DEVELOPING ELEMENTARY PRE-SERVICE TEACHERS’ MATHEMATICAL KNOWLEDGE FOR TEACHING: ENGAGING IN AN ADAPTED LESSON STUDY CYCLE

A Dissertation in Curriculum and Instruction

by

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ABSTRACT

This study examines the impact that an adapted lesson study had on the development of elementary pre-service teachers’ (PSTs) mathematical knowledge for teaching (MKT) within a professional development school (PDS) context. Research suggests that PSTs enter teacher education lacking the subject matter knowledge needed to teach mathematics. This study represents an effort to identify a model for PST learning that can be used to continue to develop PSTs’ MKT after completing coursework. Using Shulman’s pedagogical reasoning to adapt lesson study for PST learning, I created the pre-service teacher lesson study cycle (PST-LSC). This qualitative research study used case study methodology to investigate and analyze the development of four PSTs’ MKT while participating in the PST-LSC. To analyze growth in the PSTs’ MKT, pre- and post-interviews were conducted. Additionally, audio-recordings of discussions, lesson study artifacts, and PST journal responses were collected over the course of the PST-LSC. The data were analyzed thematically using a priori codes and subcodes related to the MKT framework in order to identify how the PSTs’ MKT developed throughout the PST-LSC. The data suggests the PSTs’ MKT deepened as a result of participating in the PST-LSC. More specifically, the PST-LSC provided PSTs with opportunities to develop all six domains of MKT: common content knowledge (CCK), specialized content knowledge (SCK), knowledge at the mathematical horizon, knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of curriculum. The results of the study also indicate that the sharing of mathematical strategies, cross grade-level discussions, readings, collaboration, curricular resources, observation, and reteaching involved in the PST-LSC were catalysts for the gains in MKT. This research has implications for teacher educators who teach mathematics methods courses and who supervise PSTs in field experiences.
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Chapter 1

Introduction

As a third-grade teacher in Phoenix, Arizona for ten years, I experienced many different forms of professional development, but one of them stands out more than any other. One year, the administrators and learning specialists within my school required teachers at each grade level to collaboratively observe and reflect upon the mathematics teaching of their colleagues. While this learning experience provided teachers the opportunity to see other teachers in action, the lack of content exploration and co-planning prior to the observations of instruction resulted in reflective discussions that were difficult, uncomfortable, not forthright, and, ultimately, not very reflective. Although this experience did not lead to huge gains in knowledge of how to teach mathematics, it made me wonder what knowledge is needed to teach elementary mathematics and how collaborative learning opportunities can be structured to maximize learning. My curiosities deepened as I studied, taught, and supervised in a teacher education program, where my work with elementary pre-service teachers pushed me to examine the similarities and differences between in-service and pre-service teacher learning. The study described here is the result of my wonderings and desire to improve my own knowledge and practice as a teacher educator.

Research suggests that elementary pre-service teachers (PSTs) enter teacher education programs lacking the subject matter knowledge needed to teach mathematics
for a conceptual understanding (Ball, 1990; Li & Kulm, 2008; Linsell & Anakin, 2012; Ryan & McCrae, 2005-2006; Senk et al., 2012). Ensuring PSTs enter the world of teaching with a rich knowledge of mathematics is important because teachers’ mathematical knowledge for teaching (MKT) is connected to their quality of instruction (Charalambous, 2010; Charalambous, Hill, & Mitchell, 2012; Hill and Charalambous, 2012a, 2012b; Hill, Blunk, et al., 2008; Sleep and Eskelson, 2012; Wilhelm, 2014) and may positively impact student achievement (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Ngo, 2013). Therefore, learning to teach mathematics effectively demands that PSTs deepen their subject matter knowledge and develop knowledge that is specific to teaching (Ball, 1990; Ball, Thames, & Phelps, 2008). While teacher educators have incorporated mathematics content into coursework (Burton, Daane, & Giesen, 2008; Charalambous, 2015; Smith, Swars, Smith, Hart, & Haardorfer, 2012; Welder & Simonsen, 2011; Whitacre & Nickerson, 2016; Youngs & Qian, 2013) and have researched pedagogical practices that promote the development of MKT (Lai & Lo-Fu, 2013; Laursen, Hassi, & Hough, 2016; Turner, 2012; Tyminski, Drake, & Land, 2013), PSTs often still lack the knowledge needed to effectively teach elementary mathematics. A possible solution to this dilemma is to figure out a structure for continual learning post-coursework.

Returning for a moment to my own personal experiences, I had the opportunity during my first semester of doctoral coursework to participate in a lesson study focused on developing a data lesson for an elementary mathematics methods course. Throughout the lesson study, I collaboratively explored content and co-planned, taught, reflected on, and re-taught a lesson. As a result, not only did I gain knowledge of mathematics and
teacher education, I also developed a deeper understanding of the knowledge that
elementary teachers need to teach mathematics and a potential structure to develop that
knowledge. Then, during the second year of my program, I began a new assistantship that
provided me the opportunity to teach a mathematics methods course and supervise
student teachers. In my new role as supervisor, I struggled to know how to best support
my PSTs’ learning. Thinking about my own learning while participating in lesson study, I
began to wonder how and if lesson study could be used in supervision to address PSTs’
MKT. Therefore, I developed and studied an adapted lesson study model within an
elementary teacher education program.

Considering PSTs’ limited subject matter knowledge, the adapted lesson study
included the exploration of content and ways to make the content comprehensible to
students prior to beginning the lesson study cycle. Studying the impact an adapted lesson
study has on PSTs’ MKT may benefit the field of teacher education by providing a model
that supervisors can implement with PSTs during student teaching and post-coursework.
Therefore, the current study sought to explore the following research questions: 1) How
does the MKT of elementary PSTs develop as they engage in an adapted lesson study?
and 2) How do the activities of the adapted lesson study provide opportunities for the
development of MKT?

**Theoretical Framework**

In this section, I describe the theories and models that influenced the design of my
research. First, I briefly explain MKT. Then, I explore both pre-service and in-service
teacher (IST) learning theories. The section concludes with a description of my adapted
lesson study.
Mathematical Knowledge for Teaching

Hill, Ball, and Schilling (2008) define MKT as “the mathematical knowledge that teachers use in classrooms to produce instruction and student growth” (p. 374).

Consistent with Shulman’s (1986) argument that both content and pedagogy need to be considered key components of teacher knowledge, Hill, Ball, et al.’s proposed framework (see Figure 1.1) for MKT includes two main constructs: subject matter knowledge (SMK) and pedagogical content knowledge (PCK); therefore, unlike the frameworks proposed by researchers previously, Hill, Ball, et al.’s MKT framework does not account for general pedagogical knowledge or the knowledge of context, but instead zooms in on what Shulman considered content knowledge in teaching. Both SMK and PCK encompass three more clearly defined domains of teacher mathematical knowledge.

**Figure 1.1.** Hill, Ball, et al.’s (2008) framework for mathematical knowledge for teaching
Hill, Ball, et al. (2008) theorize that SMK consists of the following three domains: common content knowledge (CCK), specialized content knowledge (SCK), and knowledge at the mathematical horizon. While CCK is mathematical knowledge which is in common with other professions, SCK is the mathematical knowledge that is unique to the teaching profession (Ball et al., 2008; Hill, Schilling, & Ball, 2004). The third domain, knowledge at the mathematical horizon, “is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (Ball et al., 2008, p. 403). Again, just as SMK is divided into three separate domains, so too is PCK. Knowledge of content and students (KCS) and knowledge of content and teaching (KCT) are constructs that combine knowing about mathematics with knowing about students and knowing about teaching. More specifically, KCS consists of knowledge of student conceptions and misconceptions, common student errors, and mathematics topics that students find challenging. While KCT requires knowing how to best remedy student errors, build on students’ mathematical thinking, and evaluate mathematical representations. Lastly, knowledge of curriculum refers to the curricula programs and materials used to teach subject matter (Ball et al., 2008; Hill et al, 2004). While MKT and other teacher knowledge frameworks suggest teachers need to understand content and pedagogy in deep and meaningful ways, there is still a need to know more about how PST knowledge develops.

Pre-Service Teacher Learning

At the beginning of this century, the National Academy of Education’s Committee on Teacher Education (CTE) considered “the implications for the curriculum of teacher education of what the field [had] learned about effective learning and teaching,
as well as about the learning of teachers” (Darling-Hammond & Bransford, 2005, p. vii). 

Darling-Hammond and Bransford (2005) describe the results of the CTE’s work in Preparing Teachers for a Changing World: What Teachers Should Know and Be Able to Do, outlining the knowledge PSTs need to teach and effective ways teacher education programs can produce well-prepared teachers.

In order to aid PSTs in obtaining the knowledge needed to be well-prepared beginning teachers, teacher education programs must address the three problems in learning to teach: the apprenticeship of observation, the problem of enactment, and the problem of complexity (Darling-Hammond & Bransford, 2005). Lortie (1975) defines the apprenticeship of observation as the preconceptions of teaching that result from being a student for many years prior to learning to teach. These preconceptions can be problematic because they likely give PSTs a skewed vision of the nature of teaching (Grossman, 1990), and PSTs must also come to think about teaching in ways fundamentally different from how they were taught (Magnusson, Krajcik, & Borko, 1999). The problem of enactment is the need to connect what one knows about teaching to the actual act of teaching (Grossman, 1990; Magnusson et al., 1999); putting what one knows into action is not an easy task. Lastly, the problem of complexity refers to the complex nature of teaching. Lampert (2010) describes the complex work of teaching as “managing problems in several domains of work while maintaining productive relationships with students and content (p. 21).

Based on the need to address the three problems of learning to teach, Darling-Hammond and Bransford (2005) propose three principles for helping PSTs learn to teach. These three principles are: 1) addressing student preconceptions, 2) learning for
understanding and enactment, and 3) metacognition. A brief description of each of these three principles follows.

**Addressing student misconceptions.** Although PSTs’ experience in classrooms as students helps them develop understandings about teaching, often this experience results in misconceptions. PSTs have only seen teaching from the viewpoint of a student, this limited perspective does not allow PSTs to observe the underlying knowledge, skills, planning, and decision making that is required of teachers (Grossman, 1990; Darling-Hammond & Bransford, 2005). One way to address PSTs’ misconceptions about teaching is to provide opportunities for teachers to examine, elaborate, and integrate new knowledge and beliefs about teaching and learning into their existing system of knowledge and beliefs. This goal can be addressed through activities such as observing, analyzing, and reflecting upon one’s own or another’s teaching. (Magnusson et al., 1999, p. 122)

**Learning for understanding and enactment.** As the apprenticeship of observation suggests, PSTs cannot learn to teach by strictly observing the act of teaching. In the words of Darling-Hammond and Bransford (2005),

Teachers must learn to weigh difficult dilemmas and to make and implement decisions on the fly; to put their plans into action effectively as well as to alter plans for unforeseen circumstances while they are in the midst of teaching; to respond to children and to represent well the material they are teaching. (p. 370)

To learn to teach, PSTs must be given opportunities to practice and reflect upon teaching (Grossman, 1990; Magnusson et al., 1999).
Metacognition. “People with high levels of metacognitive awareness have developed habits of mind that prompt them to continually self-assess their performances and modify their assumptions and actions as needed” (Darling-Hammond & Bransford, 2005, p. 376). Flavell (1979) defines two aspects of metacognition: metacognitive knowledge and metacognitive regulation. Metacognitive knowledge is understanding one’s own thinking and developing strategies for gaining more knowledge, while metacognitive regulation refers to one’s ability to define learning goals and monitor progress toward achieving them. Building such reflective skills aids PSTs in combatting the complexity of teaching and allows for improvement in practice.

A practice-based approach. Considering the suggested ways to address the problems of learning to teach include observing, practicing, analyzing, and reflecting upon the act of teaching, it is not surprising that, as previously mentioned, researchers have argued for a practice-based approach and have defined a set of pedagogies for teacher education. These pedagogies include clinical experiences, performance assessments (microteaching, performance tasks, and portfolios), analysis of teaching and learning, case methods, autobiography, and inquiry (Ball & Forzani, 2009; Darling-Hammond & Bransford, 2005; Forzani, 2014; Grossman, Compton, et al., 2009; Grossman, Hammerness, & McDonald, 2009; Hollins, 2011; Lampert, 2010; McDonald, Kazemi, & Kavanagh, 2013; Zeichner, 2012). These pedagogies of teacher education “are intended to support teachers’ abilities to learn in and from practice” (Darling-Hammond & Bransford, 2005, p. 441), but allow PSTs to do so in varying degrees. Making connections between the PST education and IST professional development worlds may help researchers further explore ways to situate PSTs’ learning in practice.
**In-Service Teacher Learning**

Pre-service teacher education is focused on using practice-based methods to prepare PSTs for the world of teaching. Because the context of PSTs’ learning will shift once they enter the profession, it is important to know the theories that drive the professional development of ISTs. IST learning occurs when the work of teachers is situated, collaborative, continuous, and inquiry-based. When teacher learning is situated in the daily work of teachers and focused on content knowledge and instructional practices, teachers grow professionally (Cochran-Smith & Lytle, 2001; Croft, Coggshall, Dolan, Powers, & Killion, 2010; Putnam & Borko, 2000; Webster-Wright, 2009; Wei, Darling-Hammond, Andree, Richardson, & Orphanos, 2009). Additionally, when teachers are invested in the learning of all students and participate collaboratively in professional communities, teacher growth occurs because teachers learn from the knowledge and experiences of others (Croft et al., 2010; Darling-Hammond & McLaughlin, 2011; Putnam & Borko, 2000; Wei et al., 2009); and, when the work of professional communities occurs continuously, teachers develop deep understandings and solve specific problems found in teaching practice (Cochran-Smith & Lytle, 2001; Darling-Hammond & McLaughlin, 2011). Moreover, when teachers actively participate in inquiry, the result is authentic teacher learning with an improvement in practice (Cochran-Smith & Lytle, 2001; Croft et al., 2010; Darling-Hammond & McLaughlin, 2011; Webster-Wright, 2009). Since IST learning results from work that is situated, collaborative, continuous, and inquiry-based, models developed to promote IST learning are typically focused on these catalysts. Two such models, pedagogical reasoning and Japanese lesson study, and the impact each has on teacher learning are described below.
**Pedagogical reasoning.** Based on studies of dozens of teachers, Shulman (1987) proposed a process teachers go through in order to transform their own knowledge into content that is comprehensible to students. Through his research, Shulman realized that understanding alone is not enough to teach. He argued that “understanding...must be linked to judgement and action, to the proper uses of understanding in the forging of wise pedagogical decisions” (Shulman, 1987, p. 14). Pedagogical reasoning involves “a cycle through the activities of comprehension, transformation, instruction, evaluation, and reflection. The starting point and terminus for the process is an act of comprehension” (Shulman, 1987, p. 14). Shulman’s model of pedagogical reasoning appears in Figure 1.2.

![Pedagogical Reasoning Model](image)

*Figure 1.2. Shulman’s (1987) model of pedagogical reasoning*

**The impact of pedagogical reasoning on teacher knowledge and practice.**

Very little research has been conducted that focuses on using Shulman’s (1987) pedagogical reasoning model of teacher learning; instead, researchers have generally looked at how participating in certain activities reveals or influences teachers’ pedagogical reasoning. Additionally, four out of the five studies explored the pedagogical reasoning of PSTs, but in the content area of science.
In a pilot study, Peterson and Treagust (1992) used problem-based learning (PBL) as a framework for developing PSTs’ pedagogical reasoning skills. Working in small groups, 20 second-year PSTs were asked to plan a resolution to a problem. After completing the PBL task, the PSTs answered a written questionnaire concerning their views on problem-based learning. The PSTs believed PBL gave them more control over their learning, which was a motivating factor. Additionally, the participants developed their understandings from their current knowledge base and developed a better understanding of the subject matter, curriculum materials, and planning. However, the questionnaire revealed the PSTs’ difficulty in transforming ideas to fit the needs of students at a particular grade level.

Based on the results of their pilot study, Peterson and Treagust (1995) argued that to develop PSTs’ pedagogical reasoning, the “teaching approach must consider the learning environment and allow PSTs to experience each of the [pedagogical] reasoning stages, and integrate the knowledge required as part of this process” (p. 292). Therefore, Peterson and Treagust adjusted their PBL approach to include opportunities for PSTs to participate in each of the pedagogical reasoning stages. The new problem-based task required PSTs to explore a set of activities on a specific science topic, plan and teach the topic to a peer, and then evaluate, reflect, and develop a new comprehension. To support the PSTs’ development, Peterson and Treagust included a journal of guiding questions that corresponded to the six stages of pedagogical reasoning.

Analysis of interviews, journals, field observations, and surveys revealed that PSTs developed their science content and curriculum knowledge during the comprehension phase. Additionally, the transformation phase allowed PSTs to realize the
importance of understanding the prior knowledge of their students when planning to teach. Too, while evaluating their lessons, PSTs recognized “the need for improved explanations of science ideas, better sequencing of ideas, having more activities and demonstrations to support science concept development, and having a clear understanding of the science concepts themselves prior to teaching” (Peterson & Treagust, 1995, p. 302). Finally, through the use of case study methods, Peterson and Treagust (1998) carefully examined two of the PSTs from the aforementioned study to further understand the knowledge that was gained by participating in the PBL activities. Peterson and Treagust found that the group discussions during the PBL allowed the two PSTs to discover issues in their knowledge of teaching; more specifically, the PSTs realized the limitations of a teacher-centered approach to instruction and a weak ability to evaluate student learning.

Similar to Peterson and Treagust (1995, 1998), Nilsson (2009) used the process of pedagogical reasoning “to systematically elucidate different critical incidents that student teachers experienced in order to develop deeper understandings of the complex task of learning to teach primary science” (p. 243). During Nilsson’s study, 22 elementary science student teachers planned, taught, and evaluated a small group science lesson. By teaching the small group lessons, the PSTs realized that to learn to teach, they must: analyze their teaching and their students’ performance, transform theoretical knowledge into teaching practice, and reflect upon how their teaching may be different in the future. As a result of these realizations, the student teachers learned they have the following needs in order to teach: content knowledge, a large repertoire of activities, knowledge of
students’ prior experiences and learning needs, and self-reflection, which were also reflected in Peterson and Treagust’s (1995) findings.

Working with five middle school language arts ISTs, Pella (2015) investigated pedagogical reasoning as teacher professional development through the use of lesson study. Pella’s research focused on nine, 4-6 week cycles of lesson study that spanned three years. By analyzing field notes, audio recordings, interviews, and reflections, Pella found that through participating in lesson study and the process of pedagogical reasoning, the five teachers were able to make pedagogical shifts in their knowledge. The common theme between each of the ISTs’ pedagogical shifts “was away from the view of writing as the isolated teaching and learning of ‘rules’…and toward the view of writing as an integrated communicative process…” (Pella, 2015, p. 93). Interviews that were conducted two years after the lesson study project ended revealed that these pedagogical shifts were sustained over time. The shifts that Pella’s participants exhibited in the lesson study project mimic Peterson and Treagust’s (1998) finding that the PSTs’ collaborative discussions allowed them to see the limitations in their own teaching knowledge.

**Japanese lesson study.** Another model that incorporates the catalysts for teacher learning is *Japanese lesson study (JLS)*, which is a collaborative effort in which teachers improve instruction through the planning, analysis, and revision of a lesson. When participating in JLS, teachers plan lessons together, watch these lessons unfold in a classroom setting, and discuss their observations, resulting in a refinement of teaching and student learning (Fernandez & Yoshida, 2004). Even though JLS is based on the planning and teaching of a lesson, its main focus is on student learning; therefore, the planning session considers expected student responses, observations are focused on
students’ actual answers, and the debriefing focuses on student learning. The JLS cycle includes the following steps: 1) define a goal for the study, 2) plan a lesson, 3) teach the lesson while all others observe, 4) reflect on the lesson, 5) revise the lesson, 6) teach and observe the altered lesson, 7) reflect on the second lesson, and 8) share the results (Cheng & Yee, 2011; Stigler & Hiebert, 2009). These steps appear in Figure 1.3.

![Lesson study cycle diagram](image)

*Figure 1.3. Stigler & Hiebert’s (2009) lesson study cycle*

**Lesson study with in-service teachers.** After the Trends in Mathematics and Science Achievement Study (TIMSS) released results revealing not only an achievement gap between Japanese and American students, but that Japanese teachers were teaching differently from their American counterparts, the Japanese model of lesson study was seen as the defining factor and U.S. mathematics educators were intrigued (Cady,
Hopkins, & Hodges, 2008; Hart, 2008; Inoue, 2011). With the success that lesson study has garnered in Japan, other countries, including the United States have begun to study the impact and benefits of lesson study on ISTs’ knowledge and practice.

Researchers have found that teachers better understand the importance of listening and responding to student thinking after participating in lesson study (Cheng & Yee, 2011; Lewis, Perry, & Hurd, 2009; Perry & Lewis, 2009). For example, Perry and Lewis (2009) conducted a 5-year lesson study project with 70 K-8 ISTs and administrators in a California school district and found that with an appropriate amount of support and structures in place during lesson study, teachers anticipated student responses to questions more often when planning. Similarly, Cheng and Yee (2011) discovered that four upper elementary ISTs participating in lesson study in Singapore more frequently expected multiple interpretations of concepts and used student mistakes to improve instruction. Involvement in lesson study can improve teacher attention to prior knowledge and student thinking, and this increase can then prompt changes in teaching knowledge and practice.

Zooming in on student thinking during lesson study can help teachers analyze the types of questions posed, the selection of tasks, and the facilitation of mathematical discussions (Dudley, 2013; Hunter & Back, 2011; Robinson & Leikin, 2011; Suh & Seshaiyer, 2015). For example, during the teaching of the second lesson, teachers not only pose questions more clearly, but also ask more open-ended questions (Robinson & Leikin, 2011). Through studying five elementary ISTs participating in a lesson study in the United Kingdom, Dudley (2013) realized this shift in teaching practice may be the result of teachers seeing students’ capabilities in answering such questions during the first
lesson. Also, an outcome of lesson study is teachers’ realization of the importance of carefully selecting and sequencing tasks to promote better learning (Hunter & Back, 2011; Suh & Seshaiyer, 2015). Carefully selecting tasks results in students solving problems that promote a deeper understanding of mathematics. Robinson and Leiken (2011) examined one Israeli teachers’ growth while participating in a lesson study with a team of elementary mathematics teachers and found that when the teacher posed more conceptually demanding tasks, the teacher’s class discussions became lengthier, revealing a need to work on facilitation skills. Improving such skills allows teachers to develop a new understanding of the importance of analyzing students’ incorrect answers and organizing students’ responses in meaningful ways (Hunter & Back, 2011). The level of analysis of student thinking that lesson study requires pushes teachers to think about teaching mathematics in more conceptual ways, revealing lesson study as an effective catalyst for teacher learning.

*The impact of collaboration.* Lesson study provides a learning community in which teachers can collaborate. When teachers participate in lesson study, they are given the opportunity to discuss subject matter, teaching, and learning, allowing them to comprehend the importance of such collaboration (Lieberman, 2009; Puchner & Taylor, 2006). Once the importance of collaboration is realized, teachers develop: a stronger commitment to instructional improvement, an inquiry stance, accountability to colleagues and students, shared long-term goals for student learning, and their processes for working with each other (Lewis et al., 2009; Lieberman, 2009). Additionally, once comfortable with collaboration, teachers allow themselves to be more open and vulnerable to different teaching and learning approaches (Lieberman, 2009). When considering all of the effects
of collaboration, one could definitely make the argument that, due to its collaborative structure, lesson study is an effective way for teachers to continue to refine their knowledge and practice.

The collaborative nature of lesson study also results in an improvement to planned lessons and instruction (Lieberman, 2009; Puchner & Taylor, 2006). Lieberman (2009) analyzed the work of seven middle school teachers in California that had been participating in lesson study for seven years. These seven teachers developed in-depth lessons as a team rather than planning instruction separately, which resulted in a higher quality lesson plan. Seeing the results of their collaboration allowed the teachers to gain confidence, which helped them to be more comfortable with adapting materials, thinking critically about curriculum, and taking risks when teaching. Similarly, Puchner and Taylor (2006) conducted two lesson studies with 17 elementary ISTs in Illinois and discovered that the teachers’ higher quality lesson plans meant instruction improved; more specifically, co-planning a lesson increased student engagement and learning. Once the teachers experienced the positive impact that developing lessons collaboratively had on instruction, their confidence and self-efficacy increased. Lesson study as a way to improve lesson plans, instruction, and teacher confidence further reveals its strengths as a catalyst for teacher learning.

**The Pre-service Teacher Lesson Study Cycle**

Although Shulman’s (1987) model of pedagogical reasoning is based on the work of ISTs, he suggests that “teacher education should provide students with the understandings and performance abilities they will need to reason their ways through and to enact a complete act of pedagogy” (p. 19). Additionally, even though Japanese lesson
study is a model designed for IST learning, many researchers have explored its use with PSTs (Amador & Weiland, 2015; Amador, Carter, & Hudson, 2016; Amador & Carter, 2018; Chassels & Melville, 2009; Fernandez & Zilliox, 2011; Leavy, 2015; Leavy & Hourigan, 2016; Leavy & Hourigan, 2018; Myers, 2012; Myers, 2013; Suh & Fulginiti, 2012; Suh & Parker, 2010). Combining aspects of the two models could be a useful practice-based approach that allows PSTs to address the three problems of learning to teach while also developing their MKT.

This Study

In this study, I developed and implemented a lesson study cycle that combines aspects of both Shulman’s (1987) model of pedagogical reasoning and Japanese lesson study. Using these two models of IST learning allowed me to not only situate PSTs’ learning in a collaborative environment, but it also provided PSTs with opportunities to address their subject matter knowledge, pedagogical content knowledge, and reflective skills. The framework appears in Figure 1.4.
Figure 1.4. The pre-service teacher lesson study cycle (PST-LSC)

The pre-service teacher lesson study cycle (PST-LSC) includes Darling-Hammond and Bransford’s (2005) three principles for helping PSTs learn to teach, addressing student preconceptions, learning for understanding and enactment, and metacognition. The first two phases help PSTs address their preconceptions about the subject matter and the teaching of the subject matter, while the opportunities to plan, teach, reflect upon, and adjust a lesson plan embed PSTs’ learning in practice and support the development of metacognition. The PST-LSC also provides an opportunity for PSTs to develop their knowledge and practice within a collaborative atmosphere. Therefore, the PST-LSC reflects current research of PST education and IST professional development.
Chapter 2

Literature Review

In this chapter, I discuss the relevant research related to the focus of this study. The chapter is organized into two main sections. First, I describe the types of opportunities in teacher education that have been studied for their impact on PSTs’ development of MKT. I continue by examining what researchers have explored when conducting lesson study with elementary PSTs. I conclude the chapter by describing what my study can potentially contribute to the research field.

The Development of Mathematical Knowledge for Teaching

A review of the literature was conducted to see how teacher educators have addressed and researched elementary PSTs’ development of MKT. Due to my overall focus on MKT development, the review only includes studies that explored elementary PSTs’ development of MKT and does not include studies that investigated a growth in the ability to enact a core practice of teaching (i.e. facilitating mathematical discussions or providing mathematical explanations). Additionally, the studies reviewed specifically used Hill, Ball, et al.’s (2008) framework for MKT as the unit of analysis and include MKT in the findings. Lastly, because my research is focused on a specific opportunity or instructional intervention, so do the studies reviewed. The results of the review showed that teacher educators have researched the development of PSTs’ MKT through the use of the following practices: incorporating mathematics content into PST education programs, incorporating inquiry-based learning into elementary mathematics content courses, conducting learning study with PSTs, using Standards-based curriculum materials, and reflection. Within each section, a description of the purpose, methods or
methodology, opportunity, and PST demographics are provided for each study discussed (see Table 2.1). In addition to the details of the research conducted in each study, a synthesis of the findings related to the impact that an opportunity had on the development of PSTs’ MKT is also reported.

Table 2.1.

Studies Exploring the Development of MKT

<table>
<thead>
<tr>
<th>Authors</th>
<th>Methods/Methodology</th>
<th>PST Demographics</th>
<th>Opportunity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burton, Daane, and Giesen (2008)</td>
<td>Quantitative, pre/post-test design</td>
<td>44 elementary PSTs at a Southeastern university</td>
<td>20 minutes of mathematics content in a methods course</td>
</tr>
<tr>
<td>Youngs and Qian (2013)</td>
<td>Quantitative, MKT survey</td>
<td>100 elementary PSTs from 4 different universities in China</td>
<td>Number of content courses or number of mathematics topics</td>
</tr>
<tr>
<td>Welder and Simonsen (2011)</td>
<td>Quantitative, pre/post-test design</td>
<td>48 elementary PSTs at a western U.S. university</td>
<td>A mathematics content course for elementary teachers</td>
</tr>
<tr>
<td>Whitacre and Nickerson (2016)</td>
<td>Mixed-methods, pre/post-test design and interviews</td>
<td>34 elementary PSTs at a large, urban university in the SW U.S.</td>
<td>The first course of a four-course sequence of mathematics content courses for elementary PSTs</td>
</tr>
<tr>
<td>Charalambous (2015)</td>
<td>Quantitative, pre/post-test design</td>
<td>3 K-8 PSTs at a large Midwestern university</td>
<td>A content-and-methods course sequence</td>
</tr>
<tr>
<td>Smith, Swars, Smith, Hart, and Haardorfer (2012)</td>
<td>Quantitative, post-test</td>
<td>276 elementary PSTs at a large, urban university in the SE U.S.</td>
<td>3 content/2 methods courses vs. 4 content/1 methods course</td>
</tr>
<tr>
<td>Laursen, Hassi, and Hough (2016)</td>
<td>Mixed-methods, pre/post-test design and interviews</td>
<td>184 elementary, middle, and secondary PSTs from two research universities</td>
<td>Inquiry-based learning in mathematics content courses</td>
</tr>
<tr>
<td>Lai and Lo-Fu (2013)</td>
<td>Case study, analysis of PSTs’ learning journals</td>
<td>32 elementary PSTs in Hong Kong, China</td>
<td>Learning study</td>
</tr>
<tr>
<td>Tyminski, Drake, and Land (2013)</td>
<td>Qualitative, student work and reflections</td>
<td>63 elementary PSTs from 2 Midwestern universities</td>
<td>Standards-based curriculum materials in a mathematics methods course</td>
</tr>
<tr>
<td>Turner (2012)</td>
<td>Qualitative, case study</td>
<td>3 elementary postgraduate PSTs at a UK university</td>
<td>Reflection</td>
</tr>
</tbody>
</table>
Mathematics Content in PST Education

Several researchers have examined the impact that the inclusion of mathematics content in PST education programs has on the development of PSTs’ MKT. According to their research, incorporating content into coursework has a positive impact on PSTs’ MKT. Teacher educators have included mathematics content in methods courses, developed content rich course sequences, and increased the number of content courses. The details of these opportunities and the impact they had on PSTs’ MKT follows.

The inclusion of content in methods courses for PSTs. Wanting to explore the impact that the inclusion of 20 minutes of mathematics content in a methods course would have on PSTs’ mathematical content knowledge, Burton et al. (2008) used a pre-test/post-test design to assess the content knowledge of 44 elementary PSTs in the southeast United States. These 44 PSTs were randomly assigned to an experimental or control group. Each group of PSTs participated in an intervention and control methods course, which were parallel to each other, aside from the 20 minutes of content included in the intervention course. The 20 minutes in the intervention course consisted of fifth and sixth grade mathematics content, including all of the National Council of Teachers of Mathematics (NCTM) content standards: operations and numbers, geometry, algebra, data analysis, and measurement. PSTs explored these content areas by analyzing problems; explaining connections and reasons behind formulas and algorithms; and evaluating mathematical formation. The first week of the methods course, the PSTs took the pre-test, the second through twelfth weeks of the course were devoted to instruction, and in the last week of the course the PSTs took the post-test. The instrument used to assess the PSTs’ knowledge was the Content Knowledge for Teaching Mathematics
Measure (CKT-M) created by the Learning Mathematics for Teaching (LMT) group of the University of Michigan, which measures knowledge of numbers and operations; algebra, patterns, and functions; and geometry.

On the pre-test, the control group scored slightly higher than the experimental group, although there was no statistically significant difference between the two groups. At the end of the semester, the post-test revealed a statistically significant increase in the experimental group’s SMK, while it showed no statistically significant increase in the control group’s content knowledge. In addition to the difference in SMK growth between the two groups, the experimental group scored statistically significantly higher than the control group on the post-test. The results of the post-test suggest that PSTs may benefit from more opportunities to explore elementary mathematics content in mathematics methods courses.

**Content courses.** To determine whether the number of mathematics courses or the number of mathematics topics addressed in methods courses is related to PSTs’ SMK, Youngs and Qian (2013) administered two surveys to 100 elementary PSTs from four different teacher preparation programs in China. To understand aspects of the teacher preparation programs, all 100 PSTs took a Teacher Preparation Survey, which included items that were modified from surveys used in research conducted by Boyd, Grossman, Kankford, Loeb, and Wyckoff (2009) and Schmidt, Blömeke, and Tato (2011). In order to measure the PSTs’ SMK, Youngs and Qian translated and adapted SMK items developed by the LMT group to accommodate the Chinese context. Through random assignment, 50 of the PSTs took an MKT survey concentrated on just number and
operations, while the other 50 took an MKT survey focused on patterns, functions, and algebra.

Using quantitative analysis, the results from the surveys revealed that the number of mathematics courses taken is not statistically significantly associated with PSTs’ MKT scores. However, PSTs’ MKT in number and operations was significantly impacted by the completion of both a number theory and mathematical reasoning course. Additionally, PSTs that had addressed more topics in methods courses performed better on the MKT assessments to a significant degree. Youngs and Qian’s (2013) findings support Burton et al.’s (2008) conclusion that incorporating content into methods courses results in a growth in PSTs’ MKT.

Examining a mathematics content course for elementary teachers, the first of two elementary-specific content courses in a teacher education program in the western United States, Welder and Simonsen (2011) studied the impact of a single undergraduate course on PSTs’ SMK. The course met for 50 minutes, four days a week for 16 weeks and covered the following content: sets, whole numbers, number theory, fractions, decimals, ratios, proportions, percentages, and integers. Using a pre-test/post-test design, Welder and Simonsen used 51 items from the LMT project to assess 48 elementary PSTs’ subject matter knowledge of pre-algebra concepts. The results of the pre- and post-tests revealed a significant growth in SMK of number concepts and equations/functions, which means a content course specifically designed for elementary PSTs can improve MKT.

Similar to Welder and Simonsen (2011), Whitacre and Nickerson (2016) analyzed the impact the first course of a four-course sequence of mathematics content courses for elementary PSTs has on their SMK. Taking place at a large, urban university in the
southwestern region of the United States, Whitacre and Nickerson studied 34 PSTs in a number and operations course focused on developing PSTs’ number sense. More specifically, this research focused on the fraction magnitude portion of the semester. Whitacre and Nickerson’s goal was to help move PSTs from the use of standard algorithms to the use of their own nonstandard strategies. To measure PSTs’ mathematical knowledge of fraction magnitude, the Number Sense Rating Scale (Hsu, Yang, & Li, 2011) was administered in a pre/post-test format. This 37-item multiple choice test measures knowledge of both whole and rational numbers, with most of the questions including fractions and/or decimals. In addition to the data collected from the assessment, seven of the PSTs participated in a pre/post-interview involving fraction comparison tasks. Whitacre and Nickerson coded the strategies PSTs used to solve the interview tasks as standard or nonstandard and for validity and correctness. The results of both the rating scale and the interviews revealed a growth in the PSTs’ SMK.

Over the course of the semester, the PSTs improved their computation and flexibility with fraction magnitude. The average pre-test score on the Number Sense Rating Scale was 65 percent, and the average post-test score was 79 percent, which is a statistically significant increase. Too, six of the seven interviewees showed an increase in the number of correct responses that used a valid strategy. Additionally, PSTs used more nonstandard strategies and often new strategies in their valid and correct answers, which means participants not only used more valid strategies during the post-interview, but they also used a wider variety. Therefore, focusing on nonstandard algorithms in a mathematics content course can improve PSTs’ mathematical computation and flexibility.
**Content-and-methods course sequences.** In a quantitative study conducted at a large Midwest university, Charalambous (2015) investigated the effect that a content-and-methods course sequence had on three K-8 PSTs’ SMK and beliefs and how knowledge and beliefs interact to inform their teaching practice. Both the content and methods courses consisted of 13 three-hour long class meetings. The content course in this sequence focused on number and operations and offered PSTs opportunities to use representations, provide explanations, and analyze thinking. While the methods course was still centered around number and operations, it shifted to improving PSTs’ PCK, focusing on leading mathematical discussions, representing mathematical ideas, providing explanations, assessing students’ mathematical knowledge, and planning lessons. To assess the three PSTs’ SMK, Charalambous applied a pre-test/post-test design, administering a 41-item written exam; the SMK items used were also from the LMT project. In addition to assessing SMK, the PSTs’ beliefs were also measured through the use of a survey.

While all three PSTs showed improvement in their SMK, the results revealed the growth in the PSTs’ knowledge occurred to varying degrees. Both Deborah and Vonda initially scored in the last quintile, but only Deborah experienced large gains in her SMK, with Vonda still scoring in the last quintile on the post-test. Conversely, Kimberley scored at the upper end of the second quintile on the pre-test and improved her score on the post-test. Charalambous (2015) attributes Deborah’s large increase of SMK to the fact that, out of the three, she held the strongest pedagogical beliefs about mathematics instruction pre- and post-intervention; therefore, focusing on the development of beliefs alongside the development of MKT may result in a deeper increase in PSTs’ MKT.
While Charalambous (2015) explored the effects of a single content course followed by one methods course on PSTs’ MKT and mathematical beliefs, Smith et al. (2012) investigated the difference between a sequence of three content courses followed by two methods courses and four content courses followed by one methods course. Smith et al.’s longitudinal, quantitative study involved 276 elementary PSTs at a large, urban university in the southeast United States. Of these 276 PSTs, 142 completed three content courses and two methods courses as part of their teacher preparation, and 134 completed what Smith et al. consider the “new program,” which exchanged the second methods course for a fourth content course. The content courses covered the following content: number and operations; geometry and spatial reasoning; probability and statistics, and the fourth content course introduced algebraic concepts. The two methods courses in the “old program” consisted of grades PreK-2 and grades 3-5 content, while the one methods course in the “new program” covered grades PreK-5 content. Similar to Charalambous, Smith et al. also used the Learning Mathematics for Teaching Instrument (LMT) to assess PSTs’ SMK, but Smith et al. only administered a post-test to compare the level of PSTs’ SMK between the “old” and “new” programs. To assess PSTs’ beliefs about the mathematics curriculum and the roles of the learner and teacher during instruction, the Mathematics Beliefs Instrument (MBI) was administered prior to coursework and at the end of each course in the sequence for a total of five administrations.

Comparing the LMT scores revealed no statistically significant difference in SMK between the PSTs who completed three content and two methods courses and those that completed four content and one methods course, which mirrors Youngs and Qian’s (2013) finding that the number of content courses does not impact PSTs’ MKT
development. Although an additional content course did not result in higher levels of SMK, LMT scores were a statistically significant factor for changes in mathematical beliefs, which further supports Charalambous’ (2015) finding that the development of MKT and mathematical beliefs are interrelated.

While incorporating content into mathematics methods courses has shown to improve PSTs’ SMK (Burton et al., 2008), content courses that were not specifically designed for PSTs, no matter the number taken, had little impact on PSTs’ SMK (Smith et al., 2012; Youngs & Qian, 2013). Although taking more content courses does not result in higher levels of subject matter knowledge, Youngs and Qian’s (2013) findings suggest that focusing on multiple topics in methods courses can positively impact PSTs’ MKT. Additionally, Charalambous (2015) discovered that simultaneously focusing on the development of PSTs’ mathematical beliefs in content and methods courses positively impacts MKT, while Smith et al. (2012) found that PSTs’ SMK was related to more cognitively oriented beliefs, both revealing the interconnectedness of MKT and teaching beliefs.

**Inquiry-Based Learning**

Using a mixed-methods design, Laursen et al. (2016) explored how taking an inquiry-based learning (IBL) approach in PST mathematics content courses impacted elementary, middle, and secondary PSTs’ MKT. The IBL courses were structured around PSTs solving difficult problems, sharing solutions, and critiquing each other’s work. The sequence of tasks slowly led PSTs to discussions around major mathematical concepts. Similar to several of the researchers who investigated the inclusion of mathematics content in teacher education courses, Laursen et al. conducted a pre-test/post-test design
using items from the LMT project. In addition to the pre-test/post-test, a survey was administered to assess PSTs’ mathematics learning, attitudes, and beliefs. There were 109 PSTs who completed the pre/post-test and 184 completed the pre/post-survey. The qualitative portion of the study consisted of individual or small focus group interviews with 24 PSTs and individual interviews with seven instructors/teaching assistants. Statistical analyses were used to analyze the quantitative data, while the qualitative data were coded using N’Vivo 8 software.

The results of the LMT assessment revealed a statistically significant gain in mathematical knowledge. PSTs with more undergraduate mathematics experience showed greater gains compared to those with less, and PSTs who scored lower initially improved more than those with higher pre-test scores. While Smith et al. (2012) and Youngs and Qian (2013) found that taking more content courses did not result in higher levels of SMK, Laursen et al.’s (2016) findings suggest that a stronger mathematical background paired with an IBL-driven PST content course does lead to greater gains in MKT. In addition to the growth seen on the LMT assessment, the qualitative data showed that PSTs’ felt they gained knowledge of how to apply mathematical ideas and teach mathematics to others, which Laursen et al. attribute to the collaborative nature of IBL and the deep engagement with mathematics.

**Learning Study**

In Hong Kong, China, Lai and Lo-Fu (2013) conducted learning study with 32 elementary PSTs. Similar to JLS, learning study is grounded in variation theory, and addresses the following: variations in students’ preconceptions and misconceptions, variations in teachers’ conceptions of mathematics, and variations in teaching a
mathematical topic. In groups of four, the PSTs in this study participated in two iterations of the following cycle: 1) selection of a topic, 2) identifying the object of learning, 3) pre-test, student interview, 4) finalizing object of learning and critical aspects, 5) lesson planning, 6) lesson implementation and observation, 7) post-test, student interview, 8) final evaluation, and 9) report and dissemination (Wong and Lo, 2008). To evaluate the PSTs’ growth in MKT as a result of the learning study, Lai and Lo analyzed the PSTs’ learning journals, which were three-page summaries of the PSTs’ reflections and learning.

Analysis of the 32 learning journals revealed a growth in PSTs’ SCK, KCS, and KCT. As a result of participating in the learning study, the PSTs gained SCK by realizing that to understand fractions, one must know that a fraction can represent a group of objects or a continuous object. In respect to KCS, the PSTs learned how to interpret and address students’ prior knowledge and misconceptions. Lastly, concerning KCT, the PSTs gained knowledge of how to use manipulatives, the impact of using real world problems, and how to sequence questions. Each of the 32 PSTs contributed their learning to the fact that the learning study supported the understanding of the relationship between theory and practice and their ability to transform knowledge into action. Lai and Lo’s (2013) findings support my belief that situating teacher education deeply in the practice of teaching develops MKT.

**Standards-Based Curriculum Materials**

In an elementary mathematics methods course, Tyminski et al. (2013) used the Addition Starter Sentences instructional module from the NSF-funded research project, Constructing Coherence: Elementary Teachers’ Strategies for Using Standards-Based
Mathematics Curriculum Materials, to explore how the activities in the module impacted PSTs’ learning. The module was designed to help PSTs focus on alternative addition strategies. Student work from the module and responses to two reflection questions were collected from 63 PSTs from two different Midwestern universities. PSTs’ responses to the two reflection questions were analyzed using the subdomains of MKT.

All 63 PSTs’ reflections contained aspects of MKT, and references to CCK and KCS occurred most frequently. Although not necessarily a reflection of growth in knowledge, 44 PSTs reported that they preferred using an alternative strategy to solve addition problems. Additionally, 8 PSTs referred to their SCK when mentioning their realization of the importance of being able to understand others’ mathematical thinking. Moving onto PCK, the PSTs’ KCS grew as a result of them learning that students solve problems in many different ways; the authors contribute this learning to the problem solving and analysis of student work included in the Addition Starter Sentences module. While not as predominant as KCS, 15 of the 63 PSTs gained KCT. Those PSTs learned: 1) the importance of introducing alternative strategies, 2) their emerging ability to analyze student work for its mathematical sophistication, and 3) what to consider when sequencing work for a classroom discussion.

Reflection

At a university in the United Kingdom, Turner (2012) explored the impact that reflection had on 3 PSTs’ MKT. Throughout the four-year study, Amy, Kate, and Jess used the Knowledge Quartet (KQ) framework (Rowland, Huckstep, and Thwaites, 2005) to reflect on the content of their teaching. The following forms of data collection were used: videotaped lessons, individual post-lesson interviews, group interviews, and written
reflections. To determine the development of MKT, Turner used the KQ framework to analyze the videotaped lessons, transcribed interviews, and written reflections.

Over the course of the study, all three interns developed their SCK, KCS, and KCT. More specifically, Amy showed growth in her knowledge of the pre-requisites for counting, common student misconceptions, the use of examples, and representations. Additionally, while Kate gained knowledge of the stages in addition and how to evaluate the conceptual appropriateness of content for her students, Jess furthered her learning of division models and differentiation. The three PSTs attributed their growth in MKT to their experiences teaching, reflection, and work with others. Because Turner’s (2012) findings suggest that teaching, reflection, and collaboration influence the development of MKT, an argument can be made for the use of lesson study with PSTs.

**Lesson Study with Pre-Service Teachers**

An additional review of the literature was conducted to see how teacher educators have conducted and researched lesson study with elementary PSTs. Due to the focus of the present study on elementary PSTs, the review only includes studies with elementary PSTs, and does not include research conducted with secondary PSTs or ISTs. The results of the review revealed that researchers have examined lesson study’s impact on elementary PSTs’ noticing, reflection, development, and MKT, while also exploring the benefits and challenges of conducting lesson study with PSTs. Within each section, a description of the purpose, methodology, and PST demographics are provided for each study discussed (see Table 2.2). In addition to the details of the research conducted in each study, a synthesis of the findings related to the impact that lesson study had on PSTs is also reported.
### Table 2.2.

*Studies Exploring Lesson Study with PSTs*

<table>
<thead>
<tr>
<th>Authors</th>
<th>Methods/Methodology</th>
<th>PST Demographics</th>
<th>Focus of Research</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chassels and Melville (2009)</td>
<td>Qualitative, participants’ reflections</td>
<td>26 PSTs and 20 ISTs at an Ontario university</td>
<td>Challenges and benefits of conducting LS</td>
</tr>
<tr>
<td>Amador and Weiland (2015)</td>
<td>Qualitative, case study</td>
<td>24 PSTs in a large Midwestern university</td>
<td>Noticing</td>
</tr>
<tr>
<td>Amador, Carter, and Hudson (2016)</td>
<td>Qualitative, case study</td>
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</tr>
<tr>
<td>Myers (2012)</td>
<td>Qualitative, group lesson study reports and individual reflections</td>
<td>20 PSTs at a small university in Hawaii</td>
<td>Reflection</td>
</tr>
<tr>
<td>Myers (2013)</td>
<td>Qualitative, group lesson study reports and individual reflections</td>
<td>20 PSTs at a small university in Hawaii</td>
<td>Reflection</td>
</tr>
<tr>
<td>Suh and Parker (2010)</td>
<td>Qualitative, interviews, reflective journals, observations, and lesson plans</td>
<td>5 PSTs and some ISTs in a professional development school</td>
<td>Development of PSTs’ knowledge and practice</td>
</tr>
<tr>
<td>Suh and Fulginiti (2012)</td>
<td>Qualitative, reflections, meeting transcripts, researcher memos, observations, lesson plans, and pre/post-surveys</td>
<td>21 PSTs and 4 clinical faculty in a professional development school</td>
<td>Development of PSTs’ knowledge and practice</td>
</tr>
<tr>
<td>Fernandez and Zilliox (2011)</td>
<td>Qualitative, field notes, memos, videotaped lessons and presentations, and surveys</td>
<td>48 PSTs</td>
<td>Development of PSTs’ knowledge and practice</td>
</tr>
<tr>
<td>Leavy (2015)</td>
<td>Qualitative, field notes, reflections, lesson plans, presentations, and a reflective assignment</td>
<td>73 PSTs in Ireland</td>
<td>Development of MKT</td>
</tr>
<tr>
<td>Leavy and Hourigan (2016)</td>
<td>Qualitative, lesson plans, reflections, and presentations</td>
<td>25 PSTs in Western Ireland</td>
<td>Development of MKT</td>
</tr>
<tr>
<td>Leavy and Hourigan (2018)</td>
<td>Qualitative, lesson plans, reflections, and presentations</td>
<td>25 PSTs in Western Ireland</td>
<td>Development of MKT</td>
</tr>
</tbody>
</table>

**Challenges and Benefits**

While exploring the development of professional learning communities and reflective practice in an elementary teacher education program at a Canadian university in
Ontario, Chassels and Melville (2009) examined the challenges and benefits that PSTs and ISTs experienced while participating in lesson study. Analyzing 26 PST and 20 IST reflections, which focused on the benefits and challenges of lesson study and its level of appropriateness or helpfulness in PST education, Chassels and Melville identified a variety of challenges and benefits of conducting lesson study with PSTs.

The pre-service and ISTs’ reflections revealed several challenges to conducting lesson study in a teacher education program. Time and school schedules made collaboration, shared observations, and debriefing difficult. In addition to time and scheduling being a challenge to the lesson study process, the PSTs in each lesson study pair were often placed in different grade levels, which resulted in many lesson study pairs planning multiple lessons and only some PSTs taught a revised lesson. Being placed at multiple grade levels was not the only challenge to the collaborative nature of lesson study; a few PSTs found the collaboration itself uncomfortable because they preferred working alone. Lastly, the ISTs’ incomplete understanding of lesson study made it hard for them to adequately support the PSTs through the process. While there were challenges to conducting lesson study with PSTs, there were also several benefits to PSTs’ growth as teachers.

Lesson study provided PSTs the opportunity to gain a deeper understanding of students, teaching, and collaboration. As a result of participating in lesson study, PSTs realized the need to know their students and were more receptive to different teaching and learning styles. Additionally, PSTs developed an understanding of the role feedback plays on the improvement of teaching. Finally, although working in a group was uncomfortable for some, the lesson study provided PSTs the opportunity to discuss
subject matter, teaching, and learning, which helped them to see the benefits of collaboration. Chassels and Melville’s (2009) findings show that, while there are many challenges to conducting lesson study with PSTs, the benefits the PSTs experienced led to a growth in their professional teacher identities.

**Noticing**

With an interest in PSTs’ ability to notice, Amador and Weiland (2015) used a case study design to investigate what 24 elementary PSTs in a large Midwestern university noticed while participating in lesson study. These PSTs were divided into four groups, with each group completing a field experience in the same elementary classroom and implementing lesson study with the classroom teacher and a university facilitator. For seven weeks, each lesson study pair met once a week after a pair of PSTs taught a lesson. During these weekly lesson analysis meetings, the focus of discussion was students’ mathematical thinking. The PSTs who taught the lesson began the debrief by sharing the strengths and weaknesses of the lesson; then, the other PSTs and classroom teacher shared their observations, feedback on effectiveness, and suggestions for improvement. The meetings ended with the entire group making revisions to the lesson plan.

To identify what PSTs noticed during the lesson study, Amador and Weiland (2015) collected and transcribed videos of all 28 lesson analysis meetings and analyzed the data using a modified version of van Es’s (2011) framework for learning to notice student thinking. The following levels of noticing were used to code PSTs’ noticing: attend to whole class environment, attend to teacher pedagogy, begin to attend to students’ mathematical thinking, attend to students’ mathematical thinking, and attend to
the relationship between students’ mathematical thinking. An analysis based on coding the lesson analysis meetings showed that PSTs’ noticing focused mainly on the classroom environment and teacher pedagogy, the two lowest levels of noticing, but 20 percent of PSTs’ comments began to attend to student thinking and six percent attended to specific student thinking. With about a quarter of the PSTs’ noticings focusing on students’ mathematical thinking, Amador and Weiland argued that lesson study provided PSTs with an opportunity to notice student thinking.

Further analyzing the data from Amador and Weiland (2015), Amador et al. (2016) coded all instances of PST noticings for the level of analytic stance. The four levels were: descriptive, evaluative, interpretive, and making connections. Only 79 occurred at the interpretive and making connections levels. In order to more clearly understand what PSTs were noticing at the higher levels, Amador et al. coded these 79 noticings a second time. Each interpretive or making connection noticing was assigned a code related to attending to students’ strategies, interpreting students’ understandings, and deciding how to respond. Of the 79 instances of interpretive and making connections noticings, 38 percent detailed student and teacher strategies with 34 percent being details about student strategies, 48 percent were instances of interpreting students’ understandings, and 14 percent were suggestions of improvement. These results indicate “that PSTs at the most accomplished level were able to remember an exact instance with a student, discuss that instance, and interpret the student’s action to make meaning related to the student’s conceptual understanding in mathematics” (p. 380). Amador et al. suggest PSTs’ focus on student thinking was a direct result of participating in several cycles of lesson study.
Due to the fact that Amador and colleagues’ (2015, 2016) findings revealed lesson study as a structure that can focus PSTs’ noticings on students’ mathematical thinking, Amador and Carter (2018) were interested in understanding how lesson study provided such opportunities. Using the data from the Amador et al. (2016) study, Amador and Carter identified aspects of the lesson study that allowed for, or hindered, the development of PSTs’ noticing. This additional analysis of the data revealed two themes that supported and two themes that reduced noticing. Facilitators’ prompts led to noticing, and when responding to these prompts, multiple PSTs analyzed and attended to students’ mathematical thinking. Although the facilitator’s prompts led to noticing, his/her presence also hindered some noticing; sometimes the classroom teacher and the facilitator shared their expertise, leading to extended periods of time without input from PSTs. Additionally, any time a participant shifted the conversation to a new topic, noticing declined. Being aware of these affordances and constraints allows teacher educators to understand the role the facilitator can play in the development of PSTs’ noticing when participating in lesson study.

Reflection

Due to the embedded analysis in the lesson study process, it is not surprising that lesson study’s impact on PSTs’ reflection has been explored. Myers (2012, 2013) examined the reflection of 20 PSTs in an elementary mathematics methods course at a small university in Hawaii. The PSTs were split into six groups to participate in lesson study. After completing the lesson study cycle, PSTs were required to write a lesson study report, which included group and individual reflections. For the group reflections, PSTs were asked to include changes in thinking, techniques or principles learned, and
impact on their future classrooms; the individual reflections were to focus on what they learned in the areas of content knowledge, teaching, and learning. The six group lesson study reports and 20 individual reflections were the data for this study.

Myers (2012, 2013) analyzed the lesson study reports by coding the purpose of writing, recurrent themes, and levels of reflection. Four recurrent themes in the PSTs writing were identified: lesson study process, teaching, learning, and academic content. Similarly, four purposes for writing were used for coding: describing; interpreting, analyzing, and inquiring; evaluating; and expressing personal voice, which were taken from Lee’s (2008) four language functions framework. These eight codes were then used to create 16 new codes, matching each of the recurrent themes with each of the purposes for writing. Finally, using Hatton and Smith’s (1995) levels of reflection, Myers coded the PSTs’ reflection using four codes: descriptive writing, descriptive reflection, dialogic reflection, and critical reflection. The results of the analysis revealed the group reflections focused on the lesson study process and the individual reflections focused on teaching, meaning PSTs’ reflections predominantly focused on teaching rather than student learning. Additionally, PSTs’ writing consisted mainly of descriptive writing and reflection and did not include critical reflection, which shows that PSTs provided some justification for the events that occurred but were unable to base their reasons on more than their own judgements. These findings show that lesson study alone does not develop reflective skills in PSTs and that additional support is necessary to ensure that PSTs reflect critically about their practice.
PST Development

Suh and Parker (2010) used lesson study to explore how collaborative reflection supported the development of PSTs’ reflective practice and mathematical knowledge. While collaborating with cooperating, special education, and English language learner teachers, Suh and Parker researched two lesson study pairs, which were comprised of a total of five PSTs within a professional development school (PDS) context. To determine what was learned throughout the lesson study, open coding and thematic analysis were used to analyze interviews, reflective journals, observations, and lesson plans. The analysis revealed three overarching themes.

Because the lesson study pairs included both pre-service and ISTs, Suh and Parker (2010) found that reciprocal learning occurred, allowing all involved to develop their knowledge of mathematical concepts, models, strategies, and representations. In addition to providing an opportunity for PSTs to develop their knowledge of mathematics, lesson study also allowed the PSTs to realize the impact being a novice teacher has on their ability to plan and teach. Having a limited amount of experience made it difficult for PSTs to anticipate student responses and misconceptions, to account for students’ prior knowledge, to pose meaningful questions, and to assess students’ mathematical understandings. Since the PSTs dealt with the challenges associated with being a novice teacher, it is no surprise that the PSTs were also exposed to the complexity of teaching. The PSTs observed ISTs’ planning of tasks and questions, management of mathematical discussions, and identification of next steps to promote mathematical understanding.
In a similar exploration of lesson study with PSTs, Suh and Fulginiti (2012) researched the impact that lesson study has on PST development. Working with 21 PSTs and 4 clinical faculty in a summer mathematics methods course that included a mathematics enrichment lab for second through sixth grade students in a Title I school, Suh and Fulginiti used PSTs’ reflections, transcriptions of planning and debrief meetings, researcher memos, classroom observations, lesson plans, and pre- and post-surveys along with open coding and thematic analysis to identify three categories of PST development. The results revealed a development of PSTs’ knowledge about teaching mathematics, knowledge about student learning, and collective inquiry into practice.

The shared experiences that occur during lesson study provided the PSTs and clinical faculty members the opportunity to develop their knowledge of teaching mathematics. The debriefing conversations after the observation of a lesson allowed the PSTs and clinical faculty members to discuss moments in the lesson that helped them learn more about how to teach mathematics. Additionally, similar to Suh and Parker’s (2010) findings, PSTs gained knowledge of teaching mathematics by observing the clinical faculty members’ selection of tasks, questioning, use of representations, and facilitation of discussions. As for the development of their knowledge of student learning, the detailed analysis of teaching that was involved during the planning discussions resulted in PSTs learning more about students’ misconceptions and potential responses to questions. Finally, PSTs developed a positive disposition toward collaboration. The collective inquiry that is lesson study allowed PSTs to see the power that communication with colleagues has on their practice and knowledge of teaching.
Also exploring how lesson study supports PSTs’ growth as mathematics teachers, Fernandez and Zilliox (2011) conducted lesson study with 48 elementary PSTs in mathematics methods courses. Collecting field notes and memos throughout the lesson study process, video of lessons and group presentations, and PST feedback surveys, Fernandez and Zilliox coded PSTs’ growth and the aspects of lesson study that led to that growth. The collaborative nature of lesson study allowed PSTs to think about, put into practice, and modify their ideas about teaching mathematics. While the collaboration prompted a growth in PSTs’ knowledge of teaching mathematics, the lack of formative feedback from course instructors throughout the process resulted in a focus on classroom management rather than mathematical ideas. In addition to the limited amount of feedback from course instructors hindering PSTs’ growth, PSTs who selected content they were familiar with or thought would not be challenging for students experienced a weaker development of knowledge and reflection.

**Development of MKT**

Desiring to know how PSTs’ MKT develops while participating in lesson study, Leavy (2015) spent three years studying 14 lesson study pairs consisting of 73 elementary PSTs in Ireland. Through the use of participant observation, Leavy used field notes, reflection prompts, revised lessons, in-class presentations, and a reflective assignment to examine the PSTs’ MKT data handling. Leavy analyzed the data by identifying the knowledge required of PSTs when planning and teaching data lessons. The knowledge was then coded using Ball, Thames, and Phelps (2008) descriptions of MKT. The data collected showed the PSTs developed their CCK, SCK, and KCT.
As a result of participating in lesson study, the PSTs’ gained CCK of the median. At the start of the study, the PSTs’ understanding of statistical concepts was instrumental; they were able to identify the rules of the measures of central tendency, but lacked the reasoning to apply the rules when problem solving. However, by the end of the study, the PSTs held a deeper conceptual understanding of the median. In addition to developing a conceptual understanding of the median, the PSTs also developed their knowledge of questioning and how wording impacts the level of mathematical reasoning. This growth in KCT occurred because PSTs had the opportunity to revise and reteach a lesson after analyzing the initial teaching. The lesson study also led to a development of the PSTs’ knowledge of sequencing instruction and ways to support students’ development of mathematical understandings. Finally, the opportunity to explore different models for representing the measures of central tendency promoted a growth in the PSTs’ SCK of the mean.

After initial research showed a growth in PSTs’ MKT, Leavy and Hourigan (2016, 2018) continued researching the use of lesson study in teacher education. Studying 25 PSTs in a mathematics education course in Western Ireland, Leavy and Hourigan examined the development of MKT as PSTs planned early number lessons, which were to be taught in preschool classrooms. Similar to Leavy (2015), data collection consisted of lesson plans, reflections, and group presentations, but, Leavy and Hourigan focused on SCK, KCT, and KCS, excluding analysis of PSTs’ growth in CCK.

Just as Leavy’s (2015) initial lesson study research revealed a growth in PSTs’ MKT, as did the study conducted by Leavy and Hourigan (2016, 2018). As for SCK, the PSTs gained deeper understandings of connections between subitizing, cardinality, rote
and rational counting, and one-to-one correspondence. Additionally, PSTs showed growth in their knowledge of student errors. An analysis of the PSTs’ KCS revealed a growth in their ability to consider context and open-endedness when writing mathematical tasks and how to analyze students’ mathematical responses. Lastly, as a result of participating in lesson study, the PSTs furthered their understandings of the sequencing of instruction, the evaluation of representations and procedures, and the selection of models that support the development of mathematical knowledge.

**Potential Contributions to the Teacher Education Research Field**

At least half of the studies that researched the development of PSTs’ MKT included the use of quantitative methods, a pre-test/post-test design, and test items from the LMT group. Additionally, because many of the studies included the content knowledge items developed by the LMT group, several studies only explored the development of PSTs’ SMK. The present study used qualitative methods and explored the development of both SMK and PCK, doing so may provide a more detailed, holistic description of how PSTs gain MKT. Too, most of the studies were conducted in a university course context, with only two of the studies including opportunities for PSTs to teach. This study was not situated within a course context and instead occurred completely in the field, which means the results of this research could provide implications for supervision in teacher education. Finally, only one group of researchers investigated how lesson study impacts PSTs’ MKT; therefore, this study seeks to further explore how lesson study develops PSTs’ MKT.
Chapter 3

Methods

The purpose of this study was to develop an understanding of how PSTs’ MKT developed while participating in the PST-LSC described in Chapter 1. I conducted the PST-LSC with two pairs of PSTs who were student teaching in elementary classrooms within a grades K-4 professional development school (PDS) partnership. My focus was on investigating the following research questions: 1) How does the MKT of elementary PSTs develop as they engage in the PST-LSC? And 2) How do the activities of the PST-LSC provide opportunities for the development of MKT? In this chapter, I describe the design of my study, including the context, participants, PST-LSC, data collection, and data analysis.

Design of the Study

According to Mason (2002), qualitative research is: 1) grounded in an interpretivist philosophy, 2) based on flexible data collection methods that are responsive to the context being studied, and 3) based on analysis methods that produce rich explanations. Due to the explanatory nature of my research questions, focused on describing how MKT develops through the PST-LSC, qualitative methods were chosen for this study. More specifically, I used case study methods because “the case study method is best applied when research addresses descriptive or explanatory questions and aims to produce a firsthand understanding of people and events” (Yin, 2006, p. 112).

Case Study

Yin (2006) states that, “the strength of the case study method is its ability to examine, in-depth, a ‘case’ within its ‘real-life’ context” (p. 111). The goal of this study
was to do exactly that, to analyze in detail two cases, a kindergarten/first grade pair of PSTs and a third/fourth grade pair of PSTs, within a PDS context. When conducting a multiple-case study, there are three purposes for analyzing more than one case: confirmation, contrast, or theoretical diversity (Yin, 2006). The two cases in this study, two pairs of PSTs participating in lesson study cycles, acted as confirmatory cases, similar cases used to replicate the same phenomenon; however, contrasts between the two cases were also explored. Additionally, case study methods are used to obtain rich data; therefore, multiple forms of data collection are used to achieve triangulation (Glesne, 2011; Yin, 2006). In order to explain the phenomena being examined, this study used the following forms of data collection common to case studies: participant observation, interviews, and documents (Glesne, 2011). Ultimately, the goal of this study was to “focus on the complexity within the case, on its uniqueness, and its linkages to the social context of which it is a part” (Glesne, 2011, p. 22).

**Context of the Study**

This study was situated in an elementary PDS partnership between a research university and a public school district in the northeast United States. This site was chosen for the study because “the groundwork for rapport [was] already established; the research would be useful for [my] professional life; and the amount of time and money needed for various research steps would be reduced” (Glesne, 2011, p. 41). As explained further later in this chapter, this selection decision was made in part due to my position in the PDS; my role in the PDS gave rise to my research questions and also made access and data collection feasible (Glesne, 2011; Maxwell, 2013). It is important to note that
although this study occurred in the context of a PDS, the PST-LSC could be used in other teacher education contexts.

The professional development school. A PDS is a school site where “new teachers learn to teach alongside more experienced teachers who plan and work together, and university- and school-based faculty work collaboratively to design and implement learning experiences for new and experienced teachers as well as for students” (Darling-Hammond & Bransford, 2005, p. 414). During the 2017-2018 school year, 39 PSTs (interns) in the university’s Elementary and Early Childhood Education major were placed in K-4 ISTs’ (mentors’) classrooms for their student teaching internships. In the PDS partnership, interns participate in the full elementary school year, from the first teacher workday in August to the last day of school in June.

During 2017-2018, the partnership had 10 Professional Development Associates (PDAs) who supervised interns and co-taught university methods courses. As a graduate assistant in the PDS, I co-taught the mathematics methods course in the Fall and supervised two interns in the same school building throughout the year. The PDS context was a suitable environment in which to explore the use of the proposed PST-LSC, because the goal of the cycle is to develop PSTs’ knowledge of teaching through collaboration and reflection, consistent with typical PDA supervision practices. Additionally, the PDS was a good fit for the PST-LSC because multiple interns complete their student teaching within the same elementary school, providing a space where collaborative relationships had previously been established.
Participants

Glesne (2011) states that “for in depth understanding, you repeatedly spend extended periods with fewer respondents and observation sites” (p. 46); therefore, this study was conducted with two pairs of interns in one school (see Table 3.1). Choosing interns from the PDS as participants was a form of purposeful selection; this sampling strategy was used for this study because the goal was to either already have an established relationship or be able to establish a productive relationship with the participants (Glesne, 2011; Maxwell, 2013). The two pairs chosen for this study were selected according to three criteria: 1) interns were in the Fall 2017 mathematics methods course section that I taught, 2) interns were student teaching in the same school, and 3) I was not the interns’ supervising PDA. These criteria were used to ensure interns participated in the same mathematics methods course section, had a previously established relationship with me, were teaching in the same school context, and were not receiving a course grade from me during the Spring 2018 semester.

Only four interns met the three criteria and were, therefore, invited as participants. All four interns agreed to participate. Each participant was given a pseudonym, which were carefully chosen to help readers remember the interns’ grade levels (Kelly-kindergarten, Finley-first, Theresa-third, and Ford-fourth).
Table 3.1.

The Study’s Participants

<table>
<thead>
<tr>
<th>Interns</th>
<th>Grade</th>
<th>PDA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kelly</td>
<td>K</td>
<td>A</td>
</tr>
<tr>
<td>Finley</td>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td>Theresa</td>
<td>3</td>
<td>B</td>
</tr>
<tr>
<td>Ford</td>
<td>4</td>
<td>B</td>
</tr>
</tbody>
</table>

Data was collected during the Spring semester of 2018 because interns were in their classrooms every day and took on more teaching responsibility. Collecting data at this time ensured the interns had at least a small amount of experience with planning, instructing, and reflecting upon lessons. It is important to reiterate that I was not the participants’ assigned supervisor. If I were the interns’ PDA, I would have been responsible for their grades in two courses during the Spring semester. This fact may have pressured interns to participate and respond in particular ways in order to please me. Additionally, my assigned interns may not have felt able to opt out of the study, if or when they desired to. I met with all participants prior to beginning the study to provide information about research procedures, confidentiality, and voluntary participation. Each participant provided a verbal consent to participate in the study.

The Pre-Service Teacher Lesson Study Cycle

The study examined the use of a lesson study cycle with elementary PSTs in a PDS context. As presented in Chapter 1, the cycle was influenced by Shulman’s (1987) pedagogical reasoning and Japanese lesson study (Fernandez & Yoshida, 2004). Shulman’s (1987) “conception of pedagogical reasoning and action is taken from the point of view of the teacher, who is presented with the challenge of taking what he or she
already understands and making it ready for effective instruction” (p. 14); therefore, “pedagogical reasoning and action involve a cycle through the activities of comprehension, transformation, instruction, evaluation, and reflection” (p. 14). With the same goal in mind, when participating in Japanese lesson study, teachers plan lessons together, watch these lessons unfold in a classroom setting, and discuss their observations, resulting in a refinement of teaching and student learning (Fernandez & Yoshida, 2004). The PST-LSC I developed melds these two concepts together and appears in Figure 3.1. A description of the activities that occurred during each phase of the cycle follows. A detailed comparison of the two intern pairs can be found in Table 3.2.

![Figure 3.1. The pre-service teacher lesson study cycle (PST-LSC)](image-url)
Table 3.2.

Details of Each Phase of the PST-LSC Cycle

<table>
<thead>
<tr>
<th>PST-LSC Phase</th>
<th>K/1 Pair</th>
<th>K/2 Pair</th>
<th>My Role</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior to starting the LSC</td>
<td>The interns completed a pre-interview. Each intern completed 8 total tasks, 2 addition, subtraction, multiplication, and division.</td>
<td>The interns wrote journal entry #1, describing what they know about addition and subtraction and drawing a concept map.</td>
<td>I facilitated the interviews and asked impromptu questions based on interns’ responses.</td>
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<td></td>
<td></td>
<td></td>
<td>I emailed the journal prompts and read the journal entries once they were completed.</td>
</tr>
<tr>
<td>A. Understand the Subject Matter</td>
<td>The 4 interns individually solved 4 addition and 4 subtraction tasks (see Table 3.2) in a variety of ways and each shared with the group the strategies and models they used.</td>
<td></td>
<td>I prompted the interns to explain their thinking, compare solution strategies, and consider their students’ thinking.</td>
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<td></td>
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<td></td>
<td>I emailed the journal prompts and read the journal entries once they were completed.</td>
</tr>
<tr>
<td>B. Make the Subject Matter Comprehensible</td>
<td>The interns read about addition and subtraction in chapters 9 and 12 in <em>Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for PK-2.</em> (pgs. 146-163, 237-261, 268-269)</td>
<td>The interns read about addition and subtraction in chapters 8, 9, and 11 in <em>Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for 3-5.</em> (pgs. 116-122, 141-151, 182-193, 215-216)</td>
<td>I reviewed the addition and subtraction sections of the <em>Teaching Student-Centered Mathematics</em> texts to determine what each intern pair would read.</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Prior to the meeting, I prepared questions to ask the interns about the text and asked impromptu questions based on the discussion. During the discussion, I led a conversation to help clarify the bar diagram and the Take from Ten strategy.</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>I reviewed the addition and subtraction modules in the <em>Math in Practice</em> texts and gave each intern a list of modules they could read in their text. The interns then chose the module they read based on the operation they planned to teach.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Prior to the meeting, I prepared questions to ask the interns about the text and asked impromptu questions based on the discussion. I also helped the interns to begin to plan the content of their lessons. With the 3/4 intern pair, I facilitated a clarifying conversation.</td>
</tr>
</tbody>
</table>

*Note:* The interns completed a module from their individual grade-level text of *Math in Practice:*
- K-Module 7-Understanding Addition
- 1-Module 1-Exploring Addition Word Problems with Sums to 20
- 3-Module 7-Fluently Subtracting Within 1,000
- 4-Module 4-Fluently Adding and Subtracting Multidigit Numbers
<table>
<thead>
<tr>
<th>C1. Plan the Lesson</th>
<th>The intern pair collaboratively planned an addition lesson. The lesson focused on students using number bonds to add 3 addends. The students were also asked to solve the word problems using an additional strategy and to identify the commutative property.</th>
<th>I read and provided feedback on the interns’ lessons.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1. Teach the Lesson</td>
<td>Kelly taught the lesson in her mentor’s classroom, while Finley observed and took notes on the observation template. Kelly completed the observation template after teaching the lesson.</td>
<td>I created the observation template, which focused the interns’ observations on student thinking. I also observed the lesson and took notes on the observation form.</td>
</tr>
<tr>
<td>E1. Analyze and Reflect</td>
<td>Kelly began the discussion with her analysis and reflection of the lesson. Then, Finley and I shared our insights. After analyzing the lesson and student learning, the three of us discussed changes to the lesson.</td>
<td>I used the observation template to facilitate the discussion. I also encouraged the interns to consider student thinking and the development of a conceptual understanding when discussing revisions to the lesson.</td>
</tr>
<tr>
<td>After the meeting, the interns completed journal entry #4, responding to the 4 reflection questions included in the PDS lesson plan template.</td>
<td></td>
<td>After the second meeting, the interns wrote journal entry #3, describing how and in what order they would present addition and subtraction to their students.</td>
</tr>
<tr>
<td>C2. Revise the Lesson</td>
<td>The intern pair collaboratively revised the lesson based on the reflections discussed in E1. Kelly and Finley adjusted their lesson to allow students to use a self-selected strategy to solve a 3-addend word problem prior to using a number bond. They also eliminated the modeling of the Commutative Property.</td>
<td>I read and provided feedback on the interns’ lessons.</td>
</tr>
<tr>
<td>After revising the lesson, the interns completed journal entry #5, describing the changes they made to their lesson.</td>
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</table>
**D2. Teach the Lesson**

<table>
<thead>
<tr>
<th>Action</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>Finley taught the revised lesson in her mentor’s classroom, while Kelly observed and took notes on the observation template. Finley completed the observation template after teaching the lesson.</td>
<td></td>
</tr>
<tr>
<td>Ford taught the revised lesson in her mentor’s classroom, while Theresa observed and took notes on the observation template. Ford completed the observation template after teaching the lesson.</td>
<td></td>
</tr>
<tr>
<td>I created the observation template, which focused the interns’ observations on student thinking. I also observed the lesson and took notes on the observation form.</td>
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</table>

**E2. Analyze and Reflect**

<table>
<thead>
<tr>
<th>Action</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>Finley began the discussion with her analysis and reflection of the lesson. Then, Kelly and I shared our insights. After analyzing the lesson and student learning, the three of us discussed changes to the lesson.</td>
<td></td>
</tr>
<tr>
<td>Ford began the discussion with her analysis and reflection of the lesson. Then, Theresa and I shared our insights. We also looked at the formative assessments to gather additional information on student learning. After analyzing the lesson and student learning, the three of us discussed changes to the lesson.</td>
<td></td>
</tr>
<tr>
<td>I used the observation template to facilitate the discussion. I also encouraged the interns to consider student thinking and the development of a conceptual understanding when discussing differences between the two lessons and revisions to the revised lesson.</td>
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<tr>
<td>After the meeting, the interns completed journal entry #6, responding to the 4 reflection questions included in the PDS lesson plan template.</td>
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<tr>
<td>I emailed the journal prompts and read the journal entries once they were completed.</td>
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</table>

**F. New Understanding of Teaching**

<table>
<thead>
<tr>
<th>Action</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>All 4 interns and I met to share our new understandings. Each pair described their lesson study and what they learned. The interns then discussed which phases prompted their learning and why.</td>
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<tr>
<td>As a facilitator of the conversation, I helped clarify or point out key aspects of both lessons and helped the interns learn from each other’s experiences.</td>
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<tr>
<td>After the meeting, the interns completed journal entry #7, listing all the ideas they now understood about addition and subtraction and then redrew their concept map.</td>
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<tr>
<td>I emailed the journal prompts and read the journal entries once they were completed.</td>
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</tbody>
</table>

**The mathematics content.** The first step in the lesson study process is to define a problem in mathematics instruction or student learning (Stigler & Hiebert, 2009).

Whereas the identification of a problem is typically decided upon by the lesson study team, I decided to confine the content for the PST-LSC in this study to addition and subtraction. This mathematical content was explored in the interns’ mathematics methods course during the Fall semester, which addressed number sense and place value, whole number computation, addition and subtraction, and multiplication and division.

Additionally, addition and subtraction were content areas that could be taught in all four of the interns’ grade levels (K, 1, 3, 4) at the time of the study.
Understand the subject matter (Phase A). “To teach is first to understand. We ask that the teacher comprehend critically a set of ideas to be taught. We expect teachers to understand what they teach and, when possible, to understand it in several ways” (Shulman, 1987, p. 14). To help develop a more flexible understanding of addition and subtraction, in this first phase of the PST-LSC, the four interns were given several addition and subtraction tasks to solve (see Table 3.2). The interns solved each mathematics task one at a time and then discussed the strategies and models they used to solve the tasks.

Table 3.3.

Addition and Subtraction Tasks

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>7+8</td>
<td>9-6</td>
</tr>
<tr>
<td>32+41</td>
<td>56-23</td>
</tr>
<tr>
<td>46+26</td>
<td>62-35</td>
</tr>
<tr>
<td>147+64</td>
<td>301-74</td>
</tr>
</tbody>
</table>

Make the subject matter comprehensible (Phase B).

Comprehended ideas must be transformed in some manner if they are to be taught. To reason one’s way through an act of teaching is to think one’s way from the subject matter as understood by the teacher into the minds and motivations of learners. (Shulman, 1987, p. 16)

This quote shows Shulman’s belief that teachers must not only understand the subject matter to be taught, but also have knowledge of how to make the subject matter comprehensible to students. To help interns develop such an understanding of addition and subtraction, the work of this phase of the cycle included the use of the textbook from
the interns’ mathematics methods course (Van de Walle, Karp, Lovin, & Bay-Williams, 2018) and the mathematics series (Akers, O’Connell, & SanGiovanni, 2016) used as part of the district’s mathematics curricula. Using these books situated the cycle in both the university and district contexts.

This phase consisted of two meetings. Prior to attending the first meeting for this phase, the kindergarten and first grade interns read assigned pages from Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades PK-2 (Van de Walle et al., 2018), while the third and fourth grade interns read assigned pages from Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades 3-5 (Van de Walle et al., 2018). The readings described the different problem types, models, and strategies for addition and subtraction (see Table 3.4 for examples). The models described in Teaching Student-Centered Mathematics are research-based and include those that were developed in the CGI project (Carpenter, Fennema, Franke, Levi, & Empson, 2015; Carpenter, Franke, & Levi, 2003; Empson & Levi, 2011), which have been shown to have implications for teacher learning (Carpenter & Fennema, 1992; Carpenter, Fennema, & Franke, 1996; Moscardini, 2014). During the first meeting, the discussion was focused around the following questions:

- Looking at the different problem types, what do you now understand about addition and subtraction? What do you not understand?
- What do you now know about writing mathematics tasks?
- What new understandings do you have of the types of models or strategies used with addition and subtraction?
Looking at the misconceptions chart, what do you now understand about student thinking?

How would you teach addition and subtraction for a conceptual understanding?

Table 3.4.

Examples of Addition and Subtraction Problem Types, Models, and Strategies

<table>
<thead>
<tr>
<th>Addition and Subtraction Problem Types</th>
<th>Join</th>
<th>Sandra had 8 pennies. George gave her 4 more. How many pennies does Sandra have altogether?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separate</td>
<td>Sandra had 12 pennies. She gave 4 pennies to George. How many pennies does Sandra have now?</td>
<td></td>
</tr>
<tr>
<td>Part-Part-Whole</td>
<td>George has 4 pennies and 8 nickels. How many coins does he have?</td>
<td></td>
</tr>
<tr>
<td>Comparing</td>
<td>George has 12 pennies and Sandra has 8 pennies. How many more pennies does George have than Sandra?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Addition and Subtraction Models</th>
<th>Bar Diagrams</th>
<th>Ty has read 17 books since the first day of school. His goal is to read 43. How many more books does Ty have to read?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number Line</td>
<td>The difference between 8 and 5.</td>
</tr>
<tr>
<td></td>
<td>Counters or Cubes</td>
<td>The difference between 8 and 5.</td>
</tr>
</tbody>
</table>
Addition and Subtraction Strategies

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Modeling</td>
<td>Student uses manipulatives, drawings, or fingers along with counting to directly represent the numbers involved and the meaning of an operation or story problem.</td>
</tr>
<tr>
<td>Split Strategy (Decomposition)</td>
<td>Student splits the numbers by place value and then adds or subtracts.</td>
</tr>
<tr>
<td>Jump Strategy</td>
<td>Student counts up or back to add or subtract.</td>
</tr>
<tr>
<td>Shortcut Strategy (Compensation)</td>
<td>38 + 69: Student adjusts the numbers to 40 + 70 which equals 110 and then takes off the extra 3 to get 107.</td>
</tr>
<tr>
<td>Making 10</td>
<td>8 + 5: Student decomposes 5 into 2 and 3, adds the 2 to the 8 which equals 10, and then adds 3 more which is 13.</td>
</tr>
<tr>
<td>Doubles and Near Doubles</td>
<td>Student knows the doubles facts (i.e. 8+8) and uses double fact to solve a near double fact (i.e. 8+9).</td>
</tr>
<tr>
<td>Think Addition</td>
<td>Student thinks of subtraction as addition, i.e. 9-6=? Is also 6+?=9.</td>
</tr>
<tr>
<td>Down Under 10</td>
<td>Similar to Making 10, 53-7: Student takes off 3 to equal 50, then 4 more is 56.</td>
</tr>
<tr>
<td>Take From 10</td>
<td>16-8: Student decomposes the 16 into 10+6, then subtracts 8 from 10 to get 2 and adds 6+2 which is 8.</td>
</tr>
</tbody>
</table>


Then, prior to coming to the second meeting, the interns read a module from their individual grade-level text of *Math in Practice* (Akers et al., 2016). The kindergarten/first grade pair read an addition module and the third/fourth grade pair read a subtraction module, which reflected the content they chose to teach throughout the lesson study.

During the second meeting, the intern pairs and I met separately to review the
instructional activities presented in the modules. We discussed how the activities explore student misconceptions and incorporate addition and subtraction models. The following questions guided the discussion:

- What do you now know about how to teach addition or subtraction?
- How would you sequence your instruction of addition or subtraction?
- How do these instructional activities address student misconceptions?
- What will you plan for your lesson and why?
- Did you learn anything new?

**Plan the lesson (Phase C1).** After studying the mathematics content, the intern pairs collaboratively planned small-group lessons. To align with the PDS context, the interns were asked to use the lesson plan template (see Appendix A) required by the program, which is a Google document, to plan their lessons. While planning, interns were asked to think about the questions to be posed during the lesson and to anticipate possible student responses as is done in lesson study (Fernandez & Yoshida, 2004). The interns shared their lesson plans with their mentors, PDAs, and me.

**Teach the lesson (Phase D1).** Once the lesson plans were complete, one intern from each pair taught the lesson in her mentor’s classroom while the partner intern and I observed and took notes. Because Perry and Lewis (2009) found that observation tools helped with data collection and the debriefing process, I created an observation template (see Appendix B) to encourage the observers to concentrate on student thinking during the lesson. The first part of the observation template was filled out during the lesson; the observers wrote down the questions posed by the teacher, the strategies or models used by students, and any misconceptions the students seemed to hold. The second part of the
form was reflective and filled out by the observers and the teacher as soon after the lesson as possible. Like the first part of the observation template, the second part asked the observers to focus on student thinking and learning by asking the following questions:

- Were there any unexpected or surprising student responses?
- What did the students learn?
- Were the tasks posed effective in meeting the objectives of the lesson?

The intern who taught the lesson made notes in the entire observation template after teaching the lesson. The completed templates were used during the Analyze and Reflect phase of the PST-LSC.

**Analyze and reflect (Phase E1).** This phase of the cycle began with an intern sharing her analysis and reflection of the lesson she taught. Once the intern shared her thoughts, the observing intern and I shared additional insights and reflections. The focus of this discussion was the data collected on the observation form and formative assessment that was given during the lesson to further analyze student learning. Again, because Perry and Lewis (2009) found that tools providing structure while discussing the data from lessons helped teachers to focus their conversation, participants created a chart (see Appendix C) that included areas to write in their data and reflections from the observation template. After discussing the students’ thinking that emerged during the lesson, the interns and I jointly identified the aspects of the lesson that seemed more or less effective in impacting students’ learning and considered changes to the lesson.

**Revise the lesson (Phase C2).** After analyzing and reflecting upon the lessons, the interns cycled through the lesson again. The second cycle through the lesson began with each intern pair adjusting their shared lesson plan. Revisions to the lesson plan
included the improvement of the timing of the lesson, the materials or manipulatives, the clarity of the focus of the lesson, and the wording of questions (Fernandez & Yoshida, 2004). Interns also made changes to the lesson sequence. Any adjustments made to the lesson were made in a duplicate copy of the original plan in order to record the changes. Just like the initial lesson plan, the interns shared their plans with their mentors, PDAs, and me.

**Teach the lesson (Phase D2).** The teaching of the revised lesson occurred in the other intern’s classroom. Just like during the initial teaching, the observers used the observation template to record notes. Then, observers reflected in the second section of the template prior to meeting for the analyze and reflect phase of the PST-LSC. The intern who taught the lesson filled out the entire observation template after teaching the lesson. While completing the second section of the observation template, observers began to think about the differences in student thinking and learning between the first and second lessons.

**Analyze and reflect (Phase E2).** The second analysis and reflection session mimicked the first. The interns began by discussing the data collected on the observation template and then analyzed the data collected through the formative assessment. During this discussion, the team filled out the analysis and reflection chart just as they did during the first session. After discussing the students’ thinking that emerged during the lesson, the interns and I identified the aspects of the lesson that seemed more or less effective in impacting students’ learning and considered further changes to the lesson.

**New understanding of teaching (Phase F).** To allow the pairs to learn from each other’s experiences during the PST-LSC, all four interns and I met to share our new
understandings of teaching addition and subtraction. Each intern pair described their lesson to the other pair and then the entire group reflected upon the PST-LSC. After meeting, the interns wrote a reflective journal response in which they considered their entire experience and learning throughout the cycle. Details of the journal response are provided in the Data Collection section that follows.

**Data Collection**

Through being a part of a social setting, you learn firsthand how the actions of research participants correspond to their words; see patterns of behavior; experience the unexpected, as well as the expected; and develop a quality of trust, relationship, and obligation with others in the setting. (Glesne, 2011, p. 63)

The purpose and benefits of participant observation that Glesne describes are the reasons participant observation was chosen as a data collection method in this study. The degree to which a researcher is an observer or a participant varies and depends upon the questions and context being investigated (Glesne, 2011; Marshall & Rossman, 2011). During this study, I was a full participant because I was a member of the context being researched (Glesne, 2011). The main reason for being a full participant within the study’s context “[was] to better understand the research setting, its participants, and their behavior” (Glesne, 2011, p. 66).

Being a full participant throughout the entire study allowed me to act as the more knowledgeable other (MKO) (Vygotsky, 1978). Acting as the MKO provided me the opportunity to support the interns’ learning in the moment. For example, being present during Phase B. Make the Subject Matter Comprehensible, resulted in me leading discussions that attempted to clarify the bar diagram, the Take from Ten strategy, and the
compensation strategy. I was familiar with these methods of solving addition and subtraction tasks and was able to not only answer the interns’ questions, but was also able to draw on my own experiences with teaching addition and subtraction. Therefore, as the MKO, I was simultaneously serving the research and the education of the interns, carrying the dual role of researcher and teacher educator. While my role as teacher educator throughout the lesson study supported the PSTs’ learning of how to teach mathematics, my hope is that the PSTs also gained knowledge of how to work with colleagues in the future to continue to develop their MKT.

The following were the primary data sources: semi-structured interviews, audio recordings, photographs, observation notes, lesson plans, and journal responses. The sections that follow will describe each type of data, when it was collected, and why it was chosen. The types of data collected during each phase of the PST-LSC, along with the timeline, appear below in Table 3.5.

Table 3.5.

*Data Collected During the Study*

<table>
<thead>
<tr>
<th>Timeline</th>
<th>K/1 LS Pair</th>
<th>3/4 LS Pair</th>
<th>Data Collected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>Pre-Interviews</td>
<td>Audio recordings of interviews</td>
<td></td>
</tr>
<tr>
<td>Apr 15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Week 2</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>Apr 22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Week 3</td>
<td>A. Understand the Subject Matter</td>
<td>Audio recording of discussion, photographs, interns’ work, and journal responses</td>
<td></td>
</tr>
<tr>
<td>Apr 29</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Week 4</td>
<td>B. Make Subject Matter Comprehensible #1</td>
<td>Audio recording of discussion and journal responses</td>
<td></td>
</tr>
<tr>
<td>May 6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Week 5</td>
<td>B. Make Subject Matter Comprehensible #2</td>
<td>Audio recordings of discussions and journal responses</td>
<td></td>
</tr>
<tr>
<td>May 13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C1. Plan the Lesson</td>
<td>Audio recording of MSMC2 and lesson plans</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Week 6</td>
<td>D1. Teach the Lesson</td>
<td>Observation notes</td>
<td></td>
</tr>
<tr>
<td>Date</td>
<td>Event</td>
<td>Notes</td>
<td></td>
</tr>
<tr>
<td>------------</td>
<td>----------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>May 20</td>
<td>E1. Analyze and Reflect</td>
<td>Audio recordings of discussion and journal responses</td>
<td></td>
</tr>
<tr>
<td></td>
<td>C2. Revise the Lesson</td>
<td>Revised lesson plans and journal responses</td>
<td></td>
</tr>
<tr>
<td>Week 7</td>
<td>D2. Teach the Revised Lesson</td>
<td>Observation notes</td>
<td></td>
</tr>
<tr>
<td>May 27</td>
<td>E2. Analyze and Reflect</td>
<td>Audio recording of discussion and journal responses</td>
<td></td>
</tr>
<tr>
<td>Week 8</td>
<td>D2. Teach the Revised Lesson</td>
<td>Observation notes</td>
<td></td>
</tr>
<tr>
<td>June 3</td>
<td>E2. Analyze and Reflect</td>
<td>Audio recording of discussion and journal responses</td>
<td></td>
</tr>
<tr>
<td></td>
<td>F. New Understanding of Teaching</td>
<td>Audio recording of discussion and journal responses</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Post-Interviews</td>
<td>Audio recordings of interviews</td>
<td></td>
</tr>
</tbody>
</table>

**Semi-structured interviews.** Two interviews were conducted with each intern.

The first interview occurred before the start of the PST-LSC, and the second occurred after the completion of the cycle. The pre-interviews were analyzed prior to beginning the cycle in order to make responsive adjustments to the first two phases. Semi-structured interviews were used to collect data that can be compared across pre/post-interviews and interns (Bogdan & Biklen, 2007). The reason for using this data collection method was to “gather descriptive data in the subjects’ own words so that [I could] develop insights on how subjects interpret some piece of the world” (Bogdan & Biklen, 2007, p. 103). In this study, the interviews were used to gather the interns’ mathematical knowledge for teaching addition and subtraction and reflections on what and when they learned throughout the PST-LSC. The semi-structured interviews included some planned questions (see Appendix D) with the flexibility to ask follow-up, probing, and clarifying questions (Marshall & Rossman, 2011). The interviews were scheduled at a convenient time and location (Glesne, 2011; Marshall & Rossman, 2011). Interviews were audio-recorded and transcribed.
PST-LSC meetings. The following phases of the PST lesson cycle were audio-recorded and transcribed: *Phase A. Understand the Subject Matter; Phase B. Make Subject Matter Comprehensible; Phases E1. and E2. Analyze and Reflect; and Phase F. New Understanding of Teaching*. The purpose of audio-recording these phases of the PST-LSC was to have documentation of changes in interns’ MKT. All recordings of these phases were transcribed. Additionally, photographs were taken of written work during *Phase A. Understand the Subject Matter*.

Photographs. According to Bogdan and Biklen (2007), “the most common use of the camera is in conjunction with participant observation” (p. 151). Taking photographs while observing was a way to remember the specifics of the context for later reflection (Bogdan & Biklen, 2007). Photographs were taken of interns’ written work to document interns’ knowledge of addition and subtraction. Photographs of interns’ work also allowed the interns to reference their work from the first stage when planning lessons.

Documents from the PST-LSC. Documents can be used “as data to try to understand the perspective of the authors, how they make sense out of their world, themselves, and others and how these meanings were shaped” (Bogdan & Biklen, 2007, p. 65). The documents collected in this study were used to analyze interns’ mathematical knowledge for teaching addition and subtraction. Due to the fact that all documents collected for the study were created by interns during the PST-LSC (Glesne, 2011; Marshall & Rossman, 2011), there was no concern with whether the documents revealed the author’s perspective or were related to the context being studied (Bogdan & Biklen, 2007). During the cycle, three types of documents were collected: lesson plans, observation notes, and journal responses.
Lesson plans. The lesson plans that interns wrote during the planning and revising phases of the cycle were collected. The interns submitted their lesson plans as Google documents; this decision was made to stay true to how lesson plans are submitted in the PDS context. Collecting the interns’ lesson plans, both initial and revised, allowed me to determine how the interns’ planning for instruction may have been influenced by the phases of the PST-LSC.

Observation notes. The observers’ notes were collected. The observation notes were used as a reference for the interns and myself during the analyze and reflect stages of the PST-LSC. These observation notes were in the form of electronic Google documents and shared with all participants.

Journal responses. Interns responded to prompts in a journal before the start of the cycle and after each of the following stages: Phase A. Understand the Subject Matter; Phase B. Make Subject Matter Comprehensible; Phases E1. and E2. Analyze and Reflect; Phase C2. Revise the Lesson; and Phase F. New Understanding of Teaching. The information collected in these journals was not only used to obtain the interns’ perspectives on their own learning, but also as a way to track interns’ growth in MKT. To see this growth, interns completed a concept map in their journals before beginning the PST-LSC and again after its completion. The prompts varied by stage and can be found in Table 3.6. Like the lesson plans and observation notes, these journals were in the form of electronic Google documents.
Table 3.6.

**Journal Response Prompts**

<table>
<thead>
<tr>
<th>Cycle Phase</th>
<th>Journal Prompts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understand the Subject Matter</td>
<td><strong>Before (#1)</strong>&lt;br&gt;• What do you know or understand about addition and subtraction?&lt;br&gt;• Draw a concept map for addition and subtraction.&lt;br&gt;• Which ideas do you fully understand?&lt;br&gt;• Which ideas do you not understand?&lt;br&gt;<strong>After (#2)</strong>&lt;br&gt;• What do you now understand about addition and subtraction?&lt;br&gt;• What ideas do you still not understand?</td>
</tr>
<tr>
<td>Make Subject Matter Comprehensible</td>
<td><strong>After (#3)</strong>&lt;br&gt;• How can addition and subtraction be presented and explained to your students?&lt;br&gt;• In what order would you present these ideas to your students?</td>
</tr>
<tr>
<td>Plan the Lesson</td>
<td>• No journal response</td>
</tr>
<tr>
<td>Teach the Lesson</td>
<td>• No journal response</td>
</tr>
<tr>
<td>Analyze and Reflect</td>
<td><strong>After (#4)</strong>&lt;br&gt;• What aspects of the lesson went well?&lt;br&gt;• What ideas did students understand?&lt;br&gt;• What was learned about planning and teaching?&lt;br&gt;• What changes would you make and why?</td>
</tr>
<tr>
<td>Revise the Lesson</td>
<td><strong>After (#5)</strong>&lt;br&gt;• What changes did you make and why?</td>
</tr>
<tr>
<td>Teach the Revised Lesson</td>
<td>• No journal response</td>
</tr>
<tr>
<td>Analyze and Reflect</td>
<td><strong>After (#6)</strong>&lt;br&gt;• What aspects of the lesson went well?&lt;br&gt;• What ideas did students understand?&lt;br&gt;• What was learned about planning and teaching?&lt;br&gt;• What changes would you make and why?</td>
</tr>
<tr>
<td>New Understanding of Teaching</td>
<td><strong>After (#7)</strong>&lt;br&gt;• List all the ideas you now have and understand on the topic. Re-draw your concept map.</td>
</tr>
</tbody>
</table>


**Data Analysis**

To determine how the PSTs’ MKT developed over the course of the PST-LSC, the interview and meeting transcripts, journal responses, lesson plans and observation notes from each case were analyzed using thematic analysis. Conducting a thematic analysis means the data were coded and then the codes were examined to determine what
was at the core of each code (Glesne, 2011). Any text coded for MKT was put in a matrix on a Google Spreadsheet.

The initial analysis began by coding for the PSTs’ mathematical knowledge for teaching addition and subtraction. Coding for the PSTs’ knowledge was done using a priori codes. These codes were taken from Hill, Ball, et al.’s (2008) framework for MKT: common content knowledge (CCK), specialized content knowledge (SCK), knowledge at the mathematical horizon, knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of curriculum. These six codes were used to mark aspects of the PSTs’ MKT evident in the interview and meeting transcripts, journal responses, lesson plans, and observation notes.

Once an instance of MKT was identified, the knowledge was then assigned a subcode to specify more closely the MKT the PSTs exhibited (see Table 3.7). Some subcodes were determined prior to analyzing the data. These codes were identified through analysis of Hill, Ball, et al.’s (2008) descriptions of each of the six components of their MKT framework. Additionally, some subcodes were developed during the analysis process. Data that was coded as an instance of MKT, but was not described by one of the pre-determined subcodes, was assigned a new subcode. A newly defined subcode was considered valid if it appeared at least three times within a single participant’s dataset or if it appeared across three or more of the participants’ datasets. Four subcodes met these criteria and are italicized in Table 3.7.
### Subcodes for Data Analysis

<table>
<thead>
<tr>
<th>SMK Codes</th>
<th>Subcodes: Interns understand…</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Content Knowledge (CCK)</td>
<td>Addition/subtraction computation</td>
</tr>
</tbody>
</table>
| Specialized Content Knowledge (SCK) | Addition/subtraction problem types  
Addition/subtraction models  
Addition/subtraction strategies  
How to give mathematics explanations  
Unusual solution methods  
*Mathematical Terminology* |
| Knowledge at the Mathematical Horizon | How addition/subtraction are related to other mathematics concepts |

<table>
<thead>
<tr>
<th>PCK Codes</th>
<th>Subcodes: Interns understand…</th>
</tr>
</thead>
</table>
| Knowledge of Content and Students (KCS) | Students’ mathematical thinking  
How students learn mathematics  
Students’ misconceptions and common errors |
| Knowledge of Content and Teaching (KCT) | How to build on students’ mathematical thinking  
Remedies for student errors  
The sequencing of content  
How to evaluate models or strategies  
*How to promote student reasoning*  
*How to write mathematical tasks* |
| Knowledge of Curriculum | Materials  
How to evaluate curricula  
The vertical curriculum  
The horizontal curriculum  
*How to adapt curricula* |

A matrix was used to organize all data by the assigned subcode (Table 3.7) and the phase of the study from which the data was collected. The matrix was then used to determine how PSTs’ MKT developed during the PST-LSC. As data about the PSTs’ knowledge was coded, data suggesting any instances of struggle or growth were highlighted. Areas of struggle were highlighted in red and areas of growth in green. Two types of data qualified as a area of growth: 1) an intern’s self-reported learning and 2) a change from a previous piece of data. This was done to track possible changes in or development of MKT. Table 3.8, provides examples of data that were highlighted as moments of struggle or growth.
Table 3.8.

Examples of Moments of Struggle and Growth

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Moments of Struggle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant Made an Addition/Subtraction Computation Error</td>
<td>Example: Kelly’s solution for 206-139: “So, I have to make this, the zero, a 10 and then cross off the [two] in the 100s, make that a one and then 16 minus 9. Then 7, 7, 77” (Pre-Interview).</td>
</tr>
<tr>
<td>Participant Referenced an Incomplete Understanding</td>
<td>Example: Journal Question: What ideas do you not understand? Finley: “Multiple strategies for subtraction” (Journal Response 2)</td>
</tr>
<tr>
<td>Participant Used Mathematical Terminology Incorrectly</td>
<td>Example: Ford: The subtrahend is the bigger number (New Understanding Transcript).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Moments of Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant Described a New Understanding</td>
<td>Example: Researcher: What do you now know about how to teach subtraction? Theresa: I feel like I never thought about the importance of place value in subtraction (Make Subject Matter Comprehensible Transcript 2).</td>
</tr>
<tr>
<td>Growth Based on a Previous Piece of Data</td>
<td>Example: Theresa did not use an open number line during her pre-interview or Phase A. Understand the Subject Matter. During her post-interview, Theresa did use an open number line to solve all four addition and subtraction tasks.</td>
</tr>
</tbody>
</table>

Focusing on moments of struggle and growth, data from each of the six MKT codes were examined separately to develop themes. The themes described the types of MKT PSTs gained and how that knowledge was developed throughout the PST-LSC.

Some themes that occurred were:

- The kindergarten/first grade intern pair gained knowledge of subtraction computation during Phase A. Understand the Subject Matter.

- The kindergarten/first grade intern pair developed their knowledge of subtraction models during Phase A. Understand the Subject Matter and Phase B. Make the Subject Matter Comprehensible.

- The third/fourth grade intern pair gained knowledge of how addition and subtraction are related to other mathematical topics during Phase B. Make the Subject Matter Comprehensible.
• The third/fourth grade intern pair gained knowledge of students’ mathematical thinking and there is no evidence to show which phase of the PST-LSC influenced that growth.

**Trustworthiness**

As Maxwell (2013) suggests, researcher bias and participant reactivity pose threats to the validity of this study. In order to ensure the findings of this study accurately portray interns’ learning and to counteract the threats of researcher bias and participant reactivity, the following strategies proposed by Maxwell (2013) were used: rich data triangulation. According to Bogdan and Biklen (2007), “multiple sources lead to a fuller understanding of the phenomena” (pp. 115-116) being studied; therefore, collecting data through participant observation, journal responses, documents, and interviews provided a rich data set and allowed for triangulation. Additionally, the use of two different pairs of interns provided another way to triangulate the data (Bogdan & Biklen, 2007). Each of these strategies helped reduce the threat of validity throughout the course of the study.

**Summary**

The purpose of this study was to develop an understanding of how interns’ MKT developed while participating in the PST-LSC. The following research questions guided the study: 1) How does the MKT of elementary PSTs develop as they engage in the PST-LSC? and 2) How do the activities of the PST-LSC provide opportunities for the development of MKT? Due to the explanatory nature of my research questions, case study methods were used. The case study examined the use of the PST-LSC with four interns who were student teaching in the PDS in which I teach and supervise. The PST-LSC combined Shulman’s (1987) pedagogical reasoning and Japanese lesson study
(Fernandez & Yoshida, 2004). In pairs, the interns went through a lesson study cycle that included the following additional phases: understanding the subject matter, making the subject matter comprehensible, and new understanding of teaching. Participant observation was used to collect data throughout the PST-LSC. The primary data sources were: semi-structured interviews, audio recordings, photographs, observation notes, lesson plans, and journal responses. Data were analyzed using thematic analysis and codes drawn from Hill, Ball, et al.’s (2008) MKT framework. Focusing on moments of struggle and growth, data from each of the six MKT domains were examined separately to develop themes.

Chapters 4, 5, and 6 present the results of the study. Findings related to the first research question for the kindergarten/first grade intern pair are presented in Chapter 4 and for the third/fourth grade intern pair in Chapter 5. Then, in Chapter 6, the second research question is addressed; the activities of each phase of the PST-LSC that contributed to growth in the MKT of both pairs of interns are explored.
Chapter 4

K/1 Intern Pair MKT Development

The purpose of this study was to examine how PSTs’ MKT developed while participating in the PST-LSC (Figure 4.1). The following research questions were posed:
1) How does the MKT of elementary PSTs develop as they engage in the PST-LSC? and
2) How do the activities of the PST-LSC create opportunities for the development of MKT? This chapter presents findings related to the first question for the kindergarten/first grade intern pair, Kelly and Finley.

![Diagram of the pre-service teacher lesson study cycle (PST-LSC)]

Figure 4.1. The pre-service teacher lesson study cycle (PST-LSC)

The Development of MKT

The kindergarten/first grade interns, Kelly and Finley, were found to show growth in five of the six domains of MKT: common content knowledge (CCK), specialized
content knowledge (SCK), knowledge at the mathematical horizon, knowledge of content and teaching (KCT), and knowledge of the curriculum. The findings are organized by those five MKT domains.

**Common Content Knowledge**

By participating in the PST-LSC, Kelly and Finley experienced growth in their knowledge of subtraction computation. This growth, from the beginning to the end of the PST-LSC, is illustrated in Figure 4.2.

*Figure 4.2. Kelly and Finley’s Growth in Knowledge of Subtraction Computation*

Prior to the PST-LSC, both Kelly and Finley expressed limited knowledge concerning subtraction computation. During the pre-interview, Finley used only one
strategy, direct modeling, to solve 12-8 and stated, “I was going to do a second one for that one, but I didn’t remember a second strategy” (Pre-Interview). Prior to participating in Phase A. Understand the Subject Matter, when asked what ideas (about addition and subtraction) she did not understand, Kelly responded, “Borrowing from different place values [and] automaticity with large numbers” (Journal Response 1). Additionally, even after discussing different subtraction strategies, such as direct modeling, decomposition, counting on and back, and compensation, during Phase A. Understand the Subject Matter, both participants still expressed a lack of confidence with subtraction. Finley desired to know more strategies for subtraction (Journal Response 2), and Kelly said, “subtraction as a whole concept is more difficult for me to understand. I do not have much confidence in subtraction compared to addition” (Journal Response 2). Although Kelly and Finley felt they lacked knowledge of subtraction after participating in the first phase of the PST-LSC, they both exhibited growth in knowledge of subtraction computation during this phase of the cycle. An example can be found in Kelly’s journal response after the phase: “I…now understand breaking numbers up into its place value group” (Journal Response 2).

Additionally, during the post-interview, Kelly shared the following:

Before all of our math thinking, I only really knew how to do the algorithm. I didn’t really think about breaking it into friendly numbers or by place value… I never really knew you could apply the same strategies for addition and subtraction, but in a different way. And, it’s not always taking away numbers, but you can also figure out addition within subtraction. It doesn’t always have to be subtraction for you to solve a subtraction problem. (Post-Interview)
Kelly’s comments reveal a growth in her knowledge of subtraction strategies (making friendly numbers, split strategy, think addition); analysis of the pre- and post-interview data further supports her claim. When comparing the strategies participants used to solve subtraction tasks during both the pre- and post-interviews (see Table 4.1), it was evident that Kelly and Finley used different or more strategies during the post-interview.

Table 4.1

*Subtraction Strategies Used*

<table>
<thead>
<tr>
<th>Participant</th>
<th>Pre-Interview</th>
<th>Post-Interview</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kelly</td>
<td>Counting on</td>
<td>Direct modeling</td>
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<td></td>
<td>Direct modeling</td>
<td>Algorithm</td>
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<td></td>
<td>Take from 10</td>
<td>Down under 10</td>
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<tr>
<td></td>
<td>Algorithm</td>
<td>Split strategy</td>
</tr>
<tr>
<td>Finley</td>
<td>Direct modeling</td>
<td>Direct modeling</td>
</tr>
<tr>
<td></td>
<td>Algorithm</td>
<td>Algorithm</td>
</tr>
<tr>
<td></td>
<td>Down under 10</td>
<td>Split strategy</td>
</tr>
</tbody>
</table>

**Specialized Content Knowledge**

Kelly and Finley’s SCK grew in the following areas: knowledge of mathematical terminology and knowledge of addition and subtraction models. Their growth in each is illustrated in Figures 4.3 and 4.4.
Mathematical terminology. Kelly exhibited an increase in her understanding of mathematical terminology (Figure 4.3). During Phase B. Make the Subject Matter Comprehensible, Kelly realized that compare is a mathematical term. When referring to the word compare being used to name a problem type, Kelly said, “I feel like it’s always like you’re comparing and contrasting a book or more of like social studies or science. I don’t really think about it in a math way” (Make the Subject Matter Comprehensible 1). Kelly’s comment reveals that even though she knew the term, compare, was a word that is used within the elementary curriculum, she had never connected the term to subtraction or mathematics in general.

When comparing Kelly’s concept maps, the mathematical terms, commutative property and regrouping, appeared on the concept map completed after the end of the PST-LSC (Concept Map 2) and were not a part of the concept map created prior to the
start of the lesson study (Concept Map 1). Kelly’s concept maps appear below in Figure 4.5. While there is no evidence to suggest these terms were new to Kelly, the fact that the term, *commutative property*, was part of the lessons this pair taught may suggest the PST-LSC deepened Kelly’s knowledge of the commutative property’s role in addition.

Although it seems Kelly’s knowledge of mathematical terms is still developing, the PST-LSC appears to have provided opportunities for her to advance her knowledge.

*Figure 4.5. Kelly’s Concept Maps*
**Knowledge of addition and subtraction models.** During *Phase A. Understand the Subject Matter*, Kelly expressed the fact that she was not familiar with the open number line. Once she was exposed to this model, she used it to solve five different addition and subtraction tasks (*Understand the Subject Matter Intern Work*). Kelly’s gain in knowledge is further evidenced by a journal response she wrote after *Phase A. Understand the Subject Matter*:

"I now understand using the number line strategy to help solve addition and subtraction problems. I never considered using a number line before to help solve equations and it really does help my math thinking in a way that I was never taught before." (Journal Response 2)

Then, during the first meeting of *Phase B. Make the Subject Matter Comprehensible*, I asked, “So were there any other models that you came across that you hadn't thought of?” Kelly’s response was:

"I think the jump strategy, like using the number line and jumping back or forward, like I said last time I never really learned that strategy, but then after learning how people were doing it, I was like, “Oh, that makes so much sense.” So, I think it was just a strategy that I was never really forced to use, so I really didn't have to use it. I always did just the traditional way, but I really thought using the number line to help me jump backwards or forward really just helped in my own math thinking." (Make Subject Matter Comprehensible 2)

Kelly’s comments reveal that exposure to the open number line helped her to develop her own understanding of addition and subtraction computation. There is further evidence that Kelly learned the open number line from Finley, who used the model to solve tasks
During Phase A. Understand the Subject Matter. During Phase F. New Understanding, the following was discussed:

**Kelly:** Like Finley’s method, I literally would have felt like I would have had so much more confidence in math if I knew that way.

**Finley:** Which one?

**Kelly:** I forget. The number line.

**Finley:** The open number line.

**Kelly:** Yeah, the open number line. (New Understanding Transcript)

Lastly, Kelly used the open number line when solving all four of the addition and subtraction tasks posed in the post-interview (Post-Interview), and she added *number line* to her final concept map (Figure 4.5).

Kelly was not the only one who gained new knowledge of addition and subtraction models. During Phase B. Make the Subject Matter Comprehensible, Finley learned how unifix cubes can be used to model compare problems. Below was Finley’s response to the question: What new understandings do you have about models that students can use to solve addition and subtraction problems?

I think it's for subtraction, the unifix cubes and stacking them, I thought it was interesting how they like had, the way that we use them in my classroom is different, but like if you want to get, um, figure out how many more you need to get to ten, like I thought that was really interesting... (Make Subject Matter Comprehensible Transcript 1)

Finley’s remarks show that although she knew unifix cubes are used to model addition and subtraction, the PST-LSC allowed Finley to learn a new way to use unifix cubes to
model subtraction. Not only did the knowledge Kelly and Finley gained about addition and subtraction models impact their specialized content knowledge, but also improved their knowledge of addition and subtraction computation and teaching.

**Knowledge at the Mathematical Horizon**

Through her participation in the PST-LSC, Finley strengthened her understanding of how addition and subtraction are related to other mathematical concepts. Finley’s growth is illustrated in Figure 4.6.

*Figure 4.6. Finley’s Growth in Knowledge of How Addition/Subtraction are Related to Other Mathematics Concepts*

While exploring addition and subtraction tasks in *Phase A. Understand the Subject Matter*, the following was discussed:

**Researcher:** So, looking at these solutions and thinking about the solutions for the previous addition tasks, what do students need to have a good understanding of in order to add numbers together?
Finley: Place Value.

Theresa, Ford, and Kelly: Yeah.

Researcher: Definitely place value, because you guys used place value a lot in your adding. Anything else?

Ford: I think not just, I guess this is still place value, but not just knowing that the four and the seven are in the ones place and the four and the six are in the tens, but it's really forty and sixty, and not just a four and a six. So, that's what we always tell the students is going that step deeper into like the conceptual understanding, not just knowing what place value is, but understanding it.

Theresa: We just had that conversation today.

Researcher: It's a conversation about what is the place of the digit and what is the value.

Ford: Yeah, yeah. (Understand the Subject Matter Transcript)

Although it is Ford and Theresa who provide details of their knowledge of place value in this excerpt, Finley and Kelly at least recognize the relationship between place value and addition/subtraction. After completing the PST-LSC, Finley added *place value* to the concept map she created in her last journal entry (Concept Map 2). Her concept maps appear in Figure 4.7. The term was not included in the concept map she created prior to participating in the PST-LSC (Concept Map 1). The addition of place value in Finley’s concept map suggests that her understanding of the relationship between place value and addition/subtraction was strengthened.
In addition to strengthening her understanding of the role that place value plays in addition and subtraction, Finley made a connection between comparison tasks and bar graphs. During Phase B. Make Subject Matter Comprehensible and after reading about
the different addition and subtraction problem types in *Teaching Student-Centered Mathematics* (Van de Walle et al., 2018), Finley made the following comment in response to a question about models for addition and subtraction:

> I think it’s for subtraction, the unifix cubes and stacking them, I thought it was interesting how [*Teaching Student-Centered Mathematics*] had, the way that we use them in my classroom is different, but like if you want to figure out how many more you need to get to ten, like I thought that was really interesting, because I thought of it as a bar graph type of thing, because we’ve been reading bar graphs, so I felt like the students probably would kind of read it like that, because we’ve been analyzing data, like who has more, who has less, how many more do they need? So, I feel like that would be something relatable to them.

(Make the Subject Matter Comprehensible 1)

Finley’s realization that using unifix cubes to model comparison tasks is similar to analyzing bar graphs resulted from her reading about and discussing addition and subtraction after seeing students’ work with bar graphs. The PST-LSC provided this unique opportunity.

**Knowledge of Content and Teaching**

Participating in the PST-LSC led to a growth in Kelly and Finley’s knowledge of how to promote student reasoning and conceptual understanding. The findings that support this claim are illustrated in Figure 4.8.
During her pre-interview, when asked what supports she would provide her students when solving addition and subtraction tasks, Kelly said, “Giving them different strategies to solve the problem I think would be really helpful” (Pre-Interview). Kelly’s comment is evidence of her limited understanding of how to promote student reasoning.

Similarly, when answering the same question, Finley responded with the following:

Guiding questions or I would give them the physical tools instead of having them draw it. I would talk them through switching around the numbers to see if that helped them, like 12 - ? = 8, to see if they could use a different strategy. I could guide them towards other strategies that they could use to get them to think about using like 10s sticks and 1s circles or something like that. (Pre-Interview)

Kelly and Finley’s statements reveal their desire to help students use and understand multiple strategies to solve addition and subtraction tasks; however, their comments also show an incomplete understanding of how to promote student reasoning.
After reading *Teaching Student-Centered Mathematics*, Kelly and Finley revealed a deepening of their understanding of how to support the development of their students’ conceptual understanding. During *B. Make Subject Matter Comprehensible*, I asked, “So thinking about everything this chapter said, how would you go about teaching addition and subtraction for a conceptual understanding?” Kelly responded, “I think any sort of manipulative really helps the students like understand, ‘Ok, it's not just ten, but it's one group of ten and how many more ones?’” (Make Subject Matter Comprehensible 1). Supporting Kelly’s thinking, Finley said, “And like with the paper strategies, especially the use of a ten frame, so they get that grouping of ten when they are adding. So, they see it in groups of tens, instead of individual ones” (Make Subject Matter Comprehensible 1). These quotes reveal a deeper understanding of how the use of manipulatives and models can help develop students’ understanding of place value and the role it plays in addition and subtraction.

During their first lesson, Kelly and Finley had students use a number bond to solve a three-addend addition word problem (Lesson Plan 1). When analyzing and reflecting upon this lesson, Finley thought that beginning the lesson by having “different strategies available, like ten frames, a number line, and others” (Analyze and Reflect 1) would help students think through the problem posed prior to introducing the number bond. Too, when mentioning the changes she would make to the first lesson, Finley wrote:

I would not start off with asking the students to place the numbers in a number bond. I would let them solve it in their own way and then ask them to use a number bond as a second or third strategy. I would like to see how the students
choose to see the problem and approach it with what they are comfortable with.

(Journal Response 4)

This quote shows Finley’s realization that providing students with a certain strategy does not allow them to explain how they think about the task. As a result, during the second lesson, Finley posed a problem, had students solve it, and then introduced the number bond instead of having students use the number bond and then an additional strategy to solve the problem (Lesson Plan 1 and 2). According to Kelly’s observation notes, she believed that the lesson “objectives were addressed more clearly because students had a choice of what strategy to use in addition to creating a number bond” (Observation Notes 2). The PST-LSC provided Kelly and Finley the opportunity to realize the impact that providing a specific model or strategy can have on students’ reasoning.

The interns’ responses during their post-interviews further provide evidence that there was a growth in their knowledge of how to promote student reasoning. When asked what supports she would provide to students when solving addition and subtraction tasks, Kelly responded with:

Allowing them to choose whatever strategy that works best for them, giving them manipulatives, giving them strategies to use, so they can really just use what works best for them during the time period that I'm with them. I also think that allowing students to explain their thinking can really help support them...solving the problem or equation and pushing their thinking, in a way, so they get that broader understanding. (Post-Interview)

Kelly’s remarks reveal how the PST-LSC helped her learn the impact allowing students to choose a strategy and explain their thinking has on their mathematical reasoning.
When answering the same question during her post-interview, Finley shared the following:

I would give them…some strategies, like 10 frames, part-part-whole, to take out things from the number problem, probably a number bond…and a number line. What else? A hundreds chart. I would just provide the options for them to choose from and then I would support them by asking to explain their thinking of why, so, why did you do like what you did or be more specific with my questioning, obviously specific to their work and try to get them to explain, that way they have that foundation of like explaining why and knowing the why behind their thinking. (Post-Interview)

These two quotes show that Kelly and Finley gained knowledge of how students explaining their thinking supports their reasoning. Also, during the post-interview I asked, “What do you think are the most important things about teaching addition and subtraction?” Finley responded by saying: “Knowing the strategies and how to use them. Asking them the why behind their thinking and to explain their thinking. And also getting them to talk about it each time and then addressing those misconceptions…” (Post-Interview). This statement further shows that the PST-LSC provided the opportunity for Kelly and Finley to understand how allowing students to solve tasks in their own way and to explain their thinking supports student reasoning.

Knowledge of Curriculum

Finally, the interns’ knowledge of curriculum advanced as they participated in the PST-LSC. Kelly and Finley gained knowledge of how to adapt curricular resources for students’ needs and of the vertical curriculum. This growth can be seen in Figure 4.9.
How to adapt curricular resources. Through the use of the mathematics methods course (Van de Walle et al., 2018) and school district (Akers et al., 2016) texts during the PST-LSC, Kelly and Finley learned to use curricula as a resource rather than a script. When analyzing the second lesson, the following discussion occurred:

Researcher: So, what did you learn about using resources to plan your instruction?

Finley: Not to follow them, like...

Kelly: Responsive to student need.

Finley: Yeah.

Kelly: Because you could have the greatest text in the world, the greatest resource, but that could be totally not what your students need. So, like it’s our job to make sure that our students are learning what needs to be learned, at whatever pace, and it doesn’t have to go right from the textbook, but just recognizing that not all students are going to have the same understanding at the same time.
Finley: And it’s a lot of trial and error, because I’m sure if we had switched, like I would have done the number bonds first for my class. So, I think just like kind of taking that critical eye and thinking, “Will this work for my class? What has worked in the past? When I was an intern, what worked? What can I test that might work for this class and what can I do to change it?” (Analyze and Reflect 2)

This discussion suggests that Kelly and Finley both grew in their understanding of how to use their students’ needs to adapt curricula. Then, when describing her lesson to the other intern pair during Phase F. New Understanding, Finley stated:

But, [the students] already understood [the commutative property] because we had talked about it earlier, so we didn’t need the cubes at all. So, we kind of were in that like we don’t, we need to cater to our class and not really, like use the resource for what it is, just kind of like an aide and test and see what works for our class and adjust for them. (New Understanding Transcript)

Finally, in the post-interview, I asked, “Can you tell me what stages you think most helped in your thinking about teaching addition or subtraction? What did we do in that stage that helped you?” Finley responded with the following:

So, co-planning [the lesson] with Kelly and then watching her teach it helped me because I was able to see like that following the resource exactly didn’t help us, and then when we analyzed and reflected, we found that out, and then we revised the lesson to fit my class and do what was best for them minus like what the resource was telling us to do. (Post-Interview)

Finley’s quotes reveal her new understanding of the need to adaptations for learning when using curricular resources. The data suggest that participating in the PST-LSC
contributed to Kelly and Finley learning that when using curricular resources to plan lessons, it is important to consider students’ knowledge and needs.

**Knowledge of the vertical curriculum.** In addition to developing their understanding of how to use curricular resources to plan their instruction, Kelly and Finley also gained knowledge of the vertical curriculum. During the second *Phase B. Make the Subject Matter Comprehensible* meeting, I asked, “After reading [the *Math in Practice*] module, did you learn anything new?” Finley’s response was: “I learned where kindergarten is coming from, so like where the kids are coming from when they come from kindergarten and come to first grade and then where they are going to be heading for second grade” (Make Subject Matter Comprehensible 2). Later in the meeting, I asked, “What do you now know about how to teach addition?” Finley answered with:

I think just like knowing where students have been and where they are going is super important when teaching because you can know, “Oh, where do I have to support the kids that aren’t exactly there and then where can I bring the kids that are above and beyond.” (Make Subject Matter Comprehensible 2)

*Phase B. Make Subject Matter Comprehensible* helped Finley learn more about the mathematics content in prior and later grade levels.

Then, during *Phase F. New Understanding*, when asked, “So was it beneficial to you and this process that all four of you were at a different grade level or do you think it would have been more beneficial if you were all teaching the same grade level?” Kelly’s opinion was:

I think…just getting that experience to see what your students have done before and what they are going to be doing in the future, I felt was really cool, because
we don’t really have that much experience going to different grade levels… (New Understanding Transcript)

Then, in her post-interview, when asked which stages helped her thinking about addition and subtraction, Kelly stated the following:

Co-planning, I thought was really helpful in this, just because we were able to bounce ideas off of each other and really work and in a way foster a better understanding for a different grade level other than our own. (Post-Interview)

Although her remarks do not clearly state how she will use the knowledge she gained, Kelly’s remarks revealed an increased understanding of other grade levels. Being able to work with interns who were student teaching in other grade levels allowed Kelly and Finley to become more familiar with the vertical curriculum.

Summary

This chapter presented the findings for the kindergarten/first grade intern pair. While participating in the PST-LSC, Kelly and Finley developed their common content knowledge, specialized content knowledge, knowledge at the mathematical horizon, knowledge of content and teaching, and knowledge of the curriculum. More specifically, Kelly and Finley grew in their understanding of subtraction computation, mathematical terminology, addition and subtraction models, how addition and subtraction are related to other mathematics concepts, how to promote student reasoning, how to evaluate curricula, and the vertical curriculum.
Chapter 5

3/4 Intern Pair MKT Development

This chapter presents the findings from the third/fourth grade intern pair, Theresa and Ford. Similar to Chapter 4, the findings respond to the first research question: 1) How does the MKT of elementary PSTs develop as they engage in the PST-LSC (Figure 5.1)?

![Diagram of the pre-service teacher lesson study cycle (PST-LSC)]

*Figure 5.1.* The pre-service teacher lesson study cycle (PST-LSC)

**The Development of MKT**

In the third/fourth grade pair, there was evidence of interns’ growth in four of the six domains of MKT: specialized content knowledge (SCK), knowledge at the mathematical horizon, knowledge of content and students (KCS), and knowledge of
content and teaching (KCT). The findings in this section are organized by these four MKT domains.

**Specialized Content Knowledge**

Theresa gained knowledge of addition and subtraction models. Specifically, she learned how to use an open number line to model addition and subtraction. This growth is illustrated in Figure 5.2.

![Diagram](image)

**Figure 5.2. Theresa’s Growth in Knowledge of Addition and Subtraction Models**

During her pre-interview, Theresa used direct modeling, an algorithm, and invented strategies to solve the eight addition and subtraction tasks (see Table 3.2). Although she did not use a model to solve any of the tasks, she did say that kindergarteners would use counters or a ten frame to solve the single-digit tasks and that base ten blocks may help students build a conceptual understanding of addition and subtraction. Additionally, Theresa used direct modeling, an algorithm, and invented and reasoning strategies to solve the eight tasks posed during Phase A. Understand the Subject Matter. Again, she did not use models to solve the addition and subtraction tasks.
Ford initially solved the two single-digit tasks using memorization; however, when she was asked what other ways she could solve the tasks, she was able to think about how to use a number line for both the addition and subtraction task. Too, Ford was able to explain how the number line could be used to model both counting up and counting down for the subtraction task. Ford’s explanation of how to use a number line can be found in the following quote:

So, for the first one [6+8], instead of just doing like the algorithm of adding, I could draw a number line and count up from 6, like count 8 hops or spaces to get to 14, where we would end up. And then for the second one [12-8], to see how many more, you could start at 8 and then count up to 12 and you'd see that you counted 4 times. Or yeah, I guess counting up from 8 would be the way or back from 12, but it would be like a number line and just counting up or down. (Pre-Interview)

During *Phase A. Understand the Subject Matter*, Ford shared how to use an open number line to solve all eight of the addition and subtraction tasks. Although Theresa never explicitly mentioned the open number line as a model that was new to her or a strategy that she learned through the PST-LSC, during her post-interview, she used an open number line to solve all four of the addition and subtraction tasks. Additionally, as shown in Figure 5.3, Theresa added *number line* to her concept map at the end of the cycle (Concept Map 2); it was not included in the concept map she drew prior to the start of the lesson study (Concept Map 1). The emphasis Ford put on the open number line and its use to solve addition and subtraction tasks, along with the selected readings included
in *Phase B. Make the Subject Matter Comprehensible*, may have at least partly influenced Theresa’s understanding of mathematical models.

*Figure 5.3.* Theresa’s Concept Maps
Knowledge at the Mathematical Horizon

Interns’ knowledge at the mathematical horizon appeared to increase through the PST-LSC. More specifically, Theresa and Ford extended their understanding of how addition and subtraction are related to other mathematical concepts. The knowledge they gained is depicted in Figure 5.4.

Figure 5.4. Theresa and Ford’s Growth in Knowledge of How Addition and Subtraction are Related to Other Mathematics Concepts

While exploring addition tasks during Phase A. Understand the Subject Matter, Ford demonstrated her understanding of connections between place value and multidigit addition and subtraction:

Ford: Just knowing that the four and the seven are in the ones place and the four and the six are in the tens, but it's really forty and sixty, and not just a four and a six. So, that's what we always tell [the students] is going that step deeper into like
the conceptual understanding, not just knowing what place value is, but understanding it. (Understand the Subject Matter Transcript)

This discussion reveals Ford’s own understanding of place value and how that understanding is beneficial to developing a conceptual understanding of addition and subtraction.

During the first meeting of Phase B. Make the Subject Matter Comprehensible, the interns answered, “What did this [the student misconceptions chart in Teaching Student-Centered Mathematics] help you understand about how students think about mathematics?” Ford responded with the following: “I think it helped me show how little details, like understanding place value for regrouping, matter. Like this one where they did the algorithm, but they didn’t regroup, and they just ignored the place value…” (Make Subject Matter Comprehensible Transcript 1). Ford realized the need to have a knowledge of place value in order to understand regrouping when using an algorithm to solve addition and subtraction tasks. Then, during the second meeting of Phase B. Make the Subject Matter Comprehensible, the following question was asked, “What do you now know about how to teach subtraction that maybe you didn’t know before?” Theresa answered, “Yeah, I feel like I never thought about the importance of place value in subtraction” (Make Subject Matter Comprehensible Transcript 2). While this comment does not reveal Theresa’s understanding of how place value is related to addition and subtraction, it does show that the PST-LSC helped Theresa better understand that place value is related to subtraction.

Additional evidence shows a growth in Theresa and Ford’s knowledge of how addition and subtraction are related to other mathematics concepts. Theresa and Ford’s
initial concept maps did not include *place value* (Concept Map 1), but the term appears in the second concept map that each created (Concept Map 2). Theresa’s concept maps are above in Figure 5.3 and Ford’s concept maps are below in Figure 5.5. The fact that the term, *place value*, was not included in the concept maps drawn before the start of the PST-LSC is evidence that Theresa and Ford gained a better awareness of the role place value plays in addition and subtraction computation.

*Figure 5.5. Ford’s Concept Maps*
Knowledge of Content and Students

Interns’ knowledge of content and students also grew. As illustrated in Figure 5.6, Theresa and Ford developed their knowledge of students’ mathematical thinking.

At the end of the lesson study, Theresa had a broadened understanding of strategies that students use to solve addition and subtraction tasks. When asked how her students would solve 6+8 and 12-8, Theresa said: “In the beginning…I think it would have been more like skip counting, like physically adding on on-by-one, but at this point, most of them have their addition facts and subtraction down. I think they could do this almost mentally” (Pre-Interview). Before the start of the PST-LSC, Theresa only identified counting as a student strategy and had limited knowledge of the developmental
process students go through before obtaining fact fluency. When answering the same question during her post-interview, Theresa responded with:

Yeah, so I think my third-grade students would probably have them memorized, but I would, um... I think a lot of the younger grades would use number lines, kind of combined with friendly numbers to using the number lines to get to friendly numbers and then building your way up or down. I think really younger grades would use manipulatives and pictures, like base ten blocks or unifix cubes or drawing pictures. (Post-Interview)

Theresa identified number lines, friendly numbers, manipulatives, and pictures as ways that students might solve tasks, which shows that Theresa gained knowledge of student thinking. Additionally, Theresa was able to think about addition and subtraction as more than just the act of counting.

Similarly, Ford also developed a deeper understanding of how students might think about addition and subtraction. When asked how her students would solve 6+8 and 12-8, Ford responded:

I think my students would, for the first one [6+8]…be able to go right into the algorithm of adding because they would know that 8 more means that you're adding on. For the second one [12-8], with the how many more mice, that wording kind of gets tricky for them, so I think that they would still be able to get to the algorithm method of just subtracting the two numbers, but they would have to think about it a little bit more and understand that we are looking for the number, like the difference between them. (Pre-Interview)
During the pre-interview, Ford was more focused on the wording in the addition and subtraction tasks and how it would help or hinder students reasoning through the task than she was on how students might solve the tasks. Then, in the post-interview, Ford stated the following:

I think the addition one [6+8]...like maybe a first grader and like a farther along kindergartener would be able to just know the fact or be able to like show it in…the traditional way, and then maybe still use their fingers to count, but could not really have to show much more work than that. But otherwise, I think they would do kind of like a skip, where they start at 6 and then…see what 8 jumps gets them to. Then, I mean, I think like the friendly numbers thing maybe... Um, and then I think the subtraction one would be harder just because of how it's worded… I don't really know how they would approach it at first. Obviously, like the traditional way if they know facts and they understand what the question is asking them completely, but, um, I keep going back to a number line sort of system. I think that would help students, so I think maybe a number line where they would jump from 8 to 12 and see how many times it took them to get there.

(Post-Interview)

Although Ford still mentioned the wording of the tasks, she did also focus on student strategies and on how students from multiple grade-levels would think through the tasks. The inclusion of number lines, friendly numbers, pictures, and counting on fingers in Theresa and Ford’s latter responses reveal a deeper understanding of how students might think about addition and subtraction.
Additionally, Ford expanded her knowledge of how students would solve three-digit addition and subtraction problems. When asked how her students would solve 145+76 and 206-139, Ford answered with the following:

The first one [145+76] they would definitely just add them together the algorithm way, and for the second one [206-139], I think because it says “how many fewer,” that word fewer helps them know that the number is going to be smaller so that they would know to subtract, but I think they would still just go right to the traditional way of subtracting. (Pre-Interview)

This quote illustrates how, at the beginning of the lesson study, Ford could only identify the algorithm as a strategy students might use to solve multi-digit addition and subtraction tasks. Also, this quote provides further evidence of Ford’s focus on the wording and not the computation. Ford expanded her response during her post-interview with the following:

I think they would just use…the algorithm, the traditional way, but I think some of them might also count, um... I think some of them would still count on and use the number line to help them solve it. I think that the way that I did this one [145+76], I kind of broke apart the place value, I don't think anyone would use it because it kind of was more work. Um, and it could get confusing if you double add or if you forget to add a number. Um, yeah, I think they would just use like the algorithm because they're bigger numbers, which is like what we saw in my lesson too, so… (Post-Interview)
Although she still felt her students would just use the algorithm, Ford considered her students’ use of a number line and the split strategy. Participation in the PST-LSC led to Theresa and Ford broadening their understanding of students’ mathematical thinking.

**Knowledge of Content and Teaching**

Additionally, participating in the PST-LSC led to a growth in Theresa and Ford’s knowledge of content and teaching. Theresa and Ford deepened their understanding of how to promote student reasoning. A summary of this growth can be found in Figure 5.7.

![Figure 5.7. Theresa and Ford’s Growth in Knowledge of How to Promote Student Reasoning](image)

Theresa developed a deeper understanding of how to support her students’ conceptual understanding of addition and subtraction. During her pre-interview, when asked what supports she would provide students when solving 6+8 and 12-8, Theresa said, “Um, like I said, manipulatives may have helped. Other supports... Honestly, I don't know, this is like an easy task for them I feel like, even from the beginning” (Pre-Interview). Theresa’s comment is not only another example of her limited knowledge of
how students develop fact fluency, but is also evidence of her limited knowledge of ways to support students in developing fluency. Then, when answering the same question about 145+76 and 206-139, Theresa responded with the following:

I think base ten blocks would help for some of these. To see like moving from the tens to the ones and like up 100s. Especially for the first one, I think base ten blocks would help and also for this one [206-139]. I think it would help with breaking down 10s into 1s. (Pre-Interview)

Even though the mention of base ten blocks reveals more about Theresa’s knowledge of manipulatives, it further supports the claim that she held limited knowledge of supports she could provide students to promote their mathematical reasoning.

Then, after Phase B. Make the Subject Matter Comprehensible, Theresa explained how addition and subtraction could be presented and explained to her students:

It helps for students to understand place value when subtracting. They should be able to recognize that hundreds are subtracted from hundreds, tens from tens and ones from ones. When subtracting, it can be presented on a number line where students can either count up or count back. It can also be presented using base ten blocks so that students gain an understanding of regrouping and the conceptual knowledge behind it. Another strategy is for students to break apart the number into expanded form to make subtracting easier. Students can use compensation to find friendlier numbers to subtract with. (Journal Response 3)

Theresa’s mention of additional strategies shows a deeper understanding of how to develop students’ flexibility with and conceptual understanding of addition and
subtraction. Then, in her post-interview, Theresa said the following about the supports she would provide her students to solve 6+8 and 12-8:

I would provide manipulatives of some sort so that they can work through the addition and subtraction, and I would scaffold these strategies, so start off with strategies like drawing pictures and manipulatives and then probably work up to like number line and then like breaking it apart and then eventually get to the point to where they are just fluent with those facts. (Post-Interview)

In addition to gaining knowledge of addition and subtraction strategies, Theresa was also able to begin to think about how these strategies scaffold students’ development of a conceptual understanding of addition and subtraction.

Theresa had a similar response when describing the supports she would provide students to solve 145+76 and 206-139:

For the last one, I would scaffold it so that you're starting with…the number line. Um, and then work the way up to like breaking numbers apart and then the algorithm, so that they understand what they're really doing. Um, and then like we did in our lesson for subtraction, eventually representing and teaching them what regrouping's actually doing. (Post-Interview)

Not only did Theresa begin to understand how to scaffold students’ learning of basic facts, but how to scaffold a conceptual understanding of multi-digit addition and subtraction. It appears that Theresa gained knowledge of how to support her students’ conceptual understanding during Phase B. Make the Subject Matter Comprehensible. Theresa went from only being able to think about manipulatives being a support for
students to understanding how pictures, number lines, and decomposition can help students reason through addition and subtraction tasks.

The process of planning, teaching, analyzing, and revising a lesson also led to a deeper understanding of how to promote student reasoning. When analyzing the first lesson, the following discussion occurred:

**Researcher:** What do you think you would change if you taught it again or for when you teach it again?

**Theresa:** I wonder what difference it would have made to have them do it individually with the base ten blocks rather than with a partner, because like [student one's] method overshadowed [student two's]. I don't know what [student two] would have done had she been doing it herself. That's something to think about, but I also feel like it gets them talking to do it with a partner. So, like I think it's beneficial because they're actually discussing what they're doing when they're doing it with a partner, but then…one strategy might take precedence over another.

**Researcher:** Mhmm. So, what would be a solution to that?

**Theresa:** Do it individually and then talk with a partner after to explain and then talk whole group. (Analyze and Reflect 1)

Theresa recognized that allowing students to think through tasks individually before discussing them with a partner ensures that all students have an opportunity to reason through the problem in their own way, but still gives them a chance to explain their thinking. In addition to thinking that time to think individually may be beneficial to student reasoning, Theresa also wondered “if it would be beneficial to, before giving [the
students] base ten blocks, to say, ‘solve it anyway you want and then represent it using base ten blocks’” (Analyze and Reflect 1). The discussion during Phase E1. Analyze and Reflect resulted in Theresa believing the lesson she co-planned and taught should be revised to have the students “solve [the subtraction task] however they want, and then work independently with base ten blocks, and then share with a partner” (Analyze and Reflect 1).

Ford shared the same belief; when describing the changes she would make to the lesson, Ford wrote, “At the start, I will have my students solve a problem using as many strategies as possible to see how much they recall/ways they see a subtraction problem” (Journal Response 4). The changes suggested by Theresa and Ford were applied to the second lesson. Ford further explains why she chose to restructure the lesson: “I also wanted to see how many ways my students could look at a problem and solve at the start to see how they view/apply place value and regrouping” (Journal Response 5). Theresa and Ford’s belief that students should choose their own strategy to solve a task shows that they are beginning to learn the impact that providing a model or strategy to students has on their mathematical reasoning. The PST-LSC allowed Theresa and Ford to develop a deeper understanding of how to promote student reasoning and conceptual understanding.

**Summary**

This chapter presented the findings for the third/fourth grade intern pair. While participating in the PST-LSC, Theresa and Ford developed their specialized content knowledge, knowledge at the mathematical horizon, knowledge of content and students, and knowledge of content and teaching. More specifically, Theresa and Ford grew in their understanding of addition and subtraction models, how addition and subtraction are
related to other mathematics concepts, students’ mathematical thinking, and how to promote student reasoning.
Chapter 6

The Activities that Developed MKT

The previous two chapters described specific changes in the interns’ MKT that occurred while participating in the PST-LSC. This chapter describes the findings responding to the second research question: How do the activities of the PST-LSC provide opportunities for the development of MKT? The chapter explores each phase of the PST-LSC, beginning with a brief summary of the activities that occurred during the phase and then exploring more deeply the knowledge development that each phase’s activities seemed to support. Note that the claims made in this chapter are based, in part, on the analysis presented in Chapters 4 and 5 in response to the first research question.

A summary of the knowledge that each intern pair gained and the PST-LSC activities that prompted that learning can be found in Table 6.1. Although the third/fourth grade pair strengthened their KCS, there was no evidence of which phase(s) contributed to that growth; therefore, KCS appears as “unknown.”

Table 6.1

The Activities of Each PST-LSC Phase that Contributed to the Development of MKT

<table>
<thead>
<tr>
<th>Phase</th>
<th>MKT Domain</th>
<th>K/1 LS Pair Activities</th>
<th>MKT Domain</th>
<th>3/4 LS Pair Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Understand the Subject Matter</td>
<td>CCK SCK</td>
<td>• Sharing multiple strategies</td>
<td>SCK</td>
<td>• Sharing multiple strategies</td>
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<tr>
<td></td>
<td></td>
<td>• Cross grade-level discussions</td>
<td></td>
<td>• Cross grade-level discussions</td>
</tr>
<tr>
<td>B. Make Subject Matter Comprehensible</td>
<td>SCK Horizon KCT Curriculum</td>
<td>• Readings</td>
<td>Horizon KCT</td>
<td>• Readings</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Cross grade-level discussions</td>
<td></td>
<td>• Discussion</td>
</tr>
<tr>
<td>C1. Plan the Lesson</td>
<td>Curriculum</td>
<td>• Collaboration</td>
<td>KCT</td>
<td>• Collaboration</td>
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<tr>
<td></td>
<td></td>
<td>• Cross grade-level discussions</td>
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<td>• Cross grade-level discussions</td>
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<td></td>
<td></td>
<td>• The curricula</td>
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<tr>
<td>D1. Teach</td>
<td>Curriculum</td>
<td>• Observation</td>
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</table>
Phase A. Understand the Subject Matter

In the first phase of the PST-LSC, the four interns and researcher met together. The interns were given four addition and four subtraction tasks to solve (see Table 3.2). The interns solved each mathematics task one at a time and then discussed the strategies and models they used to solve each task.

During Phase A. Understand the Subject Matter, Kelly and Finley developed their CCK and SCK, while Theresa and Ford developed their SCK. The sharing of addition and subtraction strategies and the cross grade-level conversations prompted the interns’ growth in MKT.

The opportunity to discuss different addition and subtraction solution strategies allowed Kelly to develop her CCK. During this phase, she became more flexible with subtraction computation. When asked how the phases helped her to understand how to teach addition and subtraction, Kelly stated the following:

I also thought sitting down, the four of us [Phase A. Understand the Subject Matter], was really beneficial, just because... We were able to see like all the different strategies, like ones we hadn't seen in a while and then relating it to our own students in our classroom. Being like, "Well, I know some students may be thinking about it this way, but students may be taking it in totally different light
and really trying to work with numbers in ways that we haven't really seen in a while. Um, so I really think that just like set the frame of being like open to welcoming a bunch of strategies to math manipulatives in a way. (Phase F. New Understanding Transcript)

Kelly’s response supports the claim that the sharing of multiple strategies helped strengthen her CCK and helped her to understand the role multiple strategies play in instruction.

In addition to prompting a growth in CCK, this phase also aided in the deepening of Kelly and Finley’s knowledge of addition and subtraction models. More specifically, the sharing of strategies helped Kelly and Finley deepen their knowledge of the open number line. In her journal response after Phase A. Understand the Subject Matter, Kelly wrote, “I never considered using a number line before to help solve equations and it really does help my math thinking…” (Journal Response 2). Not only did the sharing of strategies develop Kelly’s understanding about the use of an open number line to solve addition and subtractions tasks, but it helped develop her CCK.

During her post-interview, Finley articulated how Phase A. Understand the Subject Matter helped her develop her understanding of the open number line. She mentioned the following:

The Understanding the Subject Matter with the tasks helped me because I got to learn new strategies from people. So, like on doing the open number line where I added instead of subtracted on the number line, I took that from [Ford], because I didn't, it didn't click in my head that I could do that until I saw her do it. (Post-Interview)
Finley learning a new way to use an open number line supports the claim that the sharing of multiple strategies helped develop her understanding of mathematical models. Solving addition and subtraction tasks with others provided Kelly and Finley the opportunity to learn new strategies and models; therefore, this phase of the PST-LSC developed their CCK and SCK.

Similar to Kelly and Finley, discussing solution strategies with other interns helped develop Theresa and Ford’s SCK. Solving tasks during this phase helped Theresa learn how to use an open number line and Ford learn how to use friendly numbers.

Like Kelly, Theresa was also unfamiliar with the open number line. She did not use the model to solve any tasks during her pre-interview or in Phase A. Understand the Subject Matter. However, during her post-interview, Theresa used the model to solve all four of the addition and subtraction tasks posed (Post-Interview). Kelly and Theresa’s newly obtained knowledge of the open number line occurred as a result of Finley and Ford sharing the model during Phase A. Understand the Subject Matter. Not only did the sharing of strategies contribute to a growth in Kelly and Theresa’s SCK, but the cross grade-level conversations allowed Kelly and Theresa to see how the open number line can be used to support students’ mathematical thinking at multiple grade levels.

The cross-grade level conversations not only played a role in Kelly and Theresa’s development, but in Ford’s new understanding of friendly numbers, which is evident in the following comment made during her post-interview:

I think talking about the, for like Understand the Subject Matter, when we did the tasks together, because the friendly numbers thing was new to me, and I would
have only known that if we had talked to the different grade levels. (New Understanding Transcript)

Additional evidence to suggest how the cross grade-level conversations impacted Ford’s SCK can be found in her statement during Phase F. New Understanding:

I think [Understand the Subject Matter] helped me too, just piece together more like the building of the understanding of it because like I forgot or I don't ever realize…how much more younger grades make things more accessible than…fourth grade I guess, because of all the tips and tricks that you guys teach them, like it builds up and it helps them get to the level I teach them at, so it was just really cool to see those beginning pieces because I don't ever see that exposure and the real, like the breaking down and the thinking of the actual numbers. (New Understanding Transcript)

Ford’s remarks reveal how the cross grade-level conversations that occurred during Phase A. Understand the Subject Matter provided Ford the opportunity to strengthen her knowledge of the mathematics strategies and models used in younger grade levels.

**Phase B. Make the Subject Matter Comprehensible**

The work of Phase B. Make Subject Matter Comprehensible included the use of the textbook from the interns’ mathematics methods course and the mathematics series used as part of the district’s mathematics curricular resources. Prior to attending the first meeting for this phase, the kindergarten and first grade interns read assigned pages from Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades PK-2 (Van de Walle et al., 2018) and the third and fourth grade interns read from Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for
Grades 3-5. The readings described the different problem types, models, and strategies for addition and subtraction (see Table 3.3 for examples). During the first meeting, the discussion was focused on the interns sharing new understandings of mathematical tasks, addition and subtraction strategies and models, misconceptions and student thinking, and teaching addition and subtraction for a conceptual understanding.

Then, prior to coming to the second meeting, the interns read a module from their individual grade-level text of *Math in Practice* (Akers et al., 2016), which includes instructional activities to support students’ learning of mathematical concepts. Kelly and Finley read an addition module and Theresa and Ford read a subtraction, which reflected the content they chose to teach throughout their lesson studies. During the second meeting, each intern pair and I met to review the instructional activities presented in the modules. For example, Kelly and Finley discussed the part-part-whole map and the role of vocabulary in mathematics teaching and learning, while Theresa and Ford talked about the expanded form method, base ten blocks, and the compensation strategy. We also discussed how the activities explore student misconceptions and incorporate addition and subtraction models.

During this phase, Kelly and Finley gained SCK, knowledge of the mathematical horizon, KCT, and knowledge of the curriculum, and Theresa and Ford developed their knowledge of the mathematical horizon and KCT. The readings, discussion, and cross grade-level conversations included in this phase contributed to the interns’ growth in MKT.

During this phase, Kelly and Finley’s SCK grew in the following ways: Kelly learned new mathematical terminology and Finley learned a new way to use unifix cubes.
to model subtraction. While the readings involved in this phase resulted in Kelly gaining an understanding of how \textit{compare} is used to describe a subtraction task (Phase B Transcript 1), Finley was able to expand her knowledge of subtraction models. While Finley knew how unifix cubes could be used to model join and separate tasks, reading \textit{Teaching Student-Centered Mathematics} helped Finley understand how they could be used to model compare subtraction tasks (Make the Subject Matter Comprehensible Transcript 1). The SCK that Kelly and Finley gained during \textit{Phase B. Make Subject Matter Comprehensible} was prompted by the readings and discussions that took place during the phase.

In addition to developing Kelly and Finley’s SCK, \textit{Phase B. Make the Subject Matter Comprehensible} also helped both intern pairs gain a deeper understanding of the mathematical horizon. More specifically, Finley, Theresa, and Ford developed their knowledge of how addition and subtraction are related to other mathematical concepts. Through reading \textit{Teaching Student-Centered Mathematics}, Finley was able to connect the use of unifix cubes to solve subtraction comparison tasks to analyzing bar graphs (Make Subject Matter Comprehensible Transcript 1). \textit{Teaching Student-Centered Mathematics} also impacted Ford’s understanding of how addition and subtraction are related to other mathematical concepts. The student misconceptions chart found in one of the chapters Theresa and Ford read helped Ford understand how place value helps students understand regrouping (Make Subject Matter Comprehensible Transcript 1). For Theresa, reading the \textit{Math in Practice} (Akers et al., 2016) chapter contributed to a deeper understanding of the role place value plays in subtraction (Make Subject Matter Comprehensible Transcript 2). It is apparent that the readings in \textit{Phase B. Make the Subject Matter Comprehensible}
helped Finley, Ford, and Theresa understand how addition and subtraction are related to other mathematical concepts.

*Phase B. Make the Subject Matter Comprehensible* contributed to a development in both intern pairs’ KCT. Both pairs strengthened their understanding of how to promote student reasoning. After reading *Teaching Student-Centered Mathematics*, Kelly and Finley gained knowledge of how manipulatives and models, like a ten frame, help students understand place value and its role in addition and subtraction computation (Make Subject Matter Comprehensible Transcript 1). Further supporting the impact the readings of this phase had on the interns’ knowledge of how to promote student reasoning is evidenced by Finley’s comment in her post-interview: “…Getting the chance to revisit [Teaching Student-Centered Mathematics] and look at Math in Practice…[helped] me a lot with what questions to ask them to get them to think deeper” (Post-Interview).

Additionally, prior to *Phase B. Make the Subject Matter Comprehensible*, Theresa had a limited understanding of how to support students’ reasoning, only mentioning manipulatives as a way to support students’ addition and subtraction computation (Pre-Interview). However, after this phase, Theresa mentioned number lines, decomposition, and compensation as ways subtraction could be presented and explained to her students (Journal Response 3). Theresa also mentioned *Phase B. Make the Subject Matter Comprehensible* in her post-interview: “…It helped me understand strategies that I had never used before and how to teach them to students (Post-Interview). While Ford developed her knowledge of how to promote student reasoning, there was no data to show that growth was prompted by *Phase B. Make the Subject Matter Comprehensible*. 
Lastly, Phase B. Make the Subject Matter Comprehensible prompted a growth in the kindergarten/first grade intern pair’s knowledge of the vertical curriculum. For Finley, reading *Math in Practice* (Akers et al., 2016) helped her develop an understanding of not only first grade content, but also of the kindergarten and second grade curriculum, which also helped Finley understand how that knowledge can help her support her students’ learning of addition and subtraction (Make Subject Matter Comprehensible Transcript 2). The readings paired with the cross grade-level conversations helped Finley deepen her understanding of the vertical curriculum.

During the post-interview, Kelly mentioned the following when discussing which phases most helped her in her thinking about addition and subtraction:

The readings, so stage B, that was very helpful just because from doing those readings, I was able to become more aware of certain models and strategies for students and then also, that's where we kind of got the idea for our lesson, so without those readings, I don't really think our lesson would have been what it was. (Post-Interview)

Finley had a similar response:

When we read the resources, what helped me with that was just like going back and revisiting the text from the Fall, like I don't, honestly, I don't really remember a lot of the readings from methods, which is why I'm glad I bought all of my books because I can go back and revisit them. …So, like getting the chance to revisit that and look at *Math in Practice*…like helps me a lot with what questions to ask them to get them to think deeper. (Post-Interview)
Kelly and Finley’s reflections support the claim that discussing selected readings during *Phase B. Make the Subject Matter Comprehensible* helped to develop their SCK and KCT.

This second phase of the PST-LSC developed the interns’ knowledge at the mathematical horizon and KCT. The readings and discussion included in this phase strengthened Theresa and Ford’s understanding of how addition and subtraction are related to other mathematical concepts and how to promote student reasoning.

Reading about addition and subtraction in the two resources validated Ford’s ideas about teaching the concepts and reminded her of an activity that can be used to promote students’ reasoning of the algorithm. During *Phase F. New Understanding*, Ford mentioned the following when describing how the stages helped her to better understand how to teach addition and subtraction:

> I think it was neat for me to read through the books and [the] ideas that I had already had or ways that I already wanted to teach it were in the book, so it kind of just like, “ok, cool, I kind of know what I'm talking about,” and it just made me feel better about going to teach it, and then with the missing digit, like that was an idea that I had done but forgot about, because it was so far removed from subtraction. Um, so it was just helpful to go through those. (New Understanding Transcript)

This quote illustrates the role that revisiting the methods and classroom texts had on Ford’s knowledge of content and teaching. Theresa also believed the content of the readings helped develop her knowledge of addition and subtraction. During her post-interview, she stated:
I think the Making the Subject Matter Comprehensible because it was strategies and problems directed for, intended for the grade level that I'm teaching. And, it helped me understand strategies that I had never used before and how to teach them to students. (Post-Interview)

For Theresa, revisiting the texts helped her learn new strategies for addition and subtraction, which resulted in a deepened understanding of how to teach the concepts to students. Therefore, this phase helped strengthen Theresa’s KCT. According to Theresa and Ford’s reflections, revisiting *Teaching Student-Centered Mathematics* and reading *Math in Practice* (Akers et al., 2016) helped develop Theresa and Ford’s knowledge of content and teaching.

**Phase C1. Plan the Lesson**

After studying the mathematics content in *Phase A. Understand the Subject Matter* and *Phase B. Make the Subject Matter Comprehensible*, each intern pair planned a small-group lesson. While planning, the two pairs were asked to think about the questions to be posed during the lesson and to anticipate possible student responses. Kelly and Finley planned a lesson that would first be taught in Kelly’s kindergarten classroom. Their lesson involved students using number bonds and then a second, self-selected strategy to solve a 3-addend word problem. During the lesson, students also used unifix cubes to model the Commutative Property of Addition. Theresa and Ford planned a lesson that would first be taught in Theresa’s third-grade classroom. Their lesson had students, in pairs, use base ten blocks to solve a three-digit subtraction problem, which was followed by a discussion about regrouping. The planned lesson also included time for students to complete some missing digit subtraction problems.
Planning the lesson helped Kelly and Finley develop their knowledge of curriculum and Theresa and Ford develop their KCT. The collaboration, cross grade-level conversations, and curriculum aided in the interns’ growth of MKT.

Kelly and Finley learned how to evaluate curricula and gained knowledge of the vertical curriculum as a result of Phase C1. Plan the Lesson. Using the Math in Practice (Akers et al., 2016) text to plan their first lesson allowed Kelly and Finley to realize the need to consider students’ knowledge and needs to evaluate and adapt curricula (Analyze and Reflect 2). Additionally, in her post-interview, Kelly shared, “Co-planning, I thought was really helpful in this, just because we were able to bounce ideas off of each other and really work and in a way foster a better understanding for a different grade level other than our own” (Post-Interview). This quote shows how the collaboration and cross grade-level conversations involved in the planning of the lesson provided an opportunity for Kelly and Finley to develop their understanding of the vertical curriculum. Therefore, not only did Math in Practice aide in Kelly and Finley’s knowledge of the curriculum, but so did the collaboration and cross grade-level conversation that occurred in Phase C1. Plan the Lesson.

Collaboratively planning across two grade levels also supported Theresa’s MKT development. Discussing the third and fourth grade Math in Practice (Akers et al., 2016) texts exposed Theresa to the missing digit activity they used for their initial lesson, which was not included in the third-grade text. This is evidenced by the following: “I think the collaborating to plan [the lesson] was helpful, because I never would have thought of the missing digit, because it wasn't even in my book, but my students are fast-paced, so it makes sense for them…” (New Understanding Transcript). Further explaining how
collaboratively planning impacted her understanding of teaching subtraction, Theresa shared, “And like through that collaboration came up with ideas and…sequenced it in a different way than I probably would have done had I not talked to someone about it” (Post-Interview). The opportunity to collaboratively plan a lesson with a colleague from another grade level allowed Theresa to gain KCT. Ford also understood the effect that collaboration had on the planning of the lesson. During her post-interview, Ford described how Phase C1. Plan the Lesson helped her thinking about teaching subtraction: “Planning the lesson as well, just because I think it's better to talk it out with somebody and like have somebody else there to give, like listen to your ideas and give ideas” (Post-Interview).

**Phase D1. Teach the Lesson**

Once the lesson plan was complete, one intern from each pair taught the lesson in their classroom while the other intern and I observed and took notes. Using an observation template, the observing intern and I wrote down the questions posed by the intern teaching, the strategies or models used by students, and any misconceptions the students seemed to hold. More specifically, Kelly taught the lesson in her kindergarten classroom while Finley and I observed, and Ford and I observed Theresa teach the lesson in her third-grade classroom.

The first teaching of the lesson helped develop Finley’s knowledge of curriculum. The observation involved in this phase played a role in Finley’s MKT development. There was no data to show a growth in Kelly, Theresa, and Ford’s MKT as a result of Phase D1. Teach the Lesson.
Having the opportunity to observe a lesson prior to teaching it allowed Finley to learn how to adapt instructional plans, particularly those involving curricular resources. In her post-interview, Finley said, “Watching [Kelly] teach it helped me, because I was able to see like that following the resource exactly didn't help us…” (Post-Interview). Being able to see how the Math in Practice (Akers et al., 2016) activity explored ideas that the students were already familiar with aided in Finley’s growth in understanding the importance of using students’ knowledge and needs to adapt curricular resources.

While there was no concrete evidence to support a gain in MKT for the other three interns during Phase D1. Teach the Lesson, Ford expressed the following about being able to observe the lesson:

I really liked observing it and I kind of like that I observed it first and then I taught it, because I think by watching Theresa, I was able to see what worked and what didn't work and then being able to change it for my kids. I think being able to see her third-graders work and see what my kids should know, like in a third-grade language, to then help me. (Post-Interview)

Ford’s remarks show that she saw a benefit in being able to observe the collaboratively planned lesson in a third-grade classroom before teaching the lesson to her fourth-grade students.

**Phase E1. Analyze and Reflect**

*Phase E1. Analyze and Reflect* began with the intern who taught in *Phase D1. Teach the Lesson* sharing her analysis and reflection of the lesson. Once the intern shared her thoughts, the observing intern and I shared additional insights and reflections. The focus of this discussion was the data collected on the observation form and then
proceeded to looking at any formative assessment that was given during the lesson to further analyze student learning. After discussing the students’ thinking that emerged during the lesson, the interns and I identified the aspects of the lesson that seemed more or less effective in impacting students’ learning and considered changes to the lesson. Because Kelly and Theresa taught during Phase D1. Teach the Lesson, they began the discussions during Phase E1. Analyze and Reflect; Finley, Ford, and I continued the discussions by then sharing our reflections.

The work of Phase E1. Analyze and Reflect contributed to a growth in all four interns’ KCT, more specifically, their knowledge of how to promote student reasoning. The discussions during this phase contributed to the development in the interns’ MKT.

The growth in KCT that resulted from the discussion in Phase E1. Analyze and Reflect is evidenced by Finley’s description of which phases helped her understanding of how to teach addition and subtraction:

…Everything with the lesson, so co-planning it with Kelly and then watching her teach it helped me because I was able to see that following the resource exactly didn't help us, and then when we analyzed and reflected, we found that out…

(Post-Interview)

Finley’s remarks reveal how the time to collaboratively analyze and reflect on the first lesson provided her the opportunity to further understand and decompose her observations. During their first lesson, Kelly and Finley had students use a number bond to solve a three-addend word problem (Lesson Plan 1). After the teaching of the initial lesson, Finley realized how providing students with a particular strategy limits students’ reasoning (Analyze and Reflect 1). As a result, Finley decided that allowing students to
solve a task using their own strategy to solve an addition problem would better support
and reveal student reasoning (Journal Response 4). The analysis and reflection that
occurred through the discussion involved in this phase contributed to the growth in
Finley’s knowledge of how to promote student reasoning.

Similar to Finley, while analyzing and reflecting on the initial lesson, Theresa also
wondered if giving students an opportunity to solve a subtraction task using their own
strategy would be beneficial to student reasoning as opposed to asking the students to use
base ten blocks (Analyze and Reflect 1). Ford agreed; she believed that giving students
the opportunity to solve a subtraction task in more than one way would allow her to see
how her students think about subtraction (Journal Response 4). Theresa shared the
following about which activities of Phase E1. Analyze and Reflect aided in the
development of her knowledge of teaching: “Collaborating with [the researcher] and Ford
too, so it wasn’t only my opinion on how it went, getting an outside perspective and then
sharing ideas on how it could be improved” (Post-Interview). The collaborative
discussion that occurred during this phase aided in the interns’ development of
knowledge of how to promote student reasoning.

**Phase C2. Revise the Lesson**

After analyzing and reflecting upon the first lesson, the intern pairs revised their
lessons. Kelly and Finley adjusted their initial lesson plan in the following ways:

1) Instead of students beginning their work with a number bond, students first used a
self-selected strategy to solve a three-addend word problem and then were asked
to use a number bond.
2) Students did not use unifix cubes to model the Commutative Property of Addition, because the kindergarteners had already grasped the concept.

Theresa and Ford revised their lesson in the following ways:

1) Instead of beginning the lesson with base ten blocks, students solved a four-digit subtraction task in as many ways as possible.

2) Students individually used base ten blocks to solve a three-digit subtraction task, rather than using them with a partner.

The intern pairs felt the changes to their lessons more effectively promoted student reasoning.

Kelly and Finley developed their knowledge of curriculum while revising their initial lesson, and Theresa and Ford gained KCT. The curriculum and the collaborative nature of this phase prompted the interns’ growth in MKT.

The opportunity to revise the first lesson allowed Finley to deepen her knowledge of how to adapt curricular resources. This phase helped Finley to understand the importance of considering student needs when using a resource to plan a lesson. During her post-interview, Finley shared, “...We revised the lesson to fit my class and do what was best for them minus like what the resource was telling us to do” (Post-Interview).

Finley’s comment shows that the collaborative discussion involved in this phase and the Math in Practice (Akers et al., 2016) text prompted a growth in her knowledge of how to adapt curricular resources.

For Theresa and Ford, Phase C2. Revise the Lesson contributed to a deeper understanding of how to promote student reasoning. After realizing the initial lesson limited students’ ability to reason through the subtraction tasks, this phase provided
Theresa and Ford the opportunity to change the lesson to better support student reasoning. In the first lesson, Theresa and Ford had students use base ten blocks to solve a subtraction task with a partner (Lesson Plan 1). Desiring to increase student reasoning, Theresa thought revising the lesson to have students solve a subtraction task individually using a self-selected strategy, then use base ten blocks, would increase the students’ subtraction reasoning (Analyze and Reflect Transcript 1). Ford elaborated on this idea by suggesting a revision that asked students to solve a subtraction task in multiple ways (Journal Response 4). While Phase E1. Analyze and Reflect aided Theresa and Ford in understanding the impact that prescribing a model or strategy for students has on their reasoning, Phase C2. Revise the Lesson provided them the opportunity to think further about how to adapt the lesson to increase student reasoning, which resulted in a growth of KCT.

**Phase D2. Teach the Lesson**

The teaching of the revised lesson included the same activities as Phase D1. Teach the Lesson, the only difference was the intern who taught the lesson. During this phase, Finley taught the lesson in her first-grade classroom while Kelly and I observed, and Theresa and I observed Ford teach the lesson in her fourth-grade classroom.

The second teaching of the lesson helped develop both intern pairs’ KCT, or their knowledge of how to promote student reasoning. The opportunity to reteach the lesson contributed to the interns’ growth in MKT.

Similar to Finley’s experience during Phase D1. Teach the Lesson, Kelly and Theresa benefited from observing the revised lesson. Being able to observe Finley teach the second lesson gave Kelly the opportunity to see how the revised lesson better
promoted student reasoning. Kelly believed the lesson “objectives were addressed more clearly because students had a choice of what strategy to use…” (Observation Notes 2). Additionally, through her observations of Ford, Theresa learned more about how to question students, which she planned on applying to her future teaching (Post-Interview). The reteaching of the lesson and the collaboration that allowed the two intern pairs to teach the lesson twice contributed to Kelly and Theresa learning more about how to use open-ended tasks and questioning to promote student reasoning. It is evident that the collaborative nature of the PST-LSC deepened Kelly and Theresa’s KCT.

**Phase E2. Analyze and Reflect**

*Phase E2. Analyze and Reflect* included the same activities as the first analyze and reflect phase. Again, the interns that taught in *Phase D2. Teach the Lesson* began the discussion. Therefore, Finley started with her analysis of the lesson, then Kelly and I shared additional insights and reflections, and Ford first shared her analysis, with Theresa and I then adding our reflections.

During this phase, Kelly and Finley developed their knowledge of the curriculum. There was no evidence to support the development of Theresa and Ford’s MKT during this phase of the PST-LSC. The discussion involved in this phase, along with the curriculum, contributed to the interns’ growth in MKT.

Analyzing and reflecting after having taught the initial and revised lessons helped the kindergarten/first grade pair develop their knowledge of how to adapt curricula. After revising their first lesson to better fit the needs of Finley’s students, Kelly and Finley realized that when using curricula to plan a lesson it is important to not only consider students’ needs, but to also consider what has worked in the past (Analyze and Reflect 2).
Discussing the lessons and the curricula during *Phase E2. Analyze and Reflect* contributed to Kelly and Finley developing a deeper understanding of how to adapt curricula.

**Phase F. New Understanding of Teaching**

To determine what content knowledge interns gained, all four interns and I met to share new understandings of teaching addition and subtraction. Each intern pair described their lesson to the other pair and then the entire group reflected upon the PST-LSC. After meeting, the interns wrote a reflective journal response that answered the following prompt: List all the new ideas you now have and understand about addition and subtraction; re-draw your concept map. In the journal response, the interns reflected upon their entire experience and learning throughout the cycle.

Kelly and Finley gained knowledge of the curriculum during *Phase F. New Understanding of Teaching*. The cross grade-level conversation aided in their growth of MKT. There was no data to show a growth in Theresa and Finley’s MKT during this phase.

Having the two intern pairs share their learnings with each other resulted in Kelly and Finley learning more about the vertical curriculum. In her post-interview, Finley said, “The New Understanding of Teaching helped because I was able to see what misconceptions they're having in third and fourth grade and how what we do impacts what they're doing” (Post-Interview). As is evidenced by Finley’s statement, the opportunity to make connections across grade levels provided Finley the opportunity to think about how her teaching impacts students’ learning beyond her own classroom. Including interns from four different grade levels in the two intern pairs not only
provided Kelly and Finley with the opportunity to learn about the vertical curriculum, but it helped them to make connections between their teaching of addition and subtraction to that in other grade levels.

Summary

As described in chapters 4 and 5, both intern pairs deepened their SCK, knowledge of the mathematical horizon, and KCT. Additionally, Kelly and Finley gained CCK and knowledge of the curriculum, and Theresa and Ford strengthened their KCS. While there was no evidence of how Theresa and Ford’s KCS grew, the data showed that Phase A. Understand the Subject Matter contributed to growth in the interns’ CCK and SCK. Phase B. Make the Subject Matter Comprehensible resulted in growth of SCK, knowledge at the mathematical horizon, KCT, and knowledge of the curriculum. The interns developed their knowledge of curriculum and KCT during Phase C1. Plan the Lesson and Phase C2. Revise the Lesson. Kelly and Finley gained knowledge of the curriculum during Phase D1. Teach the Lesson, Phase E2. Analyze and Reflect, and Phase F. New Understanding of Teaching. Lastly, both intern pairs gained KCT during Phase E1. Analyze and Reflect and Phase D2. Teach the Lesson. The activities of the PST-LSC that prompted growth in the interns’ MKT were the sharing of addition and subtraction strategies, the cross grade-level discussions, the readings, the collaboration, the curricula, the observations of teaching, and the reteaching.
Chapter 7

Discussion and Implications

In this chapter, I discuss the results of the present study in light of existing research about PSTs’ MKT development and lesson study. I then consider the implications of this study’s results for both research and practice. Briefly, this study contributes to the literature by revealing the need to connect coursework to field experiences, the impact that developing an understanding of how to make subject matter comprehensible to students has on PSTs’ MKT, and the impact lesson study has on PSTs’ knowledge and teaching. Additionally, it contributes to the literature by offering a structure that supervisors can use to promote the learning of student teachers in the field.

Developing the MKT of PSTs

To gain MKT, PSTs need not only to learn how to transform mathematical content to make it comprehensible to students; they also need opportunities to use that knowledge in practice. Although the PSTs in this study gained both SMK and PCK during Phase B. Make the Subject Matter Comprehensible (the transformation phase of the PST-LSC), the phase was not the sole catalyst for the PSTs’ gain in MKT. The PSTs had a greater understanding of their students’ prior knowledge after teaching the first lesson and were better able to consider students’ needs when planning the second lesson. Phase B. Make the Subject Matter Comprehensible provided the PSTs with the knowledge of how to transform content for students, but it was the act of teaching that allowed the PSTs in this study to develop their understanding of how to use that knowledge and students’ needs to plan instruction.
These results confirm Peterson and Treagust’s findings (1992, 1995). Peterson and Treagust (1992) found that problem-based learning (PBL) resulted in PSTs developing knowledge of the subject matter, curriculum materials, and planning; however, the PSTs struggled to transform their ideas to meet the needs of their individual students. To respond to this finding, Peterson and Treagust (1995) incorporated all stages of pedagogical reasoning into their PBL approach, which included teaching a lesson to a peer. As a result of participating in all the phases of pedagogical reasoning (comprehension, transformation, instruction, evaluation, reflection, and new comprehension) the PSTs realized the importance of considering students’ prior knowledge when planning a lesson. While the transformation phase of Peterson and Treagust’s (1995) PBL approach led to this realization, the teaching phase provided the PSTs the opportunity to transform their knowledge into practice.

Lai and Lo-Fu (2013) and Turner’s (2012) findings further support the need to provide PSTs with opportunities to not only gain knowledge of how to transform content, but to also use that knowledge in practice. The PSTs in their studies gained SCK, KCS, and KCT while participating in learning study and reflective activities, respectively. Whereas the PSTs in Lai and Lo-Fu’s study attributed this gain in MKT to the fact that learning study supported their ability to transform knowledge into action, the PSTs in Turner’s study contributed their growth to their experiences teaching and reflecting. Findings from these studies together with the present study’s results about the impact of the PST-LSC on MKT development illustrate the positive affect that transforming knowledge into practice has on PSTs’ MKT development.
When teacher education programs more closely connect mathematics content to the realities of an elementary classroom, PSTs show greater gains in SMK. The results of the present study illustrate how a single one-hour session focused on content situated within a clinical experience can support improvements in PSTs’ SMK. This finding is supported by previous research exploring the inclusion of mathematics content in teacher education. While the number of content courses PSTs took did not seem to correlate with increases in SMK (Youngs & Qian, 2013; Smith et al., 2012), the number of mathematical topics included in a methods course did appear to impact PSTs’ MKT positively. Similarly, Burton et al. (2008) found that devoting just 20 minutes to mathematical content in an elementary methods course increased PSTs’ SMK. However, incorporating content into methods courses is not the only way to improve PSTs’ SMK. Adapting content courses to specifically meet the needs of elementary teachers also has a positive impact on PSTs’ MKT (Welder & Simonsen, 2011; Whitacre & Nickerson, 2016). The findings in these studies suggest that PSTs gain SMK when mathematical content is more closely connected to the work of elementary teachers.

The mathematics content in the first two phases of the PST-LSC supports the development of PSTs’ SMK. In the current study, the PSTs who gained the most SMK while participating in the PST-LSC exhibited the most limited SMK at the start of the cycle. These results are corroborated by Laursen et al.’s (2016) findings, which suggest that giving PSTs opportunities to solve difficult problems, share solutions, and critique each other’s work allows PSTs to gain SMK. More specifically, PSTs with lower initial scores on a SMK assessment improved more than those with higher pre-test scores. These
findings suggest that a focus on content in PST mathematics courses may have potential to enhance the SMK essential for PSTs to develop effective mathematics instruction.

**The Impact of an Adapted Lesson Study on PSTs’ MKT**

Including a phase in the PST-LSC that allows PSTs to explore the transformation of content influences the growth of multiple domains of MKT. *Phase B. Make the Subject Matter Comprehensible* led to a growth in the PSTs’ SCK, knowledge at the mathematical horizon, KCT, and knowledge of the curriculum. During this phase, the PSTs investigated addition and subtraction problem types, models and strategies, student misconceptions, and instructional activities, while also discussing how to write mathematical tasks and build a conceptual understanding of the content. Discovering the role *Phase B.* played in the PSTs’ MKT development is important to consider in mathematics teacher education, because when Suh and Parker (2010) examined the use of lesson study with PSTs, they found PSTs’ ability to plan and teach was impacted by their limited experience in the classroom. The PSTs in their study struggled to anticipate student responses and misconceptions, to account for students’ prior knowledge, to pose meaningful questions, and to assess students’ mathematical understandings. While the activities of *Phase B.* may not help PSTs account for students’ prior knowledge when planning, they could improve PSTs ability to anticipate student responses and misconceptions, question, and understand students’ mathematical thinking.

Participation in lesson study compels teachers to think about teaching mathematics in new ways. In the present study’s adapted lesson study cycle, after prescribing which models or strategies their students would use to solve an addition or subtraction task, the PSTs realized the teacher-centeredness of their lessons. They
subsequently adapted their second lessons to provide students with the opportunity to use a self-selected strategy to solve a mathematics task before having students use a teacher-presented method. This suggests that PSTs attempted to focus more on student thinking and to think about how to build students’ conceptual understanding of mathematics.

Similarly, other studies have reported that teachers pose more open-ended questions after participating in lesson study (Leavy, 2015; Leavy & Hourigan, 2016, 2018; Robinson & Leiken, 2011). The opportunity to revise and reteach the initial lesson allowed the teachers to see the impact that open-ended questions had on students’ mathematical thinking and learning, which resulted in a new way of teaching mathematics.

In addition to lesson study leading to more open-ended questions being posed during instruction, the collaborative nature of lesson study also seems to prompt an increased confidence in ISTs’ ability to adapt materials and critique curriculum (Lieberman, 2009). The PST-LSC in this study provided the PSTs with this opportunity by allowing them to explore the *Math in Practice* (Akers et al., 2016) curriculum before planning their lessons. After having taught a *Math in Practice* lesson, the kindergarten/first grade pair realized the need to consider their students’ prior knowledge when using curricular materials to plan lessons.

Conducting lesson study with PSTs and ISTs may result in PSTs gaining additional MKT. Due to their inexperience with teaching, throughout the PST-LSC, the PSTs struggled to predict students’ prior knowledge, misconceptions, and potential responses to questions. Suh and Parker (2010) and Suh and Fulginiti (2012) had similar findings when conducting lesson study with pre-service and in-service teachers. However, the discussions that occurred when planning the lesson provided PSTs with the
opportunity to learn more about students’ misconceptions and potential responses. Additionally, observing the ISTs’ planning of tasks and questions, use of representations, and management of mathematical discussions allowed PSTs to gain MKT. Therefore, participating in lesson study alongside ISTs, may allow PSTs to learn from the experience of veteran teachers and how to consider student influences when planning a lesson.

**Implications for Practice and Research in Teacher Education**

In light of this study’s findings about PSTs’ development of MKT and the impact of an adapted lesson study on that development, the PST-LSC may be considered an appropriate practiced-based approach to supporting PST development. The addition of *Phase A. Understand the Subject Matter* and *Phase B. Make Subject Matter Comprehensible* to the traditional Japanese lesson study cycle offers opportunities for PSTs to gain both SMK and PCK. Then, the remainder of the cycle encourages PSTs to use that newly gained knowledge in practice, which further develops their MKT. Too, the PST-LSC appears to have potential to enhance the SMK of PSTs with limited content knowledge and help PSTs teach mathematics in unfamiliar but effective ways.

The content focus of the PST-LSC in this study was addition and subtraction, but the PSTs’ gain in MKT goes beyond the two mathematical concepts. While the PSTs’ SMK development and gains in KCS are connected to addition and subtraction, their gains in KCT and knowledge of the curriculum can be more broadly applied to other content areas. The PSTs deepened their knowledge of how to promote student reasoning and adapt curricular resources. This knowledge consists of an understanding that to promote student reasoning, a teacher can provide a more student-centered learning
environment, allowing students to use their own strategies and solve mathematical tasks in multiple ways. Too, the PSTs’ knowledge of adapting curricular resources includes the knowledge of how to consider students’ prior knowledge and current needs when using resources to plan lessons. Therefore, even though the PST-LSC in this study focused on addition and subtraction, the PSTs also gained MKT that is general to the act of teaching.

It is also important to note that Phase B. Make the Subject Matter Comprehensible contributed to the development of four of the domains of MKT (SCK, knowledge of the mathematical horizon, KCT, and knowledge of the curriculum). Two key components of this phase were the Teaching Student-Centered Mathematics (Van de Walle et al., 2018) and Math in Practice (Akers et al., 2016) texts. As a teacher educator, I chose these texts for the purpose of supporting the PSTs’ knowledge of how to make mathematics content comprehensible to students, and evidence shows that the assigned readings impacted the growth of all four of the MKT domains mentioned previously. It appears that Teaching Student-Centered Mathematics’ (Van de Walle et al., 2018) focus on mathematical vocabulary, addition/subtraction models and strategies, and student misconceptions supported the PSTs’ MKT development. Additionally, the instructional activities included in Math in Practice (Akers et al., 2016) helped the PSTs to make connections between addition/subtraction and other mathematical concepts and learn more about the vertical curriculum. While this study was not designed to identify the impact of these resources on PSTs’ MKT, some evidence suggests that the content of the texts had a positive influence on their MKT. An area for future practice and research is to further explore how teacher educators and PSTs use curricular resources to gain knowledge about how to teach mathematics.
The results of the current study also revealed a limited gain in one specific domain of MKT. Only one of the lesson study pairs developed their KCS or more specifically, knowledge of students’ mathematical thinking. The growth in KCS was only found when comparing the pre- and post-interviews. There was no other evidence that alluded to a growth in the MKT domain. Additionally, although the PSTs talked about how they thought students would solve addition and subtraction tasks, they did not specifically mention learning more about students’ mathematical thinking. As a result of this finding, it could be concluded that PSTs may need additional support with developing their KCS. If I were to conduct this study again, I would consider including specific prompts to elicit KCS. This finding also suggests a need to further research how teacher educators can support PSTs’ development of KCS, especially when PSTs typically have limited classroom experience.

Due to the positive impact the PST-LSC had on PSTs’ MKT and the fact that it is completely situated within the elementary school context, it could be considered a model for supervision. Teacher educators working in the field could use the PST-LSC to learn with the PSTs they are supervising. Conducting the PST-LSC with PSTs in the field would not only improve PSTs’ knowledge and practice, but it could inform the work of supervisors and provide an opportunity for supervisors to make connections between methods courses and practice. By participating in lesson study with PSTs, field supervisors could develop their understandings of how PSTs gain the knowledge needed to teach effectively. It would be worthwhile to not only further explore the impact the PST-LSC has on PSTs’ knowledge, but to research what teacher educators learn from conducting/participating in lesson study. Additionally, because some researchers found
that PSTs gained knowledge by participating in lesson study with ISTs, further research on exactly what knowledge PSTs gain from ISTs during lesson study could further impact teacher education.

While the PST-LSC appears to be a promising practice-based approach, the logistics of conducting lesson study with PSTs, or in the United States in general, may be challenging, as Chassels and Melville (2009) found. Considering this study was conducted within a PDS context, there is a need to know whether the PST-LSC could be conducted in other teacher education settings. The present study occurred with four PSTs that were student teaching within the same school. The same opportunity might not exist in other contexts; therefore, the PST-LSC should be researched outside the PDS setting. Additionally, because of the challenges that can come with conducting lesson study, the field of teacher education could benefit from researching other practice-based approaches as ways to connect knowledge to the practice of teaching. The results of the current study suggest that connecting content to teaching, addressing PCK by exploring ways to transform content, and providing PSTs with opportunities to enact newly gained knowledge of teaching are ways PSTs gain MKT. Teacher educators should explore how these strategies can be included in teacher education programs.
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Appendix A

Lesson Plan Template

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<th>Teacher/Intern</th>
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<td>Date(s) and time to be taught</td>
<td>Duration of lesson</td>
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**Section I: Standards and Objectives**

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<td><strong>Standards</strong> (SAS)</td>
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<td><strong>PDF version links</strong></td>
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<td>Prerequisite Skills</td>
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**Section II: Assessment**

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<th>Assessment Activities</th>
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**Section III: Lesson Details**

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<tr>
<td>Use of Technology</td>
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<th>Introduction (Hook)</th>
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<tbody>
<tr>
<td>Sequence of Instruction (Step 1, Step 2…)</td>
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<td>Closure/Wrap Up</td>
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<td>Differentiation</td>
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<td>Possible Follow-up Activity</td>
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**Section IV: Lesson Analysis**

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<td>What did students learn?</td>
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<tr>
<td>What was learned about planning and teaching?</td>
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<td>What changes would I make?</td>
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Appendix B

Observation Form

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<thead>
<tr>
<th>Posed Mathematics Questions</th>
<th>Strategies/Models Used</th>
<th>Misconceptions</th>
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**Reflective Notes:**
Were there any unexpected or surprising student responses?

What did students learn?

Were the tasks posed effective in meeting the objectives of the lesson?
Appendix C

Analysis and Reflection Chart

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<tr>
<th>Strategies/Models</th>
<th>Misconceptions</th>
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<tr>
<th>Surprising Responses</th>
<th>Student Learning Objectives Met?</th>
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Appendix D

Interview Protocol and Questions

Notes/instructions to the interviewer:
Keep the focus on the research questions: 1) How does the MKT of elementary PSTs develop as they engage in an adapted lesson study cycle? and 2) How do the activities of the PST-LSC create opportunities for the development of MKT?
Redirect the conversation if it diverges from the research topic. Be sure to ask follow up questions for clarification. Limit your own interactions; listen more and talk less. If you feel a question has already been answered by the interviewee, feel free to move on to the next question. You can rephrase questions at any time to elicit more information. Begin with the script below and move into the specific questions from there.

Opening Remarks:
“I am a researcher looking at how interns’ mathematical knowledge for teaching develops. I am interested to know about your current and developing knowledge of teaching - specifically, the knowledge you have about teaching number and operations. I am going to ask you some questions, some of which are the kinds of tasks you might use in the classroom. Please share as much of your thinking with me as you can.
I want to remind you that I am not involved in the grading of your student teaching.
Anything you share with me in this interview will be used for the purposes of my research study and will not be shared with your mentor teacher, PDA, or anyone else involved with your student teaching. I will assign you, and any names you mention during the interview, a pseudonym [fake name]. I will remove any identifying information so that your identity and others’ identities are protected.
I would like to record this so that I can listen to it at a later time. You can ask me to stop the recorder at any point during the interview. Ok?
Do you have any questions?...”

1. Robin had 6 toy cars. Her parents gave her 8 more toy cars for her birthday. How many toy cars did she have then? (Knowledge of strategies for adding within 20)
2. Mark has 8 mice. Joy has 12 mice. Joy has how many more mice than Mark? (Knowledge of strategies for subtracting within 20)
   a. What other ways can you solve these tasks? (SCK)
   b. Can you explain this solution? (SCK)
   c. How do you think your students might approach these kinds of tasks? (KCS)
   d. What kinds of challenges might they face? (KCS)
   e. What misconceptions might students have? (KCS)
   f. What kinds of supports might you provide? (KCT)
   g. How might these tasks relate to their developing understandings of addition and subtraction? (KCS & KCT)
   h. Can you compare and contrast these two tasks?
3. On our walk, we found 145 pieces of sandstone and 76 pieces of granite. We didn’t find any other types of rocks. How many rocks did we find on our walk? (Knowledge of strategies for adding multi-digit numbers)

4. Ms. Wiesner’s class has 139 fossils. We have 206 fossils. How many fewer fossils does Ms. Wiesner’s class have than we do? (Knowledge of strategies for subtracting multi-digit numbers)
   a. What other ways can you solve these tasks? (SCK)
   b. Can you explain this solution? (SCK)
   c. How do you think your students might approach these kinds of tasks? (KCS)
   d. What kinds of challenges might they face? (KCS)
   e. What misconceptions might students have? (KCS)
   f. What kinds of supports might you provide? (KCT)
   g. How might these tasks relate to their developing understandings of addition and subtraction? (KCS & KCT)
   h. Can you compare and contrast these two tasks?

5. How confident are you in teaching addition, subtraction, multiplication, and division?

6. Tell me about what your experiences have been so far in teaching addition, subtraction, multiplication, and division.

VITA

Erin C. Morgart

Education
Ph.D., Curriculum and Instruction [Emphasis: Curriculum & Supervision] (August 2019), Pennsylvania State University, University Park, PA
M.Ed., Educational Leadership (August 2009), Northern Arizona University, Flagstaff, AZ
B.A., Elementary Education (December 2004), Arizona State University West, Glendale, AZ

Professional Experiences
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Graduate Assistant (August 2014-June 2018), Department of Curriculum & Instruction, Pennsylvania State University, University Park, PA
Elementary Teacher (December 2004-June 2014), Cartwright Elementary School District, Phoenix, AZ

Additional University Experiences
Assistant Mentor Coordinator (Summer 2017), Learning Edge Academic Program (LEAP), Pennsylvania State University, University Park, PA
Copy Editor (Summer 2015), Research Grant: Developing and Evaluating Instructional Scaffolds for EngageNY Mathematics Curriculum to Enhance Math Performance of Middle and High School Students with Disabilities, Pennsylvania State University, University Park, PA

Professional Service
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Registration and Technology Assistant (2017), Pennsylvania Association of Colleges and Teacher Educators 46th Teacher Education Assembly, Harrisburg, PA

Selected Presentations

Awards and Scholarships
Emerging PDS Leadership Award (2018), National Association for Professional Development Schools