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BUS TRANSIT PRIORITY: MODELING AND EVALUATING PERFORMANCE

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Urban congestion is a problem that plagues many different cities. One solution is to improve bus operations by prioritizing them to make this mode more attractive. Since buses can carry more people than private vehicles in the same amount of space, more users of the bus mode can help mitigate urban traffic congestion. Hence, bus priority strategies have been deployed in many cities. Two commonly used bus priority strategies are transit signal priority (TSP) and dedicated bus lanes (DBL). This thesis focuses TSP strategies with or without dedicated bus lanes. Dedicated bus lanes alone are not studied since this priority method can significantly reduce car capacities if a lane is taken away from general traffic.

In general, the impacts of TSP strategies are two-fold: 1) benefits to bus transit service, including reduced delays, increase in reliability and decrease passenger waiting time, and 2) impacts to car traffic, including both the detriment to cross street cars and benefits to arterial street cars. These impacts are not independent since buses and cars use the same space on the roadway and interact with each other. This thesis uses theoretical approaches, such as the kinematic wave theory (KWT) and its variational forms (variational theory (VT) and Lax-Hopf numerical scheme), to analytically model TSP impacts. These models are then used to evaluate the applicability of TSP in different scenarios.

Two levels of analysis are considered in this thesis: 1) an isolated intersection, and 2) an arterial consisting of multiple intersections in a series. At an isolated intersection, a numerical analysis utilizing variational theory unveils the trade-off between delays to cars and buses while implementing TSP, for both oversaturated and under saturated traffic conditions when buses and cars travel at the same speed. These models are also used to understand how bus stop locations and bus dwell durations can change the impacts of TSP. Next, the assumption that buses and cars travel at the same speed is relaxed and a KWT-based numerical scheme that solves the Lax-Hopf
model is used to evaluate TSP impacts. This model is used to evaluate the sensitivity of TSP to bus stop location, bus dwell duration, and bus detection technology.

At the arterial-level, the Macroscopic Fundamental Diagram is first used to quantify the change in car throughput and change in bus travel time for a combined TSP and DBL strategy. Next, the KWT-based numerical scheme to model the changes in car delays and bus travel times is used to understand the impacts of TSP in the absence of dedicated bus lanes. These analytical models are used to optimize locations of TSP implementations to maximize bus benefits and minimize increase in car delays.
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Chapter 1

Introduction

Public transit vehicles, such as buses, can utilize roadway infrastructure more efficiently than cars by carrying more passengers than private cars in the same roadway. An efficient public transit system improves urban mobility and mitigates congestion. In the United States, the number of public transportation users has been steadily increasing. Since 1995, public transit trips have increased by 34 percent, outpacing the 21 percent of population growth in the United States (Neff and Dickens, 2017). Despite the increasing needs, the roadway infrastructure and other facilities are always limited. In view of this, public transportation agencies wish to prioritize bus or other transit movement. This can help reduce bus delays and provide increased service reliability without the need for a modified infrastructure.

For buses, two common types of priority strategies that are used are dedicated bus lanes (DBLs) and transit signal priority (TSP). Dedicating a bus lane physically separates buses from cars, minimizing the delay buses encounter due to car queues. However, it entails widening the roadway or reducing the number of car lanes. This should be considered as the cost of dedicating a lane for bus-use only. TSP provides priority for buses through adaptive change of signal timing. The cost for TSP is usually less than DBL since it only requires changes to the signal operation without any changes to the pavement. However, it cannot completely eliminate bus delays due to interactions with car queues. Moreover, some novel bus transit priority strategies have been proposed, such as queue jumper lane (QJL) and pre-signal. And much efforts have been put to investigating the impacts of those bus priority strategies on bus and car.
1.1 Bus Transit Priority

Bus is the main public transportation mode in many U.S. cities. Differing from subway and light rail, buses generally share roadway space with cars. Buses are able to carry more people using less road space providing a more effective way to utilize limited road space, thus combating urban congestion. In light of this, much effort has been put into the improvement of bus operation. Some research has focused on efficient bus route and schedule design. For example, an artificial intelligence-based search procedure has been proposed for bus route network design (Baaj and Mahmassani, 1992). Some researchers have utilized genetic algorithms for solving bus routing and network design problems (Bielli et al., 2002; Cipriani et al., 2012). Some later studies developed robust models to optimize bus schedules given fixed bus routes (Yan et al., 2013, 2012). Some studies further developed a stochastic optimization model for transit timetable design considering the randomness of bus travel times (Wu et al., 2015). In addition, some research focused on bus control strategies to improve schedule reliability (Daganzo, 2009; Daganzo and Pilachowski, 2011; Xuan et al., 2011).

Moreover, there are some studies focusing on bus operation impacts on car traffic. These impacts are usually caused by a dwelling bus at a bus stop. In the book, Accessibility and the Bus System: From Concepts to Practice, the author mentions that a queue usually forms at a bus stop as bus service time increases, and the queuing time should be considered for evaluating bus priority strategies performance (Tyler et al., 2002). In urban areas, bus stops are often placed close to intersections (Gu et al., 2013). When considering the combined effect of signalized intersections and adjacent bus stops, the impacts could be more complicated than if ignoring bus stop impacts (Zhao et al., 2007). This is because the dwelling bus and intersection are both acting as bottlenecks. In this case, a bus can be delayed by car queues during red phases, and cars are
also impeded by the dwelling bus at the stop. Bus and car delays can be estimated using kinematic wave and queuing theories (Gayah et al., 2016; Gu et al., 2014; Wu et al., 2017).

Dedicated bus lanes (DBL) are usually implemented to separate bus from car traffic. Dedicating a bus lane enables bus travel in a corridor without impediments by cars, especially under congested traffic conditions. However, DBL can have negative effects on other modes whenever buses are not present. To minimize the negative effects of DBLs on other modes, some research considered using DBLs only when and where necessary. In view of this, some researchers have proposed a dynamic treatment in the corridor, termed intermittent bus lane or bus lane with intermittent priority (BLIP), which prohibits cars from entering lane segments in advance of a bus arrival (Chiabaut et al., 2012; Eichler and Daganzo, 2006; Viegas and Lu, 2004). Another option that has been proposed is to terminate the bus lane upstream of the intersection and use an additional signal, called a pre-signal, which allows buses and cars to share the space in proximity to the intersection (Guler et al., 2016; Guler and Cassidy, 2012; Guler and Menendez, 2014). Differing from DBL, TSP benefits buses relative to intersection operations, and it has also shown promise in improving the performance of buses by reducing delay at intersections (Liu et al., 2008). TSPs are not expected to provide as much delay savings as DBLs; however, they have fewer negative impacts on other modes and can be implemented without changes to the existing infrastructure. Therefore, TSP may be preferred over DBLs to improve bus operations and reliability. A combination of DBL and TSP, named queue jumper lanes (QJL), prioritizes bus movement at signalized intersections without delay to car queues (Zhou and Gan, 2005; Zlatkovic et al., 2013). QJLs allow for there to be a short bus lane upstream of the intersection while providing signal priority to buses. A recent study explored the combined effect of TSP with DBL and QJL at intersection and arterial levels. The results revealed that the combined effect is smaller than the additive effect of single priorities (Truong et al., 2017).
Dedicated bus lane has been studied as early as 1975 (Levinson et al., 1975). Early studies focused on quantifying the increase in bus speeds that result from dedicating a bus only lane using data from field experiments and simulations (Rouphail, 1984; Shalaby and Soberman, 1994; Surpremiant-Legault and El-Geneidy, 2011; Tanaboriboon and Toonim, 1983). Some later studies typically focused on the effects of dedicated bus lanes on both car and bus operations along the bus route (Arasan and Vedagiri, 2010; Currie et al., 2007; Shalaby, 1990). Dedicating a bus lane often sacrifices car lane use, making it costly to car traffic while implementing.

TSP strategies are typically categorized as: 1) passive, 2) active, or 3) adaptive (Baker et al., 2002). Passive strategies principally include changes in the signal settings (e.g., green times, offsets, and cycle lengths) (Christofa and Skabardonis, 2011). Passive priority operates continuously, regardless of whether transit is present or not, and does not require a transit detection/priority request generation system. Passive priority can be an efficient form of TSP when transit operations are predictable with a good understanding of routes, schedule, dwell times, and passenger loads (Smith et al., 2005). One such passive priority strategy is establishing signal progression for transit. This can be accomplished by setting the offsets between successive intersections based on the speed of transit vehicles and their corresponding dwell times (Skabardonis, 2000). A simulation study revealed that traffic signal coordination can reduce bus travel time by up to 8.5 percent (Estrada et al., 2009). Since the signals are coordinated for vehicles travelling along with transit vehicles, other traffic may experience unnecessary delays and stops (Smith et al., 2005). Some other examples of passive TSP strategies include shortening cycle lengths and retiming. Passive priority strategies are easy to implement; however, their success depends on low traffic volume variability and deterministic bus travel time and dwell time at bus stops, which is not realistic in general (Skabardonis and Christofa, 2011). Therefore, passive strategies have a limited application.
This limitation can be resolved by using active priority strategies since they work in real-time and change the signal timing once a bus is detected in proximity to the signal. Typical signal timing modifications are green extension, red truncation (also termed early green), phase insertion or phase rotation (Koonce et al., 2008). A green extension strategy extends the green time for a TSP-equipped vehicle when it is approaching the intersection. Green extension is one of the most effective forms of TSP since a green extension allows a transit vehicle to be served immediately and significantly reduces delay to that vehicle relative to waiting for a red truncation (Smith et al., 2005). A red truncation shortens the red time of current phase to expedite the return to green for the movement where a TSP-equipped vehicle has been detected. In general, green extension and red truncation are available together but are not applied at the same time. Another active TSP strategy is phase insertion, wherein a phase can be inserted when a transit vehicle is detected and requests priority for this phase, for example, for a left-turn only phase for transit vehicles entering a cross street bus stop.

Adaptive TSP is a strategy that takes into consideration the trade-offs between transit and traffic delay. This strategy adjusts signal timing by adapting the movement of transit vehicles and the prevailing traffic conditions. Typically, an adaptive TSP consists of following parts: 1) a bus detection system to accurately predict bus arrival time whenever a bus is within a specified range; 2) a traffic detection system to identify traffic volumes; 3) a signal control algorithm that adjusts a signal to provide priority while considering the influence on the rest of the traffic (Baker et al., 2002; Smith et al., 2005).

In current practice, the use of microscopic traffic simulation models appears to be the most common method of evaluating TSP implementations. For example, a simulation software, named WATSim, is used to evaluate a traffic-adaptive TSP algorithm along a signalized arterial (Muthuswamy et al., 2007). The simulation results revealed that TSP often reduces both transit and car travel times on the route to which TSP was applied and had varied results – but not
always adverse – on cross streets. In addition, some researchers tested the TSP impacts on both the prioritized bus and general traffic during the morning peak and midday periods using a simulation software named INTEGRATION (Dion et al., 2004). The priority considered in this study were green extensions and early greens. For the test cases, this study showed that TSP could be provided to bus along the arterial in both morning and midday periods without significantly impacting the general traffic. And the adverse impacts of TSP can be minimized by only providing priority at intersections with low traffic volumes on conflicting approaches. Another study analyzed the effects of passive TSP strategies on a simulation software – VISSIM – with the consideration of nearside bus stops (Kim and Rilett, 2005). With the presence of near-side stops, the dwell time variability alters the anticipated bus arrival time and TSP activation. Microscopic simulation tools provide the ability to model a roadway corridor in detail in terms of traffic signal operations and vehicle movements. However, these simulation models require large amounts of data to create, validate, and apply the model correctly. In addition, a substantial limitation is that it is often impossible to obtain generalizable conclusions or transfer the findings from individual studies to evaluate TSP at other locations (Balke et al., 2000; Dion et al., 2004; Ngan et al., 2004). Hence, there is a need for analytical models to evaluate TSP implementation.

In addition to simulation analysis, some studies formulated analytical models to evaluate TSP impacts. The advantage of an analytical approach lies in its simplicity and generality (Liu et al., 2008). The analytical expressions usually provide a theoretical and objective evaluation of TSP impacts on bus and car. An earlier study determined change in overall person delay at a signalized intersection due to TSP (Sunkari et al., 1995). This study used Highway Capacity Manual (HCM) delay equations to quantify a weighted overall person delay for the main and cross streets. Field data validation showed that the HCM equation-based model seemed to overestimated the delay for some cases. Later studies used queuing theory to calculate car delays with and without TSP (Abdy and Hellinga, 2011; Li, 2017; Liu et al., 2008). These studies
assumed a deterministic car arrival rate upstream of a signalized intersection. Again, green extension and early green are considered active TSP strategies. The TSP impacts on cars travelling on prioritized and non-prioritized approaches are quantified based upon the geometry of queuing diagrams. The analytical results were validated by simulation tool – VISSIM – and showed a similar trend to simulation results (Abdy and Hellinga, 2011; Liu et al., 2008). Using the analytical expression of TSP impact on car delays, some researchers proposed a TSP optimization model with an objective of minimizing overall person delays (Christofa and Skabardonis, 2011). The optimization model is developed for traffic-responsive or adaptive TSP accounting for both transit travel time and possible negative effects on auto traffic.

1.2 Research Gap

The changes in bus travel times after implementation of TSP has been studied by many researchers. This includes some simulation work for preliminary analysis of TSP impacts (Dion et al., 2004; Kim and Rilett, 2005; Muthuswamy et al., 2007). They propose to use microsimulation packages, for example VISSIM, CORSIM and WATSim, to model TSP on entire arterial with multiple intersections. A common finding is that TSP provides considerable savings to bus travel time while imposing minimal negative impacts on cars. Microsimulation are commonly used method to model TSP, however, they require large efforts on model calibration, and the incorporating of bus is difficult since bus have different kinematic characteristics from cars. However, only a few works have considered bus delay savings from an analytical perspective (Liu et al., 2008; Sunkari et al., 1995). These papers propose analytical evaluations of TSP impacts on person delay or vehicle delay basis. Once the analytical expression of TSP impacts is developed, some researchers proposed TSP optimization algorithms aiming to minimizing transit travel time or person delays (Christofa and Skabardonis, 2011). The estimation of vehicle delay
due to TSP in those analytical research are usually simple, for example, using HCM equation to estimate intersection delay (Sunkari et al., 1995). Therefore, some earlier analytical research on TSP impacts is not so accurate since bus also causes additional delay to cars, especially when there is a near-side bus stop. In addition, most of existing analytical model can only be applied to isolated intersection level and have difficulties on extending to arterial level.

The challenge is that changes to bus travel time cannot be determined independent of car operations. Moreover, the impacts of buses and TSP on car traffic is also difficult to analyze theoretically. First, bimodal operations are complex since both cars and buses are always interacting with each other. Therefore, it is hard to investigate and model their behavior simultaneously. Most work in traffic flow theory has considered a single mode, including kinematic wave theory (KWT) and variational theory (VT) (Daganzo, 2005a, 2005b; Newell, 1993). Hence, a commonly agreed on method to study bimodal traffic does not exist. Second, the inclusion of TSP complicates this study, for it requires consideration of the communication between car, bus and signal. Therefore, it is hard to develop an analytical model to account for all these considerations. Lastly, the impacts of TSP on buses and cars on an arterial level have received very little attention in the literature.

To summarize, the gaps in the existing literature are:

1) A lack of understanding of how TSP can impact both cars and buses at isolated intersections considering the interactions between the two modes, when buses travel at the same speed as cars,

2) A lack of understanding of how TSP can impact both cars and buses at isolated intersections considering the interactions between the two modes, when buses travel at lower speeds than cars leading to queueing behind them, and

3) A lack of understanding of how TSP can impact both cars and buses along arterials
1.3 Research Objective

To overcome this challenge, my dissertation work first assumes that buses and cars travel at the same speed and look at the joint impacts of TSP and bus stops on buses and cars at isolated intersections. Next, an arterial level analysis is conducted assuming physical separation between the modes. Finally, the impacts of lower bus travel speeds on TSP implementation are analyzed both at the intersection and arterial level.

The consideration of the mixed bi-modal traffic at an arterial level is difficult, that is why, most researchers have studied the network level impacts of TSP via simulation tools. The first challenge is bus speed cannot be fixed since buses are always interacting with surrounding traffic. This imposes difficulties on the prediction of TSP activation time. Another challenge is that surrounding traffic states are hard to model because a bus can impede car movement due to its lower speed. A feasible solution is to consider bus as a moving bottleneck, and extend kinematic wave theory to incorporate moving bottleneck in the time-space plane. This enables the relaxation of the commonly used assumption of fixed bus speeds making the study more practical. Hence, this work is also extended to consider the impacts of TSP along an arterial where cars and buses are mixed.

The remainder of the dissertation is organized as follows. Chapter 2 summarizes backgrounds on traffic flow theory that are used to model bus and car motions. Chapter 3 describes the study on TSP impacts at an isolated intersection. This study considers the joint impacts of TSP and nearby bus stops. Chapter 4 summarizes the application of variational theory and macroscopic fundamental diagram to evaluate arterial capacity with the impacts of TSP. Chapters 5 and 6 describe the use of kinematic wave theory and moving bottleneck approach to estimate twofold impacts of TSP: change in bus travel time, and change in cumulative car delay, at isolated intersection and along an arterial, respectively. These two chapters relax the
assumption of bus and car sharing the same kinematic characteristics and model the mixed traffic conditions. Conclusions and future plan are summarized in Chapter 7.
Chapter 2

Background on Traffic Flow Theory

2.1 Kinematic Wave and Queuing Theories

Consider a one-dimensional homogeneous section of highway. For a given time $t$ and position $x$, we define the local traffic density $k(x, t)$ in vehicles per unit of length and the instantaneous flow $q(x, t)$ in vehicles per unit time. The conservation of vehicles on the highway is written as follows (Lighthill and Whitham, 1955; Richards, 1956).

$$\frac{\partial k(t, x)}{\partial t} + \frac{\partial q(t, x)}{\partial x} = 0$$  (2-1)

For first-order traffic flow models, flow and density are related by the Fundamental Diagram (FD). The FD is a positive function defined on $[0, k_j]$, where $k_j$ is the maximal density (jam density). It ranges in $[0, q_c]$, where $q_c$ is the maximum flow (capacity). A common assumption is that flow $q$ and density $k$ follow a triangular-shaped FD (Daganzo, 1994), see Figure 2-1. As is described by the FD, the velocity $v_f$ is not sensitive to the flow until flow is close to maximum flow or capacity $q_c$. As demand exceeds the capacity $q_c$, vehicle flow is restricted and average speed will decrease. On a triangular FD, there are only two possible values for shockwave speeds (how information about traffic conditions travel), one positive ($v_f$), which represents free flow conditions and the other negative ($-w$), which represents congested conditions.
In urban street network, the presence of traffic signal causes recurrent variations of traffic states. Following triangular FD, the traffic states and kinematic waves can be drawn in a time-space plane, see Figure 2-2a. To determine car delays, a queuing diagram can be used (Makigami et al., 1971), see Figure 2-2b. In this diagram, $V(t)$ represents the virtual arrive curve with a constant slope $q$, $D(t)$ is the departure curves with presence of traffic signal. For different periods, the car flows shown on $D(t)$ vary between $q_c$, $q$, and 0, due to the variation of traffic states. Thus, the area between $A(t)$ and $D(t)$ marks the change in cumulative car delays attributable to red phases.
2.2 Variational Theory and Macroscopic Fundamental Diagram (MFD)

As per Daganzo, variational theory (VT) allows an estimate of vehicle counts at any location on the time-space plane without needing to identify any traffic states or interfaces (Daganzo, 2005a, 2005b). VT can be simplified for a homogenous highway with a triangular FD. With this simplification, a set of moving observers traveling downstream (with speed $v_f$) and upstream (with speed $-w$), and stopping at intersections or other bottlenecks (with speed 0) is sufficient to explore all traffic motions presented in a corridor, see Figure 2-3b (Daganzo and Geroliminis, 2008; Leclercq and Geroliminis, 2013). The maximum rate, also named cost $r(u)$,
that the observers could be passed by vehicles is linear and decreases from \( r(-w) = q_c(1 + w/v_f) \) to \( r(v_f) = 0 \) when the FD is triangular, see Figure 2-3a.

For a finite corridor, a variational graph can be derived to estimate analytical street MFD, see Figure 2-3b. Within the graph, let \( N,J \) denote the total number of all vertices and edges, respectively. The cost of an edge can be specified with the known passing rates \( r(v_f) \), \( r(0) \), and \( r(-w) \). For a valid path \( P \), the total cost, \( \Delta(P) \) is the summation of costs of all edges involved, see Equation 2-2a. The least-cost, \( R(V) \), and mean speed, \( V \), of all possible paths are then represented by Equation 2-2b and c, respectively. To solve for \( R(V) \), a shortest-path algorithm (e.g., Dijkstra’s) is used with an input of matrix \( R^{N \times N} \) for all \( N \) vertices. Each element of \( R \) represents the cost (passing rate) of an edge.

\[
\Delta(P) = \sum_j r(u_j)t_j, \quad j = 1, \ldots J \tag{2-2a}
\]

\[
R(V) = \inf_P \left( \frac{\Delta(P)}{T_p} \right) \tag{2-2b}
\]

\[
V = L/T_p \tag{2-2c}
\]

where \( u_j, t_j, r(u_j) \) represents the speed, travel time and cost within edge \( j \), respectively. \( L \) denotes the length of the street, and \( T_p = \sum_j t_j \) is the total travel time on all edges within path \( P \).
As per Daganzo and Geroliminis, an macroscopic fundamental diagram (MFD) relating flow with density can be mathematically represented in Equation 2-3 (Daganzo and Geroliminis, 2008). This equation defines an analytical MFD that is consistent with the experimental MFD obtained from Yokohama’s network, see Figure 1-3c and d (Daganzo and Geroliminis, 2008; Geroliminis and Daganzo, 2008).

\[ Q = \inf_V (KV + R(V)) \]  

(2-3)

This section provides a summary of traffic flow theories that are used to model bus and car traffic. Following chapters illustrate more details on how to adopt the above traffic flow
theories into specific methodologies. Chapter 3 illustrates the use of kinematic wave, variational and queuing theories to model TSP impacts on intersection operations with presence of nearby bus stops. Chapter 4 describes a variational theory and macroscopic fundamental diagram approach to model TSP impacts along arterials. The methods used in Chapter 5 and 6 are derived from kinematic wave and queuing theories.
Chapter 3

Estimating the impacts of Transit Signal Priority on Intersection Operations: Queuing and Variational Theory Approach

This chapter describes a study of TSP impacts at an isolated intersection (Wu et al., 2017). This study considers the joint impacts of TSP and nearby bus stops with random bus dwelling times. The impacts are quantified as changes in intersection car capacity, cumulative car delay and bus delay when priority is provided to bus that dwells at near- or far-side bus stop locations. The impact analysis is conducted for two set of traffic conditions: oversaturated and undersaturated. For oversaturated traffic conditions, variational and kinematic wave theories are used for car capacity and bus delay analysis. For undersaturated conditions, queuing and kinematic wave theories are used to quantify change in bus and cumulative car delays.

3.1 Scenario Description

Considered here is an isolated signalized intersection with a fixed cycle length, \( L_c \), and \( gL_c \) green time provided to the bus direction. Two candidate TSP strategies are considered to provide bus priority: green extension or red truncation. In both cases, a minimum red duration, \( r_{min}L_c \), is maintained in the direction of the bus.

An adjacent bus stop is assumed to be located a distance \( d \) away from the intersection; see Figure 3-1. The bus dwell duration, \( S \), and bus arrival time to a stop, \( t_a \), are assumed to be random and treated independently. Bus departure times from a stop are provided as \( t_e = t_a + S \).
Traffic (both buses and cars) on the homogenous road segments upstream and downstream of the intersection is assumed to obey the same kinematic characteristics with a triangular FD. The capacity flow of traffic on bus travel and the opposite directions are $q_c$, while maximum flow of traffic on cross directions are $q'_c$. Notations are illustrated in Table 3-1.

Table 3-1: List of variables and notation

<table>
<thead>
<tr>
<th>Signal Timing</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>$L$: signal cycle length</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g$: signal green time ratio in bus direction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_{min}$: minimum red phase ratio in bus direction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta T, \Delta T'$: adaptive time of signal when TSP is activated, for a near- and far-side stop</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bus Stop and Bus Dwell</th>
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</thead>
<tbody>
<tr>
<td>$d$: distance between the intersection and bus stop</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d'$: the distance from the last queued car to the intersection</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_e$: end time of bus dwelling process</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_s$: start time of bus dwelling process</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S$: duration of bus dwelling process</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_y$: time when a bus would arrive to the back of queue before crossing the intersection</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Time Range</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>$T_{a}, T_{b}$: feasible range of $t_{a}, t_{b}$, for a near- and far-side stop, respectively</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{g}, T_{g}'$: range of $t_{g}, t_{g}'$ for which green extension can be activated, for a near- and far-side stop</td>
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<td></td>
</tr>
<tr>
<td>$T_{r}, T_{r}'$: range of $t_{r}, t_{r}'$ for which red truncation can be activated, for a near- and far-side stop</td>
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<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Car Capacity Change</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta N_{gb}$: car count change at the intersection due to dwelling bus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta N_{gb}, \Delta N_{gb}'$: car throughput change due to TSP implementation, for a near- and far-side stop, respectively</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta N_{gb}'$: car count change in the opposite direction due to TSP activation</td>
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</tr>
</tbody>
</table>

Figure 3-1: Illustration of isolated intersection with bus stop
3.2 Analytical Models

This section introduces analytical models for two set of traffic conditions: oversaturated and undersaturated. Both near- and far-side bus stops are considered in these models.

### 3.2.1 Analysis of Oversaturated Conditions

For oversaturated conditions, since cumulative car delays can be infinite, here the impacts of TSP on intersection capacity and bus delays are quantified. Variational theory in a moving-time coordinate system is used to model car capacity, while kinematic wave theory is used to estimate bus delays. Starting with bus travel direction, the overall change in car capacity for all intersection approaches is considered. Both near- and far-side stops placed close to the intersection are considered (i.e., \( d \leq w g L_c \)), since a dwelling bus would not affect the intersection capacity when located further away (Gayah et al., 2016).

#### Models for near-side stops

To begin, consider the time-space diagram in Figure 3-2a where a bus trajectory is shown as a piecewise-linear bold arrow. Again, the circled letters \( C \) and \( J \) denote the two states presenting on the approach near the intersection. These two states are separated by an interface with the backward speed \(-w\). Time 0 is specified as the beginning of the first green phase before
the bus arrival. Buses can only arrive to or depart from the bus stop when traffic is in state $C$ at the intersection. As a result, the feasible range of times that buses can begin dwelling is $t_a \in T_a$; see Figure 3-2a and Equation 3-1a. It is assumed that green extension or red truncation is only activated when the bus completes its dwell process and if doing so allows the bus to cross the intersection earlier than it otherwise would. For a given cycle, $T_g$ and $T_r$ denote the ranges of the end of the bus dwell time for which green extension or red truncation can be activated, respectively; see Figure 3-2a and Equations 3-1b and c. Note that a minimum red time is maintained in the direction of interest while implementing TSP.

\[
T_a = \left[ \frac{d}{w}, gL_c + \frac{d}{w} \right] 
\]  \hspace{1cm} (3-1a)
\[
T_g = (gL_c, (1 - r_{min})L_c] 
\]  \hspace{1cm} (3-1b)
\[
T_r = \left( (1 - r_{min})L_c, L_c + \frac{d}{w} \right) 
\]  \hspace{1cm} (3-1c)

To determine change in bus delay, consider the time-space diagram in Figure 3-2b and c, which depicts the change in bus trajectory attributable to green extension and red truncation, respectively. Here, states U and D arise due to a dwelling bus. The bus delay if no TSP is implemented, $W_B$, is calculated by defining two additional time variables, $T_{e1}$ and $T_{e2}$. The former represents the duration of the bus dwell time that extends into the signal red time, i.e., time interval from the beginning of the red, $nL_c + gL_c$, to the end of the bus dwell time, $t_e$. The latter represents the time interval from the end of the bus dwell time, $t_e$, to when the last queued car at the bus stop can clear the intersection, $(n + 1)L_c + d/w$, where $n = \lfloor t_e / L_c \rfloor$. $W_B$ can be calculated using Equation 3-2a to c.

\[
W_B = \min \left( T_{e2}, (1 - g)L_c - T_{e1} \cdot (1 - \frac{w_{dL_c}}{w}) \right) 
\]  \hspace{1cm} (3-2a)
\[
T_{e1} = L_c \cdot \left( \left( \frac{t_e}{L_c} \right) + I \left( \frac{t_e}{L_c} \leq \frac{d}{wL_c} \right) - g \right) 
\]  \hspace{1cm} (3-2b)
\[ T_{e2} = L_c \cdot \left( \frac{d}{WL_c} + I \left( \frac{L_e}{L_c} \right) > \frac{d}{WL_c} \right) \left( \frac{L_e}{L_c} \right) \] (3-2c)

where, \( \{x\} = x - [x] \), and \([x]\) denotes the largest integer smaller than or equal to \(x\), and \(I(Y)\) is an indicator function which takes the value 1 if statement \(Y\) is true and 0 otherwise.
The activation of TSP (green extension of red truncation) adjusts signal time by adding \( \Delta T \) green time to current cycle, see Figures 3-2b to c and Equation 3-3. The change in the green length, \( \Delta T \), will take on a value of zero if the bus can clear the intersection without TSP. For green extension, \( \Delta T \) is equal to the duration of the bus dwell that extends into the red, \( T_{t1} \). For red truncation, \( \Delta t \) is equal to the minimum of: 1) the maximum possible change in signal timing: i.e., \( L_c (1 - g - r_{min}) \), or 2) the bus delay without TSP, \( W_B \).

\[
\Delta T = \begin{cases} 
T_{t1}, & t_e \in T_g \\
\min (W_B, L_c \cdot (1 - g - r_{min})), & t_e \in T_r \\
0, & \text{otherwise}
\end{cases} 
\tag{3-3}
\]

While the determination of bus delay savings is straightforward, the change in car capacity is estimated through VT. This recipe is extended from a previous work done by (Gayah et al., 2016). Let \( \Delta N_B \) denote the change in cumulative count of cars traveling in the bus direction attributable to a dwelling bus (without TSP) and \( \Delta N_T \) denote the additional change attributable to TSP activation. Consider the time-space diagram in Figure 3-2c. In this figure, moving observers travels upstream, downstream and stay at intersection and bus stop (if there is a dwelling bus) to record the maximum passing rate, see an example moving path \( O - A - C - C' - D \). Consider the solid bus trajectory with \( t_e \in T_g \). As is observed by a moving observer, the intersection capacity is decreased during the time that the bus dwells \( (t_{AB}) \) and \( \Delta N_B = -(q_c - q_b) t_{AB} \), and the
implementation of green extension provides additional capacity while it is active ($t_{BC}$), which increases vehicle count by $\Delta N_T = q_b t_{BC}$. Similarly, the same concepts and equations can be applied for red truncation, that is, $\Delta N_B = -(q_c - q_b) t_{EF}$ and $\Delta N_T = q_c t_{HJ}$. Here, negative numbers indicate a decreased vehicle count or reduced capacity.

In general, the change in car counts attributable to a dwelling bus and to the TSP activation can be expressed as in Equations 3-4a and b. The last term in Equation 3-4a denotes the portion of the bus dwell time that impacts intersection capacity. In Equation 3-4b, a green extension only increases the car count by $q_b \Delta T$ since the bus is still dwelling during the green time, whereas the car count increases by $q_c \Delta T$ in the case of red truncation since the bus has already completed its dwell. The changes in car counts in the opposite and conflicting directions are denoted as $\Delta N_T^O$ and $\Delta N_T^C$, respectively, and are provided in Equations 3-4c and d. Since it is assumed no bus stops located at the opposite and cross directions, the changes in car throughputs due to the change in signal timing for those directions are considered.

$$\Delta N_B = -(q_c - q_b) \cdot L_c \cdot \left\{ \max \left( g - \max \left( \frac{d}{w L_c}, \frac{t_a}{L_c} \right), 0 \right) + \max \left( \min \left( \frac{t_e}{L_c}, g \right) - \frac{d}{w L_c}, 0 \right) \right\}$$

(3-4a)

$$\Delta N_T^O = \begin{cases} q_b \Delta T & t_e \in T_g \\ q_c \Delta T & t_e \in T_r \\ 0 & \text{otherwise} \end{cases}$$

(3-4b)

$$\Delta N_T^O = \begin{cases} q_c \Delta T & t_e \in T_g \cup T_r \\ 0 & \text{otherwise} \end{cases}$$

(3-4c)

$$\Delta N_T^C = \begin{cases} -q_c \Delta T & t_e \in T_g \cup T_r \\ 0 & \text{otherwise} \end{cases}$$

(3-4d)

The total change in intersection capacity due to TSP activation can be calculated by summing the change in all directions.
For bus delay savings, consider the time-space diagrams in Figures 3-2b and 3-2c depicting the change in bus trajectory due to a green extension and red truncation, respectively.

TSP for green extension and the length of the adaptive time (\(\Delta T\)) for red truncation, see Equation 3-5.

\[
\Delta T_S = \begin{cases} 
W_B & t_e \in T_g \\
\Delta T & t_e \in T_r \\
0 & \text{Otherwise}
\end{cases}
\]  

(3-5)

Models for far-side stops

A similar approach is used for far-side stops. The main difference here is that TSP activation depends on the virtual time a bus would cross the intersection if the signal were always green, \(t_x\), instead of the end of the bus dwell time. The feasible range of \(t_x\) where there is no TSP is denoted as \(T^f_x\). Similarly, \(T^f_g\) and \(T^f_r\) are the ranges of \(t_x\) when green extension or red truncation is applicable, respectively.

\[
T^f_x = [0, g L_c] 
\]  

(3-6a)

\[
T^f_g = (g L_c, (1 - r_{min})L_c] 
\]  

(3-6b)

\[
T^f_r = ((1 - r_{min})L_c, L_c) 
\]  

(3-6c)

TSP is activated to ensure buses can clear the intersection in the current signal cycle and would not be activated if the bus would need to wait for an additional cycle. A minimum length of red phase, \(r_{min} L_c\), is maintained when implementing TSP. Thus, the change in the green time, \(\Delta T^f\), as a function of \(t_x\) can be expressed as in Equation 3-7.

\[
\Delta T^f = \begin{cases} 
L_c \cdot \left( \frac{t_x}{L_c} - g \right) & t_x \in T^f_g \\
L_c \cdot \min \left( 1 - \frac{t_x}{L_c}, 1 - g - r_{min} \right) & t_x \in T^f_r \\
0 & \text{otherwise}
\end{cases}
\]  

(3-7)
The same notation, assumptions, and methods are used to determine the car capacity change caused by TSP at far-side stops, see Equation 3-8a. Note that the equations for the opposite and conflicting directions are the same as near-side stops and only the change in car count for the bus travel direction, $\Delta N_f^f$, is different. While the bus delay savings are measured as the change in the beginning of the dwell time of the bus at its stop, see Equation 3-8b.

$$\Delta N_f^f = \begin{cases} q_c \Delta T_f^f & t_x \in T_g^f \cup T_r^f \\ 0 & \text{otherwise} \end{cases} \quad (3-8a)$$

$$\Delta T_s^f = \begin{cases} (1 - g) L_c & t_x \in T_g^f \\ \Delta T_f^f & t_x \in T_r^f \\ 0 & \text{Otherwise} \end{cases} \quad (3-8b)$$

### 3.2.3 Analysis of Undersaturated Conditions

KWT and QT in the moving-time coordinate system are used to quantify delays. The algorithm proposed borrows from that proposed in (Gu et al., 2014). For both near- and far-side stops, delays are first computed in the bus travel direction and then expanded to the other approaches at the intersection. The equations are first developed for the case when the additional queues formed due to the dwell of the bus can clear in the following cycle, and then these equations are expanded to consider queues that can persist for multiple cycles.

#### Models for near-side stops

Figure 3-3a provides a time-space diagram for the case when TSP is not activated. In this figure, $d^*$ is the distance from the last queued car to the intersection when there is no dwelling bus. Geometrically, $d^*/w^* = (1 - g) L_c$, where $1/w^* = 1/w_{Aj} - 1/w$. It can be verified that
if \(d > d^*\), no additional impacts are imposed by a dwelling bus. Hence, only bus stops placed close to the intersection with \(d \leq d^*\) are considered.

To ease the quantifications of car and bus delays, the following time variables are defined as shown in Figure 3-3a. \(T_0\) denotes the time at which the initial car queue would clear if no bus were present. \(T_1\) represents the maximum possible duration that the constrained rate of \(q_b\) that could persist, \(T_2\) and \(T_3\) are defined as the duration of the interfaces with a slope of \(-w_{DJ}\) and \(-w\), respectively, and \(T_4\) is the duration of time for which cars are dissipating from a queue in the cycle following the bus arrival. These values can be calculated using geometric relationships as shown in Equations 3-9a to e.

\[
T_0 = d^*/w \quad \text{(3-9a)}
\]

\[
T_1 = \max \left( 0, (T_0 - t_a) \left(1 + \frac{w}{w_{AU}}\right) \right) \quad \text{(3-9b)}
\]

\[
T_2 = \max \left( 0, \min \left( T_{e1}, \frac{d}{w_{IJ}}, T_1 - \max \left( 0, (gL_c - t_a) \right) \right) \right) \quad \text{(3-9c)}
\]

\[
T_3 = \begin{cases} 
0 & \text{if } T_1 = 0 \\
\max \left( 0, (T_0 - t_a) \cdot \left(1 - \frac{T_2 + \max \left( 0, (gL_c - t_a) \right)}{T_1}\right) \right) & \text{otherwise}
\end{cases} \quad \text{(3-9d)}
\]

\[
T_4 = T_2 \frac{w_{Dj}}{w} + T_3 + \frac{w^*}{w} \left( (1 - g)L_c - T_2 \left(1 - \frac{w_{Dj}}{w}\right) \right) \quad \text{(3-9e)}
\]
Figure 3-3: For a near-side bus stop, (a) time-space diagram with no TSP; (b) time-space diagram with green extension; (c) queuing diagram showing cumulative car counts change due to green extension
Figure 3-3b shows how the time-space diagrams would change if green extension were implemented. A similar figure can be drawn for red truncation, see Figure 3-4b. The TSP strategies implemented here are consistent with the over-saturated cases and the time sets $T_g$ and $T_r$ are the same as previously defined, see Equations 2-1b and c. The bus can only reach the bus stop...
stop while the stop is not engulfed by a queue of cars; thus, the feasible range of \( t_a \) can be expressed as Equation 3-10.

\[
T_a = \left[ \frac{d}{w}, gL_c + \frac{d}{w_{AJ}} \right]
\]  

(3-10)

When TSP is implemented, the change in the duration of the green length, \( \Delta T \), is still defined as in Equation 3-3, with the bus delay before TSP implementation, \( W_B \), determined using Equation 3-11a. In Figure 3-3b and 3-4b, \( T_{4t} \) is defined as the duration of cars discharging from a queue in the cycle following the TSP activation and is calculated as in Equation 3-11b. The difference in \( T_4 \) and \( T_{4t} \) is a good indicator of the change in car delay due to TSP implementation.

\[
W_B = \min \left( T_{e2}, (1 - g)L_c - T_2 \left( 1 - \frac{w_{AJ}}{w} \right) - (T_{e1} - T_2) \left( 1 - \frac{w_{AJ}}{w} \right) \right)
\]  

(3-11a)

\[
T_{4t} = \begin{cases} 
T_3 + T_0 \cdot \frac{\Delta T}{(1 - g)L_c} & t_e \in T_g \\
T_4 - T_2 \cdot \frac{w^*}{w} & t_e \in T_r 
\end{cases}
\]  

(3-11b)

With these time variables, a queuing diagram can be drawn showing the impacts of TSP on cumulative car delays. As is exhibited in Figure 3-3c and 3-4c, \( V(t) \) is the virtual arrival curve with a constant slope of \( q_v \), \( D(t) \) is the departure of cars from the intersection if TSP were not activated, and \( D_T(t) \) is the departure of cars from the intersection when TSP is activated. The change in car delay (\( \Delta W \)) is provided by the area between \( D(t) \) and \( D_T(t) \); see Equation 3-12a. For green extension, these two departure curves separate at end time of a green phase, \( \tau_1 \), until the time when the car discharge flow recovers to \( q_\alpha \), \( \tau_2 \); see Equations 3-12 a-c. Notice that it is assumed here that the car queue clears in the cycle that follows the bus arrival, i.e., \( T_4 < gL_c \).

Note that the similar approach can be used for red truncation and for a far-side bus stop.

\[
\Delta W = \int_{r_1}^{r_2} (D_T(t) - D(t))dt
\]  

(3-12a)
\[ \tau_1 = g L_c \]  
\[ \tau_2 = L_c + T_4 \]  

\[ D(t) \text{ and } D_T(t) \] can be obtained by integrating the slopes of the piecewise linear curves, \( \dot{D}_T(t) \) and \( \dot{D}(t) \) as shown in Figure 3-3c and 3-4c. Notice that cars can only discharge at one of four states: 1) zero flow, when the signal is red, 2) \( q_c \), when the signal is green and a queue exists, 3) \( q_a \), when the signal is green but no queue exists, and 4) \( q_b \), when a dwelling bus starves the flow. Equation 3-13a to c describe the slope of \( D(t) \) and \( D_T(t) \) and the duration for which that slope persists.

\[
\dot{D}(t) = \begin{cases} 
0 & t \in [gL_c, L_c) \\
q_c & t \in [L_c, L_c + \min(gL_c, T_4)) \\
q_a & t \in [L_c + T_4, L_c + gL_c) \neq \emptyset \\
0 & t \in [L_c + gL_c, 2L_c) 
\end{cases} 
\]  

\[
\dot{D}_T(t) = \begin{cases} 
q_b (q_a, \text{if } T_4 = T_0) & t \in [gL_c, gL_c + \Delta T) \\
0 & t \in [gL_c + \Delta T, L_c) \\
q_c & t \in [L_c, L_c + \min(gL_c, T_4)) \\
q & t \in [L_c + T_4, L_c + gL_c) \neq \emptyset \\
0 & t \in [L_c + gL_c, 2L_c) 
\end{cases} \text{ if } t_e \in T_g 
\]

\[
\dot{D}_T(t) = \begin{cases} 
0 & t \in [gL_c, L_c - \Delta T') \text{ if } t_e \in T_r \\
q_c & t \in [L_c - \Delta T', \min(L_c + gL_c, L_c - \Delta T' + T_4t)] \\
q_a & t \in [L_c - \Delta T' + T_4t, L_c + gL_c) \neq \emptyset \\
0 & t \in [L_c + gL_c, 2L_c) 
\end{cases} \text{ if } t_e \in T_r 
\]

For bus delay savings (\( \Delta T_s \)), Equation 3-5 can still be used here, with \( W_B \) calculated as in Equation 3-11a.

The previous solution assumed that the car queue would clear in the cycle following the bus arrival. To allow for car queues that can persist for more than 1 cycle, i.e., \( T_4 > gL_c \), Previous equations need to be modified. Bus arrival is always assumed to occur in cycle 0, i.e., \( \lfloor t_v / L_c \rfloor = 0 \), and the queue is assumed to persist until cycle \( k \).
To do so each cycle is treated separately. In each cycle, the time the queue at the bus stop clears is denoted as \( t_v(i) \) where \( i \) is the cycle index, see Equation 3-14a. Additionally, \( t_a \) is replaced with \( t'_a(i) \) in any equation that uses \( t_a \) as an input. Then, in the cycles that follow the bus arrival \( (i \geq 1) \), the time at which the car queue clears, \( T_A \), is set equal to the dissipation time of the queue calculated from the previous cycle, \( T_A \). Hence, \( T_A \) and \( T_0 \) are replaced with arrays \( T_A(i) \) and \( T_0(i) \). \( T_0(i) \) is calculated as in Equation 3-14b, and \( T_A(i) \) is calculated by replacing \( t_a \) and \( T_0 \) with \( t'_a(i - 1) \) and \( T_0'(i - 1) \) in Equations 3-9a-e, see Equation 3-14c. In addition, \( \tau_1 \) is replaced with \( \tau_1' = gL_c + kL_c \) and \( \tau_2 \) is replaced with a set of equations depending on the cycle in which the queue clears, see Equation 16d. Additionally, \( t \) in Equations 3-13a and b would be replaced with \( t' = t - kL_c \).

\[
t'_a(i) = \begin{cases} 
    t_a & i = 0 \\
    d \frac{d}{w} & i \in [1, k] \neq \emptyset
\end{cases} \tag{3-14a}
\]

\[
T_0'(i) = \begin{cases} 
    T_0 & i = 0 \\
    T_A(i) & i \in [1, k] \neq \emptyset
\end{cases} \tag{3-14b}
\]

\[
T_A'(i) = \begin{cases} 
    T_0 & i = 0 \\
    f(t'_a(i - 1), T_0'(i - 1)) & i \in [1, k] \neq \emptyset
\end{cases} \tag{3-14c}
\]

\[
\tau_2 = \begin{cases} 
    (k + 1)L_c + T_A(k) & T_A(k) \in (0, gL_c] \\
    (k + 2)L_c + (T_A(k) - gL_c + T_0) & T_A(k) - gL_c + T_0 \in (0, gL_c] \\
    \vdots & \vdots
\end{cases} \tag{3-14d}
\]

The car delay change models for the opposite and conflicting directions follow the logic used in the previous section but are much simpler. Thus, the overall delay change for all directions can be calculated using the same principle and methods as above and are not shown here for brevity.
Models for far-side stops

Figure 3-5: For a far-side bus stop, (a) time-space diagram with no TSP; (b) time-space diagram with green extension; (c) queuing diagram showing cumulative car counts change due to green extension.

Figure 3-5 and 3-6 show examples to illustrate a far-side bus stop to aid in the explanation of the models. Figure 3-5a and 3-6a shows a case when TSP is not implemented. Notice that in this figure the queue persists for more than 1 cycle. This figure illustrates all the different durations that need to be known to draw an accurate queuing diagram as given in Equations 3-15 a-f. The notation is similar to that in Figure 3-3 and 3-4. The bus delay before
TSP implementation is denoted by $W_B^f$, the arrival time of the bus to the bus stop is $t_a$, and the arrival time of the bus to the back of the queue is $t_x$. At the intersection, $T_x$ and $T_y$ denote the duration for which cars discharge at $q_b$ due to a spillback or at free-flow speed if there is no spillback, respectively; at the far-side bus stop $T_1^f$ and $T_2^f$ denote the duration for which cars discharge at $q_b$ and $q_c$, respectively; $T_z$ denotes the duration of the interface between states $D$ and $J$; and $t_x$ denotes the time the bus arrives to the back of the queue. The equations are provided for the case when the additional queue does not persist for more than one cycle. For the more general case $T_x$, $T_y$, and $T_z$ would need to be expanded to arrays $T_x(i)$, $T_y(i)$ and $T_z(i)$, see Figure 3-5 and 3-6. This general solution would be similar to that presented for near-side stops, however, the details are omitted here for brevity.

$$W_B^f = \max \left(0, (1-g)L_c \cdot \left(1 - \frac{L_c \left(\frac{t_x}{L_c} - g\right)}{T_0 + (1-g)L_c}\right)\right) \quad (3-15a)$$

$$t_a = t_x + W_B^f \quad (3-15b)$$

$$T_x = \max \left\{0, \min \left(S, gL_c - \frac{d}{w} - L_c \cdot \left\{\frac{t_a}{L_c}\right\}, \left(\frac{d}{w} - L_c \cdot \left\{\frac{t_a}{L_c}\right\}\right) \left(1 + \frac{w}{w_{AU}}\right)\right)\right\} \quad (3-15c)$$

$$T_y = gL_c - T_x - \min \left(\frac{d}{w} + L_c \cdot \left\{\frac{t_a}{L_c}\right\}\right) \quad (3-15d)$$

$$T_z = \max \left\{0, \left(\frac{d}{w} - L_c \cdot \left\{\frac{t_a}{L_c}\right\}\right) + \frac{\left(\frac{d}{w} + L_c \cdot \left\{\frac{t_a}{L_c}\right\} - gL_c\right)}{1+w/w_{AU}}\right\} \quad (3-15e)$$

$$T_1^f = \min \left\{S, \left(\frac{t_a}{L_c}\right) \left(1 + \frac{w}{w_{AU}}\right), \left(q_c \cdot \left(\frac{t_a}{L_c}\right) + q_a T_x + q_a T_y / q_b\right)\right\} \quad (3-15f)$$

$$T_2^f = \min \left\{\left(\frac{t_a}{L_c}\right) - \frac{T_1^f}{1+w/w_{AU}}, \left(q_c \cdot \left(\frac{t_a}{L_c}\right) + q_a T_y + q_b (T_x - T_1^f)\right)\right\} \quad (3-15g)$$
The left-hand side of the max or min arguments hold if the queue does not spill back to the location of the intersection, i.e., \( \frac{d}{w} + \frac{L_c}{T_a} \cdot \{t_a / L_c\} \geq T_0 \), and the right hand side holds otherwise.

Figure 3-6: For a far-side bus stop, (a) time-space diagram with no TSP; (b) time-space diagram with red truncation; (c) queuing diagram showing cumulative car counts change due to red truncation
Next, Figure 3-5b shows the time-space diagram if green extension is implemented. Note that a similar figure can be drawn for red truncation, see Figure 3-6b. The range when green extension would be feasible, $T_g^f$, is the same as in Equation 3-6b. However, the range when red truncation would be feasible, $T_r^f$, can be expressed as in Equation 3-16a. Then, the change in the green time after implementing TSP, $\Delta T^f$, can also be calculated as in Equation 3-16b. The time the bus arrives to the far-side bus stop after the implementation of TSP, $t_{at}^f$, can also be expressed as Equation 3-16c.

$$T_r^f = ((1 - r_{min})L_c, L_c + \frac{d^*}{w})$$  \hspace{1cm} (3-16a)

$$\Delta T^f = \begin{cases} 
L_c \cdot \left(\frac{t_x}{L_c} - g\right) & t_x \in T_g^f \\
\min \left((1 - g - r_{min})L_c, W_B^f\right) & t_x \in T_r^f
\end{cases}$$  \hspace{1cm} (3-16b)

$$t_{at}^f = \begin{cases} 
t_x & t_x \in T_g^f \\
t_x + W_B^f - \Delta T^f & t_x \in T_r^f
\end{cases}$$  \hspace{1cm} (3-16c)

To determine delays, cumulative car counts at the location of the bus stop from Figures 3-5a and 3-5b are used to obtain the queuing diagram shown in Figure 3-5c. The notation is the same as in Figure 3-3c. The same method is applied to the far-side cases to determine change in car delay before and after TSP implementation. Here, we show the derivatives of $D(t)$ and $D_T(t)$ as follows:

$$D(t) = \begin{cases} 
0 & t \in [gL_c, L_c) \\
q_c & t \in [L_c, t_a) \\
q_b & t \in [t_a, t_a + T_1^f) \\
q_c & t \in [t_a + T_1^f, t_a + T_1^f + T_2^f) \\
\cdots
\end{cases}$$  \hspace{1cm} if $t_x \in T_g^f$ (3-17a)

$$D_T(t) = \begin{cases} 
q_c & t \in [gL_c, gL_c + \Delta T^f) \\
0 & t \in [gL_c + \Delta T^f, L_c) \\
q_b & t \in [L_c, L_c + T_1^f) \\
q_c & t \in [L_c + T_1^f, L_c + T_1^f + T_2^f) \\
\cdots
\end{cases}$$  \hspace{1cm} if $t_x \in T_g^f$ (3-17b)
Bus delay savings can be obtained if the bus arrives to the back of the queue, $t_x$, such that activation of TSP would be possible. The same notation as before is used to denote the bus delay savings, $\Delta t_s$. The bus delay savings are calculated as shown in Equation 3-18.

$$\Delta T_s = \begin{cases} W_g^f & t_x \in T_g^f \\ \Delta T^f & t_x \in T_r^f \\ 0 & otherwise \end{cases}$$

$$D_T(t) = \begin{cases} 0 & t \in [gL_c, L_c - \Delta T^f) \\ q_c & t \in [L_c - \Delta T^f, t_{at}) \\ q_b & t \in [t_{at}, t_{at} + \Delta T^f] \\ q_c & t \in [t_{at} + T_1^f, t_{at} + T_1^f + T_2^f) \neq \phi \end{cases} \quad (3-17c)$$
3.3 Summary of Findings

A numerical analysis was performed to demonstrate how the car and bus delays would be impacted by TSP for different bus stop locations and bus dwell durations. A near-side bus stop was considered herein. The arrival time of bus at the bus stop is assumed uniformly distributed within $T_a$, and all results are estimated as average values among all possible arrivals.

3.3.1 Oversaturated Conditions

A near-side stop was considered to evaluate the impacts of TSP on car and bus delays in over-saturated conditions on all approaches. In this example, the bus stop location and dwell time were varied while holding the capacity of the intersection, the capacity of the bus stop location, and signal timings constant. The results are obtained as an expected value for all possible bus arrival times, i.e., $t_a$ is uniformly distributed within $T_a$. The results are shown as contour plots of changes in car throughputs or change in bus delay. Positive contour values indicate reductions in either car capacity or bus delay (i.e., bus delay savings). Figure 3-7 provides the numerical results. Notice that the change in total car count due to TSP, $\Delta N_T + \Delta N_f^D + \Delta N_f^C$, is only shown for green extension, since red truncation has no impact on overall car flow when the discharge flows for all directions are the same. Figure 3-7a shows the time intervals during which no changes in car counts arise due to green extension. To illustrate this case, consider $d/wL_c = 0.10$ and $S = 70$ seconds. For all possible start times, $t_a \in T_a$, the end of the bus dwell time, $t_e = t_a + S$, falls between 82 and 142 seconds, which do not fall within the green extension activation window $T_g$ (Equation 3-1b). Hence, for these values, green extension is never activated and the car count does not change.
Figure 3-7b shows the change in total car count due to both a dwelling bus and TSP activation, i.e., $\Delta N_B + \Delta N_T + \Delta N_T^D + \Delta N_T^C$. Figure 3-7b reveals that a dwelling bus’s negative impacts steadily diminish as $d$ increases and disappear for $d/wL_c > 0.40$. Furthermore, a dwelling bus’s negative impacts increase with larger $S$. Additionally, Figures 3-7c and d aim to show the impact of green extension or red truncation on bus delays, respectively. It can be seen that green extension affects the cars and buses in a similar manner, while red truncation can provide benefits to buses without impacting cars at all. Finally Figure 3-7e shows the total bus delay savings due to the simultaneous implementation of green extension and red truncation.
Figure 3-7: The impacts of TSP strategies for a near-side stop on all approaches: (a) change in car count due to green extension ($\Delta N_T$); (b) overall change in car count due to TSP and a dwelling bus ($\Delta N_B + \Delta N_T$); bus delay savings with the implementation of: (c) green extension; (d) red truncation; and (e) both strategies ($q_c = q_c' = 1, q_b = 2/3, L_c = 120, g = 0.5, r_{min} = 0.4$).
3.3.2 Undersaturated Conditions

A similar analysis was performed to evaluate TSP impacts on car and bus delays in under-saturated conditions for all directions. The bus stop location, $d$, is varied to obtain average car and bus delays for all possible arrival times of buses to a stop, i.e., $t_a$ is uniformly distributed in $T_a$. The results are shown for several ranges of uniformly distributed dwell times, $S$.

Figure 3-8 shows the impacts of TSP on bus and cumulative car delays. Figure 2-8a provides the change in total car delay on all approaches as a result of TSP. The results are shown as a function of $d/wL_c$. Positive vertical values indicate an added or increased car delay caused by TSP. In general, the average additional car delay increases as a bus stop is located further from the intersection. This is expected since red truncation can be applied in a larger range of times as the bus stop is located further from the intersection, i.e., $T_r$ increases as $d$ increases. For the values of intersection capacity and signal timing that are used, the additional queues formed due to the implementation of TSP can be dissipated within the same cycle when $d/wL_c < 0.2$. When $d/wL_c > 0.2$ a large increase in car delay is observed since some cars are forced to wait an extra cycle due to the implementation of TSP. Additionally, the average additional total car delay generally decreases as $S$ increases. While this may appear counterintuitive, note that Figure 3-8a presents the additional car delays after TSP is implemented. As the bus dwells for a longer period of time, TSP can reduce the delay to cars that have queued during the bus dwell time. When $d \rightarrow d^* = 0.33wL_c$, the average increase in car delay approaches the same value for all $S$. Recall that if $d \geq d^*$, the additional car delay imposed by a dwelling bus without TSP diminishes to zero. Therefore, the constant value at the right side of all curves in Figures 2-8a represent the impacts imposed only by TSP, which are similar regardless of the duration of the bus dwell time.

Figure 3-8b shows the bus delay savings, $\Delta T_s$. Notice that the results are very sensitive to both the dwell time and the bus stop location. The bus delay savings increase as the bus stop is
located further from the intersection for relatively large dwell times. However, for shorter dwell times the bus delay savings will first decrease and then increase as the bus stop is located further from the intersection. As the bus stop is moved further from the intersection the percentage of time $t_e \in T_g$ vs. $t_e \in T_r$ varies, leading to a different weighted average bus delay saving since green extension provides more savings on bus travel time than red truncation.

Figure 3-8c shows the critical occupancy ratio between bus and car, i.e., $Occ_{bus}/Occ_{car}$ such that the expected passenger delay would be zero. This is calculated as:

$$\frac{Occ_{bus}}{Occ_{car}} = \max \left( 0, \frac{\text{average car delay change}}{\text{average bus delay savings}} \right)$$

(3-19)

According to this figure, passengers will benefit from TSP for all $d$ and $S$ herein if

$$\frac{Occ_{bus}}{Occ_{car}} \geq 16.$$
Figure 3-8: The impacts of TSP for a near-side stop: a) total car delay change; b) bus delay savings; c) critical occupancy ratio ($q_c = q'_c = 1$, $q_b = \frac{2}{3}$, $q_a = 0.4$, $L_c = 120$, $g = 0.5$, $r_{min} = 0.4$).
Chapter 4

Optimizing TSP Implementation along an Arterial: Variational Theory and Macroscopic Fundamental Diagram Approach

A combination of transit signal priority (TSP) and dedicated bus lane (DBL) is implemented to provide bus priority within an arterial with multiple intersections. This chapter summarizes the application of variational theory (VT) and macroscopic fundamental diagram (MFD) approach to evaluate arterial capacity with the impacts of TSP (Wu and Guler, 2018). Bus stop is also considered in this study. This analytical study includes 3 parts, (1) identifying the TSP impacts on arterial car capacity using MFDs; (2) finding the optimal placement and numbers of TSP implementation, along a corridor; (3) necessity of arterial level analysis.

4.1 Scenario Description

Considered here is an arterial street with $k$ signalized intersections, e.g., 5 in Figure 4-1b. Notice that in Figure 4-1a, the signal settings of the first and last intersections (both are labeled as 1) are exactly the same. This allows an extended version of our street as an infinite corridor by placing end-to-end an infinite number of copies of the representative finite street (Daganzo and Geroliminis, 2008). Traffic on the homogenous road segments upstream and downstream of any intersection is assumed to obey kinematic wave theory with a triangular FD (Lighthill and Whitham, 1955). Again, let $v_F$ denote the free-flow speed, $-w$ the backward wave speed, and $q_c$ the capacity of the roadway for both the bus travel and the opposite directions. The capacity of the roadway in the cross direction is assumed to be $q'_c$ for all intersections. Buses travel within the bus-dedicated lane with speed $v_F$ and stop at intersections during red times and at bus stops.
The signal timings are described by cycle lengths: \( C = [c_1, c_2, \cdots, c_k] \), green times: \( G = [g_1, g_2, \cdots, g_k] \), and offset: \( O = [0, \delta_2, \delta_3, \cdots, \delta_k] \) for intersections 1, 2, \cdots, \( k \). A bus enters the arterial at intersection 1 at time \( t_a \), where \( t_a \) is uniformly distributed over a cycle (in green phases), i.e., \( t_a \in [mc_1, (m + g_1)c_1], m = 0, 1, 2, \cdots \). The bus would have departed from the arterial at time \( t_{e2} \) if TSP were not implemented. Two candidate TSP strategies are implemented to provide bus priority: green extension and red truncation, which allows the bus to exit the arterial at time \( t_{e1} \). Thus, the bus delay savings can be represented as \( \Delta T_S = t_{e2} - t_{e1} \). In this example, the activation of TSP implies that the red phases at intersections 2, 3 and 5 are shortened by \( \Delta T_2, \Delta T_3, \Delta T_5 \), respectively; see Figure 4-1b. Correspondingly, green phases for cross directions are shortened by \( \Delta T_2, \Delta T_3, \Delta T_5 \) at these three intersections, hence, car throughput on cross streets is reduced by \( \Delta N = 2q_c(\Delta T_2 + \Delta T_3 + \Delta T_5) \).
The quantification of bus delay saving $\Delta T_s$ and cross-street car throughput, $\Delta N$, are quite straightforward. However, estimating the impacts on the arterial capacity is not easy. An efficient estimation method is to consider the MFDs on arterials with TSP implementation. This method applies VT to an arterial with and without TSP (Daganzo, 2005b). The following two subsections and next section will cover this VT-based method at length.

### 4.2 MFD Estimation with TSP

As is illustrated in Figure 4-2, a systematic recipe can be used to estimate MFDs before and after TSP implementations. This method allows a moving observer to explore kinematic waves, transforming street MFD into a shortest-path problem. The matrix $R^{N \times N}$ can be derived given the original signal time settings for all $N$ vertices within a variational graph. Each element $r_{ij}$ of matrix $R$ represents the cost of the edge connecting vertices $i$ and $j$. A vertex is defined as a
point on the variational graph that represents the end of a red period, hence, vertices locations can be changed due to TSP, see Figure 4-3a and c.

The activation of TSP for bus arrivals can result in a new variational graph. This causes the change in location of some vertices if red truncations are activated, and adds costs of some edges if green extensions are activated. Correspondingly, a new matrix $R_{N \times N}$ is generated, thus results in a new MFD, see Figure 4-3b and d. In addition to the example as shown in Figure 3-3, TSP can change the shape of MFD in four different ways:

Figure 4-3: (a) and (c) original and updated variational graphs; (b) and (d) original MFD and MFD with impacts of TSP
1) TSP does not change the cost of any cuts since it does not alter any of the shortest paths;
2) TSP increases costs associated with noncritical cuts (e.g., cut 2), hence, the final MFD is not affected;
3) TSP increases costs associated with critical cuts, hence, the uncongested branch, the congested branch, or both branches of MFD moves upward. However, the capacity (maximum flow) remains constrained by another cut (e.g., a stationary cut);
4) TSP increases the capacity of the arterial (but the impact is usually very small).

Through preliminary tests of the algorithm it was found that (1) and (2) are common cases for arterials with homogeneous conditions, that is, intersections with same cycle length and green ratio, and cases (3) and (4) only occur when the arterial network is sufficiently heterogeneous.

The numerical analysis tests an arterial with 4 intersections, the signal settings are $C = [90,60,75,120]$, $\delta_i = 0$, $g_i = 0.5$. TSP is deployed in intersections 2, 3, and 4 (the maximum change in signal time, $\Delta T_{max} = 0.1C$). The lengths of 4 blocks are the same, $d = 200m$. Within each block, buses travel at free-flow speed $v_f = 16m/s$, and stop at bus stops with dwell time $S$ which is normally distributed, $S \sim N(30,10)$. Other parameters are set as follows: $q_c = 0.5veh/s$, $-w = -5m/s$, and capacity of cross directions $q'_c = 0.55 veh/s$.

7 different bus headways $h = c_1, 2c_1, \ldots, 6c_1, > 6c_1$, where $c_1 = 90$ seconds, is tested. The case where TSP is not provided is also included as a baseline and is shown as the case where number of buses is zero. Among all cases, the analysis time window is set to be $T = 720$ seconds. Hence, the total number of buses within the arterial for the different headways is $7,4,3,2,2,2,1$, respectively. This analysis assumes a uniformly distributed bus arrival time over a
green time at intersection 1. The results are shown as an average over all these possible bus arrival times.

According to the results shown in Figure 4-4, the maximum change observed in the congested branch is an increase in flow of 3.21% for the case with bus headway \( h = 90 \) seconds compared to the baseline. TSP increases arterial capacity for the cases when \( h < 540 \) seconds. As expected, the impact of TSP decreases as bus headway increases. Among all cases, the maximum change in arterial capacity is 1.75% compared to the baseline. The arterial speed when arterial
operating at capacity varies between 4.56 and 4.60 m/s. An average bus saves between 18.39 and 25.23 seconds. Correspondingly, cross-street car throughput loss varies between 3.01 and 5.74 vehicles.

4.3 Optimal TSP along Homogeneous Arterial

The goal is to find the optimal number of intersections that should be equipped with TSP (i.e., have TSP capability), and if implementation on different intersection combinations would lead to different performances. Given this, the problem can be formulated as an integer program where the decision variable is an array $n_t$ with binary variables 1 or 0 to indicate TSP being implemented or not at an intersection. The results of the previous section show that the capacity of an arterial is minimally impacted by providing TSP, especially on homogeneous arterials. Therefore, when considering the optimal location of TSP implementation, only delay savings and cross street capacity loss are considered in this paper. However, the methodology can easily account for changes in the arterial capacity. As a result, different objectives can be determined such as:

1) minimizing the impacts on cross-streets (the change in car throughput $\Delta N$);

2) maximizing bus delay savings $\Delta T_b$; and

3) a combination of 1 and 2.

Notice that the first and second objectives are normally in conflict with each other. In general, with more intersections that have TSP implemented, the cross direction will have more capacity loss while bus delay savings will increase. Therefore, a myopic study would conclude that the best solution is to implement TSP at every intersection. However, the marginal benefit or cost of each implementation may not be the same. Especially given how expensive TSP
implementation can be the tradeoff between cost of implementation and impact should be carefully considered. Given a limited budget, implementing TSP on one intersection as compared to another one can lead to very different impacts. Therefore, in this paper, the results are discussed in terms of Pareto frontiers.
To solve this optimization problem, all possible TSP strategies in a set $\mathbf{N}_T$ is enumerated. For more complicated scenarios different optimization heuristics such as branch-and-bound can also be used. An example 8-intersection homogeneous arterial with $c_i = 90, g_i = 0.5, \delta_i = 0$ is considered. Note that TSP is not furnished at the first and last intersections, hence, $n_T(1) = 0$, and $\sum_i n_T(i) \leq 6$. An example solution would be $n_T = [0,1,0,0,0,0,0] \in \mathbf{N}_T$ indicating TSP is implemented only at intersection 2. For this arterial, a total of 63 candidate solutions, $n_T$, are considered (which excludes the no TSP solution). By applying the previous recipe, all $\Delta N$ and $\Delta T_S$ corresponding to $\mathbf{N}_T$ are quantified.

Only one bus is included in this analysis. Again, bus arrival times are uniformly distributed across a cycle. The results are shown as an average of all possible bus arrival times.

Figure 4-5: (a) Total change in car throughput versus total bus delay savings; (b) marginal effect of TSP implementation on multiple intersections.
All parameters are the same as previous section: \( v_f = 16 \text{m/s} \), \( S \sim N(30,10) \), \( -w = -5 \text{m/s} \), 
\( q_c = 0.5 \text{veh/s} \), \( q'_c = 0.55 \text{veh/s} \).

Figure 4-5a shows the total change in car throughput of cross-streets, \( \Delta N \), versus the average total bus delay savings, \( \Delta T_\delta \), across all intersections. In this figure, the data points corresponding to different number of intersections where TSP is implemented are shown with different color and shape markers. As expected, in general total \( \Delta T_\delta \) and \( \Delta N \) increase simultaneously as the intersection number increases. However, there are certain combinations of intersections on which TSP can be implemented that result in smaller changes to cross-street capacity and larger bus delay savings compared to others. To this end, a Pareto frontier can be identified as shown in Figure 4-5a. Additionally, notice that for certain combinations of intersections, implementing TSP on fewer intersections can result in larger total bus delay savings than implementing on more intersections. This is especially evident when looking at larger number of intersections equipped with TSP. For example, compare implementing TSP on 3 intersections vs. 4 intersections. It can be seen that while in most cases implementing TSP on 3 intersections results in less bus delay savings, one specific combination of intersections outperforms implementing TSP on 4 intersections for 10 different cases. This Pareto graph helps decision-maker to choose best-fit local TSP locations under different circumstances. This figure clearly shows that TSP implementation at different intersections does not always result in the same benefits and costs. Notice that this discrepancy occurs despite consideration of a homogeneous arterial.

The implementation of TSP for a specific number of intersections (e.g., only 4 intersections at once) is considered to explore the marginal benefits of extending TSP, see Figure 4-5b. For a given number of intersections equipped with TSP, the optimal distribution that provides the maximum bus delay savings, \( \Delta T_\delta \), is determined using an iterative method. This can be thought of as implementing TSP using a limited budget (i.e., only on a limited number of
intersections) on intersections where it would have the largest benefits. Another finding is that both the maximum $\Delta T_5$ per intersection and $\Delta N$ per intersection decrease simultaneously as the number of intersections on which TSP is implemented increase. Hence, the marginal impact of TSP decreases as more intersections are equipped with TSP.

### 4.4 Necessity of an Arterial Level Analysis

Given the existing theoretical tools for analyzing the impacts of TSP implementation, the typical method for evaluating an arterial would be to consider it as a summation of independent intersections rather than evaluating the arterial as a whole. This section considers how evaluating an arterial as a whole could change the predicted impacts of TSP implementation as compared to evaluating the arterial as a sum of independent intersections. To do so, two separate methods are utilized. This methodology assumes that bus arrivals to the intersection have a uniform distribution, and TSP is activated only if buses would benefit from it. VT is used to estimate the average car capacity for all possible bus arrival times. Intersections along an arterial with eight intersections are individually evaluated for the expected impacts of TSP (i.e., each intersection has the same expected car capacity and bus delay since they are assumed to operate independently). To evaluate the arterial as a whole, the MFD recipe furnished in this paper is used for an arterial with $c_1 = c_2 \cdots = c_8$, and $g_1 = g_2 \cdots = g_8$. It is assumed that a bus enters an arterial with times uniformly distributed over a cycle. For simplicity the bus is assumed to have zero dwell time at bus stops. All parameters are kept the same as the previous sections.

For the two methods to be comparable, both the change in total car throughput, $\Delta N$, and bus delay savings, $\Delta T_5$, are given as an average value per intersection per cycle. When considering individual intersections, these values are just an output of the methodology and it is assumed that every intersection experiences the same change in capacity and bus delay savings.
However, when considering the arterial as a whole, the change in capacity along an arterial is zero as shown previously. Hence $\Delta N$ represents only the average change in cross-street capacity over all the intersections along the arterial. Additionally, $\Delta T_S$ represents the bus delay savings per intersection, i.e., the total bus delay savings divided by 8.

Two candidate offsets are considered, zero offset $O = 0$, and random offset $O = [0,5,20,10,20,10,15,5]$. Different scenarios with cycle length, $c_i$, varying between 60 and 120 seconds, and green ratio, $g_i$, varying between 0.3 and 0.7 are tested. Figure 3-6 shows the change in car throughput per intersection and bus delay savings per intersection using the arterial methodology with zero offset (blue circles in Figure 4-6) or a random offset (orange squares in Figure 4-6), or using the isolated intersection methodology (black diamonds in Figure 4-6).

Figures 4-6a and b show the change in $\Delta N$ and $\Delta T_S$, respectively, for varying cycle lengths, and Figures 4-6c and d show the change in $\Delta N$ and $\Delta T_S$, respectively, for varying green ratios.

According to Figures 4-6a and b, $\Delta N$ and $\Delta T_S$ have an increasing trend as the cycle length increases, specifically when considering only an isolated intersection. Intersections with longer cycle lengths provide more opportunities to provide TSP, and hence both the change in car throughput and bus delay savings increase. However, notice that considering only an isolated intersection exaggerates the benefits and costs of implementing TSP. The bus delay savings per intersection vary around 5 seconds per intersection for both considered offsets when evaluating the arterial as a whole and are much smaller than predicted with the isolated intersection methodology. When considering the arterial as a whole even though all possible bus arrival times to the upstream intersection is considered, this results in a limited combination of bus arrival times to other intersections. Hence, at some intersections it is never feasible to implement TSP. On the other hand, when considering an isolated intersection all possible bus arrival times are considered, hence, TSP can be activated at every intersection. As for green ratio, the change in car throughput decreases as the green ratio increases since more buses arrive during the green
time and do not require preemption. Hence, accordingly the bus delay savings also decrease as the green ratio increases, especially when considering the arterial as a whole. However, the trends are not as evident when looking at an isolated intersection since the results depend on the average times of bus arrivals over a cycle. Again, both $\Delta N$ and $\Delta T_S$ are over-estimated when only an isolated intersection is considered.

Notice that these results are estimated using the parameters as mentioned above. However, considering only an isolated intersection may also under-estimate the values under some parameters. This indicates that 1) the two methods are not comparable, and 2) the methodology proposed in this paper has its unique strengths in evaluating the impacts of TSP at an arterial level.
Figure 4-6: Comparative graphs showing change in car throughput and bus delay savings (per intersection per cycle) for: (a) different cycle lengths (with $g_i = 0.5$); and (b) different green ratios ($c_i = 90$).
Chapter 5


This chapter furnishes a bus-car traffic flow modeling approach to evaluate TSP performance within the context of kinematic wave theory assuming mixed traffic on the roadway. A dynamic programming algorithm is developed based on the Lax-Hopf equation (Aubin et al., 2008), which computes vehicle counts and tracks bus motion in time-space plane with and without TSP. Changes in bus travel times and cumulative car delay are both quantified. Further numerical tests demonstrate the capabilities of the algorithm to evaluate TSP performance using existing bus detector methods, along with utilizing information from connected buses or connected vehicles.

5.1. Background

In this section, we briefly summarize the main features of the generalized Lax-Hopf equation and initial, boundary and internal conditions (Aubin et al., 2008; Mazare et al., 2011; Simoni and Claudel, 2017). The generalized Lax-Hopf equation is an implicit representation of an LWR PDE. It is assumed that all roadway segments are homogeneous and obey a triangular fundamental diagram with free-flow speed \( v_f \), backward wave speed \(-w\), capacity vehicle flow \( q_c\), and jammed density \( k_j\). The results of the Lax-Hopf equation is the cumulative vehicle numbers, denoted by \( N(t, x)\) within a \( (t, x)\) domain \( D: t \in [0, T], x \in [0, X] \), which can be represented as the infimum of a set of functions as shown in Equation 5-1 (Daganzo, 2005a).

\[
N(t, x) = \inf\{C(t - \Delta T, x - u\Delta T) + \Delta TR(u)\}, s. t. u \in [-w, v_f], (t - \Delta T, x - u\Delta T) \in D
\] (5-1)
where $C(t, x)$ corresponds to the cumulative count of numbers associated with a set of boundary conditions, and $R(u)$ represents a cost function associated with a fundamental diagram. For a triangular fundamental diagram $R(u)$ is equivalent to the maximum rate (or cost rate in the VT method) that a vehicle travelling at speed $u$ could pass a so-called wave path over $(t, x)$, which can be expressed as follows:

$$R(u) = \begin{cases} 
  k_c(v_f + w) & u = -w \\
  k_c v_f & u = 0 \\
  0 & u = v_f 
\end{cases}$$  \hfill (5-2)

$C(t, x)$ represents the boundary conditions which includes: 1) initial $C_{ini}(x)$, 2) upstream $C_u(t)$, and 3) downstream $C_d(t)$:

$$C(t, x) = \begin{cases} 
  C_{ini}(x|x = x_i) = \sum_{i} k_{i-1} (x_i - x_{i-1}) & t = 0, x \in [0, X] \\
  C_u(t|t = t_j) = \sum_{j} q_{j-1}(t_j - t_{j-1}) & x = 0, t \in [0, T] \\
  C_d(t|t = t_j) = \sum_{j} p_{j-1}(t_j - t_{j-1}) + N(0, X) & x = X, t \in [0, T] 
\end{cases}$$  \hfill (5-3)

The initial condition $C_{ini}(x)$ represents the cumulative vehicle count along the arterial at $t = 0$, where, $k_{i-1}$ denotes the density within road segment $x_{i-1}$~$x_i$. Hence, the vehicle count $N_t$ at $x_i$ is derived from $N_t = C_{ini}(x_i)$. The upstream condition, $C_u(t)$ and downstream condition, $C_d(t)$, correspond to the measurements of vehicle counts at the downstream and upstream ends of an arterial of length $X$, where $q_{j-1}, p_{j-1}$ denote the known flows at $x = 0$ and $x = X$ within time interval $t_{j-1}$~$t_j$, respectively. Here it is assumed that there is no bottleneck at the downstream exit location, hence, $p_{j-1} = q_c$ for all $j$. Hence, $N_j, N'_j$ are derived from $N_j = C_u(t_j)$, and $N'_j = C_d(t_j)$, respectively.

Given a triangular fundamental diagram, the solution to Equation 5-1 is processed by minimizing a set of vehicle counts calculated by initial condition ($N_{ini}(t, x)$), upstream condition
Where,\n
1) \( N_{ini}(x) \) is predicted based on initial conditions \( C_{ini}(x) \). If the density at \( x_i \) is \( k_i \in [0, k_c] \), this imposes a free-flow state in space segment \( x_i \sim x_{i+1} \in [0, X] \), hence,\n\n\[
N_{ini}^i(t, x) = \begin{cases} \frac{N_i - k_i (x - (x_i + v_f t))}{x} \in [x_i + v_f t, x_{i+1} + v_f t] \\ \frac{N_i - k_c (x - (x_i + v_f t))}{x} \in [x_i - wt, x_i + v_f t] \end{cases}
\]

Else if \( k_i \in [k_c, k_j] \), this imposes a congested state,\n\n\[
N_{ini}^i(t, x) = \begin{cases} \frac{N_i + k_j wt - k_i (x - (x_i - wt))}{x} \in [x_i - wt, x_{i+1} - wt] \\ \frac{N_{i+1} - k_c (x - (x_{i+1} + v_f t))}{x} \in [x_{i+1} - wt, x_{i+1} + v_f t] \end{cases}
\]

The final \( N_{ini}(t, x) \) can be determined as the minimum of the free-flow and congested states.\n\n\[
N_{ini}(t, x) = \min_i \{N_{ini}^i(t, x)\}
\]

2) \( N_u(t) \) is predicted based on the upstream condition \( C_u(t) \) at \( x = 0 \) and is expressed as follows,\n\n\[
N_u^j(t, x) = \begin{cases} \frac{N_j + q_j (t - t_j - \frac{x - 0}{v_f})}{t} \in [t_j + \frac{x - 0}{v_f}, t_{j+1} + \frac{x - 0}{v_f}] \\ \frac{N_{j+1} + q_c (t - t_{j+1} - \frac{x - 0}{v_f})}{t} \in [t_{j+1} + \frac{x - 0}{v_f}, T] \end{cases}
\]

\[
N_u(t, x) = \min_j \{N_u^j(t, x)\}
\]

3) Similarly, \( N_d(t) \) is predicted based on the downstream condition \( C_d(t) \) at \( x = X \) and is expressed as follows,\n\n\[
N_d^j(t, x) = \begin{cases} \frac{N_j + k_j (X - x) + p_j (t - t_j - \frac{X - x}{w})}{t} \in [t_j + \frac{X - x}{w}, t_{j+1} + \frac{X - x}{w}] \\ \frac{N_{j+1} + k_j (X - x) + q_c (t - t_{j+1} - \frac{X - x}{w})}{t} \in [t_{j+1} + \frac{X - x}{v_f}, T] \end{cases}
\]
\[ N_d(t, x) = \min_j N_d^j(t, x) \] (5-7b)

Figures 5-1a and 5-1b show examples of the domains of influence of the initial condition, and upstream and downstream conditions, respectively. These figures show that while the initial conditions can influence both upstream and downstream cumulative counts, the upstream boundary condition only influences the downstream counts in free flow, and the downstream boundary condition only influences the upstream counts in congested conditions.

Figure 5-1: Illustration of initial, boundary and internal conditions

4) With the presence of a moving and/or stationary bottleneck, an internal condition \( C_{\text{int}}(t, x) \) is also introduced. This condition measures the cumulative vehicle numbers along a moving bottleneck’s trajectory. As shown in Figure 1c, vehicle counts \( N_{\text{int}}(t, x) \) can be predicted within the forward, backward and central domains, given the beginning and ending times \((t_b, t_e)\) and locations \((x_b, x_e)\) associated with an active moving bottleneck. The bottleneck capacity is derived from the fundamental diagram as

\[ q_r = k_c \cdot (v_f - v_b) \cdot \frac{n_t - 1}{n_l} \] (Munoz and Daganzo, 2002),

where \( v_b \) denotes the bottleneck velocity and \( n_l \) denotes the number of lanes. Additionally, the bottleneck capacity when a bus dwells at bus stop is simply equivalent to the capacity of the remaining lanes, i.e.

\[ q_r = q_c \cdot \frac{n_t - 1}{n_l} \] . The vehicle number \( N_b \)
at the beginning of a bottleneck can be predicted by the other boundary conditions. Hence, the vehicle number \( N_e \) at the ending of a bottleneck is derived from equation, \( N_e = N_b + q_r(t_e - t_b) \). The vehicle counts, \( N_{int}(t, x) \), within all three domains are calculated as follows,

\[
N_{int}(t, x) = N_b + q_r \cdot (t - t' - t_b) + k_c v' t' 
\]  

(5-8)

where, \( v' \) denotes the magnitude of relative speed using \( v_f \) as reference speed; \( t' \) denotes the duration of forward or backward waves with relative speed \( v' \). Note that the definition of \( v' \) and \( t' \) varies among three domains as exhibited in Figure 5-1c. The presence of an active bottleneck imposes a free-flow state within forward and central domains, and congested state within backward domain. Mathematically, \( v' \) and \( t' \) are given by Equations 9a-c for these three domains.

a) Within forward domain,

\[
t' = \frac{x - [x_b + v_b(t - t_b)]}{v_f - v_b}, v' = 0
\]  

(5-9a)

b) Within backward domain,

\[
t' = \frac{[x_b + v_b(t - t_b)] - x}{v_b + w}, v' = v_f + w
\]  

(5-9b)

c) Within central domain,

\[
t' = t - t_e, v' = v_f - \frac{x - x_e}{t'}
\]  

(5-9c)

Whether or not a bottleneck is active depends on the surrounding traffic conditions. Following (Munoz and Daganzo, 2002), three situations can be identified:

a) \( \frac{N_e - N_b}{t_e - t_b} \in (0, q_r) \), bottleneck is inactive because there is enough capacity for cars to overtake, MB's speed, \( v_b = v_m \);
b) \( \frac{N_{e-N_b}}{t_{e-t_b}} \in (q_r, +\infty) \), bottleneck is active because cars travel at high speed with low capacity, MB’s speed, \( v_b = v_m \); 

\( c) \frac{N_{e-N_b}}{t_{e-t_b}} \in (-\infty, 0) \), bottleneck is inactive because of congestion, MB travels at surrounding traffic speed, \( v_b = \frac{w(k_f-k_0)}{k_0} \), where \( k_0 \) is the surrounding vehicle density.

Equation 5-1 exhibits the Lax-Hopf equation, and how to update the vehicle counts \( N(t, x) \) obtained as a solution to the Lax-Hopf equation when a moving bottleneck travels along its trajectory (Simoni and Claudel, 2017). That method is adapted to develop a dynamic programming (DP) algorithm for analyzing TSP performance. An isolated intersection with traffic on the homogeneous road segments upstream and downstream of the intersection obeying kinematic wave theory with a triangular fundamental diagram is considered. The next section will describe in detail the DP algorithm and the scenario to which the algorithm can be applied.

5.2 Methodology

This section presents how to incorporate and evaluate TSP in the Lax-Hopf framework. Since signal timing varies with TSP activation, a dynamic programming (DP) algorithm is first introduced to accommodate the dynamical interactions between traffic (both buses and cars) and traffic signal. Next, how to implement the DP algorithm for different technology levels, no connected buses, connected buses, and connected cars, is discussed.

5.2.1 Description of the DP Algorithm

The DP algorithm is developed based on the Lax-Hopf equation. The goal of the algorithm is to calculate the changes in bus travel times and the changes in total car delay after implementing TSP at the intersection. The pseudocode of the DP algorithm is given in Algorithm
1. This algorithm incorporates formulations derived from the Lax-Hopf equation (Aubin et al., 2008) as described in the previous section (Simoni and Claudel, 2017). Buses are modeled as a moving bottleneck, and on-street bus stops are modeled as temporary stationary bottlenecks. It is assumed that either green extension, or red truncation are options for TSP and the signal timing can be changed by a maximum amount of $dt_m$. Other variables are defined within the algorithm in order of appearance.

### Table 5-1: Pseudocode of DP algorithm for estimating TSP performance (Algorithm 1)

<table>
<thead>
<tr>
<th><strong>Function main()</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input</strong></td>
</tr>
<tr>
<td>$L_c, g, dt_m, x_f, x_d$ //intersection parameters</td>
</tr>
<tr>
<td>$t_A, v_m, x_B, S, q_d, q_r$ //bus parameters</td>
</tr>
<tr>
<td>$v_f, w, k_f, k_r, k_{int}, q$ //FD parameters</td>
</tr>
<tr>
<td>$T, X, \Delta t, \Delta x$ //computation parameters</td>
</tr>
<tr>
<td><strong>Output:</strong> $T_B, T_C$</td>
</tr>
</tbody>
</table>

**Begin main()**

1. Calculate $N^{mxn}$ with initial and boundary conditions as described by Equations 5a-c, 6a-b, 7a-b
2. Generate initial vehicle count matrix $N^{mxn}$ where $m$ is the total number of time steps, and $n$ is the total number of space steps
3. Update $N^{mxn}$ with internal conditions as described by Equations 8, 9a-c
4. Introduce internal conditions created by the signalized intersection, dwelling bus and moving bus bottlenecks
5. Track bus motion in $(t,x)$ plane and store bus trajectory in the format of (time, position, speed)
6. Generate a matrix $mb$ where each row stores the bus time, position and speed
7. For traditional bus detection technique, TSP strategy is determined based on the time a bus is detected, $t_d$. Find the time $t_d$ when bus is detected at $x_d$
8. For traditional bus detection, the time the bus is expected to arrive at the intersection is $t_f = t_d + \frac{x_f - x_d}{v_m}$. TSP scheme for other two techniques will be described in Section 3.2
9. Based on predicted bus arrival time, update signal setting within the context of TSP
10. If bus arrives during red time, add green time ($\Delta t_{GE}$). If bus arrival time is still in the red after maximum green extension amount of $dt_m$, switch to truncate the red time ($\Delta t_{RT}$) within the limits of a maximum red truncation time, $dt_m$. Predict bus arrival time to intersection, $t_f$
11. Re-run main() function with new signal setting
12. Update internal conditions and generate new matrices $N^{mxn}$ and $mb$
To illustrate the capabilities of Algorithm 1, we present an example case for a specific bus arrival time and show the changes in traffic states with and without TSP. For the remainder of the paper, a two-lane ($n_t = 2$ for each direction) arterial of length $X = 600m$ is considered.

Traffic operation on this roadway can be represented with a triangular fundamental diagram with free-flow speed $v_f = 20m/s$, backward speed $-w = -5m/s$, and jam density $k_j = 0.2veh/m$.

It is assumed that buses travel at a free flow speed of $v_m = 10m/s$, creating a moving bottleneck with passing rate of $q_r = 0.2veh/s$. The signalized intersection, which is placed at $x_j = 500m$, has a cycle length of $L_c = 90s$ and green ratio of $g = 0.5$. Active TSP in the form of red truncation or green extension is deployed at this intersection with maximum change to the signal time equal to $dt_m = 5s$. To simplify the analysis turning is prohibited at the intersection (Wada et al., 2017; Wu et al., 2017; Wu and Guler, 2018). The time and space steps for computation are $\Delta t = 0.5s, \Delta x = 0.5m$, respectively. The abovementioned time and space steps ensure that the difference between exact and approximate vehicle counts at any point in space-time is equal to zero.

<table>
<thead>
<tr>
<th>Store the last time entry of bus trajectory with and without TSP as $t_u^0, t_u^1$, $T_B = t_u^0 - t_u^1$</th>
<th>Compute change in bus travel time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Store the vehicle counts at intersection location $x_i$, with and without TSP as $N_i^0, N_i^1$, hence, $T_C = \Delta t \cdot \left( \text{sum}(N_i^1) - \text{sum}(N_i^0) \right)$</td>
<td>Compute change in cumulative car delay of bus travel direction, following queuing theory (Wada et al., 2017), apply similar method to opposite and cross directions</td>
</tr>
<tr>
<td>Return $T_B, T_C$</td>
<td></td>
</tr>
<tr>
<td>Repeat for every $x_i$ in sequential order</td>
<td></td>
</tr>
<tr>
<td>End main()</td>
<td></td>
</tr>
</tbody>
</table>

$$T_B = t_u^0 - t_u^1$$

$$T_C = \Delta t \cdot \left( \text{sum}(N_i^1) - \text{sum}(N_i^0) \right)$$
The solutions to the Algorithm described above are shown in space-time-density diagrams in Figures 5-2a and 5-2b, representing the time-space-density diagrams without and with TSP, respectively. In these figures, the bus is assumed to arrive at $t_a = 25s$ and the bus trajectory is marked using a solid white line. A car demand of $q = 0.3veh/s$ and initial condition of $k_{ini} = 0.04veh/m$ is considered. For this example, it is assumed that there is no bus stop near the intersection. In Figure 5-2a corresponding to the no TSP case, the bus is caught in the queue of cars and cannot make the green. Hence, it has to wait until the next green phase. On the other hand, Figure 2b shows the case where the intersection is equipped with TSP capability. In this case, a bus detector is assumed to be located at $x_d = 440m$. As a result, the bus is detected after it leaves the queue of cars and the green phase is extended by $3s$. This allows for the bus to reduce its travel time by nearly the entire length of the red phase, i.e., $T_B = 43.5s$. Correspondingly, a reduction in cumulative car delay for the bus travel direction only is observed, $T_C = -147.45$ vehicle-seconds (negative indicates decreased delays). Notice that for the remainder of the paper, positive values of $T_B$ indicate reduction in bus delays, and positive values of $T_C$ indicate increase to car delays.
5.2.2 Implementing TSP for Different Levels of Connected Vehicles

The DP algorithm (see Table 5-1) can be adapted to test the performance of three types of TSP technologies using different detection techniques: 1) traditional bus detector at a fixed location; 2) connected buses; 3) connected private vehicles and buses. Three TSP detection techniques are described in detail as follows:

1) A traditional bus detector is implemented at a fixed location, see Figure 5-3a. This fixed location is either at the bus stop to determine exactly when a bus leaves the stop or further downstream of the bus stop. However, it is assumed that a detector is never located upstream of a bus stop as the information obtained from such a location would not be useful due to the bus dwell at the stop afterwards. The activation of TSP is based on the time when the bus is detected at the bus detector location, \( t = t_d \). Once a bus is detected, the time the bus will arrive to the intersection, \( t_I \), is predicted assuming free flow conditions. If \( t_I \) is shortly after the end of a green phase, green extension is activated. If this time is after the end of the green phase, but green extension cannot be used then red truncation is activated.

2) Connected buses are assumed to be in constant communication with the signal controller and can provide information on their location. Information from connected buses can be used to change signal timings of the cycle in which they are expected to arrive at the intersection, along with previous cycles to clear car queues ahead of the bus. To do so, connected buses are expected to provide location and time information to the signal controller for three cases: 1) departing from the bus stop, 2) stopping behind a queue of cars, \( t_d \), and 3) starting to accelerate after exiting from a queue of cars, \( t_L \), downstream of a bus stop. Using this information, red truncation is activated if:
a) The bus stops behind a queue of cars during a red phase. In this case red truncation is applied to the current cycle and all consecutive cycles until the bus clears the intersection. This is an example of a preemptive TSP where even if the bus is not expected to pass during the next green period, the increased green duration will help clear car queues ahead of the bus to reduce bus delays.

b) The bus is predicted to arrive at the intersection after duration $dt_m$ after the end of a green phase. The time the bus is predicted to arrive at the intersection, $t_l$, is calculated using the maximum of the bus departure time from the bus stop and bus departure time from the back of a queue.

In both cases described above, the red phase is truncated by the minimum of $dt_m$ or the remaining time within the red phase. Green extension is only activated if $t_l$ falls during a red time within $dt_m$ of the switch of the signal from green to red.

3) All private vehicles are assumed to be connected and are expected to provide their time and location to the signal controller once private vehicles arrive at a distance $x_m$ upstream of the intersection (this distance is larger than the maximum queue length). All private vehicles within $0 \sim x_m$ upstream of the intersection are tracked. When a bus is detected at $x_m$ upstream of the intersection, the controller has the necessary information (i.e., all cars within the system and the signal phasing) to determine the existence of a queue of cars, and the number of cars in that queue ahead of the bus. Using this information, the time when a bus is expected to stop behind a queue of cars – $t_q$, and the duration of how long the bus is expected to be in the car queue – $S_0$, is predicted. This is done using the algorithm as described above. If a car queue is predicted to exist ahead of the bus, red truncation will be activated to clear the car queue as quickly as possible, regardless of the position of the bus in queue. The truncated red time is determined by the minimum of $dt_m$ and $S_0$. Additionally, this allows for the controller to predict when the
bus will be arriving to the intersection (assuming uniform discharge rates) and predict whether green extension or red truncation would be required at the time of arrival. Since TSP can be planned in advance, the performance of TSP is further improved (especially for red truncation). For example, if only information from connected buses were available, the presence of a queue of cars would only be known once the bus stops behinds that queue. If this time corresponds to a green phase, it would already be too late for red truncation. However, this can be resolved by predicting the time a bus is expected to stop behind a queue of cars using information from connected private vehicles and activating red truncation in advance.

To highlight how the presence of additional information obtained from connected buses or private vehicles could improve the TSP performance, an example is provided in Figure 3. The same two-lane arterial characterized by the same free-flow and backward speeds, and jammed density is considered as described above. To simplify the exposition, only locations 200~600m are presented in Figure 5-3 (labelled 0~400m in Figure 5-3). Now, it is assumed that a bus stop exists at \( x_B = 260 \) m and the signalized intersection is at \( x_I = 300 \) m. Notice that both the bus stop and the signalized intersection act as stationary bottlenecks. A demand of \( q = 0.35 \text{veh/s} \) is used herein. Figure 3a shows the time-space-density diagram for an illustrative bus that enters the arterial at \( t_a = 20s \), and dwells for \( S = 27 \) and TSP is not deployed at this intersection. Since the bus dwells both at the stop and also has to wait in two separate car queues, the bus exits the arterial at \( t_e = 148.5s \) with a travel time of \( t_e - t_a = 128.5s \). Figures 3b through 3d show how the trajectory of this same bus and the corresponding car densities would change if TSP were implemented. In Figure 5-3b, a bus detector is located at \( x_d = 260m \) (i.e., at the bus stop location) to inform the signal of the departure time of the bus from the stop. For this specific example when the bus departs from the stop, the red phase has already begun and the length of
the car queue is unknown, hence the only TSP option is to activate red truncation with $\Delta t_{RT} = dt_m = 5s$. This saves the bus 5s of travel time as compared to Figure 5-3a.

![Diagram](image)

Figure 5-3: Space-time-densities diagrams showing TSP impacts on bus travel times with different techniques: (a) no TSP implemented; (b) bus detector deployed at $x_d = 260m$; (c) connected bus technique implemented; (d) connected vehicle technique implemented.

For the results shown in Figure 5-3c, it is assumed that buses are connected and provide information to the controller as described above. The bus is detected when it is queued behind cars at time $t_q = 42.5s$ which is before the time the red phase ends at 45 seconds. Hence a red truncation with $\Delta t_{RT} = 45 - 42.5 = 2.5s$ is used to save the bus a travel time of 2.5s before it arrives at the bus stop. Additionally, the bus departs the bus stop at time $t_b = 89.5s$ and this allows for the activation of green extension $\Delta t_{GE} = 3.5s$, allowing the bus to pass the intersection without waiting an additional cycle. These two changes to the signal timing allow the bus to save
a total delay of $148.5 - 103.5 = 45s$. Also, the activation of red truncation reduces the number of cars delayed by the stopped bus, and a significant reduction in car delay in the travel direction of the bus is observed. While the car delay increases for the cross direction, for this example this can be more than compensated by the reduction in car delay in the bus travel and opposite directions.

For the results shown in Figure 5-3d, it is assumed that all vehicles are equipped with connected vehicle technology. In this case, the presence of a car queue downstream of the bus is known before the bus arrives to the back of this queue. A maximum duration of red truncation ($\Delta t_{RT} = 5 s$) is implemented at the first cycle to minimize the car queue that the bus will encounter. As a result, there is no need for the green extension used in Figure 5-3c since the bus can make the first green without TSP activation. The bus saves a travel time of $148.5 - 101 = 47.5s$. When considering cars, a further reduction in cumulative car travel times is observed since the queue of cars caused by a dwelling bus is shortened even further. Additionally, the increase in car delay for the cross direction is also less than if only connected buses were used since green extension is not implemented.

5.3. Numerical Analysis

To demonstrate the application of the above described methodology, we run a series of numerical analysis to test the sensitivity of TSP strategies to the bus detector location, the bus stop location and dwell duration, presence of a downstream bottleneck, and the bus detection technology in four consecutive subsections. All analyses are performed considering under saturated traffic demands (i.e., $q < gq_c = 0.4veh/s$) at the intersection (Wu et al., 2017). This implies that stopped buses or the downstream bottleneck may cause over saturated traffic conditions, however, eventually the intersection will be able to resume under saturated
operations. Over saturated traffic demands are not considered since these tests would require specific assumptions about the duration of high demands, and very long time/space spans of analysis.

All possible bus arrival times within the first cycle is tested and the numerical results are presented as an average of all these bus arrival times. Both the bus delay savings and additional cumulative car delay, compared to no TSP implementation, are computed. The change in overall cumulative car delay is calculated for cars travelling in the same, opposite and cross directions as the bus. Notice that in the results shown below positive values of change in bus delay implies bus delay savings, while positive change in car delay implies increase in car delays. As a general observation, buses experience large delay savings if green extension is activated since an entire red phase of delay is avoided. This strategy also benefits the cars travelling in the same and opposite directions as the bus since they also save the same delay. On the other hand, this strategy leads to additional delays for the cross direction. However, this only adds a few seconds of delay to all the queued cars in the cross direction as it serves to extend their red duration. Generally, the reduction in car delay for the bus travel and opposite directions exceeds the additional car delay of cross directions, hence, overall total car delays are reduced by green extension. On the other hand, buses only save a small amount of delay by up to \( d\tau_m = 5s \) if red truncation is activated. This also applies to cars traveling in the same and opposite directions of the bus. For cross directions, a red truncation ends a green phase early forcing cars that would have experienced no delay to wait an entire red phase. Consequently, the changes in cross direction car delays are more than car directions travelling parallel to the bus, and overall total car delays are increased by red truncation. The results in this paper show the combined impact of green extension and red truncation, however large fluctuations in changes in delay are often observed due to the discrete nature of the switch from green extension to red truncation.
5.3.1 Sensitivity to Bus Detector Location

Various detector locations are tested for different under saturated demand flows (0.1~0.3veh/s). For this sensitivity test, the bus is assumed not to stop at the bus stop to simplify the discussion. The sensitivity of the results to the bus stop location and bus dwell duration is presented in the next subsection. The location of the detector determines which buses trigger TSP and can sometimes lead to inefficiently triggered TSP, i.e., predicting that a bus will require TSP and triggering it, but in reality not saving delay for the bus. Figures 4-4a and 4-4b show the change in bus delay and total car delay for different detector locations, and different flow of cars using three different curves representing the different demand rates considered. The change in bus and car delays are small when the detector is placed too close to the intersection, i.e., less than 50 m. In this case, buses are either detected in the early stages of the red phase (too late for GE) or are only detected after they clear the queue of cars (too late for RT). If the detector is placed too far from the intersection, the bus delay savings remain flat while the additional car delays keep increasing, e.g., the detector location is further than 50 m from the intersection for \( q = 0.1 \text{veh/s} \) or further than 140 m from the intersection for \( q = 0.2 \text{veh/s} \). For these detectors located too far, false TSP activations occur – a bus is detected and predicted to arrive to the intersection by a certain time, however, the bus then gets delayed by a queue of cars. Hence, the activation of TSP does not save any additional delay for buses, but still increases the delay for cars travelling in the cross direction. Hence, there appears to be an ideal detector location depending on the demand level. For example, for a demand level of \( q = 0.2 \text{veh/s} \) locating the detector between 50 and 140 meters would result in large bus delay savings without unnecessarily increasing car delays.
Bus stop location and dwell duration can significantly impact the benefits expected from TSP. Firstly, the location of a bus stop would affect whether or not a bus would be impeded by a queue of cars, e.g., if a bus stop is placed far away from the intersection, a bus would never be delayed by cars while approaching the bus stop. Secondly, the dwell duration could change how long the bus will be delayed for after departing the bus stop. To better understand the interaction between these two parameters and TSP, a sensitivity test for intersections with near-side on street bus stops is considered. For the bus travel lane, a bus detector is placed 50m upstream of the intersection, i.e. $x_d = 450m$ to ensure that the detector is always downstream of the bus stop. Three possible levels of bus dwell durations, short (20 seconds), medium (40 seconds) and long (60 seconds), are considered. Notice that for the given demand level, $q = 0.3\text{veh/s}$, the maximum length of the car queue is 108 meters.

Figure 5-4: Sensitivity analysis of detector location in consideration of (a) bus travel time savings, (b) the corresponding changes in cumulative total car delay

5.3.2 Sensitivity to Bus Stop Location and Dwell Duration
Figure 5-5a exhibits the changes in bus travel time due to TSP and Figure 5-5b shows the corresponding change in car delay. For fixed bus stop locations between 50~100m, the average bus delay savings and change in car delays highly depends on the bus dwell duration. However, for bus stops placed farther than 100 meters, changes in bus travel time or change in cumulative car delay remain flat and are insensitive to bus dwell duration and bus stop location since the bus stop is located outside of the car queue which only extends to 108 meters.

Also, large bus delay savings and small increase in car delay can be observed when a bus stop is located between 60~80m with a dwell duration of 20 seconds. In these cases, a green extension becomes applicable more often – saving buses large amounts of delay while not increasing the cross direction delays by much as previously described. This also indicates that under certain combination of bus dwell duration and bus stop location, bus arrivals are very likely to be delayed without TSP, thus, TSP can help save significant amount of bus delays. For these scenarios, the added car delay due to TSP is about 4 vehicle-seconds, while the bus saves up to 8 seconds of travel time. In this case, considering that buses typically carry more people than cars, it is expected that TSP would reduce overall passenger delays.

Figure 5-5: Sensitivity analysis of bus stop location and dwell duration in consideration of (a) bus travel time savings, (b) changes in cumulative total car delay
For \( S = 40s \), as the bus stop moves from 50 m to 90 m an increase in bus delay savings corresponds to an increase in total car delay. In this case, as the bus stop moves further away from the intersection the arrival times of the bus to the intersection result in the more frequent activation of red truncation. Hence, while the bus experiences delay savings, the cross direction car delays increase significantly due to the cars that would have arrived during the green having to wait an entire red phase, as described above.

For \( S = 60s \), the bus dwell duration exceeds the duration of the red time, i.e., maximum queuing duration. The probability of a bus arrival to the intersection right after the end of the following green phase (calling for green extension) remains the same, despite variations on bus stop location. Hence, activation of a green extension is insensitive to the bus stop location. However, as the bus stop moves further from the intersection the opportunity of red truncation activation increases. Therefore, the corresponding change in overall car delay increases as bus delay saving increases, similar to the medium dwell time case discussed above.

### 5.3.3 Sensitivity to the Presence of a Downstream Bottleneck

Next, the sensitivity of the results to the presence of a downstream bottleneck that can lead to a queue spillback to the intersection of interest is tested. A queue spillback due to a downstream bottleneck can impact the TSP benefits to both buses and cars travelling in the same direction. Here, only a temporary bottleneck downstream of the intersection is considered. The start time of the temporary bottleneck is fixed at the start of the green time (i.e., at \( t = 45s \)). Earlier start times would not impact intersection operations. The impacts of different bottleneck durations (45 and 135 seconds), locations (350m, 375m, and 400m) and capacities (0.3-0.7 veh/sec) are tested. The results are compared to a baseline case where there is no downstream
bottleneck, i.e. no occurrence of queue spillback. For simplicity, for these set of tests the bus is assumed not to dwell at the bus stop.

Figures 5-6a and 5-6b show the reduction in bus travel time and increase to car delays, respectively, comparing the case where TSP is implemented to the case when TSP is not implemented. Different bottleneck locations, durations and capacities are considered. The existence of a downstream bottleneck always reduces the bus delay savings compared to the baseline case if queue spillover exists. This is due to the fact that a downstream bottleneck leads to either false TSP activation or wasted TSP due to the bus getting stuck in the downstream queue spillover. As expected, the further away from the intersection the downstream bottleneck is and the shorter the duration of the temporary bottleneck is the closer the results are to the baseline case. For large downstream bottleneck capacities there is no impact on TSP operations as expected.

On the other hand, the change in overall car delay highly depends on the bottleneck location and bottleneck capacity but is insensitive to the bottleneck duration. As the bottleneck moves further away from the intersection, a smaller increase in car delay due to TSP is observed. In other words, if the bottleneck is close to the intersection activating TSP further increases car delays since the activation of TSP allows for more cars to pass the intersection and accelerates the queue build-up and spill back process.

Overall, the results suggest that, if a downstream bottleneck or queue spillback exists, providing TSP can further hinder car operations with minimal benefit to buses. Hence TSP should be used with caution in these scenarios.
Finally, a sensitivity analysis is conducted to show how the three bus detection technologies – fixed detectors (at $x_d = 450m$), connected buses and connected cars – can improve TSP performance. These results can be useful for agencies in making decisions on what type of detection technology to use, and developing benefit/cost analyses of investing in new technology.

To do so, a demand of $q = 0.3$ veh/s and a bus dwell duration of $S = 40$ seconds is used. Similar results are obtained for other demand wells and bus dwell durations. Figures 5-7a and 5-7b show the change in bus travel time and car delay, respectively, as a function of the bus stop location using three curves representing the three bus detection technologies. As expected, bus delay savings increase as buses become connected, and increase further when all vehicles become connected.

Connected bus technology increases bus delay savings as compared to traditional detectors only for buses that arrive to the back of a queue during the red phase by activating red truncation earlier. Hence, an increase in bus delay savings is consistently observed for connected

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**Figure 5-6:** (a) change in bus travel times; (b) increased car delay.

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### 5.3.4 Sensitivity to Detection Technology

Finally, a sensitivity analysis is conducted to show how the three bus detection technologies – fixed detectors (at $x_d = 450m$), connected buses and connected cars – can improve TSP performance. These results can be useful for agencies in making decisions on what type of detection technology to use, and developing benefit/cost analyses of investing in new technology.

To do so, a demand of $q = 0.3$ veh/s and a bus dwell duration of $S = 40$ seconds is used. Similar results are obtained for other demand wells and bus dwell durations. Figures 5-7a and 5-7b show the change in bus travel time and car delay, respectively, as a function of the bus stop location using three curves representing the three bus detection technologies. As expected, bus delay savings increase as buses become connected, and increase further when all vehicles become connected.

Connected bus technology increases bus delay savings as compared to traditional detectors only for buses that arrive to the back of a queue during the red phase by activating red truncation earlier. Hence, an increase in bus delay savings is consistently observed for connected
bus technology compared to the traditional bus detector. Furthermore, connected vehicles technology doubles the bus delay savings since it can predict the car queues in advance.

The change in bus travel times remains flat when bus stop is placed farther than 80m, while either connected bus technique or traditional bus detector is implemented. When the bus stop is placed further than 80 m, red truncation cannot be triggered for prioritizing the bus but can still be triggered to clear car queues ahead of the bus. This is due to the specific combination of bus dwell time and bus stop location leading to buses only being detected during a green phase. However, if all cars are connected, since car queues are predicted in advance red truncation continues to be triggered and hence some variation in bus delay savings is still observed when the bus stop is placed 80~110m upstream of the intersection. When bus stop is placed farther than 110m > 108m, the activation of TSP becomes insensitive to bus stop location as is illustrated previously.

Figure 5-7: Comparative analysis showing influence of connected vehicle technology on TSP scheme: (a) change in bus travel time; (b) change in cumulative total car delay, due to TSP.

Figure 5-7b exhibits the corresponding changes in cumulative total car delay of all directions. Again, a positive number means additional car delay. In this figure, the changes corresponding to traditional bus detector (black solid curve) and connected bus technology (orange dashed curve) were plotted based on the left vertical axis, while the results of all-vehicle
connected environment were based on the right vertical axis. For fixed bus detector and connected bus methods of implementing TSP, the change in car delay is synchronous with the change in bus delay, i.e., if the bus delay savings decrease the car delays increase, and both curves remain relatively flat when the bus stop is placed farther than 80 meters. In addition, the maximum change in cumulative car delay is relatively small (up to 5.86 and 7.69 vehicle-second, respectively) for both methods. However, if a bus stop is placed close to the intersection the total car delay can be reduced along with the bus delay if a connected bus is used in the TSP methodology. This is because a truncated red phase implemented ahead of the bus arrival to the stop can mitigate car congestion caused by a dwelling bus simply by reducing the number of queued vehicles behind the bus. The change in overall delay is negative since the reduction of car delay in the bus travel direction outweighs the combined change in delay of all other directions.

When both bus and private vehicles are connected, the bus delay savings increase even further. However, the major difference is observed in the car delays. Depending on the bus stop location, car delays could significantly be reduced or could significantly increase. Significant delay savings occur when red truncation is activated ahead of a bus arrival to the stop, reducing the number of cars queued behind a bus. Additionally, cars can save significant delays due to the shorter red times. As the bus stop moves further, naturally a smaller number of cars queue behind the bus. Hence, the implementation of red truncation does not change the number of car queued behind a bus significantly. Therefore, the delay increase experienced by the opposite direction starts to dominate the change in car delays, resulting in an increase in total car delay.
Chapter 6

Modeling and Optimizing Transit Signal Priority Implementation along an Arterial: Moving Bottleneck Approach

Bus priority strategies are usually implemented along a corridor on which buses travel. Hence, it is necessary to estimate the overall impacts of TSP at the arterial level. This chapter extends the methodology illustrated in Chapter 5.2 to further model the impacts of TSP along an arterial. To do so, the main function in Chapter 5 shown in Table 5-1 is extended to consider multiple intersections. Twofold impacts are considered, 1) change in bus travel time, 2) change in cumulative car delay (both arterial and cross streets). The numerical analysis tests the sensitivity of TSP implementation to signal settings (cycle length and offset), bus stop location (near-side, mid-block, far-side) and dwell duration and bus headway. Next, a framework of optimizing TSP implementation is proposed and optimal placements of TSP treatments are enumerated considering bus travel time, arterial street car delay and cross street car delay. Finally, the performance of combining two common priority strategies – TSP and DBL— is analyzed.

6.1 Sensitivity Analysis

Considered in this section is a 7-intersection arterial with length $X = 1300 m$. The arterials are labeled 1 through 7, with 1 denoting the upstream most intersection from which the bus and cars enter the arterial. The number of lanes in each direction is $n_l = 2$. Denote the locations (in meters) of signalized intersections by $L = \{200, 350, 500, 650, 800, 950, 1100\}$. Traffic operation on this roadway is represented with a triangular fundamental diagram with, maximum flow rate, $q_c = 0.75 \text{veh/s}$, free-flow speed $v_f = 15 \text{m/s}$, backward speed $-w =$
−5m/s, and jam density $k_f = 0.2\text{veh/m}$. It is assumed that buses travel slower than cars with speed, $v_m = 10m/s$, creating a moving bottleneck with a maximum passing rate, $q_r$.

$$q_r = \frac{q_c}{v_f} \cdot (v_f - v_m) \cdot \frac{n_f - 1}{n_t} = 0.125\text{veh/s}.$$  

The capacity of a stationary bottleneck imposed by a bus dwelling at bus stops is $q_b$.

$$q_b = q_c \cdot \frac{n_f - 1}{n_t} = 0.375\text{veh/s}.$$  

The demands for arterial and cross streets are equal and denoted, $q' = q = 0.3\text{veh/s}$, respectively. Hence, green ratio of all intersections is, $g = \frac{q}{q' + q} = 0.5$, to fairly assign the green time between conflicting directions (i.e., cross street and arterial street). TSP detectors are placed at $x_d = 60m$. Two common TSP treatments – green extension (GE) and red truncation (RT) are considered, where a maximum change in signal timing of $\Delta t_m = 5s$ is allowed.

The numerical analysis tests the sensitivity of TSP implementation to signal setting (cycle length and offset), bus stop location (near-side, mid-block, far-side) and dwell duration, and bus headway. An analysis window equivalent to three signal cycles is chosen ($T = 800s$) where the bus always arrives during the first cycle. The expected bus travel time savings and changes in car delay are shown as an average of all possible uniformly distributed bus arrival times across the first cycle at Intersection-1.

The change in bus delay is determined considering the difference in bus travel time along the arterial when TSP is activated at intersections (when necessary) and there is no TSP activation. Notice that TSP is activated at an intersection only if a bus is detected and it would save delay by TSP activation.

The change in arterial car delays is computed at the downstream end of the last intersection along the arterial. The car delay is a summation of delays experienced by cars traveling both in the same and opposite direction as the bus. Additionally, the cross street car delay is computed at an individual intersection basis considering the signal timing plan and the
total cross street delay is calculated as a summation of the 7 independent calculations. The changes in arterial and cross street delays are calculated by analyzing the arterial with and without TSP and taking the difference of the delay values obtained from the two analyses.

Note that the changes in arterial and cross streets car delays are not comparable because they are measured in different ways, i.e., the arterial calculations are done only once for every car traveling along the arterial, whereas the cross street calculations are done for different cars at every intersection.

6.1.1 Sensitivity to Signal Setting

Both the impacts of cycle length and offset are considered in this section.

Cycle Length

This test assumes a common cycle length and green ratio for all intersections. In this experiment, 1) buses do not stop at any bus stops, and 2) all 7 intersections are equipped with TSP capability. Note that the presence of bus stops would only shift the bus arrival times, and change the average speed of bus travel time and hence we would still expect similar results to hold. Different common cycle lengths, \( C = \{50, 60, 70, 80, 90, 100, 110, 120\} \) are considered, assuming all intersections at arterial street turn green simultaneously, i.e., zero offset. First, it is assumed that only red truncation (RT) is implemented among all intersections. Second, both RT and GE are considered.
The changes in bus travel times, arterial and cross streets car delays, under different common cycle lengths due to TSP, are exhibited in Figure 6-1a, c, and d, respectively. Notice

**Figure 6-1:** Sensitivity tests of cycle length ($C = \{50, 60, 70, 80, 90, 100, 110, 120\}, g = 0.5$): a) reduced bus travel times; b) ratio of RT activation to GE; c) change in arterial street car delay; d) change in cross street car delay; e) change in bus travel time per activation of single TSP treatment.

The changes in bus travel times, arterial and cross streets car delays, under different common cycle lengths due to TSP, are exhibited in Figure 6-1 a, c, and d, respectively. Notice
that in these figures positive values indicate benefits. The change in bus travel time is shown as a bus delay savings, and hence larger numbers indicate better outcome of TSP. For cumulative car delays, positive value indicates benefit to cars (i.e., reduction in car delay), and negative value indicates detriment to cars (i.e., increased car delay). The ratio of RT to GE activations is also recorded in Figure 6-1b when both RT and GE were implemented. As can be seen, impacts of TSP varies among different cycle lengths, and there is an obvious relationship between TSP impacts and cycle length. The twofold impacts on bus and car are summarized as follows:

1) **Bus travel times.** On average, buses benefit most (i.e., 45 seconds) when common cycle length is 60 seconds. The corresponding standard deviation (which is 7.86 seconds) is the lowest among all, indicating that the bus will always benefit from TSP as it travels along the arterial. Moreover, the ratio between RT and GE activation also matters. Reader can easily verify that when cycle length increases from 60 seconds to 70 seconds, the RT to GE ratio increases while the bus delay savings decrease. Another finding is that bus delay savings when GE is implemented alone is minimal. This is due to the fact that GE is not activated frequently along the corridor, and the benefits of receiving GE are mostly diminished since buses cannot receive RT at the downstream traffic signal, see Figure 6-1e.

2) **Car delay.** The reduction in arterial car delay is generally positively correlated with reduction in bus travel times, despite some exceptions. This is because cars that travel parallel with the bus also benefit from TSP. As for cross streets, increment in car delay generally increases as common cycle length increases, the highest and lowest increments occur when common cycle lengths are set to be 60 and 120 seconds, respectively. An interesting finding is that a common cycle length of 60 seconds TSP provides most benefit to bus and less detriment to cars.
Considering arterial cars, readers can find that there is a minor gap between RT&GE and RT only scenarios, indicating GE only reduces car travel times minimally. First, only a minor proportion of cars benefits from GE, i.e., those cars travelling to the intersection at the time when there is a bus expected to arrive at an intersection $d_{t_m} = 5s$ before the beginning of a red phase. Second, for those cars benefit from GE, they are likely to be delayed at downstream intersections. As a result, the gap in savings to arterial street cars is minimal.

**Offset**

In this experiment, three different offsets: zero offset; green wave offset, the offset time between two adjacent intersections is calculated as block length divided by free-flow speed, $v_f$; bus favorable offset, the offset time between two adjacent intersections is calculated as block length divided by bus maximum speed, $v_m$. Also, short (60s) and long (120s) common cycle lengths are considered. Hence, there are a total of 6 scenarios as illustrated in Table 6-1. First, buses are assumed not to dwell at any bus stops. Second, to account for bus dwelling effects, buses are assumed to dwell at mid-block bus stops placed at $x_s = \{260, 550, 850, 1150\}m$, for duration $S = 30s$ at each stop.

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>zero offset: $O = [0,0,0,0,0,0]$; common cycle length: 60s</td>
</tr>
<tr>
<td>2</td>
<td>green wave offset: $O = [10,20,30,40,50,60]$; common cycle length: 60s</td>
</tr>
<tr>
<td>3</td>
<td>bus favorable offset: $O = [15,30,45,60,75,90]$; common cycle length: 60s</td>
</tr>
<tr>
<td>4</td>
<td>zero offset: $O = [0,0,0,0,0,0]$; common cycle length: 120s</td>
</tr>
<tr>
<td>5</td>
<td>green wave offset: $O = [10,20,30,40,50,60]$; common cycle length: 120s</td>
</tr>
<tr>
<td>6</td>
<td>bus favorable offset: $O = [15,30,45,60,75,90]$; common cycle length: 120s</td>
</tr>
</tbody>
</table>

The changes in bus travel times, arterial and cross streets car delays, under different common cycle lengths due to TSP, are exhibited in Figure 6-2 a, b, and c, respectively. As can be seen, the need for TSP activation decreases as the offset becomes more favorable to bus
movement. For zero offset, both buses and cars traveling along the arterial experience frequent stops due to poor signal coordination. Hence, an activation of TSP brings significant benefits to buses and arterial street cars. Correspondingly, there is a considerable increased cross street cumulative car delay. For green wave offset, most cars experience only one stop, buses experience only small delays due to queues of cars, hence, the need for TSP activation decreases. For bus favorable offset, since most buses would travel in green at intersections, there is relatively rare activation of TSP compared to the previous two types of offset. Correspondingly, the changes in arterial and cross street cumulative car delay is smallest. The above results are also observed for different common cycle length.

With the presence of bus stops, for zero offset, the dwelling bus would starve car arrivals to downstream locations of bus stops, hence, bus would experience less delay at downstream locations, requiring less activation of TSP. Consequently, there is a decreased change in both bus travel time and cumulative car delay (both arterial and cross streets). For green wave offsets the presence of dwelling bus breaks the progression green band for free-flow travelling cars (since it creates a stationary bottleneck). This would impose queue of cars at downstream intersections, hence, buses would require increased activation of TSP. For bus favorable offset, bus would travel to a red light after dwelling at bus stops. In both cases, the changes in bus travel time and cumulative car delay (both arterial and cross streets) increases compared to the no bus stop scenarios.
Change in bus travel time (s)

Case (a)
- no bus stop
- mid-block stop

Change in arterial street car delay (veh-s)

Case (b)
- no bus stop
- mid-block stop
Bus stop placement directly impacts the convenience and accessibility of the transit system. Depending on the distance from an adjacent intersection, there are three types of bus stops, near-side, mid-block and far-side. Near-side bus stops are located immediately before an intersection, allowing for passenger unloading and loading while bus stop at a red light. Mid-block stops are located between intersections, which are less congested locations than intersection stop locations. Far-side bus stops are located immediately after an intersection. As is illustrated in the previous chapter, bus stop location and dwell duration can impact the benefits expected from TSP.

In this experiment, bus stop locations [60,140]m upstream (or [10,90]m downstream) of intersections are considered. Both short ($S = 30s$) and long ($S = 60s$) dwell duration are considered, assuming a zero offset, common cycle length of $C = 30s$ and green ratio, $g = 0.5$,
respectively. The changes in bus travel times, arterial street cumulative car delay, cross street cumulative car delay are shown in Figure 6-3 a-c.

For near-side bus stop locations (i.e., [60,90]m upstream of an intersection), the time when a bus could access to the bus stop depends on the length of the queue of cars at a red signal. TSP can be extremely helpful for some combinations of near-side bus stop placement and dwell duration. For example, if the bus stop is placed at 60m upstream of an intersection, buses would get stuck in a queue of cars during the red phase first and then arrive at the bus stop mostly during green times. And if the dwell duration is about 30 seconds (length of green phase), buses would very likely depart from the bus stop during the red phase, requiring frequent activation of TSP. Hence, the activation of TSP is sensitive to the placement of near-side bus stops and bus dwell duration. As bus stops are placed further away from the intersection, the activation rate of TSP becomes stable. Hence, the change in bus travel times and cross street car delay is almost insensitive to mid-block and far-side stops, see Figure 5-3a and c.
For arterial street cars, the TSP benefit to cars decreases as the bus stop moves from near-side to far-side locations. In this case, a dwelling bus at downstream locations causes queue spillback. This bottleneck effect diminishes the benefits of red truncation to cars, since the
vehicles following the bus would be delayed at the queue at bus stop locations. Moreover, if bused dwell longer at a bus stop, the benefits diminishes even more, see Figure 6-3b.

In urban areas, passengers do not arrive to a bus stop in deterministic fashion and hence the bus dwell times at bus stops could have some randomness. To account for this randomness, tests where the bus dwell time is assumed to follow a normal distribution, \( S \sim \mathcal{N}(30, 10^2) \), at each bus stop are conducted. In other words, the bus dwell times are drawn from this normal distribution at each bus stop, and the experiment is repeated for 30 times using different random seeds to obtain an average result. The rectangle markers and error bars in Figure 6-3 a to c exhibit the means and standard deviations of the experiment results. From these figures it can be seen that even though the impacts of TSP on cars and buses are sensitive to the dwell times, the randomness in the dwell times do not change the trends. Moreover, as the bus stop is located further from the intersection, the results with and without randomness in the dwell time start to converge.

6.1.3 Sensitivity to Bus Headway

The bus headway is defined as the periods between two consecutive buses that enter an arterial at intersection - 1. As bus headway increases, the number of buses simultaneously travelling within the arterial decreases, hence, the TSP activation frequency decreases. Hence, bus headway can also impact the benefits expected from TSP.

The same arterial with common cycle length of \( C = 60s \), green ratio of \( g = 0.5 \), and zero offset are considered. Intersections are located at \( L = \{200,350,500,650,800,950,1100\} \) meters. Bus stops are placed at \( x_s = \{260,550,850,1150\} \) meters and bus would dwell for \( S = 30s \) at each stop. Bus headways \( h = \{60,120,180,240,300\} \) are considered. Since the analysis time window is \( T = 800s \), there are at most \( \{5,3,2,2,1\} \) buses travel simultaneously within the arterial
given headways $h$. The buses are labelled by bus-1,2 ... according to buses' arrival order at the arterial.

Figure 6-4a shows the total travel time savings among all buses and per bus, respectively. As can be seen, total bus delay savings decreases as bus headway increases. Correspondingly, as
bus headway increases, the increased arterial street cumulative car delay decreases, while the reduction in cross street car delay increases, see Figure 6-4b. This is number of bus and TSP activation decreases as bus headway increases.

Discussed here are two consecutive bus arrivals, bus -1 and -2. For bus - 1, TSP is activated frequently since bused could be stopped behind in a queue of cars at the red signal. Whenever TSP is activated, either RT or GE, there is more green time for cars, hence, the residual queue at intersection is shortened. Hence, the probability of the following bus being impeded by a queue of cars decreases, thus, the need for TSP activation decreases for bus - 2. Therefore, the marginal benefit per bus increases as bus headway increases or bus frequency decreases, see Figure 6-4a: change in bus travel time (per bus).

6.2 Optimal TSP implementation along a Corridor

In this section, an optimization framework is firstly proposed to find the optimal TSP deployment along a corridor. Secondly, numerical results of a series of TSP implementation cases are enumerated and analyzed.

6.2.1 TSP Implementation Optimization

The goal is to find the optimal combinations of intersections that should be equipped with TSP (i.e., have TSP capability). The objective function incorporates the benefits: 1) weighted bus delay savings, 2) the weighted change in arterial street car delay; and the costs: 3) the weighted increase in cross street car delay, 4) TSP implementation costs, \( C_T \). The weights \( w_1, w_2, w_3 \) should be given based on engineering calibration. The decision variable is an array \( \tau \) with
elements $\tau_i = 1$ to indicate TSP being implemented at intersection - $i$, and otherwise, $\tau_i = 0$. The TSP implementation cost is generally proportional to number of intersections equipped with TSP, hence, $C_T = g(\sum_{i \in I} \tau_i)$, where $g(\cdot)$ is a linear function. All intersection parameters, bus parameters, fundamental diagram (FD) parameters can be calibrated through field experiment, and the computation parameters are given (see illustration of the parameters in Table 4-1). Given this, the problem can be formed as a bi-level optimization program: 1) the lower-level program minimizes the vehicle count matrix $N$ to dynamically propagate bus movement and TSP activation, and to finally compute the change in bus travel times and car delays (both arterial and cross streets), i.e., the DP algorithm in Table 4-1; 2) the upper level maximizes the weighted summation of benefits (positive value) and costs (negative value) considering different combinations of locations for TSP implementation.

Denote $T_B, T_C, T_C'$ the changes in bus travel time, arterial street cumulative car delay, and cross street car delay, respectively. Denote $In$ the list of given intersection, bus, FD, computation parameters as described in Table 4-1. Denote $\tau_{max}$ the maximum number of intersections equipped with TSP. The DP algorithm is represented by function $f(\cdot)$ with inputs of $\tau$ and $In$. The upper level of the bi-level optimization problem can be expressed as an integer program as follows,

$$\min \quad F = \sum w_1 T_B + w_2 T_B + w_3 T_C' - C_T$$

Subject to:

$$[T_B, T_C, T_C'] = f(\tau, In)$$

$$C_T = g(\sum_{i \in I} \tau_i)$$

$$\sum \tau_i \leq \tau_{max}$$

The objective function can be further modified to account for passenger waiting time at bus stops if average passenger arrival rate at a bus stop information is available. This bi-level
optimization problem can be solved by mixed-integer programming algorithm or some heuristic algorithm such as genetic algorithm.

6.2.2 Numerical Analysis

Since the weights and cost of TSP implementation are unknown, the bi-level optimization problem cannot be solved without making further assumptions. However, some optimal solutions (not necessarily optimum) can be found by enumerating all possible combination of TSP placements. Consider the same two lane 7-intersection arterial with common cycle length of $c = 60s$, green ratio of $g = 0.5$, and zero offset are considered. Intersections are located at $L = \{200, 350, 500, 650, 800, 950, 1100\}$ meters. Again, buses are assumed to dwell at mid-block bus stops placed at $x_s = \{260, 550, 850, 1150\} m$, for duration $S = 30s$ at each stop. It is assumed that there is no TSP implemented at the entrance and exit intersections (i.e., intersection-1 and -7), hence, $\tau_{max} = 5$.

Only one bus is included in this analysis. Bus arrival times are uniformly distributed across a cycle at Intersection-1. The results are shown as an average of all possible bus arrival times. All parameters are the same as the previous section: $q_c = 0.75veh/s$, free-flow speed $v_f = 15m/s$, backward speed $-w = -5m/s$, and jam density $k_j = 0.2veh/m$, bus maximum speed, $v_m = 10m/s$, maximum passing rate $q_r = 0.125veh/s$, $q_b = 0.375veh/s$, the demands for arterial and cross streets are, $q' = q = 0.3veh/s$, respectively.
Figure 6-5 shows the change in cross street car delay, $T_C^*$ versus bus delay savings, $T_B$, across all intersections. In this figure, the data points corresponding to different number of intersections where TSP is implemented are shown with different color and shape markers. As can be seen, change in cross street car delay is nearly linearly correlated with bus delay savings.
Looking at the results, a 1 second delay saving for buses would impose about 7 veh \cdot s increment in cross street car delay. This would correspond to the system wide (i.e., bus plus car) delay decreasing if a bus carries more than 7 times the number of passengers of a car. Since this is the case for most bus systems, TSP activation can improve the system as a whole. However, increasing the number of TSP implementation does not always improve operations. For example, in this experiment, both implementations at intersection−2,3,5,6 and −2,3,4,5,6 have the same impacts, indicating there is no need to deploy TSP at intersection − 4.

Figure 6-5b shows the change in arterial street car delay, $T_C$ versus bus delay savings, $T_B$, across all intersections. The placement of TSP with lower and upper bounds in bus delay savings of each group are labelled by intersection IDs, for example, intersection – 2&4 and – 2&5 are the lower and upper bounds if TSP is simultaneously deployed at two intersections, respectively. This shows that in general if TSP is implemented at more intersections both the bus and car delays can be reduced. However, the results are highly dependent on the location of TSP implementation, since the impacts of TSP vary in a wide range within each group with the same number of TSP implementations. These results also confirm that implementing TSP at some specific intersections can provide the largest benefits, for example, intersection−5 in this experiment since this intersection appears in most of the best TSP implementation combinations.

### 6.3 Integrated Bus Priority

In urban streets, a combination of bus priority strategies, such as TSP and DBL, could be extremely helpful when there is a high demand for bus transit. This section further explores the joint impacts of TSP and DBL on both buses and cars. Consider the same 7-intersection arterial with common cycle length of $c = 60s$, green ratio of $g = 0.5$, and zero offset are considered. Intersections are located at $L = \{200,350,500,650,800,950,1100\}$ meters. Again, buses are
assumed to dwell at mid-block bus stops placed at $x_s = \{260,550,850,1150\}m$, for duration $S = 30s$ at each stop. The number of lanes in each direction is $n_1 = 3$.

Only one bus is included in this analysis. Bus arrival times are uniformly distributed across a cycle at Intersection-1. The results are shown as an average of all possible bus arrival times. All parameters are the same as the previous section: $q_C = 1.125veh/s$, free-flow speed $v_f = 15m/s$, backward speed $-w = -5m/s$, and jam density $k_j = 0.3veh/m$, bus maximum speed, $v_m = 10m/s$, maximum passing rate $q_r = 0.1875veh/s$, $q_b = 0.5625veh/s$, the demands for arterial and cross streets are, $q' = q = 0.3veh/s$, respectively. It is assumed that DBL implementation takes away one travel lane from cars, correspondingly, the variables $q_C, k_j, q_r, q_b$ change to be two thirds of their original values. Since bus movement is affected by signal setting, a series of $C = \{50,60,70,80,90,100,110,120\}$ are also considered.
Figure 6-6a exhibits the change in bus travel time under different cycle lengths.

Surprisingly, when implemented independently the delay savings for DBL only or TSP only are
very similar to each other. Additionally, by implementing TSP together with DBL, bus delay savings can be significantly increased, and more than doubles in certain instances. Hence, the interaction of TSP and DBL can create a synergistic system which significantly benefits the bus.

Figure 6-6b exhibits the corresponding change in arterial street car delay. As can be seen, the activation of TSP can reduce delay of cars travelling along the arterial, however, the implementation of DBL increases arterial car delays. Hence, it might be not worthy to implemented DBL when bus frequency is low. Even if TSP is implemented together with DBL, the car delays are still increased significantly.

Figure 6-6c exhibits the change in cross street car delay. The implementation of DBL imposes no change in signal timing, and hence no change in cross street car travel time. When TSP and DBL are implemented together, because TSP is triggered more often the cross street delays are reduced even more than an isolated implementation of TSP.
Chapter 7

Conclusion and Future Work

7.1 Conclusions

This dissertation furnishes a set of analytical models of TSP impacts on buses and cars. To begin, this dissertation considers a case when TSP is implemented at an isolated intersection with a nearby bus stop. Two traffic conditions are considered, oversaturated and undersaturated. For oversaturated conditions, the numerical results are expressed in changes to car throughput during one signal cycle and changes in bus delay when TSP is applied. The numerical results show that the implementation of TSP reduces bus delays under most combinations of bus stop location and dwell time but generally decreases the maximum car throughput. For undersaturated conditions, the results are expressed in changes in car and bus delays. The numerical results show the trade-off between bus delay savings and increased car delays. The bus delay savings and additional car delays experienced are very sensitive to the location of the bus stop and the average dwell time.

Next, the impacts of TSP implementation along a corridor were analyzed using analytical MFD and VT. Implementation of TSP leads to changes in signal timings imposing changes to the shape of the MFD. Case studies indicate that the impact of TSP on arterial capacity varies among different signal settings. The impacts are typically dominated by bus frequency. In general, as bus headway decreases, a larger increase in arterial capacity is observed. However, these impacts are usually minimal, especially for a homogeneous street. A scheme for determining optimal locations of TSP implementation was then examined. The proposed problem uses a binary decision variable indicating whether or not TSP is equipped at an intersection, and multiple
objective functions are considered including cross-street car throughput loss and bus delay savings. The results are furnished as the change in cross-street throughput as compared to the bus delay savings for TSP implemented at different numbers of intersections. The final Pareto graph displays the optimal TSP locations, considering both buses and cars. This Pareto graph shows that investing so that more intersections have TSP capabilities will not always result in the largest bus delay savings. Another observation is that the marginal effect of TSP decreases as more intersections are equipped with TSP capabilities. As a final step, a necessity at arterial level analysis shows that considering an intersection in an isolated fashion can result in misleading conclusions regarding the benefits of TSP implementation.

To continue, this dissertation then relaxes the assumption of fixed bus speed and considers a mixed traffic condition where buses and cars are interacting with each other. This is more challenging since it requires an accurate prediction of car densities within a given \((t, x)\) plane. A dynamic programming (DP) algorithm is developed based on the Lax-Hopf equation (Aubin et al., 2008), which computes vehicle counts and tracks bus motion in the time-space plane with and without TSP. Changes in bus travel times and cumulative car delay are both quantified. Further numerical testing demonstrates the capabilities of the algorithm to evaluate TSP performance using existing bus detection methods, along with utilizing information from connected buses or connected vehicles. Bus stop and dwell duration sensitivity testing favors deployment of TSP with the given demand and signal setting. Both changes in bus travel times and total car delay becomes insensitive to bus stop location when bus is placed further upstream than the maximum car queue length. Detector location testing reveals that there exists an optimal bus detector location associated with each demand level. Sensitivity to detection technology validates that if all vehicles are connected, the most benefits to both buses and cars would be observed for bus stop locations close to the intersection. On the other hand, traditional bus detector and connected bus techniques reduce bus travel times at the expense of total car delay.
The computational efficiency of this methodology lends itself to extending the study to the arterial level, which is being performed as future work by the author.

Finally, the DP algorithm is extended to further model the impacts of TSP along an arterial. Two impacts are considered, 1) a change in bus travel time, 2) a change in cumulative car delay (both arterial and cross streets). The numerical analysis tests the sensitivity of TSP implementation to signal setting (cycle length and offset), bus stop location (near-side, mid-block, far-sider) and dwell duration, and bus headway. First, TSP impacts vary among different cycle lengths, which is mainly caused by variation on the ratio between GE and RT activations. Second, the need for TSP decreases significantly as offset becomes more favorable to buses and cars. However, the presence of a bus stop could diminish the effects of offset increasing the necessity of TSP implementation. A framework of optimizing TSP implementation is proposed and optimal placements of TSP treatments are enumerated considering bus travel time and arterial street car delay, and bus travel time and cross street car delay, respectively. The results reveal that there are large variations on TSP performance across each group of number of intersections to be equipped with TSP. Finally, the joint performance of two common priority strategies – TSP and DBL – was evaluated. The results favor the implementation of integrated TSP and DBL strategy.

Demonstrated herein are two commonly used TSP treatments, green extension and red truncation; however, the analytical methodology can be easily extended to investigated other TSP treatments such as green reallocation (Hu et al., 2014) and combined priority, such as TSP integrated bus dedicated lane (Truong et al., 2017).

### 7.2 Future Work

To continue, the existing work can be extended to account for more research questions. First, it is favorable to implement TSP in most scenarios according to this dissertation. However,
the findings cannot be guaranteed if considering TSP signal timing recovery. And it is also meaningful to consider how bus priority can affect pedestrians and other non-motorists. Second, the dissertation focuses on bus priority impacts at isolated intersection and arterials levels, however, the research findings are questionable when considering network-wide impacts.

### 7.2.1 Bus Transit Priority in Practice

First, the focus of this dissertation is on the application of analytical method to model and evaluating bus priority implementation (mainly transit signal priority). More realistic TSP implementation issue should be further considered, such as signal timing recovery after TSP activation. This is the easiest next step research since it only requires adjustments on post-TSP signal timings.

Second, it is meaningful to compare our study with some empirical or field studies. For example, a previous public transit study shows that the combination of TSP and signal optimization reduced transit signal delay about 40% in two corridors, Tacoma, Washington (Smith et al., 2005). It could be interesting to evaluate the impacts using the analytical method in this dissertation and compare with the findings in the report. In addition, to author’s knowledge, there is no study discusses the impacts of TSP on non-motorists. For example, the change in signal timing could affect the pedestrian and bicyclist signal phases, hence, it impacts their arrival rate to bus stops and access to bus.

### 7.2.2 Bus Priority Transit at Network-level

Through my previous and ongoing studies, analytical models provided a good understanding on influence of TSP at the intersection and arterial levels. However, it is really
challenging to extend the analytical method to network level. First, the problem size increases significantly from corridor to network level, which imposes more efforts on analytical modeling of traffic flow. Second, in urban network, there are usually multiple bus routes. It can be possible the multiple buses compete for priority at one single traffic signal, hence, it requires more design on TSP activation scheme.

Despite these challenges, it is still possible to expand the analytical model to network-level analysis. For example, the macroscopic fundamental diagram (MFD) approach has been validated to model traffic flow dynamics at network level (Daganzo and Geroliminis, 2008; Geroliminis and Daganzo, 2008; Leclercq and Geroliminis, 2013).

In addition, it is also recommended to use the analytical method to calibrate micro-simulation packages under simple cases, and then use the simulator to network-level study. Some simulation tools can be used for this, for example, Aimsun and VISSIM (Girault et al., 2016; Truong et al., 2017).
REFERENCES


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