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KNOWLEDGE MANAGEMENT, SOCIAL LEARNING, AND OPTIONS TO LEARN

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ABSTRACT

Knowledge plays an important role in firm decision making as it involves the selection of technological systems and understanding and executing technologies that are implemented. Knowledge management therefore is a learning behavior which is undertaken to acquire additional knowledge and improve the firm's profit capacity. The core elements of knowledge management involve i) the learner, to whom the learning behavior is attributed, ii) the learning process, which consists of acquiring information and processing information to additional knowledge, and iii) the learning outcome, which is what we obtain from knowledge management.

This research study focuses on the firm decision maker as the learner while the learning outcome is the updated knowledge base playing an important role in the firm's decision making. The learning process, where the learner obtains additional knowledge, has two phases. In the information acquisition phase, the decision maker acquires internal information by managing the internal data from past experience (for example, learning-by-doing), and/or acquires the external information by communicating with others via social learning activities (e.g., conversation, cooperation, and collaboration). On the other hand, the knowledge updating phase involves the use of a learning mechanism translating the collected information into additional useful knowledge feeding into the existing knowledge base.

This research addresses i) how the learner (the firm decision maker) learns and seeks to formalize the learning process, ii) how the decision maker chooses among different knowledge management schemes, and iii) how social learning behavior reflects on

production heterogeneity. This thesis research develops a conceptual model focusing on the definition of knowledge, the different ways of executing learning process, and the way the updated knowledge base influences future decision making. The theoretical model is investigated by using a mathematical model where the decision maker maximizes the firm's profit over time under production and knowledge management constraints. The optimization conditions point out the marginal costs and benefits of learning and guides the decision maker to allocate the physical input and the effort for knowledge management.

Learning strategies, such as *always learn*, *wait to learn*, *learn-in-bursts*, and *quit learning*, are observed in the deterministic dynamic programming model as the output price changes. Considering the learning decisions under uncertainty, two stochastic dynamic programming models are constructed where the market and technological uncertainties are represented by the stochastic properties of output price and knowledge base accumulations. The numerical results indicate that the decision maker faces several possible states because of the market and the technological uncertainty. In addition, each state has its corresponding decision, and the decision maker will not make the learning decision until the true state is revealed.

The empirical model is used to reveal the connection between social learning and production heterogeneity. A latent class stochastic frontier model (LCSFM) is introduced to estimate the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT) India data. The households' group-memberships are obtained, and the households assigned to the same group are assumed to use the same production technology. Thus, the common characteristics of the households in the same group indicate the reason behind technical heterogeneity across households. The empirical results indicate that caste rank plays an important role in households' production decisions. Since households in the same caste rank are more likely to communicate with each other, they have a greater opportunity to exchange production information. The frequent social activities within the caste rank provide the opportunity for social learning. Thus, the importance of caste rank in production behavior represents the importance of social learning in production decision.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Today's global market-driven economy imposes greater competitive pressures on firm decision makers as they balance the trade off between exploiting the full productive potential of their systems and technologies, and adopting innovations. Both avenues can lead to enhanced profitability.

Sustaining competitiveness over the long run involves attention to growth prospects associated with the innovations needed to keep pushing the competitive envelope, and the efficiency gains needed to ensure that implemented technologies can succeed. The effective management of knowledge contributes to both sources of profitability growth.

Knowledge plays an important role in firm decision making as it involves the selection of technological systems and understanding and executing technologies that are implemented. Knowledge management is a learning behavior which is undertaken to acquire additional knowledge and improve the firm's profit capacity. Whether the firm should put additional effort into knowledge management depends on the potential cost and benefit of this learning effort.

This study develops a dynamic optimization model allowing for several kinds of observed knowledge management behaviors (e.g., always learn, waiting to learn, learn-in-bursts). The decision maker's knowledge management behavior under market and technological uncertainties is investigated as the output price and the knowledge base are assumed to be stochastic. An empirical study estimates the inter-group heterogeneous production behavior in the India village which is associated with their social learning behavior.

1.2 Background

The core elements of knowledge management involve i) the learner, to whom the learning process is attributed, ii) the learning process, which consists of acquiring information and processing information to additional knowledge, and iii) the learning outcome, which is what we obtain from learning.

This research study focuses on the firm decision maker as the learner while the learning outcome is the updated knowledge base which plays an important role in the firm's decision making and can lead, potentially, to profitability growth. The learning process, where the learner obtains additional knowledge, involves two phases. The first is the information acquisition phase involving different kinds of information acquisition activities. Some information acquisition activities are generated internally and can involve lower learning costs since it usually involves information acquisition management of internal data from past experience. Learning-by-doing is an example. Other information acquisition activities involve the firm engaging in socially-oriented learning activities as external data and information are generated. This form of information acquisition activity can involve higher learning costs as the learner puts forth greater cognitive effort as he communicates with others. Cooperation (where decision makers actively discuss common issues, come to common conclusions but make their own final decisions), collaboration (where decision makers actively plan to generate data and translate it into learning information that they act on jointly), and engaging external expertise via seminars/workshops are the information acquisition activities involving socially-oriented learning.

After collecting the internal and external information from information acquisition activities, the learner still needs to filter this information through a learning mechanism as a means of translating the information set into knowledge that can support production decision making. The second phase of the learning process is the knowledge updating phase which is associated with the use of a learning mechanism that translates the information collected into additional useful knowledge feeding into the existing knowledge base. The learning process is complete when one recalls and successfully adapts already existing experiences, new experiences and information to new situations, re-indexing them and making them part of the new production operations procedures. Consequently, the knowledge base is updated so that the learner (the firm decision maker) can use it in future production decision making.

The standard procedure assumed by economists is to measure learning information as based on past decisions (say, the inputs used such as capital investment) or on realizations (the actual output). In fact, there are many opportunities to learn from within the firm and from others. In the context of an organization, the learning that can take place within and between organizations greatly expands the opportunities for learning. As a result, two firms with similar endowments and operating in the same environment may make different production decisions because they have undertaken decisions to construct and interpret different knowledge bases.

Although knowledge is an important input to the firm's decision making, it may not always be profitable for the firm to build upon the knowledge base at each opportunity. The potential marginal benefits and the associated marginal costs of learning clearly impact learning decisions. For example, the potential marginal benefit of cooperation with other firms as a means to gathering data can yield valuable information that can contribute to future growth of profits. The potential gains arising from employing a learning mechanism may come from rearranging information that can connect information nodes that were previously isolated so that the knowledge base increases which can translate into increased future profit. The associated marginal costs, on the other hand, include direct costs and indirect costs. The direct cost consists of the expenditure and the time necessary for facilitating cooperation or using the learning mechanism, while the indirect costs are associated with the opportunity costs arising from diverting resources to the learning activity and away from other productive efforts.

However, the diversity of information in the knowledge base is directly related to how useful (and thus, valued) the knowledge base will be. One more unit of information (in the form of experience) may bring some learning value to the knowledge base if the learner already has a strong foundation of past experiences. Alternatively, if the learner only has limited experiential foundation, one more unit of information may not be able to add learning value. The threshold at which additional information to the knowledge base has practical economic value to the firm also influences knowledge management decisions. Clearly, the decision maker will not allocate social learning effort if his own information set is not developed sufficiently to be improved with a marginal learning infusion. Similarly, the decision maker may not want to use the learning mechanism if he has accumulated very few experiences.

The structure of the learning benefits and costs control the way the firm manages its knowledge over time. The internal information acquisition, such as learning-by-doing, accumulates information continuously by recording the results of past production decisions which can be associated with a low cost of accumulating information. However, the information acquisition activities with social learning may not always be undertaken because its start-up cost may be prohibitive. The costs and benefits of knowledge management may also be the reason why we observe heterogeneous knowledge-managing behavior across firms. The firm manager-specific characteristics reflect different abilities to process information into useable knowledge. Consequently, different structures on the cost and benefit of knowledge management can result, leading them to make different learning decisions.

1.3 Goals and Objectives

The goals of this thesis are to address i) how the learner (the firm decision maker) learns and seeks to formalize the learning process, ii) how the decision maker chooses among different knowledge management schemes, and iii) how social learning behavior reflects on production heterogeneity.

The objectives of this research are as follows:

i) Develop a conceptual model which addresses different ways of executing the learning process and how learning process influences knowledge base accumulation.

ii) Construct a dynamic optimization model which illustrates the firm's profit maximization problem based on its production and knowledge-managing constraints.

iii) Derive the optimization conditions for the mathematical model which guide the decision maker in allocating the physical inputs and efforts for knowledge management.

iv) Obtain the learning decisions over time from the deterministic dynamic programming model and analyze possible learning strategies.

v) Investigate how economic and technical forces influence knowledge management behavior by utilizing the stochastic dynamic programming model where the decision maker faces uncertainties in the output price and the knowledge base accumulation.

vi) Obtain the learning decisions/strategies under the situation where the output price is stochastic.

vii) Obtain the learning decisions/strategies when the knowledge accumulation is stochastic.

viii) Investigate whether there is production heterogeneity across the households and interpret the role of social learning in the India village by using a latent class stochastic frontier model.

The thesis is organized as follows. The next chapter reviews the literature from both an economic and knowledge management perspective. This is followed by a conceptual model illustrating different knowledge management schemes including different options for information acquisition and knowledge base updating. Then a mathematical model demonstrating the firm decision maker's learning decision under the production and knowledge-managing constraints is developed. Both stochastic and deterministic dynamic programming model are built to analyze the decision maker's learning strategies with and without uncertainty. Finally, a latent class stochastic frontier model is estimated to examine the inter-group heterogeneous production behavior resulting from the farmers' social learning behavior. A final chapter summarizes the results and presents an agenda for future research.

CHAPTER 2

LITERATURE REVIEW

Knowledge management includes the phases of information acquisition and knowledge base updating, and can influence the firm's future production performance and the production decisions. The firm's decision to allocate additional effort to knowledge management depends on the marginal cost and the future potential profit associated with this learning effort. When the firm is viewed as a dynamic system of knowledge production and application, the issues then are extended to the sources of knowledge acquisition, the form of knowledge, and the way knowledge influences production decisions, production performance, and even product design.

The next section reviews the literature focusing on how economists incorporate knowledge into economic models and whether the empirical results support learning behavior. This is followed by a review of the literature discussing the firm's behavior from the knowledge management point of view.

2.1 The Economic Point of View

2.1.1 Learning-by-Doing

Starting from Arrow's learning-by-doing model (Arrow, 1962), knowledge entered the economic model as a firm-specific capital good generated from the firm's previous production experience. To apply this concept to empirical study, the production experience is usually represented by cumulative gross investment (Arrow, 1962; Sheshinski, 1967), cumulative output (Sheshinski, 1967; Rosen 1972), or cumulative market inputs (Rosen, 1972). Under this setting, knowledge may appear in the production function along with other production inputs or as a productivity index which is multiplied by the production function in the economic model.

Aside from its influence on the production function, the cumulative knowledge from previous production experience may also influence the production decision from the cost side. The learning curve connects the relationship between unit cost and accumulated output with the marginal cost decreasing with the firm's cumulative output representing the production experience (Spence 1981). Thus, the shadow value of the cumulative output represents the future cost saving due to current production, and together with the market price, determines the production decision (Majd and Pindyck, 1989).

Other empirical problems arise from the firm's learning-by-doing behavior. Due to the dynamic characteristics of learning, a panel data set is needed to estimate the production function when learning is involved. The heterogeneous learning abilities across firms may become the unobserved firm-specific characteristic resulting in heterogeneous production behavior across firms. To avoid such problems, Cameron (1999) employed the ICRISAT (International Crops Research Institute for the Semi-Arid Tropics) panel data to estimate farmers' adoption decisions. The cumulative profit differential between a high-yielding variety (HYV) and a traditional variety is used to present the firm's own experience for HYV cotton. The empirical results stated that the farmer's adoption decision is significantly influenced by the farmers' own experience. In addition, the significance of fixed effects reveals the estimation bias that can arise in the cross-section model.

2.1.2 Learning from Others

Economists soon realized that production information does not just come from the firm itself. The experience from other firms in the same industry may offer similar valuable information and influence each other's production behavior as well. The learning-from-others phenomenon is more easily observed when new technology is introduced to the industry because the firm's previous experience may not offer much insight as the new technology comes in. Observing the production behavior of an experienced peer may bring more valuable information to the decision whether to adopt the new technology.

Besley and Case (1993) mention the possibility of learning externalities where farmers can observe the production realization from early adopters when new technology comes in the industry. Foster and Rosenzweig (1995) use household panel data during the Indian Green Revolution to analyze the relationship between the farmer's adoption behavior and external information from other farmers. The target-input use is assumed to be stochastic, and the farmer's prior belief for the target-input use is updated according to the farmer's own information and the neighboring farmers' information represented by their cumulative land use of HYV variety. The importance of both self and neighbors' experiences to the farmer's adoption decision is verified by their empirical results.

Learning-from-others behavior raises the issues of 1) from whom does the farmer learn, and 2) the transfer of perfect information. Conley and Udry (2001) conduct a survey in Ghana and find that the information may not be perfect because most of the farmers only know "relative" information rather than the exact production actions of other farmers. Furthermore, farmers do not learn from all other farmers in the same village. Instead, each farmer learns from their peers in their social communication networks.

Ueda (2002) employed the ICRISAT data in a stochastic frontier model where production inefficiency is determined by the farmer's own experience, the experiences from the reference and non-reference group, and other socio-economic factors. The household-specific characteristics, such as farm size, household size, and caste rank are used to define the reference group. The empirical results indicated that the learning-from-others effect is significant when the others are farms who have the same household size.

2.1.3 Information Acquisition

The learning-by-doing and learning-from-others models not only present another explanation for technology choice and productivity change, but also reflect the concept that knowledge needs to be acquired (Arrow, 1962). Despite the information from own or others' experiences, the decision maker can also invest in information from other sources, such as the extension service offered by government research institutes.

Feder and Slade (1984) emphasize the differences between passive and active information acquisition activities. Both activities contribute to knowledge accumulation, but the active one is costly. Their analysis reveals that larger farmers tend to put more resources into active information acquisition during the beginning of technological diffusion. In addition, their empirical results demonstrate that the convenience of information access has a positive impact on both the extent of farmers' knowledge and the probability of technological adoption.

Danthine and Magill (1985) separate capital investment into two components: output responsive investment and the investment in information acquisition. The investment of the information acquisition gathers the information about the uncertainty of the output and indirectly influences the decisions in the output responsive investment. The analysis of their two-period utility-maximization model finds that different individuals may make different decisions for the output responsive investment even when they face the same information. A rational agent will not invest in information acquisition if the prior information only decreases prior uncertainty, but not influence the output responsive investment decisions.

When new technologies are introduced to the industry, the information acquisition behavior may be one of the reasons explaining why some of the firms adopt the new technology earlier than others. Wozniak (1987) builds logit and probit models to estimate the effects of information acquisition activities on the probability of adoption of new technology. The frequencies of contact with agricultural extension service and private agricultural supply firm's information sources are used to be the indicators of the investment that the decision maker makes to acquire external information for the new technology. The empirical results indicate that the contact with agricultural extension service has a positive and significant impact on the probabilities of adoption behavior, while the coefficient for the frequency of contact with private supply firm's information sources is positive but insignificant. Thus, producers have more incentive to acquire information from agricultural extension services which offers decision makers relevant, objective, and general information.

2.1.4 Other Aspects of Knowledge in Economic Models

Unlike the role of knowledge in the economic models mentioned in the previous sections where knowledge is referred to as what people know and is associated with production performance and decision making, knowledge has an alternative characterization in the game theory field. Knowledge is defined as an event known by an individual and the common knowledge assumption plays an important role in obtaining the individual's strategies. The idea of common knowledge is introduced by Aumann (1976) and defined as an event known by everyone, and everyone knows that everyone knows it, and so on.

Samuelson (2004) uses the standard "state-space" model to summarize several propositions proved by other researches concerning the common knowledge assumption. Where the individual obtains knowledge and how this knowledge influences this individual's performance are not the issue in this line of research. Instead, the game theory approach focuses on whether everyone has the same knowledge and whether everyone knows that everyone knows it. Common knowledge is a strong assumption in game theory and whether the individuals have common knowledge influence their strategies.

2.2 The Knowledge Management Point of View

2.2.1 The Characteristics of Knowledge

Instead of viewing knowledge as a resource which can raise the firm's competitive advantage (a resource-based approach), several studies focus on the relationships between the structure of knowledge and the structure of the firm (Spender and Grant, 1996). Firms are viewed as engaged in a dynamic system of knowledge production and application (or a knowledge-based approach). Characteristics of knowledge are defined very carefully since different types of knowledge lead to different types of economic rent, and the firm's knowledge-managing behavior are the firm's strategies for pursuing this rent (Spender, 1996).

The important characteristics of knowledge which allow the firm to create value from using knowledge are: transferability, capacity of aggregation, appropriability, specialization in knowledge acquisition, and knowledge requirements of production (Grant, 1996). Understanding these characteristics of knowledge enable decision makers to choose what the firm needs from among all available knowledge. Is knowledge helpful for production processes and making production decisions? Some explicit knowledge, such as historical input and output data, is easy to transfer and aggregate while tacit knowledge, especially the knowledge that can only be gained from application, may be difficult to do. Does the firm still profit from acquiring knowledge when the cost of storage and processing of knowledge are taken into account? In addition, the decision maker should consider whether the firm has the ability to create a return from the knowledge it creates.

2.2.2 The Knowledge Strategy

Viewing the firm as a dynamic system of knowledge management activity extends the strategies for the knowledge acquisition. The internal information gained from inside the firm not only connects with the learning from past experience, but also involves some formal activities which intend to create new knowledge, such as R&D (Malerba, 1992). The possibility of acquiring external information from other firms is raised by the appearance of various types of strategic alliances formed by R&D contracts, joint development agreements, and consumer-supplier partnerships (Mowery, Oxley, and Silverman, 1996). Strategic alliances broaden the scope of information flows. Not only the firms that produce the same product can be bound by the strategic alliances, the firms that have consumer-supplier relationships can be connected through it as well. Other activities, such as consulting experts or engaging external expertise via seminars/workshops are also ways to acquire external knowledge. Along with these various knowledge strategies, one should keep in mind that different knowledge strategies bring diversity in knowledge type, learning speed, and breadth of knowledge base (Bierly and Chakrabarti, 1996).

There are efforts presenting empirical proof of knowledge strategies and inter-firm knowledge transfer. Focusing on the access to knowledge (private versus public) and the use of knowledge (restricted versus unrestricted), Appleyard (1996) identifies several options for knowledge sharing and conducts a survey to investigate the pattern of knowledge exchange. The results indicate that inter-firm knowledge exchange exists, but it differs across industries and across countries.

Mowery, Oxley, and Silverman (1996) presents an empirical test for inter-firm knowledge transfers in the presence of different kinds of strategic alliance structures. The results show that the equity-based joint ventures have higher inter-firm knowledge transfer than contract-based alliances. On the subject of the firm's extent of absorption of technology capability from its alliances, the pre-alliance relationship presents a positive impact while the importance of the firm's size and R&D intensity is not significant.

The firm's knowledge strategy influences the firm's economic performance as well. Bierly and Chakrabarti (1996) use a cluster analysis to classify firms in the pharmaceutical industry into different groups according to their knowledge strategies.

Innovators are the firms with the high internal and external learning as measured by their R&D intensity and the average number of patent citations to the scientific literature. *Loners* are the firms contributing to the R&D more than other firms in the industry, and their learning speed is much faster. *Exploiters* are the firms with a broad knowledge base and high level of external learning but a low level of R&D intensity. The final group, *Explorers*, indicates the firms that balance internal and external learning, but they are not aggressive learners given their lower R&D intensity and science linkage. The empirical results indicate that some firms change their knowledge group while most of the firms intend to stay in the same group. In addition, the firms in the *innovators* and *explorers* groups present better financial performances than the ones in *exploiters* and *loners* group.

2.2.3 Other Impacts of Knowledge Management

The impact of knowledge management not only emerges in the firm's production or economic performance, but also appears in other aspects. The improvement of the product itself, such as its physical properties, performance characteristic, or suitability for uses, may also be the result of knowledge accumulation coming from the firm's communication with material suppliers, individual consumers, its investment in R&D, or the firm's internal learning (Malerba, 1992). Moreover, changes in the firm's product and organizational design are also the consequence of alternative concepts of knowledge management. In the traditional sequential product design, a change in the design of one component of the product influences the design of other components. It is a step-by-step process because the component design in the next stage depends on the information from the previous stage. Thus, losses and delays of information transfer are likely to be observed during this process. The modular product design, on the other hand, creates a standardized component interface requiring all components of the product to fit into this standard interface. In this case, the designs of different components can proceed at the same time and a more efficient use of the information is expected (Sanchez and Mahoney, 1996).

2.3 Concluding Comments

The firm's knowledge management behaviors include the information acquisition phase and the knowledge base updating phase. The literature identifies different options for information acquisition. Learning from past experience is a passive learning behavior which costs less but may not offer much valuable information as the new technology is introduced in the industry. Learning from others is an active learning behavior, which can be observed in various forms when firms are viewed as organizations engaging in knowledge production and application. Other sources of information acquisition include R&D investment or requiring information from the extension service or workshops.

The possibility of inter-firm information exchange raises the question of the choice of learning partners. Empirical studies (Conley and Udry, 2001; Ueda, 2002) indicate that a firm decision maker does not learn from all other decision makers, implying a decision relating from who to learn takes place prior how to incorporate additional knowledge. The dynamic character of learning also indicates that the firm might change its knowledge-managing behavior over time (Bierly and Chakrabarti, 1996). Moreover, the characteristics of knowledge indicate that some knowledge is not easily transferred or aggregated. Whether the knowledge can be used to create benefits depends on the firm's ability to handle knowledge. Finally, heterogeneous abilities and knowledge management behavior across firms result in heterogeneity in production behavior.

The following chapter constructs a conceptual model identifying options of information acquisition and describing how the updated knowledge base influences the firm's production decision. According to the conditions derived from the mathematical model, the firm chooses its optimal effort for knowledge management over time, while the empirical results indicate that the factors influencing the firm's social learning network lead to inter-group heterogeneity in production behavior.

CHAPTER 3

CONCEPTUAL MODEL

The conceptual framework addresses how the firm decision maker obtains additional knowledge by engaging in knowledge management behavior. This behavior is the learning process addressing (i) how the firm decision maker acquires more information (information acquisition phase), (ii) uses a learning mechanism by translating the information collected into additional useful knowledge, and (iii) chooses among knowledge management opportunities.

In the information acquisition phase, information is collected by different information acquisition activities. Some activities involve the internal information acquisition as the decision maker manages internal data from past experience (for example, learning-by-doing) while other activities involve the decision maker engaging in social learning activities by obtaining external data and information as he communicates with others. After all the information is collected, the decision maker can choose either to organize the information for processing via the learning mechanism or store the information for processing in a later period. Once the learning mechanism is employed to integrate and rearrange the collected information, the interpreted information becomes useful knowledge that can be added to the existing knowledge base. Thus, the learning process is complete in the sense that the knowledge base is updated and can be used for future decision making.

This chapter articulates several options for information acquisition and their respective level of social learning. Graphical frameworks are constructed to illustrate different knowledge management schemes and how the updated knowledge base enters the decision maker's future production decision.

3.1 Classification of Information Acquisition Activities

The information acquisition phase involves several kinds of information acquisition activities which can be classified by their level of social learning (Bandura, 1977). The information acquisition activity is said to engage in social learning if the decision maker not only manages the information from past experiences but also engages in the additional effort to obtain the external data and information by communicating with others. Possible information acquisition activities and their corresponding attributes are presented in Table 3.1.

Table 3.1 Information Acquisition Activities

Possible Information Acquisition Activities	Description	Level of Social Learning
1. Learning-by-doing	(1) Experiences from manipulative behavior in past production process. (2) Learning from reflecting the past production decisions and past production realization (input use and output level). Both (1) and (2) represent "learning by own experience", and relate to learning from past mistakes.	Non-Social Learning
2. Learn from others		
a. Decision and Realization	Observe others' production decision and realization directly.	Non-Social Learning

Table 3.1 Continued

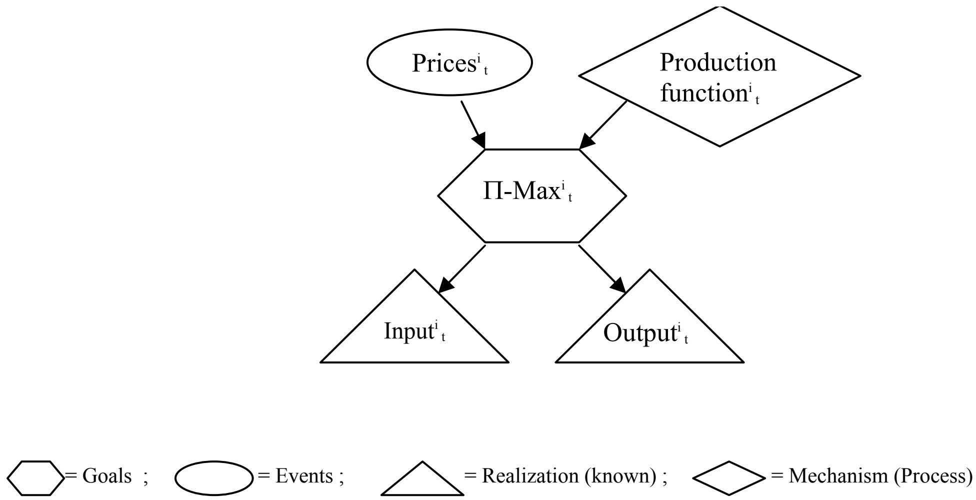
Possible Information Acquisition Activities	Description	Level of Social Learning
b. Conversation	Talk to other managers. This is the chance to compare each other's experience. We might treat this as an aggregation of their own learning information, but this learning may be imperfect because of the way they "combine" or aggregate the learning information.	Social Learning
c. Cooperating with Others	Decision makers actively discuss common issues and come to common conclusions (i.e., walk away with the same information set), but make "own" final decisions.	Social Learning
d. Collaborating with Others	Decision makers actively plan to generate data and translate it into learning information "together".	Social Learning
3. Externally Contracted	Go to Seminar/ Workshop	Social Learning
4. Directed (planned)	Decision maker knows what he wants to study / develop, so he acts on a plan to learn. Consulting experts and buying specific book/software to address specific issues are examples.	Social Learning
5. Experiments (R&D)	Decision maker sets up his own experiments. When doing so, he sacrifices short-term income for long-term understanding of the process.	Non Social Learning

Activities (1) and (5) are the information acquisition activities without social learning since the decision maker does not seek information from outside the firm. Although activity (2a) does obtain some information from outside the firm, it is characterized as non-social learning because the decision maker does not put forth effort for communicating with others and the external information is obtained only by observation. The remaining information acquisition activities are associated with social learning as the decision maker obtains the external data and information by communicating with others. Notice that in activities (2b), (2c), and (2d), decision makers not only acquire the information from other firms but also offer their own information to others; i.e., these information acquisition activities are associated with the exchange of information. The differences in these three activities stem from how decision makers collect information from these interactions and how they decide to manage their information.

3.2 Firm Decision and Learning Environment

As an orientation, consider the following illustrated characterization of production decision making for a profit maximizing firm facing perfectly competitive input and output markets. Figure 3.1 symbolically represents the firm's decision environment. The decision maker takes the production function which is a mechanism translating physical input quantities into physical yield, and takes input and output prices as predetermined at the time the profit maximization goal is executed resulting in simultaneous decisions on the levels of input to be employed and the output to be realized. The notation refers to the variables for firm *i* at time *t*.

Figure 3.1 The Decision Environment



3.3 Knowledge Management Schemes

This section illustrates the potential knowledge management schemes based on various information acquisition activities and the use of the learning mechanism (figures 3.2 to 3.5). The role of an updated knowledge base in production decision making is displayed in the figures. In each figure, a learning mechanism (LM) is specified and presents the decision maker's use of information set (IS). By employing the learning mechanism, the information set is translated into specific additional knowledge feeding into a knowledge base (KB), which is used in the future decision making.

In figure 3.2, both activities (1) and (2a) denote the information acquisition activities without social learning. The difference between them is that activity (1) is the internal information acquisition where information is generated automatically from past production decisions and realizations while activity (2a) offers some external information acquired solely by observation. Activities (3) and (4) denote the information acquisition activities where information is obtained from outside the firm. If the decision maker uses the learning mechanism, information obtained from those information acquisition activities is rearranged and the knowledge base is updated. The arrow from the updated knowledge base toward the production function in the next period reflects how the knowledge management behavior influences future decision making. From this knowledge management behavior, the decision maker may either understand more about the existing production technology, or he may acquire sufficient knowledge to decide to switch to a new production technology. Notice that the information goes only one-way in this conceptualization of knowledge management behavior, i.e., there is no information exchange.

Figures 3.3, 3.4, and 3.5 involve information collection and knowledge development by way of conversation, cooperation, and collaboration, which are information acquisition activities associated with social learning and information exchange between firms. The differences between these information acquisition activities arise from the way a decision maker collects and rearranges the information. Figure 3.3 illustrates the case where decision makers share information by having a conversation. From this conversation, decision makers construct their own information sets and employ their own learning mechanisms. Conversation may be an imperfect way to exchange information because decision makers usually bring different prior beliefs to the conversation, focus on different parts of the conversation, and therefore, come to different conclusions. Figure 3.4 illustrates the case when the decision makers decide to cooperate with each other, which results in creating a common information set, such as drafting a memorandum detailing the common conclusions resulting from the formal meetings.

Collaborations are distinguished from conversation or cooperation where the decision makers not only create the common information set and use the same learning mechanism, but they also resolve to make joint production decisions. In this case, the collaborative firms are integrated into a more extensive dynamic system which produces the physical output and manages knowledge (figure 3.5).

3.4 Choosing/Selecting a Knowledge Management Scheme

In fact, the differences among the knowledge management behaviors associating with conversation, cooperation, and collaboration are more than what is shown in these graphical frameworks. The cost associated with these knowledge management behaviors is one of the properties that are not presented in the figure. One can expect that learning from having conversations with other decision makers cost the least among these three knowledge management patterns. Collaborations, on the other hand, cost the most since the decision maker has to spend the most effort in communicating with other decision makers and arranging their joint information set and joint knowledge base so that they can come to the same production decision.

The extent of information exchange is another difference among these three knowledge management schemes. Having a conversation is an informal way of information exchange which does not offer any regulation or protection to the information sharing or receiving firms. The decision makers may not share much of the information during the conversation. In addition, the decision makers need to verify the accuracy of the information obtained from the conversation among themselves.

On the other hand, learning from cooperation or collaboration is the knowledge management behavior that can be viewed as the strategic alliances formed by some contracts or agreements. The contracts or agreements signed by both alliance partners regulate what kind of information and how much information is exchanged, and the alliance partners are expected to offer true information. There are several forms of strategic alliances and each of them has different degree of information exchange. Collaborations can be regarded as the strategic alliances with the strongest extent of information sharing (e.g., equity joint ventures) since the alliance partners are making joint production decisions. Other strategic alliances, such as R&D contract and technology sharing, are examples of learning from cooperation since the alliance partners have joint information about some specific subject, but the decision makers interpret the information independently and make separate production decisions.

3.5 Concluding Comments

Knowledge management is a learning process including information acquisition and knowledge updating. Most references related to learning theory interpret an individual's learning behavior according to the degree of cognition (Brenner, 1999). This chapter illustrates the firm's knowledge management behavior focusing on the extent of social learning. Different information acquisition and knowledge updating activities result in different knowledge management schemes which involve the way information flows (one-way information flow versus information exchange between firms), the way firms update their knowledge bases and uses them for future decision making, and differences in associated costs and the extent of information sharing. One should notice that knowledge updating is usually not a continuous process since it is associated with the use of a learning mechanism. The firm may keep collecting internal or external information, but the collected information will not be transformed into additional knowledge unless the learning mechanism is employed. The decision maker's actual learning behavior is more complex since it usually involves a mix of different knowledge management schemes. This is explored in the next chapter which outlines the construction of an optimization model of information acquisition and implementation.

Figure 3.2 Learning-by-doing and Learning from Outside the Firm

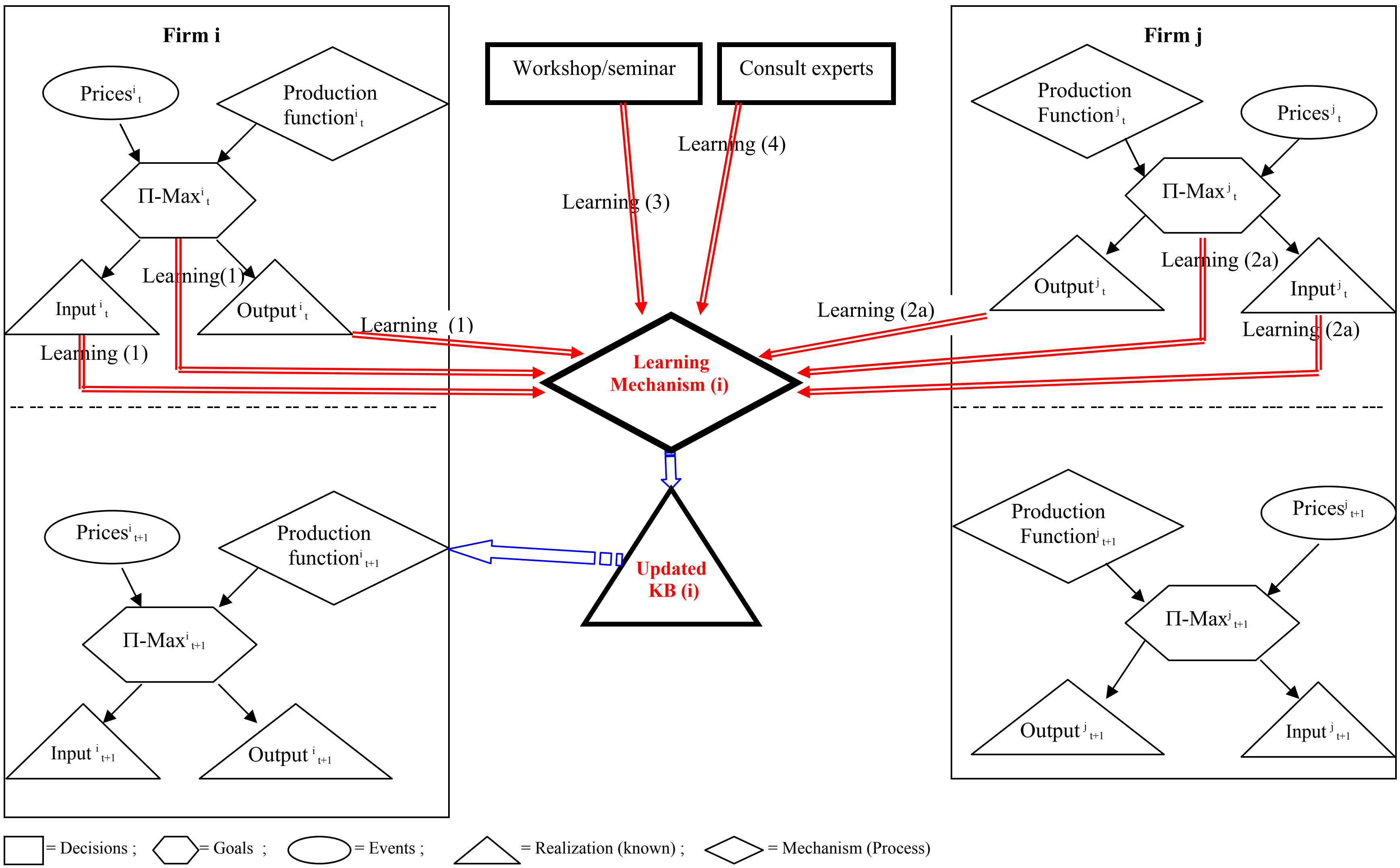


Figure 3.3 Learning-via-conversations

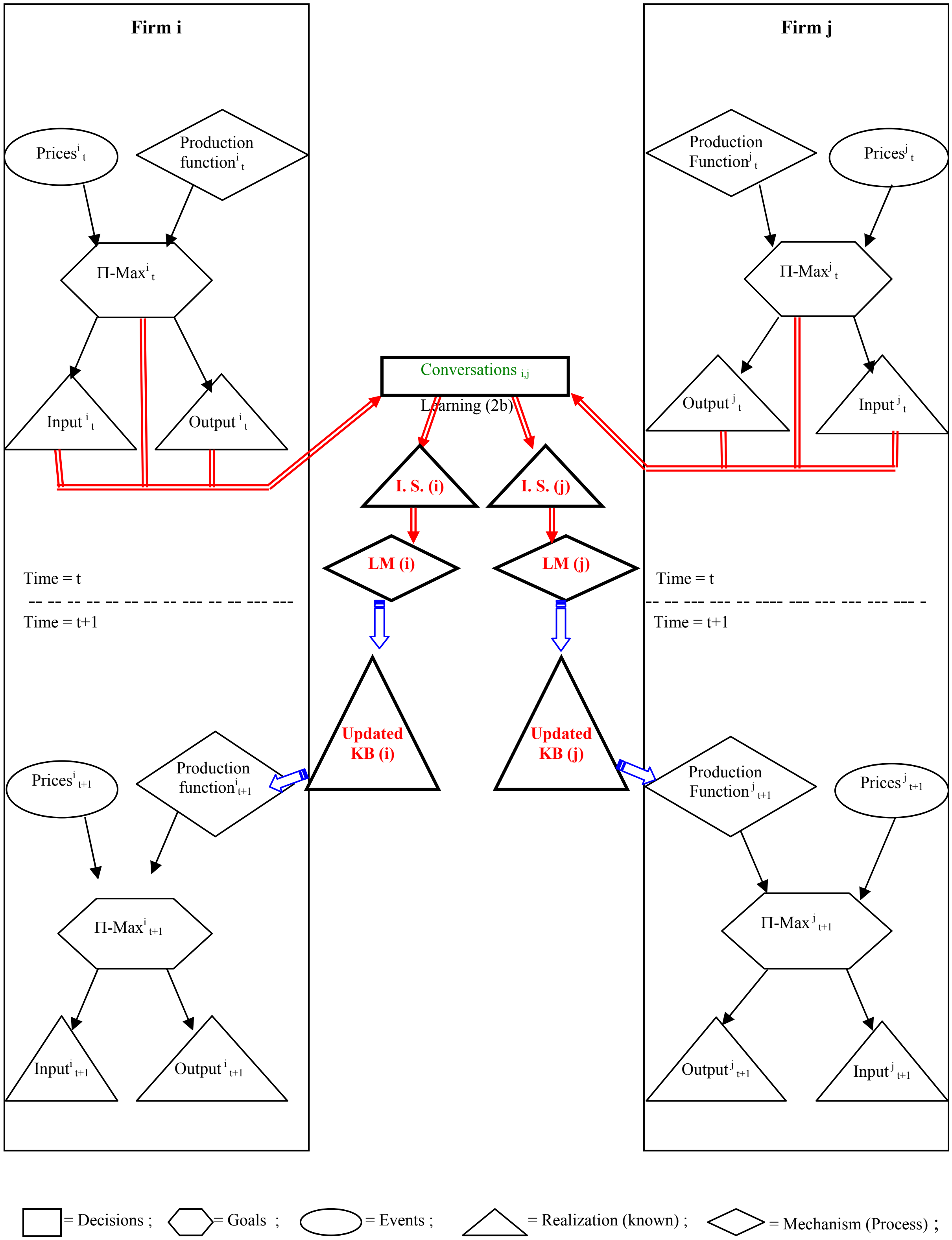


Figure 3.4 Learning-via-cooperation

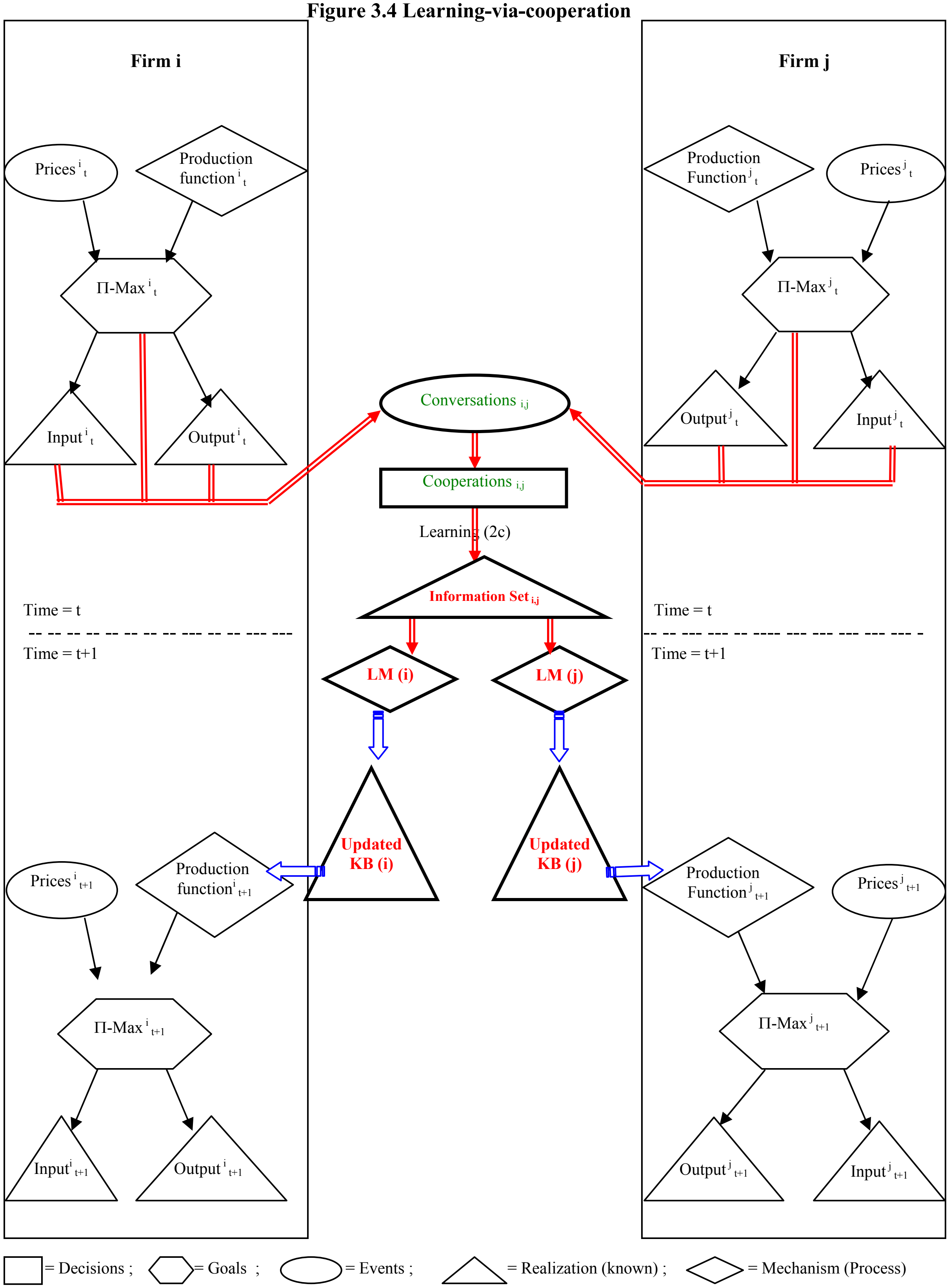
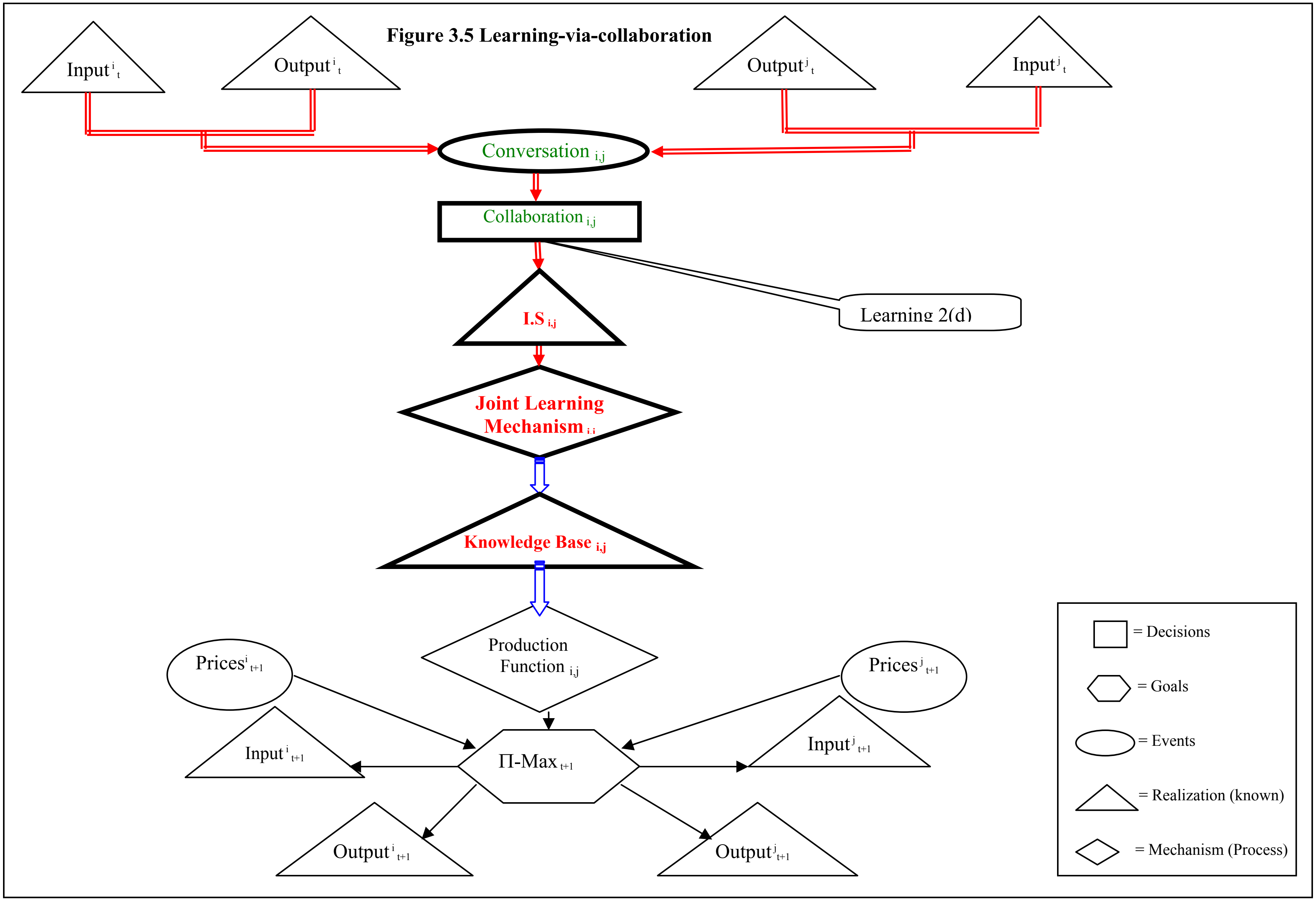


Figure 3.5 Learning-via-collaboration



CHAPTER 4

THEORETICAL MODEL

Knowledge management is a learning process involving the acquisition of information and the use of a learning mechanism, which transfers the collected information into the firm's knowledge base. The graphical analysis presented in the previous chapter shows the flow of information in different acquisition activities, the role of the learning mechanism, and how the updated knowledge base influences future production decisions. The graphical framework, however, does not reflect the costs and benefits of knowledge management behavior and therefore stops short of offering optimal rules for the decision maker regarding knowledge management decisions.

The theoretical framework is established as a dynamic optimization model involving both the cost of information acquisition and the cost of employing the firm's learning mechanism over time. The optimization conditions guide the decision maker in allocating the physical inputs and efforts for information acquisition and learning mechanism, and imply the roles of learning cost and learning benefit.

4.1 Profit Maximization Model

Assume the decision maker maximizes the net present value of firm *i*'s profit over time. In every period, the stock of knowledge or knowledge base, KB_t^i, presents how well the decision maker understands the executed production technology, and along with the physical input (X_t^i) and the effort of general management for production process $(e_t^{i.M})$, the quantity of physical output (Y_t^i) can be determined. The firm can also devote some effort to its knowledge management behavior which improves the decision maker's understanding about production technology in the future. Knowledge management is a learning process involving two phases: information acquisition and knowledge updating. The information collected in the information acquisition phase comes from two sources: internal information acquisition and external social learning. The outcome of the internal information acquisition from past production experience is denoted as firm-specific information, ω_t^i. Let $E_t^{i,\Psi}$ denote the accumulated effort for external information acquisition by social learning activities (or, effort associated with social learning activities) which equals current effort plus the depreciated effort of past social learning activities. Combining the firm-specific information (ω_t^i) and accumulated effort associated with social learning activities leads to the information set (Ω_t^i), which is what the decision maker manages in the knowledge updating phase. The accumulated effort $E_t^{i,\Psi}$ is used in the information generation function since past efforts associated with social learning still impact current information generation. Note that if the firm-specific information or the accumulated social learning effort is not greater than a stated threshold, the external information cannot be obtained, and then the information set will only reflect the firm's historical data.

All the information collected from internal information acquisition and social learning activities are stored as an information set, and the issue is whether the decision maker wants to access the information set and use the learning mechanism to interpret information so that the knowledge base can be updated. Once the information set is rearranged by the learning mechanism, additional knowledge will be generated and incorporated into the existing knowledge base. The updated knowledge base then enters the production function next period and influences future decision making. Let Λ_t^i denote the change of knowledge base; i.e., $KB_{t+1}^i = KB_t^i + \Lambda_t^i$. Knowledge generation is also viewed as a production process where the information set (Ω_t^i) and effort associated with employing the learning mechanism ($e_t^{i,LM}$) are inputs, or $\Lambda_t^i = LM_t^i(\Omega_t^i, e_t^{i,LM})$. In addition, if effort associated with the learning mechanism or the information set does not achieve the threshold level, the learning mechanism cannot work properly and therefore no additional knowledge will be generated, or $\Lambda_t^i = 0$ if $e_t^{i,LM} \le e_{\min}^{i,LM}$ or $\Omega_t^i \le \Omega_{\min}^i$.

Let $C_t^I(\omega_t^i, E_t^{i,\Psi})$ denote the cost of information generation, which is increasing in ω_t^i and $E_t^{i,\Psi}$ while $C_t^{LM}(\Omega_t^i, e_t^{i,LM})$ denotes the cost of employing the learning mechanism, which is increasing in Ω_t^i and $e_t^{i,LM}$. Considering a four-period time horizon[1],

$$
\begin{aligned}
Max \qquad V = \sum_{j=0}^{2} \frac{1}{(1+r)^j} \cdot & \begin{bmatrix} P_{t+j} \cdot F_{t+j}\left(X_{t+j}^i, KB_{t+j}^i, e_{t+j}^{i,M}\right) \\ - W_{t+j} X_{t+j}^i - C_{t+j}^I(\omega_{t+j}^i, E_{t+j}^{i,\Psi}) - C_{t+j}^{LM}(\Omega_{t+j}^i, e_{t+j}^{i,LM}) \end{bmatrix} \\
& + \frac{1}{(1+r)^3} \cdot \left[P_{t+3} \cdot F_{t+3}\left(X_{t+3}^i, KB_{t+3}^i, e_{t+3}^{i,M}\right) - W_{t+3} X_{t+3}^i\right]
\end{aligned}
$$

s.t *knowledge management constraints in table 4.1.*

---

[1] The costs of information generation and employing the learning mechanism do not appear in the last period. The reason is that the decision maker will not rearrange the information set or employ the learning mechanism at the last period since there is no future benefit for this knowledge management behavior.

Table 4.1 Knowledge Management Constraints

Knowledge Management Constraints	Description
$\Omega_s^i = \varphi_s^i(\omega_s^i, E_s^{i,\Psi})$	Information acquisition, which is determined by the firm-specific information and the accumulated effort associated with social learning.
$\Omega_s^i = \varphi_s^i(\omega_s^i) \quad if \quad E_s^{i,\Psi} \le E_{\min}^{i,\Psi} \quad or \quad if \quad \omega_s^i \le \omega_{\min}$	If the firm-specific information or the accumulated effort associated with social learning does not reach the threshold, the information set only reflects the firm's historical data.
$\omega_s^i = \omega_s^i(\sum_{\tau=t}^{s} X_\tau^i) = \sum_{\tau=t}^{s} X_\tau^i$	The firm-specific information is generated from the past input using.
$E_s^{i,\Psi} = e_s^{i,\Psi} + (1-\delta)E_{s-1}^{i,\Psi}$	The accumulated effort associated with social learning is the current effort plus the depreciated effort of past social learning activities.
$KB_{s+1}^i = KB_s^i + \Lambda_s^i$	The knowledge base updating
$\Lambda_s^i = LM_s^i(\Omega_s^i, e_s^{i,LM})$	The knowledge generation depends on the information set and the effort for learning mechanism
$\Lambda_s^i = 0 \qquad if \quad e_s^{i,LM} \le e_{\min}^{i,LM} \quad or \quad \Omega_s^i \le \Omega_{\min}^i$	No additional knowledge can be generated if the information set or the effort for learning mechanism does not exceed the threshold.
$\bar{e}_s^i = e_s^{i,M} + e_s^{i,\Psi} + e_s^{i,LM}$	Total effort constraint
$X_s^i > 0 \qquad e_s^{i,\Psi} \ge 0 \qquad e_s^{i,LM} \ge 0$	The non-negative constraint for choice variables.
$s = t, \ldots, \quad t+3$	Index for each period of time
Ψ denotes the external information acquisition associated with social learning activities (such as cooperation, collaboration, or going to a seminar).	

4.2 Properties of Information Set and Knowledge Base

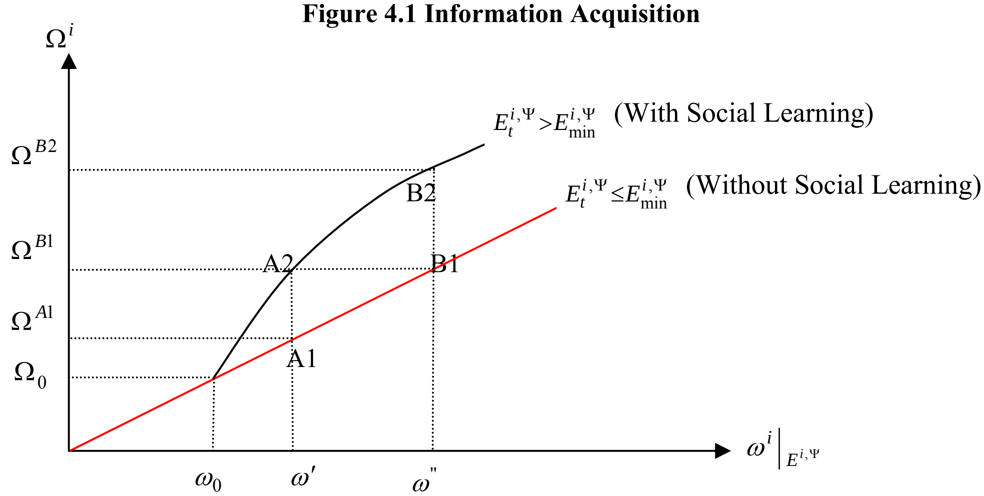
4.2.1 Production Function of Information Set and Knowledge Base

Some properties of the information set (Ω_t^i) and change of knowledge base (Λ_t^i) can be addressed by graphical analysis. The information set can be viewed as resulting from an information generation process where the firm-specific information and the accumulated effort associated with social learning activities are inputs. Figure 4.1 illustrates this production relationship. The firm-specific information is accumulated continuously from past production experience and reflected by the firm's historical input data up to the present. Further, if the firm-specific information or the accumulated effort associated with social learning does not achieve the given thresholds, ω_o and $E_{\min}^{i,\Psi}$, respectively, the decision maker cannot engage in social learning, and therefore, the external information cannot be obtained; thus, the information set will only reflect the firm's historical data.

We assume that the information set has a linear relationship with firm-specific information if either the firm-specific information or the accumulated effort associated with social learning does not achieve their respective thresholds, ω_0 and $E_{\min}^{i,\Psi}$. Notice that for a given information set (Ω^{B1}), it can be generated either by (1) $\omega = \omega'$ and $E^{i,\Psi} > E_{\min}^{i,\Psi}$ or (2) $\omega = \omega''$ and $E^{i,\Psi} \leq E_{\min}^{i,\Psi}$. The case $\omega = \omega'$ and $E^{i,\Psi} > E_{\min}^{i,\Psi}$ denotes the situation when the decision maker decides to obtain the external information by engaging in social learning activities where he puts forth the effort to communicate with the decision makers of other firms (such as cooperating with others). The case $\omega = \omega''$ and $E^{i,\Psi} \leq E_{\min}^{i,\Psi}$ denotes the situation when the decision maker decides to generate the information set by using the firm-specific information only. If the decision maker chooses to cooperate with others, he engages a cost for this social learning activity. But if the decision maker decides to generate firm-specific information set, he may need to wait some time to collect enough firm-specific data. We further assume that

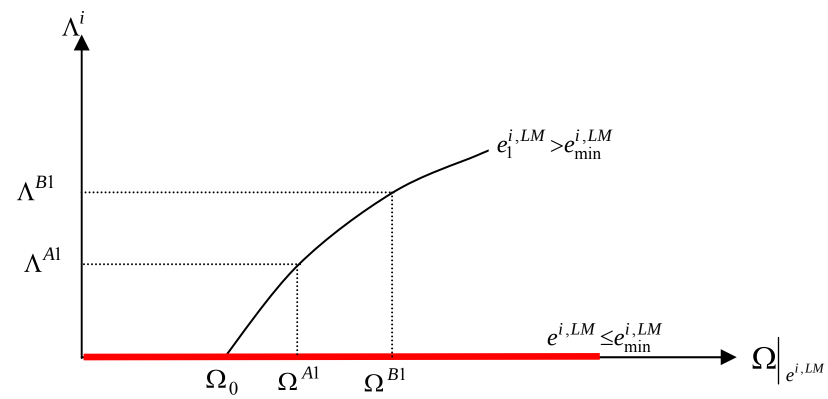
$\Omega_t^i\Big|_{w/\ social\ learning} > \Omega_t^i\Big|_{w/o\ social\ learning} \quad for \quad \omega^i > \omega_0$, implying that as the firm-specific information exceeds its threshold, ω_0, the decision maker can add to the information set from having both internal information acquisition and social learning activities than from internal information acquisition only.

Figure 4.1 Information Acquisition



Similarly, the change of knowledge base (Λ_t^i) can be produced by the learning mechanism where the information set (Ω_t^i) and current period effort allocated to the learning mechanism ($e_t^{i,LM}$) are inputs. It is also assumed that the learning mechanism cannot work if either the information set or the current period effort applied to the learning mechanism does not achieve either of their respective thresholds. In this case, the additional useful knowledge cannot be generated from the learning mechanism, and there is no change in the knowledge base. Figure 4.2 illustrates this knowledge generation relationship.

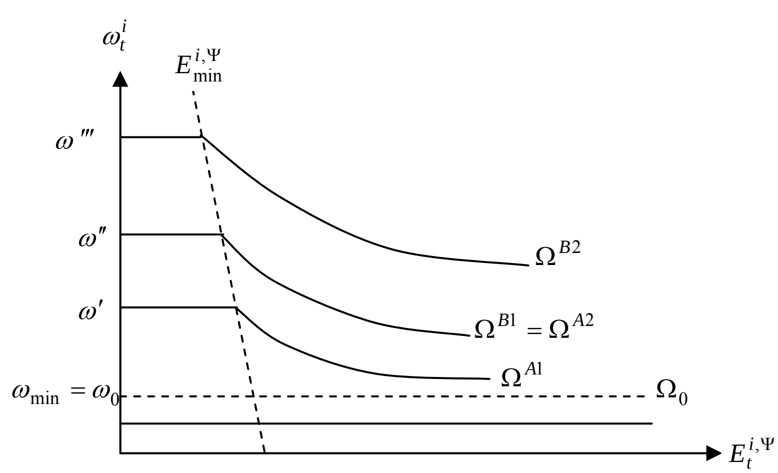
Figure 4.2 Knowledge Generation



4.2.2 The Isoquant of Information Acquisition

Figure 4.3 establishes the relationship between firm-specific information, ω_t^i, and the accumulated effort for external information acquisition, $E_t^{i,\Psi}$, which reflects the kink in the relationship between ω_t^i and Ω_t^i in figure 4.1.

Figure 4.3 The Isoquant of Information Acquisition



The thresholds of firm-specific information and accumulated efforts associated with social learning activities are represented by the $\omega_{\min}$ line and $E_{\min}^{i,\Psi}$ line, respectively. The $\omega_{\min}$ line and all the isoquants below $\omega_{\min}$ are horizontal lines indicating that the information sets only depend on firm-specific information if the firm-specific information is not greater than the threshold $\omega_{\min}$. $E_{\min}^{i,\Psi}$ line is downward sloping instead of a vertical line allowing for the case where the more firm-specific information the decision maker has, the less effort he needs to engage in social learning activity. In addition, the isoquants are horizontal lines when the accumulated effort for social learning does not achieve the threshold, $E_{\min}^{i,\Psi}$. If both firm-specific information and accumulated efforts for social learning are greater than their thresholds, the isoquant is assumed to be downward sloping and convex toward the origin. The properties of the isoquant of information set can be summarized as:

$$\frac{\partial \Omega_t^i}{\partial \omega_t^i} > 0, \qquad \frac{\partial^2 \Omega_t^i}{\partial (\omega_t^i)^2} < 0,$$

$$\begin{cases} \dfrac{\partial \Omega^i}{\partial E_t^{i,\Psi}} = 0 \quad if \quad E_t^{i,\Psi} \le E_{\min}^{i,\Psi} \quad or \quad if \quad \omega_t^i \le \omega_{\min} = \omega_0 \\ \dfrac{\partial \Omega^i}{\partial E_t^{i,\Psi}} > 0 \quad otherwise \end{cases}$$

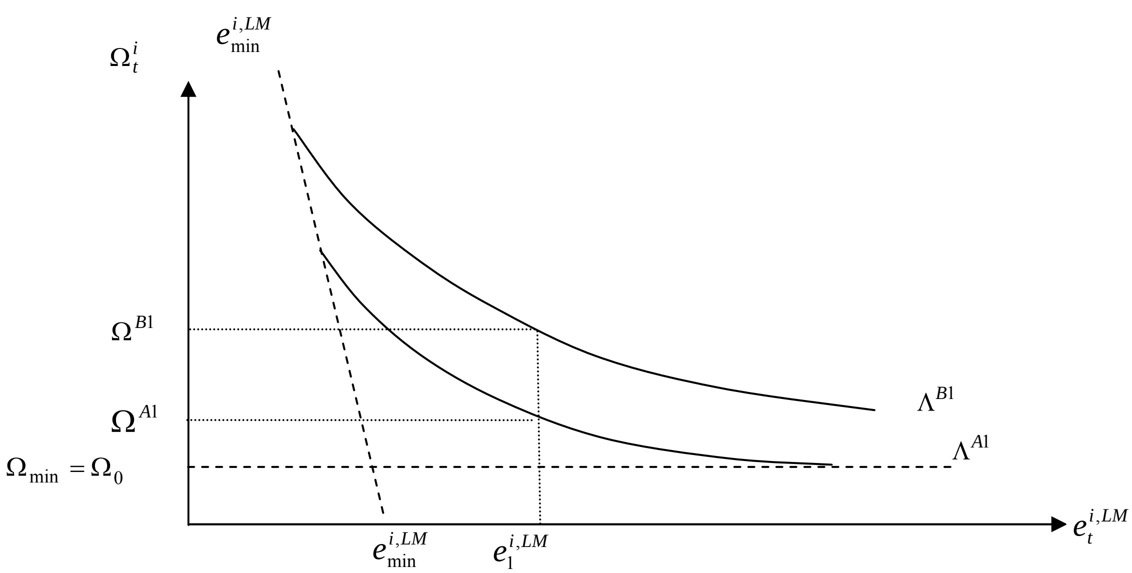
$$\begin{cases} \dfrac{\partial^2 \Omega^i}{\partial (E_t^{i,\Psi})^2} = 0 \quad if \quad E_t^{i,\Psi} \le E_{\min}^{i,\Psi} \quad or \quad if \quad \omega_t^i \le \omega_{\min} = \omega_0 \\ \dfrac{\partial^2 \Omega^i}{\partial (E_t^{i,\Psi})^2} < 0 \quad otherwise \end{cases}$$

$$\begin{cases} \dfrac{\partial}{\partial E_t^{i,\Psi}} \left(\dfrac{\partial \Omega_t^i}{\partial \omega_t^i} \right) = 0 \quad if \quad E_t^{i,\Psi} \le E_{\min}^{i,\Psi} \quad or \quad if \quad \omega_t^i \le \omega_{\min} = \omega_0 \\ \dfrac{\partial}{\partial E_t^{i,\Psi}} \left(\dfrac{\partial \Omega_t^i}{\partial \omega_t^i} \right) > 0 \quad otherwise \end{cases}$$

4.2.3 The Isoquant of Knowledge Base

The isoquant of change of knowledge base (Λ_t^i) associated with figure 4.2 is shown in figure 4.4. The thresholds of the information set and effort for the learning mechanism are denoted by $\Omega_{\min}$ line and $e_{\min}^{i,LM}$ line, respectively. As long as the information set and effort for learning mechanism exceed their thresholds, the isoquant is assumed to be downward-sloping and convex toward the origin; otherwise, the learning mechanism cannot be used and additional useful knowledge cannot be generated ($\Lambda_t^i = 0$).

Figure 4.4 The Isoquant of Knowledge Generation



The properties of the change of knowledge base can be summarized as:

$$\begin{cases} \Lambda_t^i = LM\left(\Omega_t^i, e_t^{i,LM}\right) = 0 \qquad \text{if} \quad e_t^{i,LM} \le e_{\min}^{i,LM} \quad \text{or} \quad \Omega_t^i \le \Omega_{\min}^i \\ \dfrac{\partial \Lambda_t^i}{\partial \Omega_t^i} > 0, \qquad \dfrac{\partial \Lambda_t^i}{\partial e_t^{i,LM}} > 0, \qquad \dfrac{\partial^2 \Lambda_t^i}{\partial (\Omega_t^i)^2} < 0, \qquad \dfrac{\partial^2 \Lambda_t^i}{\partial (e_t^{i,LM})^2} < 0 \quad \text{otherwise} \end{cases}$$

4.3 Optimization Conditions

The following optimization conditions of this mathematical model show how the decision maker allocates physical inputs, the effort associated with social learning activities, and the effort to allocate to the learning mechanism.

$$
\begin{aligned}
(1)\ \frac{\partial \pi}{\partial X_t^i} = {} & P_t \frac{\partial F_t}{\partial X_t^i} - W_t - \frac{\partial C_t^I}{\partial \omega_t^i}\frac{\partial \omega_t^i}{\partial X_t^i} - \frac{\partial C_t^{LM}}{\partial \varphi_t^i}\frac{\partial \varphi_t^i}{\partial \omega_t^i}\frac{\partial \omega_t^i}{\partial X_t^i} \\
& + \frac{1}{1+r}\left[P_{t+1}\frac{\partial F_{t+1}}{\partial KB_{t+1}^i}\frac{\partial LM_t^i}{\partial \varphi_t^i}\frac{\partial \varphi_t^i}{\partial \omega_t^i}\frac{\partial \omega_t^i}{\partial X_t^i} - \frac{\partial C_{t+1}^I}{\partial \omega_{t+1}^i}\frac{\partial \omega_{t+1}^i}{\partial X_t^i} - \frac{\partial C_{t+1}^{LM}}{\partial \varphi_{t+1}^i}\frac{\partial \varphi_{t+1}^i}{\partial \omega_{t+1}^i}\frac{\partial \omega_{t+1}^i}{\partial X_t^i}\right] \\
& + \frac{1}{(1+r)^2}\left[P_{t+2}\frac{\partial F_{t+2}}{\partial KB_{t+2}^i}\left(\frac{\partial LM_t^i}{\partial \varphi_t^i}\frac{\partial \varphi_t^i}{\partial \omega_t^i}\frac{\partial \omega_t^i}{\partial X_t^i} + \frac{\partial LM_{t+1}^i}{\partial \varphi_{t+1}^i}\frac{\partial \varphi_{t+1}^i}{\partial \omega_{t+1}^i}\frac{\partial \omega_{t+1}^i}{\partial X_t^i}\right)\right] \\
& + \frac{1}{(1+r)^2}\left[-\frac{\partial C_{t+2}^I}{\partial \omega_{t+2}^i}\frac{\partial \omega_{t+2}^i}{\partial X_t^i} - \frac{\partial C_{t+2}^{LM}}{\partial \varphi_{t+2}^i}\frac{\partial \varphi_{t+2}^i}{\partial \omega_{t+2}^i}\frac{\partial \omega_{t+2}^i}{\partial X_t^i}\right] \\
& + \frac{1}{(1+r)^3}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB_{t+3}^i}\left(\frac{\partial LM_t^i}{\partial \varphi_t^i}\frac{\partial \varphi_t^i}{\partial \omega_t^i}\frac{\partial \omega_t^i}{\partial X_t^i} + \frac{\partial LM_{t+1}^i}{\partial \varphi_{t+1}^i}\frac{\partial \varphi_{t+1}^i}{\partial \omega_{t+1}^i}\frac{\partial \omega_{t+1}^i}{\partial X_t^i} + \frac{\partial LM_{t+2}^i}{\partial \varphi_{t+2}^i}\frac{\partial \varphi_{t+2}^i}{\partial \omega_{t+2}^i}\frac{\partial \omega_{t+2}^i}{\partial X_t^i}\right)\right] = 0
\end{aligned}
$$

$$
\begin{aligned}
(2)\ \frac{\partial \pi}{\partial X_{t+1}^i} = {} & \frac{1}{1+r}\left[P_{t+1}\frac{\partial F_{t+1}}{\partial X_{t+1}^i} - W_{t+1} - \frac{\partial C_{t+1}^I}{\partial \omega_{t+1}^i}\frac{\partial \omega_{t+1}^i}{\partial X_{t+1}^i} - \frac{\partial C_{t+1}^{LM}}{\partial \varphi_{t+1}^i}\frac{\partial \varphi_{t+1}^i}{\partial \omega_{t+1}^i}\frac{\partial \omega_{t+1}^i}{\partial X_{t+1}^i}\right] \\
& + \frac{1}{(1+r)^2}\left[P_{t+2}\frac{\partial F_{t+2}}{\partial KB_{t+2}^i}\frac{\partial LM_{t+1}^i}{\partial \varphi_{t+1}^i}\frac{\partial \varphi_{t+1}^i}{\partial \omega_{t+1}^i}\frac{\partial \omega_{t+1}^i}{\partial X_{t+1}^i} - \frac{\partial C_{t+2}^I}{\partial \omega_{t+2}^i}\frac{\partial \omega_{t+2}^i}{\partial X_{t+1}^i} - \frac{\partial C_{t+2}^{LM}}{\partial \varphi_{t+2}^i}\frac{\partial \varphi_{t+2}^i}{\partial \omega_{t+2}^i}\frac{\partial \omega_{t+2}^i}{\partial X_{t+1}^i}\right] \\
& + \frac{1}{(1+r)^3}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB_{t+3}^i}\left(\frac{\partial LM_{t+1}^i}{\partial \varphi_{t+1}^i}\frac{\partial \varphi_{t+1}^i}{\partial \omega_{t+1}^i}\frac{\partial \omega_{t+1}^i}{\partial X_{t+1}^i} + \frac{\partial LM_{t+2}^i}{\partial \varphi_{t+2}^i}\frac{\partial \varphi_{t+2}^i}{\partial \omega_{t+2}^i}\frac{\partial \omega_{t+2}^i}{\partial X_{t+1}^i}\right)\right] = 0
\end{aligned}
$$

$$
\begin{aligned}
(3)\ \frac{\partial \pi}{\partial X_{t+2}^i} = {} & \frac{1}{(1+r)^2}\left[P_{t+2}\frac{\partial F_{t+2}}{\partial X_{t+2}^i} - W_{t+2} - \frac{\partial C_{t+2}^I}{\partial \omega_{t+2}^i}\frac{\partial \omega_{t+2}^i}{\partial X_{t+2}^i} - \frac{\partial C_{t+2}^{LM}}{\partial \varphi_{t+2}^i}\frac{\partial \varphi_{t+2}^i}{\partial \omega_{t+2}^i}\frac{\partial \omega_{t+2}^i}{\partial X_{t+2}^i}\right] \\
& + \frac{1}{(1+r)^3}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB_{t+3}^i}\frac{\partial LM_{t+2}^i}{\partial \varphi_{t+2}^i}\frac{\partial \varphi_{t+2}^i}{\partial \omega_{t+2}^i}\frac{\partial \omega_{t+2}^i}{\partial X_{t+2}^i}\right] = 0
\end{aligned}
$$

$$
(4)\ \frac{\partial \pi}{\partial X_{t+3}^i} = \frac{1}{(1+r)^3}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial X_{t+3}^i} - W_{t+3}\right] = 0
$$

$$(5)\ \frac{\partial \pi}{\partial e_t^{i,\Psi}} = -P_t \ \frac{\partial F_t}{\partial e_t^{i,M}} - \frac{\partial C_t^I}{\partial E_t^{i,\Psi}} - \frac{\partial C_t^{LM}}{\partial \varphi_t^i}\frac{\partial \varphi_t^i}{\partial E_t^{i,\Psi}} - \left(\frac{1-\delta}{1+r}\right)\frac{\partial C_{t+1}^I}{\partial E_{t+1}^{\Psi}} - \frac{1-\delta}{1+r}\left[\frac{\partial C_{t+1}^{LM}}{\partial \varphi_{t+1}^i}\frac{\partial \varphi_{t+1}^i}{\partial E_{t+1}^{i,\Psi}}\right]$$

$$-\left(\frac{1-\delta}{1+r}\right)^2\frac{\partial C_{t+2}^I}{\partial E_{t+2}^{\Psi}} - \left(\frac{1-\delta}{1+r}\right)^2\left[\frac{\partial C_{t+2}^{LM}}{\partial \varphi_{t+2}^i}\frac{\partial \varphi_{t+2}^i}{\partial E_{t+2}^{i,\Psi}}\right] + \frac{1}{1+r}\left[P_{t+1}\frac{\partial F_{t+1}}{\partial KB_{t+1}^i}\frac{\partial LM_t^i}{\partial \varphi_t^i}\frac{\partial \varphi_t^i}{\partial E_t^{i,\Psi}}\right]$$

$$+\frac{1}{(1+r)^2}\left[P_{t+2}\frac{\partial F_{t+2}}{\partial KB_{t+2}^i}\left(\frac{\partial LM_t^i}{\partial \varphi_t^i}\frac{\partial \varphi_t^i}{\partial E_t^{i,\Psi}} + (1-\delta)\cdot\frac{\partial LM_{t+1}^i}{\partial \varphi_{t+1}^i}\frac{\partial \varphi_{t+1}^i}{\partial E_{t+1}^{i,\Psi}}\right)\right]$$

$$+\frac{1}{(1+r)^3}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB_{t+3}^i}\left(\frac{\partial LM_t^i}{\partial \varphi_t^i}\frac{\partial \varphi_t^i}{\partial E_t^{i,\Psi}} + (1-\delta)\cdot\frac{\partial LM_{t+1}^i}{\partial \varphi_{t+1}^i}\frac{\partial \varphi_{t+1}^i}{\partial E_{t+1}^{i,\Psi}} + (1-\delta)^2\cdot\frac{\partial LM_{t+2}^i}{\partial \varphi_{t+2}^i}\frac{\partial \varphi_{t+2}^i}{\partial E_{t+2}^{i,\Psi}}\right)\right] \le 0,$$

$$e_t^{i,\Psi} \ge 0, \qquad e_t^{i,\Psi}\left(\frac{\partial \pi}{\partial e_t^{i,\Psi}}\right) = 0$$

$$(6)\ \frac{\partial \pi}{\partial e_{t+1}^{i,\Psi}} = \frac{1}{1+r}\left[-P_{t+1}\frac{\partial F_{t+1}}{\partial e_{t+1}^{i,M}} - \frac{\partial C_{t+1}^I}{\partial E_{t+1}^{i,\Psi}} - \frac{\partial C_{t+1}^{LM}}{\partial \varphi_{t+1}^i}\frac{\partial \varphi_{t+1}^i}{\partial E_{t+1}^{i,\Psi}}\right] - \frac{(1-\delta)}{(1+r)^2}\frac{\partial C_{t+2}^I}{\partial E_{t+2}^{\Psi}}$$

$$-\frac{(1-\delta)}{(1+r)^2}\left[\frac{\partial C_{t+2}^{LM}}{\partial \varphi_{t+2}^i}\frac{\partial \varphi_{t+2}^i}{\partial E_{t+2}^{i,\Psi}}\right] + \frac{1}{(1+r)^2}\left[P_{t+2}\frac{\partial F_{t+2}}{\partial KB_{t+2}^i}\frac{\partial LM_{t+1}^i}{\partial \varphi_{t+1}^i}\frac{\partial \varphi_{t+1}^i}{\partial E_{t+1}^{i,\Psi}}\right]$$

$$+\frac{1}{(1+r)^3}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB_{t+3}^i}\left(\frac{\partial LM_{t+1}^i}{\partial \varphi_{t+1}^i}\frac{\partial \varphi_{t+1}^i}{\partial E_{t+1}^{i,\Psi}} + (1-\delta)\cdot\frac{\partial LM_{t+2}^i}{\partial \varphi_{t+2}^i}\frac{\partial \varphi_{t+2}^i}{\partial E_{t+2}^{i,\Psi}}\right)\right] \le 0,$$

$$e_{t+1}^{i,\Psi} \ge 0, \qquad e_{t+1}^{i,\Psi}\left(\frac{\partial \pi}{\partial e_{t+1}^{i,\Psi}}\right) = 0$$

$$(7)\ \frac{\partial \pi}{\partial e_{t+2}^{i,\Psi}} = \frac{1}{(1+r)^2}\left[-P_{t+2}\frac{\partial F_{t+2}}{\partial e_{t+2}^{i,M}} - \frac{\partial C_{t+2}^I}{\partial E_{t+2}^{i,\Psi}} - \frac{\partial C_{t+2}^{LM}}{\partial \varphi_{t+2}^i}\frac{\partial \varphi_{t+2}^i}{\partial E_{t+2}^{i,\Psi}}\right]$$

$$+\frac{1}{(1+r)^3}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB_{t+3}^i}\frac{\partial LM_{t+2}^i}{\partial \varphi_{t+2}^i}\frac{\partial \varphi_{t+2}^i}{\partial E_{t+2}^{i,\Psi}}\right] \le 0, \qquad e_{t+2}^{i,\Psi} \ge 0, \qquad e_{t+2}^{i,\Psi}\left(\frac{\partial \pi}{\partial e_{t+2}^{i,\Psi}}\right) = 0$$

$$(8)\ \frac{\partial \pi}{\partial e_t^{i,LM}} = -P_t \ \frac{\partial F_t}{\partial e_t^{i,M}} - \frac{\partial C_t^{LM}}{\partial e_t^{i,LM}} + \frac{1}{1+r}\left[P_{t+1}\frac{\partial F_{t+1}}{\partial KB_{t+1}^i}\frac{\partial LM_t^i}{\partial e_t^{i,LM}}\right] + \frac{1}{(1+r)^2}\left[P_{t+2}\frac{\partial F_{t+2}}{\partial KB_{t+2}^i}\frac{\partial LM_t^i}{\partial e_t^{i,LM}}\right]$$

$$+\frac{1}{(1+r)^3}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB_{t+3}^i}\frac{\partial LM_t^i}{\partial e_t^{i,LM}}\right] \le 0, \qquad e_t^{i,LM} \ge 0, \qquad e_t^{i,LM}\left(\frac{\partial \pi}{\partial e_t^{i,LM}}\right) = 0$$

$$(9)\ \frac{\partial \pi}{\partial e_{t+1}^{i,LM}} = \frac{1}{1+r}\left[-P_{t+1}\frac{\partial F_{t+1}}{\partial e_{t+1}^{i,M}} - \frac{\partial C_{t+1}^{LM}}{\partial e_{t+1}^{i,LM}} \right] + \frac{1}{(1+r)^2}\left[P_{t+2}\frac{\partial F_{t+2}}{\partial KB_{t+2}^{i}}\frac{\partial LM_{t+1}^{i}}{\partial e_{t+1}^{i,LM}} \right]$$
$$+\frac{1}{(1+r)^3}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB_{t+3}^{i}}\frac{\partial LM_{t+1}^{i}}{\partial e_{t+1}^{i,LM}} \right] \le 0, \qquad e_{t+1}^{i,LM} \ge 0, \qquad e_{t+1}^{i,LM}\left(\frac{\partial \pi}{\partial e_{t+1}^{i,LM}} \right) = 0$$

$$(10)\ \frac{\partial \pi}{\partial e_{t+2}^{i,LM}} = \frac{1}{(1+r)^2}\left[-P_{t+2}\frac{\partial F_{t+2}}{\partial e_{t+2}^{i,M}} - \frac{\partial C_{t+2}^{LM}}{\partial e_{t+2}^{i,LM}} \right] + \frac{1}{(1+r)^3}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB_{t+3}^{i}}\frac{\partial LM_{t+2}^{i}}{\partial e_{t+2}^{i,LM}} \right] \le 0$$
$$e_{t+2}^{i,LM} \ge 0, \qquad e_{t+2}^{i,LM}\left(\frac{\partial \pi}{\partial e_{t+2}^{i,LM}} \right) = 0$$

Since the physical input is assumed to be greater than zero, conditions (1) to (4) show that the decision maker will use the physical input until its marginal profit is equal to zero. Conditions (5) to (7) reflect how the decision maker allocates the effort associated with social learning over time, while conditions (8) to (10) represent how the decision maker chooses the optimal level of effort to allocate to the learning mechanism. We can see that the way the decision maker chooses the effort associated with social learning is similar to how he allocates the learning mechanism effort. The optimization analysis shows that marginal profit of learning is equal to marginal potential benefits minus marginal cost, and the decision maker will choose positive social learning or learning mechanism effort only when their marginal potential benefit and marginal cost are balanced. The potential marginal benefit involves the increase of future profit, while the marginal cost of learning can be decomposed into the direct costs, current opportunity costs, and potential future cost.

Take condition (5) as an example. The last three components in the equation,

$\sum_{\tau=1}^{3}\frac{1}{(1+r)^{\tau}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB_{t+\tau}^{i}}\left(\sum_{d=1}^{\tau}(1-\delta)^{d-1}\cdot\frac{\partial LM_{t+d-1}^{i}}{\partial \varphi_{t+d-1}^{i}}\frac{\partial \varphi_{t+d-1}^{i}}{\partial E_{t+d-1}^{i,\Psi}} \right) \right]$, denote the potential future benefits from one more unit of the effort associated with social learning. The component, $\frac{\partial C_t^I}{\partial e_t^{i,\Psi}} + \frac{\partial C_t^{LM}}{\partial \varphi_t^i}\frac{\partial \varphi_t^i}{\partial e_t^{i,\Psi}}$, denotes the direct marginal cost which follows from the increase of current cost because of one more unit of effort associated with social learning, while the first component, $P_t \frac{\partial F_t}{\partial e_t^{i,M}}$, is the opportunity cost which is composed of the decrease of current revenue arising from diverting resources to this social learning activity and away from current productive efforts. The remaining components of the equation, $\sum_{\tau=1}^{2}\left(\frac{1-\delta}{1+r}\right)^{\tau}\left[\frac{\partial C_{t+\tau}^I}{\partial E_{t+\tau}^{\Psi}} + \frac{\partial C_{t+\tau}^{LM}}{\partial \varphi_{t+\tau}^i}\frac{\partial \varphi_{t+\tau}^i}{\partial E_{t+\tau}^{i,\Psi}}\right]$, are the potential future cost which is caused by the increasing cost for managing the information set generated at time *t*.

4.4 Learning Strategy over Time and Learning Conditions

4.4.1 The Definitions of "Learning" and the Associated Learning Strategies

From the conceptual framework and the profit maximization model, the knowledge management behavior is in fact the learning process associated with both information acquisition phase and knowledge updating phase. The meaning of "learning", however, needs to be clarified before we discuss the firm's learning strategy overtime. We can distinguish the information acquisition phase and the knowledge generation phase by the outcomes of these two knowledge management behaviors. The output of the information acquisition phase is the accumulated information set, and the output of the knowledge updating phase is the additional useful knowledge which is generated by employing the learning mechanism. No matter how many information acquisition activities the decision maker decides to undertake, if he does not use the learning mechanism to analyze the information set the decision maker learns nothing from the information collected. Therefore, *learning* is defined as the situation when the learning mechanism is employed and the knowledge base is updated, while *not learning* denotes the situation when the decision maker does not employ the learning mechanism.

The associated learning strategies from time *t* to *t+2* are listed in table 4.2 and summarized into 5 types: (i) *always learn*, (ii) *never learn*, (iii) *wait to learn*, (iv) *quit learning*, and (v) *learn-in-bursts*. We can see that the first and the second learning strategies in table 4.2 refer to a consistent learning behavior over time, illustrating the situations that the decision maker chooses to either update the knowledge base continuously or never update the knowledge base. Strategies (3) and (4) denote the *wait to learn* behavior when the decision maker does not use the learning mechanism in the early periods, but chooses to employ it later. Strategies (5) and (6) are the opposite situation where the decision maker employs the learning mechanism in the early periods, then quits using the learning mechanism later. The last two strategies are *learn-in-bursts* cases, which indicate the situation that the decision maker changes the learning decision over time.

Table 4.2 Learning Strategies

Learning Strategies	Time = t	Time = t+1	Time= t+2	Learning Type
(1)	Learn	Learn	Learn	*Always Learn*
(2)	Don’t Learn	Don’t Learn	Don’t Learn	*Never learn*
(3)	Don’t Learn	Learn	Learn	*Wait to Learn*
(4)	Don’t Learn	Don’t Learn	Learn	*Wait to Learn*
(5)	Learn	Don’t Learn	Don’t Learn	*Quit Learning*
(6)	Learn	Learn	Don’t Learn	*Quit Learning*
(7)	Don’t Learn	Learn	Don’t Learn	*Learn-in-Bursts*
(8)	Learn	Don’t Learn	Learn	*Learn-in-Bursts*

4.4.2 Learning Strategies and Corresponding Mathematical Conditions

The optimization conditions already reflect that the firm's optimal level for knowledge management efforts are determined by the marginal cost and the marginal benefit of knowledge management behavior, and the knowledge management behavior will be undertaken in each period only if its marginal cost and marginal benefit are balanced. Therefore, the conditions for the learning strategy (1) are: a) both the information set and effort for learning mechanism exceed their thresholds, and b) the marginal cost of effort for learning mechanism is equal to its marginal benefit in all three periods. The reason the decision maker chooses to never learn [i.e., learning strategy (2)] is either the information set never achieves the threshold or the marginal learning cost is always greater than the marginal learning benefit. The reason for non-consistent learning and the relationship of learning behavior over time, however, are more complicated and require an analysis of the learning conditions and the relationship of conditions across time.

(i) *Wait to Learn* Behavior

Wait to learn denotes the situation when the decision maker decides not to use the learning mechanism to analyze the information set for the first few periods, but chooses to update it later. Learning strategies (3) and (4) in table 4.2 are the cases of *wait to learn*. The optimization conditions that lead to strategy (3) are:

$$(11)\quad P_t \frac{\partial F_t}{\partial e_t^{i,M}} + \frac{\partial C_t^{LM}}{\partial e_t^{i,LM}} > \sum_{\tau=1}^{3} \frac{1}{(1+r)^{\tau}} \left[P_{t+\tau} \frac{\partial F_{t+\tau}}{\partial KB_{t+\tau}^{i}} \frac{\partial LM_t^{i}}{\partial e_t^{i,LM}} \right]$$

$$(12)\quad \frac{1}{1+r} \left[P_{t+1} \frac{\partial F_{t+1}}{\partial e_{t+1}^{i,M}} + \frac{\partial C_{t+1}^{LM}}{\partial e_{t+1}^{i,LM}} \right] = \sum_{\tau=2}^{3} \frac{1}{(1+r)^{\tau}} \left[P_{t+\tau} \frac{\partial F_{t+\tau}}{\partial KB_{t+\tau}^{i}} \frac{\partial LM_{t+1}^{i}}{\partial e_{t+1}^{i,LM}} \right]$$

and,

$$(13)\ \frac{1}{(1+r)^2}\left[P_{t+2}\frac{\partial F_{t+2}}{\partial e_{t+2}^{i,M}}+\frac{\partial C_{t+2}^{LM}}{\partial e_{t+2}^{i,LM}}\right]=\frac{1}{(1+r)^3}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB_{t+3}^{i}}\frac{\partial LM_{t+2}^{i}}{\partial e_{t+2}^{i,LM}}\right].$$

Suppose the marginal product of effort for general management ($\frac{\partial F_t}{\partial e_t^{i,M}}$) at each time are the same, the prices are constant over the time horizon and the cost structure for learning mechanism is linear. Under these assumptions, the marginal learning costs for time *t*, *t+1*, and *t+2* [i.e., the left-hand-side of equations (11), (12), and (13)] are the same. The equality of marginal learning costs is:

$$(14)\quad P_t\frac{\partial F_t}{\partial e_t^{i,M}}+\frac{\partial C_t^{LM}}{\partial e_t^{i,LM}}=P_{t+1}\frac{\partial F_{t+1}}{\partial e_{t+1}^{i,M}}+\frac{\partial C_{t+1}^{LM}}{\partial e_{t+1}^{i,LM}}=P_{t+2}\frac{\partial F_{t+2}}{\partial e_{t+2}^{i,M}}+\frac{\partial C_{t+2}^{LM}}{\partial e_{t+2}^{i,LM}}$$

Combining equations (11) to (14) leads to equation (15) which presents the conditions for the learning strategy (3):

$$(15)\quad \frac{1}{(1+r)}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB_{t+3}^{i}}\frac{\partial LM_{t+2}^{i}}{\partial e_{t+2}^{i,LM}}\right]=\sum_{\tau=2}^{3}\frac{1}{(1+r)^{\tau-1}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB_{t+\tau}^{i}}\frac{\partial LM_{t+1}^{i}}{\partial e_{t+1}^{i,LM}}\right]$$
$$>\sum_{\tau=1}^{3}\frac{1}{(1+r)^{\tau}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB_{t+\tau}^{i}}\frac{\partial LM_{t}^{i}}{\partial e_{t}^{i,LM}}\right]$$

That is, the discounted marginal benefits for using the learning mechanism are the same at time *t+1* and *t+2*, and both are greater than the discounted marginal benefit of using the learning mechanism at time *t*.

Similarly, the conditions leading to learning strategy (4) are:

$$(16)\quad \frac{1}{(1+r)}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB^{i}_{t+3}}\frac{\partial LM^{i}_{t+2}}{\partial e^{i,LM}_{t+2}}\right] > \sum_{\tau=2}^{3}\frac{1}{(1+r)^{\tau-1}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB^{i}_{t+\tau}}\frac{\partial LM^{i}_{t+1}}{\partial e^{i,LM}_{t+1}}\right]$$

and

$$\frac{1}{(1+r)}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB^{i}_{t+3}}\frac{\partial LM^{i}_{t+2}}{\partial e^{i,LM}_{t+2}}\right] > \sum_{\tau=1}^{3}\frac{1}{(1+r)^{\tau}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB^{i}_{t+\tau}}\frac{\partial LM^{i}_{t}}{\partial e^{i,LM}_{t}}\right]$$

which presents that the decision maker will choose not to update knowledge base in the first two periods, and to learn in the third period because the marginal learning benefit at the third period is greater than the marginal learning benefits in the previous two periods.

(ii) *Quit Learning* Behavior

Quit learning denotes the situation when learning mechanism is used for the first few periods, and then the decision maker chooses to stop using it. Learning strategies (5) and (6) in table 4.2 are the cases where the decision maker quits learning.

The conditions leading to this learning strategy must reflect the situations that the decision maker chooses to update knowledge base in a specific period(s) because the marginal learning benefits of learning in the first few period(s) are greater than the marginal learning benefit if he chooses to update knowledge base in later periods. The mathematical conditions for learning strategy (5) are:

$$(17)\quad \sum_{\tau=1}^{3}\frac{1}{(1+r)^{\tau}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB^{i}_{t+\tau}}\frac{\partial LM^{i}_{t}}{\partial e^{i,LM}_{t}}\right] > \sum_{\tau=2}^{3}\frac{1}{(1+r)^{\tau}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB^{i}_{t+\tau}}\frac{\partial LM^{i}_{t+1}}{\partial e^{i,LM}_{t+1}}\right]$$

and

$$\sum_{\tau=1}^{3}\frac{1}{(1+r)^{\tau}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB^{i}_{t+\tau}}\frac{\partial LM^{i}_{t}}{\partial e^{i,LM}_{t}}\right] > \frac{1}{(1+r)}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB^{i}_{t+3}}\frac{\partial LM^{i}_{t+2}}{\partial e^{i,LM}_{t+2}}\right]$$

and for learning strategy (6) are:

$$(18)\quad \sum_{\tau=1}^{3}\frac{1}{(1+r)^{\tau}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB_{t+\tau}^{i}}\frac{\partial LM_{t}^{i}}{\partial e_{t}^{i,LM}}\right]=\sum_{\tau=2}^{3}\frac{1}{(1+r)^{\tau-1}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB_{t+\tau}^{i}}\frac{\partial LM_{t+1}^{i}}{\partial e_{t+1}^{i,LM}}\right]$$
$$>\frac{1}{(1+r)}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB_{t+3}^{i}}\frac{\partial LM_{t+2}^{i}}{\partial e_{t+2}^{i,LM}}\right]$$

(iii) *Learning-in-Bursts* Behavior

Learning strategies (7) and (8) denote the situations when the decision maker switches the learning behavior over time. Under the assumption that the marginal learning costs are the same in each period, the changes in marginal learning benefits across time lead the firm to learn sporadically. The mathematical condition for strategy (7) are:

$$(19)\quad \sum_{\tau=2}^{3}\frac{1}{(1+r)^{\tau-1}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB_{t+\tau}^{i}}\frac{\partial LM_{t+1}^{i}}{\partial e_{t+1}^{i,LM}}\right]>\sum_{\tau=1}^{3}\frac{1}{(1+r)^{\tau}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB_{t+\tau}^{i}}\frac{\partial LM_{t}^{i}}{\partial e_{t}^{i,LM}}\right]$$

and

$$\sum_{\tau=2}^{3}\frac{1}{(1+r)^{\tau-1}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB_{t+\tau}^{i}}\frac{\partial LM_{t+1}^{i}}{\partial e_{t+1}^{i,LM}}\right]>\frac{1}{(1+r)}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB_{t+3}^{i}}\frac{\partial LM_{t+2}^{i}}{\partial e_{t+2}^{i,LM}}\right]$$

and the mathematical condition for learning strategy (8) is:

$$(20)\quad \sum_{\tau=1}^{3}\frac{1}{(1+r)^{\tau}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB_{t+\tau}^{i}}\frac{\partial LM_{t}^{i}}{\partial e_{t}^{i,LM}}\right]=\frac{1}{(1+r)}\left[P_{t+3}\frac{\partial F_{t+3}}{\partial KB_{t+3}^{i}}\frac{\partial LM_{t+2}^{i}}{\partial e_{t+2}^{i,LM}}\right]$$
$$>\sum_{\tau=2}^{3}\frac{1}{(1+r)^{\tau-1}}\left[P_{t+\tau}\frac{\partial F_{t+\tau}}{\partial KB_{t+\tau}^{i}}\frac{\partial LM_{t+1}^{i}}{\partial e_{t+1}^{i,LM}}\right]$$

Both conditions show that the changes of marginal learning benefits are not consistent over time. Take condition (19) as an example. The marginal learning benefits increase from time t to time $t+1$, so the decision maker chooses not to learn at time t and but learn at time *t+1*. The marginal learning benefits, however, decrease from time *t+1* to time *t+2*, which make the decision maker chooses not to learn again.

4.4.3 Learning Strategies and Graphical Analysis

The learning strategies in table 4.2 can be connected with figure 4.4 which is redrawn as figure 4.4' indicating four regions. The definitions of each region are:

I: neither information set nor the effort for learning mechanism exceeds their thresholds:

$(\Omega^i \le \Omega^i_{\min} \quad and \quad e^{i,LM} \le e^{i,LM}_{\min})$;

II: the information set does not exceed its threshold, but the effort for the learning mechanism does: $(\Omega^i \le \Omega^i_{\min} \quad and \quad e^{i,LM} > e^{i,LM}_{\min})$;

III: the information set exceeds the threshold, but the effort for learning mechanism does not: $(\Omega^i > \Omega^i_{\min} \quad and \quad e^{i,LM} \le e^{i,LM}_{\min})$; and

IV: both information set and effort for learning mechanism exceed their thresholds:

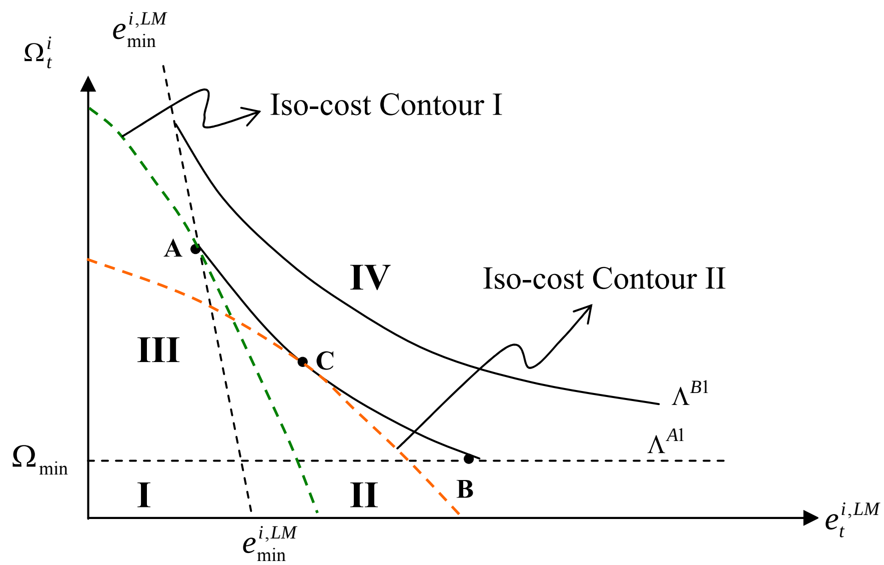
$(\Omega^i > \Omega^i_{\min} \quad and \quad e^{i,LM} > e^{i,LM}_{\min})$.

Additional useful knowledge cannot be generated if either information set or effort for learning mechanism is not greater than the threshold ($\Lambda^i_s = 0$). Therefore, Regions I, II, and III lead to the same learning decision; namely, the decision maker will allocate no learning mechanism effort since any effort applied to employing learning mechanism effort cannot generate additional knowledge in these regions. Hence, the optimal effort for learning mechanism in a given time period is zero and the decision maker chooses not to update knowledge base if the environment is in region I, II, or III.

The question about how the decision maker chooses to allocate to the learning mechanism effort is associated with the shapes of the iso-cost curve and the isoquant of additional knowledge, Λ_t^i. In general, if the output level and the prices are given, the decision maker chooses to allocate inputs so that the iso-cost line (i.e., price line) is tangent to the isoquant. In this study, since the prices of the information set construction and the learning mechanism effort are not observable, an iso-cost contour emerges that represents the relative marginal cost between the information set and the learning mechanism effort. The slope of iso-cost contour represents the ratio between marginal cost of learning mechanism effort, $e_t^{i,LM}$, and the marginal cost (i.e., shadow value) of information set, Ω_t^i. The slope of the iso-cost contour is a function of the information set and the learning mechanism effort. Therefore, the iso-cost contour is a curve instead of a line. In fact, this isocost contour shifts slope over time because the state of information set changes over time. The optimal solution then allocates at the point where the isocost contour is tangent with the isoquant in figure 4.4' (such as point C), or at the point where the isocost curve intersects with the threshold boundaries (such as point A).

If the learning decision happens at the threshold, the decision maker chooses zero effort that is associated with the learning mechanism. For example, points A and B in figure 4.4' belong to regions II and III, respectively. In both situations, the decision maker allocates zero learning mechanism effort and, therefore, the knowledge base is not updated. Region IV is where this knowledge management model obtains an interior solution (for example, at point C) resulting in an updated knowledge base. That is, the only region where the decision maker chooses to *learn* in a given period is in region IV.

Figure 4.4' Regions for Learning Decisions at Time *t*



Viewing the relationships between the regions and the learning decision, the learning strategies presented in table 4.2 are actually the decision nodes moving across the regions. Let Ω_1^i, Ω_2^i, Ω_3^i denote the information sets collected in periods 1, 2, and 3, while D_1, D_2, D_3 denote the decision nodes made by the decision maker in these same periods. The decision nodes represented in figure 4.5 illustrate the situation that the decision maker always wants to learn and stays in region IV for all three periods. Figure 4.6 represents the learning strategy reflecting the firm waiting to learn (learning strategy 3) and the decision nodes move from region I toward region III and then in region IV. There are several possible decision paths which can be drawn in this isoquant map. Each one of them represents different learning strategy executed by the decision maker for believing that this learning strategy will bring the largest net benefit to the firm.

Figure 4.5 Decision Path over Time: The *Always Learn* Strategy

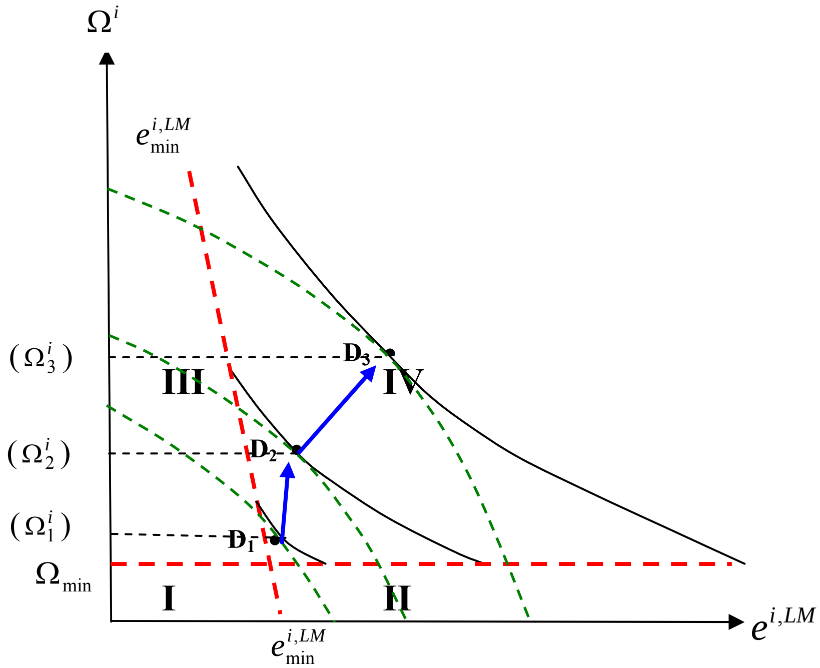
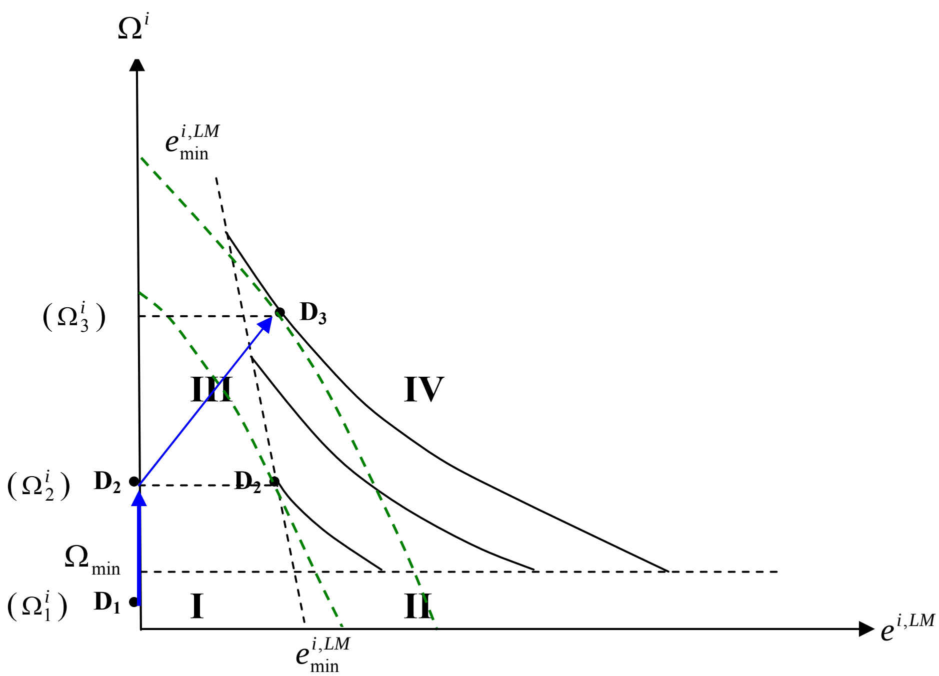
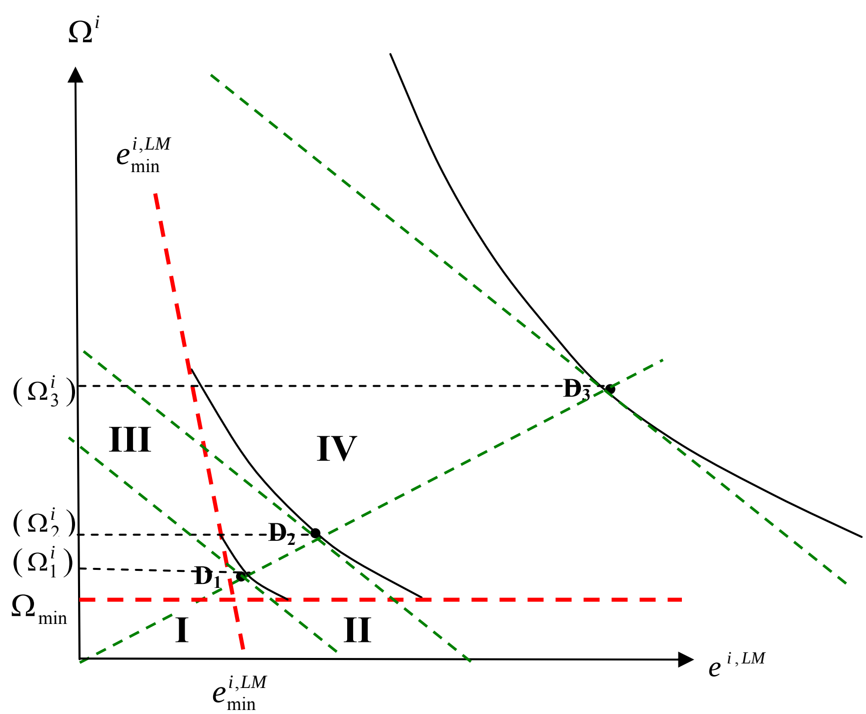


Figure 4.6 Decision Path over Time: The *Wait to Learn* Strategy



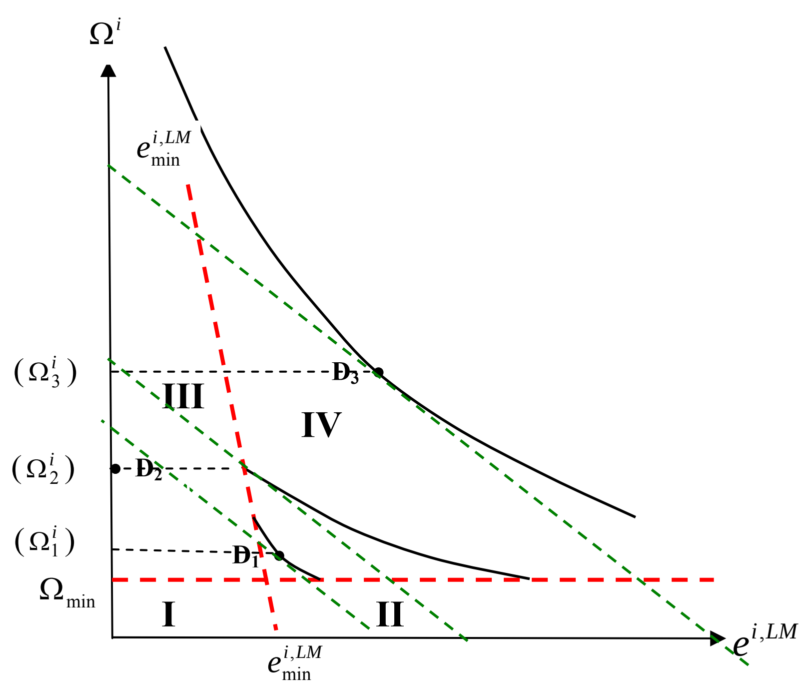
We should notice that the learning decisions (*learn* versus *don't learn*) vary across periods because either the iso-cost contour or the isoquant (or both) change slopes over time. If we simplify this concept by assuming that the marginal costs of building the information set and marginal cost of learning mechanism effort are constant (i.e., the iso-cost contour looks like a price line), the structure of the isoquant changes over time to lead to different learning decisions at each period. If the structure of the isoquant does not change, the decision maker will always choose the same learning behavior, which means the decision nodes are always interior solutions (*always learn*) or the decision nodes always hit the corner (*never learn*). Figure 4.7 presents the situation where the slope of the isoquant does not shift (say, technical change is neutral), and the decision maker chooses to learn at period 1, and keeps learning in periods 2 and 3.

Figure 4.7 Decision Path over Time: The Slope of Isoquant is Constant



Facing the same information set and relative marginal cost at each period of time, the decision maker will choose a different learning strategy if the isoquant changes slopes in each period. Figure 4.7' shows that the knowledge generation technology becomes more information-set-using (relative marginal product of information set, $\dfrac{\partial LM_t^i / \partial \Omega_t^i}{\partial LM_t^i / \partial e_t^{i,LM}}$, increases) at period 2 and then turns back to learning-mechanism-effort-using at period 3 (relative marginal product of information set decreases). This input bias associated with a growing learning capacity shows that the information set is relatively inexpensive at period 2 while the effort associated with learning mechanism becomes relatively inexpensive at period 3. Thus the decision nodes show that the decision maker chooses to *learn-in-bursts* which means he uses the learning mechanism to access the information set at period 1, and then quits using the learning mechanism at period 2 until he can accumulate more information and wait to rearrange it until period 3.

Figure 4.7' Decision Path over Time: The Slope of Isoquant Changes



4.5 Concluding Comments

This chapter presents a formal model illustrating the firm's profit-maximizing problem under its production and knowledge management constraints. The mathematical analysis points out the elements of marginal benefit and marginal cost of the knowledge management behavior and guides the decision maker's learning decision. A graphical analysis illustrates the decision maker's learning decision as well by utilizing the isoquant and iso-cost contours associated with knowledge generation. In addition, several learning strategies (such as *always learn*, *never learn*, *wait to learn*, *quit learning*, and *learn-in bursts*) can be summarized by observing the decision maker's knowledge management decision over time.

By simplifying this model, the numerical solution to a deterministic dynamic programming model is presented in the next chapter. More concrete aspects for knowledge management behavior can be provided and the value of knowledge is quantified.

CHAPTER 5

NUMERICAL MODEL I:

DETERMINISTIC DYNAMIC PROGRAMMING MODEL

Knowledge management is a learning behavior which is undertaken to acquire additional knowledge and improve the firm's profit capacity. The knowledge generation is a discrete process where the decision maker needs to accumulate sufficient information before he employs a learning mechanism to transform the collected information into additional knowledge. Whether the decision maker should allocate additional effort to knowledge management depends on the potential benefits and costs associated with the effort.

A numerical model is constructed by simplifying the theoretical model in the previous chapter. The results present the decision maker's learning behavior at each point in time. It is shown that the decision maker adopts different learning strategies (e.g. *always learn*, *wait to learn*, *learn-in-bursts*, and *quit learning*) if he knows when and how much the future output prices will change before making decisions. Finally, the shadow value of the knowledge is calculated, which is defined as the increase of the sum of the net present value of the firm's profit by having one more unit of the knowledge.

5.1 Deterministic Dynamic Programming Model

5.1.1 Model Setting

The numerical model is similar to the theoretical model where the effort for information acquisition and the effort for using the learning mechanism are combined and renamed as knowledge management effort, e_t^L. In each period, the physical input, X_t, and the effort of general management for production process, $(\bar{e}_t - e_t^L)$, is used to produce the physical output, Y_t. The knowledge management effort, e_t^L, is used to collect the production information and employ the learning mechanism so that additional knowledge can be generated in the future. Thus knowledge management effort is defined as learning effort since it represents effort employed in the learning process to generate additional knowledge.

Knowledge generation is a discrete production process since the additional knowledge is not necessarily produced in every period. If the decision maker does not accumulate sufficient production information, the learning mechanism cannot work properly and no additional knowledge can be obtained. In this numerical model, the accumulated effort for knowledge management, E_t^L, represents the decision maker's accumulated effort in learning. If the accumulated learning effort reaches the assigned threshold(s), then the decision maker devotes enough effort to learning so that the collected information is sufficient to be transformed into additional knowledge successfully. If the accumulated learning effort does not meet the assigned threshold(s), then the decision maker does not devote enough effort to learning and no additional knowledge is generated. Once the additional knowledge is generated, it will contribute to the firm's future knowledge base and influence the firm's production performance and decision making activities.

Assume that the cost associated with the learning effort is a linear function, and the deterministic dynamic programming model is constructed as following:

$$V = \underset{X_1,\ldots,X_T, e_1^L,\ldots,e_T^L}{Max} \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} \cdot \left[P_t \cdot Y_t(X_t, KB_t, e_t^M) - W_t \; X_t - \alpha_4 \cdot e_t^L \right]$$

s.t. production and knowledge management constraints listed in table 5.1.

Table 5.1 Production and Knowledge Management Constraints

Production and Knowledge Management Constraints	Descriptions
$Y_t\left(X_t, KB_t, e_t^M\right) = h(KB_t)\cdot\left(X_t^{0.3}\cdot\left(e_t^M\right)^{0.5}\right)$ $= h(KB_t)\cdot\left(X_t^{0.3}\cdot\left(\bar{e}_t - e_t^L\right)^{0.5}\right)$	Production function
$h(KB_t) = 1 + 1.7\cdot\{1 - \exp[-0.15(KB_t - 1)]\}$	Knowledge function[2]
$KB_{t=1} = KB_{\min} = 1$	Initial state of knowledge base.
$KB_{t+1} = KB_t + \Lambda_t$	Knowledge base accumulation (knowledge base updating).
$E_t^L = \sum_{\tau=1}^{\tau=t} e_\tau^L$	Accumulated learning effort at the end of time *t*.
$\Lambda_t = g_1\left(E_t^L\right) = \left(E_t^L\right)^{0.1}$ *if* $\sum_{\tau=1}^{\tau=t-1} e_\tau^L < em1$ *and* $\sum_{\tau=1}^{\tau=t} e_\tau^L = em1$ $\Lambda_t = g_2\left(E_t^L\right) = \left(E_t^L\right)^{0.9}$ *if* $\sum_{\tau=1}^{\tau=t-1} e_\tau^L < em2$ *and* $\sum_{\tau=1}^{\tau=t} e_\tau^L = em2$ *otherwise* $\Lambda_s^i = 0$	Knowledge generation as the accumulated learning effort hit the threshold(s).
$W_t = 2,\ \alpha_4 = 2,\ r = 0.1,\ X_t \geq 0,\ \bar{e}_t = 2,$ $e_t^L = 0 \ or \ 1,\ em_1 = 2,\ em_2 = 8,\ T = 25$	Given parameters.

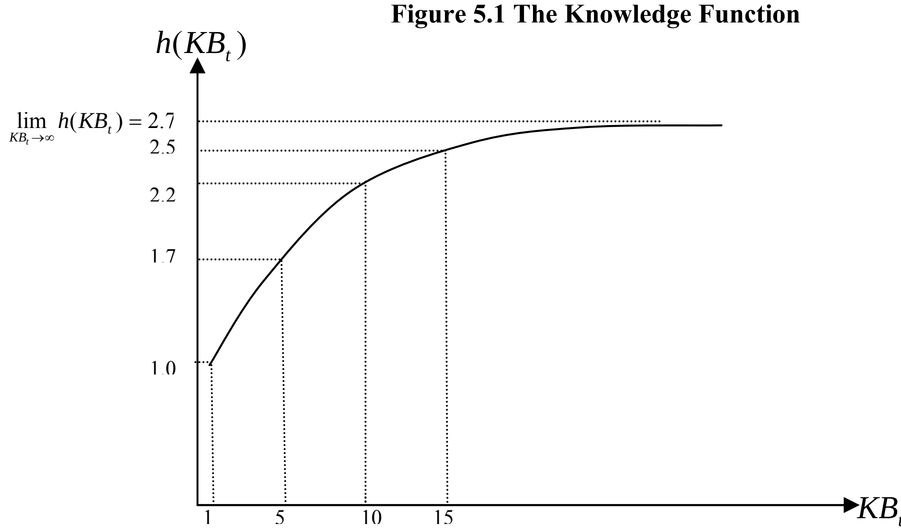
The time horizon is assumed to be 25 periods. The available effort in each period is denoted as $\bar{e}_t$ and the decision maker can choose to devote either zero or one unit of effort to learning. The thresholds for the knowledge generation process are denoted by

[2] The function, $h(KB_t)$ is called the knowledge function in Feder and Slade (1984).

em_1 and em_2 while the parameters, W_t and α_4, indicate the unit cost of the physical input and the learning effort, respectively.

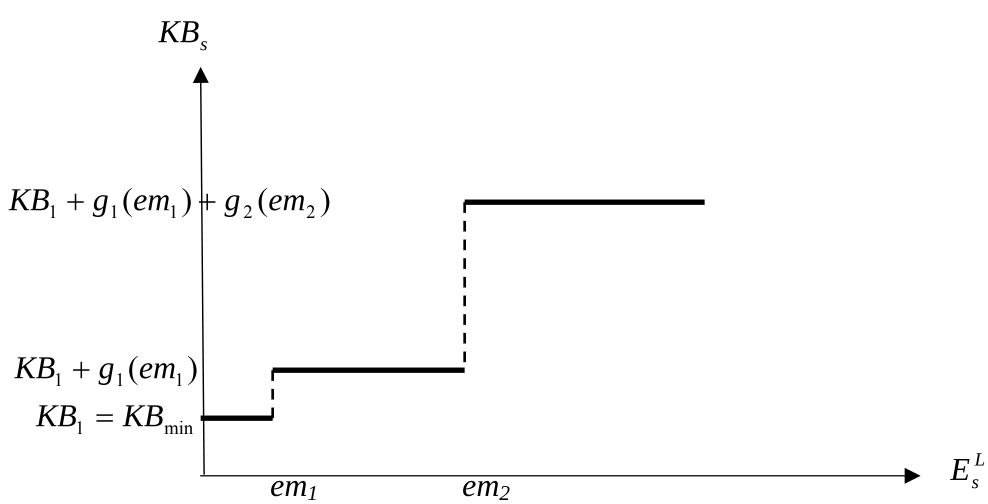
The decision maker's understanding about the executed production technology is represented by the knowledge base, KB_t, and the function, $h(KB_t)$, is defined as a knowledge function illustrating the way the knowledge base influences production performance. Although the knowledge base has a positive impact on production performance, the impact increases at a decreasing rate. Thus the firm's knowledge function converges to a limit as the knowledge base becomes very large (see figure 5.1). In this numerical model, the knowledge function converges to 2.7, which means the output quantity produced by the firm with extremely rich knowledge about the production technology is 2.7 times as much as the output quantity produced by the firm which only has basic knowledge about the production technology.

Figure 5.1 The Knowledge Function



The firm's initial knowledge base is assumed to be at the minimum level implying the decision maker only has basic understanding about the production technology at the beginning of the time horizon. In each period, the decision maker decides whether he wants to allocate learning effort ($e_t^L = 0 \;\; or \;\; 1$). If no effort is allocated to learning, all the available effort will be used for the management of production process, i.e., $e_t^L = 0 \;\; and \;\; e_t^M = \bar{e}_t = 2$. If the decision maker decides to allocate effort for knowledge management, i.e., $e_t^L = 1$, this learning effort will contribute to the improvement of the firm's knowledge base through the function $g\left(E_t^L\right)$ at the time the accumulated learning effort hits the threshold. Two thresholds, em_1 and em_2, are assigned in this numerical model to illustrate the discrete property of knowledge generation. The thresholds also represent that there are two opportunities of knowledge improvement. Figure 5.2 shows that the knowledge base increases only at the time the accumulated learning effort hits the threshold; otherwise, the knowledge base remains at the previous level. The knowledge base increases marginally once the accumulated learning effort hits the first threshold (em_1), and the knowledge base increases considerably when the second threshold (em_2) is reached.

Figure 5.2 The Improvement of Knowledge Base



There are two features to note. First, the knowledge function, $h(KB_t)$, and the knowledge improvement function, $g(E_t^L)$, are firm-specific since both involve the firm decision maker's abilities. The knowledge function can be regarded as the effective knowledge which is associated with the decision maker's ability to implement his knowledge into actual production behavior. Two decision makers with the same knowledge about the production technology may choose different ways to apply the knowledge into action. As a result, one firm may be more productive[3] than the other. On the other hand, the knowledge improvement function relates to the decision maker's learning ability. One can expect that the decision maker with stronger learning ability usually faces smaller thresholds and yet generates more knowledge than the one with less learning ability.

Second, the choice of the knowledge management scheme is another feature influencing the level of knowledge improvement. Different patterns of knowledge management involve different information acquisition activities (social activities versus nonsocial activities) and the use of a learning mechanism (an individual versus a joint learning mechanism). The firm's actual knowledge management behavior is more complex since it usually involves a mix of several knowledge management schemes. One can expect that the knowledge management behavior involving more social interactions (e.g., acquiring production information through social activities or building joint learning

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[3] The production listed in table 5.1 indicate that the physical output is determined by the input using (including physical input, X_t , and the effort for managing the production process, $e_t^M = (\bar{e} - e_t^L)$ and the knowledge function $h(KB_t)$. Because of the differences in the knowledge function, two decision makers with the same knowledge level, same production function, and same input using may have different output levels. We say that firm 1 is more productive than firm 2 if $h_1(KB_t) > h_2(KB_t)$, *ceteris paribus*.

mechanism with other firms) can generate more knowledge than the behavior involving self-learning alone.

5.1.2 The Definition of Learning and Types of Learning Strategies

Before the numerical results are presented, the definition of learning and the type of learning strategies associated with the numerical model should be clarified. In the theoretical model, *learning* is defined as the situation where the decision maker allocates effort to use the learning mechanism and update the knowledge base. In the numerical model, the effort for information acquisition and the effort for using the learning mechanism are combined and renamed as knowledge management effort, which is regarded as learning effort since it is used to acquire more knowledge. Thus, the definition of learning depends on the use of learning effort. In addition, the assumption that learning effort is either one or zero indicates whether the decision maker decides to *learn* or to *not learn*.

The types of the learning strategies need to be reconsidered as well. Two thresholds are assigned in the numerical model indicating that the decision maker faces two opportunities for knowledge base improvements. As the accumulated learning effort hits the first threshold, the decision maker gains a little more knowledge. The second threshold is assumed to be much greater than the first one, so it takes more time to reach the second one. But once the decision maker accumulates enough learning effort and reaches the second threshold, considerably more knowledge improvement can be obtained from this learning behavior. After reaching the second threshold, the decision maker will not employ any learning effort since no additional knowledge can be generated afterward.

Thus, the learning strategy category is based on the decision maker's learning behavior from the beginning until the time the second threshold is reached. *Always learn* denotes the situation where the decision maker uses the learning effort from the beginning until the accumulated learning effort hits the second threshold. *Wait to learn* denotes the situation where the decision maker does not use the learning effort at the first few periods, but once he starts to use the learning effort, he keeps using it until the second threshold is reached. *Quit learning* indicates the situation where the decision maker only uses learning effort to complete the first knowledge base improvement, and then the learning effort is never used afterward. Finally, the *learn-in-bursts* strategy states the situation in which the decision maker temporarily stops the learning process, but after a few periods, restarts the learning effort and then completes the second knowledge base improvement.

5.2 The Numerical Results

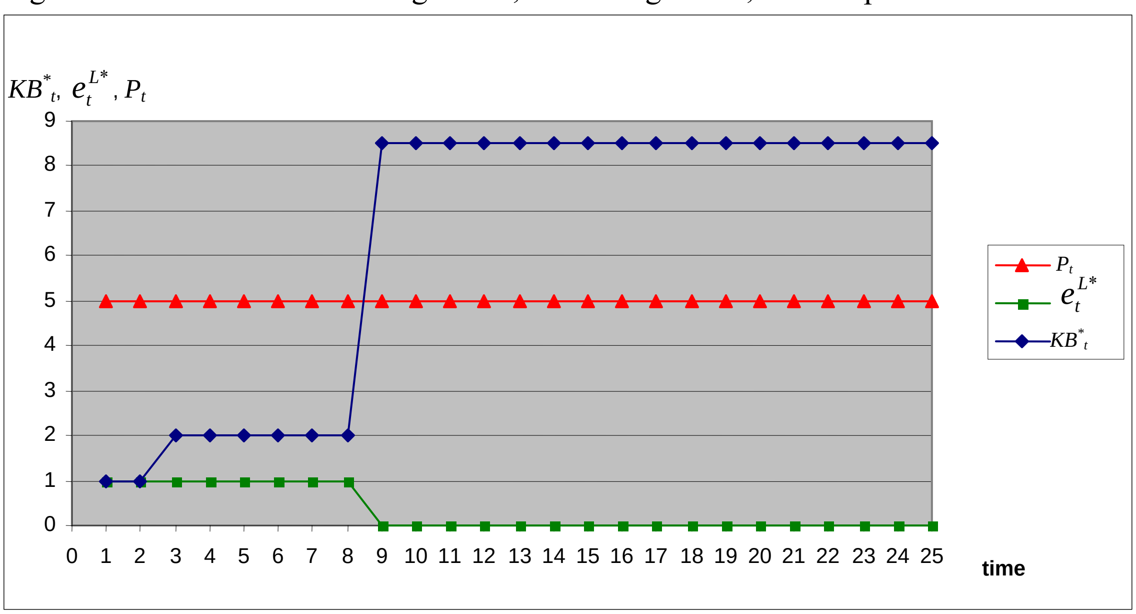
5.2.1 Basic Case

In the basic case, the output price is assumed to be constant over time, and $P_t = 5$ for t = 1,…, T. The numerical results are presented in table 5.2. The optimal learning effort, e_t^{L*}, indicates that the decision maker uses the leaning effort from the beginning until the second threshold is reached (*Always learning* strategy). The optimal level of the knowledge base at the beginning of each period t is denoted as KB_t^*. Table 5.2 shows that the firm reaches the first threshold and acquires more knowledge at the end of period 2. After six more periods of learning, the firm gains much more knowledge making the knowledge base jump from 2 to 8.5 at the end of period 8. Once the second threshold is reached, the firm cannot acquire more knowledge by employing the learning effort. Thus, no additional learning effort is employed afterward. The path of output price and the paths of optimal learning effort and knowledge base are shown in figure 5.3.

Table 5.2 Numerical Results: Basic case ($P_t = 5$)

Time(t)	KB_t^*	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}^*	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	0.5	0.812252	2	2	1.061262
3	2	2	1	1	1.236796	3	2	2.183982
4	2	3	1	1	1.236796	4	2	2.183982
5	2	4	1	1	1.236796	5	2	2.183982
6	2	5	1	1	1.236796	6	2	2.183982
7	2	6	1	1	1.236796	7	2	2.183982
8	2	7	1	1	1.236796	8	8.5	2.183982
9	8.5	8	0	3	4.223806	8	8.5	15.11903
10	8.5	8	0	3	4.223806	8	8.5	15.11903
11	8.5	8	0	3	4.223806	8	8.5	15.11903
12	8.5	8	0	3	4.223806	8	8.5	15.11903
13	8.5	8	0	3	4.223806	8	8.5	15.11903
14	8.5	8	0	3	4.223806	8	8.5	15.11903
15	8.5	8	0	3	4.223806	8	8.5	15.11903
16	8.5	8	0	3	4.223806	8	8.5	15.11903
17	8.5	8	0	3	4.223806	8	8.5	15.11903
18	8.5	8	0	3	4.223806	8	8.5	15.11903
19	8.5	8	0	3	4.223806	8	8.5	15.11903
20	8.5	8	0	3	4.223806	8	8.5	15.11903
21	8.5	8	0	3	4.223806	8	8.5	15.11903
22	8.5	8	0	3	4.223806	8	8.5	15.11903
23	8.5	8	0	3	4.223806	8	8.5	15.11903
24	8.5	8	0	3	4.223806	8	8.5	15.11903
25	8.5	8	0	3	4.223806	8	8.5	15.11903

Figure 5.3 The Paths of Learning Effort, Knowledge Base, and Output Price: Basic Case



The numerical results provided by table 5.2 and figure 5.3 indicates that the decision maker chooses the *always learn* strategy if the output price is constant over time. However, the use of the learning effort depends on the potential benefits and costs associated with the effort, and the factors affecting the potential learning benefits and costs are going to influence the decision maker's learning decision. The output price is one of the factors impacting potential learning benefits and costs. For example, a decrease in future output price implies that the opportunity cost of learning is higher in the earlier periods than it is in the later periods. In early periods (while the output price is still high), the decision maker gives up relatively more profit if he decides to divert resources away from physical output production to learning. Therefore, the change in output price alters the net learning benefits, and therefore influences learning behavior. In the next two sections, the decision maker's learning decisions are investigated under the condition that the output price is allowed to change in different periods.

5.2.2 Price-Decreasing Cases

In this section, the decision maker is assumed to make production and learning decisions under the situation where he knows that the output price is decreasing from \$5 to \$3 at time t^D. There is no uncertainty in this case because the timing of the decrease of the output price, t^D, is known. Various learning strategies can be observed as t^D changes. To see this, the numerical results for case t^D=2 to case t^D=11 are listed in appendix A. The corresponding paths of output price, optimal learning effort, and optimal knowledge base for each case are shown in figures A-1 to A-10.

Take t^D=2 as an example. The decision maker knows that the output price decreases from \$5 to \$3 at period 2, and he makes production and learning decisions based on this understanding. According to the numerical results listed in appendix A-1, the decision maker adopts a *wait to learn* strategy. Since the decision maker knows that the output price is decreasing at period 2, he decides to allocate total effort to produce the physical output while the price is still high in period 1. The use of learning effort is delayed until period 2, but once the decision maker starts to use learning effort, he keeps using it until the accumulated learning effort hits the second threshold. The path of the knowledge base shows that the first improvement of the knowledge base is completed at the end of period 3 and the second improvement is completed at the end of period 9 (figure A-1).

Observing the results offered by appendix A, and the learning strategies for the price-decreasing cases are summarized as following:

1) If the output price decreases from \$5 to \$3 before period 4, the decision maker will delay the entire learning process (the *wait to learn* strategy). The decision maker devotes the total effort to producing the physical output while the output price is still high, and starts to allocate the learning effort from the time the output price decreases. Once the decision maker starts to devote effort to learning, he keeps using the learning effort until the second threshold is reached. (Appendixes A-1, A-2 and figures A-1, A-2)

2) If the output price decreases between period 4 and period 9, the decision maker decides to *learn-in-bursts*. The learning effort is employed in the first two periods so that the decision maker can complete the first knowledge base improvement and obtain the profit gain from production enhancement. Then the learning

process stops temporarily so that the decision maker can allocate all effort to produce the physical output while the output price is still high. The decision maker starts allocating the learning effort again when the output price decreases. It is shown that the later the output price decreases, the longer the decision maker shut down the learning process. (Appendixes A-3 to A-8, figures A-3 to A-8)

3) If the output price decreases at period 10, the decision maker will adopt a *quit learning* strategy, which means the learning effort is employed only for the first two periods. The first knowledge base improvement is completed at the end of period 2, and since no more learning effort is employed afterward, the knowledge base remains at the same level for the rest of the periods (Appendixes A-9 and figure A-9). In this case, the *keeping learning* strategy from $t = 1$ to $t = 8$ or restarting the second learning process from $t = 10$ is not preferred to the *quit learning* strategy.

4) If the output price decreases at an even later period (in our case, if the output price decreases at or after period 11), the decision maker adopts an *always learn* strategy, which means he allocates the learning effort from the beginning until the accumulated effort reaches the second threshold (Appendix A-10 and figure A-10).

5.2.3 Price-Increasing Cases

Now assume that the decision maker knows that the output price will increase from \$5 to \$7 at time t^I. Again, the changes of t^I show the different timings of the output price increase. The numerical results for different t^I are shown in appendix B. Regardless of when the output price increases, the decision maker never changes the learning strategy. The *always learn* strategy is adopted, which means that the learning effort is employed from the beginning until the accumulated learning effort hits the second threshold. Since the decision maker knows that 1) the output price will never drop once it jumps to the high level, and 2) the completeness of the second knowledge base improvement brings a great enhancement of physical output production, the decision maker decides to finish the entire learning process as soon as possible so that the firm can enjoy the profit gain from the higher output price and the production improvement due to the knowledge gain.

5.3 The Value of the Knowledge

According to the model specification, there are two state variables: the knowledge base, KB_t, and the accumulated learning effort, $\sum_t e_t^L$. The initial conditions of the state variables indicate that the decision maker only has basic knowledge about the production technology ($KB_1 = 1$) and he does not have any accumulated learning effort before making production and learning decisions. The decision variables, physical input (X_t) and learning effort (e_t^L), are chosen under these initial conditions to maximize the sum of the net present value of the firm's profit over time. Once the optimal physical input and the learning effort are chosen, the optimal path for the state variables, KB_t^* and $\left(\sum_t e_t^L\right)^*$, can be drawn for each time *t*. The value function (*V*) represents the sum of net present value of the firm's profit over 25 periods as the optimal decision variables are employed. One should notice that the decision maker might make different production and learning decisions and obtain a different value function if he faces different initial conditions.

Assume that the decision maker has one more unit of knowledge at the beginning of the period. Different optimal decisions will be made to reach the maximization goal and a different value function will be obtained. Thus the shadow value of the knowledge at *t* = 1 is defined as the difference between the value functions since it represents how much the sum of the net present value will increase if the decision maker has one more unit of knowledge. Based on this concept, the shadow value of knowledge for each period *t* can be revealed.

Assume that $\left(V_\tau \Big|_{KB_\tau,\ \sum_{t=1}^{\tau-1} e_t^L} \right)$ indicates the value function at time τ, which is obtained as the decision maker employs the optimal physical input and learning effort to maximize the sum of the net present value of the firm's profit from time τ to time T under the conditions that KB_τ and $\sum_{t=1}^{\tau-1} e_t^L$ are given, i.e.,

$$V_\tau \Big|_{KB_\tau,\ \sum_{t=1}^{\tau-1} e_t^L} = \underset{X_\tau,\dots,X_T,e_\tau^L,\dots,e_T^L}{Max} \sum_{t=\tau}^{T} \frac{1}{(1+r)^{t-1}} \cdot \left[P_t \cdot Y_t(X_t, KB_t, e_t^M) - W_t\ X_t - \alpha_4 \cdot e_t^L \right].$$

The shadow value of the knowledge at time τ is represented by V_{KB_τ} which is defined as the difference of the value functions for having one more unit of knowledge:

$$V_{KB_\tau} = V_\tau \Big|_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L \right)^*} - V_\tau \Big|_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L \right)^*}$$

where KB_{τ}^{*} and $\left(\sum_{t=1}^{\tau-1} e_t^L\right)^{*}$ are the optimal level of the knowledge base and the optimal accumulated learning effort at the beginning of time τ provided by the deterministic numerical results. The shadow value of the knowledge for the basic case, the price-decreasing cases, and the price-increasing cases is presented in the following sections.

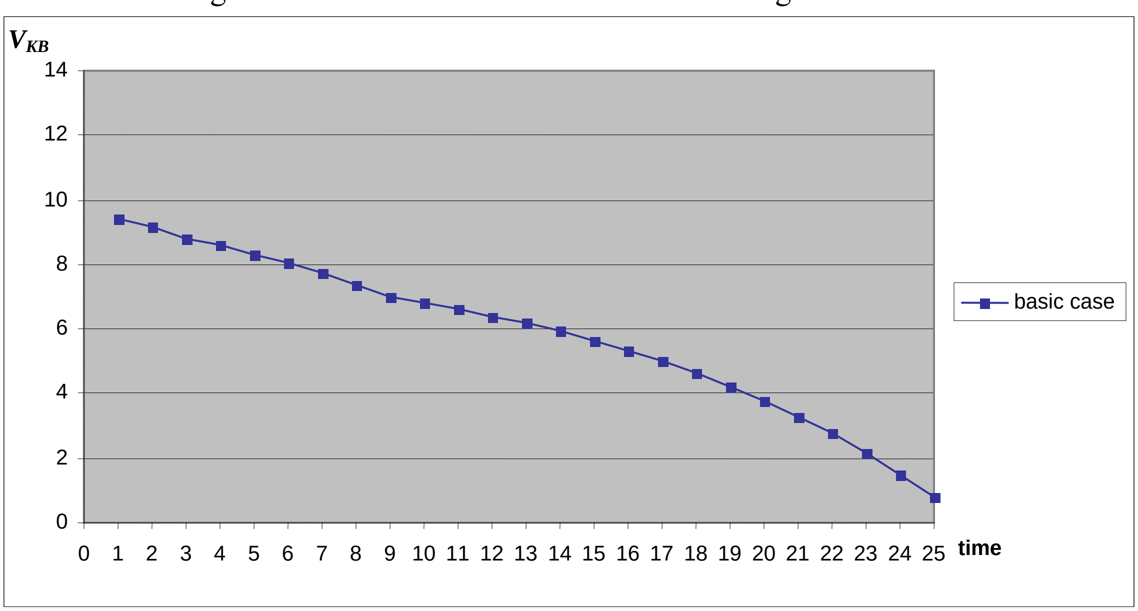
5.3.1 The Basic Case

The shadow value of the knowledge for the basic case is presented in table 5.3. As the output price is constant $(P_t = 5)$, the decision maker adopts an *always learn* strategy, which makes the knowledge base increase from 1 to 2 at the end of second period and then jumps from 2 to 8.5 when the second knowledge base improvement is completed at the end of period 8. The value of additional knowledge is the increase of the sum of the net present value at time τ when having one more unit of knowledge base than the optimal level. Figure 5.4 shows that the shadow value of the knowledge decreases smoothly overtime and the curve is slightly concave after the second knowledge base improvement is revealed at period 9.

Table 5.3 The Shadow Value of Knowledge: Basic Case

Time (τ)	KB_{τ}^{*}	$V_{\tau}\Big\vert_{KB_{\tau}=KB_{\tau}^{*},\ \sum_{t=1}^{\tau-1} e_t^L=\left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_{\tau}^{*}+1$	$V_{\tau}\Big\vert_{KB_{\tau}=KB_{\tau}^{*}+1,\ \sum_{t=1}^{\tau-1} e_t^L=\left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	72.908	2	82.343	9.435
2	1	79.031	2	88.175	9.143
3	2	85.767	3	94.590	8.823
4	2	91.942	3	100.526	8.584
5	2	98.733	3	107.055	8.321
6	2	106.204	3	114.237	8.033
7	2	114.422	3	122.137	7.715
8	2	123.462	3	130.828	7.366
9	8.5	133.406	9.5	140.387	6.981
10	8.5	130.116	9.5	136.925	6.809
11	8.5	126.496	9.5	133.116	6.620
12	8.5	122.515	9.5	128.926	6.411
13	8.5	118.135	9.5	124.317	6.182
14	8.5	113.318	9.5	119.248	5.930
15	8.5	108.019	9.5	113.672	5.653
16	8.5	102.190	9.5	107.537	5.348
17	8.5	95.778	9.5	100.790	5.012
18	8.5	88.725	9.5	93.368	4.643
19	8.5	80.966	9.5	85.203	4.237
20	8.5	72.432	9.5	76.222	3.790
21	8.5	63.044	9.5	66.343	3.299
22	8.5	52.718	9.5	55.477	2.759
23	8.5	41.359	9.5	43.523	2.164
24	8.5	28.864	9.5	30.374	1.510
25	8.5	15.119	9.5	15.910	0.791

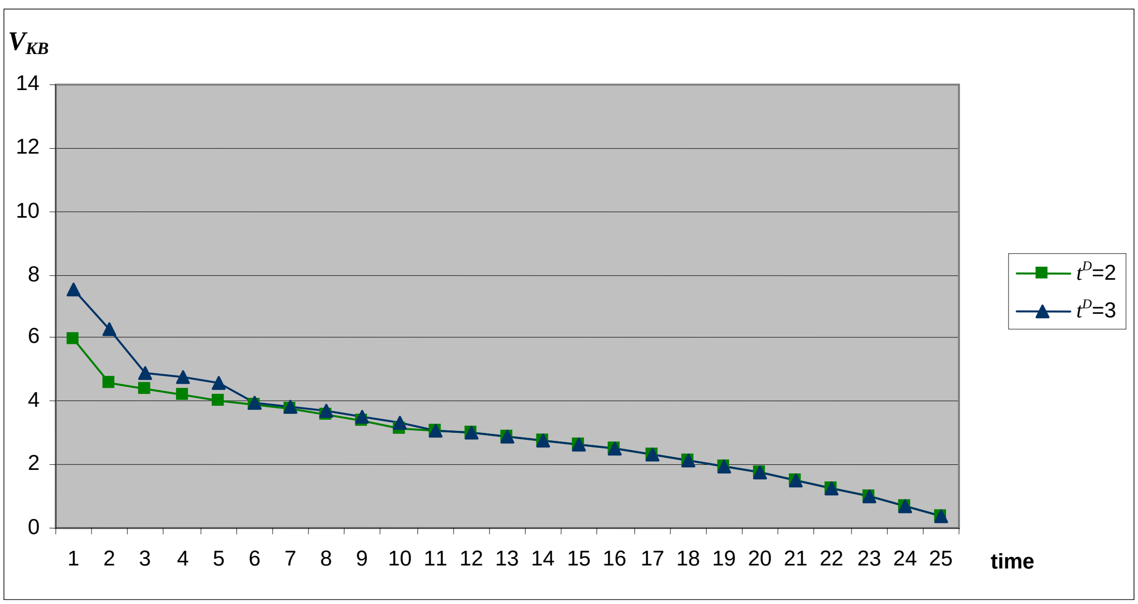
Figure 5.4 The Shadow Value of the Knowledge: Basic case



5.3.2 Price-Decreasing Cases

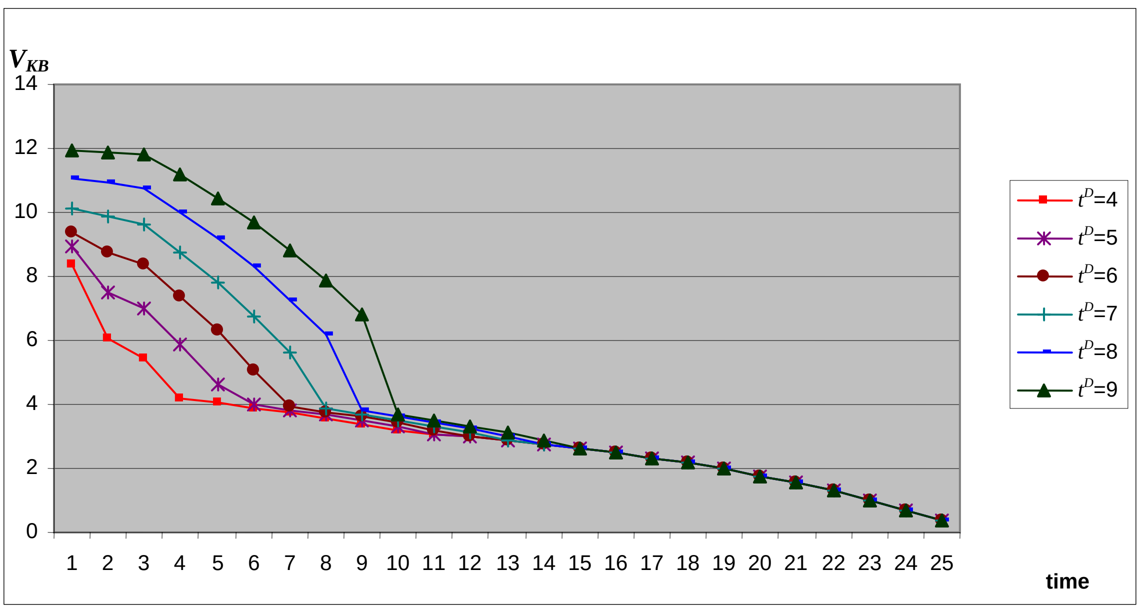
The numerical results already indicate that the change of the output price influences the firm's learning strategies. In fact, the change of the output price has an impact on the shadow value of the knowledge as well. The values of the knowledge for price-decreasing cases are calculated in appendix C. Since the decision maker adopts different learning strategies as the output price decreases in different periods, the value of the knowledge corresponding to the same learning strategy are presented in the same figure. Figures 5.5 and 5.6 show the patterns of the value of the knowledge corresponding to the *wait-to-learn* and the *learn-in-bursts* strategies, respectively. Again, t^D indicates the time the output price decreases from \$5 to \$3. In figure 5.5, cases t^D=2 and t^D=3 present the same shadow value of the knowledge after period 11. This is because both cases face the same knowledge base, accumulated learning effort, and output price after period 11. As for the periods earlier than period 11, we can see that the value of the knowledge in case t^D=3 is greater than that in case t^D=2 in every period.

Figure 5.5 The Shadow Value of the Knowledge – Price Decreasing Cases, *wait to learn*



A similar situation is observed in figure 5.6 where the decision maker adopts a *learn-in-bursts* strategy. The decision maker learns in the first two periods and then stops learning temporarily while the output price still remains at \$5. The decision maker starts learning again at the time the output price decreases. In addition, the later the output price decreases, the later the decision maker restarts the learning process. As we can see, all the cases in figure 5.6 have the same shadow value of the knowledge after period 14, which is when all the cases in figure 5.6 finish the second knowledge improvement. In addition, for the period earlier than period 14, the later the output price decreases, the greater the shadow value of the knowledge at each period.

Figure 5.6 The Shadow Value of the Knowledge - Price Decreasing Cases, *Learn-in-bursts*



The patterns of the value of the knowledge for case t^D=10 and case t^D=11 are shown in figures 5.7 and 5.8, respectively. In case t^D=10, the decision maker takes the *quit learning* strategy; the value of the knowledge increase very slightly before period 3 and then decreases over time afterwards. In case t^D=11, the output price drops at period 11, and the decision maker takes the *always learn* strategy. The shadow value of the knowledge in case 11, however, shows a very different pattern than other cases. It drops rapidly at period 4 and then decreases slowly after period 11.

Figure 5.7 The Shadow Value of the Knowledge – Price Decreasing cases, *Quit Learning*

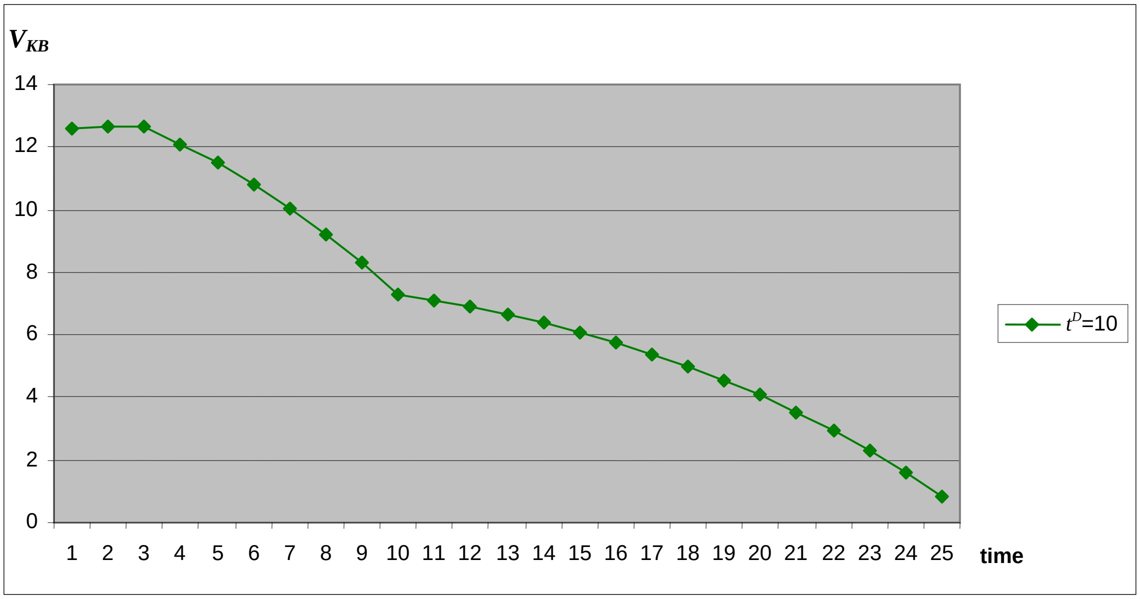
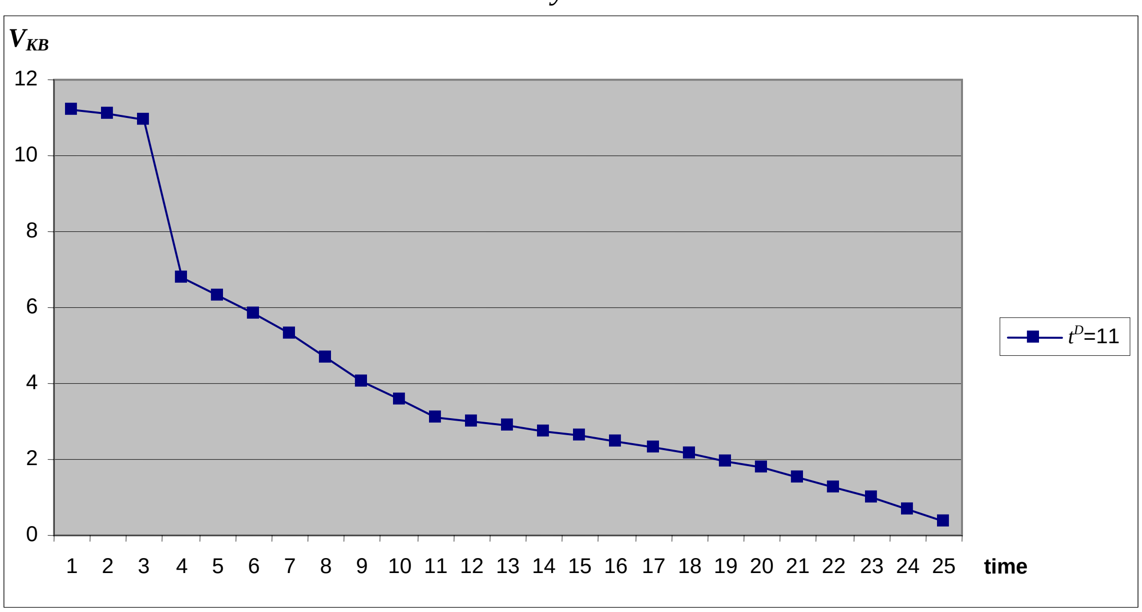


Figure 5.8 The Shadow Value of the Knowledge – Price Decreasing cases, *Always learn*

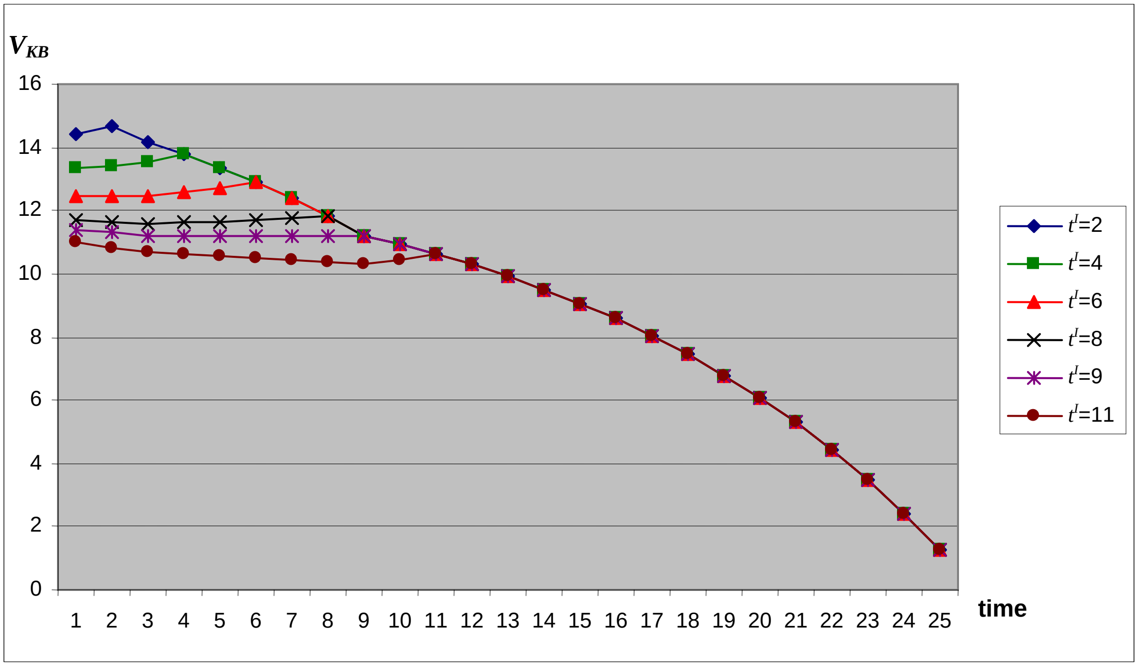


Figures 5.5 to 5.8 show the patterns of the value of the knowledge for all the price-decreasing cases. As the decision maker adopts different learning strategies, the curves are not as smooth as the value curve shown in the basic case (figure 5.4). Generally, the shadow value of the knowledge decreases at an increasing rate after the second knowledge base improvement is finished. However, the differences of the value of the knowledge among the price-decreasing cases are shown before the time the second threshold is reached. Based on the different patterns of the value of knowledge, we can conclude that the shadow value of the knowledge is influenced by both output price and the learning strategies adopted by the decision maker.

5.3.3 Price-Increasing Cases

As for the price increasing cases, the corresponding values of the knowledge are calculated in appendix D and drawn in figure 5.9. Again, the values of the knowledge are the same as the time is later than period 11 since the knowledge base, the accumulated learning effort, and the output price of these cases remain the same. In addition, since the learning strategies are the same for all the price increasing cases, the differences of the value of the knowledge only reflect the impact from the output price change. We can see that the later the output price increases, the less the shadow value of the knowledge at each period. We should also notice that the shadow value of the knowledge starts to increase a few periods earlier than the output price increases. Once the output price jumps to a higher level, the value of the knowledge starts to decrease smoothly.

Figure 5.9 The Shadow Value of Knowledge - Price Increasing Cases, *Always learn*



5.4 Concluding Comments

A deterministic dynamic programming model is specified in this chapter and provides concrete numerical results including the optimal learning and production decisions. Since the learning decision is associated with the potential learning benefits and costs, the factors influencing the learning benefits and costs are going to influence the decision maker's learning behavior. The output price is one of the factors having the impact on the learning benefits and costs. The numerical model allows the output price to change so that various learning strategies, such as *always learn*, *quit learning*, *wait to learn*, and *learn-in-bursts*, can be revealed from the numerical results. In addition, the shadow value of the knowledge is defined as the difference of the value functions if the decision maker has one more unit of the knowledge. The numerical results show that the value of the knowledge is not only associated with the changes of the output price but also the learning strategies adopted by the decision maker.

In fact, the market or technological uncertainty makes the decision maker unsure about the profitability associated with learning and therefore could result in the adjustment of the learning decision. In the next chapter, the decision maker's learning behavior is revealed from a stochastic dynamic programming model where the market and technological uncertainty are represented by the stochastic properties of the output price and knowledge base accumulation, respectively.

CHAPTER 6

NUMERICAL MODEL II:

STOCHASTIC DYNAMIC PROGRAMMING MODEL

Knowledge management is a learning behavior undertaken to acquire more information and generate additional knowledge. Whether the decision maker should devote effort to learning depends on the potential benefits and costs distributed over time associated with the learning effort. Thus, the factors influencing the learning benefits and costs affect the learning decisions as well.

Market and technological uncertainty alters the learning decision as it impacts the uncertainty of the profitability associated with learning. A stochastic dynamic programming model is constructed in this chapter where the market and technological uncertainty are represented by the stochastic properties of output price and the knowledge base evolution. The numerical results indicate that the decision maker usually faces more than one possible future state due to the market and technological uncertainty, and a different learning decision is revealed.

6.1 Market Uncertainty – The Output Price is Stochastic

6.1.1 Model Setting

Assume that the decision maker knows that the output price will change in the future but the timing of the price change is unknown. At each period in time, the decision maker knows the current output price, the possible states of the output price in the next period, and their corresponding probabilities. The conditional probabilities of the output price in the next period are:

$$\Pr\left(P_{t+1}\middle|P_t\right)=\begin{cases}0.5 & if \quad P_t = P^H \quad and \quad P_{t+1} = P^L \\ 0.5 & if \quad P_t = P^H \quad and \quad P_{t+1} = P^H \\ 1 & if \quad P_t = P^L \quad and \quad P_{t+1} = P^L \\ 0 & if \quad P_t = P^L \quad and \quad P_{t+1} = P^H \end{cases}$$

The probability assignment indicates that the output price could be at the high level (P^H) or low level (P^L). If the current price is at the high level, there is 50% probability that it drops to the low level in the next period and 50% probability that it remains at the high level. However, if the current price is at the low level, then the future output price always stays at the low level. The maximization problem at each time *t* becomes:

$$V_t\Big|_{KB_t,\sum^{t-1} e_t^L, P_t} = \max_{X_t, e_t^{LM}}\left\{\pi_t\left(KB_t, P_t, X_t, e_t^{LM}\right)+\frac{1}{1+r}E_t\left[V_{t+1}\Big|_{KB_{t+1},\sum^{t} e_t^L, P_{t+1}}\right]\right\}$$

$$=\max_{X_t, e_t^{LM}}\left\{\begin{aligned}&\pi_t\left(KB_t, P_t, X_t, e_t^{LM}\right)\\ &+\frac{1}{1+r}\left[V_{t+1}\Big|_{KB_{t+1},\sum^{t} e_t^L, P_{t+1}^H}\cdot\Pr\left(P_{t+1}^H\middle|P_t\right)+V_{t+1}\Big|_{KB_{t+1},\sum^{t} e_t^L, P_{t+1}^L}\cdot\Pr\left(P_{t+1}^L\middle|P_t\right)\right]\end{aligned}\right\}$$

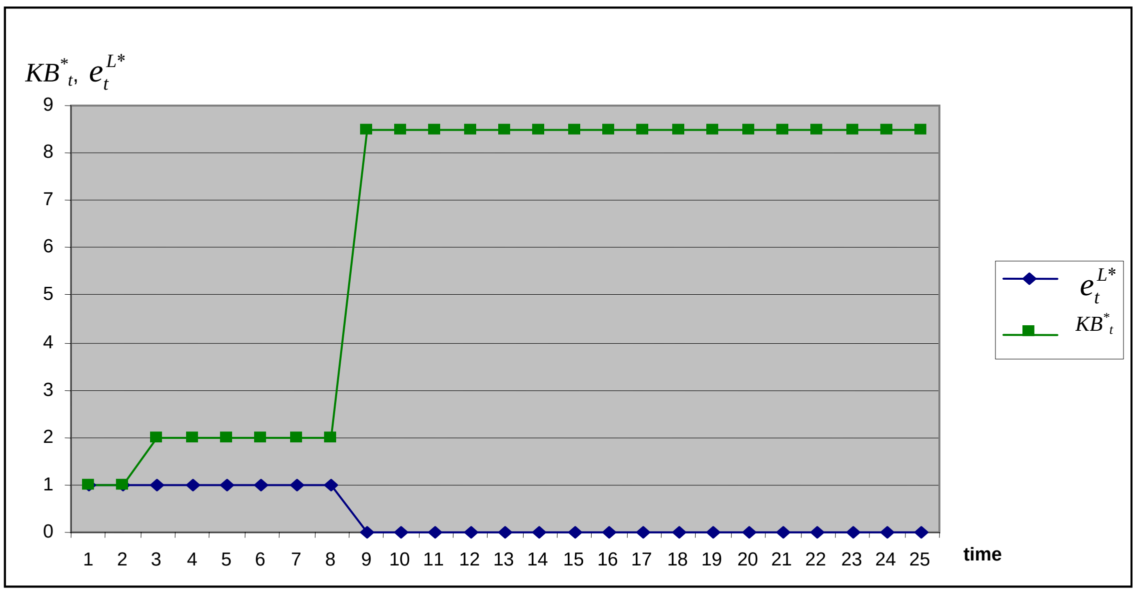
The decision maker undertakes the maximization decision based on the current knowledge base, the accumulated learning effort up to date, and the current revealed output price. Assume that the parameter settings and the knowledge base transition are the same as Chapter 5. In addition, the high level price is set to be \$5 and the low level price is \$3. The numerical results of this stochastic dynamic programming illustrating the learning decisions under the market uncertainty are presented in the next section.

6.1.2 The Optimal Learning Decisions

(a) The Initial Output Price is Low ($P_1 = 3$)

If the initial output price is at the low level, then the model is deterministic since the decision maker knows that the price will always remain at the low level. The optimal paths for the learning effort and the knowledge base are plotted in figure 6.1. It shows that the decision maker chooses an *always learn* strategy, which means he uses the learning effort from the beginning until the second threshold is reached.

Figure 6.1 The Paths of Learning Effort and Knowledge Base:
The Initial Output Price is Low



(b) The Initial Output Price is High ($P_1 = 5$)

If the initial output price is at the high level, then the decision maker faces the price uncertainty as long as the output price remains high. Let t' denote the time the output price drops from \$5 to \$3, $t' \geq 2$[4]. This model is different from the deterministic one

[4] t'=1 represents the case in section 6.1.2 (a), where the output price always stays at the low level.

from two aspects. First, the decision maker does not know when the output price will drop, i.e., t' is unknown. The time t' may not even exist if the output price always stays at the high level for the entire 25 periods. Second, in the deterministic model, the optimal learning decision for every period can be obtained from the beginning of the time horizon as long as the initial state is given. In the stochastic model, the decision maker usually faces several possible states[5] in the future, and for each state, there is one corresponding learning decision. However, the actual learning decision will not be made until the state is revealed. In this case, there are several possible learning strategies instead of one "unique" optimal learning path.

The numerical results are presented in figure 6.2. The big circle on the top of the graph is the given initial status reflecting that the initial output price is high ($P_1 = 5$), the firm only has the minimum (or basic) understanding about the current technology at the beginning ($KB_1 = 1$), and the decision maker never uses learning effort before ($Esum_1$=0)[6]. Other smaller circles in this graph present the possible states at each time *t*, and we can see that the decision maker faces more than one possible state most of the time. The box attached to each circle denotes the corresponding optimal learning decision, e_t^{*L}, under each state. Thus we know that the decision maker will not make decisions until the state is realized, and he might make different decisions when facing different circumstances.

Figure 6.2 describes that the decision maker chooses to allocate the learning effort at time $t = 1$ and $t = 2$ no matter the output price is at the high level or low level. The use of the

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[5] The "state" is determined by the revealed output price, the realized knowledge base and the accumulated learning effort.

[6] The accumulated learning effort prior to time *t* is indicated as "$Esum_t$".

learning effort at the first two periods implies the completeness of the first knowledge improvement ($KB_t = 1$ to $KB_t = 2$) and rules out the *wait to learn* strategy. However, the decision to apply the learning effort after $t = 3$ depends on the revealed output price. Take period 3 as an example. If the output price remains at the high level at period 3, the decision maker will choose not to allocate the learning effort ($e_3^{*L} = 0$ if $P_3 = 5$). The learning process could be temporarily shut down or permanently shut down depending on the timing of the output price decrease. If the output price drops before period 10, the decision maker starts to employ the learning effort again when the output price drops, and the learning effort will be employed until the second knowledge improvement is completed which makes the knowledge base increase from 2 to 8.5. If the output price drops at or after period 10, the decision maker will never allocate the learning effort again for the rest of the planning horizon.

The learning decisions illustrated in figure 6.2 are plotted in figures 6.3 to 6.10 for different $t's$, and the possible learning strategies are summarized as following.

1) If the output price drops at or before period 3, an *always learn* strategy is observed (figure 6.3). If, at the beginning of time horizon, the decision maker knows that the output price will drop from \$5 to \$3 at period 3, he chooses a *wait to learn* strategy (a deterministic case). However, in the stochastic case, the decision maker chooses to allocate the learning effort at period 1 and 2 under the circumstances that the output price is at the high level and he does not know whether the output price will drop. As the output price drops at period 3, the decision maker, however, cannot reverse his decision in the previous periods and

chooses the *wait to learn* strategy. The best choice he can make is to keep using the learning effort and finish the entire learning process.

2) If the output price drops between period 4 and period 9, then a *learn-in-bursts* strategy is observed as observed in deterministic cases. The decision maker shuts down the learning process temporarily. The later the output price drops, the longer the learning process stops (figures 6.4 to 6.9).

3) Finally, if the output price still remains at the high level at period 9, a *quit learning* strategy is observed. According to the deterministic model, the decision maker chooses an *always learn* strategy if he knows that the output prices will drop at $t \geq 10$. However, in the stochastic case, the learning decisions are made under the circumstances where the decision maker only knows about the current output price and the conditional probabilities of the output price in the next period. When the decrease of the output price is revealed at (or after) period 10, the decision maker cannot alter his previous learning decisions, and the *always learn* strategy is no longer an option. In addition, since restarting the learning process is not more beneficial, the decision maker decides to *quit learning* for the rest of the periods (figure 6.10).

Figure 6.2 The Learning Decisions under Market Uncertainty: The Output Price is Stochastic

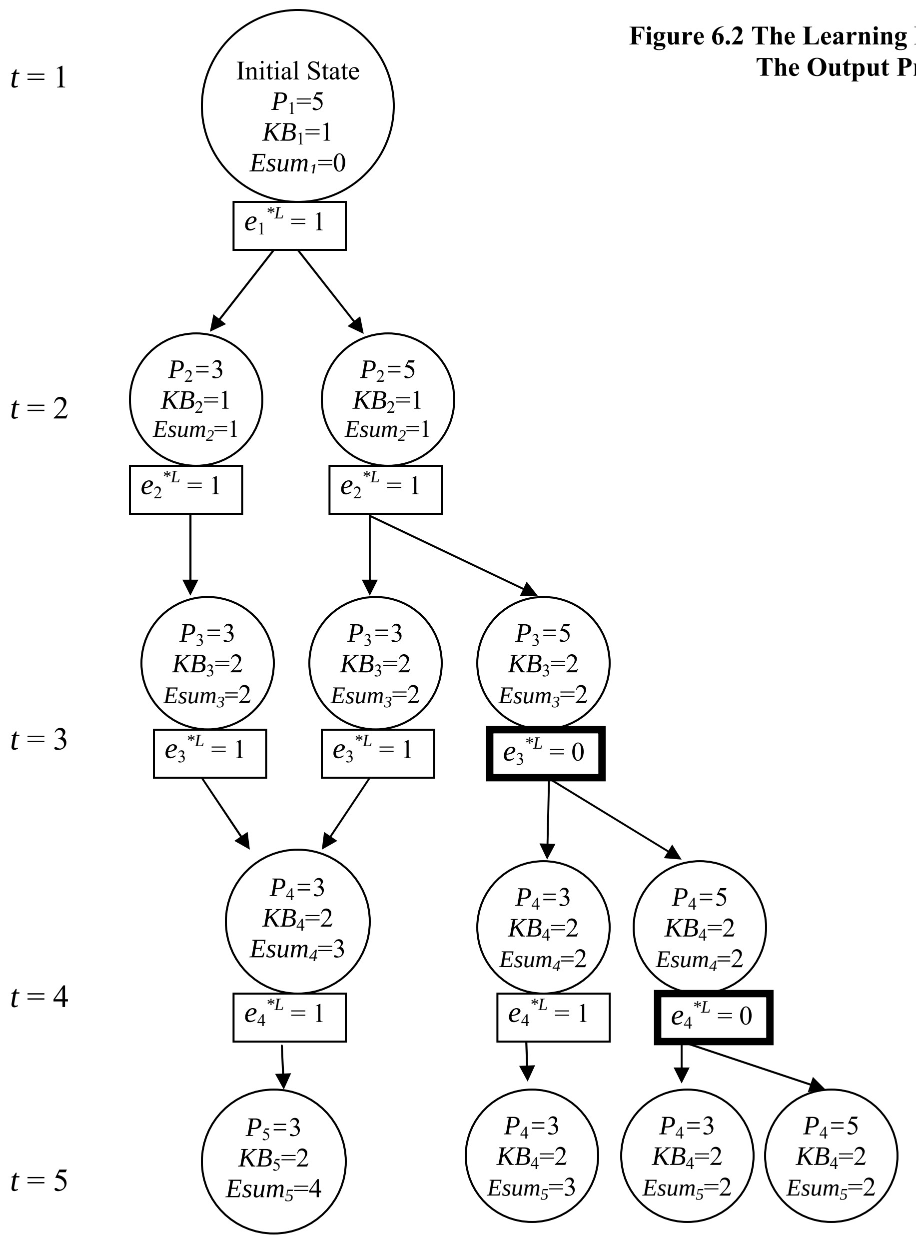


Figure 6.2 Continued

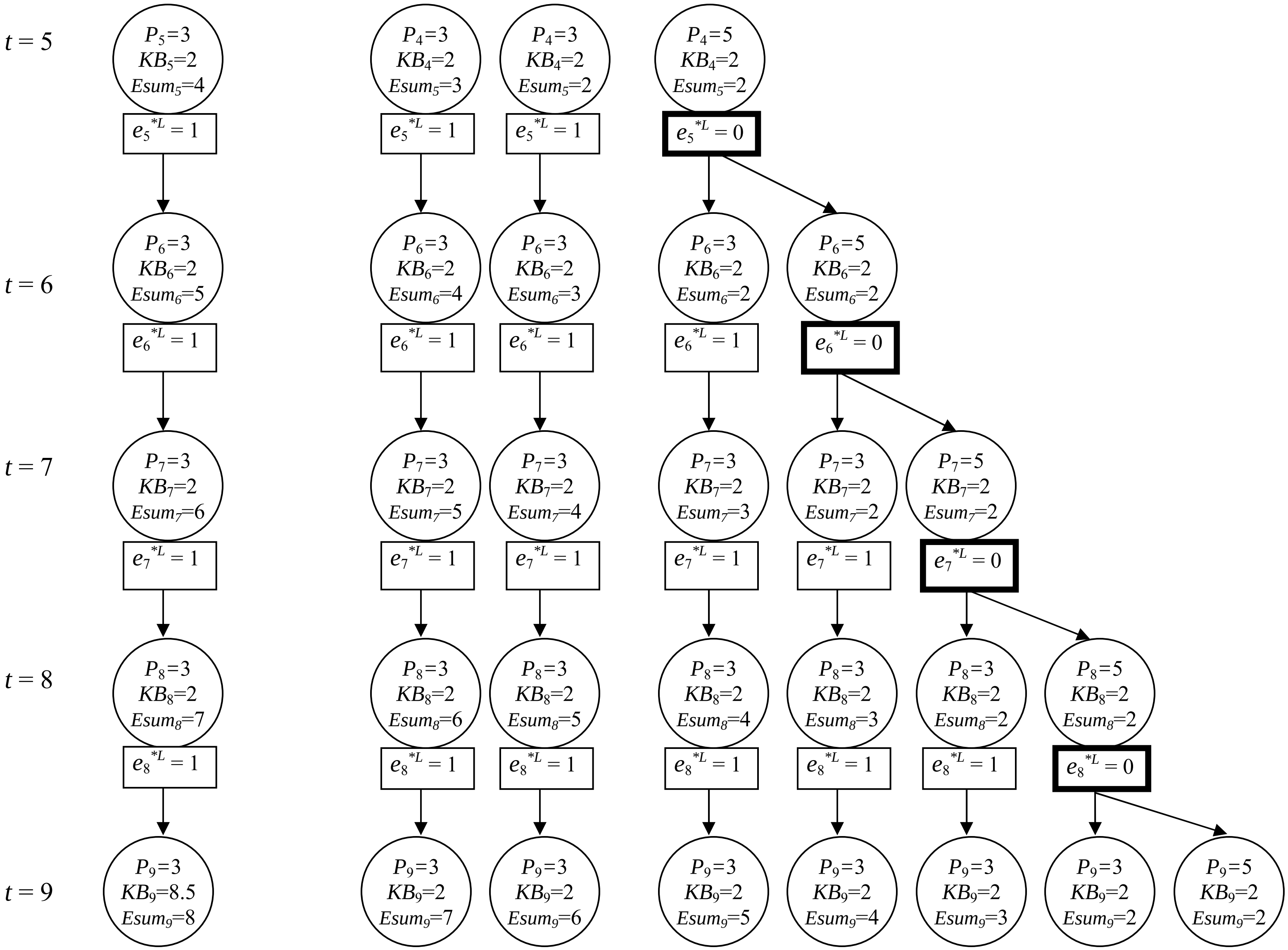


Figure 6.2 Continued

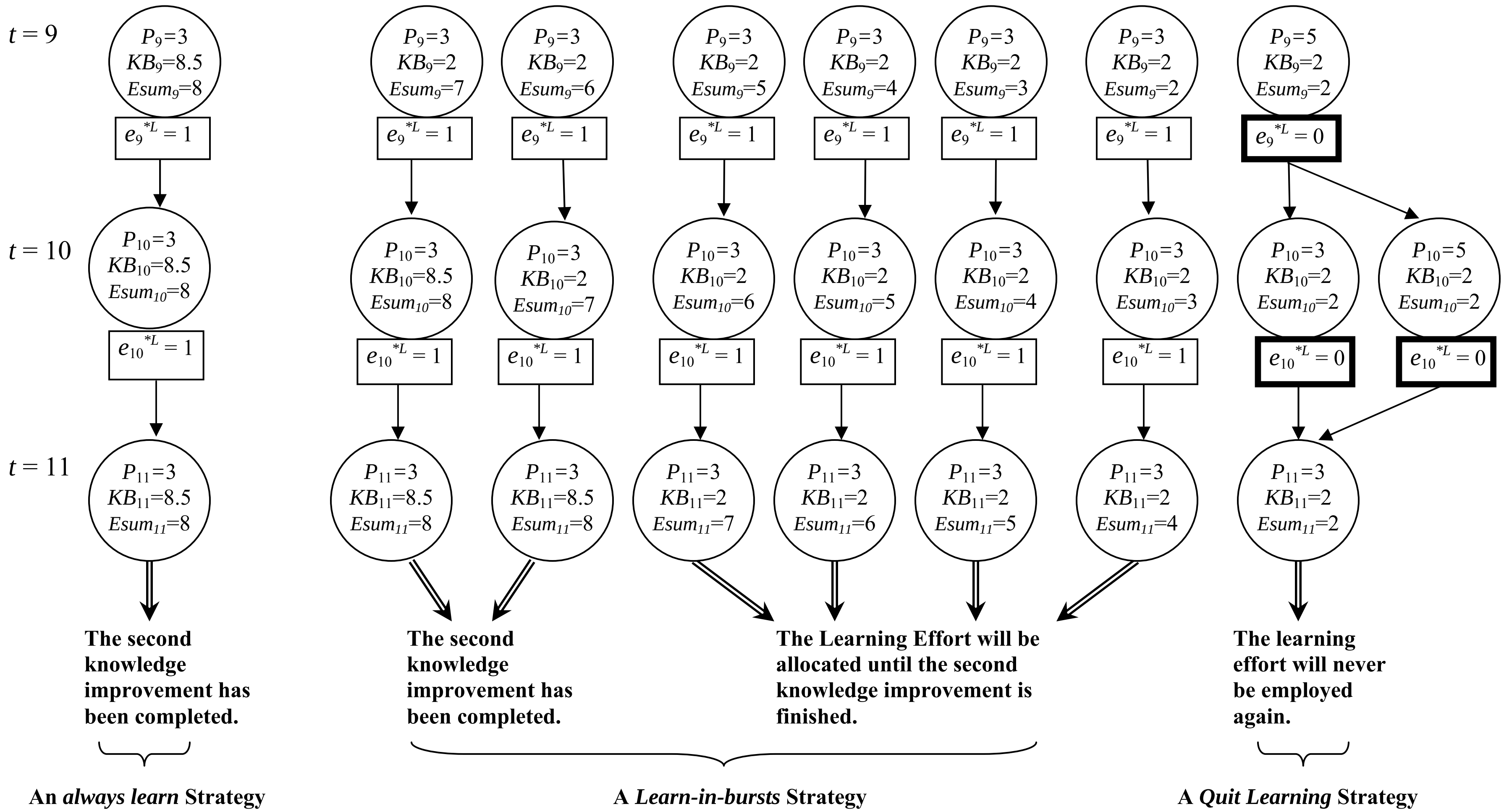


Figure 6.3 An *Always Learn* Strategy (The Output Price Drops at or before Period 3)

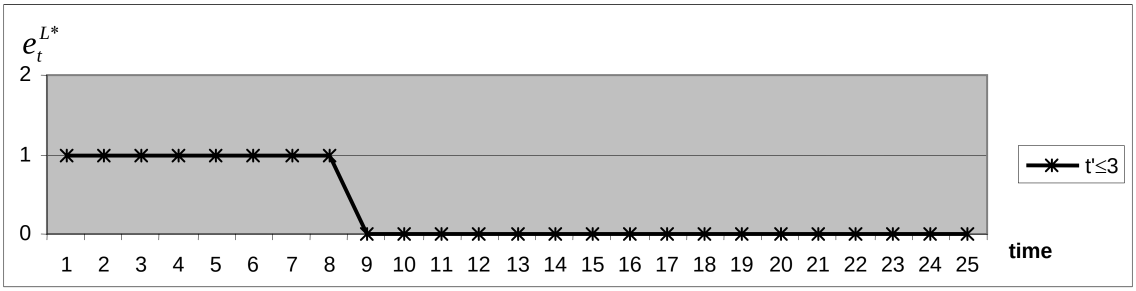


Figure 6.4 A *Learn-in-bursts* Strategy (The Output Price Drops at Period 4)

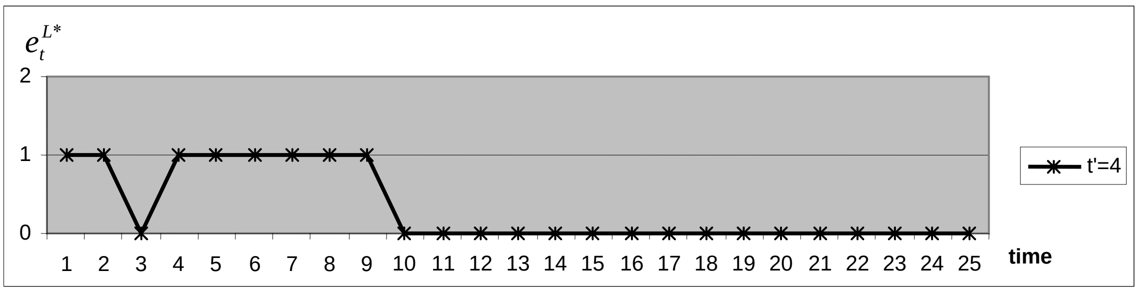


Figure 6.5 A *Learn-in-bursts* Strategy (The Output Price Drops at Period 5)

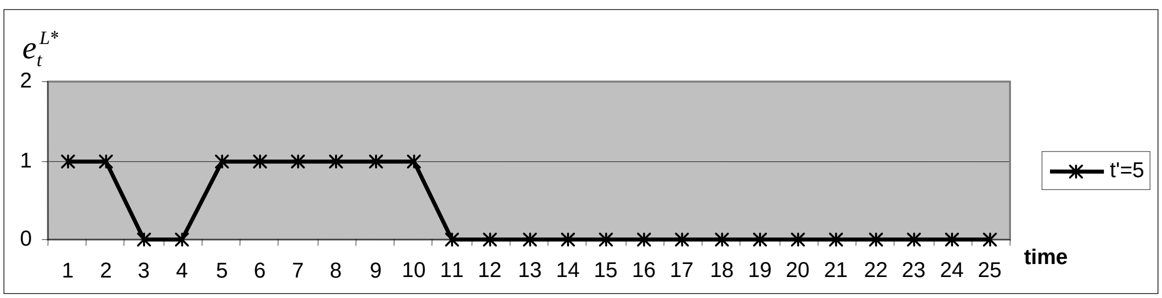


Figure 6.6 A *Learn-in-bursts* Strategy (The Output Price Drops at Period 6)

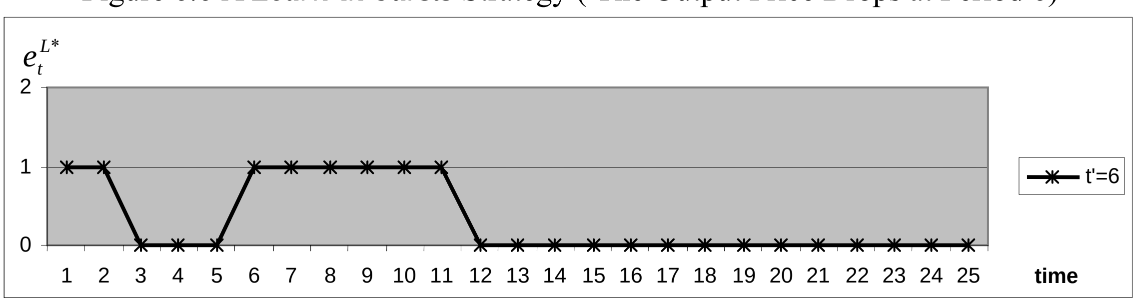


Figure 6.7 A *Learn-in-bursts* Strategy (The Output Price Drops at Period 7)

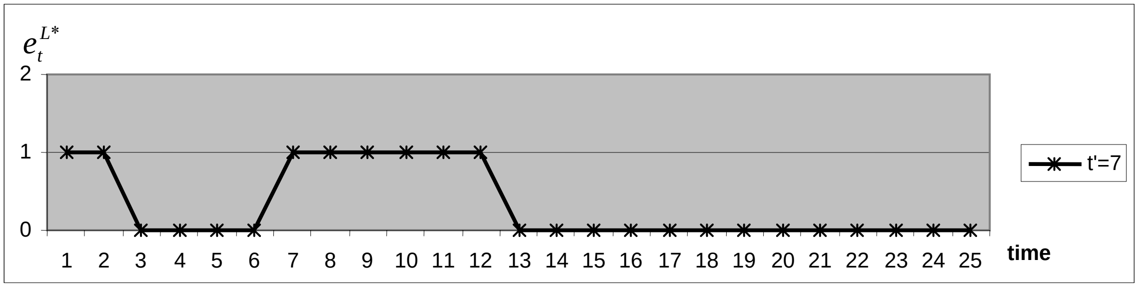


Figure 6.8 A *Learn-in-bursts* Strategy (The Output Price Drops at Period 8)

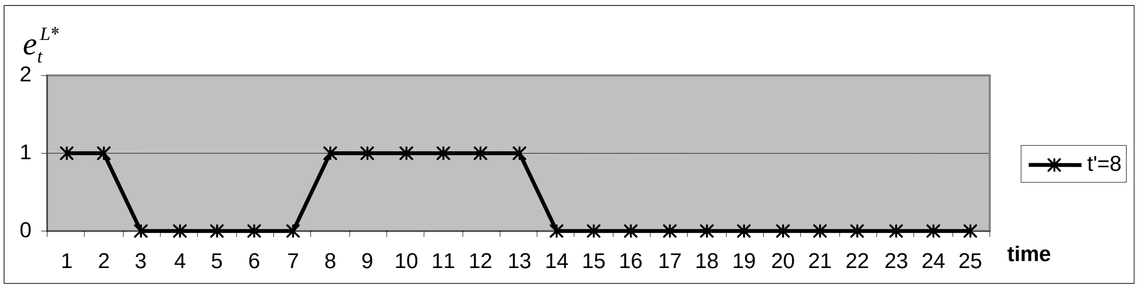


Figure 6.9 A *Learn-in-bursts* Strategy (The Output Price Drops at Period 9)

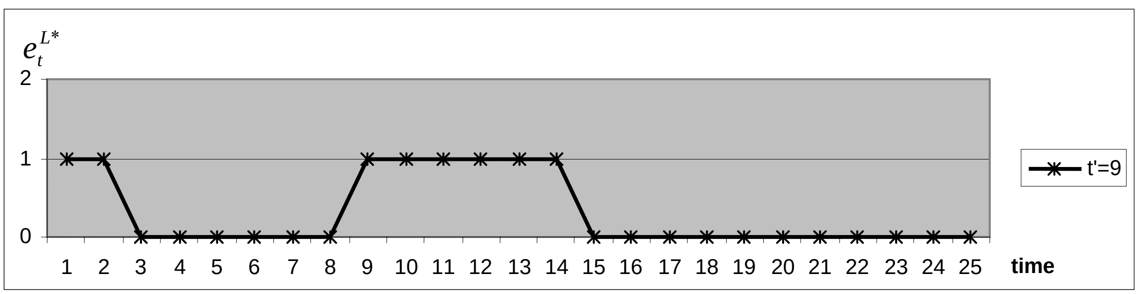
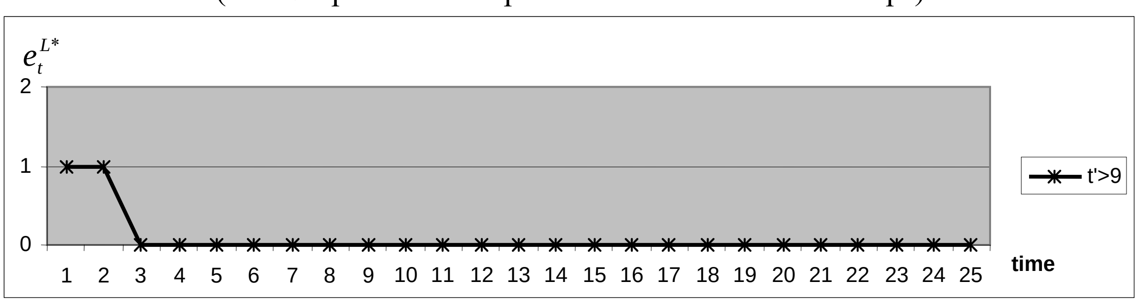


Figure 6.10 A *Quit Learning* Strategy
(The Output Price Drops after Period 9 or never Drops)



6.1.3 The Shadow Value of the Knowledge

Let $\left(V_\tau \Big|_{KB_\tau,\ \sum_{t=1}^{\tau-1} e_t^L,\ P_t} \right)$ indicate the value function at time τ, which is obtained as the decision maker makes the production and learning decisions to maximize the sum of the expected net present value of the firm's profit from time τ to time T under the conditions that 1) the current knowledge base (KB_τ), the previous accumulated learning effort ($\sum_{t=1}^{\tau-1} e_t^L$), and the current output price (P_τ) are given, and 2) the conditional probabilities of the future output price [$\Pr(P_{t+1} | P_t),\ \ t = \tau ... T$] are known. Thus, the shadow value of the knowledge is defined as the increase of the value function if the decision maker has one more unit of knowledge.

$$V_{KB_\tau} = V_\tau \Big|_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L \right)^*,\ P_\tau} - V_\tau \Big|_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L \right)^*,\ P_\tau}$$

The timing of the decrease of the output price is denoted as t', and t' is unknown until the price change is revealed. As long as the output price remains at the high level, the decision maker faces the price uncertainty resulting in several possible future states. In addition, the decision maker makes different learning decisions as different states are revealed. Therefore, instead of having one "unique" optimal learning strategy, the decision maker has several possible learning strategies, depending on when t' is revealed. Similarly, for different revealed states, there are different corresponding shadow values of the knowledge. The shadow values of knowledge corresponding to the learning decisions in figure 6.2 are presented in appendix E.

The shadow values of the knowledge corresponding to the same learning strategy are plotted in the same figure. Figure 6.11 presents the shadow value of knowledge of the cases where the output price drops at or before period 3, and an *always learn* strategy is adopted. The shadow value decreases rapidly for first two periods and then it decreases gradually after the price decrease is revealed.

Figure 6.12 presents the shadow values of knowledge as the price decreasing is revealed between period 4 and period 9, and the *learn-in-bursts* strategy is adopted. The shadow value of knowledge decreases at period 2, and then it keeps increasing until the price decrease occurs. As the output price drops, the shadow value of knowledge drops as well. The later the output price decreases, the later the shadow value drops. In addition, all the cases in figure 6.12 have the same shadow value of knowledge starting from period 14 since the decision maker faces the same state (including the same knowledge base, accumulated learning efforts, and the same output price) after period 14 in these cases.

Finally, the shadow values of knowledge corresponding to the *quit learning* strategy are plotted in figure 6.13. The trajectories in figure 6.13 present the cases where the output price drops after period 9. It is shown that the shadow value of knowledge drops from period 1 to period 2, and then it increases until period 9. After period 9, the shadow value of the knowledge has two possible trajectories. Both patterns indicate that the shadow value of the knowledge decreases at an increasing rate. As long as the output price remains at the high level, the shadow value of the knowledge follows the higher trajectory. Once the output price drops to the low level, the shadow value of knowledge drops to the lower trajectory.

Figure 6.11 Shadow Value of Knowledge under Market Uncertainty – *Always learn*

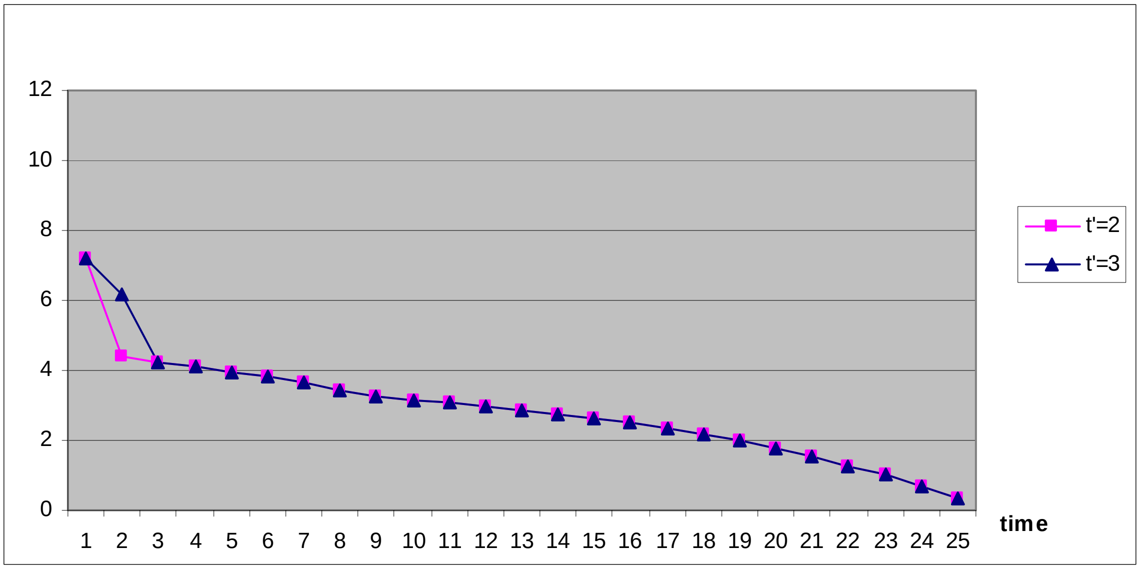


Figure 6.12 Shadow Value of Knowledge under Market Uncertainty – *Learn-in-bursts*

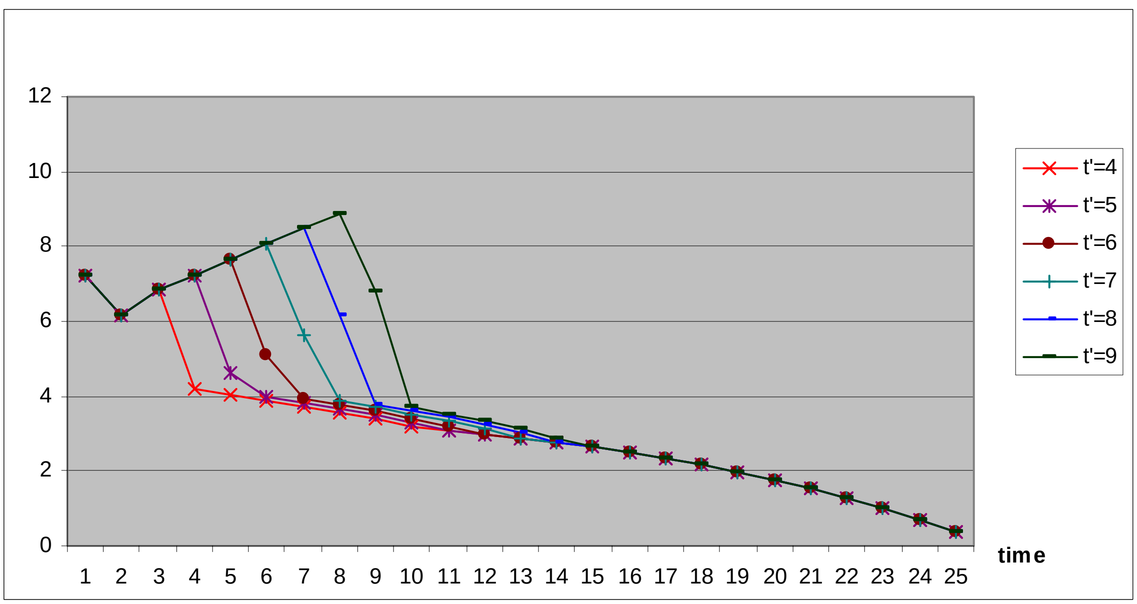
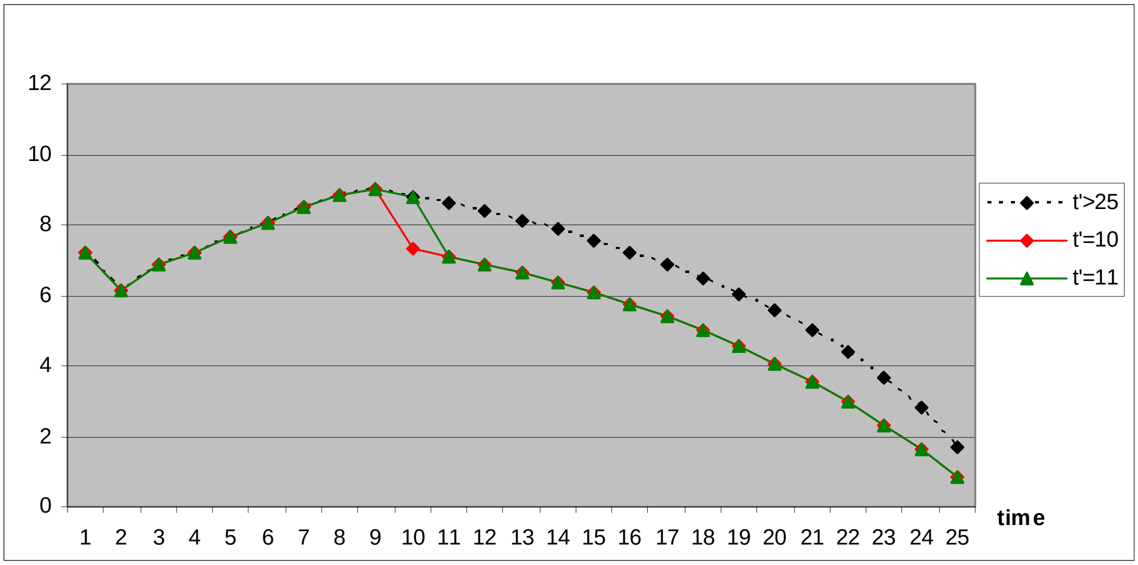


Figure 6.13 Shadow Value of Knowledge under Market Uncertainty – *Quit Learning* [7]



6.2 The Technological Uncertainty – Knowledge Base is Stochastic

6.2.1 Model Setting

The production performance of the firm is influenced by the knowledge base which reflects how well the decision maker understands the current technology. In the deterministic model, the decision maker knows that a certain amount of the knowledge base can be generated once the accumulated learning effort hits the thresholds; otherwise, the knowledge base remains at the same level. However, the knowledge base accumulation may be determined by the other factors which are not controllable. For example, the decision maker may face an erosion or depreciation of the knowledge base arising from mistakes during the knowledge transfer or the introduction of the new production technology. The uncertainty of knowledge accumulation makes the decision maker unsure about how well he understands the production technology, and thus, alters his learning and production decisions.

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[7] "t′ > 25" indicates the case where the output price stays at the high level for the entire 25 periods.

In this section, the decision maker is assumed to face the possibility of knowledge depreciation, resulting in the uncertainty of knowledge base accumulation. Thus, the technological uncertainty is represented by the stochastic property of the knowledge base accumulation and a stochastic dynamic programming model is constructed as following:

$$V_t\Big|_{KB_t,\sum e_t^L} = \underset{X_t,e_t^L}{Max}\left\{\pi_t(KB_t,X_t,e_t^L) + \left(\frac{1}{1+r}\right)\cdot E\left[V_{t+1}\Big|KB_{t+1},\sum e_{t+1}^L\right]\right\}$$

s.t. the production and knowledge management constraints listed in table 6.1.

Table 6.1 Production and Knowledge Management Constraints under Technological Uncertainty

Production and Knowledge Management Constraints	Descriptions
$Y_t\left(X_t,KB_t,e_t^M\right)=h(KB_t)\cdot\left(X_t^{0.3}\cdot\left(e_t^M\right)^{0.5}\right)$ $=h(KB_t)\cdot\left(X_t^{0.3}\cdot\left(\bar{e}_t-e_t^L\right)^{0.5}\right)$	Production function
$h(KB_t)=1+1.7\cdot\{1-\exp[-0.15(KB_t-1)]\}$	Knowledge function
$KB_{t=1}=KB_{\min}=1$	Initial state of knowledge base.
$KB_{t+1}=(1-\delta)\cdot KB_t+\Lambda_t$ $\delta=0 \;\; with \;\; prob1$; $\delta>0 \;\; with \;\; prob2=1-prob1$	Two possible transitions for knowledge base accumulation.
$E_t^L=\sum_{\tau=1}^{\tau=t}e_\tau^L$	Accumulated learning effort at the end of time *t*.
$\Lambda_t=g_1\left(E_t^L\right)=\left(E_t^L\right)^{0.1} \;\; if \;\; \sum_{\tau=1}^{\tau=t-1}e_\tau^L<em1 \;\; and \;\; \sum_{\tau=1}^{\tau=t}e_\tau^L=em1$ $\Lambda_t=g_2\left(E_t^L\right)=\left(E_t^L\right)^{0.9} \;\; if \;\; \sum_{\tau=1}^{\tau=t-1}e_\tau^L<em2 \;\; and \;\; \sum_{\tau=1}^{\tau=t}e_\tau^L=em2$ $otherwise \;\; \Lambda_s^i=0$	Knowledge generation as the accumulated learning effort hit the threshold(s).
$W_t=2,\;\; \alpha_4=2,\;\; r=0.1,\;\; X_t\geq 0,\;\; \bar{e}_t=2,$ $e_t^L=0 \;\; or \;\; 1,\;\; em_1=2,\;\; em_2=8,\;\; T=25,\;\; P_t=5$	Given parameters.

The knowledge base accumulation is associated with two factors, the knowledge base depreciation and the additional knowledge generation (Λ_t). The knowledge generation is related to the accumulated learning effort (E_t^L), and the additional knowledge can only be generated when the accumulated learning effort hits the assigned threshold(s). However, at each time *t*, the decision maker faces *prob1* that the knowledge base is accumulated without depreciation and *prob2* that the knowledge base is accumulated with depreciation rate, δ . The uncertainty of the knowledge base accumulation results in more than one possible future state. In addition, different decisions will be made when different states are revealed.

6.2.2 The Learning Decisions

Set δ =0.3, *prob*1=0.63, and *prob*2=0.37, the optimal learning behavior of each possible state at each time can be obtained. Figure 6.14 presents how the decision maker makes the learning decision while the knowledge base is stochastic. The initial states shown in the big circle on the top of the graph indicates that the decision maker has a basic understanding of the executed technology and there is no accumulated learning effort at the beginning of the time horizon. The optimal learning decision for each possible state is shown in figure 6.14 as well.

According to figure 6.14, we can see that the decision maker always employs the learning effort in the first two periods. Thus the firm's accumulated learning effort hits the first threshold and the knowledge base increases marginally at the end of period 2. The knowledge depreciation is not shown in the knowledge transition from $t = 1$ to $t = 2$ because the knowledge base cannot be less than its minimum level ($KB_{\min} = 1$). The knowledge depreciation starts from $t = 2$ to $t = 3$ and influences the learning decision. If the knowledge base accumulates without depreciation (KB_3=2.1), the decision maker stops employing the learning effort temporarily at period 3 and restarts to learn again from period 4. If the knowledge base accumulates with depreciation (KB_3=1.8), then the decision maker keeps employing the learning effort until the second threshold is reached. Therefore, there are two possible learning strategies presented in this graph: an *always learn* strategy and a *learn-in-bursts* strategy. If an *always learn* strategy is observed, the second threshold will be reached at the end of period 8 which makes a significant improvement in the knowledge base. If the *learn-in-bursts* strategy is observed, the second threshold will be reached at the end of period 9, and the second improvement of the knowledge base is realized at the beginning of period 10. Since there are many possible states after period 10, a bigger ellipse is used to present the set of possible states. We can see that no additional learning effort is employed after the second threshold is reached. Thus the knowledge base after period 10 either remains at the previous level or decreases a little because of the knowledge depreciation. Again, both *always learn* and *learn-in-bursts* strategies are possible learning strategies depending on what state the decision maker faces.

Figure 6.14 The Learning Decisions under Technological Uncertainty: The Knowledge Base is Stochastic

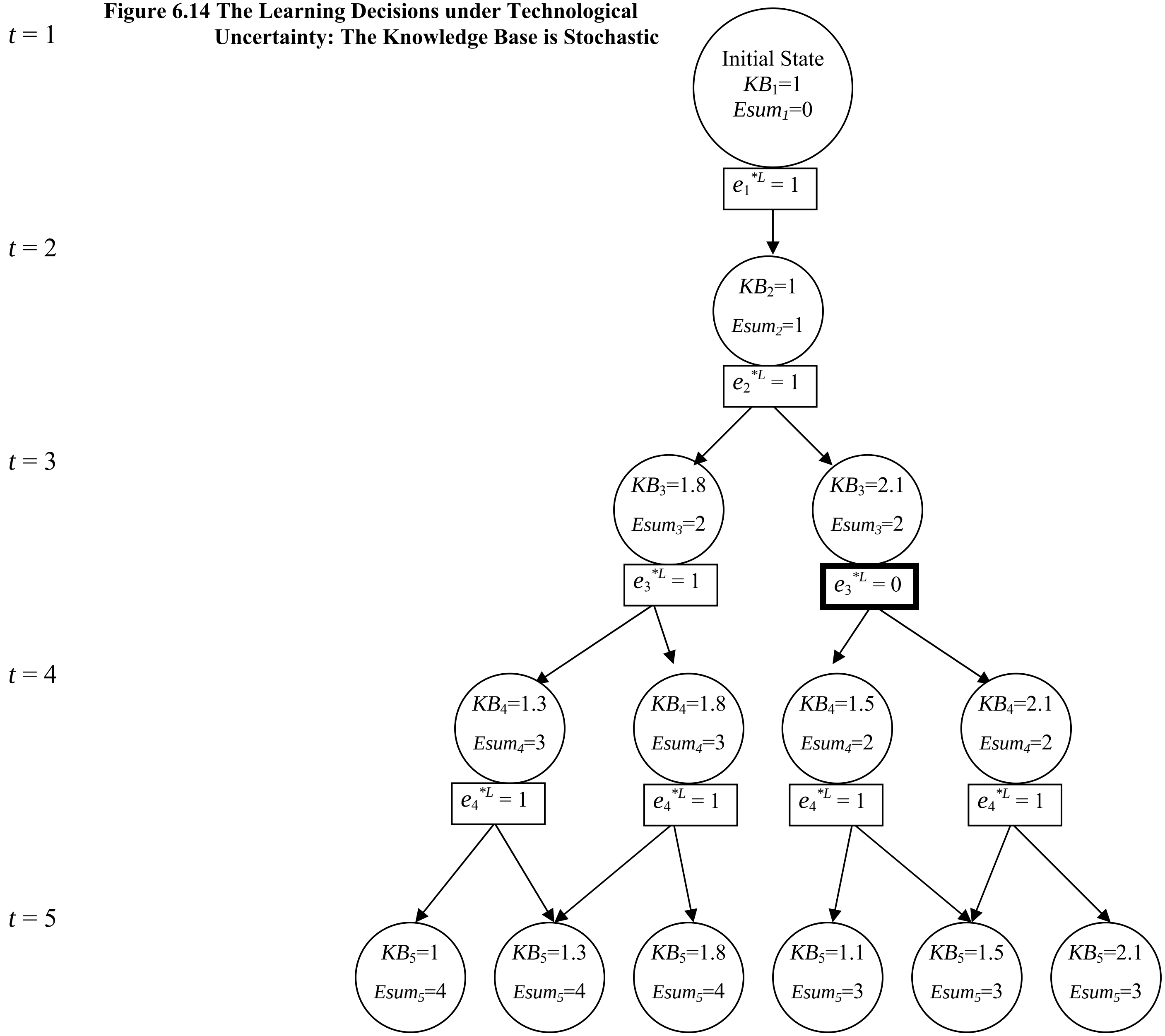


Figure 6.14 Continued

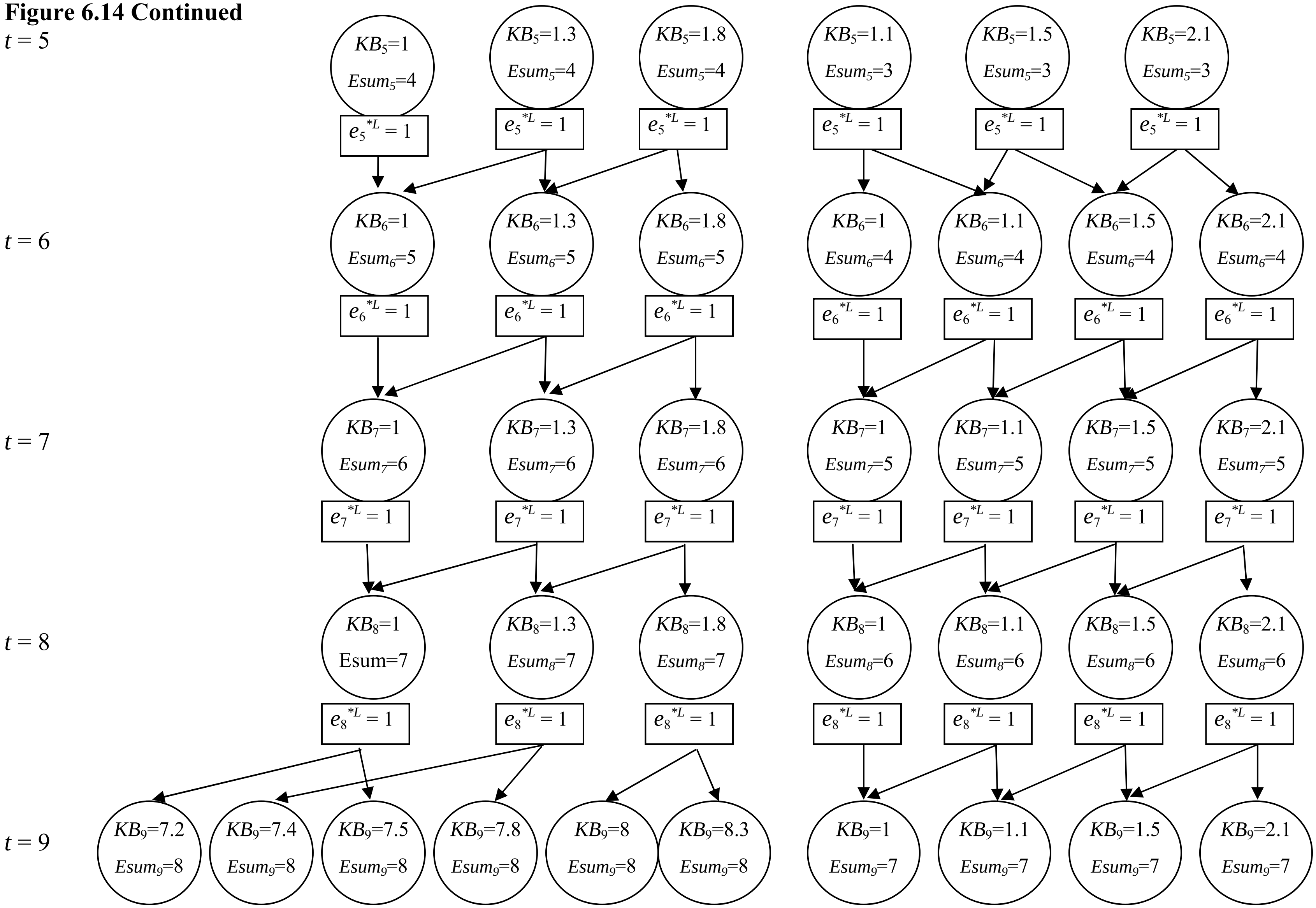
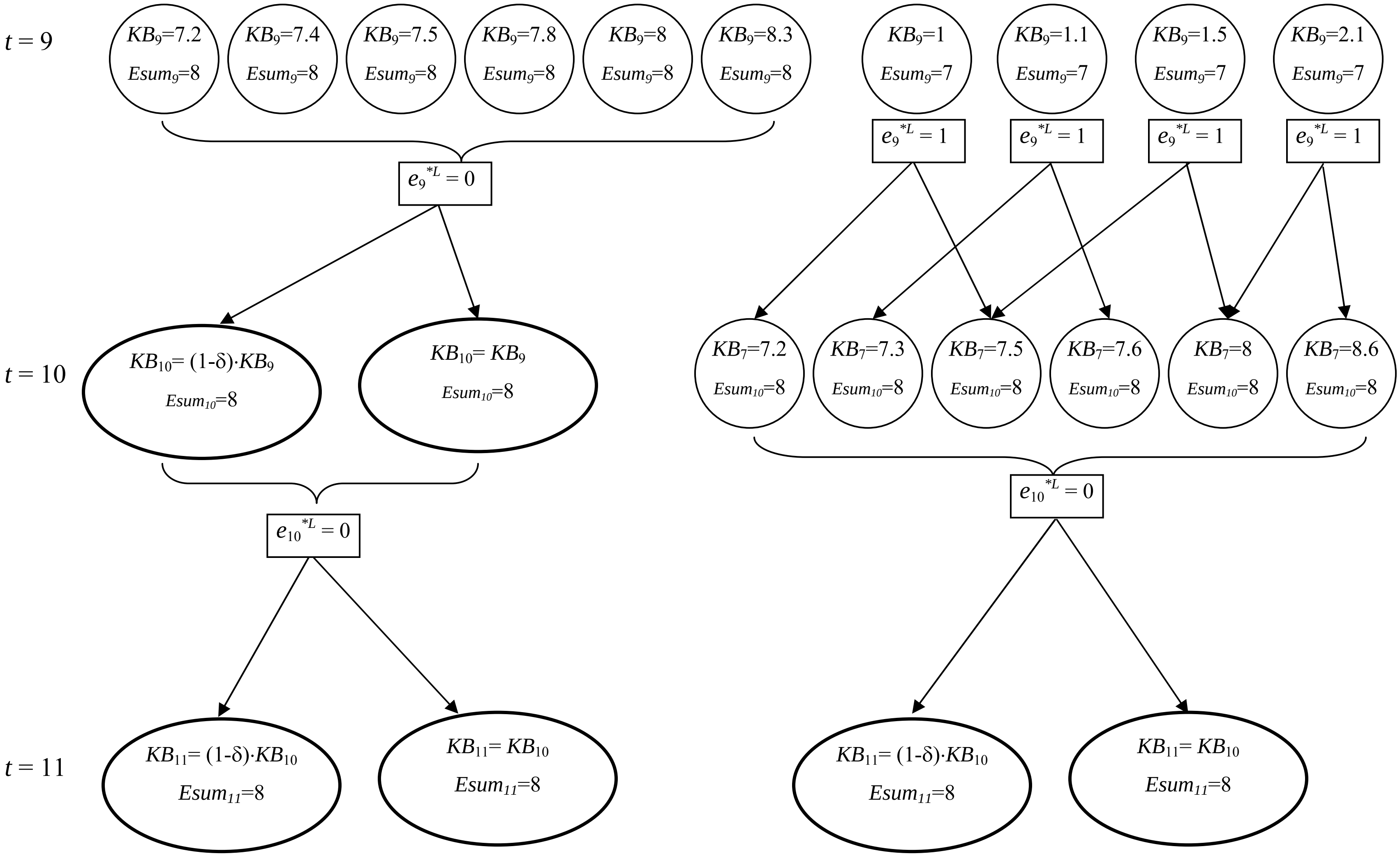


Figure 6.14 Continued



6.2.3 The Relationship between the Depreciation Rate (δ), the Probability of Knowledge Depreciation ($Prob2$), and the Learning Strategies

The learning decisions shown in figure 6.14 are obtained under a given set of parameters. In fact, alternative results are obtained when different depreciation rates (δ) and probabilities of knowledge depreciation ($Prob2$) are assigned in the model. It is reasonable to expect that the learning decisions may not be so different from those in the deterministic model in chapter 5 if the depreciation rate and the probability of the depreciation are very low. The relationships between the learning strategy, the depreciation rate, and the probability of the knowledge depreciation are shown in the table 6.2. When the depreciation rate is less than 0.1, the *always learn* strategy will be observed even if the decision maker faces an extremely high probability of knowledge depreciation. This arises because that the depreciation rate is so low that it does not affect much of the knowledge accumulation even if the knowledge depreciation occurs. As the depreciation rate increases beyond 0.1, the probability of knowledge depreciation is relatively more important to decision making. Two critical values, $Prob2^{*}$ and $Prob2^{**}$, can be assigned to the probability of knowledge depreciation to describe the relationship between the learning strategies and the probability of knowledge depreciation under a given depreciation rate.[8]

Under a given depreciation rate, δ, if the probability of knowledge depreciation is greater than the upper critical value, $Prob2^{*}$, the decision maker chooses a *never learn* strategy; if the probability of knowledge depreciation is less than or equal to the lower critical value, $Prob2^{**}$, then an *always learn* strategy is adopted. If the probability of knowledge

[8] The two critical values are assigned given the depreciation rate is greater than 0.2. For the case where the depreciation rate is less than or equal to 0.2, the two critical values are the same, i.e., $Prob2^{**} = Prob2^{*}$.

depreciation is between $Prob2^{*}$ and $Prob2^{**}$, there are several possible learning strategies. The actual learning strategy adopted by the decision maker depends on the revealed states. For example, with the depreciation rate equal to 0.3, the *never learn* strategy is observed when the probability of knowledge depreciation is greater than 0.37, and the *always learn* strategy is observed if the probability of knowledge depreciation is less than or equal to 0.36. If the probability of knowledge depreciation is between 0.36 and 0.37, both *always learn* and *learn-in-bursts* are possible learning strategies. The actual learning strategy depends on the state revealed by the decision maker at each period.[9] Thus, in the case where the depreciation rate is 0.3, the upper critical value, $Prob2^{*}$, is 0.37 while the lower critical value, $Prob2^{**}$, is 0.36.

Table 6.2: The Depreciation Rate, the Probability of knowledge Depreciation, and the Corresponding Learning Strategy

Depreciation Rate (δ)	Probability of Knowledge Depreciation (*Prob*2)	Learning Strategy
0.1	$Prob2 \leq 1$	*Always Learn*
0.2	$Prob2 \leq 0.56$	*Always Learn*
	$Prob2 > 0.56$	*Never Learn*
0.3	$Prob2 \leq 0.36$	*Always Learn*
	$0.36 < Prob2 \leq 0.37$	*Always Learn* *Learn-in-Bursts*
	$Prob2 > 0.37$	*Never Learn*
0.4	$Prob2 \leq 0.27$	*Always Learn*
	$0.27 < Prob2 \leq 0.28$	*Always Learn* *Learn-in-Bursts*
	$Prob2 > 0.28$	*Never Learn*

[9] The result presented in figure 6.14 is one example where the probability of knowledge depreciation is setting between critical values.

Table 6.2 Continued

Depreciation Rate (δ)	Probability of Knowledge Depreciation ($Prob2$)	Learning Strategy
0.5	$Prob2 \leq 21$	*Always Learn*
	$0.21 < Prob2 \leq 0.23$	*Always Learn* *Learn-in-Bursts*
	$Prob2 > 0.23$	*Never Learn*
0.6	$Prob2 \leq 0.17$	*Always Learn*
	$0.17 < Prob2 \leq 0.19$	*Always Learn* *Learn-in-Bursts* *Quit Learning*
	$Prob2 > 0.19$	*Never Learn*
0.7	$Prob2 \leq 0.14$	*Always Learn*
	$0.14 < Prob2 \leq 0.17$	*Always Learn* *Learn-in-Bursts* *Quit Learning*
	$Prob2 > 0.17$	*Never Learn*
0.8	$Prob2 \leq 0.12$	*Always Learn*
	$0.12 < Prob2 \leq 0.15$	*Always Learn* *Learn-in-Bursts* *Quit Learning*
	$Prob2 > 0.15$	*Never Learn*
0.9	$Prob2 \leq 0.10$	*Always Learn*
	$0.10 < Prob2 \leq 0.14$	*Always Learn* *Learn-in-Bursts* *Quit Learning*
	$Prob2 > 0.14$	*Never Learn*

Plotting the depreciation rates and the corresponding critical values for the probability of the knowledge depreciation in the same graph is a useful way to illustrate their relationship with learning strategies. As we can see in figure 6.15, the critical values for the probability of the knowledge depreciation are very sensitive to the depreciation rate as the depreciation rate ranges 0.1 and 0.4. When the depreciation rate is greater than 0.4, the changes in the depreciation rate no longer causes much of the changes in the critical values, $Prob2^{*}$ and $Prob2^{**}$. In addition, the curves plotted by the critical values divide the graph into three regions. Region (I) is the *always learn* region where the depreciation rate or the probability of knowledge depreciation is so low that the decision maker always chooses to learn. Region (III) is the *never learn* region where the depreciation rate or the probability of depreciation is so high than the decision maker never allocates effort to learning. Region (II) is the region between regions (I) and (III) where several possible learning strategies can be observed[10], and the actual learning strategy adopted by the decision maker in region (II) depends on how the knowledge base evolves.

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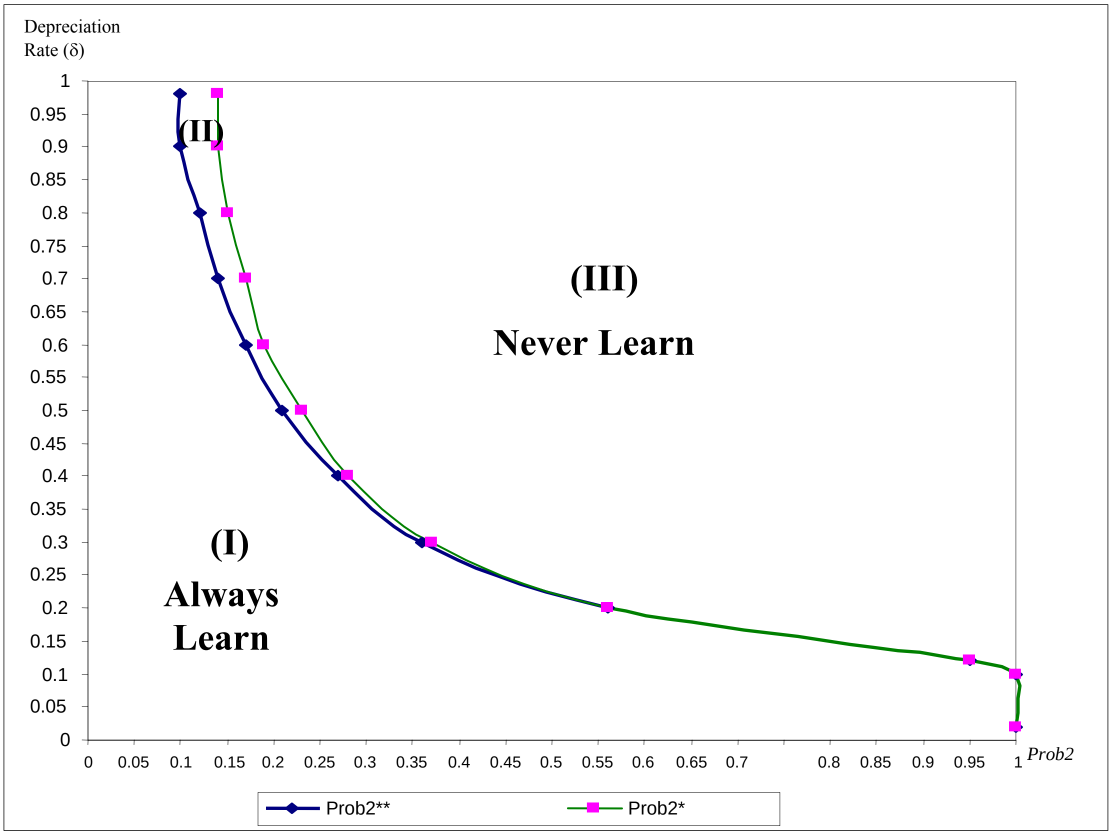
[10] According to table 6.2, the possible learning strategies which can be observed in this region are: *always learn*, *learn-in-bursts*, and *quit learning*.

Figure 6.15 The Learning Strategies, the depreciation Rate, and the Probability of Knowledge Depreciation



6.3 Concluding Comments

The market and technological uncertainty translates into the uncertainty of the potential intertemporal benefits and costs associated with learning and influences the learning decision. In this chapter, the stochastic property of the output price represents the market uncertainty while the technological uncertainty is represented by the stochastic property of knowledge base accumulation.

Under the case of market uncertainty, the decision maker makes production and learning decisions under the circumstances that he knows the current output price, the possible states of the output price in the next period, and the corresponding conditional probabilities in each state. The numerical results indicate that the decision maker usually faces several possible states because of the market uncertainty, and for each possible state, there is one corresponding learning decision. Thus, the decision maker has several possible learning strategies instead of having one "unique" strategy. The actual learning strategy adopted by the decision maker depends on which state is revealed. According to this stochastic model, the shadow value of knowledge can be calculated as well. Similarly, for each possible state, there is one corresponding shadow value of the knowledge.

For the technological uncertainty case, the knowledge base accumulation is assumed to be stochastic because the decision maker faces the possibility of knowledge depreciation. Again, the decision maker faces several possible states because of the technological uncertainty, and the learning decision will not be made until the actual state is revealed. The numerical results also reveal that there is a relationship between the depreciation rate, the probability of knowledge depreciation, and the learning decisions. As the depreciation rate and the probability of knowledge depreciation are low, only the *always learn* strategy is observed. This arises because the decision maker always allocates learning effort no matter which state he faces (as long as the accumulated learning effort has not reached the second threshold). On the other hand, if the depreciation rate and the probability of knowledge depreciation are high, only the *never learn* strategy can be observed because the decision maker never allocates learning effort no matter which state is revealed.

Table 6.3 compares model settings and numerical results for deterministic and stochastic dynamic programming models in chapters 5 and 6. These models can be compared from three aspects: the decision maker's learning decisions in response to the changes in the output price, the market uncertainty, and the technological uncertainty.

In the basic model, the output price is constant over time and the knowledge base in the next period equals to the current knowledge base plus the additional knowledge generated from learning process, i.e., the decision maker does not face knowledge depreciation. The price decreasing (or increasing) model is similar to the basic case except that the output price decreases (or increases) at time, t^D (or t^I). The basic model, the price decreasing model, and the price increasing model are deterministic because there is no market or technological uncertainty, and the decision maker makes learning decisions at the beginning of time horizon.

An *always learn* strategy is adopted in the basic model. However, whether the decision maker adopts a different learning strategy as he faces changes in the future output price depends on how the future output price changes. It is shown that the decision maker adopts different learning strategies (such as *wait-to-learn*, *learn-in-bursts*, *quit learning*, and *always learn*) as he faces a decrease of future output price, but he only adopts *always learn* strategy if he knows that the output price will increase in the future. The reason is that a decrease in future output price implies that the opportunity cost of learning is higher in the earlier periods than it is in the later periods. Thus the decision maker may decide to delay or temporarily stop the learning process to obtain profit gains from physical output while the output price is still high. On the other hand, if the decision maker knows that the output price will increase in the future, he chooses to finish the entire learning process as soon as possible so that the firm can enjoy the profit gain from the both higher output price and the production improvement due to the knowledge gain.

Comparing the price-decreasing model and the market uncertainty model, the decision maker changes his learning strategy as he faces market uncertainty. The market uncertainty model is similar to the price-decreasing model except that the timing of the price decrease becomes uncertain. The decision maker knows how much the output price decreases, but the timing is unknown. The numerical results show that the *wait-to-learn* strategy is ruled out in the market uncertainty model when the output price decreases in an early period (e.g., period 2 or 3). Similarly, in the market uncertainty model, the decision maker does not choose *always learn* strategy as the output price decreases at a much later period (e.g., after period 11). The reason is that the learning decision at each period will not be made until the actual output price at that period is revealed, and the learning decision is made under the knowledge of the current price and the probability set of possible future prices. Thus, as the actual output price is revealed, the decision maker cannot reverse his learning decision in the previous periods and choose another learning strategy.

As for the technological uncertainty model, the technological uncertainty is represented by the stochastic property of knowledge base accumulation because the decision maker faces a possibility that the knowledge base is accumulated with depreciation rate, δ. The learning strategies listed in table 6.3 represent the case where the depreciation rate (δ) equals to 0.3 and the probability of knowledge depreciation (*prob2*) is 0.37. In fact, how the decision maker responds to the technological uncertainty depends on the level of the depreciation rate and the probability that knowledge depreciation occurs. If the depreciation rate is less than 0.1, the knowledge depreciation does not affect learning decisions even if the probability of knowledge depreciation is very high. In this case, the decision maker adopts *always learning* strategy like we observed in the basic model. If the depreciation rate is higher than 0.1, different types of learning strategies (such as *always learn*, *learn-in-bursts*, *quit learning*, and *never learn*) can be observed representing the decision maker's possible learning strategies as he faces technological uncertainty.

Table 6.3 Summary of Numerical Models

Numerical Models		Model Description	Learning Strategy
Deterministic Model*	Basic Model	$P_t = 5, \ \forall t$ $KB_{t+1} = KB_t + \Lambda_t$	*Always Learn*
	Price Decreasing Model	$P_t = \begin{cases} 5 & for \quad t < t^D \\ 3 & for \quad t^D \le t \le 25 \end{cases}$ $KB_{t+1} = KB_t + \Lambda_t$	*Wait-to-learn* ($2 \le t^D \le 3$) *Learn-in-bursts* ($4 \le t^D \le 9$) *Quit Learning* ($t^D = 10$) *Always Learn* ($t^D \ge 11$)
	Price Increasing Model	$P_t = \begin{cases} 5 & for \quad t < t^I \\ 7 & for \quad t^I \le t \le 25 \end{cases}$ $KB_{t+1} = KB_t + \Lambda_t$	*Always Learn*
Stochastic Model**	Market Uncertainty Model	$P_t = \begin{cases} 5 & for \quad t < t' \\ 3 & for \quad t' \le t \le 25 \end{cases}$ $t' \ \ unknown$	*Always Learn* ($2 \le t' \le 3$) *Learn-in-bursts* ($4 \le t' \le 9$) *Quit Learning* ($t' \ge 10$)
	Technological Uncertainty Model	$P_t = 5, \ \forall t$ $KB_{t+1} = (1-\delta) \cdot KB_t + \Lambda_t$ $\delta = 0 \quad with \quad prob1$ $\delta > 0 \quad with \quad prob2$ $\Pr ob1 + \Pr ob2 = 1$	*Learn-in-bursts* *Always Learn* ($\delta = 0.3, \quad prob2 = 0.37$)

* Learning decisions are made at the beginning of time horizon.
** The learning decision at each period is made when the actual state is revealed.

CHAPTER 7

PRODUCTION HETEROGENEITY AND SOCIAL LEARNING: PANEL STUDY OF INDIAN AGRICULTURE

Knowledge management is a learning process involving two phases: (i) the information acquisition phase where the decision maker acquires more information, and (ii) the knowledge updating phase where the learning mechanism is employed to transfer the information collected into additional knowledge. The decision maker may engage in social learning during both phases. In the information acquisition phase, internal information can be collected from historical data while external information can be acquired by the decision maker communicating with others. In the knowledge updating phase, the decision maker can either employ the learning mechanism to process collected information by himself or employ a joint learning mechanism with other decision makers so that they can update the knowledge base together. The possibility that the decision maker can engage in different levels of social learning extends the options of knowledge management scheme. Different knowledge management schemes, such as learning-by-doing, learning-via-conversation, and learning-via-collaboration, not only result in different learning outcomes (i.e, the updated knowledge base) but also lead to different production decisions.

The theoretical model shows that the decision maker's learning behavior is determined by the direct and opportunity costs as well as the potential future benefits associated with learning. Firm-specific characteristics influence learning decisions since they lead to specific cost and benefit structures. Some firm-specific characteristics, such as the socioeconomic and demographic factors, not only influence the decision maker's willingness to devote effort to social learning but also influence the decision from whom the decision maker chooses to learn. Past research points out that the decision makers do learn from others, but they do not learn from all other decision makers (Conley and Udry, 2001; Ueda, 2002). The empirical questions are (i) what leads to heterogeneity of knowledge management behavior across firms and how does the decision maker choose his learning partners?, (ii) how is this heterogeneity revealed in firms' production technologies?, and (iii) how does knowledge management influence the firm's production decision and technology over time?

In this chapter, an econometric model is constructed to reflect the connection between the decision maker's knowledge management behavior and the production behavior. The latent class stochastic frontier model (LCSFM) in which the firm-specific characteristics and the group-specific production functions are defined is used to estimate the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT) India data. The determinants behind the social learning and the role of social learning in production decisions are revealed from the empirical results. In addition, the change in firm-specific production efficiency is obtained from the empirical results.

7.1 Empirical Model - A Latent Class Stochastic Frontier Model

One important task of the empirical setting is to reveal the reason why the decision makers make different learning decisions leading to heterogeneous production behavior. Considerable research deals with unobservable effects on firms being the major factor leading to the heterogeneous behaviors across firms. For example, managerial ability is one of the unobservable characteristics of the firm with many studies trying to measure this unobserved effect by calculating the residual of the production function or finding proxies for it. However, it may not be proper to use the residual term to represent the managerial ability since many other unobservable characteristics also captured in the residual term. The firm's production efficiency is another unobservable characteristic, which is usually measured by the fixed/random effect model. The fixed effect model assumes that the unobservable firm-specific characteristics are fixed and they are represented by firm-specific intercept parameters. On the other hand, the random effect model assumes that the unobservable firm-specific characteristics are randomly distributed with constant mean and variance. Unlike the fixed effect and random effect models where the unobservable firm-specific effects are reflected by the intercept term and part of error structure, Alvarez, Arias, and Greene (2002) construct a translog production function where physical input and unobservable managerial ability are independent variables and the parameters can be estimated by using a random coefficient model. The advantage of this approach is that the model setting allows technical efficiency to be time varying because of the interaction of managerial ability with input level.

Although the fixed and random effect models try to handle the unobservable firm-specific characteristics, there are some drawbacks. One is that the unobserved characteristic captured by the fixed/random effect model is assumed to be invariant over time. The other is that the unobserved characteristic is usually defined as the firm's efficiency, which becomes the only explanation of the heterogeneity across firms. Several studies, such as Kumbhakar (1990) and Battese and Coelli (1992), have offered methods to address the time-invariant efficiency problem. However, the inseparability of the firm efficiency level and the firm heterogeneity is problematic because it implies that firms in the same industry adopt the same production technology, and the only difference across firms is the efficiency level. In fact, firms in the same industry may use different production technologies, and the efficiency level may be misestimated if the technology differences are not considered.

In this study, the latent class stochastic frontier model (LCSFM) is used to estimate the ICRISAT India data because it allows the firms (households) growing the same crop to adopt different production technologies. The production functions estimated by LCSFM are group-specific production functions indicating that the households in the same group adopt the same production technology. In addition, the membership of a household is assigned by the group membership function formulated by the household-specific characteristics. Thus, each firm's production efficiency level is determined by comparing its actual production performance to the reference technology. The LCSFM model is connected with the household's social learning behavior because the decision makers in the same social learning group are likely to choose the same production technology since they are under the same learning environment and share the same production information. Via the membership assignment, we will see which households use the same production technology and whether social learning is the reason behind it.

Assume that the production function of firm *i* in group *j* is specified as:

(7.1) $$y_i = f(\beta_j, x_i) - \mu_i\big|_j + \nu_i\big|_j \quad , \qquad i = 1 \ldots I, \quad j = 1 \ldots J,$$

where y_i and x_i represent output and input, respectively, and β_j is a vector of coefficients that characterize the firm's production behavior if the household is in group *j*. For each group, the household's production inefficiency is captured by the non-negative term $\mu_i\big|_j$ which is truncated normally distributed with zero mean and variance $\sigma^2_{\mu j}$. The random statistical noise is represented by $v_i\big|_j$, which is normally distributed with mean zero and variance σ^2_{vj}. If the firm belongs to group *j*, the likelihood function of firm *i* can be written as[11]

(7.2) $$L_{i|j} = \frac{\Phi\left(\lambda_j \varepsilon_{i|j} / \sigma_j\right)}{\Phi(0)} \cdot \frac{1}{\sigma_j} \cdot \phi\left[\frac{\varepsilon_{i|j}}{\sigma_j}\right]$$

where $\varepsilon_{i|j} = y_i - f\left(\beta_j, x_i\right)$, $\sigma_j = \sqrt{\sigma^2_{\mu j} + \sigma^2_{vj}}$, and $\lambda_j = \sigma_{\mu j} / \sigma_{vj}$.

The unconditional log-likelihood function of firm *i* is:

(7.3) $$\ln L_i\left(\beta, \eta\right) = \ln\left(\sum_{j=1}^{J} L_{i|j}(\beta_j) \cdot P_{ij}(\eta_j)\right)$$

The notation $P_{ij}(\eta_j)$ represents the prior probability of the firm's group membership formulated as a multinomial logit model:

(7.4) $$P_{ij} = \frac{\exp\left(\eta_j ' q_i\right)}{\sum_{j=1}^{J} \exp\left(\eta_j ' q_i\right)} \quad , \eta_J = 0 \quad \sum_{j=1}^{J} P_{ij} = 1$$

[11] See Green (2002).

where η_j is a vector of parameters and q_i is a vector of firm-specific variables. The parameters $\beta = (\beta_1, \beta_2, ..., \beta_J)$ and $\eta = (\eta_1, \eta_2, ..., \eta_J)$ will be obtained by maximizing the overall log-likelihood function:

$$(7.5) \qquad \ln L = \sum_{i=1}^{N} \ln L_i(\beta, \eta) = \sum_{i=1}^{N} \ln\left[\sum_{j=1}^{J} L_{i|j}(\beta_j) \cdot P_{ij}(\eta_j)\right]$$

7.1.1 The Membership Assignment

Green (2002) and Orea and Kumbhakar (2003) present the posterior probabilities of the household *i*'s group membership as:

$$(7.6) \qquad P(j|i) = \frac{L_{i|j}(\beta_j) \cdot P_{ij}(\eta_j)}{\sum_{j=1}^{J} L_{i|j}(\beta_j) \cdot P_{ij}(\eta_j)}$$

We can see that the posterior probability of group *j* for a given household *i* is determined by the parameters β and η from both production and group membership functions. For each household, the membership is assigned to a group based on the highest posterior probability. Using the ICRISAT India data, the group membership indicates that the households growing the same crop may adopt different production technologies, and the households assigned to the same group adopt the same production technology. In addition, the group-memberships are addressed as the learning groups because we assume that the firms in the same learning group are under the same learning environment and share the same production information, which lead to the same production decision. The learning group might be an informal social communication network indicating that some particular households interact more with each other. The communications among households provide an opportunity for the information exchange and the social learning.

Once group membership is assigned, the common characteristics within group provide the reason why the decision makers choose that learning group.

7.1.2 The Change of Production Behavior and the Change of Group Membership

The LCSFM provides the posterior probabilities of the group membership for each household at each time. The posterior probability describes the chance of a household adopting a specific production technology. Thus, the change of the production behavior is captured by the change of the posterior probability which might result in the change of group membership assignment. If the decision maker decides to make a big change in the production behavior, such as adopting another production technology, then a significant change in the posterior probability resulting in the change of membership assignment is expected.

7.1.3 The Learning Gains

The production efficiency provided by the LCSFM is calculated for each household by using its reference technology. The production efficiency is not only household-specific but also time-varying. Since the households adopting the same production technology are assumed to be in the same social learning group, the change in the production efficiency of each household is viewed as the influence from the learning behavior. By observing the household's efficiency level over time, we may be able to figure out the decision maker's learning strategy. For example, if the firm's efficiency level improves over time, this suggests that the decision maker is adopting an *always learn* strategy, where the decision maker not only accumulates information in the learning group but also uses the information to update the knowledge base reflecting on the production performance. If the firm's efficiency level does not improved at the first few periods, but it suddenly jumps to a significantly higher level later, this suggests that the decision maker adopts a *wait to learn* strategy, where the firm keeps accumulating information in every period, but does not act on it until later. It is also possible to observe the decrease in the production efficiency since there are possibilities of the knowledge depreciation. One should notice that the decision maker's ability to implement the knowledge into production behavior influences the production efficiency as well. Even though the households in the same learning group have strong cooperation or collaboration relationships and/or they use a joint learning mechanism to update the knowledge base together, their production performance might differ because the decision makers have different abilities in implementing the knowledge into realized production.

7.2 Data Description

The dataset used in this study is the ICRISAT (International Crops Research Institute for the Semi-Arid Tropics) India data. This dataset is collected from Village Level Studies (VLS) initiated by ICRISAT Economics Program in May 1975. The original VLS covered six villages in Andhra Pradesh and Maharashtra states in India. Forty households, including 10 landless labor households[12] and 30 cultivator households, were surveyed at each village. The dataset include the characteristics of household members, plot and cultivation schedules, financial positions, farm inventory, rain data, and monthly commodity price. In 1980, the studies are extended to another four villages in Gujarat and Madhya Pradesh states. The data available for all villages in the ICRISAT database are listed at table 7.1. The data in Aurepalle village is used in this study because it has relatively long panel, which favors an econometric analysis.

[12] The labor households are those who operate less than 0.2 hectares of land, and was hired as laborers as their occupation.

Table 7.1: Data Availability of ICRISAT India Data by village

Villiage	Aurepalle	Dokur	Shirapur	Kalman	Kanzara
State	Andhra Pradesh	Andhra Pradesh	Maharashtra	Maharashtra	Maharashtra
Data Availability	1975-1984	1975-1980	1975-1984	1975-1980	1975-1984
Villiage	Kinkheda	Boriya	Rampura	Papda	Rampura Kalan
State	Maharashtra	Gujarat	Gujarat	Madhya Pradesh	Madhya Pradesh
Data Availability	1975-1980	1980-1984	1980-1984	1981-1983/84	1981-1983/84

7.2.1 Agricultural Productions in Aurepalle Village

The agricultural production in Aurepalle village is summarized in this section from the aspects of the cropping area, the crop-mixture scenario, and the crop value.

Cropping Area

From 1975 to 1984, over 60% of the cropping lands are used to grow sorghum and castor (Table 7.2). Both crops have local variety (TV) and high-yielding variety (HYV) [13]. The HYV sorghum is introduced to Aurepalle before 1976, while HYV castor is introduced in year 1981. However, the diffusions of these two HYV crops have very different results. The local variety sorghum dominated the HYV sorghum during year 1975-1984 while the HYV castor almost totally replaced the TV castor right after 1981.

Despite the production differences among crops, the production patterns are quite different among seasons as well. There are three seasons in Aurepalle, which are kharif, rabi, and summer. Kharif season covers from June to October; rabi season covers from November to February; summer season covers from March to May. Table 7.3 and figure

[13] The percentage of land use for local variety sorghum was not listed in the table because it is less than 1%.

7.1 indicate that most farming activities are operated in kharif season which accounts for over 70% of the cropping areas are operated during that season. The summer season, on the other hand, has much less farming activity accounting for less than 3% of the cropping areas operated in summer (except for 1982 and 1983). The summaries of the land use for each season are presented in tables 7.4, 7.5, and 7.6. Sorghum is usually grown during kharif and summer while castor is only grown in kharif season. In rabi season, farmers also grow safflowers and groundnuts, while the green fodder crops become relatively important during summer.

Crop Mixture

The "crop mixture" is defined as the situation where there is more than one crop growing on a single plot. Crop mixture increases the difficulty in measuring the accurate cropping area of a certain crop if it is mixed with other crops on the same plot. For the information offered in the tables 7.2 to 7.6, the cropping area of the plot is assigned to the first crop, which is designated as the dominant crop. However, this results in the overestimation of the land use for the first crop and underestimation of the land use for other crops on the same plot. Table 7.7 lists the numbers of plots operated by the farmers and the numbers of plots growing mixed crops in each season. The crop mixture is common during kharif since 43% of the plots grow mixed crops during that time. If we focus on kharif season and categorized the crop mixture plots by their dominant crops, table 7.8 indicates that TV sorghum is the crop that is always grown with other crops, such as TV pearl millet and redgram.[14] The crop mixture is not rare for castor after 1980/1981. The TV and

[14] Among the plots on which TV sorghum is the dominant crop, 270 out of 315 plots mixed TV sorghum with TV pearl millet and redgram from 1975 to 1984.

HYV castor were either mixed with redgram, or these two varieties are mixed with each other.

Crop Value

The crop value is one of the methods to avoid the overestimation/underestimation of cropping area and still enable to see the importance of the crops. During 1975 to 1980, the top 5 valuable crops are HYV paddy, TV castor, TV paddy, TV sorghum, and TV pearl millet according to their average crop value (table 7.9). After 1981, the top five valuable crops become HYV Paddy, HYV castor, TV paddy, TV sorghum, and groundnuts.[15] HYV paddy has the highest value in every year even though it accounts for 10% to 16% annual land use. The impact of the introduction of HYV castor also emerges in the crop value. The value of HYV castor is 15.28% of all crop value right after the HYV castor is introduced in the village in 1981, and it increases to over 20% after 1983. On the other hand, the value of TV castor decreases from 12.91% in 1980 to 1.43% in 1984.

The castor and sorghum data in Aurepalle village are used for the econometric analysis due to the data availability and their importance in agricultural production. The interesting change in the production pattern due to the introduction of HYV castor is another reason to use the castor data. The introduction of HYV castor makes it necessary for the farmers to learn the new knowledge about the new variety. These changes could result in the adjustments of production and learning behavior, which are expected to show in the econometric results.

[15] Although the rough dry fodder plays an important role in the crop value, it is not taken into account because it is the by-product of fodder.

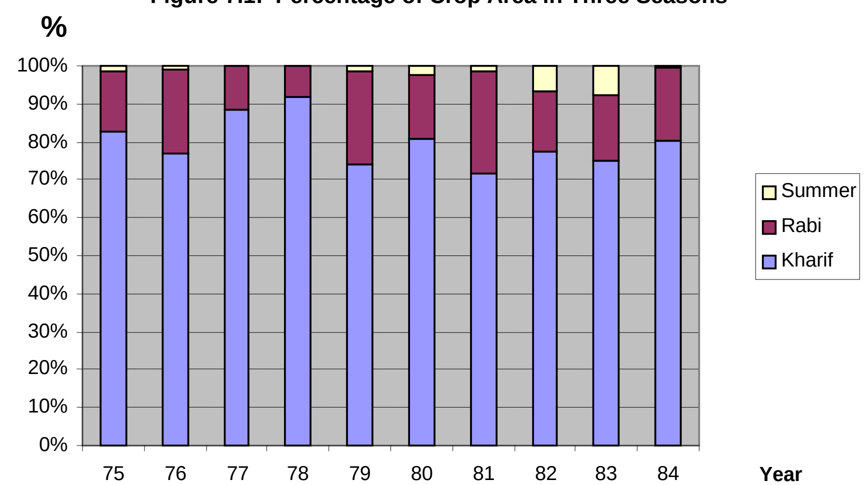
Table 7.2: The Summary of Annual Land Use (%)[16]

YEAR	TV Caster	HYV Caster	TV Sorghum	TV Paddy	HYV Paddy	Chillies	Groundnuts	Green fodder crops	HYV Pearl millet	Safflower	Other crops	total area (acres)
1975	0.292	0.000	0.422	0.057	0.099	0.049	0.001	0.000	0.000	0.043	0.038	346.790
1976	0.452	0.000	0.246	0.049	0.104	0.027	0.001	0.000	0.000	0.087	0.035	369.960
1977	0.342	0.000	0.377	0.058	0.164	0.026	0.023	0.000	0.000	0.000	0.011	259.660
1978	0.387	0.000	0.328	0.065	0.159	0.016	0.007	0.016	0.000	0.000	0.021	283.160
1979	0.345	0.000	0.286	0.081	0.154	0.024	0.035	0.009	0.000	0.017	0.050	276.230
1980	0.334	0.000	0.392	0.034	0.123	0.013	0.015	0.031	0.010	0.000	0.049	311.270
1981	0.145	0.231	0.331	0.032	0.128	0.014	0.010	0.014	0.022	0.026	0.048	342.180
1982	0.037	0.362	0.288	0.026	0.106	0.004	0.037	0.013	0.031	0.035	0.060	285.350
1983	0.040	0.372	0.245	0.022	0.161	0.006	0.009	0.006	0.035	0.054	0.050	293.350
1984	0.087	0.371	0.263	0.012	0.162	0.010	0.000	0.009	0.025	0.035	0.025	243.170

Table 7.3: Cropping Area in Different Seasons (acre)

YEAR	Khrif	Rabi	Summer	Annual
1975	286.09	55.05	5.65	346.79
1976	284.26	81.9	3.8	369.96
1977	230.26	29.4	0	259.66
1978	260.36	22.8	0	283.16
1979	204.34	68.54	3.35	276.23
1980	250.88	52.74	7.65	311.27
1981	245.59	91.04	5.55	342.18
1982	220.46	46.27	18.62	285.35
1983	219.51	51.49	22.35	293.35
1984	195.2	46.32	1.65	243.17

Figure 7.1: Percentage of Crop Area in Three Seasons



[16] It is difficult to know the accurate cropping area of a given crop if it is mixed with other crops on the same plot. If the crops are grown as mixed, the cropping area of the plot is then assigned to the first crop, i.e., the dominated crop, which occupies the larger area of the plot.

Table 7.4: Summary of Land Use in Kharif Season (%)

YEAR	TV Caster	HYV Caster	TV Sorghum	TV Paddy	HYV Paddy	Chillies	Groundnuts	Green fodder crops	HYV Pearl millet	Safflower	Other crops	total area (acres)
1975	0.353	0.000	0.481	0.010	0.083	0.046	0.000	0.000	0.000	0.000	0.027	286.090
1976	0.588	0.000	0.293	0.033	0.066	0.016	0.000	0.000	0.000	0.000	0.004	284.260
1977	0.385	0.000	0.425	0.065	0.076	0.029	0.008	0.000	0.000	0.000	0.012	230.260
1978	0.421	0.000	0.357	0.071	0.086	0.018	0.008	0.017	0.000	0.000	0.023	260.360
1979	0.466	0.000	0.251	0.078	0.128	0.032	0.009	0.005	0.000	0.000	0.032	204.340
1980	0.414	0.000	0.377	0.024	0.097	0.016	0.012	0.014	0.000	0.000	0.047	250.880
1981	0.202	0.321	0.319	0.012	0.070	0.020	0.005	0.002	0.030	0.000	0.018	245.590
1982	0.048	0.468	0.258	0.021	0.074	0.006	0.025	0.005	0.036	0.000	0.059	220.460
1983	0.053	0.497	0.246	0.029	0.100	0.008	0.011	0.005	0.047	0.000	0.005	219.510
1984	0.108	0.462	0.241	0.010	0.112	0.012	0.000	0.000	0.031	0.000	0.023	195.200

Table 7.5: Summary of Land Use in Rabi Season (%)

YEAR	TV Caster	HYV Caster	TV Sorghum	TV Paddy	HYV Paddy	Chillies	Groundnuts	Green fodder crops	HYV Pearl millet	Safflower	Other crops	total area (acres)
1975	0.000	0.000	0.065	0.303	0.193	0.073	0.009	0.000	0.000	0.269	0.088	55.050
1976	0.000	0.000	0.092	0.073	0.225	0.067	0.003	0.000	0.000	0.393	0.147	81.900
1977	0.000	0.000	0.000	0.000	0.854	0.000	0.146	0.000	0.000	0.000	0.000	29.400
1978	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000	22.800
1979	0.000	0.000	0.404	0.083	0.212	0.000	0.117	0.011	0.000	0.067	0.106	68.540
1980	0.000	0.000	0.515	0.083	0.214	0.000	0.014	0.080	0.034	0.000	0.060	52.740
1981	0.000	0.000	0.382	0.083	0.258	0.000	0.022	0.025	0.000	0.099	0.131	91.040
1982	0.000	0.000	0.549	0.015	0.000	0.000	0.110	0.022	0.000	0.217	0.088	46.270
1983	0.000	0.000	0.349	0.000	0.072	0.000	0.006	0.000	0.000	0.308	0.265	51.490
1984	0.000	0.000	0.356	0.022	0.380	0.000	0.000	0.025	0.000	0.186	0.032	46.320

Table 7.6: Summary of Land Use in Summer Season (%)

YEAR	TV Caster	HYV Caster	TV Sorghum	TV Paddy	HYV Paddy	Chillies	Groundnuts	Green fodder crops	HYV Pearl millet	Safflower	Other crops	total area (acres)
1975	0.000	0.000	0.894	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.106	5.650
1976	0.000	0.000	0.000	0.737	0.263	0.000	0.000	0.000	0.000	0.000	0.000	3.800
1979	0.000	0.000	0.000	0.224	0.567	0.000	0.000	0.209	0.000	0.000	0.000	3.350
1980	0.000	0.000	0.065	0.000	0.340	0.000	0.131	0.255	0.157	0.000	0.052	7.650
1981	0.000	0.000	0.000	0.090	0.532	0.000	0.000	0.378	0.000	0.000	0.000	5.550
1982	0.000	0.000	0.000	0.107	0.740	0.000	0.000	0.089	0.054	0.000	0.011	18.620
1983	0.000	0.000	0.000	0.000	0.960	0.000	0.000	0.036	0.000	0.000	0.004	22.350
1984	0.000	0.000	0.303	0.000	0.000	0.000	0.000	0.697	0.000	0.000	0.000	1.650

Table 7.7: The number of operated plots and crop mixture from 1975 to 1984

	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	Total
Kharif	102 (42)	115 (36)	107 (33)	96 (30)	96 (27)	136 (59)	124 (75)	123 (70)	123 (70)	104 (41)	1126 (483)
Rabi	37 (4)	50 (--)	24 (--)	10 (--)	57 (--)	38 (--)	76 (--)	41 (--)	36 (3)	35 (--)	404 (7)
Summer	9 (2)	3 (--)	-- (--)	-- (--)	6 (--)	11 (--)	8 (--)	24 (--)	20 (--)	3 (--)	84 (2)
Total	148 (48)	168 (36)	131 (33)	106 (30)	159 (27)	185 (59)	208 (75)	188 (70)	179 (73)	142 (41)	1614 (492)

* The numbers in the parentheses show the number of plots grew mixed crops.

Table 7.8: The Dominant Crops and the Number of Plots in Kharif Season[1]

	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
TV Castor	34 (2)	46 (6)	33 (2)	35 (1)	33 (5)	41 (16)	16 (12)	6 (3)	6 (6)	11 (4)
TV Sorghum	39 (39)	32 (29)	38 (31)	30 (29)	23 (22)	43 (42)	38 (38)	31 (30)	29 (29)	27 (26)
TV Paddy	4 (--)	11 (--)	10 (--)	8 (--)	7 (--)	5 (--)	4 (--)	5 (--)	6 (--)	2 (--)
HYV Paddy	14 (--)	18 (--)	14 (--)	10 (--)	20 (--)	24 (--)	17 (--)	17 (--)	20 (--)	18 (--)
Chillies	7 (--)	6 (1)	6 (--)	4 (--)	6 (--)	7 (--)	6 (--)	3 (--)	3 (--)	4 (--)
Groundnuts	0 (--)	0 (--)	2 (--)	1 (--)	3 (--)	4 (--)	1 (--)	4 (--)	2 (--)	0 (--)
Green Fodder Crops	0 (--)	0 (--)	0 (--)	3 (--)	1 (--)	3 (--)	1 (--)	2 (--)	1 (--)	0 (--)
HYV Castor	0 (--)	0 (--)	0 (--)	0 (--)	0 (--)	0 (--)	31 (18)	44 (29)	43 (35)	36 (10)
HYV Pearl millet	0 (--)	0 (--)	0 (--)	0 (--)	0 (--)	0 (--)	5 (4)	7 (4)	12 (0)	4 (1)
Other Crops	4 (1)	2 (--)	4 (--)	5 (--)	3 (--)	9 (1)	5 (3)	4 (4)	1 (--)	2 (--)
Total Plots	102 (42)	115 (35)	107 (33)	96 (30)	96 (27)	136 (59)	124 (75)	123 (70)	123 (70)	104 41

1. The numbers in the parentheses show the number of plots grew mixed crops.

Table 7.9: Summary of Annual Crop Value (%)

YEAR	TV Castor	HYV Castor	TV Sorghum	TV Paddy	HYV Paddy	Groundnuts	Redgram	Rough dry fodder	TV Pearl millet	HYV Pearl millet	Other crops	Total Crop Value (Rs)
1975	0.1305	0.0000	0.0583	0.1446	0.3387	0.0000	0.0226	0.0760	0.0505	0.0000	0.1788	90729.24
1976	0.2254	0.0000	0.0808	0.2107	0.2824	0.0055	0.0083	0.0575	0.0453	0.0000	0.0841	77944.72
1977	0.2928	0.0000	0.0541	0.1700	0.3656	0.0345	0.0097	0.0441	0.0228	0.0000	0.0064	156921.49
1978	0.1535	0.0000	0.0566	0.1701	0.3964	0.0097	0.0330	0.0990	0.0351	0.0000	0.0467	130504.4
1979	0.1025	0.0000	0.0475	0.1883	0.4161	0.0590	0.0144	0.1220	0.0105	0.0000	0.0397	153316.95
1980	0.1291	0.0000	0.0560	0.0692	0.4180	0.0356	0.0057	0.1695	0.0328	0.0156	0.0841	121919.05
1981	0.0203	0.1528	0.0402	0.0622	0.3930	0.0227	0.0288	0.1766	0.0273	0.0186	0.0762	169715.26
1982	0.0111	0.1625	0.0707	0.0689	0.3552	0.0749	0.0217	0.1489	0.0211	0.0110	0.0650	186812.75
1983	0.0052	0.2230	0.0357	0.0448	0.4261	0.0164	0.0213	0.1197	0.0163	0.0210	0.0916	255775.94
1984	0.0143	0.2113	0.0348	0.0229	0.5011	0.0000	0.0050	0.1331	0.0079	0.0038	0.0695	189238.95

7.2.2 Demographic Background

Household Caste

The caste system plays a very important role in explaining the individual behavior in India society. The original caste system was developed in ancient India including four castes and one outcaste. The four castes are Brahman, Kshatria, Vaisia, and Sudra in descending order of their hierarchy. The outcast, which is also called untouchable or Harijan is the lowest in the hierarchy and is not allowed to own land. The original caste system regulates not only individuals' behavior, such as what they eat, whom they married, but also their occupation. For example, the Brahmans are supposed to be priests and the Kshatriyas are supposed to be rulers, warriors, or landowners. The caste system becomes more complicated in real India society because each caste breaks up into several sub-castes, which also exists hierarchy among them. In addition, people in different areas have different interpretations for the sub-castes, which make their hierarchy status unclear. In modern India, the caste system is more flexible especially in the city. But in rural areas, the caste system still plays important role in regulating people's social life. In ICRISAT India data, there are three kinds of ranking system illustrating the households' caste ranking. The caste rank used for the econometric analysis is prepared based on the individual occupation and socioeconomic condition (J.G. Ryan).

Education

The education situation in India shows the division in genders, castes, and regions. In the late 1970s, for adults above age nineteen, 50% of men and 15% of women were literate (Walker and Ryan, 1990). The reason of the gender bias comes from social expectations toward women who are expected to do the housework which does not require formal education. Further, girls are not expected to wander outside the village, which prevents female from education if upper-primary schools are not within the village (Veena Das, 2003).

The regional bias indicates that the southern states of India usually have higher literacy rate. Even though most of the villages in ICRISAT data are from southern states, the education situations are different among villages. Walker and Ryan (1990) points out that the education transition is faster in villages in Maharashtra state while it is slower in villages in Andhra Pradesh state.

The disadvantaged castes have a lower literacy rate than other castes. The inequality of social status is one of the reasons discouraging people in lower castes to pursue higher education since they may not have equal job opportunity compared to people from higher castes even with the same education level.

Using the ICRISAT Indian data, the relationships between the caste rank and the household head's education level in Aurepalle village are obtained. Tables 7.10 and 7.11 describe the household heads' education level and the caste rank for the households growing castor and sorghum, respectively. For both castor and sorghum growers, there are about two-thirds of the household heads being illiterate, and only one or two household heads possessing a high school education. As for the caste ranking, generally, the households are evenly distributed among castes 1, 3, and 4. There are very few households belonging to caste 2, and it is interesting to note that they all quit growing castor after 1981. In addition, the correlation coefficient between the household head's education and the caste rank of the household indicates that the household caste has positive association with the education level of the household head for both castor and sorghum growers. [17]

Table 7.10 The Household Head's Education and Household Caste Ranks: Castor Growers

Before 1981						
	HEDU=1	HEDU=2	HEDU=3	HEDU=4	HEDU=5	Total households
# of household	21	5	6	1	2	35
	Caste=1 (highest)	Caste=2	Caste=3	Caste=4 (lowest)		Total households
# of household	10	3	11	11		35
After 1981						
	HEDU=1	HEDU=2	HEDU=3	HEDU=4	HEDU=5	Total households
# of household	20	4	4	0	1	29
	Caste=1 (highest)	Caste=2	Caste=3	Caste=4 (lowest)		Total households
# of household	8	0	11	10		29

* HEDU=1 indicates "illiterate"; HEDU=2 indicates "read and write"; HEDU=3 indicates "up to primary school"; HEDU=4 indicates "up to middle school"; HEDU=5 indicates "up to high school".

Table 7.11 The Household Head's Education and Household Caste Ranks: Sorghum Growers

1975~1984						
	HEDU=1	HEDU=2	HEDU=3	HEDU=4	HEDU=5	Total households
# of household	20	5	5	1	2	33
	Caste=1 (highest)	Caste=2	Caste=3	Caste=4 (lowest)		Total households
# of household	10	2	11	10		33

* HEDU=1 indicates "illiterate"; HEDU=2 indicates "read and write"; HEDU=3 indicates "up to primary school"; HEDU=4 indicates "up to middle school"; HEDU=5 indicates "up to high school".

[17] The correlation coefficient between the household head's education and the caste rank is -0.685 for castor growers and -0.677 for sorghum growers. Since caste ranks are denoted from level 1 to level 4 for the highest to the lowest rank, thus the negative sign of the correlation coefficient indicates that the household caste has positive association with the education level of the household head.

7.2.3 Variable Definition

The potential variables that can be used in the production function and the group membership function in the LCSFM are listed in table 7.12. The possible independent variables for production function are the inputs for castor or sorghum production and the information of the households head, such as age and education. The variables for group-membership function are household-specific characters that can influence the group membership assignment. In this study, the dummy variables for caste ranks are used to present the household's socioeconomic status. The households with similar socio-economic background are more likely to interact with each other. Via social activities, the households can exchange the production information and learn production experience from each other. Thus the similarity in the production behavior is an implied outcome of social learning.

Table 7.12 Variable Definitions

Variables	Definition
Production Function	
Y	ln Output [ln(kg/acre)]
X_1	ln Seed [ln(kg/acre)]
X_2	ln Family Labor [ln(hours/acre)]
X_3	ln Hired Labor [ln(hours/acre)]
X_4	ln Farm Animal (bullocks) [ln(hours/acre)]
X_5	ln Fertilizer [ln(kg/acre)]
X_7	ln Manure [ln(kg/acre)]
LHAGE	ln(Head Age)
HEDU	Education level of household head
Group Membership Probability Function	
C1	Caste dummy; C1=1 if the firm belongs to caste 1.
C2	Caste dummy; C2=1 if the firm belongs to caste 2.
C3	Caste dummy; C3=1 if the firm belongs to caste 3.
HYVD	HYV dummy; HYVD=1 if high yield variety is adopted.
GS	Good soil; GS=1 if the soil type is good*.

* The soil type are characterized in to 9 types in the original ICRISAT data set. The good soil in our model indicate soil type 1 to 3, which represents deep, medium, and shallow black soil, respectively.

7.3 Estimation Results of Latent Class Stochastic Frontier Model[18]

The estimation results for the castor and the sorghum, including the parameter estimation, group-membership assignment, and each household's production efficiency, are presented in the following sections. In addition, the importance of the social learning in production decision is revealed by the empirical results.

7.3.1 The Castor Production

(a) Estimation Results

The estimation reveals the presence of two groups, *CA* and *CB*, for castor growers and appendix F-1 presents the group-specific production functions estimated by the latent class stochastic frontier model. The estimation results for the production behavior of *CA* show that seeds, family labor, and hired labor all have positive and significant impacts on the castor production. Fertilizers and manures each have a positive impact as well, but they are insignificant. The negative coefficient of animal bullock indicates that the more the animal bullock is used, the less the castor production. Since the animal bullock is the property belonging to the household, it might not be adjusted as quickly as other inputs, such as seeds and fertilizers. Past studies also indicate that the animal bullock is used more extensively in years of poorer rainfall when yields are low (Battese and Coelli, 1995; Ueda, 2002). On the other hand, the results for the production behavior in group *CB* show that seeds, animal bullocks and fertilizers have positive and significant impact on growing castors while the influence of manure is negative but insignificant. The estimation results of both groups show that the output level increases with the household heads' age and their education level.

[18] The empirical results are obtained by using GAUSS program. I thank E. G. Tsionas for making the program available.

The estimation results for group membership function indicate which household characteristics are important in explaining the membership assignment. It is shown that the dummy variables for caste ranks 1 and 3, the HYV dummy and the soil type play important roles. The household belonging to caste 1 or caste 3, or grows the high-yield variety, or uses good soil to grow castor is more likely to be assigned to group *CB*, while the household from caste 2 or caste 4 is not.

(b) Group Membership – the Heterogeneity in Production Technology

The posterior probability, $P(j|i)$, describes the probability of a household to be in group j for a given household i. Since the production functions estimated by LCSFM are group-specific, the posterior probability also represents the probability of adopting a certain production technology for a given household i. In this study, the household is assigned to a group based on its highest posterior group probabilities. Once membership is assigned, the production technology of that group becomes the reference technology of the household.

Appendix F-2 lists the group membership of each household from 1975 to 1984. The households' production behavior can be roughly distinguished by the year 1981, which is the first time the HYV castor is introduced to Aurepalle village. Appendix F-3 lists the numbers of households in each group in each year and it shows that over 65% of the households adopt the production technology *CA* (production function for group *CA*) before 1981, and after 1981, only one-third of the households adopt production technology *CA*. Thus, the impact from the introduction of HYV castor is revealed in the switching of the production technologies.

Despite the impact of the introduction of high-yield variety, the household-specific characteristics play important roles in the production behavior as well. The characteristics, including caste rank and soil type, provide further explanations of the heterogeneities across households resulting in group-specific production behavior. This can be seen by reviewing the characteristics of the households in each group listed in appendix F-4 (before 1981) and appendix F-5 (after 1981). Appendix F-4 shows that there are 18 households always using production technology *CA*, 2 households always using production technology *CB*, and 11 households changing their production technology during 1975-1980 period. The results may not be able to explain much for the households who always adopt production technology *CB* because each of them has only one observation before 1981. However, the results indicate that production technology *CA* is the major production technology adopted by most of the households before HYV castor is introduced to the village. In addition, the soil type presents an important role in household's production decisions during that time. Combining the information in appendixes F-2 and F-6, we can see that 6 households changed their production behavior as the soil type changed (households 44, 45, 46, 48, 50, 56).

Considering the castor production between 1981 and 1984, there are more households adopting production technology *CB* after the HYV castor is introduced (appendix F-5). More importantly, the household's caste rank becomes the major determinant of the household's production decision. It is interesting to note the following results.

1) All of the households belonging to caste 2 exited castor production.

2) Most of the households grow HYV castor and many of them switch to production technology *CB* after 1981. However, all the households from caste 4 keep production

technology *CA* even though some of them do grow HYV castor. This indicates that the households from caste rank 4 do not adjust their production behavior for growing the new variety.

3) Although HYV castor dominate TV castor after 1981, there are four households that never adopt HYV caster (households 37, 49, 61, 70), and all of them are from low caste rank.

If we take a closer look of the households' production pattern, we will see that some households always make the same production decisions. For example, households 1, 33, 34, 36, 37, 70 adopt production technology *CA* before 1981 and keep using the same technology after 1981. These households belong to lower caste rank, and most of them are from caste 4. On the other hand, some households adopt production technology *CA* before 1981 but switch to production technology *CB* after 1981. Those households include households 53, 54, 57, and 58, and all of them are from caste 1. It is possible that these particular households communicate with each other more often than they communicate with others because they have similar socioeconomic status. These households form a learning group as the outcome of their social communication. Within the learning group, the households share the same learning environment and the same production information, and furthermore, they make the same learning decision.

(c) Heterogeneity within the Group

In the latent class stochastic frontier model, the group-specific production functions reveal the heterogeneity of the technology selection among households, while the production efficiency level represents the heterogeneity within the group. As the household is assigned to a group, the representative production technology of that group is the reference technology of the household, which is used to measure the household's production efficiency. For example, if a household is assigned to group *CA*, then its reference technology is production technology *CA*. Thus, the production efficiency of the household is obtained by comparing the actual production performance to the production frontier of the corresponding technology. The group membership of each household and its corresponding efficiency level are illustrated in appendix F-7. The efficiency levels of the household classified by their group membership and farm size before and after 1981 are shown in appendixes F-8 and F-9, respectively.

According to the results listed in appendixes F-8 and F-9, large farms are not necessarily more efficient than small or medium farms. In addition, the efficiency improvement which can represent the learning gain is shown in only few households. Appendix F-8 shows that the efficiency level of households 52 and 53 are around 0.5 before 1977, and they increased to over 0.7 afterward. The sudden increase in production efficiency implies that these two households may adopt a *wait to learn* strategy. That is, the household keeps accumulating information in every period, but not act on it until year 1977. The efficiency improvement is also shown in household 54 in appendix F-9. Since household 54 grew only HYV castor after 1981, the efficiency improvement shows that this household not only collects the information about the new variety but also acts on it immediately.

7.3.2 Sorghum Production

(a) Estimation Results

This estimation also reveals the presence of two technologies, *SA* and *SB*, and the results of group-specific production functions for sorghum growers are presented in appendix G-

1. Although the coefficients of the production function in group *SA* have positive sign on seeds, family labor, and hired labor, only seeds have significant impact on sorghum production. In addition, the animal bullock and the fertilizer impacts are negative but statistically insignificant. On the other hand, the production technology *SB* shows that family labor and hired labor have positive and significant influence on the sorghum output, while the coefficient for seeds has a negative but insignificant sign. The household head's education, again, is beneficial to the sorghum production for both production technologies.

As for the group-membership function, the dummy variable for high-yield variety is not used to explain the household's group membership because the HYV sorghum plays an insignificant role in sorghum production. The estimation results for group-membership function indicate that soil type and the dummy variable for caste 1 provide good explanatory power for membership assignment. If the household uses good soil to grow sorghum, or the household belongs to caste 1, then it is less likely for the household to be assigned to group *SB*.

(b) Group Membership – the Heterogeneity in Production Technology

The group-membership assignments for each household from year 1975 to 1984 are shown in appendix G-2. It shows a little difference in the sorghum production before and after 1981, but the change is not as obvious as what we observed in castor production. However, there is a huge decrease in the number of sorghum growers from 1980 to 1981, which then increases to around 10 households afterward (see appendix G-3). Focusing on the determinants behind the membership classification, the household characters before 1981 and after 1981 are listed in appendixes G-4 and G-5, respectively. The year

1981 is used as a separating point here because of the introduction of HYV castor which may influence sorghum production as well. According to appendix G-4, the households' production behaviors are distinguished by soil type and caste rank before 1981 since the households that always adopt production technology *SB* do not use good soil, and the households that always adopt production technology *SA* are all from caste 1.

The household characteristics after 1981 are presented in appendix G-5 by groups. During year 1981 to 1984, the households adopting production technology *SA* before 1981 either quit growing sorghum (households 50, 52, 59, 80) or stay and adopt the same production technology (households 45, 53, 54, 57, 58). For the households adopting production technology *SB* before 1981, some stay and use the same technology (households 1 and 55); some quit growing sorghum (households 39, 40, 41), and some change their production behavior (households 5, 10, 81). There are three things to note.

1) Combining the information from appendixes G-2, G-5, and G-6, soil type is still an important determinant in production decision since none of the households adopting production behavior 2 use good soil. In addition, households 81 and 82 change their production behavior as their soil type changes.

2) Households 53, 54, 57, and 58 took the same production pattern in sorghum production as they did in castor production. It is possible that these four households have more social interactions with each other because they come from the same caste rank, they operate large farms, and the household heads have similar education level. Again, the importance of social learning influences their production decision and reveals in their production behavior.

3) Some of the households from caste 3 or caste 4 adopt production technology *SA* during 1981 to 1984. However, it is hard to draw implications from their production pattern since they only have one observation during that time period.

(c) Heterogeneity within the Group

The production efficiency, which can be obtained by comparing the household's actual output to its corresponding production frontier, is used to represent the heterogeneity within the group. The production efficiencies of each household are plotted in appendix G-7. The efficiency levels of the household classified by their group membership and farm size before and after 1981 are listed in appendixes G-8 and G-9, respectively. It is interesting to note that the households adopting production behavior *SB* always have high efficiency levels. However, this does not mean that the households in group *SA* perform worse than those in group *SB*. Since the efficiency levels of the households in different groups are calculated based on different reference technologies, we can only compare the households' efficiency levels within the group, not across groups.

7.3.3 Caste Ranking versus Social Learning

Past studies point out that the decision makers do learn from others, but they only learn from some particular learning partners. When the new technology arrives to an industry, the decision maker may learn from early adopters [Besley and Case (1993)] or from the peers with similar socio-economic background. For Indian villages in ICRISAT data, Ueda (2002) finds that the farmers tend to learn from those who have the same household size, Conley and Udry (2001) points out that the farmers in Ghana learn from the ones in the same social communication networks.

The caste rank system, an indicator of the household's socioeconomic status, influences individuals' behavior in India society. Although the caste system may be more flexible in modern India, it still plays an important role in the rural areas. The data used for the empirical application in this study comes from Aurepalle village in Andhra Pradesh state, which is believed to follow the caste system more strictly than other villages in ICRISAT data (Walker and Ryan, 1990). The empirical results indicate that some households, especially the households from lower caste rank, do not adjust their production technology as quickly as other households from higher caste rank when the HYV castor is introduced to the village. In addition, there are some particular households always having the same production pattern, such as quitting castor production together (households 40, 41, 42) or switching to another production technology together (households 53, 54, 57, 58). It is interesting to note that these households with the same production pattern are from the same caste rank, which reveals the importance of caste rank in agricultural production in this community.

Since the households from the same rank of caste have more opportunities to interact with each other, they have a greater opportunity to exchange the production information including the information about production technology and the new crop variety. The frequent social activities within the caste rank provide the opportunity for social learning. Thus, the importance of the caste rank in the production behavior represents the importance of social learning in production decision.

7.3.4 Production Efficiency and Learning Gains

Although the improvement of the production efficiency is expected to reveal the learning gain in this study, it is not strongly demonstrated in the empirical results. There are two possible reasons for this. The additional knowledge, the outcome of the knowledge management behavior, is not only associated with the decision maker's learning decision in the information acquisition and knowledge updating phases, but also influenced by other factors not controlled by the decision maker, such as the possibility of knowledge depreciation. The possibility of knowledge depreciation raises the uncertainty of knowledge accumulation which reflects on the production performance. Furthermore, while production performance is associated with the decision maker's knowledge of the production technology, this does not mean that having the same knowledge base or taking the same knowledge management behavior result in the same production performance. A decision maker with a poor ability to implement his knowledge into production behavior may also lead to poor production performance even though he always engages in social learning and accumulates significant knowledge.

7.4 Concluding Comments

The latent class stochastic frontier model is introduced in this chapter to estimate the households' production functions by using ICRISAT (International Crops Research Institute for the Semi-Arid Tropics) India data. In this model, the household-specific characteristics are defined and the group-specific production functions are estimated. Thus, the households growing the same crop are allowed to adopt different production technologies, and the households classified in the same group are assumed to adopt the same production technology. The LCSFM is used to reveal the connection between the social learning and the households' production behaviors since the group-membership assignment shows which households adopt the same production technology, and the common characteristics of the households in the same group indicate whether social learning is the reason behind the heterogeneity of production technology.

In this chapter, the castor and sorghum data in Aurepalle village are estimated separately. In both cases, two group-specific production functions are distinguished, representing the heterogeneity of the production technology. Once the group-membership is assigned, the production efficiency of each household which represents the heterogeneity within the group is calculated. The empirical results indicate that the caste rank plays as important role in explaining the households' production behavior especially after the HYV caster is introduced. The households in the lower caste rank adjust their production behavior more slowly than the ones from the higher rank. Furthermore, some households in the highest caste rank always make the same production decisions. Since the caste rank is an index of the household's socioeconomic background which regulates individuals' daily behavior in rural areas in India, the households from the same caste rank are more likely to interact with each other. The social activities within the caste form a social communication network providing the opportunities for social learning. Thus, the importance of caste rank in production behavior shows the importance of the social learning in production decisions.

CHAPTER 8

CONCLUSIONS AND IMPLICATIONS

Knowledge management is a learning process involving the information acquisition phase and the knowledge updating phase. In the information acquisition phase, the decision maker collects internal information from past experience (e.g., learning-by-doing) and external information via social learning activities (e.g., by engaging in conversation, cooperation, or collaboration). After sufficient information is collected, the decision maker can employ the learning mechanism by himself or use the joint learning mechanism with other decision makers so that the information collected is transferred into additional knowledge and the knowledge base is updated. Social learning may be observed in both phases which provides the decision maker different options of learning and complex knowledge management schemes. Different patterns of knowledge management are associated with different learning costs and different intensity of social learning. In addition, they result in different learning outcomes. How the decision maker chooses among different knowledge management schemes and whether the decision maker should devote effort to learning depend on the associated learning costs and benefits.

This study focuses on investigating the decision maker's learning behavior and the connection between the production heterogeneity and the social learning. The importance of the learning benefits and costs in learning decision is revealed by the mathematical dynamic optimization model in which the firm's profit-maximizing problem is solved under its production and knowledge management constraints. The optimization conditions point out the elements of marginal benefit and marginal cost associated with the knowledge management behavior and guide the decision maker in allocating the physical inputs and efforts for knowledge management. A deterministic dynamic programming model is constructed by streamlining the mathematical model and the numerical results indicate that the decision maker adopts different learning strategies (e.g. *always learn*, *wait to learn*, *learn-in-bursts*, and *quit learning*) as he knows about the changes of the output price in different periods. However, the decision maker's learning strategies alter when the timing of the output price change is unsure (market uncertainty). The decision maker faces several possible states because of the market uncertainty, and for each possible state, there is one corresponding learning decision. Similarly, the decision maker's learning strategy under technological uncertainty is revealed from the numerical results as the knowledge base accumulation is assumed to be stochastic. Again, the decision maker faces several possible states because of the technological uncertainty, and the learning decision will not be made until the actual state is revealed.

The connection between production heterogeneity and social learning is addressed by a latent class stochastic frontier model (LCSFM) which is introduced to estimate the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT) India data, and investigate whether social learning is the reason causing the technology heterogeneity across households. The LCSFM is used to for the empirical application in this study because the households are classified into different groups by the model, and the households in the same group are assumed to adopt the same production technology. Thus, the households' group-membership assignment points out which houses adopt the same production technology, and the common characteristics among the households in the same group provide an opportunity to explain the technological heterogeneity across households. The empirical results indicate that the caste rank, an indicator for socioeconomic background, plays an important role in household's production decision especially when the high-yield-variety castor is introduced to the Aurepalle village in 1981. The households from the lower caste ranks do not adjust their production behavior for the new variety as quickly as the households from the higher cast ranks. In addition, some of the households from higher cast ranks show that they always make the same production technology decision during the years 1975-1984. Since the caste rank regulates people's social life especially in rural area in India, the households from the same caste rank are likely to interact with each other more often than they interact with the households from other castes. The social activities provide the chance for social learning and form the social communication network so that the households can exchange information and gain additional knowledge. Thus the importance of caste rank in production decision presents the importance of social learning in production behavior.

There are three conclusions from this study. First, the learning strategies observed in the numerical model indicate that the knowledge management behavior is a dynamic process since it involves the learning decisions over time. The decision maker chooses different learning strategies under different conditions indicating that both continuous learning (the *always learn* strategy) and discontinuous learning (the *wait to learn*, *quit learning*, and *learn-in-bursts* strategies) can be optimal learning decisions as long as the associated learning costs and benefits are considered. Second, the shadow value of knowledge is defined as the difference of the value functions if the decision maker has one more unit of knowledge and drives the knowledge management decisions over time. The numerical results indicate that the shadow value of knowledge over time follows different trajectories as the decision maker adopts different learning strategies. In addition, the market and technological factors influencing the learning costs and benefits alter the optimal learning strategy as well as the shadow value of knowledge. Finally, the empirical results indicate the importance between social learning and the production decisions. It is important that the government provides the agricultural extension service to offer information associated agricultural production, and encourages farmers to develop learning groups so that farmers have more opportunities to communicate with each other and the effect of social learning can be enhanced.

Suggestions for Future Research

This study is constructed under the assumptions that the output and input prices are given; a single firm's production behavior does not influence the market; in addition, the individual decision maker's learning strategy is not influenced by other decision maker's learning decisions. Future study may reconsider the decision maker's learning decisions under the case where there are very few firms in the industry (e.g., oligopoly). More assumptions need to be made before proceeding further analysis since the decision maker's knowledge level and his learning decision may play important roles in other decision makers' decision making. For example, is the learning strategy adopted by each decision maker is known by other decision makers? Can the additional knowledge generated by one decision maker's learning effort be observed or learned by other decision makers? Theses assumptions need to be considered carefully because it relates to the learning costs and benefits. If knowledge is only known by the one who generates it, then this decision maker's learning decision only benefits himself. As two firms compete with each other, one decision maker's decision to learn may encourage other decision makers to devote effort to learning.

The decision maker's willingness to exchange the information may be another research issue. Considering two decision makers, both have the intention to learn from each other and both can choose to share true or untrue information as they interact with each other. The cooperative relationship between the two decision makers exists only when both of them provide true information, which makes both decision makers benefit from this social learning. One can solve this question from a game theory perspective. Whether the decision makers choose to cooperate or not depends on the payoff functions assigned to each action. In addition, a dynamic game can be constructed to investigate the decision makers' strategies and their cooperative relationship over time.

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APPENDIX A:

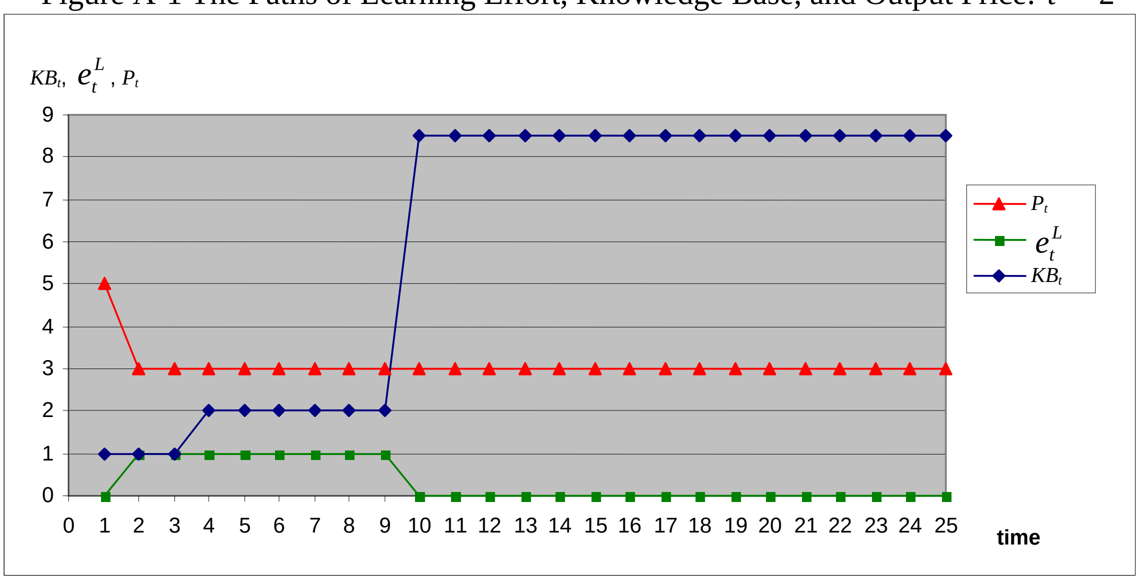
THE NUMERICAL RESULT FOR PRICE-DECREASING CASES

($P_t = 5 \quad for \quad t < t^D; \;\; P_t = 3 \quad for \quad t^D \le t \le 25$)

APPENDIX A-1 Deterministic Numerical Results: $t^D = 2$

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	0	1	1.414214	0	1	5.071068
2	1	0	1	0.5	0.812252	1	1	-0.56324
3	1	1	1	0.5	0.812252	2	2	-0.56324
4	2	2	1	0.5	1.004591	3	2	0.013773
5	2	3	1	0.5	1.004591	4	2	0.013773
6	2	4	1	0.5	1.004591	5	2	0.013773
7	2	5	1	0.5	1.004591	6	2	0.013773
8	2	6	1	0.5	1.004591	7	2	0.013773
9	2	7	1	0.5	1.004591	8	8.5	0.013773
10	8.5	8	0	1.5	3.430797	8	8.5	7.292391
11	8.5	8	0	1.5	3.430797	8	8.5	7.292391
12	8.5	8	0	1.5	3.430797	8	8.5	7.292391
13	8.5	8	0	1.5	3.430797	8	8.5	7.292391
14	8.5	8	0	1.5	3.430797	8	8.5	7.292391
15	8.5	8	0	1.5	3.430797	8	8.5	7.292391
16	8.5	8	0	1.5	3.430797	8	8.5	7.292391
17	8.5	8	0	1.5	3.430797	8	8.5	7.292391
18	8.5	8	0	1.5	3.430797	8	8.5	7.292391
19	8.5	8	0	1.5	3.430797	8	8.5	7.292391
20	8.5	8	0	1.5	3.430797	8	8.5	7.292391
21	8.5	8	0	1.5	3.430797	8	8.5	7.292391
22	8.5	8	0	1.5	3.430797	8	8.5	7.292391
23	8.5	8	0	1.5	3.430797	8	8.5	7.292391
24	8.5	8	0	1.5	3.430797	8	8.5	7.292391
25	8.5	8	0	1.5	3.430797	8	8.5	7.292391

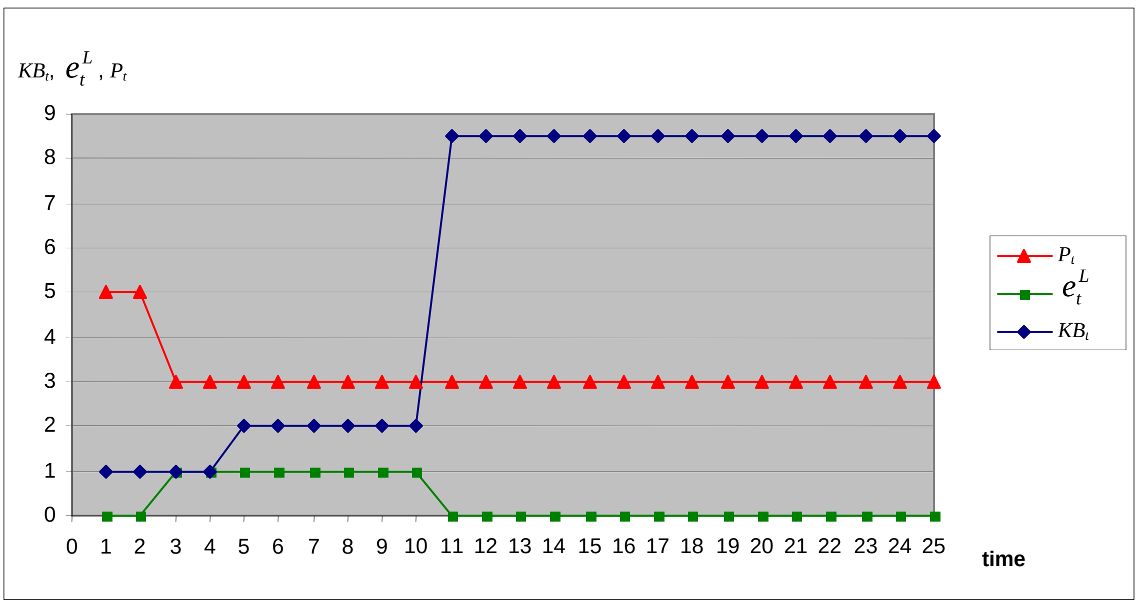
Figure A-1 The Paths of Learning Effort, Knowledge Base, and Output Price: t^D =2



APPENDIX A-2 Deterministic Numerical Results: $t^D=3$

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	0	1	1.414214	0	1	5.071068
2	1	0	0	1	1.414214	0	1	5.071068
3	1	0	1	0.5	0.812252	1	1	-0.56324
4	1	1	1	0.5	0.812252	2	2	-0.56324
5	2	2	1	0.5	1.004591	3	2	0.013773
6	2	3	1	0.5	1.004591	4	2	0.013773
7	2	4	1	0.5	1.004591	5	2	0.013773
8	2	5	1	0.5	1.004591	6	2	0.013773
9	2	6	1	0.5	1.004591	7	2	0.013773
10	2	7	1	0.5	1.004591	8	8.5	0.013773
11	8.5	8	0	1.5	3.430797	8	8.5	7.292391
12	8.5	8	0	1.5	3.430797	8	8.5	7.292391
13	8.5	8	0	1.5	3.430797	8	8.5	7.292391
14	8.5	8	0	1.5	3.430797	8	8.5	7.292391
15	8.5	8	0	1.5	3.430797	8	8.5	7.292391
16	8.5	8	0	1.5	3.430797	8	8.5	7.292391
17	8.5	8	0	1.5	3.430797	8	8.5	7.292391
18	8.5	8	0	1.5	3.430797	8	8.5	7.292391
19	8.5	8	0	1.5	3.430797	8	8.5	7.292391
20	8.5	8	0	1.5	3.430797	8	8.5	7.292391
21	8.5	8	0	1.5	3.430797	8	8.5	7.292391
22	8.5	8	0	1.5	3.430797	8	8.5	7.292391
23	8.5	8	0	1.5	3.430797	8	8.5	7.292391
24	8.5	8	0	1.5	3.430797	8	8.5	7.292391
25	8.5	8	0	1.5	3.430797	8	8.5	7.292391

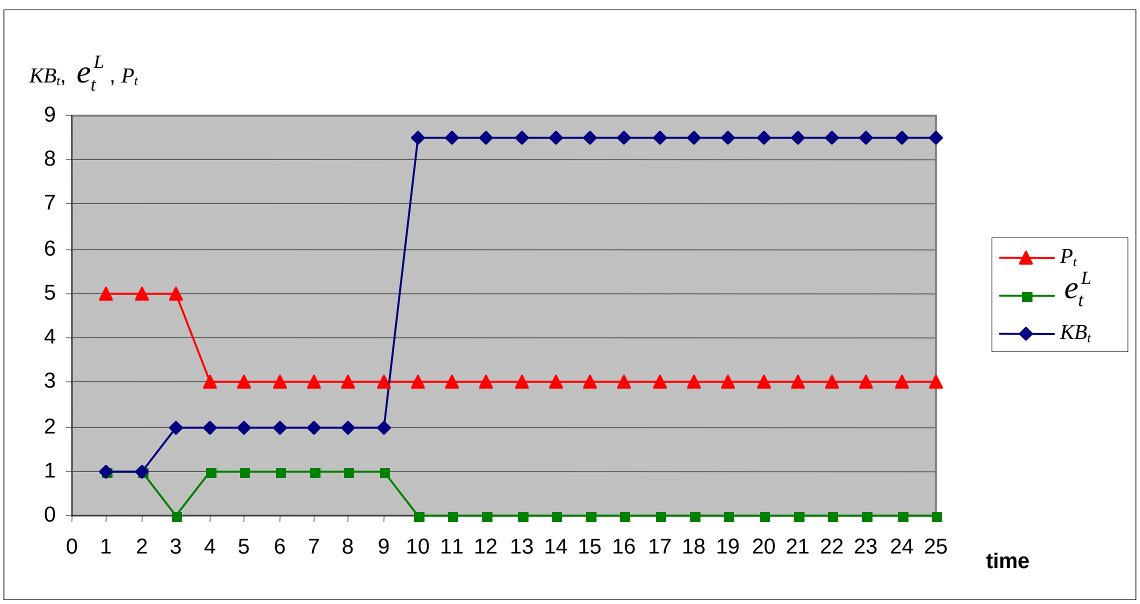
Figure A-2 The Paths of Learning Effort, Knowledge Base, and Output Price: $t^D=3$



APPENDIX A-3 Deterministic Numerical Results: t^D =4

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	0.5	0.812252	2	2	1.061262
3	2	2	0	1.5	1.975334	2	2	6.876672
4	2	2	1	0.5	1.004591	3	2	0.013773
5	2	3	1	0.5	1.004591	4	2	0.013773
6	2	4	1	0.5	1.004591	5	2	0.013773
7	2	5	1	0.5	1.004591	6	2	0.013773
8	2	6	1	0.5	1.004591	7	2	0.013773
9	2	7	1	0.5	1.004591	8	8.5	0.013773
10	8.5	8	0	1.5	3.430797	8	8.5	7.292391
11	8.5	8	0	1.5	3.430797	8	8.5	7.292391
12	8.5	8	0	1.5	3.430797	8	8.5	7.292391
13	8.5	8	0	1.5	3.430797	8	8.5	7.292391
14	8.5	8	0	1.5	3.430797	8	8.5	7.292391
15	8.5	8	0	1.5	3.430797	8	8.5	7.292391
16	8.5	8	0	1.5	3.430797	8	8.5	7.292391
17	8.5	8	0	1.5	3.430797	8	8.5	7.292391
18	8.5	8	0	1.5	3.430797	8	8.5	7.292391
19	8.5	8	0	1.5	3.430797	8	8.5	7.292391
20	8.5	8	0	1.5	3.430797	8	8.5	7.292391
21	8.5	8	0	1.5	3.430797	8	8.5	7.292391
22	8.5	8	0	1.5	3.430797	8	8.5	7.292391
23	8.5	8	0	1.5	3.430797	8	8.5	7.292391
24	8.5	8	0	1.5	3.430797	8	8.5	7.292391
25	8.5	8	0	1.5	3.430797	8	8.5	7.292391

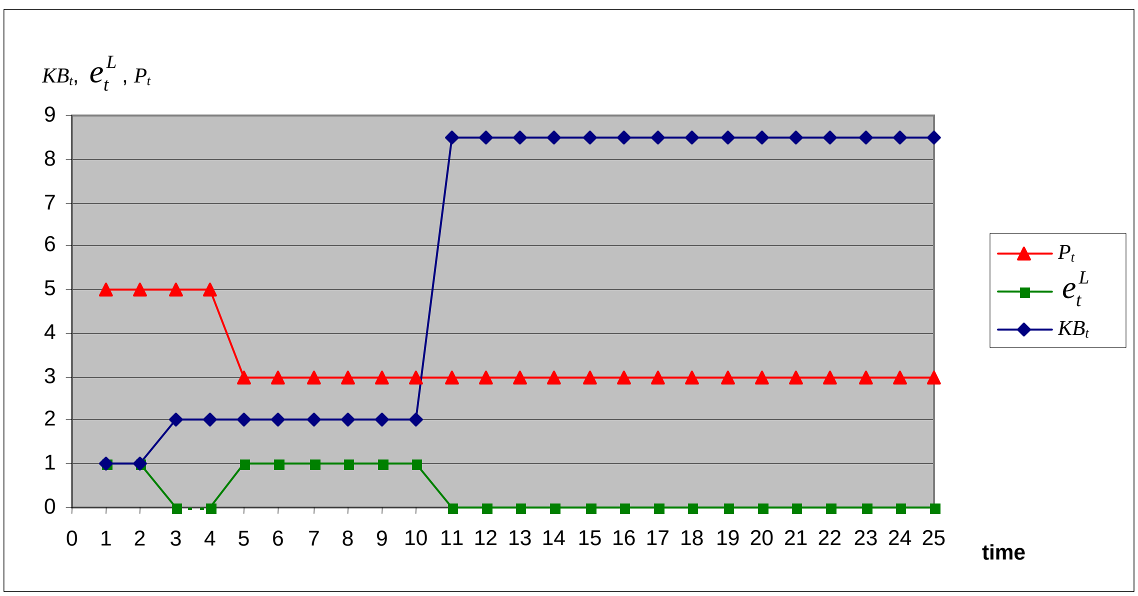
Figure A-3 The Paths of Learning Effort, Knowledge Base, and Output Price: t^D =4



APPENDIX A-4 Deterministic Numerical Results: $t^D = 5$

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	0.5	0.812252	2	2	1.061262
3	2	2	0	1.5	1.975334	2	2	6.876672
4	2	2	0	1.5	1.975334	2	2	6.876672
5	2	2	1	0.5	1.004591	3	2	0.013773
6	2	3	1	0.5	1.004591	4	2	0.013773
7	2	4	1	0.5	1.004591	5	2	0.013773
8	2	5	1	0.5	1.004591	6	2	0.013773
9	2	6	1	0.5	1.004591	7	2	0.013773
10	2	7	1	0.5	1.004591	8	8.5	0.013773
11	8.5	8	0	1.5	3.430797	8	8.5	7.292391
12	8.5	8	0	1.5	3.430797	8	8.5	7.292391
13	8.5	8	0	1.5	3.430797	8	8.5	7.292391
14	8.5	8	0	1.5	3.430797	8	8.5	7.292391
15	8.5	8	0	1.5	3.430797	8	8.5	7.292391
16	8.5	8	0	1.5	3.430797	8	8.5	7.292391
17	8.5	8	0	1.5	3.430797	8	8.5	7.292391
18	8.5	8	0	1.5	3.430797	8	8.5	7.292391
19	8.5	8	0	1.5	3.430797	8	8.5	7.292391
20	8.5	8	0	1.5	3.430797	8	8.5	7.292391
21	8.5	8	0	1.5	3.430797	8	8.5	7.292391
22	8.5	8	0	1.5	3.430797	8	8.5	7.292391
23	8.5	8	0	1.5	3.430797	8	8.5	7.292391
24	8.5	8	0	1.5	3.430797	8	8.5	7.292391
25	8.5	8	0	1.5	3.430797	8	8.5	7.292391

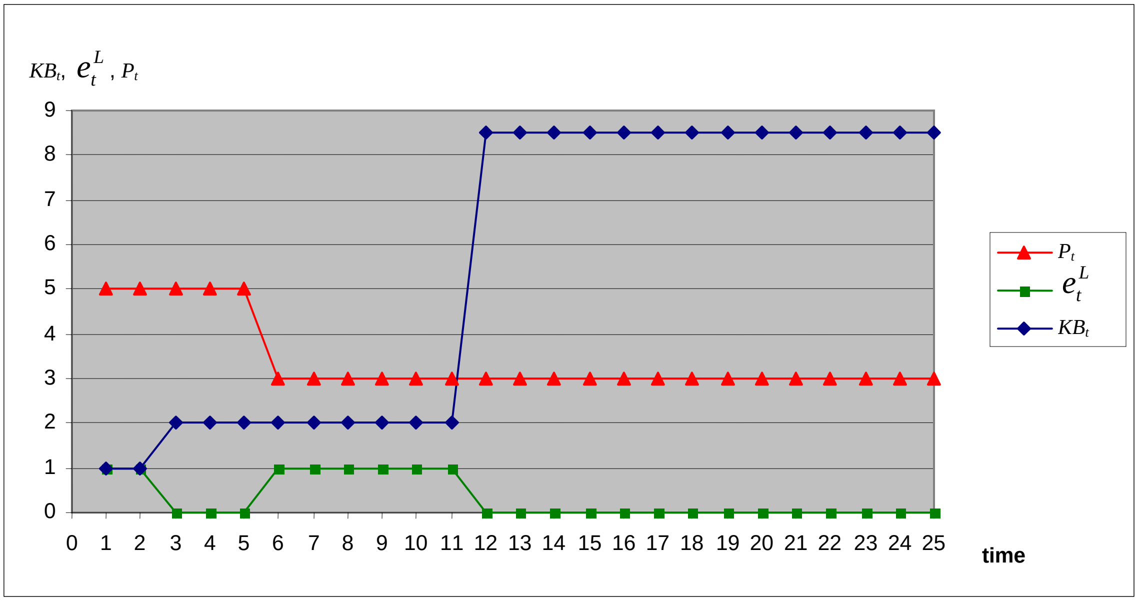
Figure A-4 The Paths of Learning Effort, Knowledge Base, and Output Price: $t^D = 5$



APPENDIX A-5 Deterministic Numerical Results: $t^D = 6$

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	0.5	0.812252	2	2	1.061262
3	2	2	0	1.5	1.975334	2	2	6.876672
4	2	2	0	1.5	1.975334	2	2	6.876672
5	2	2	0	1.5	1.975334	2	2	6.876672
6	2	2	1	0.5	1.004591	3	2	0.013773
7	2	3	1	0.5	1.004591	4	2	0.013773
8	2	4	1	0.5	1.004591	5	2	0.013773
9	2	5	1	0.5	1.004591	6	2	0.013773
10	2	6	1	0.5	1.004591	7	2	0.013773
11	2	7	1	0.5	1.004591	8	8.5	0.013773
12	8.5	8	0	1.5	3.430797	8	8.5	7.292391
13	8.5	8	0	1.5	3.430797	8	8.5	7.292391
14	8.5	8	0	1.5	3.430797	8	8.5	7.292391
15	8.5	8	0	1.5	3.430797	8	8.5	7.292391
16	8.5	8	0	1.5	3.430797	8	8.5	7.292391
17	8.5	8	0	1.5	3.430797	8	8.5	7.292391
18	8.5	8	0	1.5	3.430797	8	8.5	7.292391
19	8.5	8	0	1.5	3.430797	8	8.5	7.292391
20	8.5	8	0	1.5	3.430797	8	8.5	7.292391
21	8.5	8	0	1.5	3.430797	8	8.5	7.292391
22	8.5	8	0	1.5	3.430797	8	8.5	7.292391
23	8.5	8	0	1.5	3.430797	8	8.5	7.292391
24	8.5	8	0	1.5	3.430797	8	8.5	7.292391
25	8.5	8	0	1.5	3.430797	8	8.5	7.292391

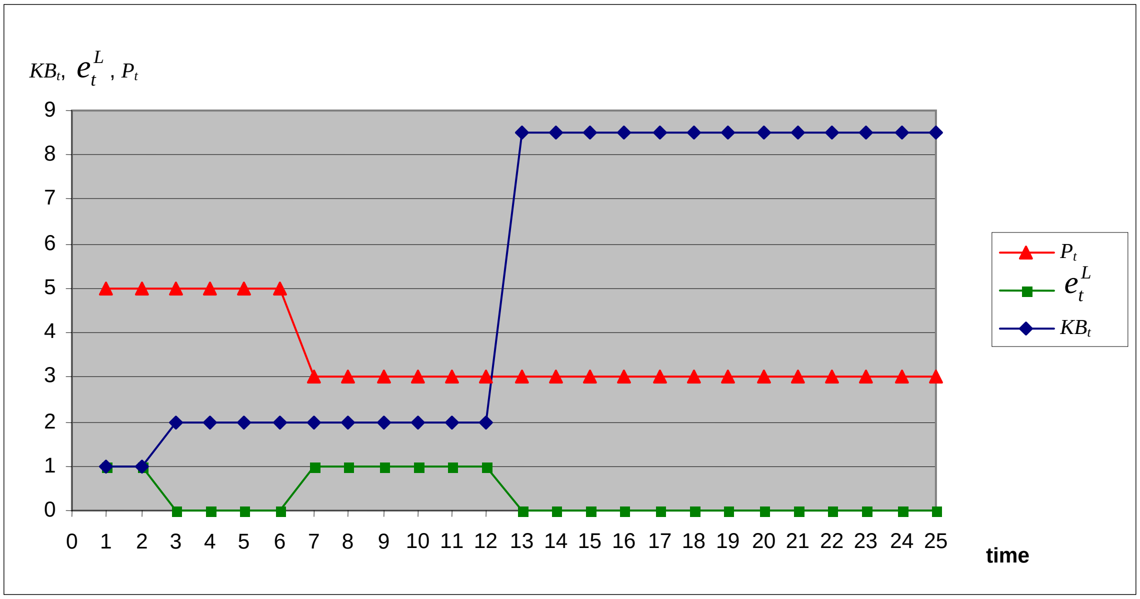
Figure A-5 The Paths of Learning Effort, Knowledge Base, and Output Price: $t^D = 6$



APPENDIX A-6 Deterministic Numerical Results: $t^D=7$

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	0.5	0.812252	2	2	1.061262
3	2	2	0	1.5	1.975334	2	2	6.876672
4	2	2	0	1.5	1.975334	2	2	6.876672
5	2	2	0	1.5	1.975334	2	2	6.876672
6	2	2	0	1.5	1.975334	2	2	6.876672
7	2	2	1	0.5	1.004591	3	2	0.013773
8	2	3	1	0.5	1.004591	4	2	0.013773
9	2	4	1	0.5	1.004591	5	2	0.013773
10	2	5	1	0.5	1.004591	6	2	0.013773
11	2	6	1	0.5	1.004591	7	2	0.013773
12	2	7	1	0.5	1.004591	8	8.5	0.013773
13	8.5	8	0	1.5	3.430797	8	8.5	7.292391
14	8.5	8	0	1.5	3.430797	8	8.5	7.292391
15	8.5	8	0	1.5	3.430797	8	8.5	7.292391
16	8.5	8	0	1.5	3.430797	8	8.5	7.292391
17	8.5	8	0	1.5	3.430797	8	8.5	7.292391
18	8.5	8	0	1.5	3.430797	8	8.5	7.292391
19	8.5	8	0	1.5	3.430797	8	8.5	7.292391
20	8.5	8	0	1.5	3.430797	8	8.5	7.292391
21	8.5	8	0	1.5	3.430797	8	8.5	7.292391
22	8.5	8	0	1.5	3.430797	8	8.5	7.292391
23	8.5	8	0	1.5	3.430797	8	8.5	7.292391
24	8.5	8	0	1.5	3.430797	8	8.5	7.292391
25	8.5	8	0	1.5	3.430797	8	8.5	7.292391

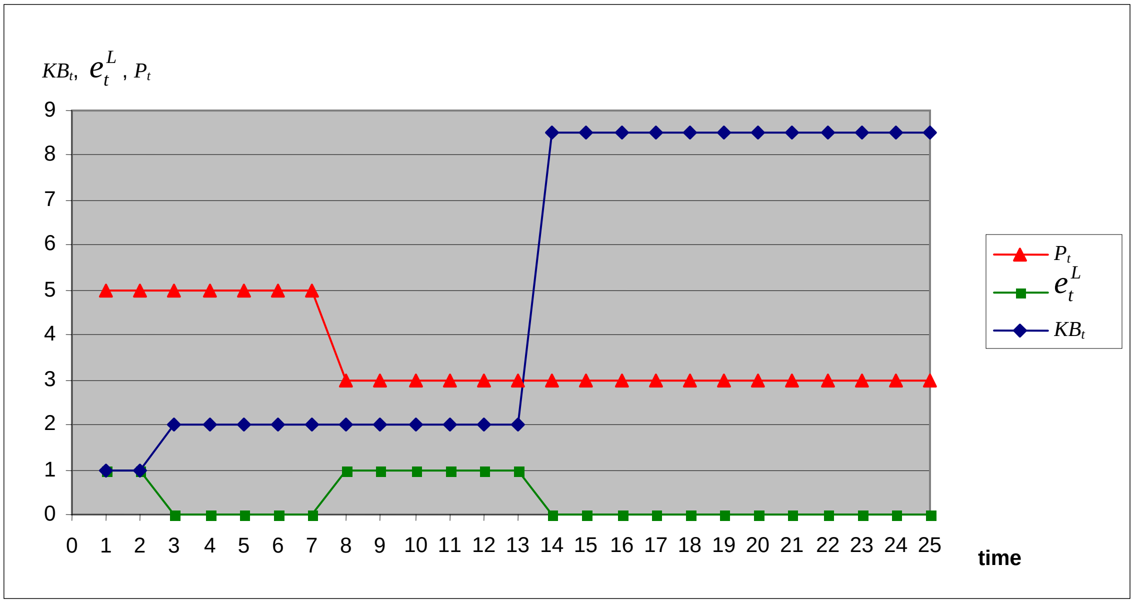
Figure A-6 The Paths of Learning Effort, Knowledge Base, and Output Price: $t^D=7$



APPENDIX A-7 Deterministic Numerical Results: $t^D = 8$

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	0.5	0.812252	2	2	1.061262
3	2	2	0	1.5	1.975334	2	2	6.876672
4	2	2	0	1.5	1.975334	2	2	6.876672
5	2	2	0	1.5	1.975334	2	2	6.876672
6	2	2	0	1.5	1.975334	2	2	6.876672
7	2	2	0	1.5	1.975334	2	2	6.876672
8	2	2	1	0.5	1.004591	3	2	0.013773
9	2	3	1	0.5	1.004591	4	2	0.013773
10	2	4	1	0.5	1.004591	5	2	0.013773
11	2	5	1	0.5	1.004591	6	2	0.013773
12	2	6	1	0.5	1.004591	7	2	0.013773
13	2	7	1	0.5	1.004591	8	8.5	0.013773
14	8.5	8	0	1.5	3.430797	8	8.5	7.292391
15	8.5	8	0	1.5	3.430797	8	8.5	7.292391
16	8.5	8	0	1.5	3.430797	8	8.5	7.292391
17	8.5	8	0	1.5	3.430797	8	8.5	7.292391
18	8.5	8	0	1.5	3.430797	8	8.5	7.292391
19	8.5	8	0	1.5	3.430797	8	8.5	7.292391
20	8.5	8	0	1.5	3.430797	8	8.5	7.292391
21	8.5	8	0	1.5	3.430797	8	8.5	7.292391
22	8.5	8	0	1.5	3.430797	8	8.5	7.292391
23	8.5	8	0	1.5	3.430797	8	8.5	7.292391
24	8.5	8	0	1.5	3.430797	8	8.5	7.292391
25	8.5	8	0	1.5	3.430797	8	8.5	7.292391

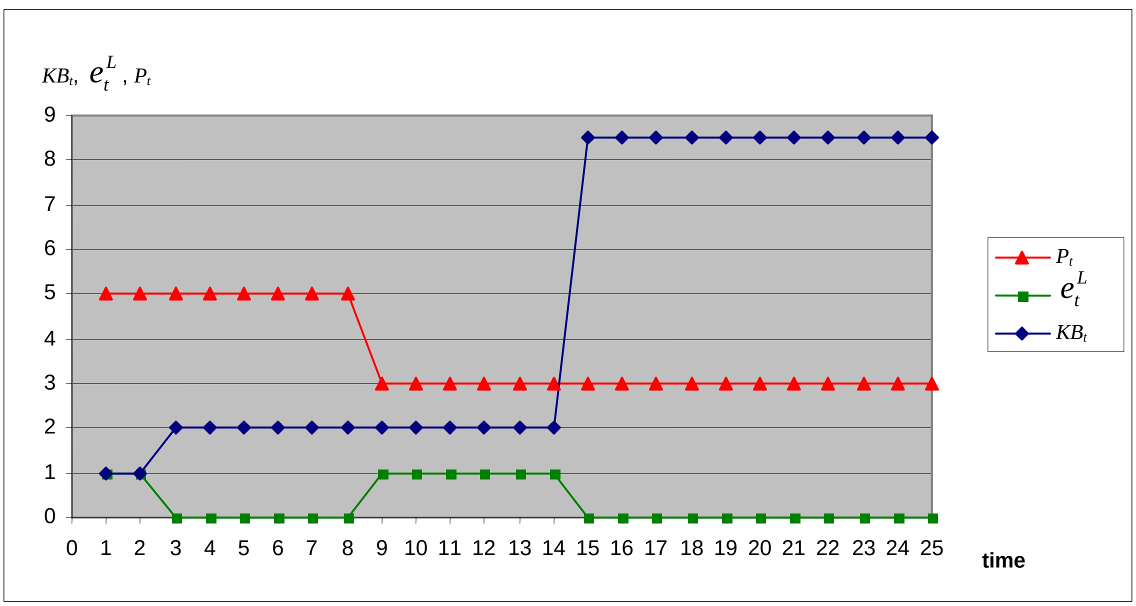
Figure A-7 The Paths of Learning Effort, Knowledge Base, and Output Price: $t^D = 8$



APPENDIX A-8 Deterministic Numerical Results: $t^D = 9$

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	0.5	0.812252	2	2	1.061262
3	2	2	0	1.5	1.975334	2	2	6.876672
4	2	2	0	1.5	1.975334	2	2	6.876672
5	2	2	0	1.5	1.975334	2	2	6.876672
6	2	2	0	1.5	1.975334	2	2	6.876672
7	2	2	0	1.5	1.975334	2	2	6.876672
8	2	2	0	1.5	1.975334	2	2	6.876672
9	2	2	1	0.5	1.004591	3	2	0.013773
10	2	3	1	0.5	1.004591	4	2	0.013773
11	2	4	1	0.5	1.004591	5	2	0.013773
12	2	5	1	0.5	1.004591	6	2	0.013773
13	2	6	1	0.5	1.004591	7	2	0.013773
14	2	7	1	0.5	1.004591	8	8.5	0.013773
15	8.5	8	0	1.5	3.430797	8	8.5	7.292391
16	8.5	8	0	1.5	3.430797	8	8.5	7.292391
17	8.5	8	0	1.5	3.430797	8	8.5	7.292391
18	8.5	8	0	1.5	3.430797	8	8.5	7.292391
19	8.5	8	0	1.5	3.430797	8	8.5	7.292391
20	8.5	8	0	1.5	3.430797	8	8.5	7.292391
21	8.5	8	0	1.5	3.430797	8	8.5	7.292391
22	8.5	8	0	1.5	3.430797	8	8.5	7.292391
23	8.5	8	0	1.5	3.430797	8	8.5	7.292391
24	8.5	8	0	1.5	3.430797	8	8.5	7.292391
25	8.5	8	0	1.5	3.430797	8	8.5	7.292391

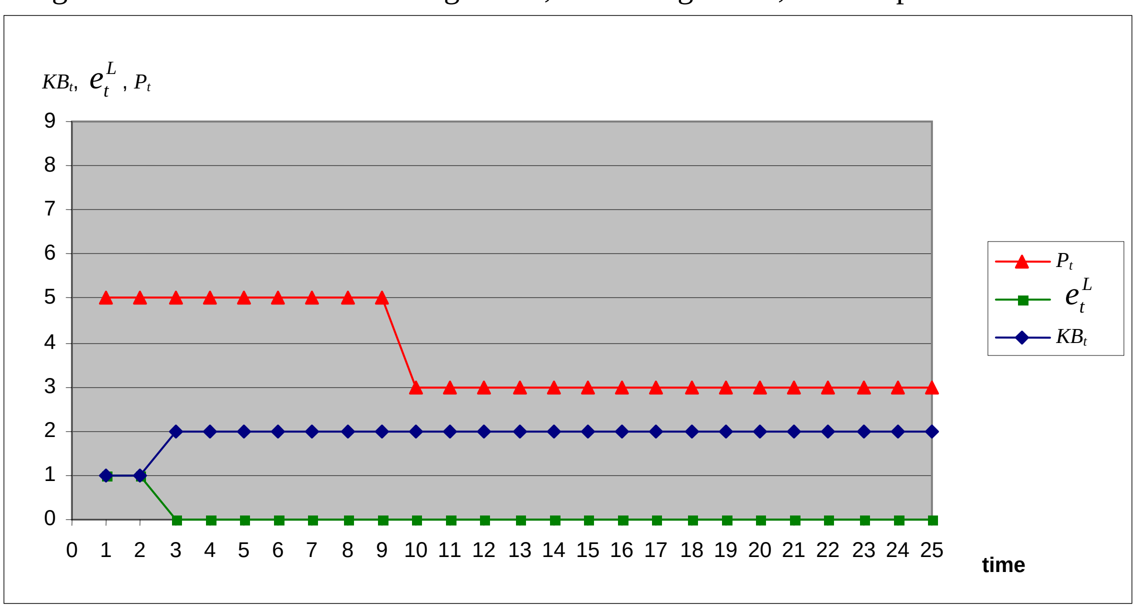
Figure A-8 The Paths of Learning Effort, Knowledge Base, and Output Price: $t^D = 9$



APPENDIX A-9 Deterministic Numerical Results: $t^D = 10$

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	0.5	0.812252	2	2	1.061262
3	2	2	0	1.5	1.975334	2	2	6.876672
4	2	2	0	1.5	1.975334	2	2	6.876672
5	2	2	0	1.5	1.975334	2	2	6.876672
6	2	2	0	1.5	1.975334	2	2	6.876672
7	2	2	0	1.5	1.975334	2	2	6.876672
8	2	2	0	1.5	1.975334	2	2	6.876672
9	2	2	0	1.5	1.975334	2	2	6.876672
10	2	2	0	0.5	1.420706	2	2	3.262118
11	2	2	0	0.5	1.420706	2	2	3.262118
12	2	2	0	0.5	1.420706	2	2	3.262118
13	2	2	0	0.5	1.420706	2	2	3.262118
14	2	2	0	0.5	1.420706	2	2	3.262118
15	2	2	0	0.5	1.420706	2	2	3.262118
16	2	2	0	0.5	1.420706	2	2	3.262118
17	2	2	0	0.5	1.420706	2	2	3.262118
18	2	2	0	0.5	1.420706	2	2	3.262118
19	2	2	0	0.5	1.420706	2	2	3.262118
20	2	2	0	0.5	1.420706	2	2	3.262118
21	2	2	0	0.5	1.420706	2	2	3.262118
22	2	2	0	0.5	1.420706	2	2	3.262118
23	2	2	0	0.5	1.420706	2	2	3.262118
24	2	2	0	0.5	1.420706	2	2	3.262118
25	2	2	0	0.5	1.420706	2	2	3.262118

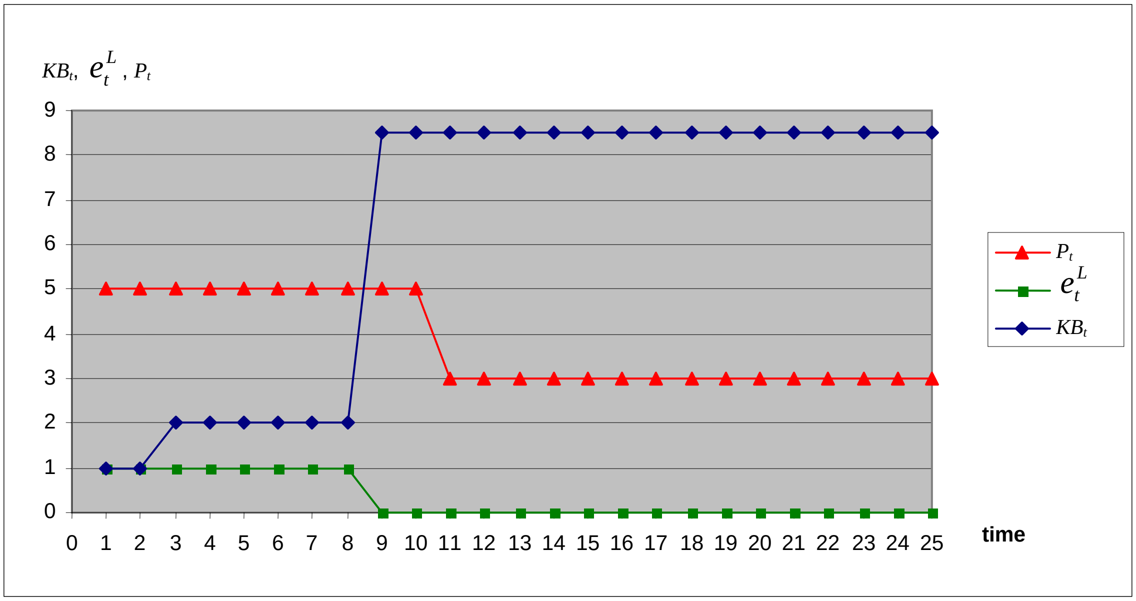
Figure A-9 The Paths of Learning Effort, Knowledge Base, and Output Price: $t^D = 10$



APPENDIX A-10 Deterministic Numerical Results: $t^D = 11$

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	0.5	0.812252	2	2	1.061262
3	2	2	1	1	1.236796	3	2	2.183982
4	2	3	1	1	1.236796	4	2	2.183982
5	2	4	1	1	1.236796	5	2	2.183982
6	2	5	1	1	1.236796	6	2	2.183982
7	2	6	1	1	1.236796	7	2	2.183982
8	2	7	1	1	1.236796	8	8.5	2.183982
9	8.5	8	0	3	4.223806	8	8.5	15.11903
10	8.5	8	0	3	4.223806	8	8.5	15.11903
11	8.5	8	0	1.5	3.430797	8	8.5	7.292391
12	8.5	8	0	1.5	3.430797	8	8.5	7.292391
13	8.5	8	0	1.5	3.430797	8	8.5	7.292391
14	8.5	8	0	1.5	3.430797	8	8.5	7.292391
15	8.5	8	0	1.5	3.430797	8	8.5	7.292391
16	8.5	8	0	1.5	3.430797	8	8.5	7.292391
17	8.5	8	0	1.5	3.430797	8	8.5	7.292391
18	8.5	8	0	1.5	3.430797	8	8.5	7.292391
19	8.5	8	0	1.5	3.430797	8	8.5	7.292391
20	8.5	8	0	1.5	3.430797	8	8.5	7.292391
21	8.5	8	0	1.5	3.430797	8	8.5	7.292391
22	8.5	8	0	1.5	3.430797	8	8.5	7.292391
23	8.5	8	0	1.5	3.430797	8	8.5	7.292391
24	8.5	8	0	1.5	3.430797	8	8.5	7.292391
25	8.5	8	0	1.5	3.430797	8	8.5	7.292391

Figure A-10 The Paths of Learning Effort, Knowledge Base, and Output Price: $t^D = 11$



APPENDIX B:

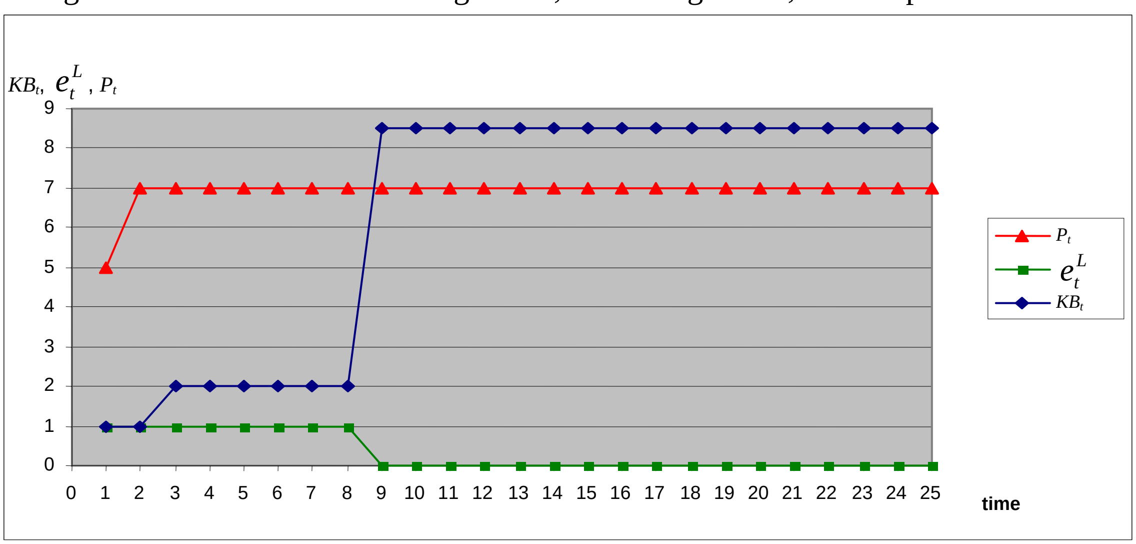
THE NUMERICAL RESULT FOR PRICE-INCREASING CASES

$(P_t = 5 \quad for \quad t < t^I; \;\; P_t = 7 \quad for \quad t^I \le t \le 25)$

APPENDIX B-1 Deterministic Numerical Results: $t^I = 2$

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	1	1	2	2	3
3	2	2	1	1.5	1.396772	3	2	4.777406
4	2	3	1	1.5	1.396772	4	2	4.777406
5	2	4	1	1.5	1.396772	5	2	4.777406
6	2	5	1	1.5	1.396772	6	2	4.777406
7	2	6	1	1.5	1.396772	7	2	4.777406
8	2	7	1	1.5	1.396772	8	8.5	4.777406
9	8.5	8	0	5	4.923326	8	8.5	24.46329
10	8.5	8	0	5	4.923326	8	8.5	24.46329
11	8.5	8	0	5	4.923326	8	8.5	24.46329
12	8.5	8	0	5	4.923326	8	8.5	24.46329
13	8.5	8	0	5	4.923326	8	8.5	24.46329
14	8.5	8	0	5	4.923326	8	8.5	24.46329
15	8.5	8	0	5	4.923326	8	8.5	24.46329
16	8.5	8	0	5	4.923326	8	8.5	24.46329
17	8.5	8	0	5	4.923326	8	8.5	24.46329
18	8.5	8	0	5	4.923326	8	8.5	24.46329
19	8.5	8	0	5	4.923326	8	8.5	24.46329
20	8.5	8	0	5	4.923326	8	8.5	24.46329
21	8.5	8	0	5	4.923326	8	8.5	24.46329
22	8.5	8	0	5	4.923326	8	8.5	24.46329
23	8.5	8	0	5	4.923326	8	8.5	24.46329
24	8.5	8	0	5	4.923326	8	8.5	24.46329
25	8.5	8	0	5	4.923326	8	8.5	24.46329

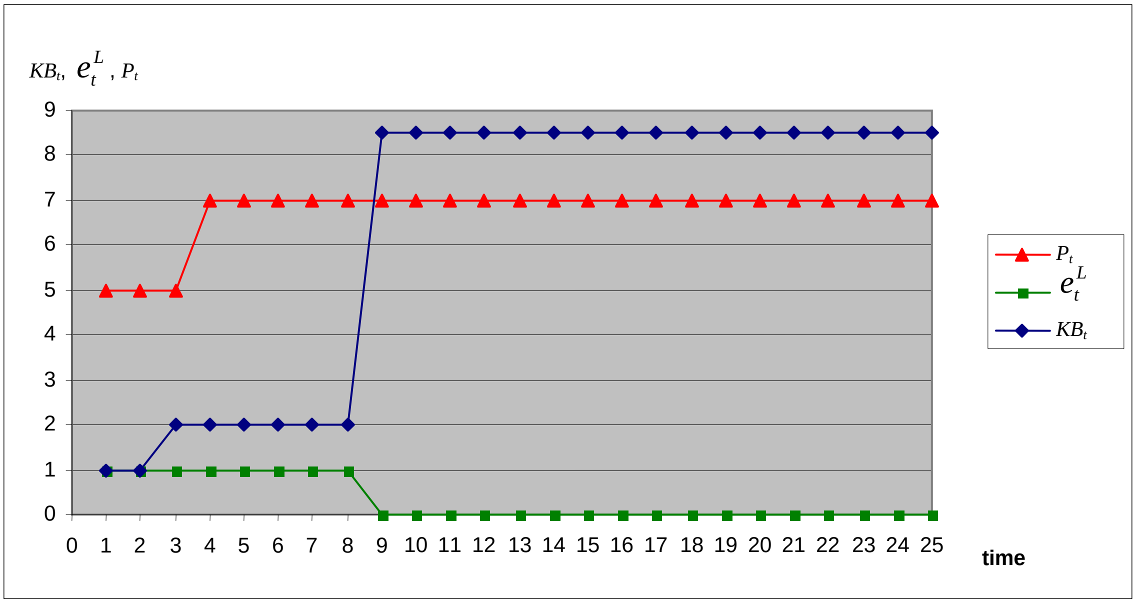
Figure B-1 The Paths of Learning Effort, Knowledge Base, and Output Price: $t^I = 2$



APPENDIX B-2 Deterministic Numerical Results: t^I =4

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	0.5	0.812252	2	2	1.061262
3	2	2	1	1	1.236796	3	2	2.183982
4	2	3	1	1.5	1.396772	4	2	4.777406
5	2	4	1	1.5	1.396772	5	2	4.777406
6	2	5	1	1.5	1.396772	6	2	4.777406
7	2	6	1	1.5	1.396772	7	2	4.777406
8	2	7	1	1.5	1.396772	8	8.5	4.777406
9	8.5	8	0	5	4.923326	8	8.5	24.46329
10	8.5	8	0	5	4.923326	8	8.5	24.46329
11	8.5	8	0	5	4.923326	8	8.5	24.46329
12	8.5	8	0	5	4.923326	8	8.5	24.46329
13	8.5	8	0	5	4.923326	8	8.5	24.46329
14	8.5	8	0	5	4.923326	8	8.5	24.46329
15	8.5	8	0	5	4.923326	8	8.5	24.46329
16	8.5	8	0	5	4.923326	8	8.5	24.46329
17	8.5	8	0	5	4.923326	8	8.5	24.46329
18	8.5	8	0	5	4.923326	8	8.5	24.46329
19	8.5	8	0	5	4.923326	8	8.5	24.46329
20	8.5	8	0	5	4.923326	8	8.5	24.46329
21	8.5	8	0	5	4.923326	8	8.5	24.46329
22	8.5	8	0	5	4.923326	8	8.5	24.46329
23	8.5	8	0	5	4.923326	8	8.5	24.46329
24	8.5	8	0	5	4.923326	8	8.5	24.46329
25	8.5	8	0	5	4.923326	8	8.5	24.46329

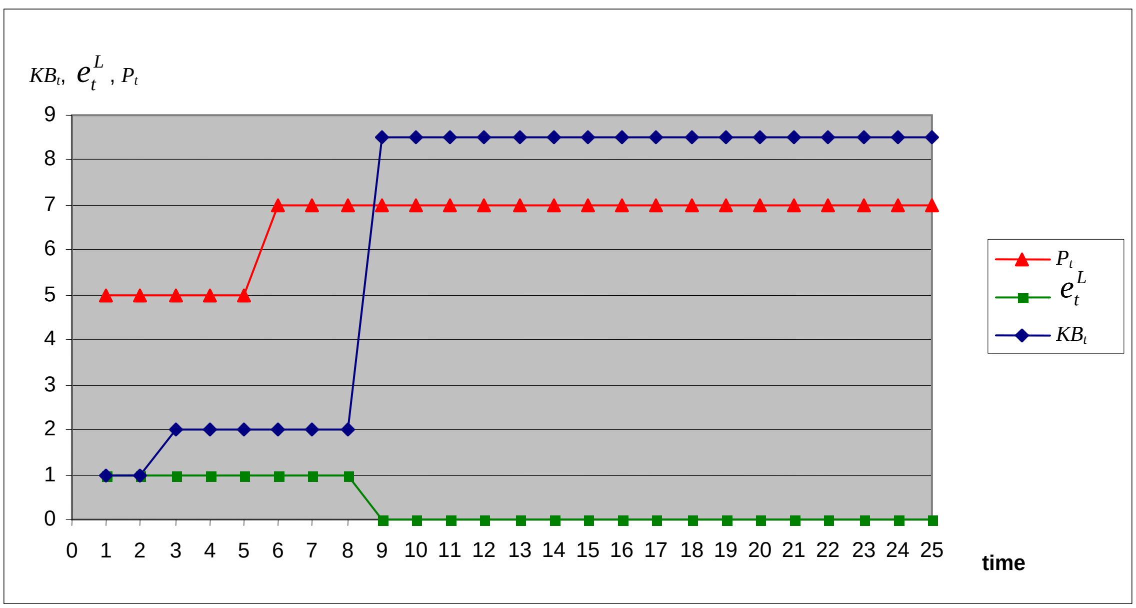
Figure B-2 The Paths of Learning Effort, Knowledge Base, and Output Price: t^I =4



APPENDIX B-3 Deterministic Numerical Results: $t^I = 6$

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	Output	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	0.5	0.812252	2	2	1.061262
3	2	2	1	1	1.236796	3	2	2.183982
4	2	3	1	1	1.236796	4	2	2.183982
5	2	4	1	1	1.236796	5	2	2.183982
6	2	5	1	1.5	1.396772	6	2	4.777406
7	2	6	1	1.5	1.396772	7	2	4.777406
8	2	7	1	1.5	1.396772	8	8.5	4.777406
9	8.5	8	0	5	4.923326	8	8.5	24.46329
10	8.5	8	0	5	4.923326	8	8.5	24.46329
11	8.5	8	0	5	4.923326	8	8.5	24.46329
12	8.5	8	0	5	4.923326	8	8.5	24.46329
13	8.5	8	0	5	4.923326	8	8.5	24.46329
14	8.5	8	0	5	4.923326	8	8.5	24.46329
15	8.5	8	0	5	4.923326	8	8.5	24.46329
16	8.5	8	0	5	4.923326	8	8.5	24.46329
17	8.5	8	0	5	4.923326	8	8.5	24.46329
18	8.5	8	0	5	4.923326	8	8.5	24.46329
19	8.5	8	0	5	4.923326	8	8.5	24.46329
20	8.5	8	0	5	4.923326	8	8.5	24.46329
21	8.5	8	0	5	4.923326	8	8.5	24.46329
22	8.5	8	0	5	4.923326	8	8.5	24.46329
23	8.5	8	0	5	4.923326	8	8.5	24.46329
24	8.5	8	0	5	4.923326	8	8.5	24.46329
25	8.5	8	0	5	4.923326	8	8.5	24.46329

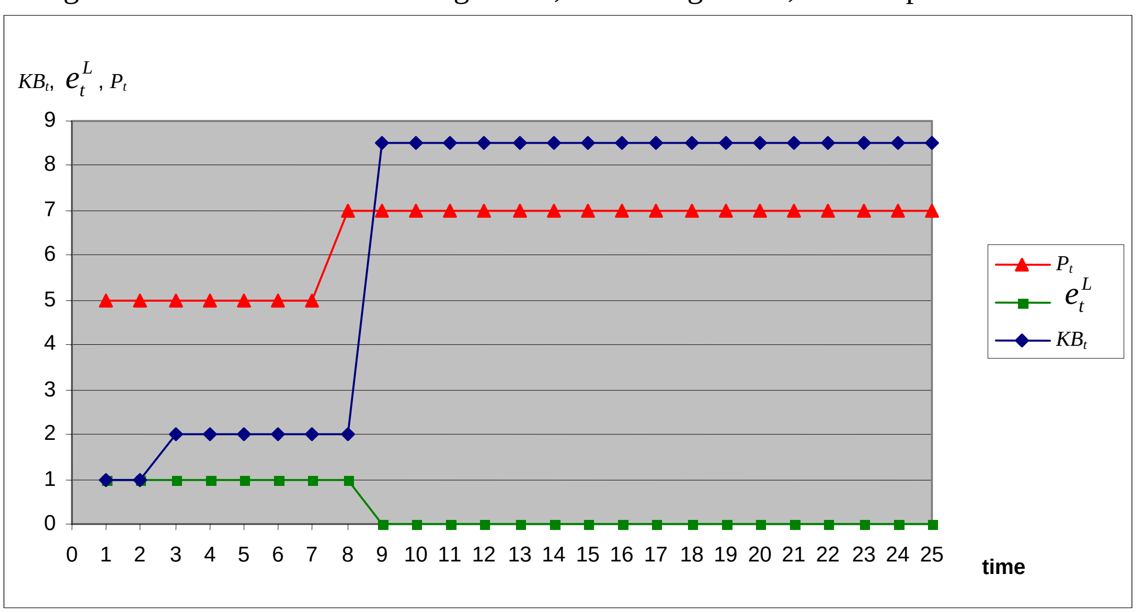
Figure B-3 The Paths of Learning Effort, Knowledge Base, and Output Price: $t^I = 6$



APPENDIX B-4 Deterministic Numerical Results: $t^I = 8$

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	0.5	0.812252	2	2	1.061262
3	2	2	1	1	1.236796	3	2	2.183982
4	2	3	1	1	1.236796	4	2	2.183982
5	2	4	1	1	1.236796	5	2	2.183982
6	2	5	1	1	1.236796	6	2	2.183982
7	2	6	1	1	1.236796	7	2	2.183982
8	2	7	1	1.5	1.396772	8	8.5	4.777406
9	8.5	8	0	5	4.923326	8	8.5	24.46329
10	8.5	8	0	5	4.923326	8	8.5	24.46329
11	8.5	8	0	5	4.923326	8	8.5	24.46329
12	8.5	8	0	5	4.923326	8	8.5	24.46329
13	8.5	8	0	5	4.923326	8	8.5	24.46329
14	8.5	8	0	5	4.923326	8	8.5	24.46329
15	8.5	8	0	5	4.923326	8	8.5	24.46329
16	8.5	8	0	5	4.923326	8	8.5	24.46329
17	8.5	8	0	5	4.923326	8	8.5	24.46329
18	8.5	8	0	5	4.923326	8	8.5	24.46329
19	8.5	8	0	5	4.923326	8	8.5	24.46329
20	8.5	8	0	5	4.923326	8	8.5	24.46329
21	8.5	8	0	5	4.923326	8	8.5	24.46329
22	8.5	8	0	5	4.923326	8	8.5	24.46329
23	8.5	8	0	5	4.923326	8	8.5	24.46329
24	8.5	8	0	5	4.923326	8	8.5	24.46329
25	8.5	8	0	5	4.923326	8	8.5	24.46329

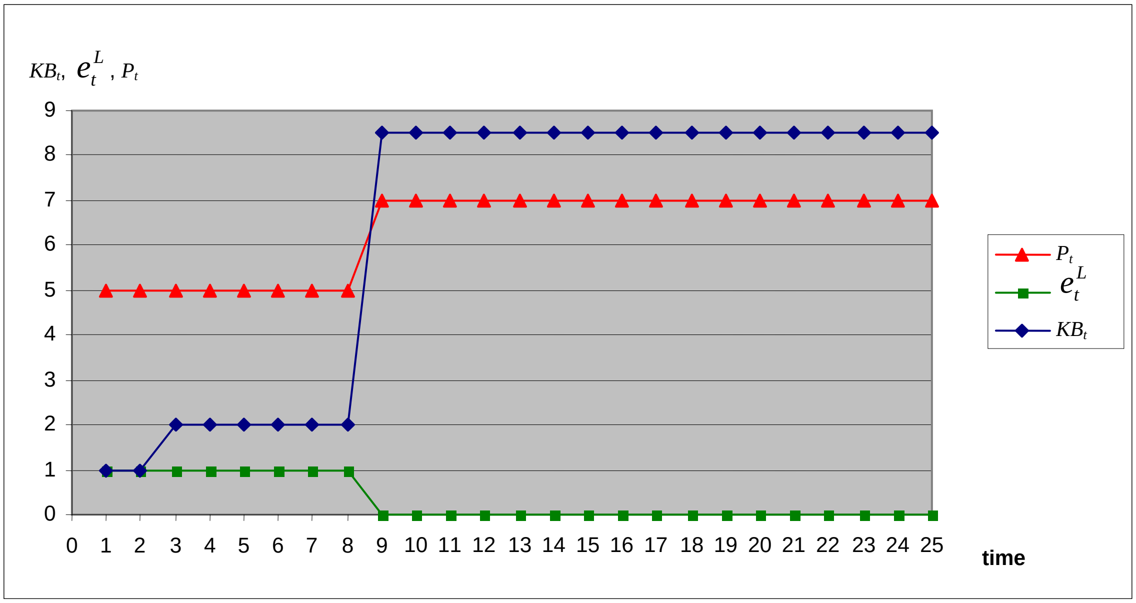
Figure B-4 The Paths of Learning Effort, Knowledge Base, and Output Price: $t^I = 8$



APPENDIX B-5 Deterministic Numerical Results: $t^I = 9$

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	0.5	0.812252	2	2	1.061262
3	2	2	1	1	1.236796	3	2	2.183982
4	2	3	1	1	1.236796	4	2	2.183982
5	2	4	1	1	1.236796	5	2	2.183982
6	2	5	1	1	1.236796	6	2	2.183982
7	2	6	1	1	1.236796	7	2	2.183982
8	2	7	1	1	1.236796	8	8.5	2.183982
9	8.5	8	0	5	4.923326	8	8.5	24.46329
10	8.5	8	0	5	4.923326	8	8.5	24.46329
11	8.5	8	0	5	4.923326	8	8.5	24.46329
12	8.5	8	0	5	4.923326	8	8.5	24.46329
13	8.5	8	0	5	4.923326	8	8.5	24.46329
14	8.5	8	0	5	4.923326	8	8.5	24.46329
15	8.5	8	0	5	4.923326	8	8.5	24.46329
16	8.5	8	0	5	4.923326	8	8.5	24.46329
17	8.5	8	0	5	4.923326	8	8.5	24.46329
18	8.5	8	0	5	4.923326	8	8.5	24.46329
19	8.5	8	0	5	4.923326	8	8.5	24.46329
20	8.5	8	0	5	4.923326	8	8.5	24.46329
21	8.5	8	0	5	4.923326	8	8.5	24.46329
22	8.5	8	0	5	4.923326	8	8.5	24.46329
23	8.5	8	0	5	4.923326	8	8.5	24.46329
24	8.5	8	0	5	4.923326	8	8.5	24.46329
25	8.5	8	0	5	4.923326	8	8.5	24.46329

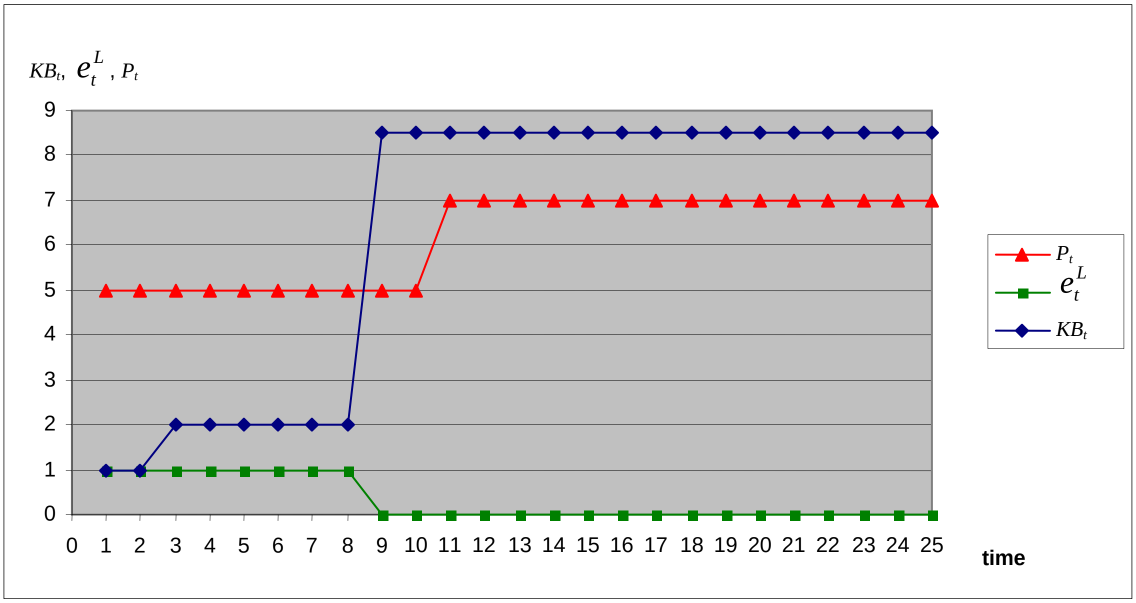
Figure B-5 The Paths of Learning Effort, Knowledge Base, and Output Price: $t^I = 9$



APPENDIX B-6 Deterministic Numerical Results: $t^I = 11$

Time(t)	KB_t	$\sum_{\tau=1}^{\tau=t-1} e_\tau^L$	e_t^{L*}	X_t^*	*Output*	$\sum_{\tau=1}^{\tau=t} e_\tau^L$	KB_{t+1}	π_t
1	1	0	1	0.5	0.812252	1	1	1.061262
2	1	1	1	0.5	0.812252	2	2	1.061262
3	2	2	1	1	1.236796	3	2	2.183982
4	2	3	1	1	1.236796	4	2	2.183982
5	2	4	1	1	1.236796	5	2	2.183982
6	2	5	1	1	1.236796	6	2	2.183982
7	2	6	1	1	1.236796	7	2	2.183982
8	2	7	1	1	1.236796	8	8.5	2.183982
9	8.5	8	0	3	4.223806	8	8.5	15.11903
10	8.5	8	0	3	4.223806	8	8.5	15.11903
11	8.5	8	0	5	4.923326	8	8.5	24.46329
12	8.5	8	0	5	4.923326	8	8.5	24.46329
13	8.5	8	0	5	4.923326	8	8.5	24.46329
14	8.5	8	0	5	4.923326	8	8.5	24.46329
15	8.5	8	0	5	4.923326	8	8.5	24.46329
16	8.5	8	0	5	4.923326	8	8.5	24.46329
17	8.5	8	0	5	4.923326	8	8.5	24.46329
18	8.5	8	0	5	4.923326	8	8.5	24.46329
19	8.5	8	0	5	4.923326	8	8.5	24.46329
20	8.5	8	0	5	4.923326	8	8.5	24.46329
21	8.5	8	0	5	4.923326	8	8.5	24.46329
22	8.5	8	0	5	4.923326	8	8.5	24.46329
23	8.5	8	0	5	4.923326	8	8.5	24.46329
24	8.5	8	0	5	4.923326	8	8.5	24.46329
25	8.5	8	0	5	4.923326	8	8.5	24.46329

Figure B-6 The Paths of Learning Effort, Knowledge Base, and Output Price: $t^I = 11$



APPENDIX C:

THE SHADOW VALUE OF THE KNOWLEDGE: PRICE-DECREASING CASES

$$\left(P_t = 5 \quad for \quad t < t^D; \;\; P_t = 3 \quad for \quad t^D \leq t \leq 25\right)$$

APPENDIX C-1 The Shadow Value of the Knowledge: $t^D = 2$

***Time* (τ)**	KB^*_τ	$V_\tau \Big\vert_{KB_\tau = KB^*_\tau,\ \sum_{t=1}^{\tau-1} e^L_t = \left(\sum_{t=1}^{\tau-1} e^L_t\right)^*}$	$KB^*_\tau + 1$	$V_\tau \Big\vert_{KB_\tau = KB^*_\tau + 1,\ \sum_{t=1}^{\tau-1} e^L_t = \left(\sum_{t=1}^{\tau-1} e^L_t\right)^*}$	***Value of Knowledge***
1	1	30.759	2	36.709	5.950
2	1	28.257	2	32.816	4.559
3	1	31.702	2	36.082	4.380
4	2	35.492	3	39.675	4.184
5	2	39.026	3	43.065	4.039
6	2	42.913	3	46.810	3.897
7	2	47.189	3	50.930	3.740
8	2	51.893	3	55.461	3.568
9	2	57.067	3	60.446	3.378
10	8.5	62.759	9.5	65.929	3.170
11	8.5	61.013	9.5	64.095	3.082
12	8.5	59.093	9.5	62.078	2.985
13	8.5	56.980	9.5	59.859	2.878
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX C-2 The Shadow Value of the Knowledge: $t^D = 3$

Time **(τ)**	KB_τ^*	$V_\tau \Big|_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big|_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	30.759	2	36.709	5.950
2	1	28.257	2	32.816	4.559
3	1	31.702	2	36.082	4.380
4	2	35.492	3	39.675	4.184
5	2	39.026	3	43.065	4.039
6	2	42.913	3	46.810	3.897
7	2	47.189	3	50.930	3.740
8	2	51.893	3	55.461	3.568
9	2	57.067	3	60.446	3.378
10	8.5	62.759	9.5	65.929	3.170
11	8.5	61.013	9.5	64.095	3.082
12	8.5	59.093	9.5	62.078	2.985
13	8.5	56.980	9.5	59.859	2.878
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX C-3 The Shadow Value of the Knowledge: $t^D = 4$

***Time* (τ)**	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	34.375	2	42.742	8.367
2	1	36.645	2	42.738	6.093
3	2	39.142	3	44.610	5.468
4	2	35.492	3	39.675	4.184
5	2	39.026	3	43.065	4.039
6	2	42.913	3	46.810	3.897
7	2	47.189	3	50.930	3.740
8	2	51.893	3	55.461	3.568
9	2	57.067	3	60.446	3.378
10	8.5	62.759	9.5	65.929	3.170
11	8.5	61.013	9.5	64.095	3.082
12	8.5	59.093	9.5	62.078	2.985
13	8.5	56.980	9.5	59.859	2.878
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX C-4 The Shadow Value of the Knowledge: t^D =5

***Time* (τ)**	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	36.444	2	45.353	8.909
2	1	38.921	2	46.399	7.478
3	2	41.646	3	48.636	6.990
4	2	38.246	3	44.105	5.858
5	2	34.506	3	39.120	4.613
6	2	37.942	3	41.926	3.985
7	2	41.721	3	45.557	3.837
8	2	45.878	3	49.552	3.674
9	2	50.450	3	53.946	3.495
10	2	55.480	3	58.779	3.298
11	8.5	61.013	9.5	64.095	3.082
12	8.5	59.093	9.5	62.078	2.985
13	8.5	56.980	9.5	59.859	2.878
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX C-5 The Shadow Value of the Knowledge: $t^D = 6$

Time (τ)	KB_τ^*	$V_\tau \Big\|_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\|_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	38.325	2	47.728	9.402
2	1	40.990	2	49.727	8.736
3	2	43.922	3	52.297	8.375
4	2	40.750	3	48.131	7.381
5	2	37.261	3	43.549	6.288
6	2	33.422	3	38.508	5.086
7	2	36.749	3	40.674	3.924
8	2	40.409	3	44.180	3.770
9	2	44.435	3	48.036	3.601
10	2	48.863	3	52.278	3.415
11	2	53.735	3	56.945	3.210
12	8.5	59.093	9.5	62.078	2.985
13	8.5	56.980	9.5	59.859	2.878
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX C-6 The Shadow Value of the Knowledge: $t^D = 7$

Time (τ)	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	**Value of Knowledge**
1	1	40.036	2	50.140	10.105
2	1	42.872	2	52.752	9.880
3	2	45.991	3	55.625	9.633
4	2	43.026	3	51.792	8.765
5	2	39.765	3	47.575	7.811
6	2	36.177	3	42.938	6.761
7	2	32.230	3	37.836	5.606
8	2	35.438	3	39.296	3.858
9	2	38.966	3	42.664	3.698
10	2	42.848	3	46.369	3.521
11	2	47.118	3	50.444	3.327
12	2	51.814	3	54.927	3.113
13	8.5	56.980	9.5	59.859	2.878
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX C-7 The Shadow Value of the Knowledge: $t^D = 8$

***Time* (τ)**	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	41.590	2	52.640	11.050
2	1	44.582	2	55.502	10.920
3	2	47.873	3	58.650	10.777
4	2	45.096	3	55.119	10.024
5	2	42.041	3	51.236	9.195
6	2	38.681	3	46.964	8.284
7	2	34.984	3	42.265	7.281
8	2	30.918	3	37.097	6.178
9	2	33.995	3	37.780	3.785
10	2	37.379	3	40.997	3.617
11	2	41.102	3	44.535	3.433
12	2	45.197	3	48.427	3.230
13	2	49.702	3	52.708	3.006
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX C-8 The Shadow Value of the Knowledge: $t^D = 9$

***Time* (τ)**	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	43.004	2	54.913	11.910
2	1	46.137	2	58.002	11.866
3	2	49.583	3	61.400	11.817
4	2	46.977	3	58.145	11.168
5	2	44.110	3	54.564	10.454
6	2	40.957	3	50.625	9.668
7	2	37.488	3	46.292	8.804
8	2	33.673	3	41.526	7.853
9	2	29.476	3	36.283	6.807
10	2	32.408	3	36.113	3.705
11	2	35.634	3	39.163	3.529
12	2	39.182	3	42.518	3.336
13	2	43.085	3	46.208	3.123
14	2	47.378	3	50.267	2.889
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX C-9 The Shadow Value of the Knowledge: $t^D = 10$

***Time* (τ)**	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	44.367	2	56.980	12.612
2	1	47.637	2	60.275	12.639
3	2	51.233	3	63.900	12.667
4	2	48.792	3	60.895	12.103
5	2	46.107	3	57.589	11.482
6	2	43.153	3	53.953	10.800
7	2	39.904	3	49.952	10.049
8	2	36.330	3	45.552	9.222
9	2	32.399	3	40.712	8.314
10	2	28.074	3	35.388	7.314
11	2	27.293	3	34.404	7.111
12	2	26.434	3	33.321	6.887
13	2	25.489	3	32.130	6.641
14	2	24.450	3	30.820	6.370
15	2	23.306	3	29.378	6.072
16	2	22.049	3	27.793	5.744
17	2	20.665	3	26.049	5.384
18	2	19.143	3	24.131	4.987
19	2	17.469	3	22.021	4.551
20	2	15.628	3	19.700	4.072
21	2	13.603	3	17.146	3.544
22	2	11.375	3	14.338	2.963
23	2	8.924	3	11.248	2.325
24	2	6.228	3	7.850	1.622
25	2	3.262	3	4.112	0.850

APPENDIX C-10 The Shadow Value of the Knowledge: $t^{D}=11$

***Time* (τ)**	KB_{τ}^{*}	$V_{\tau}\Big\vert_{KB_{\tau}=KB_{\tau}^{*},\ \sum_{t=1}^{\tau-1}e_t^L=\left(\sum_{t=1}^{\tau-1}e_t^L\right)^{*}}$	$KB_{\tau}^{*}+1$	$V_{\tau}\Big\vert_{KB_{\tau}=KB_{\tau}^{*}+1,\ \sum_{t=1}^{\tau-1}e_t^L=\left(\sum_{t=1}^{\tau-1}e_t^L\right)^{*}}$	***Value of Knowledge***
1	1	47.661	2	58.858	11.196
2	1	51.260	2	62.341	11.081
3	2	55.219	3	66.173	10.954
4	2	58.338	3	65.107	6.769
5	2	61.770	3	68.094	6.325
6	2	65.544	3	71.380	5.836
7	2	69.696	3	74.995	5.299
8	2	74.264	3	78.971	4.708
9	8.5	79.288	9.5	83.345	4.057
10	8.5	70.586	9.5	74.178	3.593
11	8.5	61.013	9.5	64.095	3.082
12	8.5	59.093	9.5	62.078	2.985
13	8.5	56.980	9.5	59.859	2.878
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX D:

THE SHADOW VALUE OF THE KNOWLEDGE:PRICE-INCREASING CASES

$$\left(P_t = 5 \quad for \quad t < t^I; \;\; P_t = 7 \quad for \quad t^I \le t \le 25\right)$$

APPENDIX D-1 The Shadow Value of the Knowledge: $t^I = 2$

***Time* (τ)**	KB_τ^*	$V_\tau\Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau\Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	123.403	2	137.849	14.446
2	1	134.576	2	149.231	14.655
3	2	144.733	3	158.899	14.166
4	2	153.951	3	167.732	13.781
5	2	164.091	3	177.449	13.357
6	2	175.245	3	188.137	12.892
7	2	187.515	3	199.894	12.379
8	2	201.011	3	212.826	11.815
9	8.5	215.857	9.5	227.052	11.195
10	8.5	210.533	9.5	221.452	10.919
11	8.5	204.677	9.5	215.292	10.616
12	8.5	198.235	9.5	208.516	10.281
13	8.5	191.149	9.5	201.063	9.914
14	8.5	183.354	9.5	192.864	9.510
15	8.5	174.780	9.5	183.845	9.065
16	8.5	165.348	9.5	173.924	8.576
17	8.5	154.973	9.5	163.011	8.038
18	8.5	143.561	9.5	151.007	7.446
19	8.5	131.007	9.5	137.802	6.795
20	8.5	117.198	9.5	123.277	6.079
21	8.5	102.009	9.5	107.299	5.291
22	8.5	85.300	9.5	89.724	4.424
23	8.5	66.920	9.5	70.391	3.471
24	8.5	46.703	9.5	49.125	2.422
25	8.5	24.463	9.5	25.732	1.269

APPENDIX D-2 The Shadow Value of the Knowledge: t^I =4

***Time* (τ)**	KB_τ^*	$V_\tau \Big\rvert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\rvert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	119.497	2	132.836	13.339
2	1	130.279	2	143.717	13.438
3	2	142.140	3	155.687	13.547
4	2	153.951	3	167.732	13.781
5	2	164.091	3	177.449	13.357
6	2	175.245	3	188.137	12.892
7	2	187.515	3	199.894	12.379
8	2	201.011	3	212.826	11.815
9	8.5	215.857	9.5	227.052	11.195
10	8.5	210.533	9.5	221.452	10.919
11	8.5	204.677	9.5	215.292	10.616
12	8.5	198.235	9.5	208.516	10.281
13	8.5	191.149	9.5	201.063	9.914
14	8.5	183.354	9.5	192.864	9.510
15	8.5	174.780	9.5	183.845	9.065
16	8.5	165.348	9.5	173.924	8.576
17	8.5	154.973	9.5	163.011	8.038
18	8.5	143.561	9.5	151.007	7.446
19	8.5	131.007	9.5	137.802	6.795
20	8.5	117.198	9.5	123.277	6.079
21	8.5	102.009	9.5	107.299	5.291
22	8.5	85.300	9.5	89.724	4.424
23	8.5	66.920	9.5	70.391	3.471
24	8.5	46.703	9.5	49.125	2.422
25	8.5	24.463	9.5	25.732	1.269

APPENDIX D-3 The Shadow Value of the Knowledge: $t^I = 6$

***Time* (τ)**	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	115.777	2	128.229	12.452
2	1	126.187	2	138.649	12.462
3	2	137.639	3	150.112	12.473
4	2	149.000	3	161.600	12.600
5	2	161.498	3	174.237	12.739
6	2	175.245	3	188.137	12.892
7	2	187.515	3	199.894	12.379
8	2	201.011	3	212.826	11.815
9	8.5	215.857	9.5	227.052	11.195
10	8.5	210.533	9.5	221.452	10.919
11	8.5	204.677	9.5	215.292	10.616
12	8.5	198.235	9.5	208.516	10.281
13	8.5	191.149	9.5	201.063	9.914
14	8.5	183.354	9.5	192.864	9.510
15	8.5	174.780	9.5	183.845	9.065
16	8.5	165.348	9.5	173.924	8.576
17	8.5	154.973	9.5	163.011	8.038
18	8.5	143.561	9.5	151.007	7.446
19	8.5	131.007	9.5	137.802	6.795
20	8.5	117.198	9.5	123.277	6.079
21	8.5	102.009	9.5	107.299	5.291
22	8.5	85.300	9.5	89.724	4.424
23	8.5	66.920	9.5	70.391	3.471
24	8.5	46.703	9.5	49.125	2.422
25	8.5	24.463	9.5	25.732	1.269

APPENDIX D-4 The Shadow Value of the Knowledge: $t^I = 8$

***Time* (τ)**	KB_τ^*	$V_\tau \Big\rvert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\rvert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	112.703	2	124.421	11.718
2	1	122.806	2	134.461	11.655
3	2	133.919	3	145.505	11.586
4	2	144.908	3	156.532	11.623
5	2	156.997	3	168.662	11.665
6	2	170.294	3	182.005	11.710
7	2	184.921	3	196.682	11.760
8	2	201.011	3	212.826	11.815
9	8.5	215.857	9.5	227.052	11.195
10	8.5	210.533	9.5	221.452	10.919
11	8.5	204.677	9.5	215.292	10.616
12	8.5	198.235	9.5	208.516	10.281
13	8.5	191.149	9.5	201.063	9.914
14	8.5	183.354	9.5	192.864	9.510
15	8.5	174.780	9.5	183.845	9.065
16	8.5	165.348	9.5	173.924	8.576
17	8.5	154.973	9.5	163.011	8.038
18	8.5	143.561	9.5	151.007	7.446
19	8.5	131.007	9.5	137.802	6.795
20	8.5	117.198	9.5	123.277	6.079
21	8.5	102.009	9.5	107.299	5.291
22	8.5	85.300	9.5	89.724	4.424
23	8.5	66.920	9.5	70.391	3.471
24	8.5	46.703	9.5	49.125	2.422
25	8.5	24.463	9.5	25.732	1.269

APPENDIX D-5 The Shadow Value of the Knowledge: $t^I = 9$

***Time* (τ)**	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	111.372	2	122.773	11.401
2	1	121.342	2	132.648	11.306
3	2	132.309	3	143.510	11.202
4	2	143.137	3	154.338	11.201
5	2	155.048	3	166.248	11.200
6	2	168.151	3	179.350	11.199
7	2	182.564	3	193.762	11.198
8	2	198.418	3	209.614	11.197
9	8.5	215.857	9.5	227.052	11.195
10	8.5	210.533	9.5	221.452	10.919
11	8.5	204.677	9.5	215.292	10.616
12	8.5	198.235	9.5	208.516	10.281
13	8.5	191.149	9.5	201.063	9.914
14	8.5	183.354	9.5	192.864	9.510
15	8.5	174.780	9.5	183.845	9.065
16	8.5	165.348	9.5	173.924	8.576
17	8.5	154.973	9.5	163.011	8.038
18	8.5	143.561	9.5	151.007	7.446
19	8.5	131.007	9.5	137.802	6.795
20	8.5	117.198	9.5	123.277	6.079
21	8.5	102.009	9.5	107.299	5.291
22	8.5	85.300	9.5	89.724	4.424
23	8.5	66.920	9.5	70.391	3.471
24	8.5	46.703	9.5	49.125	2.422
25	8.5	24.463	9.5	25.732	1.269

APPENDIX D-6 The Shadow Value of the Knowledge: $t^I = 11$

***Time* (τ)**	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	103.050	2	114.026	10.976
2	1	112.188	2	123.026	10.838
3	2	122.239	3	132.926	10.687
4	2	132.060	3	142.695	10.635
5	2	142.864	3	153.441	10.577
6	2	154.748	3	165.262	10.514
7	2	167.821	3	178.265	10.444
8	2	182.200	3	192.568	10.368
9	8.5	198.018	9.5	208.302	10.284
10	8.5	201.189	9.5	211.630	10.442
11	8.5	204.677	9.5	215.292	10.616
12	8.5	198.235	9.5	208.516	10.281
13	8.5	191.149	9.5	201.063	9.914
14	8.5	183.354	9.5	192.864	9.510
15	8.5	174.780	9.5	183.845	9.065
16	8.5	165.348	9.5	173.924	8.576
17	8.5	154.973	9.5	163.011	8.038
18	8.5	143.561	9.5	151.007	7.446
19	8.5	131.007	9.5	137.802	6.795
20	8.5	117.198	9.5	123.277	6.079
21	8.5	102.009	9.5	107.299	5.291
22	8.5	85.300	9.5	89.724	4.424
23	8.5	66.920	9.5	70.391	3.471
24	8.5	46.703	9.5	49.125	2.422
25	8.5	24.463	9.5	25.732	1.269

APPENDIX E:

THE SHADOW VALUE OF THE KNOWLEDGE:

THE STOCHASTIC CASES UNDER MARKET UNCERTAINTY

$$\left(P_t = 5 \quad for \quad t < t'; \;\; P_t = 3 \quad for \quad t' \le t \le 25\right)$$

(t' is unknown until the decrease of the output price is revealed)

APPENDIX E-1 The Shadow Value of the Knowledge: $t' = 2$

Time **(τ)**	KB_τ^*	$V_\tau \Big|_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big|_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	32.405	2	39.608	7.203
2	1	32.516	2	36.924	4.408
3	2	36.388	3	40.602	4.214
4	2	40.011	3	44.100	4.089
5	2	43.997	3	47.949	3.952
6	2	48.382	3	52.182	3.801
7	2	53.205	3	56.839	3.634
8	2	58.510	3	61.961	3.451
9	8.5	64.346	9.5	67.596	3.250
10	8.5	62.759	9.5	65.929	3.170
11	8.5	61.013	9.5	64.095	3.082
12	8.5	59.093	9.5	62.078	2.985
13	8.5	56.980	9.5	59.859	2.878
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX E-2 The Shadow Value of the Knowledge: $t'=3$

***Time* (τ)**	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	32.405	2	39.608	7.203
2	1	36.440	2	42.594	6.154
3	2	36.388	3	40.602	4.214
4	2	40.011	3	44.100	4.089
5	2	43.997	3	47.949	3.952
6	2	48.382	3	52.182	3.801
7	2	53.205	3	56.839	3.634
8	2	58.510	3	61.961	3.451
9	8.5	64.346	9.5	67.596	3.250
10	8.5	62.759	9.5	65.929	3.170
11	8.5	61.013	9.5	64.095	3.082
12	8.5	59.093	9.5	62.078	2.985
13	8.5	56.980	9.5	59.859	2.878
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX E-3 The Shadow Value of the Knowledge: $t'=4$

Time (**τ**)	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	32.405	2	39.608	7.203
2	1	36.440	2	42.594	6.154
3	2	41.445	3	48.301	6.856
4	2	35.492	3	39.675	4.184
5	2	39.026	3	43.065	4.039
6	2	42.913	3	46.810	3.897
7	2	47.189	3	50.930	3.740
8	2	51.893	3	55.461	3.568
9	2	57.067	3	60.446	3.378
10	8.5	62.759	9.5	65.929	3.170
11	8.5	61.013	9.5	64.095	3.082
12	8.5	59.093	9.5	62.078	2.985
13	8.5	56.980	9.5	59.859	2.878
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX E-4 The Shadow Value of the Knowledge: $t'=5$

Time **(τ)**	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	32.405	2	39.608	7.203
2	1	36.440	2	42.594	6.154
3	2	41.445	3	48.301	6.856
4	2	40.559	3	47.796	7.237
5	2	34.506	3	39.120	4.613
6	2	37.942	3	41.926	3.985
7	2	41.721	3	45.557	3.837
8	2	45.878	3	49.552	3.674
9	2	50.450	3	53.946	3.495
10	2	55.480	3	58.779	3.298
11	8.5	61.013	9.5	64.095	3.082
12	8.5	59.093	9.5	62.078	2.985
13	8.5	56.980	9.5	59.859	2.878
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX E-5 The Shadow Value of the Knowledge: $t'=6$

***Time* (τ)**	KB^*_τ	$V_\tau \Big\rvert_{KB_\tau = KB^*_\tau,\ \sum_{t=1}^{\tau-1} e^L_t = \left(\sum_{t=1}^{\tau-1} e^L_t\right)^*}$	$KB^*_\tau + 1$	$V_\tau \Big\rvert_{KB_\tau = KB^*_\tau + 1,\ \sum_{t=1}^{\tau-1} e^L_t = \left(\sum_{t=1}^{\tau-1} e^L_t\right)^*}$	***Value of Knowledge***
1	1	32.405	2	39.608	7.203
2	1	36.440	2	42.594	6.154
3	2	41.445	3	48.301	6.856
4	2	40.559	3	47.796	7.237
5	2	39.594	3	47.240	7.646
6	2	33.422	3	38.508	5.086
7	2	36.749	3	40.674	3.924
8	2	40.409	3	44.180	3.770
9	2	44.435	3	48.036	3.601
10	2	48.863	3	52.278	3.415
11	2	53.735	3	56.945	3.210
12	8.5	59.093	9.5	62.078	2.985
13	8.5	56.980	9.5	59.859	2.878
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX E-6 The Shadow Value of the Knowledge: $t'=7$

***Time* (τ)**	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	32.405	2	39.608	7.203
2	1	36.440	2	42.594	6.154
3	2	41.445	3	48.301	6.856
4	2	40.559	3	47.796	7.237
5	2	39.594	3	47.240	7.646
6	2	38.555	3	46.629	8.074
7	2	32.230	3	37.836	5.606
8	2	35.438	3	39.296	3.858
9	2	38.966	3	42.664	3.698
10	2	42.848	3	46.369	3.521
11	2	47.118	3	50.444	3.327
12	2	51.814	3	54.927	3.113
13	8.5	56.980	9.5	59.859	2.878
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX E-7 The Shadow Value of the Knowledge: $t'=8$

***Time* (τ)**	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	32.405	2	39.608	7.203
2	1	36.440	2	42.594	6.154
3	2	41.445	3	48.301	6.856
4	2	40.559	3	47.796	7.237
5	2	39.594	3	47.240	7.646
6	2	38.555	3	46.629	8.074
7	2	37.463	3	45.956	8.494
8	2	30.918	3	37.097	6.178
9	2	33.995	3	37.780	3.785
10	2	37.379	3	40.997	3.617
11	2	41.102	3	44.535	3.433
12	2	45.197	3	48.427	3.230
13	2	49.702	3	52.708	3.006
14	8.5	54.657	9.5	57.418	2.761
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX E-8 The Shadow Value of the Knowledge: $t'=9$

***Time* (τ)**	KB_{τ}^{*}	$V_{\tau}\Big\vert_{KB_{\tau}=KB_{\tau}^{*},\ \sum_{t=1}^{\tau-1}e_t^L=\left(\sum_{t=1}^{\tau-1}e_t^L\right)^{*}}$	$KB_{\tau}^{*}+1$	$V_{\tau}\Big\vert_{KB_{\tau}=KB_{\tau}^{*}+1,\ \sum_{t=1}^{\tau-1}e_t^L=\left(\sum_{t=1}^{\tau-1}e_t^L\right)^{*}}$	***Value of Knowledge***
1	1	32.405	2	39.608	7.203
2	1	36.440	2	42.594	6.154
3	2	41.445	3	48.301	6.856
4	2	40.559	3	47.796	7.237
5	2	39.594	3	47.240	7.646
6	2	38.555	3	46.629	8.074
7	2	37.463	3	45.956	8.494
8	2	36.370	3	45.217	8.846
9	2	29.476	3	36.283	6.807
10	2	32.408	3	36.113	3.705
11	2	35.634	3	39.163	3.529
12	2	39.182	3	42.518	3.336
13	2	43.085	3	46.208	3.123
14	2	47.378	3	50.267	2.889
15	8.5	52.101	9.5	54.733	2.632
16	8.5	49.289	9.5	51.779	2.490
17	8.5	46.197	9.5	48.530	2.333
18	8.5	42.795	9.5	44.956	2.162
19	8.5	39.053	9.5	41.025	1.973
20	8.5	34.936	9.5	36.701	1.765
21	8.5	30.408	9.5	31.944	1.536
22	8.5	25.427	9.5	26.712	1.284
23	8.5	19.949	9.5	20.956	1.008
24	8.5	13.922	9.5	14.625	0.703
25	8.5	7.292	9.5	7.661	0.368

APPENDIX E-9 The Shadow Value of the Knowledge: $t'=10$

***Time* (τ)**	KB_τ^*	$V_\tau \Big\vert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\vert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	32.405	2	39.608	7.203
2	1	36.440	2	42.594	6.154
3	2	41.445	3	48.301	6.856
4	2	40.559	3	47.796	7.237
5	2	39.594	3	47.240	7.646
6	2	38.555	3	46.629	8.074
7	2	37.463	3	45.956	8.494
8	2	36.370	3	45.217	8.846
9	2	35.411	3	44.403	8.993
10	2	28.074	3	35.388	7.314
11	2	27.293	3	34.404	7.111
12	2	26.434	3	33.321	6.887
13	2	25.489	3	32.130	6.641
14	2	24.450	3	30.820	6.370
15	2	23.306	3	29.378	6.072
16	2	22.049	3	27.793	5.744
17	2	20.665	3	26.049	5.384
18	2	19.143	3	24.131	4.987
19	2	17.469	3	22.021	4.551
20	2	15.628	3	19.700	4.072
21	2	13.603	3	17.146	3.544
22	2	11.375	3	14.338	2.963
23	2	8.924	3	11.248	2.325
24	2	6.228	3	7.850	1.622
25	2	3.262	3	4.112	0.850

APPENDIX E-10 The Shadow Value of the Knowledge: $t'=11$

***Time* (τ)**	KB_τ^*	$V_\tau\Big\vert_{KB_\tau=KB_\tau^*,\ \sum_{t=1}^{\tau-1}e_t^L=\left(\sum_{t=1}^{\tau-1}e_t^L\right)^*}$	KB_τ^*+1	$V_\tau\Big\vert_{KB_\tau=KB_\tau^*+1,\ \sum_{t=1}^{\tau-1}e_t^L=\left(\sum_{t=1}^{\tau-1}e_t^L\right)^*}$	***Value of Knowledge***
1	1	32.405	2	39.608	7.203
2	1	36.440	2	42.594	6.154
3	2	41.445	3	48.301	6.856
4	2	40.559	3	47.796	7.237
5	2	39.594	3	47.240	7.646
6	2	38.555	3	46.629	8.074
7	2	37.463	3	45.956	8.494
8	2	36.370	3	45.217	8.846
9	2	35.411	3	44.403	8.993
10	2	34.701	3	43.508	8.808
11	2	27.293	3	34.404	7.111
12	2	26.434	3	33.321	6.887
13	2	25.489	3	32.130	6.641
14	2	24.450	3	30.820	6.370
15	2	23.306	3	29.378	6.072
16	2	22.049	3	27.793	5.744
17	2	20.665	3	26.049	5.384
18	2	19.143	3	24.131	4.987
19	2	17.469	3	22.021	4.551
20	2	15.628	3	19.700	4.072
21	2	13.603	3	17.146	3.544
22	2	11.375	3	14.338	2.963
23	2	8.924	3	11.248	2.325
24	2	6.228	3	7.850	1.622
25	2	3.262	3	4.112	0.850

APPENDIX E-11 The Shadow Value of the Knowledge: The Output Price Never Drops (t' does not exist)

***Time* (τ)**	KB_τ^*	$V_\tau \Big\rvert_{KB_\tau = KB_\tau^*,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	$KB_\tau^* + 1$	$V_\tau \Big\rvert_{KB_\tau = KB_\tau^* + 1,\ \sum_{t=1}^{\tau-1} e_t^L = \left(\sum_{t=1}^{\tau-1} e_t^L\right)^*}$	***Value of Knowledge***
1	1	32.405	2	39.608	7.203
2	1	36.440	2	42.594	6.154
3	2	41.445	3	48.301	6.856
4	2	40.559	3	47.796	7.237
5	2	39.594	3	47.240	7.646
6	2	38.555	3	46.629	8.074
7	2	37.463	3	45.956	8.494
8	2	36.370	3	45.217	8.846
9	2	35.411	3	44.403	8.993
10	2	34.701	3	43.508	8.808
11	2	33.920	3	42.524	8.604
12	2	33.061	3	41.441	8.380
13	2	32.116	3	40.250	8.134
14	2	31.076	3	38.939	7.863
15	2	29.932	3	37.497	7.565
16	2	28.673	3	35.910	7.237
17	2	27.286	3	34.163	6.876
18	2	25.758	3	32.236	6.478
19	2	24.070	3	30.108	6.039
20	2	22.196	3	27.748	5.552
21	2	20.101	3	25.109	5.008
22	2	17.718	3	22.112	4.393
23	2	14.928	3	18.606	3.678
24	2	11.485	3	14.293	2.807
25	2	6.877	3	8.541	1.665

APPENDIX F:

THE LCSFM RESULTS: THE CASTOR IN AUREPALLE VILLAGE

APPENDIX F-1: Estimation Results: Castor Production

Parameters	Estimates	Std. err.	Est./s.e.
Production Function For Group *CA*			
Constant	1.0103	0.2008	5.031
Seed	0.6155	0.2286	2.692
Family Labor	0.3811	0.0704	5.414
Hired Labor	0.0497	0.0239	2.076
Animal	-0.281	0.1493	-1.883
Fertilizer	0.0557	0.0358	1.555
Manure	0.01	0.0197	0.508
ln(Head Age)	0.5248	0.1321	3.972
Head Edu	0.2378	0.0664	3.58
s.d. of random error	0.6597	0.1164	5.668
s.d. of 1-sided error	0.6031	0.301	2.003
Production Function For Group *CB*			
Constant	1.8472	0.5013	3.685
Seed	0.9967	0.2745	3.63
Family Labor	0.0185	0.0329	0.561
Hired Labor	0.03	0.0206	1.461
Animal	0.1154	0.0473	2.441
Fertilizer	0.0479	0.0177	2.713
Manure	-0.0004	0.0167	-0.021
ln(Head Age)	0.2469	0.1584	1.559
Head Edu	0.1496	0.0482	3.104
s.d. of random error	0.2515	0.072	3.494
s.d. of 1-sided error	0.5151	0.1386	3.718
Membership Probabilities for Group *CB*			
Constant	-8.0267	3.1639	-2.537
Caste 1	2.6147	1.0838	2.412
Caste 2	-3.2772	1.0178	-3.22
Caste 3	7.7953	3.0344	2.569
HYV Dummy	6.882	2.0957	3.284
Good Soil	7.5345	2.7298	2.76

* *CA* = group A for castor growers; *CB* = group B for castor growers.

APPENDIX F-2: Group Membership – Castor Growers in Aurepalle Village

Household	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
1						*CA*	*CA*	*CA*	*CA*	*CA*
5									*CA*	*CA*
10							*CB*	*CB*	*CB*	*CB*
32	*CB*	*CA*			*CB*		*CB*	*CB*	*CB*	*CB*
33	*CA*	*CA*	*CA*	*CA*	*CA*	*CA*	*CA*	*CA*	*CA*	*CA*
34		*CA*		*CA*		*CA*		*CA*		
35	*CB*	*CB*		*CB*	*CB*	*CA*	*CB*	*CB*	*CB*	*CB*
36		*CA*		*CA*		*CA*		*CA*		*CA*
37				*CA*	*CA*		*CA*			
38				*CB*				*CB*		*CA*
39				*CA*						
40	*CA*		*CA*	*CA*	*CA*					
41	*CA*	*CA*	*CA*	*CA*	*CA*	*CA*				
42	*CA*				*CA*					
43		*CB*	*CB*	*CB*	*CA*	*CB*		*CB*	*CB*	*CB*
44	*CA*	*CA*		*CB*	*CA*	*CB*		*CB*		
45	*CB*	*CA*			*CA*	*CA*	*CB*	*CA*	*CB*	*CB*
46	*CA*	*CA*	*CA*	*CB*	*CA*	*CA*	*CA*	*CA*	*CA*	*CA*
48	*CA*	*CA*	*CA*	*CB*	*CA*	*CA*	*CA*	*CA*	*CA*	*CA*
49	*CA*	*CA*	*CA*	*CA*	*CA*		*CA*			
50	*CB*	*CA*	*CA*	*CA*	*CA*	*CA*	*CA*		*CB*	*CB*
51	*CA*	*CA*	*CA*	*CB*	*CB*	*CA*	*CB*	*CB*	*CB*	*CA*
52	*CA*	*CA*	*CA*	*CA*		*CA*	*CB*	*CB*	*CA*	
53	*CA*	*CA*	*CA*		*CA*	*CA*	*CB*	*CB*	*CB*	*CB*
54	*CA*	*CA*	*CA*	*CA*	*CA*	*CA*	*CB*	*CB*	*CB*	*CB*
55	*CB*	*CB*	*CA*	*CB*	*CB*	*CA*	*CB*	*CB*	*CB*	*CB*
56	*CA*	*CA*	*CA*	*CA*	*CB*	*CA*	*CB*	*CB*	*CB*	*CB*
57	*CA*	*CA*	*CA*	*CA*	*CA*	*CA*	*CB*	*CB*	*CB*	*CB*
58	*CA*	*CA*	*CA*	*CA*	*CA*	*CA*	*CB*	*CB*	*CB*	*CB*
59	*CA*	*CA*	*CA*	*CA*	*CA*					
61								*CA*		
70						*CA*		*CA*		*CA*
80			*CA*	*CA*	*CA*					
81						*CB*	*CB*	*CB*	*CB*	*CB*
82							*CB*	*CB*	*CB*	

* *CA* = group A for castor growers; *CB* = group B for castor growers.

APPENDIX F-3: The Numbers of Households by Groups – Castor Growers

Group	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
CA	16	19	17	16	18	19	7	9	6	9
CB	5	3	1	8	5	3	14	16	15	13
TOTAL	21	22	18	24	23	22	21	25	21	22

* *CA* = group A for castor growers; *CB* = group B for castor growers.

APPENDIX F-4: Household Characteristic – Castor Growers before 1981

The households that always stay in group *CA*

Household	Household's head age	Household's head education	Soil type	Caste
1	42	1	0	4
33	57	1	0	4
34	35	1	0	4
36	55	1	0	4
37	39	1	0	4
39	43	1	0	4
40	52	3	0	2
41	53	2	0	2
42	54	3	0	2
49	49	1	0	4
52	52	3	0	1
53	65	2	0	1
54	60	2	0	1
57	60	3	0	1
58	40	2	0	1
59	24	4	0	1
70	54	1	0	3
80	26	5	0	1

Households that always stay in group *CB*

Household	Household's head age	Household's head education	Soil type	Caste
38	46	1	0	3
81	32	3	0	3

Households that change groups

Household	Household's head age	Household's head education	Soil type	Caste
32	55	1	0	3
35	58	1	0	3
43	70	1	0 or 1	3
44	43	1	0 or 1	3
45	45	2	0 or 1	1
46	61	1	0 or 1	4
48	50	1	0 or 1	4
50	39	1	0 or 1	1
51	55	1	0	3
55	60	1	0	3
56	40	5	0 or 1	1

* *CA* = group A for castor growers; *CB* = group B for castor growers.
** Education ranges from 1 to 5. Type 1 indicates "illiterate"; type 2 indicates "read and write"; type 3 indicates "up to primary school"; type 4 indicates "up to middle school"; type 5 indicates "up to high school".
*** The soil type =1 if good soil is used for production; otherwise, soil type =0. The good soil represents deep, medium, and shallow black soil from the original ICRISAT India data.
**** The caste rank ranges from 1 to 4 indicating from the highest caste to lowest caste.

APPENDIX F-5: Household Characteristic – Castor Growers after 1981

The households that always stay in group *CA*

Household	Household's head age	Household's head education	Soil type	Caste	HYV ratio
1	42	1	0	4	0.50
5	34	1	0	4	0.50
33	57	1	0	4	0.68
34	35	1	0	4	1.00
36	55	1	0	4	0.50
37	39	1	0	4	0.00
46	61	1	0	4	1.00
48	50	1	0	4	0.90
49	49	1	0	4	0.00
61	55	1	1	4	0.00
70	54	1	0	3	0.00

Households that always stay in group *CB*

Household	Household's head age	Household's head education	Soil type	Caste	HYV ratio
10	58	1	0	3	0.15
32	55	1	0	3	1.00
35	58	1	0	3	0.88
43	70	1	0	3	1.00
44	43	1	0	3	1.00
53	65	2	0	1	1.00
54	60	2	0	1	1.00
55	60	1	0	3	1.00
56	40	5	0 or 1	1	0.92
57	60	3	0	1	0.84
58	40	2	0	1	1.00
81	32	3	0 or 1	3	0.92
82	43	3	0	3	0.83

Households that change groups

Household	Household's head age	Household's head education	Soil type	Caste	HYV ratio
38	46	1	0	3	0.50
45	45	2	0 or 1	1	0.50
50	39	1	0 or 1	1	0.67
51	55	1	0	3	0.50
52	52	3	0	1	0.89

* *CA* = group A for castor growers; *CB* = group B for castor growers.

** Education ranges from 1 to 5. Type 1 indicates "illiterate"; type 2 indicates "read and write"; type 3 indicates "up to primary school"; type 4 indicates "up to middle school"; type 5 indicates "up to high school".

*** The soil type =1 if good soil is used for production; otherwise, soil type =0. The good soil represents deep, medium, and shallow black soil from the original ICRISAT India data.

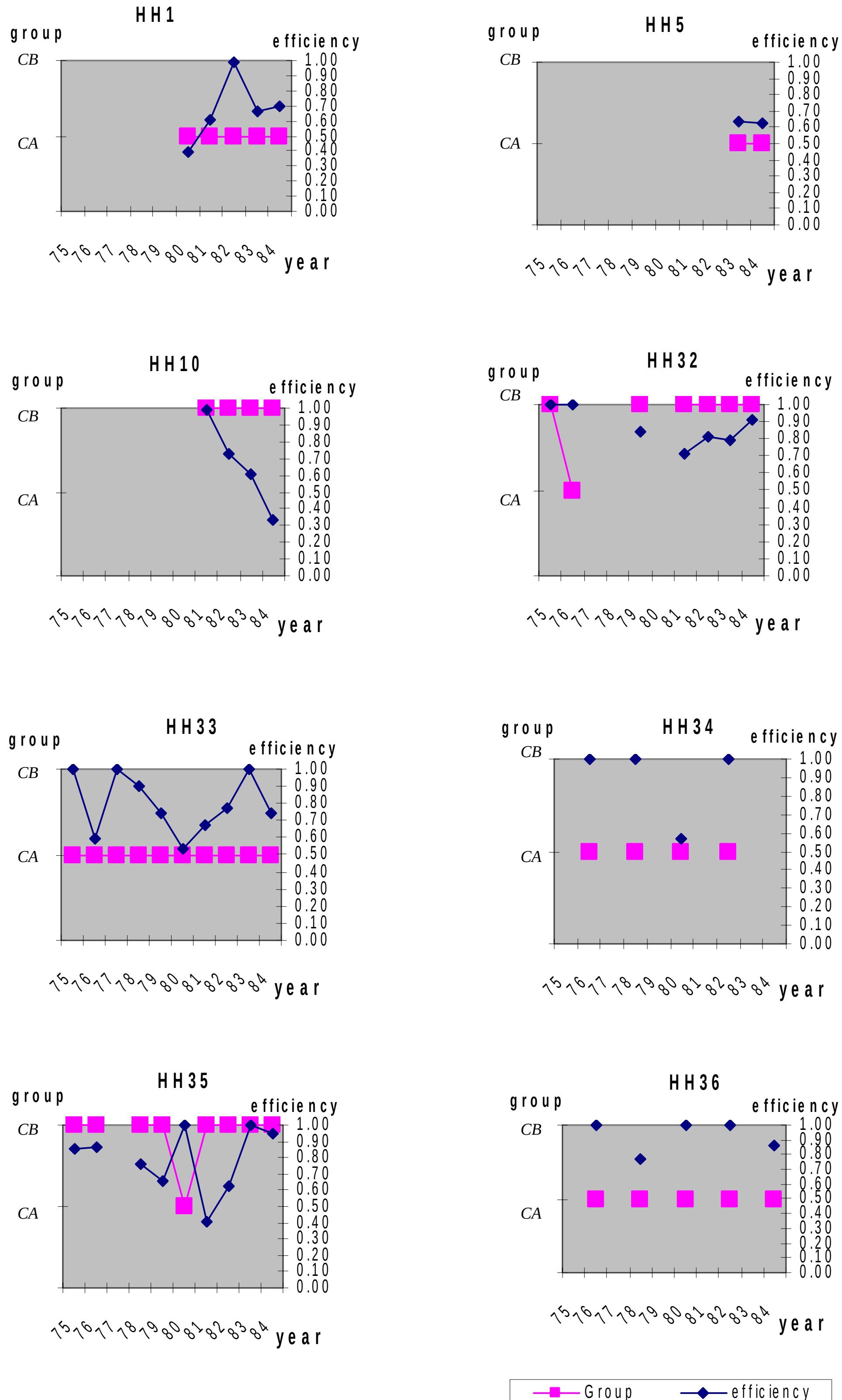
**** The caste rank ranges from 1 to 4 indicating from the highest caste to lowest caste.

APPENDIX F-6: Soil Type -- Castor

Household	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
1						0	0	0	0	0
5									0	0
10							0	0	0	0
32	0	0			0		0	0	0	0
33	0	0	0	0	0	0	0	0	0	0
34		0		0		0		0		
35	0	0		0	0	0	0	0	0	0
36		0		0		0		0		0
37				0	0		0			
38				0				0		0
39				0						
40	0		0	0	0					
41	0	0	0	0	0	0				
42	0				0					
43		0	0	1	0	0		0	0	0
44	0	0		1	0	1		0		
45	1	0			0	0	0	0	1	1
46	0	0	0	1	0	0	0	0	0	0
48	0	0	0	1	0	0	0	0	0	0
49	0	0	0	0	0		0			
50	1	0	0	0	0	0	0		1	1
51	0	0	0	0	0	0	0	0	0	0
52	0	0	0	0		0	0	0	0	
53	0	0	0		0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0
56	0	0	0	0	1	0	0	1	0	0
57	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0
59	0	0	0	0	0					
61								1		
70						0		0		0
80			0	0	0					
81						0	1	0	0	0
82							0	0	0	

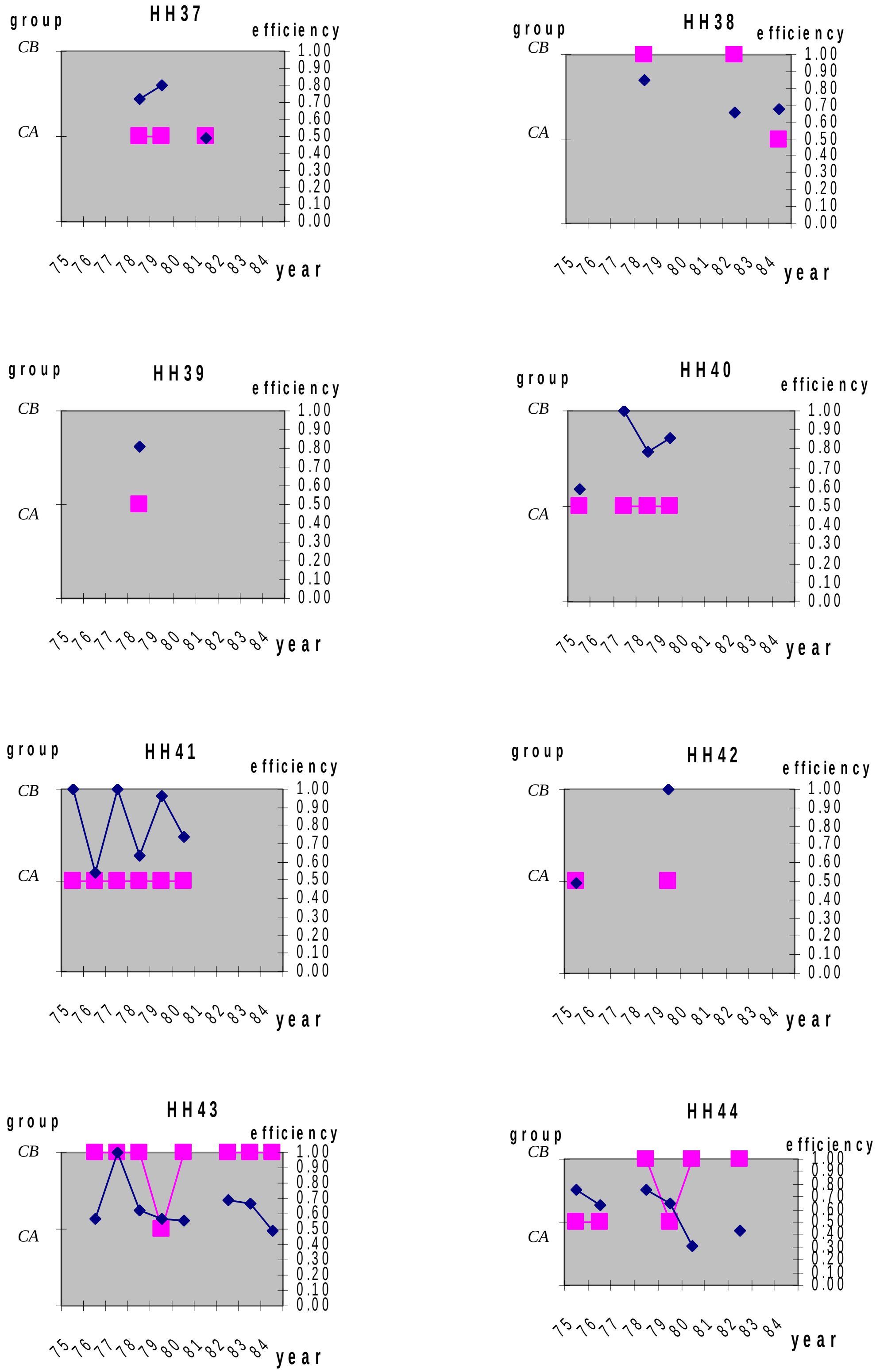
* The soil type =1 if good soil is used for production; otherwise, soil type =0. The good soil represents deep, medium, and shallow black soil from the original ICRISAT India data.

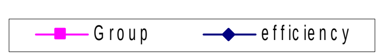
APPENDIX F-7: The Group Membership and the Corresponding Production Efficiency



* *CA* = group A for castor growers; *CB* = group B for castor growers.

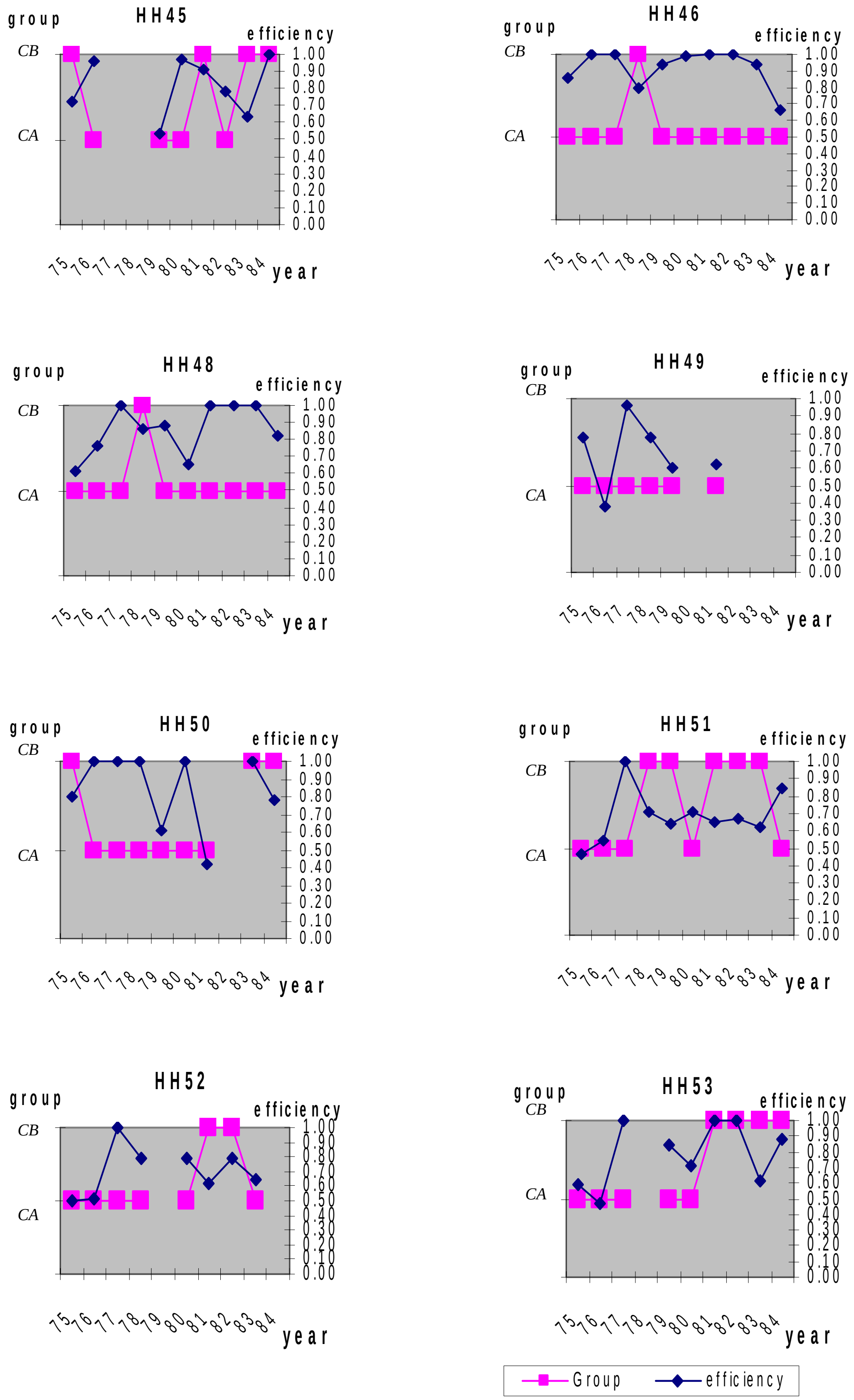
APPENDIX F-7 Continued





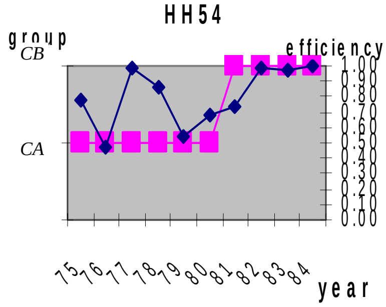
* *CA* = group A for castor growers; *CB* = group B for castor growers.

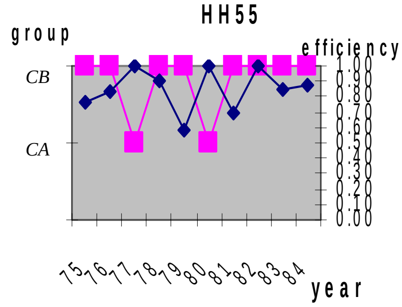
APPENDIX F-7 Continued

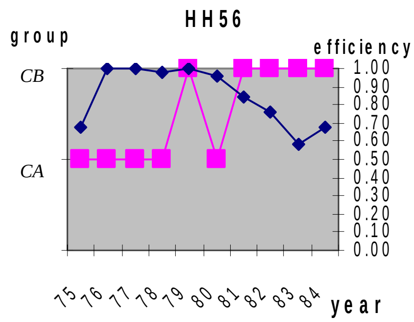


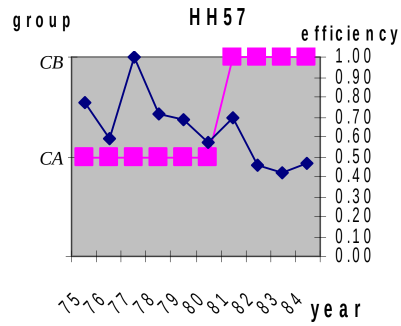
* *CA* = group A for castor growers; *CB* = group B for castor growers.

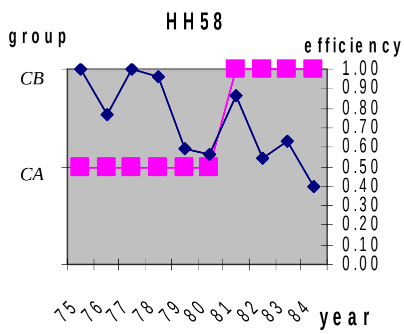
APPENDIX F-7 Continued

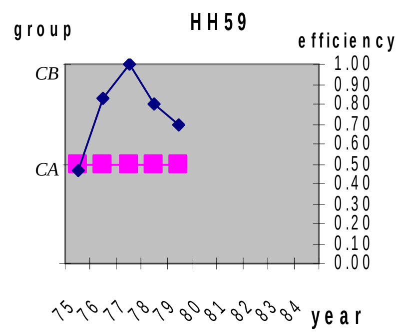


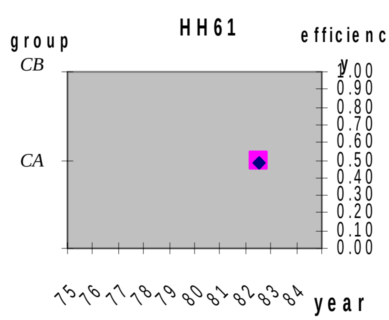


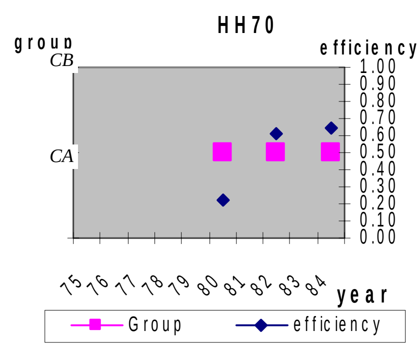






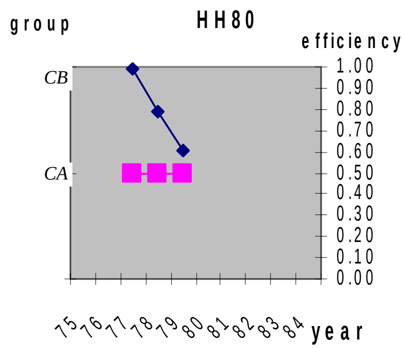


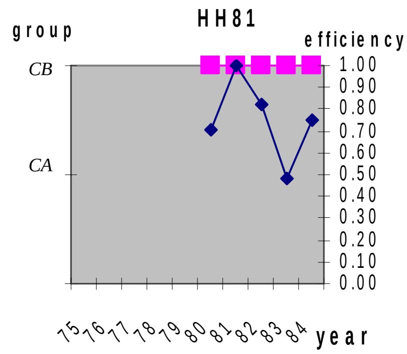


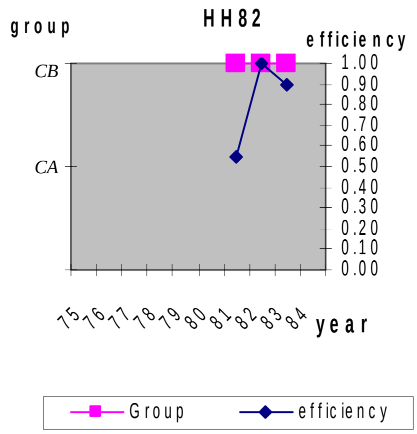


* *CA* = group A for castor growers; *CB* = group B for castor growers.

APPENDIX F-7 Continued







* *CA* = group A for castor growers; *CB* = group B for castor growers.

APPENDIX F-8: Production Efficiency – Castor Production before 1981

The households that always stay in group *CA*

Farm Size	Household	1975	1976	1977	1978	1979	1980	average
Labor households	1						0.39	0.39
Small Farm	33	1.00	0.59	1.00	0.90	0.75	0.53	0.80
	34		1.00		1.00		0.57	0.86
	36		1.00		0.77		1.00	0.92
	37				0.72	0.80		0.76
	39				0.81			0.81
	70						0.23	0.23
Medium Farm	40	0.59		1.00	0.79	0.86		0.81
	41	1.00	0.54	1.00	0.63	0.96	0.74	0.81
	42	0.49				1.00		0.74
	49	0.78	0.38	0.96	0.78	0.60		0.70
	80			0.99	0.79	0.60		0.79
Large Farm	52	0.50	0.51	1.00	0.79		0.79	0.72
	53	0.59	0.47	1.00		0.84	0.71	0.72
	54	0.77	0.47	0.99	0.86	0.54	0.68	0.72
	57	0.78	0.59	1.00	0.71	0.68	0.57	0.72
	58	1.00	0.76	1.00	0.97	0.59	0.56	0.81
	59	0.46	0.83	1.00	0.80	0.70		0.76

Households that always stay in group *CB*

Farm Size	Household	1975	1976	1977	1978	1979	1980	average
Small Farm	38				0.85			0.85
Medium Farm	81						0.71	0.71

Households that change groups

Farm Size	Household	1975	1976	1977	1978	1979	1980	average
Small Farm	32	1.00	1.00			0.84		0.95
	35	0.86	0.86		0.76	0.65	1.00	0.83
Medium Farm	43		0.56	1.00	0.62	0.56	0.55	0.66
	44	0.76	0.63		0.75	0.65	0.31	0.62
	45	0.72	0.96			0.54	0.97	0.80
	46	0.85	1.00	1.00	0.79	0.94	0.99	0.93
	48	0.61	0.77	1.00	0.86	0.88	0.66	0.80
Large Farm	50	0.80	1.00	1.00	1.00	0.61	1.00	0.90
	51	0.46	0.55	1.00	0.71	0.64	0.71	0.68
	55	0.77	0.83	1.00	0.90	0.58	1.00	0.84
	56	0.68	1.00	1.00	0.98	1.00	0.96	0.94

* *CA* = group A for castor growers; *CB* = group B for castor growers.

** Labor household indicate the households operating less than 0.2 hectares of land; small, medium, and larger farms indicate the households operating 0.2 to 2.5, 2.51 to 5.26, and over 5.26 hectares of land, respectively.

APPENDIX F-9: Production Efficiency – Castor Production after 1981

The households that always stay in group *CA*

Farm Size	Households	1981	1982	1983	1984	Average
Labor Household	1	0.60	0.99	0.67	0.70	0.74
	5			0.63	0.62	0.63
	61		0.48			0.48
Small Farm	33	0.68	0.77	1.00	0.75	0.80
	34		1.00			1.00
	36		1.00		0.86	0.93
	37	0.49				0.49
	70		0.61		0.64	0.63
Medium Farm	46	1.00	1.00	0.94	0.66	0.90
	48	1.00	1.00	1.00	0.82	0.96
	49	0.62				0.62

Households that always stay in group *CB*

Farm Size	Households	1981	1982	1983	1984	Average
Labor Household	10	0.99	0.73	0.61	0.33	0.66
Small Farm	32	0.72	0.81	0.79	0.91	0.81
	35	0.41	0.63	1.00	0.95	0.74
Medium Farm	43		0.69	0.67	0.48	0.61
	44		0.43			0.43
	81	1.00	0.82	0.49	0.75	0.76
	82	0.55	1.00	0.89		0.81
Large Farm	53	1.00	1.00	0.62	0.88	0.87
	54	0.74	0.98	0.98	1.00	0.92
	55	0.70	1.00	0.85	0.88	0.86
	56	0.84	0.76	0.58	0.67	0.71
	57	0.70	0.46	0.42	0.47	0.51
	58	0.87	0.54	0.63	0.40	0.61

Households that change groups

Farm Size	Households	1981	1982	1983	1984	Average
Small Farm	38		0.65		0.68	0.67
Medium Farm	45	0.91	0.78	0.63	1.00	0.83
Large Farm	50	0.42		1.00	0.78	0.73
	51	0.65	0.67	0.62	0.85	0.70
	52	0.62	0.79	0.64		0.69

* *CA* = group A for castor growers; *CB* = group B for castor growers.

** Labor household indicate the households operating less than 0.2 hectares of land; small, medium, and larger farms indicate the households operating 0.2 to 2.5, 2.51 to 5.26, and over 5.26 hectares of land, respectively.

APPENDIX G:

THE LCSFM RESULTS: THE SORGHUM IN AUREPALLE VILLAGE

APPENDIX G-1: Estimation Results -- Sorghum Production

Parameters	Estimates	Std. err.	Est./s.e.
Production Function For Group *SA*			
Constant	4.3429	0.5371	8.086
Seed	0.5074	0.169	3.003
Family Labor	0.1063	0.0972	1.093
Hired Labor	0.1	0.052	1.925
Animal	-0.064	0.2092	-0.306
Fertilizer	-0.0236	0.0364	-0.65
ln(Head Age)	-0.2924	0.1813	-1.613
Head Edu	0.256	0.0892	2.87
s.d. of random error	0.6061	0.1015	5.97
s.d. of 1-sided error	0.8167	0.1515	5.392
Production Function For Group *SB*			
Constant	1.6798	0.4861	3.456
Seed	-0.0238	0.0426	-0.558
Family Labor	0.3394	0.0612	5.545
Hired Labor	0.0627	0.0132	4.731
Animal	-0.0197	0.0297	-0.661
Fertilizer	-0.0475	0.0213	-2.228
ln(Head Age)	0.2504	0.1151	2.175
Head Edu	0.1371	0.0462	2.971
s.d. of random error	0.2077	0.0217	9.552
s.d. of 1-sided error	0.0023	0.0255	0.091
Membership Probabilities for Group *SB*			
Constant	0.5126	0.2626	1.952
Good Soil	-0.7227	0.3145	-2.298
Caste 1	-2.4071	1.1476	-2.098
Caste 2	1.4706	0.9604	1.531
Caste 3	-0.1644	0.2762	-0.595

* *SA* = group A for sorghum growers; *SB* = group B for sorghum growers.

APPENDIX G-2: Group Membership -- Sorghum Growers

Household	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
1						*SB*		*SB*		*SB*
5						*SB*			*SB*	*SA*
10						*SB*				*SA*
32		*SA*				*SB*			*SB*	*SB*
33	*SB*	*SA*	*SB*	*SB*		*SB*				
34			*SB*		*SB*	*SA*			*SA*	*SB*
35		*SA*	*SB*		*SB*	*SA*				*SB*
36		*SA*	*SB*		*SB*	*SB*	*SA*			
37	*SB*		*SB*			*SA*		*SB*		
38			*SB*	*SB*						*SA*
39			*SB*							
40			*SB*							
41			*SB*	*SB*	*SB*	*SB*				
43		*SA*	*SB*			*SB*	*SA*	*SA*	*SB*	
44			*SB*		*SA*	*SB*		*SA*		
45		*SA*	*SA*		*SA*		*SA*	*SA*	*SA*	
46		*SA*	*SB*	*SB*	*SB*					*SA*
48		*SA*	*SB*		*SB*	*SB*		*SB*		
49		*SA*	*SB*			*SB*				
50	*SA*	*SA*	*SA*		*SA*	*SA*				
51			*SB*	*SB*	*SA*	*SB*			*SB*	
52			*SA*			*SA*				
53		*SA*	*SA*			*SA*			*SA*	*SA*
54			*SA*		*SA*	*SA*				*SA*
55		*SB*	*SB*	*SB*		*SB*			*SB*	*SB*
56			*SB*	*SA*	*SA*	*SA*	*SA*	*SA*		
57		*SA*	*SA*	*SA*		*SA*				*SA*
58		*SA*	*SA*		*SA*			*SA*		*SA*
59		*SA*	*SA*	*SA*	*SA*					
70								*SA*		
80			*SA*	*SA*	*SA*					
81						*SB*		*SA*	*SB*	
82								*SA*		*SB*

* *SA* = group A for sorghum growers; *SB* = group B for sorghum growers.

APPENDIX G-3: The Numbers of Households by Groups – Sorghum Growers

Group	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
SA	1	14	9	4	9	9	4	8	3	8
SB	2	1	17	6	6	14	--	3	6	6
TOTAL	3	15	26	10	15	23	4	11	9	14

* *SA* = group A for sorghum growers; *SB* = group B for sorghum growers.

APPENDIX G-4: Household Characteristic – Sorghum Growers before 1981

The households that always stay in group *SA*

Household	Household's head age	Household's head education	Soil type	Caste
45	45	2	1	1
50	37	1	0 or 1	1
52	52	3	0	1
53	65	2	0	1
54	60	2	0 or 1	1
57	59	3	0 or 1	1
58	40	2	0	1
59	25	4	1	1
80	26	5	0	1

Households that always stay in group *SB*

Household	Household's head age	Household's head education	Soil type	Caste
1	42	1	0	4
5	32	1	0	4
10	57	1	0	3
38	45	1	0	3
39	42	1	0	4
40	52	3	0	2
41	54	2	0	2
55	60	1	0	3
81	32	3	0	3

Households that change groups

Household	Household's head age	Household's head education	Soil type	Caste
32	56	1	0	3
33	62	1	0	4
34	36	1	0	4
35	58	1	0	3
36	54	1	0	4
37	39	1	0	4
43	70	1	0 or 1	3
44	45	1	0 or 1	3
46	60	1	0	4
48	49	1	0 or 1	4
49	49	1	0	4
51	54	1	0	3
56	40	5	0	1

* *SA* = group A for sorghum growers; *SB* = group B for sorghum growers.
** Education ranges from 1 to 5. Type 1 indicates "illiterate"; type 2 indicates "read and write"; type 3 indicates "up to primary school"; type 4 indicates "up to middle school"; type 5 indicates "up to high school".
*** The soil type =1 if good soil is used for production; otherwise, soil type =0. The good soil represents deep, medium, and shallow black soil from the original ICRISAT India data.
**** The caste rank ranges from 1 to 4 indicating from the highest caste to lowest caste.

APPENDIX G-5: Household Characteristic – Sorghum Growers after 1981

The households that always stay in group *SA*

Household	Household's head age	Household's head education	Soil type	Caste
10	57	1	0	3
36	54	1	0	4
38	45	1	0	3
44	45	1	1	3
45	45	2	0 or 1	1
46	60	1	0	4
53	65	2	0	1
54	60	2	0	1
56	40	5	0 or 1	1
57	59	3	0	1
58	40	2	0	1
70	54	1	0	3

Households that always stay in group *SB*

Household	Household's head age	Household's head education	Soil type	Caste
1	42	1	0	4
32	56	1	0	3
35	58	1	0	3
37	39	1	0	4
48	49	1	0	4
51	54	1	0	3
55	60	1	0	3

Households that change groups

Household	Household's head age	Household's head education	Soil type	Caste
5	32	1	0	4
34	36	1	0	4
43	70	1	0	3
81	32	3	0 or 1	3
82	44	3	0 or 1	3

* *SA* = group A for sorghum growers; *SB* = group B for sorghum growers.

** Education ranges from 1 to 5. Type 1 indicates "illiterate"; type 2 indicates "read and write"; type 3 indicates "up to primary school"; type 4 indicates "up to middle school"; type 5 indicates "up to high school".

*** The soil type =1 if good soil is used for production; otherwise, soil type =0. The good soil represents deep, medium, and shallow black soil from the original ICRISAT India data.

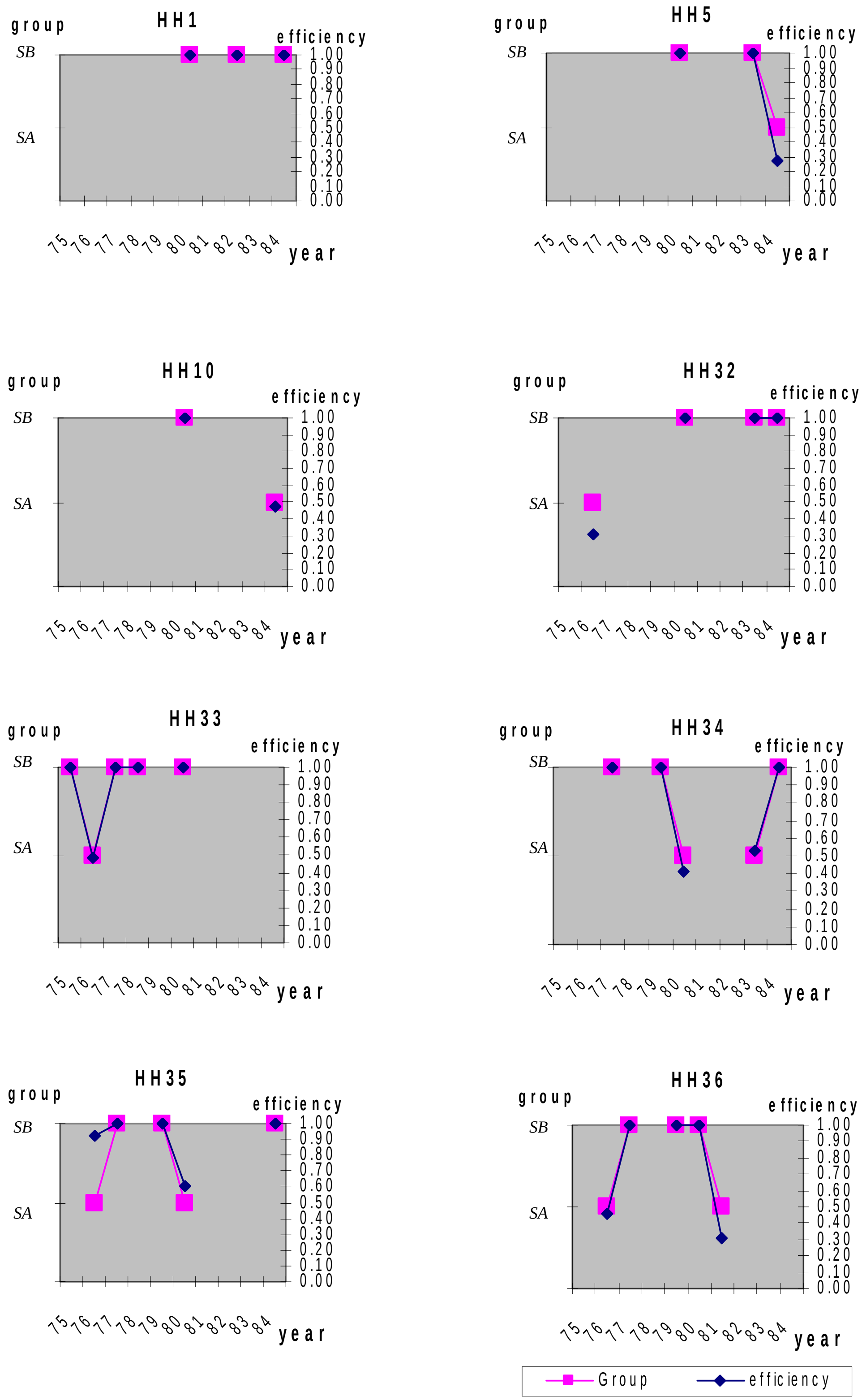
**** The caste rank ranges from 1 to 4 indicating from the highest caste to lowest caste.

APPENDIX G-6: Soil Type -- Sorghum

Household	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984
1						0		0		0
5						0			0	0
10						0				0
32		0				0			0	0
33	0	0	0	0		0				
34			0		0	0			0	0
35		0	0		0	0				0
36		0	0		0	0	0			
37	0		0			0		0		
38			0	0						0
39			0							
40			0							
41			0	0	0	0				
43		1	1			0	0	0	0	
44			1		1	0		1		
45		1	1		1		1	1	0	
46		0	0	0	0					0
48		0	1		0	0		0		
49		0	0			0				
50	0	0	0		0	1				
51			0	0	0	0			0	
52			0			0				
53		0	0			0			0	0
54			0		0	1				0
55		0	0	0		0			0	0
56			0	0	0	0	1	0		
57		1	0	0		0				0
58		0	0		0			0		0
59		1	1	1	1					
70								0		
80			0	0	0					
81						0		1	0	
82								1		0

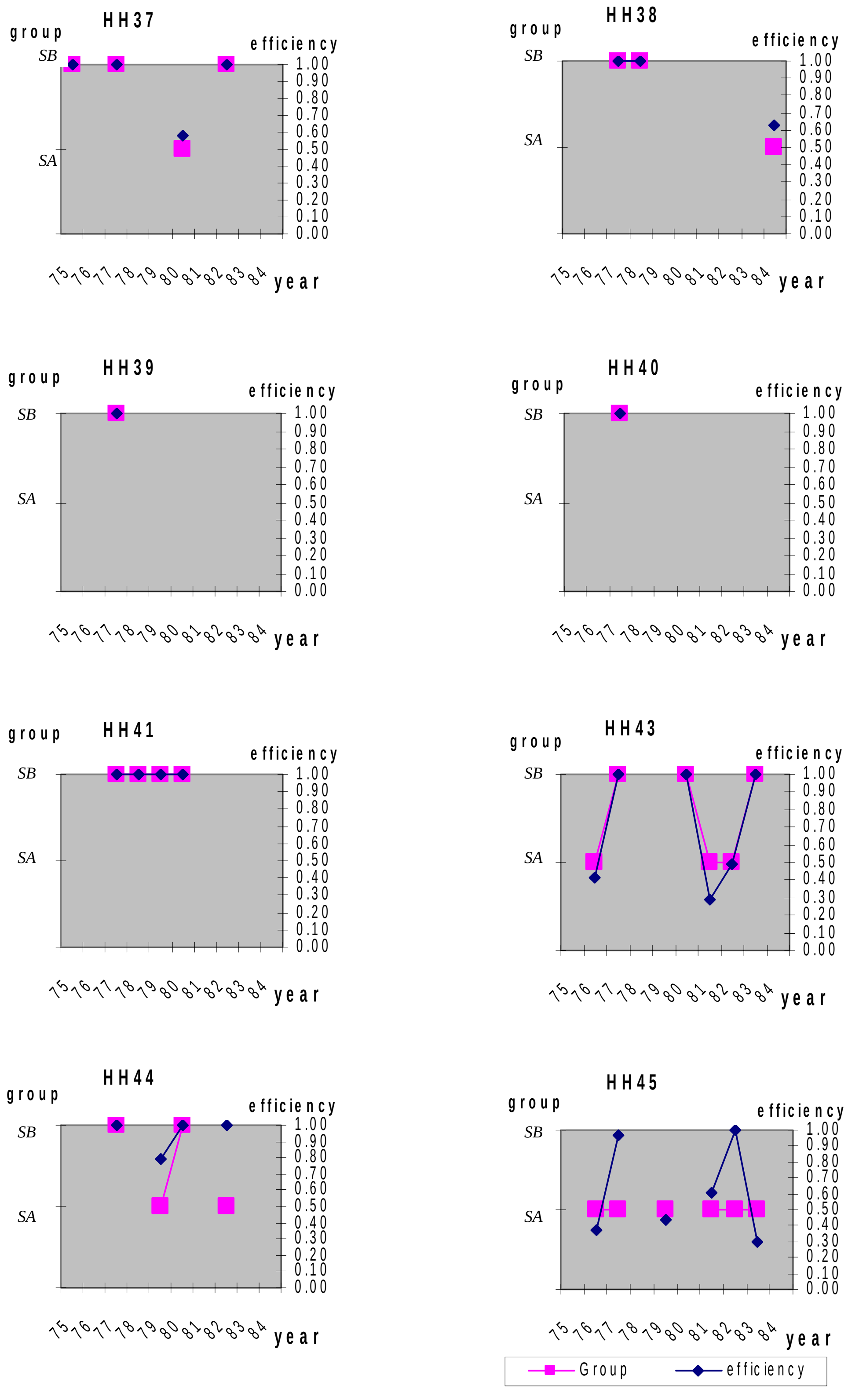
* The soil type =1 if good soil is used for production; otherwise, soil type =0. The good soil represents deep, medium, and shallow black soil from the original ICRISAT India data.

APPENDIX G-7: The Group Membership and the Corresponding Production Efficiency



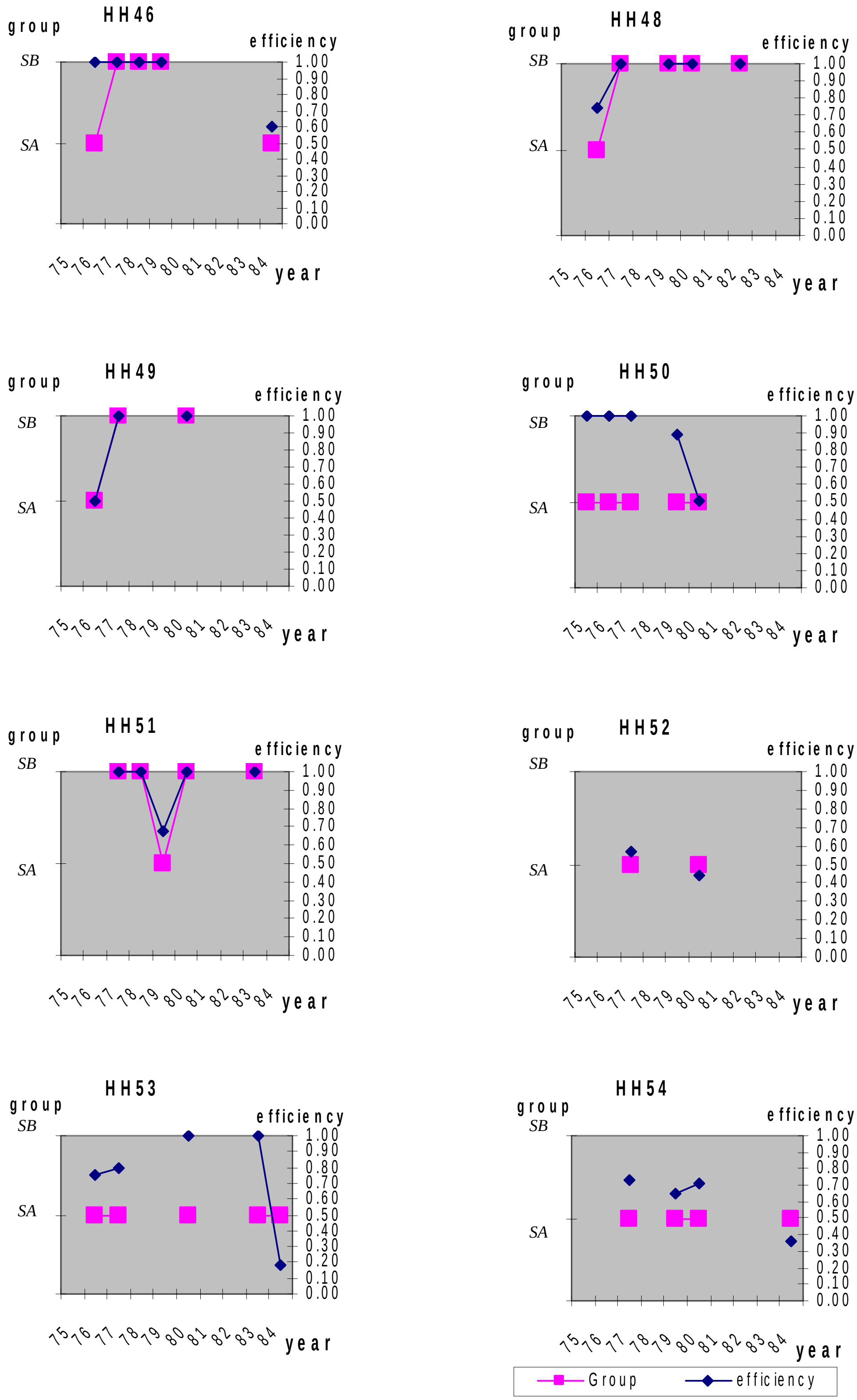
* *SA* = group A for sorghum growers; *SB* = group B for sorghum growers.

APPENDIX G-7 Continued



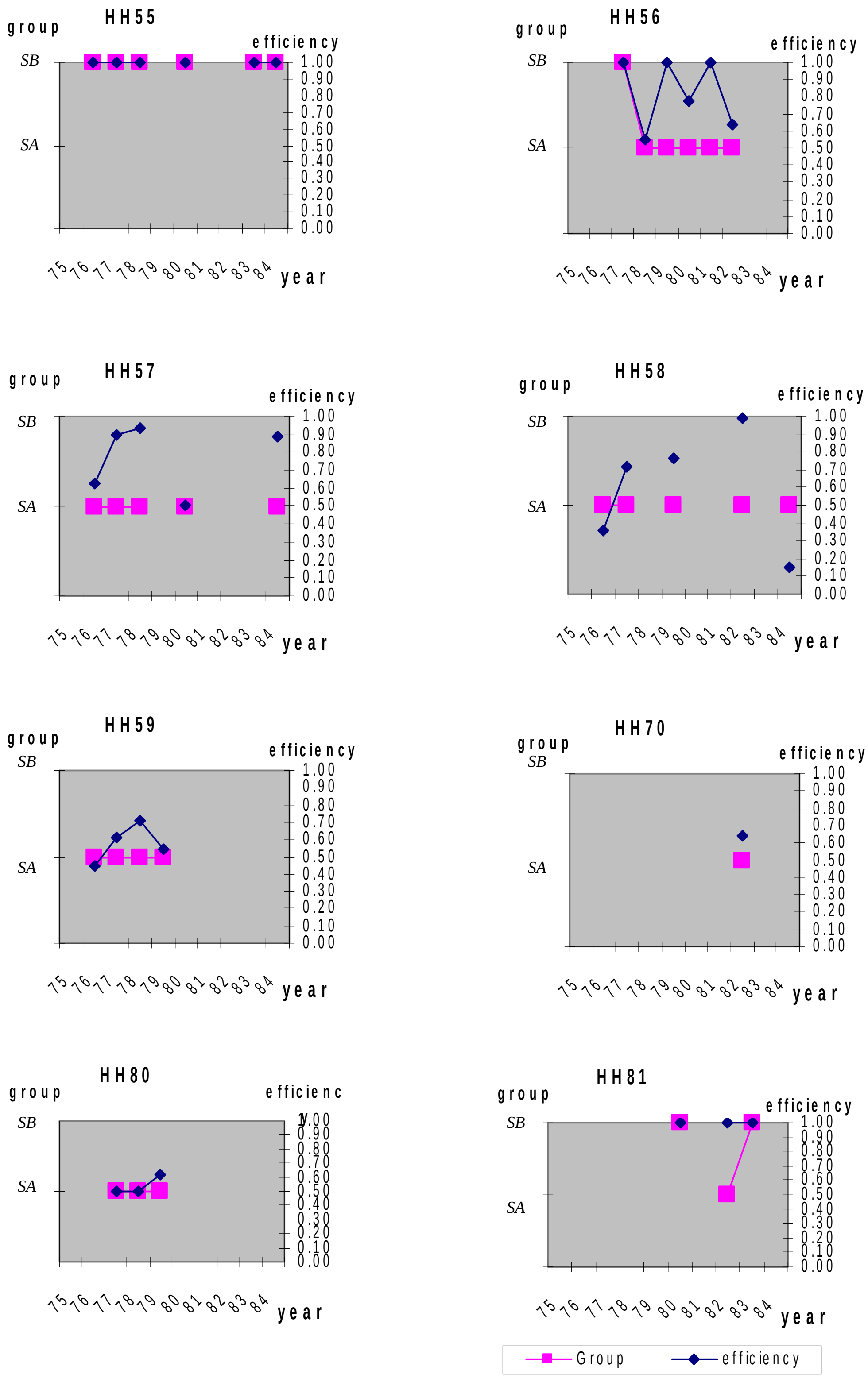
* *SA* = group A for sorghum growers; *SB* = group B for sorghum growers.

APPENDIX G-7 Continued



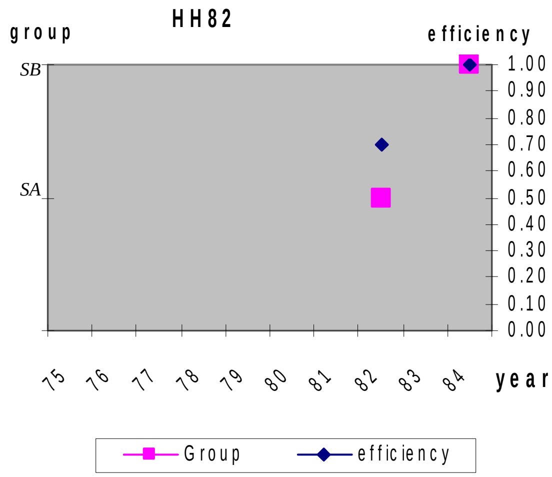
* *SA* = group A for sorghum growers; *SB* = group B for sorghum growers.

APPENDIX G-7 Continued



* *SA* = group A for sorghum growers; *SB* = group B for sorghum growers.

APPENDIX G-7 Continued



* *SA* = group A for sorghum growers; *SB* = group B for sorghum growers.

APPENDIX G-8: The Production Efficiency – Sorghum Production before 1981

The households that always stay in group *SA*

Farm Size	Household	1975	1976	1977	1978	1979	1980	average
Medium Farm	45		0.37	0.97		0.44		0.59
	80			0.50	0.50	0.62		0.54
Large Farm	50	1.00	1.00	1.00		0.89	0.51	0.88
	52			0.57			0.44	0.50
	53		0.76	0.80			1.00	0.85
	54			0.73		0.65	0.71	0.70
	57		0.63	0.90	0.93		0.51	0.74
	58		0.36	0.72		0.76		0.61
	59		0.45	0.61	0.71	0.55		0.58

Households that always stay in group *SB*

Farm Size	Household	1975	1976	1977	1978	1979	1980	average
Labor Household	1						1.00	1.00
	5						1.00	1.00
	10						1.00	1.00
Small	38			1.00	1.00			1.00
	39			1.00				1.00
Medium Farm	40			1.00				1.00
	41			1.00	1.00	1.00	1.00	1.00
	81						1.00	1.00
	55		1.00	1.00	1.00		1.00	1.00

Households that change groups

Farm Size	Household	1975	1976	1977	1978	1979	1980	average
Small Farm	32		0.31				1.00	0.66
	33	1.00	0.49	1.00	1.00		1.00	0.90
	34			1.00		1.00	0.41	0.80
	35		0.92	1.00		1.00	0.60	0.88
	36		0.46	1.00		1.00	1.00	0.86
	37	1.00		1.00			0.58	0.86
Medium Farm	43		0.41	1.00			1.00	0.80
	44			1.00		0.79	1.00	0.93
	46		1.00	1.00	1.00	1.00		1.00
	48		0.74	1.00		1.00	1.00	0.94
	49		0.50	1.00			1.00	0.83
Large Farm	51			1.00	1.00	0.67	1.00	0.92
	56			1.00	0.55	1.00	0.77	0.83

* *SA* = group A for sorghum growers; *SB* = group B for sorghum growers.

** Labor household indicate the households operating less than 0.2 hectares of land; small, medium, and larger farms indicate the households operating 0.2 to 2.5, 2.51 to 5.26, and over 5.26 hectares of land, respectively.

APPENDIX G-9: The Production Efficiency – Sorghum Production after 1981

The households that always stay in group *SA*

Farm Size	Households	1981	1982	1983	1984	Average
Labor Household	10				0.48	0.48
Small Farm	36	0.31				0.31
	38				0.62	0.62
	70		0.64			0.64
Medium Farm	44		1.00			1.00
	45	0.61	1.00	0.29		0.63
	46				0.60	0.60
Large Farm	53			1.00	0.18	0.59
	54				0.36	0.36
	56	1.00	0.64			0.82
	57				0.88	0.88
	58		0.99		0.15	0.57

Households that always stay in group *SB*

Farm Size	Households	1981	1982	1983	1984	Average
Labor Household	1		1.00		1.00	1.00
Small Farm	32			1.00	1.00	1.00
	35				1.00	1.00
	37		1.00			1.00
Medium Farm	48		1.00			1.00
Large Farm	51			1.00		1.00
	55			1.00	1.00	1.00

Households that change groups

Farm Size	Households	1981	1982	1983	1984	Average
Labor Household	5			1.00	0.27	0.64
Small Farm	34			0.53	1.00	0.76
Medium Farm	43	0.29	0.49	1.00		0.59
	81		1.00	1.00		1.00
	82		0.70		1.00	0.85

* *SA* = group A for sorghum growers; *SB* = group B for sorghum growers.

** Labor household indicate the households operating less than 0.2 hectares of land; small, medium, and larger farms indicate the households operating 0.2 to 2.5, 2.51 to 5.26, and over 5.26 hectares of land, respectively.

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