ELEMENTARY SCHOOL TEACHERS’ CONCEPTIONS OF
THE COMMON CORE STATE STANDARDS FOR MATHEMATICAL PRACTICE

A Dissertation in
Curriculum and Instruction

by
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ABSTRACT

Mathematics education in K-12 schools has focused increasingly on the development of standards for students’ learning of mathematics for the past three decades. The Common Core State Standards for Mathematics (CCSSM) define a level of quality regarding what K-12 students should know and do in mathematics. In addition to Standards for Mathematical Content, the CCSSM explicitly delineate the Standards for Mathematical Practice (SMP) that mathematics teachers should nurture in all students (NGA & CCSSO, 2010a). In the midst of widespread recognition of and attention to the CCSSM, however, questions still exist relative to teachers’ conceptions of the SMP.

Pennsylvania is one of the states that adopted the CCSSM and crafted its own versions of the standards (The PA Core State Standards). Pennsylvania teachers are expected to have a clear understanding of the PA Core Standards. It is timely and appropriate to study Pennsylvania teachers’ conceptions, as the Standards have been adopted and implemented for several years since the revision and enactment of the PA Core Standards (2014). This study examined how eight western Pennsylvania elementary school teachers displayed their understandings about the SMP.

To this end, I conducted an in-depth interview with each teacher. The in-depth interviews featured the degree of alignment of the teachers’ conceptions of the eight SMPs. The participating teachers exhibited varying degrees of conceptions aligned with the SMP. Of particular interest, the teachers’ interpretations of SMP 4 (“Model with mathematics”) was markedly different from the descriptions provided by the Standard’s authors. The findings have implications for school teachers, mathematics educators, professional development providers, and standard writers.
TABLE OF CONTENTS

LIST OF FIGURES ........................................................................................................... vii

LIST OF TABLES .............................................................................................................. viii

ACKNOWLEDGEMENTS ................................................................................................. ix

Chapter 1  Introduction to the Study .............................................................................. 1

Rationale of the study ....................................................................................................... 3
Research question .............................................................................................................. 8
Definitions of terms .......................................................................................................... 9
  Definitions of standard and standards ........................................................................... 9
  Definitions of beliefs, knowledge, and conception ....................................................... 10
  Productive beliefs ......................................................................................................... 11
Summary ............................................................................................................................ 13

Chapter 2  Literature Review ......................................................................................... 14

The Common Core State Standards .............................................................................. 15
  The Common Core State Standards for Mathematics ............................................. 18
  The standards for mathematical content .................................................................. 19
  The standards for mathematical practice ................................................................ 21
  The Pennsylvania Core Standards for Mathematics .............................................. 22
Literature on the Common Core State Standards for Mathematics ........... 26
Mathematical standards .................................................................................................. 30
  NCTM standards ....................................................................................................... 32
  Literature on the standards-based/reform-based instruction .................................. 35
  Adding It Up ............................................................................................................... 36
  Mathematical Habits of Mind .................................................................................... 39
Standards in other subjects ............................................................................................. 42
  The Common Core State Standards for English Language Arts and Literacy ....... 42
  The Next Generation Science Standards ................................................................. 43
Teacher conceptions matter ......................................................................................... 44
  Teacher beliefs in mathematics education ............................................................... 46
  Teacher knowledge in mathematics education ....................................................... 50
  Teacher conceptions in the reform movement ....................................................... 52
Summary ............................................................................................................................ 55

Chapter 3  Methodology ................................................................................................. 56

Statement of the problem ............................................................................................... 56
Research design ............................................................................................................... 57
Participants..................................................................................................................... 59
Data sources and collection .......................................................................................... 60
  Triangulation ............................................................................................................ 61
  Interviews .................................................................................................................. 62
References ........................................................................................................................................ 151

Appendix A Descriptions of the Standards for Mathematical Practice ........................................ 166

Appendix B Interview Protocol .................................................................................................... 170

Appendix C Summary Notes of Transcriptions Example ............................................................. 175

Appendix D Example of “Kid-Friendly SMP” (1) ....................................................................... 178

Appendix E Example of “Kid-Friendly SMP” (2) ....................................................................... 180

Appendix F Example of “Kid-Friendly SMP” (3) ....................................................................... 181
# LIST OF FIGURES

| Figure 1-1: | Teaching model | 4 |
| Figure 1-2: | Definitions/descriptions of terms | 10 |
| Figure 1-3: | Revised definitions of conception | 11 |
| Figure 1-4: | Beliefs about teaching and learning mathematics | 11 |
| Figure 2-1: | How to read the grade level standards: Grade 3 example | 20 |
| Figure 2-2: | How to read the grade level standards: Grade 5 example | 21 |
| Figure 2-3: | Eight standards for mathematical practice | 22 |
| Figure 2-4: | PA Core Standards for mathematical content and mathematical practice | 24 |
| Figure 2-5: | PA Core mathematical standards: development and progress | 25 |
| Figure 2-6: | SMP grade level emphasis | 26 |
| Figure 2-7: | Historical movements in mathematics education | 31 |
| Figure 2-8: | Comparison of the NCTM process standards and the SMP | 34 |
| Figure 2-9: | Comparison of the SMP and the five strands for the mathematical proficiencies | 38 |
| Figure 2-10: | Comparison of the SMP and the Habits of Mind | 41 |
| Figure 2-11: | Raymond’s model: relationship between mathematics beliefs and teaching practice | 47 |
| Figure 2-12: | Ernest’s model: relationship between mathematics beliefs and teaching practice | 48 |
| Figure 2-13: | Mathematical knowledge for teaching | 51 |
| Figure 3-1: | Verbatim transcription example | 67 |
| Figure 3-2: | Summary note example | 70 |
| Figure 4-1: | Grace’s function machine | 105 |
| Figure 4-2: | Cecilia’s placeholder for double-digit multiplication | 121 |
| Figure 5-1: | Three kid-friendly versions of SMP 4 | 141 |
LIST OF TABLES

Table 4-1: Profiles of the interviewed elementary school teachers.............................. 76
Table 4-2: Teachers’ overarching goals and the SMPs...................................................... 83
Table 4-3: Various ways to become familiar with the SMP.............................................. 87
Table 4-4: Individual teacher’s closely aligned conceptions of specific SMPs ................. 107
Table 4-5: Compiled data of teachers’ closely aligned conceptions of specific SMPs ........ 108
Table 4-6: Pareto bar chart of compiled data of teachers’ closely aligned conceptions of specific SMPs ................................................................. 109
Table 4-7: Individual teacher’s misaligned conceptions of specific SMPs ....................... 124
Table 4-8: Compiled data of teachers’ misaligned conceptions of specific SMPs............ 125
Table 4-9: Pareto bar chart of compiled data of teachers’ misaligned conceptions of specific SMPs ................................................................. 125
Table 4-10: The SMPs discussed during the interviews.................................................... 127
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Chapter 1

“We cannot hope that many children will learn mathematics unless we find a way to share our enjoyment and show them its beauty as well as its utility.”
(Mary Beth Ruskai; American mathematics and quantum physics researcher; 1944–)

Introduction to the Study

Mary Beth Ruskai’s remarks about the importance of teaching children to appreciate the beauty and utility of mathematics have been a source of inspiration for me ever since I became a mathematics educator. The ultimate goal of my profession is to help children discover the beauty intrinsic to mathematics and to appreciate the usefulness of mathematics with respect to other academic disciplines as well as to the real world. Many mathematics educators, with similar ambitions, have sought to identify ways to develop students’ interest and understanding in mathematics. Mathematics standards in the history of mathematics education is a good example of this endeavor.

The field of mathematics education in K-12 schools has focused increasingly on the development of standards for students’ learning of mathematics for the past three decades. National Council of Teachers of Mathematics (NCTM) published several Standards documents to improve mathematics education during the 1990s and 2000s (NCTM, 1989, 1991, 1995, and 2000). To complement the NCTM Standards, state education standards in a majority of the states have been developed since the 1990s. Most recently, the National Governors Association (NGA) and Center for Best Practices Council of Chief State School Officer (CCSSO) “recognized the value of consistent, real-world learning goals and launched this effort to ensure all students, regardless of where they live, are graduating high school prepared for college, career and life,”
leading the effort to develop common standards in literacy and mathematics standards (NGA & CCSSO, 2010a, Development Process Section, para. 1).

The Common Core State Standards for Mathematics (CCSSM) define a level of quality regarding what K-12 students should know and be able to do in mathematics. In particular, the CCSSM explicitly delineate the Standards for Mathematical Practice (SMP) that mathematics teachers should nurture in all students (NGA & CCSSO, 2010a). As written in the CCSSI (NGA & CCSSO, 2010a), the SMP is considerably influenced by the two leading documents in U.S. mathematics education—NCTM Process Standards (NCTM, 2000) and National Research Council (NRC) report, *Adding It Up* (NRC, 2001). The CCSSM claims to be research-based, internationally benchmarked, and developed using the best existing state standards and the NCTM standards with involvement of teachers, content experts, state leaders, parents, and students (NGA & CCSSO, 2010a).

The CCSS have received both support and opposition by the public as well as academics. Those who opposed the CCSS pointed out the lack of pilot testing, inappropriate assessments, absence of educator and public participation, less-than-transparent development process, the challenge in classroom implementation of the standards, low student performance, and federal mandates to use standardized test data (McCluskey, 2015; Ravitch, 2016; Strauss, 2014a, 2014b). The advocates argued the benefits of the CCSS such as the coherent curriculum across nations, more focused curriculum, rigor to balance conceptual and procedural understanding, efficiency of having standardized content standards, assessments, curriculum guides, and fewer floor and ceiling effects of assessments (Porter, McMaken, Hwang, & Yang, 2011). In 2010, NCTM, the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), and the Association of Mathematics Teacher Educators (AMTE) announced a joint public statement on supporting implementation of the CCSSM. These mathematics education organizations “strongly encourage[d] and support[ed] both research about
the standards themselves (e.g., research on specific learning trajectories and grade placement of specific content) and their implementation” (NCTM, NCSM, ASSM, & AMTE, 2010, para. 5).

In the midst of widespread recognition of and focus on the CCSSM, however, there exist questions relating to teachers’ beliefs about and knowledge of the CCSSM. Although there are a number of studies investigating teachers’ beliefs about and knowledge of the NCTM Standards and teachers’ standards-based mathematics instruction, some of the studies with respect to the recent CCSSM and the SMP in particular, are only emerging now (Heck et al., 2011).

**Rationale of the study**

Teaching is a complex action between and among teacher, students, and content. Previous research has used a model that described the interactions and relationships between and among teacher, students, and content (e.g., Cohen and Ball, 1999; Lampert, 2001; Murata, Bofferding, Pothen, Taylor, & Wischnia, 2012; NRC, 2001). Cohen and Ball (1999) attended to the interactions among three critical elements: teachers, students, and educational materials (content), rather than the sole function of any one element. Cohen and Ball analyzed each element’s capacity and how each interacted with and influenced other elements.

The authors of *Adding It Up* (NRC, 2001) also explained teaching as the interactions among teachers, students, and content. According to this work, the effectiveness of mathematics teaching depends on the mutual and interdependent interaction of teacher, students, and mathematical content. They assert that effective teaching and learning occurs when teachers employ cognitively demanding tasks, and when students were engaged in the mathematical tasks and opportunities teachers offer.

Lampert (2001) analyzed her own teaching as a complex practice where the “inter-actions” and collaboration between teacher and students as well as between teacher and content
were essential to teaching and learning. She stressed the importance of the teachers’ capacity to enhance the student-content relationship.

![Teaching model](image)

**Figure 1-1**: Teaching model (Murata et al., 2012, p. 620)

Further, Murata and his colleagues (2012) conceptualized the interactions among these three elements. This framework guided their study to investigate how, in the context of lesson study, teachers made sense of student learning, teaching, and content. Figure **1-1** depicts the interactions of the three components—teachers, students, and materials (content).

Along with the researchers, the standards for mathematics education such as the Principles and Standards for School Mathematics (PSSM) and the CCSSM also underscored the interaction of these three elements in teaching. The PSSM (NCTM, 2000) addressed the areas of curriculum, teaching, and assessment relating to classroom practice, focusing on the interactions between teachers and students around the mathematical content. Similarly, the CCSSM also acknowledges the three components in teaching. The CCSSM provides the guidelines of what (content) students at a different grade level should understand and how teachers should seek to develop the learning in their students.

Looking closely at the interactions of these three components in the classroom, the roles of teachers are very meaningful in school education: the roles of teachers during this era of standards-based reform have become more prominent than in previous eras. The studies
pertaining to highly-qualified teachers discussed the importance of teachers’ pedagogical content knowledge in education (Shulman, 1986) and, in particular, their knowledge for teaching in mathematics education (Ball, Thames, & Phelps, 2008). The researchers also argued teachers’ capacity of facilitating reform-based practices such as mathematical discourse (Stein, 2007), representation (Goldin, 2000), and technology (Heid, 1997). These practices were consistent with the curriculum guidelines and standards such as the NCTM Standards (NCTM, 1989, 1991, 1995, 2000), the NRC report (2001), and the CCSSM (NGA & CCSSO, 2010a). Thus, it is evident that teachers play a significant role in improving mathematics pedagogy in the classroom and creating better learning opportunities for students.

Furthermore, researchers found that the teachers’ conceptions significantly affected their implementation of teaching. As shown in a number of research studies, the teachers’ instructional decisions and implementations were closely linked to their content conception, pedagogical knowledge, and understanding of student learning (Brendefur & Frykholm, 2000; Chen, McCray, Adams, & Leow, 2014; Drake, 2006; Ernest, 1989a, 1989b; Fennema et al., 1996; Grossman, Wilson, & Shulman, 1989; Leder, Pehkonen, & Törner, 2006; Lloyd & Wilson, 1998; Wilkins, 2008). Despite the variability in levels of such relationships, the studies have affirmed that teachers’ conceptions about teaching and learning mathematics played crucial roles in determining their instructional actions.

Over the past three decades, the NCTM’s Standards documents have assumed a leading role in the reform movement in mathematics education for the purpose of improving mathematics education (NCTM, 1989, 1991, 1995, 2000). The ensuing research studied the influence of the NCTM Standards relating to teachers’ beliefs, subject knowledge, teaching practices, student learning, learning environments, teacher education, educational policy, and professional development (Battista, 1994; Borko et al., 1992; Prawat, 1992; Raymond, 1997; Smith III, 1996; Stein & Lane, 1996; Swanson & Stevenson, 2002). Specifically, the studies relevant to teachers’
beliefs and their practices have shown meaningful correlation between the two (Ball, 1990a; Ball, 1991; Brendefur & Frykholm, 2000; Bush, Lamb, & Alsina, 1990; Even, 1993; Fennema et al., 1996; Leinhardt, Putnam, Stein, & Baxter, 1991; Lloyd & Wilson, 1998; Pajares, 1992; Silver, 1985; Staub & Stern, 2002). For example, Battista (1994) asserted that teachers would not be able to understand or achieve the goals of the mathematical activity that incorporated the reform movement, no matter how well-written it was, if a teacher’s beliefs were incongruous with the intention of the activity. In other words, teachers who perceived a more traditional, procedural way of mathematics education tended to focus on following procedures rather than making sense out of mathematics.

The aforementioned studies have shown that many teachers’ beliefs seemed inconsistent with those of the standards-based movement, even after a couple decades of endeavoring to change those beliefs. Succeeding NCTM standards, the CCSSM and its practice standards, the SMP have pursued the vision of a balance of conceptual and procedural understanding of mathematics, problem solving and reasoning, strategic use of mathematical tools, mathematical discussion, and making sense of mathematics. Moreover, the recently adopted CCSS standards have required teacher’s revised or extended understanding of the standards.

However, we are unclear about how teachers’ beliefs align with the current standards, especially the SMP, and how they understand and interpret the SMP. Since the adoption of the CCSSM, efforts have been made to increase the teachers’ understanding of the standards. For example, schools and school districts switched the textbooks that had claimed to be CCSSM-aligned; mathematics education programs, along with the schools and school districts, have offered professional development opportunities for both in- and pre-service teachers. Now it is time to investigate the teachers’ conceptions of the CCSSM. In a sense, this study is appropriate in its timeliness, if you will. Studying how teachers understand the SMP is essential because teachers are critical factors to developing the students’ proficiencies through school mathematics.
After all, teachers are expected to deeply understand not only mathematics, but also mathematical pedagogy (Mewborn, 2001).

The key areas of the studies around the CCSS have been about the content, curriculum and alignment, professional development and implementation, teacher perspectives, and the effects of the CCSS on instructional practices and student outcomes. Among the main areas of the studies around the CCSS, I am interested in teacher perspectives about the CCSS, especially the SMP. Studies investigating elementary school teachers’ perspectives about the SMP are limited. Furthermore, as shown in the examples of the studies in this area above, most researchers only surveyed teacher perception about their readiness to implement the CCSSM.

Since the adoption of the CCSSM and the development of states’ own standards, school districts and individual schools have made efforts to change the curriculum materials. It is called to investigate teachers’ understanding of the CCSSM more in-depth, as the teachers have learned the new standards, have used the new curriculum, and might have implemented them in classroom settings for some years. For example, Heck and his colleagues (2011) proposed a priority research agenda to understand the impact and implementation of the CCSSM building on the NRC report (2001). One of the suggestions for qualitative study of the CCSSM is to examine teachers’ conceptions of the CCSSM as follows:

Since teachers’ knowledge, interpretations, self-efficacy, beliefs, dispositions, and skills as well as their specific intentions and plans, affect what transpires in classrooms, it is critical to understand how teachers respond to the CCSSM, and what kinds of classroom learning opportunities for their students [might] result. (Heck et al., 2011, p. 13)

The CCSSM delineates the SMP on the front page of the standards because the SMP has to permeate the entire K-12 mathematics curriculum. However, the brevity of each standard neither integrates with the mathematical content standards nor includes detailed practices for every grade level (Koestler, Felton, Bieda, & Otten, 2013). The SMP explains little about how
teachers facilitate each standard of mathematical practice. How teachers understand and interpret the SMP would imply much more.

In addition, elementary school teachers’ conceptual understanding of mathematics is superficial (Ball, 1990b; Ma, 1999; Mewborn, 2001), but their beliefs about mathematics teaching play a key role in their teaching behavior (Richardson, 1996; Thompson, 1992). Because teachers are the driving force of change—the change of mathematics teaching and learning in the U.S.—it is critical to learn teachers’ conceptions about this change. As Sheninger and Murray (2017) have stated, “to prepare students for their world of work tomorrow, we must transform their learning today” (p. 18). The actions to prepare students, to transform their learning, and to create effective learning environments require teachers who plan and implement effective teaching as suggested by the standards (NCTM, 2014). Investigating elementary school teachers’ understanding of new math standards, the SMP in particular, provides the status about how teachers perceive, know, and interpret the SMP.

**Research question**

The purpose of this study is to investigate elementary school teachers’ conceptions about the SMP as it affects their teaching practices. For this study, I conducted in-depth interviews with eight elementary school teachers in western Pennsylvania. I probed into how they interpreted the SMP and how their understanding of the SMP are aligned with the descriptions of the SMP. The intensive interviews featured the degree of teachers’ conceptions of the eight SMPs. This study was guided by the question: How do the participating elementary school teachers understand and interpret the SMP?
Definitions of terms

For clarity of meaning, I define some key words in this section. First, I define what “standard” or “standards” mean based on how prior research has defined it. Then, I outline the meanings of beliefs, knowledge, and conceptions as researchers have often used these terms interchangeably. Lastly, I define what productive beliefs are in contrast to unproductive beliefs, as such beliefs are measured in relation to their understanding of the SMP.

Definitions of standard and standards

“Standard is a statement describing what a person should know or be able to do [emphasis added].” (Weiss, 2002, p. 23). *Standards* promote teaching and learning through inclusion in school curriculum, policy decision, instructional materials, and programs. Standards influence teacher development such as teacher education and professional development. Weiss noted that standards also guide what kinds of assessment should be used as well as to guide the purposes of the assessments. Similarly, Hiebert (2003) defined standards in mathematics education as “the goals we set for our students. They are value judgments about what we would like our students to know and be able to do” (p. 6). According to the Glossary of Education Reform (Great Schools Partnership, 2014), the common attributes of the “learning standards” systems in the U.S. are as follows:

- Learning [and teaching] standards are organized by specific subject areas (e.g., mathematics)
- In each subject area, standards are typically organized by grade level or grade span.
- Standards include overarching, long-term educational goals.
- Standards are unique for each subject, but there is also commonality from system to system or state to state.
Definitions of beliefs, knowledge, and conceptions

Researchers in mathematics education have used the terms beliefs, knowledge, and conception sometimes interchangeably, with the terms often indistinguishable from another. Philipp (2007) described the meaning of these terms as shown in Figure 1-2.

| **Beliefs** | psychologically held understandings, premises, or propositions about the world that are thought to be true. Beliefs are more cognitive, are felt less intensely, and are harder to change than attitudes. Beliefs might be thought of as lenses that affect one’s view of some aspect of the world or as dispositions toward action. Beliefs, unlike knowledge, may be held with varying degrees of conviction and are not consensual. Beliefs are more cognitive than emotions and attitudes. |
| **Knowledge** | beliefs held with certainty or justified true belief. What is knowledge for one person may be belief for another, depending upon whether one holds the conception as beyond question. |
| **Conception** | a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences. |

Figure 1-2: Definitions/descriptions of terms (Philipp, 2007, p. 259)

In this study, I use the term “conception” often, because it is the term that combines the meaning of teachers’ beliefs, knowledge, views, interpretations, and preferences (Thompson, 1984). Thompson (1984, 1992) addressed the importance of considering beliefs together with knowledge and referred to this construct as teachers’ conception. Instead of distinguishing these terms, I use teacher conceptions of the SMP to encompass teachers’ beliefs, knowledge, preferences, interpretations, agreement, etc. about the SMP. When I investigate teachers’ conception, I consider the teachers’ broader perspectives that they hold about mathematics, teaching and learning mathematics, standards, and the relationship between the conceptions and instructional practices. Modifying the definition of conception by Philipp (2007) and Thompson (1984), I define conception as a general notion or mental structure that incorporates beliefs, knowledge, propositions, interpretations, preferences, views, agreement, and mental images (Figure 1-3).
Conception: a general notion or mental structure incorporating beliefs, knowledge, propositions, interpretations, preferences, views, agreement, and mental images.

Figure 1-3: Revised definitions of conception

**Productive beliefs**

The authors of *Principles to Actions* (NCTM, 2014) outlined the *productive beliefs* about teaching and learning mathematics in contrast to *unproductive beliefs* as shown in Figure 1-4. Both productive and unproductive beliefs are linked to effective practices. Productive beliefs enhance more effective instructional practice or allow student access to important mathematics content and practices. Unproductive beliefs, however, obstruct effective teaching and learning.

An example of productive belief is “mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse” as opposed to an example of unproductive belief, “mathematics learning should focus on practicing procedures and memorizing basic number combinations” (p. 11).

<table>
<thead>
<tr>
<th>Unproductive Beliefs</th>
<th>Productive Beliefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics learning should focus on practicing procedures and memorizing basic number combinations.</td>
<td>Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.</td>
</tr>
<tr>
<td>Students need only learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.</td>
<td>All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.</td>
</tr>
<tr>
<td>Students can learn to apply mathematics only after they have mastered the basic skills.</td>
<td>Students can learn mathematics through exploring and solving contextual and mathematical problems.</td>
</tr>
<tr>
<td>The role of the teacher is to tell students exactly what definitions, formulas, and rules</td>
<td>The role of the teacher is to engage students in tasks that promote reasoning</td>
</tr>
</tbody>
</table>
Among the three factors (teachers, students, materials or content) of teaching, two human factors—teachers and students—are the agents of productive beliefs (Refer to Figure 1-1 for the three factors of teaching). Teachers’ beliefs, views, and perspectives influence their decisions about lesson plans, tasks, instructional practices, and assessment. Students’ beliefs influence their perception and attitudes about mathematics and what it means to learn mathematics. Teachers who possess productive beliefs plan lessons to trigger student discourses, implement the standards-based instructions, and reflect their lessons with the goal of helping students make sense of mathematical concepts and procedures, to ensure successful mathematics learning for all students. Likewise, students who hold productive beliefs actively face challenges in problem solving, construct viable argument, critique others’ reasoning, justify their own solutions, make connections to their prior knowledge and to other subjects, and create and use multiple representations (NCTM, 2014).

<table>
<thead>
<tr>
<th>they should know and demonstrate how to use this information to solve mathematics problems.</th>
<th>and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.</td>
<td>The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.</td>
</tr>
<tr>
<td>An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.</td>
<td>An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.</td>
</tr>
</tbody>
</table>

Figure 1-4: Beliefs about teaching and learning mathematics (NCTM, 2014, p. 11)

Summary
To improve mathematics instruction in the U.S., there have been great efforts from mathematics educators, mathematicians, researchers, teachers, administrators, and others. The standards in mathematics education such as the NCTM Standards (NCTM, 1989, 1991, 1995, 2000) and the CCSSM are the fruits of such efforts. The CCSSM is newly adopted, but its mathematics standards are widespread. The SMP of the CCSSM outlines what individual students should know and be able to do as *mathematically proficient students*. The SMP guides mathematics teachers of grades K to 12 to develop all of their students’ skills and knowledge to succeed in college, career, and life. It is widely accepted that teachers play significant roles in students’ learning of mathematics. Teachers’ beliefs and knowledge about mathematics as well as about teaching and learning of mathematics influence their planning of lessons and teaching practices. It is critical, therefore, to investigate teachers’ conceptions (encompassing teachers’ beliefs and knowledge) about the SMP. It is also timely and appropriate to study teachers’ conceptions about the CCSSM (the SMP in particular) as the majority of the states in U.S. have adopted and implemented the CCSSM for several years.
Chapter 2

Literature Review

This chapter reviews research on (1) mathematical standards, including the Common Core State Standards for Mathematics (CCSSM) and the National Council of Teachers of Mathematics (NCTM) Standards, (2) teacher conceptions (focusing on beliefs and knowledge) of mathematics and teaching/learning mathematics, and (3) teacher conceptions on standards. First, I examine the CCSS (its background, content, and practice standards), Pennsylvania Core Standards for Mathematics (PA Core Standards for Mathematics), and the literature on the CCSSM. I review the CCSSM before precedent standards and principles, because this may help the readers compare and contrast the Common Core State Standards for Mathematical Practice (SMP) with other standards. Next, I inspect the historical background of the NCTM Standards and the literature on the Standards-based instruction. I also discuss other principles in mathematics education such as Adding It Up and Habits of Mind to relate them to the CCSSM. I review current standards for English/language arts and science (i.e., Common Core Standards for English Language Arts and Next Generation Science Standards respectively), focusing on their practice standards. I compare and contrast these principles and standards with the SMP. Then, I review research on teachers’ conceptions, which incorporate teachers’ beliefs, knowledge, propositions, interpretations, preferences, views, agreement, and mental images about mathematics as well as teaching and learning mathematics. I discuss the research about teachers’ beliefs and knowledge in mathematics education because teacher beliefs and knowledge (within conceptions) are the most discussed influences on classroom practice. Finally, I summarize the research on teacher conceptions in the reform movement.
The Common Core State Standards

The efforts to establish nationally unified curriculum have been issues of concern for America’s education for the past several decades. The international mathematics assessments such as the Trends in International Mathematics and Science Study (TIMSS\(^1\), 1995, 1999, 2003, 2007) demanded more coherent and focused mathematics curriculum to improve student achievement. Not only the reports of the international mathematics assessments, but also the domestic comparisons of the standards among the states called for developing better standards (e.g., National Assessment of Educational Progress [NAEP]\(^2\)). In addition, the educational policies such as No Child Left Behind Act (NCLB, 2002) and Race to the Top (2011) were the outcomes of the idea that schools were failing due to the absence of high national standards and accountability (California Alliance of Researchers for Equity in Education, 2016). The evidence and criteria to develop the CCSS standards are listed below (NGA & CCSSO, 2010e, p. 3):

- Scholarly research
- Surveys on the skills required of students entering college and workforce training programs
- Assessment data identifying college- and career-ready performance
- Comparisons to standards from high-performing states and nations
- NAEP frameworks in reading and writing for English language arts
- Findings from TIMSS and other studies, which conclude that the traditional U.S. mathematics curriculum must become substantially more coherent and focused in order to improve student achievement

The authors of the Common Core State Standards (CCSS) claimed that the CCSS would provide clear and consistent learning goals to help prepare students for college, career, and life.

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\(^2\) The National Assessment of Educational Progress (NAEP) has provided important information about how students are performing academically since 1969. Retrieved from: [https://nces.ed.gov/nationsreportcard/about/](https://nces.ed.gov/nationsreportcard/about/)
The standards demonstrate what students were expected to learn at each grade level, so that every parent and teacher could understand and support their learning.

State chiefs began to discuss developing common standards in 2007. In December 2008, the National Governors Association Center for Best Practices (NGA) and the Council of Chief State School Officers (CCSSO) released the report, *Benchmarking for Success: Ensuring U.S. Students Receive a World-Class Education*. In this report, governors, state education chiefs, and prominent education researchers articulated standards in mathematics and English language arts and literacy (ELA/Literacy) for K-12 grades “to ensure that students are equipped with the necessary knowledge and skills to be globally competitive” (NGA, CCSSO, & Achieve, 2008, p. 6). In the following year, development of common standards in ELA/Literacy and mathematics began. After the draft of the standards had been created with the input from the standards writing team, state education agency leaders, and a panel of outside education experts and practitioners, the NGA and the CCSSO also received public comments on this draft before the final revision. In June 2009, the CCSSO and the NGA announced commitment from 49 states and the U.S. territories to participate in a state-led process to develop the CCSS (NGA & CCSSO, 2010d). In June 2010, the NGA and the CCSSO released the final version of the CCSS, proposing the ultimate goal for “all American children to graduate from high school ready for college, career pathways, and success in a global economy” (NGA & CCSSO, 2010c, p. 1). Subsequently, they reported a review of the standards as follows:

- Reflective of the core knowledge and skills in ELA/Literacy and mathematics that students need to be college- and career-ready;
- Appropriate in terms of their level of clarity and specificity;
- Comparable to the expectations of other leading nations;
- Informed by available research or evidence;
- The result of processes that reflect best practices for standards development;
- A solid starting point for adoption of cross-state common core standards; and
- A sound basis for eventual development of standards-based assessments (NGA & CCSSO, 2010c, p. 3).
The CCSSM was constructed based on a set of the outstanding standards. The CCSSM, however, claimed three key concepts that were different from the previous standards: greater focus on fewer topics, coherence, and rigor. The CCSSM (NGA & CCSSO, 2010f) explained each key shift as follows:

1. Greater focus on fewer topics: Rather than racing to cover many topics in a mile-wide, inch-deep curriculum, the standards ask math teachers to significantly narrow and deepen the way time and energy are spent in the classroom.

2. Coherence (Linking topics and thinking across grades): The standards are designed around coherent progressions from grade to grade. Coherence is also built into the standards in how they reinforce a major topic in a grade by utilizing supporting, complementary topics.

3. Rigor (Pursue conceptual understanding, procedural skills and fluency, and application with equal intensity): Rigor refers to deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades. To help students meet the standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skills and fluency, and application.

Beginning in 2011, the participating states and territories reviewed, adopted, and ratified the CCSSM appropriately to their own states and territories. It is notable that there have been changes in the number of states that adopted the CCSS in ELA/Literacy and mathematics: In 2013, 45 states and the Department of Defense Education Activity (DoDEA), Washington D.C., Guam, the Northern Mariana Islands and the U.S. Virgin Islands adopted the CCSS; in 2014, 43 states, the District of Columbia, the U.S. territories, and the DoDEA; and in 2015, 42 states, the District of Columbia, the U.S. territories, and the DoDEA. The states that had initially adopted the CCSS, and repealed it later were Indiana (2013), Oklahoma (2013), and South Carolina (2014). As of this study (2019), 41 states, the District of Columbia, four territories, and DoDEA have adopted and are implementing the CCSS (NGA & CCSSO, 2010d).

The reasons that might have affected some states’ repeal of the CCSS are varied: the unfamiliar approaches to basic math might have frustrated parents; the CCSS-aligned testing and accountability of the tests might have concerned teachers (The 74 Media, n.d.; California Alliance
of Researchers for Equity in Education, 2016); or teachers’ disagreement with the idea of “federal standards and assessment” might have limited teachers’ capacity in the classroom (Schwalbach, 2018). Yet, the majority of the states have tailored the CCSS to address their specific needs and have utilized the standards as the guidelines in the education of the ELA/Literacy and mathematics.

The Common Core State Standards for Mathematics

The Common Core State Standards for Mathematics (CCSSM) was created in response to the calls for a coherent, focused, and rigorous standards for the school mathematics. The work team of the CCSSM described the standards as “what students should understand and be able to do in their study of mathematics” (NGA & CCSSO, 2010a, p. 4). The CCSSM was developed to help raise mathematically proficient students who exhibit both conceptual and procedural understanding of mathematics in a way appropriate to their mathematical maturity. The standards are grade-specific from Kindergarten to grade 12, but do not suggest specific intervention methods or materials (NGA & CCSSO, 2010a).

The CCSSM begins with the SMP that describes what knowledge and skills mathematics educators at all levels should pursue in developing students for mathematical proficiency. Including classroom teachers, all mathematics educators who design and perform curricula, assessments, and professional development should attend to the need to connect the mathematical practices to the content in mathematics teaching. This three-page document complements the CCSSM content standards by describing the expertise that K-12 teachers should develop in their students.
The standards for mathematical content

The Standards for Mathematical Content of the CCSSM explains on what specific topics and critical areas of mathematics students should learn and teachers need to focus for each grade level, with detailed examples. The CCSSM organized the concepts that students will learn in a systematic way. The standards for each grade level are clustered under appropriate domains (See Figure 2-1).

In all, there are 11 standards by domain:

1. Counting and Cardinality
2. Operations and Algebraic Thinking
3. Number and Operations in Base Ten
4. Number and Operations-Fractions
5. Measurement and Data
6. Geometry
7. Ratios and Proportional Relationships
8. The Number System
9. Expressions and Equations
10. Functions
11. Statistics and Probability

The example in Figure 2-1 shows the domain of “Number and Operations in Base Ten,” which spans from kindergarten to grade 5. The overarching cluster theme at grade 3 for this domain is “Use place value understanding and properties of operations to perform multi-digit arithmetic.” For this domain, students should

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.
   (NGA & CCSSO, 2010a, p. 24)

Figure 2-1: How to read the grade level standards: Grade 3 example (NGA & CCSSO, 2010a, p. 5)

Another example from grade 5, as shown in Figure 2-2, presents the overarching cluster theme, “use equivalent fractions as a strategy to add and subtract fractions” under the domain of Number and Operations—Fractions. The standards specify what fifth grade students should know to attain this theme. Students should:

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fraction with like denominators. For example, \( \frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} + \frac{23}{12} \) (In general, \( \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \)).

2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result \( \frac{2}{5} + \frac{1}{2} = \frac{3}{7} \), by observing that \( \frac{3}{7} < \frac{1}{2} \). (NGA & CCSSO, 2010a, p. 36)
The standards for mathematical practice

Teachers’ instructional practice is a prominent factor that causes effective teaching and learning of mathematics. A number of researchers have studied the links between particular teaching practices and student learning outcomes (e.g., Boaler, 1998; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Carpenter, Fennema, Franke, Levi, & Empson, 2000; Fuson & Briars, 1990; Grossman et al., 2009; Hiebert & Grouws, 2007; Hiebert & Wearne, 1993; Stronge, Ward, & Grant, 2011; Tchoshanov, 2011). Their findings reported that students gained better or similar learning outcomes when their teachers used student-centered approaches and standards-based pedagogy in their mathematics instructions, compared to more traditional ways of teaching. On the basis of prior mathematics education research results, this study assumes that teachers’ instructional practices are one of the most important school-based factors to account for students’ learning. Consequently, the standards for practice itself and understanding of such standards is paramount to guide teacher’s instructional practices.

Figure 2-2: How to read the grade level standards: Grade 5 example (NGA & CCSSO, 2010a, p. 36)

<table>
<thead>
<tr>
<th>Number and Operations—Fractions</th>
<th>5.NF</th>
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</thead>
<tbody>
<tr>
<td>Use equivalent fractions as a strategy to add and subtract fractions.</td>
<td></td>
</tr>
<tr>
<td>1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}$.)</td>
<td></td>
</tr>
<tr>
<td>2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} &lt; \frac{1}{2}$.</td>
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</table>
The CCSSM outlines the SMP that mathematics teachers should strive to develop in all K-12 students. The eight SMPs (Figure 2-3) advocate the NCTM’s Process Standards (2000) and the NRC report, *Adding It Up* (NRC, 2001), to complement the CCSSM content standards by describing the varieties of expertise that K-12 teachers need to develop in their students. These mathematical practices are the refinements from the five areas of the NCTM’s Process Standards: problem solving, reasoning and proof, communication, connections, and representation (NCTM, 2000) and five Strands of Mathematical Proficiency from the National Research Council (NRC)’s report, *Adding It Up*: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (NRC, 2001).

The SMP briefly explains how each of the SMPs can be implemented in the mathematics classroom in a variety of grade levels and mathematical topics. The descriptions of the SMP from the Common Core State Standards Initiative (CCSSI; NGA & CCSSO, 2010a) will guide the processes of data collection and analysis for this study. I list the eight Standards in Figure 2-3, only the title of each Standard for brevity. (See Appendix A for more in-detailed descriptions).

<table>
<thead>
<tr>
<th></th>
<th>Make sense of problems and persevere in solving them</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Reason abstractly and quantitatively</td>
</tr>
<tr>
<td>3</td>
<td>Construct viable arguments and critique the reasoning of others</td>
</tr>
<tr>
<td>4</td>
<td>Model with mathematics</td>
</tr>
<tr>
<td>5</td>
<td>Use appropriate tools strategically</td>
</tr>
<tr>
<td>6</td>
<td>Attend to precision</td>
</tr>
<tr>
<td>7</td>
<td>Look for and make use of structure</td>
</tr>
<tr>
<td>8</td>
<td>Look for and express regularity in repeated reasoning</td>
</tr>
</tbody>
</table>

Figure 2-3: Eight standards for mathematical practice (NGA & CCSSO, 2010a)

*The Pennsylvania Core Standards for Mathematics*

The CCSSM is the most recent mathematical standards that are adopted nationwide. Some states have crafted the states’ own standards based upon the CCSSM, but to meet state-
specific needs. For example, Pennsylvania created PA Core Standards for Mathematics that mirrored the CCSSM. The newly revised 2014 PA Core Standards for Mathematics emulated the content and rigor of the CCSSM. The standards were also drawn from the best of the PA Academic Standards and represent the input of Pennsylvania educators. The PA Core Standards for Mathematics were distributed to schools in the 2013-2014 academic year and professional development for teacher training followed in the 2014-2015 academic year (Commonwealth of Pennsylvania, 2018a). The Pennsylvania Department of Education (PDE) states that teachers in Pennsylvania are responsible for “a clear understanding of the instructional shifts and rigorous demands of PA Core Standards” (Commonwealth of Pennsylvania, 2018b, p. 1).

The PDE provides The Standards Aligned System (SAS), offering resources for teachers to improve student achievement. The SAS classifies and describes Standards, Assessment, Curriculum Framework, Instruction, Materials & Resources, and Safe and Supportive Schools. Above all, the Standards are accessible to the public and are downloadable. The 2014 PA Core Standards for Mathematics describes what students in Pennsylvania should understand and be able to do (PDE, 2014a). It consists of Standards for Mathematical Content and Standards for Mathematical Practice. The introductory page of the PA Core Standards for Mathematics depicts the importance of both standards as shown in Figure 2-4.

3 http://www.pacode.com/secure/data/022/chapter4/s4.12.html The Pennsylvania State Board of Education has adopted academic standards in 12 subject areas. The academic standards are benchmark measures that define what students should know and be able to do at specified grade levels beginning in grade 3. State requirements for curriculum, instruction, and assessment can be found in the Board's Chapter 4 regulations, available online at: Chapter 4.
The PA Core Standards for Mathematics specify four content areas: 1) numbers and operations, 2) algebraic concepts, 3) geometry, and 4) measurement, data, and probability. These standard areas reflect the reporting categories in the PA Core Assessment Anchors and Eligible Content. As shown in Figure 2-4, the PA Core Standards for Mathematics emphasizes not only the four content areas, but the integration of the mathematical practice as well. Figure 2-5 illustrates the development and progression in four standard areas for each grade level, framed around the SMP. It depicts the SMP as overarching standards for all grades from pre-K through high school.

Figure 2-4: PA Core Standards for mathematical content and mathematical practice (PDE, 2014a, p. 2)

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4 PSSA Mathematics Glossary to the Assessment Anchors and Eligible Content Aligned to the Pennsylvania Core Standards
The content standards in the PA Core delineate what each grade band (PreK–5, middle school, and high school) should understand. The content standards for elementary school specifies what students should understand and develop in mathematics classrooms as following:

The Pennsylvania Core Standards in Mathematics in grades PreK–5 lay a solid foundation in whole numbers, addition, subtraction, multiplication, division, fractions, and decimals. Taken together, these elements support a student’s ability to learn and apply more demanding math concepts and procedures (PDE, 2014a, p. 2).

The PA Core Standards for Mathematical Practice describes “the habits of mind” required to reach a level of mathematical proficiency. While the content standards are grade-specific, the practice standards encompass all grades and should be developed throughout students’ mathematical experiences. These standards are identical to the eight SMPs of the CCSSM (See Appendix A). However, this version of the PA Core does not explicate what each of the eight SMPs mean.

Figure 2-5: PA Core mathematical standards: development and progression (PDE, 2014a, p. 4)
<table>
<thead>
<tr>
<th>Mathematical Practice</th>
<th>K</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Make sense of problems and persevere in solving them.</strong></td>
<td>• Begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them.</td>
<td>• Realize that doing mathematics involves solving problems and discussing how they solved them.</td>
<td>• Realize that doing mathematics involves solving problems and discussing how they solved them.</td>
</tr>
<tr>
<td></td>
<td>• Explain to themselves the meaning of a problem and look for ways to solve it.</td>
<td>• Explain to themselves the meaning of a problem and look for ways to solve it.</td>
<td>• Explain to themselves the meaning of a problem and look for ways to solve it.</td>
</tr>
<tr>
<td></td>
<td>• Use concrete objects or pictures to help them conceptualize and solve problems.</td>
<td>• Use concrete objects or pictures to help them conceptualize and solve problems.</td>
<td>• Use concrete objects or pictures to help them conceptualize and solve problems.</td>
</tr>
<tr>
<td></td>
<td>• Check their thinking by asking themselves, “Does this make sense?” or they may try another strategy.</td>
<td>• Willing to try other approaches.</td>
<td>• Make conjectures about the solution and plan out a problem solving approach.</td>
</tr>
<tr>
<td><strong>Reason abstractly and quantitatively.</strong></td>
<td>• Begin to recognize that a number represents a specific quantity.</td>
<td>• Recognize that a number represents a specific quantity.</td>
<td>• Recognize that a number represents a specific quantity.</td>
</tr>
<tr>
<td></td>
<td>• Connect the quantity to written symbols.</td>
<td>• Connect the quantity to written symbols.</td>
<td>• Connect the quantity to written symbols.</td>
</tr>
<tr>
<td></td>
<td>• Create a representation of a problem while attending to the meanings of the quantities (quantitative reasoning).</td>
<td>• Create a representation of a problem while attending to the meanings of the quantities (quantitative reasoning).</td>
<td>• Create a representation of a problem while attending to the meanings of the quantities (quantitative reasoning).</td>
</tr>
</tbody>
</table>

Figure 2-6: SMP grade level emphasis (PDE, 2014b, p. 2)

There is a different version of the SMP on the SAS website. The Standards for Mathematical Practice Grade Level Emphasis (PDE, 2014b), is adapted from the CCSS SMP. In this version, the eight practices present what students in each grade level should do to meet each eight SMPs as shown in Figure 2-6 above. There is a short list of the expertise that students in each grade should develop for each SMP.

**Literature on the Common Core State Standards for Mathematics**

Since the release of the CCSS in 2010, researchers, mathematics educators, and mathematicians have studied various areas related to the CCSS. The key areas of the studies around the CCSS have been content, curriculum, and alignment; professional development and implementation; teacher conceptions (beliefs, knowledge, interpretation, understanding, etc.) about the CCSS; and the effects of the CCSS on instructional practices and student outcomes. In
this section, I review the literature in each area of research around the CCSS. Although my focus is on the CCSSM, often I refer to the literature on CCSS as one body, combining the mathematics standards and the ELA/Literacy standards together.

First, researchers have studied and analyzed the CCSS content, curriculum, and alignment (e.g., Cobb & Jackson, 2011; Dingman, Teuscher, Newton, & Kasmer, 2013; Gamson, Lu, & Eckert, 2013; Nagle & Moore-Russo, 2014; Polikoff, 2015; Porter et al., 2011; Porter et al., 2013; Williamson, Fitzgerald, & Stenner, 2013). They assessed the quality of the CCSS, compared the CCSS with other standards (e.g., state standards, NCTM Standards, etc.), examined the degree of textbook alignment to the CCSS, or compared the CCSS-aligned textbooks with former textbooks. For example, Polikoff (2015) analyzed the alignment of a textbook context of fourth grade, using an alignment tool. His study showed considerable areas of misalignment such as overemphasizing procedures and memorization compared to the CCSSM.

Some of the researchers studied whether, in their view, the CCSS were appropriate. Those who advocated the CCSS explained why integrating the CCSS in classroom instruction was necessary and how the CCSS should be implemented (e.g., Burns, 2013; Bostic & Matney, 2013; Kendall, 2011; Schoenfeld, 2015; Sztajn et al., 2012; Wu, 2011; Wu, 2014). These researchers suggested ways to assist teachers in developing an understanding of the content and practice standards of the CCSS for desirable implementation. There have been, however, voices of concern about the CCSS (e.g., Cobb & Jackson, 2011; Mathis, 2010; Tienken, 2011), critiquing the CCSS for the lack of research on the impact of the standards, little provision of necessary educational resources, and plausible implementation obstacles.

Second, there have been studies about professional development for and implementation of the CCSS (e.g., Elias, 2014; Herman, Epstein, & Leon, 2016; Holliday & Smith, 2012; Jenkins & Agamba, 2013; Kane, Owens, Marinell, Thal, & Staiger, 2016; Liebtag, 2013; Opfer, Kaufman, & Thompson, 2016; Simpson & Linder, 2014; Sztajn, Marrongelle, Smith, & Melton,
Most of the studies in the early 2010s suggested desirable implementation and professional development rather than delving into teachers’ instructional practice using the CCSSM. Since 2014, more researchers probed into teachers’ within-classroom implementation of the CCSS. For example, Kane and his colleagues reported changes in teachers’ lesson plans and instructional materials, teachers’ embracement of the CCSS, and an increase of the professional development days since the adoption of the CCSS. Opfer and others (2016) reported that a significant number of teachers responded their engagement of their students in most SMP-aligned practices “daily or almost daily” from two web-based surveys.

Third, researchers have been interested in teacher perspectives on the CCSS (e.g., Burks et. al., 2015; Davis et al., 2013; Davis et al., 2014; Troia & Graham, 2016). These studies discussed conceptions of teachers in the form of an online poll in which they surveyed how teachers think about their readiness to implement the CCSS. Moreover, there has been research examining teachers’ perception, knowledge, or beliefs about the CCSSM (e.g., Bostic & Matney, 2013; Cogan et al., 2013; Davis, Choppin, McDuffie, & Drake, 2013; Davis, Drake, Choppin, & McDuffie, 2014; McDuffie et. al., 2017; Olson, Olson, & Capen, 2014). For example, Bostic and Matney (2013) examined elementary and middle school teachers’ perceived mathematical content needs and pedagogical needs, both related to the CCSSM, and the connection between these perceptions. Olson and his colleagues (2014) analyzed teachers’ responses of the surveys associating their reading of the SMP. These studies investigated teachers’ conceptions (perception, knowledge, and beliefs) that would affect their instructional practices. However, these studies were limited either to a survey method (e.g., Bostic & Matney, 2013; Burks et. al., 2015; Choppin et al., 2013; Cogan et al., 2013; Davis et al., 2013; Davis et al., 2014; Troia & Graham, 2016) or to the setting of professional development (e.g., Carney, Brendefur, Thiede, Hughes, & Sutton, 2016; Olson et al., 2014).
Lastly, researchers have investigated the relationship of the CCSS to student achievement (e.g., Hiebert & Mesmer, 2013; Schmidt & Houang, 2012). These studies discussed the potential impact of the CCSS on students’ learning. For example, Schmidt and Houang (2012) not only compared the CCSSM and the standards of the highest achieving countries with 1995 Third International Mathematics and Science Study (TIMSS), but they also assessed the relationship between a state’s standards and its performance on the 2009 National Assessment of Educational Progress (NAEP). They found a high degree of similarity between the CCSSM and the high-achieving nations’ standards.

Later phase of this line of research analyzed the NAEP test results by relating them to the CCSSM. The results showed that the NAEP (2013) scores of the states whose standards were more similar to the CCSSM were higher than those of other states. In the report of the 2013 NAEP reading and mathematics, the U.S. Secretary of Education, Arne Duncan (USDE, 2013), applauded the improvement of student achievement in the states where the CCSS had been adopted: “While progress on the NAEP continues to vary among the states, all eight states that had implemented the state-crafted Common Core State Standards at the time of the 2013 NAEP assessment showed improvement in at least one of the Reading and/or Mathematics assessments from 2009 to 2013—and none of the eight states had a decline in scores” (U.S. Department of Education, 2013, para. 5). Burris (2015) rebuked this statement and argued that the CCSS did not work, stating, “it would appear that the solution of tough standards … is not the great path forward after all” (para 6). Loveless (2014, 2016) tested whether the states—whose standards were more aligned with the CCSS—performed better in NAEP than other states and found no evidence of those state’s greater achievement gains after the CCSS adoption. More recently, Polikoff (2017) critiqued Loveless’ (2014, 2016) methods and proposed suggestions for the studies about the standards and their effects.
Additionally, there have been research that compared the CCSSM with other standards such as state standards or standards in top performing countries (e.g., Porter et al., 2011; Dingman et al., 2013; Cogan, Schmidt, & Houang, 2013). These studies examined the differences between the standards and analyzed the changes in the CCSSM. For example, Porter and his colleagues (2011) compared the CCSSM with state standards, NCTM Standards, and the standards of other countries. They found that the alignment between state standards and the CCSSM varied, and the average of the alignment was low to moderate. The alignment between the NCTM standards and the CCSSM was moderate, not higher than the average alignment between the state standards and the CCSSM. Their comparison between the CCSSM and the standards of three high-performing countries (Finland, Japan, and Singapore) showed a high degree of similarity.

There have been various areas of research in mathematics education since the CCSS era began. During the NCTM standards period and even before, there were similar efforts to advance mathematics education. In the following section, I discuss a brief history of mathematical standards and provide descriptions about previous mathematics standards and principles such as NCTM standards, *Adding It Up*, and mathematical habits of mind in relation to the SMP.

**Mathematical standards**

The pressure and demand to improve mathematics education have been an ongoing issue in the U.S. The schools in the U.S. have been subject to many varying demands and expectations. Shown in Figure 2-7, historical movements in mathematics education field had begun with the “New Math” movement in the 1960s and continued through “Back to Basics” in the 1970s and “Standards” movement of the late 1980s-2000s that saw NCTM lead in the mathematics education. The “Math War” followed in the late 1990s. Most recently, the CCSSM has taken their places.
The “New Math” era came into being in the early 1950s and lasted through the 1970s. It is notable that mathematicians were actively involved in creating school mathematics curricula for the first time in U.S. history. The reform effort was ignited in 1958 soon after Russia successfully launched Sputnik. The low quality of mathematics and science education in K-12 schools was brought to the attention of elected officials, and the U.S. Congress passed the 1958 National Defense Education Act in an effort to affect an increase of mathematics, science, and foreign language majors. Several New Math groups introduced K-12 school curricula, focusing on coherent logical explanations for mathematical procedures. Despite the wide circulation of the curricular materials and the great financial support, the result of the New Math was considered a failure (Raimi, 2005). Critics rebuked the irrelevant topics to students’ regular experience, ignorance of the mathematics nature of cumulative development, too much pressure and demands for teachers upon new materials, and difficulty for parents to help their children (Kline, 1973; Sarason, 1971).

As a result, the “back to basics” movement swung back to prominence in the 1970s, emphasizing arithmetic computations. The public concern became increasingly concerned about declining test scores, lack of discipline, and the rising costs of education, and opinion against New Math mounted. The soft curriculum did not seem to prepare students for the world of work and a return to a more traditional curriculum became desirable with heightened emphasis on basic arithmetic. However, this direction toward a more traditional curriculum was met by the challenge that it could not hold students to high standards. During the 1970s, standardized test scores consistently decreased.
In the early 1980s, public concern about the deteriorating quality of mathematics education was expressed through various reports. The most exemplary reports were An Agenda for Action (NCTM, 1980) and A Nation at Risk (National Commission on Excellence in Education, 1983). These two reports called for a new direction in mathematics education, in the form of “standards.” An Agenda for Action emphasized problem solving and de-emphasized mastery of skills. A Nation at Risk addressed shortcomings in mathematics education and called attention to the quality of teachers and curriculum materials. Gaining public support on clearly articulated high standards, the NCTM initiated the standards movement.

NCTM standards

National Council of Teachers of Mathematics (NCTM) presented the direction of mathematics education to improve mathematics teaching and learning in Agenda for Action (NCTM, 1980). In spite of the wide dissemination of Agenda for Action, the report received little attention. In 1983, A Nation at Risk (National Commission on Excellence in Education) addressed various educational issues, including specific shortcomings in mathematics education. It compared the U.S. curriculum and teaching methods with those of other countries. This report received public attention and influenced many states to create task forces to measure their own curricula (e.g., 1985 California Model Curriculum Standards, Grades Nine Through Twelve).

As a result, the NCTM took remarkable steps to develop the quality of mathematics education by developing the Standards. The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and the Professional Standards for Teaching Mathematics (NCTM, 1991) were published. The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) focused on curriculum and evaluation, promoting the views of An Agenda for Action (NCTM, 1980) with greater refinement. The document recommended not only the topics
that should be taught in each grade, but also topics that should be de-emphasized (e.g., rote memorization of basic facts and algorithms). It also suggested teaching practices such as focusing on problem solving. Because the 1989 Standards document did not present clear teaching practices that supported the new goals for student learning, teachers could not fully implement the standards as intended. However, the 1989 Standards, as antecedent standards from which the series of the NCTM Standards stemmed, were significant in the standards-based reform movement. The *Professional Standards for Teaching Mathematics* (NCTM, 1991) put emphasis on teaching practices and professional development of the mathematics teachers. Since the publication of the 1989 and 1991 Standards, there has been consistent discussion and debate about reforming the U.S. mathematics education. NCTM later published *Assessment Standards for School Mathematics* (NCTM, 1995) and continued its effort to improve teaching and learning of mathematics. In 2000, NCTM’s publication of *Principles and Standards for School Mathematics* (PSSM) outlined key components of a high-quality school mathematics program to help all students learn mathematics effectively. The PSSM (NCTM, 2000) described the pedagogical recommendations for mathematics educators in each grade band (Pre-K-2, 3-5, 6-8, and 9-12). With much more detailed explanation about what teachers need to do and what not to do in mathematics classrooms, the PSSM provided more specific guidelines to the mathematics educators. However, some critics argued that basic mathematical skills were understated by the statements in PSSM. For example, Quirk (2002) criticized NCTM Standards’ advocacy about conceptual knowledge downplaying basic skills as follows:

> Similar to the original NCTM Standards, PSSM is vague about the major components of arithmetic mastery: 1) Memorization of basic number facts, 2) Mastery of the standard algorithms of multidigit computation, [and] 3) Mastery of fractions. The NCTM has toned down the constructivist language, but they still stress content-independent process skills and student-centered discovery learning (para. 4).
Despite such critiques that standards-based instruction lowered students’ achievement levels (e.g., Klein, 2003; Loveless, 2001) and disagreements from some parents and mathematicians (Schmid, 2000; Wu, 1997), standards-based approach to mathematics education have manifested in U.S. schools. In fact, more studies have reported that students achieved better scores, when their teachers implemented Standards-based mathematics teaching methods or curricula than students in conventional classrooms (Fuson, Carroll, & Drueck, 2000; Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000; Knapp, Shields, & Turnbull, 1992; McCaffrey et al., 2001). In addition, the TIMSS data supported the standards-based approach, reporting the U.S. students’ low performance in advanced mathematical concepts and problem solving.

The process standards of the PSSM have especially influenced the development of the SMP as stated in the CCSSI (NGA & CCSSO, 2010a): “These practices rest on important processes and proficiencies with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning, and proof, communication, representation, and connections” (p. 6). Thus, it is not surprising that the NCTM (2000) process standards of PSSM (problem solving, reasoning and proof, communication, representation, and connections) pair up with the SMPs. Koestler and her colleagues (2013) matched these two sets of standards, showing how they overlap. I have summarized this comparison below (Figure 2-8).

<table>
<thead>
<tr>
<th>NCTM Process Standards</th>
<th>CCSSM Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Solving</td>
<td>SMP 1, SMP 2, SMP 4, SMP 5</td>
</tr>
<tr>
<td>Reasoning and Proof</td>
<td>SMP 1, SMP 3, SMP 8</td>
</tr>
<tr>
<td>Communication</td>
<td>SMP 1, SMP 2, SMP 4, SMP 6</td>
</tr>
<tr>
<td>Connections</td>
<td>SMP 1, SMP 2, SMP 4, SMP 7, SMP 8</td>
</tr>
<tr>
<td>Representation</td>
<td>SMP 1, SMP 2, SMP 4, SMP 5, SMP 6, SMP 7</td>
</tr>
</tbody>
</table>

Figure 2-8: Comparison of the NCTM process standards and the SMP
The matching of these two standards sets can be different from the one above, depending on the context. How to provide students opportunities to develop the practices that are suggested in the standards is more important than cross-matching the standards is (Seeley, 2014). I pair up the SMP with the NCTM Process Standards here and with other mathematical proficiencies later because I want to emphasize the intimate relationship between the standards/principles and the SMP as well as their influence on the SMP.

**Literature on the standards-based/reform-based instruction**


Upon the release of the NCTM Standards, a succession of research ensued to investigate the influence of these Standards upon teachers’ beliefs, teaching practices, student learning, learning environments, teacher education, and educational policy (e.g., Battista, 1994; Borko et al., 1992; Prawat, 1992; Raymond, 1997; Smith III, 1996; Stein & Lane, 1996; Swanson &
Stevenson, 2002) as well as the nature of the Standards themselves (e.g., Ball, 1992; Boyer, 1990; Reys, 1992). These studies reported that the mathematical instruction characterized in most U.S. classrooms over the past few decades did not serve students well, and this finding was in direct contrast to the vision of the NCTM Standards.

This finding, however, does not mean that standards-based instruction is superior to traditional instruction. The results of the studies about the relationship between instructional practices and student performance varied in degrees of student achievements. Moreover, the correlation between teaching practices and student learning was not consistent across the studies. A number of studies, however, presented considerable results about how the non-traditional curricular programs characterized effective mathematical instruction (e.g., Boaler, 1998; Carpenter et al., 1989, 2000; Cobb et al., 1991; Fennema et al., 1996; Fuson & Briars, 1990; Hiebert & Wearne, 1993; Wood & Sellers, 1996). Upon reviewing such studies, Hiebert (2003) concluded that alternative or standards-based programs were more effective in teaching and learning mathematics appropriately. He explained that these programs let students take advantage of building directly on their initial knowledge and skills, to have both creative thinking and practice of already learned skills, and to experience multiple strategies and methods. Further, Hiebert (2003) affirmed that the NCTM Standards could provide students more opportunities to understand mathematical topics conceptually and deeply while using skills proficiently.

Adding It Up

*Adding It Up* (NRC, 2001) is a report by the Committee on Mathematics Learning. The committee reviewed and synthesized relevant research around mathematics education for pre-K through grade 8. They provided research-based recommendations for mathematics teaching in school. Furthermore, they offered advice and guidance for mathematics curriculum and teacher
education. From the results of the state, national, and international assessments, researchers discovered that U.S. students exhibited a limited understanding of basic mathematical concepts and low reasoning skills to solve even simple problems. In comparison with the curricula of high achieving countries, the U.S. school mathematics curriculum had been characterized as “a mile long and an inch deep.”

The ultimate goal of this report was to improve mathematics learning in all students by addressing the public concerns about the status of mathematics education and by describing mathematical proficiencies that K-12 students needed to develop. The mathematical proficiency encompasses mathematical knowledge, understanding, and skill for any student to learn mathematics successfully. The five suggested strands for the mathematical proficiencies are as follows:

- Conceptual understanding—comprehension of mathematical concepts, operations, and relations
- Procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- Strategic competence—ability to formulate, represent, and solve mathematical problems
- Adaptive reasoning—capacity for logical thought, reflection, explanation, and justification
- Productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy (NRC, 2001, p. 5).

These strands are not distinct from one another. They are intertwined and interdependent. The authors of the report suggested that students gain these proficiencies in a coordinated, interactive fashion. For students to be mathematically proficient, all areas such as curriculum, instructional materials, assessments, classroom practice, teacher preparation, and professional development have to be changed fundamentally (NRC, 2001). The changes cannot be made at once and independently, but continuously, coordinately, and in integrated and balanced manner.

The strands of mathematical proficiency from Adding It Up (NRC, 2001) influenced the SMP along with the NCTM’s (2000) Process Standards. Thus, there is an intimate connection
between the SMP and the proficiencies that *Adding It Up* suggested. In Figure 2-9, I present this alignment. As shown, a practice standard does not match with exactly one of the strands of the mathematical proficiencies; rather, a standard of the eight SMPs incorporates more than one strand of the mathematical proficiencies. For example, SMP 1 ("Make sense of problems and persevere in solving them") consolidates the strands of conceptual understanding, strategic competence, adaptive reasoning, and productive disposition. For students to “make sense of problems and persevere in solving them,” students should understand the meaning of the solution and explain the correspondence and relations; formulate, represent, and solve mathematical problems; make conjectures, plan a solution, and justify; and make sense of problems and understand the approaches of others to solving them.

<table>
<thead>
<tr>
<th>SMPs</th>
<th>Five Strands for the Mathematical Proficiencies</th>
</tr>
</thead>
</table>
| SMP 1 Make sense of problems and persevere in solving them. | Conceptual understanding  
Strategic competence  
Adaptive reasoning  
Productive disposition |
| SMP 2 Reason abstractly and quantitatively. | Conceptual understanding  
Strategic competence  
Adaptive reasoning  
Productive disposition |
| SMP 3 Construct viable argument and critique the reasoning of others. | Conceptual understanding  
Strategic competence  
Adaptive reasoning  
Productive disposition |
| SMP 4 Model with mathematics.             | Conceptual understanding  
Strategic competence  
Productive disposition |
| SMP 5 Use appropriate tools strategically. | Conceptual understanding  
Procedural fluency  
Strategic competence  
Productive disposition |
| SMP 6 Attend to precision.                | Conceptual understanding  
Procedural fluency  
Strategic competence |
Mathematical Habits of Mind

The SMP describes multiple dimensions of mathematical thinking as well as habits of mind that students should develop. The term, *mathematical habits of mind*, has been used to illustrate what it means to do mathematics and think mathematically. According to Levasseur and Cuoco (2003), the mathematical habits of mind enable us to reason about the world from a quantitative and spatial perspective. These habits empower students to use their “experienced” problem solving skills when confronted with problems. However, the mathematical habits of mind have not been actualized in teachers’ practices and textbooks, although the fundamental idea of building mathematics programs grounded in mathematical habits of mind has been advocated for decades (Seeley, 2014).

Cuoco and his colleagues (1996) suggested general “habits of mind.” These general habits of mind can be applied to students in all grade levels. I have summarized the general habits of mind as follows:

- Students should be pattern sniffers: Students find hidden patterns, look out for shortcuts from patterns in calculation, and search for regularity.
● Students should be experimenters: When faced with a mathematical problem, students immediately play with it. They also develop a healthy skepticism about the result of the experiment.

● Students should be describers: Students describe the steps in a process, invent notation, argue, and write their thoughts.

● Students should be tinkerers: Students take ideas apart and put them back together and think about what happens when they are doing this.

● Students should be inventors: Students create mathematics such as rules for games, algorithms, explanations, or axioms.

● Students should be visualizers: Students visualize reasoning, data, relationships, processes, changes, calculations, etc.

● Students should be conjecturers: Students make plausible conjectures.

● Students should be guessers: Students guess possible solution to a problem and find a closer approximation to the desired result.

It is interesting to note that the habits of mind and the SMP share similar expectations for the students. In fact, Seeley (2014) stated that “the Common Core State Standards’ explicit attention to mathematical habits of mind is represented by the Standards for Mathematical Practice” (p. 249). In Figure 2-10, I have matched the general habits of mind with the eight standards from the SMP. Some of these standards overlap aspects of the descriptions in the habits of mind. Thus, some habits of mind are matched with more than one SMP. For SMP 5, a match could not be found in general habits of mind that Cuoco and others (1996) have suggested. However, their explanation about geometric and algebraic approaches included more of the SMPs. I have listed them in parentheses.

Some experts and practitioners may arrive at different ways of cross-matching, because the interpretation and understanding of the standards can be different from mine. As Seeley
(2014) states, it is “far less important to identify which particular standard(s) a given problem or practice addresses than it is to look for opportunities to focus on and help students develop one or more of the practices within the context of the problem” (p. 251).

<table>
<thead>
<tr>
<th>SMP</th>
<th>Habits of Mind</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMP 1 Make sense of problems and persevere in solving them.</td>
<td>Students should be experimenters  Students should be conjecturers  Students should be guessers</td>
</tr>
<tr>
<td>SMP 2 Reason abstractly and quantitatively.</td>
<td>Students should be describers  Students should be tinkerers  (Algebraists use abstraction)</td>
</tr>
<tr>
<td>SMP 3 Construct viable argument and critique the reasoning of others.</td>
<td>Students should be describers  Students should be conjecturers</td>
</tr>
<tr>
<td>SMP 4 Model with mathematics.</td>
<td>Students should be experimenters  Students should be describers  Students should be visualizer  (Algebraists represent things)</td>
</tr>
<tr>
<td>SMP 5 Use appropriate tools strategically.</td>
<td>(Geometers love shapes)</td>
</tr>
<tr>
<td>SMP 6 Attend to precision.</td>
<td>Students should be describers</td>
</tr>
<tr>
<td>SMP 7 Look for and make use of structure.</td>
<td>Students should be tinkerers  Students should be inventors  (Algebraists break things into parts)</td>
</tr>
<tr>
<td>SMP 8 Look for and express regularity in repeated reasoning</td>
<td>Students should be pattern sniffers  Students should be inventors  Students should be guessers</td>
</tr>
</tbody>
</table>

Figure 2-10: Comparison of the SMP and the Habits of Mind

Mathematicians and mathematics educators have considered opportunities to develop students’ mathematical habits of mind in mathematics classrooms. Some encouraged teachers to provide their students with high-level, cognitively demanding tasks (e.g., Stein, Smith, Henningsen, & Silver, 2000). These tasks require students to engage with concepts and stimulate them “to make purposeful connections to meaning of relevant mathematical ideas lead[ing] to a different set of opportunities for student thinking” (Stein et al., 2000, p. 11).
Standards in other subjects

The CCSSI is a state-led effort that the NGA and the CCSSO coordinated (NGA & CCSSO, 2010a). The CCSSI suggested educational recommendations for mathematics and ELA/Literacy Standards. In the next sections, I briefly explain the CCSS of ELA/Literacy. I then explain the Next Generation Science Standards (NGSS) for K–12 science as the NGSS were developed under the influence of the CCSS.

The Common Core State Standards for English Language Arts and Literacy

Drawn upon high-performing international models, research, and input from numerous sources, CCSS—both Mathematics and ELA/Literacy Standards—was established and adopted by most of the states in the U.S. (NGA & CCSSO, 2010a). The CCSS was created to provide guidelines for K-12 curriculum and instruction with “fewer, clearer, [and] higher” (Phillips & Wong, 2010) standards. Both standards for mathematics and ELA/Literacy outline their contents explicitly.

The CCSSM delineate eight practice standards that mathematics educators at all levels should seek to develop in K-12 students using the language of what “mathematically proficient students” know and able to do. Similarly, ELA/Literacy Standards describe what skills and knowledge students should develop in ELA and literacy area. Each of the College and Career Readiness Anchor Standards for Reading, Writing, Speaking and Listening, and Language specifies the skills and understanding that all students should demonstrate. The authors of the standards noted that students should read a wide range of texts and learn literary and cultural knowledge through reading. In addition, students’ participation in various discussions and contribution to the conversations were emphasized (NGA & CCSSO, 2010b).
As the new standards were adopted, teachers of ELA/Literacy were called to understand the shifts in standards, instructional practices, curriculum materials, and assessment in accordance with the CCSS for ELA/Literacy. Their instructional practice should be responsive to these shifts and rely on deep content knowledge as well as pedagogical knowledge (Santos, Darling-Hammond, & Cheuk, 2012).

The Next Generation Science Standards

Soon after the CCSS for mathematics and ELA/Literacy was released in 2010, the writing team for the science standards began to work in 2011. The final draft of the Next Generation Science Standards (NGSS) for K–12 science was released in 2013. During the development process of NGSS, 26 lead state partners collaborated to provide rigorous standards set for high quality K-12 science education. NGSS was created in order to provide science educators flexibility in the instruction while stimulating students’ interests in science (NGSS Lead States, 2013).

The Framework for K-12 Science Education, which is the foundation of NGSS, reflects the expectations for what students should know and be able to do in science classrooms. The Framework presents three important dimensions that are reflected in each standard of NGSS (NRC, 2012):

Dimension 1: Practices
This dimension describes how students should think and do like scientists and engineers by investigating and building models, theories, and systems. The word, “scientific inquiry” is emphasized as cognitive, social, and physical practices.

Dimension 2: Crosscutting Concepts
This dimension emphasizes interrelating knowledge from various domains of science. Students should develop crosscutting concepts to deepen their understanding across a broad realm of science.

**Dimension 3: Disciplinary Core Ideas**

This dimension describes the core ideas for K-12 science curriculum, instructional practices, and assessments. Across the science domains, the core ideas should:

- have broad importance across multiple science or engineering disciplines or be a key organizing concept of a single discipline;
- provide a key tool for understanding or investigating more complex ideas and solving problems;
- relate to the interests and life experiences of students or be connected to societal or personal concerns that require scientific or technological knowledge; and
- be teachable and learnable over multiple grades at increasing levels of depth and sophistication (NGSS Lead States, 2016, para. 4)

The CCSS explains what knowledge and skills in mathematics and ELA/Literacy K-12 students should develop for success in college and careers upon high school graduation. The NGSS delineates the very similar expectations of the CCSS in the areas of science education. Researchers (e.g., Pruitt, 2014) present some challenges to implement NGSS such as development of quality materials and teachers’ understanding of the standards. Pruitt (2014) suggested 3-4 years of implementation timelines after adoption because it would take time for teachers to know and conceptualize the vision of the standards and develop teaching practices according to the revised set. He also calls for study about teacher’ efficacy regarding their understanding of scientific practices.

**Teacher conceptions matter**

In education field, it is widely accepted that teachers’ conceptions—including beliefs, knowledge, and attitudes—influence their behaviors in the classrooms and that it is necessary to
understand teachers’ conceptions structures to improve their teaching practices (Ashton & Webb, 1986; Brookhart & Freeman, 1992; Buchmann, 1984; Clark, 1988; Dinham & Stritter, 1986; Feiman-Nemser & Floden, 1986; Nespor, 1987; Weinstein, 1988, 1989). As Fenstermacher (1978) predicted several decades ago, the study of teachers’ beliefs for effective teaching has received great attention. In the early 1980s, researchers began to view more the importance of teachers’ active and cognitive factors that would affect student learning in classrooms (Peterson, Fennema, Carpenter, & Loef, 1989). The shift in research on teaching coincided with a change of the focus from examining teachers’ knowledge in quantitative terms, such as the number of years of teaching experiences, the number of college courses taken, standardized test scores, etc., to studying teachers’ cognitive process and decision-making (Lloyd & Wilson, 1998).

Mathematics education is no exception to this wave. Since the 1980s, a number of studies have probed how teachers’ mental schemas—including teacher conceptions—are related to their teaching practices in classrooms. In the last few decades, the need to study teachers’ mental processes has received increased attention and a substantial body of studies has investigated teachers’ conceptions. These studies sought better understanding of the nature of teachers’ beliefs about teaching and learning as well as the links between teacher beliefs and their teaching practice (e.g., Ball, 1990a; Ball, 1991; Bauch, 1984; Brendefur and Frykholm, 2000; Bush, Lamb, & Alsina, 1990; Even, 1993; Ferrini-Mundy, 1986; Fennema et al., 1996; Leinhardt, Putnam, Stein, & Baxter, 1991; Lloyd & Wilson, 1998; Pajares, 1992; Silver, 1985; Staub & Stern, 2002; Thompson, 1984).

Students’ learning is closely related to what and how teachers teach. Furthermore, what and how teachers teach are intimately associated to teachers’ beliefs, content knowledge, and pedagogical knowledge (Ball, 1990a; Ball, 1991; Even, 1993; Leinhardt, Putnam, Stein, & Baxter, 1991; Lloyd & Wilson, 1998). As stated in the report of the National Commission on Teaching & America’s Future (1996), “what teachers know and can do makes the crucial
difference in what teachers can accomplish” (p. 5). In mathematics education, teachers’ conceptions about mathematics and teaching/learning mathematics play a significant role in stimulating the effectiveness of student learning.

In the next sections, I review the research about teacher conceptions—focusing on beliefs and knowledge—in mathematics education. Furthermore, I discuss the relationship of teacher beliefs and knowledge with their instructional practices. Then I examine what researchers have found about teacher conceptions of the Standards.

**Teacher beliefs in mathematics education**

Since the 1980s, researchers have actively studied mathematics teachers’ beliefs. Studies have reported that teachers—including both in-service and prospective teachers—tend to view mathematics as a static body of knowledge that is skill-based, complete, and rigid (Agudelo-Valderrama, 2008; Collier, 1972; Gregg, 1995; Philipp, 2007; Steffe, 1990; Wilson, 1994). Ernest (1989a) noted that “teachers’ mental contents or schemas, particularly the system of beliefs concerning math and its teaching and learning” (p. 249) as key elements that influence teachers’ instructional practice.
Raymond (1997) regarded teacher’s mathematics beliefs as central to the relationship between beliefs and practice, as shown in the model above (Figure 2-11). This model suggests that teachers’ beliefs on mathematics are derived not only from their prior school experiences as students, but also from the influence of their prior teachers or teacher preparation programs. Raymond (1997) notes that mathematical beliefs and instructional practices are not totally consistent, although there appear to be a direct relationship between beliefs and practices in the model. In fact, researchers have found different relationships between teacher beliefs and their instructional practices. Some identified consistencies (e.g., Kaplan, 1991; Peterson et al., 1989), while others claimed inconsistencies between teacher beliefs and their mathematical practices in classrooms (e.g., Cooney, 1985; Thompson, 1984).

Ernest (1989a) also modeled the relationship between teachers’ beliefs and their impact on teaching practice as shown in Figure 2-12. He explains that a teacher’s view about nature of mathematics is the foundation of teachers’ espoused and enacted models of teaching and learning mathematics. Teachers’ aspect of what mathematics is, their beliefs about teaching and learning...
mathematics, and their teaching practice are thus closely related. For example, a teacher who holds a “problem solving view of mathematics” intends to facilitate problem posing and solving in his/her instruction. Ernest (1989a)’s model of teacher’s belief and practice seems similar to Raymond’s (1997) model illustrated in Figure 2-11. They both assert that teacher beliefs affect their (either intended or enacted) teaching practices. As Raymond (1997) noted, teachers’ beliefs and their practices are not always consistent. Ernest (1989a) explains this mismatch with constraints and opportunities provided by the social context of teaching such as expectations or pressure from students, parents, peer teachers, and school/district administrators. According to Ernest (1989a), the degree of teachers’ awareness of his or her own beliefs as well as teacher’s reflection on his or her own practice affect the enactment in mathematics classrooms.

![Ernest's model: relationship between mathematics beliefs and teaching practice](Ernest, 1989a, p. 252)

Figure 2-12: Ernest’s model: relationship between mathematics beliefs and teaching practice (Ernest, 1989a, p. 252)

Drawn upon mathematics educators and researchers’ definitions and categorization of mathematics beliefs (e.g., Cobb, 1986; Ernest, 1989a; Hart, 1989; Lester, Garofalo, Kroll, 1989; McLeod, 1989; Raymond, 1997; Skott, 2001; Thompson, 1992), I regard nature of mathematics, learning mathematics, and teaching mathematics as the essence of mathematics teachers’ beliefs.
Of these three key components of mathematics teachers’ beliefs, I focus on teachers’ beliefs on teaching mathematics. This includes a relationship between teacher beliefs and their instructional practice. The body of research examined an impact of in- or pre-service teachers’ beliefs on their instructional practices or a relationship between what teachers believe and what they do in classrooms (e.g., Beswick, 2012; Brosnan, 1994; Brosnan, Edwards, & Erickson, 1996; Ernest, 1989a; Frykholm, 1999; Leder et al, 2006; Nespor, 1987; Pajares, 1992; Polly et al., 2013; Raymond, 1997; Richardson, 1996; Skott, 2001; Stipek, Givvin, Salmon, & MacGyvers, 2001; Thompson, 1984; Wilson, 1999). These studies showed some variability in degrees of the relationships. Some of them discovered that the teachers’ beliefs and their mathematics teaching practice were not consistent (e.g., Hoyles, 1992; Raymond, 1997; Skott, 2001; Sztajn, 2003), while others found that teachers’ beliefs were closely related with their teaching of mathematics (e.g., Borko, Mayfield, Marion, Flexer, & Cumbo, 1997; Guskey, 1986). They suggested that teachers’ beliefs played important roles in deciding their instructional actions.

Researchers examined not only the relationships between teachers’ beliefs and their instructional practices, but they also probed into other areas such as teachers’ beliefs within curriculum material or curriculum reform. These studies took teachers’ beliefs about reform-based teaching of mathematics into account, based on the beliefs that “teachers are key to the success of the…reform movement in U.S.A. mathematics education” (Battista, 1994, p. 462) and “teaching reforms cannot take place unless teachers’ deeply held beliefs about mathematics and its teaching and learning change” (Ernest, 1989a, p. 249). Among these studies, some investigated teachers’ beliefs and mathematics teaching practices in relation to the NCTM standards. These studies focused on measuring teachers’ belief systems about the Standards. Some investigated in-service teachers’ beliefs of the NCTM vision. Most of such research used an instrument to measure teacher beliefs (e.g., Alba, 2001; Carter & Norwood, 1997; Futch & Stephens, 1997; Zollman & Mason, 1992). For example, Zollman and Mason (1992) developed
Standards Beliefs Instrument (SBI) to assess teachers’ beliefs underlying the 1989 Standards. SBI used 4-point Likert scale (1 = strongly agree, 2 = agree, 3 = disagree, 4 = strongly disagree) for 16 items that represent the NCTM Standards for every grade level (non-grade specific).

Researchers have continued to examine teachers’ beliefs since the CCSSM was adopted (e.g., Buehl & Beck, 2015; Carney et al., 2016; Chen et al., 2014; Davis et al., 2013; Davis et al., 2014; Davis, Chopping, Drake, & McDuffie, 2017; Felbrich, Kaiser, & Schmotz, 2014; Lui, & Bonner, 2016; Polly et al., 2013; Porter, Fusarelli, & Fusarelli, 2015; Skott, 2015). The results of these studies consistently showed the importance of teachers’ beliefs about teaching and learning mathematics as well as nature of mathematics. For example, Lui and Bonner (2016) asserted that both in-service and preservice teachers had shown more constructivists than traditional beliefs. However, the participating teachers’ beliefs and their conceptual or procedural knowledge didn’t present significant relationships.

**Teacher knowledge in mathematics education**

Researchers often distinguish knowledge from belief. They regard knowledge as “belief with certainty (Clement, 1999 as cited in Philipp, 2007), a “stronger condition than belief (Scheffler, 1965 as cited in Wilson & Cooney, 2002) and a “true belief” (Philipp, 2007).

Studies examined teachers’ knowledge to teach mathematics using various models such as Pedagogical Content Knowledge (PCK; Shulman, 1986), the Instructional Triangle (Cohen & Ball, 1999, p. 3), and Mathematical Knowledge for Teaching (MKT; Ball et al., 2008). The model of MKT by Ball and others (2008), in particular, has guided a significant number of studies of teacher knowledge to teach mathematics (See Figure 2-13). Developed specifically for mathematics teaching and built upon Shulman’s (1986) idea of pedagogical content knowledge (PCK), MKT relates mathematical content knowledge to the practice of teaching mathematics.
As depicted in the diagram below, MKT includes both the subject (mathematics) matter knowledge for those who work with mathematics and the PCK that is specialized to teach mathematics.

![Diagram of Subject Matter Knowledge and Pedagogical Content Knowledge](image)

Figure 2-13: Mathematical knowledge for teaching (Adopted from Ball, Thames, & Phelps, 2008)

The MKT comprises six areas of knowledge: common content knowledge, horizon content knowledge, specialized content knowledge, knowledge of content and students, knowledge of content and teaching, and knowledge of content and curriculum. Of these six domains of the MKT, I focus on the Knowledge of Content and Curriculum (KCC), which defines teachers’ knowledge about the curriculum and encompasses curriculum materials, assessments, and standards.

The KCC is “represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to these programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances” (Shulman, 1986, p. 10). The “set of characteristics” or “standards” guide teachers’ use of curriculum, assessment, and instructional practices.
There have been numerous studies about teacher knowledge. These studies focused on exploring the construct of teachers’ content and pedagogical knowledge of mathematics by utilizing qualitative methods (e.g., qualitative study approach conducting interviews, and classroom observations) on examining the degree of the relationship between teacher knowledge and the quality of instruction (Aubrey, 1997; Borko et al, 1992; Campbell et al., 2014; Cohen, 1990; Fennema et al., 1996; Hill et al., 2008; Lloyd & Wilson, 1998; Ngo, 2013; Putnam et al, 1992; Thompson & Thompson, 1994). For example, Campbell and her colleagues (2014) examined the relationship between elementary school teachers’ mathematical content knowledge and their students’ mathematics achievement. Hill and her colleagues (2008) examined teachers’ MKT in their series of case studies to find a strong, positive relationship between levels of teachers’ MKT and the quality of mathematical instructions. Others attempted to measure MKT and tested the validity of the measurement (e.g., Kane, 2004; Schilling, Blunk, & Hill, 2007). In their study, Schilling and his colleagues (2007) examined the “MKT interpretive argument” in several other studies to revise the measurement and to provide guidance for the future studies that would concern argument-based test validation.

**Teacher conceptions in the reform movement**

Influenced by widely adopted standard-based school mathematics curricula along with the intensive attention to teachers’ conceptions (e.g., teacher beliefs and knowledge) about teaching and learning of mathematics, researchers in mathematics education became interested in examining teachers’ conceptions about the NCTM Standards. A number of studies probed into the effects of the Standards on various areas such as teachers’ beliefs, pedagogical knowledge, instructional practices, teacher education, professional development, student learning, learning environment, and educational policy (e.g., Benbow, 1995; Battista, 1994; Borko et al., 1992;
The major aspects of the reform movement in mathematics education in 1990s and 2000s are about content and pedagogy. The contents of mathematics inclined to embrace topics that require students’ conceptual understanding and problem solving more than computations and procedures. Pedagogical view for the mathematics education also changed throughout the reform movement. Educators turned to see mathematics learning and teaching from behaviorists’ view to constructivists. This means that the focus of the mathematics teaching has turned from computational skills to constructing students’ own understanding (Battista, 1994).

Teachers’ beliefs, knowledge, and attitude toward this movement is critical as numerous studies have presented the close relationships of teachers’ conceptions and their instructional practices regardless of some variations of the degrees (e.g., Borko et al., 1992; Frykholm, 1999; Raymond, 1997; Smith III, 1996; Skott, 2001; Stipek et al., 2001; Wilson, 1994). In the extension of this line of studies, there was research about teachers’ beliefs and their practices regarding the NCTM Standards.

On one hand, some researchers have found that teachers’ beliefs with respect to the reform movement were consistent with their instructional practices in the math classrooms (e.g., Brosnan, 1994; Stipek et al., 2001; Yates, 2006). For example, Brosnan (1994) observed how teachers and their classrooms had changed as they increased the use of the NCTM Standards in their mathematics program. For this study, teachers’ motivation to implement the standards and their instructional changes were examined through a mathematical reform program (Model Mathematics Program). He explained the changes and possible effect to the changes as follows: The administrative support through the program intrigued the teachers’ motivation to implement standard- and inquiry-based instruction, although some teachers experienced difficulties in employing new methodologies; the participating teachers developed their views toward students’
conceptual understanding, student-centered instruction, communication, and appropriate use of technology; and the teachers’ behaviors in their classrooms had changed accordingly such as great reduction of teaching procedures and of assigning drill and practice work.

On the other hand, some researchers found the inconsistencies between teachers’ beliefs and their mathematics teaching practices (e.g., Borko et al., 1992; Frykholm, 1999; Raymond, 1997; Skott, 2001; Wilson, 1994). For example, Frykholm (1999) examined the impact of the NCTM Standards on prospective teachers in a teacher preparation program. He compared their perceptions of the reform movement with their teaching practice. In his study, the participating teachers exhibited some levels of optimism about implementing standards-based teaching strategies. However, they responded that they did not receive enough preparation to practice the Standards and that they could not engage in the mathematics experiences that matched with the reform movement.

Another line of study around teacher conceptions in the reform movement speculated teacher conceptions and their impact on curriculum reform. As shown in some studies, mathematics teachers’ beliefs either facilitated or inhibited a certain curriculum. When teachers held more adaptable beliefs toward innovation, they likely accepted and implemented the reform-based curriculum in their classrooms (Handal & Herrington, 2003; Haynes, 1996; Koehler & Grouws, 1992; Sosniak, Ethington, & Varelas, 1991). Likewise, teachers who held opposing beliefs seemed to resist adoption of the new curriculum (Burkhardt, Fraser, & Ridgway, 1990; Prawat, 1992). Thus, teacher conception is an important factor in predicting success of reform-based curriculum implementation.

Several studies, still, presented a mismatch between teachers' beliefs and the beliefs underlying particular curricular innovations (e.g., Anderson & Piazza, 1996; Brew, Rowley, and Leder, 1996; Frykholm, 1995; Sowell & Zambo, 1997; Watts, 1991). These studies explained that teachers’ beliefs were not well-aligned with the vision of reform movement.
Summary

In mathematics education history, there have been prominent mathematics education standards and principles such as the NCTM Standards (NCTM, 1989, 1991, 1995, 2000) as well as proficiencies (e.g., Adding It Up) and “mathematical habits of mind.” The SMP, influenced by these standards and principles, “describe varieties of expertise that mathematics educators at all levels should seek to develop in their students” (NGA & CCSSO, 2010a, p. 6).

In the field of education, it is widely accepted that teachers’ conceptions—including beliefs and knowledge—influence their behaviors in the classrooms. Researchers have used models (e.g., Ernest, 1989b; Raymond, 1997) to examine the relationships between teachers’ beliefs and their instructional practices. Furthermore, studies have examined teachers’ knowledge to teach mathematics using various models such as PCK (Shulman, 1986), the Instructional Triangle (Cohen & Ball, 1999, p. 3), and MKT (Ball et al., 2008).

There have been various areas in which researchers have studied teachers’ conceptions since the reform movement. Studies have examined the relationship between levels of teacher knowledge or beliefs and the quality of mathematical instruction, the relationship between teacher knowledge or beliefs and students’ achievement, and construct of teachers’ content and pedagogical knowledge or beliefs of mathematics.
Chapter 3

Methodology

In this chapter, I discuss the research methodology employed to conduct this study. The purpose of this study is to identify and document elementary school teachers’ conceptions (See Figure 1-3 for the definition of “conception”) relating to the Common Core State Standards for Mathematical Practice (SMP). In this qualitative study of eight elementary school teachers, I employed in-depth interviews. This chapter includes the research question, descriptions of the participants and settings, and the procedures of data collection and analysis.

Statement of the problem

Mathematics educators and researchers have put forth considerable efforts to improve teaching and learning of mathematics. Standards in mathematics education can be regarded as products of these efforts. When the nation was considered to be “at risk” of shortcomings in mathematics education in the 1980s, “standards” took on the parlance in the U.S. mathematics education field. The NCTM Standards (1989, 1991, 1995, 2000) have played the leading role to improve school mathematics education. Most recently, the majority of the states have adopted the Common Core State Standards (CCSS).

The CCSS were developed to provide consistent and achievable goals and quality of education (in mathematics and ELA/Literacy as an initial point) for all students across the nation. A greater number of the states in the U.S. has adopted the CCSS as a guidepost to prepare students for success in college and the workplace. Within the CCSS for Mathematics (CCSSM), the Standards for Mathematical Practice (SMP) delineates mathematical proficiencies and
competencies that K-12 teachers should develop in their students. The CCSSM emphasize the importance of the SMP by presenting this set of practice standards ahead of the content standards. The CCSSM recommends mathematics teachers (furthermore, designers of curricula, assessments, and professional development) to associate the content standards with the practice standards.

Teacher conceptions about the standards are important factors that affect teachers’ intended or implemented instructional practices. However, the studies that investigated teachers’ conceptions about the SMP are scarce and limited. Most are quantitative studies that utilized a survey method (e.g., Bostic & Matney, 2013; Burks et. al., 2015; Choppin et al., 2013; Cogan et al., 2013; Davis et al., 2013; Davis et al., 2014; Troia & Graham, 2016). While these quantitative studies explained teachers’ conceptions clearly using the numerical data, they did not explain teachers’ more detailed, complex parts of the conceptions such as their interpretation of each SMP and their understanding about their own implementation of the SMP. As Heck and his colleagues (2011) suggested, it would be necessary to conduct a qualitative study to examine teachers’ conceptions of the CCSSM—the SMP, in particular—that examine “teachers’ knowledge, interpretations, self-efficacy, beliefs, dispositions, and skills as well as their specific intentions and plans” (p. 13). The question that guided this study is: How do the participating elementary school teachers understand and interpret the SMP?

Research design

In this study, the center of research interest is on elementary school teachers’ conceptions of the SMP. In Chapter 1, I define “conception” as a general notion or mental structure incorporating beliefs, knowledge, propositions, dispositions, interpretations, preferences, views, agreement, and mental images.
In order to learn a more in-depth and detailed conceptions of teachers, it is best to conduct a qualitative research. My methodological orientation is rooted in in-depth interview study, focusing on the elementary school mathematics teachers in western Pennsylvania. This qualitative study analysis was conducted through in-depth interviews to identify teachers’ beliefs that might be aligned with the SMP as well as to what extent teachers understand and interpret the SMP.

According to Yin (2009), there are three conditions to consider when to using a research method: “(1) the type of research question posed, (2) the extent of control an investigator has over actual behavioral events, and (3) the degree of focus on contemporary as opposed to historical events” (p. 8). The research question I pose in this study is a ‘how’ question. While ‘who,’ ‘what,’ and ‘where’ questions are exploratory and are meant to develop hypotheses and propositions, ‘how’ and ‘why’ questions are explanatory and are more likely to involve the use of interviews and observations. The second condition to consider when deciding a research method is the extent of control an investigator has over behavioral events. This is closely related to the first condition, the type of the questions. As my questions ask ‘how,’ they are more explanatory than experimentory. In other words, I neither design an experiment nor manipulate a behavior. Lastly, a qualitative study is more acceptable in examining contemporary events. Despite the fact that both historical research and qualitative study are descriptive and rely on many of the same techniques (e.g., using some evidences such as artifacts and documents), there are some distinctions between the two. While histories deal with the past, case studies concern contemporary events. Moreover, case studies draw data from multiple sources such as interviews, observations, artifacts, and documents. Because my research question asks about “a contemporary set of events”—teacher’s understanding about the newly adopted SMP—the qualitative study method is applicable (Yin, 2009, p. 13). This qualitative study that is very
similar to a qualitative study, focuses greatly on in-depth interviews of eight in-service
elementary school teachers’ conceptions.

A number of studies that investigated teacher’ conceptions of standards employed
qualitative methods. These studies employed interviews and classroom observations including
vignettes and conversations of teachers and students, teacher talk, etc., to provide detailed
descriptions of teacher conceptions (e.g., Alba, 2001; Battista, 1994; Brosnan et al., 1996; Collier,
1972; Porter, Fusarelli, & Fusarelli, 2015; Raymond, 1997).

Participants

Selecting the proper participants is as important as other procedures in conducting a
qualitative study that is very similar to a case study. There are details to pay attention to when
choosing the research participants. For example, Saunders (2012) suggests the qualitative
researchers to consider: (1) gaining access or permission to collect data, (2) enabling the
collection of appropriate data, (3) use of non-probability sampling techniques, and (4) the number
of participants needed. The choice of eight participants for this study was made based on all four
suggestions. First, the IRB process enabled my collection of data from school teachers. Second,
the participants do not need to represent the population in non-probability sampling of qualitative
research. Third, my participants can be considered to be “purposive samples” with a small
number of participants, as in most case studies (Miles & Huberman, 1994). Lastly, the suitable
size of the participants was determined after discussing with my dissertation committee members.
Creswell (2007) suggests between 5 to 25 participants.

The participants for this qualitative research are in-service elementary school teachers
who have taught mathematics in four school districts of western Pennsylvania. The
characteristics of the schools are quite similar among these four school districts: The majority of
the enrolled students’ race/ethnicity is white, non-Hispanic; the enrollment of the students is no more than 450 per school; and the school districts serve about 300 - 1400 students from Pre-K to 12. The district facilities include 1 to 4 elementary schools (grades K-5) where about 16 - 45 teachers work\footnote{The data of each school is accessible at https://nces.ed.gov/globallocator/}.

The eight participants’ mathematics teaching experiences in elementary schools vary from 5 to 34 years. Average of mathematics teaching experience of the participating teachers is 12 years. Four teachers used to be or currently are the mathematics liaisons or mathematics coaches for their schools or school districts and all eight teachers hold a master’s degree or higher in education field. Detailed profiles for each teacher are described in Chapter 4.

It is common for elementary school teachers in western Pennsylvania provide instruction in all subjects, but some schools departmentalize in upper grade levels (grades 4-6). As opposed to self-contained instruction in which one teacher teaches all subjects, departmentalized instruction is run when each teacher of a grade teaches one academic subject in his/her area of expertise. Thus, some of the elementary school teachers teach only mathematics while others teach all subjects including mathematics. Four participants teach all subjects for a class of students and the other four teach only mathematics or focus on teaching mathematics. Both types of teachers became the participants for this study as they have taught elementary mathematics in the school districts in western Pennsylvania.

**Data sources and collection**

Data for this study includes interviews, email exchanges, teaching materials, and artifacts. The multiple data resources provided me with more credible sources than if I had been
limited to only one source. This *triangulation* allowed me to ensure the validity of my research and to gain a more secure understanding of teachers’ conceptions about the SMP by using multiple resources to support the same conclusion. In my study, the in-depth interview is the core data resource. However, other resources such as email conversations, teaching materials, and artifacts supplemented the drawn inference about teacher conceptions.

**Triangulation**

In qualitative research, collecting data using multiple methods is common. According to Patton (2014), this is one of the four types of *triangulation*. The four types of triangulation that Patton discussed are as follows: (1) data triangulation, (2) investigator triangulation, (3) theory triangulation, and (4) methodological triangulation.

This study pertains to the first of these four types, data triangulation, as I collected information from multiple sources (interviews, email exchanges, teaching materials, and artifacts), aiming for the same conclusion. I tried to analyze all sources that support my findings, despite the greater portion in the analysis of the interviews. Yin (2009) suggests using multiple sources of evidence properly to maximize the benefits of triangulation: high validity and reliability. The validity of the qualitative study could be secured when using multiple sources of evidence. However, “use of *in-depth interviews* alone, when done with skill, can avoid tensions that sometimes arise when a researcher uses multiple methods [emphasis added]” (Seidman, 2006, p. 6). During the interviews, I tried to provide as much space as the participants wanted to talk about their teaching, philosophy, and experiences. I discuss how I conducted the interviews and what I mean by “in-depth interview” in the following section.
Interviews

Qualitative interviews “are particularly well-suited for studying people’s understanding of the meanings in their lived world, describing their experiences and self-understanding, and clarifying and elaborating their own perspective on their lived world” (Kvale & Brinkmann, 2009, p. 116). The purpose of the interviews for this study was to uncover and analyze the elementary school teachers’ understanding of the SMP.

In this study, I often use the terms “in-depth interview” and “intensive interview” interchangeably. Seidman (2006) explicated in-depth interview meticulously. According to Seidman, “the purpose of in-depth interviewing is not to get answers to questions, nor to test hypotheses, and not to ‘evaluate’ as the term is normally used” (p. 9). He suggested three characteristics for in-depth questions. First, in-depth interviews use open-ended questions to give the participants capacity to reconstruct their experience. Second, in-depth interviews comprise a three-interview series, because a one-time interview may not provide the researcher a full insight of the participants’ experience. Third, the length of in-depth interviews is between 1 and 2 hours. Seidman recommended a 90-minute format. Boyce and Neale (2006) defined in-depth interviewing as “a qualitative research technique that involves conducting intensive individual interviews with a small number of respondents to explore their perspectives on a particular idea, program, or situation [emphasis added]” (p. 3). The interviews I employed for this study meets the characteristics that Seidman discussed, except for the second characteristic (series of three interviews). My interviews are intensive interviews in which only eight participants were involved, and questions were asked to gain detailed information about teachers’ thoughts and behaviors.

Before recruiting the participants, I created interview protocols. The interview protocols were carefully organized to address the research question, but not to limit the participants’
responses to a frame. Researchers recommend to pilot-test interview questions with one or two individuals close to the population of the study. Pilot test allows the researcher not only to practice the interview protocol, but also to gain important insider information, to find any flaw in the interview protocol, and to revise the interview protocol prior to the implementation (Jacob & Furgerson, 2012; Turner, 2010). I pre-tested the interview protocol with an in-service elementary school teacher. After the pilot interview, I revised the protocol based on the feedback from the teacher as well as from colleagues who were experienced mathematics educators with more than 10 years of teaching experience in western Pennsylvania. I made several revisions of the interview protocol thereafter to best fit the purpose. The interview consisted of six parts: 1) teachers’ experience of mathematics teaching, 2) teachers’ beliefs about teaching and learning of mathematics, 3) teachers’ beliefs about standards (In particular, the SMP), 4) teachers’ practices, 5) support/obstacles to learn about the SMP, and 6) overall impression about the SMP (See Appendix B for the detailed questions for each part).

I organized the interview protocols not only to address the research question, but to draw more inference by not limiting the participants’ responses. Thus, the interview questions were open-ended. Open-ended questions are useful to generate a rich understanding of each teacher’s conceptions about the SMP and to uncover the participants’ honest thoughts and individualized experiences about the implementation of the SMP.

Once the interview protocol was finalized, I began the recruiting process by sending emails to plausible candidates. In the email, I introduced myself, explained the purpose of the study, clarified the strict confidentiality of the study (e.g., using pseudonyms for names of teachers, students, and schools), and indicated the compensation for the participation (each teacher received a $25 gift card). Once I received the teachers’ replies expressing their willingness to participate in the study, I set up the date, time, and place for the interviews according to their schedules. I conducted face-to-face interviews either at the teachers’ schools or
at my office. All interviews were audio-recorded with the participants’ consent. I transcribed these recordings later.

During the interviews, I guided the questions around the interview protocol, and tried to make the conversations flow naturally by changing the order of the questions whenever appropriate to allow the participants to speak freely. Thus, I utilized semi-structured interviews that provided flexibility and allowed for follow-up questions and prompts. In this format of semi-structured interviews, I asked most of the questions in the order of the protocol, but I sometimes asked additional questions, revisited earlier questions, and changed the order slightly depending on the flow of teachers’ statements. During the interviews, I encouraged the teachers to speak out their responses which enabled me to assess their “thought processes and the value judgements they bring to bear” about the SMP (Atkins & Wallace, 2012, p. 86). The interviews took between 1.5 and 2 hours.

In addition, I made notes on what I felt was significant during the interviews. The notes were very helpful in clarifying any confusing comment due to the audio-recording without the motion. For example, when Daisy mentioned, “It’s less you have to read into this one. This is a lot easier to read [emphasis added],” I had to look at my notes to see what “this” meant. When transcribing, I inserted clarification in brackets such as “It’s less you have to read into this one [Grade Level Emphasis]. This [Grade Level Emphasis] is a lot easier to read.”

**Email exchanges**

I collected data not only from the face-to-face interviews, but also from the email exchanges. The email conversations occurred before and after the interviews. Most of the emails before the interviews were to invite the participants to the study and to schedule the date, time, and place for the interview.
The emails after the interviews were made for two reasons. One was to retrieve additional information to supplement the interview conversation. For example, when a teacher mentioned about a kid-friendly version of the SMP, I asked her to send me the scanned file of it through an email. The second reason was for member checking. Member check is fundamental when conducting qualitative research. Through member checking, a participant receives the opportunity to inspect his/her statements for accuracy, credibility, and validity of what has been recorded during a research interview. “This is the single most important way of ruling out the possibility of misinterpreting the meaning of what participants say and do and the perspective they have on what is going on, as well as being an important way of identifying [the] biases and misunderstandings of what [I] observed” (Maxwell, 2013, pp. 126-127). I sent each participant the verbatim transcript to check the accuracy of the interview conversations. Member checking was made not only via emails after the interviews, but also during the interviews. Throughout the interviews, I often restated or summarized the information from the participants and asked them to determine the accuracy.

Teaching materials

Adding to intensive interviews and email exchanges, textbooks and other teaching materials were the resources of my data. I collected the textbooks the teachers were using at the time of the interviews and other artifacts (e.g., Pennsylvania System of School Assessment examples) to support the information from the participants. I either made a physical copy of the curricular materials (e.g., McGraw Hill Everyday Mathematics) or retrieved an access to the online textbooks (e.g., Engage New York).
Data analysis

The intensive interview was the main method to collect the data for this study. I conducted a face-to-face interview with each of the eight elementary school teachers. The interviews each took 1.5 - 2 hours and all interviews were audio-recorded with participants’ consent. I transcribed all eight audio-recorded interview data verbatim. Later, I categorized the statements from the verbatim transcription around the themes in the context of the research question. In this section, I explain how I organized and analyzed the collected data in detail. I focus on the transcription process, categorization of the transcripts, and refinement of the research question.

Transcriptions of the interviews

The verbatim transcription is widely considered to be intrinsic to the analysis and interpretation of interview data. Poland (1995) defines verbatim transcription as the word-for-word written form of reproduction of verbal data, where spoken words from the recorded ones are exactly replicated in a textual form. Poland argues that “the very notion of accuracy of transcription is problematic given the inter-subjective nature of human communication, and transcription as an interpretative activity” (p. 292). The way in which interview content is both heard and perceived by a transcriber plays a vital role in both the form and accuracy of transcription (MacLean, Meyer, & Estable, 2004). To minimize different, if not wrong, interpretation from the intention of the speakers, the verbatim transcription was checked with the interviewees later.

In transcription, I used the “turn numbers” to easily keep track of the exchanges on the dialogues between the interviewee and the researcher (See Figure 3-1 for an example). The odd
turn numbers are the researcher’s questions or comments while the even turn numbers are the interviewees’ responses. Henceforth, I write an interviewee’s name (in pseudonym) and turn number in the parenthesis whenever I refer the quotes from the transcripts. For example, I put (Alicia, Turn 12) for Alicia’s statement cited from turn number 12.

<table>
<thead>
<tr>
<th>Turn</th>
<th>Speakers</th>
<th>Talk (Activity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Researcher</td>
<td>Okay, that’s cool. In terms of the practice, what do you most value?</td>
</tr>
<tr>
<td>12</td>
<td>Alicia</td>
<td>That’s a tough one. Um, I think, as far as the practice goes, again it’s all problem-based learning, teaching students through experience, and I think that the best practice is teaching students through experience, rather than…</td>
</tr>
</tbody>
</table>

Figure 3-1: Verbatim transcription example

The confidentiality or anonymity in qualitative research is necessary for ethical reasons. The terms “confidentiality” and “anonymity” have been used interchangeably in research. However, some researchers distinguish the two words from one another. Saunders and his colleagues (2015) defined confidentiality as “a generic term that refers to all information that is kept hidden from everyone except the primary research team” and anonymity as “one form of confidentiality—that of keeping participants’ identities secret” (p. 617). The most common type of anonymisation comprises using pseudonyms. When using pseudonyms, it is possible to avoid revealing too much about any information about the interviewees. When transcribing the interviews, I assigned pseudonyms for every name of individual, school, school district, college or university, and organization that can be traced to find the identity of the participants.

I used square brackets [ ] and parenthesis ( ) in the transcriptions. The square brackets were used to add clarifying words or abbreviations and to describe nonverbal communications that occurred at the moment. For example, when asked about Eligible Content, Cecilia answered, “When I need to find it, I just google PA fourth grade Eligible Content, and it comes up. I think it’s on the Say Yes website [sayyestoeducation.org]. I believe it’s on there” (Cecilia, Turn 84).
The parenthesis was barely used. When either the interviewee or I uttered an answer like “Yep,” I put them in parenthesis rather than making it different turn number. One example of the use of a parenthesis is from the interview of Alicia: “Researcher: So, when you were a student, you learned multiplication as like a fact (Alicia: Yep). So, you had to memorize that (Alicia: Yep). So, did I” (Alicia, Turn 45).

After completing the first draft of all transcriptions, I reviewed them for two reasons: (1) to check for errors and (2) to highlight the key words. First, I made any change, if needed, to ensure the words were transferred correctly by both carefully listening to the recordings and reading the verbatim transcripts at the same time. While proofreading the transcripts, I paid close attention to the content and found the key words or key statements. I highlighted the texts that disclosed the characteristics of the participants such as to identify their conceptions or misaligned conceptions of the SMP, enactment of the SMP, the experiences, the change of practices, etc. In the next section, I explain the process of how I organized the data using these key statements more in-depth.

**Organization of data**

I began data analysis immediately after conducting the interviews. Maxwell (2013) affirms to analyze the collected data immediately and to continue analyzing them. Coffey and Atkinson (1996) also suggest, “never [to] collect data without substantial analysis going on simultaneously” (p. 2). Soon after I completed transcribing an interview, I re-read the transcription and listened to the audio-recording to make sure the transcription was flawless and to begin developing the meaningful themes.

Discussing about the data analysis, Maxwell (2013) asserts that there are various ways to analyze qualitative data. From “reading and thinking about [the] interview transcripts and
observation notes” to “writing memos, developing coding categories and applying these to [the] data, analyzing narrative structure and contextual relations, and creating matrices and other displays are all important forms of data analysis” (p. 105). While listening to the audio files and reading the verbatim transcripts again, I highlighted the key statements on the transcripts and made notes on a Word document.

**Categorization**

Some researchers differentiate coding from categorization (e.g., Kvale & Brinkmann, 2009), while others consider coding as a typical categorizing strategy in qualitative research (e.g., Maxwell, 2013). Kvale and Brinkmann (2009) ratify that coding involves “attaching one or more key words to a text segment in order to permit later identification of a statement (pp. 201-202)” and categorization is a more “systematic conceptualization of a statement” (p. 202).” Maxwell (2013) distinguished categorizing from connecting strategies, explaining two different modes of relationship—similarity and contiguity. According to Maxwell, categorization is “analytical strategies that focus on relationships of similarity” as similarity-relations involves comparison and contrast to define categories (p. 106). Maxwell confirms that coding is a typical categorizing strategy. Connecting (or contextualizing) strategies, on the other hand, focus on the contiguity-based relations where identification involves connections between things. Maxwell also suggests the possibility of combining the two strategies.

In this study, I adopt Maxwell’s (2013) term of categorization, which incorporates coding, but I do not completely distinguish it from connecting strategy. That is because my analysis of data was held both inductively and deductively. Categorization is a more inductive way of analysis as a researcher identifies and marks what is of interest to later develop a theme or
a code. But it can also be a deductive analysis as a researcher catches connecting statements in the context of the transcriptions, keeping a specific theme in mind (Maxwell, 2013).

The categorizing process began by identifying segments of data that seemed to contribute meaningful findings. During the process of re-reading the verbatim transcriptions and listening to the audio files simultaneously, I was able to draw important themes. I marked most of these themes as to answer the research question. Yet, I annotated some other statements that seemed to be important as well.

On a Google Sheet file, I summarized the important statements by combining and organizing the key statements from the transcripts and the notes. In this summary sheet, I compiled the relevant statements from the transcripts into eight categories (or themes): Theme 1 The profiles of the interviewed teachers; Theme 2 The teachers’ overarching goals; Theme 3 How do the teachers’ beliefs align with the SMP?; Theme 4 How do the teachers understand the SMP?; Theme 5 To what extent are the teachers’ beliefs related to their understanding of the SMP?; Theme 6 To what extent do the teachers implement the SMP in their mathematics classrooms?; Theme 7 What factors support or hinder the teachers’ understanding and implementation of the SMP?; and Theme 8 Special notes. In this summary sheet, I used a table so that I could see what statement from which participant supported each theme at a glance (See Figure 3-2 for a part of the summary note and Appendix C for the full summary note). At the beginning, the themes were more spread-out and broader. Gradually, I re-articulated them several times to narrow down by comparing and contrasting the themes. I created subcategories under each theme that needed an additional classification.

<table>
<thead>
<tr>
<th>Themes</th>
<th>Alicia</th>
<th>Betsy</th>
<th>Cecilia</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Profiles of the interviewed teachers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1. Teaching Experiences</td>
<td>5 years</td>
<td>4 years</td>
<td>6 years</td>
</tr>
<tr>
<td>1.2. Education</td>
<td>B.S.Ed. in Early Childhood-Special</td>
<td>M.Ed. in Mathematics Education</td>
<td>M.Ed. in Special Education, B.S.Ed. in</td>
</tr>
</tbody>
</table>
Under the eight themes or categories, I added subcategories to specify and narrow down the findings. In Theme 1 The profiles of the interviewed teachers, I added “Teaching Experience” to write the number of years of each teacher’s mathematics teaching experience in elementary school and “Education” to write the level of each teacher’s academic degree in higher education. Theme 4 is directly related with the research question: How do the participating elementary school teachers understand and interpret the SMP? The subcategories for this theme include “The teachers’ familiarity of the SMP” and “Confusion between SMP 4 (Model with mathematics) and SMP 5 (Use Appropriate tools strategically).” To analyze Theme 6 To what extent do the teachers implement the SMP in their mathematics classrooms? more in detail, I subcategorized this into four parts: “6.1. The most implemented SMPs,” “6.2. The least implemented SMPs, 6.3. The teachers’ beliefs relating to the students’ mathematical need (what students need to develop more?),” and “6.3. Which SMP the teachers want to focus more.” I broke down Theme 7 What factors support or hinder the teachers’ understanding and implementation of the SMP? into two parts: “7.1. What factors support the teachers’ understanding and implementation of the SMP?”; “7.2. What factors hinder the teachers’ understanding and implementation of the SMP?” I also color-coded these two subcategories to distinguish the positive factors from the negative factors. I filled the cell with positive factors.
(which seemed to support the teachers’ understanding and implementation of the SMP) in green and the negative factors (which seemed to hinder the teachers’ understanding and implementation of the SMP) in pink. For each of the two subcategories, I branched them out to four sub-subcategories. I grouped the supporting factors into: 7.1.1. Graduate course work and professional development opportunities; 7.1.2. Students’ mathematical achievement, their attitudes toward mathematical learning, and their readiness 7.1.3. Availability of resources; and 7.1.4. Perceived Usefulness of the SMP. Similarly, I classified the hindering factors into four parts: 7.2.1. Lack of class time; 7.2.2. Lack of clarity and relevance of the SMP; 7.2.3. The teachers’ greater focus on the content standards than the SMP; and 7.2.4. The professional development programs’ greater focus on the content standards than the SMP. These categories and subcategories were helpful to analyze the data in a more systematic way.

**Refinement of the research question**

I modified the categorizations several times after re-reading the verbatim transcripts and listening to the audio files of all interviews and during the analysis, considering the emanating topics from the data and pondering subsequences of the data. Accordingly, I revised, developed, and refined my research question. Creswell (2007) noted that “questions change during the process of research to reflect an increased understanding of the problem” (p. 43). It was inevitable to revise the initial research questions, because 1) my initial question was not specific enough to describe what I noticed from the data, 2) my data spoke more important things than I had expected at the beginning of the study or 3) my data didn’t provide enough evidence to answer an original question. Agee (2009) asserted, “In qualitative studies, the ongoing process of questioning is an integral part of understanding the unfolding lives and perspectives of others” (p. 431). It is rather common that research questions are developed and refined in any stage of
research process. My research question evolved through continuous reflection of data and consideration of appropriateness of the research question.

Guided by the refined research question, I organized the statements from the verbatim transcripts under the classified themes. Whenever I found a teacher’s comments that fell under a category of a research question or its subcategories, I added them along with the linking turn numbers. For example, I added Alicia’s comment from turn 76: “[I focus on] Modeling,” turn 78: “They’re learning to represent and model their math. I did a problem the other day that worked with money, …,” and more into the cell where it crosses by 4.2. Confusion between SMP 4 (Model with mathematics) and SMP 5 (Use appropriate tools strategically), as such statements exhibit Alicia’s understanding about “modeling with mathematics.”

I kept memos, which came from the reflective thinking about the data as well as my initial thought from reading and hearing the data. Writing memos helped me “not only capture [the] analytic thinking about [the] data, but also facilitate such thinking, stimulating analytic insights” (Maxwell, 2013, p. 105). However, I didn’t limit my analysis within this memo. I revisited the transcripts again and again to deeply understand the meaning of each interview statement. In the next chapter, I reported the results of my analysis based on the summary of the data that was classified around the research question.

Summary

In order to identify elementary school teachers’ conceptions of the SMP, I conducted a qualitative study similar to a qualitative study. I utilized an intensive in-person interview with each of the eight participating teachers. To supplement and triangulate the data, I also collected data from email exchanges, teaching materials, and artifacts.
I transcribed each interview verbatim. In the transcripts, I added turn numbers to keep track of the dialogue exchanges between the researcher and the interviewee. Throughout the transcription, I used pseudonyms for every name of individual, school, and school district to protect the participants’ confidentiality.

For data analysis, I categorized the verbatim transcripts around the important themes to answer the research question. I narrowed down the themes by some subcategories for additional classification. I added quotes from the transcripts to support the themes as well as made notes and color-coded the themes for better and quicker identification of the meanings of the themes.
Chapter 4

Findings

This chapter summarizes the analysis of the data collected from eight elementary school teachers in western Pennsylvania. This study examined how the eight teachers exhibited their conceptions of the Standards for Mathematical Practice (SMP), focusing on the alignment with the SMPs. To analyze their understandings of the SMP and how closely their interpretations are aligned with the SMP written in Common Core State Standards Initiatives (CCSSI: NGA & CCSSO, 2010a), I attended to the explanation about their own in-class examples for a specific SMP.

I analyzed the verbatim transcripts of the eight participating teachers’ interviews. I examined additional sources (e.g., interviews, email exchanges, teaching materials, and artifacts) for deeper understanding of each participant’s understanding of the SMP.

Profiles of the interviewed elementary school teachers

This section provides the profiles of the eight interviewed elementary school teachers. Specifically, I documented their years of teaching elementary school mathematics, their levels and areas of educational attainment, and their teaching assignments. The interviewed teachers have varying years of elementary school mathematics teaching experiences from 5 years to 34 years with the average of 12 years. They also have wide-ranging teaching assignments from special education support to specific grade level to math specialist/coach. Every participating teacher is either currently pursuing her master’s degree or has already earned a master’s degree. One teacher (Jane) is working toward a doctorate degree in Curriculum and Instruction. Three
teachers (Alicia, Betsy, and Olivia) majored in elementary level of mathematics education, the other three (Cecilia, Jane, and Grace) have earned their master’s degree in Special Education, and the remaining two (Daisy and Rachel) pursued their master’s degree in Literature. Of the eight teachers, four teachers (Jane, Grace, Olivia, and Rachel) have experienced working in a school district as mathematics coaches or specialists. I summarized the profiles of the participating teachers in Table 4-1.

Table 4-1: Profiles of the interviewed elementary school teachers

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Years of teaching elementary school mathematics</th>
<th>Levels and areas of educational attainment</th>
<th>Teaching assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alicia</td>
<td>5</td>
<td>B.S.Ed. in Early Childhood-Special Education, Pursuing M.Ed. in Mathematics Education (Elementary-Middle School Track)</td>
<td>3rd and 4th grades reading and mathematics in the special education setting, co-teaching 6th grade mathematics intervention, English Language Arts (ELA), social studies, autistic and emotional support</td>
</tr>
<tr>
<td>Betsy</td>
<td>5</td>
<td>M.Ed. in Mathematics Education (Elementary-Middle School Track)</td>
<td>2nd grade (previously, kindergarten)</td>
</tr>
<tr>
<td>Cecilia</td>
<td>6</td>
<td>M.Ed. in Special Education, B.S.Ed. in Early Childhood-Special Education</td>
<td>4th grade mathematics</td>
</tr>
<tr>
<td>Jane</td>
<td>7</td>
<td>M.Ed. in Special Education, ABD in D.Ed. in Curriculum and Instruction</td>
<td>3rd grade general mathematics (previously, K to 3 math coach, K to 6 mathematics and reading learning support)</td>
</tr>
<tr>
<td>Daisy</td>
<td>13</td>
<td>M.Ed. in Literacy Education (Elementary-Middle School Track)</td>
<td>Kindergarten (previously, 4th grade)</td>
</tr>
<tr>
<td>Grace</td>
<td>6 (previously, middle school mathematics for 3 years)</td>
<td>M.Ed. in Special Education, B.S. Ed in Elementary Education</td>
<td>K to 5 math specialist, co-teaching K to 5 (previously, middle school mathematics)</td>
</tr>
<tr>
<td>Olivia</td>
<td>20</td>
<td>M.Ed. in Mathematics Education (Elementary-Middle School Track)</td>
<td>5th grade mathematics (previously, 4th and 6th grades), math coach</td>
</tr>
<tr>
<td>Rachel</td>
<td>34</td>
<td>M.Ed. in Literacy and Early Childhood Development</td>
<td>Recently retired (previously, 1st grade for 10 years, mathematics instructional support for 4 years, 2nd grade for 20 years)</td>
</tr>
</tbody>
</table>
Elementary school teachers’ overarching goals in teaching mathematics

To gauge each teacher’s overarching goals in teaching mathematics, I asked the following question at the beginning of the interviews: “Throughout your experience, what is your goal in teaching mathematics?” In this section, I discuss not only what each participant described explicitly as their goals in teaching mathematics, but also what they seemed to value implicitly through the interview conversations.

I also speculated which SMPs were closely associated to the teachers’ overarching goals in teaching mathematics. As teaching is a complex activity and teacher behavior cannot be defined with only one SMP, it is common that I relate several SMPs to describe a goal of teaching mathematics or an instructional activity that a teacher exemplified. The reason is that teaching (or learning) is not a clean-cut task that can be explained simplistically, and the SMP is situated in an intertwined teaching and learning setting. However, I pointed out the most directly connected SMPs to each participant’s comment, rather than analyzing each word and any latent meaning of it to link with the SMP.

Five out of eight teachers listed “love of learning” or “love of mathematics” as a goal (Betsy, Jane, Daisy, Grace, and Olivia). Other non-mathematics-specific or non-pedagogy-related goals listed were to help students overcome their unconstructive view of their mathematical ability (Daisy), to have a good attitude towards learning (Grace), and to feel confidence in themselves (Olivia).

Six teachers specifically emphasized content goals: basic mathematics (Cecilia), number sense (Daisy and Rachel), be prepared for the next grade (Jane), rigor of the selected tasks or cognitively demanding tasks (Betsy and Grace), and understanding mathematics conceptually (Olivia and Rachel). The rest of the goals are pedagogical in nature: problem-based learning (Alicia), seeing mathematics as applicable or having real-life connections (Alicia Daisy, and
Rachel), experiencing hands-on activities (Alicia, Rachel, and Daisy), cultivating students’ problem-solving ability (Betsy and Daisy), be willing to learn and make mistakes (Grace), encountering productive struggles (Betsy and Jane), sustaining a mathematically productive discourse environment (Olivia), being challenged (Jane), being engaged (Grace), persevering (Alicia), valuing differentiated instruction (Grace), explaining students’ strategies (Betsy and Jane), understanding other students’ strategies (Jane and Olivia), and emulating the pedagogical model, “I do, We do, You do” (Cecilia). In the following, I present each teacher’s mathematical teaching goals that arose during the interviews.

Alicia emphasized problem-based learning, hands-on learning, real-life connections, and stamina (perseverance). She stated that her goals in teaching mathematics were closely related with pedagogical perspectives and proficiencies. Further, they represented SMP 1. Problem-based learning, hands-on learning, and stamina are the values that SMP 1 emphasizes. SMP 1 provides the suggestions for students to understand a mathematical problem, figure out how to solve it, and keep working on it until the problem is solved. It recommends various pedagogical strategies to teachers such as “using concrete objects or pictures to help conceptualize and solve a problem” (NGA & CCSSO, 2010a, p. 6).

Betsy instilled her love of learning to her students, and valued problem solving, productive struggles, cognitively demanding tasks, and student explanations. Betsy’s teaching goals corresponded to some of the SMPs. For example, she mentioned several strategies to “teach kids to love learning” such as using writing prompts to reason, differentiating lessons, and providing problem-solving opportunities where they have to struggle to solve a problem. Betsy divulged her beliefs in SMP 1. She mentioned that she would “give [her students] problem-solving opportunities where they have to struggle through something [emphasis added]” (Betsy, Turn 20). Also, she stated her focus on “problem solving and being able to explain how they get to an answer” (Betsy, Turn 24). She emphasized writing prompts to reason and student
discussion throughout the interview. SMP 3 explicitly states that “mathematically proficient students … justify their conclusions, communicate them to others, and respond to the arguments of others” (NGA & CCSSO, 2010a, pp. 6-7).

Cecilia focused on basic mathematics, the pedagogical move of “I do, we do, you do,” and cultivating students’ independence and confidence. She believed that elementary school students’ acquiring of basic math skills (adding, subtracting, multiplying, and dividing) was essential for mathematical learning. For that purpose, she facilitated the teaching strategy of “I do, we do, you do.” Known as gradual release of responsibility, this model of teaching has been the major teaching strategy in all fields of education (Fisher & Frey, 2013). In this model, a teacher demonstrates how to solve a problem (I do) first, provides a guided practice (We do) next, then provides unprompted practice to the students (You do).

In more recent years, a movement of reversing this order has taken a place in student-centered education. This reverse order (“You do, We do, I do”) lets students attempt to solve a problem first without teachers’ guidance (You do). This requires a high level of cognitive demand of mathematical tasks that may provide multiple entry points and allow for varied solution paths (McCaffrey, 2016). In this sense, the reverse model represents the idea of the SMP better than gradual release of responsibility model of teaching.

Cecilia stated that her focus of teaching practice was students’ independent learning. She said, “Sometimes, they instantly wanna put their hand up. ‘I can’t do this. I don’t know how to do this.’ They don’t even try… ‘No, go. What’s the worst thing that could happen? You could be wrong. We’ll fix it.” (Cecilia, Turn 26). This focus on students’ independent learning is relevant to perseverance that SMP 1 emphasizes. She continued, “So, getting them to learn they don’t need to have somebody standing beside them telling them step by step, trying to think for themselves, and to not be afraid to be wrong” (Cecilia, Turn 26). This statement, however, contradicts her pedagogical focus of gradual release of responsibility (I do, We do, You do).
Jane valued her students to love mathematics, to encounter productive struggles, to be challenged, and to be prepared for the next grade. She believed in students’ learning from “confusion.” She called it confusion, and I interpreted it as productive struggle. It is the idea that “students grapple with the issues and are able to come up with a solution themselves, developing persistence and resilience in pursuing and attaining the learning goal or understanding” (Jackson & Lambert, 2010). In this manner, her goal for the students to “feel the confusion and be challenged” fits with SMP 1.

Jane mentioned another teaching goal as having students being ready for the next level. For this goal, she stated several items such as “a ton of modeling in mathematics, being able to explain, being able to teach someone else the strategy that they’ve used and what it actually means … [and to] explain why it works best for them” (Jane, Turn 8). The list of teaching approaches she mentioned assimilates SMP 1, SMP 3, and SMP 4.

Daisy strives to develop her students’ number sense, to instill her love of mathematics, to help students overcome their unconstructive view of their mathematical ability, and to incorporate hands-on learning that cultivates students’ problem-solving ability. Daisy teaches kindergarten. The CCSSM recognizes two instructional focal points for kindergarten mathematics teachers: “(1) representing, relating, and operating on whole numbers, initially with sets of objects; and (2) describing shapes and space. More learning time in kindergarten should be devoted to number than to other topics” (NGA & CCSSO, 2010a, p. 9). Thus, it is not surprising that Daisy’s teaching goal is to build a foundation (e.g., number sense) of mathematics. The key sentence for SMP 4 (Model with mathematics) would be “apply the mathematics they know to solve problems arising in everyday life (p. 7). Daisy’s emphasis on hands-on activities and application of mathematics to everyday life, thus, are linked with SMP 4.

Grace desired her students to like mathematics, to have a good attitude toward learning, and to be willing to learn and make mistakes. Furthermore, she valued differentiated instruction,
student engagement, and rigor of the selected tasks. As Grace was serving as a mathematics specialist and co-teaching K-5 in her school, her responses were more general than grade-specific. She mentioned the importance of students’ positive attitude towards mathematics. She thought students with a more optimistic attitude towards math could have better opportunity to be successful in school and in their career trajectories.

The key shifts of the CCSSM can be summarized into three words—focus, coherence, and rigor—as I explained in Chapter 2. Academic rigor, in particular, is the extent to which students are being intellectually challenged. Through well-planned instruction such as inquiry-based, problem-solving based, differentiated, and student-centered instruction, teachers and students can strive for academic rigor (NGA & CCSSO, 2010a). Grace’s focus on students’ positive attitude, differentiation in teaching, and high cognitive demanding tasks aligned with the rigor that CCSSM emphasizes.

Olivia valued her students to love mathematics, to feel confidence in themselves, to understand mathematics conceptually, and to sustain a mathematically productive discourse environment. Olivia aspired for her students’ love of mathematics, and instilling confidence in doing mathematics is her highest goal in teaching mathematics. She stated, “everybody’s a mathematician.” Helping students think like a mathematician, who think more critically and understand conceptually, is a primary focus of the CCSSM. Such rigor is one of the key shifts the CCSSM called for.

When asked about her focus of teaching practice, Olivia expressed her passion for using mathematical discourse to improve students’ reasoning. This seems to match with SMP 3, as it describes that “[students] justify their conclusions, communicate them to others, and respond to the arguments of others” (NGA & CCSSO, 2010a, pp. 6-7). She reiterated her dedication for classroom discourse later, when talking about her textbook usage. Olivia stated, “I am a huge fan of that math discourse” (Olivia, Turn 144).
Rachel focuses on her students to see mathematics as applicable and to experience hands-on activities. SMP 4 articulates application of the mathematics to solve problems arising in everyday life. This is one of the mathematical proficiencies that students should develop. She also values students’ number sense and conceptual understanding away from memorization. One example is her teaching of two-digit numbers. She indicated her effort to help the students understand the base-10 number system by perceiving the place value rather than memorizing the order of numbers. She explained that she taught 13 as one ten and three ones. In this specific example, Rachel’s instruction corresponds with a Grade 1 content standard in Number and Operations in Base Ten: CCSS.MATH.CONTENT.1.NBT.B.2 (Understand that the two digits of a two-digit number represent amounts of tens and ones). This simple example also aligns with SMP 2, as it suggests students to attend to the meaning of quantities.

I examined the participants’ responses attending to the association with the SMP. I speculated the participants’ responses to the questions: “Throughout your experience, what is your goal in teaching mathematics?” and “Can you also tell me about the teaching practices that you have valued the most?” Additionally, I looked over the entire interview to find what the teachers emphasized in their mathematics teaching. By relating the participants’ comments to the descriptions of each SMP, I found that the participating teachers’ overarching goals in mathematics education were adherent to some of the SMPs.

The SMPs that seemed to align with the participants’ teaching goals are SMP 1, SMP 2, SMP 3, and SMP 4. Four of the eight teachers’ (Alicia, Betsy, Cecilia, and Jane) goals are closely related with the substance of SMP 1. They emphasized their focus of their students’ “stamina,” “struggle to solve mathematics,” “independence,” and “learning from confusion.” There was one teacher (Rachel) whose teaching goal aligns with SMP 2, as she attends to the meaning of the quantities. Three teachers (Betsy, Jane, and Olivia) highly value the idea of SMP 3. The key words that stood out for SMP 3 are “reasoning,” “explaining the strategy,” “justifying
the conclusion,” “communicating,” “responding to the arguments of others,” and “math discourse.” Lastly, three teachers’ (Jane, Daisy, and Rachel) responses are linked to SMP 4, as they highlighted the application of mathematics in daily life (Table 4-2).

Table 4-2: Teachers’ overarching goals and the SMPs

<table>
<thead>
<tr>
<th>SMP</th>
<th>Teacher</th>
<th>Key Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMP 1</td>
<td>Alicia</td>
<td>Stamina</td>
</tr>
<tr>
<td></td>
<td>Betsy</td>
<td>Struggle through problems</td>
</tr>
<tr>
<td></td>
<td>Cecilia</td>
<td>Independence</td>
</tr>
<tr>
<td></td>
<td>Jane</td>
<td>Learning from confusion</td>
</tr>
<tr>
<td>SMP 2</td>
<td>Rachel</td>
<td>The meaning of the quantities</td>
</tr>
<tr>
<td>SMP 3</td>
<td>Betsy</td>
<td>Reasoning</td>
</tr>
<tr>
<td></td>
<td>Jane</td>
<td>Explain the strategy</td>
</tr>
<tr>
<td></td>
<td>Olivia</td>
<td>Justifying, communicating, math discourse</td>
</tr>
<tr>
<td>SMP 4</td>
<td>Jane</td>
<td>Modeling in mathematics</td>
</tr>
<tr>
<td></td>
<td>Daisy</td>
<td>Apply mathematics … in everyday life</td>
</tr>
<tr>
<td></td>
<td>Rachel</td>
<td>Mathematics as an everyday useful tool</td>
</tr>
</tbody>
</table>

In the following section, I examined whether or not the participating teachers’ conceptions are aligned with the SMP. I examined each interview transcript in a thorough manner. Some researchers may not agree completely with my interpretation of the SMP. There could be various interpretations of the SMP description based on contextual situations, because teaching is a complex activity and the eight SMPs are not distinct from one another.

**Elementary school teachers’ conceptions of the SMP**

In June 2010, the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) released the final version of the CCSSM. In the following month, the Pennsylvania State Board of Education adopted the CCSSM. In the ensuing months, the Pennsylvania Department of Education (PDE) crafted the PA Core Standards to meet the state’s “specific needs.” The PA Core Standards was fully implemented in the academic year 2014-2015.
Specifically, the PDE advocates: “In mathematics, provide professional development to strengthen teachers’ knowledge and skills in *The Standards for Mathematical Practices*” (PDE, 2013). All participants have been teaching since the fall of 2014 and should have had opportunities to become familiar with the SMP in the past four years. This section examines the various ways in which the teachers have become familiar with the SMP and their conceptions of the SMP.

**Various ways the elementary school teachers became familiar with the SMP**

When and in what contexts do teachers encounter and investigate the SMP? The PDE explicitly advocated: “In order to lay the groundwork for a smooth transition to the new standards, district/school leadership can inform teachers, school board members and parents of the PA Core standards” (PDE, 2013, p. 3).

To assess the ways the elementary school teachers have become familiar with the SMP, I asked the following questions:

- Have you heard about the Standards for Mathematical Practice?
- Have you read the SMP?
- Within what contexts did you learn more about the SMP?
- What resources helped you to become more familiar with the SMP?

Among the eight interviewed teachers, there was a spectrum of their familiarity with the SMP. Jane, Grace, and Olivia recalled some of the standards and expressed their usefulness. It appeared that there had been a great deal of their own motivation and effort to become more familiar with the SMP, and they include continued professional development and graduate course work.
Alicia initially encountered the SMP during her undergraduate years and became more familiar with the SMP in her graduate program and while utilizing the school’s adopted textbook. Betsy’s initial encounter with the SMP occurred when the school district adopted and implemented the new curriculum and textbooks. Similar to Alicia, Betsy became more familiar with the SMP in her graduate program. Betsy commented that her learning of the SMP was more from the master’s program she attended than from her school district’s support. She said, “The school district just kind of bought the updated curriculum and gave us a day to learn it…” Fortunately, I was working on my master’s degree at University B at the time in mathematics education, and that is really where I feel I learned about Common Core mathematics” (Betsy, turn 72). Olivia emphatically stated her love of the SMP and how she recently implemented SMP 1 (Persevere in Solving Problems) in a lesson. Olivia initially encountered the SMP during her master’s program. She became more familiar with the SMP through her efforts to improve teaching as well as through workshops offered by a nearby university. In particular, the latter effort has resulted in a sustained growth as a teacher.

Alicia and Betsy mentioned how their CCSSM-aligned curricula provided the idea of practicing SMP in their lessons. For example, Alicia opened her textbook and pointed out how a lesson incorporated SMP 4: “Yep, so model with math, listen and look for students who connect using a number line. So, it’s asking you to look for students who are able to model with mathematics because the Standards for Mathematical Practice number 4 was the one this lesson mainly focused on” (Turn 58).

Serving as the school’s math coach, math liaison, or math instructional coach also appeared to be related to a greater awareness about the SMP. Jane, Grace and Olivia, had served as a math coach during the transition to the Common Core and the SMP and became more familiar by providing professional development trainings for the teachers. Jane remarked: “The mathematical practices were nice in that those are the standards we would want for any
mathematician to be able to do, regardless of age. I think the inclusion of those—I think as math coach—that’s one of the biggest things I introduced a lot of teachers to” (Turn 72). For her school district, Grace has served as the math liaison (math specialist) and has provided numerous professional development sessions—some involving the Common Core and the SMP. She has diligently sought professional development training opportunities to learn more about teaching mathematics. Olivia also had served as the (Math) Instructional Coach and has provided professional development sessions about the Common Core and the SMP. Jane, Grace, and Olivia gave credit for their learning of the SMP to their role of mathematics specialist in their school districts.

The teachers had had numerous exposures to the SMP. Some used reliable online resources such as the Standards Aligned System (SAS) of PDE and NCTM websites. Daisy, Grace, and Jane said that they were familiar with the SAS resources, even though Jane concluded, “I don’t like the way that site’s set up.” In addition, Jane joined the NCTM and found useful resources on its website relating to the SMP. Grace referred to the resources found at the Achieve the Core website (https://achievethecore.org). Her serving as a per-diem trainer for the publisher of *Everyday Mathematics* also supported her to learn the SMP more in-depth. Some teachers benefitted from their serving as a curriculum committee member. Daisy has had an opportunity to read the SMP to embed the practices in their curriculum.

On the contrary, Cecilia and Rachel could not recall the SMP even though they had participated in the professional development relating to the Common Core. In teaching a standard-based curriculum, their focus seemed to be on the content standards. Even when presented with the SMP so that the teacher could recall the discussed standards, Cecilia was unable to do so.

When I asked Cecilia about her familiarity with the SMP, she responded: “I’m not a hundred percent sure. Sometimes, I know what something is, and I didn’t realize that’s what the
name was.” Asked if she had ever read the SMP, she responded, “I don’t think so.” During the interview, Cecilia was given time to read the SMP. She then responded, “No, I haven’t [read them before].” She added: “I don’t think I ever pay attention to the words up there [She points to the top of the PA Core Standards where the SMP is located]. We usually just look for the Common Core standard and go.” Despite having utilized “the textbook which follow the Common Core standards” and attended a Common Core-based workshop, Cecilia had no familiarity with the SMP. Her sole focus, in regard to the standards, was the content as she had often mentioned the Eligible Content Aligned to the PA Core Standards.

After a 34-year teaching career, Rachel retired in the summer of 2018. When asked, “How about in terms of the practice standards?” (in general), she replied, “I’m not sure what you…” When specifically asked, “How about the Standards for Mathematical Practice? It’s a part of Common Core,” she replied, “I’m probably not as familiar with that.” Like Cecilia, Rachel appeared to equate all standards to the content standards. Furthermore, she stated that she had received some professional development about the Common Core standards, had served on the school’s math committee during this transition to the standards, and had utilized a Common Core-aligned textbook.

In Table 4-3, I summarized the resources from which the teachers heard or learned about the SMP.

Table 4-3: Various ways to become familiar with the SMP

<table>
<thead>
<tr>
<th>Sources to access to the SMP</th>
<th>Teacher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undergraduate/graduate program</td>
<td>Alicia</td>
</tr>
<tr>
<td></td>
<td>Betsy</td>
</tr>
<tr>
<td></td>
<td>Olivia</td>
</tr>
<tr>
<td>CCSSM-aligned textbook</td>
<td>Alicia</td>
</tr>
<tr>
<td></td>
<td>Betsy</td>
</tr>
<tr>
<td>PDE SAS resources</td>
<td>Jane</td>
</tr>
<tr>
<td></td>
<td>Daisy</td>
</tr>
<tr>
<td></td>
<td>Grace</td>
</tr>
<tr>
<td>NCTM resources</td>
<td>Jane</td>
</tr>
<tr>
<td>Serving as a math coach</td>
<td>Jane</td>
</tr>
</tbody>
</table>
Elementary school teachers’ closely aligned conceptions of the SMP

Familiarity of the SMP does not necessarily parallel to understanding of the practice standards. During the interviews, all teachers had access to the several versions of the SMP (e.g., the Common Core State Standards Initiative [CCSSI; NGA & CCSSO, 2010a], the Standards for Mathematical Practice Grade Level Emphasis [PDE, 2014b], and the PA Core Standards for Mathematics [PDE, 2014a]). The eight participating elementary school teachers demonstrated a range of conceptions of the Standards for Mathematical Practice. This section examines the participating teachers’ conceptions of the SMP. Their conceptions aligned with the SMP are organized by each practice standard (e.g., SMP 1, SMP 2, etc.).

Teachers’ closely aligned conceptions connected to SMP 1

SMP 1 (Make sense of problems and persevere in solving them): Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help
conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches (NGA & CCSSO, 2010a, p. 6).

Among the eight mathematical habits that teachers should exhibit and nurture as well as students should develop, the teachers showed understanding of SMP 1 (“Make sense of problems and persevere in solving them”) as their most frequent. Seven of the eight teachers (Alicia, Betsy, Jane, Daisy, Grace, Olivia, and Rachel) displayed closely aligned conceptions of SMP 1.

Alicia effectively partitioned SMP 1 into two steps: she ensured that her students made sense of the problem and encouraged them to persevere in solving it. The example of her third-grade mathematics classroom illustrated her efforts to implement this standard: “For example, with the problem I posed about Annie with saving money and then buying books, the students had to make sense of the problem, breaking it into two steps and then persevere in solving them” (Turn 112). Alicia further noted how her students lacked in persevering in solving problems as follows:

…my third graders, they have no perseverance right now. They are so used to just being spoon-fed and being handed formulas and worksheets that when you pose a problem, like the one I gave them on Friday and just say, “You can solve this in whatever way you want,” they panicked. I had students that cried because they were not used to having to persevere in solving problems (Alicia, Turn 140).

Alicia asserted her focus in teaching mathematics was developing students’ “stamina,” as shown in the earlier section, ‘Elementary school teachers’ overarching goals in teaching mathematics.’ She noticed that her students usually expected her to give them answers rather than struggling with the mathematical problems. She said, “they want to have a quick and easy solution to problems, and I feel that, to become better at math, they need to build stamina and build the ability to struggle with math in order to learn it” (Turn 18).

Betsy valued her students’ perseverance as an important mathematical practice that transferred to life skills. She used the words, “struggle through problem solving” which she
equated to “persistence” or “perseverance.” When asked for further explanations, she said, “When something is hard, don’t stop. Keep working through it. See if there’s another way to think about it” (Turn 112). One of her mathematical teaching goals appeared to align with SMP 1. She emphasized students’ persevering skill and its benefit as follows:

… if they can persevere through a problem, they gonna get really good at persevering, and that’s a skill that will transfer to every single math class… Like when you say, “Just understand two addends equal a sum.” That’s not the same as struggling through a problem. Because as a grown-up, I struggle through problems, and they’re not math-based or reasoning or communicating with people and problem solving with people and deciding if their arguments valid or not. That’s not just math. It’s a bigger issue that you have to be able to do in life. So, I kind of look at the practices as needing to be vague because they will be applied everywhere, not just in math (Betsy, Turn 154).

Jane deeply cared that each student achieved to his or her full potential. Jane demonstrated her persistence about students being given time, encouragement, and reward. Jane picked “the Genius of Hard Work” each time she noticed a student who demonstrated perseverance in solving a problem. She had the students “take their work, stand up in front of the bulletin board, and take their pictures” (Turn 34). On her classroom doors, she said, there were some lines of different kids on different days of when they’ve shown they’ve been “the Genius of Hard Work. She mentioned that her mathematical teaching goal was to get kids to love mathematics. She further asserted that “whatever level the students come in at, at some point they should feel the confusion.” Jane believed that students could learn, appreciate, and come to love mathematics when they overcome the confusion.

In Daisy’s response, she provided convincing evidence to support that she was familiar with SMP 1. After reading the SMP, Daisy said that she clearly understood what she could do to implement SMP 1 in her kindergarten classroom. Further, when asked which SMP she focused the most in her teaching, she answered as follows:

I think even the first one. Although I do it, and I want to say I’m a good teacher in making sense of problems. But really, the kids do a lot of hands-on. I want them to be confident and to be able to: “Does this make sense?” and “Why does
this make sense?” They do use concrete objects and pictures, but I think we may able to move a little higher than that. I want them to do it more. We need to prepare them to show all of their work. We do a lot of talking. So, I think getting them away from guidance on that and having them do it on their own would be my push. And that’s always my push (Turn 160).

Daisy’s above assertion demonstrated her understanding of the practice. For instance, in asking, “Why does this make sense?”, she was nurturing this important practice standard.

Grace was quick to point out the second portion of SMP 1 (persevere in solving problems). To support her understanding of the standard, she remarked: “the kid won’t give up” and “if it’s something that’s difficult… they’re open to keep working on it.” In addition, the concluding observation, “they have a desire to keep trying,” appeared to be underscoring students’ intrinsic motivation (Turn 90).

In Olivia’s classroom, she emphasized to her students “to use a tape diagram” to conceptualize the bus problem, to make sense of the given conditions, and to persevere and not to give up so easily. She also incorporated a research finding to reinforce the value of students’ preserving in solving problems. All of these teaching moves actualized SMP 1. Olivia valued SMP 1 as the most important standards out of the eight SMPs, even though she advocated all SMPs. Her example of a school bus problem in the below excerpt shows how she helped her students “conceptualize and solve a problem” and “when the students did not try to make sense of the problem.

We just did a problem today where you had nine school buses that were 12.6 meters long. And you had two meters of space between these nine school buses. And what would be the entire length that these school buses would fill? We’re trying to use a tape diagram to draw this out. But so many of the children, I mean, the language… With my second class, fortunately, I get to teach math twice a day. So, I looked at their results and the struggles they were having in solving it, and just saying close your eyes and just think about the school buses parked in front of building at the end of the day. About how many buses can you imagine from the stop sign to the end of the building. And the children overwhelmingly said 5. And I said, “You’d want a little space between those two buses, right?” So, we had a meter stick out and said, “What if you had two-meter sticks between them? Can you get an idea of how long that would be” (Olivia, Turn 78)?
In solving the problem, Olivia’s students were impatient to use tape diagram or any other methods, because they just wanted to have an answer. Without thinking deeply of the problem situation, a lot of her students just multiplied 12.6 by 9. Olivia said, “But just, they’re in such a hurry. Like, I think they go straight from seeing the question to panic mode to answer. And just not taking time, making sense of it” (Turn, 80). Olivia even stated that “American children had much less time that they would try to persevere,” by mentioning Liping Ma’s (1999) comparison of elementary school mathematics in America and China (Turn 80).

Rachel freely admitted that she was not familiar with the SMP. It is important to note that she cited, “being a busy teacher and math was just one subject, I taught across the second-grade curriculum,” as a factor. Nonetheless, she established her understanding of SMP 1. In particular, she mentioned that “Drawing things like that out of it helps them to make sense that they understand the problem” and “Again, instilling that confidence is important and persevering” (Turn 82). She stated her intention of implementation of SMP 1 as follows:

I would probably have a small group of children who were struggling, and we would work together or to pull out the important information, look for key words. Or I would partner them with a more proficient student and have the proficient student explain to them how they solved the problem (Rachel, Turn 102).

One reason for this is due to the teachers’ perception that SMP 1 is “relevant” for their elementary school students. For example, Daisy remarked: “I want to say I’m a good teacher in making sense of problems. But really, the kids do a lot of hands-on. I want them to be confident and to be able to: “Does this make sense?” and “Why does this make sense?” They do use concrete objects and pictures, but I think we may able to move a little higher than that (Daisy, Turn 160).” Grace added: “Perseverance through problem solving is that the kid won’t give up, but I also think that it means you’re willing to try as well. If you see something challenging or new, you’re willing to take it on. And if it’s something that’s difficult or long or doesn’t have a clear end in sight or a precise solution, they’re open to keep working on it, and they have a desire
to keep trying (Grace, Turn 90).” Betsy shared that this particular practice standard transferred to life skills. Likewise, the teachers viewed the other more frequent SMPs (SMP 3, SMP 5, and SMP 6) as applicable in their classrooms.

The second reason is that SMP 1 can pervade most mathematical lessons. The alignment of SMP 1 with every NCTM Process Standards, shown in Figure 2-8 (Comparison of the SMP and the NCTM process standard), supports the fact that SMP 1 is an integral standard with the rest of the SMPs. Being “an integral part of mathematics, not an isolated piece of the mathematics program” (NCTM, 2000, p. 52), problem solving should involve any content areas described in the CCSSM (NGA & CCSSO, 2010a). Koestler and her colleagues (2013) also emphasized that problem solving should “be integrated into students’ experience, involve important mathematics, and connect to multiple process and content standards” (p. 2).

The third reason is that when encountering the CCSSM, the teachers may have paid the most attention to the first few standards of the SMP. It is plausible that the teachers have read SMP 1 the most frequently because it appears at the beginning of the CCSSM document. Both Betsy and Grace utilized the phrase, “problem solving” in mathematics, and the other teachers might also be familiar with the phrase as it has been accentuated not only in the previous standards (e.g., PSSM [NCTM, 2000]), but also in numerous research (Billstein, Libeskind, Lott, & Boschmans, 2004; Carpenter et al., 1999; Goldin, 2000; Hart, 1989; Hiebert et al., 1996; Lester, Garofalo, & Kroll, 1989). However, the focus on the “perseverance” is almost new to the teachers. Olivia (personal communication, December 3, 2018) stated that while it was very important in their learning, it was not easy to foster in students.

*Teachers’ closely aligned conceptions connected to SMP 2*
SMP 2 (Reason abstractly and quantitatively): Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; knowing and flexibly using different properties of operations and objects (NGA & CCSSO, 2010a, p. 6).

Two teachers (Alicia and Rachel) talked about SMP 2 during the interviews.

To develop students’ concept of multiplication, Alicia utilized the equal-groups representation, the number line, and the array arrangement. She claimed her creation of coherent representations of multiplication for students’ quantitative reasoning. She mentioned her uses of equal groups model for students’ understanding of the multiplication quantitatively. She explained five times four can be “five groups of four. They have five boxes, and then they model it with four counters in each box to get their product of twenty” (Turn 42). She commented that she would then use a number line or let her students build array models to increase their understanding of multiplication.

Moreover, Alicia connected the inverse relationship between multiplication and division using the equal-groups representation. She explained that her students would use an array model to learn $20 \div 4$, using the very same array model for $5 \times 4 = 20$. In this way, the students would develop their quantitative reasoning and they would extend their understanding of the inverse property of multiplication and division.

Rachel expressed her goal to develop her students’ reasoning quantitatively. Rachel valued SMP 2 as one of the most important practice in her mathematics classroom. In particular, she mentioned her teaching of addition by using some strategies and by representing numbers with concrete objects to make more sense of the sum as follows:
So, if they were adding two numbers, they were given something so that they were allowed to choose something—a strategy, a tool—that would help them add those two numbers. Then they would have to articulate how they got them to the right answer. And of course, we want them as they move through their developmental process, to think more abstractly (Rachel, Turn 108).

Addressed portions of SMP 2 that Alicia’s and Rachel’s statements are “Mathematically proficient students make sense of quantities and their relationships in problem situations” and “Quantitative reasoning entails habits of creating a coherent representation of the problem at hand”.

**Teachers’ closely aligned conceptions connected to SMP 3**

SMP 3 (Construct viable arguments and critique the reasoning of others): Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is (NGA & CCSSO, 2010a, pp. 6-7).

Six of eight teachers’ (Alicia, Betsy, Jane, Daisy, Grace, and Rachel) statements about SMP 3 were consistent with their wording in the CCSSI (NGA & CCSSO, 2010a). The addressed portions of SMP 3 that these teachers’ comments particularly associated with are “[Students] justify their conclusions, communicate them to others, and respond to the arguments of others”; and “students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments” (NGA & CCSSO, 2010a, pp. 6-7).
For her students, Alicia talked about her provision of time and facilitation of guided practices to discuss their reasoning. In fact, she conveyed an expectation that students “write a letter to me explaining how they solved the problem” (Alicia, Turn 112). She tried to implement SMP 3 in her classroom by focusing on this specific. Instead of a content-related goal of the lesson, she mentioned that her instructional objective in one of her recent mathematics classrooms was “constructing arguments” (Alicia, Turn 64). Incorporated throughout her lessons, she asked: “Why are you thinking that way? How do you know you’re correct? If you were to explain this to a student who disagrees with you, how can you prove your answer?”

Alicia’s responses throughout the interview displayed her great focus on SMP 3. Not only in class, but also as an assignment, Alicia endeavored to incorporate SMP 3 for her students’ development of this specific standard. For example, she asked her students to write her a letter to explain how they had solved a problem and why their solutions made sense. Despite her students’ struggle in doing this, she believed that this activity would scaffold her students’ ability to justify their conclusions and communicate with others.

In her classroom, Betsy had an expectation that her students explain how they got answers: “I wanted the kids to explore more, explain why that works.” Particularly, she mentioned about her facilitation of group work among students: “Then they have to talk to their partners or in their group and come with answers as team so that they’re listening to others talk, thinking about it, and responding.”

Jane mentioned that she knew what she should do to cultivate her students’ ability to construct viable arguments. Her interpretation of SMP 3 was: “that you are arguing for the way you solved it. This is the process before you even know if it’s wrong or right or accurate. Do you understand why you did it the way you did? Can you explain that” (Jane, Turn 164)?
Daisy briefly mentioned that the most important SMP for her kindergarten students could be SMP 3. Without further explanation or example, she said that it was very important her students to “argue in a different way” to exhibit their mathematical reasoning.

Grace, in her own words, summarized SMP 3: “I think that almost looks like an argument, which is fine. I think it’s agreeing and disagreeing. I think it’s the ability to explain someone else’s thinking and then elaborate on that or find an error or a mistake. That looks like student-to-student talk to me.”

While Rachel’s focus might have been to pair a “struggling” student with a “more proficient student” for the benefit of the struggling student, she provided valuable opportunities to communicate their thinking. Her testimonial about the successful case of pairing different levels of students inferred the effectiveness of mathematical talk among the students.

My brighter students were very good at—most of the time and depending on their patience level—were very good at explaining their thought process to another child…So, the more mature children would be more open. But I also find it’s a learned skill, so the more you do it, the more they become proficient at it. And it helps children who, perhaps, don’t understand that particular problem to realize that they can be verbal about math too (Rachel, Turn 104).

Rachel further remarked, “It’s not always about numbers. It’s about understanding. It’s about being able to articulate that.” Moreover, what can be a better teaching moment than to hear, “Oh, I get it now that Jenna explained that part” (Rachel, Turn 130)?

SMP 3 was one the most commented SMPs by the participants. Moreover, their explanations about SMP 3 aligned well with the descriptions—all six teachers’ interpretations about this standard paralleled with what is written in CCSSI (NGG & CCSSO, 2010a). There could be a couple of plausible reasons for their understanding of SMP 3. First, similar to SMP 1, SMP 3 describes less grade-specific and less subject-specific practice. It might be easier for the elementary school teachers to implement this more general standard as they usually teach more
than one subject. Another reason can be that the teachers might be familiar with SMP 3 as this standard has been emphasized since the previous standard (PSSM: NCTM, 2000). The teachers have learned about this specific standard through the courses of mathematics education program or professional development program. For example, Betsy’s utilization of “writing in math” or “writing prompts” is the practice that she had learned from her master’s program: So, one of my classes here was about writing in math, and after I took that class—it was with Dr. M actually—I really just valued doing that. That’s why I keep talking about writing prompts. I use that for…constructing viable arguments” (Betsy, Turn 210).

**Teachers’ closely aligned conceptions connected to SMP 4**

**SMP 4 (Model with mathematics):** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Even if all eight participants have talked about SMP 4, only Daisy’s practice example aligned with the SMP document (NGA&CCSSO, 2010a). The anecdote below was about counting pumpkin seeds. The problem context could have been richer if Daisy had posed the question such as “A farmer will sell the pumpkin seeds at a market. How many seeds are there?”
Demonstrating her knowledge of this SMP even further, she guided her students to improve the counting model by making “tens” and “hundreds.”

… We dug out a pumpkin, got all the seeds out the other day, and we talked about… I said, “We’re going to count these seeds.” And, you know, this is still October, and we haven’t counted very high… And they said, “We can’t count all these pumpkin seeds!” I said, “We can. What’s the easiest number we can count to?” Or, not the easiest but… And they’re like, “Well, ten.” So, we put them (seeds) all in groups of ten, and we started there. And they said, “Well, that wasn’t that hard.” And then we counted by tens, and they can count to a hundred by tens… because the one pumpkin had 600 seeds in it…they were like, “Oh, that wasn’t that hard. It didn’t take us that long.” And I said, “You are good at making groups of ten.” And we did still use our ten-frames to support. Some kids are still not good at making groups of ten, but you know, we’re learning. But doing that, they were not afraid of it anymore, and they said, “I’m going to go home and make my own ten-frame and count my own pumpkin seeds!” And I’m like, “I hope you do, but your parents might not like me. I hope you do it on a Saturday because you’ll need all day to do it. Go ahead, count your pumpkin seeds, kids.” Then even the other day, they’re like, “We found one on the floor! Did we count it?” and I’m like, “I don’t know.” So, just doing those things. That’s math. And I want them… even though that’s not what we were learning, it was the time of the year we did it. We counted to 600! I think it was 636. They were so proud (Daisy, Turn 154).

Daisy’s example of counting pumpkin seeds addressed some portions of SMP 4 such as “Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace”; “They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose”; and “They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas” (NGA & CCSSO, 2010a, p. 7).

*Teachers’ closely aligned conceptions connected to SMP 5*

SMP 5 (Use appropriate tools strategically): Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or
dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts (NGA & CCSSO, 2010a, p. 7).

SMP 5 was one of the most frequently stated SMPs. Five teachers (Alicia, Betsy, Daisy, Grace, and Olivia) interpreted this standard as it is portrayed, or the examples of their practices aligned with the description of the SMP.

Alicia’s use of the counters was entirely appropriate to enhance students’ mathematical reasoning. For example, she explained that she incorporated counters to introduce the concept of multiplication, while she was open to use different tools such as a number line to teach the same concept. She reiterated the usefulness of the practice standard with a pronouncement: “No matter how you choose to teach it, students need to be able to use the available tools. So, I need to teach them all the tool that they can use” (Alicia, Turn 222).

Students in Betsy’s 2nd grade classroom seemed to have much flexibility in selecting useful tools. Betsy had established a routine that encouraged appropriate use among available tools. She mentioned that her curricular material included a tool kit that contains “coins, a calculator, a number grid, a ruler…” (Turn 160) and she advised her students to decide if they needed a tool and what tool they needed to get. Betsy claimed that by placing all available tools on a shelf, she was able to monitor students’ selections of appropriate tools.

To Daisy, SMP 5 is one of the most essential practice standards to develop her kindergarten students’ mathematical thinking. She not only provided various tools for her
students to choose from, but also made them to argue how and why they would use the tools. The tools Daisy specifically mentioned are interlocking cubes, bears, and colorful gems. For her kindergarten students, the main purpose of her using the tools was to develop students’ number sense and counting skills.

The students know they can use our connecting cubes to show a group of ten and maybe three ones for 13. But they can also model it in a ten-frame. Some kids will go get 13 counting bears. But it’s neat to see some kids will just get counting bears and just show me 13. But some will do it and say, “Well, I got ten blue and three red.” So, it’s neat to see how they think of it. And I love to be able to show them, “Look at your friends. Why did you choose it?” “Well, Susy decided she wanted these colorful gems because she loves to do it. It took Susy forever to get to 13. It took me a second to get one ten and three ones.” So, we talk about that, and they’re like “Oh, how do you represent it in different ways?” So, we are using it… they’re learning off of each other through trial and error (Daisy, Turn 154)

In Grace’s response, she displayed conceptions of SMP 5. She recalled many of the SMPs as she remarked about SMP 5, and reference to “physical tool” firmly supported her understanding of the practice standard as follow:

Kids, assessing what they have, that could help them solve a problem, whether it be a physical tool or something that they could draw or a list and really choosing the right one. That’s really tricky—choosing the right tool. Then, when kids use tools, some are quicker and more efficient than others. So, I think those are all things that come to mind for that (Grace, Turn 92).

Olivia mentioned several tools including protractor, calculator, blocks, centimeter cubes, paper and pencil, and a computer software (Zearn). Olivia deliberately employed the Concrete, Representational, and Abstract (CRA) approach that she had learned from a workshop to her mathematics lessons. Olivia explained how she implemented SMP 5 by incorporating the CRA approach: “So, first off, we start with our blocks, and then we draw our blocks, and then we use the numbers instead of the block sets” (Turn 88).

Additionally, to enhance understanding of volumes in her geometry lesson, Olivia provided her students physical blocks as well as a software, Zearn, to supplement the curricular
materials. She tried to utilize the available tools as much as possible, because “if you don’t show them that and all you use is a piece of paper to teach volume, they’re very confused” (Turn 88).

**Teachers’ closely aligned conceptions connected to SMP 6**

SMP 6 (Attend to precision): Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions (NGA & CCSSO, 2010a, p. 7).

SMP 6 is the most frequently mentioned by the participating teachers. Some of the comments were consistent with the SMP, but others seemed somewhat different from the descriptions. Here, I present the teachers’ conceptions and their practices that seemed to address the idea.

Shown in the document, *The PA Core Standards for Mathematical Practice: Grade Level Emphasis* (PDE, 2014b), Alicia referred back to the context of the previously mentioned example and correctly facilitated SMP 6. In particular, her emphasis on the correct units and the mathematical vocabulary such as “multiply” and “product” demonstrated her understanding of the practice standard. The example problem Alicia posed to her students was “Annie saves six dollars a week for four weeks...” While her students worked on this problem, Alicia required the students not only to communicate to each other what the meanings of the solutions were, but she also made them specify the units. She said, “they have to label everything that they have” (Alicia, 112).
Betsy mentioned that her students had provided feedback to each other based on their formulated rubrics, and they set expectations for themselves by asking, “Did [we] explain it clearly and precisely? What could make [the work] better?” This example presented Betsy’s effort to encompass SMP 6 and SMP 3. It is common that teachers implement more than one SMP during a lesson and that any SMP can be employed along with other SMPs.

Cecilia stressed the importance of “using the math words” and making a transition from informal expressions to more formal ways of communicating. For example, she used “fun words” to introduce a mathematical concept, but gradually she replaced them with the “math words.” She stated, “if you do it step by step, like as they practice slowly, start taking the fun words out, putting the math words in…they have to tell me what they’re doing step by step, and you can hear them slowly telling me the right words” (Cecilia, Turn 92). She specifically emphasized to her students the need to recall definitions and to articulate “step-by-step” thinking, when “explaining it to someone else” as follows:

I always tell them to use math words when they’re trying to explain something in words. Use the math words. Use the definition. Sometimes, we’ll make up little tricks to remember it, but in the grand scheme of it, when you go to explain it to somebody else, they don’t know our little trick. You have to learn how to put it in real math words. If they have to explain something, I always tell them, “Think of the definition. You have to use the definition to help you explain it.” That seems to work pretty well for fourth graders at least—learning how to apply what they’re doing, the rules of it into words because it’s hard for a lot of them to put things into words. They can stand up there and show you how to do it but to explain is another story. So, you try to teach them how to explain it to another person. So, we always stress the precision: making sure that you’re using the definition, using the correct words (Cecilia, Turn 88).

At the outset, Jane confessed that she should devote more time for her students “to have a sentence at the end” of a story problem (Turn 170). She stated that attending to the units of the story problem could be an example of SMP 6. She said that she would like to spend more time on this specific standard for the upcoming quarter of the year. She interpreted “attend to precision” as “looking at the reasonableness of students’ answers” (Turn 170).
Regardless of the challenge that Grace experienced in implementing SMP 6, she interpreted “precision” as “accuracy” that students constantly “[check] the solutions to problems” to see “if they are accurate.” Grace consistently reminded her students of the importance in labelling units and slowing down to produce quality work. She specifically mentioned “the unit box” to reinforce the need to place students’ work and conclusions in proper context.

*Teachers’ closely aligned conceptions connected to SMP 7*

SMP 7 (Look for and make use of structure): Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the $14$ as $2 \times 7$ and the $9$ as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as $5$ minus a positive number times a square and use that to realize that its value cannot be more than $5$ for any real numbers $x$ and $y$.

Four of eight teachers have mentioned SMP 7 and their interpretations and understandings about this standard seemed to correspond to the description above.

Through a writing-prompt, Betsy’s students discover why the “turn-around rule” (commutative property) worked for addition but not for subtraction. She guided her students quite appropriately as to why the “turn-around rule” did not hold for subtraction as follows:

By second grade, we’re now looking at… and I had mentioned I just taught a lesson about the turn-around rule. Some kids look at that, and they get it right away… Our first lesson was addition, and then the next was a writing prompt. And it said, “Does the turn-around rule work for subtraction?” And that’s always fun to watch because their first answer is: “Yes, of course it does.” And then, as they work through that, they realize that it doesn’t. And one thing I’ve started doing is trying to pair to teach addition and subtraction together as a family instead of first mastering addition (Betsy, Turns 166 & 168).
Even though Cecilia stated, “looking for patterns,” she correctly described SMP 7. She specifically validated her understanding of the practice standard through the example, “seeing that 7 times 8 is the same thing as 8 times 7.” While she may not have utilized the phrase, “commutative property for multiplication,” she, nonetheless, helped her students to examine this important mathematical property.

The use of “function machines” appeared to be quite appropriate in discerning pattern or structure. For example, Grace illuminated SMP 7 by drawing of “function machines” or “in-and-out boxes” (Figure 4-1). Grace explained that “We do those a lot. They are called ‘What’s My Rule?’ in the program. I think kids automatically do look for patterns” (Turn 108). She could ask, “What’s my rule?” based on the input of 1 to the output of 3, the input of 2 to the output of 4, the input of 3 to the output of 5, etc.

![Grace’s function machine](image)

Figure 4-1: Grace’s function machine

Olivia’s emphasis on the “place value system” was wholly appropriate in regard to SMP 7. She further demonstrated the need for her students to understand that the base-ten system is both complex and elegant. This foundation is crucial to scaffold mathematical concepts. In addition, there was a nice exchange between Olivia and her students in which they paused, pondered, and shifted their perspectives about the importance of the numbers in each place value as follows:

Honestly, I probably would say, for my grade level, because we spend so much time in the place value system, being able to look at and make sense in use of the structure and patterns that we see… place value, and powers of ten, working through powers of ten, introducing exponents, and understanding when you’re
dividing or when you’re multiplying. You’re moving from place value to place value. And understanding that when you say... when we give a test and say, “What’s the value of this number?” “Hundreds.” I said, “That’s correct. That is the place. But what is the value of that digit?” And they had such a hard time with that. And they said, “Why are you asking? Why does that matter?” And I said, “Okay, …” and I did almost a hangman kind of thing with five numbers. And I said, “I’m going to ask to write a number, and let’s see if you can guess it.” And I said, “I’ll tell you what each one is.” And I said, “Hundreds.” And they’re like, “Well, how many hundreds?” And I’m like, “Oh! You just told me it didn’t matter. The value of that number doesn’t matter. Hundreds. Tenths. Hundredths.” And they’re like, “Oh, we get it. Okay, I gotcha.” But so many of them didn’t recognize that it’s the value of the place. The digit is worth this much. Because they’re so baffled by these place value titles that they just—even decimals—they just can’t understand what we’re talking about (Olivia, Turn 84).

**Teachers’ closely aligned conceptions connected to SMP 8**

SMP 8 (Look for and express regularity in repeated reasoning): Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation \((y - 2)/(x - 1) = 3\). Noticing the regularity in the way terms cancel when expanding \((x - 1)(x + 1), (x - 1)(x^2 + x + 1),\) and \((x - 1)(x^3 + x^2 + x + 1)\) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. (NGA & CCSSO, 2010a, p. 8).

Four teachers talked about SMP 8 during the interviews. Of the four teachers, Rachel’s explanation seemed closely aligned with the SMP. Rachel recalled that counting by twos, by fives, by tens, etc. provided her students opportunities to discover number patterns. For example, she stated: “Looking for those patterns and knowing that when you add four, you’re only gonna get one repetition in the ones place” (Turn 130). Starting at 4 and adding 4s lead to the sequence: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, … Her “one repetition” reference appears to be the unique repeating sequence—4, 8, 2, 6, 0—in the unit’s (or ones) place. It is possible that Rachel’s
students could change the starting number to discover another unique repeating sequence of numbers.

**Summary of teachers’ closely aligned conceptions of the SMP**

Based on the above documented evidence, the teachers demonstrated closely aligned conceptions of the Standards for Mathematical Practice. The below bar chart (Table 4-4) represents the individual teacher’s conceptions that were consistent with the words in the SMP descriptions by specific SMPs. Three teachers (Alicia, Betsy, and Grace) identified, described, and provided applicable examples relating to five SMPs (the most), while Cecilia furnished only two SMPs (the fewest).

Table 4-4: Individual teacher’s closely aligned conceptions of specific SMPs

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<tbody>
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<td>Alicia</td>
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<td>Betsy</td>
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<td>Grace</td>
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Table 4-5 summarizes the compiled data of the teachers’ closely aligned conceptions of each of the SMPs. In this chart, I indicated which teacher identified the specific SMP. Seven teachers (Alicia, Betsy, Jane, Daisy, Grace, Olivia, and Rachel) showed their understanding of SMP1; two teachers (Alicia and Rachel) showed their understanding of SMP 2; six teachers (Alicia, Betsy, Jane, Daisy, Grace, and Rachel) showed their understanding of SMP 3; one teacher (Daisy) showed her understanding of SMP 4; five teachers (Alicia, Betsy, Daisy, Grace,
and Olivia) showed their understanding of SMP 5; five teachers (Alicia, Betsy, Cecilia, Jane, and Grace) showed their understanding of SMP; four teachers (Betsy, Cecilia, Grace, and Olivia) showed their understanding of SMP; and one teacher (Rachel) showed her understanding of SMP 8. This does not necessarily mean that more teachers understood SMP 1, SMP 3, SMP 5, and SMP 6 than the rest of the SMPs. Instead of asking the teachers to explain every SMP, my questions guided the teachers to talk about the SMPs that they recalled, the SMPs they had focused the most, the SMPs they had implemented the most, the SMPs they had implemented the least, and the SMPs they would focus on more. Thus, the teachers did not make comments on every eight SMPs.

Table 4-5: Compiled data of teachers’ closely aligned conceptions of specific SMPs

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Corresponding to this compiled data, the Pareto bar chart (Table 4-6) below represents the descending order of the teachers’ understood SMPs. Both charts (Table 4-5 and Table 4-6) provide some interesting observations. First, the four most frequent SMPs that the teachers displayed their aligned conceptions to the SMP are as follows:

SMP 1 (make sense of problems and persevere in solving them): 7 teachers
SMP 3 (construct viable arguments and critique the reasoning of others): 6 teachers

SMP 5 (use appropriate tools strategically): 5 teachers

SMP 6 (attend to precision): 5 teachers

Second, the four least frequent SMPs are as follows:

SMP 7 (look for and make use of structure): 4 teachers

SMP 2 (reason abstractly and quantitatively): 2 teachers

SMP 4 (model with mathematics): 1 teacher

SMP 8 (look for and express regularity and repeated reasoning): 1 teacher

Table 4-6: Pareto bar chart of compiled data of teachers’ closely aligned conceptions of specific SMPs

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<tr>
<th>Teacher</th>
<th>SMP 1</th>
<th>SMP 3</th>
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<td>Grace</td>
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<td>Jane</td>
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<td>Alicia</td>
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<td>Alicia</td>
<td>Alicia</td>
<td>Betsy</td>
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Teachers’ conceptions of both SMP 4 and SMP 8 seemed lower than other standards. 7 teachers exhibited understanding of the SMP 4 differently from the descriptions of the standard. In addition, 3 of 4 teachers who talked about SMP 8 seemed to misunderstand it. I discuss the teachers’ misaligned conceptions of SMP 4 and SMP 8 in the following section.
Elementary school teachers’ misaligned conceptions of the SMP

This section examines the interviewed teachers’ misaligned conceptions of the SMP. The misaligned conceptions do not mean “wrong” understanding here, but rather a “different” understanding of the original description of the SMP.

The participating elementary school teachers demonstrated a minimal range of misaligned conceptions of the Standards for Mathematical Practice. However, based on their discussed SMPs, the interviewed teachers consistently demonstrated their misaligned conceptions of SMP 4 (model with mathematics). It appears there are three interpretations of this particular practice standard. First, most of the teachers transferred the concept of representation from the PSSM (NCTM, 2000). The examples of using red counters (Alicia), drawing ten frames (Betsy), visualization (Jane), using number line or fraction bars (Grace), and coloring numbers in a block paper (Rachel) characterizes representation of the PSSM (NCTM, 2000). Second, Cecilia replied: “In a lot of that model in which we teach, there’s a lot of me doing it on the board, me showing them how to do it, and then modeling how to do it together, and then trying to get them to repeat that model that you showed them” (Turn 138). For Cecilia, modeling was perceiving and imitating the teacher moves. Lastly, Olivia likened modeling to using tools. Her using manipulative such as base-10 blocks is a suitable example for SMP 5.

Despite the fact that several SMPs can be implemented in teaching one topic or in working on one task, it is significant to know what a specific SMP means and how to apply it (Heck et al., 2011). Furthermore, Davis and his colleagues (2017) stressed the importance of teachers’ understanding of “what types of knowledge and skills comprise an SMP, when those should best be taught, how those skills are sequenced, and what curricular resources embodying these activities look like” (p. 126).
One factor that hindered the teachers from understanding the SMP appears to be their “busy schedule.” Rachel reflected, “I definitely think it’s (the SMP’s) valuable. If I’m just gonna be totally honest, I think I was very aware of this, but being a busy teacher and math was just one subject, I taught across the second-grade curriculum. This became easy … And because this is what was in our textbook. That’s just an honest answer. I think it helped me to get through a very, very busy schedule. And I’m not using that as an excuse” (Turn 80, 82). More concisely, Jane revealed, “I do math, science, social studies, health” (Turn 36). Teaching multiple subjects, the interviewed elementary school teachers had not had the requisite time nor opportunities to learn the SMP deeply.

Finally, Jane appears to be pointing out one major shortcoming of the SMP: “People have actually made even more kid-friendly versions of the mathematical practices” (Turn 134). Some parts of SMP 8 is readily accessible in its meaning: “[Students] continually evaluate the reasonableness of their intermediate results” (NGA & CCSSO, 2010a, p.8). In comparison, the same practice standard also states: “Noticing the regularity in the way terms cancel when expanding (x - 1)(x + 1), (x - 1)(x^2 + x + 1), and (x - 1)(x^3 + x^2 + x + 1) might lead them to the general formula for the sum of a geometric series” (NGA & CCSSO, 2010a, p.8). Discovering the general formula for the sum of a geometric series is a challenging exercise even at the secondary school level. In response to the researcher’s question, “As a teacher, what do you think should be added, removed, or revised to the list of these standards for mathematical practice so teachers can understand better?” Daisy responded:

I like having a small… keep it to a paragraph to truly understand what each one of these means. But break it down, make there be a primary and elementary, middle school, high school, or maybe not even that many. But it needs to be broken down. The ones that were broken down, saying that… “Young students might rely on…” I was like, “Wait, here I am. You’re speaking to my students. This is what I need to do.” That’s a little bit clearer but is a nice generalization overall of just what it is. Just so you have… you understand the concepts but breaking it down would really help. Where you start? What’s our endgame? I want to see that. But it is messy (Turn 138).
In short, Daisy articulated the sentiment: The SMP needs to be in an age-appropriate language with grade-specific examples.

In the following sections, I illustrated the participating teachers’ interpretations and understanding of the SMPs that seemed to misalign with the descriptions in the CCSSI (NGA & CCSSO, 2010a).

*Teachers’ misaligned conceptions connected to SMP 2*

SMP 2 (Reason abstractly and quantitatively): Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; knowing and flexibly using different properties of operations and objects (NGA & CCSSO, 2010a, p. 6).

Jane conveyed that the concise, “academic,” and not-so “kid-friendly” wording of the SMP was an obstacle to elementary school teachers’ understanding of and their classroom use of the practice standards. She sent me the more “kid-friendly” version of the SMP via email after the interview (Appendix D). For Jane, SMP 2 was vague to implement for the elementary school classrooms:

But that’s a little more vague. I feel like there’s more room to interpret that …This, I can understand giving to someone who is a secondary math teacher because their life has been content. Do you know what I mean? But hand this to an elementary teacher and say, “They bring two complementary abilities to bear on problems involving quantitative relationships. The ability to…” Stop it. Like if you’re using the word, “decontextualize.” … People have actually made even more kid-friendly versions of the mathematical practices, and when I was math coach, I gave those to teachers to post in their rooms (Jane, Turn 134).
Teachers’ misaligned conceptions connected to SMP 4

SMP 4 (Model with mathematics): Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. (NGA & CCSSO, 2010a, p. 7)

In mathematics education, “representation” has been used interchangeably with “mathematical model,” meaning “a mathematical representation of the elements and relationships in an idealized version of a complex phenomenon. Mathematical models can be used to clarify and interpret the phenomenon and to solve problems” (NCTM, 2000, p. 70). The term “model” has various meanings. One of the common uses of “model” in mathematics education is when it means “representation.” Some of the different representations include physical displays (e.g., pattern blocks, fraction bars, and number line) as well as more-complex pictures, tables, graphs, and words to model problems and situations.

Regardless of the significant influence of the 2000 NCTM Standards (Principles and Standards for School Mathematics [PSSM]) on the SMP, the fourth standard (SMP 4) “uses language that is somewhat different from prior standards documents” (Malkevitch, 2012). When referring to “model with mathematics,” SMP 4 focuses on a great deal of “application of] the mathematics they know to solve problems arising in everyday life, society, and the workplace” (NGA & CCSSO, 2010a, p. 7). Thus, the implied meaning of “model with mathematics” is a process to create “a simplified version of the real world that employs the tools of mathematics”
rather than a representation that PSSM presents (Malkevitch, 2012, Mathematical Modeling section, para. 3). Taking the meaning of the SMP into account, I investigated the interviewed teachers’ misaligned conceptions of the SMP 4.

First, most of the interviewed teachers interpreted SMP 4 solely as representation without considering the real-world connection or application.

Alicia correctly portrayed an SMP but didn’t provide a proper example that parallels SMP 4 descriptions. She referred “using the red counters” to the described practice as “modeling.” In fact, her practice was more aligned to the representation standard of the PSSM (NCTM, 2000 as well as SMP 2 (“Reason abstractly and quantitatively”). The representation standard describes the expectation as follows:

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Create and use representations to organize, record, and communicate mathematical ideas;
- Select, apply, and translate among mathematical representations to solve problems;
- Use representations to model and interpret physical, social, and mathematical phenomena. (NCTM, 2000, p. 67).

If a number line is used or a picture is drawn to visualize the situation in a more simplified way than using real objects, it is more aligned to SMP 2. Addressed portion of SMP 2 is “Quantitative reasoning entails habits of creating a coherent representation of the problem at hand” (NGA & CCSSO, 2010a, p. 6). She did not connect her implementation of SMP 4 with solving everyday life problems or applying mathematics in the real world. Alicia’s example of using counters or drawing arrays to represent the word problem was to simplify the given situation:

It was “Annie saved $6 a week for four weeks. She wants to buy some books. If each book costs $3, how many books can Annie buy?” And the students were asking, “Why didn’t you just give us money to work with?” And I said, “Well, the counters are gonna represent money or blocks or whatever you choose. You can choose to draw it.” So, just learning to use other manipulatives to model what they’re working with. Instead of always a concrete object.
… So, for example, at the board, I said I was stuck on 8 times 5, and I spoke aloud. I said, “I’m thinking. I know I’m really strong with building arrays. So, how can I build an array? Well, since it’s 8, I’m going to make 8 rows with 5 counters in each row. So, that models 8 times 5, and if I count them, I have 40.” So, a lot of it is modeling on my part is what a good mathematician should be doing (Alicia, Turns 78 & 82).

At first glance, Betsy seemed to be interpreting SMP 4 as “apply[ing] what students know...to simplify a complicated situation” (NGA & CCSSO, 2010a, p. 7). Betsy referred to drawing the ten frames to show the situation in a problem: “Bethany knows that 8 plus 4 equals 12, but she used 8 plus 2. Explain how she thought about it” (Turn 184). This example, however, is more aligned with SMP 2 (“Reason abstractly and quantitatively”), because Betsy expected her students to attend to the meaning of quantities when using the ten frames. Addressed portion of SMP 2 here are as follows: “Quantitative reasoning entails habits of creating a coherent representation of the problem at hand”; “considering the units involved”; “attending to the meaning of quantities, not just how to compute them”; and “knowing and flexibly using different properties of operations and objects” (NGA & CCSSO, 2010a, p. 6). Likewise, Olson and Olson (2013) identified that “teachers’ use of representations to understand the meaning of quantities through the constructs of contextualizing and decontextualizing” as a key component to understanding SMP 2.

Jane stated, “I would say, as they get older and go into secondary schools, that modeling wouldn’t be as key.” On the contrary, teachers should provide real-world examples to show the utility and applications of mathematics. With an increase of the subject matter knowledge, students should encounter greater opportunities to model meaningful and consequential real-world problems. Like Alicia and Betsy, Jane displayed a similar conception of mathematical modeling with “representation” or “visualization,” which is linked with SMP 2 (“Reason abstractly and quantitatively”). It is important to note that representation is a way of modeling
with mathematics, but SMP 4 highlights the purpose of representation as to solve and apply mathematics in the context of a real-world situation.

Grace considered representation, such as “a number line or fraction bars,” as “modeling.” It is more aligned with SMP 2 (“Reason abstractly and quantitatively”), because Grace referred to using those tools to help students reason quantitatively and create a comprehensible representation of a problem. Her additional statement, “like physically drawing them or using manipulatives,” also aligns more with SMP 2 in the same sense.

When asked about the SMP that she could understand clearly, Rachel was confident about her understanding of SMP 4. She maintained, “If a child can model their answer and how they got to it, it’s pretty black-and-white whether they reach the correct answer.” As the evidence for this practice standard, she offered, “I always had many, many different tools, so they might pick… I don’t know… a block graph paper, and they might be coloring in the numbers.” Her understanding is more aligned to SMP 2 (“Reason abstractly and quantitatively”), because coloring the number of block was to represent the number in the process of solving a problem.

Second, to some teachers, SMP 4 and SMP 5 sounded very similar. For example, Alicia admitted her confusion about SMP 4 and SMP 5, as follows:

The two that I sometimes confuse is the “modeling with mathematics” and the “use appropriate tools strategically.” When I first learned about those, it was a little unclear to me what the difference between the two was because they both kind of talk about using tools and how to use them appropriately. So, I think those are the two that might need a little bit of work—just on differentiating between what they mean and how the two mathematical practices are different (Alicia, Turn 226).

When Olivia explained SMP 4, she showed some confusion about SMP 4 and SMP 5 as well. Olivia correctly underscored the need “to use manipulatives.” She specifically mentioned “the base-10 blocks” to build conceptual understanding about numbers and their values and “the little centimeter cubes” to demonstrate volumes. This is an example for teachers’ strategic use of appropriate tools that SMP 5 illuminates. Moreover, she contrasted two views of using tools in
classroom: “And the clarity I see kids have with that show me that we should be using more manipulatives like that more often… I know a lot of people think that once they get those out that the kids can become very off-task, so they choose not to get them out” (Olivia, Turn 138).

Third, Cecilia literally equated “modeling” to what the teacher and students should do: an all-encompassing ideal relating to teaching and learning of mathematics. Furthermore, Cecilia used the word, “model,” to mean “imitate.” The example of her modeling illustrates her interpretation of “model” as “me showing [the students] how to do it, then modeling how to do it together, and then trying to get them to repeat that model that you showed them” (Cecilia, Turn 138). Similar results were shown in Olson and his colleagues’ (2014) study. The participants (23 various grade level teachers) anticipated their own responsibility in classroom settings for showing students how to do mathematics and exemplifying a mathematical process so that students repeat what teachers did. In addition, Cecilia’s conception of modeling with mathematics transpired when she explained the teaching model that she focused the most—“I do, We do, You do.” Cecilia firmly believed that teachers first show students how to do mathematics.

Like Cecilia, Grace interpreted SMP 4 as her students imitating her (the teacher), or in her words, “by me modeling it—*modeling* the modeling, it sounds silly, but—then the kids kind of see what that looks like [emphasis added]” (Grace, Turn 98). The participants in Olson and others’ study (2014) presented similar understanding of SMP 4: “the participants still felt an initial compulsion to show students how such modeling is done through completing models and activities for the students as a way of exemplifying such processes” (p. 17). The concept of “modeling” as imitating, demonstrating, or exemplifying is pervasively used in professional development (Gulamhussein, 2013; Marzano, Water, & McNulty, 2005).
**Teachers’ misaligned conceptions connected to SMP 5**

SMP 5 (Use appropriate tools strategically): Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts (NGA & CCSSO, 2010a, p. 7).

Olivia explained her emphasis of SMP 5 by providing me with an example of her instruction on measurement in metric system as follows:

… As far as tools, I also think about tools to generate interest in the students. Not just a protractor, but… Early in the year, we were doing learning how to measure in the metric system. So, the first thing I look at is: what are they doing? What they showing me to use as far as my lesson? And then my question is: Well, what’s fun about that? Because our time together should be enjoyable. So, I brought in all these stuffed animals from my house. I have this stuffed giraffe that’s about yay high, and I did a metric zoo with the children. And I said, all these animals in the zoo… And I asked them to measure all of the animals’ height in centimeters. And from there, we converted all of them into meters, and then to millimeters, and we converted the m to kilometers. … So, that day, my tools for teaching this lesson were a bunch of stuffed animals. This week, … I brought in a snake and a lizard, a starfish and a dolphin. And the packaging says if you put them in water for ten days, they grow to 600%. So, I first asked the children, “What does that mean?” I showed them these things, and we measured them, and I said… We did a “smath” lesson where we took math and science and combined them. And we wrote a hypothesis after we measured them. And we said, “How big do think this is gonna get?” But “What does it mean to grow to 600%?”. And our whole thing was truth in advertising. Is this true, or is this advertiser being a liar? So, when we come back to school next week, we’ve got a big tub of water. We have these four objects. Everyone’s made their predictions. Then, daily, they measure them and weigh them on a gram scale, and they had to write their predictions on how big they think they’ll
get... So, that’s way to generate... that’s the tools I’m using, rather than, you know, technology to create interest and I hope that they’ll remember that.

This example present Olivia’s passion about helping students learn mathematics in a fun way, using interesting objects (e.g., stuffed animals and expandable animals). However, Olivia demonstrated a confusion between this standard (SMP 5) and SMP 4 (model with mathematics). For instance, she brought in “stuffed animals” to measure their heights in the metric units. In addition, she brought in expandable animals (“a snake and a lizard, a starfish and a dolphin”) to collect data about their changing sizes. These teaching and learning aides modeled “problems arising in everyday life, society, and the workplace” that SMP 4 emphasizes.

Furthermore, Olivia recognized SMP 5 by mentioning both “calculator” and “protractor.” However, she concluded that these tools were irrelevant due to her elementary school students’ needs as follows:

You’re using a calculator, you’re using a protractor, your using... This seems like more of a secondary standard to me than an elementary, and certainly not a primary at all. Because, I don’t see you talking about base-10 blocks here. I don’t see any talk about attribute shapes. I don’t see anything... It seems higher ed (Olivia, Turn 102).

She could not grasp that SMP 5 was never meant to be an all-inclusive tool list for the standard states: “Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations (emphasis added)” (NGA & CCSSO, 2010a, p. 7).

**Teachers’ misaligned conceptions connected to SMP 6**

SMP 6 (Attend to precision): Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They
are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions (NGA & CCSSO, 2010a, p. 7).

When asked to explain what SMP 7 means to her, Betsy inadequately attributed SMP 1 (“Make sense of problems and persevere in solving them”) to SMP 6. She stated, “I really ask them to focus on, ‘Does that make sense?’ So, when they write down an answer, ‘Would it make sense?’… But other than that, I don’t really know (Turn 182). Asking students if their work or answers make sense is what SMP 1 encourages teachers to do. Furthermore, she seemed to be interpreting incorrectly that “precision” to an answer must be “perfect.” SMP 6 refers “a degree of precision appropriate for the problem context.” Addressed portion of SMP 1 here is “Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, ‘Does this make sense?’” (NGA & CCSSO, 2010a, p. 6).

Like Betsy, Daisy equated “precision” to “perfection.” She stated, “I want them to make mistakes. So, I don’t think we’re attending to precision with anything” (Turn 158). Her response appears to solidify the belief that she was not familiar with the practice standard in general.

When asked the SMP she did not focus in her classroom, she answered “attend to precision” without hesitation. She explained that her focus is more students’ involvement than precision: “we’re just hoping that they’re able to talk about it. I try to carefully… they cannot create something, they’re just blurting out whatever they want. And I try to encourage that. I want them to make mistakes. So, I don’t think we’re attending to precision with anything” (Turn 158).
When asked to provide examples of SMP 6, Cecilia demonstrated her teaching of place holder and rounding numbers. In the below multiplication example, Cecilia described her “Duck Hunter” analogy to underscore the need to place 0s as place holders. It sounds more precise when using the word “placeholder,” instead of “dropping your dead duck” or “putting zero.” However, she didn’t seem to clarify the definition of placeholder. Figure 4-2 depicts her teaching a two-digit multiplication problem by drawing a duck face for zero as a placeholder. Her purpose of teaching appeared to be students’ *instrumental* understanding rather than *relational* understanding as shown in this example. Skemp (2006) defined relational understanding as more conceptual (“knowing both what and why”) and instrumental understanding as knowing about rules without reason. Instead of judging one as better than the other, Skemp suggested teachers to have a “reasoned choice” by considering the advantages and disadvantages of both relational and instrumental understanding. In Cecilia’s case, her teaching of placeholder in multiplication seemed to be aimed at students’ instrumental understanding.

Another example of Cecilia’s instruction of rounding numbers consistently corroborated that her teaching approach was instrumental. She introduced to her students imaginary Stan, “the little toolbox guy,” to help them understand the *rule* for rounding. She stated, “If ‘Stan’ was five or higher, they got to high-five their neighbor.” Her comments such as “You have to go from something that they can remember... (emphasis added)” and “At some point, they had to get to,
'Look to the place to my right,' start saying the rules (emphasis added)” affirmed her beliefs in students’ instrumental understanding (Cecilia, Turn 90)

Rachel interpreted “precision” to mean “doing it (determining the solution) the same [way].” She further stated that the practice standard hindered her students “to think outside the box,” “to come up with their own approaches,” and “to accept the approaches of others.” In addition to these misaligned conceptions, Rachel ended the exchange with “And I don’t think that’s what it means. So, it is vague to me” (Turns 110 $ 112). In Chapter 5, I address the need for teachers to overcome such “vagueness.”

**Teachers’ misaligned conceptions connected to SMP 8**

**SMP 8 (Look for and express regularity in repeated reasoning):** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation \((y - 2)/(x - 1) = 3\). Noticing the regularity in the way terms cancel when expanding \((x - 1)(x + 1)\), \((x - 1)(x^2 + x + 1)\), and \((x - 1)(x^3 + x^2 + x + 1)\) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. (NGA & CCSSO, 2010a, p. 8).

When explaining the example of SMP 8 from her third-grade mathematics class, Alicia incorrectly attributed the classroom dynamics to SMP 8 rather than SMP 7 (“Look for and make use of structure”) as follows:

I’m working with them with division right now… and I guess I did do this correctly. I was talking to the students, and I said, “I’m noticing that whenever I take 4 times 5, and I get 20. And then I take 20 divided by 5, and I get 4.” I was like, “What do you notice about that?” And they said, “It’s backwards, Mrs. K (Alicia)!” And they said, “So, that means with multiplication and division, they’re backwards.” And then we got into the discussion of “Well, that means that they’re inverse operations. Inverse operations are when they’re quote
‘backwards’.” When students are noticing things, then we look for the regularity. It’s difficult because you don’t just want to come out and say, “Multiplication and division are inverse operations.” You kind of want them to discover it on their own (Alicia, Turn 262).

Multiplication and division are inverse operations, and this is a recognition of a pattern (a structure of mathematics), rather than repeated reasoning that SMP 8 describes.

Daisy expressed the ambiguity of SMP 8 in her comment: “I wasn’t as clear in what I needed to do, how I could build. I understand, for students, I get it, but what am I going to do to prepare them for that? What’s that first baby step that we need to make as elementary teachers?” (Turn 134). Most likely, seeing the actual standards statement like, “Noticing the regularity in the way terms cancel when expanding \((x - 1)(x + 1)\), \((x - 1)(x^2 + x + 1)\), and \((x - 1)(x^3 + x^2 + x + 1)\) might lead them to the general formula for the sum of a geometric series” (NGA & CCSSO, 2010a, p. 8). She concluded, “Does it have elementary? No, there’s nothing in here that says anything about…” Nevertheless, she made an important point: The SMP needs to be articulated in an age-appropriate language with grade-specific examples.

Similar to her own conclusion that SMP 5 had less relevance to her students’ needs, Olivia seemed to be missing the general point about the SMP: Here are some important, thematic practices (mathematical habits) that teachers need to develop in students. Hence, her statement, “I don’t know because they’re giving algebraic expressions as examples,” seems to support her inability to substitute her classroom example for the ones given in the standards. She ended with: “They (students) continually evaluate the reasonableness of their intermediate results.’ Maybe that is the most vague for me.” In the interview, Olivia had mentioned her use of “the expandable animals” in the context of measuring the defined attributes such as length and weight. In the example, her students continually evaluated the reasonableness of their intermediate results to test their hypotheses to the claimed, “600% increase” (Olivia, Turn 90). In short, her students were actually practicing this particular SMP: comparing their measured values to their hypotheses.
Summary of teachers’ misaligned conceptions of the SMP

Based on the above documented evidence, the participating elementary school teachers demonstrated a minimal range of misaligned conceptions of the Standards for Mathematical Practice. The below bar chart (Table 4-7) represents the individual teacher’s misaligned conceptions of specific SMPs. The teachers displayed two to three misaligned conceptions among the practice standards.

Table 4-7: Individual teacher’s misaligned conceptions of specific SMPs

<table>
<thead>
<tr>
<th>Teacher</th>
<th>SMP 4</th>
<th>SMP 5</th>
<th>SMP 6</th>
<th>SMP 7</th>
<th>SMP 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alicia</td>
<td>SMP 4</td>
<td></td>
<td>SMP 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Betsy</td>
<td>SMP 4</td>
<td></td>
<td>SMP 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cecilia</td>
<td>SMP 4</td>
<td></td>
<td>SMP 6</td>
<td></td>
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</tr>
<tr>
<td>Jane</td>
<td>SMP 2</td>
<td></td>
<td>SMP 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Daisy</td>
<td>SMP 6</td>
<td></td>
<td>SMP 8</td>
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<tr>
<td>Grace</td>
<td>SMP 4</td>
<td></td>
<td>SMP 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Olivia</td>
<td>SMP 4</td>
<td>SMP 5</td>
<td>SMP 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rachel</td>
<td>SMP 4</td>
<td>SMP 6</td>
<td></td>
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</tbody>
</table>

The next bar chart (Table 4-8) summarizes the compiled data of the teachers’ misaligned conceptions by specific SMPs. In this chart, I indicated which teacher identified which specific SMP. I found that one teacher (Jane) showed her misaligned conceptions of SMP 2; seven teachers (Alicia, Betsy, Cecilia, Jane, Grace, Olivia, and Rachel) showed their misaligned conceptions of SMP 4; two teachers (Grace and Olivia) showed their misaligned conceptions of SMP 5; four teachers (Betsy, Cecilia, Daisy, and Rachel) showed their misaligned conceptions of SMP 6; and three teachers (Alicia, Daisy, and Olivia) showed their misaligned conceptions of SMP 8. Those who mentioned SMP 1, SMP 3, and SMP 7 seemed to have aligned conceptions as discussed earlier.
Table 4-8: Compiled data of teachers’ misaligned conceptions of specific SMPs

Table 4-9: Pareto bar chart of compiled data of teachers’ misaligned conceptions of SMPs

Corresponding to this compiled data, the Pareto bar chart (Table 4-9) below represents the descending order of teachers’ misaligned conceptions of SMPs. As shown, most of the teachers understood SMP 4 somewhat differently from the descriptions in CCSSI (NGA & CCSSO, 2010a).

Table 4-9: Pareto bar chart of compiled data of teachers’ misaligned conceptions of SMPs
Summary

I employed a qualitative study to understand eight western Pennsylvania elementary school teachers’ conceptions about the Standards for Mathematical Practice (SMP). For deeper understanding of the teachers’ conceptions of the SMP, I conducted in-depth interview with each participating teacher. I analyzed the verbatim transcripts of the in-depth interviews of the teachers as the main source, supplementing it with other resources, such as email exchanges, textbooks, and artifacts.

This study examined how the eight teachers exhibited their conceptions (both closely-aligned and misaligned conceptions) of the SMP. Before I examined the teacher’ conceptions relating to each of the SMPs, I briefly analyzed their overarching goals. The teachers had various overarching goals in teaching mathematics: from general and non-pedagogy-related goals, such as “love of learning (mathematics),” to content-related goals, such as “basic mathematics” and “number sense.” Furthermore, some of the teachers mentioned various pedagogy-related goals, such as “productive discourse environment.” In particular, the pedagogy-related goals were associated with their conceptions about the SMP. The most aligned SMPs to teachers’ overarching goals in teaching mathematics seemed to be SMP 1 (4 teachers), SMP 3 (3 teachers), and SMP 4 (3 teachers). The elementary school teachers demonstrated a range of understanding of the SMP. Based upon their discussions about the SMPs, the teachers often exhibited conceptions that were consistent to the words in the CCSSI (NGA & CCSSO, 2010a). Those who mentioned their interpretations and implementations of SMP 1 (Alicia, Betsy, Jane, Daisy, Grace, Olivia, and Rachel), SMP 2 (Alicia and Rachel), SMP 3 (Alicia, Betsy, Jane, Daisy, Grace, and Rachel), and SMP 7 (Betsy, Cecilia, Grace, and Olivia) showed closely-aligned understanding of the standards to the SMP descriptions. For other standards (SMP 4, SMP 5, SMP 6, and SMP 8), the teachers displayed varying degrees of understanding.
It is notable that only one teacher’s (Daisy) explanation of SMP 4 seemed to align with the description of SMP 4 in CCSSI (NGA & CCSSO, 2010a). The rest of the teachers exhibited misaligned conceptions of the description of the SMP. Three types of interpretation for SMP 4 surfaced. First, most of the teachers (Alicia, Betsy, Jane, Grace, and Rachel) accepted the concept of representation from the PSSM (NCTM, 2000) as “model with mathematics.” Second, one teacher (Cecilia) understood modeling as perceiving and imitating the teacher moves. Third, one teacher (Olivia) discerned modeling for using tools. Furthermore, teachers’ misaligned conceptions of the SMP occurred when they talked about SMP 4 (7 teachers), SMP 5 (2 teachers), SMP 6 (4 teachers), and SMP 8 (3 teachers).

Table 4-10: The SMPs discussed during the interview

![Bar graph](image)

The above bar graph (Table 4-10) represents the SMPs that were discussed by the teachers during the interviews regardless of their alignment to the SMP. The teachers talked about the SMPs that they recalled, the SMPs they had implemented the most, the SMPs they had implemented the least, and the SMPs they would focus more. As shown, there are some SMPs that the teachers talked more frequently than other SMPs. The majority of the participants
mentioned SMP 4 (eight teachers), SMP 6 (eight teachers), SMP 1 (seven teachers), SMP 5 (seven teachers) and SMP 3 (six teachers). However, their understanding of each of the SMP varied. For example, all seven teachers who spoke of SMP 1 exhibited their conceptions that were consistent with the SMP, while seven of the eight teachers who spoke of SMP 4 displayed misaligned conceptions.
Chapter 5

Discussion

This chapter presents a summary of the study, a summary of the findings and conclusions based on the findings presented in Chapter 4. In addition, I discuss the contribution of this study to the field of mathematics education and implications of the study on teachers’ conceptions of the Common Core State Standards for Mathematical Practice (SMP). It concludes with a discussion of the study’s limitations and recommendations for future research.

Summary of the study

In this section, I briefly summarize the study. I begin the section by restating the problem as well as the purpose of the study and the research question that I presented in Chapter 1. Next, I outline the employed research methodology for data collection and analysis. The findings of the study answer the research question. I briefly compile the findings of the elementary school teachers’ conceptions of the SMP.

A couple of decades ago, Hiebert and his colleagues (1996) asserted that teachers are as important as reform doctrines. Teacher conceptions, in particular, have been accepted as an important factor that affects teachers’ intended or enacted instructional practices. Consequently, numerous studies, with various interests, have examined teachers’ conceptions with various interests. The topics of the studies include teachers’ conceptions (beliefs, knowledge, and interpretations) of teaching and learning mathematics as well as mathematical standards.

A line of research has studied teachers’ beliefs for effective teaching (e.g., Ball, 1990a; Ball, 1991; Bauch, 1984; Brendefur and Frykholm, 2000; Bush, Lamb, & Alsina, 1990; Even, 1993; Ferrini-Mundy, 1986; Fennema et al., 1996; Leinhardt, Putnam, Stein, & Baxter, 1991;
These studies examined how teachers’ beliefs about mathematics as well as teaching and learning of mathematics were related with their practice. They showed some variability in the degrees of the relationships, suggesting that teachers’ beliefs have played important roles in deciding their instructional actions.

In addition to teachers’ beliefs, researchers have been interested in teachers’ knowledge to teach mathematics. Some researchers developed models for teachers’ knowledge to teach such as Pedagogical Content Knowledge (PCK; Shulman, 1986), the Instructional Triangle (Cohen & Ball, 1999, p. 3), and Mathematical Knowledge for Teaching (MKT; Ball et al., 2008). These models have guided a significant number of studies to understand teachers’ knowledge to teach mathematics (Aubrey, 1997; Borko et al, 1992; Campbell et al., 2014; Cohen, 1990; Fennema et al., 1996; Hill et al., 2008; Lloyd & Wilson, 1998; Ngo, 2013; Putnam et al, 1992; Thompson & Thompson, 1994).

Moreover, there have been studies examining teachers’ conceptions about mathematical standards such as the NCTM Standards (1989, 1991, 1995, 2000) and the CCSSM (NGA & CCSSO, 2010a). These studies presented varying degrees of teachers’ conceptions about the standards (e.g., Anderson & Piazza, 1996; Brew, Rowley, and Leder, 1996; Frykholm, 1995; Burks et. al., 2015; Davis et al., 2013; Davis et al., 2014; Davis et al., 2017; Sowell & Zambo, 1997; Troia & Graham, 2016; Watts, 1991) and teachers’ conceptions and their impact on curriculum reform (e.g., Burkhardt, Fraser, & Ridgway, 1990; Handal & Herrington, 2003; Haynes, 1996; Koehler & Grouws, 1992; Prawat, 1992; Sosniak, Ethington, & Varelas, 1991).

Mathematical standards such as the National Council of Teachers of Mathematics (NCTM) Standards (NCTM, 1989, 1991, 1995, 2000) and the Common Core State Standards for Mathematics (CCSSM; NGA & CCSSO, 2010a) have provided teachers with guidelines to help students learn mathematics most effectively. As shown in numerous pieces of literature, teachers
may not provide students proper opportunity to learn without corresponding conceptions of the standards.

The CCSSM was created in response to the calls for a coherent, focused, and rigorous standards for the school mathematics. Within the CCSSM, the Standards for Mathematical Practice (SMP) delineates mathematical proficiencies and competencies that K-12 teachers should develop in their students. Since the launch of the CCSSM in 2010, the majority of the states in the U.S. have adopted the CCSSM. Pennsylvania, for one, crafted its own standards—The PA Core State Standards—mirroring both content and practice standards of the CCSSM. Pennsylvania teachers are required to have a clear understanding of the PA Core Standards. It is timely and appropriate to study Pennsylvania teachers’ conceptions, as the Standards have been adopted and implemented for several years since the revision of the PA Core Standards in 2014.

Researchers have shown various interests in studying the CCSSM including the content, curriculum, and alignment (Cobb & Jackson, 2011; Dingman, Teuscher, Newton, & Kasmer, 2013; Gamson, Lu, & Eckert, 2013; Nagle & Moore-Russo, 2014; Polikoff, 2015; Porter et al., 2011; Porter et al., 2013; Williamson, Fitzgerald, & Stenner, 2013), appropriateness of the CCSSM (Burns, 2013; Bostic & Matney, 2013; Cobb & Jackson, 2011; Kendall, 2011; Mathis, 2010; Schoenfeld, 2015; Sztajn et al., 2012; Tienken, 2011; Wu, 2011; Wu, 2014), professional development for and implementation of the CCSSM (Elias, 2014; Holliday & Smith, 2012; Jenkins & Agamba, 2013; Kane, Owens, Marinell, Thal, & Staiger, 2016; Liebtag, 2013; Opfer, Kaufman, & Thompson, 2016; Simpson & Linder, 2014; Sztajn, Marrongelle, Smith, & Melton, 2012), teacher perspectives of the CCSSM (Burks et al., 2015; Davis et al., 2013; Davis et al., 2014; Troia & Graham, 2016), and the effectiveness of the CCSSM (Hiebert & Mesmer, 2013; Schmidt & Houang, 2012). Only a few studies have examined teachers’ conceptions about the mathematical standards for practice since the beginning of the Common Core State Standards
(CCSS) era. In fact, studies investigating elementary school teachers’ conceptions about the SMP are limited.

The purpose of this study was to examine elementary school teachers’ conceptions about the Standards for Mathematical Practice (SMP). As defined in Chapter 1, “conception” is a general notion or mental structure that incorporates beliefs, knowledge, propositions, interpretations, preferences, views, agreement, and mental images. I paid close attention to the participating teachers’ conceptions (attending to their knowledge and interpretations) of the SMP. To guide this study and to fulfill its purpose, I used the following question: How do the participating elementary school teachers understand and interpret the SMP?

Most of the studies on the CCSSM have been conducted quantitatively, using surveys (e.g., Bostic & Matney, 2013; Burks et. al., 2015; Choppin et al., 2013; Cogan et al., 2013; Davis et al., 2013; Davis et al., 2014; Troia & Graham, 2016). Despite the advantage of survey type of research such as the usefulness when a researcher aims to explain features of a very large group (Blackstone, 2012) and its time- and cost-efficiency, it would be more appropriate to conduct a qualitative study when investigating teachers’ conceptions. Attending to an individual’s uniqueness (e.g., conception) is the specific business of qualitative research (Maxwell, 2013).

This research study used qualitative methodology, specifically qualitative study. I employed in-depth interviews with eight elementary school teachers in western Pennsylvania. The verbatim transcripts of the interviews were the main resource for the data analysis. However, I triangulated the data with other collected data such as email exchanges and curricular materials. The email exchanges with the teachers were especially useful because they provided me with an opportunity for member checking and for gaining additional explanations to what they had said during the interviews.

At completion of the transcribing, I began analysis. It required re-reading the transcripts and listening to the audio recordings several times. In doing so, I highlighted the key statements
on the transcripts and made notes. I categorized the meaningful themes to answer the research question for this study. I narrowed down the themes to subcategories for additional classification. Quotes from the transcripts, email exchanges, and personal communication were added to support the themes. Lastly, I reviewed the categorization multiple times before writing the results section.

Summary of the findings

The purpose of this study was to examine elementary school teachers’ conceptions about the SMP in school districts of western Pennsylvania. During the interviews, the eight participating elementary school teachers talked about the SMPs that they recalled, the SMPs they had implemented the most, the SMPs they had implemented the least, and the SMPs they would focus on more. I examined how they presented their understanding (closely aligned conceptions to misaligned conceptions) about the SMP. In the following sections, I summarize the findings that answered the research question.

Elementary school teachers’ overarching goals

The eight teachers have varying years of elementary school mathematics teaching experiences: from 5 to 34 years. Their teaching assignments also vary (e.g., special education support, specific grade level, and math coach). It is notable that every teacher has earned at least a master’s degree in an education field (elementary mathematics education, special education, literature education, and curriculum and Instruction). Moreover, four teachers have mathematics coaching experience in their schools or school district.
I examined the teachers’ overarching goals in teaching mathematics and thoroughly reviewed their discussions about the SMP. Of the eight teachers, six (Betsy, Cecilia, Jane, Daisy, Grace, and Rachel) emphasized content goals such as basic mathematics and number sense. Their focus, however, was not limited to the content. The teachers (Alicia, Betsy, Jane, Daisy, Grace, Olivia, Rachel) also exhibited their attention to instructional practices to teach mathematics. Most of these were related to the SMP such as problem-based learning (Alicia), having real-life connections (Alicia, Daisy, and Rachel), experiencing hands-on activities (Alicia, Betsy, Daisy, and Rachel), encountering productive struggles (Betsy and Jane), providing the opportunity for productive discourses (Betsy, Jane, and Olivia), and persevering (Alicia).

Furthermore, I found some correlations between relating the teacher’s overarching goals to their conceptions of the SMP. The overarching goals of Alicia, Betsy, Cecilia, and Jane were consistent with the idea of the SMP 1. Alicia, Betsy, and Jane commented that they not only were familiar with SMP 1, but they also had focused on this practice in their classrooms. Cecilia, who admitted her unfamiliarity of the SMP, did not talk about SMP 1 during the interview at all. Rachel’s focus of the meaning of the quantities emerged during her discussion about SMP 2 with her classroom example of teaching addition quantitatively and abstractly. Three teachers’ (Betsy, Jane, and Olivia) overarching goals aligned with SMP 3. These three teachers either explained SMP 3 correctly or illustrated examples associated with SMP 3. Even though Olivia did not address SMP 3 particularly, she showed her implementation of SMP 3 throughout the interviews. Three of eight teachers’ overarching goals matched with SMP 4. Jane, Daisy, and Rachel stressed the importance of mathematical modeling and application of mathematics for their teaching goals. However, only Daisy’s explanations were consistent with the descriptions of the SMP. Both Jane and Rachel transferred the concept of representation from the PSSM (NCTM, 2000) for SMP 4.
Furthermore, I learned various ways that the teachers have become familiar with the SMP. First, three teachers (Jane, Grace, and Olivia) expressed their own motivation and effort to become more familiar with the SMP. Second, continued professional development and graduate course work influenced their learning of the SMP (Alicia, Betsy, and Olivia). Third, the experiences of math coaching appeared to be related to their greater awareness about the SMP (Jane, Grace, and Olivia). Fourth, their teaching materials and online resources (e.g., Pennsylvania Department of Education Standards Aligned System [PDE SAS]) expanded their knowledge of the SMP (Alicia, Betsy, Jane, Daisy, and Grace). Two of the eight teachers, Cecilia and Rachel, were not familiar with the SMP.

**Elementary school teachers’ conceptions of the SMP**

Taking the teachers’ explanations about and examples of each stated SMP into consideration, I found that the participating teachers demonstrated a range of understanding of the SMP. The SMP description in the Common Core State Standards Initiative (CCSSI; NGA & CCSSO, 2010a) guided the determination of the teachers’ understanding of each SMP. I compared teachers’ clarification about the SMP as well as their related examples to the detailed description of the SMP written in the CCSSI.

It is notable that only one teacher (Daisy) exhibited a closely aligned understanding of SMP 4 and only one teacher (Rachel) exhibited such an understanding of SMP 8, while seven of the eight teachers (Alicia, Betsy, Jane, Daisy, Grace, Olivia, and Rachel) showed their understanding of SMP 1 which were parallel to the SMP descriptions. Six teachers (Alicia, Betsy, Jane, Daisy, Grace, and Rachel) showed their understanding of SMP 3; five teachers (Alicia, Betsy, Daisy, Grace, and Olivia) showed their understanding of SMP 5, five teachers (Alicia, Betsy, Cecilia, Jane, and Grace) showed their understanding of SMP 6, four teachers
(Betsy, Cecilia, Grace, and Olivia) showed their understanding of SMP 7, and two teachers (Alicia and Rachel) showed their understanding of SMP 2.

While the participating teachers exhibited applicable understanding of the SMP, the data analysis suggested their misaligned conceptions of some SMPs. The misaligned conceptions in this paper do not mean “wrong” understanding; rather, it means a “different” understanding of the original description of the SMP in the CCSSI (NGS & CCSSO, 2010a). The foremost misaligned conception area was SMP 4. Seven of the eight teachers understood SMP 4 somewhat differently from its description.

Before I discuss how the participating teachers exhibited their misaligned conceptions, I briefly review the literature regarding mathematical modeling. Cirillo and her colleagues (2016) distinguished mathematical modeling from modeling mathematics. Modeling mathematics refers to using mathematical representations such as drawings, number lines, tables, and graphs, to connect mathematical concepts. On the other hand, mathematical modeling in SMP 4 “links mathematics and authentic real-world questions…where the main task is to translate a problem into a mathematical form” (p. 5). Pollak (1969, 1989, 2007, 2011), who is known as one of the pioneers of mathematical modeling in school education, has promoted mathematical modeling and application for many years. Emphasizing the connection between mathematics and real life, Pollak has advocated for an inclusion of mathematical modeling in school mathematics. A line of research has examined how mathematical modeling can be taught in school mathematics (e.g., Anhalt, Cortez, & Bennett, 2018; Blum & Ferri, 2009; Boaler, 2001; Heid, 1995; Hernández, Levy, Felton-Koestler, & Zbiek, 2017; Meyer, 2015; Stohlmann, Maiorca, & Olson, 2015; Swetz & Hartzler, 1991). The studies unanimously stressed out the importance of mathematical modeling in mathematics education to help students “build the understandings and abilities that are needed for success not just in school but also in daily life” (Stohlmann et al. 2015).
The teachers’ misaligned conceptions were identified when their interpretations were different from mathematical modeling that connects mathematics and real-world. I categorized three different types of their misaligned conceptions of mathematical modeling: (1) as representation, (2) as imitating, and (3) as using tools.

First, six teachers (Alicia, Betsy, Jane, Grace, and Rachel) regarded mathematical modeling as representation. Principles and Standards for School Mathematics (PSSM; NCTM, 2000) equated mathematical modeling and mathematical representation. However, Malkevitch (2012) stated that SMP 4 described modeling differently from prior standards document such as PSSM (NCTM, 2000). The greatest focus of SMP 4 is for students to apply mathematics to solve everyday problems rather than to represent the everyday problem situation into simpler forms. Pollak (2011) expected the SMP 4 would “help to bridge the gulf between reasoning in the mathematics class and reasoning about a situation in the real world” (p. 64). Second, Cecilia misunderstood “modeling with mathematics” as teacher’s demonstration of how to solve mathematical problems so that students imitate the teacher’s strategy in solving similar problems. Third, Olivia misinterpreted “using tools” as “modeling with mathematics.” Despite her appropriate use of base-10 blocks to teach place values as well as of “centimeter cubes” to demonstrate volumes, these examples are more related to SMP 5 than SMP 4. SMP 5 explains that “Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful” (NGA & CCSSO, 2010a, p. 7).

Furthermore, teachers’ misaligned conceptions of the SMP appeared in other SMPs. Four teachers (Betsy, Cecilia, Daisy, and Rachel) misunderstood SMP 6. Among the four teachers, Betsy not only attributed SMP 1 to SMP 6, but she also interpreted “precision” to “perfect” answer to a mathematics problem. Similarly, Daisy regarded “precision” as “perfection.” When Cecilia explained her classroom examples for SMP 6, she exhibited her
focus on students’ instrumental understanding. Moreover, Rachel equated “precision” to “doing it (determining the solution) the same [way].”

Considering which SMPs the teachers mentioned the most, I found that the teachers talked more about some SMPs than other SMPs (Table 4-8). The SMPs that most of the teachers made comments on were SMP 4 (eight teachers), SMP 6 (eight teachers), SMP 1 (seven teachers), SMP 5 (seven teachers) and SMP 3 (six teachers). Less teachers mentioned about the rest of the SMPs: two teachers talked about SMP 2 and four teachers talked about SMP 7 and SMP 8 each. The teachers’ understanding of each of the SMP, however, varied. For example, all seven teachers who spoke of SMP 1 exhibited aligned conceptions with the SMP, while seven of the eight teachers who spoke of SMP 4 displayed misaligned conceptions.

**Conclusions**

This study was different from other studies because it analyzed a group of elementary school teachers’ conceptions of the SMP concentrating on their understanding and interpretations of the SMPs through the in-depth interviews. As identified through the review of literature, numerous studies around the CCSSM have focused on the content of the standards, curriculum alignment, professional development and implementation, and the effects of the CCSSM on instructional practices and student outcomes. Studies investigating elementary school teachers’ conceptions about the SMP are limited. Moreover, most of these studies used survey methods to examine teachers’ conceptions on a large scale. The in-depth interviews I employed for this qualitative study was an efficient tool for deep examination into eight teachers’ conceptions of the SMP.
Contributions of this study

The findings of this study portrayed how eight western Pennsylvania elementary school teachers understood the SMP. The plentiful examples of the teachers’ practices through their own voices supported the findings. The findings contribute to the line of the studies that have investigated teachers’ “knowledge of content and curriculum” (Ball et al., 2008) and beliefs about standards. In particular, this study is significant as it bridges the gap between teacher conceptions studies of NCTM Standards and the CCSSM. As identified through the review of literature, numerous studies around the CCSSM have focused on the content of the standards, curriculum alignment, professional development and implementation, and the effects of the CCSSM on instructional practices and student outcomes. Studies investigating elementary school teachers’ conceptions about the SMP are limited. Moreover, most of these studies used survey methods to examine teachers’ beliefs on a large scale. The in-depth interviews I employed for this qualitative study was an efficient tool for deep scrutiny into eight teachers’ conceptions of the SMP.

Additionally, the findings of such literature indicate that teachers’ conceptions are the critical factors for teachers’ implementation of the SMP, which may ultimately affect students’ learning of mathematics. It is likely that examining teachers’ conceptions of the SMP could be found interesting not only to teachers, schools, and school districts, but also to other stakeholders such as mathematics educators, curriculum developers, and standards writers. In the following section, I offer several suggestions for the stakeholders based on what I found in this study.

Implications of this study

The results of this study provided valuable information about elementary school teachers’ conceptions of the SMP. The findings revealed the varying degrees of teachers’ understanding of
the SMPs. For example, most of the teachers who showed confidence in SMP 1 provided examples of classroom practice that aligned with SMP 1. However, the teachers had misaligned conceptions about SMP 4 as written in the CCSSI (NGA & CCSSO, 2010a). Taking the above two disparate cases of the SMP understanding into consideration, I suggested possible reasons of the teachers’ conceptions and misaligned conceptions shown in Chapter 4.

The teachers mentioned the *relevance* of the standard for their own grade level as one reason for their ample understanding of SMP 1. The relevance and vagueness are closely related. The teachers thought some of the SMPs were vague because the provided explanation of a particular SMP was not applicable to their grade level. The *relevance* seemed to be an important factor in the teachers’ implementation of the SMP. Similarly, the teachers’ lack of implementation of certain SMP was due to the standard’s perceived irrelevance. The teachers commented on the importance of the relevant standards to their grade level as follows:

- “Yeah, because this is a lot of stuff that wouldn’t even apply to me because it’s high school [content]. I’m looking at [the SMP] like my kids don’t have to know that. There’s a lot of content in there.” (Cecilia, Turn 76)
- “It would be much nicer just to have what would be relevant to me.” (Cecilia, Turn 116)
- “I would probably have to say that for my grade level” (Olivia, Turn 84)
- “Maybe [SMP 8] just seems a little bit more secondary math, higher learning.” (Olivia, Turn 136).

Such comments from the teachers provide several implications.

First, teachers should take ownership of the standards. The teachers should be the subjects who seek for more relevant examples for their classrooms. There are abundant resources for teachers as some of the teachers have mentioned. When the teachers seek for more resources, however, they should be attentive regarding what resources to select. The resources should not only be *relevant* to their students’ grade level and needs, but also be *consistent* to the standards. For example, several teachers suggested use of a “kid-friendly” version of the SMP and shared
some of them with me (Appendix D, Appendix E, and Appendix F). I present three versions of SMP 4 in Figure 5-1 for a clear comparison.

Figure 5-1: Three kid-friendly versions of SMP 4

All three versions use the language for students to easily understand what the SMP requires them to do with illustrations. However, there are significant differences. The first version of SMP 4 emphasizes using mathematical models to “solve real world problems.” At the bottom of this version, the phrase, “math can be found everywhere!” supports this idea. The second version underscores two most important ideas of the SMP 4: (1) Model real-world situations using graphs, drawings, tables, symbols, numbers, diagrams, and other representations
and (2) Use mathematical models to solve problems and answer questions. The first two indicate the central idea for SMP 4 as “to solve real world problems” or “model real-world situations.” However, the last one highlights representation in mathematics by denoting, “show your thinking.” This version of SMP 4 pairs with many teachers’ interpretation of SMP 4 in this study. As discussed in Chapter 4, most of the teachers (Alicia, Betsy, Jane, Grace, and Rachel) regarded mathematical modeling as representation. Without deep knowledge of the SMP, such misinterpretation can happen. Teachers should be responsible for their own knowledge as well as their own selection of the knowledge resources. The findings from this study confirm the need for teachers’ strong understanding of the SMP.

Second, professional development developers should consider SMP more deliberately. As presented in the findings of this study, the teachers became familiar with the SMP through various resources. One of the most impactful ways for the teachers’ learning of the SMP was professional development programs or post-baccalaureate programs they attended. The teachers who had never participated in professional development relating to the Common Core presented less understanding of the SMP. Such findings suggest the need for more SMP-focused professional development opportunity to elementary school teachers. The professional development developers should pay as much attention to the practice standards as they do to the content standards. The teachers in this study voiced the lack of opportunity to learn about the SMP because most professional development programs focused on the content standards. The former NCTM president, Michael Shaughnessy (2011), summarized the report of a professional development conference and suggested “action steps” for teachers, school leaders, district- and state-level personnel, and professional development providers. One of the suggestions for the professional development providers was that “professional development should be intensive, ongoing, and connected to practice” (para. 8). The professional development opportunity for teachers should be not only about the content standards, but also the practice standards.
Third, the standards writers should collect feedback on a regular basis from teachers and make improvements as needed. When the participating teachers discussed the irrelevance of the SMP, they lamented the lack of grade-specific examples in the SMP descriptions. They wished to read more practical examples that were directly related to their mathematical instruction. While the intent of the SMP is to develop every student in U.S. schools to be mathematically proficient, teachers hope to see more of what they can do for their grade-level students. The standards writers might want to hear such feedback from teachers and to reflect upon them for improving standards more often. Further, the teachers suggested a different organization of the SMP. They liked the organization of the Standards for Mathematical Practice Grade Level Emphasis (PDE, 2014b) that presented the list of the expertise for each grade level, using bullet points (Figure 2-6). The standards writers may need to consider the best organization of the SMP context so that teachers understand the standards in a more effective way.

The results of this study provide the stakeholders with the strengths and the shortcomings regarding the teachers’ understanding of the SMP collectively. Moreover, the results of this study affirm that a school-wide, professional development that specifically addresses developing and nurturing teachers’ knowledge of the SMP is appropriate. The data-driven decision making should be welcomed by all stakeholders. Furthermore, the follow-up professional development opportunities and individual or small-group planning need to be both sustainable and meaningful. In particular, a grade-level, collaborative planning that articulates the SMPs in age-appropriate language with grade-specific work samples is constructive. Having gone through this process, teachers will most likely take the ownership of the created product: a deeper understanding of the SMP. In turn, they are better prepared to develop these mathematical habits of mind—or in the Common Core language, “varieties of expertise”—in their students.

Finally, the CCSSI (NGA & CCSSO, 2010a) states: “Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices
to mathematical content in mathematics instruction.” Throughout this research, I found no evidence to support the teachers’ intentionality in planning their lessons with the SMP. A pedagogically appropriate planning might include focusing on one or two specific SMPs for each lesson. While other SMPs may arise and should be encouraged for planning a mathematical lesson, teachers must thoughtfully incorporate the SMP and purposefully develop the standards in their students throughout the lesson.

**Constraints of this study**

To better understand the elementary school teachers’ understanding of the SMP, I conducted a qualitative study by employing in-depth interviews. In this qualitative study, I found several constraints. In this section, I discuss the constraints in the selection of participants and of data collection and analysis.

**Constraints in selection of participants**

As explained in Chapter 3, this study is an interview study similar to a case study, in which eight elementary school teachers in western Pennsylvania were the participants of the study. Yin (2009) recommended a screening step for the selection of the cases for a qualitative study. The screening, however, was not applicable for this study. Because it was difficult to recruit the elementary school teachers for the study, every teacher who showed a willingness to participate in the study became the participants. Thus, the participating teachers did not totally represent the elementary school teachers in western Pennsylvania. In fact, two teachers, who had served as math coaches, mentioned the teachers’ lack of knowledge about the SMP as follows:

I think that with our knowledge level here they are familiar with them. When I’ve done trainings for this and I’ve gone to other schools, I’ve asked them if they
know them and no one knows them. They’ve never seen them. I’m like, ‘you’re kidding me’ (Grace, Turn 104).

Last year. And it was just so shocking to me. They knew the Common Core standards, but… I venture to wonder if people are even fluent in the standards per grade level – on what they’re teaching. Not to be negative, I’m just saying in my experience there is a lack of knowledge. There really is (Grace, Turn 106).

To be honest, the teachers in my school district don’t know much about the SMP. They have heard about the Common Core Standards and PA Core Standards, but Standards for Mathematical Practice? No, they don’t … (Olivia, personal communication, December 3, 2018).

Most of the interviewed teachers in this study took a stance in favor of the SMP. I am not certain if this is common for elementary school teachers in western Pennsylvania. It is only plausible to surmise that the teachers for this study were more influenced by the programs they had attended (e.g., graduate courses in mathematics education and professional development programs) or that the teachers didn’t want to speak differently from the trend of the non-traditional, “innovative” teaching that aligns with the suggestions of the SMP.

Regardless of the limitation, the participants processed varying years of elementary school mathematics teaching experiences in various grade level of teaching. Furthermore, they exhibited varying degrees of understanding or misunderstanding of the SMP.

**Constraints of data collection and analysis**

Seidman (2006) stated that the “use of in-depth interviews alone, when done with skill, can avoid tensions that sometimes arise when a researcher uses multiple methods [emphasis added]” (p. 6). One might argue how to measure “done with skill.” Seidman further suggested three characteristics for in-depth interviews as follows: (1) using open-ended questions, (2) comprising a three-interview series, and (3) being between 1 and 2 hours. The interviews I conducted for this study met the first and third qualifications for in-depth interviews. The
interviews, however, were taken place in one meeting. The one-time interview per teacher might have limited my insights into the teachers’ conceptions.

When collecting and analyzing the data, I relied extensively on the in-depth interviews of each teacher and the verbatim transcripts of the interviews. The open-ended questions of the interviews helped the participating teachers to talk freely and honestly about their opinions and experiences related to the SMP. Qualitative researchers use interview protocols. Among several types of interviews (e.g., informal conversational interview, general interview guide approach, and standardized open-ended interview), the type of interview I employed was close to the general interview guide approach. Gall and others (2003) defined this approach as being “more structured than the informal conversational interview although there is still quite a bit of flexibility in its composition” (as cited in Turner, 2010). In this semi-structured interview, I interacted with the participating teachers by flexibly changing the order of the interview protocols. In this way, I was able to develop a rapport with the teachers, and they became more open to talk about their stories (Turner, 2010). Despite the flexibility of this type of interview, the protocol guided the interviews. Nonetheless, I realized that I should have asked how the teachers interpreted and implemented each of the SMPs. The teachers did not talk about all eight SMPs. Through the questions I asked (e.g., the SMPs they recalled, the most important SMPs to the teachers, the SMPs they implemented the most and the least, and the SMPs that are most clear and vague), the teachers talked about their thinking as well as examples of their implementation of the SMP. However, one or two standards from the eight SMPs were not discussed per teacher.

**Recommendations for future research**

Reflecting on the constraints of this study, I suggest some recommendations for future studies. The following is a list of the ways that this study could be extended.
First, the findings of this study contributed to help reduce the gap in the research literature about elementary school teachers’ conceptions about the CCSSM in general and the SMP in particular. However, one of the limitations of this study was the selection of participants. When selecting teacher participants for a qualitative study, a more varied range of teachers’ background has to be considered. The screening step might be necessary to select the participants who most adequately fit the research interest. According to Yin (2009), “the goal of the screening procedures is to be sure that [the researcher] identifies the final cases properly prior to formal data collection” (p. 91). A simple survey can be used as a screening tool. In the survey, a researcher asks participant candidates a few questions that inform the researcher whether the candidate is suitable for the study. This preparatory step may reinforce the case to be viable and to represent what the researchers intend to study.

Second, I recommend for the researchers who use in-depth interviews to conduct a three-interview series or at least a two-interview series. Seidman (2006) identified a three-interview series as one of the features of the in-depth interview for phenomenological interviews. For my qualitative study, I did a couple of informal follow-up interviews, which were of a more casual and personal kind of communication, with the interviewed teachers. Also, emails were exchanged to ask additional questions and to receive their explanations. However, it would have been better had I conducted a series of three separate interviews with each participant. For example, during the first interview, I may ask the participant’s background and experiences in learning (as a K-12 student) and teaching of mathematics (as a preservice teacher) up until the time he or she became an elementary school teacher. During the second interview, I may learn about the teachers’ experiences as mathematics teachers, focusing on current teaching. I can ask questions relating to their current teaching experiences with respect to their students, their curricular materials, and their school support for professional development. This would enable my understanding of the participants’ conceptions in relation with their teaching practices. In the
last interview of the series, I may ask the teachers to reflect on their understanding of and implementation of the SMP. This kind of a series of three interviews will allow more in-depth interviews to gather detailed information gradually as shown in the above example. Seidman (2006) affirms why a three-interview series are required as follows:

- first interview establishes the context of the participants’ experience. The second allows participants to reconstruct the details of their experience within the context in which it occurs. And the third encourages the participants to reflect on the meaning their experience holds for them (Seidman, 2006, p. 17)

The three-interview series can help an interviewer create rapport with the participants, allowing them to talk about their careers, their students, their schools, their teaching, and their thoughts in a gradual manner. Furthermore, the three-interview series would enable the researcher to recognize the changes of the participating teachers’ conceptions of the SMPs over time as well as the plausible factors that affected their conceptions (either closely-aligned or misaligned conceptions).

Third, I suggest the qualitative researchers who conduct in-depth interviews to triangulate the data as much as possible. An in-depth interview itself provides a great deal of the information that a researcher wants to collect and analyze. However, multiple sources of evidence ensure the increase of the validity and reliability of the study. On many occasions, interviews are coupled with other forms of data collection so that the researcher can collect well-rounded information for analysis (Turner, 2010). I suggest a researcher to consider observation of the participants’ mathematics classrooms when investigating teachers’ conceptions of the CCSSM. The classroom observation would provide more concrete examples of the conceptions that emitted during the interviews.

To extend this line of study, I propose future studies that examine the relationship between teachers’ conceptions about the SMP and their practices. The previous research showed that teachers’ beliefs and their practices were not always consistent (Cooney, 1985; Raymond,
1997; Thompson, 1984). Studying the relationship between the teachers’ conceptions and their practices about the SMP could provide the teachers, mathematics educators, and professional development program developers how and what teachers' conceptions affect their enactment of the SMPs. Such studies could present the plausible factors that might affect the relationship (either consistent or not) between the teachers’ conceptions and their practices.

Lastly, I advise researchers who are interested in employing an interview method for their qualitative study about teachers’ conceptions of the SMP to contemplate the design of the interview. I suggest that the researchers add questions regarding the participants’ interpretation about and the relevant examples of each of the eight SMPs in the interview protocols. For example, during the third interview of a three-interview series, I may ask the participants to tell me what each of the eight SMPs means in their own words and how they can relate each SMP to their own lessons. Asking such questions will help the researchers obtain a holistic understanding of the teachers’ conceptions of the SMP. Moreover, it would assist the researchers to compare and contrast how the participants understand or misunderstand each and every SMP without missing any standard.

**Summary**

The objective of this study was to deepen the understanding of elementary school teachers’ conceptions of the SMP in western Pennsylvania. It is meaningful that the eight elementary school teachers in this study exhibited various conceptions (closely aligned to misaligned conceptions) of the SMP. They displayed significantly high understanding of SMP 1 and SMP 3; no teacher showed misaligned conceptions of them. At the same time, the teachers showed high misaligned conceptions on SMP 4. Their focus of SMP 4 seemed to be somewhat different from what the SMP 4 described. The CCSSI (NGA & CCSSO, 2010a) emphasizes
problem solving and application of real-world situations in SMP 4. To that end, students might use mathematical representations such as diagrams, tables, and graphs. However, most of the teachers concentrated considerably on mathematical representation without relating it to the context of a real-world problem. As some math coaches among the participants of this study have asserted, many teachers (not necessarily in this study) do not have sufficient knowledge about the SMP. It was revealing that the participants with high profiles and ample experience in teaching mathematics (and even coaching mathematics teachers) did not attend to the SMP vigorously.

I call for a collaboration of the work from every stakeholder including teachers, mathematics educators, professional development providers, and standards writers to improve not only the standards themselves, but also teachers’ awareness and understanding of the standards. It was hopeful to hear the participating teachers chorusing their changes of instructional practices over time. Many of the teachers appreciated far-reaching teacher education programs (specifically post-baccalaureate programs in mathematics education), abundant online resources from reputable organizations such as NCTM and PDE SAS, professional development programs offered by their schools or school districts as well as some universities, and the advice from peer teachers. Most of all, I value teachers’ efforts and self-motivation to learn more about the SMP to improve their own mathematical instructions.
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Appendix A

Descriptions of the Standards for Mathematical Practice

(NGA & CCSSO, 2010a)

1. Make sense of problems and persevere in solving them: Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively: Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the
referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; knowing and flexibly using different properties of operations and objects.

3. **Construct viable arguments and critique the reasoning of others:** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

4. **Model with mathematics:** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical
results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically:** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. **Attend to precision:** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
7. **Look for and make use of structure:** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

8. **Look for and express regularity in repeated reasoning:** Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.
Appendix B

Interview Protocol

What to Bring

- A copy of the SMP (original, PA Academic Standards for Mathematics, PA Core Standards for Mathematical Practice Grade Level Emphasis (PDE, 2014b), and The Standards for Mathematical Practice, annotated for the K-5 classroom)
- Mathematics textbook the teacher use
- Beliefs about teaching and learning mathematics cards and table.

Hi. I am Jung Colen, a Ph.D. student at Penn State, studying Mathematics Education.

I am interested in investigating elementary school teachers’ thoughts on the Common Core State Standards, in particular, the Standards for Mathematical Practice. Please feel free to say anything about my questions.

Throughout the interview, I will ask questions about your thoughts and experiences about the Standards for Mathematical Practice (SMP). The SMP is also written in the PA Core.

Do you have any question about the interview?

Can I record this interview?

Then shall we begin the interview?

Part 1: Teachers’ experience of mathematics teaching

1. How many years have you taught mathematics in elementary schools?
2. What grade have you taught mathematics in elementary schools?
3. Throughout your experience, what is your goal in teaching mathematics?
4. [If not talked about a teaching practice] Can you also tell me about the teaching practices that you have valued the most?

**Part 2: Teachers’ Beliefs about teaching and learning of mathematics**

1. Based on your experience, what do you think students need to develop in learning mathematics?

2. [Hand out the cutout beliefs cards and the sorting table]. I will hand you the cards that talk about teachers’ thinking about teaching and learning mathematics. Please sort them out into two piles (agree and disagree).

**Part 3: Teachers’ Beliefs about standards (In particular, SMP)**

1. There has been a number of mathematical standards (e.g., NCTM Standards, Common Core State Standards, and PA Core Standards…) What do the standards for mathematics teaching, in general, mean to you?

2. How do mathematical standards help you to plan, implement, and reflect your lessons?

3. Have you heard of the Common Core State Standards or PA Core Standards before this interview? How about Standards for Mathematical Practice?

4. What was the resource for your learning about the CCSSM? and SMP?

5. Have you read either content or practice standards or both?

   **If yes:**

   A. If you remember any standard or item from your reading of the practice standards, can you tell me a standard you can recall? What does this standard mean to you? Can you describe the standard in your words?

   B. Do you recall any other standard? Tell me about the standard.

   C. If you have read the Standards for Mathematical Practice, which version did you read? [by showing each version] original, PA Academic Standards for Mathematics, PA Core Standards for Mathematical Practice Grade Level Emphasis (PDE, 2014b),
and The Standards for Mathematical Practice (NGA & CCSSO, 2010a), annotated for
the K-5 classroom?

If no: Go to the question 6.

6. [Have the interviewee read the original and the PA Core Standards for Mathematical Practice
Grade Level Emphasis.] Here are the Standards for Mathematical Practice. I will let you read
them now. Please take your time to read them carefully.

7. Which version is easier for you to understand what to do in your classroom and why?

A. Both the Common Core State Standards and PA Core Standards for Mathematics
   consist of content standards and practice standards. What do these two categories
   mean to you?

B. Which standard from the SMP do you think is the most important for teaching
   mathematics? Tell me one or more standards you think very important.

8. Can you relate the standards with the grades and topics you teach? Can you give me an
   example of the standard for the grade you teach?

9. Which of the SMP is clear for you to understand what to do in your classroom? Tell me more
   about it.

10. Is there any standard or description of the SMP not clear for you to understand? Tell me
    more about it.

11. As a teacher, what do you think should be added, removed, or revised to this list of Standards
    for Mathematical Practice, so that teachers can understand and implement the practice
    standards in their teaching?

Part 4: Teachers’ Practices

1. Comparing to your beginning years of the career, do you think there is any change in your
   beliefs about teaching mathematics? How about any change in your teaching practices?

2. What do you think caused/influenced the change of your beliefs and practices?
3. [If not talked about standards] Did CCSSM influence your teaching of mathematics? Tell me more about it.

4. Think about your own math classroom. What standards from the SMP do you implement the most? Can you tell me some examples of your implementation of the practice?

5. Thinking back to your math classroom again, what standards from the SMP do you implement the least? Why do you think you emphasize this standard less?

6. What practice do you want to focus more in your teaching and why?

7. Tell me all possible reasons that you could and could not implement the SMP in your own teaching.

**Part 5: Support/Obstacles to learn about the SMP**

1. What textbook or resources are you using for your instruction? Is the textbook (or resources) CCSS-aligned?

2. Did you choose the textbook or resources? If yes, what was the main reason for you to select that textbook. If not, who chose the textbooks and why?

3. (If your textbooks or resources are CCSSM-aligned) In what ways do they direct you to implement the SMP?

4. Did your school or school district require you to know and use the CCSSM including the SMP? If yes, what kind of support from your school or school district was provided for you to learn about the standards?

5. Have you attended a workshop/conference/any type of professional development for teaching mathematics in recent years (within 5 years or since 2014)?
   
   **If yes:**

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6 As the Pennsylvania Core Standards in Mathematics was revised and disseminated in 2014. [https://static.pdesas.org/content/documents/PA%20Core%20Standards%20Mathematics%20PreK-12%20March%202014.pdf](https://static.pdesas.org/content/documents/PA%20Core%20Standards%20Mathematics%20PreK-12%20March%202014.pdf)
A. What was the topics of the professional development program (PD) focused on?

B. From the professional development program, did you hear/learn anything about the Common Core or PA Core for mathematics?

C. If the PD was about the CCSSM, what was the main focus of the PD: Content standards, practice standards, or both?

D. How did the PD affect your teaching? Have you tried anything that you learned from the PD program in your classroom?

E. Can you tell me the changes in your teaching after the attendance of the PD, if there is any?

F. How often do you attend PD programs?

G. Is it easy to implement what you learned from a PD program?

6. If not, what could be the obstacles?

Part 6: What is your overall impression about the SMP? Tell me anything that you want to talk about the SMP.

Thank you so much for your participation!
## Appendix C

### Summary Notes of Transcriptions Example

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<td><strong>4.2. Confusion between SMP 4 (Model with Mathematics) and SMP 5 (Use Appropriate Tools Strategically):</strong></td>
<td>&quot;[I focus on] Modeling&quot; (Turn 76) &quot;They’re learning to represent and model their math. I did a problem the other day that worked with money, and one of the students was asking me, “Well, why don’t you just give us money to use? Well, you always have money&quot;</td>
<td>&quot;Model with mathematics.&quot; So, today, they had a question where it gave student, “Bethany knows that 8 plus 4 equals 12, but she used 8 plus 2. Explain how she thought about it.” So, I expected them to draw the ten frames, just like we had talked about. (Turn 184)</td>
<td>&quot;Because they could visualize the problems and create a picture to match that problem, they could model it.&quot; (Turn 28) &quot;Modeling is a huge thing. We have a three-step story-problem process: read, draw, write. And they must draw for every single story-problem. So, this year more than any other year, I feel like we’ve been modeling mathematics extensively, the students know they can use our connecting cubes to show a group of ten and maybe three ones for 13. But they can also model it in a ten-frame&quot; (Turn 154)</td>
<td>When asked to explain more about her implementation of Modeling mathematics, Grace responded: &quot;So, number-lines, you know, open number-lines, physically using a number-line. We’ve done a lot with fractions: using fraction circle pieces, fraction bars. Fractions we’ve taught recently, and I’ve done a lot&quot;</td>
<td>When talking about &quot;Model with mathematics,&quot; Olivia talked about manipulatives such as base 10-blocks. &quot;We did for multiplication and division, for fraction work. You need to practice. You can’t just pull a manipulative out and put it on the table and think magically students are going to get it. You really have to think how your&quot;</td>
<td>Well, I think because it’s very clear. If a child can model their answer and how they got to it, it’s pretty black-and-white whether they reach the correct answer. (Turn 118) &quot;Let’s go back to the number story, if it’s addition and subtractition. So, if they can’t figure out the answer to their&quot;</td>
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right in front of you. Can you make drawings, can you use counters to represent money, can you draw a picture for me?" (Turn 78)

"And the students were asking, “Why didn’t you just give us money to work with?” and I said, “Well, the counters are gonna represent money or blocks or whatever you choose. You can choose to draw it.” So, just walk around and try to help them out. So, the one that we use the most is definitely the modeling." (Turn 138)

and I feel really good about that. That is something that I do feel is good about PSSA because those open-ended… The open-ended are useful in a way because they usually do break them up into chunks. That gives kids the opportunity to model what they’re doing. " (Turn 30)

"The amount of modeling that is happening in my room now is lightyears more than I had this time last year. The ability the kids have to visualize what’s happening in the story is so far modeling with that specifically." (Turn 100) and "Like, physically drawing them or using manipulatives." (Turn 102)

transitions are going to look like, how you’re going to use this tool. And I know a lot of people think that once they get those out that the kids can become very off-task, so they choose not to get them out. But, like, the base-10 blocks and stuff, we should be using those more often. And I know that there’s students who would really benefit from those. We do a lot with the little centimeter cubes: when we’re doing problem. In the classroom I always had many, many different tools, so they might pick a block graph paper and they might be coloring in the numbers. Then it becomes clear to them that, well, if he had five – then they color in five – then he added four more, but somebody took two… So, you can see a child color five, color four, then ‘X’ out two. Where children who color five, color four,
learning to use other manipulatives to model what they’re working with. Instead of always a concrete object.” (Turn 82)

beyond what I... It makes me wish I could go back in time and do this for my kids last year.” (Turn 52) Jane's modeling is drawing of RDW (Read-Draw-Write) model. Modeling = visualizing

volume, we build so many things. And the clarity I see kids have with that show me that we should be using more manipulatives like that more often. " (Turn 138)

color two and would give me an answer of 11, you can see they used a model, but they didn’t have the correct concept behind it. (Turn 120)
Appendix D

Example of “Kid-Friendly SMP” (1)

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7 Retrieved from
file:///Users/jungcolen/Desktop/StandardsforMathematicalPracticePostersFreebie.pdf
I CAN SELECT AND USE MATHEMATICAL TOOLS DESIGNED TO HELP ME UNDERSTAND MATH

It is important to know how, why, and when to use math tools.

TOOLS I MIGHT USE:

- Clock
- Ruler
- Pencil

M.P.5 USE APPROPRIATE TOOLS STRATEGICALLY

I CAN BE ACCURATE WHEN SOLVING PROBLEMS AND EXACT WHEN I SHARE MY STRATEGIES

Ways I can be precise:
- Use exact measurements
- Use math symbols correctly
- Use numbers purposefully
- Use appropriate math vocabulary
- Show accurate calculations
- Give detailed explanations

M.P.6 ATTEND TO PRECISION

I CAN FIND PATTERNS AND USE STRATEGIES I ALREADY KNOW TO HELP ME FIND THE ANSWER TO A PROBLEM

Example:

\[3 + q = 12\]

Is the same as

\[q + 3 = 12\]

AND

\[12 - q = 3\]

Because

\[3 + q = 12\]

M.P.7 LOOK FOR AND MAKE USE OF STRUCTURE

I CAN FIND REPEATED CALCULATIONS OR PATTERNS AND USE THEM AS SHORTCUTS WHEN SOLVING MATH PROBLEMS

Example:

If I know

\[8 + 5 = 13\]

Then I know

\[8 + 6 = 14\]

If I increase one of the addends by \(7\), then the sum will also increase by \(7\).

M.P.8 LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING
Appendix E

Example of “Kid-Friendly SMP”\(^8\) (2)
Appendix F

Example of “Kid-Friendly SMP”

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9 Retrieved from https://docs.google.com/file/d/0B19SejfVMU1rbW1sbEZ3Y1E1Qkk/preview
VITA

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Education

- Ph.D. Candidacy in Curriculum and Instruction (with Mathematics Education emphasis)
  The Pennsylvania State University (2013-present)
- M.Ed. in Elementary and Middle School Mathematics Education
  Indiana University of Pennsylvania (2010-2011)
- Non-degree Mathematics Courses:
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  Proofs I, Introduction to Math Proofs II
  Indiana University of Pennsylvania (2005-2010)
- English Intensive Program
  Teachers College Columbia University (1998)
- B.A. in Educational Technology
  Ewha Womans University, South Korea (1993-1998)

Teaching & Work Experience

- Indiana University of Pennsylvania (August 2015-December 2018) – mathematics instructor (teaching
  MATH 100 Intermediate Algebra, MATH 101 Foundation of Mathematics, MATH 151 Elements of
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- Penn State University (August 2013-May 2015) – instructor (taught MTHED 420 Teaching
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- Kiski School (Spring 2011) – graduate intern
- Indiana University of Pennsylvania (August 2010-May 2011) – graduate assistant
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- Sung Il Academy (1996-1997) – mathematics and English instructor (grades 5 to 10)
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