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FLUIDIC FLEXIBLE MATRIX COMPOSITE
VIBRATION TREATMENTS FOR HELICOPTER AIRFRAMES
AND ROTOR BLADES

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by
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Abstract

Vibrations caused by periodic and unsteady loading in rotorcraft must be minimized to maintain acceptable fatigue life in structural parts and ride quality for passengers and crew. Rotorcraft vibrations are typically addressed through some combination of passive and active solutions that focus on reducing steady-state vibrations at the $n/\text{rev}$ frequency, where $n$ is the number of rotor blades. Currently existing passive treatments are often heavy, bulky variations of the classical tuned vibration absorber. Active treatments either attempt to isolate the cabin from hub vibratory loads or reduce cabin vibrations using a set of actuators, but they are more difficult to implement because they require a power supply and controller. This research covers the modeling, design, and experimental verification of fluidic flexible matrix composite (F$^{2}$MC) vibration treatments for two main rotorcraft applications: airframe vibration control and rotor blade damping. The main advantages to using F$^{2}$MC tubes over conventional hydraulic devices with pistons are their high strain-induced pumping capability and high force output per unit pressure.

A laboratory-scale rotorcraft tailboom was used as a testbed for demonstrating new F$^{2}$MC vibration absorber concepts. The tailboom is modeled using Euler-Bernoulli beam finite elements and coupled to a model of the F$^{2}$MC tubes and fluidic circuit. Based on the combined structural and fluid system model, an F$^{2}$MC damped vibration absorber was designed and built using four F$^{2}$MC tubes placed near the corners of the rectangular tailboom. Experimental results showed reduction of both lateral and torsional vibrations in a 26.7 Hz coupled tailboom vibration mode by up to 80%. Three fluidic circuits were tested for performance and model verification. This single-mode F$^{2}$MC vibration absorber was then modified so that two tailboom vibration modes can be treated by the same device. A lateral absorber frequency was tuned by selecting lengths of short branch segments connecting the left and right F$^{2}$MC tube pairs. Then, a vertical absorber frequency was tuned by selecting the appropriate length of tubing to connect the top and bottom F$^{2}$MC tube pairs. The tuned multi-mode vibration absorber reduced vibration by 63% in the vertical mode and 65% in the lateral mode, whereas a comparable absorber
designed to only treat the vertical mode reduced vibration by 68% in the vertical mode but only 42% in the lateral mode. The weight penalty from modifying the circuit to treat both modes was only 2% of the original absorber weight.

New F²MC devices are proposed to augment the damping of both articulated and hingeless rotor blades. The proposed device for articulated blades dissipates energy by using an F²MC tube to pump fluid through an orifice. In contrast, the proposed device for hingeless blades uses an F²MC tube as part of a damped vibration absorber with a tuned inertia track. Models are derived for both concepts to assess the feasibility of these dampers for representative articulated, stiff-in-plane, and soft-in-plane rotor blades. Parametric studies are conducted to understand how fluidic circuit design variables impact damper performance. For the articulated blade damper, increasing orifice resistance increases the damping ratio of the blade-damper system at the cost of increased F²MC tube oscillatory pressures. Increasing the accumulator capacitance reduces the F²MC damper stiffness and also increases the achievable damping, although the benefits diminish as the accumulator becomes larger. A stiff-in-plane hingeless blade is modeled with beam finite elements, and F²MC damped absorbers are tuned for the first chordwise bending blade mode. Eigenvalue analysis predicts an increase in the first chordwise blade mode damping ratio from a baseline of 0.02 to a range of 0.059-0.066 with the F²MC damped absorber. Using a large accumulator in the absorber fluidic circuit improves the absorber effectiveness and reduces the inertance required for tuning the fluidic circuit to a specific frequency.

Benchtop tests were conducted on a 9.7-foot diameter rotor integrated with a prototype articulated blade F²MC damper. Springs were attached to the blade to simulate centrifugal stiffness, and both frequency-domain and time-domain data were collected to assess damper performance. Model predictions of blade displacement and F²MC tube pressure were verified by experiments. Blade damping ratios in frequency-domain benchtop tests increase from a range of 0.054-0.064 with the orifice fully closed to a range of 0.300-0.335 with the orifice tuned to maximize damping. In time-domain benchtop tests, measured damping ratios increase from 0.062-0.090 with the orifice fully closed to a range of 0.298-0.404 with the orifice tuned. The benchtop tests and model verification are a key first step in developing functional F²MC damper technology for rotor blade applications.
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List of Symbols

\( (\cdot) \) derivative with respect to time

\( (\ast) \) derivative with respect to \( \psi \)

\( A \) finite element cross-sectional area

\( A_{col} \) cross-sectional area of fluid column in capacitance test

\( A_{orf} \) orifice cross-sectional area

\([A_{sys}]\) combined structural/fluid system model state matrix

\([A_t]\) F^2MC tube/fluid state matrix

\([A_t], \{B_t\}, \{C_t\}^T, D_t\) F^2MC damper fluid system state-space matrices

\(\{B_{sys}\}\) combined structural/fluid system model input matrix

\(\{B_{t,i}\}\) part of F^2MC tube/fluid input matrix multiplied by \( x_i \)

\( c_1 \) F^2MC tube axial stiffness

\( c_2 \) F^2MC tube force-pressure coefficient

\( c_3 \) F^2MC tube volume change coefficient

\( c_4 \) F^2MC tube capacitance

\( c_a \) accumulator capacitance

\( c_G \) hingeless blade geometric damping coefficient

\( c_\zeta \) linear viscous damping coefficient
\([C]\) finite element global damping matrix
\(C_d\) orifice flow discharge coefficient
\([C]_e\) hingeless blade element damping matrix
\([C_m]\) finite element modal mass matrix
\(\{C_{t,i}\}^T\) part of F^2MC tube/fluid output matrix multiplied by states \(\{\xi\}\) to produce F^2MC tube force \(F_i\)
\(d\) F^2MC tube constant offset distance from neutral axis
\(d_s\) distance from springs to lag hinge
\(D_t\) feedforward matrix from F^2MC tube displacement \(x_i\) to force \(F_i\)
\(e\) lag hinge distance from axis of rotation
\(E\) Young’s modulus
\(EI_{bl}\) soft-inplane blade bending stiffness
\(EI_e\) hingeless blade element chordwise bending stiffness
\(EI_{flex}\) soft-inplane flexure bending stiffness
\(F_{ext}\) scalar external force input
\(\{F_{ext}\}\) external force vector
\(F_i\) axial force acting on F^2MC tube \(i\)
\(\{\tilde{F}_i\}\) F^2MC tube \(i\) finite element model load
\(\{F_{int}\}\) vector of internal reaction forces and moments in element
\(\tilde{F}_q\) matrix accounting for part of F^2MC finite element load multiplied by nodal degrees of freedom
\(F_t\) F^2MC damper tube force
\(\hat{F}_x\) in-plane aerodynamic force per unit span, not including aerodynamic damping forces

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\[ \hat{F}_\xi \]  matrix accounting for part of F²MC finite element load multiplied by fluid states

\( g \)  acceleration of gravity

\( G \)  shear modulus

\([H(s)]\)  transfer function matrix from F²MC tube displacements to F²MC tube forces

\( I \)  fluid inerance (subscripts: \( c=\text{circuit}, b=\text{branch}, m=\text{main}, s=\text{segment} \))

\( I_p \)  finite element polar moment of inertia

\( I_{yy} \)  finite element second area moment about \( y \)-axis

\( I_{zz} \)  finite element second area moment about \( z \)-axis

\( I_\zeta \)  blade mass moment of inertia about lag hinge

\( J \)  finite element torsion constant

\( k \)  linear spring constant

\( k_{shak} \)  torsional stiffness contribution from shaker in benchtop test

\( k_{spr} \)  torsional stiffness contribution from springs in benchtop test

\([K]\)  finite element global stiffness matrix

\( K_{a,b} \)  entry in the \( a^{th} \) row and \( b^{th} \) column of element stiffness matrix

\([K_{el}]\)  local element stiffness matrix

\([K_{nr}]_e \)  hingeless blade element nonrotating component of stiffness matrix

\([K_{rot}]_e \)  hingeless blade element rotating component of stiffness matrix

\( l \)  inertia track length (subscripts: \( c=\text{circuit}, s=\text{segment} \))

\( L_e \)  finite element length
$L_o$ initial F$^2$MC tube length

$L_{o,a}$ initial F$^2$MC tube active length

$L_t$ F$^2$MC tube length after lag displacement

$L_{t,a}$ F$^2$MC tube active length after lag displacement

$m_b$ small scale blade mass

$m_e$ hingeless blade element mass per unit length

$[M]$ finite element global mass matrix

$M_{a,b}$ entry in the $a^{th}$ row and $b^{th}$ column of element mass matrix

$[M]_e$ hingeless blade element mass matrix

$M_{ext}$ external moment applied about lag hinge

$[M_m]$ finite element modal mass matrix

$M_{nd}$ external nondimensional moment about lag hinge

$M_t$ F$^2$MC damper tube moment

$n$ number of rotor blades

$N_{el}$ number of finite elements in hingeless blade model

$N_s$ number of springs attached to rotor blade in benchtop test

$p_a$ accumulator gas absolute pressure

$p_i$ internal pressure of F$^2$MC tube $i$

$p_t$ F$^2$MC damper tube pressure

$\{q\}$ finite element degrees of freedom vector

$\{q_{loc}\}$ vector of local degrees of freedom for an element

$Q_i$ volume flow rate out of F$^2$MC tube $i$

$p_a$ accumulator pressure

$r$ rotor blade radial coordinate

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\( r \)  inertia track radius (subscripts: \( c= \)circuit, \( s= \)segment)

\( r_{cg} \)  radial distance from lag hinge to blade center of gravity

\( R \)  rotor blade radius

\( R \)  fluid resistance (subscripts: \( c= \)circuit, \( b= \)branch, \( m= \)main, \( s= \)segment)

\( s \)  Laplace variable

\( \tilde{s} \)  nondimensionalized Laplace variable

\( S_\zeta \)  blade first mass moment of inertia about lag hinge

\( t \)  time

\( T \)  blade oscillation period, released from rest as pendulum

\( u, v, w \)  finite element translation degrees of freedom

\( V_a \)  accumulator gas volume

\((x_1, y_1, z_1)\)  location of F\(^2\)MC tube attachment point closer to tailboom root

\((x_2, y_2, z_2)\)  location of F\(^2\)MC tube attachment point closer to tailboom tip

\((x_b, y_b, z_b)\)  F\(^2\)MC damper attachment point on blade

\((x_h, y_h, z_h)\)  F\(^2\)MC damper attachment point on hub

\( x_i \)  axial displacement of F\(^2\)MC tube \( i \)

\( x_t \)  F\(^2\)MC damper tube axial displacement

\(|Y(f)|\)  magnitude of hingeless blade chordwise tip force to tip displacement transfer function at frequency \( f \)

\( Z \)  objective function to minimize in circuit tuning

\( \alpha \)  F\(^2\)MC tube fiber wind angle

\( \alpha_f \)  instantaneous F\(^2\)MC tube fiber angle

\( \alpha_o \)  initial, unstrained F\(^2\)MC tube fiber angle
$\beta_\chi$ coefficient in $\{\Psi_i\}^T$ corresponding to nodal degree of freedom $\chi$

$\Delta h$ change in height of fluid column in capacitance test

$\Delta p$ pressure increment in capacitance test

$\Delta V_{out}$ volume of fluid moved out of $F^2MC$ tube in capacitance test

$\zeta$ damping ratio or lag angle

$\zeta_o$ blade operating lag angle

$\eta$ accumulator gas polytropic exponent

$\theta_x, \theta_y, \theta_z$ finite element rotation degrees of freedom

$\kappa_I$ inertance frequency dependence correction factor

$\kappa_R$ resistance frequency dependence correction factor

$\mu$ fluid dynamic viscosity

$\nu_\zeta$ blade nondimensional lag frequency (/rev)

$\{\xi\}$ vector of states for $F^2MC$ tube/fluid subsystem dynamics

$\rho$ material or fluid density

$\sigma$ $F^2MC$ damper tube force moment arm

$\{\sigma_i\}$ vector converting $F^2MC$ tube scalar force into finite element load

$[\phi]$ eigenvector matrix

$\psi$ rotor azimuth angle

$\Psi$ scalar conversion factor from lag angle to $F^2MC$ tube displacement

$\{\Psi_i\}^T$ row vector converting nodal displacements into $F^2MC$ tube $i$ axial displacement

$\omega_n$ natural frequency

$\Omega$ rotor speed
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Epigraph

“Grow old along with me, the best is yet to be.”
- Robert Browning
Chapter 1  
Introduction

1.1 Background on Helicopter Vibrations

Helicopters operate in a complex aerodynamic environment that leads to unsteady loading on the rotor. These unsteady loads cause airframe vibration that has many undesirable effects. Oscillatory strains from airframe vibration lead to fatigue of structural components, which can increase maintenance costs and reduce the availability of a given aircraft. In addition, vibrations have many adverse effects on helicopter pilots and passengers. They can make instruments hard to read, weapons hard to point, and accelerate the fatigue of pilots, crew, and passengers [1]. Although mission performance specifications such as range and endurance are often considered functions of the helicopter speed and fuel efficiency, a harsh vibration environment can also impact pilot and crew performance, indirectly limiting how long the aircraft can safely be operated. In long missions, pain in a helicopter pilot’s legs and back begins 2 to 4 hours into the flight and can persist for over 24 hours after the mission ends [2]. While current design goals for production helicopters aim for "jet smooth" vibration levels below 0.05 g [3], this is challenging to achieve over an entire flight envelope.

Although the complex, unsteady aerodynamic environment at the main rotor produces loading with many harmonics of the rotor speed, the rotor hub acts as a filter so that only 1/rev and harmonics of \( n/\text{rev} \) are transmitted to the fuselage non-rotating frame. Here, \( n \) denotes the number of blades on the main rotor. 1/rev vibration results from aerodynamic or inertial dissimilarity between blades and can be minimized with track and balancing operations. \( n/\text{rev} \) vibration and its
harmonics generally dominate in production helicopters [4]. Although most fuselage vibration is at the frequencies \( n/\text{rev} \) and its harmonics (i.e., \( 2n/\text{rev} \), \( 3n/\text{rev} \), etc.), the helicopter tailboom in particular is susceptible to broadband excitation from turbulent flow. In contrast to the steady \( n/\text{rev} \) vibration, vibrations caused by broadband excitation are dominated by the airframe natural frequencies.

The helicopter tailboom is a long, relatively flexible structure extending from the aft end of the fuselage and providing a moment arm for the tail rotor to control the helicopter yaw motion. Because helicopter tailbooms have low inherent structural damping in the range of 1-2% critical, they are prone to both high resonant amplitudes of vibration and slowly decaying transients. The tailboom and its aerodynamic surfaces are often excited by the main rotor wake, as shown in Figure 1.1. In forward flight, the main rotor wake vortices may impinge on the horizontal and vertical tail surfaces to cause \( n/\text{rev} \) vibration in the tailboom, which is transmitted into the fuselage [3]. Even with modern computational tools, interactional aerodynamics between the main rotor, hub, fuselage, and tailboom aerodynamic surfaces remain difficult to predict before flight testing of a new helicopter. Examples of helicopters that experienced problems related to interactional aerodynamics during early flight testing include the EH101, Comanche, and NH90 [5].

In addition to the harmonic forcing caused by the main rotor and its wake, the tailboom is also susceptible to broadband excitation from turbulent flow that has separated at the rotor hub or a more forward part of the fuselage. The transient 'tail shake' or 'buffeting' induced by turbulent flow has a fundamentally different cause than harmonic \( n/\text{rev} \) vibration. Tail shake is caused by a turbulent wake interacting with a tail surface to excite a low-frequency elastic fuselage mode, as illustrated in Figure 1.2. In this example, the cockpit is in front of the forward node of the fuselage first lateral bending mode. An observer inside the cockpit experiences low-frequency vibration at the fuselage first natural frequency, which is typically much lower than \( n/\text{rev} \).

Although most helicopter vibration treatments primarily aim to reduce \( n/\text{rev} \) vibration, low-frequency vibrations can be more troublesome to the pilot and crew. Exposure to vibration at low frequencies around 5 Hz can lead to muscle fatigue, decreases in height, and lower back pain [6].
1.2 Helicopter Vibration Control Approaches

This section reviews the existing literature and summarizes the current state-of-the-art in passive, active, and semi-active rotorcraft vibration control. This section primarily focuses on techniques for reducing airframe vibrations, especially tailboom vibrations.

1.2.1 Airframe Design & Passive Solutions

One of the simplest approaches to minimizing helicopter vibrations involves passive design of the airframe structure such that its natural frequencies are sufficiently spaced from the rotor 1/rev and \( n/\text{rev} \) frequencies. The process of designing the airframe so that its natural frequencies are separated from the dominant excitation frequencies is called detuning, and it is considered as a best practice even for state-of-the-art modern helicopters such as the Bell 525 Relentless [7]. Detuning is accomplished by modifying mass and stiffness properties of the airframe structure. Finite element models are often used to predict the effects of modifications on
the resulting airframe dynamics. Detuning is conceptually a simple step, but it is important that the proposed modifications do not significantly increase the overall weight of the helicopter.

After the airframe has been detuned sufficiently, the most common treatments for further vibration reduction are vibration isolators and vibration absorbers. Traditional vibration isolators use soft springs so that the natural frequency of the isolated object is much lower than the excitation frequency. However, using soft springs for an isolator will result in high static deflections that are often impractical on a helicopter [8]. To address this problem, the Dynamic Antiresonant Vibration Isolator (DAVI) was developed to provide improved \( n/\text{rev} \) isolation with a low static deflection. A schematic of the DAVI is shown in Figure 1.3. The DAVI features a mass at the end of a bar, which is pivoted to both the base being excited and the isolated mass, \( M_1 \). The bar acts as a lever to amplify the motion of mass \( M_2 \) and generate inertial forces counteracting the base excitation. Transmissibility is substantially lowered at the DAVI antiresonant frequency, which is independent of the isolated mass. The fact that the antiresonant frequency is independent of the isolated mass made it an attractive early solution for reducing vibrations transmitted to pilot and passenger seats [9].
A fluidic analog to the mechanical DAVI called the Liquid Inertia Vibration Eliminator (LIVE) unit was developed by Bell Helicopter. The LIVE unit replaces the DAVI mass $M_2$ with a mass of fluid contained inside a rubber elastomeric cylinder. A section view of the initial LIVE design is shown in Figure 1.4 to illustrate its working principle. The fluid mass pumped through the LIVE tuning port experiences oscillating accelerations, and these accelerations produce oscillatory pressures and inertial forces out of phase with the vertical forces acting on the pylon. In the LIVE unit, the ratio of the outer cylinder area to the tuning port area is analogous to the ratio of the arms on the mechanical DAVI. The main advantages of the LIVE system over the DAVI are a bearingless design, compactness, reduced cost and weight, and low maintenance requirements [10].

More recent production versions of the LIVE unit have increased static stiffness and use non-toxic, non-corrosive fluids to replace the mercury in the original unit [11]. An active augmentation of a LIVE unit using piezoelectric actuators was demonstrated on a small vertical takeoff and landing (VTOL) unmanned aerial vehicle (UAV) [12], but no active versions of the LIVE isolator are currently in production. Versions of the passive LIVE unit have been utilized on several Bell aircraft including the 427 [13], the 525 Relentless [7], and the 505 Jet Ranger X [14]. The LIVE unit for the 505 is noteworthy because it is the only one to feature an external fluid track. Because the 505 has a two-bladed rotor, its $n/\text{rev}$ frequency is
lower than the $n$/rev frequency on the other four- and five-bladed helicopters, and a longer tuning port is necessary to achieve the desired isolation frequency.

Vibration absorbers of different types are also common on modern aircraft. Frahm absorbers (named after their inventor) are often used for treating vibration in specific regions of the airframe, such as the cabin. Because the technology is over a century old [15], the theory of tuned vibration absorbers (TVAs) is well-known and covered in many vibrations textbooks [16,17]. The tuned vibration absorber consists of a mass-spring-damper system attached to a vibrating primary mass. The TVA natural frequency is very close to the natural frequency of the primary system so that when the primary mass is excited, the TVA resonates instead and absorbs some of the vibration energy from the primary mass. Comprehensive reviews of passive, active, and semi-active vibration absorber technologies are given by [18,19]. A semi-active or adaptive absorber is one that can adjust or retune its properties to accommodate changes in the system, whereas an active absorber is effectively a proof-mass actuator that has been passively tuned for increased output at its resonant frequency [18].
Sample frequency response curves for an absorber attached to a simple spring-mass system are plotted in Figure 1.5. As shown in the figure, vibration absorbers with little damping are only effective within a narrow frequency range, and vibration increases at frequencies either slightly above or below the notch frequency. In a helicopter context, this means that the effectiveness of a Frahm absorber varies even with small changes in the rotor speed. Although many vibration absorbers are undamped, damping can be added to increase the effective frequency band. However, an increase in damping reduces the antiresonant notch depth in the frequency response, as shown in Figure 1.5. Vibration absorbers designed with damping are sometimes referred to as tuned mass dampers or damped vibration absorbers. Regardless of the absorber damping ratio, the system frequency response curve for a given absorber mass always passes through two fixed points in Figure 1.5. Snowdon notes that in the optimally tuned vibration absorber, the two maximum values in the frequency response curve are equal and occur at the frequencies of these fixed points [17].

![Figure 1.5: Representative frequency response for a system with a tuned vibration absorber.](image)

Frahm absorbers have been used to reduce \( n/\text{rev} \) vibration in many production rotorcraft, since they are a simple way to reduce vibrations when the forcing is at one dominant frequency. For example, Bell Helicopter has patented a vibration absorber that uses an oscillating pendulum to reduce vibration in the cabin areas [20]. The
pendulum is pivoted about an axis, is attached to a set of springs, and comes into contact with additional auxiliary springs if its oscillations exceed some design limit. These auxiliary springs ensure that absorber performance is less sensitive to small variations in rotor speed. Despite their simplicity, Frahm absorbers are not ideal solutions for reducing rotorcraft vibration because they are not weight-efficient. They are generally most desirable when a mass that is already in the helicopter can be repurposed as the absorber mass. An example of such repurposing is seen in [21], where Airbus Helicopters engineers propose mounting an aircraft battery inside the helicopter tailboom on a spring system. The lateral and vertical stiffnesses of the springs are tuned so that the mass acts as both a lateral and vertical absorber. This could be an effective treatment if a usable mass already exists inside the tailboom. One drawback to this solution is that if the device is retrofitted into an aircraft, it would alter the helicopter longitudinal center of gravity, since it must be placed near the end of the tailboom to be effective. This center of gravity shift is generally undesirable because it could reduce aircraft control margins [22]. Space constraints could also be a factor in the viability of this absorber, since the mass would be located at the tip of the tailboom, which normally tapers into a small cross-section. A similar device that is also patented by Airbus Helicopters uses a sloshing mass of fluid contained between two plates on springs as the absorber mass [23].

A separate category of vibration absorbers called pendulum absorbers are attached to the main rotor instead of the airframe. Pendulum absorbers reduce $n/\text{rev}$ dynamic shear loads transmitted through the hub before they enter the airframe. A diagram of the pendulum absorber is shown in Figure 1.6. These pendulum absorbers are attached to the root of each rotor blade, and their stiffness is derived from centrifugal forces. The absorber frequency for a pendulum absorber, or pendab, is a function of the pendulum arm length $l_a$ and the radial position $r_a$ of the pendulum root. Pendulum absorbers have been applied to reduce out-of-plane vibratory loads [24], and a similar device called a bifilar absorber has been applied to reduce in-plane dynamic shear loads [25]. As with Frahm absorbers, the main drawback of these devices is that they can require a large mass to achieve appreciable vibration reduction. They can also result in aerodynamic drag penalties [26].
Figure 1.6: Diagram of pendulum absorber on a rotor blade, adapted from [3].
The pendulum absorber frequency is a function of pendulum arm length \( l_a \) and radial position \( r_a \) of the pendulum root.

1.2.2 Active Solutions

Active vibration treatments can be effective over a broader frequency range than passive treatments, but require a power supply and controller to operate, making them more difficult to implement than passive devices. As actuation and computing technologies have improved, more focus has been placed on developing active vibration control technology for rotorcraft. In the late 1980s and 1990s, AgustaWestland introduced the Active Control of Structural Response (ACSR) approach. The ACSR approach involves measuring vibration at several points on the fuselage using accelerometers, processing these measurements, and computing the appropriate control commands for a set of actuators placed throughout the fuselage. Although the original control law presented by Staple uses a frequency-domain approach [27], later research examined the advantages and disadvantages of a time domain approach and other adaptive algorithms [28,29]. The time-domain approach from [28] is more computationally intensive than the frequency-domain approach, but responds more quickly to changes in system parameters and forcing amplitude. The hybrid controller presented in [29] combines the faster transient response of the time-domain controller with the lower computational requirements of a frequency-domain approach. Many early active control systems used actuators centrally located in the cabin or near the main rotor, but optimization techniques were later
applied to identify the most effective and efficient actuator locations [30,31]. In recent years, fuselage-based active control research has improved algorithms for controlling the actuators [32]. Active control systems have been demonstrated on many aircraft including the Bell 429 [33] and Eurocopter EC130 and EC135 [34].

In addition to fuselage treatments, active isolators have been developed to further reduce the transmission of forces and moments through the rotor hub. Panza, McGuire, and Jones add an actuator controlled by pressure feedback to a passive fluidic isolator [35]. This results in greater isolation over a wider frequency range than the passive device, and it also improves the robustness of the isolator to parameter variations such as those caused by temperature changes. Similarly, Airbus developed an active version of the DAVI using electromagnetic actuators to drive the motion of flapping masses attached to the main gearbox. This active device improves isolation at the \( n/\text{rev} \) frequency and provides more effective isolation across a wider frequency range than a passive device [36]. Bell Helicopter has patented a version of the LIVE unit in which pistons can be actively controlled to move fluid through the tuning port at a desired frequency [37]. Another isolator developed by Sikorsky and LORD Corporation called the Hub-Mounted Vibration Suppressor (HMVS) uses motor-driven rotating eccentric masses to cancel \( 4/\text{rev} \) in-plane vibratory hub loads [38]. The HMVS was ground tested in tandem with a fuselage active vibration control system that reduces the remaining vertical hub loads, and it has also been flight tested on a UH-60A helicopter to demonstrate vibration reduction similar to or better than the previously implemented \( 4/\text{rev} \) vibration absorbers inside the airframe [39]. The authors claim that the HMVS saves 40 pounds of weight compared to the bifilar absorbers installed on the production UH-60A.

A number of active solutions have been developed specifically to target and reduce helicopter tailboom vibrations. Krysinsky patented the concept of actively controlling tail rotor blade pitch to generate forces opposing low-frequency lateral tailboom vibration [40]. Alternatively, Eglin suggests generating control forces by actively controlling the incidence angle of tail surfaces or flaps attached to the tail surfaces [41]. This avoids the increased noise and accelerated component fatigue associated with controlling tail rotor pitch. Manfredotti patented an active absorber using a mass attached to a flexible beam, the motion of which can be controlled and amplified by electromagnetic forces [42]. This device is unique
because although it is active, it has no moving parts. Strehlow et al. describe both passive and active tailboom damping devices that use the sensing and actuation capabilities of piezoelectric materials [43]. Klöppel et al. suggest using active main rotor controls such as trailing edge flaps, twist, and speed control to generate actuation forces opposing vibration [44]. Even the Zero-Vibe\textsuperscript{TM} system using the HMVS and circular force generators above the cabin had dedicated actuators for tailboom vibration control. Because it was only designed to eliminate vibratory loads entering the airframe through the main rotor pylon, additional actuators were required on the tail to eliminate vibrations caused by the main rotor wake impinging on the tail [45].

Although active rotors have been studied for decades and show promise for vibration and noise reduction, no production helicopters make use of higher harmonic control (HHC), individual blade control (IBC), or any other active rotor technology. This is largely due to the complexity of integrating such a system and the reliability requirements that must be met for production helicopters [46].

1.2.3 Semi-Active Solutions

As with active treatments, semi-active treatments require controllers and actuators, but involve adjusting one or more parameters (i.e., stiffness, mass, or damping) of an otherwise passive device. Because the active aspect is only used for parameter adjustment and not for continuously generating forces, power and actuation requirements are often less demanding for semi-active treatments than for fully active treatments. Semi-active solutions are especially attractive for helicopter applications, since the ability to vary rotor speed (and therefore, the $n/\text{rev}$ frequency) can optimize performance for different flight conditions. While a passive tuned solution might only effectively treat vibrations at one rotor speed, a semi-active solution can be re-tuned so that it remains effective at a different rotor speed.

Several researchers have demonstrated semi-active versions of the Liquid Inertia Vibration Eliminator concept. Cronjé et al. use variable stiffness circular leaf springs attached to rolling diaphragms to move fluid inside the isolator [47]. du Plooy et al. modified the basic LIVE design by adding a flexible rubber membrane to separate the fluid reservoirs from adjustable pneumatic springs. The isolation frequency of this 'tunable vibration absorbing isolator' can be controlled by adjust-
ing the pressure of the pneumatic springs [48]. Bell Helicopter has also patented modifications to its initial LIVE device, such as one with a tuning port that can be axially extended by a motor [49]. Another fluid-based semi-active isolator uses magnetorheological (MR) fluid to provide vibration isolation for crew seats while preserving the crashworthiness of traditional seats [50].

A number of airframe-based semi-active solutions can be found in the literature. Bansemir developed a vibration absorber using a cantilever beam with a mass at the end [51]. The resonant frequency of the absorber can be adjusted by either varying how much of the beam is clamped at the root end or by increasing the pre-tension of nonlinear springs attached to the beam. Du Bois et al. suggest varying the load carried by structural members and using stress-stiffening effects to shift natural frequencies of a structure in real time [52]. Du Bois et al. also developed a semi-active absorber using the distributed mass of a cable as the absorber mass [53]. Adjusting the cable tension changes the cable resonant frequency, thereby introducing a tuning mechanism for the device.

1.3 Rotor Blade Dampers

Most helicopters require lead-lag dampers on the main rotor to ensure that they remain free of aeromechanical instabilities such as ground or air resonance. Blade drag contributes little aerodynamic damping, so discrete lead-lag dampers are required to augment the stability of almost all articulated rotors. However, lead-lag dampers are complex parts that increase the maintenance time and cost associated with operating the helicopter. Effective lead-lag dampers that provide high damping with minimal required maintenance have been a long-standing need of the rotorcraft industry.

Most main rotor dampers are either elastomeric or hydraulic components. Examples of these two types of dampers are shown in Figure 1.7. Each of these options have their own unique advantages and disadvantages. Elastomeric dampers are typically stiffer and may contribute less damping than hydraulic dampers, but they often require less maintenance because of their simplicity [54]. Conversely, a hydraulic damper may be necessary if an elastomeric damper will not provide sufficient damping. The presence of sliding seals in hydraulic dampers that can wear out and leak, however, is a significant drawback [55]. A third class of lead-lag
dampers combines elastomeric and hydraulic elements, mixing the benefits and drawbacks of the two technologies. An example of this hybrid technology is the Fluidlastic® lead-lag damper developed by LORD Corporation [55–57].

![Figure 1.7: Photographs of a) elastomeric damper on the AH-64 Apache and b) hydraulic damper on the CH-47 Chinook [58].](image)

Hingeless rotor blades are split into two categories based on the first inplane natural frequency. By definition, if the first inplane frequency is less than 1/rev, the rotor is classified as soft-inplane; if the frequency is greater than 1/rev, it is classified as stiff-inplane. Conventional dampers were designed primarily for articulated and soft-inplane hingeless rotors. The damper is connected between the hub and the blade, and it is stroked as the blade leads or lags as a rigid body on an articulated rotor or bends via a flexure on a soft-inplane hingeless rotor. However, these dampers are not effective on very stiff-inplane rotors such as those found on modern advancing blade concept (ABC) rotorcraft such as the X2 Technology Demonstrator (X2TD), the S-97 Raider, and the SB-1 Defiant. This class of helicopters derives its name from the fact that the advancing blades on each of the two coaxial counter-rotating rigid rotors are used to generate lift in high-speed forward flight, as shown in Figure 1.8. In a conventional single-rotor helicopter, the advancing blade experiences a higher airspeed than the retreating blade in forward flight, and the retreating blade pitch must be increased to balance the lift from both sides. Alternatively, the coaxial advancing blade concept uses the second rotor to balance the lift instead of increasing the retreating blade pitch on each rotor. For this reason, the advancing blade concept avoids the phenomenon of retreating blade stall, which limits the forward speed of a conventional single-rotor helicopter.
Figure 1.8: Lift and moment distribution for an advancing blade concept rotorcraft, from [59].

The advancing blade concept was pioneered by Sikorsky in the 1970s with the XH-59A ABC Demonstrator. The XH-59A rotor did not display any instabilities throughout its entire flight envelope; however, the first inplane (i.e., chordwise bending) blade mode was very lightly damped, with a frequency of 1.4/rev in the rotating frame. The first chordwise mode damping ratio varied from 1 to 2% critical in forward flight, but decreased further in descending flight for the upper rotor [60] as shown in Figure 1.9. The damping ratio was measured in flight tests by exciting transient blade response with sharp cyclic pulses. The decreased damping in descent is due to a destabilizing component of the lift force in the same direction as the chordwise velocity. According to [60], there was no significant coupling between chordwise bending and flap or torsion response.

The Sikorsky X2TD rotor dynamic properties were similar to those of the XH-59A, with first chordwise and first flapwise bending modes at 1.4/rev and 1.5/rev, respectively [61]. The first chordwise mode had roughly 1.5% critical damping in hover [62] and remained between 1 and 3% critical damping throughout the tested range of forward flight speeds [63]. Neither the XH-59A nor the X2TD used an external damper, so this damping was strictly due to structural damping and blade aeromechanics. Little physical deformation occurs at the root of a stiff hingeless blade, and the resulting damper stroke would be small compared to the stroke on an
articulated or soft-inplane rotor. Although stiff-inplane rotors are not susceptible to ground resonance, additional damping is still desirable to increase aeroelastic stability margins and reduce transient rotor blade vibrations caused by pilot inputs and maneuvers.

Figure 1.9: Chordwise damping of XH-59A blade versus rate of descent, from [60].
1.4 Fluidic Flexible Matrix Composite Tubes

Fluidic Flexible Matrix Composite (F²MC) tubes are an emerging technology with the ability to provide lightweight and compact vibration control for aerospace structures. Two fabrication methods for F²MC tubes are illustrated in Figure 1.10. F²MC tubes can be made either by using composite processing methods such as filament winding or by assembling a braided fiber sheath over a rubber bladder. In either fabrication method, a set of interlocking fibers, which are much stiffer than either the matrix material or the rubber bladder, reinforce the tube and form wind angle α with respect to the longitudinal axis. F²MC tubes are functionally similar to pneumatic McKibben actuators, which contract axially when pressurized if the fiber angle is less than 54.7° and extend axially when pressurized if the fiber angle is greater than 54.7° [64]. Accordingly, F²MC tubes with wind angle less than 54.7° are referred to as contractor tubes, and tubes with wind angle greater than 54.7° are referred to as extender tubes.

Filament wound F²MC tubes can be modeled using the Lekhnitskii solution for an orthotropic cylinder under axial and pressure loading [65] as in [66,67], while there are examples of both linear [68] and nonlinear [64,69] models for braided-sheath tubes. With proper tailoring of the fiber angle, F²MC tubes exhibit high volume change when strained axially as shown in Figure 1.11, and are capable of moving fluid 1-2 orders of magnitude more efficiently than a piston of the same diameter [70]. The enhanced pumping and actuation capabilities of F²MC tubes make it possible for a small amount of actual fluid mass in the circuit to have a high effective inertia.

Early research showed that coupling F²MC tubes with a fluidic circuit can achieve vibration control effects such as isolation [68] and absorption [71] in lumped parameter systems. Zhu et al. were the first to integrate F²MC tubes into a continuous structure for vibration control, experimentally demonstrating both damping [72] and vibration absorption [73] on a small-scale cantilever beam. The F²MC tubes in Zhu’s experiments were fabricated by embedding stainless steel fibers in a polyurethane matrix. The axial forces exerted by the F²MC tubes are transmitted into the beam through attachment brackets, resulting in a net moment at the beam root that counteracts the external forcing. Zhu et al. also made the key conclusion that the optimal placement for F²MC tubes in a treatment targeting
Figure 1.10: Fabrication methods for F$^2$MC tubes: a) filament winding, b) braided sheath and rubber bladder assembly.

Figure 1.11: Volume change behavior of an F$^2$MC tube with wind angle less than 54.7°.

the first bending mode is near the beam root. This maximizes strain in the F$^2$MC tubes as the beam vibrates, moving more fluid inside the circuit and giving the treatment more authority.

More recently, Miura et al. demonstrated an F$^2$MC-based damped vibration absorber on a laboratory-scale helicopter tailboom using braided-sheath F$^2$MC tubes [74]. The Miura absorber is depicted in Figure 1.12. In contrast to Zhu’s treatment, which had F$^2$MC tubes on only one side of the beam neutral axis, Miura’s treatment uses tubes attached above and below the tailboom bending
plane and coupled through the same inertia track. Transverse bending vibration causes tubes on one side of the neutral axis to extend while tubes on the other side contract. The pressures developed as the F²MC tubes deform cause fluid to oscillate within the inertia track, and this fluid mass is analogous to the attached mass in a traditional vibration absorber. The F²MC absorber reduced the vibration amplitude of the laboratory-scale tailboom 12 Hz first vertical bending mode by 70%.

Figure 1.12: Concept for F²MC vibration absorber for transverse vibrations, as demonstrated on laboratory-scale helicopter tailboom.

1.5 Research Objectives

F²MC tubes have the potential to become simple and effective vibration solutions for rotorcraft. However, the existing techniques for modeling F²MC-integrated structures have several limitations, and the design space for F²MC treatments has still largely been unexplored. In this research, two applications of F²MC tubes are considered. The first application integrates F²MC tubes into a helicopter airframe as part of a damped vibration absorber. The objectives of this research were to advance the state-of-the-art by expanding the types of vibration modes that can be targeted by F²MC vibration absorbers, experimentally demonstrating vibration control at higher frequencies than previous research, and investigating the feasibility of a multi-axial absorber that can target both vertical and lateral bending vibrations. These objectives are addressed in Chapters 2, 3, and 4 of the
dissertation. The second application considers the integration of F\(^2\)MC tubes into rotor blades for damping and stability augmentation. Prior to this research, F\(^2\)MC tubes have not been explored as part of a rotor blade damper. The objectives of this research were to evaluate the feasibility of damper concepts for articulated and hingeless rotor blades, to understand how different fluidic circuit parameters affect damper performance, and to demonstrate a new damper concept with small-scale hardware. These objectives are addressed in Chapters 5 and 6.

1.5.1 Airframe Application

Chapter 2 outlines the development of a comprehensive model for the tailboom structure, F\(^2\)MC tubes, and fluidic circuit. Previous modeling approaches used for F\(^2\)MC-integrated structures, such as closed-form transfer functions [72, 73] and the Rayleigh-Ritz method [75], are not suitable methods for modeling more complex aerospace structures. While Miura et al. demonstrated a vertical vibration absorber using F\(^2\)MC tubes, the model assumed perfect symmetry between the tubes and their strains, and the structural model only considered vibration in one direction. This research uses the finite element method as the basis for modeling an F\(^2\)MC-integrated tailboom structure. The finite element method was chosen due to its popularity as a modern modeling technique and its ability to model complex, three-dimensional structures. The model is used to identify the most effective F\(^2\)MC tube configuration for treating a particular lateral bending/torsion mode in a laboratory-scale tailboom.

The next step in this research is to experimentally verify this comprehensive model. Chapter 3 describes the testing of an F\(^2\)MC vibration absorber for the aforementioned lateral bending/torsion mode and presents the experimental results from these tests. Several different circuits are tested to assess the benefits of using different fluids or different circuit tubing materials. These experiments make several new contributions that were not covered in previous literature. The experiments are the first demonstration of vibration control for a coupled lateral bending/torsion mode using F\(^2\)MC tubes, and they also show vibration control at higher frequencies than previous experiments. These results indicate the feasibility of F\(^2\)MC vibration treatments for reducing or damping vibrations in the n/rev range of most helicopters.
Two additional advantages of the new structural modeling approach are that the finite element model is three-dimensional and all of the fluidic circuit dynamics are explicitly modeled. In previous research studying F$^2$MC-integrated structures, the beam was treated as one-dimensional along its length, and equivalent fluidic circuit properties were chosen for the model based on the assumed direction of vibration. The new model makes it possible to consider how a single F$^2$MC treatment affects vibrations in both lateral and vertical directions. Chapter 4 covers the design and experimental demonstration of a new F$^2$MC vibration treatment that can treat vibrations in one lateral and one vertical mode with the same fluidic circuit. This is accomplished with a more thorough design of the inertia track, although there are no fundamental differences between this fluidic circuit concept and the concept previously used to reduce tailboom lateral bending and torsional vibrations.

### 1.5.2 Rotor Blade Application

Chapter 5 analyzes two different F$^2$MC damper concepts for full-scale articulated and hingeless rotor blades. The feasibility of each concept is investigated by developing a model for the in-plane dynamics of a full-scale rotor blade that has been integrated with the corresponding F$^2$MC device. Rotor blades based on the UH-60 blade, the X2 blade, and the BO105 blade are used as representative articulated, stiff-inplane, and soft-inplane blades, respectively. These models are used to guide the preliminary design of F$^2$MC dampers for each type of rotor, to assess the effectiveness of these dampers, and to understand the influence of key fluidic circuit parameters.

The final goal of this research is to build and test a small-scale prototype articulated blade F$^2$MC damper to demonstrate the concept analyzed in Chapter 5 and identify physical challenges with its implementation. Chapter 6 covers the design, implementation, and benchtop testing of this F$^2$MC damper. Springs are used to simulate an effective rotational stiffness, and both frequency- and time-domain tests are conducted to characterize the damper performance. This benchtop prototyping and testing paves the way for future rotating demonstrations of F$^2$MC blade damper technology.
Chapter 2  
Finite Element Modeling of a Tailboom with $F^2MC$ Tubes

This chapter describes the process for modeling a laboratory-scale tailboom with the finite element method and incorporating the effects of attached $F^2MC$ tubes. The tailboom finite element model is verified by comparing model predictions to experimental frequency response measurements from the lab-scale tailboom before $F^2MC$ tubes are attached. Next, the fluid system is modeled, and the structural and fluid systems are coupled together by relating deformation-induced $F^2MC$ tube elongations to the resulting tube forces. Using this comprehensive model, two $F^2MC$ absorber concepts are evaluated for their potential in reducing vibrations of a tailboom lateral bending/torsion mode. Out of the two concepts, a preferred one is selected for further design studies.

2.1 Finite Element Modeling of Laboratory-Scale Tailboom

The laboratory-scale tailboom testbed used in this research is a 0.3-scale model based on the Apache AH-64A tailboom. Its design is discussed in detail by Heverly [76], but the basic properties are summarized here. The tailboom is manufactured from aluminum and has a semi-monocoque construction. The aluminum skin is attached to the eight tailboom stringers and seven frame elements by self-threading sheet metal screws. The tailboom is approximately 6 feet long, with a rectangular cross section that is 14 in. wide by 11 in. high at the root and tapers into a square 7.25 in.
by 7.25 in. cross section at its tip. The inner construction of the Heverly tailboom stringers and frame elements is depicted in Figure 2.1. Five-pound weights are attached to each end of the horizontal tail, and 12.5 pounds of weight are attached to the vertical tail tip. Both the tailboom stiffness profile and the inertial weights were selected to achieve mode shapes similar to those on the AH-64A tailboom.

The setup of the tailboom testbed for vibration experiments is depicted in Figure 2.2. The tailboom is bolted at its root to a 1 in. thick steel plate, and the plate is joined to T-slotted aluminum framing. The T-slotted aluminum framing is reinforced by a 1/4 in. thick triangular aluminum plate on each side of the tailboom and is also bolted to a heavy vibration isolation table.

Figure 2.1: Semi-monocoque construction of the PSU tailboom testbed, from [76].

Figure 2.2: Laboratory-scale tailboom vibration test stand.
The first step to modeling a structure with F$^2$MC tubes is to model the structure itself by an appropriate method. For uniform structures such as the cantilever beam in Zhu’s experiments, exact analytical transfer function solutions can be derived [72, 73]. However, for tapered or otherwise nonuniform beams, approximate methods must be used. In prior research, Miura used a Rayleigh-Ritz approach to model vertical vibrations in both a full-scale Bell 407 tailboom [75] and the Heverly laboratory-scale tailboom [74]. One disadvantage of the Rayleigh-Ritz approach is that extending it to complex structures can be challenging. For example, to accurately predict the laboratory-scale tailboom natural frequencies, its horizontal and vertical tails with inertial weights needed to be modeled, since they significantly impact the tailboom dynamics. Additional degrees of freedom may be added and tuned to capture these dynamics, but doing so requires assumptions or prior knowledge about the modal characteristics of the structure. In addition, although the Rayleigh-Ritz approach works for modeling simple structures, it is not as widely-used as the finite element method, especially in industry settings. These shortcomings motivate the use of a finite element approach, which can be implemented easily for any three dimensional structure using well-known finite elements.

In this study, the laboratory-scale tailboom is modeled using 22 Euler-Bernoulli beam finite elements, with three translations ($u$, $v$, and $w$) and three rotations ($\theta_x$, $\theta_y$, and $\theta_z$) at each node. Fourteen of those elements are used to model the tailboom itself, and the remaining eight are used to model the tail structure as shown in Figure 2.3. When F$^2$MC tubes are added to the model, the F$^2$MC tube attachment points coincide with nodes in the finite element model to facilitate combination of the structural and fluidic models.

Element mass and stiffness matrices are developed based on material and cross-sectional properties from [76], with rotations applied to convert between the local element coordinate system and the global coordinate system [77]. The twelve degrees of freedom for each element are organized in the order ($u_1$, $v_1$, $w_1$, $\theta_{x1}$, $\theta_{y1}$, $\theta_{z1}$, $u_2$, $v_2$, $w_2$, $\theta_{x2}$, $\theta_{y2}$, $\theta_{z2}$). The local element matrix for each element is 12 by 12. The unrotated element mass matrix entries are specified according to Eqs. (2.1)-(2.20), where $M_{a,b}$ denotes the entry in the $a^{th}$ row and $b^{th}$ column of the
Figure 2.3: Coordinate system and geometry of the tailboom finite element model, with point mass locations highlighted.

Element mass matrix:

\[ M_{1,1} = M_{7,7} = \frac{\rho AL_e}{3} \]  
\[ M_{1,7} = M_{7,1} = \frac{\rho AL_e}{6} \]  
\[ M_{2,6} = M_{6,2} = \frac{11\rho AL_e^2}{210} + \frac{\rho I_{zz}}{10} \]  
\[ M_{2,8} = M_{8,2} = \frac{9\rho AL_e}{70} - \frac{6\rho I_{zz}}{5L_e} \]  
\[ M_{2,2} = M_{8,8} = \frac{13\rho AL_e}{35} + \frac{6\rho I_{zz}}{5L_e} \]  
\[ M_{2,12} = M_{12,2} = -\frac{13\rho AL_e^2}{420} + \frac{\rho I_{zz}}{10} \]  
\[ M_{3,3} = M_{9,9} = \frac{13\rho AL_e^2}{35} + \frac{6\rho I_{yy}}{5L_e} \]  
\[ M_{3,5} = M_{5,3} = -\frac{11\rho AL_e^2}{210} - \frac{\rho I_{yy}}{10} \]  
\[ M_{3,9} = M_{9,3} = \frac{9\rho AL_e}{70} - \frac{6\rho I_{yy}}{5L_e} \]  
\[ M_{3,11} = M_{11,3} = \frac{13\rho AL_e^2}{420} - \frac{\rho I_{yy}}{10} \]
In these expressions, \( \rho \) is the material density, \( A \) is the element cross-sectional area, \( L_e \) is the element length, \( I_{zz} \) and \( I_{yy} \) are the element second area moments about the \( z \)- and \( y \)-axes, respectively, and \( I_p \) is the element polar moment of inertia. To accurately model the semi-monocoque tailboom, rotational inertia terms are included in these matrix entries. Similarly, the unrotated element stiffness matrix entries are defined by Eqs. (2.21)-(2.36) as follows:

\[
K_{1,1} = K_{7,7} = \frac{EA}{L_e} \tag{2.21}
\]
\[
K_{1,7} = K_{7,1} = -\frac{EA}{L_e} \tag{2.22}
\]
\[
K_{2,2} = K_{8,8} = \frac{12EI_{zz}}{L_e^3} \tag{2.23}
\]
\[
K_{2,6} = K_{6,2} = K_{2,12} = K_{12,2} = \frac{6EI_{zz}}{L_e^2} \tag{2.24}
\]
\[
K_{2,8} = K_{8,2} = -K_{2,2} \tag{2.25}
\]
where $E$ is the material Young’s modulus, $G$ is the material shear modulus, $J$ is the cross section torsion constant, and the remaining terms are as previously defined. Relevant parameters used in the tailboom modeling are listed in Table 2.1. Detailed properties for all finite elements in the tailboom model are tabulated in Appendix A.

The local matrices are assembled into the global mass and stiffness matrices, $[M]$ and $[K]$, based on element connectivity. The tailboom root boundary conditions are prescribed as zero translation in all three directions, zero twist, and a torsional spring in both the lateral and vertical directions to reflect the fact that the tailboom is not perfectly cantilevered to its test stand. The root torsional spring stiffnesses are tuned to values that provide a good match for the tailboom static stiffness and natural frequencies. Point masses corresponding to weights on the actual tailboom
Table 2.1: Properties used in finite element model of laboratory-scale tailboom.

<table>
<thead>
<tr>
<th>Material Properties (Aluminum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus, GPa</td>
</tr>
<tr>
<td>Shear Modulus, GPa</td>
</tr>
<tr>
<td>Density, kg/m$^3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tailboom Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (in.)</td>
</tr>
<tr>
<td>Base width (in.)</td>
</tr>
<tr>
<td>Base height (in.)</td>
</tr>
<tr>
<td>Tip side length (in.)</td>
</tr>
<tr>
<td>Skin thickness (in.)</td>
</tr>
<tr>
<td>Side rectangular stringer dimensions</td>
</tr>
<tr>
<td>Corner L-stringer dimensions</td>
</tr>
<tr>
<td>Vertical Tail</td>
</tr>
<tr>
<td>Horizontal Tail</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Point Springs and Masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Root vertical torsion spring, N-m/rad</td>
</tr>
<tr>
<td>Root lateral torsion spring, N-m/rad</td>
</tr>
<tr>
<td>Tailboom tip end plate mass, kg</td>
</tr>
<tr>
<td>Vertical tail tip mass, kg</td>
</tr>
<tr>
<td>Horizontal tail tip mass, kg (each side)</td>
</tr>
</tbody>
</table>

are added at both ends of the horizontal tail and the tip of the vertical tail. The plate used for attaching a shaker at the tailboom tip is also modeled as a point mass. The global damping matrix is constructed by prescribing a damping ratio for each mode and performing an inverse modal transformation,

$$ [C] = ([\phi^T])^{-1} [C_m][\phi]^{-1}, $$

(2.37)
where the modal damping matrix $[C_m]$ is defined by

$$
[C_m] = [M_m] \begin{bmatrix}
2\zeta_1\omega_{n_1} & 0 & \cdots & 0 \\
0 & 2\zeta_2\omega_{n_2} & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & 2\zeta_j\omega_{n_j}
\end{bmatrix},
$$

(2.38)

and $[\phi]$ is a matrix of eigenvectors that are solutions to the undamped tailboom eigenvalue problem. In Eq. (2.38), $[M_m]$ is the global tailboom modal mass matrix, and $\zeta$ and $\omega_n$ are the prescribed damping ratio and frequency of each tailboom mode. The subscript $j$ denotes the total number of degrees of freedom in the tailboom model. The equations of motion for the baseline tailboom can be expressed in the traditional form

$$
[M] \{\ddot{q}\} + [C] \{\dot{q}\} + [K] \{q\} = \{F_{ext}\},
$$

(2.39)

where $\{q\}$ is the vector of tailboom nodal degrees of freedom and the vector $\{F_{ext}\}$ represents the external forcing on the tailboom.

### 2.1.1 Verification of Laboratory-Scale Tailboom Model

Because the tailboom deformation causes the F²MC tube fluid pumping and forces, it is important to verify that the finite element model captures the tailboom dynamics with reasonable accuracy. If the structural model is accurate, the fluidic circuit will be easier to tune correctly, and the overall system model will accurately predict the absorber effectiveness.

The tailboom model is verified using the experimental setup shown in Figure 2.4. The magnitude of the frequency response function (FRF) from a lateral tip force input is measured at several locations on the tailboom to capture both the lateral bending and twisting components of the mode shape. The locations A, B and C in Figure 2.4 correspond to the lateral displacement of the tailboom centerline, the displacement of the horizontal tail tip, and the displacement of the vertical tail tip, respectively. Points A and C capture lateral bending vibrations, while points B and C capture torsional vibrations. A piezoelectric load cell (model PCB 208C02, sensitivity 11.241 V/kN) measures the force signal, a laser vibrometer
(sensitivity $320 \, \mu m/V$) measures the tip lateral displacement, and accelerometers (model PCB 353B02, sensitivity $20 \, mV/g$) measure the displacement of both the horizontal and vertical tail. A LabVIEW program continuously sends the shaker a sinusoidal sweep from 0 through 40 Hz and samples data at 200 Hz. To minimize noise, the frequency response function is generated by averaging several sweeps together within LabVIEW.

Figure 2.4: Setup for measuring the tailboom baseline frequency response.

The experimentally measured frequency responses are compared to model predictions in Figure 2.5. The model damping ratios are tuned to 2.1% and 1.0% for the first and second modes, respectively, to match response amplitudes seen in the experiment. In addition to these three points near the tailboom tip, modal measurements were taken by moving the laser vibrometer along the tailboom length as shown in Figure 2.6. The element bending stiffnesses were initially calculated by summing stiffness contributions from all eight stringers and the skin; however, it was found that reducing the calculated bending stiffness in the lateral and vertical directions by 25% led to better mode shape correlation between the model prediction and experimental results. The root spring was subsequently retuned to match the predicted and experimental natural frequencies. The mass density over the tailboom section, but not the horizontal or vertical tail, is also increased by 15% to approximate the effect of the frame masses. Figures 2.7 and 2.8 compare the final model-predicted mode shapes with the experimentally measured shapes.
Figure 2.5: Tailboom frequency response, model prediction versus experiment.
Figure 2.6: Points along tailboom length where mode shape measurements were taken with laser vibrometer.

Figure 2.7: Normalized first lateral mode shape for the tailboom (bending component only).

Figure 2.8: Normalized second lateral mode shape for the tailboom (bending component only).
2.2 Integration of F$^2$MC Tubes

This section describes the process for modeling the F$^2$MC tubes and fluidic circuit. The braided-sheath fabrication method was selected because it is simple and low-cost. The method for combining the fluid model with the previously described tailboom model is summarized, and the full system model is presented with the modified equations of motion that include the effects of F$^2$MC tubes.

2.2.1 F$^2$MC Tube & Fluidic Circuit Modeling

The model for the braided-sheath F$^2$MC tube is based on [68] and is the same model used by Miura in [74]. The relationship between F$^2$MC tube axial displacement $x$, internal pressure $p$, and the axial force $F$ acting on the tube is governed by the equation

$$c_1 x_i + c_2 p_i = F_i. \quad (2.40)$$

The fluid volume flow rate $Q$ out of an individual tube is obtained by differentiating the tube volume with respect to time,

$$-c_3 x_i - c_4 p_i = Q_i. \quad (2.41)$$

In these equations, the coefficients $c_1$ through $c_4$ are linearization constants that define the F$^2$MC tube axial stiffness, pressure-to-force relationship, volume change due to axial displacement, and volume change due to internal pressure. $c_1$ through $c_3$ are all predicted by Scarborough’s F$^2$MC tube model from [68]. In general, the parameters $c_2$ and $c_3$ from this model are more relevant to the vibration control authority of the F$^2$MC absorber, since the tailboom structure contributes much more to the overall bending stiffness than the F$^2$MC tube. $c_2$ relates pressures from the oscillatory fluid flow to the forces exerted on the tailboom structure, while $c_3$ determines the volume of fluid pumped due to F$^2$MC tube elongation. These two parameters are primarily functions of the F$^2$MC tube fiber wind angle $\alpha$ and the F$^2$MC tube diameter. The parameter $c_4$ represents the effective capacitance of the mesh-reinforced bladder, and the inverse of $c_4$ is analogous to the spring stiffness in a mechanical vibration absorber [78]. If the model value for $c_4$ does not
represent the true compliance of the F2MC tube, the fluidic circuit design based on the model may be mistuned, resulting in an absorber frequency that is above or below the target mode natural frequency.

The value for the $c_4$ parameter is difficult to predict and is therefore estimated using a benchtop apparatus illustrated here in Figure 2.9. In this test, known pressures are applied to the tube through an air pressure regulator, and the height of a fluid column contained in clear, rigid tubing is monitored. As the applied pressure increases, the height of the fluid column drops as the F2MC tube expands slightly. The parameter $c_4$ can be estimated empirically according to the relationship

$$c_4 = -\frac{V_{out}}{\Delta p} = -\frac{A_{col}\Delta h}{\Delta p},$$

(2.42)

which comes from the static form of Eq. (2.41). Here, $V_{out}$ is the volume of fluid moved out of the tube, $\Delta p$ is the pressure increment, $A_{col}$ is the cross-sectional area of the fluid column, and $\Delta h$ is the change in height of the fluid column, with positive defined as the column going up and negative as the column going down. 1/8 in. inner diameter rigid plastic tubing is used for the vertical fluid column. In the experimental apparatus, the length of the F2MC tube is fixed by securing the tube fittings at each end to steel brackets. Because the F2MC tube does not change volume due to axial displacement, the other term on the left-hand side of Eq. (2.41) is neglected.

Based on the sign conventions in Eqs. (2.40)-(2.41), $c_1$, $c_2$, and $c_4$ are all positive, and $c_3$ is negative. The index $i$ refers to the tube number, which implies a numbering scheme for the tubes in a given configuration. The numbering scheme is explained in the next section.

2.2.1.1 Torsional Absorber Concept, Uncoupled Tubes

This subsection considers the modeling of a device that is primarily designed to treat torsional vibrations. The device is configured as shown in Figure 2.10. In this concept, two identical pairs of diagonally crossing F2MC tubes are attached to opposite walls inside the tailboom, and each of these tubes is connected through an inertia track to another tube on the opposite wall. The F2MC tubes are connected fluidically so that as the tailboom twists, the tube on one side of the circuit extends and the tube on the other side of the circuit contracts, giving
them opposite pressures. The result is a net torque on the tailboom that opposes torsional vibration. One of the two identical pairs in the configuration is shown schematically in Figure 2.11. For the purpose of coupling the F²MC tube and fluid model with the tailboom model, each tube axial displacement \( x \) must be related to the instantaneous force \( F \) on it and the other tubes. An underlying assumption in this derivation is that all F²MC tubes are identical and therefore have the same constants \( c_1 \) through \( c_4 \).

As mentioned previously, one can write equations for each individual F²MC tube force and flow rate as

\[
\begin{align*}
    c_1 x_1 + c_2 p_1 &= F_1, \\
    c_1 x_2 + c_2 p_2 &= F_2, \\
    -c_3 \dot{x}_1 - c_4 \dot{p}_1 &= Q_1, \\
    -c_3 \dot{x}_2 - c_4 \dot{p}_2 &= Q_2.
\end{align*}
\]
Continuity of the fluid in the inertia track dictates that the flow rates $Q_1$ and $Q_2$ must be equal and opposite based on the convention in Figure 2.11,

$$Q_1 = -Q_2. \quad (2.47)$$

The dynamics of the fluid in the inertia track are governed by

$$p_1 - p_2 = I_c \dot{Q}_1 + R_c Q_1, \quad (2.48)$$

where $I_c$ and $R_c$ are the inertia track inertance and resistance, respectively. These parameters are functions of fluid density $\rho$, fluid dynamic viscosity $\mu$, track length
Inertance and resistance in the model are defined by Eqs. (2.49)-(2.50), which include a correction factor $\kappa$ to account for frequency dependence of these parameters based on correlations published by Donovan [79].

\[
I_c = \kappa_I(r_c, \rho, \mu, \omega_n) \frac{\rho l_c}{\pi r_c^2} \tag{2.49}
\]

\[
R_c = \kappa_R(r_c, \rho, \mu, \omega_n) \frac{8\mu l_c}{\pi r_c^4} \tag{2.50}
\]

The values of the correction factors are a function of track radius, fluid density, fluid viscosity, and oscillation frequency. When calculating $\kappa_I$ and $\kappa_R$, the natural frequency of the targeted mode is used as the oscillation frequency, and inertance and resistance are treated as approximately constant around this frequency. This simplification can be made because the absorber does not affect tailboom dynamics at frequencies far away from the target mode natural frequency. The effects of sharp bends, diameter changes, and other sources of added resistance are neglected.

Eqs. (2.43)-(2.48) can be manipulated to obtain the relationship between tube displacements $x_1$ and $x_2$ and tube forces $F_1$ and $F_2$. This process can be repeated to write out equations for F2MC tubes 3 and 4 in an absorber with four tubes as shown in Figure 2.10. However, the dynamics of tubes 3 and 4 will be decoupled from the dynamics of tubes 1 and 2 since the two fluidic circuits are not connected.

### 2.2.1.2 Bending Absorber Concept

The second concept is primarily designed to treat lateral bending vibrations and is an extension of previous work done by Miura in [74, 75]. The main difference in this new concept is that F2MC tube pairs are connected fluidically from the left to the right side of the tailboom as in Figure 2.12, whereas the Miura absorber circuit is connected from the top to the bottom pair. As the tailboom vibrates laterally, the left and right F2MC tube pairs develop opposite pressures, producing a net moment at the root of the tailboom to oppose this vibration.

The process from the previous subsection is modified to model the fluidic circuit for this concept, which has two pairs of F2MC tubes connected through a common inertia track as shown in Figure 2.13. Again, it is assumed that all four tubes are identical and have the same constants $c_1$ through $c_4$. Each tube is modeled separately, and the flow rates are combined based on fluid continuity to derive the
full set of equations governing F$^2$MC tube and fluid dynamics. As with the torsional absorber, each F$^2$MC tube force and flow rate can be expressed independently,

$$c_1 x_1 + c_2 p_1 = F_1, \quad (2.51)$$
$$c_1 x_2 + c_2 p_2 = F_2, \quad (2.52)$$
$$c_1 x_3 + c_2 p_3 = F_3, \quad (2.53)$$
$$c_1 x_4 + c_2 p_4 = F_4, \quad (2.54)$$
$$-c_3 \dot{x}_1 - c_4 \dot{p}_1 = Q_1, \quad (2.55)$$
$$-c_3 \dot{x}_2 - c_4 \dot{p}_2 = Q_2, \quad (2.56)$$
$$-c_3 \dot{x}_3 - c_4 \dot{p}_3 = Q_3, \quad (2.57)$$
$$-c_3 \dot{x}_4 - c_4 \dot{p}_4 = Q_4. \quad (2.58)$$

Next, a set of equations can be written for the fluid dynamics in each separate segment of the fluidic circuit. The circuit in Figure 2.13 has five segments. Each F$^2$MC tube pair has two track branches of inertance $I_b$ and resistance $R_b$ connecting to the main inertia track segment, which has inertance $I_m$ and $R_m$. The inertance $I_s$ and resistance $R_s$ of a circuit segment are given by Eqs. (2.59)-(2.60), where the parameters $l_s$ and $r_s$ now refer to the length and radius of each individual inertia track segment.
\[ I_s = \kappa_l(r_s, \rho, \mu, \omega_n) \frac{\rho l_s}{\pi r_s^2}, \quad (2.59) \]

\[ R_s = \kappa_R(r_s, \rho, \mu, \omega_n) \frac{8\mu l_s}{\pi r_s^4}. \quad (2.60) \]

**Figure 2.13:** Schematic of fluidic circuit for bending absorber using a coupled pair of F\(^2\)MC tubes.

The five equations governing the fluid dynamics of the segments are

\[ p_1 - p_5 = I_b \dot{Q}_1 + R_b Q_1, \quad (2.61) \]

\[ p_2 - p_5 = I_b \dot{Q}_2 + R_b Q_2, \quad (2.62) \]

\[ p_3 - p_6 = I_b \dot{Q}_3 + R_b Q_3, \quad (2.63) \]

\[ p_4 - p_6 = I_b \dot{Q}_4 + R_b Q_4, \quad (2.64) \]

\[ p_5 - p_6 = I_m \dot{Q}_{56} + R_m Q_{56}. \quad (2.65) \]

The final equation can be written based on fluid continuity at the junctions between the inertia track branches and main segment,

\[ Q_{56} = Q_1 + Q_2 = -Q_{65} = -(Q_3 + Q_4). \quad (2.66) \]

Eqs. (2.51)-(2.58) and (2.61)-(2.66) can be manipulated to obtain the relationship between F\(^2\)MC tube displacements \(x_1\) through \(x_4\) and F\(^2\)MC tube forces \(F_1\) through \(F_4\).
2.2.2 Combined Structural & Fluidic Circuit Model

A general method is now presented for combining the tailboom and fluid systems into one state-space model. This procedure involves taking the Laplace transform of the relevant set of fluid system equations and generating transfer functions from each F²MC tube axial displacement to each F²MC tube axial force. This relationship can be written in matrix form as

\[
\begin{pmatrix}
F_1(s) \\
F_2(s) \\
F_3(s) \\
F_4(s)
\end{pmatrix} = [H(s)]
\begin{pmatrix}
x_1(s) \\
x_2(s) \\
x_3(s) \\
x_4(s)
\end{pmatrix},
\] (2.67)

This method applies to both previously described configurations, although the structure of the transfer function matrix \([H(s)]\) depends on the configuration. In the case where tubes 1 and 2 are decoupled from tubes 3 and 4, the transfer function matrix \([H(s)]\) will have zero entries to reflect this decoupling. For example, there will be no relationship between \(F_1(s)\) and \(x_3(s)\), since tubes 1 and 3 are not connected fluidically. The transfer functions comprising \([H(s)]\) are included in Appendix B. Using MATLAB’s state-space tools, this transfer function relationship can be converted into a set of state-space equations with inputs \(x_1\) through \(x_4\) and outputs \(F_1\) through \(F_4\),

\[
\{\dot{\xi}\} = [A_t] \{\xi\} + \{B_{t,1}\} \{B_{t,2}\} \{B_{t,3}\} \{B_{t,4}\}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{pmatrix},
\] (2.68)

\[
\begin{pmatrix}
F_1 \\
F_2 \\
F_3 \\
F_4
\end{pmatrix} =
\begin{pmatrix}
\{C_{t,1}\}^T \\
\{C_{t,2}\}^T \\
\{C_{t,3}\}^T \\
\{C_{t,4}\}^T
\end{pmatrix} \{\xi\} +
\begin{pmatrix}
D_t & 0 & \cdots & 0 \\
0 & D_t & \cdots & \vdots \\
\vdots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & D_t
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{pmatrix}
\] (2.69)
where \( \{ \xi \} \) is a vector of the fluid system state variables. The axial displacement \( x_i \) of each F2MC tube is a function of the tailboom deformation,

\[
x_i = \{ \Psi_i \}^T \{ q \},
\]

where \( i \) varies from 1 to 4, and \( \{ \Psi_i \}^T \) is a row vector used to express the relationship between tailboom finite element nodal displacements and the F2MC tube axial displacement.

The contents of \( \{ \Psi_i \}^T \) are based on the spatial attachment points of the F2MC tube with index \( i \). This equation can be expanded to show the individual components of \( \{ \Psi_i \}^T \),

\[
x_i = \beta_{u_1} u_1 + \beta_{v_1} v_1 + \beta_{w_1} w_1 + \beta_{\theta_{x_1}} \theta_{x_1} + \beta_{\theta_{y_1}} \theta_{y_1} + \beta_{\theta_{z_1}} \theta_{z_1} + \beta_{u_2} u_2 + \beta_{v_2} v_2 + \beta_{w_2} w_2 + \beta_{\theta_{x_2}} \theta_{x_2} + \beta_{\theta_{y_2}} \theta_{y_2} + \beta_{\theta_{z_2}} \theta_{z_2}
\]

where \( \beta_{\chi} \) is the coefficient for nodal degree of freedom \( \chi \), and the subscripts 1 or 2 refer to the F2MC tube attachment points closer to the root or farther aft, respectively. This equation includes all possible tailboom deformations and implies that all degrees of freedom from nodes which are not F2MC tube attachment points have \( \beta \) coefficients of zero. For a given F2MC tube \( i \), its attachment point closer to the root has coordinates \( (x_1, y_1, z_1) \) and its other attachment point has coordinates \( (x_2, y_2, z_2) \) as shown in Figure 2.14. Denoting the F2MC tube initial length as \( L_o \) and linearizing the change in length for small displacements and rotations, the twelve coefficients are as follows:

\[
\beta_{u_1} = -\beta_{u_2} = \frac{x_1 - x_2}{L_o} 
\]

\[
\beta_{v_1} = -\beta_{v_2} = \frac{y_1 - y_2}{L_o}
\]

\[
\beta_{w_1} = -\beta_{w_2} = \frac{z_1 - z_2}{L_o}
\]

\[
\beta_{\theta_{x_1}} = -\beta_{\theta_{x_2}} = \frac{y_2 z_1 - y_1 z_2}{L_o}
\]

\[
\beta_{\theta_{y_1}} = \frac{x_1 - x_2}{L_o} - z_1
\]

40
\[
\begin{align*}
\beta_{\theta z_2} &= \frac{x_2 - x_1}{L_o} \theta_2 \\
\beta_{\theta z_1} &= \frac{x_2 - x_1}{L_o} y_1 \\
\beta_{\theta z_2} &= \frac{x_1 - x_2}{L_o} y_2
\end{align*}
\]

Eq. (2.70) can be substituted into Eq. (2.69) for each \( F^2MC \) tube so that each \( F_i \) is expressed in terms of \( \{\xi\} \) and \( \{q\} \),

\[
F_i = \{C_{t,i}\}^T \{\xi\} + D_t \{\Psi_i\}^T \{q\}.
\]  

(2.80)

The scalar \( F^2MC \) tube forces are converted into finite element model loads \( \{\bar{F}_i\} \) by the relationships

\[
\{\bar{F}_i\} = \{\sigma_i\} F_i,
\]  

(2.81)

where \( \{\sigma_i\} \) is a vector based on \( F^2MC \) tube geometry that is used to generate force components in Cartesian coordinates and the effective torque and moments due to the tube force at each attachment point. The signs of entries in \( \{\sigma_i\} \) reflect that the force exerted by the tube on the tailboom is equal and opposite \( F_i \), which denotes the force exerted on the tube by the tailboom. For small displacements and rotations, \( \{\sigma_i\} = -\{\Psi_i\} \).

The equations of motion for the tailboom are now revisited to insert the \( F^2MC \) tube loads. The new equations of motion are given by

\[
\begin{bmatrix} M \end{bmatrix} \{\ddot{q}\} + \begin{bmatrix} C \end{bmatrix} \{\dot{q}\} + \begin{bmatrix} K \end{bmatrix} \{q\} = \{F_{ext}\} + \sum_{i=1}^{4} \{\bar{F}_i\}.
\]

(2.82)

The tailboom equations of motion can be converted to state-space form and combined with Eq. (2.68) to express the full system dynamics,

\[
\begin{bmatrix} \{\dot{q}\} \\ \{\ddot{q}\} \\ \{\dot{\xi}\} \end{bmatrix} = \begin{bmatrix} A_{sys} \end{bmatrix} \begin{bmatrix} \{q\} \\ \{\dot{q}\} \\ \{\xi\} \end{bmatrix} + \begin{bmatrix} B_{sys} \end{bmatrix} \{F_{ext}\},
\]

(2.83)
where the state matrix for the entire system, \([A_{sys}]\), is

\[
[A_{sys}] = \begin{bmatrix}
0 & [I] & 0 \\
[M]^{-1}(-[K] + \sum_{i=1}^{4} \bar{F}_q) & -[M]^{-1}[C] & [M]^{-1} \left( \sum_{i=1}^{4} \bar{F}_\xi \right) \\
\sum_{i=1}^{4} \{B_{t,i}\} \{\Psi_i\}^T & [0] & [A_t]
\end{bmatrix}
\] (2.84)

\(F_{ext}\) in the context of Eq. (2.83) is the scalar external force applied to the tailboom, while \(\{B_{sys}\}\) converts the external force into a finite element load by defining the location and direction of this force. In this state-space formulation, \([I]\) is an identity matrix having dimension equal to the total number of degrees of freedom in the finite element model, \([\bar{F}_q]\) is a matrix that accounts for the part of Eq. (2.80) multiplied by nodal displacements, and \([\bar{F}_\xi]\) is a matrix that accounts for the part of Eq. (2.80) multiplied by fluid system state variables. Based on Eqs. (2.80)-(2.81),

\[
[\bar{F}_q]_i = \{\sigma_i\} \left( \{\sigma_i\} D_t \{\Psi_i\}^T \right),
\] (2.85)

\[
[\bar{F}_\xi]_i = \{\sigma_i\} \left( \{\sigma_i\} C_{t,i} \right)^T.
\] (2.86)
Because they are part of the overall system state vector, tailboom nodal degrees of freedom can easily be chosen as outputs for the state-space model.

2.3 Simulation Results & Parametric Studies

In this section, some simulation results are presented to show general trends and attempt to understand how the proposed $F^2MC$ absorbers can be effectively designed. Comparisons between the two previously described configurations are made, and a parametric study is conducted to examine the effect of changing different $F^2MC$ tube or fluidic circuit properties.

2.3.1 Torsional Absorber Versus Bending Absorber Comparison

One of the main early goals of this research was to determine the most effective $F^2MC$ tube and circuit configuration for reducing vibrations in bending/torsion coupled modes. The $F^2MC$-integrated tailboom model is used to compare the effectiveness of the two proposed absorbers for reducing vibration of the second lateral bending/torsion mode, which has a natural frequency between 26 and 27 Hz. The natural frequencies of the tailboom test stand were found to change slightly with forcing amplitude; therefore, a level of forcing was selected that could effectively excite the tailboom bending modes and allow for demonstration of the $F^2MC$ vibration absorber. The natural frequency measured in experiments at the chosen forcing level was 26.7 Hz.

The 26.7 Hz lateral bending/torsion mode is chosen as the target mode for demonstrating an $F^2MC$ absorber because the 10 Hz and 38 Hz modes are dominated by tail deformation. An absorber with $F^2MC$ tubes attached to the root of the tailboom would be ineffective at reducing vibrations of these tail modes since most of the modal deformation occurs away from the root. The first two model-predicted lateral bending/torsion mode shapes are plotted in Figure 2.15 to illustrate the difference between a local tail mode and a global tailboom mode. The second mode natural frequency is significantly higher than the 12.2 Hz target mode for Miura’s tailboom absorber [74]. In Figure 2.16, the $n/\text{rev}$ frequencies of existing three-bladed and four-bladed helicopters are plotted, and 26.7 Hz is near or above most of these helicopters’ $n/\text{rev}$ frequencies. Therefore, demonstrating vibration reduction
in the 26.7 Hz tailboom mode would be a good indicator that the proposed F$^2$MC absorber is viable for treating $n$/rev vibrations on currently existing rotorcraft. The modal damping ratios are increased to 3.0% and 1.8% for the first and second lateral modes, respectively, to reflect the increased damping observed at higher forcing levels in experiments.

![Visualization of (a) first and (b) second lateral bending/torsion modes for laboratory-scale tailboom (view along tailboom axis).](image)

To make comparisons between bending and torsional absorber effectiveness, devices are designed which span equal lengths of 25 in. along the axis of the tailboom (36% of its total length). In the torsional absorber, the outer and inner tubes are positioned 0.75 and 1.25 in. inside of the tapered skin, respectively, and span 75% of the side to which they are attached. For example, the distance between the attachment points in Figure 2.10 would be 75% of the tailboom height.
at that axial location. This leaves additional space for attachment hardware and hydraulic fittings that would be included on a physical prototype. In the bending absorber, there is a vertical offset of 2 in. between each tube and the tailboom vertical midplane, and tubes are positioned 1 in. inside of the tapered skin on the left and right sides. The F$^2$MC tubes used in these simulations have an inner diameter of 9.53 mm ($\frac{3}{8}$ in.), and the fluid properties are representative of a dense fluid used in production fluidic isolators. The parameters used as inputs for generating the F$^2$MC tube coefficients are given in Table 2.2. When possible, inertia track properties are tuned for each case so that both absorber peaks are the same height in the bending response. This is the convention for an optimal tuned mass damper according to the classical equal-peak design method [17]. Properties of the tuned fluidic circuits for the devices simulated can be found in Table 2.3. Based on prior benchtop experiments, the F$^2$MC tube parameter $c_4$ is assumed to scale linearly with the F$^2$MC tube active length, which is the distance between the clamps at either end of the tube.

Frequency response results are plotted in Figure 2.17 for midplane lateral displacement and twisting at the tailboom tip from the same forcing arrangement used in Figure 2.4. This input force location is 3 inches below the tailboom midplane and is representative of the experimental setup used in Chapter 3 for experimental verification of these models. A total of four cases are plotted: 1) baseline tailboom

![Figure 2.16: 1/rev and n/rev frequencies of existing helicopters, from [4].](image-url)
Table 2.2: Properties used as inputs to the F²MC tube model.

<table>
<thead>
<tr>
<th></th>
<th>Stainless Steel Fibers</th>
<th>Rubber Bladder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus, GPa</td>
<td>180</td>
<td>1</td>
</tr>
<tr>
<td># of strands</td>
<td>312</td>
<td></td>
</tr>
<tr>
<td>Strand diameter, mm</td>
<td>0.203</td>
<td></td>
</tr>
<tr>
<td>Fiber angle, °</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Elastic Modulus, MPa</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Inner diameter, mm</td>
<td>9.53</td>
<td></td>
</tr>
<tr>
<td>Wall thickness, mm</td>
<td>0.794</td>
<td></td>
</tr>
</tbody>
</table>

with no F²MC tubes, 2) tailboom with torsional absorber attached to the tailboom top and bottom, 3) tailboom with torsional absorber attached to the tailboom left and right, and 4) tailboom with lateral bending absorber. The distinction between cases 2) and 3) must be made because the torsional absorber attached to the left and right sides also functions as two (uncoupled) bending absorbers, with one tube inclined at an angle on each side. In other words, absorber option 3) pumps fluid in response to lateral bending as well as torsion. The frequency response results in Figure 2.17 indicate that the bending absorber is the most effective option, although the torsional absorber on the left and right walls gives only slightly weaker performance.

An important result displayed in Figure 2.17 is that because the mode being targeted is a coupled bending/torsion mode, the model predicts that all three absorber concepts will reduce the amplitude of both bending and twisting vibration. This implies that an F²MC absorber designed to reduce bending vibration also reduces torsional vibration and vice versa. The device effectiveness can vary greatly depending on the configuration, though. As shown in Figure 2.17a, the proposed bending absorber reduces bending vibration amplitude by 10.7 dB (71%) at resonance, and the torsional absorber with F²MC tubes on the left and right reduces vibration amplitude by 10.2 dB (69%). The torsional absorber with tubes on the top and bottom is largely ineffective in reducing tailboom vibrations and only reduces the amplitude by about 0.6 dB (7%) at resonance.
Figure 2.17: Tailboom tip a) lateral displacement and b) twist frequency responses for tip force input, with different F²MC absorber configurations.
Table 2.3: F$^2$MC tube and tuned fluidic circuit properties for torsional and bending F$^2$MC absorbers.

<table>
<thead>
<tr>
<th>Property</th>
<th>Torsional Absorber (T/B)</th>
<th>Torsional Absorber (L/R)</th>
<th>Bending Absorber</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linearized F$^2$MC Tube Model Coefficients</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial stiffness, $c_1$, N/m</td>
<td>$7.95 \times 10^3$</td>
<td>$8.18 \times 10^3$</td>
<td>$8.68 \times 10^3$</td>
</tr>
<tr>
<td>Force-pressure coefficient, $c_2$, N/Pa</td>
<td>$2.24 \times 10^{-3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume change coefficient, $c_3$, m$^3$/m</td>
<td></td>
<td>$-1.49 \times 10^{-3}$</td>
<td></td>
</tr>
<tr>
<td>Capacitance, $c_4$, m$^3$/Pa</td>
<td>$1.50 \times 10^{-12}$</td>
<td>$1.47 \times 10^{-12}$</td>
<td>$1.38 \times 10^{-12}$</td>
</tr>
<tr>
<td><strong>Tuned Fluidic Circuit Properties</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fluid density, kg/m$^3$</td>
<td></td>
<td></td>
<td>$1800$</td>
</tr>
<tr>
<td>Fluid dynamic viscosity, Pa-s</td>
<td></td>
<td></td>
<td>$9.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>Inertia track radius, mm</td>
<td></td>
<td></td>
<td>$4.67$</td>
</tr>
<tr>
<td>Branch segment length, cm</td>
<td>N/A</td>
<td>N/A</td>
<td>$15.2$</td>
</tr>
<tr>
<td>Main segment length, cm</td>
<td>$157$</td>
<td>$157$</td>
<td>$67.7$</td>
</tr>
<tr>
<td>Inertance correction factor, $\kappa_I$</td>
<td></td>
<td></td>
<td>$1.11$</td>
</tr>
<tr>
<td>Resistance correction factor, $\kappa_R$</td>
<td></td>
<td></td>
<td>$126.1$</td>
</tr>
</tbody>
</table>

The difference in performance between placing the F$^2$MC tubes on the left and right sides versus placing them on the top and bottom sides is quite significant, but this result can be physically explained. Although option 3) is referred to here as a torsional absorber, its vibration reduction appears to stem primarily from its simultaneous functioning as a bending absorber, since option 2) does not have this benefit and is much less effective. Because the inertia track length for the torsional absorber given in Table 2.3 is for only one of two identical tracks, the torsion absorbers also require substantially more inertia track tubing to tune. Physically, this is due to the fact that only one F$^2$MC tube on each side is contributing to the flow, while the bending absorber has two F$^2$MC tubes contributing on each side, giving the effect of more inertia with a shorter track. Therefore, the bending absorber is not only a more effective solution, it is also more weight- and space-efficient.
2.3.2 Effect of Changing Inertia Track Radius

Eqs. (2.59)-(2.60) express the relationship between inertia track dimensions and the inertance and resistance of a given track segment. Before the correction factor $\kappa$ is applied, inertance is inversely proportional to the square of track radius, and resistance is inversely proportional to the fourth power of track radius. Eq. (2.60) indicates that one can design an inertia track with a wider radius to reduce damping in the fluidic circuit. However, Eq. (2.59) states that increasing the track radius lowers inertance, which would raise the effective absorber frequency if the track length is not re-tuned. Therefore, the length to tune the absorber at the same frequency must be longer for the inertia track with a wider radius, which drives up the weight of the modified circuit.

In this section, results are presented to examine the benefits and drawbacks of varying inertia track radius. Because it was the superior concept from performance and weight standpoints, the bending configuration from the previous section is used as a starting point. Four configurations are tested with inertia tracks ranging in radius from 3.09 mm (0.1215 in.) to 5.46 mm (0.215 in.). These values correspond to dimensions of commercially available tubing. The track lengths and other circuit properties of the tuned circuits are listed in Table 2.4. The same F$^2$MC tube and fluid properties from Tables 2.2 and 2.3 are carried over to this study.

Frequency responses for the four test cases are plotted in Figure 2.18. Wahile varying the inertia track radius has the expected effect, the variation in performance from using different inertia track dimensions is fairly small. Of the four test cases, the circuit with the narrowest radius yielded a 9.5 dB (67%) reduction, whereas the circuit with the widest radius yielded 11.1 dB (72%) reduction. The results from these curves show that small increases in vibration reduction for the wider inertia track designs can come with substantially longer inertia tracks. Depending on the application, it may be more practical to use a narrow inertia track to minimize the length of tubing needed in the inertia track and, consequentially, the overall weight of the F$^2$MC vibration absorber.
Table 2.4: \(F^2MC\) tube and tuned fluidic circuit properties for bending absorbers using different inertia track radii.

<table>
<thead>
<tr>
<th>Property</th>
<th>Narrow Radius</th>
<th>...</th>
<th>...</th>
<th>Wide Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearized (F^2MC) Tube Model Coefficients</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constants (c_1-c_4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tuned Fluidic Circuit Properties</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inertia track radius, mm</td>
<td>3.09</td>
<td>3.87</td>
<td>4.67</td>
<td>5.46</td>
</tr>
<tr>
<td>Track branch length, cm</td>
<td>15.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Main track length, cm</td>
<td>20.0</td>
<td>41.1</td>
<td>67.7</td>
<td>98.6</td>
</tr>
<tr>
<td>Inertance correction factor, (\kappa_I)</td>
<td>1.13</td>
<td>1.12</td>
<td>1.11</td>
<td>1.10</td>
</tr>
<tr>
<td>Resistance correction factor, (\kappa_R)</td>
<td>68.0</td>
<td>95.4</td>
<td>126.1</td>
<td>159.1</td>
</tr>
</tbody>
</table>

### 2.3.3 Effect of Changing \(F^2MC\) Tube Length

Up to this point, the length of the \(F^2MC\) tubes in the device has been treated as constant. In some cases, it might be desirable to have an \(F^2MC\) absorber that takes up a smaller part of the tailboom length so that the device becomes lighter and more compact. This subsection considers the design of such a device and evaluates the impact that shortening the \(F^2MC\) tubes has on the vibration absorber effectiveness. Three absorbers featuring \(F^2MC\) tubes of 15-, 20-, and 25-inch lengths attached at the root of the tailboom are evaluated. Here, length defines the distance between \(F^2MC\) tube attachment points and not the tube active length. As in the previous cases, an optimal fluidic circuit is designed for each absorber, and the model frequency response predictions are compared. The parameters for each of the fluidic circuits are given in Table 2.5. Note that the \(F^2MC\) tube capacitance decreases and the main inertia track segment becomes longer as the \(F^2MC\) tube length decreases. For this analysis, it is assumed that the \(F^2MC\) tube capacitance \(c_4\) scales linearly with the \(F^2MC\) tube active length. Since the shorter \(F^2MC\) tube experiences a smaller volume change per unit pressure, it is effectively stiffer, and a higher inertance is required to achieve the same tuning frequency.
Figure 2.18: Tailboom tip a) lateral displacement and b) twist frequency responses for tip force input, with different $F^2MC$ bending absorbers designed using different inertia track radii.
Table 2.5: F$^2$MC tube and tuned fluidic circuit properties for bending absorbers using different F$^2$MC tube lengths.

<table>
<thead>
<tr>
<th>Property</th>
<th>15&quot; F$^2$MC Tubes</th>
<th>20&quot; F$^2$MC Tubes</th>
<th>25&quot; F$^2$MC Tubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linearized F$^2$MC Tube Model Coefficients</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial stiffness, $c_1$, N/m</td>
<td>18.32×10$^3$</td>
<td>11.78×10$^3$</td>
<td>8.68×10$^3$</td>
</tr>
<tr>
<td>Force-pressure coefficient, $c_2$, N/Pa</td>
<td>2.24×10$^{-3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume change coefficient, $c_3$, m$^3$/m</td>
<td>-1.49×10$^{-3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capacitance, $c_4$, m$^3$/Pa</td>
<td>6.52×10$^{-13}$</td>
<td>1.01×10$^{-12}$</td>
<td>1.38×10$^{-12}$</td>
</tr>
<tr>
<td>Tuned Fluidic Circuit Properties</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inertia track radius, mm</td>
<td>4.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Track branch length, cm</td>
<td>15.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Main track length, cm</td>
<td>157.2</td>
<td>97.0</td>
<td>67.7</td>
</tr>
<tr>
<td>Inertance correction factor, $\kappa_I$</td>
<td>1.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resistance correction factor, $\kappa_R$</td>
<td>126.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Frequency response results for the three absorbers are plotted in Figure 2.19. The results indicate that the absorbers with longer F$^2$MC tubes are slightly more effective. This makes physical sense because as the F$^2$MC tubes span more of the tailboom, they elongate more as the tailboom vibrates, thereby pumping more fluid and generating higher pressures. In addition, the resulting control moments are applied over a longer length of the tailboom. These results predict that with proper tuning, the F$^2$MC absorber performance does not change significantly with F$^2$MC tube length. Decreasing the F$^2$MC tube length in this design from 25 in. to 15 in. only reduces the vibration reduction at resonance from 10.7 dB (71%) to 9.5 dB (66%) after re-tuning the circuit.

While there may be benefits to having a more compact absorber, the weight reduction from having shorter fluid-filled F$^2$MC tubes could be offset by a longer, heavier inertia track, so it is not clear whether making the F$^2$MC tubes shorter reduces the overall device weight. However, a longer track may not always be necessary if the F$^2$MC tubes are shortened in a given design. The track radius was held constant when generating these plots, but as discussed in the previous section, the same inertance can be achieved by using a smaller inertia track radius to reduce the required track length without sacrificing much performance.
Figure 2.19: Tailboom tip a) lateral displacement and b) twist frequency responses for tip force input, with different F^2MC bending absorbers designed using different F^2MC tube lengths.
2.3.4 Summary of Design Study Results

Based on the predicted performance of the different F²MC absorber concepts, the proposed bending absorber is superior to the proposed torsional absorber at reducing vibrations for this particular tailboom mode. The torsional F²MC absorber concept may be effective for reducing vibrations in structures with low torsional stiffness or mode shapes containing more torsional deformation. However, the laboratory-scale tailboom seems unsuitable for verifying the performance of this particular absorber concept. Simulation results predict that F²MC bending absorbers can reduce both bending and torsional vibration by around 70% for a target coupled mode of this laboratory-scale tailboom.

The model predicts that this level of vibration reduction can be achieved using a variety of F²MC tube lengths and inertia track dimensions, which provides flexibility in how an F²MC treatment can be designed to meet different objectives. Shortening the F²MC tubes or using a narrower inertia track radius may reduce the weight and/or size of the treatment without significantly reducing the treatment effectiveness.
In this chapter, experimental results are presented to verify the model described in
Chapter 2 for coupled pairs of tubes pumping through the same fluidic circuit. A
set of four F²MC tubes is fabricated and attached to the laboratory-scale tailboom
along with fluidic circuits designed using the model. To assess the F²MC absorber
effectiveness, the resulting vibration frequency responses are measured with a laser
vibrometer and accelerometers. The model accuracy is verified by experiments using
several different fluidic circuit designs. Circuits utilizing two different fluids (water
and a high-density, low-viscosity fluid) and two different tubing materials (copper
and plastic) are tested. With proper tuning, all of these circuits are demonstrated
successfully and exhibit similar performance.

3.1 Fabrication of F²MC Tubes

The braided-sheath F²MC tubes fabricated for the prototype absorber are similar
to those used by Miura in [74]. The tubes use a 1/32 in. thick, 3/8 in. inner diameter
latex rubber bladder reinforced by a corrosion-resistant stainless steel mesh. A
thin bladder is chosen to minimize the effects of wall compliance in the F²MC
tube. Thicker bladders increase tube capacitance $c_4$, which can limit the absorber
effectiveness but may also drive required inertia track lengths down. In experiments,
the F\textsuperscript{2}MC tube is pulled into tension so that the fibers squeeze down on the bladder to reduce its diameter, while the bladder is pressurized from the inside to push it outward. These two factors ensure that there is strong engagement between the bladder and the fibers.

End fittings are important parts of the F\textsuperscript{2}MC tube because they are responsible for ensuring that the fluid is sealed and that forces are transmitted to the structure being controlled. To accomplish these two tasks, the end fitting design from Miura’s F\textsuperscript{2}MC tubes is reused here. This end fitting concept is shown in Figure 3.1. Each fitting is manufactured by drilling a hole through a \(\frac{5}{8}\)-18 stainless steel threaded rod to serve as a fluid passageway, and then female National Pipe Taper (NPT) thread is tapped into both ends. The NPT threads allow the steel fitting to be used as an adapter for other commercially available fittings. In the fluidic model, these end fittings are included when calculating the inertance and resistance of each branch segment.

![Diagram of the braided-sheath F\textsuperscript{2}MC tube and end fitting design.](image)

To fabricate an F\textsuperscript{2}MC tube, the rubber bladder is pulled over barbed adapters at either end of the tube. These barbed adapters thread into one of the female NPT pipe threads on the stainless steel fitting. The stainless steel mesh is then pulled over the rubber bladder. At one end of the tube, the bladder is clamped to secure it in place, and then a sawed-off hydraulic fitting is crimped over the mesh, securing it to the stainless steel threaded fitting. This process is repeated at the other end of the F\textsuperscript{2}MC tube. The hydraulic crimper used in this process is a
Parker Karrykrimp, and a 5/8 in. die is used to crimp the hydraulic fitting onto the threaded fitting.

The end fittings of each F²MC tube are secured to the tailboom by hex nuts at both the root steel plate and an L-bracket further down the length of the tailboom. The L-bracket attachment is depicted in Figure 3.2. Six bolts fasten each L-bracket to the tailboom stringers beneath the skin so that the F²MC tube loads are transferred through the tailboom main structural members. The L-brackets are attached approximately 53.3 cm (21 in.) from the tailboom root. The threaded rod end fittings provide a way to tension each F²MC tube by adjusting the hex nuts at one or both ends of the tube. The four F²MC tubes are installed onto the tailboom with a 2.5 cm (1 in.) vertical offset from the skin and a horizontal offset of 13.7 cm (5 3/8 in.) from the tailboom lateral midplane. The horizontal offset does not vary along the tailboom length. In other words, the F²MC tubes do not follow the tailboom lateral taper, but they do follow its vertical taper.

![Figure 3.2: Two hex nuts used to tension the F²MC tube and secure the end fitting to the L-bracket.](image)

Figure 3.3 is a photograph illustrating the complete experiment with the F²MC absorber including the F²MC tubes, attachment hardware, fluidic circuit, and the circuits for filling and bleeding the device. The fluid in Figure 3.3 has been colored green for visibility. Figure 3.4 is a closer side view of the tailboom root with attached F²MC tubes and fluidic circuit. During the filling process, fluid comes in from the pump on the right side of the figure, flows through the fluidic circuit made from copper tubing, and then exits through the clear plastic bleed circuit. Fluid is continuously pumped through the circuit until no more air bubbles are exiting through the bleed circuit. The cycling of fluid helps to flush entrapped air out of the circuit, which can alter the effective bulk modulus of the working fluid [67,80] and cause the circuit to become mistuned since it was designed for a specific capacitance as characterized by the parameter $c_4$. Once air has been
sufficiently removed from the circuit, the four valves leading from each F\textsuperscript{2}MC tube into the bleed circuit are closed, and the operating pressure is set by continuing to pump fluid into the circuit until the desired pressure has been reached. Once the desired pressure is reached, the pump valve is closed so that the entire fluidic circuit is contained.

Figure 3.3: Full setup for tailboom vibration experiment with F\textsuperscript{2}MC tubes, fluidic circuit, and fill/bleed circuits.

The shaker setup, measurement points, and LabVIEW measurement program are identical to those from the baseline experiment described in Chapter 2. The LabVIEW program drives the shaker while recording frequency response measurements from a laser vibrometer aimed at the tailboom tip and two accelerometers on the horizontal and vertical tails. In some iterations of the fluidic circuit, a valve is included in the main segment of the inertia track. Including this valve makes it easy to check properties such as actuation authority or capacitance of each F\textsuperscript{2}MC tube pair and ensure that all four F\textsuperscript{2}MC tubes are roughly equivalent. It should be noted that this valve in the main inertia track segment serves no function in the F\textsuperscript{2}MC absorber and is merely included to perform useful checks during testing.
3.2 Tailboom Experiments with F\textsuperscript{2}MC Tubes

In this section, results are presented which compare the full system model predictions to results measured on the tailboom test stand with the corresponding F\textsuperscript{2}MC vibration absorber. Three individual test cases are described here. The first test uses water as the working fluid and copper tubing for the inertia track. The second test uses a high-density, low-viscosity working fluid and copper tubing for the inertia track. Finally, the third test uses water as the working fluid and rigid plastic tubing for the inertia track. Following the results, comparisons are made between the different test cases, and the distinct advantages of each case are summarized.

3.2.1 Model Verification: Copper/Water Circuit

Copper is chosen as the first inertia track material since it is bendable with the proper tooling, compatible with common off-the-shelf fittings, and rigid enough that inertia track wall compliance is negligible. The F\textsuperscript{2}MC tube model input properties and the resulting tube coefficients are given in Table 3.1. The value for $c_4$ is estimated through a combination of benchtop testing on a single F\textsuperscript{2}MC tube with the apparatus from Figure 2.9 and tuning based on experimental results from the treated tailboom vibration tests. Since the same F\textsuperscript{2}MC tubes are used in all three test cases, these properties and inputs also apply to the remaining sections.
of this chapter. Although water is a convenient fluid for demonstrating feasibility
of this concept and verifying the model, it would likely not be appropriate in a
realistic rotorcraft application since the fluid would become very viscous or even
freeze at low temperatures. In a production F$^2$MC absorber, the fluid properties
would ideally be insensitive to temperature to ensure that the F$^2$MC absorber
remains effective in any operating environment.

Table 3.1: Fabricated F$^2$MC tube properties and coefficients.

<table>
<thead>
<tr>
<th>General Properties</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Length, cm</td>
<td>53.3</td>
</tr>
<tr>
<td>Active length between clamps, cm</td>
<td>36.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stainless Steel Fibers</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus, GPa</td>
<td>180</td>
</tr>
<tr>
<td># of strands</td>
<td>312</td>
</tr>
<tr>
<td>Strand diameter, mm</td>
<td>0.203</td>
</tr>
<tr>
<td>Fiber angle, °</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rubber Bladder</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus, MPa</td>
<td>1</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.5</td>
</tr>
<tr>
<td>Inner diameter, mm</td>
<td>9.53</td>
</tr>
<tr>
<td>Wall thickness, mm</td>
<td>0.794</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Linearized F$^2$MC Tube Model Coefficients</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial stiffness, $c_1$, N/m</td>
<td>1.92×10$^4$</td>
</tr>
<tr>
<td>Force-pressure coefficient, $c_2$, N/Pa</td>
<td>2.91×10$^{-3}$</td>
</tr>
<tr>
<td>Volume change coefficient, $c_3$, m$^3$/m</td>
<td>-1.94×10$^{-3}$</td>
</tr>
<tr>
<td>Capacitance, $c_4$, m$^3$/Pa</td>
<td>8.5×10$^{-13}$</td>
</tr>
</tbody>
</table>

Frequency response magnitudes are plotted in Figure 3.5 for both the model
predictions and experimental measurements at the tailboom tip, horizontal tail tip,
and vertical tail tip. These results were obtained with a preset 100 psi operating
pressure inside the F$^2$MC tubes and fluidic circuit. The frequency response plots
show good correlation for the predicted absorber frequency and the level of vibration
reduction. The experiments also verify the key model prediction from Chapter
2 that one F$^2$MC absorber can simultaneously reduce both bending and torsion
components of a coupled vibration mode. The 22 Hz vertical mode seen in the
horizontal tail frequency response is actually predicted by the finite element model, although the model does not predict it to be observable with a lateral excitation. The mode is likely excited due to asymmetry of the tailboom and/or alignment errors in the shaker setup.

The effect of operating pressure on the F$^2$MC absorber performance is illustrated in Figure 3.6. Note that while the circuit is tuned well at higher operating pressures of 80 and 100 psi, it becomes slightly mistuned at lower pressures such as in the 50 psi case. The fact that the absorber frequency seems to decrease for the 50 psi test case indicates that either the fluidic circuit or F$^2$MC tubes become more compliant at lower pressures. This can be explained by factors related to both the fluid and the F$^2$MC tube itself. At higher operating pressures, the volume of entrapped air in the circuit is reduced, the fluid effective bulk modulus increases, and tighter engagement is achieved between the rubber bladder and the stainless steel fibers. The main drawback to operating the F$^2$MC absorber at higher pressures is that it may create a higher risk for F$^2$MC tube static or fatigue failure.

### 3.2.2 Predicted Tailboom Dynamic Load Reductions

While the laser vibrometer and accelerometer measurements in the previous section indicate that the tailboom vibration is reduced by the F$^2$MC absorber, another important consideration when assessing the performance of an F$^2$MC absorber is its impact on the tailboom internal reaction loads. The local shear and moment at a node can be calculated from [81]

$$\{F_{int}\} = [K_e] \{q_{loc}\}, \quad (3.1)$$

where $[K_e]$ is the local element stiffness matrix and $\{q_{loc}\}$ is a vector containing the twelve local degrees of freedom for that element. $\{F_{int}\}$ is the vector of internal loads, with the first six entries being the $x$, $y$, and $z$ internal forces and moments at the node closer to the root end of the tailboom, and the second six entries being the internal forces and moments at the node closer to the tail end. The frequency response magnitudes of reaction moments and lateral shear forces at the 2$^{nd}$ node, 4$^{th}$ node, and 7$^{th}$ node are plotted in Figures 3.7 through 3.9. The 2$^{nd}$ and 7$^{th}$ nodes are the F$^2$MC attachment point nodes, and the 4$^{th}$ node is located between the two attachment points.
Figure 3.5: Tailboom frequency response for a tip force input, with and without F²MC absorber (copper/water circuit).
Figure 3.6: Variation in a) tailboom tip and b) horizontal tail tip frequency response with preset operating pressure of F²MC absorber.
In general, these internal reaction frequency responses are similar to the displacement frequency responses presented in Figure 3.5. The model predicts that even over the area spanned by the F\textsuperscript{2}MC tubes, internal reaction loads are reduced. This is consistent with the results observed by Heverly [76], where an active vibration control installation reduces the vibratory stresses in the laboratory-scale tailboom corner stringers. However, Heverly also observed that with the actuator installation active, stresses in the actuator attachment frames increased locally. Depending on the complexity of the attachment hardware in a given absorber, the stresses induced by F\textsuperscript{2}MC tube forces could be estimated using hand calculations or a finite element model to ensure that attachment hardware is sized appropriately.

### 3.2.3 Model Verification: Copper/Dense Fluid Circuit

The second test examines performance of an F\textsuperscript{2}MC absorber using a dense fluid with viscosity comparable to water. Its properties are representative of a fluid used in production fluidic vibration isolators, and its specific gravity is approximately 1.88. This second circuit is built specifically to investigate the benefits of using a denser working fluid. Based on Eq. (2.59), the same effective inertance can be achieved in a shorter segment of tubing if a denser working fluid is chosen. Also, if frequency-dependent effects are neglected, Eq. (2.60) indicates that if the same tubing radius is maintained in the new inertia track, it will have a lower resistance because of its shorter length.

Figures 3.10 and 3.11 illustrate the difference in track tubing required between the fluidic circuit using water and the fluidic circuit using the denser fluid. The circuit using the dense fluid is more compact, which makes denser fluids attractive in applications where space constraints are important. However, the experimentally measured frequency response plotted in Figure 3.12 shows similar performance to the F\textsuperscript{2}MC absorber using water as the working fluid. In fact, the F\textsuperscript{2}MC absorber using water is slightly more effective than the absorber using the dense fluid. Again, a 100 psi operating pressure is set prior to the test, and there is good correlation between model predictions and experimental measurements for both the absorber frequency and the level of vibration reduction.
Figure 3.7: Frequency response magnitude of internal (a) moment and (b) lateral shear force at 2nd tailboom node.
Figure 3.8: Frequency response magnitude of internal (a) moment and (b) lateral shear force at 4th tailboom node.
Figure 3.9: Frequency response magnitude of internal (a) moment and (b) lateral shear force at 7th tailboom node.
Figure 3.10: Photograph of copper fluidic circuit using water as working fluid.

Figure 3.11: Photograph of copper fluidic circuit using dense, low-viscosity working fluid.
Figure 3.12: Tailboom frequency response for a tip force input, with and without F²MC absorber (copper/dense fluid circuit).
3.2.4 Model Verification: Plastic/Water Circuit

The final circuit in this set of experiments replaces the copper tubing from the two previous inertia tracks with rigid PEX plastic tubing. Although this has the benefit of reducing F²MC absorber weight, it was unclear whether using a softer tubing material would introduce enough unmodeled compliance to affect the absorber performance or the inertia track tuning. The plastic fluidic circuit shown in Figure 3.13 is schematically similar to the copper fluidic circuit, and is pressurized to 100 psi before vibration testing. One difference between this circuit and previous ones is that the inner radii of the copper and plastic tubing could not be matched identically, since the plastic tubing has thicker walls for the same outer diameter of tubing. Due to the smaller inner radius of the plastic tubing, this circuit may have slightly more flow resistance, although this may be offset somewhat by its more gradual curvature compared to the 90° bends in the copper circuits. The measured frequency responses for the tailboom with the plastic circuit F²MC absorber are displayed in Figure 3.14.

Figure 3.13: Photograph of plastic fluidic circuit using water as working fluid.
Figure 3.14: Tailboom frequency response for a tip force input, with and without F\textsuperscript{2}MC absorber (plastic/water circuit).
3.3 Comparisons & Summary of Results

While it is encouraging that all three F²MC absorbers in the previous section produce appreciable vibration reduction, this section focuses on identifying the main differences between the three absorbers and the results obtained from them. Properties of the three fluidic circuits used in the tests, as well as the corresponding absorber weight estimates, are contained in Table 3.2. The weights of various individual components that contribute to the overall F²MC absorber weight are listed in Table 3.3. The weights of the two L-brackets are not included in the absorber weight estimate, as they are considered part of the tailboom structure being controlled. Component weights were obtained by weighing each item on a balance scale. It should be noted that across all three absorbers, over half the absorber weight is fixed due to the stainless steel threaded fittings, valves, and off-the-shelf compression fittings. For a production application, the weight of these components could likely be reduced by selecting lighter components intended for use in aerospace applications.

The frequency response curves for tailboom tip displacement and horizontal tail tip displacement with all three absorbers tested at 100 psi are compiled in Figure 3.15. By defining two metrics to characterize performance, the small gap between the results can be quantified. The first metric, notch depth, is defined as the difference between the resonant peak height of the uncontrolled tailboom and the magnitude of the F²MC-controlled displacement frequency response at the same resonant frequency. The second metric, peak-to-peak depth, measures the difference between the uncontrolled peak height and the higher of the two F²MC-controlled absorber peaks on either side of the tailboom natural frequency. A performance comparison of the three absorbers according to these metrics is provided in Table 3.4.
Table 3.2: Fluidic circuit and other properties for the three test cases.

<table>
<thead>
<tr>
<th>Property</th>
<th>Copper/Water</th>
<th>Copper/Dense Fluid</th>
<th>Plastic/Water</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tuned Fluidic Circuit Properties</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fluid density, kg/m³</td>
<td>1000</td>
<td>1880</td>
<td>1000</td>
</tr>
<tr>
<td>Fluid dynamic viscosity, Pa-s</td>
<td>$9.0 \times 10^{-4}$</td>
<td>$9.0 \times 10^{-4}$</td>
<td>$9.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>Inertia track diameter, mm</td>
<td>7.04</td>
<td>7.04</td>
<td>6.35</td>
</tr>
<tr>
<td>Branch segment length, cm</td>
<td>21.6</td>
<td>21.6</td>
<td>26.7</td>
</tr>
<tr>
<td>Main segment length, cm</td>
<td>96.5</td>
<td>40.6</td>
<td>71.1</td>
</tr>
<tr>
<td><strong>Absorber Weight Properties</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total F(^2)MC tubes weight, kg</td>
<td>0.55</td>
<td>0.64</td>
<td>0.55</td>
</tr>
<tr>
<td>Circuit weight, kg</td>
<td>0.73</td>
<td>0.64</td>
<td>0.22</td>
</tr>
<tr>
<td>Overall absorber weight, kg</td>
<td>3.5</td>
<td>3.5</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Table 3.3: Weights of F\(^2\)MC tube and circuit components.

<table>
<thead>
<tr>
<th>F(^2)MC Tube, Valves, &amp; Fittings</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Item</strong></td>
<td><strong>Weight, Qty.</strong></td>
</tr>
<tr>
<td>Stainless steel mesh</td>
<td>98.7 g/m</td>
</tr>
<tr>
<td>Stainless steel fitting (w/ brass barbed tube fitting)</td>
<td>152 g each, ×8</td>
</tr>
<tr>
<td>Brass ball valve</td>
<td>106.5 g each, ×5</td>
</tr>
<tr>
<td>Individual brass tee compression fitting</td>
<td>60 g each, ×3</td>
</tr>
<tr>
<td>Individual brass straight compression fitting</td>
<td>25 g each, ×13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Circuit Tubing Materials</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Material</strong></td>
<td><strong>Density</strong></td>
</tr>
<tr>
<td>Copper</td>
<td>8960 kg/m(^3)</td>
</tr>
<tr>
<td>PEX plastic</td>
<td>1300 kg/m(^3)</td>
</tr>
</tbody>
</table>
Figure 3.15: Comparison of a) tailboom tip and b) horizontal tail tip frequency responses for the three F²MC absorbers tested.
Table 3.4: Performance comparison between the three F$^2$MC absorbers in this chapter.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Copper/Water</th>
<th>Copper/Dense Fluid</th>
<th>Plastic/Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Notch depth, dB</td>
<td>14</td>
<td>12.1</td>
<td>12.9</td>
</tr>
<tr>
<td>% reduction</td>
<td>80</td>
<td>75</td>
<td>77</td>
</tr>
<tr>
<td>Peak-to-peak depth, dB</td>
<td>10.8</td>
<td>10.2</td>
<td>11.3</td>
</tr>
<tr>
<td>% reduction</td>
<td>71</td>
<td>69</td>
<td>73</td>
</tr>
</tbody>
</table>

Despite the similarity in performance across all three vibration absorbers, each possesses its own unique advantage. Because all three absorbers perform similarly, F$^2$MC absorber design choices such as the working fluid, inertia track geometry, and inertia track tubing material can be made based on other factors, such as minimizing weight or the required length of track tubing, with little impact on the absorber effectiveness. The main advantage to using the higher-density fluid is that it lowers the required inertia track lengths and can result in a more compact overall absorber. This could be especially important for lower frequency modes which require more inertance to tune the circuit. Using a plastic circuit reduced the total absorber weight by 14% compared to the other two absorbers. Because of its high density, the copper tubing is 5.6 times heavier per unit length than the PEX plastic tubing, even though the plastic tubing has thicker walls. However, the absorbers share several common components such as the stainless steel fittings, brass connectors, and brass valves. Because these components make up a significant fraction of the total absorber weight, a large weight reduction in the inertia track only results in a modest weight reduction for the overall absorber on a percent basis. The plastic circuit absorber also displayed a slightly weaker absorber notch in the frequency response, although it is unclear how much of this can be attributed to the narrower plastic circuit tubing radius, just as it is unclear how much resistance in the copper circuits is due to the 90° bends.

The main takeaway from this chapter is that the comprehensive structural/fluidic system model is an effective tool for designing and tuning this particular class of F$^2$MC absorbers. The absorbers demonstrated in this chapter are designed to reduce bending vibrations directly, but they also reduce torsional vibrations if the target mode shape contains both bending and torsional deformations. These three
F^2MC absorbers are demonstrated on a 26.7 Hz lateral bending/torsion mode, making this the highest frequency for which an F^2MC absorber has been successfully designed and tested on a continuous structure. All three F^2MC absorbers tested reduced vibrations by over 75% at this 26.7 Hz mode, resulting in similar frequency responses and levels of vibration reduction.

These experimental results are also important because they are the first demonstration of F^2MC vibration control on a representative aerospace structure using a working fluid other than the dense isolator fluid, as Miura did not test his absorber with other fluids [74]. Being able to obtain similar performance using a relatively common fluid such as water as opposed to a specialized, dense fluid could significantly reduce the cost of an F^2MC absorber or make it easier for companies to develop F^2MC vibration absorbers in the future.
Chapter 4 | Multi-Mode Vibration Control Using F$^{2}$MC Tubes

In the previous chapter, the focus when designing the F$^{2}$MC absorber fluidic circuit was to treat bending vibrations in one specific vibration mode. This was done by utilizing the fluid flow from a pair of F$^{2}$MC tubes on one side of the tailboom to the pair on the opposite side. The fluid flow is driven by a pressure difference from one pair to the other that occurs naturally as the tailboom vibrates in the direction being treated. However, this design approach does not consider that F$^{2}$MC tubes in the same pair can have a pressure difference between each other, which would move fluid between tubes in the same pair instead of from one pair to the other. One advantage of explicitly modeling each fluidic circuit segment is that previously unmodeled fluid dynamics are now included in the model. These dynamics can be harnessed to treat vibrations in more than one tailboom mode with the same fluidic circuit. In this chapter, a new multi-axial F$^{2}$MC vibration absorber is presented and demonstrated experimentally on the laboratory-scale tailboom. The new circuit uses two different "fluid modes" acting as tuned absorbers for two separate tailboom modes.

The new fluidic circuit is not fundamentally different from the one that was experimentally verified in Chapter 3 or the one used in Miura’s tailboom absorber [74]. The main difference is that tailboom vibrations in both the lateral and vertical directions are now considered, and effort is made to design an F$^{2}$MC absorber that reduces vibrations in both of these directions. To illustrate how this new absorber differs from Miura’s, consider the schematic of his vertical absorber circuit shown in Figure 4.1. In this absorber, the top set of F$^{2}$MC tubes and the bottom set of
F$^2$MC tubes are connected fluidically through short branch segments leading into a much longer vertical segment. As the tailboom vibrates vertically, both tubes in the top pair have equal pressures, and both tubes on the bottom have pressures that are equal in magnitude to the top tubes but opposite in sign. This pressure difference pumps fluid from the top pair of tubes to the bottom pair or vice versa. However, the possibility of lateral vibration was not considered in the design of this absorber.

The modified F$^2$MC absorber concept considered here uses the same basic configuration as the Miura vertical absorber, but tunes the lengths of the branch segments so that a usable absorber mode exists in the lateral direction in addition to the normal vertical absorber mode. The working principle of this new absorber is illustrated in Figure 4.2. A higher frequency fluid mode illustrated by the black arrows is excited by lateral vibration, while a lower frequency fluid mode illustrated by blue arrows is excited by vertical vibration. The lengths of the branch segments and the long vertical segment are tuned so that both fluid modes in the circuit have frequencies that coincide with tailboom vibration modes.

An important requirement for this design is that all F$^2$MC tubes have sufficient offset from both the lateral and vertical bending tailboom neutral axes. This offset ensures that the F$^2$MC tubes are strained by tailboom vibrations in each direction and also creates a moment arm for the resulting F$^2$MC tube control moments.

### 4.1 Experimental Demonstration of Concept

This section summarizes the process used to experimentally demonstrate performance of the new multi-mode F$^2$MC absorber. A single-mode vertical absorber is designed and tested to compare performance of the absorbers in the vertical direction and to evaluate the benefits of the multi-mode absorber in the lateral direction. Conclusions are then drawn about the benefits of this new multi-mode configuration relative to the previous single-mode configuration.
Figure 4.1: Schematic of Miura’s single-mode vertical vibration absorber using F²MC tubes.

Figure 4.2: Schematic of new multi-mode, vertical and lateral vibration absorber using F²MC tubes.
4.1.1 Single-Mode Vertical $F^2MC$ Absorber

The single-mode vertical $F^2MC$ absorber uses the same $F^2MC$ tubes and tube locations from the lateral bending/torsion mode absorbers in Chapter 3. However, the tube pairs are connected vertically by the fluidic circuit instead of laterally, because the 12.2 Hz vertical tailboom mode has a lower frequency than the 26.7 Hz lateral mode. Connecting the $F^2MC$ tubes with a vertical track segment as in Figure 4.2 ensures that the vertical fluid mode has a lower absorber frequency than the lateral fluid mode in the resulting multi-mode absorber. Each pair of $F^2MC$ tubes has 27.3 cm (10\text{\textfrac{3}{4}}\text{ in.}) of horizontal separation, although theoretically, only the $F^2MC$ tube offset from the vertical bending plane is important for reducing vertical vibrations. As in the absorbers tested in Chapter 3, each $F^2MC$ tube is offset 2.5 cm (1 in.) vertically from the skin as shown in Figure 4.3.

![Figure 4.3: Positioning of $F^2MC$ tubes for single-mode and multi-mode vibration absorbers. Only the top set of $F^2MC$ tubes are shown here, but the configuration is both vertically and laterally symmetric.](image)

In this single-mode vertical absorber, the fluidic circuit branch segments are made as short as possible, which has two main effects. First, making the branch segments shorter reduces the total length of tubing required in the inertia track. Constructing the inertia track with a slightly longer vertical segment to compensate for shorter branches is more weight-efficient, since the vertical segment combines flow from both $F^2MC$ tubes in each pair and therefore contributes more to the overall inertia in the vertical direction. Second, using short branch segments keeps the lateral fluid mode frequency higher than the tailboom natural frequencies by ensuring that the effective fluid inertia in the lateral direction is low. Because the circuit is not properly tuned for a tailboom lateral vibration mode, this particular treatment is less effective at reducing vibration in the lateral direction. Until this
research, the effectiveness of an F²MC vibration absorber in multiple directions had not been examined, so it was unclear whether an absorber designed to reduce bending vibrations in one direction affects vibrations in a secondary direction, and if so, how significantly the dynamics in that direction change. To determine how much improvement is achieved by using a multi-mode tuned fluidic circuit, the single-mode vertical absorber performance in the lateral direction is compared to the new multi-mode absorber performance in the lateral direction.

The fluidic circuit for the single-mode vertical F²MC absorber uses 5.7 mm (0.225 in.) inner diameter, high-pressure nylon tubing with a wall thickness of 1.9 mm (0.075 in.). The rigid, thick-walled plastic tubing is rated up to 900 psi and chosen to minimize wall compliance effects in the circuit. Each branch has a length of 20.3 cm (8 in.), and the overall length of the vertical inertia track segment is 1.83 m (72 in.). The vertical segment length includes the length of a valve in the middle, which is not required for this test but is utilized later in the multi-mode experiments. The high-density, low-viscosity fluid from Chapter 3 is used in this absorber primarily because the 12.2 Hz vertical mode being targeted has a lower natural frequency than the 26.7 Hz lateral bending/torsion mode targeted previously. Because this mode has a lower natural frequency, more inertia is required to achieve the desired vertical absorber frequency. While it may be possible to use an even narrower inertia track radius to achieve high inertance with a lower density fluid, this would come with the trade-off of increased flow resistance. Using a dense working fluid makes it possible to shorten the required inertia track length without using a narrower inertia track and potentially sacrificing performance.

Although the tailboom has a vertical mode near 22 Hz that could have been treated with a shorter inertia track, its mode shape has less bending strain near the root than the 12.2 Hz mode shape. This means that the F²MC tubes would not be strained as much at 22 Hz, and less fluid would be pumped as the tailboom vibrates. For this reason, an F²MC absorber with the tubes placed at the tailboom root would not be as effective at reducing vibrations in the 22 Hz mode as it would be at reducing vibrations in the 12.2 Hz mode.

In the experiment, the tailboom is forced at its tip using a shaker oriented vertically as shown in Figure 4.4. Vertical displacement is measured at the tailboom tip using a laser vibrometer. The comprehensive finite element, F²MC tubes, and fluid dynamics model is used to predict the tailboom frequency response with and
without the F²MC absorber. Small modifications are made to the finite element model to closely match model-predicted natural frequencies in both the vertical and lateral directions with the experimentally measured natural frequencies. Relevant properties for the target vibration modes and changes from model parameters in Table 2.1 are summarized in Table 4.1.

Table 4.1: Revised finite element model properties and target mode properties for laboratory-scale tailboom.

<table>
<thead>
<tr>
<th>Tailboom Geometry Changes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Tail</td>
<td></td>
</tr>
<tr>
<td>40 in. length,</td>
<td></td>
</tr>
<tr>
<td>1.87 in. above x-axis</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Point Spring and Mass Changes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Root vertical torsion spring, N-m/rad</td>
<td>1.10×10⁶</td>
</tr>
<tr>
<td>Root lateral torsion spring, N-m/rad</td>
<td>1.25×10⁶</td>
</tr>
<tr>
<td>Vertical tail tip mass, kg</td>
<td>5.97</td>
</tr>
<tr>
<td>Horizontal tail tip mass, kg (each side)</td>
<td>2.41</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Target Mode Properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical mode natural frequency, Hz</td>
<td>12.2</td>
</tr>
<tr>
<td>Vertical mode damping, % critical</td>
<td>3.0</td>
</tr>
<tr>
<td>Lateral mode natural frequency, Hz</td>
<td>26.5</td>
</tr>
<tr>
<td>Lateral mode damping, % critical</td>
<td>2.2</td>
</tr>
</tbody>
</table>
Experimentally-measured and model-predicted frequency response results from the baseline untreated tailboom as well as the tailboom with the single-mode vertical F²MC absorber are plotted in Figure 4.5. The model accurately predicts the vertical absorber frequency and the magnitude of vibration reduction. However, the limitations of the model can be seen in Figure 4.6, where the lateral tailboom tip displacement frequency response is plotted for the lateral force input depicted in Figure 2.4. Although the model predicts that the single-mode vertical absorber will have almost no effect on the tailboom lateral bending/torsion mode, the experimental results show that the F²MC absorber does change the lateral dynamics. In this case, the model overestimates the sensitivity of the single-mode vertical absorber performance to inertia track mistuning.

4.1.2 Multi-Mode F²MC Absorber

The multi-mode F²MC absorber presented in this section is designed to reduce vibrations in two target tailboom modes: the 12.2 Hz vertical bending mode and the 26.7 Hz lateral bending/torsion mode that was targeted in Chapter 3. An interesting feature of this multi-mode absorber is that the lateral bending/torsion mode absorber frequency is theoretically unaffected by the length of the long inertia track segment connecting the top and bottom sets of F²MC tubes. The lateral bending/torsion mode absorber frequency should depend only on the lengths of the lateral branch segments connecting the left tube to the right tube in a pair. This is because if all four F²MC tubes are identical and placed symmetrically, then as the tailboom vibrates laterally, the top-left and bottom-left F²MC tubes will have the same pressure, and the top-right and bottom-right F²MC tubes will have the equal and opposite pressure. This causes fluid to flow purely through the branches at either end of the long vertical segment, while little or no fluid flows through the vertical segment itself. On the other hand, the vertical fluid mode frequency depends strongly on the length of the long inertia track segment, since the vertical segment is much longer than the branch segments and combines flow from both F²MC tubes in each pair.

Based on this information, a procedure for tuning the multi-mode fluidic circuit is developed. Since the lateral absorber frequency depends primarily on the branch lengths, the first step is to determine the branch lengths that optimally tune the
Figure 4.5: Vertical tailboom tip displacement frequency response for tailboom tip forcing, single-mode vertical absorber, model and experiment.

Figure 4.6: Lateral tailboom tip displacement frequency response for tailboom tip forcing, single-mode vertical absorber, model and experiment.
fluidic circuit for the target lateral bending/torsion mode. After the lateral branch lengths are set, the vertical segment length is then adjusted to optimally tune the fluidic circuit for the target vertical mode. In theory, the absorber frequency for the vertical mode could depend on both the branch segment and vertical segment lengths, which is why the vertical segment should be tuned after the proper lateral branch lengths have been identified.

Both the single-mode vertical and multi-mode $F^2MC$ absorbers are illustrated side-by-side for comparison in Figure 4.7. The main difference between the multi-mode absorber and the single-mode vertical absorber is that each of the fluidic circuit branches is 35.6 cm (14 in.) long in the multi-mode absorber, while they are each 20.3 cm (8 in.) long in the vertical absorber. The shape of the single-mode $F^2MC$-treated frequency response in Figure 4.6 is an indication that more inertia is needed to optimally tune the previous fluidic circuit for the lateral mode. The lone peak in the $F^2MC$-treated frequency response is at a lower frequency than the untreated tailboom natural frequency, suggesting that the lateral absorber frequency with this particular circuit is higher than the lateral tailboom mode frequency. From this design point, the branches are made incrementally longer until the measured lateral frequency response indicates that the circuit is properly tuned. The vertical segment of the tuned multi-mode inertia track is exactly identical to the vertical segment in the single-mode vertical fluidic circuit. Although the model predicts that a multi-mode absorber circuit using longer branches would need a slightly shorter vertical segment to obtain the same vertical absorber frequency, this was not found to be the case in experiments. The effective inertia for the vertical mode in this fluidic circuit configuration is primarily governed by the long inertia track segment, so the model may overpredict the true contribution of the branch segments. This may be a limitation of using lumped-parameter inertance and resistance models that do not fully capture the complex flow in the inertia track.

The tailboom vertical and lateral frequency responses with a tuned multi-mode $F^2MC$ absorber, along with the frequency responses from the tailboom with the 12.2 Hz single-mode vertical absorber, are shown in Figures 4.8 and 4.9. As in previous tests, both absorbers are initialized with a 100 psi operating pressure after filling and bleeding the circuit. The corresponding horizontal tail frequency responses are plotted in Figure 4.10. The tail motion is also reduced with the
F²MC multi-mode absorber, so this absorber retains the full control of coupled bending/torsion vibration modes that was demonstrated in Chapter 3. Although the single-mode vertical absorber is slightly more effective at reducing 12.2 Hz vertical vibration, as shown in Figure 4.8, the response at the 26.7 Hz lateral mode is much improved with proper tuning of the branch lengths, as shown in Figure 4.9. The properties and performance of the single-mode vertical and multi-mode F²MC absorbers tested in this chapter are summarized in Table 4.2. These experimental results indicate that using the multi-mode absorber instead of the single-mode absorber trades a small amount of effectiveness in the vertical mode for a much greater effectiveness in the lateral mode.

A side experiment was also conducted in which a valve in the long vertical inertia track segment is closed before measuring the lateral tailboom tip frequency response. Closing the valve prevents fluid from flowing between the top set and the bottom set of F²MC tubes; however, this theoretically does not happen anyway, since the lateral vibration should only induce a pressure difference that causes
Figure 4.8: Vertical tailboom tip displacement frequency response, multi-mode absorber versus single-mode vertical absorber (experiment).

Figure 4.9: Lateral tailboom tip displacement frequency response, multi-mode absorber versus single-mode vertical absorber (experiment).
fluid to flow back and forth laterally through the branch segments. The tailboom frequency responses measured with the valve in both open and closed states are plotted in Figure 4.11 and are almost identical. This supports the hypotheses that almost all flow at this frequency is between the left and right F\textsuperscript{2}MC tubes and that the circuit absorber frequency for the lateral mode depends only on the branch lengths.
Table 4.2: Comparison between single-mode vertical and multi-mode F²MC absorbers.

<table>
<thead>
<tr>
<th>Tuned Fluidic Circuit Properties</th>
<th>Single-Mode Vertical</th>
<th>Multi-Mode Vertical/Lateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid density, kg/m³</td>
<td></td>
<td>1880</td>
</tr>
<tr>
<td>Inertia track radius, mm</td>
<td>5.7</td>
<td>5.7</td>
</tr>
<tr>
<td>Branch segment length, cm</td>
<td>20.3</td>
<td>35.6</td>
</tr>
<tr>
<td>Vertical segment length, m</td>
<td>1.83</td>
<td>1.83</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Performance</th>
<th>Single-Mode Vertical</th>
<th>Multi-Mode Vertical/Lateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical notch depth, dB</td>
<td>9.8</td>
<td>8.2</td>
</tr>
<tr>
<td>% reduction</td>
<td>68</td>
<td>63</td>
</tr>
<tr>
<td>Vertical peak-to-peak depth, dB</td>
<td>6.6</td>
<td>6.6</td>
</tr>
<tr>
<td>% reduction</td>
<td>53</td>
<td>53</td>
</tr>
<tr>
<td>Lateral notch depth, dB</td>
<td>N/A</td>
<td>10.2</td>
</tr>
<tr>
<td>% reduction</td>
<td>N/A</td>
<td>69</td>
</tr>
<tr>
<td>Lateral peak-to-peak depth, dB</td>
<td>4.7</td>
<td>9.2</td>
</tr>
<tr>
<td>% reduction</td>
<td>42</td>
<td>65</td>
</tr>
</tbody>
</table>

4.1.3 Weight Analysis of Single-Mode & Multi-Mode Absorbers

A key factor in assessing the value of the multi-mode F²MC absorber is how much additional weight is required to achieve vibration reduction in two target modes compared to just one. The weights of individual components and their quantities in the single-mode vertical absorber are summarized in Table 4.3. In the overall absorber weight estimate, the valve in the inertia track is not included, since it is not required for the absorber to function and is only in the circuit to run the open/closed valve experiment from the previous section. The attachment brackets are again counted as part of the tailboom structural weight instead of the absorber weight. The five valves counted are the one valve where fluid is pumped into the circuit and the four valves exiting the four F²MC tubes.
Figure 4.11: Lateral tailboom tip displacement frequency response, with valve in circuit vertical segment open and closed.

Table 4.3: Weights of individual components in F\textsuperscript{2}MC absorbers.

<table>
<thead>
<tr>
<th><strong>Component</strong></th>
<th><strong>Unit Wt. (g)</strong></th>
<th><strong>Quantity</strong></th>
<th><strong>Total Wt. (kg)</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Brass ball valve</td>
<td>106.5</td>
<td>5</td>
<td>0.533</td>
</tr>
<tr>
<td>Stainless steel threaded fitting</td>
<td>152</td>
<td>8</td>
<td>1.216</td>
</tr>
<tr>
<td>Brass tee compression fitting</td>
<td>60</td>
<td>3</td>
<td>0.180</td>
</tr>
<tr>
<td>Brass straight compression fitting</td>
<td>25</td>
<td>13</td>
<td>0.325</td>
</tr>
</tbody>
</table>

Since the bladder is thin, the F\textsuperscript{2}MC tube weight (excluding the end fittings) depends primarily on the choices of fiber mesh material and working fluid. For this particular set of F\textsuperscript{2}MC tubes, approximately two-thirds of the F\textsuperscript{2}MC tube weight comes from the stainless steel mesh, which is much denser than both the rubber bladder and the enclosed fluid. Each of the four F\textsuperscript{2}MC tubes in this absorber has an estimated weight of about 0.16 kg. The density of the circuit nylon tubing was measured as roughly 1200 kg/m\textsuperscript{3}. In the single-mode vertical absorber, the combined weight of the inertia track fluid and tubing is around 0.4 kg. The total weight estimate for the single-mode vertical F\textsuperscript{2}MC absorber is 3.3 kg, and this weight estimate is broken down by a pie chart in Figure 4.12. 2.9 kg out of the
overall 3.3 kg single-mode absorber weight is composed of the F$^2$MC tubes, valves, and fittings that are required for both the single-mode and multi-mode absorbers. The remaining contributors to the absorber weight are the inertia track tubing and the fluid inside the inertia track. Only 12% of the single-mode vertical F$^2$MC absorber weight comes from the fluidic circuit, including the fluid inside the 4-inch long stainless steel fittings.

For comparison, the weight estimate of the multi-mode F$^2$MC absorber is broken down in Figure 4.13. The longer branches in the multi-mode absorber result in a 16% increase in the fluidic circuit weight; however, the fluidic circuit makes up such a small fraction of the overall absorber weight that the multi-mode circuit is only 63 g (0.14 lb) heavier than the single-mode vertical circuit. This represents less than a 2% increase in the overall absorber weight, so the benefit of treating multiple modes likely outweighs the small additional weight penalty from this configuration. The single-mode and multi-mode F$^2$MC absorbers tested in this research have nearly identical weights, but it should be noted that the weight penalty associated with targeting the second vibration mode could depend on a number of factors such as the fluid, tubing material, and the difference in natural frequencies between the two modes being targeted.

It is also important to note that when designing an F$^2$MC absorber to target only one vibration mode, one may wish to alter the spacing between the F$^2$MC tubes, which was not considered as a design variable here. For example, in a single-mode vertical absorber, the F$^2$MC tubes could be grouped together with less horizontal spacing, allowing a shorter and lighter bracket to be used. Alternatively, the horizontal spacing could be preserved, but each F$^2$MC tube could be attached to the tailboom with its own small bracket. Both of these potential modifications are shown in Figure 4.14 and could result in lighter attachment hardware. In other cases, especially if the absorber is being retrofitted onto an existing structure, it may not be feasible to place the four F$^2$MC tubes such that they will be strained sufficiently by vibrations in both directions and effectively transmit both vertical and lateral bending moments into the tailboom structure. In general, the added weight penalty for a multi-mode F$^2$MC absorber will likely be small as long as the track tubing material is lightweight, but a more detailed weight assessment should consider how each F$^2$MC vibration absorber is most practically and effectively realized on a given structure.
Figure 4.12: Weight breakdown for single-mode vertical F$^2$MC vibration absorber.

Figure 4.13: Weight breakdown for multi-mode vertical and lateral F$^2$MC vibration absorber.
Figure 4.14: Examples of F$^2$MC tube attachment schemes: a) the long bracket used in the laboratory-scale tailboom absorber, b) one short bracket for a vertical absorber, and c) individual brackets for each F$^2$MC tube in a vertical or multi-mode absorber.
Chapter 5  |  
F\textsuperscript{2}MC Rotor Blade Damper Modeling

The previous three chapters focused on analyzing and developing F\textsuperscript{2}MC vibration absorbers for airframe vibration control. In contrast, this chapter investigates how F\textsuperscript{2}MC tubes can be used as part of a damper for rotor blade in-plane motion. Two different F\textsuperscript{2}MC dampers are presented for applications in articulated and hingeless rotors. The articulated blade damper functions by pumping fluid through an orifice to dissipate energy, while the hingeless blade damper functions as a damped vibration absorber tuned to reduce vibration at the first chordwise bending blade mode. Parametric studies are used to assess the effect of different fluidic circuit parameters on damper performance and to identify characteristics of effective F\textsuperscript{2}MC dampers for each type of rotor blade.

5.1 Articulated Blade F\textsuperscript{2}MC Damper

In this section, the F\textsuperscript{2}MC lead-lag damper for an articulated rotor blade is presented and analyzed. The concept for this F\textsuperscript{2}MC damper is illustrated in Figure 5.1. A contractor (wind angle \( \alpha < 54.7^\circ \)) F\textsuperscript{2}MC tube is connected between the rotor blade, which pivots as a rigid body around a lag hinge, and the rotating hub assembly. The F\textsuperscript{2}MC tube is attached at the trailing edge rather than the leading edge because lead-lag dampers are more commonly placed on the trailing edge in existing articulated rotors. As the blade leads and lags, the F\textsuperscript{2}MC tube stretches and compresses axially, causing a respective decrease or increase in the F\textsuperscript{2}MC tube
volume. This tube volume change pumps fluid through an inertia track between an F²MC tube and a hydraulic accumulator. Energy is dissipated as this fluid flows through an orifice in the inertia track. The goal for this new damper is to harness the strain-induced F²MC tube pumping to create a simple, low-stiffness device that generates high damping. Due to the enhanced pumping capability of F²MC tubes, a significant amount of fluid can be pumped through the orifice without relying on a piston of large cross-sectional area. The F²MC tube may therefore enable the development of a low-profile lead-lag damper that is less bulky than current solutions.

Figure 5.1: Top view of F²MC lead-lag damper for an articulated rotor blade.

5.1.1 Articulated Blade & Circuit Modeling

The blade is modeled as a rigid blade of radius $R$ that is able to lead and lag around a hinge located distance $e$ from the axis of rotation. Flapping and torsional dynamics are neglected in this analysis. It is assumed that the hinge has no stiffness and there is some initial damping due to aerodynamics. The blade lag dynamics are governed by the nondimensionalized equation [4]

$$\ddot{\zeta} + \frac{c}{I_\zeta \Omega} \dot{\zeta} + \nu^2 \zeta = \frac{1}{I_\zeta \Omega^2} \int_e^R \tilde{F}_x(r - e)dr. \quad (5.1)$$
where \( (\ast) \) denotes a nondimensional derivative with respect to rotor azimuth angle \( \psi \), \( \zeta \) denotes blade lag angle, \( I_\zeta \) is the blade mass moment of inertia about the lag hinge, \( \nu_\zeta \) is the blade lag frequency nondimensionalized by rotor speed \( \Omega \), \( c_\zeta \) is a linear viscous damping coefficient, and \( r \) is the radial coordinate from the axis of rotation. The symbol \( \tilde{F}_x \) represents the aerodynamic force on the blade per unit span, excluding aerodynamic damping terms. The initial damping due to aerodynamics and other sources except for the \( \text{F}\text{MC} \) damper are modeled by the effective linear viscous damping coefficient so that only one parameter defines the baseline blade damping.

From [68], the linearized relationship between \( \text{F}\text{MC} \) tube axial displacement \( x_t \) (positive for extension), internal tube pressure \( p_t \), and axial force \( F_t \) acting on the tube is given by

\[
c_1 x_t + c_2 p_t = F_t, \tag{5.2}
\]

and the fluid volume flow rate \( Q \) out of the \( \text{F}\text{MC} \) tube is given by

\[
-c_3 \dot{x}_t - c_4 \dot{p}_t = Q. \tag{5.3}
\]

The fluidic circuit for the articulated blade damper is shown in Figure 5.2. The fluidic circuit is modeled by two equations which express the inertia track and accumulator dynamics. The inertia track dynamics are governed by the equation

\[
p_t - p_a = I_c \dot{Q} + R_c Q, \tag{5.4}
\]

where \( p_a \) is the pressure in the accumulator, \( I_c \) is the circuit fluid inertance, and \( R_c \) is the circuit fluid resistance. This fluid resistance \( R_c \) captures both losses due to fluid viscosity as described by Eq. (2.50) and losses due to an orifice in the circuit. Modeling the pressure difference across an orifice with a linear relationship between pressure and flow rate as in Eq. (5.4) is not a precise model for flow through an orifice; however, using this simple equation keeps the overall system dynamics linear. This is convenient because linear system tools such as frequency response and eigenvalue analysis can be applied to assess the \( \text{F}\text{MC} \) articulated blade damper performance.
The accumulator dynamics are governed by the equation

\[
p_a = \frac{Q}{c_a},
\]

where \( c_a \) is the accumulator capacitance. The parameter \( c_a \) is similar to the constant \( c_4 \) in the F\(^2\)MC tube model, but instead \( c_a \) defines how much fluid enters the accumulator per unit of pressure. The accumulator is a key component in this circuit because it acts as a compliant element that receives fluid pumped by the F\(^2\)MC tube. In the tailboom vibration absorber discussed in Chapters 2-4, multiple F\(^2\)MC tubes are connected together by a common circuit, and the tubes pump fluid back and forth between antagonistic pairs. However, because this F\(^2\)MC lag damper only has one F\(^2\)MC tube, a second compliant element is needed in the circuit. If another compliant element is not present, the F\(^2\)MC tube cannot pump fluid, so no damping can be generated.

![F\(^2\)MC tube and fluidic circuit model for articulated and hingeless blade damper circuits.](image)

A second-order transfer function from F\(^2\)MC tube extension \( x_t \) to F\(^2\)MC tube force \( F_t \) can be obtained by algebraically manipulating the Laplace transforms of Eqs. (5.2)-(5.5). This transfer function can be written in the form

\[
\frac{F_t(s)}{x_t(s)} = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0},
\]

(5.6)
where the coefficients are as follows:

\[ b_2 = (c_1c_4 - c_2c_3)I_c c_a \]  
(5.7)
\[ b_1 = (c_1c_4 - c_2c_3)R_c c_a \]  
(5.8)
\[ b_0 = c_1c_4 - c_2c_3 + c_1c_a \]  
(5.9)
\[ a_2 = c_4c_a I_c \]  
(5.10)
\[ a_1 = c_4c_a R_c \]  
(5.11)
\[ a_0 = c_4 + c_a \]  
(5.12)

Nondimensionalizing Eq. (5.6) using \( \hat{s} = \Omega s \) yields

\[ \frac{F_t(\hat{s})}{x_t(\hat{s})} = \frac{g_2 \hat{s}^2 + g_1 \hat{s} + g_0}{f_2 \hat{s}^2 + f_1 \hat{s} + f_0}, \]  
(5.13)

with

\[ g_2 = \Omega^2(c_1c_4 - c_2c_3)I_c c_a \]  
(5.14)
\[ g_1 = \Omega(c_1c_4 - c_2c_3)R_c c_a \]  
(5.15)
\[ g_0 = c_1c_4 - c_2c_3 + c_1c_a \]  
(5.16)
\[ f_2 = \Omega^2(c_4c_a I_c) \]  
(5.17)
\[ f_1 = \Omega(c_4c_a R_c) \]  
(5.18)
\[ f_0 = c_4 + c_a \]  
(5.19)

The transfer function in Eq. (5.13) can be converted into state-space form using MATLAB’s state-space tools,

\[ \left\{ \dot{\xi} \right\} = [A_t] \{ \xi \} + [B_t] x_t, \]  
(5.20)
\[ F_t = [C_t]^T \{ \xi \} + [D_t] x_t, \]  
(5.21)

where \( \{ \xi \} \) is the vector of states for the F\(^2\)MC tube and fluid subsystem.

To relate F\(^2\)MC tube axial displacement to the blade lead-lag displacement, a rotating coordinate system is defined as shown in Figure 5.3. The origin is located
at the lag hinge, and the F$^2$MC tube extends from a point on the hub $(x_h, y_h, z_h)$ to a point on the blade $(x_b, y_b, z_b)$. The F$^2$MC tube initial length $L_o$ is given by

$$L_o = \sqrt{(x_b - x_h)^2 + (y_b - y_h)^2 + (z_b - z_h)^2}. \quad (5.22)$$

If the blade rotates by angle $\zeta$ about the lag hinge, with positive lag angle opposing the direction of rotation, then the new location of the blade attachment point is $(x_b \cos(\zeta) + y_b \sin(\zeta), y_b \cos(\zeta) - x_b \sin(\zeta), z_b)$, and the F$^2$MC tube length $L_t$ becomes

$$L_t = \sqrt{(x_b \cos(\zeta) + y_b \sin(\zeta) - x_h)^2 + (y_b \cos(\zeta) - x_b \sin(\zeta) - y_h)^2 + (z_b - z_h)^2}. \quad (5.23)$$

Linearizing F$^2$MC tube length $L_t$ about zero lag angle yields the kinematic coupling equation

$$x_t = \frac{x_b y_h - x_h y_b}{L_o} \zeta = \Psi \zeta. \quad (5.24)$$

Eq. (5.24) is substituted into Eq. (5.21) to express the F$^2$MC tube force in terms of the blade lag displacement,

$$F_t = \{C_t\}^T \{\xi\} + D_t \Psi \zeta. \quad (5.25)$$
The $F^2MC$ tube moment $M_t$ generated about the lag hinge is

$$M_t = \sigma \left( \{C_t\}^T \{\xi\} + D_t \Psi \zeta \right),$$

(5.26)

where $\sigma = -\Psi$.

After nondimensionalizing the $F^2MC$ damper moment $M_t$ by dividing by $I_\zeta \Omega^2$, inserting it into Eq. (5.1), and expressing all external forcing as an equivalent nondimensional moment $M_{nd}$ about the lag hinge, the equation of motion for the articulated rotor blade with $F^2MC$ damper becomes

$$\ddot{\zeta} + \frac{C_\zeta \zeta^*}{I_\zeta \Omega^2} + \nu^2 \zeta = \frac{\sigma}{I_\zeta \Omega^2} \left( \{C_t\}^T \{\xi\} + D_t \Psi \zeta \right) + M_{nd}.$$  

(5.27)

As in Chapter 2, the blade and fluid dynamic equations can be converted to state-space form,

$$\begin{bmatrix} \dot{\zeta} \\ \ddot{\zeta} \\ \dot{\xi} \\ \ddot{\xi} \end{bmatrix} = [A_{sys}] \begin{bmatrix} \zeta \\ \dot{\zeta} \\ \xi \\ \dot{\xi} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} M_{nd},$$

(5.28)

where

$$[A_{sys}] = \begin{bmatrix} 0 & 1 & \{0\}^T \\ -\nu^2 - \frac{\sigma D_t \Psi}{I_\zeta \Omega^2} & -\frac{C_\zeta}{I_\zeta \Omega^2} & \frac{\sigma \{C_t\}^T}{I_\zeta \Omega^2} \\ \{B_t\} \Psi & 0 & [A_t] \end{bmatrix}.$$  

(5.29)

### 5.1.2 Case Study: Representative Articulated Blade

A representative articulated blade based on the UH-60 rotor is modeled, and a corresponding $F^2MC$ damper is integrated into the blade to analyze its ability to damp the lag mode. The properties of the articulated blade are listed in Table 5.1. For simplicity, the rotor is assumed to have uniform properties along its length, and the $F^2MC$ tube initial position is parallel with the $x$-axis. Properties of the $F^2MC$ tube used in the damper simulated with this blade are given in Table 5.2. Note that the $F^2MC$ tube active length is shorter than the distance between the two attachment points to account for the presence of fittings at both ends of the tube.
Table 5.1: Properties for representative articulated blade based on UH-60.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor radius, m</td>
<td>8.17</td>
</tr>
<tr>
<td>Rotor speed, RPM</td>
<td>258</td>
</tr>
<tr>
<td>Chord length, m</td>
<td>0.531</td>
</tr>
<tr>
<td>Lag inertia, kg-m²</td>
<td>1840</td>
</tr>
<tr>
<td>Lag hinge distance $e$</td>
<td>0.045R</td>
</tr>
<tr>
<td>Lag frequency (without damper), /rev</td>
<td>0.27</td>
</tr>
<tr>
<td>Initial lag damping ratio</td>
<td>0.02</td>
</tr>
</tbody>
</table>

One key difference between the articulated blade damper and the tailboom vibration absorber is the fact that applying an initial pressure to generate engagement between the fibers and bladder produces free strain in the F²MC tube when the rotor is at rest. This does not occur on the tailboom because the four F²MC tubes are positioned symmetrically such that they exert no net vertical or lateral bending moment when the initial pressure is applied. F²MC tube axial contraction is accompanied by an increase in the fiber angle $\alpha$ as the tube expands radially. The F²MC tube contraction under an initial pressure pulls the blade to an initial lag angle while the rotor is at rest, but the act of spinning up the rotor generates a centrifugal moment that re-straightens the blade and pulls the F²MC tube into tension, as illustrated in Figure 5.4. If the fibers are inextensible and the F²MC tube is cylindrical, the instantaneous fiber angle $\alpha_f$ is related to the initial fiber angle $\alpha_o$ and the ratio of the F²MC tube instantaneous active length $L_{t,a}$ to its original active length $L_{o,a}$ by the equation [82]

$$\alpha_f = \cos^{-1} \left( \frac{L_{t,a}}{L_{o,a}} \cos \alpha_o \right).$$

(5.30)

The ratio $\frac{L_{t,a}}{L_{o,a}}$ is sometimes referred to in literature as the contraction ratio.

The dependence of F²MC tube fiber angle on the tube length means that an F²MC damper would exhibit different behavior as the blade lag angle changes in different helicopter flight conditions. To address this, the current research linearizes the F²MC tube model about a given operating lag angle $\zeta_o$ and analyzes the dynamics of the linearized system. Physically, this operating lag angle corresponds to the steady, nonharmonic part of the blade lag displacement in a given flight.
Figure 5.4: Change in F²MC tube length and fiber angle $\alpha$ as tube shortens to its free strain active length $L_{fs,a}$ with fiber angle $\alpha_{fs}$ when initially pressurized, then extends back to final active length $L_{t,a}$ with fiber angle $\alpha_f$ under centrifugal tensioning. $L_{fs,a} < L_{t,a} < L_{o,a}$ and $\alpha_o < \alpha_f < \alpha_{fs}$.

condition. In this analysis, the value of the capacitance parameter $c_4$ is assumed to remain constant as the F²MC tube changes length. For the F²MC damper considered here, the offset distance between the lag hinge and F²MC tube as illustrated in Figure 5.1 is 50% of the blade chord. Fluid inertance is neglected because the inertia track length is assumed to be short, and all fluid resistance is assumed to come from the orifice instead of from fluid viscosity.

### 5.1.2.1 Effects of Varying Lag Angle & Orifice Resistance

The dynamics of the articulated blade with an F²MC damper are analyzed for different blade operating lag angles and values of orifice resistance. In flight, the operating lag angle is determined by a static moment balance about the lag hinge.
Aerodynamic drag and the F\textsuperscript{2}MC tube generate moments opposite the direction of rotation, while the centrifugal restoring moment acts in the direction of rotation. In this research, the operating lag angle is varied by prescribing a range of combined aerodynamic and F\textsuperscript{2}MC tube moments and calculating the equilibrium lag angle in each case. This is a simple way of examining how the damper performance varies in different flight conditions without having to perform a full trim analysis.

The variation in blade damping ratio with the orifice resistance parameter for different sized F\textsuperscript{2}MC dampers is plotted in Figure 5.5. F\textsuperscript{2}MC dampers with tubes having outer diameters of \(\frac{3}{4}\) in., 1 in. and 1\(\frac{1}{4}\) in. are simulated, and their properties are summarized in Table 5.2. These tube properties are generated by scaling up properties from the F\textsuperscript{2}MC tubes fabricated for the tailboom absorber in Chapters 3 and 4. The F\textsuperscript{2}MC tube active length, fiber angle as calculated by Eq. (5.30), and model coefficients at each operating lag angle are listed in Table 5.3. The F\textsuperscript{2}MC tube capacitance \(c_4\) is estimated assuming it is proportional to the initial volume of the F\textsuperscript{2}MC tube active length, and the accumulator in each damper circuit has a capacitance of \(1\times10^{-10}\) m\(^3\)/Pa.

The three different plots in Figure 5.5 correspond to operating lag angles of 0\(^\circ\), 3\(^\circ\), and 6\(^\circ\). The damping ratio is extracted from the system eigenvalues using the MATLAB function “damp.” Note that an operating lag angle of 0\(^\circ\) implies that the F\textsuperscript{2}MC tube moment acting against the direction of rotation is balanced by an equivalent aerodynamic moment in the direction of rotation. This is not physically realistic because drag opposes the direction of rotation. For the dampers analyzed in this chapter, the expected operating lag angle at full rotor speed and zero thrust is around 1.5-3\(^\circ\) depending on F\textsuperscript{2}MC tube diameter and initial pressurization. The cases where \(\zeta_o\) is lower than this range provide a basis to illustrate how much fluid pumping and damper performance is lost as the F\textsuperscript{2}MC tube fiber angle increases.

For each F\textsuperscript{2}MC tube size, the damping ratio initially increases with orifice resistance, but reaches a peak value and then begins to decrease with further orifice resistance. Modern lag dampers produce damping ratios as high as 0.35 [83], and it is neither necessary nor desirable to choose the orifice resistance such that the damping ratio is much higher than this. The rotor will likely already have a sufficient stability margin, and excessively high orifice resistance results in high oscillatory F\textsuperscript{2}MC tube pressures and high loads transmitted to the rotor hub. As the orifice resistance increases, the damper contributes both damping and stiffness.
For very high values of orifice resistance, the F²MC damper contributes more stiffness and less damping, as it becomes harder for the F²MC tube to pump fluid through the orifice. An F²MC damper with orifice resistance to the right of the peak in these plots would also be infeasible because of high oscillatory F²MC tube pressures and high hub loads.

While the resistance value is on the left side of the peak in Figure 5.5, smaller diameter F²MC tubes need more orifice resistance to achieve the same amount of damping. Based on Eq. (5.4), if inertance is neglected, the pressure difference between the F²MC tube and accumulator is a product of the flow rate Q and the orifice resistance $R_c$. A smaller diameter F²MC tube with the same fiber angle pumps less fluid per unit displacement; in other words, the magnitude of $c_3$ decreases with tube diameter. For this reason, a smaller diameter F²MC tube requires more orifice resistance to generate the same pressure difference.

Frequency responses are plotted in Figure 5.6 for the blade integrated with the 1 in. diameter F²MC tube, using an orifice resistance of $2 \times 10^9$ kg/s-m⁴, and with the blade at operating lag angles of 0°, 3°, and 6°. At low operating lag angles,

| Table 5.2: Properties for different diameter F²MC tubes in articulated blade damper. |
|--------------------------------------------------|------------|----------|------------|
| Property                                         | 3/4” Tube  | 1” Tube  | 1 1/4” Tube |
| Stainless Steel Fibers                          |            |          |            |
| Elastic Modulus, GPa                            |            |          | 180        |
| # of strands                                    | 312        |          |            |
| Strand diameter, mm                             | 0.348      | 0.464    | 0.580      |
| Initial fiber angle, °                          |            |          | 20         |
| Rubber Bladder                                   |            |          |            |
| Elastic Modulus, MPa                            |            | 1        |            |
| Poisson’s ratio                                  | 0.5        |          |            |
| Inner diameter, cm                              | 1.63       | 2.17     | 2.72       |
| Wall thickness, cm                              | 0.136      | 0.181    | 0.227      |
| F²MC Tube Geometry (coordinate system from Figure 5.3) |             |          |            |
| Hub attachment point                            | (-0.331, -0.266, 0) m |          |
| Blade attachment point                          | (0, -0.266, 0) m |          |
| Initial active length, cm                       | 16.5       |          |            |
Figure 5.5: Variation of blade damping with orifice resistance at operating lag angles of a) $\zeta_o=0^\circ$, b) $\zeta_o=3^\circ$, and c) $\zeta_o=6^\circ$. 
Table 5.3: Properties for F\textsuperscript{2}MC dampers at each operating angle.

<table>
<thead>
<tr>
<th>F\textsuperscript{2}MC Tube Properties</th>
<th>$\zeta_o = 0^\circ$</th>
<th>$\zeta_o = 3^\circ$</th>
<th>$\zeta_o = 6^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active length, cm</td>
<td>16.5</td>
<td>15.1</td>
<td>13.8</td>
</tr>
<tr>
<td>Final fiber angle, $^\circ$</td>
<td>20.0</td>
<td>30.6</td>
<td>38.6</td>
</tr>
</tbody>
</table>

\textit{3/4” Diameter F\textsuperscript{2}MC Tube Damper}

| Axial stiffness, $c_1$, N/m             | $1.19 \times 10^4$  | $6.61 \times 10^4$  | $5.77 \times 10^4$  |
| Force-pressure coefficient, $c_2$, N/Pa| $5.15 \times 10^{-3}$| $3.83 \times 10^{-3}$| $2.61 \times 10^{-3}$|
| Volume change coefficient, $c_3$, m\textsuperscript{3}/m| $-3.44 \times 10^{-3}$| $-2.56 \times 10^{-3}$| $-1.75 \times 10^{-3}$|
| Tube capacitance, $c_4$, m\textsuperscript{3}/Pa | $1.32 \times 10^{-12}$ |  |  |

\textit{1” Diameter F\textsuperscript{2}MC Tube Damper}

| Axial stiffness, $c_1$, N/m             | $1.99 \times 10^4$  | $1.74 \times 10^4$  | $1.025 \times 10^4$ |
| Force-pressure coefficient, $c_2$, N/Pa| $9.16 \times 10^{-3}$| $6.81 \times 10^{-3}$| $4.64 \times 10^{-3}$|
| Volume change coefficient, $c_3$, m\textsuperscript{3}/m| $-6.12 \times 10^{-3}$| $-4.56 \times 10^{-3}$| $-3.11 \times 10^{-3}$|
| Tube capacitance, $c_4$, m\textsuperscript{3}/Pa | $2.35 \times 10^{-12}$ |  |  |

\textit{1 1/4” Diameter F\textsuperscript{2}MC Tube Damper}

| Axial stiffness, $c_1$, N/m             | $3.13 \times 10^4$  | $1.85 \times 10^4$  | $1.601 \times 10^4$ |
| Force-pressure coefficient, $c_2$, N/Pa| $1.43 \times 10^{-3}$| $1.064 \times 10^{-3}$| $7.25 \times 10^{-3}$|
| Volume change coefficient, $c_3$, m\textsuperscript{3}/m| $-9.57 \times 10^{-3}$| $-7.12 \times 10^{-3}$| $-4.86 \times 10^{-3}$|
| Tube capacitance, $c_4$, m\textsuperscript{3}/Pa | $3.67 \times 10^{-12}$ |  |  |

The F\textsuperscript{2}MC damper performs very well, but in flight conditions where blade drag increases the operating lag angle, the device does not generate as much damping. As the F\textsuperscript{2}MC tube shortens, its fiber angle increases as shown previously in Figure 5.4. When the fiber angle increases, the F\textsuperscript{2}MC tube becomes less effective at generating force and pumping fluid than it is at low operating lag angles, where the F\textsuperscript{2}MC tube fiber angle is closer to its initial value of 20°. The decreased force generation and pumping capabilities of the F\textsuperscript{2}MC tube are reflected by the fact that in Table 5.3, the magnitude of coefficients $c_2$ and $c_3$ decreases as the operating lag angle and tube fiber angle increase. This also explains why the curves in Figure 5.5 shift to the right as the blade lag angle increases, reinforcing the observation that more orifice resistance is needed to generate the same amount of damping as the F\textsuperscript{2}MC tube pumps less fluid.
The increase in F$^2$MC tube fiber angle as the blade lags is a consequence of placing the F$^2$MC tube on the blade trailing edge in this damper design. The continuous variation in F$^2$MC tube fiber angle as the blade lag angle increases is plotted in Figure 5.7a, and the corresponding decrease in damping with increasing lag angle is plotted in Figure 5.7b for several values of orifice resistance. For the frequency responses in Figure 5.6, the damping ratio is 0.247 when the operating lag angle is 0°, but the damping ratio drops down to 0.156 at a 3° operating lag angle and 0.086 at a 6° operating lag angle. The dashed line in Figure 5.7a highlights the theoretical fiber angle of 54.7° where the F$^2$MC tube does not change volume as it changes length. The point where the fiber angle reaches 54.7° is also where the F$^2$MC tube enters compressive axial loading. Note that the relationship depicted in Figure 5.7a between lag angle and F$^2$MC tube fiber angle changes depending on the F$^2$MC tube attachment points for a particular damper, but this curve will always have an asymptote at $\alpha = 54.7^\circ$.

From a helicopter dynamics standpoint, having lower damping at higher lag angles may not necessarily be a drawback. The steady lag angle of an articulated blade is most likely to be high in flight conditions where high lag damping is not necessary, such as high-speed forward flight or during maneuvers. Having high
damping in these situations would simply increase loads transmitted to the hub and reduce the fatigue life of hub components. In contrast, lag damping is needed to stabilize the rotor from ground resonance, when the blade lag angle is low because the helicopter is not in forward flight and the rotor is not generating full thrust. In this condition, the F\(^2\)MC tube fiber angle would be closer to its original value, and the F\(^2\)MC damper would be more efficient at pumping fluid to dissipate energy and stabilize the rotor.

Figure 5.7: Variation of a) F\(^2\)MC tube fiber angle and b) blade damping ratio with operating lag angle.
5.1.2.2 Effect of Varying Accumulator Capacitance

It is also worthwhile to examine the effect of accumulator capacitance on the F²MC damper behavior. Physically, a lower capacitance value $c_a$ means that more pressure is needed to move a given unit of fluid into the accumulator. In Figure 5.8, blade damping ratio is plotted versus accumulator capacitance for a wide range of capacitances and the same four orifice resistances used in Figure 5.7b. In general, the damper is ineffective for very low values of capacitance, but becomes more effective as the capacitance increases before the damping ratio starts to plateau. Physically, these curves help estimate the necessary accumulator size for a given damper circuit. The instantaneous capacitance of a piston accumulator separating an incompressible fluid from a compressible volume of gas $V_a$ at absolute pressure $p_a$ is [84]

$$c_a = \frac{V_a}{p_a \eta},$$  \hspace{1cm} (5.31)

where $\eta$ is the polytropic exponent and typically varies between 1 and 1.4 depending on the rate of gas compression.

Blade frequency responses at operating lag angles of 0°, 3°, and 6° are plotted in Figure 5.9, based on a damper using the same 1 in. diameter F²MC tube from before, an orifice resistance of $2 \times 10^9$ kg/s-m⁴, and various accumulator capacitances. The frequency responses show that increasing the accumulator capacitance by an order of magnitude from $1 \times 10^{-10}$ m³/Pa to $1 \times 10^{-9}$ m³/Pa slightly decreases the blade lag frequency and slightly increases damping. This reinforces the diminishing returns from increasing accumulator size shown in Figure 5.8. However, decreasing the accumulator capacitance by an order of magnitude from $1 \times 10^{-10}$ m³/Pa to $1 \times 10^{-11}$ m³/Pa causes the lag frequency to increase from around 0.3/rev to as high as 0.56/rev at the lag angle of 0°. The stiffness increase is most significant when the F²MC tube fiber angle is near its unstrained value of 20°, since the lower fiber angle corresponds to a higher magnitude of the $c_2$ and $c_3$ coefficients. These results illustrate the importance of using a large enough accumulator in the circuit to ensure that the blade lag frequency remains well under 1/rev and that the damper is able to sufficiently stabilize the blade.
Figure 5.8: Variation of blade damping ratio with accumulator capacitance for different levels of orifice resistance at operating lag angles of a) $\zeta_o=0^\circ$, b) $\zeta_o=3^\circ$, and c) $\zeta_o=6^\circ$. 
Figure 5.9: Blade frequency response magnitude in degrees per unit nondimensional moment, for varying accumulator capacitance at operating lag angles of a) $\zeta_o=0^\circ$, b) $\zeta_o=3^\circ$, and c) $\zeta_o=6^\circ$. 

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5.1.2.3 1/rev Steady State Damper Behavior

One practical consideration for any rotor blade damper is that it will be subjected to cyclic loading at the 1/rev frequency in forward flight. The F²MC tube damper must therefore be able to withstand the normal 1/rev pressure fluctuations from the harmonic blade lag motion in addition to the pressures that occur as it damps blade motion caused by perturbations or pilot inputs. For the damper to be effective, the F²MC tube should also remain above some minimum pressure that keeps the bladder and fibers engaged. The F²MC tube must be initially pressurized so that throughout the flight envelope, its pressure remains above this minimum pressure for engagement, but below an upper limit determined by the F²MC tube fatigue strength. This section explains how the F²MC tube pressure fluctuation can be extracted from the linearized system model, and how the magnitude of the 1/rev tube pressure varies with the fluidic circuit parameters of orifice resistance and accumulator capacitance.

The F²MC tube pressure is derived by setting Eqs. (5.2) and (5.25) equal to one another,

\[ F_t = \{C_t\}^T \{\xi\} + D_t \Psi \zeta = c_1 x_t + c_2 p_t. \]  

After substituting in \( x_t = \Psi \zeta \), this equation can be rearranged to express the F²MC tube pressure as

\[ p_t = \frac{\{C_t\}^T \{\xi\} + (D_t - c_1) \Psi \zeta}{c_2}, \]

(5.33)

The F²MC tube pressure can now be used as an output for the state space model in Eq. (5.28).

Figures 5.10 and 5.11 plot 1/rev pressure amplitudes versus operating lag angle for several different values of orifice resistance and accumulator capacitance. These pressure amplitudes are normalized per degree of 1/rev lag motion. It is important to note that the model behind Eq. (5.33) is still linear, and actual 1/rev pressure amplitudes may depend on the amplitude of the cyclic blade motion due to the aforementioned nonlinear F²MC tube behavior. The actual amplitude of the 1/rev blade lag motion would vary with flight condition.
In Figure 5.10, the accumulator capacitance is held constant at \( c_a = 1 \times 10^{-10} \text{ m}^3/\text{Pa} \) while the orifice resistance is varied. According to Figure 5.7, increasing orifice resistance increases the effectiveness of the F\(^2\)MC damper, but according to Figure 5.10, this increased damping comes at the cost of higher 1/rev pressures for the same F\(^2\)MC tube. Because the damping force comes directly from the F\(^2\)MC tube pressure, increasing orifice resistance increases the damper authority by generating larger tube pressures. However, for the same 1/rev lag motion, the F\(^2\)MC tube will pump roughly the same amount of fluid, since the fluid pumping is directly caused by the tube elongation. Based on Eq. (5.4), higher 1/rev tube pressures will be generated if the orifice resistance \( R_c \) is increased as \( Q \) remains constant. These results indicate that there is a trade-off between the amount of damping produced by the F\(^2\)MC damper and the magnitude of 1/rev oscillatory tube pressures that could decrease fatigue life of both the hub and damper.

In Figure 5.11, the orifice resistance is held constant at \( R_c = 2 \times 10^9 \text{ kg/s-m}^4 \) while the accumulator capacitance is varied. Increasing the accumulator capacitance reduces the 1/rev pressures generated in the F\(^2\)MC tube, although as in Figure 5.8, the benefit diminishes as the accumulator becomes more compliant. Based on Figures 5.8 and 5.11, increasing accumulator capacitance is shown to make the damper more effective and decrease 1/rev oscillatory pressures in the F\(^2\)MC tube. The only drawback to using a larger accumulator is that the damper becomes larger and bulkier, so the benefits of using a larger accumulator must be weighed against practical size constraints in a given application.

### 5.2 Hingeless Blade F\(^2\)MC Damper

Unlike articulated rotor blades, stiff-inplane rotor blades such as those found on advancing blade concept rotors typically do not have external dampers. Stiff-inplane rotors are not prone to ground resonance; however, they may be prone to other aeroelastic instabilities at high angle of attack or at high advance ratios [85]. Currently existing elastomeric and hydraulic dampers are not suitable for stiff-inplane rotor blades, which experience very small deformation at the blade root as shown in Figure 5.12. On the other hand, devices based on F\(^2\)MC tubes can successfully reduce structural vibrations even when the F\(^2\)MC tube elongation is very small, as demonstrated by the tailboom vibration absorber. The increased
Figure 5.10: Variation in 1/rev F\textsuperscript{2}MC tube pressure amplitude with operating lag angle for different orifice resistances, accumulator capacitance $c_a = 1 \times 10^{-10}$ m\textsuperscript{3}/Pa.

Figure 5.11: Variation in 1/rev F\textsuperscript{2}MC tube pressure amplitude with operating lag angle for different accumulator capacitances, orifice resistance $R_c = 2 \times 10^9$ kg/s-m\textsuperscript{4}.
fluid pumping and force generation of an F²MC tube may enable the development of a compact damper with enough authority to increase the damping of stiff-inplane hingeless rotor blades.

The F²MC damper concept analyzed in this section is illustrated in Figure 5.13. The device functions as a damped vibration absorber, with an F²MC tube integrated into the hingeless blade root so that chordwise bending vibration strains the F²MC tube and pumps fluid into a circuit consisting of a tuned inertia track and a hydraulic accumulator. A damped vibration absorber is more suitable than a pure damper for this application because large forces are required to influence the dynamics of the stiff blade. An absorber harnesses the oscillating fluid inertia so that a given F²MC tube develops higher pressures and exerts larger forces on the blade. As in the tailboom absorber, the circuit must be tuned for a specific blade mode. In this section, a model is developed for a hingeless rotor blade integrated with an F²MC damped absorber, and simulation results are presented for several absorber designs.

![Diagram illustrating articulated blade versus hingeless blade with damper stroke comparison.](image)

**Figure 5.12: Large stroke for damper on articulated blade versus smaller stroke on hingeless blade.**

### 5.2.1 Hingeless Blade & Circuit Modeling

The hingeless blade is modeled as a one-dimensional beam discretized with Euler-Bernoulli beam finite elements. To simplify the analysis, only chordwise bending is considered in the model. This simplification is justified because the primary goal of
this study is to assess the feasibility of the proposed absorber, and its effectiveness would be determined largely by the chordwise blade dynamics. Stiff-inplane blades such as those found on the X2 rotor have little coupling between torsion, flap, and chordwise bending because of the high blade torsional stiffness [61]. The four degrees of freedom for each element are $(v_1, \theta_{z_1}, v_2, \theta_{z_2})$. The mass matrix for a given finite element is

$$
[M]_e = m_e \begin{bmatrix}
\frac{13L_e}{35} & \frac{11L_e^2}{210} & \frac{9L_e}{70} & -\frac{13L_e^2}{420} \\
\frac{11L_e^2}{210} & \frac{L_e^3}{105} & \frac{13L_e^2}{420} & -\frac{L_e^3}{140} \\
\frac{9L_e}{70} & \frac{13L_e^2}{420} & \frac{13L_e}{35} & -\frac{11L_e^2}{210} \\
-\frac{13L_e^2}{420} & -\frac{L_e^3}{140} & -\frac{11L_e^2}{210} & \frac{L_e^3}{105}
\end{bmatrix},
$$

(5.34)

where $m_e$ is the element mass per unit length and $L_e$ is the element length. The mass of the F$^2$MC damper is neglected when generating these matrices, since it is located near the blade root where it experiences small centrifugal acceleration. The stiffness matrix for a given finite element is a sum of the nonrotating beam stiffness matrix $[K_{nr}]_e$ and an additional matrix $[K_{rot}]_e$ that captures the rotational effects of centrifugal stiffening and “spin softening,”

$$
[K]_e = [K_{nr}]_e + [K_{rot}]_e.
$$

(5.35)
The matrix \([K_{nr}]_e\) is

\[
[K_{nr}]_e = \frac{EI_e}{L_e^3} \begin{bmatrix}
12 & 6L_e & -12 & 6L_e \\
6L_e & 4L_e^2 & -6L_e & 2L_e^2 \\
-12 & -6L_e & 12 & -6L_e \\
6L_e & 2L_e^2 & -6L_e & 4L_e^2
\end{bmatrix}, \quad (5.36)
\]

and the matrix \([K_{rot}]_e\) is

\[
[K_{rot}]_e = \frac{\Omega^2 A_e}{2} - m_e \Omega^2 \begin{bmatrix}
\frac{6}{5L_e} \cdot 10 & -\frac{6}{5L_e} \cdot 10 \\
\frac{1}{10} - \frac{2L_e}{15} & -\frac{1}{10} - \frac{L_e}{30} \\
-\frac{6}{5L_e} & -\frac{1}{10} - \frac{6L_e}{15} \\
\frac{1}{10} - \frac{L_e}{30} & -\frac{1}{10} - \frac{2L_e}{15}
\end{bmatrix}
\]

\[
- \frac{3r_i}{5} + \frac{6L_e}{35} \begin{bmatrix}
\frac{L_e r_i}{10} + \frac{L_e^2}{28} & -\frac{3r_i}{5} & -\frac{6L_e}{35} & -\frac{L_e^2}{70}
\end{bmatrix}
\]

\[
- \frac{L_e^2}{70} - \frac{L_e^2 r_i}{60} - \frac{L_e^2}{140}
\]

\[
- m_e \Omega^2 \begin{bmatrix}
\frac{13L_e}{35} & \frac{11L_e^2}{210} & \frac{9L_e}{70} & -\frac{13L_e^2}{420} \\
\frac{11L_e^2}{210} & \frac{L_e^3}{105} & \frac{13L_e^2}{420} & -\frac{L_e^3}{140} \\
\frac{9L_e}{70} & \frac{13L_e^2}{210} & \frac{L_e^3}{35} & \frac{11L_e^2}{420} \\
-\frac{13L_e^2}{420} & -\frac{L_e^3}{140} & -\frac{11L_e^2}{210} & \frac{L_e^3}{105}
\end{bmatrix}, \quad (5.37)
\]

where

\[
A_e = \sum_{j=e}^{N_{el}} m_e \left( (r_{j+1})^2 - (r_j)^2 \right).
\]

In these equations, \(m_e\) is the element mass per unit length, \(L_e\) is the element length, \(EI_e\) is the element chordwise bending stiffness, \(\Omega\) is the rotation speed, \(N_{el}\) is the total number of finite elements, and \(r_i\) is the radial position of the finite element.
inboard node. The variables $r_j$ and $r_{j+1}$ in the summation correspond to the radial locations of the inboard and outboard node, respectively, for element number $j$. In this model, the aerodynamic and structural damping contributions are combined into one damping term, and geometric viscous damping \[86,87\] is used to create the damping matrix for the finite element model. The damping matrix $[C]_e$ for an element is given by

$$[C]_e = \frac{c_G}{30L_e} \begin{bmatrix} 36 & 3L_e & -36 & 3L_e \\ 3L_e & 4L_e^2 & -3L_e & -L_e^2 \\ -36 & -3L_e & 36 & -3L_e \\ 3L_e & -L_e^2 & -3L_e & 4L_e^2 \end{bmatrix}, \quad (5.38)$$

where $c_G$ is the geometric damping coefficient. The element matrices are assembled into the blade global mass, stiffness, and damping matrices. Blade root boundary conditions are prescribed as zero translation and zero slope, and the blade root is located at the axis of rotation for simplicity. The equations of motion for the rotating blade before including the F$^2$MC damper are

$$[M] \{\ddot{q}\} + [C] \{\dot{q}\} + [K] \{q\} = \{F_{ext}\}, \quad (5.39)$$

where $\{F_{ext}\}$ denotes a vector of external forcing on the blade.

The fluidic circuit for the hingeless blade damped absorber is the same circuit shown in Figure 5.2. However, when modeling this circuit for the hingeless blade application, the fluid inertance is no longer neglected, and the flow resistance comes from the fluid viscosity according to Eq. (2.50) instead of an orifice. The transfer function from F$^2$MC tube extension to F$^2$MC tube force (in dimensional form) is defined by Eq. (5.6). Using MATLAB, this transfer function can be expressed in state-space form as

$$\{\dot{\xi}\} = [A_t] \{\xi\} + [B_t] x_t, \quad (5.40)$$

$$F_t = \{C_t\}^T \{\xi\} + D_t x_t. \quad (5.41)$$
Similar to the process in Chapter 2, the \( F^2MC \) tube axial extension can be expressed in terms of the hingeless blade finite element model nodal degrees of freedom,

\[
x_t = \beta_{v_1}v_1 + \beta_{\theta_{z_1}}\theta_{z_1} + \beta_{v_2}v_2 + \beta_{\theta_{z_2}}\theta_{z_2},
\]

(5.42)

where the coefficients \( \beta_{\chi} \) were defined in Eqs. (2.73), (2.78), and (2.79). In shorthand notation, Eq. (5.42) is written as

\[
x_t = \{\Psi\}^T\{q\}.
\]

(5.43)

Eq. (5.41) can now be expressed in terms of the blade nodal degrees of freedom,

\[
F_t = \{C_t\}^T\{\xi\} + D_t\{\Psi\}^T\{q\}.
\]

(5.44)

The scalar \( F^2MC \) tube force \( F_t \) is then converted into the equivalent finite element load vector \( \{\bar{F}_t\} \),

\[
\{\bar{F}_t\} = \{\sigma\} F_t = \{\sigma\}\left(\{C_t\}^T\{\xi\} + D_t\{\Psi\}^T\{q\}\right),
\]

(5.45)

where \( \{\sigma\} = -\{\Psi\} \). After including the \( F^2MC \) tube finite element load vector, Eq. (5.39) becomes

\[
[M] \{\ddot{q}\} + [C] \{\dot{q}\} + [K] \{q\} = \{\sigma\}\left(\{C_t\}^T\{\xi\} + D_t\{\Psi\}^T\{q\}\right) + \{F_{ext}\}.
\]

(5.46)

Eqs. (5.40) and (5.46) govern the dynamics of the hingeless rotor blade integrated with the \( F^2MC \) damped absorber. These equations can be written in state-space form as

\[
\begin{bmatrix}
\{\ddot{q}\} \\
\{\ddot{\xi}\}
\end{bmatrix} = [A_{sys}] \begin{bmatrix}
\{q\} \\
\{\dot{q}\}
\end{bmatrix} + [B_{sys}] \{F_{ext}\},
\]

(5.47)
where

\[
A_{\text{sys}} = \begin{bmatrix}
\begin{bmatrix} 0 \\ I \end{bmatrix}
& [I] & [0] \\
[M]^{-1} \left( -[K] + \{\sigma\} D_t \{\Psi\}^T \right) & -[M]^{-1} [C] & [M]^{-1} \left( \{\sigma\} \{C_t\}^T \right) \\
\{B_t\} \{\Psi\}^T & [0] & [A_t] \\
\end{bmatrix}
\begin{bmatrix} 0 \\ I \end{bmatrix}^{-1} 
\end{bmatrix}.
\]

\(5.48\)

5.2.2 Case Study: Representative Stiff-Inplane Hingeless Blade

In this section, a representative stiff in-plane coaxial rotor blade based on the Sikorsky X2 rotor is modeled, and an F²MC absorber is designed to reduce vibrations in its first chordwise bending mode. The model is used to calculate the blade chordwise response due to a given forcing input. The blade properties are based on information found in [61] and [88] and are summarized in Table 5.4. A total of 12 finite elements are used to discretize the blade. To simplify the analysis, the blade mass and stiffness properties are uniform. First, the mass per unit length is tuned to yield an approximate Lock number of 5, which is representative of a stiff hingeless blade. Next, the blade bending stiffness is tuned such that the first chordwise bending frequency is 1.40/rev at the rotor speed of 446 RPM. Finally, the geometric damping coefficient is tuned so that the first chordwise blade mode has 2% critical damping (damping ratio = 0.02).

Table 5.4: Representative stiff-inplane hingeless rotor blade properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius, m</td>
<td>4.02</td>
</tr>
<tr>
<td>Rotation speed, RPM</td>
<td>446</td>
</tr>
<tr>
<td>Mass per unit length, kg/m</td>
<td>3.44</td>
</tr>
<tr>
<td>Chordwise bending stiffness, N-m²</td>
<td>2.94×10⁵</td>
</tr>
<tr>
<td>Chord length, m</td>
<td>0.203</td>
</tr>
<tr>
<td>Estimated Lock number</td>
<td>5</td>
</tr>
<tr>
<td>First chordwise frequency, Hz</td>
<td>10.4</td>
</tr>
<tr>
<td>First chordwise mode damping ratio</td>
<td>0.02</td>
</tr>
</tbody>
</table>
The F\textsuperscript{2}MC tube is a constant offset distance \(d\) forward of the blade neutral axis as shown in Figure 5.14. The F\textsuperscript{2}MC tube is placed forward of the neutral axis for two reasons. Firstly, this avoids adding mass behind the neutral axis, which could make the blade more susceptible to flutter instabilities. Secondly, the spar carries most of the structural load, and it is important to integrate the F\textsuperscript{2}MC tube into the main structural member so that it effectively transmits a moment into the blade. Two possible configurations for the F\textsuperscript{2}MC absorber are considered. The first is an internal design where the F\textsuperscript{2}MC tube and fluidic circuit are enclosed inside the blade, as shown in Figure 5.14. The second is an external design where the F\textsuperscript{2}MC tube and fluidic circuit are attached outside of the blade and covered by a fairing, as shown in Figure 5.15. The main advantage of the internal concept is that it would be compact and would not generate any parasitic drag in high-speed flight; however, it may be challenging to fabricate, install, and maintain such a damper inside the blade. On the other hand, an external damper may be bulkier and increase the complexity of the rotor hub, but it could also be developed as a modular unit for easier integration and maintenance. Note that while initially pressurizing the F\textsuperscript{2}MC tube in this concept generates a moment that bends the blade toward the F\textsuperscript{2}MC tube, the blade deformation and F\textsuperscript{2}MC tube fiber angle change due to this initial pressurization are very small because of the high chordwise bending stiffness.

The dense, low-viscosity working fluid used for the tailboom absorber is selected for this study. Because a compact device is highly desired in this application, using a denser fluid has the benefit of generating the same inertance with a shorter track length. The inertia track length is tuned by first selecting a track diameter and then choosing the track length that minimizes the objective function

\[
Z = \int_{0}^{20} |Y(f)|^2 df, \tag{5.49}
\]

where \(|Y(f)|\) is the magnitude of the blade tip force to tip displacement transfer function at frequency \(f\) (in Hz). A total of three F\textsuperscript{2}MC absorbers are studied and compared in this section. The properties of the F\textsuperscript{2}MC tube in each of these absorbers are given in Table 5.5, and the properties of the fluidic circuit in each absorber are given in Table 5.6. F\textsuperscript{2}MC tube capacitance \(c_4\) is again assumed proportional to the initial volume of the F\textsuperscript{2}MC tube active length. The three cases
are designed as follows:

- Case 1 is an internal $F^2MC$ absorber with a 3/4 in. outer diameter $F^2MC$ tube placed 25% of the chord length from the neutral axis.
Case 2 is an external F$^2$MC absorber with the same F$^2$MC tube from Case 1 placed 50% of the chord length from the neutral axis. This naturally improves performance, because the increased offset leads to more F$^2$MC tube strain-induced pumping and also gives the F$^2$MC tube a larger moment arm.

Case 3 is an external F$^2$MC absorber with the same offset distance as Case 2, but which uses a $\frac{1}{2}$ in. diameter F$^2$MC tube. In contrast, Cases 1 and 2 used a $\frac{3}{4}$ in. diameter F$^2$MC tube.

Table 5.5: Properties of F$^2$MC tubes for stiff-inplane hingeless blade damped absorbers.

<table>
<thead>
<tr>
<th>Property</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stainless Steel Fibers</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic Modulus, GPa</td>
<td>180</td>
<td>180</td>
<td>180</td>
</tr>
<tr>
<td># of strands</td>
<td>312</td>
<td>312</td>
<td>312</td>
</tr>
<tr>
<td>Strand diameter, mm</td>
<td>0.348</td>
<td>0.348</td>
<td>0.232</td>
</tr>
<tr>
<td>Fiber angle, °</td>
<td>16</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td><strong>Rubber Bladder</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elastic Modulus, MPa</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Inner diameter, mm</td>
<td>16.3</td>
<td>16.3</td>
<td>10.9</td>
</tr>
<tr>
<td>Wall thickness, mm</td>
<td>1.36</td>
<td>1.36</td>
<td>0.907</td>
</tr>
<tr>
<td><strong>Linearized F$^2$MC Tube Model Coefficients</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial stiffness, $c_1$, N/m</td>
<td>$8.31\times 10^4$</td>
<td>$8.31\times 10^4$</td>
<td>$3.69\times 10^4$</td>
</tr>
<tr>
<td>Force-pressure coefficient, $c_2$, N/Pa</td>
<td>$8.54\times 10^{-3}$</td>
<td>$8.54\times 10^{-3}$</td>
<td>$3.80\times 10^{-3}$</td>
</tr>
<tr>
<td>Volume change coefficient, $c_3$, m$^3$/m</td>
<td>$-5.70\times 10^{-3}$</td>
<td>$-5.70\times 10^{-3}$</td>
<td>$-2.53\times 10^{-3}$</td>
</tr>
<tr>
<td>Capacitance, $c_4$, m$^3$/Pa</td>
<td>$1.89\times 10^{-12}$</td>
<td>$1.89\times 10^{-12}$</td>
<td>$8.41\times 10^{-13}$</td>
</tr>
<tr>
<td><strong>F$^2$MC Tube Placement</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inboard attachment point</td>
<td>0.02$R$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outboard attachment point</td>
<td>0.12$R$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F$^2$MC tube offset, cm</td>
<td>5.08</td>
<td>10.16</td>
<td>10.16</td>
</tr>
</tbody>
</table>
Table 5.6: Properties of tuned fluidic circuits for stiff-inplane hingeless blade damped absorbers.

<table>
<thead>
<tr>
<th>Property</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid density, kg/m$^3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fluid dynamic viscosity, Pa-s</td>
<td></td>
<td>9.0×10$^{-4}$</td>
<td></td>
</tr>
<tr>
<td>Track radius, mm</td>
<td></td>
<td>1.59</td>
<td></td>
</tr>
<tr>
<td>Track length, cm</td>
<td>46.4</td>
<td>39.1</td>
<td>90.1</td>
</tr>
<tr>
<td>Accumulator capacitance, m$^3$/Pa</td>
<td></td>
<td>1.0×10$^{-11}$</td>
<td></td>
</tr>
</tbody>
</table>

Frequency responses for a blade tip input force to a tip output displacement are presented in Figure 5.16 for the three damped absorber cases studied here. As expected, Case 2 is a stronger absorber than Case 1 because of the increased moment arm. Despite having a smaller diameter F$^2$MC tube, Case 3 performs comparably to the other designs because of the large moment arm. However, using the smaller diameter tube comes with the tradeoff of requiring a longer inertia track to tune the absorber. Physically, the smaller F$^2$MC tube has a lower capacitance, because the F$^2$MC tube volume changes less for the same unit of pressure compared to a tube with a larger diameter. This increases the effective absorber stiffness and means that more inertance is required to achieve the same tuning frequency.

As another means for evaluating the amount of damping provided by each of the F$^2$MC absorbers, the damping ratio and natural frequency can be extracted from eigenvalues of the three blade-absorber systems. These characteristics are presented in Table 5.7. Because the absorber adds a degree of freedom to the system, there are now two modes near the blade natural frequency of 10.4 Hz, and the lowest mode damping ratio increases from the baseline of 0.02 to a range of 0.059-0.066 once the F$^2$MC absorber is added. It should be noted that Case 2 appears to be the most effective absorber out of the three based on the frequency response, yet one of its modes has the lowest modal damping ratio of the blade-absorber systems. While damping ratio is a convenient and easily calculated parameter, it may not be the most effective metric to evaluate and compare damped absorbers because of the fact that the two modes combine to determine the overall system dynamics. The usefulness of damping ratio as a metric may depend on whether the primary goal of adding the F$^2$MC damped absorber is to augment blade stability or simply to reduce transient blade vibration.
Figure 5.16: Frequency responses from blade tip chordwise force input to tip chordwise displacement for baseline stiff-in-plane hingeless rotor blade and three blade-absorber systems.

Table 5.7: Characteristics from eigenvalue analysis of baseline blade and blade-$F^2$MC absorber systems.

<table>
<thead>
<tr>
<th></th>
<th>Baseline Blade</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1 damping ratio</td>
<td>0.02</td>
<td>0.0716</td>
<td>0.0783</td>
<td>0.0753</td>
</tr>
<tr>
<td>Mode 1 natural frequency, Hz</td>
<td>10.4</td>
<td>9.92</td>
<td>9.38</td>
<td>9.62</td>
</tr>
<tr>
<td>Mode 2 damping ratio</td>
<td></td>
<td>0.0658</td>
<td>0.0589</td>
<td>0.0624</td>
</tr>
<tr>
<td>Mode 2 natural frequency, Hz</td>
<td></td>
<td>11.19</td>
<td>12.18</td>
<td>11.60</td>
</tr>
</tbody>
</table>

In these three designs, the accumulator capacitance value was held constant. To demonstrate the effect of varying this parameter, three more absorbers are designed with the same $F^2$MC tube, working fluid, and offset distance as in Case 3 from the previous analysis. The first new absorber uses an accumulator that is 10 times softer than the one used in Case 3. The other two absorbers use accumulators that
are 10 times and 100 times stiffer than the one in Case 3. The inertia tracks are again tuned by selecting the track length that minimizes the objective function in Eq. (5.49).

Properties of the fluidic circuits for the new absorbers are summarized in Table 5.8. The blade tip frequency responses of these three systems, in addition to the one previously analyzed as Case 3, are plotted in Figure 5.17. The damping ratios and natural frequencies of these systems are presented in Table 5.9. The results indicate that F$^2$MC absorbers with softer accumulators require shorter track lengths to tune the fluidic circuit appropriately. If the accumulator becomes stiffer – i.e., it takes more pressure to move the same amount of fluid into it – then the F$^2$MC absorber becomes less effective, and the inertance required to tune the absorber increases. As a general design guideline for good performance and minimal inertia track size, the accumulator capacitance $c_a$ should be near or greater than the F$^2$MC tube capacitance $c_4$.

Table 5.8: Properties of tuned fluidic circuits for stiff-inplane hingeless blade damped absorbers with different accumulator capacitances.

<table>
<thead>
<tr>
<th>Property</th>
<th>10× softer</th>
<th>original case</th>
<th>10× stiffer</th>
<th>100× stiffer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Track radius, mm</td>
<td></td>
<td>1.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Track length, m</td>
<td>0.833</td>
<td>0.901</td>
<td>1.58</td>
<td>8.36</td>
</tr>
<tr>
<td>Accumulator capacitance, m$^3$/Pa</td>
<td>1.0×10$^{-10}$</td>
<td>1.0×10$^{-11}$</td>
<td>1.0×10$^{-12}$</td>
<td>1.0×10$^{-13}$</td>
</tr>
</tbody>
</table>

Table 5.9: Characteristics from eigenvalue analysis of blade-absorber systems with different accumulator capacitances.

<table>
<thead>
<tr>
<th>Property</th>
<th>10× softer</th>
<th>original case</th>
<th>10× stiffer</th>
<th>100× stiffer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1 damping ratio</td>
<td>0.0760</td>
<td>0.0753</td>
<td>0.0713</td>
<td>0.0341</td>
</tr>
<tr>
<td>Mode 1 natural frequency, Hz</td>
<td>9.55</td>
<td>9.62</td>
<td>9.99</td>
<td>10.65</td>
</tr>
<tr>
<td>Mode 2 damping ratio</td>
<td>0.0620</td>
<td>0.0624</td>
<td>0.0649</td>
<td>0.1007</td>
</tr>
<tr>
<td>Mode 2 natural frequency, Hz</td>
<td>11.64</td>
<td>11.60</td>
<td>11.32</td>
<td>10.78</td>
</tr>
</tbody>
</table>
Figure 5.17: Frequency responses from blade tip chordwise force input to tip chordwise displacement for stiff-inplane blade with tuned absorbers and different accumulator capacitances.

5.2.3 Soft-Inplane Hingeless Blade F$^2$MC Damper Options

Soft-inplane hingeless rotors are designed so that a soft flexure acts as a virtual hinge, allowing the blade to effectively flap and/or lag as a rigid body about a point near the blade root. This means that for an F$^2$MC damper to be effective on a soft-inplane blade, the F$^2$MC tube must span at least part of the flexure, since little strain occurs throughout the rest of the rotor blade. While the lower stiffness of the soft-inplane blade means that the F$^2$MC damped absorber may have more authority than it does on a stiff-inplane blade, it also means that the soft-inplane chordwise natural frequency will be lower, and more inertance will be required to tune the absorber for the lower lag frequency.

A uniform blade based on the BO105 rotor is modeled with 12 finite elements using the properties in Table 5.10. The rotor speed, radius, chord, and lag frequency are found in [89]. The blade mass distribution is tuned to yield a representative Lock number of 6, the bending stiffness is tuned to set the lag frequency of 0.66/rev, and then the geometric damping coefficient is tuned to provide the blade with an initial 2% critical damping. In addition to this uniform blade, two additional blades
are modeled which have an identical lag frequency of 0.66/rev but have non-uniform bending stiffness distributions. The flexure is approximated as a section over the first 12% of the blade radius having bending stiffness $EI_{flex}$ that is a fraction of the main blade stiffness $EI_{bl}$. The properties of all three soft-inplane rotor blades are presented in Table 5.10.

Table 5.10: Representative uniform and non-uniform soft-inplane hingeless rotor blade properties.

<table>
<thead>
<tr>
<th>Property</th>
<th>Uniform Blade</th>
<th>$EI_{flex}/EI_{bl}$ = 0.5</th>
<th>$EI_{flex}/EI_{bl}$ = 0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius, m</td>
<td>4.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotation speed, RPM</td>
<td>425</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass per unit length, kg/m</td>
<td>4.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bending stiffness $EI_{bl}$, N-m$^2$</td>
<td>$1.47\times10^5$</td>
<td>$2.34\times10^5$</td>
<td>$3.98\times10^5$</td>
</tr>
<tr>
<td>Chord length, m</td>
<td>0.264</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First chordwise frequency, Hz</td>
<td>4.7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

An $F^2$MC damped absorber using an $F^2$MC tube with 1/2 in. outer diameter and fiber angle of 16° is tuned for each blade. The track length is again chosen to minimize the objective function in Eq. (5.49), although a narrower inertia track radius is used in these circuits to achieve the required inertance for the low absorber frequency without significantly increasing the track length. $F^2$MC tube and fluidic circuit properties of the three tuned absorbers for the soft-inplane blades are given in Tables 5.11 and 5.12, respectively. The tip force to tip displacement frequency responses for the three undamped blades, as well as the frequency responses for each blade integrated with its $F^2$MC damped absorber, are plotted in Figure 5.18. Table 5.13 lists the dynamic characteristics of each blade-absorber system as extracted from eigenvalue analysis.

Although the natural frequencies and static stiffnesses of the three undamped blades are very similar, the frequency responses with the $F^2$MC damped absorber show that the absorber effectiveness varies as the blade stiffness distribution is altered. Changes in the blade mode shape impact the $F^2$MC damped absorber effectiveness. Figure 5.19 compares the mode shapes for each of the three rotor blade configurations. As the flexure becomes softer relative to the rest of the blade, the mode shape strain becomes more localized at the blade root where the $F^2$MC tube is attached. Consequently, the $F^2$MC damped absorbers become more effective.
as the flexure becomes softer. Since the lag frequency is already 0.66/rev, one must be careful to design the F\textsuperscript{2}MC damped absorber so that it does not produce a new mode near the 1/rev frequency. Figure 5.18 indicates that the blade frequency response magnitude actually increases at the 1/rev frequency of 7.1 Hz when the F\textsuperscript{2}MC damped absorber is added, which could result in undesirable 1/rev blade motion and hub load increases.

Table 5.11: Properties of F\textsuperscript{2}MC tube for all three soft-inplane hingeless blade damped absorbers.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stainless Steel Fibers</strong></td>
<td></td>
</tr>
<tr>
<td>Elastic Modulus, GPa</td>
<td>180</td>
</tr>
<tr>
<td># of strands</td>
<td>312</td>
</tr>
<tr>
<td>Strand diameter, mm</td>
<td>0.232</td>
</tr>
<tr>
<td>Fiber angle, °</td>
<td>16</td>
</tr>
<tr>
<td><strong>Rubber Bladder</strong></td>
<td></td>
</tr>
<tr>
<td>Elastic Modulus, MPa</td>
<td>1</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.5</td>
</tr>
<tr>
<td>Inner diameter, mm</td>
<td>10.9</td>
</tr>
<tr>
<td>Wall thickness, mm</td>
<td>0.907</td>
</tr>
<tr>
<td><strong>Linearized F\textsuperscript{2}MC Tube Model Coefficients</strong></td>
<td></td>
</tr>
<tr>
<td>Axial stiffness, $c_1$, N/m</td>
<td>$2.73 \times 10^4$</td>
</tr>
<tr>
<td>Force-pressure coefficient, $c_2$, N/Pa</td>
<td>$3.80 \times 10^{-3}$</td>
</tr>
<tr>
<td>Volume change coefficient, $c_3$, m$^3$/m</td>
<td>$-2.53 \times 10^{-3}$</td>
</tr>
<tr>
<td>Capacitance, $c_4$, m$^3$/Pa</td>
<td>$1.16 \times 10^{-12}$</td>
</tr>
<tr>
<td><strong>F\textsuperscript{2}MC Tube Placement</strong></td>
<td></td>
</tr>
<tr>
<td>Inboard attachment point</td>
<td>$0.02R$</td>
</tr>
<tr>
<td>Outboard attachment point</td>
<td>$0.12R$</td>
</tr>
<tr>
<td>F\textsuperscript{2}MC tube offset, cm</td>
<td>13.20</td>
</tr>
</tbody>
</table>
Table 5.12: Properties of tuned fluidic circuits for soft-inplane hingeless blade damped absorbers.

<table>
<thead>
<tr>
<th>Property</th>
<th>Uniform Blade</th>
<th>$EI_{flex}/EI_{bl}$ = 0.5</th>
<th>$EI_{flex}/EI_{bl}$ = 0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid density, kg/m$^3$</td>
<td>1880</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fluid dynamic viscosity, Pa-s</td>
<td>$9.0 \times 10^{-4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Track radius, mm</td>
<td>0.794</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Track length, cm</td>
<td>60.1</td>
<td>57.5</td>
<td>56.2</td>
</tr>
<tr>
<td>Accumulator capacitance, m$^3$/Pa</td>
<td>$1.0 \times 10^{-11}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.18: Frequency responses from blade tip chordwise force input to tip chordwise displacement for soft-inplane rotor blades with tuned F$^2$MC absorbers.
Table 5.13: Characteristics from eigenvalue analysis of soft-inplane blades with F²MC damped absorbers.

<table>
<thead>
<tr>
<th>Property</th>
<th>Uniform Blade</th>
<th>$EI_{flex}/EI_{bl}$ = 0.5</th>
<th>$EI_{flex}/EI_{bl}$ = 0.25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1 damping ratio</td>
<td>0.1363</td>
<td>0.1467</td>
<td>0.1561</td>
</tr>
<tr>
<td>Mode 1 natural frequency, Hz</td>
<td>4.24</td>
<td>4.09</td>
<td>3.97</td>
</tr>
<tr>
<td>Mode 2 damping ratio</td>
<td>0.1014</td>
<td>0.0916</td>
<td>0.0836</td>
</tr>
<tr>
<td>Mode 2 natural frequency, Hz</td>
<td>5.65</td>
<td>6.03</td>
<td>6.38</td>
</tr>
</tbody>
</table>

Figure 5.19: Mode shapes for three soft-inplane hingeless blades.
It is interesting to explore whether or not an F$^2$MC “pure damper,” which would only require an orifice and would not require a tuned inertia track, could also be an effective way to add damping to a soft-inplane hingeless rotor blade. Although the effectiveness of the pure damper is limited on a stiff-inplane rotor blade because of its high bending stiffness, the presence of a soft flexure creates an area of high local strain in the soft-inplane blade. Frequency response results for the three soft-inplane hingeless blades with F$^2$MC dampers are plotted in Figure 5.20. For these plots, the fluidic circuit consists solely of resistance $R_c = 2.2 \times 10^{10}$ kg-s/m$^4$. The F$^2$MC tube properties are identical to those in Table 5.11, and the accumulator capacitance is still $c_a = 1.0 \times 10^{-11}$ m$^3$/Pa.

Again, the damper becomes more effective when the mode shape has more strain in the region where the F$^2$MC tube is attached. For the case where the flexure bending stiffness is 25% of the blade stiffness, which is indicated by the green dashed curve, the blade damping ratio with this circuit is 0.0862. However, for the case where the blade is uniform, which is indicated by the red dashed curve, the blade damping ratio is only 0.0503. The variation in frequency response with slight increases or decreases in the orifice resistance is plotted in Figure 5.21. Based on the figure, $R_c = 2.2 \times 10^{10}$ kg-s/m$^4$ is approximately the optimal resistance value already; increasing the resistance further will increase the lag frequency and decrease damping, while decreasing the resistance will lower both the lag frequency and damping. By inspecting Figure 5.18 and 5.20, one can also see that a subtle benefit of the pure F$^2$MC damper is that it causes less of an increase in the 1/rev displacement frequency response than the F$^2$MC damped absorber does.

Overall, the results presented in this section indicate that both the F$^2$MC damped absorber and pure damper could be viable solutions for stabilizing soft-inplane hingeless rotor blades. The preferred method out of these two options could depend on the specific blade properties, the ease of integrating each option into a given blade, and the required amount of blade damping.
Figure 5.20: Frequency responses from blade tip chordwise force input to tip chordwise displacement for soft-inplane rotor blades with F$^2$MC dampers.

Figure 5.21: Frequency responses for the case where flexure bending stiffness is 25% of blade stiffness, with three different orifice resistances.
Chapter 6  
Small-Scale Articulated Blade Damper Prototyping

This chapter discusses the design and implementation of a small-scale experiment to demonstrate the feasibility of the articulated blade F²MC damper concept described in Chapter 5. A small-scale hub is fabricated to facilitate benchtop testing on a 9.7-foot diameter rotor. A prototype F²MC damper is built and tested, and model predictions are verified using the experimental data. These benchtop experiments help to refine the concept and understand key challenges in physically implementing the F²MC articulated blade damper.

6.1 Articulated Rotor Hub Design

The steel hub fabricated out of 1 in. thick steel plate for these experiments allows a rotor blade to rotate around a shoulder bolt that functions as the lag hinge. The hub design, which is developed using SolidWorks, is illustrated in Figure 6.1. For future rotating tests, the six small bolt holes around the center can be used to attach an adapter with a spline pattern matching the driveshaft in the Penn State Adverse Environment Rotor Test Stand (AERTS) facility. Additional holes are included so that objects such as the damper accumulator and displacement sensors can be mounted to the hub. The different attachment holes on the rotor hub are labeled in Figure 6.2. Engineering drawings for the hub can be found in Appendix E.

A SolidWorks finite element analysis is conducted with a fixed boundary condition at the rotor shaft hole and representative blade centrifugal loads applied at the
shoulder bolt holes as shown in Figure 6.3. Using 1018 carbon steel (yield strength $= 53,700 \text{ psi}$) as the material, this analysis predicts a safety factor of 2 up to a rotor speed of 625 RPM. The finite element analysis predicts that the von Mises stress is highest at the hole where the shoulder bolt is being pulled radially outward. The distance from the hub center to this hole, which defines the parameter $e$ introduced in the previous chapter, is 10.8 cm (4.25 in). The rotor blades in these experiments are Schweizer 300 blades that have been cut to fit inside the AERTS facility. Each blade has a length of 1.37 m (4.5 ft) from the shoulder bolt hole to the blade tip, weighs 4.4 kg (9.7 lbm), and has a chord of 17.5 cm (6 7/8 in.). A photograph of one blade is shown in Figure 6.4.

Figure 6.1: CAD image of hub for articulated blade experiments.
Thrust and roller bearings are used to ensure that the shoulder bolt functions as a low-friction pin joint. A schematic of the bearing assembly is shown in Figure 6.5. A total of four thrust bearings are placed between the bolt head and the blade, between the blade and each hub surface, and between the blade and a lock nut. Roller bearings are press-fitted into holes in the blade root. To prevent the blade from flapping out-of-plane as a rigid body, the bolt is machined to a tight diameter tolerance, and the nut is tightened to close the gap between the blade and the thrust bearings. Care is taken not to overtighten the nut, which would place compressive loading on the thrust bearings and could increase friction. The hub is not truly articulated because the blade cannot pitch or flap; however, these motions are not necessary to demonstrate the F2MC lag damper concept. Constraining the blade to only undergo in-plane rigid-body motion greatly simplifies the blade analysis and the damper hardware design.
Figure 6.3: Finite element analysis result showing von Mises stress distribution in articulated rotor hub. Applied centrifugal loading is a bearing load representative of a 9.7-lb blade with uniform mass distribution spinning at 625 RPM.

Figure 6.4: Shortened Schweizer 300 rotor blade for small-scale experiments.
6.2 Benchtop Experiment Design & Modeling

The benchtop tests described in this section are developed with the goal of simulating a rotating environment to evaluate the prototype F$^2$MC damper performance. To replicate the effective centrifugal stiffness from a rotating environment, springs are attached on both sides of the blade at a distance $d_s$ from the lag hinge as shown in Figure 6.6. A perforated strap is wrapped around the blade, and a turnbuckle is used to connect the springs on each side between the strap and a rod that is threaded into a vibration-isolating table. The turnbuckles are used to pretension the springs so that they remain within their linear range as the blade moves laterally. To attach two springs at once on each side of the blade, a coupler is attached to the turnbuckle as shown in Figure 6.7.

Before adding the F$^2$MC damper, the (dimensional) blade equation of motion for the benchtop test is

$$I_\zeta \ddot{\zeta} + c_\zeta \dot{\zeta} + k_\zeta \zeta = M_{ext},$$  \hspace{1cm} (6.1)

where $I_\zeta$ is the blade inertia about the lag hinge, $c_\zeta$ and $k_\zeta$ are linear damping and stiffness coefficients, and $M_{ext}$ is an applied moment about the lag hinge. The
blade mass moment of inertia about the lag hinge is estimated using the equation for the period $T$ of a physical pendulum,

$$T = 2\pi \sqrt{\frac{I_c}{m_b g r_{cg}}},$$  \hspace{1cm} (6.2)$$

where $m_b$ is the total blade mass (including the trailing edge bracket and rod end), $g$ is the acceleration of gravity, and $r_{cg}$ is the distance from the blade lag hinge to its center of gravity. To determine the blade radial center of gravity, the blade is placed on a set of two scales as shown in Figure 6.8, with an L-shaped piece of metal placed underneath the blade at each end to minimize the contact area. Based on the two scale readings and the location of each support point, a static moment balance is used to calculate the blade center of gravity, which is estimated to be 47.9 cm (18.9 in.) from the lag hinge. The blade is then vertically suspended from
its lag hinge by a bolt and released from rest. Its oscillation period is timed with a stopwatch as 1.92 s, resulting in an estimated inertia of 2.14 kg·m² as calculated from Eq. (6.2).

Figure 6.7: Coupler for attaching two springs on one side of the rotor blade (total of four springs attached).

Figure 6.8: Process for determining blade radial center of gravity using scale measurements.
The coefficient $k_\zeta$ is determined to be a combination of the stiffness due to the springs as well as some additional torsional stiffness due to the fact that the attached shaker is constrained to purely linear motion,

$$k_\zeta = k_{spr} + k_{shak}. \quad (6.3)$$

The total torsional stiffness of $N_s$ springs, each with spring constant $k$ and attached to the blade a distance $d_s$ from the lag hinge, is

$$k_{spr} = N_s kd_s^2. \quad (6.4)$$

The spring constant $k$ is measured statically by hanging weights from one end of each spring and measuring the resulting changes in length. The springs are nearly identical, and the average measured spring constant is 2540 N/m. The torsional stiffness due to the shaker attachment is determined by experimentally measuring the baseline blade frequency response and then tuning the shaker stiffness parameter in the model so that the baseline blade model predicts the baseline blade lag frequency. The magnitude of the baseline blade frequency response is measured with $N_s = 2$ springs and $N_s = 4$ springs at a distance $d_s = 29.8$ cm (11.75 in.) from the lag hinge. These frequency responses from a unit torque input to an output angular displacement are plotted in Figures 6.9 and 6.10. The measured baseline blade natural frequencies are 3.6 Hz with two springs attached and 4.3 Hz with four springs attached. Finally, $c_\zeta$ is tuned based on the damping observed in the blade frequency response. The empirically-determined values of $k_{shak}$ and $c_\zeta$ from these tests are 675 N-m/rad and 8.56 N-m-s, respectively.
The $F_2^{MC}$ tube and damper fluidic circuit are modeled using the process outlined in Section 5.1. The predicted blade operating lag angle is calculated by performing a static moment balance between the $F_2^{MC}$ tube moment due to initial pressurization and the spring moments resisting the $F_2^{MC}$ tube contraction. In benchtop tests, the shaker is attached after setting the circuit pressure to its desired value, so the shaker torsional stiffness is not factored into this moment balance. The $F_2^{MC}$ tube coefficients $c_1$, $c_2$, and $c_3$ at the predicted operating point are
then calculated by the model from [68]. The coefficient $c_4$ is estimated with the previously described benchtop apparatus shown in Figure 2.9. When characterizing the F$^2$MC tube capacitance, its tension is set by pressurizing the tube and then adjusting the hex nuts to shorten its active length until its diameter matches the measured diameter of the F$^2$MC tube once it has been pumped up to the same pressure on the corresponding blade benchtop test. In the modeling for this benchtop test, dimensional derivatives with respect to time are used instead of the nondimensional derivatives with respect to angle $\psi$. The state-space representation of Eq. (5.6) with time derivatives is

$$
\{\dot{\xi}\} = [A_t] \{\xi\} + \{B_t\} x_t, \quad (6.5)
$$

$$
F_t = \{C_t\}^T \{\xi\} + D_t x_t. \quad (6.6)
$$

With the F$^2$MC damper moment included, Eq. (6.1) becomes

$$
I_\zeta \ddot{\zeta} + c_\zeta \dot{\zeta} + k_\zeta \zeta = \sigma \left( \{C_t\}^T \{\xi\} + D_t \Psi \zeta \right) + M_{\text{ext}}. \quad (6.7)
$$

After rearranging the dimensional equation of motion, the full system dynamics can be expressed in state-space form,

$$
\begin{bmatrix}
\dot{\zeta} \\
\ddot{\zeta}
\end{bmatrix} = \begin{bmatrix} A_{\text{sys}} \end{bmatrix} \begin{bmatrix}
\zeta \\
\dot{\zeta}
\end{bmatrix} + \begin{bmatrix} 0 \cr 1 \cr 1 \end{bmatrix} \begin{bmatrix} 0 \cr \{0\} \cr M_{\text{ext}} \end{bmatrix}, \quad (6.8)
$$

where

$$
\begin{bmatrix} A_{\text{sys}} \end{bmatrix} = \begin{bmatrix}
0 & 1 & \{0\}^T \\
-k_\zeta - c_\zeta \sigma D_t \Psi & -c_\zeta & \sigma (C_t)^T \\
\{B_t\} \Psi & 0 & [A_t]
\end{bmatrix}. \quad (6.9)
$$
6.3 Damper Experimental Results

Figure 6.11 is a photograph of the F\textsuperscript{2}MC damper hardware installed on the rotor blade. The damper consists of the F\textsuperscript{2}MC tube, an inertia track, an adjustable orifice, and an accumulator. AW32 hydraulic oil is used as the working fluid. The inertia track is made with a length of 4.57 mm (0.180 in.) inner diameter stiff nylon tubing, and the accumulator is a 6.35 cm (2.5 in.) length of 1.91 cm (3/4 in.) inner diameter fiber-reinforced PVC tubing clamped to brass barbed fittings at each end. To achieve the required accumulator compliance, compressible air is trapped behind the orifice and inside the accumulator tubing. First, the valve leading into the accumulator is closed, and fluid is continuously cycled through the circuit to bleed air out of the F\textsuperscript{2}MC tube and inertia track. To facilitate the air bleeding process, the accumulator is detached from the hub until the filling process is completed, and the accumulator is oriented vertically as fluid is pumped through the circuit. Once air has been removed from the rest of the circuit, the valve leading into the accumulator is opened, and additional fluid is pumped into the accumulator. As fluid enters, the air in the accumulator is compressed until the desired operating pressure is reached.

The F\textsuperscript{2}MC tube is fabricated by pulling a polyethylene plastic sleeve over a 6.35 mm (1/4 in.) inner diameter rubber bladder with a wall thickness of 2.38 mm (3/32 in.). Its active length between the clamps at each end is 8.26 cm (3.25 in.). The end fittings are fabricated by hollowing out a 5/8”-18 stainless steel threaded rod and tapping 1/4” female NPT threads into each end. These steel fittings are 11.75 cm (4.625 in.) long, which is longer than the fittings used on the tailboom vibration absorber in Chapters 3 and 4 because more hardware is included between the hex nuts in the F\textsuperscript{2}MC damper. A spherical bearing rod end is used at each end of the F\textsuperscript{2}MC tube so that the F\textsuperscript{2}MC tube remains straight as the blade undergoes large angular displacements. One rod end is threaded into the steel hub, and the other rod end is threaded into a bracket that is bolted onto the blade trailing edge. These spherical bearings were not necessary on the tailboom absorber because the F\textsuperscript{2}MC tube length changes caused by tailboom vibration were much smaller and did not have the potential to misalign or bend the tube. A 1/4 in. diameter steel sliding guide rod is also integrated into the design to constrain the F\textsuperscript{2}MC tube to purely axial displacement and prevent it from bending in-plane or out-of-plane. At
one end, the guide rod is threaded and secured to a plate with two nuts, and at the other end, it passes through a linear ball bearing inside another plate. The locations of the spherical bearings, guide rod, and linear ball bearing are highlighted in Figure 6.12.

The instrumentation for the benchtop experiment is depicted in Figure 6.13. As in the experiments to measure the baseline blade frequency response, the springs are attached 29.8 cm (11.75 in.) from the lag hinge. A LabVIEW data acquisition program continuously samples data at 200 Hz while sending a sinusoidal sweep to the shaker ranging from 0 to 25 Hz over a 25 second interval. The final frequency response is obtained by averaging measurements after five cycles of this sweep. The shaker is located 47.6 cm (18.75 in.) from the blade lag hinge, and it is attached to the blade via a threaded adapter that is fixed to the blade using two set screws. The displacement of a point 1.22 m (48 in.) from the lag hinge is measured using a laser vibrometer (sensitivity: 1280 µm/V), and the input forcing is measured using a PCB 208C01 load cell (sensitivity: 112.4 V/kN). These forces and linear displacements are converted into moments and angular displacements, respectively, to generate the blade frequency response. The steel hub is rigidly bolted to the table throughout the duration of these benchtop experiments. In addition to these force and displacement measurements, the pressure at the $F^2$MC tube exit is measured using a WIKA A-10 pressure transducer with a range of 0 to 200 psi.

Figure 6.11: Top view of $F^2$MC damper in benchtop test.
6.3.1 Model Verification

To simulate different rotation speeds and centrifugal stiffnesses, tests are performed with either two springs or four springs connected to the strap via the turnbuckles. From Eq. (5.1), an articulated blade rotating at angular speed $\Omega$ has lag stiffness

$$k_\zeta = I_\zeta (\Omega \nu_\zeta)^2 .$$

(6.10)
The lag frequency of an articulated blade is given by [4] as

\[ \nu_\zeta = \sqrt{\frac{S_\zeta}{I_\zeta}}, \]  

(6.11)

where \( S_\zeta = m_b r_{cg} \) is the blade first mass moment about the lag hinge. By substituting Eq. (6.4) into Eq. (6.3) and then setting the result equal to Eq. (6.10), one can determine the rotor speed simulated by a given benchtop test. Including the stiffness due to the shaker constraint, the two-spring and four-spring cases correspond to speeds of 639 RPM and 757 RPM, respectively. If the shaker is detached and therefore not included as part of the overall torsional stiffness, then the two-spring and four-spring cases correspond to speeds of 405 RPM and 573 RPM, respectively.

To verify the model for the small-scale benchtop experiment, predicted frequency responses are compared to benchtop test results obtained at initial operating pressures of 40 psi and 60 psi. Each experiment is conducted by pumping the circuit to the desired operating pressure, then closing the valve leading from the pump into the F\textsuperscript{2}MC tube and generating the frequency response for a variety of different orifice positions ranging from fully open to fully closed. The behavior of these two extreme cases is illustrated in Figures 6.14a and 6.14b. When the orifice is fully open, some damping is achieved by pumping the viscous hydraulic oil between the F\textsuperscript{2}MC tube and the accumulator. On the other hand, when the orifice is fully closed, the blade frequency increases, and the damping decreases as indicated by the sharper resonant peak. In the closed orifice case, the accumulator capacitance \( c_a \) approaches zero, and the overall system stiffness increases because the incompressible fluid in the circuit has nowhere to travel as the F\textsuperscript{2}MC tube volume changes. As the orifice is adjusted from fully open to fully closed, the shape of the blade frequency response changes as shown in Figures 6.15a and 6.15b. When two springs are attached to the blade, the frequency increases from 4.3 Hz with an open orifice to 6.0 Hz with a closed orifice. Similarly, when four springs are attached, the frequency increases from 5.2 Hz with an open orifice to 7.4 Hz with a closed orifice. The open orifice natural frequencies from the 2-spring and 4-spring experiments represent increases of 19% and 21%, respectively, from the baseline blade frequencies. In each case, the damping can be improved from the fully-open case by partially closing the orifice; however, at some point, continuing
to close the orifice starts to reduce damping and increase the damper stiffness.

The F²MC tube input parameters used in verifying the blade-damper model are listed in Table 6.1. The input inner diameter value is larger than the 6.35 mm initial bladder inner diameter because the mesh and rubber bladder are slightly disengaged at zero pressure. The inner diameter input is determined by measuring the outer diameter of the F²MC tube when it is pressurized just enough for the bladder to engage the fibers, and then subtracting twice the initial bladder wall thickness from this measured diameter. The change in bladder wall thickness as it expands to engage with the fibers is negligible.

The model circuit input parameters for each test case are listed in Table 6.2. The optimal orifice position is identified empirically in each test. Resistance values are tuned by starting with the expression in Eq. (2.50) and then further increasing the resistance to account for fitting diameter changes, sharp turns, and other losses in the circuit. The accumulator capacitance is tuned to achieve similar frequencies in the fully open orifice cases, since its compliance determines the lower bound for the blade frequency with the F²MC damper installed. To ensure that the model capacitance value is reasonable, static benchtop tests similar to the ones used in measuring the F²MC tube parameter $c_4$ are used to characterize the capacitance of the accumulator on its own. Finally, the blade linear damping constant $c_\zeta$ is estimated to be 10.7 N-m-s by selecting a value that gives closed-orifice resonant peak amplitudes that are similar to the experimental results.

Frequency responses from a unit moment input to a unit angular blade displacement are experimentally measured and compared to model predictions. The lag displacement frequency response model correlations at 40 and 60 psi for the cases with two springs attached to the blade are shown in Figure 6.16. Likewise, the lag displacement frequency response model correlations for the cases with four springs attached to the blade are shown in Figure 6.17. In general, the model correlation is good for cases when the orifice is completely open and when the orifice is tuned to maximize damping. There is some discrepancy when it comes to the closed-orifice natural frequency prediction, as the model-predicted closed-orifice frequency deviates from experimentally measured frequencies by as much as 17% in these plots. There are a couple potential reasons for this discrepancy. The closed-orifice stiffness increase comes from the fact that the F²MC tube has nowhere to pump fluid, making it harder to stretch the tube axially. Especially at the lower operating
Figure 6.14: Blade frequency responses generated at 60 psi with orifice a) fully open and b) fully closed.
Figure 6.15: Variation of blade frequency response with a) 2 springs attached and b) 4 springs attached, as orifice position ranges from fully open to fully closed. Tests conducted at 60 psi.
pressure of 40 psi, there may be times when the tube axial elongation is transferred from the fibers to the bladder, but its compression is not transferred as well. If the pressure drops low enough as the shaker sweeps through the closed-orifice frequency, the fibers may not be engaging well enough with the bladder to receive the full expected stiffness increase. Another possible reason for the discrepancy is that the model predictions are sensitive to the initial fiber wind angle input, because the $F^2MC$ tube coefficients $c_2$ and $c_3$ are functions of the fiber angle. In practice, it is hard to measure the fiber wind angle with high precision, and small amounts of hysteresis in the $F^2MC$ tube or variation in the process of pre-loading springs could also lead to variability between tests. The damping in each test case as predicted by the blade-damper model is quantified in Table 6.3.

Table 6.1: Model $F^2MC$ tube parameters for verification cases, 31.5° initial fiber angle.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Polyethylene Fibers</strong></td>
<td></td>
</tr>
<tr>
<td>Elastic Modulus, GPa</td>
<td>3</td>
</tr>
<tr>
<td># of strands</td>
<td>70</td>
</tr>
<tr>
<td>Strand diameter, mm</td>
<td>0.350</td>
</tr>
<tr>
<td>Initial fiber angle, °</td>
<td>31.5</td>
</tr>
<tr>
<td><strong>Rubber Bladder</strong></td>
<td></td>
</tr>
<tr>
<td>Elastic Modulus, MPa</td>
<td>1</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.5</td>
</tr>
<tr>
<td>Inner diameter, mm</td>
<td>8.83</td>
</tr>
<tr>
<td>Wall thickness, mm</td>
<td>2.38</td>
</tr>
<tr>
<td><strong>$F^2MC$ tube geometry (coordinate system from Figure 5.3)</strong></td>
<td></td>
</tr>
<tr>
<td>Hub attachment point</td>
<td>(-0.308, -0.175, 0) m</td>
</tr>
<tr>
<td>Blade attachment point</td>
<td>(-0.0127, -0.181, 0) m</td>
</tr>
<tr>
<td>Initial active length, cm</td>
<td>8.26</td>
</tr>
</tbody>
</table>
Table 6.2: Model circuit parameters for verification cases, 31.5° initial fiber angle.

<table>
<thead>
<tr>
<th>Fluid Properties (AW32 hydraulic oil, room temperature)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, kg/m³</td>
</tr>
<tr>
<td>Dynamic viscosity, Pa-s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inertia Track</th>
</tr>
</thead>
<tbody>
<tr>
<td>Track radius, mm</td>
</tr>
<tr>
<td>Track length, cm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2-Spring Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>40 psi, fully open</td>
</tr>
<tr>
<td>40 psi, tuned orifice</td>
</tr>
<tr>
<td>60 psi, fully open</td>
</tr>
<tr>
<td>60 psi, tuned orifice</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4-Spring Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>40 psi, fully open</td>
</tr>
<tr>
<td>40 psi, tuned orifice</td>
</tr>
<tr>
<td>60 psi, fully open</td>
</tr>
<tr>
<td>60 psi, tuned orifice</td>
</tr>
</tbody>
</table>
Figure 6.16: Model correlation for lag displacement frequency response, 2-spring test, 31.5° initial fiber angle, a) 40 psi operating pressure, b) 60 psi operating pressure.
Figure 6.17: Model correlation for lag displacement frequency response, 4-spring test, $31.5^\circ$ initial fiber angle, a) 40 psi operating pressure, b) 60 psi operating pressure.
Table 6.3: Model-predicted damping ratios for open, tuned, and closed orifice cases, 31.5° initial fiber angle.

<table>
<thead>
<tr>
<th>Operating Pressure</th>
<th>Open Orifice Damping Ratio</th>
<th>Tuned Orifice Damping Ratio</th>
<th>Closed Orifice Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-Spring Cases</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40 psi</td>
<td>0.197</td>
<td>0.319</td>
<td>0.0635</td>
</tr>
<tr>
<td>60 psi</td>
<td>0.185</td>
<td>0.335</td>
<td>0.0613</td>
</tr>
<tr>
<td>4-Spring Cases</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40 psi</td>
<td>0.214</td>
<td>0.300</td>
<td>0.0565</td>
</tr>
<tr>
<td>60 psi</td>
<td>0.190</td>
<td>0.308</td>
<td>0.0535</td>
</tr>
</tbody>
</table>

In addition to the lag displacement frequency responses, the frequency responses from a unit moment input to a unit F²MC tube pressure output are experimentally measured and plotted against model predictions. The steps for extracting the F²MC tube pressure from the system model are outlined in Section 5.1.2.3. For the tests corresponding to the lag displacement frequency responses presented in Figures 6.16 and 6.17, the corresponding F²MC tube pressure frequency responses are shown in Figures 6.18 and 6.19. The experimentally-measured frequency responses closely follow model-predicted trends, with the main discrepancy again being the closed-orifice natural frequency predictions. These F²MC tube pressure frequency response curves are also used to support tuning of the resistance values given in Table 6.2. The fact that the model is able to accurately predict F²MC tube pressures supports the claim that the resulting energy dissipation comes primarily from the F²MC tube pressure and not from other sources such as friction or axial deformation of the elastomeric bladder.

6.3.1.1 Alternate Model Tuning Approach

A slightly better correlation between the experimentally-measured and model-predicted closed orifice natural frequencies can be achieved by tuning the fiber angle parameter in the F²MC tube model for each of the four test cases individually instead of using the same value of 31.5° as the input for all four cases. By allowing this parameter to vary just a couple degrees and subsequently re-tuning the fluid resistance for each case, a much better agreement is obtained between the measured
Figure 6.18: Model correlation for $F^2$MC tube pressure frequency response, 2-spring test, 31.5° initial fiber angle, a) 40 psi operating pressure, b) 60 psi operating pressure.
Figure 6.19: Model correlation for F$^2$MC tube pressure frequency response, 4-spring test, 31.5° initial fiber angle, a) 40 psi operating pressure, b) 60 psi operating pressure.
and predicted closed-orifice frequencies. All other F$^2$MC tube and circuit parameter values remain the same as given in Tables 6.1 and 6.2. The newly tuned fiber angles and resistance values are given in Table 6.4. The drawback of this model tuning approach is that because model parameters are tuned to match experimental results after they are obtained, this approach cannot make performance predictions in advance of experiments.

Figures 6.20 and 6.21 show the lag displacement model correlations as alternatives to Figures 6.16 and 6.17 using this tuning approach. Similarly, Figures 6.22 and 6.23 show the F$^2$MC tube pressure model correlations as alternatives to Figures 6.18 and 6.19 using this tuning approach. This exercise is helpful because it reinforces the sensitivity of model predictions to small changes in the F$^2$MC tube fiber angle.
Figure 6.20: Model correlation for lag displacement frequency response, 2-spring test, 40 psi operating pressure, a) 34.0° initial fiber angle, b) 60 psi operating pressure, 32.5° initial fiber angle.
Figure 6.21: Model correlation for lag displacement frequency response, 4-spring test, a) 40 psi operating pressure, 32.5° initial fiber angle, b) 60 psi operating pressure, 31.5° initial fiber angle.
Figure 6.22: Model correlation for F²MC tube pressure frequency response, 2-spring test, 40 psi operating pressure, a) 34.0° initial fiber angle, b) 60 psi operating pressure, 32.5° initial fiber angle.
Figure 6.23: Model correlation for F\textsuperscript{2}MC tube pressure frequency response, 4-spring test, a) 40 psi operating pressure, 32.5° initial fiber angle, b) 60 psi operating pressure, 31.5° initial fiber angle.
Table 6.4: Re-tuned parameters and model-predicted damping ratios for alternative tuning approach.

<table>
<thead>
<tr>
<th>Test Case</th>
<th>New Fiber Angle, °</th>
<th>New Resistance, kg/s-m$^4$</th>
<th>Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2-Spring Cases</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40 psi, fully open</td>
<td>34.0</td>
<td>$3.93 \times 10^9$</td>
<td>0.189</td>
</tr>
<tr>
<td>40 psi, tuned orifice</td>
<td>34.0</td>
<td>$7.43 \times 10^9$</td>
<td>0.258</td>
</tr>
<tr>
<td>40 psi, fully closed</td>
<td>34.0</td>
<td>–</td>
<td>0.0716</td>
</tr>
<tr>
<td>60 psi, fully open</td>
<td>32.5</td>
<td>$3.43 \times 10^9$</td>
<td>0.173</td>
</tr>
<tr>
<td>60 psi, tuned orifice</td>
<td>32.5</td>
<td>$13.43 \times 10^9$</td>
<td>0.304</td>
</tr>
<tr>
<td>60 psi, fully closed</td>
<td>32.5</td>
<td>–</td>
<td>0.0645</td>
</tr>
<tr>
<td><strong>4-Spring Cases</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40 psi, fully open</td>
<td>32.5</td>
<td>$3.43 \times 10^9$</td>
<td>0.194</td>
</tr>
<tr>
<td>40 psi, tuned orifice</td>
<td>32.5</td>
<td>$7.43 \times 10^9$</td>
<td>0.291</td>
</tr>
<tr>
<td>40 psi, fully closed</td>
<td>32.5</td>
<td>–</td>
<td>0.0594</td>
</tr>
<tr>
<td>60 psi, fully open</td>
<td>31.5</td>
<td>$3.43 \times 10^9$</td>
<td>0.190</td>
</tr>
<tr>
<td>60 psi, tuned orifice</td>
<td>31.5</td>
<td>$12.43 \times 10^9$</td>
<td>0.308</td>
</tr>
<tr>
<td>60 psi, fully closed</td>
<td>31.5</td>
<td>–</td>
<td>0.0535</td>
</tr>
</tbody>
</table>

### 6.3.2 Sensitivity of Results to Operating Pressure

The sensitivity of F$^2$MC damper performance to the initial operating pressure is examined by comparing the measured blade frequency responses with the circuit initial pressure set at 40 psi, 60 psi, and 80 psi. The fully-open orifice cases at each pressure are compared in Figure 6.24, partially-closed orifice cases are compared in Figure 6.25, and fully-closed orifice cases are compared in Figure 6.26. In general, increasing the operating pressure tends to slightly increase the stiffness and frequency of the blade-damper system. The 60 and 80 psi cases in Figure 6.25 also have slightly higher damping than the 40 psi case. Both of these effects can be attributed to the improved bladder/fiber engagement at higher pressures, resulting in a less compliant F$^2$MC tube (i.e., lower $c_4$) that more efficiently converts F$^2$MC tube elongation into fluid pumping.
Figure 6.24: Fully-open orifice frequency response results at different circuit pressures for blade with a) 2 springs attached, b) 4 springs attached.

Figure 6.25: Partially-closed orifice frequency response results at different circuit pressures for blade with a) 2 springs attached, b) 4 springs attached.
6.3.3 Sensitivity of Results to Forcing Amplitude

To assess the linearity of the blade-damper system, frequency response results are also obtained at different forcing levels by adjusting the amplitude of the voltage sweep sent to the shaker by the LabVIEW program. The orifice fully-open, partially-closed, and fully-closed orifice cases for three different forcing amplitudes are compared in Figures 6.27, 6.28, and 6.29. When forced at higher amplitudes, the system static stiffness and natural frequency decrease slightly, but in general, the results are not very amplitude-dependent for the displacement ranges tested here.

6.3.4 Effect of Accumulator Compliance

In Chapter 5, the importance of having a compliant accumulator in the F²MC damper fluidic circuit was examined analytically. Model predictions indicated that using a stiffer accumulator would increase the blade lag frequency and reduce the blade damping ratio. To verify this phenomenon, another set of experiments is conducted using a different procedure to fill and bleed the F²MC damper. The
plug at the end of the accumulator is replaced with a valve so that air can be bled out of the entire circuit, including the accumulator. Instead of fluid being cycled through the rest of the circuit before trapping air inside the accumulator, it is now continuously pumped through the accumulator to evacuate as much air as possible before testing the damper. The only remaining source of accumulator compliance is radial expansion of the fiber-reinforced PVC tubing. By conducting a benchtop test similar to the one used to estimate $c_4$, the accumulator without air is estimated to be between 30 and 40 times statically stiffer at 60 psi than the accumulator with entrapped air.

The benefits of using an accumulator with entrapped air instead of using the accumulator full of fluid are illustrated in Figure 6.30. The orifice fully-open and fully-closed frequency responses for these two accumulator options in the two-spring test configuration are compared in Figure 6.30a. Similarly, the orifice fully-open and fully-closed frequency responses in the four-spring test configuration are compared in Figure 6.30b. The closed-orifice frequencies are nearly identical because there is no air between the F²MC tube and orifice in either case. The fact that the

![Graph](image)

Figure 6.27: Fully-open orifice frequency response results at different forcing amplitudes for blade with a) 2 springs attached, b) 4 springs attached. Tests conducted at 60 psi operating pressure.
Figure 6.28: Partially-closed orifice frequency response results at different forcing amplitudes for blade with a) 2 springs attached, b) 4 springs attached. Tests conducted at 60 psi operating pressure.

Figure 6.29: Fully-closed orifice frequency response results at different forcing amplitudes for blade with a) 2 springs attached, b) 4 springs attached. Tests conducted at 60 psi operating pressure.
fully-open and fully-closed frequencies are so closely spaced when the accumulator
does not have entrapped air indicates that the accumulator without any air is
already very stiff. The significant difference in how much damping can be obtained
by adjusting the orifice with each of these accumulators is illustrated in Figures
6.31 and 6.32. The F^2MC damper is much more effective with the entrapped air
(Figures 6.31a and 6.32a) than it is when the accumulator does not have any air
and is completely filled with fluid (Figures 6.31b and 6.32b). Again, the physical
cause of this phenomenon is that it is easier for the F^2MC tube to pump fluid into
a compliant accumulator than a stiff one.

The benefits of using the compliant accumulator can also be seen in the time
domain. Time response results are obtained by exciting blade lag vibration using a
modal hammer and measuring blade displacement with the laser vibrometer. The
resulting time responses are plotted in Figures 6.33 through 6.35. Figure 6.33 is
generated with a 40 psi operating pressure, Figure 6.34 is generated with a 60 psi
operating pressure, and Figure 6.35 is generated with an 80 psi operating pressure.
Rather than scaling experimental data to match the response amplitudes, the blade
was repeatedly excited until the measured amplitude of the first peak was nearly
identical in all three curves. Note that the shaker is detached before obtaining
these time responses.

The green curves representing the closed orifice cases are the most lightly damped.
The tuned orifice cases are generated by setting the orifice in the position that
maximizes damping, as determined by shaker testing. The logarithmic decrement
method is used to quantify the differences in damping by examining how much the
vibration amplitude decreases over its first period. The results of these calculations
are summarized in Table 6.5. Tuning the orifice to maximize damping increases
the damping ratio from a range of 0.062-0.090 with the orifice closed to a range
of 0.079-0.114 with only fluid in the accumulator. However, in the case where
there is compressible air inside the accumulator, properly tuning the orifice allows
the F^2MC damper to produce damping ratios between 0.298 and 0.404 in the six
test cases. These results illustrate and quantify the tangible improvement from a
well-designed F^2MC damper with a compliant accumulator and a tuned orifice.
Figure 6.30: Difference between blade frequency responses with and without air in accumulator, a) 2 springs attached, b) 4 springs attached. Tests conducted at 60 psi operating pressure.
Figure 6.31: Effect of tuning orifice with 2 springs attached to blade, a) with and b) without air in accumulator. Tests conducted at 60 psi operating pressure.

Figure 6.32: Effect of tuning orifice with 4 springs attached to blade, a) with and b) without air in accumulator. Tests conducted at 60 psi operating pressure.
Figure 6.33: Blade time responses for damper configurations with and without air in accumulator, 40 psi operating pressure, a) 2 springs attached, b) 4 springs attached.
Figure 6.34: Blade time responses for damper configurations with and without air in accumulator, 60 psi operating pressure, 
a) 2 springs attached, b) 4 springs attached.
Figure 6.35: Blade time responses for damper configurations with and without air in accumulator, 80 psi operating pressure, a) 2 springs attached, b) 4 springs attached.
Table 6.5: Damping ratio as calculated by logarithmic decrement from time response data in Figures 6.33 through 6.35.

<table>
<thead>
<tr>
<th>Test Case</th>
<th>Closed Orifice</th>
<th>Tuned Orifice, No Air</th>
<th>Tuned Orifice, With Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 psi, 2 springs</td>
<td>0.0802</td>
<td>0.0992</td>
<td>0.340</td>
</tr>
<tr>
<td>40 psi, 4 springs</td>
<td>0.0621</td>
<td>0.0793</td>
<td>0.298</td>
</tr>
<tr>
<td>60 psi, 2 springs</td>
<td>0.0895</td>
<td>0.1139</td>
<td>0.404</td>
</tr>
<tr>
<td>60 psi, 4 springs</td>
<td>0.0818</td>
<td>0.1031</td>
<td>0.351</td>
</tr>
<tr>
<td>80 psi, 2 springs</td>
<td>0.0679</td>
<td>0.0824</td>
<td>0.344</td>
</tr>
<tr>
<td>80 psi, 4 springs</td>
<td>0.0684</td>
<td>0.0923</td>
<td>0.370</td>
</tr>
</tbody>
</table>

6.4 Experimental Results with Stainless Steel Mesh Tube

The F²MC damper tested in the previous section used a polyethylene fiber mesh when fabricating the F²MC tube. In this section, a second damper is built and tested using a stainless steel fiber mesh when fabricating the F²MC tube. The fluidic circuit is identical to the one used in the first damper, and the F²MC tube uses the same bladder and wall thickness as the polyethylene mesh tube, although the initial fiber angle of the stainless steel mesh is about 22°. The lower fiber angle makes the stainless steel mesh F²MC tube more effective at pumping fluid and generating force than the plastic mesh tube, which starts with a fiber angle of about 31.5°. A side effect of this increased force generation is that for the same spring configuration, the stainless steel mesh F²MC tube contracts more under the same initial pressure than the plastic mesh F²MC tube. Because the fibers are inextensible, the F²MC tube outer radius increases as the active length decreases. However, the rubber bladder is clamped at both ends of the F²MC tube, meaning that its radius cannot change at the end fittings. This creates a situation where the F²MC tube becomes less cylindrical as its ends transition from the clamped bladder diameter at the end to a larger diameter at the center of the F²MC tube. The difference between the pressurized shapes of the plastic mesh and stainless steel mesh F²MC tubes is illustrated in Figure 6.36.

Many F²MC tube models assume cylindrical tubes of infinite length and do not
account for these end effects. Accurate modeling of an F²MC damper with a short F²MC tube may require a more advanced tube model, such as [69], to predict the tube coefficients $c_1$, $c_2$, and $c_3$. However, for the sake of comparing performance of the stainless steel mesh F²MC damper to performance of the plastic mesh F²MC damper, frequency response results are obtained for the blade with the stainless steel mesh F²MC damper by using the experimental procedure outlined in the previous sections. Figure 6.37a and 6.37b plot the frequency response for the blade with the stainless steel mesh F²MC damper for the cases where 2 springs and 4 springs are attached. Both of these figures are generated from data obtained at a 60 psi operating pressure. Additional benchtop test results for the blade with the stainless steel mesh damper can be found in Appendix F.

Several interesting observations can be made by looking at the frequency responses plotted in Figure 6.37. As with the plastic mesh F²MC damper, varying the orifice position causes the blade to gradually shift from a lower lag frequency with moderate damping when the orifice is completely open to a higher lag frequency when the orifice is completely closed. The closed-orifice frequencies are much higher for the F²MC damper with the stainless steel mesh than they are with the plastic mesh. For example, the closed-orifice frequency with 2 springs at 60 psi for the plastic mesh F²MC tube is about 6.0 Hz, but the closed-orifice frequency with 2 springs at 60 psi for the stainless steel mesh F²MC tube is 10.7 Hz. The higher closed-orifice frequencies with the stainless steel mesh tube are due to the aforementioned lower fiber angle. With orifice tuning, the damper is able to achieve a frequency response that is virtually flat over a certain frequency range, such as with the curves labeled “intermediate” in Figure 6.37a and “mostly open” in Figure 6.37b. However, the cases where the orifice is fully open display an even more unique behavior. There appears to be a notch around 15 Hz in both the 2-spring in 4-spring frequency responses, despite the fact that the blade closed-orifice natural frequency changes by 2.2 Hz between these two tests. This suggests that the F²MC damper in the fully-open orifice configuration could actually be functioning as a damped vibration absorber with an absorber frequency of about 15 Hz. Although these tests were not intentionally designed to demonstrate this behavior, this result opens the door for future work that explores the concept of an F²MC damped absorber for articulated blade applications and compares the benefits and drawbacks of using an absorber as an alternative to the damper configuration explored here.
Figure 6.36: Comparison between pressurized shapes of a) plastic mesh and b) stainless steel mesh F$^2$MC tubes.
Figure 6.37: Blade frequency responses for different orifice positions with stainless steel mesh F²MC damper, a) 2 springs and b) 4 springs attached, 60 psi operating pressure.
Chapter 7  |  Conclusions and Future Work

Through a combination of analytical modeling and experiments, this research investigates the viability of vibration treatments utilizing F$^2$MC tubes for several different rotorcraft vibration control applications. The first application focuses on reducing tailboom vibration with a damped F$^2$MC vibration absorber. A laboratory-scale tailboom is used as a testbed for the design and testing of F$^2$MC vibration absorbers that reduce vibrations in lateral, vertical, and torsional modes. This research is the first use of the finite element method to model a structure integrated with F$^2$MC tubes, and it is also the first time vibration reduction has been demonstrated experimentally in a coupled vibration mode using an F$^2$MC absorber. Furthermore, a new absorber configuration that can treat both a vertical and lateral bending tailboom mode with one fluidic circuit is demonstrated experimentally. This advances the state-of-the art for F$^2$MC vibration absorbers, which had previously only attempted to control one mode at a time.

The research into the second application of rotor blade damping is the first attempt to model rotor blades integrated with F$^2$MC tubes for damping of either rigid-body lag motion in the case of articulated rotors or in-plane bending motion in the case of hingeless rotors. Initial sizing and parameter studies are performed to assess the effectiveness of these F$^2$MC dampers on full-scale rotor blades. Following this analysis, an articulated blade F$^2$MC damper is retrofitted onto a small-scale rotor blade, and its damping performance is successfully demonstrated in benchtop tests. This research is the first integration of an F$^2$MC tube-based damper into realistic rotor hardware.
7.1 Single-Mode & Multi-Mode Tailboom Vibration Absorber

A comprehensive model is developed for a structure integrated with a set of F²MC tubes and a tuned fluidic circuit. The approach couples the dynamics of a three-dimensional finite element structural model and a dynamic model of the F²MC tubes and fluidic circuit. As tailboom bending strains the F²MC tubes, a fluid mass oscillates within the circuit. The effective inertia of this fluid mass is tuned to create a fluidic vibration absorber that reduces vibrations in a target tailboom mode. As the fluid oscillates, energy is dissipated due to the fluid viscosity.

A structural model is developed and experimentally verified for a 6 foot long laboratory-scale tailboom. The structural model uses 22 Euler-Bernoulli beam finite elements and adequately predicts the tailboom low-frequency dynamics. The F²MC tube model characterizes the tube force and volume as a function of its axial displacement and internal pressure, while the fluidic model captures the inertia track dynamics through the lumped parameters of inertance and resistance. The structural and fluidic domains are coupled by relating nodal displacements in the finite element model to the resulting F²MC tube displacements. Equations of motion are derived for the full system including tailboom, F²MC tube, and fluid dynamics.

Using the comprehensive system model, two possible configurations for reducing vibrations in a 26.7 Hz lateral bending/torsion tailboom mode are considered. According to model predictions, a damped vibration absorber designed to directly reduce lateral bending vibrations in this coupled mode will also reduce torsional vibrations. Conversely, the model predicts that an absorber designed to directly reduce torsional vibrations in this mode can also reduce lateral bending vibrations. Model results suggest that the absorber designed to directly reduce bending vibrations is more effective at treating the target 26.7 Hz mode. The bending absorber is also more practical from an engineering standpoint because it requires less inertia track tubing. The F²MC absorber designed to treat lateral bending vibration uses four F²MC tubes arranged in two pairs symmetrically about the lateral bending plane. As the tailboom bends laterally, F²MC tubes on one side extend axially to pump fluid, and tubes on the opposite side shorten axially to receive fluid. The
lengths of inertia track segments connecting the left and right side F²MC tube pairs are tuned to set the absorber frequency based on the given F²MC tubes and choice of working fluid.

Parametric studies are conducted to study the effect of changing inertia track radius and F²MC tube length in the lateral bending absorber. Decreasing inertia track radius has the benefit of reducing the inertia track length needed to tune the F²MC absorber, making the absorber more compact. However, a narrower track also increases losses due to fluid viscosity, resulting in a slightly less effective vibration absorber. Performance for a range of different track radii varies from 67% to 72% vibration reduction at resonance with proper inertia track tuning. Reducing the length of the tailboom spanned by the F²MC tubes increases their effective stiffness as quantified by the capacitance parameter $c_4$. As the F²MC tube is shortened, more inertance is needed to tune the circuit for the same absorber frequency. The model predicts that with proper tuning of the fluidic circuit, shortening the F²MC tubes from 25 in. to 15 in. long only reduces the F²MC absorber effectiveness from 71% reduction at resonance to 66% reduction, although the tuned inertia track main segment length becomes 2.3 times longer if the track radius remains unchanged.

An F²MC absorber is built and tested on the laboratory-scale tailboom to verify the model prediction that bending and torsional vibrations can be reduced with the same absorber. The absorber uses four F²MC tubes fabricated by pulling a stainless steel mesh over a $\frac{3}{8}$ in. inner diameter, $\frac{1}{32}$ in. thick rubber bladder. Three different fluidic circuits are tested: one with rigid copper tubing and water as the working fluid, one with rigid copper tubing and a high-density working fluid (specific gravity $\approx 1.88$), and one with rigid plastic tubing and water as the working fluid. Frequency response results are experimentally obtained by measuring the input force from a shaker and output displacements at the tailboom tip and on the horizontal and vertical tails. Reductions of up to 80% in both bending and torsional vibration are demonstrated in the coupled 26.7 Hz lateral bending/torsion tailboom mode. Good correlation is observed between model-predicted and experimentally-measured frequency responses with the F²MC absorber. The model also predicts that the tailboom vibration reduction is accompanied by a reduction in the internal shear force and moment.
Although the absorber performs similarly with all three fluidic circuits, each of them have unique advantages. The circuit with rigid copper tubing and water achieves slightly better vibration reduction than the circuit with the high-density fluid, although the tuned inertia track in the latter absorber is shorter and more compact because of the higher fluid density. Substituting lightweight plastic tubing for the copper tubing does not significantly degrade the absorber performance; the maximum vibration reduction observed at resonance is 80% with the copper tubing and 77% with the plastic tubing.

After successfully demonstrating a tailboom F$^2$MC vibration absorber that controls a single coupled lateral bending/torsion mode, its fluidic circuit design is slightly modified to treat one lateral bending and one vertical bending mode with the same circuit. This multi-mode concept is evaluated on the tailboom by testing a fluidic circuit designed to reduce vibration in a 12.2 Hz vertical bending mode and a 26.7 Hz lateral bending/torsion mode. The circuit is tuned in two steps. First, the branch length that optimally tunes the fluidic circuit for the 26.7 Hz lateral/bending torsion mode is identified. Next, the length of the segment connecting the top pair of F$^2$MC tubes to the bottom pair of F$^2$MC tubes is adjusted until the circuit is tuned for the 12.2 Hz vertical mode as well. The tuned multi-mode absorber reduces vibration by 63% and 65% in the vertical and lateral modes, respectively, whereas the absorber tuned only for a vertical mode reduces vibration by 68% in the vertical mode but just 42% in the lateral bending/torsion mode. Tuning the circuit to reduce vibrations in both the 26.7 Hz lateral bending/torsion mode and the 12.2 Hz vertical bending mode increases the overall absorber weight by less than 2%, since an inertia track with rigid plastic tubing only makes up a small fraction of the overall absorber weight.

### 7.2 Rotor Blade Dampers

Two different concepts are introduced for damping rotor blade in-plane motion with F$^2$MC tubes. The first concept is intended for use on articulated rotors. The piston in a conventional hydraulic lead-lag damper is replaced with an F$^2$MC tube, and energy is dissipated by pumping fluid through an orifice. The second concept is intended for use on hingeless rotors. The hingeless blade concept functions as a damped vibration absorber, harnessing blade chordwise bending strain to pump
fluid through a tuned inertia track and into an accumulator. Models that couple blade dynamics with F$^2$MC tube and fluid dynamics are derived for both concepts. The feasibility of each concept is assessed by modeling a representative blade with the corresponding F$^2$MC damper. An articulated blade based on the UH-60 rotor is modeled and integrated with an F$^2$MC damper, and parametric studies are performed to understand the effect of varying orifice resistance and accumulator capacitance. As the blade lag angle increases, the F$^2$MC tube shortens, and the fiber angle $\alpha$ increases. This reduces the amount of fluid pumped by the F$^2$MC tube, since it is most effective at pumping fluid when the fiber angle is low. Up to a certain point, increasing orifice resistance results in an increased blade damping ratio for the linearized blade-damper system, although this also results in higher F$^2$MC tube oscillatory pressures. Increasing accumulator capacitance results in increased blade damping and reduced F$^2$MC damper stiffness.

A stiff-inplane hingeless blade based on the X2 rotor is used as a baseline to evaluate the F$^2$MC damped absorber concept for stiff-inplane hingeless rotor blades. The rotating blade is modeled using 12 Euler-Bernoulli beam finite elements with chordwise bending degrees of freedom. In the dampers analyzed by this research, the F$^2$MC tube is installed either inside the blade spar or outside the blade leading edge. Eigenvalue analysis predicts that first chordwise bending mode (frequency = 1.4/rev) damping ratio can be increased from a baseline of 0.02 to a range of 0.059-0.066 by integrating an F$^2$MC absorber into the blade root, with the F$^2$MC tube spanning 10% of the blade radius. Choosing an accumulator with high capacitance makes the absorber more effective and can reduce the corresponding inertia track length needed to tune the absorber.

Soft-inplane hingeless rotors represent an intermediate configuration between articulated and stiff-inplane rotors. A soft-inplane blade (frequency = 0.66/rev) based on the BO105 rotor is analyzed using the same finite element approach, and both the F$^2$MC damper and the F$^2$MC damped absorber circuit configurations are evaluated. Based on model results, both circuits are feasible options for augmenting the damping of soft-inplane rotors. The effectiveness of either the F$^2$MC damper or the F$^2$MC damped absorber depends on the flexure stiffness and the blade mode shape. Making the flexure softer relative to the rest of the blade changes the blade mode shape to localize more strain at the blade root where the F$^2$MC tube is attached, thereby making the damper more effective.
To experimentally demonstrate the articulated blade F\textsuperscript{2}MC damper concept, a damper consisting of an F\textsuperscript{2}MC tube, an inertia track, a tunable orifice, and an accumulator is integrated into a small-scale blade that is free to pivot about a lag hinge. The prototype damper uses a combination of spherical and linear bearings to constrain the F\textsuperscript{2}MC tube to axial motion as the blade leads and lags. A compliant accumulator is realized by entrapping compressible air inside the accumulator after bleeding air from the rest of the fluidic circuit.

The damper is tested on the benchtop by using springs to simulate the centrifugal stiffness at a given rotor speed. In general, good correlation is observed between model predictions and experimentally-measured frequency responses. Based on model predictions, the blade damping ratios in these frequency-domain benchtop tests increase from a range of 0.054-0.064 with the orifice fully closed to a range of 0.300-0.335 with the orifice tuned to maximize damping. Benchtop tests also verify the importance of having a compliant accumulator. Experimental results show that a damper tested with the accumulator completely full of fluid is much less effective than one containing compressible air. In time-domain results, measured blade damping ratios are in the range of

- 0.062-0.090 with the orifice fully closed,
- 0.079-0.114 using an accumulator filled with fluid and no air, and
- 0.298-0.404 using an accumulator with entrapped air.

7.3 Recommendations for Future Work

This section briefly describes some suggestions for future work that would build upon this research and further mature F\textsuperscript{2}MC tubes as a technology. These suggestions are broken down into two main categories: research that would be valuable for general vibration control applications with F\textsuperscript{2}MC tubes, and research focused on the rotor blade damping applications introduced in Chapters 5 and 6.

7.3.1 General F\textsuperscript{2}MC Tube Research

The ability to predict the F\textsuperscript{2}MC tube capacitance parameter $c_4$ in advance would greatly improve the accuracy of early inertia track tuning and the prediction of F\textsuperscript{2}MC treatment performance. In an F\textsuperscript{2}MC vibration absorber, the parameter $c_4$
determines the effective absorber stiffness. It is important to quantify this parameter so that the model accurately predicts the inertia track length and/or radius that will yield a desired absorber frequency. However, this research frequently approximated the $c_4$ parameter based on previous empirical measurements, and the validity of these approximations have not yet been confirmed. The linearized model [68] in this research only calculates $F^2$MC tube coefficients $c_1$-$c_3$. Both analytical and experimental research could be done to improve the fidelity of $c_4$ approximations for new $F^2$MC tubes, rather than relying on benchtop experiments to determine $c_4$ after an $F^2$MC tube has already been sized and fabricated. Finite element analysis could be used to model a bladder reinforced by a fiber mesh and compute the $F^2$MC tube volume change due to an applied internal pressure. Alternatively, benchtop experiments could be conducted to characterize the capacitance of $F^2$MC tubes having different lengths, diameters, and bladder thicknesses to build correlations between $F^2$MC tube parameters and empirically measured $c_4$ values.

Another topic for further $F^2$MC tube research could be more detailed design to develop “production-ready” end fittings and assess the static and fatigue strength of $F^2$MC tubes built using realistic hardware. Off-the-shelf components such as barbed fittings and hose clamps were used for ease of fabrication in this research. However, designing custom hardware would likely result in a more reliable $F^2$MC tube and may also reduce the weight and/or size of end fittings. For example, Woods et al. [90] used finite element analysis to support the design of end fittings for pneumatic artificial muscles (PAMs), and this specialized end fitting was demonstrated to last over 120,000,000 pressurization cycles. Static testing was also conducted, with both ends of the actuator fixed and internal pressure applied. The observed failure mode in all three specimens was rupture of the polyethylene mesh at an applied pressure of about 270 psi, and the authors note that the polyethylene mesh limited the actuator static strength rather than the bladder. Follow-up research could evaluate whether using different mesh or bladder materials improves the actuator strength.

Woods et al. [90] mentions several failure modes noted by other researchers including the fiber mesh pulling out of the clamps, pinhole leaks in the bladder, and bladder rupture. Placing emphasis on studying and improving $F^2$MC tube durability could identify which failure modes are most likely to occur and reveal valuable information about the practical limitations of $F^2$MC tubes. As Chapters 5 and 6 illustrated, the models presented in this research can be used to estimate the
pressures that an F^2MC tube will develop at a given vibration level. If information about the typical vibratory loading for a structure is known ahead of time, then the F^2MC tube in a given treatment can be sized to reduce vibrations by a desired amount and also to ensure that it will have an adequate fatigue life and/or safety factor for the application.

### 7.3.2 Blade Damper Research

Because this is the first research to consider the idea of an F^2MC rotor blade damper, there are many paths for future work on this subject. Improvements can be made to both the damper models presented in Chapter 5 and the hardware designed in Chapter 6.

#### 7.3.2.1 Articulated Blade and Damper Modeling

The articulated blade model presented in this research is very simple. The model treats the blade as a rigid body and only considers the lead-lag degree of freedom, neglecting both flap and pitch motions and eliminating any effects of coupling between different blade modes. While this research recognizes that the blade lag angle significantly affects the F^2MC damper behavior, it does not attempt to predict what the steady blade lag angle will be in different flight conditions such as hover, forward flight, or maneuvers. Both the aerodynamic forcing as well as damper design parameters such as the F^2MC tube diameter, active length, initial pressurization, and moment arm about the lag hinge will affect the equilibrium lag angle, which can be calculating by balancing moments about the lag hinge. This moment balance should consider the fact that as the rotor spins up, the circuit pressure will gradually increase from its pressure when the rotor is at rest. As the blade straightens out due to the centrifugal restoring moment, the F^2MC tube stretches axially, decreases in volume, and pushes fluid into the accumulator to compress the gas until equilibrium is reached. Because this changing pressure during rotor spin-up will continuously change the moment exerted by the F^2MC tube on the blade, an iterative solution may be necessary to determine the equilibrium lag angle and circuit pressure.

While the linearized model presented in Chapter 5 suggested the initial feasibility of the F^2MC damper, several simplifications were made to maintain linearity of
the fluidic circuit model. As Chapter 5 mentioned, the blade lead-lag motion causes the F$^2$MC tube to undergo large axial displacements and exhibit nonlinear behavior as the fiber wind angle changes. A fully nonlinear F$^2$MC tube model that is able to continuously predict the F$^2$MC tube state, including the tube volume and force exerted on the blade for a given displacement and pressure, would allow for more refined simulation of the blade-damper system. In addition, modeling end effects due to the nonuniform F$^2$MC tube diameter illustrated in Figure 6.36 may improve predictions and reveal information about how the F$^2$MC behavior differs from the linear model predictions as the end effects become more significant. Theoretically, the end effects would become more significant for shorter F$^2$MC tubes and at higher contraction ratios. Some nonlinear McKibben actuator models have included geometric end effects [64,69,91]; however, these models typically focus on the relationship between input pressure and the resulting tube length with an applied axial load. For a nonlinear F$^2$MC tube model, the tube internal volume is also of interest, since that dictates how much fluid is pumped by the F$^2$MC tube as it changes length. End effects are much less of a concern for the hingeless rotor blade and tailboom applications, since the F$^2$MC tube length does not change as significantly as it does in the articulated blade damper.

Modeling the damper orifice as a linear fluid resistance is another approximation from this research which keeps the system dynamics linear, although it is not physically precise. Eq. (5.4) could have been modified to incorporate the nonlinear relationship [92]

$$Q = A_{orf} C_d \frac{2(p_t - p_a) (p_t - p_a)}{\rho |p_t - p_a|}, \quad (7.1)$$

where $A_{orf}$ is the orifice cross-sectional area and $C_d$ is the orifice discharge coefficient. This equation captures the so-called “V-squared” resistance of a hydraulic orifice and could more accurately predict the complex pressures and forces generated over an F$^2$MC damper cycle. The orifice cross-sectional area is also more useful as a design variable than orifice resistance $R_c$, since the area has physical significance whereas orifice resistance was effectively a tuning parameter in this research.

Finally, although the articulated blade damper concept with one F$^2$MC tube, an orifice, and an accumulator was successfully demonstrated in benchtop tests, this is not guaranteed to be the most effective way to implement an F$^2$MC damper on an
articulated rotor blade. It may be worthwhile to conceptualize, analyze, and test other F^2MC damper configurations in future research. For example, one alternate damper configuration could attach a second F^2MC tube on the leading-edge side of the blade so that fluid is pumped between an antagonistic pair of F^2MC tubes instead of one F^2MC tube and an accumulator.

7.3.2.2 Articulated Blade Damper Hardware & Circuit

One of the main reasons for building the benchtop F^2MC damper prototype in Chapter 6 was to start understanding the practical challenges of implementing this concept. Early testing revealed that without any mechanism for aligning the two end fittings, the F^2MC tube would bend out-of-plane due to the weight of its end fittings and the attached valves or pressure transducers. The spherical bearings allow the end fittings to rotate, and the F^2MC tube has very low bending stiffness, especially if the tube is not under significant tension. The problem was addressed adequately in this research by adding a linear bearing and a guide rod as illustrated in Figure 6.12. However, because the linear bearing can accommodate some guide rod misalignment, it is possible for the F^2MC tube to twist slightly between the two end fittings. To prevent this, the hardware could have been redesigned so that two guide rods are used for each F^2MC tube as shown in Figure 7.1. With this new design, the entire F^2MC tube is still free to spin around its own axis, but the two ends would not be able to twist relative to each other. The alignment mechanism used in this damper prototype was conveniently designed and implemented using off-the-shelf hardware, but there may be better ways to constrain the F^2MC tube in the damper to purely axial motion.

The experiments in Chapter 6 confirmed the importance of using a soft accumulator to achieve an F^2MC damper with low stiffness and high damping. In the benchtop experiment, this is accomplished by trapping air behind the orifice in the circuit. For the best possible damper performance, all of the compressibility in the circuit should remain behind the orifice. If any air were to remain stuck in the F^2MC tube or the inertia track, this would lower the effective bulk modulus of the hydraulic oil and reduce the amount of fluid pumped through the orifice. In a rotating environment, the air may have a different equilibrium position within the fluidic circuit. Movement of this entrapped air could cause the damper to perform differently when rotating than it does in benchtop tests, especially if air travels past
the orifice and into the circuit. On a production damper or in future rotating tests of the articulated blade damper, a more robust accumulator option that physically separates the fluid from the gas, such as a piston or a diaphragm accumulator, could be utilized.

7.3.2.3 Hingeless Blade Damper Modeling

As with the articulated blade model, the hingeless blade model in this research is simple and considers only the in-plane degree of freedom. This approach allowed for a preliminary assessment of the treatment effectiveness, but leaves much more room for detailed modeling of both the blade and the F^2MC damped absorber. The static deflection of the stiff hingeless blade under high-speed aerodynamic loading could be calculated to determine whether blade elastic deformation has any impact on the F^2MC tube length and fiber angle. Future work could incorporate additional degrees of freedom, such as flapwise bending and torsion, into the blade structural model. Modeling other elastic degrees of freedom introduces the possibilities of coupled vibration modes and aeroelastic instabilities with certain sets of blade parameters. Although the F^2MC damped absorber would be attached near the blade root, Coriolis forces may impact fluid flow within the circuit. Future research should assess whether it is necessary to model the effects of rotation on the F^2MC
absorber, and it should also determine under which conditions or with which choices of circuit design parameters these effects become significant. The use of F²MC damped absorbers to stabilize an unstable rotor or to further improve the stability margin of a stable rotor could be another area of future research.

If a passive device cannot provide sufficient blade damping, or if the chordwise bending frequency is sensitive to rotor speed and a more robust device is needed to provide damping across a wider frequency range, then an active F²MC damper using hydraulic power may also be considered as a potential damping solution.

7.3.2.4 Testing of Articulated & Hingeless Blade Dampers in Rotating Environment

The eventual goal of this F²MC damper research is to successfully demonstrate increased damping for both articulated and hingeless rotor blades in a rotating frame. For a number of reasons, this is much more complicated than demonstrating damper performance on a benchtop test. Some factors to consider when planning and conducting these tests include:

- **Instrumentation:** A displacement sensor for each rotor blade must be integrated into the hub. For the articulated blade application, these sensors must be capable of sensing low frequency (0-10 Hz) vibration, and they must also have suitable sensitivity and measurement range to observe blade lag displacement in the rotating frame. Having at least one pressure transducer in the fluidic circuit would be useful for real-time pressure monitoring. As mentioned earlier, spinning up the rotor from rest will cause the pressure in the fluidic circuit to increase as the blade straightens out. This testing would be complemented well by an analytical prediction of final rotating F²MC tube pressure as part of the moment balance discussed in Section 7.3.2.1. This prediction could be verified by the experiment and could also help determine the initial pressurization that results in the desired circuit pressure when the rotor is at full speed. High-speed cameras can be used to visualize the F²MC damper as it is rotating and aid in troubleshooting any issues that arise.

- **Signal Conditioning:** Conducting tests in a rotating frame requires signals to be passed back into the fixed frame through a slip ring, which may result in increased noise that disrupts sensor measurements. Signal processing
methods such as digital filtering may be necessary to separate unwanted signal components, such as 1/rev and other steady-state harmonic vibration, from the transient blade motion that more clearly illustrates damping. When possible, low-noise sensors that are not affected by the expected centrifugal accelerations should be selected for these tests.

- **Excitation:** To prove that the F²MC damper is effective in a rotating environment, the blade lag mode must be perturbed sufficiently to measure the difference between a slowly-decaying baseline vibration case and a case where the F²MC damper causes the lag mode to decay faster. The lightly-damped case could either be the baseline rotor blade or the blade with the F²MC damper installed and the orifice fully closed. In helicopter flight testing, rotor aeromechanical stability can be assessed by applying cyclic blade pitch to excite the lag mode and then measuring the resulting decay. However, the hub fabricated for this research was not designed to accommodate blade pitch, and implementing this excitation method in a rotating test would be complex. Exciting the lag mode in a reliable and repeatable manner may require installation of one or more actuators on the rotor blade, with appropriate actuator control signals passed through the slip ring.

- **Rotor Speed Regulation:** The benchtop tests in Chapter 6 simulate the centrifugal stiffness due to a constant rotational speed. If the rotor speed is not constant in the actual test, energy from the blade lag motion may be transferred from the blade into the driveshaft, making it harder to observe a low-damping baseline case. Ideally, the rotor speed can be precisely maintained by an electronic control system with strong control gains. Adding more mass to the rotor hub would also increase the rotor inertia and reduce any rotor speed fluctuations. Appropriate instrumentation should be added to the rotor system to measure its rotation speed in real time and ensure that it remains constant as blade lag motion is perturbed.

The Penn State Adverse Environment Rotor Test Stand has been identified as one candidate facility for this research, although other options may be explored if necessary.
Appendix A  
Element Properties in Laboratory-Scale Tailboom Model

The table below contains cross-section information for all fourteen tailboom elements as calculated based on properties in [76]. This cross-section information is used in generating element mass and stiffness matrices for the fourteen tailboom elements. Elements are numbered such that element 1 is the one closest to the root and element 14 is the one closest to the tip. In this research, the tailboom finite element mesh changes slightly as the tube length varies, since the model requires both F²MC tube attachment points to be nodes. However, these changes in the mesh have minimal effect on the predicted tailboom natural frequencies and mode shapes. Point masses and root spring values also change slightly for different models as discussed in the text. Tuning these values helps match certain tailboom natural frequencies more closely to experimental results depending on which ones are most important for the research at hand.

The cross-sectional area moment values in this table are before stiffness modifications made for model tuning purposes. Specifically, the second area moments $I_{yy}$ and $I_{zz}$ in the model are reduced by 25% from the values listed in Table A.1 when calculating the element stiffness matrix.
Table A.1: Finite element properties for laboratory-scale tailboom model.

<table>
<thead>
<tr>
<th>Element Number</th>
<th>Length $L_e$, m</th>
<th>Cross-sectional area $A$, m$^2$</th>
<th>Area Moment $I_{yy}$, m$^4$</th>
<th>Area Moment $I_{zz}$, m$^4$</th>
<th>Torsion Constant $J$, m$^4$</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>0.00159</td>
<td>0.00173</td>
<td>$2.41 \times 10^{-5}$</td>
<td>$3.70 \times 10^{-5}$</td>
<td>$2.35 \times 10^{-5}$</td>
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<td>$2.33 \times 10^{-5}$</td>
<td>$3.54 \times 10^{-5}$</td>
<td>$2.25 \times 10^{-5}$</td>
</tr>
<tr>
<td>3</td>
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<td>0.00169</td>
<td>$2.17 \times 10^{-5}$</td>
<td>$3.24 \times 10^{-5}$</td>
<td>$2.05 \times 10^{-5}$</td>
</tr>
<tr>
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<td>$2.02 \times 10^{-5}$</td>
<td>$2.96 \times 10^{-5}$</td>
<td>$1.86 \times 10^{-5}$</td>
</tr>
<tr>
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<td>$2.69 \times 10^{-5}$</td>
<td>$1.69 \times 10^{-5}$</td>
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<td>$2.44 \times 10^{-5}$</td>
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<td>$2.19 \times 10^{-5}$</td>
<td>$1.36 \times 10^{-5}$</td>
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<tr>
<td>8</td>
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<td>$1.46 \times 10^{-5}$</td>
<td>$1.94 \times 10^{-5}$</td>
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<tr>
<td>9</td>
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<td>0.00150</td>
<td>$1.33 \times 10^{-5}$</td>
<td>$1.71 \times 10^{-5}$</td>
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<tr>
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<td>0.00147</td>
<td>$1.21 \times 10^{-5}$</td>
<td>$1.49 \times 10^{-5}$</td>
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</tr>
<tr>
<td>11</td>
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<td>$1.29 \times 10^{-5}$</td>
<td>$0.79 \times 10^{-5}$</td>
</tr>
<tr>
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<td>0.00140</td>
<td>$0.98 \times 10^{-5}$</td>
<td>$1.11 \times 10^{-5}$</td>
<td>$0.68 \times 10^{-5}$</td>
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<td>$3.20 \times 10^{-5}$</td>
<td>$1.53 \times 10^{-5}$</td>
<td>$0.51 \times 10^{-5}$</td>
</tr>
</tbody>
</table>
Appendix B
Transfer Functions for F\textsuperscript{2}MC Tubes & Fluidic Circuit Model

This appendix contains the full form of entries in the transfer function matrix \([H(s)]\) from Eq. (2.67). These are used in conjunction with the MATLAB state-space tools to generate an equivalent state-space system for the F\textsuperscript{2}MC tubes and fluid dynamics.

Uncoupled Pairs of Tubes/Torsion Treatment

This derivation is for one of the two uncoupled pairs. The same process can be applied to the other pair since the two pairs are not connected fluidically. Eqs. (2.43)-(2.48) are the basis for the transfer functions and are reproduced here for reference.

\[
\begin{align*}
c_1x_1 + c_2p_1 &= F_1 \\
c_1x_2 + c_2p_2 &= F_2 \\
-c_3\dot{x}_1 - c_4\dot{p}_1 &= Q_1 \\
-c_3\dot{x}_2 - c_4\dot{p}_2 &= Q_2 \\
Q_1 &= -Q_2 \\
p_1 - p_2 &= I_c\dot{Q}_1 + R_cQ_1
\end{align*}
\]
After manipulation of Eqs. (B.1)-(B.6), the resulting transfer functions from input F^2MC tube displacements to output tube forces are:

\[
\frac{F_1(s)}{x_1(s)} = \frac{F_2(s)}{x_2(s)} = \frac{(c_1c_4^2 - c_2c_3c_4)I_c s^2 + ((c_1c_4^2 - c_2c_3c_4)R_c)s + (2c_1c_4 - c_2c_3)}{(c_4^2I_c)s^2 + (c_4^2R_c)s + 2c_4}
\]

(B.7)

\[
\frac{F_1(s)}{x_2(s)} = \frac{F_2(s)}{x_1(s)} = \frac{-c_2c_3}{(c_4^2I_c)s^2 + (c_4^2R_c)s + 2c_4}
\]

(B.8)

**Coupled Pairs of Tubes/Bending Treatment**

For the case where both pairs of tubes are coupled together through a common inertia track, a total of sixteen equations are needed to characterize the complete system of four F^2MC tubes and the fluidic circuit. Eqs. (2.51)-(2.66) define the system, but they are not reproduced here for brevity. Note that Eq. (2.66) actually expresses more than one equation in the text.

There are only three unique transfer functions that populate the entire transfer function matrix from input F^2MC tube displacements to output tube forces. The three unique transfer functions are:

\[
\frac{F_1(s)}{x_1(s)} = \frac{F_2(s)}{x_2(s)} = \frac{b_4s^4 + b_3s^3 + b_2s^2 + b_1s + b_0}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0},
\]

(B.9)

where

\[
b_4 = 4c_1c_4^2I_b^2 + 4c_1I_mc_4^3I_b - 4c_2c_3c_4^2I_b^2 - 4c_2c_3I_mC_4^2I_b
\]

\[
b_3 = 8c_1c_4^3I_bR_b + 4c_1c_4^3I_bR_m + 4c_1c_4^3I_mR_b - 8c_2c_3c_4^2I_bR_b
\]

\[-4c_2c_3c_4^2I_bR_m - 4c_2c_3c_4^2I_mR_b
\]

\[
b_2 = 8c_1c_4^2I_b + 4c_1c_4^2I_m + 4c_1c_4^3R_b^2 - 4c_2c_3c_4^2R_b^2 - 5c_2c_3c_4I_b
\]

\[-2c_2c_3c_4I_m + 4c_1c_4^3R_bR_m - 4c_2c_3c_4^2R_bR_m
\]

\[
b_1 = 8c_1c_4^2R_b + 4c_1c_4^2R_m - 5c_2c_3c_4R_b - 2c_2c_3c_4R_m
\]
\[ b_0 = 4c_1c_4 - c_2c_3 \]
\[ a_4 = 4c_4^2I_b(c_4I_b + c_4I_m) \]
\[ a_3 = 4c_4^2R_b(c_4I_b + c_4I_m) + 4c_4^2I_b(c_4R_b + c_4R_m) \]
\[ a_2 = 4c_4^2I_b + 4c_4(c_4I_b + c_4I_m) + 4c_4^2R_b(c_4R_b + c_4R_m) \]
\[ a_1 = 4c_4^2R_b + 4c_4(c_4R_b + c_4R_m) \]
\[ a_0 = 4c_4 \]

\[
\frac{F_1(s)}{x_2(s)} = \frac{d_4s^4 + d_3s^3 + d_2s^2 + d_1s + d_0}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0}, \quad (B.10)
\]

where

\[
d_4 = 0
\]
\[
d_3 = 0
\]
\[
d_2 = -c_2c_3(c_4I_b + 2c_4I_m)
\]
\[
d_1 = -c_2c_3(c_4R_b + 2c_4R_m)
\]
\[
d_0 = -c_2c_3
\]

\[
\frac{F_1(s)}{x_3(s)} = \frac{f_4s^4 + f_3s^3 + f_2s^2 + f_1s + f_0}{a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0}, \quad (B.11)
\]

where

\[
f_4 = 0
\]
\[
f_3 = 0
\]
\[
f_2 = -c_2c_3c_4I_b
\]
\[
f_1 = -c_2c_3c_4R_b
\]

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\[ f_0 = -c_2 c_3 \]

The remaining terms are all identical to one of these three transfer functions. Note that the transfer function matrix is symmetric and all entries on the diagonal are identical.

\[
\begin{align*}
\frac{F_1(s)}{x_1(s)} &= \frac{F_2(s)}{x_2(s)} = \frac{F_3(s)}{x_3(s)} = \frac{F_4(s)}{x_4(s)} \quad \text{(B.12)} \\
\frac{F_1(s)}{x_2(s)} &= \frac{F_2(s)}{x_1(s)} = \frac{F_3(s)}{x_4(s)} = \frac{F_4(s)}{x_3(s)} \quad \text{(B.13)} \\
\frac{F_1(s)}{x_3(s)} &= \frac{F_1(s)}{x_4(s)} = \frac{F_2(s)}{x_3(s)} = \frac{F_2(s)}{x_4(s)} = \frac{F_3(s)}{x_1(s)} = \frac{F_3(s)}{x_2(s)} = \frac{F_4(s)}{x_1(s)} = \frac{F_4(s)}{x_2(s)} \quad \text{(B.14)}
\end{align*}
\]
Appendix C
Relevance of Inertia Track Symmetry

The fluidic circuit model presented in Chapter 2 assumes that all four branch segments are equivalent; in other words, the T-junction in the fluidic circuit has exactly the same length of tubing from the junction to the F\textsuperscript{2}MC tube on either side. If the junction is moved away from this spot such that the two branches in a pair are not equal lengths, then the effective inertance of the two branches will decrease for the absorber mode where fluid is moving from one pair of tubes, through the main segment, and into the other pair of tubes. Therefore, the two fluidic circuit designs depicted in Figure C.1 will not produce the same frequency response despite using the same total length of track tubing. This phenomenon is mathematically proven here.

Branch segments with inertance $I_1$ and $I_2$ are in parallel. Assuming that the pressures for both F\textsuperscript{2}MC tubes in a pair are equal, their equivalent inertance is given by

$$I_{eq} = \frac{I_1 I_2}{I_1 + I_2}.$$

If the branches are symmetric such that $I_1 = I_2 = I_b$, then the equivalent inertance of the two branches is

$$I_{eq} = \frac{I_b I_b}{I_b + I_b} = \frac{I_b}{2}.$$
Figure C.1: Two non-equivalent inertia track designs using symmetric (left) and asymmetric (right) branch arrangements.

It can be shown that this is the maximum inertance for two branches whose individual lengths add up to a fixed length. Consider two branches whose lengths sum up to $L_{tot}$, with one branch having length $l$ and the other branch having length $L_{tot} - l$. The track length is used here in place of the inertance, but these two quantities are proportional according to Eq. (2.59). The “equivalent length” of the two branches is

$$L_{eq} = \frac{(l)(L_{tot} - l)}{L_{tot}} = l - \frac{l^2}{L_{tot}}. \quad (C.1)$$

Differentiating Eq. (C.1) with respect to $l$ gives

$$\frac{dL_{eq}}{dl} = 1 - \frac{2l}{L_{tot}}. \quad (C.2)$$

Setting Eq. (C.2) equal to zero reveals that $l = L_{tot}/2$ is a critical point. The second derivative of $L_{eq}$ with respect to $l$ is negative, which means that $l = L_{tot}/2$ produces a maximum value of $L_{eq}$. The value of $L_{eq}$ is equal to $L_{tot}/4$ for this particular choice of branch lengths, and this is the maximum equivalent length for two branch segments adding up to a constant length. Consequently, this is the optimal way to design the branches of a fluidic circuit such that the total amount
of tubing in the fluidic circuit is minimized. Any other placement of the T-junction would result in the two branch segments having a lower inertance contribution and therefore requiring a longer main segment to achieve the desired inertance for tuning.
Appendix D | Mechanical Analogy for Rotor Blade with F$^2$MC Damper

The dynamics of a rotor blade integrated with an F$^2$MC treatment can be physically understood by considering an analogous mechanical system consisting of masses, springs, and dampers. The following section explains how to derive the equations of motion for this equivalent mechanical system. If a modal reduction is used to reduce the hingeless blade dynamics to one mode, then the same mechanical analogy applies to both the hingeless or articulated rotor cases.

Consider a system with the single degree of freedom $q$ governing blade dynamics and the properties $m_b$, $c_b$, and $k_b$ defining the blade modal mass, damping, and stiffness:

$$m_b \ddot{q} + c_b \dot{q} + k_b q = BF_{ext} + \sigma F_t.$$  \hfill (D.1)

$B$ and $\sigma$ are scalars which convert the applied external forcing and F$^2$MC tube force into the appropriate loading for the chosen modal coordinates. As described in Chapter 5, the transfer function from F$^2$MC tube extension $x_t$ to F$^2$MC tube force $F_t$ for a circuit with an F$^2$MC tube, inertia track, and accumulator is given by

$$\frac{F_t(s)}{x_t(s)} = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0},$$  \hfill (D.2)

where the coefficients are as follows:
\begin{align*}
b_2 &= (c_1c_4 - c_2c_3)I_c c_a \\
b_1 &= (c_1c_4 - c_2c_3)R_c c_a \\
b_0 &= c_1c_4 - c_2c_3 + c_1c_a \\
a_2 &= c_4c_a I_c \\
a_1 &= c_4c_a R_c \\
a_0 &= c_4 + c_a
\end{align*}

Alternatively, Eq. (D.2) can be written in the time domain as two equations:

\begin{align*}
F_t &= b_2 \ddot{\xi} + b_1 \dot{\xi} + b_0 \xi \quad (D.3) \\
x_t &= a_2 \ddot{\xi} + a_1 \dot{\xi} + a_0 \xi \quad (D.4)
\end{align*}

Substituting the kinematic relation \( x_t = \Psi q \) into Eq. (D.4), the second equation becomes

\[ \Psi q = a_2 \ddot{\xi} + a_1 \dot{\xi} + a_0 \xi. \quad (D.5) \]

Eq. (D.5) can be solved for \( \ddot{\xi} \) and substituted back into Eq. (D.3) to yield a new expression for the dynamics of the F\(^2\)MC tube force \( F_t \) in terms of the degree of freedom \( q \) and the state variable \( \xi \),

\[ F_t = \left( b_1 - \frac{b_2a_1}{a_2} \right) \ddot{\xi} + \left( b_0 - \frac{b_2a_0}{a_2} \right) \xi + \frac{b_2}{a_2} \Psi q. \quad (D.6) \]

This expression can be substituted into Eq. (D.1) to produce the full system equation of motion,

\[ m_b \dddot{q} + c_b \ddot{q} + k_b q - \sigma \left( \left( b_1 - \frac{b_2a_1}{a_2} \right) \ddot{\xi} + \left( b_0 - \frac{b_2a_0}{a_2} \right) \xi + \frac{b_2}{a_2} \Psi q \right) = BF_{ext}. \quad (D.7) \]

Note that the coefficient of the \( \dot{\xi} \) term in the above equation reduces to 0. Eqs. (D.5) and (D.7) can be written in matrix form as
Next, perform a change of variables so that the units of the new state variable \( \lambda \) have the same units as \( q \),

\[ \xi = \frac{\Psi}{a_0} \lambda. \]  

Eq. (D.8) now becomes

\[
\begin{bmatrix}
mb & 0 \\
0 & a_2 \\
\end{bmatrix} \begin{bmatrix}
\ddot{q} \\
\ddot{\lambda} \\
\end{bmatrix} + \begin{bmatrix}
cb & 0 \\
0 & a_1 \\
\end{bmatrix} \begin{bmatrix}
\dot{q} \\
\dot{\lambda} \\
\end{bmatrix} + \\
\begin{bmatrix}
k_b - \frac{b_2 a_2 \Psi}{a_0} & -\sigma \left( \frac{b_0 - \frac{b_2 a_0}{a_2}}{a_0} \right) \\
-\Psi & -\frac{\Psi}{a_0} \\
\end{bmatrix} \begin{bmatrix}
q \\
\xi \\
\end{bmatrix} = \begin{bmatrix}
B \\
0 \\
\end{bmatrix} F_{ext} \quad \text{(D.8)}
\]

From here, multiply the bottom equation by \( \frac{a_0}{b_0 - \frac{b_2 a_0}{a_2}} \) so that the new matrix system of equations is

\[
\begin{bmatrix}
m_b & 0 \\
\frac{\sigma \Psi a_2}{a_0} \left( b_0 - \frac{b_2 a_0}{a_2} \right) & \frac{\sigma \Psi a_1}{a_0} \\
0 & \frac{\sigma \Psi a_1}{a_0} \left( b_0 - \frac{b_2 a_0}{a_2} \right) \\
\end{bmatrix} \begin{bmatrix}
\ddot{q} \\
\ddot{\lambda} \\
\end{bmatrix} + \begin{bmatrix}
c_b & 0 \\
0 & \frac{\sigma \Psi a_1}{a_0} \left( b_0 - \frac{b_2 a_0}{a_2} \right) \\
\end{bmatrix} \begin{bmatrix}
\dot{q} \\
\dot{\lambda} \\
\end{bmatrix} + \\
\begin{bmatrix}
k_b - \sigma \Psi \frac{b_2 a_2}{a_0} & -\sigma \Psi a_0 \left( b_0 - \frac{b_2 a_0}{a_2} \right) \\
\frac{\sigma \Psi}{a_0} \left( b_0 - \frac{b_2 a_0}{a_2} \right) & \frac{\sigma \Psi}{a_0} \left( b_0 - \frac{b_2 a_0}{a_2} \right) \\
\end{bmatrix} \begin{bmatrix}
q \\
\lambda \\
\end{bmatrix} = \begin{bmatrix}
B \\
0 \\
\end{bmatrix} F_{ext}. \quad \text{(D.10)}
\]

Now, consider the two-degree-of-freedom mechanical system depicted in Figure D.1. The matrix equation for this mechanical system is

\[
\begin{bmatrix}
m_m & 0 \\
0 & m_f \\
\end{bmatrix} \begin{bmatrix}
\ddot{z}_1 \\
\ddot{z}_2 \\
\end{bmatrix} + \begin{bmatrix}
c_m & 0 \\
0 & c_f \\
\end{bmatrix} \begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2 \\
\end{bmatrix} + \begin{bmatrix}
k_m + k_f & -k_f \\
-k_f & k_f \\
\end{bmatrix} \begin{bmatrix}
z_1 \\
z_2 \\
\end{bmatrix} = \begin{bmatrix}
F \\
0 \\
\end{bmatrix}. \quad \text{(D.12)}
\]

The system defined by only \( m_m, c_m, \) and \( k_m \) is analogous to the baseline blade system, and the addition of coupling spring \( k_f \), absorber mass \( m_f \), and damper \( c_f \) is analogous to the integration of an F\(^2\)MC damper or damped absorber. By equating terms in the mass, damping, and stiffness matrices of Eqs. (D.11) and
Figure D.1: Analogous two-degree-of-freedom mechanical system representing dynamics of blade integrated with F$^2$MC tube and fluidic circuit.

(D.12), the properties of the analogous mechanical system can be identified in terms of properties from the blade integrated with the F$^2$MC damper. These analogous system properties are as follows:

\[
\begin{align*}
    m_m &= m_b \\
    c_m &= c_b \\
    m_f &= \sigma \Psi \frac{a_2}{a_0^2} \left( b_0 - \frac{b_2 a_0}{a_2} \right) = \sigma \Psi \frac{c_2 c_3 c_a^2 I_c}{(c_4 + c_a)^2} \\
    c_f &= \sigma \Psi \frac{a_1}{a_0^2} \left( b_0 - \frac{b_2 a_0}{a_2} \right) = \sigma \Psi \frac{c_2 c_3 c_a^2 R_c}{(c_4 + c_a)^2} \\
    k_f &= \sigma \Psi \frac{1}{a_0} \left( b_0 - \frac{b_2 a_0}{a_2} \right) = \sigma \Psi \frac{c_2 c_3 c_a}{c_4 (c_4 + c_a)} \\
    k_m &= k_b - k_f - \sigma \Psi \frac{b_2}{a_2} = k_b - \sigma \Psi \frac{c_1 ((c_4 + c_a) - c_2 c_3)}{c_4 + c_a}
\end{align*}
\]

The value of $c_3$ is negative for a contractor tube, and the product $\sigma \Psi$ must be negative. Therefore, the analogous properties $m_f$, $c_f$, and $k_f$ are all positive. From this mechanical analogy, a number of observations can be made. These observations can be explained physically and support phenomena observed when using the full model discussed in Chapter 5:
1. The analogous mass and damping terms \( m_f \) and \( c_f \) are directly proportional to fluid inertance and resistance.

2. The stiffness of the F\(^2\)MC-integrated blade is higher than the baseline blade stiffness, because \( k_m > k_b \). \( \frac{c_1((c_4+c_a)-c_2c_4)}{c_4+c_a} \) is a positive quantity and the product \( \sigma \Psi \) is a negative quantity. Physically, the F\(^2\)MC tube has axial stiffness defined by the parameter \( c_1 \), and additional stiffness comes from the F\(^2\)MC tube pumping incompressible fluid into an accumulator with some stiffness.

3. The coupling spring stiffness \( k_f \) increases with decreasing accumulator capacitance \( c_a \). The coupling spring in the analogy transfers energy from the mass \( m_m \) into the absorber mass \( m_f \). In the F\(^2\)MC system, a stiff accumulator resists the F\(^2\)MC tube fluid pumping and allows less fluid mass to oscillate within the circuit, reducing the F\(^2\)MC damper effectiveness. As the absorber mass \( m_f \) decreases to zero, the analogous mechanical system becomes a spring-mass-damper connected to a “pure damper” governed by a spring and damper in series.

4. The undamped natural frequency of the spring-mass absorber system is

\[
\sqrt{\frac{k_f}{m_f}} = \sqrt{\frac{c_4 + c_a}{c_4c_aI_c}}.
\] (D.13)

This equation can be useful to estimate the inertance that will tune the circuit for a given absorber frequency based on a given F\(^2\)MC tube capacitance \( c_4 \) and accumulator capacitance \( c_a \). For very large values of accumulator capacitance \( c_a \), the absorber frequency asymptotically approaches \( \sqrt{\frac{1}{c_4I_c}} \).

5. The magnitude of the terms \( k_f, m_f, \) and \( c_f \) are all proportional to \( \sigma \) and \( \Psi \). Increasing all of these terms while holding the properties \( m_m, c_m, \) and \( k_m \) constant increases the authority of the damper or damped absorber. For the hingeless blade model, the magnitudes of both \( \sigma \) and \( \Psi \) increase as the F\(^2\)MC tube distance from the neutral axis increases. Physically, the increased separation means that the F\(^2\)MC tube exerts a larger moment and strains more for the same blade deformation. For the articulated blade model, the
magnitudes of both $\sigma$ and $\Psi$ increase as the distance between the lag hinge and the F$^2$MC tube increases.

6. The F$^2$MC tube properties $c_2$ and $c_3$ define how much force the F$^2$MC tube exerts for a given unit pressure and how much fluid is pumped for a given unit displacement. Physically, increasing both of these properties also increases the magnitude of $k_f$, $m_f$, and $c_f$. 
Appendix E
Specifications for Articulated Rotor Hub
Appendix F
Additional Benchtop Damper Test Results

This section contains additional frequency-domain and time-domain results from the F²MC damper testing in Chapter 6. Results from both the plastic mesh and stainless steel mesh F²MC tubes are included.
Plastic Mesh $F^2MC$ Tube Results

(a)

(b)

Blade frequency responses with a) 2 springs attached and b) 4 springs attached for tests conducted with plastic mesh $F^2MC$ tube at 80 psi.
Fully-open orifice frequency response results at different forcing amplitudes for blade with a) 2 springs attached, b) 4 springs attached. Tests conducted at 40 psi operating pressure.

Partially-closed orifice frequency response results at different forcing amplitudes for blade with a) 2 springs attached, b) 4 springs attached. Tests conducted at 40 psi operating pressure.
Fully-closed orifice frequency response results at different forcing amplitudes for blade with a) 2 springs attached, b) 4 springs attached. Tests conducted at 40 psi operating pressure.

Fully-open orifice frequency response results at different forcing amplitudes for blade with a) 2 springs attached, b) 4 springs attached. Tests conducted at 80 psi operating pressure.
Partially-closed orifice frequency response results at different forcing amplitudes for blade with a) 2 springs attached, b) 4 springs attached. Tests conducted at 80 psi operating pressure.

Fully-closed orifice frequency response results at different forcing amplitudes for blade with a) 2 springs attached, b) 4 springs attached. Tests conducted at 80 psi operating pressure.
Effect of tuning orifice with 2 springs attached to blade, a) with and b) without air in accumulator. Tests conducted at 40 psi operating pressure.

Effect of tuning orifice with 4 springs attached to blade, a) with and b) without air in accumulator. Tests conducted at 40 psi operating pressure.
Effect of tuning orifice with 2 springs attached to blade, a) with and b) without air in accumulator. Tests conducted at 80 psi operating pressure.

Effect of tuning orifice with 4 springs attached to blade, a) with and b) without air in accumulator. Tests conducted at 80 psi operating pressure.
Stainless Steel Mesh F$^2$MC Tube Results

Blade frequency responses with a) 2 springs attached and b) 4 springs attached for tests conducted with stainless steel mesh F$^2$MC tube at 40 psi.
Blade frequency responses with a) 2 springs attached and b) 4 springs attached for tests conducted with stainless steel mesh F^2MC tube at 80 psi.
Blade time responses for damper configurations with and without air in accumulator, using stainless steel mesh $F^2MC$ tube with a) 2 springs attached, b) 4 springs attached. Tests conducted at 40 psi operating pressure.
Blade time responses for damper configurations with and without air in accumulator, using stainless steel mesh F²MC tube with a) 2 springs attached, b) 4 springs attached. Tests conducted at 60 psi operating pressure.
Blade time responses for damper configurations with and without air in accumulator, using stainless steel mesh F²MC tube with a) 2 springs attached, b) 4 springs attached. Tests conducted at 80 psi operating pressure.
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Selected Conference Publications


Selected Journal Publications

