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MATHEMATICS TEACHER EDUCATORS' ROLES, TALKS, AND KNOWLEDGE IN COLLABORATIVE PLANNING PRACTICE: OPPORTUNITIES FOR PROFESSIONAL DEVELOPMENT

A Dissertation in

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Nursen Konuk

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The dissertation of Nursen Konuk was reviewed and approved* by the following:

E. Frances Arbaugh
Associate Professor of Education (Mathematics Education)
Dissertation Advisor
Chair of Committee

Andrea McCloskey
Associate Professor of Education (Mathematics Education)

George E. Andrews
Evan Pugh Professor of Mathematics

Gwendolyn Lloyd
Henry J. Hermanowicz Professor of Teacher Education
Director of Graduate Studies in Curriculum and Instruction

*Signatures are on file in the Graduate School

ABSTRACT

Mathematics Teacher Educators (MTEs) argue that mathematics education needs more research that investigates and characterizes MTEs' practice (Tirosh et al., 2014), particularly to conceptualize pedagogies of teacher education (Ghousseni & Herbtz, 2016) and to support MTEs in their professional development (Doerr & Thompson, 2004). Researchers have used different perspectives to describe and analyze the work of MTEs and have focused primarily on documenting MTEs' knowledge, practices, and roles. Regardless of their perspectives, however, researchers agree that it is important for MTEs' professional growth to be part of collaborative communities (Wilson & Franke, 2008). One practice identified in the literature that provides an opportunity for teacher educators to work in collaborative learning communities is co-planning (Albrecht, 2003). Despite the role of collaborative planning in professional development (Bleiler, 2015), the field knows little about the structure and nature of co-planning (Wilson, 2016), particularly in teacher education (Nevin, Thousand, & Villa, 2009). To better understand the collaborative practices of MTEs, this study investigated MTEs' practice in the context of co-planning a methods course for secondary mathematics PTs organized around iterative Cycles of Enactment and Investigations (CEIs; Lampert et al., 2013). Using data from co-planning meetings of the communities of practice formed by four MTEs, this single embedded case study presents findings of a study of MTEs' co-planning through the lenses of their *talk,* their *roles*, and their *knowledge*. Results show that collaborative planning in mathematics teacher education provides opportunities for both advancing MTEs' teaching practices by bringing together varying skills, knowledge, and roles (Sztajn, Ball, & McMahon, 2006) and supporting MTEs' professional growth by offering rich learning experiences (i.e., analysis, inquiry, and reflection) (Jaworski, 2008). Findings from this study support further investigations of MTEs' collaborative practices by providing guiding frameworks for such studies (i.e., using *type of talk* as a tool for analysis) as

well as providing practical co-planning activities that can be used to ground professional development of future and current faculty involved in mathematics teacher education.

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Chapter 1

Introduction and Study Rationale

> If it is true that teaching is specialized, professional work that requires knowledge and skill (and I believe that it is), then teaching teachers must also be a kind of professional work that requires knowledge and skill (and I believe that it is). (Wilson, 2006, p. 316)

To understand the nature of teacher educators' (TEs) professional work, there is an increasing effort in making pedagogies of teacher education and mathematics teacher educators' (MTEs) practice more visible and accessible (e.g., Standards for Preparing Teacher of Mathematics; AMTE, 2017) to the community. The work of teacher educators is very complex and requires multiple skills and areas of expertise (Knight et al., 2014; Zaslavsky, 2008); however, very little work has investigated and characterized MTEs' practice (Appova & Taylor, 2017; Even 2008; Tirosh et al., 2014). The field of mathematics education still needs a theorization of the work of MTEs to understand how those practices and skills facilitate pedagogies of teacher education (Ghousseni & Herbtz, 2016) and to support MTEs in their professional development (Doerr & Thompson, 2004; Superfine & Li, 2014).

Addressing this request, researchers characterize the intricate work of MTE by focusing on different aspects of the role. While some focus on MTEs' knowledge (e.g., Chauvot, 2009), others focus on MTEs' practice (e.g., Zaslavsky, 2007). In general, the existing literature focuses on four main dimensions of MTEs' work a) knowledge of MTEs (ATE, 2008; Approva & Taylor, 2017; Chauvot, 2008; 2009; Garcia, Sanchez, & Escudero, 2007; Superfine & Li, 2014; Zbiek & Hirsh, 2008), b) practices of MTEs (Doerr & Thompson, 2004; Ghousseni & Herbtz, 2016; Llinares & Krainer, 2006; Taylor, 2013), c) professional growth of MTEs (Campbell & Malkus, 2014; Peled & Hershkovitz, 2004; Tzur, 2001; Wilson & Franke, 2008; Zaslavsky & Leikin, 2004;), and d) roles and responsibilities of MTEs (Jowarski & Huang, 2014; Li & Superfine, 2018; Zaslavsky, 2007).

Researchers have developed different perspectives to describe and analyze these four dimensions of MTEs work and their professional growth. While some take more a cognitive approach (Superfine & Li, 2014) to investigate MTEs' learning, others take a socio-constructive and socio-cultural stand in doing so (Jaworski & Huang, 2014). Researchers who take a sociocultural approach to studying the professional development of MTEs argue that in addition to completing an appropriate amount of theory and research related coursework, it is critical to engage in reflective practice (Zaslavsky, 2008), collaboration (Bleiler, 2015; Jaworski, 2008), and community building (Llinares & Krainer, 2006).

The professional learning community could be named as the community of inquiry (Tirosh et al., 2014) or community of practice (COP) (Wilson & Franke, 2008) depending on the nature of the collaboration. Studies have shown the positive impact of communities of inquiry and practice in MTEs' professional growth (Even, 2008) particularly where teachers and MTEs work collaboratively (Kieran et al., 2013; Zaslavsky, 2008). Wilson and Franke (2008) emphasize the importance of MTEs engaging in apprenticeship opportunities within their communities of practice. They suggest building a collaborative environment for MTEs by bringing new faculty with diverse background and experiences into the program. Wenger (1999) supports their argument by addressing the benefits of collaborative communities.

> Collaboration in COP provides resolutions to conflicts and contradictions; supports a communal memory, allows individuals to their work without needing to know everything; helps newcomers to join the community by participating in practice; generates specific perspectives, terms enable to accomplishing what needs to get done; and creates rituals, customs, stories, events, rhythms of community life. (p. 46)

One practice that enables the establishment of collaborative communities that facilitate professional development is co-teaching and co-planning in teacher education.

Research on team planning shows that collaborative teaching and planning supports faculty professional learning (Albrecht, 2003; Bleiler, 2015). Co-planning would provide opportunities for a rich collaborative setting (George & Davis-Wiley, 2000; Gray & Halbert, 1998). Combining different skills, areas of expertise, and perspectives not only supports MTEs' professional growth (Sztajn et al., 2014), but also functions as assisted performance for mentoring novice MTEs (Feiman-Nemser & Beasley, 1997). Although the literature in the field of education emphasizes the importance of collaboration, very little of this existing work examines the nature of collaboration in higher education, particularly collaborative efforts leading to successful instruction (Bleiler, 2015).

The purpose of this study is to examine and describe the collaboration among four mathematics teacher educators as they co-plan for teaching a secondary mathematics methods course in which course activities are designed around pedagogies of practice. A number of studies have examined the professional growth of teachers and MTEs when they work collaboratively (e.g., Campbell & Malkus, 2014; Huang et al., 2014; Jowarski & Huang, 2014; Kieran et al., 2013; Krainer 1999; Sakonidis & Potari, 2014; Sztajn et al., 2014; Zaslavsky, 2008), but there are no well-established models or guidelines for when teacher educators work collaboratively (Nevin, Thousand, & Villa, 2009). Specifically, we do not know much about the structure and nature of co-planning (Lynch, 2017; Wilson, 2016) when MTEs co-plan mathematics methods courses.

This study addresses the following questions:

1. What types of *knowledge* surfaced and were used during the co-planning sessions?
2. What *types of talk* did the MTEs engage in during co-planning sessions?
3. What *roles* did the MTEs adopt during co-planning sessions?
4. In what ways are these three phenomena connected?

This study will enable mathematics teacher educators and researchers in teacher education to better understand the practice of MTEs in the context of co-planning a methods course. In addition, this study will contribute to the field by proposing ways to think about the professional development (PD) of future and current faculty involved in mathematics education.

Theoretical Framework

My theoretical framework consists of three main parts. The first part introduces a communities of practice perspective to describe the practice of MTEs. The second part expands on existing models for studying MTEs' knowledge and roles. The third discusses the structure of co-planning activities and how those activities facilitate MTEs' professional development (See Figure 1-1). My description of development draws from theories of communities of practice (COP) (Lave & Wenger, 1991; Wenger 2008) and communities of inquiry (Cochran-Smith, 2003) underpinned by social constructivist theories of learning (Vygotsky, 1978). These theories emphasize reflection, inquiry, and professional dialogue between colleagues. Taking a sociocultural standpoint, I examine the practice, co-planning, and social interaction in which COP members engage. I also consider MTEs' participation in joint practice and the reification they produce, both indicators of professional development (Engeström, Miettinen, & Punamäki, 1999; Farnsworth & Wenger-Trayner, 2016; Lave & Wegner, 1991; Wenger 2008; Rogoff, 1994). In describing knowledge for MTEs, teachers' knowledge models (e.g., Grossman, 1990; Hill, Ball, & Shilling, 2008; Shulman, 1986), TE knowledge (e.g., Cochran-Smith; 2003; 2005), and MTEs' knowledge models (e.g., Jaworski, 2008b; Superfine & Li, 2014), particularly Chauvot's (2009) model, inform my framework. Similarly, models for the roles of TEs and MTEs in COP (e.g., Cochran-Smith, 2005; Jaworski & Huang, 2014; Zaslavsky, 2007) informed my conceptualization of MTEs' roles in co-planning practice.

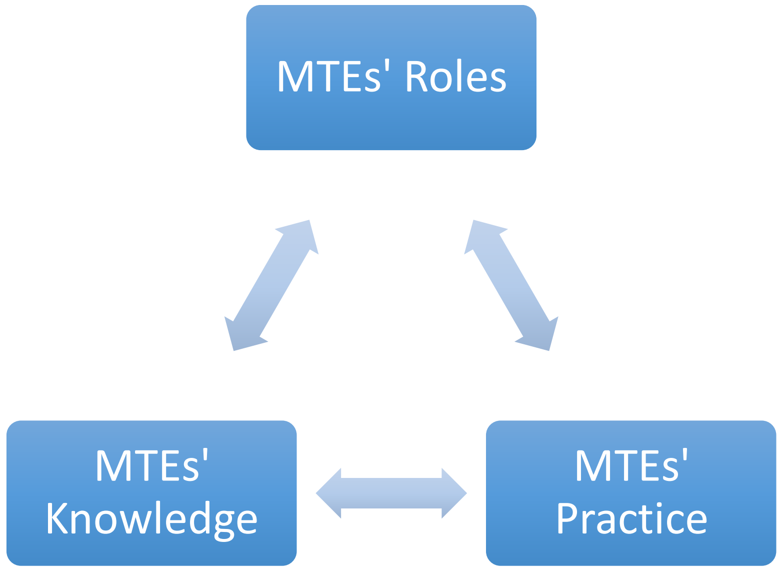

Figure **1-1**: MTEs' communities of practice and professional development.

This theoretical framework section is organized in the following way. First, I describe core concepts and ideas from communities of practice theory and explain how those constructs assist us in understanding MTE practice and professional development. Next, I summarize important concepts and ideas to examine and describe the structure of co-planning activities. Finally, I discuss models for the knowledge and roles of mathematics teacher educators and emphasize important ideas from studies of collaborative practices, co-teaching, and educator learning from those studies.

Communities of Practice

In this section, I introduce my approach to examining what ideas or practices facilitate MTEs' learning. First I introduce core concepts from the communities of practice (COP) (Wenger, 2008) perspective, which help assist me to describe and analyze the COP. Then, I explain how and why the COP framework informs my study.

Although other socio-constructivist and socio-cultural theories (SCT) informed my research, I mostly base my study on the COP framework (Farnsworth, Kleanthous, & Wenger-Trayner, 2016; Wenger, 2008;). This model is appropriate for my study due to its emphasis on collaboration and engaging in practices, which are determined as two significant contributors to MTEs' professional development (Jaworski, 2008; Wilson & Franke, 2008). This model does not focus just on initial and end products of learning processes, but rather a new understanding formed by collaborative effort. This model allows individuals to contribute to the collaborative practice instead of being passive recipients in the group. Also, this model prioritizes the investigation of professional development in authentic settings (Rogoff, 1994). The COP perspective emphasizes our experiences and complex and dynamic relations (Wenger, 2008). Building on other SCT, and Wenger's model of Legitimate Peripheral Participation (LPP) which describes how newcomers become experienced members of collaborative practice, Wenger developed COP model (Omidvar & Kislov, 2014). In his model of COP, Wenger uses dual processes such as participation and reification — conceptual and physical artifacts produced by a COP as they engage in a joint practice — to examine professional development and identities (Farnsworth, Kleanthous, & Wenger-Trayner, 2016).

Taking this approach, one could say the more MTEs participate in the joint activities of MTE, the more they produce reifications[1]. The group produced conceptual and physical artifacts as they engaged in planning and enacting a methods course. This dynamic relationship enabled the group to learn about teaching teachers and becoming MTEs.

Community of Practice. One fundamental construct in COP model is community. With varying levels of participation, individuals are involved in multiple communities of practice (Wenger, 2008). Lave and Wenger define community of practice as a group of people who "share an understanding of what they are doing and what that means for their lives and communities"

(1991, p. 98), and learn how to do it better as they interact regularly (Wenger, 2008). In this theory, communities are where learning actually occurs as individuals participate in practices and produce boundaries of practice. Through participation in collaborative practices, members of the community develop a shared understanding and repertoire that bind these individuals together as a social entity (Wenger, 2008). Although social constructivists debate whether researchers could talk about the mind of an individual or a society, they all agree that learning occurs through individuals interacting with each other (Norton & D'ambrosio, 2008). Each group member, regardless of his or her expertise, participates in joint activities to some degree, and the group continues to develop new knowledge as they interact each other and work collaboratively.

Wenger defines two audiences in describing the social interactions and relations in COP: individuals and community. These two audiences facilitate each other's professional growth through the processes of participation and reification. On the one hand, we engage directly in activities, conversations, reflections, and other forms of personal participation in social life, and learn from those interactions. On the other hand, we produce physical and conceptual artifacts—words, tools, concepts, methods, stories, documents, links to resources, and other forms of reification—that reflect our shared experience and around which we organize our participation in the COP's joint practice. Meaningful learning in social contexts requires both participation and reification to be in interplay (Wenger, 2008).

In this study, our community of practice includes a group of MTEs working together in various group practices. Below, I explain what constitutes those group practices.

Joint Practice. Wenger (2008) describes that collaboration in COP provides resolutions to conflicts and contradictions, supports a communal memory, allows individuals to their work without needing to know everything, helps newcomers to join the community by participating in practice, generates specific perspectives, terms enable to accomplishing what needs to get done, and "creates rituals, customs, stories, events, rhythms of community life" (p. 46). While

Vygotsky takes individual behavior as the unit of analysis, activity theorists see joint activity or practice as the unit of analysis for analyzing human development (Engeström, Miettinen, & Punamäki, 1999).

In this study, which views the community as central to the process of making and interpreting meaning, I take the joint practice as the unit of analysis for examining a subject's growth in professional knowledge. Specifically, this study examines the joint activity of co-planning. Co-planning is generally defined as a process where instructors who teach together "decide how they will implement instruction to meet the needs of all students" (Wilson, 2016, p. 120). Here, I define co-planning as a collective activity that involves the participants in designing the mathematics methods course. By collective activity, I do not mean an activity where the MTEs were solely physically together and engaged in an activity. Rather, all worked towards a particular goal while continuously interacting with each other.

Within this context, I define an MTE's knowledge development as a transformation and increase in his or her participation in the co-planning a methods course which focused on teaching practices. Their shared activity included a mentor, beginning MTEs, and PTs, and every participant has a role.

MTE Knowledge in Co-planning

Chauvot's (2008; 2009) model on MTEs' knowledge informed my description of knowledge in this study. Chauvot (2009) conducted a self-narrative inquiry to identify the knowledge she drew upon to fulfill her role as an MTE and researcher (MTE-R). After teaching several mathematics methods and content courses, Chauvot examined professional development from her doctoral program into her third year of a faculty position. She used multiple frameworks of teacher knowledge to investigate the knowledge content, structure, and growth of a novice

MTE-R. Chauvot's model (See Table 1-1) combined research on teacher knowledge, explicitly drawing on the frameworks of Shulman (1986) and Grossman (1990), and on the three-layer model of professional growth through practice that Zaslavsky and Leikin (2004) develop. She used Shulman's (1986) categories of knowledge and the notion of knowledge of context (Grossman 1990) to identify and organize knowledge for MTE. This study provides a useful structure for studying MTEs' knowledge.

To identify and organize the components of each category, Chauvot adopted a three-layer model proposed by Zaslavky and Leikin (2004) consisting of children in grades K-12, PTs and in-service teachers, and doctoral students she taught in her classes and mentored (See Figure 1-2). Chauvot argued that teaching each layer of students required different components of knowledge. Chauvot's model of MTE-R consists of knowledge of SMCK-MTE (subject matter knowledge for mathematics teacher educators), PCK-MTE (pedagogical knowledge for mathematics teacher educators), CK-MTE (curriculum knowledge for mathematics teacher educators), and CXK-MTE (context knowledge for mathematics teacher educators).

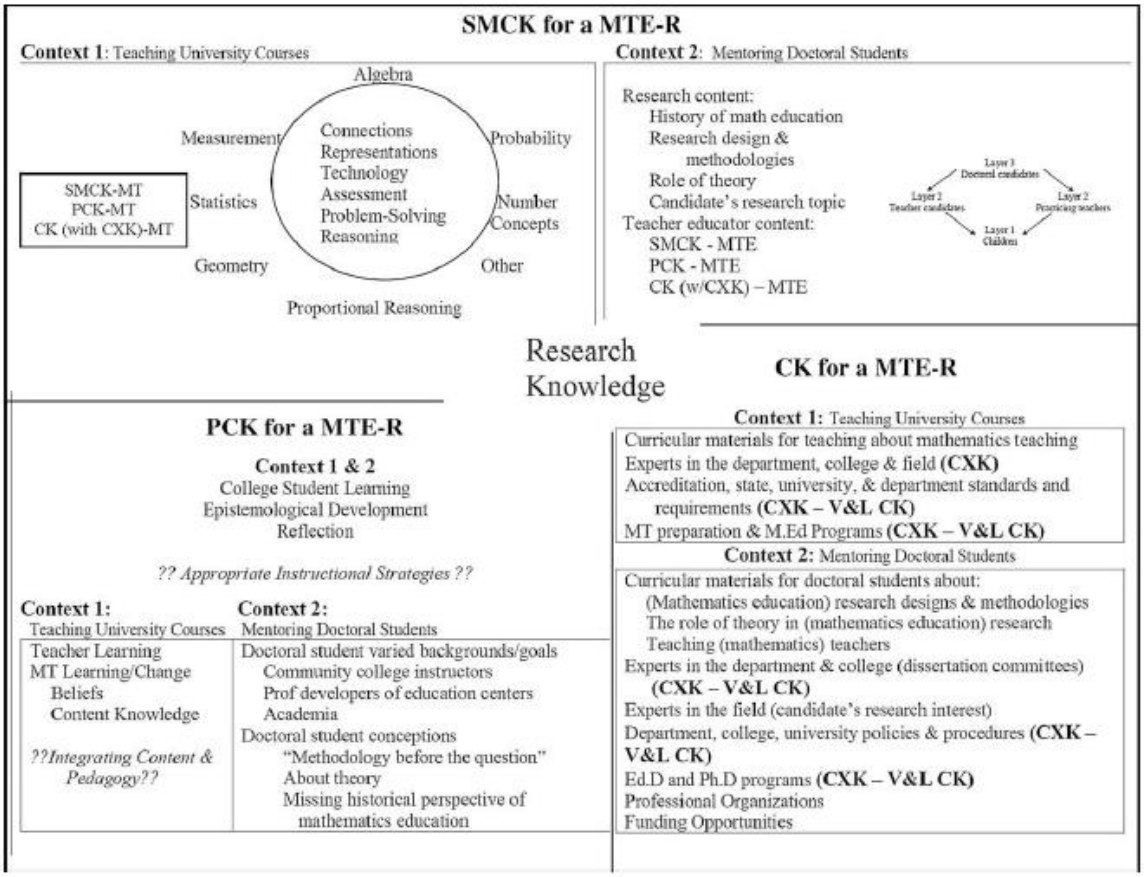

Figure 1-2: Knowledge map of SMCK, PCK, CK, and knowledge of context (CXK) for MTE-R) where V & L CK refers to vertical and lateral curricular knowledge. Reprinted from "Grounding practice in scholarship, grounding scholarship in practice: Knowledge of a mathematics teacher educator–researcher" by J. B. Chauvot, 2009, *Teaching and Teacher Education, 25,* p. 363. Copyright 2008 by Elsevier Ltd.

I chose Chauvot's model because, similar to my research question, her work investigates the knowledge an MTE needs to have in order to design a particular course (in her case a "proportional reasoning" course). In addition, building upon an existing model allows me to formulate an initial idea of the specific knowledge I should look for. Although Chauvot describes the knowledge of MTE-R required as a mentor for doctoral students, for my study, I specifically focus on her categories for and descriptions of instructor knowledge that doctoral students seek and use to plan learning activities for preservice teachers (PTs). I also use her study to identify what knowledge the mediators (i.e., expert MTEs) bring to the table as teacher educators. Below, I explain the knowledge areas I will use to identify, observe, and analyze MTEs' professional knowledge.

Table 1-1: Adapted from Chauvot's model for MTE knowledge (2008) & (2009)

SMK-MTE Impact the content we teach	Teaching the course *Anything could be taught in the course*	Knowledge about relevant math Knowledge about how children develop math Knowledge about curricular materials for students (Vertical & Lateral CK)
	Teaching doc students	Mentoring doc students: a) history of mthed, b) research design, c) role of theory Knowledge about the work of MTEs (assisting TEs in their work & design PDs and curriculum for TEs)
PCK-MTE Impact the way we teach	Teaching the course *Retrieved from research & practice*	Knowledge about how PTs (college students/teachers) develop math understanding Knowledge about how PTs develop PCK Knowledge about PTs beliefs/conceptions Knowledge about instruction, teaching models
	Mentoring doctoral students	Knowledge about doctoral student's conceptions, belief, /background, goals Knowledge about problems/issues/needs emerged in the conversations Knowledge about TEs education, doctoral programs, academia
CK-MTE Impact the materials we use to teach *Interpreting research about how the learner interacts with curricular materials*	Teaching the course	Curricular materials for teaching about mathematics teaching Human resources – experts in the department, college & field (CX) Accreditation, state, university, & department standards and requirements (CX– V&L CK) MT preparation Programs (CX – V&L CK) –what PTs are doing & what PTs have learned and will learn
	Mentoring doctoral students	Curricular materials about (mathematics education) research designs & methods Curricular materials about the role of theory in (mathematics education) research Curricular materials about teaching (mathematics) teachers Human resources: Experts in the department & college (dissertation committees) (CXK – V&L CK) Experts in the field (candidate's research interest) Department, college, university policies & procedures (CXK – V&L CK) Ed.D. and Ph.D. programs (CXK – V&L CK) & Professional Organizations & Funding Opportunities

Subject matter knowledge (SMK-MTE). The first category of MTE-R knowledge is SMCK-MTE. Chauvot found that as an instructor—where she taught a mathematics content course—that she needed SMCK-MTE, which she defined as a combination of the knowledge that a mathematics teacher should have (SMCK-MT, PCK-MT, and CK-MT, with MT indicating Mathematics Teacher). She describes the subject matter of her course under two areas: mathematics and mathematics education. Regarding mathematics, an MTE should know axiomatic systems and rules of logic. Regarding mathematics education, an MTE should know theories, research, and practice in mathematics education. Some potential questions this knowledge addresses are: what does a particular concept mean in mathematics? What are crucial concepts and what skills children should learn about those concepts? What ideas are these concepts and skills related to within the domain of mathematics and across other disciplines? What instructional strategies and curricular materials are effective, and how do they support student learning?

Pedagogical content knowledge (PCK-MTE). Another knowledge category for MTE is PCK. Chauvot (2009) describes PCK in two domains: a) knowledge of teaching mathematics in K-12 contexts, and 2) knowledge of teaching preservice teachers (PTs). Knowledge of mathematics teaching includes understanding how children learn mathematics and how they develop mathematics concepts. Chauvot describes this area as the knowledge of research about children's learning of mathematics. Knowledge about mathematics teacher education includes how PTs—and college students more generally—learn about mathematics education concepts. Chauvot explains this knowledge as a) the knowledge of research about PTs' and in-service teachers' conceptions and beliefs about mathematics, mathematics teaching, and mathematics learning and how their conceptions and beliefs impact students learning; b) the knowledge of PTs' mathematical experiences and understanding and how educators can advance PTs' experiences and understanding of mathematics. In addition, Chauvot's theory of this knowledge includes knowing appropriate instructional studies that aim to support PTs' learning from their practice and to assist them as they access and build upon their instruction on students' thinking (Hill, Ball, & Shilling, 2008).

Curriculum knowledge for MTE. Chauvot's third category of MTE knowledge is CK-MTE. Although CK for teachers is sometimes considered a component of PCK (Cochran, 1991; Grossman, 1990; Hill, Sleep, Lewis, & Ball, 2007), CK is specialized for teacher educators due to its historical, social, and political aspects in addition to the different theories upon which curricular designs are based. Chauvot described CK in a context where the teacher is an MTE-R. She identified three hypothetical cases to redefine CK where the MTE is a university-based faculty member teaching mathematics methods courses, a university-based faculty member serving as a mentor for doctoral students, and a curriculum supervisor within a school district. She describes four components of CK across three different roles of MTE. Although the knowledge of case 1 and case 2 are not distinguishable, I will use her description of CK for case 1 (See Appendix A).

In general, CK for mathematics teacher education has been described as how mathematicsteachers learn about mathematics education concepts, which emphasize research-based knowledge. Some specific examples of CK for teaching teachers include; knowledge of available textbook and curricular materials for teaching a methods course; knowledge of teachers' interaction and use of curricular materials (e.g. Lloyd, 1999; Remillard, 1999); and knowledge of effectiveness of training programs, PDs, workshops, teacher education programs, and field experiences.

Chauvot included another component of CK that is specific to the educational setting, institutions, teacher education programs, and school districts. This knowledge includes knowing the previous, current, and future coursework that PTs take. Lateral CK is knowing the coursework that PTs take simultaneously with a methods course and determining if those other courses are addressing any common or related topics such as instructional models for teaching. Vertical CK, in contrast includes the knowledge of PST courses completed and whether they have discussed—or will have discussed—joint or related topics such as assessing students' thinking.

Context Knowledge for MTE. The notion of knowledge of context (CX) was first introduced by Grossman (1990). She described CX for teachers as the knowledge about the roles of students and teachers in the classroom. Chauvot expands the scope of CX for teacher educators. In addition to the context of a classroom, CX-MTE means knowing external resources of TE knowledge. She argues that TEs should know subject curricula in a broader context that includes law, guidance, and counseling (O'Sullivan, 2010). Thus, TEs need to familiarize themselves with state certification requirements, teacher education program requirements, and departmental requirements.

Overall, Chauvot (2008) argued that engaging in course development activities and curricular materials for PSTs furthered her CX-MTE. As Van Zoest, Moore, & Stockero (2006) recommend, Chauvot also agreed that doctoral students participating in activities where they analyze PSTs' thinking enhances their PCK-MTE (i.e., how PSTs learn and think). Chauvot emphasized that social interactions, particularly collaborations (Pellegrino, Sweet, Kastner, Russell, & Reese, 2014; Swennen, Shagrir, & Cooper, 2009; Van Zoest, Moore, & Stockero, 2006) and reflective analysis (Cochran-Smith, 2003; Murray & Male, 2005), play a significant role in the professional growth of MTEs. In addition, she needed and sought new knowledge whenever she was faced with a challenge while designing or teaching a course. Based on her arguments, three attributes come forward in the professional development of an MTE: social interactions, participation (and/or collaboration), and reflection. I discuss how these three attributes support MTEs professional growth in the following chapters.

MTEs Roles in Co-planning

Individuals' roles in a COP influence the joint practice and interaction between members. Wenger (2008) defines knowledge as not only our experiences of meaning and competence, but also our positions that orient our practice. Members of a group work toward a shared a goal with everyone taking different roles depending on their expertise. The participants switch roles as they become more independent in delivering the task (Rogoff, 1994). Wenger emphasizes that identity formation as an ongoing progress and they form trajectories. As Wenger explains, "An identity is a layering of events of participation and reification by which our experiences and its social interpretation inform each other" (2008, p. 151). Wenger defines three modes of belonging to the COP:

- engagement ("active involvement in negotiation of meaning");
- imagination ("creating images of the world and seeing connections through time and space by extrapolating from our own experience"- they may be doing the same thing but their perception, meaning –image- of what they are doing and how they are doing is different);
- alignment ("coordinating our energy and activities in order to fit within broader structures and contribute to broader enterprise") (2008, p. 173-174).

In his model, the relationship between members can take multiple forms such as agreement, disagreement, and conflict. Wenger finds it challenging to separate boundaries between collective and individual identities. Thus, he suggests focusing on the mutual constitution between two instead of one. Wenger addresses these roles as complementary communities, where individuals' competence and experiences shape the contributions of individuals. He argues that belonging to the COP (participating the work of COP and interacting with members) shapes identities, yet members retain a "unique identity" and "unique experience" (Wenger, 2008). Those "unique identities" are determined by many attributes, particularly their characteristics and experiences in addition to belonging to multiple COPs either in the past or currently.

In this study, despite diverging roles individuals might play, each MTE contributes to the joint activity of co-planning. Activity theorists use the term "division of labor," which means "the shared participation responsibilities in the activity determined by the community" (Yamagata-Lynch & Haudenschild, 2009, p. 508), to describe group members' contributions. Thus, while the community works toward a particular goal, individuals can take separate actions. The division of labor in a COP can be more or less structured and explicit or implicit.

The COP model introduces three main roles individuals play: old-timers, new-comers, and brokers. As researchers study the practice of MTEs using COP perspectives, they define additional roles. While Cochran-Smith (2005) and Jaworski (2008) describe the MTE as an inquirer, Zaslavsky (2007 & 2008) describes MTEs as designers and facilitators of a learning activity in which they engage in reflective practice. In a community of inquiry formed by MTEs and teachers, MTEs develop professional knowledge through their practice as they engage in the joint activity. In their model, MTEs design an instructional activity that facilitates PTs engagement in the activity, and reflect on the practice.

Jowarski and Huang (2014) used the term 'didactician' to characterize the role of an MTE, including university faculty, education researcher, curriculum developer, or mathematics coach. They define 'didactics' as "transformation of disciplinary knowledge into forms through which learners can develop their versions of that knowledge" (p. 175). Jowarski and Huang explain that "Didacticians who are teacher–educators work with practicing or prospective teachers to enable a transformation of theoretical ideas and research findings into modes of teaching that are informed by theory and research" (p. 175). In this description, the authors emphasize the MTEs role is not only to transfer theoretical perspectives, but to facilitate teachers in learning how to implement those perspectives in practice. The roles introduced by MTE researchers (Jowarski, 2008; Zaslavsky, 2008) in addition to COP model (Wenger, 2008) informed my characterization of COP members' roles and guided my investigation of how they contributed to the community.

Other Constructs used in this Study

Co-planning. In co-planning, two or more educators decide together how they will implement instruction to meet the needs of all students (Wilson, 2016). Although different structures and models exist in the literature, all suggest that co-planning is complex, time-consuming work (Albrecht, 2003; Waters & Burcoff, 2007) and should follow a routine to lead a successful practice (Villa, Thousand, & Nevin, 2013). Collaborative planning supports MTEs' professional learning (Albrecht, 2003; Bleiler, 2015) by surfacing expert knowledge so that it is visible to a novice (George & Davis-Wiley, 2000; Gray & Halbert, 1998) and providing opportunities for MTEs to engage in professional discussions (Goodchild, Fuglestad, & Jaworski, 2013)

Feiman-Nemser and Beasley (1997) define co-planning as an assisted performance where novice teachers learn from an expert as the mentor shares her knowledge, thinking, and decision-making. The authors identified patterns of activities, namely *kind of talk,* dominating co-planning episodes: exploring content, designing learning activities, coaching, and clarifying roles. Likewise, Lynch (2017) investigated MTEs co-planning practices and defined three main types of activities MTEs engage in: establishing goals, determining instructional details, and brainstorming.

Three models inform my analysis in determining types of talking taking place in the co-planning meetings: Feiman-Nemser and Beasley (1997), Lynch (2017), and Wilson (2016).

Defining those patterns as sub joint activities COP engage in, I describe their relation to MTEs' expertise and examine how those types of talk facilitate MTE learning.

Broker. Communities of practice approach defines a broker as an individual that introduces new constructs to the community. Wenger states, "brokering requires the ability to manage carefully the coexistence of membership and non-membership, yielding enough distance to bring a different perspective, but also enough legitimacy to be listened to" (2008, p. 110). In Wenger's study, the broker examples he provides are non-members of the focus COP. However, Wenger did not specifically define that the broker has to be a non-member of the focus group. Based on his definition of brokering as "connections provided by people who can introduce elements of one practice into another" (2008, p. 105), and the broker's role as translating and coordinating knowledge from one COP to another, I identify the actual members of the COP as brokers as well. A broker in this study brings new knowledge and acts dominantly in negotiating the meaning of core concept. Although they are part of the COP examined in the study, they are also members of different COPs with varying expertise in different areas.

Chapter Conclusion

In this study, I analyze the work of MTEs as they co-plan a methods course. I consider the group of MTEs as a COP since in this study they engage in a joint practice of, co-planning. Taking a communities of practice standpoint derived from studies on MTEs' practice, I test three different lenses to examine MTE work: 'knowledge', 'roles', and 'type of talk.' I believe looking at work of MTEs through multiple lenses provides us with a thicker description (Geertz, 1973) of MTEs practice. Considering social interaction, collaboration, reflective practices, and productive learning experiences play a crucial role in the professional development of MTEs (Krainer, 2008;

Sanchez, 2011; Van Zoest, Moore, & Stockero, 2006; Wilson & Franke, 2008; Zaslavsky & Leikin, 2004), an analysis of the work of MTEs in a collaborative setting and describing their co-planning through the lenses of their talk, their roles, and their knowledge provides opportunities to understand the practice of MTEs. These findings could open a door for possibilities to think about the professional development of future and current faculty involved in mathematics education.

Chapter 2

Relevant Literature

For my literature synthesis, I discuss main approaches and major findings from the areas of MTE knowledge, practices, and roles, and I explain how they help me to frame my study. First, I investigate literature on TE and the subarea of MTE knowledge. Next, I review the literature on TEs and the subarea of MTEs' professional development and how to facilitate that development. Then, I summarize the previous research on MTEs' roles and practices. Last, I focus on MTEs' co-planning practice. After each section, I discuss emerging questions in the literature and how this study will address those questions.

Characterization of TEs Knowledge

Educators emphasize the need for research about educating teacher educators, yet little empirical work has been conducted on what specific knowledge TEs need (Lunenberg, 2002; Cochran-Smith, 2003). Overall, researchers have based their conceptualization of the knowledge of TEs on five main resources: a) the Standards for TEs (ATE, 2008; Koster & Dengering, 2008; 2008; Lunenberg, 2002), b) literature on professional knowledge (Labaree, 2004; Wilson, 2006), c) TEs' experiences and practices (Doerr & Thompson, 2004; John, 2002) d) frameworks about teacher knowledge (Chauvot, 2008; 2009; Superfine & Li, 2014), and e) TEs' perceptions about essential TEs' knowledge (Murray & Male, 2005; Shagrir, 2007; 2010). In the next section, I elaborate on the details of each approach, explain what they are looking at and how, discuss the differences among the approaches, and highlight common themes across all.

Standards-based approaches for determining essential knowledge for TE are being used globally. Researchers use standards for teaching teachers to identify required knowledge for TEs. For instance, Lunenberg (2002) used VELON-Dutch standards for TEs to describe the professional competences a TE should have when designing a curriculum for TE. He identified competencies for TEs in six different domains: subject matter, pedagogy, communication, organization, reflective practice, and curriculum (See Figure 2-1). Using these competencies as measures, Lunenberg et al., (Lunenberg, Dengering, & Korthagen, 2014) compared TEs' training in three different countries: Netherlands, Israel, and England. Despite the cultural differences, he found the need for competences in similar areas.

TABLE I.

Groups of competences	Examples mentioned above
Subject competences	The teacher educator is able: • To maintain knowledge and skills concerning the own subject • To integrate the complexity of the school practice with the content of the own subject
Pedagogical and didactical competences	The teacher educator is able: • To adapt a series of lessons to a new curriculum, together with colleagues • To create a stimulating learning environment for student teachers • To differentiate between individual students; he or she can coach students with different competencies towards the teaching profession • To explain the didactical choices he or she makes to student teachers • To stimulate students to reflect on their experiences and to self-assess their suitability for the teaching profession
Organisational competences	The teacher educator is able: • To follow actively the vision and policy development of the own organisation
Communicative competences	The teacher educator is able: • To deal with student teachers and with experienced teachers in schools
Competences for learning and growing	The teacher educator is able: • To reflect on his or her own teaching in relation with students and colleagues
Institute specific competences	The teacher educator is able: • To carry out a problem-centred curriculum • To carry out a computer-steered curriculum

Figure **2-1**: The categorization of TEs' competences. Reprinted from "Designing a Curriculum for Teacher Educators" by M. Lunenberg, 2002, *European Journal of Teacher Education, 25* (2&3), p.269. Copyright 2002, by Association for Teacher Education in Europe.

The Association of Teacher Educators [ATE] (2008) identified nine standards in different domains for accomplished TEs in addition to indicators and artifacts that measure how much a TE meets those standards (See Appendix B). Those standards are products of several research findings, evaluations, theoretical analyses, and ongoing discussions, and they address various aspects/requirements of being a teacher educator. The US standards, except for the last three bullet points, address competencies for TEs that are very similar to the Dutch standards (Lunenberg, Dengering, & Korthagen, 2014).

The two examples illustrate some crucial components of being a teacher educator not only for teaching teachers, but also when adopting various roles as a leader, researcher, and educator. Although the standards are more skill-oriented than knowledge-oriented, we could use standards to inform our understanding of knowledge for TEs. Similar to Lunenberg's (2002) classification of competences, by looking across TE standards proposed by The Association of Teacher Educators, we could identify essential knowledge to fulfill requirements for each category. For instance, the Teaching standard requires TEs to develop knowledge about content, instruction, technology, and assessment; the Cultural Competence standard requires knowledge about students learning and instructional models; the Scholarship standard requires knowledge about research methodologies and inquiry; the Professional Development standard requires knowledge about reflective practice; the Program Development standard requires knowledge about teacher education programs; the Collaboration standard requires knowledge about relevant stakeholders to education, and the Vision standard requires broader knowledge about different issues relevant to education. In sum, they are standards generated for educating TEs to address crucial aspects of TEs' professional knowledge.

Some researchers offer a more flexible description of TEs' knowledge that contrasts with a standards-based approach. For example, Wilson (2006) suggested raising inquiry questions regarding essential knowledge for TEs before generating assumptions about it. She categorized what knowledge TEs need in two main areas: a) understanding the practice of teacher education, and b) the practice of teacher education research, which includes: knowledge of theory, knowledge of the discipline, knowledge (and practice in) of teacher education, and knowledge of research methodologies. Likewise, Labaree (2004) categorized essential knowledge for future education researchers in four main areas: a) any knowledge of the methods and theories of relevant disciplines (Labaree, 2004), b) knowledge of theory, c) expertise and knowledge in research methodologies: depth and breadth, and d) expertise and knowledge in teacher education. Although these two suggest taking a more inquiry-based stand in studying TEs' knowledge, they both consider educating teachers and research as complementary and emphasize the role of theory in both.

In contrast, theoretical and practice-oriented approaches have identified vital aspects of TEs' knowledge that emerged from TEs' practices. John (2002) investigated the knowledge and practices of six TEs as they worked with student teachers and provided a detailed description of an MTEs' experiences and knowledge. Although John's categorization has four dimensions—intentionality, practicality, subject specificity, and ethicality—they all highlight the knowledge that is necessary to analyze TEs' practices critically. All of these approaches aim to create a framework for professional knowledge that is specific to TEs.

Rather than generating new constructs from scratch to conceptualize TEs' knowledge, some researchers use existing models about teacher knowledge (Abell et al., 2009; Arbaugh, Nolan, Mark, & Burns, 2012; Liu, 2013; Superfine & Li, 2014). Focusing on the core concepts such as PCK and CK that the two practices share, they extend teacher knowledge frameworks in the context of K-12 to college and upper level courses. Taking this approach, Abell and her colleagues (2009) proposed a trajectory model for the knowledge of science teacher educators. They believed that explicit attention should be given to developing components of PCK for teaching science teachers. Their definition of PCK, similar to Cochran's (1991), is a bigger umbrella that consists of other categories of teacher educator knowledge. The researchers argue that a TE's PCK must include knowledge of curriculum, instruction, and assessment to teach and supervise PSTs. Also, a science teacher educator should know about K-12 teachers' conceptualization of science teaching, potential struggles they might have while developing those concepts, and alternative strategies to promote their understanding of science education (See Figure 2-2).

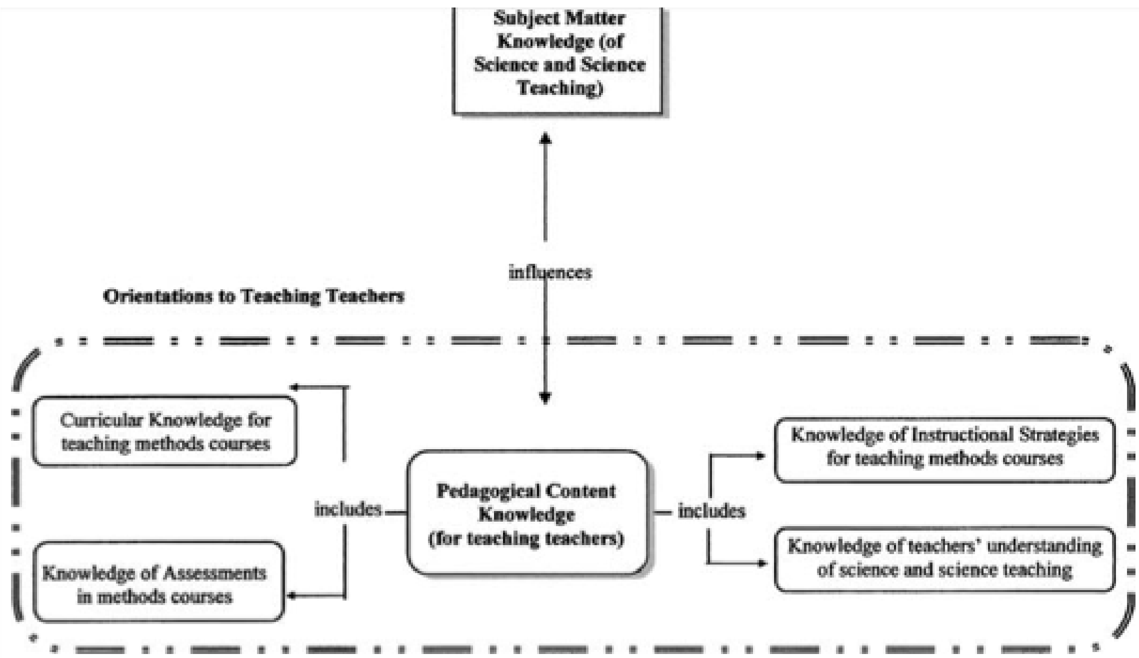

Figure 2-2: A model of PCK for teaching science teachers. Reprinted from "Preparing the Next Generation of Science Teacher Educators: A Model for Developing PCK for Teaching Science Teachers" by S. K. Abell et al., 2009, *Journal of Science Teacher Education, 20*, p. 80. Copyright 2008 by Springer.

Framing his study on the concept of PCK, Liu (2013) conducted an empirical single-case study on what PCK an expert English as a Second Language (ESL) TE has, how he or she developed that PCK, and what components of PCK were actively used. The researcher identified what had been covered in the course content both theoretically and practically and what strategies and pedagogies the TE used while teaching the course. Similar to Abell et al. (2009), Liu (2013) concludes that, although formal education is essential, PCK is gained mostly by experiences in classroom teaching. He identified pedagogical knowledge as the most active component of PCK..

Although the research on teacher knowledge is informative and useful in framing the necessary knowledge for TEs, the nature of teacher knowledge and teacher educator knowledge is different (Cochran-Smith, 2003). For instance, many studies that analyze identity transition from being a teacher to a TE have concluded that "Being a good teacher does not prepare you to be a good teacher educator" (Dinkelman, Margolis, & Sikkenga, 2006; Murray & Male, 2005; Zeichner, 2005). Murray and Male (2005) use the phrase "experts become novice" (p. 135) to emphasize that teaching experience does not guarantee success as a TE. Thus, framing TEs' knowledge using only teacher knowledge models limits addressing the unique and inclusive (Ferrini-Mundy & Floden, 2007; Fey, 2001) nature of TEs' knowledge.

Apart from studies that base their frameworks on what researchers consider vital knowledge, some researchers identify essential knowledge for TEs by focusing on what TEs foresee as essential knowledge. As a part of PD in Israel, Shagrir, (2007; 2010) investigated what knowledge and skills the beginning teacher educators wanted to develop. Analyzing the questionnaires filled by novice TEs, Shaqrir concluded that TEs would like to learn the language essential for the profession. They want to develop a repertoire of solutions for personal problems and difficulties that arise in practical work. They would like to develop a rich understanding of research, theories, and existing approaches to teacher education. Their identification of essential TE knowledge reflects central aspects of TE skills identified by ATE's standards (Shaqrir, 2010). Similarly, Smith (2006) asked Israeli and Swedish novice teachers and TEs about the characteristics of good TEs, the professional knowledge of TEs, and the professional knowledge of teachers. She found that the participants see distinct differences between teacher knowledge and TE knowledge. They classified essential knowledge for TEs in seven categories: a) interpersonal communication knowledge, b) SMK, c) PCK-propositional, d) knowledge about assessment, e) knowledge about research, f) knowledge about the school system, and g) knowledge about students and adult learning. Despite slight differences, both studies illustrate that TEs consider practical and didactical knowledge about learning, teaching, and research essential in educating teachers.

Although they have taken different approaches, these studies all illustrate that TEs need to develop knowledge in multiple domains that extend beyond the knowledge teachers need. In my study, I provide details about the nature of TEs' knowledge in each domain. Having described essential knowledge for TEs in general, in the next section, I review literature in the context of mathematics education and identify essential knowledge for MTEs.

Characterization of MTEs Knowledge

TEs, teachers, and MTEs share overlapping domains of knowledge. While considering MTEs' knowledge as an expansion of mathematics teacher knowledge, Perks and Prestage (2008) describe MTEs knowledge as specialized knowledge of TEs (Jaworski, 2008b). Despite the intersecting domains, the nature of the knowledge domains is different from other TEs (Chazan & Lewis, 2008; Zaslavsky, 2004). The essential knowledge for MTEs is also different from the knowledge of mathematics teachers (Jowarski & Huang, 2014; Tirosh et al., 2014). In this section, I focus on existing literature that describes the knowledge MTEs should develop, including SMK, CK, and PCK.

In recent decades, researchers in the fields of mathematics education have worked to identify crucial knowledge for MTEs (e.g., Appova & Taylor, 2017; Chazan & Lewis, 2008; Doerr & Thompson, 2004; Ferrini-Mundy & Floden, 2007; Superfine & Li, 2014; Sztajn, Ball, & McMahon, 2006; Zbiek & Hirsch, 2008). While some have proposed essential mathematical knowledge, others have investigated overall knowledge MTEs develop in order to teach teachers.

This group of studies contends that the first type of knowledge that is essential for MTEs is the content knowledge of the discipline. Despite the amount of work around knowledge (e.g., Ball, Thames, & Phelps, 2008; Hill et al., 2008) for teachers, few studies have sought to conceptualize content knowledge for teacher educators. AMTE's (2002) report, *Principles to Guide the Design and Implementation of Doctoral Programs in Mathematics Education*, emphasizes the importance of developing mathematical knowledge for doctoral students of mathematics education. The document states, "MTEs need a broad and deep mathematical knowledge both to identify the big ideas in the Pr-K-14 mathematics curriculum and to examine how these ideas develop throughout the curriculum" (p. 4). Van Zoest, Moore, and Stockero (2006) similarly emphasize that the mathematical knowledge required for teaching teachers is beyond the knowledge for teaching K-12. Chazan and Lewis (2008) provide more details about the depth of essential content knowledge and described the degree of mathematics knowledge MTEs need to have for teaching teachers of different grade bands. They suggest developing a strong CK in K-12 curriculum for teaching elementary teachers. For teaching middle school teachers, Chazan and Lewis (2008) suggest that TEs have a strong mathematical background, while for teaching high school teachers they argue that an equivalent to a master's degree in mathematics knowledge is essential. Last, in order to teach mathematics at the undergraduate level, the authors suggest acquiring mathematical knowledge at least at a MS level. Superfine and Li (2014) also investigated what mathematical knowledge TEs should have in order to support PTs' thinking at a high level of cognitive complexity. As a part of the Mathematical Knowledge for Teaching Teachers (MKTT) Project, the authors proposed a professional development model for teacher educators who teach mathematics content courses for elementary teachers. They argue, "mathematics teacher educators need to understand mathematical knowledge for teaching for themselves and should be knowledgeable about ways to connect preservice teachers' mathematical learning to the practice of teaching K-12 students" (p.129-130). Their model combines work on teachers' knowledge, particularly specialized content knowledge from Mathematical Knowledge for Teaching (Hill et al., 2008) model, and features high-quality professional development programs (PDs) (Stein, Smith, & Silver, 1999). Superfine and Li (2014) concluded that, in addition to learning the content that PTs need to know, MTEs need to learn how PTs use that knowledge in teaching so that they could anticipate challenges PTs might encounter when learning to teach mathematics. Mason (2008) conceptualizes this additional knowledge as mathematical knowledge for teaching (MKT), which relates learning to teaching.

In addition to a strong math background, MTEs need rich curricular knowledge (Beswick & Chapman, 2013; Chauvot, 2008; Zbiek & Hirsch, 2008). ATE (2008) defines essential components of curricular knowledge as knowledge of curriculum frameworks for designing and implementing programs, knowledge of how integrated curricula and technology support mathematics learning, and knowledge of relevant topics in the curriculum and how they develop across grades. Zbiek and Hirsch (2008) offer a more detailed description and model for core curriculum knowledge required in mathematics education, including multiple perspectives, principles, and models for designing curricula, and understanding the developmental, enacted and evaluative processes of a curriculum. Also, the authors highlight that TEs should be able to conceptualize, design, conduct, and evaluate research on mathematics curriculum development, which requires knowledge beyond theory. Despite overlapping domains, MTEs' curricular knowledge is different from teachers of mathematics. For instance, MTEs do not need to know the daily implementation of a particular curriculum but should know the theory and design that determined the context and the structure of that particular curriculum (Beswick & Chapman, 2013).

Similar to teachers and teacher educators, MTEs also need to have strong PCK, including knowing their students' (PSTs') development, challenges and inquiries to support their learning (Chauvot, 2009). PCK is considered fundamental knowledge for novice TEs in particular due to their limited experience in teaching teachers (Cochran-Smith, 2003). Researchers describe the nature of PCK for MTEs using what MTEs consider and report as essential knowledge (Appova & Taylor, 2017), and by examining MTEs practice (Jaworski & Huang, 2014). Arbaugh, Nolan, Mark, and Burns (2012) interviewed seven elementary school teachers' mentors and discussed how they conceptualized mentoring using teaching practices. The mentors emphasized knowing the strengths/weaknesses and the needs of their co-teachers and knowing instructional strategies to support their planning and noticing skills. Appova and Taylor (2017) also interviewed ten expert MTEs as they designed and implemented K-8 mathematics content courses. Researchers examined the purposes and reflections associated with their PCK. Their findings suggest four components of PCK: knowledge of instructional strategies, knowledge of curriculum, knowledge of student understanding, and knowledge of assessment.

While some studies have focused on one aspect of MTEs' knowledge deeply, others tried to capture various dimensions of it. Chauvot (2009) for instance, conducted a self-study to identify the knowledge she drew from to fulfill her role as an MTE and researcher. She used multiple frameworks (i.e., Grossman, 1990; Leinhart & Smith, 1985; Ma, 1999a; Shulman, 1986) of teacher knowledge to investigate knowledge structures and the development of a mathematics teacher educator-researcher (MTE-R). Her work provides a detailed investigation of the knowledge an MTE needs to have while teaching a "proportional reasoning" course, mentoring doctoral students, and researching PCK. Jaworski (2008b), described three types of essential MTE knowledge: knowledge of secondary schools and students, knowledge about instruction, and knowledge about PTs and research in mathematics education (See Figure 2-3).

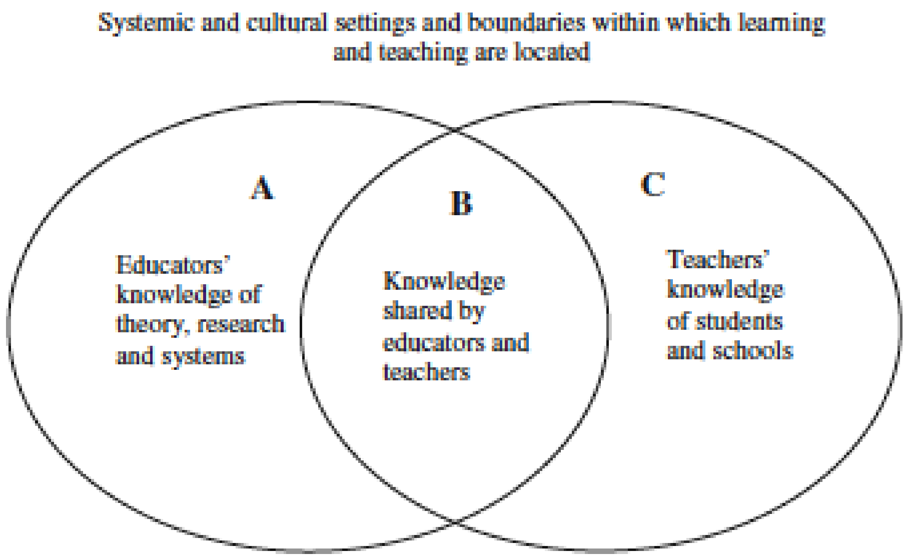

Figure 2-3: Knowledge in teacher education. Reprinted from *The international handbook of mathematics teacher education volume 4: The mathematics teacher educator as a developing professional* (p. 336) by B. Jaworski, 2008b, Rotterdam, Copyright by Sense Publishers.

In addition, the previous literature discusses several other aspects of MTEs' knowledge, including:

- Knowledge of research design and methodologies (See Ferrini-Mundy, 2008; Murray & Male, 2005)
- Knowledge of technology in teaching, learning, and doing mathematics (See Heid & Lee, 2008)
- Knowledge of strategies for promoting diversity and equity (See Taylor & Kitchen, 2008)
- Knowledge of policy regarding teacher education, curriculum, assessment, technology, professional development, and research (See Silver & Walker, 2008).

In conclusion, the work of MTEs is very complicated (Cochran-Smith, 2005) and requires advanced knowledge in multiple domains (Chauvot, 2009; Jaworski, 2008; Zaslavsky, 2008). Although research has identified some aspects of essential knowledge for MTEs, there is less empirical research that examines the specific knowledge MTEs need to design a mathematics methods course for PTs (Even, 2008). Recent studies offer innovations and new constructs, such as pedagogies of practice (Grossman & McDonald, 2008) and rehearsals (Lambert et al., 2013) to facilitate teachers' preparation. To integrate these innovations into their practice, TEs need to acquire new knowledge. "We know very little about the work TEs must do to design and implement what Grossman and McDonald (2008) have called *pedagogies of enactment*" (Kazemi, Ghousseini, Cunard, & Turrou, 2016, p. 189). With this study, I aim to expand Chauvot's model for MTEs' knowledge based on these novel ideas and implications in teacher education.

Knowledge Development for TEs

Experts identified professional skills and expertise TEs should develop during training. "Schools of education that train doctoral students for careers in education research should articulate the competencies those graduates should know and be able to do and design their programs to enable students to develop them." (NRC, 2005, p. 6). To meet these criteria that the National Research Council suggest, how could doctoral programs support teacher educators? Scholars have been discussing how we could improve educational research in general (e.g., Labaree, 2004; Lagemann, 2002; NRC, 2002; 2005), but have focused primarily on doctoral programs for educating researchers (Lagemann, 2002). Different approaches have been offered to support initial and continuing teacher education based on the context where TEs develop professional knowledge: school-based TE development and university-based TE development. Studies investigating school-based TEs are interested in how TEs professionally grow in a school environment, such as programs designed to work with teachers who will lead professional learning groups in their schools (Loughran, 2014). Although the school-led programs are informative, I think university-based programs, particularly at the doctoral level, provide more structured training for future TEs. Thus, in this study, I focus on TEs' professional development in a university-based environment.

Professional Development of TEs

As discussed in the previous section, the knowledge of TEs influences the outcomes of teacher education (Feuer, Floden, Chudowsky, & Ahn, 2013). Nonetheless, not many studies investigate how we educate TEs and how we could improve the quantity of that professional training (Murray & Male, 2005; Cochran-Smith, 2003). In addition, beginning TEs report that it is challenging to establish their professional identities as teachers of teachers in higher education, particularly when becoming an educator-researcher (Murray & Male, 2005), and developing higher education pedagogy (Lunenberg & Hamilton, 2008). For TEs to develop essential expertise and knowledge, they need to go through particular experiences, including research and teaching. However, as Wilson (2006) argues, most TEs are not provided with opportunities to learn about teaching teachers.

> I do not think that many scholars of this new generation have opportunities to learn to teach teachers in structured and scholarly apprenticeships; instead, they are thrown into the practice of teacher education, either as doctoral students or as newly minted Ph.D.s. (p. 315)

Similar to Wilson (2006), some TEs (e.g., Abell et al., 2009; Murray & Male, 2005; O'Sullivan, 2010) highlight the lack of experiences TEs have through their doctoral programs:

> Even though 100% of the doctoral program heads expected their graduates to be able to teach methods courses and supervise student teaching, only 34% required their graduates to be involved in the mentored teaching of a methods course, student teaching, or in-service workshops. Forty-two percent said the students could do this as an elective and 24% said their graduates had no opportunity to be mentored in any of these skills (Jablon, 2002, p. 17)

The literature discusses two primary resources that contribute to sustainable support for TEs: theory and experience. In addition to theoretical support for educating TEs, research on the development of teacher educators has focused on TEs' practices of reflection and collaboration (Llinares & Krainer, 2006). Practice-based learning opportunities would benefit TEs more when reflective practices (Cochran-Smith 2003; Gallego, 2014; Murray & Male, 2005; Zaslavsky, 2007) and collaboration (Graziano & Navarrete, 2012; Van Zoest, Moore, & Stockero, 2006) become a natural aspect of their practice (Cochran-Smith, 2005; Lunenberg, 2002). For instance, in a hybrid case study, Dinkelman, Margolis, and Sikkenga (2006) analyzed two beginning TEs' experiences—doing, thinking, reflecting, and interacting—and documented the resources of their knowledge in two main categories: theory and experience. They concluded that reflecting on the interactions between TEs and preservice teachers and challenges that emerged in practice were powerful tools for professional development. Their findings support the role of reflective practice in TEs' learning.

In addition to the reflective nature of learning in practice, research shows that collaboration among the TE community increases the acquisitions from those experiences. Graziano and Navarrete (2012) developed and implemented a Language Acquisition, Development, and Learning course for PTs. The authors argue that collaboration between instructors with different areas of expertise and perspectives on teaching supported their professional learning, particularly in the areas of students' needs and teaching strategies. Vogler and Long (2003) examined their experience as they were team teaching a social studies/art methods course for teacher candidates. They found that instructors learn from one another as they plan together and observe one another's practice. In another study, Swennen, Shagrir, and Cooper (2009) reviewed the self-study, interview, and narrative case literature of TEs to synthesize the rewards and challenges of beginning teacher educators. They found the common theme across studies is the positive impact of building a community of practice during the transition from being teachers to TEs. Likewise, Pellegrino, Sweet, Kastner, Russell, and Reese (2014) investigated the journey of three doctoral students while becoming music teacher educators within a professional development community. Parallel to previous findings, these researchers highlight the importance that establishing a community of practice holds for enriching TEs' experiences. Abell et al. (2009) argue that doctoral programs should function as communities of practice where TEs develop crucial knowledge and expertise for teaching teachers. These studies all agree that support for TEs should not be limited to the doctoral programs but should be sustained throughout their careers (Abell et al., 2009; Cochran-Smith, 2003; Shaqrir, 2010).

Abell et al. (2009) also argue that doctoral students, through their doctoral program and into the beginning years of being a TE, should be provided with a sequence of meaningful opportunities to develop PCK. It is very crucial for TEs to engage in a diversity of experiences, such as observer of methods instruction, co-teaching or independent teaching experiences, supervising field experiences, and work in science teacher education research to develop adequate knowledge for teaching teachers (Abell et al., 2009). Van Zoest, Moore, and Stockero (2006) suggest that these educational experiences should follow a trajectory. For instance, they suggest that doctoral students co-design methods courses only after they develop a robust understanding of PTs' thinking, similar to the following suggestion by Cochran-Smith:

> I suggest that the education of teacher educators in different contexts and at different entry points over the course of the professional career is substantially enriched when inquiry is regarded as a stance on the overall enterprise of teacher education and when teacher educators inquire collaboratively about assumptions and values, professional knowledge and practice, the contexts of schools as well as higher education, and their own as well as their students' learning. (Cochran-Smith, 2003, p. 21)

Abell et al.'s work is different from others in this area due to its broader description of essential learning experiences for TEs. While most studies focus on one particular experience, Abell and her colleagues suggested a list of necessary experiences TEs should have during their doctoral programs. Emphasizing the role of various experiences in doctoral programs, they offer a model, which explains TEs' learning trajectories through different phases of their career based on various learner roles they take (See Figure 2-4).

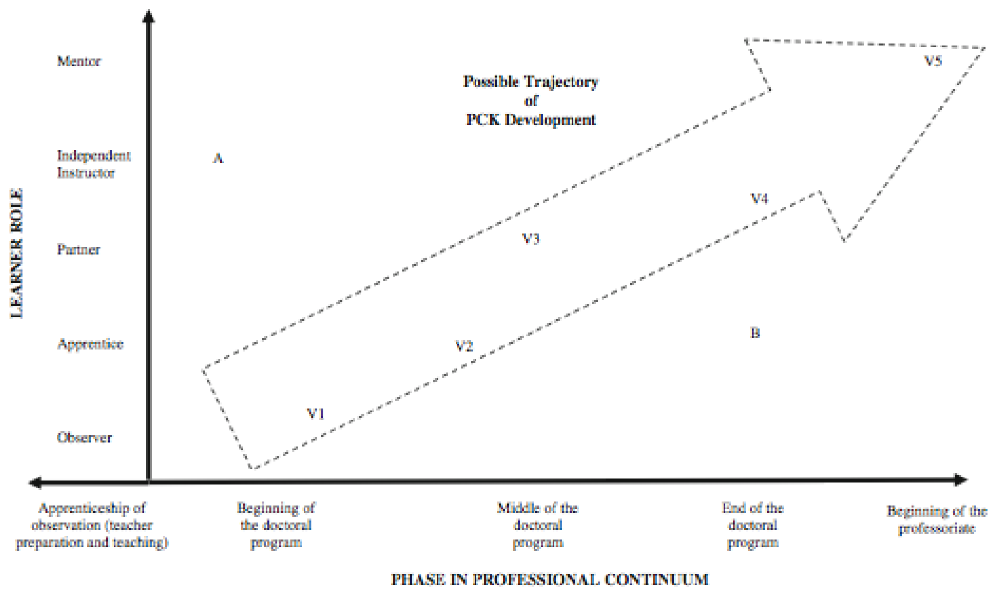

Figure 2-4: A model for the professional growth of TEs. Reprinted from "Preparing the Next Generation of Science Teacher Educators: A Model for Developing PCK for Teaching Science Teachers" by S. K. Abell et al., 2009, *Journal of Science Teacher Education*, *20*, p. 87. Copyright 2008 by Springer.

Identifying essential learning experiences for TEs raises another question: how could TE programs provide TEs with rich learning experiences?

Various studies have been conducted on how to design PDs (i.e., university-led, school-led and partnership models) and doctoral programs for TEs to address different needs of TEs (Koster & Dengerink, 2008; Lunenberg, 2002; Shagrir, 2007; 2010) and offer continuing support for them. Programs that center on practice and reflection in educating TEs received positive feedback. Korthagen, Kessels, Koster, Lagerwerf, and Wubbels (2001) designed a course for TEs using the Realistic Approach. The model focuses on gaining expertise by working with real context problems and systematically reflecting on practice. They analyzed the impact of a training course on TEs using a reflection model, the ALACT Model (Action, Looking back, Awareness of essential aspects, Creating alternatives methods of action, Trial) for supervision. Lunenberg (2002), in the Netherlands, designed a curriculum for beginning TEs both in the context of university programs and schools. Instead of taking a particular educational approach, this design was informed by various resources including the Dutch professional standard for TEs, literature review, case studies, and ongoing conversations with experts.

Despite the existence of various training programs for TEs around the world, the vast majority of TEs in the US and UK are graduates of doctoral programs related to teacher education (Lunenberg & Hamilton, 2008). While some of the programs specialize in teaching in one discipline, such as Mathematics or Science Education, some programs, such as one at Stanford University, offer a general Ph.D. in teacher education. Similar to PDs and training programs, doctoral programs should offer multiple experiences to support TEs' learning. In my dissertation study, I aim to describe the nature of learning experiences for MTEs and investigate in what ways those experiences support MTEs' development. In the next section, I review how researchers conceptualize professional growth of MTEs and what they suggest to promote that development.

Professional Development of MTEs

The majority of the studies conceptualizing professional growth of MTEs reflect MTEs' own experiences and practices (Bleir, 2015; Garcia, Sánchez, & Escudero, 2007; Krainer, 2008; Mohammed, 2008; Tzur, 2001; 2008; Watson & Mason, 2008; Zaslavsky, 2007; Zeichner, 2005). Zeichner (2005) for instance, worked with teachers from various disciplines including mathematics and reflected on his transition from being a classroom teacher to TE. He identified specific experiences that helped him to grow professionally in four categories: a) teaching PTs, b) supervising PTs in their field experiences, c) inquiring into his practice as TE d) developing a rich repertoire of TE literature. Similarly, Garcia, Sánchez, and Escudero (2007), looked at their knowledge growth as they engaged in teaching and research. They analyzed and reflected on their actions after teaching a methods course for PTs. The authors concluded that ongoing reflection and analysis of their practices in addition to knowledge of theory were the leading resources for them to develop their professional knowledge. Likewise, Zaslavsky (2007) found that an iterative process of designing, enacting, and reflecting on challenging mathematics tasks improved their selection and implementation of tasks as they support PTs learning. Zaslavsky formed a community of inquiry with a group of MTEs and teachers. MTEs develop professional knowledge through the processes of designing an instructional activity, facilitate PTs engagement in the activity, and reflect on the practice (See Figure 2-5). Similarly, Wu, Huang, and Cai (2017) found that MTEs develop strategies to deal with challenges that emerge as they fulfill their responsibilities by reflecting on their practice.

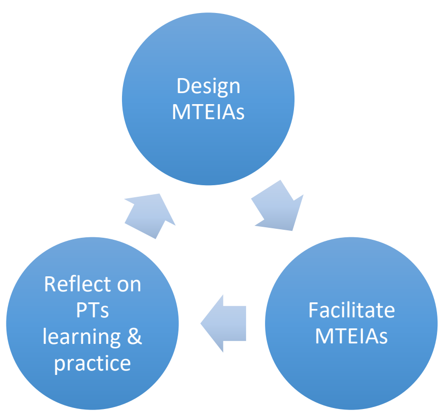

Figure 2-5: MTEs' iterative process of designing, enactment, and reflecting on mathematics task (Zaslavsky, 2007).

Krainer (2008) and Tzur (2001) wrote a reflective analysis of their professional growth as MTEs. Based on his reflection on his experiences, Tzur (2001) conceptualized MTEs' development in four main domains. He added two more developmental areas to Zeichner's (2005) list: learning mathematics and mentoring MTEs. In another self-study, Mohammed (2008) wrote her personal story of becoming an MTE in Pakistan. Different from previous studies, she defined knowledge growth as gaining skills to deal with conflicts emerging from her interactions with teachers and schools. Tirosh et al. (2014) used videos as a tool to support teacher learning and inquired about their uses of video in professional development. The authors documented their own professional growth as MTEs in using video analysis to support teachers' development. These reflective studies show us, as in teacher education (Hiebert et al., 2007), inquiry and reflection plays an essential role in understanding MTEs' practice and professional growth. Similarly, Doerr and Thompson (2004) looked at understandings of MTEs as they work with PTs. Unlike the previous studies, these authors did not reflect on their own practice but analyzed the practices of four expert MTEs as they used video case analysis with PTs. The participants stated that video case analysis helped them to develop a better understanding of PTs' content and pedagogical knowledge in addition to unpacking the concept of professional knowledge for MTEs. They concluded that video case analysis was a useful tool for supporting the professional development of MTEs. Another PD designed in the Netherlands (Dolk, den Hertog, & Gravemeijer, 2002) also used video case analysis to support MTEs for learning mathematics and teaching mathematics. These researchers documented the advancement in MTEs' "noticing" skills. Based on their findings, the authors proposed a six-stage model for MTEs to learn from their own practice: a) observing, noticing multiple aspects of teaching; b) sharing and discussing their observations; c) analyzing the practice; d) reflecting; e) developing narrative knowledge (generating stories of classroom practices); and f) generating statements about teaching. Two common themes arise across all these reflective studies: the need for MTE's development in multiple areas and the correlation between essential experiences and advancement of MTEs' growth.

Similar to other TE programs, experts emphasize the role of experiences in developing essential knowledge and skills for MTEs (Superfine & Li, 2014). Wilson and Franke (2008) offered a model named “tension in preparing MTEs to teach” (p. 106) on how to prepare MTEs. In their model, they emphasize the importance of MTEs engaging in apprenticeship opportunities within their community of practice in addition to completing an appropriate amount of theory and research related coursework. They also point out the importance of building a collaborative environment for MTEs by bringing candidates with diverse backgrounds and experiences into the program. In the words of Roth (1998) “Knowledgeability comes from participating in a community’s ongoing practices. Through this participation, newcomers come to share community’s conventions, behaviors, viewpoints, and so forth; and sharing comes through participation” (p. 12).

A number of empirical studies in the mathematics education literature have shown the positive impact of communities of practice in MTEs professional development (Beiler, 2015; Kieran, Krainer, & Shaughnessy, 2013; Tirosh et al, 2014). The communities of practices described in the literature vary. While some documented MTEs’ professional growth as they work with teachers collaboratively, mostly in PDs (e.g., Kieran, Krainer, & Shaughnessy, 2013; Sakonidis & Potari, 2014), some researchers examined their learning as teaching a mathematics course (Bleir, 2015; Rowland, Turner, & Thwaites, 2014). Bleir (2015) studied experiences of a mathematics education and a mathematics faculty as they team-teach a content and methods course for PTs. The author argues that crossing boundaries of communities of practice offered rich opportunities for both educators and professional learning. While mathematics faculty developed a better understanding about students and gained new instructional for his practice, the mathematics education faculty learned the skills of rationalizing her instructional moves and reflecting on her practice. Tirosh et al. (2014) designed a PD for sixteen preschool practicing teachers. The authors used video analysis as a tool to support teachers learning of teaching mathematics. Throughout the PD, five MTEs established communities of inquiry where they examine their practice. Researchers reported increasing professional learning about using representations of practices in teaching teachers in addition to developing preschools teachers' communities of practice. Even (2008) designed a two year long PD, the MANOR Program– a national program for mathematics teachers– for MTEs. This project intended to educate a professional group of mathematics educators (75 experienced mathematics teachers) whose role was to support professional development of in-service teachers. Even's (2008) findings illustrate that through engaging in an active community of practice, MTEs gained knowledge, skills and practices that are required to teach teachers. Similarly, Zaslavsky and Leikin (2004) investigated the conditions that contributed to their own training and professional growth within the community of mathematics educators. They attended a PD for MTEs (consisting of 120 teachers, 20 TEs and a TE educator) as a part of a 5-year long project. In their findings, Zaslavsky and Leikin (2004) introduced a three-layer model of growth through practice to describe the professional development of beginning MTEs. They highlighted the role of reflective practices and collaboration in the development of professional growth.

In a different PD setting, Van Zoest, Moore, and Stockero (2006) examined the professional growth of three beginning MTE-Rs under the mentorship of a more experienced MTE in the context of teaching a middle-school mathematics methods course. These researchers suggested two main categories where MTEs need support: a) understanding PTs' thinking, and b) balancing "telling" —sharing their experiences as teachers —and "inquiry" — promoting PTs' ability to inquire and reflect on mathematics and teaching practices. Providing support for novice MTEs is essential to change their perceptions of the instructional activities and to use those activities purposefully. In this study, for instance, the mentor used video cases to engage PTs' productive discourse around practice and teaching analysis. However, the novice MTE initially assumed the goal of the activity was to model an ideal instruction. To develop required expertise, Van Zoest, Moore, and Stockero (2006) recommend doctoral programs provide opportunities for

MTEs to engage in conversations about the professional identities and practices of their colleagues, where they could work collaboratively with other novice and experienced MTEs.

The literature on team planning shows that collaborative teaching and planning supports faculty professional learning (Albrecht, 2003; Bleiler, 2015). Experts highlight the need for collaboration among mathematics education and mathematics faculty (CBMS, 2001, 2012). Bleiler (2015) explored mathematics and mathematics education instructors' perceptions of development during their team-teaching collaboration. The instructors found themselves deeply engaged in contemplation and rationalization of their practice and increasing their reflective practices. While mathematics education faculties found "participation led her to reflect on the importance of being able to provide explicit justifications for her instructional decision making", the mathematics faculty found "participation in the team-teaching collaboration led to his increased understanding of student needs and a renewed vision for mathematics instruction in his classroom" (p. 242-243).

Another attribute that supports MTEs' professional growth is bringing research into teaching. Adopting research-based intervention in instruction increases TEs capacity to solve problems in the classroom (Villa, Thousand, & Nevin, 2013). One example of empirical work is a study conducted during a PD in which 65 MTEs with different backgrounds worked together to design a mathematics content course for PTs (Sztajn, Ball, & McMahon, 2006). The researchers reported that the framework of Mathematical Knowledge for Teaching (MKT) — which was particular to the focus content course — provided a common language for participants to work productively in designing and teaching the course. Similarly, Rowland et al. (2014) investigated their professional development as they plan and enact a course for novice teachers that is designed around the Knowledge Quartet (KQ), a theoretical framework to analyze mathematics teaching. Their findings show that using a KQ in their design and teaching supported MTEs' own professional learning.

To summarize, there is an increasing interest in the field about how to educate MTEs and design programs to support their professional growth. They all address the need for MTEs to engage in rich learning opportunities including collaborative practices and reflective practices to develop expertise and skills to teach PTs. I believe there is a need for more studies that describe those experiences in detail. Masingila et al. (2012) found that many novice MTEs who teach future teachers felt unprepared and reported receiving limited support from their institutions. By providing a detailed description of the nature of MTEs' practice as they design and enact a methods course, I aim to contribute to the ongoing discussion of facilitating TEs' education, particularly MTEs' professional development.

MTEs' Practice

As discussed in the introduction chapter, the work of MTEs is complex and there is limited research on the practices of mathematics teacher–educators (Doerr & Thompson, 2004; Tirosh et al., 2014). Educators suggest different ways to increase our knowledge about MTEs practice. Even (2008) argues that researchers should study mathematics teacher–educators' practices cross-culturally while Bergsten and Grevholm (2008) and Superfine and Li (2014) emphasize the importance of studying the relation between MTEs' knowledge and their practices. In order to study and describe the work of MTEs, researchers focus on various decompositions/aspects of their practice, such as: MTEs noticing student learning (Amador, 2016), MTEs determining goals (Appova & Taylor, 2017; Li & Superfine, 2018), MTEs selecting and implementing mathematics tasks (Zaslavsky, 2007), MTEs facilitating PTs' knowledge of students (Taylor, 2013), MTEs promoting equity (Han, Vomvoridi-Ivanović, Jacobs, Karanxha, & Feldman, 2017), MTEs using video analysis in teaching (Doerr & Thompson, 2004; Tirosh et al., 2014); MTEs preparing novice teachers to lead discussions (Baldinger, Selling, & Virmani, 2016; Moss, 2011), and MTEs facilitating PTs posing purposeful question (Arbaugh, Freeburn, Graysay, & Konuk, 2018).

Addressing various aspects of MTEs' practice, these studies illustrate some limitations in current practices and suggest ways to improve MTEs' instruction. Zaslavsky (2007) demonstrates that MTE's practices of selecting, designing, and implementing challenging mathematics tasks facilitate PTs' learning of deeper mathematics and teaching of mathematics. These findings show that designing a high quality task that offers rich learning opportunities for PTs is a challenging task and requires time for MTEs to develop this skill. Vomvoridi-Ivanovic and McLeman (2015) investigated the instructional practices of MTEs who adopted equity lenses to promote equity in their classrooms. The authors presented twenty-three MTEs' self-reports about the challenges they encountered and the resolutions they implemented when teaching mathematics methods courses. Similarly, their findings suggest that MTEs' practices are limited in implementing equity principles and MTEs need more support to build a robust understanding of equity and how to implement in their practice. Another aspect of MTEs practice is noticing students learning. Kazemi et al. (2011), during a PD on mathematical tasks for teacher leaders, examined what expert MTEs notice and how their noticings impact their practices. The authors then modified the structure of PD to address teacher leaders' interests and needs. Different from Kazemi et al.'s work, Amador (2016) examined the professional noticings of novice MTEs as they taught PTs or conduct PDs for practicing teachers. Each MTE selected one student and focused on students' mathematical thinking. They observed and collected data about their selected student and shared their noticings and analysis with other MTEs. They found limitations in MTEs' noticings the connection of mathematical thinking and general principles about learning. Based on observed trends and levels, the authors suggest that novice MTEs should engage in more noticing activities followed by reflections.

Appova and Taylor studied how MTEs articulate goals and what challenges they encounter executing those purposes as they design and enact elementary content courses. The authors reported six expert MTEs' perspectives and purposes, and how they impacted instructional decisions through the course. They documented differences in MTEs purposes and found how these differences influence the opportunities to learn for PTs. This study shows us the impact of MTEs goals on their choices, instructional decisions, and practice.

In addition to these studies, MTEs also shared different aspects of their work in practitioner's journals. For instance, Lee, Ive, Starling, and Hollebrands (2010) shared their experiences as they designed a five-week unit on data analysis and probability in the methods course focusing on teaching statistics with technology. They listed the elements of technological, pedagogical, and statistical knowledge that MTEs used in designing a statistics method course. Similarly, Steele (2008a) designed a mathematics content method course on geometry and measurement. Although he focused on the context knowledge, he addressed other types of knowledge (i.e., PCK) that MTEs need in order to design a mathematics content course. Likewise, Mathews (2004) reported her experiences and observations in teaching a calculus course for middle school mathematics teachers. She shared her analysis of student thinking processes (inductive more than deductive), offered strategies to promote deductive thinking (exploring connections between concepts) and suggested ways for MTEs to gain expertise (whom to consult for course designing and implementing course activities and experiments). This study contributes to the field by sharing MTEs' experiences and different aspects of their practice as they co-plan a methods course.

Collaborative Practices in Higher Education

The studies around collaborative teaching in higher education aim to provide theoretical lenses for collaboration, describe team teaching models and implementations, and evaluate the collaborative experiences (Nevin, Thousand, & Villa, 2009). In the literature there exists various models of team teaching in higher education: collaboration of a special educator and general instructor (e.g, Kluth & Straut, 2003), faculty developing and teaching a course together (e.g, Waters & Burcroff, 2007), co-planning and teaching sections of the same course (e.g., Albrecht, 2003; Vogler. & Long, 2003), faculty team teaching with doctoral students (e.g., George & Davis-Wiley, 2000; Gray & Halbert 1998), and faculty collaborating with teachers (e.g., Kieran et al., 2013).

Collaboration in mathematics teacher education is strongly recommended by experts, particularly among mathematics education and mathematics instructors (CBMS, 2001; 2012). As discussed in the professional development section, collaboration provides plentiful opportunities for professional development of teacher educators. In addition, collaborative work between faculty and their graduate assistants (George & Davis-Wiley, 2000; Gray & Halbert, 1998) illustrates that co-planning and co-teaching can be used for the promotion of doctoral student professional development. Although the importance of the collaboration is emphasized in the literature, there is not much work that examines the nature and process of collaboration, particularly the ones that lead to successful practices for future teachers (Bleiler, 2015). Nevin, Thousand, and Villa addressed the limited conceptualization and resources for collaborative work in higher education:

> Within the social psychological framework of cooperative group learning, there are two major processes, goal and resource interdependence. That is, there exists no curriculum for teacher educators to become co-teachers with others in higher education. There is no

> information about how department chairs or deans might work together to establish the culture for co-teaching to thrive. There are no models for research that assess the impact on student achievement when professors co-teach. (2009, p. 572)

Even though it is given less attention, collaboration in teacher education is essential because it models team teaching strategies and models for future teachers to meet the needs of an increasing diversity of students (Graziano & Navarrete, 2012; Nevin, Thousand, & Villa, 2009). This is the case especially if modeling different co-teaching structures and making the collaboration explicit to the students, while sharing different perspectives and controversies, in addition to achieving consensus with the students (Kluth & Straut, 2003). Collaboration between instructors with different skills, expertise, and perspectives provides a rich learning experience for students (Vogler & Long, 2003) and provides effective differentiated instruction (Graziano & Navarrete, 2012).

To provide details about the nature of collaborative practices, experts use different lenses. One way to describe the collaboration type is by identifying the roles each member takes in teaching teams.

Roles in Collaborative Practices

A majority of collaborative teaching takes place in educational settings where educators focus on meeting the individual needs of students. Studies of these settings describe two primary roles: the general instructor who plans a lesson and a special educator who makes some accommodation based on students need (Graziano & Navarrete, 2012). With the recent emphasis on collaborative practices in education, team teaching finds implications in other educational settings, especially teacher education (Cochran-Smith, 2005). Wilson (2016) categorizes five different co-teaching models based on the types of roles team members take: one teach/one support, teaming ("ping-pong"), alternative ("back-table") model, parallel (two heterogeneous groups of learners), and station (rotation of learning groups). Her model has been widely implemented in various co-teaching settings including higher and teacher education. Villa, Thousand, and Nevin (2013) point out that the roles team members take vary based on the group of learners and collaboration approach that dominates the teamwork. The authors identify individual members of the groups such as teacher, special educator, paraprofessional, speech and language therapist, and supervisor. They also introduce four main types of collaboration models: supportive co-teaching (i.e., one takes the lead, the others provide support for learners, mostly one-one), parallel co-teaching (i.e., two or more instructors work with different groups of learners), complementary co-teaching (i.e., one teacher supports the instruction with expending ideas and providing additional strategies), and team teaching (i.e., instructors share responsibilities and authority in making decisions, planning instruction, assessing equally). The roles they describe are very similar to Wilson's model (2016). The authors argue although the flexibility of roles enables teachers to adapt their instruction based on students' needs but also creates confusion for students. They argue it is essential to make the roles of team members clear. In another study, Waters and Burcoff (2007) described three models of co-teaching: parallel teaching, station teaching, and one teach/one assist teaching. In all these models, collaboration approaches are dominantly shaped by the roles and responsibilities team members are assigned. In the next section, I will discuss the literature about MTEs' roles in collaborative practices.

MTEs' Roles in Collaborative Practices

Researchers discuss multiple roles MTEs take under the categories of their responsibilities, identities, and function in their professional communities. An MTE could be a university faculty member/instructor, education researcher, curriculum developer, supervisor, or mathematics coach (Jowarski & Huang, 2014) with the primary role of facilitating teacher learning (Zaslavsky, 2007). Based on surveys gathered from seventy-seven Chinese MTEs, Wu and Huang (2017) described four main responsibilities for MTEs: teaching pedagogical courses, teaching problem-solving courses, teaching college mathematics courses, and supervising student teaching. Two common identities that have been used to describe MTEs in literature are experts and novices. Despite varying characteristics, an MTE with a depth of knowledge in the profession and expertise in teaching PTs is usually defined as an expert. Appova and Taylor (2017) describe the characteristics of an expert MTE as: (a) having at least a Master's degree in mathematics or mathematics education (b) having at least fifteen (15) years of combined K-12 teaching experience and teaching mathematics content courses for PTs at the university level and (c) being professionally active in the field by attending/presenting at local, state, and national professional meetings in addition to teaching mathematics courses for PTs.

Another term used to characterize the role of an MTE is "didactician" (Jowarski, 2008a; 2008b). Jowarski and Huang (2014) define didacticians as "teacher–educators work with practicing or prospective teachers to enable a transformation of theoretical ideas and research findings into modes of teaching that are informed by theory and research" (p. 175). Coles (2014) studied mathematics teachers learning with video analysis and added another role for the didactician: a heightened listener. In the literature, MTEs are also identified as designers. While Zaslavsky (2007) describes the role of MTEs as designers of mathematics tasks as they work to provide rich learning opportunities for PTs, Li and Superfine (2018) identify MTEs as designers of the learning goals for instructional activities. As the authors conduct a cross-case analysis of six expert MTEs who design elementary mathematics content course, they define MTEs as "designers who leverage their understanding of the domain (i.e., mathematics), their knowledge of learners (i.e., preservice teachers), as well as their beliefs about teaching and learning to create learning experiences that meet their instructional goals" (p. 181-182). Two other terms commonly used to describe MTEs work are inquirer and reflective practitioner. These two roles are mostly emphasized in the literature that takes a COP approach to teacher education. MTEs, as inquirers and reflective practitioners, learn together and from one another (Cochran-Smith 2003; Jaworski 2004). Zavlasky (2007) adds one final role to communities of inquiry—critics, in which MTEs analyze their own and the practice of others with a critical lens. With this study, I describe the predominant roles MTEs take as they co-plan a course for PTs.

Collaborative Planning

Co-teaching experts find co-planning an essential component for co-teaching. They argue that without co-planning, lessons often remain unchanged (Albert, 2003). Combining different skills, areas of expertise, and perspectives provided a rich learning experience for students (Vogler & Long, 2003). Collaborative planning provides the opportunity for teachers to engage in natural discussions of pedagogical content knowledge. Collaborative planning creates the environment for teachers to discuss and broaden their pedagogical content knowledge because they may be asked to make their knowledge and understanding knowable to others (e.i., Goodchild, Fuglestad, & Jaworski, 2013; Roth McDuffie, Mather, & Reynolds, 2004). The vast amount of time co-planning requires is a challenge for the instructor (Villa, Thousand, & Nevin, 2013; Waters & Burcoff, 2007). One way to minimize the time devoted for co-planning is establishing productive routines.

Examining daily routines and establishing roles, responsibilities, and co-teaching models increase efficient co-planning. No matter how good the intentions of the co-teachers, co-planning every aspect of every lesson is daunting—if not impossible. By concentrating on the class elements that are routine, and then identifying ways to make the co-teaching of these elements routine, co-teachers minimize the amount of co-planning that is needed and can thus maximize the effectiveness and efficiency of their partnership (Wilson & Blendick, 2011).

The literature discusses different formats (models) for co-planning and provides planning tools such as co-planning templates. Most previous studies focused on the co-planning structure between a general instructor and special educator. These models explain different routines and suggest strategies to increase the efficiency of co-planning meetings as well as meeting each student's need. Wilson (2016) suggests that "Co-planning must be done routinely and strategically" (p. 39) to have a productive and effective co-planning. The authors propose a co-planning routine where instructors create, organize, plan, look, anticipate, and notice (See Figure 2-6). Successful co-planning practices require explicit, clear determined objectives and rationale to for co-planners to communicate (Villa, Thousand, & Nevin, 2013).

The Road To Co-Planning

Create a co-teaching team based on trust, hard work, reflection, and an openness to new ideas, and understand that a co-taught inclusive classroom is substantially different from a solo-taught classroom.

Organize teaching time by analyzing routines and incorporating powerful co-teaching models such as parallel and station.

Plan to scrutinize the efficacy of the strategies that you teach and that students use.

Look at technology as a way for students to access the complex components of learning.

Anticipate increased attainment of goals by students.

Notice that by varying the co-teaching models used, adjusting strategies, and incorporating technology into routines, student learning and knowledge is enhanced.

Figure 2-6: A model for co-planning. Reprinted from *Co-planning for Co-teaching: Time-Saving Routines That Work in Inclusive Classrooms* by Wilson, 2016, p. 608, Alexandria, VA: ASCD. Copyright 2016 by ASCD.

Feiman-Nemser and Beasley (1997) see co-planning as an assisted performance, a form of mentoring for novice teachers. They argue,

> Through joint planning, a mentor can model an approach to planning, make explicit her thinking and decision making, share practical knowledge about students, subject matter and teaching. By participating with the mentor in the activity of planning, a novice can gradually construct a framework for planning." (p. 110).

Their work shows that co-planning practice provided novice teachers with an opportunity beyond observing a working model of planning but rationalizing instructional decisions and developing their own co-planning practice.

Table **2-1**: Components of co-planning episode.

Table 6.1: Components of a Co-planning Episode

Segment	Focus	Kind of Talk
1	Sharing 1st impressions	C
2	How to begin the unit	D
3	Studying *Up in the Air*	C
4	Summarizing key ideas	D
5	Reading *Storm on the Jetty*	C/D
6	Gathering up ideas	D
7	Studying *Boxes*; how to use it	C/D
8	Studying *Number Art*	C
9	Designing culminating activity	D
10	Clarifying roles	R
11	Planning 1st lesson	D/T
12	Blocking out unit/anthol. project	D/T

Notes: C = Exploring content D = Designing learning activities T = Coaching for teaching R = Clarifying roles

Note. Reprinted from Mentoring as assisted performance: A case of co-planning. *Constructivist teacher education*, by Feiman-Nemser & Beasley, 1997, p. 112. Washington, DC: The Falmer Press. Copyright 1997 by Taylor Francis Inc.

As they describe the structure of co-planning, they identified patterns of activities taking place in co-planning episodes (See Table 2-1), and how those components assisted novices' learning. The authors broke the conversation between teachers into segments based on the focus and the purpose of their dialogue and identified patterns, namely kind of talk. Lynch (2017) investigated MTEs' co-planning practices by gathering data from planning sessions and interviewing the members of the co-planning group. In her description of the co-planning structure, Lynch (2017) defined three main types of activities that MTEs perform as members of COP: establishing goals, determining instructional details, and brainstorming.

Similar to the characterization of co-planning activities by Feiman-Nemser and Beasley (1997) and Lynch (2017), in this study, I describe co-planning activities in a different context. I describe patterns of talk taking place in the co-planning meetings. Studying patterns of talk COP engage in their practice not only enables educators to establish effective co-planning structures but also helps them learn more about MTE practice as they design methods courses.

Chapter 3

Methods and Procedures

The purpose of the study is to investigate the work of MTEs in the context of co-planning a methods course for mathematics PTs. The group of MTEs forms a COP since the group engages in a joint practice, in this study, co-planning. Taking a COP standpoint and derived from studies on MTEs practice, I test three different lenses to examine MTE work: ‘knowledge,’ ‘roles,’ and ‘type of talk.’ I begin this chapter with a description of the design of the study and how it informed data collection and analysis. Then, I explain the context of the study followed by the methods for data collection and data analysis. I conclude with a description of how I established trustworthiness for this study.

Design of the Study

In order to document ‘knowledge,’ ‘roles,’ and ‘type of talk,’ I use existing models for TEs’ knowledge, roles and responsibilities, and practice. In addition, I analyze potential opportunities for MTEs’ professional development as they engage in co-planning activities, basing my analysis in a COP perspective. My research questions are:

In the context of planning a secondary mathematics method course organized around iterative Cycles of Enactment Instructions (CEIs) (as described below in the “setting” and Appendix A),

1. What types of *knowledge* surfaced and were used during the co-planning sessions?
2. What *types of talk* did the MTEs engage in during co-planning sessions?
3. What *roles* did the MTEs adopt during co-planning sessions?
4. In what ways are these three phenomena connected?

I collect and analyze data from a single embedded case study with four participants, and present findings from this study. Case study is a methodology used in descriptive research (Svensson, 1984). A case study is defined as "the documentation of some particular phenomenon or set of events, which has been assembled with the explicit end in view of drawing theoretical conclusions from it" (Stake, 1995, p. 6).

A case study approach allows me to address my research questions in this particular context for three reasons. First, a case study "aims to delineate the nature of contemporary phenomena through detailed investigation of a case or cases and within a specific context" (Yin, 2013, p. 18). Researching participants in their natural settings is an essential aspect of qualitative studies. Creswell and Creswell (2018) explain, "seeing participants behave and act within their context is a major characteristic of a qualitative approach" (p. 181). Case studies seek to document events, rather than abstract concepts, within real life situations (Yin, 2013) and they enable me to observe, describe, and document dimensions MTE practice as it naturally occurs. In my study, I investigate aspects of MTEs' professional practice (phenomena) while they are co-planning a methods course for preservice teachers (real-life context). As in all descriptive approaches, case study enabled me to "discover new meanings, describe what exists, and determine the frequency with which something occurs and/or categorizing information" (Dulock, 1993).

Second, I use a case study because one of the most powerful characteristics of this method is to illuminate relationships between constructs that are impossible to discern from large-scale correlational research (Stake, 1995; Yin, 2013). A case study reveals associations or relationships among selected variables, in this case MTEs' 'knowledge,' 'type of talk,' and 'roles' and answers "what", "how" and "why" research questions rather than cause-and-effect validation. I am more interested in the processes of MTEs' practices than in evaluating the effectiveness or success of their work. Because case studies are non-experimental but observational designs requiring limited researcher manipulation (Yin, 2013), I have little control over the occurrence of events and attributes in this study. Although conducting non-experimental studies may seem less scientific, they provide “thick description”[2] about an action, or construct and generate theories and hypotheses that are valuable even though they are not universal (Yin, 2013). Thick description explains a phenomenon in detail within its context (Geertz, 1973). It offers multiple perspectives about a theme and makes the findings richer and more realistic and (Creswell, 2018). Thus, case studies’ descriptive and explanatory power (Stake, 1995) allows me to describe characteristics of MTEs co-planning practice from multiple lenses in my study. Furthermore, a case study approach enables me to discuss the behavior of a group of MTEs instead of one individual in that group (Yin, 2013).

Last, a case study can gather data from a wide variety of sources, including documentation, direct & participant observation, interviews, archives, and artifacts (Yin, 2013). Instead of relying on one source of data, its findings rely on triangulation of data. In this study, I use multiple sources of data, including audio- and video-recordings and field notes, which work to reveal a deeper meaning of the data (Patton, 2002), and facilitate the verification of my results (Creswell & Creswell, 2018).

Yin (2013) details five crucial components for research design in a case study: a) identifying the research question, b) formulating a hypothesis, c) defining the case and boundaries, d) connecting data and the initial hypothesis, e) interpreting the results. In this study, I followed an approach similar to Yin’s suggestion. First, Yin (2013) recommends selecting a case that either “(a) predicts similar results (a literal replication) or (b) produces contrasting results but for predictable reasons (a theoretical replication)” (p. 46). I predicted the overall structure of MTEs’ professional knowledge by relying on theoretical propositions based upon existing literature and empirical studies, which is a preferred strategy for case study analysis (Yin, 2013). Yin (2013) also advises defining a unit of analysis —the case— that can answer the core research questions. Stake (1995) further clarifies this, adding that the case should be an object (event, action, or construct) that takes place in a certain time and specific location. Here, I defined the intrinsic case[3] as the 'MTEs' nature of work as they collaboratively plan and enact a methods course for secondary mathematics preservice teachers,' which is bounded by the method course.

In the following section, I first describe the context of the study. Next, I describe data resources. Then, I explain how I analyze the data. I conclude with a description of limitations of the study and how I establish trustworthiness.

Context of Study

Setting

This study took place in a collaborative secondary mathematics methods course planning/enactment group that met on a weekly basis at a large Mid-Atlantic university during the Fall 2013 and Spring 2014 semesters. The members of the group were three mathematics education doctoral students and an expert MTE. The group's joint activity was to plan and re-design one of three secondary mathematics courses that PTs at this institution take in their mathematics teacher preparation program. This particular methods course for teaching mathematics at the secondary level aims to introduce and support PTs' learning of high leverage practices for ambitious mathematics teaching and focuses particularly on eliciting student thinking and posing purposeful questions (NCTM, 2014). The participants of the study adopted a pedagogies of practice approach (McDonald, Kazemi, & Kavanagh, 2013) and worked

[3] intrinsic case: a case study where "a researcher wants to understand a particular case (Stake, 1995, p. 437)

collaboratively to design instructional activities to support PTs learning to teach. They used a Cycles of Enactment and Investigation (CEIs; Lampert et al., 2013) structure for engaging PSTs in learning to teach through the use of representations, approximations, and decompositions of practice (Grossman et al., 2009) (See Appendix C). The group met weekly to discuss and plan the content, materials, and activities for the course. During these regular meetings, individuals took various responsibilities as they planned, evaluated, and revised the instructional activities. These meetings were audio-recorded through two consecutive semesters. This group, which I define as the community of practice in this study, consists of four MTEs, which I describe in the next section (using pseudonyms for the participants).

Participants

Dr. Finn (F) is an experienced and award-winning mathematics teacher educator. She taught high school mathematics for 11 years before completing her doctoral work in Curriculum & Instruction with an emphasis in mathematics education. She was awarded a Ph.D. in December 2000 and has been a faculty member in mathematics education since that time. Although she has taught numerous methods courses for secondary mathematics education majors, Fall 2013 represented the first semester that she designed the course around pedagogies of practice (McDonald, Kazemi, & Kavanagh, 2013). Her area of scholarship is teacher education; she regularly publishes her scholarship in research journals, practitioner mathematics journals, and books. Dr. Finn recently completed a five-year term as co-editor of the leading research journal in teacher education.

In the semesters during which this group did their work, Bruce (B) was pursuing a Ph.D. in mathematics education. He completed his degree in 2016. He earned a BS in Secondary Education and a M.Ed. in Curriculum and Instruction. Prior to enrolling in the doctoral program, Bruce taught mathematics at the secondary level for five years. In addition to his teaching experience in high school, Bruce was an instructor for the same mathematics methods course prior to this study's focus semesters, and it was with his urging that Dr. Finn adopted a pedagogies of practice approach for the Fall 2013 offering of the course. While Bruce worked actively in other research groups with different foci during his doctoral studies, his primary research interest was in pre-service mathematics teacher education.

Dan (D), the third participant, was also a doctoral student in mathematics education. He earned a BS in Secondary Education and a M.Ed. in Curriculum and Instruction prior to his arrival in the PhD program, and taught multiple mathematics courses in a public high school for 11 years. He also completed his MA in mathematics concurrently with his Ph.D. in Curriculum and Instruction/Mathematics Education. Like Bruce, Dan also contributed to several research groups during his doctoral program, worked as an assistant editor for a leading mathematics education research journal, and taught courses for pre-service teachers. Dan's research interest mainly focuses on the mathematical understandings of undergraduate and graduate students. Dan completed his Ph.D. in the Summer of 2016.

My Role as Participant-researcher

I (Norah) am the fourth participant in this study and I define my role as a participant-researcher (Creswell & Creswell, 2018). I was a second-year doctoral student in the same program as B and D. I earned a BS in secondary mathematics education at a different university. I earned M.Ed. in mathematics education prior to arrival in the program I taught mathematics at the secondary level. During my doctoral program, I taught a mathematics methods course for elementary preservice teachers. My primary interest is in teacher education, specifically educating preservice teachers of mathematics. I address participant-researcher biases in the trustworthiness section at the end of this chapter.

Data Collection

The data collection occurred in Fall 2013 and Spring 2014 as a component of a larger research project, designed to address the broad question: What outcomes occur for PTs and MTEs when a mathematics methods course is designed from a pedagogies of practice perspective? It is important to note that this group not only planned/designed and enacted the mathematics methods courses across these two semesters, members of the group have also conducted a number of research studies from this project. As a result, my dissertation study draws from the data corpus that was generated by the group in those semesters. Specifically, my study focuses on data that was collected as the participants engaged in the co-planning meetings. Below, I describe each data source.

Table 3-1: Data sources

Type	**’13 Fall**	**’14 Spring**
Course syllabus and a list of instructional activities with detailed descriptions (See Appendix C)	1 document	1 document
Audio recordings of group meetings	10 meetings	9 meetings
Meeting minutes and memos from planning sessions	10 meetings	9 meetings
Course materials and artifacts including assignment descriptions and PTs work (See Appendix C)		
Math tasks & Student solutions generated by MTEs for rehearsals problems, including their memos (See Appendix D)	2 sets of solutions for each problem	2 sets of solutions for each problem

Audio-recordings of planning meetings: The doctoral students and Dr. Finn met weekly or biweekly to design the curriculum for the course. These meetings typically lasted 60-90 minutes. Each of the meetings was audio-recorded. I transcribed all the recordings.

Planning meeting notes. During each meeting, participants recorded main points of the discussion on a Google word document and took notes about their analyses of the activities to plan instructional activities for the following week.

Course materials and artifacts. Two types of course materials and artifacts were used: curricular documents including instructions for activities (i.e., collective analysis), reading prompt reaction questions, mathematics tasks, assignments, and Code Window; artifacts created during and outside of classroom activities, including coded instances (i.e., StudioCode timelines) and criteria for proof.

Data Analysis

As described in previous chapters, my goal was to provide a thick description of the co-planning practice of MTEs using different perspectives. As qualitative research experts recommend, I used a recursive and iterative process for analyzing data. Patton (2002) describes inductive analysis as "discovering patterns, themes, and categories in one's data" and deductive analysis as generating categories beforehand "according to an existing framework" (p. 453). I used both inductive and deductive thinking in my analysis. I used *predetermined codes* from existing literature, identified *expected codes* based on existing literature and common sense (Creswell & Creswell, 2018). Then, I worked inductively, looking for patterns, and categories, organizing data into a meaningful unit of analysis. As I continued to develop patterns, I worked back and forth between codes and categories until I established a comprehensive set of codes and themes. Later switching into deductive lenses again, I used emerging codes to go back to the data and to seek more evidence for themes (Creswell & Creswell, 2018).

Data analysis for this study occurred in five phases (see Figure 3-1) (Creswell & Creswell, 2018). In this section, I explain those five stages of data analysis. First, I describe my procedure for identifying and organizing relevant data sources for this study. Then, I report my initial analysis to make sense of data and generate a structure for coding. Next, I explain my procedure for coding data sources, variations in coding process, and products generated from this stage of analysis. Next, I report the fourth stage, assembling categories, clustering codes, and creating definitions for patterns. I conclude with my fifth stage, which interprets data.

Phase 1	Reviewing, organizing and preparing data for analysis
Phase 2	Summarizing data and planning for coding
Phase 3	Coding data, using predetermined & expected codes and generating additional codes.
Phase 4	Assembling data & writing descriptions for categories
Phase 5	Looking for patterns, Interpreting data, & generating statements

Figure **3-1**: Data analysis phases in the study.

Phase 1: Review and Prepare

First I organized and prepared data for analysis. I reviewed the data sources and arranged data into different sources of information. The course syllabus and course agenda informed data selection. I identified the segments in the group meeting audio-recordings that were dedicated to planning the course activities and transcribed all those sections.

Phase 2: Pre-Analysis

To create a general impression of the data, I summarized the focus and the content of each meeting in a table. I documented the activities taking place and questions being addressed and broke each meeting into segments based on the primary focus of the conversation. I followed a chronological order in analyzing the co-planning meetings. This stage helped me to plan a structure for coding data.

Phase 3: Coding data

I started coding with the transcript. I marked data segments in the word document, added comments, and wrote a word or phrase that represented the category in the margin (Rossman & Rallis, 2012). For each construct (i.e., knowledge, type of talk, and roles) I used a different strategy for coding.

Data Analysis for Type of Talk (Co-planning structure)

I used Type of Talk model to describe the structure and activities in co-planning. My analytical strategy was to break the conversation during each meeting into segments based on the focus and the purpose of the dialogue and look for patterns in the structure of the episode (See Table 3-2). I used a similar strategy to Feiman-Nemser and Beasley (1997) in the analysis. I used *predetermined codes* for the type of talk introduced in the literature on co-planning: designing learning activity[4] (Feiman-Nemser & Beasley, 1997), determining instructional details[5] (Lynch, 2017), and content & assigning responsibilities (Friend, 2014). I also generated additional categories in my analysis based on the emerging talk patterns, such as discussing curricular materials and analysis of PTs' learning.

Table **3-2**: Examples of type of talk from Co-planning F2

Event: Planning the collective analysis node for the CEI - preparing materials,	**Type of Talk**
Collective analysis of F's practice -What is the goal of the activity (both for the research and instructional purposes) -Instructions for collective analysis (e.g., demonstrating coding, forming student groups)	Establishing goals & Determining Instructional details
Preparing content material (e.g., representation of practice to analyze) for the class: Discussed; -How to code on StudioCode (i.e., codes to be used A.A.T. & other) -Selecting segments of the representation of practice for coding (the group listened excerpts B chose for students to analyze from F's practice)	Preparing & discussing curricular material

[4] starting with "pieces of ideas" and creating activities and "the work involved in creating them" (Feiman-Nemser & Beasley, 1997, p. 116)
[5] discussing "how the activity would play out in the classroom" (Lynch, 2017, p. 108)

Data Analysis for Professional Knowledge

To characterize MTEs' professional knowledge, I first used the main categories of Chauvot's model. I coded the transcripts of audio recordings and written documents based on Chauvot's knowledge categories: PCK-MTE, SMK-MTE, CX-MTE, and CK-MTE and other *expected codes* based on existing literature (Creswell & Creswell, 2018) such as Research Knowledge for MTEs. I used an additional code as "other" in order to indicate professional knowledge that has not been captured in Chauvot's study. After the first cycle coding (Saldana, 2013), described above, I used descriptive coding[6] (Miles, Hubarman, & Saldan, 2013) to identify subcategories of knowledge domains that emerge when designing and teaching the course. In the process, I revised my codes multiple times. I also noted what type of knowledge contributes to producing group *reifications*, such as course materials, artifacts, and discourse, in addition to which member brought that knowledge to the surface. Some of the knowledge for MTE has been coded twice based on the contexts and depending on the audience: one coding is in the context of designing and enacting a course for PTs; the other coding is in the context of teaching doctoral students. For instance, the *brokers* of the COP introduce new constructs to the group, such as advancing questions, which counts as SMK in the context of course teaching. This also shows the broker's expertise in the course content, thus coded as CK. In another example, as the veteran MTE shares her knowledge of the role of instructors in engaging PTs in the analysis of student-generated mathematical argument, she mentors doctoral students to focus on monitoring PTs' ideas rather than intervening to change their conceptions of proof.

First, I created tables with data segments, a summary of data, codes, potential subcodes, and contributors for each meeting (See Table 3-3). I clustered and combined similar codes to have fewer categories (Creswell & Creswell, 2018) (Table 3-4). I also combined relevant research and

[6] type of open coding: assigning labels to data that describes it in short phrases

CX Knowledge under CK.

Table 3-3. Example analysis table of a planning meeting with clustered codes

131003	**Event: Creating students' solutions for Rehearsal 2, Length: 66 Minutes**			
Meeting	**Knowledge**	**Type**	**Details**	**Member**
F7	PCK	PTs' learning to teach & instruction	If solutions could create opportunities for PTs to practice A.A.T. & Predictions for what A.A.T. PTs would ask & How to create opportunities for PTs to ask A.A.T. in the induction proof	B D
F7	SMK	Pedagogy Constructs	Examples of assessing and advancing questions are…	B
F7	CK	Tasks	The mathematical goal of the task is …	B
F7	PCK	PTs' understanding of math	Knowledge of PTs' conception of proof (e.g., PTs do not pay attention to defining variables as they write proofs)	D
F7	PCK	PTs' understanding of math	PTs' possible struggle: They might struggle understanding students' solutions if they are too obscure	D
F7	SMK	Students' math understanding	Students' difficulties; where students might get stuck in writing proofs	D
F7	SMK	Math	Knowledge about math representation that can be used in writing proofs	N
F7	PCK	PTs' understanding of math	What representations students are likely to use in calculus	D

Data Analysis for Roles

To characterize the roles MTEs took as they collaboratively planned the course, I looked for patterns of actions and responsibilities of group members. I coded the transcripts of audio recordings using *expected codes* based on existing literature (Creswell & Creswell, 2018). Three of the expected categories are derived from COP framework; expert, novice, and broker (Wenger, 1999). Some codes are based on common knowledge about social group structures, such as leader. The rest of the codes emerged from data through the processes of inductive analysis based upon repeated actions of group members, such as constructive critic and analyzer. Then, I created tables for each meeting that include possible roles and evidence that supports each role (See Table 3-4)

Table 3-4. Roles of MTEs during co-planning F9 (Peer teaching planning)

D as OBSERVER	Shares Observations about PTs' perceptions of mathematics tasks' and their purposes Observations about PTs' understanding of teaching mathematics Observations about PTs' perceptions of proof (e.g., proving is not a math content)
D as CONSTRUCTIVE CRITIC	Shares; Concerns about the structure of the rehearsals (e.g., the time given to PTs for the rehearsals is not realistic Concerns about the selected tasks; they are not being problematic enough for PTs
F as EXPERT TE:	Knows learning goals of instructional activities Makes final revisions in the learning activities based on reflections Expert in instructional models, knows what model works best for the purpose instructional activity (i.e. launch-explore-summarize) Expert in secondary mathematics textbooks; assists the group in selecting appropriate tasks for PTs (i.e., The cognitive demand of the tasks in CMP & CORE+ textbooks & tasks' appropriateness for the peer teaching activity
F as LEADER	Assigns responsibilities to the group for preparing materials (i.e., assigns to find reasoning tasks for the rehearsals)

Phase 4: Assembling Coded Data

In this phase, I assembled the data that belongs to the same category. I transferred data from each co-planning session based on categories. Next, I wrote a description for each category and created abbreviations for codes (See Appendix E). As a result, I generated a list of categories for knowledge, roles, and type of talk.

Phase 5: Interpretation of Data

At this stage, I sought patterns and explanations. I taught what I could say about this data and how those statements could be supported either from literature or theory. I transferred data tables to Excel and searched for patterns, (i.e. created frequency tables (See Figure 3-2). Patton (2002) describes interpretation as "attaching significance to what was found, making sense of the findings, offering explanations, drawing conclusions, extrapolating lessons, making inferences, considering meanings, and otherwise imposing order" (p. 480) Looking for patterns enabled me not only to revise my definitions for emerging categories but also to make sense of and rationalize the relations between them.

	A	B	C	D	E
1	meet	knowledge	type	details	person
2	F1	PCK	PTs understanding of math	Knowing PTs experiences in learning math	F
3	F1	SMK	Students math understanding	Strands of math proficiencies	N
4	F1	PCK-M	instruction	Strategies: how to give feedback to PTs	F
5	F1	PCK	Instruction	Debriefing solutions for staircase -Analyzing staircase using math proficiency framework-Sketches from history of math & intro to the history assignment	F
6	F1	PCK-M	instruction	How to communicate the goal of an assignment: share experiences as TEs how history of math supports math teaching	F
7	F1	CK	Materials	Knowledge of course readings (assessing & advancing)	F
8	F1	CK	Resources	Knowledge of equipment available and whom to ask for	F
9	F1	CK	resources	Facilities around/rooms to use for analysis	F
10	F1	CK	resources	knowing whom to consult for tech support	B
11	F1	CK	Tools	knowledge about using studiocode to analyze practice	B
12	F1	CK	Tools	knowledge of tools to teach the content: how to analyze practice using studiocode	B
13	F1	CK	Tools	Use flash drive: Loading and sharing studiocode files	B
14	F1	RK-M	Research	what data to collect and how	F
15	F2	CK	Resources & Tools	Tools to use and whom to ask for support: Laptops with studio codes, speakers	B F
16 17	F2	CK	tools	Instructions for video analysis activity	B
18	F2	CK	tools	Software: studiocode –creating code window and coding	B
19	F2	CK	tools	Efficient way to set up studiocode coding customs	B
20	F2	CK	tools	Writing instructions for the coding activity: How to code	B
21	F2	CK	tools	Knowledge of a unit analysis in studiocode	B
22	F2	CK	Goals & Design	Knowledge of the purpose of the coding activity	F B
23 24	F2	CK-M	Tools	Knowledge about different analysis tools, nvivo	F
25	F2	CK	Materials	Artifacts: How to use coded instances later in the course	F
26	F2	CK	tools	Knowledge of editing instances in studiocode	N
27	F2	CK	tools	Identifying a problem with the curricular material: Studiocode instances are not saved	B
28	F2	CK	tools	Knowledge of studiocode: setting up hot keys	B

Figure 3-2. Knowledge analysis assemble on Excel.

Sharing Findings

The final part of the case study is the reporting of the results and findings. To report the case I used the linear-analytic approach (Yin, 2013), which starts with introducing the problem and reviewing relevant literature, proceeds with the methods used and the findings from the analysis, and ends with the conclusions and implications from the findings. The final product of this study consist of three main parts: a concept map for MTEs' knowledge necessary in designing and teaching a method course, a list and description of roles MTEs take as they plan collaboratively, and type of talk that describes the structure of MTEs co-planning practice.

Trustworthiness & Credibility

Two major categories of problems could emerge in this study due to: a) potential researcher bias and b) general concerns about the rigor of the research. In this section, I describe my role and explain how I addressed these issues.

I was a participant-observer in this study, meaning that as the researcher I was involved in the culture and the practices of the participants to some degree (Collins, 2013). As a participant-observer, my observation role was secondary to my participant role (Creswell & Creswell, 2018). Although observations can be both direct and participant-level in a case study (Yin, 2013), the level of involvement in the context correlates to the quality of the data being collected (Kawulich, Garner, & Wagner, 2009). Being a member of the community allowed me to understand the events in the context more clearly (Geertz, 1973). While being an “insider” assists in describing the context, this raises concerns regarding the objectivity of the study and researcher bias (Denzin & Lincoln, 2005; Yin, 2013).

My interest in studying the co-planning of MTEs started after being a part of the COP. The co-planning meetings were recorded as a part of a bigger study designed to investigate the development and implementation of a secondary mathematics methods course grounded in iterative cycles of enactment and investigation. For my dissertation study, I investigated the co-planning meetings and materials that were already gathered. Being a participant, but not a researcher (yet), in the co-planning minimized the researcher bias in the data collection process. During the analysis process, one technique I used to address the researcher bias issue was reflexivity.[7] I wrote memos about my experiences in the study and I recorded and reflected upon my thoughts, decisions, and actions during the research process (Marshall & Rossman, 2006) (See Appendix E). Reflexivity not only enabled me to make my decisions visible to myself—to consider how my experiences and relationships with the participants influences my interpretation (Marshall & Rosmann, 2006)—but also to make these decisions visible to the audience and distinguish evidence from interpretation (Yin, 2013).

Despite precautions, I accept that I might have minimized the researcher bias but not removed it completely. However, as Mehra (2002) points out regardless of researcher's relation with participants, researcher bias is an inevitable aspect of research:

> The researcher can't separate himself or herself from the topic/people he or she is studying; it is in the interaction between the researcher and researched that the knowledge is created. So the researcher bias enters into the picture even if the researcher tries to stay out of it. (p. 1).

In addition to the concerns regarding participant–observer bias, there exist other potential limitations on the trustworthiness and credibility of the research.

I established the trustworthiness of the findings by triangulating[8] data, which means that I looked at the relationships, overlaps, and discrepancies between different data types (Yin, 2013). Evidence analyzed from one set of data, such as audio recordings, was checked using meeting notes (Bogdan & Biklen, 2006). Comparing multiple sources allowed me to display multiple realities of the context simultaneously (Denzin & Lincoln, 2005).

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[7] "researchers reflect about how their role in the study and their personal background, culture, and experiences hold potential for shaping their interpretation such as the themes they advance and meaning they ascribe to the data (Creswell & Creswell, 2018, p. 182)

Chapter 4

Types of Talk in Co-planning

In this chapter, I discuss the structure of co-planning meetings. I categorize co planning activities into ten groups based on the kind of talk around which the conversation centers. Although the group addressed various topics, they engaged in ten general types of talk that formed a routine for the co-planning meetings: discussing curricular materials, determining instructional details, analyzing PTs' learning, reflecting on practice, evaluating & revising activities, designing learning activities, discussing goals, organizing, assigning responsibilities, and discussing course content (See Table 4-1). While the first five types of talk dominated the conversation, the other five took place less frequently and were mostly nested in the first five. In this section, I first describe the types of talk categories. Next, I explain the relationship between types of talk and MTEs' professional development. Findings suggest that engaging in different types of talk provide opportunities for MTE knowledge development (Bleiler, 2012) (See Figure 4-1). The types of talk provided a structure for MTEs' participation in the co-planning. Furthermore, co-planning offered natural opportunities for collaboration and reflection, which are described as core experiences for MTEs professional growth (Krainer, 2008).

Categories of Types of Talk

MTEs engaged in ten types of talk during the co-planning meetings, but their talk predominantly fell under five types: *analyzing of PTs learning*, *discussing curricular materials*, *determining instructional details*, *reflecting on practice*, and *evaluating and revising activities*. Although the sequence of the talk categories varied, they established a structure for co-planning. Below, I describe the nature of each type of talk that emerged as the COP engaged in co-planning.

Table 4-1. Co-planning types of talk

Types of Talk	**Description**
Discussing Curricular Materials (DCM)	Discussing resources & course content Creating, selecting, preparing, and evaluating curricular materials
Analyzing PTs Learning (APTsL)	i) Sharing observations & analysis & noticings about PTs' learning, understanding, and performance in the course (both in class and in the assignments) ii) Discussing PTs learning in general; their experiences, perceptions, knowledge, and struggles
Reflecting on practice (RoP)	Analyzing teaching and reflecting on practice
Determining instructional details (DID)	Determining the structure for the instructional activities and planning the details of the enactment
Evaluating & Revising Activities (ERA)	Evaluating the course activities & materials Suggesting revisions and modifying course activities
Discussing Goals (DG)	Setting/revisiting/revising the learning goals of the course, assignments, and instructional activities
Designing Learning Activities (DLA)	Designing additional learning activities based on analysis of PTs' learning and reflections on practice
Discussing Content (DC)	Discussing the mathematical & pedagogical course content
Assigning (A)	Listing things to do and assigning/clarifying responsibilities
Organizing (O)	Writing & presenting meeting agenda and meetings' goals Organizing the structure of meeting and summarizing meeting decisions

Analysis of PTs' Learning

The first predominant focus of the conversation in co-planning was sharing observations and analysis of PTs' learning both in the course and general. This type of talk generally takes place at the beginning of each meeting and also throughout the session. Meetings often began with Finn inviting MTEs to share their observations about PTs' engagement and performances during instructional activities. MTEs shared their overall impression about PTs' performance, themes, and issues they noticed in PTs' small group conversations, and analyses of their work. This type of talk included but was not limited to observations about PTs' understandings and uses of teachers' questions, PTs' attitudes toward learning activities, analysis of PTs' understandings of mathematical concepts, and PTs' dispositions toward teaching and learning mathematics. These types of talk most often started with first-hand observations from the course but were then enriched with research-based knowledge and MTEs' previous experiences with PTs. Below are two example of episodes after PTs analyzed Finn's practice in small groups. Finn invited the members of the MTE group to share their experiences with the PTs' analysis of the representation of practice.

> Finn: So what do you have to say about what you experienced this week?
> Norah: I heard this comment. I realized that the assessing and advancing questions are highly tied to the context. So having one person who was present when Finn posed the assessing or advancing question in the group helped the group identify which one was what. They gave more in depth thought what was happening in the context.
> Dan: I heard a very similar comment in the group that I was observing. For them the effect of the question is part of what they were using to determine what type of question. Not just their perception of what the instructor purpose was but what actually happened. That sounds to me what you were described. (Co-planning F3)

In this excerpt, MTEs shared what they had noticed about PTs understanding of A.A.T., which was highly tied to the context. Two members noticed that PTs determined type of teacher question as A.A.T. based not on the intention of the instructor, but the consequences of the question—they were most attuned to how the question impacted students' thinking.

Similarly, in the following meeting, Finn opened the conversation again by asking "Any general impressions you would like to talk about?" This question initiated a discussion of reflections on PTs' understandings and implementation of A.A.T in the first rehearsal.

> Dan: I was hearing a lot of questions: assessing and some advancing. It seems to be oriented to not telling to the students so the example I was thinking of was Ranold. It was during the first half, when Pat was acting like having difficulties; and whoever was working with that asked “does this equation you have written, is it correct if x=0?” she said no. The response was “what could you do to make it correct?” she said “ok, let me try that.” I recognize how this is different from a teacher approaching and telling to students that “this equation does not hold for 0, so you need to find a way to fix it.” It is just a slight shift in the approach. But it caught my attention and I started noticing it in other people as well. (Co-planning F5)

Here, Dan noticed that teacher questions started to shape PTs’ practices. Rather than pointing out student mistakes directly, PTs tended to prompt students to figure out their own mistakes and think about ways to correct their errors. The group found this interesting due to their knowledge about PTs’ overall experiences with instructional models. They agreed that a majority of PTs had seen mostly direct instruction rather than inquiry-based models.

Discussing Curricular Materials

Another primary type of talk emerged during the planning meetings that were aimed to prepare curricular materials for the course. During these conversations, the group created, discussed, and revised course artifacts. The MTE group talked about PT assignments and readings, selected mathematical tasks and activities for PTs, created student solutions for rehearsals, and discussed resources and tools available for the course activities. The group also talked about and prepared materials for assignments and assessments, including the assigned readings and prompting questions, and assessment instruments. They also discussed the focus for each assignment and assessment. As they create curricular materials, the group revisited the learning goals of instructional activities.

Two other main topics related to materials were: Creating representations of practices for PTs to analyze and preparing written instructions to guide PTs as they analyze representations of practices. The group decided what the PTs would need to code, what codes they should have in the CodeWindow, and what portions of video PTs should focus on. For instance, in preparation for the 1st collective analysis of a Staircase problem, the COP selected episodes where Finn used A.A.T. and created four codes for A.A.T. and Others (See Figure 4-1). The group generated written instructions to hand PTs for collective analysis, which asked PTs to code 5-10 minute of Finn's practice and code A.A.T. (See Figure 4-2). MTEs asked PTs to write the question they heard and determine the purpose of the question, and then analyze how it impacted students' thinking in the practice. Below is an example where the group watched and selected episodes from a representation of practice for PTs to analyze and worked on writing instructions for PTs to code representation of practice on StudioCode.

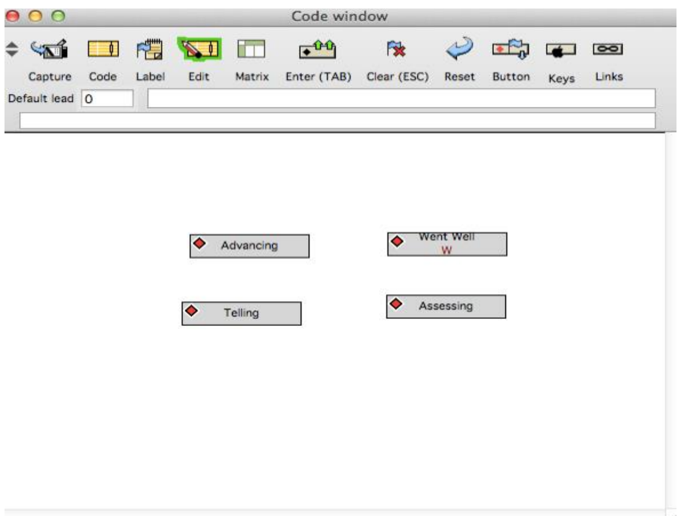

Figure 4-1: Code Window for the collective analysis activity (codes: *Assessing*, *Advancing*, *Telling*, and *Other*.

Read the Case of Edith Hart. While reading, "code" for *Telling Statements*, *Assessing Questions*, and *Advancing Questions.*

One way Hiebert and Wearne (2003) described problematic mathematics is "students need to struggle with challenging problems if they are to learn mathematics deeply" (p. 6). Based on your previous reading of their chapter, our recent work in class, and your analysis of the Case of Edith Hart, answer the following question:

What is the instructional purpose that **telling statements, assessing questions,** and **advancing questions** serve for a teacher whose goal is to allow mathematics to be problematic for students?

a. The instructional purpose of **telling statements** is…

b. The instructional purpose of **assessing questions** is…

c. The instructional purpose of **advancing questions** is…

Figure **4-2**: Instructions for analyzing a representation of practice.

Finn: So they are going to pause and talk to each other?
Dan: In most cases, I think they have to go back to the point they think they heard an assessing question. So it might make sense just to say roughly get to the middle of the where that happened and then click to code it. Because the code should capture that, but if they wait until to the end. They might be ready at the end for the statement to code.
Finn: How do we instruct them to when to click the button?
Bruce: I was going to instruct them along the lines of similar instruction. Essentially I was going to have them listen to telling statements, assessing questions, and after hearing that pause move the slider back and forth and move to the middle of the statement, move to the beginning, and move to the end, and hit it.
Finn: So, listen to it, pause, decide what it is, go back to the middle of it, and hit the right code. Is that what we want to tell them? I am asking authentic questions. I have never coded using these. (Co-planning F2)

Determining Instructional Details

Another dominant type of talk was determining instructional details for learning activities. These discussions included focus, structure, timeframe, launching, forming groups, arranging settings, and discussing tools to be used in activities. Most of the time was devoted to planning the details of rehearsals and joint analysis of representations of practice. The group talked about the scenario for the rehearsal, the length of each rehearsal, and the necessary and available rooms and equipment for the activity. Also, the COP discussed how to set up the rehearsal, such as forming student and feedback groups, presenting student work, and targeting chosen pedagogical ideas. The conversation also addressed the actions of TEs in rehearsals and considered how to create a realistic student-teacher conversation and yet more opportunities for PTs to practice A.A.T. The MTEs discussed the TEs' roles in rehearsals, including how much to say/explain as a student, how to respond to PTs' questions, and what to attend to so as to best support PTs' learning. These meetings also planned very detailed descriptions of the setting. For instance, the group discussed where PTs and students would stand or the order of presenting student solutions.

In one episode, MTEs brainstormed a sequence of student solutions to be displayed in rehearsals. The group decided that each TE would share one of the three solutions to the same task in varying order.

> Norah: My question would be whether those groups plan together for each question, are you proposing that one student show all 3 responses to the same question?
> Finn: Yes.
> Dan: We could do that. We are not sure it is necessary. Let students A, B, C work together to plan together. For question 1. Student A does the first rehearsal then 6 rehearsals later B gets up and C after 6 rehearsals.
> Finn: Ahh.
> Dan: They do not have to go in a row
> Finn: I agree with that but it would be nice to get it in order the questions in order content wise.
> Dan: Then it would be wise to switch the student because one student representing 3 different ways of thinking is challenging and might cause audience to think that is one person's thinking. (Co-planning SP2)

Both instructors of the course started the conversation by listing his/her initial plans and inviting the other members to answer specific questions about the instructions, followed up by the group brainstorming ideas.

> Dan: On Tuesday. Debrief with them about where they are in the process of preparing for this. Deal with some logistics, such as how are you getting out there, what time are you supposed to be there? Remind them that ... No, I'm not even going to do that. I'm not going to remind them about professional dress, because we already had the conversation. Then if they need more time to work on the planning and the after part, then I'm going to spend that time because the goal is they'd written it. I'm not sure that all of them really could say what it would look like for that

> goal to be met, so that's where we need to dig on on Tuesday. Then, if there's time, to bring in a list of teaching actions and start trying to kick that back to the list of principles that they started to develop a couple of weeks ago. There's plenty to do on Tuesday, but it's not so much that we can't make 15 minutes for you to come in, talk to them about those, and get them to ... Could we just do the SRTEs at that same time? (Co-planning SP7)

As they planned the analysis of practice, MTEs first talked about the learning goal of collective analysis, how it was situated within the CIE model, on what PTs would focus, and what MTEs expected PTs to learn from this experience. Next, we brainstormed the logistics for the analysis activity: resources and equipment needed and available, including labs with computers that support StudioCode. Also, we planned details of how PTs would analyze representations of practice on StudioCode, including start and stop coding an instance using hotkeys, editing instances, and adding memos to codes. Once the analysis activity was complete, the group brainstormed how to disseminate videos, Code Windows and StudioCode timelines. They needed to find platforms to share large size of coding files with PTs. Bruce’s expertise and knowledge about analysis tools enabled the group to think about best possible ways to engage PTs in the collective analysis.

Analysis and Reflection on Practice

In addition to the analysis of PTs learning, the group engaged in analyzing their own practices as they co-planned. MTEs talked about what they did and did not do well, what challenges they faced, what they learned from this experience, and what they might repeat and change in the future. One example of this type of talk was comparing the alignment between course goals and instructor enactment. For instance, Dan talked about the extent to which he addressed the goals written in the syllabus and course description. He shared his experiences in writing a syllabus and reflected on his decision-making.

> Dan: Well, no. I think there are a couple that we're really not addressing. As an example, one of the first items was learning something about skills and concepts that are important for secondary mathematics. We really don't address that very well.

Dan: I can now understand better why ... I'm just thinking about how to frame this mathematically. While one instructor might have chosen a different set of goals to emphasize than another, and then maybe wonder why those goals are no longer going to be emphasized. Like if you really set that first one as a big part of understanding skills and concepts, as an example, and suddenly somebody else is emphasizing a different part of it, then it can seem a little surprising.

Dan: They probably are because I copied and pasted it, but what I'm saying is, I haven't been deliberate about paying attention to whether those goals are really framing what I'm doing because I believe in what I'm doing, and that's, with me, always been the problem with syllabus writing, is the syllabus outline is something that is a bureaucratic document and what we do in the classroom is not often driven by this. It's not a statement of my beliefs as an instructor about what matters, it's a statement of the stuff that I've been told I have to make sure I say (Co-planning SP6.

The other focus of this type of talk was sharing analyses of other MTEs' practices. They provided feedback on the instructors' enactment. For instance, Dan shared his observations about Finn's use of her voice. He noticed that the variations of her tone challenged PTs and assisted them in moving toward mathematical goals.

Dan: Which is one of the things one group was arguing. That is started as assessing and eventually become advancing. I was actually surprised when you actually said "I had a little sharp tone." When I was listening to the groups what I was hearing was a tone of challenge. How do you know that is true? Which to me sounded like an advancing question, challenging students to explain. None of them came up. It was about what was the question and what was the response not at all about how the question was asked. (Co-planning F3)

Evaluating & Revising Course Activities

Since this was a design study and a new approach implemented in a methods course, the group continuously engaged in evaluating and revising course activities. They discussed the implementation of instructional activities, what worked well, what did not, and what possible modifications could be made. This type of talk mostly took place right after analysis of PTs' learning or analysis of practice. The revisions addressed the structure, length, setting, tasks, groups, equipment, or instructions used in the CEI model, particularly around the rehearsals.

For instance, seeing that three minutes was not enough for PTs to analyze student work and pose teacher questions, the MTE group decided to change the scenario for the rehearsal and post student solutions the night before the rehearsal (Co-planning F6). In the following semester, based on PTs' feedback, the group decided to extend rehearsal length to 5 minutes (Co-planning SP2). The group also talked about possibly revising the focus of each rehearsal based on the knowledge about recent studies in the next iteration of the course. They considered implementing two consecutive rehearsals, one focused on assessing questions and the other focusing on advancing (Co-planning SP7). A couple of other suggestions concerned rehearsals, such as changing the language for observation sheets to missed opportunities and successful moments, while another revision for the rehearsals scrutinized the tasks used in the rehearsals. The group noted that creating valid arguments was too big for PTs to practice A.A.T. and decided to select tasks that relate to PTs' experiences in other courses, such as connecting representation and generalizations (Co-planning SP3).

Based on the evaluation, MTEs discussed changing structures and instructions for activities and assignments in the next iterations of the course. For instance, experiencing the heavy content of the course and observing the rehearsals serves better to support PTs' learning practice instead of peer teaching. Since they are coached and receive immediate feedback, MTEs decided to approach peer teaching as extended rehearsal (Co-planning F9, SP2, & SP3). Observing PTs' limited attention to student thinking, MTEs discussed revising PT engagement with student work. MTEs suggested analyzing student thinking in other teaching episodes, bringing real secondary students for one rehearsal, or assigning readings about students' misconceptions and errors (Co-planning SP3). Observing PTs' frustration, the group also discussed breaking assignments (i.e., a mathematics history assignment) into parts (i.e. reading, solving sketches, and reflection) and providing feedback along the way (Co-planning SP6).

In addition to these five major types of talk, the group engaged in five other less frequent types. As MTEs **discussed learning goals**, they established goals for a new activity or made the purpose of learning activities explicit to other MTEs. Members of the group did not always know the learning goals of the planned activities. The group discussed the purpose of an activity and how it serves in CEI, such as rehearsal or collective analysis. In addition to talking about specific activities, they discussed the overall goals of the course and evaluated the match between learning activities and the course goals. Based on the evaluations of IAs, MTEs sometimes made revisions and narrowed the focus or decided to emphasize one or more aspects. This conversation was embedded in mostly determining instructional details.

As MTEs **assigned responsibilities**, they discussed TEs roles and responsibilities in teaching, how they could contribute to PTs' learning, what materials TEs needed to prepare for an instructional activity, and what work TEs needed to complete before the next meeting. **Organizing** was when the group talked about the agenda for each and purpose for each meeting. This type of talk also included debriefing meetings and group decisions. Although the CIE model and course activities were mostly planned before the semester began, MTEs **designed** additional **learning activities** to support PT learning based on performance, reflections on practice, and emerging concerns. As MTEs designed learning activities, they discussed what PTs needed to practice more, how to engage them in that practice, the crucial concepts and skills, the theory that should be embedded in the activity, and how the new activity fits the course structure.
The group also **discussed** both mathematical and pedagogical **course content.** This type of talk was primarily nested in discussing curricular materials. The group needed to revisit and review the definition of decompositions of practices like assessing and advancing questions frequently, in addition to revisiting and reviewing mathematical ideas such as representations and forms of proofs.

In this study, I observed two significant relations between types of talk and MTEs professional development, which I assert in the next section.

Assertion 1: Co-planning provided rich opportunities for MTEs to make their knowledge knowable to others.

Aligning with the literature, my analysis found that collaborative planning provided the opportunity for MTEs to engage in natural discussions involving knowledge and expertise necessary for teaching teachers, particularly PCK (Goodchild, Fuglestad, & Jaworski, 2013; Roth McDuffie & Mather, 2009). Collaborative planning created the environment for MTEs to discuss and make their knowledge and understanding knowable to others. Particularly, analysis of PT learning and practice allowed members place to make their PCK visible to other members of the COP. The participants engaged in contemplation and rationalization of their practices. In particular, instructors were pushed to be able to provide explicit justifications for their instructional decisions. The results in this study show that analysis of PT learning and teaching led to an increased understanding of PTs, their needs, and their behaviors. Below, I discuss how different types of talk provided opportunities for MTEs' professional growth.

Figure **4-3**: The relation between types of talk & MTEs' professional development in the co-planning.

Analysis of PTs' learning provided opportunities for MTEs knowledge and professional development

Analysis of PT learning enables MTEs to share and hear one another's experiences and knowledge about PTs; this practice makes PCK visible for the group members and supports

MTEs' understanding of PTs. This ongoing analysis of PTs' performances and engagement provided opportunities for MTEs to develop better understandings of PT learning to monitor their developmental progress regarding learning about teaching practices; particularly teacher questions.

The analysis was not only about PTs' performances and engagement but also their dispositions toward engaging in particular activities. For instance, as PTs completed the collective analysis using StudioCode, Finn asked COP members about their impressions of PTs' reactions to the activity. Although members agreed that PTs were mostly non-demonstrative at showing their interest in StudioCode analyses of their own rehearsals, they seemed to have different opinions regarding their engagement in the activity.

> Finn: What are your impressions about their engagement with this and their interest in this?
>
> Dan: Engagement was there. They seem to be completely engaged with the task, kind of willing to do it. Whether they are interested in it; is a little different. Seems a little bit like this was the task they are being asked to do, they have to do it. I am not sure they got the sense they are doing the task and saying "this is kind of cool" they are also fairly non-demonstrative in that sense.
>
> Finn: Because they are students, right? They are not supposed to be interested in what they are learning.
>
> Norah: It is obvious to see the difference when I compare their engagement with the staircase task, how motivating and interesting it was for PTs. They spend a lot of time on this task. I do not think it was due to the time spent or the nature of the task.
>
> Bruce: I cannot really tell, agree with you Dan. But as far as the whole group, you saw them hands are going up every single time. Seems like they wanted to get in to the conversation at a couple of different locations. (Co-planning F5)

With these conversations, MTEs practiced analyzing PTs' work and different indicators for PTs' learning. Sharing the analysis of PTs' performance gave opportunities for expert MTE to mentor doctoral students, particularly during the spring semester when the primary instructor of the course switched. For instance, MTEs noticed that PTs had limited understanding of what constitutes as a proof. PTs seemed to consider a mathematical argument with mostly an algebraic structure as a valid proof. Hearing that observation, Finn suggested designing an instructional activity to challenge PTs' perceptions about proofs. She proposed postponing rehearsals and devoting more time to PTs' understanding of what constitutes a proof so that PTs would be ready for the rehearsal. She suggested showing PTs a non-algebraic valid proof, discussing why it is accurate, and revising the criteria for proof based on the discussion.

> Finn: So take them back to that one and say, this is a valid proof. It is a proof by exhaustion. Now, does this have formulas in it? Does this use corollaries, does this use, I mean you don't have time to, like in the CORP class, we would unpack this and unpack it and unpack it. You need to now just confront the things that are on their list that don't make any difference.
> Dan: Mm-hmm (affirmative). Yeah, that works. Yeah, I'll go back through what they said was and was not and come up with some ways of pushing on that so that they start to look a little differently at the role of formulas and theorems play.
> Finn: And that you postpone rehearsals for a day and go back in and reschedule the ... I just, don't rush into this, because they won't get what they need to without unpacking this a little bit. I didn't realize it was quite that serious. (Co-planning SP2)

As MTEs shared their analyses of PTs learning, they also made assumptions about underlying reasons behind PTs' thinking. During meeting SP7, Dan shared with the group his observations about PTs' understandings of percent-decimal relationships.

> Dan: The more complicated ... The one that's embedded in the case where they're describing the faction percent and decimal areas for shaded regions, not where they're trying to shade a region because we didn't get to that in class, I'm not sure that that's even on their radar (Co-planning SP7)

Beyond practicing the analysis of PT learning, these conversations helped MTEs to see how to use those analyses as tools for their instruction. Finn continuously commented that instructors should revise the lesson plans and change the schedule of activities based on PTs' progress. Her suggestions emphasized the importance of PT–centered learning and providing opportunities for PTs to question their perception instead of "telling." Below is an example where Finn mentored MTEs to use their observation of PTs' learning, performance, and engagement in class as a pedagogical tool to support their instruction.

> Finn: I am wondering if. Are we keeping notes/recording of these observations?
> Dan: Should we start keeping records of these observations?

> Finn: I think we could because we could use that as a pedagogical tool. So Christy, seems to making an argument of bila bila. Talk about that in your tables a little bit. Or Nursen wondered if I had purposefully arranged the groups? I did it totally alphabetically. It just happened that they got good mix up. I am not sure if we have a person from each staircase group in the analysis group. Let's make a google doc that has observations on it. I might regularly look at and add to that document. I think that could be nice collective things we might be noticing. (Co-planning F3)

In the meetings as seen above, the MTEs regularly analyzed PTs' learning and thinking. Based on their analyses and their prior experiences with PTs, MTEs also made generalizations about PTs' thinking. They discussed PTs' experiences in mathematics and learning, and what research says about PT's perceptions, expertise, and struggles. Both the analysis of PTs' learning in the course and conversation on common PTs' thinking assisted MTEs in anticipating PTs' performances and struggles and determining instruction to support their learning. The group constantly talked about PTs' mathematics experiences and how that would impact their practice. For instance, during meeting F7, as the group planned the second rehearsal, MTEs discussed PTs' understanding of proofs: their understanding of what constitutes as a proof and where they might struggle as they write proofs. Also, the group discussed PTs' lack of experiences with exhaustion and pictorial proofs, and their familiarity with induction and algebraic proofs rooted in their college mathematics courses. Similarly, in meeting SP4, they talked about PTs' experiences with mathematical ideas such as the quadratic formula, Cramer's rule, Series, and integrals. One MTE shared research findings on PTs' struggles with understanding series convergence as well as the connections between integral of variations and R–squared. These conversations again helped the group members in selecting tasks, anticipating PTs' actions, and making instructional decisions. Discussions around PT learning also provided an opportunity for the veteran MTE to mentor doctoral students. For instance, as the group shared their 'noticings' about PTs' reactions to a challenging but seemingly easy mathematics problem, Finn shared a common perception PTs hold about what secondary students can do. That is, PTs tend to think “If I struggle with this task, 6th graders will struggle too.”

> Finn: One of the things that typically comes out about pre-service and in service teachers, when they are challenged mathematically by what they think is simple mathematics. Like percent and decimals and fractions, when they get challenged conceptually one of the byproducts of that sometimes is, well if this is so hard for me, there's no way a sixth grader could do it. Could think this way. You might bring that up, “How many of you are thinking, if I can't understand this how will a sixth grader understand this?” To make the point and what we know from research is when people learn the algorithm first, it is very hard for them to then come back and build in the conceptual pieces. (Co-planning SP5)

Discussing Curricular Materials provided opportunities for MTEs’ knowledge and professional development

Within each episode of preparing curricular materials, the group benefitted from the multiple areas of expertise the members offer. These conversations invited the COP to share and use their knowledge and experiences in teaching and research. The MTEs’ teaching experiences in secondary settings, as well as their SMK as MTEs, helped them to think about secondary students and curricula. For instance, as the group worked on generating authentic student solutions number theory proof tasks (See Figure 4-3), they benefitted from their knowledge about secondary students’ mathematical understandings and secondary curricula. In writing those solutions they discussed the following:

- How secondary students would attempt to solve number theory tasks; what representations they might use; and what pictorial solutions, numerical solutions, and algebraic solutions they might generate.
- How secondary students’ mathematical thinking develops, possible mistakes and misconceptions they might have, and the mathematical ideas they struggle with.
- The mathematical experiences students engage in from 7th grade to 9th grade, and the experiences their curriculum would address.

1. The product of any two perfect squares is a perfect square.
2. The sum of any two positive consecutive odd numbers is divisible by 4.
3. The product of two positive even numbers is even.
4. The sum of any three positive consecutive odd numbers is divisible by 3.
5. The product of any three positive consecutive numbers is always a multiple of 6.
6. For every counting number N (1, 2, 3, ….), N^2 + N is always even.

Figure **4-4**: The list of number theory statements for the rehearsal planning.

The group benefitted from its members' mathematical knowledge of number relations, algebra, generalizing, reasoning, and multiple representations in generating incomplete or inaccurate proofs. While creating incomplete student solutions, the group members needed to consider what opportunities each solution provides PTs to implement teaching practices. In doing so, they benefitted from both their knowledge and experiences of PT learning and teacher questions literature. The MTEs discussed what an incomplete or inaccurate student solution would look like so that it would be realistic and still would provide opportunities to implement A.A.T. for PTs.

Discussing appropriate mathematical tasks for PTs opened conversations about not only the nature and cognitive demands of tasks but also mathematical and social learning goals targeted in those activities. For instance, as the MTEs selected tasks for peer teaching, they talked about available resources to find tasks, such CLP, Core+ textbooks, and practitioner journal articles (e.g., Calendar problems in the *Mathematics Teacher*). In selecting the task, determining the target learning was fundamental. The group had an ongoing conversation about the features of the desired task in a particular instructional activity: being mathematically challenging and novel for PTs or being most suitable to practice talk moves such as having multiple solution strategies. In addition, the group discussed what opportunities tasks should offer for PTs to explore and use pedagogical tools the course introduced. One suggestion was to investigate a mathematics procedure PTs already knew (e.g., the quadratic formula) but had not explored in the conceptual meaning. In deciding the tasks, the MTEs benefitted from their knowledge about PTs' experiences with mathematics algorithms. For instance, PTs learned formulas such as Cramer's rule in matrices but did not explore why they work.

The enactment and analysis of the rehearsal portion of the CEI model pushed the group to seek out and use multiple tools and equipment for presenting and recording rehearsals, and then saving, disseminating, and analyzing recordings. The conversation included access to available computers, rooms, labs outlets in the classrooms, external drives, recorders, cameras, and dissemination platforms. The platform used for analysis of practice was StudioCode. The group interacted with the program occasionally to learn how to use the program and how PTs should use it in their analysis. During their engagement, the members discussed and learned about functions of StudioCode such as creating code windows with labels and timelines, coding instances and adding memos, and stacking and comparing timelines. During this type of talk, Bruce's CK on analysis tools became apparent to the group members. Below is an episode where the group worked on producing curricular materials for the collective analysis activity: a video, a Coding Window, and instructions. Bruce instructed group members how to code on StudioCode.

> Bruce: With the actual studio I have created a movie. Audio from your practice. What they would do is: we have each one of the laptop will have unique code: 1-4. It is the same code but this allows us to do when we come together as a whole group, we can stack all the time lines on each other, they have that individual codenames, otherwise it just blinks together.
> 3 codes; telling assessing and advancing. We can call telling something else.
> Finn: I think telling is just fine.
> Bruce: 3 basic codes and Dan, have you used StudioCode before?
> Dan: I have not. I think I will be learning as they [students] do.
> Bruce: Ok. That will be fine. After getting the program, they will open up their code window. I am on laptop 1, I will use codes 1. We will go to the file, we will go to the new and we want to do is to create a timeline and
> Finn: Will you write up these directions?
> Bruce: I will [took a note]. So will choose class 1 for the timeline. That is the movie I created.

This is our time and it allows us to code. And what we do to begin the coding process to press code on our code window

Finn: You have a video. It will show up here. If we do not have a video what will show up?

Bruce: I think it is going to be blank we could move anywhere on the audio

Dan: Is the audio already attached to the timeline?

Bruce: No it is not. Let’s start coding, press code and right now you cannot hear anything because the volume is not turned on. (Co-planning SP2)

As the group continued discussing curricular materials, the members gained experience in creating purposeful curricular materials for the course. This type of talk invited group members to explain their pedagogical decisions and to make their rationalization apparent to the whole group. For instance, as the COP planned the rehearsals, they developed observation sheets for PTs to fill out as they watched their peers. The group identified the purpose of observation sheets as, a) to keep PTs attentive to their peers’ teaching, and b) to support PTs’ understandings of teacher questions as they hear examples of A.A.T. and observe how they work out. Based on these goals, the group decided what questions to pose and what language to use in the observation sheets (See Appendix C).

During discussions on curricular materials, MTEs encountered various resources to teach PTs and engaged in designing and selecting curricular materials. Thus, it provided opportunities for MTEs to develop CK and PCK.

Determining instructional details provided opportunities for MTEs to develop professional knowledge.

As the group talked about the specifics of instructions, the conversation provided opportunities for the veteran MTE to mentor doctoral students about teaching PTs. Across this conversations, her PCK became visible to the members of the group. For instance, the group sought mathematics tasks that would facilitate PTs developing PCK but also would be appealing for PTs. Finn suggested designing an activity where PTs could first develop conceptual understandings about a mathematics topic and then generate an algorithm related to that topic.

While doing so, she advised encouraging PTs to see how course constructs like procedural fluency and conceptual understanding play out in mathematics tasks (Co-planning SP4). Another mentoring opportunity emerged when the group planned a trip to a high school as the third approximation of practice. Finn shared her experiences in organizing school visits and different ways to integrate school visits to support PT learning practice. By doing so, she informed the MTEs about getting permission for recording kids in schools (Co-planning F5).

As MTEs discussed ways to support PTs' analyses of teaching, Finn stated that a TE instructor would support PTs' learning of reflective practices more by monitoring progress with less instructional intervention. She suggested that MTEs should provide opportunities for PTs to learn to interrogate their own teaching (Co-planning F6). Finn explained how she used peer feedback as a learning tool to support PTs practicing inquiry into teaching. As PTs filled out rehearsal observation sheets and provided feedback to their peers about things that went well and things they needed to work on, PTs not only gained experience in analyzing a teaching episode but also saw different examples of teacher questions (Co-planning SP1). In another example, MTEs were discussing PTs who were not ready for rehearsing proof tasks. They were seeking ways to support PTs' understanding of what constitutes as a proof. Finn stated that, based on PTs' progress, a TE should revise the lesson plans. Finn suggested generating a whole group discussion and provided details for the activity: go back to class criteria for proof, show a non-algebraic valid proof, discuss with the group why it is valid, and revise their criteria for valid proof based on the discussion.

> Finn: And that you postpone rehearsals for a day and go back in and reschedule the ... I just, don't rush into this, 'cause they won't get what they need to without unpacking this a little bit. I didn't realize it was quite that serious. So take them back to that one and say, this is a valid proof. It is a proof by exhaustion…Now, does this have formulas in it? Does this use corollaries, does this use, I mean you don't have time to, like in the Corp class, we would unpack this and unpack it and unpack it. You need to now just confront the things that are on their list that don't make any difference.

Dan: Mm-hmm (affirmative). Yeah, that works. Yeah, I'll go back through what they said was and was not and come up with some ways of pushing on that so that they start to look a little differently at the role of formulas and theorems play. (Co-planning SP2)

She provided a number of instructional suggestions for teaching PTs such as, a) assign independent work for making up a missed class meeting (Co-planning F6), b) model implementing A.A.T. in your own practice (Co-planning SP3), and c) engage PTs in creating their own lists of teaching practices by analyzing a rich narrative case (Co-planning SP4). In addition, Finn listed practical ideas based on her experiences in teaching methods courses such as a) use the last 10 minute of class for getting SRTEs done (Co-planning SP6 & SP7), and b) design an interesting activity for PTs during the scheduled final time. She also shared her knowledge about the university and department policy regarding syllabi, finals, signature assignments, and grading (Co-planning SP6). For instance, Finn shared her experiences in teaching a scripted course—a data analysis course for secondary mathematics PTs. She explained how she negotiated with the department:

Finn: I got challenged about the way I was teaching it and the activities I was designing. I had to go back to the course goals and highlight things and say, "In the documents, I was given, these are the course goals." (Co-planning SP7)

She mentored the doctoral students about how to negotiate their personal vision and the institution's vision for teaching PTs.

These example excerpts show that the different expertise of MTEs enabled the group to consider multiple perspectives and to plan rich learning activities for PTs. In addition, these conversations invited Finn to share her knowledge and made her PCK visible to the members of the group.

Assertion 2: Co-planning provided natural opportunities for MTEs to engage in reflecting on practice.

The collaboration led to rich and in-depth reflection on practice. Supporting Bleiler's (2012) argument, I found co-planning served as a "catalyst for reflection on practice" (p. 239). MTEs reflected on their own teaching, PTs' performances, and design and enactment of instructional activities. In addition, they provided feedback for other MTEs' instruction.

Two instructors of the course reflected on their instructional decisions regularly. For instance, as the first semester was about to end, Finn reflected on her focus as an instructor during peer teaching activities. She was not pleased with her choice of focusing on PTs' mathematical learning rather than their development of PCK in the previous semesters of the methods course.

> Finn: I feel like I focus so much on what the audience is going to get this mathematically. That I have not been focusing on the teaching and I think that is why I have not been satisfied with what I have done so far with peer teaching (Co-planning F9)

This realization made her purposefully choose moments to support PTs learning of practice rather than their learning of mathematics. These reflections allowed novice MTEs to access an instructor's decision-making process in addition to notice the importance of reflective practice.

The other instructor of the course, Dan, critically reflected on his practice as well. His reflections were more activity specific and considered how those experiences shaped his instruction. He found himself doing a limited job in coaching PTs during their rehearsals (Co-planning S3). He talked about how his learning from coaching the rehearsal would influence his upcoming pedagogical decisions.

> Dan: I am thinking one of the things I learn from here is if I am doing another rehearsal, I might tell them what math goal is. This is the goal and you design your questions toward that…to me the math goal was getting an argument based on the criteria that developed. It is clear that their enactment goal is getting symbolic expression argument. (Co-planning F8)

Dan found himself enacting teacher moves that were not desirable to the mathematics education experts. He realized his tendency to make tasks easier for PTs when they felt frustrated working on a mathematics problem. For instance, as PTs were challenged by a task that asked them to create a mathematical model to explain the relationship between decimals and percentages, Dan shared his desire to make the task less problematic.

> Dan: Not only for them, but as they started to get more frustrated, I started wanting to make it easier. Had to try to let them know, "This is okay, this is we mean by a lot of mathematics can be problematic. It's all right to be frustrated, without saying here's how to do it." (Co-planning SP6)

Initially, he was not expecting PTs would struggle in making sense of percentage and decimal conversion. But, as he worked on the mathematics task, he realized that seeing the connection between decimals and percentage on a particular model is not an easy task and requires some time.

> Dan: Honestly, Bruce warned me about it and I looked at it and went, "I don't understand what it is that Bruce thinks is going to be so hard about this." Then I started reading through the case and Randy said, "I want them to not just convert between these but to understand why." I thought, "What would it mean to actually understand what a percent is through this context and how to compute it without.". I sat in here before class for 20 minutes finally realizing that this was going to be difficult. Finally realizing that we weren't going to get through both sets of tasks in one class period. (Co-planning SP6)

The members also provided feedback on the instructors' enactments. For instance, after PTs' collective analysis of Finn's representation of practice, MTEs talked about what PTs take on from the activity. The group shared their observations about the instructor's use of her tone and its impact on students' responses. As Finn reflected on her tone being sharp, Dan observed that Finn's use of variations of her tone pushed students to move toward the mathematical goal.

The group also discussed what challenges they faced as they co-planned the course. The challenges varied from the cognitive demand of teaching to the vague course goals. For instance, as MTEs planned acting as "students" during the rehearsals, they talked about the possible difficulties of the task:

> Dan: This is going to be difficult for us.
> Bruce: I think this is going to be difficult and I think what we are planning will probably go out the door in 6-7 minutes. I think that will just hold throught to..we just need to react what they are saying. I think that will be the big thing. (Co-planning F4)

After the first semester, the COP discussed the content that needed to be covered in the course. Finn found that the existing content was a lot to capture in one semester and decided to focus on fewer ideas.

> Finn: I think we don't have time to do it all. 'Cause we missed some pretty big chunks of content last semester. Well, I mean just the principles and trying to get, you know, focusing on teaching moves and trying to get it pulled together a little better. (Co-planning SP2)

Similarly, Dan found the work demand required for teaching this course very high. He noticed that the heavy content limited opportunities for PTs to generate examples of A.A.T. as well as to make connections between other constructs taught in the course. Dan noted, "There's no time to really go back and look in any detail at what they do in coding" (Co-planning SP3).

Another challenge was the limited literature on teaching practices; without this, PTs did not have enough opportunities to learn about different decompositions of practice. The group was able to benefit from the literature around teacher questions. However, to communicate other decomposition of teaching practices, the group decided they needed a broader repertoire of resources and tools.

> Dan: We're still in the process of trying to settle. But I don't ... Okay so Gatz said all we've been doing so far is that these readings really help them to build a framework that allows them to make sense of this idea of teacher talking, or make sense of the types of future questions, not by generating their own typology, but by giving them one somebody else has created. But then if I were to shift to teaching practices teaching roles, would do some other focus, I don't believe I have a sense of what that framework would look like, in a way that would allow me to communicate. (Co-planning SP4)

As seen in the excerpt, the group found it challenging to have limited resources and tools on decompositions of practice in the field of mathematics education.

Chapter Summary

In this chapter, I described ten types of talk the COP engaged in as they co-planned the course. Five types of talk occurred consistently and frequently across two semesters: discussing curricular materials, determining instructional details, analyzing PT learning, reflecting on practice, and evaluating and revising activities. After discussing those five types of talk, I explained the contribution of types of talk to MTE professional development. Results show that the types of talk—mainly discussing curricular materials, determining instructional details, and analyzing PTs learning—provided opportunities for MTEs to develop knowledge. In addition, the results show that co-planning enabled MTEs to engage in collaboration and reflection practices naturally, which can help to support their professional development. A conclusion I return to in the discussion chapter is that compared to the 'knowledge' and 'roles' framework, 'types of talk' provided a more detailed description of MTEs' work in co-planning practice.

Chapter 5

MTEs' Roles in Co-planning

In this chapter, I describe six different roles MTEs took within the COP: *leader, organizer, critic-inquirer, expert (and broker), analyzer,* and *novice* (See Table 5-1), and discuss how those different roles contributed to MTEs' co-planning practices and supported their professional knowledge.

Assertion 1: The complex work of MTEs could be less demanding and more efficient with MTEs taking on different roles in collaborations that contribute to their practice.

Planning and designing a methods course is multi-faceted work and demands multiple roles. In this study, individuals taking different responsibilities not only reduced the complexity of the work but also enabled opportunities to enhance the practice. In a collaborative setting, members took on multiple roles and responsibilities over the semester. However, MTEs presented some of the roles more dominantly based on their individual expertise, experiences, and competence (Wenger, 2009). Each role that emerged in the co-planning meetings advanced the COP's practice. Taking different roles enabled each member to contribute to the co-planning practice in multiple ways. Wenger (2009) contends that development of collaborative engagement requires diversity: "What makes engagement in practice possible and productive is as much a matter of diversity as it is a matter of homogeneity" (p. 75). While experts and brokers brought their expertise, skills, and knowledge to inform the practice, critics-inquirers extended ideas, offered alternative ways, and made the thought processes visible; analyzers evaluated the instruction and contributed to revisions, and leaders and organizers made decisions to maintain the work progress. In the next section, I describe these roles and discuss how the work of each role owner contributed to the MTEs' co-planning practice.

Table 5-1: The roles that emerged in co-planning

Leader	Defined the learning goals for the course and activities, decided meeting agendas, proposed ideas and solutions to emerging issues, established bridges between communities, made final instructional decisions, distributed the work among the group members.
Organizer	Scheduled group meetings; reminded members of upcoming meetings; kept meeting notes; collected, organized, and shared PTs' work and course artifacts; noted works needed to be completed, debriefed previous meetings, and decisions made; and informed the group about upcoming events and important deadlines.
Expert	Negotiated their expertise by sharing personal experiences, stories, and knowledge, which contributed to the COP's knowledge and assisted the group in negotiating the practice between MTEs' implementation and professional models of practice *Broker:* Brought new knowledge, offered novel ideas and tools, and acted dominantly in negotiating the meaning of course concepts and practices of teaching.
Critic–Inquirer	Voiced possible issues and limitations of instruction he/she noticed and provided alternative forms of instruction. Posed questions about instruction, particularly the purpose and rational behind pedagogical decisions.
Analyzer	Monitored PTs' performance and focused on the factors that might have led to certain PTs' learning and disposition.
Novice	Showed limited knowledge, contributed least to the practice, and asked verification and clarification questions mostly

Leader. The leader of the COP provided general guidance for the practices of the group. He/she defined and redefined the instructional goals for the course, determined the agenda for group meetings, suggested solutions to emerging issues, bridged the COP with other communities, proposed new ideas, made final instructional decisions, assigned responsibilities to the group members, and scheduled group meetings. Except for a few sessions, Finn held the leader position as the veteran MTE. If Finn was not present, Bruce took this role.

One of the practices of the leader was to determine the group work and divide the work labor. The leader initiated the discussion by defining the purpose of the meeting. Knowing how each COP member could contribute, in addition to being an expert practitioner, enabled the leader to distribute the labor among group members. She/he assigned responsibilities to individuals based on their expertise and the work that needed to be completed. These assignments were either preparing curricular materials for the course or arranging course activities. For instance, during the 3rd planning meeting, Finn asked two of her mentees to construct a list of features of A.A.T. based on PTs' definitions and examples.

> Finn: I am sending you the chart today [talking about the charts students filled out where they [PTs] wrote their definition and rationale and purpose of assessing advancing and telling questions from my practice. See if you can pull 4-5 bullets that are pretty consistent. Capture what they had to say. Put it all one document that I just send. Dan and Norah, you could work on that and get that done by Tuesday. They [PTs] printed out what they have written and they wrote on that today. So you will be able to see what they come up with themselves and what they added to their charts. We need to get their notebooks.
> Bruce: If they submit it electronically?
> Norah: Do you want us to have rationale and purposes separately?
> Finn: They [PTs] should have just done the features. I guess feature also includes purpose, right? Let's just see what they have and decide what kind of doctrine we need to do later. (Co-planning F3)

In another meeting, Finn assigned doctoral students to generate so-called student solutions. She instructed them about the desired features of student responses for the rehearsal and explained how those solutions would be used in the instruction.

> Finn: I'd like you to prepare a response to each of these tasks and put them on a big poster. Now the response needs to have something that they can advance you. So it needs to be not perfect, right. There needs to be some kind of flow in your response. So, what I envision is for each 6 minutes we have two students' responses on posters. Instructor 1 and Dan [drawing] instructor 1 talks to you about 3 minutes about your response. Then instructor 2 talks to you about 3 minutes about your response. (Co-planning F6)

As the primary instructor, she decided how to structure the collective analysis activity, what materials were needed for the activity, and what another member would do to facilitate PTs' learning.

> Finn: No. We are going to use the 4 computers that we have StudioCode on and we will put students in groups, we probably need. I think that we can gather students around 4 computers. Two groups of 5 and 2 groups of 4. If we have 4 computers, they can be looking at those computers while you [Bruce] are running a demo. (Co-planning F2)

The leader also invited members to share their reflections and observations regularly, mostly at the beginning of group meetings. She encouraged members to evaluate both instructors' and PTs' performances and the instructional activities. For instance, after the peer teaching, she started the group conversation by asking the group members to share their impressions about the peer teaching activity. Finn: "Any general impressions you would like to talk about?" (Co-planning F10). She devoted a considerable amount of time for listening to MTEs' noticings about PTs. Finn posed questions such as; "So what do you have to say about what you experienced this week?" and "What are your impressions about their [PTs] engagement with this [analysis of practice activity] and their interest?" (Co-planning F2). Emphasizing the importance of reflective practice, she requested an ongoing analysis of course activities and students' performance and suggested strategies for recording these reflections and observations.

When the primary instructor of the course was absent, the leader position switched to another group member, Bruce. As the leader, he initiated the conversation and decided the agenda for the group meeting. For instance, as the COP worked on generating student solutions for the rehearsal, he started the session briefly with listing the work to be completed.

> Bruce: We need to decide what roles we are going to play for this activity. I work through the problems as if I was trying to do this to teach it. I started something that I think might be worthwhile to, there might be good ones we can try perturbing whoever is teaching with some students' error or what problems may be fruitful for students' errors and misconceptions. (Co-planning F4)

His decisions assisted the group in completing the work. For instance, after creating visual solutions to number theory statements the group wondered who would be presenting each solution in the rehearsals. Bruce suggested deciding that later:

> Dan: Whose work is this?
> Bruce: Let's decide that at the end. What I am thinking is whoever is the visual person will stay visual.
> Dan: Sounds good. (Co-planning F7)

Bruce decided when to pause and when to move on to the next topic using statements such as "Before moving into the next question, Let's…" and "We will move on number 2 now" in addition to scheduling another meeting if necessary: "Let's plan a five-minute meeting after the class, and you will be ready to pack up right afterwards" (Co-planning F7).

Organizer. The organizer scheduled group meetings and sent reminders; collected, organized, and shared PTs' coursework, analysis, and course artifacts; and recorded meeting notes. During the co-planning meetings, the organizer listed the work that needed to be completed, debriefed decisions made in previous meetings, and reminded the group about upcoming events and important deadlines. The organizer's identity did not vary much across two consecutive semesters. Being the primary designer of the course activities and expert on the analysis of practice, Bruce did most of the organizing. He saved assignments, course artifacts, and student work, and shared these items with the group members:

> Bruce: Norah, I need to show you how the data is organized on the computer, so that more than one of us will know where they are located. What I am going to do is on flash drive 3 there will be folder called Torris folder.

He organized those documents to be easily accessible to the other members of the COP. He informed MTEs about how to read his recordings and charts and how to find specific materials and notes. Below is an example of Bruce explaining where and how he recorded course materials and artifacts. He shared his organization strategy for the documents:

> Bruce: In the Fall 2013 study folder, there's an Excel Spreadsheet called class activities. That has a list of the class activity and the homework assignment if you want to pull that up to look to see what data might be. What I can do is for this chart, what I'm going to put into this chart is going to be the name for the specific activity. (Co-planning F7)

He stored the course materials (e.g., readings, assignment descriptions, mathematics tasks) and made them accessible to the group members. As the lead instructor of the course changed, he served as a mediator between two instructors. Knowing the course design and materials enabled him in informing and guiding the COP to gather necessary curricular materials and tools. For instance, Bruce reminded of the group things to be prepared for the first rehearsal. “Norah and I need to decide on the seventh problem. I don't think that's going to take too long, Dan, I'll shoot you the list of rehearsal problems that we're going to use” (Co-planning SP4). His remainders assisted the group in preparing all necessary materials for upcoming activities.

In addition to organizing documents and materials, Bruce assisted in leading the co-planning meetings and kept the group on task. For instance, during the rehearsal-planning meeting, the COP members began an in-depth conversation about students’ conceptual vs. procedural ways of thinking. He prompted the group to choose and follow a structure for the group conversation.

> Bruce: How do we want to do this? I guess since we are looking problem #4. Do we want to go through this as a problem set? 1-4 and decide who wants to play the role I guess, first of all decide what are the different possible solutions we will prepare for them. (Co-planning F4)

His organizing skills assisted the group in realizing the purpose of meetings on time. As the organizer, he gave reminders to the group members preparing and keeping the course materials for the next instructors of the course. For instance, he suggested keeping copies of the student responses they created for future semesters:

> Bruce: We have to remember on a side note that the pdf what we are going to create here and the ones for the 1st round of rehearsals you need copies of that for your future class; considering the work we put in these solutions. (Co-planning F7)

In addition to bringing the group’s attention to works they needed to complete, he helped the group to solve logistical issues such as findings computers for analysis. He also reserved rooms for course activities (i.e., collective analysis). He identified necessary equipment for the course activities and searched for how to access those tools and facilities.

Critic – Inquirer. A critic - inquirer’s role in the COP was to address possible issues and limitations of the MTEs’ instruction he noticed in addition to providing alternative ways of thinking. Although other group members took this role once in a while, it was mostly represented by Dan. The critic pushed the COP to inquire into their practice and to analyze it from a critical perspective. His inquiry enabled the group to revisit their plans and design, clarify objectives, modify activities, and to improve the group's practice. The critic – inquirer did share not only his concerns about present or possible issues but also proposed alternative ways. For instance, as the group was discussing the rehearsal time for each PT, Dan addressed possible problems and raised the question of how to arrange the classroom so that all PTs could see the "student solution" and also be captured by the cameras.

> Dan: Do we have enough time? Because the core problem is we need to make everyone see work being done and the people we are doing it facing toward the voices projected toward the camera. Either that or we need to come up with probably a wireless microphone that allows us to record what they are saying to the recording device. (Co-planning F6)

In addition to addressing problems, Dan pointed out the limitations of instructional activities. For instance, based on his experiences with PTs, he pointed out that the proposed structure and selected multi-step mathematics task was not appropriate for the rehearsal. He said the original length planned for the Rehearsal 2 was not enough for PTs to generate and practice asking purposeful questions. The content of student work was different from Rehearsal 1 and PTs needed to understand both presented solutions and connect them in the given time.

> Dan: I am not being stubborn but I am still stuck with the idea of trying to choose these problems carefully. So there is enough for to fill in, but only enough for to fill in.
> Finn: It might not be. As you said before, after 3 minutes we are going to stop where you are and you are going to debrief what they have done.
> Dan: The concern I had was about more the audience, there is a potential that you are hopping from question to question with no resolution, they all run out of time. Then we just summarize where we are at this point and we are up to a new thing
> Finn: What is the point of peer teaching? It is not about the audience. It is about the teachers to practice teaching
> Dan: OK, good point. (Co-planning F9)

Based on the learning objective of the rehearsal, Dan voiced a similar concern several times over the semester. He found that three minutes was not feasible for PTs to understand students' solutions and pose teacher questions. Thus, he suggested the MTEs post the students solutions beforehand and assign PTs to plan one assessing question to initiate the conversation.

> Dan: I was wondering if we want to . . . Is the goal of this activity to have them thinking on what advancing question to ask in the moment? Because if they have the solution in advance it makes it easier to decide what question to ask to start and deciding what question to ask based on what the student says a little bit easier but some situations they cannot know what to ask enough to get something out there within these 3 minutes. It is my concern. (Co-planning F6)

Most of Dan's concerns were about the alignment between the expectations for PTs and what they actually can achieve. In addition to addressing the possible limitations of particular instructional activities, Dan highlighted general issues with the course design and content. Being the instructor of the course, he pointed out that the time and work required to teach this course was demanding and limited meaningful discussions with PTs.

> Dan: Part of it's the design, because so, to be frank, part of why it feels like I'm on a train hurtling forward is because I'm having the mod coding activities, and if I'm going to keep this course within the boundaries of a quarter of my time…There's no time to really go back and look in any detail at what they do in coding. (Co-planning SP6)

Although Dan predominantly played the role of critic, other members of the COP sometimes voiced their concerns and addressed problems in the instruction as well. Bruce expressed similar concerns regarding the mismatch between expectations from PTs and what they can achieve. Reflecting on PTs' performance during collective analysis, Bruce found it very time-consuming to ask PTs to recode existing coding instances of A.A.T. as nine types of teacher questions.

> Dan: Maybe we were asking too much.
> Bruce: Plus, we never unpacked those nine- types of questions. I am concerned about that. I was thinking the time they go through and code A/A/T and code for these other ones.
> Dan: You were describing their first task to code types of teacher talk and then going back for the second pass with the 9 questions?
> Bruce: Yes. (Co-planning F8)

The role of the critic–inquirer was essential for the improvement of the practice. Wenger (2009) argues that disagreement is a mutual engagement and a productive part of the enterprise in COP. Wenger highlighted the strong relationship between perturbation and learning by stating "Learning involves a close interaction of order and chaos" (p. 97). He defined perturbability and resilience as being two characteristics of adaptability. In this study, the critic and inquirer challenged the group's decisions and perturbed the practice. For instance, as the group discussed a possible mathematics task for the peer teaching, Dan disagreed with the previous selection of tasks. He pointed out that choosing a mathematics task that was not challenging for the PTs would limit the authenticity of the peer teaching because PTs had to act like struggling students. Below is an example episode from the conversations on task selection.

> Dan: If the content that group is teaching if they are already familiar with that so is there student group.
> Finn: Right but…
> Dan: We have to artificially create a student group who does not understand… it is difficult because the sense making arise when you keep the mathematics problematic. If we cannot make the task problematic, if it is not equally problematic for the instructor group as it is the student group; then, that becomes a challenge. (Co-planning SP6)

He argued that forming an artificial group of students from the PTs who acted like struggling students not only would reduce the authenticity but also would make the activity less appealing for PTs. This disagreement enabled the group to reconsider selecting a task that would allow for PTs to implement decomposition of practices but also retain the authenticity of the setting. In addition to creating artificial students, he argued that the length of peer teaching was not reflecting a real secondary classroom setting:

> Dan: That is something that bothers me that the launch, explore and summarize model doing that in 10 minute. That is the length of quick presentation on a conference not a mathematics lesson. (Co-planning F9)

The group also extended the length of peer teaching after hearing similar criticisms and inquiries.

Expert. Similar to COP models, experts in this study negotiated their expertise by sharing their personal experiences, stories, and knowledge. This contributed to the COP's knowledge and assisted the group in negotiating the practice between MTEs' implementation and professional models of practice (Wenger, 2009; Yukawa, 2010). During the co-planning meetings, each member of the COP brought different expertise to the table. They shared their experiences and reflections and areas of research expertise in addition to narrating their personal stories. Those lived experiences played a considerable role in developing the group's knowledge. Some expertise MTEs had and offered to the COP's practice were:

- mentoring and teaching MTEs
- teaching different iterations of the same methods course
- teaching PTs
- supervising PTs
- experiences in conducting research
- expertise in design of the course; including CEIs and rehearsals
- expertise in teaching secondary students
- expertise in secondary and college mathematics

For instance, Dan's expertise in teaching college mathematics guided the group in selecting mathematics tasks and understanding PTs' mathematics experiences. While all MTEs had taught secondary mathematics, three of the COP members taught methods course for PTs.

While the group benefitted from the different areas of expertise the members offered, two experts shaped the MTEs' practices significantly: an expert on CEI models and decompositions of practice and an expert in designing and teaching courses for PTs. Bruce was expert in overall course design, particularly in the instruments that were used for PTs' analysis of practice. He addressed the group's questions regarding analysis tools, proposed ways for PTs to reflect on their practice, and took a dominant role in preparing curriculum materials for analyzing activities. Bruce created the analysis window on StudioCode, demonstrated to the MTEs how to code on StudioCode, and proposed ways PTs could code. Below is an episode from group conversation where Bruce informed the COP about his plans to instruct the PTs to code on StudioCode:

Bruce: I was going to instruct them along the lines of similar. Essentially I was going to have them listen to telling statements, assessing questions and after you hear that pause, you can move the slider back and forth and move to the middle of the statement move to the beginning and move to the end and hit it…the way I set up is to use hot keys, so they do not actually physically go to the code window and press that button but they can just click t for telling (Co-planning F4)

He also proposed a way for the group to use PTs' StudioCode Timeline (See Figure 5-1) analysis as an instructional tool to facilitate their learning of A.A.T.

Bruce: With the actual studio I have created a movie. Audio from your practice. What they would do is: we have each one of the laptop will have unique code: 1-4. It is the same code but this allows us to do when we come together as a whole group, we can stack all the time lines on each other, they have that individual codenames, otherwise it just blinks together. 3 codes; telling assessing and advancing: we can call telling something else. (Co-planning F2)

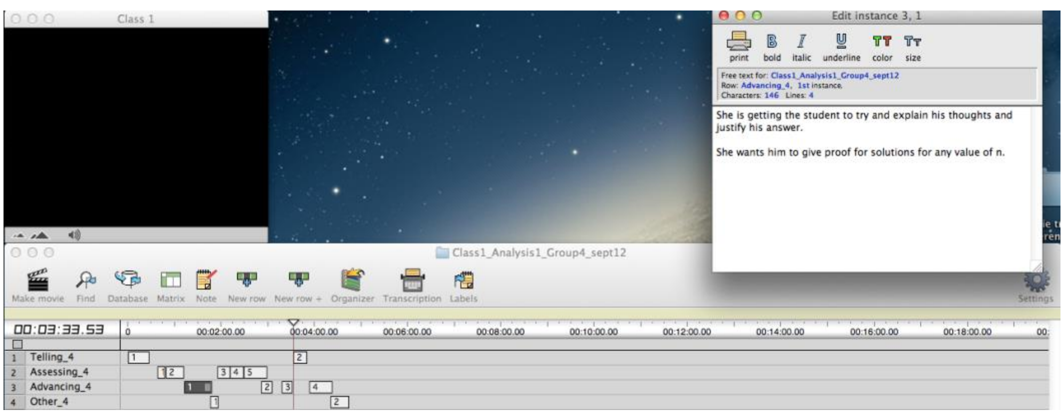

Figure 5-1: An example of StudioCode timeline.

Bruce used his expertise in analysis of practice to plan details such as size video recordings and capacity of external drives required. He assisted the group in deciding how to disseminate videos and PTs' analyses of practice. Bruce used his expertise in analysis of practice to plan details such as size video recordings and capacity of external drives required. He assisted the group in deciding how to disseminate videos and PTs' analysis of practice. "It is going to take them 2-3 minutes to download. That is something we could do; we can disseminate that among themselves. It is too big to upload to box angel I think" (Co-planning F5).

In addition to the analysis tools, Bruce also was an expert on the literature that discusses core course concepts such as rehearsals. He not only taught the A.A.T. framework to the group but also provided resources for the COP to learn more about them. When the COP sought more information about the assessing/advancing framework for the debriefing of collective group analysis, he recommended possible resources to read more about their definitions:

> Finn: Is there any other resources Bruce? I want this to be about not we think assessing and advancing questions are but what the theory says about. We cannot come up with our definitions of them. The staff is already in the literature. I want to make sure I understand. I may have a conversation with Peg some point. There may not be enough in that article.
> Bruce: It is very short.
> Norah: I heard students asking question like "assessing / advancing what?" You mentioned advancing toward a math goal. Was the goal discussed in the article?
> Finn: I think It is. Because it part of the 5 practices is identifying the math goal, right Bruce?
> Bruce: 5 practices or the TTLP?
> Finn: It is all connected together.
> Bruce: In both articles they (Smith & Hughes, 2008; Smith & Stein, 2011) talk about the math goal. Math goal I think is mentioned a little more explicit but both mentioned. In the book (Smith, Silver, & Stein, 2005), they spend more time then they have in the MTS article on the math goal. They actually stated that it is practice 0 of 5 practices.
> Finn: Do you have the practices to actions book? Is there anything mentioned about assessing and advancing in that book?
> Bruce: There is. I cannot say yes or no. I will check on that this evening.
> Finn: The book might be helpful in unpacked assessing and advancing a little more in there that we can copy and give to them.
> Bruce: I do not think they have but let me check. Staples (2007) article I know. That is not related to Peg's but I think she does talk about "extending the work of students"
> Finn: That might help us unpack that a little more.
> Bruce: I have a couple sources I can go to. (Co-planning F3)

As seen in this excerpt, Bruce used his expertise in analysis tools and course constructs to assist and lead the group in structuring and setting up learning activities such as collective analysis.

Another expertise that benefitted the group was the designing and enacting of instructional activities for PTs. As the primary instructor of the course, Finn determined learning goals and planned the course activities to realize those goals. She shared not only her plans for lessons but also the purpose and rationale for instructional activities. Her expertise guided the group in preparing curricular materials for the course activities. In an excerpt from the group conversation, Finn described an activity PTs would engage in, her rationale for the activity, and preparations needed for that activity: “I want us to talk about the first part of your framework, assessing, telling and advancing questions. I want to unpack those things by analyzing some of my practice in the staircase problems. I want you to pick off a couple of ones that they can analyze using StudioCode” (Co-planning F2). As seen in the quote, Finn explained how she intended to unpack pedagogy concepts for PTs and asked the doctoral students to identify instances for PTs to analyze.

As the veteran MTE, Finn mentored doctoral students with her expertise in teaching PTs. While during the fall Finn provided more direct instructions for MTEs, the following semesters she coached MTEs with her experiences and suggestions. Below is an example episode where Finn told Dan how to lead a class activity and discussion in fall.

> Finn: For the last half hour on Thursday, what I want you to do with them is the chapter 2 materials[9]. I am going to ask them (PTs) to do “odd + odd = even” (i.e., Show that odd + odd = even is true) task but probably not to prove; convince someone odd+odd is always even. Write a convincing argument that shows…
> Dan: None of these students were in the reasoning course[10]?
> Finn: No. I do not want you to unpack this very much. Walk around and see what they are doing. I just want to get them in this thinking. We suspect what will happen is that they will create algebraic proofs. Even though I do not say proof. It will be interesting to see if they write algebraic proofs. Then I want you to give them student solutions A-J [See CORP book for more details], and ask them to go through them and discuss whether they are valid arguments.
> Dan: This is during class?
> Finn: Yes. If they do not finish it will be homework. (Co-planning F6)

She pointed out things to consider, suggested pedagogies for instructions, and shared her reasoning behind her suggestions. For instance, as Dan struggled with PTs not developing necessary understandings of proofs, she advised him to postpone upcoming activities and address issues that would support PTs’ limited understanding of valid mathematical arguments. Below,

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[9] A chapter from the Cases of Reasoning and Proving (CORP) book (Smith et al., n.d.)
[10] A content course offered for PTs in the program

she proposed to engage PTs in a whole group discussion of the question "what constitutes a proof?"

> Finn: If you debriefed the case, and then you put up their what makes a valid argument, what they said, and said do you have anything now to add to this?
> Dan: Mm-hmm (affirmative). I think that seems reasonable.
> Finn: Then get them to add to it and then say, okay, I've got some problems with some of these things…, 'cause I think you'll get more out of pushing on what makes a valid argument if you've made ... if you have pointed out a couple of things that's happening in the case of Nancy Edwards[11], which is who has a valid argument here? (Co-planning SP6)

Broker. Similar to experts, a broker in the co-planning informed and guided the COP in particular aspects of their practice. Being a member of other COPs, (i.e., Dan as mathematician & Finn as a faculty member), they translated and coordinated knowledge from one COP to another (Wenger, 2009). A broker brought new knowledge, offered novel ideas and tools, and acted dominantly to negotiate the meanings of course concepts and practices of teaching PTs. For instance, Bruce took the role of broker in the planning of the course since he was the one who introduced analysis tools and activity structures to the group. In addition, nonmembers of the COP took the role of broker once in a while. These outside members introduced and instructed the use of tools in the analysis of practice and provided resources for instruction. Two educational technology experts contributed to the COP's use of technology in engaging PTs in an analysis of practice: one suggested tools for instructions and recording instruction and the other suggested efficient ways to use StudioCode in instruction. For instance, when the group was trying to solve how to project students' solutions so that they could capture both student work and PTs in the camera, the tech expert suggested a solution:

> Finn: Here is the another possible solution, Mark suggesting.
> Bruce: Yes. He has got ideas.
> Finn: It is built on something he said. We could do it on paper and scan it. Make a pdf and just project to the screen through a computer. You could stand next to the computer screen and talk to each other. (Co-planning F6)

[11] A narrative teaching episode where secondary students work on "Show that odd + odd is always even"

Another time when the group was struggling with how to set up StudioCode to make coding easier for PTs, they consulted a faculty member who is an expert in using analysis tools for practice:

> Bruce: What it captured here is based on Scott's suggestion. He is noticed that sometimes students caught up with figuring out when to start and finish stop. The way it was set up is when you click it, it will capture 5 seconds before and after the instance. (Co-planning F2)

In addition to members of the group, the research on CEIs and decompositions of practice acted as a broker during the co-planning. The research provided the group with new repertoire of practices and tools such as *posing purposeful questions* and *eliciting and use of students thinking* in addition to models to design the course to support PTs learning of practice such as rehearsals and CEI cycles. In this study, CEI models, decomposition, approximation, representations of practice, and teaching practices themselves provided a shared repertoire of tools and language for the group. Having a systematic vocabulary, particularly a familiarity with Assessing, Advancing, and Telling, assisted the group when talking about shared experiences and making sense of MTEs practice (Wenger, 2009). In addition, those constructs served the group in planning and enacting instructions for the methods course.

Analyzer. An analyzer in this study monitored PTs' performance and focused on the factors that might have led to certain PTs' learning and disposition. The analyzer observed PTs, recorded his noticings about PTs and shared them with the COP, and sought for and aimed to explain the dynamics between PTs' learning and instruction. MTEs in this study engaged in analysis of PTs' learning regularly throughout the year. Bruce and Dan acted this role dominantly. This course was a part of Bruce's research, so he engaged in a regular analysis of PTs' work. He shared his analysis and noticings regarding PTs' learning and progress constantly. For instance, analyzing participants' interviews, he notified the group about the contribution of the StudioCode collective analysis on PTs' learning (Co-planning F10). In another meeting, the

COP consulted him on the improvement in PTs' conceptualizations and uses of teacher questions. Based on PTs' rehearsals and collective analysis, Bruce found that PTs were not learning and implementing assessing questions and eliciting student thinking as the COP initially expected. Based upon his evaluation of the PTs' learning, he suggested additional instructional activities for PTs to analyze and engage in more students' thinking.

> Bruce: That they're acknowledging that these particular verbal moves that Nancy Edwards is doing with her students are situated in the larger context of moves. That those are starting to come to the surface and that those are explicitly being pointed to. (Co-planning SP3)

Similarly, he informed the group about PTs' learning of mathematics. Below is an excerpt where Bruce shared his observation about PTs' understanding of mathematical arguments. He saw that their judgment for the validity of a proof is vague:

> Bruce: A big one on the list is the use of examples. They did recognize that examples alone don't constitute a proof. But one of the things that they didn't recognize with student B, who uses a generic argument, they didn't recognize that as being a proof. (Co-planning F7)

Likewise, being a novice instructor of the course, Dan engaged in analyzing PTs' learning frequently. He shared his observations about PTs' perceptions of mathematics and teaching. For instance, during the CEI cycle, he noticed that PTs' mathematical goal was "getting symbolic expression argument" rather than students generating arguments based on the proof criteria they created (Co-planning F8). Dan observed that PTs did not consider proving as mathematics content. He also noticed that PTs did not see the purpose and value of the instructional activities:

> Dan: Since student solutions are coming through in an authentic way PTs think they are not likely to happen in an actual class. We are just trying to make things difficult for them which gives an excuse to PTs not to learn. (Co-planning F8)

Novice. The novice in this study was the novice doctoral student. She contributed least to the COP's practice. Although she shared her analysis of PTs' learning and contributed to preparing students' solutions, she rarely made suggestions about instructions. She asked questions to verify her understanding of the conversation and to clarify tasks to be completed. Her questions showed her lack of experience and limited professional knowledge. For instance, as the COP discussed mathematics problems to be selected for the first rehearsal, she struggled in identifying the mathematical goal of a mathematics task:

> Norah: I do not know the learning goal of the some of the questions. For example, the question that is asking to obtain three different representations of the same number. I do not know, what is the goal of that question. Is it just to generate three different representations, is it asking them to make connections between those representations? (Co-planning F4)

Her comments here illustrate her limited understanding of a mathematical goal. In another episode, the COP members were assigned to generate “student solutions” for the rehearsal; she was not sure what a secondary school student’s incomplete solution would look like and made inaccurate assumptions about students’ proof writing. For instance, she created student solutions using induction and modular arithmetic. This indicated her limited knowledge of student understandings of mathematics and secondary curricula.

Assertion 2: The critic-inquirer assisted in making some aspects of PCK accessible to the members of the group.

Some of the roles, notably the critic-inquirer, helped MTEs’ professional knowledge (i.e., PCK) to be visible and knowable to members of the group. The critic-inquirer’s questions invited the COP members (mostly the expert) to open up their minds and make their thought processes more accessible to others. The critic-inquirer pushed the experts to seek and explain the theoretical or practical ground for their instructions. For instance, the COP discussed how to assign PTs to code their rehearsal videos collectively. Dan wondered why, if each person has StudioCode on their laptop, the analysis groups were formed in threes. Finn explained that collective analysis was part of the course and the study design. Dan asked “Why have we been doing this in groups of 3? We have StudioCode installed each of the laptops.” Finn replied “Because it is the group analysis, the part of the study.” (Co-planning F8). Dan’s curiosity invited

Finn and Bruce to explain the purposes for instructional activities. For instance, as the COP were planning a school visit for PTs where they worked with a group of high school students, Dan questioned the purpose of the school visit and its relation to other course activities.

> Dan: I am missing the point of what they are doing with Pat's students if they are not doing the rehearsal 3, how it fits within?
> Finn: It is the final project. It fits in because it is the thing we collect as the final assignment (Co-planning F7).

Here, his question enabled Finn's justification of an assignment visible to members of the group. Dan's questions enabled Finn to communicate "hidden" aspects of pedagogical decisions that were less likely to be surfaced/apparent in solely mentee/mentor conversations. For instance, as the group selected tasks for the peer teaching, Dan wondered aloud about the criteria two instructors of the course had previously used in selecting tasks.

> Dan: I have a question. If we are not focused on the whether the audience would learn anything and what are they learning, then what are the features of the task we are looking for? You asked if the combinatory was too easy, does is matter if it is too easy?
> Bruce: What I am looking for a task is whether or not there is a possibility for students to approach in different ways whether the way they represent the problem in multiple ways so that we come back to the whole class discussion maybe some of them have actually done in a different way and there is more when we come back to the whole class.
> Finn: Even if there is not different ways to do it, what is the sense making to do it, right? What I want teachers to do is to is that I want them to make sure that their students are making sense of what they are doing not just getting the right answers. So even if there is only one answer, there are right answers to the problems and you can only get there in one way, there is still, "how do you know the mathematics underneath that, how could you justify that is the right answer?" I think that is the more authentic. Because when they go to teach, they are going to teach out of materials where is one answer and one way to get it and the point is not "Ohh, you get the right answer, great" the point is "I want you to practice understand how students thought about it and if the student is building understanding of some mathematics content becoming procedurally competent.
> Dan: It is difficult because the sense making arise when you keep the mathematics problematic. If we cannot make the task problematic… it is equally problematic for the instructor group as it is the student group; then that becomes a challenge.
> Finn: In some ways yes in some ways no. There is a culture of doing peer teaching and that is what we do. But there are so many, it causes us to over think it and try to make it perfect. Basically I want PTs to plan some instructions, I want them to get up in front of class and practice some of the staff we have talked about, right?
> Dan: Right.

Finn: We give them the task to use, right? We make them ask questions that probe students understanding of the mathematics that is in the task, right? We let them answer some of the questions students pose to them about the mathematics, "I do not understand how you got that. How did you get it?" it is not they give the right explanation or a perfect explanation. How would you answer that question? What would you pull from it?" did you tell too soon? It is not…we are trying to create a perfect situation where really it is all about get them in front and give enough scaffolding to get up in front of their peers and lead a mathematics discussion. Lead a mathematical discussion that the students did from the problem. That does not mean that the problem has to be problematic. I can give you one of these questions right now and could have a very rich mathematics discussion about what you just did.

Dan: Maybe this is what is missing. b/c of the problems I am looking at. I am having a hard time to envisioning you could have a rich discussion around these problems.

Finn: That is what you cannot do it with any problem. You have to really careful about the problems you pick. (Co-planning F9)

This episode reveals very detailed information about an expert MTE's preference in selecting mathematics tasks to facilitate PT learning; he focuses on determining what specific characteristics of the tasks they were searching for and why. Based on the learning goals of the peer teaching activity, the expert MTE was looking for mathematics tasks that required making sense of the mathematics involved, promoted rich mathematical discussions, and rich opportunities to practice understanding students' thinking and posing teacher questions. Finn also pointed out that she chose tasks from secondary textbooks she wanted PTs to get familiar with. Even though those tasks might look less sophisticated than advanced level mathematics tasks, PTs would still learn something new about something they already know. Finn described this process:

Finn: Both the students and the teachers will learn something new. The teachers because they kind of already know something about this and then can look at these problems and think about how does this deepen what I already know, way that I do not ever see them doing in the core+ staff. For peer teaching. I am using the core+ textbook for the function course and they learn a lot. But because my focus on them as learners. (Co-planning F9)

Zaslavky (2007) argues that task selection to facilitate PT learning is a complex process and requires the MTE to consider various factors. Dan's questions helped to reveal those factors influential in task selection. He continued to ask about the features of mathematics tasks across the semesters. His questions revealed not only Finn's justification for selecting tasks but also those different factors to be considered in task choices. For instance, as he questioned the feasibility of PTs using the launch-explore-summarize model in their peer teaching, Finn pointed out the need for alignment between the instructional tool and the mathematics task:

> Dan: Is launch, explore, and summarize a thing one does only when they have a class period?
>
> Finn: No. I do launch explore and summarize, you could do 3-4 cycles of it in a class. In fact, I see teachers get into trouble with a 90-minute class period when they launch and let them explore for 60 minutes. And summarize for 25 minutes. It is out of control.
>
> Dan: I am wondering and I do not have the experience to answer this question: Is doing launch, explore, and summarize within the amount of time we have worth?
>
> Finn: As opposed to?
>
> Dan: I am not sure what the alternative would be. But if you are saying we are focusing on their learning of teaching, are they going to learn something from trying to do that cycle in that short amount of time and is it consistent with the other things we have done in the course up to this point to focus on that way of framing the activity.
>
> Finn: I think the answer is yes if we make the tasks small enough to be able to it. (Co-planning F9)

As seen in the excerpt, Finn stated that the instructional model being used for the mathematical activity was important in selecting appropriate tasks. Dan's question here also enabled the group to hear more about the features of an instructional model, launch-explore-summarize and how the iterations of that model could be used in different contexts.

Chapter Summary

In this chapter, I explain MTEs' roles as they co-planned a course together. Based on MTEs responsibilities, expertise, and contributions, I grouped these roles in six categories: *leader, organizer, critic-inquirer, expert (and broker), analyzer,* and *novice*. Findings suggest that variations in roles contributed to the MTEs' co-planning practices and supported their professional knowledge. Because they facilitated different contributions to the MTEs' practices, these roles increased the efficiency of co-planning. In addition, those different roles assisted in bringing important aspects of PCK out into the open for the group to access and provided opportunities for MTEs to increase their knowledge.

Chapter 6

MTEs' Knowledge that Surfaced in Co-Planning

In this chapter, I present the knowledge that was surfaced in MTEs' co-planning meetings. I describe the knowledge sought and used by a group of MTEs in co-planning a methods course that focused on facilitating PTs' learning of teaching practices. I expand Chauvot's model (2008; 2009) by providing a detailed description for Pedagogical Content Knowledge (PCK), Curricular Knowledge (CK), and Subject Matter Knowledge (SMK) (See Table 6-2). This chapter illustrates how different forms of knowledge inform MTEs' decisions and practices. Findings show that co-planning allows space for MTEs' knowledge to be visible and provides opportunities for novice MTEs' to develop professional knowledge.

Knowledge Categories, Distributions, and Contributions

In this section, I describe the knowledge domains and their subcategories that emerged in MTEs' co-planning practice. I demonstrate the distribution of knowledge domains across co-planning meetings and explain their contributions to MTEs' co-planning practices. The group used CK while they designed learning activities, created course artifacts, and selected course readings. While PCK shaped a majority of the instructional decisions, SMK mostly assisted in selecting tasks and creating student solutions for rehearsals. A brief description of each knowledge domain from Chauvot's model (2008; 2009) is provided for the reader in Table 6-1.

Table **6-1**: Brief descriptions for CK, PCK, & SMK for teaching PTs

Curricular Knowledge CK	*Addresses the knowledge about curricular materials & resources in mathematics teaching.* This domain includes knowledge of available resources and curricular materials for teaching a methods course; knowledge of teachers' interaction and use of curricular materials; and knowledge of effectiveness of training programs, PDs, teacher education programs, and field experiences. In addition to knowing teacher education programs what PTs are doing CK-L & what PTs have learned and will learn CK-V
Pedagogical Content Knowledge PCK	*Addresses any knowledge about the teaching work of MTEs.* Knowledge about mathematics teacher education includes how PTs—and college students more generally—learn about mathematics education concepts. Chauvot explains this knowledge as the knowledge of research about PTs' and in-service teachers' conceptions and beliefs about mathematics, mathematics teaching, and mathematics learning; how conceptions and beliefs impact students learning; knowing PTs' mathematics knowledge and their experiences with mathematics concepts and procedures; and envisioning how educators can advance PTs' experiences and understanding of mathematics. In addition, this knowledge includes knowledge about instructional models and approaches.
Subject Matter Knowledge SMK	*Addresses anything that could be taught in the course.* Regarding mathematics, knowing axiomatic systems, structures, and rules of logic. Regarding mathematics education, knowing theories, research, and practice in mathematics education. It is also the knowledge of mathematics teaching about children learning mathematics and secondary curriculum.

Table 6-2: MTEs' knowledge surfaced in co-planning a methods course

CK teaching the course)	Knowledge of Resources	Knowledge of facilities, rooms, and equipment
	Knowledge of (Curricular) Materials	Knowledge of tools Knowledge of course materials & artifacts Knowledge about mathematics tasks
	Knowledge about Assignments & Assessments	Knowledge of exams and projects Knowledge of creating assignments & assessments
	Knowledge about the Course Design	Knowledge of the course activities and instructional goals Knowledge about CEIs
	CK-L & CK-V (Other courses)	Knowledge of PTs' learning in other course Knowledge of the other courses PTs take
CK-M general	(Mentoring) General CK for teaching PTs	Knowledge of the resources for MTEs; books, journals, & tools
PCK teaching the course	Knowledge about PTs' understanding of mathematics	Knowledge about PTs' mathematics experiences & their struggles Knowledge of PTs' learning of mathematics
	Knowledge about PTs' learning to teach	Knowledge of PTs' learning about analysis of teaching Knowledge of PTs' learning about students thinking Knowledge of PTs' learning about responding to students thinking Knowledge of PTs' perceptions about teaching
	Knowledge about instructional methods and strategies	Knowledge of strategies to support PTs' learning Knowledge of instruction models Knowledge for determining details of instruction
PCK-M general	(Mentoring) General PCK for teaching PTs	Knowledge of MTEs' teaching practices Knowledge about instruction (teaching methods and strategies)
SMK Teaching course	Knowledge of Pedagogical Constructs	Knowledge of teaching practices Knowledge of the components of teaching mathematics
	Knowledge of Students mathematical understanding	Students' approaches Students' mistakes Students' difficulties
	Knowledge of Students	Students' characteristics and behaviors
	Knowledge of Secondary Curriculum	Tasks, tools, content, textbooks
	Knowledge of Mathematics (related to the course)	Proofs, Functions, Representations, Number relations, Calculus
CXK Knowledge	Knowledge of Educational institutions	Secondary schools Department, university policy & regulations

Assertion 1: Co-planning provided opportunities for important MTEs' professional knowledge (i.e., CK, PCK, and SMK) to be brought into light.

Curricular Knowledge for Teaching PTs

Regardless of the depth and length of the conversation episode, CK occurred the most frequently during the co-planning meetings. In my analysis, 144 out of 327 coded knowledge instances from the co-planning meetings were about the use of CK (See Figure 6-1). The CK used mostly by the group was about the structure and use of the curricular material in teaching PTs. This knowledge also included how to engage PTs with the particular curricular material that best aligns with the learning goals. The subcategories of CK are; *knowledge of tools*, *knowledge of course materials*, *knowledge of tasks*, *knowledge of resources*, *knowledge of assignments* and *knowledge of other courses* (See Figure 6-1). In the following sections, I describe each category and provide specific knowledge examples. The subcategories of knowledge are presented based on the frequencies of their occurrences in the meetings.

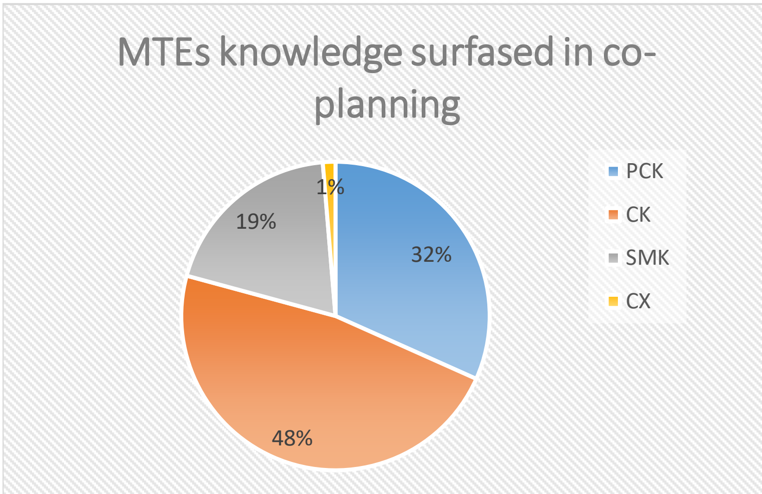

Figure 6-1: The distribution of knowledge domains (i.e., CK, PCK, SMK, CX[12]) emerged in MTEs' co-planning practice.

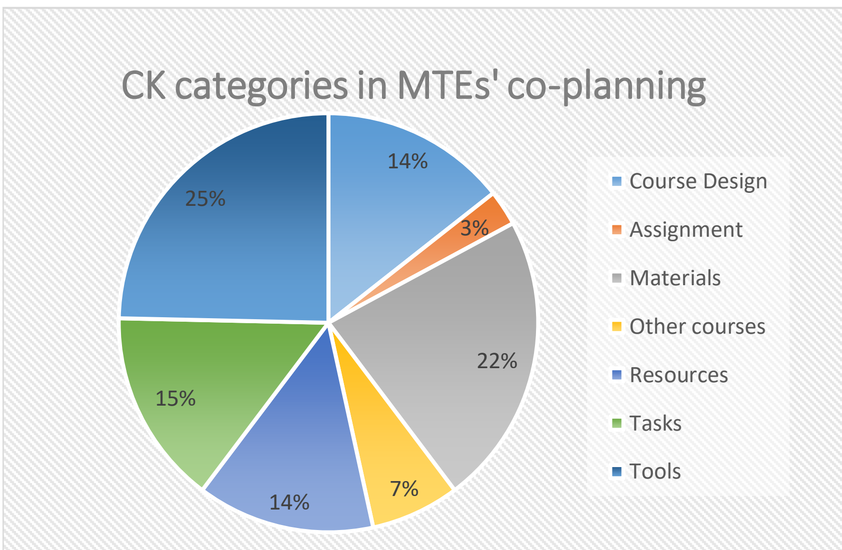

Figure 6-2: Subcategories of CK surfaced in the MTEs' co-planning.

Knowledge of tools

The CK knowledge that predominantly emerged in the conversations concerned tools that would be necessary, ideal, and available for an instructional activity, and how to use those

[12] *Note:* Only 1% of the group conversations involve CXK. Thus, it was purposefully left out of the discussion in this chapter.

particular tools in the practice that best aligns with the learning objectives. For instance, the group discussed how to set up for the rehearsal, what tools PTs would need, and ways to present student work. Another specific example of knowledge is the resources needed for collective analysis of a representation of practice and how to use those resources. These resources include but are not limited to software for video analysis, computers, and file sharing platforms. The group also identified problems with the current tools. For instance, StudioCode instances are not saved if used in a different laptop. They compared the use of multiple tools—such as the document sharing tools Google doc, Dropbox, and Box—and selected the best option among the available ones. After careful selection, the group wrote instructions for the coding activity. This knowledge requires expertise in using specific tools, such as familiarity with the storage size of the videos or how to link codes to labels and code simultaneously.

Table **6-3**: Knowledge of tools & examples

Knowledge about available and needed tools for an activity	Tools needed for the collective analysis of a representation of practice and how to use them: StudioCode, computers, file sharing tools Materials PTs need for Rehearsal 1: Graph paper, rulers, calculators, and tools to present student work; notepad & projector
Knowledge about features of tools (pros & cons)	Problems with a tool: StudioCode instances are not saved if used in a different laptop Comparing file sharing tools: Box is more convenient compared to Dropbox and Google Drive Comparing platforms to keep course materials posted: Angel vs Google doc
Knowledge about how to engage PTs with a tool	Writing instructions for coding on StudioCode: How to link codes to labels and code both simultaneously

Knowledge of course materials and artifacts

After the tools, knowledge of the course materials was the second most frequent knowledge category discussed in the co-planning meetings. The group used this knowledge as they decided on and created curricular materials for learning activities. This subcategory of knowledge includes knowledge about course-related materials and how to use them in instruction. The group knew the content of course readings, the teaching moves addressed in the Edith Hart case, and materials needed for each node. They also knew the prompt questions for PTs to reflect on during their practice, the multiple purposes of curricular materials in practice, and the format for observation sheets used for practicing A.A.T. and keeping PTs' attentive in peers' rehearsals. This knowledge also includes documents that could be beneficial for course activities, such as *Moving Straight Ahead* textbooks, *Mathematics Teachers* articles on fraction division, and artifacts PTs created in the course, such as criteria for valid proof and StudioCode timelines.

Table **6-4**: Knowledge of course materials and artifacts

Knowledge about appropriate curricular materials	*Mathematics Teachers'* articles on fraction division Criteria for valid proof
Knowledge of creating materials	Writing partial students' solutions for proof tasks Stacking StudioCode timelines
Knowledge about purpose and use of curricular materials in practice	PTs fill out observations protocols while listening to others in rehearsals in order to categorize types of A.A.T. & to pay more attention to their peers
Knowledge about course related materials	Eight teaching practices described in *Principles to Actions* book

Knowledge of mathematics tasks to be used in course activities

The group used task knowledge in selecting appropriate mathematical tasks for different instructional activities and assignments. This knowledge included several resources for finding tasks, such as textbooks, websites, and practitioner journals. It also included identifying desired characteristics of tasks, such as having high cognitive demands and prompting mathematical proficiency. The group decided that a task for peer teaching should be doable with multiple solutions. Also, the task should have a high potential to prompt rich mathematics discussion such as generalizations or connecting representations. Another possibility is to choose tasks that focus on developing the conceptual understanding for a known procedure, such as the quadratic formula, R squared function, fraction decimal conversion, or Cramer's rule with matrices. As the group decided on tasks, they benefitted from their CK-V as well. For instance, they knew PTs had learned about induction proofs, Cramer's rule, and symmetry in undergraduate mathematics courses.

Table **6-5**: Knowledge of mathematics tasks & examples

Resources for tasks	Eg: Number proof problems used for reasoning course Eg: Calendar Tasks in the *Mathematics Teacher* Journal
Tasks' characteristics	Eg: Generalizations and connecting representations can generate rich mathematical discussions during the rehearsals

Knowledge of resources and facilities

This knowledge category is not about specific tools for teaching the course but general instruction-related resources in teaching institutions. Chauvot (2008) categorized these types of resources under CK. This knowledge category is similar to Contextual Knowledge (CX) defined by Grossman (1990), yet is still related to the use of tools in the course. This knowledge includes knowledge of space, available facilities, technical support, human resources, and how to access and benefit from them in teaching the course. The group used the knowledge in reserving rooms for collective analysis activities, accessing technical consultations, and available equipment in the department.

Knowledge of the course design and CEI models

This knowledge helped the group to determine instructional details for activities that fit in the course design, and also helped the group reflect on and revise course activities. This category includes overall knowledge about the CIE models and rehearsals and the literature on the approximation of practice and teaching practices. The category also includes instructional activities in CEI nodes and their relationship to other course activities. Examples of specific knowledge for this category include the role of mathematical goals in A.A.T. framework and the differences between micro-teaching models like rehearsals, peer teaching, and mini-lessons. The group benefitted from this knowledge as they enacted and revised the CEI models in the course. The group also used this knowledge to compare the theoretical perspectives with the course design and activities. For instance, MTEs noted that peer teaching in the course did not match the description of a coached rehearsal. In addition, teaching high school students was not rehearsal due to the lack of coaching, yet this experience counted as the last approximation of practice. This knowledge captures the goal of each node in the CEI, the activities that PTs would engage in, and how CEI nodes would support PTs learning to teach. For instance, while the goal of completing the TTLP was to anticipate students' approaches to generating A.A.T., the StudioCode activity was designed to engage PTs in collective and reflective analysis on practice.

In addition to CEI models and decompositions of practice, this category involves knowing the research related to course content, such as *Principles to Actions* (NCTM, 2014) (i.e., a book on teaching practices), Munter's (2014) framework on PTs' visions for high-quality mathematics instruction. For instance, the group talked about a framework for supporting PTs' practice they discovered in research conferences, namely *Conceptualizing Mathematically Significant Pedagogical Opportunities to Build on Students' Thinking* (Leatham, Peterson, Stockero, & Van Zoest, 2015). MTEs discussed ways this framework could inform the course activities; specifically, they discussed ways to facilitate PTs' understanding and their responses to students' thinking.

Table 6-6: Knowledge of CEI models & decompositions of practice

Knowledge about the course design	The role of mathematical goals in A.A.T. framework Difference between rehearsals, peer teaching, mini lesson (i.e., Peer teaching corresponds to the last coached rehearsal while teaching to high school kids not rehearsal but the last approximation of practice)
Knowledge about CEI models and nodes	Approximation of practice: how to increase authenticity in each cycle (e.g., The rehearsals are artificial because they are controlled for the learning purposes)
Knowledge about the course activities & their learning goals	The goal of the analysis of representation of practice for PTs in the 2nd rehearsal is …. PTs complete the TTLP to predict student approaches in order to generate A.A.T., StudioCode activity is designed to engage PTs in collective/reflective analysis on practice

Two other subcategories of knowledge surfaced in the co-planning meetings: knowledge of assessment and assignments, and knowledge of other courses. The group benefitted from the **knowledge of assignment and assessment** in creating assignments and assessment instruments to support and evaluate PTs' learning. For instance, the group decided that PTs would write reflections of their uses of A.A.T. and examples of Finn's coaching to show their understandings of teacher questions. The group needed to know university policy for finals and grades, and signature assignment requirements for the course in order to identify expectations from PTs. The group benefited from their **knowledge of** mathematics tasks and tools used in **other** methods **courses**, which Chauvot defines (2008) as CK-L. For instance, knowing PTs are required to bring an external drive for another methods course helped the group to make decisions about how to disseminate video recordings.

Pedagogical Content Knowledge for Teaching PTs

PCK follows CK in frequency, occurring ninety-six times in the meetings (See Figure 6-1). PCK for MTEs refers to overall use of the knowledge about learners, content, and instruction in teaching PTs. Here, I describe PCK in four domains; *knowledge about PTs' understandings of mathematics*, *knowledge about PTs learning to teach*, *knowledge about instructional methods*, and *general knowledge for teaching PTs.* As seen Figure 6-3 shows, the majority of the PCK that surfaced in the co-planning sessions was related to instructional strategies for teaching the course and PTs' understandings of mathematics. Although PCK regarding PTs' understandings of mathematics occurred less frequently, the conversations around this topic were lengthy and in depth. Below, I explain each category and its contribution to MTEs' practices.

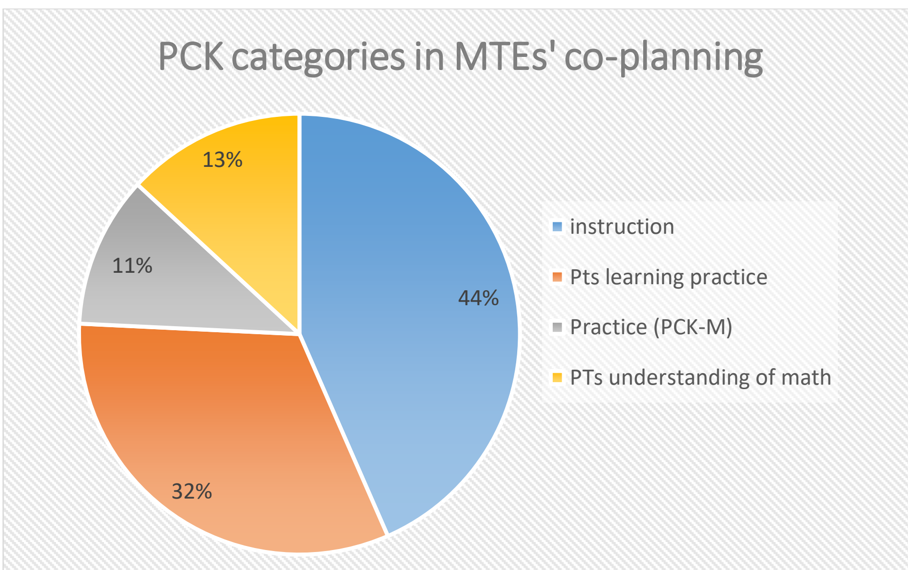

Figure 6-3: Subcategories of PCK surfaced in MTEs' co-planning.

Knowledge about instructional methods and strategies

Knowledge about instructional methods and strategies surfaced as the MTE group determined instructional details and reflected on practice. This category requires knowing multiple instructional models and teaching techniques. This knowledge informed many pedagogical decisions that MTEs made during the semester. MTEs benefitted from this knowledge as they wrote instructions for rehearsals and coding activities. These activities included planning the rehearsal structure, arranging student groups, and writing scenarios for the rehearsals. They determined the expectations for PTs as they plan, enact, analyze, and reflect on practice and designed additional learning activities to support PTs' learning. Using their knowledge of instruction, MTEs made critical decisions about when to make student solutions available for PTs, how to respond to PTs during the rehearsals as acting students, and when to coach PTs in the rehearsals. Knowing the complexity of teaching practice, MTEs worked to reduce the work demand for PTs. For instance, to decrease the difficulty of co-planning, MTEs formed smaller groups for competing TTLPs. Knowing PTs would struggle with generating teacher questions, MTEs demonstrated where and how to use A.A.T. in their own practice.

Several instructional ideas informed MTEs' pedagogical decisions such as "productive struggle" (NCTM, 2014), "funneling and focusing questions," (Franke et al., 2009), "constructive feedback" (Dweck, 2010), and "orchestrating discussion" (Smith & Stein, 2011). For example, MTEs decided to inform PTs ahead of time that student solutions are based on realities and designed to give opportunities for A.A.T. To support PTs' analyzing skills, MTEs asked PTs to give their peers feedback on things that went well and things to work on. MTEs decided to use the language of "missed opportunity" and "successful moment" in the observation protocol. Furthermore, MTEs designed additional activities if they saw a need to highlight some ideas. For instance, to support PTs understanding the connection between pedagogical ideas, MTEs asked PTs to analyze mathematics tasks through the lenses of "strands of mathematical proficiency" (NCTM, 2001). To challenge PTs' limited understandings of proof, MTEs engaged PTs in analyzing and discussing what makes a mathematical argument a valid proof.

Knowledge about PTs' learning to teach

This category addresses MTEs' knowledge of PTs' learning in four domains: how PTs learn to analyze teaching; how PTs learn to attend to student thinking; how PTs learn to respond to student thinking, and PTs' perceptions about teaching. Below, I provide a description and specific examples for each domain. MTEs benefit from this knowledge as they design learning activities and determine instructional details. I provided a "thicker description" (cite and maybe define) for this category of knowledge due to the increasing demand for teacher educators to learn more about how to support PTs' learning of decompositions of practice. In addition, this type of knowledge was not discussed in Chauvot's model. Describing this category extends her model regarding MTEs knowledge in order to facilitate PTs learning of repertoire of practices and tools for teaching.

Table 6-7: Knowledge about PTs' learning to teach

Knowledge of PTs' learning about analysis of teaching	Eg: How far PTs can code in a class period, what would be efficient to assign PTs in terms of coding
Knowledge of PTs' learning about students' thinking	Eg: Knowledge about what PTs can recognize in terms of students thinking (e.g., PTs will ask questions only if something looks incorrect at this level)
Knowledge of PTs' learning about responding to students' thinking	Eg: Knowledge about what PTs would take as an advancing question (e.g., PTs probably will pose the same type of questions)
Knowledge of PTs' perceptions about teaching	Eg: PTs' perceptions of what teaching is: stand in front and talk to the class & problem based learning does not afford that

i) *Knowledge about PTs' learning to analyze teaching*

This knowledge captures PTs' learning trajectory for analyzing and reflecting on their own practice or that of a colleague. MTEs used their expertise of PTs' noticing and analysis skills in addition to their expertise with how to support these two areas. For instance, MTEs needed to determine instructional details for the collective analysis of teaching activity. As they wrote the instructions for coding, they needed to consider how much video PTs could analyze individually and collectively and what PTs should focus on for the coding to be efficient and feasible. This knowledge assisted MTEs in deciding the coding structure, coding labels, and allocated time needed for the activity and daily writing assignments. As MTEs designed the instructions for the rehearsals, they needed to determine what PTs pay attention to when observing teaching, and what kind of peer feedback would be most efficient for PTs. For instance, feeling nervous would inhibit PTs from focusing on both analyzing and enacting teaching simultaneously, MTEs asked PTs to write a few observations about their peers' teaching.

ii) Knowledge of PTs' learning about students' thinking

This knowledge includes MTEs' knowledge regarding PTs' assumptions about students' thinking in addition to their attention to students' thinking. This knowledge was derived from MTEs' prior experiences with PTs and research on PTs' understanding of students' learning. MTEs shared two specific observations: PTs learn better by eliciting students' thinking from analyzing their own practice, and PTs assume that mathematics tasks they find challenging would be too difficult for secondary students. This knowledge informed MTEs in trying alternative ways to increase PTs attending and responding to students thinking. Knowing PTs are likely to ask questions if something looks incorrect in students' solutions, MTEs, 1) assigned PTs to analyze teaching episodes that included student thinking, 2) had actual students for one rehearsal, and 3) assigned readings about student thinking.

iii) Knowledge of PTs' learning about responding to students' thinking

This knowledge includes awareness of PTs' prior experiences with teacher talk moves and awareness of how to support PTs learning by posing purposeful questions. MTEs used this knowledge to create opportunities for PTs to practice A.A.T. in a limited time. For instance, knowing PTs have experienced models of telling, MTEs emphasized practicing assessing and advancing in rehearsals. MTEs discussed how the course shaped PTs' understandings of eliciting students' thinking and posing purposeful questions. For instance, hearing one type of advancing questions in the rehearsals limited PTs' understandings of the purpose of advancing questions and led them to pose that same type of question. MTEs tried to support PTs' use of student thinking in their instruction by connecting teaching practices to pedagogical ideas such as the iceberg model (Webb, Boswinkel, & Dekker, 2008), problematizing mathematics (Hiebert et al., 1997), and conceptual and procedural understanding (NCTM, 2001).

iv) Knowledge of PTs' perceptions about teaching

MTEs used their knowledge of PTs' beliefs about teaching in order to challenge and change their assumptions. Many PTs consider teaching to be standing and lecturing to a class. This belief was rooted in PTs' prior experiences where teachers demonstrate and students practice the script. MTEs knew this belief impacted PTs' expectations and their practices. For instance, MTEs discussed that PTs did not see the benefits of problem-based learning and took the pedagogical ideas introduced in the course less seriously. They have seen PTs' tendencies to 'tell' mathematics in front of a class in previous semesters. Based on this knowledge, MTEs assigned readings and had group discussions to challenge PTs' perceptions about teaching.

Knowledge of PTs' understandings of mathematics

This knowledge surfaced more as the group members shared their analyses of PTs learning and generated student solutions for rehearsals. It includes the knowledge of PTs' experiences with and understandings of mathematical concepts.

Table **6-8**: Knowledge of PTs' understanding of mathematics & examples

Knowledge about PTs' mathematics experiences	The dinner problem is easy for them because they are familiar Typical representations PTs are likely to use in calculus are…
Knowledge about PTs' struggles	PTs' struggle to understand the connection between integral and average. Studies show that PTs' understanding of series convergence & R is limited PTs struggle with making sense of percentage and decimal conversion using a particular model
Knowledge of PTs' perceptions	PTs' conceptions of proof (e.g., PTs do not pay attention to defining variables) PTs' limited conception of proof impacts their practice of asking purposeful questions.

As Table 6-8 shows, MTEs used their knowledge of PTs' mathematical experiences from other courses. They benefitted from their awareness of PTs' common difficulties in particular mathematics areas such as integrals and averages. This knowledge was derived from MTEs' experiences with PTs and their research on PTs' understandings of mathematics. MTEs used this knowledge particularly for creating student solutions and selecting appropriate mathematics tasks. For instance, knowing PTs' possible struggle with understanding proofs, MTEs avoided proposing "too obscure student solutions" (Co-planning F4). In addition, MTEs predicted that PTs' limited perceptions about a valid proof would impact the advancing questions they could pose.

Knowledge of MTEs' teaching practices

This is a general knowledge that captures information about the work of an MTE as an instructor. This knowledge was introduced as SMK for doctoral students in Chauvot's framework. In this study, I define this knowledge as PCK for Mentoring MTEs (PCK-M), having been brought to the group's attention by Dr. Finn, the mentor MTE, because it provided practical ideas for novice MTEs. This category includes knowledge of designing instructional activities in teacher education: how to plan, enact and record rehearsals as a sole instructor; how to instruct PTs to make up a missed class; how to facilitate an instructional activity, and how to receive constructive feedback from PTs about instruction. Some fundamental ideas the mentor shared during co-planning were:

Table 6-9: Examples of PCK-M from the co-planning

MTEs should practice more monitoring than intervention as PTs work on mathematics tasks
MTEs should provide either immediate feedback on PTs practice or address them later collectively
based on PTs' progress, MTEs should revise the lesson plan, change the sequence of activities, or add design additional learning opportunities
MTEs should provide opportunities for PTs to learn reflecting on their own teaching
MTEs should give PTs time to get SRTEs done in class to receive a higher return rate
As an MTE instructor, identify and focus 3 big goals per semester
The primary expectations from PTs depends on the learning goal of the activity.
Collective inquiry is more productive compared to individual inquiry
PTs can develop conceptual understanding by exploring and discussing a well-known mathematics procedure

Subject Matter Knowledge for Teaching PTs

Compared to other knowledge domains, conversations including SMK occurred less. SMK captures knowledge of content that could be taught in a methods course. A majority of this category consisted of knowledge of secondary students' mathematical understandings. Other subcategories of SMK are pedagogical constructs, mathematics content knowledge, and secondary mathematics curriculum.

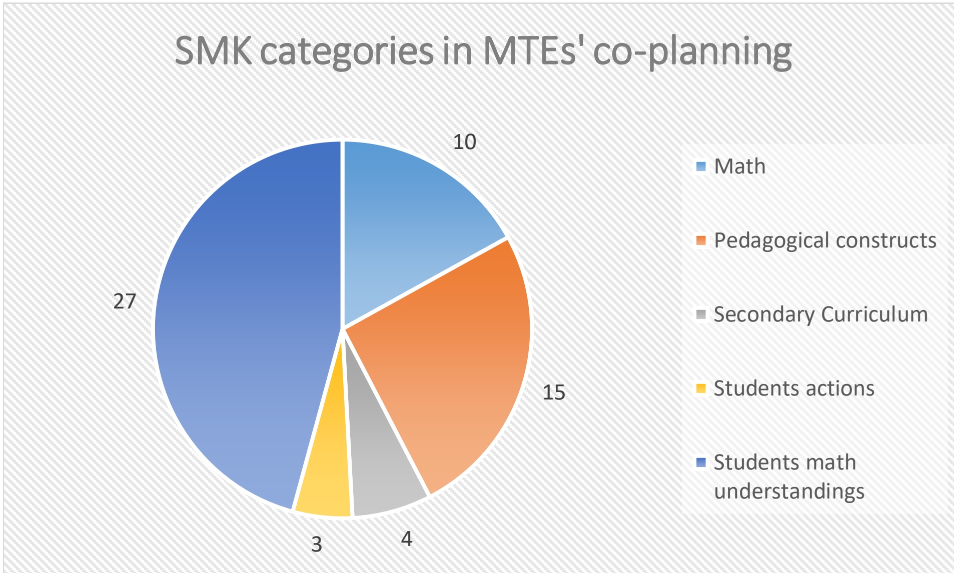

Figure 6-4: Subcategories of SMK surfaced in MTEs' co-planning.

Knowledge of students' mathematical understanding

Secondary students' understandings of mathematics were the dominant part of the group's conversation. This category includes knowledge of variations in students' mathematical understanding in addition to students' development of mathematical proficiency. This knowledge intersects with teachers' content knowledge yet differs slightly due to its scope and function. This knowledge assisted MTEs in generating student solutions for rehearsals that provided rich opportunities for PTs to practice posing purposeful questions and eliciting students' thinking. MTEs knowledge of students' mathematical understandings includes secondary students' approaches, their uses of mathematical representations and tools, and common student misconceptions and errors.

i) Students' approaches

MTEs used their knowledge about different students' approaches to predict secondary students' mathematical thinking and use of different representations (e.g., symbols, graphs, number lines) in solving mathematics problems. This knowledge was derived from MTEs' experiences in teaching mathematics and their research on students' thinking. For instance, as MTEs created proofs for a number theory statement (i.e., The sum of any two positive consecutive odd numbers is divisible by 4.), they talked about typical student approaches like guessing a formula and checking if it works, or showing that the statement holds for example cases. Due to their experiences in designing and enacting a reasoning methods course, all three doctoral students were able to contribute to writing student proofs. Bruce's and Dan's knowledge of students' uses of representations in proving number relations proved most helpful in creating authentic students' solutions. Figure 6-5 shows examples of students' inductive thinking that

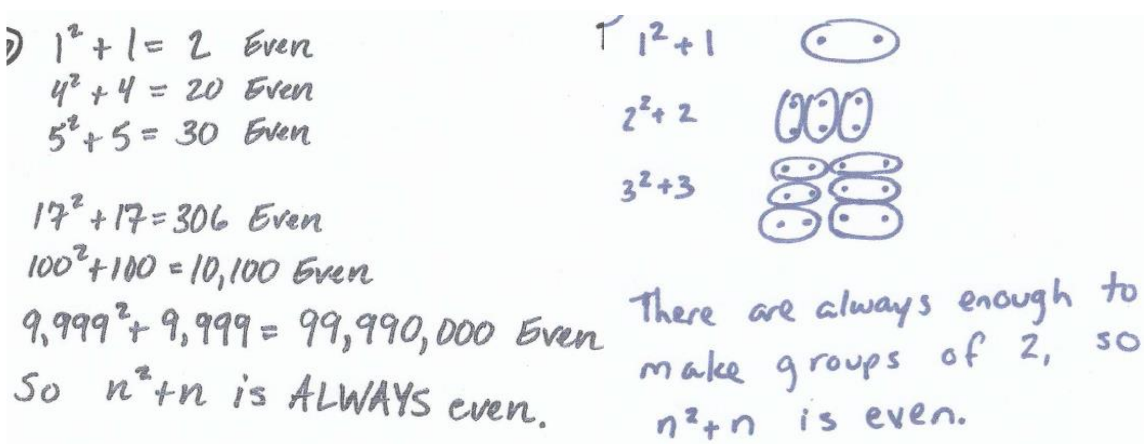

Figure 6-5: Student solutions generated by MTEs to prove n^2+n is always even.

MTEs generated using different representations (i.e., numerical and contextual representations).

ii) *Students' difficulties*

MTEs used their knowledge of secondary students' difficulties, misconceptions, and common errors to generate partial, incomplete, or inaccurate student solutions to a mathematics task. In order to make the student solutions authentic, the COP used their knowledge of students' understandings of patterns, functions, variables, graphs, and representations in addition to common students' mistakes about linear relationships. Two specific examples of knowledge assisted MTEs in creating students' solutions for a linear relation problem (see Appendix D):

- Middle school students create unscaled graphs for rational values due to their experiences with number lines where they label only whole numbers
- Students can switch from additive reasoning to multiplicative reasoning discarding the relationship.

Knowledge of pedagogical constructs

This knowledge includes pedagogical ideas that could be taught in a methods course, including mathematics standards, teaching practices, tools for practice, and teacher roles. Two experts on the course content (Bruce and Finn) dominated the conversation with their knowledge of A.A.T., while Dan, the second instructor of the course, contributed with general pedagogical terms such as mathematical and social goals of tasks. The group engaged in in-depth conversations about advancing and assessing questions to develop a better understanding of those concepts and to design learning activities to support PTs understandings and uses of those tools. MTEs discussed the meaning of assessing and advancing questions, their purpose and function, and specific examples of A.A.T. questions. For instance, Finn stated several times that "advancing was not just advancing the students' answer, but was moving students forward to reach a mathematical goal." MTEs also discussed the 'mathematical goal of a task,' which is the mathematical point to reach at the end; they reflected that a teacher's role is not to pre-teach but to understand students' thinking to assist them in making connections and moving forward. MTEs also talked about implementation of other pedagogical tools they introduced to PTs (e.g., problem-based instructional models, a launch-explore-summarize model). Finn informed the group that a teacher could implement cycles of launch-explore-summarize in one class.

Knowledge of mathematics content related to the course

MTEs also benefitted from their knowledge of mathematics in explaining the student solutions they generated for rehearsal tasks, such as induction and exhaustion proofs, in addition to selecting tasks that were challenging and could generate rich mathematical discourse for PTs.

MTEs used multiple approaches and representations in generating students' solutions. For instance, MTEs used residual thinking in proving the sum of 3 consecutive odds is divisible by 3. That is, adding the residuals of 3 consecutive numbers after dividing 3 makes a multiple of 3 (see Figure 6-6). MTEs also used their knowledge to propose tasks for PTs' peer teaching activity, such as knowing algorithms to test numerical series convergence and knowing the relationship between the integral of variations and R-squared.

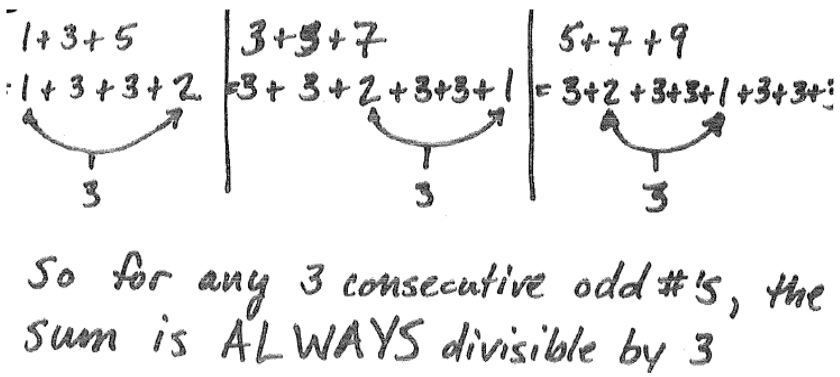

Figure 6-6: A student solution generated by MTEs to prove the sum of 3 consecutive odds is divisible by 3.

Table 6-10: MTEs' knowledge of mathematics areas and examples

Generalizations	How to construct and present a mathematical generalization
Representations	Use of and connecting multiple mathematical representations
Calculus	Derivatives as average change The relation between R-squared and integral
Reasoning & Proof	What constitutes as a proof Constructing and evaluating the validity of a mathematical argument
Functions	Function concepts: algebraic expressions, domain, range, etc Writing and solving linear equations Drawing graphs: scope, steepness Additive & multiplicative relations
Numbers	Divisibility, modular arithmetic, number domains, fractions, quantitative reasoning, and decimals
Terminology	Mathematical conventions and axioms

Knowledge of secondary curriculum

This knowledge is related to secondary students' mathematical understanding but different due to its focus on the mathematics content in the secondary curricular materials. It includes the progression of mathematics concepts across grade levels. For example, the group decided that the Sigma summation symbol would not occur prior to pre-calculus. Based on this category of knowledge, MTEs introduced PTs' curricular materials such as textbooks and online resources and how to use those resources in teaching secondary students. In addition, MTEs discussed how to use mathematical tasks in a secondary classroom to support student understanding. For instance, they saw that number relations and divisibility rules are two possible context to integrate odd +odd = even task into the middle school.

Knowledge of students' behaviors

This is the least discussed knowledge category within SMK, including secondary students' general actions and characteristics. For instance, the group discussed that students could copy the right answer even if their solution did not lead to that answer. One common characteristic of an average middle school student is her/his willingness to respond to teacher questions. This knowledge informed MTEs acting as students during the rehearsals. MTEs intentionally avoided acting like stubborn students in the rehearsals to enrich PTs' opportunities to practice A.A.T.

Members' Knowledge and Participation in Practice

Taking a COP perspective, I define knowing as participation in practice. As Wenger (1998) argues "every practice is in some sense a form of knowledge, and knowing is participating in that practice" (p.9). Based on this definition, I identify members' participation in the co-planning practice as an indication for their 'knowing.' In this study, the group members contributed to the co-planning using their expertise. Table 6-11 illustrates individuals' participation in the co-planning talk. Two experts in decompositions of practice and the co-designers of the course, Bruce and Finn, contributed with their CK to the group's practice dominantly. While Bruce participated in 42% of the conversations involving CK, Finn contributed to 49% of co-planning talk. They used this knowledge mostly in "discussing curricular materials" and "determining instructional details" types of talk. PCK was surfaced mostly in Finn's and Dan's talk, since they were the instructors of the course. While Finn participated in 58% of the talk involving PCK, Dan's knowledge contributed to 33% of the conversation. The group members all contributed to talk with their SMK depending on their expert areas. Dan, as the primary contributor of SMK (i.e., 59), mostly brought his knowledge of mathematics and students' mathematical understanding, while Finn and Bruce participated in the group talk mostly with their knowledge of pedagogy constructs. Being a newcomer, Norah participated in the co-planning least.

Table **6-11**: Members' knowledge participation in the co-planning

MTEs	Knowledge domains & Participation ratio in co-planning talk
Bruce	CK 60/144 (PCK 20/96, SMK 18/59)
Finn	CK 71/144 & PCK 56/96
Dan	SMK 35/59 (PCK 32/96 CK 24/144)
Norah	CK 8/144 (PCK 3/96, SMK 6/59)

Assertion 2: Co-planning provided a space for MTEs gaining knowledge and for their professional development.

Group members reported that they gained professional knowledge as they collaboratively planned the course. Starting from the first meeting, the MTEs had high expectations from the collaborative regarding their professional learning. The group was excited about trying out a new approach to teaching PTs:

Dan: I am actually excited about this. I cannot wait to see how it turns out. I think it is a lot of fun and a very good learning experience.
Norah: Yes. For me as well.
Finn: I am really excited about this as teacher educator pedagogically. I would not be figuring this out if you were not doing the study. (Co-planning F1)

MTEs continued to comment on their overall learning and enthusiasm. They used phrases like "I am really curious what is going to happen" and "It is really interesting, and I think we learn a lot." As the semester progressed, MTEs reflected on how this experience contributed to their professional development. All four, and particularly Dan, commented that their engagement in the co-planning practice, along with their research, informed them about the work of teaching.

During collaborative work, Dan had an opportunity to go through most of the learning experiences discussed in the trajectory of TEs' professional growth model that Abell et al. (2009) describe. He started as an observer, acted as an apprentice and partner for the first semester, and became almost an independent instructor at the end of the second semester. His participation in the co-planning increased over the two semesters. While he contributed to 11% of co-planning talk involving CK, this ratio increased to 24% in the spring semester. Likewise, his participation regarding PCK increased, particularly in the instruction subcategory. He reflected on his development of PCK a couple of times. In one instance he reflected:

Dan: Yeah, I was thinking too that one of the things that I have mentioned this to Bruce probably more times than he wants to hear, what I really valued about this experience last semester and this semester is that I really think that this gave me as a teacher educator a way to think about how to engage preservice teachers in helping them to be reflective…To be thoughtful about the teaching. Honestly, my way of thinking about it has changed as a result of being a part of this project, and I would argue that I was already someone who had thought a lot about it….But now it's like; oh, I've got the structure in that thinking that has changed my way of teaching.
Finn: Yay. That makes me very happy.
Dan: Well good. That's what I keep telling Bruce. (Co-planning SP2).

As the excerpt shows, Dan found the collaborative planning facilitated his learning. He talked about how his understanding of PTs' thinking changed as he engaged in the collaborative work.

For instance, he learned that making sense of simple mathematics procedures (i.e., explaining with a model decimal percentage conversion) might be a challenging task for PTs.

Chapter Summary

In this chapter, I provide a detailed description of the knowledge that surfaced as MTEs co-planned the course and I explain how knowledge in different areas informed their decisions and practices. I expand Chauvot's (2008; 2009) knowledge framework for MTEs, particularly in the areas of knowledge that assist designing a course that focuses on supporting PTs' learning of decompositions of practice (See Table 6-1). I also analyze members' participation in the practice. Findings from my analysis suggest that co-planning provided an opportunity for MTEs' knowledge (i.e., CK, PCK, and SMK) to become visible and for MTEs development of professional knowledge. The scope of the knowledge the group benefitted from confirms that being an MTE for secondary PTs requires advanced knowledge in multiple domains (Jaworski, 2008; Superfine & Li, 2014), particularly teaching courses that aim to support PTs learning of decompositions of practice (Ghousseni & Herbtz, 2016; Kazemi, Ghousseini, Cunard, & Turrou, 2013). It is difficult to be an expert in all these knowledge categories, but collaborative practices enable instructors to generate a collective MTE knowledge in different domains.

Chapter 7

Discussion, Implications, and Conclusion

In this study, I analyze the work of MTEs as they co-planned a methods course. I define the group of MTEs as a Community of Practice (COP) since they engage in a joint practice (Wenger, 2009), in this case, co-planning. Taking a communities of practice standpoint and deriving my theoretical perspectives from studies on MTE practice, I test three different lenses to examine MTEs' co-planning practice: 'type of talk,' 'roles,' and 'knowledge.' This study contributes to the field of mathematics education by conceptualizing MTEs' co-planning practice and its impact on MTEs' professional growth.

MTEs argue that mathematics education needs more work that investigates and characterize MTEs' practice (Appova & Taylor, 2017; Even 2008; Tirosh et al., 2014). In particular, scholars have called for research that conceptualizes pedagogies of teacher education (Ghousseni & Herbtz, 2016) and supports MTEs in their professional development (Doerr & Thompson, 2004; Superfine & Li, 2014). Researchers have used different perspectives to describe and analyze the work of MTEs. While some focused on MTEs' knowledge (Chauvot, 2009; Superfine & Li, 2014) and practices of MTEs (Doerr & Thompson, 2004; Llinares & Krainer, 2006), others focused on MTEs professional development (Tzur, 2001; Zaslavsky & Leikin, 2004) and the multiple roles MTEs take (Jowarski & Huang, 2014; Zaslavsky, 2007). Regardless of their perspectives, researchers agree that taking part in collaborative communities is important for MTEs' professional growth (Tirosh et al., 2014; Wilson & Franke, 2008).

One practice that provides an opportunity for TEs to work in collaborative learning communities is co-planning (Albrecht, 2003). Despite the role of collaborative planning in professional development (Bleiler, 2015), educators do not know much about the structure and nature of co-planning (Wilson, 2016), particularly in teacher education (Nevin, Thousand, & Villa, 2009). To better understand the collaborative practices of MTEs, I studied their work in the context of co-planning a methods course for mathematics PTs organized around iterative CEIs. I collected and analyzed data from the co-planning meetings of the COP formed by four MTEs. The analysis of MTEs' work in co-planning through the lenses of their talk, their roles, and their knowledge provided a thicker description (Geertz, 1973) of the practices of MTEs.

In Chapters 4 – 6, my findings from a single embedded case study show that collaborative planning in teacher education provides opportunities for not only advancing MTEs' practices by bringing together varying skills, knowledge, roles, and perspectives (Bleiler, 2015; Sztajn, Ball, & McMahon, 2006) and but also supporting MTEs' professional growth by offering rich learning experiences (Tirosh et al., 2014; Van Zoest, Moore, and Stockero, 2006).

In this chapter, I report the ways in which my findings contribute to the existing body of literature discussed in Chapter 1 and Chapter 2. I organize this chapter around four assertions I made in Chapters 4-6. For each assertion, I provide a brief description and present its relation to my findings. In addition, I explain the ways in which findings associated with the assertion contribute to the existing research on MTEs' collaborative practices and professional development. I also summarize the relationship between the five core constructs in this study: types of talk, roles, knowledge, practice, and professional development (Assertion 4). I conclude this chapter by outlining implications for TE education and research and proposing directions for future research.

Assertion 1: The complex nature of MTEs' co-planning work could be less demanding and more efficient with MTEs taking on different roles and contributing different areas of expertise to their practice.

As discussed in Chapter 1 and Chapter 2, the work of MTEs is very complex and demanding. It requires MTEs to take on multiple responsibilities and asks them to combine multiple skills and areas of expertise (Jaworski & Huang, 2014; Zaslavsky, 2008). Findings from this study confirm that members with different roles and expertise contributed to the MTEs' co-planning practices in multiple ways.

Researchers list multiple roles and responsibilities that MTEs take in teaching PTs, with the primary role being to facilitate teacher learning (Zaslavsky, 2007). Some discuss those roles under titles such as curriculum developer and coach (Jowarski & Huang, 2014), or, instructor and supervisor (Wu & Huang, 2017). Others focus on specific characteristics of MTEs work such as didactician — transfers theoretical ideas and research findings into modes of teaching — (Jowarski, 2008a; 2008b), task designer (Li & Superfine, 2018; Zaslavsky, 2007), heightened listener (Coles, 2014), and inquirer and reflective practitioner (Jaworski, 2004). Regardless of their approach, MTEs all agree on the demanding work of MTE.

In chapter 5, I discussed MTEs' roles as they co-planned the methods course. Based on MTEs' responsibilities, expertise, and contributions, I grouped these roles in six categories: *leader, organizer, critic-inquirer, expert (and broker), analyzer,* and *novice*. Despite the diverging roles individuals play, each MTE contributed to the joint activity of co-planning. As activity theorists describe, individuals contributed to the practice with different skills, areas of expertise, and perspectives while the community worked toward a particular goal (Yamagata-Lynch & Haudenschild, 2009). Defining these communities as complementary communities, Wenger (2008) argues that members' contributions are determined primarily by their personalities and experiences from previous or concurrent COPs. This study confirms the arguments made by both Wenger (2008) and Yamagata-Lynch and Haudenschild (2009). With their diverse expertise, experience, and competence, members impacted the COP's shared practice. Taking on different roles enabled each member to contribute to the development of the practice: experts and brokers brought their expertise, skills, and knowledge to inform the practice; critics-inquirers extended ideas, offered alternative ways, and made the thought processes visible; analyzers evaluated the instruction and contributed to revisions; leaders and organizers made decisions to maintain the work in progress.

In addition to varying roles, being an MTE for secondary PTs requires advanced knowledge in multiple domains (Chauvot, 2009; Jaworski, 2008). Some of the knowledge areas required to teach PTs are; strong mathematical knowledge (Chazan & Lewis, 2008; Superfine and Li, 2014), rich curricular knowledge (Chauvot, 2008; Zbiek & Hirsch, 2008), knowledge of instructional strategies (Arbaugh, Nolan, Mark, & Burns; 2012; Steele, 2008), knowledge of students' learning (Appova & Taylor, 2017); knowledge of tasks (Zaslavky, 2007), knowledge of goals (Superfine & Li, 2018), knowledge of technology for teaching, learning, and doing mathematics (Heid & Lee, 2008), and knowledge of educational policy (Silver & Walker, 2008). It is challenging for an individual to become an expert in all these knowledge categories. However, collaborative practices enable MTEs to generate a collective knowledge that includes different domains.

In chapter 4, I described knowledge domains (i.e., CK, PCK, SMK) that emerged in MTEs' co-planning practice and I explained individuals' knowledge contributions to the group's practice. Being experts in decompositions of practice and co-designers of the course, Bruce and Finn exhibited CK that assisted the group mostly in creating course artifacts, selecting course reading, and designing activities. Finn and Dan contributed with their PCK dominantly as the group made instructional decisions and analyzed PTs' learning and their practices. Dan and

Bruce’s knowledge of mathematics and students’ mathematical understanding—their SMK—assisted the group in writing students’ solutions and selecting tasks.

Even though co-planning could be demanding for educators due to amount of time it requires (Wilson, 2016), it still pays off by: a) contributing to the development of the MTEs’ teaching practices; b) building professional communities of practice (Albrecht, 2003; Zaslavsky, 2007); c) modeling a collaborative instruction example to future teachers (Villa, Thousand, & Nevin, 2013), and d) supporting the professional growth of practitioners (Bleiler, 2015). The collaborative practice as Wenger (2008) states;

> provided resolutions to conflicts and contradictions; supported a communal memory, allowed individuals to their work without needing to know everything; helped newcomers to join the community by participating in practice; generated specific perspectives, terms enable to accomplishing what needs to get done; and created rituals, customs, stories, events, rhythms of community life. (p. 46)

MTEs forming the COP with diverse roles and areas of expertise in this study contributed to their collaborative practice. Having members with different knowledge, roles, and perspectives participating in the co-planning enabled to generate a collective MTE expertise. This collective expertise led to a successful instruction and course design. Results from Freeburn (2015) show that the activities in the course supported PTs’ understanding and use of specific decompositions of practice. The MTEs’ collective expertise supported the instruction by combining/comparing multiple ideas, analyzing and reflecting on practice, and revising the design based on the reflections (Zaslavsky, 2007). Also, in reflecting on their experience as the members of the COP, the participants anecdotally stated that the co-planning assisted them to grow professionally. Further research in this area is warranted to establish what participants learned from this kind of collaborative planning experience.

Although this was not part of this study, participating in a co-planning practice provides an opportunity for the MTEs to show their PTs that educators value collaboration and work collaboratively. More importantly, co-planning supports practitioners' professional growth. In the following sections, I discuss how co-planning promoted MTEs' development.

Assertion 2: Co-planning provided rich opportunities for MTEs to make their knowledge (i.e., CK, PCK, and SMK) knowable to others.

Co-planning practice enabled the COP to engage in a discourse that reveals various aspects of MTEs' professional knowledge. In my analysis, I found that ten types of co-planning talk occurred in co-planning sessions. In chapter 4, I described the five types of talk that appeared consistently and frequently over two semesters: discussing curricular materials, determining instructional details, analyzing PT learning, reflecting on practice, and evaluating and revising activities. In addition, I explained how types of talk contributed to MTEs' professional knowledge. Results show that the types of talk—primarily discussing curricular materials, determining instructional details, and analyzing PT learning—provided opportunities for MTEs to demonstrate their knowledge to the group.

Aligning with the literature, my analysis finds that collaborative planning provides the opportunity for teachers to engage in natural discussions of pedagogical content knowledge (Beleiler, 2015; Goodchild, Fuglestad, & Jaworski, 2013). Collaborative planning creates the environment for teachers to discuss and broaden their pedagogical content knowledge because they may be asked to make their own knowledge and understanding knowable to others (Roth, McDuffie, & Mather, 2009). In this co-planning experience, the analysis of practice allowed members a space to make their PCK visible to other members of the COP. In particular, instructors were pushed to provide explicit justifications for their instructional decision-making. The results show that analysis of PTs' learning and teaching led to increased understanding of PTs' thinking, needs, and behaviors. Similar findings were presented by Doerr and Thompson

(2004) as they investigated four expert MTEs analyzing video cases. The authors conclude that analysis of PTs' learning provide opportunities for unpacking the concept of professional knowledge for MTEs and for developing a better understanding of PTs' content and pedagogical knowledge. This study confirms that engaging in the analysis of PTs' learning plays a fundamental role in MTEs' knowledge development (Krainer 2008; Van Zoest, Moore, & Stockero, 2006).

In addition to types of talk, variations in MTEs' roles assisted them in bringing essential MTE knowledge to the surface. As discussed in chapter 5, while experts provided the group with their knowledge and expertise in different domains, the critic-inquirer assisted in making some aspects of PCK accessible to the members of the group. His questions particularly invited the mentor MTE to explain the rationale and justification for her pedagogical moves.

Assertion 3: Co-planning provided natural opportunities for MTEs to engage in learning experiences that support their professional growth.

Educators studying the professional development of TEs argue that, in addition to completing theory and research related coursework, it is critical for novice TEs to engage in other learning experiences that help them to grow professionally (Abel et al. 2009; Cochran-Smith, 2005; Zeichner, 2005). In this study, the group of MTEs engaged in learning experiences recommended by expert MTEs: community of inquiry (Even, 2008; Tirosh et al., 2014), reflective practice (Krainer, 2008; Zaslavsky, 2008), apprenticeship opportunities in communities of practice (Wilson & Franke, 2008), and analyses of PTs' learning (Van Zoest, Moore, & Stockero, 2006; Zaslavsky and Leikin, 2004).

The co-planning experience in this study enabled MTEs to engage in collaborative and reflective practices, in addition to an analysis of PTs learning naturally, which supported their professional development. As discussed in Chapter 4, the MTEs regularly analyzed PTs' performances and shared their noticings (Amador, 2016) about PTs' learning. Based on their analyses and their prior experiences with PTs, MTEs derived generalizations about PT thinking. In addition to PTs' experiences in mathematics and learning, they discussed research findings of PTs' perceptions, thinking, and struggles. The analysis of specific PTs' learning in the course, and conversations about PT thinking in general, both assisted MTEs in anticipating PTs' performances and struggles. These experiences also determined pedagogical moves to support their learning (Dolk, den Hertog, & Gravemeijer, 2002).

In addition, the group regularly reflected on the practice. Similar to Zaslavsky's (2007) iterative process of designing tasks, MTEs in this study had an opportunity to experience the processes of designing instructional activities, facilitating PTs engagement in the activities, and reflecting on their practice. Slightly different from reflective practice, inquiring and critiquing (Cochran-Smith, 2003) were common practices the group was engaged in. The critical lenses enabled participants to engage in contemplation and rationalization of their practice in addition to comparing their work with theoretical models. These querying perspectives also allowed space for the veteran MTE to mentor doctoral students (George & Davis-Wiley, 2000). All these learning experiences in the co-planning provided rich opportunities for MTEs to grow professionally. This study confirms the literature (Jaworski & Huang, 2014; Tzur 2001; Zaslavsky, 2008) on the importance of reflection, inquiry, and collaboration in supporting MTEs' professional development.

Assertion 4: Types of talk provided a structure for MTEs' participation in COP and mediated the interactions between MTEs' roles, knowledge, and practice which support MTEs' professional development.

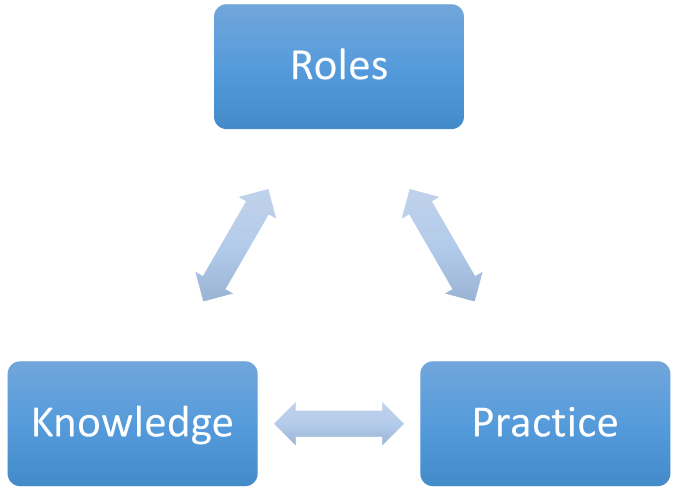

Figure 7-1: MTEs' professional development in their co-planning practice**.**

Note. The arrows represent the two way interaction between the constructs.

In this section, I summarize the dynamic relationship between three constructs (i.e., roles, knowledge, and practice) and how they support MTEs' professional growth. The findings in this study suggest that MTEs' expertise in various fields enabled them to contribute to the co-planning through taking different roles. Further, the variations in their roles and responsibilities led MTEs to gain knowledge. Diverse roles and varied knowledge both informed and enhanced the teaching practices for this methods course. Experiences from the teaching practices promoted MTEs' knowledge and expertise as well. The MTEs' reflections on practice during co-planning brought new knowledge to the table. In addition, the evaluation of activities invited MTEs to take on additional responsibilities. The arrows in Figure 7-1 represent the dynamic interaction between these phenomena**. The types of talk provided a structure for MTEs' participation in the co-planning practice**.

The types of talk identified in this study represent both the COP's participation and reification (Wenger, 2008) and they also mediated the relationship between roles, knowledge, and participation. For example, the group designed a learning activity using their knowledge, and the members took different roles in the co-planning and assisted in preparing the materials. After the enactment, the group shared their analysis of PTs' performances and reflected on the teaching practices, which led to revisions in teaching practices for that activity. Also, MTEs developed knowledge of PTs' learning through sharing their observations and noticings about PTs' performances and perceptions. These activities led MTEs to build knowledge of instruction through analysis of, and reflection on, practice. Thus, the co-planning structure generated a cycle of knowledge-sharing and knowledge-building for MTEs. In co-planning, the group participated in the practice with diverse roles and expertise, and they produced reifications such as course artifacts, assignments, learning activities, and "a shared repertoire of ways of doing things" (Wenger, 2008). In this context, the type of talk prompted the primary participation and reification.

Implications for MTEs' professional development

This study confirms that collaborative practices not only support MTEs' professional growth (Sztajn et al., 2014), but also function as assisted performance for mentoring novice MTEs (Feiman-Nemser & Beasley, 1997). The collaborative practice could take different forms depending on its purpose and function. It could be a collaboration between MTEs (e.g., Doerr & Thompson, 2004; Tirosh et al., 2014), mathematics education and mathematics experts (e.g., Bleiler, 2015), teachers and MTEs (e.g., Kieran et al., 2013; Zaslavsky, 2008), or doctoral students and MTEs (e.g., Van Zoest, Moore, & Stockero, 2006; Vogler & Long, 2003). Regardless of its structure and participants' levels of expertise, collaborative practices offer opportunities for MTEs' professional growth (Jaworski & Huang, 2014; Wilson & Franke, 2008). In addition to building collaborative spaces for MTEs, educators have identified specific experiences that helped MTEs to grow professionally. Zeichner (2005) and Wilson and Franke (2008) argue that these learning experiences should vary from supervising to teaching PTs, and novices should engage in apprenticeship opportunities across diverse learning activities. Yet, those experiences should involve three essential processes: reflection (Garcia et al., 2007; Zaslavsky, 2008), inquiry (Tirosh et al., 2014; Zeichner, 2005), and analysis of learning (Dolk, den Hertog, and Gravemeijer, 2002; Van Zoest, Moore, & Stockero, 2006; Zaslavsky & Leikin 2004). Co-teaching, co-designing, and co-planning could all provide opportunities for rich learning experiences that involve reflection, inquiry, and analysis.

Van Zoest, Moore, and Stockero (2006) argue that involving MTEs directly in the work of TE instructors limits their understandings and analyses of PT learning. They recommend breaking away from doctoral students becoming co-designers of methods courses as interns because that makes doctoral students focus on generating PTs' thinking more than analyzing their thinking. However, as my study shows, if doctoral students regularly engage in reflective practices and analyses of PTs thinking as they co-design the courses, they could focus on both analyzing and generating PTs thinking simultaneously. Making analysis of PTs' learning a routine talk can offer a solution for TEs to learn more about PTs' thinking. These analyses could also be verified and justified by research and theory about PT learning and can facilitate developing MTEs' professional noticing skills (Amador, 2016). In conclusion, the findings in this study suggest that it is important for MTEs' professional growth to engage in collaborative planning/designing practices where reflection, inquiry, and analysis of PTs learning become a routine.

Collaborative practices are not essential only for MTEs' professional development but facilitating TE learning in other fields. Research shows that novice TEs find it challenging to become educator-researchers (Lunenberg & Hamilton, 2008; Murray & Male, 2005). Yet, most TEs are not provided with practice-based learning opportunities that would prepare for them to establish professional identities as teachers of teachers (Wilson, 2006). Besides involving TEs in collaborative and reflective practices (Cochran-Smith 2003; 2005; Gallego, 2014), it is important to create communities of practices where TEs could take different roles to facilitate their professional growth. Members could form these communities with complementary competencies where experts in different areas could learn from one another (Wenger, 2008), or among members with different levels of expertise so that novices start as observers and become sole instructors over time (Abel et al., 2009).

Implications for Research

This study contributes to the field of teacher education by providing a tool for analyzing the social interactions in a collaborative TEs practice: types of talk. In research, co-planning has classically been studied within a broader context, such as co-teaching (e.g., Villa, Thousand, & Nevin, 2013; Wilson, 2016), coaching (e.g., Gleason, Livers, & Zelkowski, 2015; Hansen, 2015), or lesson study (e.g., Fernandez & Yoshida, 2012). The study extends this body of research by focusing on the co-planning practices of TEs. Informed by the studies that investigated the co-planning structures (Feiman-Nemser & Beasley, 1997; Lynch, 2017), I described the co-planning discourse of MTEs in a university context. I used the patterns of talk in describing their co-planning routines. Although the analytical tools ‘knowledge,’ ‘roles,’ and ‘type of talk’ all provided a *thick description* (Greetz,1973) of MTEs’ collaborative planning, ‘type of talk’ enabled me to examine and better explain the social interactions and relations in the COP. Studying patterns of talk in collaborative practices could not only enable researchers to establish effective co-planning models but also develop a better understanding of MTEs’ work.

This study, extending Chauvot’s knowledge framework, contributes to the body of research in MTEs’ knowledge, and offers an in-depth knowledge analysis model for studying MTEs professional knowledge for teaching.

The work of MTEs requires advanced knowledge in multiple domains (Chauvot, 2009; Jaworski, 2008b; Zaslavsky, 2008). Although research has identified some aspects of essential knowledge for MTEs, there is less empirical research that examines the specific knowledge MTEs need in order to design a mathematics methods course for PTs (Even, 2008). Recent studies offer innovations and new constructs, such as pedagogies of practice (Grossman & McDonald, 2008) and rehearsals (Lambert et al., 2013) to facilitate teacher preparation. To integrate these innovations into their practice, MTEs need to acquire new knowledge (Kazemi, Ghousseini, Cunard, & Turrou, 2013). In chapter 6, I provided a detailed description of the knowledge that surfaced as MTEs co-planned the course and explained how knowledge in different areas informed their decisions and practice. I expanded Chauvot's (2009) knowledge framework for MTEs, particularly in the areas of knowledge that assist in designing a course that focuses on supporting PTs learning of decompositions of practice. This study contributes to building a knowledge base for pedagogies of teacher education (Ghousseni & Herbtz, 2016; Li & Superfine, 2018). Conceptualizing the structure of knowledge of MTEs as they planned a course informs the field about designing curriculum and PD to support MTEs learning.

This study offers new roles such as critic-inquirer and organizer to study responsibilities and the work of MTEs in collaborative practices. From a social-cultural perspective, the findings suggest that other forms of mediators could take roles in communities of practice. With changing modes of communication, today the COPs consist not only of people but also other mediators such as online tools and platforms (e.g., social networks & Wikipedia) that contribute to practice (Murillo, 2008). In this study, one of the dominant brokers of the COP was research in approximations/decompositions of practice. Those systematic ways of talking contribute to creating a common ground for COP shared practices (Wenger, 2008). Other researchers who investigated collaborative practices of MTEs reported similar findings. Sztajn, Ball, and McMahon (2006) found that the framework of Mathematical Knowledge for Teaching (by Ball, Hill, & Bass, 2005) provided a common ground for educators with different backgrounds to work together to design a mathematics content course for PTs to work productively in designing and teaching the course. Likewise, Lynch (2017), studies the Mathematical Tasks Analysis framework (by Henningsen & Stein, 1997) and provides a common intellectual language in planning instructional activities for a methods course. In our collective practice of planning, constructs such as ‘teaching practices,’ ‘rehearsals,’ and ‘CIE,’ provided a common language for the group. More importantly, research around these ideas served as a broker in planning activities for PTs.

Areas for Further Research

Given that this is one of the few studies focusing on the co-planning practice of TEs, it raises several important questions about TEs’ co-planning practices and TEs’ professional development.

One research area for teacher education is to investigate co-planning practices in different contexts. Although the literature in the field of education emphasizes the importance of collaboration (Cochran-Smith, 2005), very little of this existing work examines the nature of collaboration in higher education (Villa, Thousand, & Nevin, 2013). Specifically, this field has said little about collaborative efforts leading to successful instruction (Waters & Burcroff, 2007). Despite the vital role of co-planning in co-teaching, it rarely finds a place in collaborative teaching practices due to the amount of vast time co-planning requires (Albert, 2003; Bleiler, 2015). One way to minimize the time devoted to co-planning is to establish effective routines (Wilson, 2015). I believe there is a need for more studies to examine co-planning of TEs in order to determine not only effective routines but also practices that lead to successful instruction. In addition, investigating collaborative planning in other contexts would raise interesting questions. What would co-planning look like if participants were all experts; would they compromise or tradeoff if necessary? What would be the difference between COPs where members take on the same role vs COP members taking on different roles? How would co-planning impact the professional learning of members with different backgrounds and expertise? For instance, what would it look like if mathematics graduate students co-plan a course or a unit together with mathematics education faculty? How would PTs and mathematics faculty co-plan a calculus lesson together?

After studying the co-planning practice between novice and expert educators, it would be an interesting follow up to investigate the residues novices take well into their practice. By conducting interviews with the participants and observing their practice, they could provide details about the impact of co-planning in their professional growth. For instance, how would the co-planning experience affect their instruction or dispositions? Do they continue to participate in collaborative planning in their profession? Studying the residues of co-planning could provide feedback to structure co-planning activities that would promote enhanced novice learning.

A broader question for the field of teacher education to investigate is how to support TEs in their professional learning. Beginning TEs report feeling unprepared and receiving limited support from their institutions (Masingila et al., 2012; Murray & Male, 2005; O'Sullivan, 2010). There is an increasing interest in the field concerning how to educate TEs and design programs to support their professional growth. They all address the need for TEs to engage in rich learning opportunities that include collaborative practices and reflective practices to develop expertise and skills to teach PTs. However, educators do not know how and when to integrate those productive learning experiences into TE training. I believe there is a need for more studies that provide guidance and structure for TE programs to promote TE education.

Conclusion

This study confirms that analysis, inquiry, and reflection are important learning experiences to facilitate MTEs' professional growth; and collaborative practices provide rich opportunities to engage in those learning experiences (Krainer, 2008; Sanchez, 2011; Van Zoest, Moore, & Stockero, 2006; Wilson and Franke, 2008; Zaslavsky and Leikin, 2004). Yet, educators do not have robust models to frame the structure of collaboration that leads to successful practice. In this study, I used different tools to examine the collaborative practices of MTEs. I found that analyzing MTEs' co-planning through the lenses of their talk, their roles, and their knowledge provided an opportunity to understand the work of MTEs more fully. Findings from this study support further investigations of MTEs' collaborative practices by providing guiding frameworks for such studies (i.e., using *type of talk* as a tool for analysis) as well as providing practical co-planning activities that can be used to ground professional development of future and current faculty involved in mathematics teacher education.

REFERENCES

Abell, S. K., Rogers, M. A. P., Hanuscin, D. L., Lee, M. H., & Gagnon, M. J. (2009). Preparing the next generation of science teacher educators: A model for developing PCK for teaching science teachers. *Journal of Science Teacher Education*, *20*(1), 77-93.

Albrecht, N. M. R. (2003). University faculty collaboration and its impact on professional development (Doctoral dissertation, Kansas State University.) Available from ProQuest Dissertations and Theses database (Publication No. AAT 3090338).

Amador, J. (2016). Professional noticing practices of novice mathematics teacher educators. *International Journal of Science and Mathematics Education*, *14*(1), 217-241.

Appova, A., & Taylor, C. E. (2017). Expert mathematics teacher educators' purposes and practices for providing prospective teachers with opportunities to develop pedagogical content knowledge in content courses. *Journal of Mathematics Teacher Education*, 1-26.

Arbaugh, F., Nolan, J., Mark, K., & Burns, R. (2012). The practices and knowledge of school-based teacher education practitioners. In M. M. Latta & S. Wunder (Eds.) *Placing Practitioners Knowledge at the Center of Teacher Education: Re thinking the Policies and the Practices of the Educational Doctorate*, pp. 105-123. IAP.

Arbaugh, F., Freeburn, B., Graysay, D., & Konuk, N. (2018). The *Three-Minute-Rehearsal Cycle of Enactment and Investigation*: Preservice Secondary Mathematics Teachers Learning to Elicit and Use Evidence of Student Thinking. Manuscript in preparation.

Association of Mathematics Teacher Educators. (2017). *Standards for Preparing Teachers of Mathematics.* Available online at amte.net/standards.

Association of Mathematics Teacher Educators (AMTE) (2002). *Principles To Guide the Design and Implementation of Doctoral Programs in Mathematics Education. A Task Force Report*. Author. Retrieved from https://amte.net/sites/all/themes/amte/resources/publications/DoctoralProgramsinMathematicsEducation.pdf

Association of Teacher Educators (ATE) (2008). *Standards for teacher educators*. The Association of teacher educators in US. Retrieved from http://www.ate1.org / pubs/Standards_for_Teac.cfm

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407.

Baldinger, E. E., Selling, S. K., Virmani, R. (2016). Supporting novice teachers in leading discussions that reach a mathematical point: Defining and clarifying mathematical ideas. *Mathematics Teacher Educator, 5*(1), 8-28.

Beswick, K., & Chapman, O. (2013). Mathematics teacher educators'knowledge. In A. M. Lindmeier & A. Heinze (Eds.), Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education (Vol. 1, p. 215). Kiel, Germany: PME.

Bergsten, C., & Grevholm, B. (2008). Knowledgeable teacher educators and linking practices. In B. Jaworski & T. Woods (Eds.), *The International Handbook of Mathematics Teacher Education, Volume 4: The Mathematics Teacher Educator as a Developing Professional*. (pp. 221-246). Rotterdam: Sense Publishers.

Bleiler, S. K. (2015). Increasing awareness of practice through interaction across communities: The lived experiences of a mathematician and mathematics teacher educator. *Journal of Mathematics Teacher Education*, 18(3), 231–252.

Bogdan, R., & Biklen, S. K. (2006). *Qualitative research for education: An introduction to theories and methods (5th Ed.).* New York: Pearson Education.

Campbell, P. F., & Malkus, N. N. (2014). The mathematical knowledge and beliefs of elementary mathematics specialistcoaches. ZDM—*The International Journal on Mathematics*

Education, 46(2).

Chauvot, J. B. (2008). Curricular knowledge and the work of MTEs. *Issues in Teacher Education, 17*(2), 83-99.

Chauvot, J. B. (2009). Grounding practice in scholarship, grounding scholarship in practice: Knowledge of a mathematics teacher educator-researcher. *Teaching and Teacher Education*, *25*(2), 357–370.

Chazan D. & W. J. Lewis (2008). The mathematical education of doctorates in mathematics education In R. E. Reys & J. A. Dossey (Eds.), *U.S. Doctorates in Mathematics Education: Developing Stewards of the Discipline (pp. 75-85)*. (Vol. 15). Providience, RI: American Mathematical Society.

Cochran, K. F. (1991, April). *Pedagogical Content Knowledge: A Tentative Model for Teacher Preparation*. Paper presented at the annual meeting of the American Educational Association, Chicago.

Cochran-Smith, M., & Fries, M. K. (2001). Sticks, stones, and ideology: The discourse of reform in teacher education. *Educational Researcher*, *30*(8), 3-15.

Cochran-Smith, M. C. (2003). Learning and unlearning: The education of teacher educators. *Teaching and Teacher Education, 19*,5–28.

Cochran-Smith, M. (2005). Teacher educators as researchers: Multiple perspectives. *Teaching and Teacher Education*, *21*(2), 219–225.

Collins, K. M. (2013). *Ability profiling and school failure: One child's struggle to be seen as competent, 2nd edition.* New York, NY: Routledge.

Conference Board of Mathematical Sciences (CBMS). (2001). *The mathematical education of teachers* (Vol. 2). Washington, DC: American Mathematical Society.

Conference Board of the Mathematical Sciences. (2012). The mathematical education of teachers II (Issues in mathematics education, Vol. 17). Providence, RI: American Mathematical Society.

Creswell, J. W., & Creswell, J. D. (2017). *Research design: Qualitative, quantitative, and mixed methods approaches*. Los Angeles, CA: Sage Publications.

Denzin, N. and Lincoln, Y. (2005). Introduction: The Discipline and Practice of Qualitative Research. In N. Denzin and Y. Lincoln *The SAGE Handbook of Qualitative Research, third edition.*

Dinkelman, T., Margolis, J., & Sikkenga, K. (2006). From Teacher to Teacher Educator: Reframing knowledge in practice. *Studying Teacher Education*, *2*(2), 119–136. http://doi.org/10.1080/17425960600983155

Doerr, H. M., & Thompson, T. (2004). Understanding teacher educators and their pre-service teachers through multi-media case studies of practice. *Journal of Mathematics Teacher Education, 7*(3), 175-201. doi:http://dx.doi.org/10.1023/B: JMTE.0000033048.97096.39

Dolk, M., den Hertog, J., & Gravemeijer, K. (2002). Using multimedia cases for educating the primary school mathematics teacher educator: a design study. *International Journal of Educational Research, 37*(2), 161-178. doi: 10.1016/S0883-0355(02)00058-7

Dulock, H. L. (1993). Research design: Descriptive research. *Journal of Pediatric Oncology Nursing*, *10*(4), 154-157.

Dweck, C. S. (2010). Mindsets: How praise is harming youth and what can be done about it. *The Essays of Leonard Michaels*, 302.

Engestro¨m, Y. (1999). Activity theory and individual and social transformation. In Y. Engestro¨m, R. Miettinen & R-L. Punama¨ki (Eds.), Perspectives on activity theory (pp. 19–38). Cambridge: Cambridge University Press.

Even, R. (2008). Facing the challenge of educating educators to work with practicing mathematics teachers. In T. Wood, B. Jaworski, K. Krainer, P. Sullivan & T. Tirosh (Eds.), *The international handbook of mathematics teacher education: The mathematics teacher educator as a developing professional* (Vol. 4, pp. 57-74). Rotterdam, Netherlands: Sense Publishers.

Farnsworth, V., Kleanthous, I., & Wenger-Trayner, E. (2016). Communities of practice as a social theory of learning: A conversation with Etienne Wenger. *British Journal of Educational Studies*, *64*(2), 139-160.

Feiman-Nemser, S., & Beasley, K. (1997). Mentoring as assisted performance: A case of co-planning. *Constructivist teacher education*, 108-126. Washington, DC: The Falmer Press.

Fernandez, C., & Yoshida, M. (2012). *Lesson study: A Japanese approach to improving mathematics teaching and learning*. Routledge.

Ferrini-Mundy, J., & Floden, R. E. (2007). Educational policy research and mathematics education. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 1247-1279). Charlotte, NC: National Council of Teachers of Mathematics.

Ferrini-Mundy, J. (2008). What core knowledge do doctoral students in mathematics education need to know? In R. E. Reys & J. Kilpatrick, *One field, many paths: U.S. doctoral programs in mathematics education* (pp. 63-75). Providence, RI: American Mathematics Society.

Feuer, M.J., Floden, R.E., Chudowsky, N., & Ahn, J. (2013). *Evaluation of teacher preparation programs: Purposes, methods, and policy options*. Washington, DC: National Academy of Education.

Fey, J. T. (2001). Doctoral programs in mathematics education: Features, options, and challenges. In R. E. Reys & J. Kilpatrick, *One field, many paths: U.S. doctoral programs in mathematics education* (pp. 55-62). Providence, RI: American Mathematics Society.

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education*, *60*(4), 380-392.

Freeburn, B. L. (2015). *Preservice Secondary Mathematics Teachers' Learning of Purposeful Questioning and Judicious Telling for Promoting Students' Mathematical Thinking* (Doctoral dissertation). The Pennsylvania State University.

Friend, M. (2014). *Co-teach! Building and sustaining effective classroom partnerships in inclusive schools* (2nd ed.). Greensboro, NC: Marilyn Friend, Inc.

Gallego, M. (2014). Professional development of graduate teaching assistants in faculty-like positions: Fostering reflective practices through reflective teaching journals. *Journal of the Scholarship of Teaching and Learning*, *14*(2), 96–110. http://doi.org/10.14434/josotl.v14i2.4218

García, M., Sánchez, V., & Escudero, I. (2007). Learning through reflection in mathematics teacher education. *Educational Studies in Mathematics*, *64*(1), 1-17.

Geertz, C. (1973). Thick description: Toward an interpretive theory of culture. In *The interpretation of cultures*: *Selected essays* (pp. 3–30). New York: Basic Books. Retrieved from http://hypergeertz.jku.at/GeertzTexts/Thick_Description.htm

George, M. A., & Davis-Wiley, P. (2000). Team teaching a graduate course: Case study: A clinical research course. *College teaching*, *48*(2), 75-80.

Ghousseini, H., & Herbst, P. (2016). Pedagogies of practice and opportunities to learn about classroom mathematics discussions. *Journal of Mathematics Teacher Education*, *19*(1), 79-103.

Gleason, J., Livers, S.D., & Zelkowski, J. (2015). Mathematics classroom observation protocol for practices: Descriptors manual. Retrieved from http://jgleason.people.ua.edu/mcop2.html

Goodchild, S., Fuglestad, A. B., & Jaworski, B. (2013). Critical alignment in inquiry-based practice in developing mathematics teaching. *Educational Studies in Mathematics*, *84*(3), 393-412.

Gray, T., & Halbert, S. (1998). Team teach with a student: New approach to collaborative teaching. *College Teaching*, *46*(4), 150-153.

Graziano, K. J., & Navarrete, L. A. (2012). Co-teaching in a teacher education classroom: Collaboration, compromise, and creativity. *Issues in Teacher Education*, *21*(1), 109-126.

Grossman, P. L. (1990). *The making of a teacher: Teacher knowledge and teacher education*. New York: Teachers College Press.

Grossman, P., & McDonald, M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Educational Research Journal*, *45*(1), 184-205. doi:http://dx.doi.org/10.3102/0002831207312906\

Han, H. S., Vomvoridi-Ivanović, E., Jacobs, J., Karanxha, Z., & Feldman, A. (2017). *Using Collaborative Self-Study Methods to Explore Culturally Responsive Pedagogy in Higher Education*. Sage Publications Limited.

Hansen, P. (2015). *The Mathematics Coaching Handbook: Working with K-8 Teachers to Improve Instruction*. New York: Routledge.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for research in mathematics education*, 524-549.

Heid, M. K. & Lee, H. S. (2008). Using technology in teaching and learning mathematics: What should doctoral students in mathematics education know? In R. E. Reys & J. Kilpatrick, *One field, many paths: U.S. doctoral programs in mathematics education* (pp. 117-125). Providence, RI: American Mathematics Society.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., ... & Wearne, D.

(1997). Making mathematics problematic: A rejoinder to Prawat and Smith. *Educational Researcher*, *26*(2), 24-26.

Hiebert, J., Morris, A. K., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of Teacher Education*, *58*(1), 47-61. doi: 10.1177/0022487106295726

Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing teachers' mathematical knowledge: What knowledge matters and what evidence counts? In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 111–156). Charlotte, NC: Information Age.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 372-400.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*, *26*(4), 430-511.

Jablon, P. C. (2002). The status of science education doctoral programs in the United States: The need for core knowledge and skills. *Electronic Journal of Science Education*, *7(1)*. Avaliable online at http://wolfweb.unr.edu/homepage/crowther/ejse/jablon.pdf.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, *9*, 187–211.

Jaworski, B. (2008a). Building and sustaining inquiry communities in mathematics teaching development: Teachers and didacticians in collaboration. In K. Krainer & T. Wood (Eds.), *International handbook of mathematics teacher education: Participants in mathematics teacher education: Individuals, teams, communities and networks* (Vol. 3, pp. 309–330). Rotterdam: Sense.

Jaworski, B. (2008b). Development of the mathematics teacher educator and its relation to teaching development. In B. Jaworski & T. Wood (Eds.), *The international handbook of mathematics teacher education volume 4: The mathematics teacher educator as a developing professional* (pp. 335–361). Rotterdam: Sense Publishers.

John, P. D. (2002). The teacher educator's experience: Case studies of practical professional knowledge. *Teaching and Teacher Education*, *18*(3), 323-341.

Kazemi, E., Elliott, R., Mumme, J., Carroll, C., Lesseig, K. & Kelley-Petersen, M. (2011). Noticing leaders' thinking about videocases of teachers engaged in mathematics tasks in professional development. In M. G. Sherin, V. Jacobs & R. Philipp (Eds.), *Mathematics teacher noticing* (pp. 188–203). New York: Routledge.

Kazemi, E., Ghousseini, H., Cunard, A., & Turrou, A. C. (2016). Getting inside rehearsals insights from teacher educators to support work on complex practice. *Journal of Teacher Education*, *67* (1), 18-31. http://doi: 10.1177/0022487115615191

Kieran, C., Krainer, K., & Shaughnessy, J. M. (2013). Linking research to practice: Teachers as key stakeholders in mathematics education research. In M. A. Clements, A. J. Bishop, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Third international handbook of mathematics education* (pp. 361–392). New York: Springer.

Kluth, P., & Straut, D. (2003). Do as we say and as we do: Teaching and modeling collaborative practice in the university classroom. *Journal of Teacher Education, 54*(3), 228-240.

Knight, S. L., Lloyd, G. M., Arbaugh, F., Gamson, D., McDonald, S. P., & Nolan, J. (2014). Professional development and practices of teacher educators. *Journal of Teacher Education*, *65*(4), 268-270.

Korthagen, F. A., Kessels, J., Koster, B., Lagerwerf, B., & Wubbels, T. (2001). *Linking practice and theory: The pedagogy of realistic teacher education*. Routledge.

Koster, B., Dengerink, J.J., (2008). Professional Standards for teacher educators: How to deal with complexity, ownership and function. Experience from the Netherlands. *European Journal of Teacher Education* 31 (2), 135–149.

Krainer, K. (2008). Reflecting the development of a mathematics teacher educator and his discipline. In B. Jaworski & T. Wood (Eds.), *The international handbook of mathematics teacher education: Vol. 4 The mathematics teacher educator as a developing professional* (Vol. 4, pp. 177-199). Rotterdam, the Netherlands: Sense Publishers.

Labaree, D. F. (2004). *The trouble with ed schools*. New Haven, CT: Yale University Press.

Lagemann, E. C. (2002). *An elusive science: The troubling history of education research*. University of Chicago Press.

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., ... & Crowe,K. (2013). Keeping It Complex Using Rehearsals to Support Novice Teacher Learning of Ambitious Teaching. *Journal of Teacher Education*, *64*(3), 226-243.

Lave, J. & Wenger, E. (1991) *Situated Learning: Legitimate Peripheral Participation*. Cambridge, UK: Cambridge University Press.

Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education*, *46*(1), 88-124.

Lee, H. S., K. F. Hollebrands, and P. H. Wilson. 2010. *Preparing to Teach Mathematics with Technology: An Integrated Approach to Data Analysis and Probability*. Dubuque, IA: Kendall Hunt.

Leinhardt, G., & Smith, D. A. (1985). Expertise in mathematics instruction: Subject matter knowledge. *Journal of Educational Psychology*, *77*, 247–271.

Lewis, C. C., Perry, R. R., & Hurd, J. (2009). Improving mathematics instruction through lesson study: A theoretical model and North American case. *Journal of mathematics teacher* *education*, *12*(4), 285-304.

Li, W., & Superfine, A. C. (2018). Mathematics teacher educators' perspectives on their design of content courses for elementary preservice teachers. *Journal of Mathematics Teacher Education*, *21*(2), 179-201.

Llinares, S., & Krainer, K. (2006). Mathematics (student) teachers and teacher educators as learners. *Handbook of research on the psychology of mathematics education: Past, present and future*, 429-459.

Liu, S. (2013). Pedagogical content knowledge: A case study of ESL teacher educator. *English Language Teaching*, *6*(7), 128–138. http://doi.org/10.5539/elt.v6n7p128

Lloyd, G. M., & Wilson, M. (1998). Supporting innovation: The impact of a teacher's conceptions of functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education*, 248-274.

Loughran, J. (2014). Professionally Developing as a Teacher Educator. *Journal of Teacher Education*, *65*(4), 271–283. http://doi.org/10.1177/0022487114533386

Lunenberg, M. (2002). Designing a Curriculum for Teacher Educators. *European Journal of Teacher Education*, *25*(June 2015), 263–277. http://doi.org/10.1080/0261976022000044872

Lunenberg, M., & Hamilton, M. L. (2008). Threading a golden chain: An attempt to find our identities as teacher educators. *Teacher Education Quarterly*, *35*(1), 185-205.

Lunenberg, M., Dengerink, J., & Korthagen, F. (2014). *The professional teacher educator: Roles, behaviour, and professional development of teacher educators*. Rotterdam: Sense Publishers.

Lynch, C. M. (2017). *Collaborative Planning for an Elementary Mathematics Methods Course in a Third Space: The Role of Expertise in a Community of Practice* (Doctoral dissertation). The Pennsylvania State University.

Ma, L. (1999a). Introduction and chapter 1: Subtraction with regrouping *Knowing and teaching elementary mathematics* (pp. 17-27). Mahwah, NJ: Lawrence Erlbaum Associates.

Marshall, C., & Rossman, G. (2006). The how of the study: Building the research design. *Designing qualitative research*, 55-101.

Munter, C. (2014). Developing visions of high-quality mathematics instruction. *Journal for Research in Mathematics Education, 45*(5), 584-635.

Masingila, J. O., Olanoff, D. E., & Kwaka, D. K. (2012). Who teaches mathematics content courses for prospective elementary teachers in the United States? Results of a national survey. *Journal of Mathematics Teacher Education, 15*(5), 347–358.

Mason, J. (2008). PCK and beyond. In P. Sullivan & T. Wood (Eds.), *The International Handbook of Mathematics Teacher Education* (Vol. 1, pp. 301-322). Rotterdam: Sense Publishers.

Mason, J., & Watson, A. (2008). Mathematics as a constructive activity: Exploiting dimensions of possible variation. In M. P. Carlson & C. Rasmussen (Eds.), *Making the connection: Research and practice in undergraduate mathematics*, 191-204. Mathematical Association of America.

Mathews, S. M. (2004). The experiences in a concepts in calculus course for middle school mathematics teachers. *The work of mathematics teacher educators: Exchanging ideas for effective practice (AMTE Monograph 1)*, 67-85.

McDonald, M., Kazemi, E., & Kavanagh, S. S. (2013). Core practices and pedagogies of teacher education a call for a common language and collective activity. *Journal of Teacher Education*, *64*(5), 378-386.

Mehra, B. (2002). Bias in qualitative research: Voices from an online classroom. *The Qualitative Report*, *7*(1), 1-19.

Miles, M. B., Huberman, A. M., & Saldana, J. (2013). *Qualitative data analysis*. (3rd ed.)

Thousand Oaks, CA: Sage Publications.

Mohammad, R. F. (2008). Becoming a mathematics teacher educator: Processes and issues. In B. Jaworski & T. Wood (Eds.), *The international handbook of mathematics teacher education: Vol. 4 The mathematics teacher educator as a developing professional* (pp. 157-175). Rotterdam, the Netherlands: Sense Publishers.

Murillo, E. (2008). Searching Usenet for virtual communities of practice: using mixed methods to identify the constructs of Wenger's theory. *Information Research: An International Electronic Journal*, *13*(4).

Murray, J., & Male, T. (2005). Becoming a teacher educator: Evidence from the field. *Teaching and Teacher Education*, *21*(2), 125–142. http://doi.org/10.1016/j.tate.2004.12.006

National Research Council, & Mathematics Learning Study Committee. (2001). *Adding it up: Helping children learn mathematics*. National Academies Press.

National Research Council. (2002). *Scientific research in education* (Committee on Scientific Principles for Education Research, R. J. Shavelson, & L. Towne, Eds.). Washington, DC: National Academy Press.

National Research Council. (2005). *Advancing scientific research in education*. Washington, DC: NRC.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

Nevin, A. I., Thousand, J. S., & Villa, R. A. (2009). Collaborative teaching for teacher educators—What does the research say? Teaching and Teacher Education, 25(4), 569–574.

Norton, A., & D'Ambrosio, B. S. (2008). ZPC and ZPD: Zones of teaching and learning. *Journal for Research in Mathematics Education*, 220-246.

Omidvar, O., & Kislov, R. (2014). The evolution of the communities of practice approach: Toward knowledgeability in a landscape of Practice—An interview with Etienne Wenger-Trayner. *Journal of Management Inquiry*, *23*(3), 266-275. doi:10.1177/1056492613505908

O'Sullivan, M. C. (2010). Educating the teacher educator—A Ugandan case study. *International journal of educational development*, *30*(4), 377-387. http://doi.org/10.1016/j.ijedudev.2009.12.005

Patton, M. Q. (2002). Two decades of developments in qualitative inquiry: A personal, experiential perspective. *Qualitative social work*, *1*(3), 261-283.

Peled, I., & Hershkovitz, S. (2004). Evolving research of mathematics teacher educators: The case of non-standard issues in solving standard problems. *Journal of Mathematics Teacher Education*, *7*(4), 299-327. Retrieved from http://search.proquest.com/docview/61929305?accountid=13158

Pellegrino, K., Sweet, B., Derges Kastner, J., Russell, H. a, & Reese, J. (2014). Becoming music teacher educators: Learning from and with each other in a professional development community. *International Journal of Music Education*, *32*(4), 462–477.

Perks, P., & Prestage, S. (2008). Tools for learning about teaching and learning. In B. Jaworski & T. Wood (Eds.), *International handbook of mathematics teacher education: The mathematics teacher educator as a developing professional* (Vol. 4, pp. 265–280). Rotterdam: Sense.

Remillard, J. T. (1999). Curriculum materials in mathematics education reform: A framework for examining teachers' curriculum development. *Curriculum Inquiry*, *29*(3), 315-342.

Rogoff, B. (1994). Developing understanding of the idea of communities of learners. *Mind, culture, and activity*, *1*(4), 209-229.

Rossman, G. B., & Rallis, S. F. (2012). *Learning in the field: An introduction to qualitative research*. (3rd ed.) Thousand Oaks: California. Sage Publications.

Roth, W. M. (1998). *Designing communities* (Vol. 3). Netherlands: Kluwer Academic Publications.

Roth McDuffie, A., Mather, M., & Reynolds, K. (2004). Professional development for teacher and teacher educator through sustained collaboration. *The Work of Mathematics Teacher Educators: Exchanging Ideas for Effective Practice*, 189-204.

Rowland, T., Turner, F., & Thwaites, A. (2014). Research into teacher knowledge: A stimulus for development in mathematics teacher education practice. *ZDM—The International Journal on Mathematics Education*, *46*(2). doi:10.1007/s11858-013-0564-9.

Sakonidis, H., & Potari, D. (2014). Mathematics teachers educators'/researchers' collaboration with teachers as a context for professional learning. *ZDM—The International Journal on Mathematics Education*, *46*(2). doi:10.1007/s11858-014-0569-z.

Saldaña, J. (2013). *The coding manual for qualitative researchers*. (2nd ed.) Thousand Oaks, CA: Sage Publications.

Shagrir, L. (2007). *Building identities as teacher educators – What works?* Paper presented at the AERA Annual Meeting, Chicago, 2007.

Shagrir, L. (2010). Professional Development of Novice Teacher Educators: Professional Self, Interpersonal Relations and Teaching Skills. *Professional Development in Education*, *36*(1-2), 45–60. http://doi.org/10.1080/19415250903454809

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Silver E. & Walker, E. (2008). Making policy issues visible in the doctoral preparation of mathematics educators. In In R. E. Reys & J. Kilpatrick, *One field, many paths: U.S.*

doctoral programs in mathematics education: Developing Stewards of the Discipline. (pp. 63-75). Providence, RI: American Mathematics Society.

Smith, T. (2006). Becoming a teacher of mathematics: Wenger's social theory of learning perspective. *Identities, cultures and learning spaces*, 619-622.

Smith, M. S., Bill, V., & Hughes, E. K. (2008). Thinking through a lesson: Successfully implementing high-level tasks. *Mathematics Teaching in the Middle School*, *14*(3), 132-138.

Smith, M. & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. Reston, VA: NCTM.

Smith, M.S., Arbaugh, F., Boyle, J., Stylianides, G., Steele, M.D., Greeno, J., & Hillen, A.F., (n.d.). Cases of reasoning-and-proving in secondary mathematics: A curriculum for the professional education for teachers. *Unpublished manuscript.*

Stake, R. E. (1995). *The art of case study research.* Thousand Oaks, CA: Sage Publications.

Staples, M. (2007). Supporting whole-class collaborative inquiry in a secondary mathematics classroom. *Cognition and Instruction*, *25*(2-3), 161-217. doi: http://dx.doi.org/10.1080/07370000701301125

Stein, M. K., Smith, M. S., & Silver, E. (1999). The development of professional developers: Learning to assist teachers in new settings in new ways. *Harvard educational review*, *69*(3), 237-270.

Stein, M. K., & Brown, C. A. (1997). Teacher learning in a social context: Integrating collaborative and institutional processes with the study of teacher change. *Mathematics teachers in transition*, 155-191.

Steele, M. D. (2008a). Building bridges: Cases as catalysts for the integration of mathematical and pedagogical knowledge. In M. S. Smith & S. N. Friel (Eds.), *AMTE Monograph 4:*

Cases in mathematics teacher education: Tools for developing knowledge needed for teaching (pp. 57-72). San Diego, CA: Association of Mathematics Teacher Educators.

Stylianides, A. J., & Ball, D. L. (2008). Understanding and describing mathematical knowledge for teaching: Knowledge about proof for engaging students in the activity of proving. *Journal of Mathematics Teacher Education*, *11*(4), 307-332.

Superfine, A. C., & Li, W. (2014). Developing Mathematical Knowledge for Teaching Teachers: A Model for the Professional Development of Teacher Educators. *Issues in Teacher Education*, *23*(1), 113-132.

Svensson, L. (1984). Three approaches to descriptive research. Report No 1984: 01. Sweden: Department of Education, University of Gotenborg.

Swennen, A., L. Shagrir, and M. Cooper. (2009). Becoming a teacher educator: Voices of beginning teacher educators. In A. Swennen and M. van der Klink (Eds), *Becoming a teacher educator: Theory and practice for teacher educators*, (pp. 91–102). Dordrecht: Springer.

Sztajn, P., Ball, D. L., & McMahon, T. A. (2006). Designing learning opportunities for mathematics teacher developers. In K. Lynch-Davis & R. L. Rider (Eds.), *The work of mathematics teacher educators: Continuing the conversation*, AMTE monograph (Vol. 3, pp. 149–162). San Diego, CA: Association of Mathematics Teacher Educators.

Sztajn, P., Wilson, P. H., Edgington, C., & Myers, M. (2014). Mathematics professional development as design for boundary encounters. *ZDM - International Journal on Mathematics Education*, *46*(2), 201–212.

Taylor E. V. & Kitchen, R. (2008). Doctoral programs in mathematics education: Diversity and equity. In R. E. Reys & J. Kilpatrick, *One field, many paths: U.S. doctoral programs in mathematics education: Developing Stewards of the Discipline* (pp. 111-116). Providence, RI: American Mathematics Society.

Taylor, C. E. (2011). *Facilitating the development of elementary prospective teachers' pedagogical content knowledge: A case study of a mathematics teacher educator's actions and purposes* (Order No. 3533859). Available from ProQuest Dissertations & Theses A&I. (1262380945). Retrieved from http://search.proquest.com/docview/1262380945?accountid=13158

Tirosh, D., Tsamir, P., Levenson, E., Barkai, R., & Tabach, M. (2014). Using video as a tool for promoting inquiry among preschool teachers and didacticians of mathematics. *ZDM—The International Journal on Mathematics Education, 46*(2). doi:10.1007/s11858-013-0563-x.

Tzur, R. O. N. (2001). Becoming a mathematics teacher-educator: conceptualizing the terrain through self-reflective analysis. *Journal of Mathematics Teacher Education*, *4*, 259–283. http://doi.org/10.1023/A:1013314009952

Tzur, R. (2008). Profound awareness of the learning paradox (PALP): A journey towards epistemologically regulated pedagogy in mathematics teaching and teacher education. In B. Jaworski & T. Wood (Eds.), *The International Handbook of Mathematics Teacher Education: The Mathematics Teacher Educator's as a Developing Professional* (Vol. 4, pp. 137-156). Rotterdam, the Netherlands: Sense.

Van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education*, *10*(4), 571-596.

Van Zoest, L. R., Stockero, S. L., & Edson, A. J. (2010). Multiple uses of research in a mathematics methods course. In J. W. Lott & J. Luebeck (Eds.), *AMTE Monograph 7: Mathematics teaching: Putting research into practice at all levels* (pp. 157- 171). San Diego, CA: Association of Mathematics Teacher Educators.

Villa, R. A., Thousand, J. S., & Nevin, A. I. (2013). *A guide to co-teaching: Practical* *tips for facilitating student learning* (3rd ed.). Thousand Oaks, CA: Corwin Press and Council for Exceptional Children.

Vogler, K. E., & Long, E. (2003). Team teaching two sections of the same undergraduate course: A case study. *College Teaching*, *51*(4), 122-126.

Vomvoridi-Ivanovic, E., & McLeman, L. (2015). Mathematics Teacher Educators Focusing on Equity: Potential Challenges and Resolutions. *Teacher education quarterly*, *42*(4), 83-100.

Vygotsky, L. (1978). Interaction between learning and development. *Readings on the development of children*, *23*(3), 34-41.

Wagner, C., Garner, M., & Kawulich, B. (2011). The state of the art of teaching research methods in the social sciences: Towards a pedagogical culture. *Studies in Higher Education*, *36*(1), 75-88.

Waters, F. H., & Burcroff, T. L. (2007). Collaborative teaching at the university level: Practicing what is preached. *The Teacher Educator*, *42*(4), 304-315.

Webb, D. C., Boswinkel, N., & Dekker, T. (2008). Beneath the Tip of the Iceberg: Using Representations to Support Student Understanding. *Mathematics teaching in the middle school*, *14*(2), 110-113.

Wenger, E. (2008). *Communities of practice: Learning, meaning, and identity* (1st ed. 18th print). New York; Cambridge, U.K;: Cambridge University Press.

Wilson, S. M. (2006). Finding a Canon and Core: Meditations on the Preparation of Teacher Educator-Researchers. *Journal of Teacher Education*, *57*(3), 315–325. http://doi.org/10.1177/0022487105285895

Wilson, P. S & Franke, M. (2008). Preparing teachers in mathematics education doctoral programs: Tensions and strategies. In R. E. Reys & J. Kilpatrick, *One field, many paths:*

U.S. doctoral programs in mathematics education (pp. 103-109). Providence, RI: American Mathematics Society.

Wilson, G. L. (2016). *Co-planning for Co-teaching: Time-Saving Routines That Work in Inclusive Classrooms.* Alexandra, VA: Association for Supervision & Curriculum Development (ASCD).

Wilson G. L. & Blendick, J. (2011). Teaching in tandem: Effective co-teaching in the inclusive classroom. Alexandria, VA: ASCD.

Wu, Y., Hwang, S., & Cai, J. (2017). Being a Mathematics Teacher Educator in China: Challenges and Strategic Responses. *International Journal of Science and Mathematics Education*, *15*(7), 1365-1384.

Yamagata-Lynch, L. C., & Haudenschild, M. T. (2009). Using activity systems analysis to identify inner contradictions in teacher professional development. *Teaching and Teacher Education*, *25*(3), 507-517.

Yin, R. K. (2013). *Case study research: Design and methods* (Fifth ed.). Thousand Oaks, CA: Sage publications, Inc.

Yukawa, J. (2010). Communities of practice for blended learning: Toward an integrated model for LIS education. *Journal of Education for Library and Information Science*, , Vol. 51, No. 2, pp. 54-75.

Zaslavsky, O., & Leikin, R. (2004). Professional Development of Mathematics Teacher Educators: Growth Through Practice. *Journal of Mathematics Teacher Education*, *7*(1), 5–32. http://doi.org/10.1023/B:JMTE.0000009971.13834.e1

Zaslavsky, O. (2007). Mathematics-related tasks, teacher education, and teacher educators. *Journal of Mathematics Teacher Education*, *10*(4-6), 433-440.

Zaslavsky (2008). Meeting the challenges of mathematics teacher education through design and use of tasks that facilitate teacher learning. In B. Jaworski & T. Wood (Eds.), International handbook of mathematics teacher education: The mathematics teacher educator as a developing professional (Vol. 4, pp. 93–114). Rotterdam: Sense.

Zbiek, R. M., & Hirsch, C. R. (2008). Curriculum as Core Knowledge. In In R. E. Reys & J. Kilpatrick, *One field, many paths: U.S. doctoral programs in mathematics education: Developing Stewards of the Discipline*. (pp. 63-75). Providence, RI: American Mathematics Society.

Zeichner, K. (2005). Becoming a teacher educator: A personal perspective. *Teaching and Teacher Education*, *21*(2), 117–124. http://doi.org/10.1016/j.tate.2004.12.001

Appendix A

Curricular Knowledge of MTEs for Teaching PTs

	Case I University-based faculty and instructor of a secondary mathematics methods course for undergraduates
1 Programs & Materials	• Models of mathematics teacher preparation programs • Textbooks/materials for secondary mathematics methods courses • Materials for teaching about [cooperative learning, equity, manipulatives, technology, etc.] in a mathematics classroom
2 Indications & Contradindications	• Pros and cons of different mathematics teacher preparation programs • How pre-service teachers interact with and learn from mathematics methods curriculum materials • Research knowledge
3 Lateral Curricular Knowledge	• The institution's mathematics teacher preparation program • Other courses pre-service teachers are enrolled in at the time
4 Vertical Curricular Knowledge	• The institution's mathematics teacher preparation program • Other courses pre-service teachers are enrolled in before and after the methods course

Figure A-1. Curricular Knowledge of MTEs. Reprinted from "Curricular knowledge and the work of MTEs" by J. B. Chauvot, 2008, *Issues in Teacher Education, 17* (2), p. 86.

Appendix B

ATE Standards for Teacher Educators

Table B-1. *Standards for teacher educators* (Association of Teacher Educators [ATE], 2008).

STANDARDS	Indicators
Teaching: Model teaching that demonstrates content and professional knowledge, skills, and dispositions reflecting research, proficiency with technology and assessment, and accepted best practices in teacher education.	• Model effective instruction to meet the needs of diverse learners • Demonstrate and promote critical thinking and problem solving among teacher educators, teachers, and/or prospective teachers • Revise courses to incorporate current research and/or best practices • Model reflective practice to foster student reflection • Demonstrate appropriate subject matter content • Demonstrate appropriate and accurate professional content in the teaching field • Demonstrate a variety of instructional and assessment methods including use of technology • Mentor novice teachers and/or teacher educators • Facilitate professional development experiences related to effective teaching practices • Ground practice in current policy and research related to education and teacher education
Cultural competence: Apply cultural competence and promote social justice in teacher education.	• Exhibit practices that enhance both an understanding of diversity and instruction that meets the needs of society • Engage in culturally responsive pedagogy • Professionally participate in diverse communities • Model ways to reduce prejudice for pre-service and in-service teachers and/or other educational professionals • Engage in activities that promote social justice • Demonstrate connecting instruction to students' families, cultures, and communities • Model how to identify and design instruction appropriate to students' stages of development, learning styles, linguistic skills, strengths and needs • Foster a positive regard for individual students and their families regardless of differences such as culture, religion, gender, native language, sexual orientation, and varying abilities • Demonstrate knowledge of their own culture and aspects common to all cultures and foster such knowledge in others • Promote inquiry into cultures and differences • Teach a variety of assessment tools that meet the needs of diverse learners • Recruit diverse teachers and teacher educators

Scholarship: Engage in inquiry and contribute to scholarship that expands the knowledge base related to teacher education.	• Investigate theoretical and practical problems in teaching, learning, and/or teacher education • Pursue new knowledge in relation to teaching, learning, and/or teacher education • Connect new knowledge to existing contexts and perspectives • Engage in research and development projects • Apply research to teaching practice and/or program or curriculum development • Conduct program evaluation • Acquire research-based and service-based grants • Disseminate research findings to the broader teacher education community • Engage in action research • Systematically assess learning goals and outcomes
Professional Development Inquire systematically into, reflect on, and improve their own practice and demonstrate commitment to continuous professional development.	• Systematically reflect on own practice and learning • Engage in purposeful professional development focused on professional learning goals • Develop and maintain a philosophy of teaching and learning that is continuously reviewed based on a deepening understanding of research and practice • Participate in and reflect on learning activities in professional associations and learned societies • Apply life experiences to teaching and learning
Program Development Provide leadership in developing, implementing, and evaluating teacher education programs that are rigorous, relevant, and grounded in theory, research, and best practice.	• Design, develop, or modify teacher education programs based on theory, research, and best practice • Provide leadership in obtaining approval or accreditation for new or modified teacher education programs • Lead or actively contribute to the ongoing assessment of teacher education courses or programs • Provide leadership that focuses on establishing standards for teacher education programs or on developing, approving, and accrediting teacher education programs at the local, state, national, or international level • Contribute to research that focuses on effective teacher education programs
Collaboration: Collaborate regularly and in significant ways with relevant stakeholders to improve teaching, research, and student learning.	• Engage in cross-institutional and cross-college partnerships • Support teacher education in the P-12 school environment • Participate in joint decision making about teacher education • Foster cross-disciplinary endeavors • Engage in reciprocal relationships in teacher education • Initiate collaborative projects that contribute to improved teacher education • Acquire financial support for teacher education innovation to support collaboration

Public Advocacy Serve as informed, constructive advocates for high quality education for all students	• Promote quality education for all learners through community forums, activities with other professionals, and work with local policy makers • Inform and educate those involved in making governmental policies and regulations at local, state, and/or national levels to support and improve teaching and learning • Actively address policy issues which affect the education profession
Teacher Education Profession Contribute to improving the teacher education profession.	• Actively participate in professional organizations at the local, state, national, or international level • Edit/review manuscripts for publication or presentation for teacher education organizations • Review resources designed to advance the profession • Develop textbook or multimedia resource for use in teacher education • Recruit promising pre-service teachers • Recruit future teacher educators • Mentor colleagues toward professional excellence • Design and/or implement pre-service and induction programs for teachers • Support student organizations to advance teacher education • Advocate for high quality teacher education standards
Vision: Contribute to creating visions for teaching, learning, and teacher education that take into account such issues as technology, systemic thinking, and world views.	• Actively participate in learning communities that focus on educational change • Demonstrate innovation in the field of teacher education • Demonstrate qualities of an early adopter of technology and new configurations of learning • Actively pursue new knowledge of global issues • Support innovation adoption with research • Relate new knowledge about global issues to own practice and K-12 classroom teaching

Appendix C

The Methods Course Design and Materials

Course Description:

This is the one of three secondary mathematics methods courses in the SECED Mathematics Education Option. In this course, pre-service teachers consider mathematics teaching and learning from a teacher's perspective as well as from a student's perspective. Course participants engage in mathematical problem solving and in the study of the history and nature of mathematics as the foundation for understanding current curriculum and standards. Lesson planning follows from the consideration of different types of mathematical content, including skills and concepts. Looking specifically at the learning of mathematics and questioning to promote higher-level thinking prepares students for field experiences in subsequent semesters.
The goals for the course are:
- To improve understanding of some of the mathematical concepts which are important in secondary school mathematics.
- To improve understanding of the nature of mathematics: what is important, how it is practiced, how mathematical validity is determined.
- To improve understanding of the historical development of selected topics from secondary school mathematics.
- To develop a vision of good school mathematics.
- To see mathematics, mathematics learning, and mathematics teaching as problematic and to develop an inquiry approach to and an ability to reflect on these domains.
- To increase understanding of secondary school students' mathematical thinking and understanding.
- To increase ability to specify subject matter involved in a specific mathematics topic and make distinctions among them.
- To improve understanding of various teaching strategies and their strengths and weaknesses.
- To increase ability to choose among lessons and curriculum materials based on the intended mathematical subject matter and the current understandings of the students.
- To increase insight into creating a thriving, supportive mathematics classroom culture.

Table C-1: The modified CEI during MTHED 411.

Nodes of the Cycle	**Description**
Node 1. Doing the mathematical task	PTs do a mathematical task, identify learning goals for a teacher using the task.
Node 2-3. Analyzing *representations of practice*	PTs individually and collectively *analyze a representation of practice* involving the mathematical task from Node 1 using *A.A.T framework*.
Node 4. Planning for *approximation of practice* (i.e., rehearsal)	Groups of two-three PTs plan to teach a rehearsal involving similar tasks in Node 1.
Node 5. Enacting the *approximation of practice* (i.e., rehearsal)	PTs rehearse their plans with doctoral students, peers, or actual students. The lead instructor coach during the rehearsal.
Node 6. Analyzing rehearsals using *decompositions of practice*	Small groups of PTs collectively and individually analyze the rehearsal from Node 4 using a *A.A.T.*

Thinking Through a Lesson Protocol (TTLP): Abbreviated

The main purpose of the Thinking Through a Lesson protocol is to prompt you in thinking deeply about a specific lesson that you will be teaching. The goal here is to move beyond the structural components associated with lesson planning (e.g., listing the materials you will need, describing the way students will be grouped, determining teacher actions during the lesson) to a deeper consideration of how you are going to advance students' mathematical understanding during the lesson. This is not to say that structural components of a lesson are not important, but rather that a focus on structural components alone is *not sufficient* to ensure that students learn mathematics.

<u>The Mathematical Task(s)</u>

- What mathematical content and processes do you hope students will learn from their work on these tasks? (What are the mathematical learning goals for this task?)

- What specific *Standards for Mathematical Practice* from the Common Core State Standards - Mathematics (CCSS-M) will your students have the opportunity to engage in during their work on this task?

- What are all the ways the task can be solved?
  - Which of these methods do you think your students will use?
  - What misconceptions might students have?
  - What errors might students make?

<u>Supporting Students' Exploration of the Task(s)</u>

- In your rehearsal:
  - What questions will you ask to *assess* students' understanding of key mathematical ideas, problem-solving strategies, or the representations?
  - What questions will you ask to *advance* students' understanding of the mathematical ideas?
  - What mathematical concepts/definitions/constructs might you need to introduce to the students by *telling*?

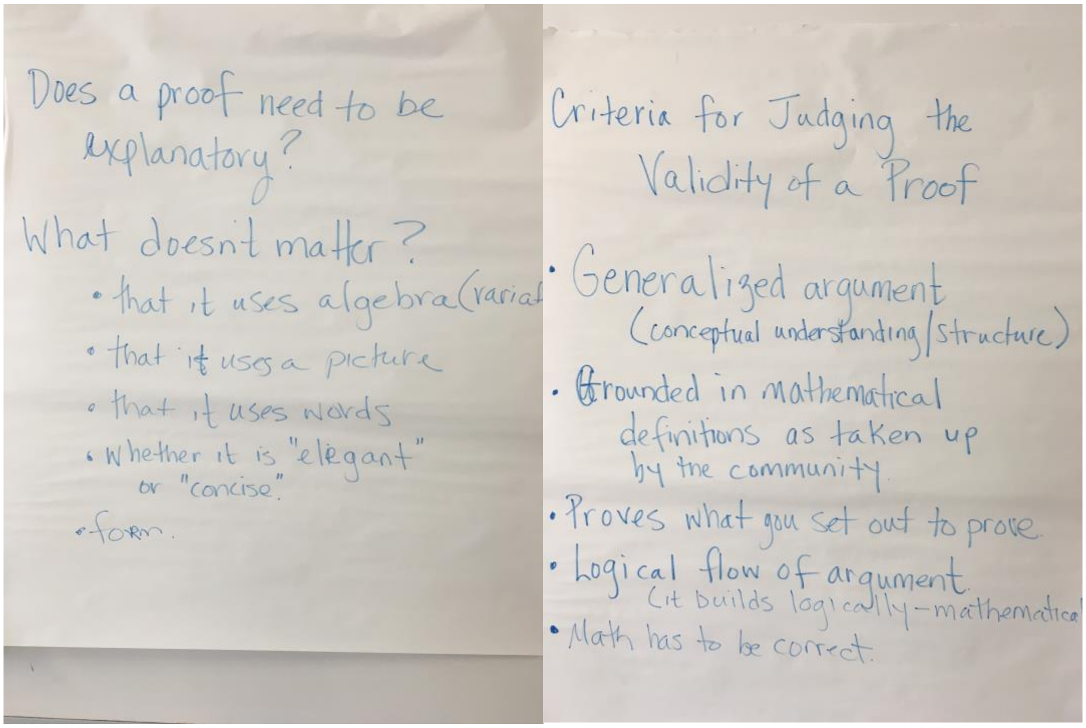

Figure C-1: PTs' criteria for judging the validity of a proof.

Appendix D

Examples of Mathematics Task and Student Solutions from the Course

In 1–3, use the following information: José, Mario, and Melanie went on a weeklong cycling trip. The table below gives the distance each person traveled for the first 3 hours of the trip. The table shows only the time when the riders were actually biking, not when they stopped to rest, eat, and so on.

	Distance (miles)		
Cycling time (hours)	José	Mario	Melanie
0	0	0	0
1	5	7	9
2	10	14	18
3	15	21	27

1. a. How fast did each person travel for the first 3 hours? Explain how you got your answer.

b. Assume that each person continued at this rate. Find the distance each person traveled in 7 hours.

2. a. Graph the time and distance data for all three riders on the same coordinate axes.

b. Use the graphs to find the distance each person traveled in $6\frac{1}{2}$ hours.

c. Use the graphs to find the time it took each person to travel 70 miles.

d. How does the rate at which each person rides affect the graphs?

3. a. For each rider, write an equation that can be used to calculate the distance traveled after a given number of hours.

b. Use your equations from part a to calculate the distance each person traveled in $6\frac{1}{2}$ hours.

c. How does a person's biking rate show up in his or her equation?

Figure D-1: First rehearsal problem set, page 1, from Lappan et al., (2002).

4. Mike was on the bike trip with José, Mario, and Melanie (from questions 1–3). He made the following table of the distances he traveled during day 1 of the trip.

Time (hours)	Distance (miles)
0	0
1	6.5
2	13
3	19.5
4	26
5	32.5
6	39

a. Assume Mike continued riding at this rate for the entire bike trip. Write an equation for the distance Mike traveled after *t* hours.

b. Sketch a graph of the equation.

c. When you made your graph, how did you choose the range of values for the time axis? For the distance axis?

d. How can you find the distance Mike traveled in 7 hours and in $9\frac{1}{2}$ hours, using the table? The graph? The equation?

e. How can you find the number of hours it took Mike to travel 100 miles and 237 miles, using the table? The graph? The equation?

f. For parts d and e, give the advantages and disadvantages of using each form of representation—a table, a graph, and an equation—to find the answers.

g. Compare the rate at which Mike rides with the rates at which José, Mario, and Melanie ride. Who rides the fastest? How can you determine this from the tables? From the graphs? From the equations?

Figure D-2: First rehearsal problem set, page 2, from Lappan et al., 2002.

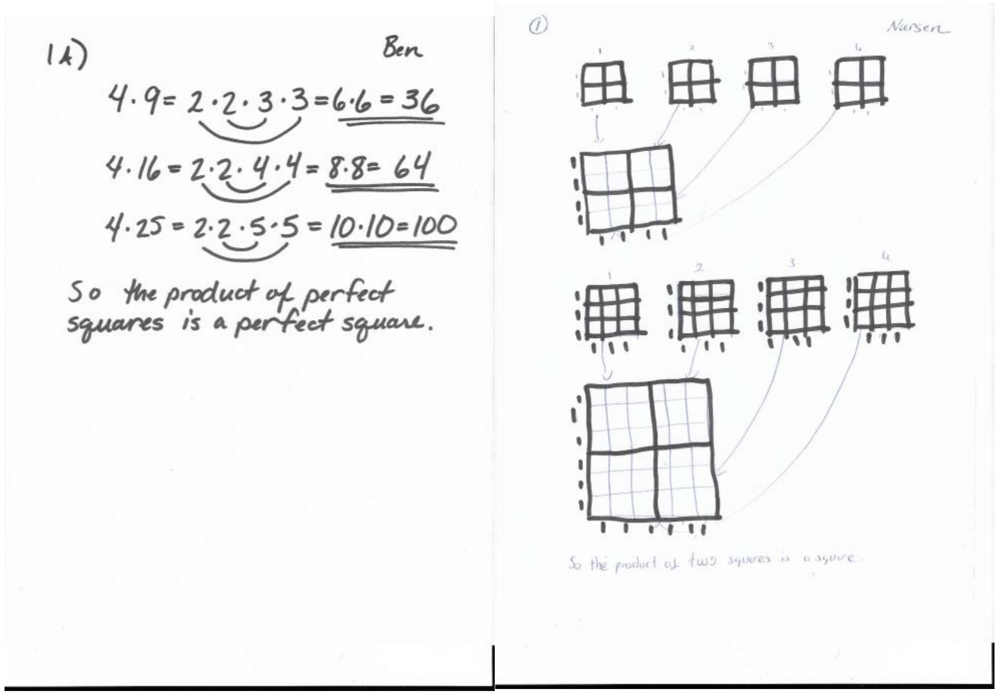

Figure D-3: Student solution examples for proving “A perfect square times a perfect square is perfect square”

Appendix E

Data Analysis

Table E-1. An example of relationship table between MTEs' types of talk and knowledge

	Type of Talk	Knowledge
F1	Discussing curricular materials Logistics, science computers, reserving rooms, outlets in the classroom, hard drive and thumb drives to save audio files along with analysis How to use studio code: code window to make, stack students analysis	CK - resources, tools
F2	Discussing curricular materials What laptops to use and how to access those -innovation studio, math lab, mthed computers, science education laptops	CK - resources, tools
F3	Determining instructional details, Discussing curricular materials Preparing content material (representation of practice to analyze) for class -How to code on StudioCode and decide what will the students code -what portions of video, what codes to use A.A.T. & other	CK – course material SMK – content PCK – PTs' learning
F4	Determining instructional details, Discussing curricular materials, Discussing Content Creating student solutions for Rehearsal 2 -Creating incomplete or inaccurate student solutions, -what to put on poster as solution to create a realistic conversations and yet opportunities for PTs to practice A.A.T -types of learning goals: math vs socio learning goal	CK – secondary curriculum SMK –students' math understandings PCK – PTs lerning
F5	Discussing curricular materials Resources to select tasks for peer teaching -textbooks : CLP , Core+	CK - tasks
F5	Discussing curricular materials, Determining instructional details Writing observation protocol -word choice, instructional details	CK- course material PCK - instruction
F5	Discussing curricular materials Saving and accessing and disseminating videos	CK - tools
F5	Discussing curricular materials, content, reflection on practice Generate authentic student solution for odd + odd = even task - What secondary students know/ do in math - What was done in previous semesters, use of those materials, comparisons, tasks (in reasoning class)	SMK – students' math understanding, math CK - course materials
F6	Analysis of PTs, Discussing curricular materials -Details of HW: -What PTs read: cases and articles, and prompt questions	CK – assignment, course materials
F6	Reflections on practice, Designing learning activity, Discussing curricular materials, Analysis of PTs learning Identifying tasks for RH2: number theory generalizations - Planning a follow up activity (and possible assignments) for the CIE2	CK- tasks PCK – instruction PCK – PTs' learning of math
F7	Discussing curricular materials, Content, Determining instructional details, Analysis of PTs thinking Creating student solutions -What secondary students would do to solve odd=odd task -what pictorial solution, numerical solution, algebraic solution they will generate	CK – course materials, SMK –students' math understandings,

	- how secondary kids think -what mistakes, struggles, understanding they have - grade level, curriculum, and what kids are familiar Provide opportunities to ask A.A.T for PTs - how much to put on the student solution so that it will be realistic yet still -what is the math goal of odd + odd task	approaches, secondary curriculum PCK – PTs learning, instruction
F9	Discussing curricular materials, Analysis of PTs thinking Deciding what tasks PTs use for peer teaching -Available resources, secondary textbooks: their structure and cognitive demand of task for PTs -practitioner journal articles, Calendar problems offered for math teachers -Features of a desired task: mathematically challenging for PTs? Appropriate to practice talk moves? Multiple approaches, rich discussion	CK-Tasks, resources PCK – PTs' learning
F10	Deciding what to and how to assess final project, exam schedule	CK-assignments CXK - department
SP1	Discussing curricular materials, Analysis of PTs' thinking, Content -student solutions - Write full solution, let's decide after partial students solutions Observation sheets -purpose & function of observation sheets: to keep PTs attended +categorizing types of A.A.T -language to be used in the observation sheet (match with course constructs)	CK & SMK– secondary curriculum, materials SMK –students' math understandings
SP4	Reflecting on practice "a need for curricular materials: writing journals that explains the thinking of each "student solutions" for next MTE instructor	CK- course artifacts
SP4	Reflecting on practice, Evaluation of activities, Analysis of PTs' thinking, Discussing curricular materials Tasks selection Developing conceptual understanding for quadratic formula using core+ tasks Resources to find tasks/activities to develop conceptual understanding for R- squared - Resources: Tasks for Developing conceptual understanding: NCTM journals - Algorithms to test numerical series convergence (used in MidAtlantic project) -NCTM journal: fraction division	PCK- PTs math experiences CK- resources, tasks
SP4	Evaluation of activities, Analysis of PTs' thinking, Discussing curricular materials Selecting tasks for rehearsal Support PTs understanding of course constructs: Conceptual understanding -tasks: quadratic formula, Cramer's rule with matrix, Series -explore a procedure they knew conceptually	PCK- PTs math experiences, instruction CK- resources, tasks

Examples of Memos from My Research Journal

January

- I started first analyzing using StudioCode audio files. I realized that was not efficient at least for the first couple of meetings. I was not able to capture the episodes that could be potential evidence for knowledge. I needed to see them as a text in order to actually think through for the first coding. So I decided to first transcribe the meeting audios. I used ExpressScribe and Word files for transcribing. I wanted to use Studio Code but it was not convenient to make comments, or play backwards on StudioCode.
- While transcribing, I also coded some instances. Sometimes just as PCK, sometimes more specific like knowledge about technology, sometimes in more details as, knowledge about how to analyze practice using StudioCode.
- I also summarized activities taking place in each planning meeting: what are the main topics discussed, questions asked and problems addressed.

February

- I have started to code the problems emerged and discussed during meetings as well to see how a particular knowledge assists in addressing that problem.
- I decided to separate the codes for problems and activities (so what is the "big issue" and what activities the group engage in –to address that problem- and what suggestions people make – knowledge or expertise they offer-

  These are the questions in the long run I struggled & I needed to face.

1) To identify if something is an indication of expertise or knowledge.
2) Also, if coding some instances as a particular knowledge, or just brainstorming a thought or idea
3) When should I code a knowledge domain:

    a) Is it when one shares the knowledge (bring to the table)?

    a. Or if the group seeking for knowledge: Who initiates the conversation around a particular knowledge [person ask the questions that targets a particular knowledge domain]
    b. Or the shared knowledge of the group?
    c. Or if there is an Indication of lack of/limitation of a particular knowledge

March

- I struggled deciding what category Knowledge of tech (KT) fits in: Is it in the intersection of CX (knowledge of available tools/ support) and CK (knowledge of analysis, rehearsals)?
- As I progressed in transcribing and coding, I went back to the meeting notes from the same meeting. I checked the course agenda and course materials (to see the nature of activities took place) and course readings.
- I decided to use NVivo to create sub-codes that will help me to characterize and ls it different types of knowledge that belongs to each knowledge category. I created the nodes: SMK, CK, CXK, PCK AND TK and others on Nvivo and cases for each participant. copied and saved my memos and data (interview transcriptions and meeting notes from my laptop)
- I started coding interviews again, identify the type of knowledge emerged creating a new node, adding a description for the node: such as giving examples or more details then use the hierarcy option on Nvivo (while creating the node define and locate under which big category of knowledge node it belongs to)

Note: As needed go back to the Nvivo tutorials on Youtube:
https://www.youtube.com/watch?v=KzLcFA0mWDQ&index=7&list=PLNjHMRgHS4FfTN-GoztTaPLshavAb0NxR

March 21st

- I decided to code only on Word docs not Nvivo due to its limitations in writing comments and developing subcodes.
- As I continued coding. I went back to the Shulman's Grossman and Chauvot's article to create a brief list of characteristics of each knowledge category.

Note: Think about data resources and course artifacts, what are the reificiations in this study?

1) the documents generated during meetings such as student works for the odd+odd task?
2) syllabus, course agenda, discourse?

- Chauvot documented her actions and coded them as either using or seeking for a particular knowledge and marked which ones are also considered teacher knowledge. (I plan to go back once in a while to reexamine my categories of knowledge to check/verify and if any emerging "other" category exist)
- I started to generate a list & table for knowledge sub- categories

Note: Think about Reliability/validity/generalizability issues

A useful quote to consider:

Kilpatrick (1995) "No one should expect to draw strong implications for practice from the results of a single research. Research in mthed gains its relevance to practice or to further research by its power to cause us stop an think. It equips us not with results we can apply but rather with tools for thinking about our work" (p. 25)

- Studies contribution to the field: conceptualizing the structure and the growth of knowledge has potential to help us designing curriculum and PD for TEs.
- I also started seeing patterns in the conversations taking place in the planning meetings: such as
  1-instructional decisions for each class & IAs
  2-the learning goal behind IAs and revisiting the learning goals (the focus) of this course
  3-evaluations Of IAs and possible revisions for the next semester
  4-discussions about students' interactions
  5-other: more general things about teacher ed, not directly course related

April 12th

- After our conversation with Lynch, I started thinking;

1) what is the boundary in this context of study? What is non boundary?
2) what knowledge is considered as boundary -knowledge that have some meaning for everyone in the group and provides common language although might be understood differently by individuals)
3) who or what is boundary broker (the one introduces new ideas to the community)

May 13th

- I started labeling the patterns in the co-planning talk using phrases such as: Pre plan (upcoming content), Review, Reflect, Evaluate, Plan instruction (big picture, content, design activities, content delivery, assign roles and responsibilities,

May 21st

- Final decision for coding:
  Sharing a certain knowledge (could be an individual from the group or a more than one member)
  Code what certain knowledge the group is seeking for planning an activity.

June 13th

Some initial interpretations of data:

- Himm! Van Zoest recommended (2006) breaking away from doctoral students becoming codesigners of method courses as interns b/c that makes doctoral students focus on *generating* PTs thinking more than *analyzing* their thinking.
  If the doctoral students engage in reflective practices and analysis of PTs thinking regularly as they co-design the courses, I do not agree with this statement.
- (Ball Hill & Bass, 2005) found MTEs used MKT as "common intellectual space" in organizing and implementing instructions for content courses. Sztajn (2006) MKT served for productive focus for diverse group in designing.
  In our design, "teaching practices" "rehearsals" and "CIE" served as the same purpose to organize and implement instructions for a method course. These might have served as "brokers."
- Brokering: the member makes connections b/w COPs, translating knowledge from one to another
- Communities of practice BROKERING – making connections b/w different COPs and translating knowledge from one COP to another; then the research served that function for us.
- Question: Can I also describe two separate boundaries; one as MTE mentor, the other as doctoral students?
- Question: Are these below BOUNDRY OBJECTS or REIFICATIONS we produced as we engage in practice?
  - artifacts: IAs, student solutions, analysis tools (code windows) assignments, etc
  - discourses - common language: teaching practices, strands of math proficiency, cognitive demand, etc
  - processes-routines: meeting protocols, recording meetings notes, observations,

Vita

Nursen Konuk

Education

Ph.D., Curriculum & Instruction (Mathematics Education). The Pennsylvania State University. State College, Pennsylvania. August 2018.
Ph. D. Dissertation: MTEs' roles, talk, and knowledge in co-planning & Opportunities co-planning offers for professional development. Directed by Dr. Fran Arbaugh.

M.Ed., Mathematics Education. The Pennsylvania State University. State College, Pennsylvania. December 2011.

B.S., Secondary Mathematics Education. Centennial University, Van, Turkey. August 2006.

Experiences

Research Assistant: Department of Curriculum & Instruction. The Pennsylvania State University. University Park, Pennsylvania. 2010 – Present.
NSF-funded University Teaching Experience for Mathematics Pre-Service Teachers (UTEMPT) Project. Supervised by Dr. Fran Arbaugh. 2018.
NSF-funded Penn State Secondary Mathematics Noyce Scholarship Program. Supervised by Dr. M. Kathleen Heid. 2018.
Arbaugh Research Group (ARG). Supervised by Dr. Fran Arbaugh. 2014-2018.
NSF-funded Cases of Reasoning-and-Proving (CORP) in Secondary Mathematics Project. Supervised by Dr. Fran Arbaugh. 2010 – 2012.

Teaching Assistant & Instructor: Department of Curriculum & Instruction. The Pennsylvania State University. University Park, Pennsylvania. 2015 –2017.

Data Analysis Associate: University of Pittsburgh's Learning Research & Development Center. Supervised by Dr. Mary Kay Stein. 2017.

Professional Development Facilitator for the Pennsylvania Mathematics Initiative. 2017.

Mathematics Instructor in the Upward Bound Summer Academy. The Pennsylvania State University. 2017.

Secondary Mathematics Teacher: Pi-Analytic Academy, Kutahya, Turkey. 2006 – 2008.

Publications

Arbaugh, F., Freeburn, B., Graysay, D., & Konuk, N. (in preparation). The three-minute rehearsal: Implementation and learning outcomes. To be submitted to *Mathematics Teacher Educator*.

Karunakaran, S., Freeburn, B., Konuk, N., & Arbaugh, F. (2014). Improving pre-service secondary mathematics teachers' capability with generic example proofs. *Mathematics Teacher Educator*, *2*(2), 158-171.

Grants, Honor, & Awards

Donald B. and Mary Louise Elder Tait Scholarship in Mathematics Education. The Pennsylvania State University (2016)

Full-Scholarship for graduate study from National Education Ministry of Turkey (2007).

Honor student in BS Program in Secondary Mathematics Education. Centennial University, Van, Turkey (2006)