure was achieved by incorporating wall and interfacial shear force calculations during momentum conservation for pipe flow and accounting for additional momentum losses due to flow split at Tee junction. Appendix B summarizes these shear forces for pipe flow and additional losses for Tee split corresponding to each flow pattern.

4.11.1 Wall Shear Force ($F_{kwall}$)

The change in momentum due to frictional losses at wall-fluid interface is given by wall shear force $F_{kwall}$ for phase ‘k’ (where ‘k’ is either gas ‘g’ or liquid ‘l’). The wall shear force is calculated as:

$$F_{kwall} = A_{kwall} f_{kwall} \frac{\rho_k |U_k| U_k}{2} \quad (4.35)$$

Where,

$A_{kwall} =$ Wall-phase ‘k’ contact area (m$^2$)

$f_{kwall} =$ Fanning friction factor for phase ‘k’ (dimensionless)

$\rho_k =$ density of phase ‘k’ (kg/m$^3$)

$U_k =$ velocity of phase ‘k’ (m/sec$^2$)

The gas and liquid wetted areas ($A_{kwall}$) vary for different flow patterns and must be calculated separately for each case. For laminar flow fanning friction factor ($Re<2100$) is calculated as:

$$f_{kwall} = \frac{16}{Re} \quad (4.36)$$
For turbulent flow, Chen (1979) proposed a fanning friction factor explicit in Reynolds number as:

$$\frac{1}{\sqrt{f_{kwall}}} = -4.0\log \left[ \frac{\varepsilon}{3.7065d_{hk}} - \frac{5.0452}{\text{Re}_k} \log \left( \frac{(\varepsilon / d_{hk})^{1.1098}}{2.8257 + \frac{5.8506}{\text{Re}_k^{0.8981}}} \right) \right]$$  \hspace{1cm} (4.37)

Where,

- \(\varepsilon\) = roughness parameter (dimensionless)
- \(d_{hk}\) = equivalent hydraulic diameter for phase ‘k’ (m)
- \(\text{Re}_k\) = Reynolds Number for phase ‘k’ (dimensionless)

An alternative form of fanning friction factor was presented by Blausius as:

$$f_{kwall} = C_a \text{Re}_a^{-n}$$  \hspace{1cm} (4.38)

For turbulent flow, \(C_a = 0.046\) and \(n = 0.2\) whereas for laminar flow, \(C_a = 16\) and \(n=1\). The Blausius expression assumes smooth pipeline or no roughness and is thus unrealistic. In this study Blausius was used during model development stages and was later replace by Chen’s correlation, given by Eq. (4.37), for better representation of actual system being modeled.

Reynolds number (\(\text{Re}_k\)) for each phase ‘k’ is obtained from the following expression:

$$\text{Re}_k = \frac{\rho_k U_k d_{hk}}{\mu_k}$$  \hspace{1cm} (4.39)

Where,

- \(d_{hk}\) = equivalent hydraulic diameter for phase ‘k’ (m)
- \(\mu_k\) = viscosity of phase k (kg/m.sec)

Expressions for equivalent hydraulic/wetted diameter depend on the wall area wetted by phase ‘k’ and are distinctly calculated for each flow pattern. Hydraulic diameter for a phase k is given by:

$$d_{hk} = 4 \frac{\text{Volume of phase 'k'} }{\text{Area wetted by phase 'k'} }$$  \hspace{1cm} (4.40)
4.11.2 Interfacial Shear Force ($F_{\text{int}}$)

For two phase flow, momentum losses are encountered at gas liquid interface and are accounted in momentum conservation as interfacial shear forces. The magnitude of interfacial forces is determined by the relative velocity of two phases and interfacial contact area.

$$F_{\text{int}} = A_{\text{int}} f_{\text{int}} \frac{\rho_c |U_g - U_l| (U_g - U_l)}{2}$$  \hspace{1cm} (4.41)

Where,

- $A_{\text{int}}$ = gas/liquid interfacial contact area (m$^2$)
- $f_{\text{int}}$ = interfacial fanning friction factor (dimensionless)
- $U_{g/l}$ = gas/liquid velocities (m/sec$^2$)
- $\rho_c$ = density of the continuous phase (kg/m$^3$)

Since for low liquid loading systems gas is the predominant phase, it is assumed to be the continuous phase as suggested by Ayala (2001). A comparison between Eq. (4.37) and Eq. (4.41) shows that interfacial shear force is similar to wall shear except for, phase velocity is replaced by gas/liquid relative velocity and a different interfacial fanning friction factor. Interfacial contact area calculations are done on the basis of liquid holdup values for each flow pattern separately. Similarly, interfacial fanning friction is flow pattern dependent and will be discussed in later sections when considering each one separately.

4.11.3 Tee Junction Losses

4.11.3.1 Energy Dissipation Approach

The calculation of additional losses at Tee junction can be based upon frictional loss factors ‘$k_{12}$’ and ‘$k_{13}$’ for run and branch arms respectively. Hart (1989) suggested
using single phase correlations presented by Gardel (1957) for calculation of these factors. A mechanical energy balance or pressure energy form of Bernoulli’s equation incorporating at the tee junction was then used to affect single phase split. However, for two phase split flow Hart (1989) proposed an energy dissipation factor given by:

$$\omega_k = \frac{1}{2}(1 + k_{12,k} + k_{13,k})$$  \hspace{1cm} (4.42)

Where,
- $\omega_k$ = Energy dissipation factor for phase ‘k’
- $k_{12,k}$ = Inlet to run frictional loss factor for phase ‘k’
- $k_{13,k}$ = Inlet to branch frictional loss factor for phase ‘k’

In order to account for liquid phase, the gas and liquid energy dissipation factors were assumed to be equal i.e,

$$\omega_g = \omega_l$$  \hspace{1cm} (4.43)

Ottens (2001) proposed separate friction factor correlations each for gas and liquid phase split at Tee junctions. These expressions were adapted to the proposed model formulation as a part of this study. Although, they were originally proposed for wavy stratified flow, they were implemented to study both stratified smooth and wavy flow patterns. This assumption is valid for low liquid loading systems since the difference in friction factors for the two flow patterns under low liquid loading conditions is fairly small.

**Gas Phase Friction Factor Correlations**

*Inlet to Run*

$$k_{12,g} = 0.03(1 - \lambda_g)^2 + 0.35\lambda_g^2 - 0.2\lambda_g(1 - \lambda_g)$$  \hspace{1cm} (4.44)

*Inlet to Branch*
\[ k_{13,g} = 0.95(1 - \lambda_g)^2 + \lambda_g^2 \left[ 1.3 \tan \left( \frac{1}{2} \varphi \right) (1 - 0.9 \sqrt{r}) \right] \]

\[ + 0.8 \lambda_g (1 - \lambda_g) \tan \left( \frac{1}{2} \varphi \right) \] \hspace{1cm} (4.45)

**Liquid Phase Friction Factor Correlations**

**Inlet to Run**

\[ k_{12,l} = \left\{ 11.48 + 1063(1 + \text{Re}_{l,\text{run}})^{-1} - 12.67 \kappa \right\} k_{12,g} \] \hspace{1cm} (4.46)

**Inlet to Branch**

\[ k_{13,l} = \left\{ 1.247 + 69.26(1 + \text{Re}_{l,\text{branch}})^{-1} - 0.198 \kappa \right\} k_{13,g} \] \hspace{1cm} (4.47)

Where,

- \( \lambda_g \) = branch gas mass fraction
- \( \varphi \) = angle between branch and run arms
- \( r \) = radius of radiansation

The value of ‘\( \kappa \)’ in Eq. (4.46) and Eq. (4.47) is calculated as:

\[ \kappa = \frac{\beta_{g,\text{inlet}} \rho_{g,\text{inlet}} U_{g,\text{inlet}}^2 \alpha_{g,\text{inlet}}^2}{\beta_{l,\text{inlet}} \rho_{l,\text{inlet}} U_{l,\text{inlet}}^2 \alpha_{l,\text{inlet}}^2} \] \hspace{1cm} (4.48)

Where,

- \( \beta_{g,\text{inlet}} \) = Inlet gas/liquid velocity profile correction factor (dimensionless)
- \( \rho_{g,\text{inlet}} \) = Inlet gas/liquid density (kg/m\(^3\))
- \( U_{g,\text{inlet}} \) = Inlet gas/liquid velocities (m/sec\(^2\))

The value of ‘\( \beta \)’ varies with its corresponding Reynolds number for each phase and was given by Ottens (2001) as:

\[
\beta = \begin{cases} 
1.54 & \text{for } R_e < 1500 \\
1.54 - (1.08)10^{-3}(R_e-1500) & \text{for } 1500 < R_e < 2000 \\
1 & \text{for } R_e > 2000 
\end{cases} \] \hspace{1cm} (4.49)

An adapted force form of losses encountered at Tee junction was developed to attain closure in the Tee momentum balance for the case of smooth stratified flow.
pattern. Since, this expression cumulatively accounts for all momentum losses, the wall and interfacial shear force components were set to zero. Expressions for losses along inlet-run and inlet-branch flow directions for phase ‘k’ can be written as:

**Inlet-Run Momentum Loss**

\[ F_{\text{Tee, inlets-run}} = k_{12,k} \frac{1}{2} \rho_{k,\text{inlet}} A_{k,\text{flow}} \beta_k U_{k,\text{inlet}}^2 \alpha_{k,\text{inlet}}^2 \] (4.50)

**Inlet-Branch Momentum Loss**

\[ F_{\text{Tee, inlets-branch}} = k_{13,k} \frac{1}{2} \rho_{k,\text{inlet}} A_{k,\text{flow}} \beta_k U_{k,\text{inlet}}^2 \alpha_{k,\text{inlet}}^2 \] (4.51)

Where,

\[ A_{k,\text{flow}} = \text{Flow cross sectional area for phase ‘k’ given by following expression:} \]

\[ A_{k,\text{flow}} = \alpha_k \pi \left( \frac{D}{2} \right)^2 \] (4.52)

### 4.11.3.2 Interfacial Drag Force Approach

An alternative to the above approach involves accounting of additional momentum loss at Tee junction through an equivalent interfacial shear force. Lahey (1990) proposed a modification in his existing drag force model for bubbly flow regime to accommodate for stratified flow split phenomena at a Tee. The average interfacial drag force for bubbly flow regime is given by Eq. (4.53). A wavy stratified drag force coefficient proposed by Hanratty et al (1987) was used given by Eq. (4.54).

\[ F_{\text{int,eqv}} = \frac{1}{8} \rho_i C_D |U_g - U_i| |(U_g - U_i) A_{\text{int,eqv}}| \] (4.53)
Where,

- $F_{\text{int,eqv}}$ = Equivalent interfacial drag force
- $A_{\text{int,eqv}}$ = Equivalent interfacial area
- $C_D$ = Interfacial drag force coefficient
- $h_l$ = liquid phase depth for stratified flow
- $\rho_{gst}$ = gas phase density at standard temperature and pressure conditions
- $D$ = pipe diameter

A stratified flow pattern was assumed for pipe flow in the inlet, run and branch arms as a part of this study. The interfacial area for pipe flow presented by Lahey (1980) is given by Eq. (4.55). A separate expression was suggested for interfacial area at Tee junction, given by Eq. (4.56), assuming well agitated flow so that a bubbly flow regime existed.

$$A_{\text{int,pipe}} = 2R\sin\left(\frac{\phi}{2}\right)$$  \hspace{1cm} (4.55)

$$\alpha_g = 1 - \frac{1}{2\pi}(\phi - \sin\phi)$$  \hspace{1cm} (4.56)

Where,

- $A_{\text{int,pipe}}$ = Interfacial area for pipe flow
- $\alpha_g$ = Void fraction or gas holdup

Thus, a modified form of interfacial surface area at tee was suggested at T-junction given by Eq. (4.57).

$$A_{\text{int,Tee}} = 0.12\left(\frac{\alpha_g}{D_h}\right) + 0.88A_{\text{int,pipe}}$$  \hspace{1cm} (4.57)
Where,

\[ A_{\text{int,tee}} = \text{Equivalent interfacial area for tee flow} \]
\[ D_b = \text{liquid droplet diameter}. \]

The liquid droplet diameter calculations were adopted from mist flow study presented by Ayala (2001) which will be discussed later in section 4.10.6.

4.11.4 Stratified Smooth Flow Pattern

The geometrical representation of smooth stratified flow is depicted in Fig. 4.18 with liquid occupying a sector of the circular pipe cross section. An angle of 2\( \theta \) is subtended by the sector at the centre of circle allows us to correlate wetted areas and liquid holdup using geometrical relationships. Using geometrical properties, expressions for gas and liquid wetted areas and hydraulic diameters were formulated. A detailed derivation of these expressions was presented by Ayala (2001).
4.11.4.1 Wall Shear Force Coefficients

The wall contact area per unit volume for gas($A_{g,wall}$) and liquid($A_{l,wall}$) phases can be calculated using Eq. (4.58) and Eq. (4.59) based upon geometrical considerations given by Ayala (2001) below:

\[
A_{l,wall} = \left( \frac{\theta}{180} \right) \frac{4}{D}
\]  \hspace{1cm} (4.58)

\[
A_{g,wall} = \left( \frac{180 - \theta}{180} \right) \frac{4}{D}
\]  \hspace{1cm} (4.59)

Where,

$D = $ Diameter of cylindrical pipe

A correlation between liquid holdup ($\alpha_l$) and $\theta$ can be derived using geometrical considerations as shown in Fig. 4.18.

\[
\alpha_l = \left( \frac{\theta}{180} - \frac{\sin 2\theta}{2\pi} \right)
\]  \hspace{1cm} (4.60)

The value of $\theta$ is calculated using an iterative scheme from Eq. (4.60). Liquid holdup ($\alpha_l$) and $\theta$ can thus be used interchangeably as primary unknowns during calculation and numerical solution of the governing equations.

4.11.4.2 Interfacial Shear Force Coefficients

Baker et al (1988) proposed an expression for equivalent roughness parameter by treating the liquid as an additional roughness. Under the assumption of low liquid holdup interfacial velocity can be approximated to liquid phase velocity. This forms the rationale behind roughness treatment of slow moving liquid phase. The expression set for equivalent roughness parameter as proposed by Baker (1988) is as follows:
An equivalent interfacial fanning friction factor can be determined following calculation of interfacial hydraulic radius by incorporating the equivalent roughness parameter \( \varepsilon_{eqv} \) calculated in Eq. (4.37) as applied to gas phase. Correlations for gas/liquid hydraulic diameters for the case of smooth stratified flow derived in the work presented by Ayala (2001) were used.

\[
W_e N_l = \frac{\rho_g v_i^2 \mu_i^2}{\rho_l \sigma^2} \quad (4.61)
\]

If \( W_e N_l \leq 0.005 \Rightarrow \varepsilon_{eqv} = \frac{34 \sigma}{\rho_g v_i^2} \quad (4.62) \]

If \( W_e N_l > 0.005 \Rightarrow \varepsilon_{eqv} = \frac{170 \sigma (W_e N_l)^{0.30}}{\rho_g v_i^2} \quad (4.63) \]

An equivalent interfacial fanning friction factor can be determined following calculation of interfacial hydraulic radius by incorporating the equivalent roughness parameter \( \varepsilon_{eqv} \) calculated in Eq. (4.37) as applied to gas phase. Correlations for gas/liquid hydraulic diameters for the case of smooth stratified flow derived in the work presented by Ayala (2001) were used.

\[
d_{hg} = \left( \frac{180 \alpha_i}{\theta} \right) D \quad (4.64)
\]

\[
d_{hg} = \left( \frac{\pi \alpha_g}{\left( \frac{180 - \theta}{180} \right) \pi + \sin \theta} \right) D \quad (4.65)
\]

Where,

\( d_{hg/l} = \) Gas liquid hydraulic diameters (m)
\( \alpha_{g/l} = \) Gas/liquid holdup (dimensionless)
\( \theta = \) liquid holdup parameter, shown in Fig. 4.18 (degrees)

Gas/liquid interfacial contact area derived from geometrical consideration in smooth stratified flow shown in Fig. 4.18, presented by Ayala (2001), can be written as:

\[
A_{int} = \left( \frac{\sin \theta}{\pi} \right) \frac{4}{D} \quad (4.66)
\]
4.11.5 Wavy Stratified Flow Pattern

A ‘double circle’ model was put forth by Chen (1997) which accounted for liquid creeping along pipe wall creating a wavy stratified flow pattern. A geometrical description of Chen’s model is given in Fig. 4.19.

4.11.5.1 Wetted Shear Coefficients

The determination of gas/liquid wetted wall area was based upon geometrical considerations proposed by Chen et al (1977). A detailed discussion on ‘double circle’ model and ‘climb up’ effect which is a characteristic of wavy flow pattern, was presented by Ayala (2001) suggesting the use of Grolman and Fortuin (1997a) correlation for determining wetted wall fraction ($W_f$) as:

$$W_f = 0.624 \alpha_i^{0.374} + We_{sl}^{0.25} Fr_g^{0.8} \frac{\rho_g}{(\rho_l - \rho_g) \cos \beta}$$  \hspace{1cm} (4.67)
We_{ef} = \frac{\rho_l(U_l\alpha_l)^2 D}{\sigma}, \text{ equivalent Weber number}

Fr_g = \left(\frac{U_g\alpha_g}{\alpha_g^2 gD}\right), \text{ gas-phase Froude number}

Where,

\beta = \text{pipe line inclination} = 0 \text{ (for this study)}

g = \text{acceleration due to gravity (m/sec}^2)\)

Grolman and Fortuin (1997b) indicated that if \(W_f > 0.80\), an annular flow pattern be considered instead of wavy stratified. The liquid wetted pipe angle ‘\(\theta\)’ shown in Fig. 4.19 can then be calculated as:

\[\theta = 180 \cdot W_f\] (4.68)

The interface angle ‘\(\theta_i\)’ can then be calculated iteratively from an implicit expression based upon the geometry defined in Fig. 4.19 as:

\[\theta_i = \left(\frac{\sin \theta_i}{\sin \theta}\right)^2 \left(\theta + \frac{\sin^2 \theta - \sin 2\theta}{\tan \theta_i} - \frac{\pi \alpha_i}{2}\right)\] (4.69)

A correlation between interface radius (\(R_i\)) and pipe radius (\(R\)) can be derived as:

\[R_i \sin \theta_i = R \sin \theta\] (4.70)

The gas and liquid wetted wall areas per unit volume are calculated using Eq. (4.58) and Eq. (4.59), similar to stratified smooth flow pattern. However, hydraulic diameters remain are distinct and are defined as:

\[d_{hg} = \left(\frac{180\alpha_i}{\theta}\right)D\] (4.71)

\[d_{hg} = \left(\frac{\alpha_g \sin \theta_i}{\left(\frac{180 - \theta}{180}\right) \sin \theta_i + \frac{\theta_i}{180} \sin \theta}\right)D\] (4.72)
4.11.5.2 Interfacial Shear Force Coefficients

An expression for gas liquid interfacial contact surface area per unit volume was derived from geometrical considerations shown in Fig. 4.19.

\[ A_{\text{int}} = \left( \frac{\theta_i \sin \theta}{180 \sin \theta_i} \right) \frac{4}{D} \]  \hspace{1cm} (4.73)

The interfacial contact area in a control volume can then be determined by multiplying Eq. (4.73) above expression with the block volume. The ‘double circle’ model proposed by Chen (1997) used the following correlation for calculation of interfacial friction factor:

\[ f_i = \left[ 1 + 3.17 \left( \frac{\alpha_i}{W_f} \right)^{0.2} \left( \frac{\rho_l \rho_g U_i}{\mu_l (\rho_l - \rho_g) g} \right)^{0.5} - 8.165 \right]^{0.08} f_{g,\text{wall}} \]  \hspace{1cm} (4.74)

Where,
- \( f_{k,\text{wall}} \) = fanning friction factor for phase ‘k’ (dimensionless)
- \( \alpha_i \) = liquid holdup (dimensionless)
- \( \mu_k \) = viscosity of phase ‘k’ where, \( k = \text{gas (g) / liquid (l)} \)
- \( U_{g/l,\text{inlet}} \) = Inlet gas/liquid velocities (m/sec²)
- \( W_f \) = Wetted wall fraction (dimensionless)
- \( g \) = acceleration due to gravity (m/sec²)
4.10.6 Mist Flow Pattern

The geometry of mist flow pattern is shown in Fig. 4.20 with liquid droplets dispersed in vapor phase. The droplets were assumed to be of same size and homogeneously distributed throughout the circular cross-section. A maximum droplet size was determined using a critical Weber number given by the expression:

\[
We = \frac{2 \rho_g (U_g - U_1)^2 r_{\max}}{\sigma} \quad (4.75)
\]

Where,

- \( \sigma \) = interfacial surface tension (N/m)
- \( \rho_g \) = gas phase density

Wallis (1969) suggested a critical Weber number of \( We =12 \) to determine maximum droplet size ensuring stability for non viscous fluids. A modified critical Weber number was proposed for viscous fluid as:

\[
We = 12 \left[ 1 + \left( \frac{\mu_i}{2 \rho_i r_{\max} \sigma} \right)^{0.36} \right] \quad (4.76)
\]
Where, the maximum droplet radius ($r_{\text{max}}$) is calculated using expression for nonviscous Weber number from Eq. (4.75). A mean droplet to maximum droplet radius of 0.06147 was used by Ayala (2001) to calculate mean droplet radius as:

$$r_p = 0.06147r_{\text{max}} \quad (4.77)$$

### 4.11.6.1 Wall Shear Force Coefficients

The gas and liquid wall contact areas assuming homogeneous distribution of liquid droplets were given by Ayala (2001) as:

$$A_{k,\text{wall}} = \alpha_k \frac{4}{D} \quad (4.78)$$

Where,

$$\alpha_k = \text{hold-up value for phase 'k'}$$

Since, a homogeneous phase distribution was considered gas and liquid hydraulic radii are equal to the pipe diameter ‘D’.

### 4.11.6.2 Interfacial Shear Force Coefficients

A detailed derivation of interfacial surface area per unit volume for mist/dispersed liquid flow was presented by Ayala (2001) as:

$$A_{\text{int}} = 3 \frac{\alpha_k}{r_p} \quad (4.79)$$

The liquid droplets were assumed to be perfect spheres with gas flowing around it at a relatively different velocity. A drag force expression presented by Hughes (1976) was used to calculate interfacial shear force to account for flow of gas phase across spherical liquid droplets. The drag force ($F_{\text{drag}}$) per unit volume is given by:
Where,

\[ A_{\text{int}} = \text{Interfacial contact area} \]
\[ C_d = \text{drag coefficient} \]

Since, an area averaged unidirectional flow formulation is being considered, the radial lift force normal to flow direction has no significance. Drag coefficients corresponding to each Reynolds number range were presented by Cliff et al (1978) tabulated below.

\[
Re = \frac{\rho_g (2r_p)(U_g - U_i)}{\mu_g}
\]

Table 4.9: Drag Coefficients, \( w = \log_{10} Re \), (Cliff et al., 1978), (Ayala, 2001)

<table>
<thead>
<tr>
<th>Range</th>
<th>Drag Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Re&lt;0.01 )</td>
<td>( C_d = \frac{24}{Re} \left[ 1 + \frac{3}{16} Re \right] )</td>
</tr>
<tr>
<td>0.01&lt;( Re&lt;20 )</td>
<td>( C_d = \frac{24}{Re} \left[ 1 + 0.1315 Re^{0.82-0.05w} \right] )</td>
</tr>
<tr>
<td>20&lt;( Re&lt;260 )</td>
<td>( C_d = \frac{24}{Re} \left[ 1 + 0.0.1935 Re^{0.6305} \right] )</td>
</tr>
<tr>
<td>260&lt;( Re&lt;1500 )</td>
<td>( C_d = 10^{1.6435-1.1242w+0.1558w^2} )</td>
</tr>
<tr>
<td>1500&lt;( Re&lt;12000 )</td>
<td>( C_d = 10^{-2.4571+2.558w-0.9295w^2+0.1049w^3} )</td>
</tr>
<tr>
<td>12000&lt;( Re&lt;44000 )</td>
<td>( C_d = 10^{-1.9181+0.637w-0.0636w^2} )</td>
</tr>
<tr>
<td>44000&lt;( Re&lt;338000 )</td>
<td>( C_d = 10^{-4.339+1.5809w-0.1546w^2} )</td>
</tr>
<tr>
<td>338000&lt;( Re&lt;400000 )</td>
<td>( C_d = 29.78 - 5.3w )</td>
</tr>
<tr>
<td>400000&lt;( Re&lt;10^6 )</td>
<td>( C_d = 0.1w - 0.49 )</td>
</tr>
<tr>
<td>( Re&gt;10^6 )</td>
<td>( C_d = 0.19 - \frac{8 \cdot 10^4}{Re} )</td>
</tr>
</tbody>
</table>
Chapter 5
Numerical Treatment

The governing equations developed for single and two phase flow cases in previous discussions (section 4.4) must be solved simultaneously for determining desired flow parameters characterizing flow conditions. An iterative solver employed for this purpose allows study of influence of interconnectivity between pipeline network entities (annulus and T-joint) on the final solution. Since, constitutive equations along with boundary conditions posses a high degree of non linearity; a non linear simultaneous solver was thus implemented. In this study, Generalized Newton-Raphson (GNR) technique was used in conjugation with LU decomposition matrix-inversion technique to obtain a solution. The choice Newton-Raphson (NR) is evident because of its quadratic convergence thereby reducing computational time required.

5.1 Newton-Raphson Method

The Newton-Raphson method, first proposed by Issac Newton, and later developed upon by Joseph Raphson is used to calculate roots of a non linear function (TJ Ypma, 1995). For a non linear function \( f(x) \) in one variable ‘x’, the roots are given by the function \( f(x) = 0 \). Assuming an initial value \( x = x_n \) far from the root of function \( f(x) \) and using Taylor series to expand \( y = f(x_n) \) we obtain:

\[
f(x_n + h) = f(x_n) + h \frac{df(x)}{dx}\bigg|_{(x_n, y_n)} + \frac{h^2}{2!} \frac{\partial^2 f(x)}{\partial x^2}\bigg|_{(x_n, y_n)} + ..... \tag{5.1}
\]

Truncating second and higher order terms in Eqn. (5.1) and rearranging:

\[
\frac{f(x_n + h) - f(x_n)}{df(x)}\bigg|_{(x_n, y_n)} = h \tag{5.2}
\]
If ‘$x_n+h$’ is a root of the equation $f(x) = 0$, $f(x_n+h)$ can be approximated as zero thus Eq. (5.2) reduces to:

$$h = \frac{-f(x_n)}{\frac{df(x)}{dx}_{(x_n,y_n)}}$$  \hspace{1cm} (5.3)

An iterative procedure is thus obtained wherein an updated value of ‘$x$’ closer to actual root is calculated at each iteration step as:

$$x_{n+1} = x_n + h$$  \hspace{1cm} (5.4)

### 5.1.1 Geometrical Interpretation

![Geometrical Representation – Newton Raphson](image)

Figure 5.1: Geometrical Representation – Newton Raphson

For the purpose of simplicity, a two dimensional Cartesian co-ordinate system is considered. Assuming a non linear equation $f(x) = 0$ in one variable ‘$x$’ such that function $f(x)$ is continuous and differentiable in the closed interval $[a,b]$. A graphical representation of the function $f(x)$ is shown in Figure 5.1. The roots or solution of the non-linear function then, is the value of independent variable ‘$x$’ in the closed interval $[a,b]$ for which dependent variable ‘$y$’ becomes zero (shown as red circle in Figure 5.1). An algorithm for Newton-Raphson solution to obtain root ‘$x$’ is given below:
1. A tangent to the curve at any point \( (x_n, f(x_n)) \) is given by Eqn. (5.5) below:

\[
y - y_n = \left( \frac{dy}{dx} \right)_{x_n} (x - x_n)
\]  

(5.5)

2. The value of ‘x’ for next iteration ‘n+1’ level is obtained from the point of intersection of tangent and x-axis as:

\[
x_{n+1} = x_n + \frac{-y_n}{\left( \frac{dy}{dx} \right)_{x_n}}
\]  

(5.6)

3. The corresponding y co-ordinate is calculated from the given equation definition as: Eq. (5.7)

\[
y_n = f(x_n)
\]  

(5.7)

4. The preceding three steps are repeated till desired convergence is achieved.

A criterion for convergence can either be based upon a near zero value for ‘\( y_n \)’ calculated in Eqn. (5.7) or a difference between ‘x’ for the current (n) and previous (n-1) iteration level given by Eqn. (5.8), where ‘m’ is a positive integer referred to as convergence exponent.

\[ |x_n - x_{n-1}| < 10^{-m} \]  

(5.8)

Similarly, solution to a system of ‘n’ non-linear equations \( f_1(x_1, x_2, x_3, \ldots x_n) = f_2(x_1, x_2, x_3, \ldots x_n) = \ldots = f_n(x_1, x_2, x_3, \ldots x_n) = 0 \) in ‘n’ independent variables \( (x_1, x_2, x_3, \ldots x_n) \) is equivalent to a vector \( (x_1^*, x_2^*, x_3^* \ldots x_n^*) \) in an n-dimensional hyper-plane such that:

\[
\begin{align*}
f_1(x_1^*, x_2^*, \ldots, x_n^*) &\equiv 0 \\
f_2(x_1^*, x_2^*, \ldots, x_n^*) &\equiv 0 \\
&\vdots \\
f_n(x_1^*, x_2^*, \ldots, x_n^*) &\equiv 0
\end{align*}
\]  

(5.9)
5.1.2 Generalized Newton Raphson

The procedure outlined above is used when a single non linear equation in one independent variable is to be solved. However, in this study we encounter multiple equations in more than one independent variable. A modification in NR procedure known as Generalized Newton Raphson must then be followed to simultaneously solve for ‘n’ non linear equations in ‘n’ unknowns (independent variables). Generalized Newton Raphson (GNR) allows a non linear system to be reduced to an equivalent linear algebraic system which can be demonstrated through a simple example. Assume a system of two non linear equation in two unknowns ‘x’ and ‘y’ given by Eqn. (5.10) as:

\[ f_1(x, y) = 0 \]
\[ f_2(x, y) = 0 \]  

(5.10)

Using Taylor series expansion Eqn. (5.10) can be expressed as:

\[ f_1(x + \Delta x, y + \Delta y) = f_1(x, y) + \Delta x \frac{df_1(x, y)}{dx} + \Delta y \frac{df_1(x, y)}{dy} + \Delta x^2 \frac{\partial^2 f_1(x, y)}{\partial x^2} + \frac{\Delta y^2 \partial^2 f_1(x, y)}{2!} + \ldots \]

\[ f_2(x + \Delta x, y + \Delta y) = f_2(x, y) + \Delta x \frac{df_2(x, y)}{dx} + \Delta y \frac{df_2(x, y)}{dy} + \Delta x^2 \frac{\partial^2 f_2(x, y)}{\partial x^2} + \frac{\Delta y^2 \partial^2 f_2(x, y)}{2!} + \ldots \]  

(5.11)

Truncating second and higher order derivatives in Eqn. (5.11) we obtain:

\[ f_1(x + \Delta x, y + \Delta y) = f_1(x, y) + \Delta x \frac{df_1(x, y)}{dx} + \Delta y \frac{df_1(x, y)}{dy} \]
\[ f_2(x + \Delta x, y + \Delta y) = f_2(x, y) + \Delta x \frac{df_2(x, y)}{dx} + \Delta y \frac{df_2(x, y)}{dy} \]  

(5.12)

Assuming \((x+\Delta x,y+\Delta y)\) to be close enough to final solution, \(f_1(x+\Delta x,y+\Delta y)\) and \(f_2(x+\Delta x,y+\Delta y)\) is approximated as zero. Thus, rearranging Eqn. (5.12) we obtain:
An iterative procedure similar to NR technique, as discussed in section 5.1.1 can be outlined for GNR:

1. Assuming initial values of \( x = x_n \) and \( y = y_n \) Eq. (5.13) reduces to two linear algebraic equations in two unknowns ‘\( \Delta x \)’ and ‘\( \Delta y \)’ given by:

\[
\Delta x \left( \frac{df_1(x, y)}{dx} \right)_{(x_n, y_n)} + \Delta y \left( \frac{df_1(x, y)}{dy} \right)_{(x_n, y_n)} = -f_1(x, y)
\]

\[
\Delta x \left( \frac{df_2(x, y)}{dx} \right)_{(x_n, y_n)} + \Delta y \left( \frac{df_2(x, y)}{dy} \right)_{(x_n, y_n)} = -f_2(x, y)
\]

(5.14)

Where R.H.S in Eqn. (5.14) is collectively referred to as residuals and is given by:

\[
R_1 = -f_1(x_n, y_n)
\]

\[
R_2 = -f_2(x_n, y_n)
\]

A Jacobian matrix containing differential coefficients is constructed along with two column matrices containing unknowns and residual. Eqn. (5.14) can then be expressed in matrix format as:

\[
\begin{pmatrix}
\frac{df_1(x, y)}{dx} & \frac{df_1(x, y)}{dy} \\
\frac{df_2(x, y)}{dx} & \frac{df_2(x, y)}{dy}
\end{pmatrix}
\begin{pmatrix}
\Delta x \\
\Delta y
\end{pmatrix}
= \begin{pmatrix}
-R_1 \\
-R_2
\end{pmatrix}
\]

(5.15)

2. The values in differential matrix are calculated using numerical differentiation as:

\[
\frac{df_1(x, y)}{dx} \bigg|_{(x_n, y_n)} = \frac{f_1(x_n + h, y_n) - f_1(x_n, y_n)}{h}
\]

(5.16)
3. The values of corrections in ‘x’ and ‘y’ given by ‘Δx’ and ‘Δy’ respectively are then obtained from Eqn. (5.15). In this study, LU decomposition method was employed for matrix inversion. The updated values of unknowns ‘x’ and ‘y’ are given by Eqn. (5.17)

\[
\begin{align*}
    x_{n+1} &= x_n + \Delta x \\
    y_{n+1} &= y_n + \Delta y
\end{align*}
\] (5.17)

4. The above procedure is repeated until the desired convergence is achieved. Converge criterion can be based upon a near zero value for corrections, column vector on L.H.S in Eqn. (5.15).

\[
    \text{max}(\Delta x, \Delta y) < 10^{-m} \] (5.18)

Another criterion can be formed if the residual vector, R.H.S in Eqn. (5.15) tends to zero i.e.

\[
    \text{max}(-R_1, -R_2) < 10^{-m} \] (5.19)

Where, \( m \) = convergence exponent.

Eq. (5.18) requires that Eq. (5.19) be true for the solution to be physically realizable. However, Eq. (5.19) is a sufficient condition for convergence to be achieved. The non linear algebraic governing equations presented in section 4.4, for single and two phase fluid flow, (FVM equations) can be solved on lines of the procedure outlined above.

5.1.3 Convergence Issues

The NR procedure outlined in section 5.1 the function \( f(x) \) to be continuous if a solution is to be obtained. The presence of points of inflection, maximums or minimums even in a continuous domain might also lead to convergence issues. These issues can be understood if we undertake a similar example as in section 5.1.1.
Assuming a non-linear function \( f(x) \) in one variable ‘x’ continuous and differentiable in the closed interval \([a,b]\), shown in figure. Mathematically, maximums and minimums of function \( f(x) \) are points on the X-Y plane (shown as ‘A’ in Figure 5.2) where slope or first derivative of \( f(x) \) is equal to zero i.e.;

\[
\frac{df(x)}{dx} = 0
\] (5.20)

Since the above differential occurs in the denominator of Eqn. (5.6), substitution of Eqn. (5.20) in Eqn. (5.6) results in an infinite expression. Thus, an initial guess far from actual solution is feasible only if the non-linear equations vary monotonically with each independent variable.

Cases where NR might converge to a wrong solution must also be addressed. Assuming unknowns acquire values corresponding to ‘C’ (shown in Figure 5.2) during an NR iteration step, the method might converge to a wrong solution. This issue is resolved if a large value of convergence exponent is selected in Eq. (5.18) and Eq. (5.19). However, largest value of the exponent is determined by machine precision.

The errors introduced due to numerical differentiation and machine-precision (number of significant digits) are some other factors affecting convergence. This can be understood by considering a simple example. An expression for analytical differentiation of a simple non-linear function in single variable ‘x’ represented by Eq. (5.21) is given by Eq. (5.22).
An analytical differential expressions for constitutive equations formulated in this study cannot be obtained due to high degree of non linearity. A numerical differentiation of Eq. (5.21) is achieved by using Taylor series truncating second and higher order differentials given by Eq. (5.22). The dependency of numerical differentiation on delta (or ‘h’) is demonstrated by Fig. 5.3. An increased difference between numerical and analytical differentiation (ε) at large delta (or ‘h’) values was attributed to error introduced due to Taylor series truncation and at very small ‘h’ to machine precision.

\[ f(x) = (x - 2)^2 \]  \hspace{2cm} (5.21)

\[ \frac{df(x)}{dx} = 2(x - 2) \]  \hspace{2cm} (5.22)

Thus, appropriate selection of delta values is of utmost importance while solving a system of non-linear equations. In this study, delta values corresponding to each unknown or independent variable was selected on the basis of pre-sampling and desired precision corresponding to the unknown parameter.
5.2 Initialization of Variables

Simultaneous solution of a non-linear set of equations via NR requires an initial value be defined for each independent variable or unknown. Thus, variables are initialized either by setting them equal to a specification or approximated from specifications indirectly assuming ideal cases. However, NR may face convergence problems if initial conditions are far from actual solution as discussed in the previous section. A proper variable initialization is therefore necessary to circumvent these issues.

5.2.1 Single Phase Flow-Variable Initialization

Single phase pipe flow requires phase mass flow rate and either of inlet or outlet pressures to be specified. However, a single phase T-split requires two pressures to be specified at any two of inlet, run and branch arms. Table 5.1 below provides an inventory of unknowns for a pipe and tee-junction FV cell block.

Table 5.1: Single Phase Gas Flow Unknowns

<table>
<thead>
<tr>
<th>Non Tee Cell</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>X direction Gas</td>
<td>U\textsubscript{g}</td>
<td></td>
</tr>
<tr>
<td>Velocity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Unknowns</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tee Cell</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>P</td>
<td></td>
</tr>
<tr>
<td>X and Y direction</td>
<td>U\textsubscript{g}, V\textsubscript{g}</td>
<td></td>
</tr>
<tr>
<td>Gas Velocities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Unknowns</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Pressure unknowns are directly inferred from the pressure specifications as:

\[ \text{P}_{\text{pipe}} = \text{P}_{\text{inlet/outlet}} \]

(any on of inlet or outlet is specified)

\[ \text{P}_{\text{inlet}} = \text{P}_{\text{branch}} + \text{P}_{\text{run}} \]

(any two of inlet, run and branch P is specified)
Gas velocity unknowns are calculated from gas mass flow rate \( \dot{m}_{gas} \) and pressure specifications as:

\[
\rho_{n,gas} = \frac{P_n M}{ZRT} \\
U_{n,gas} = \frac{\dot{m}_{gas}}{\rho_n A_{pipe}}
\]

Where,

- \( n = \) inlet/outlet, run, branch
- \( \rho = \) Density
- \( M = \) Molecular weight of gas
- \( Z = \) Gas phase super-compressibility (constant)
- \( R = \) Universal Gas Constant
- \( T = \) Temperature
- \( \dot{m}_{gas} = \) Gas mass flow-rate (specified)

### 5.2.2 Two Phase Flow-Variable Initialization

Table 5.2: Two Phase Gas-Liquid Flow Unknowns

<table>
<thead>
<tr>
<th>Non Tee Cell</th>
<th></th>
<th>Tee Cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>P</td>
<td>Pressure</td>
</tr>
<tr>
<td>Liquid Holdup</td>
<td>( \alpha_l )</td>
<td>Liquid Holdup</td>
</tr>
<tr>
<td>Gas Velocity</td>
<td>( U_g )</td>
<td>Gas Velocity</td>
</tr>
<tr>
<td>Liquid Velocity</td>
<td>( U_l )</td>
<td>Liquid Velocity</td>
</tr>
<tr>
<td><strong>Total Unknowns</strong></td>
<td><strong>4</strong></td>
<td><strong>Total Unknowns</strong></td>
</tr>
</tbody>
</table>

For two phase gas-liquid flow both gas \( \dot{m}_{gas} \) and liquid \( \dot{m}_{liquid} \) mass flow rates must be specified in addition to pressure specifications in single phase flow. Table
5.2 below provides an inventory of unknowns for two phase flow in a pipe and tee-junction FV cell block. Initialization of pressure and phase velocity unknowns is similar to single phase given by Eqn. (5.23) and Eqn. (5.24). However, liquid holdup variable initialization is based upon ‘no-slip’ assumption at the inlet i.e. the gas and liquid phase travel with same velocities at the inlet. An initial value of liquid holdup can thus be calculated from gas ($\dot{m}_{gas}$) and liquid ($\dot{m}_{liquid}$) mass flow rate specifications and densities ($\rho_{gas/liquid}$) obtained from Eqn. (5.24) as:

$$\alpha_{liq} = \frac{\dot{q}_{liq}}{\dot{q}_{liq} + \dot{q}_{gas}}$$

(5.25)

$$\dot{q}_{k} = \frac{\dot{m}_{k}}{\rho_{k}}$$

Where,

$\dot{m}_{k}$ = mass flow rate of phase ‘k’

$\dot{q}_{k}$ = volumetric flow-rate of phase ‘k’

$\alpha_{liq}$ = liquid holdup

$\rho_{k}$ = density of phase ‘k’

k = gas/liquid
Chapter 6

Results and Discussion

A number of experimental studies on two phase air water flow at tee junctions were conducted by Shoham et al (1987, 1989), Ballyk and Shoukri (1991), Azzopardi et al (1990), Hart et al (1990), Penmatcha (1993) and Azzopardi et al (2006). Hart et al (1990) presented a mechanistic approach to predict two phase splitting at T-junctions using 'double stream model'. A modified form of Bernoulli's equation as applied to T-junctions, accounting for appropriate T-junction energy losses was used to predict two phase air water. Another approach was put forth by Shoham et al (1996) to predict phase splitting behavior using a geometrical model. The model presented by Shoham (1996) operated on predefined flow geometry to generate two phase split profiles at T-junctions. A number of input parameter combinations were used to generate pressure, velocity and holdup profiles for the purpose of validation and parametric studies.

6.1 System Description

In this study an isothermal, viscous, constant density liquid phase and constant super-compressibility gas (air) system was considered. An assumption of steady state condition maintained during model formulation is relayed herein. The calculations were based upon international systems of units (SI). Table 6.1 provides invariable system properties i.e., pipe line, gas and liquid property data which is kept similar to experimental conditions used by Hart (1990). The study was restricted to the case of near horizontal flow thus effect of gravity force was neglected. An equal diameter and length was assumed for inlet, run and branch arms however, proposed model is not restricted by the preceding assumption. The gas and liquid properties were determined assuming air as lighter phase and water as denser phase.
6.2 Single Phase Steady State Analysis

In order to gain a better understanding of two phase flow predictions, it is necessary to discuss model predictions and behavior at a crude level. Therefore, current study proceeds through a number of cases for single phase pipe and tee junction flow to analyze model predictions. A single phase flow study served as a preliminary validation tool before additional complexities were incorporated. A number runs were used to parameterize pressure and velocity trends for gas flow in pipelines and T-junctions.

6.2.1 Single Phase Pipe Flow

An inlet pressure specification for backward and an outlet pressure specification for forward staggering approaches is required as discussed previously in Chapter 4. However, the two formulations can be used interchangeably for pipe flow with negligible difference in profiles. Table 6.1 below provides input parameters to generate generic profiles using the two model formulations. Fig. 6.1 thru Fig. 6.3 represent variation of pressure, phase velocity and wall shear force for gas flow through a pipeline. As distance from inlet increases, pressure decreases due to increasing frictional losses at fluid-wall contact (given by Fig. 6.3). An increase in gas phase velocity with distance is the
consequence of continuity equation. Similar profiles can be generated for forward staggered formulation shown by Fig. 6.4 thru Fig. 6.6.

Table 6.2: Single phase forward and backward staggering input parameters

<table>
<thead>
<tr>
<th>Backward Staggering Inputs</th>
<th>Forward Staggering Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of pipe (m)</td>
<td>1000</td>
</tr>
<tr>
<td>Inlet Gas mass flow rate (kg/s)</td>
<td>0.05</td>
</tr>
<tr>
<td>Inlet pressure (Pa)</td>
<td>400000</td>
</tr>
<tr>
<td>Roughness parameter (ε)</td>
<td>0.008</td>
</tr>
<tr>
<td>Number of grid blocks</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of pipe (m)</td>
<td>1000</td>
</tr>
<tr>
<td>Inlet Gas mass flow rate (kg/s)</td>
<td>0.05</td>
</tr>
<tr>
<td>Outlet pressure (Pa)</td>
<td>148438</td>
</tr>
<tr>
<td>Roughness parameter (ε)</td>
<td>0.008</td>
</tr>
<tr>
<td>Number of grid blocks</td>
<td>20</td>
</tr>
</tbody>
</table>

Figure 6.1: Single phase pipe flow pressure profile - Backward Staggering

Figure 6.2 Single phase pipe flow velocity profile - Backward Staggering
Figure 6.3: Single phase pipe flow wall shear force profile - Backward Staggering

Figure 6.4: Single phase pipe flow pressure profile - Forward Staggering

Figure 6.5: Single phase pipe flow velocity profile - Forward Staggering
A comparison between forward and backward staggering profiles for single phase gas flow suggests that the two formulation approaches do not possess a significant difference. Thus backward and forward formulation for single-phase pipe-flow will be used interchangeably hereafter.

6.2.1.1 Variation with Pipeline Roughness

Table 6.3 below provides input parameters for studying variation in pressure and velocity profiles at different roughness parameters ($\varepsilon$) while remaining input parameters were held constant to isolate profile behavior with roughness. The plots (Fig. 6.7 thru Fig. 6.9) are color coded with corresponding roughness parameter value in Table 6.3.

Table 6.3: Input parameters for single phase pipe flow with varying roughness (Forward Staggered Approach)

<table>
<thead>
<tr>
<th>Length of pipe (m)</th>
<th>100</th>
<th>100</th>
<th>100</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas mass flow rate (kg/s)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Inlet pressure (Pa)</td>
<td>400000</td>
<td>400000</td>
<td>400000</td>
<td>400000</td>
</tr>
<tr>
<td>Roughness parameter ($\varepsilon$)</td>
<td>0.002</td>
<td>0.004</td>
<td>0.006</td>
<td>0.008</td>
</tr>
</tbody>
</table>

Figure 6.6: Single phase pipe flow wall shear force profile - Forward Staggering
Fig. 6.7: Single phase pipe flow pressure profile variation with roughness parameter

Fig. 6.8: Single phase pipe flow velocity profile variation with roughness parameter

Fig. 6.7 shows variation of pressure with distance wherein zero x co-ordinate represents the inlet. The profiles suggest that as roughness of the pipeline increases, pressure losses increase due to increased momentum dissipation. As roughness increases the rate of gas velocity increase decreases as shown by Fig. 6.8 which was attributed to lower densities thereby higher velocities due to steady state mass continuity of the model formulation. A higher roughness parameter corresponds to a higher value of gas wall shear force shown by Fig. 6.9.
6.2.1.2 Effect of Grid Resolution (Number of Grid Blocks)

Table 6.4: Input parameters for single phase pipe flow - Grid block resolution

<table>
<thead>
<tr>
<th></th>
<th>Length of pipe (m)</th>
<th>1000</th>
<th>1000</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Gas mass flow rate (kg/s)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Inlet pressure (Pa)</td>
<td>400000</td>
<td>400000</td>
<td>400000</td>
<td></td>
</tr>
<tr>
<td>Roughness parameter (ε)</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>Number of grid blocks</td>
<td>20</td>
<td>100</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

Table 6.4 provides input parameters used to generate pressure profiles (corresponding to the color code) for three different grid resolutions for the same pipeline. Fig. 6.10 presents a semi-log plot of pressure v/s distance demonstrating pressure profile deviation at the outlet as the length of grid block increases. Although, changes in the profile are rather insignificant, selection of a larger grid block will result in reduced accuracies. On the contrary a smaller block size will increase the computation time. Thus grid block size selection must be based upon a break-even point between computational time and desired precision.

Figure 6.9: Single phase pipe flow wall shear force profile variation with roughness
6.2.2 Single Phase Flow at Tee

A number of parametric model runs for single phase flow at T-junctions revealed insights into the predicted tee split behavior. In this study a forward staggering approach was implemented due to convenience of input parameters. Although, current study was restricted to diverging tee case, the model is capable of handling impacting tee junction. Table 6.5 provides input parameters for model formulation used to generate single phase pressure profile for comparison with Hart's single phase pressure profile. Hart suggested
that a static pressure gain and pressure loss (Fig. 6.11) in run and branch arms respectively, is a direct consequence of Bernoulli’s equation.

Table 6.5: Input parameters - Model generated single phase pressure profile

<table>
<thead>
<tr>
<th>Inlet/ Run / Branch Length (m)</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of FVM Cell Blocks</td>
<td>31</td>
</tr>
<tr>
<td>Inlet gas flow-rate (kg/s)</td>
<td>0.032</td>
</tr>
<tr>
<td>Branch arm angle (β)</td>
<td>90</td>
</tr>
<tr>
<td>Roughness parameter(ε)</td>
<td>0.001</td>
</tr>
<tr>
<td>Prun (Pa)</td>
<td>101635</td>
</tr>
<tr>
<td>Pbranch (Pa)</td>
<td>101370</td>
</tr>
<tr>
<td>λg (dimensionless)</td>
<td>0.6375</td>
</tr>
</tbody>
</table>

A comparison with single-phase pressure profile presented by Hart (1990) (Fig. 6.11) was found to be in good agreement with model predicted profile (Fig. 6.12). Hart(1990) suggests a separate procedure for pressure profile calculation in inlet, run and branch arms whereas phase split at T-junction is affected on the basis of inlet gas and liquid phase velocities and pressure difference between branch and run arms. Thus, T-junction remains independent of upstream pressure conditions.

Figure 6.12: Model generated pressure profile
The proposed model solves inlet, run, branch and T-junction governing equations simultaneously thus incorporating an inherent interconnectivity study. Furthermore, available models (Hart, 1990; Shoham, 1996; Ottens, 2001) are not able to generate a continuous profile which is instrumental in characterizing pressure spike phenomenon due to tee split. The proposed model is also capable of generating continuous velocity profiles Fig. 6.13 not discussed by prior studies.

6.2.2.1 Effect of Grid Resolution (Number of Grid Blocks)

At this point, it is necessary to discuss the impact of grid block resolution on tee split characterization. Table 6.6 presents input data for two cases for varying number of grid blocks used to discretize T-junction while maintaining constant values for all other input parameters. Pressure profiles in Fig. 6.14 correspond to color coded values represented in Table 6.6. Fig. 6.14 demonstrates variation in pressure profile with grid resolution for a regular straight tee with equal length inlet, run and branch arms. As can be seen in Fig. 6.14, a lower grid resolution (Case 2) dampens the characteristic pressure spiking due to tee split when compared to a higher grid resolution. In this study, a maximum grid block size equal to five times the pipe diameter was found to result in reasonable pressure spike resolution.
Table 6.6: Input parameters for single phase tee split - Grid resolution

<table>
<thead>
<tr>
<th>Case No.</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet/Run/Branch Length (m)</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>No. of FVM Cell Blocks</td>
<td>46</td>
<td>16</td>
</tr>
<tr>
<td>Inlet gas flow-rate (kg/s)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Branch arm angle (β)</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Roughness parameter(ε)</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Run Arm Outlet Pressure (Pa)</td>
<td>300000</td>
<td>300000</td>
</tr>
<tr>
<td>Branch Arm Outlet Pressure(Pa)</td>
<td>270000</td>
<td>270000</td>
</tr>
</tbody>
</table>

Figure 6.14: Grid block resolution - Single phase tee flow

6.2.2.2 Variation with Branch Arm Angle (β)

Table 6.7 provides input parameters for varying angles between branch and run arms for single phase split at T-junction. Fig. 6.15 illustrates the effect of branch arm angle on pressure profiles corresponding to color code presented in preceding table. Fig. 6.16 presents a magnified cross section of the characteristic pressure spike. It can be inferred that magnitude of pressure spiking is a function of angle (β) between run and branch arms, the larger the branch arm angle more prominent is the pressure spike. Thus a pressure spike is characteristic of change in flow direction rather than T-junction losses.
Table 6.7: Input parameters for single phase tee split with varying branch arm angle

<table>
<thead>
<tr>
<th>Case No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet/ Run / Branch Length (m)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>No. of FVM Cell Blocks</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
</tr>
<tr>
<td>Inlet gas flow-rate (kg/s)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Branch arm angle (β)</td>
<td>90</td>
<td>80</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>Roughness parameter (ε)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Prun (Pa)</td>
<td>300000</td>
<td>300000</td>
<td>300000</td>
<td>300000</td>
</tr>
<tr>
<td>Pbranch (Pa)</td>
<td>290000</td>
<td>290000</td>
<td>290000</td>
<td>290000</td>
</tr>
<tr>
<td>λg (dimensionless)</td>
<td>0.5218</td>
<td>0.5266</td>
<td>0.5414</td>
<td>0.5476</td>
</tr>
</tbody>
</table>

Figure 6.15: Effect of branch arm angle on pressure spiking - Single Phase

Figure 6.16: Effect of branch arm angle on pressure spiking - Single Phase (Magnified Section)
Fig. 6.17 demonstrates variation in mass fraction of inlet gas turning to branch arm ($\lambda$) with branch arm angle ($\beta$). Although an increase in branch arm angle results in a decrease in branch gas mass fraction, the relatively small magnitude of change indicates that branch gas mass fraction ($\lambda$) is a weak function of branch arm angle ($\beta$).

![Graph showing variation in mass fraction with branch arm angle](image)

Figure 6.17: Effect of branch arm angle on branch gas mass fraction - Single Phase

### 6.2.2.3 Variation with Branch Arm Pressure

Table 6.8: Input parameters for single phase tee split with varying branch arm outlet pressure

<table>
<thead>
<tr>
<th>Run</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet/ Run / Branch Length (m)</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>No. of FVM Cell Blocks</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
<td>46</td>
</tr>
<tr>
<td>Inlet gas flow-rate (kg/s)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\beta$ (degrees)</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>$\varepsilon$ (dimensionless)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Prun (Pa)</td>
<td>300000</td>
<td>300000</td>
<td>300000</td>
<td>300000</td>
<td>300000</td>
</tr>
<tr>
<td>$P_{\text{branch}}$ (Pa)</td>
<td>300000</td>
<td>270000</td>
<td>260000</td>
<td>250000</td>
<td>220000</td>
</tr>
<tr>
<td>$\lambda_g$ (dimensionless)</td>
<td>0.4618</td>
<td>0.6341</td>
<td>0.6863</td>
<td>0.7358</td>
<td>0.8675</td>
</tr>
</tbody>
</table>

Table 6.8 below provides single phase model input parameters for varying branch arm outlet pressure keeping all other parameters constant. Fig. 6.18 presents variation in
branch gas mass fraction with branch arm pressure. It can be easily seen that as branch arm pressure decreases, branch arm gas mass fraction increases. Since a large change in gas mass fraction is registered due to a small change in pressure suggesting branch gas mass fraction is a strong function of branch outlet pressure.

![Graph showing the effect of branch arm pressure on branch gas mass fraction.](image)

Figure 6.18: Effect of branch arm pressure on branch gas mass fraction - Single Phase

### 6.3 Comparison between Single and Two Phase Flow in Pipelines

A comparison between single phase (gas) and two phase (mist flow pattern) flow suggests that the presence of a liquid phase increases drastically increases pressure losses as shown in Fig. 6.19.

Table 6.9: Input parameters for two-phase mist flow: Comparison between single and two phase pressure losses in pipeline

<table>
<thead>
<tr>
<th>Backward Staggering Inputs</th>
<th>Forward Staggering Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe length (m)</td>
<td>Pipe length (m)</td>
</tr>
<tr>
<td>Gas mass flow-rate (kg/s)</td>
<td>Gas mass flow-rate (kg/s)</td>
</tr>
<tr>
<td>Liquid mass flow-rate (kg/s)</td>
<td>Liquid mass flow-rate (kg/s)</td>
</tr>
<tr>
<td>Inlet liquid holdup (α_l)</td>
<td>Outlet liquid holdup (α_l)</td>
</tr>
<tr>
<td>Inlet pressure (Pa)</td>
<td>Outlet pressure (Pa)</td>
</tr>
<tr>
<td>Roughness parameter (ε)</td>
<td>Roughness parameter (ε)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>0.05</td>
<td>0.0418</td>
</tr>
<tr>
<td>295000</td>
<td>292113</td>
</tr>
<tr>
<td>0.008</td>
<td>0.008</td>
</tr>
</tbody>
</table>
For the purpose of this analysis a mist flow pattern was assumed. Table 6.9 provides input parameters for mist flow in a pipeline. The inlet gas mass flow rate, inlet pressure, pipeline roughness for single and two phase cases were assumed to be constant for flow in same pipeline.

Figure 6.19: Comparison between single and two phase flow in pipelines
6.4 Two Phase Steady State Analysis

A similar study of two phase flow was conducted for air water flow through pipelines and diverging tee junctions with low liquid holdups. The pressure, phase velocities and liquid holdup profiles were analyzed to draw inferences into general behavior for different flow patterns. The assumption of near horizontal flow during model formulation entails neglecting effects of gravity force.

6.4.1 Two Phase Pipe Flow - Smooth Stratified Flow Pattern

![Figure 6.20: Determination of initial estimate for implicit liquid holdup calculation](image)

The calculation of liquid holdup for smooth stratified flow is based upon an implicit expression given by Eq. (6.1) which adds an additional degree of non-linearity to the system. A Newton-Raphson (NR) approach to obtain a value of liquid holdup
corresponding to a given wetted angle (θ), shown in Fig. 4.18, usually requires an initial estimate close to final solution for NR technique to converge.

\[ f_{objective} = \alpha_l - \left( \frac{2\theta}{360} - \sin \frac{2\theta}{2\pi} \right) \]  

(6.1)

Table 6.10: Two phase forward and backward staggering input parameters

<table>
<thead>
<tr>
<th>Backward Staggering Inputs</th>
<th>Forward Staggering Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe length (m)</td>
<td>Pipe length (m)</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Gas mass flow-rate (kg/s)</td>
<td>Gas mass flow-rate (kg/s)</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Liquid mass flow-rate (kg/s)</td>
<td>Liquid mass flow-rate (kg/s)</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Inlet liquid holdup (α_l)</td>
<td>Outlet liquid holdup (α_l)</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1414</td>
</tr>
<tr>
<td>Inlet pressure (Pa)</td>
<td>Outlet pressure (Pa)</td>
</tr>
<tr>
<td>500000</td>
<td>478989</td>
</tr>
<tr>
<td>Roughness parameter (ε)</td>
<td>Roughness parameter (ε)</td>
</tr>
<tr>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

The geometrical and flow pattern calculations were based upon formulations presented by Ayala (2001). Table 6.10 provides input parameters for two phase pipe flow model formulation. An inlet pressure and liquid holdup specification was used for backward staggering whereas an outlet pressure and holdup specification is required for forward staggering approach.

Figure 6.21: Comparison between forward and backward staggered pressure profile - Smooth stratified pipe flow
Fig. 6.21 thru Fig. 6.24 represent a negligible difference between forward and backward staggered pressure and phase velocity profiles, similar to single phase pipe flow case, suggesting that the two can be used interchangeably for two phase pipe flow.

![Graph showing pressure difference between forward and backward staggering](image1.png)

**Figure 6.22:** Comparison between forward and backward gas velocity profile - Smooth stratified pipe flow

![Graph showing liquid velocity difference between forward and backward staggering](image2.png)

**Figure 6.23:** Comparison between forward and backward liquid velocity profile - Smooth stratified pipe flow
A parametric study investigating the effect of inlet liquid holdup on pressure, phase velocity and liquid holdup profiles suggested an increase in pressure losses due to presence of an additional liquid phase. This was attributed to higher magnitude of liquid wall shear losses. Table 6.11 presents input parameters for two phase pipe flow model formulation. The resulting profiles are given by Fig. 6.25 thru Fig. 6.27.
However, it can be seen from Fig. 6.25 that an increasing liquid holdup while keeping other parameters constant results in a decrease in pressure loss. An increasing liquid holdup value at the inlet implies a lower inlet liquid velocity and a higher inlet gas velocity which can be seen clearly in Fig. 6.26 and Fig. 6.27.
A closer look at Fig. 6.29 and Fig. 6.30 suggests a lower liquid wall shear and a higher gas wall shear at higher liquid holdups. A sum of gas and wall shear forces at a particular location for a specific liquid holdup value decreases as liquid holdup increases. Thus, it can be inferred from above discussion that a derivative behind increased pressure losses due to presence of an additional liquid phase flowing at relatively higher velocities. Although higher liquid holdups result in lower pressure losses for two phase flow, the pressure losses are always higher when compared to single phase gas flow.
6.4.2 Two Phase Tee Split - Smooth Stratified Flow

Fig. 6.31 illustrates nomenclature for a generic diverging T-junction and a definition of branch gas and liquid mass fractions used by previous studies to characterize phase splits at tees.
Where,

\( \dot{m}_{k,in} \) = Mass flow rate of phase ‘k’ in inlet arm

\( \dot{m}_{k,br} \) = Mass flow rate of phase ‘k’ in branch arm

\( \lambda_{k,br} \) = Mass fraction of phase ‘k’ entering branch arm

Figure 6.31: Branch arm gas and liquid mass fractions

Table 6.12: Input and output parameters for two phase tee split for smooth stratified flow: Effect of junction losses (Forward Staggered Approach)

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet/Run/Branch (m)</td>
<td>15</td>
<td>15</td>
<td>Input</td>
</tr>
<tr>
<td>Inlet Gas Flow Rate (kg/sec)</td>
<td>0.05</td>
<td>0.05</td>
<td>Input</td>
</tr>
<tr>
<td>Inlet Liquid Flow Rate (kg/sec)</td>
<td>0.1</td>
<td>0.1</td>
<td>Input</td>
</tr>
<tr>
<td>Pressure in run arm (Pa)</td>
<td>295000</td>
<td>295000</td>
<td>Input</td>
</tr>
<tr>
<td>Pressure in branch arm (Pa)</td>
<td>294500</td>
<td>294500</td>
<td>Input</td>
</tr>
<tr>
<td>Roughness Parameter (( \varepsilon ))</td>
<td>0.001</td>
<td>0.001</td>
<td>Input</td>
</tr>
<tr>
<td>Inlet Liquid Holdup (( a_l ))</td>
<td>0.0905</td>
<td>0.0891</td>
<td>Input</td>
</tr>
<tr>
<td>Branch angle (( \beta ))</td>
<td>90</td>
<td>90</td>
<td>Input</td>
</tr>
<tr>
<td>Branch gas mass fraction (( \lambda_g ))</td>
<td>0.5996</td>
<td>0.6148</td>
<td>Output</td>
</tr>
<tr>
<td>Branch liquid mass fraction (( \lambda_l ))</td>
<td>0.6102</td>
<td>0.6236</td>
<td>Output</td>
</tr>
</tbody>
</table>
The impact of tee junction losses on phase split was studied by considering cases wherein junction losses were neglected for Case 2. Table 6.12 presents input parameters for two cases - Case 1 considers junction losses whereas in Case 2 junction losses are set to zero. Fig. 6.32 depicts a lower pressure difference between run and branch arms in the vicinity of tee. Since profile variations for “negligible junction losses” case were expected to be small, a smaller pressure difference between branch and run arms was specified so that impact of junction losses on tee could be studied.

Figure 6.32: Pressure profile for two phase tee split for stratified flow - Effect of junction losses

Figure 6.33: Liquid phase velocity profile for two phase tee split for stratified flow - Effect of junction losses
Variation with Inlet Liquid Mass Flow-rate

Fig. 6.36 thru Fig. 6.38 show variation of inlet liquid holdup, branch gas and liquid mass fraction respectively with inlet liquid mass flow rate. Table 6.13 and Table 6.14 present a list of input parameters and output parameters respectively used to study profile variation with inlet liquid mass flow rate.
Table 6.13: Input parameters for two-phase tee split for smooth stratified flow:
Variation with inlet liquid mass flow-rate (Forward Staggered Approach)

<table>
<thead>
<tr>
<th>Inlet/Run/Branch arms (m)</th>
<th>15</th>
<th>15</th>
<th>15</th>
<th>15</th>
<th>15</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Gas Flow-Rate (kg/sec)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Inlet Liquid Flow-Rate (kg/sec)</td>
<td>0.05</td>
<td>0.07</td>
<td>0.1</td>
<td>0.15</td>
<td>0.2</td>
<td>0.25</td>
</tr>
<tr>
<td>Pressure in run arm (Pa)</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
<td></td>
</tr>
<tr>
<td>Pressure in branch arm (Pa)</td>
<td>294500</td>
<td>294500</td>
<td>294500</td>
<td>294500</td>
<td>294500</td>
<td></td>
</tr>
<tr>
<td>Roughness Parameter (ε)</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>Inlet Liquid Holdup (α_l)</td>
<td>0.0117</td>
<td>0.0158</td>
<td>0.0218</td>
<td>0.0315</td>
<td>0.0409</td>
<td>0.0511</td>
</tr>
<tr>
<td>Branch angle (β)</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 6.14: Output parameters for two-phase tee split for smooth stratified flow:
Variation with inlet liquid mass flow-rate (Forward Staggered Approach)

| Branch gas mass fraction (λ_g) | 0.5208 | 0.5199 | 0.5162 | 0.5135 | 0.5114 | 0.5103 |
| Branch liquid mass fraction (λ_l) | 0.5074 | 0.5097 | 0.5280 | 0.5252 | 0.5229 | 0.5441 |

Figure 6.36: Inlet liquid holdup v/s inlet liquid mass flow rate - Smooth stratified flow
It can be seen from Fig. 6.37 and Fig. 6.38 that as inlet liquid mass flow rate increases, branch gas decreases. This was attributed to higher gas momentum entering tee junction reducing its tendency to change direction and enter the branch arm. The branch liquid mass fraction varies depending upon liquid holdup variation. A low liquid holdup results in higher liquid momentum at tee thereby decreasing branch liquid fraction and vice versa.

Figure 6.37: Branch gas mass fraction v/s inlet liquid mass flow rate - Smooth stratified flow

Figure 6.38: Branch liquid mass fraction v/s inlet liquid mass flow rate - Smooth stratified flow
6.4.2.2 Variation with Branch Arm Pressure

Similar comparative analysis was conducted by varying branch arm outlet pressure with respect to run arm outlet pressure. Table 6.15 provides input and output parameters for different branch arm outlet pressures keeping other input parameters constant for inlet liquid mass flow rate of 0.05 kg/sec.

Table 6.15: Input parameters for two phase tee split for smooth stratified flow: Variation with branch arm pressure for \( m_{\text{inlet}} = 0.05 \) kg/sec (Forward Staggered Approach)

<table>
<thead>
<tr>
<th>Inlet/Run/Branch arms (m)</th>
<th>15</th>
<th>15</th>
<th>15</th>
<th>15</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Gas Flow Rate (kg/sec)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Inlet Liquid Flow Rate (kg/sec)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Pressure in run arm (Pa)</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
</tr>
<tr>
<td>Pressure in branch arm (Pa)</td>
<td>297000</td>
<td>294700</td>
<td>292000</td>
<td>290000</td>
<td>289000</td>
</tr>
<tr>
<td>Roughness Parameter (( \varepsilon ))</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>Inlet Liquid Holdup (( \alpha_l ))</td>
<td>0.0117</td>
<td>0.0117</td>
<td>0.0117</td>
<td>0.0117</td>
<td>0.0117</td>
</tr>
<tr>
<td>Branch angle (( \beta ))</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 6.16: Output parameters for two phase tee split for smooth stratified flow: Variation with branch arm pressure for \( m_{\text{inlet}} = 0.05 \) kg/sec (Forward Staggered Approach)

| Branch gas mass fraction (\( \lambda_g \)) | 0.4162 | 0.5022 | 0.61378 | 0.6543 | 0.7338 |
| Branch liquid mass fraction (\( \lambda_l \)) | 0.4704 | 0.5008 | 0.5399 | 0.5535 | 0.5780 |

Figure 6.39: Branch liquid mass fraction v/s branch arm pressure - Smooth stratified flow
Fig. 6.39 and Fig. 6.40 illustrate a decrease in branch gas and liquid mass fractions as the branch arm outlet pressure increases. A large magnitude of decrease suggests that the phase split at T-junctions is a strong function of pressure difference between difference between run and branch outlet pressures as suggested by Hart et al (1990), Shoham et al (1996) for stratified flow. Fig. 6.41 depicts variation in mass fraction of liquid turning into branch arm corresponding to a mass fraction of gas turning into branch arm for different inlet superficial liquid velocities ($U_{sl}$) at a constant inlet superficial gas velocity ($U_{sg}$).

Figure 6.40: Branch gas mass fraction v/s branch arm pressure - Smooth stratified flow

Figure 6.41: Hart’s branch liquid mass fraction v/s gas mass fraction for $U_{sg} = 5.1\text{m/s}$ – Stratified Flow (Hart, 1990)
Table 6.17: Input parameters for two phase tee split for smooth stratified flow: Variation with branch arm pressure for $m_{\text{inlet}} = 0.005$ kg/sec (Forward Staggered Approach)

<table>
<thead>
<tr>
<th>Inlet/Run/Branch arms (m)</th>
<th>15</th>
<th>15</th>
<th>15</th>
<th>15</th>
<th>15</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Gas Flow Rate (kg/sec)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Inlet Liquid Flow Rate (kg/sec)</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>Pressure in run arm (Pa)</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
</tr>
<tr>
<td>Pressure in branch arm (Pa)</td>
<td>295200</td>
<td>295000</td>
<td>294900</td>
<td>294800</td>
<td>294725</td>
<td>294715</td>
</tr>
<tr>
<td>Roughness Parameter ($\varepsilon$)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Inlet Liquid Holdup ($\alpha_l$)</td>
<td>0.0121</td>
<td>0.0136</td>
<td>0.0136</td>
<td>0.0132</td>
<td>0.0126</td>
<td>0.0125</td>
</tr>
<tr>
<td>Branch angle ($\beta$)</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 6.18: Output parameters for two phase tee split for smooth stratified flow: Variation with branch arm pressure for $m_{\text{inlet}} = 0.005$ kg/sec (Forward Staggered Approach)

| Branch gas mass fraction($\lambda_g$) | 0.4035 | 0.4770 | 0.5134 | 0.5497 | 0.5771 | 0.5807 |
| Branch liquid mass fraction($\lambda_l$) | 0.1769 | 0.4086 | 0.5539 | 0.6899 | 0.7743 | 0.7842 |

Table 6.17 above tabulates model input information for a different inlet liquid mass flow rate of 0.005 kg/sec. It was observed for smooth stratified flow that a ‘flip flop’ (Oranje, 1970; Hart et al, 1990; Shoham et al, 2006) behavior is more predominant at low liquid loading.

As liquid holdup of system increases the above mentioned phenomenon does not affect flow split at T-junction as demonstrated in Fig. 6.42. The profiles obtained in Fig. 6.42 were in agreement with generic trends branch gas v/s liquid mass fraction curves. However, it is important to note that Hart’s ‘double stream model’ is based upon inlet liquid and gas superficial velocities and pressure difference between branch and run arms. The proposed model inherently takes into account all pertinent flow parameters due to implementation of a simultaneous solution technique. A more realistic profile prediction is thus expected.
Two Phase Pipe Flow - Mist Flow Pattern

6.4.3.1 Variation with Inlet Liquid Holdup

A parametric study on the effect of inlet liquid holdup on pressure, phase velocity and liquid holdup profiles as done previously in section 6.4.1.1 for stratified flow in pipeline, yields similar results. The rationale behind decrease in pressure losses with increasing liquid holdup for a constant inlet liquid mass flow rate is same as in smooth stratified flow.

Table 6.19: Input parameters for two phase pipe flow with varying inlet liquid holdup: Mist flow (Backward Staggered Approach)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe Length (m)</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Inlet Gas Flow Rate (kg/sec)</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Inlet Liquid Flow Rate (kg/sec)</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Inlet Pressure (Pa)</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
</tr>
<tr>
<td>Roughness Parameter (ε)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Inlet Liquid Holdup (a_l)</td>
<td>0.05</td>
<td>0.1</td>
<td>0.15</td>
</tr>
</tbody>
</table>
However, mist flow differs from stratified flow pattern due to a relatively homogeneous distribution of liquid phase (mist) in gas phase. Thus gas and liquid phase velocities tend to become equal as liquid holdup decreases shown in Fig. 6.43 and Fig. 6.44. The input parameters used are listed in Table 6.19 for different values of inlet liquid holdup keeping other parameters constant.

Figure 6.43: Two phase pipe flow pressure profile variation with inlet liquid holdup-Mist flow pattern

Figure 6.44: Two phase pipe flow gas velocity profile variation with inlet liquid holdup-Mist flow pattern
Two Phase Tee Split – Mist Flow

Table 6.20 lists input and output parameters (liquid holdup, branch gas and liquid mass fractions) used to generate generic tee split profiles for mist flow pattern. The liquid holdup profile (Fig. 6.50) is indicative of a stronger split at T-junction when compared to smooth stratified flow pattern.
Table 6.20: Input parameters for two phase tee split for mist flow: Generic profiles (Forward Staggered Approach)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet/Run/Branch Arms (m)</td>
<td>15</td>
</tr>
<tr>
<td>Gas Flow Rate (kg/sec)</td>
<td>0.05</td>
</tr>
<tr>
<td>Liquid Flow Rate (kg/sec)</td>
<td>0.2</td>
</tr>
<tr>
<td>Pressure in run arm (Pa)</td>
<td>295000</td>
</tr>
<tr>
<td>Pressure in branch arm (Pa)</td>
<td>294500</td>
</tr>
<tr>
<td>Roughness Parameter (ε)</td>
<td>0.001</td>
</tr>
<tr>
<td>Inlet Liquid Holdup (α_l)</td>
<td>0.0238</td>
</tr>
<tr>
<td>Branch angle (β)</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 6.21: Output parameters for two phase tee split for mist flow: Generic profiles (Forward Staggered Approach)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch gas mass fraction (λ_g)</td>
<td>0.5494</td>
</tr>
<tr>
<td>Branch liquid mass fraction (λ_l)</td>
<td>0.5680</td>
</tr>
</tbody>
</table>

Figure 6.47: Pressure profile for two phase tee split – Mist flow pattern

Fig. 6.48 and Fig. 6.49 represent gas and liquid phase velocity profiles. Table 6.21 lists values of branch gas and liquid mass fractions predicted by the proposed model formulation.
Figure 6.48: Gas phase velocity profile for two phase tee split – Mist flow pattern

Figure 6.49: Liquid phase velocity profile for two phase tee split – Mist flow pattern

Figure 6.50: Liquid holdup profile for two phase tee split – Mist flow pattern
6.4.4.1 Variation with Branch Arm Pressure

Table 6.22 lists input and output parameters for different branch arm pressures keeping other parameters constant. A decrease in branch gas and liquid mass fractions with increasing branch arm outlet pressures (Fig. 6.51) is similar to profiles for smooth stratified flow pattern. The large variation in branch gas and liquid mass fractions indicate a stronger dependency on branch and run arm pressure difference as for the case of smooth stratified flow split.

Table 6.22: Input parameters for two phase tee split for mist flow: Variation with branch arm outlet pressure (Forward Staggered Approach)

<table>
<thead>
<tr>
<th>Inlet/Run/Branch (m)</th>
<th>15</th>
<th>15</th>
<th>15</th>
<th>15</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Gas Flow Rate (kg/sec)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Inlet Liquid Flow Rate (kg/sec)</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Pressure in run arm (Pa)</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
</tr>
<tr>
<td>Pressure in branch arm (Pa)</td>
<td>295000</td>
<td>294900</td>
<td>294500</td>
<td>294000</td>
<td>293000</td>
</tr>
<tr>
<td>Roughness Parameter (ε)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Inlet Liquid Holdup (αl)</td>
<td>0.0238</td>
<td>0.0238</td>
<td>0.0238</td>
<td>0.0238</td>
<td>0.0238</td>
</tr>
<tr>
<td>Branch angle (β)</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 6.23: Output parameters for two phase tee split for mist flow: Variation with branch arm outlet pressure (Forward Staggered Approach)

| Branch gas mass fraction (λg) | 0.4929 | 0.5041 | 0.5494 | 0.6055 | 0.7188 |
| Branch liquid mass fraction (λl) | 0.4902 | 0.5058 | 0.5680 | 0.6449 | 0.7901 |

Variation of branch liquid mass fraction with branch gas mass fraction is demonstrated in Fig. 6.52. Although profile obtained is similar to smooth stratified flow case, a flip-flop behavior was not observed. This was attributed to a well dispersed (homogeneous) liquid phase due to flow pattern considerations.
Variation with Branch Arm Angle

Table 6.24 lists input and output parameters for different branch arm inclinations with respect to run arm keeping other parameters constant. Fig. 6.53 and Fig. 6.54 demonstrate variation in branch gas ($\lambda_g$) and liquid mass ($\lambda_l$) fractions with branch arm angle ($\beta$). The relatively small magnitude of change indicates that branch mass fractions
are weak functions of branch arm angle ($\beta$) similar to single phase split at tee in section 6.2.2.2.

Table 6.24: Input parameters for two phase tee split for mist flow: Variation with branch arm inclination (Forward Staggered Approach)

<table>
<thead>
<tr>
<th>Inlet/Run/Branch (m)</th>
<th>15</th>
<th>15</th>
<th>15</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Gas Flow Rate (kg/sec)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Inlet Liquid Flow Rate (kg/sec)</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Pressure in run arm (Pa)</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
</tr>
<tr>
<td>Pressure in branch arm (Pa)</td>
<td>294500</td>
<td>294500</td>
<td>294500</td>
<td>294500</td>
</tr>
<tr>
<td>Roughness Parameter ($\varepsilon$)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Inlet Liquid Holdup ($\alpha_l$)</td>
<td>0.0238</td>
<td>0.0238</td>
<td>0.0238</td>
<td>0.0238</td>
</tr>
<tr>
<td>Branch angle ($\beta$)</td>
<td>90</td>
<td>70</td>
<td>50</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 6.25: Output parameters for two phase tee split for mist flow: Variation with branch arm inclination (Forward Staggered Approach)

| Branch gas mass fraction ($\lambda_g$) | 0.5494 | 0.5511 | 0.5525 | 0.5536 |
| Branch liquid mass fraction ($\lambda_l$) | 0.5680 | 0.5703 | 0.5722 | 0.5737 |

Figure 6.53: Branch gas mass fraction v/s branch arm angle – Mist flow pattern
6.4.4.3 Variation with Inlet Liquid Flow-rate

Table 6.26 presents a list of input parameters and output parameters (inlet liquid holdup, branch and liquid gas mass fraction) used to parameterize profile variation with inlet liquid mass flow rate. Fig. 6.55 thru Fig. 6.57 show variation of inlet liquid holdup, branch gas and liquid mass fraction respectively with inlet liquid mass flow rate.

Table 6.26: Input parameters for two phase tee split for mist flow: Variation with inlet liquid mass flow-rate (Forward Staggered Approach)

<table>
<thead>
<tr>
<th>Pipe Length (m)</th>
<th>30</th>
<th>30</th>
<th>30</th>
<th>30</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Gas Flow Rate (kg/sec)</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Inlet Liquid Flow Rate (kg/sec)</td>
<td>0.8</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>Pressure in run arm (Pa)</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
<td>295000</td>
</tr>
<tr>
<td>Pressure in branch arm (Pa)</td>
<td>294500</td>
<td>294500</td>
<td>294500</td>
<td>294500</td>
<td>294500</td>
</tr>
<tr>
<td>Roughness Parameter (ε)</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Inlet Liquid Holdup (α_l)</td>
<td>0.0884</td>
<td>0.0572</td>
<td>0.0463</td>
<td>0.0352</td>
<td>0.0238</td>
</tr>
<tr>
<td>Branch angle (β)</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Branch gas mass fraction (λ_g)</td>
<td>0.5138</td>
<td>0.5223</td>
<td>0.5275</td>
<td>0.5354</td>
<td>0.5494</td>
</tr>
<tr>
<td>Branch liquid mass fraction (λ_A)</td>
<td>0.5190</td>
<td>0.5308</td>
<td>0.5381</td>
<td>0.5492</td>
<td>0.5680</td>
</tr>
</tbody>
</table>

Figure 6.54: Branch liquid mass fraction v/s branch arm angle – Mist flow pattern
Table 6.27: Output parameters for two phase tee split for mist flow: Variation with inlet liquid mass flow-rate (Forward Staggered Approach)

<table>
<thead>
<tr>
<th>Inlet Liquid Mass Flow Rate (kg/sec)</th>
<th>Branch gas mass fraction ($\lambda_g$)</th>
<th>Branch liquid mass fraction ($\lambda_l$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.5138</td>
<td>0.5190</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5223</td>
<td>0.5308</td>
</tr>
<tr>
<td>0.4</td>
<td>0.5275</td>
<td>0.5381</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5354</td>
<td>0.5492</td>
</tr>
<tr>
<td>0.6</td>
<td>0.5494</td>
<td>0.5680</td>
</tr>
</tbody>
</table>

Figure 6.55: Branch gas mass fraction v/s inlet liquid mass flow rate – Mist flow pattern

Figure 6.56: Branch liquid mass fraction v/s inlet liquid mass flow rate – Mist flow pattern

It can be seen from Fig. 6.55 and Fig. 6.56 that as the inlet liquid mass flow rate increases both inlet branch gas and liquid mass fractions decrease similar to stratified flow and was attributed to higher gas and liquid momentums entering tee junction.
Fig. 6.57 illustrates increase in liquid holdup as inlet liquid flow rate increases which is an expected behavior owing to steady state continuity equation.

![Inlet liquid holdup v/s inlet liquid mass flow rate – Mist flow pattern](image)

**Figure 6.57:** Inlet liquid holdup v/s inlet liquid mass flow rate – Mist flow pattern

### 6.4.5 Wavy Stratified Flow

Fig. 4.19 represents an ideal geometry representative of wavy stratified flow proposed by Chen (1997) in his ‘double circle model. The calculation of liquid holdup for mist flow is based upon an implicit expression given by Eqn. (4.71).

\[ f_{objective} = \left( \frac{\sin \theta_i}{\sin \theta} \right)^2 \left( \theta + \frac{\sin^2 \theta}{\tan \theta_i} - \frac{\sin 2\theta}{2} - \frac{\pi \alpha_i}{\theta} \right) - \theta_i \]  

(6.2)

Although an initial estimate of \( \theta = \theta_i \) converges to a plausible solution for \( \theta < 60 \) (blue), a higher value faces convergence issues which is depicted in Fig. 6.58. Since in this study a Generalized Newton Raphson (GNR) approach was used to solve governing equations small oscillations often lead to \( \theta > 60 \) (blue). Therefore, in this study wavy stratified flow was not pursued extensively for Tee split studies. During parametric studies only a few converging cases for two phase pipe flow were identified.
As discussed previously, predictions and profiles were generated for the case of diverging Tee. However, the proposed model is capable of handling issues related to converging T-junctions. An inconsistent set of (Table 6.28) specifications for diverging T-junction formulation leads the solution towards a converging T-junction. Fig. 6.60 thru Fig. 6.63 represents profiles for smooth stratified flow case wherein an incorrect pressure specification yields a converging Tee solution. Fig. 6.59 depicts a converging tee along with inlet and outlet specifications used to generate pressure, velocity and liquid holdup profiles.

**Figure 6.58**: Objective function v/s $(\theta_i)$ – red $(\theta = 15)$, green $(\theta = 30)$, blue $(\theta = 60)$

---

### 6.5 Additional Model Capabilities: Converging Tees

As discussed previously, predictions and profiles were generated for the case of diverging Tee. However, the proposed model is capable of handling issues related to converging T-junctions. An inconsistent set of (Table 6.28) specifications for diverging T-junction formulation leads the solution towards a converging T-junction. Fig. 6.60 thru Fig. 6.63 represents profiles for smooth stratified flow case wherein an incorrect pressure specification yields a converging Tee solution. Fig. 6.59 depicts a converging tee along with inlet and outlet specifications used to generate pressure, velocity and liquid holdup profiles.
Inlet/Run/Branch Arms (m) | 15
---|---
Inlet Gas Flow Rate (kg/sec) | 0.05
Inlet Liquid Flow Rate (kg/sec) | 0.1
Pressure in run arm (Pa) | 295000
Pressure in branch arm (Pa) | 290000
Roughness Parameter (ε) | 0.001
Inlet Liquid Holdup (α_1) | 0.0117
Branch angle (β) | 90

Figure 6.59: Converging T-junction Example

Table 6.28: Converging T-junction: Mist Flow Pattern (Forward Staggered Approach)

![Pressure Profile Graph](image)

Figure 6.60: Converging tee – pressure profile
Figure 6.61: Converging tee – gas phase velocity profile

Figure 6.62: Converging tee – liquid phase velocity profile

Figure 6.63: Converging tee – liquid holdup profile
Chapter 7

Summary and Conclusions

A comprehensive and generic model formulation for two-phase, air-water flow split at T-junctions has been developed. The mass and momentum conservation principles with appropriate closure relations are applied on each phase in a finite volume discretized system to obtain constitutive equations representative of the system. Numerical solution of governing two-fluid finite-volume equations is accomplished using Generalized Newton-Raphson (GNR) technique. Since T-junctions are relatively smaller as compared to other network entities in large pipeline networks, gas-liquid inter-phase mass transfer is neglected. An incompressible liquid phase is considered along with the assumption of constant super-compressibility gas phase. The length and diameter of inlet run and branch arms are kept equal to isolate and study impact of flow conditions, such as inlet (or outlet pressure), inlet mass rates and T-junction geometry on phase split.

A number of parametric studies on single and two phase flow in pipelines helped to develop an understanding of model working and so that characteristic features are isolated before discussing additional intricacies. The two-phase flow system is studied for low liquid-loading systems wherein stratified smooth and mist flow patterns were considered. Some of the model capabilities and significant features developed over and above previous studies are summarized below:

1. T-junction losses are usually neglected for single phase gas flow in large pipeline networks. However, these losses are crucial if two-phase, gas liquid flow is encountered due to a characteristic preferential route selectivity of liquid phase at T-junctions. The proposed model is able to capture this inherent system peculiarity suggested by a number of previous studies.

2. The effect of T-junction geometry such as angle between branch and run arms, radius of radiasation due to machining or fabrication of tee and,
different diameters for inlet, run and branch arms is incorporated. The model formulation can thus be used to predict flow conditions for a wider range of non-reactive, two-phase pipeline flow systems.

3. The model provides continuous pressure, phase velocities and liquid holdup profiles which allow identification of characteristic variations in these profiles at T-junctions. Presence of liquids in a gas-pipeline network due to moisture loading drastically increases pressure losses thereby increasing compression costs. The profile predictions can be used to locally diagnose pipeline networks wherein two-phase flow is encountered to predict location of liquid-phase in the network so that possible removal scenarios can be suggested.

4. A comprehensive study on mist flow at T-junction is not available in literature. Flow conditions for different flow pattern such as stratified smooth and mist flow patterns can be predicted. Parametric studies on two phase flow at T-junction substantiated a ‘flip-flop’ behavior as suggested by experimental observations in previous studies (Hart, 1990; Shoham, 2006) is characteristic of stratified flow patterns.

5. The proposed model is modular in nature and can therefore function as a standalone formulation for predicting flow conditions at a T-junction. Additionally, it can be integrated in existing pipeline network soft-ware to predict liquid pathways in large scale gas-pipeline networks. The formulation is flexible and can be incorporated in network models, for interconnectivity analyses, with relative ease.

6. A comparative study revealed that forward and backward staggering can be used interchangeably to translate from one set of specifications to another. The model is therefore capable of handling a wider range input specifications. This is of significance in field applications, since most of the available models require specifications which are either not available or entail some modifications in available field data.
7. An extension to converging T-junction is possible assuming proper correlations for junction losses are available. This is a significant development since most of the prior studies in this field are restricted to a single predisposed flow direction specification in inlet, run and branch arms. The model can shift from converging to diverging or vice versa depending on input specifications.

7.1 Future Work and Recommendations

The proposed model formulation relies upon a staggered grid approach for developing governing equations representative of the system. During developmental phase it is suggested that a staggered approach be followed to formulate a physically consistent constitutive equation set. However, solution of these equations might be achieved utilizing an information translation approach. This will ensure better convergence while using a simultaneous solver such as GNR (Generalized Newton Raphson), implemented in this study, at the same time maintaining physical consistency of model predictions.

Consideration of different fluid flow distributions and flow-pattern transitions, corresponding to transition criteria, in each of the inlet, run and branch arms will further improve model predictions. Further, better representation of actual physical system can be achieved by accounting for inter-phase mass transfer between gas and liquid phases. This may enhance model capability to generate more realistic flow condition predictions while simultaneously increasing its range of applicability. A suitable equation of state representing gas and liquid phases along with an appropriate equilibrium phase behavior model can be utilized for the purpose.

An actual implementation of the proposed steady state tee split model in a pipeline network model is necessary to numerically quantify the impact of characteristic phase splitting at T-junctions on liquid route selectivity. An interconnectivity study on small pipeline networks must be conducted followed by comparative field data analysis. This will allow researchers to identify areas where future efforts must be concentrated.


Appendix A

Three Block Problem

The three block problem allows comprehension of two fluid FV model formulation and working. Consider two phase pipe flow in a small pipe section discretized into three finite volume blocks as shown in Fig. A1.2. Mass and momentum conservation principles applied to each finite volume cell block yield constitutive equations representative of the system. Fig. A1.1 represents grid block nomenclature used for mass and momentum balance grids.

![Figure A1.1: Three block problem: Grid block nomenclature](image)

A.1 Forward Staggered Approach: Two Phase Pipe Flow

![Figure A1.2: Three block problem: Two phase pipe flow (Forward Staggered Approach)](image)

Mass Balance for Block 1

\[ \dot{m}_k - \rho_{k,l} A_{f,k,l} U_{k,l} = 0 \]  

(1.1)

Momentum Balance for Block 1

...
\[ \rho_{k,1} A_{f,k,1} U_{k,1} | U_{k,1} | - \rho_{k,2} A_{f,k,2} U_{k,2} | U_{k,2} | + P_0 A_{f,k,2} - P_2 A_{f,k,2} \pm F_{k\text{int},1} - F_{kw\text{all},1} = 0 \] (1.2)

Mass Balance for Block 2
\[ \rho_{k,1} A_{f,k,1} U_{k,1} - \rho_{k,2} A_{f,k,2} U_{k,2} = 0 \] (1.3)

Momentum Balance for Block 2
\[ \rho_{k,2} A_{f,k,2} U_{k,2} | U_{k,2} | - \rho_{k,3} A_{f,k,3} U_{k,3} | U_{k,3} | + P_2 A_{f,k,2} - P_3 A_{f,k,3} \pm F_{k\text{int},2} - F_{kw\text{all},2} = 0 \] (1.4)

Mass Balance for Block 3
\[ \rho_{k,2} A_{f,k,2} U_{k,2} - \rho_{k,3} A_{f,k,3} U_{k,3} = 0 \] (1.5)

Momentum Balance for Block 2
\[ \rho_{k,3} A_{f,k,3} U_{k,3} | U_{k,3} | - \rho_{k,4} A_{f,k,4} U_{k,4} | U_{k,4} | + P_3 A_{f,k,3} - P_4 A_{f,k,4} \pm F_{k\text{int},3} - F_{kw\text{all},3} = 0 \] (1.6)

Boundary Conditions at Outlet Block
\[ U_{k,3} = 2U_{k,2} - U_{k,1} \] (1.7)

Auxiliary Equations for Dependent Variables
\[ F_{k\text{int}} = f_1(\alpha_k, U_k, P_k, \varepsilon) \] (1.8)
\[ F_{kw\text{all}} = f_2(\alpha_k, U_k, P_k) \] (1.9)
\[ A_{f,k} = f_3(\alpha_k) \] (1.10)
\[ \rho_g = f_4(P) \] (1.11)
A.2 Backward Staggered Approach: Two Phase Pipe Flow

Figure A1.3: Three block problem: Two phase pipe flow (Backward Staggered Approach)

Mass Balance for Block 1

\[ \dot{m}_k - \rho_{k,1} A_{f_k,1} U_{k,1} = 0 \]  

(1.12)

Momentum Balance for Block 1

\[ \dot{m}_k \left( \frac{\dot{m}_k}{\rho_{k,in} A_{f_k,in}} \right) - \rho_{k,1} A_{f_k,1} U_{k,1} | U_{k,1} | + P_{sp} A_{f_k,sp} - P_{1} A_{f_k,1} \]

\[ \pm F_{kint,1} - F_{kwall,1} = 0 \]  

(1.13)

Mass Balance for Block 2

\[ \rho_{k,1} A_{f_k,1} U_{k,1} - \rho_{k,2} A_{f_k,2} U_{k,2} = 0 \]  

(1.14)

Momentum Balance for Block 2

\[ \rho_{k,1} A_{f_k,1} U_{k,1} | U_{k,1} | - \rho_{k,2} A_{f_k,2} U_{k,2} | U_{k,2} | + P_{1} A_{f_k,1} - P_{2} A_{f_k,2} \]

\[ \pm F_{kint,2} - F_{kwall,2} = 0 \]  

(1.15)

Mass Balance for Block 3

\[ \rho_{k,2} A_{f_k,2} U_{k,2} - \rho_{k,3} A_{f_k,3} U_{k,3} = 0 \]  

(1.16)

Momentum Balance for Block 2

\[ \rho_{k,2} A_{f_k,2} U_{k,2} | U_{k,2} | - \rho_{k,3} A_{f_k,3} U_{k,3} | U_{k,3} | + P_{2} A_{f_k,2} - P_{3} A_{f_k,3} \]

\[ \pm F_{kint,2} - F_{kwall,2} = 0 \]  

(1.17)
Auxiliary Equations for Dependent Variables

\[
F_{k,\text{int}} = f_1(\alpha_k, U_k, P_k, \varepsilon) \quad (1.18)
\]
\[
F_{k,\text{wall}} = f_2(\alpha_k, U_k, P_k) \quad (1.19)
\]
\[
A_{f_k} = f_3(\alpha_k) \quad (1.20)
\]
\[
\rho_g = f_4(P) \quad (1.21)
\]

A.3 Inventory of Unknowns and Specifications: Two Phase Pipe Flow

Table A1.1: Inventory of unknowns: Two phase pipe flow

<table>
<thead>
<tr>
<th>Primary Unknowns</th>
<th>Independent Variables</th>
<th>Dependent Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>(P_{\text{in}}, P_1, P_2, P_3)</td>
<td>(F_{k,\text{int}}, F_{k,\text{wall}}, \rho_k)</td>
</tr>
<tr>
<td>Gas Phase Velocity</td>
<td>(U_{g1}, U_{g2}, U_{g3})</td>
<td>(F_{k,\text{int}}, F_{k,\text{wall}})</td>
</tr>
<tr>
<td>Liquid Phase Velocity</td>
<td>(U_{l1}, U_{l2}, U_{l3})</td>
<td>(F_{k,\text{int}}, F_{k,\text{wall}})</td>
</tr>
<tr>
<td>Liquid Holdup</td>
<td>(\alpha_{l1}, \alpha_{l2}, \alpha_{l3})</td>
<td>(F_{k,\text{int}}, F_{k,\text{wall}}, A_{f_k})</td>
</tr>
</tbody>
</table>

Table A1.2: Specifications: Two phase pipe flow

<table>
<thead>
<tr>
<th>Forward Staggered Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Set</strong></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Backward Staggered Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Set</strong></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>
A.4 Two Phase Flow at T-junction: Forward Staggered Approach

Similarly, consider a two phase flow at a T-junction discretized into three finite volume blocks as shown in Fig. A1.4. Mass and momentum conservation principles applied to each finite volume cell block yield constitutive equations peculiar to the system. Conservation principles applied to blocks other than those discussed here yield similar equations as for two phase pipe flow.

Figure A1.4: Three block problem :Two phase tee split (Forward Staggered Approach)

Mass Balance for Block 1
\[ m_k - \rho_{k,1} A_{f_k,1} U_{k,1} = 0 \] (1.22)

Momentum Balance for Block 1
\[ \rho_{k,1} A_{f_k,1} U_{k,1} |U_{k,1}| - \rho_{k,2} A_{f_k,2} U_{k,2} |U_{k,2}| + P_1 A_{f_k,1} - P_2 A_{f_k,2} \]
\[ \pm F_{kint,1} - F_{kwall,1} - \rho_{k,2} A_{f_k,2} V_{k,2} |V_{k,2}| \cos \beta = 0 \] (1.23)

Mass Balance for Block 2
\[ \rho_{k,1} A_{f_k,1} U_{k,1} - \rho_{k,2} A_{f_k,2} U_{k,2} - \rho_{k,2} A_{f_k,2} V_{k,2} = 0 \] (1.24)

Momentum Balance for Block 2
X-direction
\[ \rho_{k,2} A_{f_k,2} U_{k,2} |U_{k,2}| - \rho_{k,3} A_{f_k,3} U_{k,3} |U_{k,3}| + P_2 A_{f_k,2} - P_3 A_{f_k,3} \]
\[ -F_{tee-loss(x)} = 0 \] (1.25)
\[ Y\text{-direction} \]

\[ \rho_{k,2} A_{jk,2} V_{k,2} |V_{k,2}| - \rho_{k,4} A_{jk,4} V_{k,4} |V_{k,4}| + P_2 A_{jk,2} - P_4 A_{jk,4} \]

\[ -F_{\text{tee-loss}(y)} = 0 \]  

(1.26)

**Mass Balance for Block 4**

\[ \rho_{k,2} A_{jk,2} V_{k,2} - \rho_{k,4} A_{jk,4} V_{k,4} = 0 \]  

(1.27)

**Momentum Balance for Block 4**

\[ \rho_{k,4} A_{jk,4} V_{k,4} |V_{k,4}| - \rho_{k,br} A_{jk,br} V_{k,br} |V_{k,br}| + P_4 A_{jk,4} - P_{br} A_{jk,br} \]

\[ \pm F_{\text{int},4} - F_{\text{wall},4} = 0 \]  

(1.28)

**Boundary Conditions at Outlet Blocks**

\[ U_{k,\text{rn}} = 2U_{k,2} - U_{k,1} \]  

(1.29)

\[ V_{k,\text{br}} = 2V_{k,br} - V_{k,br} \]  

(1.30)

\[ A_{jk,br} = 2A_{jk,4} - A_{jk,2} \]

Or

\[ A_{jk,\text{rn}} = 2A_{jk,4} - A_{jk,1} \]  

(1.31)

**Auxiliary Equations for Dependent Variables**

\[ F_{\text{int}} = f_1(\alpha_k, U_k, (or V_k), P_k, \varepsilon) \]  

(1.32)

\[ F_{\text{wall}} = f_2(\alpha_k, U_k, (or V_k), P_k, \varepsilon) \]  

(1.33)

\[ A_{jk} = f_3(\alpha_k) \]  

(1.34)

\[ \rho_g = f_4(P) \]  

(1.35)

\[ F_{\text{tee-loss}(x)} = f_1(\alpha_k, U_k, P_k) \]  

(1.36)

\[ F_{\text{tee-loss}(y)} = f_1(\alpha_k, V_k, P_k) \]  

(1.37)
A.5 Two Phase Flow at T-junction: Backward Staggered Approach

Figure A1.5: Three block problem : Two phase tee split (Backward Staggered Approach)

Mass Balance for Block 1

\[ \dot{m}_k - \rho_{k,1} A_{f_k,1} U_{k,1} = 0 \]  

Momentum Balance for Block 1

\[ \dot{m}_k \left( \frac{\dot{m}_k}{\rho_{k,in} A_{f_k,in}} \right) - \rho_{k,1} A_{f_k,1} U_{k,1} |U_{k,1}| + P_{in} A_{f_k,in} - P_1 A_{f_k,1} \]

\[ \pm F_{kint,1} - F_{kwall,1} = 0 \]

Mass Balance for Block 2

\[ \rho_{k,1} A_{f_k,1} U_{k,1} - \rho_{k,2} A_{f_k,2} U_{k,2} - \rho_{k,2} A_{f_k,2} V_{k,4} = 0 \]

Momentum Balance for Block 2

X-direction

\[ \rho_{k,1} A_{f_k,1} U_{k,1} |U_{k,1}| - \rho_{k,2} A_{f_k,2} U_{k,2} |U_{k,2}| + P_1 A_{f_k,1} - P_2 A_{f_k,2} \]

\[ - \rho_{k,2} A_{f_k,2} V_{k,4} |V_{k,4}| \cos \beta - F_{\text{tee-loss(x)}} = 0 \]

Mass Balance for Block 4

\[ \rho_{k,2} A_{f_k,2} V_{k,4} - \rho_{k,4} A_{f_k,4} V_{k,br} = 0 \]
Momentum Balance for Block 4

\[ \rho_{k,2} A_{f,k,2} V_{k,4} | V_{k,4} | - \rho_{k,4} A_{f,k,4} V_{k,br} | V_{k,br} | + P_2 A_{f,k,2} - P_4 A_{f,k,4} \]
\[ \pm F_{\text{int},4} - F_{\text{wall},4} - F_{\text{tee-loss}(y)} = 0 \]  \(1.43\)

Boundary Conditions at Outlet Blocks

\[ V_{k,br} = 2V_{k,4} - V_{k,2} \]  \(1.44\)

Auxiliary Equations for Dependent Variables

\[ F_{\text{int}} = f_1(\alpha_k, U_k (or V_k), P_k, \varepsilon) \]  \(1.45\)
\[ F_{\text{wall}} = f_2(\alpha_k, U_k (or V_k), P_k, \varepsilon) \]  \(1.46\)
\[ A_{f,k} = f_3(\alpha_k) \]  \(1.47\)
\[ \rho_g = f_4(P) \]  \(1.48\)
\[ F_{\text{tee-loss}(x)} = f_1(\alpha_k, U_k, P_k) \]  \(1.49\)
\[ F_{\text{tee-loss}(y)} = f_1(\alpha_k, V_k, P_k) \]  \(1.50\)

A.5.1 Inventory of Unknowns and Specifications: Two Phase Tee Split

Table A1.3: Inventory of unknowns: Two phase tee split

<table>
<thead>
<tr>
<th>Primary Unknowns</th>
<th>Independent Variables</th>
<th>Dependent Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>(P_{in}, P_1, P_2, P_3)</td>
<td>(F_{\text{int}}, F_{\text{wall}}, \rho_k)</td>
</tr>
<tr>
<td>Gas Phase Velocity</td>
<td>(U_{g1}, U_{g2}, V_{g2}, U_{g3})</td>
<td>(F_{\text{int}}, F_{\text{wall}}, F_{\text{tee-loss}(x \text{ or } y)})</td>
</tr>
<tr>
<td>Liquid Phase Velocity</td>
<td>(U_{l1}, U_{l2}, V_{l2}, U_{l3})</td>
<td>(F_{\text{int}}, F_{\text{wall}}, F_{\text{tee-loss}(x \text{ or } y)})</td>
</tr>
<tr>
<td>Liquid Holdup</td>
<td>(\alpha_{l1}, \alpha_{l2}, \alpha_{l3})</td>
<td>(F_{\text{int}}, F_{\text{wall}}, A_{f,k})</td>
</tr>
</tbody>
</table>
Table A1.4: Specifications: Two phase tee split

<table>
<thead>
<tr>
<th>Set</th>
<th>Inlet</th>
<th>Run</th>
<th>Branch</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \rho_k, A_{fk}/\alpha_k, U_k )</td>
<td>( P_{out} )</td>
<td>( P_{out} )</td>
</tr>
<tr>
<td>2</td>
<td>Inlet mass flow rates</td>
<td>( P_{out} )</td>
<td>( P_{out} )</td>
</tr>
</tbody>
</table>

**Forward Staggered Approach**

**Backward Staggered Approach**

<table>
<thead>
<tr>
<th>Set</th>
<th>Inlet</th>
<th>Run</th>
<th>Branch</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( A_k/\alpha_k, U_k, P_{in} )</td>
<td>( U_{k,out} )</td>
<td>NA</td>
</tr>
<tr>
<td>2</td>
<td>( A_k/\alpha_k, U_k, P_{in} )</td>
<td>NA</td>
<td>( V_{k,out} )</td>
</tr>
<tr>
<td>3</td>
<td>( P_{in}, \text{inlet mass flow rates} )</td>
<td>( U_{k,out} )</td>
<td>NA</td>
</tr>
<tr>
<td>3</td>
<td>( P_{in}, \text{inlet mass flow rates} )</td>
<td>NA</td>
<td>( V_{k,out} )</td>
</tr>
</tbody>
</table>

A.6 Nomenclature

\( m_k \) = Inlet mass flow-rate of phase ‘k’
\( U_k \) = Velocity of phase ‘k’ in x-direction
\( V_k \) = Velocity of phase ‘k’ in y-direction
\( F_{\text{tee-loss}(x)} \) = T-junction losses in x-direction
\( F_{\text{tee-loss}(y)} \) = T-junction losses in y-direction
\( A_{fk} \) = Cross-sectional flow area of phase ‘k’
\( P_{in} \) = Inlet pressure
\( \rho_{k,\text{in}} \) = Inlet density of phase ‘k’
\( A_{fk,\text{in}} \) = Inlet flow area of phase ‘k’
Appendix B

Programs and Subroutines

B.1 Two Phase Pipe Flow

Main Program

clear all
clc

%%% Input %%%
% Outlet pressure
% Inlet gas and liquid mass flow rates

%%% Output %%%
% Pressure at each block
% Gas and liquid velocities at each block
% Liquid holdup at each block

format long e

%% Gas liquid property data %
Zgas = 0.9;  %% gas supercompressibility
R = 8.314;  %% universal gas constant
T = 303;  %% ambient temperature -- isothermal
Mgas = 29/1000;  %% gas molecular weight
mugas = 0.01827*10^-3;  %% gas viscosity
muliq = 1.787*10^-3;  %% liquid viscosity
rholiq = 1000;  %% liquid density
sig = 7.12*10^-2;  %% Gas liquid surface tension

%% Pipeline property data %
eps = input('Enter roughness parameter = ');  %% roughness parameter
D = 0.051;  %% Diameter -- inlet, run and branch arms ---
Regular Straight Tee

pipelen = input('Enter the length of pipe = ');
loo = input('Enter number of blocks = ');
delx = pipelen/loo;  % length of an FVM block
loo = loo + 1;

%%% Supply specifications, boundary conditions, diagnostic inputs %%%
P(1,loo+1) = input('Enter Outlet Pressure = ');
rhogasout = P(1,loo+1)* Mgas/(Zgas * R * T);

n0 = input ('Enter n for numerical differentiation "10^-n" = ');
delta = 10^-n0;
gasflo = input('Enter inlet gas mass flow rate = ');
liqflo = input('Enter inlet liquid mass flow rate = ');

user = input('Specify liquid holdup -- "Enter NO = 0 YES = 1" = ');
if user==0
    lambda_l_guess = (liqflo/rholiq)/(liqflo/rholiq+gasflo/rhogasout);
    %% no slip condition at inlet
else
    lambda_l_guess = input('Enter outlet liquid holdup = ');
end

Apipe = (3.14/4) * D ^ 2;
Aliq = Apipe * lambda_l_guess;
Agas = Apipe - Aliq;

Vgas(1,loo+1) = gasflo / (rhogasout * Agas);
Vliq(1,loo+1) = liqflo / (rholiq * Aliq);
lambda_l(1,loo+1) = lambda_l_guess;

for i = 2:loo
    ind(1,i) = 1;
P(1,i) = P(1,loo+1);
    Vgas(1,i) = Vgas(1,loo+1);
    Vliq(1,i) = Vliq(1,loo+1);
    lambda_l(1,i) = lambda_l(1,loo+1);
end
ind(1,loo) = 2;
ind(1,1) = 0;
ind(1,2) = 3;

%P(1,10) = 295000 - 2000; % Diagnostic mode : Perturbation

%% ind=1 -- variables unknown
%% ind=2 -- zero curvature
%% ind=0 -- specifications

len = length(ind);
unkn=0;
for i=1:len
if \( \text{ind}(1,i) \neq 0 \) 
    \( \text{unkn} = \text{unkn} + 4; \)  \( \%\) four unknowns at each grid block assuming two extra unknowns at Tee blocks are specified. 
end 
end 

%%%% pre-allocation of matrices %%%
jac = zeros(\text{unkn}, \text{unkn});
res = zeros(\text{unkn}, 1);
del = zeros(\text{unkn}, 1);

%%%% numbering blocks %%%
n = 1;
num = zeros(1, \text{len});
for \( i = 1 : \text{len} \)
    if \( \text{ind}(1,i) \neq 0 \)
        \( \text{num}(1,i) = n; \)
        \( n = n + 1; \)  \( \%\) two unknowns at each block
    else
        \( \text{num}(1,i) = 0; \)
    end
end

iter = 0;
convg = 1;
convg_res = 1;
while (convg_res > 10^{-12})

    \% Calculate Residual matrix \%
    res = residual(ind, num, P, Vgas, Vliq, lambda_l, Zgas, R, T, Mgas, D, mugas, muliq, rholiq, eps, sig, delx, gasflo, liqflo);

    \% Calculate Jacobian \%
    for \( i = 1 : \text{len} \)
        if \( \text{ind}(1,i) \neq 0 \)
            \( m = 4 \times \text{num}(1,i) - 3; \)
            \( P(1,i) = P(1,i) + \text{delta}; \)
            \( \text{res1} = \) residual(ind, num, P, Vgas, Vliq, lambda_l, Zgas, R, T, Mgas, D, mugas, muliq, rholiq, eps, sig, delx, gasflo, liqflo);
            \( \text{for} \ r = 1 : \text{unkn} \)
                \( \text{jac}(r,m) = (\text{res1}(r,1) - \text{res}(r,1))/\text{delta}; \)
            end
            \( P(1,i) = P(1,i) - \text{delta}; \)
            \( \text{lambda}_l(1,i) = \text{lambda}_l(1,i) + \text{delta}; \)
            \( \text{res2} = \) residual(ind, num, P, Vgas, Vliq, lambda_l, Zgas, R, T, Mgas, D, mugas, muliq, rholiq, eps, sig, delx, gasflo, liqflo);
            \( \text{for} \ r = 1 : \text{unkn} \)
jac(r,m+1) = (res2(r,1) - res(r,1))/delta;
end
lambda_l(1,i) = lambda_l(1,i) - delta;

Vgas(1,i) = Vgas(1,i) + delta;
res3 = residual(ind,num,P,Vgas,Vliq,lambda_l,Zgas,R,T,Mgas,D,mugas,muliq,rholiq,eps,sig,delx,gasflo,liqflo);
for r = 1 : unkn
  jac(r,m+2) = (res3(r,1) - res(r,1))/delta;
end
Vgas(1,i) = Vgas(1,i) - delta;

Vliq(1,i) = Vliq(1,i) + delta;
res4 = residual(ind,num,P,Vgas,Vliq,lambda_l,Zgas,R,T,Mgas,D,mugas,muliq,rholiq,eps,sig,delx,gasflo,liqflo);
for r = 1 : unkn
  jac(r,m+3) = (res4(r,1) - res(r,1))/delta;
end
Vliq(1,i) = Vliq(1,i) - delta;
end
end

%%%% Check to ensure main diagonal elements are non zero %%%%
for i = 1 : unkn
  if jac(i,i) == 0
    i iter
    pause
  end
end

%%%% Matrix inversion %%%%
[L,U] = lu(jac);
inver = inv(U) * inv(L);
del = inver * (-res);
err = max(abs(jac * del + res));
convg = max(abs(del));
convg_res = max(abs(res));
fprintf('\n---------------- Iteration = %d ----------------
',iter)
fprintf('\nconvg = %d ----- convg_res = %d ----- Error = %d
',convg,convg_res,err)

%%%% Update values %%%%
for i=1:len
  if ind(1,i) == 0
    m = 4 * num(1,i) - 3;
P(1,i) = P(1,i) + del(m,1);
lambda_l(1,i) = lambda_l(1,i) + del(m+1,1);
Vgas(1,i) = Vgas(1,i) + del(m+2,1);
Vliq(1,i) = Vliq(1,i) + del(m+3,1);
  end
end
end
    iter = iter + 1;
end

for i = 2 : len+1
    Pplot(1,i-1) = P(1,i);
    pl1(1,i-1) = delx * (i-2);
end

for i = 2: len
    Vgpl(1,i-1) = Vgas(1,i);
    Vlpl(1,i-1) = Vliq(1,i);
    lamplot(1,i-1) = lambda_l(1,i);
    pl2(1,i-1) = delx * (i-2);
end

%%%% Plot pressure v/s node %%%%
hold off
plot(pl1(1,:),Pplot(1,:),'-or')
hold on
xlabel('Distance in meters','fontsize',20)
ylabel('Pressure in Pascals','fontsize',20)
grid on
pause

%%%% Plot gas velocity v/s node %%%%
hold off
plot(pl2(1,:),Vgpl(1,:),'-or')
hold on
xlabel('Distance in meters','fontsize',20)
ylabel('Gas velocity in m/s','fontsize',20)
grid on
pause

%%%% Plot liquid velocity v/s node %%%%
hold off
plot(pl2(1,:),Vlpl(1,:),'-or')
hold on
xlabel('Distance in meters','fontsize',20)
ylabel('Liquid velocity in m/s','fontsize',20)
grid on
pause

%%%% Plot liquid holdup v/s node %%%%
hold off
plot(pl2(1,:),lamplot(1,:),'-or')
hold on
xlabel('Distance in meters','fontsize',20)
ylabel('Liquid Holdup (\lambda_l)','$\lambda_l$','fontsize',20)
grid on

%%%%!!!!!!! Caution !!!!!!!!!%%%  
% Theta is in degrees  
% lambda is calculated from no slip condition at outlet  
% eps -- roughness parameter  
% Upwind scheme form mass balance and downstream staggering for  
% momentum balance  
% Implementing Blasius for diagnosis -- normal mode uses Chen's  
% friction factor correlation %

len = length(ind); % ind = 0 since 1st block is specification block  
reslen = (len-1) * 4; % 4 unknowns at each block  
out = zeros(reslen,1);

for i = 1:len
   if ind(1,i)~=0
      %%%% Flow Areas Calculation %%%
      Apipe = (pi / 4) * D^2;

      if ind(1,i)~=3
         Aliql = Apipe * lambda_l(1,i-1);
         Agasl = Apipe - Aliql;
         rhogasl = P(1,i-1) * Mgas / (Zgas * R * T);
      end
      rhogasc = P(1,i) * Mgas / (Zgas * R * T);
      rhogasr = P(1,i+1) * Mgas / (Zgas * R * T);

      if ind(1,i) == 2
         Vgasout = 2 * Vgas(1,i) - Vgas(1,i-1);
         Vliqout = 2 * Vliq(1,i) - Vliq(1,i-1);
      end

      Aliqc = Apipe * lambda_l(1,i);
      Agasc = Apipe - Aliqc;
      Aliqr = Apipe * lambda_l(1,i+1);
      Agasr = Apipe - Aliqr;
   end
end
% Diagnostic mode -- constant density system %
% rhogasl = P(1,2) * Mgas / (Zgas * R * T);
% rhogasr = rhogasl;
% rhogasc = rhogasl;

%% Equation Building Blocks %
if lambda_l(1,i) <= 0
    disp('****Error - liquid holdup negative or zero****')
    pause
end

theta = newrap(lambda_l(1,i));

%%% Wall Shear Contact Areas %
Awl = pi * D * delx * theta / 180;
Awg = pi * D * delx * (1 - theta / 180);

%%% Equivalent Diameters for liquid and gas wetted wall areas %
dhl = 180 * lambda_l(1,i) * D / theta;
dhg = pi * (1 - lambda_l(1,i)) * D / (((180 - theta) * pi / 180) + sind(theta));

%%% Gas and Liquid Phase Reynolds Number %
Regas = rhogasc * abs(Vgas(1,i)) * dhg / mugas;
Reliq = rholiq * abs(Vliq(1,i)) * dhl / muniq;

up = 2500;
low = 1700;
if Regas < low
    fwgas = 16 / Regas;
end
if Regas >= low && Regas < up
    R1 = (up - Regas) / (up - low);
    R2 = (Regas - low) / (up - low);
    fwgas1 = 16 / Regas;
    fwgas2 = (-4 * log10((eps / (3.7065 * dhg)) -
    (5.0452/Regas) * log10(((eps/dhg)^1.1098) / 2.8257 + (5.8506 / (Regas^0.8981))))))^(2);
    fwgas = 0.046 * Regas ^ (-0.2); % Diagnosis mode - Blasius friction factor
endif
if Regas >= up
    fwgas = (-4 * log10((eps / (3.7065 * dhg)) -
    (5.0452/Regas) * log10(((eps/dhg)^1.1098) / 2.8257 + (5.8506 / (Regas^0.8981))))))^(2);
    fwgas = 0.046 * Regas ^ (-0.2); % Diagnosis mode - Blasius friction factor
endif

if Reliq < low
    fwliq = 16 / Reliq;
end
if Reliq >= low && Reliq < up
  R1 = (up - Reliq) / (up - low);
  R2 = (Reliq - low) / (up - low);
  fwliq1 = 16 / Reliq;
  fwliq2 = (-4 * log10 ((eps / (3.7065 * dhl)) - (5.0452/Reliq) * log10(((eps/dhl)^1.1098) / 2.8257 + (5.8506 / (Reliq^0.8981))))^(-2));
  %    fwliq2 = 0.046 * Reliq ^ (-0.2);    %Diagnosis mode - Blasius friction factor
  fwliq = R1 * fwliq1 + R2 * fwliq2;
end
if Reliq >= up
  fwliq = (-4 * log10 ((eps / (3.7065 * dhl)) - (5.0452/Reliq) * log10(((eps/dhl)^1.1098) / 2.8257 + (5.8506 / (Reliq^0.8981))))^(-2));
  %    fwliq = 0.046 * Reliq ^ (-0.2);    %Diagnosis mode - Blasius friction factor
end

%%%% Wall Shear Forces %%%%
Fwgas = Awg * fwgas * rhogasc * abs(Vgas(1,i)) * Vgas(1,i) / 2;
Fwliq = Awl * fwliq * rholiq * abs(Vliq(1,i)) * Vliq(1,i) / 2;

%%%% Interfacial Contact Area %%%
Aint = D * sind(theta) * delx;

%%%% Equivalent roughness parameter %%%
eps_par = rhogasc * (Vliq(1,i) * muli)^2 / ( rholiq * sig^2 );
if eps_par <= 0.005 
  eps_eqv = 34 * sig / (rhogasc * Vliq(1,i)^2);
else  
  eps_eqv = 170 * sig * ( eps_par ^ 0.3 )/( rhogasc * Vliq(1,i)^2 );
end

%%%% Interfacial Friction Factor %%%
fi = (-4 * log10 ((eps_eqv / (3.7065 * dhg)) - (5.0452/Regas) * log10(((eps_eqv/dhg)^1.1098) / 2.8257 + (5.8506 / (Regas^0.8981))))^(-2));
%    fi = 0.046 * Regas ^ (-0.2);    %Diagnosis mode - Blasius friction factor

%%%% Interfacial Shear Force %%%
Fi = Aint * fi * rhogasc * abs(Vgas(1,i) - Vliq(1,i)) * (Vgas(1,i) - Vliq(1,i)) / 2;
%--------------------------------------------------------------------------------
%%%% Diagnostic mode -- Shear forces zero %%%%%%%%
%    Fi = 0;
%    Fwgas = 0;
%    Fwliq = 0;

% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Check to ensure liquid velocity is non-negative %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if Vliq(i)<0;
  fprintf(’\nNegative Liquid Velocity at Block = %d’,i)
pause

%---------------------------------------------------------------------%
end

%%%%%%%%%%%%%%%%%%%%%%%%%%% Equation Engine %%%%%%%%%%%%%%%%%%%%%%%%%%%%
m = 4 * num(1,i) - 3 ; % Residual placement

if ind(1,i) == 3
  %%% Mass Balance %%%
  out(m+2,1) = gasflo - rhogasc * Vgas(1,i) * Agasc;
  out(m+3,1) = liqflo - rholiq * Vliq(1,i) * Aliqc;

  %%% Momentum Balance %%%
  out(m,1) = rhogasc * Agasc * abs(Vgas(1,i)) * Vgas(1,i) -
             rhogasr * Agasr * abs(Vgas(1,i+1)) * Vgas(1,i+1) + ...
             (P(1,i) * Agasc - P(1,i+1) * Agasr) - Fi - Fwgas;
  out(m+1,1) = rholiq * Aliqc * abs(Vliq(1,i)) * Vliq(1,i) -
             rholiq * Aliqr * abs(Vliq(1,i+1)) * Vliq(1,i+1) + ...
             (P(1,i) * Aliqc - P(1,i+1) * Aliqr) + Fi - Fwliq;
end

if ind(1,i) == 2
  %%% Mass Balance %%%
  out(m+2,1) = rhogasl * Vgas(1,i-1) * Agasl - rhogasc *
               Vgas(1,i) * Agasc;
  out(m+3,1) = rholiq * Vliq(1,i-1) * Aliql - rholiq *
               Vliq(1,i) * Aliqc;

  %%% Momentum Balance %%%
  out(m,1) = rhogasc * Agasc * abs(Vgas(1,i)) * Vgas(1,i) -
             rhogasr * Agasr * abs(Vgasout) * Vgasout + ...
             (P(1,i) * Agasc - P(1,i+1) * Agasr) - Fi - Fwgas;
  out(m+1,1) = rholiq * Aliqc * abs(Vliqout) * Vliqout + ...
             (P(1,i) * Aliqc - P(1,i+1) * Aliqr) + Fi - Fwliq;
end

if ind(1,i) == 1
  %%% Mass Balance %%%
  out(m+2,1) = rhogasl * Vgas(1,i-1) * Agasl - rhogasc *
               Vgas(1,i) * Agasc;
  out(m+3,1) = rholiq * Vliq(1,i-1) * Aliql - rholiq *
               Vliq(1,i) * Aliqc;

  %%% Momentum Balance %%%
  out(m,1) = rhogasc * Agasc * abs(Vgas(1,i)) * Vgas(1,i) -
             rhogasr * Agasr * abs(Vgas(1,i+1)) * Vgas(1,i+1) + ...
             (P(1,i) * Agasc - P(1,i+1) * Agasr) - Fi - Fwgas;
  out(m+1,1) = rholiq * Aliqc * abs(Vliq(1,i)) * Vliq(1,i) -
             rholiq * Aliqr * abs(Vliq(1,i+1)) * Vliq(1,i+1) + ...
             (P(1,i) * Aliqc - P(1,i+1) * Aliqr) + Fi - Fwliq;
end
%---------------------------------------------------------------------%
end
end
Mist Flow Residual Calculation Subroutine

function [out]=residual(ind,num,P,Vgas,Vliq,lambda_l,Zgas,R,T,Mgas,D,mugas,muliq,rholiq,eps,sig,delx,gasflo,liqflo)

%%%%!!!!!!! Caution !!!!!!!!!%%%%
% Theta is in degrees
% lambda is calculated from no slip condition at outlet
% eps -- roughness parameter
% Upwind scheme form mass balance and downstream staggering for
% momentum balance
% Implementing Blasius for diagnosis -- normal mode uses Chen's
% friction factor correlation%

len = length(ind);  % ind = 0 since 1st block is specification
block
reslen = (len-1) * 4;  % 4 unknowns at each block

out = zeros(reslen,1);

for i = 1:len
    if ind(1,i)~=0

        %% Flow Areas Calculation %%%
        Apipe = (pi/4) * D^2;

        if ind(1,i)~=3
            Aliql = Apipe * lambda_l(1,i-1);
            Agasl = Apipe - Aliql;
            rhogasl = P(1,i-1) * Mgas / (Zgas * R * T);
        end
        rhogasc = P(1,i) * Mgas / (Zgas * R * T);
        rhogasr = P(1,i+1) * Mgas / (Zgas * R * T);

        if ind(1,i) == 2
            Vgasout = 2 * Vgas(1,i) - Vgas(1,i-1);
            Vliqout = 2 * Vliq(1,i) - Vliq(1,i-1);
        end

        Aliqc = Apipe * lambda_l(1,i);
        Agasc = Apipe - Aliqc;
        Aliqr = Apipe * lambda_l(1,i+1);
        Agasr = Apipe - Aliqr;
    end
Diagnostic mode -- constant density system

```matlab
rhogasl = P(1,2) * Mgas / (Zgas * R * T);
rhogasr = rhogasl;
rhogasc = rhogasl;
```

Equation Building Blocks

```matlab
if lambda_l(1,i)<=0
    disp('****Error-liquid holdup negative or zero****')
    pause
end

Awl = lambda_l(1,i) * pi * D * delx;
Awg = (1-lambda_l(1,i)) * pi * D * delx;

dhl = D;
dhg = D;

Regas = rhogasc * abs(Vgas(1,i)) * dhg / mugas;
Reliq = rholiq * abs(Vliq(1,i)) * dhl / muliq;

up = 2500;
low = 1700;
if Regas < low
    fwgas = 16 / Regas;
end
if Regas >= low && Regas<up
    R1 = (up-Regas)/(up-low);
    R2 = (Regas-low)/(up-low);
    fwgas1 = 16 / Regas;
    fwgas2 = (-4 * log10 ((eps / (3.7065 * dhg)) - (5.0452/Regas) * log10(((eps/dhg)^1.1098) / 2.8257 + (5.8506 / (Regas^0.8981))))^(-2));
    fwgas = R1 * fwgas1 + R2 * fwgas2;
end
if Regas >= up
    fwgas = (-4 * log10 ((eps / (3.7065 * dhg)) - (5.0452/Regas) * log10(((eps/dhg)^1.1098) / 2.8257 + (5.8506 / (Regas^0.8981))))^(-2));
    fwgas = 0.046 * Regas ^ (-0.2); %Diagnosis mode - Blasius friction factor
end
if Reliq < low
    fwliq = 16 / Reliq;
end
if Reliq >= low && Reliq<up
    R1 = (up-Reliq)/(up-low);
```

```matlab
Aw = lambda_l(1,i) * pi * D * delx;
Awg = (1-lambda_l(1,i)) * pi * D * delx;
```

```matlab
Regas = rhogasc * abs(Vgas(1,i)) * dhg / mugas;
Reliq = rholiq * abs(Vliq(1,i)) * dhl / muliq;

up = 2500;
low = 1700;
if Regas < low
    fwgas = 16 / Regas;
else
    if Regas >= low && Regas<up
        R1 = (up-Regas)/(up-low);
        R2 = (Regas-low)/(up-low);
        fwgas1 = 16 / Regas;
        fwgas2 = (-4 * log10 ((eps / (3.7065 * dhg)) - (5.0452/Regas) * log10(((eps/dhg)^1.1098) / 2.8257 + (5.8506 / (Regas^0.8981))))^(-2));
        fwgas = R1 * fwgas1 + R2 * fwgas2;
    end
    if Regas >= up
        fwgas = (-4 * log10 ((eps / (3.7065 * dhg)) - (5.0452/Regas) * log10(((eps/dhg)^1.1098) / 2.8257 + (5.8506 / (Regas^0.8981))))^(-2));
        fwgas = 0.046 * Regas ^ (-0.2); %Diagnosis mode - Blasius friction factor
    end
end
if Reliq < low
    fwliq = 16 / Reliq;
else
    if Reliq >= low && Reliq<up
        R1 = (up-Reliq)/(up-low);
```
\[ R2 = (\text{Reliq-low})/(\text{up-low}); \]
\[ \text{fwliq1} = 16 / \text{Reliq}; \]
\[ \text{fwliq2} = (-4 * \log_{10}((\text{eps} / (3.7065 * \text{dhl})) - (5.0452/\text{Reliq}) * \log_{10}((\text{eps} / (\text{dhl})^{1.1098}) / 2.8257 + (5.8506 / (\text{Reliq}^{0.8981})))))^{(-2)}; \]
\%
\[ \text{fwliq2} = 0.046 \times \text{Reliq}^{(-0.2)}; \]
\%
\text{Blasius friction factor}
\[ \text{fwliq} = R1 \times \text{fwliq1} + R2 \times \text{fwliq2}; \]
end
if Reliq >= up
\[ \text{fwliq} = (-4 * \log_{10}((\text{eps} / (3.7065 * \text{dhl})) - (5.0452/\text{Reliq}) * \log_{10}((\text{eps} / (\text{dhl})^{1.1098}) / 2.8257 + (5.8506 / (\text{Reliq}^{0.8981})))))^{(-2)}; \]
\%
\[ \text{fwliq} = 0.046 \times \text{Reliq}^{(-0.2)}; \]
\%
\text{Blasius friction factor}
end

%%% Wall Shear Forces %%%
Fwgas = Awg * fwgas * rhogasc * abs(Vgas(1,i)) * Vgas(1,i) / 2;
Fwliq = Awl * fwliq * rho(liq) * abs(Vliq(1,i)) * Vliq(1,i) / 2;

%%% Mean Droplet Size %%%
We_non_visc = 12;
rmmax = sig * We_non_visc / (2 * rhogasc * (Vgas(1,i) - Vliq(1,i)) ^ 2);
We_visc = 12 * (1 + ({muliq^2}) / (2 * rholiq * rmax * sig)^0.36);
rmmax = sig * We_visc / (2 * rhogasc * (Vgas(1,i) - Vliq(1,i))^2);
rm_mean = 0.06147 * rmmax;

%%% Interfacial Contact Area %%%
\text{Aint} = (3 * lambda_l(1,i) / r_mean) * pi * (D/2)^2 * delx;

%%% Interfacial Friction Factor Calculation %%%
\text{Re_int} = rhogasc * 2 * \text{r_mean} * abs(Vgas(1,i) - Vliq(1,i)) / mugas;
\text{w} = \log_{10} \left( \text{Re_int} \right);
if Re_int < 0.01
\[ \text{Cd} = (24 / \text{Re_int}) \times (1 + 3 \times \text{Re_int} / 16 ); \]
end
if Re_int > 0.01 \&\& Re_int < 20
\[ \text{Cd} = (24 / \text{Re_int}) \times (1 + 0.1315 \times \text{Re_int}^{(0.82 - 0.05 \times \text{w})}); \]
end
if Re_int > 20 \&\& Re_int < 260
\[ \text{Cd} = (24 / \text{Re_int}) \times (1 + 0.1935 \times \text{Re_int}^{(0.6305)}); \]
end
if Re_int > 260 \&\& Re_int <= 1500
\[ \text{Cd} = 10 \times (1.6435 - 1.1242 \times \text{w} + 0.1558 \times \text{w}^2); \]
end
if Re_int > 1500 \&\& Re_int <= 12000

\[ Cd = 10^{\left( -2.4571 + 2.558 \times w - 0.9295 \times w^2 + 0.1049 \times w^3 \right)}; \]

\begin{verbatim}
 end
 if Re_int > 12000 && Re_int <= 44000
   Cd = 10^{\left( -1.9181 + 0.6370 \times w - 0.0636 \times w^2 \right)};
 end
 if Re_int > 44000 && Re_int <= 338000
   Cd = 10^{\left( -4.3390 + 1.5809 \times w - 0.1546 \times w^2 \right)};
 end
 if Re_int > 338000 && Re_int <= 400000
   Cd = 29.78 - 5.3 \times w;
 end
 if Re_int > 400000 && Re_int <= 10^6
   Cd = 0.1 \times w - 0.49;
 end
 end

%% Interfacial Friction Factor %%%
fi = Cd / 4;

%% Interfacial Shear Force %%%
Fi = Aint * fi * rhogasc * abs(Vgas(1,i) - Vliq(1,i)) * (Vgas(1,i) - Vliq(1,i)) / 2;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Diagnostic mode -- Shear forces zero %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Fi = 0;
% Fwgas = 0;
% Fwliq = 0;

%%%%%%%%%%%%%%%%%%%%%%%% Check to ensure liquid velocity is non negative %%%%%%%%%%%%%%%%%
if Vliq(1,i)<0;
  fprintf('\nNegative Liquid Velocity at Block = %d',i)
  pause
end

%%%%%%%%%%%%%%%%%%%%%%%%% Equation Engine %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
\( m = 4 \times \text{num}(1,i) - 3 ; \) % Residual placement

if ind(1,i) == 3
  \text{Mass Balance} \%%
  \begin{align*}
  \text{out}(m+2,1) &= \text{gasflo} - \text{rhogasc} \times Vgas(1,i) \times \text{Agasc}; \\
  \text{out}(m+3,1) &= \text{liqflo} - \text{rholiq} \times Vliq(1,i) \times \text{Aliqc};
  \end{align*}

  \text{Momentum Balance} \%%
  \begin{align*}
  \text{out}(m,1) &= \text{rhogasc} \times \text{Agasc} \times \text{abs}(Vgas(1,i)) \times Vgas(1,i) - \text{rhogasr} \times \text{Agasr} \times \text{abs}(Vgas(1,i+1)) \times Vgas(1,i+1) + \ldots \\
  &\quad (P(1,i) \times \text{Agasc} - P(1,i+1) \times \text{Agasr}) - \text{Fi} - \text{Fwgas}; \\
  \text{out}(m+1,1) &= \text{rholiq} \times \text{Aliqc} \times \text{abs}(Vliq(1,i)) \times Vliq(1,i) - \text{rholiq} \times \text{Aliqr} \times \text{abs}(Vliq(1,i+1)) \times Vliq(1,i+1) + \ldots
  \end{align*}
\end{verbatim}
(P(1,i) * Aliqc - P(1,i+1) * Aliqr) + Fi - Fwliq;
end

if ind(1,i) == 2
%%% Mass Balance %%%
out(m+2,1) = rhogasl * Vgas(1,i-1) * Agasl - rhogasc * Vgas(1,i) * Agasc;
out(m+3,1) = rholiq * Vliq(1,i-1) * Aliql - rholiq * Vliq(1,i) * Aliqc;
%%% Momentum Balance %%%
out(m,1) = rhogasc * Agasc * abs(Vgas(1,i)) * Vgas(1,i) - rhogasr * Agasr * abs(Vgasout) * Vgasout +...
    (P(1,i) * Agasc - P(1,i+1) * Agasr) - Fi - Fwgas;
out(m+1,1) = rholiq * Aliqc * abs(Vliq(1,i)) * Vliq(1,i) - rholiq * Aliqr * abs(Vliqout) * Vliqout +...
    (P(1,i) * Aliqc - P(1,i+1) * Aliqr) + Fi - Fwliq;
end

if ind(1,i) == 1
%%% Mass Balance %%%
out(m+2,1) = rhogasl * Vgas(1,i-1) * Agasl - rhogasc * Vgas(1,i) * Agasc;
out(m+3,1) = rholiq * Vliq(1,i-1) * Aliql - rholiq * Vliq(1,i) * Aliqc;
%%% Momentum Balance %%%
out(m,1) = rhogasc * Agasc * abs(Vgas(1,i)) * Vgas(1,i) - rhogasr * Agasr * abs(Vgasout) * Vgasout +...
    (P(1,i) * Agasc - P(1,i+1) * Agasr) - Fi - Fwgas;
out(m+1,1) = rholiq * Aliqc * abs(Vliq(1,i)) * Vliq(1,i) - rholiq * Aliqr * abs(Vliqout) * Vliqout +...
    (P(1,i) * Aliqc - P(1,i+1) * Aliqr) + Fi - Fwliq;
end

%---------------------------------------------------------------------%
end
end
B.2 Two Phase Flow at T-junction

Main Program

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Two Fluid Finite Volume Two Phase Tee Junction %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Upwind Scheme Downstream Staggering %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% clear all
clc
format long
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Notes %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% delta values for numerical differentiation are problem dependent
%-------------------------------------------------------------------

%%%% Input %%%%
% Inlet gas and liquid mass flow rates
% run outlet pressure
% branch outlet pressure

%%%% Output %%%%
% Pressure at each block
% Gas and liquid velocities at each block

pipelen = input ('Enter length of pipe = ');
run = input ('Enter number of FVM blocks -- an odd number = ');
gasflo = input ('Enter inlet gas mass flow rate = ');
liqflo = input ('Enter inlet liquid mass flow rate = ');
lambda_l_in = input('Enter inlet liquid holdup = ');
angle = input ('Enter angle between branch and run arms = ');

%% Gas liquid property data %%
Zgas = 1; % gas supercompressibility
R = 8.314; % universal gas constant
T = 303; % ambient temperature -- isothermal
Mgas = 29/1000; % gas molecular weight
mugas = 0.01827*10^-3; % gas viscosity
muliq = 1.78 *10^-3; % liquid viscosity
rholiq = 1000; % liquid density
sig = 7.12*10^-2; % gas liquid surface tension

%% Pipeline property data %%
eps = input('Enter roughness parameter = '); % roughness parameter
D = 0.051; % Diameter -- inlet, run and branch arms ---
Regular Straight Tee
delx = pipelen/run; % length of an FVM block

%%% Supply specifications, boundary conditions, diagnostic inputs %%%%
mid = (run + 1)/2; % number of blocks in inlet + run arms
bran = (run + 1)/2;  % number of blocks in the branch arm

Prunout = input('Enter run outlet pressure = ');
Pbranout = input('Enter branch outlet pressure = ');

P(1,run+1) = Prunout;
P(bran+1,mid) = Pbranout;

%% Pre Allocation %
Ugas = zeros(bran,run);
Uliq = zeros(bran,run);
Vgas = zeros(bran,run);
Vliq = zeros(bran,run);
lambda_l = zeros(bran,run);
ind = zeros(bran,run);

%% Flow Area Calculations %
Apipe = (pi/4) * (D)^2;
Aliq = Apipe * lambda_l_in;
Agas = Apipe - Aliq;

rhogas = P(1,run+1) * Mgas / (Zgas * R * T);
Ugas(1,run) = gasflo/(rhogas * Agas);
Uliq(1,run) = liqflo/(rholiq * Aliq);

for i = 1:mid
    ind(1,i) = 2;
    P(1,i) = P(1,run+1)+1000;
    Ugas(1,i) = Ugas(1,run);
    Uliq(1,i) = Uliq(1,run);
    lambda_l(1,i) = lambda_l_in;
end

masgas = gasflo/2;
masliq = liqflo/2;
Ugas(1,mid) = masgas/((P(1,run+1) * Mgas / (Zgas * R * T)) * Agas);
Uliq(1,mid) = masliq/(rholiq * Aliq);
Vgas(mid,bran) = masgas/((P(bran+1,mid) * Mgas / (Zgas * R * T)) * Agas);
Vliq(mid,bran) = masliq/(rholiq * Aliq);

for i = mid:run
    ind(1,i) = 2;
    P(1,i) = P(1,run+1);
    Ugas(1,i) = Ugas(1,mid);
    Uliq(1,i) = Uliq(1,mid);
    lambda_l(1,i) = lambda_l_in;
end

for i = 1:bran
    ind(i,mid) = 4;
    P(i,mid) = P(bran+1,mid);
    Vgas(i,mid) = Vgas(mid,bran);
Vliq(i,mid) = Vliq(mid,bran);
lambda_l(i,mid) = lambda_l_in;
end

ind(1,1) = 1;
ind(1,mid-1) = 7;
ind(1,mid) = 3;
ind(1,run) = 5;
ind(bran,mid) = 6;

%% Diagnostic mode : Perturbation %
% P(1,10) = 295000 - 200000;
% vgas(1,10) = 10 + 10000; %% Diagnostic mode : Perturbation
% Vliq(1,10) = 1 + 100;

%%%%%% Index Nomenclature %%%%%%
% ind=1 -- inlet
% ind=2 -- generic x direction block
% ind=3 -- tee block
% ind=4 -- generic y direction block
% ind=5 -- x direction outlet block
% ind=6 -- y direction outlet block
% ind=7 -- block before tee junction

[leny,lenx] = size(ind);
unkn=0;
for i=1:leny
    for j=1:lenx
        if ind(i,j)~=0 && ind(i,j)~=3
            unkn = unkn + 4;  %%two unknowns at each grid block
        end
        if ind(i,j) == 3
            unkn = unkn + 6;  %% three unknowns at Tee junction block
        end
    end
end

%%%% pre-allocation of matrices %%%
jac=zeros(unkn,unkn);
res=zeros(unkn,1);
del=zeros(unkn,1);

%% numbering blocks %
n=1;
for i=1:leny
    for j=1:lenx
        if ind(i,j)~=0 && ind(i,j)~=3
            num(i,j)=n;
            n=n+4;  %% two unknowns at each block
        end
        if ind(i,j) == 3
            num(i,j) = n;
        end
    end
end
\begin{verbatim}

n = n + 6;
if ind(i,j) == 0
    num(i,j)=0;
end
end

iter=0;
convg=1;
convg_res = 1;

while (convg_res > 10^-12)

%%%%%% Modified Delta for Numerical Differentiation %%%%%%
delta1 = 10^-5 * max(max(abs(P)));
delta2 = 10^-5 * max(max(abs(lambda_1)));
delta3 = 10^-4 * max(max(abs(Ugas)));
delta4 = 10^-4 * max(max(abs(Uliq)));
delta5 = 10^-4 * max(max(abs(Vgas)));
delta6 = 10^-4 * max(max(abs(Vliq)));

%--------------------------------------------------------%

%% Calculate Residual matrix %%
res = residuals(ind,num,unkn,P,Vgas,Vliq,Ugas,Uliq,lambda_1,Zgas,R,T,Mgas,D,mugas,muliq,rholiq,eps,sig,delx,gasflo,liqflo,angle);

%% Calculate Jacobian %%
for i=1:leny
    for j=1:lenx

        if ind(i,j) == 1 || ind(i,j) == 2 || ind(i,j) == 5 || ind(i,j) == 7

            m = num(i,j);

            P(i,j) = P(i,j) + delta1;
            res1 = residuals(ind,num,unkn,P,Vgas,Vliq,Ugas,Uliq,lambda_1,Zgas,R,T,Mgas,D,mugas,muliq,rholiq,eps,sig,delx,gasflo,liqflo,angle);
            for r = 1 : unkn
                jac(r,m) = (res1(r,1) - res(r,1))/delta1;
            end
            P(i,j) = P(i,j) - delta1;

            lambda_1(i,j) = lambda_1(i,j) + delta2;
            res2 = residuals(ind,num,unkn,P,Vgas,Vliq,Ugas,Uliq,lambda_1,Zgas,R,T,Mgas,D,mugas,muliq,rholiq,eps,sig,delx,gasflo,liqflo,angle);
            for r = 1 : unkn
                jac(r,m+1) = (res2(r,1) - res(r,1))/delta2;
            end

        end
    end
end

\end{verbatim}
\[ \lambda_l(i,j) = \lambda_l(i,j) - \delta_2; \]

\[ U_{gas}(i,j) = U_{gas}(i,j) + \delta_3; \]

\[ res_3 = - \]

\[ residuals(ind, num, unk, P, V_{gas}, V_{liq}, U_{gas}, U_{liq}, \lambda_l, Z_{gas}, R, T, M_{gas}, D, m_{gas}, m_{liq}, \rho_{liq}, \epsilon, sig, delx, gasflo, liqflo, angle); \]

\[ \text{for } r = 1 : unk \]

\[ jac(r, m+2) = (res_3(r,1) - res(r,1))/\delta_3; \]

\[ \text{end} \]

\[ U_{gas}(i,j) = U_{gas}(i,j) - \delta_3; \]

\[ U_{liq}(i,j) = U_{liq}(i,j) + \delta_4; \]

\[ res_4 = - \]

\[ residuals(ind, num, unk, P, V_{gas}, V_{liq}, U_{gas}, U_{liq}, \lambda_l, Z_{gas}, R, T, M_{gas}, D, m_{gas}, m_{liq}, \rho_{liq}, \epsilon, sig, delx, gasflo, liqflo, angle); \]

\[ \text{for } r = 1 : unk \]

\[ jac(r, m+3) = (res_4(r,1) - res(r,1))/\delta_4; \]

\[ \text{end} \]

\[ U_{liq}(i,j) = U_{liq}(i,j) - \delta_4; \]

\[ \text{end} \]

\[ \text{if } \text{ind}(i,j) == 4 || \text{ind}(i,j) == 6 \]

\[ m = \text{num}(i,j); \]

\[ P(i,j) = P(i,j) + \delta_1; \]

\[ res_1 = - \]

\[ residuals(ind, num, unk, P, V_{gas}, V_{liq}, U_{gas}, U_{liq}, \lambda_l, Z_{gas}, R, T, M_{gas}, D, m_{gas}, m_{liq}, \rho_{liq}, \epsilon, sig, delx, gasflo, liqflo, angle); \]

\[ \text{for } r = 1 : unk \]

\[ jac(r, m) = (res_1(r,1) - res(r,1))/\delta_1; \]

\[ \text{end} \]

\[ P(i,j) = P(i,j) - \delta_1; \]

\[ \lambda_l(i,j) = \lambda_l(i,j) + \delta_2; \]

\[ res_2 = - \]

\[ residuals(ind, num, unk, P, V_{gas}, V_{liq}, U_{gas}, U_{liq}, \lambda_l, Z_{gas}, R, T, M_{gas}, D, m_{gas}, m_{liq}, \rho_{liq}, \epsilon, sig, delx, gasflo, liqflo, angle); \]

\[ \text{for } r = 1 : unk \]

\[ jac(r, m+1) = (res_2(r,1) - res(r,1))/\delta_2; \]

\[ \text{end} \]

\[ \lambda_l(i,j) = \lambda_l(i,j) - \delta_2; \]

\[ V_{gas}(i,j) = V_{gas}(i,j) + \delta_5; \]

\[ res_3 = - \]

\[ residuals(ind, num, unk, P, V_{gas}, V_{liq}, U_{gas}, U_{liq}, \lambda_l, Z_{gas}, R, T, M_{gas}, D, m_{gas}, m_{liq}, \rho_{liq}, \epsilon, sig, delx, gasflo, liqflo, angle); \]

\[ \text{for } r = 1 : unk \]

\[ jac(r, m+2) = (res_3(r,1) - res(r,1))/\delta_5; \]

\[ \text{end} \]

\[ V_{gas}(i,j) = V_{gas}(i,j) - \delta_5; \]

\[ V_{liq}(i,j) = V_{liq}(i,j) + \delta_6; \]
res4 = residuals(ind, num, unkn, P, Vgas, Vliq, Ugas, Uliq, lambda_l, Zgas, R, T, Mgas, D, mugas, muliq, rholiq, eps, sig, delx, gasflo, liqflo, angle);
    for r = 1 : unkn
        jac(r, m+3) = (res4(r, 1) - res(r, 1))/delta6;
    end
    Vliq(i, j) = Vliq(i, j) - delta6;
end

if ind(i, j) == 3

    m = num(i, j);
    P(i, j) = P(i, j) + delta1;
    res1 = residuals(ind, num, unkn, P, Vgas, Vliq, Ugas, Uliq, lambda_l, Zgas, R, T, Mgas, D, mugas, muliq, rholiq, eps, sig, delx, gasflo, liqflo, angle);
    for r = 1 : unkn
        jac(r, m) = (res1(r, 1) - res(r, 1))/delta1;
    end
    P(i, j) = P(i, j) - delta1;
    lambda_l(i, j) = lambda_l(i, j) + delta2;
    res2 = residuals(ind, num, unkn, P, Vgas, Vliq, Ugas, Uliq, lambda_l, Zgas, R, T, Mgas, D, mugas, muliq, rholiq, eps, sig, delx, gasflo, liqflo, angle);
    for r = 1 : unkn
        jac(r, m+1) = (res2(r, 1) - res(r, 1))/delta2;
    end
    lambda_l(i, j) = lambda_l(i, j) - delta2;
    Ugas(i, j) = Ugas(i, j) + delta3;
    res3 = residuals(ind, num, unkn, P, Vgas, Vliq, Ugas, Uliq, lambda_l, Zgas, R, T, Mgas, D, mugas, muliq, rholiq, eps, sig, delx, gasflo, liqflo, angle);
    for r = 1 : unkn
        jac(r, m+2) = (res3(r, 1) - res(r, 1))/delta3;
    end
    Ugas(i, j) = Ugas(i, j) - delta3;
    Uliq(i, j) = Uliq(i, j) + delta4;
    res4 = residuals(ind, num, unkn, P, Vgas, Vliq, Ugas, Uliq, lambda_l, Zgas, R, T, Mgas, D, mugas, muliq, rholiq, eps, sig, delx, gasflo, liqflo, angle);
    for r = 1 : unkn
        jac(r, m+3) = (res4(r, 1) - res(r, 1))/delta4;
    end
    Uliq(i, j) = Uliq(i, j) - delta4;
    Vgas(i, j) = Vgas(i, j) + delta5;
    res5 = residuals(ind, num, unkn, P, Vgas, Vliq, Ugas, Uliq, lambda_l, Zgas, R, T, Mgas, D, mugas, muliq, rholiq, eps, sig, delx, gasflo, liqflo, angle);
for \( r = 1 : \text{unkn} \)
\[
jac(r,m+4) = (\text{res5}(r,1) - \text{res}(r,1))/\text{delta5};
\]
end

\( \text{Vgas}(i,j) = \text{Vgas}(i,j) - \text{delta5}; \)

\( \text{Vliq}(i,j) = \text{Vliq}(i,j) + \text{delta6}; \)

\( \text{res6} = \text{residuals}(\text{ind},\text{num},\text{unkn},P,\text{Vgas},\text{Vliq},Ugas,\text{Uliq},\text{lambda}_l,2\text{gas},R,T,Mgas,D,mugas,muliq,rholiq,\text{eps},\text{sig},\text{delx},\text{gasflo},\text{liqflo},\text{angle}); \)

for \( r = 1 : \text{unkn} \)
\[
jac(r,m+5) = (\text{res6}(r,1) - \text{res}(r,1))/\text{delta6};
\]
end

\( \text{Vliq}(i,j) = \text{Vliq}(i,j) - \text{delta6}; \)

end

\%\%\% Check to ensure main diagonal elements are non zero \%\%\%
for \( i = 1 : \text{unkn} \)
\[
\text{if} \quad \text{jac}(i,i)==0
\]
i iter
    pause
end

\%\%\% Matrix inversion \%\%\%
\[
[L,U] = \text{lu}(\text{jac});
\]
\text{inver} = \text{inv}(U) * \text{inv}(L);
inver = \text{inv}(U) * \text{inv}(L);
\text{del} = \text{inver} * (-\text{res});
\text{err} = \text{max(abs}(\text{jac} * \text{del} + \text{res}));
\text{convg} = \text{max(abs}(\text{del}));
\text{convg_res} = \text{max(abs}(\text{res}));
\text{fprintf}(''\n
-----------------------------Iteration = %d-------------------------------'','iter)
\text{fprintf}(''\n
convg = %d ---- convg_res = %d ----- Error = %d\n'',\text{convg},\text{convg_res},\text{err})

\%\%\% Update values \%\%\%
for \( i=1:\text{leny} \)
    for \( j=1:\text{lenx} \)
        \text{m} = \text{num}(i,j);
        \text{if} \quad \text{ind}(i,j) == 1 || \text{ind}(i,j) == 2 || \text{ind}(i,j) == 5 || \text{ind}(i,j) == 7
            \text{P}(i,j) = \text{P}(i,j) + \text{del}(m,1);
            \text{lambda}_l(i,j) = \text{lambda}_l(i,j) + \text{del}(m+1,1);
            \text{Ugas}(i,j) = \text{Ugas}(i,j) + \text{del}(m+2,1);
            \text{Uliq}(i,j) = \text{Uliq}(i,j) + \text{del}(m+3,1);
        \end
        \text{if} \quad \text{ind}(i,j) == 4 || \text{ind}(i,j) == 6
            \text{P}(i,j) = \text{P}(i,j) + \text{del}(m,1);
            \text{lambda}_l(i,j) = \text{lambda}_l(i,j) + \text{del}(m+1,1);
            \text{Vgas}(i,j) = \text{Vgas}(i,j) + \text{del}(m+2,1);
            \text{Vliq}(i,j) = \text{Vliq}(i,j) + \text{del}(m+3,1);
        \end
if ind(i,j) == 3
    P(i,j) = P(i,j) + del(m,1);
    lambda_1(i,j) = lambda_1(i,j) + del(m+1,1);
    Ugas(i,j) = Ugas(i,j) + del(m+2,1);
    Uliq(i,j) = Uliq(i,j) + del(m+3,1);
    Vgas(i,j) = Vgas(i,j) + del(m+4,1);
    Vliq(i,j) = Vliq(i,j) + del(m+5,1);
end
end
end
iter = iter + 1;
end

rhogasout_bran = P(bran,mid) * Mgas / ( Zgas * R * T);
rhogasout_run = P(1,run) * Mgas / (Zgas * R * T);
Apipe = (pi/4) * D ^ 2;

Aliqc_run = Apipe * lambda_1(1,run);
Agasc_run = Apipe - Aliqc_run;

Aliqc_bran = Apipe * lambda_1(bran,mid);
Agasc_bran = Apipe - Aliqc_bran;

gas_mas_bal = (rhogasout_bran * Agasc_bran * Vgas(bran,mid) +
rhogasout_run * Agasc_run * Ugas(1,run))/gasflo
liq_mas_bal = (rholiq * Aliqc_bran * Vliq(bran,mid) + rholiq *
Aliqc_run * Uliq(1,run)) /liqflo

gas_frac = rhogasout_bran * Agasc_bran * Vgas(bran,mid)/gasflo;
liq_frac = rholiq * Aliqc_bran * Vliq(bran,mid)/liqflo;

fprintf(‘\nlambda (g) = %d ’,gas_frac)
fprintf(‘\nlambda (l) = %d ’,liq_frac)

for i = 1 : mid
    pl1(1,i) = delx * i;
    Pin(1,i) = P(1,i);
    Ugin(1,i) = Ugas(1,i);
    Ulin(1,i) = Uliq(1,i);
    lambda_in(1,i) = lambda_1(1,i);
end

for i = mid : run+1
    k = i-mid+1;
    pl2(1,k) = delx * i;
    Prun(1,k) = P(1,i);
end

for i = 1 : bran+1
    k = mid + i - 1;
\[ pl3(i,1) = \text{delx} \times k; \]
\[ \text{for } i = \text{mid} : \text{run} \]
\[ k = i - \text{mid} + 1; \]
\[ pl4(l,k) = \text{delx} \times i; \]
\[ U\text{run}(l,k) = U\text{gas}(l,i); \]
\[ U\text{lrun}(l,k) = U\text{liq}(l,i); \]
\[ \lambda\text{run}(l,k) = \lambda\text{bda}_l(l,i); \]
\[ \text{end} \]
\[ \text{for } i = 1 : \text{bran} \]
\[ k = \text{mid} + i - 1; \]
\[ pl5(i,1) = \text{delx} \times k; \]
\text{end}

plot(pl1(:,1),Pin(:,1),'-ob',pl2(:,1),Prun(:,1),'-ob',pl3(:,1),P(:,mid),'-b',pl5(:,1),pl5(:,mid),'-b');
hold on
grid on
h=legend('Inlet','Run','Branch');
xlabel('Distance x in meters','fontsize',20);
ylabel('Pressure in Pascals','fontsize',20);
hold off
pause
plot(pl1(:,1),Ugin(:,1),'-ob',pl4(:,1),Ugrun(:,1),'-ob',pl5(:,1),Vgas(:,mid),'-b');
hold on
grid on
h=legend('Inlet','Run','Branch');
xlabel('Distance x in meters','fontsize',20);
ylabel('Gas Velocity in m/s','fontsize',20);
hold off
pause
plot(pl1(:,1),Ulin(:,1),'-ob',pl4(:,1),Ulrun(:,1),'-ob',pl5(:,1),Vliq(:,mid),'-b');
hold on
grid on
h=legend('Inlet','Run','Branch');
xlabel('Distance x in meters','fontsize',20);
ylabel('Liquid Velocity in m/s','fontsize',20);
hold off
pause
plot(pl1(:,1),lambda_in(:,1),'-ob',pl4(:,1),lambda_run(:,1),'-ob',pl5(:,1),lambda_l(:,mid),'-b');
hold on
grid on
h=legend('Inlet','Run','Branch');
xlabel('Distance x in meters','fontsize',20);
ylabel('Liquid holdup (\alpha)','fontsize',20);
hold off
Stratified Smooth Flow Residual Calculation Subroutine

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%% Residual Calculation Subroutine
-- Smooth Stratified Tee Flow
%%%% Upwind Scheme Downstream Staggering
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [out] = residuals(ind,num,unkn,P,Vgas,Vliq,Ugas,Uliq,lambda_l,Zgas,R,T,Mgas,D,mugas,muliq,rholiq,eps,sig,delx,gasflo,liqflo,angle)

% Avoid the staggering and upwind scheme mistake at all costs
% eps -- roughness parameter
% Upwind scheme for mass balance and downstream staggering for momentum balance
% Implementing Blasius for diagnosis -- normal mode uses Chen's friction factor correlation
% Diagnostic mode -- make changes in shear subroutine as well

[leny,lenx] = size(ind);
out = zeros(unkn,1);
format long

for i=1:leny
    for j=1:lenx
        if ind(i,j)~=0
            rhogasc = P(i,j) * Mgas / (Zgas * R * T);

            if ind(i,j) == 1
                rhogasr = P(i,j+1) * Mgas / (Zgas * R * T);
            end

            if ind(i,j) == 2 || ind(i,j) == 5 || ind(i,j) == 7
                rhogasl = P(i,j-1) * Mgas / (Zgas * R * T);
                rhogasr = P(i,j+1) * Mgas / (Zgas * R * T);
            end

            if ind(i,j) == 3
                rhogasl = P(i,j-1) * Mgas / (Zgas * R * T);
                rhogasr = P(i,j+1) * Mgas / (Zgas * R * T);
                rhogasb = P(i+1,j) * Mgas / (Zgas * R * T);
            end

            if ind(i,j) == 4 || ind(i,j) == 6
                rhogast = P(i-1,j) * Mgas / (Zgas * R * T);
                rhogasb = P(i+1,j) * Mgas / (Zgas * R * T);
            end

            if ind(i,j) == 5
                Ugasout = 2 * Ugas(i,j) - Ugas(i,j-1);
                Uliqout = 2 * Uliq(i,j) - Uliq(i,j-1);
            end

            if ind(i,j) == 6

        end
    end
end
Vgasout = 2 * Vgas(i,j) - Vgas(i-1,j);
Vligout = 2 * Vliq(i,j) - Vliq(i-1,j);
end

%%%%%%%%%%%%%%%%% Diagnostic mode -- constant density system %%%%%%%%%%%%%%%%%
%  rhogasc = P(1,lenx+1) * Mgas / (Zgas * R * T);
%  rhogasl = rhogasc;
%  rhogasr = rhogasc;
%  rhogast = rhogasc;
%  rhogasb = rhogasc;
%----------------------------------------------------------

%%%%%%%%%%%%%%%% Equation Building Blocks %%%%%%%%%%%%%%%%%

%% Flow Areas Calculation %%
Apipe = (pi/4) * D ^ 2;
Aliqc = Apipe * lambda_1(i,j);
Agasc = Apipe - Aliqc;

if ind(i,j) == 1
  Aligr = Apipe * lambda_1(i,j+1);
  Agasr = Apipe - Aligr;
end
if ind(i,j) == 2 || ind(i,j) == 5 || ind(i,j) == 7
  Aliql = Apipe * lambda_1(i,j-1);
  Agasl = Apipe - Aliql;
  if ind(i,j) == 5
    lambda_l_run_out = 2 * lambda_l(i,j) - lambda_l(i,j-1);
    Aligr = Apipe * lambda_l_run_out;
    Agasr = Apipe - Aligr;
  else
    Aligr = Apipe * lambda_1(i,j+1);
    Agasr = Apipe - Aligr;
  end
end
if ind(i,j) == 3
  Aliql = Apipe * lambda_1(i,j-1);
  Agasl = Apipe - Aliql;
  Aligr = Apipe * lambda_1(i,j+1);
  Agasr = Apipe - Aligr;
  Aligb = Apipe * lambda_1(i+1,j);
  Agasb = Apipe - Aligb;
end
if ind(i,j) == 4 || ind(i,j) == 6
  Aliqt = Apipe * lambda_1(i-1,j);
  Agast = Apipe - Aliqt;
  if ind(i,j) == 6
    lambda_l_bran_out = 2 * lambda_l(i,j) - lambda_l(i-1,j);
    Aligb = Apipe * lambda_l_bran_out;
    Agasb = Apipe - Aligb;
  else

\[ \text{Aliqb} = \text{Apipe} \times \lambda_{l(i+1,j)}; \]
\[ \text{Agasb} = \text{Apipe} - \text{Aliqb}; \]

end

end

if \( \lambda_{l(i,j)} \leq 0 \)
    disp('****Error-liquid holdup negative or zero****')
    pause
end

theta = newrap(\( \lambda_{l(i,j)} \));

%%%% Wall Shear Contact Areas %%%%
Awl = \( \pi \times \text{D} \times \text{delx} \times \theta / 180 \);
Awg = \( \pi \times \text{D} \times \text{delx} \times (1 - \theta / 180) \);

%%%% Equivalent Diameters for liquid and gas wetted wall areas %%%%
dhl = \( 180 \times \lambda_{l(i,j)} \times \text{D} / \theta \);
dhg = \( \pi \times (1 - \lambda_{l(i,j)}) \times \text{D} / ((180 - \theta) \times \pi / 180) + \text{sind} (\theta) \);

% % % Gas and Liquid Phase Reynolds Number % %
if \( \text{ind}(i,j) = 1 \) || \( \text{ind}(i,j) = 2 \) || \( \text{ind}(i,j) = 5 \) || \( \text{ind}(i,j) = 7 \)
    Regas = \( \text{rhogasc} \times \text{abs(Ugas}(i,j)) \times \text{dhg} / \text{mugas;} \)
    Reliq = \( \text{rholiq} \times \text{abs(Uliq}(i,j)) \times \text{dhl} / \text{muliq}; \)
end

if \( \text{ind}(i,j) = 4 \) || \( \text{ind}(i,j) = 6 \)
    Regas = \( \text{rhogasc} \times \text{abs(Vgas}(i,j)) \times \text{dhg} / \text{mugas;} \)
    Reliq = \( \text{rholiq} \times \text{abs(Vliq}(i,j)) \times \text{dhl} / \text{muliq;} \)
end

if \( \text{ind}(i,j) = 1 \) || \( \text{ind}(i,j) = 2 \) || \( \text{ind}(i,j) = 5 \) || \( \text{ind}(i,j) = 7 \)
    gasvel = \text{Ugas}(i,j);
    liqvel = \text{Uliq}(i,j);
end

if \( \text{ind}(i,j) = 4 \) || \( \text{ind}(i,j) = 6 \)
    gasvel = \text{Vgas}(i,j);
    liqvel = \text{Vliq}(i,j);
end

up = 2500;
low = 1700;

if \( \text{Regas} < \text{low} \)
    \( \text{fwgas} = 16 / \text{Regas;} \)
end

if \( \text{Regas} \geq \text{low} \) \&\& \( \text{Regas} < \text{up} \)
    \( \text{R1} = (\text{up} - \text{Regas}) / (\text{up} - \text{low}); \)
    \( \text{R2} = (\text{Regas} - \text{low}) / (\text{up} - \text{low}); \)
    \( \text{fwgas1} = 16 / \text{Regas;} \)
fwgas2 = (-4 * log10 ( (eps / (3.7065 * dhg)) -
(5.0452/Regas) * log10(((eps/dhg)^1.1098) / 2.8257 + (5.8506 / (Regas^0.8981))))))^(−2);  
\% fwgas2 = 0.046 * Regas ^ (-0.2);  
fwgas = R1 * fwgas1 + R2 * fwgas2;  
end  
if Regas >= up  
fwgas = (-4 * log10 ( (eps / (3.7065 * dhg)) -
(5.0452/Regas) * log10(((eps/dhg)^1.1098) / 2.8257 + (5.8506 / (Regas^0.8981))))))^(−2);  
\% fwgas = 0.046 * Regas ^ (-0.2);  
end

if Reliq < low  
fwliq = 16 / Reliq;  
end  
if Reliq >=low && Reliq<up  
R1 = (up−Reliq)/(up−low);  
R2 = (Reliq−low)/(up−low);  
fwliq1 = 16 / Reliq;  
fwliq2 = (-4 * log10 ( (eps / (3.7065 * dhl)) -
(5.0452/Reliq) * log10(((eps/dhl)^1.1098) / 2.8257 + (5.8506 / (Reliq^0.8981))))))^(−2);  
\% fwliq2 = 0.046 * Reliq ^ (-0.2);  
fwliq = R1 * fwliq1 + R2 * fwliq2;  
end  
if Reliq >=up  
fwliq = (-4 * log10 ( (eps / (3.7065 * dhl)) -
(5.0452/Reliq) * log10(((eps/dhl)^1.1098) / 2.8257 + (5.8506 / (Reliq^0.8981))))))^(−2);  
\% fwliq = 0.046 * Reliq ^ (-0.2);  
end

%%%% Wall Shear Forces %%%%  
Fwgas = Awg * fwgas * rhogasc * abs(gasvel) * gasvel / 2;  
Fwliq = Awl * fwliq * rholiq * abs(liqvel) * liqvel / 2;

%%%% Interfacial Contact Area %%%%  
Aint = D * sind(theta) * delx;

%%%% Equivalent roughness parameter %%%%  
eps_par = rhogasc * (liqvel * muliq)^2 / ( rholiq * sig^2 );  
if eps_par <= 0.005  
eps_eqv = 34 * sig / (rhogasc * liqvel^2);  
else  
eps_eqv = 170 * sig * ( eps_par ^ 0.3 )/( rhogasc * liqvel^2 );  
end

%%%% Interfacial Friction Factor %%%%  
fi = (-4 * log10 ( (eps_eqv / (3.7065 * dhg)) -
(5.0452/Regas) * log10(((eps_eqv/dhg)^1.1098) / 2.8257 + (5.8506 / (Regas^0.8981))))))^(−2);
\[
\text{fi} = 0.046 \times \text{Regas}^\left(-0.2\right);
\]

\text{TARGET INTERFACIAL SHEAR FORCE}

\[
\text{Fi} = A_{\text{int}} \times \text{fi} \times \rho_{\text{gasc}} \times \text{abs(\text{gasvel} - \text{liqvel})} \times (\text{gasvel} - \text{liqvel}) / 2;
\]

\text{TEE JUNCTION LOSS CALCULATION - Ottens (2001)}

\[
\text{if ind(i,j) == 3}
\]

\[
\theta_{\text{in}} = \text{newrap}(\lambda_{\text{l}}(i,j-1));
\]

\text{Tee inlet gas and liquid hydraulic diameters}

\[
\text{dhl}_{\text{in}} = 180 \times \lambda_{\text{l}}(i,j-1) \times D / \theta_{\text{in}};
\]

\[
\text{dhg}_{\text{in}} = \pi \times (1 - \lambda_{\text{l}}(i,j-1)) \times D / ((180 - \theta_{\text{in}}) \times \pi / 180) + \sin(\theta_{\text{in}});
\]

\text{Tee Inlet gas and liquid reynolds number}

\[
\text{Regas}_{\text{in}} = \rho_{\text{gasc}} \times \text{dhg}_{\text{in}} \times \text{abs(Ugas(i,j-1))} / \mu_{\text{gasc}};
\]

\[
\text{Reli}_{\text{q}_{\text{in}}} = \rho_{\text{liq}} \times \text{dhl}_{\text{in}} \times \text{abs(Uliq(i,j-1))} / \mu_{\text{liq}};
\]

\text{Tee branch and run arm liquid reynolds number}

\[
\text{Reliqv} = \rho_{\text{liq}} \times \text{abs(Vliq(i,j))} \times \text{dhl} / \mu_{\text{liq}};
\]

\[
\text{Reliqu} = \rho_{\text{liq}} \times \text{abs(Uliq(i,j))} \times \text{dhl} / \mu_{\text{liq}};
\]

\text{Velocity Profile Correction Factor}

\text{for con = 1 : 2}

\[
\text{if con == 1}
\]

\[
\text{Re} = \text{Regas}_{\text{in}};
\]

\[
\text{else}
\]

\[
\text{Re} = \text{Reli}_{\text{q}_{\text{in}}};
\]

\text{end}

\[
\text{if Re} \leq 1500
\]

\[
\beta = 1.54;
\]

\text{end}

\[
\text{if Re} > 1500 \&\& \text{Re} \leq 2000
\]

\[
\beta = 1.54 - (1.08) \times (10^{-3}) \times (\text{Re} - 1500);
\]

\text{end}

\[
\text{if Re} > 2000
\]

\[
\beta = 1;
\]

\text{end}

\[
\text{if con == 1}
\]

\[
\beta_{\text{gas}} = \beta;
\]

\[
\text{else}
\]

\[
\beta_{\text{liq}} = \beta;
\]

\text{end}

\text{end}

\[
\kappa = (\beta_{\text{gas}} \times \rho_{\text{gasc}} \times (\text{Ugas}(i,j-1) \times (1-\lambda_{\text{l}}(i,j-1)))^2) / \ldots
\]

\[
(\beta_{\text{liq}} \times \rho_{\text{liq}} \times (\text{Uliq}(i,j-1) \times \lambda_{\text{l}}(i,j-1))^2);
\]

\text{Calculate branch gas mass fraction lambda}

Different from liquid holdup λ_{l1} %
\[
\lambda = \frac{\rho_{gas} \cdot A_{gas} \cdot V_{gas(i,j)}}{\rho_{gas} \cdot A_{gas} \cdot U_{gas(i,j-1)}}
\]

% Modified Gardel's Correlations - Ottens(2001) %
\%
%% Inlet to Run %%%
\[
\text{k}_{in2rn} \cdot g = 0.03 \cdot (1-\lambda)^2 + 0.35 \cdot \lambda^2 - 0.2\cdot \lambda \cdot (1-\lambda) + 0.1267 \cdot \kappa
\]
\[
\text{k}_{in2rn} \cdot l = \left(11.48 + 1063 \cdot (1 + \text{Reliqu})^2\right) - 12.67 \cdot \kappa \cdot \text{k}_{in2rn} \cdot g;
\]
%% Inlet to Branch %%%
\[
\text{k}_{in2br} \cdot g = 0.958 \cdot (1-\lambda)^2 + 1.3 \cdot \lambda^2 \cdot \tan(\text{angle}/2) + 0.8 \cdot \lambda \cdot (1-\lambda) \cdot \tan(\text{angle}/2);
\]
\[
\text{k}_{in2br} \cdot l = \left(1.247 + 69.26 \cdot (1 + \text{Reliqv})^2\right) - 0.198 \cdot \kappa \cdot \text{k}_{in2br} \cdot g;
\]
%% Tee Junction Loss Force %
\[
\text{F}_{tee} \cdot \text{rn} \cdot g = \text{k}_{in2rn} \cdot g \cdot 0.5 \cdot \rho_{gas} \cdot A_{gas} \cdot \beta_{gas} \cdot (U_{gas(i,j)} \cdot (1 - \lambda_{l(i,j)})^2);
\]
\[
\text{F}_{tee} \cdot \text{rn} \cdot l = \text{k}_{in2rn} \cdot l \cdot 0.5 \cdot \rho_{liq} \cdot A_{liq} \cdot \beta_{liq} \cdot (U_{liq(i,j)} \cdot \lambda_{l(i,j)})^2;
\]
\[
\text{F}_{tee} \cdot \text{br} \cdot g = \text{k}_{in2br} \cdot g \cdot 0.5 \cdot \rho_{gas} \cdot A_{gas} \cdot \beta_{gas} \cdot (U_{gas(i,j)} \cdot (1 - \lambda_{l(i,j)})^2);
\]
\[
\text{F}_{tee} \cdot \text{br} \cdot l = \text{k}_{in2br} \cdot l \cdot 0.5 \cdot \rho_{liq} \cdot A_{liq} \cdot \beta_{liq} \cdot (U_{liq(i,j)} \cdot \lambda_{l(i,j)})^2;
\]
%----------------------------------------------%  %

% Diagnostic mode -- Shear forces zero %
\[
\text{F}_{wgas} = 0;
\]
\[
\text{F}_{wliq} = 0;
\]
\[
\mathbf{F}_{\text{tee} \cdot \text{rn} \cdot g} = 0;
\]
\[
\mathbf{F}_{\text{tee} \cdot \text{rn} \cdot l} = 0;
\]
% Check to ensure liquid velocity is non negative %
\[
\text{if } V_{gas(i,j)} < 0 || U_{gas(i,j)} < 0;
\]
\[
\text{fprintf('Negative velocity at row = %d -- column = %d',i,j)}
\]
\[
\text{pause}
\]
% Equation Engine %
\[
\text{out} = \text{num}(i,j);
\]
\[
\text{if } \text{ind}(i,j) = 1
\]
\[
\text{Gas Phase Mass Balance} \%
\]
\[
\text{out}(m+2,1) = \text{gasflo} - \rho_{gas} \cdot U_{gas(i,j)} \cdot A_{gas};
\]
\[
\text{Liquid Phase Mass Balance} \%
\]
\[
\text{out}(m+3,1) = \text{liqflo} - \rho_{liq} \cdot U_{liq(i,j)} \cdot A_{liq};
\]
%%%% Gas Phase Momentum Balance %%%%
out(m,1) = rhogasc * Agasc * abs(Ugas(i,j)) * Ugas(i,j)
- rhogasr * Agasr * abs(Ugas(i,j+1)) * Ugas(i,j+1) + ...
(P(i,j) * Agasc - P(i,j+1) * Agasr) - Fwgas - Fi;
%%%% Liquid Phase Momentum Balance %%%%
out(m+1,1) = rholiq * Aliqc * abs(Uliq(i,j)) * Uliq(i,j)
- rholiq * Aliqr * abs(Uliq(i,j+1)) * Uliq(i,j+1) + ...
(P(i,j) * Aliqc - P(i,j+1) * Aliqr) - Fwliq + Fi;
end

if ind(i,j) == 2 || ind(i,j) == 5
%%%% Gas Phase Mass Balance %%%%
Ugas(i,j) * Agasc;
%%%% Liquid Phase Mass Balance %%%%
out(m+3,1) = rholiq* Uliq(i,j-1) * Aliql - rholiq *
Uliq(i,j) * Aliqc;
end

%%%% Gas & Liquid Phase Momentum Balances %%%%
if ind(i,j) == 5
% Gas Phase Mass Balance %%%
Ugas(i,j) * Agasc;
% Liquid Phase Mass Balance %%%
out(m+3,1) = rholiq* Uliq(i,j-1) * Aliql - rholiq *
Uliq(i,j) * Aliqc;
else
% Gas Phase Mass Balance %%%
Ugas(i,j) * Agasc;
% Liquid Phase Mass Balance %%%
out(m+3,1) = rholiq* Uliq(i,j-1) * Aliql - rholiq *
Uliq(i,j) * Aliqc;
end

if ind(i,j) == 7
% Gas Phase Mass Balance %%%
Ugas(i,j) * Agasc;
% Liquid Phase Mass Balance %%%
out(m+3,1) = rholiq* Uliq(i,j-1) * Aliql - rholiq *
Uliq(i,j) * Aliqc;
% Gas & Liquid Phase Momentum Balances %%%
out(m,1) = rhogasc * Agasc * abs(Ugas(i,j)) * Ugas(i,j)
- rhogasr * Agasr * abs(Ugas(i,j+1)) * Ugas(i,j+1) + ...
(P(i,j) * Agasc - P(i,j+1) * Agasr) - Fwgas - Fi
- rhogasr * Agasr * abs(Vgas(i,j+1)) * Vgas(i,j+1) * cosd(angle);
\text{out}(m+1,1) = rholiq * Aliqc * abs(Uliq(i,j)) * Uliq(i,j) - rholiq * Aliqr * abs(Uliq(i,j+1)) * Uliq(i,j+1) + ... (P(i,j)) * Aliqc - P(i,j+1) * Aliqr - Fwliq + Fi - rholiq * Aliqr * abs(Vliq(i,j+1)) * Vliq(i,j+1) * \cos(angle); \\
\text{end}

\text{if} \ \text{ind}(i,j) == 4 \ || \ \text{ind}(i,j) == 6 \\
%%%% Gas Phase Mass Balance %%%%%
\text{Vgas}(i,j) * Agasc;
%%%% Liquid Phase Mass Balance %%%%%
\text{out}(m+3,1) = rholiq * Vliq(i,j) * Aliqt - rholiq * Vliq(i,j) * Aliqc;

%%% Gas & Liquid Phase Momentum Balances %%%%
\text{if} \ \text{ind}(i,j) == 6
\text{out}(m,1) = rhogasc * Agasc * abs(Vgas(i,j)) * Vgas(i,j) - rhogasb * Agasb * abs(Vgasout) * Vgasout + ...
(P(i,j)) * Agasc - P(i,j+1) * Agasb) - Fwgas - Fi;
\text{out}(m+1,1) = rholiq * Aliqc * abs(Vliq(i,j)) * Vliq(i,j) - rholiq * Aliqb * abs(Vliqout) * Vliqout + ...
(P(i,j)) * Aliqc - P(i,j+1) * Aliqb) - Fwliq + Fi;
\text{else}
\text{out}(m,1) = rhogasc * Agasc * abs(Vgas(i,j)) * Vgas(i,j) - rhogasb * Agasb * abs(Vgas(i,j)) * Vgas(i,j) + ...
(P(i,j)) * Agasc - P(i,j+1) * Agasb) - Fwgas - Fi;
\text{out}(m+1,1) = rholiq * Aliqc * abs(Vliq(i,j)) * Vliq(i,j) - rholiq * Aliqb * abs(Vliqout) * Vliqout + ...
(P(i,j)) * Aliqc - P(i,j+1) * Aliqb) - Fwliq + Fi;
\text{end}
\text{end}

\text{if} \ \text{ind}(i,j) == 3 \\
%%%% Gas Phase Mass Balance %%%%%
\text{Vgas}(i,j) * Agasc - rhogasc * Ugas(i,j) * Agasc;
%%%% Liquid Phase Mass Balance %%%%%
\text{out}(m+3,1) = rholiq * Uliq(i,j-1) * Aliql - rholiq * Uliq(i,j) * Aliqc;

%%% X Direction Momentum Balance %%%%
\text{out}(m,1) = rhogasr * Agasr * abs(Ugas(i,j)) * Ugas(i,j) - rhogasr * Agasr * abs(Ugas(i,j)) * Ugas(i,j) + ...
(P(i,j)) * Agasr - P(i,j+1) * Agasr) - F_{tee\_rn\_g}; \\
\text{out}(m+1,1) = rholiq * Aliqr * abs(Uliq(i,j)) * Uliq(i,j+1) + ...
(P(i,j)) * Aliqr - P(i,j+1) * Aliqr) - F_{tee\_rn\_l};

%%% Y Direction Momentum Balance %%%
\[ \text{out}(m+4,1) = \rho_{\text{gasc}} \times A_{\text{gasc}} \times |V_{\text{gas}}(i,j)| \times V_{\text{gas}}(i,j) - \rho_{\text{gasb}} \times A_{\text{gasb}} \times |V_{\text{gas}}(i+1,j)| \times V_{\text{gas}}(i+1,j) + \ldots \]
\[ \text{out}(m+5,1) = \rho_{\text{lqic}} \times A_{\text{lqic}} \times |V_{\text{lqic}}(i,j)| \times V_{\text{lqic}}(i,j) - \rho_{\text{lqib}} \times A_{\text{lqib}} \times |V_{\text{lqic}}(i+1,j)| \times V_{\text{lqic}}(i+1,j) + \ldots \]
\[ (P(i,j) \times A_{\text{gasc}} - P(i+1,j) \times A_{\text{gasb}}) - F_{\text{tee_br_g}} ; \]
\[ (P(i,j) \times A_{\text{lqic}} - P(i+1,j) \times A_{\text{lqib}}) - F_{\text{tee_br_l}} ; \]
\%--------------------------------------------------------
\% end
\%--------------------------------------------------------
\% end
\% end
\% end
function [out]=residuals(ind,num,unkn,P,Vgas,Vliq,Ugas,Uliq,lambda_l,Zgas,R,T,Mgas,D,mugas,muliq,rholiq,eps,sig,delx,gasflo,liqflo,angle)

%!!!!!!! Caution !!!!!!!!%
% Avoid the staggering and upwind scheme mistake at all costs
% eps -- roughness parameter
% Upwind scheme for mass balance and downstream staggering for momentum balance
% Implementing Blasius for diagnosis -- normal mode uses Chen's friction factor correlation %
% Diagnostic mode -- make changes in shear subroutine as well

[leny,lenx] = size(ind);
out = zeros(unkn,1);
format long

for i=1:leny
    for j=1:lenx
        if ind(i,j)==0
            rhogasc = P(i,j) * Mgas / (Zgas * R * T);
            if ind(i,j) == 1
                rhogasr = P(i,j+1) * Mgas / (Zgas * R * T);
            end
            if ind(i,j) == 2 || ind(i,j) == 5 || ind(i,j) == 7
                rhogasl = P(i,j-1) * Mgas / (Zgas * R * T);
                rhogasr = P(i,j+1) * Mgas / (Zgas * R * T);
            end
            if ind(i,j) == 3
                rhogasl = P(i,j-1) * Mgas / (Zgas * R * T);
                rhogasr = P(i,j+1) * Mgas / (Zgas * R * T);
                rhogasb = P(i+1,j) * Mgas / (Zgas * R * T);
            end
            if ind(i,j) == 4 || ind(i,j) == 6
                rhogast = P(i-1,j) * Mgas / (Zgas * R * T);
                rhogasb = P(i+1,j) * Mgas / (Zgas * R * T);
            end
            if ind(i,j) == 5
                Ugasout = 2 * Ugas(i,j) - Ugas(i,j-1);
                Uliqout = 2 * Uliq(i,j) - Uliq(i,j-1);
            end
            if ind(i,j) == 6
Vgasout = 2 * Vgas(i,j) - Vgas(i-1,j);
Vliqout = 2 * Vliq(i,j) - Vliq(i-1,j);
end

Diagnostic mode -- constant density system

rhogasc = P(1,lenx+1) * Mgas / (Zgas * R * T);
rhogasl = rhogasc;
rhogasr = rhogasc;
rhogast = rhogasc;
rhogasb = rhogasc;

Equation Building Blocks

%%% Flow Areas Calculation %%%
Apiped = (pi/4) * D ^ 2;
Aliqc = Apiped * lambda_l(i,j);
Agasc = Apiped - Aliqc;

if ind(i,j) == 1
    Aliqr = Apiped * lambda_l(i,j+1);
    Agasr = Apiped - Aliqr;
end
if ind(i,j) == 2 || ind(i,j) == 5 || ind(i,j) == 7
    Aliql = Apiped * lambda_l(i,j-1);
    Agasl = Apiped - Aliql;
    if ind(i,j) == 5
        lambda_l_run_out = 2 * lambda_l(i,j) - lambda_l(i,j-1);
        Aliqr = Apiped * lambda_l_run_out;
        Agasr = Apiped - Aliqr;
    else
        Aliqr = Apiped * lambda_l(i,j+1);
        Agasr = Apiped - Aliqr;
    end
end
if ind(i,j) == 3
    Aliql = Apiped * lambda_l(i,j-1);
    Agasl = Apiped - Aliql;
    Aliqr = Apiped * lambda_l(i,j+1);
    Agasr = Apiped - Aliqr;
    Aliqb = Apiped * lambda_l(i+1,j);
    Agasb = Apiped - Aliqb;
end
if ind(i,j) == 4 || ind(i,j) == 6
    Aliqt = Apiped * lambda_l(i-1,j);
    Agast = Apiped - Aliqt;
    if ind(i,j) == 6
        lambda_l_bran_out = 2 * lambda_l(i,j) - lambda_l(i-1,j);
        Aliqb = Apiped * lambda_l_bran_out;
        Agasb = Apiped - Aliqb;
    else

else
\[ A_{\text{lb}} = \text{A}_{\text{pipe}} \times \lambda_{(i+1,j)}; \]
\[ A_{\text{gsb}} = \text{A}_{\text{pipe}} - A_{\text{lb}}; \]

end

% if \( \lambda_{(i,j)} \leq 0 \)
% disp('****Error-liquid holdup negative or zero****')
% pause
% end

%%%% Wall Shear Contact Areas %%%%
\[ A_{\text{wl}} = \lambda_{(i,j)} \times \pi \times D \times \Delta x; \]
\[ A_{\text{wg}} = (1-\lambda_{(i,j)}) \times \pi \times D \times \Delta x; \]

%%%% Equivalent Diameters for liquid and gas wetted wall areas %%%
\[ d_{\text{hl}} = D; \]
\[ d_{\text{hg}} = D; \]

% Gas and Liquid Phase Reynolds Number
if \( \text{ind}(i,j) == 1 \) \( \| \) \( \text{ind}(i,j) == 2 \) \( \| \) \( \text{ind}(i,j) == 5 \) \( \| \) \( \text{ind}(i,j) == 7 \)
\[ \text{Regas} = \rho_{\text{gasc}} \times \text{abs}(U_{\text{gas}}(i,j)) \times d_{\text{hg}} / \mu_{\text{gas}}; \]
\[ \text{Reliq} = \rho_{\text{holiq}} \times \text{abs}(U_{\text{liq}}(i,j)) \times d_{\text{hl}} / \mu_{\text{liq}}; \]
\[ \text{gasvel} = U_{\text{gas}}(i,j); \]
\[ \text{liqvel} = U_{\text{liq}}(i,j); \]
end

if \( \text{ind}(i,j) == 4 \) \( \| \) \( \text{ind}(i,j) == 6 \)
\[ \text{Regas} = \rho_{\text{gasc}} \times \text{abs}(V_{\text{gas}}(i,j)) \times d_{\text{hg}} / \mu_{\text{gas}}; \]
\[ \text{Reliq} = \rho_{\text{holiq}} \times \text{abs}(V_{\text{liq}}(i,j)) \times d_{\text{hl}} / \mu_{\text{liq}}; \]
\[ \text{gasvel} = V_{\text{gas}}(i,j); \]
\[ \text{liqvel} = V_{\text{liq}}(i,j); \]
end

up = 2500;
low = 1700;

if \( \text{Regas} < \text{low} \)
\[ \text{fwgas} = 16 / \text{Regas}; \]
end
if \( \text{Regas} >= \text{low} \) \&\& \( \text{Regas} < \text{up} \)
\[ R1 = (\text{up} - \text{Regas}) / (\text{up} - \text{low}); \]
\[ R2 = (\text{Regas} - \text{low}) / (\text{up} - \text{low}); \]
\[ \text{fwgas1} = 16 / \text{Regas}; \]
\[ \text{fwgas2} = (-4 \times \log10( (\text{eps} / (3.7065 \times d_{\text{hg}})) - (5.0452/\text{Regas}) \times \log10((\text{eps/dhg})^{1.1098} / 2.8257 + (5.8506 / (\text{Regas}^{0.8981})))))^{-2}; \]
% \[ \text{fwgas2} = 0.046 \times \text{Regas} ^ (-0.2); \]
\[ \text{fwgas} = R1 \times \text{fwgas1} + R2 \times \text{fwgas2}; \]
end
if \( \text{Regas} >= \text{up} \)
\[ \text{fwgas} = (-4 \times \log10( (\text{eps} / (3.7065 \times d_{\text{hg}})) - (5.0452/\text{Regas}) \times \log10(((\text{eps/dhg})^{1.1098} / 2.8257 + (5.8506 / (\text{Regas}^{0.8981})))))^{-2}; \]
% \[ \text{fwgas} = 0.046 \times \text{Regas} ^ (-0.2); \]
end

if Reliq < low
    fwliq = 16 / Reliq;
end
if Reliq >= low && Reliq < up
    R1 = (up - Reliq) / (up - low);
    R2 = (Reliq - low) / (up - low);
    fwliq1 = 16 / Reliq;
    fwliq2 = (-4 * log10 (eps / (3.7065 * dhl)) -
              (5.0452 / Reliq) * log10(((eps / dhl)^1.1098) / 2.8257 + (5.8506 / (Reliq^0.8981))))^(-2);
    % fwliq2 = 0.046 * Reliq ^ (-0.2);
    fwliq = R1 * fwliq1 + R2 * fwliq2;
end
if Reliq >= up
    fwliq = (-4 * log10 (eps / (3.7065 * dhl)) -
              (5.0452 / Reliq) * log10(((eps / dhl)^1.1098) / 2.8257 + (5.8506 / (Reliq^0.8981))))^(-2);
    % fwliq = 0.046 * Reliq ^ (-0.2);
end

%%%% Wall Shear Forces %%%
Fwgas = Awg * fwgas * rhogasc * abs(gasvel) * gasvel / 2;
Fwliq = Awl * fwliq * rholiq * abs(liqvel) * liqvel / 2;

%%%% Mean Droplet Size %%%
We_non_visc = 12;
remax = sig * We_non_visc / (2 * rhogasc * (gasvel - liqvel) ^ 2);
We_visc = 12 * (1 + ((muliq ^ 2) / (2 * rholiq * remax * sig)) ^ 0.36);
remax = sig * We_visc / (2 * rhogasc * (gasvel - liqvel) ^ 2);
rm = 0.06147 * remax;

%%%% Interfacial Contact Area %%%
Aint = (3 * lambda_l(i,j) / rm) * pi * (D/2)^2 * delx;

%%%% Interfacial Friction Factor Calculation %%%
Re_int = rhogasc * 2 * rm * abs(gasvel - liqvel) / mugas;
w = log10 (Re_int);
if Re_int <= 0.01
    Cd = (24 / Re_int) * (1 + 3 * Re_int / 16);
else
    if Re_int > 0.01 && Re_int <= 20
        Cd = (24 / Re_int) * (1 + 0.1315 * Re_int ^ (0.82 - 0.05 * w));
    else
        if Re_int > 20 && Re_int <= 260
            Cd = (24 / Re_int) * (1 + 0.1935 * Re_int ^ (0.6305));
        else
            Cd = (24 / Re_int) * (1 + 0.1935 * Re_int ^ (0.82 - 0.05 * w));
    end
end
else
    Cd = (24 / Re_int) * (1 + 3 * Re_int / 16);
end
if Re_int > 260 && Re_int <= 1500
    Cd = 10 ^ (1.6435 - 1.1242 * w + 0.1558 * w ^ 2);
end
if Re_int > 1500 && Re_int <= 12000
    Cd = 10 ^ (-2.4571 + 2.558 * w - 0.9295 * w ^ 2 + 0.1049 * w ^ 3);
end
if Re_int > 12000 && Re_int <= 44000
    Cd = 10 ^ (-1.9181 + 0.6370 * w - 0.0636 * w ^ 2);
end
if Re_int > 44000 && Re_int <= 338000
    Cd = 10 ^ (-4.3390 + 1.5809 * w - 0.1546 * w ^ 2);
end
if Re_int > 338000 && Re_int <= 400000
    Cd = 29.78 - 5.3 * w;
end
if Re_int > 400000 && Re_int <= 10 ^ 6
    Cd = 0.1 * w - 0.49;
end
if Re_int > 10 ^ 6
    Cd = 0.19 - (8 * 10 ^ 4) / Re_int;
end

%%%% Interfacial Friction Factor %%%%
fi = Cd / 4;

%%%% Interfacial Shear Force %%%%
Fi = Aint * fi * rhogasc * abs(gasvel - liqvel) * (gasvel - liqvel) / 2;

%%%% Tee Junction Loss Calculation - Ottens (2001) %%%%
if ind(i,j) == 3
    % Tee inlet hydraulic diameters %
    dhl_in = D;
    dhg_in = D;

    % Tee inlet gas and liquid reynolds number %
    Regas_in = rhogasl * dhg_in * abs(Ugas(i,j-1)) / mugas;
    Reliq_in = rholiq * dhl_in * abs(Uliq(i,j-1)) / muliq;

    % Tee branch and run arms reynolds number %
    Reliqv = rholiq * abs(Vliq(i,j)) * dhl / muliq;
    Reliqu = rholiq * abs(Uliq(i,j)) * dhl / muliq;

    %%% Velocity Profile Correction Factor %%%%
    for con = 1 : 2
        if con == 1
            Re = Regas_in;
        else
            Re = Reliq_in;
        end
        if Re<=1500
            beta = 1.54;
        elseif Re>1500 && Re<=4500
            beta = 1.54 - 0.007 * Re;
        elseif Re>4500 && Re<=6000
            beta = 1.54 - 0.0045 * Re - 0.000125 * Re * Re;
        end
    end
end
if Re>1500 && Re<=2000
    beta = 1.54 - (1.08)*(10^-3)*(Re-1500);
end
if Re >2000
    beta = 1;
end
if con == 1
    beta_gas = beta;
else
    beta_liq = beta;
end

kappa = (beta_gas * rhogasl * ( Ugas(i,j-1) * (1-
lambda_l(i,j-1)) ) ^ 2 ) /... ( beta_liq * rholiq * ( Uliq(i,j-1) *
lambda_l(i,j-1)) ) ^ 2 );

% Calculate branch gas mass fraction lambda %
% Different from liquid holdup lambda_l %
lambda = (rhogasc * Agasc * Vgas(i,j)) / (rhogasl * Agasl * Ugas(i,j-1));

%% Modified Gardel's Correlations - Ottens(2001) %%
%%% Inlet to Run %%%
kin2rn_g = 0.03 *(1-lambda)^2+0.35*lambda^2-
0.2*lambda*(1-lambda); kin2rn_l = ( 11.48 + 1063 * (( 1 + Reliquu) ^(-1)) -
12.67 * kappa ) * kin2rn_g;
%%% Inlet to Branch %%%
kin2br_g = 0.958 * (1-lambda)^2+ 1.3 * (lambda^2) *
tand(angle/2) + 0.8 * Lambda * (1-lambda) * tand(angle/2);
kin2br_l = ( 1.247 + 69.26 * (( 1 + Reliquv) ^ (-1)) -
0.198 * kappa ) * kin2br_g;

%%% Tee Junction Loss Force %%
F_tee_rn_g = kin2rn_g * 0.5 * rhogasl * Agasc *
beta_gas * ( Ugas(i,j-1) * (1 - lambda_l(i,j-1)) )^2; F_tee_rn_l = kin2rn_l * 0.5 * rholiq * Aliqc *
beta_liq * ( Uliq(i,j-1) * lambda_l(i,j-1) )^2;
F_tee_br_g = kin2br_g * 0.5 * rhogasl * Agasc *
beta_gas * ( Ugas(i,j-1) * (1 - lambda_l(i,j-1)) )^2;
F_tee_br_l = kin2br_l * 0.5 * rholiq * Aliqc *
beta_liq * ( Uliq(i,j-1) * lambda_l(i,j-1) )^2;
end

%%%%%%%%%%%%%%%%%% Diagnostic mode -- Shear forces zero %%%%%%%%%%%%%%%%%%%
Fwgas = 0;
% Fwliq = 0;
% Fi = 0;
% F_tee_rn_g = 0;
% F_tee_rn_l =0;
F_tee_br_g = 0;
F_tee_br_l =0;

m = num(i,j); % Residual placement

if ind(i,j) == 1
  %%% Gas Phase Mass Balance %%%
  out(m+2,1) = gasflo - rhogas * Ugas(i,j) * Agas;
  %%% Liquid Phase Mass Balance %%%
  out(m+3,1) = liqflo - rholiq * Uliq(i,j) * Aliq;

  %%% Gas Phase Momentum Balance %%%
  out(m,1) = rhogas * Agas * abs(Ugas(i,j)) * Ugas(i,j) - rhogasr * Agasr * abs(Ugas(i,j+1)) * Ugas(i,j+1) + ...
              (P(i,j) * Agas - P(i,j+1) * Agasr) - Fwgas - Fi;
  %%% Liquid Phase Momentum Balance %%%
  out(m+1,1) = rholiq * Aliq * abs(Uliq(i,j)) * Uliq(i,j) - rholiqr * Aliqr * abs(Uliq(i,j+1)) * Uliq(i,j+1) + ...
              (P(i,j) * Aliq - P(i,j+1) * Aliqr) - Fwliq + Fi;
end

if ind(i,j) == 2 || ind(i,j) == 5
  %%% Gas Phase Mass Balance %%%
  out(m+2,1) = rhogasl * Ugas(i,j-1) * Agas - rhogas * Ugas(i,j) * Agas;
  %%% Liquid Phase Mass Balance %%%
  out(m+3,1) = rholiq* Uliq(i,j-1) * Aliql - rholiq * Uliq(i,j) * Aliq;

  %%% Gas & Liquid Phase Momentum Balances %%%
  if ind(i,j) == 5
    out(m,1) = rhogas * Agas * abs(Ugas(i,j)) * Ugas(i,j) - rhogasr * Agasr * abs(Ugas(i,j+1)) * Ugas(i,j+1) + ...
                (P(i,j) * Agas - P(i,j+1) * Agasr) - Fwgas - Fi;
    out(m+1,1) = rholiq * Aliq * abs(Uliq(i,j)) * Uliq(i,j) - rholiqr * Aliqr * abs(Uliq(i,j+1)) * Uliq(i,j+1) + ...
                (P(i,j) * Aliq - P(i,j+1) * Aliqr) - Fwliq + Fi;
  else
    out(m,1) = rhogas * Agas * abs(Ugas(i,j)) * Ugas(i,j) - rhogasr * Agasr * abs(Ugas(i,j+1)) * Ugas(i,j+1) + ...
                (P(i,j) * Agas - P(i,j+1) * Agasr) - Fwgas - Fi;
    out(m+1,1) = rholiq * Aliq * abs(Uliq(i,j)) * Uliq(i,j) - rholiqr * Aliqr * abs(Uliq(i,j+1)) * Uliq(i,j+1) + ...
                (P(i,j) * Aliq - P(i,j+1) * Aliqr) - Fwliq + Fi;
  end
end

if ind(i,j) == 7
  %%% Gas Phase Mass Balance %%%
\[
\text{out}(m+2,1) = \text{rhogasl} \ast \text{Ugas}(i,j-1) \ast \text{Agasl} - \text{rhogasc} \ast \\
\text{Ugas}(i,j) \ast \text{Agasc};
\]

%%%% Liquid Phase Mass Balance %%%

\[
\text{out}(m+3,1) = \text{rholiq} \ast \text{Uliq}(i,j-1) \ast \text{Aliql} - \text{rholiq} \ast \\
\text{Uliq}(i,j) \ast \text{Aliqc};
\]

%%%% Gas & Liquid Phase Momentum Balances %%%

\[
\text{out}(m,1) = \text{rhogasc} \ast \text{Agasc} \ast \text{abs}(\text{Ugas}(i,j)) \ast \text{Ugas}(i,j) - \text{rhogasc} \ast \text{Agasr} \ast \text{abs}(\text{Ugas}(i,j+1)) \ast \text{Ugas}(i,j+1) + \ldots \\
(P(i,j) \ast \text{Agasc} - P(i,j+1) \ast \text{Agasr}) - Fwgas - Fi - \\
\text{rhogsr} \ast \text{Agasr} \ast \text{abs}(\text{Ugas}(i,j+1)) \ast \text{Vgas}(i,j+1) \ast \text{cosd} \ast (\text{angle}) ; \\
\text{out}(m+1,1) = \text{rholiq} \ast \text{Aliqc} \ast \text{abs}(\text{Uliq}(i,j)) \ast \text{Uliq}(i,j) - \text{rholiq} \ast \text{Aliqr} \ast \text{abs}(\text{Uliq}(i,j+1)) \ast \text{Uliq}(i,j+1) + \ldots \\
(P(i,j) \ast \text{Aliqc} - P(i,j+1) \ast \text{Aliqr}) - Fwliq + Fi - \text{rholiq} \ast \text{Aliqr} \ast \text{abs}(\text{Vliq}(i,j+1)) \ast \text{Vliq}(i,j+1) \ast \text{cosd} \ast (\text{angle}) ;
\]

end

if \text{ind}(i,j) == 4 || \text{ind}(i,j) == 6

%%%% Gas Phase Mass Balance %%%

\[
\text{Vgas}(i,j) \ast \text{Agasc};
\]

%%%% Liquid Phase Mass Balance %%%

\[
\text{Vliq}(i,j) \ast \text{Aliqc};
\]

%%%% Gas & Liquid Phase Momentum Balances %%%

\[
\text{if} \text{ind}(i,j) == 6 \\
\text{out}(m+2,1) = \text{rhogast} \ast \text{Vgas}(i-1,j) \ast \text{Agast} - \text{rhogasc} \ast \\
\text{Vgas}(i,j) \ast \text{Agasc} - \text{rhogasc} \ast \text{Agasb} \ast \text{abs}(\text{Vgas}(i,j)) \ast \text{Vgasout} + \ldots \\
(P(i,j) \ast \text{Agasc} - P(i+1,j) \ast \text{Agasb}) - Fwgas --Fi; \\
\text{out}(m+1,1) = \text{rholiq} \ast \text{Aliqc} \ast \text{abs}(\text{Vliq}(i,j)) \ast \text{Vliqout} + \ldots \\
(P(i,j) \ast \text{Aliqc} - P(i+1,j) \ast \text{Aliqb}) - Fwliq + Fi; \\
\text{else} \\
\text{out}(m,1) = \text{rhogasc} \ast \text{Agasc} \ast \text{abs}(\text{Vgas}(i,j)) \ast \text{Vgas}(i,j) - \text{rhogasc} \ast \text{Agasb} \ast \text{abs}(\text{Vgas}(i+1,j)) \ast \text{Vgas}(i+1,j) + \ldots \\
(P(i,j) \ast \text{Agasc} - P(i+1,j) \ast \text{Agasb}) - Fwgas -Fi; \\
\text{out}(m+1,1) = \text{rholiq} \ast \text{Aliqc} \ast \text{abs}(\text{Vliq}(i,j)) \ast \text{Vliq}(i,j) - \text{rholiq} \ast \text{Aliqb} \ast \text{abs}(\text{Vliq}(i+1,j)) \ast \text{Vliq}(i+1,j) + \ldots \\
(P(i,j) \ast \text{Aliqc} - P(i+1,j) \ast \text{Aliqb}) - Fwliq + Fi; \\
\text{end}
\]

end

if \text{ind}(i,j) == 3

%%%% Gas Phase Mass Balance %%%

\[
\text{Vgas}(i,j) \ast \text{Agasc} - \text{rhogasc} \ast \text{Vgas}(i,j) \ast \text{Agasc};
\]

%%%% Liquid Phase Mass Balance %%%

\[
\text{Vliq}(i,j) \ast \text{Aliqc} - \text{rholiq} \ast \text{Uliq}(i,j) \ast \text{Aliqc};
\]
%%% X Direction Momentum Balance %%%%
out(m,1) = rhogasc * Agasc * abs(Ugas(i,j)) * Ugas(i,j)
  - rhogasr * Agasr * abs(Ugas(i,j+1))* Ugas(i,j+1) +...
  (P(i,j) * Agasc - P(i,j+1) * Agasr) - F_tee_rn_g;
out(m+1,1) = rholiq * Aliqc * abs(Uliq(i,j)) * Uliq(i,j)
  - rholiq * Aliqr * abs(Uliq(i,j+1))* Uliq(i,j+1) +...
  (P(i,j) * Aliqc - P(i,j+1) * Aliqr) - F_tee_rn_l;

%%% Y Direction Momentum Balance %%%%
Vgas(i,j) = rhogasc * Agasc * abs(Vgas(i,j+1))* Vgas(i,j+1) +...
  (P(i,j) * Agasc - P(i+1,j) * Agasb) - F_tee_br_g;
Vliq(i,j) = rholiq * Aliqc * abs(Vliq(i,j+1))* Vliq(i,j+1) +...
  (P(i,j) * Aliqc - P(i+1,j) * Aliqb) - F_tee_br_l;

end
end
end