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ENSEMBLE SENSITIVITY ANALYSIS OF ICE FLOW SIMULATIONS WITH DIFFERENT PARAMETRIC MODEL UNCERTAINTIES

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by
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ABSTRACT

Large ensembles of forward model simulations with well-constructed parametric model perturbations can provide useful insights to assessing different sources of uncertainties in ice-stream modeling. Ensemble sensitivity analysis of these simulations can further provide guidance to retrieve and improve these uncertain parameters through data assimilation, as well as to assess what potential observations will be more valuable for reducing such uncertainties.

In this thesis, we use a 2-d flowline model to study the ensemble sensitivity of ice-flow speed and ice thickness to the spatially varying basal friction profile, bottom topography and upstream flux. We found that ice surface velocity is highly correlated to the basal drag at the downstream end. The results also show that a 5% uncertainty in the initial upstream mass flux will have a strong impact on the surface ice velocity simulation (their correlations are 0.8 or greater everywhere). This upstream flux uncertainty can be more dominant than the uncertainty due to the basal friction coefficient. Thus, previous retrieval results that did not consider uncertainty in the upstream flux may be overconfident.
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Chapter 1

Introduction

Background and Rationale for Research

In recent decades, climate change has been the focus of many studies. There are many internal and external factors that cause climate change, including ocean–atmosphere variability, solar output, glaciers and human activity. In recent decades, the Greenland ice sheet has become a significant contributor to the global sea level rise due to anthropogenic signal emergence (Fyke et al., 2014). The removal of the entire Greenland ice sheet (GrIS) would result in ~7.3 meters of global sea level rise (Bamber et al., 2001; Gregory et al., 2004; Rignot et al., 2011; Bamber et al., 2013). The production and transport of meltwater is the main driving mechanism of this increasing contribution with outlet glacier discharge accounting for roughly the other half of ice mass loss from GrIS, with a combined contribution of 0.63 ± 0.07 mm annually to the mean global sea level rise (Enderlin et al., 2014). Thus, predicting the evolution of ice streams and outlet glaciers is of vital importance to the study of climate change. In addition, it can also benefit decision makers by helping them determine future sea-level-rise related strategies. In order to make more reliable prediction of ice-stream evolution, we need to understand how ice sheets respond to their surrounding conditions, including bed lubrication, surface mass balance, basal sediments and other controlling ingredients. Therefore, it is necessary to study the behavior associated with parameter uncertainties when ice sheets are coupled with an ice-flow model.

Numerical models have become a useful tool for simulating and predicting complex natural systems. They can also provide profound knowledge about the relationships among the parameters in these systems, which can improve our understanding of the natural phenomena and
give direction to further research. Typically, there are two types of parameters in numerical models: physical parameters and process parameters. Physical parameters can be measured directly, while process parameters can only be derived from physical parameters, but are frequently set arbitrarily. Therefore, parameter uncertainties can appear due to the poor estimation of the process parameters of numerical models.

Ensemble sensitivity analysis is a commonly used method for understanding the interaction among the parameters and variables of numerical models (Nielsen-Gammon et al., 2010). It can be used to reveal the hidden correlation among model parameters, which can provide intuitive insight into the numerical model structure and physical system dynamics. Ensemble sensitivity experiments can also provide guidance on observational system designs and find the strongest correlation between the output variables and the physical parameters.

The kinematic properties of ice sheets are best described by ice flow speed and thickness. Forecasting the evolution of these variables requires knowledge of the natural physical schemes and the values of several parameters, including the upstream mass flux, basal topography and the basal friction profile. In this study, we look at one process parameter, basal friction. Basal friction cannot be measured directly, and it can affect the flow speed significantly. Consequently, understanding the basal frictional properties of ice sheets is important to forecasting the future states of the ice sheets accurately.

Many different methods have been used on estimating the basal shear stress of ice sheets, for example, control method inversions (Joughin et al., 2004), ideal model simulation (Børling et al., 2016) and data reconstruction (Thorp, 1991). However, to the extent of the authors’ knowledge, all earlier studies assume constant upstream mass flux. However, the uncertainties of upstream mass flux may have non-negligible influence on the state variables. In this thesis, we study the ensemble sensitivity of ice flow speed and ice thickness to several uncertain parameters using the PSU 2-D flowline model (Parizek and Walker, 2010; Parizek et al., 2010; Parizek et al.,
2013), which is a 2-d thermomechanical finite element model of ice flow based on a higher order treatment of the momentum balance. The parameters that we focused on have relatively significant effects on ice flow properties, and can be measured or estimated from physical parameters. We conduct a large number of ensemble forward simulations while perturbing these parameters within reasonable and realistic ranges. The nonlinearities, interactions, correlations and influences among model parameters are studied and assessed.
Chapter 2

Problem formulation and tools

The Governing Equation

The two-dimensional (in \( x \) and \( z \) coordinates, assuming the ice has a parameterized width \( \beta \) in the \( y \) direction, and \( z \) is with respect to sea level) ice flow assumes a power law \((n = 3)\) rheology for ice (Glen, 1955; Budd et al., 1989):

\[
\sigma'_{ij} = 2\nu\varepsilon_{ij},
\]

\[
\nu = \frac{B(T^*)^{\frac{1-n}{2}}}{{\varepsilon_*}^n},
\]

\[
\varepsilon_{ij} = A(T^*)\tau_i^n - \tau_{ij}',
\]

where \( \sigma'_{ij} \) is the deviatoric stress component, \( \nu \) is the effective viscosity, \( \varepsilon_{ij} \) and \( \tau_{ij}' \) are the strain rate and deviatoric stress components, respectively, with the direction of \( j \) in the plane orthogonal to \( i \). \( \varepsilon_* \) is the effective strain rate, \( \tau_* \) is the effective shear stress, which is defined as

\[
\tau_* = \sum_{ij} \varepsilon_{ij}\varepsilon_{ij}.
\]

\( A(T^*) \) is an ice softness prefactor influenced by some physical parameters, including temperature, crystal orientation and others. \( B(T^*) \) is the ice hardness parameter, which is a function of space but does not vary with time, and is defined as \( B(T^*) = A(T^*)^{-1/n} \). These equations sometimes are referred to Glen’s Flow Law.

The continuity equation for ice flow is

\[
\frac{\partial h}{\partial t} = -\frac{\partial (h\bar{u})}{\partial x} - \frac{h\bar{u}}{\beta} \frac{\partial \beta}{\partial x} + \dot{A} - \dot{B},
\]
where \( h \) is the ice thickness, \( \bar{u} \) is the depth-averaged horizontal velocity, \( \beta \) is the flowband width, \( \dot{A} \) is the net ice-equivalent surface precipitation rate, and \( \dot{B} \) is the basal melting rate. The width-averaged horizontal momentum balance for ice flow is

\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = \frac{2}{\beta} \tau_s,
\]

where \( \sigma_x \), \( \tau_{xz} \) and \( \tau_s \) are the longitudinal stress, vertical shear stress and parameterized lateral drag, respectively.

The longitudinal stress can be written in terms of longitudinal and transverse deviatoric stresses as well as the vertical normal stress \( \sigma_z = 2\sigma_x' + \sigma_y' + \sigma_z \). By assuming depth-dependent glaciostatic pressure \( \sigma_z = -\rho g (s - z) \), where \( \rho \) is the density of ice and \( s \) is the surface elevation) and using Glen’s Flow Law, the momentum balance can be rewritten (MacAyeal, 1989; Parizek et al., 2010) as

\[
\frac{\partial}{\partial x} \left( 2v \left( 2 \frac{\partial \bar{u}}{\partial x} + \frac{u \partial \beta}{\beta \partial x} \right) \right) + \frac{\partial}{\partial z} \left( v \frac{\partial \bar{u}}{\partial z} \right) = \rho g \frac{\partial (s-z)}{\partial x} + \frac{2}{\beta} \tau_s,
\]

The mass (momentum) balance is solved using the Petrov-Galerkin (Galerkin) method of weighted residuals with linear (bilinear) basis functions (Parizek et al., 2013).
Model Introduction

In this thesis, the experiments are designed to assess the ensemble sensitivity of ice surface horizontal velocity and ice thickness to uncertain parameters using a 2-D flowline model. For all the simulations, the horizontal domain size is 50 km. The horizontal resolution is mostly 0.5 km and the model has 35 nodes vertically, but for the momentum balance both horizontal and vertical resolutions vary at domain boundaries in order to minimize numerical artifacts (Vieli and Payne, 2005; Durand et al., 2009; Goldberg et al., 2009; Parizek et al., 2010). The flowband width is set to a constant value ($\beta = 100 \text{ km}$) without temporal evolution. Basal elevation decreases linearly at -0.003 in the positive x direction from an upstream depth of $-200 \text{ m}$, while a Gaussian bump with constant height is located at the middle of the domain, which is at 25 km, and the deviation of the Gaussian bump is 2 km.

Each ensemble contains 51 members and is integrated for 100 years. The simulation time is extended to 200 years in some cases for better understanding of the simulation results. The ensemble outputs are saved every 20 years and we examine the ensemble correlations between the perturbed model parameters and the state variables (ice surface horizontal velocity and ice thickness). The basal friction is parameterized assuming an exponential relation $\tau_b = B_b u^m$, where $\tau_b$ is the basal friction (or basal drag), $B_b$ is the basal friction coefficient, and $m = 3$ in all of our simulations.

The perturbed parameters in our experiments include the bottom terrain height ($A$), basal friction coefficient ($B_b$) and the upstream mass flux ($q_o$). The basal friction coefficient is further divided beneath different segments of ice, every 10 km ($B_1 - B_5$). All the parameters are perturbed independently.

For the bottom terrain height and basal friction coefficient, parameters in each ensemble member are perturbed from 0.5 to 1.5 times the control value with an interval of 0.02 times the
control value, which is chosen randomly for all joint sensitivity experiments. The control values are: \( A = 50m \) and \( B_{bo} = 1.3 \times 10^6 \ Pa \ s^{1/3} \ m^{-1/3} \). The perturbation of other parameters is randomly chosen from a Gaussian distribution with zero mean and given values of standard deviation. All the perturbed ensemble parameters in different experiments are listed in Table 2.1.
Table 2. List of perturbation conditions of ensemble experiments, where $q_o$, $q_{speed}$ and $q_{depth}$ stand for initial upstream mass flux, initial upstream ice velocity, and initial upstream ice thickness, respectively. Blank cells mean there is no perturbation of this parameter in the corresponding experiment. For terrain height ($A$) and basal friction coefficient ($B_i$), we perturbed them from 0.5 to 1.5 times their control value in steps of 0.02 times its control value. In experiment 1b and 1c, the basal friction coefficient is uniform for the whole domain. For the initial upstream mass flux, ice velocity and ice thickness, we perturbed them with values randomly selected from a Gaussian distribution with mean zero and standard deviation of the labeled value in the table.

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Chapter 3

Simulation results

Single Parameter Sensitivity

Before the ensemble sensitivity studies, we start with testing the single parameter sensitivity for terrain height \((A)\) and domain basal friction coefficient \((B)\). The results for single parameter sensitivity experiment are shown in Figures 3.1 and 3.2. The single parameter sensitivity experiment of terrain height shows that the impact of terrain height is mainly located between 21-33 km, with a negative correlation with respect to ice thickness and positive correlation with respect to ice surface velocity (Figure 3.1a and 3.1c). There is also a little impact on the upstream side of the terrain (0-21 km), but almost no influence on the time-evolving downstream side, based on the standard deviation results (Figure 3.1b and 3.1d). The existence of the terrain will generate form drag, blocking ice from the upstream and thus have an accumulated potential and accelerate downstream ice. With higher terrain height, it will cause thicker ice upstream and decelerate the ice to maintain the upstream flux balance, and accelerate the thinner ice at the downstream side, thus leading to the correlation results as shown in Figure 3.1.

The results for single parameter sensitivity of domain basal friction coefficient are shown in Figure 3.2. Results indicate that the impact of domain basal friction coefficient is global. The basal friction coefficient has a fully positive correlation with ice thickness and a fully negative correlation with ice surface velocity. Larger basal friction coefficient will decrease domain ice velocity and increase domain accumulated ice, which is what friction is supposed to do. Besides, we can see that the standard deviation of ice thickness decreases rapidly at the downstream side of the terrain. It also increases with time, which indicate that the impact of basal friction on ice thickness is initially on the upstream end, and it requires time to propagate in the downstream
direction. However, the standard deviation of ice surface velocity does not monotonously increase with time, as what the standard deviation of ice thickness shows. The standard deviation of ice surface velocity reaches a relatively large value at 20 years, especially in the second half of the domain, and it drops at 40 years, then increases back with time. At 20 years, it also increases in the downstream direction, which is caused by the initial model spin-up with different values of the model parameters.
Figure 3.1 The correlation and standard deviation results as a function of flowline length for experiment 1a. Panel (a)(b) are the correlation results between terrain height ($A$) and state variables. Panel (c)(d) are the standard deviation results of the state variables. The unit of standard deviation is $m$ for ice thickness and $m/yr$ for ice surface velocity.
Figure 3.2 The correlation and standard deviation results as a function of flowline length for experiment 1b. Panel (a)(b) are the correlation results between domain basal friction coefficient \((B)\) and state variables. Panel (c)(d) are the standard deviation results of the state variables. The unit of standard deviation is \(m\) for ice thickness and \(m/yr\) for ice surface velocity.
Joint parameter sensitivity for terrain and domain basal friction coefficient

In order to see how the correlation is going to change with additional uncertainties, we run an experiment with adding perturbation for both parameters simultaneously, which is our experiment 1c. The results for joint parameter sensitivity (Figure 3.3) show that both parameters are visible in correlation and standard deviation. Comparing to experiment 1a and 1b, the correlation results change after introducing other uncertainties, but there is an order to which uncertainty is more dominant. In this case, the more dominant one is the domain basal friction coefficient. From Figure 3.3a to 3.3d, we find that the domain basal friction coefficient has larger influence than the terrain height with the same amplitude of perturbation (both ±50%), and the tendency remains the same for both ice thickness and ice surface velocity. The correlation and standard deviation results for domain basal friction coefficient are similar to the results from experiment 1b, except in the terrain height region. The impact of terrain height on the upstream side almost disappears, while some regional impact still remains, especially on ice thickness. As time increases, the terrain height becomes less important with decreasing correlation values, while the domain basal friction coefficient becomes more and more dominant. The standard deviation is almost the same as results in experiment 1b, with a little larger standard deviation within the terrain region.
Figure 3.3 The correlation and standard deviation results as a function of flowline length for experiment 1c. Panel (a)(b) are the correlation results between terrain height ($A$) and state variables. Panel (c)(d) are the correlation results between domain basal friction coefficient ($B$) and the state variables. Panel (e)(f) are the standard deviation results. The unit of standard deviation is $m$ for ice thickness and $m/yr$ for ice surface velocity.
Joint parameter sensitivity for terrain and regional basal friction coefficient

To test the ensemble sensitivity with non-uniform basal friction coefficient, different values of basal friction coefficient are applied along different segments of ice flow (values are different for every 10km and are defined as $B_1, B_2, B_3, B_4,$ and $B_5$), which is our experiment 2. The results of experiment 2 are shown in Figure 3.4. Consistent with the results in previous experiments, higher basal friction coefficients lead to a thicker ice-stream and slower ice velocity. A segments of regional basal friction coefficients have a large impact on their own region and everywhere upstream, but almost no downstream impact (Figure 3.4a to 3.4e).

Among all five segments, $B_5$ is the most dominant and it becomes even more dominant as time increases, while the impact of other parameters becomes less dominant or stay the same. This result is similar to a buttressing effect. The standard deviation for surface velocity increases in the downstream direction (Figure 3.4l), which results from the increasing impact of basal friction in the downstream direction (in general, the correlation value for $B_5 > B_4 > B_3 > B_2 \approx B_1$). The standard deviation for ice thickness decreases in the downstream direction (Figure 3.4f), which indicates that the impact of basal friction on ice thickness is initially on the upstream end, and the impact needs time to propagate to the downstream end. The initial model spin-up patterns for standard deviation of ice surface velocity still exists, but it takes less time to grow back to its values at 20 years than experiment 1b and 1c.

At 100 years, the correlation for $B_5$ with ice thickness is larger than 0.8 everywhere and reaches 0.95 at the downstream end, while the impacts of $B_1 - B_4$ remain small with correlations less than 0.6 in the whole domain. Again, this indicates the increasing impact for basal friction in the downstream direction.

Moreover, to find the correlation difference with additional terrain height uncertainty, we repeat experiment 2 with additional perturbations on the terrain height in experiment 3. In order
to observe how the correlation evolves with longer time, the simulation time for experiment 3 is extended to 200 years. The results are shown in Figure 3.5 and 3.6, which are similar to the results from experiment 2.

As we know from experiment 1c, the impact of terrain height is less dominant than the impact of domain basal friction coefficient. Thus, we would expect the correlation for basal friction coefficient probably doesn’t change much with additional terrain height uncertainty.

However, it may not be the case when we compare the impact of terrain height with basal friction coefficient at different segments of ice flow instead of the domain-wide basal friction coefficient. The results suggest that initial correlation for basal friction coefficient changes significantly with additional terrain height uncertainty, especially for parameter $B_5$ (dropping from 0.8 to 0.2 at the upstream end with $t = 20$ year). The terrain height plays an important role at $t = 20$ year, especially at the terrain region (Figure 3.5f). As time increases, the impact for basal friction coefficient will approach the same values as in experiment 2, while the terrain height still retains some regional impact. The buttressing effect shows again, $B_5$ becoming dominant and reaching 0.75 over the whole domain at the end of simulation time. Meanwhile, with extended simulation time, the time-decreasing impact for $B_1$-$B_4$ and $A$ is more obvious than experiment 2.
Figure 3.4 The correlation and standard deviation results as a function of flowline length for experiment 2. Panel (a)-(e) are the correlation results between different segments of basal friction coefficient and ice thickness. Panel (g)-(k) are the correlation results between different segments of basal friction coefficient and ice surface velocity. Panel (f) and (l) are the standard deviation of ice thickness and ice surface velocity. The unit of standard deviation is m for ice thickness and m/yr for ice surface velocity.
Figure 3.5 The correlation and standard deviation results as a function of flowline length for experiment 3. The simulation time is extended to 200 years. Panel (a)-(e) are the correlation results between different segments of basal friction coefficient and ice thickness. Panel (g)-(k) are the correlation results between different segments of basal friction coefficient and ice surface velocity. Panel (f) and (l) are the correlation results between terrain height and ice thickness/ice surface velocity. Panel (m) and (n) are the standard deviation of ice thickness and ice surface velocity. The unit of standard deviation is m for ice thickness and m/yr for ice surface velocity.
Figure 3.6 The correlation results for experiment 3 at 200 years. 50% uncertainties are added on both terrain height and segments of basal friction coefficient, with color-coded results with respect to parameters.
Moreover, the standard deviations for both state variables increase with time with approximately constant speed (Figure 3.5m and 3.5n), which suggests the model still hasn’t reached steady state after 200 years. The initial model spin-up pattern is less obvious compared to the previous experiments, and the standard deviation of ice surface velocity at 40 years is already larger than the values at 20 years in the whole domain. We also plot the correlations with both ice surface velocity and ice thickness at 200 years in Figure 3.6. The correlation value for \( B_5 \) is greater than 0.8 everywhere and reaches 0.95 at the downstream end, while \( B_1 \) almost has no impact except for within the upstream region, which again indicates the increasing impact for basal friction within further downstream segment. Additionally, the local impact of terrain height still has a correlation of 0.4, which is larger than the correlation of \( B_1 \) and \( B_2 \) parameters.

**Joint parameter sensitivity for upstream flux**

In experiments 4 and 5, we introduce additional uncertainty on the upstream flux to study the joint sensitivity to influx. As mentioned above, \( B_5 \) is more dominant than other basal friction coefficients, which is also confirmed by experiment 4 and 5. In experiment 4a, with 5\% uncertainty in the upstream mass flux, the results of ice surface velocity are mostly dominated by the uncertainty of the upstream mass flux, with a correlation greater than 0.8 over the whole domain (Figure 3.7l), while \( B_5 \) only has a correlation value of around 0.5. The correlation results to ice surface velocity lose most of their small-scale features and become largely uniform over the whole domain. However, due to the model implementation scheme, there is almost no impact of the upstream mass flux on the ice thickness (Figure 3.7k), and the correlation results for basal friction coefficients are similar to those from experiment 2. The fact that the impact of the upstream mass flux eventually all goes into velocity with almost no impact on ice thickness is
likely caused by the way the model is set up. The assigned influx first influence the calculated flow speeds, and with well-lubricated streaming ice flow across the entire domain, the mass balance is thereby largely accommodated without significant changes in ice thickness.

In experiment 4b, we reduce the uncertainty of the initial upstream mass flux to 1%. The impact of uncertainty in the upstream mass flux is largely weakened due to the decreasing uncertainty and it only has a correlation value of 0.2 with the ice surface velocity. The correlation results of basal friction coefficient do not vary much from the previous results for both state variables, and the standard deviation is similar as well, which suggests that the uncertainty of the upstream mass flux does not affect other correlations much and all the conclusions still hold once we control the uncertainty of the initial upstream flux under some threshold, in this case 1%.

We also take the perturbation of terrain height into account in experiment 5 and extended the simulation time to 200 years. The results show that even though initially there is some small correlation between initial flux and ice thickness, all the impacts from initial flux will eventually go into ice surface velocity with a correlation greater than 0.8 everywhere (Figure 3.9o), which is not changed with extra perturbation on the terrain height. As always, the terrain height will have a local impact, with a -0.45 correlation on ice thickness (Figure 3.9f) and a 0.25 correlation on ice surface velocity (Figure 3.9n).
Figure 3.7 The correlation and standard deviation results as a function of flowline length for experiment 4a. 5% uncertainty is added on the upstream mass flux. Panel (a)-(e) are the correlation results between different segments of basal friction coefficient and ice thickness. Panel (g)-(k) are the correlation results between different segments of basal friction coefficient and ice surface velocity. Panel (f) and (l) are the correlation results between initial upstream mass flux and ice thickness/ice surface velocity. Panel (m) and (n) are the standard deviation of ice thickness and ice surface velocity. The unit of standard deviation is \( m \) for ice thickness and \( m/\text{yr} \) for ice surface velocity.
Figure 3.8 The correlation and standard deviation results as a function of flowline length for experiment 4b. 1% uncertainty is added on the upstream mass flux. Panel (a)-(e) are the correlation results between different segments of basal friction coefficient and ice thickness. Panel (g)-(k) are the correlation results between different segments of basal friction coefficient and ice surface velocity. Panel (f) and (l) are the correlation results between initial upstream mass flux and ice thickness/ice surface velocity. Panel (m) and (n) are the standard deviation of ice thickness and ice surface velocity. The unit of standard deviation is m for ice thickness and m/yr for ice surface velocity.
Figure 3.9 The correlation and standard deviation results as a function of flowline length for experiment 5. 5% uncertainty is added on the upstream mass flux. Panel (a)-(e) are the correlation results between different segments of basal friction coefficient and ice thickness. Panel (i)-(m) are the correlation results between different segments of basal friction coefficient and ice surface velocity. Panel (f) and (n) are the correlation results between terrain height and ice thickness/ice surface velocity. Panel (g) and (o) are the correlation results between initial upstream mass flux and ice thickness/ice surface velocity. Panel (h) and (p) are the standard deviation of ice thickness and ice surface velocity. Correlation results at 200 years are shown in panel (q) and (r). The unit of standard deviation is m for ice thickness and m/yr for ice surface velocity.
Figure 3.10 The correlation and standard deviation results as a function of flowline length for experiment 6a, with distributing the impact of upstream mass flux into ice velocity and ice thickness. 5% uncertainty is added on the upstream mass flux. Panel (a)-(e) are the correlation results between different segments of basal friction coefficient and ice thickness. Panel (j)-(n) are the correlation results between different segments of basal friction coefficient and ice surface velocity. Panel (f) and (o) are the correlation results between terrain height and ice thickness/ice surface velocity. Panel (g)-(i) and (p)-(r) are the correlation results between initial upstream variables (mass flux, ice velocity and ice thickness) and ice thickness/ice surface velocity.
Figure 3.11 Panel (a)(b) are the correlation results for experiment 6a at 200 years. 50% uncertainties are added on both terrain height and segments of basal friction coefficient, and 5% uncertainties are added on upstream variables, with color-coded results with respect to parameters. Panel (c)(d) are the standard deviation for ice thickness and ice surface velocity. The unit of standard deviation is $m$ for ice thickness and $m/yr$ for ice surface velocity.
Figure 3.12 The correlation and standard deviation results as a function of flowline length for experiment 6b, with distributing the impact of upstream mass flux into ice velocity and ice thickness. 2.5% uncertainty is added on the upstream mass flux. Panel (a)-(e) are the correlation results between different segments of basal friction coefficient and ice thickness. Panel (f) and (o) are the correlation results between terrain height and ice thickness/ice surface velocity. Panel (g)-(i) and (p)-(r) are the correlation results between initial upstream variables (mass flux, ice velocity and ice thickness) and ice thickness/ice surface velocity.
Figure 3.13 Panel (a)(b) are the correlation results for experiment 6b at 200 years. 50% uncertainties are added on both terrain height and segments of basal friction coefficient, and 2.5% uncertainties are added on upstream variables, with color-coded results with respect to parameters. Panel (c)(d) are the standard deviation for ice thickness and ice surface velocity. The unit of standard deviation is m for ice thickness and m/yr for ice surface velocity.
Joint parameter sensitivity for initial upstream ice velocity and thickness profile

In order to distribute the impact of initial upstream flux to both ice surface velocity and ice thickness, we change the model setup and the parameters that we perturbed. In experiment 6, we use the same experiment setup as in experiment 3, with additional perturbation on both initial upstream velocity and initial domain ice thickness instead of initial upstream mass flux and integrate it for 200 years.

The results are shown in Figure 3.10 to 3.13. In experiment 6a (Figure 3.10 and 3.11), the uncertainty on both initial upstream velocity and initial domain thickness is 5%. As we expected, the impact of initial uncertainty has been distributed to both state variables and all the results that we found before still hold. The impact of $B_5$ effect still increases with time, while others decrease or stay the same. Besides, we find that most of the impacts are still from the initial velocity and thickness uncertainty, with a correlation of 0.6 from initial flux and 0.8 from the corresponding initial uncertainty (initial upstream velocity for ice surface velocity and initial upstream thickness for ice thickness). There is almost no cross-correlation between initial and final velocity/thickness profile (their absolute correlation value is less than 0.2).

When we decrease the uncertainty on both initial variables by half (2.5% in experiment 6b, Figure 3.12 and 3.13), the dominance of $B_5$ effect is more apparent than experiment 6a (with correlation values changing from 0.45 to 0.6 at the upstream end), but the initial profile uncertainty still has a slightly larger impact on both state variables than the $B_5$ effect. The results of experiment 6b indicate that the uncertainty in initial upstream velocity and domain thickness profile, even if only 2.5% uncertainty, may cause large difference in the results which is comparable to or larger than 50% uncertainty in the basal friction coefficient and terrain height profile.
Chapter 4

Concluding Remarks

In this thesis, we have conducted several sets of ice model simulations with ensemble perturbation to study some uncertain key model parameters, including basal terrain height, basal friction coefficient, upstream mass flux and upstream variables. The correlations of these parameters to ice surface velocity and ice thickness are calculated and compared.

In joint parameter sensitivity, correlation relationships may change significantly when extra parameter uncertainties are introduced, which is shown to be the case in our experiments. We found that the perturbation of basal terrain height would have a significant impact on the local region, but less dominant than the impact of basal friction profile on both ice surface velocity and ice thickness. The perturbation of basal friction coefficient remains relatively strong locally as well as on the upstream side, but has little influence on the downstream side. As for uncertainty in the upstream mass flux, a 5% uncertainty seems to have a relatively large correlation with state variables, while a 1% uncertainty is safer to ignore in our current setup. The results still hold after we distribute the impact of upstream mass flux into initial upstream ice velocity and initial domain ice thickness.

In the real world, a 5% uncertainty is a fair estimation for the upstream flux, so we have reasons to believe that previous retrieval results may be overconfident without considering the uncertainties of the upstream fluxes. In order to isolate system behavior, we note that the initial conditions of the control experiment use ideal data and do not try to match any real-world setting, but we have reasons to believe the results of this study still can be applied to help improve constraints and reduce potential uncertainties from inversion results.
Bibliography


