AN ESTIMATION OF GEOMETRIC CHANGES IN THE PROXIMAL
FEMURS OF US ADULTS FROM 1988-2014

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Abstract

Total hip replacement (THR) is one of the most common operations in the world. As a result, there are a number of studies focused on improved femur implant design. Among these are studies focused on the measurement of femur geometry. Many of these use data from cadavers and/or scanned data to collect information about femur geometry. However, these data are not widely available and are typically from a convenience sample. As a result, data on the range of geometries across a population are scarce. This project aims to mitigate this by using a statistical shape model to quantify the range of variability. Body mass index (BMI, a measure of weight-for-stature), age, gender, and stature are predictor variables in the model. To generate a representative sample of femurs, the data from NHANES (National Health and Nutrition Examination Survey) III and NHANES 07-14 were used. A total of 36,019 femurs were generated using NHANES population demographics. A sample population containing 216,509 individuals was generated using weighted data in NHANES 07-14. Measurements of the proximal femur (head diameter, neck diameter, shaft diameter and neck-shaft angle) were made on the synthesized femurs using shape fitting algorithms. Gender different were investigated with controlling input variables. The males measurements were found significantly bigger than females except for neck-shaft angle. The measurements collected from synthesized NHANES III are compared with data from the real NHANES III proximal femur measurements gathered from 2D x-ray images. The synthesized NHANES III measurements were compared with synthesized NHANES 1314 and, the recent population measurements were found slightly bigger except neck-shaft angle. Correlation tests between input demographics and output measurements were performed. The neck-shaft angle was found strongly correlated and decreasing with age. The changes of measurement were investigated between years and races pairs. The significant changes were found in most of the year pairs except head diameter. Most of the race pairs were also found significantly different. Additionally, the relationship between bone geometry and hip fracture was investigated using NHANES 07-10 & 13-14 questionnaire data. The neck shaft angle was found significant different between non-fractured and fractured individuals. The results can be used to improve implant design for the US and other populations.
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Chapter 1

Introduction

The hip joint is the ball socket connection between the femur head and the pelvic acetabulum (Figure 1.1). This joint connects the lower body to the upper body and thus bears a lot of weight. The femur is described as a bone of the thigh or upper hind limb. It is the longest, heaviest, and strongest bone in the entire human body. Moreover, the femur supports all of the body’s weight during many activities such as running, jumping, walking, and standing. As such, extreme forces act on the femur and it is classified structurally as a long bone and as a major component of the appendicular skeleton. These loads, especially that on the hip joint, cause a lot of health problems. Also, for individuals suffering from a bone disease such as osteoporosis, the hip joint is more likely to be injured and it will wear faster than other joints such as knee, shoulder. These individuals have a greater tendency to require medical treatment for their hip joint. Total hip replacement (THR) or total hip arthroplasty (THA) are a medical terms used to describe the procedure of hip joint replacement due to the wear and tear. According to the National Center for Health Statistics (NCHS) [7], the number of THAs among patients aged over 45 was 138,700 in 2000. However, this number increased to 326,100 in 2010 [7]. Moreover, in 2010, around 73,000 hip surgeries were performed in the United Kingdom [8]. Between 1999 and 2014 in Australia, 364,427 individual patients had a hip replacement operation of which 92,855 died, equivalent to 25.5 percent [9]. Clearly, many people undergo the THA operation globally.

Many studies have been conducted, mainly focusing on the femur, to build a better understanding of the hip joint, and many of these studies have involved measurements on the femurs of cadavers. Walensy in 1965 [10], found that people have a different amount of femoral curvature in different locations and Noble et al. [11] found a correlation of standard femur dimensions. Additionally, many other studies have looked the population effect on femur design for specific populations [12–16].
Most of these studies used cadaver femurs and/or scan of the cadaver femurs or scan of the real patient femurs. However, these scanned data are not widely available or not in convenient sample. Additionally, most of the previously cited studies did not consider the gender effect due to a lack of samples and/or lack of information. As a result, most of the measurement results were given without gender separation.

In this study, femurs were generated using statistical shape modeling to mitigate sample size problem. Changes to the measurements of proximal femur geometry were analyzed. Femurs were generated using a statistical shape model developed by Klein et al. [5]. The model utilizes stature, BMI (Body Mass Index), age, and gender to create the entire femur surface mesh. The measurements were collected only from the proximal part of the femur in order to build a better understanding for THA in the US population. National Health and Nutrition Examination Survey (NHANES) data demographics were used to create population-based femurs and the measurements were taken programmatically using geometrical shape fitting algorithms.

Figure 1.1. Anterior view of hip joint [1].
1.1 Research Goals

The research goals of this effort are to

- Gather population demographics.

- Generate patient specific 3D femur by using BMI, stature, gender, and age with a statistical model [5].

- Measure the generated femurs.

- Analyze the measurements and build a better understanding for the population.

This analysis will result in

- Understanding the range in variability of femurs.
- Understanding how that range in variability of femurs has changed over time.
- Understanding how geometry might vary with other factors such as gender and ethnicity.

Additional analyses were also conducted on the measured data. Most of the NHANES data include a fracture questionnaire requesting information about the person’s hip-related history, as well as their knee and spine fracture history. The demographics of people with a history of hip fracture were then recorded. The relevant measurements of the fractured population were compared with those of the non-fractured population. Results were also compared with the NHANES III hip measurement dataset. The NHANES III data include femur measurements taken from x-ray images.
Chapter 2

Literature Review

This section introduces general information about the femur bone and the studies which are concerned with the statistical shape modeling and femur measurement.

2.1 Orientation

Standard anatomical definitions are used to describe femur orientations.

- **Superior** is a direction that towards the head end of human body [2].
- **Interior** is opposite direction of superior [2].
- **Anterior** is a direction that toward the front of human body [2].
- **Posterior** is opposite direction of anterior [2].
- **Medial** is a direction that toward the midline [2].
- **Lateral** is opposite direction of medial [2].
- **Proximal** is used for limps that indicates the direction close to skeleton axis [2].
- **Distal** is opposite direction of proximal [2].

These are explained in Figure 2.1.
2.2 Femur Geometry

The femur is the longest and strongest bone in the human body and supports more weight than any other bone especially during major activities such as standing, running, and walking [2]. The femur consists of several components, shown in Figure 2.2. These include:
⇒ **Head** is the rounded upper part of the bone connected to acetabulum of pelvis and has a spherical shape [2].

⇒ **Neck** is the connection between the head and the shaft of the femur [2].

⇒ **Greater trochanter** is large rounded part of lateral proximal femur [2].

⇒ **Lesser trochanter** is the rounded protrusion located under femoral head and the point where the neck and the shaft join [2].

⇒ **Intertrochanteric line** is the line connects the lesser and greater trochanter [2].

⇒ **Shaft** is the long part between the upper and lower ends of the femur [2].
Figure 2.2. Right femur [2]
2.3 Femur Studies

Studies have been contacted to improve understanding of femur geometry. Most of the studies used cadaver femurs to collect measurements. Recently, some studies used CT scanned real patient data. Also, there are some studies that have measured CT scanned patient and/or cadaver femurs in specific populations. However, there are not many studies that use statistical shape modeling to create specific population femurs.

Walensky [10] undertook research relating to femurs in 1965 and used 814 cadaver femurs for measurement and investigation. It was determined that the location of the femoral curvature changes with race but that effects relating to gender that would not be accounted for stature differences were found to be insignificant in regard to femoral curvature. Also, femoral shaft curvature increases with age and African American femurs are longer than those of Eskimos, American Indians, and Whites.

Noble et. al [11] collected 200 cadaver femurs with an average age of 69.9 and measured them using renephelographic imaging. A correlation analysis was performed on the dimensions of the femurs. It was found that the femoral length and head diameter are correlated with each other with a coefficient of 0.76. The correlation coefficient between the femoral length and femoral neck length was also determined to be 0.70. Moreover, the femur head diameter was determined to have a minimum value of 23.6mm and a maximum of 61mm with the average being 43mm, with a standard deviation of 6.8mm.

Feldesman et. al [17] used 51 femurs from different populations to identify a correlation between stature and femur length for a given population. The sample size was 13,149 in the study and the mean ratio of femur length and stature was found to be 26.74%. There are no significant differences between male and female ratios, however, there are significant race differences.

Furthermore, Feldesman and Fountain [18] undertook the following study to assess the race effect on the femur/stature ratio. Mean absolute deviation, mean squared error, and Pitman’s measure of closeness for 798 known sample ratios were used in the validation process. Results indicate that the “Black” femur/stature ratio significantly differs from other populations (“Whites,” “Asians”). However, the results did not carry a high statistical significance due to weak group coherence. Therefore, it was suggested that using a generic femur/stature ratio is better than using a race-specific ratio.

Mahaisavariya et. al [13] used 108 Thai cadaver femurs to generate a proximal femur geometry. Femurs (26 male, 22 female, 12 unknown) were CT scanned. Then, a reverse engineering technique (shape fitting) was used to create a 3D CAD model. Sphere fitting was used for the femur head, a circle and ellipse fit was used for the femoral neck, and a circle fit was used for the femur shaft. Head diameter, neck axis, shaft axis, and neck-shaft angle were calculated for each femur. Head diameter was found to be smaller in comparison to the value reported in [11] using Caucasian data.

Atilla et. al [12] measured 114 Turkish cadaver femurs to build a national data base. Measurements were collected using x-ray imaging of the proximal femur, as used by Noble et. al,
and compared with reported Western population data by Noble et al. Both similarities and differences within the Western data were identified. The femoral head diameter was found to be similar, but the head position and neck-shaft angle were higher than the Western population.

Zheng and Schumann [19] investigated pathologic and non-pathologic cases involving 22 cadaver bones to create patient-specific proximal femurs. A statistical shape model was used to generate a 3D surface model using calibrated x-ray radiographs. A point distribution model and principal component analysis were used for the statistical model. The model works interactively with radiograph images, and has a surface error distance of 0.95 mm on average.

Bailey et al. [20] used computer navigation systems without an image to achieve 37 hip resurfacing operations. Patient-specific proximal femurs were created with a computer navigation system which helps the placement of the femoral component to be accurate and yields information about the implant alignment during the surgery processes.

Bryan et al. [21] developed a statistical model that generates the whole femur to assist in computer-aided medicine. Principal component analysis was used to generate a statistical model using 21 individual CT scanned femurs and 1000 femurs were generated using the statistical model. Finite element analysis (FEA) was conducted on these models to understand risks factor relating to femoral neck fracture. A “falling to the side” scenario was used in the FEA since it is frequently seen in elderly patients and also it has a lower load compared to other approaches. In this scenario, 28 of the 1000 femurs had the highest fracture risk. Moreover, fracture location was shown to be more likely intertrochanteric, as supported by previous studies.

Schumann et al. [22] used 16 cadaver bones to validate a statistical shape model and a 2D surface model of the proximal femur. Unlike their previous study [19], these authors used clinically related morphometric parameters to calculate the error between the real and the generated femur. No significant difference between the generated and the cadaver CT scanned femurs was found. Moreover, the model succeeded in generating femurs that are both normal and outliers.

Bryan et al. [23] also created a statistical model that can be used in finite element analysis. The high-resolution model was developed to be fully automatic and 46 CT scans were used to generate the models. It was also determined in this work that 35 eigenvectors were needed for accurate reproduction of the femur. Moreover, 1000 femurs were generated via this technique and compared with data in the National Health and Nutrition Examination Survey III (NHANES III) [4]. The comparison is made by observing the distribution of measurements. Additionally, the correlation between stature and femur length was examined.

Galibarov et al. [24] generated a model that uses 2D x-ray images of the proximal femur to create patient-specific 3D proximal femur geometry. The model detects the counter of the 2D x-ray image and finds the closest match in the generic 3D model library, warping the selected generic model to determine the best fit.

Zhu and Li [25] used a statistical shape model (SSM) to generate 3D knee joints using 2D joint images of 40 distal femurs. The prediction uses 2D fluoroscopic images to generate the distal femur. The model can be used in navigating total knee arthroplasty and is also helpful for designing patient-specific knee implants.

Umer et al. [16] conducted a morphologic study of the measurement of proximal femurs in the
Pakistani population. They used x-ray images of 166 males and 20 females who were healthy to measure the proximal femurs. Twelve measurements were collected in this study and compared with Noble et al.’s study [11].

Cho et al. [14] generated a 3D model of femurs gathered from 202 Koreans and employed a 3D model with a geometrical computation program. Twenty-eight different measurements were collected from the 3D femurs, which were all CT scanned and healthy. The femurs from 88 men and 114 women were included in the data. The mean age for the males was 50 years and for the females 54 years. The mean stature is given in the paper; mean stature for the males was 1670 mm and for the females 1564 mm. The study also used two different neck-shaft angles; the first was the 2D angle which is calculated when the femur is aligned by the osteometric board and the second was the 3D angle.

Klein et al. [5] generated a model based on data from 62 male and 36 female CT scans of right femurs. Principal component analysis was used to define relationship between femur geometry and patient demographics. Principal scores are generated as a function of age, BMI, and femur length through a linear regression. Instead of using stature as an input, Klein et al. [5] utilized femur length due to the strong correlation [17] of these variables. The average absolute error is 4.57 mm for males and 4.23 mm for females.

2.4 Data Acquisition

There are several data acquisition techniques in medicine.

2.4.1 Computed Tomography (CT)

Computed tomography (CT) was first used in 1971 in England [26]. A CT scan is an imaging technique that uses a narrow beam of x-rays at several different angles to obtain the measurement. These measurements are then reverse-engineered using a computer algorithm, creating an object layer by layer using slices [26].

The most prevalent Computer Aided Orthopedic Surgery (CAOS) procedure uses the CT scan [27]. It is used for gathering clear information of bone geometry and pre-operative planning [27]. Due to radiation exposure and its potential risk, there are some concerns about using CT scans [28].

2.4.2 Magnetic Resonances Imaging (MRI)

Magnetic resonance imaging is a commonly used imaging technique that provides a lot of information that is not available when using other imaging methods. MRI machines use a magnetic field to manipulate atomic nucleons, especially hydrogen atoms, monitoring the changes to generate the images. Tissues are mostly made of water which contains hydrogen nucleons. In medicine, hydrogen atoms are used in MRI to generate high-quality images, although some problems with this approach exist. For example, the movement of the patient during imaging can make an MRI
image useless. Also, machine problems and/or radio frequency leaks may cause similar problems. This can make the images useless or diagnostics very difficult [26].

2.4.3 3D Ultrasound Imaging (US)

Ultrasound imaging is used in medicine for generating a real-time image of a patient using sound waves. Usage of ultrasound in the medical field is ubiquitous due to its advantages, which include [26]:

- Ultrasound probes are small and easy to move. The user manipulates the orientation do obtain real-time tomographic images [26].
- Generates images are good enough to display the body structure in detail [26].
- Inexpensive and mobile [26].

However, ultrasound images are in 2D, which is insufficient for imaging specific organs. Also, the diagnostics are subjective and based on the user and/or practitioner decision which may not be accurate. The technique of 3D ultrasound imaging was developed to mitigate these problems. To obtain 3D ultrasound images, mechanical sensing devices are used to control a slicing mechanism and generate 3D ultrasound images. Free-hand scanning with or without position sensing is another technique for 3D imaging [26].

2.5 Reconstructing

Beitsel et al. developed a method for bone detection which is fully automated and works with ultrasound. CT scans were used to build patient-specific femurs for a joint-detection model in their study and the model gives the geometric limits of the bone which helps robust and accurate detection. B-spline fits were used to define the contact surfaces based on the brightest pixel [27].

2.6 Statistical Shape Analysis

Shape models are widely used in anatomy [3]. Geometric objects are represented by a finite set of points, such as boundary points, in the medical image. This approach was proposed in the 1970s and 80s [3, 29–32]. In the mid-1990s, a point distribution model, which uses landmarks as a base for the statistical shape model, was introduced and is still widely used in medical image analysis today [3, 33]. Statistical shape models have been used in many different areas and especially in physical anthropology [3] where it is used for identifying anatomical differences between species using both modern and fossil records [3, 34–37]. The statistical shape model is also used for identifying age, race, gender, and other characteristics from bone pieces in forensic investigations [3, 38–41].
2.6.1 Principal Component Analysis (PCA)

Principal components analysis (PCA) influenced the development of population-based statistical modeling significantly [3]. PCA is used in all scientific areas because it can retrieve the relevant information from elaborate data sets [42]. PCA is used to reduce the dimensions of the data set to make it simple to understand [42].

An entire human body was generated by Allen et al. [43] using template mesh fitting. In this study, PCA was used to generate random human body shapes related to scanned data of 250 bodies.

Park and Reed [44] developed a statistical shape model for children’s bodies using demographic information as input. Principal component analysis was used to analyze landmark points and principal scores were used for the prediction model.

Tsai et al. [28] generated SSM to predict the 3D knee geometry using 2D images. PCA was used to predict the 3D surface geometry.
2.7 Radial Basis Function

Radial basis functions (RBF) are values based on distances with an initial location and last location [45]. The 3D surfaces can be generated with a system of linear functions by using RBF [45, 46]. They are faster compared to mesh-based techniques [45, 47, 48]. RBF can also change and/or deform the existing surface [45, 49].

Researchers use RBF and PCA together to obtain better results. Li et al. used RBF and PCA to obtain a better prediction of the infant skull and adult torso shape [50]. Park and Reed used RBF and PCA to build a statistical body shape model for children’s bodies [51].

2.8 Other Studies

Mahfouz et al. [52] analyzed 1000 normal adult knees to determine the relationship between gender and knee geometry. Every bone was represented as a 3D surface model. Statistical shape analysis was performed, combining both principal component and multiple discriminate analyses. Eleven measurements of the distal femur and nine measurements from the proximal tibia were collected. The differences between gender and ethnicity were found from the mean measurements. According to the results, males have larger knees than females in all ethnicities.

Dai and Bischoff [53] analyzed the proximal tibia in order to define the gender and ethnicity differences to improve total knee arthroplasty. In this study, 347 right tibias, including 97 Indian, 99 Japanese, and 151 Caucasians, were investigated. Principal component analysis was used to determine the differences in the populations. The results demonstrate that proximal tibia size (dimension, area, and radius) is affected by gender and ethnicity at the level of total knee replacement.

de Vries [45] conducted a shoulder bone study to recreate a defect bone geometry via statistical shape modeling. CT scanned shoulder bones collected from arthritis and intact were used to build a statistical shape model. The model uses the RBF to create a consistent mesh and principal component analysis was performed to analyze and regenerate the glenoid surface.

Baldwin et al. [54] developed a subject-specific knee model using segmentation and mesh morphing. PCA was used to characterize the knee model.

Bone dimensions are widely used in anthropology to estimate the actual geometry. Femur length has been used to estimate stature [55–58]. Genoves [55] analyzed cadaver femurs of Mexican Hispanics from the early 20th century and built a regression model using the femur length to estimate stature for each gender. Oliver [56] used the femoral bicondylar length and the tibia full length to define stature. Sciulli and Giesen [57] created a regression model to estimate stature via femur length among a population of early Woodland Ohio Valley Native Americans for each gender. Trotter and Gleser [58] used US whites and blacks cadaver femurs and created four estimation models for each gender and race.

Moreover, femur dimensions have been used to estimate body mass [59–61]. Ruff et al. [59] used the femoral head breath to estimate body mass using The femurs of 80 individuals (from Baltimore, MD USA) to create three different regression models (one for each gender and one for
the combined sample). McHenry [60] combined North American, African Pymies, and Khoisan’s data and created one regression model to estimate body mass using the femur head. Grine et al. [61] combined African American, European American, and Native American data and generated a single regression model for body mass estimation.

2.9 The National Health and Nutrition Examination Survey

The National Health and Nutrition Examination Survey (NHANES) is a health and nutrition evaluation program of United States’ adults and children [62]. NHANES data are two-year weighted data that represent all of the United States. The survey consists of a combination of physical exams and interviews. NHANES is an ongoing survey and every two years some of the survey questions and the examination change. Every other year the Centers for Disease Control and Prevention (CDC) publishes a data set that was completed two years ago. For example, a recently published data set is NHANES 2013-2014. Despite the changes in the survey, major human demographics (stature, weight, age, gender) have been collected in all NHANES surveys. Moreover, NHANES includes questionnaires. Most of the NHANES surveys include the hip fracture history, except for the NHANES 2011-2012 data.

2.9.1 NHANES III

NHANES III is a survey that was conducted between 1988 and 1994 [6]. The total number of people examined was 30,818. NHANES III has proximal femur dimensions which are calculated from x-ray images, with the total number of scanned people at 13,562. 2D image processing is used to gather dimensions. Figure 2.4 demonstrates the one of the measurement steps [4].

![Image Analysis](image.png)

Figure 2.4. One of the measurement steps [4].
2.10 The State of Art of Femur Geometry Measurements

The hip joint connects the lower body to the upper body and thus bears a lot of weight. The femur is a bone between your hip and knee. It supports human body in many activities (walking, running). The femur wears a lot if there is bone disease. Total hip replacement (THR) or total hip arthroplasty (THA) are medical terms used to describe the procedure of hip joint replacement due to the wear and tear. The statistics show that THR is increasing all around the world. Many studies have been conducted, mainly focusing on the femur, to build a better understanding of the hip joint, and many of these studies have involved measurements on the femurs of cadavers. Many of these use data from cadavers and/or scanned data to collect information about femur geometry. These data are not widely available and are typically from a convenience sample. As a result, data on the range of geometries across a population are scarce. Moreover, most of the previously cited studies did not consider the gender effect due to a lack of samples and/or lack of information. As a result, most of the measurement results were given without gender separation.

This project aims to mitigate those by using a statistical shape model to quantify the range of variability. Body mass index (BMI, a measure of weight-for-stature), age, gender, and stature are predictor variables in the model. To generate a representative sample of femurs, the data from NHANES (National Health and Nutrition Examination Survey) was utilized. This study focuses on understanding the range in variability of femurs and how that range has changed over time. Also, to investigate the effects of other factors (gender, race) on proximal femur measurements.
Chapter 3

Statistical Shape Model

Statistical modeling relates to building a model using the relation(s) between variables. Collected data sets are used to define these relations which are then utilized to create a statistical model. The model can be used to identify characteristics within the data and also to generate a new data set. Statistical analysis is mostly used to define body measurement and/or geometrical shape changes of bones. To build a model, data need to be collected. The second step is to process the data and analyze them. The third step is conduct the statistical analyses. The last step is to apply the statistical model. Statistical modeling is widely used in many different areas. Anatomy is one of the areas in which statistical shape models are broadly used [3]. Procedures used for statistical shape modeling are almost the same in each medical field. Data are collected with scans (CT, MRI, US) followed by global registration and point transformation. A statistical model is created with principal component analysis. The last step is to apply the shape model (Figure 3.1).
3.1 Data Acquisition

Initially, a statistical shape model (SSM) needs data to generate a model. The 3D anatomical shapes can be collected via CT scan, MRI, and/or ultrasound imaging. Nowadays, computer tomography can gather data with an accuracy of less than a millimeter [28, 63]. For this study, CT scans are very useful for collected bone geometrical information. A CT scan of the femur is represented in Figure 3.2.
CT scanned data need to be post-processed because the CT scan generates an image for every slice and in all three directions (x,y,z). The machine generates DICOM image files and images need to be combined using software such as 3D Slicer or OsiriX. These programs generate 3D representations of the femur geometry. The different file formats include STL, OBJ, PYD. These file types are mostly represented using triangle and/or point cloud data. Even though the generated files include lots of geometrical information, the data can suffer from too much noise (Figure 3.3).
Processing noisy point cloud data can be difficult because these the points are not distributing in the same pattern for every scan. To make a consistent point cloud, templates are used. Templates are ideal point meshes that involve enough points to represent a good quality femur. Figure 3.4 shows the template femur that was used. To obtain consistent results and/or to analyze the data set, all of the data need to be in same space and/or distribution.

**3.2 Landmarks**

Landmarks can be described as noticeable points and/or objects in a specific area. The lesser trochanter point, greater trochanter point, and points on the head of the femur are some of the
landmark points in the proximal femur. These landmarks and a number of other points are used to generate a specific femur geometry using template femur point clouds. In this way, points have consistent location coordinates. For instance, if a point represents the lesser trochanter, the point location will shrink or expand with respect to a specific femur geometry. A radial basis function is used to generate the template femurs using landmark points of a specific femur geometry.

### 3.3 Radial Basis Function (RBF)

To generate template-based femurs, a radial basis function (RBF) is used to transfer the template point to a new location using the displacement from its matching landmark point [45]. Using landmark points \( p(X) \) and distance \( F(X) \) the RBF can be created. A general RBF interpolation function is given in Equation 3.1.

\[
F(X) = \sum_{i=1}^{n} \lambda_i \varphi(\| X - X_i \|) + p(X) \tag{3.1}
\]

Every point in the template is transferred using the RBF displacement field \( F(X) \). \( X \) represents the \( x, y, z \) coordinates of the points. The defined polynomial in null space \( p(X) \) is equal to \( c_0 + c_1 x + c_2 y + c_3 z \). \( \| \cdot \| \) is the Euclidean norm and \( \varphi(\| X - X_i \|) \) or \( \varphi(r) \) is a distance function between given points. In this research, a thin-plate spline was used as shown in Equation 3.2.

\[
\varphi(r) = r^2 \log(r) \tag{3.2}
\]

\( \lambda_i \) are coefficients and \( c_i \) are real numbers. To find the coefficients of the RBF in Equation 3.1, a linear system of equations is to be solved. The linear system equation is given in Equation 3.3.

\[
\begin{bmatrix}
B & T \\
T^T & 0
\end{bmatrix}
\begin{bmatrix}
\lambda \\
c
\end{bmatrix}
= 
\begin{bmatrix}
F \\
0
\end{bmatrix} \tag{3.3}
\]

The \( B \) matrix is created using the distance function between each landmark position and target position.

\[
B_{i,j} = \varphi(\| X_i - X_j \|) \tag{3.4}
\]

The \( T \) matrix represents the template points locations for \( i^{th} \) row \( (1, x_i, y_i, z_i) \) where “0” is a matrix with zeros. The \( F \) matrix is the last location and/or desired location of a certain point.

\[
T = 
\begin{bmatrix}
1 & x_1 & y_1 & z_1 \\
\vdots & \vdots & \vdots & \vdots \\
1 & x_n & y_n & z_n
\end{bmatrix} \tag{3.5}
\]

The linear system equation becomes similar to Equation 3.6. The equation need to be solved for coefficients \( \lambda_i \) and real numbers \( c \).
To find the \((\lambda, c)^T\) matrix, the \(F\) matrix is set as the displaced values of selected landmarks on the CT scanned data and the corresponding points in the template femur are set as the \(T\) matrix. After solving the equation, weights, and constants, all points can be transformed using Equation 3.1.

### 3.4 Principal Component Analysis

After obtaining clean and consistent patient-specific femur point clouds, the femurs are ready for analysis. Principal component analysis (PCA) is used because of its ability to find relations in big data sets by transferring data in PCA space. Principal component analysis has influenced the development of population-based statistical modeling significantly [3].

Principal component analysis transforms a data set to a space where all of the components are orthogonal to each other. PCA generates a series of components (loadings) and scores (weights). With two variables, PCA is similar to linear regression. For a multidimensional data set, such as the range of landmarks, new locations can be generated for points using uncorrelated PCA components.

To perform PCA analysis, data should be organized. Each variable should be placed into separate columns, and every iteration is in a different row. Let the data set be matrix \(X\) of size \(k\) by \(n\), and the variance of each column be a summation of rows from the mean of that row.

\[
\text{var}(X_j) = \frac{\sum_{i=1}^{k}(X_{i,j} - \bar{X}_j)^2}{k-1} \quad (3.7)
\]

The covariance is calculated from the sum of the deviations of one row multiplied by the deviation of another.

\[
\text{cov}(X_{ja}, X_{jb}) = \frac{\sum(X_{i,ja} - \bar{X}_{ja})(X_{i,jb} - \bar{X}_{jb})}{n-1} \quad (3.8)
\]

Each of the columns are centered around zero, then data set \(X\) multiple with its transpose and the \(n\) by \(n\) covariance matrix is created.

\[\text{cov}(X) = X^TX \quad (3.9)\]

The eigenvectors of the covariance matrix represent principal components, and every eigenvector is orthogonal to every other. Each eigenvalue involves a fractional share of the total amount.
of variance.

\[
[W, D] = eig(X^TX)
\]  

(3.10)

The eigenvalues are arranged from the greatest to the smallest. The transpose of the sorted eigenvectors is multiplied by the centered data and new reference frame \( Y \) created.

\[
Y = (W_s^T X)^T
\]  

(3.11)

The columns of the \( Y \) matrix represent the scores (weights) of the principal components (loadings), and the rows represent each iteration. PCA is used in many different areas for several different reasons. One of the usages of PCA is in regenerating a data set by using the principal components and weights. Any member can be recreated by the following equation.

\[
X_{rc(i)} = W_{s1} Y_{(i,1)} + W_{s2} Y_{(i,2)} + \cdots + W_{sn} Y_{(i,n)} + \bar{X}
\]  

(3.12)

### 3.4.1 Generate New Femurs

PCA can be performed in the manner explained in Allen et al. [43] and Turk et al. [64]. All specific femurs, which are cleaned and organized with RBF, are analyzed. If we have \( k \) scanned sample and every sample involves \( n \) vertices which our template has. Then the vertices are stacked over into the \( k \) row vector. Our table will have \( k \) rows and \( 3n \) columns.

### Table 3.1. Sample data set format for PCA analysis

<table>
<thead>
<tr>
<th>No</th>
<th>BMI</th>
<th>Stature</th>
<th>Age</th>
<th>Gender</th>
<th>( x_1 )</th>
<th>( y_1 )</th>
<th>( z_1 )</th>
<th>( \cdots )</th>
<th>( x_n )</th>
<th>( y_n )</th>
<th>( z_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25.5</td>
<td>1788</td>
<td>21</td>
<td>M</td>
<td>12.45</td>
<td>5.35</td>
<td>-113.37</td>
<td>( \cdots )</td>
<td>-7.25</td>
<td>33.80</td>
<td>153.35</td>
</tr>
<tr>
<td>2</td>
<td>23.4</td>
<td>1622</td>
<td>32</td>
<td>F</td>
<td>13.13</td>
<td>5.45</td>
<td>-111.66</td>
<td>( \cdots )</td>
<td>-13.19</td>
<td>30.05</td>
<td>150.71</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( k )</td>
<td>27.6</td>
<td>1569</td>
<td>48</td>
<td>F</td>
<td>11.84</td>
<td>5.23</td>
<td>-108.74</td>
<td>( \cdots )</td>
<td>-6.95</td>
<td>33.00</td>
<td>146.94</td>
</tr>
</tbody>
</table>

PCA analysis is performed for the point vectors defined as \( \vec{s} \). It is run separately for every gender. Centered data around zero \( u \) is created by subtracting the mean of \( s \) from each \( s_i \).

\[
u_i = \vec{s}_i - \bar{s}
\]  

(3.13)

PCA analysis is performed on \( (u) \) and the covariance matrix is created by multiplying the \( u \) transpose with \( u \).

\[
cov(u) = u^T u
\]  

(3.14)

The principal components are obtained from the eigenvectors of the covariance matrix.

\[
[W_u, D_u] = eig(u^T u)
\]  

(3.15)
The principal components ($W$) are sorted from greatest to smallest and represent the eigenvalues. This sorted components are used for simplifying the data so the least representative eigenvalues can be excluded from data without loosing precious information. The ($Y$) scores matrix is created using Equation 3.11.

$$Y = (W_s^T u^T)^T$$ (3.16)

As previously mentioned, the data set can be regenerated using the principal components and scores. Also, it is possible to generate a new data set by manipulating the principal scores. New landmarks can be generated by using the scores in Equation 3.17.

$$X_{new} = W_{s1}Y_{new1} + W_{s2}Y_{new2} + \cdots + W_{sn}Y_{newn} + \bar{X}$$ (3.17)

Instead of generating random femur landmarks, specific femur landmarks can be generated by building a prediction model. The prediction model is generated by building a regression analysis between the user demographics and principal scores (weights). Age, stature, and BMI are used as input demographics. The coefficient matrix ($C$) is found from Equation 3.18 and is calculated for males and females separately.

$$Y_{si} = C_{1i} Age + C_{2i} Stature + C_{3i} BMI + C_{4i}$$ (3.18)

Using the coefficient matrix $C$ and any demographics, the new principal scores can be created. As a result, a patient-specific femur can be generated using the new scores. The general equation for a male is given by Equation 3.19 and the equation for females is similar.

$$[X_{new}^\text{male}] = [\bar{X}\text{male}] + [W_s^\text{male}]^T \begin{bmatrix} C_{\text{male}} & \text{Age} & \text{BMI} & \text{Stature} & 1 \end{bmatrix}^T$$ (3.19)

In this way, specific femur landmark points are generated. Figure 3.5 demonstrates two different specific femurs. On the left is the estimated average femur geometry of a 21-year-old male who has a stature of 1788 mm and a 25.5 BMI. The femur on the right a 48-year-old female who has a stature of 1569 mm and a 27.6 BMI.
When the landmarks are ready, they are used to generate a fine mesh and cleaned to reveal the full femur point cloud. To generate the full femur, RBF is used (see Equation 3.1).
In this study, a statistical shape model created by Klein et al. [5] was used. Their model is based on data from 62 male and 36 female CT scans of right femurs. Principal scores are generated as a function of age, BMI, and femur length through a linear regression [5]. Instead of using stature as an input, Klein et al. [5] utilized femur length due to the strong correlation [17] of these variables. Klein et al. calculated the error of the model with several techniques. The average euclidean distance error was calculated between the fitted meshes to the real data and synthesize meshes [5] (Figure 3.7). The average absolute error is 4.57 mm for males and 4.23 mm for females [5]. Also, Klein et al. calculated the cross-sectional area error in shaft area across five location. The average error between synthesize cross-sectional areas and validation set of cadaver cross-sectional areas across five location was 2.9% [5] (Figure 3.8).
Figure 3.7. Average absolute error between nodal coordinate location [5].

Figure 3.8. The five cross-sectional location across shaft area [5].
Chapter 4

Measurements

After generating a series of femurs using SSM, four inputs, namely age, gender, BMI, and stature, are used to generate the whole femur 3D model. Matlab is used to create 3D point clouds of the femur. These points have \((x,y,z)\) coordinate information. The model creates 3062 single points, each of which has their own \((x,y,z)\) coordinates. The mesh can be visualized using 3D meshing programs such as Mesh Lab, Netfabb. The generated 3D geometries can be used in, for example, finite element analysis. Additionally, the statistical shape model creates only the surface of the femur and a quality analysis requires additional information such as bone thickness and material properties. Obtaining the bone geometry measures is easy and fast for both generating and analyzing the data set.

4.1 Dimensions

Since the model creates a 3D surface, any number of measures can be calculated from the estimated femur geometry. NHANES III measured the proximal femur dimensions between 1988 to 1994 \cite{5}. Recent NHANES data sets do not have femur measurements only NHANES III has. Using real measurements gathered from the population helps us to compare the results with actual data. NHANES III contains all the demographic information and femur dimensions which are calculated from x-ray images including (Figure 4.1):

- **Narrow neck diameter** obtained from the location where the femur neck has its minimum diameter.
- **Intertrohenteric distance** which is located between the lesser trochanter and the greater trochanter.
- **Shaft diameter** which is located 20 mm below the lesser trochanter landmark point toward the femur shaft.
- **Neck shaft angle** angle between the neck head axis and the shaft axis which is toward the long direction of the femur.
Measurements and the locations that gathered in NHANES III form xray images [6]. Measurements are calculated using an x-ray image processing program that simplifies the x-ray image and allows the user to adjust the location of the points [6]. However, adjusting the measurements for a better fit is subjective and leads to bias. In this study, measurements are calculated which are widely used in the literature. They are:

- **Femur head diameter**.
  
- **Femur narrow neck diameter**.

- **Femur neck shaft angle**.

- **Femur shaft diameter**.

### 4.1.1 Femur head diameter

The shape of the femur head is close to spherical. Therefore, obtaining the femur head diameter is easy from a sphere closely fitted to the head area. Information about the femur head is needed to define the sphere location and radius. This information has been obtained from the generated femur 3D points. One of the generated femurs is selected as a template model to define the measurements. For femur head measurement, landmarks are selected to make a sphere fit, see Figure 4.2.
Figure 4.2. Landmark points of femur head diameter.

The spherical fit function was used to determine the diameter of the femur head. The function uses a series of points that are spread around the spherical surface and generates the radius and center location of the best-fit sphere. A total of 78 landmark points are used for the sphere fit function, which is given in Appendix C.
4.1.2 Femur neck diameter

The femur neck is a narrow section of the proximal femur. In NHNAES III, the narrow neck diameter is measured from x-ray images [6]. Due to the angle of the femur head with both XY and YZ directions, the 2D image may lead to incorrect measurements. Also, the patients pose may change the measurement value. Obtaining the neck diameter from a 3D generated model may lead to better results.

Femur neck diameter is not as easy to measure as the femur head. However, the same methodology was used to obtain this measurement. Several points were selected as landmarks from a specific area where the neck is located. A total of 43 points were selected to define the diameter of the narrow neck in Figure 4.4.
The neck appears as cylindrical in shape from the perspective of Figure 4.4. and the sphere fit function can be used to define the neck diameter. However, the sphere fit does not cover all of the defined landmark points. Figure 4.5 demonstrates a fitted sphere and the landmarks that do not coincide with the sphere.
The cross-section of the neck has a shape close to an ellipse. To cover this elliptic shape an ellipsoid fit is used instead of using spherical fit. The ellipsoid fit function is given in Appendix C. The fitting function generates the best-fit ellipsoid equation and gives the center location and three radius values. Figure 4.6 demonstrates that the ellipsoid fit covers the landmark points. The ellipsoid has three different diameters. The biggest one represents the larger width of neck cross-section. The second largest diameter of ellipsoid represents the diameter of minimum narrow neck diameter on width. The last one comes from the thickness of the landmarks along the neck axis. Hence, the second largest diameter of the ellipsoid fit is considered the actual narrow neck diameter.
4.1.3 Femur shaft diameter

Femur shaft diameter is one of the measurements that was collected in NHANES III. Shaft width is defined from a distance between two points at the location 20 mm below the lesser trochanter [6]. Landmarks need to be selected from a specific location. First, the anatomic landmark location of the lesser trochanter is defined and then points are selected which are 20 mm away from the lesser trochanter. The point that has z coordinates between 21 mm and 19 mm below the lesser trochanter is selected as a landmark point because there may not be enough points at the location 20 mm lower than the lesser trochanter to make a measurement.

\[
\text{LesserTrochanter} = l.th
\]

\[
l.th - 21 \geq Shaft_w(z) \geq l.th - 19
\]
The femoral shaft has a circular cross-section. The spherical fit is also applicable in this area but 2 mm distance in landmark points may make the spherical fit a little bit larger than the shaft width. With this information, the selected landmarks, which are between 2 mm range and 20 mm away from the lesser trochanter (19 mm and 21 mm), are used for a circular fit. Points are projected onto a 2D plane, and the circle fitting algorithm is applied to speed up the calculations. The circle fitting function is provided in Appendix C. Figure 4.8 demonstrates the circle fit for this location.
4.1.4 Femur neck shaft angle

Femoral neck angle plays a major role in the literature. Many studies present the neck-shaft angle, which is the angle between the shaft axis and the neck axis.

**Neck axis** is created using the femoral head center and neck diameter center. The femoral head center location is found by using the center of the fitted sphere, as explained in section 4.1.2.

**Shaft axis** is an axis that is the best-fit line into the femur shaft. The axis is defined by using the centers of the upper and lower shafts.
Figure 4.9. Shaft axis and neck axis.
4.1.5 Nodal system

Landmarks are selected points from one of the generated femurs. MeshLab was used to select the points shown previously. These points possess only (x,y,z) coordinate information for the generated femur. However, every femur has its own location information for their landmarks. To measure the different femurs, landmarks need to be selected. This process requires time and effort and leads to subjective results. The nodal system is used to mitigate this problem. The statistical shape model generates the new locations for the points by using a B-spline fitting operation. As explained in Chapter 3, SSM uses the node’s first location and generates a new location for the specific femur. This means that the node does not change but just shifts to the new location with respect to the new femur dimensions. Knowing this leads us to use nodal base landmark selection. To do so, node numbers are obtained for every landmark point. After this, the locations of the nodes are obtained. In this way, the dimensions were generated and collected for every generated femur geometry automatically. The Matlab code is given in Appendix C.
Results

In this chapter, two models were implemented using the various demographic data sets. The femurs of the US population in the previous decade were generated and measured in a fully automated manner.

5.1 Gender Effect

Most of the previously cited studies did not consider the gender effect due to a lack of samples and/or lack of information. As a result, most of the measurement results were given without gender separation. Recent studies show that males and females have differences in bone measurements. To verify this in our model, 100 male and female femurs were generated. Demographic information (stature, BMI, and age) were randomly generated with the mean and standard deviation of NHANES (13-14). The created 100 statures, BMIs, and ages were utilized to generate femurs for each gender separately. This analysis aims to identify any differences between males and females while controlling for differences in body size across gender. The same stature, BMI, and age were used as inputs to each model. Even though they are two different statistical models, the analysis of the femur shape may give the similar results. The femurs were generated and measured. Two sample t-test analyses were conducted. Table 5.6 shows the results of the test. The measurements (neck-shaft angle, shaft width, head diameter, narrow neck width) were found to be significantly different for each gender.

<table>
<thead>
<tr>
<th>Table 5.1. T-test for gender differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welch Two Sample t-test</td>
</tr>
<tr>
<td>t  df  p value</td>
</tr>
<tr>
<td>Neck-Shaft Angle  -4.93  157.7  &lt; 0.001</td>
</tr>
<tr>
<td>Shaft Width    18.08  197.93  &lt; 0.001</td>
</tr>
<tr>
<td>Head Diameter  19.51  173.56  &lt; 0.001</td>
</tr>
<tr>
<td>Narrow Neck Width  32.34  173.86  &lt; 0.001</td>
</tr>
</tbody>
</table>

38
Additionally, density plots were generated to understand the differences between the genders (Figure 5.1). Except for the neck-shaft angle, males have slightly bigger measurements than females. Neck-shaft angles were found to be slightly bigger in females.

![Gender density plots](image)

**Figure 5.1.** Gender density plots.

### 5.2 Femurs of NHANES III

Generating femur and collecting the measurement can be applied almost all NHANES data set. NHANES III was used to generate and collect femurs to compare the results since NHANES III includes proximal femur measurements collected with x-ray image. The NHANES III data set is relatively older but includes some femur measurements. Over 31,000 people participated in the survey but just 13,562 participants had an x-ray image taken. For the analysis in this thesis, the measurement data and demographic information of these data are merged into one matrix. Participants who have missing data were removed from the list and the total number was reduced to 13,484. Table 5.2 demonstrates some NHANES III demographics and the femur measurements which were gathered from the x-ray images.
Table 5.2. Some of NHANES III (x-ray) demographics and femur measurements (mm).

<table>
<thead>
<tr>
<th>SEQN</th>
<th>Stature</th>
<th>BMI</th>
<th>Age</th>
<th>Sex</th>
<th>NSA</th>
<th>FSWID</th>
<th>NNWID</th>
<th>FNWID</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1788</td>
<td>25.5</td>
<td>21</td>
<td>M</td>
<td>139.6</td>
<td>30.5</td>
<td>30.1</td>
<td>34.7</td>
<td>1523</td>
</tr>
<tr>
<td>4</td>
<td>1622</td>
<td>23.4</td>
<td>32</td>
<td>F</td>
<td>143.7</td>
<td>27.7</td>
<td>29.4</td>
<td>33.9</td>
<td>1566.1</td>
</tr>
<tr>
<td>9</td>
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<td>27.6</td>
<td>48</td>
<td>F</td>
<td>127</td>
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<td>27.1</td>
<td>28.6</td>
<td>18155</td>
</tr>
<tr>
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<td>33.8</td>
<td>35.7</td>
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<td>22.6</td>
<td>42</td>
<td>F</td>
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<td>29</td>
<td>29.7</td>
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</tr>
<tr>
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<td>F</td>
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<td>39.5</td>
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</tr>
<tr>
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<td>19.1</td>
<td>82</td>
<td>F</td>
<td>131</td>
<td>30.1</td>
<td>31.5</td>
<td>37.5</td>
<td>3838.9</td>
</tr>
<tr>
<td>52</td>
<td>1782</td>
<td>25.1</td>
<td>50</td>
<td>M</td>
<td>136.8</td>
<td>31.3</td>
<td>34.3</td>
<td>38.2</td>
<td>1292.7</td>
</tr>
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<td>53</td>
<td>1549</td>
<td>21.2</td>
<td>36</td>
<td>M</td>
<td>132.1</td>
<td>30.2</td>
<td>32.3</td>
<td>36.6</td>
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<tr>
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<td>1726</td>
<td>37.5</td>
<td>48</td>
<td>M</td>
<td>128.4</td>
<td>33.5</td>
<td>37.7</td>
<td>43.3</td>
<td>756.95</td>
</tr>
</tbody>
</table>

NSA = Neck Shaft Angle, FSWID = Shaft Width, NNWID = Narrow Neck Width, FNWID = Neck Width. All measurements are in millimeter.

The statistical shape model was used to generate right femurs from NHANES III. A total of 13,484 femurs were created using the demographic information of NHANES III. Then, the measurements of the femurs were made, as explained in Chapter 4. The NHANES III result and our results (SSM) are compared on a percentage basis using the weighted quantile method. Percentile comparison is given in Table 5.3.

Table 5.3. NHANES III (x-ray and synthesize) Weighted quantile comparison.

<table>
<thead>
<tr>
<th>Weighted Quantile</th>
<th>1&lt;sup&gt;st&lt;/sup&gt;</th>
<th>5&lt;sup&gt;th&lt;/sup&gt;</th>
<th>10&lt;sup&gt;th&lt;/sup&gt;</th>
<th>25&lt;sup&gt;th&lt;/sup&gt;</th>
<th>50&lt;sup&gt;th&lt;/sup&gt;</th>
<th>75&lt;sup&gt;th&lt;/sup&gt;</th>
<th>90&lt;sup&gt;th&lt;/sup&gt;</th>
<th>95&lt;sup&gt;th&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neck-Shaft Angle (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NHANES III</td>
<td>122</td>
<td>125.4</td>
<td>127.4</td>
<td>130.1</td>
<td>133</td>
<td>136</td>
<td>138.8</td>
<td>140.4</td>
</tr>
<tr>
<td>SSM</td>
<td>125</td>
<td>128.6</td>
<td>130.7</td>
<td>133.3</td>
<td>135.9</td>
<td>137.8</td>
<td>139.5</td>
<td>140.3</td>
</tr>
<tr>
<td>Shaft Width (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NHANES III</td>
<td>25.4</td>
<td>27.1</td>
<td>28.1</td>
<td>29.9</td>
<td>32.2</td>
<td>34.8</td>
<td>37.4</td>
<td>38.8</td>
</tr>
<tr>
<td>SSM</td>
<td>27</td>
<td>27.5</td>
<td>27.8</td>
<td>28.4</td>
<td>30.5</td>
<td>32.2</td>
<td>32.8</td>
<td>33.1</td>
</tr>
<tr>
<td>Narrow Neck Width (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NHANES III</td>
<td>25.4</td>
<td>27</td>
<td>28</td>
<td>30</td>
<td>32.4</td>
<td>35.2</td>
<td>37.4</td>
<td>38.7</td>
</tr>
<tr>
<td>SSM</td>
<td>24.8</td>
<td>25.3</td>
<td>25.6</td>
<td>26.3</td>
<td>30</td>
<td>31.2</td>
<td>31.7</td>
<td>31.9</td>
</tr>
</tbody>
</table>

Further, to understand the model accuracy compared to x-ray measurements, additional analyses were conducted. As mentioned previously, the NHANES III measurements were taken from on 2D x-ray. Q-q plots were generated to compare the model measurements and the NHANES III measurements. Results are given in Figure 5.2.
As seen in the q-q plots, excepting the neck-shaft angle, the model’s measurements were not
close to each other. The reason for this result might be the differences between the 2D image and the 3D model since the orientation of the femur will affect the 2D measurement drastically. Also, the narrow neck width is more likely to have differences due to it having an elliptic cross-sectional area. However, the shaft width should have given similar results since a circular diameter does not change much with axial rotation either in 2D or 3D.

To investigate this, the 3D model of the femur was utilized to replicate the 2D x-ray imaging technique. General hip measurements were measured from the x-ray images. The images require a consistent orientation of the femur. Many studies used cadaver femurs and gathered the measurements from the view that is perpendicular to the femur head plane. In real NHANES III, proximal femur measurements were collected using the same method. The shaft width is taken to be 20 mm from the lesser trochanter. Width is calculated based on pixels.

Figure 5.3. A sample x-ray image used in real NHANES III measurements (left) [6]. 2D simulation of 3D generated via model (right).

Required rotations were performed and the 2D femur image generated. A sample of the real x-ray image and simulated images shown in Figure 5.3. The lesser trochanter point is defined in the 3D model but visually the location of the lesser trochanter seems changed due to the conversion. A 4 mm offset was performed to adjust the lesser trochanter point. The result seems more reasonable (seen in Figure 5.4).
The shaft measurements were close to each other in the upper tail, but the lower tail is not covered well. This might be due to fewer landmark points around the specific shaft area in the model. This might be also due to error in SSM. The error rate was added to model to 2D with 4 mm offset. The errors were consist of randomly generated numbers has mean of zero and 3 mm and 2 mm standard deviation for male and female respectively. The results were demonstrated in Figure 5.5. These indicates that the model can represent the 2D measurements well with error addition. However, the error can not be used for all measurements since the model has different error rates for different locations.

Figure 5.4. Q-Q plots of 2D shaft width measurements and NHANES III measurements.

Figure 5.5. Q-Q plots of 2D shaft width measurements and NHANES III measurements with offsets and errors.
5.3 NHANES 13-14

The old population femurs generated using NHANES III. To observe the measurement of recent population, the femurs were generated using recently released NHANES. A total of 5,520 femurs were generated using NHANES 2013-2014 demographic information. The NHANES data set is over-sampled in the tails, and statistical weights must be considered in any analysis. The weighted quantile test was used to observe the population measurements. Combined data means are given in Table 5.4.

Table 5.4. NHANES 1314 (synthesize) Weighted Mean Values.

<table>
<thead>
<tr>
<th>Stature (mm)</th>
<th>BMI</th>
<th>Age</th>
<th>Neck-Shaft Angle (mm)</th>
<th>Shaft Width (mm)</th>
<th>Narrow Neck Width (mm)</th>
<th>Head Diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighted Mean</td>
<td>1667</td>
<td>27.9</td>
<td>48</td>
<td>135</td>
<td>30.5</td>
<td>29.6</td>
</tr>
</tbody>
</table>

Generated femurs and measurements need to be analyzed. Analyzing a weighted data set can be difficult since not all statistical tests can be readily conducted on weighted data. Therefore, weighted data were re-sampled to perform the statistical analysis. Sample size plays a significant role in re-sampling since the results may alter when re-sampling is performed again. Sample size should cover the sum of the weights to account for this.

\[
\sum weights_{nh1314} = 226,849,425.00
\]

However, the sum of the weights is too big, as seen. Sampling that amount of data is time consuming and also the calculation time for the results will scale with the sample size. Instead of using a large number of samples, weights were scaled down such that the smallest weight becomes 1. To do so, all weights are divided by the minimum weight. Then, the new sum of the weights is used as a sample size. The sum of the weights was rounded to remove fractions.

\[
\sum_{i=1}^{n} \left( \frac{weights}{min(weight)} \right) \quad (5.1)
\]

\[
sample_{nhanes1314} = \sum_{i=1}^{5520} \left( \frac{weights_{nh1314}}{5735.122} \right) \approx 39,440
\]

A sample size of 39,440 was randomly selected using individual weights using Equation 5.1. The sample population consists of 18,874 males and 20,566 females. Quantile analysis was performed to understand the measurements changes for each sex. Table 5.5 demonstrates the male and female measurement percentiles (5, 10, 25, 50, 75, 90, 95).
Table 5.5. Quantiles of NAHNES 13-14 (synthesize) demographics and proximal femur dimensions.

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>5th</th>
<th>10th</th>
<th>25th</th>
<th>50th</th>
<th>75th</th>
<th>90th</th>
<th>95th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stature (mm)</td>
<td>1637</td>
<td>1666</td>
<td>1706</td>
<td>1760</td>
<td>1811</td>
<td>1855</td>
<td>1879</td>
</tr>
<tr>
<td>BMI</td>
<td>20.7</td>
<td>22.1</td>
<td>24.8</td>
<td>27.8</td>
<td>32</td>
<td>36.5</td>
<td>40.7</td>
</tr>
<tr>
<td>Age</td>
<td>22</td>
<td>24</td>
<td>32</td>
<td>46</td>
<td>59</td>
<td>70</td>
<td>76</td>
</tr>
<tr>
<td>Neck-Shaft Angle (mm)</td>
<td>126.9</td>
<td>128.6</td>
<td>131.5</td>
<td>134.6</td>
<td>137.2</td>
<td>139.1</td>
<td>140.6</td>
</tr>
<tr>
<td>Shaft Width (mm)</td>
<td>31.1</td>
<td>31.4</td>
<td>31.9</td>
<td>32.5</td>
<td>33</td>
<td>33.6</td>
<td>33.9</td>
</tr>
<tr>
<td>Narrow Neck Width (mm)</td>
<td>30.6</td>
<td>30.7</td>
<td>31</td>
<td>31.4</td>
<td>31.8</td>
<td>32.3</td>
<td>32.8</td>
</tr>
<tr>
<td>Head Diameter (mm)</td>
<td>46</td>
<td>46.3</td>
<td>46.9</td>
<td>47.6</td>
<td>48.2</td>
<td>48.8</td>
<td>49.1</td>
</tr>
<tr>
<td>Female</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Stature (mm)</td>
<td>1499</td>
<td>1528</td>
<td>1567</td>
<td>1618</td>
<td>1662</td>
<td>1700</td>
<td>1734</td>
</tr>
<tr>
<td>BMI</td>
<td>19.7</td>
<td>21.1</td>
<td>23.7</td>
<td>28</td>
<td>33.4</td>
<td>39.8</td>
<td>43.8</td>
</tr>
<tr>
<td>Age</td>
<td>22</td>
<td>25</td>
<td>33</td>
<td>47</td>
<td>61</td>
<td>72</td>
<td>79</td>
</tr>
<tr>
<td>Neck-Shaft Angle (mm)</td>
<td>131.7</td>
<td>132.4</td>
<td>133.9</td>
<td>136</td>
<td>138</td>
<td>139.5</td>
<td>140.2</td>
</tr>
<tr>
<td>Shaft Width (mm)</td>
<td>27.4</td>
<td>27.6</td>
<td>28</td>
<td>28.7</td>
<td>29.3</td>
<td>30</td>
<td>30.5</td>
</tr>
<tr>
<td>Narrow Neck Width (mm)</td>
<td>25.2</td>
<td>25.5</td>
<td>26</td>
<td>26.7</td>
<td>27.6</td>
<td>28.6</td>
<td>29.4</td>
</tr>
<tr>
<td>Head Diameter (mm)</td>
<td>38.9</td>
<td>39.4</td>
<td>40.2</td>
<td>41.2</td>
<td>42.1</td>
<td>43</td>
<td>43.7</td>
</tr>
</tbody>
</table>

5.4 NAHNES III and NHANES 13-14

The generated NHANES (13-14) were then compared with generated NHANES III. The aim of this is to investigate that the measurements have changed over time. If there is change the femur implant should be designed based on that change.

Welch two-sample t-tests were performed for each gender separately. The results indicate that the NHANES 13-14 male shaft width, narrow neck width, and head diameter are significantly different ($p < 0.01$) from the corresponding NHANES III measurements. However, the neck-shaft angle was found not to be significantly ($p < 0.27$) different. Furthermore, the NHANES 13-14 female measurements (neck-shaft angle, shaft width, narrow neck width, head diameter) were found significantly different ($p < 0.01$). Figure 5.6 shows density plots of the NHANES III and NAHNES 13-14 data.
<table>
<thead>
<tr>
<th></th>
<th>t</th>
<th>df</th>
<th>p value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Male</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neck-Shaft Angle</td>
<td>1.105</td>
<td>4467.9</td>
<td>0.2689</td>
</tr>
<tr>
<td>Shaft Width</td>
<td>15.931</td>
<td>4310.44</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>Head Diameter</td>
<td>7.742</td>
<td>4613.98</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>Narrow Neck Width</td>
<td>7.742</td>
<td>3881.61</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td><strong>Female</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neck-Shaft Angle</td>
<td>-9.898</td>
<td>5628.83</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>Shaft Width</td>
<td>16.276</td>
<td>4668.89</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>Head Diameter</td>
<td>10.427</td>
<td>4892.7</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>Narrow Neck Width</td>
<td>17.578</td>
<td>4352.75</td>
<td>&lt; 0.001</td>
</tr>
</tbody>
</table>
Figure 5.6. Density plots of NHANES III and NAHNES 13-14 measurements.
5.5 NHANES 07-14

Modern NHANES data sets are published every other year. For analysis, new femurs were generated using the NHANES data demographics and the SSM model [5]. A total of 5,706 femurs were generated using NHANES 2007-2008, 5,994 femurs were generated using NHANES 2009-2010, and 5,237 femurs were generated using NHANES 2011-2012 data. Measurements for each year are calculated using the technique explained in Chapter 4. The sample population are selected for each year. The sample size is calculated using Equation 5.1. Therefore, every two-year data set has its sample size. In Table 5.7, sample sizes are shown. Each year’s data were combined and analyzed.

<table>
<thead>
<tr>
<th>NHANES</th>
<th>Sample Size</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td>07-08</td>
<td>36779</td>
<td>39500</td>
</tr>
<tr>
<td>09-10</td>
<td>24301</td>
<td>26304</td>
</tr>
<tr>
<td>11-12</td>
<td>23992</td>
<td>26193</td>
</tr>
<tr>
<td>13-14</td>
<td>18946</td>
<td>20494</td>
</tr>
<tr>
<td>Total</td>
<td>104018</td>
<td>112491</td>
</tr>
</tbody>
</table>

5.5.1 Year Differences

The differences between old population and recent population was observed previously. To understand the measurement changes in every other year population. If there is a change, the femur implant design should be adjusted. The combined data were analyzed with ANOVA. Later, Tukey’s post hoc test was applied to see which pairs of years show differences. As done previously, male and female analyses were performed separately.

5.5.1.1 Neck-Shaft Angle

Neck-shaft angle shows a significant difference between years for both genders. Male neck-shaft angles were found to be significantly different for the (11-12) - (07-08), (09-10) - (11-12), (07-08) - (13-14), and (09-10) - (13-14) year pairs ($p < 0.01$). However, there were no significant differences between (09-10) and (07-08), and (11-12) and (13-14). Furthermore, female measurements were found to be significantly different for each year pair, except 09-10 and 11-12. Figure 5.7 demonstrates the Tukey results for the neck-shaft angle.
Figure 5.7. Year difference Tukey analysis of Neck-Shaft angle
5.5.1.2 Shaft Width

Shaft width also had significant changes with year. Male shaft widths were found to be significantly different for almost all year pairs ($p < 0.01$) except the (07-08)/(11-12) and (09-10) (13-14) pairs ($p > 0.05$). Female shaft width differences between (07-08) and (09-10) peer were not significant ($p > 0.05$). Other pairs were significantly different ($p < 0.05$) (Figure 5.8).
Figure 5.8. Year difference Tukey analysis of Shaft Width
5.5.1.3 Narrow Neck Width

Female narrow neck widths were found to be significantly different between year pairs \((p < 0.05)\) except for year \((07-08) (09-10)\) pairs \((p > 0.05)\). Year pairs \((07-08) (11-12)\) and \((13-14) (09-10)\) were not significantly different in regard to male shaft width \((p < 0.05)\). The other pairs for the male shaft width \((p < 0.01)\) (Figure 5.9).
Figure 5.9. Year difference Tukey analysis of Narrow Neck Width
5.5.1.4 Head Diameter

Head diameter was found to be significantly different in pairs (07-18) (09-10) ($p < 0.05$), and (09-10) (11-12) ($p < 0.01$). However, the other pairs were not significantly different ($p > 0.05$) (Figure 5.10).
Figure 5.10. Year difference Tukey analysis of Head Diameter
5.5.2 Race Differences

In the literature, scholars look for race differences. NHANES data also contain ethnicity information. In this study the measurement differences between races was investigated. To aim this to how the race affects the measurements and to understand how many race specification is needed in implant design.

<table>
<thead>
<tr>
<th>Table 5.8. NHANES Race Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Code</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
</tbody>
</table>

5.5.2.1 Neck-Shaft Angle

Male neck-shaft angles were found to be significantly different between each race pair ($p < 0.05$). However, the female neck-shaft angle was not found to be significantly different for any of the race pairs. Other Hispanics neck-shaft angles were not found to be significantly different compared to those of Mexican Americans, Blacks, and Other Race. Also, female Mexican American neck-shaft angles were not found to be significantly different from Other Races. The other race pairs were found to be significantly different ($p < 0.05$) (Figure 5.11).
Figure 5.11. Race difference Tukey analysis of Neck-Shaft angle (1 Mexican American, 2 Other Hispanic, 3 Non-Hispanic White, 4 Non-Hispanic Black, 5 Other Race - Including Multi-Racial)
5.5.2.2 Shaft Width

In the male data, Mexican Americans’ shaft width was found to be not significantly different from Other Races’. However, other race pairs were found to be significantly different from each other ($p < 0.01$) (Figure 5.12). In the female, Mexican Americans’ shaft width was found not to be significantly different from Other Hispanics’. However, other race pairs were found to be significantly different from each other ($p < 0.01$) (Figure 5.12).
Figure 5.12. Race difference Tukey analysis of Shaft Width (1 Mexican American, 2 Other Hispanic, 3 Non-Hispanic White, 4 Non-Hispanic Black, 5 Other Race - Including Multi-Racial)
5.5.2.3 Narrow Neck Width

In the male population, Mexican Americans’ narrow neck width was found not to be significantly different from Other Hispanics’. Also, Whites and Blacks were not significantly different. However, other pairs were significantly different ($p < 0.01$)(Figure 5.13). In addition, female narrow neck widths were found to be significantly different in all race pairs ($p < 0.01$)(Figure 5.13).

5.5.2.4 Head Diameter

Other races’ head diameters were not significantly different from Other Hispanics’ in the male population, and not significantly different from Mexican Americans’ in the female population. Other pairs were found to be significant ($p < 0.01$)(Figure 5.14).
Figure 5.13. Race difference Tukey analysis of Narrow Neck Width (1 Mexican American, 2 Other Hispanic, 3 Non-Hispanic White, 4 Non-Hispanic Black, 5 Other Race - Including Multi-Racial)
Figure 5.14. Race difference Tukey analysis of Head Diameter (1 Mexican American, 2 Other Hispanic, 3 Non-Hispanic White, 4 Non-Hispanic Black, 5 Other Race - Including Multi-Racial)
5.6 Correlation Test Between Measurements

Femur shape is related to demographic information since statistical shape models is designed using this relation. Correlation tests were conducted between the input variables and output measurements to understand how the input variables effects the proximal femur measurements. These information may be useful for better understanding. Pearson’s product moment correlation was used. Correlation tests were applied separately for each gender for synthesized NHANES 1314 and real NHANES III.

In the synthesized NHANES 1314 correlation, neck-shaft angle is weakly correlated with stature in males but moderately correlated in females. However, the neck-shaft angle is moderately correlated with BMI in males but weakly correlated in females. There is a strong correlation between neck-shaft angle and age in both genders (Table 5.9).

<table>
<thead>
<tr>
<th></th>
<th>Correlation</th>
<th>Stature</th>
<th>BMI</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neck-Shaft Angle</td>
<td>0.26</td>
<td>0.45</td>
<td>-0.85</td>
<td></td>
</tr>
<tr>
<td>Shaft Width</td>
<td>0.58</td>
<td>0.69</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>Narrow Neck Width</td>
<td>0.48</td>
<td>0.88</td>
<td>-0.12</td>
<td></td>
</tr>
<tr>
<td>Head Diameter</td>
<td>0.91</td>
<td>0.30</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neck-Shaft Angle</td>
<td>0.46</td>
<td>-0.28</td>
<td>-0.93</td>
<td></td>
</tr>
<tr>
<td>Shaft Width</td>
<td>0.36</td>
<td>0.87</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>Narrow Neck Width</td>
<td>0.05</td>
<td>0.94</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>Head Diameter</td>
<td>0.76</td>
<td>0.60</td>
<td>0.08</td>
<td></td>
</tr>
</tbody>
</table>

0 = no correlation
0 to −0.3 and 0 to 0.3 → weak correlation
−0.3 to −0.7 and 0.3 to 0.7 → moderate correlation
−0.7 to −1 and 0.7 to 1 → strong correlation

In the real NHANES III, there is no strong correlation found between demographics and x-ray measurements. Shaft width is found moderately correlated with stature and age. Narrow neck width is also moderately correlated with stature and age.
Table 5.10. NHANES III (x-ray) Measurements Correlation Test

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Stature</th>
<th>BMI</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Male</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neck-Shaft Angle</td>
<td>0.14 ± 0.09</td>
<td>0.03 ± 0.10</td>
<td>-0.10 ± 0.10</td>
</tr>
<tr>
<td>Shaft Width</td>
<td>0.45 ± 0.09</td>
<td>0.23 ± 0.10</td>
<td>0.28 ± 0.09</td>
</tr>
<tr>
<td>Narrow Neck Width</td>
<td>0.42 ± 0.08</td>
<td>0.22 ± 0.10</td>
<td>0.34 ± 0.09</td>
</tr>
<tr>
<td><strong>Female</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neck-Shaft Angle</td>
<td>0.10 ± 0.10</td>
<td>0.03 ± 0.10</td>
<td>-0.06 ± 0.10</td>
</tr>
<tr>
<td>Shaft Width</td>
<td>0.36 ± 0.09</td>
<td>0.23 ± 0.10</td>
<td>0.33 ± 0.09</td>
</tr>
<tr>
<td>Narrow Neck Width</td>
<td>0.32 ± 0.10</td>
<td>0.22 ± 0.10</td>
<td>0.38 ± 0.09</td>
</tr>
</tbody>
</table>

Standard divisions of correlation is given
0 = no correlation
0 to −0.3 and 0 to 0.3 → weak correlation
−0.3 to −0.7 and 0.3 to 0.7 → moderate correlation
−0.7 to −1 and 0.7 to 1 → strong correlation

5.7 Fracture Probabilities

The NHANES data set also includes a questionnaire. Some data sets have historical information about Person’s bone injury. One of the questions is about hip fracture history. Goal of this analysis is to focus on prospected hip replacement patients. If there is a significantly different measurement the design team may focus on that.

The NHANES 07-08, 09-10, and 13-14 sets have this information. Statistical analysis was performed to seek any relation between femur measurements and fracture rate. Individuals who did not answer the question were removed from the data sets. Additionally, enough sample data were selected from the sets using Equation 5.1. Sample sizes of the individuals who experienced a hip fracture and who did not, are given in Table 5.11.

Table 5.11. Sample size and fractured and non-fractured individuals

<table>
<thead>
<tr>
<th>NHANES</th>
<th>Sample size</th>
<th>Fracture</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td>07-08</td>
<td>36757</td>
<td>39473</td>
</tr>
<tr>
<td>09-10</td>
<td>24349</td>
<td>26225</td>
</tr>
<tr>
<td>13-14</td>
<td>11738</td>
<td>13322</td>
</tr>
<tr>
<td>Total</td>
<td>72844</td>
<td>79020</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
<th>Yes</th>
<th>No</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>07-08</td>
<td>312</td>
<td>3645</td>
<td>440</td>
<td>39033</td>
<td>752</td>
<td>75478</td>
</tr>
<tr>
<td>09-10</td>
<td>321</td>
<td>24028</td>
<td>341</td>
<td>25884</td>
<td>662</td>
<td>49912</td>
</tr>
<tr>
<td>13-14</td>
<td>157</td>
<td>11581</td>
<td>270</td>
<td>13052</td>
<td>427</td>
<td>24633</td>
</tr>
<tr>
<td>Total</td>
<td>790</td>
<td>72054</td>
<td>1051</td>
<td>77969</td>
<td>1841</td>
<td>150023</td>
</tr>
</tbody>
</table>

The analysis was applied to every year data separately. Also, all data were combined and analyzed with ANOVA and Tukey’s test. Every test was performed for each gender.
In NHANES 07-08, head diameter is not significantly different for both genders. While the male narrow neck width and shaft width are significantly different, female measurements are not. Additionally, the neck-shaft angle is significantly different in both genders (Figure 5.15).

In NHANES 09-10, all measurements are significantly different for both sexes (Figure 5.16).

NHANES 13-14, head diameter is significantly different in the male population, but it is not significantly different in the female population. While male narrow neck width and shaft width are not significantly different, female measurements are. Additionally, the neck-shaft angle is significantly different in both genders (Figure 5.17).

In combined data, each measurement was found to be significantly different in the male population. However, the female head diameter was not found to be significantly different. Other female measurements are significantly different between individuals who experienced a hip fracture and those who did not (Figure 5.18).

The results show that some measurements are not consistent with each year. Male head diameter is not significant in (07-08) but is significant in the other two sets while the female head diameter is not significantly different in each year. Moreover, the male narrow neck width and shaft width are significantly different in (07-08) and (09-10) but not in (13-14). On the contrary, the female narrow neck width and shaft width are not significantly different in (07-08) and (09-10) but are significantly different in (13-14).
Figure 5.15. NHANES 07-08 Fracture relation to femur measurements.
Figure 5.16. NHANES 09-10 Fracture relation to femur measurements.
Figure 5.17. NHANES 13-14 Fracture relation to femur measurements.
**Figure 5.18.** NHANES 07-10 & 13-14 Fracture relation to femur measurements.
Chapter 6

Conclusion

Hip fracture is one of the most common injuries in today’s world. Total hip replacement (THR) is performed on patients who have experienced a hip fracture. THR is used all over the world and is increasing in number each year. THR applies to the two major bone elements, femur head and pelvis acetabulum. This study focused on measurements of the proximal femur which would be helpful in the pre-surgical planning of THR.

Many studies have been conducted on the femur to improve our understanding of hip joint. They use data from cadavers and/or scanned data to collect information about femur geometry. However, these data are not widely available and are typically from a convenience sample. As a result, data for the range of geometries across a population are scarce. This project mitigated this by generating population femurs using a statistical shape model and collecting measurements using shape fitting algorithms.

Right femurs were generated using the statistical shape modeling technique created by Klein et al. [5]. Model inputs were stature, BMI, age, and gender. Proximal femur dimensions were successfully generated for the US population using NHANES. A total of 36,019 femurs were generated using NHANES population demographics. A sample population containing 216,509 individuals was generated using weighted data in NHANES 07-14. Analysis has been done to improve understanding of proximal femur measurements.

A gender difference was found to be significant. Gender differences were not often included in the previous studies due to lack of sample details and/or sample size. This study found that males have slightly bigger proximal femur measurements, except for the neck-shaft angle. This difference may cause changes in mean value in the combined population. Thus, measurements should be considered separately for each gender.

Furthermore, a correlation between dimensions was found. Noble et al. [11] found a 76% correlation between femur length and head diameter. In this study, a 76% correlation was found in female femurs between stature and head diameter (SSM using femur length as a linear function of stature). The correlation is found to be 91% for the male population. Additionally, age is strongly correlated with neck-shaft angle for both sexes. Neck-shaft angle decreases with age.
Femur head loads may cause this change.

A comparison between synthesized NHANES III and NHANES 13-14 was performed, and measurements were found to be significantly different except for the neck-shaft angle. Head diameter, shaft diameter, and narrow neck width were slightly bigger in the recent population. Having higher BMI may cause this result. Shaft width and narrow neck width were calculated in 3D in our model and in 2D in NHANES III. The orientation of a femur causes differences between 3D and 2D measurements. Even though the 2D image technique replicated shaft width it is still not close to the NHANES III measurements. The upper tail is close, but the lower tail is not. This may be due to the lack of a landmark in the specific shaft width location in the SSM.

Femurs of the US population were generated from NHANES 07-08 to 13-14. A sample population was generated consisting of 104,018 males and 112,491 females. ANOVA analysis was applied to seek differences between year pairs (07-08, 09-10, 11-12, 13-14). Male and female narrow neck width and shaft width show the same results for pairs since both measurements are strongly correlated with BMI. Head diameter and year comparison is consistent for in both sexes. The two following year pairs, (07-08) - (09-10) and (09-10) - (11-12), were found to be significantly different from each other.

Race differences were found for each measurement. In the literature, studies have indicated a race effect on femur geometry [10]. However, the SSM does not have any ethnicity information and does not have race input. Therefore, we were unable to say precisely that the race differences are verified. Stature and BMI changes between races could result in femur geometrical significance.

Fracture analysis results are not consistent except for the neck-shaft angle. The other three measurements are significant for one year and not significant for the other years. However, the neck-shaft angle is significantly different between those who experienced a hip fracture at least once in their lifetime. Fractured femurs have a slightly lower neck-shaft angle. Statistics [7] demonstrate that the elderly are having hip replacements more than the others. As we pointed out previously, the neck-shaft angle decreases with age. People who have a low neck-shaft angle are more likely to experience a fracture. Mechanically, a decrease of the neck-shaft angle will increase the shear stress on the femur neck. Our results support the mechanical statement.

All analysis and results are coming from the synthesized data set. The significant differences between years can be occurred the changes in people demographics (stature, BMI). The significant measurement change can be observed since stature and BMI is used for creating the femur geometry with statistical shape model. The correlation between demographics and femur measurements is very strong in synthesized data but these correlations are weak or moderate in real NHANES III data set. This means that the models strong correlation may misleading.

In the model PCA scores are predicted with stature, BMI and age for each gender. However, the error of the prediction model is not considered. For future studies, statistical shape model can be created with errors so the geometrical prediction would become more accurate for population studies.

The utilized model has approximately 4.5 mm error that can mislead the results especially in low sample data. The model may not be enough for making decision on patient-specific implants.
This study can be used to determine the design range and/or boundaries of femur implants for the US population.
Appendix A
Statistical Shape model model
Matlab Codes

Matlab code was claimed from Klein et. al [5].

function fem = RFemurFit (x)
addpath(genpath(cd));

%% Load statistical data for RFemur=f(Age, Height, BMI,Gender)

height = x(1,1); %mm
BMI = x(1,2);
age = x(1,3);
gender = x(1,4); % 1 male , 2 female

if gender==1
InFemurLength = 0.2382*height+1.0136; % From regression results
meanVectorRFemur = xlsread('meanVectorRFemurNew2.xls');
evecsRFemur = csvread('evecsRFemurNew2.csv');
CoeffRFemur = xlsread('CoeffMatrixRFemurNew2.xls');
meanVectorRFemurThickness = xlsread('meanVectorRFemurThicknessNew2.xls');
evecsRFemurThickness = csvread('evecsRFemurThicknessNew2.csv');
CoeffRFemurThickness = xlsread('CoeffMatrixRFemurThicknessNew2.xls');
else
InFemurLength = 0.2026*height+60.395; % From regression results
meanVectorRFemur = xlsread('meanVectorRFemurFNew2.xls');
evecsRFemur = csvread('evecsRFemurFNew2.csv');
CoeffRFemur = xlsread('CoeffMatrixRFemurFNew2.xls');
end
CoeffRFemur = xlsread('CoeffMatrixRFemurFNew2.xls');
meanVectorRFemurThickness = xlsread('meanVectorRFemurThicknessFNew2.xls');
evecsRFemurThickness = csvread('evecsRFemurThicknessFNew2.csv');
CoeffRFemurThickness = xlsread('CoeffMatrixRFemurThicknessFNew2.xls');
end
%
%% Import RFemur THUMS files
RFemurIn = GetKfileNodes('THUMSRFemurReady.k');
RFemurNodesSet = textread('RFemurNodesSet.txt','%s');
RFemurLinkedNodes = cell2mat(RFemurNodesSet);
a= [];
for i=1:length(RFemurLinkedNodes)
aa=str2double(RFemurLinkedNodes(i,17:24));
bb=str2double(RFemurLinkedNodes(i,25:32));
cd=str2double(RFemurLinkedNodes(i,33:40));
dd=str2double(RFemurLinkedNodes(i,41:48));
ee=str2double(RFemurLinkedNodes(i,49:56));
ff=str2double(RFemurLinkedNodes(i,57:64));
gg=str2double(RFemurLinkedNodes(i,65:72));
hh=str2double(RFemurLinkedNodes(i,73:80));
a = [a aa bb cd dd ee ff gg hh];
end
RFemurLinkedNodes = reshape(a,8,[]);'
clear a aa bb cd dd ee ff gg hh i;

CoordsRFemur = xlsread('CoordsRFemur.xls');
RFemurAllInfo = csvread('THUMSRFemurAll.csv');

OrigRFemurShaft = CoordsRFemur(1:1440,:);
OrigRFemurEnds = CoordsRFemur(1441:3062,:);
OrigRFemurEndsFew = OrigRFemurEnds(1:10:end,:); %%% delute /10
OrigLandRFemur = [OrigRFemurShaft(:,2:4);OrigRFemurEndsFew(:,2:4)];
OrigRFemur = RFemurAllInfo(:,2:4);

RFOutInfo = csvread('THUMSRFemurOut.csv');
RFInInfo = csvread('THUMSRFemurIn.csv');
RFOutInfoNodes = RFOutInfo(:,2:4);
RFInInfoNodes = RFInInfo(:,2:4);
RFOutID = RFOutInfo(:,1);
RFInID = RFInInfo(:,1);
% Format for a specific RFemur
PredRFemur = meanVectorRFemur+evecsRFemur'*((CoeffRFemur*[age,InFemurLength,BMI,1]));
PredRFemur = (reshape(PredRFemur',3,[1]))';

% Predict a specific RFemur Shape
PRFemurInt = RBFsInterpolationWithSmooth(OrigLandRFemur,PredRFemur,OrigRFemur,0);

% Link inner nodes based on outer nodes
PRFemurIntInfo = RFemurAllInfo;
PRFemurIntInfo(:,2:4) = PRFemurInt(:,2:4);
RFemurAllIDPool = PRFemurIntInfo(:,1);

[Lia,Locb] = ismember(RFemurAllIDPool,RFOutID);
[r,c,v] = find(Locb);
PRFOutIntInfo = PRFemurIntInfo(r,1:4);

% Export Tri mesh
fem = PRFOutIntInfo(:,2:4);
fem = PRFOutIntInfo(:,2:4);
Matlab code was claimed form Klein et. al [5].

```matlab
function MorphedNodesAddNumber = TPSInterpWithSmooth(SourceLandmarks, 
            TargetLandmarks, SourceModel, SmoothPar)
    %--------------------------------------------
    % SourceLandmarks are the landmarks on the baseline model, the matrix is 
    % n*3, including [x,y,z] coordinate;
    %
    % TargetLandmarks are the landmarks on the target model, the matrix is 
    % n*3, including [x,y,z] coordinate;
    %
    % SourceModel are the nodes on the baseline model, the matrix is 
    % n*3 or n*4, including [x,y,z] coordinate or [NodeNumber, x, y, z];
    %
    % SmoothPar is the parameter to smooth the morphed model, and the 
    % parameter means the changed distance of the source landmarks. the default 
    % value is 0.
    %--------------------------------------------

    % Detect the input parameters
    SourceSurfaceLandmarks = SourceLandmarks;
    TargetSurfaceLandmarks = TargetLandmarks;

    [~, WholeWidth] = size(SourceModel);
    if WholeWidth == 3
        WholeBodyCord = ((1:length(SourceModel)))', SourceModel);
    else
        WholeBodyCord = SourceModel;
    end
```
% RBFs interpolation

% calculate the weight matrix
if length(SmoothPar) == 1
    SmoothMatrix = (SmoothPar+1)*ones(1,length(SourceSurfaceLandmarks));
else SmoothMatrix = SmoothPar+1;
end

DistanceMatrix = pdist2(SourceSurfaceLandmarks,SourceSurfaceLandmarks);
% following 2 formul are for TPS

P = [ones(length(SourceSurfaceLandmarks),1) SourceSurfaceLandmarks];
O = zeros(4);
PP = P';
DistanceMatrix(:,end+1:end+4) = P;
DistanceMatrix(end+1:end+4,:) = [PP O];
clear P PP

V = [TargetSurfaceLandmarks;O(:,1:3)];
W = DistanceMatrix\V;
clear DistanceMatrix V

w1 = W(1:end-4,:);
a = W(end-3:end,:);
clear W

% Morph the baseline model

% calculate every nodes
for i = 1:length(WholeBodyCord)
    FirstNode = WholeBodyCord(i,2:4);
    SingleDistanceMatrix = pdist2(SourceSurfaceLandmarks,FirstNode);
    % following 2 formul are for TPS
    % U_single = (SingleDistanceMatrix.^2).* (log(SingleDistanceMatrix));
    % U_single(isnan(U_single)) = 0;
    MorphedNodesCordinate(i,:) = [1 FirstNode]*a+ SingleDistanceMatrix'*w1;
end
68  %-----------------------------------------------Export the model-----------------------------------------------%
69  MorphedNodesAddNumber = [WholeBodyCord(:,1), MorphedNodesCordinate];
70  % WriteKfile('FastMorphedNodes.k',MorphedNodesAddNumber);
71  end
C.1 Measurements Code

```matlab
function [a,b,c,d,f,g] = getresults(x);

% generated femurs mesuremts are done automatially here
%

fem=x;

%% Points input

% node nummbers which found in sample femur measuremets that will%
% consistance troughtout all template

% node numbers for shaft wide
points.ss_w = [31;547;548;549;550;551;552;553;554
  555;556;963;965;1017;1069;1070;1171;1172
  1173;1192;1193;1194;1195;1424];

points.ss_alt = [69;760;761;796;797;814;903;904
  905;906;907;979;1010;1012;1051;1052;1232;1240;1241;1430];

points.nn = [2329;2330;2368;2369;2385;2403
  2472;2473;2485;2494;2550;2557;2559;2560
  2582;2583;2597;2610;2611;2658;2662;2675]
```
2681;2682;2780;2783;2827;2828;2831
2833;2858;2860;2863;2975];
points.nn2 = [2315; 2316; 2328; 2329
2331;2368;2369;2372;2385;2395;2403
2414;2485;2494;2547;2553;2560;2581
2593;2601;2657;2658;2659;2662;2675
2681;2683;2783;2784;2786;2805;2806
2826;2827;2847;2858;2860;2861;2862
2863;2978;2981];
points.head = [2505;2506;2516;2520;2521
2522;2525;2526;2527;2528;2529
2530;2531;2532;2533;2571;2573
2576;2578;2579;2612;2614;2617
2621;2627;2633;2634;2635;2636
2637;2639;2643;2645;2651;2652
2653;2654;2886;2887;2888;2889
2891;2892;2900;2901;2906;2908
2909;2910;2924;2925;2927;2928
2929;2930;2931;2932;2934;2935
2936;2937;2938;2939;2941;2943
2949;2951;2952;2953;2954;2961
2964;2985;2988;2991;2992;2994;2996];
points.ss.a2 = [86;344;345;346;347;348
349;350;351;352;353;354;355;356;357
358;359;360;1117;1118;1329;1330;1331;1399];
points.ndist = [2560;2863];
points.lesserth = [2764];

%%
% cut half for visual
k=1;
for i=1:length(fem) ;
   if fem (i,3) > 120 ;
      femc(k,:) = fem(i,:);
      k=k+1 ;
%% shaft wide and axis
lesser.th = fem(points.lesserth,3);
k=1;
for i=1:length(fem) ;
    if fem (i,3) > lesser.th 21 & fem (i,3) < lesser.th 19 ;
        shaft.th(k,:) = fem(i,:);
        k=k+1 ;
    else
end
end

[xc,yc,r] = circfit(shaft.th(:,1),shaft.th(:,2));
shaft.lth_cp = [xc,yc,lesser.th 20];
shaft.lth_r = r;
clear xc yc r

shaft.wide_cicle = zeros( length(points.ss_w),3);
for i=1:length(points.ss_w);
    shaft.wide_cicle (i,:) = fem(points.ss_w (i),:);
end

[xc,yc,r] = circfit(shaft.wide_cicle(:,1),shaft.wide_cicle(:,2));
shaft.w_cp = [xc,yc,fem(points.ss_w(i),3)];
% axis

shaft.alt_cicle = zeros(length(points.ss_alt),2);
for i=1: length(points.ss_alt)
    shaft.alt_cicle (i,:) = fem(points.ss_alt (i),1:2);
end

[xc,yc,r] = circfit(shaft.alt_cicle(:,1),shaft.alt_cicle(:,2));
shaft.a_cp = [xc,yc,fem(points.ss_alt (i),3)];
shaft.a_r = r;

clear xc yc r

shaft.a2_cicle = zeros(length(points.ss_a2),2);
for i=1: length(points.ss_a2)
    shaft.a2_cicle (i,:) = fem(points.ss_a2 (i),1:2);
end

[xc,yc,r] = circfit(shaft.a2_cicle(:,1),shaft.a2_cicle(:,2));
shaft.a2_cp = [xc,yc,fem(points.ss_a2 (i),3)];
shaft.a2_r = r;

clear xc yc r

% visuals

% pcshow(femc)
% hold on
% plotCircle3D(shaft.a_cp,[0 0 1],shaft.a_r)
% plotCircle3D(shaft.a2_cp,[0 0 1],shaft.a2_r)
% plotCircle3D(shaft.lth_cp,[0 0 1],shaft.lth_r)
% scatter3(shaft.th(:,1),shaft.th(:,2),shaft.th(:,3),'*','b')
% scatter3(shaft.wide_cicle(:,1),shaft.wide_cicle(:,2),shaft.wide_cicle(:,3),'*','p')
% scatter3(fem(points.lesserth,1),fem(points.lesserth,2),fem(points.lesserth,3)
%   ,'*','k')

% narrow neck
neck.coord = zeros (length(points.nn),3);
for i=1: length(points.nn);
    neck.coord (i,:)= fem(points.nn (i),:);
end

[neck.c, neck.r] = sphereFit(neck.coord);

% neck 2
neck.coord2 = zeros (length(points.nn2),3);
for i=1: length(points.nn2);
    neck.coord2 (i,:)= fem(points.nn2 (i),:);
end

[neck.c2, neck.r2] = sphereFit(neck.coord2);
[neck.c21, neck.r21,v,evec] = ellipsoid_fit_new(neck.coord2);
%dist

neck.dist = norm(fem(points.ndist(1,1),:) - fem(points.ndist(2,1),:));

% visual
[x, y, z] = meshgrid(-30:30, -40:40, 130:230);

% pcshow(fem)
hold on
p = patch(isosurface(x, y, z, v(1,1)*x.^2 + v(2,1)*y.^2 + v(3,1)*z.^2 + 2*v(4,1)*x.*y + 2*v(5,1)*x.*z + 2*v(6,1)*y.*z + 2*v(7,1)*x + 2*v(8,1)*y + 2*v(9,1)*z , -v(10,1)));

% scatter3(neck.coord2(:,1),neck.coord2(:,2),neck.coord2(:,3),'*','r')

% [x,y,z]= sphere(30);
% mesh(x*neck.r2+neck.c2(1,1),y*neck.r2+neck.c2(1,2),z*neck.r2+neck.c2(1,3))

% clear x y z p

% head

head.coord = zeros(length(points.head),3);
for i=1:length(points.head);
  head.coord (i,:) = fem(points.head (i),:);
end

[head.c, head.r] = sphereFit(head.coord);

% visual aid (added later for visual)

% pcshow (fem) ;
% hold on
% scatter3(head.coord(:,1),head.coord(:,2),head.coord(:,3),'*','b')
% hold on
% [x,y,z]= sphere(30);
% mesh(x+head.r+head.c(1,1),y+head.r+head.c(1,2),z+head.r+head.c(1,3))
% hold off

% angle
% nn.v= head.c - neck.c;
% ss.v= shaft.w_cp- shaft.a2_cp;
% angle = atan2(norm(cross(nn.v,ss.v)),dot(nn.v,ss.v));
% angle = 180- rad2deg(angle);

% axis intersection
th.cstartpoints= [head.c ; shaft.w_cp];
th.cendpoints= [neck.c21' ; shaft.a_cp];
axis.int = lineIntersect3D(th.cstartpoints,th.cendpoints);

nn.v2= head.c - axis.int;
ss.v2= axis.int- shaft.a2_cp;
angle1 = atan2(norm(cross(nn.v2,ss.v2)),dot(nn.v2,ss.v2));
angle2 = 180- rad2deg(angle1);

% visuals
% nn.vv= [head.c ; axis.int];
% ss.vv= [axis.int; shaft.a2_cp];
%
% pcshow(fem)
% hold on
% plot3(nn.vv(:,1) ,nn.vv(:,2),nn.vv(:,3), 'LineWidth',2 )
% hold on
% plot3(ss.vv(:,1) ,ss.vv(:,2),ss.vv(:,3),'LineWidth',2 )

%% Results

a = angle2 ;
b = shaft.lth_r*2;
c = neck.r2*2 ;
d = head.r*2;
f = neck.r21(1,1)*2 ;
g = neck.r21(2,1)*2 ;

end

C.2 Circular fit

www.mathworks.com/matlabcentral/fileexchange/5557-circle-fit?focused=5059278&tab=

function

function [xc,yc,R] = circfit(x,y)

%CIRCFIT Fits a circle in x,y plane
%
% [XC, YC, R, A] = CIRCFIT(X,Y)
% Result is center point (yc,xc) and radius R. A is an optional
% output describing the circle's equation:
%
% x^2+y^2+a(1)*x+a(2)*y+a(3)=0

% by Bucher izhak 25/oct/1991

n=length(x); xx=x.*x; yy=y.*y; xy=x.*y;
A=[sum(x) sum(y) n; sum(xy) sum(yy) sum(y); sum(xx) sum(xy) sum(x)];
B=[-sum(xx+yy) ; -sum(xx.*y+yy.*y) ; -sum(xx.*x+xy.*y)];
a=A\B;
xc = -.5*a(1);
yc = -.5*a(2);
\[ R = \sqrt{\frac{(a(1)^2 + a(2)^2)}{4}} - a(3); \]

### C.3 Spherical Fit


```matlab
function [Center,Radius] = sphereFit(X)
% this fits a sphere to a collection of data using a closed form for the
% solution (opposed to using an array the size of the data set).
% Minimizes Sum((x-xc)^2+(y-yc)^2+(z-zc)^2-r^2)^2
% x,y,z are the data, xc,yc,zc are the sphere's center, and r is the radius

% Assumes that points are not in a singular configuration, real numbers, ...
% if you have coplanar data, use a circle fit with svd for determining the
% plane, recommended Circle Fit (Pratt method), by Nikolai Chernov
% http://www.mathworks.com/matlabcentral/fileexchange/22643

% Input:
% X: n x 3 matrix of cartesian data
% Outputs:
% Center: Center of sphere
% Radius: Radius of sphere
% Author:
% Alan Jennings, University of Dayton

A=[mean(X(:,1).*(X(:,1)-mean(X(:,1)))), ...
    2*mean(X(:,1).*(X(:,2)-mean(X(:,2)))), ...
    2*mean(X(:,1).*(X(:,3)-mean(X(:,3))))]; ...
0, ...
mean(X(:,2).*(X(:,2)-mean(X(:,2)))), ...
2*mean(X(:,2).*(X(:,3)-mean(X(:,3))))]; ...
0, ...
0, ...
mean(X(:,3).*(X(:,3)-mean(X(:,3))));
A=A+A.';
B=[mean((X(:,1).^2+X(:,2).^2+X(:,3).^2).*(X(:,1)-mean(X(:,1))));
    mean((X(:,1).^2+X(:,2).^2+X(:,3).^2).*(X(:,2)-mean(X(:,2))));
    mean((X(:,1).^2+X(:,2).^2+X(:,3).^2).*(X(:,3)-mean(X(:,3))));
Center=(A\B).';
Radius=sqrt(mean(sum([X(:,1)-Center(1),X(:,2)-Center(2),X(:,3)-Center(3)].^2,2)));
```

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C.4 Ellipsoid Fit

https://www.mathworks.com/matlabcentral/fileexchange/24693-ellipsoid-fit

```matlab
function [ center, radii, evecs, pars ] = ellipsoid_fit_new( X, equals )
%
% Fit an ellipsoid/sphere/paraboloid/hyperboloid to a set of xyz data points:
% 
% [center, radii, evecs, pars ] = ellipsoid_fit( X )
% [center, radii, evecs, pars ] = ellipsoid_fit( [x y z] );
% [center, radii, evecs, pars ] = ellipsoid_fit( X, 1 );
% [center, radii, evecs, pars ] = ellipsoid_fit( X, 2, 'xz' );
% [center, radii, evecs, pars ] = ellipsoid_fit( X, 3 );
%
% Parameters:
% * X, [x y z] — Cartesian data, n x 3 matrix or three n x 1 vectors
% * flag — ' ' or empty fits an arbitrary ellipsoid (default),
% — 'xy' fits a spheroid with x- and y- radii equal
% — 'xz' fits a spheroid with x- and z- radii equal
% — 'xyz' fits a sphere
% — '0' fits an ellipsoid with its axes aligned along [x y z] axes
% — '0xy' the same with x- and y- radii equal
% — '0xz' the same with x- and z- radii equal
%
% Output:
% * center — ellipsoid or other conic center coordinates [xc; yc; zc]
% * radii — ellipsoid or other conic radii [a; b; c]
% * evecs — the radii directions as columns of the 3x3 matrix
% * v — the 10 parameters describing the ellipsoid / conic algebraically:
%       Ax^2 + By^2 + Cz^2 + 2Dxy + 2Exz + 2Fyz + 2Gx + 2Hy + 2Iz + J = 0
% * chi2 — residual sum of squared errors (chi^2), this chi2 is in the
% coordinate frame in which the ellipsoid is a unit sphere.
%
% Author:
% Yury Petrov, Oculus VR
% Date:
% September, 2015
%
%marginchk( 1, 3 ); % check input arguments
if nargin == 1
    equals = ''; % no constraints by default
```

88
if size( X, 2 ) ~= 3
    error( 'Input data must have three columns!' );
else
    x = X(:, 1);
    y = X(:, 2);
    z = X(:, 3);
end

% need nine or more data points
if length( x ) < 9 && strcmp( equals, '' )
    error( 'Must have at least 9 points to fit a unique ellipsoid' );
end
if length( x ) < 8 && ( strcmp( equals, 'xy' ) || strcmp( equals, 'xz' ) )
    error( 'Must have at least 8 points to fit a unique ellipsoid with two equal radii' );
end
if length( x ) < 6 && strcmp( equals, '0' )
    error( 'Must have at least 6 points to fit a unique oriented ellipsoid' );
end
if length( x ) < 5 && ( strcmp( equals, '0xy' ) || strcmp( equals, '0xz' ) )
    error( 'Must have at least 5 points to fit a unique oriented ellipsoid with two equal radii' );
end
if length( x ) < 4 && strcmp( equals, 'xyz' )
    error( 'Must have at least 4 points to fit a unique sphere' );
end

% fit ellipsoid in the form Ax^2 + By^2 + Cz^2 + 2Dxy + 2Exz + 2Fyz + 2Gx + 2Hy + 2Iz + J = 0 and A + B + C = 3 constraint removing one extra parameter
if strcmp( equals, '' )
    D = [ x .* x + y .* y - 2 * z .* z, ...
         x .* x + z .* z - 2 * y .* y, ...
         2 * x .* y, ...
         2 * x .* z, ...
         2 * y .* z, ...
         2 * x, ...
         2 * y, ...
         2 * z, ...
         ];
78    1 + 0 * x ]; % ndatapoints x 9 ellipsoid parameters
79 elseif strcmp( equals, 'xy' )
80    D = [ x .* x + y .* y - 2 * z .* z, ...
81        2 * x .* y, ...
82        2 * x .* z, ...
83        2 * y .* z, ...
84        2 * x, ...
85        2 * y, ...
86        2 * z, ...
87        1 + 0 * x ]; % ndatapoints x 8 ellipsoid parameters
88 elseif strcmp( equals, 'xz' )
89    D = [ x .* x + z .* z - 2 * y .* y, ...
90        2 * x .* y, ...
91        2 * x .* z, ...
92        2 * y .* z, ...
93        2 * x, ...
94        2 * y, ...
95        2 * z, ...
96        1 + 0 * x ]; % ndatapoints x 8 ellipsoid parameters
97 % fit ellipsoid in the form Ax^2 + By^2 + Cz^2 + 2Gx + 2Hy + 2Iz = 1
98 elseif strcmp( equals, '0' )
99    D = [ x .* x + y .* y - 2 * z .* z, ...
100       x .* x + z .* z - 2 * y .* y, ...
101        2 * x, ...
102        2 * y, ...
103        2 * z, ...
104        1 + 0 * x ]; % ndatapoints x 6 ellipsoid parameters
105 % fit ellipsoid in the form Ax^2 + By^2 + Cz^2 + 2Gx + 2Hy + 2Iz = 1,
106 % where A = B or B = C or A = C
107 elseif strcmp( equals, '0xy' )
108    D = [ x .* x + y .* y - 2 * z .* z, ...
109        2 * x, ...
110        2 * y, ...
111        2 * z, ...
112        1 + 0 * x ]; % ndatapoints x 5 ellipsoid parameters
113 elseif strcmp( equals, '0xz' )
114    D = [ x .* x + z .* z - 2 * y .* y, ...
115        2 * x, ...
116        2 * y, ...
117        2 * z, ...
118        1 + 0 * x ]; % ndatapoints x 5 ellipsoid parameters
% fit sphere in the form $A(x^2 + y^2 + z^2) + 2Gx + 2Hy + 2Iz = 1$

if strcmp(equals, 'xyz')
    D = [ 2 * x, ...
    2 * y, ...
    2 * z, ...
    1 + 0 * x ]; % ndatapoints x 4 ellipsoid parameters
else
    error( [ 'Unknown parameter value ' equals '!'] );
end

% solve the normal system of equations

d2 = x .* x + y .* y + z .* z; % the RHS of the llSQ problem (y's)

u = ( D' * D ) \ ( D' * d2 ); % solution to the normal equations

% find the residual sum of errors

chi2 = sum( ( 1 - ( D * u ) ./ d2 ).^2 ); % this chi2 is in the coordinate frame
in which the ellipsoid is a unit sphere.

% find the ellipsoid parameters
% convert back to the conventional algebraic form

if strcmp( equals, '' )
    v(1) = u(1) + u(2) - 1;
    v(2) = u(1) - 2 * u(2) - 1;
    v(3) = u(2) - 2 * u(1) - 1;
    v( 4 : 10 ) = u( 3 : 6 );
elseif strcmp( equals, 'xy' )
    v(1) = u(1) - 1;
    v(2) = u(1) - 1;
    v(3) = -2 * u(1) - 1;
    v( 4 : 10 ) = u( 2 : 8 );
elseif strcmp( equals, 'xz' )
    v(1) = u(1) - 1;
    v(2) = -2 * u(1) - 1;
    v(3) = u(1) - 1;
    v( 4 : 10 ) = u( 2 : 8 );
elseif strcmp( equals, '0' )
    v(1) = u(1) + u(2) - 1;
    v(2) = u(1) - 2 * u(2) - 1;
    v(3) = u(2) - 2 * u(1) - 1;
    v = [ v(1) v(2) v(3) 0 0 0 u( 3 : 6 )' ];
elseif strcmp( equals, '0xy' )

91
v(1) = u(1) - 1;
v(2) = u(1) - 1;
v(3) = -2 * u(1) - 1;
v = [ v(1) v(2) v(3) 0 0 0 u( 2 : 5 )' ];

elseif strcmp( equals, '0xz' )
v(1) = u(1) - 1;
v(2) = -2 * u(1) - 1;
v(3) = u(1) - 1;
v = [ v(1) v(2) v(3) 0 0 0 u( 2 : 5 )' ];

elseif strcmp( equals, 'xyz' )
v = [ -1 -1 -1 0 0 0 u( 1 : 4 )' ];
end
v = v';

% form the algebraic form of the ellipsoid
A = [ v(1) v(4) v(5) v(7); ...
v(4) v(2) v(6) v(8); ...
v(5) v(6) v(3) v(9); ...
v(7) v(8) v(9) v(10) ];

% find the center of the ellipsoid
center = -A( 1:3, 1:3 ) \ v( 7:9 );

% form the corresponding translation matrix
T = eye( 4 );
T( 4, 1:3 ) = center';

% translate to the center
R = T * A * T';

% solve the eigenproblem
[ evecs, evals ] = eig( R( 1:3, 1:3 ) / -R( 4, 4 ) );
radii = sqrt( 1 ./ diag( abs( evals ) ) );
sgns = sign( diag( evals ) );
radii = radii .* sgns;

% calculate difference of the fitted points from the actual data normalized by the conic radii
d = [ x - center(1), y - center(2), z - center(3) ]; % shift data to origin
d = d * evecs; % rotate to cardinal axes of the conic;
d = [ d(:,1) / radii(1), d(:,2) / radii(2), d(:,3) / radii(3) ]; % normalize to the conic radii
chi2 = sum( abs( 1 - sum( d.^2 .* repmat( sgns', size( d, 1 ), 1 ), 2 ) ) );
if abs( v(end) ) > 1e-6
\[ v = -v / v(\text{end}); \text{ \% normalize to the more conventional form with constant term} \]
\[ = -1 \]

else
\[ v = -\text{sign}( v(\text{end}) ) * v; \]
end
Appendix D
Some Generated Right Femurs of the US Population
Figure D.1. Some male Right femurs.
Bibliography


