The Pennsylvania State University

The Graduate School

Department of Industrial and Manufacturing Engineering

LATTICE STRUCTURE MODELING AND OPTIMIZATION

A Thesis in

Industrial Engineering

by

Yuqin Liu

Submitted in Partial Fulfillment

of the Requirements

for the Degree of

Master of Science

December 2017

The thesis of Yuqin Liu was reviewed and approved* by the following:

Sanjay Joshi
Professor of Industrial and Manufacturing Engineering
Thesis Advisor

Saurabh Basu
Assistant Professor of Industrial and Manufacturing Engineering

Janis Terpenny
Peter and Angela Dal Pezzo Department Head and Professor
Head of the Department of Industrial and Manufacturing Engineering

*Signatures are on file in the Graduate School

ABSTRACT

An algorithm for designing and optimizing homogeneous lattice structure to support given load condition is presented. The approach is to homogenize the safety factor distribution by altering trusses parameters defining the lattice structure. The algorithm combines theoretical mechanics in the initial design section, and FEA method to optimize and verify the tests result. The Octahedral lattice structure is utilized as an example of the design and optimization algorithm. The initial design decides the lattice structure node locations, and the optimization process redistributes structure material to reach higher strength without adding or removing material at the whole structure level, this means each truss may have a different cross-section due to their functions. Modern additive manufacturing technology made the fabrication of such lattice structure to be possible. The reference parameters in optimization iterations are Safety Factor distribution and the Von-Mises stress. Material stress limit and design safety factor are determined outside the iteration loops. The optimized lattice structure and the traditional lattice structures are physically tested in an MTS compress test machine. The result for a 3×3×3 octahedral cells lattice structure shows the optimized lattice structure support 60% more load than the traditional homogeneous lattice structure at the same weight. The optimize/design algorithm could be applied to other lattice structures formed by nodes and or general structures.

Keywords: Lattice structure, Homogeneous, Safety Factor, Truss,

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACKNOWLEDGEMENTS

This work could not be achievable without the advice and support from Dr. Sanjay Joshi and Dr. Saurabh Basu and The Harold and Inge Marcus Department of Industrial and Manufacturing Engineering. Especially Dr. Joshi, as my advisor and mentor, who is supportive for my academic goal and career development. We had been worked closely to derive optimization theory and algorithm; we also discussed how to build lattice structure with Additive Manufacturing Process. I had learned how to do research, and that is more precious than doing research. Dr. Joshi is excellent in Additive Manufacturing and Computer Aid Process planning, those are firmly connected with this research. Dr. Basu is studying in advanced manufacturing; we had co-operated in compression test and data analysis.

All the manufacturing process and experiment tests are based on the equipment in Industrial and Manufacturing Engineering Department. All the faculties and staffs had given me great help in perusing the research. They provide tremendous help in contributing academic environment and research development around the community.

Nobody could be more critical of me than my family. I would like to thank my parents for their love and kindness in my life. I wish I can be as good as them in the future.

Chapter 1. Introduction

1.1 Background

Lattice structures have attracted much research attention because of their high strength/weight ratio and multi-functionality, such as thermal conduction and energy absorption. Lattice structures can divide large load into smaller stress through multiple-trusses or support structure. Traditional manufacturing processes are not capable of building these kinds of complex structure. Additive manufacturing provides an easy way to fabricate complex lattice structures since Additive manufacturing process brings free-form manufacturing possibility to industry and research. Many researchers in material science have studied the strength of lattice structures based on the lattice structure's relative density, lattice cell type, size and other parameters [1][2]. Mathematical models for lattice structures and analysis of their strength with theoretical mechanics have been presented [3] [4]. However, none of them demonstrate on general lattice structure or optimized lattice structure topology at cell (truss) level. The lattice structure design/optimization algorithm introduced in this research can work on large lattice structures to optimize detail topology. This optimization algorithm is demonstrated using an octahedral lattice structure, while it is also applicable to other lattice structures or general structure strength optimization. It is based on the assumption of evenly distributed safety factor/stress.

1.2 Octahedral Lattice structure

The octahedral lattice structure contains nodes and trusses (Fig 1.1). As the figure shows, the trusses are designed to show the highest strength in Z-axis direction, the cell's property in X and Y direction is lower than the strength in the Z direction. When a Z-direction load/force is applied to the lattice structure, the majority of the load will distribute into smaller loads through the trusses and nodes. It allows the octahedral lattice structure or other trusses used to form lattice structure to resist a large amount of load. The octahedral lattice structure is also robust to vibration and small damages as stretching in different directions, which also provides support in multi-directions. These trusses are not only beneficial in lattice structure application but also lead to interesting design and optimization problems. The amount of load that is transmitted by each truss under given loading condition can be used to determine trusses' parameters. Engineers also demand an appropriate way to build a lattice structure with clear parameters and explicit logic.

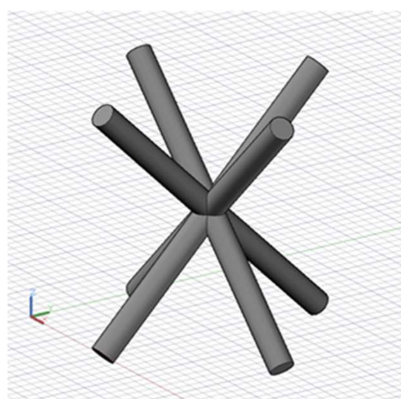


Figure 1.1. Octahedral Lattice cell

Six primary design parameters are used to define a cellular lattice: truss length, truss diameter, height of the lattice cell (H), length of the lattice cellular(L), width of the lattice cellular(W), the truss-ground angles, the truss-truss angles and the circular cross-section areas (Fig 1.2).

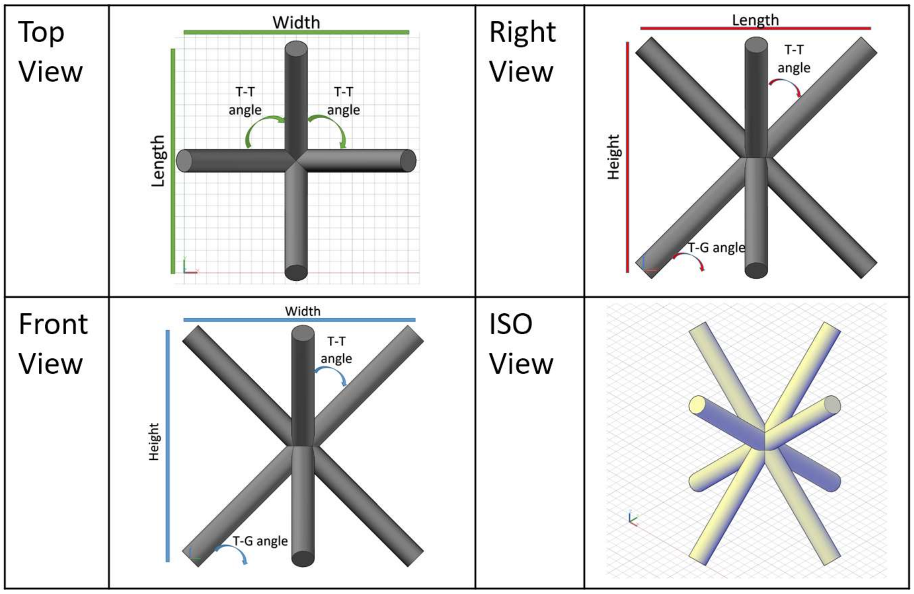


Figure 1.2. Octahedral lattice cell parameter definition

Previous research has been focused on studying the properties of homogeneous lattice structures. This means that all the lattice cells/trusses have the same parameters through the whole structure. The optimization algorithm demonstrated in this work will alter each lattice truss diameter based on their load condition and create a non-homogeneous lattice. This requires each truss to have individual parameter and calculation. The objective function and algorithm to design and optimize lattice structure proposed here is based on the assumption of evenly distributed safety factor.

1.3 Evenly distributed safety factor assumption

The evenly distributed safety factor/stress assumption is based on von-Mises Stress [5][22] (also called maximum distortion energy theory of failure) and safety factor. The von-Mises yield criterion also called the maximum distortion energy theory of failure; it describes when one point's equivalent stress/strain reaches a specific value, the material will begin to yield. The equivalent stress/strain in this description is called second deviatoric stress invariant J2. Once the stress is the same as von-Mises stress, the safety factor equals to one. Thus, if the whole lattice structure has evenly distributed safety factor, the whole lattice structure will break down at the same time. It eliminates the weak points in the traditional lattice structure and increases material utilization.

Safety factor formula:

$$\eta = \frac{S_y}{\sigma_e}$$

S_y is the ultimate stress/material allowance stress.

$\sigma_e$is the von-Mises stress.

Based von Mises yield criterion, the trusses will not break down until the safety factor is lower than 1. Using evenly distributed safety factor can avoid the occurrence of weak points. The primary way to achieve evenly distributed safety factor is by the re-distributing material. Thus maintaining structure weight while also achieving the highest strength.

Utilizing optimization based on von-Mises stress and afety factor requires the calculation of these two parameters. Because the calculation process of safety factor in the complex structure is time-consuming and complicated, simulation is used as the means to extract the safety factor and von-Mises stress distribution. Modern simulation process provided agreeable accuracy on strength, thermal, acoustic and other areas. A brief overview of the optimization process is shown below (Fig 1.3).

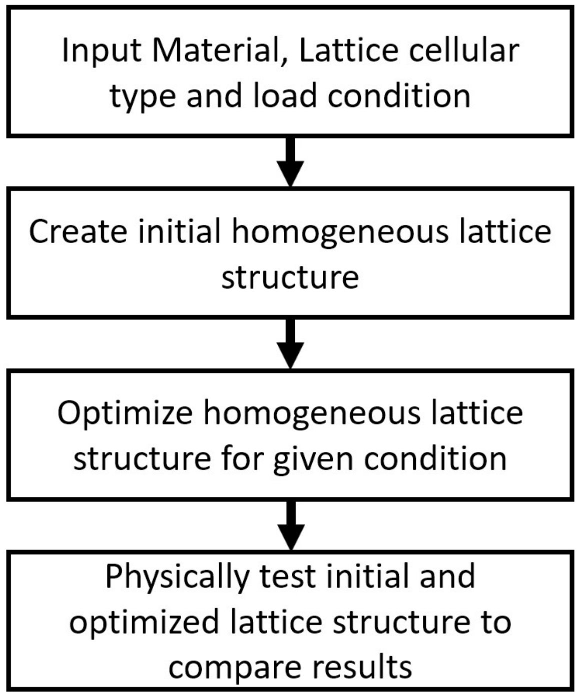


Figure 1.3. Brief optimization process

1.4 Problem Defenition and Research Objective

Lattice structures are expected to fulfill the following design and load requirements:

1. To meet the functional design claims, for instance, weight, strength or stiffness.

2. To improve material utilization and reinforce the strength of, lattice structure with trusses, by using evenly distribute safety factor design/optimization theory.

3. To reduce the material consumption of lattice structure in supporting given load condition, by removing redundant material from trusses overqualified.

To achieve the requirements of a lattice structure design, the problem addressed in this research is stated as follows:

Given an entire homogeneous lattice structure, develop an approach to optimize the design of the structure byvarying te design parameters. For the purpose of this research, only a single parameter (diameter of truss) will be optimized. The approach developed should be generate enough to allow other parameters to be optimized.

Research Objective:

1. To demonstrate that evenly distributed safety factor theory can be used for structure design/optimization problems.

2. To develop a step-by-step algorithm and related formulas for calculating lattice structure truss diameter.

3. To create an optimization framework and prove its effectiveness by compression testing.

1.5 Outline of this thesis

Fig. 1.4 describes a flowchart outlining this thesis. The research findings and results have been present in three major parts. Part 1: Introduction and literature Review. Part 2: Methods, design and generation of the initial homogeneous lattice structure, lattice structure optimization and experimental compression test results. Part 3. Discussion and conclusion.

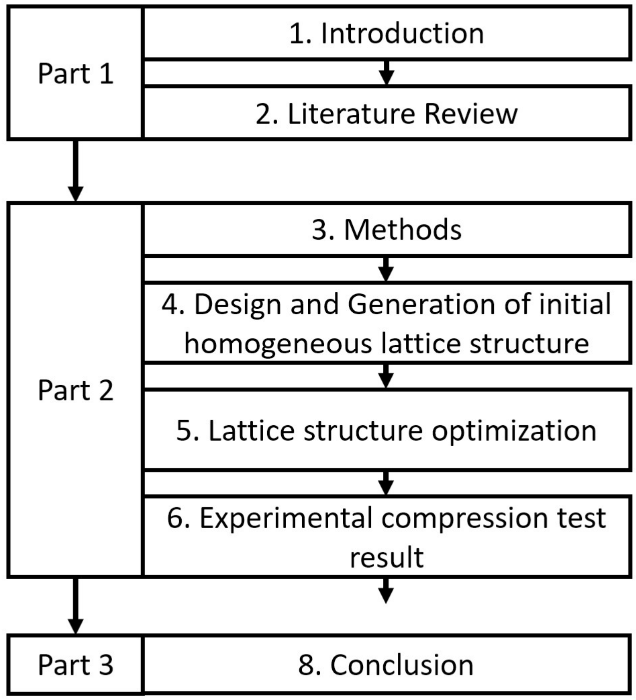


Figure 1.4. Flowchart of the Thesis

Chapter 1. Provides a brief background and introduction to lattice structures and the theory going into this research. It also provides the research definition and objectives for the design and optimization of the lattice structure. Chapter 2. Discusses state of the art in lattice structure modeling and optimization. Chapter 3. Describes the software, material, manufacturing machine, calculation formulas and lattice structure parameter used in this research. Chapter 4. Uses the previous method to generate initial homogeneous lattice structure and example calculation process. Chapter 5. Presents the homogeneous lattice structure's optimization process and simulation result, using simulation result for further optimization iterations.Chapter 6. Evaluate optimization result through physically performance of the compression tests and compares initial lattice structure property with the optimized lattice structure. Then discusses the difference between this work and previous studies. Chapter 8. Illustrates the significant contributions of this work, propose other possible areas that can use this research's result.

Chapter 2. Literature review

2.1 Introduction

The first and foremost step to optimize a structure or specifically---lattice structure in the past is to build a mathematical model for it. However, considering its complex geometry all over the structure; it is tough to build a mathematical model for a whole lattice structure, especially for the lattice structure filled with complex shells. Most of the researchers focus on only modeling a single cell of the lattice structure. For example, Li Yang[6] has studied how to model Octahedral lattice structure and built different size octahedral lattice structures with Ti-6Al-4V in SLM process.His research shows at the model of lattice structure could not predict their property. The lattice structures strength in the physical test is far away from calculation predictions. The more complex the structure is, the larger the difference is, this trend also exists in other lattice structure modeling researches[7][8]. Moreover, there is no sign any new mathematical theory has or will be developed to fix the problem. Hence, it is time to looking for a new way to optimize lattice structure.

2.2 Lattice structure modeling and simulation

It is obvious many new developed technology or methods could integrate with lattice structure design and optimization, Jason Nam Nguyen [9] established Size Matching and Scaling (SMS) method. The lattice structure design was simplified into variables D_{MAX} and D_{min} (Truss Diameter) problems. D_{max} and D_{min} for lattice structure trusses were manually set up and their effect on lattice structure strength was studied. However, the research does not show how to find the optimal D_{max} and D_{min}. Thus, this research inapplicable to the real design.

Gregory C Graf [10] studied how the D_{max} and D_{min} relationship affected the lattice structure property; it was reported that when D_{min} equals to 28% of the D_{MAX}, the lattice structure reaches the optimal property. It may seem reasonable at first, but lattice structure's property may be affected by other parameters. The relationship of D_{max} and D_{min} are the representatives of trusses diameters. Nguyen[9] has studied the efficiency of different topology optimization methods and shows that the 28% theory does not work well in extensive experiments. The most efficient topology optimization methodologies Nguyen[9] found is Ground Truss method [11], although it produces the best result, it is also the most time-consuming one of all the optimization methods.

Jikai Liu et al. [12] Xiangyang Zhou [13] et al. use SIMP (Solid Isotropic Material with Penalization) method in strut optimization. They also alter the lattice structure in their research, and the examples in their research show better property than the non-optimized lattice structures. However, their research did not address how to save material from the areas that do not require strengthening. Thus, missing the opportunity to reduce material cost.

Pu Zhang et al. [14] tested Homogenization, Optimization, and Construction (HOC) method (Similar to SIMP) in a honeycomb structure. Zhang[14] and Jikai Liu[12], Xiangyang Zhou[13] only focus on the general structure and ignore detail topology. Thus, their research is not applicable to lattice structures in different sizes. Because the stress condition on a single truss or unit can be very complicated, for example, one truss may have different stress condition at the beginning, middle, and end.

Modern AI technology has introduced more flexible optimization method. Feng Ruo-qiang [15] and James Norman Richardson [16] have introduced AI method in single layer grid optimization; their algorithm can be attractive to future designers and optimization algorithms. Their method still lacks simulation and stochastic arithmetic. Although a lattice structure is much more complicated than single layer grid, the study of single layer grid can be the base for lattice structure study.

J. Zhou et al. [17] presented a study of pyramidal core structures with triangulated planar truss faces. Lattice structure's behavior under other effects, such as compression and distortion was studied. Because these effects can significantly affect lattice structure property in multi-functional applications and there is not much research about them.

J.C Wallach and L.J Gibson [18] studied the mechanical behavior of Three-Dimensional lattice structure. Their study introduced the node perturbations concept, which shows failure always begins around the truss connecting area (nodes). Aremu et al.[19] studied the capacity of Self Supported Metallic lattice structures. They found D-gyroid and Gyroid lattice structures have much lower Von-Mises stress than the lattice structures formed with trusses under same load condition. It means they can support more load with the same weight. These lattice structures have shorter research history compare with octahedral lattice structure (Figure 1.1), which has attracted much attention [20].

With the development of additive manufacturing, scientists have also found other manufacturing methods for lattice structures, D.J. Sypeck and H.N.G. Wadley [21] studied multifunctional micro-truss laminates; their research shows a potential way to manufacturing an inexpensive textile-based periodic lattice structure. Compared to using FDM [22] process and ABSplus-P430[23] material to build lattice structures; their method is simple and cheaper in mass production.

All the above researchers have developed many mathematical models and methods for lattice structure optimization and manufacturing. Some FEA software such as ANSYS and SolidWorks fusion also provides topology optimization tools for structure optimization problems. However, these tools cannot deal with complex structures [24] (including lattice structure), their outputs could not exceed original geometry boundary, that means these tools could only subtract material from designs, but not add material to the design.

Chapter 3. Methods

3.1 Design and Optimization process

All of the design and optimization processes in this research depend on CAD (Computer Aid Design) and FEA (Finite Element Analysis) software, the optimization step also requires calculation tools such as MS Excel or MATLAB to manipulate data. A brief description of design/optimization steps process is listed below (Fig 3.1).

1. Decide lattice cellular type and size, fill the part shape with lattice structure skeleton.
2. Calculate trusses diameters and create a homogeneous lattice structure.
3. Create a 3D model of the initial lattice structure.
4. Put the 3D model into ANSYS under given load and support conditions.
5. Run FEA simulation and extract safety factor distribution from simulation result.
6. Calculate necessary trusses diameters based on truss safety factor (Evenly distributed safety factor).
7. Create a new 3D model based on new trusses diameters.
8. Run through the step.4 to step.7 again until strength/deflection does not change or reach the requirement.

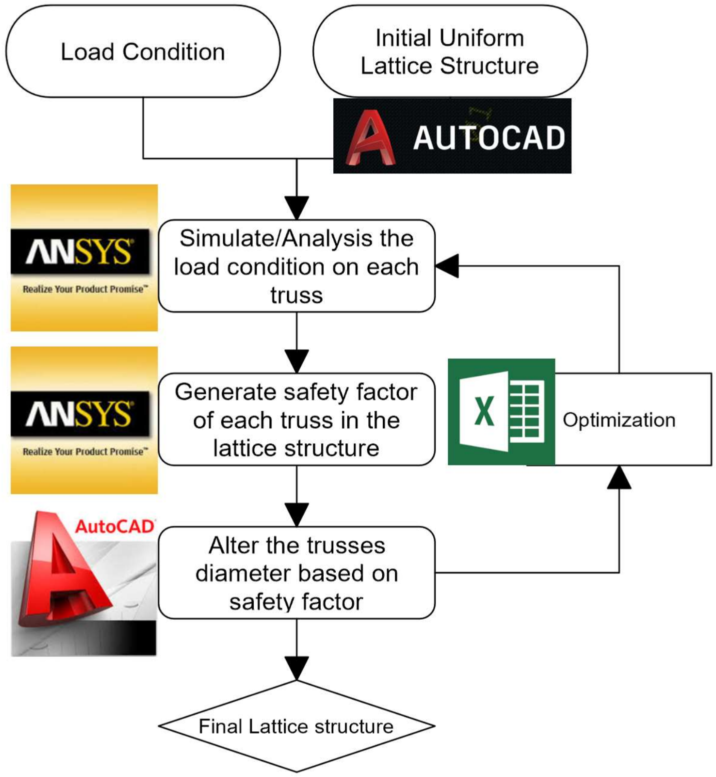


Figure 3.1. Lattice structure design/optimization general process

Lattice structure design and optimization starts by replacing a bulk part with lattice cells. In this thesis, the octahedral lattice structure is used to fill the bulk part. According to some researchers [2] [25], octahedral lattice structure can align in two different ways---Abreast and Oblique crossing (Fig 3.2). This research uses oblique crossing alignment of the octahedral lattice structure. The first step is to build a skeleton for the lattice structure---use lines and points to represent the trusses and nodes in the lattice structure, these points and lines represent coordinates and vectors. Furthermore, each line(truss) will be filled with its diameter, which determines the structure mass and volume.

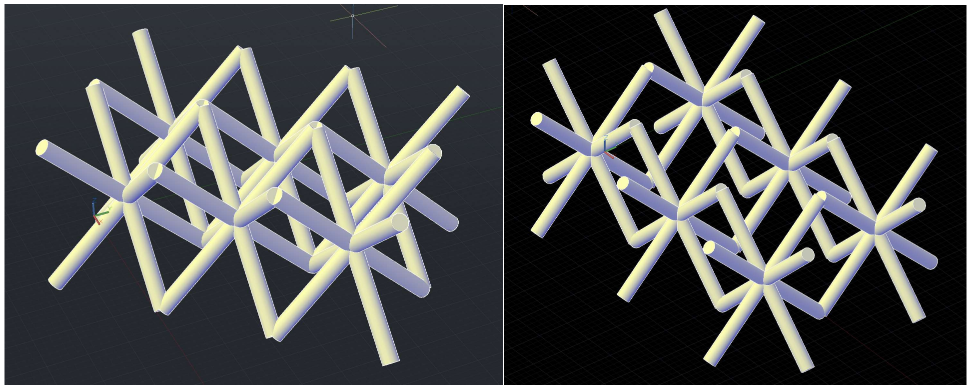


Figure 3.2.lattice cellular alignment method 1. Abreast (Left) 2. Oblique crossing(Right)

3.2 Lattice structure modeling

In this research, the work is to optimize a 3×3×3 inches lattice structure to support a given load in the Z direction. The first step is to fill the 3×3×3 part with uniform lattice structure cells.

Since the lattice structure will be manufactured using additive manufacturing also requires additional support structures when the build angle under self-support angle (around 45°). The machine used in this work is Fortus 250mc, using material Stratasys P430, The self-support angle is 45°. The trusses-ground angles are designed to be both 45° (self-supportable angle in additive manufacturing [26]). The lattice structure cells' size designed to be 1×1×1 inch in the length, width, and height. Thus, made it easier to calculate the whole lattice structure size by periodically arranging the lattice cells in X, Y, Z directions. The lattice structure cell parameters are shown below (Fig 3.3).

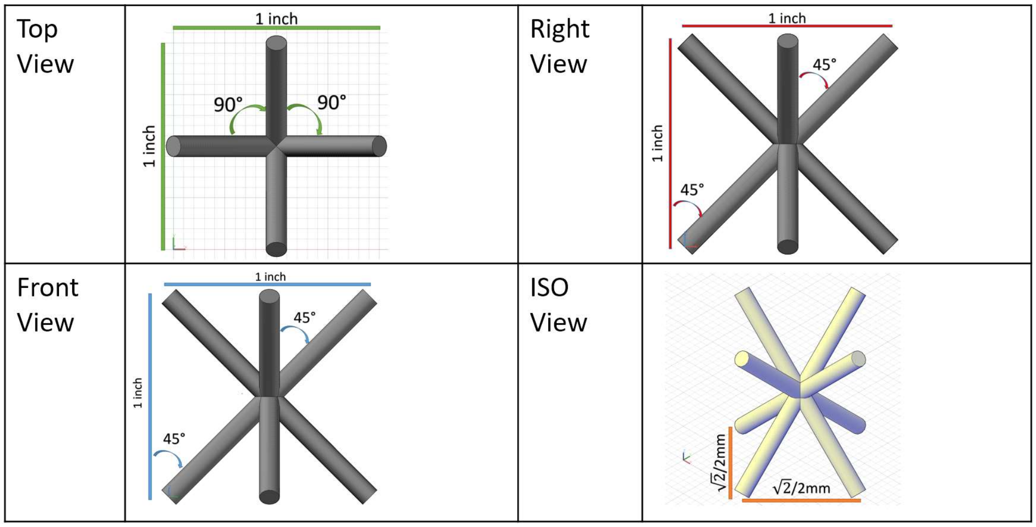


Figure 3.3. Octahedral lattice cell used in the research

Based on Newton's Third Law of motion-force and acceleration, the total loads and forces at the nodes are zero. Thus, the force at any node of the lattice structure is 0. The equation is:

$$\sum_{1}^{n} \vec{F_i} = 0$$

$\vec{F_i}$ means the forces at one node and $\vec{F_1}$ to $\vec{F_n}$ includes all the forces at that node.

3.3 Lattice structure parameter calculation

The calculation formulas to determine lattice structure parameters depend on theoretical material mechanics and the optimization algorithm re-distributes based on Von-Mises Stress[27], and the safety factor derived from it. Based on Wen-Yea Jang's research [28], while the lattice structure is under compression load, the trusses' deflection obeys the compression deflection/strength formula (Formula 1-4). For each truss, it may bear a different force; this requires the trusses to have different diameters. However, the first lattice structure the design/optimization process build is a homogeneous lattice structure. Homogeneous lattice structure's trusses and nodes all have the same parameters (Length, diameter, and angles).

Because $\sum_1^n \vec{F_i} = 0$ and the homogeneous lattice structure trusses all have the same diameter. Thus, it is straightforward to calculate the truss diameter in evenly load situation using the material's property[18]. Every truss supports the same load, and their parameters will be the same. The calculation formulas are listed below.

$$\sigma = \frac{F}{A} \tag{1}$$

$$\epsilon = \frac{l - l_0}{l_0} \tag{2}$$

Formula (1) and (2) yields to

$$\Delta \mathrm{l} = \frac{Fl}{EA} \tag{3}$$

Formula (3) yields to

$$A = \frac{Fl}{\Delta l \cdot E} \tag{4}$$

A is the cross-section area of lattice structure truss, *l* is the length of single truss, *F* is the stress translated into the truss, ϵ is the Tensile/Compression modulus of the material, E is the material tensile strength at yield and compression strength, The material Poisson ratio is 2%, the ultimate deflection boundary of lattice structure is 2%, design deflection rate of each truss is 2%, too. There's another important design parameter---the design safety factor, based on T. V. Galambos[29] design theory, 3 is a regular choice for structure designation.

As has been mentioned above, Von-Mises stress is used as the parameter to evaluate structure failure. The Maximum Allowance Stress shown below means the yield/compression strength of the material.

$$\text{Safety Factor} = \frac{Maximum\ Allowance\ stress}{Von{-}Mise\ \ Stress}$$

Based on the safety factor on each truss and general safety factor through whole structure, it is straightforward to determine whether a truss need strengthening or not. The trusses' safety factors that are under the design safety factor need to be strengthened, the trusses' safety factor above design safety factor can be weakened to save material. The principle is to make the safety factor evenly distributed through lattice structure by altering trusses diameters. Alongside the extracted safety factor, the final target is to re-distribute material. That will require re-calculating the truss cross-section areas based on each truss safety factor. The calculation process is listed below:

$$New\ truss\ cross\ section\ area = \frac{initial\ truss\ cross\ section\ area}{Safety\ factor\ scale}$$

The safety factor scale can be calculated by design safety factor and simulation truss safety factor.

$$Safety\ factor\ scale = \frac{Simulation\ Truss\ Safety\ factor}{Design\ safety\ factor}$$

The detail steps for altering truss diameter are shown below:

1. Extract average safety factor of each truss from simulation result (ANSYS).
2. Calculate each truss's cross-section area based on design safety factor and simulation safety factor (Theory: change cross section area to fulfill load/stress), finally, make all the trusses have same theoretical safety factor.
3. Calculate structure weight after altered lattice trusses diameters.
4. Expand/shrink all the cross sections with the same ratio, let the optimized lattice structure weight to be the same as initial lattice structure weight.

Theoretically, the optimization process will not stop until the safety factor has been evenly distributed throughout the whole lattice structure. The termination criteria for the optimization process to stop can be the standard deviation of safety factors, the deflection rate, or the maximum-minimum safety factors ratio. This research uses deflection rate as the termination criteria. Once the deflection rate reaches a limit or it does not change after optimization iteration, the optimization process will stop.

There are other issues in lattice structure optimization, such as trusses with different diameters having gaps and sudden shape change at their joint connections. A sudden change of shape in a joint usually causes dangerous stress concentration. Thus, this work introduces joint ball (Fig 3.4) to connect two trusses with different diameters. The ball's radius is same as the truss which has a larger radius at the connection.

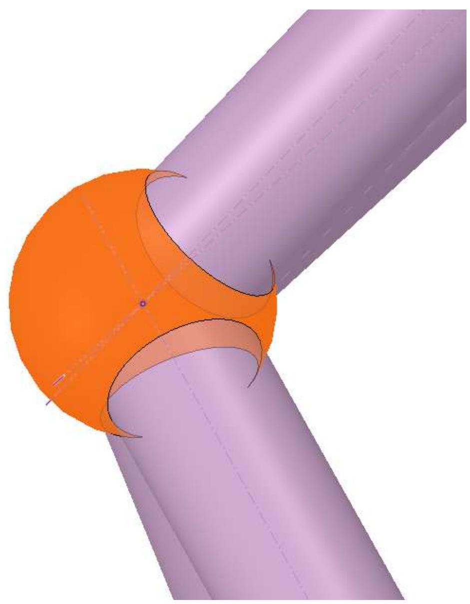


Figure 3.4. Trusses Joint ball

Last but not least, the lattice structures are never applied independently, there are surface plates at the bottom and the top of the lattice structure, to constrain the trusses displacement and also to evenly spread the load into each truss. The lattice structure used in this research has both top-surface-plate and bottom-surface plate.

Chapter 4. Design and generation of initial lattice structure

4.1 Example lattice structure parameters

This chapter will present the calculation and generation process of the initial lattice structure. Before manufacturing a lattice structure, there must be a 2D/3D model for it. The modeling process is straightforward with CAD. The process is presented below.

What calls for particular attention is the simulation parameters, such as lattice structure size, load condition and support condition. They are listed below (Table 4.1):

Table. 4.1 Lattice structure parameters

Simulation Parameter	Parameters/Value
Lattice cell type	Octahedral lattice cell
Lattice cell alignment type	Oblique crossing
Design Safety factor	3
Deflection Rate limits	2%
Lattice structure size	3×3×3 inches
Lattice cell size	1×1×1 inch (W×L×H)
Load condition	900 Newton
Support Condition	Fixed support
Material	Stratasys P430 ABS
Material Youngs Modulus	31Mpa
Material Compress Strength	35MPa
Material Poison ratio	2%

The initial lattice structure is designed to contain 3×3×3=27 octahedral lattice cells, which means it will have 3×3=9 cells on each side. With top and bottom surface plates, the initial 900 Newton load could evenly divide into nine cells, and each cell will bear 100 Newtons. Based on equations 1-4 in chapter 3 and the material property (Stratasys P430), the initial homogeneous lattice structure trusses diameters are all 2.5mm (Calculation result).

The lattice structure 3D model is shown below (Fig 4.1). It is built in AutoCAD, the detail steps of modeling are:

1. Create single lattice cell skeleton with lines.
2. Form 3×3×3 lattice structure skeleton with single lattice cell skeleton.
3. Sweep all the skeleton with calculated truss diameter.

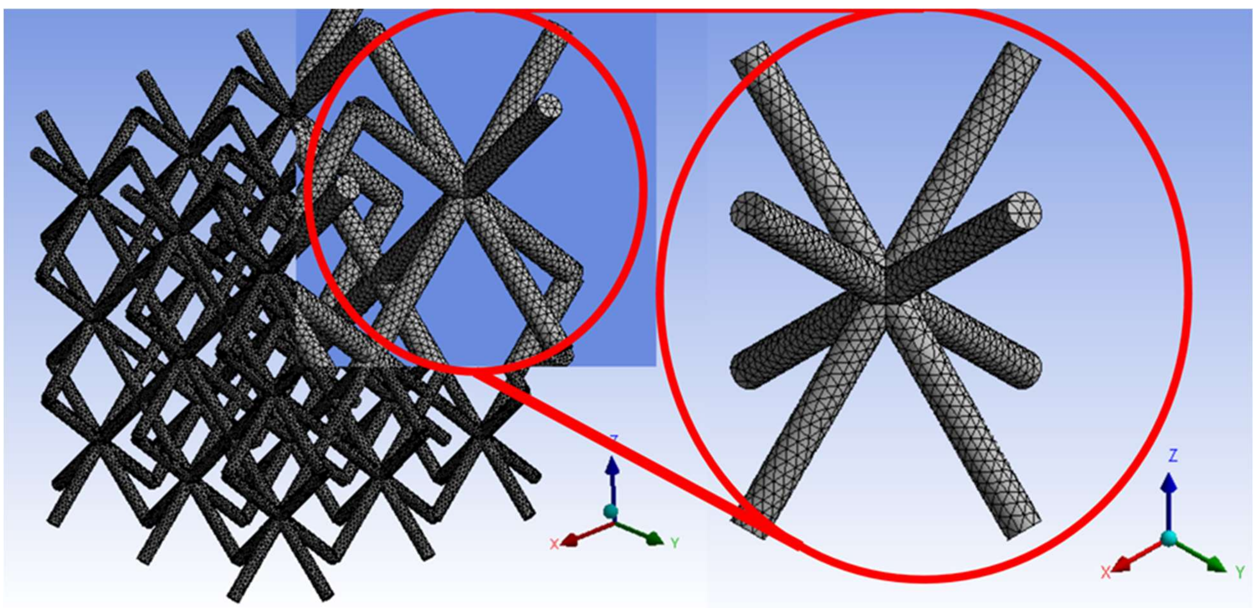


*Figure 4.1. 3*3*3 lattice structure and independent lattice cell*

4.2 Simulation in ANSYS

After building the lattice structure 3D model in AutoCAD, it is straightforward to translate it into ANSYS with AutoCAD-ANSYS interface. There are many simulation settings to take care before the FEA process can begin. The material property is set in ANSYS engineering data window. The mesh type and the quality setting is in the Model section and affects the simulation accuracy and time, the more precise the mesh is, the longer time it will take for simulation. There is a balance between mesh quality and running time. In this research project, the mesh quality is set to be medium; the mechanical physics relevance is set to be 100% (it means the part simulation is related to material property). The mesh still contains more than 4×10^6 nodes and 2×10^6 elements making the simulation result very precise. The load conditions and support condition are set in static structural part.

Chapter 5. Lattice structure optimization process

5.1 Initial lattice structure simulation result

The first step is to simulate the homogeneous lattice structure. The initial simulation result for the example lattice structure is shown below (Fig.5.1).

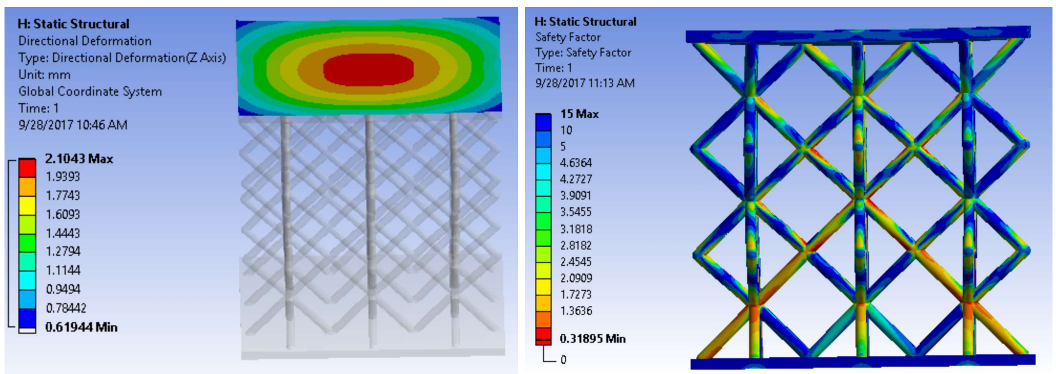


Figure 5.1. Homogeneous lattice structure Simulation result (1.0 True Scale), (Left): Deformation in the Z direction, (Right): Safety factor distribution.

The average deflection of the top surface plate in negative Z axis is 1.549mm, the average safety factor through whole lattice structure except the top-bottom surfaces plates is 5.96. It can be noticed that most of the area of the lattice structure has safety factor that is significantly higher than 3, while there are still some areas that have a significantly low safety factor. The red and yellow colors show areas where the part's safety factor is lower than 3. This means some areas will break down much earlier than the other areas. That is the reason and motivation to make safety factor homogeneously distributed through design optimization.

5.2 Optimization Calculation

After the initial lattice structure simulation, researcher/designers can export the safety factor and calculate for the next iteration lattice structure parameters (trusses diameters). The calculation process is presented in Chapter three. A new cross-section area is calculated using the process presented below.

Table 5.1 Initial truss diameter, simulation safety factor, and lattice structure volume

	Truss diameter	Truss safety factor	Whole lattice structure volume
Initial parameters	2.5mm	9	50000mm^3

Because the design safety factor is 3, then the trusses safety factor is three times of what they need to be. Hence, the initial cross-section is three times of the cross-section needed.

$$Safety\ factor\ scale = \frac{Simulation\ Truss\ Safety\ factor}{Design\ safety\ factor} = \frac{9}{3} = 3$$

The next step is to calculate the following iterations on truss diameters based on the previous calculation. As all trusses have circular cross-sections and the initial diameter is 2.5mm, the calculation of new cross-section is shown below;

$$New\ truss\ cross\ section\ area = \frac{initial\ truss\ cross\ section\ area}{Safety\ factor\ scale} = \frac{\pi \cdot \frac{2.5mm^2}{4}}{3} = 0.52\pi$$

Based on the relationship between circular area and diameter, the diameter for the next iteration should be

$$\text{New truss daimeter} = \sqrt{4 \cdot \frac{0.52\pi}{\pi}} = 1.44mm$$

Other trusses can follow the same steps to calculate new diameters. By doing so, the calculation can also get the next iteration's lattice structure volume by summing up all the trusses volumes. To fairly compare the lattice structures property changes through optimization, volume and weight need to be stable in the optimization process. There's another step to keep the volume to be the same with the initial lattice structure. Here is one more example of altering lattice structure volume/weight to the initial weight/volume by altering truss diameter:

If the lattice structure volume after homogenization of safety factor scaling is 20000mm^3 and the initial lattice structure volume is 50000mm^3, then each truss diameter need to be multiplied by $\sqrt{\frac{50000}{20000}} = 1.58$. If the truss diameter before multiplying is 1.44mm, then the final truss diameter should be 1.44 ×1.58=2.2752mm. (Because trusses' volumes have proportional relationship with their cross-section area and the truss cross-section area is affect by the diameter). Due to the change in truss diameter, the trusses strength is also changed. The load transformation characteristic will also change. It is necessary and essential to simulate the new lattice structure with same load and support condition again.

5.3 Optimization process

This section will present how the lattice structure optimization process increases the lattice structure property and finally reaches the termination criteria.

5.3.1 First Optimization iteration

Once the new lattice structure parameters are calculated, the lattice structure is simulated to test its capability.

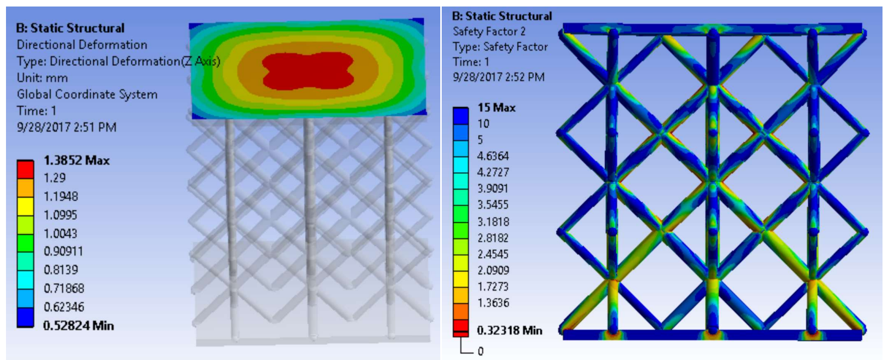


Figure 5.2. Simulation results after first optimization iteration of the lattice structure, Z direction deflection (Left), Safety factor distribution (Right).

The result (Fig 5.2) shows that after the first iteration lattice structure's Z direction deflection is now 1.05mm, it is 30% lower than the 1.5mm Z direction deflection of the initial lattice structure. This means the lattice structure stiffness has increased. Based on the safety factor distribution, there are fewer places where the safety factor is under design safety factor. However, the lattice structure is still facing the issue of uneven safety factor distribution. Therefore, another optimization iteration is needed.

5.3.2 Second Optimization Iteration

From the first simulation and optimization iteration, a new lattice structure has been generated and simulated under the same load and support condition. The simulation result of new lattice structure under given load is shown below.

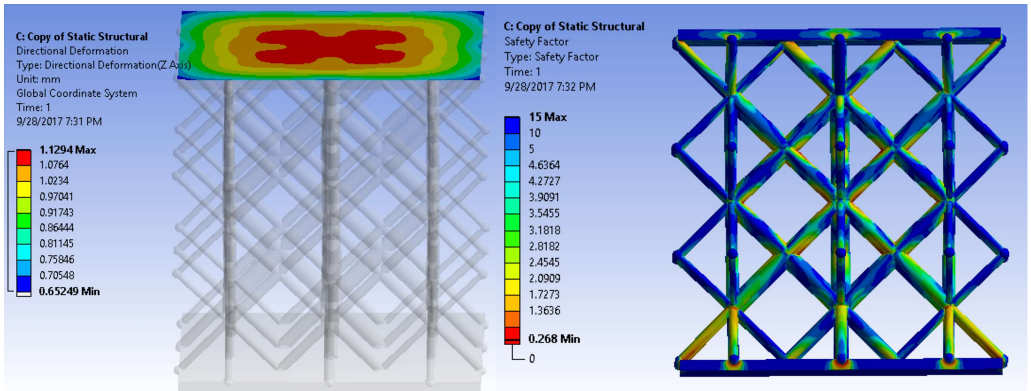


Figure 5.3 Simulation result after the second optimization iteration of the lattice structure, Z direction deflection (Left), Safety factor distribution (Right).

The average top surface-plate deflection in the negative Z direction is 0.96mm, which is lower than the previous 1.05mm deflection, and it also means a further iteration is necessary because it has not reached the stable deflection boundary.

Moreover, there is still some area where the safety factor is lower than one and the lattice structure has not reached the termination criteria. The optimization process needs another iteration.

5.3.3 Third Optimization Iteration

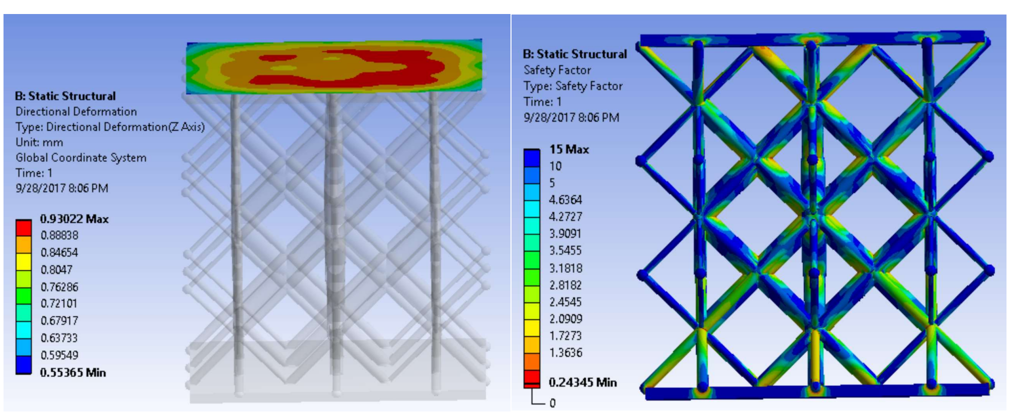


Figure 5.4 Simulation result after the third optimization iteration of the lattice structure, Z direction deflection (Left), Safety factor distribution (Right).

The average top surface-plate deflection is now 0.84mm, that is smaller than 0.96mm deflection in the previous optimization iteration.

It is evident that the trusses diameter at the side begins to shrink and the trusses in the middle begin to expand. Although this lattice structure's safety factor distribution situation is better than the initial homogeneous lattice structure, but this lattice structure still didn't reach the termination point.

5.3.4 Fourth Optimization Iteration

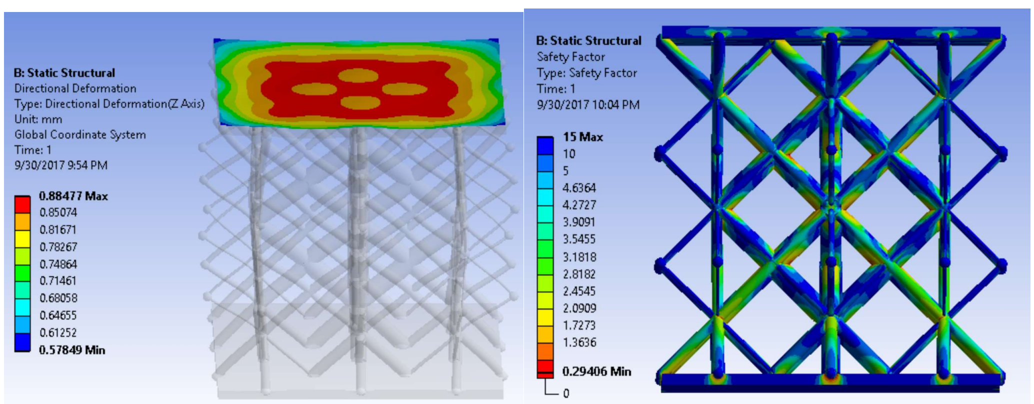


Figure 5.5 Simulation result after the fourth optimization iteration of the lattice structure, Z direction deflection (Left), Safety factor distribution (Right).

The average top surface-plate deflection is now 0.80mm. The outside trusses keep shrinking, and the inner trusses are expanding. Because this step did not reach stable deflection point, another optimization iteration is required.

5.3.5 Fifth Optimization Iteration

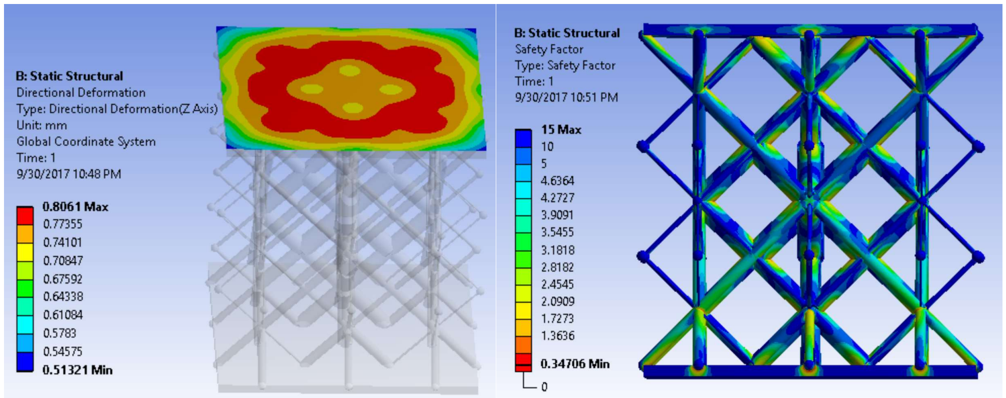


Figure 5.6 Simulation result after the fifth optimization iteration of the lattice structure, Z direction deflection (Left), Safety factor distribution (Right).

The average top surface-plate deflection in the negative Z direction is 0.74mm. The safety factor distribution (Fig 5.6) shows there is less area with a safety factor lower than one compared to initial lattice structure. Since the outside trusses shrunk to a tiny diameter, there is another problem, is it necessary for the lattice structure to keep these trusses. Although their diameters are smallest, their safety factors are highest. That means their weight efficiency is not as high as the other trusses.

5.3.6 Sixth Optimization Iteration

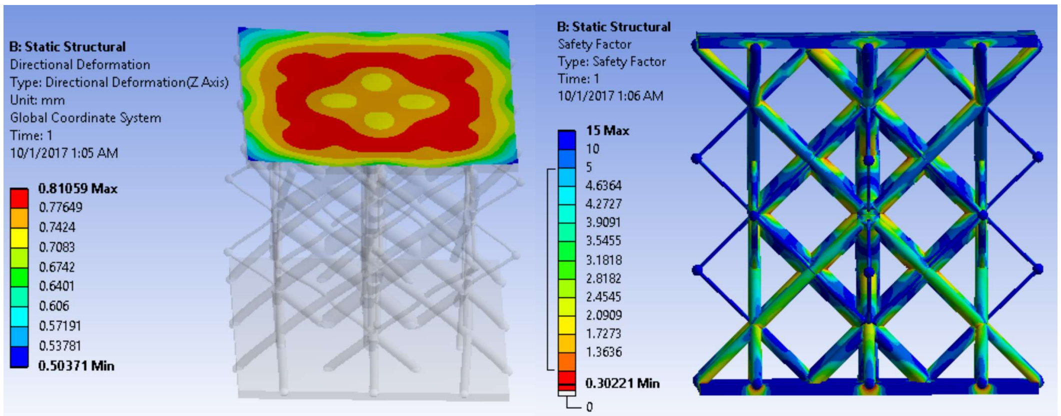


Figure 5.7. Simulation result after the Sixth optimization iteration of the lattice structure, Z direction deflection (Left), Safety factor distribution (Right).

The average top surface-plate deflection is still 0.74mm

As we can see, the safety factor does not distribute evenly. However, the deflection remain the same after this optimization iteration. It means the optimization does not bring as much strength increase as before. The lattice structure deflection reaches a stable point. The lattice structure optimization process reaches termination point.

Fig 5.8 shows the first optimization iteration reduced deflection from 1.549mm to 1.05mm, which is the most efficient, then the following optimization iterations efficiency continually decrease the deflection until stopped.

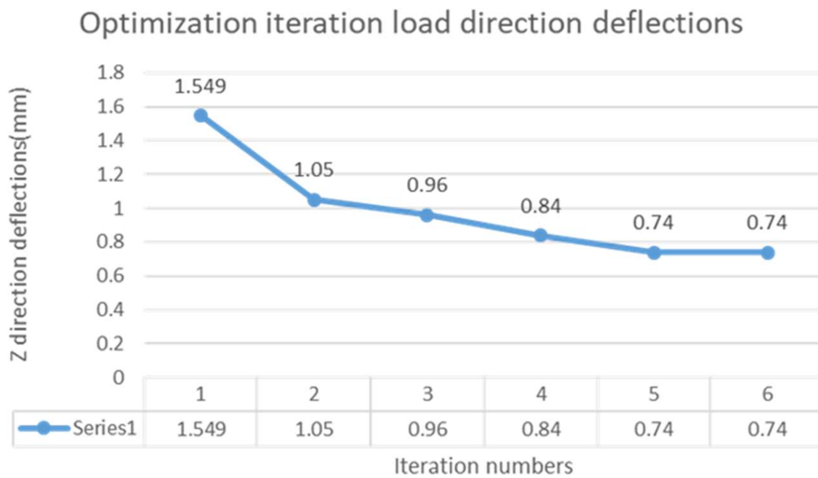


Figure 5.8. Lattice structure deflection in Negative Z direction through optimization processes

Another index in optimization iterations is the safety factor distribution. The comparison of initial lattice structure safety factor distribution and the final optimize lattice structure safety factor distribution (Fig. 5.9) shows the final design has much more evenly spread safety factors. The trusses in the middle have been strengthened, and the trusses at the side are weakened.

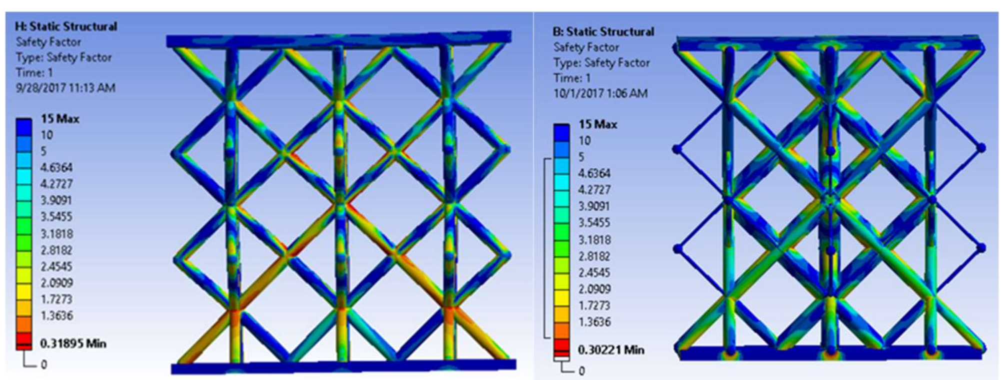


Figure 5.9. Comparison of initial lattice structure safety factor distribution(left) and final optimized lattice structure safety factor distribution (Right).

5.4 Optimization of Octet Lattice Cell

To show that the optimization can work on other lattice structure. An Octet truss structure is used for demonstration. The objective is to increase its strength and compare the optimized lattice structure with the original lattice structure to show the effectiveness of evenly distributed safety factor assumption.

The Octet truss structure is formed with trusses, similar to the Octahedral lattice structure.

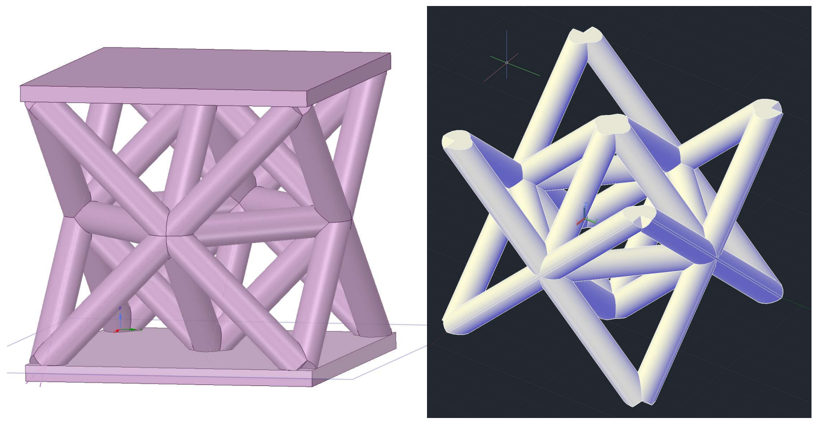


Figure 5.10. Octet-truss structure, With Surface Plate(Left), Without Surface Plate(Right).

All the trusses in the Octet-truss structure are in the same length; the cell has the same parameter with Octahedral lattice cells: Structure length, Structure width, Structure height, Truss diameter and Truss angle.

Holly D. Carlton et al.[30] in Lawrence Livermore National Laboratory had studied the property of Octet-truss structure property; they built three Octet-truss structures with different relative density, 10%,20%, and 30%. Because their research shows more details in the 20% relative density Octet cell, the optimization in this work will focus on 20% relative density Octet cell to compare with their compression test result.

Holly D.Carlton and his teammates built Octet Cells with Ti-6Al-4V (material) using the SLM (Selective Laser Melting) process. The material property is based on ASM (Aerospace Specification Metals Inc) datasheet. All of the Octet Cells they built have a same basic skeleton but different trusses diameters. All the trusses in the 20% relative density Octet cell have 0.52mm diameter and 3.02mm trusses length. In this optimization, the load condition will be 1000Newton

Load in the Negative Z direction, Holly D.Carlton et al.'s research shows their Octet-truss structure's deflection under 1000 Newton is 140µm, which is 0.14mm.

Table 5.2. Ti-6Al-4V Properties[46]

Properties	Density	Young's Modulus (Gpa)	Shear Modulus (Gpa)	Poisson's Ratio	Yield Strength (Mpa)	Ultimate Strength (Mpa)
Values	4.43 g/cc	113.8	44	0.342	880	950

5.4.1 Initial Octet-truss structure simulation result

The initial simulation result of 20% relative density Octet-truss structure is shown below.

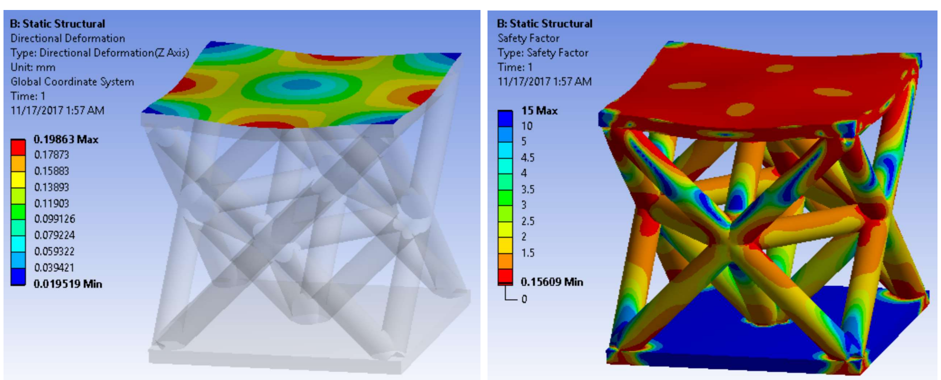


Figure 5.11. Simulation Result of the initial 20% relative density Octet-truss structure. Octet-truss structure deflection in the Z direction (Left), Safety factor distribution (Right).

The simulation result shows the Octet-truss structure deflection under 1000 Newton load in the negative Z direction is 0.123mm. That is smaller than Holly's research result. It may be caused by the property difference or simulation defects. The optimization process of the Octet-truss structure is same as the optimization process of the Octahedral lattice structure.

5.4.2 Simulation result of optimized Octet-truss structure

The first optimization iteration result of the Octet-truss structure was then put into simulation using the same load condition. The simulation result is shown below.

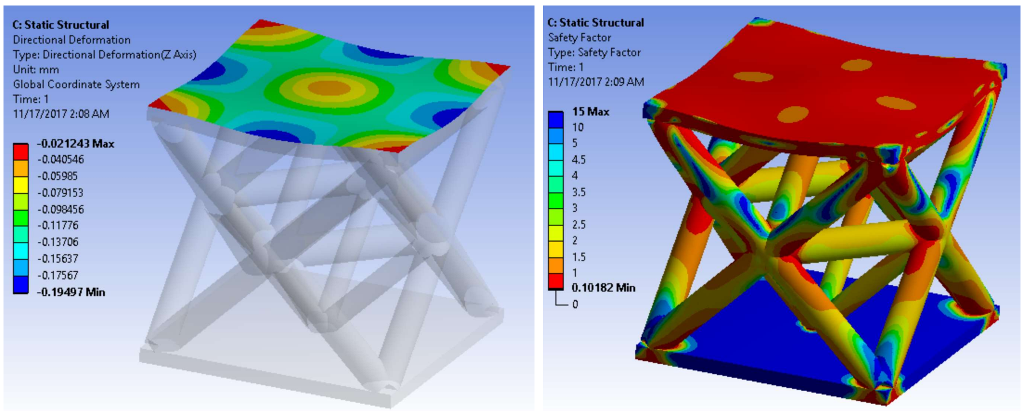


Figure 5.12. Simulation result of the 20% relative density Octet-truss structure after the first optimization iteration. Deflection in the Z direction (Left), Safety factor distribution (Right).

The simulation result shows the Octet-truss Structure deflection in Z-axis after the first optimization is 0.120mm. That is 2.4% less than the initial result. Since the optimization does not reach the termination point, another optimization iteration is needed.

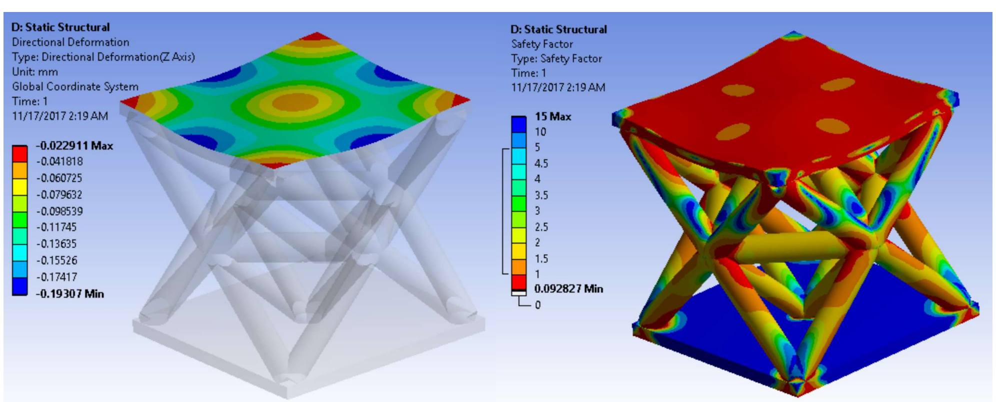


Figure 5.13. Simulation result of the 20% relative density Octet-truss structure after the second optimization iteration. Deflection in the Z direction (Left), Safety factor distribution (Right).

The simulation result shows the Octet-truss Structure deflection in Z-axis after the second optimization is 0.118mm. That is 4.06% less than the initial deflection. Since there is no sign showing the optimization process reached termination point, another optimization iteration is needed.

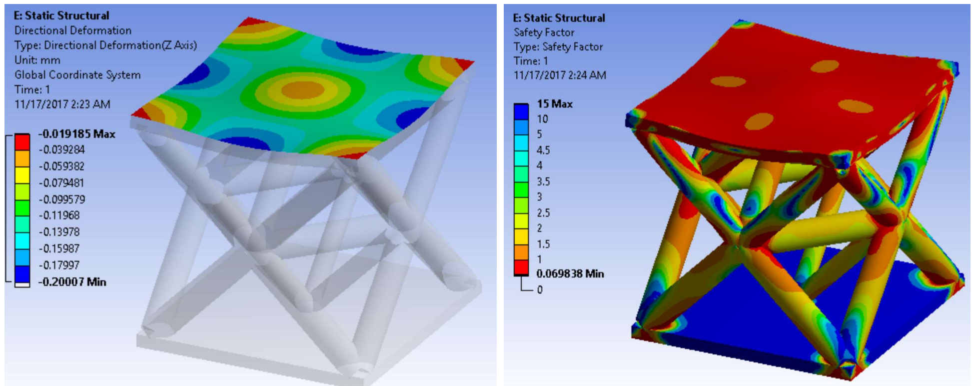


Figure 5.14. Simulation result of the 20% relative density Octet-truss structure after the third optimization iteration. Deflection in the Z direction (Left), Safety factor distribution (Right).

The simulation result shows the Octet-truss Structure deflection in Z-axis after the second optimization is 0.118mm. Since it is the same as the previous optimization iteration result, there is no need for another optimization iteration. Although the 4% less deflection is small compared to the previous Octahedral lattice structure optimization result, it is a considerable amount for a single cell, and it is achieved within two optimization steps. It also shows the evenly distributed safety factor does work for other structure. Future researchers can study how the number of lattice cells affects the lattice structure general property.

Chapter 6. Verification simulation result

6.1 Manufacturing and Physical Test machine

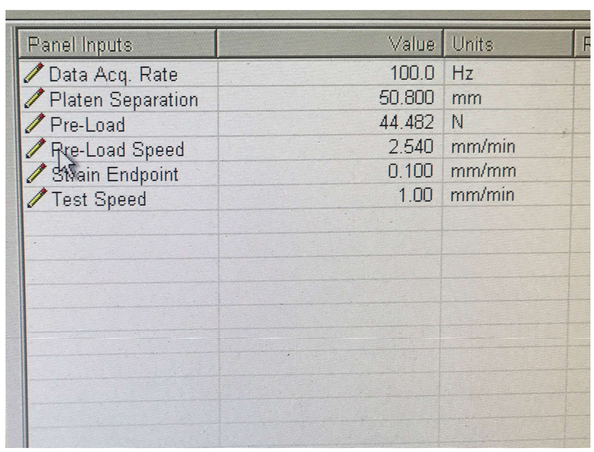
Four 3×3×3 Octahedral lattice structure (2 for initial lattice structures, 2 for optimized lattice structure) parts were built with Fortus 250 machine. The additive manufacturing process is straightforward, translate the CAD file into STL format file, then the AM machine can directly build the part with/without a support structure.

Manufacturing the lattice structure through traditional method is difficult. However, additive manufacturing technology, including SLM (Selective laser melting)/EBM (Electronic beam melting) process for metal materials and SLA (Stereo Lithography Apparatus)/FDM (Fused Deposition Modeling) for non-metal materials are ideal choices for complex structure manufacturing. For example, R. Gumruk G et al.[7] utilized SLM method to manufacture complex support structure, they built a mathematical model for single lattice cell, and their simulation result shows the model result fit with the test result. However, they did not study how the force or load will transfer through the structure. That made their research not applicable to optimization; their model could only predict lattice cell mechanical behavior.

The parts were tested in an MTS compression test machine (Fig 6.1) to verify the effect of the lattice structure optimization algorithm and optimized lattice structure property. These lattice structures may have different material distribution because of optimization iterations, but both of the structures have the same bottom and top surface plates. The compression test result is used to verify the previous simulation result and effectiveness of the optimization.

Figure 6.1. MTS Stress-Strain Test machine

During the compression test, the machine is set up with default testing parameters (Fig. 6.2). Data refresh rate is 100Hz; it means every second the test machine will receive100 data points. The test speed is 1.00mm/min, that means the compression plates move closer to each other at 1mm/min.



Panel Inputs	Value	Units
Data Acq. Rate	100.0	Hz
Platen Separation	50.800	mm
Pre-Load	44.482	N
Pre-Load Speed	2.540	mm/min
Strain Endpoint	0.100	mm/mm
Test Speed	1.00	mm/min

Figure 6.2. Compression test parameters

6.2 Compression Test Result

The test result is shown in (Fig 6.3). Fig 6.3 (A) (C) show the lattice structures before the compression test begins. The Fig6.3 (B) (D) indicate the lattice structures' status after break down, which is after the Stress-Strain Curve peak point. As the figures show, both of the lattice structures experienced structure failures. The entire homogeneous lattice structure exhibits a more catastrophic break down when compared to the designed/optimized lattice structure. Compared to the homogeneous lattice structure, the optimized lattice structure retains a better shape after compression and break down. Furthermore, the optimized lattice structure delayed lattice structure break down, due to even-safety factor.

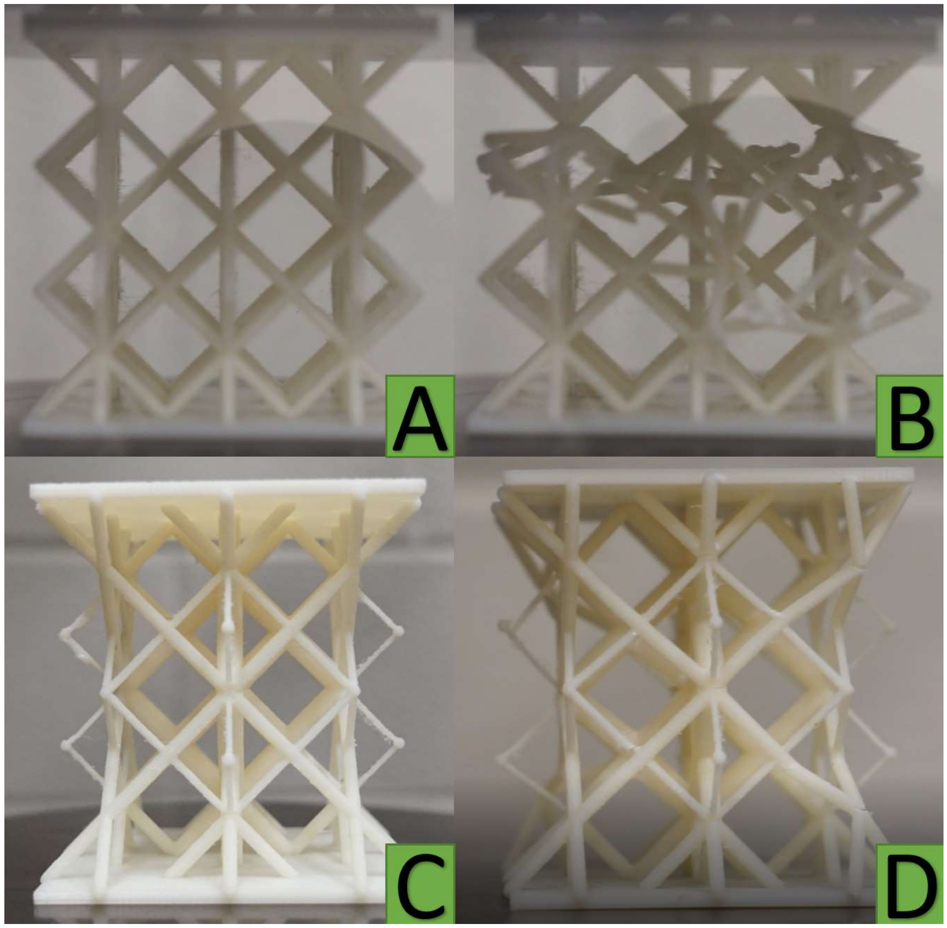
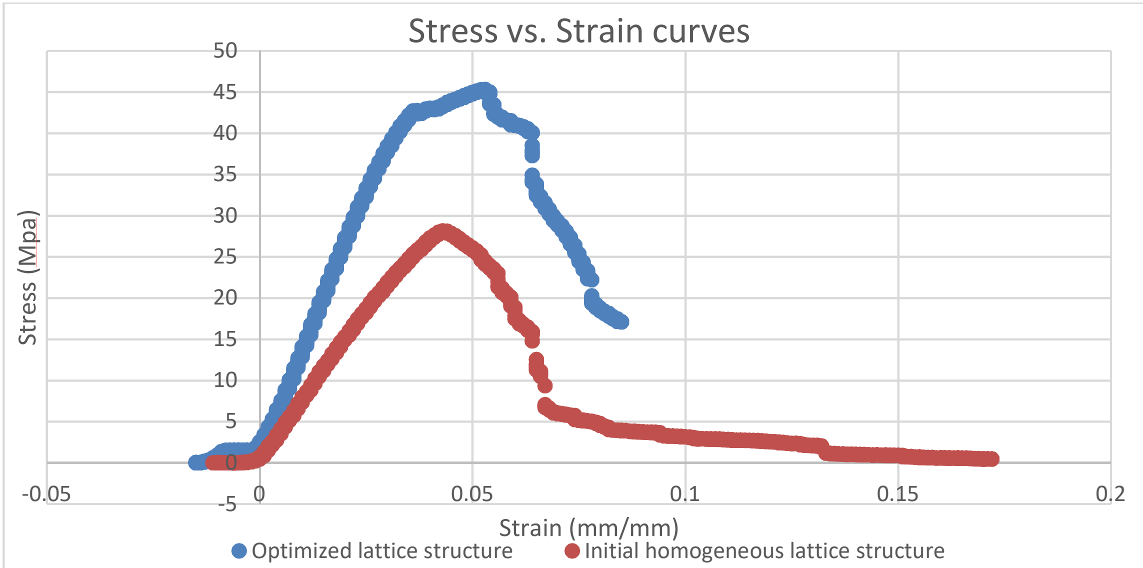


Figure 6.3. Homogeneous Lattice Structure in compression test, Before (A) After (B), Designed Lattice Structure in compression test, Before (C) After (D)

After the compression test, the test data was exported for analysis. It is also essential to analyze their stress vs. strain behavior and Deflection vs. Load curves. Since the lattice structure was designed and built to support the load in the Z axis, The compression test was only conducted in the Z axis, the test lattice structure strength and deflection are shown in Fig. 21. These curves show the lattice structure properties, strength, modulus, and stiffness.



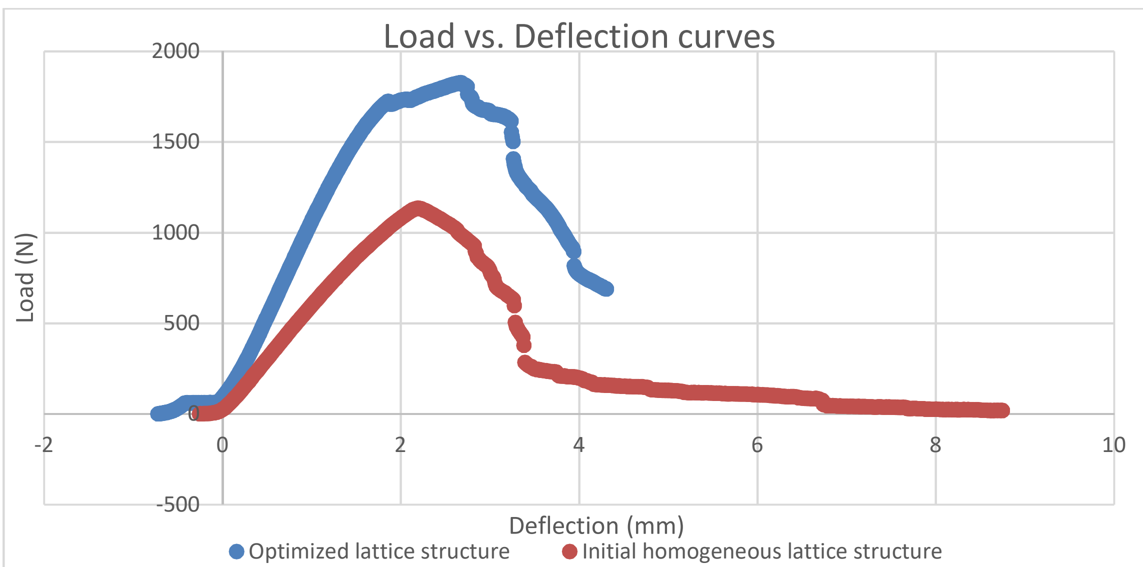


Figure 6.4.Lattice structure Stress-Strain curve (Upper), Load vs. Deflection Curve(Lower)

Fig. 6.4 shows both the optimized lattice structure and the initial homogeneous lattice structure could support 900 Newton load without yielding. In the Load vs. Deflection curve, there is no break down before the 900 Newton load. The original homogeneous lattice structure begins failure at 1135Newton's load, while the optimized lattice structure supports more load before break down (1827Newton load), that is 60% more than the initial homogeneous lattice structure. The optimized lattice structure also shows a higher slope in Load vs. Deflection curve before failure begins. That means it has a higher stiffness. The optimized lattice structure strength is maintained even after failure begins.

In the Stress-Strain curves, the designed/optimized lattice structure exhibited a higher modulus; its modulus is even higher after failure begins, this is similar to material's yield strength and ultimate strength. Moreover, the optimized lattice structure's peak strength point has more considerable strain than the homogeneous lattice structure's peak strength point. That means the even-safety factor design algorithm does delay break down and the algorithm works for sample lattice structure. Considering these two lattice structures were designed and built with the same amount of material and cost, the optimized lattice structure shows better properties.

There was another discovery made during the test. In the compression test of the optimized lattice structure, there is a stage where the strain/deflection changes while the stress/load is maintained the same (Fig 6.5). This stage occurred in more than one tests, and it needs further study to discover why it occurs.

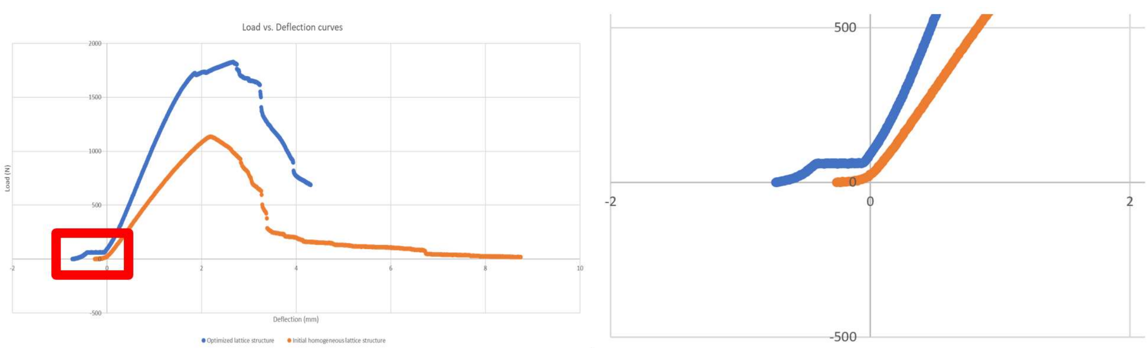


Figure 6. Abnormal stage Deflection growing while load maintain the same

6.3 Compression Test Result Comparison

Table 6.1 compares the simulation result with the physical compression test result. The simulation deflection and the test result deflection are similar in their values. The differences between simulation and test data are maintained within 10%; this is acceptable considering the final products is not as perfect as the one in the simulation. Therefore, the lattice structure deflection under given load condition is smaller in simulation than in real test.

The optimized lattice structure not only deflects less than the original lattice structure under a given load but also exhibit a more significant deflection and supports more load when it breaks down. The test result shows the optimized lattice structure could bear 60.09% more load.

Table. 6.1 Comparison of Simulation and Compression Test Result

	Deflection at 900Newtons load		Peak Strength Load and Deflection	
Homogeneous Lattice structure	Simulation	Test	Load (Test)	Deflection (Test)
	1.52mm	1.643mm	1135N	2.207mm
Designed Lattice structure	Simulation	Test	Load (Test)	Deflection (Test)
	0.74mm	0.84mm	1827N	2.67mm

In table 6.1, it is evident that the simulation results show lower deflection in given load condition when compared with the test result. That may come from Additive Manufacturing product defects (Rough surface, porosity). Because FDM process was used to build the part, there are layers and gaps, which may reduce product strength. The smaller the part's size is, the more the AM layer and gaps will affect the strength.

6.4 Discussion

This study demonstrated an even safety factor distribution and an optimization algorithm based on it to optimize a lattice structure in compression loading condition and demonstrate that it can work for other truss structure. In the past, researchers only focused on modeling or optimizing different lattice structures[31] by selecting lattice structures for specific load conditions. The optimization algorithm in this work was initially applied to the given octahedral lattice structure[32] and mainly focused on optimization rather than the selection of lattice cells. The approach is general enough to work with different lattice structures[33]. The optimization algorithm increased 60% strength on the example presented in this paper; researchers could study its effects on other lattice structures or general structures. This algorithm depends on evenly distribute safety factor assumption; the theory tends to make the whole structure breakdown at the same time. During the optimization, the trusses at the side and the trusses at the middle of lattice structure show different properties in supporting given load [34]. Although this research only studied Octahedral lattice structures and the optimization algorithm kept the same basic skeleton of the lattice. Some other lattice structure optimization studies[36] [37] [38] seems more flexible since they have more examples optimized although there is no verification their work is effective. However, the optimization algorithm in this work could work in other lattice structures as well, as demonstrated by the octet truss cell.Because any other lattice structure will have uneven safety factor distribution under given load condition and that is the parameter this algorithm could optimize.

The lattice structures in this research were built by Fused Deposition Modeling method [39]. While other additive manufacturing methods such as SLS, SLM, EBM or PBF are also capable of building such lattice structures [40] [41] [42]. The AM process may bring some manufacturing defects to the part, and it may reduce product strength. However, traditional manufacturing techniques such as molding and casting are not capable of building such a complex structure. Additive manufacturing technique is the only choice to fabricate such kind of lattice structure.

Chapter 7. Conclusion and Future work

The optimization algorithm described in this research delayed the lattice structure break down with optimized lattice structure by re-distributing material at the whole structure level. The areas with higher safety factor will redistribute material to the areas with a lower safety factor, which could make an optimized lattice structure maintain the same amount of material. The major discoveries and advances are presented below:

1. The design algorithm for initial lattice structure and optimization algorithm for optimized lattice structures are both efficient. The early stage optimized lattice structure property can be improved even further through the same optimization algorithm. Once the optimization algorithm could not reduce deflection anymore or reaches strength limitation, the optimization steps will stop. This algorithm utilized AutoCAD, ANSYS, and Excel.

2. For the example 3×3×3 lattice structure, the overall relative density of the optimized lattice structure and the homogeneous lattice structure are within 4.157%. While the Ultimate Strength of the homogeneous lattice structure is only 28MPa, the optimized lattice structure achieved 45MPa Ultimate Strength. Moreover, the optimized lattice structure reaches peak strength point slower than the initial homogeneous lattice structure, which shows that the evenly distributed safety factor does delay the breakdown. The new optimized lattice structure not only can support more load but also can be lighter when supporting the same load.

3. From the physical compression test, it is apparent the breakdown of lattice structure often begins at the nodes connection areas, the simulation result also shows the nodes areas always have the lowest safety factor. That means the current optimization should focus on even smaller units for future optimization, such as divide one single truss into several parts and then optimize their topology independently. The nodes area topology optimization still needs further

research, because the sudden change shape of nodes joints area can also cause stress concentration and lower safety factor.

4. Future research can focus on applying this optimization algorithm on different lattice structures. Although this design algorithm does work in truss formed lattice structure, no evidence shows how efficient it is for other lattice structure optimizations. Moreover, different structures may have different characteristics, such as the negative Poisson's ratio lattice structure [43] [44] [45]. Future researchers can also focus on how to choose proper optimization parameters, such as truss angles and cell sizes other than truss diameters. Besides strength optimization, other property could also be optimized through homogeneous optimization algorithms.

5. Considering the additive manufactured lattice structure rough surface, advancing additive manufacturing technology can reduce trusses surfaces roughness and increase lattice structure property.

References:

[1]. Chunze Yan, “Advanced lightweight 316L stainless steel cellular lattice structures fabricated via selective laser melting”, Material and Design 55 (2014)533-541

[2]. Ian Maskery, “An investigation into reinforced and functionally graded lattice structures,” Journal of Cellular Plastics, 2017. Vol. 53(2) 151-165

[3]. Raj Kumar Pal, “A continuum model for nonlinear lattices under large deformations,” International Journal of solids and Structures 96(2016) 300-319

[4]. A.M. Guzman, “Natural Vibrations and buckling of a Spatial Lattice Structure Using a Continuous Model Derived from an Energy Approach,” International Journal of Steel Structures 17(2): 565-578(2017)

[5]. von Mises, R. (1913). Mechanik der festen Körper im plastisch deformablen Zustand. Göttin. Nachr. Math. Phys., vol. 1, pp. 582–592.

[6] Li Yang, 2015, “Experimental-assisted design development for an octahedral cellular structure using additive manufacturing,” Rapid Prototyping Journal

[7] R. Gumruk R.A.W. Mines, 2012, “Compressive behavior of stainless steel micro-lattice structures,” International JournalofMechanicalSciences68(2013)125–139

[8] Yuichiro Koizumi, Arata Okazaki, Akihiko Chiba, Takahiko Kato, Akihiro Takezawa, 2016,” Cellular lattices of biomedical Co-Cr-Mo-alloy fabricated by electron beam melting with the aid of shape optimization” Additive Manufacturing 12 (2016) 305–313

[9] Jason Nam Nguyen, 2012, “A Heuristic Optimization method for the Design of Meso-scale Truss Structure for Complex-Shaped parts,” Georgia Institute of Technology

[10] Gregory C Graf, 2009, “*Development of Specialized Base Primitives for Meso-Scale Conforming truss structures*,” Georgia Institute of Technology

[11] T. Hagishita, M. Ohsak, 2009 "*Topology optimization of trusses by growing ground structure method*" Structural and Multidisciplinary Optimization, DOI 10.1007/s00158-008-0237-4

[12] *Jikai Liu,* Albert C. To, 2017, "Topology optimization for hybrid additive-subtractive manufacturing," Struct Multidisc Optim (2017) 55:1281–1299

[13] Xiangyang Zhou, Liping Chen, Zhengdong Huang, "The SIMP-SRV Method for Stiffness Topology Optimization of Continuum Structures," CAD Center, School of Mechanical Science & Engineering, HuaZhong University of Science and Technology

[14] Pu Zhang, Jakub Toman, and Yiqi Yu, 2015, Efficient Design-Optimization of Variable-Density Hexagonal Cellular Structure by Additive Manufacturing: Theory and Validation, Journal of Manufacturing Science and Engineering

[15] Feng Ruo-Qiang, Liu Feng-cheng, Xu Wei-Jia, Ma Min, and Liu Yang, 2016, "Topology Optimization Method of Lattice Structures Based on a Genetic Algorithm," International Journal of Steel Structures 16(3): 743-753 (2016)

[16] James Norman Richardson, Sigrid Adriaenssens, Rajan Filomeno Coelho, Philippe Bouillard，2013, "Coupled form-finding and grid optimization approach for single layer grid shells" Engineering Structures

[17] J Zhou, P Shrotriya, W.O Soboyejo, 2002, "On the deformation of aluminum lattice block structures: from struts to structures" Mechanics of Materials 36 (2004) 723–737

[18] J.C. Wallacha, L.J. Gibson, 2001, "Mechanical behavior of a three-dimensional truss material" International Journal of Solids and Structures Volume 38, Issues 40–41, October 2001, Pages 7181–7196

[19] A.O. Aremu, I. Maskery, C. Tuck, I. A., Ashcroft, R.D., Wildman, and R.I.M, Hague. 2014, "A Comparative Finite Element Study of Cubic Unit Cells For Selective Laser

Melting" International Solid Freeform Fabrication Symposium – An Additive Manufacturing Conference

[20] A. Saigal, J. Tumbleston, "Stress-Strain Behavior of an Octahedral and Octet-Truss Lattice Structure Fabricated Using the CLIP Technology," Advanced Materials Research, Vol. 1142, pp. 245-249, 2017

[21] D.J. Sypeck and H.N.G. Wadley, 2000, "Multifunctional microtruss laminates: Textile synthesis and properties," Materials Research Society J. Mater. Res., Vol. 16, No. 3, Mar 2001

[22] Ian Gibson· David Rosen, Brent Stucker, "Additive Manufacturing Technologies" 2015

[23] Product-Grade Thermoplastic for Design Series 3D Printers, "http://usglobalimages.stratasys.com/Main/Files/Material_Spec_Sheets/MSS_FDM_ABSplusP430.pdf."

[24] Kurowski, Paul, "More errors that mar FEA results," Machine Design; Cleveland74.6 (Mar 21, 2002): 51-56.

[25] Takezawa, A.,"Development of porous material using topology optimization and metal 3D printing", Bull. Jpn. Soc. Mech. Eng. Volume 117, Page 697 2003

[26]Andrew T. Gaynor, James K. Guest2, 2016, "Topology optimization considering overhang constraints: Eliminating sacrificial support material in additive manufacturing through design" Struct Multidisc Optim (2016) 54:1157–1172

[27] Samuel Amstutz· Antonio A. Novotny, "Topological optimization of structures subject to Von Mises stress constraints," Struct Multidisc Optim (2010) 41:407–420

[28] Wen-Yea Jang, Stelios Kyriakides, Andrew M. Kraynik, 2010 "On the compressive strength of open-cell metal foams with Kelvin and random cell structures," International Journal of Solids and Structures

[29] T. V. GALAMBOS, M. K. RAVINDR, 1977 “The basis for load and resistance factor design criteria of steel building structures” Can. J. Civ. Eng.. 4, 178 (1977)

[30] Holly D. Carlton, Jonathan Lind, Mark C. Messner, Nickolai A. Volkoff-Shoemaker, Harold S. Barnard, Nathan R. Barton, Mukul Kumar, “Mapping local deformation behavior in single cell metal lattice structures,” Acta Materialia 129 (2017) 239e250

[31] Christiane Beyer, Dustin Figueroa, 2016, “Design and Analysis of Lattice Structures for Additive Manufacturing,” Journal of Manufacturing Science and Engineering DECEMBER 2016, Vol. 138

[32] L. Dong, V. Deshpande and H. Wadley, “Mechanical Response of Ti-6Al-4V Octet-Truss Lattice Structures, International Journal of Solids and Structures”, Vols. 60-61 (2015), pp.107-124.

[33] Clémence Petit, Eric Maire, Sylvain Meille, Jérôme Adrien, Shingo Kurosu, Akihiko Chiba “CoCrMo cellular structures made by Electron Beam Melting studied by local tomography and finite element modeling,” Materials Characterization 116 (2016) 48–54

[34] Wen-Ming Chen, Sung-Jae Lee, Peter Vee Sin Lee, “Failure Analysis of An Additive Manufactured Porous Titanium Structure for Orthopedic Implant Applications”, 2016, Materials Science Forum, ISSN: 1662-9752, Vol. 863, pp 45-49

[35] Qingcheng Yang, Pu Zhang, Lin Cheng, Zheng Min, Minking Chyu, Albert C. To, “Finite element modeling and validation of thermomechanical behavior of Ti-6Al-4V in directed energy deposition additive manufacturing”, Additive Manufacturing 12 (2016) 169–177

[36] A. V. Balakrishnan, “Combined Structures-Controls Optimization of Lattice Trusses”, NAS1-18585, 1991

[37] Jun Yan, Suxia Yang, Zunyi Duan, and Chunqiu Yang, “Minimum Compliance Optimization of a Thermoelastic Lattice Structure with Size-Coupled Effects”, Journal of Thermal Stresses, 38: 338–357, 2015

[38] Tommaso Ingrassiaa, Antonio Mancusoa, Giorgio Paladinoa, “Influence of the evolutionary optimization parameters on the optimal topology”, Springer International Publishing AG 2017, B. Eynard et al. (eds.), Advances on Mechanics, Design Engineering and Manufacturing, Lecture Notes in Mechanical Engineering

[39] O.S. Carneiro, A.F. Silva, R. Gomes, “Fused deposition modeling with polypropylene”, Materials & Design 83 (2015) 768–776

[40] T. S. Srivatsan and T. S. Sudarshan, “Additive manufacturing, Chapter 1”, 2015, CRC Press

[41] David Ian Wimpenny • Pulak M. Pandey L. Jyothish Kumar, “Advances in 3D Printing & Additive Manufacturing Technologies by Springer”, ISBN: 978-981-10-0811-5 (Print) 978-981-10-0812-2 (Online)

[42] Chunze Yan, Liang Hao, Ahmed Hussein, Philippe Young David Raymont, “Advanced lightweight 316L stainless steel cellular lattice structures fabricated via selective laser melting”, 2014, Materials and Design 55 (2014) 533–541

[43] Y.J. Chen, F. Scarpa,Y.J. Liu,J.S. Leng, “Elasticity of anti-tetrachiral anisotropic lattices,” International Journal of Solids and Structures 50 (2013) 996–1004

[44] Sahab Babaee, Jongmin Shim, James C. Weaver, Elizabeth R. Chen, Nikita Patel, Katia Bertoldi, “3D Soft Metamaterials with Negative Poisson’s Ratio”, Adv. Mater. 2013, 25, 5044–5049

[45] Mustapha Bouakba, Abderrezak Bezazi, Fabrizio Scarpa, “FE analysis of the in-plane mechanical properties of a novel Voronoi-type lattice with positive and negative Poisson’s ratio configurations,” International Journal of Solids and Structures 49 (2012) 2450–2459

[46] Materials Properties Handbook: Titanium Alloys, R. Boyer, G. Welsch, and E. W. Collings, eds. ASM International, Materials Park, OH, 1994.