SPEED SENSOR-LESS CONTROL OF INDUCTION MACHINE
BASED ON CARRIER SIGNAL INJECTION AND
SMOOTH-AIR-GAP INDUCTION MACHINE MODEL

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Abstract

The standard induction machine model will lose its observability at DC excitation, so the rotor speed can not be estimated if only based on the fundamental frequency variables. Many speed estimation methods that are still effective at DC excitation either use second order effects or require modification of the rotor structure of the induction machine. This thesis presents one speed estimation scheme that can work at fundamental DC excitation based on the standard smooth-air-gap induction machine model and carrier signal injection.

The carrier signals used for speed estimation are selected to rotate in the opposite direction of the fundamental frequency signals at a sufficiently high frequency, so even if the fundamental exciting frequency is zero, the rotor speed can still be estimated based on the injected carrier signals.

In the stator flux reference frame, the locus of steady-state stator currents as a function of rotor speed is a circle. Using the difference between the stator current and the center of this locus as an auxiliary vector, we can define a correction term for the rotor speed as the cross product of the vector based on measured stator current, which is related with the actual rotor speed, and the estimated stator current vector, which is related to the estimated rotor speed. The stability of the scheme is analyzed using the two-time-scale method and classic control stability theory. This estimation is implemented in the stator flux reference frame of the carrier frequency. The estimated rotor speed is then used in the torque controller, which is at fundamental frequency.
Simulation and experiments are carried out on a 3-phase, 4-pole induction machine rated at 1.5 HP, 60 Hz, 230 V line-line, and 4.7 A to verify the feasibility of the scheme.

The carrier signal will tend to cause torque ripple. The magnitude of the carrier signal can be selected relatively small compared to the fundamental frequency signals to minimize the ratio of the torque ripple to the rated torque. However, one can use different methods to reduce or even eliminate this torque ripple. Experimental results are given to illustrate these ideas.
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Preface

To

My parents who care and support me the most
Chapter 1

Notation and Smooth-air-gap Induction Machine Model

This chapter provides the mathematical notation that is used throughout the thesis and the standard smooth-air-gap induction machine model.

1.1 Notation

The variables used throughout the thesis are presented in Table 1.1.

The induction machine used in the experiment is 3-phase wye-connected. For control convenience, we use the standard conversion (see Appendix A) to transform the three-phase model to a two-phase model. In the two-phase model, the motor has two sinusoidally-distributed windings (denoted as direct windings and quadrature windings) which are oriented 90 degrees apart. The two-phase model can be in different rotating reference frames, such as the stationary stator reference frame, the electrical stator flux-linkage reference frame and so on. The electrical variables (here we use flux linkage as one example) will be written in the form [15]:

\[
\begin{bmatrix}
\lambda_{xx}^y \\
\lambda_{yd}^y \\
\lambda_{yq}^y
\end{bmatrix}
\] (1.1)

The subscript \(y\) will be either ‘s’ for a stator quantity or ‘r’ for a rotor quantity. The superscript \(xx\) represents the reference frame of the electrical variable. The common
reference frames are listed in Table 1.2 [15], where $\rho$ is the angle of the direct axis of the chosen reference frame to the direct axis of the stator, $\theta_r$ is the angle between the direct axes of the rotor and the stator, $\omega_e$ is the angular electrical frequency as seen by the stator, $\omega_r$ is the angular rotor speed, and $\omega_{re}$ is the electrical angular rotor speed.

For a P-pole induction machine, $\omega_{re} = P \omega_r$.

In the analysis of induction machines, it’s common to transform the variables from one reference frame to another. This transformation is provided in Appendix A.

1.2 The Induction Machine Model

The induction machine model is the basis for our control scheme. Here we have some assumptions:

1. Neglect high-order winding and slot harmonics
2. Assume a linear magnetics model and
3. Neglect core loss

At ordinary operating conditions, the above assumptions are appropriate. Figure 1.1 shows the cross sectional view of the smooth-air-gap induction machine. We choose the stator-flux-linkage and rotor-flux-linkage two-phase vectors as the electromagnetic state variables. With the above assumptions, in a reference frame $xx$ ($xx$ can be $ss$, $sr$, $es$, $er$ or $e$ as in Table 1.2), the state equations are given by [15]:

$$\dot{\lambda_s^{xx}} = \left(-\frac{R_s L_r}{\sigma^2} \mathbf{I} - \mathbf{J}\omega_e\right)\lambda_s^{xx} + \frac{R_s L_m}{\sigma^2}\lambda_r^{xx} + \vec{u}_s^{xx}$$ (1.2)
\[ \dot{\lambda}_{xx}^r = \frac{R_r L_m}{\sigma^2} \dot{\lambda}_{xx}^s - \frac{R_r L_s}{\sigma^2} \dot{\lambda}_{xx}^r + (\omega_{r'e} - \omega_x) J \dot{\lambda}_{xx}^r \] (1.3)

where \( \omega_x \) is \( \rho \) in Table 1.2. Vector notation for a variable \( \bar{y} \) is as follows:

\[ \bar{y} = \begin{bmatrix} y_d \\ y_q \end{bmatrix}, \] (1.4)

the orthogonal rotation matrix \( J \) is as:

\[ J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \] (1.5)

The flux-linkage/current relations of the machine are given by:

\[ \lambda_{xx}^s = L_{s1} \bar{y}_{s} + L_{m1} \bar{y}_{r} \] (1.6)

\[ \lambda_{xx}^r = L_{r1} \bar{y}_{r} + L_{m1} \bar{y}_{s} \] (1.7)
The electromagnetic torque of a three-phase, P-pole machine is given by:

\[
\tau_e = 3P \frac{\tau_{xx}}{4} T \bar{J}_{\lambda_{s}^{xx}}
\]

\[
= 3P \frac{L_m}{\sigma^2} \bar{\lambda}_{s}^{xx} T \bar{J}_{\lambda_{r}^{xx}}
\quad (1.8)
\]

The mechanical dynamics of the machine are given by:

\[
\dot{\omega}_r = \frac{1}{H} [(\tau_e - \tau_l) - B \omega_r]
\quad (1.9)
\]

This is a standard two-phase smooth-air-gap induction machine model. Hofmann and Sanders’ paper [15] analyzed the **observability** of the smooth-air-gap induction machine model using a two-time-scale approach and showed that this model is observable for all operating points except for DC excitation.
**Electrical Variables**

\[
\begin{align*}
\vec{\lambda}_s &= \begin{bmatrix} \lambda_{sd} ; \lambda_{sq} \end{bmatrix}^T \quad \text{Stator Flux-linkage Vector} \\
\vec{\lambda}_r &= \begin{bmatrix} \lambda_{rd} ; \lambda_{rq} \end{bmatrix}^T \quad \text{Rotor Flux-linkage Vector} \\
\vec{i}_s &= \begin{bmatrix} i_{sd} ; i_{sq} \end{bmatrix}^T \quad \text{Stator Current Vector} \\
\vec{u}_s &= \begin{bmatrix} u_{sd} ; u_{sq} \end{bmatrix}^T \quad \text{Stator Voltage Vector} \\
\omega_e &= \text{Electrical Frequency} \\
R_s &= \text{Stator Resistance} \\
R_r &= \text{Rotor Resistance} \\
L_s &= \text{Stator Inductance} \\
L_r &= \text{Rotor Inductance} \\
L_m &= \text{Mutual Inductance} \\
T_r &= L_r/R_r \quad \text{Rotor Time Constant} \\
\sigma^2 &= L_s L_r - L_m^2 \quad \text{Leakage Term} \\
L_{ls} &= L_s - L_m \quad \text{Stator Leakage Inductance} \\
L_{lr} &= L_r - L_m \quad \text{Rotor Leakage Inductance} \\
P &= \text{Number of poles}
\end{align*}
\]

**Mechanical Variables**

\[
\begin{align*}
\omega_r &= \text{Rotor Angular Velocity} \\
\omega_{re} &= \text{Rotor Electrical Angular Velocity} \\
\omega_s &= \omega_r - \omega_{re} \quad \text{Slip Frequency} \\
\tau_e &= 3P L_m^2 J \vec{\lambda}_r \quad \text{Electromagnetic Torque} \\
\tau_l &= \text{Load Torque} \\
H &= \text{Moment of Inertia of Rotor and Load} \\
B &= \text{Mechanical Damping Constant of Rotor}
\end{align*}
\]

**Matrix Notation**

\[
\begin{align*}
I &= 2 \times 2 \text{ Identity Matrix} \\
J &= \text{Orthogonal Rotation Matrix} \\
0_{m \times n} &= \text{Matrix of zeros with } m \text{ rows and } n \text{ columns}
\end{align*}
\]

Table 1.1. Notation of the induction machine.
Reference Frame | Superscript | \( \rho \) | \( \dot{\rho} \) |
--- | --- | --- | --- |
Stationary Stator | ss | 0 | 0 |
Stationary Rotor | sr | \( \theta_r \) | \( \omega_{re} \) |
Electrical Stator Flux-linkage | es | \( \angle \lambda_s^{ss} = \tan^{-1}\left( \frac{\lambda_{sq}^{ss}}{\lambda_{sd}^{ss}} \right) \) | \( \omega_e \) |
Electrical Rotor Flux-linkage | er | \( \angle \lambda_r^{ss} = \tan^{-1}\left( \frac{\lambda_{rq}^{ss}}{\lambda_{rd}^{ss}} \right) \) | \( \omega_e \) |
Arbitrary Electrical Frame | e | | \( \omega_e \) |

Table 1.2. Superscripts and their corresponding reference frames
Fig. 1.1. The cross sectional view of a two-phase smooth-air-gap induction machine model.
Chapter 2

Introduction of Speed Estimation Techniques

Various techniques have been used to estimate rotor speed, rotor angle and flux linkages. In this chapter, first we will describe the details of these techniques [2, 9, 13, 19, 22, 23, 46, 53, 55], then classify them and show their disadvantages. Finally, we will talk about the difference between these methods and the proposed method.

2.1 Open-loop speed estimators

In this section, the rotor speed and slip frequency estimators are obtained by considering the voltage equations of the induction machine. No feedback is used to check the correctness of the estimation. From the literature, we have different rotor speed estimators listed below.

2.1.1 Rotor speed estimation scheme 1

The equations of this scheme are given in [5, 17, 18, 28, 55]:

\[
\omega_{re} = \frac{-\lambda_{rd} - \frac{\lambda_{rd}}{R_{r}} + L_{m} i_{sd}}{\lambda_{rq}}
\]  

(2.1)
\[
\dot{\lambda}_r = \frac{L_r}{T_m} \left( \bar{u}_s - R_s \bar{i}_s - L_s^r \bar{\bar{i}}_s \right) \tag{2.2}
\]

The block diagram is shown in Figure 2.1.

Fig. 2.1. Rotor speed estimator using scheme 1.

2.1.2 Rotor speed estimation scheme 2

The equations of this scheme are given by [5, 17, 18, 55]:

\[
\omega_{TE} = -\frac{u_{sd} - (R_s + \frac{L_s}{T_p}) i_{sd} - L_s^r \dot{i}_{rd} + \dot{\lambda}_{sd}}{\lambda_{sq} - L_s^r \dot{i}_{sq}} \tag{2.3}
\]
\[
\dot{\lambda}_S = \ddot{u}_S - \ddot{i}_s R_s - K \dot{\lambda}_S
\]  
(2.4)

where $K$ is a decay constant.

The block diagram is shown in Figure 2.2. The stator flux-linkage estimator is based on (2.4).

![Block diagram](image)

**Fig. 2.2.** Rotor speed estimator using scheme 2.

### 2.1.3 Rotor speed estimation scheme 3

The equations of this scheme are given by [5, 17, 18, 28, 55]:

\[
\dot{\lambda}_S = \ddot{u}_S - \ddot{i}_s R_s - K \dot{\lambda}_S
\]
\[ \omega_{rc} = \frac{u_{sq}^*}{||\lambda_s|| - L_s' i_{sd}} \] (2.5)

\[ u_{sd}^* + j u_{sq}^* = e^{-j \rho_s} \left[ \vec{u}_s - \left( R_s + \frac{L_s}{T_r} \right) \vec{i}_s - L_s' \vec{i}_s' \right] \] (2.6)

\[ i_{sd} + j i_{sq} = e^{-j \rho_s \vec{i}_s} \] (2.7)

where \( ||\lambda_s|| \) is the magnitude of the stator flux-linkage, \( \rho_s \) is the angle of the stator flux-linkage vector in the stator reference frame and \( i_{sd} \) is the real part of the stator current in the stator flux-linkage reference frame.

The block diagram is shown in Figure 2.3.

### 2.1.4 Rotor speed estimation scheme 4

The equations of this scheme are given by [5, 17, 18, 28, 55]:

\[ \omega_{rc} = \frac{\lambda_{rd} \lambda_{rq} - \lambda_{rq} \lambda_{rd}}{||\lambda_r||^2} - \frac{L_m}{T_r ||\lambda_r||^2} \left( -\lambda_{rq} i_{sd} + \lambda_{rd} i_{sq} \right) \] (2.8)

The block diagram is shown in Figure 2.4.
2.1.5 Rotor speed estimation scheme 5

The equations of this scheme are given by [17, 18, 55]:

\[
\omega_{re} = \frac{\dot{\lambda}_{rq}\lambda_{sd}-\dot{\lambda}_{rd}\lambda_{sq}}{\lambda_{rd}\lambda_{sd}+\lambda_{rq}\lambda_{sq}}
\]  

(2.9)

where

\[
\dot{\lambda}'_s = \ddot{\lambda}_s - L_s \tilde{i}_s
\]  

(2.10)

\[
\ddot{\lambda}_s = \ddot{u}_s - R_s \dot{i}_s - K\lambda_s
\]  

(2.11)

\[
\ddot{\lambda}_r = \frac{L_p}{L_m} \left( \ddot{\lambda}_s - L'_s \ddot{i}_s \right)
\]  

(2.12)

where \( K \) is a decay constant.

The block diagram is shown in Figure 2.5.
2.1.6 Summary

These open-loop speed estimators are simple and easy to implement. They are based on the fundamental induction machine model. Their advantages and disadvantages will be discussed at the end of this chapter.

2.2 Estimators using spatial saturation third-harmonic voltage

2.2.1 General description of the algorithm

In a symmetrical three-phase induction motor with stator windings without a neutral connection, the sum of the stator voltages, which is the third-harmonic zero-sequence stator voltage modulated by the high frequency slot harmonics ($\bar{u}_{s0} = \bar{u}_{s3} + \bar{u}_{sh}$), is monitored. Due to stator and rotor teeth saturation, which is normal in a standard motor, a spatial saturation third-harmonic voltage is generated. The third-harmonic flux-linkage can be obtained by integrating the third-harmonic voltage. Through a saturation function, which is obtained experimentally by performing the conventional no-load test, the fundamental component of the magnetizing flux-linkage is then determined. Through the monitored currents, one can obtain the angle between the stator current space vector and the stationary reference frame, and also between the stator current space vector and the magnetizing flux linkage, and therefore the angle between the magnetizing flux linkage and the direct-axis of the stationary reference frame can be obtained. One can therefore obtain the rotor and stator flux linkage, and also the rotor speed [18, 24, 25, 30, 55, 57].

To use this scheme, access to the neutral point of the stator windings is required.
The equations are given by:

\[
\int \tilde{u}_{s0} dt = \int (\tilde{u}_{s3} + \tilde{u}_{sh}) dt \\
= \int \tilde{u}_{s3} dt \\
= \tilde{\lambda}_{m3}
\]  

(2.13)

where \(\tilde{u}_{sh}\) is filtered by a low-pass filter with high cut-off frequency, and \(\tilde{\lambda}_{m3}\) is the third-harmonic magnetizing flux-linkage. From its magnitude and the saturation function, we can obtain the magnitude of the fundamental frequency magnetizing flux-linkage. After the fundamental frequency magnetizing flux-linkage is obtained, we can just follow the descriptions in section 2.1 and get the estimation of the stator flux linkage, rotor flux linkage, and also the rotor speed.

2.3 Estimators using saliency (geometrical, saturation) effects

There are different types of geometrical effects, (e.g., normal slotting, inherent air-gap asymmetry, intentional rotor magnetic saliency created by spatial modulation of rotor-slot leakage inductance, etc.), that can be exploited to estimate rotor speed. These geometrical saliency effects or saliency effects created by saturation can be used to estimate the rotor speed, rotor position, and various flux linkages of a squirrel-cage induction machine.

2.3.1 Estimators using rotor slot harmonics

The rotor slot harmonics can be detected by using two different techniques:
• Utilizing stator voltages

• Utilizing stator currents

2.3.1.1 Estimate rotor speed using stator voltages [1, 4, 55]

The space harmonics of the air-gap flux-linkage in a symmetrical three-phase induction motor are generated because of the non-sinusoidal distribution of the stator windings and the variation of the reluctance due to stator and rotor slots, which are called m.m.f. space harmonics, stator slot harmonics, and rotor slot harmonics, respectively. The rotor slot harmonics can be utilized to determine the rotor speed of induction machines.

When the air-gap m.m.f. contains slot harmonics, slot-harmonic voltages are induced in the primary windings when the rotor rotates. The magnitude and the frequency of the slot-harmonic voltages depends on the rotor speed, so they can be utilized to estimate the slip frequency and rotor speed. Generally we only use the frequency of the slot-harmonic voltages since the magnitude depends not only on the rotor speed, but also on the magnitude of the flux-linkage level and the loading conditions.

It can be proven that if the stator voltages of the induction machine ($\bar{u}_{sA}$, $\bar{u}_{sB}$ and $\bar{u}_{sC}$) are added, and if the m.m.f. distribution is assumed to be sinusoidal, then the resulting stator voltage $\bar{u}_{s0} = \bar{u}_{sA} + \bar{u}_{sB} + \bar{u}_{sC}$ will contain the rotor slot harmonic voltages ($\bar{u}_{sh}$). Due to main flux saturation, it will also contain a third-harmonic component $\bar{u}_{s3}$, and if an inverter supplies power to the induction machine, extra time-harmonic voltages, $\bar{u}_{shk}$, will be present as well. In general,
The frequency of the dominant component (fundamental slot-harmonic frequency) of the slot harmonic voltages is given by [54]:

\[ f_{sh} = N_r f_r \pm f_1 \]

\[ = 3N f_1 - N_r f_{sl} \quad N_r = 3N \pm 1 \]

\[ = \left[ \frac{Z_r (1-s)}{P} \pm 1 \right] f_1 \]  

where:

- \( f_{sh} \) is the fundamental slot-harmonic frequency;
- \( f_r \) is the rotor rotational frequency;
- \( f_{sl} \) is the slip frequency;
- \( s \) is the slip;
- \( f_1 \) is the stator electrical frequency;
- \( N_r \) is the number of rotor slots per pole-pair;
- \( Z_r \) is the number of rotor slots;
- \( P \) is the number of pole-pairs.
\( \vec{u}_{s0} \) can be measured as shown in Figure 2.6 or Figure 2.7, or by measuring the three phase voltages separately and adding them together in the controller. Using various circuits [54], the voltage components \( \vec{u}_{s3} \) and \( \vec{u}_{shk} \) can be removed from \( \vec{u}_{s0} \). From equation (2.14), we get \( \vec{u}_{sh} \) and also its frequency. Then from equation (2.15), the rotor speed can be obtained by:

\[
\omega_r = 2\pi \frac{f_{sh} \pm f_1}{N_r P} \tag{2.16}
\]

Because at low speeds the magnitude of the slot-harmonic voltage decreases, special considerations are required in the low speed range.

Estimate the rotor speed by monitoring the stator voltages is not as preferred as by monitoring the stator currents, since it’s always necessary to monitor the stator currents in a high-performance induction machine control system and, if we can estimate the rotor speed only from the stator currents, we can reduce the number of sensors required.

### 2.3.1.2 Estimate rotor speed using stator currents [39, 40, 55]

The basic steps to estimate rotor speed using stator currents are almost the same with those using stator voltages. A Fast Fourier Transform (FFT) is used to detect the slot-harmonic frequency \( f_{sh} \), and \( f_1 \) can be obtained from the angle of the rotor flux linkage vector. So from equation (2.16), the rotor speed can be estimated [55].
2.3.2 Estimators using saliency introduced by special rotor construction and carrier signal injection [20, 34, 55]

To estimate the rotor speed at low or even zero stator frequency, some techniques use special rotor constructions. For example, periodically varying the rotor-slot opening widths or varying the depths of the rotor-slot openings, is done to create a physical saliency in the rotor structure. When special (asymmetrical) rotor constructions are used, the stator transient inductances due to asymmetry are position dependent. When the stator windings are excited by carrier-frequency voltages, position-dependent currents are generated. By measuring these stator currents, we can extract the information on the rotor position and therefore the rotor speed.

2.3.3 Estimators using saturation-induced saliency with high-frequency voltages injected

In an induction machine, due to the magnetic saturation of the stator and rotor teeth, the stator inductances depend not only on the level of saturation but also on the position of the main flux, and so saliency is created and the stator direct- and quadrature-axis inductances become asymmetrical [55]. Different implementation techniques have been presented to extract the rotor speed information [10, 23, 34, 50, 55]. In [10], a scheme which allows sensor-less vector control at zero fundamental flux frequency is presented and the test results reported proved its validity.
2.4 Observers

In section 2.1, some open-loop estimators are presented. In this section, some closed-loop estimators, which contain a correction form involving an estimation error to adjust the response of the estimator, are introduced. These closed-loop estimators are referred to as observers.

Compared to open-loop estimators, observers are more robust against parameter mismatch and also signal noise.

The most commonly used observers are the Luenberger observer and the Kalman Observer. Among them, the basic Luenberger observer (LO) is applicable to a linear, time-invariant deterministic system, while the extended Luenberger observer (ELO) is applicable to a non-linear time-varying deterministic system. The basic Kalman observer (KO) is only applicable to linear stochastic systems, while the extended Kalman filter (EKF) is applicable to nonlinear stochastic systems.

2.4.1 Luenberger Observer

When an error compensator is added to the equations of the induction machine in the stationary reference frame, a full-order adaptive state observer can be constructed. The rotor speed is considered as a state variable. For example, the dynamic equations can be given by [12, 35, 36, 38, 55]:

\[
\begin{align*}
\dot{\tilde{\mathbf{x}}}_r &= \tilde{\mathbf{A}} \tilde{\mathbf{x}}_r + \mathbf{B} \tilde{\mathbf{u}} + \mathbf{G} \left( \tilde{i}_s - \tilde{i}_s \right) \\
\tilde{i}_s &= \mathbf{C} \tilde{\mathbf{x}}_r
\end{align*}
\] (2.17)
where

\[ \ddot{\bar{x}} = \begin{bmatrix} \dot{\bar{x}} \\ \dot{i}_s \\ \dot{\bar{\lambda}}_r \end{bmatrix} \quad (2.18) \]

\[ \dot{i}_s = [i_{sd}, i_{sq}]^T \quad (2.19) \]

\[ \ddot{\bar{\lambda}}_r = [\dot{\lambda}_{rd}, \dot{\lambda}_{rq}]^T \quad (2.20) \]

\[ \bar{u} = [\bar{u}_{sd}, \bar{u}_{sq}]^T \quad (2.21) \]

\[ \ddot{\omega}_{re} = K_p (\dot{\lambda}_{rq} e_{sd} - \dot{\lambda}_{rd} e_{sq}) + K_i \int (\dot{\lambda}_{rq} e_{sd} - \dot{\lambda}_{rd} e_{sq}) \, dt \quad (2.22) \]

\[ \ddot{e} = \dot{i}_s - \dot{i}_s \quad (2.23) \]
\[ \dot{A} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 1/T_s + (1-\sigma)T_r' \
 \end{bmatrix} I_2 & \begin{bmatrix} I_2/T_r \end{bmatrix} \left( L_m/L_sL_r' \right) \end{bmatrix} \left[ I_2/T_r - \omega_r(t)J \right] \\
 L_mI_2/T_r & -I_2/T_r + \omega_r(t)J 
 \end{bmatrix} \] (2.24)

\[ B = \begin{bmatrix} I_2/L_s \bar{u}_{sd} \\
 O_2 
 \end{bmatrix} \] (2.25)

\[ C = [I_2, O_2] \] (2.26)

\[ I_2 = \begin{bmatrix} 1 & 0 \\
 0 & 1 
 \end{bmatrix} \] (2.27)

\[ O_2 = \begin{bmatrix} 0 & 0 \\
 0 & 0 
 \end{bmatrix} \] (2.28)

\[ J = \begin{bmatrix} 0 & -1 \\
 1 & 0 
 \end{bmatrix} \] (2.29)
\begin{equation}
\sigma = 1 - L_m^2 / (L_s L_r) \tag{2.30}
\end{equation}

\( G \) is the observer gain matrix, which is selected so that the system will be stable. 

\( K_p \) and \( K_i \) are the proportional and integral gain constants respectively.

The block diagram is shown in Figure 2.8.

The basic Luenberger observer is applicable to a linear, time-invariant deterministic system.

### 2.4.2 Extended Luenberger Observer (ELO)

The extended Luenberger observer can be applied to the estimation of the states of a non-linear time-invariant system, whose state equations are given by [21, 44, 51, 52]:

\begin{equation}
\begin{align*}
\dot{x} & = f[x(t)] + Bu(t) \\
\tilde{y}(t) & = Cx(t)
\end{align*} \tag{2.31}
\end{equation}

One example of the full-order extended Luenberger observer is given by:

\begin{equation}
\dot{x}(t) = A \left[ \dot{x}(t - \tau) \right] + Bu(t) + G \left[ \dot{x}(t - \tau) \right] \left[ \tilde{y}(t) - Cx(t - \tau) \right] + g \left[ \dot{x}(t - \tau) \right] \tag{2.32}
\end{equation}
where

$$A = \frac{df}{dx}$$ (2.33)

is the system Jacobian matrix. $\tau$ is the step length or the sampling time. $G$ is the gain matrix to make the system stable. $G$ is not constant. It depends on the past estimates of the system state vector.

$$g(\bar{x}) = f(\bar{x}) - A(\bar{x})$$ (2.34)

By adjusting the gain matrix, the performances of the extended Luenberger observer (such as speed of response, speed of convergence, robustness against parameter drift, and so on.) can be altered. It’s applicable to most of the industrial systems to produce unbiased estimates.

2.4.3 Extended Kalman filter (EKF)

The extended Kalman filter is a recursive stochastic state estimator which can be used for the joint state and parameter estimation of a nonlinear dynamic system in real time by using noisy monitored signals that are disturbed by random noise [8, 11, 31, 42, 55, 56]. In EKF, the rotor speed is considered as a state variable.

With the measured stator voltages (or the DC link voltage) and stator currents, the states are firstly predicted using a mathematical model of the induction machine, then the weighted difference between the estimated and measured states is added to the next estimation, and so on, the states are estimated recursively. The structure of EKF
is given by Figure 2.9, where \( v \) is the noise vector of the states, \( w \) is the noise in the measured stator currents. We assume both \( v \) and \( w \) are zero-mean, white Gaussian.

**EKF** can be used under both steady-state and transient conditions of the induction machine for the estimation of rotor speed. With **EKF**, rotor speed can be estimated in a very wide range, down to very low speed, but not zero speed. **EKF** has some inherent disadvantages. When the noise content of the system and associated measurements are too low, **EKF** will be difficult to use. Also there is no means in the **EKF** design and implementation which can be utilized to tune its dynamic performance without affecting its steady-state accuracy. The computational burden of **EKF** is very heavy.

### 2.5 Estimation with artificial intelligence

Different from mathematical-model-based analysis techniques, artificial-intelligence-based techniques, such as artificial neural networks (ANN), fuzzy-logic systems, fuzzy-neural networks, etc., do not require a precise analytical expression of the machine and drive system. Moreover, they have the advantages of fast parallel computation, immunity from input harmonic ripple, and fault tolerance characteristics [7, 29, 47, 55].

One example of the neural network speed estimator is shown in Figure 2.10 [29].

Neural network estimator can take the role as the adaptive model in a Model Reference Adaptive System (MRAS), or directly generate a rotor speed estimate as its output. Artificial-intelligence-based speed estimation techniques can obtain a speed estimation that is not based on the mathematical model of the controlled system. It is shown that this algorithm can work in a wide speed range and has good dynamic performance and stability [29, 47]. It is believed that this type of approach will find
increasing application in the future. But it needs to be trained or has the knowledge base to understand the model of a plant or a process. The training algorithm decides the learning speed, the stability and the dynamic performance of the system. This method is also computationally intensive.

2.6 Model Reference Adaptive Systems (MRAS)

Tamai et al. [48] described one speed estimation technique based on the Model Reference Adaptive System (MRAS) in 1987. Later Schauder [49] presented an alternative MRAS scheme which is less complex and more effective. The Model Reference Adaptive Systems (MRAS) approach uses two models. The model that does not involve the quantity to be estimated (the rotor speed $\omega_{re}$ in our case) is considered as the reference model. The model that has the quantity to be estimated involved is considered as the adaptive model (or adjustable model). The output of the adaptive model is compared with that of the reference model, and the difference is used to drive a suitable adaptive mechanism whose output is the quantity to be estimated (rotor speed in our case). The adaptive mechanism should be designed to assure the stability of the control system. Figure 2.11 illustrates the basic structure of MRAS [6, 37, 48, 49, 58].

Different approaches have been developed using MRAS, such as rotor-flux-linkage-estimation-based MRAS, back-emf-based MRAS, reactive-power-based MRAS, artificial-intelligence-based MRAS, etc. In the following we will give a basic description of these schemes.
2.6.1 MRAS based on rotor flux-linkage estimation

The reference model is given by [49]:

\[
\dot{\lambda}_r = \frac{L_r}{L_m} \left( \ddot{u}_s - R_s \dot{i}_s - L_s \dot{i}_s \right) \tag{2.35}
\]

The adjustable model is given by:

\[
\dot{\lambda}_{rd} = \frac{1}{T_r} \left( L_m \dot{i}_{sd} - \dot{\lambda}_{rd} - \omega_r T_r \dot{\lambda}_{rq} \right) \tag{2.36}
\]

\[
\dot{\lambda}_{rq} = \frac{1}{T_r} \left( L_m \dot{i}_{sq} - \dot{\lambda}_{rq} - \omega_r T_r \dot{\lambda}_{rd} \right) \tag{2.37}
\]

The adaptive mechanism is given by:

\[
\dot{\omega}_{re} = \left( K_p + \frac{K_i}{T_p} \right) \left( \lambda_{rq} \dot{\lambda}_{rd} - \lambda_{rd} \dot{\lambda}_{rq} \right) \tag{2.38}
\]

Figure 2.12 illustrates the structure of this scheme.

The presence of the pure integrators brings the problems of initial conditions and drift. In [49], a low pass filter was used to replace the pure integrator, but the
performance in the low speed range is not satisfying, for reasons which will be explained later.

2.6.2 MRAS based on back emf estimation

The reference model is given by [46]:

\[
\vec{e} = \vec{u}_s - R_s \vec{i}_s - L_s \frac{d}{ds} \vec{i}_s
\]  

(2.39)

The adjustable model is given by [46, 55]:

\[
\dot{\hat{e}}_d = \frac{L_m}{L_r} \frac{L_n i_{sd} - \dot{\hat{\lambda}}_{rd} - \omega_r T_r \dot{\hat{\lambda}}_{rq}}{T_r}
\]  

(2.40)

\[
\dot{\hat{e}}_q = \frac{L_m}{L_r} \frac{L_n i_{sq} - \dot{\hat{\lambda}}_{rq} - \omega_r T_r \dot{\hat{\lambda}}_{rd}}{T_r}
\]  

(2.41)

The adaptive mechanism is given by [46, 55]:

\[
\dot{\omega}_{re} = \left( K_p + \frac{K_i}{p} \right) \left( e_q \dot{e}_d - e_d \dot{e}_q \right)
\]  

(2.42)

Figure 2.13 illustrates the structure of this scheme.
This scheme does not have pure integrators in the reference model.

### 2.6.3 MRAS based on reactive power estimation

The reference model is given by [37]:

\[
q_m = \hat{i}_s^T J \left( \hat{u}_s - \sigma L_s \hat{i}_s \right)
\]

(2.43)

The adjustable model is given by [55]:

\[
\hat{q}_m = L_m' \left( \left( \hat{i}_m^T \hat{i}_s \right) \hat{\omega}_r + \frac{1}{L_r} \hat{i}_m^T J \hat{i}_s \right)
\]

(2.44)

The adaptive mechanism is given by [37, 55]:

\[
\hat{\omega}_{re} = \left( K_p + \frac{K_i}{p} \right) (q_m - \hat{q}_m)
\]

(2.45)

Figure 2.14 illustrates the structure of this scheme.

### 2.6.4 MRAS based on artificial intelligence [26, 27, 55]

The reference model is given by:
\[ \hat{\lambda}_r = \frac{L_r}{T_m} \left( \bar{u}_s - R_s i_s - L_s \dot{i}_s \right) \] (2.46)

The adjustable model is given by:

\[ \hat{\lambda}_{rd}(k) = w_1 \hat{\lambda}_{rd}(k-1) - w_2 \hat{\lambda}_{rq}(k-1) + w_3 i_{sd}(k-1) \] (2.47)

\[ \hat{\lambda}_{rq}(k) = w_1 \hat{\lambda}_{rq}(k-1) + w_2 \hat{\lambda}_{rd}(k-1) + w_3 i_{sq}(k-1) \] (2.48)

where

\[ w_1 = 1 - c \] (2.49)

\[ w_2 = \omega_r T \] (2.50)

\[ w_3 = cL_m \] (2.51)
\[ c = T/T_r \]  

(2.52)

T is the sampling time.

The adaptive mechanism is given by:

\[
\dot{\omega}_r (k) = \dot{\omega}_r (k - 1) + \frac{\eta}{T} \left\{ - \left[ \omega_{rd} (k) - \dot{\omega}_{rd} (k) \right] \dot{\omega}_{rq} (k - 1) \\
+ \left[ \omega_{rq} - \dot{\omega}_{rq} (k) \right] \dot{\omega}_{rd} (k - 1) \right\} + \frac{\alpha}{T} \Delta w_2 (k - 1)
\]  

(2.53)

where \( \eta \) is the learning rate, \( \alpha \) is a positive constant called the momentum constant and usually is in the range between 0.1 and 0.8.

\[
\Delta w_2 (k) = \eta \left[ -\epsilon_d (k) \dot{\omega}_{rq} (k - 1) + \epsilon_q (k - 1) \right]
\]  

(2.54)

\[
\epsilon_d (k) = \omega_{rd} (k) - \dot{\omega}_{rd} (k) \\
\epsilon_q (k) = \omega_{rq} (k) - \dot{\omega}_{rq} (k)
\]  

(2.55)

The structure of the ANN is given by Figure 2.15 and Figure 2.16 illustrates the structure of this scheme.

Some artificial intelligence schemes require an off-line supervised training stage before the ANN can be used, and this is usually a slow process. This simple one doesn’t
need a training stage. This technique should be combined with other artificial intelligence schemes.
Fig. 2.3. Rotor speed estimator using scheme 3.
Fig. 2.4. Rotor speed estimator using scheme 4.

Fig. 2.5. Rotor speed estimator using scheme 5.
Fig. 2.6. Monitoring of the zero-sequence stator voltage using three potential transformers.

Fig. 2.7. Monitoring of the zero-sequence stator voltage using three identical external resistors.
Fig. 2.8. Adaptive speed observer (Luenberger observer).

Fig. 2.9. Structure of the extended Kalman filter (EKF).
Fig. 2.10. Speed estimation with neural network.

Fig. 2.11. The basic structure of MRAS.
Fig. 2.12. MRAS based on rotor flux-linkage estimation.

Fig. 2.13. MRAS based on back emf.
Fig. 2.14. MRAS based on back emf.

Fig. 2.15. The estimation of the rotor flux linkage with ANN.
Fig. 2.16. MRAS based on ANN.
2.7 Discussion of the previous art

In this chapter, we described the main types of rotor speed estimation techniques in the literature.

Among them, the open-loop estimator, estimators using spatial third-harmonic voltage, observers, and model reference adaptive systems are based on the fundamental induction model. According to Hofmann and Sanders’ analysis [15], the observability of the induction machine vanishes at DC excitation, so these methods can not work at zero stator frequency.

Estimators using rotor slot harmonics have not been directly used for rotor speed estimation in a high-performance torque control scheme due to the measurement bandwidth limitation. It has only been used as a tool to tune the speed estimators of MRAS [55]. Estimators using the saliency introduced by special rotor construction and high-frequency voltage injection work well at low speed range, but require rotor modification, which is not preferred by the manufacturers. Estimators using saturation-induced saliency and high-frequency carrier signal injection have shown good performance at low speed or even DC excitation, but it’s based on the saturation effect, which is nonexistent at low flux levels in the machine.

Based on the information above, the only way to estimate rotor speed at zero fundamental frequency is to inject high frequency carrier signals and estimate the rotor speed based on these carrier signals. Two methods, the estimators using the saliency introduced by special rotor construction, and the estimators using saturation-induced
saliency, belong to this class, but they are not based on the fundamental induction machine model.

Sng and Lipo [33] presented a speed estimation scheme based on carrier signal injection and the fundamental smooth-air-gap induction machine model. Their technique is based on the rotor flux dynamics of the system and only works for systems with a high moment of inertia. The implementation of their technique requires numerical differentiation, which is subject to errors caused by noise. Only locked rotor experimental results are provided. No methods are given to reduce the torque ripple caused by the injected carrier signal. Based on Sng and Lipo’s work, Hinkkanen, Leppänen and Luomi [32, 41] also proposed one speed estimation method based on carrier signal injection and fundamental smooth-air-gap induction machine model. Their method is based on the mechanical dynamics of the system and assumes that the total inertia of the system is relatively low. The stability of the method is shown only experimentally. No rigorous proof is provided.

The approach pursued here is also to combine the fundamental induction machine model and the injection of high frequency carrier signals in the estimation of rotor speed. The estimation is based on the injected carrier signals, hence even if the fundamental excitation frequency is zero, we can still have the speed information contained in the induction machine model at the carrier signal frequency. To increase the accuracy and robustness of the estimation, we choose MRAS as the implementation method. This is an alternative method of that proposed by Jorma Luomi [32, 41]. But rigorous stability analysis is provided for our method, which makes our proposed method clearer in its scientific meaning.
2.8 Overview of Thesis

This thesis presents a speed estimator that is based on carrier signal injection and the smooth-air-gap induction machine model. This speed estimator can work at fundamental DC excitation. Therefore a full range speed estimator can be obtained. Based on the obtained speed information, a torque controller was developed and tested. This technique can be used on electric vehicles so that, even when the excitation is DC, a torque output can still be provided.

Chapter 3 analyzes the stability of the speed estimation scheme using a two-time-scale method and basic control stability theory. Chapter 4 provides simulation results to show the feasibility of the scheme, and discusses different possible ways to implement the scheme. Chapter 5 presents the experiment setup and the supporting experimental results. Chapter 6 compares two ways to reduce or eliminate the torque ripple. Chapter 7 gives conclusions about the thesis work, and discusses possible future work.
Chapter 3

Stability Analysis with Two Time Scale Approach and Control Stability Theory

The stability of the speed estimation scheme is analyzed in this chapter with the two time scale approach and basic control stability theory. First, the two-time-scale approach is briefly introduced, then the stability of our speed estimation scheme is analyzed using this technique. Two possible speed estimation techniques are given in this chapter along with their stability analysis. Simulation and experimental results will be provided in the next two chapters. The characteristics of the stability analysis constrain the application of this speed estimation technique to systems with high moments of inertia.

3.1 Brief Introduction of Two Time Scale Approach

For control engineers, singular perturbation methods are often used to simplify the dynamic models of complicated systems. Generally one way is to neglect "small" time constants, masses, capacitances, and similar "parasitic" parameters which increase the dynamic order of the model. The prerequisite condition is that the dynamics of the variables with "small" time constants should be stable and converge to their quasi-steady-state values quickly. If this is satisfied, the systems can be decomposed into two different subsystems, one with slow time scales ("outer" series or reduced model) and one with faster time scales ("inner" series or boundary layer model) [45]. The reduced
model mostly represents the slow and dominant phenomena, while the boundary layer model represents the deviations from the predicted slow behavior. If the boundary layer models are asymptotically stable, the deviations will rapidly decay.

When interest is in "local" or "small-signal" approximations of more realistic nonlinear models of dynamic systems, the linear time-invariant models of two-time-scale systems in the fast time frame are generally represented as follows,

\[
\frac{d}{dt}f = A_{11} f + A_{12} s + B_1 u, \tag{3.1}
\]

\[
\frac{d}{dt}s = \epsilon \left( A_{21} f + A_{22} s + B_2 u \right), \tag{3.2}
\]

where the positive scalar \( \epsilon \) represents all the small parameters to be neglected, and it emphasizes that \( f \) evolves on a much faster time scale than \( s \). When \( \epsilon \) approaches 0, the slow variables are approximately constant. Provided that the dynamics of the fast variables are stable, in other words, when the eigenvalues of \( A_{11} \) all have negative real parts, the fast variables will converge to their quasi-steady-state values.

As seen in the slow time frame, a new time variable, \( \tau = \epsilon t \), can be introduced to transform the equations (3.1) and (3.2) into,

\[
\epsilon \frac{d}{d\tau} f = A_{11} f + A_{12} s + B_1 u \tag{3.3}
\]
\frac{d}{d\tau} x_s = A_{21} x_f + A_{22} x_s + B_2 u \tag{3.4}

When \( \epsilon \) approaches 0 and all eigenvalues of \( A_{11} \) have negative real parts, the fast variables will converge to their quasi-steady-state values, i.e.:

\[ x_f = -A_{11}^{-1} A_{12} x_s - A_{11}^{-1} B_1 u, \tag{3.5} \]

and at the same time the slow variables can be obtained by substituting equation (3.5) into equation (3.4).

\[
\frac{d}{d\tau} x_s = -A_{21} A_{11}^{-1} A_{12} x_s + A_{22} x_s + \left( -A_{21} A_{11}^{-1} B_1 + B_2 \right) u
= \left( -A_{21} A_{11}^{-1} A_{12} + A_{22} \right) x_s + \left( -A_{21} A_{11}^{-1} B_1 + B_2 \right) u \tag{3.6}
\]

In our case, the mechanical variables, (e.g., the rotor speed), are considered as those evolving on a slow time scale, and the electromagnetic variables, (e.g., the rotor flux linkage) are considered as those evolving on a fast time scale. Provided the moment of inertia of the mechanical system is sufficiently high (i.e., \( \frac{1}{I_H} \) is less than an upper bound \( \epsilon_0 \), where the lower bound for \( \epsilon_0 \) can be derived with equation (2.18) in [43]. However, to meet the objective of good system performance, \( \epsilon_0 \) should be determined by simulation and experimental results), the electrical system has a much faster response.
than the mechanical system, so that in the electrical system time frame the mechanical variables look essentially constant (e.g., $\omega_{re} \approx 0$). Also, in the mechanical system time frame, the electrical system is so fast that for each speed point it can be assumed that the electrical variables have already reached their quasi-steady-state values. Hence we can use the steady state stator current locus to analyze the stability of the proposed speed estimation technique. It should be noted, however, that this approach places limitations on the convergence rate of the speed estimator [15, 45].

3.2 Speed Estimation Schemes

The stator voltages and currents can be separated through filters into two components: a fundamental component and a carrier-signal component. In the following the carrier-signal components are denoted with a subscript $c$ and the fundamental frequency signals are denoted with a subscript $f$. The stator voltages and currents are therefore given by:

$$
\tilde{u}_s = \tilde{u}_{sf} + \tilde{u}_{sc}, \quad \text{(3.7)}
$$

$$
\tilde{i}_s = \tilde{i}_{sf} + \tilde{i}_{sc} \quad \text{(3.8)}
$$

The estimated variables are represented with $\tilde{}$. The speed estimation scheme is based on MRAS (Model Reference Adaptive System). Two flux observers, one of which is not explicitly related with the rotor speed while the other one is, are built. The
derivative of the estimated rotor speed is constructed based on the outputs of the two observers. The stator flux-linkage is obtained by:

\[ \dot{\lambda}_{sc} = -R_s \tilde{i}_{sc} + \tilde{u}_{sc} - K_1 \dot{\lambda}_{sc} \]  

(3.9)

where \( K_1 \) is a decay constant that makes the stator flux-linkage integration stable. As this estimate is accurate provided the electrical frequency is sufficiently higher than the decay constant \( K_1 \), we will consider this value to correspond to the actual stator flux-linkage. The obtained \( \lambda_{sc} \) is used in both of the two observers. The two rotor flux-linkage observers are therefore reduced-order observers.

The rotor flux-linkage associated with the carrier signal in the observer 1 is therefore determined by:

\[ \dot{\lambda}_{rc} = \frac{L_r}{L_m} \lambda_{sc} - \frac{\sigma^2}{L_m} \tilde{\lambda}_{sc} \]  

(3.10)

In the following analysis this will also be considered as a known quantity.

The dynamics of rotor flux-linkage with the carrier signal in observer 2 is determined by:

\[ \ddot{\lambda}_{rc} = \frac{R_r L_m}{\sigma^2} \lambda_{sc} - \frac{R_r L_s}{\sigma^2} \tilde{\lambda}_{rc} + J \dot{\omega}_{re} \dot{\lambda}_{rc} \]  

(3.11)
The carrier signal is set to rotate in the opposite direction of the fundamental frequency signals so that the frequency difference between them can be guaranteed, so that they can be separated with filters. The rotor speed is then estimated based on the extracted carrier frequency signals. Different variables from the two observers can be used to estimate the rotor speed. Here we propose two options. Speed estimation scheme 1 utilizes an auxiliary vector constructed based on the stator current, while speed estimation scheme 2 utilizes the rotor flux-linkage. The two speed estimation schemes are shown below.

3.2.1 Speed Estimation Scheme 1

Assuming that the motor parameters are accurate, observer variables that are not explicitly determined from the rotor speed will be considered equivalent to the actual induction machine variables, provided the frequency of excitation is sufficiently high. In the stator flux-linkage reference frame, equation (1.3) becomes

\[
\dot{\lambda}_{rc} = \frac{R_r L_m}{\sigma^2} \lambda_{sc} - \frac{R_r L_s}{\sigma^2} \lambda_{rc} + (\omega_{rc} - \omega_c) J \lambda_{rc}
\]

\[
= - \left( \frac{R_r L_s}{\sigma^2} I + \omega_{sc} J \right) \lambda_{rc} + \frac{R_r L_m}{\sigma^2} \lambda_{sc}
\]

The quasi-steady-state value of rotor flux-linkage is given by,

\[
\lambda_{rc} = \left( \frac{R_r L_s}{\sigma^2} + \omega_{sc} J \right)^{-1} \frac{R_r L_m}{\sigma^2} \lambda_{sc}
\]
Substituting the equation above to the equation below,

\[
\vec{i}_{sc} = \frac{1}{\sigma^2} \begin{bmatrix} L_r I - L_m I \end{bmatrix} \begin{bmatrix} \vec{\lambda}_{sc} \\ \vec{\lambda}_{rc} \end{bmatrix}
\] (3.14)

we can express the stator current as a function of stator flux-linkage and slip frequency:

\[
i_{sdc} = \frac{L_r}{\sigma^2} - \frac{R^2 L_m^2 L_s}{\sigma^4} \left( \frac{R_r L_s}{\sigma^2} \right)^2 + \omega_{sc}^2 \right) \lambda_{sdc}
\] (3.15)

\[
i_{sqc} = \frac{R_r L_s^2}{\lambda^4 m \omega_{sc}} \lambda_{sdc}^2
\] (3.16)

The locus of steady-state stator currents as a function of slip frequency, and hence rotor speed, is shown in Figure 3.1 [14]. It is evident that the rotor speed is uniquely related with the steady-state stator current vector. For a given stator flux-linkage magnitude and excitation frequency, both \( \vec{\lambda}_{sc} \) and \( \vec{\lambda}_{sc}^2 \) will be on the same locus, while \( \omega_{re} \) and \( \omega_{re}^2 \) determine their positions on the locus, respectively. The geometric center of the locus in the stator flux-linkage reference frame is given by:

\[
i_{center} = \frac{1}{2} \left( \frac{1}{L_s} + \frac{L_r}{\sigma^2} \right) \vec{\lambda}_{sc}
\] (3.17)
A new vector is defined as \( \vec{y} = \vec{i}_{sc}^s - \vec{i}_{center}^s \), which is shown in Figure 3.1. The positions of \( \vec{y} \) and \( \vec{y} \) will depend on \( \omega_{re} \) and \( \dot{\omega}_{re} \), and the angle \( \theta \) between the two vectors will have a monotonic dependence on the estimated rotor speed error. Therefore, the cross product of the two vectors is used as a correction term for the rotor speed estimate.

\[
\dot{\omega}_{re} = K \vec{y}_1^T \vec{y}_2 \\
= K ||\vec{y}_1|| ||\vec{y}_2|| \sin \theta
\]  

(3.18)

The constant \( K \) is related to the convergence rate of the rotor speed estimation. The determination of \( K \) is up to simulation and experimental results. Provided \( |\theta| < 180^\circ \), \( \dot{\omega}_{re} \) has the same sign as \( \theta \). The estimated rotor speed is then fed back into observer 2. The stability of this method will be proven with the two-time-scale approach and basic control stability theory in the next section.

3.2.2 Speed Estimation Scheme 2

Based on Schauder’s work [49], the cross product of the two carrier frequency rotor flux-linkage vectors can be selected as the correction term for the estimated rotor speed,

\[
\dot{\omega}_{re} = K_2 \lambda^T_{rc} \dot{\lambda}_{rc} \\
= K_2 ||\lambda_{rc}|| ||\dot{\lambda}_{rc}|| \sin \theta
\]  

(3.19)
The selection of constant $K_2$ will be described in the next section because it involves the
stability analysis of the system. Figure 3.2 illustrates the relationship of the two rotor
flux-linkage vectors.

The stability of this speed estimation scheme will be discussed in the next section.

3.3 Stability Analysis

3.3.1 Stability Analysis for Speed Estimation Scheme 1

In the stator flux-linkage reference frame the dynamic equation of rotor flux-
linkage in observer 2 becomes:

$$
\ddot{\lambda}_{sc} = \frac{R_r L_m}{\sigma^2} \lambda_{sc} - \frac{R_r L_s}{\sigma^2} \dot{\lambda}_{sc} + (\dot{\omega}_r - \omega_c) J \dot{\lambda}_{sc}
$$

$$
= \frac{R_r L_m}{\sigma^2} \lambda_{sc} - \frac{R_r L_s}{\sigma^2} \dot{\lambda}_{sc} - \dot{\omega}_s c J \dot{\lambda}_{rc} \tag{3.20}
$$

where $\omega_{sc}$ is the slip frequency between the rotor speed and the carrier signal.

Similarly, in the stator flux-linkage reference frame the dynamic equation of rotor
flux-linkage in observer 1 becomes:

$$
\ddot{\lambda}_{sc} = \frac{R_r L_m}{\sigma^2} \lambda_{sc} - \frac{R_r L_s}{\sigma^2} \dot{\lambda}_{sc} + (\dot{\omega}_r - \omega_c) J \dot{\lambda}_{sc}
$$

$$
= \frac{R_r L_m}{\sigma^2} \lambda_{sc} - \frac{R_r L_s}{\sigma^2} \dot{\lambda}_{sc} - \dot{\omega}_s c J \dot{\lambda}_{rc} \tag{3.21}
$$
We define error terms for the estimated rotor flux-linkage and rotor speed as follows:

\[ \tilde{e}_r = \tilde{\lambda}_{rc} - \bar{\lambda}_{rc}, \]  
\[ \delta \omega_{re} = \tilde{\omega}_{re} - \omega_{re} \]  
\[ \delta \omega_{re} = \tilde{\omega}_{re} - \omega_{re} \]  
\[ \delta \omega_{re} = \tilde{\omega}_{re} - \omega_{re} \]  

From equation (3.20) and (3.21), the error dynamics in the stator flux-linkage reference frame are therefore given by:

\[ \dot{\tilde{e}}_s = - \left( \frac{R_r L_s}{\sigma^2} \mathbf{I} + \omega_{sc} \mathbf{J} \right) \tilde{e}_s + \delta \omega_{re} \mathbf{J} \tilde{\lambda}_{rc} \]  
\[ \dot{\delta \omega}_{re} = \delta \omega_{re} \]  
\[ \dot{\delta \omega}_{re} = \delta \omega_{re} \]  
\[ \dot{\delta \omega}_{re} = \delta \omega_{re} \]  

Using two-time-scale theory, in the slow time scale we can assume that the electrical dynamics have converged to their quasi-steady-state value; i.e., \( \dot{\tilde{e}}_s \approx 0 \). Under these conditions the rotor flux-linkage error can be written as follows:

\[ \ddot{\tilde{e}}_s = \left( \frac{R_r L_s}{\sigma^2} \mathbf{I} + \omega_{sc} \mathbf{J} \right)^{-1} \delta \omega_{re} \mathbf{J} \tilde{\lambda}_{rc} \]  
\[ \ddot{\delta \omega}_{re} = \delta \omega_{re} \]  
\[ \ddot{\delta \omega}_{re} = \delta \omega_{re} \]  

where we have used the relation:

\[ (a \mathbf{I} + b \mathbf{J})^{-1} = \frac{1}{a^2 + b^2} (a \mathbf{I} - b \mathbf{J}) \]  

Likewise, from equation (3.21), using two-time-scale theory, the rotor flux-linkage due to the carrier signal can be assumed to have converged to its quasi-steady-state
value, hence

\[
\tilde{\chi}_{re}^{\lambda_s} = \frac{1}{\left( \frac{R_r L_s}{\sigma^2} \right)^2 + \omega_{sc}^2} \left( \frac{R_r L_s}{\sigma^2} I - \omega_{sc} J \right) \frac{R_r L_m}{\sigma^2} \tilde{\chi}_{sc}^{\lambda_s} \tag{3.27}
\]

In the following we assume the rotor speed is fixed, i.e.:

\[
\dot{\omega}_{re} = 0 \tag{3.28}
\]

The dynamics of the rotor speed error are therefore:

\[
\delta \dot{\omega}_{re} = \dot{\omega}_{re} = K \tilde{y}_1^T J \tilde{y}_2 \tag{3.29}
\]

where

\[
\tilde{y} = \tilde{\chi}_{sc}^{\lambda_s} - \tilde{\gamma}_{center}^{\lambda_s} \tag{3.30}
\]

\[
\tilde{\gamma}_{center}^{\lambda_s} = \frac{1}{2} \left( \frac{1}{L_s} + \frac{L_r}{\sigma^2} \right) \tilde{\chi}_{sc}^{\lambda_s} \tag{3.31}
\]
Hence:

\[ \bar{y}_1 = \frac{1}{2} \left( \frac{L_r}{\sigma^2} - \frac{1}{L_s} \right) \bar{\lambda}_{sc} \lambda_s - \frac{L_m}{\sigma^2} \bar{\lambda}_{rc} \]

\[ = \frac{L^2_m}{2L_s\sigma^2} \bar{\lambda}_{sc} \lambda_s - \frac{L_m}{\sigma^2} \bar{\lambda}_{rc} \]  
(3.32)

\[ \bar{y}_2 = \frac{1}{2} \left( \frac{L_r}{\sigma^2} - \frac{1}{L_s} \right) \bar{\lambda}_{sc} \lambda_s - \frac{L_m}{\sigma^2} \bar{\lambda}_{rc} \]

\[ = \frac{L^2_m}{2L_s\sigma^2} \bar{\lambda}_{sc} \lambda_s - \frac{L_m}{\sigma^2} \bar{\lambda}_{rc} \]  
(3.33)
The dynamics of the rotor speed error in the slow time scale are therefore given by:

\[
\delta \omega_{re} = K \left[ \frac{L_m^2}{2L_s \sigma^2} \bar{\lambda}_{sc} \bar{\lambda}_{sc} - \frac{L_m}{\sigma^2} \bar{\lambda}_{rc} \bar{\lambda}_{rc} \right]^T J \left[ \frac{L_m^2}{2L_s \sigma^2} \bar{\lambda}_{sc} \bar{\lambda}_{sc} - \frac{L_m}{\sigma^2} \bar{\lambda}_{rc} \bar{\lambda}_{rc} \right]
\]

\[
= K \left[ -\frac{L_m^3}{2L_s \sigma^4} \left( \bar{\lambda}_{sc}^2 T J \bar{\lambda}_{sc} \bar{\lambda}_{sc} + \bar{\lambda}_{sc}^2 T J \bar{\lambda}_{rc} \bar{\lambda}_{sc} \right) + \left( \frac{L_m}{\sigma^2} \right)^2 \bar{\lambda}_{rc}^2 \bar{\lambda}_{rc} \bar{\lambda}_{rc} \right]
\]

\[
= K \left[ -\frac{L_m^3}{2L_s \sigma^4} \bar{\lambda}_{sc}^2 \bar{\lambda}_{sc} T J \bar{\lambda}_{rc}^2 + \left( \frac{L_m}{\sigma^2} \right)^2 \bar{\lambda}_{rc}^2 \bar{\lambda}_{rc} \bar{\lambda}_{rc} \right]
\]

\[
= K \left[ -\frac{L_m^3}{2L_s \sigma^4} \bar{\lambda}_{sc}^2 \bar{\lambda}_{sc} T J \bar{\lambda}_{rc} \right] \bar{\lambda}_{rc}^2 + \left( \frac{L_m}{\sigma^2} \right)^2 \bar{\lambda}_{rc}^2 \bar{\lambda}_{rc} \bar{\lambda}_{rc} \right]
\]

\[
= K \left[ -\frac{L_m^3}{2L_s \sigma^4} \bar{\lambda}_{sc}^2 \bar{\lambda}_{sc} T J \bar{\lambda}_{rc} \right] \bar{\lambda}_{rc}^2 \bar{\lambda}_{rc} \bar{\lambda}_{rc} \right]
\]

\[
= K \left\{ \left[ -\frac{L_m^3}{2L_s \sigma^4} \bar{\lambda}_{sc}^2 \bar{\lambda}_{sc} + \left( \frac{R_f L_m}{\sigma^2} \right)^2 \bar{\lambda}_{sc}^2 \bar{\lambda}_{sc} \right] \right\}^T \bar{\lambda}_{rc}^2 \bar{\lambda}_{rc} \bar{\lambda}_{rc} \right]
\]

\[
= K \left\{ \left[ -\frac{L_m^3}{2L_s \sigma^4} \bar{\lambda}_{sc}^2 \bar{\lambda}_{sc} + \left( \frac{R_f L_m}{\sigma^2} \right)^2 \bar{\lambda}_{rc}^2 \bar{\lambda}_{rc} \right] \right\}^T \bar{\lambda}_{rc}^2 \bar{\lambda}_{rc} \bar{\lambda}_{rc} \right]
\]

\[
= K \left\{ \left[ -\frac{L_m^3}{2L_s \sigma^4} \bar{\lambda}_{sc}^2 \bar{\lambda}_{sc} + \left( \frac{R_f L_m}{\sigma^2} \right)^2 \bar{\lambda}_{rc}^2 \bar{\lambda}_{rc} \right] \right\}^T \bar{\lambda}_{rc}^2 \bar{\lambda}_{rc} \bar{\lambda}_{rc} \right]
\]

\[
= K \left\{ \left[ -\frac{L_m^3}{2L_s \sigma^4} \bar{\lambda}_{sc}^2 \bar{\lambda}_{sc} + \left( \frac{R_f L_m}{\sigma^2} \right)^2 \bar{\lambda}_{rc}^2 \bar{\lambda}_{rc} \right] \right\}^T \bar{\lambda}_{rc}^2 \bar{\lambda}_{rc} \bar{\lambda}_{rc} \right]
\]

\[
= \frac{(R_f L_s I - \omega_{sc} J)}{(R_f L_s I - \omega_{sc} J)} \bar{\lambda}_{rc}^2 \bar{\lambda}_{rc} \bar{\lambda}_{rc} + h.o.t.
\]

(3.34)

where we have made extensive use of the fact that:

\[
\bar{\alpha}^T J \bar{\alpha} = 0
\]

(3.35)
By focusing on small-signal analysis, we neglect the higher-order terms and focus on the first-order dynamics.

\[
\delta \omega_{rc} \approx K \left\{ -L_m^3 \frac{1}{2L_s \sigma^4} \mathbf{I} + \frac{R_r L_m^3}{\sigma^6} \left( \frac{R_r L_s}{\sigma^2} \right) ^2 + \omega_{sc}^2 \left( \frac{R_r L_s}{\sigma^2} \mathbf{I} - \omega_{sc} \mathbf{J} \right) \right\}^T \mathbf{J} \]

\[
= K \left\{ \frac{L_m^3}{2L_s \sigma^4} \mathbf{I} - \frac{R_r L_m^3}{\sigma^6} \left( \frac{R_r L_s}{\sigma^2} \right) ^2 + \omega_{sc}^2 \left( \frac{R_r L_s}{\sigma^2} \mathbf{I} - \omega_{sc} \mathbf{J} \right) \right\}^T \tilde{\lambda}_{sc}^{\lambda_s} \]

\[
= K \left\{ \frac{L_m^3}{2L_s \sigma^4} \mathbf{I} - \frac{R_r L_m^3}{\sigma^6} \left( \frac{R_r L_s}{\sigma^2} \right) ^2 + \omega_{sc}^2 \left( \frac{R_r L_s}{\sigma^2} \mathbf{I} - \omega_{sc} \mathbf{J} \right) \right\}^T \tilde{\lambda}_{sc}^{\lambda_s} \]

\[
= K \left\{ \frac{L_m^3}{2L_s \sigma^4} \mathbf{I} - \frac{R_r L_m^3}{\sigma^6} \left( \frac{R_r L_s}{\sigma^2} \right) ^2 + \omega_{sc}^2 \left( \frac{R_r L_s}{\sigma^2} \mathbf{I} - \omega_{sc} \mathbf{J} \right) \right\}^T \tilde{\lambda}_{sc}^{\lambda_s} \]

\[
\delta \omega_{rc} = \frac{R_r L_s}{\sigma^2} \mathbf{J} \tilde{\lambda}_{sc}^{\lambda_s} \delta \omega_{rc}
\]

where we have utilized the following relations:
\[ J^2 = -I, \] (3.37)

\[ (aI + bJ)^T = aI - bJ, \] (3.38)

\[ (aI + bJ)(aI - bJ) = \left( a^2 + b^2 \right) I \] (3.39)

continuing, we achieve:

\[
\delta \omega_{re} \approx K \left\{ \frac{R_r L_m^4}{2L_s \sigma^6} \left( \frac{R_r L_s}{\sigma^2} \right)^2 + \omega_{sc}^2 \right\}^2 \tilde{\lambda}_{sc}^T \left( \frac{R_r L_s}{\sigma^2} I - \omega_{sc} J \right)^2 \tilde{\lambda}_{sc}^T \\
- \frac{R_r L_m^3}{\sigma^6} \left[ \left( \frac{R_r L_s}{\sigma^2} \right)^2 + \omega_{sc}^2 \right]^2 \tilde{\lambda}_{sc}^T \left( \frac{R_r L_s}{\sigma^2} I - \omega_{sc} J \right) \frac{R_r L_m}{\sigma^2} \tilde{\lambda}_{sc}^T \delta \omega_{re} \\
= \frac{K \| \tilde{\lambda}_{sc} \|^2}{\left[ \left( \frac{R_r L_s}{\sigma^2} \right)^2 + \omega_{sc}^2 \right]^2} \left\{ \frac{R_r L_m^4}{2L_s \sigma^6} \left[ \left( \frac{R_r L_s}{\sigma^2} \right)^2 + \omega_{sc}^2 \right] - \frac{R_r^3 L_s L_m^4}{\sigma^{10}} \right\} \delta \omega_{re} \\
= - \frac{K \| \tilde{\lambda}_{sc} \|^2}{\left[ \left( \frac{R_r L_s}{\sigma^2} \right)^2 + \omega_{sc}^2 \right]^2} \left\{ \frac{R_r L_m^4 \omega_{sc}^2}{2L_s \sigma^6} + \frac{R_r^3 L_s L_m^4}{2\sigma^{10}} \right\} \delta \omega_{re} \\
= - \frac{K R_r L_m^4 \| \tilde{\lambda}_{sc} \|^2}{2\sigma^6 L_s \left[ \left( \frac{R_r L_s}{\sigma^2} \right)^2 + \omega_{sc}^2 \right]} \delta \omega_{re} \] (3.40)

where we have utilized the following relation:

\[ (aI - bJ)^2 = \left( a^2 - b^2 \right) I - 2abJ \] (3.41)
The solution for a dynamic equation,

\[ \dot{x} = ax \]  

(3.42)

is

\[ x(t) = x(0)e^{at} \]  

(3.43)

As long as \( a \) has negative real part, \( x \) will exponentially converge to zero. And this is the case for this speed estimator, since from equation (3.40), we can see that the coefficient

\[-\frac{KR_rL_m^4\|\lambda_{sc}\|^2}{2\sigma_b L_s \left[ \left( \frac{R_rL_s}{\sigma_b} \right)^2 + \omega_{sc}^2 \right]}\]

is always negative when \( K > 0 \) regardless of operating point. \( \delta\omega_{re} \) will therefore exponentially converge to zero. Figure 3.3 shows the value of the coefficient with the induction machine parameters when \( \omega_{sc} \) changes from 0 to 1000 rad/s.

When the mechanical dynamics are considered, the cross product of the two vectors \( \vec{y}_1 \) and \( \vec{y}_2 \) can be used as the correction term \( -\frac{2}{P} \frac{K}{H} f_{ex} \) in equation (24) of [15], and from equation (38) and (39) of [15],

\[
\begin{bmatrix}
\dot{\delta\omega}_r \\
\delta\tau_{\ell}
\end{bmatrix} = \frac{1}{H} \Lambda_s \begin{bmatrix}
\delta\omega_r \\
\delta\tau_{\ell}
\end{bmatrix},
\]  

(3.44)
Provided the induction machine is operating in the open-loop stable operating range (i.e., \( g_\tau < 0 \)), it can be easily shown that the matrix \( A_s \) is exponentially stable since \( h_{cx} > 0 \) regardless of the operating point when \( K' > 0 \). The stability of the proposed scheme is proven when assuming the total moment of inertia of the induction motor and load is sufficiently large.

### 3.3.2 Stability Analysis for Speed Estimation Scheme 2

The stability of speed estimation scheme 2 is almost the same as that of speed estimation scheme 1. In this speed estimation scheme, the derivative of rotor speed estimate is defined as
\[ \dot{\omega}_{re} = K_2 \lambda_{rs} T J \lambda_{rs} \]

The dynamics of the rotor speed error in the slow time scale are therefore given by:

\[
\delta \dot{\omega}_{re} = \dot{\omega}_{re} - \omega_{re} = K_2 \lambda_{rs} T J \lambda_{rs} = K_2 \lambda_{rs} T J \left( \lambda_{rs} + \delta \lambda_{rs} \right) = K_2 \lambda_{rs} T J \delta \lambda_{rs} = K_2 \lambda_{rs} T J \frac{1}{(R_s L_s)^2 + \omega_{sc}^2} \left( R_s L_s I - \omega_{sc} J \right) J \lambda_{rs} \delta \omega_{re}
\]

\[ = K_2 \lambda_{rs} T J \frac{1}{(R_s L_s)^2 + \omega_{sc}^2} \left( R_s L_s I - \omega_{sc} J \right) J \lambda_{rs} \delta \omega_{re} + \text{h.o.t.} \]

\[
\approx \frac{K_2}{(R_s L_s)^2 + \omega_{sc}^2} \left[ - \frac{R_s L_s}{\sigma^2} \lambda_{rs} T \lambda_{rs} + \omega_{sc} \lambda_{rs} T J \lambda_{rs} \right] \delta \omega_{re}
\]

\[ = - \frac{K_2 R_s L_s}{(R_s L_s)^2 + \omega_{sc}^2} \| \lambda_{rs} \|^2 \delta \omega_{re} \quad (3.48)
\]

As long as \( K_2 > 0 \), the coefficient of equation (3.48) is always negative regardless of operating point. When mechanical dynamics are considered, the cross product of the two vectors \( \tilde{y}_1 \) and \( \tilde{y}_2 \) can be used as a correction term for the mechanical dynamics and it is shown in [15] that, provided the induction machine is operating in the open-loop
stable operating range (i.e., $g_r < 0$), the system is exponentially stable. Therefore the stability of the proposed scheme 2 is also proven when assuming the total moment of inertia of the induction motor and load is sufficiently large.

3.4 Conclusion

The stability of two proposed speed estimation schemes are analyzed with the two-time-scale approach and basic control stability theory. It is shown that when the system has a sufficiently large moment of inertia, both of the proposed speed estimation schemes are stable. The next chapter will provide the simulation results to show the feasibility of these speed estimation schemes.
Fig. 3.1. Steady-state stator current as a function of slip frequency in stator flux-linkage reference frame with constant stator flux-linkage magnitude.
Fig. 3.2. The cross product of two rotor flux-linkage vectors used as the derivative of the estimated rotor speed.
Fig. 3.3. The coefficient of the differential equation for $\omega_{pe}$ in speed estimation scheme 1.
Simulation of the Proposed Speed Estimation Schemes

Compared to the ideal models of induction machine and control system, the actual motor and the control system, including the hardware and software, are more complicated and might have some parasitic parameters that influence the performance of the estimator. It is therefore difficult to determine whether difficulties arise from the proposed schemes or whether they are from defects of the hardware or software that are used to implement the schemes. Also for a control system, the control theory gives clues for the appropriate range of control gains, but the actual control gains need to be adjusted in the real system to achieve the best performance. Generally before attempting a real system, simulations are performed to obtain initial values for the control gains and then, starting with these initial values, modifications to these gains will be done in the experiments.

For the induction machine control system, the proposed speed estimation scheme needs to be tested first to see if the idea really works. Furthermore, the control gains that are used in the speed estimation block, flux-linkage controller, and torque controller can be determined in the simulations and used as initial values in the experiments.
In this chapter the simulation environment, including the computer system and the software, is introduced; then the simulation structures will be described, and finally the simulation results will be given to show the feasibility of our proposed speed estimation schemes.

4.1 Simulation Environment Introduction

4.1.1 Hardware

The computer that is used for the simulation is a Dell OptiPlex GX1 desktop. The specific information is shown in Table 4.1.

4.1.2 Software

The simulation is implemented with MATLAB Simulink. We can use the provided standard blocks build our simulation code. When we change the induction machine model in the simulation code to a real induction machine (i.e. send the inputs of the induction machine model to the real induction machine through D/A, digital I/O or PWM interface, measure the stator voltages and currents of the real induction machine and feed these data to the computer through A/D converters instead of getting the numerical data from the model), we can implement the experiment without significant additional work through the use of MATLAB Real-Time Workshop.

4.2 The Simulation for Speed Sensor-less Control of Induction Machine

The control algorithm contains a torque regulator, a speed estimator and a flux-linkage controller. The signals in the torque control loop and the flux-linkage control loop
are only the fundamental frequency signals, while the signals in the speed estimation loop are the carrier signals. The carrier signals are added after the main controller, i.e., the carrier signals are added to the calculated voltage commands of the main controller. So in the induction machine, the fundamental signals and the carrier signals are together and will operate simultaneously, thus generally we will have some oscillation on the torque due to the addition of the high frequency carrier signals. Means of reducing or eliminating the torque ripple will be discussed in a separate chapter. The carrier signals are set to rotate in the opposite direction of the fundamental frequency signals. With this approach, there will always be a frequency difference between the two sets of signals. If we transform everything to the carrier signal reference frame (which is the case used in this simulation), we can use filters to separate the carrier signal from fundamental frequency signals and process them in different loops respectively. The rotor speed is estimated with the carrier signals, while the torque and flux-linkage regulators are based on the fundamental frequency signals. The reason that the torque control and the flux-linkage control loops should only work on the fundamental frequency signals is to avoid the cancellation of the added carrier signals. The torque and flux-linkage controllers will use the rotor speed estimation to estimate the rotor flux-linkage and the electromagnetic torque.

The block diagram of the system is shown in Figure 4.1.

In the simulation the induction machine is represented by the mathematical model presented in chapter 1.
4.2.1 MRAS used in our technique

The speed estimation method proposed uses MRAS as the specific implementation tool. It is implemented on the injected carrier signals. The selected high frequency of the carrier signals makes the speed estimation feasible even at sustained low or zero fundamental stator electrical frequency. Figure 4.2 illustrates the structure of MRAS for speed estimation. There are two observers with the carrier signal frequency variables as inputs. Observer 1 is not explicitly related with rotor speed, whilst observer 2 is. The cross product of the outputs of the two observers will be used as the derivative of rotor speed, and the estimated rotor speed will be fed back to observer 2. With the right adaptive mechanism, the rotor speed will be estimated with high accuracy. Based on the rotor speed estimation, the fundamental frequency rotor flux-linkage can be estimated, as can the electromagnetic torque. Thus torque control can be accomplished.

We process the estimated fluxes with filters, which are assumed here as ideal, to extract the carrier frequency components. When the carrier frequency is high enough, with careful selection of the cut-off frequency, the influence of the filters on the carrier signals is negligible. So the carrier frequency components of the rotor flux-linkage can be considered separately from the fundamental frequency components with the superposition property. The signal extraction processes are shown in Figure 4.3, 4.4 and 4.5.

4.2.2 Induction Machine Model used in the Simulation

In the actual control system, the controller calculates the desired command voltages and sends corresponding signals to the inverter to control the actual voltages applied
to the induction machine. The terminal voltages and currents are measured and sent to the controller to calculate the command voltages of the next cycle. In the simulation, the actual induction machine is replaced by an induction machine model. With the calculated command voltages as the inputs, the induction machine model should be able to provide the terminal voltages, currents, electromagnetic torque, and rotor speed. The correctness of the induction machine model is very important to the effectiveness of the simulation. With an appropriate induction machine model, the simulation can provide useful information for the proof of the proposed speed estimation scheme and for the determination of the initial values of the control gains. The induction machine model used in the simulation is given in this section.

With the command voltages and the rotor speed, in the stationary stator reference frame, the stator and rotor fluxes are calculated with the following equations:
The terminal currents are calculated with the equations below,
\[
\begin{bmatrix}
i_{sd} \\
i_{sq}
\end{bmatrix} = \frac{1}{\sigma^2} \begin{bmatrix}
L_r & 0 & -L_m & 0 \\
0 & L_r & 0 & -L_m
\end{bmatrix} \begin{bmatrix}
\lambda_{sd} \\
\lambda_{sq} \\
\lambda_{rd} \\
\lambda_{rq}
\end{bmatrix}
\] (4.2)

With the stator and rotor fluxes in the same stationary stator reference frame, the electromagnetic torque is calculated as,

\[
\tau_e (\lambda_s, \lambda_r) = \frac{3P}{4} \left[ \frac{L_m}{\sigma^2} \mathbf{J} \lambda_r \right]^T \lambda_s
\]

\[
= \frac{3P}{4} \frac{L_m}{\sigma^2} \lambda_r^T \mathbf{J} \lambda_r
\]

\[
= \frac{3P}{4} \frac{L_m}{\sigma^2} \left( \lambda_{sq} \lambda_{rd} - \lambda_{sd} \lambda_{rq} \right)
\] (4.3)

where \(P\) is the number of poles (in this case, \(P = 4\)).

The rotor speed dynamics are represented with the equation below:

\[
\omega_r = \frac{1}{H} \left[ (\tau_e - \tau_l) - \mathbf{B} \omega_r \right]
\]

\[
= \frac{1}{H} \left[ \left( \frac{3P}{4} \frac{L_m}{\sigma^2} \lambda_r^T \mathbf{J} \lambda_r - \tau_l \right) - \mathbf{B} \omega_r \right]
\] (4.4)

where \(H\) is the moment of inertia of the rotor and the load, and \(\mathbf{B}\) is the damping constant. In the simulation, \(\mathbf{B}\) is set as 0.1.
With the equations above, the terminal currents and the rotor speed can be calculated. The terminal voltages are set to be the same as the command voltages, assuming the inverter used for the control of the induction machine is ideal. In the experiments this is not true, largely due to the dead-time effect [16]. The induction machine model provides terminal voltages and currents for the speed estimator, speed controller, flux-linkage controller and torque controller. The calculated rotor speed can be used to compare with the estimated rotor speed.

4.2.3 Simulation Based on the Speed Estimation Scheme 1

First we implement the simulation based on the speed estimation scheme 1 introduced in section 3.2.1.

4.2.3.1 Speed Estimator

In the following estimated variables will be denoted with a $\hat{\cdot}$. A stator flux-linkage estimator that is valid at high electrical frequencies is given by:

$$\dot{\tilde{\lambda}}_{sc} = \tilde{u}_{sc} - \tilde{i}_{sc} R_s - K \tilde{\lambda}_{sc}$$

(4.5)

where $K$ is a decay constant that makes the observer asymptotically stable. We transform the estimated stator fluxes and stator currents to the carrier signal reference frame, and use low-pass filters to extract the carrier signals, as shown in Figure 4.3 for the stator flux-linkage. In the following, extracted carrier signal variables will be denoted by a subscript 'c'.
A rotor flux-linkage carrier-signal estimator that is dependent upon rotor speed in the stationary stator reference frame is given by:

\[
\dot{\tilde{\lambda}}_{rc} = \frac{R_r L_m}{\sigma^2} \tilde{\lambda}_{sc} - \frac{R_r L_s}{\sigma^2} \tilde{\lambda}_{rc} + J \hat{\omega}_r \tilde{\lambda}_{rc}
\]  

(4.6)

However, in our approach, instead of estimating rotor flux linkage we estimate a scaled version of rotor flux linkage:

\[
\tilde{x} = \frac{\sigma^2}{R_r L_m} \tilde{\lambda}_{rc}
\]  

(4.7)

The dynamics of this new vector are given by:

\[
\dot{\tilde{x}} = \frac{\sigma^2}{R_r L_m} \tilde{\lambda}_{rc}
\]  

(4.8)

This vector is chosen as it reduces the number of machine parameters necessary for speed estimation.

The approach for estimating the rotor speed can be determined by inspection of the steady-state stator current carrier-signal locus as a function of the carrier-signal slip frequency \((\omega_s = \omega_c - \omega_{rc})\), where \(\omega_c\) is the carrier signal frequency) in the reference frame of the carrier signal stator flux-linkage, as shown in Figure 3.1. As the carrier frequency
is constant, the locus can also be thought of as being a function of rotor speed. Using a two-time-scale approach [15], assuming that the mechanical inertia of the system is large enough, the dynamics of the rotor speed are much slower than the electrical dynamics of the induction machine.

In the stator flux-linkage reference frame, the two vectors are determined by:

\[
\begin{align*}
\tilde{y}_1 &= \tilde{\lambda}_s - \tilde{\lambda}_s^{\text{center}}, \\
\tilde{y}_2 &= \frac{L_r}{\sigma^2} \tilde{\lambda}_s - \frac{L_m}{\sigma^2} \tilde{\chi}_r^{\lambda_s} - \frac{1}{2} \left( \frac{1}{L_s} + \frac{L_r}{\sigma^2} \right) \tilde{\lambda}_s^{\text{sc}} \\
&= \frac{1}{2} \left( \frac{L_r}{\sigma^2} - \frac{1}{L_s} \right) \tilde{\lambda}_s^{\text{sc}} - \frac{R_r L_m^2}{\sigma^4 \tilde{y}} \tag{4.9}
\end{align*}
\]

The cross product of these two vectors will be used as the input to a Proportional-Integral (PI) regulator. The output of the PI regulator is considered as rotor speed and used in model 2. This will form a closed loop for the estimation of rotor speed. The phase-locked loop will bring the speed estimation to the right value. The equation of the speed estimator is therefore given by:

\[
\tilde{\omega}_{rc} = K_p \left( \tilde{y}_1^T \boldsymbol{J} \tilde{y}_2 \right) + K_i \int \left( \tilde{y}_1^T \boldsymbol{J} \tilde{y}_2 \right) dt \tag{4.10}
\]

The structure of the speed estimator is shown in Figure 4.6. It should be noted that the resolution of this speed estimator is decreased with increasing rotor speed (i.e. high slip frequency). However, under these conditions the same estimation technique
can be used by replacing carrier-signal variables with fundamental frequency variables at a specified rotor speed, using hysteresis to avoid jittering about the transition point.

### 4.2.3.2 The Torque Controller

With the rotor speed obtained from the rotor speed estimator, the fundamental frequency rotor flux-linkage in the stationary stator reference frame is given by:

\[
\tilde{\lambda}_r = J_\omega r c_\omega r \tilde{\lambda}_r - \frac{R_r}{L_r} \tilde{\lambda}_r + \frac{R_r L_{mm}}{L_r} \tilde{i}_s
\]  

(4.11)

By transforming the variables into the rotor flux-linkage reference frame, the electromagnetic torque is given by:

\[
\tau_e = \frac{3P}{4} \frac{L_{mm}}{L_r} \tilde{i}_s \tilde{\lambda}_r^T J \tilde{\lambda}_r
\]

\[
= \frac{3P}{4} \frac{L_{mm}}{L_r} \tilde{i}_s \| \tilde{\lambda}_r \| \tilde{\lambda}_r
\]  

(4.12)

where \( \| \tilde{\lambda}_r \| \) is the magnitude of rotor flux linkage vector and \( \tilde{i}_s \tilde{\lambda}_r \) denotes the quadrature stator current in the rotor flux-linkage reference frame.

By using a PI controller as a regulator of the quadrature stator current in the rotor flux-linkage reference frame, the quadrature voltage command in the rotor flux-linkage reference frame is obtained. Also, using the PI controller as a rotor flux-linkage regulator, the direct voltage command in the rotor flux-linkage reference frame is determined. We then transform the command voltages back to the stationary stator reference frame. The carrier signals are added to the voltages calculated by the main controller, and then the final voltage commands are sent to the induction machine model.
The flux-linkage regulator is given by:

\[ K_f(s) = K_{pf} + K_{if} \frac{1}{s} \]  \hspace{1cm} (4.13)

where \( K_{pf} \) and \( K_{if} \) are the proportional and integral gains of the controller respectively.

The torque-current regulator is given by:

\[ K_c(s) = K_{pc} + K_{ic} \frac{1}{s} \]  \hspace{1cm} (4.14)

where \( K_{pc} \) is the proportional gain and \( K_{ic} \) is the integral gain of the torque-current regulator.

The current regulator is an inner loop of the torque regulator, so its response should be faster than the torque regulator. For the simulation, the trial and error method is used to obtain the gains, which are shown below:

\[ \begin{align*} K_{pc} &= 50, \quad K_{ic} = 2000 \quad (4.15) \\
K_{pf} &= 170, \quad K_{if} = 1000 \quad (4.16) \end{align*} \]
4.2.3.3 Simulation Results and Conclusion

The induction machine used in the simulation is a 3-phase, 4-pole machine. The parameters are provided in Table 4.2.

The simulation results are given in Figures 4.7, 4.8, 4.9 and 4.10. In Figure 4.7, the torque waveform shows both the command value and the actual value. The speed waveform shows both the actual value and the estimated value. Figure 4.7 shows that the torque controller works well at very low speed range. Figure 4.8 gives the corresponding rotor flux-linkage magnitude and fundamental frequency stator flux-linkage components in the stationary stator reference frame. Figure 4.9 shows that when the load torque changes, the speed estimator and the torque controller still work well, which means the algorithm is not sensitive to load conditions. Figure 4.10 gives the corresponding rotor flux-linkage magnitude and fundamental frequency stator flux-linkage components in the stationary stator reference frame.
### General Information

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<th>Description</th>
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<td>Microprocessor type</td>
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<td>Microprocessor speeds</td>
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<td>L2 cache memory</td>
<td>512-KB pipeline burst, 4-way set-associative, write-back SRAM</td>
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<td>Microprocessor slot</td>
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### System Information

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<td>Address bus width</td>
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<td>Flash EPROM (BIOS)</td>
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<td>I/O controller chip</td>
<td>National PC87309</td>
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### Expansion Bus

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<td>Bus type</td>
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<tr>
<td>Bus speeds</td>
<td>PCI: 33 MHz, ISA: 8.33 MHz</td>
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<tr>
<td>PCI data transfer rate</td>
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<tr>
<td>ISA connector data width</td>
<td>16 bits</td>
</tr>
<tr>
<td>PCI connector data width</td>
<td>32 bits (maximum)</td>
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<tr>
<td>ISA expansion-card connector size</td>
<td>98 pins</td>
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</table>

### System Clocks

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<th>Details</th>
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</thead>
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<td>Diskette/communications ports</td>
<td>48 MHz from system clock</td>
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<tr>
<td>System clock</td>
<td>66/100 MHz</td>
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<tr>
<td>Keyboard controller</td>
<td>48 MHz</td>
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Table 4.1. Information of the computer for simulations and experiments
Fig. 4.1. The block diagram of the torque control system

<table>
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<tr>
<th><strong>Electrical Parameters</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s = 1.59 \Omega$</td>
<td>Stator Resistance</td>
</tr>
<tr>
<td>$R_r = 1.86 \Omega$</td>
<td>Rotor Resistance</td>
</tr>
<tr>
<td>$L_s = 0.1165 H$</td>
<td>Stator Inductance</td>
</tr>
<tr>
<td>$L_r = 0.1167 H$</td>
<td>Rotor Inductance</td>
</tr>
<tr>
<td>$L_m = 0.1095 H$</td>
<td>Mutual Inductance</td>
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</table>

<table>
<thead>
<tr>
<th><strong>Mechanical Parameters</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$H = 0.1 kg \ m^2$</td>
<td>Moment of Inertia of Rotor and Load</td>
</tr>
<tr>
<td>$B = 0.1 \frac{kg \ m^2}{s}$</td>
<td>Mechanical Damping Constant of Rotor</td>
</tr>
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Table 4.2. Parameters of the induction machine model
Fig. 4.2. Structure of speed estimation technique by means of MRAS

Fig. 4.3. Carrier frequency extraction of stator flux-linkage
Fig. 4.4. Carrier frequency extraction of stator currents

Fig. 4.5. Fundamental frequency extraction of stator currents
Fig. 4.6. Structure of the speed estimator.
Fig. 4.7. Torque and speed waveforms at very low speed range. Simulation performed on a 3-phase, 4-pole induction machine.
Fig. 4.8. Rotor flux-linkage magnitude and stator flux-linkage waveforms at very low speed range. Simulation performed on a 3-phase, 4-pole induction machine.
Fig. 4.9. Torque and speed waveforms when load torque changes. Simulation performed on a 3-phase, 4-pole induction machine.
Fig. 4.10. Rotor flux-linkage magnitude and stator flux-linkage waveforms when load torque changes. Simulation performed on a 3-phase, 4-pole induction machine.
The simulation results show that the speed estimation scheme 1 works well even at very low speed range. Based on the estimation of rotor speed, torque control can be accomplished.

### 4.2.4 Simulation Based on the Speed Estimation Scheme 2

Here the simulation is based on the speed estimation scheme 2 introduced in section 3.2.2.

#### 4.2.4.1 Speed Estimator

This speed estimation is almost the same as the first carrier signal, except that the derivative of the rotor speed is set as the cross product of the two rotor flux-linkage vectors. One of the two rotor flux-linkage vectors is from observer 1 and is not explicitly related with the rotor speed. The other one is from observer 2 and is explicitly related with the rotor speed. The cross product of these two vectors will be used as the input to a Proportional-Integral (PI) regulator. The output of the PI regulator is considered as rotor speed and used in model 2. This will form a closed loop for the estimation of rotor speed. The phase-locked loop will bring the speed estimation to the right value. The stability based on the two-time-scale approach and basic control stability theory is given in section 3.3.2. The equation of the speed estimator is given by:

\[
\dot{\omega}_{rc} = K_p \left( \hat{\lambda}_{rsc}^s T \hat{\lambda}_{rsc}^s \right) + K_i \int \left( \hat{\lambda}_{rsc}^s T \hat{\lambda}_{rsc}^s \right) dt\]  

(4.17)
The stator flux-linkage is first calculated. Rotor and stator fluxes can be expressed in term of stator current and rotor current as,

\[ \tilde{\lambda}_{rc}^{x} = L_{r} \tilde{i}_{rc}^{x} + L_{m} \tilde{i}_{sc}^{x} \]  \hspace{1cm} (4.18) 

\[ \tilde{\lambda}_{sc}^{x} = L_{s} \tilde{i}_{sc}^{x} + L_{m} \tilde{i}_{rc}^{x} \]  \hspace{1cm} (4.19) 

where the superscript \( ^{x} \) represents the reference frame. The stator current can be obtained by cancelling the rotor current in the above two equations as,

\[ \tilde{i}_{sc}^{x} = \frac{L_{s}}{\sigma^{2}} \tilde{\lambda}_{sc}^{x} - \frac{L_{m}}{\sigma^{2}} \tilde{\lambda}_{rc}^{x} \]  \hspace{1cm} (4.20) 

With the same method, the rotor current can be obtained as,

\[ \tilde{i}_{rc}^{x} = \frac{L_{r}}{\sigma^{2}} \tilde{\lambda}_{rc}^{x} - \frac{L_{m}}{\sigma^{2}} \tilde{\lambda}_{sc}^{x} \]  \hspace{1cm} (4.21) 

The rotor flux-linkage in model 1 therefore can be calculated by substituting equation (4.20) to equation (4.18) as,
\[
\dot{\lambda}_{rc}^x = L_r \left[ \frac{L_s}{\sigma^2} \dot{\lambda}_{rc}^x - \frac{L_m}{\sigma^2} \dot{\lambda}_{sc}^x \right] + L_m \ddot{\lambda}_{sc}^x \\
= \frac{L_r L_s}{\sigma^2} \ddot{\lambda}_{rc}^x - \frac{L_r L_m}{\sigma^2} \ddot{\lambda}_{sc}^x + L_m \dddot{\lambda}_{sc}^x
\]

\[
\frac{L_m^2}{\sigma^2} \dddot{\lambda}_{rc}^x = \frac{L_r L_m}{\sigma^2} \ddot{\lambda}_{sc}^x - L_m \dddot{\lambda}_{sc}^x
\]

\[
\dddot{\lambda}_{rc}^x = \frac{L_r}{L_m} \dot{\lambda}_{sc}^x - \frac{\sigma^2}{L_m} \dddot{i}_{sc}^x
\]

The rotor flux-linkage dynamic equation in reference frame x can be expressed as,

\[
\dot{\lambda}_{rc}^x = -J (\omega_x - \omega_{re}) \dot{\lambda}_{rc}^x - R_i \dddot{i}_{rc}^x
\]
So the rotor flux-linkage in model 2 can be obtained by substituting equation (4.21) into equation (4.23),

\[
\hat{\chi}^x_{rc} = -J (\omega_x - \omega_{re}) \hat{\chi}^x_{rc} - R_r \left( \frac{L_s}{\sigma^2} \chi^x_{rc} - \frac{L_m}{\sigma^2} \chi^x_{sc} \right)
\]

\[
= \frac{R_r L_m}{\sigma^2} \hat{\chi}^x_{sc} - \frac{R_r L_s}{\sigma^2} \chi^x_{rc} + J (\omega_{re} - \omega_x) \hat{\chi}^x_{rc}
\]

(4.24)

\(\omega_x\) is the speed of the reference frame, so in the carrier signal reference frame, \(\omega_x = -2\pi f_c\), while in the stationary reference frame, \(\omega_x = 0\). As mentioned above, the speed estimator is implemented in the carrier signal stator flux-linkage reference frame, so in the simulation, \(\omega_x = \omega_c\).

The structure of the speed estimator here is given in Figure 4.11.

4.2.4.2 Simulation Results and Conclusion

The simulation results are given in Figures 4.12, 4.13, 4.14 and 4.15. In Figure 4.12, the torque waveform shows both the command value and the actual value. The speed waveform shows both the actual value and the estimated value. Figure 4.12 shows that the torque controller works well at very low speed range. Figure 4.13 gives the corresponding rotor flux-linkage magnitude and fundamental frequency stator flux-linkage components in the stationary stator reference frame. Figure 4.14 shows that when the load torque changes, the speed estimator and the torque controller still work well, which means the algorithm is not sensitive to load conditions. Figure 4.15 gives the corresponding rotor flux-linkage magnitude and fundamental frequency stator flux-linkage components in the stationary stator reference frame.
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Fig. 4.14. Torque and speed waveforms when load torque changes. Simulation performed on a 3-phase, 4-pole induction machine.
Fig. 4.15. Rotor flux-linkage magnitude and stator flux-linkage waveforms when load torque changes. Simulation performed on a 3-phase, 4-pole induction machine.
The simulation results show that the speed estimation scheme 2 works well even at very low speed range. Based on the estimation of rotor speed, torque control is accomplished.

4.2.5 Discussion and Conclusion

(1) To have the system working properly, the rotor speed estimation should be fast and accurate enough so that the calculation of torque can follow the real value quickly. To achieve this, the difference between the fundamental frequency and the carrier frequency should be big enough, so that the two sets of signals can be separated and reconstructed correctly. The carrier signals are added after the main controllers, but to make sure that the signals are really added, the control of the induction machine should only be based on the fundamental frequency signals, otherwise the added carrier signals will be cancelled out by the closed-loop controller, as the carrier signals will be treated as noise and the feedback control loops have the ability to eliminate noise. Based on this, we should use two sets of signals, one for carrier signals and one for fundamental frequency signals, as shown in Figure 4.16.

Loop 1 is for carrier signals. The signals are extracted from the outputs of the induction machine model. The purpose of this loop is to estimate the rotor speed. Loop 2 is for the control of the fundamental frequency signals, such as the fluxes, the rotor speed, and the electromagnetic torque. The controllers in loop 2 utilize the rotor speed information obtained from loop 1. The controllers in loop 2 should only use the fundamental frequency signals. This can be done by transforming the terminal variables of the induction machine model to the carrier frequency reference frame, and using high
pass filters to extract the fundamental frequency signals and then transforming them back to the stationary stator reference frame.

Since the carrier signals are rotating in the opposite direction of the fundamental frequency signals, the frequency difference between the fundamental frequency signals \( \omega_f \) and the carrier signals is \( f_d = f_c - (-f_f) = f_c + f_f \). In the simulation we use \( f_c = 100 \text{Hz} \), so \( f_d \geq 100 \text{Hz} \). To separate the carrier signals from the fundamental frequency signals, we can transform the total signal into the carrier signal reference frame and then the carrier signals become DC signals, and therefore can be extracted with low-pass filters. The fundamental frequency signals become high frequency signals and can be obtained with high-pass filters. Or, we can transform the total signal into the fundamental frequency signal reference frame and get the fundamental frequency signals with low-pass filters and the carrier signals with high-pass filters. Both methods work but the former one will be easier to implement since \( f_c = 100 \text{Hz} \) is constant so that the carrier signal reference frame is easy to determine while \( f_f \) is always changing and it’s therefore not as simple to determine the fundamental frequency signal reference frame. Therefore the former method of separating signals is used in both the simulation and the experiments.

The separation of the carrier signals from the fundamental frequency signals requires that the frequency difference between them be sufficiently large, therefore \( f_c \) should not be too small, otherwise it will be difficult to separate the fundamental frequency signals from the carrier signals. But when the carrier frequency is large, the carrier slip frequency \( \omega_{sc} = \omega_c - \omega_{re} \) will be large and therefore the coefficient of the equation (3.40) will be too small, and the convergence speed of the speed estimator will
be slow. Therefore an optimal carrier frequency needs to be determined. Different carrier frequencies were tried with the simulation code, and finally $f_c$ was set at 100 Hz. With this carrier frequency, it’s easy to separate carrier signals from fundamental frequency signals, and also the rotor speed can be estimated quickly and accurately.

(2) In order to use carrier signals to estimate the rotor speed, the carrier signals need to be introduced to the induction machine. There are different ways to introduce the carrier signals. Here a carrier signal is introduced directly into the calculated voltage commands, as shown in Figure 4.17 and Figure 4.18.
Fig. 4.16. The two sets of signals in the control system.
Fig. 4.17. Injection of carrier signals that are rotating in the opposite direction of the fundamental frequency signals.

Fig. 4.18. Simulink model for injecting the carrier signals rotating in the opposite direction of the fundamental frequency signals.
In this way, the carrier signal voltage is considered as a vector with a given magnitude and rotating in the opposite direction of the fundamental frequency signals. The simulation results of this carrier signal injection method are given in Figures 4.7, 4.8, 4.9, 4.10, 4.12, 4.13, 4.14 and 4.15.
Chapter 5

Experimental Results

Based on the simulation results shown in chapter 4, experiments are implemented to test the feasibility of the speed estimation schemes. Positive results have been obtained. This chapter will present the details of the experimentation, including the hardware, software, experimental results, and discussion of the control system.

5.1 Induction Machine Used in the Experiment

The induction machine used in the experiment is a product of US Motor. The rated data of is shown in Table 5.1.

<table>
<thead>
<tr>
<th>Phase:</th>
<th>3 phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>V/I:</td>
<td>230 Volt/4.7 Amps</td>
</tr>
<tr>
<td>Frequency:</td>
<td>60 Hz</td>
</tr>
<tr>
<td>Power:</td>
<td>1.5 Hp</td>
</tr>
<tr>
<td>Nominal speed:</td>
<td>1745 rpm</td>
</tr>
<tr>
<td>Torque:</td>
<td>4.5 lb-ft</td>
</tr>
</tbody>
</table>

Table 5.1. Rated information of the induction machine for the experiments
To get an accurate estimation of the rotor speed, the parameters of the induction machine need to be appropriately measured. One way to measure the parameters is through no-load and blocked-rotor tests.

5.1.1 Parameter Measurement through No-load and Blocked-rotor Tests

The parameters needed for computing the performance of an induction machine can be obtained from the results of a no-load test, a blocked-rotor test, and measurements of the DC resistances of the stator windings [3]. With this method, the parameters for the induction machine used in the experiment are measured as in Table 5.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$</td>
<td>1.3667 Ω/phase</td>
</tr>
<tr>
<td>$R_r$</td>
<td>1.8267 Ω/phase</td>
</tr>
<tr>
<td>$L_s$</td>
<td>0.1058 H/phase</td>
</tr>
<tr>
<td>$L_r$</td>
<td>0.1058 H/phase</td>
</tr>
<tr>
<td>$L_m$</td>
<td>0.1003 H/phase</td>
</tr>
</tbody>
</table>

Table 5.2. Parameters of the induction machine measured with the no-load and blocked-rotor tests

5.1.2 Parameter Measurement through Recursive Method

The above method for parameters estimation determines five parameters: $R_r$, $R_s$, $L_r$, $L_s$ and $L_m$. However, the speed estimation scheme that is used in our system only requires four parameters: $\frac{R_r L_s}{\sigma^2}$, $\frac{R_r L_m^2}{\sigma^4}$, $\frac{L_r}{\sigma^2}$ and $\frac{1}{L_s}$. In the adaptive model, the dynamics of the auxiliary variable $\hat{x}$ is
\[
\dot{\tilde{x}} = \left( - \frac{R_r L_s}{\sigma^2} \mathbf{I} + \mathbf{J} \omega_{re} \right) \tilde{x} + \tilde{\lambda}_{sc} \tag{5.1}
\]

The calculation of \( \tilde{i}_s \) in term of \( \tilde{x} \) involves \( \frac{L_r}{\sigma^2} \) and \( \frac{R_r L_m^2}{\sigma^4} \) as

\[
\tilde{i}_{sc} = \frac{L_r}{\sigma^2} \tilde{\lambda}_{sc} - \frac{R_r L_m^2}{\sigma^4} \tilde{x} \tag{5.2}
\]

Another auxiliary vector \( \tilde{y} \) is

\[
\tilde{y} = \tilde{i}_{sc} - K_1 \tilde{\lambda}_{sc} \tag{5.3}
\]

where

\[
K_1 = \frac{1}{2} \left( \frac{1}{L_s} + \frac{L_r}{\sigma^2} \right) \tag{5.4}
\]

so that finally the derivative of the rotor speed is obtained as

\[
\dot{\omega}_{re} = K \tilde{y}^T_1 \mathbf{J} \tilde{y}_2 \tag{5.5}
\]
In order to estimate these four parameters, we use the locus of the stator current in the stator flux-linkage reference frame, which is shown in Figure 5.2. In the stator flux-linkage reference frame, the stator currents can be expressed in term of the magnitude of the stator flux as,

\[
\lambda_s^{i_{scd}} = \left( \frac{L_y}{\sigma^2} - \frac{R_r^2 L_m^2 L_s}{\sigma \omega_s} \right) \|\vec{\lambda}_{sc}\| \tag{5.6}
\]

\[
\lambda_s^{i_{scq}} = \frac{R_r L_m^2 \omega_s}{\left( \frac{R_r L_s}{\sigma^2} \right)^2 + \omega_s^2} \|\vec{\lambda}_{sc}\| \tag{5.7}
\]

Voltages at constant frequency are applied to the induction machine while the magnitude of the stator flux is kept approximately constant, and the rotor speed is determined by a dyne (shown in Figure 5.3) linked with the induction machine. The dyne is controlled by an UNI1405 inverter. The stator currents in the stator flux reference frame at different speed are recorded. A recursive program (in Appendix C) will calculate the four parameter groups.

Through this method, the four parameter groups are obtained as

\[
\frac{R_r L_s}{\sigma^2} = 88.7864 \ \frac{\Omega}{H}
\]
\[
\frac{R_r L_m^2}{\sigma^4} = 6699.3 \frac{\Omega}{H^2}
\]

\[
\frac{L_r}{\sigma^2} = 87.8229 \frac{1}{H}
\]

\[
\frac{1}{L_s} = 0.8547 \frac{1}{H}
\]

5.2 Inverter

With the calculated voltage commands, the duty cycles are calculated as

\[
D = \frac{v}{V_{bus}} + 0.5
\]  \hspace{1cm} (5.8)

A schematic of a three phase inverter is shown in Figure 5.4.

The duty cycles are sent to the inverter and the six IGBTs’ switching conditions are controlled by them. To avoid the shorting of the DC bus voltage, there is a delay between the moment of one switch going off and another one in the bridge going on. This time delay is called dead time, and the influence of the introduction of dead time is called dead time effect. During the dead time, both switches in one bridge are off,
so the potential of the output terminal is determined by the direction of the current. If
the positive direction of the current is set as flowing out of the bridge, then when the
current is positive, the output voltage is \( \frac{V_{bus}}{2} \), otherwise when the current is negative,
the output voltage is \(-\frac{V_{bus}}{2}\). Figure 5.5 [16] shows the waveforms of the PWM, the
output current and the dead time voltage.

The average dead time voltage is

\[
<V_{\text{dead}}(t)> = -\text{sgn}(i_{out}) \frac{2}{T_s} \int_{0}^{t_d} \frac{V_{bus}}{2} dt
\]

\[
= -\text{sgn}(i_{out}) \frac{T_s}{T_d} V_{bus}
\]

The average output voltage is

\[
<V_{\text{out}}(t)> = \left( D - \frac{1}{2} \right) V_{bus} - \text{sgn}(i_{out}) \frac{2}{T_s} \int_{0}^{t_d} \frac{V_{bus}}{2} dt
\]

(5.10)

The average dead time voltage will be a square wave 180° out of phase with the
(sinusoidal) output current. The dead time voltage will make the actual voltages applied
to the inverter different from the command voltages. When the command voltages are
very small, the dead time voltages will even overwhelm the command voltage. Also the
dead time voltages will add harmonics to the output voltages.

To minimize the influence of the dead time voltages, we can either
detect the direction of the output currents and add the compensation voltages
directly to the calculated command voltages before they are sent out to the inverter.
The compensation voltages will be in phase with the output current, which is

\[ v_{\text{compensation}} = \text{sgn}(i_{\text{out}}) \frac{t_d}{T_d} V_{\text{bus}} \]  

So after the influence of the dead time effect, the output voltage will be exactly
what we want. The Simulink code for this method is attached in Appendix D.

- minimize the dead time \( t_d \). From equation (5.9), the dead time average voltage
  is proportional to \( t_d \), so if we can minimize the dead time providing the safe
  operation of the inverter, the influence of the dead time effect can be controlled to
  some tolerable extent.

In the experiment, at first we used method 1 (software dead time compensation
to the command voltages). This method works well when the command voltages are big
enough which means at the same time, the phase currents are big enough. Since the
software compensation is based on the polarity of the phase currents, when the phase
currents are very small or distorted, the performance of the control system is poor.
Considering this project mainly focuses on the low speed range where both the command
voltages and phase currents are very small, the software dead time compensation method
is not the one suitable for our control system. However, it still can be used for the
medium-speed and high-speed range. In the experiment, the first inverter used was one
whose dead time can not be adjusted, so although we tried different software dead time compensation methods, including the one described above and some other methods, the speed estimation results were not satisfactory until we changed to a new inverter whose dead time was reduced to 1.2\(\mu s\).

5.3 Experimentation Results

Experiments have been implemented on a 3-phase, 4-pole induction machine rated at 1.5 HP, 60 Hz, 230 V line-line and 4.7 A. We use a dSpace 1103 card to implement the control algorithm. Hall-effect sensors measure the stator currents and differential voltage probes measure the stator voltages. A dyne is axially linked with the induction machine to serve as a load. The block diagram of the control system is shown in Figure 4.1. Figure 5.6 shows the picture of the entire control system. The sampling frequency of the control algorithm is 15 kHz.

5.3.1 Speed Estimation Scheme 1

The experimental results are shown in Figure 5.7 and 5.8. Figure 5.7 shows the actual and estimated rotor speed. The estimated rotor speed follows the actual rotor speed very well. Even when the fundamental frequency is zero, the rotor speed can still be estimated.

For a fixed torque \(\tau_e\), the slip frequency \(\omega_s\) is fixed. When decreasing the rotor speed \(\omega_{re}\) to a negative value whose magnitude is the same as the required \(\omega_s\), the electrical frequency \(\omega_e = \omega_s + \omega_{re}\) becomes zero. The small-signal model of the smooth air gap induction machine loses observability for this case if utilizing the fundamental
frequency signals to estimate the rotor speed, but based on the injected carrier signal (30 Hz), we can still estimate the rotor speed well and implement the torque regulation.

From the state of DC excitation, the torque command steps from 1.6 Nm to 2.0 Nm. The estimated rotor speed remains the same, and the electromagnetic torque follows the command well. Figure 5.8 illustrates the waveforms. The fundamental frequency rotor flux linkage and torque follow their command values very well so that the lines representing them in Figure 5.8 are almost exactly on the lines representing their command values. But the total rotor flux and torque contain ripple around the command value. The peak-to-peak value of the flux ripple is around 0.04 V.s. The peak-to-peak value of the torque ripple is around 1 N.m. Figure 5.9 shows the transient performance of the speed estimator.

So the rotor speed estimation, Scheme 1 as shown in section 3.2.1, works well at wide speed range even when the fundamental exciting frequency is DC.

5.3.2 Speed Estimation Scheme 2

Although the stability of the second estimation scheme has been proven and the simulation results show its feasibility, at this point, no positive experimental results have been achieved.

5.4 Conclusion

This chapter shows that the proposed speed estimation scheme 1 works well in a wide rotor speed range even when the exciting frequency is DC. The injected carrier signal also adds ripple to the torque. The peak-to-peak value of the torque ripple is
around 1 N.m. In the next chapter we will discuss how to reduce or eliminate the torque ripple.
Fig. 5.1. The induction motor used in the experiment

Fig. 5.2. The stator currents locus in the stator flux reference frame.
Fig. 5.3. The Dyne used to control the rotor speed of the induction machine.

Fig. 5.4. Three phase inverter.
Fig. 5.5. Output voltage waveforms considering dead time effect.
Fig. 5.6. The experimental setup.
Fig. 5.7. Real rotor speed and estimated rotor speed. Experiments performed on a 3-phase, 4-pole induction machine.

Fig. 5.8. Rotor flux and torque control based on the estimated rotor speed. Experiments performed on a 3-phase, 4-pole induction machine.
Fig. 5.9. Rotor speed steps from 270 rpm to 250 rpm to 220 rpm.
Chapter 6

Torque Ripple Minimization

With the proposed method of carrier signal injection, torque ripple exists in the electromagnetic torque. This chapter will discuss methods of minimizing the torque ripple.

In the rotor flux-linkage reference frame, the electromagnetic torque of the induction machine is given by:

$$\tau_e = \frac{3P L_m}{4 L_r} \|\tilde{\lambda}_r\| \|i^r\|_s q$$  \hspace{1cm} (6.1)

In section 5.3.1, the carrier signals are injected directly to the voltage commands, while the flux and torque controllers are only controlling the fundamental frequency components. The injected carrier signals generate carrier frequency flux and current, and these will distort the fundamental fluxes and currents and generate ripple in the electromagnetic torque. One method to eliminate the torque ripple is to apply the flux and torque controllers on the total signals, not only on the fundamental frequency signals, and to inject the carrier signals in the magnitude of the total rotor flux magnitude,

$$\|\tilde{\lambda}_r\| = \|\tilde{\lambda}_{rf}\| + A \cos (\omega_1 t)$$  \hspace{1cm} (6.2)
where \( ||\bar{\lambda}_{r,f}|| \) is the command for the fundamental frequency rotor flux-linkage and \( A\cos(\omega_1 t) \) is the injected carrier signal. To make the total torque constant, the total stator current in the total rotor flux-linkage reference frame is set as:

\[
\bar{i}_{rsqref} = \frac{\tau_{ref}}{3P L_m} \frac{1}{L_r} \bar{\lambda}_r \tag{6.3}
\]

In this proposed method, the carrier signals are equivalent to two sets of signals rotating from the fundamental frequency rotor flux-linkage in two opposite directions. The one in the opposite direction with the fundamental frequency signals is used for rotor speed estimation and its frequency in the stationary stator reference frame is \( \omega_c = \omega_{ef} - \omega_1 \). \( \omega_{ef} \) is the fundamental electrical frequency, and is always changing during the operation of the induction machine, which means \( \omega_c \) will be always changing as well. This is not encouraging for the rotor speed estimation and actually, experimental results show that, with this idea, the rotor speed can barely be estimated.

Other ideas have been tried to eliminate the torque ripple. At this point only two methods have been successful.

- **Application of torque regulator to the total torque.** This method is almost the same as the method presented in chapter 5.3.1. The carrier signals still are still added directly to the voltage commands. The flux controller is still regulating the fundamental frequency flux but the torque regulator now regulates the total torque. With a constant torque command, the total torque is well controlled and
torque ripple is significantly reduced. The experimental results are shown in Figure 6.1 and 6.2.

Figure 6.3 compares the total torques with and without this torque ripple reduction technique.

- **Compensation of torque ripple.** The injected carrier signals generate carrier frequency rotor flux-linkage and stator current vectors, therefore totally there are four vectors influencing the total electromagnetic torque: the fundamental frequency rotor flux-linkage vector, the fundamental frequency stator current vector, the carrier frequency rotor flux-linkage vector and the carrier frequency stator current vector. When the four vectors interact with each other, four torque components are generated:

\[
\tau_{ef} = \frac{3P}{4} \frac{L_m}{L_r} \|\bar{\lambda}_{rf}\| \|i_{sfq}\| \lambda_{rf}
\]

\[
\tau_{ec} = \frac{3P}{4} \frac{L_m}{L_r} \|\bar{\lambda}_{rc}\| \|i_{scq}\| \lambda_{rc}
\]

\[
\tau_{efc} = \frac{3P}{4} \frac{L_m}{L_r} \|\bar{\lambda}_{rf}\| \|i_{scq}\| \lambda_{rf}
\]
\[ \tau_{ecf} = \frac{3PL_m}{4L_r} \vec{\lambda}_{rc} \| i_{sfq} \]  

(6.7)

where \( \tau_{ef} \) and \( \tau_{ec} \) are the self-torques for the fundamental frequency signals and the carrier frequency signals respectively. \( \tau_{efc} \) is the torque generated by the fundamental frequency rotor flux-linkage and the carrier frequency stator current. \( \tau_{ecf} \) is the torque generated by the carrier frequency rotor flux-linkage and the fundamental frequency stator current. The torque ripple is mainly from \( \tau_{efc} \) and \( \tau_{ecf} \). If we compensate \( \tau_{efc} \) and \( \tau_{ecf} \) in the command for \( \tau_{ef} \), i.e.,

\[ \tau_{ef-new}^* = \tau_{ef}^* - (\tau_{efc} + \tau_{ecf}) \]  

(6.8)

the total torque will be constant. In the experiment, we use

\[ \tau_{ef-new}^* = \tau_{ef}^* - K (\tau_{efc} + \tau_{ecf}) \]  

(6.9)

where \( K \) is a constant to control the magnitude of the compensation. If \( K = 1 \), ideally the torque ripple will be eliminated completely. However, in the experiment, when \( K = 1 \), the rotor speed estimation performance is poor, so we choose \( K = 0.3 \) in the experiment. Thus the torque ripple is reduced and the rotor speed can still be estimated correctly. The experimental results are shown in Figure 6.4 and 6.5.
Figure 6.6 compares the total torques with and without this torque ripple reduction technique.
Fig. 6.1. The estimated speed and the real speed under torque control.

Fig. 6.2. Starting from DC fundamental frequency, torque command steps from 0.8 N.m. to 1.2 N.m. with total torque control.
Fig. 6.3. Total torque with and without torque ripple compensation.

Fig. 6.4. The estimated speed and the real speed with torque command compensation when $K = 0.3$. 
Fig. 6.5. Starting from DC fundamental frequency, torque command steps from 1.0 N.m. to 1.5 N.m. with torque command compensation when $K = 0.3$.

Fig. 6.6. Total torque with and without torque ripple compensation.
Chapter 7

Conclusion

This thesis presents a speed sensor-less control system of induction machine based on carrier signal injection and smooth-air-gap induction machine model.

Based on Hofmann and Sanders’ work [15], the observability of induction machine will vanish when the excitation frequency is zero. To obtain the rotor speed of an induction machine at DC excitation frequency, a carrier signal should be injected and used to estimate the rotor speed. Sng [33], Hinkkanen [41] and Leppänen [32] have developed a speed estimator based on the injected carrier signal and smooth-air-gap induction machine model. Sng’s speed estimator only works for systems with a high moment of inertia. Numerical differentiation is required for the implementation of this speed estimator, and is therefore subject to errors created by noise. Only locked-rotor experimental results are provided. Methods to reduce the torque ripple caused by the injected carrier signal are not provided. Hinkkannen and Leppänen’s speed estimation scheme only works for systems with low moment of inertia. The stability of their scheme is shown with simulation and experimental results but not rigorously proven. Our proposed speed estimation scheme works for systems with high moment of inertia. Stability of the proposed scheme is proven with two-time-scale approach with the assumption that the time constants of the electrical dynamics are much smaller than those of the mechanical dynamics. Experimental results show the proposed scheme works over a wide rotor speed range, even
at zero speed or zero fundamental frequency excitation. Transient waveforms are also
provided. Methods of reducing torque ripple caused by the injected carrier signal are
also provided.

The dead time effect should be considered in the control system. To obtain
optimal performance, dead time should be set as small as possible while ensuring safe
operation.

The speed estimation is based on the injected carrier signal. The carrier frequency
in our experiments was set at 30 Hz. If the carrier frequency is too small, it is difficult
to separate the carrier signals from the fundamental frequency signals. If the carrier
frequency is too big, however, the error signal is reduced convergence speed of the speed
estimation will get slow. For a specific control system, the optimal carrier frequency
should be determined with a trial-and-error method.

With the estimated rotor speed, rotor flux and torque controllers are developed
and experimental results show they work well. The work proposed in this thesis can be
used as the basis for future industrial applications.
Appendix A

Transformations

A.1 Reference Frame Transformation

Variables in the stationary stator reference frame can be transformed to another reference frame with the following relation [15]:

\[
x^{yz} = e^{-J \rho} x^{ss} = \begin{bmatrix}
\cos(\rho) & \sin(\rho) \\
-\sin(\rho) & \cos(\rho)
\end{bmatrix} x^{ss}
\]

(A.1)

where \( \rho \) is the angle of the reference frame of interest with respect to the stator. The reverse transformation is given by:

\[
x^{ss} = e^{J \rho} x^{yz} = \begin{bmatrix}
\cos(\rho) & -\sin(\rho) \\
\sin(\rho) & \cos(\rho)
\end{bmatrix} x^{yz}
\]

(A.2)

Variables in the stationary rotor reference frame can be transformed to another reference frame with the following relation [15]:

\[
x^{yz} = e^{-J (\rho - \theta)} x^{sr} = \begin{bmatrix}
\cos(\rho - \theta) & \sin(\rho - \theta) \\
-\sin(\rho - \theta) & \cos(\rho - \theta)
\end{bmatrix} x^{sr}
\]

(A.3)
where $\rho$ is the angle of the reference frame of interest with respect to the stator and $\theta$ is the angle between the direct axis of the stationary rotor reference frame and that of the stationary stator reference frame. The reverse transformation is given by:

$$x^{sr} = e^{J(\rho-\theta)}x^{yz} = \begin{bmatrix} \cos(\rho - \theta) & -\sin(\rho - \theta) \\ \sin(\rho - \theta) & \cos(\rho - \theta) \end{bmatrix} x^{yz} \quad \text{(A.4)}$$

### A.2 Transformations between two- and three-phase variables

Three-phase variables can be transformed to two-phase variables with the relation [15, 3]:

$$\begin{bmatrix} S_d \\ S_q \\ S_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} \quad \text{(A.5)}$$

Under balanced three-phase conditions there will be no zero-sequence components $S_0$, so in our implementation we assume it to be zero and do not calculate it. The reverse transformation is given as [15, 3]:

$$\begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{bmatrix} \begin{bmatrix} S_d \\ S_q \\ S_0 \end{bmatrix} \quad \text{(A.6)}$$
If we ignore $S_0$, the above equation can be written as [15, 3]:

\[
\begin{bmatrix}
S_a \\
S_b \\
S_c
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
-\frac{1}{2} & \frac{\sqrt{3}}{2} \\
-\frac{1}{2} & -\frac{\sqrt{3}}{2}
\end{bmatrix}
\begin{bmatrix}
S_d \\
S_q
\end{bmatrix}
\] (A.7)
Appendix B

Simulink Codes for Simulation

The simulink codes for simulation are illustrated. The codes are in the layer structure. Different group of functions are put together as a sub-system. In Simulink, when the sub-system is double clicked, the detail of them will be displayed in a separate window. Here both of the main structure and the detail of the sub-systems are presented.

B.1 Speed Estimator 1

This is the simulation code for the speed estimator based on the stator currents locus, i.e., $\dot{\omega}_r = K\dot{y}_1^T J\dot{y}_2$. 
Fig. B.1. Simulation code of 3-phase, 4-pole induction machine.
Fig. B.2. The induction machine model used in the main simulation code. Simulation performed on a 3-phase, 4-pole induction machine.
Fig. B.3. The flux calculation sub-system in the induction machine model.
Fig. B.4. The terminal currents and electromagnetic torque calculation sub-system in the induction machine model sub-system.
Fig. B.5. The rotor speed calculation sub-system in the induction machine model sub-system.

Fig. B.6. The controller sub-system in the main code of the simulation.
Fig. B.7. The rotor speed estimator sub-system in the controller sub-system.

Fig. B.8. The stator flux estimation sub-system in the speed estimator sub-system.
Fig. B.9. The first auxiliary vector $y_1$ calculation sub-system in the speed estimator sub-system.
Fig. B.10. The second auxiliary vector $y_2$ calculation sub-system in the speed estimator sub-system.
Fig. B.11. The block in the speed estimator sub-system to calculate the rotor speed.

Fig. B.12. The subsystem to implement the speed, flux and torque control in the controller sub-system.
Fig. B.13. The rotor speed regulator in the main control loop sub-system.

Fig. B.14. The flux regulator in the main control loop sub-system.
Fig. B.15. The torque regulator in the main control loop sub-system.
Fig. B.16. The sub-system to generate the voltage command with the injection of carrier signal in the controller sub-system.
B.2 Speed Estimator 2

This is the simulation code for the speed estimator based on the rotor fluxes, i.e.,
\[ \dot{\omega}_{re} = K \vec{\lambda}_r^T J \vec{\lambda}_r. \]
Basically the code for speed estimator 2 is the same as that for speed estimator 1, including the induction machine model, the main controller, the carrier signals injection block. The only difference is for the rotor speed estimator block, which is illustrated here.
Fig. B.17. The rotor speed estimator sub-system in the controller sub-system.
Fig. B.18. The first rotor flux vector calculation sub-system in the speed estimator sub-system.
Fig. B.19. The second rotor flux vector calculation sub-system in the speed estimator sub-system.

Fig. B.20. The block in the speed estimator sub-system to calculate the rotor speed.
Appendix C

The Recursive Code for Parameter Measurement

The Matlab code for the parameter measurement is attached here.

%*********************************************************************
% parametertest.m
%*********************************************************************

%This program is designed to measure the parameters of the induction machine
that is used in our speed sensorless control project. The original version is from Heath,
but there is some defection in that one. Here is the modified one from me.

%The reason why I did this experiment again is because that I have solved the
problem of the protection triggering of the IGBT’s at low rotor speed and measured
the parameters with the traditional methods (No-load test and blocked-rotor test). I
want to compare the two results of these two different methods.

%Guanghui Wang 12/05/2003

%*********************************************************************

%Clear the workspace

%*********************************************************************

clear

%*********************************************************************

%Experiment conditions
wrpm = [0 50 100 150 200 250 300 350 400 450 500 550 600 650 700 750 800 850 900 950 1000 1050 1100 1150 1200 ...
1250 1300 1350 1400 1450 1500 1550 1600 1650 1700 1750 1800];

wr = 4*pi*wrpm/60;

we = 2*pi*60;

ws = we - wr;

isde = [5.02 4.99 4.97 4.94 4.91 4.87 4.84 4.81 4.77 4.73 4.68 4.64 4.59 4.54 4.47 4.41 4.34 4.27 4.18 4.09 3.99 3.88 3.76 3.62 ...
3.47 3.30 3.12 2.91 2.68 2.43 2.16 1.87 1.57 1.27 0.99 0.75 0.58];

isqe = [1.56 1.57 1.59 1.61 1.63 1.65 1.67 1.69 1.71 1.74 1.76 1.79 1.82 1.85 1.87 1.91 1.94 1.97 2.01 2.04 2.08 2.12 2.15 2.18 ...
2.21 2.22 2.24 2.23 2.21 2.16 2.07 1.94 1.75 1.48 1.13 0.68 0.13];

flux = [0.0605 0.0605 0.0604 0.0604 0.0604 0.0603 0.0603 0.0603 0.0602 0.0602 0.0601 0.0600 0.0600 0.0599 0.0598 0.0598 0.0597 ...
0.0597 0.0596 0.0596 0.0595 0.0595 0.0594 0.0594 0.0594 0.0594 0.0594 0.0594 0.0594 0.0594 0.0594 0.0594 0.0594 0.0594 ...
0.0596 0.0597 0.0597 0.0599 0.0603 0.0603 0.0608 0.0614 0.0623 0.0635 ...
0.0650 0.0663 0.0657];
isdem = isde./flux;

isqem = isqe./flux;

% ******************************************************************************
% Options for the function
% ******************************************************************************

options = [];  
options(2) = 1e-8;
options(3) = 1e-8;

% ******************************************************************************
% initial conditions
% ******************************************************************************
a0 = 61.6486;
b0 = 7736.1;
c0 = 58.6314;
x0 = [a0 b0 c0];

length(ws);
length(isde);
length(isqe);
x = fnmins('parest', x0, options, [], isde, isqe, ws, flux);

parest(x, isde, isqe, ws, flux);
a = x(1)
b = x(2)
c = x(3)
isdc=\(c-(a*b)/(a^2+ws^2)\);

isqc=b*ws/(a^2+ws^2);

wsv=4*pi/60*linspace(-100,2000,10000);

isdcv=\(c-(a*b)/(a^2+wsv^2)\);

isqcv=b*wsv/(a^2+wsv^2);

plot(isdcv,isqcv,'-',isdc,isqc,'*',isdem,isqem,'x')

axis('equal')

hold on

for i=1:length(isde)

    xlinee=[0.5*(c+isdc(length(isdc))) isdem(i)];

    ylinee=[0 isqem(i)];

    xlinec=[0.5*(c+isdc(length(isdc))) isdc(i)];

    ylinec=[0 isqc(i)];

    plot(xlinee,ylinee,xlinec,ylinec)

end

grid on;

Lsinverse=isdc(length(isdc))

K=0.5*(c+Lsinverse)

K1=0.5*(c-Lsinverse)
function err = parest(x, isde, isqe, ws, flux)

a = x(1);

b = x(2);

c = x(3);

isdc = (c - (a * b) ./ (a ^ 2 + ws.^ 2)) .* flux;

isqc = b * ws ./ (a ^ 2 + ws.^ 2) .* flux;

err = norm(isde - isdc) + norm(isqe - isqc);
Appendix D

Simulink Codes for Dead Time Compensation
Fig. D.1. Voltage generation with dead time compensation.
Fig. D.2. Dead time compensation voltage generation.
References


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Guanghui Wang’s research interest includes power electronics, circuit design, control and energy management.