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Abstract

This dissertation consists of four chapters. The first two chapters examine the roles of crowdfunding platform and regulator in designing information transmission rules from innovators/entrepreneurs to investors, on crowdfunding markets. An important function of a crowdfunding platform is to reduce the asymmetric information between entrepreneurs and investors. In the first chapter, I ask whether the the platform can be trusted this role. Investors believe the platform only to the extent that it's incentives align with their own. Using data from Kickstarter, a leading crowdfunding platform, I develop a statistical test confirming that platform's incentives undermine credibility of its signals, propose regulations that would curb those incentives, and quantify their welfare consequences. These regulations allow the platform to commit to an information disclosure rule and lead to Pareto improvements. Finally, I show that the platform's reputation could substitute for commitment and provide a rationale for why this does not occur in the real world.

In the second chapter, I study the tradeoff between innovation and investor protection on crowdfunding platforms. Informing investors about the potential risks of a given investment opportunity protects them from failure, but comes at the cost of dissuading innovation. I show that a regulator, who values investor protection, may find it optimal to choose disclosure requirements that are not fully informative about projects. Partial disclosure enables investors to commit to sometimes fund bad projects encouraging further innovation. I provide necessary and sufficient conditions, under which a profit motivated platform would set investor-optimal disclosure requirements. Optimal dynamic regulatory experiment also is studied.

The third chapter constructs an electoral competition model in which political candidates can choose to microtarget voters with campaign messages. Microtargeting refers to collect-

ing information on voters' characteristics and using it to approach them with personalized messages. I study implications of microtargeting campaigns for voter turnout and political awareness. I provide conditions on a class of campaigning technologies, under which better informed voters are more likely to vote, but higher political awareness of an electorate (larger size of informed electorate) does not necessarily lead to a higher voter turnout. These conditions can be interpreted as requiring either a sufficiently high political awareness in the electorate, before the campaigning starts, or a low level of elite polarization. To address the debate on the importance of understanding why people vote, I consider two models of voter's behavior: instrumental and expressive voting. I identify conditions, under which the candidates' equilibrium campaigning behavior is the same across the two models.

The last chapter constitutes a normative study of society's role to discipline public officials whose interests may be misaligned with society's interests. Completion of large-scale public projects require effort from a sequence of office-motivated public officials. Since the effort exerted by each official is often difficult to verify, it is hard to say who is responsible for a project's outcome. How should a society discipline public officials in such situations? In a dynamic citizen-candidate model with multi-period projects and persistent agency friction, I characterize the unique public official firing rule that achieves the first best, for the highest possible misalignment of interests between society and appointed officials, is Markovian and trembling-hand perfect. The rule completely ignores the midterm project signals and conditions punishment (firing) only on the end of the project outcomes. Under this rule, it is always the case that only one official is in charge of a given project from its commencement to the completion. This eliminates the need for the society to figure out who is responsible for a given project's outcome.

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Chapter 1

Commitment vs. Flexibility in Information Disclosure: the Case of Kickstarter

1.1 Introduction

On a reward-based crowdfunding platform, entrepreneurs presell their products to buyers (small investors).[1] A key characteristic of this market is asymmetric information, between entrepreneurs and investors, about product quality. Moreover, small investors have insufficient incentives to screen products individually. These create a need for a crowdfunding platform to serve as an information intermediary - facilitating information transmission from entrepreneurs to investors. In this chapter, I study this role for the largest reward-based crowdfunding platform in the USA - Kickstarter.com. More specifically, I ask whether the crowdfunding platform can serve as a credible information intermediary; try to understand the content of platform's incentives that would endanger its credibility; and study mechanisms that would discipline such incentives.

I start with two assumptions: a crowdfunding platform is better informed about en-

[1] More specifically, reward-based crowdfunding involves funding of innovations/new products, where a large number of small investors contribute small amounts of money in return for non-financial rewards (often the product itself).

trepreneurs' products than individual investors are, and the platform is profit-motivated. These vividly suggest that if the platform tried to transmit information about products to investors, then it would have incentives to transmit only the information that would help it earn higher profits. Thus, there should be either some exogenous mechanism (e.g. regulation) or endogenous incentives (e.g. platform's long-run reputation concerns) that would discipline the incentives of the platform to misrepresent information, as otherwise information transmitted would not be credible.

To see more clearly what the incentives of a platform to misrepresent information are, consider a platform that publicly announces an information disclosure rule before learning the actual quality of a product. An information disclosure rule is a joint distribution over signals and product quality. Consequently, the platform privately observes a draw of quality and signal from that distribution. At this point, the platform might be tempted to misrepresent the realized signal, as some other signal, if it expects higher revenues under this alternative signal. If one somehow ties the platform's hands to manipulate with the actually realized signals, then I say that the platform can commit to information disclosure.

The first set of my results concerns the welfare effects of platform's (Kickstarter's) incentives to misrepresent information. I develop a statistical test and confirm that those incentives constrain the platform from achieving better outcomes for itself (i.e. incentive compatibility constraints, ensuring that platform transmits credible information to investors, bind), implying that it does not have commitment power. Moreover, the welfare effects of granting the platform commitment power (e.g. through regulation) are positive for entrepreneurs and investors.

My second set of results concerns the role of public monitoring for resolving the platform's incentive issues. Public monitoring is the ability of public to figure out a product's quality on its own. I estimate that if public monitoring were sufficiently good, then the platform's incentives to misrepresent information would be curbed and commitment outcome would be implemented. Hence, public monitoring could substitute for commitment.

Formally, I study an interaction between a long-lived crowdfunding platform (Kickstarter in the data, henceforth, "it"), and a sequence of short-lived project creators (entrepreneurs, henceforth, "he") and investors (henceforth, "she"). At the beginning of each period, a creator may arrive at the platform. If so, he decides whether to post his request for funding or not. A creator is characterized by his project's observable characteristics and privately known quality. If a project is posted, the platform learns the quality and can send a cheap-talk message to the investors, who form expectations about the quality and make decisions whether to invest or not. The platform maximizes expected discounted revenues under the All or Nothing (AON) funding mechanism.[2] Creator cares about the probability with which he gets funded and incurs a cost if posts his request for funding. When deciding whether to invest or not, an investor cares about project's quality and incurs an investment cost (e.g. opportunity cost of money). The intensive margin of investor's decision is modeled in a reduced form.

The model has many equilibria. To deal with this, I assume that at the beginning of period 0 the platform announces a stationary rule according to which it promises to send signals about projects' qualities (information transmission/disclosure rule). This rule is a joint distribution over project's quality and signals. However, since the platform is profit-motivated it wants to induce as many successfully funded projects as possible and has an incentive to send signals that induce better beliefs about project's quality (better beliefs in the sense of higher posterior expected quality). If there is nothing that would discipline such incentives, then the platform would not be able to follow its promised information transmission rule - negating its role as an effective information intermediary.

There are at least two ways to mitigate platform's incentives to misrepresent signals. The first is an exogenous mechanism that forces the platform to follow its promised rule. One such mechanism could be a regulator that makes sure that the platform sends signals according to the rule that was announced. The second way is via the platform's long-run incentives i.e. if it violates the rule then there will be consequences in the future. For example, public may detect that the platform misrepresented a signal and in such a case the platform would lose credibility - leading to lower future profits. Therefore, public monitoring may be another way to discipline the platform's incentives to misrepresent information.

[2]Under the AON funding mechanism a project creator gets funded only if the total amount of investment raised is greater than or equal to the goal amount announced by the creator at the beginning of starting the fundraising campaign. In addition, that amount has to be raised during the time-frame that is also set by the creator. Otherwise, investors take back whatever they pledged. The platform takes a fixed share of the invested amount from the successfully funded projects.

The data suggest that investors respond to Kickstarter's signals meaning that Kickstarter's incentives are somehow disciplined. To check whether incentives constrain the platform from achieving the commitment outcome, I develop a statistical test. The logic of the test is as follows. I prove that one can identify and estimate all the parameters of the model, that are necessary to evaluate platform's discounted revenues, without using the platform's optimization problem. If there was commitment, then the platform would be maximizing its revenues without any incentive constraints. This would imply that the estimated transmission rule is a global maximizer of platform's revenues. Under no commitment, there would be incentive constraints and some constraints might bind. This would imply that if we considered small perturbations of the estimated transmission rule, we might have increased revenues. The test poses under the null hypothesis that small perturbations of the estimated rule do not increase revenues (there is commitment). The test rejects the null.

Once I know that the platform does not have commitment power, I estimate welfare effects of granting it this power. It turns out that creators, investors and the platform would benefit. The platform's revenues would increase by 7 percent, the creators' welfare by 4 percent, and the investors' welfare by 0.5 percent. That commitment is Pareto improving is not clear before estimating the model, as there are parameter values for which this would not be the case.

To obtain the second set of my results, I solve the platform's dynamic problem. I define public monitoring technology to be the probability with which the public verifies (learns) project's quality independently across projects and over time. I allow this probability to vary with the information transmission rule that the platform promises. Using the platform's optimality conditions, I recover the public monitoring technology to be such that on average less than 1.2 months is required for the public to verify a project's quality. In addition, the model rationalizes that the public monitoring technology under the commitment information transmission rule would be such that on average more than 1.5 months would be required for the public to verify a project's quality. In addition, if one would somehow improve the public monitoring technology, so that less than 1.5 months were required for the public to verify a project's quality under the counterfactual commitment world, then public monitoring would substitute for commitment. In practice, creating a forum for the investors to exchange their experience with the previous projects would be one way of improving public monitoring technology.

The main message of this chapter is as follows. Granting the platform commitment power would be Pareto improving. A regulator could accomplish this by verifying that the platform follows a promised information disclosure rule. If such a regulation is too costly, then the platform could try to improve public monitoring and achieve the commitment outcome.

Related Literature. The literature on reward-based crowdfunding has focused on studying the funding mechanisms on the platforms. One strand of the literature has argued for the ability of crowdfunding platforms to facilitate the learning of consumer-demand (Strausz 2016, Ellman & Hurkens 2015). Entrepreneur can learn demand by observing the outcome of his listing on the platform and the platform can affect the learning by its choice of a funding mechanism. Another strand of the literature has focused on social learning on crowdfunding platforms (Kim, Newberry & Qiu 2018, Kuppswamay and Bayus 2018, Marwell 2015). Investors learn about some payoff relevant state by observing previous history of number of project backers (investors who have already invested) and pledged amount. Unlike this work, the papers cited above either abstract away from the privately informed entrepreneur or model it in a reduced way. This work tries to fill in this gap by explicitly modeling asymmetric information and studying a platform's potential for serving as an information intermediary.

It is believed that financial intermediaries can deal with information asymmetry between lenders and borrowers more effectively compared to individual lenders, and their role as information intermediaries has been well studied (Diamond 1984, Hirschleifer & Riley 1979, Leland & Pyle 1977). The basic idea is that there is economies of scale in screening borrowers and if a financial intermediary has incentives sufficiently aligned with lenders then it can serve as an information intermediary. Similarly, reward-based crowdfunding platforms involve small investors who are not well incentivized to individually screen projects suggesting that there might be a role for a platform to serve as an information intermediary. This work tests this conjecture.

The structural model borrows its structure and assumptions from the literature on

Bayesian Persuasion (Kamenica & Gentzkow 2011, Rayo & Segal 2010, Tamura 2016) and dynamic cheap talk (Best & Quigley 2017, Margaria & Smolin 2015). Most closely related to my model is Best & Quigley 2017. They study a repeated cheap-talk model with public monitoring in which a long-run sender tries to persuade short-run receivers to take certain actions and the state of the world is i.i.d. over time. They ask whether the sender's long-run incentives can substitute for full commitment a la Kamenica & Gentzkow 2011, in terms of sender's payoffs. In their framework, this is possible under very special cases. To my knowledge, my work is the first one addressing this same question empirically.

Among other papers, summarized in Bar-Isaac & Tadelis 2008, the role of reputation on two-sided markets has been studied by Saeedi 2014. She estimates the value of reputation mechanisms on Ebay. She models sellers as having private information about their product quality and buyers getting signals about the quality from the platform (Ebay) via existing reputation mechanisms. She does not have data that would directly measure quality and hence relies on a quality index that is recovered by estimating a structural model. Unlike me, Saeedi does not assume parametric distribution over quality. This constrains her in considering counterfactuals in which beliefs about quality change endogenously. Such counterfactuals are at the core of my work.

In summary, this chapter contributes to the above literature in four ways. First, by taking an initial step towards the structural identification and estimation of information design models. Second, by revisiting the question of whether information intermediation is possible and posing that question in the non-traditional context. Third, by exposing how commitment power, a la Kamenica & Gentzkow 2011, can be granted in practice. Fourth, by empirically examining the question of whether public monitoring can substitute for exogenous commitment.

Outline. The rest of the chapter is organized as follows: the basic facts about the industry and Kickstarter are presented in Section 1.2; Section 1.3 sets up the model, discusses equilibrium selection strategy, and the key assumption of the model; in Section 1.4, I solve the model and present basic theoretical results; Section 1.5 describes the data set and provides some reduced form evidence on the ability of the platform to serve as an information intermediary; Section 1.6 proves the identification of model parameters; Section 1.7 discusses the estimation procedure. Finally, Section 1.8 presents all the main results in this paper. Proofs, estimation details, tables and figures are relegated to the Appendix A.

1.2 Kickstarter.com Background

Kickstarter.com was launched on April 18, 2009.[3] Currently, it is the largest reward-based crowdfunding platform on the US reward-based crowdfunding market, with the market share of around 80 percent.[4] Its revenue model is based on the All or Nothing (AON) funding mechanism. Kickstarter takes 5 percent of the invested amount from the successfully funded projects.[5]

There are two important features that distinguish Kickstarter from a traditional venture capital firm:

1. The bulk of the investors on the Kickstarter are inexperienced and make only small stake investment decisions. Those investors have insufficient incentives or resources to conduct due diligence and screen the new ventures individually.[6]

2. There are no regulatory measures that would help resolve asymmetry of information between project creators and investors. Because rewards are not classified as financial instruments or securities, reward-based crowdfunding does not fall under the securities law. A candidate agency for regulating reward-based crowdfunding could be the Consumer Product Safety Commission. However, because crowdfunding is a form of pre-selling products to the investors, the product quality certification is required only after a project is funded.

Those two features, poorly informed investors and no regulation, enable Kickstarter to intermediate information transmission from project creators to investors and more importantly, to choose what information to transmit.

[3] https://techcrunch.com/2009/04/29/kickstarter-launches-another-social-fundraising-platform/

[4] https://trends.builtwith.com/widgets/Kickstarter/Market-Share

[5] https://help.kickstarter.com/hc/en-us/categories/115000499013-Kickstarter-basics

[6] http://www.finance-watch.org/hot-topics/blog/1182-take-care-of-the-crowd-crowdfunding

Currently, Kickstarter intermediates information transmission through "Projects We Love" - a way to feature projects that kickstarter deems promising. "Projects We Love" was launched on February 2, 2016 and is an evolution of a similar feature previously known as "Staff Pick".[7] The difference from the "Staff Pick" is that "Projects We Love" assigns a badge to the featured projects that cannot be falsified by the creators.[8]

The website claims that every project posted on it is pre-screened by either a complicated algorithm or a staff member.[9] Consequently, if a project meets certain quality standards, it is assigned a badge.

The structural model presented in the next section is motivated by the features of the Kickstarter outlined above.

1.3 The Model

The model involves a long-lived crowdfunding platform, one-period lived investors and project creators. Time is infinite and discrete, indexed by $t = 0, 1, 2, ...$

1.3.1 Creators

A creator is characterized by a type of his project, that is a triplet (q, m, l), where q stands for a quality of the project, m stands for the funding goal, and l stands for the length of the time during which the creator will be collecting funds from the investors on the platform (also referred to as the length of the project).

In a given period, the probability that a creator with a project type (q, m, l) arrives to the platform is denoted $f(q, m, l)$, and its CDF is denoted $F(q, m, l)$. I assume that the sets of all possible funding goals and project lengths, M and L, are finite. The set of all possible qualities is normalized to be the $[0, 1]$ interval. Let $\oslash$ stand for the event in which no creator arrives. Its probability is $1 - \int_0^1 \sum_{(m,l) \in M \times L} f(q, m, l) dq$.

After arriving to the platform, a creator decides whether he wants to post his project

[7] https://www.kickstarter.com/blog/introducing-projects-we-love-badges

[8] https://thenextweb.com/insider/2016/01/11/kickstarter-kills-staff-picks-in-favor-of-official-badges-to-avoid-confusion/

[9] https://www.kickstarter.com/blog/how-projects-launch-on-kickstarter

on the platform or not. To post the project, he must incur the cost c_{cr} that is distributed according to CDF F_{cr}, independently across creators. He gets utility of 1 if his project is funded and 0 otherwise. A project is said to be funded if and only if the total amount invested in the project by investors, denoted x, is greater than or equal to the project goal, m. This funding mechanism is the All Or Nothing (AON) funding mechanism, currently used by the Kickstarter.

1.3.2 Investors

If a project is posted by a creator, the number of investors that view its web-page, k, is distributed Poisson with the mean n. The parameter n is also random and has a Gamma distribution with the shape parameter β and rate parameter α. After an investor views a project, she needs to decide whether to invest or not. Investor's payoff per dollar invested is $q - c_b$, whenever she invests and the project is funded, and 0 otherwise. The opportunity cost of investing, c_b, is distributed according to CDF F_b. It follows that the probability that an investor viewing the project backs it is $F_b(E(q|I))$ where E stands for the expectation operator and I is the information set of the investor. It further follows that the number of investors that back the project, k', has a Negative Binomial distribution. Its density is denoted $g(k'|F_b(E(q|I)))$ and is derived in the Appendix.

I model the intensive margin of investor's decision in a reduced form by assuming that the total amount pledged towards a project is drawn from an exponential distribution with the parameter $\lambda(m, l, k, E(q|I))$. It follows that, conditional on k' backers investing in a project with the goal m, length l, and expected quality $E(q|I)$, the probability that the project is successfully funded is

$$h(m, l, k', E(q|I)) = e^{-\lambda(m,l,k',E(q|I))m}$$

.

The following assumptions are maintained throughout the paper,

Assumption 1.1. *F_{cr} and F_b are continuous CDFs. $F(q, m, l)$ is continuous in q.*

Assumption 1.2. *$\lambda(m, l, k', E(q|I))$ is continuous in $E(q|I)$.*

Assumption 1.3. *$\lambda(m, l, k', E(q|I))$ is increasing in k' and $E(q|I)$.*

Assumptions 1.1 and 1.2 are needed to prove the existence of an equilibrium. Assumption 1.3 implies that the expected amount invested into a project, $1/\lambda(m, l, k', E(q|I))$, must be increasing in the number of backers and in the expected quality of a project. In the estimation, I verify that assumption 1.3 holds.

1.3.3 The Platform

The platform maximizes discounted sum of revenues with the discount factor δ. It gets a share, r, of the invested amount into the funded projects (recall, a project is funded if the invested amount is greater than the goal amount). Hence, its revenue in a given period is simply rx if the project is funded and the invested amount is x, and 0 otherwise.

At the beginning of each period t, if a creator posts a project the platform privately learns project's quality, q. The platform can communicate information about the project's quality to the potential investors. For this, I assume the platform has an access to a fixed message space, $S = \{0, 1\}$, and can send a message, s, to the potential investors after learning the project's quality. Throughout the paper, message $s = 1$ is interpreted as assigning a badge to a project.

A rule determining how signals are correlated with the private information of the platform is announced by the platform at the beginning of period 0. Such a rule is called an Information Transmission Standard (ITS).

Definition 1.1. *Information Transmission Standard (ITS) is a threshold quality, q^*, such that a project is promised to be assigned a badge if and only if $q \geq q^*$.*

Even though the platform announces ITS at the beginning of period 0 and promises to adhere to it forever, such a promise should be credible as otherwise the platform's signals would not be informative for the investors. Particularly, project creators' and investors' decisions depend on the beliefs about the platform's strategy. Since an investor does not observe q at the moment of making the decision of whether or not to back the project, if the platform deviates from the announced ITS by sending a signal that is not supposed to be sent under the existing ITS, such a deviation would not be verifiable by the investors.

Investors understanding this would deem signals sent by the platform uninformative unless the platform had sufficient incentives not to deviate from the announced ITS. To generate such incentives, making sure that there exist ITSs that are both informative and credible, I assume that there is an imperfect public monitoring of projects' qualities.

A public monitoring technology maps ITS (q^*) to the probability with which a project's quality becomes public information (probability that the quality is verified) - independently across projects and over time. Formally, it is a function $\pi : [0,1] \to [0,1]$. This monitoring technology is one way to discipline the platform to adhere to the announced ITS - if the public verifies that the platform deviated from its announced ITS then the platform may be punished by switching to an equilibrium that ensures lower continuation value to the platform.

I do not make parametric assumptions on $\pi(q^*)$ and hence allow for all possible ways a public monitoring structure could depend on ITS. For instance, one mechanism could be that whenever q^* increases, that is ITS sets higher quality standard for a project to get badged, competitors of the Kickstarter decrease the efforts to verify badged project's qualities as they expect that those projects are of pretty high quality and hence expect less benefit from negative campaigning. This would deteriorate public monitoring. I do not take a stance on a specific mechanism. Finally, I assume that $\pi(q^*)$ is continuous in q^*.

1.3.4 Timeline

At the beginning of period 0, the platform announces ITS. Within each period, t, the timeline of the game is depicted in Figure 1.1.

1.3.5 Strategies and Solution Concept

Let h_t denote a public history at the beginning of period t. It includes the following information up to time t: whether or not a project was posted in each period, conditional on a project being posted its goal amount, length, total amount pledged, total number of backers and signal sent by the platform, whether or not a project's quality has been verified and projects quality if it has been verified. Let H_t denote the set of all period t public histories.

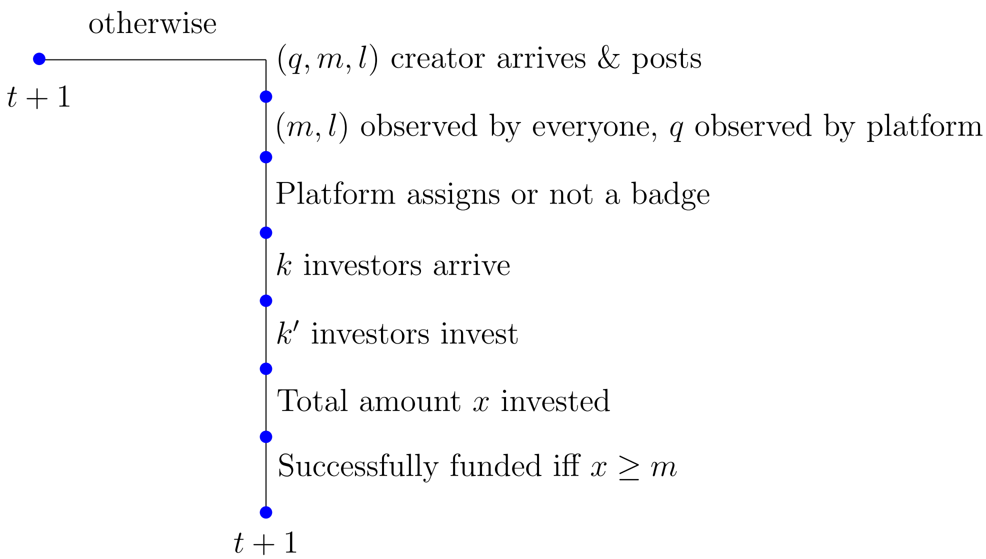

Figure 1.1: Time-line

The platform, in addition to h_t, observes realization of q for each project that was posted on it up to time t. A history observed by the platform is, $h_t^P = h_t \cup \{q_t\}_{t=0}^{t-1}$. I drop time subscripts whenever a reference to a specific time period is clear from the context.

A strategy of a creator who arrives at the platform is a probability of posting his project given h_t, his project's type, (q, m, l), and cost of posting the project, c_{cr}. A strategy of an investor arriving at a posted project's page is a probability of investing given h_t, current project's goal and length, (m, l), signal sent by the platform, s, and cost of investing, c_b. A strategy of the platform consists of the choice of q^* at the beginning of period 0 and a probability of sending $s = 1$ given h_t^P and type of the current project, (q, m, l).

I study Perfect Bayesian Equilibria (PBE) that survive the intuitive criterion of Cho & Kreps, 1987. For the estimation, I need to further narrow down the set of equilibria. Since this is a repeated game there are many equilibria. I assume that the platform announces a two signal communication rule that is partitional (I refer to it as Information Transmission Standard (ITS) - see section 1.3.3) which is history independent. I further assume the worst off-equilibrium punishment, if the platform is detected to have deviated from the announced ITS. I will elaborate on this punishment-phase selection further in the next section. Henceforth, I will refer to a PBE that survives intuitive criterion, involves history independent

ITS, and prescribes the worst punishment for the platform (in case its deviation is detected) as an *equilibrium*.

1.3.6 Discussion of the Model

Project's Quality: I assume that a project's quality is exogenous. Quality can be interpreted as something that is determined by the ability of a creator and uniqueness of his idea, and hence is difficult to change. Since quality differentiates projects on the vertical dimension, I estimate the model only for the projects in the technology category. In this category, quality has a more or less straightforward interpretation. For example it could be an index of endurance, energy usage, size and other characteristics of a new technology piece.

Alternatively we can think about one of two possible qualities, high or low, associated with each project. In such a case q can be interpreted as a probability of high quality. This would be an observationally equivalent way of representing the model.

Finally, instead of thinking about the platform observing certain signals about a product that investors do not, one could equivalently think about the platform having an expertise in interpreting certain publicly observable signals about a product. In such a case, the signals platform transmits to investors would summarize information contained in the publicly observable signals (e.g. creator posts a blueprint and the platform interprets its content to the potential investors).

Project's Goal and Length: I abstract away from modeling creator's choice of project's goal and length. One would expect that a creator would choose these strategically if such choices signal something about his project's quality. One reason for why I do not model strategic choice of goal and length is that it would require me to answer very specific modelling questions that I believe would make things way too artificial. For instance, what would happen if a creator sets lower goal than he actually needs to develop the project? Would the project quality be affected by this? If so, how? Would this mean that the creator would be less likely to deliver his promises and do we translate this into the lower quality of the project? If so, how? If not so, do we explicitly model what it means to deliver promised project? To at least partially compensate for answering all these modelling questions, I allow project's quality distribution to depend on its goal and length. In addition, in the estimation, I coarsely partition projects' length and goal (e.g. I partition project's goal into two bins - from 5000 USD to 24999 USD and, from 25000 USD to 50000 USD). Since 5000 USD projects are very likely to be different from 50000 USD projects it would be difficult for a creator to manipulate investors' beliefs based solely on setting project's goal.

Cost of Investment: The parameter c_b can be interpreted as the opportunity cost of a dollar invested. Ideally c_b would be modeled to be explicitly correlated with the actual amount invested, allowing for non-linearity of opportunity cost in amount invested. However, I cannot afford this as I do not observe the individual donation amounts (see Marwell 2015 for the similar modelling choice).

It is also noteworthy that an investor does not incur c_b in case she invests but the project ends up unfunded. The reason is that under the AoN funding mechanism if the project ends up unfunded all the invested amount is returned to an investor. In addition, since in theory the investor could wait up until the deadline of the project to make the investment decision investing should be a costless action in case a project ends up unfunded.

Investor Arrival: I do not have data that would let me recover investor arrival process. I assume that investor arrival is governed by a mixture of Poisson and Gamma. First, these distributional assumptions give me a good fit for the number of backers observed conditional on the characteristics of a project. Second, I end up with the number of backers having Negative Binomial distribution that is a simple distribution to work with.

The parameters α and β govern the arrival of investors. The investors' cost and belief parameters govern behavior of individual investors. My identification strategy allows α and β to vary across projects based on whether a project has a badge or not. This allows me to capture the fact that projects that have badges are usually also on the front page and easier for the investors to find. This directly affects arrival of investors on a project's page. In addition, the platform may take other measures to increase the visibility of badged projects like sending out newsletters. Such measures affect visibility but not beliefs.

Information Transmission Standard: In general, a communication protocol could be a complicated rule involving more than two signals. ITS restricts communication protocols in two ways. First, by allowing for at most two signals. Second, allowing only for threshold rules. The first restriction replicates the "Projects We Love" type of communication protocols. There are reasons why Kickstarter does not implement a protocol that is more complicated than "Projects We Love" and those reasons are out of the scope of my model. In the absence of such reasons, my model rationalizes the use of communications protocols that involve more than two signals (the platform could do better under such protocols). Since, we do not observe Kickstarter implementing such types of protocols my model does not capture those reasons and hence it would be misleading to talk about what would happen if the platform implemented more complicated protocols. The second restriction is due to making the analysis tractable. It is also intuitively appealing as it means that if a project of a certain quality gets a badge then a higher quality project must also get a badge. In practice, violating this might lead to the public accusing the platform of showing favoritism.

Equilibrium Selection: I select equilibria in which on-equilibrium path behavior is history independent.

In general, ITS could be announced by the platform in any period and it could be history dependent. The model and identification strategy can be straightforwardly extended to Markovian or even more history dependent ITSs. However, estimating such a model would be more data intensive.

One reason for why I do not consider history dependent ITSs is that, to my knowledge, there is no evidence of it being such. Kickstarter provides guidelines for becoming a "Project We Love".[10] Those guidelines discuss only the quality of a project that is determined by creativity, uniqueness of an idea and implementability. There is no mention that the decision of assigning a badge is history dependent. Moreover, there is no mention that project's goal or length determine the decision of assigning a badge. This leaves us with a simple communication protocol that depends only on the quality of a given project.

Another assumption that helps to reduce the number of potential equilibria is prescribing the worst punishment for the platform if it is detected to have deviated from its announced ITS. In the Appendix, I provide a more detailed discussion of the theoretical foundation for my equilibrium selection strategy and show that the worst punishment for the platform arises naturally. At this point, I deem it sufficient to state that most of the results (including the main results) are insensitive to how we punish the platform or even what kind of public

[10] `https://www.kickstarter.com/blog/how-to-get-featured-on-kickstarter`

monitoring technology we assume. The reason is that all the parameters that are sufficient for reaching certain results can be estimated independently of the platform's dynamic problem. This point will become more clear when I get to the details of the model identification.

1.4 Solving the Model

Let $I_t = \{m_t, l_t, s_t, q^*\}$ be the information that an investor has about the period t project when making the investment decision. I will drop time subscripts whenever this does not cause any confusion. Note that q^* stands for the ITS that the players believe the platform is following.

We can write the probability that the posted project gets funded conditional on the information contained in I_t as,

$$Z(I) = \sum_{k'=1}^{\infty} g(k'|F_b(E(q|I)))h(m, l, k', E(q|I)) \tag{1.1}$$

Let I_+ denote I with $s = 1$ and I_- denote I with $s = 0$. The following probabilities follow from 1,

$$\begin{aligned} &\text{For } q \geq q^* \\ &Z(I_+) \equiv Z(I_+, q) = \sum_{k'=1}^{\infty} g(k'|F_b(E(q|I_+)))h(m, l, k', E(q|I_+)) \end{aligned} \tag{1.2}$$

$$\begin{aligned} &\text{For } q < q^* \\ &Z(I_-) \equiv Z(I_-, q) = \sum_{k'=1}^{\infty} g(k', |F_b(E(q|I_-)))h(m, l, k', E(q|I_-)) \end{aligned} \tag{1.3}$$

Expressions 1.2 and 1.3 are probabilities of getting funded from the perspective of a creator. Note that, for a fixed (m, l), for all creators with $q \geq q^*$ the probability of getting funded is the same and similarly for all creators with $q < q^*$. The reasons is that, given (m, l), the only thing that differentiates projects in the eyes of investors is whether a project was assigned a badge or not. This would not be the case, for instance, if I allowed investors to get private signals on the top of whatever information the platform provides. However, since investors on reward-based platforms are usually unaccredited and have little incentive and ability to scree the projects on their own, I consider the assumption of not privately informed investors a good approximation to the reality.

It follows that the probability that a (q, m, l) type creator posts a project with $q \geq q^*$ is $F_{cr}(Z(I_+))$ and for $q < q^*$ it is $F_{cr}(Z(I_-))$. We can further account for the equilibrium expectations of project quality, $E(q|I_+)$ and $E(q|I_-, q^*)$,

$$E(q|I_+) = \frac{\int_{q^*}^{1} f(m,l|q)f(q)qdq}{\int_{q^*}^{1} f(m,l|q)f(q)dq} \tag{1.4}$$

$$E(q|I_-) = \frac{\int_{0}^{q^*} f(m,l|q)f(q)qdq}{\int_{0}^{q^*} f(m,l|q)f(q)dq} \tag{1.5}$$

Notice that $E(q|I_+)$ and $E(q|I_-)$ do not enter right hand sides of 1.4 and 1.5. This is again because the entry decision of a creator does not provide information over and above what investors already have at the stage of making the investment decision. If instead, I allowed investors to get private signals about creator's type then this would no longer hold and those expectations would be fixed points of certain equations. We would likely have multiple $E(q|I_+)$ and $E(q|I_-)$ satisfying those equations.

One more object that is useful to account for before turning to the platform's problem is the expected funds that a creator posting a (q, m, l) project obtains conditional on k' investors deciding to invest, $o(m, l, k', E(q|I))$,

$$o(m, l, k', E(q|I)) =$$

$$Pr(x \geq m|m, l, k', E(q|I))E(x|x \geq m, l, k', E(q|I)) =$$
$$e^{-\lambda(m,l,k',E(q|I))m}\left(m + \frac{1}{\lambda(m, l, k', E(q|I))}\right)$$

Note that $o(m, l, k', E(q|I))$ does not depend on q as the investors see s, not q.

Using the notation I developed so far, I can write down the platform's expected per period payoff before a creator posts a project and given that the creators and backers believe that the platform is using q^* as its ITS,

$$U(q^*) = r \sum_{(m,l)\in M\times L} f(m,l) \sum_{k=1}^{\infty}$$
$$\Big[o(m,l,k',E(q|I_+))(1 - F(q^*|m,l))F_{cr}(Z(I_+))g(k'|F_b(E(q|I_+)))$$
$$+ o(m,l,k',E(q|I_-))F(q^*|m,l)F_{cr}(Z(I_-))g(k'|F_b(E(q|I_-)))\Big] \tag{1.6}$$

In the expression 1.6, $F(q^*|m,l) = \frac{\int_0^{q^*} f(q,m,l)dq}{f(m,l)}$.

We can also account for the platform's payoffs after a project is posted on the platform. Recall that if a creator posts a project, (m,l) becomes a public information and in addition, the platform privately observes q. Suppose the players believe that the platform is using the standard q^*. If the platform assigns a badge to the project, $s = 1$, its current period profits are,

$$U_1(m,l,q^*) = r\sum_{k=1}^{\infty} o(m,l,k',E(q|I_+))g(k'|F_b(E(q|I_+))) \tag{1.7}$$

If the platform does not assign a badge, $s = 0$, its expected profits are,

$$U_0(m,l,q^*) = r\sum_{k=1}^{\infty} o(m,l,k',E(q|I_-))g(k'|F_b(E(q|I_-))) \tag{1.8}$$

Comparing the payoffs from 1.7 and 1.8 we can see the platform's short-run incentive to always assign a badge to a project. To see why note that for $q^* \in (0,1]$ we have $E(q|I_+) > E(q|I_-)$ (I will consider the case $q^* = \underline{q}$ shortly) which further implies that $G(k'|F_b(E(q|I_+)))$ first order stochastically dominates $G(k'|F_b(E(q|I_-)))$. Also, assumption 3 implies that $o(m,k',E(q|I))$ is increasing in k' and $E(q|I)$. It is easy to see that $U_1(m,l,q^*) > U_0(m,l,q^*)$ follows from those observations. Hence, if the platform did not have any dynamic incentives then it would always assign a badge to a project thus deeming any $q^* \in (0,1]$ non-credible. The only candidate for the equilibrium would be $q^* = 0$ which is the uninformative equilibrium. This equilibrium involves $s = 0$ being off-path. Recall that I require equilibrium to be sequential and to satisfy intuitive criterion. In case of $q^* = 0$, these requirements select the unique equilibrium in which signal $s = 0$ is also uninformative.

Now we can set up a dynamic problem for the platform. Recall that the platform's incentive to always assign a badge is disciplined by the threat that the public will detect the deviation and punish the platform by switching to the worst equilibrium for the platform. The worst punishment that can be supported as an equilibrium is babbling equilibrium of a stage game played forever. In such an equilibrium, the public loses trust in the platform and deems all signals uninformative. This is an equilibrium because given that current investors expect that future investors will be regarding platform's signals uninformative, they know that the platform does no have any dynamic incentives (no reputation to lose) and hence cannot be disciplined to follow any ITS in the current period other than the uninformative one.

To see that babbling forever is the worst punishment for the platform, suppose there is a punishment that can be supported as an equilibrium and such a punishment generates lower value to the platform compared to the babbling equilibrium forever. Since the alternative punishment is different from babbling forever, after some histories the platform's signals must be informative. After such histories, the platform could simply deviate to sending a signal that induces expected quality higher than $E(q|m, l)$ and hence obtain higher expected revenue. Note that such a signal always exists after the histories where the platform is supposed to send informative signals. This shows that the platform can always guarantee a payoff that is at least as large as the payoff from the babbling equilibrium.

Let $V(q^*)$ be the platform's discounted average expected revenues (value) evaluated at the beginning of period 0 and assuming that it never deviates from the announced ITS. The discounted average expected revenues is defined as the expected discounted revenues times $1 - \delta$. We have,

$$V(q^*) = U(q^*) \tag{1.9}$$

Let $V(q^*, d)$ be the platform's value when the platform has deviated d times from its announced standard but has not been detected yet.

The platform's dynamic problem can be formulated as,

$$Max_{q^* \in [0,1]} U(q^*) \tag{1.10}$$

$$s.t.$$

For $q^* > 0$

$$\delta\big[U(q^*) - (1-\pi(q^*))V(q^*,1) - \pi(q^*)V\big] \geq (1-\delta)Max_{(m,l)\in M\times L}\big[U_1(m,l,q^*) - U_0(m,l,q^*)\big] \tag{1.11}$$

In 1.11, V stands for the platform's value in the permanent babbling equilibrium. 1.11 ensures that the platform does not want to deviate from its announced ITS. The right hand side is the best possible short-run gain from sending $s = 1$ whenever $q < q^*$ and the left hand side is the long run loss from doing so.

Given the values of the parameters we already know how to calculate all the quantities in the problem defined by 1.10 and 1.11 except for $V(q^*, 1)$. To evaluate $V(q^*, 1)$, we need to solve for the dynamic optimal deviation strategy of the platform given that it has already deviated once from the ITS. Note that one shot deviation principle does not hold in this model as we have a privately informed long-run player.

To find $V(q^*, 1)$ we need to keep track of the number of times that platform deviates from its standard, denoted d. This is because more times the platform deviates the probability that at least one deviation will be detected by the public increases. Hence, the number of times the platform deviates becomes payoff relevant private information for it.

Consider some $d \geq 1$. Suppose the platform learns that the type of the creator who posted the project in the current period is (q, m, l) with $q \geq q^*$. The platform sends $s = 1$. By doing so it does not increase the probability that at least one of its deviations will be detected in the future and in the current period gets higher expected profits compared to sending $s = 0$. The platform's value in this case is,

$$W_1(m,l,q^*,d) = (1-\delta)U_1(l,m,q^*) + \delta[(1-(1-\pi(q^*))^d)V + (1-\pi(q^*))^d V(q^*,d)] \tag{1.12}$$

If $q < q^*$, then the platform needs to decide whether it wants to deviate from its standard by sending $s = 1$ and increasing the probability that its deviation will be detected in the future, or sticking to the standard and not increasing that probability. Platform's value in this case solves,

$$W_0(m,l,q^*,d) = Max_{j\in\{0,1\}}\Big\{(1-\delta)U_j(l,m,q^*)+ \\ \delta[(1-(1-\pi(q^*))^{d+j})V+(1-\pi(q^*))^{d+j}V(q^*,d+j)]\Big\} \tag{1.13}$$

If a creator does not post a project then the platform's value is,

$$W(q^*,d) = \delta[(1-(1-\pi(q^*))^{d})V+(1-\pi(q^*))^{d}V(q^*,d)] \tag{1.14}$$

Using 1.12, 1.13 and 1.14 we can write $V(q^*,d)$ as,

$$V(q^*,d) = \\ \sum_{(m,l)\in M\times L} f(m,l)\Big[(1-F(q^*|m,l))F_{cr}(Z(I_+,q^*))W_1(m,l,q^*,d)+ \\ F(q^*|m,l)F_{cr}(Z(I_-,q^*))W_0(m,l,q^*,d)\Big]+ \\ \Big[1-\sum_{(m,l)\in M\times L} f(m,l)+\sum_{(m,l)\in M\times L} f(m,l)\big[(1-F(q^*|m,l))(1-F_{cr}(Z(I_+,q^*)))+ \\ F(q^*|m,l)(1-F_{cr}(Z(I_-,q^*)))\big]\Big]W(q^*,d) \tag{1.15}$$

We have the following proposition,

Proposition 1.1. *There exists unique $V(q^*,d)$ that solves equation 15. $V(q^*,d)$ is continuous in q^* on $[\epsilon,1]$ for any $1>\epsilon>0$, decreasing and convex in d.*

Consequently, to show that an equilibrium exists, it is sufficient to prove that the problem defined by 1.10 and 1.11 has a solution. Let q^{**} denote an ITS that solves the problem.

Proposition 1.2. *There exists a q^{**} that solves the problem defined by 1.10 and 1.11.*

The proofs of Proposition 1.1 and 1.2 can be found in the Appendix A.

1.5 Data

The data are publicly available[11] and include all the projects that were posted on Kickstarter.com since its commencement till February 15, 2018.

I select data from February 2, 2016 (when "Projects We Love" was commenced) till December 15, 2017. There are two reasons for restricting the data to this time-frame. First, I do not want to consider projects that were posted under the "Staff Pick" regime as "Staff Pick" badges were being falsified by the creators. Second, I want each project to be associated with one of the three outcomes, "funded", "not funded" or "canceled". Those outcomes are only assigned after the project's funding deadline. According to the Kickstarter's rules, the funding period can last maximum for 60 days.[12] This means that all the projects launched before December 15, 2017 have one of those status.

For each project, I observe its unique id, category, location of the creator, project launch time, funding deadline, goal amount, total amount pledged, total backers, final funding status, and whether or not the project had a "Projects We Love" badge. I consider only the projects from the technology category. Since quality differentiates projects on the vertical dimension, I need project quality to be roughly comparable across projects. I drop all the project that have status "canceled", as I do not observe a reason why a project was canceled during a campaign. Anecdotal evidence (e.g. forums) suggests that a creator usually cancels a project if he wants to adjust the goal amount, or some other characteristics of his project. Because of this, I expect that the projects that are canceled in the data (10 percent of the data) are re-posted later.

Finally, I restrict the data to the projects with goal amounts within $[5000, 50000]$ USD and number of backers within $[0, 800]$. This, along with dropping canceled projects, leaves me with 47 percent of the original data. The projects with too little goals are dropped because they are likely to get funded from family and friends, while the project with high goals and large number of backers may involve experienced investors and strong social learning effects. The final sample size totals 5411 projects. The sample is summarized in Table 1.1.

One thing worth noting from Table 1 is that 9 percent of projects get a badge while 38

[11] `https://webrobots.io/kickstarter-datasets/`

[12] `https://www.kickstarter.com/help/handbook/funding`

Table 1.1: Data Summary

Variable	Mean	Std. Dev.	25%	50%	75%	Min	Max
Goal	19,931	13,497	10,000	15,000	28,334	5,000	50,000
Length (days)	34.85	10.57	30	30	39.3	1	64
Badged	0.09	0.29	1	1	1	0	1
Pledged	14,508	32,321	50	1,799	17,028	0	667,311
Backers	101.89	161.42	2	20	136	0	800
Funded	0.38	0.48	1	1	1	0	1

percent get funded, that is over four times more projects are getting funded than the number of projects getting a badge. This suggests that platform's badging decisions do not blindly follow the simple rule under which it monitors pledges in the real time and calibrates its badging decisions accordingly. In such a case, one would expect the shares of badged and funded projects to be nearly the same.

Figure 1.2 shows the distribution of the number of backers for badged and non-badged projects. It is vivid that those distributions differ. I also verify this using the Kolmogorov-Smirnov test for the equality of distributions. About 55 percent of the non-badged projects end up with less than 20 backers while for the badged projects this number is about 3 percent. Figure 1.3 shows the distributions of total pledged amount for badged and non-badged projects. The Kolmogorov-Smirnov test for the equality of distributions rejects that these distributions are the same. Finally, 87 percent of the badged projects get funded while only 33 percent of the non-badged projects get funded.

The above suggest that badged projects are different from the non-badged projects. In theory, if badges affect investors' behavior then it should happen due to a combination of the following effects:

i) Badged projects are easier to spot on the website and hence, badging reduces search costs for the investors;

ii) Badges transmit unique information to the investors that affects their beliefs;

iii) A badge summarizes some other public information i.e. it is correlated with other public signals and serves as a summary statistic for the investors;

Because, in the data, I do not observe all the public information that the investors do, it is not possible to perfectly separate those three effects. However, still we can partially separate ii and iii from i. The logic is as follows: if badges only serve as the means for decreasing search frictions for the badged projects, then we should have that the effect of badges on the investor behavior does not vary with project characteristics. This should be so because Kickstarter does not make projects more or less visible based on their characteristics, other than whether they are badged or not (on the website, once you select badged projects, further filtering with respect to other characteristics is effortless). Consider the following regression equation,

$$outcome_{t,i} = \gamma_0 P^n(const, goal_{t,i}, length_{t,i}) + \gamma_1 E(q|goal_{t,i}, length_{t,i}, Badged_{t,i}, \psi_{t,i}) +$$
$$\gamma_2 t + \gamma_3 Badged_{t,i} + \epsilon_{ti}$$

In the regression equation, outcome is a variable that measures some aspect of investors' behavior (e.g. outcome can be number of investors that back a project), i denotes project and t time when it was posted. Const. stands for constant and $P^n(const, goal, length)$ is a collection of polynomial features of const, goal and length, up to (and including) degree n. Badged is a dummy variable taking value 1 for a badged project, γ's are coefficients and ϵ is error term independent of all the regressors. Finally, $E(q|goal, length, Badged, \psi)$ is expected quality of a project, as perceived by investors, conditional on information that investors have, and ψ stands for some signals that investors observe but an econometrician does not. Note that my discussion above implies that γ_3 entirely captures the effect i. So, if we added interactions of *Badged* with *goal* or *length*, then their coefficients would not involve any search cost effects.

Even though I do not observe $\psi_{t,i}$ and it affects investors' beliefs, I can still get consistent estimates. To see how, note that we can make the following decomposition of expected quality,

$$E(q|goal, length, Badged, \psi) = E(q|goal, length, Badged) + \chi$$

where $E(\chi|goal, length, Badged) = 0$ (this is a property of conditional expectations).

Further, we can use polynomial approximation of $E(q|goal, length, Badged)$ leading to,

$$E(q|goal, length, Badged) \approx \beta^{Badged} P^n(const, goal, length)$$

where β^{Badged} is a coefficient vector of length H, where the first coefficient, β_0^{Badged}, corresponds to $const$. Finally, we can rewrite our regression equation as,

$$outcome_{t,i} \approx (\gamma_0 + \gamma_1 \beta_{1:\,H}^{Badged}) P^n(goal_{t,i}, length_{t,i}) + \gamma_2 t + \gamma_3 Badged_{t,i} +$$
$$(\gamma_0 + \gamma_1 \beta_0^{Badged}) + (\gamma_1 \chi_{t,i} + \epsilon_{ti})$$

The error term, $\gamma_1 \chi_{t,i} + \epsilon_{ti}$, has conditional zero mean. Hence, we can get consistent estimates of the parameters. It follows that we can identify γ_2, $\gamma_0 + \gamma_1 \beta_{1:\,H}^{Badged}$, $\gamma_0 + \gamma_1 \beta_0^{Badged=0}$ and $\gamma_0 + \gamma_1 \beta_0^{Badged=1} + \gamma_3$. Most importantly, we can identify $\gamma_1(\beta_{1:\,H}^{Badged=1} - \beta_{1:\,H}^{Badged=0})$. If any element of this vector is nonzero then it means that badges affect investors' decisions via effects ii or iii. Finally, note that by increasing n we can make the approximation in the above equation arbitrarily good. In addition, it is straightforward to generalize the above arguments for the case where outcome depends on higher moments of beliefs (e.g. variance).

Table 1.2(a) provides implementation of the identification strategy outlined above. It presents two regression results. In the first, the outcome variable is the number of backers and in the second, the outcome variable is total pledged amount. In the first regression, coefficients on $Badged * Goal$ and $Badged * Goal^2$ are significant. In the second regression, coefficient on $Badged * Goal^2$ is significant. These imply that the platform's badging decision affects investors' behavior via the effects ii or iii. However, per our discussion above, the coefficients on those interactions may be underestimating the effects of ii or iii.

If we want to further understand whether ii is important for determining outcomes, observing all the information that investors do would be sufficient. Alternatively, we could implement the following strategy. Suppose we observe projects across two different information transmission rules and we are able to identify $\gamma_1((\beta_{1:\,H}^{Badged=1} - \beta_{1:\,H}^{Badged=0}) - (\hat{\beta}_{1:\,H}^{Badged=1} - \hat{\beta}_{1:\,H}^{Badged=0}))$, where $(\beta_{1:\,H}^{Badged=1} - \beta_{1:\,H}^{Badged=0})$ determines difference in beliefs across badged and non-badged projects under one transmission rule, and $(\hat{\beta}_{1:\,H}^{Badged=1} - \hat{\beta}_{1:\,H}^{Badged=0})$ - under another transmission rule. If any element of the vector, $\gamma_1((\beta_{1:H}^{Badged=1} - \beta_{1:H}^{Badged=0}) - (\hat{\beta}_{1:H}^{Badged=1} - \hat{\beta}_{1:H}^{Badged=0}))$, is non-zero and in addition it is the case that the joint distribution of ψ and project quality is unchanged across those rules (this is a paralell trends assumption), then we would know that ii is true (because ii not being true means that *Badged* is perfectly correlated with ψ and hence, $\beta_{1:H}^{Badged=1} - \beta_{1:H}^{Badged=0} = \hat{\beta}_{1:H}^{Badged=1} - \hat{\beta}_{1:H}^{Badged=0}$ must be true if the joint distribution of ψ and project quality is unchanged across those rules).

Since I have data from the "Staff-Pick" epoch (before February 2, 2016), I can check whether investors' behavior (number of backers and amount pledged) varies across "Staff-Pick" and "Projects We Love" (PWL) projects. I run diff-in-diff regressions. I include interactions of all variables from Table 1.2(a) with the dummy variable PWL, that equals 1 if a project was posted in PWL epoch. Table 1.2(b) presents the regression results. In the first column, coefficients on $PWL * Badged * Goal$ and $PWL * Badged * Goal^2$ are significant. In the second column, $PWL * Badged * Goal$ and $PWL * Badged * Gial * Length$ are significant. These mean that badges affect investors' beliefs differently across the two rules. Finally, assuming that the joint distribution of ψ and project quality is fixed across "Staff-Pick" and "Projects We Love" epochs (placebo tests indicate that this assumption is true), the change in the information content of badges across the two epochs is due to the effect ii.

Even though the above regressions provide some evidence that the platform can persuade, this is not enough for the purposes of my paper. Since my main objective is to see how certain counterfactual changes in beliefs would affect investors', creators' and platform's behavior, I need to structurally model each player's behavior and quantify the meaning of "changing beliefs".

1.6 Parameterization and Identification

I set F_{cr} and F_b to be Frechet distributions with location parameters set to 0 and shape parameters set to 1. The scale parameters will be estimated and are denoted, β_{cr} and β_b.

The distribution of quality conditional on (m, l) project arriving, $F(q|m, l)$, is set to be truncated exponential on $[0, 1]$ with the parameter $\gamma(m, l)$. I normalize $\gamma(m, l) = 1$ for some

$(m, l) \in M \times L$. Let such (m, l) be denoted $(\underline{m}, \underline{l})$.

I allow the parameters of Negative Binomial, α and β, to depend on whether a project is badged or not. The investor arrival parameters for the badged projects are denoted α_1 and β_1 and, α_0 and β_0 for non-badged projects. Let $\alpha = (\alpha_0, \alpha_1)$ and $\beta = (\beta_0, \beta_1)$.

Finally, I assume

$$\lambda(m, l, k', E(q|I)) = a_1 + a_2 k'^{a_3} + a_4 m + a_5 l + a_6 E(q|I)$$

Let $\theta = \{\theta_1, \theta_2, \theta_3\}$ where $\theta_1 = (q^{**}, \{\gamma(m, l)\}_{(m,l) \in M \times L \setminus (\underline{m}, \underline{l})}, \beta_b, \alpha, \beta)$, $\theta_2 = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\theta_3 = (\{f(m, l)\}_{(m,l) \in M \times L}, \beta_{cr})$. I need to identify θ and $\pi : [0, 1] \rightarrow [0, 1]$. In what follows, I argue that θ is point identified and develop my identification strategy. Even though I cannot identify the function $\pi : [0, 1] \rightarrow [0, 1]$, I discuss what information I am able to recover about this function.

1.6.1 Identification of θ_1

The parameters in θ_1 are identified from the distribution of the number of backers conditional on the observable characteristics of the posted projects (goal amount and length) and badge assignment decision of the platform.

First, recall the distribution of project backers as implied by the model is Negative Binomial with the probability mass function,

$$g(k'|F_b(E(q|I))) =$$

$$\binom{k' + \beta_s - 1}{k'} \left(\frac{\alpha_s}{\alpha_s + F_b(E(q|I))} \right)^{\beta} \left(\frac{F_b(E(q|I))}{\alpha_s + F_b(E(q|I))} \right)^{k'}$$

The mean of that distribution conditional on $\underline{I}_- \equiv \{\underline{m}, \underline{l}, 0, q^{**}\}$ is,

$$\frac{\beta_0}{\alpha_0} F_b(E(q|\underline{I}_-))$$

Let $h(\underline{I}_-)$ be the mean from the data as the sample size approaches infinity. Then we must have,

$$h(\underline{I}_-) = \frac{\beta_0}{\alpha_0} F_b(E(q|\underline{I}_-)) \tag{1.17}$$

Similarly we have,

$$h(\underline{I}_+) = \frac{\beta_1}{\alpha_1} F_b(E(q|\underline{I}_+)) \tag{1.18}$$

We also have similar identities for the variances,

$$h'(\underline{I}_-) = \frac{\beta_0(\alpha_0 + F_b(E(q|\underline{I}_-))}{\alpha_0^2} \tag{1.19}$$

$$h'(\underline{I}_+) = \frac{\beta_1(\alpha_1 + F_b(E(q|\underline{I}_+)))}{\alpha_1^2} \tag{1.20}$$

We can rewrite the identities, 17, 18, 19 and 20, as follows,

$$\beta_0 = \frac{h(\underline{I}_-)^2}{h'(\underline{I}_-) - h(\underline{I}_-)} \tag{1.21}$$

$$\beta_1 = \frac{h(\underline{I}_+)^2}{h'(\underline{I}_+) - h(\underline{I}_+)} \tag{1.22}$$

$$\alpha_0 = \frac{F_b(E(q|\underline{I}_-))h(\underline{I}_-)}{h'(\underline{I}_-) - h(\underline{I}_-)} \tag{1.23}$$

$$\alpha_1 = \frac{F_b(E(q|\underline{I}_+))h(\underline{I}+)}{h'(\underline{I}_+) - h(\underline{I}_+)} \tag{1.24}$$

From 1.21 and 1.22 we have identified β as those expressions depend only on the data. If we knew the right hand sides of 1.23 and 1.24 we would also identify α. However, right hand sides of 1.23 and 1.24 depend on q^{**} and β_b.

Consider some $I' \equiv \{m', l', s, q^{**}\}$ with $(m', l') \neq (\underline{m}, \underline{l})$. We can write down similar identities as above for the mean and variance of the number of backers conditional on I'_- and the mean of the number of backers for I'_+. We get,

$$h(I'_{-}) = \frac{\beta_0}{\alpha_0} F_b(E(q|I'_{-})) \tag{1.25}$$

$$h(I'_{+}) = \frac{\beta_1}{\alpha_1} F_b(E(q|I'_{+})) \tag{1.26}$$

$$h'(I'_{-}) = \frac{\beta_0(\alpha_0 + F_b(E(q|I'_{-}))}{\alpha_0^2} \tag{1.27}$$

Using 1.21, 1.22, 1.23 and 1.24 into 1.25, 1.26 and 1.27 and the fact that $F_b(E(q|I)) = e^{-\beta_b/E(q|I)}$ we arrive at the following,

$$\beta_b = ln\left(\frac{h(I'_{-})}{h(\underline{I}_{-})}\right) \frac{E(q|\underline{I}_{-})E(q|I'_{-})}{E(q|I'_{-}) - E(q|\underline{I}_{-})} \tag{1.28}$$

$$\left(ln\left(\frac{h(I'_{-})}{h(\underline{I}_{-})}\right)\right)^{-1} \frac{h(I'_{+})}{h(\underline{I}_{+})} = \frac{E(q|\underline{I}_{-})E(q|I'_{-})}{E(q|I'_{-}) - E(q|\underline{I}_{-})} \frac{E(q|I'_{+}) - E(q|\underline{I}_{+})}{E(q|\underline{I}_{+})E(q|I'_{+})} \tag{1.29}$$

$$\left(ln\left(\frac{h(I'_{-})}{h(\underline{I}_{-})}\right)\right)^{-1} \frac{h(I'_{-})h'(I'_{-})}{(h(I'_{-}) + h'(\underline{I}_{-}) - h(\underline{I}_{-}))h(\underline{I}_{-})^2} = \frac{E(q|\underline{I}_{-})}{E(q|I'_{-}) - E(q|\underline{I}_{-})} \tag{1.30}$$

In 1.28, the right hand side depends only on q^{**} and $\gamma(m', l')$. The left hand sides of the equations 1.29 and 1.30 are functions of only data and the right hand sides depend only on q^{**} and $\gamma(m', l')$. If I verify that only a unique $(q^{**}, \gamma(m', l'))$ can solve 1.29 and 1.30 then this will imply that $(\alpha, \beta, \beta_b, q^{**}, \gamma(m', l'))$ is identified. I verify this numerically.

From Figure 1.4 one can see that the level curves of the right hand sides of the equations 1.29 and 1.30 intersect only once. Even though Figure 1.4 is constructed for $q^{**} \in [0.3, 1]$ and $\gamma(m', l') \in [0, 10]$, I have verified that level curves intersect only once for other ranges of the parameters, as well.

From θ_1, it remains to identify $\{\gamma(m, l)\}_{(m,l) \in M \times L \setminus \{(\underline{m}, \underline{l}), (m', l')\}}$. This can be done by considering the mean number of backers for each $(m, l) \in M \times L \setminus \{(\underline{m}, \underline{l}), (m', l')\}$. The only unknown parameter in the mean number of backers for a project with observable characteristics (m, l) is now $\gamma(m, l)$. In addition, it is easily verified that the mean is monotonic in

$\gamma(m,l)$. Hence, $\{\gamma(m,l)\}_{(m,l)\in M\times L\setminus\{(\underline{m},\underline{l}),(m',l')\}}$ is identified.

1.6.2 Identification of θ_2

Identification of θ_2 is straightforward. Variations in the goal, length, number of backers and $E(q|I)$ identify θ_2. The effect of beliefs on the intensive margin is captured by incorporating $E(q|I)$ in the definition of $\lambda(m,l,k',E(q|I))$. Alternatively, I could have used dummy variables for all combinations of (m,l,s) instead of using $E(q|I)$. However, I did not choose that route as the coefficients on those dummies would be fixed in my counterfactuals and I would not be able to capture the effect of changing beliefs on the intensive margin.

I allow the number of backers, k', to enter $\lambda(m,l,k',E(q|I))$ non-linearly. The data suggests that the expected pledges is increasing and concave in k'. The expected pledges implied by the model is $1/\lambda(m,l,k',E(q|I))$ and to make sure that it allows for the pattern observed in the data, we must allow k' to enter $\lambda(m,l,k',E(q|I))$ non-linearly.

1.6.3 Identification of θ_3

Creator's cost parameter, β_{cr}, is identified from the variation in the frequencies of the posted projects across badged and non-badged projects. We have already identified the probabilities of getting funded from the perspective of a creator - those probabilities depend only on θ_1 and θ_2. Those probabilities pin down creator's cost thresholds for badged and non-badged projects. Given the thresholds, the difference in the frequencies of projects across badged and not badged projects identifies creator's cost parameter.

After identifying β_{cr}, the arrival probabilities of (m,l) projects, $f(m,l)$, are identified by the variation in the frequencies of posted projects with different characteristics.

Formally, the probability that a creator with (m,l) project would post it, conditional on the project quality being above q^{**}, is $f(m,l)F_{cr}(Z(I_+))$ and conditional on his project quality being less than that threshold, it is $f(m,l)F_{cr}(Z(I_-))$. Recall that $Z(I_+)$ and $Z(I_-)$ are probabilities of getting funded. Then the probability of observing a posted project with the goal and length (m,l) that was badged by the platform is $f(m,l)F_{cr}(Z(I_+))(1-F(q^{**}))$ and the joint probability of observing a posted project with the goal and length (m,l) that was not badged by the platform is $f(m,l)F_{c_{cr}}(Z(I_-))F(q^{**})$. From the data we can identify those two probabilities. We also know that $Z(I_+)$, $Z(I_-)$ and $F(q^{**})$ depend only on the parameters that we already identified, θ_1 and θ_2. This means that we can identify β_{cr} from the ratio, $\frac{(1-F(q^{**}))F_{cr}(Z(I_+))}{F(q^{**})F_{cr}(Z(I_-))}$. Consequently, $f(m,l)$ is identified from $f(m,l)F_{cr}(Z(I_-))F(q^{**})$.

1.6.4 Identification of $\pi(q^*)$

So far, my identification strategy did not use the platform's optimality conditions, that is I have not used any information from the solution of the problem defined by 1.10 and 1.11. This means that any result that would depend only on the knowledge of θ_1, θ_2 and θ_3 would be insensitive to how one models the platform's incentives. This goes back to my earlier claim that some results in this paper would not change if I assumed any other public monitoring technology or off-equilibrium punishment for the platform.

The public monitoring technology function, $\pi(q^*)$, is the only object that requires me to solve the platform's problem. Even though $\pi(q^*)$ cannot be identified, I am able to identify certain bounds on the function. Those bounds have an economic content that will be discussed later in the text.

1.7 Estimation

The estimation procedure parallels my identification strategy and is comprised of four steps:

1. Estimate θ_1, calculate and save $E(q|I)$;
2. Estimate θ_2 using $E(q|I)$;
3. Estimate θ_3 using the parameters estimated in steps 1 and 2;
4. Solve the platform's problem using parameters estimated in the prior steps and recover needed information about $\pi(q^*)$.

Since I am allowing prior beliefs about project's quality to depend on (m,l), finer partition of $M \times L$ space means we need to estimate more parameters. To avoid the incidental parameter problem, I partition M in two subsets - projects with goals in $(0, 25,000)$ and projects with goals in $[25,000, 50,000]$. The reason for choosing this partition is that it roughly divides the sample in equal parts and at $25,000$ there is a peak in the distribution of goal amount (see Figure 1.5). I also partition L in two subsets - projects with length in $(0, 30]$ days and projects with length in $(30, 64]$ days. About 50 percent of the projects choose 30 days length. The other peaks in the distribution of project length occur above 30 days (see Figure 1.6). In what follows, I denote a project from the lower part of a partition as lo and from the upper part as hi. For example, a project with $m \in (0, 25,000)$ and $l \in (30, 64]$ is denoted as (lo, hi) project. I choose to normalize $\gamma(lo, lo) = 1$.

I also need to choose a unit of time because for estimating θ_3 I need to calculate frequencies with which projects with different characteristics are being posted and need to assume how often the platform makes decisions when solving its dynamic problem. I choose one second as a unit of time because there are several projects in the data that were posted one second apart.

In what follows, I will discuss some details of the estimation procedure outlined above. The parameters, θ, are estimated using the three stage conditional maximum likelihood estimator. In the first stage, I calculate the probability of observing k' backers conditional on observing a project with certain characteristics (including, whether it is badged or not) posted on the platform (note that conditional on project not being posted, probability of observing $k' = 0$ is always 1). Let the natural logarithm of that probability be denote $L_{1i}(\theta_1) \equiv L_1(k'_i, s_i, m_i, l_i, \theta_1)$ where i indexes observation. I need to solve,

$$max_{\theta_1} \sum_{i=1}^{n} L_{1i}(\theta_1)$$

. The solution gives me the estimate of θ_1 denoted, $\hat{\theta}_1$.

In the second stage, I construct log-likelihood using the distribution of amount pledged to a project conditional on observing a posted project with a certain goal, length, number of backers and whether it is badged or not. This likelihood depends on $\hat{\theta}_1$ via $E(q|I)$ that enters the parameter of the distribution of pledges, $\lambda(m, l, k', E(q|I))$. Let this log-likelihood for observation i be denoted $L_{2i}(\hat{\theta}_1, \theta_2)$. I need to solve,

$$max_{\theta_2} \sum_{i=1}^{n} L_{2i}(\hat{\theta}_1, \theta_2)$$

. The solution gives me the estimate of θ_2 denoted, $\hat{\theta}_2$.

In the third stage, I construct log-likelihood using the probability of observing a posted project with the goal, length and signal (m, l, s). Let this log-likelihood for observation i be denoted $L_{3i}(\hat{\theta}_1, \hat{\theta}_2, \theta_3)$. I need to solve,

$$max_{\theta_3} \sum_{i=1}^{n} L_{3i}(\hat{\theta}_1, \hat{\theta}_2, \theta_3)$$

. The solution gives me the estimate of θ_3 denoted, $\hat{\theta}_3$.

Finally, using $\hat{\theta}$ and setting the annual discount rate to 2 percent, I solve the platform's problem defined by 1.10 and 1.11. For each $\pi(\hat{q}^{**}) \in [0, 1]$, I find $V(\hat{q}^{**}, 1)$ using value function iteration. Then, I check whether 11 is true. I know that 11 becomes more relaxed as $\pi(\hat{q}^{**})$ increases. Intuitively, better public monitoring induces platform to abide to its announcement as otherwise, it can lose its future value easily. I find the threshold, $\bar{\pi}(\hat{q}^{**})$, such that whenever the probability with which the public verifies project's quality is below that threshold, 11 is violated meaning that the platform does not have an incentive to follow its announced ITS and hence, $\hat{q}^{**}$ could not have been an equilibrium ITS. So, any $\pi(\hat{q}^{**})$ above that threshold is rationalized by the model and no $\pi(\hat{q}^{**})$ below that threshold is rationalized. I do the same exercise for the counterfactual ITSs that are of interest and discuss the economic content of the results.

In order to find the global maxima of the likelihoods, I use Mesh Adaptive Direct Search Algorithm (MADS) from Audet & Dennis, 2006 along with the Simplex Search Method from Lagarias et al., 1998. I use those methods for various initial points and then choose the parameters that achieve the highest objective value.

1.8 Results

1.8.1 Estimate of θ

The estimates of θ_1, θ_2 and θ_3 are presented in Tables 1.3, 1.4 and 1.5, respectively. Standard errors are in parenthesis and are calculated using the estimator that is a generalization of Murphy & Topel 1985 two-stage estimator and is derived in the Appendix A. To compute reliable estimates of standard errors, I set $\beta_{cr} = 0$ as the estimated β_{cr} is $1.431e-15$, that is close to zero. Such a low value for β_{cr} means that the cost of posting a project is virtually zero i.e. the creator's cost distribution is concentrated around zero. This makes sense because once a creator has developed his idea to some extent, posting it on the Kickstarter does not require much effort - he just needs to register, upload few photos, describe his project and set goal and length of the project. Moreover, there is no fee for posting a project.

The estimate of backers' cost parameter, β_b, is 1.624. This suggests that backers are responsive to their beliefs about project's quality. The estimates of $\gamma(m,l)$ imply that the quality distribution of (hi, hi) projects first order stochastically dominates (FOSD) the quality distribution of (lo, hi) projects which FOSD the quality distribution of (hi, lo) projects which FOSD the quality distribution of (lo, lo) projects. This ranking, implied by the model, is consistent with the ranking, implied by the data, of the share of badged projects across projects with different characteristics. In the data, 12.6 percent of (hi, hi) projects are badged, and this number is 9.8 for (lo, hi) projects, 8.3 for (hi, lo) projects and 7.3 for (lo, lo) projects. The ranking of quality distribution also implies that high goal projects are of better quality (in terms of FOSD) than low goal projects, and high length projects are of better quality than low length projects.

In Table 1.6, I calculate first two moments of the distribution of the number of backers from the data and compare them to the same moments as implied by the model. In the data, mean and variance of the number of backers are always higher for badged projects compared to the non-badged projects. This same pattern is also replicated by the model. To further elaborate on the model fit, in Figure 1.7 I visually show how the model implied distribution of the number of backers fits the distribution in the data. For the non-badged projects, the model does a good job in matching the shape of the distribution except for consistently overestimating the probability of observing zero backers. For the badged projects, the fit does not look so good. However, note that the number of badged projects is small in the data. It varies from the minimum 78 for the (hi, lo) projects to the maximum 159 for the (lo, hi) projects. The abundance of the spikes of the same height in the distributions for the badged projects is due to a given number of backers being observed at most 3 times (usually only once).

The estimated values for a_2, a_4, a_5 and a_6 are negative meaning that expected pledged amount is increasing in the number of backers, expected quality (validating assumption 1.3), goal and length. Table 1.7 compares mean pledges from the data to the mean pledges as implied by the model for each $(m, l) \in \{lo, hi\} \times \{lo, hi\}$ project. Figure 1.8 plots mean pledges against the number of backers, for each $(m, l) \in \{lo, hi\} \times \{lo, hi\}$ project. One can see that the fitted curves are convex up to roughly 50 backers and then concave.

Finally, the estimates of $f(m, l)$ (backer arrival rates) are very close to the probability of observing a posted project (see Table 1.8). This is not surprising given that estimated value of β_{cr} is close to zero implying that creator's decision to post a project is insensitive to his private information. Hence, whenever a creator arrives, he posts a project with probability close to one leading to the arrival rate being close to the probability of observing a posted project.

1.8.2 Commitment vs. Flexibility

The main objective of this paper is to quantify the welfare benefits and losses of granting the platform commitment power to information disclosure.

To see how commitment to information disclosure can arise, think about a regulator who decides to form a committee that closely monitors the process through which the platform evaluates the projects and assigns badges on the grounds of information it obtains. Given the ITS announced by the platform, the committee would make sure that the platform follows the announcement. The platform's incentive to misrepresent information, would no longer be a threat to the platform's reputation. Hence, any ITS announced by the platform would be credible. In terms of the formal model, platform having the commitment power is equivalent to solving the relaxed version of the problem defined by 10 and 11 i.e. the constraint 11 is dropped.

Comparing commitment to no commitment (flexibility) is appealing at least for four reasons. First, we know exactly what incentives are curbed under commitment, that is we can understand how commitment works. Second, we have an idea of how commitment can actually be implemented through regulation. Third, we do not request the platform to design complicated information transmission rules under commitment (e.g. involving more than two signals), like it would be the case if we were quantifying welfare effects of asymmetric information - in such a case we would need to see what would happen if we provided the investors with complete information about projects. In practice, providing complete information may be too time consuming in terms of communicating it to the investors, requiring us to consider modeling such details. Lastly, to my knowledge, this paper provides the first empirical examination of whether the outcomes under commitment, in the sense of Bayesian Persuasion (see Kamenica & Gentzkow 2011), can be achieved in a dynamic cheap talk setting in which exogenous commitment is replaced by the platform's long-run incentives (reputation concerns).

Testing for Commitment

Before comparing commitment to flexibility, I investigate which one is more likely to be the factual world. Given that information intermediation by Kickstarter is not regulated, we should expect that the factual world is a flexibility world. To formally validate this claim, I construct a statistical test to check whether the platform's cheap-talk incentives (incentives to misrepresent information) are binding it to achieve higher revenues.

Recall that if the factual is commitment then the platform solves,

$$Max_{q^* \in [0,1]} U(q^*)$$

If the argmax, q^{**}, is interior (which is the case according to the data) then the necessary condition for an optimum is,

$$\frac{\partial U(q^{**})}{\partial q^*} = 0$$

If the factual is no commitment, then 11 may bind at an optimum and it may be the case that,

$$\frac{\partial U(q^{**})}{\partial q^{*}} \neq 0$$

The test is as follows:

$$H0 : \frac{\partial \hat{U}(\hat{q}^{**})}{\partial q^{*}} = 0$$

$$H1 : \frac{\partial \hat{U}(\hat{q}^{**})}{\partial q^{*}} \neq 0$$

Under the alternative hypothesis, the test rejects that the factual is commitment.

It is worthwhile to note that the test does not use any particular information about 11, that is we just need to know that there are some incentive constraints under no commitment - exactly what those constraints are is irrelevant for this test. In addition, all the parameters, including $\hat{q}^{**}$, were estimated without any reference to either 1.10 or 1.11. These imply that the result of this test would be insensitive to whatever public monitoring technology or off-equilibrium path punishments one assumes.

To conduct the test, we need to know the asymptotic distribution of $\frac{\partial \hat{U}(\hat{q}^{**})}{\partial q^{*}}$. Since $\frac{\partial \hat{U}(\hat{q}^{**})}{\partial q^{*}}$ depends only on $\hat{\theta}$ and we already derived the asymptotic distribution of $n^{1/2}(\hat{\theta} - \theta)$, which is normal with mean 0 and variance-covariance matrix Σ (see the Appendix A), applying the delta-method we have,

$$n^{1/2}\left(\frac{\partial \hat{U}(\hat{q}^{**})}{\partial q^{*}} - \frac{\partial U(q^{**})}{\partial q^{*}}\right) \xrightarrow{D} \mathcal{N}\left(0, \left(\frac{\partial^2 U(q^{**})}{\partial q^{*}\partial \theta}\right)' \Sigma \frac{\partial^2 U(q^{**})}{\partial q^{*}\partial \theta}\right)$$

Using the above distribution, I conduct a Z-test. The resulting p value is 0.0272, meaning that we can reject the commitment assumption at 3 percent significance level.

Welfare effects of Commitment

So far, I have argued that we have reasons to think that the platform does not have commitment power and we know how to grant it commitment power through regulation. The only thing that remains is to check for the welfare effects of commitment.

Table 1.9 provides percentage change in the welfare of the platform, investors and creators from granting the platform commitment power. It is noteworthy that everyone benefits from commitment, even though this is not clear before estimating the model parameters (except for the platform). The platform's revenues would increase by 7.4 percent, investors' welfare would increase by 0.5 percent and creators' - by 4 percent. This result implies that a regulation that pushes the platform towards commitment should be well-received by everyone.

Table 1.9: Welfare Effects of Commitment

	$\Delta Percent$
Platform	7.4
Investor	0.5
Creator	4.0

Further Regulation

In practice, in addition to making sure that an intermediary abides to a disclosure rule, a regulation of disclosure rule involves regulator dictating such a rule. For example, Securities and Exchange Commission regulates disclosure requirements (rule) on the equity-based crowdfunding platforms by requiring a platform to make sure that certain information is transmitted from entrepreneurs to the investors. In this model, such regulation would mean that a regulator is choosing an ITS, q^*. What are the potential gains of regulating q^* after the platform's commitment problem is resolved?

I start by finding the best ITS for the investors and the best ITS for the creators. For example, if a regulator is maximizing only investors' welfare, it would be willing to set investor-best ITS on the top of granting the platform commitment power.

Tables 1.10 and 1.11 provide welfare changes from setting investor-best ITS and creator-best ITS, respectively. The platform's revenue gain (compared to the factual ITS) is 6.9 percent under the creator-best ITS and 1.6 percent under the investor-best ITS. This means that under any regulation the platform is better-off compared to the factual. The investor is also better-off under any regulation compared to the factual with the 0.7 percent improvement in investors' welfare under the investor-best ITS. Creators' welfare is reduced by 5.5

percent under the investor-best ITS.

It is noteworthy that compared to the platform-best ITS, the investor-best ITS increases investors' welfare further by only 0.2 percent and creator-best ITS increases creators' welfare further by only 0.5 percent. These numbers suggest that if one granted the platform commitment power to information disclosure, more stringent regulation would not be necessary, as investors' and creators' welfare would already be near their global optima.

Figure 1.9 depicts investors', creators' and platform's welfare as functions of q^*. Platform's and creators' welfare functions have very similar shapes. For the creators, this shape is driven by the probability that a project gets funded. This is so because creators' project posting costs are concentrated near zero and the shape of the creators' welfare function is driven only by the benefits which come about in the form of the probability that a project gets funded. The shape of the platform's revenue function is also driven by the probability that a project gets funded, in addition to the intensive margin of investment.

The shape of the investors' welfare function is driven by how well an ITS can partition quality space in order to maximize expectation of $q - c_b$, conditional on the event that a project was posted; and the probability that a project is posted on the platform. The latter is the channel through which investors' welfare depends indirectly on creators' welfare. As one increases the probability with which a project gets successfully funded, creators expect higher benefits from using the platform and this encourages more creators to post projects. Consequently, everything else equal, more projects posted translates into higher investors' welfare. However, this effect turns out to be approximately non-existent since β_{cr} is close to zero.

1.8.3 Public Monitoring

In this subsection, I discuss the role of public monitoring in curbing the platform's incentives to misrepresent information.

To start with, I ask whether public monitoring can substitute for commitment. This same question was asked by Best & Quigley 2017, who posed it in a repeated cheap-talk model with public monitoring in which a long-run sender tries to persuade short-run receivers to take certain actions and the state of the world is i.i.d. over time. They ask whether the sender's long-run incentives can substitute for full commitment a la Kamenica & Gentzkow 2011, in terms of sender's payoffs. It turns out that this is possible under very special cases. Even though the question I am asking is the same, I do this under the restricted strategy space of the sender, special case of monitoring and quantitatively.

To see whether public monitoring could substitute for commitment, I set $\pi(\hat{q}^{**}_{comm}) = 1$, where $\hat{q}^{**}_{comm}$ denotes the estimate of platform-optimal ITS under commitment, and see if 11 holds. It is easily verified that this is so. Hence, under extremely good public monitoring, the platform has reputation concerns strong enough to let it achieve commitment outcome. Can we be more specific about how good is enough for the substitution to take place?

We can recover a threshold, $\bar{\pi}(\hat{q}^{**}_{comm})$, such that the dynamic incentives (reputation concerns) substitute for commitment for all $\pi(\hat{q}^{**}_{comm}) \geq \bar{\pi}(\hat{q}^{**}_{comm})$ and do not substitute for commitment for all $\pi(\hat{q}^{**}_{comm}) < \bar{\pi}(\hat{q}^{**}_{comm})$. I recover this threshold to be $2.53e-7$. Since this number is the probability with which the public verifies project's quality in a given second independently over time and across projects, we can calculate the average time that the public would need to verify the quality of a project at this threshold. It turns out to be 1.5 months.

Since the observed ITS is not the same as $\hat{q}^{**}_{comm}$, we know that under the true $\pi(\hat{q}^{**}_{comm})$ it would not be credible for the platform to announce $\hat{q}^{**}_{comm}$ implying that under commitment, on average, more than 1.5 months would be needed for the public to verify quality. Hence, if the public monitoring was such that it required, on average, less than 1.5 months for a project's quality to be verified by the public - the reputation concerns would substitute for commitment.

I also recover the threshold under the factual, $\bar{\pi}(q^{**})$, which is $3.1e-7$, meaning that currently, on average, less than 1.2 months is needed for the public to verify quality. Taken together, the thresholds under commitment and factual imply that there is a better public monitoring (in the sense of higher true π) under factual ITS compared to what would have been under the commitment-ITS, $\hat{q}^{**}_{comm}$. To improve public monitoring under the commitment ITS, in practice, Kickstarter could create a special web-page/forum on which backers would share their experience with the projects. This would likely increase the probability that a project's quality is verified by the public after people invest in it.

Rationalizing the result, that in the factual world the platform is not achieving its commitment payoffs, depends on how one models the platform's long-run incentives. My mechanism suggests that for whatever reasons, the public would be more restricted or reluctant to conduct monitoring on its own under the commitment ITS compared to the factual ITS. Even though I do not take a stance on a specific story behind this, one scenario could be as follows: because $\hat{q}^{**}_{comm} > \hat{q}^{**}$, that is the ITS under the commitment sets higher quality standard for a project to get badged, competitors of the Kickstarter would decrease the efforts to verify badged projects' qualities, as they would expect that those projects are of very high quality, and hence negative campaigning has little value.

1.9 Conclusion

This chapter is an empirical study of the role that crowdfunding platforms play as information intermediaries. I identified incentives of a platform that are detrimental to this role, and proposed mechanisms that could mitigate such incentives.

Using the data from Kickstarter, I estimated a repeated cheap talk model and developed a statistical test to confirm that incentives to misrepresent information are present and that they are constraining the platform in achieving better outcomes for itself, entrepreneurs and investors. Complete annihilation of such incentives can be achieved via granting the platform commitment power to information disclosure. Regulation is one way to grant this power. Another way turns out to be public monitoring.

Public monitoring creates long-run reputation concerns for the platform and disciplines platform's cheap-talk incentives. I showed how strengthening public monitoring can substitute for commitment.

This work contributes to the existing literature by taking a step towards the identification and estimation of dynamic cheap talk and information design models; posing the question of whether information intermediation is possible, in the non-traditional context of crowdfunding; exposing how commitment power, a la Kamenica & Gentzkow 2011, can come about in practice; empirically examining whether public monitoring could substitute for exogenous commitment. I introduce two new questions in the empirical IO literature - What is the value of commitment to information disclosure? How can one extract that value? - and shows that it is feasible to empirically answer those questions, and provides a framework for doing so.

The summary message for a policy maker is as follows. Granting the platform commitment power would be Pareto improving. A regulator could accomplish this by verifying that the platform follows a promised information disclosure rule. If such a regulation is too costly, then the platform could try to improve public monitoring and achieve the commitment outcome. A traditional meaning of regulating disclosure requirements on the financial markets is that a regulator chooses information that has to be transmitted from entrepreneurs (borrowers) to investors (lenders). As long as the platform is granted commitment power to information disclosure, regulation of disclosure requirements, in the traditional sense, would not be of much additional value.

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Appendix A

Distribution of the Number of Backers

We have the following sequence of equalities,

$$g(k'|F_b(E(q|I))) =$$

$$\sum_{k=k'}^{\infty}\int_0^{\infty}\frac{e^{-n}n^k}{k!}\frac{\alpha^\beta}{\Gamma(\beta)}n^{\beta-1}e^{-\alpha n}dn\binom{k}{k'}F_b(E(q|I))^{k'}(1-F_b(E(q|I)))^{k-k'} =$$

$$\int_0^{\infty}\left[\sum_{k=k'}^{\infty}\frac{e^{-n}n^k}{k!}\binom{k}{k'}F_b(E(q|I))^{k'}(1-F_b(E(q|I)))^{k-k'}\right]\frac{\alpha^\beta}{\Gamma(\beta)}n^{\beta-1}e^{-\alpha n}dn =$$

$$\int_0^{\infty}\frac{e^{-nF_b(E(q|I))}(nF_b(E(q|I)))^{k'}}{k'!}\frac{\alpha^\beta}{\Gamma(\beta)}n^{\beta-1}e^{-\alpha n}dn =$$

$$\frac{F_b(E(q|I))^{k'}\alpha^\beta}{k'!\Gamma(\beta)}\int_0^{\infty}n^{k'+\beta-1}e^{-(\alpha+F_b(E(q|I)))n}dn =$$

$$\frac{\Gamma(k'+\beta)}{\Gamma(k'+1)\Gamma(\beta)}\frac{F_b(E(q|I))^{k'}\alpha^\beta}{(\alpha+F_b(E(q|I)))^{k'+\beta}} =$$

$$\binom{k'+\beta-1}{k'}\left(\frac{\alpha}{\alpha+F_b(E(q|I))}\right)^{\beta}\left(\frac{F_b(E(q|I))}{\alpha+F_b(E(q|I))}\right)^{k'}$$

Here $\Gamma(\cdot)$ is the gamma function. For the third equality see Myerson 1998. In the fifth equality I integrate out the density of Gamma distribution. The final expression is the probability mass function of the Negative Binomial distribution.

Proof of Proposition 1.1

Proof. Under the assumptions 1.1 and 1.2, $F_{cr}(Z(I))$, $F(q|m,l)$, $U_1(m,l,q^*)$ and $U(q^*)$ are continuous in q^*. However, $U_0(m,l,q^*)$ is discontinuous at $q^*=0$. This is due to $E(q|I_-)$ jumping to $\frac{\int_0^1 f(m,l|q)f(q)qdq}{\int_0^1 f(m,l|q)f(q)dq}$ at $q^*=0$. For this reason, I show continuity of $V(q^*,d)$ only on $[\epsilon,1]$ for any $1>\epsilon>0$. The arguments for why the unique $V(q^*,d)$ satisfying 15 exists and is continuous in q^* are then standard (see, Stokey & Lucas 1989). Here I sketch the arguments. First we define the operator T as follows,

$$
\begin{aligned}
&T(V'(q^*,d)) = \\
&\sum_{(m,l)\in M\times L} f(m,l)\Big[(1-F(q^*|m,l))F_{cr}(Z(I_+,q^*))W_1'(m,l,q^*,d)+ \\
&F(q^*|m,l)F_{cr}(Z(I_-,q^*))W_0'(m,l,q^*,d)\Big]+ \\
&\Big[1-\sum_{(m,l)\in M\times L} f(m,l)+\sum_{(m,l)\in M\times L} f(m,l)\big[(1-F(q^*|m,l))(1-F_{cr}(Z(I_+,q^*)))+ \\
&F(q^*|m,l)(1-F_{cr}(Z(I_-,q^*)))\big]\Big]W'(q^*,d)
\end{aligned}
\tag{1.16}
$$

In 1.16, W_1', W_0' and W' depend on $V'(q^*,d)$ and are given by the expressions 1.12, 1.13 and 1.14. We can easily verify that T satisfies Blackwell's sufficient conditions for contraction and maps bounded and continuous functions in q^* to bounded and continuous functions in q^*. This proves that unique $V(q^*,d)$ satisfying 15 exists and is continuous in q^*.

Further, I show that $V(q^*,d)$ is decreasing in d. Again, replicating the arguments from the Stokey & Lucas 1989, we need to show that if we have $V'(q^*,d)$ decreasing in d then $T(V'(q^*,d))$ is also decreasing in d. Inspecting 1.12, 1.13 and 1.14 we can easily verify that this is indeed the case.

To show that $V(q^*,d)$ is convex in d we need to verify that if $V'(q^*,d)$ convex in d then $T(V'(q^*,d))$ is also convex. Take a convex $V'(q^*,d)$. If we verify that W_1', W_0' and W' are all convex in d then this will imply the result. Let's first investigate W_1'. We need to show that $W_1'(m,l,q^*,d)-W_1'(m,l,q^*,d+1)\geq W_1'(m,l,q^*,d')-W_1'(m,l,q^*,d'+1)$ for all $(q,m,l)\in[0,1]\times M\times L$ and $d'>d$. Evaluating the inequality, this is equivalent to showing that $(1-\pi(q^*))^d(V'(q^*,d)-(1-\pi(q^*))V'(q^*,d+1)-\pi(q^*)V)$ is decreasing in d which is further equivalent to showing that $V'(q^*,d)-V'(q^*,d+1)+\pi(q^*)V'(q^*,d+1)$ is decreasing in d. By assumption $V'(q^*,d)$ is convex and decreasing in d implying that the last expression is decreasing in d.

The same arguments apply to show that W' is convex. As for the W'_0, it is maximum of convex functions and hence is also convex.

□

Proof of Proposition 1.2

Proof. Lets first make $U_0(m,l,q^*)$ continuous at $q^*=0$ by setting $E(q|I_-)=0$ whenever $q^*=0$. Note that under such a modification none of the quantities in 1.11 are affected for $q^*\neq 0$. This claim is obvious for $U_1(m,l,q^*)$, $U(q^*)$ and $U_0(m,l,q^*)$. As for $V(q^*,d)$, note that q^* is a fixed parameter of the dynamic problem so changing something for certain values of q^* does not affect $V(q^*,d)$ for other values of q^*.

Under this modification, the left and right hand sides of 1.11 are continuous in q^* for $q^*\in[0,1]$. It follows that the set of q^* that satisfy the inequality in 1.11 is a compact set. Call this set A.

Now A is not an actual feasible set that we need. We have to discard the assumption that $E(q|I_-)=0$ at $q^*=0$. The only thing that can potentially change, from discarding this assumption, is $q^*=0$ to be added to the set A. If $0\notin A$ then $0\cup A$ would be compact because we know A is compact and also one point set is compact. If $0\in A$ then $0\cup A=A$ that is compact. Hence, the actual constrained set must be compact. Then, by the Bolzano-Weierstrass theorem there exists q^{**} that solves the problem defined by 1.10 and 1.11.

□

Theoretical Foundation for the Worst Punishment

I modify the game and discuss how under this modified game the platform would choose to announce ITS only at the beginning of period 0 and optimally chooses the worst punishment if it is ever detected to have deviated from the announced ITS.

Suppose that the platform, at the beginning of each period, can announce a plan of continuation play. In this modification of the game, the public history is amended with the platforms' decision whether or not to announce the plan in each period and the announced plan. A plan announced at the beginning of t prescribes the platform's, creators' and investors' behavioral strategies for each realization of public history from t onward. This modification is closely related to Safronov & Strulovici 2014 and, can be interpreted as the platform having the renegotiation power.

The platform and only it having the ability to announce a plan of continuation play at each date is justified as follows. First, even if in practice we do not observe the platform announcing the entire continuation play, it could have done so if this would be beneficial for it. Hence, the platform not announcing such plans should mean that it cannot benefit by doing so and hence it cannot be in the equilibrium that is worse for it than having explicitly announced a plan. Second, since creators and investors are short lived on the platform and there is a large population of them, they cannot coordinate to compete with the platform by rejecting the platform's plans and suggesting their own.

In this modified game, the platform's behavioral strategy in each period in addition to the mixing over signals includes mixing over the continuation plans. I restrict the plan announcement decisions, plans and ITSs to depend only on the quality of the last project that was verified by the public and last announced plan.

Note that the value to the platform (expected discounted sum of revenues) at the beginning of period 0, must be maximized across all the equilibria in this modified game because such a plan would be chosen by the platform at the beginning of period 0.

First, since the platform is maximizing its value at the beginning of period 0 by announcing a plan, it is clear that it should announce the worst punishment for itself if it takes off the equilibrium actions.

Second, I argue that we can without loss of generality focus on equilibria in which the platform makes the plan announcement only at the beginning of period 0 and any further announcement is punished by permanent babbling. To see this, suppose the platform announces a plan at date 0, other plan at date t after some history and maybe yet another plan after some history at $t' > t$. Since those plans do not depend on previous project qualities observed by the platform, they cannot signal platform's private information to the investors and backers. Consider plan announced at t. It prescribes certain ITS from t onward. We can prescribe exactly same ITS after that history at t according to the plan in period 0.

So, we end up with the model in the main text i.e. ITS is announced only at the beginning of period 0 and any deviation by the platform is punished by the permanent babbling equilibrium.

Derivation of the Variance-Covariance Matrix of θ

Here I sketch the derivation of the asymptotic distribution of $\hat{\theta}$. The three stage maximum likelihood estimator satisfies,

$$\sum_{i=1}^{n} \frac{\partial L_{1i}(\hat{\theta}_1)}{\partial \theta_1} = 0 \tag{1.31}$$

$$\sum_{i=1}^{n} \frac{\partial L_{2i}(\hat{\theta}_1, \hat{\theta}_2)}{\partial \theta_2} = 0 \tag{1.32}$$

$$\sum_{i=1}^{n} \frac{\partial L_{3i}(\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3)}{\partial \theta_3} = 0 \tag{1.33}$$

Under standard regularity conditions, $\hat{\theta}$ is consistent (see Murohy & Topel 1985). Using mean-value theorem we can expand 1.31, 1.32 and 1.33 about θ, yielding,

$$-\frac{1}{n^{1/2}} \sum_{i=1}^{n} \frac{\partial L_{1i}(\theta_1)}{\partial \theta_1} = \frac{1}{n} \sum_{i=1}^{n} \frac{\partial^2 L_{1i}(\bar{\theta}_1)}{\partial \theta_1 \partial \theta_1'} n^{1/2}(\hat{\theta}_1 - \theta_1) \tag{1.34}$$

$$\begin{aligned} -\frac{1}{n^{1/2}} \sum_{i=1}^{n} \frac{\partial L_{2i}(\theta_1, \theta_2)}{\partial \theta_2} = \frac{1}{n} \sum_{i=1}^{n} \frac{\partial^2 L_{2i}(\bar{\theta}_1, \bar{\theta}_2)}{\partial \theta_2 \partial \theta_1'} n^{1/2}(\hat{\theta}_1 - \theta_1) + \\ \frac{1}{n} \sum_{i=1}^{n} \frac{\partial^2 L_{2i}(\bar{\theta}_1, \bar{\theta}_2)}{\partial \theta_2 \partial \theta_2'} n^{1/2}(\hat{\theta}_2 - \theta_2) \end{aligned} \tag{1.35}$$

$$
\begin{aligned}
&-\frac{1}{n^{1/2}}\sum_{i=1}^{n}\frac{\partial L_{3i}(\theta_1,\theta_2,\theta_3)}{\partial\theta_3} = \frac{1}{n}\sum_{i=1}^{n}\frac{\partial^2 L_{3i}(\bar\theta_1,\bar\theta_2,\bar\theta_3)}{\partial\theta_3\partial\theta_1'}n^{1/2}(\hat\theta_1-\theta_1)+ \\
&\frac{1}{n}\sum_{i=1}^{n}\frac{\partial^2 L_{3i}(\bar\theta_1,\bar\theta_2,\bar\theta_3)}{\partial\theta_3\partial\theta_2'}n^{1/2}(\hat\theta_2-\theta_2)+\frac{1}{n}\sum_{i=1}^{n}\frac{\partial^2 L_{3i}(\bar\theta_1,\bar\theta_2,\bar\theta_3)}{\partial\theta_3\partial\theta_3'}n^{1/2}(\hat\theta_3-\theta_3)
\end{aligned}
\tag{1.36}
$$

In 1.34, 1.35 and 1.36 bars above variables denote their values between true value and estimate. Using the law of large numbers, the fact that $\bar\theta \xrightarrow{p} \theta$ and properties of Fisher Information we have,

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\partial^2 L_{1i}(\bar\theta_1)}{\partial\theta_1\partial\theta_1'} \xrightarrow{p} -E\frac{\partial L_1(\theta_1)}{\partial\theta_1}\left(\frac{\partial L_1(\theta_1)}{\partial\theta_1}\right)' \equiv R_1(\theta_1)$$

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\partial^2 L_{2i}(\bar\theta_1,\bar\theta_2)}{\partial\theta_2\partial\theta_1'} \xrightarrow{p} -E\frac{\partial L_2(\theta_1,\theta_2)}{\partial\theta_2}\left(\frac{\partial L_2(\theta_1,\theta_2)}{\partial\theta_1}\right)' \equiv R_2(\theta)'$$

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\partial^2 L_{2i}(\bar\theta_1,\bar\theta_2)}{\partial\theta_2\partial\theta_2'} \xrightarrow{p} -E\frac{\partial L_2(\theta_1,\theta_2)}{\partial\theta_2}\left(\frac{\partial L_2(\theta_1,\theta_2)}{\partial\theta_2}\right)' \equiv R_2(\theta_2)$$

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\partial^2 L_{3i}(\bar\theta_1,\bar\theta_2,\bar\theta_3)}{\partial\theta_3\partial\theta_1'} \xrightarrow{p} -E\frac{\partial L_3(\theta_1,\theta_2,\bar\theta_3)}{\partial\theta_3}\left(\frac{\partial L_3(\theta_1,\theta_2,\bar\theta_3)}{\partial\theta_1}\right)' \equiv R_3(\theta)'$$

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\partial^2 L_{3i}(\bar\theta_1,\bar\theta_2,\bar\theta_3)}{\partial\theta_3\partial\theta_2'} \xrightarrow{p} -E\frac{\partial L_3(\theta_1,\theta_2,\bar\theta_3)}{\partial\theta_3}\left(\frac{\partial L_3(\theta_1,\theta_2,\bar\theta_3)}{\partial\theta_2}\right)' \equiv R_4(\theta)'$$

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\partial^2 L_{3i}(\bar\theta_1,\bar\theta_2,\bar\theta_3)}{\partial\theta_3\partial\theta_3'} \xrightarrow{p} -E\frac{\partial L_3(\theta_1,\theta_2,\bar\theta_3)}{\partial\theta_3}\left(\frac{\partial L_3(\theta_1,\theta_2,\bar\theta_3)}{\partial\theta_3}\right)' \equiv R_3(\theta_3)$$

Then, from equation 1.34 we have the following asymptotic equivalence:

$$n^{1/2}(\hat\theta_1-\theta_1) = -R_1(\theta_1)^{-1}\frac{1}{n^{1/2}}\sum_{i=1}^{n}\frac{\partial L_{1i}(\theta_1)}{\partial\theta_1} \tag{1.37}$$

Substituting 1.37 into 1.35 we have the following asymptotic equivalence:

$$n^{1/2}(\hat{\theta}_2 - \theta_2) = -R_2(\theta_2)^{-1}\frac{1}{n^{1/2}}\sum_{i=1}^{n}\frac{\partial L_{2i}(\theta_1,\theta_2)}{\partial\theta_2} +$$
$$R_2(\theta_2)^{-1}R_2(\theta)'R_1(\theta_1)^{-1}\frac{1}{n^{1/2}}\sum_{i=1}^{n}\frac{\partial L_{1i}(\theta_1)}{\partial\theta_1} \tag{1.38}$$

Substituting 1.37 and 1.38 into 1.36 we have the following asymptotic equivalence:

$$n^{1/2}(\hat{\theta}_3 - \theta_3) = -R_3(\theta_3)^{-1}\frac{1}{n^{1/2}}\sum_{i=1}^{n}\frac{\partial L_{3i}(\theta_1,\theta_2,\theta_3)}{\partial\theta_3} +$$
$$R_3(\theta_3)^{-1}R_3(\theta)'R_1(\theta_1)^{-1}\frac{1}{n^{1/2}}\sum_{i=1}^{n}\frac{\partial L_{1i}(\theta_1)}{\partial\theta_1} +$$
$$R_3(\theta_3)^{-1}R_4(\theta)'R_2(\theta_2)^{-1}R_2(\theta)'R_1(\theta_1)^{-1}\frac{1}{n^{1/2}}\sum_{i=1}^{n}\frac{\partial L_{1i}(\theta_1)}{\partial\theta_1} - \tag{1.39}$$
$$R_3(\theta_3)^{-1}R_4(\theta)'R_2(\theta_2)^{-1}\frac{1}{n^{1/2}}\sum_{i=1}^{n}\frac{\partial L_{2i}(\theta_1,\theta_2)}{\partial\theta_2}$$

The only random vectors that the right hand sides of 1.37,1.38 and 1.39 involve are,

$$\frac{1}{n^{1/2}}\sum_{i=1}^{n}\frac{\partial L_{1i}(\theta_1)}{\partial\theta_1}$$

$$\frac{1}{n^{1/2}}\sum_{i=1}^{n}\frac{\partial L_{2i}(\theta_1,\theta_2)}{\partial\theta_2}$$

$$\frac{1}{n^{1/2}}\sum_{i=1}^{n}\frac{\partial L_{3i}(\theta_1,\theta_2,\theta_3)}{\partial\theta_3}$$

By the central limit theorem, as n approaches infinity, the distribution of those vectors converges to joint normal. This implies that $n^{1/2}(\hat{\theta}-\theta)$ is asymptotically normally distributed with the mean 0 and some variance-covariance matrix, that I denote Σ. To find Σ, I calculate variance-covariance of the right hand sides of 1.37, 1.38 and 1.39. Let me introduce some additional notation,

$$R_5(\theta) \equiv \frac{\partial L_1(\theta_1)}{\partial\theta_1}\left(\frac{\partial L_{2i}(\theta_1,\theta_2)}{\partial\theta_2}\right)'$$

$$R_6(\theta) \equiv \frac{\partial L_1(\theta_1)}{\partial\theta_1}\left(\frac{\partial L_3(\theta_1,\theta_2,\theta_3)}{\partial\theta_3}\right)'$$

$$R_7(\theta) \equiv \frac{\partial L_2(\theta_1,\theta_2)}{\partial \theta_2}\left(\frac{\partial L_3(\theta_1,\theta_2,\theta_3)}{\partial \theta_3}\right)'$$

Let $\Sigma_{\theta_i,\theta_j}$ denote variance-covariance of (θ_i, θ_j) for $i = 1, 2, 3$ and $j = 1, 2, 3$. We have the following:

$$\Sigma_{\theta_1,\theta_1} = R_1(\theta_1)^{-1}$$

$$\Sigma_{\theta_2,\theta_2} = R_2(\theta_2)^{-1}+$$
$$R_2(\theta_2)^{-1}[R_2(\theta)'R_1(\theta_1)^{-1}R_2(\theta) - R_5(\theta)'R_1(\theta_1)^{-1}R_2(\theta) - R_2(\theta)'R_1(\theta_1)^{-1}R_5(\theta)]R_2(\theta_2)^{-1}$$

$$\Sigma_{\theta_1,\theta_2} = R_1(\theta_1)^{-1}[R_5(\theta) - R_2(\theta)]R_2(\theta_2)^{-1}$$

$$\Sigma_{\theta_3,\theta_3} = R_3(\theta_3)^{-1} + R_3(\theta_3)^{-1}R_3(\theta)'R_1(\theta_1)^{-1}R_3(\theta)R_3(\theta_3)^{-1}+$$
$$R_3(\theta_3)^{-1}R_4(\theta)'R_2(\theta_2)^{-1}R_2(\theta)'R_1(\theta_1)^{-1}R_2(\theta)R_2(\theta_2)^{-1}R_4(\theta)R_3(\theta_3)^{-1}-$$
$$R_3(\theta_3)^{-1}[R_4(\theta)'R_2(\theta_2)^{-1}R_4(\theta) + R_6(\theta)'R_1(\theta_1)^{-1}R_3(\theta)]R_3(\theta_3)^{-1}-$$
$$R_3(\theta_3)^{-1}R_6(\theta)'R_1(\theta_1)^{-1}R_2(\theta)R_2(\theta_2)^{-1}R_4(\theta)R_3(\theta_3)^{-1}+$$
$$R_3(\theta_3)^{-1}[R_7(\theta)'R_2(\theta_2)^{-1}R_4(\theta) - R_3(\theta)'R_1(\theta_1)^{-1}R_6(\theta)]R_3(\theta_3)^{-1}+$$
$$R_3(\theta_3)^{-1}R_3(\theta)'R_1(\theta_1)^{-1}R_2(\theta)R_2(\theta_2)^{-1}R_4(\theta)R_3(\theta_3)^{-1}-$$
$$R_3(\theta_3)^{-1}R_3(\theta)'R_1(\theta_1)^{-1}R_5(\theta)R_2(\theta_2)^{-1}R_4(\theta)R_3(\theta_3)^{-1}-$$
$$R_3(\theta_3)^{-1}R_4(\theta)'R_2(\theta_2)^{-1}R_2(\theta)'R_1(\theta_1)^{-1}R_6(\theta)R_3(\theta_3)^{-1}+$$
$$R_3(\theta_3)^{-1}R_4(\theta)'R_2(\theta_2)^{-1}R_2(\theta)'R_1(\theta_1)^{-1}R_3(\theta)R_3(\theta_3)^{-1}-$$
$$R_3(\theta_3)^{-1}R_4(\theta)'R_2(\theta_2)^{-1}R_2(\theta)'R_1(\theta_1)^{-1}R_5(\theta)R_2(\theta_2)^{-1}R_4(\theta)R_3(\theta_3)^{-1}+$$
$$R_3(\theta_3)^{-1}[R_4(\theta)'R_2(\theta_2)^{-1}R_7(\theta) - R_4(\theta)'R_2(\theta_2)^{-1}R_5(\theta)'R_1(\theta_1)^{-1}R_3(\theta)]R_3(\theta_3)^{-1}-$$
$$R_3(\theta_3)^{-1}R_4(\theta)'R_2(\theta_2)^{-1}R_5(\theta)'R_1(\theta_1)^{-1}R_2(\theta)R_2(\theta_2)^{-1}R_4(\theta)R_3(\theta_3)^{-1}$$

$$\Sigma_{\theta_1,\theta_3} = R_1(\theta_1)^{-1}[R_6(\theta) - R_3(\theta) + (R_5(\theta) - R_2(\theta))R_2(\theta_2)^{-1}R_4(\theta)]R_3(\theta_3)^{-1}$$

$$\begin{aligned}
\Sigma_{\theta_2,\theta_3} &= R_2(\theta_2)^{-1}[R_7(\theta) - R_5(\theta)'R_1(\theta_1)^{-1}R_3(\theta)]R_3(\theta_3)^{-1}+ \\
&R_2(\theta_2)^{-1}[R_4(\theta) - R_5(\theta)'R_1(\theta_1)^{-1}R_2(\theta)R_2(\theta_2)^{-1}R_4(\theta)]R_3(\theta_3)^{-1}+ \\
&R_2(\theta_2)^{-1}R_2(\theta)'R_1(\theta_1)^{-1}[R_3(\theta) - R_6(\theta)]R_3(\theta_3)^{-1}+ \\
&R_2(\theta_2)^{-1}R_2(\theta)'R_1(\theta_1)^{-1}[R_2(\theta) - R_5(\theta)]R_2(\theta_2)^{-1}R_4(\theta)R_3(\theta_3)^{-1}
\end{aligned}$$

Calculating $V(q^*, d)$

The state space for $V(q^*, d)$ is the set of all natural numbers. I take $750/\pi$ as the upper bound on the state space when finding the fixed point of $V(q^*, d)$. For a given probability, π, of verifying project quality, the probability of detecting at least one deviation becomes numerically zero whenever $d \approx 750/\pi$. Hence, for all states above $750/\pi$ we know that the platform will be detected for sure in the following period and hence we can pin down the value for such states. Pinning down the value for very high states enables me to, alternatively, solve the problem using backward induction on the states starting from the state $750/\pi$. However, the value function iteration is more time efficient as it allows me to avoid loops. For the results presented in the current version of the paper, for each π, I consider at most $100,000$ equally spaced points in the state space.

Tables and Figures

Table 1.2(a): Regression Results

	# of Backers	Amount Pledged
Intercept	-369.17*	-1.3e+05***
	(204.63)	(3.31e+04)
Goal	1.7e-03**	0.51***
	(1.0e-3)	(0.13)
Length	11.56***	248.96
	(1.18)	(291.72)
Goal^2	-2.57e-08**	-2.55e-06
	(1.24e-08)	(2e-06)
Goal*Length	-9.43e-06	-1.48e-03
	(1.51e-05)	(2e-3)
Length^2	-0.14***	-2.62
	(0.01)	(2.3)
Badged	63.87	9,202
	(107.6)	(1.74e+04)
Badged*Goal	7.32e-03**	0.16
	(3e-03)	(0.5)
Badged*Length	-1.86	-402.81
	(5.3)	(857.8)
Badged*Goal^2	-8.7e-08**	1.18e-05*
	(4.02e-08)	(6.5e-06)
Badged*Goal*Length	2.16e-05	-1.42e-04
	(6.28e-05)	(0.01)
Badged*Length^2	0.03	4.79
	(0.06)	(10.41)
Time	1.52e-07	8.06e-05***
	(1.37e-07)	(2.21e-05)
# of Backers		111.42***
		(2.2)
R^2	0.111	0.421

Notes: ***, ** and * stand for the significance at the 1, 5 and 10 percent level, respectively.

Table 1.2(b): Regression Results: PWL vs. Staff-Pick

	# of Backers	Amount Pledged
Intercept	3,270*	3.61e+05
	(1,974)	(2.95e+05)
Goal	2.4e-03	0.25
	(2.0e-3)	(0.25)
Length	7.78***	111.96
	(2.42)	(362.7)
Goal^2	-5.46e-08**	3.19e-06
	(2.45e-08)	(3.67e-06)
Goal*Length	6.72e-06	-4.2e-03
	(2.83e-05)	(4e-03)
Length^2	-0.09***	-0.95
	(0.029)	(4.34)
Badged	81.75	2.01e+04
	(235.33)	(3.52e+04)
Badged*Goal	-2.6e-03**	-4.57***
	(0.01)	(1.17)
Badged*Length	5.71	628.26
	(12.41)	(1,853)
Badged*Goal^2	1.53e-07**	9.74e-06
	(1.03e-07)	(1.54e-05)
Badged*Goal*Length	-7.97e-05	0.13***
	(1.52e-04)	(0.02)
Badged*Length^2	-0.08	-31.53
	(0.16)	(23.15)
Time	-2.32e-06	-3e-04*
	(1.36e-06)	(2.4e-04)
# of Backers		101.62***
		(4.34)

Table 1.2(b) continued:

PWL	-3,324*	-6.98e+05**
	(2,042)	(3.05e+05)
PWL*Goal	-8e-04	0.37
	(2.0e-03)	(0.35)
PWL*Length	2.49	127.84
	(3.24)	(486.7)
PWL*Goal^2	4.25e-08	-8.05e-06
	(3.46e-08)	(5.17e-06)
PWL*Goal*Length	-7.93e-06	0.01
	(4.17e-05)	(0.01)
PWL*Length^2	-0.03	-2.56
	(0.04)	(5.86)
PWL*Badged	-132.86	-4.78e+04
	(296.11)	(4.42e+04)
PWL*Badged*Goal	0.02*	6.59***
	(0.01)	(1.41)
PWL*Badged*Length	-6.08	-70.49
	(15.56)	(2,323)
PWL*Badged*Goal^2	-3.44e-07***	1.83e-05
	(1.27e-07)	(1.9e-05)
PWL*Badged*Goal*Length	5.62e-05	-0.2***
	(2e-04)	(0.03)
PWL*Badged*Length^2	0.12	37.74
	(0.2)	(30)
PWL*Time	2.28e-06	5e-04**
	(1.41e-06)	(2.2e-04)
PWL*# of Backers		10.44*
		(5.72)
R^2	0.111	0.456

Table 1.3: Estimate of θ_1

$\hat{q}^{**}$	$\hat{\gamma}(lo,hi)$	$\hat{\gamma}(hi,lo)$	$\hat{\gamma}(hi,hi)$	$\hat{\beta}_b$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_0$	$\hat{\beta}_1$
0.3003	0.0025	0.9713	1.003e-5	1.624	5.778e-8	4.653e-4	0.3305	1.509
(0.094)	(0.1198)	(0.0936)	(3.695e-6)	(0.1659)	(1.943e-7)	(1.5e-4)	(0.0072)	(0.1045)

Table 1.4: Estimate of θ_2

$\hat{a}_1$	$\hat{a}_2$	$\hat{a}_3$	$\hat{a}_4$	$\hat{a}_5$	$\hat{a}_6$
3.096e-5	0.0275	-1.533	-1.668e-5	-1.476e-6	-9.545e-6
(1.449e-6)	(0.0015)	(0.0964)	(1.750e-5)	(2.588e-5)	(3.259e-5)

Table 1.5: Estimate of θ_3

$\hat{f}(lo, lo)$	$\hat{f}(lo, hi)$	$\hat{f}(hi, lo)$	$\hat{f}(hi, hi)$	$\hat{\beta}_{cr}$
3.435e-5	2.754e-5	1.594e-5	1.400e-5	1.431e-15
(8.344e-6)	(7.657e-6)	(3.829e-6)	(3.734e-6)	

Table 1.6: Model Fit: # of Backers

	Mean		Std. Dev.	
	Data	Model	Data	Model
$(lo, lo, 0)$	65.80	65.04	125.60	113.42
$(lo, lo, 1)$	194.30	226.09	158.14	184.66
$(lo, hi, 0)$	116.81	114.72	168.47	199.83
$(lo, hi, 1)$	232.89	266.76	196.01	217.77
$(hi, lo, 0)$	64.28	66.17	136.37	115.38
$(hi, lo, 1)$	281.06	227.22	201.07	185.58
$(hi, hi, 0)$	111.06	114.83	171.52	200.01
$(hi, hi, 1)$	324.64	266.87	199.88	217.86

Notes: The table presents the model fit of the first two moments of the distribution of the number of backers for all combinations of (m, l, s).

Table 1.7: Model Fit: Pledges

	Mean	
	Data	Model
$(lo, lo, 0)$	6,701	7,560
$(lo, lo, 1)$	31,229	23,848
$(lo, hi, 0)$	12,450	12,368
$(lo, hi, 1)$	29,267	26,761
$(hi, lo, 0)$	12,048	11,352
$(hi, lo, 1)$	61,437	62,474
$(hi, hi, 0)$	21,143	22,286
$(hi, hi, 1)$	69,187	82,101

Table 1.8: Model Fit: Probability of Posting a Project (per second)

	Mean	
	Data	Model
(lo, lo)	3.435e-5	3.435e-5
(lo, hi)	2.754e-5	2.754e-5
(hi, lo)	1.594e-5	1.594e-5
(hi, hi)	1.40e-5	1.40e-5

Table 1.10: Best ITS for the Investors

	$\Delta Percent$
Platform	1.6
Investor	0.7
Creator	-5.5

Notes: The percentage changes are from the factual ITS.

Table 1.11: Best ITS for the Creators

	$\Delta Percent$
Platform	6.9
Investor	0.4
Creator	4.5

Notes: The percentage changes are from the factual ITS.

Figure 1.2: Distribution of the Number of Backers

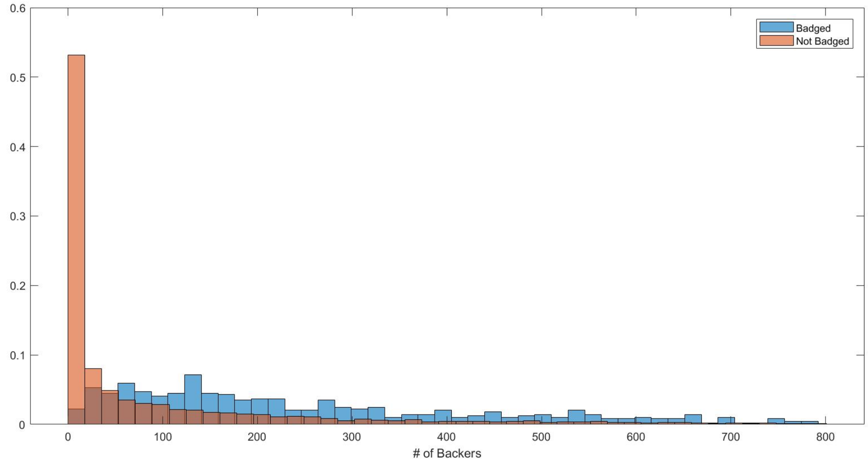


Figure 1.3: Distribution of the Amount Pledged

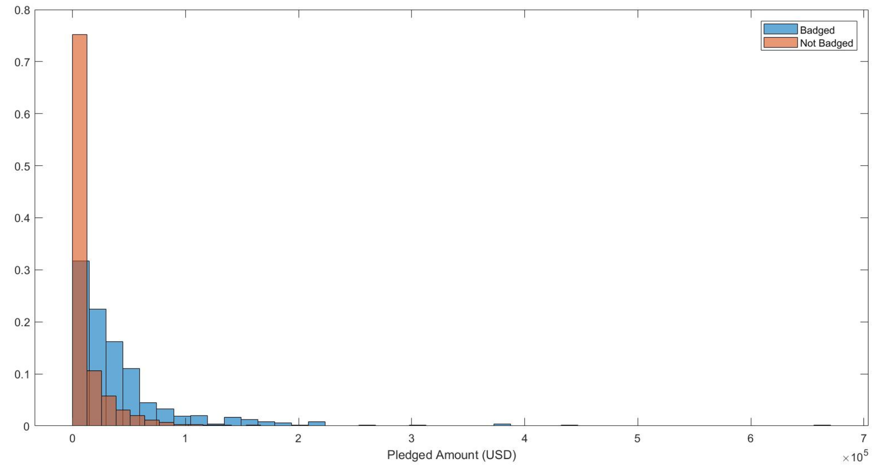


Figure 1.4: Level Curves for the Right Hand Sides of the Equations 1.29 and 1.30

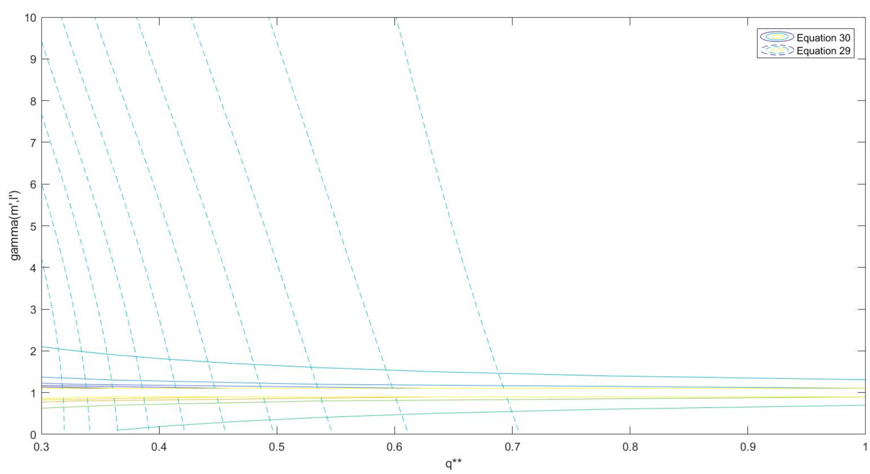


Figure 1.5: Distribution of the Goal

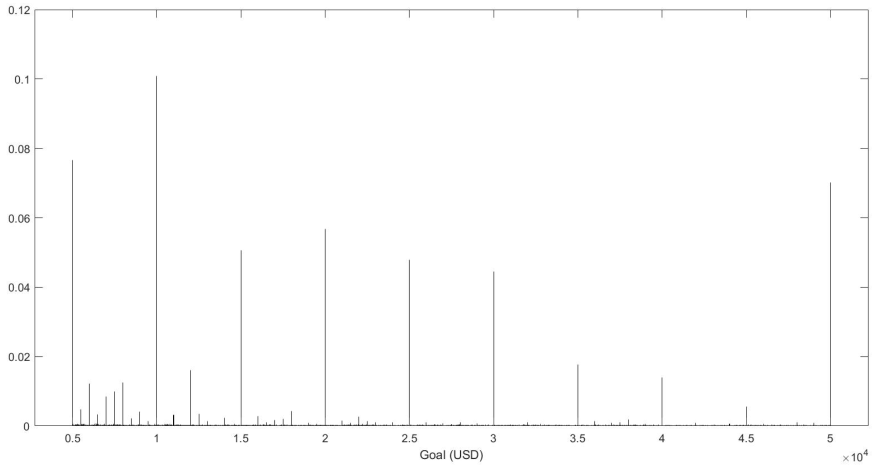


Figure 1.6: Distribution of the Project Length

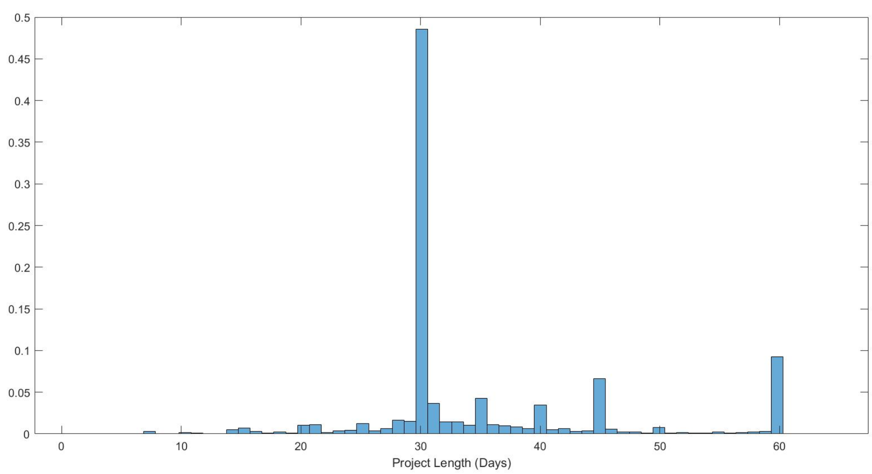


Figure 1.7: Model Fit of the Distr. of # of Backers

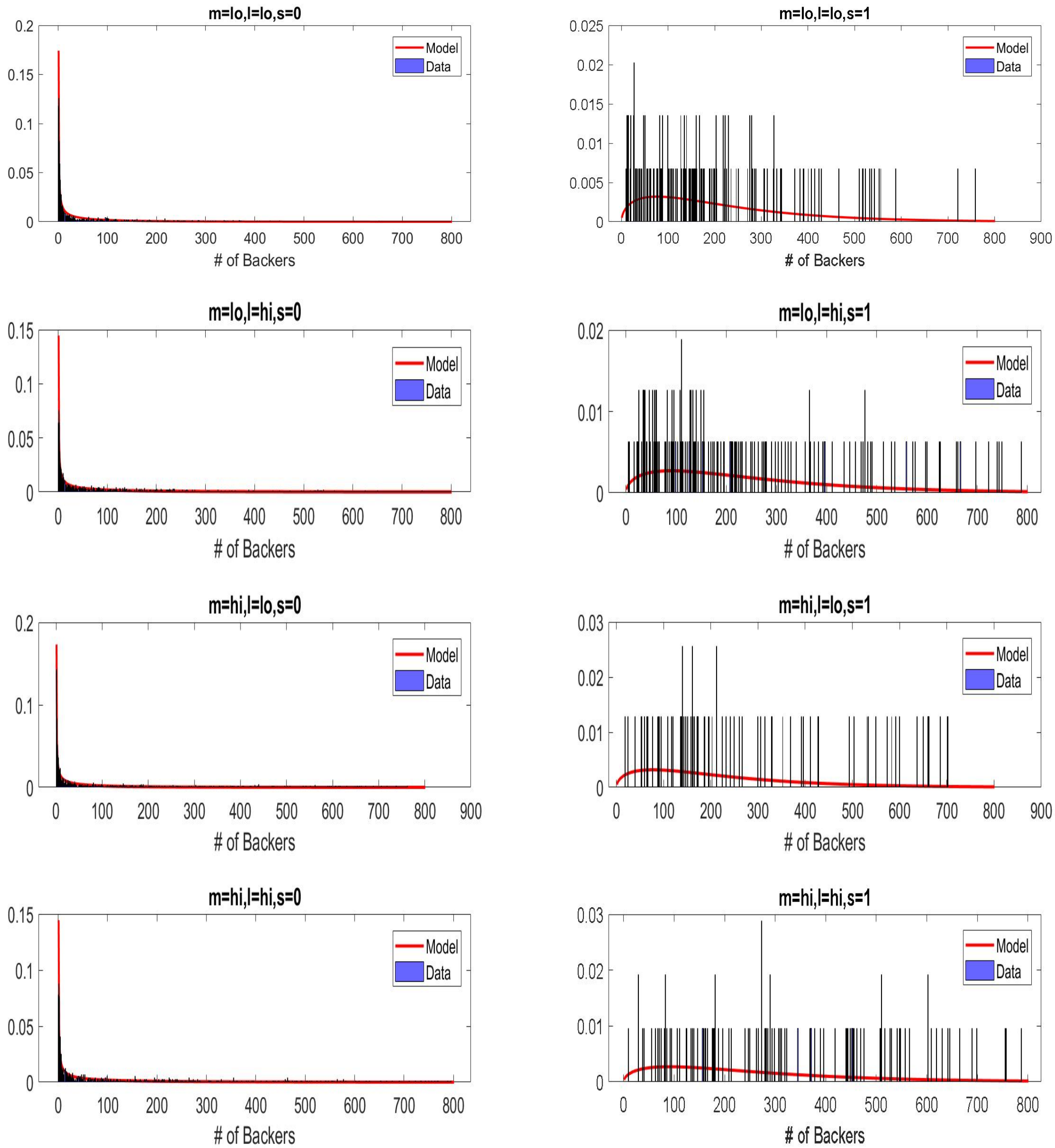


Figure 1.8: Model Fit of the Mean Pledges Against # of Backers

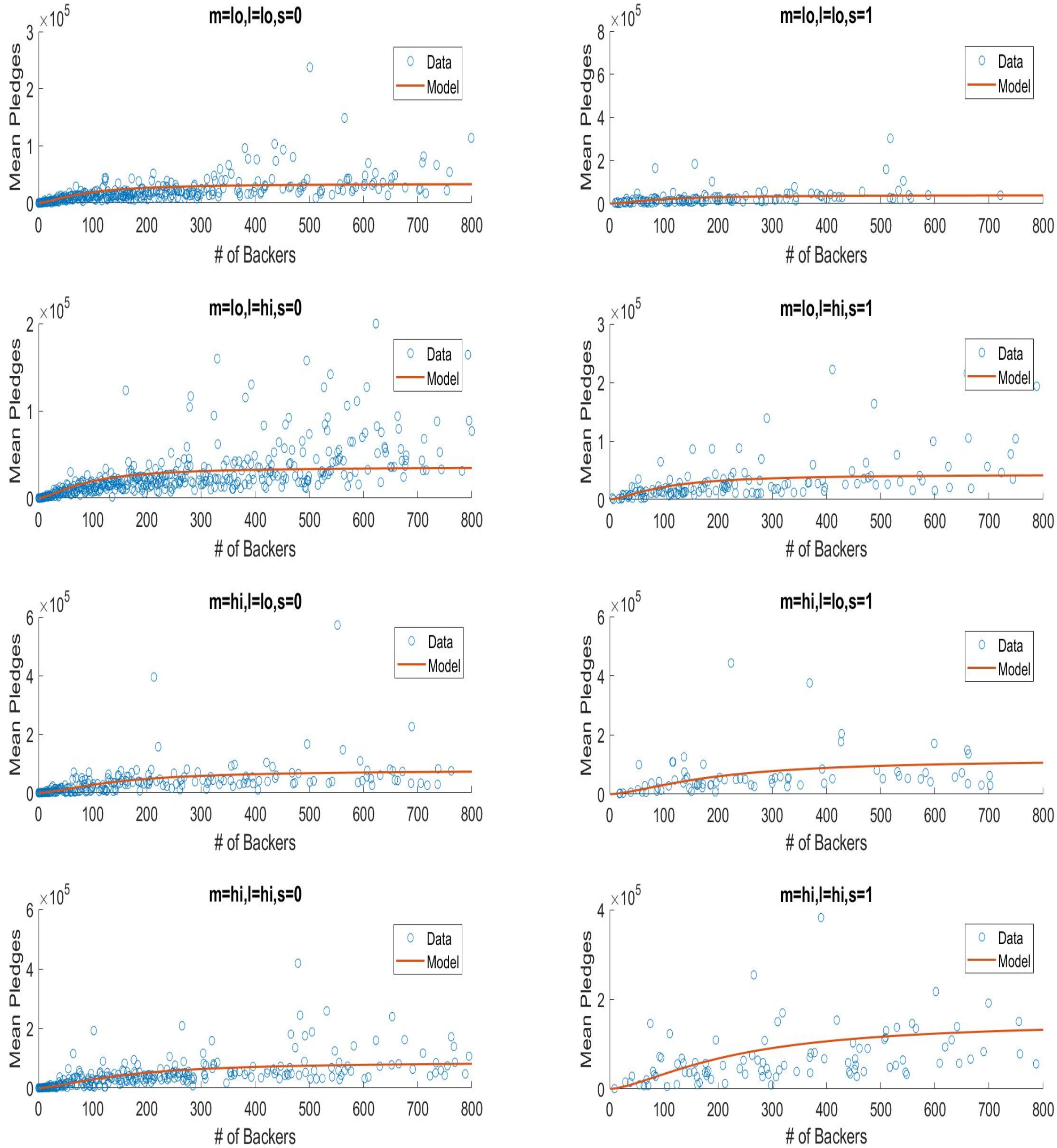
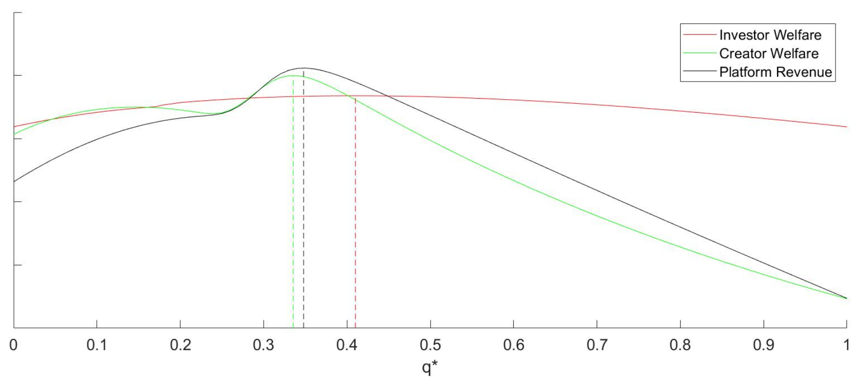


Figure 1.9: Investor, Creator and Platform Welfare



Chapter 2

Optimal Disclosure on Crowdfunding Platforms

2.1 Introduction

Crowdfunding is a form of financing startups that is believed to complement traditional venture capital investing by motivating further innovation. As the Wall Street Journal put it, "Crowdfunding has the potential to revolutionize the financing of small business, transforming millions of users of social media such as Facebook into overnight venture capitalists, and giving life to valuable business ideas that might otherwise go unfunded (WSJ, 2013)."

Crowdfunding platforms match innovators (entrepreneurs) with investors who are typically less experienced, than those involved in venture capital funding (see Lenz 2015). For this reason, investor protection is an important aspect of this new market. The JOBS act, passed by the U.S. House of Representatives on March 8, 2012, sets out the rules for the crowdfunding platforms and ensures investor protection. Although its details are still controversial, one of the main objectives of the JOBS act is to ensure adequate disclosure of entrepreneur information to the potential investors.

Imposing stringent disclosure requirements on innovators directly benefits investors by helping them screen innovators. However, it comes at the cost of dissuading innovation. How should disclosure requirements be designed to protect investors from failure while encouraging innovation?

I show that the optimal disclosure requirements may be partial. To see why, consider the decision of an investor (henceforth, "she") who obtains perfect information about an innovator's (hentheforth, "he") project quality. Because the investor cannot commit to invest in low quality projects, investment takes place if and only if the project is of high quality. The innovator, who initially is unsure about the quality of his project, expects then that his project is funded if and only if it is of high quality. If he attributes sufficiently low probability to this contingency, he has no incentives to innovate at an early stage or enter the crowdfunding platform. Full disclosure disincentivizes innovation.

This issue can be addressed by using milder disclosure requirements. For example, the platform may commit to occasionally hide evidence from the investor that the project is of low quality, and thereby increase the chance of being successfully funded. While concealing information from the investor makes it more difficult for her to screen innovators, it alleviates her commitment problem. This leads to the following insight:

> *Insight 1: Optimal disclosure may be partial. Partial disclosure compensates for the investor's lack of commitment and improves social welfare (including making the investor strictly better off).*

Formally, I study an interaction between an innovator and an investor on a platform. The platform is committed to disclosure requirements and chooses a fee structure (platform usage fees for the innovator and investor) to maximize expected profits. The innovator decides whether to incur a fixed cost of innovation, pay the fees, and enter the platform. The potential quality of the innovation (project) is unknown to the innovator when making the decision. If he innovates and enters the platform, a signal is revealed to the investor according to the disclosure requirements. The investor observes the signal realization and decides whether to fund the project. The investor only values high quality projects. The innovator enjoys a weakly higher payoff in the contingency when his project gets funded and is realized to be of high quality compared to the contingency when the project is funded and is of low quality.

Motivated by the current regulatory practice in this market, I consider two cases for who holds the authority to set disclosure requirements on a platform:

- *Regulated Disclosure Requirements:* The regulator chooses disclosure requirements to maximize investor's welfare net of platform fees. [1]

- *Unregulated Disclosure Requirements:* The profit-motivated platform chooses disclosure requirements.

The regulator's optimal disclosure requirements depend on the probability that the innovator's project is of high quality (success rate of innovation). For an intermediate success rate, the optimal is a partial disclosure policy that minimizes the probability that the investor is recommended to invest whenever the innovation is of low quality, subject to incentivizing innovation. Full disclosure is optimal if the success rate of innovation is sufficiently high, while for sufficiently low success rate of innovation, there is no way to incentivize innovation and ensure non-negative expected payoff to the investor.

Under the unregulated disclosure requirements, in addition to the success rate of innovation, the platform's optimal disclosure requirements depend on the investment level required to fund the project and the difference in the innovator's payoff from the contingency when the project gets funded and turns out to be of low quality and the contingency when the project is funded and turns out to be of high quality. This difference is interpreted as a cost of bad reputation to the innovator (henceforth, "reputation cost"). Reputation cost may arise due to the loss of trust in the innovator's ability to deliver good projects if he fails to deliver the current project. The above mentioned difference in payoffs captures such concerns in a reduced form and turns out to be an important determinant of the platform's optimal disclosure requirements.

Whenever the reputation cost is sufficiently high relative to the investment level required to fund the project, the platform's profit maximization problem becomes equivalent to the investor's welfare maximization. The platform chooses the regulator's optimal disclosure requirements and extracts investor surplus using the project fees. The reason is that increasing the probability of investment in the low quality project increases the innovator's welfare less than it decreases the investor's welfare. Hence, decreasing that probability enables the platform to extract higher surplus from the investor compared to the loss in the innovator's

[1] Some examples of regulators are: the Securities and Exchange Commission (SEC) in the USA, Ontario Securities Commission in Canada, Financial Conduct Authority in the UK.

surplus extraction. Whenever the reputation cost is sufficiently low relative to the investment level required to fund the project, the platform sets the disclosure requirements that maximize innovator's welfare. The intuition is the mirror image of the previous case. This leads to the following insight:

> *Insight 2: Regulation is unnecessary if and only if the reputation cost is sufficiently high relative to the required investment - the platform implements the same disclosure requirements as the regulator would under the regulated disclosure requirements.*

Insight 2 suggests that any regulatory tool that would be able to control reputation cost or required investment is a potential substitute for regulating disclosure requirements. Studying the regulations affecting required investment (e.g. income or capital gains tax) is out of the scope of this paper. I focus on regulations affecting reputation cost.

The Securities and Exchange Commission (SEC) in the USA, by rule, can require an equity based platform to implement certain types of online reputation systems (e.g. innovator rating systems). A reputation system ties innovator's performance to his reputation. An effective reputation system induces high reputation cost for the innovator. When should a regulator request a platform to implement more effective reputation system? I show the following,

> *Insight 3: From a regulator's perspective, an effective reputation system can benefit if and only if disclosure requirements are not regulated.*

Insight 3 follows from two observations. First, an effective reputation system discourages innovation for a given disclosure requirements. The reason is straightforward - higher reputation cost means lower expected benefit for the innovator from using the platform and can only discourage him to innovate. This makes a regulator more constrained in choosing optimal disclosure requirements leading to the result that an effective regulation system can only hurt whenever disclosure requirements are regulated. Second, to see how an effective reputation system can benefit when disclosure requirements are not regulated, consider a case in which the condition from the insight 2 does not hold. By implementing sufficiently effective reputation system a regulator would induce sufficiently high reputation cost relative to the cost of investment - inducing the platform to implement the regulator-optimal disclosure requirements. Of course, this would also discourage innovation. I show that under certain conditions the benefits from inducing the platform to implement the regulator-optimal disclosure requirements out-weights the losses from discouraging innovation.

Setting out the rules for crowdfunding is not a one-shot problem. Experimentation is a key component that drives evolution of rules. The SEC is involved in continuous rulemaking while the JOBS act has been subject to several amendments. Moreover, since crowdfunding is a relatively new phenomenon, there is much uncertainty surrounding it. This further stresses importance of experimentation. Gubler (2013) proposes a regulatory experiment in order to better understand crowdfunding potential and at the same time to adjust disclosure requirements correspondingly:

> "SEC, in adopting its rules, could treat crowdfunding with a relatively light regulatory touch: for example, by not requiring audited financials but specifying that the rule will expire after three to five years. If the evidence over that period suggests the incidence of **fraud is high**, then the agency might impose **stricter** and more permanent requirements."

At the heart of such an approach lies the task of designing disclosure requirements that ensure investor protection, incentivize innovation and at the same time allow regulator to learn some features of the market with the hope of designing even better disclosure requirements as learning takes place.

To understand the role of experimentation, I study a dynamic extension of the static model. The uncertainty about the market is captured by the aggregate uncertainty on the success rate of innovation. Innovators have private information about the true success rate.

The optimal regulatory experiment boils down to choosing either to learn from the observed quality of implemented projects or to learn from the actions of innovators. It is a threshold policy in beliefs on the success rate of innovation, prescribing to set more informative disclosure requirements when beliefs become sufficiently in favor of high success rate - precluding entry of innovators in case there is a low success rate and thus learning the true success rate from the decisions of the innovators. Otherwise, it keeps setting less stringent requirements and slowly learns from the market outcomes (realizations of project qualities).

The dynamics of the optimal regulatory experiment reverses Gubler's proposal if we think about *failure* instead of *fraud*.

> *Insight 4: If the evidence suggests that the incidence of project* ***failure is high****, the optimal regulatory experiment prescribes to set* ***milder*** *disclosure requirements.*

Outline The remainder of this chapter is organized as follows. Section 2.2 gives an account of the related work. Section 2.3 sets up the model. Section 2.4 discusses the case of regulated disclosure requirements, identifies the key tradeoff between innovation and investor protection, and culminates in the insight 1. Section 2.5 studies unregulated disclosure requirements and culminates in the insight 2. Section 2.6 studies the value of reputation systems to a regulator and culminates in the insight 3. Section 2.7 sets up the dynamic model and discusses optimal dynamic regulatory experimentation culminating in the insight 4. Section 2.8 discusses several extensions. All formal derivations and proofs are relegated to the Appendix B.

2.2 Literature Review

Most models of crowdfunding study the merits of crowdfunding that arise due to the availability of pre-selling, facilitating learning of consumer demand.

Strausz (2016) considers a model in this spirit. He studies a design of Pareto efficient crowdfunding platform in the presence of moral hazard and asymmetric information on the part of entrepreneur. The entrepreneur, once he collects funds on the platform, can run away without implementing a project. He also possesses private information on his production costs. The principal uses a mechanism that mitigates bad incentives arising from asymmetric information and moral hazard. Chang (2016) and Chemla & Tinn (2016) compare all-or-nothing and take-it-all funding schemes also in the presence of moral hazard and demand uncertainty. Ellman & Hurkens (2015) abstract from moral hazard and interpret crowdfunding platform as a commitment device for entrepreneurs. Threshold funding rules, that are prevalent, can be thought of as entrepreneurs committing to the production only in case sufficient funds are extracted from consumers and threatening not to produce otherwise.

Unlike these papers, I study a less explored aspect of crowdfunding design - disclosure requirements. I abstract away from the demand uncertainty and focus on how much information should be given to investors. Given their inexperience, investors value information about the quality of projects.

This work is also related to the Bayesian persuasion literature pioneered by Kamenica & Gentzkow (2011). Among the descendents of that paper, the most closely related to the structure of my model are Boleslavsky & Kim (2017) and Barron et al (2017). Those papers study the baseline persuasion model structure (as in Kamenica & Gentzkow (2011)) with the additional agent acting at the beginning of the game. The twist in such classes of games is that the first mover's decision depends on the persuasion rule rather than the realization of posterior beliefs.

The literature on two-sided markets (Rochet & Tirole (2003)) has emphasized the importance of externalities for the optimal pricing structures chosen by platforms. Roughly, the message is that one should jointly take into account the demand from both sides of the market for a service/product offered by a platform. The intuition that my work offers for the optimal fee structures verify some of the lessons learned from that literature in a context where the platform has an additional tool (disclosure requirements) that could be helpful in controlling externalities between the two sides of the market.

The dynamic extension of the model is related to the papers that merge Bayesian persuasion and experimentation. Kremer, Mansour & Perry (2014) study the dynamic interaction of a principal with myopic uninformed agents. The principal privately learns information from agents' actions and commits to an information disclosure to subsequent agents. Like them, I also study to what extent the principal should induce experimentation by the agents. Unlike them, in my model the principal can always induce full experimentation (i.e. learning the true state instantly) - rather than finding out how a principal can induce as much experimentation as possible the interesting angle in my model is to see to what extent the principal should deter experimentation. Other important distinctions are that in my model all the agents' actions are public information, the state on which experimentation takes place is privately known by certain players (innovators) and information disclosure is public. Che & Hörner (2015) is in the spirit of Kremer, Mansour & Perry (2014) but, unlike them and like me, Che & Hörner (2015) study public disclosure and focus on the dynamics of learning that happens gradually in their model. Like in Kremer, Mansour & Perry (2014), the second best in Che & Hörner (2015) always involves under-experimentation while in my model under-experimentation is not an issue; on the contrary, the principal may have to deter over-experimentation.

2.3 The Model

An innovator needs to decide whether to use the crowdfunding platform for financing his innovation (project). He needs an amount $m > 0$. To use the platform, he has to incur the cost $c + c_E \geq 0$, where c_E is the platform entry fee, and c is the initial cost of developing a product. For instance, c can be the cost of creating a product prototype, doing initial research, or cost of some minimal level of effort that is needed to obtain the evidence that the innovator indeed has something to offer on the platform.

The innovator's project can be one of two qualities - high (H) or low (L). The project is H with probability $p \in [0, 1]$. At the time of making the platform entry decision, the innovator does not know the quality of his project.[2]

If the innovator does not enter the platform he gets payoff 0 from the outside option. Otherwise, he must incur the cost $c + c_E$. The set of available actions for the innovator is $A = \{E, NE\}$, where E denotes entry and NE - no entry.

Once the innovator enters the platform, he must comply with the disclosure requirements that are present. The platform commits to a disclosure rule (requirements) that generates signals about the project's quality to the investor who is uninformed about the project's quality. After the investor observes information shared by the platform, she decides whether to invest an amount $m > 0$ in the project. If investment takes place, the innovator's payoff

[2] More realistic timing where first, the innovator decides on whether to do the initial development, then observing certain signals about project quality makes the platform entry decision, is developed in Section VIIA. None of the results are altered.

is 1 from the high quality project and payoff $k \equiv 1 - c_r$ from the low quality project, where c_r captures reputation cost to the innovator. If the project turns out to be of low quality ex post then the innovator may suffer a reputation cost arising from the loss of investors' trust in innovator's ability to deliver good results. Also, note that the payoff of 1 from the high quality project is a normalization. We can always normalize payoff to the innovator from the high quality project to 1 by appropriately adjusting c. Throughout the paper I assume that reputation costs are sufficiently low relative to the innovator entry cost i.e. $k \geq c$.[3]

The investor gets payoff of 1 from the high quality project and 0 from the low quality project. The set of available actions to the investor is $A_I = \{IN, NI\}$ where IN denotes investment and NI - no investment. The investor may also have to pay fees to the platform upon investment (explained in more details below).

The following expression shows the payoffs to the innovator (row) and the investor (column) from each combination of project qualities and actions of the investor net of the platform fees and conditional on the innovator playing E:

	IN	NI
H	$1 - c, 1 - m$	$-c, 0$
L	$k - c, -m$	$-c, 0$

Formally, a disclosure rule is a set of signals S and a pair of conditional distributions over S, $f(s|H)$ and $f(s|L)$. I assume that S is finite. Let $\Theta = \{H, L\}$ be the set of project qualities and θ stand for an element from this set. The set of all possible disclosure rules is denoted D and $d = (S,\ f(s|\theta)_{\theta\in\Theta})$ denotes a particular element from D.

A given disclosure rule can be thought of as arising from a set of audited financial documentation, specifications about business plan and risks, questionnaires, how securities offered are being valued and other information that signals quality of the offering and is verifiable either by a regulator (e.g. SEC), the platform, investors or any other credible third parties. A disclosure rule can also be interpreted as an experiment that provides

[3] This assumption is made for the clarity of exposition. All qualitative features of all results remain true for $k \in (0, c)$. For $k \leq 0$, the analysis is trivial as investor and innovator have aligned preferences in the sense that they both prefer that L projects not be implemented. Hence, optimal disclosure (both, for SEC and the platform) is always full disclosure.

information about some features of a product prototype.[4]

As already noted, the platform sets fees for its services provided to investors and innovators. The platform fee structure consists of $c_E \geq 0$ and $(c_I(s))_{s\in S}$ such that $c_I(s) \geq 0$ for each $s \in S$. Here c_E stands for the platform usage fee paid by innovator and $(c_I(s))_{s\in S}$ are the platform usage fees for the investor conditional on each realization of the signal. I assume that the investor observes the realization of the signal, s, before deciding whether she wants to use the platform for making the investment. Let a realization of s be denoted by $\bar{s}$.

Let $c_P(d) = (c_E, (c_I(s))_{s\in S})$ denote a fee structure for a given d and $C_P(d)$ denote all possible fee structures for a given d. Note that the set of possible fee structures depends on d only through S.

I study two cases of the model - one in which disclosure requirements is regulated and hence chosen by a regulator that maximizes investor's welfare net of platform fees and another in which the profit maximizing platform chooses disclosure requirements.

Timeline of the Game Under the Regulated Disclosure Requirements:

1. A regulator chooses $d \in D$;
2. Platform chooses $c_P(d) \in C_P(d)$ and commits to it along with d;
3. Innovator chooses action from A not knowing the true θ

   a) If $a = NE$ is chosen game ends and everyone gets 0 payoff;

   b) If $a = E$ is chosen, innovator incurs $c + c_E$, $\bar{s}$ is realized according to d;
4. In case 3(b) - investor observes $\bar{s}$, updates beliefs on the project quality and chooses action from A_I

   a) If $a_I = NI$ is chosen game ends;

   b) If $a_I = IN$ is chosen, she pays $c_P(\bar{s}) + m$ and project payoffs are realized.

[4]For example, Kickstarter requires the projects that involve manufacturing gadgets to provide demos of working prototypes. Photorealistic renderings are prohibited.

Under the unregulated disclosure requirements, platform chooses $d \in D$ instead of a regulator. The payoffs to the innovator, investor, regulator and profit motivated platform are respectively,

$$U = 1_{\{a=E\}}(1_{\{\theta=H,a_I=IN\}} + 1_{\{\theta=L,a_I=IN\}}k - c - c_E)$$

$$U_I = 1_{\{a=E,a_I=IN,\theta=H\}} - 1_{\{a_I=IN\}}(m + \sum_{s\in S} c_I(s)1_{\{s=\bar{s}\}})$$

$$U_R = 1_{\{a=E,a_I=IN,\theta=H\}} - 1_{\{a_I=IN\}}m$$

$$U_{PL} = 1_{\{a=E\}}c_E + 1_{\{a_I=IN\}} \sum_{s\in S} c_I(s)1_{\{\bar{s}=s\}}$$

2.4 Regulated Disclosure Requirements

In the USA, investor protection is mandated by the Congress to the SEC. The JOBS act explicitly states that investor protection is the objective that the SEC needs to be targeting. On the other hand, the JOBS act was created with the aim of incentivizing innovation. This means that investor protection and venture capital formation are the two objectives of the SEC. Investor welfare net of transfers to the platform is one specification of a regulator's payoffs that would capture those two objectives in my framework.[5]

A regulator sets a disclosure rule, with which the platform has to comply, to maximize the investor's ex-ante welfare net of payments to the platform. The platform is profit motivated and hence maximizes the sum of expected fees collected from the innovator and investor by choosing a fee structure in response to the choice of a disclosure rule by the regulator.

In what follows, first I discuss full disclosure and no disclosure. I identify investor com-

[5]More generally, one could define a regulator's payoffs as

$$W_1 1_{\{\theta=H,a_I=IN\}} - W_2 1_{\{\theta=L,a_I=IN\}}$$

where $(W_1, W_2) \in R^2_{++}$ and $1_{\{\theta,a_I=IN\}}$ takes the value of 1 when investment is made in the project of quality θ. For any such specification of regulator's objective, none of the results in this paper would be altered.

mitment problem and show how no disclosure can mitigate it. Then, I state the main result of this section and provide the intuition. For the sake of exposition, in subsections 2.4.1 and 2.4.2 I assume that the platform is an inactive player (i.e. restricted to setting zero fees for innovators and investors).

2.4.1 Full Disclosure

Suppose the regulator sets a disclosure rule that fully reveals the project's quality to the investor. She invests if and only if the quality of the project is high. Knowing this, the innovator enters the platform if and only if $p \geq c$ (recall that p is the probability that the project is of high quality) because otherwise the cost of initial development, c, is too high relative to the expected benefit from using the platform. Hence, the expected payoff to the investor, $Z_I^{fd}(p)$, is

$$Z_I^{fd}(p) = \begin{cases} 0 & \text{if } p \in [0, c) \\ p(1-m) & \text{if } p \in [c, 1] \end{cases}$$

2.4.2 No Disclosure

It is straightforward to show that the investor invests and the innovator enters if and only if $p \geq m$. Investor's welfare from the no disclosure rule is

$$Z_I^{nd}(p) = \begin{cases} 0 & \text{if } p \in [0, m) \\ p - m & \text{if } p \in [m, 1] \end{cases}$$

Observing $Z_I^{fd}(p)$ and $Z_I^{nd}(p)$ more closely, we see that whenever $c > m$, for all $p \in (m, c)$ we have $Z_I^{nd}(p) > Z_I^{fd}(p)$ that is, no disclosure is strictly better than full disclosure for the investor. This highlights the commitment problem of the investor. Because the investor cannot commit to invest in a low quality project if she learns that the project is of low quality, under full disclosure the innovator expects to be financed if and only if his project turns out to be of high quality. Thus, his expected payoff is p and, since $p < c$, he will not incur the initial cost of innovation.

No disclosure rule solves this commitment issue if p is above m and below c - the investor gets strictly positive expected payoff from investing and hence, if the innovator enters she invests. Knowing this, the innovator knows that he will be financed with probability 1. His expected payoff from incurring c and entering the platform is $p+(1-p)k-c$ which is strictly greater than 0 (recall that we are maintaining that $k \geq c$).[6] So, the innovator enters and the investment takes place.

However, often we can do even better than no disclosure. Optimal disclosure is not generically a no disclosure rule.

2.4.3 Optimal Disclosure

Proposition 2.1 describes the regulator's optimal disclosure rule and the resulting fee structures set by the platform as functions of p. Let $T^{opt} \equiv \frac{mc}{(1-m)k+m}$.

Proposition 2.1. *i) The optimal disclosure is any disclosure for* $p \in [0, T^{opt})$*; a disclosure rule with 2 signal realizations* $\{\sigma^H, \sigma^L\}$*, where* $f(\sigma^H|H) = 1$ *and* $f(\sigma^H|L) = \frac{c-p}{(1-p)k}$*, for* $p \in [T^{opt}, c)$ *and; full disclosure for* $p \in [c, 1]$*.*

ii) The value to the regulator and the investor (net of payments to the platform) is,

$$Z_I^{opt}(p) = \begin{cases} 0 & \text{if } p \in [0, T^{opt}) \\ \frac{p[k(1-m)+m]-mc}{k} & \text{if } p \in [T^{opt}, c) \\ p(1-m) & \text{if } p \in [c, 1] \end{cases}$$

iii) The optimal fee structure is any fee structure for $p \in [0, T^{opt})$*;* $c_E = 0$ *and* $c_I(\sigma^L) = 0$ *for all* $p \in [T^{opt}, 1]$*;* $c_I(\sigma^H) = \frac{p[k(1-m)+m]-mc}{k}$ *for* $p \in [T^{opt}, c)$ *and;* $c_I(\sigma^H) = p(1-m)$ *for* $p \in [c, 1]$*.*

Proof. See Appendix B. □

Figure 2.1 depicts the regulator's payoffs under the full and optimal disclosure rules.

[6] If $c > k$, no disclosure would solve the commitment issue for p above $max\{m, \frac{c-k}{1-k}\}$

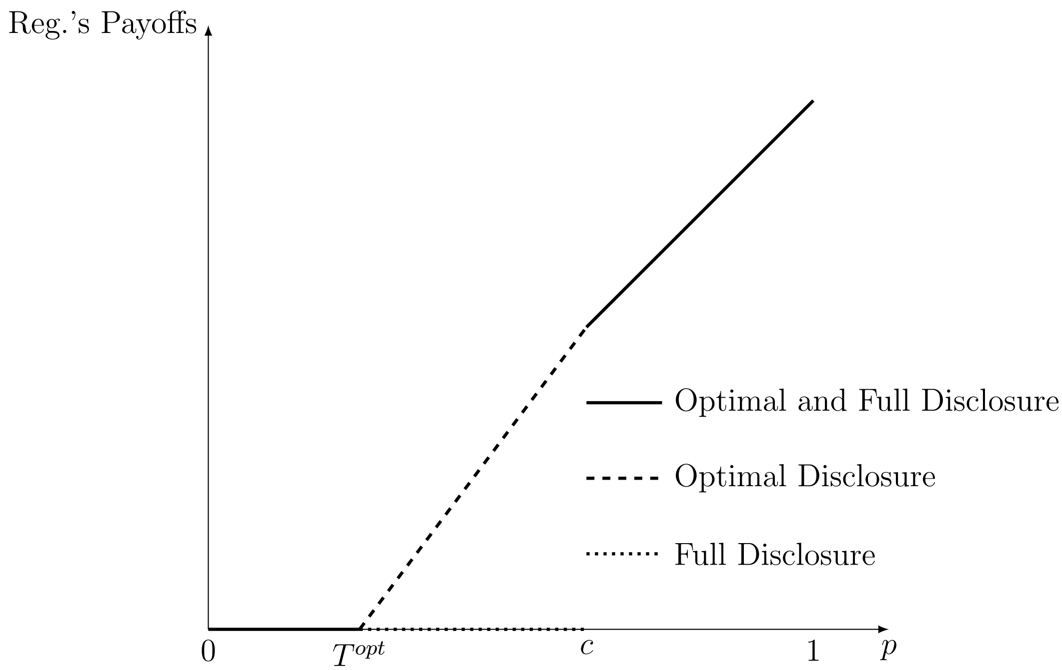


Figure 2.1: Full vs Optimal Disclosure

The difference in the regulator's payoffs between the optimal and the full disclosure occurs for $p \in (T^{opt}, c)$. In that intermediate region, partial disclosure solves the investor commitment problem and induces innovation. The intuition for the result is as follows. Suppose there is no problem in incentivizing the innovator to enter ($p \geq c$). Then, full disclosure maximizes investor welfare. For $p < T^{opt}$, there is no way to incentivize entry and at the same time to guarantee positive expected payoff to the investor. Hence, any disclosure is optimal as the market shuts down in any case and the investor gets 0 payoff from the outside option.

For $p \in (T^{opt}, c)$, we have the optimal disclosure being partial. We can use a two signal mechanism where one signal recommends the investor to finance the project and another recommends not to finance the project. Making sure that after seeing the recommendation the investor indeed wants to follow it, we can increase the probability that the innovator is financed by sometimes recommending her to invest when the project is of low quality. This "bad" recommendation is given with the lowest probability that incentivizes the innovator to enter.

We also need to make sure that the platform's optimal fee structure does not distort incentives of the players under the disclosure rule in proposition 2.1. Whenever $p \in [0, T^{opt})$ the platform's choice of fee structure is redundant as the market still shuts down for any fee structure. Otherwise, the platform's optimal fee structure is always such that it makes the innovator indifferent between entry and no entry and whenever the investor is recommended to invest, the platform extracts all her surplus and makes the investor indifferent between investing and not investing. Thus, the platform's optimal fee structure does not affect the innovator's incentive to enter and the investor's incentive to invest whenever recommended to do so.

2.4.4 Discussion

Proposition 2.1 highlights how crowdfunding can be useful for realizing untapped potential for innovation. If we think of the traditional venture capital investors as being effective in screening projects then it would be likely that the innovators with intermediate potential for success ($p \in (T^{opt}, c)$) would not be incentivized to realize their ideas. If the investors are less effective in screening projects on their own (like crowdfunders) then disclosure requirements become powerful regulatory tool with the potential for dealing with the investor's commitment problem and hence further incentivizing innovation.

In practice, institutional investors (e.g. venture capital firms) employ investment professionals and perform due diligence on entrepreneurs. This process involves interviewing former customers, competitors, employees, experts and conducting intense financial and legal work. This level of screening would not be feasible for a small retail investor simply because of the costs associated with it. Also, a lion's share of venture capital goes to Silocon Valley, New York, Boston and Los Angeles. In 2016, only 22 percent of investment went to companies outside those hubs.[7] In addition, institutional investors mostly target ventures in post-startup stages.[8] The innovators having their projects in later development stages and coming from those talent hubs can be regarded as high p innovators. A relatively good screening ability of institutional investors does not disincentivize such innovators to pursue

[7] http://www.thirdway.org
[8] https://www.forbes.com

their ideas. On the other hand, innovators who are believed not to have sufficiently high success probability would be disincentivized to pursue their ideas. Crowdfunding could help by allowing for a new segment of investors where each investor is relatively small, unable to effectively screen potential investment opportunities and hence dependent on whatever disclosure is provided by a regulator or a platform. It follows that the retail investors being less experienced and less able to screen the projects compared to the institutional investors is not necessarily a disadvantage of crowdfunding. On the contrary, it can facilitate innovation and improve the social welfare if a regulator chooses correct disclosure requirements.

2.5 Unregulated Disclosure Requirements

Regulation is costly. It is useful to understand when one could avoid those costs by deregulating disclosure requirements.

In the US, the JOBS act is tailored for loan and equity based crowdfunding while reward-based crowdfunding is left relatively unregulated. Because rewards are not classified as financial instruments or securities, its remedies fall under the traditional consumer protection law. This gives a high degree of freedom to reward-based platforms to set their own disclosure requirements.[9] Since the reward-based platforms involve pre-selling products to consumers a candidate agency for regulating those platforms is the Consumer Product Safety Commission which requires manufacturers to conduct certain product certifications before selling final good to consumers. However, since crowdfunders receive final product only after the investment takes place, the certification is required only after the project is financed and production takes place. This means that innovators are not required to do certification when advertising their product prototypes on the platforms. This leaves disclosure requirements on the reward-based platforms unregulated.

In this section I analyze the variant of the model in which the platform chooses disclosure rule. In the Appendix B, I derive the platform optimal disclosure rules and fee structures when the platform's objective function is a weighted sum of innovator, investor and platform

[9] See http://www.europa.eu, or http://www.europarl.europa.eu for the current regulation practices concerning reward based crowdfunding in EU. See https://lawreview.law.ucdavis.edu for how the consumer protection law has been applied in case of Kickstarter disputes.

welfare and summarize the result in proposition 2.2. Here I state the result only about the optimal disclosure rule for purely profit motivated platform.

Proposition 2.2 (a). *i) If $m \geq k$, the optimal disclosure is the same as in the proposition 1.*

ii) If $m < k$, the optimal disclosure is any disclosure for $p \in [0, T^{opt})$; a disclosure rule with 2 signal realizations $\{\sigma^H, \sigma^L\}$, where $f(\sigma^H|H) = 1$ and $f(\sigma^H|L) = min\{\frac{p(1-m)}{(1-p)m}, 1\}$, for $p \in [T^{opt}, c)$ and; full disclosure for $p \in [c, 1]$.

Proof. See Appendix B.

□

Whenever the reputation cost is sufficiently low relative to the investment required, $m < k$, and $p \in [T^{opt}, c)$ the platform sets the disclosure rule that recommends investment in the low quality project with strictly higher probability compared to the regulator's optimal disclosure rule. To understand what drives optimal behavior of the platform it is useful to distinguish among different effects of changing fee structure.

Increasing c_E has a positive first order effect on profits, a negative effect on the innovator's utility and a negative effect on the investor's utility. The intuition for the latter is that increasing c_E means the platform should sacrifice informativness of signal σ^H in order to incentivize the innovator to enter the platform but this is detrimental to the investor's expected payoff because now there is a higher chance that the investor is recommended to invest in state L.

Increasing $c_I(\sigma^H)$ has a positive first order effect on profits, a negative effect on the investor's payoffs and a negative effect on the innovator's payoffs. The intuition for the latter is that if the platform wants to increase $c_I(\sigma^H)$ then it has to provide better information to the investor in order to increase the value of investment to the investor. However, this is detrimental to the innovator's utility as he expects lower probability of his project being financed.

Now, one could think about the following decomposition of the problem for the platform. First, for a fixed disclosure rule, it is definitely optimal for the platform to increase both, c_E and $c_I(\sigma^H)$, until the innovator keeps playing E and the investor keeps playing I when recommended. The platform would do this for each possible disclosure rule. Second, given the fee structure associated with each disclosure rule that was found in the first stage, the platform needs to choose the optimal disclosure rule. If it would like to increase c_E it would need to increase the probability with which the innovator is recommended to invest in state L, in order to keep inducing entry of the innovator. But then the platform would also need to decrease $c_I(\sigma^H)$ because otherwise the investor would no longer follow the recommendation to invest. Similarly, if the platform decided to increase $c_I(\sigma^H)$, he would need to decrease c_E. Thus, for instance, if increasing c_E hurts the investor more than increasing $c_I(\sigma^H)$ hurts the innovator then the platform would increase $c_I(\sigma^H)$ and provide better information to the investor (decrease probability with which the investor is recommended to invest in state L). In case $m \geq k$, this is exactly what happens and the platform chooses the disclosure rule that maximizes investor welfare subject to incentivizing innovation. In case $m < k$, the opposite happens.

As a corollary to proposition 2.2, we see that regulation is not necessary if and only if $m \geq k$. This means that if the regulator has a reason to believe that the innovator reputation cost is sufficiently high relative to the cost of investment then deregulating disclosure requirements would not threaten investor protection. Below, I provide an evidence suggesting that for the reward based platforms in the US the condition for unnecessity of regulation is likely to hold.

Is regulation necessary on the reward-based platforms?

First I state the result that will be the key in answering this question. Recall that c_E denotes the platform entry fee for the innovator and $c_I(\sigma^H) > 0$ denotes payment from the investor to the platform in case investment is recommended (in practice, referred to as a project fee).

Proposition 2.2 (b)**.** *Under the deregulated disclosure requirements, the purely profit motivated platform sets $c_E = 0$ and $c_I(\sigma^H) > 0$ if and only if $m \geq k$ and $p < c$.*

Proof. See Appendix B.

□

Propositions 2.2 (a) and 2.2 (b) imply that regulation is not necessary if and only if a fee structure that sets $c_E = 0$ and $c_I(\sigma^H) > 0$ is optimal for the platform.

It turns out that most US based reward-based platforms indeed charge 0 fees to the innovators and strictly positive project fees. According to the data obtained from www.crowdsurfer.com, out of the 171 reward-based crowdfunding platforms that were in the active status in the US as of April 2017, 107 of them provide information on their fee structures. From 107, 97 charge only project fees, 2 charge only entry fees, 5 charge both and 3 charge no fees at all.

This observation along with the propositions 2.2 (a) and 2.2 (b) imply that the model rationalizes the following claims:

i) regulation of disclosure requirements on the US reward-based platforms is not necessary;

ii) partial disclosure is optimal (since $p < c$ is rationalized).

2.6 Reputation Systems

Disclosure requirements is not the only regulatory tool that is available to a regulator. For instance, SEC can influence innovator's reputation cost by requesting the platform to implement certain online reputation system. Indeed, the SEC retains authority under the JOBS Act to require the platforms to "meet other requirements as the Commission may, by rule, prescribe, for the protection of investors and in the public interest".[10] The JOBS act (Title III), in the current form, requires equity and lending based crowdfunding platforms to obtain and publicize information such as innovator's name, legal status, physical address and the names of the directors and officers. In addition, the SEC can, by rule, require publicization of innovator's online information (e.g. Facebook account) or implementation of a certain type of online reputation system. All such measures would ensure that performance of an innovator is closely tied to his reputation.[11]

Here I make the distinction between reputation systems and disclosure requirements clear. Disclosure requirements concern information that signal project quality to potential investors. Reputation systems control visibility of ex post project quality realizations to parties in the aftermarket (not modeled in this paper) and tie innovator's identity to his

[10]See 15 U.S.C. § 77d-1(a) (2012).

[11]see https://www.sec.gov.

project results.

Several platforms are already using some forms of online reputation systems. For example, on Indiegogo innovator can link Facebook and Indiegogo accounts and obtain verified Facebook badge. Feedback reputation systems similar to ones on Ebay and Amazon have also been proposed. [12]

The problem with all such online reputation systems is that most innovators do not go to a platform repeatedly. For example, more than 90 percent of project creators propose only one campaign on Kickstarter. [13] This weakens the effect of reputation systems on innovator incentives.

I suggest that there may not even be a need for a reputation system if disclosure requirements are regulated.

Proposition 2.3 (a). *Under regulated disclosure requirements it is optimal for a regulator to set $k = 1$.*

Proof. See Appendix B.

□

The intuition behind proposition 2.3 (a) is straightforward. The only thing that increasing k does is to relax the innovator's incentive for entry. This is desirable for a regulator as under regulation a regulator would be able to decrease the probability of financing the project in state L and would improve investor welfare.

But there still is a role for reputation systems. I argue that under deregulated disclosure requirements, from a regulator's perspective, requesting a platform to implement more effective reputation system is sometimes beneficial.

Suppose disclosure requirements are deregulated and $k > m$. From proposition 2.2 (a) we know that the platform would set the disclosure rule that recommends the investor to invest in state L with too high probability. It can be verified that if $p > \frac{c}{2-m}$ then a regulator would respond to this by requesting the platform to decrease k to $k' = m$, inducing the platform to choose lower probability of recommending investment in state L and strictly improving

[12] see Schwartz (2015).
[13] See Kuppuswamy, V., Bayus, B.L., (2013).

investor welfare. One can also verify that for the rest of the combinations of parameters a regulator would never request the platform to implement more effective reputation system. This discussion and proposition 2.3 (a) lead to the following,

Proposition 2.3 (b)**.** *A regulator would benefit from a more effective reputation system if and only if disclosure requirements are not regulated,* $k > m$ *and* $p > \frac{c}{2-m}$*.*

Proof. See Appendix B.

□

To summarize, sufficiently high reputation cost (inducing $m \geq k$) relieves a regulator from the need for regulating disclosure requirements on the platform - the profit maximizing platform maximizes investor welfare and extracts surplus using project fees. If the reputation cost is not sufficiently high then the platform considers extracting innovator welfare by setting a disclosure rule that favors the innovator. Such a disclosure rule increases the probability of financing the project as much as possible and hence induces too high probability of investment in state L. By regulating disclosure requirements, the regulator can directly decrease the probability of financing the L project while maintaining incentives for innovation. Alternatively, if in addition $p > \frac{c}{2-m}$, the regulator could keep the disclosure requirements deregulated and request the platform to increase the reputation cost for the innovator by implementing a more effective reputation system. Which of those ways the regulator chooses would depend on the costs associated with implementing each type of regulatory tool and effectiveness of reputation systems (i.e. is there a reputation system that would be able to increase reputation cost?).

2.7 Experimentation

Regulation is associated with a great deal of experimentation. The SEC is involved in continuous rule-making and the JOBS act has been subject to several amendments. Being a relatively new phenomenon, one could think of many features of the crowdfunding that are surrounded by uncertainty. One such feature is the probability of success of the average innovation. That probability (denoted p) was assumed to be known by the regulator in the previous sections. If p were known then previous sections suffice for describing optimal disclosure rules for each possible p under regulation and deregulation. If p were unknown then a regulator could start experimenting with one regulation, learning about p and subsequently possibly redesigning initial rules taking the updated information into account. The goal of this section is to understand how the experimentation should be done by a regulator when p is unknown.

2.7.1 Model Without A Platform

Time is infinite and discrete, indexed by t. There is a large pool of myopic innovators and investors. At the beginning of each period, one of the innovators has an idea and needs to decide whether to use the platform to fund his idea. An investor decides whether to invest or not in a project that is posted (if posted) on the platform each period. In this subsection, I assume the platform is an inactive player. Otherwise, the game within each period with stage payoffs is the same as in the static model. Once the stage game ends the innovator and investor quit the platform forever.

There is an aggregate uncertainty about p (success rate of innovation). The true p is known to the innovators but not to the investor or the regulator. True p can take one of two values p^H or p^L, where $p^H > p^L$, and is distributed $p \sim G$. Let g denote the probability of p^H. The aggregate uncertainty about the success rate of projects captures the fact that the market is new, it serves a new segment of potential innovators and it is not known what the average innovation potential is on the market.

Conditional on the true p, each innovator's project quality (conditional on entry) is drawn independently of others. Let $a_t \in A$ and $a_{t,I} \in A_I$ denote actions of innovator and investor, respectively, that were taken in period t. Let θ_t stand for a quality of the project in period t (conditional on $a_t = E$).

A regulator commits to a dynamic disclosure rule on the platform. Its objective is investor protection and hence maximizes discounted payoffs of the investors, with discount factor δ,

$$E\left[\sum_{t=0}^{\infty} \delta^t \mathbf{1}_{\{a_t=E, a_{t,I}=IN\}}(\mathbf{1}_{\{\theta_t=H\}} - m)\right]$$

All past actions and signals are publicly observable. The regulator transmits information about θ_t through a disclosure rule in place in period t. In addition, if investment takes place in a given period then θ_t becomes public information immediately after that.

Let S_t be a signal realization space at t, chosen by the regulator. I distinguish 3 special signals that are necessarily part of this space - $\oslash, s^H, s^L$.

Let $h^t = (a_0, a_{0,I}, s_0, ..., a_{t-1}, a_{t-1,I}, s_{t-1})$ be a public history of previous actions and signal realizations. Let H^t be the set of such t period histories. $H = \cup_{t=0}^{\infty} H^t$. Let $h^t_+ = (h^t, a_t, \theta_t)$ and H^t_+, H_+ defined in an obvious manner. Let $h^t_{++} = (h^t_+, a_{t,I})$ and H^t_{++}, H_{++} defined in an obvious manner.

Definition 2.1. *Dynamic disclosure rule is a set of history-dependent distributions* $\mathbf{f} : H_+ \to \Delta(S_t)$ *and a signal transition function* $\chi : H_{++} \to \Delta(\{\oslash, s^H, s^L\})$ *such that*

i) $f(\oslash|h^t_+) = 1, \chi(\oslash|h^t_{++}) = 1$ *for all* $h^t_+ \in H_+, h^t_{++} \in H_{++}$ *with* $a_t = NE$

ii) $\chi(s^H|h^t_{++}) = 1$ *for all* $h^t_{++} \in H_{++}$ *with* $a_t = E, \theta_t = H$ *and* $a_{t,I} = IN$

iii) $\chi(s^L|h^t_{++}) = 1$ *for all* $h^t_{++} \in H_{++}$ *with* $a_t = E, \theta_t = L$ *and* $a_{t,I} = IN$

iv) $\chi(s|h^t_{++}) = f(s \mid h^t_+)$ *for all* h^t_{++} *with* $a_t = E, a_{t,I} = NI$ *and* $S_t \setminus \{\oslash, s^H, s^L\}$

v) $f(s \mid h^t_+) = 0$ *for all* $s \in \{s^H, s^L\}$

Function χ is a signal transition function from the end of period signals to the beginning of the next period signals. The definition states that θ_t becomes public knowledge if investment takes place; if the innovator posted a project on the platform but investment did not take place then whatever signal was realized according to $\mathbf{f}$ remains in place and; if no project was posted then $\mathbf{f}$ just generates null signal.

The functions $\mathbf{f}$ and χ are common knowledge.

Let $G(h^t)$ be the distribution of p at the beginning of period t and let $Z_{G(h^t)}$ be its support. $G(h^t)$ is public knowledge. $Z_{G(h^0)} = \{p^L, p^H\}$ is the support of G at the beginning of the game.

Lemma 2.2 in the Appendix shows that the optimal dynamic disclosure rule would involve end of experimentation and shutdown of the market (no innovation from there on) if at some time t, $Z_{G(h^t)} \subseteq [0, \frac{mc}{(1-m)k+m})$. Lemma 2.3 shows that the optimal would be permanent full disclosure after time t such that $Z_{G(h^t)} \subseteq [c, 1]$. After that time, beliefs would evolve without further interference of the regulator in the experimentation process.

I focus on the non-trivial case where $p^H, p^L \in (\frac{mc}{(1-m)k+m}, 1]$.

Let

$$d_{p^H} = (\{\sigma^H, \sigma^L\}, f(\sigma^H|H) = 1, f(\sigma^H|L) = \frac{c - p^H}{(1-p^H)k})$$

$$d_{p^L} = (\{\sigma^H, \sigma^L\}, f(\sigma^H|H) = 1, f(\sigma^H|L) = \frac{c - p^L}{(1-p^L)k})$$

Note that I drop the arguments h^t and $a_t = E$ from $f(s|h^t, E, \theta)$ whenever it is clear that a rule conditions on public history before time t and realizes informative signals only if entry takes place. Proposition 2.4 characterizes the optimal dynamic disclosure rule.

Proposition 2.4. *The optimal dynamic disclosure rule is a threshold rule in $g(h^t)$ with threshold g^* such that for all histories, h^t, inducing $g(h^t) \geq g^*$, d_{p^H} is set and otherwise d_{p^L} is set.*

Proof. See Appendix B.

□

The following is a sketch of the proof of proposition 2.4. Let $G(s)$ denote the posterior public belief given prior G and signal s. First, I consider a relaxed problem where I ignore investor's constraints i.e. constraints that ensure that investor indeed wants to invest when receiving a signal recommending investment. This simplifies the problem because now the only thing that depends on the public beliefs, G, is the regulator's objective function. Note that necessary conditions for $G(h^t, E)$ to be a consistent belief after history h^t and innovator's entry are

$$\sum_{s \in S^{tr}} f(s|h^t, E, H)p^j + k \sum_{s \in S^{tr}} f(s|h^t, E, L)(1 - p^j) \geq c \text{ if type } p^j \in Z_{G(h^t,E)}$$

which do not depend on the public beliefs other than through the support of the beliefs, $Z_{G(h^t,E)}$. Those are the only constraints that we need to keep track of.

I consider the dynamic programming formulation of the relaxed problem. Assuming that the value function exists, I prove necessary conditions that the value function must satisfy. Claims 2.1, 2.2 and 2.3 prove those conditions - the value function must be convex in g and the optimal dynamic disclosure rule must be choosing a disclosure rule from the set $\{d_{p^H}, d_{p^L}\}$ in any given period. Further, lemma 2.4 verifies that investor's constraints are not violated for any dynamic disclosure rule in this class.

The dynamic formulation of the regulator's problem simplifies to

$$V(G) =$$
$$\max \left\{ \begin{array}{c} g\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta(1-g)\frac{[p^L((1-m)k+m)-mc)]}{k}, \\ (1-\delta)[E_G(p)(1-m) - \frac{c-p^L}{(1-p^L)k}E_G(1-p)m]+ \\ \delta[E_G(p)V(G(s^H)) + E_G(1-p)V(G(s^L))] \end{array} \right\}$$

For each G, the regulator needs to decide whether to set $f(\sigma^H|G,E,L) = \frac{c-p^H}{(1-p^H)k}$ or $f(\sigma^H|G,E,L) = \frac{c-p^L}{(1-p^L)k}$. In the former case, only high type innovator (p^H) enters the platform thus revealing the type and from the next period onwards static optimal rule is chosen. This generates expected discounted payoff $g\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta(1-g)\frac{[p^L((1-m)k+m)-mc)]}{k}$ for the regulator. I will call such a rule *separating*. In the latter case, the regulator sets a milder disclosure rule. Both types of the innovator enter and learning about the success rate takes place via observing realization of the project outcome after investment takes place. I call such a rule *pooling*. This information comes in the form of observing θ_t after investment takes place or observing only σ^L if the investor does not invest. Moreover, since σ^L fully reveals that $\theta_t = L$ it is always the case that optimal rule induces full revelation of θ_t.

Using standard arguments from Stokey & Lucas (1989), lemma 2.5 proves that the value function satisfying above equation exists, is unique, continuous and strictly increasing in g. I also verify that it is convex in g.

Now, we are ready to say more about the optimal dynamic rule. The idea of the proof of the proposition 2.4 can be conveniently represented in the figure 2.2.

The region inside the triangle depicts all possible values that V can take. The lower bound curve on these values is the function $g\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta(1-g)\frac{[p^L((1-m)k+m)-mc)]}{k}$

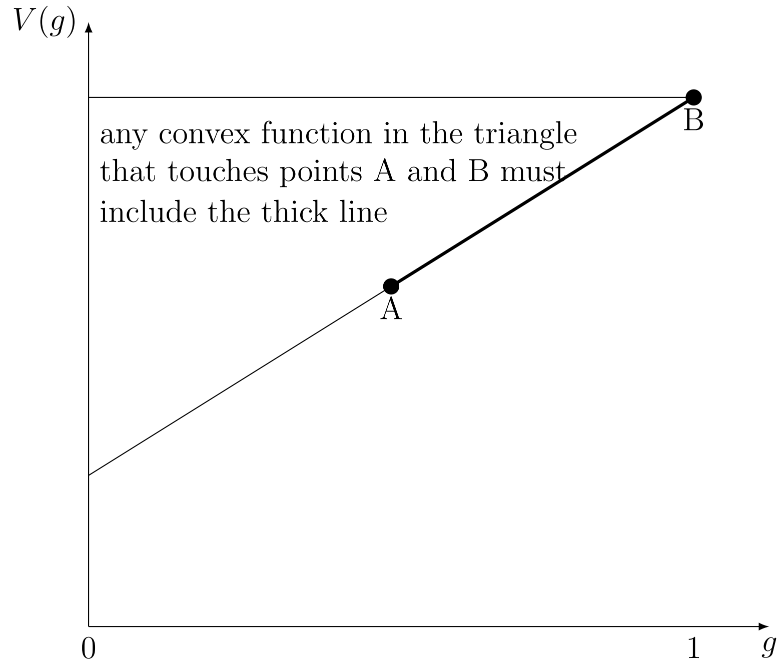


Figure 2.2: Proposition 2.4 Proof

(hypotenuse of the triangle). We know that point B is the optimal value when $g = 1$. Suppose that we also have point A as a part of the value function. Then, because we know that V is globally convex, the only way it can happen is if it includes the entire thick line from A to B. This proves that the optimal dynamic rule must be a threshold rule.

If g is above the threshold, g^*, then the optimal thing to do is to use separating rule (d_{p^H}). If g is below the threshold then the pooling rule is optimal.

In particular, what the optimal experimentation prescribes is to set a milder disclosure requirement when the incidence of **project failure** is high. This is in contrast with the Gubler's (2013) proposal which suggests a stricter disclosure requirement when the incidence of **fraud** is high.

Fraud arises due to an agent taking (or not taking) some action. Project failure, as it is in my model, does not arise due to actions taken by the agent - the probability of a project failure is determined by the ability of the innovator or some other exogenous factors. In practice, we usually observe unsuccessful projects and it is difficult to distinguish whether no success is due to the innovator's actions or some other factors not under his control. We need to carefully investigate the reasons behind unsuccessful projects as otherwise we may end up implementing disclosure requirements that are opposite to what is indeed optimal.

2.7.2 Model With A Platform

In this subsection I discuss some implications for the regulator's optimal dynamic rule in the presence of an active profit motivated platform. The platform sets a fee structure in each period to maximize discounted sum of profits with the discount factor δ^{PL}. As before, the regulator commits to the dynamic disclosure rule. The platform observes the same history as the regulator does. In particular, it does not know the true p. I prove the following proposition,

Proposition 2.5. *For all $(\delta, \delta^{PL}) \in [0, 1]^2$, the optimal dynamic disclosure rule is the same as in the proposition 4 if and only if $m \geq k$.*

Proof. Consider the threshold g^* implied by the optimal disclosure rule in the proposition 4. Suppose the regulator wants to implement the same rule. Whenever $g \geq g^*$ there is no problem with providing incentives to the platform not to affect equilibrium actions of innovators and investors: the regulator separates types in the first period such that only p^H innovates in the current period and moreover p^H gets 0 ex ante payoff. Hence, the platform sets $c_E = 0$ and collects positive fees from the investor. If it sets $c_E > 0$ no innovation would take place and the platform would lose potential payments from the investor in the current period. After the initial period, everyone learns the true p and static optimal disclosure is set thereafter that induces the platform to set $c_E = 0$ forever.

Whenever $g < g^*$ the regulator may need to distort its pooling disclosure rule. More specifically, if the regulator decides to implement the pooling rule, d_{p^L}, the platform could set $c_E > 0$ and induce only p^H type to innovate. However, this would not be optimal for the regulator. The regulator would like the platform to set $c_E = 0$ as this is the only case that would induce pooling. To deal with the platform's possible incentive to set $c_E > 0$, the regulator could commit to shut down the market forever if it sees that the platform sets $c_E > 0$ in the period when pooling is optimal for the regulator. It would simply set a sufficiently stringent disclosure rule forever after the deviation in order to induce shut down.

Under such a threat of punishment, the platform would compare the expected profits of setting $c_E = 0$ this period and continuing the business thereafter to setting $c_E > 0$, getting larger current period profit and then getting 0 profits thereafter. For lower δ^{PL} it would be more difficult for the regulator to provide such incentives to the platform. Hence to prove the proposition, it is enough to consider the most difficult case for providing dynamic incentives to the platform that is, when $\delta^{PL} = 0$.

In this worst case we can solve for a threshold $g^{PL} \in (0, 1)$ such that below this threshold the platform would want to set $c_E = 0$ whenever the regulator sets a pooling rule. Above the threshold, platform would deviate to setting $c_E > 0$ under the pooling rule. To solve for the threshold first consider the platform's payoff from setting $c_E = 0$. In this case, profits come only through project fees which is,

$$E_G(p)(1-m) - \frac{c - p^L}{(1-p^L)k} E_G(1-p)m$$

This is the amount that platform extracts from investors, in expectation. If the platform sets $c_E > 0$ then the optimal c_E would be such that it makes p^H type indifferent between innovating and not. Also, if innovation happens, investor knows that market potential is p^H and the platform sets fees to extract entire investor surplus whenever investors have that information. Hence, in expectation, the platform gets,

$$g[p^H(1-m) - \frac{c - p^L}{(1-p^L)k}(1-p^H)m + p^H - c + (1-p^H)\frac{c-p^L}{(1-p^L)}]$$

Using those two expressions, I solve for the threshold

$$g^{PL} = \frac{(p^L((1-m)k+m) - mc)(1-p^L)}{(p^L((1-m)k+m) - mc)(1-p^L) + k(p^H - p^L)(1-c)}$$

If $g^* \leq g^{PL}$ then there is no problem in incentivizing the platform to set fees that comply with the regulators optimal dynamic disclosure rule. This is because below g^* the platform sets $c_E = 0$ and this is indeed what the regulator would like. Above g^*, the platform's incentives do not matter as the regulator is choosing separating disclosure. To complete the proof, I show that $m \geq k$ if and only if $g^* \leq g^{PL}$.

To do this, I provide an upper bound for g^* denoted $\bar{g}$. Recall the value function for the regulator without the presence of the platform,

$$V(G) =$$

$$\max \left\{ \begin{array}{c} g\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta(1-g)\frac{[p^L((1-m)k+m)-mc)]}{k}, \\ (1-\delta)[E_G(p)(1-m) - \frac{c-p^L}{(1-p^L)k}E_G(1-p)m]+ \\ \delta[E_G(p)V(G(s^H)) + E_G(1-p)V(G(s^L))] \end{array} \right\}$$

Note that $g\frac{[p^H((1-m)k+m)-mc)]}{k} + (1-g)\frac{[p^L((1-m)k+m)-mc)]}{k} \geq E_G(p)V(G(s^H)) + E_G(1-p)V(G(s^L))]$ because the left hand side in the inequality is the upper bound on the expected continuation value. Hence, if the stage payoff from choosing the separating rule is higher than the stage payoff from choosing the pooling rule then separating rule is chosen. This provides the following upper bound on g^*,

$$\bar{g} = \frac{(p^L((1-m)k+m)-mc)(1-p^L)}{(p^L((1-m)k+m)-mc)(1-p^L)+m(p^H-p^L)(1-c)}$$

Now, it is easy to verify that $\bar{g} \leq g^{PL}$ if and only if $m \geq k$. This proves the result. ■

□

The content of the proposition 2.5 is that the platform's incentives to induce over-experimentation (instant learning) do not distort the regulator's optimal experimentation if and only if $m \geq k$. The basic idea of the proof is to find conditions under which whenever the regulator wants to implement the pooling rule the platform also prefers the pooling rule to the separating rule.

Whenever $m < k$ extracting surplus from the innovators becomes tempting for the platform and the regulator may need to distort its rule in order to discipline the platform's incentive to set a fee structure that would separate types whenever the pooling rule is optimal for the regulator.

Solving the case with $m < k$ is open. A solution would illustrate how to deter over-experimentation rather than how to encourage experimentation.

2.8 Extensions

2.8.1 Privately Informed Innovator

So far, I have abstracted away from any kind of private information on the part of the innovator. Particularly, it was maintained that the innovator does not learn the project quality after incurring innovation cost, c, and before the platform entry decision. In addition, he conducts required experiment only after entering the platform. However, in practice, initial phase of the project development may provide information about the potential quality of the project to the innovator. In addition, if entering the platform is not free, the innovator could conduct the experiment required by the platform prior to the entry decision and then make more informed decision.

It turns out that introducing those features do not affect qualitative features of the results. In particular, propositions 2.1-2.5 are still true.

First, consider the case of regulated disclosure requirements in the static model. Whenever $p > c$ the optimal disclosure is still full disclosure as there is no problem with incentivizing innovation. The platform sets $c_E = 1 - c/p$ as setting higher entry fee would lead to no innovation.

Whenever, $p < c$ full disclosure does not induce entry. Moreover, for any disclosure rule set by the regulator, that incentivizes innovation for $c_E = 0$, if the platform decides to set $c_E > k$ then the innovator would not use the platform whenever he learns that the project is of quality L. But this would mean that ex-ante he would not be willing to incur c. So the platform would be better off by setting $c_E \leq k$ as in this case, it would at least be getting positive fees from the investor.

Formally, the problem is the same as formulated in the appendix (see the proof of proposition 2.1) with the addition of 2 more constraints: $c_E \leq k$ and $pf(\sigma^H|H) + (1-p)f(\sigma^H|L)k - c - c_E(pf(\sigma^H|H) + (1-p)f(\sigma^H|L)) \geq 0$. We know the solution to the relaxed problem. Particularly, $pf(\sigma^H|H) + (1-p)f(\sigma^H|L)k - c = 0$ under the solution to the relaxed problem. Under this solution additional 2 constraints are satisfied if and only if $c_E = 0$ but this is exactly what the platform would do as otherwise, innovation would not take place. Hence, proposition 2.1 is still true.

To see that propositions 2.2 (a) and (b) are still true, consider the problem of the profit motivated platform choosing both, disclosure rule and a fee structure.

$$\max_{c_P(d),d} \left\{ \sum_{s\in S^{tr}} (pf(s|H) + (1-p)f(s|L))(c_I(s) + c_E) \right\}$$

$$s.t.$$

$$\frac{f(s|H)p}{f(s|H)p + f(s|L)(1-p)} \geq m + c_I(s) \ \forall \ s \in S^{tr}$$

$$\sum_{s\in S^{tr}} f(s|H)p + \sum_{s\in S^{tr}} f(s|L)(1-p)k \geq$$

$$c + c_E(\sum_{s\in S^{tr}} f(s|H)p + \sum_{s\in S^{tr}} f(s|L)(1-p))$$

$$c_E \leq k$$

Following similar arguments as in the lemma 2.1 one can show that we can restrict attention to the disclosure rules with at most 2 signal realizations. One can rewrite the problem as,

$$\max_{f(\sigma^H|\theta^L)} \{(p + (1-p)f(\sigma^H|L))c_E + p(1-m) - m(1-p)f(\sigma^H|L)\}$$

$$s.t.$$

$$\frac{p}{p + (1-p)f(\sigma^H|L)} - m \geq 0$$

$$p + (1-p)f(\sigma^H|L)k - c \geq 0$$

$$c_E \leq min\{k, \frac{p + (1-p)kf(\sigma^H|L) - c}{p + (1-p)f(\sigma^H|L)}\}$$

For $p \geq \frac{c}{1-k}$ the right hand side of the third constraint equals k. Substituting $c_E = k$ into the objective we see that if $k > m$, setting $f(\sigma^H|L)$ such that it makes investor indifferent between investing and not investing is optimal. If $p < \frac{c}{1-k}$ then we substitute $c_E = \frac{p+(1-p)kf(\sigma^H|L)-c}{p+(1-p)f(\sigma^H|L)}$ into the objective and again, if $k > m$, the same disclosure would be optimal. Hence, we get the same disclosure rule as in the Case 3(a) of the proposition 2.2

(see Appendix B). If $k \leq m$ one can similarly verify that we get exactly same disclosure rule as in Case 3(b) in the proposition 2.2. Moreover, it is easily verified that whenever $k \leq m$ and $p \leq c$ the platform sets 0 entry fees for the innovator.

A little inspection reveals that the propositions 2.3, 2.4 and 2.5 are also true.

Alternatively, one could also consider the innovator getting some signal about his project quality before incurring the cost c. This case can be modeled as a special case of the dynamic model with $T = 1$. Before incurring c, the innovator observes a private signal that drives his posterior belief either to p^H or to p^L. The innovator knows p^j while the rest of the players believe that with probability g the innovator gets a signal that induces p^H.

Recall $\bar{g}$ and g^{PL} from Section 2.7.2 proof of the proposition 2.5. One can verify that the regulator would like to choose the separating rule (d_{p^H}) if and only if $g \geq \bar{g}$ and otherwise chooses the pooling rule (d_{p^L}). If the regulator chooses the pooling rule, the platform sets $c_E = 0$ if and only if $g \leq g^{PL}$. Hence, whenever $m \geq k$, the regulator chooses the separating rule if and only if $g \geq \bar{g}$ and otherwise chooses the pooling rule.

Whenever $k > m$, the regulator chooses the separating rule for $g \geq \bar{g}$ and pooling for $g \leq g^{PL}$. For $g \in (g^{PL}, \bar{g})$ the regulator needs to distort d_{p^L} in such a way that both types of the innovator are incentivized to enter. To this end, it would need to set a disclosure rule that induces higher probability of investment in state L compared to d_{p^L}. This way, the platform would be able to set $c_E > 0$ without disincentivizing the entry of p^L.

Under the deregulated disclosure requirements, the platform sets the same disclosure rule as the regulator would set under regulation if and only if $m \geq k$. The intuition is the same as it was for the proposition 2.2. The platform wants to extract as much welfare as possible from the investor. For this, it maximizes investor welfare and hence sets the same disclosure rule as the regulator would. For $k < m$, the platform sets disclosure rule that induces the probability of investment in state L which is higher than the regulator would set under regulation.

To summarize, private information on the part of the innovator does not overturn the key messages of the paper - partially informative disclosure requirements are optimal, if reputation cost is sufficiently high relative to the investment cost regulation is not necessary and optimal dynamic regulation prescribes to set milder disclosure requirements when the incidence of project failure is high.

2.8.2 Innovator Moral Hazard

Proposition 2.3 (a) states that under the regulated disclosure requirements, reputation systems do not benefit the regulator. Lowering the reputation cost enables the regulator to provide better investor protection by making it easier to encourage innovation. As k increases, investment starts taking place for lower values of p and the investor welfare strictly increases for all such p.

One may argue that this result would not be immune to introducing moral hazard on the part of the innovator. Moral hazard would create a force in favor of lower k. For instance, if we allow the innovator to control p by exerting unobserved effort we would expect that lowering the reputation cost would weaken incentives for exerting higher effort. Exerting lower effort increases the probability that a project will turn out to be of low quality but if the reputation cost is low, payoff to the innovator in case investment takes place in the low quality project is high and thus it is more difficult to incentivize him to exert high effort. It turns out that weakening incentives for exerting high effort need not necessarily overweight benefits from the lower reputation cost.

To make this point clear, suppose that the innovator in addition chooses how much effort, $e \in [0,1]$, to exert on developing the idea. The choice of e is private information to the innovator. The project development has two stages: first, the innovator needs to exert a minimum level of effort that is observable and costs c to the innovator. This stage can be interpreted as creating an actual prototype or blueprint existence of which can be easily verified by the platform conducting a due diligence. If c is incurred, the innovator can exert additional hidden effort, e, that is not easily verifiable.

If e is exerted, probability of the project ending up being H is e and the cost of effort is $c(e) = e^2$.

Using similar arguments as in the proof of proposition 2.1, one can verify that we can restrict attention to the disclosure rules with at most 2 signal realizations and that setting $f(\sigma^H|H) = 1$ is optimal. The only thing we need to find is the optimal $f(\sigma^H|L)$. We need to solve the following problem,

$$\max_{f(\sigma^H|\theta^L)} (1-m)(1-kf(\sigma^H|L)) + (1+kf(\sigma^H|L))f(\sigma^H|L)m$$

$$s.t.$$

$$\frac{(1+kf(\sigma^H|L))^2}{4} \geq c$$

The solution gives, $f(\sigma^H|L) = 0$ for $c \leq \frac{1}{4}$; $f(\sigma^H|L) = \frac{2\sqrt{c}-1}{k}$ for $c \in [1/4, \frac{(1+k)^2}{4}]$ and; the market shutting down for $c > \frac{(1+k)^2}{4}$.

Increasing k has similar implications as in the model without moral hazard. It increases range of c for which innovation takes place and thus strictly increases the investor welfare for all such c while not affecting the investor welfare for the rest of the values of c. Setting $k = 1$ would still be optimal for the regulator.

2.9 Conclusion

I have shown how a regulator of crowdfunding can leverage the feature of the market, that retail investors can (do) not comprehensively screen investment opportunities, to incentivize innovation from entrepreneurs who might had been unserved by the traditional venture capitalists. I identified commitment problem on the part of the investors and argued that disclosure requirements can serve as a tool for resolving this problem. Sometimes partial disclosure is needed to address the commitment problem.

I analyzed regulated and unregulated disclosure requirements scenarios and derived the conditions under which the platform's optimal choice of the disclosure requirement would coincide with the regulator's choice. Whenever those conditions are violated, online reputation systems (e.g. innovator ratings) can substitute for regulating disclosure requirements to a certain extent.

Observing that regulation involves a great deal of experimentation, I also studied the optimal dynamic regulatory experiment. The results indicate that we need to be careful in distinguishing fraud from project failure - if a regulator observes an unsuccessful project it is important to investigate whether the innovator's actions (fraud) led to no success or other factors (innovator's ability, shocks to the market) led to no success. Otherwise, a regulator may end up doing the opposite of whatever is optimal.

Lastly, over-experimentation turns out to be a challenge to a regulator in the presence of a long-run profit maximizing platform. In contrast to this, the previous literature has mainly focused on under-experimentation (Kremer, Mansour & Perry (2014), Che & Hörner (2015)). One possible direction for future work would be to further understand this phenomenon.

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Appendix B

Proposition 2.1. *i) The optimal disclosure is any disclosure for* $p \in [0, T^{opt})$*; a disclosure rule with 2 signal realizations* $\{\sigma^H, \sigma^L\}$*, where* $f(\sigma^H|H) = 1$ *and* $f(\sigma^H|L) = \frac{c-p}{(1-p)k}$*, for* $p \in [T^{opt}, c)$ *and; full disclosure for* $p \in [c, 1]$*.*

ii) The value to the regulator and the investor (net of payments to the platform) is,

$$Z_I^{opt}(p) = \begin{cases} 0 & \text{if } p \in [0, T^{opt}) \\ \frac{p[k(1-m)+m]-mc}{k} & \text{if } p \in [T^{opt}, c) \\ p(1-m) & \text{if } p \in [c, 1] \end{cases}$$

iii) The optimal fee structure is any fee structure for $p \in [0, T^{opt})$*;* $c_E = 0$ *and* $c_I(\sigma^L) = 0$ *for all* $p \in [T^{opt}, 1]$*;* $c_I(\sigma^H) = \frac{p[k(1-m)+m]-mc}{k}$ *for* $p \in [T^{opt}, c)$ *and;* $c_I(\sigma^H) = p(1-m)$ *for* $p \in [c, 1]$*.*

Proof of Proposition 2.1

Proof. To derive the optimal disclosure rule first note that while the investor's decision depends only on the realized posterior, the innovator's decision depends on the entire signal rule (i.e. induced distribution of posteriors) and hence we cannot readily apply Kamenica & Genzkow (2011) concavification result.

I first solve for the optimal disclosure assuming the platform sets 0 fees for the investor and innovator for any disclosure rule chosen by SEC. Then, I will argue that even if the platform is free to choose any $c_P(d) \in C_P(d)$, the optimal disclosure rule chosen by the regulator under the relaxed problem remains the same.

Lets first account for the expected payoffs for the innovator and investor in case the innovator plays E, denoted $Z(p)$ and $Z_I(p)$, respectively.

$$Z(p) = \begin{cases} -c & \text{if } p \in [0, m) \\ p + (1-p)k - c & \text{if } p \in [m, 1] \end{cases}$$

$$Z_I(p) = \begin{cases} 0 & \text{if } p \in [0, m) \\ p - m & \text{if } p \in [m, 1] \end{cases}$$

A given disclosure rule induces distribution over posteriors, μ. I argue that μ can always be replaced by some μ' that contains at most 2 posteriors in its support. To see this, suppose μ contains more than 2 posteriors. Then there must be at least 2 posteriors $q_1 < q_2$ such that they are both either on $[0, m)$ or both on $[m, 1]$. Suppose $q_1 < q_2$ are both on $[0, m)$. We can find a new μ' such that it pools q_1 , q_2 into a single posterior q' and leaving the other posteriors the same. That is, $q' = \frac{\mu(q_1)}{\mu(q_1)+\mu(q_2)} q_1 + \frac{\mu(q_2)}{\mu(q_1)+\mu(q_2)} q_2$ and $\mu'(q') = \mu(q_1) + \mu(q_2)$. Because on $[0, m)$ we have $Z_1(p), Z_2(p)$ both linear this modification of the original disclosure gives same expected value to both players, conditional on posterior being on $[0, m)$, as the original disclosure rule. It is also easy to see that if a disclosure rule induces all the posteriors on either side of m then it is equivalent to no disclosure policy.

Hence, to solve for the optimal disclosure rule, first we need to consider only the rules with exactly two signal realizations where one signal recommends the investor to invest and another recommends not to invest. This will give us expected payoff $Z_I^*(p)$ for the investor. Then the investor's expected payoff from the optimal disclosure rule will be $Z_I^{opt}(p) = \max\{Z_I^*(p), Z_I^{nd}(p)\}$. Recall that $Z_I^{nd}(p)$ is the value to the investor from the no disclosure rule.

To find $Z_I^*(p)$ we need to solve the following linear program

$$Z_I^*(p) = \max_{f(\sigma^H|H), f(\sigma^H|L)} \{f(\sigma^H)(p(H|\sigma^H) - m)\}$$

$$s.t.$$

$$pf(\sigma^H|H) + (1-p)f(\sigma^H|L)k \geq c$$

$$1 \geq p(H|\sigma^H) \geq m$$

$$0 \leq p(H|\sigma^L) \leq m$$

σ^H, σ^L are two signal realizations. $f(\sigma^H|H), f(\sigma^H|L)$ are respective probabilities of drawing those realizations. $p(H|\sigma^H)$ is updated probability on state H after observing σ^H. The first constraint is to make sure that the innovator plays E. The second and third constraints make sure that the investor follows respective recommendations. Rewriting the problem,

$$Z_I^*(p) = \max_{f(\sigma^H|\theta^H), f(\sigma^H|\theta^L)} \{f(\sigma^H|H)p(1-m) - (1-p)f(\sigma^H|L)m\}$$

$$s.t.$$

$$pf(\sigma^H|H) + (1-p)f(\sigma^H|L)k \geq c$$

$$f(\sigma^H|H)/f(\sigma^H|L) \geq \frac{(1-p)m}{p(1-m)}$$

$$(1-f(\sigma^H|H))/(1-f(\sigma^H|L)) \leq \frac{(1-p)m}{(1-m)p}$$

$$(f(\sigma^H|H), f(\sigma^H|L)) \in [0,1]^2$$

Notice that all the constraints are relaxed and the objective increases if we increase $f(\sigma^H|H)$. Hence, $f(\sigma^H|H) = 1$ is part of the solution to the problem.

Using this fact, the first constraint becomes $f(\sigma^H|L) \geq \frac{c-p}{(1-p)k}$. Observing that decreasing $f(\sigma^H|L)$ relaxes all the other constraints and increases the objective function we must have $f(\sigma^H|L) = \frac{c-p}{(1-p)k}$. If $p \geq c$ then $f(\sigma^H|L) = 0$ and hence full disclosure is strictly optimal. $Z_I^*(p) \geq 0$ if and only if $p \geq \frac{mc}{(1-m)k+m}$ implying that if $p < \frac{mc}{(1-m)k+m}$ then any disclosure rule leads to the innovator playing NE and the market shutting down. For $p \in [\frac{mc}{(1-m)k+m}, c)$ partial disclosure is optimal where $f(\sigma^H|L) = \frac{c-p}{(1-p)k}$ and $f(\sigma^H|H) = 1$.

Thus we have,

$$Z_I^*(p) = \begin{cases} 0 & \text{if } p \in [0, T^{opt}) \\ \frac{p[k(1-m)+m]-mc}{k} & \text{if } p \in [T^{opt}, c) \\ p(1-m) & \text{if } p \in [c, 1] \end{cases}$$

It is straightforward to verify that $Z_I^{opt}(p) = \max\{Z_I^*(p), Z_I^{nd}(p)\} = Z_I^*(p)$ for all $p \in [0, 1]$.

Now we need to argue that the same rule is optimal if the platform is free to choose a fee structure. To see this, consider any choice of d by the regulator. If d is such that it induces the innovator to play NE then there is no fee structure that the platform could choose and induce E as $c_E \geq 0$. If d induces the innovator to play E then it is weakly optimal for the platform to choose c_E such that the innovator still wants to play E. As for the investor, d induces two types of signal realizations - one that induces the investor to invest and another that induces her not to invest. For the signals that do not induce investment, there is no $c_I(s) \geq 0$ which would induce the investor to invest. For the signals that induce investment, the platform would set $c_I(s)$ such that investment is still induced. Otherwise, the platform would induce no investment for such signal and get 0 rents from the investor.

Thus, for any d, the optimal fee structure would induce the same distribution over innovator's and investor's equilibrium actions as a fee structure that sets $c_E = 0$ and $c_I(s) = 0$ for all $s \in S$. In addition it would extract all surplus from the innovator and investor. This completes the proof of proposition 2.1.

□

I derive the optimal disclosure rule and fee structure under the deregulated disclosure requirements. The derivation is done for the general linear preferences of the platform. Recall that the platform's utility is a weighted sum of the investor welfare, innovator welfare and profits with the weights α_I, α_E and $1 - \alpha_I - \alpha_E$, respectively. Purely profit motivated platform is the special case.

Fix a finite set of signal realizations S and distributions $f(s|\theta)$. Recall that $(S, f(s|\theta)_{\theta \in \Theta})$ is a disclosure rule. Let $S^{tr} \subset S$ be set of signal which induce entry and investment and let $S^{trc} = S/S^{tr}$. Recall also that, $c_P(d) = (c_E, c_I(s)_{s \in S})$ is a fee structure. The problem for the platform is formulated as follows,

$$\max_{c_P(d)\in C_P(d), d\in D} \left\{ \begin{array}{c} \alpha_I[p\sum_{s\in S^{tr}} f(s|H)(1-m-c)_I(s))- \\ (1-p)\sum_{s\in S^{tr}} f(s|L)(m+c_I(s)]+ \\ \alpha_E[\sum_{s\in S^{tr}}(pf(s|H)+(1-p)f(s|L)k)-c-c_E]+ \\ (1-\alpha_I-\alpha_E)[c^E+\sum_{s\in S^{tr}}(pf(s|H)+ \\ (1-p)f(s|L))(c_I(s)) \end{array} \right\} \tag{2.1}$$

$$s.t.$$

$$\frac{f(s|H)p}{f(s|H)p+f(s|L)(1-p)} \geq m + c_I(s)\ \forall\ s\in S^{tr} \tag{2.2}$$

$$\sum_{s\in S^{tr}} f(s|H)p + \sum_{s\in S^{tr}} f(s|L)(1-p)k \geq c + c_E \tag{2.3}$$

Lemma 2.1. *We can restrict attention to the disclosure rules with at most 2 signal realizations.*

Proof. The platform's objective function can be rewritten as

$$\left\{ \begin{array}{c} \alpha_I[p\sum_{s\in S^{tr}} f(s|H)(1-m)-(1-p)\sum_{s\in S^{tr}} f(s|L)m]+ \\ \alpha_E[\sum_{s\in S^{tr}}(pf(s|H)+(1-p)f(s|L)k)-c-c_E]+ \\ (1-\alpha_I-\alpha_E)c_E+(1-2\alpha_I-\alpha_E)\sum_{s\in S^{tr}}(pf(s|H)+ \\ (1-p)f(s|L))(c_I(s)) \end{array} \right\} \tag{2.4}$$

Case 1: Suppose $(1-2\alpha_I-\alpha_E)\leq 0$. Then it is optimal to set $c_I(s)=0$ for all $s\in S^{tr}$ because decreasing $c_I(s)$ would increase objective and would relax constraint 2.2. 2.4 becomes

$$\left\{ \begin{array}{c} \alpha_I[p\sum_{s\in S^{tr}} f(s|H)(1-m)-(1-p)\sum_{s\in S^{tr}} f(s|L)m]+ \\ \alpha_E[\sum_{s\in S^{tr}}(pf(s|H)+(1-p)f(s|L)k)-c-c_E]+ \\ (1-\alpha_I-\alpha_E)c_E \end{array} \right\} \tag{2.5}$$

2.5 now depends only on $\sum_{s\in S^{tr}} f(s|H)$ and $\sum_{s\in S^{tr}} f(s|L)$. Hence, we can take two signal realizations, (σ^H,σ^L), and set the new disclosure rule as follows: $p(\sigma^H|H)=\sum_{s\in S^{tr}} f(s|H)$,

$p(\sigma^H|L) = \sum_{s\in S^{tr}} f(s|L)$. The value of the objective remains the same and the inequalities in 2.3 hold.

To see that 2.2 still holds, observe that 2.2 implies $\sum_{s\in S^{tr}} f(s|H)p \geq m \sum_{s\in S^{tr}}(f(s|H) + (1-p)f(s|L))$. Hence, by construction of our disclosure rule 2.2 still holds.

Case 2: Suppose $(1 - 2\alpha_I - \alpha_E) > 0$. This implies that an optimal rule must have 2.2 binding for each $s \in S^{tr}$. Summing over S^{tr} and rearranging 2.2, we have

$$p \sum_{s\in S^{tr}} f(s|H) - \sum_{s\in S^{tr}} (pf(s|H) + (1-p)f(s|L))m = \sum_{s\in S^{tr}} (pf(s|H) + (1-p)f(s|L))(c_I(s)) \tag{2.6}$$

Let $r \equiv \frac{p\sum_{s\in S^{tr}} f(s|H) - \sum_{s\in S^{tr}}(pf(s|H)+(1-p)f(s|L))m}{\sum_{s\in S^{tr}}(pf(s|H)+(1-p)f(s|L))}$. Constraint 2.2 implies that the numerator is positive so r is well defined. Substituting this into 2.4 we get,

$$\left\{\begin{array}{c} \alpha_I[p\sum_{s\in S^{tr}} f(s|H)(1-m) - (1-p)\sum_{s\in S^{tr}} f(s|L)(m)] + \\ \alpha_E[\sum_{s\in S^{tr}}(pf(s|H) + (1-p)f(s|L)k) - c - c_E] + \\ (1-\alpha_I-\alpha_E)c_E + (1-2\alpha_I-\alpha_E)[p\sum_{s\in S^{tr}} f(s|H) - \\ \sum_{s\in S^{tr}}(pf(s|H) + (1-p)f(s|L))m] \end{array}\right\} \tag{2.7}$$

2.7 depends only on $\sum_{s\in S^{tr}} f(s \mid H)$ and $\sum_{s\in S^{tr}} f(s \mid L)$. Hence, we can apply the same modification as in case 1.

□

From now on, I will call r a project fee. Given 2 signal realizations, (σ^H, σ^L) where σ^H is a recommendation for trade, the problem can now be rewritten as

$$\max \left\{\begin{array}{c} \alpha_I[pf(\sigma^H|H)(1-m-r) - (1-p)f(\sigma^H|L)(m+r)] + \\ \alpha_E[pf(\sigma^H|H) + (1-p)f(\sigma^H|L)k - c - c_E] + \\ (1-\alpha_I-\alpha_E)[c_E + (pf(\sigma^H|H) + (1-p)f(\sigma^H|L))r] \end{array}\right\} \tag{2.8}$$

$$s.t.$$

$$\frac{f(\sigma^H|H)p}{f(\sigma^H|H)p + f(\sigma^H|L)(1-p)} \geq m + r \tag{2.9}$$

$$f(\sigma^H|H)p + f(\sigma^H|L)(1-p)k \geq c + c_E \tag{2.10}$$

The optimal disclosure rule involves $f(\sigma^H|H) = 1$ as increasing $f(\sigma^H|H)$ increases objective and relaxes both constraint. The optimization problem reduces to

$$\max_{c_E, r, f(\sigma^H|L)} \left\{ \begin{array}{c} \alpha_I[p(1-m) - (1-p)f(\sigma^H|L)m] + \\ \alpha_E[p + (1-p)f(\sigma^H|L)k - c] + (1-\alpha_I - 2\alpha_E)c_E + \\ (1-2\alpha_I - \alpha_E)(p + (1-p)f(\sigma^H|L))r \end{array} \right\} \tag{2.11}$$

$$s.t.$$

$$\frac{p}{p + f(\sigma^H|L)(1-p)} \geq m + r \tag{2.12}$$

$$p + f(\sigma^H|L)(1-p)k \geq c + c_E \tag{2.13}$$

$$c_E \geq 0, r \geq 0 \tag{2.14}$$

Proposition 2.2. *The deregulated optimal disclosure rules and fee structures are as follows:*

Case 1: if $\frac{m\alpha_I}{k} \geq \alpha_E \geq \max\{\frac{1-\alpha_I}{2}, 1-2\alpha_I\}$ *then* $c_E^{case1} = r_{case1} = 0$ *and* $f_{case1}(\sigma^H|L) = \max\{0, \frac{c-p}{(1-p)k}\}$;

Case 2: if $\alpha_E \geq \max\{\frac{1-\alpha_I}{2}, 1-2\alpha_I, \frac{m\alpha_I}{k}\}$ *then* $c_E^{case2} = r_{case2} = 0$ *and* $f_{case2}(\sigma^H|L) = \min\{\frac{p(1-m)}{(1-p)m}, 1\}$;

Case 3 (a): if $\alpha_E < \min\{\frac{1-\alpha_I}{2}, 1-2\alpha_I\}$ *and* $k > m$ *then* $f_{case3(a)}(\sigma^H|L) = \min\{\frac{p(1-m)}{(1-p)m}, 1\}$. *If, in addition,* $m > p$ *then* $c_E^{case3(a)} = \frac{p(m+(1-m)k)}{m} - c$, $r_{case3(a)} = 0$. *If, in addition,* $m \leq p$ *then* $r_{case3(a)} = p - m$, $c_E^{case3(a)} = p + (1-p)k - c$;

Case 3 (b): if $\alpha_E < \min\{\frac{1-\alpha_I}{2}, 1-2\alpha_I\}$ *and* $k \leq m$ *then* $f_{case3(b)}(\sigma^H|L) = \max\{\frac{c-p}{(1-p)k}, 0\}$. *If, in addition,* $c > p$ *then* $c_E^{case3(a)} = 0$, $r_{case3(a)} = \frac{kp}{c-(1-k)p} - m$. *If, in addition,* $c \leq p$ *then* $r_{case3(a)} = 1 - m$, $c_E^{case3(a)} = p - c$;

Case 4: if $\max\{1-2\alpha_I, 1-\frac{(k+m)}{k}\alpha_I\} \leq \alpha_E < \frac{1-\alpha_I}{2}$ *then* $f_{case4}(\sigma^H|L) = \max\{0, \frac{c-p}{(1-p)k}\}$. *If, in addition,* $c > p$ *then* $c_E^{case4} = r_{case4} = 0$. *If, in addition,* $c \leq p$ *then* $r_{case4} = 0$, $c_E^{case4} = p - c$;

Case 5: if $1 - 2\alpha_I \leq \alpha_E < \min\{\frac{1-\alpha_I}{2}, 1 - \frac{(k+m)}{k}\alpha_I\}$ *then* $f_{case5}(\sigma^H|L) = \min\{\frac{p(1-m)}{(1-p)m}, 1\}$. *If, in addition,* $p > m$ *then* $c_E^{case5} = p + (1-p)k - c$, $r_{case5} = 0$. *If, in addition,* $p \leq m$ *then*

$r_{case5} = 0$, $c_E^{case5} = \frac{p(m+(1-m)k)}{m} - c$;

Case 6 (a): if $\frac{1-\alpha_I}{2} \leq \alpha_E < 1 - 2\alpha_I$ *and* $k > m$ *or* $\frac{1-\alpha_I}{2} \leq \alpha_E < 1 - 2\alpha_I$, $k \leq m$ *and* $\frac{1-\alpha_I}{1+k/m} \leq \alpha_E$ *then* $f_{case6(a)}(\sigma^H|L) = \min\{\frac{p(1-m)}{(1-p)m}, 1\}$. *If, in addition,* $p \leq m$ *then* $c_E^{case6(a)} = 0$, $r_{case6(a)} = 0$. *If, in addition,* $p > m$ *then* $r_{case6(a)} = p - m$, $c_E^{case6(a)} = 0$;

Case 6 (b): if $\frac{1-\alpha_I}{2} \leq \alpha_E < 1 - 2\alpha_I$, $k \leq m$ *and* $\frac{1-\alpha_I}{1+k/m} > \alpha_E$ *then* $f_{case6(b)}(\sigma^H|L) = \max\{\frac{c-p}{(1-p)k}, 0\}$. *If, in addition,* $p \leq c$ *then* $c_E^{case6(b)} = 0$, $r_{case6(b)} = \frac{pk}{c-(1-k)p} - m$. *If, in addition,* $p > c$ *then* $r_{case6(a)} = p - m$, $c_E^{case6(a)} = 0$.

Under all cases, $f(\sigma^H|H) = 1$.

Proof of Proposition 2.2

Proof. First, recall that if $p < \frac{mc}{(1-m)k+m}$ then there is no way to jointly induce entry and investment. Adding fees to the model can only harm incentives for entry and investment. The following cases focus on $p \geq \frac{mc}{(1-m)k+m}$.

Also, recall that under the assumption 1 we have $k \geq c$.

Case 1:

Under this case, 2.11 is always decreasing in c_E and r. Because decreasing c_E and r also relaxes 2.12 and 2.13 we have $c_E = r = 0$. $m\alpha_I > k\alpha_E$ also implies that 2.11 is decreasing in $f(\sigma^H|L)$. Since decreasing $f(\sigma^H|L)$ relaxes 2.12 and tightens 2.13 the optimal would be $f(\sigma^H|L) = \max\{0, \frac{c-p}{(1-p)k}\}$. Under this rule we can verify that 2.12 is satisfied.

Case 2:

We have $c_E = r = 0$. $m\alpha_I \leq k\alpha_E$ implies 2.11 is nondecreasing in $f(\sigma^H|L)$. Increasing $f(\sigma^H|L)$ tightens 2.12 and relaxes 2.13. Hence we set, $f(\sigma^H|L) = \min\{\frac{p(1-m)}{(1-p)m}, 1\}$. We can verify that 2.13 is satisfied under this rule.

Case 3:

Objective is strictly increasing in both, r and c_E. Hence, at the optimum 2.12 and 2.13 must bind. We then substitute the constraints into the objective, simplify and get the following problem

$$\max_{f(\sigma^H|\theta^L)} \{(k-m)f(\sigma^H|L)\}$$

$$s.t.$$

$$\frac{p}{p+(1-p)f(\sigma^H|L)} - m \geq 0$$

$$p+(1-p)f(\sigma^H|L)k - c \geq 0$$

Case 3 (a):

We can increase $f(\sigma^H \mid L)$ until the first constraint binds. We get, $f(\sigma^H|L) = \min\{\frac{p(1-m)}{(1-p)m}, 1\}$. Substituting this into second constraint we can verify that it is satisfied.

Case 3 (b):

We can decrease $f(\sigma^H|L)$ until the second constraint binds. We get, $f(\sigma^H|L) = \max\{\frac{c-p}{(1-p)k}, 0\}$. Under this solution, we can verify that the first constraint is also satisfied.

Case 4:

A11 is strictly increasing in c_E and nonincreasing in r. Hence, at the optimum we must have $r = 0$ and $c_E = p + (1-p)f(\sigma^H|L)k - c \geq 0$. Using these, we can rewrite the problem as

$$\max_{f(\sigma^H|\theta^L)} \{(k(1-\alpha_I-\alpha_E) - \alpha_I m)f(\sigma^H|L)\}$$

$$s.t.$$

$$p+(1-p)f(\sigma^H|L)k - c \geq 0$$

$$\frac{p}{p+f(\sigma^H|L)(1-p)} - m \geq 0$$

The objective is decreasing in $f(\sigma^H|L)$ and hence we set $f(\sigma^H|L) = \max\{0, \frac{c-p}{(1-p)k}\}$. One can check that both constraints are satisfied.

Case 5:

A11 is strictly increasing in c^E and nonincreasing in r. Hence, at the optimum we must have $r = 0$ and $c_E = p + (1-p)f(\sigma^H|L)k - c \geq 0$. Using these, we have similar problem as in case 4

Objective is increasing in $f(\sigma^H|L)$ and hence we set $f(\sigma^H|L) = \min\{\frac{p(1-m)}{(1-p)m}, 1\}$.

Case 6:

2.11 is strictly increasing in r and nonincreasing in c_E. At the optimum we must have $c_E = 0$ and $r = \frac{p}{p+f(\sigma^H|\theta^L)(1-p)} - m$. The problem is rewritten as

$$\max_{f(\sigma^H|\theta^L)} \{-(1-\alpha_I-\alpha_E(1+\frac{k}{m}))m(1-p)f(\sigma^H|L)\}$$

$$s.t.$$

$$p+(1-p)f(\sigma^H|L)k-c \geq 0$$

$$\frac{p}{p+f(\sigma^H|L)(1-p)} - m \geq 0$$

Case 6 (a):

The objective is increasing in $f(\sigma^H|L)$. So, $f(\sigma^H|L) = \min\{\frac{p(1-m)}{(1-p)m}, 1\}$. One can verify that both constraints are satisfied.

Case 6(b):

The objective is decreasing in $f(\sigma^H|L)$. So, $f(\sigma^H|L) = \max\{\frac{c-p}{(1-p)k}, 0\}$. ■

□

Proposition 2.4. *The optimal dynamic disclosure rule is a threshold rule in $g(h^t)$ with threshold g^* such that for all histories, h^t, inducing $g(h^t) \geq g^*$, d_{p^H} is set and otherwise d_{p^L} is set.*

I will prove several intermediary lemmas and claims that will lead to the proof of the proposition 2.4.

Let,

$$W_{\mathbf{f}}(G(h^{t'})) = (1-\delta)E_{\mathbf{f},\mathbf{z},t'}\left[\sum_{t=t'}^{\infty}\delta^t \mathbf{1}_{\{a_t=E,a_{t,I}=IN\}}(\mathbf{1}_{\{\theta_t=H\}}-m)\right]$$

be the average expected discounted payoffs for the regulator at the beginning of period t' under the dynamic disclosure rule $\mathbf{f}$. The following lemmas show how the problem can be simplified.

First, recall that $Z_{G(h^t)}$ is the support of G at the history h^t.

Lemma 2.2. *If $Z_{G(h^t)} \subseteq [0, \frac{mc}{(1-m)k+m})$ there is no policy which would avoid shutting down the market.*

Proof. If a policy induces trade for some realizations of $S^{tr} \subseteq S/\{\oslash, s^H, s^L\}$ it must be the case that for those realizations the following ICs for the innovator and investor are satisfied

$$\frac{f(s|h^t,E,H)E_{G(h^t,E)}(p)}{f(s|h^t,E,H)E_{G(h^t,E)}(p)+f(s|h^t,E,L)(1-E_{G(h^t,E)}(p))} \geq m \;\forall\; s \in S^{tr} \tag{2.15}$$

$$\sum_{s\in S^{tr}} f(s|h^t,E,H)p^j + k\sum_{s\in S^{tr}} f(s|h^t,E,L)(1-p^j) \geq c \text{ iff type } p^j \text{ plays } E \tag{2.16}$$

2.15 ensures that investor invests where $E_{G(h^t,E)}(p)$ is expectation of p conditional on observing innovator playing E in period t along with the history h^t. 2.16 says that type p^j innovator would play E iff his expected payoff from doing this is no less than playing NE.

Let $\bar{p}_{Z_{G(h^t)}}$ be an upper bound of $Z_{G(h^t)}$. The left hand side of the inequality in 2.15 is nondecreasing in $E_{G(h^t,E)}(p)$ and 2.16 is independent of $E_{G(h^t,E)}(p)$. Hence, we can fix $E_{G(h^t,E)}(p) = \bar{p}_{Z_{G(h^t)}}$. Then, 2.15 and 2.16 are relaxed if we increase $f(s|h^t,E,H)$ hence, we can set $f(s|h^t,E,H) = 1$ for some $s^{tr} \in S^{tr}$ and take $|S^{tr}| = 1$.

$$\frac{\bar{p}_{Z_{G(h^t)}}(1-m)}{m(1-\bar{p}_{Z_{G(h^t)}})} \geq f(s^{tr}|h^t,E,L) \tag{2.17}$$

$$p^j + k(1-p^j)f(s^{tr}|h^t,E,L) \geq c \tag{2.18}$$

2.18 is further relaxed if we increase p^j. Since the maximum value p can take is $\bar{p}_{Z_{G(h^t)}}$ substituting we get,

$$f(s^{tr}|h^t,E,L) \geq \frac{c-\bar{p}_{Z_{G(h^t)}}}{(1-\bar{p}_{Z_{G(h^t)}})k} \tag{2.19}$$

Combining 2.17 and 2.19 we obtain that $f(s^{tr}|h^t,E,L)$ satisfying both, 2.17 and 2.19, exists iff $\bar{p}_{Z_{G(h^t)}} \geq \frac{mc}{(1-m)k+m}$, contradicting that $Z_{G(h^t)} \subseteq [0, \frac{mc}{(1-m)k+m})$. □

Lemma 2.3. *If $Z_{G(h^t)} \subseteq [c,1]$ full disclosure is the optimal policy.*

Proof. If $Z_{G(h^t)} \subseteq [c,1]$ then fully disclosing information induces all types $p \in Z_{G(h^t)}$ to play E i.e. there is no problem of incentivizing the innovators to realize ideas. The investor invests iff state is H. This maximizes the expected stage payoffs of the investor. Since this is true for all $Z_{G(h)} \subseteq [c,1]$ it implies that for all possible beliefs paths the optimal static policy would be the same (full disclosure). Hence, no matter how beliefs evolve fully disclosing information period by period is the optimal dynamic rule, as well. □

Now I formulate the dynamic programming problem for the regulator. In order to prove that the problem is well defined and to characterize the optimal dynamic disclosure I take the following approach: first, I consider a problem where constraints for the innovator are ignored (the relaxed problem). Assuming that the value function exists, I derive necessary conditions that it must satisfy (claims 2.1, 2.2 and 2.3). Then I show that under those necessary conditions, investor's constraints are satisfied. This step enables me to drastically simplify the problem. In particular, I reduce the space of disclosure rules over which I need to optimize. Given this, it becomes easy to prove existence, uniqueness, monotonicity and continuity of the value function. In the end, I show that the optimal rule is a threshold rule in the belief about high success rate.

The problem is formulated as follows,

$$V(G(h^t)) = \max_{(S_t, f(\cdot|h^t,E,\theta_t)_{\theta_t\in\Theta})} \tag{2.20}$$

$$\left\{\begin{array}{c}(1-\delta)\sum_{s\in S^{tr}}[f(s|h^t,E,H)P(Z_{G(h^t,E)})E_{G(h^t,E)}(p)(1-m)-\\ f(s\mid h^t,E,L)P(Z_{G(h^t,E)})E_{G(h^t,E)}(1-p)m]+\\ \delta[\sum_{s\in S^{tr}} f(s|h^t,E,H)P(Z_{G(h^t,E)})E_{G(h^t,E)}(p)V(G(h^t,E,I,s^H))+\\ \sum_{s\in S^{tr}} f(s|h^t,E,L)P(Z_{G(h^t,E)})E_{G(h^t,E)}(1-p)V(G(h^t,E,I,s^L))+\\ [\sum_{s\in S^{trc}} f(s|h^t,E,H)P(Z_{G(h^t,E)})E_{G(h^t,E)}(p)+\\ \sum_{s\in S^{trc}} f(s|h^t,E,L)P(Z_{G(h^t,E)})E_{G(h^t,E)}(1-p)]V(G(h^t,E,NI,s))+\\ (1-P(Z_{G(h^t,E)}))V(G(h^t,NE,NI,\oslash))]\end{array}\right\}$$

$$s.t.$$

$$\frac{f(s\mid h^t,E,H)E_{G(h^t,E)}(p)}{f(s|h^t,E,H)E_{G(h^t,E)}(p)+f(s|h^t,E,L)(1-E_{G(h^t,E)}(p))} \geq m \ \forall\ s\in S^{tr}$$

$$\sum_{s\in S^{tr}} f(s|h^t,E,H)p^j + k\sum_{s\in S^{tr}} f(s|h^t,E,L)(1-p^j) \geq c \text{ if type } p^j\in Z_{G(h^t,E)}$$

Here $P(Z_{G(h^t,E)})$ is the probability of $Z_{G(h^t,E)}$ under $G(h^t)$.

Claim 2.1. *$V(G)$ is convex in the relaxed problem.*

Proof. I consider the problem 2.20 without the first constraint. Fix $\lambda \in [0,1]$ and $G = \lambda G' + (1-\lambda)G''$. I denote by $\mathbf{f^G}$ an optimal dynamic rule when we start from public belief G. I use the notation $V_{\mathbf{f^G}}(G)$. We know that $\sup_{\mathbf{f}}\{\lambda W_{\mathbf{f}}(G') + (1-\lambda)W_{\mathbf{f}}(G'')\} \leq \lambda V_{\mathbf{f^{G'}}}(G') + (1-\lambda)V_{\mathbf{f^{G''}}}(G'')$ is true. Hence, if we show that there exists some $\mathbf{f}$ such that $V_{\mathbf{f^G}}(G) \leq \lambda W_{\mathbf{f}}(G') + (1-\lambda)W_{\mathbf{f}}(G'')$ then we will obtain the convexity. I will construct such $\mathbf{f}$.

Start from some h^t such that $G = G(h^t)$. Define $\mathbf{f}$ in the following manner: set $(S_{t'}, f(\cdot|h^{t'}, E, \theta_{t'})_{\theta_{t'}\in\Theta}) = (S^G_{t'}, f^G(\cdot|h^{t'}, E, \theta_{t'})_{\theta_{t'}\in\Theta})$ for all histories which proceed from h^t.

In particular, at h^t I set $(S_t, f(\cdot|h^t, E, \theta_t)_{\theta_t\in\Theta}) = (S^G_t, f^G(\cdot|h^t, E, \theta_t)_{\theta_t\in\Theta})$. Since $G = \lambda G' + (1-\lambda)G''$, the first period expected stage payoff to the regulator is linear in G and the expression in the second constraint does not depend on G, we must have that the first stage payoff to the regulator under $\mathbf{f^G}$ equals first stage payoff under $\mathbf{f}$.

Now consider any history $h^{t'}$. Let $prob_G(h^{t'})$ be its probability under G. I claim that $prob_G(h^{t'})$=$\lambda prob_{G'}(h^{t'}) + (1-\lambda)prob_{G''}(h^{t'})$. Conditional on the true success rate of innovation, p^j, the history is determined by disclosure rules and agents' actions which are the same under $\mathbf{f}$ and $\mathbf{f_G}$, by construction. This fact and $G = \lambda G' + (1-\lambda)G''$ imply the claim. Now we see that $V_{\mathbf{f^G}}(G) = \lambda W_{\mathbf{f}}(G') + (1-\lambda)W_{\mathbf{f}}(G'')$ as after any given history $\mathbf{f_G}$ and $\mathbf{f}$ are the same, by construction, first period expected stage payoff to the regulator is linear in G and the expression in the second constraint does not depend on G. □

Claim 2.2. *In the relaxed problem, the optimal dynamic rule uses at most two signal realizations (σ^H, σ^L) in each period and sets $f(\sigma^H|h^t, E, H) = 1$ for all h^t.*

Proof. The second part follows because the second constraint in problem 2.20 is relaxed and stage payoff increases if we increase $f(s \mid h^t, E, H)$ for some $s \in S^{tr}$. In addition, since $V(G)$ is convex and increasing $f(s|h^t, E, H)$ for such s is the same as providing a better information (in the Blackwell sense) for the continuation game, we have that expected continuation payoffs also go up with $f(s|h^t, E, H)$ for any $s \in S^{tr}$.

Since, $f(\sigma^H|h^t, E, H) = 1$ for all h^t, the objective function and the second constraint now only depend on $\sum_{s\in S^{tr}} f(s|h^t, E, L)$ (not $f(s|h^t, E, L)$ separately). Hence, we can restrict attention to rules with at most 2 signal realizations. ■ □

Using implications of claim 2.2 in the objective function, we see that, increasing $f(\sigma^H|h^t, E, L)$ strictly decreases the objective and is only useful for incentivizing innovation (relaxing constraint 2.2). Hence, the optimal dynamic rule in the relaxed problem would use one of the 2 disclosure rules in each period. One disclosure rule induces, both, p^H and p^L to innovate. Another, induces only p^H to innovate. Thus, we can rewrite the relaxed problem as,

$$V(G) =$$

$$\max \left\{ \begin{array}{c} g\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta(1-g)\frac{[p^L((1-m)k+m)-mc)]}{k}, \\ \max_{f(\sigma^H|G,E,L)}\{(1-\delta)[E_G(p)(1-m)- \\ f(\sigma^H \mid G, E, L)E_G(1-p)m]+ \\ \delta[E_G(p)V(G(s^H)) + E_G(1-p)V(G(s^L))]\} \end{array} \right\}$$

$$s.t.$$

$$p^L + kf(\sigma^H|G, E, L)(1-p^L) \geq c$$

$G(s)$ denotes posterior public belief given prior G and signal s. g denotes $g(p^H)$ and $1-g$ denotes $g(p^L)$.

Claim 2.3. $f(\sigma^H|G, E, L) = \frac{c-p^L}{k(1-p^L)}$

Proof. Continuation values are not affected by $f(\sigma^H|G, E, L)$ and stage payoff strictly decreases in $f(\sigma^H|G, E, L)$. Hence, we make the innovator's constraint to bind. □

The relaxed problem can be rewritten as

$$V(G) = \tag{2.21}$$

$$\max \left\{ \begin{array}{c} g\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta(1-g)\frac{[p^L((1-m)k+m)-mc)]}{k}, \\ (1-\delta)[E_G(p)(1-m) - \frac{c-p^L}{(1-p^L)k}E_G(1-p)m]+ \\ \delta[E_G(p)V(G(s^H)) + E_G(1-p)V(G(s^L))] \end{array} \right\}$$

Lemma 2.4. *The investor's incentive constraints are never violated in the relaxed problem.*

Proof. We need to verify that $\frac{E_G(p)}{E_G(p)+\frac{c-p^L}{(1-p^L)k}(1-E_G(p))} \geq m$ for all G. Since the expression increases in g we take the worst case where $g = 0$. The condition reduces to $p^L \geq \frac{mc}{(1-m)k+m}$ which is true for $p^L \in (\frac{mc}{(1-m)k+m}, 1]$ (otherwise, apply Lemma 2.2). □

Now, we are ready to prove the existence and uniqueness of V that satisfies 2.21. I also show that V is continuous, strictly increasing and convex in g.

Lemma 2.5. *There exists unique V that satisfies 2.21. Moreover, it is continuous, strictly increasing and convex in g.*

Proof. Define operator T as follows,

$$T(W)(g) = \tag{2.22}$$

$$\max \left\{ \begin{array}{c} (1-\delta)g\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta[gW(1) + (1-g)W(0)], \\ (1-\delta)[E_G(p)(1-m) - \frac{c-p^L}{(1-p^L)k}E_G(1-p)m]+ \\ \delta[E_G(p)W(g^H) + E_G(1-p)W(g^L)] \end{array} \right\}$$

It is straightforward to verify that T satisfies Blackwell's sufficient conditions for the contraction mapping. Also, it maps continuous and bounded functions to continuous and bounded functions as upper envelope of continuous functions is continuous and current stage payoffs are bounded. Also, if we start with W weakly increasing in g, T will generate a function that is strictly increasing in g because stage payoffs from choosing either d_{p^H} or d_{p^L} are strictly increasing in g. Hence, we have a contraction mapping on the space of continuous, bounded and weakly increasing functions. Moreover, mapping happens from weakly increasing functions to the strictly increasing functions. Using standard arguments from Stokey & Lucas (1989) leads to the result.

To verify that V is convex in g one may use similar argument as in claim 2.1. □

Now we are ready to conclude the proof of the proposition 2.4. I argue that the optimal dynamic disclosure rule is a threshold rule in g.

Proof of Proposition 2.4

Proof. If $g = 1$ then the optimal thing to do is to set static optimal rule for $p^j = p^H$. This gives the average discounted payoffs of $\frac{[p^H((1-m)k+m)-mc)]}{k}$. This observation will be useful.

We need to show that if for some $g'' < 1$ we have

$$V(G'') = g''\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta(1-g'')\frac{[p^L((1-m)k+m)-mc)]}{k}$$

then for all $g > g''$ we also have

$$V(G) = g\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta(1-g)\frac{[p^L((1-m)k+m)-mc)]}{k}$$

By contradiction, suppose that there is some $g' > g''$such that $V(G') = (1-\delta)[E_{G'}(p)(1-m) - \frac{c-p^L}{(1-p^L)k}E_{G'}(1-p)m] + \delta[E_{G'}(p)V(G(s^H)) + E_{G'}(1-p)V(G(s^L))] > g'\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta(1-g')\frac{[p^L((1-m)k+m)-mc)]}{k}$.

Since $g' > g''$ we have that $g'\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta(1-g')\frac{[p^L((1-m)k+m)-mc)]}{k}$ $> V(G'')$ and hence $V(G'') < V(G')$. Since $g' > g''$, $V(G'') < V(G')$, $V(G)$ is convex and $V(G) \geq g\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta(1-g)\frac{[p^L((1-m)k+m)-mc)]}{k}$ for all G, we must have that for all $g > g'$ it is the case that $V(G)$ is strictly increasing. Suppose this was not the case, then there would be some $\bar{g} > g \geq g'$ with $g\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta(1-g)\frac{[p^L((1-m)k+m)-mc)]}{k} \leq V(G), \bar{g}\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta(1-\bar{g})\frac{[p^L((1-m)k+m)-mc)]}{k} \leq V(\bar{G})$ and $V(\bar{G}) \leq V(G)$. If we take $\lambda \in (0,1)$ such that $G = \lambda G'' + (1-\lambda)\bar{G}$ then, $V(G) > \lambda V(G'') + (1-\lambda)V(\bar{G})$ which violates convexity of V.

Since for all $g > g'$ it is the case that V is strictly increasing, and $V(G) \geq g\frac{[p^H((1-m)k+m)-mc)]}{k} + \delta(1-g)\frac{[p^L((1-m)k+m)-mc)]}{k}$ for all G, it must be the case that $V(g=1) > \frac{[p^H((1-m)k+m)-mc)]}{k}$ which contradicts that the value from the optimal rule is $\frac{[p^H((1-m)k+m)-mc)]}{k}$ at $g = 1$. □

Proposition 2.6. *Consider the dynamic model with inactive platform, $T = 2$ and $k = 1$. The regulator's optimal dynamic rule is a threshold rule where threshold g^* is given by*

$$g^* = \frac{(1-p^L)(p^L - mc)}{(1-p^L)(p^L-mc) + (1+\delta)(p^H-p^L)(1-c)m}$$

Proof of Proposition 2.6

Proof. In the second period, the regulator solves

$$\max\{g(p^H - mc), (1-g)(p^L - mc) + g((1-m)p^H - m(1-p^H)\frac{c - p^L}{(1-p^L)})\}$$

I use g^θ for the updated g after learning θ.

Regulator separates in the second period iff $g \geq Z_0$ where Z_0 is

$$Z_0 \equiv \frac{(1-p^L)[p^L - mc)]}{(1-p^L)[p^L - mc)] + (p^H - p^L)(1-c)m}$$

Z_0 completely pins down behavior of the regulator in the second period.

If the regulator chooses not to separate in the first period, then in the second period there are 3 cases:

Case 1- $g^H, g^L \geq Z_0$,

Case 2- $g^H, g^L < Z_0$

Case 3- $g^H \geq Z_0$ and $g^L < Z_0$.

Note that $g^L \geq Z_0$ and $g^H < Z_0$ can never arise as $g^H > g^L$.

In addition, I account for two more thresholds for g. Z_1 will be the threshold above which $g^H \geq Z_0$ and Z_2 threshold above which $g^L \geq Z_0$. Those thresholds are

$$Z_1 \equiv \frac{p^L(1-p^L)(p^L - mc)}{p^L(1-p^L)(p^L - ic) + p^H(p^H - p^L)(1-c)m}$$

$$Z_2 \equiv \frac{(1-p^L)^2(p^L - mc)}{(1-p^L)^2(p^L - mc) + (1-p^H)(p^H - p^L)(1-c)m}$$

Now for the three cases, we account for thresholds, Z_{case1}, Z_{case2} and Z_{case3}. Z_{case1} for instance would say that if prior g is above Z_{case1} and in the second period Case 1 obtains then regulator would separate. These thresholds are calculated as follows- for each case, we know what is the second period optimal rule under each realization of θ which we substitute as a continuation value and find the optimal rule in the first period. Those thresholds are given by

$$
\begin{aligned}
Z_{case1} &\equiv \frac{(1-\delta)(1-p^L)(p^L-mc)}{(1-\delta)(1-p^L)(p^L-mc)+(p^H-p^L)(1-c)m} \\
Z_{case2} &\equiv \frac{(1-p^L)(p^L-mc)}{(1-p^L)(p^L-mc)+(1+\delta)(p^H-p^L)(1-c)m} \\
Z_{case3} &\equiv \frac{(1-\delta p^L)(1-p^L)(p^L-mc)}{(1-\delta p^L)(1-p^L)(p^L-mc)+(1+\delta(1-p^H))(p^H-p^L)(1-c)m}
\end{aligned}
$$

One can verify that $Z_2 > Z_1 > Z_{case2} > Z_{case3} > Z_{case1}$. Case 2 can obtain only for $g \in [Z_{case2}, Z_1)$ and increasing g beyond Z_1 gets us to the case 3. Because $Z_{case3} < Z_1$ the regulator still separates as we increase g further. Beyond Z_2 we go to case 1 and since $Z_{case1} < Z_2$ the regulator still separates. Hence, we have a threshold rule and the threshold is given by $g^* = Z_{case2}$.

□

We can conduct the following comparative static exercises as corollaries to the proposition 2.6,

i) $\frac{\partial g^*}{\partial m} < 0$. Higher investment cost incentivizes learning (separation) as under no separation the quality of the information provided to the investors is low (higher probability of investment in state L) and as m increases the investor becomes more vulnerable to low quality information.

ii) $\frac{\partial g^*}{\partial c} < 0$ if $m > p^L$ and otherwise $\frac{\partial g^*}{\partial c} \geq 0$.

iii) $\frac{\partial g^*}{\partial \delta} < 0$. Higher patience means that there is more weight on the gains from learning through separation.

iv) $\frac{\partial g^*}{\partial p^L} > 0$ and $\frac{\partial g^*}{\partial p^H} < 0$. Low p^L or high p^H means that the first period gain from pooling (increased probability of trade in the first period) is low.

Chapter 3

Microtargeting Rational Voters

3.1 Introduction

Consumer targeting strategies have long been utilized in marketing a wide variety of products. Segmenting consumers based on their preferences and advertising product features that are most relevant to a given segment of consumers, are at the core of such strategies. Recently, it has become a trend to apply similar strategies in political campaigns. This has been termed as *microtargeting.* It implies collecting information on potential voters' preferences, habits and demographic characteristics, and using that information to approach voters with messages that are tailored to their interests.[1]

Drastic developments in information technologies and easier access to consumer data has made microtargeting more applicable. However, similar strategies have been used to some extent at least since 1892, when Republican National Committee chairman James Clarkson boasted:

> "(I) with two years of hard work, secured a list of the names of all the voters in all the important States of the North, in 20 or more states, and lists with the age, occupation, nativity, residence and all other facts of each voters' life, and had

[1]Another method for tailoring messages for a particular audience is known as *dog whistle* politics. It means sending messages that can only be decoded by certain members of an audience. Such technique is often used in the context of racism e.g. in 2012, Obama's Deputy Campaign Manager sent out an email stating that Paul Ryan was "making a pilgrimage" to Las Vegas to "kiss the ring" of Adelson. This message was interpreted as a dog whistle targeted to antisemite audience, that would be pleased by such a tone.

> them arranged alphabetically, so that literature could be sent constantly to each voter directly, dealing with every public question and issue from the standpoint of his personal interest."[2]

The importance and potential of microtargeting in political campaigns is widely acknowledged by political campaign management firms and experts in the field, and many see it at the core of the future of political campaigning.[3]

This chapter studied the benefits and costs of microtargeting, for political candidates, and its implications for political awareness and voter turnout. More specifically, I ask the following questions: *To what extent would political candidates use microtargeting strategies? How does the availability of better targeting technologies influence political awareness and voter turnout? How sensitive is a candidate's campaigning behavior to voters' strategic sophistication, and how does this depend on the quality of microtargeting? What are the effects of negative campaigning on political awareness?*

To answer the questions, I study a model of electoral competition with two candidates. Each candidate has a fixed policy position unknown to the voters. A candidate can use campaigning strategies in order to influence voters' behavior by informing the voters about the policy position. The information is hard - a voter is approached either by a message conveying the whole truth, or not approached by a message. The intensity with which a voter is approached by a message is determined by a candidate's campaigning strategy. There are 2 policy positions and each voter has a strict policy preferences. There is an aggregate uncertainty about the distribution of voters' preferences in the population, and conditional on the resolution of that uncertainty, each voter's preference is drawn independently. Voting is costly and costs are drawn independently across the voters. The candidates simultaneously choose their campaigning actions. Voters receive messages from the campaigns and independently decide which candidate to vote for. The candidate with the majority of votes wins the election. I consider two models of voter behavior. One with *rational* (instrumental) voters.[4] Another with *boundedly rational* (expressive) voters.[5]

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[2]See Michael McGerr (1986)

[3]See http://www.winningcampaigns.org

[4]A voter is rational, when he takes into account pivotality of his vote.

[5]A boundedly rational voter cannot do equilibrium calculation of nontrivial pivotal probabilities and takes

To allow for microtargeitng, for a given candidate, I model a feasible set of campaigning strategies (this set is called *campaigning technology*) as a set of pairs of probabilities, where one element in a given pair corresponds to the probability that a voter whose most preferred policy is the policy position held by the candidate is approached by the campaign's message. Another element in a pair corresponds to the probability that a voter whose most preferred policy is not the policy position held by the candidate, is approached by the campaign's message. For example, if this set is a 45 degree line, then the candidate cannot microtarget at all. On the other extreme, if it is the entire unit square, then the candidate has *unlimited targeting power*. Given the choice of the pair of probabilities, voters are independently approached by a message (revealing the candidate's position, if approached).

3.1.1 To What Extent Would Political Candidates Use Microtargeting Strategies?

To understand the merits of microtargeting to a candidate, fix a policy position of a given candidate, and note that conditional on this candidate winning, there are two types of voters in a population - potential winners and potential losers. If the candidate is somehow able to discriminate between potential winners and losers (by segmenting voters), she would like not to inform potential losers about her position, as losers would learn that stakes are higher and would vote against this candidate. She would like to inform only the winners. It turns out that, fixing voters' behavior in any equilibrium, this is indeed how a candidate who has unlimited power in targeting would behave. If a candidate has limited targeting power (e.g. information provided to potential winners may leak to losers through social networks), then even though she can target to some degree, in this process there is a chance that losers will also learn about her policy, and hence voter alienation will take place. Limited targeting power means that campaigning technology has a built in trade-off between informing winners and achieving better targeting - better information provided to potential winners may lead to information leakage to potential losers further leading to more aggressive voting by losers.

It is often argued that whenever voters perceive higher difference in candidates' policies

them as given. None of the results is affected if instead we assume expressive voting.

they are turned on to vote.[6] The reason is that voters perceive higher stakes in the election and would like to avoid loss in payoff, in case a candidate who they dislike wins. Gains to a candidate from microtargeting work through a similar channel - a candidate would like potential supporters to be informed about her position, as this increases stakes for such voters, but would like potential opponents to be uninformed - keeping stakes low for such voters.

3.1.2 Microtargeting and Voter Turnout

The empirical research on the effects of campaigning on voter turnout addresses two questions: how does intensity of campaigning (campaign spending) affect turnout for a fixed campaigning method (e.g. TV ads)? How does turnout change across different methods of campaigning (e.g. ground vs mass campaigning)? Answers to the first question are mixed that makes it difficult to argue for a stylized relationship between campaign spending and turnout. With regard to mass campaigning, some studies find no or negligible impact of campaign spending on turnout.[7] Others find a significant positive relationship.[8] The answer to the second question is more settled. Roughly, the empirical literature on campaigns and voter mobilization divides political campaigning into two categories: mass campaigning and ground/local campaigning. Ground campaigning involves face to face contacts and direct mails, while mass campaigning is mostly televised and other types of mass media advertising. It has been argued that personal canvassing and direct mails are much more effective than mass campaigning in increasing voter turnout.[9] This is mostly attributed to the ground campaigning being more effective in targeting voters. Arguably, the switch from traditional face to face campaigning to mass campaigning has contributed to the decreased turnout rates in the US since 1960s.

Feddersen & Pesendorfer (1999) show that improving information environment (increasing the fraction of the population that is informed) may lead to lower turnout in a rational voter electoral competition model. This happens because less informed voters may tend to

[6]See Stromberg (2004), Matsusaka (1995), Krasno & Green (2008).
[7]See Green et al. (2008)
[8]See Freedman et al. (2004), Hillygus (2005)
[9]See Gerber & Green (2000), Enos & Fowler (2016)

delegate voting power to more informed voters by abstaining, in order to avoid by mistakenly voting for a wrong candidate. This suggests, that even if one believes that more intense campaigning improves information environment, this should not be sufficient to conclude that turnout will increase. So, even if switching to mass campaigning in 1960s led to increased political awareness in the society, Feddersen & Pesendorfer (1999) rationalize decreasing turnout along with increased political awareness. However, switching to mass campaigning also meant less targeting. Can we rationalize less targeting, along with increased political awareness and decreasing turnout?

My model allows for a framework rich enough to study relationship between campaign spending (political awareness) and voter turnout for various specifications of campaigning technologies, and rationalizes the phenomenon where political awareness improves, targeting becomes less effective and turnout decreases. I focus on a class of campaigning technologies (linear technologies) and find conditions under which a result with the similar flavor as in Feddersen & Pesendorfer holds. I show that higher spending leading to a lower aggregate turnout is an equilibrium phenomenon - equilibria with higher campaign spending may involve lower aggregate turnout. In addition, higher spending equilibria always involve worse trageting. In particular, if the voters are boundedly rational, aggregate turnout is always nondecreasing in campaign spending. This happens because higher spending increases expected number of better informed voters who are more likely to vote compared to their less informed peers. In the model with the rational voters additional strategic force arises: higher spending decreases probability with which a particular voter is pivotal since a voter knows that there is a better information environment, more voters are informed and vote with higher probability. This decreases the probability with which both, informed and uninformed voters vote. It turns out that this concern is sufficiently strong to decrease aggregate turnout whenever there is a sufficiently good status-quo information environment in the population (probability with which a voter is informed in the absence of campaigning).

3.1.3 How Sensitive are Campaigns to Voters' Strategic Sophistication?

The question - why people vote? - has long been a subject of debate. It is believed that in order to have an electoral competition model capable of matching and explaining empirical regularities, it is important to understand why and how people vote. I consider two models of voter behavior. In the *model 1* voters are rational. In the *model 2* voters are boundedly rational, in the sense that they cannot calculate non-trivial probabilities of being pivotal in the election. If a boundedly rational voter expects that ex-ante homogeneous candidates are playing symmetric strategies, then he understands that receiving the same message from both candidates makes his vote redundant, in expectation. Otherwise, the voter assigns some exogenous probability to the event that his vote is pivotal. In equilibrium, bundedly rational voter has correct beliefs about candidates' strategies.

I compare equilibrium behavior of candidates across these two models of voting behavior, when candidates have unlimited power in targeting, and when candidates have limited targeting power (there is a tradeo-ff between providing more information to the potential supporters, and leaking that information to the potential opponents). It turns out, that equilibrium behavior of a candidate is the same across the models, if candidates have unlimited targeting power. On the other hand, if they have limited targeting power, behavior is same only under certain condition (explained shortly) on the aggregate uncertainty about voters' preference distribution in the population.

When candidates have unlimited targeting power, it is irrelevant whether the voters are rational or boundedly so, because this distinction does not change the aspects of voter behavior that are relevant for the candidates' incentives. Those aspects are: conditional on voting, a voter votes for the preferred candidate; conditional on having same signal about each candidate, a voter does not vote (saves on the cost of voting); and conditional on having different signals about the candidates, more informed voters vote with higher probability, compared to the less informed counterparts. The only distinction between the behavior of rational and boundedly rational voters arises in the event when a voter receives different signals about 2 candidates. Under this event, the probability with which a voter votes for a preferred candidate is exogenous for a boundedly rational voter, but is endogenous for a rational voter. When voters are boundedly rational, as long as it is the case that a more informed voter (rational or boundedly so) votes with higher probability, candidates' incentives are the same as in the model with rational voters.

Whenever candidates have limited targeting power, concerns about how voters vote becomes relevant via the *underdog effect*.[10] To see how this effect works, suppose we are interested in the pure strategy interior equilibria, meaning that candidates' equilibrium spending are interior in some compact space. Conditional on candidates having the same policy stance, a candidate can increase her probability of winning the election through the following channel: if she increases spending to inform more voters, she will be able to reach potential supporters and increasing their probability of voting for her. However, on the other side, some potential opponents will also become informed, and hence such votes will be lost. Spending more than your competitor amounts to gaining potential supporter votes and losing potential opponent votes. Spending less than your competitor, amounts to gaining potential opponent votes (because they will be more more likely to be uninformed about your policy than your competitor's policy, and will vote for you with higher probability) and losing potential supporter votes. Suppose potential supporters are underdogs. This implies that the supporters vote with higher probability than opponents, and you want to gain their votes. The other candidate also wants to do this. If such gains are high enough, both candidates would always want to increase their spending, and thus no interior equilibrium would exist. One way to deter such behavior, is to have a condition on the aggregate uncertainty about preference distribution in the population. The condition makes sure that even though an underdog votes with higher probability, the likelihood of a voter being an underdog is low enough. The underdog effect is present only in the model 1. Boundedly rational voters do not generate that effect, because they do not calculate endogenous probabilities of being pivotal.

[10] See Goeree & Großer (2007), Krishna & Morgan (2003), Myatt (2015)

3.1.4 Negative Campaigning and Political Awareness

People usually focus on cons of negative campaigning and view it as being detrimental to political process. Geer (2006) and Schipper & Woo (2016) are some exceptions. Geer (2006), provides evidence that shows that negative campaign ads have higher information content, than positive campaign ads. Schipper & Woo (2016) study a Downsian model of electoral competition with the possibility of microtargeting, and conclude that negative campaigning improves information environment of elections. The way it works in their model is as follows: providing information is beneficial for one candidate, whenever it is not beneficial for another. Since candidates can target voters, they will always try to provide information about own policies whenever it is beneficial for them, and about other candidate's policy whenever it is not beneficial for the other candidate. Hence, maximum possible information will be provided in the equilibrium.

Introducing the possibility of negative campaigning, I obtain a similar result. Possibility of negative campaigning leads to the more politically aware electorate. The intuition is similar to the Schipper & Woo (2016).

The chapter is organized as follows: section 3.2 reviews related theoretical literature; section 3.3 constructs and analyses the model with rational voters, and under linear campaigning technology; section 3.4 studies the model with boundedly rational voters; section 3.5 discusses voter turnout and its comparative statics; section 3.6 includes results on general campaigning technologies; and section 3.7 about negative campaigning.

3.2 Literature Review

3.2.1 Rational Voters

In the electoral competition models with discrete number of rational voters it is usually intractable to obtain sharp results unless one considers limiting results for large societies. Myatt (2015), Krishna & Morgan (2011), and Myerson (1998) are some papers that take this route. The closest to my model is Myatt (2015). The distinction from his model is that I allow for strategic candidates and campaigning.

3.2.2 Political campaigning

There is a vast strand of theoretical literature on political campaigning. Austen-Smith (1987) construct a model where candidates obtain contributions from interest groups in order to inform voters about their policy positions. Candidates' objective is to maximize probability of winning simple majority rule elections. They assume exogenous functions for voting probabilities and discuss implications of campaign contributions for elite polarization. Coate (2004) proposes similar model and analyses welfare implications of limits on campaign contributions. Matsusaka (1995) abstracts away from strategic considerations of candidates, assumes that voters have uncertainty about fixed candidates' types; assumes that voters vote probabilistically and that before voting they receive informative signals with some precision. He shows that better precision of signals increases turnout and interprets better precision as candidates spending more on advertising. All those models assume verifiable messages from candidates. Unlike my model, those papers do not allow for the possibility of negative campaigning, microtargeting or rational (instrumental) voters.

3.2.3 Microtargeting

Recently, some authors realized that microtargeting has a potential for explaining various phenomena arising in the choice of policy positions, turnout and electorate information environment. Glaeser, Ponzetto and Shapiro (2005) study implications of microtargeting for observed extremism in the policy choices between Democrats and Republicans. They build on Downsian model where candidates choose policy positions. They assume that a candidate has a set of citizen types (insiders) who observe her position with higher probability relative to the other types of citizens. They show how this leads to candidates catering to insiders and thus not necessarily choosing ideal position for a median voter. In contrast to my model, they assume probabilistic voting and exogenous targeting technology i.e. candidates do not choose which voters and to what extent to inform. Prummer (2016) builds on the Downsian model with probabilistic voters and studies how elite polarization can arise in an environment with targeted campaigning strategies. The intuition is similar to Glaeser, Ponzetto and Shapiro (2005). They also show that improving campaigning technologies (that is to be able to target messages narrowly to a certain segment of voters) increases polarization. Schipper & Woo (2016) abstract away from the policy choice of candidates (as in my model). Positions are fixed but unknown to voters. Candidates have complete information about the voters' preferences (unlike in my model) and choose which voters to target i.e. which voters to inform about policy positions. Information is hard. Position of a candidate is a multidimensional vector from a space of $[0, 1]^n$ where n is number of possible issues. It is assumed that if the voters are not informed about a given dimension, they are not able to perceive it and thus beliefs about unobserved dimensions are irrelevant. It is shown that election outcomes in this games are same as election outcomes would be under full awareness of issues and complete information about the candidates' positions. It is also shown that such a complete information equivalence is violated under some alternative specifications of the model but introducing possibility of negative campaigning can restore the equivalence in some situations. My result on the benefits of negative campaigning verifies their finding and suggests that benefits of negative campaigning is not specific to their modelling choice. Kasamatsu (2016) is yet another paper that shows benefits of not restricting negative campaigning. My work departs from the above mentioned papers at least in two important dimensions: 1. none of those papers considers rational voters; 2. none of those papers studies campaigning technologies with limited targeting power.

3.3 Model 1

In this section, the model with rational voters is constructed and analyzed.

There are 2 candidates and $n + 1$ citizens. Candidates have fixed policy positions. A candidate's position is her private information. The candidates can use campaigning strategies in order to influence voters' behavior by informing the voters about policy positions. The campaigning technology (defined formally later) for each candidate is fixed. Each candidate chooses how much resources to allocate to campaigning. This choice leads to a certain probability that a given voter learns about the candidate's position.

A voter's preference type is his private information. There is an aggregate uncertainty on the electorate size and preference distribution. Voters are rational and have privately known cost of voting. After the voters receive messages (privately) about candidates' policies, they cast their votes taking into account equilibrium behavior of other voters and candidates. Voting rule is a simple majority rule and ties are broken fairly[11]. Candidates maximize probability of winning the election. I do not explicitly model costs of exerting effort or spending resources for the candidates. One can think about a candidate having predetermined fixed level of allowance for the campaign that is solely devoted to campaign and cannot be diverted for other reasons.

Formally, there is a population of size $n+1$ and two candidates, labelled A and B, competing in a majority rule election. There are 2 ideologies (issue positions, policy stances) labelled $p \in P = \{Y, N\}$. Candidate j is of type p_j with probability $\mu(p_j)$.

Throughout, without loss of generality, I assume that $\mu(Y) \geq \mu(N)$. There are two possible types of voter preferences, denoted $p_v \in P = \{Y, N\}$ where subscript v stands for a voter. Y voter prefers Y to N and N voter prefers N to Y (e.g. one can think about Y voters as potential winners from policy Y and N voters as potential losers from policy Y). A randomly chosen voter is of the type Y with probability λ. Conditional on λ, preference types are independently assigned to the voters. There is an aggregate uncertainty about λ. It is a common knowledge that λ is drawn from the distribution with the density $f(\lambda)$. There is also an aggregate uncertainty about electorate size - with probability a given citizen is eligible to vote. It is a common knowledge that a drawn from the distribution with the density $g(a)$. I denote by $\bar{a}$ the mean of a and $\bar{\lambda}$ denotes the mean of λ.

Assumption 3.1. *$F(\lambda), G(a)$ are strictly increasing and continuously differentiable with bounded first and second order derivatives.*

Voting is voluntary and costly. A voter gets net benefit u whenever his preferred candidate wins and incurs a cost c if he votes.

Assumption 3.2. *$c \sim U[0,1]$ and $u \in (0,1)$.*

Assumption 3.2 is made for normalization. Candidates are ex-ante symmetric. Each candidate maximizes the probability of winning the election. Candidate j privately observes

[11] results are immune to tie breaking rules

her type p_j. Type of a candidate can be thought of as a position on an issue or a policy. Once candidates learn their types (policy positions), they can inform potential voters about the policy positions by conducting political campaigns. I assume that information to a voter is conveyed based on a hard evidence and thus a candidate can inform either about her true type or can hide it.

3.3.1 Campaigning Technologies

A campaigning strategy of candidate j is a pair of probabilities,

$$(\phi_{opp}(p_j), \phi_{own}(p_j)) \in [0,1]^2$$

The first element in the pair is the probability with which a given voter of type different than p_j is approached by the campaign message of j (that is, perfectly informed about j's position) and the second element is the probability with which a voter with the same preference type as p_j is approached. A campaigning technology is a set of strategies that a candidate can choose from.

Definition 3.1. *A campaigning technology is a set $P_{CT} \in [0,1]^2$ such that P_{CT} is compact.*

Definition 3.1 allows for a wide class of technologies. In this work, I focus on a certain class of campaigning technologies that I call linear campaigning technologies.

Here I define linear campaigning technologies. In order to conduct a campaign, a candidate will need to spend resources (time, effort, money). Let the amount of the resources a candidate spends be denoted $m \in [0, \bar{m}]$. Choice of m determines the probability with which a candidate succeeds to approach a voter informing him about the policy stance. However, not all voters are approached in a symmetric way. For instance, if A is of type Y and incurs m_A^Y on campaigning then type Y voters have different probability of getting informed about $A's$ type than type N voters. This captures the possibility of microtargeting: $Y(N)$ type voter has a higher likelihood of getting informed about type $Y(N)$ candidate's policy than type $N(Y)$ voter.

More formally, campaign spending by candidate j who is of ideology type p_j is denoted

$m(p_j)$. A function $\phi_{p_v}(p_j, m(p_j)) : [0, \bar{m}] \to [0, 1]$ outputs the probability of approaching a $p_v \in \{Y, N\}$ voter when candidate j of type p_j spends $m(p_j)$.

Definition 3.2. *A linear campaigning technology is a function*

$$\phi_{p_v}(p_j, m(p_j)) = k_{p_v,p_j} + (1 - k_{p_v,p_j})m(p_j)$$

such that

i) $k_{p_v,p_j} \geq k_{p'_v,p'_j}$ *if* $p_v = p_j$ *and* $p'_v \neq p'_j$

ii) $k_{p_v,p_j} = k_{p'_v,p'_j}$ *if* $p_v = p_j$ *and* $p'_v = p'_j$ *or if* $p_v \neq p_j$ *and* $p'_v \neq p'_j$

iii) $k_{p_v,p_j} \in (0, 1]$ *for all* (p_v, p_j)

iv) $\bar{m} \in [0, 1)$

Part ii) of definition 3.2 ensures that campaigning technologies are symmetric across Y and N. Part i) ensures that the technology involves microtargeting. Thus, we have two parameters (k_H, k_L) with $k_H \geq k_L > 0$ and $k_H \equiv k_{p_v,p_j}$ if $p_v = p_j$; $k_L \equiv k_{p_v,p_j}$ if $p_v \neq p_j$. Part iv) defines bounds on m and implies that even if a candidate spend maximum amount, there is still a small probability that a voter will not be reached. Thus, off equilibrium events that could be observed by a voter are ruled out.

In order to interpret the linear campaigning technology parameters consider choice of m by a candidate of type Y. If $m = 0$, voters of type Y get informed with probability k_H and voters of type N get informed with probability k_L. Hence, (k_H, k_L) captures what voters can learn without campaigning. If the number of voters is sufficiently large, those parameters approximate political awareness in the society (share of informed voters) prior to the campaigning. Also, for any spending level m, the difference between Y and N types' probabilities of getting informed is $(k_H - k_L)(1 - m) \geq 0$ meaning that for any spending level, Y types have weakly higher likelihood of getting informed about Y type candidate than N types do. In addition, $(k_H - k_L)$ can be though of as a proxy for the highest possible level of targeting feasibly for a candidate. Note that there is a trade-off between achieving better targeting and informing voters i.e. $(k_H - k_L)(1 - m)$ is decreasing in m.

3.3.2 Strategies

Since I will be studying only symmetric pure strategy equilibria, I drop voter subscripts and define only pure strategies. Let $(r_A, r_B) \in P \cup \varnothing \times P \cup \varnothing$ be messages received by a voter from candidates A and B. Available voter's type is a quadruple $t = (r_A, r_B, p_v, c) \in T = P \cup \varnothing \times P \cup \varnothing \times P \times [0,1]$, that is private information to the voter. Candidate $j's$ type is p_j. A pure strategy for a candidate of type p_j is $m : P \to [0, \bar{m}]$ and a pure strategy for a voter of type t is $\sigma : T \to \{V_A, V_B, Abs\}$ where V_j denotes a decision to vote for candidate j and Abs denotes a decision to abstain. I will use bold symbols to denote strategy profiles.

3.3.3 Time-line and Solution Concept

Time-line of the game is as follows:

1. Each player is independently assigned a type - candidate j is assigned p_j, citizen i becomes a voter with probability a and is assigned type Y with probability λ (a and λ unknown) and assigned cost c from the uniform distribution. Types are privately known.
2. Candidates simultaneously choose spending levels, $m(p_j)$.
3. A voter of the preference type p_v receives a hard message from candidate $A(B)$ with the probability $\phi_{p_v}(p_A, m(p_A))(\phi_{p_v}(p_B, m(p_B)))$.
4. Voters simultaneously choose whether to cast a vote for A, for B or to abstain.
5. A candidate with the majority of votes wins the election and in case of a tie outcome is determined by a toss of a fair coin.

I study pure strategy ϵ-PBE in type symmetric strategies. From here on, I will refer to it as *equilibrium.*

3.3.4 Review of Myatt (2015) Lemma 1

Consider an election with $n + 1$ citizens, 2 candidates and a possibility of abstention. Let $v \in \Delta$ such that $\Delta = \{v \in R^3_+ : \sum_{i=0}^{2} v_i = 1\}$. v_i is the probability with which a randomly selected voter casts vote for candidate i and v_0 is the probability of abstention. Let $b \in B_n$ such that $B_n = \{b \in N_+^3 : \sum_{i=0}^2 b_i = n\}$ be a possible outcome of the election. If v is known then b has a multinomial distribution. If there is uncertainty about v that is represented by some density $h(v)$ then from the perspective of a given voter we have

$$P(b \mid h(\cdot)) = \int_{\Delta} \frac{\Gamma(n+1)}{\prod_{i=0}^{2} \Gamma(b_i+1)} \left[\prod_{i=0}^{2} v_i^{b_i} \right] h(v) dv \tag{3.1}$$

Lemma 1 in Myatt 2015 claims that if n is sufficiently large the expression in 1 is approximated by

$$\frac{1}{n^2} h(\frac{b}{n}) \tag{3.2}$$

The idea is as follows: as n becomes large, the expression in 1 becomes concentrated around the maximum of $\prod_{i=0}^{2} v_i^{b_i}$ that is maximized at $\frac{b}{n}$ and hence only the value of the density $h(\cdot)$ at $\frac{b}{n}$ matters.

3.3.5 Equilibria

Voter's Problem

I start by solving the voters' part of the game given a belief about some symmetric strategies plaid by the candidates.

An eligible voter has the following private information: the fact that he was drawn to be an eligible voter, his preference type, his cost of casting a vote and messages observed from the candidates. This information determines a voter's beliefs about the types of the candidates and types of other voters.

Remark 1: In any equilibrium, a voter who receives same messages from both candidates abstains. This is a direct consequence of type symmetry of strategies. Since same messages induces same beliefs across candidates' types, probability of a voter's vote being pivotal is 0 *and hence he abstains for all* $c \in (0, 1]$.

Remark 2: $\sigma(N, Y, Y, c) \neq V_A$, $\sigma(Y, N, N, c) \neq V_A$, $\sigma(\varnothing, N, N, c) \neq V_A$, $\sigma(\varnothing, Y, Y, c) \neq$

V_A, $\sigma(Y,\varnothing,N,c) \neq V_A$, $\sigma(N,\varnothing,Y,c) \neq V_A$ for all c and symmetrically for B. To see why this is true, consider a voter who learns that the candidates have opposite types, then give that he votes, it is always strictly better for him to vote for the candidate with $p_j = p_v$. Suppose a voter learns that candidate A has type $p_A = p_v$ and does not learn other candidate's type. Then, his vote is pivotal if and only if the other candidate is of the opposite type. This happens with some probability and the voter considers only this event when making the decision. However, this event occurs if and only if the candidates have opposite types and thus, given that he votes, he strictly prefers to vote for A. Same argument applies for the other cases.

Now I account for the probabilities of being pivotal when a voter receives different messages from candidates. Let $n_A, n_B, n-(n_A+n_B)$ denote the numbers of voters who vote for A, B and abstain. First, I consider a voter of a type with $r_A \neq r_B, r_A = p_v$ and $r_A \neq \varnothing, r_B \neq \varnothing$. This defines a subset of types $\bar{T} = ((Y,N,Y) \cup (N,Y,N)) \times [0,1]$. An element from $\bar{T}$ is denoted $\bar{t}$. Probability of $\bar{t}$ being pivotal for sufficiently large n, using 3.2, is approximated by

$$\begin{aligned} &\Pr(piv \mid r_A, r_B, p_v, c) = \\ &\frac{P(n_A = n_B - 1 \mid r_A, r_B, p_v, c) + P(n_A = n_B \mid r_A, r_B, p_v, c)}{2} \\ &\approx \frac{1}{n^2}\sum_{z=0}^{[n/2]} h(\frac{z}{n}, \frac{z}{n}, 1-\frac{2z}{n} \mid r_A, r_B, p_v, c) \end{aligned} \tag{3.3}$$

For n large, 3.3 defines a Riemann integral[12]

$$\Pr(piv \mid r_A, r_B, p_v, c) \approx \frac{1}{n}\int_0^{1/2} h(x, x, 1-2x \mid r_A, r_B, p_v, c)dx \tag{3.4}$$

Now I find distributions $h(x, x, 1-2x \mid r_A, r_B, p_v, c)$. For this, first I need to account for the probabilities of a randomly chosen voter voting for A, B and abstaining, from the perspective of a type $\bar{t} \in T$ voter. Because types of players are drawn independently, these probabilities depend only on (r_A, r_B). Moreover, because a voter believes that the candidates

[12]See Lemma 2 in Myatt (2015)

play symmetric strategies, we only need to consider the case where candidate A is of type Y and B is of type N. The opposite case is symmetric.

Let $z_A(Y,N)(z_B(Y,N))$ denote the probability that a randomly chosen voter votes for candidate $A(B)$ from the perspective of a voter who received message Y from candidate A and message N from candidate B. These probabilities are as follows

$$z_A(Y,N) = \lambda a \times \left\{ \begin{array}{c} \phi_Y(Y,m(Y))\phi_Y(N,m(N))C(Y,N,Y)+ \\ \phi_Y(Y,m(Y))(1-\phi_Y(N,m(N)))C(Y,\varnothing,Y)+ \\ (1-\phi_Y(Y,m(Y)))\phi_Y(N,m(N))C(\varnothing,N,Y) \end{array} \right\}$$

$$z_B(Y,N) = (1-\lambda) a \times \left\{ \begin{array}{c} \phi_N(Y,m(Y))\phi_N(N,m(N))C(Y,N,N)+ \\ \phi_N(Y,m(Y))(1-\phi_N(N,m(N)))C(Y,\varnothing,N)+ \\ (1-\phi_N(Y,m(Y)))\phi_N(N,m(N))C(\varnothing,N,N) \end{array} \right\}$$

$C(r_A, r_B, p_v)$ denotes a cost threshold below which a voter with (r_A, r_B, p_v) votes. Since costs are drawn from the uniform distribution, this is also the probability with which a voter of preference type p_v receiving messages r_A and r_B votes. To see that every equilibrium involves such threshold strategies note that a voter solves $Max\{\Pr(Piv \mid t)u - c, 0\}$ where Piv is the event when a voter's vote is pivotal. This, along with the remark 2, implies the threshold structure.

Let $z_A(Y,N) \equiv \lambda a z'_A(Y,N)$ and $z_B(Y,N) \equiv (1-\lambda) a z'_B(Y,N)$. Density $h(x,x,1-2x \mid Y,N,Y,c)$ is evaluated at $x = z_A(Y,N) = z_B(Y,N)$. The determinant of the Jacobian of $(x_A = z_A(Y,N),\ x_B = z_B(Y,N))$ is $a z'_A(Y,N) z'_B(Y,N)$. Transforming variables we obtain the following expressions,

$$\begin{aligned} h(x,x,1-2x \quad &| \quad Y,N,Y) = \frac{f(\lambda^* \mid p_v = Y)g(a \mid av)}{a z'_A(Y,N) z'_B(Y,N)} \\ \text{where } \lambda^* \quad &= \quad \frac{z'_B(Y,N)}{z'_B(Y,N) + z'_A(Y,N)} \\ \text{and } a \quad &= \quad \frac{x(z'_B(Y,N) + z'_A(Y,N))}{z'_A(Y,N) z'_B(Y,N)} \end{aligned}$$

Integrating $h(x, x, 1-2x \mid Y, N, Y)$ from 0 to 1/2 and noting that $f(\lambda \mid p_v = Y) = \frac{f(\lambda)\lambda}{\bar{\lambda}}$ and $g(a \mid av) = \frac{g(a)a}{\bar{a}}$ we get,

$$\int_0^{1/2} h(x, x, 1-2x \mid Y, N, Y, c)dx = \frac{f(\lambda^*)\lambda^*}{\bar{\lambda}\bar{a}(z'_B(Y, N) + z'_A(Y, N))}$$

Similar derivations lead to,

$$\int_0^{1/2} h(x, x, 1-2x \mid Y, N, N, c)dx = \frac{f(\lambda^*)(1-\lambda^*)}{(1-\bar{\lambda})\bar{a}(z'_B(Y, N) + z'_A(Y, N))}$$

Given the expressions above, it is now straightforward to derive formulas for pivotal probabilities for voters who receive information only about one candidate's type. First I calculate $P^{p_v}(p_j \mid \varnothing)$ for all $(p_v, p_j) \in \{Y, N\} \times \{Y, N\}$ where $P^{p_v}(p_j \mid \varnothing)$ is p_v voter's belief that a candidate is of type p_j conditional on not receiving a campaign message from that candidate. Note that this belief does not depend on other candidate's message because candidates play independent strategies and their types are drawn independently. These beliefs also do not depend on c. Using Baye's rule we get,

$$P^{p_v}(p_j \mid \varnothing) = \frac{\mu(p_j)(1-\varphi_{p_v}(p_j, m(p_j)))}{\mu(p_j)(1-\varphi_{p_v}(p_j, m(p_j)) + \mu(p_j^c)(1-\varphi_{p_v}(p_j^c, m(p_j^c))}$$

where p_j^c is complement of p_j in P. For large n,

$$\Pr(piv \mid Y, N, Y) \approx \frac{1}{n}\frac{f(\lambda^*)\lambda^*}{\bar{\lambda}\bar{a}(z'_B(Y, N) + z'_A(Y, N))} \tag{3.5}$$

$$\Pr(piv \mid Y, N, N) \approx \frac{1}{n}\frac{f(\lambda^*)(1-\lambda^*)}{(1-\bar{\lambda})\bar{a}(z'_B(Y, N) + z'_A(Y, N))} \tag{3.6}$$

$$\Pr(piv \mid Y, \varnothing, Y) \approx \frac{1}{n}P^Y(N \mid \varnothing)\frac{f(\lambda^*)\lambda^*}{\bar{\lambda}\bar{a}(z'_B(Y, N) + z'_A(Y, N))} \tag{3.7}$$

$$\Pr(piv \mid Y, \varnothing, N) \approx \frac{1}{n} P^N(N \mid \varnothing) \frac{f(\lambda^*)(1-\lambda^*)}{(1-\bar{\lambda})\bar{a}(z'_B(Y,N) + z'_A(Y,N))} \tag{3.8}$$

$$\Pr(piv \mid \varnothing, N, Y) \approx \frac{1}{n} P^Y(Y \mid \varnothing) \frac{f(\lambda^*)\lambda^*}{\bar{\lambda}\bar{a}(z'_B(Y,N) + z'_A(Y,N))} \tag{3.9}$$

$$\Pr(piv \mid \varnothing, N, N) \approx \frac{1}{n} P^N(Y \mid \varnothing) \frac{f(\lambda^*)(1-\lambda^*)}{(1-\bar{\lambda})\bar{a}(z'_B(Y,N) + z'_A(Y,N))} \tag{3.10}$$

A voter's best response is determined by voting for the preferred candidate if and only if $\Pr(piv \mid r_A, r_B, p_v) \geq \frac{c}{u}$. Hence, given voters' belief that candidates are playing some type-symmetric pure strategy profile $\mathbf{m}$, voter equilibrium is characterized by the following equations,

$$\frac{u}{n} \frac{f(\lambda^*)\lambda^*}{\bar{\lambda}\bar{a}(z'_B(Y,N) + z'_A(Y,N))} = C(Y, N, Y)$$

$$\frac{u}{n} \frac{f(\lambda^*)(1-\lambda^*)}{(1-\bar{\lambda})\bar{a}(z'_B(Y,N) + z'_A(Y,N))} = C(Y, N, N)$$

$$\frac{u}{n} P^Y(N \mid \varnothing) \frac{f(\lambda^*)\lambda^*}{\bar{\lambda}\bar{a}(z'_B(Y,N) + z'_A(Y,N))} = C(Y, \varnothing, Y)$$

$$\frac{u}{n} P^N(N \mid \varnothing) \frac{f(\lambda^*)(1-\lambda^*)}{(1-\bar{\lambda})\bar{a}(z'_B(Y,N) + z'_A(Y,N))} = C(Y, \varnothing, N)$$

$$\frac{u}{n} P^Y(Y \mid \varnothing) \frac{f(\lambda^*)\lambda^*}{\bar{\lambda}\bar{a}(z'_B(Y,N) + z'_A(Y,N))} = C(\varnothing, N, Y)$$

$$\frac{u}{n} P^N(Y \mid \varnothing) \frac{f(\lambda^*)(1-\lambda^*)}{(1-\bar{\lambda})\bar{a}(z'_B(Y,N) + z'_A(Y,N))} = C(\varnothing, N, N)$$

Solution to the set of equations is unique. Doing some algebra we obtain,

$$C(Y, \varnothing, Y) = P^Y(N \mid \varnothing) C(Y, N, Y)$$

$$C(\varnothing, N, Y) = P^Y(Y \mid \varnothing) C(Y, N, Y)$$

$$C(Y, \varnothing, N) = P^N(N \mid \varnothing) C(Y, N, N)$$

$$C(\varnothing, N, N) = P^N(Y \mid \varnothing) C(Y, N, N)$$

$$C(Y,N,Y) = \left(\frac{u}{n\bar{a}}\right)^{1/2} \left(\frac{1-\bar{\lambda}}{\bar{\lambda}}\right)^{1/4} f\left(\frac{(z''_B\bar{\lambda})^{1/2}}{(z''_B\bar{\lambda})^{1/2} + ((1-\bar{\lambda})z''_A)^{1/2}}\right)^{1/2} \times \frac{z''^{1/4}_A z''^{3/4}_B}{z''_B(\bar{\lambda}z''_A)^{1/2} + z''_A((1-\bar{\lambda})z''_B)^{1/2}}$$

$$C(Y,N,N) = \left(\frac{u}{n\bar{a}}\right)^{1/2} \left(\frac{\bar{\lambda}}{1-\bar{\lambda}}\right)^{1/4} f\left(\frac{(z''_B\bar{\lambda})^{1/2}}{(z''_B\bar{\lambda})^{1/2} + ((1-\bar{\lambda})z''_A)^{1/2}}\right)^{1/2} \times \frac{z''^{3/4}_A z''^{1/4}_B}{z''_B(\bar{\lambda}z''_A)^{1/2} + z''_A((1-\bar{\lambda})z''_B)^{1/2}}$$

where $z'_B(Y,N) = z''_B C(Y,N,N)$ and $z'_A(Y,N) = z''_A C(Y,N,Y)$.

Remark 3: For large enough n, $C(\cdot,\cdot,\cdot) \in (0,1)$. Also, $C(Y,N,Y) > C(\varnothing,N,Y)$; $C(Y,N,Y) > C(Y,\varnothing,Y)$; $C(Y,N,N) > C(Y,\varnothing,N)$ and $C(Y,N,N) > C(\varnothing,N,N)$. $C(Y,N,Y), C(Y,N,N)$ are always bounded and for large n they are strictly between zero and one. Also, $P^{p_v}(p_j \mid \varnothing) \in (0,1)$.

It is often claimed that more informed voters are more likely to vote.[13] One explanation for this is that voters become informed about the stakes involved. Leighley and Nagler (2014) provide evidence on 1972 to 2008 US presidential elections suggesting that "individuals who perceived a difference between the policy positions and ideological positions of the Republican and Democratic presidential candidates were more likely to vote than those who were not aware of these differences". The intuition they provide is similar to the predominant intuition in the literature - if a potential voter has a strong belief that all the candidates will enact same policies then there is no benefit from electing one candidate over others. My model exhibits similar intuition. Conditional on voting with positive probability, voters with better information are more likely to vote. This is due to the fact that a voter is pivotal if and only if the candidates have opposite types. Moreover, conditional on candidates having opposite types, probability of being pivotal is higher for the voter who knows both candidates' types

[13]See Freedman et al. (2004), Stromberg (2004), Matsusaka (1995), Krasno & Green (2008)

than for the voter who knows only the type of 1 candidates. For such a partially informed voter, there is some chance that a candidate whose type the voter does not observe has same type as the candidate whose type is observed and thus casting a vote will not change election outcome. However, a positive relationship between being informed and probability of voting is true only conditional on voter voting with positive probability. Otherwise it fails. For instance, if both candidates happen to have same types then observing both types leads to abstention while observing only one candidate's type may leads to a positive probability of voting.

Candidate's Problem

For each belief about candidates' strategy profile, a candidate knows how voters behave. That behavior is pinned down by the previous section. Each candidate cares only about the probability of winning the election and chooses a strategy that maximizes this probability taking voters' beliefs, behavior and other candidate's strategy as given. I start by approximating the probability of winning the election for large enough n. By symmetry, I focus on candidate A. In what follows, $\mathbf{m}_{-}$ denotes the strategy (spending) of candidate B that is believed, by voters and candidate A, to be plaid with probability 1 by candidate B; $\mathbf{m}$ denotes strategy profile of the candidates that is believed, by voters, to be plaid the candidates. Suppose $p_A = Y$. $A's$ payoff is

$$\begin{aligned} U(Y, m, \mathbf{m}_{-}) &= \mu(Y)\Pr(n_A \geq n_B \mid p_A = Y, p_B = Y, m, \mathbf{m}_{-}) \\ &+ \mu(N)\Pr(n_A \geq n_B \mid p_A = Y, p_B = N, m, \mathbf{m}_{-}) \end{aligned} \tag{3.11}$$

If $p_A = N$

$$\begin{aligned} U(N, m, \mathbf{m}_{-}) &= \mu(Y)\Pr(n_A \geq n_B \mid p_A = N, p_B = Y, m, \mathbf{m}_{-}) \\ &+ \mu(N)\Pr(n_A \geq n_B \mid p_A = N, p_B = N, m, \mathbf{m}_{-}) \end{aligned} \tag{3.12}$$

Using 3.2, $\Pr(n_A \geq n_B \mid p_A = Y, p_B = N, m, \mathbf{m}_-)$ is approximated as

$$\begin{aligned} &\Pr(n_A \geq n_B \mid p_A = Y, p_B = N, m, \mathbf{m}_-) \\ &\approx \frac{1}{n^2} \sum_{n_A=n_B}^{n-n_B} \sum_{n_B=0}^{n/2} h(\frac{n_A}{n}, \frac{n_B}{n}, 1 - \frac{n_A + n_B}{n} \mid p_A = Y, p_B = N, m, \mathbf{m}_-) \end{aligned} \tag{3.13}$$

For n large, 3.13 defines a double Riemann integral and gives

$$\begin{aligned} &\Pr(x_1 \geq x_2 \mid p_A = Y, p_B = N, m, \mathbf{m}_-) \\ &\approx \int_{x_2=0}^{1/2} \int_{x_1=x_2}^{1-x_2} h(x_1, x_2, 1 - (x_1 + x_2) \mid p_A = Y, p_B = N, m, \mathbf{m}_-) \end{aligned} \tag{3.14}$$

In 3.14, the variables of integration are

$$x_1 = a\lambda[P^Y(N \mid \varnothing)\phi_Y(Y, m) + P^Y(Y \mid \varnothing)\phi_Y(N, m(N))]C(Y, N, Y)$$

and

$$x_2 = a(1-\lambda)[P^N(N \mid \varnothing)\phi_N(Y, m) + P^N(Y \mid \varnothing)\phi_N(N, m(N))]C(Y, N, N)$$

Here, $m(Y)$ and $m(N)$ denote strategies that are believed to be played by Y and N types of candidates. m is actual strategy plaid by candidate A. After doing transformation of variables and integrating, one gets

$$\begin{aligned} &\Pr(x_1 \geq x_2 | p_A = Y, p_B = N, m, \mathbf{m}_-) \approx 1 - \\ &F\left(\frac{[P^N(N|\varnothing)\phi_N(Y,m) + P^N(Y|\varnothing)\phi_N(N,m(N))]C(Y,N,N)}{[P^N(N|\varnothing)\phi_N(Y,m) + P^N(Y|\varnothing)\phi_N(N,m(N))]C(Y,N,N) + [P^Y(N|\varnothing)\phi_Y(Y,m) + P^Y(Y|\varnothing)\phi_Y(N,m(N))]C(Y,N,Y)}\right) \end{aligned} \tag{3.15}$$

Similarly one can derive the following,

$$\begin{aligned} &\Pr(x_1 \geq x_2 | p_A = N, p_B = Y, m, \mathbf{m}_-) \approx \\ &F\left(\frac{[P^N(N|\varnothing)\phi_N(Y,m(Y)) + P^N(Y|\varnothing)\phi_N(N,m)]C(Y,N,N)}{[P^N(N|\varnothing)\phi_N(Y,m(Y)) + P^N(Y|\varnothing)\phi_N(N,m)]C(Y,N,N) + [P^Y(N|\varnothing)\phi_Y(Y,m(Y)) + P^Y(Y|\varnothing)\phi_Y(N,m)]C(Y,N,Y)}\right) \end{aligned} \tag{3.16}$$

Substituting for $C(Y,N,N), C(Y,N,Y)$ in 3.15 and 3.16,

$$\Pr(x_1 \geq x_2 | p_A = Y, p_B = N, m, \mathbf{m}_-) \approx 1 - F\left(\frac{[P^N(N|\varnothing)\phi_N(Y,m) + P^N(Y|\varnothing)\phi_N(N,m(N))] z_A''^{1/2}}{[P^N(N|\varnothing)\phi_N(Y,m) + P^N(Y|\varnothing)\phi_N(N,m(N))] z_A''^{1/2} + [P^Y(N|\varnothing)\phi_Y(Y,m) + P^Y(Y|\varnothing)\phi_Y(N,m(N))] z_B''^{1/2} (\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}} \right) \tag{3.17}$$

$$\Pr(x_1 \geq x_2 | p_A = N, p_B = Y, m, \mathbf{m}_-) \approx F\left(\frac{[P^N(N|\varnothing)\phi_N(Y,m(Y)) + P^N(Y|\varnothing)\phi_N(N,m)] z_A''^{1/2}}{[P^N(N|\varnothing)\phi_N(Y,m(Y)) + P^N(Y|\varnothing)\phi_N(N,m)] z_A''^{1/2} + [P^Y(N|\varnothing)\phi_Y(Y,m(Y)) + P^Y(Y|\varnothing)\phi_Y(N,m)] z_B''^{1/2} (\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}} \right) \tag{3.18}$$

Note that z''_A, z''_B and $P^{p_v}(p_j \mid \varnothing)$ depend on $m(Y)$ and $m(N)$ and not on m. Recall that $\mathbf{m} = (m(Y), m(N))$.

Remark 4: approximations in 3.17 and 3.18 do not depend on the expected size of the electorate $\bar{a}n$, unlike voters' strategies. For sufficiently large n, candidates care only about shares of the votes they are receiving and not about the absolute size of the electorate.

I also account for the probability of winning conditional on both candidates having the same position,

For $m > m(Y)$,

$$\Pr(x_1 \geq x_2 | p_A = Y, p_B = Y, m, \mathbf{m}_-) \approx 1 - F\left(\frac{P^N(N|\varnothing)[\phi_N(Y,m) - \phi_N(Y,m(Y))] z_A''^{1/2}}{P^N(N|\varnothing)[\phi_N(Y,m) - \phi_N(Y,m(Y))] z_A''^{1/2} + P^Y(N|\varnothing)[\phi_Y(Y,m) - \phi_Y(Y,m(Y))] z_B''^{1/2} (\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}} \right) \tag{3.19}$$

For $m < m(Y)$

$$\Pr(x_1 \geq x_2 | p_A = Y, p_B = Y, m, \mathbf{m}_-) \approx F\left(\frac{P^N(N|\varnothing)[\phi_N(Y,m) - \phi_N(Y,m(Y))] z_A''^{1/2}}{P^N(N|\varnothing)[\phi_N(Y,m) - \phi_N(Y,m(Y))] z_A''^{1/2} + P^Y(N|\varnothing)[\phi_Y(Y,m) - \phi_Y(Y,m(Y))] z_B''^{1/2} (\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}} \right) \tag{3.20}$$

For $m > m(N)$

$$\Pr(x_1 \geq x_2 | p_A = N, p_B = N, m, \mathbf{m}_-) \approx F\left(\frac{P^N(Y|\varnothing)[\phi_N(N,m) - \phi_N(N,m(N))] z_A''^{1/2}}{P^N(Y|\varnothing)[\phi_N(N,m) - \phi_N(N,m(N))] z_A''^{1/2} + P^Y(Y|\varnothing)[\phi_Y(N,m) - \phi_Y(N,m(N))] z_B''^{1/2} (\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}} \right) \tag{3.21}$$

For $m < m(N)$

$$\Pr(x_1 \geq x_2 | p_A = N, p_B = N, m, \mathbf{m}_-) \approx 1 - F\left(\frac{P^N(Y|\varnothing)[\phi_N(N,m) - \phi_N(N,m(N))]z_A''^{1/2}}{P^N(Y|\varnothing)[\phi_N(N,m) - \phi_N(N,m(N))]z_A''^{1/2} + P^Y(Y|\varnothing)[\phi_Y(N,m) - \phi_Y(N,m(N))]z_B''^{1/2}(\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}\right) \tag{3.22}$$

Proposition 3.1. *When candidates are endowed with the linear campaigning technologies, there exists equilibrium if and only if* $\frac{\bar{\lambda}^{1/2}}{\bar{\lambda}^{1/2} + (1-\bar{\lambda})^{1/2}} = median(F)$. *Moreover, the set of equilibria is all* $(m^*(Y), m^*(N))$ *that satisfy* $m^*(Y) = 1 - \frac{\mu(N)}{\mu(Y)}(1 - m^*(N))$

Proof. First, I show that expressions 3.19, 3.20, 3.21 and 3.22 are constant in m. Using linear technologies and doing algebraic manipulations we get,

For $m > m(Y)$,

$$\Pr(x_1 \geq x_2 \mid p_A = Y, p_B = Y, m, \mathbf{m}_-) \approx$$
$$1 - F\left(\frac{P^N(N \mid \varnothing)(1-k_L)z_A''^{1/2}}{P^N(N \mid \varnothing)(1-k_L)z_A''^{1/2} + P^Y(N \mid \varnothing)(1-k_H)z_B''^{1/2}(\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}\right)$$

For $m < m(Y)$,

$$\Pr(x_1 \geq x_2 \mid p_A = Y, p_B = Y, m, \mathbf{m}_-) \approx$$
$$F\left(\frac{P^N(N \mid \varnothing)(1-k_L)z_A''^{1/2}}{P^N(N \mid \varnothing)(1-k_L)z_A''^{1/2} + P^Y(N \mid \varnothing)(1-k_H)z_B''^{1/2}(\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}\right)$$

For $m > m(N)$,

$$\Pr(x_1 \geq x_2 \mid p_A = N, p_B = N, m, \mathbf{m}_-) \approx$$
$$F\left(\frac{P^N(Y \mid \varnothing)(1-k_H)z_A''^{1/2}}{P^N(Y \mid \varnothing)(1-k_H)z_A''^{1/2} + P^Y(Y \mid \varnothing)(1-k_L)z_B''^{1/2}(\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}\right)$$

For$m < m(N)$,

$$\Pr(x_1 \geq x_2 \mid p_A = N, p_B = N, m, \mathbf{m}_-) \approx$$
$$1 - F\left(\frac{P^N(Y \mid \varnothing)(1-k_H)z_A''^{1/2}}{P^N(Y \mid \varnothing)(1-k_H)z_A''^{1/2} + P^Y(Y \mid \varnothing)(1-k_L)z_B''^{1/2}(\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}\right)$$

Now I argue that a necessary condition for an equilibrium is

$$F\left(\frac{P^N(N|\varnothing)(1-k_L)z_A''^{1/2}}{P^N(N|\varnothing)(1-k_L)z_A''^{1/2}+P^Y(N|\varnothing)(1-k_H)z_B''^{1/2}(\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}\right) \tag{3.23}$$
$$= F\left(\frac{P^N(Y|\varnothing)(1-k_H)z_A''^{1/2}}{P^N(Y|\varnothing)(1-k_H)z_A''^{1/2}+P^Y(Y|\varnothing)(1-k_L)z_B''^{1/2}(\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}\right)$$
$$= 1/2$$

Since, by assumption, F is strictly increasing, 3.17 and 3.18 are continuous in m and 3.19, 3.20, 3.21 and 3.22 are constant in m, the only way to make sure there are no profitable local deviations is to make sure that 3.23 holds. Otherwise, one can choose m sufficiently close to $m(p_j)$ and guarantee that change in 3.17 or 3.18 is small enough. Since either $m > m(p_j)$ or $m < m(p_j)$ would be the case, the probability of winning, conditional on the candidates' having the same type, would increase discontinuously and hence the deviation would be profitable for some candidate. Note that we know the change would be discontinuous because we know that 3.19, 3.20, 3.21 and 3.22 are constant in m.

In addition, it can be easily verified that,

$$\begin{aligned} & F\left(\frac{P^N(N \mid \varnothing)(1-k_L)z_A''^{1/2}}{P^N(N \mid \varnothing)(1-k_L)z_A''^{1/2} + P^Y(N \mid \varnothing)(1-k_H)z_B''^{1/2}(\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}\right) \\ = \quad & F\left(\frac{P^N(Y \mid \varnothing)(1-k_H)z_A''^{1/2}}{P^N(Y \mid \varnothing)(1-k_H)z_A''^{1/2} + P^Y(Y \mid \varnothing)(1-k_L)z_B''^{1/2}(\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}\right)\end{aligned}$$

Hence, we need,

$$F\left(\frac{P^N(Y \mid \varnothing)(1-k_H)z_A''^{1/2}}{P^N(Y \mid \varnothing)(1-k_H)z_A''^{1/2} + P^Y(Y \mid \varnothing)(1-k_L)z_B''^{1/2}(\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}\right) = 1/2$$

Under this condition, we know that deviations from equilibrium strategies would not affect the probability of winning conditional on candidates having the same type. We need to also ensure that deviations are not profitable conditional on candidates having opposite policy stances. For this, we look at 3.17 and 3.18. Differentiating 3.17 and 3.18 with respect to m,

$$\frac{d\Pr(x_1 \geq x_2 \mid p_A = Y, p_B = N, m, \mathbf{m}_-)}{dm} =$$
$$J(m)[\mu(Y)(1 - m(Y)) - \mu(N)(1 - m(N))]$$
$$\frac{d\Pr(x_1 \geq x_2 \mid p_A = N, p_B = Y, m, \mathbf{m}_-)}{dm} =$$
$$J'(m)[\mu(N)(1 - m(N)) - \mu(Y)(1 - m(Y))]$$

where $J(m) > 0$ and $J(m)' > 0$ for all m. The signs of the derivatives are determined only by $[\mu(Y)(1-m(Y)) - \mu(N)(1-m(N))]$ and $[\mu(N)(1-m(N)) - \mu(Y)(1-m(Y))]$ that do not depend on m.

Also, $\frac{d\Pr(x_1 \geq x_2 | p_A = Y, p_B = N, m, \mathbf{m}_-)}{dm} > 0$ if and only if $\frac{d\Pr(x_1 \geq x_2 | p_A = N, p_B = Y, m, \mathbf{m}_-)}{dm} < 0$ which means that in equilibrium we must have $\mu(Y)(1-m(Y)) - \mu(N)(1-m(N)) = 0$. Otherwise, one type of candidate is better-off by some deviation. This gives,

$$m^*(Y) = 1 - \frac{\mu(N)}{\mu(Y)}(1 - m^*(N))$$

Substituting $m^*(N), m^*(Y)$ into z''_B, z''_A, one verifies that $z''_B = z''_A$. Substituting in $\frac{P^N(Y|\varnothing)(1-k_H)z''^{1/2}_A}{P^N(Y|\varnothing)(1-k_H)z''^{1/2}_A + P^Y(Y|\varnothing)(1-k_L)z''^{1/2}_B(\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}$ and manipulating, we obtain

$$\frac{\bar{\lambda}^{1/2}}{\bar{\lambda}^{1/2} + (1 - \bar{\lambda})^{1/2}} = median(F)$$

□

3.4 Model 2

Model 2 is the same as Model 1 except that in model 2 voters are boundedly rational. Boundedly rational voters understand simple pivotal events - if a voter receives same message from both candidates then he understands that the probability his vote being pivotal is 0; if a boundedly rational voter does not receive any signals then he understands that because of symmetric strategies, the probability his vote being pivotal is 0. Conditional on receiving different messages from the candidates (this includes no message from one candidate and a message from another) the voter understands that there is some probability that his vote is pivotal but he cannot calculate it and hence assigns some non-equilibrium belief to this event. I assume this belief to set probability 1 on being pivotal.[14]

Alternatively, we could assume voters being expressive i.e. conditional on not abstaining, a voter votes for the candidate who has the highest probability of having a policy stance that a voter likes. None of the results is altered under this interpretation. This kind of voting behavior leads to the following cutoffs, $C^{ni}(r_A, r_B, p_v)$, where superscript ni stands for non-instrumental (boundedly rational),

$$\begin{aligned} C^{ni}(Y, \varnothing, Y) &= P^Y(N \mid \varnothing)u \\ C^{ni}(\varnothing, N, Y) &= P^Y(Y \mid \varnothing)u \\ C^{ni}(Y, \varnothing, N) &= P^N(N \mid \varnothing)u \\ C^{ni}(\varnothing, N, N) &= P^N(Y \mid \varnothing)u \\ C^{ni}(Y, N, Y) &= u \\ C^{ni}(Y, N, N) &= u \end{aligned}$$

Analogously to 3.14, one can follow same steps to derive candidates' payoffs. 3.14 is now integrated over

$$x_1^{ni} = a\lambda[P^Y(N \mid \varnothing)\phi_Y(Y, m) + P^Y(Y \mid \varnothing)\phi_Y(N, m(N))]u$$

and

$$x_2^{ni} = a(1-\lambda)[P^N(N \mid \varnothing)\phi_N(Y, m) + P^N(Y \mid \varnothing)\phi_N(N, m(N))]u$$

where x_1^{ni} is the probability that a randomly chosen citizen votes for candidates A of type Y conditional on the candidates having different types and x_2^{ni} is the probability that a

[14]Note that one could set this probability as some function of the model parameters but this would not affect equilibrium spending and would only affect voter's equilibrium strategies and turnout in trivial ways.

randomly chosen citizen votes for candidates B. We obtain,

$$\Pr\left(x_1^{ni} \geq x_2^{ni} | p_A = Y, p_B = N, m, \mathbf{m}_-\right) \approx 1 -$$
$$F\left(\frac{[P^N(N|\varnothing)\phi_N(Y,m) + P^N(Y|\varnothing)\phi_N(N,m(N))]}{[P^N(N|\varnothing)\phi_N(Y,m) + P^N(Y|\varnothing)\phi_N(N,m(N))] + [P^Y(N|\varnothing)\phi_Y(Y,m) + P^Y(Y|\varnothing)\phi_Y(N,m(N))]}\right)$$

$$\Pr\left(x_1^{ni} \geq x_2^{ni} | p_A = N, p_B = Y, m, \mathbf{m}_-\right) \approx$$
$$F\left(\frac{[P^N(N|\varnothing)\phi_N(Y,m(Y)) + P^N(Y|\varnothing)\phi_N(N,m)]}{[P^N(N|\varnothing)\phi_N(Y,m(Y)) + P^N(Y|\varnothing)\phi_N(N,m)] + [P^Y(N|\varnothing)\phi_Y(Y,m(Y)) + P^Y(Y|\varnothing)\phi_Y(N,m)]}\right)$$

Similarly, we can calculate probability of winning conditional on the candidates having the same type.

For $m > m(Y)$,

$$\Pr\left(x_1^{ni} \geq x_2^{ni} | p_A = Y, p_B = Y, m, \mathbf{m}_-\right) \approx$$
$$1 - F\left(\frac{P^N(N|\varnothing)[\phi_N(Y,m) - \phi_N(Y,m(Y))]}{P^N(N|\varnothing)[\phi_N(Y,m) - \phi_N(Y,m(Y))] + P^Y(N|\varnothing)[\phi_Y(Y,m) - \phi_Y(Y,m(Y))]}\right)$$

For $m < m(Y)$,

$$\Pr\left(x_1^{ni} \geq x_2^{ni} | p_A = Y, p_B = Y, m, \mathbf{m}_-\right) \approx$$
$$F\left(\frac{P^N(N|\varnothing)[\phi_N(Y,m) - \phi_N(Y,m(Y))]}{P^N(N|\varnothing)[\phi_N(Y,m) - \phi_N(Y,m(Y))] + P^Y(N|\varnothing)[\phi_Y(Y,m) - \phi_Y(Y,m(Y))]}\right)$$

For $m > m(N)$,

$$\Pr\left(x_1^{ni} \geq x_2^{ni} | p_A = N, p_B = N, m, \mathbf{m}_-\right) \approx$$
$$F\left(\frac{P^N(Y|\varnothing)[\phi_N(N,m) - \phi_N(N,m(N))]}{P^N(Y|\varnothing)[\phi_N(N,m) - \phi_N(N,m(N))] + P^Y(Y|\varnothing)[\phi_Y(N,m) - \phi_Y(N,m(N))]}\right)$$

For $m < m(N)$,

$$\Pr\left(x_1^{ni} \geq x_2^{ni} | p_A = N, p_B = N, m, \mathbf{m}_-\right) \approx$$
$$1 - F\left(\frac{P^N(Y|\varnothing)[\phi_N(N,m) - \phi_N(N,m(N))]}{P^N(Y|\varnothing)[\phi_N(N,m) - \phi_N(N,m(N))] + P^Y(Y|\varnothing)[\phi_Y(N,m) - \phi_Y(N,m(N))]}\right)$$

Proposition 3.2. *When candidates are endowed with the linear campaigning technologies, there exists equilibrium if and only if* $1/2 = median(F)$*. Moreover, the set of equilibria is all* $(m^{*ni}(Y), m^{*ni}(N))$ *that satisfy* $m^{*ni}(Y) = 1 - \frac{\mu(N)}{\mu(Y)}(1 - m^{*ni}(N))$

Proof. similar to proposition 3.1. □

The condition $1/2 = median(F)$ is a special case of $\frac{\bar{\lambda}^{1/2}}{\bar{\lambda}^{1/2}+(1-\bar{\lambda})^{1/2}} = median(F)$ when $\bar{\lambda} = median(F) = 1/2$. Whenever equilibrium exists in the models 1 and 2, the set of candidates' equilibrium strategies is the same across two models.

3.5 Voter Turnout

We can calculate expected voter turnout rates in models 1 and 2 under the condition $\bar{\lambda} = median(F) = 1/2$. Turnout rate in the model 1, (T), is defined as the expected turnout divided by $\bar{a}n$.

$$\begin{aligned}
&T = \mu(Y)\mu(N)(z'_A + z'_B) + \\
&\mu(Y)^2 \sum_{p_{v \in P}} \phi_{p_v}(Y, m(Y))(1 - \phi_{p_v}(Y, m(Y)))C(Y, \varnothing, p_v) \\
&+ \mu(N)^2 \sum_{p_{v \in P}} \phi_{p_v}(N, m(N))(1 - \phi_{p_v}(N, m(N)))C(N, \varnothing, p_v)
\end{aligned}$$

We can evaluate the above expression at the equilibria of the model 1. We obtain,

$$\begin{aligned}
&T = \left[\frac{f(1/2)u}{n\bar{a}z^*}\right]^{1/2} 2\mu(N)\times \\
&\left[(1-\mu(N))z^* + \frac{(1-k_H)(1-k_L)(1-m(N))}{2-k_L-k_H}[1 + 2\mu(N)(m(N)-1)]\right]
\end{aligned}$$

where,

$$z^* = \frac{1 - k_H k_L + (1-k_L)(1-k_H)(m(N) - \frac{\mu(N)}{\mu(Y)}(1 - m(N)))}{2 - k_L - k_H}$$

Similarly, we can calculate T^{ni},

$$T^{ni} = u2\mu(N)\times$$
$$\left[(1-\mu(N))z^* + \frac{(1-k_H)(1-k_L)(1-m(N))}{2-k_L-k_H}[1+2\mu(N)(m(N)-1)]\right]$$

Proposition 3.3. *For any parameters, T^{ni} is increasing in equilibrium spending. If $(2-k_L-k_H)\mu(N)(1-m(N)) \leq 1/4$ then T is decreasing in equilibrium spending.*

Proof. Differentiating T^{ni} with respect to $m(N)$ we obtain,

$$\frac{dT^{ni}}{dm(N)} = 4u\mu^2(N)(1-k_H)(1-k_L)(1-m(N)) > 0$$

Differentiating T with respect to $m(N)$ we obtain,

$$\frac{dT}{dm(N)} = 4(2-k_L-k_H)\mu(N)(1-m(N))-$$
$$\left[1+\frac{(1-k_H)(1-k_L)(1-m(N))[1+(2-k_H-k_L)\mu(N)(m(N)-1)]}{(1-\mu(N))(1-k_Hk_L)+(1-k_H)(1-k_L)[m(N)-\mu(N)]}\right]$$

One can easily verify that the second term in the brackets is strictly more than 1 for any $m(N)$ and any parameter values. The first term is less than 1 if $(2-k_L-k_H)\mu(N)(1-m(N)) \leq 1/4$. □

Proposition 3.3 shows that in general, aggregate turnout may respond differently to campaign spending across the two models. Comparison between the two models makes it possible to isolate two channels through which campaigning affects turnout. First channel is that higher spending increases expected number of better informed voters in the population that is, it increases political awareness. We know that better informed voters vote with higher probability than their less informed peers, and hence increased spending increases turnout in model 2. Model 1 allows for the additional strategic channel that works via pivotal probabilities - higher spending decreases the probability that a particular voter is pivotal as a voter knows that there is better knowledge in the population and it is more likely that others are better informed. For high enough $k_L, k_H, m(N)$ or low enough $\mu(N)$ this channel dominates the first channel and hence we get that the turnout is decreasing in equilibrium spending.

This result is similar to the Feddersen and Pesendorfer (1999) result. In their model voting is costless and the only channel that generates abstention is the "fear" of making mistakes by voting for a candidate who in fact is not a good choice. Less informed voters may tend to delegate voting power to more informed voters by abstaining. They show in a specific example that if a fraction of better informed voters becomes sufficiently large then, even though it is always the case that better informed voters vote with higher probability than their less informed peers, it may lead less informed voters to abstain with sufficiently high probability (by delegating voting power) so that the net effect leads to the lower aggregate turnout. Here, in contrast, without voting costs, it would be a weakly dominant strategy to vote for a prefered candidate. There is no fear of making mistakes. The only use of considering the probability of pivotal event for a voter is to compare its magnitude to the private voting cost and to save on that cost whenever the probability of being pivotal is low enough.

3.6 General Campaigning Technologies

So far, the analysis has been done for the linear campaigning technologies with the interpretation that candidates spend resources, m, that determines probabilities with which voters with different preference characteristics are approached. In this section, I consider general campaigning technologies in which candidates directly choose the probabilities of approaching different types of voters from some fixed feasible subset of probabilities.

Recall the notation for a candidate's strategy under general technologies, $(\phi_{opp}(p_j), \phi_{own}(p_j))$, where the first (second) element is the probability with which a voter of the type different (same) from p_j is approached. I assume symmetric campaigning technologies across candidates. I could allow campaigning technologies to depend on the candidate's type but as this does not alter anything for the result to be stated, I assume symmetric technologies across candidate's types.

The voters' problem is the same as in the models 1 and 2. The candidates' problem is also the same with the exception that now each candidate chooses two dimensional action

$(\phi_{opp}(p_j), \phi_{own}(p_j))$. Hence, each candidate may now control $\phi_{opp}(p_j)$ and $\phi_{own}(p_j)$ independently in order to determine the optimal level of targeting in addition to the level of information provided. I denote by $(\phi_{opp}(p_j), \phi_{own}(p_j))$ a candidate's strategy that is beleived to be plaid, by other players. I denote by $(\bar{\phi}_{opp}(p_j), \bar{\phi}_{own}(p_j))$ the actual actions taken. Let $P_{CT}(\phi_{opp}(p))$ be the projection of $\phi_{own}(p)$ on P_{CT}. I define the following sets,

$$P^*_{CT} = \{(\phi_{opp}(p), \phi_{own}(p)) \in P_{CT} : \phi_{own}(p) = \max_{\phi'_{own}(p) \in P_{CT}(\phi_{opp}(p))} \phi'_{own}(p)\}$$

$$P^{**}_{CT} = \{(\phi_{opp}(p), \phi_{own}(p)) \in P^*_{CT} : (\phi_{opp}(p), \phi_{own}(p)), (\phi'_{opp}(p), \phi'_{own}(p)) \in P^{**}_{CT} \leftrightarrow \phi_{opp}(p) < \phi'_{opp}(p) \rightarrow \phi_{own}(p) < \phi'_{own}(p)\}$$

Proposition 3.4. *If $(\phi_{opp}(p), \phi_{own}(p))$ is an equilibrium strategy profile for the candidates, in the model 1, then $(\phi_{opp}(p), \phi_{own}(p)) \in P^{**}_{CT}$.*

Proof. I show that the proposition is true when the candidates know each others' types. This will imply the result in case they don't know each others' types. Suppose, $p_A = Y, p_B = N$ then the probabilities of winning for the candidates A and B are,

$$\Pr(x_1 \geq x_2 | p_A = Y, p_B = N, m, \mathbf{m}_-) \approx 1 - F\left(\frac{[P^N(N|\varnothing)\bar{\phi}_{opp}(Y) + P^N(Y|\varnothing)\phi_{own}(N)] z''^{1/2}_A}{[P^N(N|\varnothing)\bar{\phi}_{opp}(Y) + P^N(Y|\varnothing)\phi_{own}(N)] z''^{1/2}_A + [P^Y(N|\varnothing)\bar{\phi}_{own}(Y) + P^Y(Y|\varnothing)\phi_{opp}(N)] z''^{1/2}_B (\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}\right) \tag{3.24}$$

$$\Pr(x_2 \geq x_1 | p_B = N, p_A = Y, m, \mathbf{m}_-) \approx F\left(\frac{[P^N(N|\varnothing)\phi_{opp}(Y) + P^N(Y|\varnothing)\bar{\phi}_{own}(N)] z''^{1/2}_A}{[P^N(N|\varnothing)\phi_{opp}(Y) + P^N(Y|\varnothing)\bar{\phi}_{own}(N)] z''^{1/2}_A + [P^Y(N|\varnothing)\phi_{own}(Y) + P^Y(Y|\varnothing)\bar{\phi}_{opp}(N)] z''^{1/2}_B (\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}\right) \tag{3.25}$$

3.24 is trictly increasing in $\bar{\phi}_{own}(Y)$ and strictly decreasing in $\bar{\phi}_{opp}(Y)$. 3.25 is strictly increasing in $\bar{\phi}_{own}(N)$ and strictly decreasing in $\bar{\phi}_{opp}(N)$. Now consider $p_A = p_B = Y$ (by symmetry, same will be true for $p_A = p_B = N$). We have the following, For $\bar{\phi}_{opp}(Y) > \phi_{opp}(Y)$, $\bar{\phi}_{own}(Y) > \phi_{own}(Y)$,

$$\Pr(x_1 \geq x_2 | p_A = Y, p_B = Y, m, \mathbf{m}_-) \approx 1 - F\left(\frac{P^N(N|\varnothing)[\bar{\phi}_{opp}(Y) - \phi_{opp}(Y)]z_A''^{1/2}}{P^N(N|\varnothing)[\bar{\phi}_{opp}(Y) - \phi_{opp}(Y)]z_A''^{1/2} + P^Y(N|\varnothing)[\bar{\phi}_{own}(Y) - \phi_{own}(Y)]z_B''^{1/2}(\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}\right) \tag{3.26}$$

For $\bar{\phi}_{opp}(Y) < \phi_{opp}(Y)$, $\bar{\phi}_{own}(Y) < \phi_{own}(Y)$,

$$\Pr(x_1 \geq x_2 | p_A = Y, p_B = Y, m, \mathbf{m}_-) \approx F\left(\frac{P^N(N|\varnothing)[\bar{\phi}_{opp}(Y) - \phi_{opp}(Y)]z_A''^{1/2}}{P^N(N|\varnothing)[\bar{\phi}_{opp}(Y) - \phi_{opp}(Y)]z_A''^{1/2} + P^Y(N|\varnothing)[\bar{\phi}_{own}(Y) - \phi_{own}(Y)]z_B''^{1/2}(\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/2}}\right) \tag{3.27}$$

For $\bar{\phi}_N(Y) > \phi_N(Y)$, $\bar{\phi}_Y(Y) \leq \phi_Y(Y)$ or $\bar{\phi}_N(Y) \geq \phi_N(Y)$, $\bar{\phi}_Y(Y) < \phi_Y(Y)$,

$$\Pr(x_1 \geq x_2 \mid p_A = Y, p_B = Y, m, \mathbf{m}_-) \approx 0 \tag{3.28}$$

For $\bar{\phi}_N(Y) < \phi_N(Y)$, $\bar{\phi}_Y(Y) \geq \phi_Y(Y)$ or $\bar{\phi}_N(Y) \leq \phi_N(Y)$, $\bar{\phi}_Y(Y) > \phi_Y(Y)$,

$$\Pr(x_1 \geq x_2 \mid p_A = Y, p_B = Y, m, \mathbf{m}_-) \approx 1 \tag{3.29}$$

3.26 and 3.28 imply that whenever $\bar{\phi}_{opp}(Y) > \phi_{opp}(Y)$, increasing $\phi_{own}(Y)$ is strictly improving the chances of winning for a candidate. 3.27 and 3.29 imply that whenever $\bar{\phi}_{opp}(Y) < \phi_{opp}(Y)$, increasing $\phi_{own}(Y)$ is strictly improving the chances. 3.28 and 3.29 imply that whenever $\bar{\phi}_{opp}(Y) = \phi_{opp}(Y)$ increasing $\phi_{own}(Y)$ is strictly improving. 3.26 and 3.29 imply that whenever $\bar{\phi}_{own}(Y) > \phi_{own}(Y)$, decreasing $\phi_{opp}(Y)$ is strictly improving. 3.27 and 3.28 imply that whenever $\bar{\phi}_{own}(Y) < \phi_{own}(Y)$, decreasing $\phi_{opp}(Y)$ is strictly improving. 3.28 and 3.29 imply that whenever $\bar{\phi}_{own}(Y) = \phi_{own}(Y)$, decreasing $\phi_{opp}(Y)$ is strictly improving. This completes the proof. □

The content of proposition 3.4 is the following: a candidate would always increase the probability of approaching potential supporters if this does not require her to change the probability with which potential opponents are approached. She would also decrease the probability of approaching potential opponents if this does not alter the probability with which she approaches potential supporters. These lead us to the set P^{**}_{CT}. Hence, any equilibrium involves a trade-off between improving the political awareness in the society and targeting voters more effectively (increasing polarization in political awareness).

Proposition 3.5. *If $(\phi^{ni}_{opp}(p), \phi^{ni}_{own}(p))$ is an equilibrium strategy profile for the candidates, in the model 2, then $(\phi^{ni}_{opp}(p), \phi^{ni}_{own}(p)) \in P^{**}_{CT}$.*

3.6.1 Campaigning Technologies with Unlimited Targeting Potential

Here, I study one more class of campaigning technologies that ensure unique equilibrium in models 1 and 2.

Definition 3.3. *P_{CT} is a campaigning technology with unlimited targeting potential if $P_{CT} = [k_L, \bar{m}_L] \times [k_H, \bar{m}_H]$ where $\bar{m}_L, \bar{m}_H < 1, k_L, k_H > 0$.*

The condition $k^H, k^L > 0$ means that there is some prior level of political awareness in the population. The condition $\bar{m}_L, \bar{m}_H < 1$ means that there is always some chance that a voter will be uninformed. These conditions guarantee that no off-path events are ever observed by the voters. The campaigning technologies are unlimited in targeting voters in the sense, that increasing the probability that one type of voter receives message can be done without affecting probability that the other type of a voter receives the message.

The following are corollaries to propositions 3.4 and 3.5.

Corollary 3.1. *$\phi_{own}(Y) = \phi_{own}(N) = \bar{m}_H, \phi_{opp}(N) = \phi_{opp}(Y) = k_L$ is the unique equilibrium, in model 1, under the campaigning technologies with unlimited targeting potential.*

Corollary 3.2. *$\phi^{ni}_{own}(Y) = \phi^{ni}_{own}(N) = \bar{m}_H, \phi^{ni}_{opp}(N) = \phi^{ni}_{opp}(Y) = k_L$ is the unique equilibrium, in model 2, under the campaigning technologies with unlimited targeting potential.*

Corollaries 3.1 and 3.2 show that the candidates' equilibrium behavior does not change across models 1 and 2 for any $f(\cdot)$ and $g(\cdot)$. Moreover, the equilibrium behavior of a candidate is belief free with regard to the other candidate's type, which implies that even if candidates knew each others' types, their equilibrium behavior would be unchanged.

3.7 Negative campaigning

In this section, I allow for negative campaigning when the candidates are endowed with the campaigning technologies with unlimited targeting potential. I assume that candidates know each others' types before the campaigning phase. A candidate's strategy is now a quadruple

$$(\phi_Y(p_j, p_{-j}), \phi_N(p_j, p_{-j}), \phi^o_Y(p_j, p_{-j}), \phi^o_N(p_j, p_{-j}))$$

where $\phi^o_{p_v}(p_j, p_{-j})$ stands for the probability with which a voter is approached by candidate p_j who sends message p_{-j} about candidate $-j$. Hard information is assumed with regard to sending a message about own or competitor's type. It is straightforward to redefine all model variables for this specification. Following the similar lines of arguments as before, we have the following proposition,

Proposition 3.6. $\phi_{p_v}(p_j, p_{-j}) = \phi^{ni}_{p_v}(p_j, p_{-j}) = \bar{m}^H$ *for* $p_j = p_v$,
$\phi_{p_v}(p_j, p_{-j}) = \phi^{ni}_{p_v}(p_j, p_{-j}) = k^L$ *for* $p_j \neq p_v$,
$\phi^o_{p_v}(p_j, p_{-j}) = \phi^{ni}_{p_v}(p_j, p_{-j}) = \bar{m}^L$ *for* $p_v = p_j \neq p_{-j}$,
$\phi^o_{p_v}(p_j, p_{-j}) = \phi^{ni}_{p_v}(p_j, p_{-j}) = k^H$ *for* $p_v = p_{-j} \neq p_j$,
$\phi^o_{p_v}(p_j, p_{-j}) = \phi^{ni}_{p_v}(p_j, p_{-j}) = k^H$ *for* $p_v = p_j = p_{-j}$,
$\phi^o_{p_v}(p_j, p_{-j}) = \phi^{ni}_{p_v}(p_j, p_{-j}) = \bar{m}^H$ *for* $p_v \neq p_j = p_{-j}$
is the unique equilibrium in models 1 and 2.

In words, candidates' behavior when informing voters about own types is the same as in corollaries 3.1 and 3.2. As for negative campaigning, if candidates have different types, then they are inclined to inform own type voters about other candidate's type while they do not inform opposite type voters about other candidate's type. If candidates have same types, then they do not inform own type voters about other candidate's type and inform opposite type voters about other candidate's type.

In the equilibrium, there is a higher political awareness in the society compared to the case where negative campaigning is not allowed. Each voter is informed about each candidate's position with the probability $\bar{m}_H$. The reason is that even though each candidate wants to inform only own type voters about her type the other candidate always wants to do the opposite and uses negative campaigning to achieve that. In the equilibrium, this make targeting ineffective in polarizing information environment and leads to every type of voter being informed about both candidates' types with the probability $\bar{m}_H$.

We can also calculate the turnout rates in the models 1 and 2,

$$T = \left[\frac{u}{n\bar{a}\bar{m}^H}\right]^{1/2} (\frac{1-\bar{\lambda}}{\bar{\lambda}})^{1/4} \times$$
$$f(\frac{\bar{\lambda}^{1/2}}{\bar{\lambda}^{1/2}+(1-\bar{\lambda})^{1/2}})^{1/2} 2\mu(N)(1-\mu(N))\left[\bar{m}^H+\bar{m}^H(1-\bar{m}^H)\right]$$

$$T^{ni} = u2\mu(N)(1-\mu(N))\left[\bar{m}^H+\bar{m}^H(1-\bar{m}^H)\right]$$

To do the comparative statics on the turnout rates, first I define what it means to say that the elite polarization is higher (lower) in the model.

Definition 3.4. *We say that the elite polarization is higher for $\mu(N)$ than $\mu'(N)$ if* $\mid \mu(N) - 1/2 \mid < \mid \mu'(N) - 1/2 \mid$

This definition of the elite polarization is motivated by the following: Let $np(\mu(N)) = \mu(N)^2 + (1-\mu(N))^2$ and $p(\mu(N)) = 2\mu(N)(1-\mu(N))$ where $np(\mu(N))$ is the probability that the candidates have the same policy position and $p(\mu(N))$ is the probability that the candidates the different policy position. $np(\mu(N))$ is convex minimized at $\mu(N) = 1/2$ and $p(\mu(N))$ is concave maximized at $\mu(N) = 1/2$. Hence, as we move $\mu(N)$ further away from $1/2$, $p(\mu(N))$ decreases and $np(\mu(N))$ increases.

Proposition 3.7. *T^{ni} is strictly increasing in $\bar{m}_H$; T is stirctly increasing in $\bar{m}_H$ if $\bar{m}^H < 2/3$ and is nonincreasing otherwise; T^{ni}, T are increasing in the elite polarization.*

Here I discuss the intuition behind proposition 3.7. Note that $\bar{m}_H$ measures the informativeness of campaigning technologies. The intuition for the first two parts of the statement is the same as for proposition 3.3. Here we can see the two channels discussed in the context of proposition 3.3 more vividly. The term $[\bar{m}_H + \bar{m}_H(1-\bar{m}_H)]$ in the expressions for turnout rates increases in $\bar{m}_H$ is due to the channel 1. The term $\frac{1}{\bar{m}_H^{1/2}}$ is present only in the expression for the turnout rate of model 1 and is due to the channel 2.

Elite polarization drives up turnout in both models. This is consistent with Hetherington (2008) who argues that elite polarization has stimulated voter participation in US presidential elections.[15] Dodson (2010) argues that rebound of voter turnout in US presidential elections since 1988 has been mainly driven by increased party polarization. The mechanism that is thought to be at work is that as polarization increases so does stakes for the voters and hence participation increases. Here, I formalize this intuition. Increasing polarization increases the probability with which voters are getting the signals about opposite types of the candidates and decreases the probability with which voters are getting the signals about the same types of candidates. Since the former type of the voters vote with positive probability (because if they do not then there is a chance that the opposite type candidate wins with higher probability and the voters forego some payoff i.e. stake effect) while the latter types do not vote (because there are no stakes), expected turnout increases.

[15] Also see, Epstein and Graham (2007), Brooks and Geer (2008), Abramowiz (2008).

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Chapter 4

A Fair Punishment

4.1 Introduction

Responsibility for the outcomes following politicians' and public officials' actions is often a subject of debate. Politicians often boast about the projects and policies that generate good outcomes during their term of office, and tend to blame bad outcomes on the misdeeds of the politicians who held offices before them. Given all that, it is usually difficult for a society to figure out who is responsible for what. This is one source of uncertainty that makes it difficult for the society to discipline the behavior of political actors through the threat of losing office. Another source of uncertainty is that political actors usually have inside information about other political actors' actions, as well as the ability to hide their own actions. There are two questions to be scrutinized by the society - **what was done, and who did what.** Answers to those questions determine who is punished and who is rewarded. I ask, *to what extent, should the society care about answering each of those questions, in order to make sure that the politicians behave in the interests of the society.*

In a dynamic citizen-candidate model with multi-period projects and persistent agency friction, I characterize the unique politician firing rule (**disciplining device**) that achieves the first best, for the society, for the highest possible misalignment of interests between the society and the appointed officials. In addition, I require the disciplining device to be Markovian and trembling-hand perfect. The disciplining device completely ignores midterm project signals and conditions punishment only on the end of project outcomes. For the society, this disciplining device eliminates the need for asking a question - who did what? It never happens that a public official is in charge of only a part of the project, and hence the society knows that all the responsibility for a given project's outcome is on one public official. The question - what was done? - is asked only after the project is completed and only the final outcome of the project is considered for making the public official replacement decision.

Formally, there is a continuum of citizens with homogeneous preferences over a sequence of projects. Time is discrete and horizon is infinite. At the start of each period, there is an incumbent politician who initiates a project that needs 2 periods (stages) to be completed. Citizens and politicians care about the project outcome at the completion stage (no payoffs are accrued, from the project, at the interim stage). A politician also receives benefits from holding the office and may care about project outcome less than a citizen (**misalignment of interests**). An incumbent politician can exert a hidden costly effort when he is in the office. In the first stage of the project development, citizens receive a signal about the effort that was exerted and decide whether to keep the politician in the office or replace him with a challenger drawn randomly from the pool of citizens. At the end of the second stage of the project, outcome is generated that depends on the effort exerted in both stages of the project. After observing the outcome, citizens decide whether to keep the incumbent or replace him with a challenger.

Restricting contract space to the Markovian contracts, the main purpose is to find a disciplining device (contract) that implements first best, for the society, for the highest possible misalignment of interests between society and politicians. It turns out that there may be more than one such disciplining device. However, if we refine our search to the disciplining devices that are also trembling-hand perfect, we end up with the unique device.

The intuition for why termbling-hand refinement leads to uniqueness is as follows. We need to solve for the first best allocation and derive conditions for its implementability. The first best is when the politicians always exert effort on the equilibrium path. Because we have persistent effort, the presence of private histories can cause divergence of beliefs between the society and politicians. For instance, in an equilibrium where political actors always exert effort on the equilibrium path, after any public history, the society believes that with probability 1 the effort was exerted by the politicians. But after a private history (private to the politician) where effort was not exerted in the previous period, because of persistence, the society has wrong evaluation of probabilities of the continuation histories (without persistence this issue does not arise). Because not exerting effort is a zero probability event from the perspective of the society, it does not necessarily have to incentivize politicians to exert effort after such event. Hence, for implementing the first best, after the private histories where a politician has exerted no effort in the previous period, we have some degree of freedom in designing firing rules. This creates multiplicity of disciplining devices that can implement first best for the highest misalignment of interests. Trembling hand refinement ensures that, to implement the first best, disciplining device has to incentivize the politicians to exert effort after EVERY public or private history.

Derivation of the disciplining device achieving the first best for the highest misalignment of interest proceeds as follows. To implement the first best, we have incentive campatibility (IC) constraints, that guarantee that a politician is exerting effort in the first stage (first stage IC constraint). In addition, we have second stage IC constraints guaranteeing that a politician is exerting effort in the second stage, conditional on his privately known effort exerted in the first stage and publicly observed signal from the first stage. First, I look at the relaxed problem where first stage IC constraints are ignored. I show that second stage IC constraints are relaxed if we never fire a politician in the first stage. The intuition is that this way, from the perspective of second stage, continuation value from staying in the office is maximized, and hence the politician has better incentives to exert effort in the second stage. Consequently, I show that under Monotone Likelihood Ratio Property assumption on the distribution of project outcomes conditional on effort exerted, the second stage firing rule is a threshold rule and that it is history independent. The fact that it has to be history independent comes from requesting effort to be exerted after every history. Finally, I verify that first stage IC constraint is not violated by the constructed disciplining device. In summary, we have a unique Markovian disciplining device, that achieves first best for the highest misalignment of interests and is trembling-hand perfect. The device never fires politicians due to the first stage results and prescribes history independent threshold firing rule, that is based only on the project's final outcome.

4.2 Related Literature

There are at least two strands of literature that are related to this chapter: principal-agent models with persistent moral hazard problem and citizen-candidate models.

4.2.1 Principal-Agent Models with Persistent Moral Hazard Problem

Kwon (2015) constructs a model with persistent Markovian effort where the second best contract is a tenure contract that eventually becomes history independent. Ogawa (2011) shows that optimal contract doesn't depend on the past history of outcomes in a 2 period principal agent model with persistent effort. Mukoyama (2005) provides numerical exercises for a finite horizon principal-agent model with 2 period persistent effort and concludes that persistence makes compensation less responsive to the project's first stage signals. Hopenhayen (2007) and Jarque (2008) draw some parallels between traditional repeated principal agent models without persistence and models with persistence. They find sufficient conditions on the payoff functions and the structure of effort persistence that ensure that optimal contracts resemble those characterized in Rogerson (1985). Fernandes and Phelan (2000) provide recursive formulation for a class of principal agent models with Markovian effort persistence. In all those papers there is a tendency that optimal contracts are or eventually become history independent i.e. even though there is persistence (exogenous links between the periods), principal tends not to use that for screening or incentivizing the agents.

Most of the papers in this strand of literature allow for monetary transfers. This gives a flexibility in characterizing optimal contracts however, on the other hand, it also generates redundancy in the problem which leads to the existence of multiple contracts that are payoff equivalent. Multiplicity of optimal contracts does not allow one to say much about qualitative aspects of such contracts. Unlike this, I shut down monetary transfers. This reduces the principal's flexibility for disciplining agents but buys us a unique contract for our purposes.

4.2.2 Citizen-Candidate Models

This class of models was introduced by Osborne, Martin and Slivinsky (1996) and Besley and Coate (1997) independently. In those models, similarly to ours, there are no preexisting political entities that compete for the office. Politicians come from the pool of citizens and rejoin the pool once they are fired. Banks (1999) is closely related to my model. The difference is that he has iid one period projects, politicians do not return to the pool of society once fired and politicians only live for 2 periods. He restricts analysis to the Markov perfect equilibria in cutoff strategies. Duggan (2015) studies the same model as in Banks (1999) but assumes that once a politician is fired he returns to the pool of the citizens. Rothert (2015), building on Banks (1999), adds persistent effort to the model and considers only moral hazard problem under technically more simplified version of the model. His setup is also different from ours in several respects most important of which is the modeling of persistence. None of the above models allows for monetary transfers and all of them restrict analysis to Markov equilibria.

4.3 The Model

4.3.1 Setup

Time is discrete, indexed by $t = 0, 1..$, and horizon is infinite. There is a continuum of citizens and a sequence of homogenous 2 stage projects. Without loss of generality, I consider discounting from the end of project to the end of the following project. There are two stages within each period t, t_0 and t_1, where 0 stands for the first stage of the project development and 1 stands for the second stage of the project development.

At each stage, incumbent politician chooses how much effort to exert. Together, effort choices in the first and the second stages determine the outcome for the society. Effort choices are not observed by the society. When taking the office a politician observes what effort was exerted in the previous period by his forerunner.

Stage t_0 : At the start of stage t_0 there is an incumbent politician i who chooses effort level $e_{t_0} \in \{0, 1\}$. Society's utility is independent of politician's actions in this stage and is normalized to 0. i gets utility $z - c(e_{t_0})$. z is the payoff from holding office and $c(\cdot)$ is the cost of effort. A signal θ from some compact set Θ is drawn according to the distribution $\Phi^{e_{t_0}}(\cdot)$ where the distribution is commonly known and θ is commonly observed. Society updates beliefs about e_{t_0}. A challenger j is randomly drawn from the pool of citizens and the society decides whether to keep i at the office or to fire him and appoint j.

Stage t_1 : Incumbent politician, j ($j = i$ or $j \neq i$), observes e_{t_0} (in addition to public history) and chooses $e_{t_1} \in \{0, 1\}$. Let $e \equiv e_{t_0} + e_{t_1}$. The project outcome, y_t, is drawn from $F(y_t \mid e)$. The society gets utility of y_t and updates beliefs about e_{t_0} and e_{t_1} given belief about the strategies of the politicians, realizations of θ, y_t and term of j in the office. Let I_t denote all the information that the society has at this point (including belief about the strategies chosen by the politicians). I denote the society's beliefs about effort levels exerted conditional on I_t by $\Pr(e_{t_0} \mid I_t)$ and $\Pr(e_{t_1} \mid I_t)$. A politician's utility function is $(1 - \alpha)y_t + z - c(e_{t_1})$ where α measures how aligned politician's and society's preferences are, in terms of outcomes of the project. At the end of the period, a new challenger, k, is drawn. Society chooses whether to keep j or to appoint k. A new project is commenced in the next period and the above game repeats.

The society and the politicians use a common discount factor, δ.

4.3.2 Discussion of Model

Large scale public projects and reforms take long time to be completed.[1] In addition, for such projects it is usually the case that members of a society have homogeneous preferences over outcomes. Take for instance public school reforms. People may have different definitions of what better public school system means and thus they may have different preferences on the approaches to the reforms. But in the end, public officials working hard on such reforms is desirable for everyone[2] This model is suited for such examples.

The main parameter of the model is the degree of alignment of preferences, α, between

[1] Education system reforms, infrastructural projects etc.

[2] One could alternatively assume heterogeneous preferences in the society and a majority voting rule in which case only the median voter's preferences would matter. Another scenario for the homogeneous preferences would be a problem of local public good provision where residents of a constituency want their representative to work hard in order to divert as much public funding as possible for the local public good.

the politicians and the society. Misalignment of preferences may arise due to various reasons. One reason is that a politician may have better outside option compared to citizens, that arises due to privileges from holding the office. For example, having a private security guard, a politician may discount the importance of public security.

An important assumption is that the society cannot discipline politicians using monetary transfers. This assumption is common in the models of dynamic elections. A way one could interpret this feature is that politicians are the ones who usually dictate property rights, that constraints the society from using monetary incentives for disciplining the politicians.

4.3.3 Technical Assumptions

Let $E_e \equiv E_F(y_t \mid e)$ and $F_e \equiv F(y_t \mid e)$, where E stands for the expectation operator. I make the following assumptions on the model parameters.

Assumption 4.1. $c(e_{t_0} = 1) = c(e_{t_1} = 1) = c$, $c(e_{t_0} = 0) = c(e_{t_1} = 0) = 0$.

Assumption 4.2. $E_2 - E_1 = E_1 - E_0 \equiv \bar{E} > c$.[3]

Assumption 4.3. $z - c > E_2$.

Assumption 4.4. $F_0, F_1, F_2, \Phi^0(\cdot), \Phi^1(\cdot)$ *are continuous.* F_0, F_1, F_2 *have common support* $Y \subseteq R_+$.

Assumption 4.5. $F_2 \succ_{MLRP} F_1 \succ_{MLRP} F_0$.

4.3.4 Strategies

Here I define the strategies for the players. In order to avoid unnecessary notation, I define only Markovian strategies.

Let $Term = \{f, s\}$ where f stands for a politician working on a given project for the first time after he was appointed to the office and s stands for a politician working on a given project for the second time since he was appointed to the office, respectively. With some abuse of language I call f the first term and s the second term. During the first stage of

[3]The main result goes through with $E_2 - E_1 \neq E_1 - E_0$. The assumption is made for simplifying the exposition.

the project development, in any period, incumbent politician is in the state f while at the beginning of the second stage, if incumbent was reelected then he is in the state s and if a new politician was appointed then this new politician is in the state f. Elements of $Term$ are called *term*. Let $E = \{(e_{t-1}, e_t)\}$ where $(e_{t_0}, e_{t_1}) \in \{1, 0\}^2$. Let the first stage part of $i's$ strategy be $(\pi, 1-\pi)$ where $\pi \equiv \Pr(e_{t_0} = 1)$. Let the second stage part be $p(term, e_{t_0}, \theta)$ which is the probability of playing $e_{t_1} = 1$ given incumbent's term, his or his forerunners first stage action and public signal from the first stage. For the society, let the first stage part of a strategy be $k(\theta) \equiv k_{t_0}(\theta)$ which is the probability of keeping the incumbent given the observed signal. Let the second stage part of the society's strategy be $k_{t_1}(\theta, y, term)$ which is also the probability of keeping the incumbent at the office given the signal realizations in both stages and the term of the incumbent.

Here I note that the class of strategies defined above are absorbing in the sense that if the society uses such strategies then the politicians cannot benefit by using strategies that dependent on richer histories. Conversely, if all politicians use symmetric Markovian strategies then the principal cannot gain by responding with strategies from other class of strategies.

Definition 4.1. *A Symmetric Markov Disciplining Device with the second stage disciplining device restricted to the intervals in* Y *(MDD) is a pair* $(k(\theta), Y_1(\theta, term))$ *where* $k : \Theta \longrightarrow [0, 1]$ *and* $Y_1 : \Theta \times Term \rightrightarrows (a, b)$*, where* $b \geq a$ *and* $(a, b) \in Y^2$.[4]

A politician's strategy is a pair $P = (\pi,\, p(term, e_{t_0}, \theta))$ where π is the first stage probability of playing $e_{t_0} = 1$ and $p(term, e_{t_0}, \theta)$ is the second stage probability of playing $e_{t_1} = 1$.

I assume that all the indifferences are resolved in favor of the outcome that benefits the society.

A note about the restriction of the second stage disciplining device to the intervals in R_+ is in order here. Generally, the literature restrict attention to the firing rules that depend on the updated probabilities about agents' types or actions. Equilibria usually turn out to be of the cutoff structure in beliefs.[5] It is easy to see that my restriction to the connected

[4]Let $Y_0(\theta, term) \equiv Y_1(\theta, term)^c$. Note that for a fixed θ, $Y_0(\theta, term) \cup Y_1(\theta, term) = Y$.

[5]See Banks & Sundaram (1998), Duggan (2015) and Rothert (2015).

intervals is strictly weaker than restricting second term disciplining device to depend on the beliefs. Appendix C provides a more detailed discussion.

4.3.5 Values

Note that the only relevant state variable for a player at the beginning of t_0 is whether he is a politician or a citizen. Also, because of the continuum of citizens each citizen has 0 probability of becoming a challenger. I write * to indicate the equilibrium variables and values that are taken as given by a player making a decision. Also, I assume that whenever effort is exerted there is a probability $1-l$ with which it does not succeed. For example, if at the first stage, effort was exerted but it did not succeed then the signal will be drawn from the distribution $\Phi^0(\cdot)$. Let $H(term, \theta, e_{t_0})$ denote the expected payoff to the politician in power at the beginning of t_1,

$$\begin{aligned}
&H(term, \theta, e_{t_0}) = \qquad (4.1)\\
&z + lp(term, \theta, e_{t_0})(1-\alpha)E_{1+e_{t_0}} - p(term, \theta, e_{t_0})c + \\
&(1 - lp(term, \theta, e_{t_0}))(1-\alpha)E_{e_{t_0}} + \\
&\delta lp(term, \theta, e_{t_0})[F_{1+e_{t_0}}(Y_1^*(\theta, term))V^*(pol) + F_{1+e_{t_0}}(Y_0^*(\theta, term))V^*(cit)] + \\
&\delta(1 - pl(term, \theta, e_{t_0}))[F_{e_{t_0}}(Y_1^*(\theta, term))V^*(pol) + F_{e_{t_0}}(Y_0^*(\theta, term))V^*(cit)]
\end{aligned}$$

The term $V^*(pol)$ is the value for a politician at the beginning of stage 1 in any given period and $V^*(cit)$ is the respective value for the society. Note that, none of those values depend on the histories from the previous projects as per restriction on the strategies. At the beginning of t_1 the incumbent gets benefit from holding the office, z, and incurs c only if exerts effort. Conditional on exerting the effort, he succeeds with the probability l and has instant enjoyment of social outcome which in expectation is $(1-\alpha)E_{1+e_{t_0}}$ where e_{t_0} is previous stage's effort and subscript 1 indicates that we are conditioning on $e_{t_1} = 1$. If he either does not put the effort or puts the effort and does not succeed, he gets $(1-\alpha)E_{e_{t_0}}$. This summarizes the expected payoff for the politician from stage t_1. After t_1, the new period starts and discounting with the factor δ takes place. If at t_1 effort was exerted and it succeeded, the joint probability of which is $lp(term, \theta, e_{t_0})$, then starting from $(t+1)_0$ the politicians will get the expected continuation value $[F_{1+e_{t_0}}(Y_1^*(\theta, term))V^*(pol) + F_{1+e_{t_0}}(Y_0^*(\theta, term))V^*(cit)]$ where he gets $V^*(pol)$ if he maintains the office (is not fired) and $V^*(cit)$ if he is fired. Probability of being fired depends on the effort exerted and the disciplining device. For instance, $F_{1+e_{t_0}}(Y_1^*(\theta, term))$ is the probability of not being fired conditional on: in the stage t_0 the effort level e_{t_0} was exerted and succeeded (if the effort was exerted), in stage t_1 $e_{t_1} = 1$ was exerted and succeeded and the society's strategy is not to fire the politician if and only if $y \in Y_1^*(\theta, term)$. If at t_1 effort was not exerted or it was and did not succeed then the politician gets expected continuation value of $[F_{e_{t_0}}(Y_1^*(\theta, term))V^*(pol) + F_{e_{t_0}}(Y_0^*(\theta, term))V^*(cit)]$.

Now I account for the first stage value of the politician (that is, at the beginning of each project),

$$
\begin{aligned}
&V(pol) = z - \pi c + \sum_{j=0}^{1}(j\pi l + (1-\pi l)(1-j)) \times \qquad (4.2)\\
&\int \phi^j(\theta)\{k^*(\theta)H^*(s,\theta,j) + (1-k^*(\theta))[lp^*(f,\theta,j)E_{j+1}(J^*(y_t,\theta_t,term)) + \\
&(1-lp^*(f,\theta,j))E_j(J^*(y_t,\theta_t,term)))]\}d\theta
\end{aligned}
$$

In any given period t, at the beginning of the first stage, incumbent gets z and if exerts effort then pays c. Given that exerted effort succeeds and conditional on the signal realization, θ, he is kept in the office with the probability $k^*(\theta)$ and gets $H^*(s, \theta, 1)$ as a continuation value from the second stage. This accounts for $\pi l \int \phi^1(\theta)k^*(\theta)H^*(s, \theta, 1)d\theta$. Note that we do not have discounting between the stages. If the politician exerted effort and succeeded in the first stage, which happens with the probability πl, and given the realization of θ he is fired, which happens with probability $1 - k^*(\theta)$, then from the second stage he goes back to the citizen pool. Once he becomes a citizen, he enjoys the outcome from the end of stage 2 that also depend on the strategies of a new politician who exerts effort with the probability $p^*(f, \theta, 1)$ and succeeds with l. In this case, the ex-politician, who is now a citizen, gets $J^*(y_t, \theta_t, term) = y_t + \delta V^*(cit)$ expectation of which is taken with respect to F_2. If the new politician does not succeed or does not exert the effort then the ex-politician gets $J^*(y_t, \theta_t, term)$ expectation of which is taken with respect to F_1.

All these account for the term $\pi l \int \phi^1(\theta)(1-k^*(\theta))[lp^*(f,\theta,1)E_2(J^*(y_t,\theta_t,term)) + (1 - lp^*(f,\theta,1))E_1(J^*(y_t,\theta_t,term)))]d\theta$. Other terms in 4.2 are interpreted similarly. The second stage value for a citizen is,

$$J(y_t,\theta_t,term) = y_t + \delta V^*(cit)$$

First stage value for a citizen is,

$$\begin{aligned} V(cit) = \sum_{j=0}^{1}(jl\pi^* + (1-l\pi^*)(1-j)) \times \qquad (4.3)\\ \int \phi^j(\theta)\{k(\theta)[l(p^*(s,\theta,j)E_{j+1}(J^*(y_t,\theta_t,term)) + \\ (1-lp^*(s,\theta,j))E_j(J^*(y_t,\theta_t,term)))] + \\ (1-k(\theta))[l(p^*(f,\theta,j)E_{j+1}(J^*(y_t,\theta_t,term)) + \\ (1-lp^*(f,\theta,j))E_j(J^*(y_t,\theta_t,term)))]\}d\theta \end{aligned}$$

In 4.3, $H^*(term, e_{t_0})$ is $H(term,\theta,e_{t_0})$ with $p(term,\theta,e_{t_0}) = p^*(term,\theta,e_{t_0})$; $V^*(pol)$ is $V(pol)$ with $\pi = \pi^*$; and $V^*(cit)$ is $V(cit)$ with $k(\theta) = k^*(\theta)$. Index j in the summation indicates whether the effort was exerted and succeeded in stage 1 or not. For instance, if it was exerted and did not fail, which happens with the probability $l\pi^*$, then a signal θ is drawn from Φ^1. Conditional on that signal, the probability of not firing an incumbent is $k(\theta)$. The society gets utility zero at the end of stage one (a normalization). At the end of stage 2, y is drawn from F_2 given that at the second stage additional effort is exerted and succeeds, which happens with the probability $l(p^*(s,\theta,1)$. If at the second stage effort is not exerted or it is exerted without a success, which happens with the probability $1-lp^*(s,\theta,1)$, then y is drawn from F_1. Other terms are interpreted similarly.

4.3.6 Solution Concept and Refinement

The society can commit to a Markovian Disciplining Device. However, it turns out that the restriction to the Markovian contracts along with the assumption that the society does not have any direct costs of firing or hiring the politicians makes the commitment assumption redundant. Hence, I can focus on the dynamic game between the society and the politicians and search for the best Markov perfect equilibrium for the society.

Definition 4.2. *A Symmetric Markov Perfect Equilibrium is the tuple* $(\pi^*, p^*(term, \theta, e_{t_0}), k^*(\theta), Y_1^*(\theta, term))$ *satisfying the following,*

i) $p^*(term, \theta, e_{t_0}) \in \arg\max_{p(term,\theta,e_{t_0})\in[0,1]} H(term, \theta, e_{t_0})$ *for all term,* θ *and* e_{t_0}*;*

ii) $\pi^* \in \arg\max_{\pi\in[0,1]} V(pol)$*;*

iii) $k^*(\theta) \in \arg\max_{k(\theta)\in[0,1]} V(cit)$ *for all* θ*;*

iv) $Y_1^*(\theta, term) \in \arg\max_{Y_1(\theta,term)} J(y, \theta, term)$ *for all term,* θ*.*

I introduce several definitions that culminate into defining the main problem to be solved.

Definition 4.3. *Optimal MDD (OMDD) is MDD that implements the socially optimal outcome in Symmetric Markov Perfect Equilibrium.*

The social welfare in a given period can be defined either by $E_e - c(e_{t_0}) - c(e_{t_1})$ or by E_e. Assumptions 4.1 guarantees that in both cases, society wants the politicians to exert effort in both stages.

Definition 4.4. *The best OMDD (BOMDD) is OMDD that implements the socially optimal outcome in Symmetric Markov Perfect Equilibrium for the highest possible* α *(denoted* α^{sol}*) among all OMDDs.*

BOMDD is an OMDD that implements the social optimum for the highest possible divergence between the society's and the politicians' interests. One scenarion where we would care about BOMDD would be if the society has the following lexicographic preferences: in the first place the society is concerned with the optimality of the outcome and then it is concerned with ensuring that the worst possible type of a politician would behave so that to achieve this optimum. Other reason for why we would be interested in BOMDD is to figure out when it is possible to implement the first best. Yet another reason would be to find out when the belief-free (with regard to the politicians' preferences) implementation of the first best would be possible.

Solving for BOMDD is complicated by the presence of the persistence of private information (first stage effort also affects the final outcome of the project). To deal with this complication I introduce the following refinement. Let $(1-l)$ be a small probability that even if effort is exerted it will be in vain i.e. exerted effort will not affect the probability distribution of y. Let $\alpha^{sol}(l)$ be the worst type of the politician for which the first best can be implemented, given the tremble of magnitude $1-l$.

Definition 4.5. *$l-$tremble BOMDD is BOMDD under l. The set of $l-$tremble BOMDDs is denoted by $B(l)$ and its elements are denoted by $b(l)$.*

Definition 4.6. *$b^*(1)$ is a trembling hand perfect BOMDD (TBOMDD) if there exist some sequences $\{l_k\}_{k=1}^{\infty}$ and $\{b(l_k)\}_{k=1}^{\infty}$ such that i) for each k, $l_k < 1$ and $\lim_{k\to\infty} l_k = 1$; ii) for each k, $b(l_k) \in B(l_k)$; iii) $\lim_{k\to\infty} b(l_k) = b^*(1)$ a.s.*

TBOMDD is a trembling hand refinement with a specific type of a tremble and is the object of the consequent analysis.

4.4 The Main Result

In this section I solve for the TBOMDD. Technical details of the proofs are relegated to the Appendix C. Substituting for $J(y_t)$, 4.2 and 4.3 can be further reduced to,

$$V(pol) = z - \pi c + \sum_{j=0}^{1}(j\pi l + (1-\pi l)(1-j)) \times \int \phi^j(\theta)\{k^*(\theta)H^*(s,\theta,j) + (1-k^*(\theta))[lp^*(f,\theta,j)E_{j+1} + (1-lp^*(f,\theta,j))E_j + \delta V^*(cit))]\}d\theta \tag{4.4}$$

$$V(cit) = \sum_{j=0}^{1}(jl\pi^* + (1-l\pi^*)(1-j)) \times \int \phi^j(\theta)\{k(\theta)[l(p^*(s,\theta,j)E_{j+1} + (1-p^*(s,\theta,j))E_j + \delta V^*(cit))] + (1-k(\theta))[l(p^*(f,\theta,j)E_{j+1} + (1-p^*(f,\theta,j))E_j + \delta V^*(cit))]\}d\theta \tag{4.5}$$

First, I discuss some properties of OMDD. We definitely need effort exerted with probability 1 at every stage and in every period on the equilibrium path. Suppose that $l = 1$. Since the choice of effort is unobserved by the society and persistent we could achieve the first best with a kind of strategy that does not incentivize the politicians to exert effort in stage 2 if stage 1 effort was 0. This can be done because effort is a private history for a politician and in OMDD, from the society's perspective, stage 1 effort being 0 is a probability 0 event. Hence, the society is not restricted to implementing a certain effort choice after that probability 0 history.[6] However, when $1-l > 0$, 0 effort in t_0 is no more a 0 probability event even if effort is exerted after every history. Thus, for OMDD we need to ensure that effort is exerted after any private history. Substituting $\pi^* = p^*(term, \theta, e_{t_0}) = 1$ for all $term, \theta$ and e_{t_0} we get,

$$V^*(cit) = \frac{l^2 E_2 + 2l(1-l)E_1 + (1-l)^2 E_0}{1-\delta} \tag{4.6}$$

From 4.6 we can now explicitly see why commitment issues do not arise on the part of the society. As $V^*(cit)$ is independent of the society's strategies the problem, $k^*(\theta) \in \arg\max_{k(\theta)\in[0,1]} V(cit)$ for all θ, is redundant.

For now, I drop the dependence of values and strategies on $term$ and solve this relaxed problem. Later I will argue that the society cannot achieve better outcomes by conditioning on $Term$. We need IC's for the politicians ensuring that $\pi^* = p^*(\theta, e_{t_0}) = 1$ for all θ and e_{t_0}. First, we need $H(\theta, e_{t_0})$ evaluated at $p(\theta, e_{t_0}) = 1$ to be more than $H(\theta, e_{t_0})$ evaluated at $p(\theta, e_{t_0}) = 0$. This translates into

$$\begin{aligned}
& l(1-\alpha)E_{e_{t_0}+1} - c + (1-l)(1-\alpha)E_{e_{t_0}} + \\
& \delta l[F_{e_{t_0}+1}(Y_1^*(\theta))V^*(pol) + F_{e_{t_0}+1}(Y_0^*(\theta))V^*(cit)] + \\
& \delta(1-l)[F_{e_{t_0}}(Y_1^*(\theta))V^*(pol) + F_{e_{t_0}}(Y_0^*(\theta))V^*(cit)] \\
\geq \quad & (1-\alpha)E_{e_{t_0}} + \delta[F_{e_{t_0}}(Y_1^*(\theta))V^*(pol) + F_{e_{t_0}}(Y_0^*(\theta))V^*(cit)]
\end{aligned}$$

$$\iff$$

[6] This kind of a solution would be in the spirit of Fernandes & Phelan (2000).

$$\alpha \leq \alpha^*(\theta) \equiv \min\left\{1, 1 + \frac{\delta(F_2(Y_1^*(\theta)) - F_1(Y_1^*(\theta)))V^*(pol, \alpha^*(\theta))}{\bar{E}} + \frac{\delta(F_2(Y_0^*(\theta)) - F_1(Y_0^*(\theta)))V^*(cit) - c/l}{\bar{E}}\right\}$$

$$\alpha \leq \alpha^{**}(\theta) \equiv \min\left\{1, 1 + \frac{\delta(F_1(Y_1^*(\theta)) - F_0(Y_1^*(\theta)))V^*(pol, \alpha^*(\theta))}{\bar{E}} + \frac{\delta(F_1(Y_0^*(\theta)) - F_0(Y_0^*(\theta)))V^*(cit) - c/l}{\bar{E}}\right\}$$

We can further simplify the expressions by noting that $F_2(Y_0^*(\theta)) = 1 - F_2(Y_1^*(\theta))$ and $F_1(Y_0^*(\theta)) = 1 - F_1(Y_1^*(\theta))$ imply $F_2(Y_0^*(\theta)) - F_1(Y_0^*(\theta)) = F_1(Y_1^*(\theta)) - F_2(Y_1^*(\theta))$ and similarly for $F_1(Y_1^*(\theta)), F_0(Y_1^*(\theta))$. Also, I will drop dependence of $V^*(pol, \alpha)$ on α and will make it explicit only when it is important for the arguments. Using these simplifications the above conditions reduce to,

$$\alpha \leq \alpha^*(\theta) \equiv \min\left\{1, 1 + \frac{\delta(F_2(Y_1^*(\theta)) - F_1(Y_1^*(\theta)))(V^*(pol) - V^*(cit)) - c/l}{\bar{E}}\right\} \tag{4.7}$$

$$\alpha \leq \alpha^{**}(\theta) \equiv \min\left\{1, 1 + \frac{\delta(F_1(Y_1^*(\theta)) - F_0(Y_1^*(\theta)))(V^*(pol) - V^*(cit)) - c/l}{\bar{E}}\right\} \tag{4.8}$$

We also need to ensure that $\pi^* = 1$. For this we need $V^*(pol)$ evaluated at $\pi^* = 1$ to be more than $V^*(pol)$ evaluated at $\pi^* = 0$,

$$\begin{aligned} &\int \phi^1(\theta)k^*(\theta)H^*(\theta,1)d\theta - \int \phi^0(\theta)k^*(\theta)H^*(\theta,0)d\theta + \\ &\int \phi^1(\theta)(1-k^*(\theta))[lE_2 + (1-l)E_1 + \delta V^*(cit))]d\theta - \\ &\int \phi^0(\theta)(1-k^*(\theta))[lE_1 + (1-l)E_0 + \delta V^*(cit))]d\theta \geq c/l \end{aligned} \tag{4.9}$$

4.7, 4.8 and 4.9 ensure that the politicians exert effort after every history. The problem of finding an $l - tremble$ BOMDD reduces to finding a disciplining device that satisfies 4.7, 4.8 and 4.9 for the highest possible α for a given l. In the end, we need to ensure that there is some BOMDD for $l = 1$ to which some sequence of solutions converge.

Now I state the main result. Let $(\mathbf{1}, Y_1^* = (y^*, \infty))$ a.e. be a disciplining device where $\mathbf{1}$ a.e. means that $k^*(\theta) = 1$ a.e. θ and $Y_1^* = (y^*, \infty)$ a.e. means that $Y_1^*(\theta, term) = (y^*, \infty)$

a.e. θ. Let the functions $\alpha^*(y), \alpha^{**}(y)$ be defined by

$$\alpha^*(y) = \min\left\{1, 1 + \frac{\delta(F_1(y) - F_2(y))2(z-c) - c(1-\delta(1-F_2(y)))}{\bar{E}(1-\delta(1-F_2(y))) + \delta(F_1(y) - F_2(y))E_2}\right\} \tag{4.10}$$

$$\alpha^{**}(y) = \min\left\{1, 1 + \frac{\delta(F_0(y) - F_1(y))2(z-c) - c(1-\delta(1-F_2(y)))}{\bar{E}(1-\delta(1-F_2(y))) + \delta(F_0(y) - F_1(y))E_2}\right\} \tag{4.11}$$

Definition 4.7. *I say that a disciplining device is almost unique TBOMDD if it is unique for almost all histories.*

Proposition 4.1. *The disciplining device* $(\mathbf{1}, Y_1^* = (y^*, \infty))$ *a.e. with*

$$y^* = \arg\max_{y \in Y} \min\{\alpha^*(y), \alpha^{**}(y)\}$$

is the almost unique TBOMDD.

TBOMDD is very simple. It never fires a politician in the midterm of the project no matter what the signal is and at the end of the project it is simply a cutoff rule independent of the midterm signals and the term of the politician. Moreover TBOMDD is almost unique.

Not conditioning the disciplining device on the midterm signals completely eliminates the need for asking - who did what? Because it never happens that two different politicians work on the same project, all the responsibility for a given project outcome is due to the politician working on that project.

Other interesting aspect of the TBOMDD is that it does not punish or reward a politician on the basis of whether the politician in the office at the second stage was the one who initiated the project at the stage one or not. This happens mainly because a politician would never internalize such distinction given that disciplining device is Markovian. At the beginning of the first stage a politician cares about his future promised value as a politician only under the contingency in which tomorrow he is still the one who started the project. At stage 2, a politician knows that things will start afresh from the following period and thus if at the beginning of stage 2 an f politician is incentivized "better" than s politician to exert effort this does not make the outcome for the society superior to the outcome under which both, f and s politicians, are incentivized as f type.

Throughout, I assumed that α is the same across politicians and it is known to the society. What if we allow α to vary across politicians and in addition, assume that it is privately known by a politician. Recall that $\alpha^{sol} = \min\{\alpha^*(y^*), \alpha^{**}(y^*)\}$ is the worst possible type of a politician for which the first best can be implemented. Proposition 4.1 immediately implies the following,

Corollary 4.1. *In the model where α is allowed to vary across politicians and in addition, it is privately known by a politician, the belief-free implementation of the first best is possible if and only if the support of the politician types is restricted to $[0, \alpha^{sol}]$.*

Finally, we have the following comparative static on $\alpha(l)$,

Corollary 4.2. *$\alpha^{sol}(l)$ is non-decreasing in l.*

Remark 4.1. *One modification of the model would be to allow for the overlapping projects, that is to allow a politician to choose whether to continue the current project or to start the new one. If it is optimal for the society that no politician interrupts a project in progress (scrapping a project is wasteful), then it is not difficult to see that the main result remains true. Claim 4.1 in Appendix C shows that a politician would be willing to be a politician rather than returning to the pool of the citizens, for every possible disciplining device. To guarantee that a politician does not scrap a project, the society could punish a politician scrapping a project by immediately replacing him. This punishment would be sufficient to ensure that no wasteful scrapping takes place.*

This model is tailored for the particular application. However, small modifications would make it relevant for other applications. For instance, we could consider a manager (instead of a politician) and an owner (instead of a society) of a firm. The firm owner has multiperiod projects and needs a manager for those. A disciplining device consists of a firing decision, in addition to a linear profit sharing, as in Berhold (1971) i.e. $(1 - \alpha)$ is the share of y that goes to the manager and α is the share of y that goes to the owner. We also can change outside option for a manager from $V^*(cit)$ to 0. It is easy to verify, that if $\bar{E} > \sqrt{c}$, then the main result goes through.

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Appendix C

Proof of Proposition 4.1

First, through the series of claims and lemmas, I prove proposition 4.1. Let

$$M \equiv 1 - \delta l^2 \int \phi^1(\theta)k^*(\theta)F_2(Y_1^*(\theta))d\theta - \delta l(1-l) \int \phi^1(\theta)k^*(\theta)F_1(Y_1^*(\theta))d\theta -$$

$$\delta l(1-l) \int \phi^0(\theta)k^*(\theta)F_1(Y_1^*(\theta))d\theta - \delta(1-l)^2 \int \phi^0(\theta)k^*(\theta)F_0(Y_1^*(\theta))d\theta.$$

Some algebra leads us to,

$$\begin{aligned} V^*(pol) \quad = \quad & \frac{z - c + l \int \phi^1(\theta)(1-k^*(\theta)) \left[lE_2 + \delta V^*(cit) + (1-l)E_1\right] d\theta}{M} + \\ & \frac{(1-l) \int \phi^0(\theta)(1-k^*(\theta)) \left[lE_1 + \delta V^*(cit) + (1-l)E_0\right] d\theta}{M} + \\ & \frac{l \int \phi^1(\theta)k^*(\theta)[z + l(1-\alpha)E_2 - c + (1-l)(1-\alpha)E_1]d\theta}{M} + \\ & \frac{l \int \phi^1(\theta)k^*(\theta)[\delta l F_2(Y_0^*(\theta))V^*(cit) + \delta(1-l)F_1(Y_0^*(\theta))V^*(cit)]d\theta}{M} + \\ & \frac{(1-l) \int \phi^0(\theta)k^*(\theta)[z + l(1-\alpha)E_1 - c + (1-l)(1-\alpha)E_0]d\theta}{M} + \\ & \frac{(1-l) \int \phi^0(\theta)k^*(\theta)[\delta l F_1(Y_0^*(\theta))V^*(cit) + \delta(1-l)F_0(Y_0^*(\theta))V^*(cit)]d\theta}{M} \end{aligned}$$

I start by defining the problem to be solved for $l-tremble$ BOMDD. I drop the dependence of strategies on $Term$ and later will prove that by conditioning on $Term$ the society cannot do better. The problem is formulated as follows,

$$\sup_{Y_1(\theta)} \inf_{\theta\in\Theta} \min\{\alpha^*(\theta), \alpha^{**}(\theta)\} \tag{4.12}$$
$$\text{subject to } IR_1, IR_2(\theta, e_{t_0}), IC_1$$

$IR_1, IR_2(\theta, e_{t_0})$ are politicians' individual rationality constraints for stages 1,2, respectively. They ensure that a politician does not want to return to the pool of citizens. IC_1 is given by 4.9, $\alpha^*(\theta)$ is given by 4.7 and $\alpha^{**}(\theta)$ is given by 4.8. Problem 4.12 states that given these constraints, we need to find a correspondence $Y_1(\theta)$ such that under this correspondence the infrimum of α over all relevant private and public histories is as high as possible. Rephrasing, we are looking for the second stage disciplining device that induces exertion of second stage effort for the worst possible type of a politician uniformly over all relevant private and public histories given that constraints for participation and exerting effort in the first stage are satisfied. Note that, in addition to the public histories, we also need this for all relevant private histories because of trembling hand perfection. Otherwise, only on-equilibrium path private histories would matter. Let the solution to 4.12 be denoted as α^{sol} which of course may not be attained. We must have $\alpha^{sol} \leq \min\{\alpha^*(\theta), \alpha^{**}(\theta)\}$ for all $\theta \in \Theta$.

Note that 4.7 and 4.8 depend on $\alpha^*(\theta)$ and $\alpha^{**}(\theta)$ through $V^*(pol)$. However, I do not make the dependence of $V^*(pol)$ on $\alpha^*(\theta)$ and $\alpha^{**}(\theta)$ explicit whenever this would not affect the arguments.

The proof of proposition 4.1 proceeds in 7 steps. Claim 4.1 deals with the IR constraints. For claims 4.2, 4.3 and 4.4 we drop IC_1 constraint and consider the relaxed problem. Claims 4.2 and 4.3 show that $k^*(\theta) = 1$ for almost all θ is the first stage $l-tremble$ BOMDD. Claim 4.4 shows that $Y_1^*(\theta)$ is independent of θ and is of the form (y^*, ∞) a.e. Claim 4.5 shows that under the solution suggested by claims 4.2 to 4.4, IC_1 is satisfied. Claim 4.6 shows that the $l-tremble$ BOMDD implied by previous claims is TBOMDD. Lastly, claim 4.7 argues that conditioning disciplining device on $Term$ cannot improve our solution (i.e. cannot increase α^{sol}).

Claim 4.1. *IR for a politician is satisfied for all $l \in [0,1]$ and disciplining devices.*

Proof. $V^*(pol)$ can be rewritten as

$$\begin{aligned} V^*(pol) &= V^*(cit) + \frac{z - c + l \int \phi^1(\theta)(1-k^*(\theta))\,[lE_2 + (1-l)E_1]\,d\theta}{M} + \\ &\frac{(1-l)\int \phi^0(\theta)(1-k^*(\theta))\,[lE_1 + (1-l)E_0]\,d\theta}{M} + \\ &\frac{l\int \phi^1(\theta)k^*(\theta)[z + l(1-\alpha)E_2 - c + (1-l)(1-\alpha)E_1]d\theta}{M} + \\ &\frac{(1-l)\int \phi^0(\theta)k^*(\theta)[z + l(1-\alpha)E_1 - c + (1-l)(1-\alpha)E_0]d\theta}{M} \end{aligned} \tag{4.13}$$

IR for a politician is $V^*(pol) \geq V^*(cit)$. This is indeed the case in 4.11 because $z > c$ by assumption 4.2 and also $M \geq 0$. As for the second stage IR, dosing some algebra, it is easy to see that assumption 4.2 implies that it is also always satisfied for all histories, disciplining devices and strategies of the politicians. □

Claim 4.2. $k^*(\theta) = 1$ *almost everywhere maximizes* $V^*(pol) - V^*(cit)$ *for all* l

Proof. Let $K \subseteq \Theta$ be the set of θ where $k^*(\theta) < 1$. Let k^{**} be such that $k^{**}(\theta) = k^*(\theta) = 1$ for all $\theta \in \Theta$. Then, comparing $V^*(pol) - V^*(cit)$ under these alternative strategies one immediately sees that the expression under $k^{**}(\theta)$ is greater if K does not have zero measure. This is because for a fixed θ, the numerator increases and M decreases in $k(\theta)$. □

Claim 4.3. $(F_2(Y_1^*(\theta)) - F_1(Y_1^*(\theta))) \geq 0$ *and* $(F_1(Y_1^*(\theta)) - F_0(Y_1^*(\theta))) \geq 0$

Proof. Otherwise we would just set $Y_1^*(\theta) = R_+$ which would ensure that both, 4.7 and 4.8 are higher. □

Observing 4.7, 4.8 and 4.11, claims 4.2 and 4.3 immediately imply that $k^*(\theta) = 1$ a.e is the first stage disciplining device that maximizes $\alpha^*(\theta)$ and $\alpha^{**}(\theta)$ for each θ.

Claim 4.4. $Y_1^*(\theta)$ *is independent of* θ.

Proof. $V^*(pol)$ depends on $Y_1^*(\theta)$ only through M. We can write the following

$$\begin{aligned} \alpha^*(\theta) &\equiv \min\{1, 1 + \frac{\delta(F_2(Y_1^*(\theta)) - F_1(Y_1^*(\theta)))U(\alpha^*(\theta)) - Mc/l}{\bar{E}M}\} \\ \alpha^{**}(\theta) &\equiv \min\{1, 1 + \frac{\delta(F_1(Y_1^*(\theta)) - F_0(Y_1^*(\theta)))U(\alpha^{**}(\theta)) - Mc/l}{\bar{E}M}\} \end{aligned}$$

$U(\alpha)$ is the collection of terms that are independent of $Y_1^*(\theta)$ and it is decreasing in α. Hence in our problem, $Y_1^*(\theta)$ enters only through $(F_2(Y_1^*(\theta)) - F_1(Y_1^*(\theta)))/M$ and $(F_1(Y_1^*(\theta)) - F_0(Y_1^*(\theta)))/M$. Thus, if we show that some second stage disciplining device maximizes those terms for each θ then that device would also maximize $\alpha^*(\theta)$ and $\alpha^{**}(\theta)$ for each θ. Assumption 4.4 guarantees that each pair of densities (f_j, f_i), $i \neq j$ cannot intersect multiple times in such a way that those intersection points are disconnected. Given this observation and claim 4.3, there must be a set $C_\theta \subseteq Y_1^*(\theta)$ such that $f_2 \geq f_1$ and $f_1 \geq f_0$ for each θ on C_θ. To see this, suppose this was not the case then $f_2 < f_1$ and $f_1 < f_0$ for all subsets of $Y_1^*(\theta)$ cannot be the case because then this would contradict claim 4.3. Suppose for every subset it is the case that $f_2 < f_1$ whenever $f_1 \geq f_0$ and vice versa. Take $y' = \min\{\inf(y \in Y_1^*(\theta) : f_1 \geq f_0), \inf(y \in Y_1^*(\theta) : f_2 \geq f_1)\}$. Suppose $y' = \inf(y \in Y_1^*(\theta) : f_1 \geq f_0)$ then for all $y < y'$ we have $f_2 < f_1$ and for all $y \geq y'$ we have $f_1 \geq f_0$. By MLRP there is some $y'' > y'$ such that $f_2 \geq f_1$ for all $y \geq y''$. Take infrimum of such y'' then given that we are restricted to connected sets and given claim 4.3 then we must have $(y', y''] \in Y_1^*(\theta)$ and hence at least on y'' we must have $f_2 \geq f_1$ and $f_1 \geq f_0$.Contradicting that for every subset of $Y_1^*(\theta)$ it can be the case that $f_2 < f_1$ whenever $f_1 \geq f_0$ and vice versa. Now because of assumption 4.4, and observing that M decreases in the length of $Y_1^*(\theta)$ this set must be going all the way to the upper support of the distributions for almost all θ.Thus, for a.e. θ we have $Y_1^*(\theta) = (y_\theta, \infty)$. Now we are ready to see that $Y_1^*(\theta)$ must be independent of θ for a.e. θ. Take some subset $K \subseteq \Theta$ of nonzero measure and suppose y_θ depends on θ on that set. Take $y^* = \inf(y_\theta : \theta \in K)$ then we can set $y_\theta = y^*$ for all $\theta \in K$ that would decrease M as $y^* \leq y_\theta$ and because $\alpha^{sol} \leq \min\{\alpha^*(\theta), \alpha^{**}(\theta)\}$ for all $\theta \in \Theta$, $\alpha^{sol\prime}$ under this modification would be such that $\alpha^{sol\prime} \geq \alpha^{sol}$. Hence, $Y_1^*(\theta)$ is independent of θ a.e. □

So far, I have shown that $l - tremble$ BOMDD is $(1, Y_1^* = (y^*, \infty))$ a.e. That is, the disciplining device is completely independent of midterm signals and the midterm disciplining is never used. Now, I will verify that 4.9 is satisfied.

Claim 4.5. *Under $l - tremble$ BOMDD 4.9 is satisfied*

Proof. Substituting $l - tremble$ BOMDD in 4.9

$$(1-\alpha)\bar{E} + [l(F_2(Y_1^*) - F_1(Y_1^*)) + (1-l)(F_1(Y_1^*) - F_0(Y_1^*))]\delta(V^*(pol) - V^*(cit)) \geq c/l$$

Let $m \equiv \delta(V^*(pol) - V^*(cit))(F_2(Y_1^*) - F_1(Y_1^*))$ and $m' = \delta(V^*(pol) - V^*(cit)(F_1(Y_1^*) - F_0(Y_1^*))$. Suppose without loss of generality that solution to P is such that $\alpha^* \leq \alpha^{**}$ then, this implies that $m \leq m'$. Note that if we ensure that 4.9 holds for α^* then it holds for all $\alpha \leq \alpha^*$. Substituting α^* in above inequality, one gets

$$c/l - m + lm + (1-l)m' \geq c/l$$

The inequality reduces to $m \leq m'$ which is what we started with. □

We can rewrite problem 4.12 as

$$\max_{y \in Y} \min\{\alpha^*(l), \alpha^{**}(l)\}$$

We can write max because $\min\{\alpha^*, \alpha^{**}\}$ is a continuous function in y and Y is compact or can be made compact by defining the maximization problem on $[y_l, \arg\min_{y',y''}\{f_2(y') = f_1(y'), f_1(y'') = f_0(y'')\}$ where y_l is the lower bound of the support Y. Also we know that $IR_1, IR_2(\theta, e_{t_0}), IC_1$ are satisfied and we drop them.

Claim 4.6. *The disciplining device* $(\mathbf{1}, Y_1^* = (y^*, \infty))$ *a.e. with*

$$y^* = \arg\max_{y \in Y} \min\{\alpha^*(y), \alpha^{**}(y)\}$$

is the almost unique TBOMDD.

Proof. As we saw all the IR's and IC's are satisfied under $l - tremble$ BOMDD. Also, first stage disciplining device is not used for all $l \in [0,1]$ and for almost all histories. $y^*(l)$ a.e. is also continuous in l by the Berge's Maximum theorem and thus converges to the limit $y^*(1)$. One more thing to ensure is that $\lim_{l \to 1} \alpha^{sol}(l) = \alpha^{sol}(1)$ but this also follows the Berge's Maximum theorem. □

I have solved for the TBOMDD assuming that strategies don't depend on $Term$. Now I show that making the strategies $Term$ dependent cannot increase $\alpha^{sol}(l)$ for any l.

Claim 4.7. *Making the strategies $Term$ dependent cannot increase $\alpha^{sol}(l)$ for any l*

Proof. The social optimum is to incentivise agents to exert effort in every period and during any term and hence the social optimum is independent of the $Term$. The only channel through which $Term$ could make a difference is if one could relax incentives of politicians for exerting effort by making strategies dependent on $Term$. $Term$ affects only $H(term, \theta, e_{t_0})$. Making the disciplining device $Term$ dependent would add 2 more constraints that is we would have $\alpha^*(term), \alpha^{**}(term)$ and of course adding more constraints cannot increase $\alpha^{sol}(l)$. □

Proof of Corollary 4.2

Proof. We need to show that $V^*(pol, \alpha^{sol}(l)) - V^*(cit)$ is nondecreasing in l under $l-tremble$ BOMDD. Note that $V^*(pol, \alpha^{sol}(l))$ depends on $\alpha^{sol}(l)$ as well. For now, for the sake of arguments, it is important to make this dependence explicit. Differentiating numerator, we have $(1 - \alpha^{sol}(l))\bar{E} \geq 0$. Differentiating denominator - $l(F_1(Y_1^*) - F_2(Y_1^*)) + (1 - l)(F_0(Y_1^*) - F_0(Y_1^*)) \leq 0$ where the inequality follows from claim 4.3. Also the term c/l decreases in l. Note that in the differentiation I fixed Y_1^*, and $\alpha^{sol}(l)$ in $V^*(pol, \alpha^{sol}(l))$. Hence, $\min\{\alpha^*(l', Y_1^*, \alpha^{sol}(l)), \alpha^{**}(l', Y_1^*, \alpha^{sol}(l))\} > \alpha^{sol}(l)$ for $l' > l$ if we fix $Y_1^*(l)$, and $\alpha^{sol}(l)$ in $V^*(pol, \alpha^{sol}(l))$. Now let us consider what happens when we vary $\alpha^{sol}(l)$ in $V^*(pol, \alpha^{sol}(l))$. Because $V^*(pol, \alpha^{sol}(l))$ is decreasing in α, we can increase $\alpha^{sol}(l)$ to some α' such $\alpha' = \min\{\alpha^*(l', Y_1^*, \alpha^{sol}(l)), \alpha^{**}(l', Y_1^*, \alpha^{sol}(l))\}$ and hence $\alpha' > \alpha^{sol}(l)$. Now, if we also allow Y_1^* to change with l then the new solution at higher l' cannot be lower than α'. □

Second Stage Disciplining Device: Intervals vs Threshold Strategies in Beliefs

I argue that the restriction to the second stage interval firing strategies (disciplining devices) is strictly weaker than restricting second term disciplining device to depend on beliefs about effort exerted. The posterior beliefs about politician's actions in period t are given by

$$Pr(e_{t_0} = 1 \mid y_t, \theta_t, term, equilibrium\ strategies) = \frac{1}{1 + \frac{(1-\pi)[(1-p(term,0,\theta_t))f_0(y)+p(term,0,\theta_t)f_1(y)]\phi^0(\theta)}{\pi[p(term,1,\theta_t)f_2(y)+(1-p(term,1,\theta_t))f_1(y)]\phi^1(\theta)}} \tag{4.14}$$

and

$$Pr(e_{t_1} = 1 \mid y_t, \theta_t, term, equilibrium\ strategies) = \frac{1}{1 + \frac{\pi(1-p(term,1,\theta_t))f_1(y)\phi^1(\theta)+(1-\pi)(1-p(term,0,\theta_t))f_0(y)\phi^0(\theta)}{\pi p(term,1,\theta_t)f_2(y)\phi^1(\theta)+(1-\pi)p(term,0,\theta_t)f_1(y)\phi^0(\theta)}} \tag{4.15}$$

Differentiating 4.14 with respect to y_t we obtain,

$$(1-p(term,1,\theta_t))(1-p(term,0,\theta_t))(f_1'(y)f_0(y) - f_1(y)f_0'(y)) +$$

$$p(term,1,\theta_t)p(term,0,\theta_t)(f_2'(y)f_1(y) - f_2(y)f_1'(y)) +$$

$$p(term,1,\theta_t)(1-p(term,0,\theta_t))(f_2'(y)f_0(y) - f_2(y)f_0'(y))$$

This derivative is positive because the MLRP assumption implies that

$$(f_1'(y)f_0(y) - f_1(y)f_0'(y)), (f_2'(y)f_1(y) - f_2(y)f_1'(y)), (f_2'(y)f_0(y) - f_2(y)f_0'(y))$$

are all positive.

Similarly, for expressions 4.15. Hence, beliefs are strictly increasing in the second period outcome. So, if we have a cutoff strategy, p_1^*, p_2^*, this is the same as not firing agent for $y \geq y^*$ where y^* is the outcome that solves $\min\{p_1^*, p_2^*\}$. Hence we have a corresponding interval firing rule, $(y^*, upperboundofdistr\sup port)$. To see that I allow for strictly more with the interval strategies suppose the politician always exerts effort with probability 1 then, equilibrium beliefs assign probability 1 that effort was exerted in any stage. In such case, cutoff strategies in beliefs would be redundant. However, with the interval strategies we can still condition on observed outcomes in the second stage, and punish a politician on the basis of realization of project outcome.

Vita

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Giorgi Mekerishvili was born on September 29, 1986, in Tbilisi, Georgia. He earned his B.A. degree in International Relations in 2007, from I. Chavchavadze State University, Georgia. He earned his M.A. degree in Economics in 2012, from International School of Economics at Tbilisi State University, Georgia. Since 2013, he has continued his studies at Penn State University Economics Department, as a graduate student. His Ph.D. dissertation has focused on applied theoretical and empirical aspects of information design.