The Pennsylvania State University

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MODELING THE FREQUENCY RESPONSE OF A BRIDGE SUPERSTRUCTURE

A Thesis in

Civil Engineering

by

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ABSTRACT

With the evolution in technology there has been a rise of remote electronic monitoring of structures in the transportation industry. These advanced monitoring systems are usually non-destructive, meaning there is no need to disrupt the structure by drilling into it or taking samples, which is a major benefit to use. One of the most easily available non-destructive testing systems is accelerometers that can measure the dynamic response of a structure in real-time. Another is a strain gage, which measures the flexure of a member. Using that strain, and known values of the system such as length, depth, and centroid, total bridge deflection can be calculated. There are different methods by which the test could be conducted. It should be apparent by the results. In this research, the purpose was to determine if data from a non-destructive testing system could be used to calculate the acceleration, velocity, and displacement at the testing location, as well as determine an impulse response model to estimate these reactions to future loading.

It is complicated and time consuming to design bridges to resist dynamic loads produced by high speed motor vehicles. It is strongly advisable to reduce the degrees of freedom in the structural system to a single one.

An approach of a single degree of freedom system is used for different simple loading cases. It focuses on comparing the response of displacement and bridge span for selected load combinations according to the governing standards and specifications. Oscillations and their effect on the bridge structural integrity are examined to provide some basic understanding on how to reduce and eliminate these undesirable outcomes. Using input data and system output based on the real-world testing of I-80E over Interstate I-287N in New Jersey, it was possible to determine an impulse response model for the system. Using that model, it was possible to convolve a new system output using a new input function converted into a vector. This showed

how different load cases affected the structure. After seeing the long settling times, it was determined that the dynamical behavior of the bridge could be improved by using tuned mass dampers.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1

Introduction

1.1 Aim of Study

The purpose of this research is to study and raise the data of the dynamic response of bridges using a purely mathematical approach for modeling dynamic response of a bridge located in New Jersey based on non-destructive testing system output and a known input. Many research studies have already been done to analyze bridge vibrations induced by high speed moving cars, however; most of these studies used computer software that was designed for local use only and most design engineers couldn't have access to the software unless they work for the company who owns it. This research uses computer codes generated by Matlab and Simulink software which are available and accessible to most engineers. This study should improve the way bridge dynamic response is analyzed by understanding how different methods of applying the load on a bridge can affect the outcome of the bridge design and thereby help engineers better predict the dynamic behavior of the structure. When a car drives over a bridge, it gradually loads the bridge girder to only a portion of the total load as the front axle approaches and passes the node, at which point both wheels would be contributing until the rear axle finally passes the node, at which point the loading would fall back to zero [5]. So, will changing the way the load is applied change the results of the dynamic response of the bridge? How significantly will the response results change? AASHTO specifications accounts for these dynamic vibrations by using a larger impact factor which is based on span length only [2]. Nevertheless; there are other factors that affect the dynamic response of a bridge structure such as the motorcar's properties, shock absorber reduce the amount of impact load on the bridge and pavement roughness affects the acceleration and breaking. Regardless of the fact that design codes used today provide a safe and strong design results, they may, in many cases, overlook the importance of actual bridge response As a result, a high speed moving load could induce stresses that were both unexpected and unaccounted for by current design codes [5].

1.2 Methodology

Researches and studies have been done in order to find, understand and compile unique simple methods used when analyzing the behavior of structures under moving and vibrant loads. In order to fully understand the internal deformations within a bridge structure, a finite element model is investigated and the results are compared with previous studies from the literature. The finite element software called RAM ELEMENT by Bentley Institute is used to analyze a simply supported beam under different combinations of moving loads. After collecting necessary data using non-destructive testing system, it is then transferred to Matlab to be analyzed and to clean any unexpected noise that could be the result of electronic interference from nearby devices or other loads such as wind load for example. After the noise is removed and the data is clean and ready for simulations, it was then transferred to Simulink to run simulations and analyze the results. The load was initially introduces as a sawtooth type of loading, meaning the load is gradually increasing as the car moves towards the girder, then goes back to zero as soon as it passes. The results of this type of loading are then compared with a sinusoidal loading, in which the load gradually increases as the car approaches the bridge girder and gradually decreases the car passes.

1.3 Research Limitations

The strategy referred to through this research is used so that simpler design approach of bridge structures exposed to dynamic forces could be developed, and could be used to analyze the dynamic and impulse response of any beam subjected to such forces. However, only a simple span beam which was pinned at one end and roller supported at the other end was analyzed. This method is purely mathematical and is based on computer simulations and analysis only, which means that it hasn't been beta tested yet. This study did not include vehicle properties such as suspensions and shock absorbers stiffness as well as bridge parameters such as pavement roughness and composite deck stiffness. The overall stiffness of the bridge composite section could benefit from impact load reduction due to the stiffness of the shock absorbers [4]. In order to avoid complex calculations produced by sophisticated behaviors of materials used in bridges construction, only linear elastic and viscous elastic behaviors are analyzed to idealize the system using traffic loads generated by moving vehicles.

Chapter 2

Materials

2.1 Linearly Elastic Material

Dynamic forces can induce high stresses in beams. Every material has a limit to how much stress it can withstand without any permanent deformations. Stress and strain are important in understanding the way materials deform. Steel is the main material used in this research and Hook's law describes the relationship between stress and strain in linearly elastic materials. The stress is linearly proportional to the strain and is described by equation (2.1) [1]:

$$\sigma = E\varepsilon \qquad (2.1)$$

The proportional constant E is called the modulus of elasticity. Figure 2.1(a) shows the principle relation between stress and strain of a linear elastic material. The spring in Figure 2.1(b) is to describe a linear elastic material. A loading of the structure with the stress $\sigma=\sigma_0$ gives the response of a strain ε_0. Figure 2.1(c-d) describes how the strain will disappear when the load is removed at time t_1 [1].

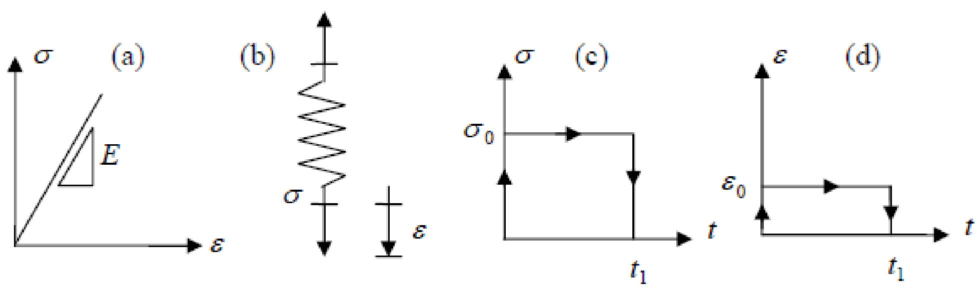

Figure 2.1: The Behavior of a Linear Elastic Material [1].

A 1D structure is one that only allows displacement in one direction and the elastic force F_E in a 1D structure subjected to a load will thus be linearly proportional to the displacement u, i.e.:

$$F_E = \sigma A = EA\varepsilon = \frac{EA}{L}u = ku \qquad (2.2)$$

Where the stiffness of the 1D spring is represented by k. Figure 2.2.shows the principle relation between the elastic force and the displacement for a 1D linear elastic material [1].

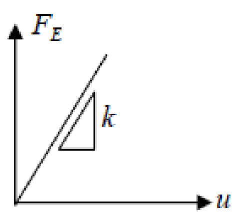

Figure 2.2: The Principal Relation Between The Elastic Force and The Displacement for a 1D Linear Elastic Material [1].

2.2 Viscous Elastic Material

The deformations that arise from a linearly elastic material are modeled to be time independent, but in reality all deformations in materials are time dependent. Time dependent elastic materials are called viscous elastic materials and the deformation can be divided into two different types of phenomena's; creep and relaxation. Figure 2.3 (a-b) shows how the strain increases with a constant stress in a creep situation, and in a relaxation situation the stress decreases with a constant strain as shown in Figure 2.3 (c-d). In this research, the creep and relaxation are not discussed any deeper. Viscous elastic materials can be described by different types of models, but in this thesis only models for a Newton material and a Kelvin material are discussed [1].

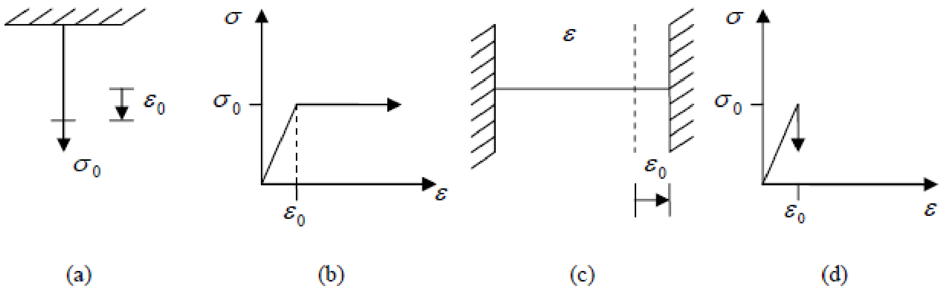

Figure 2.3: The Creep and Relaxation Phenomena for a Viscous Elastic Material [1].

2.2.1 Newton Material

A material is considered Newtonian when it is able to exhibit a linear relationship between stress and strain rate [1]. The constitutive relation for a Newton material is stated as:

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon} = \dot{\varepsilon}_N = \frac{\sigma}{\eta} \tag{2.3}$$

The stress is linearly proportional to the time dependent strain and η is the constant of viscosity, see Figure 2.4 (a). An instantaneous loading of the structure with the stress $\sigma=\sigma_0$ gives the response of a time dependent strain. If the load is removed at time t_1 the structure receives a remaining strain, see Figure 2.4 (b-c), and the remaining strain is derived as:

$$\varepsilon_r = \int_0^{t_1} \left(\frac{\sigma_0}{\eta} \right) dt = \frac{\sigma_0 t_1}{\eta} \tag{2.4}$$

A model of a damper is used to describe the Newton material, see Figure 2.4(d).

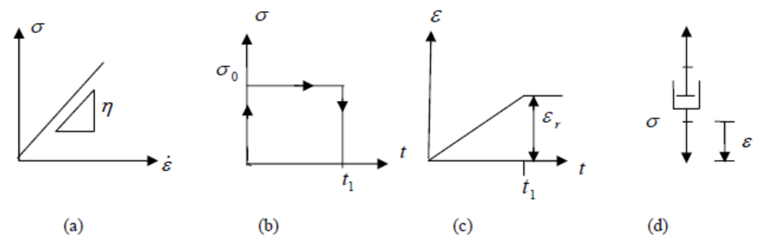

Figure 2.4: The Behavior for a Newton Material [1]

The damping force FD in a 1D structure with area A subjected to a load will thus be linearly proportional to the velocity u& , i.e.:

$$F_D = A\eta\dot{\varepsilon} = c\dot{u} \tag{2.5}$$

where *c* is the damping of the 1D structure. A schematic relation between the damping force and the velocity for a viscous elastic material is shown in Figure 2.5.

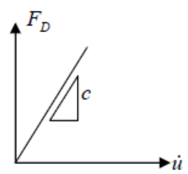

Figure 2.5: The Principle Relation Between Damping Force and The Velocity for a Viscous Elastic Material [1].

The Kelvin material consists of a parallel coupling between elastic materials and a viscous elastic material, i.e. Hooke and Newton material respectively, see Figure 2.6.

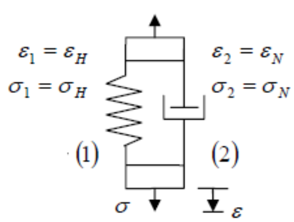

Figure 2.6: The Model for a Kelvin Material [1]

The differential equation and the constitutive relation for the Kelvin material can be derived as [1]:

$$\sigma = \sigma_H + \sigma_N = E\varepsilon_H + \eta\varepsilon_N = E\varepsilon + \eta\dot{\varepsilon} \quad \text{where } \dot{\varepsilon} + \frac{E}{\eta}\varepsilon = \frac{\sigma}{\eta} \tag{2.6}$$

To be able to describe instantaneous loading of the structure with a stress σ=σ0, the derivative of the strain has to be rewritten by use of the chain rule as [1]:

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{d\varepsilon}{d\sigma}\frac{d\sigma}{dt} = \frac{d\varepsilon}{d\sigma}\dot{\sigma} \tag{2.7}$$

By combining Equation (2.6) and Equation (2.7) gives:

$$\frac{d\varepsilon}{d\sigma} + \frac{1}{\dot{\sigma}}\frac{E}{\eta}\varepsilon = \frac{1}{\dot{\sigma}}\frac{\sigma}{\eta} \tag{2.8}$$

For instantaneous loading at time t=0 it holds that if σ& →∞ Equation (2.8) gives:

$$\frac{d\varepsilon}{d\sigma} = 0 \Rightarrow \varepsilon(\sigma) = \text{Constant} \tag{2.9}$$

The solution of Equation (2.6) with t>0 and σ=σ0 is:

$$\varepsilon = Ce^{\frac{-E}{\eta}t} + \frac{\sigma_0}{E} \tag{2.10}$$

The initial condition of ε(0)=0 gives C=-σ0/E and the strain is then:

$$\varepsilon = \frac{\sigma_0}{E} - \frac{\sigma_0}{E}e^{\frac{-E}{\eta}t} \tag{2.11}$$

The instantaneous loading at t=0 affects at first the viscous part only which carries the whole stress σ=σ0. When t→∞ the strain limit is obtained:

$$\varepsilon_\infty = \frac{\sigma_0}{E} \tag{2.12}$$

The strain at time t1 is:

$$\varepsilon_1 = \frac{\sigma_0}{E} - \frac{\sigma_0}{E}e^{\frac{-E}{\eta}t_1} = \frac{\sigma_0}{E}\left(1 - e^{\frac{-E}{\eta}t_1}\right) \tag{2.13}$$

If the loading is released at time t=t1, such that σ(t<t1)≠0 and σ(t1)=0, see Figure 2.4(b), this will give no jump in the strain ε according to Equation (2.9). The differential equation in equation (2.6) can now be solved with the condition σ(t1)=0.

$$t \geq t_1: \quad \dot{\varepsilon} + \frac{E}{\eta}\varepsilon = 0 \tag{2.14}$$

The differential equation has the solution:

$$\varepsilon = Ce^{\frac{-E}{\eta}t} \tag{2.15}$$

The initial condition is ε(t1)=ε1 according to Equation (2.14) which gives the solution of Equation (2.16):

$$\varepsilon = \varepsilon_1 e^{\frac{-E}{\eta}(t-t_1)} \quad \text{for } t > t_1 \tag{2.16}$$

When t → ∞ means that ε → 0 and there is no remaining deformation, see Figure 2.7.

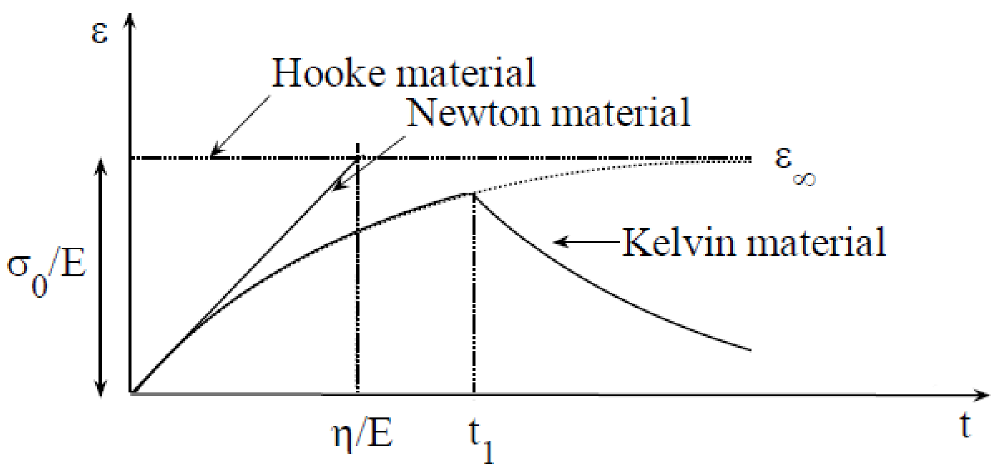

Figure 2.7: The Behavior of a Kelvin Material [1].

Chapter 3

Basic Dynamics

The basic theory of dynamics includes both the study of kinematics and kinetics. Kinematics describes the geometrical movement of a particle or a body in terms of displacement, velocity and acceleration, while kinetics is the science of a body movement caused by a force [1]. This chapter treats free and forced vibrations, both damped and undamped. In the case of forced vibrations, only the case with systems excited by harmonic loads are treated and derived analytically.

3.1 Kinematics

Kinematics is the study of particle motion regardless of its shape and weight and is used in this research in developing equation of motion that will be later used for simulations and analysis. The linear motion of a particle is the simplest way to describe a movement of the particle. A particle P, see Figure 3.1, is restricted to move along the s-axis and the position is described by a function f (t), where t is the time. At time t the particle has the position s and with a provided time step of Δt the particle moves a distance Δs [1].

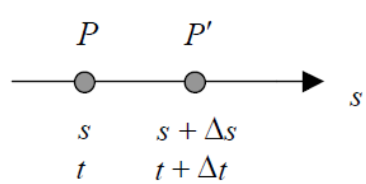

Figure 3.1: Linear Motion of a Particle.

3.1.1 Velocity

The velocity for the same particle as described in Figure 3.1 is derived from studying how fast the position of the particle is changing. When the time changes from t to t+Δt the particle moves a distance Δs and by that the mean velocity during the movement can be stated as [1]:

$$\overline{v} = \frac{\Delta s}{\Delta t} \tag{3.1}$$

The velocity of the particle is stated by letting the time step Δt go towards zero. That will lead to P′ moves closer to P and the mean velocity will approach a boundary value. Therefore the velocity of the particle at time t is defined by the boundary value as:

$$v(t) = v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \equiv \dot{s} \tag{3.2}$$

When $v>0$, the particle movement is defined to be positive along the s-axis and negative when $v<0$.

3.1.2 Acceleration

It is important to know how fast the velocity varies as the particle is moving; therefore the velocity of the particle is studied in the points P and P′, see Figure 3.1. The particle in these points has a velocity of v and v+Δv respectively. The mean value of the acceleration is defined as the mean velocity change per time unit with a particle movement from point P to P′. The mean value of the acceleration can be stated as [1]:

$$\overline{a} = \frac{\Delta v}{\Delta t} \tag{3.3}$$

The acceleration can be derived from the equation of velocity by calculating the change in velocity when time approaches zero. The acceleration formula can be written as follow:

$$a(t) = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \equiv \dot{v} = \frac{d^2 s}{dt^2} \equiv \ddot{s} \tag{3.4}$$

It is important to note that several combinations of sign changes of both the velocity and acceleration occur as the particle is moving along the s-axis. Negative acceleration leads to a decrease in the magnitude of the velocity; however, negative velocity does not necessary lead to negative acceleration or deceleration [1].

3.2 Kinetics

The response of bodies subjected to dynamic forces can be described by means of differential equations abbreviated as DE. This chapter will only describe linear vibrations with a single degree of freedom, which is the case of the beam being studied in this research. In a single degree of freedom system the position for the body is defined by one coordinate only [1].

3.2.1 Newton's Second Law

Newton's formulation of the second law is: "The change in the quantity of motion is proportional to the pushing force and occurs along a straight line, where the force is acting". This is defined by formula 3.5:

$$F_I = k \frac{d}{dt}(mv) \tag{3.5}$$

The quantity of motion corresponds to mv, the "change" corresponds to the derivative d/dt and k is a constant of proportionality. By using SI units, which will give k=1, and assuming that the mass is constant, Equation (3.5) can be rewritten as:

$$F_I = \frac{d}{dt}(mv) = ma = m\ddot{s} \tag{3.6}$$

3.2.2 Single Degree of Freedom System

A simple single degree of freedom system consists of a vertical spring and damper attached to a rigid body with a mass m, see Figure 3.2. The mass can move only in the vertical direction and therefore has only one degree of freedom. The spring is assumed to be massless and linearly elastic, with the stiffness *k* and damping coefficient *c*.

In a state of equilibrium, the force in the spring is equal to the gravity force generated by the mass. While the force of gravity is independent of the position of the mass, the force within the spring can change with the deformation of the spring [1].

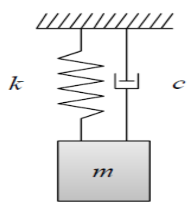

Figure 3.2: Mass-Spring System With Single Degree of Freedom [1].

Dynamic vibrations occur when the system is disturbed from its stable equilibrium position. The disturbance creates internal forces that try to bring back the system to its equilibrium position and this phenomena causes oscillations. The system will oscillate around its equilibrium position until the damping has reduced the oscillation to zero and finally a new stable equilibrium has occurred [1].

3.2.3 Free Vibration – Undamped

Consider a mass attached to a spring as illustrated in Figure 3.3. The unloaded equilibrium position for the system is noted as "*ue*" and is the static equilibrium position when the dead weight is the only presented load. "*u*" is the coordinate describing the distance from the unloaded equilibrium position to the current position. The elastic force FE for the system, described in Section 2.1, can be expressed as:

$$F_E = ku \tag{3.7}$$

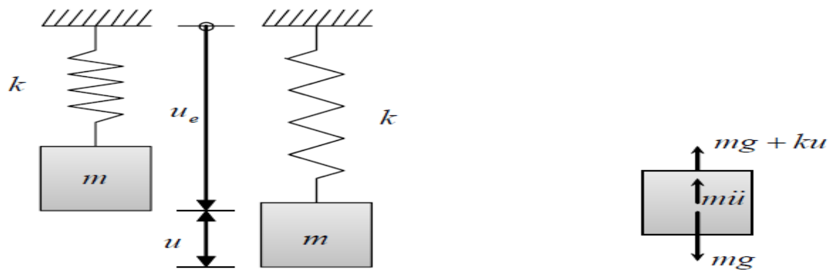

Figure 3.3: System with Undamped Free Vibration [1].

When the body is moved a distance u from the unloaded equilibrium position and then released the system will undergo an undamped free vibration about the unloaded equilibrium position. The forces acting on the isolated body is shown in Figure 3.3 where mg is the dead weight of the system [1].

Due to dynamic equilibrium conditions the sum of the forces shall be zero.

$$mg - (mg + ku) - m\ddot{u} = 0 \tag{3.8}$$

where the displacement u varies in time i.e. *u=u(t)*.

The DE of motion is linear, homogenous and it has constant coefficients. The DE is defined as:

$$m\ddot{u} + ku = 0 \tag{3.9}$$

By introducing the circular frequency ω, Equation (3.9) can be written as:

$$\ddot{u} + \omega^2 u = 0 \quad \text{where} \quad \omega = \sqrt{\frac{k}{m}} \quad \text{(Circular frequency)} \tag{3.10}$$

The general solutions of the differential Equation (3.10) are:

$$u(t) = A\sin(\omega t + \theta) \text{ or } u(t) = C_1 \sin \omega t + C_2 \cos \omega t \tag{3.11}$$

where the A and θ respectively C_1 and C_2 are constants of integration and they are determined from the boundary conditions [1].

When the system has started to oscillate, it will oscillate endlessly with the same amplitude A since the system is not affected by any kind of damping, see Figure 3.4.

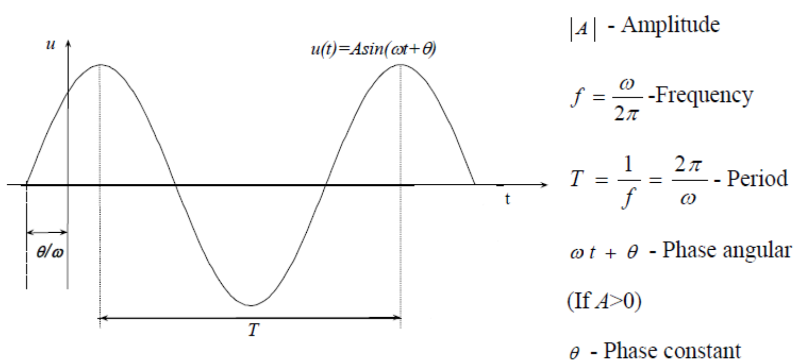

Figure 3.4: Oscillation of an Undamped System [1].

The oscillation can be described by terms of frequency "*f*", amplitude "*A*", phase constant "*θ*" and period "*T*". The frequency for the system describes how often an occurrence appears during a time period. The phase constant determines the amount that $u(t)$ lags the function $\sin \omega t$ and the period describes the time for an oscillation to move from one position and return to the same position. It should be remembered that this undamped case is a solely theoretical state. All structures in reality have some kind of damping [1].

3.2.4 Free Vibration - Damped

Using the same notations as in the case of undamped free vibrations, see Section 3.2.3, and also taking the damping into consideration, the differential equation of motion of a damped free system can be derived. The system in Figure 3.5, recalls a lot about the Kelvin material described in Section 2.2.2. So, here the properties for the spring and damper are combined together. The damping of the system is noted as *c* and the damping force *FD* for the system, described in section 2.2.1, can be expressed as:

$$F_D = c\dot{u} \tag{3.12}$$

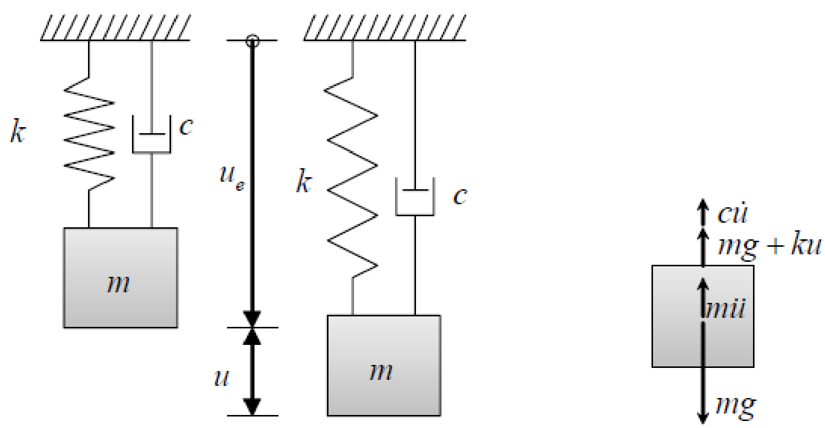

Figure 3.5: System with Damped Free Vibration [1]

When the body is moved a distance u from the unloaded equilibrium position and then released the system will undergo a damped free vibration about the unloaded equilibrium position. The forces acting on the isolated body are shown in figure 3.5.

Due to dynamic equilibrium conditions the sum of the forces shall be zero [1].

$$mg - (mg + ku) - m\ddot{u} - c\dot{u} = 0 \tag{3.13}$$

where the displacement *u* varies in time i.e. *u*=*u*(*t*).

The DE of motion is linear, homogenous and it has constant coefficients. The DE is defined as:

$$m\ddot{u} + c\dot{u} + ku = 0 \tag{3.14}$$

By introducing the damping coefficient ξ and the circular frequency ω, Equation (3.14) can be written as:

$$\ddot{u} + 2\xi\omega\dot{u} + \omega^2 u = 0 \quad \text{where} \quad \xi = \frac{c}{c_{cr}} = \frac{c}{2\sqrt{km}} \quad \text{and} \quad \omega = \sqrt{\frac{k}{m}} \tag{3.15}$$

As can be seen in Equation (3.15), ξ is a percentage of the critical damping c_{cr}, see Section 3.2.4.1. Setting $u=e^{\lambda t}$ gives the characteristic equation as:

$$\lambda^2 + 2\xi\omega u\lambda + \omega^2 = 0 \quad \text{with roots} \quad \lambda_{1,2} = \left(-\xi \pm \sqrt{\xi^2 - 1}\right)\omega \tag{3.16}$$

Hence the general solution of the differential Equation (3.16) is:

$$u(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \tag{3.17}$$

Depending on whether $\sqrt{\xi^2 - 1}$ is imaginary, real or zero, the value of *u(t)* has different mathematical form: Critical damping: ξ =1 Strong damping: ξ >1 Weak damping: ξ <1

3.2.4.1 Critical Damping ξ =1

The two roots of Equation (3.16) have the same value and that leads to a solution that contains a polynomial. In this case of a first order equation as [1]:

$$u(t) = (At + B)e^{-\omega t} \tag{3.18}$$

where *A* and *B* is constants of integration and they are determined from the boundary conditions. This function is also a non-periodic and has the same principal shape as in Figure 3.6.

3.2.4.2 Strong Damping ξ >1

If the roots for Equation (3.16) are both real this result in a general solution as:

$$u(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \quad (3.19)$$

where C_1 and C_2 are constants of integration that are real and determined from the boundary conditions [1].

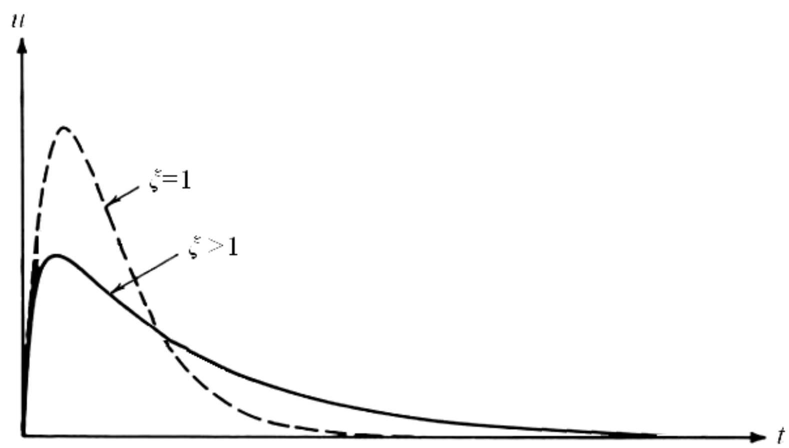

Figure 3.6: Example of Typical Oscillation of a Damped System with Critical and Strong Damping [1].

The boundary conditions for the system determine the curvature of the oscillation and a period cannot be found. The amplitude approaches exponentially towards zero with time due to the roots of Equation (3.16) being negative, see Figure 3.6.

3.2.4.3 Weak Damping ξ <1

If the roots for Equation (3.16) are imaginary, then the general solutions of the differential equation (3.14) are:

$$u(t) = e^{-\xi\omega t}\left(B_1 \sin\omega_d t + B_2 \cos\omega_d t\right) \quad \text{or} \quad u(t) = Ae^{-\xi\omega t}\sin\left(\omega_d t + \theta\right) \quad (3.20)$$

where $\omega_d = \omega\sqrt{1-\xi^2}$ - Damped circular frequency and B_1 and B_2 are constants of integration that are real and determined from the boundary conditions.

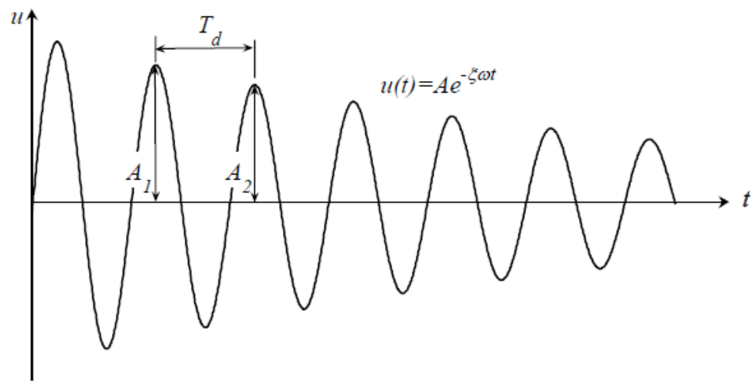

Figure 3.7: Oscillation of a Damped System with Weak Damping [1].

The difference between the undamped and the weak damped case is that the amplitude is decreasing exponentially with time and the oscillation has a lower circular frequency ωd, which leads to a longer period T_d, see Figure 3.7, the case of critical and strong damping rarely or never occurs in real structures and therefore only weak damping will be treated further in this thesis. Whenever damping is discussed or mentioned it is the case of weak damping [1].

3.2.5 Forced Vibration – Undamped with a Harmonic Load

The system shown in Section 3.2.3 did not include any external dynamic load. Now the system is subjected to an external dynamic load *p*(*t*) and in this case the damping is neglected as shown in Figure 3.8.

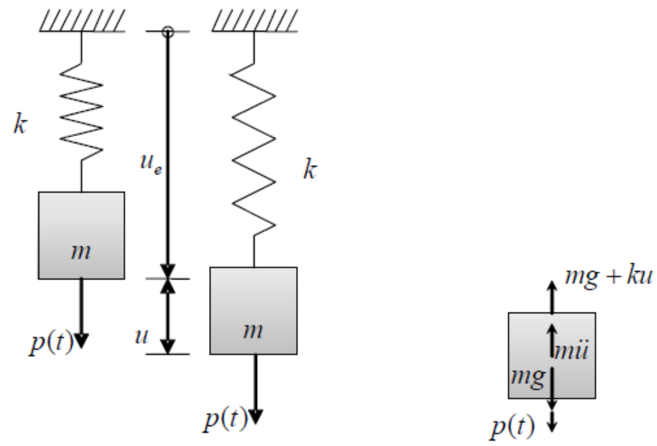

Figure 3.8: System with Undamped Forced Vibration [1].

Due to dynamic equilibrium conditions the sum of the forces shall be zero.

$$mg + p(t) - (mg + ku) - m\ddot{u} = 0 \tag{3.21}$$

where the displacement *u* varies in time i.e. *u*=*u*(*t*).

The DE of motion is linear, inhomogeneous and it has constant coefficients. The DE is defined as:

$$m\ddot{u} + ku = p(t) \tag{3.22}$$

Assume that the load in this case is harmonic and therefore periodic and has a shape of sinus function. The load is defined as:

$$p(t) = p_0 \sin \omega_p t \quad \text{where } \omega_p \text{ is the load circular frequency} \tag{3.23}$$

The DE with a harmonic load can thus be written as:

$$m\ddot{u} + ku = p_0 \sin \omega_p t \tag{3.24}$$

The general solution to the DE consists of a homogenous solution $u_h(t)$ and a particular solution $u_p(t)$. The system is undamped and therefore the homogenous solution is the same as in Equation (3.11). The general solution is defined as [1]:

$$u(t) = u_h(t) + u_p(t) \text{ where } u_h(t) = C_1 \sin \omega t + C_2 \cos \omega t \tag{3.25}$$

Assume the particular solution as:

$$u_p(t) = C_3 \sin \omega_p t \tag{3.26}$$

The constant $C3$ is solved by combing Equation (3.24) and Equation (3.26).

$$\left(-m\omega_p^{\ 2} + k\right) C_3 \sin \omega_p t = p_0 \sin \omega_p t \tag{3.27}$$

Where $C_3 = \dfrac{p_0}{k - \omega_p^{\ 2} m} = \dfrac{p_0}{k} \dfrac{1}{1 - \left(\dfrac{\omega_p}{\omega}\right)^2} = \dfrac{p_0}{k} \dfrac{1}{1-\beta^2}$ and $\beta = \dfrac{\omega_p}{\omega}$

The general solution for the DE is then:

$$u(t) = (C_1 \sin \omega t + C_2 \cos \omega t) + \frac{p_0}{k} \frac{1}{1-\beta^2} \sin \omega_p t \tag{3.28}$$

where $C1$ and $C2$ are constants of integration that are determined from the boundary conditions.

3.2.6 Forced Vibration – Damped with a Harmonic Load

Forced vibration system is one that is exposed to a force of motion that keeps repeating throughout and interval of time. Consider the damped mass-spring system in Section 3.2.4 but now subjected to an external dynamic load *p*(*t*) as shown in Figure 3.9 [1].

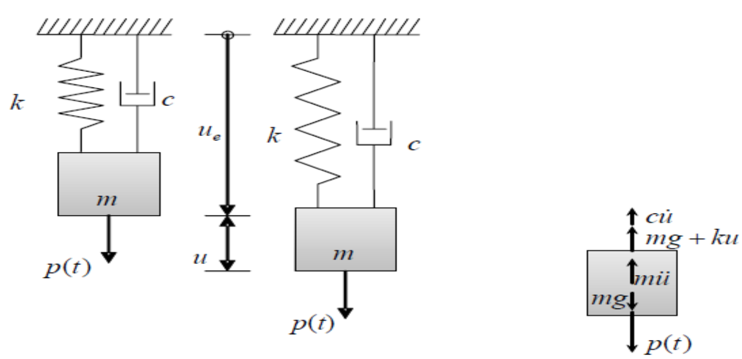

Figure 3.9: System with damped forced vibration [1]

Due to dynamic equilibrium conditions the sum of the forces shall be zero.

$$mg + p(t) - (mg + ku) - m\ddot{u} - c\dot{u} = 0 \tag{3.29}$$

where the displacement u varies in time i.e. $u=u(t)$.

The DE of motion is linear, inhomogeneous and it has constant coefficients. The DE is defined as:

$$m\ddot{u} + c\dot{u} + ku = p(t) \tag{3.30}$$

Assume that the load in this case is harmonic and therefore periodic and has a shape of a sinus function. The load is defined as:

$$p(t) = p_0 \sin \omega_p t \tag{3.31}$$

The DE with a harmonic load can be written as:

$$m\ddot{u} + c\dot{u} + ku = p_0 \sin \omega_p t \tag{3.32}$$

The general solution to the DE consists of a homogenous solution and a particular solution. The damping is assumed to be weak and therefore the homogenous solution is same as in Equation (3.20). The general solution is defined as [1]:

$$u(t) = u_h(t) + u_p(t) \quad \text{Where} \quad u_h(t) = e^{-\xi\omega t}(B_1 \sin \omega_d t + B_2 \cos \omega_d t) \tag{3.33}$$

Assume the particular solution as:

$$u_p(t) = C_1 \sin \omega_p t + C_2 \cos \omega_p t \tag{3.34}$$

3.3 Resonance and Dynamic Amplification Factor

When the load circular frequency ω_p intersect with the natural circular frequency ω of the system, then resonance occurs. This leads to in a noticeable increase of the amplitude for the oscillation in the system. The dynamic amplification factor (*DAF)* describes the effects of resonance for the system, which is dependent on the damping coefficient ξ, see Figure 3.10 [1].

3.3.1 Undamped System

Any dynamic system is always affected by damping due to its own weight or some external force that acts against the system's motion such as wind load for example. This leads to that the homogenous solution in Equation (3.28) will die out and the particular solution will dominate. Assume that the boundary conditions for the undamped system are:

$$u(0) = u_0 = 0 \text{ and } \dot{u}(0) = \dot{u}_0 = 0 \tag{3.35}$$

Then the constant of integration C_1 and C_2 are solved from Equation (3.28) and the total solution is derived as:

$$u(t) = \left(\frac{\dot{u}_0}{\omega} - \frac{p_0}{k}\frac{\beta}{1-\beta^2}\right)\sin\omega t + u_0\cos\omega t + \frac{p_0}{k}\frac{1}{1-\beta^2}\sin\omega_p t \tag{3.36}$$

Since the boundary conditions in Equation (3.28) describe that the system initially was in a stable equilibrium position, i.e. $u(0) = u_0 = 0$ and $\dot{u}(0) = \dot{u}_0 = 0$ the response can be stated as:

$$u(t) = u_{static}\frac{1}{1-\beta^2}\left(\sin\omega_p t - \beta\sin\omega t\right) \tag{3.37}$$

where $u_{static}=p_0/k$ describes the static displacement of the system. The dynamic amplification factor is derived by:

$$DAF = \left|\frac{u_p(t)_{\max}}{u_{static}}\right| = \left|\frac{1}{1-\beta^2}\right| \text{ where } \quad \beta = \frac{\omega_p}{\omega} \tag{3.38}$$

The dynamic amplification factor increases rapidly when the load frequency closes the natural frequency of the system and decreases when it has larger frequencies.

3.3.2 Damped System

In a similar way as for the undamped system the dynamic amplification factor is derived from Equation (3.38) and the static response of the loaded system.

$$DAF = \left|\frac{u_{\max}}{u_{static}}\right| = \left|\frac{1}{\sqrt{\left(1-\beta^2\right)^2 + \left(2\xi\beta\right)^2}}\right| \tag{3.39}$$

The dynamic amplification factor has the same effects as in the undamped case, but it will be smaller when the system is damped, see Figure 3.10.

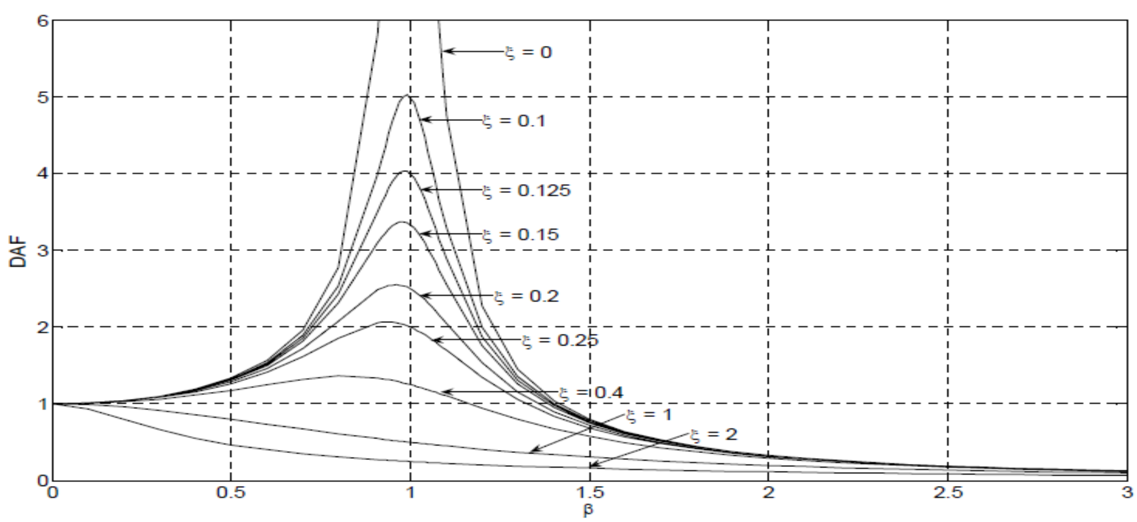

Figure 3.10 Illustrate The Effects of DAF, Damping Coefficient and Relationship Between the Load Frequency and The Natural Frequency of The System [1]

Chapter 4

Beam Dynamics

4.1 Eigenmodes and Frequencies for a Uniform Beam

For a structure, in this case a beam, the eigenfrequencies are the frequencies for which the structure will vibrate of its own accord when exposed to a perturbation. The different shapes of the structure for the different eigenfrequencies are called eigenmodes, and each eigenmode is related to one specific eigenfrequency [1].

To determine the beam response versus time the eigenmodes for a simply supported beam can be computed, by verifying the eigenvalue equation with the homogeneous boundary conditions. In Figure 4.1, the three first eigenmodes for a simply supported beam are shown. The first eigenmode corresponds to the lowest eigenfrequency [1].

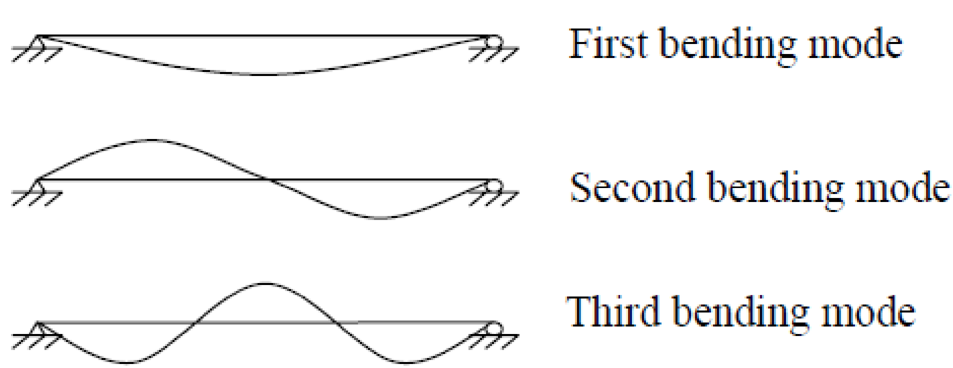

Figure 4.1: The Three First Eigenmodes for a Simply Supported Beam [1].

Normally, when a beam is subjected to a dynamic load, the load frequency will not coincide with the eigenfrequencies and therefore the resulting shape of deformation will not be the same as any of the eigenmodes. However, the dominating shape of deformation is usually the first eigenmode but it is influenced by higher modes. SDOF systems have only one eigenmode and hence there are no influences from higher modes [1].

4.2 Transformation from Deformable Body to SDOF System

To be able to simplify analyses of continuous systems, which have an infinite number of degrees of freedom, the system needs to be discretized to a finite number of elements and degrees of freedom. In practice, beams and plates have a limited possibility to move and this makes it possible to transform the structures into a single degree of freedom system, see Figure 4.2 [1].

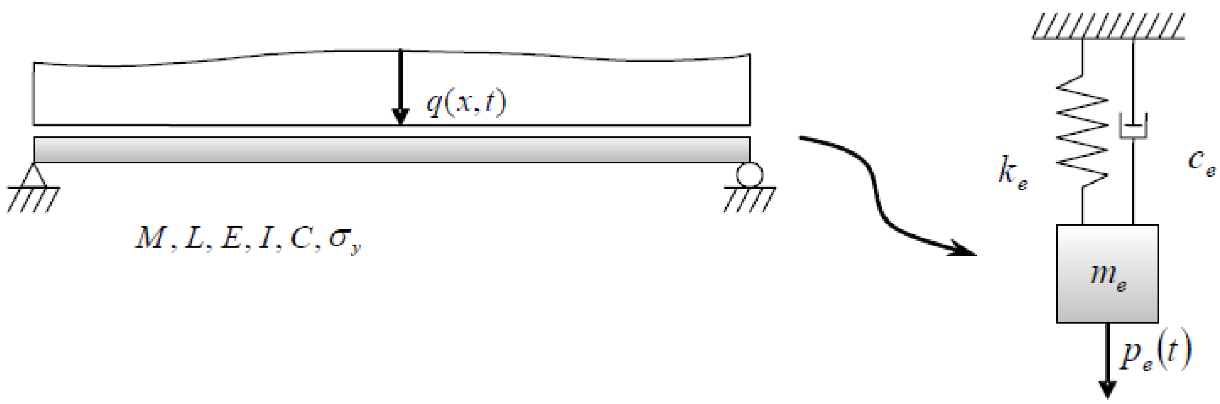

Figure 4.2: Transformation from Continuous System to a SDOF System [1].

The simplification to a SDOF system implies that the properties of the continuous system has to be assigned with equivalent quantities for the mass *m*, the internal force *FI*, the damping force *FD* and the load *p*(*t*) applied to a certain system point. The deflection in the system point is assumed to be described by the same function as for the SDOF system. The system point is chosen to coincide with the point that normally will achieve the largest displacement, i.e. the midspan in the case of a simply supported beam, see Figure 4.3. One condition, for the transformation of the properties to be possible, is that a uniform change of the deformation is assumed. This means that if the displacement increases in one point the displacements in all other points will increase proportional to this displacement.

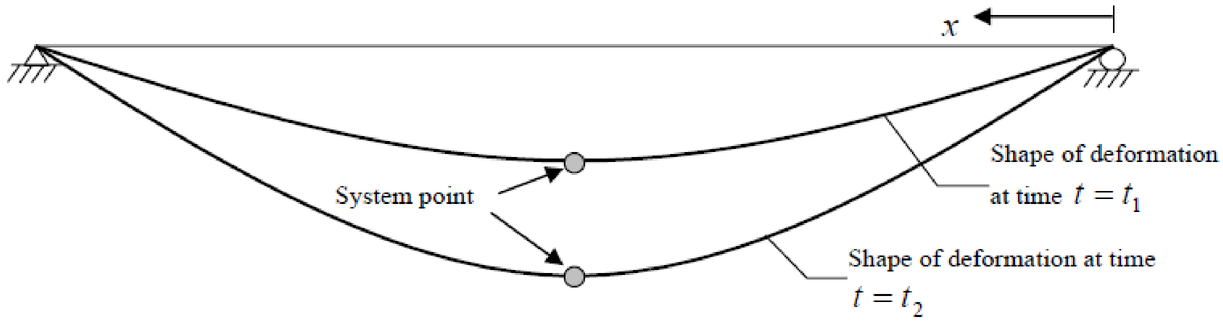

Figure 4.3: Illustration of the System Point Chosen to Appear in Midspan [1].

The transformation of the properties for the real structure to equivalent properties used in the SDOF system is made by use of transformation factors. The equivalent quantities and the transformation factors are derived from the condition that the energy exerted by the equivalent SDOF system must be equal to the energy exerted by the beam, when exposed to a certain load. Hence, the transformation factors will depend on the applied load and the deflection shape of the beam [1].

4.3 Transformation Factor for The Internal Force

In order to derive the transformation factor for the internal load, the condition that the equivalent internal force shall perform a work that is equivalent to the work of the deformation of the beam, when following the oscillation of the system point *us*, can be used [1].

The internal force and the work it performs are depending on the behavior of the material. For the SDOF system this is shown in Figure 4.4, where the shaded areas represent the total internal work for the material. $F_{I,\mathrm{e,max}}$ is the maximum value of the equivalent internal force. In the case of linear elastic material the maximum internal force is corresponding to

$F_{I,e,\max}=K_{e}\cdot u_{s,\max}$.

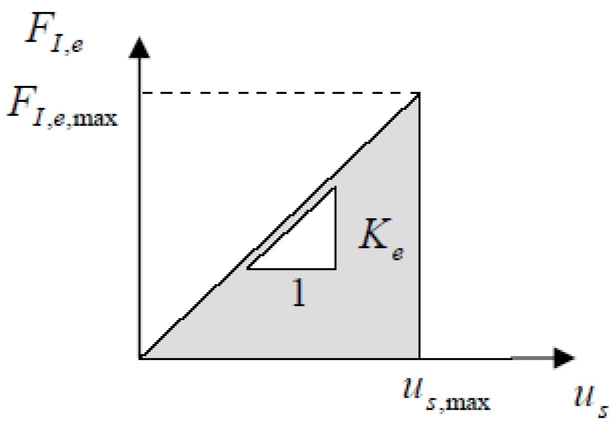

Figure 4.4: Work for a SDOF System for a Linear Elastic Material [1].

Chapter 5

Finite Elements Model

5.1 RAM Modeler

RAM Elements is a finite element-based general analysis program with an integrated design toolkit. In RAM Elements, you can design complex three dimensional structures or individual structural components. In the RAM Elements main application, a model is built by entering information, such as element properties or loading, into a series of data sheets. Since RAM Elements performs a finite element analysis, certain model properties must be defined prior to performing an analysis of the structure, such as section or material properties of all elements, in order for the loads to be distributed through the model properly. The design process in RAM Elements is iterative. In RAM Elements, elements (nodes, members, and shells) are generated through the global coordinate system represented by X, Y, and Z. In RAM Elements, the global Y axis is set at the vertical axis of the structure and the X and Z axes as the horizontal axes. If the Y axis is not specified as the vertical axis, several features of the program, such as the rigid floor diaphragm, will not function properly .

A model is essentially an assembly of components. These components can be plan sub-assemblies, such as floor structures; individual frames consisting of linear beams, columns, and braces; or even 2D or 3D elements such as slabs or walls. To represent these components in RAM Elements, you will create nodes, members, and shells. Nodes define the coordinates or boundary locations of all of the elements in a structure. Shell elements are defined based on the following assumptions shell elements are defined based on the following assumptions:

- Rectangular Plates

• Elastic material, isotropic and homogeneous

• Strength developed by a combination of bending and membrane actions

Nodes on each plan, parallel to the global X-Z plane, can be assigned to a rigid diaphragm which will simulate the inplane rigidity produced by a slab and make all of the nodes assigned to that diaphragm move or rotate together.

5.2 Thesis Data

For this project, the "system data" was not provided by an actual NDT device placed in the field. It was instead based on a dynamic model developed using accelerometers placed on I-80E over Interstate I-287N in New Jersey. The properties of this structure are show in Table 1 below.

Table 1 – Provided Values of Structure [6]

Property	Value
Span Length	87.75 ft
Structure Width	51 ft
# of Girders	7
Deck Thickness	8 in
Haunch Thickness	1.5 in
Moment of Inertia of Composite Section	72,488 in^4

The finite element model shown in Figure 5.1 was constructed in Ram Elements to visualize the static responses of the system. The beams and deck were modeled as shell elements which allow responses in the X, Y and Z directions. Figure 5.2 shows the static deflection of the bridge under gravity load, we used these system reactions to help determine if the dynamic model's behavior was reasonable.

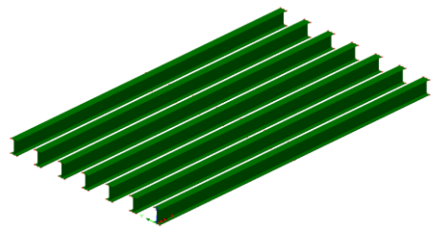

Figure 5.1 – Finite Element Model of I-80E over Interstate I-287N

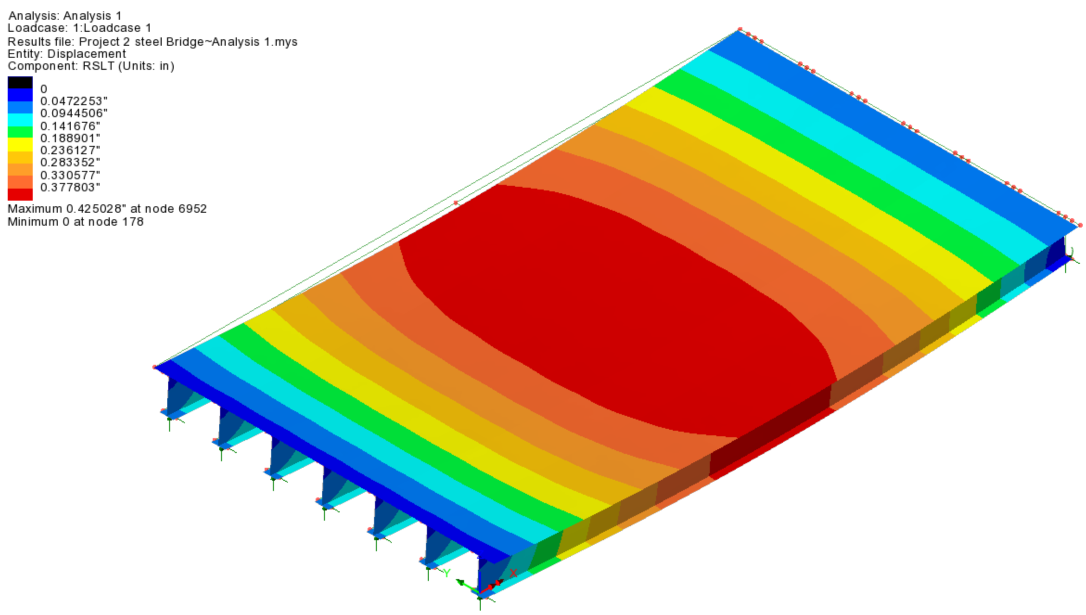

Figure 5.2 – FEA of Static Gravity Load on Bridge

5.3 System Identification

Once the input and NDT output data were provided, it was imported into MATLAB to be analyzed. Figure 5.3 shows the plot of the provided data, with the known sampling time of six seconds and sampling rate of $3x10^{-3}$ seconds (for a total of 2,000 sampling points). "*u1*" is the known input, a sawtooth representing the passing of a load which linearly increases as it approaches the testing equipment, then immediately falls to zero as it passes. "*y1*" is the raw system data. The structure's dynamic response is evident as a sine wave after the load passes, but the data is noisy.

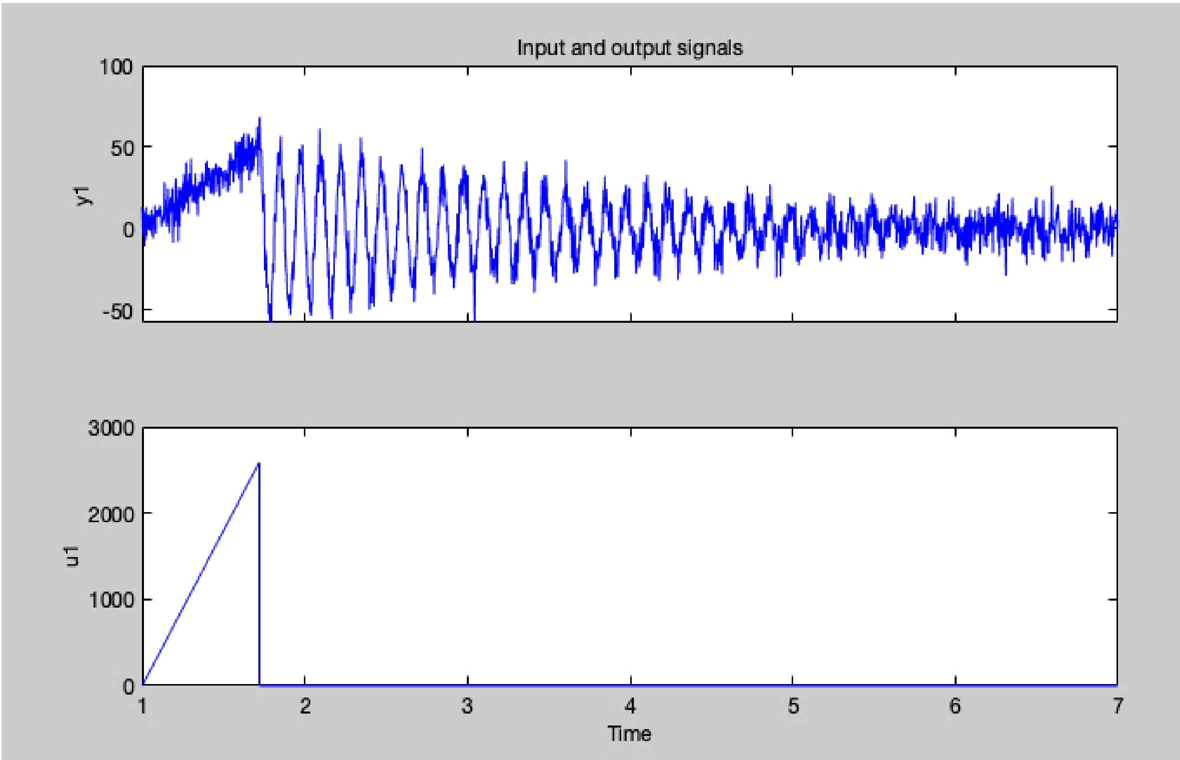

Figure 5.3 – Output and Input Data

This noise could be attributed to additional loading unaccounted for, such as wind loading, as well as electronic interference from nearby utilities or within the measuring equipment itself. On inspection, it appears as though the data is coming from a strain gauge. The amplitude rises linearly with load, then oscillates sinusoidally until it settles to zero. We can see the same happen with a guitar string and its displacement. As the load increases, so too does the displacement. If this were an accelerometer, the readings would peak after the load is sharply removed rather than gradually increase with the load [4]. This may also explain the noise in the system. Strain gauges are extremely fragile and applying the sensor could have introduced "locked in" stresses[6]. Using MATLAB's built in PID tool to remove the noise, refer to Figure 5.4 for the PID identification window; it was possible to obtain a clean version of the output data, Figure 5.6. PID also calculated a transfer function, Figure 5.5.

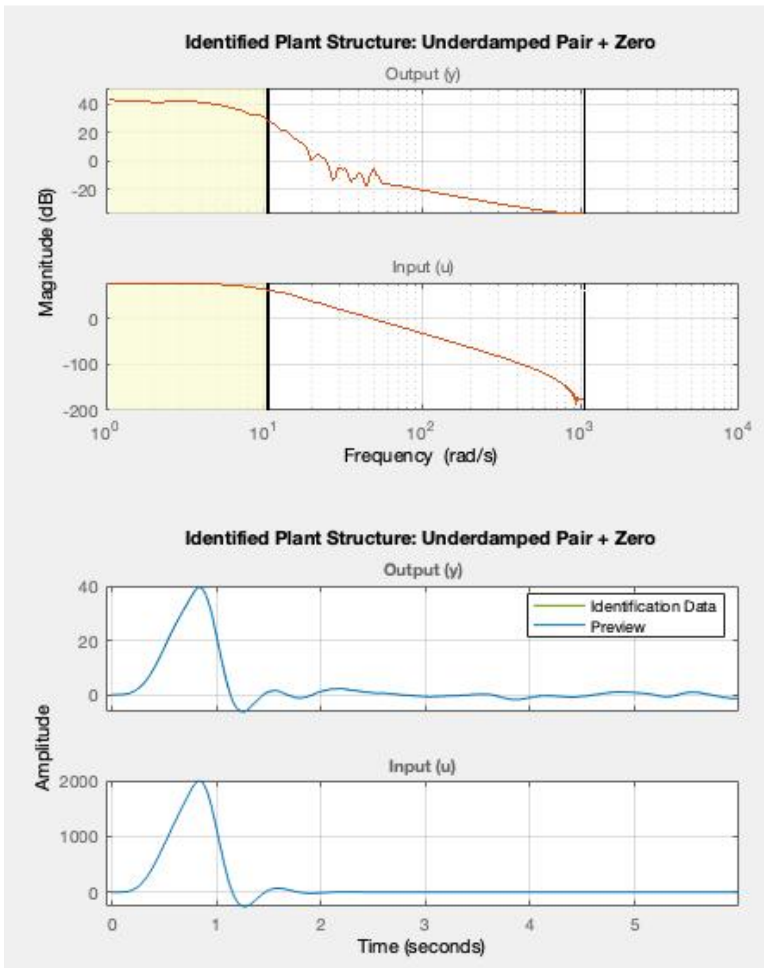

Figure 5.4 – PID Analysis Output Window

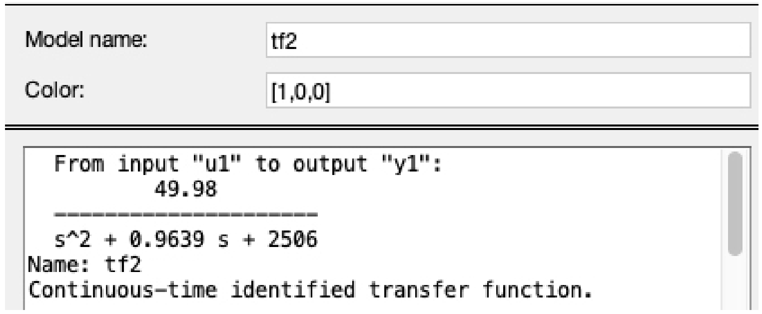

Figure 5.5 – Transfer Function Calculated by System Identification Tool

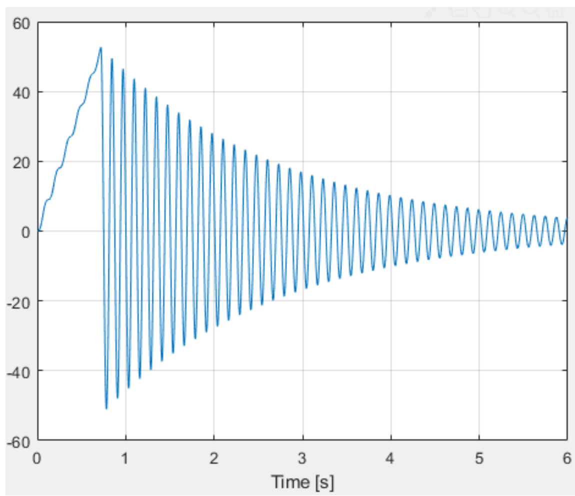

Figure 5.6 – Clean System Plot

With the cleaned output data, a system data matrix was created using the known time sampling. MATLAB's system identification toolbox was used to obtain the transfer function for this system matrix, as shown in Equation 5.1.

$$H(t) = \frac{49.98}{s^2 + 0.9639s + 2506} \tag{5.1}$$

Given the accuracy of the data, it was assumed the transfer function could be simplified as Equation 5.2 for use in subsequent calculations.

$$H(t) = \frac{50}{s^2+s+2500} \quad (5.2)$$

The transfer function, Equation 5.2, can be easily converted into an impulse response using MATLAB's "impulse" function on a given transfer function. PID can also produce this impulse response, which is provided in Equation 5.3 (again rounded for simplification). Figure 5.7 is a plot of this impulse function. Figure 5.8 is a zoomed in plot of a single period of the sinusoid.

$$h(t) = e^{-0.5t}\sin(50t) \quad (5.3)$$

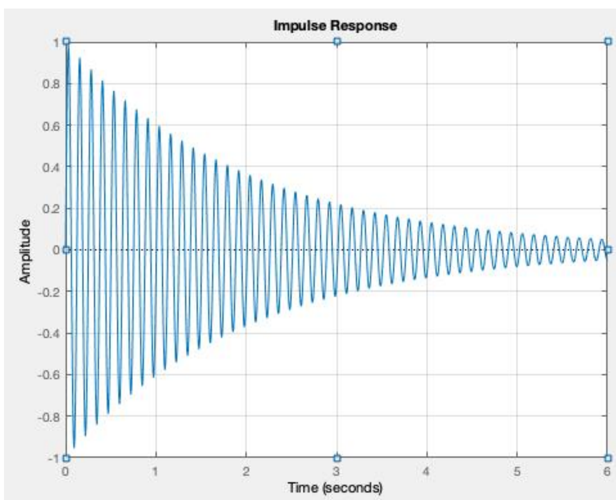

Figure 5.7 - Impulse Response

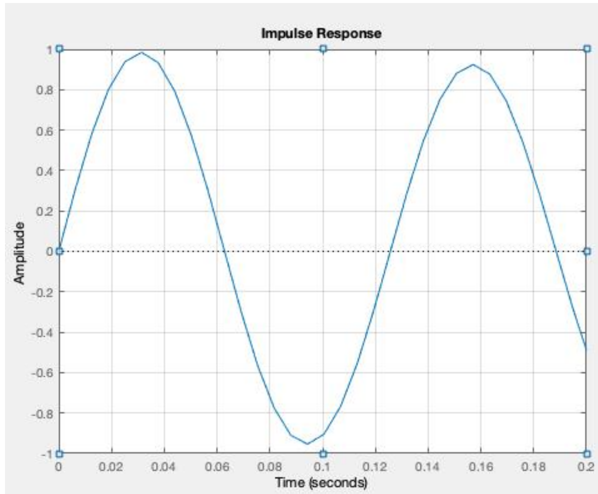

Figure 5.8 – Enlarged Impulse Response

The impulse response above is what would be expected for a steel beam, which is the construction material of the I-80 structure that this model is based on. The response is quick, symmetrical, and quickly dampens. Normal simple span bridge beams are “fixed” on one end and have a “roller” type bearing on the other end. The fixed bearing has directional restraint in all directions and the roller is normally a neoprene pad that only has stiffness in the negative vertical direction [3]. Given these parameters it is possible that the response is similar to a cantilevered beam.

5.4 Testing System Response to New Inputs.

The input sawtooth “*u1*” seen in Figure 5.3 was used to induce a response in the initial data, but this loading is not ideal. Looking at the load at a single point on a structure, a vehicle would initially load to only a portion of the total load of the vehicle, as the front axle approaches and passes the node, at which point both wheels would be contributing until the rear axle finally passes the node, at which point the loading would fall back to zero [8]. Typically, for ease of analysis, we would model this as a trapezoid, but it was decided to try to model as a smooth curve. Equation 5.4 and Figure 5.9 show the proposed loading as a function of time. The sin wave is limited to t = 1 second so it is only a positive force and does not repeat.

$$y(t) = 5000\sin(\pi t) \tag{5.4}$$

This function was used to achieve the wave type input but also having it dampen to zero. This new input is shown in the plot below.

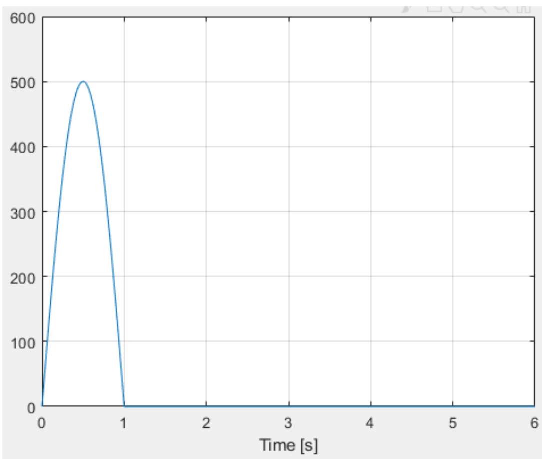

Figure 5.9 – New Impulse

The original and new inputs were applied to the model in Simulink as shown in the Figure 5.10.

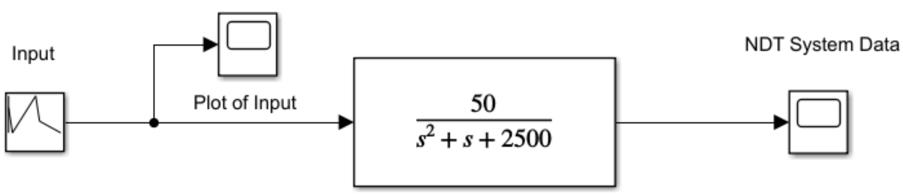

Figure 5.10 - Simulink Diagram

In Simulink the transfer function was used to model the two inputs. Figure 5.12 replicates the original system output data, while Figure 5.14 is the acceleration due to the new sinusoid input function. Figures 5.11 and 5.13 are the Original Input and New Input, respectively.

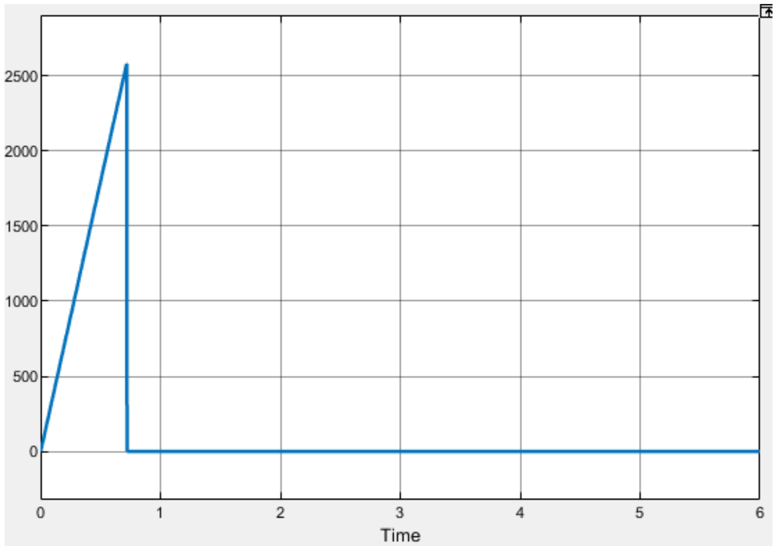

Figure 5.11 – Original Input Data

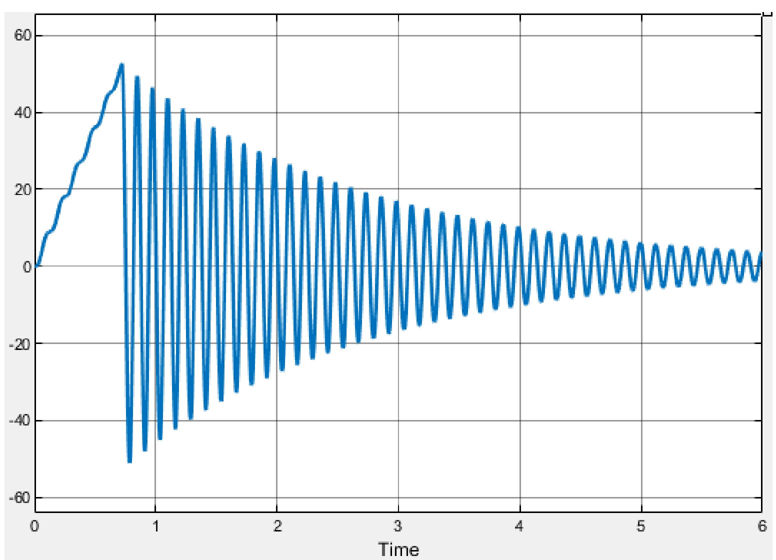

Figure 5.12 - Strain Plot Based on Original Input Data

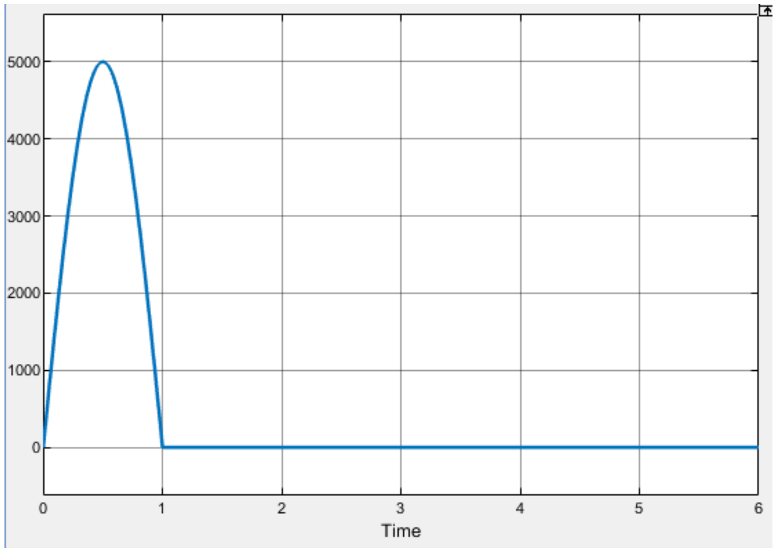

Figure 5.13 – New Input Data

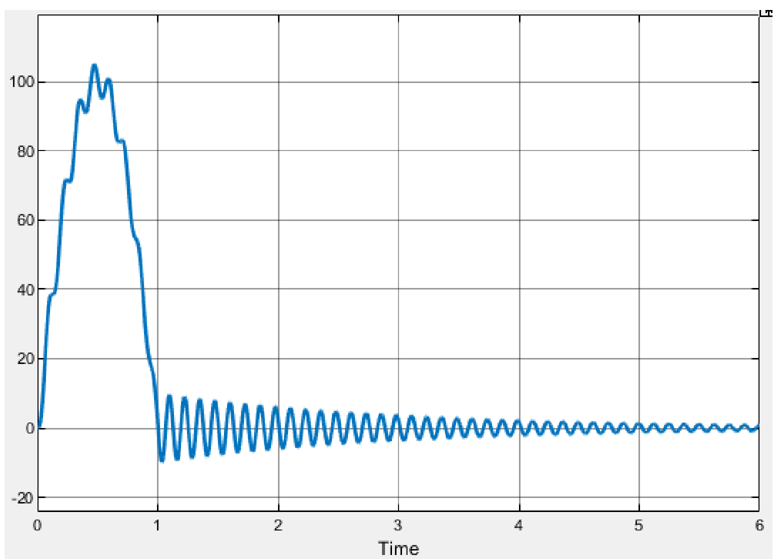

Figure 5.14 - Strain Plot Based on New Input Data

To verify the results in Simulink, MATLAB discrete convolution was used to apply the new input, Equation 5.4, with the determined impulse, Equation 5.3. Two vectors needed to be created of equal length in order to convolve them. In Figure 5.16, the MATLAB code is provided. Figure 5.15 is the output plot which matches Figure 5.14, giving a verification of the Simulink model.

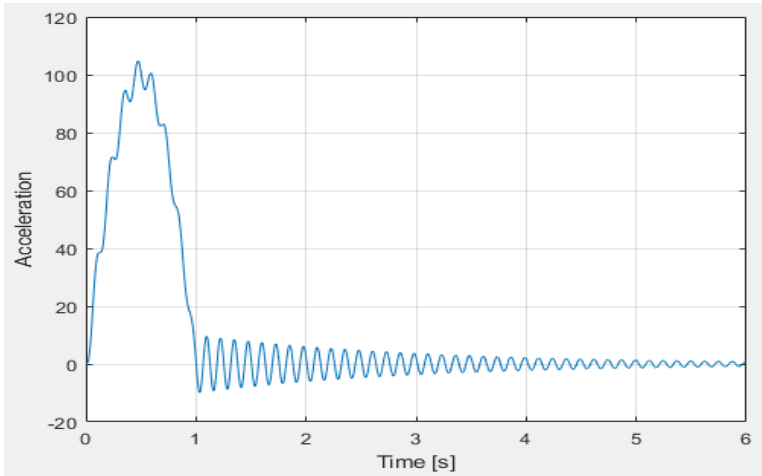

Figure 5.15 – MATLAB Plot of Acceleration with New Input

```
clearvars;
close;
%sample time is 6 seconds
t_final=6;
numpts = 2000;
t_sample = t_final/(numpts-1);
t = linspace(0,t_final,numpts);

%Impulse Response equation
h = exp(-0.5.*t).*sin(50.*t);

t_on = t(1:0.1667*numpts);
%New Input equation, limited to 1 second
u(1:0.1667*numpts)=5000.*sin(pi.*t_on);
zero = zeros(1,numpts);
u = [u zero];
%Adds zeroes to end of the sin impluse
u = u(1:numpts);
figure (1)
plot(t,u);

%Convolve the new Impluse and Response
y = conv(h,u)*t_sample;
y = y(1:numpts);
figure (2)
plot(t,y);
xlabel ("Time [s]");
ylabel ("Acceleration");
grid on;
```

Figure 5.16 – MATLAB Convolution Code

One important thing to note based on the response from the new input was the drastically reduced response amplitude of the structure after the load has passed in relation to the load itself. Without the sharp sawtooth input, which assumes 3,000 units of force is removed from the node in an instant, this more realistically assumes a gradual removal of the load, allowing for the structure to stabilize before the load is removed. This highlights why so called “impact” loading is a concern in bridge design. Impact loads occur as a high speed, high mass load meets the ends of the structure and impart a load [6]. Changing the phase of the new input as to reduce the travel time over the node from one second to 0.2 second, refer to Figure 5.17, demonstrates an increase in the relative amplitude of the post loading vibrations.

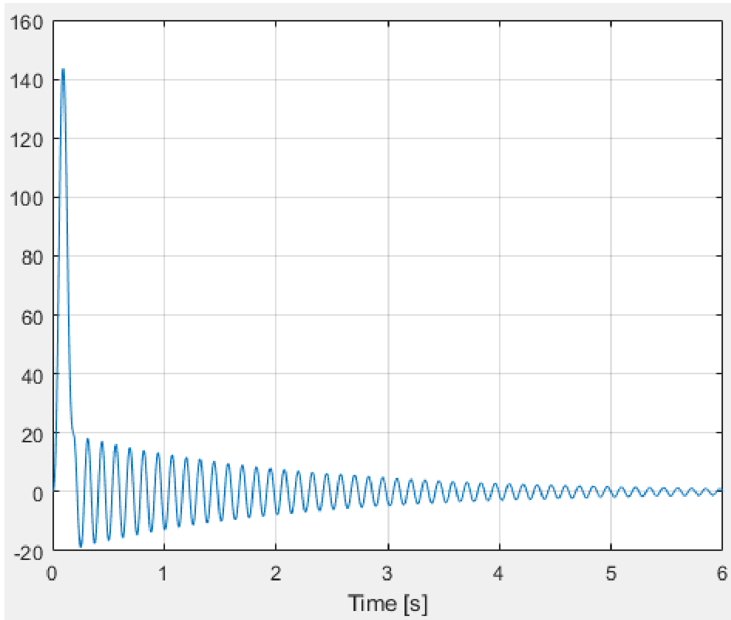

Figure 5.17 – Strain Plot with Sinusoidal Load Over 0.2 Second.

To evaluate how these differing methods of input would affect the traveling public, the strain gauge information has to be converted to an acceleration. The human body cannot perceive movement in terms of displacement or velocity without visual cues [6]. Without knowing the precise location of the strain gauge from the neutral axis of the composite beam and deck, or what scale the data is being reported at, the best that could be done is to derive a unitless acceleration from the strain gauge data. This was accomplished using the Simulink model and a series of derivative blocks. Velocity could also be found by using a single derivation of the resulting system data, but this information is not reported for brevity.

Figure 5.19 is a plot of the acceleration from the provided system data. As expected, a little to no acceleration was found as the load is applied linearly, then peak acceleration as the load is removed and the system is left to vibrate to equilibrium. Figure 5.20 is a plot of the sinusoidal loading. Notice the vibrations begin immediately, settle somewhat while the load is at the node, then begin again as the load is removed. This is again as expected. Figure 5.18 is the Simulink diagram showing the placement of the differentiators.

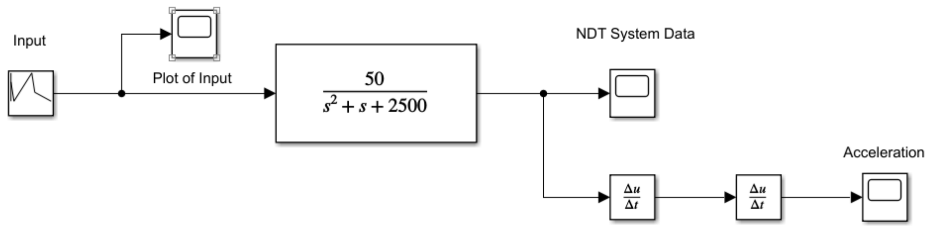

Figure 5.18 – Simulink Diagram

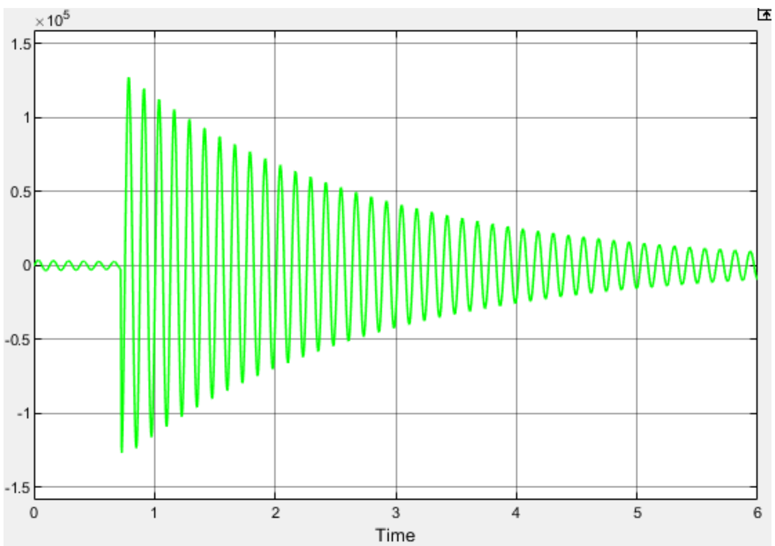

Figure 5.19 – Acceleration due to Sawtooth Loading

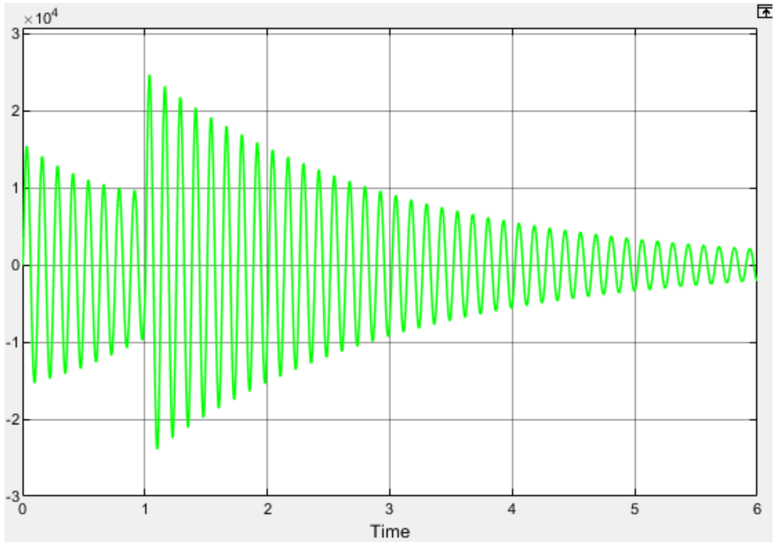

Figure 5.20 – Acceleration due to Sine Loading

5.5 Improving System Response

To address the long settling time, which is beyond the 6 seconds that the system measured, the damping of the system had to be increased. Rather than pick a new damping ratio at random, the British Standard (BD 37/01, Table 21) was used and it recommends a damping ratio of a composite bridge of no more than .04 [7]. A Bode plot, Figure 5.21, was used to determine the damping ratio.

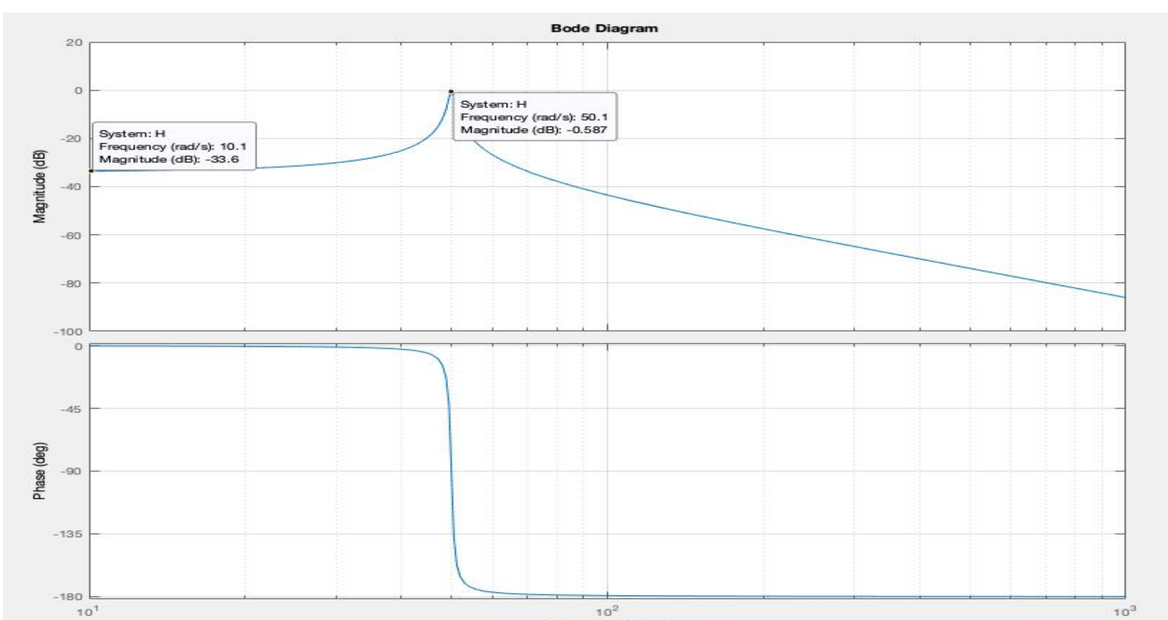

Figure 5.21 - Bode Diagram of System

Using the bode diagram above the damping ratio was found to be 0.011, which is very low.

Hdb(0)=-33.6   Hdb(12)=-0.587   ΔHdb= -0.587-(-33.6) = 33 db

Covert to rad/s $10^{\frac{33}{20}} = 44 \text{ rad/s}$

$$\zeta = \frac{1}{2*44} = 0.011 \quad (5.4)$$

The calculated damping ratio, Equation 5.4, is 0.01, which means the bridge in this study needs to have its damping ratio increased by four times its current ratio. Knowing the denominator in the transfer function is the characteristic Equation 5.5, and the damping ratio, Equation 5.6, we could determine what the new transfer function, Equation 5.7, needed to be. Below are the calculations for the revised increasing the dampening ratio and adjusted transfer function for the system.

There are a few options for retro-fits on the bridge that could stiffen the bridge (essentially damping its frequency). The two most reasonable solutions are adding stiffeners which will create a more rigid cross section or add cover plates to the bottom flange which will increase the moment of inertia increasing the natural frequency. There are options as well for increasing the damping coefficient. Among these are changing the type and material properties of the bearings the beams are sitting on and installing tuned mass damper systems on the beams [6]. Rather than increasing the stiffness of the structure, which would adversely impact the fatigue capacity of the connection details, it was decided to change the damping in the model. Additional testing of the bridge would have to be conducted to determine which method of reducing the damping ratio to an acceptable level would be best.

$$p^2 + 2\sigma p + {\omega_n}^2 = p^2 + a_1 p + a_0 \quad (5.5)$$

$$\zeta = \frac{\sigma}{\omega_n} = \frac{\frac{a_1}{2}}{\sqrt{2500}} = 0.04 \qquad (5.6)$$

$$H(t) = \frac{50}{s^2+4s+2500} \qquad (5.7)$$

Simulating the revised equation showed a significant improvement in settling time. Figure 5.22 shows the original sawtooth input run through the new transfer function in Simulink. Notice the structure now settles roughly at the four second mark, whereas it continued to vibrate off the graph prior to the additional damping.

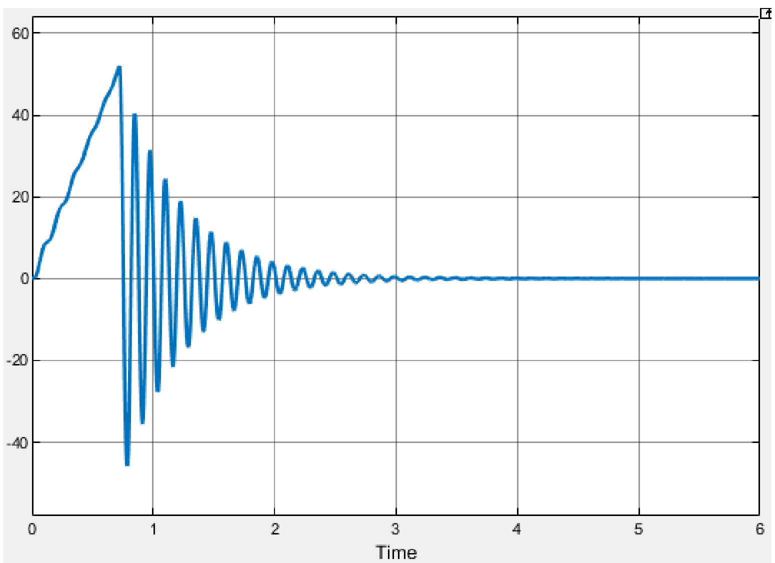

Figure 5.22 - Adjusted Dampening Results

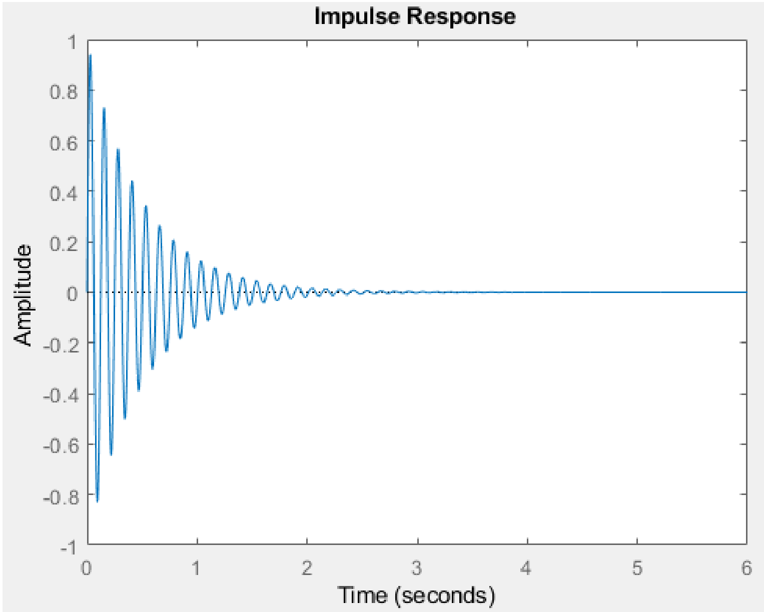

Figure 5.23 - Adjusted Dampening Impulse Response

The improvements to the results can also be shown in the acceleration plot Figure 5.24.

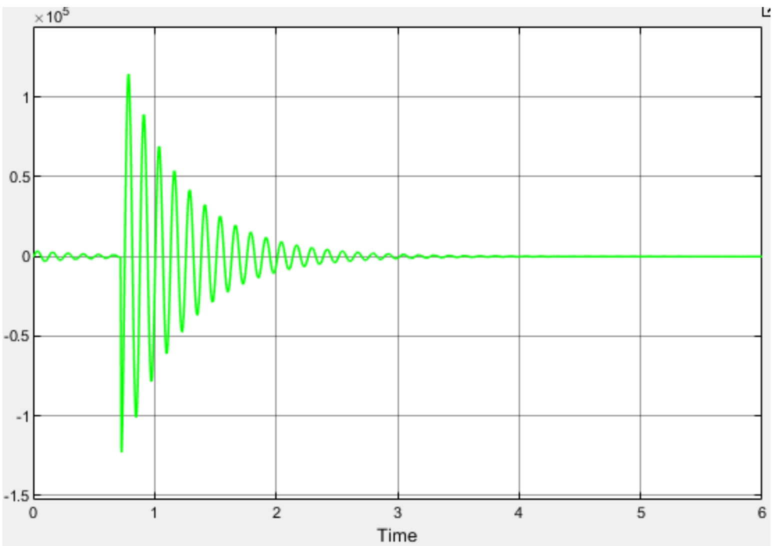

Figure 5.24 – Damped System Acceleration

5.6 Conclusion:

The purpose of this thesis was to determine a mathematical model, based on NDT system output and a known input, for the response of the system. This is done to prove that model could be used to estimate the response of any load vector we could think to apply, and to use that response to find important data like deflection, velocity, and displacement. Using input data and system output based on the real-world testing of I-80E over Interstate I-287N in New Jersey, it was possible to determine an impulse response model for the system. Using that model, it was possible to convolve a new system output using a new input function converted into a vector. This showed how different load cases affected the structure. Seeing the long settling times, it was determined that the dynamical behavior of the bridge could be improved by using tuned mass dampers. The specifications of the dampers like the mass, stiffness of the spring elements and required damping capability can be estimated by the computational method presented in this thesis given additional system properties. Existing structural guidelines were used to improve on the structure’s impulse response, then it was re-introduced to an original input to show a greatly improved settling time. Based on these findings, the impulse response model is a success. The most important factors affecting dynamic response are the basic flexibility of the structure and, more specifically, the relationship between the natural frequency of the structure and the exciting frequency of the vehicle.

Recommendations

The results of the study provided a comparatively simple numerical process of representing moving loads within the framework of a commercial finite element code and demonstrated the usage of this process for evaluating the consequences of particular loadings a bridge might undergo. An understanding of the parameters that take up a significant part in bridge response and their relative importance should help designers of bridges in developing new methods that may minimize unwanted response features. Some of the issues that would require further consideration would be to come up with a procedure for finding the response of a bridge to an actual moving vehicle that includes the characteristics of such vehicle as suspension stiffness and shock absorber damping as well as developing a procedure for including parameters such as pavement roughness, which was not considered in this study [2].

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