A SIMULATION OF MAGNETIC SUSPENSION AND BALANCE SYSTEM
FOR WIND TUNNEL

A Thesis in
Mechanical Engineering
by
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Wind tunnels are used to examine the airflow around the model and measure the aerodynamic forces and moments acting on this body. In many studies, the aerodynamic model is supported by mechanical supporting systems which can cause some disturbances on the flow around the model and reduce the accuracy of results. On the other hand, magnetic suspension and balance systems (MSBS) are designed for suspension the model without any mechanical support in wind tunnel tests, which is a very ideal way to deal with undesirable effects of support structures. This study presents the modeling and simulation of the MSBS by COMSOL Multiphysics, AC/DC module, which provides opportunities to generate and analyze magnetic fields and forces produced by magnets and coils for testing different geometries and configurations. It is carried out with an airfoil type object, NACA 0012, for wind tunnel having a 0.47-meter width and 0.3-meter height test section. The system was designed for both the 2-D and 3-D geometry and consisted of four permanent magnets and four electromagnets to control airfoil in the vertical direction. The current through the electromagnets was rearranged for each simulation to determine magnetic force on the levitation of airfoil to balance gravitational and aerodynamic forces on it, which was measured by COMSOL as well. Electromagnetic force on z-direction for the 3-D study was validated with magnetic dipole calculation. Replicated studies were used to verify general the simulation methodology using the same boundary condition for different geometries.
# TABLE OF CONTENTS

LIST OF FIGURES ........................................................................................................... vi

LIST OF TABLE ............................................................................................................... viii

ACKNOWLEDGEMENTS ............................................................................................. ix

Chapter 1 INTRODUCTION ....................................................................................... 1

1.1 Motivation ................................................................................................................ 1
1.2 Magnetic Suspension and Balance System .............................................................. 3
   1.2.1 Coil Configurations ......................................................................................... 3
   1.2.2 Model Position Sensing ................................................................................ 5
1.3 Background .............................................................................................................. 7

Chapter 2 BASIC THEORY ....................................................................................... 13

2.1 Basic Electromagnetic Theory .............................................................................. 13
   2.1.1 Maxwell Equations ....................................................................................... 13
   2.1.2 Magnetic and Electric Fields ....................................................................... 14
   2.1.3 Electromagnets and Permanent Magnets ................................................... 16
   2.1.4 Magnetic Substance ....................................................................................... 17
2.2 COMSOL Multiphysics ....................................................................................... 18
   2.2.1 Introduction .................................................................................................... 18
   2.2.2 AC/DC Module ............................................................................................. 19

Chapter 3 MODELING AND SIMULATION ...................................................... 22

3.1 System Description .............................................................................................. 22
   3.1.1 Model .......................................................................................................... 22
   3.1.2 Aerodynamic Forces and Moments ............................................................. 23
      3.1.2.1 2D NACA 0012 Airfoil Simulation ....................................................... 25
   3.1.3 The Simulation Procedure .......................................................................... 27
   3.1.4 Boundary Conditions .................................................................................. 28
   3.1.5 Model Examination ...................................................................................... 29
3.2 2D and 3D Simulations ....................................................................................... 32
   3.2.1 Inertial Force and Moment ......................................................................... 32
   3.2.2 Electromagnets ............................................................................................ 32
   3.2.3 2D Analysis .................................................................................................. 33
      3.2.3.1 Geometry ............................................................................................... 33
      3.2.3.2 Meshing .................................................................................................. 34
      3.2.3.3 Analysis Results .................................................................................... 35
   3.2.4 3D Analysis .................................................................................................. 37
      3.2.4.1 Geometry ............................................................................................... 37
      3.2.4.2 Meshing .................................................................................................. 38
3.2.4.3 Results of Analysis ........................................................................................................39
3.2.4.4 Validation of Results ........................................................................................................42

Chapter 4 VALIDATION STUDIES .........................................................................................45

Chapter 5 CONCLUSION AND FUTURE WORK ..................................................................49

APPENDIX A .........................................................................................................................51

REFERENCES .........................................................................................................................53
LIST OF FIGURES

Figure 1-1: Schematics of L system on the left and Double-L system on the right [9]..........................4
Figure 1-2: Schematics of V system [9]. ..........................................................................................5
Figure 1-3: ONERA’s system design L configuration (left) and V configuration (right) [11]..............7
Figure 1-4: University of Southampton 7-inch MSBS coil arrangement [5]......................................8
Figure 1-5: One degree of freedom MSBS of NASA Langley [5].......................................................9
Figure 1-6: Schematic of the superconducting MSBS [18]. ...............................................................10
Figure 1-7: Coil configuration of JAXA 60 cm [19] ........................................................................11
Figure 1-8: Coil configuration of 1-m MSBS [27] ...........................................................................12
Figure 2-1: Magnetic fields on solenoid(left) and permanent magnet (right) [53]..............................14
Figure 2-2: Hysteresis loop [53]. ....................................................................................................17
Figure 2-3: All modules of COMSOL Multiphysics [56]..................................................................19
Figure 3-1: Cross-section of airfoil in COMSOL for c=20 cm.......................................................23
Figure 3-2: Forces on the airfoil (left) and axes of the airfoil (right)..............................................24
Figure 3-3: Pressure distribution for the simulation results of flow around NACA 0012 airfoil when
angle of attack equals 0° and 10° respectively. ..............................................................................25
Figure 3-4: The comparison of lift coefficients for NACA 0012 between numerical results by
COMSOL and experimental data from Ladson, NASA TM 4074 for Re=6 million [61].............26
Figure 3-5 The comparison of drag coefficients for NACA 0012 between numerical results by
COMSOL and experimental data from Ladson [61]......................................................................27
Figure 3-6: The arrangement of coils for model examination .........................................................29
Figure 3-7: Examined models ........................................................................................................30
Figure 3-8: Geometry of 2D MSBS ................................................................................................34
Figure 3-9: Meshed geometry of 2D study ......................................................................................35
Figure 3-10: The magnetic flux density [T] distribution on the airfoil and magnets for different angle of
attack ..............................................................................................................................................36
Figure 3-11: Geometry of 3D MSBS ................................................................................................38
Figure 3-12: 3D Mesh structure generated by COMSOL ............................................................38
Figure 3-13: The magnetic flux density norm [T] distribution on airfoil and magnets for different (α)...40

Figure 3-14: Lift and electromagnetic force on z direction for different (α).................................41

Figure 3-15: Magnetic dipole analysis model.............................................................................43

Figure 4-1: System layout of real study on the left and the general simulation result on the right........46

Figure 4-2: Geometry (left) and magnetic flux density results (right) for simulation study.............47

Figure 4-3: Distribution of magnetic flux density in the horizontal direction A) The simulation result from real study [65] B) COMSOL analysis result.............................................................................................................48
LIST OF TABLES

Table 2-1: Magnetic properties of four materials [53] ................................................................. 18
Table 3-1: Lift and drag coefficient of present study for the different angle of attack ...................... 26
Table 3-2: Properties of coils ........................................................................................................ 29
Table 3-3: The dimensions of permanent magnets on the model .................................................. 30
Table 3-4: The comparison of models .......................................................................................... 31
Table 3-5: The comparison of permanent magnets for model 3 .................................................. 31
Table 3-6: Mass of the model ........................................................................................................ 32
Table 3-7: Electromagnetic force on z direction and currents taken from the simulation ............... 37
Table 3-8: The amount of coil currents for different angle of attack ............................................ 39
Table 3-9: Electromagnetic forces and moments on airfoil for all directions ................................. 41
Table 3-10: The comparison of results between COMSOL and Equation (3-4) ............................. 44
Table 4-1: System and coil features [63] ....................................................................................... 45
Table 4-2: Parameters used in the magnetic field analysis of our case taken by real study [65] ....... 47
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CHAPTER 1
INTRODUCTION

1.1 Motivation

Magnetic levitation, a type of levitation that depends on magnetic fields, is one of the most notable phenomena over the last decades. Magnetic levitation (maglev) has led to comprise of a wide range of research areas and has been supported by many innovators to design new industrial technologies. It has been well-known with an advanced transportation system like a high-speed train. Moreover, several systems have been successfully developed and demonstrated in different fields, such as conditioning technologies (magnetic bearing, elevator, fan, compressor, and pumps), spacecraft, analyzing food and beverages, military purposes (rockets and guns), automotive, biomedical devices (heart pumps) and household appliances (lamps, toys) [1].

For magnetically levitated devices, magnetic fields are used to generate mechanical forces against the gravitational force to keep the balance of moving objects. Any support structure is not used during this process. This feature makes essential the maglev systems that can be used for various purposes to reduce limitations of mechanical contacts. Airplane tests in wind tunnels become one of the significant examples of these applications. Magnetic suspension and balance systems (MSBS) are used in the wind tunnels for testing the model that is balanced against gravitational force and unknown aerodynamic forces and torques, which is generated by the relative velocity of the wind on the body, by magnetic forces and torques that are calculated in terms of electric currents. That provides an opportunity to examine the aerodynamic forces and torques on the body.

Using MSBS can eliminate problems of mechanical supporting systems such as struts or stings in the flow field and then increases the accuracy of measuring data. The principal advantage of utilizing magnetic systems is that they prevent distortions near the support rods when fluid flows over the model. The simplest examples of these undesirable consequences are that errors formed while measuring the base
pressure coefficient because of the large base area of support model. Secondary errors can be observed due to disturbance in the wake flow region.

Support errors can occur while areas of separation with coherent structures are getting larger. As shown in reference [2], when a missile is at a high angle of attack, a cylinder-like flow field takes place. The measured coefficient of this study changes based on the support location. Another example of this cases in reference [3], a sharp-edged wing model was supported by three schemes of supporting to compare them. The rolling moment coefficient of this case varies depending on the support scheme as a function of a yaw angle.

Moreover, during operation at the high angle of attacks, the error grows higher as a result of tunnel blockage. Some studies (from reference 4) demonstrate additional errors as results of flow failures in the transonic flow regime. Although there is not supportive free data to compare, researchers determined problems for various support arrangements. Consequently, the errors become larger and in reverse direction by increasing of Mach number within the transonic range.

One of the other advantages of MSBS is an opportunity to use the model for different positions and orientations in the wind tunnel test section. Also, it is capable of lifting various bodies in terms of shape and size without additional mechanic base requirements.

In a dynamic wind tunnel test, the aerodynamic forces and moments of a model are determined while the model is oscillating in the wind tunnel. One of the benefits is that the center of oscillation has the limited location where forced oscillation mechanism is accommodated for mechanically supported systems, on the contrary, oscillation center can be chosen in different locations for magnetic suspension systems. Besides, we can apply simultaneous motions to the model, and these motions can be combined with angle of attack during dynamic-stability testing.

Furthermore, while using MSBS, we can improve the productivity of the wind tunnel. Testing time may become shorter because changing the model position, reopening test section and building
support tools are eliminated for testing different configuration. The quality of data per hour becomes higher owing to the lack of support interference. [5,6,7]

Based on these advantages, the objective of this thesis is to model and simulate the magnetic suspension and balance system for an airfoil in the wind tunnel. COMSOL Multiphysics is used to generate simulation models to examine geometries to decide on forces and moments examined in the test section for different positions and orientations of the magnetized body.

1.2 Magnetic Suspension and Balance System

Magnetic Suspension and Balance Systems (MSBS) provide an ideal method to support models electromagnetically and measure aerodynamic characteristic on the model in the wind tunnel. The technique has the ability to use in wind tunnels for the different type of speeds such as very low subsonic speeds, the transonic speed regime and up to hypersonic speeds. This system can be applied fundamentally to examine drag and lift coefficients, and disruptive model support effects.

Even though there have been various models of magnetic suspension systems over the past years, the simplest MSBS includes a model, coils, a controller, power supply, magnets, an amplifier, and sensors. [8,9] The function of each component can be summarized as follows.

It is probable to magnetize the model in a variety of ways. Models can be asymmetrical or symmetrical shapes and commonly made from non-magnetic materials. For wind tunnel, aerodynamic test models are suspended or levitated in the test section by magnetic fields which are classically generated from ferromagnetic materials in the models like soft iron or a permanent magnet. Applied fields are produced by a design of electromagnets (coils) surrounding the test section. [9,10]

1.2.1 Coil Configurations

Magnetic forces and torques are results of the variations in the magnetic field over the levitated body; hence, coils are differently arranged and constructed to produce and control necessary forces like the drag, lift and lateral. Moreover, required systems can affect the number of coils used in the study. Some examples of previously used coil configuration can explain the situation as indicated below. [9,10]
One of the common illustrations for the coil arrangement in the wind tunnel test section is "L" configuration. There are two electromagnets over the model and two electromagnets on the side as shown in figure 1-1 (left). Each coil is named with the force name used for the counterattack; for instance, the forward lift, aft lift, forward lateral and aft lateral. Also, drag forces are balanced by an air core solenoid which is accommodated upstream of the model and wound concentric with the wind tunnel. [9,11]

The double-L system, in figure 1-1 (right), has been preferred mostly for dynamic stability testing and permanent magnet roll control. It has an extra coil group on the opposite side and can become more beneficial thanks to the ability of positive roll control. Roll torques can be generated via those extra coils without any combination of both lift and pitch by application of a rectangular core, or a double-finned model. [9]

**Figure 1-1:** Schematics of L system on the left and Double-L system on the right [9]

Figure 1-2 illustrates the coils configuration of MSBS that is mostly known as "V" configuration. The V system was invented as an enhancement of the L system to reduce the requirement of lateral bias currents or a two-sided (plus or minus current) lateral power supply. Four electromagnets symmetrically accommodated in pairs at 45 degrees. While doing this, the lift and gravitational forces on the body is provided at the same time. Drag forces are counteracted by an air-core solenoid as in the L configuration. Generally, measuring of force data from coil currents for the V system is more complicated than the L configuration, but it provides a clear observation of models through the test section. [9,11]
Since coils configuration is open-loop unstable in at least one degree-of-freedom, optimally up to the six possible, some automatic control technologies and position sensing equipment are essential to resistance to flow distortions and provide stability in magnetic suspension technologies. Whenever the model moves away from the equilibrium point, the current has to be adjusted to keep the object in the same position/orientation and maintain stability. The controller operates the functions of signal processing and excites the current in the magnet coil according to the Biot-Savart law, the magnetic field produced by a coil is proportional to the current. The specifications of the amplifier define maximum output current which is demanded as high as possible to supply sufficient power besides a minimum loss for the MSBS. [8,9,10]

1.2.2 Model Position Sensing

It is very significant to detect the position, velocity, and acceleration of the model under forces and moments. Sensors are used to sense the attitude of the object and ensure the desired position of it. Some features are demanded from the position sensing systems, as follows:

- They should work without interference from the magnetic fields and without mechanical contact with the model.
- Resolution, repeatability, and frequency response should be adequate to provide accurate results.
• Compact systems are necessary and should not interfere with the test section.
• They should be easily adjustable to different geometries.
• Translational and angular displacements of the model axes about the wind tunnel should be converted to independent high-level electrical signals. [8,9]

Although there are various types of position detectors, optical, capacitive and Hall effect sensors are extensively utilized in magnetic suspension systems.

The position of the levitating object can be determined optically. The optical sensor works according to the amount of the ray of light. While the levitating object is moving away from the electromagnet, more light is detected via the sensor, and the current of the electromagnet is increased. Optical sensors have high sensitivity, and enough resistance to work at varied temperatures. For proper installation of these systems, exposing linear light is vital when the object rises and falls to take the best result. Therefore, the way of the light source, light sensor and the shape of the model are very significant.

One of the most popular sensing systems is capacitive which is used widely for maglev because of its simple technique and linearly operation. A conducting plate can be placed between the levitating object and the reference point. The capacitance connecting the model and plate might be sensed and used to determine the distance between the plate and coils. These sensors are used to determine the separation or gap between a model and reference point, but not absolute position.

Another common position sensor is a Hall effect. The work function of Hall effect element is based on measuring the magnetic field of a specific point. When the fixed magnet model approaches the Hall sensor, the magnetic field strength raises, and the output of the Hall device increases. As the changes in magnetic field strength, the Hall device generates a voltage which changes according to the location of the model position. These sensors also have some disadvantages such as less thermal ability, slow responses, less durability, and sensitivity. [12,13,14]

The changes in the exact position and attitude of models led to the development of further technology during testing in the wind tunnel. For instance, absolute model position monitors can be used in addition to regular sensors to confirm the certain position of the model. [9]
1.3 Background

There are significant examples of magnetic suspension and balance systems (MSBS) for the testing of aircraft in wind tunnels. MSBS have been constructed in many countries including England, France, United States of America and Japan. Some of remarkable research centers working on these systems are University of Southampton, MIT, AEDC/NASA Langley, Oxford University.

The very first MSBS was developed in France by ONERA (Office National d'Etudes et deRecherches Aerospatiales) in 1951. Measuring the drag of rotating sphere was the first aim of that study. After then, using that idea in wind tunnel was found and, Tournier and Laurenceau published the first report in 1957. Several MSBS were constructed by the French for many years. They built the system which had “L” configuration and then “V” configuration system illustrated in Fig.1-3. Both of these arrangements could test the body in five degrees of freedom. An optical position sensor with photocells was used to determine model position. These systems worked adequately for simple models, for instance, wake and drag experiments up to intermediate supersonic Mach numbers; furthermore, they measured base pressure and heat transfer rate. [5,15].

**Figure 1-3:** ONERA’s system design L configuration (left) and V configuration (right) [11]
University of Southampton, England, has become a very influential foundation for MSBS. The first research was launched in 1959, and the production of MSBS was started in 1962. The 7-inch MSBS at Southampton has been worked on many studies for subsonic and supersonic speeds since 1964. It was used for static and dynamic wind tunnel tests. Figure 1-4 illustrates that the test section consists of ten electromagnets to control forces and moments on the body. The electromagnets 1, 3, 5 and 7 were used to control lift and gravity forces on the body. The drag force in the way of the wind was controlled with coil 9 and 10. The electromagnets 2, 4, 6 and 8 were applied to control side forces and moments. [5,15,16]

![Coil Arrangement](image)

**Figure 1-4:** University of Southampton 7-inch MSBS Coil Arrangement [5]

Researchers from the Langley Research Center started to study the application of magnetic suspension for the support of a body to simulate free flight after French researchers published their studies in 1957. A pilot suspension system started to develop in 1962 in order to achieve some purposes. The first objective was the demonstration of the feasibility of magnetic suspension system which can be used as the experimental investigation of airfoil. The second object was control of large electrical currents to supply enough magnetic field for using MSBS. The First operating system built in the Langley Research Center in 1964 illustrated in figure 1-5. This model consists of single air-cored, dual-wound coil. Beside
the levitated model was located right under this coil. The system has one degree of freedom and the control was supplied only in vertical direction. [5]

![Diagram of wind tunnel setup](image)

**Figure 1-5: One degree of freedom MSBS of NASA Langley [5]**

Johnson and Dress from the NASA Langley Research Center introduced a 13-inch for subsonic wind tunnel operating at Mach numbers up to 0.5. It contained the four lift electromagnets, arranged in a "V" configuration and above the wind tunnel test section, which supply the lift force, pitching moment, side force, and yawing moments. The drag force was provided by the solenoid. The 13-inch MSBS had a capability of lifting a few pounds that were based on the size and shape of the iron core in the system [5, 17].

In the beginning stages, the majority of MSBS were constructed for small applications working low level of Reynolds numbers and at Mach numbers no higher than low supersonic (< 2.5). After a while, it has been required building new systems in the large wind tunnel to operate for ultra-high Reynolds numbers and different range of speeds testing.

To work in the wind tunnel at ultra-high Reynolds number, using liquid helium as the working fluid has been under examination for several years. In [18], Smith et al designed a superconducting
magnetic suspension system to measure drag on rotationally symmetric bodies while using liquid helium. The general configuration contains vertical support coils, drag coils and sensor coils in the test section. The vertical support coils, a counter-wound racetrack configuration, are used to generate a quadrupole field on the model perpendicular to the flow. The drag coils, Helmholtz pairs, are arranged to produce a particular magnetic field and field gradient the same way of the flow. Sensor coils positioned through the flow direction to identify a flux change at the time when the levitated model is enabled to deflect a small amount of distance under the applied drag force. These coils could be counter-wound to enhance the flux difference through the circuit and reduce the entire inductance of the sensor coil pair.

Figure 1-6: Schematic of the superconducting MSBS [18]

One of the largest magnetic suspension and balance systems (MSBS) all around the world was developed at the National Aerospace Laboratory (NAL) in Japan. Sawada and his group built and operated firstly the 60cm MSBS in 1993 and was controlled in 3 degrees of freedom in the longitudinal direction. Magnetic forces and moments on the model are controlled with coil currents through 8 electromagnets and 2 air cored coils which are shown in Figure 1-7 [19].

Following this, the system has been used in many experiments; for example, MSBS was used for low-speed wind tunnel in 1999 and the model can be controlled in 5 or 6 degrees of freedom [20]. Different types of shafts without fletching and many kinds of Japanese arrows were investigated to
understand the aerodynamic characteristics of Japanese arrows in a low-speed wind tunnel, in [21]. Sawada et al claimed that the viscous drag is the main drag component action on the arrows and could be estimated analytically. A simple model was suggested to describe the results of fletching on a rotating arrow, and the lift acting on an arrow. They found that lift depends on the angle of attack but not rotation angle or speed. Moreover, fletching and the drag from the fletching produced principally lift and pitching moment. Other studies by Sawada and his group are in references [22], [23], [24], [25], [26].

![Coil configuration of JAXA 60 cm](image)

**Figure 1-7:** Coil configuration of JAXA 60 cm [19]

The largest MSBS in the world was introduced by the Institute of Fluid Science (IFS), Tohoku University, in 2015. The system comprises 1-m MSBS with a 1.01-m wide test section. This perfect wind tunnel works at relatively high Reynolds number and low turbulence intensity. Ten coils are accommodated in the test section as illustrated in figure 1-8. Due to the large section, the coils are far away from each other. Hence, coils are connected to each other (# 1 to # 4 and # 5 to # 8) using the yoke to generate an effective magnetic field. A large permanent magnet, main magnet of the arrangement, is used with the model for testing at the 1-m MSB. This magnet has a capacity of five-degrees-of-freedom
control. Small magnets, known as auxiliary magnets, are combined with the main magnet to control system in six-degrees-of-freedom. Five- or six-line sensor cameras using LED light sources and optical systems are used to control position and orientation of the body. [27]

![Figure 1-8: Coil configuration of 1-m MSBS](image)

There are many other examples of magnetic suspension-based wind tunnel studies and further information about MSBS found in references between [28] and [52].
CHAPTER 2

BASIC THEORY

2.1 Basic Electromagnetic Theory

The design and implementation of magnetic suspension systems are primarily based on the concepts of electromagnetism. Therefore, fundamental principles and expressions of electromagnetism summarized in this part. The governing equations associated with our simulation by COMSOL are presented as well.

2.1.1 Maxwell's Equations

Although all electromagnetic phenomena can be described as being derived from the forces acting between still or moving point charges (electrons and protons) at the macroscopic level for engineering science, four short equations as known Maxwell equations are adequate to explain easily all identified macroscopic field interactions.

Maxwell’s equations are set of partial differentials or integrals used for describing the relationships between the fundamental electromagnetic terms; for instance, the creation of magnetic fields, electrical currents, and point charges.

For general time-varying fields, differential form of Maxwell’s equations can be stated as:

- Maxwell-Ampere’s law

\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \]  \hspace{1cm} (2.1)

- Faraday’s law

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]  \hspace{1cm} (2.2)

- Gauss’ law, electric

\[ \nabla \cdot \vec{D} = \rho \]  \hspace{1cm} (2.3)
• Gauss’ law, magnetic

$$\nabla \cdot \vec{B} = 0$$  \hspace{1cm} (2.4)

To explain the force of electromagnetism, we need to introduce two vector fields which are the magnetic field \(H \) and electric field \(E \).

### 2.1.2 Magnetic and Electric Fields

Magnetic Field: \(H \) represents magnetic field which is vector field generated by moving charge (q) and has a relationship with electric current. \(H \) is measured in the unit of ampere per meter (A/m).

Permanent magnets and solenoids are used commonly to create magnetic fields by electric charges inside them. Magnetic field lines start from the North pole (N) and end the South pole (S) continuously, as in Figure 2-1.

![Magnetic fields on solenoid (left) and permanent magnet (right)](image)

**Figure 2-1:** Magnetic fields on solenoid (left) and permanent magnet (right) [53]

Magnetic Induction: \(\vec{B} \) is the magnetic induction, it is response of the material when magnetic field is applied on it. It is also known as a magnetic flux density. The units of the magnetic induction in the SI system are tesla (T) or newtons per ampere (N/A). The correlation between magnetic field and magnetic flux density:

$$\vec{B} = \mu \vec{H}$$  \hspace{1cm} (2.5)
\[ \mu = \text{magnetic permeability} \] is the measure of magnetizable material degree. It is unit in henries per meter (H/m), or in newtons per ampere squared (N/A²).

Magnetization: The magnetization vector \( \vec{M} \) also describes the material which is magnetized under the magnetic field. \( M \) is generally a function of \( H \) and can be calculated by equation 2.6,

\[ \vec{B} = \mu_0 (\vec{H} + \vec{M}) \]  

(2.6)

\( \mu_0 = \) the magnetic permeability of free space and it is the value \( 4\pi \times 10^{-7} \)

\( \mu_r = \) denotes the relative permeability which is dimensionless ratio. It is the ratio of the permeability of a material to that of space (or vacuum). Relation between magnetic and relative permeability is,

\[ \mu_r = \frac{\mu}{\mu_0} \]  

(2.7)

Magnetization \( M \) is also described as the total magnetic moment (m) per volume(V) unit:

\[ M = \frac{m}{V} \]  

(2.8)

Electric Field: The vector \( \vec{E} \), the electric field strength, is related to voltage and is generated by the presence of electric charges (electrons). Electric field intensity must be measured by the unit newtons per coulomb (N/C), the force per unit charge or volts per meter (V/m).

Electric flux density: The electric flux density \( \vec{D} \) is related to the electric field by the constitutive relation:

\[ \vec{D} = \varepsilon \vec{E} \]  

(2.9)

\( \varepsilon = \) is the electric permittivity which is related with medium that the field exists. The values of this quantity in free space (vacuum), it is also known as the electric constant, are important physical constants as known in SI units,

\[ \varepsilon_0 = \frac{1}{\mu_0 c^2} \]  

(2.10)

where \( c \) is the speed of light in vacuum. It is measured in Farads per meter (F/m) and it has a value of approximately \( 8.854 \times 10^{-12} \) F/m. The permittivity of a material identifies the response of that material to the electric field. In order to simplify models, the permittivity and permeability are usually considered as
constants for a given material, but in actuality, their values depend on the fields which are present. Note that the relative permittivity $\varepsilon_r$, which is dimensionless quantity, are also generally used and defined by equation 2.11,

$$
\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}
$$

(2.11)

The quantities $\rho$ and $J$ are the electric charge density and electric current density respectively. Charge density illustrates electric charge per volume and the unit of its coulombs per cubic meter. Electric current density describes the electric current per unit cross-section area and is measured in amperes per square meter. It can be related proportionally to the electric field, as given by Ohm’s law,

$$
\vec{J} = \sigma \vec{E} = \frac{\vec{E}}{\rho}
$$

(2.12)

$\sigma$ : is the electrical conductivity which is inverse of electrical resistivity and units of conductivity is siemens per meter (S/m). Conductivity describes the ability of materials to conduct an electric current.

### 2.1.3 Electromagnets and Permanent Magnets

The solenoid is a long cylindrical coil which has a large number of closely spaced of wire which electric current flows in order to create a magnetic field similar to a bar magnet. The magnetic field is nearly uniform inside the solenoid and close to zero outside and its direction depends on the direction of the current. Ampere Laws are used to calculating the magnetic effect in free space caused by a current in a wire.

An electromagnet, used in magnetic levitation systems most widely, is a type of solenoid which has a central magnetic core made from ferromagnetic materials such as iron. The ferromagnetic property concentrates the magnetic flux and makes a more powerful magnet.

Magnets are magnetic materials having permanent and aligned atoms to generate its own persistent magnetic field when magnetized by an external magnetic field. This magnetic field is usually generated by a current which carries through a coil in a magnetic circuit. The remarkable characteristic of a permanent magnet is that continuous power supply is not necessary. Also, it beneficial for using MSBS because of very simple configuration and lower maintenance costs.
2.1.4 Magnetic Substance

Some materials such as iron, nickel, cobalt and most of their alloys have the ability of magnetization during the attraction of magnetic field. These materials are also known as a ferromagnetic material and they have the greatest capability for magnetism. A magnetic attitude of ferromagnetic materials is not constant due to the permeability. Paramagnetic materials, aluminum, gold, and copper are metals attracted weakly by magnets. Diamagnetic materials do not respond to an applied magnetic field and their relative permeability is less than one. A few examples of diamagnetic materials are almost all organic substances and metals like mercury.

![Hysteresis Loop Diagram](image)

**Figure 2-2:** Hysteresis loop [53]

Magnetic materials are identified by magnetic hysteresis loop, the result of H-B relationship for all applied magnetic field, called as the main loop in the figure 2-2. When an external magnetic field is exposed to these materials, they need to be magnetized continuously. For some materials, there is a nonlinear relationship between magnetic field intensity H and the flux density in the materials B. When H increases, B raises through the initial magnetization curve until a value that is saturation flux density (Bsat). When magnetic field strength decreases to H=0, magnetic flux density drops to the remanence point, Br in Tesla (T), also known as a remanent flux density. Briefly, it is the amount of magnetic flux density when the external magnetic field strength is zero and the highest magnetization of magnetic
materials changing from 0.2 to 1.4T as shown in table2-1. Demagnetization curve occurs when the value of reverse magnetic field strength $H$ increases to reduce the magnetization to zero. This value of $H$ is intrinsic coercivity, $H_c$ (A/m). Moreover, increases of $H$ cause that magnetization reaches saturation in the reverse direction. Ferromagnetic materials are divided into two types of materials. Soft ferromagnetic materials, such as iron, have very narrow hysteresis loop and the magnetic flux density $B$ can be easily changed by external magnetic field strength $H$. Hard magnetic materials, like permanent magnets, are materials that the magnetic flux density $B$ is hardly influenced by the magnetic field. [53,54,55]

**Table 2-1:** Magnetic properties of four materials[53]

<table>
<thead>
<tr>
<th>Material</th>
<th>$B_r$ (tesla)</th>
<th>$H_c$ (tesla)</th>
<th>Relative Recoil Permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neodymium magnet (Nd-Fe)</td>
<td>1.0-1.5</td>
<td>1.11</td>
<td>1.9-7</td>
</tr>
<tr>
<td>Samarium Cobalt Magnet (SmCo)</td>
<td>0.7-1.05</td>
<td>0.69</td>
<td>1.05-1.15</td>
</tr>
<tr>
<td>Ferrite</td>
<td>0.35-0.53</td>
<td>0.29</td>
<td>1.02-1.07</td>
</tr>
<tr>
<td>Alnico</td>
<td>0.6-1.35</td>
<td>0.16</td>
<td>1.04-1.1</td>
</tr>
</tbody>
</table>

2.2 COMSOL Multiphysics

2.2.1 Introduction

Computer simulations are very significant for scientists and engineers while developing and designing new products to reduce prototyping costs. COMSOL Multiphysics, a finite element software, gives an opportunity of simulation of extensive variety of scientific and engineering problems based on partial a differential equation. It can be used not only for core modules but also for a combination of these modules such as electromagnetics, structural mechanics, acoustics, fluid flow, heat transfer, and chemical engineering behavior while modeling. Users can easily define the necessary parameters, boundary conditions, and initial conditions for the projects. COMSOL offers several study types such
as stationary, time-dependent and eigenvalue. We can employ different solver algorithms depending on the problems to solve either linear systems or non-linear systems.

Further, COMSOL’s graphical user interference supply creating various models for all direction in 1D, 2D or 3D. It consists of an advanced mesh generation capability. It obtains quite useful material library with physical properties and several examples of models to compare and verify techniques. [56]

![Figure 2-3: All modules of COMSOL Multiphysics](image)

### 2.2.2 AC/DC Module

The AC/DC Module includes many sub-categories of physics interfaces in order to solve problems about electrostatics, electric currents, magnetostatics, and time-varying electromagnetic fields including induction effects. Moreover, this module can merge with different interfaces to achieve extensive modeling functions. When we consider all reasons above, the COMSOL Multiphysics software is advantageous in terms of modeling and testing advanced magnetic levitation devices. [57]

The governing equations behind the AC/DC module are highlighted, especially applied equations to solve the analysis of our problem. Maxwell equations are fundamental to analyze
electromagnetic problems on macroscopic level depending on exact boundary conditions, in addition to that, continuity equation is essential to solving problems for AC/DC Module.

- The equation of continuity:

\[ \nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \]  \hspace{1cm} (2.9)

Designing the Multi-Turn Coil is a requirement for MSBS. Features of coils can implement to COMSOL software like real coil consisting of numerous tightly-wound conducting wires, separated by an electrical insulator. Therefore, parameters including the number of wires and the cross-section area are needed to calculate current density by equation 2.10;

\[ J_e = -\frac{NI_{coil}}{A} \]  \hspace{1cm} (2.10)

\( I_{coil} \) is a total current of the component and \( N \) the number of turns which are specified by a user. \( A \) is the total cross-section area of the coil domain.

There are many methods for computation of electromagnetic forces in COMSOL Multiphysics. One way is that the computation of magnetic forces on nonmagnetic, current-carrying domains utilizes a predefined physics interface variable for the Lorentz force distribution in a magnetic flux density. Electromagnetic forces’ calculation includes the computation of volume forces applying a model, and of surface forces comes from electromagnetic fields on the boundaries. The most general way is Maxwell stress tensor which is an integral computed over the exterior surfaces that are an arbitrary closed path (2D) or closed surface area (3D). This is the default method for COMSOL Multiphysics along with the AC/DC Module. The force calculation includes both magnetic and electric force. Maxwell’s stress tensor might be used generally for electromagnetic contexts such as electrostatics, electric currents, magnetic fields, and magnetic and electric fields interfaces for force and torque calculations. [57]

The magnetic force \( F \) and torque \( \tau \) as a boundary integral of the stress tensor in vacuum (\( \mu=1 \)) can be computed based on the following equations:
\[ F = \int_{\partial \Omega} nT dS \quad (2.11) \]

\[ \tau = \int (r - r_0) \times (nT) dS \quad (2.12) \]

\[ \tau_{ax} = \frac{r_{ax}}{|r_{ax}|} \cdot \tau \quad (2.13) \]

“\( T \)” is Maxwell’s stress tensor and “\( n \)” is a normal vector on surface area. “\( r \)” is a direction vector which indicates coordinates of a material position and “\( r_0 \)” is a point on the axis of rotation. “\( r_{ax} \)” describes a vector for the torque axis.

In the Magnetic Fields interface, the expression of stress tensor [57]

\[ nT = -\frac{1}{2} n(H \cdot B) + (n \cdot H)B^T \quad (2.14) \]

or,

\[ nT = \frac{1}{\mu_0} \left( -\frac{1}{2} nB^2 + B(B \cdot n) \right) \quad (2.15) \]
CHAPTER 3

MODELING AND SIMULATION

3.1 System description

3.1.1 Model

An aerodynamic test model is located in the test section of the wind tunnel to be suspended magnetically against a flow without mechanical supports for the magnetic suspension and balance systems (MSBS). This study has been performed for the wind tunnel whose test section dimensions are identified approximately a 0.47-meter width and 0.3-meter height. As illustrated in figure 3-1, we have employed an airfoil type object, which is the geometry of aircraft wings.

The National Advisory Committee for Aeronautics (NACA) developed a database for different shape airfoils. The features of these airfoils define with a series of numbers following the word "NACA" to create the cross-sectional geometry of airfoil and calculate its properties. The particular airfoil geometry, NACA 0012, is chosen to carry out this research. NACA 00 has a meaning that it has a symmetrical structure and does not have camber. For the NACA 0012, the maximum thickness is equal to 12% chord length. In order to create the symmetrical NACA airfoil geometry in COMSOL, following equation (3.1) is applied,

\[
y = \pm c \cdot 0.594689181 \left[ 0.298222773 \sqrt{\frac{x}{c}} - 0.127125232 \left( \frac{x}{c} \right) \right. \\
\left. - 0.357907906 \left( \frac{x}{c} \right)^{2} + 0.291984971 \left( \frac{x}{c} \right)^{3} - 0.105174696 \left( \frac{x}{c} \right)^{4} \right] \tag{3.1}
\]

In this equation, \( c \) is chord length, \( x \) is the coordinate value between 0-\( c \) and \( y \) is the half thickness of airfoil given \( x \) value [58]. For this research, chord-length is defined as \( c=20 \text{cm} \).
3.1.2 Aerodynamic Forces and Moments

It is significant that understanding airfoil and its relationship with working flow which is necessary for the modeling and simulation of MSBS to indicate proper design of components. Particularly, whole-body forces and moments on the winged model should be evaluated in order to calibrate and design electromagnets to provide suspension and balance function.

When fluid flows over the airfoil, it creates two mechanical forces (Figure 3.2 on the left) on the surface of the body. One of them acts throughout a perpendicular to wings and it is called a lift force whose pressure distribution on the surface causes it. The drag force acts on the airfoil in the opposite direction of flow and it consists of pressure and friction forces.

\[
\text{Lift force } = L = 0.5 \times C_L \times A \times \rho \times U^2 \tag{3.1}
\]

\[
\text{Drag Force } = D = 0.5 \times C_D \times A \times \rho \times U^2 \tag{3.2}
\]
- $A =$ projected frontal area of the object in the direction of the flow
- $\rho =$ density of the fluid
- $u =$ velocity of the fluid in the direction of the fluid flow
- $C_D =$ The drag coefficient is quantity of the drag on objects in the fluid.
- $C_L =$ The lift coefficient is derived from pressure distribution on the top and bottom surface of the airfoil and is related to the angle of attack ($\alpha$), the angle between the body's reference line and the oncoming flow, and the flow velocity.

In addition to controlling lift and drag force in the test section, we require adjusting aerodynamic moments on the airfoil. These moments are defined by the names of aircraft rotational axes, perpendicular to each other, as a roll, pitch and yaw movements showed in fig. 3-2 on the right. The pitching moment of an airfoil is the torque generated by aerodynamic forces on the y-axis, parallel to the airfoil. The yaw moment is a torque about the z-axis and when the nose of the airfoil moves to the right, the yaw moment is a positive direction. For basic calculations, it can be assumed that the rolling moment is about the roll axis, the longitudinal axis or x-axis. This axis goes from the nose to the tail of the airfoil and moment is positive for a right turn. [59]

Figure 3-2: Forces on the airfoil (left) and axes of the airfoil (right)
3.1.2.1 2D NACA 0012 Airfoil Simulation

This section presents a simulation of turbulent flow with COMSOL Multiphysics around NACA 0012 airfoil.

In order to get the lift and drag data, the air-flow around a two-dimensional (2-D) NACA 0012 airfoil was simulated using the SST turbulence model. For getting further information, CFD Module User’s Guide in reference [60]. It was considered that at the flow relative free-stream velocity is \( U_\infty = 37 \text{[m/s]} \). The properties of air are defined as density 1.2043\([\text{kg/m}^3]\) and dynamic viscosity \( 1.81 \times 10^{-5} \text{[kg/(m.s)]} \). As a result, Reynold number based on the chord length, \( c = 20 \text{ cm} \), is calculated almost \( 5 \times 10^5 \).

The study was performed a Parametric Sweep with the angle of attack (\( \alpha \)) while taking the values, \( \alpha = 0^\circ, 2^\circ, 4^\circ, 6^\circ, 8^\circ, 10^\circ \).

![Figure 3-3](image)

**Figure 3-3**: Pressure distribution for the simulation results of flow around a NACA 0012 airfoil when angle of attack equals 0\(^\circ\) and 10\(^\circ\) respectively.

The plots generated by COMSOL in Fig.3-3 demonstrate the pressure distribution on the surface of the model. According to results, angle of attack has ability to control the distribution of pressure top and bottom of the airfoil. While alpha is changing from 0\(^\circ\) to 10\(^\circ\), the amount of pressure on the airfoil increases. Understanding pressure distribution is necessary since all the forces acting on the airfoil is
results of it. In order to use calculation of aerodynamic forces and moments on the airfoil, numerical results of drag and lift coefficients are listed in table 3.1. These forces are considered to balance with magnetic forces which changes in the amount and direction of the currents of electromagnets for our case.

Table 3-1: Lift and drag coefficient of present study for the different angle of attack

<table>
<thead>
<tr>
<th>Angle of Attack</th>
<th>Lift coefficient($C_L$)</th>
<th>Drag Coefficient ($C_D$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.0026</td>
</tr>
<tr>
<td>2</td>
<td>0.2075</td>
<td>0.0033</td>
</tr>
<tr>
<td>4</td>
<td>0.4136</td>
<td>0.0054</td>
</tr>
<tr>
<td>6</td>
<td>0.6157</td>
<td>0.0089</td>
</tr>
<tr>
<td>8</td>
<td>0.8105</td>
<td>0.0139</td>
</tr>
<tr>
<td>10</td>
<td>0.9919</td>
<td>0.0207</td>
</tr>
</tbody>
</table>

Figure 3-4: The comparison of lift coefficients for NACA 0012 between numerical results by COMSOL and experimental data from Ladson, NASA TM 4074 for Re=6 million [61]
Figure 3-5: The comparison of drag coefficients for NACA 0012 between numerical results by COMSOL and experimental data from Ladson [61]

3.1.3 The Simulation Procedure

The below-mentioned steps are performed in order to simulate both 2D and 3D systems based on the finite element method.

- In the Model Wizard, select model space dimension (“2D” for two-dimensional and “3D” for three-dimensional study).
- Select physic “magnetic fields (mf) interface” under the AC/DC module.
- From the study window, select a stationary study type.
- Define parameters which are necessary for study.
- Create geometry
- Add materials and define additional the required properties of material such as the permeability or remanent flux density of materials that are assigned for the geometry.
- For magnetic fields interface, adjust default boundary conditions and add additional boundary conditions.
3.1.4 Boundary Conditions

The most proper method for simulation of MSBS is "Magnetic Interface (mf)" under the AC/DC Module. This interface supplies opportunity of computation for the magnetic field and current distributions in and around coils, conductors and magnets.

While adding this physics interface to the model window, three default boundary conditions (Ampère’s Law, Magnetic Insulation and Initial Values) come out automatically under the interface part to define the basic principles and properties to calculate the magnetic field. Using this interface in simulations, the geometry domains of the system should be assigned with suitable boundary conditions.

The magnetization of magnets is indicated by adding additional “Ampere’s law” nodes. Additionally, Constitutive relation, under the setting windows, should be changed to remanent flux density and Br entered in the z-direction.

In the setting window of coils, we can arrange coil type (single or multi-turn coil), coil excitation (voltage or current), coil current, conductor parameters like the number of coil wire and coordinate system. Coil module sets up for the multi-turn coil for the copper to generate the external magnetic field. Constitutive relation of coils is selected as the relative permeability. In here, the domain of coils should link to correct material because permeability is taken from material properties. The boundary condition for each coil is built separately to control the current of them independently. In order to calculate magnetic forces on the model, “Force calculation” node is added to the model builder window. The geometry domain of magnets is selected for calculation.
3.1.5 Model Examination

To create the most proper design for magnetic suspension and balance system, several models have been examined when angle of attack equals zero, and some of them are demonstrated in this part. All systems consist of ten electromagnets as shown in figure 3-6.

![Diagram of coil arrangement](image)

**Figure 3-6:** The arrangement of coils for model examination

The drag force in the way of the x-axis was controlled with air-core coil 1 and 2. The electromagnets 3, 4, 5 and 6 operated to control side forces and moments. The coils 7, 8, 9 and 10 were used to balance lift and gravity forces on the body. All properties of coils are listed in table 3-2.

<table>
<thead>
<tr>
<th>Table 3-2: Properties of coils</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Coil Number</strong></td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>Drag Coil</td>
</tr>
<tr>
<td>Side Coil</td>
</tr>
<tr>
<td>Lift Coil</td>
</tr>
</tbody>
</table>
All models have the same features of coils and permanent magnets. Four models are illustrated in figure 3-7. The material of permanent magnets for models is neodymium magnet, which its remanent flux density is 1 Tesla. Dimensions of the magnets on the airfoil are shown in table 3-3.

**Figure 3-7**: Examined models

<table>
<thead>
<tr>
<th>Magnet's</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length [cm]</td>
<td>12</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Depth [cm]</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Heights [cm]</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Mass and center of gravity can be measured easily by COMSOL Multiphysics. A mass properties module under definitions in the model builder window is added to study. This module works with the integration of geometry domain and requires additional density information by the user. The mass of all models is calculated to evaluate all models according to their weight, and electromagnetic forces on the model for all direction.

**Table 3-4:** The comparison of the models

<table>
<thead>
<tr>
<th></th>
<th>Weight (N)</th>
<th>Lift Force (N)</th>
<th>Magnetic force, z component (N)</th>
<th>Magnetic force, x component (N)</th>
<th>Magnetic force, y component (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>18.565</td>
<td>0</td>
<td>17.898</td>
<td>26.795</td>
<td>-3.889</td>
</tr>
<tr>
<td>Model 2</td>
<td>13.962</td>
<td>0</td>
<td>13.584</td>
<td>-8.264</td>
<td>4.123</td>
</tr>
<tr>
<td>Model 3</td>
<td>13.549</td>
<td>0</td>
<td>13.455</td>
<td>-4.733</td>
<td>0.291</td>
</tr>
<tr>
<td>Model 4</td>
<td>12.292</td>
<td>0</td>
<td>12.144</td>
<td>-8.622</td>
<td>-4.025</td>
</tr>
</tbody>
</table>

When we analyze all results, the first model is the heaviest model and creates a large amount of magnetic force on the x-direction. On the other hand, the model 3 and 4 are the lightest models and model 4 generates a higher magnetic force than model 3. As a result, the third model shows most proper performance and is used for the further investigation which neodymium magnet was replaced with ferrite magnet and then results in table 3-5 are compared.

**Table 3-5:** The comparison of the permanent magnets for model 3

<table>
<thead>
<tr>
<th></th>
<th>Total Mass (kg)</th>
<th>Magnetic force, x component (N)</th>
<th>Magnetic force, y component (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neodymium magnet</td>
<td>1.381</td>
<td>-4.473</td>
<td>0.291</td>
</tr>
<tr>
<td>Ferrite Magnet</td>
<td>1.009</td>
<td>0.311</td>
<td>-0.157</td>
</tr>
</tbody>
</table>
To study on the 2D and 3D simulation in the next parts, the third model is the most appropriate design. Besides, ferrite magnet is used as permanent magnet and these magnets are relocated for the best results.

### 3.2 2D and 3D Simulations

#### 3.2.1 Inertial Force and Moment

Determination of inertial force and moments is necessary to design MSBS in addition to aerodynamical forces and moments. It is important that mass of the model should be lightweight to levitate it easily. Therefore, material type of the airfoil should be chosen carefully. In our study, we assume our simulation model is made by one of the most popular plastics which is ABS (Acrylonitrile Butadiene Styrene).

It is assumed that airfoil has the density of 1040 kg/m$^3$ and the density of magnets is 5000 kg/m$^3$. The values of the mass for the 3-dimensional aerodynamic model are shown in table 3-6. The weight of the model is calculated by using Newton’s Second Law as approximately 9.91 Newton.

<table>
<thead>
<tr>
<th>Contents</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of airfoil [kg]</td>
<td>0.829</td>
</tr>
<tr>
<td>Mass of magnets [kg]</td>
<td>0.180</td>
</tr>
<tr>
<td>Total Mass [kg]</td>
<td>1.009</td>
</tr>
</tbody>
</table>

#### 3.2.2 Electromagnets

For our simulation models, we apply the classical approach to achieve levitation of the airfoil. In this method, the magnetic field and deflecting or attractive force between the poles are created by magnets and electromagnets (ferromagnetic core coils). It is expected the MSBS should have the ability to
counterbalance the weight and aerodynamic force, especially lift force model with the magnetic forces generated by the four permanent magnets.

The test section is surrounded by four coils where located above and under the airfoil. Coils are designed as a combination of iron core and multi-turn copper wires. The iron core is supposed to have the relative permeability of $\mu_r = 5000$. The cross-section area of the copper wires is defined as “User Defined” and entered to 1.12 [mm] as a wire diameter. There are 1000 number turns around the iron core and the properties of copper chosen as default by COMSOL. Coils have a length of 3 [cm] copper wire and 3 [cm] iron core, and 6 [cm] in width and their heights are 6 [cm]. The size of coils and properties of materials are maintained constant for all simulation studies.

Ferrite is material of permanent magnets that are placed on the top and bottom of the airfoil. The remanent flux density of the magnets remains constant at 0.35 Tesla for magnetization. They are identified by setting remanent flux density values in the z-direction. After performing with the different size and geometry of permanent magnets, we consider the dimensions of magnets 3 [cm] in length, 3 [cm] in width and 1 [cm] in height in order to reduce the mass of the system.

### 3.2.3 2D Analysis

#### 3.2.3.1 Geometry

This section is about modeling and analyzing of the two-dimensional model examined using magnetic field (mf) modules. The model geometry of the 2D MSBS is shown in Fig. 3-8. It was modeled in an r-z plane where the z-direction shows the heights of the geometries and the r-direction denotes the width of objects. All the length units used in geometry are in cm, while angular units are in degree. The Geometry of MSBS consists of four basic domains which are magnets, airfoil, coils and surrounding air. The airfoil model was suspended in the center of the test section parallel to the airflow.

The aerodynamic model included four permanent magnets in order to react to the magnetic field generated by the four coils. Permanent magnets were placed on the airfoil by considering the pressure
distribution and gravitational force which were taken from simulated results. Big rectangles are used for building the geometry of the multi-turn coil and small rectangles denote magnets. These pairs of coils provide the necessary control of the model in all z directions. The parameters of the system such as the number of coil turns, and the length of coils and magnets were described in previous sections.

![Figure 3-8: Geometry of 2D MSBS](image)

3.2.3.2 Meshing

The next step is mesh creation after defining geometry, material and boundary conditions. Meshing geometry is a significant part of the simulation process to achieve reasonable results. COMSOL provides two mesh sequence types: the physics-controlled mesh and user-controlled mesh. For two-dimensional model of our study, physic-controlled mesh is preferred as a sequence type because of the simplicity of the geometry. The mesh is modified automatically to the current physics settings in the geometry model. The mesh element size is selected extremely fine size for the mesh. Figure 3-9 illustrates the mesh of the system for 2D MSBS.
3.2.3.3 Analysis Results

Analysis results are obtained to achieve the required magnetic force against lift force and gravity. The magnetic flux distribution of system in $x - z$ plane is separately illustrated in Fig.3-10 for all angle of attacks for 2D study. Magnetic flux density is expressed by the surface plot and color bar. Besides arrows display the direction of magnetic flux. We can see from the plots that the amount of magnetic field increases while alpha and lift is getting larger. The bottom-left coil has the higher amount of current than the others, and it creates more magnetic field to the magnet placed on the bottom of airfoil as is shown in plots.

Figure 3-9: Meshed geometry of 2D study
Figure 3-10: The magnetic flux density [T] distribution on the airfoil and magnets for different angle of attack
Electromagnetic forces and torques are calculated as a function of currents in coils and results obtained through the analysis are listed in table 3-7. The estimated value of the current changes from 0 A to +120 A. It is clearly seen that the direction and amount of current changes with alpha since the magnetic force and field between the magnetized object and coils are directly related to it.

**Table 3-7**: Electromagnetic force on z direction and currents taken from the simulation

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.5870</td>
<td>15</td>
<td>33</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.7790</td>
<td>0</td>
<td>30</td>
<td>28</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>-7.4284</td>
<td>0</td>
<td>27</td>
<td>45</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>-16.3449</td>
<td>0</td>
<td>30</td>
<td>60</td>
<td>20</td>
</tr>
<tr>
<td>8</td>
<td>-24.2372</td>
<td>0</td>
<td>48</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>-31.9279</td>
<td>0</td>
<td>58</td>
<td>120</td>
<td>20</td>
</tr>
</tbody>
</table>

3.2.4 3D Analysis

3.2.4.1 Geometry

A similar coil arrangement with 2D can be modeled for a 3-dimensional magnetic suspension system. Figure 3-11 indicates schematically the configuration of electromagnets used in 3D. The system is composed of four ferrite magnets and four electromagnets. The electromagnets are modeled as circular coils. In this case, the coil model simplifies to a cylinder. For each coil, two cylinders are placed on the same axis to indicate copper wires and iron core. The copper wires are wound in the outer cylinder around the inner cylinder (core). 3D geometry gives an opportunity to control current direction which edges of outer cylinder are selected to identify the direction of current flow.
3.2.4.2 Meshing

In the case of our MSBS model, we are going to apply again the physics-controlled mesh which consists of tetrahedral mesh nodes as default, and element size adjusts to finer. Figure 3-12 is from the model in the COMSOL software, presenting the resulting mesh that consists of enough number of elements to solve the problem quite well.
3.2.4.3 Results of Analysis

3D analysis has been carried out by changing the coil currents to observe the magnetic force and torques variation of the system to balance both aerodynamic and gravitational force and torques.

The required amount of current in upper coils, as well as the lower coils, are determined to levitate the airfoil and they are illustrated in table 3-8. To be able to generate the large magnetic field and force on our system design, we need to supply a high total amount of current, especially for high lift force as result of the bigger angle of attack. The maximum value of demanded current in the entire system is 120 A.

**Table 3-8:** The amount of coil currents for different angle of attack

<table>
<thead>
<tr>
<th>$\alpha$ [°]</th>
<th>top-left coil current [A]</th>
<th>top-right coil current [A]</th>
<th>bottom-left coil current [A]</th>
<th>bottom-right coil current [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-35</td>
<td>-40</td>
<td>-35</td>
<td>-35</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>-38</td>
<td>20</td>
<td>-38</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>-40</td>
<td>80</td>
<td>-40</td>
</tr>
<tr>
<td>6</td>
<td>115</td>
<td>-20</td>
<td>116</td>
<td>-20</td>
</tr>
<tr>
<td>8</td>
<td>120</td>
<td>20</td>
<td>120</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>120</td>
<td>50</td>
<td>120</td>
<td>50</td>
</tr>
</tbody>
</table>

3D surface plots in figure 3-13 provide an understanding of the general distribution of the magnetic flux density norm from the 3D study results which are generated by the electromagnets. The magnetic field patterns show that there is significant flux changing on magnets. It is stronger when $\alpha = 10$ as we expected. The observation of the magnetic field on the left and right surfaces of the magnets is different each other for the reason that applied currents through the left coils are greater than right coils to provide the balance of the moments on the system.
Figure 3-13: The magnetic flux density norm [T] distribution on the airfoil and magnets for different (α)
Obtained results of the forces on airfoil are demonstrated in fig.3-14 while the angle of attack is changing from $0^\circ$ to $10^\circ$. The figure compares the magnetic forces along the $z$-axis with the lift forces. These results show that the increases in current create higher electromagnetic forces while the lift force is increasing in the opposite direction. Measured electromagnetic torques and forces by simulation respect to all-axes are described in table 3-9.

**Table 3-9:** Electromagnetic forces and moments on airfoil for all directions

<table>
<thead>
<tr>
<th>$\alpha$ [°]</th>
<th>Electromagnetic force, x component (N)</th>
<th>Electromagnetic force, y component (N)</th>
<th>Electromagnetic force, z component (N)</th>
<th>Torque, x component (N*m)</th>
<th>Torque, y component (N*m)</th>
<th>Torque, z component (N*m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3110</td>
<td>0.1570</td>
<td>9.9476</td>
<td>0.0005</td>
<td>0.1573</td>
<td>-0.0148</td>
</tr>
<tr>
<td>2</td>
<td>0.3578</td>
<td>0.1657</td>
<td>0.7203</td>
<td>-0.0021</td>
<td>-0.2587</td>
<td>0.0114</td>
</tr>
<tr>
<td>4</td>
<td>3.8205</td>
<td>4.3166</td>
<td>-7.3094</td>
<td>-0.1186</td>
<td>-0.5960</td>
<td>-0.5553</td>
</tr>
<tr>
<td>6</td>
<td>0.7043</td>
<td>-0.0853</td>
<td>-15.7131</td>
<td>0.0039</td>
<td>-0.8898</td>
<td>0.0002</td>
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<tr>
<td>8</td>
<td>1.2257</td>
<td>0.1978</td>
<td>-24.0175</td>
<td>-0.0038</td>
<td>-0.8812</td>
<td>-0.0088</td>
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<tr>
<td>10</td>
<td>1.5222</td>
<td>0.6106</td>
<td>-33.5995</td>
<td>-0.0135</td>
<td>-0.9558</td>
<td>-0.0086</td>
</tr>
</tbody>
</table>
3.2.4.4 Validation of Results

In the previous parts, we present the numerical results for the magnetic suspension system, particularly electromagnetic forces and torques on the airfoil. In order to determine the degree of accuracy for our simulation results, we can validate them with representations of the results from analytical calculations. These analytical expressions were derived to calculate the magnetic force between two magnetic dipoles.

In the numerical scheme, the system consists of four coils and four permanent magnets. We consider that each domain has the magnetic dipole moment. $\vec{m}_a$ denotes the magnetic dipole of coils and $\vec{m}_b$ is the magnetic dipole of magnets. Both of them can be calculated with following equations,

$$m_a = INS$$  
$$m_b = MV$$

$I$ is the current of coils

$N$ is the number of turns

$S$ is the cross sectional area of coils

$V$ is the volume of magnet ($m^3$)

$M$ is the magnetization of magnet.

The equation is given by Yung et al. (1999) [62] contains analytical expressions for calculating the magnetic force between two magnetic dipoles. According to this definition, the force exerted by dipole $a$ on dipole $b$ is derived from as follows:

$$F_{ab} = \frac{3\mu_0}{4\pi r^4} \left((\hat{r} \times \vec{m}_a) \times \vec{m}_b + (\hat{r} \times \vec{m}_b) \times \vec{m}_a - 2\hat{r}(\vec{m}_a \cdot \vec{m}_b) \right)$$

$$+ 5\hat{r}((\hat{r} \times \vec{m}_a) \cdot (\hat{r} \times \vec{m}_b)))$$

In this equation, $\hat{r}$ is the vector from the center of magnetic dipole “a” to the center of dipole “b”.
The model of magnetic dipoles for our case is shown in Fig. 3-15; however, the direction of magnetic dipoles for coils can adjust with the current direction for different alpha values.

The radius of the magnetic dipoles has been considered in a coordinate system with the z-axis. $\vec{m}_a$ and $\vec{m}_b$ were also examined only in the z-direction. Equation (3.3) can be reduced to equation (3.4) for our case,

$$\vec{F}_{ab} = -\frac{3\mu_0}{2\pi r^4} \vec{r}(\vec{m}_a \cdot \vec{m}_b)$$

We calculate both magnetic moments of a and b with the same parameters in use COMSOL simulation such as the current of coils, magnetization value of magnets and the distance between magnets and coils. The results of numerical simulations and analytic solutions calculated by using equation (3-4) are listed in table 3-10.
Table 3-10: The comparison of results between COMSOL and Equation (3-4)

<table>
<thead>
<tr>
<th>$\alpha$ [°]</th>
<th>COMSOL Calculated Magnetic Force (N)</th>
<th>Analytical Solution Magnetic Force (N)</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.9476</td>
<td>9.6248</td>
<td>3.35</td>
</tr>
<tr>
<td>2</td>
<td>1.1900</td>
<td>1.2289</td>
<td>3.16</td>
</tr>
<tr>
<td>4</td>
<td>-7.3094</td>
<td>-7.9466</td>
<td>8.01</td>
</tr>
<tr>
<td>6</td>
<td>-15.7131</td>
<td>-16.6606</td>
<td>5.68</td>
</tr>
<tr>
<td>8</td>
<td>-24.0175</td>
<td>-24.9024</td>
<td>3.55</td>
</tr>
<tr>
<td>10</td>
<td>-33.5995</td>
<td>-33.0561</td>
<td>1.64</td>
</tr>
</tbody>
</table>

As we can see from the table chart, the analytical solution closely matches the numerical solution for magnetic force on z direction. The maximum error is approximately less than 8 %.
CHAPTER 4
VALIDATION STUDIES

Extensive applications of different geometries for simulation models are required to validate using some approaches including replication methodology to evaluate how the simulation model is reasonable. Replication methodology is applied in previously published studies for the repeatable process to investigate the generality of the simulation. In particular, data from experiments or simulation of real studies are worked to build simulation models and compared quantitatively with the new simulation results.

This section illustrates replicating of some studies by COMSOL while using the same model structures of original studies to validate how well our simulation model of magnetic suspension and balance system for the wind tunnel rather than build on the experimental data. Basically, previous studies were chosen to examine coils and magnets relation, magnetic field and force calculation to extend our simulation model to verify data by using same boundary conditions and geometries.

The first replicated study has been worked in reference [63]. Wong illustrated a magnetic levitation control system which was built for the undergraduate project. This project based on an electromagnetic attractive force was produced by the coil to hold a steel ball. The simulation was designed as 2D study and geometry as showed in figure 4.1 on the left. Measured parameters of both system and coil feature same as the real study, as listed in table 4.1.

**Table 4-1:** System and coil features [63]

<table>
<thead>
<tr>
<th>Components</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of ball [kg]</td>
<td>0.068</td>
</tr>
<tr>
<td>Coil wire turn</td>
<td>3600</td>
</tr>
<tr>
<td>Coil ampere [A]</td>
<td>0.76</td>
</tr>
<tr>
<td>Gap between ball and coil [m]</td>
<td>0.008</td>
</tr>
</tbody>
</table>
After simulation of the system by the 2D-axisymmetric module of COMSOL Multiphysics, the electromagnetic force on the ball is calculated as nearly 0.672 Newton. According to Newton law, gravitation force on the model is 0.667 N. When we compare both forces, the results are really close to each other.

The other study in reference [64] is replicated to understand again how levitation device can be modeled well with COMSOL. Magnetic levitation device for levitating steel ball is simulated similar to the previous validation study. Three-dimensional geometry is used for obtaining results for this case. This system was designed for steel ball whose mass is 0.8 kg. According to Newton Law, gravitational force can be calculated as roughly 7.848 [N]. Parameters of the system are that number of turns is 1000, predicted current is 1.69 [A].

The analysis result of electromagnetic force on the ball is 7.870 (N) that is pretty much near to gravitational force.
Figure 4-2: Geometry (left) and magnetic flux density results (right) for simulation study [64]

A thin plate is widely used in the industrial world and using magnetic levitation technique make essential to solving problems related with this industry. The model from [65] is replicated to compare simulation results from real study and COMSOL to understand how the different geometries study well.

Table 4-2: Parameters used in the magnetic field analysis of our case taken by real study [65]

<table>
<thead>
<tr>
<th>Components</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions of steel plate</td>
<td>150 mm (width) x 50 mm (length)</td>
</tr>
<tr>
<td>Thickness of steel plate</td>
<td>0.24 mm</td>
</tr>
<tr>
<td>Steady Current</td>
<td>1.00 [A]</td>
</tr>
</tbody>
</table>

The following surface plots compare the results of magnetic flux densities which are taken from the real study and from the COMSOL simulation. For both study magnetic field shows approximately the same behavior. For real study, the magnetic flux density is around 2 Tesla, however, in our study shows a bit higher density. Differences between the two results can be expressed with inadequate information of material properties.
A) The simulation result from real study [65]

B) COMSOL analysis result

**Figure 4-3:** Distribution of magnetic flux density in the horizontal direction
CHAPTER 5

CONCLUSION AND FUTURE WORK

In this thesis, we have studied the simulation of magnetic suspension balance system by using the commercial software, COMSOL Multiphysics®. The system has been designed as two and three-dimensional model. Simulations are carried out by supplying rearranged current through the electromagnets to achieve magnetic force for the levitation of airfoil. Some conclusions from this work and some suggestions for future work are presented in the final section.

When we compare the results of both the 2-D model and 3-D model, the data show that when lift forces increase, we need to apply more magnetic field and force to the airfoil in the opposite direction. Therefore, the current of coils should change with the angle of attack. It is observed that there is a similar tendency between the results of the 2D and 3D simulation for the flux density distribution on the airfoil for all angle of attack. However, the value of current data is different because the 2D model has some limitations of using the AC/DC module for our case. One of the limitations is related the calculation of magnetic force on whole airfoil shape since we cannot manage the length of the airfoil and magnets. The other important issue is that the current in coils cannot be managed in the demanded direction.

On the other hand, the 2D analysis can be more reasonable in some perspectives. For example, it is a faster method of analyzing the flux distribution, estimating the current and force compared to the 3D analysis. The 2D analysis can be used to reduce calculation time for the symmetric model like levitation of ball as in example validation studies. For our non-symmetric structure model, the 3D simulation should be more logical and accurate, as we show that there is a similarity between the results of magnetic forces for both analytical and COMSOL calculation.

Additionally, we examined some real studies by using COMSOL in order to validate our simulation. These geometries are modeled with the same boundary conditions and geometry domains.
Consequently, results demonstrate that AC/DC module works for the acceptable calculation of magnetic force.

There are several different ways to maintain the improvement of this work. Following steps can be applied for future works.

- To create enough magnetic field for high angle of attack without using a high amount of currents in coils, a stronger permanent magnet can be used for the system instead of a ferrite magnet or the displacement between coils and magnets can be reduced.
- The material of the model can be replaced with a relatively low-density material to reduce the mass of the system.
- Development of experimental design based on the 3-D simulation analysis.
Appendix A

Detailed Results of Magnetic Dipole Calculations

<table>
<thead>
<tr>
<th>Value</th>
<th>Alpha=0</th>
<th>Alpha=2</th>
<th>Alpha=4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole moment of Magnets [A m^2]</td>
<td>0.002954304</td>
<td>0.002954304</td>
<td>0.002954304</td>
</tr>
<tr>
<td>The current of top left coil [A]</td>
<td>-35</td>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td>The current of top right coil [A]</td>
<td>-40</td>
<td>-38</td>
<td>-40</td>
</tr>
<tr>
<td>The current of bottom left coil [A]</td>
<td>-35</td>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td>The current of bottom right coil [A]</td>
<td>-35</td>
<td>-38</td>
<td>-40</td>
</tr>
<tr>
<td>Dipole moment of top left coil [A m^2]</td>
<td>98.91</td>
<td>-56.52</td>
<td>-226.08</td>
</tr>
<tr>
<td>Dipole moment of top right coil [A m^2]</td>
<td>113.04</td>
<td>107.388</td>
<td>113.04</td>
</tr>
<tr>
<td>Dipole moment of bottom left coil [A m^2]</td>
<td>98.91</td>
<td>-56.52</td>
<td>-226.08</td>
</tr>
<tr>
<td>Dipole moment of bottom right coil [A m^2]</td>
<td>98.91</td>
<td>107.388</td>
<td>113.04</td>
</tr>
<tr>
<td>r bottom left coil to left magnet [m]</td>
<td>0.085</td>
<td>0.09</td>
<td>0.095</td>
</tr>
<tr>
<td>r bottom left coil to right magnet [m]</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
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<tr>
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<td>0.097</td>
<td>0.099</td>
</tr>
<tr>
<td>r bottom right coil to left magnet [m]</td>
<td>0.13</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>r top left coil to left magnet [m]</td>
<td>0.095</td>
<td>0.09</td>
<td>0.085</td>
</tr>
<tr>
<td>r top left coil to right magnet [m]</td>
<td>0.15</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>r top right coil to right magnet [m]</td>
<td>0.105</td>
<td>0.103</td>
<td>0.101</td>
</tr>
<tr>
<td>r top right coil to left magnet [m]</td>
<td>0.14</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>F bottom left coil to left magnet [N]</td>
<td>2.8512</td>
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<td>-4.1767</td>
</tr>
<tr>
<td>F bottom left coil to right magnet [N]</td>
<td>0.4112</td>
<td>-0.2257</td>
<td>-0.8676</td>
</tr>
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<td>F bottom right coil to right magnet [N]</td>
<td>1.8273</td>
<td>1.8253</td>
<td>1.7707</td>
</tr>
<tr>
<td>F bottom right coil to left magnet [N]</td>
<td>0.5016</td>
<td>0.4932</td>
<td>0.4699</td>
</tr>
<tr>
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<td>1.8273</td>
<td>-1.2962</td>
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</tr>
<tr>
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<td>-0.2002</td>
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</tr>
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<td>1.6346</td>
</tr>
<tr>
<td>F top right to left magnet [N]</td>
<td>0.4699</td>
<td>0.4932</td>
<td>0.5733</td>
</tr>
<tr>
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<td>-7.9466</td>
</tr>
<tr>
<td>Value</td>
<td>Value Alpha=6</td>
<td>Value Alpha=8</td>
<td>Value Alpha=10</td>
</tr>
<tr>
<td>---------------------------------------------------------</td>
<td>--------------------------------</td>
<td>--------------------------------</td>
<td>--------------------------------</td>
</tr>
<tr>
<td>Dipole moment of Magnets [A m^2]</td>
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<td>0.002954304</td>
<td>0.002954304</td>
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<tr>
<td>The current of top left coil [A]</td>
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<td>120</td>
<td>120</td>
</tr>
<tr>
<td>The current of top right coil [A]</td>
<td>-20</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>The current of bottom left coil [A]</td>
<td>116</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>The current of bottom right coil [A]</td>
<td>-20</td>
<td>15</td>
<td>50</td>
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<tr>
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<td>-56.52</td>
<td>-141.3</td>
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<tr>
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<td>-42.39</td>
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</tr>
<tr>
<td>r bottom left coil to left magnet [m]</td>
<td>0.102</td>
<td>0.0105</td>
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</tr>
<tr>
<td>r bottom left coil to right magnet [m]</td>
<td>0.14</td>
<td>0.14</td>
<td>0.15</td>
</tr>
<tr>
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<td>0.103</td>
<td>0.105</td>
</tr>
<tr>
<td>r bottom right coil to left magnet [m]</td>
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<td>0.15</td>
<td>0.15</td>
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<tr>
<td>r top left coil to left magnet [m]</td>
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<td>0.07</td>
</tr>
<tr>
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<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
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<td>0.097</td>
<td>0.095</td>
</tr>
<tr>
<td>r top right coil to left magnet [m]</td>
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<td>0.13</td>
<td>0.12</td>
</tr>
<tr>
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<td>-4.1982</td>
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</tr>
<tr>
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<td>F top left coil to right magnet [N]</td>
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<td>-1.4098</td>
</tr>
<tr>
<td>F top right to right magnet [N]</td>
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<tr>
<td>F top right to left magnet [N]</td>
<td>0.3162</td>
<td>-0.3483</td>
<td>-0.9577</td>
</tr>
<tr>
<td>TOTAL FORCE [N]</td>
<td>-16.6606</td>
<td>-24.9024</td>
<td>-33.0561</td>
</tr>
</tbody>
</table>
REFERENCES


[22] Sawada, Hideo, Hiroshi Kanda, And Hisashi Suenaga. "The 10cm x 10cm Magnetic Suspension and Balance System at the National Aerospace Laboratory."


