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# A MULTI-CRITERIA OPTIMIZATION MODEL FOR A FOUR-STAGE MULTI-PRODUCT SUPPLY CHAIN

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by

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# ABSTRACT

The goal of this thesis is to develop a supply chain model that integrates several common supply chain modeling techniques into a realistic and comprehensive general model. The model addresses production, transportation, and inventory decisions, which are often considered separately. Supply chain modeling problems tend to be inherently multi-criteria problems in that the optimization of several objectives simultaneously is required in order to develop the most efficient supply chain. In this thesis up to three criteria are considered simultaneously and are used to develop a multi-criteria optimization model. The three criteria are profit, customer service, and inventory capital. The solution to the model can be used as a supply chain management tool to determine the quantities and scheduling related to the production and distribution of products across a supply chain in order to best optimize the objectives.

The model developed is for a four-stage, centralized supply chain in which two products are being produced and distributed. There are several suppliers, manufacturers, warehouses, and retailers at each stage and each manufacturer has three production lines. Two modes of transportation are available between each stage of the supply chain and a freight rate function is utilized. The freight rate function follows an All-Units discount cost structure that depends on the weight of shipments and the transportation mode being used. The demand at the retailers drives the supply chain and is deterministic and independent among the retailers. The supply chain is modeled as a mixed integer linear program that is solved using GAMS (General Algebraic Modeling System). The multi-criteria model is solved using goal programming, a multi-criteria modeling technique that aims to minimize the total deviation of the objective values from their goals. The problems are solved either as

a preemptive goal program or non-preemptive goal program, depending on how the criteria preference information is specified.

An illustrative example is used to demonstrate the implementation of goal programming and the overall use of the model. Two bi-criteria cases are modeled in which profit and customer service are maximized. Additionally, a third case addresses changes in the solution when a third objective, the minimization of inventory capital, is added to the model. The solutions to all three cases are compared and differences among preemptive and non-preemptive solutions are identified, as well as the differences between the bi-criteria and multi-criteria models. The model not only incorporates several supply chain modeling techniques into a single model, it also addresses multiple criteria simultaneously in an attempt to develop a realistic supply chain model that can be used as a tool to improve supply chain efficiency.

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# Chapter 1 Introduction

## 1.1 The General Supply Chain

Maximizing profit is often thought to be the key to success for businesses in a competitive market. Although a simple concept to grasp, achieving a maximum profit tends to be a daunting task that requires the synergy of all components of a company. From new product development and the product's supply chain, to effective management, marketing, and advertising; all aspects need to be operating efficiently to maximize profit. In dealing with straightforward revenue and cost though, supply chain management is imperative to success. The largest contributors to supply chain costs are those associated with inventory and transportation. Ideally the direct revenue of the product balances these costs resulting in a profit for a company. Although inventory and transportation costs are the leading supply chain drivers, there is limited literature and modeling for supply chains that include both important components due to the complexity of the cost functions.

With supply chain management at the forefront of industry today, the development and analysis of realistic models is important when dealing with profit and responsiveness of a supply chain. A supply chain includes all stages in the production and distribution of a product in response to customer demand. A supply chain is typically broken down into stages where a four stage supply chain typically includes suppliers, manufacturers, warehouses, and retailers. A serial supply chain will have one party in each of these stages as shown in Figure 1.1.

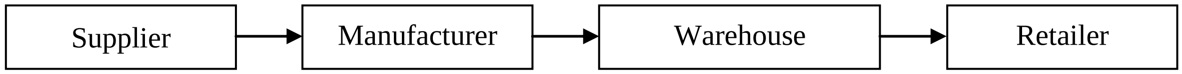



**Figure 1.1: A four stage serial supply chain**

However, it is often more realistic to include multiple parties at each stage and multiple modes of transportation between stages to create a supply chain network. This thesis will model a supply chain network consisting of multiple suppliers, manufacturers, warehouses, and retailers with multiple modes of transportation between each stage. A general supply chain with *'k'* suppliers, *'m'* manufacturers, '*n'* warehouses, and *'r'* retailers is shown in Figure 1.2.

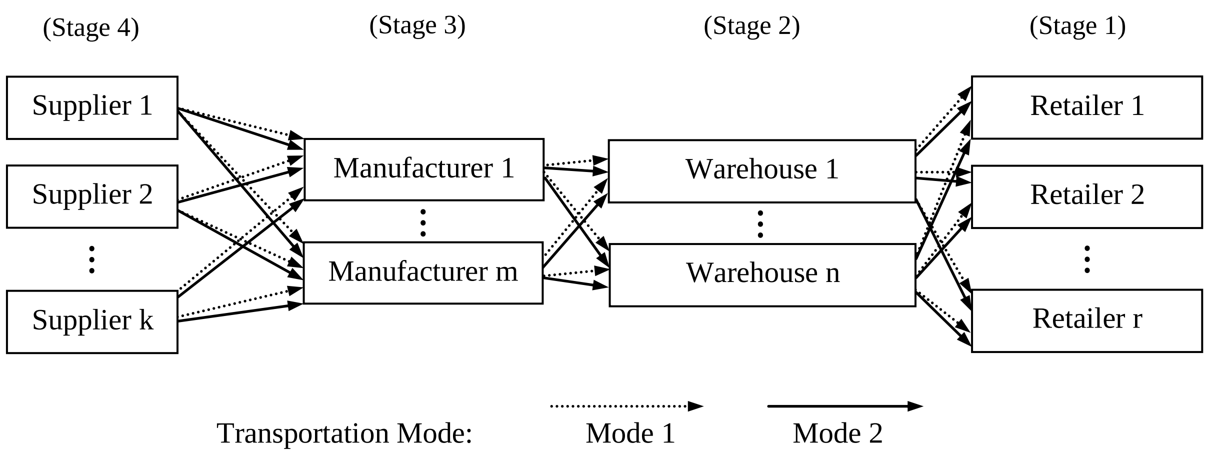



**Figure 1.2: Four stage supply chain network with two modes of transportation**

Supply chain management (SCM) involves determining the appropriate quantities to ship to and from each stage, the right time for each shipment, and the best mode of transportation for the product while still turning a profit. Chopra and Meindl explain: "Effective supply chain management involves the management of supply chain assets and product, information, and fund flows to maximize total supply chain profitability"

(2007, pp. 6). By optimizing a supply chain, the best quantities are determined for both product flows and inventory levels to meet a certain objective like maximizing profit.

In multiple stage supply chains, ownership of each stage may be shared by the same company or a different party may control each stage. In this thesis, a centralized supply chain is considered implying one company has ownership of all stages and can access information including inventory, supply, and demand levels across the entire supply chain. Although the suppliers can be a second party vendor in the model included in this thesis, the materials for the final product require only the supplies available and therefore the decisions regarding the distribution of the supplies are made by the single owner of the other three stages.

## 1.2 Multi-Criteria Optimization

It is common for optimization models to consider just one objective at time. However, to accurately model a problem, multiple objectives, or criteria, are more appropriate. In addition to the maximization of profit, the responsiveness of a supply chain is also a key to a successful business. The responsiveness of a supply chain reflects how well it can respond to demand by providing the products on time to customers regardless of demand uncertainty. This can be controlled in several ways including the location of facilities and the amount of product in inventory. Maximization of responsiveness aims to balance the costs of customer dissatisfaction and the cost of holding extra products in inventory while limiting the number of lost sales. Profit typically considers the difference between the revenue and the costs of the supply chain. When focusing on just customer responsiveness, transportation and holding costs may be disregarded in order to maintain a high enough inventory level to accommodate for unexpected spikes in demand. Consequently the idea of limiting lost sales can increase transportation and inventory costs, resulting in a conflicting objective with profit.

When dealing with a supply chain problem, multiple criteria such as the maximization of profit and maximizing responsiveness produce conflicting solutions for consideration. Masud and Ravindran (2008) describe the concept of Multiple Criteria Decision Making (MCDM), where several conflicting criteria are considered simultaneously and the optimal solution is the alternative that best balances the tradeoffs among the conflicting criteria. For example, with the two objectives of maximizing profit and responsiveness at the same time, one alternative may be to produce a large amount of a product to ensure demand can always be satisfied. Although this alternative may maximize responsiveness, it may not be the most profitable solution due to holding costs of inventory and the cost of overproduction of a product. The opposite case, which is another alternative, is to maximize profit by producing as little as possible so minimal inventory is held. However, this increases the risk of not satisfying demand which decreases customer responsiveness and can result in the loss of customers and goodwill. Consequently, both profit and responsiveness should be considered simultaneously to produce the best compromise solution.

Constraints on the model are also considered along with the objectives in a decision making problem. The constraints may include limited capacity at a warehouse, a transportation cost budget, and demand requirements, among others. The goal of a supply chain optimization problem is to best satisfy all of the objectives while adhering to the constraints. The objectives and constraints are included in the general form of a **Multiple Criteria Mathematical Program** (MCMP) as follows:

$$\text{Maximize } [f_1(x), f_2(x), \dots, f_k(x)]$$

$$\text{Subject to: } g_j(x) \leq 0, j = 1, 2, \dots, m$$

$$x \geq 0$$

Where $f_1$, $f_2, \dots$, $f_k$ are the objective functions and $g_j(x)$ is the set of constraints. Each alternative is represented as an x vector where $x = \{x_i \mid i = 1,2,\dots,n\}$.

**Definitions**

**Ideal Solution**: The ideal solution is the best value achievable for each objective ignoring other objectives. This is the solution to the problem when only one objective function is considered.

**Efficient Solution**: An efficient solution is an alternative for which an objective can only be improved at the expense of at least one other objective. An efficient solution is also known as a non-dominated solution.

**Dominated Solution**: A dominated solution is an alternative for which an objective can be improved without losing achievements in other objectives. This solution will not be the optimal solution since this solution is dominated by other solutions that have improved objective values.

## Methodology for MCMP

When solving MCMPs, a decision maker (DM) is identified to provide information about their preferences for how the problem is to be solved. This information includes how important each criterion is in relation to the others, as well as what level of satisfaction certain constraints must meet, among other preferences. There is typically only one DM in a centralized supply chain while decentralized supply chains can involve input from several DMs before a final solution is reached.

There are three categories for problem solving under Multiple Criteria Mathematical Programming: All information about the objectives and the problem is pre-specified by the DM; All information from the DM is retrieved after a set of efficient solutions has been found; Interactive approaches that use information and preferences that are revealed progressively by the DM or DMs throughout the solution and analysis processes. Since the focus of this thesis is a centralized supply chain, it will be assumed that all of the DMs preferences have been pre-specified. Under this category, there are several solution methods available (Masud & Ravindran, 2008).

## Goal Programming

Goal programming is a solution method commonly used in practice to convert a multiple objective program into a solvable single objective program. Goal programming requires that the DM specifies a goal for each criterion. It may not be possible to achieve all the specified goals simultaneously. Therefore, the sum of the weighted deviations from the preset goals is minimized during the optimization. Depending on how the DM is asked to specify their goal preferences, the goal program can be identified as either a non-preemptive or preemptive goal programming.

Non-preemptive goal programming uses pre-specified weights for the criteria. These weights represent the decision maker's tradeoffs among the goals. Sometimes assigning a value to the tradeoffs can be a difficult task for the DM so deriving the weights is complicated. These weights are used to reduce the formulation to a single objective problem that can then be solved with linear programming optimization techniques. Single objective optimization problems can be solved rather quickly and efficiently with current optimization software. However, non-preemptive goal programming requires scaling of all criteria and assumes a linear utility function. Alternatively, preemptive goal programming allows the decision maker to assign priority to certain goals over others, which is sometimes easier than assigning weights to criteria. Then, goals with higher priority must be satisfied as much as possible before lesser priority goals are considered. This type of problem is solved in a sequential fashion with multiple single objective problems being solved one at a time. Scaling of criteria values is not required. However, the drawback of this method is the intensity and duration of the solution process (Masud & Ravindran, 2008; Charnes & Cooper, 1977).

## 1.3 Thesis Overview

The goal of this thesis is to expand on existing supply chain models to develop a multi-criteria, multi-period model that more closely resembles a real world centralized supply chain with a focus on the maximization of profit and maximization of responsiveness simultaneously. In addition to the model development, several scenarios will be solved and analyzed with a focus on tradeoffs and changes in inventory and shipment quantities while the satisfaction of the objectives is considered. This analysis will be possible due to the fact that both inventory and transportation cost structures will be modeled and included together in the problem. This thesis will be an extension and integration of theses published by Vijayaragavan (2008), Mysore (2005), and Difilippo (2003).

In this thesis, a four stage centralized supply chain will be modeled with the inclusion of inventory and transportation decisions. The two main objectives of maximizing cost while maximizing responsiveness will be used and a third objective to minimize the capital in inventory will be added in certain cases. Two modes of transportation, ground and air, will be available along all possible paths for transportation of material and finished goods. This supply chain structure will be based on the general four stage supply chain with two modes of transportation that was presented in Figure 1.2.

Quantity discounts will be available on the shipment of the products so tradeoffs between transportation costs and holding costs of inventory will need to be balanced. The demand at each retailer will be deterministic and predefined for several periods. Lastly, the most unique aspect of this thesis is the inclusion of two products flowing along the supply chain. The products have some shared raw materials, and all other parties in the supply chain will handle both products, but each retailer will have a separate demand for each product.

The goal will be to determine the optimal values for the quantities of each product moving from stage to stage and the identification of what transportation mode to use when. Consequently, the optimal production schedule for all periods will be determined along with inventory levels at the warehouse. Each manufacturing facility will have multiple production lines available for the production of each product. The model will also determine dedicated lines which will be used for certain products and flexible lines which will be turned on or off or need to transfer from product to product depending on production needs.

Chapter 2 of this thesis will include a detailed literature review of work related to the topic and model presented in this thesis. In Chapter 3, the general model will be developed in detail. Here, all decision variable definitions will be provided, along with a presentation of the objective goal specifications and objective function construction. In addition, the constraints of the model will be explained and presented. Chapter 4 will include an illustrative example of the model where all costs, selling prices, and constraint values such as demands for each product will be quantified. Lastly, Chapter 5 will be a conclusion including a complete analysis and discussion of the results from the illustrative example as well as suggestions for future work.

# Chapter 2 Literature Review

## 2.1 Supply Chain Performance Measures

The acknowledgement of Supply Chain Management has made it apparent for many companies that their goal is to create the most efficient supply chain possible. However, it is often unclear how to evaluate a supply chain. In order to reach the goal of achieving an efficient system, supply chain performance measures are used to evaluate the effectiveness of a supply chain and where there is room for improvement. Beamon (1999) discussed the difficulties faced in developing and utilizing performance measures. Two common quantitative performance measures that have been used extensively in the literature are cost and customer responsiveness. Costs can include a wide range of categories such as inventory and operating costs, whereas customer responsiveness includes lead times, stockout probability, and fill rate. Other qualitative performance measures such as customer satisfaction, information flow, and risk management may be valuable but are hard to model and measure and therefore literature surrounding them is much more limited.

Many models only consider cost and perhaps one other measure and therefore have been declared non inclusive and may not provide all of the information necessary for a company to achieve its strategic goals. Consequently, research available tends to focus on the development of a complete framework for measuring supply chain performance. Beamon (1999) stressed the importance of the performance measure selection step in developing a supply chain model and emphasized a focus on complete and accurate analysis. A comprehensive overview of previous literature surrounding supply chain performance measures was also presented throughout the article.

Gunasekaran et al. (2001) stressed the development of a balanced approach for developing supply chain measures and metrics. The approach covered strategic, tactical, and operational levels as well as covering financial and non-financial measures. The supply chain was then addressed from beginning to end and various types of measures were presented for each stage. For example, the total order cycle time was the focus for the procurement stage of the supply chain. In addition, measures for the production and delivery processes were presented, as well as how to address customer service and satisfaction. Gunasekaran et al. (2001) stated that "perhaps the most important research concerning logistics that is going on is in the area of designing efficient and cost-effective distribution systems" (2001, p, 78). Distribution costs stem from inventory, transportation costs, and distribution methodology, including transportation mode selection. Thomas and Griffin (1996) stated that transportation costs can account for 50% of the total logistics costs. This emphasis on the reduction of transportation and inventory costs as supported by the consideration of both costs structures in the model presented in this thesis.

## 2.2 Inventory Control and Transportation Decisions

With the large role of transportation costs incurred throughout a supply chain, the efficient movement and storage of materials and products is imperative to limiting the total costs for a company. When considering the movement of materials, Cohen and Lee (1989) explained that the locations and capacities of each site in the whole supply chain network must be considered. Cohen and Moon (1990) extended this work and presented results surrounding the influence of supply chain transportation costs on the overall supply chain strategy for a firm. The location of manufacturing facilities and the distribution policy of a supply chain network are directly affected by transportation costs. Supply chain design with a constrained cost minimization problem that allows for variation of inputs such as fixed costs, transportation costs, and production costs was analyzed. The model then provided information regarding the opening and closing of sites, order quantities, production quantities, and specifics about the transportation modes and quantities to operate at minimum costs. This analysis provided an overall evaluation of supply chain strategies with a focus on transportation modes and costs (Cohen and Moon, 1990).

Langley (1980) was one of the first researchers to use estimated and actual freight rate functions to more accurately calculate transportation costs. An enumeration technique was used to determine break points and an appropriate value for Q, the optimal order quantity, in an Economic Order Quantity (EOQ) inventory model. A similar technique was implemented by Carter and Ferrin (1996) while actual freight rate functions were used. The focus of the model was again the minimization of transportation costs.

In the work by Swenseth and Godfrey (2002), the derivation of an accurate freight rate function in models that incorporate inventory and transportation decisions was discussed. The freight rate function was used to model the transportation costs based on weight of shipments. Companies often offer discounts on larger or heavier shipments. Because of this, it is important that the cost of a shipment depends directly on the weight or quantity shipped from site to site in a supply chain. Five freight rate models from literature were presented and analyzed in an attempt to determine which functions best emulate actual costs. The strategy of identifying breakpoints where it is appropriate to over-declare a shipment in order to incur a lower per-unit cost is a common practice in industry and an appropriate strategy for doing so was presented in the paper.

Terminology surrounding inventory control, as well as a description of common modeling practices, as presented in a review of inventory control papers through 1971 by Clark (1972). The idea of multi-echelon inventory problems considers supply chains that have two or more facilities involved in the response to customer demand. Several variations of multi-echelon inventory models were compared such as deterministic vs. stochastic demand, and continuous vs. period review policies. Chopra and Meindl (2007) emphasized the importance of inventory and transportation cost tradeoffs. Inventory aggregation is achieved by physically aggregating inventories in one location to reduce the safety inventory and the number of warehouses. Although this reduces facility and holding costs, there is a tradeoff with the increase in transportation costs caused by moving all product quantities through fewer locations as opposed to having more warehouses closer to the final destinations. Consequently, a recent focus on incorporating both inventory and transportation decisions into the same model has created an extensive library of work available in the literature.

The following is a review of key work in the area of simultaneous inventory and transportation decisions with a focus on relevant work for this thesis. Baumol and Vinod (1970) developed the inventory theoretic model. This was the first attempt to integrate both transportation and inventory costs in the same model. Transportation costs were calculated as the constant shipping rate multiplied by the number of products shipped. As expected, when multiple modes of transportation were available, the selection of a mode that will not only be dependable, but will be most efficient in cost and time was important. The model addressed the question, what is the advantage of speed in transportation? Choosing a fast transportation mode limited the amount of freight-in-transit and the lead-time of an order, this in turn limited cost. However, there are trade-offs between speed, dependability, and cost of transportation which Baumol and Vinod (1970) discussed in detail. The inventory held in warehouses in case demand exceeds expectation or to compensate for a lack of dependability of transportation is referred to as safety stock (Chopra and Meindl, 2007). To compensate for the lack of speed and dependability of a cheaper form of transportation, the amount of safety stock must increase which increases holding costs. All costs and tradeoffs were considered together in the inventory theoretic model for which deterministic demand was used to derive an optimal solution for quantities and costs (Baumol and Vinod, 1970).

Constable and Whybark (1978) further extended the work of Baumol and Vinod to include backorders in a model that again, included multiple modes of transportation, transportation costs, and inventory costs when determining the minimum cost solution. By focusing on the interaction of transportation and inventory costs within a model, it was identified that the lowest cost transportation mode was favored as the lowest cost alternative. Therefore it was concluded that it is rational to use the transportation cost ratio for separating the transportation and inventory decisions.

The joint determination of an appropriate transportation mode and an optimal inventory control policy in supply chain management was presented by Tyworth and Zeng (1998). The transit time was dependent upon the transportation mode and was part of lead time which is the time it takes to process an order and ship it to its destination. Sensitivity analysis of the effect of transit time on the logistics costs was performed and discussed in the article.

The inventory theoretic model by Baumol and Vinod (1970) is used as a basis for a model in which freight rate discounts are combined with the both transportation and inventory decisions by Tersine et al. (1989). Algorithms were developed to use freight rate discounts while determining optimal inventory and transportation decisions and order sizes to minimize the total cost. The assumptions of the model included a single product with independent, deterministic demand, no stockouts allowed, infinite replenishment rates, and constant lead times. Two different algorithms were developed to solve the model with either an all-units freight-rate discount cost structure or an incremental freight rate discount cost structure based on shipment weight. In the all-units structure, the per-unit cost that corresponds to the bracket that the total quantity falls within is applied to the entire quantity. In the incremental cost structure, the per-unit cost corresponding to a bracket is applied to the portion of the quantity that falls within that cost bracket. The value of algorithms that can solve the minimum cost problems with the inclusion of these structures in the market today was stressed due to the increasing demand for improved operations and the best utilization of a company's assets.

The savings resulting from logistics cost savings initiatives at General Motors (GM) were presented by Blumenfeld et al. (1987). GM has a large network consisting of 2500 suppliers worldwide, 130 GM parts plants, and 30 assembly plants. A typical GM vehicle will require approximately 13,000 parts total. Due to the complexity of this supply chain problem, reduction in costs is very important to companies with distribution networks of this size. A research group at GM incorporated transportation and inventory decisions with an estimated freight rate function in a model of the supply chain network. The tool resulted in a

26% reduction in total annual logistics costs which was equal to a $2.9 million savings per year at a single division of GM.

A more general description of distribution strategies that optimized the tradeoffs between transportation and inventory costs was presented by Burns et al. (1985). Researchers at the General Motors Research Laboratories conducted analysis on distribution methods comparing costs of direct shipping routes to peddling routes. In the direct shipping routes a separate truck is sent directly to each customer, whereas "peddling routes," or "milk runs," utilize one truck to service several customers. The order quantities derived using the EOQ model were most efficient in a direct route distribution network. Conversely, a full truck load was the optimal shipment for peddling routes. This conclusion was contingent upon the number of customers being serviced though. Formulas for the necessary calculations to draw these conclusions were developed and presented in an attempt to simplify distribution problems that consider both transportation and inventory costs.

Mysore (2005) developed a single objective model for a three stage centralized supply chain for which profit was to be maximized. The model was unique in that it incorporated both inventory and transportation cost structures while also implementing all-unit and incremental freight rate shipping discount structures. Typically supply chain models only incorporate one type of discount structure whereas in the thesis by Mysore (2005), a different cost structure was used for transportation and inventory quantities. The model offered an option for a leased warehouse if the company owned warehouse was used to capacity and multiple modes of transportation were available between each stage of the supply chain. A mixed integer nonlinear program was developed and was then converted to a linear program to determine the optimal shipping modes, quantities, and timing of orders over multiple periods. With the program developed, two scenarios were compared. One scenario modeled a low dollar value product while the second scenario considered a high dollar value product. The influence of this change on transportation and inventory decisions was analyzed. This thesis utilizes the all-units discount structure implemented by Mysore (2005). However, a four stage supply chain with multiple objectives and multiple products is modeled.

## 2.3 Multi-Criteria Optimization

The work reviewed thus far has considered formulations focused on single objective models. The field of multiple criteria optimization focuses on finding the best alternative while considering multiple conflicting criteria, or objectives, at the same time. Most decisions are inherently multi-objective, however early mathematical models in literature simply chose one main objective and relied on the constraints of a model to account for the other goals of the model. Some examples of inherently multi-objective models include: project management problems, inventory planning problems, capacity expansion problems, and facility location problems (Evans, 1984). Terminology and problem definitions surrounding multi-criteria mathematical problems were presented in full by Evans (1984) and Masud and Ravindran (2008). Both sources also discussed the three options for the elicitation of information from the decision maker (DM): before the problem is modeled, throughout the solution process, or after all efficient solutions have been identified. With each of these categories came several solution methodologies.

When objective targets and preferences are pre-specified by the Decision Maker (DM), the problem can be solved using Goal Programming. Preemptive Goal Programming allows the DM to assign a priority level to each objective and goals with higher priorities must be satisfied as far as possible before the next lower priority goal is considered. Non-preemptive Goal Programming requires that the DM assigns a weight to each goal which reflects its importance and the DM's trade-offs among the goals. The weights can be derived using ranking methods such as Borda Count, pair wise comparison, or the Analytical Hierarchy Process (Masud and Ravindran, 2008). The Partitioning Algorithm for Goal Programming (PAGP), used for solving preemptive Goal Programming problems, utilizes the objective priorities to partition the goal constraints. This constraint partitioning, along with variable elimination, facilitates the efficient derivation of the solution to the original problem (Arthur and Ravindran, 1980). A variation of the Simplex Method for goal programming was developed by Lee (1972). In this approach, the preemptive goal priorities specified by the DM are incorporated into the single objective function and the deviational variables indicating the distance from the goal, along with the decision variables, are manipulated iteratively until the optimal solution is found.

When the DM is not asked for information before the solution process then all efficient solutions are generated first and presented to the DM. Then the DM is asked for information regarding which, of all of the solutions, they prefer. Geoffrion's Theorem is used for parametric programming for this class of problems (Geoffrion, 1968). Compromise Programming, developed by Zeleny, can also be used to derive all of the possible solutions to a Multi-Criteria Mathematical Program (MCMP) (Zeleny, 1982).

Interactive methods for solving MCMPs require that the DM progressively articulates their preferences as solutions are presented to them until the best compromise solution is reached. One alternative is the STEM/STEP method for which an initial efficient solution is derived. From there the DM is asked how much they are willing to give up on the objectives that have been previously satisfied. This flexibility was introduced by Benyayoun et al. (1971) and allowed additional objectives to be satisfied until an acceptable compromise solution had been reached. The Ziont-Wallenius Method and the Paired Comparison Method are other interactive solution algorithms that utilize a utility function to represent the DMs preferences (Zionts and Wallenius, 1976; Sadagopan and Ravindran, 1982). A complete overview of interactive methods is available in the work by Shin and Ravindran (1991).

## 2.4 Multi-Criteria Optimization in Supply Chain

Many supply chain optimization problems are focused on one objective, minimizing cost. However, a more realistic representation of overall supply chain management is modeled with multi-criteria mathematical programs. Bookbinder and Chen (1992) made the original contribution to literature by applying multi-criteria techniques to a two-stage serial supply chain consisting of a retailer and warehouse. The distribution and inventory systems were optimized and both deterministic and stochastic demand cases were evaluated. In the deterministic case, the first objective was the minimization of inventory costs which included holding and ordering costs. The second objective was the minimization of transportation costs. This problem was formulated as a non-linear multi-criteria optimization problem and was solved using an analytical algorithm to arrive at the optimal non-dominated solution. In the stochastic case, the third objective of maximizing customer service measured by the number of stockouts a year was introduced. Non-linear optimization algorithms were developed for two cases with this multi-criteria structure. The solutions to these models were presented while tradeoffs between various supply chain measures were identified.

An interactive approach to a stochastic multi objective problem was presented by Agrell (1995). Here the need for multiple criteria decision making (MCDM) in inventory control systems was explained. In the system presented, a continuous review inventory policy was in place for a single product with deterministic lead time and stochastic lead time demand. The three objectives were the minimization of the expected total annual cost, the number of stockouts annually, and the number of items stocked out. An algorithm called IDEM (Interactive Decision Exploration Method) was employed to derive the optimal solution based on the DMs preferences. First the ideal solutions for each objective individually were obtained. Then the multi-criteria problem was optimized. The optimal solution was then presented to the DM and the DM expressed their opinions on improvements, sacrifices, and tradeoffs related to each objective. These preferences were then implemented in the mathematical program to derive the final optimal solution.

Difilippo (2003) incorporated transportation costs into an inventory model for a two-stage supply chain and a multi-criteria problem was solved. The supply chain consisted of one wholesaler and multiple retailers, for which deterministic demand for a single product was known and lead time was fixed. The three objectives to be minimized were the capital invested in inventory, the annual number of orders, and the annual transportation costs. In addition, an interactive method specific to the problems was developed to solve both the centralized and decentralized supply chain cases. Various inventory policies such as common replenishment and different replenishment times were also compared. A fixed order policy was implemented into the decentralized system and it outperformed both of the centralized policies, a conclusion that was attributed to the freight rate structure of the problem. This thesis will also use these criteria for minimizing the capital invested in inventory. However, goal programming will be used to solve the multiple criteria model for a four stage supply chain instead.

Natarajan (2007) published work on several supply chain models that focused on inventory planning including transportation costs. A freight rate function and a quantity discount model were incorporated in which breakpoints and indifference points were used to determine what per-unit cost was applied to each shipment. Shipments could be full truck load (TL) or less than truck load (LTL) quantities and each type had different cost structures. The supply chain modeled consisted of a warehouse servicing several customers under decentralized control. Initially, a single objective model was used to manage inventory at the warehouse while meeting customer demand. That model was then extended into a multi-criteria model in which the DM evaluated the optimal solution and the tradeoffs necessary to reach alternative solutions. The DM's preferences among these solutions were used to derive the optimal solution that best satisfied the DM. The three objectives were to minimize the capital invested in inventory, the annual number of orders, and the annual transportation costs. This model was then extended to a third model in which stochastic demand and random lead time were incorporated as well as a fourth criterion, fill rate, which reflected the customer satisfaction. All of the models were solved using Excel so the models can serve as a tool in industry. The transportation cost modeling used by Natarajan is incorporated into this thesis where a similar freight rate function is used and indifference points are calculated and applied in the model.

The work done by Mysore (2005) was extended by Vijayaragavan (2008) to include a supplier stage in the supply chain and add a second criterion to the model. Vijayaragavan (2008) modeled a single-product centralized four stage supply chain with multiple suppliers, one manufacturer, one warehouses, and multiple retailers. The two objectives were to minimize cost while maximizing customer responsiveness. Customer responsiveness was quantified as the total number of backorders that occurred over the entire time horizon. One product composed of raw materials from the suppliers would travel along the supply chain where four transportation modes were available between each member of each stage. Incremental quantity discounts were available for each transportation mode. The model was solved with both preemptive and non-preemptive goal programming. The optimal solution minimized the sum of the deviations from the goals specified by the DM. The solution indicated the optimal shipment quantities, transportation modes, and inventory levels over multiple periods. This thesis will produce a similar type of solution through goal programming as well and the objective of maximizing the customer responsiveness will also be included. It is assumed that customer responsiveness is measured as the total number of lost sales across the time horizon. Any amount of demand that cannot be satisfied at the retailer is assumed to result in lost potential sales so the terms "customer responsiveness" and "lost sales" are interchangeable throughout this thesis. The model presented in this thesis will extend the work done by Vijayaragavan (2008) by including multiple products, multiple manufacturers with several manufacturing lines, and multiple warehouses. In addition, the second and third objectives will differ. Lastly, an all-units discount structure will be used for the freight rate function instead of an incremental quantity discounts structure.

## 2.5 Multiple Product Supply Chain Modeling

Although extensive work has been done on modeling single product system, it is common in industry for one company to own several products. Similar to the combination of inventory and transportation cost management into one model, it can be advantageous to merge multiple products into one supply chain to save costs. However, there is limited literature on multiple product supply chain management. This thesis will address a multiple product supply chain and will utilize previously presented modeling techniques, as well as techniques developed in the following work, to optimize the supply chain.

When dealing with multiple products in one manufacturing facility, it is important to implement a production strategy that promotes efficiency. Graves and Tomlin (2003) discussed the importance of flexible configurations in manufacturing facilities for multiproduct supply chains. By analyzing the entire supply chain, the presence of bottlenecks specific to multi-stage supply chains was observed to decrease the effectiveness of supply chain configuration. Therefore new supply chain configurations unique to multi-stage supply chains were presented and it was observed that the overall efficiency was improved. Federgruen and Katalan (1998) focused on the development of minimum cost production schedules in manufacturing facilities that produce multiple products. In the model several products competed for production time on shared machines which had a fixed capacity. The explanation and inclusion of the setup costs when a machine was converted to produce a different product was an important part of the model that influenced production costs. A heuristic was developed for the production sequencing required to minimize system wide costs for the system presented.

Aggarwal and Dhavale (1975) modeled a complex multi-echelon inventory distribution system and analyzed the effects of variations on several input parameters. The system consisted of one warehouse that received material from multiple suppliers and then distributed the material to five regional distribution centers. From there the products were distributed to their respective retailers. To keep the problem within reasonable size limits, four products were modeled. The independent variables in the model were demand, lead time and cost-rate structure while the dependent variables included five management criteria. These criteria, or performance measures, were average inventory investment, average annual number of reorders within the system, average shortage cost per year, average reordering cost per year, and average annual inventory carrying cost. Analysis of variance was used to investigate the influence of each independent variable on the performance criteria and the findings were presented.

Lenard and Roy (1995) focused on inventory control in systems where multiple items were managed simultaneously. An original approach was developed that did not focus solely on costs, but also on the grouping of items in to "families" that share attributes to more effectively model inventory. Several different attributes were used for these groupings such as how the items were store and the minimum lead-times required for each item. Each family was then assigned an aggregated item. For example, there may be a functional group, or family, of items that are all used for one activity and without all of them available, the activity cannot commence. These items would be assigned one aggregate item to represent the group. The suggested strategy involved significant input from the decision maker regarding the standards for each family. Once the families were formed, the decision maker was asked to specify the number of allowable shortages and a suggested period length for each family. Their knowledge of the existing system to not only group the items, but also to define the attributes of the family was needed to develop the improved inventory management structure. Subsequently, an efficient policy surface based on the number of shortages and the average stock levels was identified for the aggregated items and was presented to give the manager a better view of the complete inventory system and aid in their decisions.

Viswanathan and Mathur (1997) addressed a single warehouse, multiple retailer distribution system. There were several products with independent demands at each retailer. A peddling routing system was in place where vehicles left from the warehouse and distributed products to several retailers via an efficient route. The goal was to determine a replenishment policy that minimized the long run average inventory and transportation costs. They developed a new heuristic that could handle multiple products and generate the optimal policy which was a stationary nested joint replenishment policy.

Qu et al. (1999) extended distribution system management models based on inventory and transportation decisions, to incorporate more than one product. A traveling-salesman approach was used to develop a modified periodic-review inventory policy for a system. The demand was stochastic, and inventory and transportation decisions were made simultaneously in the model. The model considered a central warehouse that obtained materials from several suppliers. A heuristic that decomposed the problem into a separate inventory and transportation problem first, and then iteratively merged the two decisions to minimize the long-run total average costs was proposed.

This thesis will address a multiproduct supply chain. The modeling techniques for efficient production of multiple products at a manufacturer are incorporated as well as combined shipment methods. Combining multiple products in one shipment provides a way to reduce transportation costs to promote overall supply chain efficiency.

# Chapter 3 Problem Statement and Model Formulation

## 3.1 Problem Statement

The problem solved in this thesis deals with a centralized, four stage supply chain for which the production plan, transportation modes and quantities, and inventory levels over several periods are solved for and specified in the presented solution. In the interest of illustration during the model development, specific values will be assigned to the parameters of the problem. In section 3.6, the general model will then be presented. An illustrative example of the general model will then be presented in Chapter 4 with results and conclusions in Chapter 5.

The centralized supply chain used to illustrate the model has five suppliers, two manufacturers, two company owned warehouses, and six retailers and fits the general supply chain network shown in Figure 3.1. Each product, ‘p’, is made up of up to ‘k’ raw materials, where k is also equal to the number of suppliers in the supply chain. Each supplier supplies on unique raw material to contribute to the manufacturing of the products. The number of each raw material required for each product is defined as a ratio. Production of a product will commence when all of the required raw material quantities have reached the manufacturer. Each manufacturer has three production lines available to produce the two products that have independent demands at each retailer. There are two modes of transportation, air and ground, available between each stage of the supply chain. Each transportation mode takes a specified number of time periods, called lead time, to reach a location. A freight-rate function that follows an All-Units discount cost structure, specific to each mode of transportation, is available for each transportation path. In this cost structure, the per-unit cost that corresponds to the cost bracket that the total quantity falls within is applied to the total quantity.

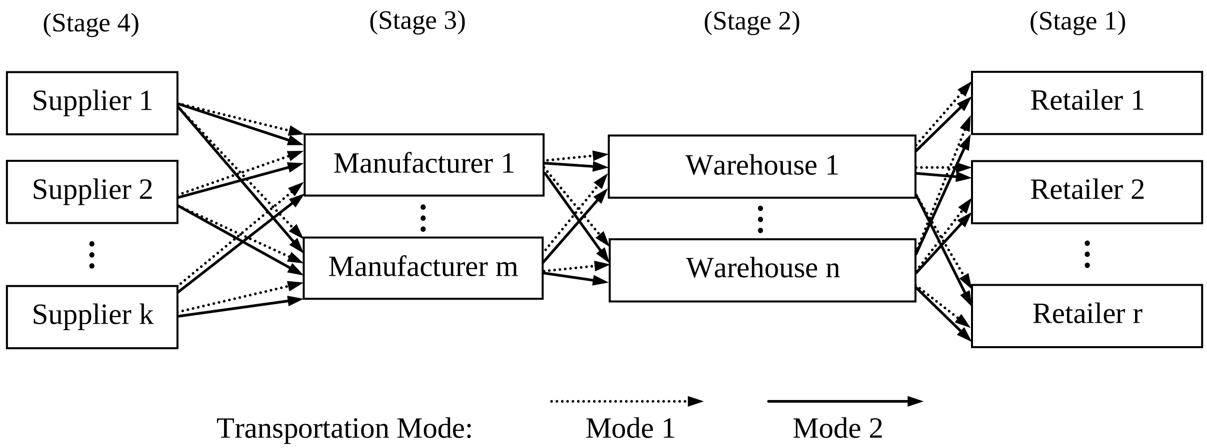



**Figure 3.1: The general supply chain network modeled in this thesis**

The goal is to determine the optimal quantities of material or product to ship between each stage, the most efficient mode of transportation for these shipments, and the optimal number of products to produce at each manufacturer and with which production lines. These decisions are all driven by the deterministic demand for each product at each retailer. The resulting inventory levels from this production and transportation schedule are specified. The schedule covers $T$ time periods of production and demand. The objectives of the model are to maximize profit and maximize customer responsiveness simultaneously. Customer responsiveness is measured as the total number of lost sales across the entire time horizon in this thesis. A third objective to minimize the amount of capital in inventory is then added and the problem is solved again. This problem is modeled as a mixed-integer linear program that is solved using Goal Programming. Preemptive and non-preemptive methods are used to construct the objective functions. The weighted sum of the deviational variables that represent the distance between the solution and the goal are minimized to optimize the problem. The solutions for the preemptive and non-preemptive methods are then compared.

## 3.2 Model Assumptions

- A multi-period, four-stage, centralized supply chain is considered
- The production of 'P' products is modeled where $p = \{1,2,\ldots,P\}$
- There are '*T*' time periods considered where $t = \{1,2,\ldots,T\}$
- The supply chain has '*k*' suppliers, '*m*' manufacturer, '*n*' warehouses, and '*r*' retailers
- '*K*' raw materials are available, one supplied by each supplier where $k = \{1,2,\ldots,K\}$
- Each product is made up of up to '*K*' raw materials in a specified ratio represented by $\{r_{1p},r_{2p},\ldots,r_{kp}\}$
- Two modes of transportation (e.g. air and ground) are available, denoted by $i = \{1,2\}$ which also represents their respective lead times
- The transportation modes are used to transport materials and products between each supplier and each manufacturer, each manufacturer and each warehouse, and each warehouse and each retailer
- Shipment and delivery times vary depending on the transportation mode and are represented by the value of '*i*'
- Each transportation mode has a unique freight rate function that utilizes the All-Unit discount cost structure
- Each manufacturing facility has '*L*' production lines available where $l = \{1,2,\ldots,L\}$

- Manufacturing of a product does not begin until all required quantities of each necessary raw material, based on the raw material ratios, have arrived at the manufacturer
- Manufacturing of each product can be completed in 1 time period
- Once a product has been manufactured it is shipped to one of the company owned warehouses
- Inventory holding costs occur at the manufacturers and warehouses and vary at each location
- Inventory holding costs are applied to products once they arrive at the location until they are shipped
- Inventory costs are calculated based on the inventory levels at the end of each period
- Inventory holding costs are no longer incurred after the product leaves the warehouse (i.e. FOB – Origin for the customer)
- Products are retrieved from a warehouse and shipped to a retailer to satisfy expected demand
- Demand for each product at each retailer is independent and deterministic
- Lost sales can only occur at the retailers

## 3.3 Model Variables and Parameters

The index sets used in the model formulation are:

$c$ - index for transportation weight cost brackets for air shipments

$d$ - index for adjusted transportation weight cost brackets for air shipments used for the All-Units discount cost structure

$e$ – index for transportation weight cost brackets for ground shipments

$f$ – index for adjusted transportation weight cost brackets for ground shipments used for the All-Units discount cost structure

$p$ – set of products being modeled

$i$ - set of transportation modes defined by their lead time being '$i$' time periods

$k$ - set of suppliers (also equal to the number of raw materials available)

$l$ – set of production lines available at each manufacturer

$m$ - set of manufacturers

$n$ - set of warehouses

$r$ - set of retailers

$t$ - set of time periods

$u$ – set of objective functions

$T$ – total length of the planning horizon

**VARIABLES**

The following variables are used in the model:

$X_{k\,m\,i\,t}$ – quantity of raw material 'k' shipped from supplier 'k' to manufacturer 'm' using transportation mode 'i' in period 't'

$Y_{p\,m\,n\,i\,t}$ – quantity of finished product 'p' shipped from manufacturer 'm' to warehouse 'n' using transportation mode 'i' in period 't'

$TY_{m\,n\,i\,t}$ – total quantity of finished products shipped from manufacturer 'm' to warehouse 'n' using transportation mode 'i' in time period 't'

$Z_{p\,n\,r\,i\,t}$ – quantity of finished product 'p' shipped from warehouse 'n' to retailer 'r' using transportation mode 'i' in period 't'

$TZ_{n\,r\,i\,\mathrm{t}}$ – total quantity of finished products shipped from warehouse 'n' to retailer 'r' using transportation mode 'i' in time period 't'

$TRM_{k\,m\,t}$ – total amount of raw material 'k' from supplier 'k' at manufacturer 'm' in time period 't'

$RM_{k\,p\,m\,t}$ – total amount of raw material 'k' from supplier 'k' used to manufacture product 'p' at manufacturer 'm' starting in time period 't'

$FP_{p\,m\,l\,t}$ – total number of finished product 'p' produced at manufacturer 'm' on production line '*l*' at the end of time period 't'

$\alpha_{\,p\,m\,l\,t}$ – binary variable (0,1) indicating whether line '*l*' at manufacturing facility 'm' is used for production of product 'p' in time period 't'

$TFPM_{p\,m\,t}$ – total amount of finished product ‘p’ available to ship from manufacturer ‘m’ at the end of time period ‘t’

$WINV_{p\,n\,t}$ – inventory of product ‘p’ stored at warehouse ‘n’ at the end of time period ‘t’

$TFPW_{p\,n\,t}$ – total amount of finished product ‘p’ received from all manufacturers at warehouse ‘n’ in time period ‘t’

$TFPR_{p\,r\,t}$ – total amount of finished product ‘p’ shipped to retailer ‘r’ in time period ‘t’ from all the warehouses

$LOST_{p\,r\,t}$ – number of lost sales or unfilled demand of product ‘p’ at retailer ‘r’ at the end of time period ‘t’

$\rho_{k\,m\,i\,t}$ – binary variable (0,1) indicating whether a shipment of raw material ‘k’ to manufacturer ‘m’ using transportation mode ‘i’ in time period ‘t’ occurs

$\tau_{m\,n\,i\,t}$ – binary variable (0,1) indicating whether a shipment from manufacturer ‘m’ to warehouse ‘n’ using transportation mode ‘i’ in time period ‘t’ occurs

$\sigma_{n\,r\,i\,t}$ – binary variable (0,1) indicating whether a shipment from warehouse ‘n’ to retailer ‘r’ using transportation mode ‘i’ in time period ‘t’ occurs

$\delta_d$ – binary variable (0,1) indicating whether a portion of the weight of an air transportation shipment falls within cost bracket ‘d’

$TRMWA_{k\,m\,t}$ – total weight of raw material shipment using air transportation from supplier ‘k’ to manufacturer ‘m’ in time period ‘t’

$TRMWAS_{k\ m\ t\ d}$ – portion of total weight of raw material shipment using air transportation from supplier ‘k’ to manufacturer ‘m’ in time period ‘t’ that lies in cost bracket ‘d’

$\delta 0_{k\ m\ t\ d}$ – binary variable (0,1) indicating whether a portion of the weight of an air transportation shipment of raw materials from supplier ‘k’ to manufacturer ‘m’ in time period ‘t’ falls within cost bracket ‘d’

$TRMWG_{k\ m\ t}$ – total weight of raw material shipment using ground transportation from supplier ‘k’ to manufacturer ‘m’ in time period ‘t’

$TRMWGS_{k\ m\ t\ f}$ – portion of total weight of raw material shipment using ground transportation from supplier ‘k’ to manufacturer ‘m’ in time period ‘t’ that lies in cost bracket ‘f’

$\theta 0_{k\ m\ t\ f}$ – binary variable (0,1) indicating whether a portion of the weight of a ground transportation shipment of raw materials from supplier ‘k’ to manufacturer ‘m’ in time period ‘t’ falls within cost bracket ‘f’

$TFPWA1_{m\ n\ t}$ – total weight of finished product shipment using air transportation from manufacturer ‘m’ to warehouse ‘n’ in time period ‘t’

$TFPWAS1_{m\ n\ t\ d}$ – portion of total weight of finished product shipment using air transportation from manufacturer ‘m’ to warehouse ‘n’ in time period ‘t’ that lies in cost bracket ‘d’

$\delta 1_{m\,n\,t\,d}$ – binary variable (0,1) indicating whether a portion of the weight of an air transportation shipment of finished products from manufacturer ‘m’ to warehouse ‘n’ in time period ‘t’ falls within cost bracket ‘d’

$TFPWG1_{m\,n\,t}$ – total weight of finished product shipment using ground transportation from manufacturer ‘m’ to warehouse ‘n’ in time period ‘t’

$TFPWGS1_{m\,n\,t\,f}$ – portion of total weight of finished product shipment using ground transportation from manufacturer ‘m’ to warehouse ‘n’ in time period ‘t’ that lies in cost bracket ‘f’

$\theta 1_{m\,n\,t\,f}$ – binary variable (0,1) indicating whether a portion of the weight of a ground transportation shipment of finished products from manufacturer ‘m’ to warehouse ‘n’ in time period ‘t’ falls within cost bracket ‘f’

$TFPWA2_{n\,r\,t}$ – total weight of finished product shipment using air transportation from warehouse ‘n’ to retailer ‘r’ in time period ‘t’

$TFPWAS2_{n\,r\,t\,d}$ – portion of total weight of finished product shipment using air transportation from warehouse ‘n’ to retailer ‘r’ in time period ‘t’ that lies in cost bracket ‘d’

$\delta 2_{n\,r\,t\,d}$ – binary variable (0,1) indicating whether a portion of the weight of an air transportation shipment of finished products from warehouse ‘n’ to retailer ‘r’ in time period ‘t’ falls within cost bracket ‘d’

$TFPWG2_{n\,r\,t}$ – total weight of finished product shipment using ground transportation from warehouse ‘n’ to retailer ‘r’ in time period ‘t’

$TFPWGS2_{n\,r\,t\,f}$ – portion of total weight of finished product shipment using ground transportation from warehouse ‘n’ to retailer ‘r’ in time period ‘t’ that lies in cost bracket ‘f’

$\theta 2_{n\,r\,t\,f}$ – binary variable (0,1) indicating whether a portion of the weight of a ground transportation shipment of finished products from warehouse ‘n’ to retailer ‘r’ in time period ‘t’ falls within cost bracket ‘f’

$\mu a_c$ – indifference point for cost bracket ‘c’

$\mu b_e$ – indifference point for cost bracket ‘e’

$d$ – the number of cost brackets for air shipments in the cost structure including indifference points

$DMAX$ – the number of adjusted cost brackets that consider indifference points for air shipments

$f$ – the number of cost brackets for ground shipments in the cost structure including indifference points

$FMAX$ – the number of adjusted cost brackets that consider indifference points for ground shipments

$CAPA_d$ – capacity of cost bracket ‘d’ used for air shipment weights

$CAPG_f$ – capacity of cost bracket ‘f’ used for ground shipment weights

$TOTALC_d$ – Total cost of an air shipment of total weight that falls into cost bracket ‘d’

$TSOLD_{p\,r\,t}$ – total number of units of product ‘p’ sold at retailer ‘r’ in time period ‘t’

$IDEAL_u$ – ideal solution found when objective ‘u’ is solved independently from the other conflicting objectives

$TARGET_u$ – target value, or goal, for objective ‘u’ used in the goal programming formulation

**INPUT DATA**

The input parameters, which remain constant, of the problem are:

$INITRM_{k\,m}$ – initial quantity of raw material ‘k’ assumed to be available at manufacturer ‘m’ at the beginning of the planning horizon

$r_{k\,p}$ - quantity of raw material ‘k’ required to produce one unit of product ‘p’

$OPERC_{p\,m\,l}$ – operating cost of production of product ‘p’ on production line ‘*l*’ at manufacturer ‘m’ for one time period

$PRODC_{p\,m\,l\,t}$ – the per-unit cost of production of product ‘p’ at manufacturer ‘m’ on line ‘*l*’ in time period ‘t’

$MCAP_{p\,m\,l}$ – maximum production capacity for product ‘p’ on line ‘*l*’ at manufacturer ‘m’ in a single time period

$INVCRM_k$ – inventory holding cost of raw material ‘k’ held at a manufacturer per unit per time period

$INITFPW_{p\,n}$ – initial quantity of finished product ‘p’ assumed to be available at warehouse ‘n’ at the beginning of the planning horizon

$WCAP_n$ – total finished product inventory capacity at warehouse ‘n’

$RD_{p\,r\,t}$ – demand for product ‘p’ at retailer ‘r’ at the beginning of time period ‘t’

$INVCFP_p$ – inventory holding cost of finished product ‘p’ held at a warehouse per unit per time period

$INITFPR_{p\,r}$ – initial quantity of finished product ‘p’ assumed to be available at retailer ‘r’ at the beginning of the planning horizon

$MINRM_i$ – minimum quantity for shipments of raw material using transportation mode ‘i’

$MAXRM_i$ – maximum quantity for shipments of raw material using transportation mode ‘i’

$MINFP_i$ – minimum quantity for shipments of finished product using transportation mode ‘i’

$MAXFP_i$ – maximum quantity for shipments of finished product using transportation mode ‘i’

$aLB$ – lower bound for weight of air shipments

$a_c$ – upper bound on air shipments in cost bracket ‘c’

$bLB$ - lower bound for weight of ground shipments

$b_e$ –upper bound on ground shipments in cost bracket ‘e’

$AIRC_c$ – the per hundredweight cost of air shipments in cost bracket ‘c’

$GROUNDC_e$ – the per hundredweight cost of ground shipments in cost bracket ‘e’

$RMW_k$ – per-unit weight (in CWT) of raw material ‘k’

$FPW_p$ – per-unit weight (in CWT) of finished product ‘p’

$CMAX$ – the number of cost brackets available for air shipments

$EMAX$ – the number of cost brackets available for ground shipments

$REV_p$ – revenue generated per unit sold of product ‘p’

## 3.4 Constraints and Objectives of the Model

### 3.4.1 Manufacturer Constraints and Costs

#### 3.4.1.1 Product Flow Constraints at the Manufacturers

Two modes of transportation, air and ground, are available. It is assumed that air has a lead time of one time period, and ground has a lead time of two time periods. The time periods can take any base period such as days, weeks, or months depending on the problem being modeled. The manufacturing and transport times are an integer multiple of this base period. Transportation mode ‘i’ takes ‘i’ time periods to arrive at its destination. Therefore the raw materials shipped with transportation mode ‘i’ in time period ‘t-i’ will arrive at the manufacturer at time period ‘t’.

It is assumed that this is a model of an existing supply chain. Therefore it is unrealistic to assume that there aren't existing materials in-transit on their way to the suppliers before time period 2, when the first scheduled shipments by this model arrive by air. Therefore in time period 1, an initial amount of each raw material is assumed to be available at each manufacturer. $INITRM_{k\ m}$ is assumed to be equal to the total quantity of each raw material in-transit and scheduled to arrive at manufacturer 'm' in time period 1.

Accordingly:

$$TRM_{k\ m\ t} = INITRM_{k\ m} \quad \forall k, m, t \quad when\ t = 1$$

When t = 2, the only raw material that would be available at the manufacturers would have been shipped in time period 1 by air to the manufacturer. Any other shipments would not have arrived yet. Therefore:

$$TRM_{k\ m\ t} = X_{k\ m\ 1\ (t-1)} \quad \forall k, m, t \quad when\ t = 2$$

After enough time has passed so all transportation modes are possible, then the material shipped with each mode is summed to determine how much material is arriving in time period 't'. Since there are only two modes of transportation available, the total amount of raw material 'k' at manufacturer 'm' in time period 't' had to be shipped either one or two time periods previously. Accordingly, when there are two modes of transportation available,

$$TRM_{k\ m\ t} = X_{k\ m\ 1\ (t-1)} + X_{k\ m\ 2\ (t-2)} \qquad \forall\ m, k, t \quad when\ t \geq 3$$

To simplify the notation for shipments leaving each manufacturing facility, the following constraints define the total amount of finished product 'p' available to leave manufacturing facility 'm' at the end of time period 't' which is equal to the sum of the finished product ‘p’ finished on each production line ‘*l*’. It is assumed that it takes one time period to manufacture finished products so the first time period products are available is time period 2. Before that the total finished product available is set to zero.

$$FP_{p\,m\,l\,t} = 0 \quad \forall p, m, l, t \quad when\; t = 1$$

$$TFPM_{p\,m\,t} = \sum_{l} FP_{p\,m\,l\,t} \quad \forall p, m, t$$

When three production lines are available (*l* = 3), the preceding equation would become:

$$TFPM_{p\,m\,t} = FP_{p\,m\,1\,t} + FP_{p\,m\,2\,t} + FP_{p\,m\,3\,t} \quad \forall p, m, t$$

It is assumed that shipments throughout the supply chain can consist of combinations of products. The following constraints assure that once a product has completed manufacturing it is sent to a warehouse immediately since the manufacturers do not have storage room for finished product inventories. The total amount of finished product ‘p’ available for shipment from manufacturer ‘m’ in time period ‘t’ is distributed among quantities shipped to all of the warehouses ‘n’ using all transportation modes ‘i’.

$$TFPM_{p\,m\,t} = \sum_{n} \sum_{i} Y_{p\,m\,n\,i\,t} \quad \forall p, m, t$$

### 3.4.1.2 Manufacturing Constraints

It will be necessary to break up the raw material allocation among the products. $RM_{k\,p\,m\,t}$ is used to represent the amount of raw material ‘k’ used to produce product ‘p’ at manufacturer ‘m’ starting in time period ‘t’. Therefore the following constraints define the total amount of raw material ‘k’ at manufacturer ‘m’ in time period ‘t’ that is to be used to manufacture each product.

$$TRM_{k\,m\,t} = \sum_{p} RM_{k\,p\,m\,t} \qquad \forall k, m, t$$

When two products are being modeled,

$$TRM_{k\,m\,t} = RM_{k\,1\,m\,t} + RM_{k\,2\,m\,t} \qquad \forall k, m, t$$

Each product requires a unique blend of raw materials necessary for production at the manufacturers. The variable $r_{k\,p}$ represents the quantity of raw material ‘k’ required to produce one unit of product ‘p’. Let the raw material ratio for product 1 be represented by $r_{11}:r_{21}:\ldots:r_{K1}$. For example, if the ratio for product 1 was 1:2:3:0:0 for the five raw materials available, then in order to produce 100 units of product 1, there would need to be 100 units of raw material 1, 200 units of raw material 2, and 300 units of raw material 3 available for product 1 production at a particular manufacturing facility. It will be assumed that the manufacturing facilities operate under a Just In Time (JIT) environment. Therefore, as soon as materials arrive at the manufacturer they are sent for production immediately.

The products being modeled can all be manufactured within one time period. Since it takes 1 time period to manufacture product ‘p’, the raw material that arrives at the manufacturer in time period ‘t’ and is used for product ‘p’ will result in the finished product ‘p’ at ‘t+1’. Once the materials arrive at the manufacturer, they are assigned a production line. Each manufacturing facility is assumed to have three production lines ($l = 1, 2, 3$), all capable of producing all products, however only one product can be produced on a line at a time. Therefore binary variables are used to control when materials can be processed on certain lines depending on whether the product has been allotted production time on the production line. The combination of the production ratios, and the total number of finished products possible create the constraints that implement the raw material ratios and ensure that a JIT environment is maintained at the manufacturing facilities. The total quantity of each raw material required in period ‘t’ can be written as follows, assuming three production lines are available at each manufacturer:

$$RM_{1\,p\,m\,t} = r_{1\,p} * FP_{p\,m\,1\,(t+1)} + r_{1\,p} * FP_{p\,m\,2\,(t+1)} + r_{1\,p} * FP_{p\,m\,3\,(t+1)}$$

$$RM_{2\,p\,m\,t} = r_{2\,p} * FP_{p\,m\,1\,(t+1)} + r_{2\,p} * FP_{p\,m\,2\,(t+1)} + r * FP_{p\,m\,3\,(t+1)}$$

$$\vdots$$

$$RM_{K\,p\,m\,t} = r_{K\,p} * FP_{p\,m\,1\,(t+1)} + r_{K\,p} * FP_{p\,m\,2\,(t+1)} + r_{K\,p} * FP_{p\,m\,3\,(t+1)}$$

In general, all of the preceding constraints can be simplified with the following expression where T is the index for the last time period being modeled:

$$RM_{k\,p\,m\,t} = \sum_{l} (r_{k\,p} * FP_{p\,m\,l\,(t+1)}) \;\; \forall k, p, m, t \; for \; t = 1,2, \ldots, T-1$$

Binary variables are used in conjunction with the finished product quantities to limit production to only lines that have been turned on. The variable $\alpha_{p\,m\,l\,t}$ is a binary variable that is equal to one when production line ‘$l$’ at manufacturer ‘m’ is used to manufacture product ‘p’ in time period ‘t’, and equal to 0 otherwise. Since manufacturing of each product can be completed within one time period, the decision whether a production line is set up for a particular product ‘p’ must be made at the beginning of each time period ‘t’. When a line is turned off, or not available for production of product ‘p’, the finished product quantity on that line is forced to zero. It is assumed that each manufacturer has a fixed production capacity for each product on each line in a single time period. Therefore, the number of finished products produced in time period t cannot exceed the production capacity of that line for that product, $MCAP_{p\,m\,l\,t}$.

The following constraint relates the line operation and capacity to the finished product quantity:

$$\alpha_{p\,m\,l\,t} * MCAP_{p\,m\,l} \geq FP_{p\,m\,l\,t} \quad \forall p, m, l, t$$

Each line can only be set up to produce at most one product at a time in each time period. Consequently the following constraint is necessary:

$$\sum_{p} \alpha_{p\,m\,l\,t} \leq 1 \qquad \forall m, l, t$$

The above constraint allows a line to be idle if necessary.

### 3.4.1.3 Manufacturing Costs

A fixed operating cost exists for each line, $OPERC_{p\,m\,l\,t}$, independent of the amount produced. Therefore, the total operating cost of the production lines at the manufacturing facilities across the entire time horizon is:

$$\sum_{p}\sum_{m}\sum_{l}\sum_{t}(OPERC_{p\,m\,l\,t} * \alpha_{p\,m\,l\,t})$$

A variable operating cost per unit of product ‘p’ produced at manufacturer ‘m’ on line ‘*l*’ in time period ‘t’ exists at the manufacturing facilities. This cost, $PRODC_{p\,m\,l\,t}$, is applied to the number of finished products, to determine the total production cost across the entire time horizon:

$$\sum_{p}\sum_{m}\sum_{l}\sum_{t}(PRODC_{p\,m\,l\,t} * FP_{p\,m\,l\,t})$$

#### 3.4.1.4 Inventory Holding Costs at the Manufacturer

Although the manufacturing facilities operate in a JIT environment, the raw materials remain at the manufacturer throughout the manufacturing process. During this time an inventory holding cost is incurred for the work in process. Since the manufacturing process only takes one time period the inventory holding cost at manufacturer ‘m’ incurred in time period ‘t’ is given by:

$$\sum_{k}\sum_{p}(RM_{k\,p\,m\,t} * INVCRM_{k})$$

The total raw material inventory holding cost at all manufacturers across the entire time horizon is:

$$\sum_{k}\sum_{p}\sum_{m}\sum_{t}(RM_{k\,p\,m\,t} * INVCRM_{k})$$

### 3.4.2 Warehouse Constraints and Costs

#### 3.4.2.1 Product Flow Constraints at the Warehouses

All shipments of product ‘p’ arriving at warehouse ‘n’ in time period ‘t’ that were shipped using transportation mode ‘i’ had to be shipped in time period ‘t-i’. Similar to the initial raw material quantities, initial finished product inventories are available at the warehouses at $t$ = 1 and 2. The first shipments to the warehouses can begin from t = 2 since it takes one time period to manufacture products. In addition, the shortest lead time is one time period by air, so the earliest shipments scheduled by this model will arrive at the warehouses in time period 3. Therefore, shipments leaving the manufacturers can be summarized by the following general constraints, which follow a similar format as the product flow constraints at the manufacturer:

$$TFPW_{p\,n\,t} = INITFPW_{p\,n} \quad \forall p,n,t \quad when\ t = 1,2$$

$$TFPW_{p\,n\,t} = \sum_{m} Y_{p\,m\,n\,1\,t-1} \quad \forall p,n,t \quad when\ t = 3 \quad \text{(Received by air)}$$

$$TFPW_{p\,n\,t} = \sum_{m}\sum_{i} Y_{p\,m\,n\,i\,t-i} \quad \forall p,n,t \quad when\ t \geq 4 \quad \text{(Received by air and ground)}$$

The total amount of finished product ‘p’ available for shipment from warehouse ‘n’ in time period ‘t’, including the existing inventory of product ‘p’ at warehouse ‘n’ from the end of time period ‘t-1’, is distributed among quantities shipped to all of the retailers ‘r’ using all transportation modes ‘i’ as follows:

$$TFPW_{p\,n\,t} = \sum_{r}\sum_{i}(Z_{p\,n\,r\,i\,t}) + WINV_{p\,n\,t} \quad \forall p,n,t \quad when\ t = 1$$

Then the following general formula can be used:

$$TFPW_{p\,n\,t} + WINV_{p\,n\,t-1} = \sum_{r}\sum_{i}(Z_{p\,n\,r\,i\,t}) + WINV_{p\,n\,t} \quad \forall p,n,t \quad when\ t \geq 2$$

Each warehouse has limited space, or capacity, for its inventory. Therefore a maximum capacity constraint on the total inventory held over a time period at a warehouse is required.

$$\sum_{p} WINV_{p\,n\,t} \leq WCAP_{n} \quad \forall n, t$$

### 3.4.2.2 Inventory Holding Costs at the Warehouse

The finished product stored at the warehouses incurs a per-period inventory holding cost. The inventory holding cost is applied at the end of each time period to the number of products held in inventory until the next time period. Therefore the total inventory holding cost at warehouse ‘n’ at the end of time period ‘t’ is:

$$\sum_{p}\sum_{t}(WINV_{p\,n\,t} * INVCFP_{p}) \quad \forall n$$

The total inventory holding cost at all warehouses across the entire time horizon is:

$$\sum_{p}\sum_{n}\sum_{t}(WINV_{p\,n\,t} * INVCFP_{p})$$

## 3.4.3 Retailer Constraints and Costs

### 3.4.3.1 Product Flow Constraints at the Retailers

Initial inventory of finished products is available at the retailers before the first shipment scheduled by this model from the warehouses arrives. Therefore the following constraints ensure the product flow out of the warehouses to the retailers.

$$TFPR_{p\,r\,t} = INITFPR_{p\,r} \quad \forall p, r, t \quad when\; t = 1$$

$$TFPR_{p\,r\,t} = \sum_{n} Z_{p\,n\,r\,1\,t-1} \quad \forall p, r, t \quad when\; t = 2$$

$$TFPR_{p\,r\,t} = \sum_{n} \sum_{i} Z_{p\,n\,r\,i\,t-i} \quad \forall p, r, t \quad when\; t \geq 3$$

The demand at the retailers drives shipments from the warehouses in each time period. It is assumed that inventory cannot be held at the retailers, however retailer demands may not be fulfilled during a time period. If there is not enough product at a retailer to satisfy the demand, the quantity of demand that is not filled is classified as lost sales. In a supply chain model, sometimes unfulfilled demand is intentional because it is not profitable to satisfy the demand. In other cases the lack of product at the retailer is not intentional and therefore results in potential sales being lost. This quantity, regardless of whether the unfulfilled demand was intentional or not, is called lost sales in this model. Thus, the demand constraint at the retailers is written as follows:

$$TFPR_{p\,r\,t} + LOST_{p\,r\,t} = RD_{p\,r\,t} \quad \forall p, r, t$$

### 3.4.4 Transportation Quantity Constraints

The transportation modes in this model have a specified minimum and maximum shipment quantity due to available capacity. These limits depend on the transportation mode and whether raw material or finished products are being shipped. This setup is used because it is assumed that the raw materials are all of similar size and weight. The same assumption holds true for the finished products. A minimum shipment is often imposed by transportation companies to ensure that they carry enough product to charge a reasonable per-unit cost to offset their fixed operating costs. The maximum shipment quantity is necessary because of capacity constraints of the transportation mode as well as weight limits. Tables 3.1 and 3.2 summarize the parameters regarding transportation mode and quantity limits for raw materials and finished products respectively.

**Table 3.1: Shipping limits on raw material shipments**

| Transportation Mode | Minimum Shipping Quantity for Raw Material k | Maximum Shipping Quantity for Raw Material k |
|---|---|---|
| $i$ | $MINRM_i$ | $MAXRM_i$ |

**Table 3.2: Shipping limits on finished product shipments**

| Transportation Mode | Minimum Finished Product Shipping Quantity | Maximum Finished Product Shipping Quantity |
|---|---|---|
| $i$ | $MINFP_i$ | $MAXFP_i$ |

The following constraints ensure that all shipping quantities for each transportation mode meet the appropriate levels for transport from suppliers to manufacturers. $X_{k\ m\ i\ t}$ represents the quantity of raw material 'k' shipped from supplier 'k' to manufacturer 'm' using transportation mode 'i' in time period 't'. The constraint is enforced using the binary variable $\rho_{k\ m\ i\ t}$, when the shipment quantities are larger than 0. Otherwise the quantity remains 0. Therefore the constraints on the quantity of raw material shipped from suppliers to manufacturers are:

$$\rho_{k\,m\,i\,t} * MINRM_i \leq X_{k\,m\,i\,t} \leq \rho_{k\,m\,i\,t} * MAXRM_i \qquad \forall k, m, i, t$$

M is assumed to be a large integer:

$$X_{k\,m\,i\,t} \leq M * \rho_{k\,m\,i\,t} \quad \forall k, m, i, t$$

Finished products are produced at the manufacturing stage and are then shipped to the warehouses and then to the retailers. Although the production process and demands are independent for each product 'p', it is assumed that the finished products can be combined during shipments. Therefore, the limits regarding finished product shipments must consider the total number of finished products in a shipment regardless of the quantity of each individual product. $TY_{m\,n\,i\,t}$ represents the total number of finished products shipped from manufacturer 'm' to warehouse 'n' using transportation mode 'i' in time period 't' and is represented by:

$$TY_{m\,n\,i\,t} = \sum_{p} Y_{p\,m\,n\,i\,t} \quad \forall\, m, n, i, t$$

The constraints to enforce the shipping quantity limits on these quantities are represented by:

$$\tau_{\mathrm{m\,n\,i\,t}} * MINFP_i \leq TY_{m\,n\,i\,t} \leq \tau_{\mathrm{m\,n\,i\,t}} * MAXFP_i \qquad \forall m, n, i, t$$

M is assumed to be a large integer:

$$TY_{m\,n\,i\,t} \leq M * \tau_{m\,n\,i\,t} \qquad \forall m, n, i, t$$

Shipments from the warehouses to retailers can consist of multiple product types as well. The total amount of finished products being shipped from warehouse 'n' to retailer 'r' using transportation mode 'i' in time period 't' is defined by:

$$TZ_{n\,r\,i\,t} = \sum_{p} Z_{p\,n\,r\,i\,t} \quad \forall\, n, r, i, t$$

The constraints restricting the shipping quantities from stage 2 to stage 1 in the supply chain are:

$$\sigma_{n\,r\,i\,t} * MINFP_i \leq TZ_{n\,r\,i\,t} \leq \sigma_{n\,r\,i\,t} * MAXFP_i \qquad \forall n, r, i, t$$

M is assumed to be a large integer:

$$TZ_{n\,r\,i\,t} \leq M * \sigma_{n\,r\,i\,t} \qquad \forall n, r, i, t$$

## 3.4.5 Freight Rate Function

### 3.4.5.1 Cost Structure Notation

The freight rate function for the transportation modes in this model follows an *All-Units discount cost structure*. Similar to a minimum shipment quantity requirement, many transportation companies try to offset fixed costs by making larger shipments more cost effective for companies purchasing the transportation. The transportation companies often incur the same large fixed cost for a shipment regardless of the weight or quantity. For example, the difference in cost to drive an empty truck compared to a full truck from New York to Ohio is minimal compared to the fixed cost of fuel, the truck, the driver, and the time. Therefore, many companies offer an *All-Units discount cost structure* to try to force full truck load (TL) shipments rather than less than truck load (LTL) shipments in order to incur the large fixed cost less often. It is common to define a TL shipment as a shipment that achieves the maximum weight allowance for the transportation mode. In addition, a minimum weight can be used to offset the fixed cost of the transportation mode. Shipment weights in this thesis are measured in hundredweight units (CWT), or 100lbs of shipment.

Transportation companies give incentives to shippers by offering a lower per CWT cost for heavier, and therefore fuller, shipments. Shippers will then opt to aggregate their orders into fewer complete orders rather than ordering just a few items rather frequently. In the *All-Units discount cost structure,* the per-CWT cost is applied to the entire order quantity. This is different than the *Incremental discount cost structure,* in which the lower cost is only applied to the units that fall within a certain cost bracket. An example of a general *All-Units discount cost structure* where X represents the total shipment weight in CWT is presented in Table 3.3. In the example, if 4,000 CWT are shipped, the shipping cost will be (4000)*(0.75) = $3,000.

**Table 3.3: An example of an All-Units discount cost structure**

| Per-Unit Cost | Weight Limits (CWT) |
|---|---|
| $1.00 | $0 \leq X \leq 999$ |
| $0.90 | $1,000 \leq X \leq 2,999$ |
| $0.75 | $3,000 \leq X \leq 5,999$ |
| $0.55 | $6,000 \leq X \leq 10,000$ |

It is more common to have weight restrictions with air shipments than ground. Binary indicator variables used in the constraints for shipments between each stage of this model are $\delta 0_c$, $\delta 1_c$, $\delta 2_c$, $\theta 0_e$, $\theta 1_e$, and $\theta 2_e$, depending on which stages the shipments are between. The indicator variables are equal to 1 when the shipment weight falls within the weight limits for cost bracket 'c' or 'e'. Tables 3.4 and 3.5 display the notation for the parameters surrounding the All-Units cost structure for air and ground transportation respectively. In this notation, the index denotes the cost bracket. The indicator variables are generalized as just $\delta_c$ or $\theta_e$ in the tables. The variable $\delta E_c$ is used for the example.

**Table 3.4: All-Units discount cost structure for air transportation**

| Bracket Index (c) | Per-CWT Cost | Air Shipment Weight Limits (CWT) | Indicator Variable ($\delta_c$) |
|---|---|---|---|
| 1 | $AIRC_1$ | $\text{aLB} \leq X \leq \text{a}_1$ | $\delta_1$ |
| 2 | $AIRC_2$ | $(\text{a}_1+1) \leq X \leq \text{a}_2$ | $\delta_2$ |
| 3 | $AIRC_3$ | $(\text{a}_2+1) \leq X \leq \text{a}_3$ | $\delta_3$ |
| 4 | $AIRC_4$ | $(\text{a}_3+1) \leq X \leq \text{a}_4$ | $\delta_4$ |

**Table 3.5: All-Units discount cost structure for ground transportation**

| Bracket Index ($e$) | Per-CWT Cost | Ground Shipment Weight Limits (CWT) | Indicator Variable ($\theta_e$) |
|---|---|---|---|
| 1 | $GROUNDC_1$ | $\text{bLB} \leq \text{X} \leq \text{b}_1$ | $\theta_1$ |
| 2 | $GROUNDC_2$ | $(\text{b}_1+1) \leq \text{X} \leq \text{b}_2$ | $\theta_2$ |
| 3 | $GROUNDC_3$ | $(\text{b}_2+1) \leq \text{X} \leq \text{b}_3$ | $\theta_3$ |
| 4 | $GROUNDC_4$ | $(\text{b}_3+1) \leq \text{X} \leq \text{b}_4$ | $\theta_4$ |

As incentive for the company purchasing the transportation mode, heavier shipments will incur a smaller per-CWT cost, so $\text{AIRC}_1 > \text{AIRC}_2 > \text{AIRC}_3 > \text{AIRC}_4$ in this cost structure. The costs for ground shipments, represented by $\text{GROUNDC}_e$, follow the same decreasing structure as weights get higher. The cost structure is represented graphically in Figure 3.2.

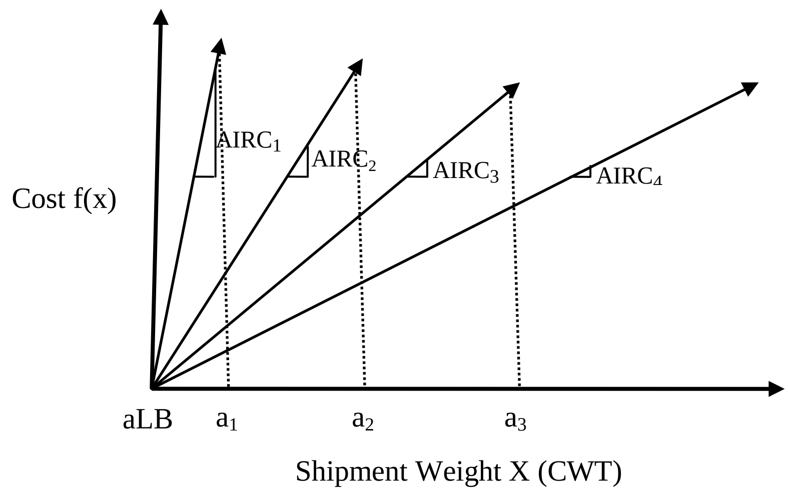



**Figure 3.2: Graphical representation of an All-Units discount cost structure**

The cost function for the All-Units discount cost structure summarized in Table 3.4 is represented with the following cost function:

$$f(x) = \begin{cases} X * AIRC_1 & when \quad aLB \leq X \leq a_1 \\ X * AIRC_2 & when \quad (a_1 + 1) \leq X \leq a_2 \\ X * AIRC_3 & when \quad (a_2 + 1) \leq X \leq a_3 \\ X * AIRC_4 & when \quad (a_3 + 1) \leq X \leq a_4 \end{cases}$$

Consequently, the total cost of an air shipment of weight X would be:

$$Shipment\ cost = \sum_c (AIRC_c * X * \delta E_c)$$

This equation for the shipment cost is now nonlinear. Nonlinear problems are difficult to solve and there is a risk of identifying a local maxima or minima as the optimal solution instead of the global maxima or minima. Consequently, the above equation must be converted to a linear function so linear optimization software and solution procedures can be used to identify the global solution. This transformation is discussed in section 3.4.6.3.1.

### 3.4.5.2 Shipment Weight Constraints

The cost brackets in this model will depend on shipments weights in CWT units. The constant $RMW_k$ represents the per-unit weight (in CWT) of raw material 'k' and $FPW_p$ represents the per-unit weight (in CWT) of finished product 'p'. These per-unit weights are multiplied by the shipment quantity to define the total weight (in CWT) of shipments. These total weights are used to define the value of X, the total shipment weight, which determines the cost bracket a shipment falls into. Since shipments in this model consist of just raw materials or just finished products, two different sets of transportation constraints are necessary for each transportation mode. The total weight of a raw material shipment of material 'k' from supplier 'k' to manufacturer 'm' using air transportation in time period 't' is defined as follows:

$$TRMWA_{k\ m\ t} = (RMW_k * X_{k\ m\ 1\ t}) \qquad \forall k, m, t$$

Similarly, the total raw material weight of a ground shipment is:

$$TRMWG_{k\ m\ t} = (RMW_k * X_{k\ m\ 2\ t}) \qquad \forall k, m, t$$

Finished product shipments can consist of combinations of quantities of all products 'p'. Consequently the total weight of a finished product shipment requires the summation of each product's weight multiplied by the number of that product in a shipment. Therefore the total weight of finished product shipments from either manufacturers to warehouses, or warehouses to retailers, are defined as follows:

$$TFPWA1_{m\,n\,t} = \sum_{p} (FPW_p * Y_{p\,m\,n\,1\,t}) \qquad \forall m, n, t$$

$$TFPWG1_{m\,n\,t} = \sum_{p} (FPW_p * Y_{p\,m\,n\,2\,t}) \qquad \forall m, n, t$$

$$TFPWA2_{n\,r\,t} = \sum_{p} (FPW_p * Z_{p\,n\,r\,1\,t}) \qquad \forall n, r, t$$

$$TFPWG2_{n\,r\,t} = \sum_{p} (FPW_p * Z_{p\,n\,r\,2\,t}) \qquad \forall n, r, t$$

The cost brackets will depend on the previously defined total weight quantities.

### 3.4.5.3 Breakpoints and Indifference Points

In the interest of illustration, the discussion in sections 3.4.6.2 and 3.4.6.3 will refer mostly to the example of the All-Units cost structure presented in Table 3.3. It will be assumed that an air shipment of weight X CWT is being modeled and there are four original cost brackets. The extension of the modeling to the variables used in this model is presented in section 3.4.6.4.

Since larger shipments yield a lower per-unit cost, the practice of over-declaring shipments becomes very relevant in an All-Units discount model. At times it may be advantageous to artificially inflate a shipment weight to achieve a lower per-CWT cost for the entire shipment. To determine when it is economical to over-declare a shipment, the company purchasing the shipments must decide where the indifference and breakpoints are. The indifference point is the weight which, when multiplied by the per-CWT cost in the next cost bracket, yields the same total cost as the total cost of the minimum weight at the next breakpoint. The breakpoint is the minimum weight of the next cost bracket and is the weight that should be declared once a shipment weight surpasses the indifference point. It is appropriate to over-declare a shipment when the actual shipment weight lies between the indifference point and the breakpoint. The following notation corresponds to the example provided in Tables 3.4 and 3.5. The indifference point for air shipments is defined with the following equation:

$$\mu a_{(c-1)} = \frac{(AIRC_c * (a_{c-1} + 1))}{AIRC_{(c-1)}} \quad \forall c \quad where\ c > 1$$

$c$ = cost bracket index

$AIRC_c$ = per-hundredweight cost of an air shipment in cost bracket ‘c’

$a_{c-1}$+1 = lower quantity breakpoint for bracket ‘c’

$\mu a_c$ = indifference point for cost bracket ‘c’

The indifference points for ground shipments are similarly defined.

$$\mu b_{(e-1)} = \frac{(GROUNDC_e * (b_{e-1} + 1))}{GROUNDC_{(e-1)}} \quad \forall e\ \ where\ e > 1$$

$e$ = cost bracket index

$GROUNDC_e$ = per-hundredweight cost of a ground shipment in cost bracket ‘e’

$b_{e-1}$+1 = lower quantity breakpoint for bracket ‘e’

$\mu b_e$ = indifference point for cost bracket ‘e’

Table 3.6 summarizes the same costs and weight limits presented in Table 3.3 along with total cost of the orders and assumes they correspond with air transportation.

**Table 3.6: Example of an All-Units cost structure for air shipments**

| Cost Bracket Index | Per-CWT Cost | Weight Limits (CWT) | Fixed cost calculated at lower limit |
|---|---|---|---|
| 1 | \$1.00 | $0 \leq X \leq 999$ | \$0 + part thereof |
| 2 | \$0.90 | $1{,}000 \leq X \leq 2{,}999$ | \$900 + part thereof |
| 3 | \$0.75 | $3{,}000 \leq X \leq 5{,}999$ | \$2,250 + part thereof |
| 4 | \$0.55 | $6{,}000 \leq X \leq 10{,}000$ | \$3,300 + part thereof |

For example, using the values from Table 3.3 presented earlier, the following equations would be used to calculate the indifference point for an air shipment and cost bracket 2:

$$\mu a_1 = \frac{(AIRC_2 * (a_1 + 1))}{AIRC_1} = \frac{(0.9 * 1{,}000)}{1.00} = 900$$

Therefore, if the air shipment weight is between 900 and 1,000 it should be declared as 1,000 to get the lower per-unit cost of cost bracket 2. The total cost of a shipment weight that falls between 900 and 1,000 will then always be \$0.9*1,000 = \$900. This is proven to be advantageous to the company purchasing the shipment because if 901 CWT's was declared, the total cost would be \$1.00*901 = \$901, which is more expensive than the cost of the over-declared weight of 1,000. This is true for any shipment weight from 901 to 999 that is not over-declared. The identification and use of the indifference points yield significant savings on transportation costs to the shippers. When calculating indifference points, if the resulting weight is a decimal it should always be rounded down to the nearest integer. The indifference points and total cost for the example are presented in Table 3.7.

**Table 3.7: Example of indifference points and costs for All-Units discounts model**

| Weight Limits (CWT) | Indifference Point |
|---|---|
| $0 \le X \le 999$ | 900 |
| $1,000 \le X \le 2,999$ | 2,250 |
| $3,000 \le X \le 5,999$ | 3,300 |
| $6,000 \le X \le 10,000$ | N/A |

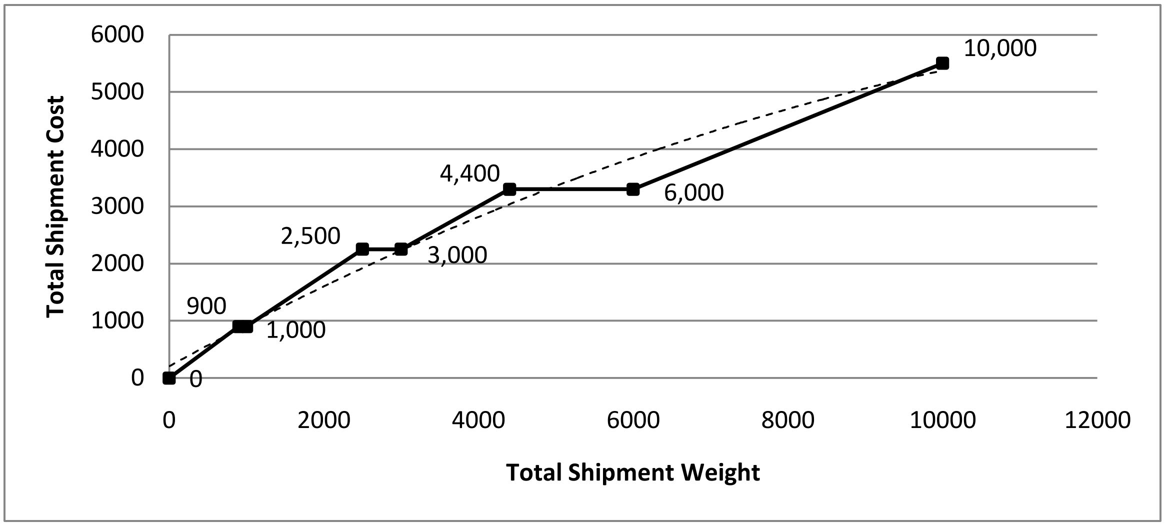
Table 3.8 summarizes the final cost structure of the example with consideration for the indifference points. The capacity of each cost bracket, denoted by $CAPA_d$ for air shipments, is also included.

**Table 3.8: Example of final cost structure with indifference points**

| Cost Bracket Index | Per-CWT Cost ($AIRCadj_d$) | Weight Limits (CWT) | Bracket Capacity ($CAPA_d$) | Total Cost ($TotalC_d$) | Indicator Variable ($\delta E_d$) |
|---|---|---|---|---|---|
| 1 | $1.00 | $0 \le X \le 900$ | 900 | $1.00*X | $\delta E_1$ |
| 2 | $0.90 | 901≤ X ≤999 | 98 | $900 | $\delta E_2$ |
| 3 | $0.90 | $1,000 \le X \le 2,500$ | 1,400 | $0.9*X | $\delta E_3$ |
| 4 | $0.75 | 2,501 ≤ X ≤ 2,999 | 498 | $2,250 | $\delta E_4$ |
| 5 | $0.75 | $3,000 \le X \le 4,400$ | 1,400 | $0.75*X | $\delta E_5$ |
| 6 | $0.55 | 4,401 ≤ X ≤ 5,999 | 1,598 | $3,300 | $\delta E_6$ |
| 7 | $0.55 | $6,000 \le X \le 10,000$ | 4,000 | $0.55*X | $\delta E_7$ |

The resulting cost structure from Table 3.8 with the weights labeled is displayed in Figure 3.3.

**Figure 3.3: Graphical representation of cost structure including indifference points**




The graphical representation is a piece-wise linear function. As weight increases costs still decrease, but in a piecewise manner which reflects the 'economies-of-scale' trend often implemented in discount structures. The dashed line represents the trend line of the graph. The slope of the trend line decreases as quantity increases supporting the lower per-CWT costs of higher weight shipments. The flat portions of the graph represent the quantities between the indifference point and the following breakpoint. The indifference point is the left point of a flat portion and the next breakpoint is the right point. The total cost in this range is constant by over-declaring the weight as the lower weight limit of the subsequent cost bracket therefore represented by a flat curve. If the shipment weight is greater than or equal to the breakpoint but less than the next indifference point, the cost function is increasing depending on the variable cost which accounts for the weight above the breakpoint. However, if the weight falls between the indifference point and the cost bracket breakpoint, then the weight should be over-declared and the total cost defined as a constant.

For example, if the shipment weight is 4,000 units, then the exact shipment weight is declared and the total cost is ($0.75)*(4,000) = $3,000. However, if the shipment weight is 2,800 units, it is economical to over-declare the shipment as 3,000 CWT's and incur a cost of ($0.75)*(3,000) = $2,250.

### 3.4.5.4 Modeling the Freight Rate Function

As seen in the example, the number of cost brackets necessary to model the All-Units discount model is larger than the original number of cost brackets. Given that the index 'c' was used to represent the original number of cost brackets for air transportation, the following constraint defines the number of cost brackets necessary to accurately model the All-Units discount cost structure while taking into account the indifference points. *CMAX* is the largest value of the index 'c' and *DMAX* is the largest value of the index 'd'.

$$d = 2c - 1$$

And so,

$$DMAX = (2 * CMAX) - 1$$

Given 'e' represents the original cost brackets for ground transportation, the number of cost brackets necessary for the All-Units discount cost structure is 'f'. *EMAX* is the largest value of the index 'e' and *FMAX* is the largest value of the index 'f'.

$$f = 2e - 1$$

Similarly,

$$FMAX = (2 * EMAX) - 1$$

The original data presents the costs and breakpoints for each transportation mode. However, the costs incurred must depend on the indifference points. Therefore new variables are dedicated to the adjusted costs and weight limits to include the indifference points. Table 3.9 represents the same information as Table 3.8, however the values are displayed as variable names and air transportation is still assumed.

**Table 3.9: Variables for example of cost structure with indifference points**

| Bracket Index ($d$) | Per-CWT Cost ($AIRCadj_d$) | Weight Limits (CWT) | Bracket Capacity ($CAPA_d$) | Total Cost ($TotalC_d$) | Indicator Variable ($\delta E_d$) |
|---|---|---|---|---|---|
| 1 | $AIRC_1$ | $aLB \leq X \leq \mu a_1$ | $\mu a_1 - aLB$ | $AIRC_1$*X | $\delta E_1$ |
| 2 | $AIRC_2$ | $(\mu a_1+1) \leq X \leq a_1$ | $a_1 - (\mu a_1+1)$ | $(a_1+1)$*AIRC | $\delta E_2$ |
| 3 | $AIRC_2$ | $(a_1+1) \leq X \leq \mu a_2$ | $\mu a_2 - (a_1+1)$ | $AIRC_2$*X | $\delta E_3$ |
| 4 | $AIRC_3$ | $(\mu a_2+1) \leq X \leq a_2$ | $a_2 - (\mu a_2+1)$ | $(a_2+1)$*AIRC | $\delta E_4$ |
| 5 | $AIRC_3$ | $(a_2+1) \leq X \leq \mu a_3$ | $\mu a_3 - (a_2+1)$ | $AIRC_3$*X | $\delta E_5$ |
| 6 | $AIRC_4$ | $(\mu a_3+1) \leq X \leq a_3$ | $a_3 - (\mu a_3+1)$ | $(a_3+1)$*AIRC | $\delta E_6$ |
| 7 | $AIRC_4$ | $(a_3+1) \leq X \leq a_4$ | $a_4 - (a_3+1)$ | $AIRC_4$*X | $\delta E_7$ |

The capacity of each cost bracket in the adjusted model can be defined generally as follows:

$$CAPA_{2c-1} = \mu a_c - aLB \quad when\ c = 1$$

$$CAPA_{2c-1} = \mu a_c - (a_{c-1} + 1) \quad \forall\ c\ when\ 2 \leq c \leq CMAX - 1$$

$$CAPA_{2c-1} = a_c - (a_{c-1} + 1) \quad when\ c = CMAX$$

$$CAPA_{2c} = a_c - (\mu a_c + 1) \quad when\ c = 1$$

$$CAPA_{2c} = a_c - (\mu a_c + 1) \quad \forall\ 2 \leq c \leq CMAX - 1$$

The variable $CAPG_f$ is used to represent the capacity of the cost brackets for ground shipments. Therefore the following constraints define the capacities:

$$CAPG_{2e-1} = \mu b_e - bLB \quad when\ e = 1$$

$$CAPG_{2e-1} = \mu b_e - (b_{e-1} + 1) \quad \forall e\ when\ 2 \le e \le EMAX - 1$$

$$CAPG_{2e-1} = b_e - (b_{e-1} + 1) \quad when\ e = EMAX$$

$$CAPG_{2e} = b_e - (\mu b_e + 1) \quad when\ e = 1$$

$$CAPG_{2e} = b_e - (\mu b_e + 1) \quad \forall e\ when\ 2 \le e \le EMAX - 1$$

As noted previously, the transportation cost is represented by:

$$\sum_c (AIRC_c * X * \delta_c)$$

$X_d$ is used to represent the shipment weight (in CWT) that falls within the weight limits of the adjusted cost bracket 'd'. Therefore, using the adjusted cost structure presented in Table 3.9, the total cost of an air shipment of weight $X$ is:

$$AIRC_1 * X_1 * \delta E_1 + AIRC_2 * X_2 * \delta E_2 + AIRC_2 * X_3 * \delta E_3 + AIRC_3 * X_4 * \delta E_4 + AIRC_3 * X_5 * \delta E_5 + AIRC_4 * X_6 * \delta E_6 + AIRC_4 * X_7 * \delta E_7$$

Subject to,

$$X = \sum_d X_d$$

$$\sum_d \delta E_d = 1$$

$$X_d \geq 0 \quad \forall d$$

$$\delta E_d \,\epsilon\, (0,1) \quad \forall d$$

Since the cost function is nonlinear due to the multiplication of variables, the linearization of the cost structure using indifference points is necessary.

### 3.4.5.4.1 Linearization of Freight Rate Function

Each cost bracket presented in Table 3.8 has an adjusted weight range based on the breakpoints and indifference points. The difference between the upper and lower limits of the range is referred to as the capacity of the bracket, $CAPA_d$. The cost bracket itself will be considered to be a ‘bucket’ of fixed capacity. The shipment weight is distributed among the buckets, filling them in order and to capacity before moving on to the next bucket. Based on the example, the first cost bracket would have a bucket of capacity 900 - 0 = 900 CWT’s ($CAPA_1$). The price per CWT in this bucket is \$1.00, or $AIRCadj_1$. The total cost of a shipment that has a weight that can be held by this bucket is equal to the actual weight multiplied by \$1.00 for every CWT. The second cost bracket would have a bucket of capacity 999 – 901 = 98 CWT’s ($CAPA_2$). However, unlike bucket 1, the total cost of a weight that extends into this bucket is fixed at $900, regardless of how full bucket 2 gets. It is important to note that bucket 2 cannot be filled until bucket 1 is filled to capacity. Therefore, there would be a set of buckets lined up next to each other, which are to be filled sequentially. Once one bucket is filled to its maximum weight, or capacity, the remaining weight pours over into the next bucket and so on. For example bucket 4 cannot begin to be filled until buckets 1, 2, and 3 are filled completely.

The total cost of shipments then depends on what bucket the weight quantity reaches. Some buckets, where the index ‘d’ is even, have a fixed cost for the shipment. The other buckets, where the index ‘d’ is odd, have a cost that is calculated by multiplying the quantity in that bucket ($X_d$) by the price per hundredweight ($AIRCadj_d$) of items in that bucket. The indicator variables $\delta E_d$ are equal to 1 when bucket ‘d’ contains a portion of the total shipment went, and 0 otherwise. This is true for the indicator variables $\delta 0_c$, $\delta 1_c$, $\delta 2_c$, $\theta 0_e$, $\theta 1_e$, and $\theta 1_e$, in the actual model. With this structure, the transportation of a shipment using the cost structure in Table 3.8 can be represented linearly as follows:

$$Shipment\ Cost = X_1 * AIRCadj_1 + 900 * \delta E_2 + X_3 * AIRCadj_3 + 2250 * \delta E_4 + X_5 * AIRCadj_5 + 3300 * \delta E_6 + X_7 * AIRCadj_7$$

Subject to,

$$X_1 \leq 900 * \delta E_1$$

$$X_1 \geq 900 * \delta E_2$$

$$X_2 \leq 98 * \delta E_2$$

$$X_2 \geq 98 * \delta E_3$$

$$X_3 \leq 1400 * \delta E_3$$

$$X_3 \geq 1400 * \delta E_4$$

$$X_4 \leq 498 * \delta E_4$$

$$X_4 \geq 498 * \delta E_5$$

$$X_5 \leq 1400 * \delta E_5$$

$$X_5 \geq 1400 * \delta E_6$$

$$X_6 \leq 1598 * \delta E_6$$

$$X_6 \geq 1598 * \delta E_7$$

$$X_7 \leq 4000 * \delta E_7$$

$$X = \sum_{d=1}^{7} X_d$$

$$X_d \geq 0 \quad \forall d = 1,2, \dots ,7$$

$$\delta E_d \ \epsilon \ (0,1) \quad \forall d = 1,2, \dots ,7$$

Generally, the preceding constraints are represented by:

$$Shipment\ Cost = \sum_{c}^{CMAX} (X_{2c-1} * AIRCadj_{2c-1}) + \sum_{c}^{CMAX-1} (TotalC_{2c} * \delta E_{2c})$$

Subject to,

$$X_d \leq (CAPA_d * \delta E_d) \quad \forall d$$

$$X_d \geq (CAPA_d * \delta E_{d+1}) \quad \forall d \leq DMAX - 1$$

$$X = \sum_{d} X_d$$

$$X_d \geq 0 \quad \forall d$$

$$\delta E_d \in (0,1) \quad \forall d$$

The shipment cost equation can be revised to implement the original per hundredweight costs and the fixed costs in the adjusted model to include the indifference point as follows:

$$Shipment\ Cost = \sum_{c}^{CMAX} (X_{2c-1} * AIRC_c) + \sum_{c}^{CMAX-1} (\mu a_c * AIRC_c * \delta E_{2c})$$

### 3.4.5.5 Cost Structure for Shipments from Suppliers to Manufacturers

As presented earlier, the total weight (in CWT) of a raw material air shipment is represented by $TRMWA_{k\,m\,t}$. This total weight is broken down into the amount of the weight distributed in each cost bracket 'd' as follows:

$$TRMWA_{k\,m\,t} = \sum_{d} TRMWAS_{k\,m\,t\,d} \quad \forall k, m, t$$

For example, the total weight of material 3 shipped to manufacturer 2 using air transportation in time period 5 is split up into the weight shipped in each cost bracket. Assuming there were 4 original cost brackets (c = 4) that resulted in 7 (d = 7) adjusted cost brackets, the following constraint summarizes the breakdown of the total shipment weight

$$TRMWAS_{3\,2\,1\,5}$$
$$= TRMWAS_{3\,2\,5\,1} + TRMWAS_{3\,2\,5\,2} + TRMWAS_{3\,2\,5\,3} + TRMWAS_{3\,2\,5\,4} + TRMWAS_{3\,2\,5\,5} + TRMWAS_{3\,2\,5\,6} + TRMWAS_{3\,2\,5\,7}$$

Since each shipment from each supplier to each manufacturer during each time period is unique, a different binary indicator variable is required for each instance. Therefore the binary variables used in this portion of the model are $\delta 0_{k\,m\,t\,d}$ and $\theta 0_{k\,m\,t\,f}$. Using this notation, the total cost of all air shipments of raw materials is expressed as follows:

$$Shipment\ Cost = \sum_{k}\sum_{m}\sum_{t}\left[\sum_{c}^{CMAX}\left(TRMWAS_{k\,m\,t\,(2c-1)} * AIRC_c\right) + \sum_{c}^{CMAX-1}\left(\mu a_c * AIRC_c * \delta 0_{k\,m\,t\,2c}\right)\right]$$

The following constraints are necessary to ensure the All-Units discount cost structure is implemented correctly.

$$TRMWAS_{k\,m\,t\,d} \le (CAPA_d * \delta 0_{k\,m\,t\,d}) \quad \forall k, m, t, d$$

$$TRMWAS_{k\,m\,t\,d} \ge (CAPA_d * \delta 0_{k\,m\,t\,(d+1)}) \quad \forall k, m, t, d \quad where\ d \le DMAX - 1$$

$$TRMWAS_{k\,m\,t\,d} \ge 0 \quad \forall k, m, t, d$$

$$\delta 0_{k\,m\,t\,d} \,\epsilon\, (0,1) \quad \forall k, m, t, d$$

Similarly, the total raw material weight of a ground shipment is broken down as follows:

$$TRMWG_{k\ m\ t} = \sum_{f} TRMWGS_{k\ m\ t\ f} \quad \forall k, m, t$$

These variables are used in the equation for the total cost of ground shipments of raw materials across the entire time horizon.

$$Shipment\ Cost = \sum_{k}\sum_{m}\sum_{t}\left[\sum_{e}^{EMAX}(TRMWGS_{k\ m\ t\ (2e-1)} * GROUNDC_{e}) + \sum_{e}^{EMAX-1}(\mu b_{e} * GROUNDC_{e} * \theta 0_{k\ m\ t\ 2e})\right]$$

The following constraints are necessary to ensure the total cost incorporates the All-Units discount cost structure appropriately:

$$TRMWGS_{k\ m\ t\ f} \leq (CAPG_{f} * \theta 0_{k\ m\ t\ f}) \quad \forall k, m, t, f$$

$$TRMWGS_{k\ m\ t\ f} \geq (CAPG_{f} * \theta 0_{k\ m\ t\ (f+1)}) \quad \forall k, m, t, f \quad where\ f \leq FMAX - 1$$

$$TRMWGS_{k\ m\ t\ f} \geq 0 \quad \forall k, m, t, f$$

$$\theta 0_{k\ m\ t\ f} \in (0,1) \quad \forall k, m, t, f$$

### 3.4.5.6 Cost Structure for Shipments from Manufacturers to Warehouses

The binary indicator variables used for shipments from manufacturers to warehouses are $\delta 1_{m\ n\ t\ d}$ and $\theta 1_{m\ n\ t\ f}$. The total weight of finished product shipments from manufacturers to warehouses is broken down as follows:

$$TFPWA1_{m\,n\,t} = \sum_{d} TFPWAS1_{m\,n\,t\,d} \quad \forall m, n, t$$

$$TFPWG1_{m\,n\,t} = \sum_{f} TFPWGS1_{m\,n\,t\,f} \quad \forall m, n, t$$

Accordingly, the total cost of shipments from all manufacturers to warehouses is:

$$\begin{aligned} Shipment\ Cost &= \sum_{m}\sum_{n}\sum_{t}\left\{\left[\sum_{c}^{CMAX}\left(TFPWAS1_{m\,n\,t\,(2c-1)} * AIRC_c\right)\right.\right. \\ &+ \left.\sum_{c}^{CMAX-1}\left(\mu a_c * AIRC_c * \delta 1_{m\,n\,t\,2c}\right)\right] \\ &+ \left[\sum_{e}^{EMAX}\left(TFPWGS1_{m\,n\,t\,(2e-1)} * GROUNDC_e\right)\right. \\ &+ \left.\left.\sum_{e}^{EMAX-1}\left(\mu b_e * GROUNDC_e * \theta 1_{m\,n\,t\,2e}\right)\right]\right\} \end{aligned}$$

The following constraints ensure the All-Units discount cost structure is properly implemented:

$$TFPWAS1_{m\,n\,t\,d} \leq \left(CAPA_d * \delta 1_{m\,n\,t\,d}\right) \quad \forall m, n, t, d$$

$$TFPWAS1_{m\,n\,t\,d} \geq \left(CAPA_d * \delta 1_{m\,n\,t\,(d+1)}\right) \quad \forall m, n, t, d \quad where\ d \leq DMAX - 1$$

$$TFPWAS1_{m\,n\,t\,d} \geq 0 \quad \forall m, n, t, d$$

$$\delta 1_{m\,n\,t\,d}\ \epsilon\ (0,1) \quad \forall m, n, t, d$$

$$TFPWGS1_{m\,n\,t\,f} \leq \left(CAPG_f * \theta 1_{m\,n\,t\,f}\right) \quad \forall m, n, t, f$$

$$TFPWGS1_{m\,n\,t\,f} \geq \left(CAPG_f * \theta 1_{m\,n\,t\,(f+1)}\right) \quad \forall m, n, t, f \quad where\ f \leq FMAX - 1$$

$$TFPWGS1_{m\,n\,t\,f} \geq 0 \quad \forall m, n, t, f$$

$$\theta 1_{m\,n\,t\,f}\ \epsilon\ (0,1) \quad \forall m, n, t, f$$

### 3.4.5.7 Cost Structure for Shipments from Warehouses to Retailers

The binary indicator variables used for shipments from warehouses to retailers are $\delta 2_{n\,r\,t\,d}$ and $\theta 2_{n\,r\,t\,f}$. The total weight of finished product shipments from warehouses to retailers is broken down as follows:

$$TFPWA2_{n\,r\,t} = \sum_{d} TFPWAS2_{n\,r\,t\,d} \quad \forall n, r, t$$

$$TFPWG2_{n\,r\,t} = \sum_{f} TFPWGS2_{n\,r\,t\,f} \quad \forall n, r, t$$

Accordingly, the total cost of shipments from all warehouses to retailers is:

$$\begin{aligned} &Shipment\ Cost \\ &= \sum_{n}\sum_{r}\sum_{t}\left\{\left[\sum_{c}^{CMAX}(TFPWAS2_{n\,r\,t\,(2c-1)} * AIRC_c)\right.\right. \\ &\left. + \sum_{c}^{CMAX-1}(\mu a_c * AIRC_c * \delta 2_{n\,r\,t\,2c})\right] \\ &+ \left[\sum_{e}^{EMAX}(TFPWGS2_{n\,r\,t\,(2e-1)} * GROUNDC_e)\right. \\ &\left.\left. + \sum_{e}^{EMAX-1}(\mu b_e * GROUNDC_e * \theta_{2n\,r\,t\,2e})\right]\right\} \end{aligned}$$

The following constraints ensure the All-Units discount cost structure is properly implemented:

$$TFPWAS2_{n\ r\ t\ d} \leq (CAPA_d * \delta 2_{n\ r\ t\ d}) \quad \forall n, r, t, d$$

$$TFPWAS2_{n\ r\ t\ d} \geq (CAPA_d * \delta 2_{n\ r\ t\ (d+1)}) \quad \forall n, r, t, d \quad where\ d \leq DMAX - 1$$

$$TFPWAS2_{n\ r\ t\ d} \geq 0 \quad \forall n, r, t, d$$

$$\delta 2_{n\ r\ t\ d} \in (0,1) \quad \forall n, r, t, d$$

$$TFPWGS2_{n\ r\ t\ f} \leq (CAPG_f * \theta 2_{n\ r\ t\ f}) \quad \forall n, r, t, f$$

$$TFPWGS2_{n\ r\ t\ f} \geq (CAPG_f * \theta 2_{n\ r\ t\ (f+1)}) \quad \forall n, r, t, f \quad where\ f \leq FMAX - 1$$

$$TFPWGS2_{n\ r\ t\ f} \geq 0 \quad \forall n, r, t, f$$

$$\theta 2_{n\ r\ t\ f} \in (0,1) \quad \forall n, r, t, f$$

## 3.5 Objective Functions

Two objective functions are implemented into the first model solved in this thesis. The first objective, $Obj_1$, is to maximize profit and the second objective, $Obj_2$, is to maximize the customer responsiveness. Each product has a fixed revenue per unit sold. Profit is defined as total revenue minus total cost. It is assumed that all shipments that arrive at a retailer are sold in that time period since inventory storage is not available at the retailer. Therefore the total number of units of product ‘p’ sold at retailer ‘r’ in time period ‘t’ is equal to the variable $TFPR_{p\ r\ t.}$ The revenue generated per unit sold of product ‘p’ is defined as $REV_p$. Therefore, the total revenue generated across the entire time horizon is:

$$\sum_p \sum_r \sum_t (TFPR_{p\ r\ t} * REV_p)$$

The costs in this model come from manufacturing costs, inventory costs, and shipping costs between each stage. The total profit is then defined as follows:

$$OBJ_1 = \sum_p \sum_r \sum_t (TFPR_{p\,r\,t} * REV_p) - [manufacturing\ costs] - [total\ inventory\ holding\ costs] - [total\ shipping\ costs]$$

Using the previously presented cost functions, *Obj₁* is expanded to the following expression:

$$\begin{aligned}
Obj_1 = {} & \sum_p \sum_r \sum_t (TFPR_{p\,r\,t} * REV_p) - \left[\sum_p \sum_m \sum_l \sum_t \left(OPERC_{p\,m\,l\,t} * \alpha_{p\,m\,l\,t}\right) + (PRODC_{p\,m\,l\,t} * FP_{p\,m\,l\,t})\right] \\
& - \left[\sum_k \sum_p \sum_m \sum_t (RM_{k\,p\,m\,t} * INVCRM_K) + \sum_p \sum_n \sum_t (WINV_{p\,n\,t} * INVCFP_p)\right] \\
& - \Bigg[\Bigg(\sum_k \sum_m \sum_t \Bigg\{\left[\sum_c^{CMAX} (TRMWAS_{k\,m\,t\,(2c-1)} * AIRC_c) + \sum_c^{CMAX-1} (\mu a_c * AIRC_c * \delta 0_{k\,m\,t\,2c})\right] \\
& + \left[\sum_e^{EMAX} (TRMWGS_{k\,m\,t\,(2e-1)} * GROUNDC_e) + \sum_e^{EMAX-1} (\mu b_e * GROUNDC_e * \theta 0_{k\,m\,t\,2e})\right]\Bigg\}\Bigg) \\
& + \Bigg(\sum_m \sum_n \sum_t \Bigg\{\left[\sum_c^{CMAX} (TFPWAS1_{m\,n\,t\,(2c-1)} * AIRC_c) + \sum_c^{CMAX-1} (\mu a_c * AIRC_c * \delta 1_{m\,n\,t\,2c})\right] \\
& + \left[\sum_e^{EMAX} (TFPWGS1_{m\,n\,t\,(2e-1)} * GROUNDC_e) + \sum_e^{EMAX-1} (\mu b_e * GROUNDC_e * \theta 1_{m\,n\,t\,2e})\right]\Bigg\}\Bigg) \\
& + \Bigg(\sum_n \sum_r \sum_t \Bigg\{\left[\sum_c^{CMAX} (TFPWAS2_{n\,r\,t\,(2c-1)} * AIRC_c) + \sum_c^{CMAX-1} (\mu a_c * AIRC_c * \delta 2_{n\,r\,t\,2c})\right] \\
& + \left[\sum_e^{EMAX} (TFPWGS2_{n\,r\,t\,(2e-1)} * GROUNDC_e) + \sum_e^{EMAX-1} (\mu b_e * GROUNDC_e * \theta 2_{n\,r\,t\,2e})\right]\Bigg\}\Bigg)\Bigg]
\end{aligned}$$

The second objective is to maximize the customer responsiveness which is achieved by minimizing the number of lost sales, or unsatisfied demand, over the entire time horizon. A lost sale implies that the company not only misses out on potential profit, but they also suffer goodwill loss due to unsatisfied customers. As discussed earlier, unsatisfied demand may be intentional or unintentional. In this model, regardless of intent, it is assumed that unsatisfied demand is quantified as the number of lost sales. Therefore minimizing the number of lost sales is essential to achieving good customer responsiveness.

$$Obj_2 = \sum_p \sum_r \sum_t LOST_{p\,r\,t}$$

The goal of this thesis is to find the solution that optimizes both objectives at the same time.

## Solution Procedure

Solving for both objectives at the same time creates a bi-criteria mathematical programming problem. In this case, maximizing profit is a conflicting objective with maximizing responsiveness because limiting the number of lost sales requires more inventory which is costly. This competes with the goal of maximizing profit. Therefore, a single optimal solution that maximizes both objectives simultaneously does not exist. Goal Programming is an approach for multi-criteria mathematical programs that allows a decision maker to set goals or target levels for each objective. Then the problem is solved by finding an optimal solution that comes as close as possible to each of those goals. It is required that the goal or target information is specified before the solution procedure begins. Although some goals may not be achievable, they will still offer a direction to work towards when deriving the optimal solution. The following problem **(3.1)** is an example of a multi-criteria mathematical program, where it is assumed that there are ‘s’ objectives and ‘t’ constraints.

$$\text{Maximize } [f_1(x), f_2(x), \dots, f_s(x)] \quad \textbf{(3.1)}$$

$$\text{Subject to: } g_v(x) \leq 0, v = 1, 2, \dots, t$$

$$x \geq 0$$

Where $f_1$, $f_{2}, \dots$, $f_s$ are the objective functions and $g_v(x)$ is the set of constraints. Each alternative is represented as an x vector, where x = ($x_1$, $x_2$, ..., $x_m$). We will assign the variable $TARGET_u$ to the value of the goal for objective 'u', for u = 1, 2, …, S. In addition, $w_u$ will represent the weight of each goal and $d_u^+$ and $d_u^-$ will represent the positive and negative deviations from goal 'u'. $d_u^+$ represents the over achievement of the u$^{th}$ goal and $d_u^-$ represents the underachievement. The objective of the goal programming formulation is minimize the sum of the deviational variables, in other words, find a solution that minimizes the total weighted distance from all of the goals. Thus, problem **(3.1)** is converted to the following goal programming problem **(3.2)**:

$$Minimize\ Z = \sum_u (w_u * (d_u^+ + d_u^-)) \quad \textbf{(3.2)}$$

Subject to,

$$f_u(x) + d_u^- - d_u^+ = TARGET_u \quad for\ u = 1, \dots, s$$

$$g_v(x) \leq 0, \quad for\ v = 1, \dots, t$$

$$x, d_u^+, d_u^- \geq 0 \quad for\ u = 1, \dots, s$$

For example, if we are maximizing the responsiveness of the supply chain, which is equivalent to minimizing the number of lost sales (objective 2: $u$ = 2), the goal for this objective could be 0, or $TARGET_2=0$. Therefore an optimal solution would minimize the number of lost sales by minimizing the deviational variable, $d_u^+$, which represents the deviation from the actual number of lost sales from zero. If there were 50 lost sales, then $f_2$ =

50 and $d_2^+$ would be equal to 50 and $d_2^-$ would be 0. The $w_u$ assigned to this goal would indicate how important it was to satisfy this goal when addressing the problem as a whole. The weights, $w_u$, can be either ordinal or cardinal, depending on whether the goal programming problem is preemptive or non-preemptive respectively.

Preemptive goal programming compares goals to each other and assigns a priority variable, or an ordinal weight, to the goal. The priorities are represented by the vector ($P_1, P_2, \ldots, P_s$). Therefore the goals are put in order of most important first and least important last and are solved sequentially with the most important goal being satisfied as far as possible before moving on to the next goal, or objective. First, just the goals affiliated with $P_1$ are satisfied. Then, attempts are made to improve $P_2$ goals without destroying the achievement levels of P1 goals. Essentially, preemptive goal programming solves a sequence of single objective problems.

Non-preemptive goal programming uses cardinal weights, where a value is assigned to the goal. These weights should reflect the decision maker's tradeoffs between the objectives. For example, if $w_1 = 2$ and $w_2 = 1$, this indicates that goal one is two times more important than goal two. These weights are multiplied by the deviational variables in the objective so a deviation from goal one will carry more weight and influence in the problem, than a deviation from goal two. The notation ($w_1, w_2, \ldots, w_u$) is used to represent the weights of each goal. These weights are used in the objective to reduce the problem to a single objective optimization problem. Since the units of measure may differ between the objectives, their values have to be scaled in this method so the weights are applied to similar metrics in the objective. To do this, the optimal solutions are first identified for each objective optimized individually while ignoring the other objectives. This solution is defined as the ideal value for that objective. Then the objective value in the goal programming problem is divided by the ideal solution of that objective to scale it on a 0 to 1 scale.

In the multi-criteria problem modeled in this thesis some objectives are to be maximized while others require minimization. Instead of choosing goals arbitrarily, the ideal solutions are used to set a reasonable upper or lower bound for each objective. Then the targets are set values within these ranges. A certain percent increase or decrease is used to relax the ideal values to the target values of the goal program. For example, if *IDEAL* is set equal to the ideal solution for a maximum objective, and the percent relaxation for objective 1 is set to 10%, then the goal for objective 1 would be to come within 10% of the ideal solution. Since the objective is a maximization, the target would be lower than the ideal solution, so the target is set to 0.9**IDEAL*. However, for the minimization objectives, the target would be 10% larger than the ideal value. Therefore the target for minimization objectives is set equal to 1.1**IDEAL*. The following steps are used to solve the preemptive and non-preemptive goal programs:

1. Solve for the ideal solutions by solving for each objective individually while ignoring the other objectives.
2. The upper or lower bounds for the objectives are identified by finding the ideal solutions for the objective. The upper bound for maximization objectives is their ideal solution while the lower bound is the lowest value for that objective among the other ideal solutions. The lower bound for minimization objectives is the ideal solution for that objective. That same objective's upper bound is the largest value for that objective in the other ideal solutions. For example, suppose we have two objectives, objective one is to be maximized and objective two is to be minimized.

The value of the objectives is presented as ($Obj_1$, $Obj_2$). Assume the ideal solution for objective one is (100,50) and the ideal solution for objective two is (60,0). Objective one would have a lower bound of 60 and an upper bound of 100. Similarly, objective two would have a lower bound of 0 and an upper bound of 50. The ideal solution to the entire problem is identified as (100,0). The optimal solution would minimize the deviation of objective one from the target based on 100 and objective two from the target based on 0.

3. Solve the preemptive goal program by implementing the pre-specified goal priorities. Solve the non-preemptive goal program by first scaling the objective function values using the ideal solutions and then incorporating the weights to the single objective problem.

**Step 1: Identify the ideal solutions**

Objective 1: Maximize profit while ignoring customer responsiveness,

$$
\begin{aligned}
Maximize\ Obj_1 = & \sum_{p}\sum_{r}\sum_{t}(TFPR_{p\,r\,t} * REV_p) \\
& -\left[\sum_{p}\sum_{m}\sum_{l}\sum_{t}\big(OPERC_{p\,m\,l\,t} * \alpha_{p\,m\,l\,t}\big) + (PRODC_{p\,m\,l\,t} * FP_{p\,m\,l\,t})\right] \\
& -\left[\sum_{k}\sum_{p}\sum_{m}\sum_{t}(RM_{k\,p\,m\,t} * INVCRM_K) + \sum_{p}\sum_{n}\sum_{t}(WINV_{p\,n\,t} * INVCFP_p)\right] \\
& -\Bigg[\Bigg(\sum_{k}\sum_{m}\sum_{t}\Bigg\{\Bigg[\sum_{c}^{CMAX}(TRMWAS_{k\,m\,t\,(2c-1)} * AIRC_c) + \sum_{c}^{CMAX-1}(\mu a_c * AIRC_c * \delta 0_{k\,m\,t\,2c})\Bigg] \\
& + \Bigg[\sum_{e}^{EMAX}(TRMWGS_{k\,m\,t\,(2e-1)} * GROUNDC_e) + \sum_{e}^{EMAX-1}(\mu b_e * GROUNDC_e * \theta 0_{k\,m\,t\,2e})\Bigg]\Bigg\}\Bigg) \\
& + \Bigg(\sum_{m}\sum_{n}\sum_{t}\Bigg\{\Bigg[\sum_{c}^{CMAX}(TFPWAS1_{m\,n\,t\,(2c-1)} * AIRC_c) + \sum_{c}^{CMAX-1}(\mu a_c * AIRC_c * \delta 1_{m\,n\,t\,2c})\Bigg] \\
& + \Bigg[\sum_{e}^{EMAX}(TFPWGS1_{m\,n\,t\,(2e-1)} * GROUNDC_e) + \sum_{e}^{EMAX-1}(\mu b_e * GROUNDC_e * \theta 1_{m\,n\,t\,2e})\Bigg]\Bigg\}\Bigg) \\
& + \Bigg(\sum_{n}\sum_{r}\sum_{t}\Bigg\{\Bigg[\sum_{c}^{CMAX}(TFPWAS2_{n\,r\,t\,(2c-1)} * AIRC_c) + \sum_{c}^{CMAX-1}(\mu a_c * AIRC_c * \delta 2_{n\,r\,t\,2c})\Bigg] \\
& + \Bigg[\sum_{e}^{EMAX}(TFPWGS2_{n\,r\,t\,(2e-1)} * GROUNDC_e) + \sum_{e}^{EMAX-1}(\mu b_e * GROUNDC_e * \theta 2_{n\,r\,t\,2e})\Bigg]\Bigg\}\Bigg)\Bigg]
\end{aligned}
$$

Subject to all previously mentioned constraints as listed in the general model in section 3.6

Let $IDEAL_1$ be the ideal solution for the first objective.

Objective 2: Maximize customer responsiveness by minimizing the total number of lost sales, or unsatisfied demand, ignoring profit.

$$Minimize\ Obj_2 = \sum_p \sum_r \sum_t LOST_{p\ r\ t}$$

Subject to all previously mentioned constraints as listed in the general model in section 3.6

Let $IDEAL_2$ be the ideal solution for the second objective.

**Step 2: Identify Targets for each objective**

Allow $dec_1$ be the percentage decrease from the ideal solution for the objective 1 target.

$$TARGET_1 = \frac{(100 - dec_1)}{100} * IDEAL_1$$

Let $inc_2$ be the percentage increase from the ideal solution for the objective 2 target.

$$TARGET_2 = \frac{(100 + inc_1)}{100} * IDEAL_2$$

**Step 3: Solve the goal programs**

<u>Method 1: Preemptive Goal Programming</u>

Goal constraint for profit,

$$Obj_1 + d_1^- - d_1^+ = TARGET_1$$

Goal constraint for customer responsiveness,

$$Obj_2 + d_2^- - d_2^+ = TARGET_2$$

Since goal 1 is a maximization goal, $d_1^-$ will be the deviational variable to be minimized. Whereas for goal 2, which is ultimately a minimization goal, $d_2^+$ is to be minimized.

Goal Programming Objective:

Case 1: Prioritize profit over customer responsiveness

$$Minimize\ Z = P_1(d_1^-) + P_2(d_2^+)$$

Case 2: Prioritize customer responsiveness over profit

$$Minimize\ Z = P_1(d_2^+) + P_2(d_1^-)$$

<u>Method 2: Non-Preemptive Goal Programming</u>

The objective values and goals are divided by the ideal values to normalize the data so the weights are applied to similar metrics. If the objective is equal to the target, the goal has been reached and the fractions will be equal and the deviational variables will be equal to zero. The deviational variables in this model represent the percentage that the actual value of the objective misses the goal by.

Goal constraint for the profit,

$$\frac{Obj_1}{IDEAL_1} + d_1^- - d_1^+ = \frac{TARGET_1}{IDEAL_1}$$

Goal constraint for customer responsiveness,

$$\frac{Obj_2}{IDEAL_2} + d_2^- - d_2^+ = \frac{TARGET_2}{IDEAL_2}$$

Goal Programming Objective:

$$Minimize\ Z = w_1(d_1^-) + w_2(d_2^+)$$

A third objective, which is added as an option to the model, is to minimize the amount of capital in inventory. It is important to minimize the amount of money tied up in inventory because inventory holding costs are usually low enough that the marginal profit of each item in inventory reveals monetary assets to the company that can be applied elsewhere in the supply chain. In addition, items held in inventory can become outdated and consequently less profitable the longer they sit in inventory unused. Consequently, the minimization of capital in inventory is a goal of supply chain management. However, this objective conflicts with the responsiveness of the supply chain because limiting inventory often increases the number of lost sales. Due to cost of over producing products and holding extra products that don't get used, the objective of minimizing capital in inventory also conflicts with maximizing profit. The capital in inventory in this model is simplified to the selling price, or revenue, of the finished products multiplied by the number of that finished product being stored at the warehouses. Therefore the third objective is defined as:

$$Obj_3 = \sum_p \sum_n \sum_t (WINV_{p\,n\,t} * REV_p)$$

Since this is a minimization objective, the ideal solution for objective 3 while ignoring all other objectives is,

$$Minimize\ Obj_3 = \sum_p \sum_n \sum_t (WINV_{p\,n\,t} * REV_p)$$

Subject to all previously mentioned constraints as listed in the general model in section 3.6

Let $IDEAL_3$ be the ideal solution for this objective and $inc_3$ be the percent increase allowed for the target value for objective 3. The ideal solution is set as the goal for this objective,

$$TARGET_3 = \frac{(100 + inc_1)}{100} * IDEAL_3$$

<u>Method 1: Preemptive Goal Programming</u>

Additional goal constraint for capital in inventory,

$$Obj_3 + d_3^- - d_3^+ = TARGET_3$$

Since objective 3 is a minimization objective, the deviational variable $d_3^+$ will be minimized and appear in the objective functions.

Goal Programming Objective:

Scenario 1: Prioritize profit > customer responsiveness > capital in inventory

$$Minimize\ Z = P_1(d_1^-) + P_2(d_2^+) + P_3(d_3^+)$$

Scenario 2: Prioritize profit > capital in inventory > customer responsiveness

$$Minimize\ Z = P_1(d_1^-) + P_2(d_3^+) + P_3(d_2^+)$$

Scenario 3: Prioritize customer responsiveness > profit > capital in inventory

$$Minimize\ Z = P_1(d_2^+) + P_2(d_1^-) + P_3(d_3^+)$$

Scenario 4: Prioritize customer responsiveness > capital in inventory > profit

$$Minimize\ Z = P_1(d_2^+) + P_2(d_3^+) + P_3(d_1^-)$$

Scenario 5: Prioritize capital in inventory > profit > customer responsiveness

$$Minimize\ Z = P_1(d_3^+) + P_2(d_1^-) + P_3(d_2^+)$$

Scenario 6: Prioritize capital in inventory > customer responsiveness> profit

$$Minimize\ Z = P_1(d_3^+) + P_2(d_2^+) + P_3(d_1^-)$$

Method 2: Non-Preemptive Goal Programming

Additional goal constraint for the inventory in capital,

$$\frac{Obj_3}{IDEAL_3} + d_3^- - d_3^+ = \frac{TARGET_3}{IDEAL_3}$$

Goal Programming Objective:

$$Minimize\ Z = w_1(d_1^-) + w_2(d_2^+) + w_3(d_3^+)$$

## 3.6 General Model

Maximize Profit:

$$
\begin{aligned}
Maximize\ Obj_1 = & \sum_{p}\sum_{r}\sum_{t}(TFPR_{p\ r\ t} * REV_p) \\
& - \left[\sum_{p}\sum_{m}\sum_{l}\sum_{t}\left(OPERC_{p\ m\ l\ t} * \alpha_{p\ m\ l\ t}\right) + (PRODC_{p\ m\ l\ t} * FP_{p\ m\ l\ t})\right] \\
& - \left[\sum_{k}\sum_{p}\sum_{m}\sum_{t}(RM_{k\ p\ m\ t} * INVCRM_K) + \sum_{p}\sum_{n}\sum_{t}(WINV_{p\ n\ t} * INVCFP_p)\right] \\
& - \Bigg[\Bigg(\sum_{k}\sum_{m}\sum_{t}\Bigg\{\left[\sum_{c}^{CMAX}(TRMWAS_{k\ m\ t\ (2c-1)} * AIRC_c) + \sum_{c}^{CMAX-1}(\mu a_c * AIRC_c * \delta 0_{k\ m\ t\ 2c})\right] \\
& + \left[\sum_{e}^{EMAX}(TRMWGS_{k\ m\ t\ (2e-1)} * GROUNDC_e) + \sum_{e}^{EMAX-1}(\mu b_e * GROUNDC_e * \theta 0_{k\ m\ t\ 2e})\right]\Bigg\}\Bigg) \\
& + \Bigg(\sum_{m}\sum_{n}\sum_{t}\Bigg\{\left[\sum_{c}^{CMAX}(TFPWAS1_{m\ n\ t\ (2c-1)} * AIRC_c) + \sum_{c}^{CMAX-1}(\mu a_c * AIRC_c * \delta 1_{m\ n\ t\ 2c})\right] \\
& + \left[\sum_{e}^{EMAX}(TFPWGS1_{m\ n\ t\ (2e-1)} * GROUNDC_e) + \sum_{e}^{EMAX-1}(\mu b_e * GROUNDC_e * \theta 1_{m\ n\ t\ 2e})\right]\Bigg\}\Bigg) \\
& + \Bigg(\sum_{n}\sum_{r}\sum_{t}\Bigg\{\left[\sum_{c}^{CMAX}(TFPWAS2_{n\ r\ t\ (2c-1)} * AIRC_c) + \sum_{c}^{CMAX-1}(\mu a_c * AIRC_c * \delta 2_{n\ r\ t\ 2c})\right] \\
& + \left[\sum_{e}^{EMAX}(TFPWGS2_{n\ r\ t\ (2e-1)} * GROUNDC_e) + \sum_{e}^{EMAX-1}(\mu b_e * GROUNDC_e * \theta 2_{n\ r\ t\ 2e})\right]\Bigg\}\Bigg)\Bigg]
\end{aligned}
$$

Maximize Customer Responsiveness (Minimize the number of lost sales):

$$Minimize\ Obj_2 = \sum_{p}\sum_{r}\sum_{t} LOST_{p\ r\ t}$$

Minimize Capital in Inventory (Optional objective):

$$Minimize\ Obj_3 = \sum_{p}\sum_{n}\sum_{t}(WINV_{p\ n\ t} * REV_p)$$

Subject to,

Manufacturer Constraints:

$$TRM_{k\ m\ t} = INITRM_k \quad \forall k, m, t \quad when\ t = 1$$

$$TRM_{k\ m\ t} = X_{k\ m\ 1\ (t-1)} \quad \forall k, m, t \quad when\ t = 2$$

$$TRM_{k\ m\ t} = \sum_i X_{k\ m\ i\ (t-i)} \quad \forall\ k, m, t\ when\ t \geq 3$$

$$FP_{p\ m\ l\ t} = \ 0 \quad \forall p, m, l, t \quad when\ t = 1$$

$$TFPM_{p\ m\ t} = \ \sum_l FP_{p\ m\ l\ t} \quad \forall p, m, t$$

$$TFPM_{p\ m\ t} = \sum_n \sum_i Y_{p\ m\ n\ i\ t} \quad \forall p, m, t$$

$$TRM_{\ k\ m\ t} = \sum_p RM_{k\ p\ m\ t} \quad \forall k, m, t$$

$$RM_{k\ p\ m\ t} = \sum_l (r_{k\ p} * FP_{p\ m\ l\ (t+1)})\ \forall k, p, m, t\ for\ t = 1,2, \dots, T-1$$

$$\alpha_{p\ m\ l\ t} * MCAP_{p\ m\ l} \geq FP_{p\ m\ l\ t} \quad \forall p, m, l, t$$

$$\sum_p \alpha_{p\ m\ l\ t} \leq 1 \quad \forall m, l, t$$

$$\alpha_{p\ m\ l\ t} \in (0,1) \quad \forall p, m, l, t$$

Warehouse Constraints:

$$TFPW_{p\ n\ t} = INITFPW_{p\ n} \quad \forall p, n, t \quad when\ t = 1, 2$$

$$TFPW_{p\ n\ t} = \sum_m Y_{p\ m\ n\ 1\ t-1} \quad \forall p, n, t \quad when\ t = 3$$

$$TFPW_{p\,n\,t} = \sum_{m}\sum_{i} Y_{p\,m\,n\,i\,t-i} \quad \forall p,n,t \quad when\ t \geq 4$$

$$TFPW_{p\,n\,t} = \sum_{r}\sum_{i} (Z_{p\,n\,r\,i\,t}) + WINV_{p\,n\,t} \quad \forall p,n,t \quad when\ t = 1$$

$$TFPW_{p\,n\,t} + WINV_{p\,n\,t-1} = \sum_{r}\sum_{i} (Z_{p\,n\,r\,i\,t}) + WINV_{p\,n\,t} \quad \forall p,n,t \quad when\ t \geq 2$$

$$\sum_{p} WINV_{p\,n\,t} \leq WCAP_{n} \quad \forall n,t$$

Retailer Constraints:

$$TFPR_{p\,r\,t} = INITFPR_{p\,r} \quad \forall p,r,t \quad when\ t = 1$$

$$TFPR_{p\,r\,t} = \sum_{n} Z_{p\,n\,r\,1\,t-1} \quad \forall p,r,t \quad when\ t = 2$$

$$TFPR_{p\,r\,t} = \sum_{n}\sum_{i} Z_{p\,n\,r\,i\,t-i} \quad \forall p,r,t \quad when\ t \geq 3$$

$$TFPR_{p\,r\,t} + LOST_{p\,r\,t} = RD_{p\,r\,t} \quad \forall p,r,t$$

Transportation Quantity Constraints:

$$\rho_{k\,m\,i\,t} * MINRM_{i} \leq X_{k\,m\,i\,t} \qquad \forall k,m,i,t$$

$$X_{k\,m\,i\,t} \leq \rho_{k\,m\,i\,t} * MAXRM_{i} \qquad \forall k,m,i,t$$

$$X_{k\,m\,i\,t} \leq M * \rho_{k\,m\,i\,t} \qquad \forall k,m,i,t$$

$$\rho_{k\,m\,i\,t} \in (0,1) \quad \forall k,m,i,t$$

$$TY_{m\,n\,i\,t} = \sum_{p} Y_{p\,m\,n\,i\,t} \quad \forall\ m,n,i,t$$

$$\tau_{m\,n\,i\,t} * MINFP_i \leq TY_{m\,n\,i\,t} \qquad \forall m,n,i,t$$

$$TY_{m\,n\,i\,t} \leq \tau_{m\,n\,i\,t} * MAXFP_i \qquad \forall m,n,i,t$$

$$TY_{m\,n\,i\,t} \leq M * \tau_{m\,n\,i\,t} \qquad \forall m,n,i,t$$

$$\tau_{m\,n\,i\,t} \in (0,1) \quad \forall m,n,i,t$$

$$TZ_{n\,r\,i\,t} = \sum_p Z_{p\,n\,r\,i\,t} \quad \forall\, n,r,i,t$$

$$\sigma_{n\,r\,i\,t} * MINFP_i \leq TZ_{n\,r\,i\,t} \qquad \forall n,r,i,t$$

$$TZ_{n\,r\,i\,t} \leq \sigma_{n\,r\,i\,t} * MAXFP_i \qquad \forall n,r,i,t$$

$$TZ_{n\,r\,i\,t} \leq M * \sigma_{n\,r\,i\,t} \qquad \forall n,r,i,t$$

$$\sigma_{n\,r\,i\,t} \in (0,1) \quad \forall n,r,i,t$$

Shipment Weight Constraints:

$$TRMWA_{k\,m\,t} = (RMW_k * X_{k\,m\,1\,t}) \qquad \forall k,m,t$$

$$TRMWG_{k\,m\,t} = (RMW_k * X_{k\,m\,2\,t}) \qquad \forall k,m,t$$

$$TFPWA1_{m\,n\,t} = \sum_p (FPW_p * Y_{p\,m\,n\,1\,t}) \qquad \forall m,n,t$$

$$TFPWG1_{m\,n\,t} = \sum_p (FPW_p * Y_{p\,m\,n\,2\,t}) \qquad \forall m,n,t$$

$$TFPWA2_{n\,r\,t} = \sum_p (FPW_p * Z_{p\,n\,r\,1\,t}) \qquad \forall n,r,t$$

$$TFPWG2_{n\,r\,t} = \sum_{p} (FPW_p * Z_{p\,n\,r\,2\,t}) \qquad \forall n, r, t$$

All-Units Discount Model Constraints:

$$\mu a_{(c-1)} = \frac{(AIRC_c * (a_{c-1} + 1))}{AIRC_{(c-1)}} \quad \forall c, \;\; when\; c > 1$$

$$\mu b_{(e-1)} = \frac{(GROUNDC_e * (b_{e-1} + 1))}{GROUNDC_{(e-1)}} \quad \forall e, \;\; when\; e > 1$$

$$d = 2c - 1$$

$$DMAX = (2 * CMAX) - 1$$

$$f = 2e - 1$$

$$FMAX = (2 * EMAX) - 1$$

$$CAPA_{2c-1} = \mu a_c - aLB \quad when\; c = 1$$

$$CAPA_{2c-1} = \mu a_c - (a_{c-1} + 1) \quad when\; 2 \le c \le CMAX - 1$$

$$CAPA_{2c-1} = a_c - (a_{c-1} + 1) \quad when\; c = CMAX$$

$$CAPA_{2c} = a_c - (\mu a_c + 1) \quad when\; c = 1$$

$$CAPA_{2c} = a_c - (\mu a_c + 1) \quad \forall \quad when\; 2 \le c \le CMAX - 1$$

$$CAPG_{2e-1} = \mu b_e - bLB \quad when\; e = 1$$

$$CAPG_{2e-1} = \mu b_e - (b_{e-1} + 1) \quad when\; 2 \le e \le EMAX - 1$$

$$CAPG_{2e-1} = b_e - (b_{e-1} + 1) \quad when\; e = EMAX$$

$$CAPG_{2e} = b_e - (\mu b_e + 1) \quad when\ e = 1$$

$$CAPG_{2e} = b_e - (\mu b_e + 1) \quad \forall \;\; 2 \leq e \leq EMAX - 1$$

$$TRMWA_{k\ m\ t} = \sum_{d} TRMWAS_{k\ m\ t\ d} \quad \forall k, m, t$$

$$TRMWAS_{k\ m\ t\ d} \leq (CAPA_d * \delta 0_{k\ m\ t\ d}) \quad \forall k, m, t, d$$

$$TRMWAS_{k\ m\ t\ d} \geq (CAPA_d * \delta 0_{k\ m\ t\ (d+1)}) \quad \forall k, m, t, d \quad when\ d \leq DMAX - 1$$

$$TRMWAS_{k\ m\ t\ d} \geq 0 \quad \forall k, m, t, d$$

$$\delta 0_{k\ m\ t\ d}\ \epsilon\ (0,1) \quad \forall k, m, t, d$$

$$TRMWG_{k\ m\ t} = \sum_{f} TRMWGS_{k\ m\ t\ f} \quad \forall k, m, t$$

$$TRMWGS_{k\ m\ t\ f} \leq (CAPG_f * \theta 0_{k\ m\ t\ f}) \quad \forall k, m, t, f$$

$$TRMWGS_{k\ m\ t\ f} \geq (CAPG_f * \theta 0_{k\ m\ t\ (f+1)}) \quad \forall k, m, t, f \quad when\ f \leq FMAX - 1$$

$$TRMWGS_{k\ m\ t\ f} \geq 0 \quad \forall k, m, t, f$$

$$\theta 0_{k\ m\ t\ f}\ \epsilon\ (0,1) \quad \forall k, m, t, f$$

$$TFPWA1_{m\ n\ t} = \sum_{d} TFPWAS1_{m\ n\ t\ d} \quad \forall m, n, t$$

$$TFPWAS1_{m\ n\ t\ d} \leq (CAPA_d * \delta 1_{m\ n\ t\ d}) \quad \forall m, n, t, d$$

$$TFPWAS1_{m\ n\ t\ d} \geq (CAPA_d * \delta 1_{m\ n\ t\ (d+1)}) \quad \forall m, n, t, d \quad when\ d \leq DMAX - 1$$

$$TFPWAS1_{m\ n\ t\ d} \geq 0 \quad \forall m, n, t, d$$

$$\delta 1_{m\,n\,t\,d} \in (0,1) \quad \forall m,n,t,d$$

$$TFPWG1_{m\,n\,t} = \sum_{f} TFPWGS1_{m\,n\,t\,f} \quad \forall m,n,t$$

$$TFPWGS1_{m\,n\,t\,f} \leq (CAPG_f * \theta 1_{m\,n\,t\,f}) \quad \forall m,n,t,f$$

$$TFPWGS1_{m\,n\,t\,f} \geq (CAPG_f * \theta 1_{m\,n\,t\,(f+1)}) \quad \forall m,n,t,f \quad when\ f \leq FMAX - 1$$

$$TFPWGS1_{m\,n\,t\,f} \geq 0 \quad \forall m,n,t,f$$

$$\theta 1_{m\,n\,t\,f} \in (0,1) \quad \forall m,n,t,f$$

$$TFPWA2_{n\,r\,t} = \sum_{d} TFPWAS2_{n\,r\,t\,d} \quad \forall n,r,t$$

$$TFPWAS2_{n\,r\,t\,d} \leq (CAPA_d * \delta 2_{n\,r\,t\,d}) \quad \forall n,r,t,d$$

$$TFPWAS2_{n\,r\,t\,d} \geq (CAPA_d * \delta 2_{n\,r\,t\,(d+1)}) \quad \forall n,r,t,d \quad when\ d \leq DMAX - 1$$

$$TFPWAS2_{n\,r\,t\,d} \geq 0 \quad \forall n,r,t,d$$

$$\delta 2_{n\,r\,t\,d} \in (0,1) \quad \forall n,r,t,d$$

$$TFPWG2_{n\,r\,t} = \sum_{f} TFPWGS2_{n\,r\,t\,f} \quad \forall n,r,t$$

$$TFPWGS2_{n\,r\,t\,f} \leq (CAPG_f * \theta 2_{n\,r\,t\,f}) \quad \forall n,r,t,f$$

$$TFPWGS2_{n\,r\,t\,f} \geq (CAPG_f * \theta 2_{n\,r\,t\,(f+1)}) \quad \forall n,r,t,f \quad when\ f \leq FMAX - 1$$

$$TFPWGS2_{n\,r\,t\,f} \geq 0 \quad \forall n,r,t,f$$

$$\theta 2_{n\,r\,t\,f} \in (0,1) \quad \forall n,r,t,f$$

# Chapter 4 Illustrative Example and Analysis

## 4.1 Illustrative Example

The model presented in Chapter 3 was implemented with an example for this chapter to illustrate the use of the model and results. The goal of the model is to develop a production and distribution schedule for two products in a four-stage centralized supply chain. Multiple objectives were implemented to drive the solution process.

### 4.1.1 Parameters

Since real time data is not readily available, the values in this example are either assumed or based on the data used in the work by Mysore (2005). The demand pattern across the 24 time periods is identical to the pattern used in Mysore's work. However, the number of retailers in this example is six, compared to the two that Mysore used, so assumed demand values were created for the remaining retailers. The products in this example generate higher revenue than those used in Mysore's work. Therefore the transportation and operating costs were scaled to realistically balance the total revenue. All other values were assumed and scaled appropriately for the example. In the illustrative example, the following specific values for inputs and the supply chain configuration are used:

- 2 products: $p = \{1,2\}$
- 2 transportation modes: $i = \{1,2\}$
    - $i = 1$ = air transportation with a lead time of 1 time period
    - $i = 2$ = ground transportation with a lead time of 2 time periods

- 5 suppliers and raw materials: $k = \{1,2,\ldots,5\}$
- 3 manufacturing lines at each manufacturer: $l = \{1,2,3\}$
- 2 manufacturing facilities: $m = \{1,2\}$
- 2 warehouses: $n = \{1,2\}$
- 6 retailers: $r = \{1,2,\ldots,6\}$
- 24 time periods: $t = \{1,2,\ldots,24\}$
    - 3 total objectives: $u = \{1,2,3\}$
- Product 1 has a raw material ratio of: 2:3:5:0:0
- Product 2 has a raw material ratio of: 0:0:1:4:2

This configuration results in the following model size:

- 17,432 variables
    - 10,904 continuous variables
    - 6,528 binary variables
- 34,376 constraints

A diagram of the supply chain network used in the illustrative example is presented in Figure 4.1.

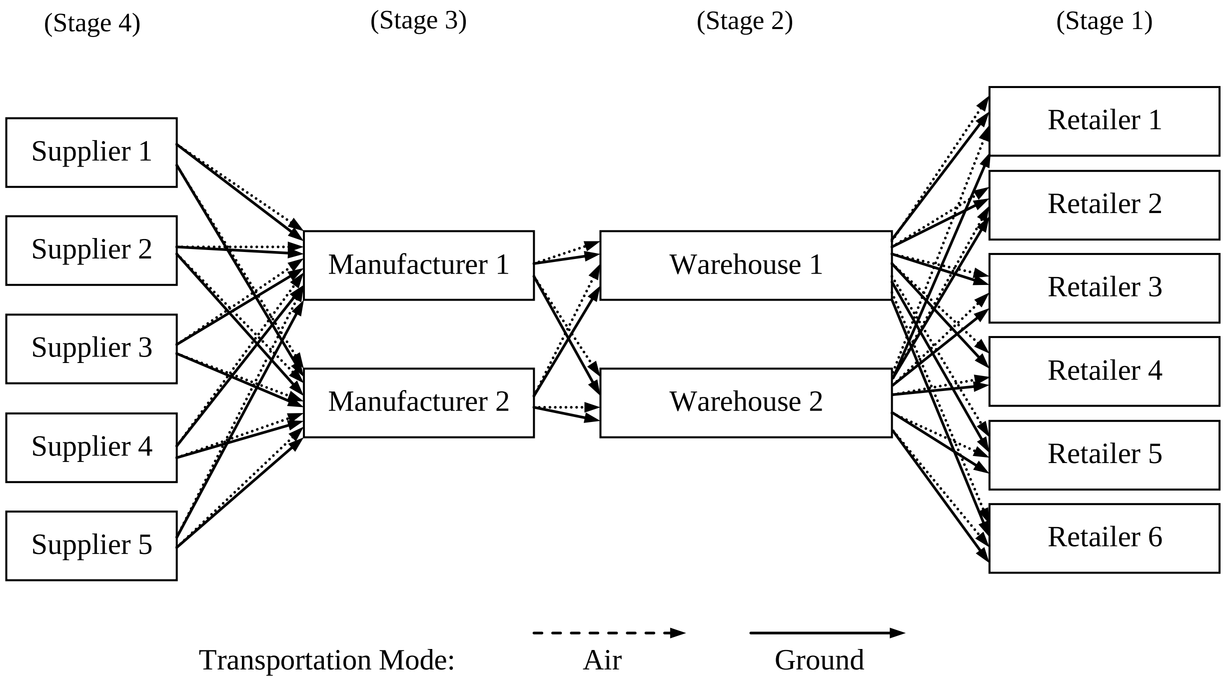



**Figure 4.1: The supply chain network for the illustrative example**

The following assumed inputs are used in the illustrative example:

**Table 4.1: Retailer demands for product 1**

| Time period | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Retailer 1 | 3000 | 2000 | 1000 | 1500 | 1800 | 2000 | 750 | 1300 | 1300 | 2200 | 4200 | 5000 |
| Retailer 2 | 3500 | 2500 | 1000 | 1200 | 2000 | 1500 | 1250 | 1300 | 1200 | 2000 | 4500 | 5500 |
| Retailer 3 | 3200 | 2000 | 1200 | 1300 | 1600 | 1700 | 1250 | 1000 | 1500 | 2000 | 4000 | 5200 |
| Retailer 4 | 3000 | 2200 | 1300 | 1000 | 1700 | 1700 | 800 | 1500 | 1250 | 2100 | 4300 | 5500 |
| Retailer 5 | 3100 | 2500 | 1500 | 1200 | 1500 | 1500 | 1000 | 1300 | 1000 | 2000 | 4400 | 5000 |
| Retailer 6 | 3000 | 1400 | 1400 | 1200 | 1450 | 1400 | 900 | 1200 | 1000 | 1900 | 4200 | 4800 |

| Time period | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Retailer 1 | 3000 | 2000 | 1000 | 1500 | 1800 | 2000 | 750 | 1300 | 1300 | 2200 | 4200 | 5000 |
| Retailer 2 | 3500 | 2500 | 1000 | 1200 | 2000 | 1500 | 1250 | 1300 | 1200 | 2000 | 4500 | 5500 |
| Retailer 3 | 3200 | 2000 | 1200 | 1300 | 1600 | 1700 | 1250 | 1000 | 1500 | 2000 | 4000 | 5200 |
| Retailer 4 | 3000 | 2200 | 1300 | 1000 | 1700 | 1700 | 800 | 1500 | 1250 | 2100 | 4300 | 5500 |
| Retailer 5 | 3100 | 2500 | 1500 | 1200 | 1500 | 1500 | 1000 | 1300 | 1000 | 2000 | 4400 | 5000 |
| Retailer 6 | 3000 | 1400 | 1400 | 1200 | 1450 | 1400 | 900 | 1200 | 1000 | 1900 | 4200 | 4800 |

**Table 4.2: Retailer demands for product 2**

| Time period | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Retailer 1 | 1800 | 1500 | 1650 | 1000 | 1200 | 1000 | 600 | 950 | 1000 | 1500 | 1800 | 2000 |
| Retailer 2 | 1700 | 1400 | 1800 | 900 | 1000 | 1200 | 650 | 1000 | 900 | 1100 | 1750 | 2100 |
| Retailer 3 | 1600 | 1500 | 1800 | 1200 | 1200 | 1100 | 500 | 900 | 950 | 1200 | 1900 | 2000 |
| Retailer 4 | 1500 | 1300 | 1600 | 1000 | 1100 | 1000 | 500 | 850 | 900 | 1400 | 1800 | 2100 |
| Retailer 5 | 1600 | 1200 | 1500 | 900 | 1000 | 800 | 400 | 800 | 800 | 1200 | 1800 | 2000 |
| Retailer 6 | 1500 | 1100 | 1400 | 900 | 1000 | 700 | 400 | 700 | 750 | 1100 | 1700 | 1900 |

| Time period | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Retailer 1 | 1800 | 1500 | 1650 | 1000 | 1200 | 1000 | 600 | 950 | 1000 | 1500 | 1800 | 2000 |
| Retailer 2 | 1700 | 1400 | 1800 | 900 | 1000 | 1200 | 650 | 1000 | 900 | 1100 | 1750 | 2100 |
| Retailer 3 | 1600 | 1500 | 1800 | 1200 | 1200 | 1100 | 500 | 900 | 950 | 1200 | 1900 | 2000 |
| Retailer 4 | 1500 | 1300 | 1600 | 1000 | 1100 | 1000 | 500 | 850 | 900 | 1400 | 1800 | 2100 |
| Retailer 5 | 1600 | 1200 | 1500 | 900 | 1000 | 800 | 400 | 800 | 800 | 1200 | 1800 | 2000 |
| Retailer 6 | 1500 | 1100 | 1400 | 900 | 1000 | 700 | 400 | 700 | 750 | 1100 | 1700 | 1900 |

**Table 4.3: Operating costs for each manufacturing line per period**

| Product ($p$) | Manufacturer ($m$) | Line 1 ($OPERC_{pm1}$) | Line 2 ($OPERC_{pm2}$) | Line 3 ($OPERC_{pm3}$) |
|---|---|---|---|---|
| 1 | 1 | $4,000 | $4,000 | $5,000 |
| | 2 | $4,500 | $4,000 | $5,000 |
| 2 | 1 | $3,000 | $3,500 | $4,000 |
| | 2 | $3,000 | $4,000 | $3,000 |

The production costs in this example are assumed to only depend on the product.

**Table 4.4: Per-unit production costs**

| Product ($p$) | Per-unit Production Cost ($PRODC_p$) |
|---|---|
| 1 | $10 |
| 2 | $15 |

**Table 4.5: Manufacturing production line capacities per period**

| Product ($p$) | Manufacturer ($m$) | Line 1 ($MCAP_{pm1}$) | Line 2 ($MCAP_{pm2}$) | Line 3 ($MCAP_{pm3}$) |
|---|---|---|---|---|
| 1 | 1 | 10000 | 7000 | 8000 |
| | 2 | 6000 | 5000 | 5000 |
| 2 | 1 | 10000 | 7000 | 8000 |
| | 2 | 5000 | 6000 | 6000 |

**Table 4.6: Raw material inventory holding costs per unit per period**

| Raw Material ($k$) | Cost ($INVCRM_k$) |
|---|---|
| 1 | $2.00 |
| 2 | $1.00 |
| 3 | $0.50 |
| 4 | $1.00 |
| 5 | $2.00 |

Each warehouse has a capacity of 40,000 units.

Inventory holding cost for product 1 = $5.00 per unit per time period

Inventory holding cost for product 2 = $10.00 per unit per time period

**Table 4.7: Raw material shipment quantity requirements**

| Transportation Mode ($i$) | Minimum Quantity ($MINRM_i$) | Maximum Quantity ($MAXRM_i$) |
|---|---|---|
| 1 | 1 | 100,000 |
| 2 | 1 | 100,000 |

**Table 4.8: Finished product shipment quantity requirements**

| Transportation Mode ($i$) | Minimum Quantity ($MINFP_i$) | Maximum Quantity ($MAXFP_i$) |
|---|---|---|
| 1 | 200 | 7,000 |
| 2 | 180 | 7,000 |

**Table 4.9: All-Units cost structure for shipping**

| Transportation Mode | Mode Index (Lead Time) ($i$) | Cost Bracket | Weight Range (CWT) | Cost | Cost Variable |
|---|---|---|---|---|---|
| Air | 1 | 1 | 0-50 | $35/CWT | $AIRC_1$ |
| | | 2 | 51-200 | $25/CWT | $AIRC_2$ |
| Ground | 2 | 1 | 0-100 | $20/CWT | $GROUND_1$ |
| | | 2 | 101-400 | $15/CWT | $GROUND_2$ |
| | | 3 | 401-800 | $10/CWT | $GROUND_3$ |

**Table 4.10: Raw material weights per unit**

| Raw Material ($k$) | Weight (CWT) ($RMW_k$) |
|---|---|
| 1 | 0.02 |
| 2 | 0.01 |
| 3 | 0.01 |
| 4 | 0.02 |
| 5 | 0.02 |

**Table 4.11: Finished product weights per unit**

| Finished Product ($p$) | Weight (CWT) ($FPW_p$) |
|---|---|
| 1 | 0.12 |
| 2 | 0.13 |

Selling price of product 1 = $100/unit

Selling price of product 2 = $150/unit

#### 4.1.1.1 Initialization of the model

It is unrealistic to assume that the supply chain being modeled in this thesis only functions from time period 1 to 24. A company with this supply chain is most likely already processing materials and finished products and is looking to improve its efficiency with optimization modeling. Therefore, initial quantities of raw materials at each manufacturer and initial quantities of finished products at each warehouse and retailer were used to initialize the model. It was assumed that the initial quantities of raw material and finished product at the retailer allowed for the demand in time period 1 to be satisfied. The initial quantities at the warehouse were sufficient for the satisfaction of the demand in the second, third, and fourth time periods with extra supply in some cases available for storage at the warehouse. This was because the first products produced, using the initial shipments of raw material scheduled by the model, do not reach the retailer until time period 5 because of the one period for manufacturing time and the minimum of three total periods of transportation time between each stage.

For example, if the fastest mode of transportation is used for all shipments, then the raw material shipments that are scheduled in time period 1 will not arrive at the manufacturer until time period 2. These raw materials are converted into finished products available for shipment from the manufacturer at the beginning of time period 3. These finished product shipments can then reach the warehouse at time period 4 and can then be shipped to the retailers to arrive at the beginning of time period 5.

It was assumed that any excess inventory from the initialization quantities represented the finished product inventory that already existed in the supply chain before the model was implemented. In addition, unless the company goes completely bankrupt and the supply chain ceases to exist after time period 24, the quantities in the last few time periods will most likely be inaccurate because they do not account for demand after the $24^{th}$ time period.

A solution to the complications of a fixed time horizon is to solve the model on a moving time horizon basis. With this option the initialization of the supply chain is still necessary for the first model run, but thereafter the initial quantities and input parameters of the supply chain, such as demand, can be updated based on previous results and the supply chain model can be run again. This process continues throughout the time period of interest. The moving time horizon approach is more realistic and useful in eliminating any bias or unrealistic initialization values, as well as giving a company the option to update inputs such as demand values that may become more accurate with forecasting as time progresses. In addition, supply chain models take several time periods to get product moving and therefore do not reach steady state immediately. The values of the variables when the supply chain is in steady state are usually the most accurate and realistic solution values. Therefore, the initialization time period is required but does not necessarily produce accurate variable values for the solution immediately. For the purpose of illustration and solution time, only one model run for time periods 1 through 24 was analyzed and the initial values were based solely on first period demand and demand averages. However the influence of the initial values and the immediate end of the time horizon are addressed in the discussion.

The initial values for raw materials and finished products at the retailer were derived based on the demand in time period 1. The total demand at all six retailers in time period 1 was divided in two, to assume that both manufacturing facilities share equal responsibility. Therefore, the initial raw material quantities only depend on the raw material, $k$, and were equal for each production facility. Since the total quantity of product 1 in period 1 at all of the retailers is 18,800, the initial quantity of product 1 assumed to be manufactured at each manufacturer was calculated to be 9,400 units. The following equation from the model was used to calculate the assumed initial values for raw materials for manufacturing each product.

$$RM_{k\,p\,m\,t} = r_{k\,p} * TFPM_{p\,m\,t} \quad \forall k, p, m, t$$

For example, the equation to calculate the amount of raw material 1 necessary to manufacture 9,400 units of product 1 is:

$$RM_{1\,1\,m\,t} = 2 * 9{,}400 = 18{,}800$$

The remaining initial raw materials quantities for manufacturing were calculated similarly and were as follows:

**Table 4.12: Initial raw material requirements to produce 9,400 units of product 1**

| Raw Material | Required Quantity |
|---|---|
| 1 | 18,800 |
| 2 | 28,200 |
| 3 | 47,000 |
| 4 | 0 |
| 5 | 0 |

**Table 4.13: Initial raw material requirements to produce 4,850 units of product 2**

| Raw Material | Required Quantity |
|---|---|
| 1 | 0 |
| 2 | 0 |
| 3 | 4,850 |
| 4 | 19,400 |
| 5 | 9,700 |

These quantities were added together to derive the total final initialization values:

**Table 4.14: Total raw material initialization values**

| Raw Material ($k$) | Initial Quantity ($INITRM_k$) |
|---|---|
| 1 | 18,800 |
| 2 | 28,200 |
| 3 | 51,850 |
| 4 | 19,400 |
| 5 | 9,700 |

The initial quantities of finished products at each warehouse were used to cover the demand for the second, third, and fourth time periods. The total demand of finished product 1 over those three time periods was 27,400 and the total demand for product 2 was 23,650.

These values were rounded up to 30,000 and assumed to be divided equally among the two warehouses. Therefore 15,000 units of each product were assumed to be available at each warehouse initially. Lastly, the initial quantities of finished products at each retailer were assumed to be equal to the demand so lost sales were not incurred at the end of the first time period. All subsequent time periods were solved by using the optimization model.

## 4.2 Solution Procedure and Results

The illustrative example model was solved using the General Algebraic Modeling System (GAMS) with a CPLEX processor. The problems were solved on a cluster of eight computers with a total of four AMD Opteron 8222 SE Dual-Core 3.0 GHz processors. The total memory available in the cluster was 128 GB and all problems solved in less than three minutes. First, the bi-criteria model, with Profit and Customer Responsiveness as objectives, was solved. Customer Responsiveness is measured in terms of Lost Sales in the model.

### 4.2.1 Bi-Criteria Example

To find the ideal solution, the problem was solved as a single objective optimization problem. Solving for just Maximize Profit produced *Solution 1*. For *Solution 1* the objective value was set equal to the ideal for objective 1, so the max profit is: $IDEAL_1$ = \$43,300,606. The maximization of profit for *Solution 1* resulted in a total number of lost sales of 19,285. Solving for just Minimize Lost Sales produced a total number of lost sales for *Solution 2* as $IDEAL_2$ = 2,008 lost sales. When Lost Sales was the only objective the total profit was \$42,023,284. These two solutions were used to set the bounds for each objective. The maximum profit was \$43,300,606, and the minimum profit was set to \$42,023,284.

Similarly, the minimum number of lost sales was 2,008, and the maximum number of lost sales was set to 19,285. The Ideal Solution to the bi-criteria problem, expressed as (*Obj1*, *Obj2*), was ($43300606, 2008).

In *Solution 1*, profit was the only objective included in the optimization. Therefore the profit was maximized regardless of lost sales. Consequently the number of lost sales neared 20,000. However, when the minimization of lost sales was the only objective, regardless of profit, the number of lost sales was minimized to 2,008. This required a sacrifice of profit as it decreases by over one million dollars from $43,300,606 to $42,023,384. This trade-off behavior is typical of multi-criteria problems when each objective is solved for individually. When both objectives are included at the same time in the multi-criteria model, the ideal solution could not be achievable if the two objectives conflicted with each other. Many multi-criteria problems have conflicting criteria so the Best Compromise Solution is identified as the optimal solution. The Best Compromise Solution is the efficient solution to the multi-criteria problem that maximizes the Decision Maker's Utility/Value function.

For the goal programming model, the bounds on each objective are used to determine the targets for each objective. In order to stay within the range of the profit values, the profit target was set to 98% of the Ideal value. Thus, $dec_1$ was set to 2 and,

$$TARGET_1 = \frac{(100 - dec_1)}{100} * IDEAL_1 = \frac{(100 - 2)}{100} * 43{,}300{,}606 = \$42{,}434{,}594$$

Similarly, objective 2 was relaxed by 2% also, so $inc_2$ = 2. (Note: Objective 2 is a minimization objective)

$$TARGET_2 = \frac{(100 + inc_1)}{100} * IDEAL_2 = \frac{(100 + 2)}{100} * 2{,}008 = 2{,}048$$

The deviational variables, $d_u^-$ and $d_u^+$, were used to represent the deviations between the actual objective value and its target in the goal programming models. The overall goal was then to minimize the total deviations from the goals, or targets. For the preemptive goal programming model, the objectives were assigned a priority.

The preemptive goal programs were solved for two cases for which the importance of one objective in relation to the other were as follows:

<u>Case 1</u>: Profit has a higher priority over Lost Sales

<u>Case 2</u>: Lost Sales have a higher priority over Profit

<u>Preemptive Goal Programming Solution to Case 1:</u>

The preemptive goal program was solved first for the two cases. The additional goal constraints in preemptive goal programming model were:

$$Obj_1 + d_1^- - d_1^+ = TARGET_1$$

$$Obj_2 + d_2^- - d_2^+ = TARGET_2$$

To solve for <u>Case 1</u> the objective function was:

$$Minimize\ Z = P_1(d_1^-) + P_2(d_2^+)$$

In this formulation, $d_1^-$ represented the amount that the profit deviated, or underachieved, its goal of $TARGET_1$. The deviational variable $d_2^+$ was equal to the number of lost sales that exceeded the goal for the minimum number of lost sales. The problem was solved iteratively so the following single objective problem was solved first.

$$Minimize\ Z = d_1^-$$

Subject to,

$$Obj_1 + d_1^- - d_1^+ = 42{,}434{,}594$$

$$Obj_2 + d_2^- - d_2^+ = 2{,}048$$

$$d_1^-, d_1^+, d_2^-, d_2^+ \geq 0$$

[All real constraints in the general model]

The objective values and deviational variables for the solution to the first single objective model for Case 1 in the preemptive model are summarized in Table 4.15.

**Table 4.15: First solution – Case 1: Preemptive model**

| | Value | $d_u^+$ | $d_u^-$ | $TARGET_u$ | $IDEAL_u$ |
|---|---|---|---|---|---|
| $Obj_1$: Max Profit | 42,434,594 | 0 | 0 | 42,434,594 | 43,300,606 |
| $Obj_2$: Min Lost Sales | 19,285 | 17,237 | 0 | 2,048 | 2,008 |

As expected, the profit goal was achieved, but the lost sales (19,285) were higher than the target value (2,048). The solution to this single objective problem was then implemented in the next single objective problem by adding a constraint that restricted the value for Objective 1 to be larger than or equal to $42,434,594. This ensured that the value of objective 1, the Profit, was not worsened from the preceding optimal value. This led to the derivation of the best compromise solution. The principle that the first goal is satisfied as far as possible before the second goal is implemented is imperative to preemptive goal programming. The best compromise solution will not take away from the previous achievements of the higher priority objectives. The second single objective problem for the preemptive goal programming solution was:

$$Minimize\ Z = d_2^+$$

Subject to,

$$\boldsymbol{Obj_1 \geq 42{,}434{,}594}$$

$$Obj_2 + d_2^- - d_2^+ = 2{,}048$$

$$d_2^-, d_2^+ \geq 0$$

[All real constraints in the general model]

The solution to this second single objective problem was the overall solution to the preemptive goal program and was labeled *Solution 3*. *Solution 3* was the optimal solution to the goal program and the objective and deviational variable values are presented in Table 4.16.

**Table 4.16: Final Solution 3 – Case 1: Preemptive model**

| | Value | $d_u^+$ | $d_u^-$ | $TARGET_u$ | $IDEAL_u$ |
|---|---|---|---|---|---|
| $Obj_1$: Max Profit | 42,875,386 | 440,792 | 0 | 42,434,594 | 43,300,606 |
| $Obj_2$: Min Lost Sales | 2,048 | 0 | 0 | 2,048 | 2,008 |

The progression of the solutions in this iterative approach to a bi-criteria model was logical. The first solution disregarded the lost sales and only optimized profit. Therefore, the target profit was achieved and the number of lost sales was far from its target. However, when the lost sales were added to the model objective, the number of lost sales went down and achieved the target value.

The deviational variables indicating a target was not met in this model are $d_1^-$ and $d_2^+$. Since both of these were 0, the targets for this model were met. However, $d_1^+$ became positive in the final solution for Case 1 in the preemptive model. This indicates that the profit exceeded the target, although it was still less than the ideal value. Since both targets were met with this model and profit exceeded its target, the solution may be a dominated solution. If it is, at least one objective can be improved upon without losing achievements in the other objectives. In addition, dominated solutions are not optimal for a model. Therefore, the targets for this model were reset to the ideal values, $IDEAL_u$, and the problem was solved again using the same solution procedure. The final results are presented in Table 4.17.

**Table 4.17: Final Solution 3 – Case 1: Preemptive model with IDEAL targets**

| | Value | $d_u^+$ | $d_u^-$ | $TARGET_u$ | $IDEAL_u$ |
|---|---|---|---|---|---|
| $Obj_1$: Max Profit | 43,300,606 | 0 | 0 | 43,300,606 | 43,300,606 |
| $Obj_2$: Min Lost Sales | 2,008 | 0 | 0 | 2,008 | 2,008 |

Ultimately, the ideal solution for the problem was achieved and is optimal. This is an indication that the two objectives in this model, maximize profit and minimize the number of lost sales, are not conflicting. This can most likely be attributed to the large profit margin since the individual product revenues are high. Therefore the ideal profit can still be met even when lost sales are limited. In this model profit is generated by selling enough product to counteract the total costs of the model. Therefore maximizing profit would involve minimizing the number of lost sales and so the ideal solution would be optimal. This solution is not a dominated solution because each objective was at its ideal, which is the optimal solution for each individual objective value. Therefore, neither objective can be improved upon. Ultimately, for the preemptive model for Case 1, the ideal solution was achievable and identified as the optimal solution. Because of this, the targets were set to the ideal values in the remaining models for the example.

<u>Preemptive Goal Programming Solution to Case 2:</u>

The same solution process with the ideal values for the targets for the preemptive goal programming was implemented for <u>Case 2</u>. The first single objective linear program was:

$$Minimize\ Z = d_2^+$$

Subject to,

$$Obj_1 + d_1^- - d_1^+ = 43{,}300{,}606$$

$$Obj_2 + d_2^- - d_2^+ = 2{,}008$$

$$d_1^-, d_1^+, d_2^-, d_2^+ \geq 0$$

[All real constraints in the general model]

The solution is summarized in Table 4.18.

**Table 4.18: First Solution – Case 2: Preemptive model with IDEAL targets**

| | Value | $d_u^+$ | $d_u^-$ | $TARGET_u$ | $IDEAL_u$ |
|---|---|---|---|---|---|
| $Obj_1$: Max Profit | 42,729,990 | 0 | 570,616 | 43,300,606 | 43,300,606 |
| $Obj_2$: Min Lost Sales | 2,008 | 0 | 0 | 2,008 | 2,008 |

This solution was used in the next single objective problem as follows:

$$Minimize\ Z = d_1^-$$

Subject to,

$$\boldsymbol{Obj_2 \leq 2{,}008}$$

$$Obj_1 + d_1^- - d_1^+ = 43{,}300{,}606$$

$$d_1^-, d_1^+ \geq 0$$

[All real constraints in the general model]

The solution to this second single objective problem was identical to the solution for the preemptive problem in Case 1. The solution for Case 2, which is again the ideal solution, is summarized in Table 4.17. This was expected since both targets were achievable and reversing priorities did not affect the optimal solution.

Non-Preemptive Goal Programming Solution

Non-preemptive goal programming requires a numerical weight to be assigned to the deviational variables to form the objective function so the model can be formulated as a single objective problem. However, the use of numerical weights requires that the objectives are scaled properly. We scaled the objectives using Ideal values. Since the targets in this problem were set equal to the ideal values, the right side of the equation became 1. The following goal constraints were used in the non-preemptive goal program:

$$\frac{Obj_1}{IDEAL_1} + d_1^- - d_1^+ = \frac{TARGET_1}{IDEAL_1} = 1$$

$$\frac{Obj_2}{IDEAL_2} + d_2^- - d_2^+ = \frac{TARGET_2}{IDEAL_2} = 1$$

The non-preemptive goal programming objective was:

$$Minimize\ Z = w_1(d_1^-) + w_2(d_2^+)$$

For Case 1 the weights for the objectives were $w_1 = 5$ and $w_2 = 1$ indicating that $Obj_1$, or Profit, is five times as important as $Obj_2$, which is Lost Sales. Therefore the Case 1 goal program became:

$$Minimize\ Z = (5 * d_1^-) + (d_2^+)$$

Subject to,

$$\frac{Obj_1}{43{,}300{,}606} + d_1^- - d_1^+ = 1$$

$$\frac{Obj_2}{2{,}008} + d_2^- - d_2^+ = 1$$

$$d_1^-, d_1^+, d_2^-, d_2^+ \geq 0$$

[All real constraints in the general model]

The solution to the goal program with the preceding objective was the same as the solution to the preemptive solution for Case 1 and is summarized in Table 4.17. Once again, this was expected since both targets were achievable.

In Case 2 the weights were reversed and $w_1$ was 1 while $w_2$ was 5. The goal programming problem for this case was:

$$Minimize\ Z = (d_1^-) + (5 * d_2^+)$$

Subject to, the goal constraints and real constraints as before.

The optimal solution was the same as both targets were achievable and changing the weights had no impact on the optimal solution.

**Discussion of solutions for Case 1 and Case 2 using IDEAL values for targets**

Case 1 and Case 2 both resulted in the achievement of the ideal solution as the optimal solution using the preemptive approach. However, the progression of the solutions differed. The first solution for Case 1 allowed the profit, which was the prioritized objective, to reach its ideal value. In this first solution, the number of lost sales was large. Then, when the second objective was added to the model, the number of lost sales decreased to its ideal value and profit remained optimal. The opposite progression occurred when Case 2 was solved. In Case 2, lost sales were prioritized over profit. In this case, the first solution optimized just objective 2 to 2,008 lost sales and profit was less than its ideal. Then objective 1 was added to the model and the profit was maximized to its ideal, thereby resulting in the overall ideal solution.

In the non-preemptive model the same final solution was derived for Cases 1 and 2 using the weights 5 and 1, depending on which objective had priority. It was found that the same solution was derived when the weights are 2 and 1 as well.

Since the ideal solution was feasible, the two objectives, Profit and Lost Sales, are not completely conflicting. Neither of them had to be compromised to reach optimality. Hence priority ranking of the objectives in the preemptive case and the choice of weights in the non-preemptive case did not make any difference on the final optimal solution. Since revenue is generated in this model by selling as many products as possible, it is not surprising that minimizing the number of lost sales would also maximize profit. A third objective, which wants to minimize inventory capital, was then added to the model and the solutions were derived with a similar solution process.

### 4.2.2 Multi-Criteria Model

The results for all three objectives for *Solution 1*, where Profit was the only objective and *Solution 2*, where Customer Responsiveness was the only objective, are presented in Table 4.19. *Solution 4* was identified as the solution to the problem when Inventory Capital was the single objective. The objective values that resulted when the objectives were solved individually are included in Table 4.19.

**Table 4.19: Results for all objectives for single objective models: *Solutions 1, 2* and *4***

| | *Solution 1* | *Solution 2* | *Solution 4* |
|---|---|---|---|
| *(max)* $Obj_1$ | **$43,300,606** | $42,023,284 | $17,614,024 |
| *(min)* $Obj_2$ | 19,285 | **2,008** | 313,120 |
| *(min)* $Obj_3$ | $50,984,172 | $81,583,502 | **$47,732,917** |

The ideal solution, in bold face, for the three objectives was ($43300606, 2008, $47732917). A graphical representation of the data in Table 4.19 is presented in Figure 4.2.

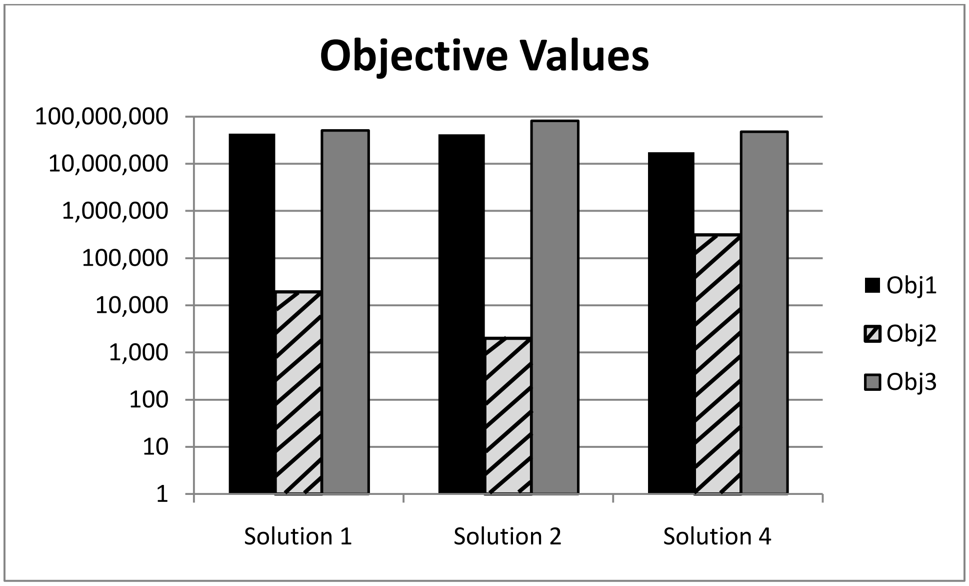



**Figure 4.2: Graphical representation of results for all objectives for *Solutions 1, 2,* and *4***

For all three objectives, the ideal value was set as the target. Therefore, $IDEAL_3$ was set to $47,732,917. The bounds on the objectives, which were the overall maximum or minimum values from *Solutions 1*, *2*, and *4*, were: *MINObj1* = $17,614,024, *MAXObj2* = 313,120, *MAXObj3* = $81,583,502. Additionally, the objective expressions in the general model, including the expression for $Obj_3$ were written as goal constraints in the model and set equal to the variables $Obj_1$, $Obj_2$, $Obj_3$.

**Discussion of solutions for single objective problems: *Solutions 1, 2,* and *4***

It can be noted that the bounds on the objectives were relaxed in this model compared to the two-objective model. This was a result of the solution when Inventory Capital was the only objective. Minimizing Inventory Capital resulted in more lost sales because there was less product available in inventory to compensate for unexpected spikes in the demand. In the solution for minimizing the inventory capital, there were not any shipments of raw material from the suppliers. Therefore, the only finished products manufactured or shipped in this model were a direct result of the initialization values. This minimized the inventory quantities and inventory capital. Although inventory holding costs were limited, profit was lowest in this solution because the revenues were significantly decreased due to a lack of product at the retailers. This resulted in the maximum number of lost sales across the three solutions as well. The inventory level progression across the time horizon for each solution is summarized in Figure 4.3.

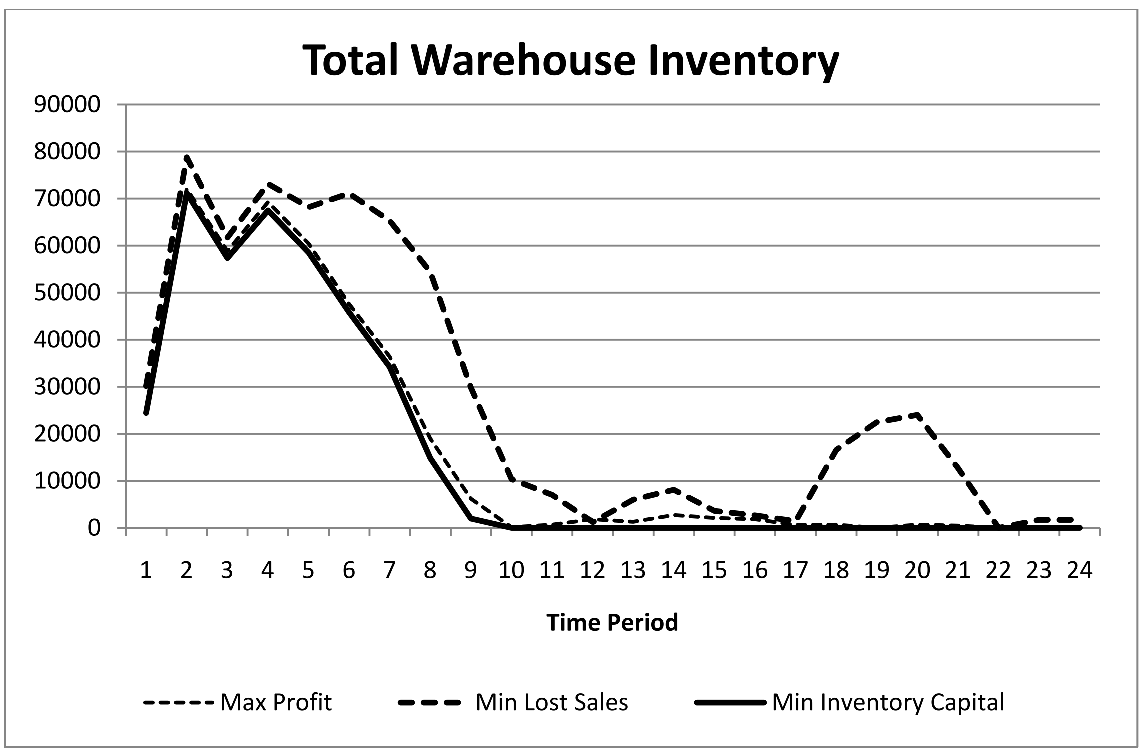



**Figure 4.3: Total warehouse inventory levels when objectives were solved individually**

Figure 4.3 shows the total inventory maintained throughout the model in which lost sales were minimized (*Solution 2*), exceeds the inventory in the other two solutions. Additionally, when inventory capital was minimized (*Solution 3*), the total warehouse inventory levels were the lowest compared to the other solutions. When profit was maximized (*Solution 1*), the total inventory levels were similar to those when inventory capital was minimized. This can be attributed to the inclusion of an inventory holding cost and not a cost for lost sales in the model in the profit equation.

When three objectives were considered, just one case was analyzed. The optimal solutions found so far have all utilized the warehouse inventory frequently. Therefore we will examine the changes in the solutions when Inventory Capital is included as an objective and prioritized over the other two objectives. The following case indicates the importance of each objective over the others.

Case 3: Inventory Capital > Profit > Lost Sales

The objective for Case 3 was represented as Scenario 5 in Chapter 3 and is:

$$Minimize\ Z = P_1(d_3^+) + P_2(d_1^-) + P_3(d_2^+)$$

Preemptive Goal Programming:

In the preemptive goal programming problem, Case 3 was implemented by solving three single objective problems. The first single objective only minimized the deviation from the third objective, Inventory Capital, as follows:

$$Minimize\ Z = d_3^+$$

Subject to,

$$Obj_1 + d_1^- - d_1^+ = 43{,}300{,}606$$

$$Obj_2 + d_2^- - d_2^+ = 2{,}008$$

$$Obj_3 + d_3^- - d_3^+ = 47{,}732{,}917$$

$$d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0$$

[All real constraints in the general model]

The solution to the first single objective problem in the preemptive goal programming is summarized in Table 4.20.

**Table 4.20: First solution – Case 3: Preemptive model with IDEAL targets**

| | Value | $d_u^+$ | $d_u^-$ | $TARGET_u$ |
|---|---|---|---|---|
| $Obj_1$: Max Profit | 17,613,210 | 0 | 25,686,396 | 43,300,606 |
| $Obj_2$: Min Lost Sales | 312,777 | 310,769 | 0 | 2,008 |
| $Obj_3$: Min Inventory Capital | 47,732,917 | 0 | 0 | 47,732,917 |

The ideal solution for objective three was achieved in the first problem and the objective value of the first single objective program was implemented as a constraint in the second single objective program as follows:

$$Minimize\ Z = d_1^-$$

Subject to,

$$\boldsymbol{Obj_3 \leq 47{,}732{,}917}$$

$$Obj_1 + d_1^- - d_1^+ = 43{,}300{,}606$$

$$Obj_2 + d_2^- - d_2^+ = 2{,}008$$

$$d_1^-, d_1^+, d_2^-, d_2^+ \geq 0$$

[All real constraints in the general model]

The solution to the second single objective problem in the preemptive goal program is summarized in Table 4.21.

**Table 4.21: Second solution – Case 3: Preemptive model with IDEAL targets**

| | Value | $d_u^+$ | $d_u^-$ | $TARGET_u$ |
|---|---|---|---|---|
| $Obj_1$: Max Profit | 43,300,606 | 0 | 0 | 43,300,606 |
| $Obj_2$: Min Lost Sales | 16,690 | 14,682 | 0 | 2,008 |
| $Obj_3$: Min Inventory Capital | 47,732,917 | 0 | 0 | 47,732,917 |

The ideal solutions for objectives 1 and 3 were achieved. The solution from this previous problem was implemented as a constraint in the third, and last, single objective problem for which the deviational variable for lost sales was minimized.

$$Minimize\ Z = d_2^+$$

Subject to,

$$\boldsymbol{Obj_3} \leq \mathbf{47{,}732{,}917}$$

$$\boldsymbol{Obj_1} \geq \mathbf{43{,}300{,}606}$$

$$Obj_2 + d_2^- - d_2^+ = 2{,}008$$

$$d_2^-, d_2^+ \geq 0$$

[All real constraints in the general model]

The resulting solution was *Solution 5* and is presented in Table 4.22.

**Table 4.22: Final Solution 5 – Case 3: Preemptive model with IDEAL targets**

| | Value | $d_u^+$ | $d_u^-$ | $TARGET_u$ |
|---|---|---|---|---|
| $Obj_1$: Max Profit | 43,300,606 | 0 | 0 | 43,300,606 |
| $Obj_2$: Min Lost Sales | 2,129 | 121 | 0 | 2,008 |
| $Obj_3$: Min Inventory Capital | 47,732,917 | 0 | 0 | 47,732,917 |

The progression of the objective values is displayed in Figure 4.4. As each objective was added to the model, the objective value improved. The initial solution for the inventory in capital remained unchanged as the first and second objectives were added to the model. Initially profit was lower than the ideal value but once included as part of the objective, it reached the ideal level. Lastly, the number of lost sales was very large initially, but it continued to decrease as the preemptive model progressed. Ultimately, the number of lost sales came closer to the ideal value of 2,008, but did not achieve the ideal. It was not possible to achieve all three ideal values simultaneously therefore the ideal solution for Case 3 was not feasible. The best compromise solution was derived with the preemptive case and the smallest number of lost sales possible was 2,129 while the ideal solutions were achieved for objectives 1 and 3, at $43,300,606 and $47,732,917 respectively.

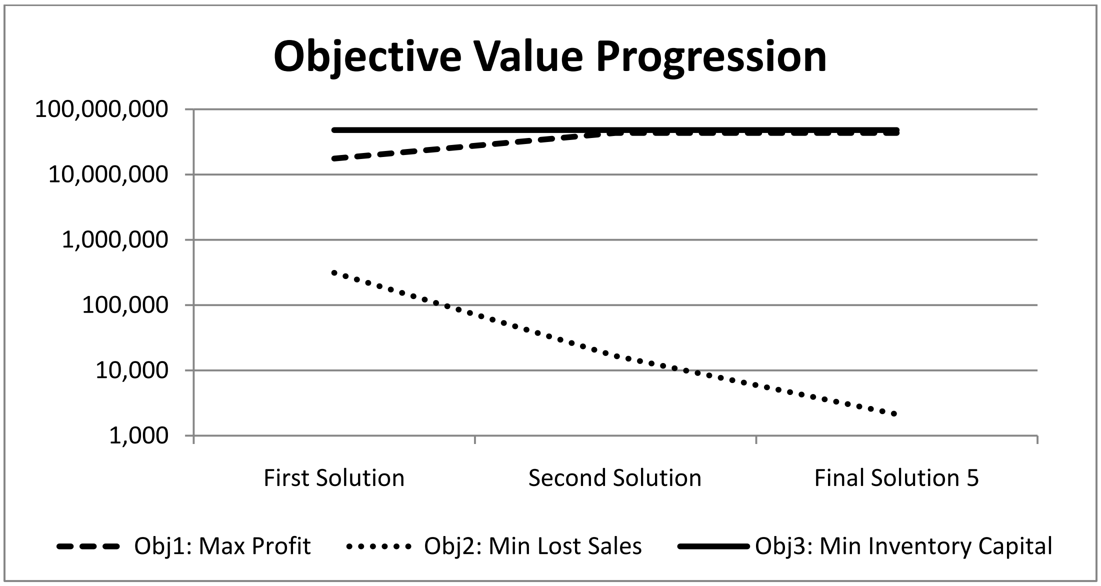



**Figure 4.4: Progression of objective values for Case 3 – Preemptive Model**

Non-Preemptive Goal Programming

In the non-preemptive goal programming problem, the first set of weights used in the objective was: $w_1 = 2$, $w_2 = 1$, $w_3 = 4$. These weights indicated that Profit, $Obj_1$, was two times as important as Lost Sales, $Obj_2$. Additionally, Inventory Capital, which was $Obj_3$, was twice as important as Profit, and consequently four times as important as Lost Sales. Since the target was set to the ideal value, the following goal constraint was necessary for objective 3:

$$\frac{Obj_3}{IDEAL_3} + d_3^- - d_3^+ = \frac{TARGET_3}{IDEAL_3} = 1$$

The objective for the non-preemptive program with three objectives was:

$$Minimize\ Z = w_1(d_1^-) + w_2(d_2^+) + w_3(d_3^+)$$

The following problem formulation was implemented for this scenario:

$$Minimize\ Z = 2(d_1^-) + (d_2^+) + 4(d_3^+)$$

Subject to,

$$\frac{Obj_1}{43{,}300{,}606} + d_1^- - d_1^+ = 1$$

$$\frac{Obj_2}{2{,}008} + d_2^- - d_2^+ = 1$$

$$\frac{Obj_3}{47{,}732{,}917} + d_3^- - d_3^+ = 1$$

$$d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0$$

[All real constraints in the general model]

The final solution to the non-preemptive problem was slightly different than the optimal solution obtained in the preemptive case. The final solution to the non-preemptive model for Case 3 with the corresponding objective weights was identified as *Solution 6* and is summarized in Table 4.23. The solution to the non-preemptive model for Case 3 underachieved the targets for both Objective 2 and Objective 3. However, profit was maximized to its ideal value.

**Table 4.23: Final Solution 6 – Case 3: Non-Preemptive model with IDEAL targets**

| | Objective Weight ($w_u$) | Value | $d_u^+$ | $d_u^-$ | $TARGET_u$ |
|---|---|---|---|---|---|
| $Obj_1$: Max Profit | 2 | 43,300,606 | 0 | 0 | 43,300,606 |
| $Obj_2$: Min Lost Sales | 1 | 2,119 | 0.0554 | 0 | 2,008 |
| $Obj_3$: Min Inventory Capital | 4 | 47,788,462 | 0.0012 | 0 | 47,732,917 |

**Discussion of solutions for Case 3 using IDEAL values for targets**

The solution to the non-preemptive model was different than the solution to the preemptive model; however both models achieved the ideal value for the first objective. Although Objective 3, the minimization of inventory capital, had the largest objective weight, it did not reach its ideal value in the optimal solution. The value of $d_3^+$ was 0.0012 indicating the value of objective 3 was 0.12% larger than the ideal value. Additionally, the optimal value for lost sales exceeded the ideal value by 5.54%.

It is often challenging to match the solution of the preemptive goal program with the solution to a non-preemptive goal program. This is because the determination of the objective weights is challenging and can produce varying results. Table 4.24 summarizes the preemptive solution for Case 3 and several non-preemptive solutions for Case 3 with different objective weights. The weights for the non-preemptive solutions are summarized as ($w_1$, $w_2$, $w_3$) in the table.

**Table 4.24: Final Solutions for Case 3**

| | | | Non-Preemptive Solutions | | |
|---|---|---|---|---|---|
| | *TARGET* Value | Preemptive Solution | (2,1,3) | (2,1,4) | (5,1,10) |
| $Obj_1$ | 43,300,606 | 43,300,606 | 43,300,606 | 43,300,606 | 43,300,606 |
| $Obj_2$ | 2,008 | 2,129 | 2,008 | 2,119 | 2,129 |
| $Obj_3$ | 47,732,917 | 47,732,917 | 48,578,200 | 47,788,462 | 47,732,917 |
| $d_1^+$ | - | 0 | 0 | 0 | 0 |
| $d_1^-$ | - | 0 | 0 | 0 | 0 |
| $d_2^+$ | - | 121 | 0 | 0.0554 | 0.0602 |
| $d_2^-$ | - | 0 | 0 | 0 | 0 |
| $d_3^+$ | - | 0 | 0.0177 | 0.0012 | 0 |
| $d_3^-$ | - | 0 | 0 | 0 | 0 |

The difficulty in choosing weights accurately caused differences in the optimal solutions for the non-preemptive model. As shown in Table 4.24, different weight combinations for the objectives produced different solutions. The ideal solution for objective 3, the objective with the highest weight, was not achieved with the objective weight combinations (2, 1, 3) or (2, 1, 4). However, it was achieved when the weight combination was (5, 1, 10). This combination produced the same solution as the preemptive solution for Case 3. Therefore, the weights of the prioritized objectives have to exceed the weight of the least important objective by a significant amount in this model to derive the same solution as an iterative approach to the implementation of objective priorities.

It is also observed that the number of lost sales achieved the ideal value if a sacrifice on inventory capital was allowed to be incurred. In the solution with the (2, 1, 4) weight combination, the only objective that achieved the ideal value was objective 1, while the other two objectives underachieved their goals by a small percentage. Although the non-preemptive approach may be faster, since just one single objective problem needs to be solved, the derivation of appropriate weights is a difficult task. Not only does the Decision Maker (DM) have to quantify their preference for each objective, but the determination of an appropriate range or scale for the weights is ambiguous. In the preemptive case, the DM just had to choose which objectives had priority over the others and ordinal scaling is used. This is usually easier than attempting to quantify a preference. It can be noted that none of the solutions presented in Table 4.24 were dominated solutions because each time a single objective was improved a sacrifice was made on the achievement of another objective.

Since the total inventory capital was the third objective being added to the model, the total warehouse inventory levels for the bi-criteria model were compared to the multiple-criteria model with three objectives. *Solution 3* was the optimal solution for the preemptive and non-preemptive models for Cases 1 and 2. It is compared to *Solution 5*, which was the optimal solution for Case 3 for which the preemptive and non-preemptive cases matched. This required that the non-preemptive weights were $w_1 = 5$, $w_2 = 1$, $w_3 = 10$. As displayed in Figure 4.5, the total inventory was consistently lower when the inventory capital objective was added to the model and was the most prioritized objective.

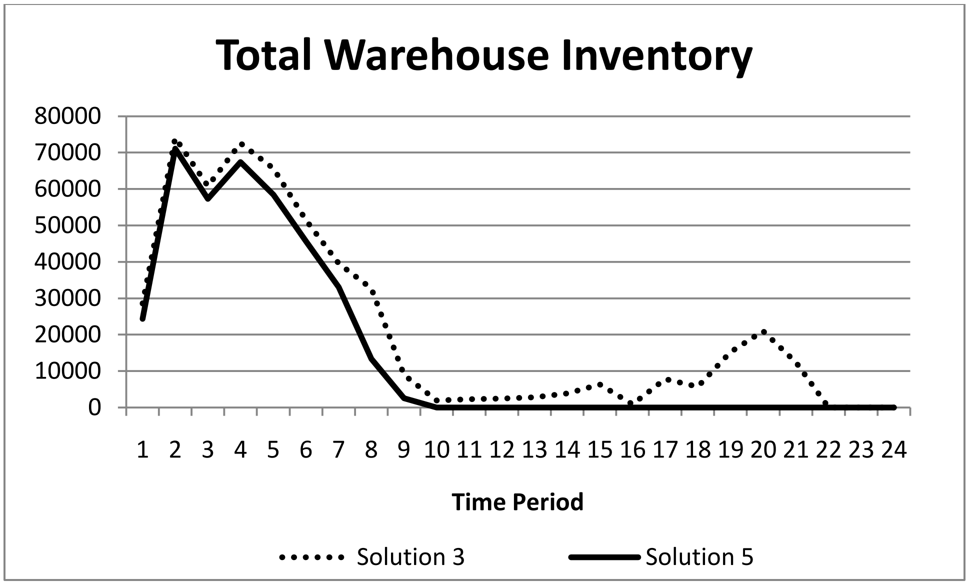



**Figure 4.5: Inventory levels for bi-criteria solution and multi-criteria solution**

It is important to note the total warehouse inventory levels were significantly larger in the first few time periods and then diminish significantly in both solutions. Additionally, the inventory level after time period 22 is zero for both models. These results can most likely be attributed to the effects of the initialization of the model and the lack of demand after time period 24. As discussed previously, these skewed results could be remedied with a moving time horizon solution process.

The values of all of the variables defined in the model are part of the solution when the model is solved. The variable values define the production and shipping schedule, which modes and cost brackets to use for transportation, and the inventory and product quantities at each stage of the supply chain. With all of these values a company can identify the exact schedules and quantities necessary to obtain the objective values desired.

# Chapter 5 Conclusion

Supply Chain Management (SCM) is used to develop and utilize supply chain models to maximize the efficiency of supply chains. Accurate supply chain modeling requires consideration of all aspects of the production and distribution of a product. The integration of inventory and transportation decisions in one model is not covered extensively in the literature. In addition, multiple production lines at manufacturing facilities and multiple products flowing along a supply chain are aspects of real-world supply chains that are not the focus of many existing supply chain models. In this thesis, a model that aims to reflect a real-world supply chain accurately and extensively is developed.

The model was for a two product, four-stage, centralized supply chain. Each stage had more than one entity, as well as three available production lines at each manufacturing facilities. Multiple modes of transportation were available between each stage and a freight rate function that reflected the all-units quantity discount cost structure on shipments was included in the model. The model considered transportation and inventory decisions, as well as manufacturing decisions related to the utilization of production lines. The supply chain model was used to determine the appropriate production and distribution quantities and the timing of production and shipments that optimize the objectives of the model.

The goals implemented in the model were the maximization of profit, maximization of customer responsiveness, and minimization of the capital in inventory. Multi-Criteria mathematical programming was used to either optimize a bi-criteria model or to optimize all three criteria simultaneously. In multi-criteria models, the criteria are often conflicting, so the best compromise solution which best satisfies all of the objectives while satisfying the constraints of the model must be identified.

The methods implemented to solve the mathematical programs developed for the model were variations of goal programming. In goal programming, a goal for each objective is defined. Then the total deviation from all of the goals is minimized. Preemptive and non-preemptive goal programming were used to solve the multi-criteria models developed in this thesis. In the preemptive problems, the priority of each objective compared to the others was specified. Then single objective models were solved iteratively to arrive at the final optimal solution. On the contrary, in the non-preemptive problems, weights are assigned to each objective and a single objective model was solved. The weights reflected the preference of each objective and how much it was to be prioritized over the others. Weights in non-preemptive programming are difficult to derive since preference information is hard to quantify. In addition, the determination of the range and scale of the weights for multi-criteria problems is ambiguous.

Three different cases were modeled preemptively and non-preemptively. The first two cases were applied to the bi-criteria model in which one criterion was prioritized over the other. For example, in the first case, profit was prioritized over customer service, whereas in the second case, customer service was prioritized over profit. The third case identified preferences among three criteria for the multi-criteria problem. In case 3, the amount of capital in inventory was the highest priority objective, then profit, and lastly customer responsiveness. The problems in this thesis were solved using GAMS (General Algebraic Modeling System) optimization software. The optimal solutions to the models were derived and key aspects of the solutions were identified.

In the bi-criteria models, the ideal solution was achievable with preemptive and non-preemptive goal programming for cases 1 and 2. This indicated that the two objectives, maximize profit and maximize customer responsiveness, were not completely conflicting. The optimal objective value identified when each criteria was solved independently was defined as the ideal value for the objective. In the ideal solution, each criterion was equal to its ideal value even though both criteria were included in the model indicating the objectives were not conflicting. The ideal solution was not feasible for the model that incorporated all three objectives though. In this solution, the objectives for profit and inventory in capital reached were the two prioritized objectives and they achieved their ideal values. However, customer responsiveness was identified as the least important objective and did not equal its ideal value. This indicated that there were conflicts among the ideal solutions for each objective when solved individually so when all three were incorporated into one model, some sacrifices were necessary to reach optimality.

The third objective that was added to form the multi-criteria model was the minimization of capital in inventory which reflected the maximization of customer responsiveness. It was observed that the addition and prioritization of this objective led to overall lower inventory levels. Even with minimal inventory, the profit was still maximized. However a sacrifice to the customer responsiveness was made, resulting in more lost sales. In supply chain management, it is common to observe increased lost sales as a result of minimal inventory levels since excess inventory, or safety stock, is not available to compensate for unexpected spikes in demand.

The complete solutions to the model identify when and where to ship materials and finished products between each stage of the supply chain and which transportation modes to use for the shipments. Additionally, the scheduling of the production lines to a particular product during each time period is specified. These solutions, along with the objective values for the model developed in this thesis, can be a useful supply chain management tool. The efficient operation of the supply chain can lead to increased profits and savings, as well as the elimination of excessive production or spending. Ultimately, the goal is to use the model developed as a management tool to optimize a supply chain so it operates efficiently. The model developed in this thesis extends existing models by adding a second product and more entities at each stage of the supply chain in addition to solving the model with multiple criteria to more realistically depict a real-world supply chain. By extending existing models in literature, this model provides a better fit for real-world supply chains, and therefore provides a better supply chain management tool to derive a complete solution to a realistic supply chain problem.

Despite the extensions and additions to this model, there are several ways the model can be extended further to enhance it or create an even more realistic representation of an existing real-world supply chain. For example, inflation rate can be added to the model to reflect the progression of prices and costs over time. Although this thesis extends previous models by adding facilities at each stage of the model, further extension of this concept could result in a direct replication of a real-world supply chain since they typically include many more suppliers, manufacturers, warehouses, and retailers. Another attempt to accurately model a realistic supply chain would be to generalize the model for 'n' products, and 'm' transportation modes. The manufacturing and lead times are fixed in this model whereas realistically the manufacturing times may depend on the product and multiple lead times could be available for each mode of transportation (i.e. 2-day and 5-day ground shipping).

On this same note, the model requires that products are shipped from the manufacturer as soon as they complete manufacturing. However, realistically the time period may be set as small as several hours and shipments may be aggregated throughout a day before a shipment goes out. Therefore revisions on product flow constraints at the manufacturer would be required.

Lastly, goal programming was used to solve the multi-criteria aspect of the model. Alternative solution methods for goal programming are available such as the Partitioning Algorithm and the Simplex Method for goal programming. Goal programming requires that all preference information is specified before the solution process. Other multi-criteria methods such as Compromise Programming and the STEM/STEP method utilize preference information differently. All of these methods could be used to solve the multi-criteria model developed in this thesis a different way.

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