DYNAMIC PHASOR MODELING AND ANALYSIS OF AN OFFSHORE AC NETWORK CONNECTED TO VSC-HVDC LINKS

A Thesis in
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by
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Abstract

The thesis addresses the challenges of integrating offshore Wind Power Plants (WPPs) to the onshore AC grids. The research is focused on investigating the offshore AC network concept. The proposed method involves the interconnection of nearby WPPs using High Voltage Alternating Current (HVAC) cables to form the offshore AC network. Point-to-point Voltage Source Converter - High Voltage Direct Current (VSC-HVDC) links are established to connect the offshore AC network to the onshore AC grids. The underlying concept enables system protection without the use of DC circuit breakers.

Grid-forming control of the system is investigated with simulations using the detailed model of the VSCs. Voltage and frequency regulation of the system is achieved despite sudden injections of wind currents. The Dynamic Phasor (DP) theory is proposed to enable simulation-based studies of the system with relatively lower memory and computation requirements. The impact on network stability due to the variation in the offshore HVAC cables' length, VSC reactors' impedance, filter capacitance and the control gain values of the inner current and outer voltage controllers is investigated. The DP-based model of the system is used to perform stability studies of the system based on the eigenvalue analysis. The results of this analysis are validated with detailed model-based simulations of the system. The accuracy of the DP modeling technique is verified against the detailed model with simulation-based comparative studies.
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∗ - conjugate

$e$ - exponential.

$i$ - imaginary

$\infty$ - infinity.

$f$ - integral.

$\pi$ - pi.

$r$ - real

$\tau$ - tau.

$\omega$ - omega.

$\sum$ - summation.
First and foremost, praises and thanks to God Almighty for providing me the strength and opportunity to pursue this research. None of my success would have been possible without His countless Blessings.

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Chapter 1
Introduction

1.1 Background and Motivation

The increasing need of climate-friendly, cost-efficient and endless power production has necessitated the exploitation of renewable resources. Wind is regarded to be among the most reliable sources of renewable energy [1]. In order to harness greater amounts of wind for larger power generation, Wind Power Plants (WPPs) are increasingly being installed offshore [2].

The objective of integrating offshore WPPs to the onshore AC grids poses several challenges. The integration may severely impact the stability, power quality and distribution efficiency of the onshore grids [3]. These potential consequences entail the employment of an effective transmission system to ensure reliable transfer of generated power from the offshore WPPs to the onshore grids. The cost of design, installation and operation of the transmission system is a relevant factor in determining the suitable integration scheme. The High Voltage Direct Current (HVDC) transmission system enables adequate power transfer in an efficient manner for the afore-mentioned purposes [4, 5]. Stability studies regarding the integration of offshore WPPs using HVDC transmission are discussed in [6–15].

The studies in [6–9] employed classical point-to-point HVDC systems based on Line Commutated Converters (LCCs). In [9], a small-signal stability analysis (SSSA) was applied to design the controllers of the LCC based HVDC system. The simulation-based studies of the system reported stable responses with very high frequency components. Such systems require the use of large filters due to the production of large amounts of harmonics. Furthermore, the applied technology cannot provide...
independent control of the active and reactive powers [16]. Since LCC-HVDC systems require an ac voltage source for commutation, it can only transfer power between two AC networks [16].

The point-to-point Voltage Source Converter (VSC) based HVDC systems were proposed in [10–12]. The impact of a VSC-HVDC connection to an offshore wind farm on power system stability and control was simulated in [10]. The research study reported stable system responses. Independent control of the system reactive power, inverter DC voltage, and AC voltage and frequency of the wind farm were observed in the case studies. This technology does not require an active commutation voltage since it employs IGBTs, which can switch off currents [16]. Therefore, it does not require strong ac networks and has the ability to start up against a dead network [16].

The studies in [6–12] involved a point-to-point HVDC link between a single offshore WPP and an onshore AC grid. Increased efficiency, reliability, flexibility, reduced costs, and improved power management can be achieved by the interconnection of multiple offshore WPPs [17,18]. The multi-terminal VSC based HVDC system was proposed for this purpose in [13–15]. SSSA was used to design the controllers of the MTDC system in [13]. The proposed method provided stable network operation as well as enhanced dynamic performance. The droop control strategy was employed in [14] to control the DC bus voltage of the MTDC system. Root locus was applied to determine the range of droop parameter values to ensure stable system operation. The system responses were shown to be stable, and the control objectives were achieved. The pole-placement scheme was used in [15] to design the power oscillation damping (POD) controllers for the corresponding VSCs of the MTDC system. The proposed system was shown to be effective in suppressing large variations of the considered systems’ quantities.

The fault clearance in point-to-point VSC-HVDC systems is undertaken by the AC side circuit breakers of the VSCs. This is not practical for MTDC transmission since the entire DC system would need to be de-energized in case of single cable failures. HVDC circuit breakers would be required to isolate the faulted lines individually. The impact of these breakers was not investigated in [13–15]. According to [19], such components rely on large DC reactors which influence the stability and dynamic performance of the MTDC systems. It is also quite costly to install large DC reactors [19].

The offshore AC network concept is another means to perform the interconnection
of offshore WPPs. The underlying concept proposes the interconnection of nearby offshore WPPs using HVAC cables. This forms an offshore AC network, which is connected to onshore AC grids via point-to-point VSC-HVDC transmission links [20]. This enables a faulted HVDC link to be isolated using only AC side circuit breakers. Therefore, system protection can be ensured without the use of DC reactors. The control and operation of the offshore AC network has been studied in [21–27]. In all of these studies, grid-forming control was employed as the primary control scheme in the offshore VSCs to regulate the voltage and frequency of the offshore network.

The grid-forming control scheme is applied to stabilize a system in case of sudden change in renewable energy generation, cable failures, etc. It is widely employed in applications which operate in island mode including UPS, micro-grid, PV and wind-battery hybrid systems, etc. [28–33]. The purpose of the grid-forming converter is to enable the VSCs to operate autonomously, and generate a voltage waveform [34]. Therefore, the converter can be modeled as an ideal AC voltage source with low output impedance, setting the voltage and frequency of the grid based on a control loop [35]. The voltage and current dual closed-loop structure is usually employed in such converters [35,36]. The inner current loop ensures fast dynamic compensation for disturbances, and the outer voltage control enables set-point tracking of the reference AC voltage [37]. Therefore, it effectively improves the output waveform quality of the converters.

In addition to the primary control scheme, power sharing among HVDC links was enabled by varying the operating frequency of the offshore AC grid in [21]. DC overvoltage control methods and the frequency droop controller were successfully tested with the proposed system in [22]. The research objectives of [21–26] aimed at analyzing the performance of the proposed control systems. None of these studies focused on investigating the stability of the offshore network. In [27], SSSA was applied to determine the stable range of operation for the frequency and voltage droop gain values. The impact of the rest of the control and network parameters on system stability was not investigated. Non-linear simulations were performed to validate the proposed concepts of the study. The averaged model of the converters, proposed in [38], was used in these simulations. The accuracy of the system responses is not guaranteed with this approach. The system may need to be simulated using the detailed model of the converters to ensure validation of the suggested methods.

Generally, the performance of power system generation, control and operation
is investigated using computer software tools. The HVDC studies in [6–15, 21–23, 25–27] were performed using computer software to simulate the system responses for different test scenarios. Due to the constraints associated with computer storage and computation time, it is very difficult to simulate complete representations of large power systems. The corresponding computational burden and analytic complexity makes it infeasible to perform simulation-based studies on such systems [39, 40].

An alternative modeling approach based on the Dynamic Phasor (DP) theory can be applied to enable simpler simulations. Several studies have derived DP-based models to study HVDC systems [41–50]. Such models are computed with much larger time steps resulting in a drastic reduction of simulation time. In [41], low-frequency dynamics of a LCC-HVDC system were modeled using the DP technique. Lower computation requirements and high accuracy of the proposed model were verified against its detailed model, based on the Electromagnetic Transient (EMT) approach. In [42], extended-frequency DPs were used to model the LCC-HVDC system. The results of the case studies showed very high accuracy of the system responses, and significantly accelerated simulation speeds compared to those of the detailed models. LCC switching functions operating under balanced conditions were modeled using a DP-based approach proposed in [43]. The presented model described the system responses considering only their fundamental frequency components. The accuracy and effectiveness of the proposed method were verified with simulation results of test examples. This methodology was also applied in [48] to model a 2-area AC/DC power system and a multi-infeed HVDC power system, which were considered to be operating under balanced conditions. The paper reported simulation results which demonstrated the high accuracy and effectiveness of the proposed model. The same approach was applied for a balanced VSC-based HVDC system in [50]. Accurate system responses in addition to reduced computation requirements were observed with the proposed model. Although the DP theory has been used to model a large variety of power system applications, the offshore AC network has not yet been considered with this modeling approach.

This thesis will focus on studying the stability and dynamic performance of the offshore AC network connected to multiple VSC-HVDC links. A simulation model of the system having constant frequency will be developed, considering the detailed model of the VSCs. The control performance of the system will be analyzed with simulations of different test cases. Then, a DP-based state space model of the system
will be derived. The accuracy of the proposed model will be assessed with simulation-based studies. The impact of the network and control parameters on the stability of the system will be investigated using the eigenvalue analysis of the proposed model. The results of the stability studies will be validated with simulations of the detailed model. For the purpose of adding frequency and voltage droop controllers, the small-signal $dq$ model of the system with variable network frequency will be derived. The validated DP-based modeling techniques will be applied for the derivation of the DP-based model of this system. The responses of the controlled variables will be simulated using this model to analyze the dynamic performance of the system under consideration.

1.2 Research Objectives

In view of the proposed studies for this thesis, the research objectives to be completed during the course of the work are summarized as follows:

1. Modeling and simulation of the offshore AC network connected to VSC-HVDC links under grid-forming control mode using MATLAB SimScape Power Systems. A constant network frequency of the system will be assumed. The detailed model of the VSCs will be used in the simulations to obtain fully accurate system responses for validating the proposed concepts.

2. Deriving the DP-based mathematical model of the system with constant network frequency.

3. Simulation-based validation of the proposed model. The derived equations will be simulated using MATLAB Simulink. The accuracy of the simulation response will be analyzed by comparing its results with those obtained from the detailed model using MATLAB SimScape Power Systems. The simulation time for both models will also be compared to assess the trade-off between response accuracy and memory requirements for simulations.

4. Performing stability studies of the system. The eigenvalue analysis will be performed using the DP-based model to assess the impact of varying the HVAC cable length and VSC reactors’ impedances, and the effect of tuning the outer voltage and inner current control gains. The simulations of the detailed model of the system will be performed to verify the conclusions of the stability studies.

5. Deriving the small-signal $dq$ model of the system having variable network
frequency.

6. Deriving the DP-based mathematical model of the system with variable network frequency.

7. Performing simulation-based studies using the DP-based model to analyze the primary control performance of the system. Finally, the voltage and frequency droop controllers will be employed to investigate their control performance.

The fulfillment of these thesis objectives will establish the viability of the presented methods, paving the way for further research and development of the proposed concepts. Furthermore, the presented DP-based models will considerably simplify the simulation-based studies and stability analysis of the offshore AC network for future works.

1.3 Thesis Structure

The modeling and control of the system with constant network frequency is discussed in the second chapter. The detailed model of the system is simulated using MATLAB SimScape Power Systems in the third chapter. The fourth chapter introduces the DP theory. In this chapter, the proposed techniques are applied to obtain the DP-based model of the system with constant network frequency. In the fifth chapter, the accuracy of the proposed modeling technique is validated by performing a comparative analysis with the results obtained from the detailed simulation model in the third chapter. The sixth chapter involves the use of the derived model to perform stability studies of the system under varied cable parameters and altered VSC reactor/control values. The seventh chapter presents a small-signal $dq$ model of the system with variable network frequency. The primary and droop controllers of the system are also included in this chapter. The DP-based model of this system is presented in the eighth chapter. In the ninth chapter, the DP-based model of the system with variable network frequency is simulated to analyze the dynamic performance of the considered system. The conclusion of the thesis as well as the recommended future work are presented in the final chapter.
Chapter 2  
Modeling and Control of Offshore AC Network connected to VSC-HVDC Links assuming Constant Network Frequency

2.1 Introduction
In this chapter, a generic configuration of the system under consideration will be introduced along with its grid-forming control strategy. The design of the system presented in [27] will be used for the purpose of developing the time-domain model of the considered system. A constant network frequency will be assumed for the modeling process in this chapter. The dynamics of the control scheme will also be presented.

2.2 System Design
The system configuration is shown in Fig. 2.1. In the presented design, HVAC cables are used to interconnect the nearby offshore WPPs. Type IV wind turbines are considered for the WPPs [27]. The offshore network is connected to the onshore AC grids using VSC-based HVDC links [20].

The VSC-HVDC transmission system isolates the offshore AC network from the inertia of the onshore synchronous generators. Additionally, the wind turbines’
full power converters isolate the inertia of the turbine generators from the offshore network. Therefore, the frequency of the network is the average of all the grid-forming VSCs’ reference frequencies [51]:

\[
\omega = \frac{1}{z} \sum_{i=1}^{z} \omega_{ri}
\]  

(2.1)

where \( \omega \) is the frequency of the network, \( \omega_{ri} \) is the reference frequency of the \( i \)th VSC and \( z \) is the total number of VSCs in the network.

### 2.3 Time-domain Modeling of the System under Consideration

In this section, the time-domain model of the system will be derived. The connection of each VSC to the offshore network is presented in Fig. 2.2.

In order to simplify the modeling process, the overall system will be represented in terms of the models of its sub-systems; the VSC and the offshore AC network. The VSC model will include differential equations describing the dynamics of the VSC connection to its corresponding AC filter busbar. The offshore AC network model will involve differential equations defining the variable dynamics emanating from the
HVAC cables’ interconnection with the WPPs [27].

2.3.1 VSC Model

The VSC substation is connected to the offshore network at the Point of Common Coupling (PCC). This was illustrated in Fig. 2.2. The differential equations describing the PCC bus voltage can be defined in space phasor form as:

\[
\frac{d\tilde{v}_{si}}{dt} = \frac{1}{L_i} \tilde{v}_{ti} - \frac{1}{L_i} \tilde{v}_{si} - \frac{R_i}{L_i} \tilde{i}_{si} \tag{2.2}
\]

\[
\frac{d\tilde{v}_{si}}{dt} = \frac{1}{c_{fi}} \tilde{i}_{ni} + \frac{1}{c_{fi}} \tilde{i}_{si} \tag{2.3}
\]

For the \(i^{th}\) VSC, \(R_i\), \(c_{fi}\) and \(L_i\) are the VSC’s resistance, capacitance and inductance respectively, \(\tilde{i}_{si}\) is the VSC current, \(\tilde{i}_{ni}\) is the network current, \(\tilde{v}_{ti}\) is the voltage at the VSC terminal, \(\tilde{v}_{si}\) is the voltage at the VSC controlled AC busbar. Based on (2.2) and (2.3), the differential equations can be obtained in time-domain as:

\[
\frac{di_{sai}}{dt} = \frac{1}{L_i} v_{taai} - \frac{1}{L_i} v_{sai} - \frac{R_i}{L_i} i_{sai} \tag{2.4}
\]

\[
\frac{di_{sbi}}{dt} = \frac{1}{L_i} v_{tha} - \frac{1}{L_i} v_{sbi} - \frac{R_i}{L_i} i_{sbi} \tag{2.5}
\]

\[
\frac{di_{sci}}{dt} = \frac{1}{L_i} v_{tci} - \frac{1}{L_i} v_{sci} - \frac{R_i}{L_i} i_{sci} \tag{2.6}
\]
\[ \frac{dv_{sai}}{dt} = \frac{1}{c_{fi}} i_{nai} + \frac{1}{c_{fi}} i_{sai} \quad (2.7) \]
\[ \frac{dv_{sbi}}{dt} = \frac{1}{c_{fi}} i_{nbi} + \frac{1}{c_{fi}} i_{sbi} \quad (2.8) \]
\[ \frac{dv_{sci}}{dt} = \frac{1}{c_{fi}} i_{nci} + \frac{1}{c_{fi}} i_{sci} \quad (2.9) \]

In this form, the subscripts \( a, b \) and \( c \) are used to denote the corresponding phases of the respective variables.

The VSC time-domain model can be represented in state space form as: \( \dot{X}_{vi} = A_1 X_{vi} + B_1 U_{vi} \), where

\[
X_{vi} = \begin{bmatrix} i_{sai} & i_{sbi} & i_{sci} & v_{sai} & v_{sbi} & v_{sci} \end{bmatrix}
\]

\[
U_{vi} = \begin{bmatrix} v_{tai} & v_{tbi} & v_{tci} & i_{nai} & i_{nbi} & i_{nci} \end{bmatrix}
\]

The state and input matrices of the model are defined as:

\[
A_1 = \begin{bmatrix}
-\frac{R_i}{L_i} & 0 & 0 & -\frac{1}{L_i} & 0 & 0 \\
0 & -\frac{R_i}{L_i} & 0 & 0 & -\frac{1}{L_i} & 0 \\
0 & 0 & -\frac{R_i}{L_i} & 0 & 0 & -\frac{1}{L_i} \\
\frac{1}{c_{fi}} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{c_{fi}} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{c_{fi}} & 0 & 0 & 0
\end{bmatrix}
\quad (2.10)
2.3.2 Offshore AC Network

The modeling of the offshore AC Network will involve the combination of the differential equations describing the network currents and voltages. The modeling approach will be based on the reference diagram presented in Fig. 2.3.

\[
B_1 = \begin{bmatrix}
\frac{1}{L_i} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{L_i} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{L_i} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{c_{fi}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{c_{fi}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{c_{fi}}
\end{bmatrix}
\]  \quad (2.11)

**Figure 2.3.** Reference diagram for modeling the offshore network.
2.3.2.1 AC Hub Bus Voltage

The voltage differential equation describing the dynamics of the AC hub bus is derived in space phasor form as:

$$\frac{d\vec{v}_j}{dt} = \frac{1}{c_1 + c_2} \vec{i}_w + \frac{1}{c_1 + c_2} \vec{i}_{k1} + \frac{1}{c_1 + c_2} \vec{i}_{k2}$$

(2.12)

where $\vec{i}_w$ is the equivalent current from the WPPs, $\vec{i}_{k1}$ and $\vec{i}_{k2}$ are the currents flowing from the network branches, $c_1^*$ and $c_2^*$ represent half of the total capacitance of each cable respectively.

The time-domain form of the voltage differential equation can be derived as:

$$\frac{dv_{ja}}{dt} = \frac{1}{c_1 + c_2} i_{wa} + \frac{1}{c_1 + c_2} i_{ka1} + \frac{1}{c_1 + c_2} i_{ka2}$$

(2.13)

$$\frac{dv_{jb}}{dt} = \frac{1}{c_1 + c_2} i_{wb} + \frac{1}{c_1 + c_2} i_{kb1} + \frac{1}{c_1 + c_2} i_{kb2}$$

(2.14)

$$\frac{dv_{jc}}{dt} = \frac{1}{c_1 + c_2} i_{wc} + \frac{1}{c_1 + c_2} i_{kc1} + \frac{1}{c_1 + c_2} i_{kc2}$$

(2.15)

The bus voltage equations can be represented in state space form, $\dot{X}_g = A_2 X_g + B_2 U_g$, where:

$$X_g = \begin{bmatrix} v_{ja} & v_{jb} & v_{jc} \end{bmatrix}$$

$$U_g = \begin{bmatrix} i_{wa} & i_{wb} & i_{wc} & i_{ka1} & i_{kb1} & i_{kc1} & i_{ka2} & i_{kb2} & i_{kc2} \end{bmatrix}$$

(2.16)

The state and input matrices of the model are:

$$A_2 = 0$$

(2.17)
\[
B_2 = \begin{bmatrix}
B_{11} & 0 & 0 & B_{12} & 0 & 0 & B_{13} & 0 & 0 \\
0 & B_{21} & 0 & 0 & B_{22} & 0 & 0 & B_{23} & 0 \\
0 & 0 & B_{31} & 0 & 0 & B_{32} & 0 & 0 & B_{33}
\end{bmatrix}
\] (2.18)

where:

\[
B_{11} = B_{12} = B_{13} = \frac{1}{c_1^* + c_2^*}
\]

\[
B_{21} = B_{22} = B_{23} = \frac{1}{c_1^* + c_2^*}
\]

\[
B_{31} = B_{32} = B_{33} = \frac{1}{c_1^* + c_2^*}
\]

2.3.2.2 Branch Currents

The current flow across each branch of the offshore network can be defined in space phasor form as:

\[
\frac{d\vec{i}_{ki}}{dt} = \frac{1}{l_{ki}}\vec{v}_{ki} - \frac{1}{l_{ki}}\vec{v}_j - \frac{r_{ki}}{l_{ki}}\vec{i}_{ki}
\] (2.19)

For the \(i\)th branch, \(r_{ki}\) and \(l_{ki}\) are the branch resistance and inductance, \(\vec{i}_{ki}\) is the network current, \(\vec{v}_{ki}\) is the voltage across the PCC and \(\vec{v}_j\) is the voltage across the AC hub bus.

In time-domain form, the current differential equations can be defined as:

\[
\frac{d i_{kai}}{dt} = \frac{1}{l_{ki}} v_{kai} - \frac{1}{l_{ki}} v_{ja} - \frac{r_{ki}}{l_{ki}} i_{kai}
\] (2.20)

\[
\frac{d i_{kbi}}{dt} = \frac{1}{l_{ki}} v_{kbi} - \frac{1}{l_{ki}} v_{jb} - \frac{r_{ki}}{l_{ki}} i_{kbi}
\] (2.21)

\[
\frac{d i_{kci}}{dt} = \frac{1}{l_{ki}} v_{kci} - \frac{1}{l_{ki}} v_{jc} - \frac{r_{ki}}{l_{ki}} i_{kci}
\] (2.22)

The model can be represented in state space form, \(\dot{X}_p = A_3 X_p + B_3 U_p\), where:

\[
X_p = \begin{bmatrix}
i_{kai} & i_{kbi} & i_{kci}
\end{bmatrix}
\]

\[
U_p = \begin{bmatrix}
v_{kai} & v_{kbi} & v_{kci} & v_{jai} & v_{jbi} & v_{jci}
\end{bmatrix}
\] (2.23)
The state and input matrices are defined as:

\[
A_3 = \begin{bmatrix}
-\frac{r_{ki}}{l_{ki}} & 0 & 0 \\
0 & -\frac{r_{ki}}{l_{ki}} & 0 \\
0 & 0 & -\frac{r_{ki}}{l_{ki}} \\
\end{bmatrix}
\]

(2.24)

\[
B_3 = \begin{bmatrix}
\frac{1}{l_{ki}} & 0 & 0 & -\frac{1}{l_{ki}} & 0 & 0 \\
0 & \frac{1}{l_{ki}} & 0 & 0 & -\frac{1}{l_{ki}} & 0 \\
0 & 0 & \frac{1}{l_{ki}} & 0 & 0 & -\frac{1}{l_{ki}} \\
\end{bmatrix}
\]

(2.25)

### 2.4 Control Strategy

The grid-forming control scheme is employed within each offshore VSC in order to achieve adequate network operation. The control strategy is applied for the purpose of achieving two main objectives. Firstly, it is used to enable set-point tracking of the network frequency based on the reference frequencies. Secondly, the voltage at the corresponding AC bus of an offshore VSC is regulated according to the reference voltages provided to the control system. Both of the controlled variables can be adjusted independently, without having an impact on other system variables \[24,52\].

The applied control scheme is designed as shown in Fig. 2.4 \[53\]. It consists of an inner current controller and an outer voltage controller for each axis in the \(dq\) frame.

The measured values of the control system are based on the variables in Fig. 2.2, which can be defined in terms of their \(dq\) components as \(\tilde{v}_{si} = v_{tdi} + jv_{tqi}\), \(\tilde{i}_{si} = i_{sdi} + ji_{sqi}\), \(\tilde{v}_{si} = v_{sdi} + jv_{sqi}\) and \(\tilde{i}_{ni} = i_{ndi} + ji_{nqi}\), where the subscripts \(d\) and \(q\) denote the direct and quadrature components respectively \[53\]. The system variables are measured in the \(abc\) reference frame. Therefore, the control system involves the conversion of the variables from \(abc\) to the \(dq\) frame and then vice versa. The reference frequency, \(\omega_{ri}\) is used in the conversions. The network frequency tracks the reference value as a result of the presented control process.
2.4.1 Voltage Controller

The design of the outer voltage controller is as shown in Fig. 2.5. The purpose of the controller is to enable set-point tracking of the bus $dq$ voltages based on the provided reference values. This is achieved by employing the Proportional-Integral (PI) control loop feedback mechanism. The PI controller determines the difference between the measured and reference values of the $dq$ voltages and applies a responsive correction based on the proportional and integral control gains. The voltage controller generates $dq$ current values as a result of the control formulation. The dynamics of the controller can be described as:

$$i_{sdi}^* = (v_{sdi}^* - v_{sdi})(k_{opi} + \frac{k_{oii}}{s}) - i_{ndi} - \omega r_i f_i v_{sqi} \tag{2.26}$$

$$i_{sqi}^* = (v_{sqi}^* - v_{sqi})(k_{opi} + \frac{k_{oii}}{s}) - i_{nqi} + \omega r_i f_i v_{sdi} \tag{2.27}$$

where $i_{sdi}^*$ and $i_{sqi}^*$ are the outputs of the controller, $v_{sdi}$ and $v_{sqi}$ are the $dq$ components of the voltage measured at the VSC’s corresponding AC bus, $k_{opi}$ and $k_{oii}$ are the proportional and integral control gains, and $i_{ndi}$ and $i_{nqi}$ are the $dq$ components of the current flowing across the HVAC cable link.
2.4.2 Current Controller

The inner current controller is shown in Fig. 2.6. In this controller, the $dq$ currents produced by the voltage controller are used as the reference input values. For the purpose of achieving an improved set-point tracking performance of the overall control system, this control loop ensures that the bus $dq$ currents track the reference currents.
This is achieved using PI-based control as well. The current controller dynamics can be represented as:

\[ v_{tdi} = (i_{sdi}^* - i_{sdi}) (k_{pi} + \frac{k_{ii}}{s}) + v_{sdi} - \omega_{ri} L_i i_{sqi} \]  
\[ (2.28) \]

\[ v_{tqi} = (i_{sqi}^* - i_{sqi}) (k_{pi} + \frac{k_{ii}}{s}) + v_{sqi} + \omega_{ri} L_i i_{sdi} \]  
\[ (2.29) \]

where \( v_{tdi} \) and \( v_{tqi} \) are the outputs of the controller, \( i_{sdi} \) and \( i_{sqi} \) are the \( dq \) components of the current flowing across the VSC, and \( k_{pi} \) and \( k_{ii} \) are the proportional and integral control gains.

### 2.5 Conclusion

The design of the considered system has been introduced in this chapter. The time-domain model of the system, considering a constant network frequency, has been derived during the course of this chapter. The control system for a VSC in grid-forming mode, along with its dynamics and control objectives, has also been presented.
Chapter 3  
System Simulations considering Detailed VSC Models

3.1 Introduction

The purpose of this chapter is to validate the proposed design and control of the offshore AC network. To this end, simulations are performed on MATLAB Simscape Power Systems considering the detailed model for the VSCs.

3.2 Case Studies of System Configuration

The simulation model of the system will be developed based on the research objectives of this thesis. Since the study is focused on the offshore AC network stability, the DC side of the system is not considered in the model. It can be assumed that the DC voltage control of the onshore VSC ensures the stability of the DC side, with its parameters set within their operational limits [27]. Considering this assumption, the simulation model includes two DC voltage sources, with amplitude of 225 kV each, connected to the DC side of the offshore VSC.

Due to the employment of the type IV wind turbines for the offshore WPPs, it can be assumed that their active and reactive power is controlled independently [27]. Therefore, the dynamics of the wind turbine are not included in the WPP model. Controlled current sources, synchronized with the network frequency, are used to represent the equivalent of the interconnected WPPs.

The network configuration of the test system is shown in Fig. 3.1. It consists
of two VSCs and the equivalent of the WPPs. The offshore VSCs are connected to the AC hub bus using HVAC cables. Three pi section lines are used to model each HVAC connection. The network and control parameters for the study are provided in Appendix .1. The proportional and integral gain values for the controllers are determined using the trial-and-error method. A phase-locked loop (PLL) is employed to measure the frequency response of the network.

Both VSCs are configured with the same operating parameters and equal reference values. Furthermore, both HVAC cables have the same line parameters. Under such considerations, the VSC controlled buses will have identical responses. Therefore, it would be sufficient to analyze the voltage response of either one of the buses.

![Test System for Case Studies](image)

**Figure 3.1.** Test System for Case Studies

### 3.2.1 Step change in reference voltages

The independent voltage control aspect of the proposed control scheme is investigated in this case. To this end, the system is simulated with a stepped change in the VSC $dq$ voltages. The reference value for the direct voltages of the VSCs is stepped down from 220 kV to 150 kV at $t = 3.0$ s, whereas the quadrature voltages’ reference value is stepped up from 0 to 50 kV at $t = 6.0$ s. The reference frequency for each VSC is set to 50 Hz.
The $dq$ voltages at the corresponding AC busbar of VSC$_1$, and the network frequency response are provided in Fig. 3.2. It can be observed that the $dq$ voltages track their reference input values. The voltage adjustments cause minimal transients in the system response, which are quickly cleared in order to maintain the control.

![Network response involving step change in VSC reference voltages](image)

Figure 3.2. Network response involving step change in VSC reference voltages

### 3.2.2 Step change in reference frequencies

In this case study, the independent frequency control feature of the system is analyzed. The control validation is performed by simulating the system with the reference frequencies of the VSCs stepped up from 50 Hz to 55 Hz at $t = 7.0$ s. The $dq$ voltages are provided with reference input values of 220 kV and 0 respectively.
The frequency response of the network, and the \( dq \) voltages of VSC\(_1\) are measured as shown in Fig. 3.3. The set-point tracking control of the network frequency is achieved. The response has an acceptable rise time and a negligible overshoot. The control is achieved without affecting the rest of the system variables. The regulation of the \( dq \) voltages is maintained despite the change in network frequency.

![Figure 3.3. Network response involving step change in VSC reference frequencies](image)

**Figure 3.3.** Network response involving step change in VSC reference frequencies

### 3.2.3 Injection of WPP currents

In this case, the system response is analyzed by applying a step change in the WPP active current from 0 to 2000 A at \( t = 7.0 \) s, and then stepping up the WPP inductive reactive current from 0 to 1500 A at \( t = 10.0 \) s. In order to regulate the voltages of the VSC controlled busbars, the reference \( dq \) voltages for each VSC are set at 220 kV.
and 0 respectively. For the purpose of regulating the network frequency, a reference frequency of 50 Hz is provided to each VSC.

The network frequency and the $dq$ voltages of VSC$_1$ are measured as provided in Fig. 3.4. It can be observed that the regulation of the frequency and voltages is maintained despite the injection of the wind currents.

**Figure 3.4.** Network response involving wind current injections
3.3 Conclusion

A simulation-based study of the considered system has been performed using the VSCs’ detailed models. From the results of these studies, it is concluded that the system provides adequate operation and control of the offshore network. Set-point tracking of the network frequency and VSC voltages was maintained despite the sudden injection of large wind currents.


4.1 Introduction

The DP theory will be introduced in this chapter. Key properties of the proposed modeling technique will be covered here as well. The derivation of the DP-based model of the system will be performed. Finally, the complete DP-based state space representation of the system will be presented towards the end of this chapter.

4.2 Dynamic Phasor Theory

The proposed DP concept is based on the generalized averaging theory [54]. DPs are the time-varying coefficients of a time domain-based signal’s Fourier Series (FS) representation. In the FS concept, a signal is decomposed into a series of frequency components. From this series, the higher order harmonics can be truncated, leaving behind only the significant DPs [55]. These are sufficient to simulate the dynamic behavior of the original abstruse model [56]. Therefore, the DP-based model may be simulated with lower computation and memory capacities while maintaining good accuracy [57].

For the time interval, \( \tau \in (t - T, t] \), a complex time-domain waveform, \( x(\tau) \) can be represented by its Fourier series as:

\[
x = \sum_{k=-\infty}^{\infty} X_k(t)e^{jk\omega\tau}
\] (4.1)
From (4.1), \( \omega \) is the frequency of the signal which can be represented as \( \frac{2\pi}{T} \). The \( X_k(t) \) term is the time-varying Fourier coefficient, which has been defined earlier as the DP. At the time instant, \( t \), the \( k \)th DP can be determined as:

\[
X_k(t) = \frac{1}{T} \int_t^{t+T} x(\tau)e^{-jk\omega \tau} d\tau = \langle x \rangle_k(t)
\]  

(4.2)

The angled brackets are used to show that the variable is in phasor form.

Since the DP is a complex variable, it can be represented as:

\[
\langle x \rangle_k = \langle x \rangle_k^r + j\langle x \rangle^i_k = \langle x \rangle^*_{-k} = (\langle x \rangle^r_{-k} + j\langle x \rangle^i_{-k})^*
\]  

(4.3)

In (4.3), \( r \) represents the real component of the phasor and \( i \) represents the imaginary part. The asterisk denotes the complex conjugate of the phasor.

The DP-based representation of a complex time-domain waveform can be derived by applying a key property of DPs. This pertains to the relationship between the derivatives of the variable \( x(\tau) \), and the derivatives of the \( k \)th Fourier coefficient, which can be described as:

\[
\langle \frac{dx}{dt} \rangle_k(t) = \frac{dX_k}{dt}(t) + j\omega X_k(t)
\]  

(4.4)

The differential term on the right side of (4.4) represents the transient dynamic of the variable. The differential property of traditional phasors does not include this term. Thus, the key advantage of DPs is their ability to model a signal’s dynamic characteristics.

### 4.3 Dynamic Phasor Modeling of Considered System

The DP-based model of the system is developed based on the presented DP properties. For simplicity of the analysis, the modeling process will assume balanced conditions of the considered system. Under this consideration, the AC variables of the system can be accurately described by their respective fundamental frequency components \( (k = 1) \) [50]. When assuming balanced conditions, the relationships between the variable phases, \( x_{abc} \) can be described as:

\[
x_b = x_a e^{-j\frac{2\pi}{3}}
\]  

(4.5)
\[ x_c = x_a e^{-j \frac{4\pi}{3}} \]  

(4.6)

Therefore, any equations describing the dynamics of the variables will provide the same magnitude for all of its three phases. Thus, it would be sufficient to derive the DP-based model of the system based on any single phase. In this research, the variables’ phase \( a \) is considered as the reference phase. It can be represented in terms of its real and imaginary parts as:

\[ x_a = x_{a}^r + j x_{a}^i \]  

(4.7)

According to (4.3), the DP of \( x_a \) can be represented in terms of its real and imaginary parts as:

\[ \langle x_a \rangle_1 = \langle x_{a}^r \rangle_1 + j \langle x_{a}^i \rangle_1 \]  

(4.8)

Based on (4.4), the relationship between the derivatives of \( x_a \), and those of \( \langle x_a \rangle_1 \), can be described as:

\[ \langle \frac{dx_a}{dt} \rangle_1 = \frac{d\langle (x_a^r + j x_a^i) \rangle_1}{dt} + j \omega \langle x_a^r + j x_a^i \rangle_1 \]  

(4.9)

The proposed modeling process to describe all the system variables will be based on (4.8) and (4.9).

The simulation-based study in the previous chapter involved the analysis of the responses of variables in the \( dq \) frame. The relationship between the variables of the detailed models and their DPs is determined in order to compare the responses of these two modeling approaches. This would also simplify the derivations of the DPs of the variables represented in the \( dq \) reference frame.

A variable in space phasor form, \( \vec{x} \), is considered for the purpose of establishing the aforementioned relationship. It can be represented in terms of its \( dq \) components as [53]:

\[ \vec{x} = x_d + j x_q \]  

(4.10)
The differentiation of the variable is related to that of its $dq$ components as [53]:

\[
\frac{d\vec{x}}{dt} = \frac{d(x_d + jx_q)}{dt} + j\omega(x_d + jx_q)
\]  

(4.11)

Based on these characteristics, the complete method to convert a variable, represented in terms of its direct and quadrature components, $y_{dq}$, to the space phasor form and then finally into DPs can be performed as:

\[
y_d + jy_q = \vec{y}
\]  

(4.12)

\[
\vec{y} \Rightarrow \langle y_a \rangle_1
\]  

(4.13)

\[
\langle y_a \rangle_1 = \langle y'_a \rangle_1 + j\langle y^i_a \rangle_1
\]  

(4.14)

For variables in the first order differential form, the derivation can be performed as:

\[
\frac{d(y_d + jy_q)}{dt} + j\omega(y_d + jy_q) = \frac{d\vec{y}}{dt}
\]  

(4.15)

\[
\frac{d\vec{y}}{dt} \Rightarrow \langle \frac{dy_a}{dt} \rangle_1
\]  

(4.16)

\[
\langle \frac{dy_a}{dt} \rangle_1 = \frac{d(\langle y'_a \rangle_1 + j\langle y^i_a \rangle_1)}{dt} + j\omega(\langle y'_a \rangle_1 + j\langle y^i_a \rangle_1)
\]  

(4.17)

In these steps, the DPs of the variables represented in the $dq$ reference frame are derived as:

\[
y_d + jy_q \Rightarrow \langle y'_a \rangle_1 + j\langle y^i_a \rangle_1
\]  

(4.18)

\[
\frac{d(y_d + jy_q)}{dt} + j\omega(y_d + jy_q) \Rightarrow \frac{d(\langle y'_a \rangle_1 + j\langle y^i_a \rangle_1)}{dt} + j\omega(\langle y'_a \rangle_1 + j\langle y^i_a \rangle_1)
\]  

(4.19)

Therefore, the $dq$ components of the detailed model’s variables can be converted to their DPs as:

\[
y_d \Rightarrow \langle y'_a \rangle_1
\]  

(4.20)

\[
y_q \Rightarrow \langle y^i_a \rangle_1
\]  

(4.21)
4.3.1 VSC DP Model

Based on the presented DP-based modeling process, (2.4) and (2.7) can be converted to DPs as:

\[
\frac{d\langle isai \rangle}{dt} = \frac{1}{L_i} \langle v_{tai} \rangle - \frac{1}{L_i} \langle v_{sai} \rangle - \frac{R_i}{L_i} \langle i_{sai} \rangle - j\omega \langle i_{sai} \rangle \tag{4.22}
\]

\[
\frac{d\langle v_{sai} \rangle}{dt} = \frac{1}{c_{fi}} \langle i_{nai} \rangle + \frac{1}{c_{fi}} \langle i_{sai} \rangle - j\omega \langle v_{sai} \rangle \tag{4.23}
\]

The real and imaginary components of (4.22) are defined in (4.24) and (4.25), whereas those of (4.23) are defined in (4.26) and (4.27):

\[
\frac{d\langle i^r_{sai} \rangle}{dt} = \frac{1}{L_i} \langle v^r_{tai} \rangle - \frac{1}{L_i} \langle v^r_{sai} \rangle - \frac{R_i}{L_i} \langle i^r_{sai} \rangle + \omega \langle i^i_{sai} \rangle \tag{4.24}
\]

\[
\frac{d\langle i^i_{sai} \rangle}{dt} = \frac{1}{L_i} \langle v^i_{tai} \rangle - \frac{1}{L_i} \langle v^i_{sai} \rangle - \frac{R_i}{L_i} \langle i^i_{sai} \rangle - \omega \langle i^r_{sai} \rangle \tag{4.25}
\]

\[
\frac{d\langle v^r_{sai} \rangle}{dt} = \frac{1}{c_{fi}} \langle i^r_{nai} \rangle + \frac{1}{c_{fi}} \langle i^r_{sai} \rangle + \omega \langle v^i_{sai} \rangle \tag{4.26}
\]

\[
\frac{d\langle v^i_{sai} \rangle}{dt} = \frac{1}{c_{fi}} \langle i^i_{nai} \rangle + \frac{1}{c_{fi}} \langle i^i_{sai} \rangle - \omega \langle v^r_{sai} \rangle \tag{4.27}
\]

The VSC DP model can be represented in state space form: \( \dot{X}_{dvi} = A_4 X_{dvi} + B_4 U_{dvi} \), where:

\[
X_{dvi} = \begin{bmatrix} \langle i^r_{sai} \rangle & \langle i^i_{sai} \rangle & \langle v^r_{sai} \rangle & \langle v^i_{sai} \rangle \end{bmatrix}
\]

\[
U_{dvi} = \begin{bmatrix} \langle v^r_{tai} \rangle & \langle v^i_{tai} \rangle & \langle i^r_{nai} \rangle & \langle i^i_{nai} \rangle \end{bmatrix}
\]
The state and input matrices of the state space representation can be defined as:

\[ A_4 = \begin{bmatrix} \frac{R_i}{L_i} & \omega & -\frac{1}{L_i} & 0 \\ -\omega & -\frac{R_i}{L_i} & 0 & -\frac{1}{L_i} \\ \frac{1}{c_{fi}} & 0 & 0 & \omega \\ 0 & \frac{1}{c_{fi}} & -\omega & 0 \end{bmatrix} \]  

(4.29)

\[ B_4 = \begin{bmatrix} \frac{1}{L_i} & 0 & 0 & 0 \\ 0 & \frac{1}{L_i} & 0 & 0 \\ 0 & 0 & \frac{1}{c_{fi}} & 0 \\ 0 & 0 & 0 & \frac{1}{c_{fi}} \end{bmatrix} \]  

(4.30)

### 4.3.2 Offshore AC Network DP Model

The DP-based model of the offshore AC network will also be derived on the basis of the modeling techniques presented earlier in the chapter.

#### 4.3.2.1 AC Hub Bus Voltage

According to the DP characteristics, (2.13) can be converted to its DP form as:

\[
\frac{d\langle v_{ja}\rangle_1}{dt} = \frac{1}{c_1 + c_2} \langle i_{wa}\rangle_1 + \frac{1}{c_1 + c_2} \langle i_{ka1}\rangle_1 + \frac{1}{c_1 + c_2} \langle i_{ka2}\rangle_1 + \omega \langle v_{ja}\rangle_1
\]

\[ -j\omega \langle v_{ja}\rangle_1 \]  

(4.31)

The real and imaginary parts of (4.31) are presented in (4.32) and (4.33) respectively:

\[
\frac{d\langle v_{jar}\rangle_1}{dt} = \frac{1}{c_1 + c_2} \langle i_{wa}^r\rangle_1 + \frac{1}{c_1 + c_2} \langle i_{ka1}^r\rangle_1 + \frac{1}{c_1 + c_2} \langle i_{ka2}^r\rangle_1 + \omega \langle v_{ja}\rangle_1
\]

(4.32)
\[
\frac{d\langle v_{ja}^i \rangle_1}{dt} = \frac{1}{c_1^* + c_2^*} \langle i_{wa}^i \rangle_1 + \frac{1}{c_1^* + c_2^*} \langle i_{ka1}^i \rangle_1 + \frac{1}{c_1^* + c_2^*} \langle i_{ka2}^i \rangle_1 - \omega \langle v_{ja}^r \rangle_1
\] (4.33)

The DP equations describing the AC hub bus voltage can be represented in state space form, \( \dot{X}_{dg} = A_5 X_{dg} + B_5 U_{dg} \), where:

\[
X_{dg} = \begin{bmatrix} \langle v_{ja}^r \rangle_1 & \langle v_{ja}^i \rangle_1 \end{bmatrix}
\]

\[
U_{dg} = \begin{bmatrix} \langle i_{wa}^r \rangle_1 & \langle i_{wa}^i \rangle_1 & \langle i_{ka1}^r \rangle_1 & \langle i_{ka1}^i \rangle_1 & \langle i_{ka2}^r \rangle_1 & \langle i_{ka2}^i \rangle_1 \end{bmatrix}
\] (4.34)

The state and input matrices of this model are:

\[
A_5 = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}
\] (4.35)

\[
B_5 = \begin{bmatrix} B_{41} & 0 & B_{42} & 0 & B_{43} & 0 \\ 0 & B_{51} & 0 & B_{52} & 0 & B_{53} \end{bmatrix}
\] (4.36)

where,

\[
B_{41} = B_{42} = B_{43} = \frac{1}{c_1^* + c_2^*}
\]

\[
B_{51} = B_{52} = B_{53} = \frac{1}{c_1^* + c_2^*}
\]

4.3.2.2 Branch Currents

The DP properties defining the proposed modeling process can be applied to convert (2.20) to its DP-based form:

\[
\frac{d\langle i_{kai} \rangle_1}{dt} = \frac{1}{l_{ki}} \langle v_{kai} \rangle_1 - \frac{1}{l_{ki}} \langle v_{ja} \rangle_1 - \frac{r_{ki}}{l_{ki}} \langle i_{kai} \rangle_1 - j\omega \langle i_{kai} \rangle_1
\] (4.37)

The real and imaginary parts of the derived equation are defined in (4.38) and (4.39):

\[
\frac{d\langle \dot{i}_{kai} \rangle_1}{dt} = \frac{1}{l_{ki}} \langle \dot{v}_{kai} \rangle_1 - \frac{1}{l_{ki}} \langle \dot{v}_{ja} \rangle_1 - \frac{r_{ki}}{l_{ki}} \langle \dot{i}_{kai} \rangle_1 + \omega \langle \dot{i}_{kai} \rangle_1
\] (4.38)

\[
\frac{d\langle \ddot{i}_{kai} \rangle_1}{dt} = \frac{1}{l_{ki}} \langle \ddot{v}_{kai} \rangle_1 - \frac{1}{l_{ki}} \langle \ddot{v}_{ja} \rangle_1 - \frac{r_{ki}}{l_{ki}} \langle \ddot{i}_{kai} \rangle_1 - \omega \langle \ddot{i}_{kai} \rangle_1
\] (4.39)
These equations can be presented in state space form, \( \dot{X}_{dp} = A_{6}X_{dp} + B_{6}U_{dp} \), where:

\[
X_{dp} = \begin{bmatrix} \langle i_{r kai}^r \rangle_1 & \langle i_{kai}^i \rangle_1 \end{bmatrix}
\]

\[
U_{dp} = \begin{bmatrix} \langle v_{kai}^r \rangle_1 & \langle v_{kai}^i \rangle_1 & \langle v_{ja}^r \rangle_1 & \langle v_{ja}^i \rangle_1 \end{bmatrix}
\]

(4.40)

The state and input matrices of this model are defined as:

\[
A_{6} = \begin{bmatrix} -\frac{r_{ki}}{l_{ki}} & \omega \\ -\omega & -\frac{r_{ki}}{l_{ki}} \end{bmatrix}
\]

(4.41)

\[
B_{6} = \begin{bmatrix} \frac{1}{l_{ki}} & 0 & -\frac{1}{l_{ki}} & 0 \\ 0 & 1 & 0 & -\frac{1}{l_{ki}} \end{bmatrix}
\]

(4.42)

### 4.3.3 Grid-Forming Control in DP form

The grid-forming control scheme will be defined in DP form as well. The derivations of the control equations will be performed based on the DP characteristics applied throughout this chapter.

#### 4.3.3.1 Voltage Controller

The dynamics of the voltage controller have been defined in (2.26) and (2.27). The DPs of those equations can be derived as:

\[
\langle i_{sai}^{r*} \rangle_1 = (\langle v_{sai}^{r*} \rangle_1 - \langle v_{sai}^r \rangle_1)(k_{opi} + \frac{k_{oii}}{s}) - \langle i_{nai}^r \rangle_1 - \omega_{ri}c_{fi} \langle v_{sai}^i \rangle_1
\]

(4.43)

\[
\langle i_{sai}^{i*} \rangle_1 = (\langle v_{sai}^{i*} \rangle_1 - \langle v_{sai}^i \rangle_1)(k_{opi} + \frac{k_{oii}}{s}) - \langle i_{nai}^i \rangle_1 + \omega_{ri}c_{fi} \langle v_{sai}^r \rangle_1
\]

(4.44)

The design of the voltage controller in DP form is presented in Fig. 4.1.
4.3.3.2 Current Controller

The DP form of the current control equations, defined in (7.3) and (7.4), can be derived as:

\[
\langle v_{tai}^r \rangle_1 = (\langle i_{sai}^* \rangle_1 - \langle i_{sai} \rangle_1)(k_{pi} + \frac{k_{ii}}{s}) + \langle v_{sai}^r \rangle_1 - \omega_r L_i \langle i_{sai}^i \rangle_1 \tag{4.45}
\]

\[
\langle v_{tai}^i \rangle_1 = (\langle i_{sai}^* \rangle_1 - \langle i_{sai} \rangle_1)(k_{pi} + \frac{k_{ii}}{s}) + \langle v_{sai}^i \rangle_1 + \omega_r L_i \langle i_{sai}^r \rangle_1 \tag{4.46}
\]

The DP-based current controller design is shown in Fig. 4.2.
The DPs of the grid-forming control system have been derived in this section. The presented equations can be used to describe the control of the DP-based variables of the considered system. The design of the overall control scheme in DP form is provided in Fig. 4.3.

Figure 4.2. Current controller in DP form

Figure 4.3. DP-based design of grid-forming control scheme
4.4 Conclusion

In this chapter, the key characteristics of the proposed modeling technique have been introduced. The DP-based model of the system has been derived. The grid-forming control scheme has also been defined in terms of DPs.
Chapter 5
Simulation-based Validation of Proposed Model

5.1 Introduction

The purpose of this chapter is to perform simulation-based studies of the DP-based model of the considered system. The system responses of the proposed model will be compared to those of the detailed model. The analysis will assess the accuracy of the proposed model and deduce the reduction in the real time taken for performing the simulations.

5.2 Comparative Analysis

In order to determine the accuracy of the proposed model, the responses of the variables of the detailed model will be compared with the responses of their DPs. The system configuration, and the network and control parameters used for the simulation-based studies of the detailed model will be kept unchanged for the simulations of the proposed model. The network frequency will be calculated using (2.1). The DP-based model will be investigated with the same case studies considered for the detailed model of the system. Therefore, the system responses of the detailed model, reported in the third chapter, will serve as the reference for the comparisons. The DP-based model will be simulated using only MATLAB Simulink components, whereas the Simscape PowerSystems tool of MATLAB Simulink was used to simulate the detailed model.
5.2.1 Step change in reference voltages

The voltage step response of the considered system is simulated using the DP-based model. The reference values of both the VSCs’ direct voltages are stepped down from 220 kV to 150 kV at $t = 3.0$ s, whereas the reference quadrature voltages of the VSCs are stepped up from 0 to 50 kV at $t = 6.0$ s. The reference frequencies of the VSCs are kept at 50 Hz. The simulation results include the network frequency response, and the $dq$ voltage responses at the corresponding busbar of VSC$_1$. They are presented in Fig. 5.1, along with the results from the detailed model. The transient responses of the DP-based model have a comparatively lower rise time. The steady state responses of both models are alike.

![Graphs showing system response](image)

**Figure 5.1.** System response involving step change in VSC reference voltages
5.2.2 Step change in reference frequencies

This case analytically compares the models’ system responses emanating from a step change in the reference frequencies. The DP-based model is simulated with the reference frequencies stepped up from 50 Hz to 55 Hz at \( t = 7.0 \) s. The reference \( dq \) voltages are set at 220 kV and 0 respectively. The network frequency and the \( dq \) voltage responses of VSC\(_1\) are presented in Fig. 5.2, where the respective responses from the detailed model are also included. The transient response of the network frequency has a higher rise time with the detailed model, whereas the steady state response of the variable is similar with both modeling approaches. The dynamic and steady state voltage responses are alike for both models.

![Diagram](image)

**Figure 5.2.** System response involving step change in VSC reference frequencies
5.2.3 Injection of WPP currents

In this case study, the system responses to wind current injections are simulated with the proposed model. A step change in the WPP active current from 0 to 2000 A is applied at $t = 7.0$ s, whereas the WPP inductive reactive current is stepped up from 0 to 1000 A at $t = 10.0$ s. The $dq$ voltages of both VSCs are provided with reference values of 220 kV and 0 respectively. The reference frequencies of the VSCs are set at 50 Hz. In Fig. 5.3, the simulation results of the considered models are provided, which include the network frequency and $dq$ voltage responses of VSC$_1$. It is observed that both modeling approaches provide similar system responses.
A detailed comparison of simulation times for both the detailed and DP-based models is presented in Table 5.1. The DP-based model simulates relatively faster as compared to the detailed model.
Table 5.1. Comparison of computation time between two models

<table>
<thead>
<tr>
<th>Modeling Approach</th>
<th>Case Study I</th>
<th>Case Study II</th>
<th>Case Study III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Detailed</td>
<td>DP</td>
<td>Detailed</td>
</tr>
<tr>
<td>Simulation time (Second)</td>
<td>82.63</td>
<td>19.98</td>
<td>94.42</td>
</tr>
<tr>
<td>Acceleration factor</td>
<td>1</td>
<td>4.14</td>
<td>1</td>
</tr>
</tbody>
</table>

5.3 Conclusion

In this chapter, the system responses were simulated with the proposed DP-based modeling approach. The results of the simulations were sufficiently accurate. Furthermore, the DP-based model was shown to simulate relatively faster than the detailed model. Therefore, it can be concluded that the proposed model has provided an accurate and efficient alternative to simulating the response of the offshore network.
Chapter 6  
Stability Studies of the Considered System

6.1 Introduction

In this chapter, the stability of the offshore AC network will be analyzed. The impact of varying the network parameters and controller gains on the stability of the system will be investigated. This will involve an eigenvalue analysis with the DP-based representation of the considered system. The results of the analysis will then be validated with simulations of the system’s detailed model.

6.2 Stability Studies

The system is analyzed under varied parameter/gain values. The system eigenvalues are computed for each case to assess the system stability. The system is concluded to be unstable if any of its eigenvalues has a positive real part. Otherwise, the system is regarded to be stable. The response of the system is simulated to validate the analysis.

The case studies are performed with the test system presented in the second chapter. The DP model of each VSC includes four states. The DP-based control scheme of each VSC is represented with four states as well. Since two VSCs are considered in the test system, there are a total of sixteen states based on the VSCs. In the offshore network, two states are used to describe the AC hub bus voltage. The current of an offshore branch is modeled with two states as well. The test system
includes a single AC hub bus and two HVAC branches. Therefore, the offshore network model includes a total of six states. The complete system model, inclusive of the VSCs and the offshore network, has twenty-two states to describe its dynamics.

The simulation model based on the DP-based representation of the system was developed in the previous chapter. Based on this model, the `linmod` function is entered in the MATLAB command window to obtain the state space model of the system. Then, the `eig` command is applied, with the A matrix as its argument, to compute the eigenvalues of the system. The damping ratio of an eigenvalue, $\lambda = \sigma \pm j\omega$ can be calculated as $\zeta = -\sigma / \sqrt{\sigma^2 + \omega^2}$ [58]. The system eigenvalues, along with their frequencies and damping ratios are listed in Table 6.1. The system can be concluded to be stable since all of the eigenvalues have negative real parts. This was observed in the stable system responses from the simulation results of the third and fifth chapters.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>Value</th>
<th>Frequency(Hz)</th>
<th>Damping Ratio(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$ &amp; $\lambda_2$</td>
<td>$-39.25 \pm 24766.84i$</td>
<td>3941.76</td>
<td>0.158</td>
</tr>
<tr>
<td>$\lambda_3$ &amp; $\lambda_4$</td>
<td>$-39.25 \pm 24138.52i$</td>
<td>3841.76</td>
<td>0.163</td>
</tr>
<tr>
<td>$\lambda_5$ &amp; $\lambda_6$</td>
<td>$-37.24 \pm 18042.77i$</td>
<td>2871.60</td>
<td>0.206</td>
</tr>
<tr>
<td>$\lambda_7$ &amp; $\lambda_8$</td>
<td>$-37.24 \pm 17414.46i$</td>
<td>2771.60</td>
<td>0.214</td>
</tr>
<tr>
<td>$\lambda_9$ &amp; $\lambda_{10}$</td>
<td>$-0.15 \pm 3552.68i$</td>
<td>565.43</td>
<td>0.00426</td>
</tr>
<tr>
<td>$\lambda_{11}$ &amp; $\lambda_{12}$</td>
<td>$-0.024 \pm 2924.36i$</td>
<td>465.43</td>
<td>0.000825</td>
</tr>
<tr>
<td>$\lambda_{13}$ &amp; $\lambda_{14}$</td>
<td>$-5.44 \pm 314.18i$</td>
<td>50.00</td>
<td>1.73</td>
</tr>
<tr>
<td>$\lambda_{15}$ &amp; $\lambda_{16}$</td>
<td>$-0.0019 \pm 0.39i$</td>
<td>0.062</td>
<td>0.493</td>
</tr>
<tr>
<td>$\lambda_{17}$ &amp; $\lambda_{18}$</td>
<td>$-0.088 \pm 0.37i$</td>
<td>0.058</td>
<td>23.48</td>
</tr>
<tr>
<td>$\lambda_{19}$ &amp; $\lambda_{20}$</td>
<td>$-0.67 \pm 1.28i$</td>
<td>0.20</td>
<td>46.09</td>
</tr>
<tr>
<td>$\lambda_{21}$ &amp; $\lambda_{22}$</td>
<td>$-0.67 \pm 1.28i$</td>
<td>0.20</td>
<td>46.09</td>
</tr>
</tbody>
</table>

6.2.1 Network stability under increased HVAC cable length

In this case study, the length of the offshore HVAC cables is increased to determine its impact on the system stability. As a result of this variation, the system eigenvalues, $\lambda_{11}$ to $\lambda_{18}$, are displaced from their positions on the s-plane. The movement of these eigenvalues is shown in Fig. 6.1.
It can be observed that some of the eigenvalues shift from the left side of the imaginary axis to its right side. Therefore, it can be concluded that the system becomes unstable as a result of this variation. In order to validate this conclusion, the system is simulated with the length of the cables increased from 10 km to 2500 km at \( t = 11.0 \) s. The \( dq \) voltage responses of VSC\(_1\) as well as the network frequency response are presented in Fig. 6.2. The network instability is reflected in the responses.
6.2.2 Network stability under increased filter capacitance

The filter capacitance is increased in this study to analyze the resulting effect on system stability. The trajectory of the eigenvalues, $\lambda_{11}$ to $\lambda_{14}$, is plotted in Fig. 6.3.
Figure 6.3. Shift in eigenvalue positions due to increased filter capacitance

It is observed that the new eigenvalues include those with positive real parts. Therefore, the system has been destabilized. This analysis is validated by simulating the system with the filter capacitance stepped up from 0.00329 $\mu$F to 0.165 mF at $t = 11.0$ s. The system responses are presented in Fig. 6.4. The simulation results validate the analysis.
6.2.3 Network stability under increased VSC reactor inductance

This case study analyzes the system stability under increased inductance of the VSC reactors \((L_i)\). The trajectories of \(\lambda_{11}\) and \(\lambda_{12}\) emanating from this variation are provided in Fig. 6.5.
Figure 6.5. Shift in eigenvalue positions due to increased VSC reactor inductance

It can be noted that the pair of eigenvalues are shifted to the right side of the imaginary axis. Thus, it is concluded that the system has become unstable. The system response is simulated for validating the analysis. A step change in the reactors’ inductance from 55 mH to 100 mH is applied at \( t = 11.0 \) s. In Fig. 6.6, the network frequency and \( dq \) voltage responses of VSC\(_1\) are included. It can be observed that the variation causes network instability.
6.2.4 Network stability under tuned current control gains

The influence of tuning the current controller gains is investigated in this study. Firstly, the increase of the proportional gain is analyzed. The system eigenvalues are shifted along the complex plane due to the increase in the gain value. The eigenvalues’ trajectories are shown in Fig. 6.7.

Figure 6.6. Network response under increased reactor inductance
The system eigenvalues are shown to move towards the imaginary axis. Therefore, the stability of the system is decreased. However, the system has not become unstable since none of the eigenvalues have been shifted to the right side of the imaginary axis. In order to verify this, the system is simulated with the gain value stepped up from 0.1 to 5 at $t = 5.0$ s. The simulation results are shown in Fig. 6.8. The change in the gain value has caused a transient response of the system. Eventually, the transients are cleared and the system reaches steady state. Although the system response maintains a steady-state error, it does not reflect network instability.
The second part of this study involves the tuning of the considered controller’s integral gain value. An increase in this gain causes the system eigenvalues, $\lambda_9$, $\lambda_{10}$, $\lambda_{11}$ and $\lambda_{12}$ to move along the s-plane as shown in Fig. 6.9.

**Figure 6.8.** Network response under increased current controller proportional gain
Figure 6.9. Shift in eigenvalue positions due to increased current controller integral gain

It can be observed that these eigenvalues are shifted to the right side of the imaginary axis. Therefore, it can be concluded that the system has become destabilized. This is verified by simulating the system with the gain value stepped up from 200 to 8000 at t = 5.0 s. The system responses are provided in Fig. 6.10. Because of the change, the system becomes unstable. Therefore, the simulation results are in agreement with the analysis results.
6.2.5 Network stability under tuned voltage control gains

This case study investigates the impact of tuning the voltage controller gains. The influence of increasing the proportional gain value is analyzed first. This change results in the movement of eigenvalues, $\lambda_9$, $\lambda_{10}$, $\lambda_{11}$ and $\lambda_{12}$ along the s-plane. The trajectories of these eigenvalues are as presented in Fig. 6.11.

Figure 6.10. Network response under increased current controller integral gain
Figure 6.11. Shift in eigenvalue positions due to increased voltage controller proportional gains

The eigenvalues have been shifted to the right side of the imaginary axis. Thus, the analysis suggests system instability due to the gain adjustment. For validation, a simulation of the system is performed, in which the gain is stepped up from 7 to 100 at \( t = 11.0 \) s. The responses of the system are provided in Fig. 6.12. The network instability is reflected in these results.
Finally, the impact of varying the integral gain of the voltage controller is investigated. The increase in this gain results in movement of the system eigenvalues along the s-plane. This is presented in Fig. 6.13.
It is noted that the real parts of some eigenvalues become positive. Consequently, the system is deemed to have become unstable. In order to verify this, the system is simulated with the voltage integral gain stepped up from 11 to 440 at $t = 11.0$ s. The results of the simulation are presented in Fig. 6.14. The conclusion of the analysis is validated based on these results.
6.3 Conclusion

Various stability studies of the offshore AC network have been undertaken in this chapter. The system eigenvalue analysis was performed using its DP-based representation. In order to verify each analysis, simulations of the system detailed model were performed. Based on the results of the case studies, it is concluded that the proposed model can be used to accurately investigate the stability of the system. Furthermore, it was observed that the length of the HVAC cables, capacitance of the filters, inductance of the VSC reactors and the gains of the current and voltage controllers are required to be configured within their range of stable operation. An unreasonable setting of any of these parameters may affect the network stability.
Chapter 7  
Modeling and Control of System with Variable Network Frequency

7.1 Introduction

This chapter will introduce the modeling and control of the system having a variable network frequency. The purpose of the modeling in this chapter is to enable the use of droop controllers for the considered system, since the network frequency becomes a variable when frequency droop control is enabled. The design of the system, which was presented in the second chapter, will be considered for the modeling process in this chapter as well. The primary and droop control schemes for the system will be discussed. The model and control schemes of the system are based on the small-signal derivation of the offshore AC network presented in [27].

7.2 Primary Control Strategy

For the purpose of enabling adequate network operation, the employed control scheme of the system is based on the grid-forming control strategy. It is presented in Fig. 7.1. In the control method, there are a few modifications as compared to scheme introduced in the second chapter. The proposed voltage controllers have the following dynamics:

\[ i_{sdi}^* = (v_{sdi}^* - v_{sdi})(k_{opi} + \frac{k_{vii}}{s}) - i_{ndi} - \omega_c f_i v_{sqi}(\tau_i s + 1) \] (7.1)
\[
\dot{i}_{sqi} = (v_{sqi}^* - v_{sqi})(k_{opi} + \frac{k_{oii}}{s}) - i_{nqi} + \omega c_f i_{sdi}(\tau_i s + 1) \quad (7.2)
\]

where \(\tau_i\) is the time constant of the current response. The dynamics of the proposed current controllers are:

\[
v_{tdi} = (i_{sdi}^* - i_{sdi})(k_{pi} + \frac{k_{ii}}{s}) + v_{sdi} - \omega L_i i_{sqi} \quad (7.3)
\]

\[
v_{tqi} = (i_{sqi}^* - i_{sqi})(k_{pi} + \frac{k_{ii}}{s}) + v_{sqi} + \omega L_i i_{sdi} \quad (7.4)
\]

However, the application of the control laws, purposes of the control loops and objectives of the overall control scheme remain unchanged. Therefore, it is intended for the applied scheme to achieve the same operational control of the offshore AC network.

### 7.3 Modeling of the Considered System

The modeling process of the system is covered in this section. The small-signal \(dq\) model of the complete system will be derived in terms of the models of its sub-systems;
the VSC and the offshore AC network. The common reference frame of the system is considered to be based on the network frequency. In order to interconnect the VSC and offshore network models, the variables rotating with the angular frequency of the VSC will be converted to the common reference frame. The conversion between the two frames can be performed as:

\[
\begin{bmatrix}
  x_{d_i}^v \\
  x_{q_i}^v
\end{bmatrix} =
\begin{bmatrix}
  \cos(\delta_i) & \sin(\delta_i) \\
  -\sin(\delta_i) & \cos(\delta_i)
\end{bmatrix}
\begin{bmatrix}
  x_{d_i}^v \\
  x_{q_i}^v
\end{bmatrix}
\]  

(7.5)

where \(x_{d_i}\) and \(x_{q_i}\) are the variables rotating with the network frequency, \(x_{d_i}^v\) and \(x_{q_i}^v\) represent the variables rotating with the VSC frequency, and \(\delta_i\) is the phase difference between the two reference frames. The frames are considered to be initially aligned \((\delta_0 = 0)\). Accordingly, the transformation equations can be linearized as:

\[
\tilde{x}_{d_i}^v = \tilde{x}_{d_i} + x_{q0}\tilde{\delta}_i
\]  

(7.6)

\[
\tilde{x}_{q_i}^v = \tilde{x}_{q_i} - x_{d0}\tilde{\delta}_i
\]  

(7.7)

For simplicity in the modeling process, a flat-start of the network is considered. Thus, the initial values of the voltages, \(v_{sd0}\) and \(v_{sq0}\), are set to \(1.0\) p.u and 0 respectively, whereas the current initial values are set to \(i_{sd0} = 0\), and \(i_{sq0} = 0\). The \(dq\) voltage variables from the voltage and current controllers can be transformed to the common frame using (7.8) and (7.9):

\[
\tilde{v}_{sdi} = \tilde{v}_{sdi}
\]  

(7.8)

\[
\tilde{v}_{sqi} = \tilde{v}_{sqi} - v_{sd0}\tilde{\delta}_i
\]  

(7.9)

Moreover, the current variables from the controllers can be transformed to the common frame using (7.10)-(7.13):

\[
\tilde{i}_{sdi} = \tilde{i}_{sdi}
\]  

(7.10)

\[
\tilde{i}_{sqi} = \tilde{i}_{sqi}
\]  

(7.11)

\[
\tilde{i}_{ndi} = \tilde{i}_{ndi}
\]  

(7.12)

\[
\tilde{i}_{nqi} = \tilde{i}_{nqi}
\]  

(7.13)
A variation in the difference of the network and VSC frequencies would result in a phase angle deviation,
\[ \frac{d\delta_i}{dt} = \omega_{ri} - \omega \]  
(7.14)

### 7.3.1 VSC Model

The VSC model describes the PCC bus voltage and the VSC primary control scheme. The space phasor form of the voltage differential equation describing the VSC connection to its corresponding busbar was presented in (2.3). It can be converted to the \(dq\) reference frame as:

\[ \frac{dv_{sdi}}{dt} = \frac{1}{c_{fi}} \tilde{i}_{sdi} + \omega v_{sqi} + \frac{1}{c_{fi}} \tilde{i}_{ndi} \]  
(7.15)

\[ \frac{dv_{sqi}}{dt} = \frac{1}{c_{fi}} \tilde{i}_{sqi} - \omega v_{sdi} + \frac{1}{c_{fi}} \tilde{i}_{nqi} \]  
(7.16)

Based on the initial values of the system, (7.15) and (7.16) can be linearized as:

\[ \frac{d\tilde{v}_{sdi}}{dt} = \frac{1}{c_{fi}} \tilde{i}_{sdi} + \omega_0 \tilde{v}_{sqi} + \frac{1}{c_{fi}} \tilde{i}_{ndi} - v_{sq0}(\omega_0 - \omega) \]  
(7.17)

\[ \frac{d\tilde{v}_{sqi}}{dt} = \frac{1}{c_{fi}} \tilde{i}_{sqi} - \omega_0 \tilde{v}_{sdi} + \frac{1}{c_{fi}} \tilde{i}_{nqi} + v_{sdi0}(\omega_0 - \omega) \]  
(7.18)

The current controller is designed with the objective of enabling a faster response as compared to the voltage controller. To this end, the VSC current differential equations can be defined using equivalent first-order transfer functions instead of the PI control equations with current decoupling scheme [27]:

\[ \frac{di_{sdi}}{dt} = \frac{1}{\tau_i}(i_{sdi}^* - i_{sdi}) \]  
(7.19)

\[ \frac{di_{sqi}}{dt} = \frac{1}{\tau_i}(i_{sqi}^* - i_{sqi}) \]  
(7.20)

where \(\tau_i\) is the time constant of the VSC current response. Based on the tunability of \(\tau_i\), the current response time of the VSC becomes a design choice when (7.19) and (7.20) are used to model the variables. A smaller value of \(\tau_i\) can be selected to enable a faster response. However, it should be adequately large enough such that the
bandwidth of the control loop, $1/\tau_i$, is considerably smaller than the VSC switching frequency $[53]$.

In the voltage control loops, the integral outputs of the PI controllers can be defined as:

$$\frac{do_i^v}{dt} = k_{oi}(v_{sdi}^* - v_{sdi})$$ (7.21)

$$\frac{do^q_i}{dt} = k_{oi}(v_{sqi}^* - v_{sqi})$$ (7.22)

where $o_i^v$ and $o^q_i$ represent the PI control integral outputs. These equations can be converted to the common reference frame and represented in their linear forms as:

$$\frac{d\tilde{o}_i^v}{dt} = k_{oi}(v_{sdi}^* - \tilde{v}_{sdi})$$ (7.23)

$$\frac{d\tilde{o}^q_i}{dt} = k_{oi}(v_{sqi}^* - \tilde{v}_{sqi}) + k_{oi}v_{sd0}\tilde{\delta}_i$$ (7.24)

The complete outputs of the voltage control loops’ PI controllers are defined as:

$$i_{odi}^v = k_{opi}(v_{sdi}^* - v_{sdi}) + o_{di}^v$$ (7.25)

$$i_{oqi}^v = k_{opi}(v_{sqi}^* - v_{sqi}) + o_{qi}^v$$ (7.26)

where $i_{odi}^v$ and $i_{oqi}^v$ represent the outputs of the PI controllers. The voltage control loops generate the reference currents for the current controllers. They can be defined as:

$$i_{sdi}^{v*} = i_{odi}^v - i_{ndi}^v - \omega c_f i v_{sqi}^v - \omega c_f \tau_i \frac{d v_{sqi}^v}{dt}$$ (7.27)

$$i_{sqi}^{v*} = i_{oqi}^v - i_{nqi}^v + \omega c_f i v_{sdi}^v + \omega c_f \tau_i \frac{d v_{sdi}^v}{dt}$$ (7.28)

The linearized forms of (7.27) and (7.28) are then obtained as:

$$\tilde{i}_{sdi}^{v*} = \tilde{i}_{odi}^v - \tilde{i}_{ndi}^v + c_f i v_{sq0}(\omega_0 - \omega) - \omega_0 c_f i \tilde{v}_{sqi}^v - \omega_0 c_f \tau_i \frac{d \tilde{v}_{sqi}^v}{dt}$$ (7.29)

$$\tilde{i}_{sqi}^{v*} = \tilde{i}_{oqi}^v - \tilde{i}_{nqi}^v - c_f i v_{sd0}(\omega_0 - \omega) + \omega_0 c_f \tilde{v}_{sdi}^v + \omega_0 c_f \tau_i \frac{d \tilde{v}_{sdi}^v}{dt}$$ (7.30)
The complete first-order differential equations of the VSC currents can be derived by substituting (7.25), (7.26), (7.29) and (7.30) in (7.19) and (7.20). The derived equations are presented as:

\[
\begin{align*}
\frac{d\tilde{i}_{sdi}}{dt} &= \frac{1}{\tau_i} \tilde{d}_{sdi} - \frac{1}{\tau_i} \tilde{i}_{sdi} - \frac{\omega_0 c_{fi}}{\tau_i} \tilde{v}_{sdi} - \omega_0 c_{fi} \frac{d\tilde{v}_{sdi}}{dt} - \frac{1}{\tau_i} \tilde{v}_{ndi} \\
&\quad + \frac{c_{fi} \omega_0}{\tau_i} (\omega_0 - \omega) + \frac{k_{opi}}{\tau_i} (v_{sdi}^* - \tilde{v}_{sdi}) \\
\frac{d\tilde{i}_{sqi}}{dt} &= \frac{1}{\tau_i} \tilde{d}_{sqi} - \frac{1}{\tau_i} \tilde{i}_{sqi} + \frac{\omega_0 c_{fi}}{\tau_i} \tilde{v}_{sqi} + \omega_0 c_{fi} \frac{d\tilde{v}_{sqi}}{dt} - \frac{1}{\tau_i} \tilde{v}_{nqi} \\
&\quad - \frac{c_{fi} \omega_0}{\tau_i} (\omega_0 - \omega) + \frac{k_{opi}}{\tau_i} (v_{sqi}^* - \tilde{v}_{sqi})
\end{align*}
\]  

(7.31)  

(7.32)

Subsequently, (7.31) and (7.32) are converted to the common reference frame using (7.8)-(7.13):

\[
\begin{align*}
\frac{d\tilde{d}_{sdi}}{dt} &= \frac{1}{\tau_i} \tilde{d}_{sdi} - \frac{1}{\tau_i} \tilde{i}_{sdi} - \omega_0 \tilde{i}_{sqi} + \frac{\omega_0^2 c_{fi} \tau_i}{\tau_i} - \frac{k_{opi}}{\tau_i} \tilde{v}_{sdi} - \omega_0 c_{fi} \frac{d\tilde{v}_{sdi}}{dt} + \frac{\omega_0 c_{fi} v_{sdi0}}{\tau_i} \Delta \omega \\
&\quad + \frac{k_{opi}}{\tau_i} v_{sdi}^* - \frac{1}{\tau_i} \tilde{i}_{ndi} - \omega_0 \tilde{i}_{nqi} + \omega_0 c_{fi} v_{sdi0} \omega_i - \omega_0 c_{fi} v_{sdi0} \omega \\
&\quad + \frac{c_{fi} \omega_0}{\tau_i} - \omega_0 c_{fi} v_{sdi0} \tau_i \Delta \omega
\end{align*}
\]  

(7.33)

\[
\begin{align*}
\frac{d\tilde{d}_{sqi}}{dt} &= \frac{1}{\tau_i} \tilde{d}_{sqi} + \omega_0 \tilde{i}_{sdi} - \frac{1}{\tau_i} \tilde{i}_{sqi} + \frac{\omega_0 c_{fi}}{\tau_i} \tilde{v}_{sqi} + \omega_0^2 c_{fi} \tau_i - \frac{k_{opi}}{\tau_i} \tilde{v}_{sqi} + \frac{k_{opi} v_{sdi0}}{\tau_i} \Delta \omega \\
&\quad + \frac{k_{opi}}{\tau_i} v_{sqi}^* + \omega_0 \tilde{i}_{nqi} - \frac{1}{\tau_i} \tilde{i}_{nqi} - \frac{c_{fi} v_{sdi0}}{\tau_i} + \frac{c_{fi} \omega_0 v_{sdi0} \tau_i}{\tau_i} \Delta \omega
\end{align*}
\]  

(7.34)

where \(\Delta \omega\) is the deviation of the network frequency from its initial value. It represents a linear change in the network impedences as a result of a variation in the network frequency, and it can be calculated as:

\[
\Delta \omega = \omega_0 - \omega
\]  

(7.35)

The state space representation of the VSC model can be derived as \(\dot{X}_{oi} = A_{7} X_{oi} + B_{7} U_{oi}\), where:

\[
X_{oi} = \begin{bmatrix} \tilde{d}_{sdi} & \tilde{d}_{sqi} & \tilde{i}_{sdi} & \tilde{i}_{sqi} & \tilde{v}_{sdi} & \tilde{v}_{sqi} & \tilde{\delta}_{i} \end{bmatrix}
\]
\[ U_{oi} = \begin{bmatrix} v^*_{sdi} & v^*_{sqi} & i_{ndi} & i_{nqi} & \omega_{ri} & \omega & \Delta \omega \end{bmatrix} \]

The state and input matrices of the model are:

\[
A_7 = \begin{bmatrix}
0 & 0 & 0 & 0 & -k_{oii} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -k_{oii} & k_{oii}v_{sd0} \\
\frac{1}{\tau_i} & 0 & -\frac{1}{\tau_i} & -\omega_0 & Y_{11} & Y_{12} & Y_{13} \\
0 & \frac{1}{\tau_i} & \omega_0 & -\frac{1}{\tau_i} & Y_{21} & Y_{22} & Y_{23} \\
0 & 0 & \frac{1}{c_{fi}} & 0 & 0 & \omega_0 & 0 \\
0 & 0 & 0 & \frac{1}{c_{fi}} & -\omega_0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \quad (7.36)
\]

\[
B_7 = \begin{bmatrix}
k_{oii} & 0 & 0 & 0 & 0 & 0 & 0 \\
k_{oii} & 0 & 0 & 0 & 0 & 0 & 0 \\
k_{opi} & 0 & -\frac{1}{\tau_i} & -\omega_0 & \omega_0c_{fi}v_{sd0} & -\omega_0c_{fi}v_{sd0} & Y_{31} \\
0 & \frac{k_{opi}}{\tau_i} & \omega_0 & -\frac{1}{\tau_i} & 0 & 0 & Y_{41} \\
0 & 0 & \frac{1}{c_{fi}} & 0 & 0 & 0 & -v_{sq0} \\
0 & 0 & 0 & \frac{1}{c_{fi}} & 0 & 0 & v_{sd0} \\
0 & 0 & 0 & 0 & 1 & -1 & 0
\end{bmatrix} \quad (7.37)
\]

Where \( Y_{11} = \frac{\omega_0^2c_{fi}\tau_i - k_{opi}}{\tau_i}, Y_{12} = -\frac{\omega_0c_{fi}}{\tau_i}, Y_{13} = \frac{\omega_0c_{fi}v_{sd0}}{\tau_i}, Y_{21} = \frac{\omega_0c_{fi}}{\tau_i}, Y_{22} = \frac{\omega_0^2c_{fi}\tau_i - k_{opi}}{\tau_i}, Y_{23} = \frac{k_{opi}v_{sd0}}{\tau_i}, Y_{31} = \frac{c_{fi}v_{sq0} - \omega_0c_{fi}v_{sd0}\tau_i}{\tau_i}, \) and \( Y_{41} = -\frac{c_{fi}v_{sd0} + c_{fi}\omega_0v_{sq0}\tau_i}{\tau_i}. \)

### 7.3.2 Offshore AC Network

The modeling of the offshore AC Network includes differential equations which describe the network currents and voltages. This modeling process is based on the reference diagram which was presented in Fig. 2.3.
7.3.2.1 AC Hub Bus Voltage

The space phasor form of the voltage differential equation describing the dynamics of the AC hub bus was presented in (2.12). In the \(dq\) reference frame, it can be represented as:

\[
\frac{dv_{jd}}{dt} = \omega v_{jq} + \frac{1}{c_1 + c_2} \tilde{i}_{wd} + \frac{1}{c_1^* + c_2^*} \tilde{i}_{kd1} + \frac{1}{c_1^* + c_2^*} \tilde{i}_{kd2}
\]

(7.38)

\[
\frac{dv_{jq}}{dt} = \omega v_{jd} + \frac{1}{c_1 + c_2} \tilde{i}_{wq} + \frac{1}{c_1^* + c_2^*} \tilde{i}_{kq1} + \frac{1}{c_1^* + c_2^*} \tilde{i}_{kq2}
\]

(7.39)

where \(i_{wd}\) and \(i_{wq}\) are the equivalent \(dq\) currents from the WPPs, \(v_{jd}\) and \(v_{jq}\) represent the \(dq\) components of the AC hub bus voltage, and \(i_{kd1}, i_{kq1}, i_{kd2}\) and \(i_{kq2}\) are the \(dq\) currents flowing from the network branches. Based on the initial values of the system, (7.38) and (7.39) can be linearized as:

\[
\frac{d\tilde{v}_{jd}}{dt} = \omega_0 \tilde{v}_{jq} + \frac{1}{c_1 + c_2} \tilde{i}_{wd} + \frac{1}{c_1^* + c_2^*} \tilde{i}_{kd1} + \frac{1}{c_1^* + c_2^*} \tilde{i}_{kd2} - v_{jq0}(\omega_0 - \omega)
\]

(7.40)

\[
\frac{d\tilde{v}_{jq}}{dt} = -\omega_0 \tilde{v}_{jd} + \frac{1}{c_1 + c_2} \tilde{i}_{wq} + \frac{1}{c_1^* + c_2^*} \tilde{i}_{kq1} + \frac{1}{c_1^* + c_2^*} \tilde{i}_{kq2} + v_{jd0}(\omega_0 - \omega)
\]

(7.41)

The state space representation describing the AC hub bus voltage can be derived based on (7.40) and (7.41). It can be defined as \(\dot{X}_t = A_8 X_t + B_8 U_t\), where:

\[
X_t = \begin{bmatrix} \tilde{v}_{jd} & \tilde{v}_{jq} \end{bmatrix}
\]

\[
U_t = \begin{bmatrix} \tilde{i}_{wd} & \tilde{i}_{wq} & \tilde{i}_{kd1} & \tilde{i}_{kq1} & \tilde{i}_{kd2} & \tilde{i}_{kq2} & \Delta\omega \end{bmatrix}
\]

(7.42)

The state and input matrices of the model are:

\[
A_8 = \begin{bmatrix} 0 & \omega_0 \\ -\omega_0 & 0 \end{bmatrix}
\]

(7.43)

\[
B_8 = \begin{bmatrix} B_{61} & 0 & B_{62} & 0 & B_{63} & 0 & -v_{jq0} \\ 0 & B_{71} & 0 & B_{72} & 0 & B_{73} & v_{jd0} \end{bmatrix}
\]

(7.44)
where:

\[
B_{61} = B_{62} = B_{63} = \frac{1}{c'_1 + c'_2}
\]

\[
B_{71} = B_{72} = B_{73} = \frac{1}{c''_1 + c''_2}
\]

### 7.3.2.2 Branch Currents

The space phasor form of the differential equation describing the current flow across each branch of the offshore network was presented in (2.20). It can be represented in the \(dq\) reference frame as:

\[
\frac{di_{ki}i_{kdi}}{dt} = -\frac{r_{ki}}{l_{ki}}i_{kdi} + \omega i_{kqi} + \frac{1}{l_{ki}}v_{kdi} - \frac{1}{l_{ki}}v_{jd}
\]

(7.45)

\[
\frac{di_{kqi}}{dt} = -\omega i_{kdi} - \frac{r_{ki}}{l_{ki}}i_{kqi} + \frac{1}{l_{ki}}v_{kqi} - \frac{1}{l_{ki}}v_{jq}
\]

(7.46)

For the \(i\)th branch, \(i_{kdi}\) and \(i_{kqi}\) are the \(dq\) components of the network current, and \(v_{kdi}\) and \(v_{kqi}\) represent the \(dq\) voltages across the PCC. The linearized forms of (7.45) and (7.46) are presented in (7.47) and (7.48) respectively:

\[
\frac{d\tilde{i}_{kdi}}{dt} = -\frac{r_{ki}}{l_{ki}}\tilde{i}_{kdi} + \omega_0\tilde{i}_{kqi} + \frac{1}{l_{ki}}\tilde{v}_{kdi} - \frac{1}{l_{ki}}\tilde{v}_{jd} - i_{kq0}(\omega_0 - \omega)
\]

(7.47)

\[
\frac{d\tilde{i}_{kqi}}{dt} = -\omega_0\tilde{i}_{kdi} - \frac{r_{ki}}{l_{ki}}\tilde{i}_{kqi} + \frac{1}{l_{ki}}\tilde{v}_{kqi} - \frac{1}{l_{ki}}\tilde{v}_{jq} + i_{kd0}(\omega_0 - \omega)
\]

(7.48)

The model describing the branch currents can be represented in state space form:

\[
\dot{X}_n = A_9X_n + B_9U_n,
\]

where:

\[
X_n = \begin{bmatrix} \tilde{i}_{kdi} & \tilde{i}_{kqi} \end{bmatrix}
\]

\[
U_n = \begin{bmatrix} \tilde{v}_{kdi} & \tilde{v}_{kqi} & \tilde{v}_{jd} & \tilde{v}_{jq} & \Delta \omega \end{bmatrix}
\]

(7.49)

The state and input matrices are defined as:

\[
A_9 = \begin{bmatrix} -\frac{r_{ki}}{l_{ki}} & \omega_0 \\ \frac{1}{l_{ki}} & -\frac{r_{ki}}{l_{ki}} \end{bmatrix}
\]

(7.50)
\[
B_9 = \begin{bmatrix}
\frac{1}{l_{ki}} & 0 & -\frac{1}{l_{ki}} & 0 & -i_{kq0} \\
0 & \frac{1}{l_{ki}} & 0 & -\frac{1}{l_{ki}} & i_{kd0}
\end{bmatrix}
\] (7.51)

7.4 Droop Control

In this section, the frequency and voltage droop controllers are introduced. They can be employed to complement the overall control scheme. The offshore network is connected to multiple onshore AC grids. Therefore, droop control can be applied to enable power sharing among the VSC-HVDC links connected to the onshore grids. Frequency droop control is used for the purpose of achieving active power sharing, whereas voltage droop control can be used to enable reactive power sharing. The dynamics of both droop controllers will be discussed in this section.

7.4.1 Frequency Droop Controller

The frequency droop controller is designed as presented in Fig. 7.2. In this figure, \( p_i \) represents the active power measured at the PCC, \( \omega_s \) is the nominal grid frequency, and \( k_{fi} \) is the frequency droop gain. The control dynamics are described using (7.52):

\[
\omega_{ri} = \omega_s - k_{fi} p_i
\] (7.52)

The measurement of the active power at the PCC of a VSC can be performed as:

\[
p_i = v_{sdi} i_{sdi} + v_{sqi} i_{sqi}
\] (7.53)
Based on the system initial values, (7.53) can be linearized as:

\[ \tilde{p}_i = \tilde{i}_{sdi} \quad (7.54) \]

Accordingly the linear form of the frequency droop controller can be defined as:

\[ \omega_{ri} = \omega_s - k_f \tilde{i}_{sdi} \quad (7.55) \]

The active power is shared according to the value of the frequency droop gain. Therefore, power sharing can be adjusted by tuning this gain. The controller generates the reference frequency for the VSC.

### 7.4.2 Voltage Droop Controller

The voltage droop control design is presented in Fig. 7.3. In this figure, \( q_i \) is the measured reactive power at the PCC, \( v_s \) is the nominal grid voltage, and \( k_{ui} \) represents the voltage droop gain. The dynamics of the voltage droop controller can be described as:

\[ v_{sdi}^* = v_s + k_{ui} q_i \quad (7.56) \]

The reactive power at the corresponding AC busbar of the VSC can be measured as:

\[ q_i = v_{sdi} i_{sdi} - v_{sdi} i_{sqi} \quad (7.57) \]

Based on the initial values of the system, (7.57) can be linearized as:

\[ \tilde{q}_i = -\tilde{i}_{sqi} \quad (7.58) \]
Thus, the linear form of the frequency droop controller can be defined as:

\[ v_{sdi}^* = v_s - k_{ui} \tilde{i}_{sqi} \quad (7.59) \]

The voltage droop gain value can be tuned according to the desired reactive power sharing among the VSC-HVDC links. The reference direct voltage of the VSC is generated from the output of the voltage droop controller.

### 7.5 Conclusion

The modeling and control of the system having variable network frequency has been presented in this chapter. The small-signal \( dq \) model of the considered system has been derived. Furthermore, the designs of the frequency and voltage droop controllers have been introduced and discussed along with the control dynamics.
Chapter 8  
Dynamic Phasors of System having Variable Frequency-based Network Configuration

8.1 Introduction

The DP-based model of the offshore AC network with variable frequency will be derived in this chapter. The proposed approach enables simpler model-based studies of the system since droop controllers can be included in the simulation model to develop the complete model of the system. The key properties of the DP modeling technique, presented in the fourth chapter, will be applied here. The DPs of the overall system will be represented in state space form.

8.2 Dynamic Phasor Modeling of Variable Frequency System

The theory of DPs has been introduced earlier in the thesis. This chapter discusses the methods of deriving the DPs of a system with variable frequency. For this purpose, the DP technique is studied with the complex time-domain waveform presented in (4.1). Here, the network frequency $\omega$ is considered as a variable. At time, $t$ the $k$th Fourier coefficient can be determined as [59]:

$$X_k(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} x(t(\delta))e^{-jk\delta}d\delta = \langle x \rangle_k(t) \quad (8.1)$$
where $\delta$ is the phase angle of the system \[59\],

$$
\delta = \int_0^t \omega(\tau) d\tau \quad (8.2)
$$

As the DP is a complex variable, it can also be represented in terms of its real and imaginary parts as presented in \(4.3\). According to \[59\], the relationship between the derivatives of the considered waveform, and the derivatives of the $k$th DP can be described as presented in \(4.4\). In the fourth chapter, this key property was used to derive the DPs for all of the system variables. Balanced conditions were assumed for the system with constant frequency. Such conditions are also assumed for the system under consideration in this chapter. Therefore, \(4.5\)-\(4.21\) will apply for this case as well. The DP-based model of the system will be derived accordingly.

### 8.2.1 DP Representation of Control Scheme

The equations used to describe the dynamics of the control strategy for the considered system are converted to their respective DP forms in this section. Firstly, the DPs of the voltage and current controllers are derived. Based on the characteristics of DP-based modeling, \(7.1\) and \(7.2\) can be converted to their DP forms as presented in \(8.3\) and \(8.4\) respectively:

\[
\langle i_{sai}^r \rangle_1 = (\langle v_{sai}^r \rangle_1 - \langle v_{sai} \rangle_1) (k_{opi} + \frac{k_{oii}}{s}) - \langle i_{nai}^r \rangle_1 - \omega L_i \langle i_{sai}^i \rangle_1 (\tau_i s + 1) \quad (8.3)
\]

\[
\langle i_{sai}^i \rangle_1 = (\langle v_{sai}^i \rangle_1 - \langle v_{sai} \rangle_1) (k_{opi} + \frac{k_{oii}}{s}) - \langle i_{nai}^i \rangle_1 + \omega L_i \langle i_{sai}^r \rangle_1 (\tau_i s + 1) \quad (8.4)
\]

The equations representing the dynamics of the current controllers, \(7.3\) and \(7.4\), are converted into DPs as:

\[
\langle v_{tai}^r \rangle_1 = (\langle v_{sai}^r \rangle_1 - \langle i_{sai}^r \rangle_1) (k_{pi} + \frac{k_{ii}}{s}) + \langle v_{sai}^r \rangle_1 - \omega L_i \langle i_{sai}^i \rangle_1 \quad (8.5)
\]

\[
\langle v_{tai}^i \rangle_1 = (\langle i_{sai}^i \rangle_1 - \langle i_{sai} \rangle_1) (k_{pi} + \frac{k_{ii}}{s}) + \langle v_{sai}^i \rangle_1 + \omega L_i \langle i_{sai}^r \rangle_1 \quad (8.6)
\]

The DP-based equations describing the current and voltage controllers can be combined to describe the considered system’s primary control scheme in DP form. This is presented in Fig. 8.1.
The same DP techniques are applied for the derivation of the DPs of frequency and voltage droop controllers as well. The equation representing the dynamics of the frequency droop control was presented in (7.52). Based on the DP properties, it can be converted to DP form as:

$$\omega_{ri} = \omega_s - k_f \langle p_{ai} \rangle_1$$

(8.7)

Accordingly, the DP-based representation of the frequency droop control design can be presented as shown in Fig. 8.2.

The active power measurement in small-signal form was presented in (7.54). This is converted to DP form as:

$$\langle p_{ai} \rangle_1 = \langle i_{sai}^r \rangle_1$$

(8.8)
Using (8.7) and (8.8), the frequency droop control loop can be defined in a simplified DP form as:

\[ \omega_{ri} = \omega_s - k_f \langle i^r_{sai} \rangle_1 \] (8.9)

The voltage droop control dynamics were described using the equation, (7.56). The DP-based form of the equation is derived as:

\[ \langle v^r_{sai} \rangle_1 = v_s + k_{ui} \langle q_{ai} \rangle_1 \] (8.10)

In Fig. 8.3, the design of the DP-based voltage droop controller is shown.

Figure 8.3. DP-based voltage droop controller

A linearized reactive power calculation was presented in (7.58). In DP-form, this can be represented as:

\[ \langle q_{ai} \rangle_1 = -\langle i^r_{sai} \rangle_1 \] (8.11)

Based on (8.10) and (8.11), the simplified DP form of the voltage droop controller can be defined as:

\[ \langle v^r_{sai} \rangle_1 = v_s - k_{ui} \langle i^r_{sai} \rangle_1 \] (8.12)

### 8.2.2 VSC DP Model

In order to describe the VSC with DP equations, the presented modeling techniques can be used to convert the small-signal VSC model to DP form. Accordingly, (7.17) and (7.18) are converted to their DP forms as presented in (8.13) and (8.14):

\[ \frac{d\langle v^r_{sai} \rangle_1}{dt} = \frac{1}{c_{fi}} \langle i^r_{sai} \rangle_1 + \omega_0 \langle v^i_{sai} \rangle_1 + \frac{1}{c_{fi}} \langle i^r_{nai} \rangle_1 - v_{sq0}(\omega_0 - \omega) \] (8.13)
\[
\frac{d\langle v^i_{\text{sai}} \rangle_1}{dt} = \frac{1}{c_{fi}} \langle i^i_{\text{sai}} \rangle_1 - \omega_0 \langle v^r_{\text{sai}} \rangle_1 + \frac{1}{c_{fi}} \langle i^i_{\text{nai}} \rangle_1 + v_{sd0}(\omega_0 - \omega)
\] (8.14)

Similarly, the DPs of (7.23) and (7.24) are derived as:

\[
\frac{d\langle o^r_{\text{ai}} \rangle_1}{dt} = k_{oii} \langle v^r_{\text{sai}} \rangle_1 - k_{oii} \langle v^r_{\text{sai}} \rangle_1
\] (8.15)

\[
\frac{d\langle o^i_{\text{ai}} \rangle_1}{dt} = k_{oii} \langle v^i_{\text{sai}} \rangle_1 - k_{oii} \langle v^i_{\text{sai}} \rangle_1 + k_{oii} v_{sd0}\tilde{\delta}_i
\] (8.16)

For (7.33) and (7.34), the respective DP-based forms can be derived as:

\[
\frac{d\langle i^r_{\text{sai}} \rangle_1}{dt} = \frac{1}{\tau_i} \langle o^r_{\text{ai}} \rangle_1 - \frac{1}{\tau_i} \langle i^r_{\text{sai}} \rangle_1 - \omega_0 \langle i^r_{\text{sai}} \rangle_1 + \frac{\omega_0^2 c_{fi} \tau_i - k_{opi}}{\tau_i} \langle v^r_{\text{sai}} \rangle_1
\]

\[- \frac{\omega_0 c_{fi}}{\tau_i} \langle i^r_{\text{sai}} \rangle_1 + \frac{\omega_0 c_{fi} v_{sd0}\tilde{\delta}_i}{\tau_i} + \frac{k_{opi}}{\tau_i} \langle v^r_{\text{sai}} \rangle_1 - \frac{1}{\tau_i} \langle i^r_{\text{nai}} \rangle_1 - \omega_0 \langle i^r_{\text{nai}} \rangle_1
\]

\[+ \frac{v_{sd0} k_{opi}}{\tau_i} \tilde{\delta}_i + \frac{k_{opi}}{\tau_i} \langle v^i_{\text{sai}} \rangle_1 + \omega_0 \langle i^r_{\text{nai}} \rangle_1 - \frac{1}{\tau_i} \langle i^r_{\text{nai}} \rangle_1
\]

\[+ \frac{c_{fi} v_{sd0} + c_{fi} \omega_0 v_{sd0}\tau_i}{\tau_i}(\omega_0 - \omega)
\] (8.17)

\[
\frac{d\langle i^i_{\text{sai}} \rangle_1}{dt} = \frac{1}{\tau_i} \langle o^i_{\text{ai}} \rangle_1 + \omega_0 \langle i^r_{\text{sai}} \rangle_1 + \frac{1}{\tau_i} \langle i^i_{\text{sai}} \rangle_1 + \frac{\omega_0 c_{fi}}{\tau_i} \langle v^r_{\text{sai}} \rangle_1 + \frac{\omega_0^2 c_{fi} \tau_i - k_{opi}}{\tau_i} \langle v^i_{\text{sai}} \rangle_1
\]

\[+ \frac{v_{sd0} k_{opi}}{\tau_i} \tilde{\delta}_i + \frac{k_{opi}}{\tau_i} \langle v^i_{\text{sai}} \rangle_1 + \omega_0 \langle i^r_{\text{nai}} \rangle_1 - \frac{1}{\tau_i} \langle i^r_{\text{nai}} \rangle_1
\]

\[+ \frac{c_{fi} v_{sd0} + c_{fi} \omega_0 v_{sd0}\tau_i}{\tau_i}(\omega_0 - \omega)
\] (8.18)

The state space representation of the VSC model in DP form can be presented as

\[\dot{X}_{ji} = A_{10} X_{ji} + B_{10} U_{ji},\]

where:

\[X_{ji} = [\langle o^r_{\text{ai}} \rangle_1 \langle o^i_{\text{ai}} \rangle_1 \langle i^r_{\text{sai}} \rangle_1 \langle i^i_{\text{sai}} \rangle_1 \langle v^r_{\text{sai}} \rangle_1 \langle v^i_{\text{sai}} \rangle_1 \tilde{\delta}_i]
\]

\[U_{ji} = [\langle v^r_{\text{sai}} \rangle_1 \langle v^i_{\text{sai}} \rangle_1 \langle i^r_{\text{nai}} \rangle_1 \langle i^i_{\text{nai}} \rangle_1 \omega_{ri} \omega \Delta \omega]
\]
The state and input matrices of the model are:

\[
A_{10} = \begin{bmatrix}
0 & 0 & 0 & 0 & -k_{oii} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -k_{oii} & k_{oii}v_{sd0} \\
\frac{1}{\tau_i} & 0 & -\frac{1}{\tau_i} & -\omega_0 & Y_{51} & Y_{52} & Y_{53} \\
0 & \frac{1}{\tau_i} & \omega_0 & -\frac{1}{\tau_i} & Y_{61} & Y_{62} & Y_{63} \\
0 & 0 & 0 & \frac{1}{c_{fi}} & 1 & -\omega_0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0
\end{bmatrix}
\]  

(8.19)

\[
B_{10} = \begin{bmatrix}
k_{oii} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & k_{oii} & 0 & 0 & 0 & 0 & 0 \\
k_{opi} & \frac{1}{\tau_i} & 0 & -\frac{1}{\tau_i} & \omega_0 & \omega_0c_{fi}v_{sd0} & -\omega_0c_{fi}v_{sd0} & Y_{71} \\
0 & k_{opi} & \omega_0 & -\frac{1}{\tau_i} & 0 & 0 & 0 & Y_{81} \\
0 & 0 & \frac{1}{c_{fi}} & 0 & 0 & 0 & 0 & -v_{sq0} \\
0 & 0 & 0 & \frac{1}{c_{fi}} & 0 & 0 & 0 & v_{sd0} \\
0 & 0 & 0 & 0 & 1 & 1 & -1 & 0
\end{bmatrix}
\]  

(8.20)

Where \(Y_{51} = \frac{\omega_0^2c_{fi}\tau_i - k_{opi}}{\tau_i}, Y_{52} = -\frac{\omega_0c_{fi}}{\tau_i}, Y_{53} = \frac{\omega_0c_{fi}v_{sd0}}{\tau_i}, Y_{61} = \frac{\omega_0c_{fi}}{\tau_i}, Y_{62} = \frac{\omega_0^2c_{fi}\tau_i - k_{opi}}{\tau_i}, Y_{63} = \frac{k_{opi}v_{sd0}}{\tau_i}, Y_{71} = \frac{c_{fi}v_{sq0} - \omega_0c_{fi}v_{sd0}\tau_i}{\tau_i}, \) and \(Y_{81} = -\frac{c_{fi}v_{sd0} + c_{fi}\omega_0v_{sq0}\tau_i}{\tau_i}.

### 8.2.3 Offshore AC Network DP Model

The DP-based model of the offshore AC network is derived in this section. The complete model includes the combination of the DP-based equations describing the AC hub bus voltage and the branch currents.
8.2.3.1 AC Hub Bus Voltage

The AC hub bus voltage dynamics can be described using DPs. This is achieved by deriving the DP forms of (7.40) and (7.41), as presented in (8.21) and (8.22) respectively:

\[
\frac{d}{dt} \langle v_{ja}^r \rangle_1 = \omega_0 \langle v_{ja}^i \rangle_1 + \frac{1}{c_1^* + c_2^*} \langle i_{wa}^r \rangle_1 + \frac{1}{c_1^* + c_2^*} \langle i_{kai}^r \rangle_1 + \frac{1}{c_1^* + c_2^*} \langle i_{ka2}^r \rangle_1 - v_{jq0}(\omega_0 - \omega)
\] (8.21)

\[
\frac{d}{dt} \langle v_{ja}^i \rangle_1 = -\omega_0 \langle v_{ja}^r \rangle_1 + \frac{1}{c_1^* + c_2^*} \langle i_{wa}^i \rangle_1 + \frac{1}{c_1^* + c_2^*} \langle i_{kai}^i \rangle_1 + \frac{1}{c_1^* + c_2^*} \langle i_{ka2}^i \rangle_1 + v_{jd0}(\omega_0 - \omega)
\] (8.22)

The model can be represented in state space form, \( \dot{X}_{dgo} = A_{11}X_{dgo} + B_{11}U_{dgo} \), where:

\[
X_{dgo} = \begin{bmatrix} \langle v_{ja}^r \rangle_1 & \langle v_{ja}^i \rangle_1 \end{bmatrix}
\]

\[
U_{dgo} = \begin{bmatrix} \langle i_{wa}^r \rangle_1 & \langle i_{wa}^i \rangle_1 & \langle i_{kai}^r \rangle_1 & \langle i_{kai}^i \rangle_1 & \langle i_{ka2}^r \rangle_1 & \langle i_{ka2}^i \rangle_1 & \Delta \omega \end{bmatrix}
\] (8.23)

The state and input matrices of this model are:

\[
A_{11} = \begin{bmatrix} 0 & \omega_0 \\ -\omega_0 & 0 \end{bmatrix}
\] (8.24)

\[
B_{11} = \begin{bmatrix} B_{s1} & 0 & B_{s2} & 0 & B_{s3} & 0 & -v_{jq0} \\ 0 & B_{g1} & 0 & B_{g2} & 0 & B_{g3} & v_{jd0} \end{bmatrix}
\] (8.25)

where,

\[
B_{s1} = B_{s2} = B_{s3} = \frac{1}{c_1^* + c_2^*}
\]

\[
B_{g1} = B_{g2} = B_{g3} = \frac{1}{c_1^* + c_2^*}
\]
8.2.3.2 Branch Currents

The DP techniques can be used in the same manner to convert (7.47) and (7.48) to their respective DP-based forms:

\[
\frac{d\langle i_{r kai}^i \rangle_1}{dt} = -\frac{r_{ki}}{l_{ki}}\langle i_{r kai}^r \rangle_1 + \omega_0 \langle i_{kai}^i \rangle_1 + \frac{1}{l_{ki}}\langle v_{kai}^r \rangle_1 - \frac{1}{l_{ki}}\langle v_{ja}^r \rangle_1 - i_{kq0}(\omega_0 - \omega) \quad (8.26)
\]

\[
\frac{d\langle i_{kai}^i \rangle_1}{dt} = -\omega_0 \langle i_{r kai}^i \rangle_1 - \frac{r_{ki}}{l_{ki}}\langle i_{r kai}^r \rangle_1 + \frac{1}{l_{ki}}\langle v_{kai}^i \rangle_1 - \frac{1}{l_{ki}}\langle v_{ja}^i \rangle_1 + i_{kd0}(\omega_0 - \omega) \quad (8.27)
\]

The derived equations can be presented in the form of a state space model, \( \dot{X}_{ep} = A_{12}X_{ep} + B_{12}U_{ep} \), where:

\[
X_{ep} = \begin{bmatrix} \langle i_{r kai}^r \rangle_1 & \langle i_{kai}^i \rangle_1 \end{bmatrix}
\]

\[
U_{ep} = \begin{bmatrix} \langle v_{kai}^r \rangle_1 & \langle v_{kai}^i \rangle_1 & \langle v_{ja}^r \rangle_1 & \langle v_{ja}^i \rangle_1 & \Delta\omega \end{bmatrix}
\]

(8.28)

The state and input matrices of this model are defined as:

\[
A_{12} = \begin{bmatrix} -\frac{r_{ki}}{l_{ki}} & \omega_0 \\ \omega_0 & -\frac{r_{ki}}{l_{ki}} \end{bmatrix} \quad (8.29)
\]

\[
B_{12} = \begin{bmatrix} \frac{1}{l_{ki}} & 0 & -\frac{1}{l_{ki}} & 0 & -i_{kq0} \\ 0 & \frac{1}{l_{ki}} & -\frac{1}{l_{ki}} & 0 & i_{kd0} \end{bmatrix} \quad (8.30)
\]

8.3 Conclusion

This chapter has presented the DP-based modeling properties for a system with variable frequency. DP-based equations have been derived to describe the dynamics of the primary and droop controllers of the system under consideration. The VSC and offshore network models have also been converted to DP form and then represented in state space form.
Chapter 9
Case Studies of System with Variable Network Frequency

9.1 Introduction

In this chapter, the control performance of the system with variable network frequency is investigated. Simulation-based studies of the considered system are performed using its DP-based model. MATLAB Simulink is used as the simulation tool for the case studies.

9.2 Case Studies of System under Consideration

A simulation model is developed using the DP-based representation of the system. It is based on the test system which was used for previous case studies, and presented in Fig. 3.1. The control and network parameters of the system are provided in Appendix 2. The purpose of the case studies is to validate the primary and droop control schemes for the considered system.

9.2.1 Step change in reference values of voltages and frequencies

In this case, the independent control of the network frequency and VSC $dq$ voltages is investigated. The following inputs are provided as the set-points in the VSCs’ control systems:

1. The reference frequencies of the VSCs are stepped up from 50 Hz to 55 Hz at $t = 3.0$ s.
2. A step change from 220 kV to 150 kV is applied with the reference values of the direct voltages after 5 seconds of simulation.

3. The reference values for the quadrature voltages are stepped up from 0 to 50 kV at t = 8.0 s.

Droop controllers are not employed in this case. Therefore, there will be no active and/or reactive power sharing between the VSC-HVDC links. The responses at both of the VSC controlled buses will be identical. Thus, it will be sufficient to measure the variable responses at either PCC.

The responses of the $dq$ voltages at the controlled bus of VSC$_1$, as well as the network frequency response are provided in Fig. 9.1.

![Figure 9.1. Simulation-based responses involving step change in VSC reference frequencies and voltages](image-url)
The variables are controlled as intended since the set-point tracking of their reference values is achieved. It is also observed that the adjustment of either variable does not impact the responses of other variables. Therefore, it can be concluded that the variables are controlled independently.

9.2.2 System response to injection of WPP currents

This case study will involve the analysis of the system response to injection of wind currents. Droop controllers are not used in this study. Reference values for the voltages and frequencies are set at their rated and nominal values respectively. Thus, the direct voltage reference value is 220 kV, whereas the quadrature voltage reference value is 0. Furthermore, the reference frequency is set at 50 Hz. The active WPP current is stepped up from 0 to 650 A at $t = 3.0$ s, whereas a step change from 0 to 300 A in the reactive WPP current is applied at $t = 6.0$ s. The responses of the $dq$ voltages, as well as the network frequency, and active and reactive power of VSC$_1$ are provided in Fig. 9.2.
It is observed that the wind current injection has a negligible impact on the network frequency and VSC voltage responses. Therefore, it can be concluded that the control system successfully regulates these variables. The wind currents produce a flow of active and reactive power across the VSC-HVDC links. The power flow is equal among both links due to the absence of droop controllers.
9.2.3 Droop controlled response to injection of wind currents

In this case study, the frequency and voltage droop controllers are employed to enable and active and reactive power sharing among the VSC-HVDC links. The step changes in the wind currents, which were considered for the previous case, are applied in this study as well. The voltage droop gains are set to $k_{u1} = 0.002$ and $k_{u2} = -0.002$, whereas the frequency droop gains are tuned to $k_{f1} = 0.00165$ and $k_{f2} = 0.00231$. The responses of the system are provided in Fig. 9.3. In this figure, the measured variables include the $dq$ voltages of VSC$_1$ and VSC$_2$, the network frequency, and the active and reactive power responses of both VSCs.
Figure 9.3. Simulation-based responses involving injection of wind currents with droop control enabled

It can be observed from the responses that the regulation of the network frequency and VSCs’ voltages is maintained according to their nominal and rated values respectively. Furthermore, it is noted that the active and reactive powers are shared among the VSC-HVDC links according to the assumed requirements of the onshore
AC grids.

9.3 Conclusion

Simulation-based studies of the system with variable network frequency has been performed in this chapter. The results of the study validate the primary and droop control objectives of the proposed controllers.
Chapter 10
Conclusions and Future Work

The research objectives of the thesis have focused on investigating the offshore AC network as a reliable method for integrating offshore WPPs to the onshore AC grids. The grid-forming control scheme was employed to regulate the frequency and voltages of the considered system. The proposed design and control of the system with constant network frequency have been validated by performing detailed model-based simulations.

The DP modeling approach has been proposed to simplify the system analysis. The proposed model of the system has been derived and simulated. It has shown high accuracy, and increased simulation speed. It has also been used to perform an eigenvalue analysis to study the stability of the offshore AC network. The impact on system stability due to the variation of the HVAC cable length, VSC reactor impedance, filter capacitance, and current and voltage control gains has been investigated. The results of the analysis have been validated with detailed model-based simulations.

The small-signal $dq$ and DP-based models of the system with variable network frequency have also been derived in this research. The primary and droop controllers were introduced and then validated with DP model-based simulations.

This research has laid the groundwork for future works pertaining to the studies of the offshore AC network. Some suggestions to extend the work include:

1. Considering additional harmonics in the derivation of the DP-based model and assessing the resulting trade-off between the simulation precision and simulation scale.

2. Deriving and analyzing DP-based models for an offshore AC network under unbalanced and/or faulted conditions.

3. Implementing a systematic procedure to determine the PI control gains of
the current and voltage controllers, for the purpose of achieving optimal control performance of the presented system.

4. Employing modern control methods such as Linear Quadratic Regulator (LQR) and Model Predictive Control (MPC) for a comparative analysis of different control schemes of the presented system.
Appendix
Network and Control Parameters

The operating parameters for the case studies assuming a system with constant network frequency are set as provided in Table 1.

Table 1. Operating parameters for system with constant network frequency

<table>
<thead>
<tr>
<th>Variables</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cable resistance</td>
<td>0.032</td>
<td>Ω/km</td>
</tr>
<tr>
<td>Cable inductance</td>
<td>0.4</td>
<td>mH/km</td>
</tr>
<tr>
<td>Cable capacitance</td>
<td>0.17</td>
<td>µF/km</td>
</tr>
<tr>
<td>Length of cables 1, and 2</td>
<td>10</td>
<td>km</td>
</tr>
<tr>
<td>Filter capacitance</td>
<td>0.00329</td>
<td>µF</td>
</tr>
<tr>
<td>VSC reactor resistance</td>
<td>0.5</td>
<td>mΩ</td>
</tr>
<tr>
<td>VSC reactor inductance</td>
<td>55</td>
<td>mH</td>
</tr>
<tr>
<td>VSC voltage control gains</td>
<td>$5 + \frac{20}{s}$</td>
<td>1</td>
</tr>
<tr>
<td>VSC current control gains</td>
<td>$0.1 + \frac{200}{s}$</td>
<td>1</td>
</tr>
</tbody>
</table>

The control and network parameters for the system with variable network frequency are provided in Table 2.
Table 2. Control and network parameters for system with variable network frequency

<table>
<thead>
<tr>
<th>Variables</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated voltage</td>
<td>220</td>
<td>kV</td>
</tr>
<tr>
<td>Rated apparent power</td>
<td>500</td>
<td>MVA</td>
</tr>
<tr>
<td>Nominal angular frequency</td>
<td>314.16</td>
<td>rad/s</td>
</tr>
<tr>
<td>Cable resistance</td>
<td>0.032</td>
<td>Ω/km</td>
</tr>
<tr>
<td>Cable inductance</td>
<td>0.4</td>
<td>mH/km</td>
</tr>
<tr>
<td>Cable capacitance</td>
<td>0.17</td>
<td>µF/km</td>
</tr>
<tr>
<td>Length of cables 1, and 2</td>
<td>10</td>
<td>km</td>
</tr>
<tr>
<td>Filter capacitance</td>
<td>3.29</td>
<td>µF</td>
</tr>
<tr>
<td>VSC reactor resistance</td>
<td>0.5</td>
<td>mΩ</td>
</tr>
<tr>
<td>VSC reactor inductance</td>
<td>55</td>
<td>mH</td>
</tr>
<tr>
<td>VSC voltage control gains</td>
<td>$2.5 + \frac{1.25}{s}$</td>
<td>1</td>
</tr>
<tr>
<td>VSC current time constant</td>
<td>0.002</td>
<td>s</td>
</tr>
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</table>
Bibliography


