AUTONOMOUS CONTROL MODES AND OPTIMIZED PATH GUIDANCE
FOR SHIPBOARD LANDING IN HIGH SEA STATES

A Dissertation in
Aerospace Engineering
by
Junfeng Yang

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The dissertation of Junfeng Yang was reviewed and approved* by the following:

Amy Pritchett
Department Head of Aerospace Engineering

Joseph F. Horn
Professor of Aerospace Engineering
Dissertation Advisor, Committee Chair

Edward C. Smith
Professor of Aerospace Engineering
Director of Penn State Rotorcraft Center

Jack W. Langelaan
Associate Professor of Aerospace Engineering
Director of Graduate Programs

Christopher D. Rahn
Professor of Mechanical and Nuclear Engineering

*Signatures are on file in the Graduate School
Abstract

Rotorcraft, due to their unique vertical take-off and landing capability, are well-suited for maritime applications. The capability of shipboard launch and recovery of a rotorcraft allows extension of the operational envelope of a single ship as well as the whole fleet. However, due to the cross-axes coupling, inherent instability and sluggish response, accurate and soft landing of rotorcraft is a challenging task, especially when the flight deck is moving in high sea states and in the presence of adverse factors such as the gusty airwake and limited space. In low to medium sea state, an experienced pilot can designate a window of quiescent ship motion to perform the landing while also using an intuitive predictive strategy. In very high sea states, the workload and control precision become unacceptable, and therefore motivate the development of an automated landing system.

This thesis contributes both theoretical investigations and technical solutions to the guidance, navigation and control for an autonomous shipboard recovery mode in high states. Using the high-fidelity modeling software FLIGHTLAB together with ship airwake and motion models, the specific aspects of rotorcraft flight dynamics and the characteristics of the shipboard landing environment were studied and provided guidelines for the formulation of design requirements. The Dynamic Inversion method was applied to design an inner-loop attitude control system and then an outer-loop trajectory following system, and the associated design problems such as control parameter optimization, robustness testing are discussed. In order to provide a fully autonomous capability, a parameterization and optimization algorithm was developed for approach path generation. The resulting path geometry and velocity profile can ensure fundamental flight safety but also provide enough flexibility for console operators to specify approach azimuth and steepness. A landing path generator incorporating predictive deck state has been developed to complete the last stage of shipboard recovery, both forecasting and instantaneous measurement of deck state are used to construct the commanded descent trajectory through a hybrid implementation. The technical adequacy of high-grade vehicle location and motion detection has been proven by an integrated navigation system incorporating information from GPS, inertial measurement unit and shipboard tracking system. Stand-alone testing of system components, as well as comprehensive testing of the entire system from approach entry to touchdown have been carried out using FLIGHTLAB simulations. As justified by the simulation results: the scientific concept and engineering approach developed in this thesis show great potential to an overall solution to the challenging problems of shipboard recovery of rotorcraft in high sea states in a fully autonomous mode.
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\( x \) \hspace{1cm} \text{state vector of a linear/nonlinear system}
\( u \) \hspace{1cm} \text{input vector of a linear/nonlinear system}
\( y \) \hspace{1cm} \text{output vector of a linear/nonlinear system}
\( A, B, C \) \hspace{1cm} \text{Stability, control and output matrices of a linear/non-linear system}
\( \phi, \theta, \psi \) \hspace{1cm} \text{Roll, pitch and yaw attitude angles}
\( p, q, r \) \hspace{1cm} \text{Roll, pitch and yaw angular rates}
\( u, v, w \) \hspace{1cm} \text{Components of velocity vector in body axis}
\( X, Y, Z \) \hspace{1cm} \text{Components of force vector in body axis}
\( L, M, N \) \hspace{1cm} \text{Roll, Pitch and Yaw moment in body axis}
\( X_{X}, Y_{X}, Z_{X} \) \hspace{1cm} \text{Non-dimensional derivative of force w.r.t variable “x”}
\( L_{X}, M_{X}, N_{X} \) \hspace{1cm} \text{Non-dimensional derivative of moment w.r.t variable “x”}
\( K_p, K_i, K_{ii} \) \hspace{1cm} \text{Proportional, integral and double integral gains}
\( X_{lon}, Y_{lat}, Z \) \hspace{1cm} \text{Distance of longitudinal, lateral and vertical translation}
\( V_{lon}, V_{lat}, V_{z} \) \hspace{1cm} \text{Speed of longitudinal, lateral and vertical translation}
\( A_{lon}, A_{lat}, A_{z} \) \hspace{1cm} \text{Acceleration of longitudinal, lateral and vertical translation}
\( \delta_{lon}, \theta_{lon}, \delta_{lat}, \delta_{ped} \) \hspace{1cm} \text{Longitudinal, lateral, collective and pedal stick position}
\( \theta_{lon}, \theta_{lat}, \theta_{col}, \theta_{tr} \) \hspace{1cm} \text{Longitudinal, lateral cyclic and collective pitch of swashplate, collective pitch of tail rotor}
\( \delta_{3} \) \hspace{1cm} \text{Flap-feather coupling factor}
\( X_{N}, Y_{E}, H \) \hspace{1cm} \text{Coordinates to the North and East, and Height}
\( V_{N}, V_{E}, V_{z} \) \hspace{1cm} \text{Inertial velocity to the North, East and Upward}
\( A_{N}, A_{E}, A_{z} \) \hspace{1cm} \text{Inertial acceleration to the North, East and Upward}
\( V_{climb} \) \hspace{1cm} \text{Climbing velocity}
\( V_d \) \hspace{1cm} \text{Descent velocity}
\( \omega_{e,il} \) \hspace{1cm} \text{Frequency parameter of error dynamics of inner-loop DI controller}
\( \zeta_{e,il} \) \hspace{1cm} \text{Damping parameter of error dynamics of inner-loop DI controller}
\( \omega_{e,ol} \) \hspace{1cm} \text{Frequency parameter of error dynamics of outer-loop DI controller}
\( \zeta_{e,ol} \) \hspace{1cm} \text{Damping parameter of error dynamics of outer-loop DI controller}
\( p_{ol} \) \hspace{1cm} \text{Integrator pole of error dynamics of outer-loop controller}
\( \omega_{e,\phi} \) \hspace{1cm} \text{Frequency parameter of roll axis DI controller}
\( \omega_{e,\theta} \) \hspace{1cm} \text{Frequency parameter of pitch axis DI controller}
\( G_r \) \hspace{1cm} \text{Transfer function matrix of reduced order model}
\( G_f \) \hspace{1cm} \text{Transfer function matrix of full order model}
\( \Delta G \) \hspace{1cm} \text{Error transfer function matrix due to order reduction}
\( K(s) \) \hspace{1cm} \text{Controller transfer function matrix}
\( \bar{\sigma} \) \hspace{1cm} \text{Upper bound of singular value}
\( \sigma \) \hspace{1cm} \text{Lower bound of singular value}
\( V_0 \)  
Asymptotic speed of Heffley approach profile

\( V_{app} \)  
Approach speed

\( A_{app} \)  
Approach acceleration

\( R_{pd} \)  
Range of peak deceleration

\( \tau \)  
Time to close the gap of a parameter

\( \tau_{pos} \)  
Time to close the gap of position

\( \tau_{spd} \)  
Time to close the gap of speed

\( \psi_{app} \)  
Approach azimuth angle

\( y_{app} \)  
Approach glide slope angle

\( H_{hel} \)  
Height of helicopter

\( V_{hel} \)  
Vertical speed of helicopter

\( H_{deck} \)  
Height of deck center

\( V_{z_{deck}} \)  
Vertical speed of deck center

\( \Sigma_w \)  
Covariance of process noise  \( w \)

\( \Sigma_v \)  
Covariance of measurement noise  \( v \)

\( R_{rel} \)  
Relative range of helicopter in ship frame

\( \psi_{rel} \)  
Azimuth angle of helicopter in ship frame

\( \theta_{rel} \)  
Elevation angle of helicopter in ship frame

Subscripts

cmd  
Command

ref  
Reference

helo  
Helicopter

lon  
Longitudinal

lat  
Lateral

pred  
Prediction

des  
Desired value

e2b  
Earth to Body

rel  
Relative

fcast  
Forecast

lim  
Limit

Abbreviations

S&C  
Stability and Control

SISO  
Single Input and Single Output

MIMO  
Multiple Input and Multiple Output

CFD  
Computational Fluid Dynamics

FEM  
Finite Element Method

SHOL  
Ship-Helicopter Operating Limit
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<tr>
<td>MCA</td>
<td>Minor Component Analysis</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Position System</td>
</tr>
<tr>
<td>INS</td>
<td>Inertial Navigation System</td>
</tr>
<tr>
<td>STS</td>
<td>Shipboard Tracking system</td>
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<td>CVs</td>
<td>Controlled Variables</td>
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<td>FCS</td>
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<td>T.F.</td>
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<td>Gain Margin</td>
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<td>PM</td>
<td>Phase Margin</td>
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<td>DR</td>
<td>Disturbance Rejection</td>
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<td>Disturbance Rejection Bandwidth</td>
</tr>
<tr>
<td>DRP</td>
<td>Disturbance Rejection Peak</td>
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<tr>
<td>DI</td>
<td>Dynamic Inversion</td>
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<tr>
<td>LQR</td>
<td>Linear Quadratic Regulator</td>
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<tr>
<td>EXP</td>
<td>Exponential function</td>
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<tr>
<td>C.G.</td>
<td>Center of gravity</td>
</tr>
<tr>
<td>R.P.</td>
<td>Reference Point</td>
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<tr>
<td>ACAH</td>
<td>Attitude Command Attitude Hold</td>
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<tr>
<td>TRC</td>
<td>Translational Rate Command</td>
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Acknowledgments

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Chapter 1. Introduction

Contents

1.1 Relevant Work on Shipboard Launch and Rotorcraft Recovery
1.2 Adequacy Survey of Sensor and Control Technologies
1.3 Research Objective
1.4 Contributions
1.5 Thesis Overview
1.6 Reference

For maritime operations, the helicopter is often the primary weapon system and logistic asset. Ship-helicopter operating limits (SHOL) have a first order effect on sortie generation rate and hence mission effectiveness, as well as the individual ship operational envelope and overall fleet disposition. Restrictions on the SHOL result from ship motion, unsteady aerodynamic environment, power and pedal margin limits. Launch/Recovery become particularly challenging in degraded visual environment and high sea state, in undesirable weather, as reported by naval pilots; the workload is too high to maintain a fair control precision with which the safety of recovery can be ensured. Hence, the application of automatic control system towards enhancing the ease and accuracy of a manual operation and ultimately a full automation of this process has been a long-term objective sought after by academia and industry.

1.1 Relevant Work on Shipboard Launch & Recovery of Rotorcraft

In the worldwide fleet practice, the recovery of rotorcraft still relies on the experience of pilots on identifying and capturing a quiescent period of ship motion. On particularly equipped vessels, auxiliary systems such as “harpoon rapid deck lock” are used to aid post-landing safety. Typical harpoon systems can fasten on board rotorcraft so tight as to resist a rocking roll angle up to 30° in rough sea[1]. The application of auxiliary systems can be acknowledged as an elementary level of automation (the deployment is automatic), to achieve a higher level or even a full level of automation, the physical and statistical laws of the concurrent phenomenon must be thoroughly understood.

Figure 1.1.1 Harpoon Deck Lock and Grid Plate [1]
Understanding the nature of deck motion and ship airwake laid the foundations for studying the aerodynamics of a rotorcraft flying behind a ship under disturbance. Ship motion dynamics have been widely studied in the past decades, seakeeping theory [2][3] investigates the ship motion from a detailed ship dynamics point of view. Schwartz et al in Ref [4] systematically investigated the wave motion of deck coordinates in different sea states. Time history of ship angular and translational motion have been encapsulated into a data package SCONE (Standard Deck Motion Data for a Generic Surface Combatant), which can be easily used in simulation of landing tasks. The disturbances from ship airwake to the rotor aerodynamics are known to have significant impact on the position hold accuracy. Both experimental and numerical methods have been employed to study the complexity of unsteady flow behind ship body and the interaction with rotor downwash. Ref [5] is a representative work aimed to design a coupling interface between a CFD solver and a rotorcraft flight simulation code. Airwake-rotor downwash interaction was modeled in a scheme where the CFD solver accepts downwash information from the flight simulation code and calculates the flow field of the ship body; at the same time flight simulation code takes into account the local flow field information from CFD solver. For real-time performance, flow field data are generated and stored off-line and the flow velocity database is accessed by the simulation.

Deep study into the mathematical model of ship motion have been shown to provide reliable prediction of deck state in the near future. Triantafyllou et al [6] attempted to use Kalman Filtering technique to predict the 6-DOF motion of vessels represented by a state-space model. Lainiotis et al in [7] derived another kind of state-space forecasting model based on the knowledge of inherent ship dynamics, however, this model was strongly dependent on the oceanic wave measurement and thus less practical in the implementation. Ra et al in [8] represented the ship motion as a special sinusoidal form and obtained a recursive robust least square algorithm to estimate the frequency by assuming that the ship motion period changes slowly. Although the deck motion exhibits quite random characteristics under wave action, its time series are still full of patterns. Time series theory provides another powerful solution for the prediction of deck state. The trend, seasonality and irregular fluctuation can be extracted through separation algorithms, then a time series model can be determined given the model order and corresponding coefficients, the model determination is a typical big-data problem and can be potentially solved using machining learning techniques [9]. Based on the decomposition of time series, each of the component can be predicted individually. Principle/Minor component analysis (MCA) is another forecast algorithm, Oja in Ref [10] presented an overall picture of this method. An initial prediction algorithm using MCA to deck state can be found in Ref [11], as a part of this study, a comparison of actual and predicted deck state in time history was given in Ref [12]; the prediction model has shown satisfactory fidelity particularly in those region with apparent motion pattern. Continual research on control strategy also opened new possibilities of aided landing. Ref [13] suggested an assistant algorithm inspired by human pilots’ strategy to mitigate the impact of deck motion by identifying a landing period with quiescent ship motion. The algorithm evaluates a synthetic index consisting of ship displacement, velocity and acceleration to inform pilots about the upcoming landing period. However, such a grace period may not always exist especially in high
states. On the other hand, a pioneering work in automated deck landing can be found in Ref [14], advanced concepts such as deck motion compensation, relative position measurement and deck state prediction were introduced as innovative solutions, albeit in a primitive manner: In Ref [14], guidance is conducted using the relative position reconstructed from range, azimuth and elevation angle observed from flight deck. The range is measured using microwave range finder, angles are measured by a scanning beam system, and both sensors are mounted on an attitude-stabilized platform to provide stable measurement. The system error is expressed in angular terms, which in turn results in an error that is approximately proportional to range. As the aircraft approaches the landing pad the location error can be reduced to less than 1 foot at touchdown. The control law in Ref [14] follows a classical scheme, i.e. position error and velocity error were used to drive a PID controller, which generates swashplate pitch command. The deck velocity measurements were injected into the PID controller to constitute a “deck motion compensation”. The deck state prediction was implemented like a “trigger function”; it simply forecasts the roll angle of flight deck at eighth seconds in the future, as soon as a level deck attitude is expected, the landing procedure starts the final descent. The above workframe achieved only part of the technical anticipation due to the inherent weakness of information acquisition and low-bandwidth of autopilot.

1.2 Adequacy Survey of Sensor and Control Technologies

In recent years, both the sensor technologies and rotorcraft control design methodologies have gained great progress. GPS, INS, Infrared/Optical or similar compound tracking system have been developed to provide unprecedented accuracy in aircraft and vessel location, which is adequate to support a precision approach and landing. Ref [15] summarized the British effort on shipboard recovery automation. Experience of British navy established 0.3 meters (approximately 1 ft) as the navigation precision requirement. To achieve this goal, Raytheon. Ltd provided GPS receivers with Kinematic Carrier Phase Tracking (KCPT), Common Mode Error Elimination, together with a tightly coupled Kalman Filter based GPS/INS solution. The product of this project is alleged to show centimetric accuracy sufficient for any recovery requirement. Ref [16] introduced the product of Thales. Ltd - Microwave and GPS Integrated Cooperative Automatic Take-off and Landing System (MAGIC ATLOS), the locationing module is alleged to provide the relative position of aircraft with respect to the mount platform with a better accuracy than merely provided by GPS. As a cheaper positioning solution, Ref [17] and [18] suggested using a dual-GPS-INS system. One set is carried by the ship, the other is mounted on the air vehicle, the ship measurement is transmitted to the air vehicle for flight navigation reference. Besides traditional navigation device, the vision-based sensors provide another solution to ship-helicopter relative measurement, in Ref [19] the authors described a method for ship deck estimation using camera sensing from rotorcraft aboard, the vision-based sensing was used to supplement classic navigation hardware in a Kalman Filter frame, thus enabling the ability to autonomous land.

In the area of flight control design, high fidelity rotorcraft modeling software succeeded to provide nonlinear and linear mathematical models, based on which modern control theory can be adopted to design multi-mode fly-by-wire control laws with great capability in addressing the response decoupling, closed-loop frequency shaping, and command-tracking issues.
Representative rotorcraft codes such as GENHEL \cite{20} and FLIGHTLAB \cite{21} are featured by blade element rotor models, wide-range airfoil data, dynamic inflow models, tabulated data of fuselage and empennage aerodynamics. State of the art includes Viscous Particle Model (VPM) of rotor wake, CFD modeling of aerodynamics and FEM of structural coupling. In any case, an adequate mathematical model should provide traditional rigid fuselage modes, and additional rotor modes for rigorous control system design.

Given rotorcraft mathematical model, various methods can be used to design control laws. Classical transfer-function based method had been widely used to design medium bandwidth autopilots, such as attitude hold, speed hold and altitude hold. Chapter 3 of Ref \cite{22}, Chapter 4 of Ref \cite{23} and Chapter 12 of Ref \cite{24} provided enriched materials of classical controller design. In practice, simple PID controllers have been designed for various unmanned rotorcraft \cite{25,26}, Park in Ref \cite{27} designed three independent SISO controllers for attitude control of a model helicopter, flight test data verified the author’s simple PD control strategy. Dzul et al in \cite{28} used a classical pole-placement technique for the yaw axis control and an adaptive pole-placement method for the altitude control. In Ref \cite{29} Mettler et al studied the negative effect of stabilizing bar fitted on small helicopters to PD controller and introduced notch filter of 2\textsuperscript{nd} order system to suffice the gain and phase margin. However, the SISO nature of classical method of control design essentially limited its effectiveness in application of rotorcraft due to the inherent plant instability and cross coupling. Those issues are better solved by modern synthesis techniques such as eigenstructure assignment, LQR, and Dynamic Inversion(DI). To match the sophisticated design methods, elaborative design objectives were established in regulatory documents such as U.S. ADS-33, and U.K. Def.Stan.00-970. The former defines ideal response in frequency domain, while the later in time domain. In either way, the control system performance can be readily translated into close-loop eigenstructure, therefore the eigenstructure assignment method firstly obtained extensive study. Ref \cite{30}-\cite{36} elaborated the theory of eigenstructure assignment and its application in helicopter control design. Major topics cover the construction of ideal close-loop eigenstructure, rate command system, attitude hold system, altitude hold system, heading hold system, inner-outer loop organization of multi-mode controller and robustness margin.

LQR is another powerful method for designing MIMO plant controller. Ref \cite{37}-\cite{39} provided theoretical foundation and outlined the general way of LQR method in designing regulator and tracking controller. In Ref \cite{40}-\cite{42}, the authors presented the application of LQR method to the design of Nz-U controller in the longitudinal axis and side-slip angle/roll rate controller in the lateral and directional axes. The advantage of LQR method is its guaranteed stability margin, e.g. for a scalar system LQR feedback designs always ensure gain margin more than 6dB, phase margin more than 60°. Although this conclusion cannot be extended to MIMO case rigorously, the robustness is always a preferred property of linear quadratic controllers.

Dynamic Inversion (DI) as a novel method won a lot of attention in the past decades. Thorny control problems such as stability augmentation, command tracking, control decoupling and close-loop bandwidth tuning can be readily handled by this method. As a unique feature of DI controller, the platform-agnostic control gains render a fast transplant from one class of rotorcraft to another. At Penn State, the rotorcraft application of DI controller has been extensively studied for different
purpose, including for shipboard handling quality evaluation. Ref [43] discovered the gust rejection function as an augmentation to DI controller, the model following control architecture was proven to help reject disturbance from a ship airwake and thus reduce pilot workload for shipboard operations. Ref [44] presented the study on different response types designed using DI controller in context of sea-based application, non-linear DI based ACAH and TRC/PH control law were evaluated through pilot-in-the-loop simulation, pilot HQR proved the advantage of ACVH and TRC mode in approach and landing respectively. The above researches paved the path to apply DI control strategy for autonomous ship landing system.

The guidance law is another critical issue in autonomous flight. Guidance law is responsible for generating reference position, velocity even acceleration for a vehicle to follow. An early stage work in the area was reported in Ref [45] where the authors conducted flight investigation to determine the characteristic shapes of the altitude, ground-speed and deceleration profiles of visual approaches for helicopters. Mathematical relationships were developed that describe the characteristic profiles which are applicable to improve handling qualities in approaching flight. Another remarkable work in Ref [46] by Heffley proposed analytical expressions inspired by pilot control strategy that generate approach profiles for speed and acceleration as functions of range to hover spot. A lot of effort such as Ref [47], [48] and [49] have been devoted to developing Tau-based guidance. The parameter tau is the time to close the gap using current rate for any variable of interest $x$:

$$\tau = \frac{x}{\dot{x}}$$

(1.1)

Tau theory builds such guidance laws around parameter $\tau$ that lead to a safe perch or touchdown. In Ref [50], a high order Tau-guidance was suggested by Holmes to support modeled shipboard landing, in his guidance algorithm the position, speed and acceleration profile are generated as functions of time with non-impact entry but also satisfy the velocity and attitude matching constraints at touchdown.

1.3 Research Objectives

Implementing the newly emerged technologies for a competitive GNC solution for shipboard landing is a focus of R&D in many nations. In fact, The FireScout MQ-8B and MQ-8C unmanned rotorcraft have performed autonomous landing at sea [51], French Naval Group and Thales. Ltd had been awarded UAV deck landing project, their product SADA had successfully landed a rotor-wing UAV in automatic mode on a navy frigate [52]. Korean Aerospace Research Institute is also running a project studying the automatic landing technology on a tilt rotor UAV [53]. Despite there is not a lot of public domain information on the details of the control laws or the operating limits of these autonomous modes, in any case, autonomous and piloted landing in high sea states and high winds continue to be a problem for the U.S. Navy. In 2014 ONR funded a program to investigate the potential of sea-based automated launch and recovery system. The project is a joint effort from three parties: Advanced Rotorcraft Technologies. Ltd (ART) was to provide rotorcraft modeling software, deck landing simulation environment and deck state prediction algorithm; PSU takes the responsibility to develop system architecture, multi-mode flight control law, path generation algorithm for approaching and landing, implementation of deck state prediction and
also assist ART in software validation; Naval Air Systems Command (NAVAIR) is responsible to perform path parameter optimization and evaluate the overall performance of autonomous control system. This dissertation primarily reflects the research activity carried out at Penn State in the past three years. Major research objectives include the following:

1. The development of a high-performance autonomous trajectory-following control law. The system architecture and control parameter selection should involve feasible measurements, guarantee the tracking performance under environmental disturbance and preserve certain close-loop stability margin against modeling error.

2. The development of an approach path planer that provides recovery guidance in terms of reference position, velocity and acceleration. The path planer should utilize flexible path geometry, tunable kinematic profile and optimization algorithm so that the corresponding vehicle state will fall within operational limits.

3. The development of landing trajectory algorithms that make use of predicted deck motion in order to improve the accuracy of rotorcraft landing on a moving, heaving, pitching and rolling platform while minimizing control and power usage.

4. Verifications of the method using non-linear simulation with high level of fidelity including high-order rotorcraft model, embedded navigation system, unified controller for entire landing process and mode transition logics.

5. Demonstrations of the scalability and generality of the method by applying it on three different classes of rotorcraft.

The system was designed using the plant model generated by FLIGHTLAB software. Linear controller design and analysis are performed in Matlab/Simulink. Test and evaluation of overall system performance are conducted in the non-linear simulation environment of FLIGHTLAB with extended functions modeling the ship motion, ship airwake and flight deck - landing gear interactions.

1.4 Contributions

- Path Generator/Optimizer for Autonomous Approach: A B-spline parameterization and separate processing of horizontal/vertical path were implemented. As opposed to other path solutions using fixed pattern of geometry, the B-spline geometry enables a larger space to cover those solutions that cannot be captured by fixed angle approach [15] or Tau-based approach [50]. The separation of path geometry as well as the path criteria leads to a physically meaningful and straightforward objective function, which together with the B-spline parameterization has been incorporated in an optimization algorithm and can sort out the optimal approach trajectory from the solution space.

- Predictive Landing Program: Using polynomial representations, the descent programs based on deck state forecasting were generated and optimized subject to certain kinematic constraints. This technology allows for high-order initial/terminal conditions to match a moving landing deck, also can utilize interim control points to achieve trajectory shaping. Compared to existing methods using trigonometric functions [54] or Tau-guidance [55], the landing program developed in this research can not only provide monotonically descending trajectories to save control energy, but also guarantee terminal position, velocity and
acceleration matching and envelop protection. The hybrid implementation of live-time measurement and prediction of deck state succeeded to compensate the forecasting error.

- **Trajectory Following Control Law Design:** A Dynamic Inversion based controller was designed that regulates the three-axial position and velocity of rotorcraft. Using inner-outer loop concept and acceleration feedforward, this controller can drive the rotorcraft to follow spatial trajectory command with responsive quickness. The capability of simultaneous regulation of position, velocity and acceleration enables precision approach with better tracking errors than applying a traditional airspeed/altitude/heading hold autopilot [16]. A corresponding control parameter optimization approach was developed to seek for a balance between robustness criteria and demanding performance metrics required by deck landing tasks.

- **Demonstration of the autonomous system:** using high fidelity simulation models of three different classes of rotorcraft with realistic high order dynamics, sensor models and data fusion, the autonomous shipboard recovery system has been proven of high mission accuracy, low vehicle dependency and strong design robustness against model roughness and navigation error. Despite of the complicated rotor modes and inflow dynamics, the designed performance exhibited slight degradation in the full order model. The GNC solution can be quickly transplanted from one platform to another by replacing the plant model of Dynamic Inversion, while using the same set of compensator design, no apparent loss of designed performance was observed.

### 1.5 Thesis Overview

To produce a more accessible document, efforts have been made to write self-contained chapters. As a result, each chapter has its own introduction, content, summary and reference. A brief outline of the chapters follows:

**Chapter 2** is devoted to introduce the software used for modeling and simulation. The methods for modeling ship airwake and deck prediction are briefed. A simple description of the rotorcraft mathematical model and flight mechanics is presented and exemplified on a typical helicopter.

**Chapter 3** details the control law design approach. It begins with the theory of Dynamic Inversion controller and develops an implementation on inner-loop attitude controller and outer-loop trajectory controller of rotorcraft. The second half of this chapter presents the techniques developed to measure the robustness and disturbance rejection of flight control system. Control parameter optimization is discussed in the end.

**Chapter 4** will introduce the path generation and guidance for autonomous approach. Focusing on path geometry parameterization and optimization, this chapter will study the homing trajectory and the associated profiles of velocity and acceleration.

**Chapter 5** presents the trajectory planning techniques for descent and landing. Specifics of landing path will be discussed and a timed path generation and optimization algorithm using predicted deck state will be demonstrated.

**Chapter 6** will describe the sensor fusion, signal processing and testing of integrated GNC solution.

### 1.6 Reference


Nanjing University of Aeronautics and Astronautics, Nanjing, PRC, 1999


Any worthwhile attempt at helicopter control law design must be based on a sound understanding of the system dynamics. This chapter gives an overview of the rotorcraft flight dynamics. In practice, engineers employ computer simulations, trim and linearization algorithms, in this study a commercial software FLIGHTLAB is used as the modeling tool. A useful insight into the system dynamics will be obtained from modal analysis and frequency characteristics.

2.1 Philosophy of Rotorcraft Modeling in FLIGHTLAB

Among the few rotorcraft modeling codes, FLIGHTLAB [1] is one that found wide application in research institutions as well as industry due to its high fidelity and comprehensive toolbox. FLIGHTLAB employs a module-assembly method for fast prototyping. Users can select necessary components of the desired rotorcraft, as well as specify the mass, geometry properties etc., meanwhile the method of modeling can be customized for each component. A conventional rotorcraft can be modeled using the following parts: the main rotor, the fuselage, the tail rotor, the vertical fin and the horizontal tailplane. The fuselage is a pivot of all components, the force and moment generated by each component are conducted to the fuselage for the evaluation of the flight dynamics equations. The well-known rigid body equations of aircraft motion can be found in standard texts such as Ref [2]. The equations of motion are shown below in Eqns. 2.1.1-2.1.9.

\[
\begin{align*}
\dot{u} &= \frac{X}{m} - g \cdot \sin \theta \cdot qw + rv \\
\dot{v} &= \frac{Y}{m} - g \cdot \cos \theta \sin \phi \cdot ru + pw \\
\dot{w} &= \frac{Z}{m} + g \cdot \cos \theta \cos \phi \cdot pv + qu \\
\dot{\phi} &= \frac{I_x L + I_{xz} N + I_{xx} \left( I_x - I_y + I_z \right) pq - \left( I_z^2 - I_x I_y + I_{xx}^2 \right) qr}{\left( I_x I_z - I_{xx}^2 \right)} \\
\dot{\theta} &= \frac{\left[ M - \left( I_x - I_z \right) r p - I_{xz} \left( p^2 - r^2 \right) \right] / l_y}{l_y} \\
\dot{\psi} &= \frac{I_x N + I_{xz} L - I_{xx} \left( I_x - I_y + I_z \right) qr - \left( I_x^2 - I_y I_z + I_{xx}^2 \right) pq}{\left( I_x I_z - I_{xx}^2 \right)}
\end{align*}
\]
\[ \dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \]  
(2.1.7)

\[ \dot{\theta} = q\cos\phi - r\sin\phi \]  
(2.1.8)

\[ \dot{\psi} = \frac{q\sin\phi + r\cos\phi}{\cos\theta} \]  
(2.1.9)

The main rotor is a predominant part of a rotorcraft model, it generates the lift, propulsion and primary control effects. For flight simulation, FLIGHTLAB utilizes rigid blade with flapping and lead-lag DOF to model rotor dynamics. The aerodynamic force and moment are calculated using well-proven blade element method. The airloads are evaluated on each blade segment based on a 2D airfoil flow assumption. A major feature of rotor wing airfoil flow is the unsteadiness and wide-range variation of AOA: unsteadiness is caused by the flapping motion of rotor blade and maneuvering flight; extraordinarily high AOA occurs around the reverse flow region. FLIGHTLAB handles AOA variation by using a wide range of airfoil data which covers AOA from -180° to 180° and Mach number from 0 to 1.0. In addition to static airfoil data, a “quasi-unsteady” correction is used to account for the effect of unsteadiness. FLIGHTLAB provides a gradation of inflow model for induced velocity calculation, the Peters-He finite state inflow model \[3\] is a reasonable compromise available in FLIGHTLAB between fidelity and computational cost.

The tail rotor is intended to provide anti-torque and directional control. FLIGHTLAB has different levels of fidelity for tail rotor. Similar to main rotor it can be modeled by blade element method, but often a simpler model such as actuator disk or Bailey’s model \[4\] are used for flight simulation. In Bailey’s technical report \[4\] an analytical expression of thrust and torque of a lifting rotor in forward flight was derived considering \( \delta_3 \) effect, tail fin blockage, tip loss, main rotor interference and fuselage interference.

The vertical fin and horizontal tailplane in FLIGHTLAB are modeled as lifting surfaces. For simplicity, tabulated aerodynamic data are used to calculate the force and moment. Usually the aerodynamic force and moment are functions of AOA of empennage in local flow. The calculations of AOA of empennage must account for the fuselage maneuvering speed, its angular motion, ambient wind speed, main rotor and fuselage interference.

The fuselage is the most massive body of a rotorcraft as well as the pivot connecting all other components. In FLIGHTLAB the aerodynamic force and moment are calculated by interpolating tabulated data through AOA and sideslip angle, the aerodynamic angles of a fuselage are determined by path speed, ambient wind speed and main rotor downwash.

The landing gear model calculates the reactive force and moment when the rotorcraft interacts with ground or flight deck. In FLIGHTLAB, landing gears consist of two parts – strut and tire. Both can be approximated by spring-damper systems. The equivalent stiffness and damping property are stored in look-up table as functions of LG compression travel and rate to account for non-linear properties.

The engine and drive train model are intended to account for the RPM variation. The power consumption of rotor system changes as the control input and flight condition change. The power variation coupled with engine and drive train dynamics leads to RPM fluctuation. Modern
rotorcraft has RPM governors designed to keep the rotor system working at a desirable speed, thus a constant RPM is assumed in a preliminary study. For research of flight dynamics, an ideal engine model is commonly used to supply as much power as the rotor system consumes instantaneously, thus a constant RPM is supposed to be true. For a detailed survey of RPM variation on overall control system performance, FLIGHTLAB provides engine and drive train models with different fidelities. A 1st order system model will be used to exam the impact of power plant on the controller performance.

The above components together with their properties and modeling methods are organized in a structure tree in FLIGHTLAB. Figure 2.1.1 shows the user’s interface of the model editor.

![Figure 2.1.1. Structure Tree of a Light Class Helicopter Model Built in FLIGHTLAB](image)

### 2.2 Rotorcraft Plant and Ship Motion Models

To validate the effectiveness and universal appliance of autonomous system, three FLIGHTLAB models were developed corresponding to light, medium and heavy class rotorcraft representative of the MQ-8C, UH-60 and CH-53 respectively. Their general parameters and operational
environment are summarized in Table 2.2.1 and 2.2.2.

<table>
<thead>
<tr>
<th>Main Rotor</th>
<th>Light Class</th>
<th>Medium Class</th>
<th>Heavy Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blades</td>
<td>4</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>Rotor RPM</td>
<td>395</td>
<td>258</td>
<td>180</td>
</tr>
<tr>
<td>Rotor radius (ft)</td>
<td>17.5</td>
<td>27</td>
<td>39</td>
</tr>
<tr>
<td>Mast forward tilt angle (deg)</td>
<td>5</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Nominal vehicle mass (lbm)</td>
<td>4000</td>
<td>17000</td>
<td>50000</td>
</tr>
<tr>
<td>Roll moment of inertia (slug \cdot ft^2)</td>
<td>2000</td>
<td>12000</td>
<td>80000</td>
</tr>
<tr>
<td>Pitch moment of inertia (slug \cdot ft^2)</td>
<td>3500</td>
<td>45000</td>
<td>250000</td>
</tr>
<tr>
<td>Yaw moment of inertia (slug \cdot ft^2)</td>
<td>3000</td>
<td>45000</td>
<td>360000</td>
</tr>
<tr>
<td>Tail Rotor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of blades</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Rotor RPM</td>
<td>2560</td>
<td>1191</td>
<td>700</td>
</tr>
<tr>
<td>Rotor radius (ft)</td>
<td>2.6</td>
<td>5.5</td>
<td>10</td>
</tr>
<tr>
<td>Engine Power (hp)</td>
<td>1x420</td>
<td>2x1890</td>
<td>2x3925</td>
</tr>
</tbody>
</table>

Table 2.2.2: Operational Environment for Rotorcraft

<table>
<thead>
<tr>
<th>Light Class</th>
<th>Medium Class</th>
<th>Heavy Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>Similar Operational Rotorcraft</td>
<td>MQ-8C</td>
<td>UH-60</td>
</tr>
<tr>
<td>Generic Model Similar to DDG-51. Motion Based on SCONC Small Deck Motion Model. Airwake Based on CFD Solutions of SFS2 [4]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Generic Model Similar to LHA-1. Motion based on SCONC. Airwake based on CFD Solutions from PSU [5]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

15
The bare-frame rotorcraft has four control inputs – longitudinal cyclic $\delta_{lon}$, lateral cyclic $\delta_{lat}$, collective $\delta_{col}$ and pedal $\delta_{ped}$ measured in percentage of full travel. The four control inputs are rescaled and mixed in a mechanical rigging system, the output of which are connected to swashplate actuators as illustrated by the I/O definition in Figure 2.2.1. Hence the non-linear rotorcraft model in FLIGHTLAB is a set of nonlinear equations resulting from flight dynamics, rotor dynamics, inflow dynamics and any other dynamic systems if specified in the model tree. Eqn 2.2.1 is the most general form of non-linear rotorcraft models:

$$f(\dot{x}, x, u, t) = 0$$  (2.2.1)

Where $x$ is the state vector including ridged fuselage state, rotor state and state of other high order dynamics, $u$ is the input vector including the four control inputs explained above and also in Figure 2.2.1.

![Figure 2.2.1. Input-Output Definition of a Rotorcraft Model](image)

The ship motion in FLIGHTLAB is modeled using prescribed data. Office of Naval Research and the Naval Surface Warfare Center released a package of standard deck motion time histories under the SCONE program for relevant research. The motion data included a generic surface combatant similar to DDG class ship and a generic helicopter-carrying assault similar to LHA class ship. The former is used as the carrier for light and medium class helicopter, while the later for heavy class helicopter. FLIGHTLAB’s ship motion module outputs the inertial position, velocity and acceleration of the deck center as well attitude angles and angular rates at each moment of simulation. Figure 2.2.2 illustrates a fragment of deck state time series representative for DDG class with medium amplitude of dynamic ship motion. Time plots exhibit a relatively steady forward motion along with random-like oscillation on all other axes. In the given example, the ship sails to the North direction, thus $X_N$ coordinate has a time series of ramp shape; in more common cases, both $X_N$ and $Y_E$ coordinates can have ramp property if the direction of sailing is not aligned with either of the inertial axes.
Figure 2.2.2 Ship Motion Data Representative of DDG Class
DDG-similar ship motion data include “low”, “medium” and “high” deck motion cases for both roll-dominated and heave-dominated conditions. The SCONE “low” and “medium” heave-dominated motion exhibit relatively large dynamic property. Table 2.2.3 summarizes the motion for the medium case, the +/-13 ft and 12 ft/sec maximum heave displacement and velocity are of specific interest. Those cases cast a significant challenge to the automatic landing problem.

Table 2.2.3. Ship Motion Characteristics

<table>
<thead>
<tr>
<th>DOF</th>
<th>Displacement</th>
<th>Rate (deg/sec or ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMS</td>
<td>Max/Min</td>
</tr>
<tr>
<td>Roll</td>
<td>0.94°</td>
<td>3.5°/-4.1°</td>
</tr>
<tr>
<td>Pitch</td>
<td>0.91°</td>
<td>3.7°/-3.4°</td>
</tr>
<tr>
<td>Yaw</td>
<td>0.21°</td>
<td>1.2°/-0.7°</td>
</tr>
<tr>
<td>Sway</td>
<td>2.1 ft</td>
<td>4.3/-13 ft</td>
</tr>
<tr>
<td>Heave</td>
<td>2.5 ft</td>
<td>25/-3.5 ft</td>
</tr>
</tbody>
</table>

Despite the random appearance of ship motion implied by the time sequence, hidden characteristics can be drawn from spectrum analysis of deck motion following a discrete Fourier transformation (DFT), Figure 2.2.3 illustrated the frequency component of heave motion from “medium” DDG-similar ship data. Spectrum analysis unveiled that the majority of signal energy is concentrated around 0.15 Hz corresponding to a period of 7 sec, which can be regarded as the characteristic period of ship motion.

![Figure 2.2.3. Spectrum Property of Deck’s Heave Motion](image)

Since the ship oscillation frequency may not be significantly lower than rotorcraft response bandwidth, predictive control strategy can be adopted to reduce the demand on maneuverability. A forecasting algorithm based on Minor Component Analysis (MCA) has been integrated into FLIGHTLAB to provide deck state prediction. With the knowledge of historical data MCA algorithm provides the predicted deck state within a horizon of 5 sec, Figure 2.2.4 is a comparison of the actual and predicted deck state. The major trend has been captured by MCA algorithm except some minor disagreement in peak amplitude, generally the accuracy and reliability of MCA prediction are adequate to support a predictive landing.
In order to account for the ship airwake’s affection on rotor aerodynamic loading, FLIGHTLAB uses table loop-ups of velocity raised by gust and turbulence at sampling points surrounding flight deck. CFD solutions of the SFS2 generic frigate shape were used together with the small deck ship motion. The first round of CFD solutions were generated at Penn State and reported by Oruc et al in Ref [6] for 20 knots, 0° WOD. Later a flow field data for 20 knots, 30° WOD was also incorporated into FLIGHTLAB to support simulation with cross-wind. The large deck airwake was based on CFD solutions of an LHA-class ship previously developed at Penn State by
Sezer-Uzol\cite{7}. Figure 2.2.5 demonstrate the turbulence and gust induced air velocity sensed by a reference point on rotor disk as the rotorcraft approaches and enters the zone of ship airwake.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure225.png}
\caption{The Turbulence and Gust-Raised Air Velocity}
\end{figure}

### 2.3 Trim and Linearization

For a thorough assessment of rotorcraft fight dynamics, FLIGHTLAB solves the non-linear differential equations using an implicit numerical integration algorithm\cite{8}. However, an insightful understanding of aircraft fight physics is still gained from a linear system approach, which will be later used to support control law design. The linear model is extracted from a nonlinear model around a trim flight through numerical perturbations. From the pilot’s point of view, a trimmed flight is a steady flight state with the interested parameters being kept constant. When this equilibrium is achieved the helicopter is said to be trimmed. Common trim conditions include:

**Hover:** Translational velocities, angular rates are arranged to be zero. To hold this trim condition the helicopter will have to adopt left roll attitude to balance the tail rotor thrust and often a nose-up pitch to accommodate the main rotor mast tilt and aft center of gravity if any.

**Straight and level flight:** All angular and linear accelerations are zero and the forward velocity is constant. At low speed pilots tend to trim for zero side-slip and accept a small roll attitude; As speed increases a “wing level” attitude is to be adopted with a necessary, but usually small side
slip angle.

**Coordinated turn:** Helicopter’s constant turn rate is purely provided by centripetal component of rotor thrust maintained through roll angle. This ensures passengers and pilots do not feel the side-to-side acceleration as the helicopter turns.

**Steady climb/descent:** Similar to level flight, the only difference is steady forward flight is maintained in a straight climbing or descending course.

The trimming algorithm is used to find the numerical values of state variables and control inputs so that the equations of motion corresponding to given steady flight are satisfied as well as certain constraints are kept. Mathematically this is amount to find roots of a set of simultaneous nonlinear equations under constraints. For fix-wing aircraft trimming, a direct application of Newton-Raphson iteration on the equations of motion has been proven to succeed \cite{9}. However, this strategy encounters difficulties upon defining a rotorcraft trim, even in the simplest case - a steady level flight, a constant equilibrium of force and moment cannot be achieved due to the periodic fluctuation of rotor airloads. FLIGHTLAB employs an averaged trim concept in which the force and moment used for the equilibrium test are their average values over a rotor revolution. Under this definition, the bare-frame model of three classes of rotorcraft are evaluated in trim test. The test scenario is a level flight at an altitude 300ft above sea level, forward speed sweeps from 0 to 125 ft/sec. Trim variables are the four control inputs $[\delta_{lon}, \delta_{lat}, \delta_{col}, \delta_{ped}]$ and two attitude angles $[\phi, \psi]$. The trim curves are plotted in Figure 2.3.1- 2.3.3.
FLIGHTLAB uses a numerical perturbation for linear model generation with a selectable level of model order. A low order linear model incorporates only nine rigid body states \( x = [\phi, \theta, \psi, u, v, w, p, q, r] \) and uses stability and control derivatives for studying helicopter flight dynamics. The exclusion of high-order dynamics is achieved by allowing the fast mode (including rotor dynamics, inflow dynamics and etc.) to reach their equilibrium during calculation of S&C derivatives, hence it is also called “quasi-static” model. The use of low order model is adequate for many applications ranging from eigen-analysis of flight modes, medium-bandwidth autopilots design to high-bandwidth tracking control law. However, the approximation treatment of fast modes can cause problems when the analysis is extended to high frequency region. In the literature, Chen\(^{[10]}\), Ellis\(^{[11]}\), Hall and Bryson\(^{[12]}\) revealed that use of the low order linear model would result in overly optimistic estimation of stability margin for high-gain feedback systems and thus an optimistic appraisal of true controller performance. In a control system design effort for the AH-56A, Heimbold\(^{[13]}\) concluded that using low-order linear model tends to give a deceptive impression of satisfactory closed-loop system stability due to the neglected regressive flapping mode. Towards this end, FLIGHTLAB generates high-order linear models which apart from rigid body states also incorporate rotor dynamics, inflow dynamics and structural mode. A low-order linear model can also be obtained through order reduction of the high-order model, residulization is one of the approaches elaborated in Raymond and Hansen’s report\(^{[14]}\). Partitioning the state vector of high-order model into two parts – rigid body states and fast mode states, the full-order linear model can be regrouped as:

\[
\begin{bmatrix}
X_b \\
X_{fm}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
X_b \\
X_{fm}
\end{bmatrix} +
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} u
\]

(2.3.1)

Where \( X_b \) is the rigid body state vector, \( X_{fm} \) is the fast mode state vector, using steady state assumption, let \( \dot{X}_{fm} = 0 \), the steady state of fast mode can be found in Eqn 2.3.2

\[
X_{fm} = -A_{22}^{-1}A_{21}X_b - A_{22}^{-1}B_2 u
\]

(2.3.2)

After substitution the reduced order model can be obtained:

\[
\dot{X}_b = A_b X_b + B_b u
\]

(2.3.3)

Where \( A_b = A_{11} - A_{12}A_{22}^{-1}A_{21} \), \( B_b = B_1 - A_{12}A_{22}^{-1}B_2 \)
2.4 Modal Analysis

Understanding the dynamics of the plant is a prerequisite for designing a control system. In aerospace application, modal analysis plays an essential part in understanding the plant dynamics. The eigenvalues and eigenvectors indicate the stability of different modes and their contribution to state variable responses. For modal analysis, both the high-order model and low-order model may be used. The high-order linear model comprises fast dynamics thus can provide insight into the frequency separation of fast modes (flap, lead-lag, inflow and structural) and slow modes (rigid body). However, lacks of direct expression of control input in the aircraft response make the high-order model less effective for flight S&C analysis. In the past decades of industry practice, low order models or the S&C derivatives models are commonly used to study the stability of rotorcraft flight and its response to control input. Although FLIGHTLAB can provide numerical values of S&C matrices, it’s useful sometimes to refer to the physical meaning of each term in the S&C matrices summarized in Table 2.4.1-Table 2.4.3.

Table 2.4.1 System Definition of S&C Model

<table>
<thead>
<tr>
<th>S&amp;C model / 9th order model:</th>
<th>$\dot{X}_b = A_9X_b + B_9u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>State vector:</td>
<td>$X_b = [\phi, \theta, \psi, u, v, w, p, q, r]$</td>
</tr>
<tr>
<td>Input vector:</td>
<td>$u = [\delta_{lon}, \delta_{lat}, \delta_{col}, \delta_{ped}]$</td>
</tr>
</tbody>
</table>

Table 2.4.2 Stability Derivatives in $A_9$

<table>
<thead>
<tr>
<th>Linearized Euler Kinematic Equations At trim</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{\phi}$</td>
</tr>
<tr>
<td>$Y_{\phi}$</td>
</tr>
<tr>
<td>$Z_{\phi}$</td>
</tr>
</tbody>
</table>

Table 2.4.3 Control Derivatives in $B_9$

| $M_{\phi}$ | $M_{\theta}$ | $M_{\psi}$ | $M_p$ | $M_q$ | $M_r$ |
| $N_{\phi}$ | $N_{\theta}$ | $N_{\psi}$ | $N_p$ | $N_q$ | $N_r$ |

---

23
The following example given by FLIGHTLAB is a low-order linear model of the medium class helicopter flying at 20 knots at 300 ft above the sea level. Numerical values of S&C derivatives are listed in Table 2.4.4 and Table 2.4.5.

Table 2.4.4 Numerical Example of Stability Matrix $A_9$

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<td>0.0000</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>1.0000</td>
<td>-0.0008</td>
<td>0.0213</td>
<td></td>
</tr>
<tr>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.9993</td>
<td>0.0382</td>
<td></td>
</tr>
<tr>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.0382</td>
<td>0.9995</td>
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</tr>
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<td>-32.1667</td>
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<td>-1.2961</td>
<td>1.7036</td>
<td>-0.1562</td>
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<td>32.1432</td>
<td>0.0262</td>
<td>0.0000</td>
<td>0.0241</td>
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<td>-1.4541</td>
<td>-1.3468</td>
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<td>3.0163</td>
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<td>0.0040</td>
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<td>-0.0567</td>
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</tr>
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<td>0.0000</td>
<td>-0.0055</td>
<td>0.0048</td>
<td>-0.0004</td>
<td>-0.2577</td>
<td>0.0191</td>
<td>-0.2570</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.4.5 Numerical Example of Control Matrix $B_9$

<p>| | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
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<td>0.0000</td>
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<td></td>
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<tr>
<td>0.0000</td>
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<td>0.0000</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>-0.1600</td>
<td>0.0042</td>
<td>0.0274</td>
<td>0.0004</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0150</td>
<td>0.0945</td>
<td>0.0370</td>
<td>-0.0486</td>
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<td></td>
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<tr>
<td>-0.0592</td>
<td>0.0160</td>
<td>-1.6963</td>
<td>0.0257</td>
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<tr>
<td>-0.0223</td>
<td>0.0917</td>
<td>0.0010</td>
<td>-0.0132</td>
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<tr>
<td>0.0317</td>
<td>0.0041</td>
<td>0.0012</td>
<td>0.0071</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>-0.0011</td>
<td>0.0049</td>
<td>0.0191</td>
<td>0.0165</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Theoretically, examination of the eigenvector can reveal which mode is predominately observed in each state. In practice, different physical scales and units obscure the evaluation of mode dominance; thus, a normalization transform was suggested in Ref [15] to remove this ambiguity. The scale factors for normalization are selected based on the nominal range of state variation. For example, during a perturbated motion, attitude angles may vary within 20 degrees, angular rates may vary within 30 deg/sec and 10 deg/sec on yaw axis, while linear velocities vary within 15 ft/sec and 5 ft/sec on vertical axis. The normalization can be expressed by a linear transformation: $X_p = T \cdot X_T$. Where $X_T$ is the normalized state vector, the transform matrix $T$ is diagonal as follow:

$$T = \text{diag} \begin{bmatrix} 20 & 20 & 20 & 20 \\ 57.3 & 57.3 & 57.3 & 57.3 \\ 15,15,5 & 20 & 20 \\ 57.3 & 57.3 & 57.3 \end{bmatrix}$$

The state-space model of non-dimensional state vectors can be obtained by:

$$X_T = A_T X_T + B_T u \quad (2.4.1)$$
Where \( A_T = T^{-1} \cdot A_B \cdot T \), \( B_T = T^{-1} \cdot B_B \). The eigenvalue and eigenvector of non-dimensional model are summarized in Table 2.4.6 and visualized on the complex plane in Figure 2.4.1, the highlighted numbers in Table 2.4.6 indicate the predominate mode.

<table>
<thead>
<tr>
<th>Roll Subsidence</th>
<th>Pitch Subsidence</th>
<th>Heave Subsidence</th>
<th>Yaw Subsidence</th>
<th>Lateral Phugoid</th>
<th>Longitudinal Phugoid</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda = -3.98 )</td>
<td>( \lambda = -1.00 )</td>
<td>( \lambda = -0.48 )</td>
<td>( \lambda = -0.08 )</td>
<td>( \lambda = -0.20 \pm 0.58i )</td>
<td>( \lambda = 0.13 \pm 0.39i )</td>
</tr>
<tr>
<td>( \phi )</td>
<td>(-0.2408)</td>
<td>0.0652</td>
<td>0.049</td>
<td>0.0421</td>
<td>0.0366\pm0.1707i</td>
</tr>
<tr>
<td>( \theta )</td>
<td>-0.0044</td>
<td>(-0.1893)</td>
<td>-0.0234</td>
<td>-0.0011</td>
<td>-0.1329\pm0.0638i</td>
</tr>
<tr>
<td>( u )</td>
<td>0.0065</td>
<td>-0.151</td>
<td>-0.0353</td>
<td>-0.0111</td>
<td>0.0216\pm0.1825i</td>
</tr>
<tr>
<td>( v )</td>
<td>0.0665</td>
<td>-0.0801</td>
<td>-0.0908</td>
<td>0.0176</td>
<td>0.4582\pm0.079i</td>
</tr>
<tr>
<td>( w )</td>
<td>-0.0628</td>
<td>-0.9287</td>
<td>(-0.9929)</td>
<td>0.0346</td>
<td>0.0167\pm0.2169i</td>
</tr>
<tr>
<td>( p )</td>
<td>0.9574</td>
<td>-0.055</td>
<td>-0.0234</td>
<td>-0.0044</td>
<td>-0.1079\pm0.0088i</td>
</tr>
<tr>
<td>( q )</td>
<td>0.0152</td>
<td>0.1921</td>
<td>0.0115</td>
<td>-0.0015</td>
<td>0.061\pm0.0569i</td>
</tr>
<tr>
<td>( r )</td>
<td>0.1256</td>
<td>-0.1137</td>
<td>-0.0127</td>
<td>(0.083)</td>
<td>0.1399\pm0.3963</td>
</tr>
</tbody>
</table>

Figure 2.4.1 Open-Loop Eigenvalues
Table 2.4.6 implies that the state variables are heavily coupled through common mode, significant coupling are between roll rate and yaw rate, lateral velocity and yaw rate, pitch rate and longitudinal velocity, yaw rate and heave rate. From the other hand, coupled response can also be observed by time domain evaluation with zero input but initial value of each state at a time. For instance, heave velocity, roll rate, pitch rate and yaw rate are initially set as 5 ft/sec, 5 deg/sec, 5 deg/sec, 5 deg/sec. The time evaluation of interested states are plotted in Figure 2.4.5-2.4.8, those figures exhibit an exact coherence with eigenstructure analysis.
Figure 2.4.8 Time Evaluation with Initial Value Arranged to Yaw Rate

Figure 2.4.6 and 2.4.7 seem to indicate a light coupling between roll rate and pitch rate. In fact, the above test only revealed the internal coupling of system states, another sort of coupling raised by input may also be significant. The input coupling can be demonstrated by setting zero initial states but deploying pulse signal to each input at a time. Figure 2.4.9-2.4.12 are the time response of states of interest when the four inputs are deployed to excite the low-order linear model. 5% of full control travel is applied on each input for a single test.
Figure 2.4.10 State Excitation of Lateral Pulse Input

Figure 2.4.11 State Excitation of Collective Pulse Input

Figure 2.4.12 State Excitation of Pedal Pulse Input
Scrutiny of the short-term response unveiled the fact that longitudinal control input has a noticeable excitation to the off-axis response, in contrast the lateral control results in a purer excitation of the on-axis variables. Collective control unavoidably gives rise to yaw response as a secondary effect, while the pedal input has less effect on heave motion but much on roll rate.

2.5 Frequency Analysis
Bare-frame rotorcraft is said to be of “rate response type”, which means with longitudinal, lateral, collective and pedal control input a rotorcraft will have steady pitch rate, roll rate, heave speed and yaw rate response respectively, the simulation results have partially proved this point with only impulsive excitation. More enriched information can be provided by looking at the frequency characteristics of transfer functions: \( w(s)/\delta_{\text{col}}(s), p(s)/\delta_{\text{lat}}(s), q(s)/\delta_{\text{lon}}(s), r(s)/\delta_{\text{ped}}(s) \) generated from 9\textsuperscript{th} order state-space model as well as the singular value when the linear model is treated as a MIMO plant. The SISO and MIMO frequency characteristics are illustrated in Figure 2.5.1 and 2.5.2. From the frequency plot we can see that rate response type of rotorcraft is well exhibited except in low frequency range where the longitudinal and lateral rate response are very small, this is explained by the “blow-back effect” which damps the long-term effectiveness of angular rate command. This indicates that it will be difficult to design a tracking control law working with low frequency command on pitch and roll axes.

![Figure 2.5.1 Magnitude Characteristics of SISO T.F.](image1)

![Figure 2.5.2 Singular Value Plot of MIMO Plant of 9\textsuperscript{th} Order Model](image2)
2.6 Summary
This chapter introduced the modeling approach of FLIGHTLAB and the natures of rotorcraft flight as well as the necessary tools for flight dynamics research such as trim, linearization and mode reduction. The mode and frequency analysis are conducted on linear model generated from low speed forward flight which represents the major flight condition during shipboard recovery of rotorcraft. Typical motion modes and response types were carefully scrutinized, from the preceding modal and frequency analysis, we can conclude that the helicopter is highly cross-coupled and has an unstable bare airframe. It is thus a very difficult vehicle to fly and in practice a flight control system is required to aid the pilot.

2.7 Reference
No.2, April, 1972, pp. 55-65


Chapter 3. Guidance and Control Laws

Contents

3.1 Design Objectives of Rotorcraft Automatic Control Systems
3.2 Theoretical Aspect of Dynamic Inversion
3.3 Application of DI to Inner-Loop ACAH Controller Design
3.4 Application of DI to Outer-Loop Trajectory Following Design
3.5 Stability Margin and Disturbance Rejection of Augmented Rotorcraft
3.6 Parameter Optimization of DI Control Laws
3.7 MIMO Robustness of DI Controller
3.8 Summary
3.9 Reference

The ultimate goal of this chapter is to equip the open-loop rotorcraft with a set of trajectory tracking control laws, which will be then used to guide the rotorcraft to a moving ship and land on the flight deck. The control law was constructed in an inner-loop & outer-loop structure, the ACAH controller of inner-loop serves as the basis for outer-loop trajectory tracking. Dynamic Inversion techniques will be used to formulate the inner and outer loop controller. Design trade-off between controller performance and robustness will be discussed at the end of this chapter.

3.1 Design Objectives of Rotorcraft Automatic Control Systems

Stability & Control Augmentation Systems (SCAS) is aimed to address the following design objectives:

1. Stability augmentation of unstable bare frame rotorcraft
2. Command tracking
3. Response decoupling
4. Response bandwidth tuning
5. Closed-loop robustness

Depending on different commanded variables, three response types are defined for a rotorcraft.

Rate Command (RC): a fixed stick position commands an appropriate constant rate that holds for more than four seconds. In this mode, the longitudinal control is used to command pitch rate, lateral control input is used to command roll rate, pedal control is used to command yaw rate.

Attitude Command Attitude Hold (ACAH): a fixed stick position commands an appropriate constant body attitude. In this mode, the longitudinal control is used to command pitch angle, lateral control input is used to command roll angle, pedal control is used to command yaw angle.

Translational rate command (TRC): a fixed stick position commands a steady translational rate. In this mode, the longitudinal control is used to command forward speed, lateral control input is used to command sideward speed, collective lever is used to command vertical rate.

From the first glance, only the last control mode TRC has a direct correlation with the goal of trajectory tracking, however a TRC controller is never designed along, in fact it is always designed as an outer-loop of ACAH controller. Thus, the control design effort breaks into two parts: For the
first part, an inner loop controller is designed corresponding to a ACAH mode, the second part will be dedicated to the trajectory tracking control law. The control law design must be able to satisfy the vehicle level requirements in Table 3.1.1 and controller level requirements in Table 3.1.2.

### Table 3.1.1. Vehicle Level Requirements

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Relaxed Objective</th>
<th>Primary Objective</th>
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</thead>
<tbody>
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<td></td>
<td></td>
</tr>
<tr>
<td>- On approach</td>
<td>10 ft</td>
<td>5 ft</td>
</tr>
<tr>
<td>- Station Keeping</td>
<td>5 ft</td>
<td>3 ft</td>
</tr>
<tr>
<td>- Touchdown</td>
<td>Level 3</td>
<td>Level 2</td>
</tr>
<tr>
<td>Relative velocities at</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Touchdown</td>
<td>12 ft</td>
<td>8 ft</td>
</tr>
<tr>
<td>- Sink Rate</td>
<td>6 ft</td>
<td>4 ft</td>
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<tr>
<td>- Horizontal Velocity</td>
<td>6 ft</td>
<td>4 ft</td>
</tr>
<tr>
<td>Control Margins</td>
<td>10% from full throw</td>
<td>20% from full throw</td>
</tr>
<tr>
<td>Engine Torque Margins</td>
<td>5% from powerplant limits</td>
<td>10% from powerplant limits</td>
</tr>
</tbody>
</table>

### Table 3.1.2. Controller Level Requirements

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Relaxed Objective</th>
<th>Primary Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stability Margins</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- 30° PM</td>
<td></td>
<td>45° PM</td>
</tr>
<tr>
<td>- 4 dB GM</td>
<td></td>
<td>6 dB GM</td>
</tr>
<tr>
<td>Attitude Angle Transients</td>
<td></td>
<td></td>
</tr>
<tr>
<td>During Mode Switch</td>
<td>Less than 10°</td>
<td>Less than 5°</td>
</tr>
<tr>
<td>Sensors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Actuators</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control system must be stable and fulfill performance requirements with actuator bandwidth limit - 5Hz, rate limit - 100%/sec</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In extensive research activity and experience at PSU [1][2][3][4], the Dynamic Inversion (DI) controller has been implemented on rotorcraft simulations for different purposes ranging from deck landing handling qualities to ship airwake/gust rejection, it shows great potential in achieving the above mentioned technical objectives. Therefore, the core stability and control augmentation will be constructed using the DI approach.

The DI approach has the advantage that the design process can essentially be automated if a comprehensive set of linear models of the aircraft are readily available. This is our case since the FLIGHTLAB allows for fast and accurate linearization of the model at any operating point. Using scripting tools in FLIGHTLAB the linear models can be programatically generated for a range of flight condition, and then they can be reduced and implemented in the controller loops. The
control design with the above convenience reduces to the selection of a few frequency parameters that govern command models (response to commands) and disturbance rejection.

In the course of this research, FLIGHTLAB scripts were developed to automate the DI control law design. These scripts were first developed for the medium class helicopter as a benchmark case, then re-used for light and heavy class models. Following major updates to the various simulation models of each class, the design scripts could be run again to reflect any changed properties in the rotorcraft. This rapid design process greatly expedited the control design study.

3.2 Theoretical Aspect of Dynamic Inversion

Dynamic Inversion (DI) has evolved into an important tool for flight control law design\[5\][6], great convenience can be obtained from this family of control laws in addressing control-response decoupling, response bandwidth tuning, command tracking etc. Using dynamic inversion, the natural dynamics of the plant can be subtracted off and augmented with desired dynamics to meet handing qualities requirements. The DI algorithm needs full-envelope plant model carried on-board, this became less an obstacle as modeling methods progress. With the modeling power of FLIGHTLAB, this technique is highly appropriate to achieve the demanding technical requirements.

DI controllers can be formulated as linear or nonlinear, a complete derivation of linear DI controller is presented in this section. A square linear system as the plant to be controlled is represented by state-space model in Eqn 3.2.1

\[
\frac{dx}{dt} = Ax + Bu, \quad A \in \mathbb{R}^{n \times n} \quad \text{and} \quad B \in \mathbb{R}^{n \times m} \tag{3.2.1-a}
\]

\[
y = Cx, \quad C \in \mathbb{R}^{m \times n} \tag{3.2.1-b}
\]

The derivation of DI controller requires the plant to be linearly square, which means the number of inputs is equal to the number of outputs or equivalently the input vector \(u\) and the output vector \(y\) have the same dimension. This is particularly true for rotorcraft systems, since it has four inputs and for any response type there can be defined four outputs accordingly. If there are redundant control inputs (like a compound rotorcraft), control dimension can be reduced to obtain a square system by allocating control effectiveness using techniques such as ganging, pseudo control or daisy chaining\[5\]. For a square system each output can be assigned to one control input to form a “command-response” pair, the outputs are known as the controlled variables (CVs). Differentiating the output Eqn 3.2.1-b once with respect to time results in:

\[
y = Cx = C(Ax + Bu) \tag{3.2.2}
\]

The concept of feedback linearization\[7\] developed by Hunt and Su \[8\] and Jacubczuk and Respondek\[9\] states that with any reference function \(y_r\) if we select Eqn 3.2.3 as the input and substitute it into Eqn 3.2.2,

\[
u = (CB)^{-1}(y_r - CAx) \tag{3.2.3}
\]

then the output of augmented system in Figure 3.2.1 will have an interesting property indicated in Eqn 3.2.4
\[
\frac{dy}{dt} = C \frac{dx}{dt} = CB(CB)^{-1} y_r = \dot{y}_r
\]  \hspace{1cm} (3.2.4)

It is clear that for the closed-loop system, the derivatives of the controlled variables are identical to those of the commanded reference signal, in other words the feedback linearization loop of DI effectively converts the output response into a system of completely decoupled integrators. From a modal point of view, a mode is a linear combination of state variables whose dynamics can be evaluated independently. Therefore, each of the outputs \( y \) of the augmented system corresponds to a mode with zero eigenvalue. In addition to the modes associated with the output, the remaining \( n - m \) modes of the augmented system represent the zero dynamics. Unlike the integrator modes, the stability of zero dynamics is not definite and need to be addressed later.

![Figure 3.2.1. Augmented System with Linearization Loop](image)

Note that the linearization loop requires that matrix \( CB \) is invertible, since control matrix \( B \) is always of full rank, the non-singularity of \( CB \) can be guaranteed if the derivatives of the output vector are directly affected by the input, this is an important rule for selecting CVs. Also, the linearization loop requires full state feedback through the \( x \) vector in the equations, thus the order of linear plant model should be selected in such a way that all states are available for feedback (e.g. a 6-DOF rigid body model of an aircraft).

Merely applying the linearization loop renders the output tracking accurate in special case where the initial conditions of the output \( y(0) \) match the initial command \( y_r(0) \) and when there is neither external disturbance nor modeling error. To correct the tracking for non-zero IC and disturbance, additional compensation denoted by \( G \) is added as illustrated in Figure 3.2.2.
The sum of the reference signal and the compensation is known as the pseudo-command, along with the state feedback they form the total input of the augmented system:

$$u = (CB)^{-1} \left( \dot{y}_r + G \frac{\text{pseudo-command}}{CA} \right)$$

Substituting Eqn 3.2.5 into Eqn 3.2.2 we found that the tracking error of output $e_y \equiv y_r - y$ is subject to the differential equation in 3.2.6

$$\frac{de_y}{dt} = -G$$

This is the error dynamics of the output governed by the compensator $G$. Eqn 3.2.6 implies that a purposeful design of compensator $G$ can be used to shape the error dynamics, so that any error arising from external disturbances or modelling error can decay rapidly. However, it must be kept in mind that the linear model used in DI is approximate, and thus overly aggressive (expressed as high gain) compensation must also be avoided, this field will be discussed in detail in the gain optimization section. Note that the linearization loop has decoupled the output into individual integrators, thus the compensator can be designed independently for each output variable with SISO techniques. The most widely used compensators are of PID class, which can easily specify the error dynamics as 2nd order or 3rd order linear systems. The PID compensator may incorporate the differential, proportional, integral or even double integral of the tracking error $e_y$, depending on how the controlled variable is defined. Equivalent formulation of the compensator can be derived as either proportional-integral-derivative (PID) or proportional-integral-double integral (PII). For example, in the pitch attitude DI controller design, $q, \theta, \theta_f$ will be incorporated in the error compensator, if the controlled variable is defined as pitch attitude $\theta$ the compensator is then a PID formulation, however if pitch rate $q$ is selected to be controlled variable then the compensator is a PII formulation. In fact, the two formulations are exactly equivalent. To be consistent in form with rotorcraft DI control law, the PII formulations are presented below.
A 3rd order error dynamics can be obtained by applying the following proportional plus integral plus double integrator compensator:

\[ G = K_p e_y + K_i \int e_y dt + K_u \int \int e_y dt^2 \]  

(3.2.7)

Thereby the error dynamics is governed by:

\[ \dot{e}_y + K_p e_y + K_i \int e_y dt + K_u \int \int e_y dt^2 = 0 \]  

(3.2.8)

Differentiate the above equation twice:

\[ \ddot{e}_y + K_p \dot{e}_y + K_i \dot{e}_y + K_u e_y = 0 \]  

(3.2.9)

Using a Laplace transform the characteristic equation of error dynamics can be derived as:

\[ s^3 e_y + K_p s^2 e_y + K_i s e_y + K_u e_y = \Delta I_c(s) \]  

(3.2.10)

The right-hand side of Eqn 3.2.10 represents the perturbation due to initial conditions. The poles of the desired error dynamics can be solved using the following factorized form of the characteristic equation:

\[ \left( s^2 + 2\zeta \omega_n s + \omega_n^2 \right) \left( s + p \right) = 0 \]  

(3.2.11)

The frequency parameter \( \omega_n \), damping parameter \( \xi \), and integrator parameter \( p \) can be selected to provide stable error dynamics with reasonable time and frequency domain properties. In practice, these can be set to be in a similar frequency range as the ideal response models (described below). If the compensator gains need to be tuned, the following guideline provides the direction of tuning: increasing \( \omega_n \) also increases the fastness of error decaying; increasing \( \xi \) reduces the overshoot and oscillation of error, far location of \( -p \) on the LHP tends to reduce the steady state value of the error. Comparing Eqn 3.2.10 and 3.2.11, PID gains are found to be calculated by Eqn 3.2.12.

\[ \begin{cases} 
K_p = 2\xi \omega_n + p \\
K_i = 2\xi \omega_n p + \omega_n^2 \xi \\
K_u = \omega_n^2 \xi 
\end{cases} \]  

(3.2.12)

Similarly, for 2nd error dynamics, the PID compensator is expressed by Eqn 3.2.13

\[ G = K_p e_y + K_i \int e_y dt \]  

(3.2.13)

For desired error dynamics of the following form:

\[ (s + p_1)(s + p_2) = 0 \]  

(3.2.14)

The PID gains are calculated by Eqn 3.2.15:

\[ \begin{cases} 
K_p = p_1 + p_2 \\
K_i = p_1 p_2 
\end{cases} \]  

(3.2.15)

Or the error dynamics can be designed with complex poles for given natural frequency and
damping ratio:

\[
\begin{align*}
K_p &= 2\zeta \omega_n \\
K_i &= \omega_n^2
\end{align*}
\]  

(3.2.16)

Higher PID gains obviously lead to faster error decay rate and thus higher close-loop frequency, better tracking performance and disturbance rejection. However, necessary stability margins must always be retained. The techniques for analyzing the stability margin will be described in section 3.4.

Until now, only the error dynamics has been discussed. The linearization loop and the output error compensator removed the inherent dynamics of the plant and rebuilt preferred stability to the output modes, this can be well demonstrated in the mode evolution diagram in Figure 3.2.3.

![Modal Evolution Diagram](image)

Figure 3.2.3. Modal Evolution Diagram

The stability of the modes associated with the CVs are affected by the addition of the compensators, while the stability of the modes of zero dynamics remained the same as they were shaped by the linearization loop. Thus, the stability of the zero dynamics can be checked by calculating the poles of closed-loop system in Figure 3.2.1. The system equation of zero dynamics is:

\[ \dot{x} = \left[I - B(CB)^{-1}C\right]Ax = A_zx \]  

(3.2.17)

The stability matrix \( A_z \) obviously has \( m \) zero eigenvalues corresponding to the \( m \) integrator mode, the remaining \( n - m \) eigenvalues must have negative real part to maintain stable zero dynamics. Appearance of unstable zero dynamics may be the consequence of many reasons, the unstable mode in original plant can remain unstable after loop-linearization, sometimes an improper selection of controlled variable produces a linearization loop destabilizing the already stable mode in original plant. Thus, an afterward check of zero dynamics is definitely necessary. If any instability arises in zero dynamics, a reselection of controlled variables is required. A general
guideline is to analyze which mode is unstable and try to introduce the mode's representative state variable into CVs. The underlying philosophy is to blend the unstable mode into integrator modes and then they can be stabilized by adding compensator. Of course, determining the amount of extra state variable is very tricky, in many ways it depends on the designer's knowledge of plant dynamics and requires a lot of trial-and-error.

The pseudo-command of DI controller needs \( \dot{y}_r \) for the feedforward loop and other differential-integral relations of reference signal to feed the compensator. Those parameters are usually generated by a command model. For example, in a simple pitch angle controller, pitch rate \( q \) is selected to be the CV (this may seem non-intuitive, but if one can enforce a tracking of \( q \) one obviously can control \( \theta \) since they are in an approximate integral relation), the feedforward loop requires \( \dot{q}_r \) or its approximate equivalent \( \dot{\theta}_r \), the PID compensator will incorporate the integral and double integral of \( q_r \) which are approximated by \( \theta_r \) and \( \theta_{rf} \), those signals can be extracted from command model in Figure 3.2.4,

![2nd Order Command Model Diagram](image)

**Figure 3.2.4. Reference Signals Extracted from Command Model**

The command model in this example is a standard 2nd order low-pass filter formulated in Eqn 3.2.18, another frequently used type of command model is 1st order low-pass filter in Eqn 3.2.19

\[
\frac{y_r}{y_{cmd}} = \frac{\omega_m^2}{s^2 + 2\zeta_m\omega_m s + \omega_m^2} \tag{3.2.18}
\]

\[
\frac{y_r}{y_{cmd}} = \frac{\omega_m}{s + \omega_m} \tag{3.2.19}
\]

![1st Order Command Model Diagram](image)

**Figure 3.2.5. Implementation of 1st Order Command Model**
In some literature the command model is also called a command filter, because it filters out the noisy and fast changing component of command signal so that the plant can adequately track. It is also called ideal response model, as the controller enforces the plant to track the model output i.e. behave like the ideal response model. The order of command model thus matches the response type, e.g. yaw and heave axes are usually designed to exhibit rate response like a 1st order system, so are their command models; while pitch and roll attitude are usually designed to behave like a 2nd order system, thus 2nd order command models are used. With the command model appeared in the DI controller, the DI controller is said to be formulated in an Explicit Model Following (EMF) form.

3.3 Application of DI for Inner Loop ACAH controller design

The theory introduced in the previous section will be implemented to design an ACAH controller based on medium class rotorcraft model. Under this control mode, the augmented rotorcraft tracks pitch and roll attitude command, the yaw and heave axes are designed to track rate command. In the second half of this section practical aspects of controller implementation will be discussed where corrections will be made to eliminate some idealized design assumptions.

3.3.1 8th Order DI Control Law

From the feedback availability point of view, low-order linear models with S&C derivatives are more suited for control law synthesis. The benchmark design model is the 9th order linear model of medium class helicopter trimmed at 20 knots at 300 ft above sea level, the state variable \( \psi \) is related only to the Euler rate equation which is approximately \( \dot{\psi} = r \); in other words, yaw angle is not coupled with other state variables, hence can be truncated to yield an 8th order linear model.

Table 3.3.1 Definition of 8th order linear model

<table>
<thead>
<tr>
<th>System equation: ( x = A_8x + B_8u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output equation: ( y = C_8x )</td>
</tr>
<tr>
<td>State vector: ( x = [\phi, \theta, u, v, w, p, q, r]^T )</td>
</tr>
<tr>
<td>Input vector: ( u = [\delta_{lon}, \delta_{lat}, \delta_{col}, \delta_{ped}]^T )</td>
</tr>
</tbody>
</table>

Stability matrix \( A_8 \) and control matrix \( B_8 \) have been shown in Table 2.4.4 and Table 2.4.5, output matrix \( C_8 \) is to be determined in the following discussion.

Step 1. Select Controlled Variables

Simply selecting \( \phi \) and \( \theta \) as the controlled variables for pitch and roll attitude controller leads to a failure of DI design, since the \( C_8B_8 \) matrix is found to be singular. The physical reason is the lack of input effectiveness in \( \phi \) and \( \theta \) derivatives. A better selection of the controlled variables or the output vector is:

\[
y = \begin{bmatrix} w & p & q & r \end{bmatrix}^T = C_8 \cdot x
\] (3.3.1)
Where,
\[
\begin{align*}
C_8(1,:) &= [0,0,0,0,1,0,0,0] \\
C_8(2,:) &= [0,0,0,0,0,1,0,0] \\
C_8(3,:) &= [0,0,0,0,0,0,1,0] \\
C_8(4,:) &= [0,0,0,0,0,0,0,1]
\end{align*}
\] (3.3.2)

The advantage of this set of CVs is firstly their derivatives are directly affected by input, namely collective input gives rise to thrust increment which turns into \( \dot{w} \), same control effects happen between lateral input and \( \dot{p} \), longitudinal input and \( \dot{q} \), pedal input and \( \dot{r} \); the second reason is \( p \) and \( q \) have a simple integral relation with \( \phi \) and \( \theta \), at low attitude angles they are: \( \int p \cdot dt = \phi \) and \( \int q \cdot dt = \theta \), thus the controllers can be easily extended to regulate attitude angles, in Step 2 we will see the regulation of attitude angles is through compensator design.

Upon the closure of linearization loop with above defined \( C_8B_B \) and \( (C_8A_B)^{-1} \), the derivatives of the CVs at any instant are supposed to be equal to their desired values injected at the feedback pivot. Figure 3.3.1 is a schematic of the closure of linearization loop, subscript “des” denotes the desired derivatives of each CV.

![Figure 3.3.1 Closure of Linearization Loop Designed Using 8th Model](image)

**Step 2. PID compensator design**

Individual PID compensators are designed to the tracking error of each CV. For vertical speed a PI compensator is used:
\[
G_w = K_{p_w} e_w + K_{i_w} \int e_w dt
\] (3.3.3)

The proportional and integral gains are selected so that the error dynamics decays elegantly. For vertical speed \( w \), according to handling quality regulation for piloted rotorcraft an initial guess of target error dynamics has two real pole both at -0.5. The corresponding PID gains are:
\[
K_{p_w} = 1.0, \quad K_{i_w} = 0.25
\] (3.3.4)

Another PI compensator is set for tracking error of yaw rate \( r \),
\[ G_r = K_p e_r + K_i \int e_r dt \]  
(3.3.5)

A fair guess of target error dynamics has two real poles both at -2.5, hence the PID gains are:

\[ K_p = 5.0, \quad K_i = 6.25 \]  
(3.3.6)

For roll rate error dynamics, a PII compensator is used:

\[ G_p = K_p p e_p + K_i \int e_p dt + K_{ii} \int \int e_p dt^2 \]  
(3.3.7)

A fair guess of target error dynamics has a frequency parameter 2.5 rad/sec, damping parameter 1.0, and an integral pole at -0.5. Hence the compensator gains are:

\[ K_p = 8.75, \quad K_i = 5.5, \quad K_{ii} = 3.125 \]  
(3.3.8)

For pitch rate error dynamics, the same PII compensator is used with the same gain setting as an initial guess:

\[ G_q = K_p q e_q + K_i \int e_q dt + K_{ii} \int \int e_q dt^2 \]  
(3.3.9)

\[ K_p = 8.75, \quad K_i = 5.5, \quad K_{ii} = 3.125 \]  
(3.3.10)

**Step 3. Implementation of DI controller**

A complete DI controller has three loops: linearization loop, compensator loop and feedforward loop. The totality of the last two is the "pseudo-command". As opposed to the linearization loop which is simply a state feedback, the pseudo-command of each control axis may be tricky and need certain transformation for implementation. This part will be dedicated to the construction of pseudo-command.

The vertical heave axis and yaw axis have straightforward implementation illustrated in Figure 3.3.2-3.3.3.

![Diagram of Heave axis Pseudo-command](image)

Figure. 3.3.2. Implementation of Heave axis Pseudo-command
The implementation of pitch and roll axis needs some sort of transformation. For ease of explanation, small attitude assumption is made. The integral relation between $p$ and $\phi$ can be propagated to their error:

$$
e_p = p_r - p$$

$$
\int e_p dt = \int p_r dt - \int p dt = \phi_r - \phi = e_{\phi}
$$

$$
\int \int e_p dt^2 = \int (\phi_r - \phi) dt = \int e_{\phi} dt
$$

Thus, the roll rate error compensator can be generalized to a roll-axis compensator by replacing the integral-double integral loop of $p$ with a proportional-integral loop of $\phi$.

$$G_{roll} = K_p e_p + K_p e_{\phi} + K_i \int e_{\phi} dt$$

(3.3.12)

Where the PID gains are exactly inherited from Eqn. 3.3.8

$$K_p = K_{i_p} = 5.5, \quad K_i = K_{i_p} = 3.125$$

(3.3.13)

Similarly, a pitch-axis pseudo-command can be derived in terms of $q$ and $\theta$.

$$G_{pitch} = K_p e_q + K_p e_{\theta} + K_i \int e_{\theta} dt$$

(3.3.14)

Where the compensator gains are inherited from Eqn. 3.3.10

$$K_p = K_{i_q} = 5.5, \quad K_p = K_{i_\theta} = 3.125$$

(3.3.15)
Figure 3.3.5. Implementation of pitch-axis pseudo-command

Assembling the linearization loop and the pseudo-command generators, one can have the implementation schematic of the overall DI controller in Figure 3.3.6.

Figure 3.3.6. Assembly of Linear Model and 8th Order Controller

The linear model together with DI controller can be readily tested in Matlab/Simulink, Figure 3.3.7 demonstrates the tracking performance of pitch angle, roll angle, yaw rate and heave speed when square wave command were simultaneously injected into four control axes.
3.3.2 4th Order DI Control Law

The 8th order DI controller needs the feedback of outer-loop variable $u$ and $v$, this is undesirable from the view point of variable grouping. On the other hand, the measurement and
estimation of \( u \) and \( v \) are difficult and less reliable, therefore designers tend to rule out their feedback in the Inner-loop to improve the safety of critical FCS.

Careful scrutiny of the \( C_8A_8 \) matrix in the 8th order design reveals that the first four columns have many zero elements. In fact, with the output matrix defined in Eqn 3.3.1, each element of the first four columns of \( C_8A_8 \) has clear physical significance, they are the non-dimensional derivatives of net normal force and 3-axes moment w.r.t attitude angles and forward/sideward speed as in Table 3.3.3

Table 3.3.2 Numerical Value of \( C_8A_8 \) matrix

\[
\begin{array}{cccccc}
1.2293 & -0.6842 & -0.1937 & 0.0023 & -0.4345 & 3.0163 & 33.2979 & 2.1805 \\
0.0000 & 0.0000 & 0.0161 & -0.0212 & 0.0098 & -3.9167 & 0.0187 & 0.0494 \\
0.0000 & 0.0000 & 0.0021 & 0.0040 & -0.0016 & -0.0567 & -1.0099 & 0.0697 \\
0.0000 & 0.0000 & -0.0055 & 0.0048 & -0.0004 & -0.2577 & 0.0191 & -0.2570 \\
\end{array}
\]

Table 3.3.3 Physical Explanation of first four columns of \( C_8A_8 \) matrix

\[
\begin{array}{cccc}
Z_\phi & Z_\theta & Z_u & Z_v \\
L_\phi & L_\theta & L_u & L_v \\
M_\phi & M_\theta & M_u & M_v \\
N_\phi & N_\theta & N_u & N_v \\
\end{array}
\]

Around a trim point, \( Z_\phi = -g \cdot \sin\phi_{trim} \), \( Z_\theta = -g \cdot \sin\theta_{trim} \), since trim attitude angles have small magnitude, \( Z_\phi \) and \( Z_\theta \) should be small. The nature of rotorcraft flight establishes that the 3-axes moment are insensitive to attitude changes, numerically FLIGHTLAB gave out moment to attitude derivatives of order \( 10^{-7} \). Thus, the contributions of \( \phi \) and \( \theta \) in the linearization loop are readily negligible. However, ignoring \( Z_\phi \) and \( Z_\theta \) removes from the plant model the contribution to heave speed of attitude change, this will cause some disturbance on heave axis control particularly during maneuvering flight unless some sort of compensation is added.

The third and fourth columns are derivatives w.r.t to forward/sideward speed, they have moderate magnitude and are very essential parameters unique to rotorcraft caused by “blow back effect”, and other aerodynamic phenomena. Considering that the forward/sideward speed are developed by phugoid mode in a relatively long term, while inner-loop ACAH controller works in a short term; the speed variation and thus the contributions of speed variation in the linearization loop should be small.

Based on above analysis, it’s reasonable to truncate state variables \( \phi, \theta, u, v \) in the linearization loop design, in other words, the linearization loop can be designed using a 4th order model described below.
Table 3.3.4 Definition of 4\(^{th}\) order linear model

<table>
<thead>
<tr>
<th>State vector:</th>
<th>( x = [w, p, q, r] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input vector:</td>
<td>( u = [\delta_{lon}, \delta_{lat}, \delta_{col}, \delta_{ped}]^T )</td>
</tr>
<tr>
<td>System equation:</td>
<td>( \dot{x} = A_4x + B_4u )</td>
</tr>
<tr>
<td>Output equation:</td>
<td>( y = C_4x )</td>
</tr>
</tbody>
</table>

The stability and control matrix \( A_4 \) and \( B_4 \) are obtained from 8\(^{th}\) order model through truncation, output matrix is then: \( C_4 = \text{diag}[1,1,1,1] \). Using the same compensator loop, the 4\(^{th}\) order DI controller can be implemented on linear model as illustrated in Figure 3.3.8.

Figure 3.3.8. Implementation of 4\(^{th}\) order DI Inner-Loop ACAH Controller on Linear Model

The implementation in Figure 3.3.8 was tested in Matlab/Simulink with the 8\(^{th}\) order linear model, each control axis was tested independently to clarify the success and defect of single control axis. Figure 3.3.9-3.3.12 are the time history plot of representative flight parameters for test cases on roll, pitch, yaw and heave axes respectively. The command on axes other than the axis being tested are kept zero to indicate suppression of inter-axis coupling.
Remarkable observations on roll-axis test include the following:

1. The inter-axes coupling was weak in this case.
2. A degraded tracking performance on roll-axis was demonstrated.
3. Roll attitude produced a significant lateral speed, this is obviously resulted by the natural dynamic relation between roll angle and lateral translation.
Remarkable observations on pitch-axis test include the following
1. The good tracking performance on pitch-axis was preserved
2. A moderate cross-coupling shows up on roll axis, an apparent cross-coupling on heave axis
3. A significant forward speed was produced due to thrust tilting
Figure 3.3.11 Linear Model Test of 4th Order DI Controller

Test No 3. Square wave command on yaw rate, commands on other axes are all zero

Remarkable observations on pitch-axis test include the following
1. Some loss of tracking performance of yaw rate
2. Moderate disturbance to roll-axis regulation
3. Significant sideward speed was produced
Remarkable observations on pitch-axis test include the following

1. The on-axis tracking performance is satisfactory
2. Disturbances on other axes are adequately small
The degraded performance of 4th order DI controller is not surprising: ignoring the state feedback of $u, v$ in the linearization loop can explain many of the unwanted phenomenon, also provide a hint towards improving the controller. The roll angle has a worse tracking performance than pitch angle, this can be explained by the large difference between $L_v = -0.0212$ and $M_u = 0.0021$, in other words, the “blow back effect” on lateral axis is ten times more intensive than on longitudinal axis, this is largely attributed to the tail rotor. The lateral “blow back effect” strongly hinders the development and hold of roll angle. In 8th order DI controller this was not a problem since lateral speed $v$ was fed back in the linearization loop to counteract the “blow back effect”. We can also add a $v$ feedback to the 4th order DI controller, but this is not an elegant solution given that lateral speed $v$ is not easy to measure nor estimate. In practice, designers tend to increase roll compensator gains, particularly increase the integrator gain, because the tracking error behaves like a steady state error. Even a high-gain compensator will not completely enforce a long-term roll angle hold, however this is not a warrior in real flight due to the following fact:

- In near to hover flight, roll angle is needed when pilots intend to do a lateral translation. In this case, the nominal lateral speed will not exceed 35 knots or equivalently 58.8 ft/sec, holding 30° roll angle for 5 sec can raise that much lateral speed, i.e. no need for a long-term roll angle hold. Secondly, the lateral cyclic and tail rotor actuators are likely to be saturated. i.e. no technical adequacy for a long-term roll angle keeping.

- In forward flight, roll angle is needed when pilot wants to make a turn. In this case, turn-coordination technique is used to produce yaw rate of such an amount that the side slip is totally eliminated. A rough design of a turn coordinator is:

$$r = \frac{g \cdot \phi}{V_{forward}} \quad (3.3.14)$$

The turn-coordinator can be easily implemented in 4th order DI controller in Figure 3.3.13, linear simulation of roll angle command is illustrated in Figure 3.3.14.

Figure 3.3.13. 4th Order DI Controller with Turn-Coordinator
The introduction of turn-coordinator successfully removes the lateral speed hence the roll.
angle regained a nice tracking to its command. Also, the pure 4\textsuperscript{th} order DI controller lost some tracking performance of yaw rate. This is because: in forward flight, any yaw motion exposes rotorcraft side to the coming flow, the emerging lateral speed hinders a further yaw motion and also produces a disturbance to roll angle regulation. 8\textsuperscript{th} order DI controller however does not have this issue since \(v\) feedback in the linearization loop provided effective compensation. Towards this end, a practical amendment for 4\textsuperscript{th} order DI controller is still the turn-coordination, Figure 3.3.14 demonstrated that the tracking performance of both yaw rate and roll angle are largely improved at the same time.

### 3.3.3 Correction of Euler Rate Non-linearity

The pseudo-command of \(r, w\) (denoted as \(\dot{r}_{\text{des}}, \dot{w}_{\text{des}}\)) are the desired value of 1\textsuperscript{st} derivative of yaw rate and vertical speed which is supposed to guarantee their command tracking. They are directly allocated to \(\dot{r}\) and \(\dot{w}\) at the feedback pivot of linearization loop without losing any effectiveness even at high attitude angles. However, the pitch and roll axes control become complicated when the rotorcraft flies with a large attitude. The following derivation provides an explanation and a solution.

**Euler rate equation:**

\[
\begin{align*}
\dot{\phi} &= p + (q \sin \phi + r \cos \phi) \tan \theta \\
\dot{\theta} &= q \cos \phi - r \sin \phi 
\end{align*}
\]  

(3.3.15)

Differentiating Eqn 3.3.15 once w.r.t time yields:

\[
\begin{align*}
\dot{\phi} &= \dot{p} + (\dot{q} \sin \phi + q \phi \cos \phi + r \cos \phi - r \dot{\phi} \sin \phi) \tan \theta + (q \sin \phi + r \cos \phi) \frac{\dot{\theta}}{\cos^2 \theta} \\
\dot{\theta} &= q \cos \phi - \dot{q} \phi \tan \phi - r \sin \phi - r \dot{\phi} \cos \phi 
\end{align*}
\]  

(3.3.16)

Assume in moderate maneuvering flight \(p, q, r, \dot{\phi}, \dot{\theta}\) are small, neglecting the high-order small quantities in Eqn 3.3.16:

\[
\begin{align*}
\dot{\phi} &= \dot{p} + (\dot{q} \sin \phi + r \cos \phi) \tan \theta \\
\dot{\theta} &= \dot{q} \cos \phi - r \sin \phi 
\end{align*}
\]  

(3.3.17)

Under a small attitude assumption, Eqn 3.3.17 can be furtherly simplified

\[
\begin{align*}
\dot{\phi} &= \dot{p} \\
\dot{\theta} &= \dot{q}
\end{align*}
\]  

(3.3.18)

The pseudo-command of \(\theta\) (denoted as \(\ddot{\theta}_{\text{des}}\)) is the desired value of 2\textsuperscript{nd} derivative of pitch angle, which is required to guarantee roll angle command tracking. Under the small attitude assumption, Eqn 3.3.18 has been used for a simple design where \(\ddot{\theta}_{\text{des}}\) was completely allocated to \(\dot{q}\) of rotorcraft at the feedback pivot of linearization loop. At large attitude angles, however, Eqn 3.3.17 suggests

\[
\dot{q}_{\text{des}} = \frac{\ddot{\theta}_{\text{des}}}{\cos \phi} + r \tan \phi 
\]  

(3.3.19)
However, $\dot{r}$ is not measurable, but it is known to be equal to $\dot{r}_{\text{des}}$ as a result of perfect dynamic inversion, hence Eqn 3.3.20 is used for implementation

$$\dot{q}_{\text{des}} = \frac{\dot{\theta}_{\text{des}}}{\cos \phi} + \dot{r}_{\text{des}} \tan \phi$$  \hspace{1cm} (3.3.20)

For roll angle, similarly, Eqn 3.3.17 suggests to use:

$$\dot{p}_{\text{des}} = \dot{\phi}_{\text{des}} - \left( \dot{q} \sin \phi + \dot{r} \cos \phi \right) \tan \theta$$ \hspace{1cm} (3.3.21)

For the same reason that $\dot{q}$ is not measurable but is known to be equal to its desired value, after substitution Eqn 3.3.21 became:

$$\dot{p}_{\text{des}} = \dot{\phi}_{\text{des}} - \dot{\theta}_{\text{des}} \tan \phi \tan \theta + \dot{r}_{\text{des}} \cos \phi \tan \theta$$ \hspace{1cm} (3.3.22)
The ultimate control objective of rotorcraft in low-speed is to regulate the translational motion on longitudinal, lateral axes in Helicopter Heading Frame (HHF) and vertical axes. However, the linear model (8th order or 4th order) generated by FLIGHTLAB or other code uses the body-axis velocity \([u, v, w]\) in the state vector. It is thus desirable to replace \([u, v, w]\) with \([V_{l\text{on}}, V_{l\text{at}}, V_D]\) in the control law synthesis model. For the 4th order model, focus is only casted on replacing the heave speed \(w\) with vertical speed \(V_D\).

The original 4th order model is represented by the following state-space equations:

\[
\frac{d}{dt} \begin{bmatrix} w \\ p \\ q \\ r \end{bmatrix} = A_4 \begin{bmatrix} w \\ p \\ q \\ r \end{bmatrix} + B_4 \begin{bmatrix} \delta_{\text{lon}} \\ \delta_{\text{lat}} \\ \delta_{\text{col}} \\ \delta_{\text{ped}} \end{bmatrix}
\]  

(3.3.23)

Where,

\[
A_4 = \begin{bmatrix} Z_w & Z_p & Z_q + U_{\text{trim}} & Z_r \\ L_w & L_p & L_q & L_r \\ M_w & M_p & M_q & M_r \\ N_w & N_p & N_q & N_r \end{bmatrix}, \quad B_4 = \begin{bmatrix} Z_{\delta\text{lon}} & Z_{\delta\text{lat}} & Z_{\delta\text{col}} & Z_{\delta\text{ped}} \\ L_{\delta\text{lon}} & L_{\delta\text{lat}} & L_{\delta\text{col}} & L_{\delta\text{ped}} \\ M_{\delta\text{lon}} & M_{\delta\text{lat}} & M_{\delta\text{col}} & M_{\delta\text{ped}} \\ N_{\delta\text{lon}} & N_{\delta\text{lat}} & N_{\delta\text{col}} & N_{\delta\text{ped}} \end{bmatrix}
\]

In particular, the linearized equation for \(w\) is

\[
\dot{w} = Z_w w + Z_p p + (Z_q + U_{\text{trim}})q + Z_r r + Z_{\delta\text{lon}} \delta_{\text{lon}} + Z_{\delta\text{lat}} \delta_{\text{lat}} + Z_{\delta\text{col}} \delta_{\text{col}} + Z_{\delta\text{ped}} \delta_{\text{ped}}
\]

(3.3.24)

In order to obtain the equation for \(V_D\), Eqn (3.3.25) is substituted in to Eqn (3.3.24) and yields Eqn (3.3.26), the substitution only uses Eqn (3.2.25) to replace \(w\) as a kinematic parameter, the stability derivatives w.r.t \(w\) are all preserved for \(V_D\) as long as the vehicle attitude is not too large, for the same reason heave-axis force \(Z\) is used as an approximation to vertical axis force.
\[ w = V_D + V_{lon.trim} \cdot \theta \]  

\[ \dot{V_D} = Z_w \cdot (V_D + V_{lon.trim} \cdot \theta) + Z_p p + Z_q q + Z_r r + Z_{slon} \delta_{lon} + Z_{slat} \delta_{lat} + Z_{scol} \delta_{col} + Z_{speed} \delta_{ped} \]  

\[ (3.3.26) \]

The state-space model with state vector \([V_D, p, q, r]^T\) in matrix form is then Eqn (3.3.26)

\[
\begin{bmatrix}
V_D \\
p \\
q \\
r
\end{bmatrix}
= A' 
\begin{bmatrix}
P \\
q \\
r
\end{bmatrix}
+ B_4
\begin{bmatrix}
\delta_{lon} \\
\delta_{lat} \\
\delta_{col} \\
\delta_{ped}
\end{bmatrix}
+ Z_w \cdot V_{lon.trim}
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\theta
\end{bmatrix}
\]

\[ (3.3.26) \]

Where,

\[
A' =
\begin{bmatrix}
Z_w & Z_p & Z_q & Z_r \\
L_w & L_p & L_q & L_r \\
M_w & M_p & M_q & M_r \\
N_w & N_p & N_q & N_r
\end{bmatrix}
\]

The disturbance to vertical speed rate \( \dot{V_D} \) due to pitch angle \( \theta \) is now expressed as an external input, compensation to this disturbance is simple in dynamic inversion controller: the measurement of the disturbance is injected with a negative sign to the pseudo-command and inversion loop Eqn (3.3.27). The vertical axis compensation depends on the availability of longitudinal speed signal and pitch angle signal. The estimation of those signals will be elaborated in Chapter 6.

\[
\begin{bmatrix}
\delta_{lon} \\
\delta_{lat} \\
\delta_{col} \\
\delta_{ped}
\end{bmatrix}
= C_4 B_4^{-1}
\begin{bmatrix}
\dot{V_D} \\
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix}_{pseudo-cmd}
- C_4 A' \begin{bmatrix}
V_D \\
p \\
q \\
r
\end{bmatrix}
- C_4 A_\theta \theta
\]

\[ (3.3.27) \]

Figure 3.3.18 Implementation of Vertical Axis Non-Linear Compensation
3.3.5 Non-Linear Test of ACAH control law in FLIGHTLAB

The designed above ACAH control law together with the preliminary gains setting has been tested in FLIGHTLAB on the medium class rotorcraft model. For a fair comparison, the flight condition for non-linear testing is the same as used for trim, linearization and linear model testing, i.e. 20 knot forward speed at 300 ft altitude. Individual tests have been carried out on each axis. Figure 3.3.19-3.3.23 present the time evaluation of the major parameters during each test.

Figure 3.3.19: Nonlinear Test. No1. Doublet Command of 5° on roll angle, commands of other axes are zero
Figure 3.3.20: Nonlinear Test No 2, Doublet Command of 5° on pitch angle, commands of all other axes are zero.
Figure 3.3.21: Nonlinear Test No.3 Doublet Command of 5 deg/sec on yaw rate, commands of other axes are zero
Figure 3.3.22: Nonlinear Test No.4 Doublet Command of 5 ft/sec on vertical speed, commands of all axes are zero.
3.4 Application of DI to Outer Loop Trajectory Following Design

Longitudinal and lateral speed of rotorcraft can be regulated given the inner-loop ACAH controller. The translation of rotorcraft is raised by thrust tilting as illustrated in Figure 3.4.1-a and 3.4.1-b. Thrust tilting inevitably causes fuselage attitude change, thus the pitch and roll angle of fuselage can be used to indicate the amount of longitudinal and lateral component of rotor thrust
tilting, the linear equations in Eqn 3.4.1 represent the quantitative relation between translational rate and vehicle attitude.

\[
\begin{align*}
\dot{V}_{\text{lon}} &= X_{\text{vol}} \cdot V_{\text{lon}} + X_{\text{vol}} \cdot V_{\text{lat}} - g \cdot \theta \\
\dot{V}_{\text{lat}} &= Y_{\text{vol}} \cdot V_{\text{lon}} + Y_{\text{vol}} \cdot V_{\text{lat}} + g \cdot \phi
\end{align*}
\tag{3.4.1}
\]

In the above dynamic system, attitude angles play the role of system input. In an Equivalent formulation, rotorcraft deploy attitude angle variation to generate translational acceleration. In Eqn 3.4.1 the thrust vectoring terms are predominate, hence a simplified version of governing equations is usually used instead.

\[
\begin{align*}
\dot{V}_{\text{lon}} &= -g \cdot \theta \\
\dot{V}_{\text{lat}} &= g \cdot \phi
\end{align*}
\tag{3.4.2}
\]

Since the longitudinal and lateral motion are decoupled in Eqn. 3.4.2, controllers are individually designed for each channel with the same approach, resulting in an application of DI controller to a SISO system. Without losing generality, the longitudinal outer-loop controller is elaborated.

Selecting \( V_{\text{lon}} \) as the controlled variable yields \( C = 1, B = -g \) and \((CB)^{-1} = -1/g\), the \( A \) matrix of this particular system is a scalar zero, thus the linearization loop vanishes due to a zero feedback gain: \( CA = 0 \), i.e. simply scaling the input by \((CB)^{-1} = -1/g\) the original system can become a unit integrator. Next step is to add a compensator to the integrator for desirable error decaying dynamics. The longitudinal speed error and its integral, double integral are used to drive the compensator in Eqn 3.4.3

\[
G_{V_{\text{lon}}} = K_{\text{p}} e_{\text{lon}} + K_{\text{i}} \int e_{\text{lon}} dt + K_{\text{d}} \frac{d}{dt} \int \frac{e_{\text{lon}}}{dt}^2
\tag{3.4.3}
\]

This compensator shapes the error dynamics into a 3rd order system whose characteristic equation has been derived in Eqn 3.2.11. Given the structure (frequency parameter, damping parameter and integral pole) of the roots of the characteristic equation, one can immediately determine the compensator gains. From the frequency separation point of view, the inner-loop error dynamics must be times faster (a factor of 5 is a fair guess justified by experience) than that of the outer-loop. Hence the relations in Table 3.4.1 are used to provide an initial guess of outer-loop
compensator gains. The integral pole is also 5 times slower than the natural frequency as a ball-
park value.

### Table 3.4.1 Outer-Loop Gain Setting

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Parameter of Error Dynamics:</td>
<td>(\omega_{ol} = 0.2\omega_{il})</td>
</tr>
<tr>
<td>Damping Parameter of Error Dynamics:</td>
<td>(\xi_{ol} = 1.0)</td>
</tr>
<tr>
<td>Integral Pole of Error Dynamics:</td>
<td>(p_{ol} = -0.2\omega_{ol})</td>
</tr>
</tbody>
</table>

Using the integral relation between speed error \(e_{vlon}\) and position error \(e_{xlon}\) in Eqn 3.4.4,

\[
\int e_{vlon} \, dt = e_{xlon}
\]

The longitudinal speed compensator becomes a generic longitudinal compensator whose gains are exactly inherited from the preceding design.

\[G_{lon} = K_{P_{vlon}} e_{v,lon} + K_{P_{xlon}} e_{xlon} + K_{I_{xlon}} \int e_{xlon} \, dt\] (3.4.5)

The generic longitudinal compensator renders the outer loop controller no longer a speed regulator, but rather regulate the speed and position simultaneously. Summing the acceleration feedforward and compensator contribution, the pitch angle needed to achieve the desired longitudinal speed and position is quantified in Eqn 3.4.6. The required pitch angle by outer-loop DI controller is then fed into the inner-loop as the pitch attitude command.

\[
\theta_{cmd} = \frac{1}{g} \left[ K_{P_{vlon}} (V_{lon,des} - V_{lon}) + K_{P_{xlon}} (X_{lon,des} - X_{lon}) + K_{I_{xlon}} \int (X_{lon,des} - X_{lon}) \, dt + A_{lon,des} \right]
\]

(3.4.6)

Similarly, the lateral outer-loop DI controller can be designed,

\[
\phi_{cmd} = \frac{1}{g} \left[ K_{P_{vlon}} (V_{lat,des} - V_{lat}) + K_{P_{xlat}} (X_{lat,des} - X_{lat}) + K_{I_{xlat}} \int (X_{lat,des} - X_{lat}) \, dt + A_{lat,des} \right]
\]

(3.4.7)

Joint working of inner-loop and outer-loop controller are demonstrated in Figure 3.4.2-3.4.3

---

**Figure 3.4.2.** Joint working of inner-loop and outer-loop DI controllers of Longitudinal Channel
In practice, the desired acceleration, speed and destination may not be aligned with either axis, in this case the total desired motion parameters must be resolved and allocated to each control axis as demonstrated in Figure 3.4.4.

During the trajectory tracking, the yaw axis works in a mode called “low speed coordination” which simply line-up the rotorcraft heading to the horizontal speed vector by giving a proper command to the inner-loop yaw rate controller, this command is generated by an outer-loop yaw angle DI controller illustrated in Figure 3.4.5. Since the yaw rate and yaw angle have an integral relation, linearization loop is not needed but only the pseudo-command in Eqn 3.4.8:

$$ r_{cmd} = \frac{d\psi_{des}}{dt} + K_{\psi} (\psi_{des} - \psi) + K_{v} \int (\psi_{des} - \psi) dt \tag{3.4.8} $$

To avoid spike from taking derivative of desired yaw angle, the outer-loop of yaw axis applies a command model to extract derivative. The command model introduces an additional phase delay, if the yaw angle response behaves sluggish, a larger frequency parameter of command model can be used. Upon designing the PI compensator gains, the principle of frequency separation between inner and outer loops should be followed, a rough guess is still the outer-loop error dynamics.
should be 5 times slower than the inner-loop error dynamics.

Figure 3.4.5. Yaw Angle Controller for “Low Speed Coordination”

The outer-loop controller of vertical axis is assigned to enforce tracking of vertical speed and position, a DI design of such an outer-loop is illustrated in Figure 3.4.6, similar to the case of yaw rate, the linearization loop vanishes leaving only a pseudo-command which is originated from inertial vertical displacement but fed into body heave rate command in Eqn 3.4.9

\[
\begin{align*}
    w_{cmd} &= V_{z,des} + K_p (Z_{des} - Z) + K_i \int (Z_{des} - Z) dt \\
    &\quad (3.4.9)
\end{align*}
\]

The desired motion parameters \( A_{z,des}, V_{z,des}, Z_{des} \) are computed outside by the guidance algorithm. The immediate availability of acceleration leads to some change in the inner-loop, namely a command model is no-longer needed to extract the commanded vertical acceleration, but instead the desired vertical acceleration can be directly injected. This modification was clearly shown in Figure 3.4.7. The removal of command model also removes the phase delay; hence a tighter vertical axis tracking can be expected.

Figure 3.4.6. Outer-Loop Vertical Axis DI Controller
The trajectory tracking control law has been tested in FLIGHTLAB on the non-linear model. To demonstrate the tracking performance of four control axes, a helical path is used as the reference trajectory parameterized by time. The commanded position, velocity and acceleration in the inertial frame are functions of time formulated in Eqn 3.4.10-3.4.18.

\[
X_{cmd} = R_{spiral} \sin \left( \frac{V_{forward}}{R_{spiral}} \cdot (t - t_{ini}) \right) \\
Y_{cmd} = R_{spiral} \cos \left( \frac{V_{forward}}{R_{spiral}} \cdot (t - t_{ini}) \right) \\
H_{cmd} = V_{cimb} \left( (t - t_{ini}) + EXP\left[ -(t - t_{ini}) \right] - 1 \right) \\
V_{cmd} = V_{forward} \cos \left( \frac{V_{forward}}{R_{spiral}} \cdot (t - t_{ini}) \right) \\
V_{cmd} = -V_{forward} \sin \left( \frac{V_{forward}}{R_{spiral}} \cdot (t - t_{ini}) \right) \\
V_{cmd} = V_{cimb} \left( 1 - EXP\left[ -(t - t_{ini}) \right] \right) \\
A_{cmd} = -V_{forward}^{2} \sin \left( \frac{V_{forward}}{R_{spiral}} \cdot (t - t_{ini}) \right) \\
A_{cmd} = V_{forward}^{2} \cos \left( \frac{V_{forward}}{R_{spiral}} \cdot (t - t_{ini}) \right)
\]
The rotorcraft model was trimmed at $V_{\text{forward}} = 20$ ft/sec, at an initial height of 300 ft. Level flight was maintained until the onset of helical climbing at instant $t_{\text{ini}} = 5$ sec. The spiral radius was selected as $R_{\text{spiral}} = 67$ ft, so that a roll angle about 30° is needed to make the turn. Figure 3.4.8 illustrates the spatial flight trajectory and the time history of major flight parameters.
Despite the complex geometry and moderate maneuvers, the steady tracking errors of position and velocity have been confined within 3 ft and 2 ft/sec respectively. Non-linear simulation of trajectory following justifies the inner-outer loop architecture as well as the preliminary gains setting.

3.5 Stability Margin and Disturbance Rejection of Augmented Rotorcraft
Stability Margin and Disturbance Rejection are two very important metrics of augmented rotorcraft, the former is used to quantify the robustness of close-loop system, the latter is used to
quantify the tracking performance. The trade-off between those two metrics will inspire a design methodology of controller gains.

3.5.1 Stability Margin

Stability Margin (SM) is a SISO concept used to evaluate the Nyquist stability criterion which is derived in standard control theory textbooks such as Franklin’s [10]. SM is a measure of how far away the closed loop system is approaching instability (Characterized by $-1 + 0j$). For an already stable feedback design, there are traditionally two ways of destroying the stability, increasing feedback gain or introducing phase delay. Correspondingly, two types of SM are defined, Gain Margin (GM) and Phase Margin (PM). On Nyquist plot, gain and phase margin are the smallest phase or gain shift that will render the curve crossing $-1 + 0j$. While stability margin analysis is more rigorously analyzed with Nyquist diagrams, it is more easily observed using Bode plots of the broken loop transfer function. Phase Margin is seen as the difference between phase and $-180^\circ$ at the gain crossover frequency (where magnitude passes through 0 dB) and gain margin as the difference between magnitude and 0 dB where phase passes through $-180^\circ$. Geometrically they are illustrated on the Nyquist and Bode diagram in Figure 3.5.1, on each of the plot the frequency characteristics is tested on the open-loop transfer function $G_{ol}(s)$.

![Figure 3.5.1 Geometrical Explanation of GM and PM on Nyquist and Bode Plot]({link})

Certain level of stability margin is needed for a linear control design so that the controller can resist the nonlinearity, unmodeled dynamics and external disturbance when implemented on a real plant. For a flight control system, a widely used SM regulation is provided by MIL-STD-8785, which recommends 6dB of gain margin and 45° of phase margin. However, there are examples of digital rotorcraft flight control systems known to violate 45° phase margin specifications in certain axes, as discussed by Einthoven [11]. Thus, the SM for rotorcraft control system is enacted to be $6\text{dB} + 45^\circ$ as the priority objective, $3\text{dB} + 30^\circ$ as a compromised plan.

Technically, the Nyquist criterion applies to SISO feedback systems, but in practice the stability margin method can also be applied to MIMO aircraft flight controls systems by breaking the feedback of a single axis at a time, then observing the frequency response from broken loop input to broken loop output. The loop break is normally applied right before the actuator, as illustrated for this application in Figure 3.5.2. The unmodeled dynamics are accounted for by using high-order linear plant model with 46 states and actuator model (1st order system with time-
constant 0.02 sec). Dynamic modules are assembled with the inner-loop and outer-loop flight control systems in Matlab/Simulink environment. The “linearize” command of Matlab can be used to extract the SISO transfer function of a complex Simulink model once the in-port and out-port are specified.

Figure 3.5.2 Block Diagram of Simulink Model Used to Generate T.F. for SM analysis

Figure 3.2.3-3.2.4 are typical Bode plot for the medium class helicopter model linearized at 20 knots, 300 ft. Those diagram shows the frequency characteristics of T.F. with loop break at longitudinal and lateral cyclic actuator respectively.

Figure 3.5.3. Stability Margin of Longitudinal axis  Figure 3.5.4. Stability Margin at Lateral axis

The broken loop T.F of the helicopter tend to be “conditionally stable systems” with at least two phase-crossovers. This is anticipated, since the open-loop helicopter dynamics are normally unstable, so reducing gains also tends to destabilize the system. Thus, there are typically both a positive GM indicating maximum allowable loop gain and a negative GM indicating minimum allowable loop gain. Other observations taken from the diagrams include:

1. The gain crossover frequency is around 3 rad/sec, which is a frequency within the normal range of flight control compensation.
2. The magnitude dB curve is fairly linear around this frequency (from 1 to 10 rad/sec) with slope about -20 dB/decade, and the phase is hovering just below -90°. These are the properties of an integrator. This is a characteristic of the DI feedback linearization which converts the plant to integrators on each axis. This ensures desired dynamic properties when the axis loop is closed.

3. There are two phase crossings, one below and one above the gain crossover frequency, these define upper and lower bounds on loop gain. This indicates that the helicopter dynamics with the DI compensator represent a conditionally stable system. Generally, GM and PM tend to degrade when higher control gains are used. Thus, the SM requirements set an upper bound of control gain design.

3.5.2 Disturbance Rejection

The Disturbance Rejection Bandwidth (DRB) parameter was developed to provide a measurable metric for assessing the capability of a control system to hold trim in the presence of external disturbances. Like stability margins, the DRB metric is defined in the frequency domain, where one observes the closed loop frequency response from a disturbance applied to a sensor and the resulting sensor measurement as in Figure 3.5.5. The resulting transfer function of sensor disturbance to sensor output in Eqn 3.5.1 is known as the Sensitivity transfer function described in any classical control text[10].

\[
\frac{y(s)}{y_d(s)} = \frac{1}{1 + G(s)H(s)}
\]  

(3.5.1)

Figure 3.5.5 Disturbance Rejection Analysis of a Generic System

Since the sensor disturbance is direct feedthrough, one can expect that the high frequency gain for the system is 1 (0 dB), while the low frequency gain is 0 (-\infty dB) as the “hold” function of the control system should reject the disturbance in steady-state. The frequency where the curve passes through -3 dB represents the Disturbance Rejection Bandwidth (DRB) or the maximum frequency at which the controller effectively rejects disturbances. The curve will typically overshoot 0 dB, and the peak of the curve is known as Disturbance Rejection Peak (DRP). Excessive peak can indicate lack of damping in the closed loop system.

A simple example of scalar system can be used to demonstrate the relation between DI controller gains and the disturbance rejection property. Assuming a linear plant model and perfect inversion in Figure 3.5.6, we can analyze the output response due to a disturbance applied at the sensor as shown in the figure below. The DR analysis considers the frequency response of \(y(s)/y_d(s)\).
Using simple transforming techniques, the disturbance dynamics can be derived as a 3rd order transfer function of sensor disturbance to sensor output:

\[
\frac{y}{y_d}(s) = \frac{s^3}{s^3 + K_p s^2 + K_i s + K_d} = \frac{s^3}{(s^2 + 2\zeta \omega_n s + \omega_n^2)(s + p)}
\]  

(3.5.2)

The frequency response of 3.5.2 is plotted in Figure 3.5.7 with the frequency, damping and pole parameters selected as: \(\omega_n = 0.5, \zeta = 0.5, p = 0.1\)

Eqn 3.5.2 also implies that the disturbance response follows the error dynamics specified by the gains or alternatively by the equivalent frequency, damping, and real pole parameters. Which means, increasing frequency parameter \(\omega_n\) yields larger DRB; more damping \(\zeta\) yields less DRP; far location of \(-p\) on LHP renders a steeper dropping of magnitude curves at low frequency.
Although the above example is conducted on a scalar system, the pattern of analysis can be reached out to a complex plant with multi-loop controllers. An obvious difference is one can no-longer derive the disturbance transfer function by hand, but rather use Matlab/Simulink. Since the controller of interest is designed to follow a commanded trajectory, the disturbance rejection properties of the position hold are analyzed following the diagram in Figure 3.5.8.

![Diagram](image)

**Figure 3.5.8 Disturbance Analysis of Position Hold System**

Focusing on the longitudinal axis, details of the longitudinal position control are shown in Figure 3.5.9. For clarity the figure also includes the notional disturbance input used for longitudinal position shown as the red dashed signal (this is notional as it is used in DRB analysis only and not part of the actual control law). Unlike the simple scalar system, the position hold disturbance rejection properties will be affected by numerous components of the control law: 1) The inversion, 2) The inner loop PID compensation, 3) The inner loop command filters, 4) The outer loop PID compensation.

![Diagram](image)

**Figure 3.5.9 Details of Disturbance Analysis of Longitudinal Position Control**

With the preliminary design of control gains, the disturbance rejection frequency characteristics is plotted in Figure 3.5.10. Note that the DRB is 0.21 rad/sec which is a little above the minimum recommended for ADS-33 design guidelines [15] (no DRB requirements in ADS-33 have been officially defined). The peak 3.8 dB is slightly higher than the maximum recommended peak of 3dB. The same analysis has been performed on lateral position controller as demonstrated.
The fitting of disturbance rejection properties is achieved through the tuning of multiple compensator gains in the inner-loop and outer loop and also the command model parameters. The correlation between compensator gains and DR properties drawn from the simple scalar example can also be propagated to complex system, generally using high frequency parameters of inner or outer loop compensator gains can yield better DRB or equivalently a tighter command tracking. Using DR properties as a criterion for handling quality design has been studied in Ref 4, while in Ref 12 the DR properties were optimized towards a better stabilization of rotorcraft hovering over flight deck under airwake disturbances. Due to the complexity of rotorcraft trajectory control law, the multiplicity of tunable parameters posts a challenging design issue.

3.6 Parameter Optimization of DI control Laws
The previous section roughly implied that gain tuning of a helicopter flight control system can be framed as a designed trade-off between high gain designs for disturbance rejection versus lower gain design for more robust stability of the close loop system. In Ref 13, this idea has been
incorporated into a gain optimizing tool such as CONDUIT\textsuperscript{[14]}. The following sections present a more rigorous gain study conducted for the longitudinal and lateral position hold properties of the control law.

### 3.6.1 Assumptions Made to Reduce the Multiplicity of Tunable Gains

Although there are design tools that can handle the tuning of multiple control gains with given criteria. In practice designers do not have to treat each control gain as an individual design DOF, as some of the design parameters may have narrow range of variation, others may be associated by certain design constraints. The inner-outer loop structure and the specifics of DI control low established several principles to simplify the tuning procedure, such as a critical damping ratio for error dynamics, a factor of “5” for integrator pole location and a factor of “5” for frequency separation between loops. Applying the above principles, the only tunable parameters are the frequency parameter of roll angle error dynamics $\omega_{e\phi}$ for the lateral channel and the pitch angle error dynamics $\omega_{e\theta}$ for the longitudinal channel. Once those two factors are specified, all the others can be determined accordingly. Using the techniques developed in the previous section, SM vs. DRB analysis can be perform against a range of control parameter setting by following the steps below:

1. Select the inner loop natural frequency parameter for the attitude control (roll or pitch). Use this value for natural frequency of both the command filter and the error dynamics.
2. Set the damping ratio for the inner loop gain selection to $\zeta = 1.0$, and the pole to $p = 0.2\omega_e$. Then set inner loop gains based on formula in 3.2.12.
3. Set the outer loop natural frequency parameter for the position control (longitudinal or lateral) to be 1/5 that of the inner loop $\omega_{eолуч} = 0.2\omega_{e,ит}$. Use this value for natural frequency of the error dynamics.
4. Set the damping ratio for the outer loop gain selection to $\zeta_{pecific} = 1.0$, and the pole to $p_{олуч} = 0.2\omega_{eолуч}$. Then set outer loop gains based on formula in 3.2.12.
5. Implement in linearization diagram and extract stability margins and DRB/DRP.
6. Repeat for range of inner loop frequency from 1.5 to 4.0 rad/sec.

### 3.6.2 Parametric Study of Longitudinal and Lateral Trajectory Control Law

Using the procedures developed in the previous section, parameter sweeping study has been conducted on longitudinal and lateral axis controller independently. The control parameters of other axes keep constant when those of the axis of interest are undergoing variation. The on-axis Bode plots for SM&DR analysis with sweeping frequency parameters are juxtaposed in Figure 3.6.1-3.6.2 for lateral axis, and in Figure 3.6.3-3.6.4 for longitudinal axis.
Figure 3.6.1. Bode Plot for SM analysis of Lateral Axis Broken Loop with Frequency Parameters Sweeping from 1.5 rad/sec to 4.0 rad/sec

Figure 3.6.2. Bode Plot for DR analysis of Lateral Axis Broken Loop with Frequency Parameters Sweeping from 1.5 rad/sec to 4.0 rad/sec
Figure 3.6.3. Bode Plot for SM analysis of Longitudinal Axis Broken Loop with Frequency Parameter Sweeping from 1.5 rad/sec to 4.0 rad/sec

Figure 3.6.4. Bode Plot for DR analysis of Longitudinal Axis Broken Loop with Frequency Parameter Sweeping from 1.5 rad/sec to 4.0 rad/sec
The metrics to be monitored are the DRB/DRP of longitudinal/lateral position controller, and also the GM&PM of the four actuator loops. Table 3.6.1 and 3.6.2 summarized the metrics variation vs. sweeping of $\omega_{e\theta}$ and $\omega_{e\phi}$ from 1.5 to 4.0 rad/sec. As the frequency parameter is increased, the DRB goes up in the primary axis of interest. At the same time, we find that the stability margins tend to go down and the DRP goes up, indicating less damping/stability of the closed loop system. Physically this can be explained as the augmented rigid dynamics starts to couple with the low frequency mode of rotor dynamics, which in extreme case can turn into air-resonance. Due to the decoupling achieved by the DI control law, it can be observed that the stability and disturbance rejection of the other control axes are basically unaffected by the gain variations. This is one of the main advantages of the DI control architecture.

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{e\phi}$ (rad/s)</td>
<td>$\omega_{e\theta}$ (rad/s)</td>
<td>DRB (rad/s)</td>
<td>DRP (db)</td>
<td>PM (deg)</td>
<td>GM (db)</td>
<td>PM (deg)</td>
</tr>
<tr>
<td>2.5</td>
<td>1.5</td>
<td>0.17</td>
<td>3.8</td>
<td>2.9</td>
<td>38.4</td>
<td>+24.3</td>
</tr>
<tr>
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<td>2.0</td>
<td>0.17</td>
<td>3.7</td>
<td>3.3</td>
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<td>+24.1</td>
</tr>
<tr>
<td>2.5</td>
<td>2.5</td>
<td>0.17</td>
<td>3.7</td>
<td>0.21</td>
<td>3.9</td>
<td>38.5</td>
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<tr>
<td>2.5</td>
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<td>0.17</td>
<td>3.7</td>
<td>0.25</td>
<td>4.4</td>
<td>38.3</td>
</tr>
<tr>
<td>2.5</td>
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<td>0.17</td>
<td>3.7</td>
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<td>38.0</td>
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<tr>
<td>2.5</td>
<td>4.0</td>
<td>0.17</td>
<td>3.6</td>
<td>0.33</td>
<td>4.8</td>
<td>37.4</td>
</tr>
</tbody>
</table>

Table 3.6.1 Parametric Study by Sweeping the Longitudinal Frequency Parameter
### Table 3.6.2 Parametric Study by Sweeping the Lateral Frequency Parameter

<table>
<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_{e\phi} ) (rad/s)</td>
<td>( \omega_{e\theta} ) (rad/s)</td>
<td>DRB (rad/s)</td>
<td>DRP (dB)</td>
<td>DRB (rad/s)</td>
<td>DRP (dB)</td>
<td>PM (deg)</td>
</tr>
<tr>
<td>1.5</td>
<td>2.5</td>
<td>0.11</td>
<td>3.2</td>
<td>0.21</td>
<td>3.9</td>
<td>35.5</td>
</tr>
<tr>
<td>2.0</td>
<td>2.5</td>
<td>0.14</td>
<td>3.5</td>
<td>0.21</td>
<td>3.9</td>
<td>38.3</td>
</tr>
<tr>
<td>2.5</td>
<td>2.5</td>
<td>0.17</td>
<td>3.7</td>
<td>0.21</td>
<td>3.9</td>
<td>38.5</td>
</tr>
<tr>
<td>3.0</td>
<td>2.5</td>
<td>0.20</td>
<td>4.0</td>
<td>0.21</td>
<td>3.9</td>
<td>35.8</td>
</tr>
<tr>
<td>3.5</td>
<td>2.5</td>
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<td>0.21</td>
<td>3.9</td>
<td>31.4</td>
</tr>
<tr>
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<td>0.26</td>
<td>4.6</td>
<td>0.21</td>
<td>3.9</td>
<td>27.2</td>
</tr>
</tbody>
</table>

In Ref [15] it is proposed that position hold DRB achieve at least 0.17 rad/sec on the longitudinal and lateral axes, while DRP<3dB. It can be seen that the DRB is achievable and can even be exceeded. However, DRP slightly exceeds these guidelines. It is possible that the DRP requirement is not as significant for fully autonomous modes as long as the aircraft is stable. The design objective established 45°/6 dB phase and gain margin. The gain margin is easily achieved on all axes. The phase margin is readily achieved on all axes except roll. Some relaxation in the phase margin is generally acceptable, especially for high levels of control augmentation, hence for the roll axis PM=30° will be viewed as a satisfactory design. Since the GMS on each axis hold an adequate margin throughout the sweeping range, the design trade-off goes primarily between PM and DRB as demonstrated on Figure 3.6.5 and 3.6.6. From the design chart one can easily pick a reasonable compromise between SM and DRB by specifying \( \omega_{e\phi} = 3.0 \) rad/sec, \( \omega_{e\theta} = 2.5 \) rad/sec.
The impact of controller gains on tracking performance and close-loop stability can also be observed from simulation. The medium-class rotorcraft with nominal controller design \((\omega_{e\phi} = 3.0, \omega_{e\theta} = 2.5)\) has been modified to generate a low-gain lateral controller \((\omega_{e\phi} = 1.5, \omega_{e\theta} = 2.5)\) and a high-gain lateral controller \((\omega_{e\phi} = 4.0, \omega_{e\theta} = 2.5)\). Deck-tracking performance and corresponding actuation are plotted in Figure 3.6.7-3.6.9.
The non-linear simulation test implies that: with low-gain design the position tracking error is poor (consistent with low DRB), as control gains start to increase the tracking performance improves significantly in the beginning (as to 3.0 rad/sec), then slowly with further control gains increment at a cost of a large amount of high-frequency actuation (consistent with low SM). The above tendency is demonstrated in Figure 3.6.10 against the frequency parameter.
If an extreme high-gain design (e.g. $\omega_{\phi}=7.5$) is employed, the rotorcraft exhibits a self-induced instability, which is caused by the tightened coupling of rotor dynamics and fuselage dynamics. Simulation results are illustrated in Figure 3.6.11.

**Figure 3.6.10 Spectrum of Rotorcraft Response vs. Frequency Parameter**

**Figure 3.6.11 Example of Extremely High Controller Gains – Air resonance**

### 3.7 MIMO Robustness of DI Controller

The SISO method of robustness analysis has been widely applied in the industry because of the enormous experience on SISO controller and the well-established criteria. However, the helicopter controller is essentially a MIMO system, whose robustness to modeling error, if analyzed using SISO technique, must be tested on each loop to guarantee an overall assessment of stability. The MIMO frequency method provides a rigorous mathematical tool to evaluate the stability of controller designed on synthesis model but implemented on actual model. The difference between synthesis and actual models is the modeling error. There are two categories of modeling error, structured error (model structure is correct but certain parameters are incorrect) and unstructured error (primarily because of unmodeled dynamics). For helicopter controller design, the synthesis model is obtained from full order model by ignoring the fast dynamics, thus
belongs to the unstructured modeling error. There are two ways modeling errors can happen – additive error and multiplicative error, mathematically they are formulated in Eqn 3.7.1-3.7.2

\[ G_f = G_r + \Delta G \]  
(3.7.1)

\[ G_f = (1 + \Delta G) \cdot G_r \]  
(3.7.2)

Schematically they are shown in Figure 3.7.1, note that the multiplicative modeling error must occur after nominal model as a prerequisite in deriving the robust stability criterion.

In case of rotorcraft modeling, the reduced order model is generated by ignoring the fast mode such as flap/lag dynamics, structural dynamics, inflow dynamics, actuator dynamics, sensor dynamics and digital bus delay, those unmodeled dynamics act almost in series with rigid body flight dynamics, and therefore the modeling error can be expected primarily to be unstructured multiplicative type. As a consequence, the “significance” of modeling error is logically to be small in low frequency range where the nominal model is close to actual model, but significant in high frequency range where nominal model is notably different from actual model. This property is exemplified in Figure 3.7.2. The term “significance” is in sense of magnitude frequency response for SISO system, while singular value for MIMO system.

Robust control theory \[^{19,20}\] states that once we specify a design model \(G_r\), and accept the existence of unstructured modeling error in the form of Eqn (3.7.2), the output feedback controller
in Figure 3.7.3 would work stably with $G_f$, if

1. The nominal feedback close-loop system $G_r(s)K(s)[I + G_r(s)K(s)]^{-1}$ is stable
2. The full-order system $G_f(s)$ and the reduced-order system $G_r(s)$ have the same number of unstable poles but they don’t have to be identical.
3. Condition $\sigma[I + (G_rK)^{-1}] > \sigma(\Delta G)$ is satisfied

Figure 3.7.3. The Output Feedback Control System

To demonstrate the MIMO stability criterion, both full-order model (46 states) and reduced order model (9 states) of medium-class helicopter are generated from a trimmed flight with 20 knot forward speed, 300 ft height above the sea. The poles of reduced and full order plant model are illustrated in Figure 3.7.4-3.7.5, which proved that the reduced order model captured all the unstable dynamics of full order model. The singular value plot of $\tilde{G}_r(s)$ and $\tilde{G}_f(s)$ are illustrated in Figure 3.7.6, where we can see that the reduced order model is a fair approximation of full order model in low frequency range, where their singular values overlay. Singular value difference emerges only in the high frequency range, this is consistent with the prior analysis: the unmodeled dynamics has in fact high frequency feature.

Figure 3.7.4 Poles of Reduced Order model

85
Figure 3.7.5 Poles of Full Order Model

Figure 3.7.6 Comparison of the Singular Value of $G_r(s)$ and $G_f(s)$
According to foregoing analysis, the modeling error is multiplicative and can be calculated using Eqn 3.7.3 given the reduced-order and full order models.

\[ \Delta \tilde{G} = \tilde{G}_f(\tilde{G}_r)^{-1} - I \]  

(3.7.3)

The singular value plot of multiplicative modeling error shows good agreement with the typical shape in Figure 3.7.2, however not in all frequency range can the correctness be observed, when frequency is lower than 0.3 rad/sec, two branches of singular value significantly deviate from the normal course. This is partly because the modeling error is not purely multiplicative, a fraction of additive error also contributes to the model difference. Nevertheless, the low frequency range is not a primary concern in stability analysis, we will use the multiplicative modeling error and confine our investigation to the frequency range above 0.3 rad/sec.

Figure 3.7.7 Singular Value of Multiplicative Modeling Error

Another necessary step is to reconstruct the controller, the MIMO stability criteria demand the feedback controller to be based on output, namely \( y = [V_D, p, q, r] \), whereas the position hold DI controller involves additionally \( u, v, \phi, \theta, X_{lon}, Y_{lat}, H \), the feedback variables, if not in the output list, can be reconstructed using the following linear relations:

\[
\frac{d\phi}{dt} = p + q\sin \phi_{trim} \tan \theta_{trim} + r\cos \phi_{trim} \tan \theta_{trim} 
\]  

(3.7.4)

\[
\frac{d\theta}{dt} = q\cos \phi_{trim} - r\sin \phi_{trim} 
\]  

(3.7.5)
\[
\frac{d\psi}{dt} = q \frac{\sin \phi_{trim}}{\cos \theta_{trim}} + r \frac{\cos \phi_{trim}}{\cos \theta_{trim}} \quad (3.7.6)
\]
\[
\frac{dV_{lon}}{dt} = X_u \cdot V_{lon} - g \cdot \theta + X_q \cdot q \quad (3.7.7)
\]
\[
\frac{dV_{lat}}{dt} \approx Y_v \cdot V_{lat} + g \cdot \phi + Y_p \cdot p \quad (3.7.8)
\]
\[
\frac{dX_{lon}}{dt} = V_{lon} \quad (3.7.9)
\]
\[
\frac{dY_{lat}}{dt} = V_{lat} \quad (3.7.10)
\]

Having the reconstructed feedback variables, the singular value of DI controller can be evaluated and compared to the singular value of modeling error. A range of gain setting are investigated by sweeping the frequency parameters on longitudinal and lateral axes. Three pairs frequency parameter \([\omega_{e\phi}, \omega_{e\theta}] = [2,2], [3,3] \text{ and } [4,4] \text{ rad/sec}\) are studied for demonstration, their robustness criteria \(\sigma[I + (\hat{G}_r K)^{-1}] > \sigma(\Delta \hat{G})\) are illustrated in Figure 3.7.8, the plot reveals the same trend of robustness vs. control gains, i.e. high-gain design yields a low robustness margin. To some level when the controller minimum singular value intersects modeling error singular value, the robustness is no longer ensured.

![Figure 3.7.8. MIMO Robustness Criterion with Different Controller Gains](image)

The MIMO robustness criterion is a worst-case rule, i.e. even in case of criterion violation, an instability may not occur if the excitation is not the most of all destabilizing. Leading to an overly conservative design, the MIMO robustness criterion is not suited to guide engineering design before a quantitative regulation becomes available. A proper definition of MIMO stability margin and the associated regulation is yet to be established for a full exploration of MIMO method.
3.8 Summary
The Dynamic Inversion theory has been implemented in the inner-loop ACAH mode and outer-loop trajectory following design, extensive linear and non-linear simulations proved the capability of DI controller in achieving the design objectives. A SISO-based Method for measuring the closed-loop stability margin and disturbance rejection has been developed and used for the optimization of control parameters tuning. The stability analysis has been extended to a MIMO criterion which is mathematically rigorous yet less practical for implementation, however showing the same trend given by SISO method.

3.9 Reference.


http://www.roymech.co.uk/Related/Control/Bode.html


Chapter 4. Approach Path Generation

Contents

4.1 Design Objective of Approach Path
4.2 Parameterization of Approach Path
4.3 Optimization of Approach Path
4.4 Sample Approach Profile
4.5 Guidance Command Generation
4.6 Simulation Test of Approach Path and Guidance Law
4.7 Summary
4.8 Reference

As formulated in the “Shipboard-Helicopter Operational Procedures Manual”, shipboard recovery of rotorcraft to a moving vessel usually include the following phases in sequence:
1. Pre-Approach phase. The operation completed in this phase is to regulate the rotorcraft with a desirable state for approach, which is usually characterized by a level flight with recommended initial altitude, speed, heading angle and distance to the landing pad.
2. Approach phase. In this phase a rotorcraft chases the sailing vessel, decelerates and flares into a stable hovering over the flight deck.
3. Station-Keeping. In this phase the rotorcraft maintains the stable hovering by following the mean center or dynamic motion of flight deck despite the disturbance of gusty environment. In the same time, deck motion data are analyzed and predicted to find a preferable time window for landing.
4. Descent and touchdown. Once a time window for landing is designated, rotorcraft is triggered to close the vertical gap between LG and deck. In the meantime, horizontal position error should also be controlled within tolerance.

4.1 Design Objective of Approach Path

Approach path is a smooth curve defined in ship-relative frame, along which a helicopter is supposed to fly close to the landing pad. Apart from the geometry of flight path, the velocity and acceleration must also be defined at each waypoint to make a whole approach profile and provide full guidance information to feed the path following controller. The design objective of approach profile includes: ensuring smooth entry and exit of approach flight, and achievable flight maneuvering. Two approach parameters are traditionally used to specify the terminal shape – glide slope angle $\gamma_{app}$ and azimuth angle $\psi_{app}$, a comprehensive path planner must be able to specify those two parameters during path generation.

4.2 Parameterization of Approach Path

Intuitively, the path generation can be simplified if the spatial curve breaks down into projections on horizontal plane and vertical plane. The vertical projection uses the curve length of horizontal projection as its abscissa as illustrated in Figure 4.2.1-4.2.2.
Based on the above geometric breakdown, the path planning can be handled in horizontal and vertical plane separately as a 2-D problem. B-spline was selected to parameterized the planar curves for its excellent smoothness and adequate flexibility. B-spline is usually defined as a piecewise cubic spline with 1st and 2nd order derivative continuity in the conjunctions. Two methods exist to define a B-spline depending on their different ways of using control points; first method generates B-spline that exactly interpolates the control points; second method uses the control points to form a polygon spanning the B-spline. To reduce the sensitivity of spline
geometry to the control points movement, the control polygon approach is used for planar curve parameterization. To generate a L-piece B-spline, L+3 control points need to be given which are called de’Boor control points, using de’Casteljau algorithm \(^1\) the coordinates of as many as needed sampling points on the B-spline can be calculated. Figure 4.2.3-a and 4.2.3-b demonstrate two examples of using control polygon to generate a 3-piece B-spline.

\[
V_{app}(R) = R \left( \frac{V_0}{2R_{pd}} \right) \left( 1 + \frac{R}{2R_{pd}} \right)
\]

(4.2.1)

\[
A_{app}(R) = R \left( \frac{V_0}{2R_{pd}} \right)^2 \left( 1 + \frac{R}{2R_{pd}} \right)^3
\]

(4.2.2)

It is reasonable to use range to hover spot rather than time to parameterized the speed and
acceleration profile, because the approach is not a time-sensitive process and according to Ref [3]-[5], an intuitive approach profile can be summarized as “τ coupling” strategy, where τ is the time to close the gap of position or speed calculated using the instantons speed and acceleration), in case of Heffley profile:

\[ \tau_{\text{pos}} = \frac{R}{V_{\text{app}}(R)} = \left( 1 + \frac{R}{2R_{pd}} \right) \left( \frac{V_0}{2R_{pd}} \right) \]  

\[ \tau_{\text{spd}} = \frac{V_{\text{app}}(R)}{A_{\text{app}}(R)} = \left( 1 + \frac{R}{2R_{pd}} \right)^2 \left( \frac{V_0}{2R_{pd}} \right) \]  

The “τ coupling” of Heffley profile is then reveal by Eqn 4.2.5

\[ \tau_{\text{spd}} = \tau_{\text{pos}} \left( 1 + \frac{R}{2R_{pd}} \right) \]  

The Heffley profile can be justified comparing the deceleration of computer-generated and flight test data in Figure 4.2.5. Red dots are the deceleration profile sampled from a pilot-flown visual approach with initial speed 80 knots and initial altitude at 500 ft [5]. Using the same initial speed and an estimated peak-deceleration range of 200 ft, the blue line represents the deceleration generated by Heffley profile, which has a very similar pattern to the flight test data.

![Figure 4.2.5 Comparison of Heffley Profile and Flight Test Data](image)

4.3 Optimization of Approach Path

Knowing spatial path geometry along with velocity and acceleration profile, the path property can be rated using the following criteria:

- The trajectory slope should comply with the helicopter flight direction at the start and with the prescribed approach angles at the end. This criterion is designed to ensure smooth entry and exit.
- Trajectory curvature together with kinematic profile must satisfy the constraints of helicopter maneuverability.
- Trajectories should be as short as possible to minimize approach time.
- Trajectories should be friendly in saving power consumption and actuator duty cycles.
- Trajectories should avoid obstacles and hazardous region.

The above path criteria are also quantified separately in horizontal and vertical plane as summarized in Table 4.3.1

<table>
<thead>
<tr>
<th>Path Criteria</th>
<th>Horizontal Plane Specification</th>
<th>Vertical Plane Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Angle</td>
<td>Equal to flight heading angle</td>
<td>Equal to flight path angle</td>
</tr>
<tr>
<td>Final Angle</td>
<td>Equal to approach azimuth angle $\psi_{app}$</td>
<td>Equal to approach glide slope $\gamma_{app}$</td>
</tr>
<tr>
<td>Maneuverability Limitation</td>
<td>Turn rate less than designed maximum, typically corresponding to 30° roll angle or equivalently lateral load factor 0.5$g$</td>
<td>Descent rate to avoid VRS (typically less than 10 ft/sec) and acceleration less than designed maximum $\pm 7$ ft/sec$^2$</td>
</tr>
<tr>
<td>Actuator Duty Cycle</td>
<td>Total variation of curvature</td>
<td>Total variation of curvature</td>
</tr>
</tbody>
</table>

Based on the above discussion, a generalized optimization algorithm was designed for both horizontal and vertical path generation. Six de'Boor control points $(d_1, d_2, d_3, d_4, d_5, d_6)$ are used to parameterize a 3-pieceed B-spline: the end points $d_1$ and $d_6$ are naturally fixed at helicopter position and hover point over landing pad which is defined as the origin of ship relative frame. The four internal points $d_2$ - $d_5$ are initialized as equally spaced on a straight line between $d_1$ and $d_6$, then the internal points migrate subject to an optimization algorithm to satisfy the path criteria. Control points migration in horizontal plane and vertical plane are schematized in Figure 4.2.4 and 4.2.5

![Horizonal Guidance](image)

Figure 4.3.1. Typical Control Points Migration during Optimization in Horizontal Plane
A gradient-based optimization algorithm is applied to drive the internal de’Boor points $d_2 - d_5$ towards a minimal value of the following objective function:

$$J = w_1 \mid \theta_{ini} - \theta_{ini.des} \mid + w_2 \mid \theta_{end} - \theta_{end.des} \mid + w_3 \cdot L_{tot} + BarrFcn(w_4, mvio) + w_5 \cdot TVC$$

(4.3.1)

Where:
- $\theta_{ini}$: initial slope angle of path
- $\theta_{ini.des}$: desired initial slope angle, set equal to the flight heading angle in horizontal plane and flight path angle in vertical plane
- $\theta_{end}$: end slope angle of path
- $\theta_{end.des}$: desired end slope angle, set equal to the approach azimuth angle in horizontal plane and glide slope angle in vertical plane
- $L_{tot}$: total length of path
- $BarrFcn$: Barrier function to penalize the maneuverability violation
- $mvio$: amount of the maneuverability violation, for horizontal plane $mvio = \phi_{max} - 30^\circ$, for vertical plane $mvio = V_D - 10 \text{ ft/sec}$
- TVC: total variation of curvature
- $w_1 - w_5$: weighting factors, $w_3 - w_5$ are tuned so that in terms of optimization priority: $mvio > TVC > L_{tot}$

### 4.4 Sample Approach Profile

Firstly, the horizontal and vertical path planner are tested independently to verify if the optimization algorithm can sort out those feasible paths with given criteria priority. Figure 4.4.1 illustrates approach paths in horizontal plane with initial speed sweeping from 60 ft/sec to 130 ft/sec. Figure 4.4.2 illustrates the maneuvering roll angle along the path length. Results proved that the path planner firstly tends to satisfy maneuverability limitation, if this is satisfied the end-to-end path length and TVC criteria are more considerable.
The vertical plane path planner was tested in scenario where the rotorcraft is released 2800 ft behind the ship stern at 300 ft height with initial speed sweeping from 40ft/sec to 100 ft/sec. Figure 4.4.3 illustrates the approach path in vertical plane, Figure 4.4.4 illustrates the descent rate along approach path as opposed to its limit against VRS.
A spatial path can be generated by combining the results of horizontal and vertical path planner. Figure 4.4.5-4.4.9 illustrated the spatial geometry of an approach path together with the SHF components of velocity and acceleration.
Figure 4.4.5 Ground Track of Optimized Approach Path

Figure 4.4.6 Vertical Profile of Optimized Approach Path

Figure 4.4.7 3D Plot of Optimized Approach Path
In the above testing, the initial position, flight heading as well as the speed and $R_{pd}$ for Heffley profile are preliminary specified and thus not involved in the optimization. It is entirely possible that the path planner cannot come out with a feasible trajectory if the pre-specified parameters define a severe approach situation, the selection of proper pre-approach parameters to increase the possibility of a successful path planning belongs to the procedural study and has been thoroughly investigated in NAVAIR as an external optimization [7].

4.5 Guidance Command Generation

Observed in ship-heading frame, the rotorcraft is commanded to follow the approach path together with the velocity and acceleration profile. At any moment, the reference point is picked from approach path by a projection algorithm schematized in Figure 4.5.1. Path sampling points $L(X_L, Y_L, Z_L)$ and $R(X_R, Y_R, Z_R)$ are the two closest points to the rotorcraft; reference point $P(X_p, Y_p, Z_p)$ is defined as the projection of the rotorcraft to the straight line connecting $L$ and $R$, the coordinates of reference point are determined by Eqn (4.5.1)
Corresponding reference velocity and acceleration are determined by a linear interpolation in Eqn (4.5.2)

\[ f_{ref}^{SHF} = f_L \frac{PR}{LR} + f_R \frac{LP}{LR} \]  

The above reference position, velocity and acceleration are all defined in ship-heading frame, command in inertial frame can be obtained by adding ship motion:

\[ X_{Ncmd} = X_{ref}^{SHF} \cos \psi_{ship}^{ref} - Y_{ref}^{SHF} \sin \psi_{ship}^{ref} + X_{deck} \]  
\[ Y_{Ecmd} = X_{ref}^{SHF} \sin \psi_{ship}^{ref} + Y_{ref}^{SHF} \cos \psi_{ship}^{ref} + Y_{deck} \]  
\[ Z_{Dcmd} = Z_{ref}^{SHF} + Z_{deck} \]  
\[ V_{Ncmd} = V_{ref}^{SHF} \cos \psi_{ship}^{ref} - V_{ref}^{SHF} \sin \psi_{ship}^{ref} + V_{deck} \]  
\[ V_{Ecmd} = V_{ref}^{SHF} \sin \psi_{ship}^{ref} + V_{ref}^{SHF} \cos \psi_{ship}^{ref} + V_{deck} \]  
\[ V_{Dcmd} = V_{ref}^{SHF} + V_{deck} \]  
\[ A_{Ncmd} = A_{ref}^{SHF} \cos \psi_{ship}^{ref} - A_{ref}^{SHF} \sin \psi_{ship}^{ref} + A_{deck} \]  
\[ A_{Ecmd} = A_{ref}^{SHF} \sin \psi_{ship}^{ref} + A_{ref}^{SHF} \cos \psi_{ship}^{ref} + A_{deck} \]  
\[ A_{Dcmd} = A_{ref}^{SHF} + A_{deck} \]  

In the above equations, \( \psi_{ship} \) is the ship heading angle. Since the approach path is rigidly carried on the ship, an alternating value of \( \psi_{ship} \) would be amplified by the range of helicopter and thus turn into the large variation of reference coordinates. Therefore \( \psi_{ship} \) uses the signal processed by a low-pass filter. Similarly, if the deck motion parameters...
The rotorcraft has to do a lot of unnecessary maneuvering during approach and station-keeping. However, just like human pilots would do, the rotorcraft does not have to follow the dynamic deck motion but rather the mean center. In this case the deck motion parameters added to trajectory command should be processed by a low-pass filter too. Several deck filters have been designed to extract the mean motion, the simplest type is a 1st order analogue filter in Figure 4.5.2 with variable frequency parameter to change the effectiveness of filtering.

![Figure 4.5.2. 1st order deck filter](image)

The 1st order deck filter can effectively saturate the oscillation around a fixed mean value, thus is suitable for processing the deck height. Figure 4.5.3 illustrates the deck height processed by 1st order filter with frequency parameter $\omega = 0.01$ rad/sec, the low value of frequency parameter is demanded to attenuate the ship motion characterized by frequency 0.15 Hz, expected filtering effectiveness has been proven in spite of relatively long settling time.

![Figure 4.5.3. Deck Height Processed by 1st order Deck Filter](image)

However, the 1st order deck filter fails to extract the steady trend when applied to deck state with ramp features because of having steady state tracking error. A remedy can be a 2nd order filter obtained by introducing an integral mechanism into the 1st order filter as illustrated in Figure 4.5.4, an empirical selection of the integral gain is suggested be 1/3 of the frequency parameter.
Figure 4.5.4. 2nd order deck filter

The 2nd order filter was tested on the East coordinate of deck center which has a relatively steady growing component on the top of random oscillation. However, the general trend was extracted as to hit the middle of the original trajectory.

Figure 4.5.5. East-Coordinate of Deck Center Processed by 2nd order Filter

4.6 Simulation Test of Approach Path and Guidance Law

In the non-linear simulation, the commanded inertial position, velocity and acceleration were generated by the guidance law and injected into the trajectory following control law. A demonstration was performed on medium-class helicopter trying to approach and hover above the ship sailing to towards North. The rotorcraft was maintaining a level flight towards deck center with a speed of 125 ft/sec at an altitude of 300 ft from the starboard with an azimuth angle of 45°, until it hit the range of 2500 ft where the approach begins. The approach parameters were set as follow:

- Approach azimuth angle: \( \psi_{app} = 45° \)
- Approach glide slope angle: \( \gamma_{app} = 0° \)
- Peak deceleration range: \( R_{pd} = 200 \text{ ft} \)

The flight path and command tracking history are plotted in Figure 4.6.1-4.6.8 for approach and station-keeping.
Figure 4.6.1. 3D Trajectory of Approach and Station Keeping

Figure 4.6.2. Top view of Terminal Approach and Station Keeping
Figure 4.6.3. Command Tracking of North Coordinates

Figure 4.6.4. Command Tracking of East Coordinates

Figure 4.6.5. Command Tracking of Altitude

Figure 4.6.6. Command Tracking of Velocity to the North
Figure 4.6.7. Command Tracking of Velocity to the East

Figure 4.6.8. Command Tracking of Climbing Velocity

Figure 4.6.9-a Approach and Station-Keeping Trajectories, $\psi_{app} \in [-45^\circ, 45^\circ]$

Figure 4.6.9-b Zoom-In plot of 4.6.9-a
The maximum positional tracking error during approach is less than 2 ft, while the velocity tracking error is less than 1.5 ft/sec, therefore the precision of approach and station-keeping satisfies the level-1 design objective.

4.7 Conclusion
This chapter introduced a B-spline based parameterization method which breakdown the spatial path geometry onto horizontal plane and vertical plane. A coverage of path criteria has also been assigned into horizontal and vertical partition. The associated kinematic profile is calculated using Heffley’s equation. The geometry along with kinematic profiles are optimized by gradient descent algorithm. A point-to-curve projection method has been applied to find target way point and generate reference speed and acceleration. Simulation

4.8 Reference
Chapter 5. Landing Path Generation

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5.1 Methodology of Predictive Landing
5.2 Landing Path Generation with Position and Velocity Matching
5.3 Guidance Command Generation
5.4 Attitude Mismatch Avoidance
5.5 Landing Path Generation with Attitude Matching
5.6 Summary
5.7 Reference

Landing is definitely the most challenging part of a shipboard helicopter recovery by either human piloting or automatic control. The recovery safety depends on accuracy touchdown position as well as soft impact which can be essentially undermined by the disturbance arising from the maneuvering ship in high sea states, gusty aerodynamic environment and clustered deck surrounding in the vicinity of ship structure. In a human-controlled operation, pilots always tend to find a quiescent period of ship motion and drive the helicopter to touchdown with a simple descent law and minor corrective control. When it comes to high sea states, the adverse factors can cause fairly heavy workload to human pilot, in worst case a quiescent period may not exist at all. Some rudimentary level automatic landing simply commands the heave axis to have a small enough inertial sink rate $^{[1]}$, hopefully this strategy may land softly if the deck is not violently plunging. With the help of trajectory following system, rotorcraft can keep constant tracking of dynamic deck motion and perform descent at a desirably small relative speed $^{[2]}$. However, with this strategy of landing the rotorcraft will have to do a lot of maneuvering tracking the ship motion. In extreme case, the rotorcraft can be commanded to deploy excessive agility, engine power and torque. Another control strategy comes to feasible and less demanding in control actuation if some degree of prediction is available to the future state of moving target. The difference between tracking and predictive landing strategies are schematically indicated in Figure 5.1.1

Figure 5.1.1-a Landing with Deck Tracking

Figure 5.1.1-b Landing with Deck Prediction
Unlike approach, the landing quality is time-sensitive thus the flight program must be stringently defined as a time sequence. This chapter seeks to develop an algorithm generating inertial path with certain kinematic constraints based on predicted future deck state. The basic approach includes a polynomial representation of path geometry and an optimization algorithm used to form the shape of landing path. Accordingly, a guidance law will be developed based on the specifics of landing problem.

5.1 Methodology of Predictive Landing

Predictive control has a long history of application in chasing problem to reduce the maneuverability requirements. In missile rendezvous solution, the target trajectory prediction method has replaced the tail-chasing method since the second generation of self-homing air-to-air missile [3]. Many attempts have been made to incorporate predicted deck state into control, some of the authors use deck forecasting only for decision making of descent initiation [1][4]. The philosophy of a full use of deck prediction lies in that: forecasting algorithm keeps monitoring the deck state at a horizon (typically 5 seconds), once a desirable for touchdown deck state is predicted (a downward deck motion is regarded desirable for touchdown), a descent time-sequence should be generated for position and velocity on three axes with terminal values matching the deck state. The time-sequence of kinematic parameters are then used for descent guidance, however, since forecasting always has errors, slight changes of the descent program must be made according to the latest measurement and prediction. Since the trajectory following control law developed in Chapter 3 manages to regulate the three inertial coordinates in a decoupled manner, the time-sequence of the position, velocity and acceleration on each axis can be generated independently. Also, because the forward ship motion has few dynamic content and is almost steady, the predictive landing strategy will be applied only on lateral and vertical axes.

5.2 Landing Program Generation with Position and Velocity Matching

In the following discussion, landing program will be referred to as the time-sequence of position, velocity. A shipboard landing is initiated from station-keeping and terminated upon the touchdown: a reasonable landing program must be a smooth sequence with the initial states matching the rotorcraft’s and terminal state matching the deck’s, while the internal value of landing sequence should not exceed the maneuverability limit. The descent program of altitude along with the deck trajectory are demonstrated in Figure 5.2.1.

![Figure 5.2.1 Descent Program of Altitude](image)
Without losing generality, the algorithm is explained on the vertical axis. The time-sequence of altitude is denoted by $H(t)$; the descent is initiated at $t = 0$ and terminated at $t = t_f$ by a touchdown. The time-sequence can be approximated as a fixed-end polynomial in Eqn 5.2.1 with six coefficients to be determined.

$$H(t) = at^5 + bt^4 + ct^3 + dt^2 + et + f$$  \hspace{1cm} (5.2.1)$$

The consistent velocity and acceleration can be obtained by differentiating Eqn 5.2.1

$$V_z(t) = 5at^4 + 4bt^3 + 3ct^2 + 2dt + e$$  \hspace{1cm} (5.2.2)$$

$$A_z(t) = 20at^3 + 12bt^2 + 6ct + 2d$$  \hspace{1cm} (5.2.3)$$

At the two ends, the trajectory and its derivative are specified to match the current helicopter state and deck state at touchdown. Quantitatively this can be expressed as the following conditions:

i. At $t = 0$, the position $H(0)$ is equal to the current helicopter height $H_{helo}$, the velocity $V_z(0)$ is equal to zero or current helicopter velocity $V_{z_helo}$ whichever is smaller.

ii. At $t = t_f$, the position $H(t_f)$ is equal to the predicted deck height $H_{deck}$, the velocity $V_z(t_f)$ is equal to the deck velocity corrected by a desired relative sink rate e.g. 1 ft/sec

In the meantime, the altitude descent program should not have intersections with deck trajectory until touchdown, also the velocity and acceleration should not exceed their operational limits $V_{lim}$ and $A_{lim}$.

From condition i,ii four mathematical constraints can be constructed,

$$H_{helo} = f$$  \hspace{1cm} (5.2.4)$$

$$V_{z_helo} = e$$  \hspace{1cm} (5.2.5)$$

$$H_{deck}(t_f) = at_f^5 + bt_f^4 + ct_f^3 + dt_f^2 + et_f + f$$  \hspace{1cm} (5.2.6)$$

$$V_{z_{deck}}(t_f) - 1 = 5at_f^4 + 4bt_f^3 + 3ct_f^2 + 2dt_f + e$$  \hspace{1cm} (5.2.7)$$

another two constraints are needed for a close-form solution of the six unknown coefficients. The additional degree of freedom should be able to affect the internal shape of the position profile which consequently turns into the velocity and acceleration profile and serves as the variables for optimization. A straightforward selection of such constraints is to specify $H(t)$ at $t = t_f/3$ and $t = 2t_f/3$, this condition can be quantified as in Eqn 5.2.8-5.2.9

$$H_1 = a\left(\frac{1}{3}t_f\right)^5 + b\left(\frac{1}{3}t_f\right)^4 + c\left(\frac{1}{3}t_f\right)^3 + d\left(\frac{1}{3}t_f\right)^2 + e\left(\frac{1}{3}t_f\right) + f$$  \hspace{1cm} (5.2.8)$$

$$H_2 = a\left(\frac{2}{3}t_f\right)^5 + b\left(\frac{2}{3}t_f\right)^4 + c\left(\frac{2}{3}t_f\right)^3 + d\left(\frac{2}{3}t_f\right)^2 + e\left(\frac{2}{3}t_f\right) + f$$  \hspace{1cm} (5.2.9)$$

The six equations solving for polynomial coefficients then can be expressed in matrix form
So far, $H_1$ and $H_2$ are to be determined, an optimization task is set to find out those two variables in pursuit of the satisfaction of the following constraints:

1. $H(t) > H_{fcast}$ over $t \in [0, t_f]$ to avoid premature contacts
2. $\max|V_Z(t)| < V_{lim}$
3. $\max|A_Z(t)| < A_{lim}$

The inequality constraints can be implemented in objective function through barrier functions. The barrier functions should be designed as to increase quickly if the constraints are violated but hold a negligible magnitude when the constraints are satisfied. Exponential functions are usually modified to provide barrier, e.g. the velocity constraint can be implemented by adding barrier function (5.2.11) into the optimization objective.

$$J = \lambda_1 e^{-(V_{lim} - \max|V_Z(t)|)^{0.1}}$$

(5.2.11)

The tunable parameter $\lambda$ is intended to change barrier steepness. Inadequate barrier effect may lead the optimizer to putting out cases with violated constraints, which implies to apply a larger value of $\lambda$. The overall objective function is in Eqn. 5.2.12, where $w_1, w_2, w_3$ are the weighting factors providing trade-off between criteria.

$$J = w_1 e^{-\lambda_1 \max(H(t) - H_{fcast})^{0.1}} + w_2 e^{-\lambda_2 (V_{lim} - \max|V_Z(t)|)^{0.1}} + w_3 e^{-\lambda_3 (A_{lim} - \max|A_Z(t)|)^{0.1}}$$

(5.2.12)

Using conjugate gradient-descent optimizer to solve for $H_1$ and $H_2$ and therefore the landing program can be determined. The above algorithm was tested with the condition sampled from station-keeping flight of medium-class helicopter over “S Cone2” deck motion, necessary for landing program generation quantities are summarized in table 5.2.1

Table 5.2.1 Initial and Terminal Conditions for Landing Path Generation

<table>
<thead>
<tr>
<th>Duration of landing program</th>
<th>$t_f=5$ sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial helicopter altitude</td>
<td>$H_{helo}=45$ ft</td>
</tr>
<tr>
<td>Initial helicopter vertical speed</td>
<td>$V_{z_{helo}}=0$ ft/sec</td>
</tr>
<tr>
<td>Predicted deck altitude at terminal time</td>
<td>$H_{deck}^{pred}=16$ ft</td>
</tr>
<tr>
<td>Predicted deck vertical speed at terminal time</td>
<td>$V_{z_{deck}}^{pred}=-3$ ft/sec</td>
</tr>
<tr>
<td>Forecasting time array</td>
<td>$T_{fcast}=[0, 1, 2, 3, 4, 5]$ sec</td>
</tr>
</tbody>
</table>
Forecasted deck position array \( H_{\text{cast}} = [15.0, 20.0, 22.12, 21.95, 19.46, 16] \) ft

Maximum allowed vertical speed \( V_{\text{lim}} = 6.7 \) ft/sec

Maximum allowed vertical acceleration \( A_{\text{lim}} = 11.3 \) ft/sec²

The optimized position, velocity and acceleration profiles are plotted in Figure 5.2.2. It is shown that the initial guess of landing path violated the velocity constraint, while the optimization algorithm successfully addressed this problem and yielded a feasible for guidance trajectory.

As the landing sequence elapses, the forecasting horizon becomes shortened and the prediction accuracy improved. To make full use of the latest prediction the landing program has to
regenerate several times until touchdown. In order to save computational expense and facilitate real-time control effect, the landing program should not regenerate too frequently, in practice a 1 second interval is adequately frequent to remedy forecasting errors. The same algorithm can be applied to landing program generation on lateral axis with slight change in that lateral axis has no concern with premature deck contact, thus the corresponding penalty can be eliminated in the objective function.

5.3 Guidance Command Generation

The guidance law is supposed to supply the trajectory follower with commanded position, velocity and acceleration. Theoretically, the landing sequence generated in section 5.2 can be readily injected into the trajectory command. However, the direct implementation of predictive landing path renders the landing quality solely dependent on the forecasting accuracy, which is risky and can jeopardize the recovery safety, even though frequent predictions are made to drive landing program update. A hybrid implementation of predictive landing program incorporating real-time deck measurement was developed, where the relative landing program is obtained by subtracting the predicted deck profile from the optimized inertial landing program, at any moment during landing the inertial command can be recovered by adding the real-time deck state to the relative landing program. The advantage of this method is in that: if the prediction is accurate, the inertial command recovery exactly replicates the optimized landing program; in case of off-prediction, the command recovery maybe distorted from the optimized version, however, the safety and landing quality can still be guaranteed thanks to the nature of relative motion control.

\[
X_{Ncmd}(t) = X_{Ndeck}(t)
\]

\[
Y_{Ecmd}(t) = Y_{Eopt}(t) - Y_{Efast} + Y_{Edeck}(t)
\]

\[
H_{cmd}(t) = H_{opt}(t) - H_{Edeck}(t) + H_{Edeck}(t)
\]

\[
V_{Ncmd}(t) = V_{Ndeck}(t)
\]

\[
V_{Ecmd}(t) = V_{Eopt}(t) + V_{Edeck}(t)
\]

\[
V_{Zcmd}(t) = V_{Zopt}(t) - V_{Zdeck} + V_{Zdeck}(t)
\]

\[
A_{Ncmd}(t) = A_{Ndeck}(t)
\]

\[
A_{Ecmd}(t) = A_{Edeck} + A_{Edeck}(t)
\]

\[
A_{Zcmd}(t) = A_{Zopt}(t) - A_{Zdeck} + A_{Zdeck}(t)
\]

Once any of the landing gears contacts the flight deck, the aircraft dynamic property changes significantly due to the supportive force and friction acting on the landing gear, no airborne control law would be applicable under such condition. The controller simply reduce collective at first
contact to allow the aircraft weight to immediately push the airframe on the deck, this action is called “lift-dumping”. The predictive landing program together with guidance law were tested in FLIGHTLAB non-linear simulation. Figure 5.3.1-5.3.4 illustrated a representative case tested on medium-class helicopter model with “SCONE2” deck motion. The landing metrics are summarized in Table 5.3.1. The plots and metrics imply that the path optimization algorithm together with the hybrid control strategy resulted in level-1 landing quality for this particular case. However, the vertical impact on the front landing gear at touchdown was larger than expected or desired.

Table 5.3.1 Landing Quality Metrics

<table>
<thead>
<tr>
<th>Longitudinal Position Error (ft)</th>
<th>Lateral Position Error (ft)</th>
<th>Lateral Share Velocity (ft/sec)</th>
<th>Vertical Impact Velocity (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tail Gear</td>
<td>Front Gear 1</td>
<td>Front Gear 2</td>
<td>Tail Gear</td>
</tr>
<tr>
<td>0.433</td>
<td>0.499</td>
<td>0.226</td>
<td>0.6651</td>
</tr>
<tr>
<td>0.679</td>
<td>0.679</td>
<td>0.679</td>
<td>4.648</td>
</tr>
<tr>
<td>4.648</td>
<td>4.919</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.3.1 Top View of Trajectory of Approach, Station Keeping and Landing
Figure 5.3.2 Lateral and Vertical Position of Vehicle C.G.

Figure 5.3.3 Lateral and Vertical Velocities of Vehicle C.G.
Additional flight cases were tested with randomized initial conditions in order to evaluate the robustness and reliability of the path optimization algorithm and landing control law. The statistics for 30 cases are summarized in Figures 5.3.5-5.3.6 which show the landing scatter of the aircraft R.P., and the velocities of each landing gear relative to the ship deck point of contact at touchdown. These results prove again that the vertical impact on the front landing gear was higher than desired. On the positive side, the final position tolerance for the aircraft R.P. at touchdown was excellent, and the tail gear touched down softly in all cases.
5.4 Attitude Mismatch Avoidance

Detailed study unveiled that the hard landing of the front landing gears was caused by the "lift-dumping" operation and attitude mis-match. In station-keeping flight the rotorcraft is trimmed at a pitch-up attitude angle, in case of rotorcraft-deck attitude mismatch, the tail gear always
contacts first. Following the lift-dumping after tail gear contact is a fast drop of front gears, this is particularly true for the medium-class helicopter model with its tail gear mounted on the tail boom.

This problem is a consequence of attitude mismatch at touchdown; where the aircraft attitude does not comply with the deck attitude. Attitude mismatch is common for helicopter; even a land-based helicopter faces the same problem during landing since the helicopter attitude is not level in hover trim and they sometime need to land on a sloped surface. However, the attitude mismatch is much more complex with a moving landing deck. In high sea states, a quick sealing of touchdown is required to avoid the complex deck-aircraft interactions with landing gear contact dynamics.

A possible solution to this problem is to place the aircraft parallel to deck plane just before touchdown so that all three landing gears contact simultaneously, thereafter quickly dump the rotor lift to press aircraft tightly on flight deck. This can be achieved by a quick level-out operation at a moment before anticipated contact. The so called “level-out” maneuver cannot occur too early, as the helicopter begins to translate as soon as the attitude changes away from the trim. The start time of level-out depends on the quickness of response to attitude command. Since the control law switches from trajectory tracking to attitude command, the rotorcraft during level-out is expected to drift from the landing center. The level-out must be initiated at a proper time, which cannot be too early to avoid notable drifting, nor to be too late to avoid premature contact. Based on the characteristics of the UH-60 model being investigated, the level-out is initiated 1.0 second before contact.

The level-out control law commands attitude corrections quantitated for removing the difference among heights of three landing gears over deck. The schematic of the geometric parameters of the landing gear is shown in Figure 5.4.1:

![Figure 5.4.1 Geometric Parameters of Landing Gears](image)

The amount of pitch and bank angle corrections required to level out the front and tail gear is based on a small angle assumption quantified in Eqn. (5.4.1) and (5.4.2) respectively:

\[ \Delta \theta = \left( \frac{H_{LG1} + H_{LG2}}{L_g} \right) / 2 - H_{LG3} \]

\[ \Delta \phi = \left( \frac{H_{LG1} - H_{LG2}}{L_r} \right) \]

The real-time values of the above corrective attitude angles are fed into longitudinal and lateral ACAH controller for prompt driving the helicopter to the desired attitude. The maximum response
quickness allowed by the command filter in the ACAH controller is used. In the meantime, the vertical axis continues descent along the trajectory planned by algorithm presented before. Since three landing gear are designed to hit deck at the same time, the lift-dumping takes place at a fast rate – 20% per second to seal the touchdown as soon as one gear strikes deck. Figures 5.4.2 to 16 represent a typical landing with the level-out strategy designed and tested on medium class helicopter.

Figure 5.4.2 Top View of Approach and Landing Trajectory
Figure 5.4.3 Lateral and Vertical Position vs. Time

Figure 5.4.4 Lateral and Vertical Velocities vs. Time
Figure 5.4.5 Height Over Deck and Impact Velocity of Landing Gears

Figure 5.4.6 Helicopter Attitude Angles vs. Time
Figure 5.4.5 clearly shows the elevation of tail gear during level out, as it is expected. In Figure 5.4.6, the vehicle attitude experienced a step-command in the beginning of level-out action, namely - pitch down to elevate tail gear and bank right to elevate left gear. However, the undesired drifting can also be observed in the lateral position plot of Figure 5.4.3-5.4.4, although the amount was acceptable. Quantitative metrics of landing quality are summarized in Table 5.4.1.

Table 5.4.1 Landing Metrics with Level-out maneuver

<table>
<thead>
<tr>
<th>Longitudinal Position Error (ft)</th>
<th>Lateral Position Error (ft)</th>
<th>Lateral Share Velocity (ft/sec)</th>
<th>Vertical Impact Velocity (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tail Gear</td>
<td>Front Gear 1</td>
<td>Front Gear 2</td>
<td>Tail Gear</td>
</tr>
<tr>
<td>2.1</td>
<td>1.1</td>
<td>1.635</td>
<td>0.553</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.610</td>
<td>Front Gear 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.625</td>
<td>Front Gear 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.672</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.732</td>
</tr>
</tbody>
</table>

In order to verify the control strategy and reasonability of parameter selections, extensive test case with randomized flight conditions were performed on three classes of rotorcraft model along with the lodging ship using FLIGHTLAB non-linear simulation. Simulation results shown in Figures 5.4.7-5.4.8. The level-out control law successfully addressed the impact on front gears due to sequence of touchdown at a tolerable cost in terms of landing location drifting and lateral speed with respect to the deck.
Figure 5.4.7 Scatters of Vehicle Ref Point upon Touchdown (Medium Class)

Figure 5.4.8 Scatters of Touchdown Velocity Error (Medium Class)
Figure 5.4.7 Scatters of Vehicle Ref Point upon Touchdown (Heavy Class)

Figure 5.4.8 Scatters of Touchdown Velocity Error (Heavy Class)
Figure 5.4.9 Scatters of Vehicle Ref Point upon Touchdown (Light Class)

Figure 5.4.10 Scatters of Touchdown Velocity Error (Light Class)
5.5 Landing Path Generation with Attitude Matching

In the previous section, position & velocity matching was the solely concern but not attitude, thus to avoid impact on single landing gear a special “level out” operation was required prior to touchdown. This strategy has an obvious disadvantage: during “level out” maneuver the rotorcraft is flying in attitude command mode, thereby velocity is not controlled albeit for a short time (1 second as designed), as a consequence a noticeable drifting in position & velocity were observed. In this section the predictive landing algorithm is extended with the capability of “attitude matching”. The key concept making attitude-matching possible is the mapping from rotorcraft attitude to its acceleration

\[
\begin{align*}
A_{\text{lon}} &= -g \cdot \Delta \theta \\
A_{\text{lat}} &= g \cdot \Delta \phi
\end{align*}
\]  

(5.5.1)

The $\Delta \theta$ and $\Delta \phi$ are perturbations of pitch and roll angle around trim attitude, thus the desired attitude angles can be calculate based on the station-keeping attitude of rotorcraft and predicted attitude of deck at terminal moment in Eqn (5.5.2).

\[
\begin{align*}
A_{\text{lon.des}} &= -g \cdot (\theta_{\text{deck.pred}} - \theta_{\text{helo}}) \\
A_{\text{lat.des}} &= g \cdot (\phi_{\text{deck.pred}} - \phi_{\text{helo}})
\end{align*}
\]  

(5.5.2)

In light of Eqn (5.5.2), a matching to deck rotorcraft’s attitude angle is achieved as long as the corresponding longitudinal/lateral acceleration is achieved. Therefore, the terminal “to be matched” variable list is expanded from “position+velocity” to “position+velocity+acceleration” for longitudinal and lateral axes. Nevertheless, on the longitudinal axis the station-keeping control law continues during landing phase for safety consideration; after all, any maneuvering on longitudinal axis is risky upon touchdown. The following derivation is primarily applicable to lateral axis to incorporate roll angle matching.

To impose the terminal acceleration specification, one more DOF must be added to the parameterization, reasonably this can be achieved by using a sixth polynomial Eqn. (5.5.3).

\[
P(t) = a \cdot t^6 + b \cdot t^5 + c \cdot t^4 + d \cdot t^3 + e \cdot t^2 + f \cdot t + g
\]  

(5.5.3)

The consistent velocity and acceleration

\[
V(t) = 6a \cdot t^5 + 5b \cdot t^4 + 4c \cdot t^3 + 3d \cdot t^2 + 2e \cdot t + f
\]  

(5.5.4)

\[
A(t) = 30a \cdot t^4 + 20b \cdot t^3 + 12c \cdot t^2 + 6d \cdot t^2 + 2e
\]  

(5.5.5)

In the initial end, trajectory and its derivative are specified to match the current rotorcraft state; in the terminal end, position and velocity must be specified as equal to predicted deck state, additionally the acceleration must be specified corresponding to predicted deck attitude. Quantitatively, this can be expressed as the following conditions

I. At $t = 0$ end, $P(0) = P_{\text{helo}}$, $V(0) = V_{\text{helo}}$, where $P_{\text{helo}}$ and $V_{\text{helo}}$ are the initial coordinate and velocity of rotorcraft

II. At $t = t_f$ end, $P(t_f) = P_{\text{deck}}$, $V(t_f) = V_{\text{deck}}$, $A(t_f) = A_{\text{map}}$, where $P_{\text{deck}}$ and $V_{\text{deck}}$
are the predicted coordinate and velocity of deck, $A_{map}$ is the acceleration mapped from predicted deck attitude angle.

To make close-form solution of the polynomial, another two conditions are required, like before two interpolation points are set at $t = t_f/3$ and $t = 2t_f/3$, once the corresponding position are specified, the last two constraints can be expressed in Eqn. 5.5.6 and Eq. 5.5.7

\[
H_1 = a\left(\frac{1}{3}t_f\right)^6 + b\left(\frac{1}{3}t_f\right)^5 + c\left(\frac{1}{3}t_f\right)^4 + d\left(\frac{1}{3}t_f\right)^3 + e\left(\frac{1}{3}t_f\right)^2 + f\left(\frac{1}{3}t_f\right) + g \tag{5.5.6}
\]

\[
H_2 = a\left(\frac{2}{3}t_f\right)^6 + b\left(\frac{2}{3}t_f\right)^5 + c\left(\frac{2}{3}t_f\right)^4 + d\left(\frac{2}{3}t_f\right)^3 + e\left(\frac{2}{3}t_f\right)^2 + f\left(\frac{2}{3}t_f\right) + g \tag{5.5.7}
\]

The six equations solving for polynomial coefficients then can be expressed in matrix form

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
t_f^6 & t_f^5 & t_f^4 & t_f^3 & t_f^2 & t_f & 1 \\
6t_f^5 & 5t_f^4 & 4t_f^3 & 3t_f^2 & 2t_f & 1 & 0 \\
30t_f^4 & 20t_f^3 & 12t_f^2 & 6t_f & 2 & 0 & 0 \\
\left(\frac{1}{3}t_f\right)^6 & \left(\frac{1}{3}t_f\right)^5 & \left(\frac{1}{3}t_f\right)^4 & \left(\frac{1}{3}t_f\right)^3 & \left(\frac{1}{3}t_f\right)^2 & \left(\frac{1}{3}t_f\right) & 1 \\
\left(\frac{2}{3}t_f\right)^5 & \left(\frac{2}{3}t_f\right)^4 & \left(\frac{2}{3}t_f\right)^3 & \left(\frac{2}{3}t_f\right)^2 & \left(\frac{2}{3}t_f\right) & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c \\
d \\
e \\
f \\
g
\end{bmatrix}
=
\begin{bmatrix}
P_{helo} \\
V_{helo} \\
P_{deck} \\
V_{deck} \\
A_{map} \\
H_1 \\
H_2
\end{bmatrix} \tag{5.5.8}
\]

Using the same gradient optimization solver to find out $H_1$ and $H_2$ and thus the polynomial coefficients can all be determined.

5.6 Summary

In the light of the decoupled trajectory following controller designed on each axis, this chapter introduced a predictive timing trajectory parameterized by under-determined cubic polynomials. Additional DOF are used to impose the kinematic constraints using a gradient descent optimizer. The optimized descent trajectory together with the hybrid implementation can lead to an accurate in position and soft in speed landing without doing dynamic maneuver. Additional operations level-out and lift-dumping are suggested to avoid attitude mismatch.

5.7 Reference


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===========================================================================

The Guidance and Control strategy of the autonomous system presented in the foregoing chapters requires the information of inertial position and velocity from both ship and rotorcraft. Furthermore, the crowded landing deck environment implies there is a fairly small tolerance for recovery error \[1\]. Hence the accuracy and precision of navigation in many ways govern the success of automated landing. For air, maritime even ground vehicles Inertial Navigations System (INS) is a common equipment usually built in form of a strap-down navigation system. However, the performance of a pure INS is undermined by a time-dependent drifting in the accuracy of the velocity and position calculation. The rate at which navigation error grows over a long period is governed predominately by the accuracy and perfection of inertial sensor manufacturing and assembly. Although accuracy improvement can be achieved by using finer sensors, there are limits to the performance that can reasonably be achieved before the cost of INS becomes prohibitively expensive. An alternative approach that has gained much attention in recent years, and is capable to compensate the INS accumulative error, is known as the integrated navigation. This technique employs external from inertial source of navigation information to improve the overall navigation accuracy, reliability and also lead to lower cost. This chapter proposes an architecture of integrated navigation using three different sensors (INS, GPS and shipboard tracking sensor) and also discusses the impact of estimation error on the autonomous system performance.

6.1 Principle of Integrated Navigation Systems

Integrated navigation systems usually use two independent sources of information with complementary characteristics; it is common that one source provides data with short term/ high frequency accuracy, while the other source long term/low frequency credibility \[2\]. For example, the inertial measurement unit (IMU) provides low noise signal of vehicle angular rate, attitude angle and body-axis acceleration, based on which the strap-down navigation algorithm computes the vehicle velocity and position. However, the integral nature of the algorithm indicates that the estimation errors will accumulate. Even a small error in the acceleration cause unbounded growth in the integrated quantities. The global positioning system (GPS) is a radio positioning system fed by satellite which has reached full operational status to provide a worldwide navigation capability \[3\]. GPS consists of 24 satellites in near-circular orbit around the Earth, the satellites orbit the Earth
with a period of 12h and are distributed in six orbital planes which are inclined at 55° to equatorial plane at a height of 20180 km. The spacing of the satellites is designed so that at any instant of time at least six satellites are in view. GPS is intended to provide three-dimensional position and velocity data to users anywhere near to the Earth. Position is calculated by measuring the time it takes for a radio signal to travel from each satellite to the GPS receiver, while the velocity is calculated from the Doppler effect. Many GPS receivers provide higher quality velocity information than position. The GPS measurement is noisy owing to a variety of reasons:
- Weak signal strength
- Limited length of the pseudo-random code
- Limited resolution of the code-tracking loop
- Multi-path noise particularly prevalent on moving vehicles
- Receiver-clock instability

Special treatment such as differential GPS or relative GPS are widely used to remove the large common errors between two GPS receivers at a specific time, enabling relative accuracy estimates of less than 1 foot. The bias-free but noisy GPS signal thus can be used to complement INS by providing periodic resets. Another signal source particular to marine time carrier is the shipboard tracking system. Based on the working principle, shipboard tracking system can be microwave-based, radar-based, vision-based or Electro-Optical Tracking System (EOTs). EOT uses a combination of electronic and optical sensor to locate and follow air-object surrounding ship in terms of range, azimuth and elevation. EOTs are often mounted on a gyro-stabilized platform to provide stable sensing. In any case, shipboard tracking system is able to measure the relative position of air-object to ship. A data fusion algorithm therefore is needed to combine the multiple signal sources providing high quality estimation and also increase the overall system reliability.

A simple data fusion technique often used to combine measurements is the complementary filter. The basic complementary filter is schematized in Figure 6.1, where \(x\) and \(y\) are noisy measurements of the same signal \(z\) and \(\hat{z}\) is the estimation produced by the filter. Assume \(y\) is corrupted by high frequency noise \(n_2\), and the noise \(n_1\) in \(x\) has low frequency characteristics. If \(G(s)\) is a low-pass filter, \(1 - G(s)\) is a high-pass filter then the most credible part of each information source is extracted and assembled into an optimal estimation. No detailed descriptions of the noise process need to be specified in the complementary filter design, but instead only the cross-over frequency or equivalently the time constant need to be tuned during simulation or flight test.
The simplicity of the CF structure indicates it is applicable to a single-parameter estimation. When the estimation is needed for multiple parameters governed by a coupled system equation, and the measurements are expressed in a linear/non-linear function of to-be estimated parameters (like in the navigation problem where both position and velocity need to be estimated simultaneously), Kalman filters are used.

### 6.2 A Quick Review of Kalman Filter Algorithm

Kalman filter is designed to provide the optimal estimation of the state variables of a stochastic dynamic process with noisy measurements. To demonstrate a linear discrete-time Kalman filter, the following discrete-time systems are considered:

\[
x_{k+1} = Ax_k + Bu_k + Bw_k
\]  
(6.2.1)

\[
y_{k+1} = Cx_k + Du_k + Dv_k
\]  
(6.2.2)

The dynamic process is corrupted by a Gaussian noise \( w_k \sim N(0, \Sigma_w) \) which is called the process noise. While the measurement is corrupted by another Gaussian noise \( v_k \sim N(0, \Sigma_v) \). The state variable vector \( x \) is thus stochastic whose mean value \( \bar{x} \) and covariance \( P \) propagate through linear system as quantified in Eqn 6.2.3-6.2.4

\[
\bar{x}_{k+1} = A\bar{x}_k + Bu_k
\]  
(6.2.3)

\[
P_{k+1} = AP_kA^T + Bw\Sigma_w B_w^T
\]  
(6.2.4)

The obtained above state estimate and covariance can then be combined with measurement through least mean-square error estimation. To clarify the Kalman filter algorithm, the following notation is adopted:

- \( \hat{x}_{k|m} \) The best estimate of \( x \) at the time step \( k \) given best measurement up to time step \( m \)
- \( P_{k|m} \) The associated covariance

The discrete time Kalman filter consists of two steps

1. **Time update (prediction step)**, this step propagates the estimate forward in time by one step from the previous best estimate

![Figure 6.1.1 Basic Complementary Filter](image-url)
\hat{x}_{k|k-1} = A\hat{x}_{k-1} + B_w u_{k-1} \quad (6.2.5)

\quad p_{k|k-1} = Ap_{k-1}A^T + B_w \sum_w B_w^T \quad (6.2.6)

2. Measurement update (correction step), based on the prior information from step 1, this step drives a minimum variance estimator to incorporate the latest measurement

\quad K_k = p_{k|k-1}C^T \left(C^T p_{k|k-1}C^T + D_v \sum_v D_v^T \right)^{-1} \quad (6.2.7)

\quad p_{k|k} = (I - K_kC) p_{k|k-1} \quad (6.2.8)

\quad \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \left(y_k - C \cdot \hat{x}_{k|k-1}\right) \quad (6.2.9)

In case of non-linear system defined in 6.2.10-6.2.11, a variation called Extended Kalman Filter (EKF) algorithm can be applied to handle the non-linearity.

\quad x_k = f(x_{k-1}, u_{k-1}, w_{k-1}) \quad (6.2.10)

\quad y_k = g(x_k, v_k) \quad (6.2.11)

The essence of EKF is to supply the linear Kalman filter with the system matrices generated from linearizing Eqn 6.2.10-6.2.11 around the current best estimate of the state.

\quad \frac{\partial f}{\partial x} \bigg|_{x_{k-1}} \quad (6.2.12)

\quad B_w = \frac{\partial f}{\partial w} \bigg|_{x_{k-1}} \quad (6.2.13)

\quad C = \frac{\partial g}{\partial x} \bigg|_{x_{k-1}} \quad (6.2.14)

\quad D_v = \frac{\partial g}{\partial v} \bigg|_{x_{k-1}} \quad (6.2.15)

In contrast to linear Kalman filter, the EKF algorithm uses non-linear relation in the state propagation and innovation. In the computation of state covariance and Kalman gain, however, localization matrices are used.

1. Time update (prediction step)

\quad \hat{x}_{k|k|k-1} = f(\hat{x}_{k-1}, u_{k-1}, 0) \quad (6.2.16)

\quad p_{k|k-1} = Ap_{k-1}A^T + B_w \sum_w B_w^T \quad (6.2.17)

2. Measurement update (correction step)
\[ K_k = p_{kj-1}C^T \left( C \cdot p_{kj-1}C^T + D_c \sum D_c^T \right)^{-1} \]  
(6.2.18)

\[ p_{kj} = \left( I - K_kC \right)p_{kj-1} \]  
(6.2.19)

\[ \hat{x}_{kj} = \hat{x}_{kj-1} + K_k \left[ y_k - g(\hat{x}_{kj-1}, 0) \right] \]  
(6.2.20)

### 6.3 Mathematical model of Navigation

Depending on the installation of Inertial Measurement Unit (IMU), the Inertial Navigation System can be platformed (if IMU is mounted on a gimbaled frame) or strap-down (if IMU is mounted rigidly on vehicle body). The IMU on modern air or marine time vehicles are usually mounted on vehicle body to measure the body-axis acceleration, thus the mathematical model is the same set used for strap-down navigation \(^5\)\(^6\). Since the helicopter application is short-range, and short-duration, application of strict round earth model is not necessary. Assuming the GPS position of both helicopter and ship are preliminarily converted into a local inertial frame with collocated NED coordinate system, the navigation algorithm will be derived from the equations 6.3.1-6.3.2 according to Flat-Earth model. This set of equations is general and thus can be applied to both ship and helicopter.

\[
\begin{bmatrix}
    \dot{X}_N \\
    \dot{Y}_E \\
    \dot{Z}_D
\end{bmatrix}
= T_{e2b}^T
\begin{bmatrix}
    u \\
    v \\
    w
\end{bmatrix}
\]

\[ \dot{u} = vr - wq + a_{bx} \]

\[ \dot{v} = wp - ur + a_{by} \]

\[ \dot{w} = uq - vp + a_{bz} \]

According to basic strap-down navigation theory, the body-axis acceleration \(a_{bx} \), \(a_{by} \), \(a_{bz} \) are measured by IMU and integrated in time to obtain velocity \([u, v, w]\) and position \([X_N, Y_E, Z_D]\). The equations of navigation are non-linear and involve external variables \([p, q, r, \phi, \theta, \psi]\) that are also measured by IMU. Theoretically the angular rate and attitude angle should also be estimated as a part of the navigation system, however, the commercially available IMUs have imbedded algorithms that compensate for offsets and other gyro errors, thus further estimation algorithm is not warranted for the attitude and angular rate. A 1\(^{st}\) order linearized discrete-time version of the navigation equations is presented in Eqn 6.3.3-6.3.4.

\[
\begin{bmatrix}
    u(k) \\
    v(k) \\
    w(k)
\end{bmatrix}
= \Delta T \cdot
\begin{bmatrix}
    s \Delta T & r(k-1) & -q(k-1) \\
    -r(k-1) & s \Delta T & p(k-1) \\
    q(k-1) & -p(k-1) & s \Delta T
\end{bmatrix}
\begin{bmatrix}
    u(k-1) \\
    v(k-1) \\
    w(k-1)
\end{bmatrix}
+ \Delta T \cdot
\begin{bmatrix}
    a_{bx}(k-1) \\
    a_{by}(k-1) \\
    a_{bz}(k-1)
\end{bmatrix}
\]

(6.3.3)
\[
\begin{align*}
X_n(k) &= X_n(k-1) + u(k-1) \\
Y_n(k) &= Y_n(k-1) + \Delta T \cdot T_e^{(k-1)} v(k-1) \\
Z_n(k) &= Z_n(k-1) + w(k-1)
\end{align*}
\] (6.3.4)

Shipboard Tracking System (STS) measures the relative range, azimuth and elevation angle of helicopter in the Ship Heading Frame, thus produces the following outputs:

\[
R_{rel} = \sqrt{(X_{N,ship} - X_{N,helo})^2 + (Y_{N,ship} - Y_{N,helo})^2 + (Z_{N,ship} - Z_{N,helo})^2}
\] (6.3.5)

\[
\psi_{rel} = \sin^{-1} \left( \frac{(X_{N,helo} - X_{N,ship}) \sin \psi_{ship} + (Y_{N,helo} - Y_{N,ship}) \cos \psi_{ship}}{\sqrt{(X_{N,ship} - X_{N,helo})^2 + (Y_{N,ship} - Y_{N,helo})^2}} \right)
\] (6.3.6)

\[
\theta_{rel} = \sin^{-1} \left( \frac{-Z_{D,helo} - Z_{D,ship}}{\sqrt{(X_{N,ship} - X_{N,helo})^2 + (Y_{N,ship} - Y_{N,helo})^2 + (Z_{N,ship} - Z_{N,helo})^2}} \right)
\] (6.3.7)

An assumption is made for ease of the linearization of \( \psi_{STS} \) and \( \theta_{STS} \), which states that the denominators (range) hold constant during evaluation, for STS working at an adequate distance this is fairly true. Therefore, the linearization of measurement incremental are in Eqn 6.3.8-6.3.10

\[
\Delta R_{rel} = \frac{(X_{N,ship} - X_{N,helo}) \Delta X_{N,helo} + (Y_{N,ship} - Y_{N,helo}) \Delta Y_{N,helo} + (Z_{N,ship} - Z_{N,helo}) \Delta Z_{N,helo}}{\sqrt{(X_{N,ship} - X_{N,helo})^2 + (Y_{N,ship} - Y_{N,helo})^2 + (Z_{N,ship} - Z_{N,helo})^2}}
\] (6.3.8)

\[
\Delta \psi_{rel} = -\sin \psi_{ship} \frac{\Delta X_{N,helo}}{\cos \psi_{ship}} + \frac{\Delta Y_{N,helo}}{\cos \psi_{ship}} - \cos \psi_{ship} \Delta Y_{N,ship}
\] (6.3.9)

\[
\Delta \theta_{rel} = \frac{-\Delta Z_{D,helo} + \Delta Z_{D,ship}}{\cos \theta_{rel}[k-1] \sqrt{(X_{N,ship} - X_{N,helo})^2 + (Y_{N,ship} - Y_{N,helo})^2 + (Z_{N,ship} - Z_{N,helo})^2}}
\] (6.3.10)

### 6.4 EKF Applied to Shipboard Navigation

The design of EKF requires the knowledge of the noise characteristics of the dynamic process (introduced by acceleration in Eqn 6.3.2) as well as that of the measurement (GPS Position, velocity and STS range, azimuth and elevation). The measurement noises are assumed to be mutually independent \(^7\) and subject to Gaussian distribution with the statistic properties summarized in Table 6.4.1

#### Table 6.4.1 Statistical Properties of Sensors

<table>
<thead>
<tr>
<th>Sensor Type</th>
<th>Channel</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shipboard Tracking System (STS)</td>
<td>Range</td>
<td>0.5 ft</td>
</tr>
<tr>
<td></td>
<td>Azimuth</td>
<td>0.1°</td>
</tr>
<tr>
<td></td>
<td>Elevation</td>
<td>0.1°</td>
</tr>
<tr>
<td>GPS Position</td>
<td>North</td>
<td>1 ft</td>
</tr>
<tr>
<td></td>
<td>East</td>
<td>1 ft</td>
</tr>
<tr>
<td></td>
<td>Height</td>
<td>2 ft</td>
</tr>
</tbody>
</table>
In order to formulate a canonical EKF problem, the following definitions of state vector and output vector are employed:

\[
X_{\text{est}} = [X_{N,\text{ship}} \quad Y_{N,\text{ship}} \quad Z_{D,\text{ship}} \quad u_{\text{ship}} \quad v_{\text{ship}} \quad w_{\text{ship}} \quad \ldots]
\]

\[
y_{\text{sens}} = [X_{\text{gps,ship}} \quad Y_{\text{gps,ship}} \quad Z_{\text{gps,ship}} \quad V_{\text{gps,ship}} \quad V_{\text{gps,ship}} \quad V_{\text{gps,ship}} \quad \ldots]
\]

Because of the coupling effect of STS measurement, the ship states and rotorcraft states must be estimated simultaneously in an integrated frame of EKF. The overall system equations are represented in a matrix form in Eqn 6.4.1, additional variables are the process noise introduced by accelerometer and thus have the statistic properties given in Table 6.4.1.

\[
X_{N,\text{ship}}(k) = A_{N,\text{ship}} X_{N,\text{ship}}(k-1) + B_{N,\text{ship}} + W_{N,\text{ship}}
\]

The non-zeros block of matrices A and B are listed as following

\[
A(1:3,1:3) = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
A(1:3,4:6) = \Delta T \cdot T_{\text{EKF}}^{T} (\phi_{\text{ship}}(k-1), \theta_{\text{ship}}(k-1), \psi_{\text{ship}}(k-1))
\]
The measurement equations are listed in Eqn 6.4.10-6.4.16, additional variables are measurement noises of GPS and STS whose stochastic properties are given in Table 6.4.1.

\[
\begin{align*}
X_{N,\text{gps,ship}}(k) &= X_{N,\text{ship}}(k) + V_{Xu,\text{ship}}(k) \\
Y_{E,\text{gps,ship}}(k) &= X_{E,\text{ship}}(k) + V_{Ye,\text{ship}}(k) \\
Z_{D,\text{gps,ship}}(k) &= Z_{D,\text{ship}}(k) + V_{Zd,\text{ship}}(k)
\end{align*}
\]

\[
\begin{bmatrix}
V_{N_{\text{gps,ship}}}(k) \\
V_{E_{\text{gps,ship}}}(k) \\
V_{D_{\text{gps,ship}}}(k)
\end{bmatrix} = T\varepsilon_{2b} \begin{bmatrix}
\phi_{\text{ship}}(k) \\
\theta_{\text{ship}}(k) \\
\psi_{\text{ship}}(k)
\end{bmatrix} \begin{bmatrix}
u_{\text{ship}}(k) \\
v_{\text{ship}}(k) \\
w_{\text{ship}}(k)
\end{bmatrix} + \begin{bmatrix}
u_{\text{u,ship}}(k) \\
u_{\text{v,ship}}(k) \\
u_{\text{w,ship}}(k)
\end{bmatrix}
\] (6.4.11)

\[
\begin{align*}
X_{N,\text{gps,helo}}(k) &= X_{N,\text{helo}}(k) + V_{Xu,\text{helo}}(k) \\
Y_{E,\text{gps,helo}}(k) &= X_{E,\text{helo}}(k) + V_{Ye,\text{helo}}(k) \\
Z_{D,\text{gps,helo}}(k) &= Z_{D,\text{helo}}(k) + V_{Zd,\text{helo}}(k)
\end{align*}
\]

\[
\begin{bmatrix}
V_{\text{gps,helo}}(k) \\
V_{E_{\text{gps,helo}}}(k) \\
V_{D_{\text{gps,helo}}}(k)
\end{bmatrix} = T\varepsilon_{2b} \begin{bmatrix}
\phi_{\text{helo}}(k) \\
\theta_{\text{helo}}(k) \\
\psi_{\text{helo}}(k)
\end{bmatrix} \begin{bmatrix}
u_{\text{helo}}(k) \\
v_{\text{helo}}(k) \\
w_{\text{helo}}(k)
\end{bmatrix} + \begin{bmatrix}
u_{\text{u,helo}}(k) \\
u_{\text{v,helo}}(k) \\
u_{\text{w,helo}}(k)
\end{bmatrix}
\] (6.4.13)
\[ R_{STS}(k) = \sqrt{(X_{N,ship}(k) - X_{N,helo}(k))^2 + (Y_{N,ship}(k) - Y_{N,helo}(k))^2 + (Z_{N,ship}(k) - Z_{N,helo}(k))^2} + V_r(k) \] (6.4.14)

\[ \psi_{STS}(k) = \sin^{-1} \left( \frac{(X_{N,helo}(k) - X_{N,ship}(k))\sin\psi_{ship}(k) + (Y_{N,helo}(k) - Y_{N,ship}(k))\cos\psi_{ship}(k)}{\sqrt{(X_{N,ship}(k) - X_{N,helo}(k))^2 + (Y_{N,ship}(k) - Y_{N,helo}(k))^2} + V_\psi(k)} \right) \] (6.4.15)

\[ \theta_{STS}(k) = \sin^{-1} \left( \frac{-(Z_{D,helo}(k) - Z_{D,ship}(k))}{\sqrt{(X_{N,ship}(k) - X_{N,helo}(k))^2 + (Y_{N,ship}(k) - Y_{N,helo}(k))^2} + V_\theta(k)} \right) \] (6.4.16)

The localization matrix of measurement C in Eqn 6.4.17 can be calculated using the linearization relation in Eqn 6.3.8-6.3.10

\[ C = \frac{\Delta v_{\text{sens}}}{\Delta x_{\text{est}}} \] (6.4.17)

The non-zero elements of C matrix are:

\[ C(1:3,1:3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \] (6.4.18)

\[ C(4:6,4:6) = T_{x2b}^T \left( \phi_{ship}(k), \theta_{ship}(k), \psi_{ship}(k) \right) \] (6.4.19)

\[ C(7:9,7:9) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \] (6.4.20)

\[ C(10:12,10:12) = T_{x2b}^T \left( \phi_{helo}(k), \theta_{helo}(k), \psi_{helo}(k) \right) \] (6.4.21)

\[ \text{Rang}_{\text{est}}(k-1) = \sqrt{(X_{N,ship}(k-1) - X_{N,helo}(k-1))^2 + (Y_{N,ship}(k-1) - Y_{N,helo}(k-1))^2 + (Z_{D,ship}(k-1) - Z_{D,helo}(k-1))^2} \] (6.4.22)

\[ C(13,1) = \frac{X_{K,ship}(k-1) - X_{K,helo}(k-1)}{\text{Rang}_{\text{est}}(k-1)} \] (6.4.23)

\[ C(13,2) = \frac{Y_{E,ship}(k-1) - Y_{E,helo}(k-1)}{\text{Rang}_{\text{est}}(k-1)} \] (6.4.24)

\[ C(13,3) = \frac{Z_{D,ship}(k-1) - Z_{D,helo}(k-1)}{\text{Rang}_{\text{est}}(k-1)} \] (6.4.25)

\[ C(13,7) = \frac{X_{K,helo}(k-1) - X_{K,ship}(k-1)}{\text{Rang}_{\text{est}}(k-1)} \] (6.4.26)

\[ C(13,8) = \frac{Y_{E,helo}(k-1) - Y_{E,ship}(k-1)}{\text{Rang}_{\text{est}}(k-1)} \] (6.4.27)
\[ C(13, 9) = \frac{Z_{D.helo}(k-1) \cdot Z_{D.ship}(k-1)}{Rang_{est}(k-1)} \]  

(6.4.28)

\[ C(14, 1) = \frac{\sin \psi_{ship}(k)}{\cos \psi_{ship}(k)} \]  

(6.4.29)

\[ C(14, 2) = -\frac{\cos \psi_{ship}(k)}{\cos \psi_{est}(k)} \]  

(6.4.30)

\[ C(14, 7) = -\frac{\sin \psi_{ship}(k)}{\cos \psi_{ship}(k)} \]  

(6.4.31)

\[ C(14, 8) = \frac{\cos \psi_{ship}(k)}{\cos \psi_{est}(k-1)} \]  

(6.4.32)

\[ C(15, 3) = \frac{1}{\cos \theta_{est}(k-1) \cdot Rang_{est}(k-1)} \]  

(6.4.33)

\[ C(15, 9) = -\frac{1}{\cos \theta_{est}(k-1) \cdot Rang_{est}(k-1)} \]  

(6.4.34)

Substituting the non-linear system equations, measurement equations and localization matrices together with the sensors noise covariance matrices into the EKF algorithm in section 6.2, the deck state and rotorcraft state can be estimated with an optimal noise suppression. The EKF algorithm is illustrated by the flow chart in Appendix B.

### 6.5 Off-line Test of the EKF Algorithm

Using the recorded data of flight simulation, the EKF algorithm has been tested then compared with GPS and INS output. To fabricate the signal with realistic error properties, the actual signals are corrupted by white noise to produce GPS and IMU reading. Besides, a pure INS is subject to unbounded integral error not only because the accelerometers are corrupted by white noise, but also by bias-type of errors. Two significant types of bias contribute primarily, first is the bias variation from one turn-on to another depending on the thermal condition of application, for a moderate accuracy IMU this bias is well controlled within 0.003g [8]. In simulation, the bias variation is usually modeled as a random variable given in the beginning but keeps constant during the simulation. The other type of bias is bias-drifting caused by the null-shift in the electrical pick-off device [8], this kind of error can be model as a random walk generated by integrating a white noise with its standard deviation to the order of 10^{-2} g/\sqrt{h}. The marching of EKF algorithm requires an initial guess of the estimated state along with the covariance matrix, the GPS measured data and noise level are used for this purpose. The testbed setup of EKF algorithm is schematized in Appendix C. Figure 6.5.1-6.5. demonstrated the effectiveness of EKF in estimating the rotorcraft position and inertial velocity, same trend can be found for the estimation of deck state.
Figure 6.5.1-a Estimation of Rotorcraft Position

Figure 6.5.1-b Zoom-in Plot of 6.5.1-a

Figure 6.5.2-a Estimation of Rotorcraft Inertial Velocity

Figure 6.5.2-b Zoom-in Plot of 6.5.2-a
Figure 6.5.3-a Convergence History of Standard Deviation of Estimated Rotorcraft Position

Figure 6.5.3-b Zoom-in Plot of 6.5.3-a

Figure 6.5.4-a Convergence History of Standard Deviation of Estimated Rotorcraft Velocity

6.5.4-b Zoom-in Plot of 6.5.4-a

Figure 6.5.5 EKF Estimation Error of Rotorcraft Position

Figure 6.5.6 INS Estimation Error of Rotorcraft Position
Unlike the INS estimated data, EKF estimation is free from integral error (unbounded growth in Figure 6.5.6 and 6.5.8); in the meantime, EKF estimation reduced the standard deviation of position noise from 1 ft original of GPS to 0.1 ft, the standard deviation of velocity noise from 0.5 ft/sec original of GPS to 0.21 ft/sec. The average estimation error of position and velocity are therefore adequate to support a precision approach and accurate recovery.

6.6 On-line Test of EKF algorithm
The EKF algorithm has been integrated into FLIGHTLAB model to evaluate its impact on control accuracy and overall task performance. Extensive tests and evaluations have been performed on helicopter models of different platform (light, medium, heavy) using the autonomous approach and predictive landing techniques. Figure 6.6.1-6.6.4 summarized the major results of Monte-Carlo test.
Figure 6.6.1 Scatters of Vehicle Ref Point upon Touchdown (Heavy Class)

Figure 6.6.2 Scatters of Touchdown Velocity Error (Heavy Class)
Figure 6.6.3 Scatters of Vehicle Ref Point upon Touchdown (Medium Class)

Figure 6.6.4 Scatters of Touchdown Velocity Error (Medium Class)
Figure 6.6.5 Scatters of Vehicle Ref Point upon Touchdown (Light Class)

Figure 6.6.6 Scatters of Touchdown Velocity Error (Light Class)
6.7 Summary
An Extended Kalman Filter was developed to combine three different signal sources (GPS, INS and STS) based on an achievable data set of instrumentations typical for commercial-grade products. the data fusion demonstrated a largely improved navigation error and noise level, simulations also approved a tolerable performance deterioration of designed autonomous system working with the non-ideal navigation signal.

6.8 Reference
Chapter 7. Conclusions and Future Work

Contents

7.1 Conclusions
7.2 Future Work

7.1 Conclusions

An autonomous shipboard recovery system was designed to perform rotorcraft landing on flight deck in rough sea. The achievements of this study include: A Dynamic Inversion trajectory following controller enabling simultaneous regulation of acceleration, velocity and position on three axes. A representation and optimization algorithm to yield criteria-abiding approach path. Guidance law for approach path following. Snap-to-grid guidance law for station keeping. Representation and optimization of descent program. Hybrid guidance law for landing path following. Level out maneuver for attitude mismatch avoidance. EKF data fusion for multi-sensor integration. Major conclusions from this study are summarized below:

1. Linear DI controller with gain scheduling can be used to design ACAH mode as well as trajectory following controller with designated stability and control augmentation. Both linear analysis and non-linear simulation have approved the effectiveness of DI controller in addressing challenging helicopter control design requirements.

2. The results of this study support that the traditional stability margin and disturbance rejection bandwidth though established on SISO theory, are still giving valuable measure to a MIMO close-loop system robustness and performance. The regulations on stability margin and disturbance rejection properties are still a good initial criterion to satisfy towards designing a capable for shipboard operation controller. The stability regulation due to its conservativeness may be relaxed in case of necessity, and with proven safety. MIMO stability margin criteria suggested a similar stability disposition, thus, can be used to provide a double proof.

3. The handling quality regulation of piloted rotorcraft can be adopted to guide the controller design in the first iteration. This study used the response quickness suggested by HQ regulatory document for initial design, yielding a fair compromise between stability margin and tracking performance, which is very close to the optimized results. Only a few tuning is needed on top of the initial guess. However, designers should keep in mind that autonomous system does have to abbey the same regulation enacted for piloted vehicle; If it is necessary and safe, certain relaxation is tolerable.

4. For in line-of-sight fight, the path breakdown and B-spline representation is adequate to handle the parameterization for upcoming optimization, the smoothness and simplicity provide robustness to avoid strange shapes which is a usual worry of other high-order methods. The many path criteria can also break down onto each plane. The point-to-curve projection guidance law assigned the path following into two parts: position controller keeps the vehicle to stay on the track; the velocity and acceleration command drive the vehicle to fly forward. This type of path following strategy is particularly suitable for controllers that can
simultaneously regulate acceleration, speed and position.

5. Predictive strategy succeeded to reduce the demand on rotorcraft agility for landing on moving deck. Since the deck state is changing over time, the landing program has been implemented strictly in time using a timed trajectory. A cubic polynomial has enough degree of freedom to represent a landing path with position matching and velocity matching simultaneously. Additional degree of freedom can be introduced to incorporate the maneuverability constraints. Hybrid implementation of landing path can be used to remove the impact of prediction error.

6. With current sensor technologies, the integrated system can provide a high-grade position signal to support rotorcraft recovery. A EKF data fusion of INS, GPS and STS in this study demonstrated a ft-level estimation although using only a 1st order scheme. The redundancy of multi sensors also provide damage-safety when part of the system was disabled. Simulation demonstrated that with the realistic sensor measurement, the overall system performance is still reserved to an acceptable level.

7.2 Future Work
There are several studies that can improve the coverage and depth of this investigation and are listed below

1. Future plant may include an engine and drive-train model to account for the RPM variation so that the heave axis can be evaluated for useful stability margin information
2. Future plant may include a tail-boom elastic model to account for the anti-torque delay so that the yaw axis can be evaluated for useful stability margin
3. Suitable for maritime rotorcraft stability and response criteria may be explored based on the product of this study
4. The impact of modeling error including architecture error and parameter error on performance degradation may be researched using the product of this study.
5. More complex criteria can be added into the approach profile, such as possible hazardous zone avoidance.
6. Advanced approach profile apart from the Heffley profile may be developed, since this is a computer-controlled approach, intuitive profile is not a must.
7. Ship motion with larger amplitude may be used to test the system performance. The quantification of applicable sea state should be theoretically established.
8. A flight test on UAV may be arranged to validate the design concept
## Appendix A

### Table A.1. Reference Frame and Coordinate Systems

<table>
<thead>
<tr>
<th>Reference Frame</th>
<th>Coordinate Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat-Earth Inertial Frame</td>
<td>North-East-Down (NED), X-axis aligned with North direction, Y-axis aligned with East direction, Z-axis aligned with Down direction</td>
</tr>
<tr>
<td>Body frame</td>
<td>Vehicle body-fixed system (BODY), origin in vehicle center of gravity, X-axis aligned with structural longitudinal axis going forward, Z-axis located in structural vertical plane and perpendicular to X-axis going downward, Y-axis perpendicular to X-Z plane going rightward</td>
</tr>
<tr>
<td>Helicopter Heading Frame (HHF)</td>
<td>Helicopter Heading System, origin in helicopter center of gravity, X-axis located in horizontal plane and pointing to vehicle heading direction; Z-axis pointing to inertial down, Y-axis perpendicular to X-Z plane going rightward</td>
</tr>
<tr>
<td>Ship Heading Frame (SHF)</td>
<td>Ship Heading System, origin in ship center of gravity, X-axis located in horizontal plane and pointing to vehicle heading direction; Z-axis pointing to inertial down, Y-axis perpendicular to X-Z plane going rightward</td>
</tr>
</tbody>
</table>

![Figure A.1 Relation between Coordinate Systems](image)

**Figure A.1 Relation between Coordinate Systems**
Appendix B

Linear Simulink Model
The linear model implemented in Matlab/Simulink is a replication of FLIGHTLAB controller with the non-linear operators and feed-forward loops omitted. The linear analysis model uses 46 state full order model generated from a level flight trimmed at 20 knots, 300 ft. Matlab command \textit{linmod} was invoked to extract transfer function from in-port to out-port using perturbation techniques. Script in Matlab m-file has been created to programmatically scan over all channels of interest and generate figure plot.
Figure B.1: Inner-loop and Outer-loop Controller Connection
Figure B.2: Longitudinal and Lateral Position Controller
Figure B.3: Vertical Speed Command Model

Figure B.4: Vertical Speed Compensator

Figure B.5: Roll Angle Command Model
Figure B.6: Roll axis PID Compensator

Figure B.7: Pitch Angle Command Model

Figure B.8: Pitch axis PID Compensator

Figure B.9: Yaw Rate Compensator
Figure B.10: Yaw axis PID Compensator

Figure B.11: Non-Linear Euler Conversion
Figure B.12: Plant model and Dynamic Inversion Loop

Figure B.13: Reconstruction of State Variables from Output Vector
Figure B.14: Reconstructed Vertical PID Compensator

Figure B.15: Reconstructed Roll axis PID Compensator

Figure B.16: Reconstructed Pitch axis PID Compensator
Figure B.17: Reconstructed yaw axis PID Compensator
Figure C.1 Flow chart of EKF algorithm of Integrated Navigation System

Initial Guess: $\hat{x}_{0|0}$, $P_{0|0}$

$k=1$

Calculate localization matrix:
$A$, $B_w = I_{12}$

Time update estimated variable and error covariance matrix:
$\hat{x}_{k|k-1} = \text{nonlinear system equation}(\hat{x}_{k-1|k-1}, a_{bx,ship}[k-1], a_{by,ship}[k-1], a_{bx,helo}[k-1], a_{by,helo}[k-1], A_{k|k-1}, b_{Wp,\epsilon}, b_{Wp})$

$P_{k|k-1} = AP_{k-1|k-1}A^T + BW_pS_{wP}BW_p^T$

Prediction step

$k=k+1$

Calculate linear measurement matrix:
$C$, $D_v = I_{12}$

Calculate Kalman gain:
$K_k = P_{k|k-1}C^T(CP_{k|k-1}C^T + D_vS_{v}D_v^T)^{-1}$

Measurement update of estimated variable and error covariance matrix:
$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - \text{nonlinear measurement equation}(\hat{x}_{k|k-1}))$

$P_{k|k} = (I-K_k)P_{k|k-1}$

Correction step

$(\phi[k-1], \theta[k-1], \psi[k-1])$

$(p[k-1], q[k-1], r[k-1])_{\text{ship/helo}}$

$(a_{bx}[k-1], a_{by}[k-1], a_{bz}[k-1])_{\text{ship/helo}}$
Figure D.1 Setup of Extended Kalman Filter Test Bed
VITA

Junfeng Yang

Date of Birth: December 17, 1986
Place of Birth: Chongqing, People’s Republic of China

Education
PhD in Aerospace Engineering at Pennsylvania State University, January 2015 – August 2018
MSc in Aerospace Engineering at Moscow Institute of Physics and Technology, September 2009 – June 2012.
BEng in Aerospace Engineering at Beijing University of Aeronautics and Astronautics, September 2004 – June 2008

Professional Activities
Rotorcraft simulation and control, automatic control modes and path optimization of shipboard recovery in high sea states, Pennsylvania State University, 2015-2018
C919 flight performance analysis, stability and handling quality analysis, flight simulation and control law development, Commercial Aircraft of China, 2013-2015
C919 supercritical wing design, high-lift device analysis and design, airframe-engine integration, Commercial Aircraft of China, 2012-2013
Aerodynamic design of Superjet-100, MS-21, TU-204 stability and control, Central Aero-hydrodynamics Institute of Russia, 2010-2012

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