The Pennsylvania State University

The Graduate School

College of Engineering

MODELING DYNAMIC INSTABILITY OF
OFF-HIGHWAY MINING DUMP TRUCKS

A Thesis in
Mechanical Engineering

by

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ABSTRACT

In this research, a three-degree-of-freedom (3-DOF) mathematical model is developed to predict the onset of unstable pitch/bounce-type vibrations in heavy, off-highway, mining haul trucks. This unstable vibration phenomenon can be described as an instance of “power hop”, a type of dynamic instability also seen in agricultural tractors, wheeled construction vehicles, and other off-highway vehicles (OHVs). A model that can adequately predict dynamic instability is a necessary first step in developing controls solutions that might be able to mitigate or even prevent unstable motion from occurring.

The 3-DOF model describes a rear-wheel-drive (RWD) mining haul truck operating on a non-deformable, sloped surface. Analytical stability criteria are derived through closed-form eigenvalue analysis of the equations of motion while neglecting all damping terms. Numerical stability analysis studies parameter sensitivity for a representative haul truck with a 250 ton payload capacity in two payload scenarios: operation at empty vehicle weight (EVW) and operation at 100% payload capacity, or gross vehicle weight (GVW).

Results show that the 3-DOF predicts dynamic instability. By utilizing a stability margin, the results further show that there are numerous different operating point configurations that could potentially lead to dynamic instability. Of the 33 million cases tested in each of the two different payload scenarios, 21% are deemed potentially unstable for the empty truck and 38% are deemed potentially unstable for the loaded truck. Furthermore, potentially unstable cases exist over the full range of parameters simulated. The analytical stability criteria derived from the undamped equations of motion do not always correlate with the numerical eigenvalue analysis of the full set of equations of motion that include all damping terms.

Instability is shown to be heavily dependent on road grade, tire stiffness and damping properties, the location of the vehicle’s center of gravity, and forward travel speed. The model mainly predicts instability on severe road grades and at high speeds. A result not seen in previous studies of power hop in agricultural tractors is the prediction
of dynamic instability when a vehicle travels downhill. While instability on uphill grades primarily occurs when the rear axle is stiffer than the front axle, the opposite is observed on downhill grades. Increased tire damping is shown to reduce the likelihood of instability in both loading scenarios and on all grades. Like tire stiffness, longitudinal center of gravity location also plays a role in whether the truck is more likely to become unstable on positive or negative road grades.
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NOMENCLATURE

Symbols

\(a\)  Longitudinal distance from the front axle to the center of gravity
\(b\)  Longitudinal distance from the rear axle to the center of gravity
\(c\)  Damping coefficient
\(C\)  Wheel numeric
\(D\)  Discriminant – analytical stability criteria
\(e\)  Moment arm of the normal force acting on a tire contact patch
       Base of the natural logarithm
\(F\)  Gross tractive force
\(g\)  Acceleration of gravity (32.2 ft/s\(^2\), 9.81 m/s\(^2\))
\(h\)  Height/normal distance of the center of gravity from the road surface
\(h_f\)  Vertical distance from the front axle to the center of gravity
\(h_r\)  Vertical distance from the rear axle to the center of gravity
\(\bar{H}\)  Analytical stability criteria #2
\(l_{yy}\)  Mass moment of inertia about the y-axis (pitch)
\(j\)  \(\sqrt{-1}\)
\(k\)  Stiffness coefficient
       Rubber hardness term (units: kN/m\(^2\))
\(L\)  Wheelbase
\(m\)  Mass of the vehicle
\(n\)  Number of tires on a given axle
\(N\)  Total normal force acting on a given axle
\(P\)  Power
\(r\)  Rolling radius of a tire
\(R\)  Normal force acting on an individual wheel
\(s\)  Wheel slip
       Eigenvalue
\(SLR\)  Static loaded radius of a tire
\(t\)  Time
\(T\)  Torque
\(TF\)  Rolling resistance force or “tow force”
\(V\)  Forward travel speed
\(W\)  Weight of the vehicle
\(x\)  Longitudinal direction or fore-aft displacement of a vehicle
\(z\)  Vertical direction or bounce/heave displacement of the vehicle
\(\beta\)  Road grade/incline
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>Radial deflection of a tire</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Damping ratio</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Engine-to-axle powertrain efficiency</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Pitch rotational direction or pitch angle of the vehicle</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Slope of the gross tractive coefficient vs. slip curve</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Tractive coefficient</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Rolling resistance coefficient</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Real part of an eigenvalue (growth rate)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Period of oscillation</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Rotational speed</td>
</tr>
</tbody>
</table>

**Recurring Subscripts and Accents**

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$(_0)$</td>
<td>Operating point value</td>
</tr>
<tr>
<td>$(_{EVW})$</td>
<td>Empty vehicle weight</td>
</tr>
<tr>
<td>$(_f)$</td>
<td>Front axle</td>
</tr>
<tr>
<td>$(_g)$</td>
<td>Gross</td>
</tr>
<tr>
<td>$(_{GVW})$</td>
<td>Gross vehicle weight</td>
</tr>
<tr>
<td>$(_i)$</td>
<td>On any given axle, i.e., front ($f$) or rear ($r$)</td>
</tr>
<tr>
<td>$(_{max})$</td>
<td>Maximum value of a parameter</td>
</tr>
<tr>
<td>$(_r)$</td>
<td>Rear axle</td>
</tr>
<tr>
<td>$(_t)$</td>
<td>Tire</td>
</tr>
<tr>
<td>$(_v)$</td>
<td>Variation of a value from its operating point value</td>
</tr>
<tr>
<td>$(_x)$</td>
<td>Longitudinal/fore-aft translational direction</td>
</tr>
<tr>
<td>$(_z)$</td>
<td>Vertical/bounce translational direction</td>
</tr>
<tr>
<td>$(_\theta)$</td>
<td>Pitch rotational direction</td>
</tr>
<tr>
<td>$(')$</td>
<td>Velocity</td>
</tr>
<tr>
<td>$('')$</td>
<td>Acceleration</td>
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**Matrix/Vector**

<table>
<thead>
<tr>
<th>Matrix/Vector</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[C]$</td>
<td>Damping matrix</td>
</tr>
<tr>
<td>$[I]$</td>
<td>Identity matrix</td>
</tr>
<tr>
<td>$[K]$</td>
<td>Stiffness matrix</td>
</tr>
<tr>
<td>$[M]$</td>
<td>Mass matrix</td>
</tr>
<tr>
<td>${q}$</td>
<td>Vector of generalized coordinates</td>
</tr>
</tbody>
</table>
### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D</td>
<td>Two-dimensions/two-dimensional</td>
</tr>
<tr>
<td>3-DOF</td>
<td>Three-degree-of-freedom</td>
</tr>
<tr>
<td>4WD</td>
<td>Four wheel drive</td>
</tr>
<tr>
<td>CCW</td>
<td>Counter-clockwise</td>
</tr>
<tr>
<td>CG</td>
<td>Center of gravity</td>
</tr>
<tr>
<td>DOF(s)</td>
<td>Degree(s) of freedom</td>
</tr>
<tr>
<td>EOM(s)</td>
<td>Equation(s) of motion</td>
</tr>
<tr>
<td>EVW</td>
<td>Empty vehicle weight/mass</td>
</tr>
<tr>
<td>GVW</td>
<td>Gross vehicle weight/mass</td>
</tr>
<tr>
<td>HISLO</td>
<td>High impact shovel loading operations</td>
</tr>
<tr>
<td>OHV(s)</td>
<td>Off-highway vehicle(s)</td>
</tr>
<tr>
<td>OTR</td>
<td>Off-the-road</td>
</tr>
<tr>
<td>QEP</td>
<td>Quadratic eigenvalue problem</td>
</tr>
<tr>
<td>RWD</td>
<td>Rear wheel drive</td>
</tr>
<tr>
<td>WBV(s)</td>
<td>Whole body vibration(s)</td>
</tr>
</tbody>
</table>
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To my parents, Panos and Deborah Papavizas, thank you for loving me unconditionally and being the constant source of support, motivation, and wisdom that has always helped me through the most difficult of times.
CHAPTER 1

INTRODUCTION

1.1 Background

Large, rigid-frame dump trucks commonly referred to as “haul trucks” are used in high-production surface mining operations for overburden and orebody haulage. Some of the largest trucks in the world today can haul payloads of 400 short tons (363 metric tons) while reaching top speeds of around 40 mph (64.5 km/h). The typical haul truck is a two-axle, rear wheel drive (RWD) vehicle. Propulsion comes in the form of a front diesel engine coupled to either a mechanical powertrain or an electric drive. The first electric drive haul trucks utilized DC generators, but most modern haul trucks run more efficient AC inverters and alternators with traction motors housed directly in the rear axles. A common rear suspension arrangement is a pivoting trailing arm/solid rear axle with two hydropneumatic struts that suspend the vehicle chassis. Two independent hydropneumatic struts suspend the vehicle chassis in the front, which allow the front tires to move independently.

These massive off-highway vehicles (OHVs) are regularly subjected to loads that cause significant vibration and severely reduce their efficiency. With their high rate of usage, mine haul roads quickly deteriorate, developing roughness and defects that result in increased vehicle maintenance and operation costs and decreased productivity (Hugo et al., 2008; Anzabi et al., 2012). For their operators, haul truck vibrations can cause discomfort ranging from slight annoyances to serious health risks. High-frequency shockwaves generated by impact forces during stationary shovel loading operations propagate through the truck, inducing whole-body vibrations (WBVs) in the operator. These WBVs can have debilitating long-term effects on musculoskeletal health (Frimpong et al., 2011, Aouad and Frimpong, 2013; Aouad and Frimpong, 2014). The
sheer inertia of a haul truck is also enough to cause significant pitching during routine acceleration and braking (Ichinose et al., Apr. 2014; Ichinose et al., Oct. 2014).

The focus of this research is on a less common but equally problematic vibration that is thought to be the result of the inherent stability of the truck itself, rather than external influences like severe road roughness or shovel loading impact forces. Haul trucks will sometimes experience an unstable, combined pitch/bounce-type oscillation under normal operating conditions, i.e., while traveling forward at normal speeds, empty or loaded, and on both flat and sloped sections of well-maintained haul roads. This vibration phenomenon can perhaps be best described as an instance of “power hop.” Power hop is type of dynamic instability most commonly seen in agricultural tractors, but it has been known to occur in other OHVs.

### 1.2 Power Hop Instability

Much of the current research into power hop has been performed on agricultural tractors. Zoz (2007) describes power hop in tractors as the onset of oscillatory motion with fore-aft pitch and bounce modes where the amplitude of the oscillation increases until it attains a steady-state limit. It typically occurs during field operations when a tractor is exerting tractive effort to tow an agricultural implement. Once the tractor begins hopping, the only corrective for an operator to take is to reduce power and/or stop the tractor completely. Dessevre (2005) notes that power hop is more likely to occur when soil is dry or has been recently worked.

Perhaps the most comprehensive look into the power hop phenomenon is given by Wiley and Turner (2008). Wiley and Turner (2008) describes the characteristic pitch/bounce oscillation of power hop as an unstable “porpoising” that is superimposed on the forward motion of tractors. They also note that it is most often observed for tractors having two powered axles, namely mechanical front wheel drive (MFWD) tractors and articulated four-wheel drive (4WD) tractors, while pulling towed implements in moderate to high draft field operations on dry and firm soils.
Wiley and Turner (2008) recounts the decades-long efforts of many people that eventually led to a mathematical model that could predict power hop. As shown in the free-body diagram of Figure 1.1, the mathematical model analyzes a 4WD or MFWD tractor pulling a horizontal draft load $P_0$ from its drawbar on a level concrete or soil surface. The tractor is supported by pneumatic tires on the front and rear axles, which serve as the only means of suspension. Tractive forces are generated at the contact patches of the front and rear tires. The tractor is modeled as a single rigid body supported by two sets of parallel spring and damper combinations, which represent the tires. The model has three degrees of freedom (DOFs) describing the motion of the tractor chassis: longitudinal translation ($x$), bounce/heave translation ($z$), and pitch rotation ($\theta$).

![Free-body diagram of a 3-DOF, 4WD/MFWD tractor-implement system. Source: Wiley and Turner, 2008.](image)

Following the development of a set of linearized differential equations of motion (EOMs) for the tractor-implement system, a classical stability analysis is carried out. Wiley and Turner show that, depending on the values of the system parameters, certain levels of drawbar pull or wheel slip result precisely in the flutter-type dynamic instability known as power hop. It is shown that inclusion of the fore-aft $x$-DOF and the associated traction and rolling resistance forces were necessary for producing a set of EOMs that could exhibit dynamic instability.
It is important to note that this model includes no external periodic forcing inputs, meaning that power hop is not a resonance phenomenon. Zoz (2007) notes that power hop is often confused with *road lope*, a forced vibration caused by “out-of-round tires or wheels rolling at a speed that generates a rotational input frequency near one of the vehicle (vertical) natural frequencies.” Instead, power hop is an example of a *self-excited vibration* where the source of input energy is the engine driving the wheels, which generates the traction forces that directly influence the pitching motion of the chassis (Wiley and Turner, 2008). An unstable tractor, or other wheeled vehicle operating in similar conditions, may run normally until, at the right level of pull or slip, small variations in the tractive forces induce small oscillations. These small oscillations will steadily grow into severe pitching and bouncing unless the energy source—the power delivered by the engine—is removed.

The 3-DOF model was validated with extensive experimental testing of numerous tractor configurations on concrete test tracks and soil fields. The tests showed that operating surface conditions play a major role in determining whether or not power hop occurs—the model proved reliable in predicting power hop on firm surfaces like concrete where the stiffness and damping characteristics are primarily due to the tires alone, but did not work as well on softer soil surfaces where the combined tire/soil traction, stiffness, and damping effects are highly variable and extremely difficult to model. These tests led to passive measures for controlling power hop, which includes making adjustments to fore-aft weight distribution via ballasting weights and tire stiffnesses via inflation pressures or tire size. To this day, no definitive fix for power hop in tractors exists because it largely depends on soil condition, which can change from day to day and from field to field (Grainews, 2009).

Wiley and Turner (2008) makes two brief observations about power hop that are of particular interest. First, while it is extremely rare in field applications, power hop has been observed under test conditions in RWD tractors pulling heavy draft loads on soil and concrete. Second, power hop has also been observed in a 4WD tractor climbing a steep hill on loose soil without pulling any implement, where the “downslope component of the tractor weight was enough of a draft load to induce instability.” Such scenarios seem to more closely relate to the power hop problem seen in haul trucks.
1.3 Haul Truck Vibration Models

Published research into the power hop phenomenon as it occurs in mining haul trucks could not be found. Available vibration models address the problems mentioned in Section 1.1. Frimpong et al. (2011), Aouad and Frimpong (2013), and Aouad and Frimpong (2014) model truck vibrations during high-impact shovel loading operations (HISLO) with the focus placed on WBVs. They develop a 9-DOF pitch-plane, rigid body model (including cab, operator seat, and operator body motions) using Newtonian and Lagrangian methods and implement it in MSC ADAMSTM. Dindarloo (2016) utilizes the truck model of the preceding works to examine the same HISLO/WBV problem, but with a focus on the effects of aging hydropneumatic suspension struts. Hugo et al. (2008) describes efforts to identify and classify haul road defects by measuring front wheel vibration response to road inputs. Experimental characterization of haul truck tires and front suspension struts is also described. Anzabi et al. (2012) uses a standard lumped mass, quarter-vehicle model to simulate dynamics of haul truck tires in response to road profiles, the goal being to identify tire defects. Siegrist and McAree (2006) develop a state-space formulation of a four-wheel truck model for real-time tire force estimation by Kalman inverse filtering.

The following U.S. Patents are related to haul truck vibrations. U.S. Patent 8,195,351 and its continuations, U.S. Patent 8,380,381 and U.S. Patent 8,706,338, disclose an electric vehicle having a control device that implements a method to actively control the pitch of vehicles, particularly those which experience significant weight changes in normal operation, i.e., haul trucks (Ichinose et al., April 2014). However, the pitching motion addressed by this patent is that which occurs during acceleration and braking events due to dynamic weight transfer. U.S. Patent 8,862,300 discloses a pitch control device that suppresses pitching motion due to abrupt changes in acceleration of an electric vehicle during braking, specifically when the vehicle comes to a full stop. The control method also accounts for weight variations due to changing loads as well as changes in motion resistance due road gradient (Ichinose et al., October 2014). U.S. Patent 9,079,502 and U.S. Patent 9,315,116 disclose an electric drive vehicle which can control wheel speed to achieve an appropriate slip ratio of the wheels even when pitching
vibration of the vehicle is large (Kikuchi et al., July 2015; Kobayashi et al., April 2016). These two patents explicitly mention the pitching problem in heavy electric vehicles for which the pitching vibrational frequency spectrum can change in accordance with presence or absence of a carried load.

1.4 Overview of the Present Work

The aim of this research is to develop a mathematical model that is able to capture the dynamic instability observed in heavy, off-highway, mining haul trucks. A model that can adequately predict dynamic instability is a necessary first step in developing controls solutions that might be able to mitigate or even prevent unstable motion from occurring.

In Chapter 2, a three-degree-of-freedom (3-DOF) model of a haul truck is formulated. Given that this instability behavior is not addressed by current haul truck literature, the lessons taught by Wiley and Turner (2008) are used as the foundation for transferring the power hop model from the realm of agricultural tractors to that of mining haul trucks. Static equilibrium and traction equations are used to determine the steady-state operating point of the truck. Then, a set of three differential equations that govern small perturbation motions about the operating point are linearized and put into standard linear algebra form. Closed-form stability results are obtained by performing a classical eigenvalue analysis of the EOMs where all velocity-dependent (damping) terms are neglected. Alterations of the Wiley and Turner (2008) model lead to set of dynamic equations that have the same in general structure as the tractor equations, but yield different analytical stability criteria.

In Chapter 3, the model developed in Chapter 2 is implemented in MATLAB® and numerical stability analysis of the full set of EOMS is performed on a representative haul truck with a 250 ton payload capacity. The code uses a sensitivity study approach, wherein millions of parameter combinations are tested in order to gain a broad sense of the dependence of stability on the system parameters. The results of this broad parameter study show that the present 3-DOF model predicts dynamic instability in circumstances
not previously addressed by previous power hop models, namely for a RWD vehicle with no towed implements while climbing up or descending an inclined road surface.

Finally, in Chapter 4 improvements for the 3-DOF model are recommended, possible avenues for developing higher-fidelity models to address the haul truck instability problem are discussed, and experiments for obtaining more accurate haul truck parameter are proposed.
CHAPTER 2

3-DOF MODEL

A 3-DOF model of the dynamic instability behavior observed in off-highway mining haul trucks is developed following the process described in Wiley and Turner (2008). Stability, or instability, is determined in part by the specific operational state of the vehicle in addition to the inherent parameters of the system. Therefore, development of a model for stability analysis is carried out in three major steps. First, the steady-state operating point of the vehicle must be determined in order to identify values of external forces and other performance variables. Second, a set of linearized differential equations that govern small motions of the system about the operating point must be developed. These “small motions” are perturbations that, depending on the configuration of the system, either grow or decay with time, with or without oscillation. The third major step is to determine the exact nature of the system’s response to perturbations via an eigenvalue analysis of the differential equations of motion (EOMs). The 3-DOF model for mining haul trucks presented in this research is a modification of the power hop model for agricultural tractors presented in Wiley and Tuner (2008); various aspects are altered to suit the different vehicle configuration.

2.1 Operating Point Determination

2.1.1 Assumptions

The 3-DOF model assumes that the motion of the haul truck can be analyzed in two-dimensions (2D). The longitudinal plane of the vehicle in which fore-aft motion, bouncing motion, and pitching motion occurs is the plane of interest. As shown in Figure
2.1, the truck is initially in static equilibrium, moving forward at a constant velocity $V_0$ along an inclined road of positive or negative grade $\beta$ with respect to the horizontal. This static equilibrium configuration is called the operating point. The road surface is assumed to be flat and rigid (non-deformable), which is consistent with typical mine haul roads (Anzabi, 2012) and a common assumption for simplifying vibration analyses of OHVs (Goering et al., 2003). Haul roads must be able to support massive, cyclical loads of several hundreds of tons as trucks travel between shovel loading locations and remote dump sites, so assuming a rigid road surface is reasonable. This also eliminates the need to incorporate combined stiffness and damping effects of specific haul road/haul truck tire combinations, for which there are no available data or mathematical models.

![Figure 2.1. Diagram of a haul truck operating on a non-deformable haul road.](image)

The truck has three DOF, all of which are measured from a fixed reference located at the operating point position of the center of gravity (CG). The first DOF is $x$, the fore-aft translational motion of the CG, with forward motion taken as positive. The second DOF is $z$, the bouncing motion of the CG, with downward motion taken as positive—a standard convention for vehicle coordinate systems. The third DOF is $\theta$, the pitch rotational motion of the CG, with upward rotation of the nose of the vehicle taken as positive. In Figure 2.1, positive rotation is counter-clockwise (CCW).
This 3-DOF model considers the motion of rear-wheel-drive (RWD) vehicles only. Electric drive haul trucks have independently driven left and right rear tires, but for the purposes of this 2D simplification all rear wheels are assumed to rotate at the same speed. While all-wheel-drive haul trucks exist, RWD remains an industry standard and is the focus of this work. It is noted that the Wiley and Tuner (2008) power hop model considers tractors in which both the front and rear axles are powered, so extension to an all-wheel-drive haul truck is certainly possible.

The truck is modeled as a single rigid body supported by pneumatic tires on its front and rear axles. Although modern haul trucks have independent front and rear suspension systems that utilize hydropneumatic struts, it is postulated that a majority of the suspension effects come from the tires. Most of the deflection seen during pitch and bounce oscillation is seen in the tires, as opposed to extension and compression of the suspension struts. The implications of choosing a 3-DOF model and considering the truck as a single rigid body are two-fold. First, it helps keep the model simple; the equations of motion (EOMs) are relatively easy to formulate. Second, it eliminates a great deal of uncertainty in terms of the number of parameters that must be known.

Other assumptions are made with regard to the geometry of the vehicle as shown in Figure 2.1. The relevant haul truck geometries are summarized in Table 2.1.

“Longitudinal” distances are measured parallel to the road surface, or in the $x$-direction, and “vertical” distances are measured normal to the road surface, or in the $z$-direction.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheelbase</td>
<td>$L$</td>
</tr>
<tr>
<td>Longitudinal distance from front axle to CG</td>
<td>$a$</td>
</tr>
<tr>
<td>Longitudinal distance from rear axle to CG</td>
<td>$b$</td>
</tr>
<tr>
<td>Height of the CG from the ground</td>
<td>$h$</td>
</tr>
<tr>
<td>Vertical distance from front axle center to CG</td>
<td>$h_f$</td>
</tr>
<tr>
<td>Vertical distance from rear axle center to CG</td>
<td>$h_r$</td>
</tr>
<tr>
<td>Rolling radius of front tires</td>
<td>$r_f$</td>
</tr>
<tr>
<td>Rolling radius of rear tires</td>
<td>$r_r$</td>
</tr>
</tbody>
</table>
Values with subscript \( f \) refer to geometries corresponding to the front axle, and values with subscript \( r \) refer to geometries corresponding to the rear axle. As can be inferred from Figure 2.1, the wheelbase \( L \) can be expressed as the sum \( a + b \). Likewise, the distance \( h \) can be expressed as either \( h_f + r_f \) or \( h_r + r_r \). Haul trucks normally have identical tires on the front and rear axles, which are inflated to the same pressures, so the rolling radii \( r_f \) and \( r_r \) can nominally be assumed the same. As a result, the distances \( h_f \) and \( h_r \) will also be the same. However, for the purposes of clarity and generality, these distances are kept distinct and thus retain their subscripts.

Figure 2.2 shows the forces acting on a haul truck operating in a steady-state configuration as previously established. Operating point forces are denoted by the subscript 0. Again, the truck travels forward at constantly velocity \( V_0 \) along an inclined road at positive or negative grade \( \beta \) with respect to the horizontal. Due to the road incline, the weight of the vehicle \( W \) has a component acting parallel to the road surface, \( W \sin \beta \). In the Wiley and Turner (2008) model, the road surface is always horizontal, so \( W \) always acts normal to the road surface. Here, \( W \) always acts downslope when \( \beta \) is nonzero, which can be toward the front or rear of the vehicle depending on the sign of \( \beta \).

![Figure 2.2. Operating point forces acting on a haul truck.](image-url)
An internally developed torque $T_{r0}$ is applied to the rear axle (not shown), causing a gross tractive force $F_{r0}$ that acts at a distance $r_r$ from the wheel center to propel the truck forward. The rear wheels are assumed to rotate a constant angular velocity $\omega_{r0}$ while the rotational motion of the front tires is neglected because they are unpowered. It is important to note that $\omega_{r0}$ would not be assumed constant if additional DOFs corresponding to dynamics of the powertrain were included (Wiley and Turner, 2008).

Rolling resistance forces, or alternatively “tow forces”, due to tire/road surface interactions always act opposite of the direction of motion. Primary mechanisms for rolling resistance include energy loss due to tire deflection and contact patch scrubbing and can be affected by numerous factors like road surface conditions, tire temperature, inflation pressure, tire material, and slip (Gillespie, 1992). There is a rolling resistance force $TF_{f0}$ acting through the front axle wheel center and a rolling resistance force $TF_{r0}$ acting through the rear axle wheel center. The component of $W$ acting perpendicular to the road surface is counteracted by normal ground reaction forces $N_{f0}$ and $N_{r0}$, which act on the tires through the front and rear wheel centers, respectively. Because haul trucks operate at low speeds (typically up to 40 mph), aerodynamic drag is neglected.

### 2.1.2 Traction Mechanics

Gross tractive force, rolling resistance force, and normal force are all components of a single ground reaction force acting on a tire. Figure 2.3 is adopted from Goering et al. (2003), which shows free-body diagrams of an individual towed wheel and an individual drive wheel. In the present 3-DOF model, the front wheels are towed and the rear wheels are driven. The following is presented solely as a review of traction mechanics for OHVs as given in Chapter 13 of Goering et al. (2003) and hence uses the same notation.

On a towed wheel, the ground reaction force $G$ acts on the tire contact patch. $G$ can be resolved into a vertical component $R$ and a horizontal component $TF$. $R$ is the normal force exerted by the ground, which must be equal and opposite the load $W$ placed on the wheel axle. $TF$ is the rolling resistance force, which is opposed by an equivalent
Figure 2.3. Free-body diagram of a towed wheel (left) and a drive wheel (right). Source: Goering et al., 2003.

reaction force acting through the wheel center. Since \( TF \) acts at the contact patch, which is located at a distance \( r \) from the wheel center, it results in a clockwise moment \( TF \cdot r \). Therefore, \( R \) must necessarily be shifted ahead of the wheel center by a distance \( e \) in order to maintain moment equilibrium. Summing moments acting on the wheel, we find that

\[
TF \cdot r - R \cdot e = 0
\]

(2.1)

or \( e = (TF/R) \cdot r = (TF/W) \cdot r \).

On a driven wheel, the situation is similar except now an applied torque \( T \) results in a gross tractive force \( F \). The horizontal component of \( G \) must then be equal to \( F - TF \). Summing forces in the horizontal direction shows that the reaction force \( H \) acting at the wheel center must also be \( F - TF \). Summing moments about the wheel center yields

\[
T - (F - TF) \cdot r - R \cdot e = 0
\]

(2.2)

Using Eq. (2.1), we find that \( T = F \cdot r \).

Note that for the haul truck in Figure 2.2 the rolling resistance forces \( TF_{f0} \) and \( TF_{r0} \) and normal forces \( N_{f0} \) and \( N_{r0} \) act directly through the wheel centers. This seems to
contradict what is dictated by Eq. (2.1). However, this is a dynamically equivalent scenario where force and moment equilibrium is maintained (Wiley and Turner, 2008). Referring back to Figure 2.3, by shifting the line of action of the rolling resistance force $TF$ to pass through the wheel center, this requires that the distance $e$ be equal to zero, and so the normal force $R$ must also act through the wheel center. For both the towed and driven wheel, the sums of vertical and horizontal forces are identical and the result of Eq. (2.2) will be the same, $T = F \cdot r$.

Gross tractive force $F$ can be related to the normal force $R$ acting on a wheel by a gross tractive coefficient $\mu_g$ through the friction-like relationship

$$F = \mu_g R$$  \hfill (2.3)

Likewise, rolling resistance force $TF$ can be related to the normal force $R$ acting on a wheel by a rolling resistance coefficient $\rho$ through the friction-like relationship

$$TF = \rho R$$  \hfill (2.4)

Eq. (2.3) and (2.4) are written for individual wheels, but can be applied to the total gross tractive force and rolling resistance force acting on a given axle by replacing the individual wheel load $R$ with the total axle load, namely $N_{f0}$ or $N_{r0}$. If we sum the horizontal forces acting on the drive wheel of Figure 2.3, divide through by the normal force acting on the wheel $R = W$, and utilize Eq. (2.3) and (2.4), we obtain the following:

$$\frac{H}{W} = \frac{F}{W} - \frac{TF}{W} \iff \mu = \mu_g - \rho$$  \hfill (2.5)

where $\mu$ is called the net tractive coefficient. Expressed alternatively,

$$\mu_g = \mu + \rho$$  \hfill (2.6)
A crucial point to note is that for many traction models the coefficients $\mu_g$ and $\rho$ are not constants. Instead, they are typically functions of wheel slip, tire geometry, and soil parameters, as described in the literature (Wismer and Luth, 1973; Wolf et al., 1996; Goering et al., 2003; Olson et al., 2005; Wiley and Turner, 2008). The relations given by Eq. (2.3), (2.4) and (2.6) will be absolutely essential in forming the linearized EOMs. While traction models for certain types of OHVs operating on firm soil or concrete are readily available, mathematical models that describe the traction characteristics of haul truck tires on mining haul roads are not. Therefore, certain assumptions and limitations will have to be accepted in incorporating traction mechanics into the 3-DOF model.

A traction prediction model developed by Zoz and Brixius (1979) for agricultural tractors on concrete is suggested by Wiley and Turner (2008) and is used in the present work. In Zoz and Brixius (1979), the gross tractive coefficient $\mu_g$ is given as a function of wheel slip and other tire properties. Wheel slip is a dimensionless measure of the difference between the actual vehicle speed $V$ and the circumferential speed $\omega r$ of the wheel relative to the wheel center, where $\omega$ is the angular velocity of the wheel and $r$ is its rolling radius (Olson et al., 2005). Wheel slip can be defined in several ways, but, for the purposes of the present work, wheel slip for a vehicle in forward motion is given by

$$s = \frac{\omega r - V}{\omega r} = 1 - \frac{V}{\omega r} \quad (2.7)$$

If $s = 0$, this implies that $V = \omega r$. If $s = 1$ and $\omega \neq 0$, then $V = 0$, which implies that the wheels are spinning with no forward motion. The equation for $\mu_g$ as a function of slip presented by Zoz and Brixius (1979) is

$$\mu_g(s) = 1.02(1 - e^{-Cs}) + 0.02 \quad (2.8)$$

where $C$ is a dimensionless parameter herein referred to as the wheel numeric, which is given by

$$C = \frac{k b d}{W} \quad (2.9)$$

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The term $k$ is a constant relating to rubber hardness and takes on a value of 400 kN/m$^2$ (58.4 psi). The terms $b$ and $d$ are tire geometries that can be obtained from tire manufacturer data sheets, where $b$ is the tire section width and $d$ is the unloaded overall diameter of the tire. The usage of the term $W$ here again represents the dynamic load acting on an individual tire, which is determined through static equilibrium analysis, as will be detailed in Section 2.1.3. The factor 0.02 (2% of the load $W$) in Eq. (2.8) is the rolling resistance coefficient $\rho$, which was based on a series of tests performed by John Deere (Zoz and Brixius, 1979). This implies that the exponential term $1.02(1 - e^{-Cs})$ in Eq. (2.8) is the net tractive coefficient $\mu$ of Eq. (2.5) and (2.6).

The rationale behind choosing this model is as follows. Like haul trucks, agricultural tractors use pneumatic off-the-road (OTR) tires. Given the unavailability of traction prediction equations for haul truck tires, traction models for agricultural tires are likely to be more representative of haul truck tires than those for automobiles, heavy commercial trucks, or other paved road vehicles. Additionally, the rolling resistance value of 2% proposed by Zoz and Brixius (1979) aligns with typical haul road rolling resistance values (Kaufman and Ault, 1978). Rolling resistance is typically assumed constant on hard, firm surfaces and is independent of wheel slip (Zoz and Brixius, 1979; Goering et al., 2003). Wolf et al. (1996) cites Zoz and Brixius (1979) as well as others in stating that the rolling resistance coefficient of agricultural tires is a constant typically “in the range of 2 to 5%, with negligible effect of inflation pressure or other tire parameters.” For these reasons, it is reasonable to assume that the rolling resistance coefficient on haul roads is also a constant in this range with the potential for slight leeway depending on road conditions.

### 2.1.3 Static Equilibrium Equations

Referring again to Figure 2.2, three static equilibrium equations can be written for the haul truck, which are given by Eq. (2.10), (2.11), and (2.12):
\[ \sum F_x = 0 = F_{r0} - TF_{f0} - TF_{r0} - W \sin \beta \]  \hspace{1cm} (2.10)

\[ \sum F_z = 0 = W \cos \beta - N_{f0} - N_{r0} \]  \hspace{1cm} (2.11)

\[ \sum M_{CG,y} = 0 = N_{f0} a - N_{r0} b + F_{r0} h - TF_{r0} h_r - TF_{f0} h_f \]  \hspace{1cm} (2.12)

The major differences between Eq. (2.10) through (2.12) and the static equilibrium equations derived in Wiley and Turner (2008) is the removal of a drawbar load, the removal of a front gross tractive force, and the addition of road incline \( \beta \). The result of this modification is three equations in five unknowns: \( F_{r0}, TF_{f0}, TF_{r0}, N_{f0}, \) and \( N_{r0} \). However, Eq. (2.3) and (2.4) can be used to rewrite \( TF_{f0} \) and \( TF_{r0} \) in terms of \( N_{f0} \), \( N_{r0} \), and the rolling resistance coefficients of the front and rear axles:

\[ TF_{i0} = \rho_{i0} N_{i0}, \quad (i = r, f) \]  \hspace{1cm} (2.13)

Furthermore, based on the assumptions associated with the traction prediction model Eq. (2.8), the rolling resistance coefficients of the front and rear axles are taken to be constants of the same value: \( \rho_{f0} = \rho_{r0} = \rho \). Therefore, Eq. (2.10) through (2.12) become

\[ F_{r0} - \rho N_f - \rho N_{r0} - W \sin \beta = 0 \]  \hspace{1cm} (2.14)

\[ W \cos \beta - N_{f0} - N_{r0} = 0 \]  \hspace{1cm} (2.15)

\[ N_{f0} a - N_{r0} b + F_{r0} h - \rho N_{r0} h_r - \rho N_{f0} h_f = 0 \]  \hspace{1cm} (2.16)

This is now three equations in three unknowns \( (F_{r0}, N_{f0}, \) and \( N_{r0} \)), which is a linear formulation that can be solved analytically, as opposed to the fourteen equation/fourteen unknown nonlinear system in Wiley and Turner (2008). It might appear that these equations are nonlinear because the rear axle gross tractive force can be rewritten as
\[ F_{r0} = \mu_{r0}(s_{r0}) N_{r0} \]  \hspace{1cm} (2.17)

where the gross tractive coefficient \( \mu_{r0}(s_{r0}) \) is a function of slip and thus also a variable that is not readily known. However, it can be shown that reformulating Eq. (2.14) through (2.16) as nonlinear equations with \( N_{f0}, N_{r0}, \mu_{r0}, \) and \( s_{r0} \) as unknowns, adding Eq. (2.8) in order to relate \( \mu_{r0} \) to \( s_{r0} \), and solving the four equation/four unknown system via a Newton-Raphson iterative method will yield the same exact results as the linear formulation. This is because the base forms of Eq. (2.14) through (2.16) dictate what the “top level” values \( (F_{r0}, N_{f0}, \text{and} \ N_{r0}) \) must be. Taking the linear formulation, Eq. (2.14) through (2.16) can be arranged into matrix form:

\[
\begin{bmatrix}
1 & -\rho & -\rho \\
0 & -1 & -1 \\
h & a - \rho h_f & -(b + \rho h_r)
\end{bmatrix}
\begin{bmatrix}
F_{r0} \\
N_{f0} \\
N_{r0}
\end{bmatrix}
= 
\begin{bmatrix}
W \sin \beta \\
-W \cos \beta \\
0
\end{bmatrix}
\hspace{1cm} (2.18)

The solution of Eq. (2.18) is given by Eq. (2.19) through (2.21):

\[ F_{r0} = W(\sin \beta + \rho \cos \beta) \]  \hspace{1cm} (2.19)

\[ N_{f0} = W \frac{b \cos \beta - h \sin \beta - (h - h_r)\rho \cos \beta}{a + b + (h_r - h_f)\rho} \]  \hspace{1cm} (2.20)

\[ N_{r0} = W \frac{a \cos \beta + h \sin \beta + (h - h_f)\rho \cos \beta}{a + b + (h_r - h_f)\rho} \]  \hspace{1cm} (2.21)

Various simplifications of this solution can be made if quick hand calculations are needed. For example, if only operation on gradual road grades is analyzed, then the small angle approximation could be used so that \( \cos \beta \approx 1 \) and \( \sin \beta \approx \beta \). The rigorous form Eq. (2.18) is employed for all operating point calculations in the present work.
2.1.4 Performance Variables

Once the operating point forces are calculated via Eq. (2.18), other performance variables must be calculated in order to proceed with stability analysis via the linearized differential EOMs. Individual tire loads can be calculated by dividing the total axle normal forces by the number of tires on each axle:

\[ R_{f0} = \frac{N_{f0}}{n_f}, \quad R_{r0} = \frac{N_{r0}}{n_r} \]  

(2.22)

where \( R_{f0} \) is the normal force acting on an individual front tire, \( R_{r0} \) is the normal force acting on an individual rear tire, \( n_f \) is the number of wheels on the front axle, and \( n_r \) is the number of wheels on the rear axle. For a typical haul truck, there are two wheels in the front and four wheels in the back.

With \( F_{r0} \) and \( N_{r0} \) known, the gross tractive coefficient of the rear axle can be calculated as

\[ \mu_{r0} = \frac{F_{r0}}{N_{r0}} \]  

(2.23)

Then with \( \mu_{r0} \) known, rear wheel slip \( s_{r0} \) can be calculated by rearranging Eq. (2.8) and replacing the factor 0.02 with the general rolling resistance coefficient \( \rho \), as the traction model certainly supports the possibility of using other constant values:

\[ s_{r0} = -\frac{1}{C} \ln \left( 1 - \frac{\mu_{r0} - \rho}{1.02} \right) \]  

(2.24)

where in this case the wheel numeric \( C = \frac{k b_r d_r}{R_{r0}} \). The \( r \) subscripts attached to the terms \( b \) and \( d \) to denote the section width and overall diameter of the rear tires, respectively. This is a major departure from the Wiley and Turner (2008) model, where the slip of both the front and rear wheels must be calculated simultaneously with the operating point forces.
If \( s_{r0} \) is nonzero, then the actual forward speed \( V \neq r_r \omega_{r0} \). If \( \omega_{r0} \) is treated as prescribed parameter, then the actual forward velocity of the vehicle can be calculated by rearranging the expression for slip Eq. (2.7) into

\[
V = \omega_{r0} r_r (1 - s_{r0})
\]

(2.25)

### 2.1.5 Limiting Factors

In accordance with Goering et al. (2003), the operating point forces and performance variables in the previous sections may be physically limited by one of three factors: (1) longitudinal stability, (2) traction, or (3) power. These checks need to be performed before beginning stability analysis to ensure that the scenario being analyzed is physically possible.

**Longitudinal Stability**

The normal forces acting on the front and rear tires must be greater than zero. Anything otherwise would suggest that tires have lifted off the ground, at which point the system would not be in static equilibrium. Therefore, longitudinal stability requires that

\[
N_{f0} > 0 \quad \text{and} \quad N_{r0} > 0
\]

(2.26)

**Traction**

The vehicle cannot generate more traction than what is allowed by the conditions of the road surface. Looking at Eq. (2.8), it is clear that the maximum possible gross tractive coefficient occurs when \( s = 1 \), in which case \( \mu_{g,max} = 1.02(1 - e^{-C}) + \rho \). Therefore, the traction limitation imposed on a given operating point is

\[
\mu_{r0} \leq \mu_{g,max} = 1.02(1 - e^{-C}) + \rho
\]

(2.27)
**Power**

The torque developed at the rear axle must not exceed the maximum torque that can be supplied by the haul truck’s engine (or electric motors). The torque developed at the rear axle is \( T_{r0} = F_{r0} r_r \), so then the required power at the axle is \( P_{r0} = T_{r0} \omega_{r0} \).

Therefore, the power limit can be written as

\[
P_{r0} = \frac{T_{r0} \omega_{r0}}{\eta} \leq P_{\text{max}}, \quad \eta \leq 1
\]  

(2.28)

where \( P_{\text{max}} \) is the maximum power that can be delivered to the axle by the engine or motor and \( \eta \) is an overall engine-to-axle powertrain efficiency.

### 2.2 Linearized Equations of Motion

#### 2.2.1 Base Forms of the Equations

Small force variations perturb the system from the operating point, causing accelerations in the directions of the system’s DOFs. Following Wiley and Turner (2008), base forms of the longitudinal and bounce EOMs are obtained by utilizing Newton’s Laws and summing the forces in the x- and z-directions:

\[
\sum F_x = m\ddot{x} = F_r - TF_r - TF_r - W \sin \beta
\]  

(2.29)

\[
\sum F_z = m\ddot{z} = W \cos \beta - N_f - N_r
\]  

(2.30)

where \( m \) represents the total mass of the vehicle, \( \ddot{x} \) is the longitudinal acceleration of the vehicle, and \( \ddot{z} \) is the vertical acceleration of the vehicle with respect to the road surface.

The 0 subscripts have been removed to indicate that the forces in the dynamic case are no longer equal to the operating point forces.
Wiley and Turner (2008) notes that a common assumption when formulating EOMs for rotational DOFs is that rotations are small enough that moment arms do not vary with the rotational DOF, but that “this assumption has the effect of omitting some linear terms that should be included for a rigorous analysis.” Therefore, the pitch EOM is formulated considering the effects of varying moment arms for the normal forces $N_f$ and $N_r$ as shown in Eq. (2.31):

$$\sum M_{CG,y} = I_{yy} \ddot{\theta} \cong N_f (a + h_f \theta) - N_r (b - h_r \theta) + F_r h - TF_f h_r - TF_r h_f \quad (2.31)$$

where $I_{yy}$ is the pitch mass moment of inertia of the vehicle and $\ddot{\theta}$ is the pitch acceleration of the vehicle chassis. The normal force moment arm variation terms $h_f \theta$ and $h_r \theta$ are derived using the classical small angle approximation. A more detailed derivation of these varying moment arm terms can be found in the appendix of Wiley and Tuner (2008). It is noted that the moment arm of $F_r$ will technically vary with $z$ as the vehicle bounces. The moment arms of $TF_f$ and $TF_r$ will also vary with $\theta$ as the vehicle pitches. However, it can be shown that including these variations has no effect on the structure of the linearized EOM, adding only a few additional stiffness terms with minimal influence. Here they are omitted for simplicity, but may be easily included to consider all varying moment arm effects. A brief description of how these varying moment arm effects may be included is given in the Appendix.

From here, Eq. (2.29) through (2.31) can be written in terms of the DOFs $x$, $z$, and $\theta$, which requires expanded expressions for the normal forces $N_f$ and $N_r$, the rear gross tractive force $F_r$, and the rolling resistance forces $TF_f$ and $TF_r$.

### 2.2.2 Force Expressions

Figure 2.4 shows a haul truck in a state of perturbed motion, which is induced by small variations in the external forces. In the perturbed state, each force acting on the truck can be expressed as the sum of its operating point value (subscript 0) and variation of that force about the operating point value (subscript $\nu$).
Expressions for the force variations $N_{fv}$, $N_{rv}$ and $F_{rv}$ are adopted from Wiley and Turner (2008). For the sake of brevity, only the key details and final results of the derivations of these expressions are presented here. These derivations are fully explained in the appendix of Wiley and Turner (2008). Because of the assumption of a constant rolling resistance coefficient $\rho$, the forces $TF_f$ and $TF_r$ take on different forms from their tractor model counterparts, as will be described below.

**Normal Forces**

The normal forces $N_f$ and $N_r$ are the result of the pneumatic tires supporting the vehicle on the road surface. Therefore, the normal forces can be modeled as a spring and damper in parallel to reflect tire characteristics. Because the road surface is assumed to be rigid/non-deformable, the stiffness and damping effects can be considered due to the tires alone (Wiley and Turner, 2008). The tire stiffness coefficient of an individual tire is given by $k_{ti}$ and the tire damping coefficient is given by $c_{ti}$, where the subscript $t$ stands for “tire” and the subscript $i = f$ or $r$ for either front or rear as before. The total stiffness and
damping for a given axle, $k_i$ and $c_i$, respectively, can be obtained by multiplying the individual tire stiffness and damping values by the number of tires on that axle:

$$k_i = n_i k_{ti} \quad (2.32)$$

$$c_i = n_i c_{ti} \quad (2.33)$$

As is done in Wiley and Turner (2008), it can then be shown that total axle normal forces are given by

$$N_f = N_{f0} + N_{fv} = N_{f0} + k_f (z - a\dot{\theta}) + c_f (\dot{z} - a\dot{\theta}) \quad (2.34)$$

$$N_r = N_{r0} + N_{rv} = N_{r0} + k_r (z + b\dot{\theta}) + c_r (\dot{z} + b\dot{\theta}) \quad (2.35)$$

**Gross Tractive Force**

The gross tractive force $F_r$ is the product of the rear gross tractive coefficient $\mu_r$ and the real axle normal force $N_r$. Because $\mu_r$ is a function of slip $s$, then by the definition of slip, Eq. (2.7), it must also be a function of perturbation velocities. An example gross tractive coefficient vs. slip relationship is shown in Figure 2.5.

![Figure 2.5. General form of a typical gross tractive coefficient vs. slip relationship. Source: Wiley and Turner, 2008.](image)
For small variations in slip, \( F_r \) can be expressed as

\[
F_r = \mu_r N_r = (\mu_r + \lambda_r \Delta(\text{slip})_r)(N_{r0} + N_{rv})
\] (2.36)

where \( \lambda_r \) is the slope of the \( \mu_r \) versus slip relationship taken at the operating point value of slip \( s_{r0} \) obtained from Eq. (2.24), as illustrated in Figure 2.5. Using Eq. (2.8),

\[
\lambda_r = \frac{d\mu_r}{ds} = 1.02 Ce^{-Cs}
\] (2.37)

The term \( \Delta(\text{slip})_r \) represents the change in slip about the operating point value, which is obtained by first expanding Eq. (2.7) to include perturbation velocity terms that affect fore-aft motion \( (\dot{x} \text{ and } \dot{\theta}) \) and then taking partial derivatives with respect to those perturbation velocities. As Wiley and Turner (2008) show,

\[
\Delta(\text{slip})_r = -\frac{\dot{x}}{r_r \omega_{r0}} - \left( \frac{h_r}{r_r \omega_{r0}} + \frac{V}{r_r \omega_{r0}^2} \right) \dot{\theta}
\] (2.38)

**Rolling Resistance Forces**

In the Wiley and Turner (2008) power hop model for a tractor operating on soil, the rolling resistance coefficients of the front and rear axles, \( \rho_f \) and \( \rho_r \), respectively, are also function of slip. Hence, \( TF_f \) and \( TF_r \) take on similar forms to the gross tractive force as described by Eq. (2.36). However, it was previously established here that the Zoz and Brixius (1979) traction model assumes \( \rho_f = \rho_r = \rho = \text{constant} \). Therefore, using Eq. (2.34) and (2.35), we can write

\[
TF_f = \rho N_f = \rho(N_{f0} + N_{fv}) = \rho[N_{f0} + k_f(z - a\theta) + c_f(\dot{z} - a\dot{\theta})]
\] (2.39)

\[
TF_r = \rho N_r = \rho(N_{r0} + N_{rv}) = \rho[N_{r0} + k_r(z + b\theta) + c_r(\dot{z} + b\dot{\theta})]
\] (2.40)
2.2.3 Matrix Form of the Linearized Equations of Motion

Full EOMs in terms of the system’s DOFs are obtained by first substituting Eq. (2.34) through (2.40) into Eq. (2.29), (2.30), and (2.31). This results in the appearance of second-order, nonlinear terms such as $\lambda_r \Delta \text{(slip)}_r N_{rv}, \dot{\theta} \dot{\theta},$ and $\dot{\theta}^2.$ All second-order terms are neglected in order to linearize the equations. Finally, the equations can be simplified and rearranged into standard form. For the sake of brevity, only the derivation of the $z$-direction equilibrium equation is presented in full to illustrate the process:

**$z$-Direction Equation**

From Eq. (2.30), (2.34), and (2.35),

$$\sum F_z = m \ddot{z} = W \cos \beta - N_f - N_r = W \cos \beta - (N_{f0} + N_{fr}) - (N_r + N_{rv})$$

$$= W \cos \beta - N_{f0} - k_f (z - a\theta) - c_f (\ddot{z} - a\dot{\theta}) - N_{r0} - k_r (z + b\theta) - c_r (\ddot{z} + b\dot{\theta})$$

From Eq. (2.15) for $z$-direction static equilibrium,

$$W \cos \beta - N_{f0} - N_{r0} = 0$$

Therefore,

$$m \ddot{z} = -k_f (z - a\theta) - c_f (\ddot{z} - a\dot{\theta}) - k_r (z + b\theta) - c_r (\ddot{z} + b\dot{\theta})$$

Rearranging the above equation to put all terms on the left-hand side yields the $z$-direction equation in standard form:

$$m \ddot{z} + k_f (z - a\theta) + c_f (\dot{z} - a\dot{\theta}) + k_r (z + b\theta) + c_r (\dot{z} + b\dot{\theta}) = 0 \quad (2.41)$$
\( \theta \)-Direction Equation

\[
I_{yy} \ddot{\theta} - (N_f h_f + N_r h_r) \theta
\]

\[
- [k_f(z - a\theta) + c_f(\dot{z} - a\dot{\theta})] a + [k_r(z + b\theta) + c_r(\dot{z} + b\dot{\theta})] b
\]

\[
+ \rho h_f [k_f(z - a\theta) + c_f(\dot{z} - a\dot{\theta})]
\]

\[
-(\mu_r h - \rho h_r) [k_r(z + b\theta) + c_r(\dot{z} + b\dot{\theta})]
\]

\[
+ \lambda_r h \left[ \frac{\dot{x}}{r_r \omega_{r0}} + \left( \frac{h_r}{r_r \omega_{r0}} + \frac{V}{r_r \omega_{r0}^2} \right) \dot{\theta} \right] N_{r0}
\]

\( x \)-Direction Equation

\[
m \dddot{x} + \rho [k_f(z - a\theta) + c_f(\dot{z} - a\dot{\theta})]
\]

\[
-(\mu_r h - \rho h_r) [k_r(z + b\theta) + c_r(\dot{z} + b\dot{\theta})]
\]

\[
+ \lambda_r \left[ \frac{\dddot{x}}{r_r \omega_{r0}} + \left( \frac{h_r}{r_r \omega_{r0}} + \frac{V}{r_r \omega_{r0}^2} \right) \ddot{\theta} \right] N_{r0} = 0
\]

Expressed in matrix-vector form, the final linearized differential EOMs are

\[
[M] \begin{bmatrix} \dddot{x} \\ \ddot{\theta} \end{bmatrix} + [C] \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} + [K] \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

(2.44)

with
\[
[M] = \begin{bmatrix}
m & 0 & 0 \\
0 & I_{yy} & 0 \\
0 & 0 & m
\end{bmatrix},
\]

\[
[C] = \begin{bmatrix}
c_f + c_r & -c_f a + c_r b & 0 \\
-c_f a + c_r b & c_f a^2 + c_r b^2 & -\rho h_f c_f a \\
+\rho h_f c_f & -(\mu_{r0} h - \rho h_r) c_r & -{(\mu_{r0} h - \rho h_r) c_r}_b \\
-(\mu_{r0} h - \rho h_r) c_r & \lambda_r h \left(h_r + \frac{V}{\omega_{r0}}\right) N_{r0} & \frac{\lambda_r h N_{r0}}{r_r \omega_{r0}} \\
\rho c_f - (\mu_{r0} - \rho) c_r & \lambda_r \left(h_r + \frac{V}{\omega_{r0}}\right) N_{r0} + \frac{\lambda_r N_{r0}}{r_r \omega_{r0}} & 0
\end{bmatrix},
\]

and

\[
[K] = \begin{bmatrix}
k_f + k_r & -k_f a + k_r b & 0 \\
-k_f a + k_r b & k_f a^2 + k_r b^2 & -\rho h_f k_f a \\
+\rho h_f k_f & -(\mu_{r0} h - \rho h_r) k_r & -{(\mu_{r0} h - \rho h_r) k_r}_b \\
-(\mu_{r0} h - \rho h_r) k_r & -(N_{r0} h_f + N_{r0} h_r) & 0 \\
\rho k_f - (\mu_{r0} - \rho) k_r & -\rho k_f a - (\mu_{r0} - \rho) k_r b & 0
\end{bmatrix}
\]

As expected, the EOMs retain the same general structure as those presented in Wiley and Turner (2008), but with certain terms removed or changed to reflect (1) the unpowered front wheels, (2) the lack of a towed implement mass and associated drawbar load terms, and (3) the constant rolling resistance coefficients due to operation on a firm surface. Consequently, similar observations can be made about the EOMs. The EOMs are coupled and exhibit no dependence on time. There are no forcing functions present that
could induce large amplitude motions or resonance, i.e., the right hand side of Eq. (2.44) is zero. Furthermore, the mass matrix is diagonal and positive definite, the damping matrix is non-symmetric, and the stiffness matrix is non-symmetric and positive semi-definite, as indicated by the null third column. This is because the $x$ DOF is not constrained by spring elements like the $\theta$ and $z$ DOFs (Wiley and Turner, 2008).

It is this non-symmetric structure of the EOMs that lends to the possibility of dynamic instability. Depending on the system parameters, free vibration analysis of the 3-DOF model could yield eigenvalues that are complex conjugate pairs with positive real parts. This would indicate that the system exhibits an oscillatory response with exponential growth, which is precisely the flutter-type dynamic instability exhibited by both agricultural tractors and, theoretically, mining haul trucks.

### 2.3 Closed-Form Stability Analysis

#### 2.3.1 Eigenvalue Analysis

A conservative estimate of stability behavior can be made by looking at the system’s EOMs in the case of *undamped* free vibration. Closed-form results for eigenvalues and eigenvectors can be found by completely neglecting the damping matrix $[C]$ in Eq. (2.44). Neglecting all damping terms and normalizing each equation to its respective mass/inertia term leaves the following system of coupled equations:

\[
\ddot{z} + \frac{k_{11}}{m} z + \frac{k_{12}}{m} \theta = 0 \quad (2.45)
\]

\[
\ddot{\theta} + \frac{k_{21}}{I_{yy}} z + \frac{k_{22}}{I_{yy}} \theta = 0 \quad (2.46)
\]

\[
\ddot{x} + \frac{k_{31}}{m} z + \frac{k_{32}}{m} \theta = 0 \quad (2.47)
\]

where the terms $k_{ij}$ are the non-zero elements of the stiffness matrix $[K]$. 
\begin{align*}
k_{11} &= k_f + k_r \\
k_{12} &= -k_f a + k_r b \\
k_{21} &= -k_f a + k_r b + \rho h_f k_f - (\mu r_0 h - \rho h_r)k_r \\
k_{22} &= k_f a^2 + k_r b^2 - \rho h_f k_f a - (\mu r_0 h - \rho h_r)k_r b - (N_f h_f + N_r h_r) \\
k_{31} &= \rho k_f - (\mu r_0 - \rho)k_r \\
k_{32} &= -\rho k_f a - (\mu r_0 - \rho)k_r b \\
\end{align*}

Written in matrix form, Eq. (2.45) through (2.47) become

\begin{equation}
[I][\ddot{q}] + [K/M][q] = \{0\} \tag{2.49}
\end{equation}

where \(\{q\} = \{z \quad \theta \quad x\}^T\) is the DOFs organized into a generalized coordinates vector, \([I]\) is a 3x3 identity matrix and

\begin{equation}
[K/M] = \begin{bmatrix}
k_{11}/m & k_{12}/m & 0 \\
k_{21}/I_{yy} & k_{22}/I_{yy} & 0 \\
k_{31}/m & k_{32}/m & 0
\end{bmatrix} \tag{2.50}
\end{equation}

Given the form of Eq. (2.49), we assume a periodic solution of the form

\begin{equation}
\{q(t)\} = \{\bar{q}\}e^{st} \tag{2.51}
\end{equation}

where \(\{\bar{q}\}\) is an eigenvector corresponding to an eigenvalue \(s\). Taking the required derivatives of Eq. (2.51), Eq. (2.49) can be rewritten as

\begin{equation}
(s^2[I] + [K/M])\{\bar{q}\} = \{0\} \tag{2.52}
\end{equation}

A non-trivial solution \((\{\bar{q}\} \neq 0)\) to Eq. (2.52) requires that the determinant of the coefficients of \(\{\bar{q}\}\) be zero:

\begin{equation}
\det(s^2[I] + [K/M]) = 0 \tag{2.53}
\end{equation}
Expansion of Eq. (2.53) yields the characteristic equation of the undamped system:

\[
s^4 + \left(\frac{k_{11}}{m} + \frac{k_{22}}{I_{yy}}\right)s^2 + \frac{k_{11}k_{22} - k_{21}k_{12}}{ml_{yy}} = 0 \tag{2.54}
\]

This is a quadratic equation in \(s^2\), so the quadratic formula can be used to obtain

\[
s_{1,2}^2 = -\frac{1}{2}\left(\frac{k_{11}}{m} + \frac{k_{22}}{I_{yy}}\right) \pm \frac{1}{2} \sqrt{\left(\frac{k_{11}}{m} + \frac{k_{22}}{I_{yy}}\right)^2 - 4\left(\frac{k_{11}k_{22} - k_{21}k_{12}}{ml_{yy}}\right)} \tag{2.55}
\]

With some algebra, this expression for the eigenvalues can be simplified to

\[
s_{1,2}^2 = -\frac{1}{2}\left(\frac{k_{11}}{m} + \frac{k_{22}}{I_{yy}}\right) \pm \frac{1}{2} \sqrt{\left(\frac{k_{11}}{m} - \frac{k_{22}}{I_{yy}}\right)^2 + 4\frac{k_{21}k_{12}}{ml_{yy}}} \tag{2.56}
\]

2.3.2 Analytical Stability Criteria

Let the discriminant inside the radical of Eq. (2.56) be

\[
D \doteq \left(\frac{k_{11}}{m} - \frac{k_{22}}{I_{yy}}\right)^2 + 4\frac{k_{21}k_{12}}{ml_{yy}} \tag{2.57}
\]

which has units of \(\text{sec}^{-4}\). The sign of \(D\) determines whether the roots of the system will be stable or unstable. If \(D > 0\), the two roots \(s_{1}^2\) and \(s_{2}^2\) will be negative numbers and the four eigenvalues will be purely imaginary complex conjugate pairs. Each pair would correspond to a vibration mode without decay or growth (Wiley and Turner. 2008). If \(D < 0\), then the four eigenvalues will be two complex conjugate pairs where one pair will have positive real parts. This scenario would indicate an oscillatory response that grows exponentially with time, i.e., dynamic instability. Therefore, the rigorous requirement for stability of the undamped system is \(D > 0\).
Following the example of Wiley and Turner (2008), a simpler and more direct condition for stability can be found by observing that Eq. (2.57) is guaranteed to be positive if the product $k_{12}k_{21}$ is also positive. In other words, if $k_{12}$ and $k_{21}$ are the same sign, then the system is stable. If they are opposite signs, then $D$ is not guaranteed to be positive, i.e., the undamped system is no longer guaranteed to be stable. Therefore, a sufficient but not necessary condition for stability of the undamped system is

$$
\tilde{H} \triangleq k_{12}k_{21} > 0
$$

which can be divided by $ml_{yy}$ in order to put it into the same units as $D$. Eq. (2.58) is analogous to but ultimately different from the Hop Function stability criteria developed by Wiley and Turner:

$$
H \triangleq k_f a - k_r b > 0
$$

where if $H > 0$ “throughout the pull-slip range of interest for a tractor/implement system with negligible velocity-dependent forces, power hop cannot occur” (Wiley and Turner, 2008). It can be seen from Eq. (2.48) that $k_{12}$ and $k_{21}$ contain the same elements as the Hop Function as well as some additional terms. According to this result, stability of the undamped system is a function of (1) longitudinal location of the CG given by $a$ and $b$; (2) vertical location of the CG relative to ground and the wheel axles given by $h$, $h_f$ and $h_r$; (3) the radial stiffness of the axles $k_f$ and $k_r$, which are also dependent on tire loads and inflation pressure; and (4) traction conditions given by $\mu_{r0}$ and $\rho$.

The utility of Eq. (2.58) in predicting the stability of a haul truck is limited. First, it is not as simple or intuitive as the Hop Function because requires more information, namely vertical location of the CG and traction terms. Because of this, an operating point calculation must still be carried out to determine the value of $\mu_{r0}$, which appears in $k_{21}$. It also assumes that traction conditions of the tire/haul road combination can be adequately predicted by the chosen model. As noted previously, mathematical models to describe the traction characteristics of haul truck tires on mine haul roads are not currently available.
Second, both of the above analytical stability criteria Eq. (2.57) and Eq. (2.58) do not consider the effects of damping. The presence of damping can drastically change the characteristics of a system, especially in this case where the non-symmetric structure of the EOMs lends to the possibility of negative net damping. Therefore, the full quadratic eigenvalue problem (QEP) presented by Eq. (2.44), including all damping terms, needs to be examined. Standard computer algorithms for carrying out numerical eigenvalue analysis are particularly useful in this case.

It is also important to remember that the 3-DOF model is derived without considering the effects of the hydropneumatic struts and trailing arm/solid rear axle geometry that make up a typical haul truck’s suspension system. The 3-DOF will clearly fail to capture some dynamical features, but it serves as a starting point for understanding the nature of the dynamic instability phenomenon seen in mining haul trucks.
Chapter 3

Numerical Stability Analysis

Chapter 2 showed that the non-symmetric structure of the EOMs Eq. (2.44) leads to the possibility of dynamic instability. The question is whether the 3-DOF model can predict instability for the specific configuration of parameters that describe a RWD mining haul truck operating on firm surface, which can have upward, downward, or zero slope. Given that there is a degree of uncertainty in many of the model parameters, whether due to a lack of available data or inherent variability with loading conditions, a MATLAB® program was written to carry out stability analysis of the 3-DOF model as a sensitivity study.

3.1 Code Structure

A flowchart illustrating the structure of the MATLAB® program is shown in Figure 3.1. The program follows the same general sequence of calculations described in Chapter 2. First, all model parameters are initialized, which includes general constants, nominal parameters corresponding to the specific vehicle and tires under consideration, the variable parameters for the sensitivity study, and a stability margin. A stability margin is used to classify eigenvalues that are very near the unstable region of the complex plane as being “potentially unstable”. This will be explained in more detail shortly. For each variable parameter, the range of values and number of increments are defined and then the total number of possible parameter combinations, or test cases, is calculated.
Once the inputs have been defined, nested for-loops are employed to perform operating point and stability calculations. Parameters are incremented one-at-a-time until every combination has been tested. The physical plausibility of each case is determined via the limiting factors as described in Section 2.1.5. Any case that meets the limiting factors is passed on to the stability calculator where the mass, stiffness, and damping
matrices of Eq. (2.44) are populated with the operating point values. The system’s eigenvalues are computed numerically using standard QEP routines. Because of the structure of the EOMs, the QEP will always yield three pairs of eigenvalues, two of which will be complex conjugate pairs. The null third column in the stiffness matrix $[K]$ means that the third eigenvalue pair will always be non-oscillatory (having zero-valued imaginary parts). This pair always corresponds to the fore-aft $x$-DOF, which is not constrained by any spring elements.

The system is then classified as stable, unstable, or potentially unstable depending on where the eigenvalues lie in the complex plane. Only one pair of eigenvalues needs to cross into the right half of the complex plane (have positive real parts) in order for the entire system to be unstable. The classification of “potentially unstable” is reserved for roots that are mathematically stable (lying in the left half of the complex plane), but fall within a predetermined stability margin. The purpose of the stability margin is to account for the inherent uncertainty in the assumptions of the 3-DOF model. It is well understood that the model may predict certain cases to be stable when in the real world they may not be, or vice versa. The stability margin also establishes a threshold for cases that might be “worrisome” in the real world. In the present work, the stability margin is defined as follows:

**Stability Margin**

The three linearized EOMs given by Eq. (2.44) will yield three pairs of roots (eigenvalues) corresponding to the three DOFs. If all the roots for a given system configuration have real parts less than or equal to zero, the system is stable in a mathematical sense. However, if any one pair of such roots is both non-zero and yields a damping ratio less than or equal to some predefined value $\zeta_{SM} \ll 1$, then the system can be considered nearly or potentially unstable. The predefined damping ratio $\zeta_{SM}$ is called the **stability margin**.

Figure 3.2 shows an arbitrary eigenvalue $s$ with negative real part $\sigma$ and imaginary part $j\omega$ in the complex plane. The damping ratio can be calculated as the cosine of the acute angle $\theta$ that the root makes with the real axis. Therefore,
Figure 3.2. Diagram of how damping ratio is computed from the location of an eigenvalue in the complex plane.

\[
\zeta = \cos \theta = \frac{\text{Re}(s)}{\sqrt{[\text{Re}(s)]^2 + [\text{Im}(s)]^2}} = \frac{\sigma}{\sqrt{\sigma^2 + \omega^2}} \tag{3.1}
\]

and the criteria for any eigenvalue \( s \) lying in the left half of the complex plane that makes the system potentially unstable is

\[
|\zeta| \leq \zeta_{SM} \ll 1 \tag{3.2}
\]

It is noted that stability margin can be defined in a few different ways. One alternative method to identify potentially unstable cases is to look at only the real parts of the eigenvalues. For example, an eigenvalue \( s \) with a negative real part \( \sigma \) could be considered potentially unstable if \( \sigma \) is greater than some very small negative number.

Following the determination of stability, the eigenvalues and parameters of the unstable and potentially unstable cases are saved for further analysis. This includes plotting a root locus to identify vibration frequencies and migration paths of the eigenvalues. Also plotted are histograms that show the frequencies with which unstable and potential unstable cases occur for specific values of the variable parameters.
3.2 Simulation Results: 250 Ton Truck

3.2.1 Model Parameters

The 3-DOF model program was tested using parameter data corresponding to a representative haul truck with a 250 ton (227 metric ton) payload capacity. Two loading scenarios were simulated separately:

- **Scenario 1**: Operation at the truck’s empty vehicle weight (EVW), i.e., carrying no payload.

- **Scenario 2**: Operation at the truck’s target gross vehicle weight (GVW), i.e., hauling 100% of its rated payload capacity.

The nominal vehicle masses, power, wheelbase, and tire geometries used are listed in Table 3.1. These parameters are constants that were common to both scenarios with the exception of the masses, where $m_{EVW}$ was used for Scenario 1 and $m_{GVW}$ was used for Scenario 2.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{EVW}$</td>
<td>Vehicle mass when empty (EVW)</td>
<td>153,000 kg</td>
</tr>
<tr>
<td>$m_{GVW}$</td>
<td>Vehicle mass at 100% payload (GVW)</td>
<td>380,000 kg</td>
</tr>
<tr>
<td>$P_{max}$</td>
<td>Maximum power available at the axle</td>
<td>1,864 kW</td>
</tr>
<tr>
<td>$L$</td>
<td>Wheelbase</td>
<td>5,900 mm</td>
</tr>
<tr>
<td>$b_i$</td>
<td>Unloaded tire section width</td>
<td>1,117 mm</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Unloaded tire overall diameter</td>
<td>3,562 mm</td>
</tr>
<tr>
<td>$r_i$</td>
<td>Tire rolling radius</td>
<td>1,687 mm</td>
</tr>
<tr>
<td>$n_f$</td>
<td>Number of tires on the front axle</td>
<td>2</td>
</tr>
<tr>
<td>$n_r$</td>
<td>Number of tires on the rear axle</td>
<td>4</td>
</tr>
</tbody>
</table>
Table 3.2 shows the variable parameters used in the 250 ton truck simulations. Again, wherever the subscript “EVW” appears denotes parameters values used in Scenario 1 and the subscript “GVW” denotes parameters values used in Scenario 2. For each variable parameter, a range of likely values is defined. Because the same tires are used on the front and rear axles, $k_{tf}$ and $k_{tr}$ take on the same range of values, as do $c_{tf}$ and $c_{tr}$. Though the Zoz and Brixius (1979) traction models specifies a nominal rolling resistance coefficient of $\rho = 0.02$, it is treated as a variable parameter here to cover the range rolling resistances discussed in Section 2.1.2 (pg. 16). Obviously, the rotational inertia of the truck and longitudinal location of the CG change with load conditions, so different ranges of $I_{yy}$ and $a$ were used for Scenario 1 and Scenario 2. Directly below the values of $a_{EVW}$ and $a_{GVW}$ in parentheses are the corresponding static weight distributions between the front and rear axles, which are expressed as % front/% rear.

Table 3.2. Variable model parameters for simulation of a 250 ton payload haul truck.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Min. Value</th>
<th>Max. Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{tf}$</td>
<td>Front tire stiffness</td>
<td>1,000 kN/m</td>
<td>6,000 kN/m</td>
</tr>
<tr>
<td>$k_{tr}$</td>
<td>Rear tire stiffness</td>
<td>1,000 kN/m</td>
<td>6,000 kN/m</td>
</tr>
<tr>
<td>$c_{tf}$</td>
<td>Front tire damping coefficient</td>
<td>5 kN.s/m</td>
<td>55 kN.s/m</td>
</tr>
<tr>
<td>$c_{tr}$</td>
<td>Rear tire damping coefficient</td>
<td>5 kN.s/m</td>
<td>55 kN.s/m</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Rolling resistance coefficient</td>
<td>0.01 (1%)</td>
<td>0.05 (5%)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Road grade</td>
<td>-16% (-9.1°)</td>
<td>16% (9.1°)</td>
</tr>
<tr>
<td>$I_{yy,EVW}$</td>
<td>Pitch moment of inertia at EVW</td>
<td>5×10⁵ kg.m²</td>
<td>3×10⁶ kg.m²</td>
</tr>
<tr>
<td>$I_{yy,GVW}$</td>
<td>Pitch moment of inertia at GVW</td>
<td>2×10⁶ kg.m²</td>
<td>8×10⁶ kg.m²</td>
</tr>
<tr>
<td>$a_{EVW}$</td>
<td>Longitudinal distance of CG behind front axle at EVW</td>
<td>2,950 mm</td>
<td>3,186 mm</td>
</tr>
<tr>
<td></td>
<td>(50/50)</td>
<td></td>
<td>(46/54)</td>
</tr>
<tr>
<td>$a_{GVW}$</td>
<td>Longitudinal distance of CG behind front axle at GVW</td>
<td>3,540 mm</td>
<td>4,130 mm</td>
</tr>
<tr>
<td></td>
<td>(40/60)</td>
<td></td>
<td>(30/70)</td>
</tr>
<tr>
<td>$h$</td>
<td>Normal distance of CG from road surface</td>
<td>2,531 mm</td>
<td>5,905 mm</td>
</tr>
<tr>
<td>$V_0$</td>
<td>Initial forward travel speed</td>
<td>8 km/h</td>
<td>56.3 km/h</td>
</tr>
<tr>
<td></td>
<td>(5 mph)</td>
<td></td>
<td>(35 mph)</td>
</tr>
</tbody>
</table>
As can be seen from Table 3.2, there are ten variable parameters for each scenario. Each variable parameter was given five linearly-spaced values from “min” to “max” to be simulated except for road grade $\beta$, which was taken at 2% increments for a total of seventeen possible values from -16% to +16%. Therefore, the total number of cases simulated in each scenario was $17 \times 5^9 = 33,203,125$.

### 3.2.2 Results Overview

A breakdown of the simulation results for the EVW and GVW scenarios is given in Table 3.3.

**Table 3.3.** Overview of simulation results for a 250 ton payload haul truck at empty vehicle weight and gross vehicle weight (100% payload capacity).

<table>
<thead>
<tr>
<th>Row</th>
<th>EVW</th>
<th>GVW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Number of possible cases</td>
<td>33,203,125</td>
</tr>
<tr>
<td>2</td>
<td>Stability margin enforced</td>
<td>$\zeta_{SM} = 0.05$ (5%)</td>
</tr>
<tr>
<td>3</td>
<td>Number of stable cases: $</td>
<td>\zeta</td>
</tr>
<tr>
<td>4</td>
<td>Number of cases that exceeded max. power limit</td>
<td>5,390,625</td>
</tr>
<tr>
<td>5</td>
<td>Number of cases that exceeded max. traction limit</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>Number of cases that resulted in zero/negative front axle load</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>Number of cases that resulted in zero/negative rear axle load</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>Number of potentially unstable cases</td>
<td>7,043,913</td>
</tr>
<tr>
<td>9</td>
<td>Number of unstable cases</td>
<td>178</td>
</tr>
</tbody>
</table>

Row 1 shows the total number of cases simulated. Row 2 shows the stability margin enforced. In both scenarios, a stability margin of 5% damping was chosen; any cases that yielded negative-real-part roots all with damping ratios greater than 5% were considered stable. The number of cases that are considered stable are shown in Row 3. Rows 4
through 7 show the number of cases that failed to meet the limiting factors checks described in Section 2.1.5. The fact that zero cases exceeded maximum traction limits or resulted in zero axle loads shows that the variable parameters ranges are reasonable. A significant number of cases exceeded maximum power limits because of the broad ranges of road grade and travel speed chosen. Obviously, a 250 ton truck would not have sufficient power to climb a 16% grade at a speed of 35 mph, so any such similar cases were discarded by the program. Finally, rows 8 and 9 show the number of potentially unstable (falling within the stability margin) and unstable cases (positive real parts) that were saved for further analysis.

A few inferences can be made from Table 3.3 about the 3-DOF model. First, the 3-DOF does in fact predict dynamic instability. While actual instances of instability were rare for these specific simulations, with only 178 and 9,086 unstable cases in the EVW and GVW scenarios, respectively, it is nonetheless encouraging that the model was able to predict dynamic instability. Second, the 3-DOF model predicts that there are a significant number of operating configurations where the truck has very little positive net damping and exists in a near-unstable state. Over 7 million (21%) of the EVW cases and almost 12.6 million (38%) of the GVW cases were found to be potentially unstable. Again, enforcing a stability margin can help identify cases that are “worrisome”—cases where instability might actually be possible in the real world, either due to other dynamical features of the truck not captured by the 3-DOF model, or other forcing functions that might help “nudge” the truck over the edge into the unstable region. Severe defects in haul roads or geometric out-of-roundness of the wheels or tires would certainly not help the stability situation. Third, the 3-DOF model suggests that a loaded truck is more likely than an empty truck to become unstable. This implies that, in general, the heavier the truck the more likely it is to experience instability.

The analytical stability criteria $D$, Eq. (2.57), and $\bar{H}$, Eq. (2.58), were also computed for every case simulated. Recall that $D$ and $\bar{H}$ fall out of the closed-form eigenvalue equation of the undamped system, Eq. (2.56). It was noted that presence of damping can drastically change the characteristics of a system, especially in this case where $[C]$ is non-symmetric and lends to the possibility of negative net damping. For this reason, it was unclear how well $D$ and $\bar{H}$ would correlate to the numerical QEP results.
found from the simulations. For the unstable cases in the EVW scenario, \( \hat{H} \) predicted instability (\( \hat{H} < 0 \)) in every single one of the unstable cases while \( D \) predicted instability (\( D < 0 \)) only 63.5% of the time. \( \hat{H} \), being a sufficient but not necessary condition for stability, is more conservative than \( D \), so the fact that \( \hat{H} < 0 \) more often is not surprising. For the GVW scenario, \( D \) predicted instability for only 50.5% of the unstable stable cases while \( \hat{H} \) again proved to be much more reliable, predicting instability for all but 2 of the 9,806 unstable cases. A quick inference from these results is that \( \hat{H} \) may be a good predictor of instability insofar as it correlates well with the numerical stability results. However, the fact that \( D \) did not show good agreement with the numerical results proves that further work may be needed.

### 3.2.3 Root Locus Plots

The root loci corresponding to the potentially unstable and unstable cases are plotted in Figure 3.3. Figures 3.3(a) and 3.3(c) focus on the region of the complex plane where potentially unstable roots exist; the entire range of eigenvalues is not shown. Figure 3.3(a) shows the EVW scenario root locus and Figure 3.3(c) shows the GVW scenario root locus. The 5% damping stability margin is indicated by symmetric, dashed lines extending radially from the origins to the (x,y) coordinates (-1, 20) and (-1, -20). All eigenvalues in the left half of the complex plane that lie to the right of the constant 5% damping line are potentially unstable. Another steeper line of constant 2.5% damping is drawn for reference. This shows that there would still be a significant number of potentially unstable cases even if a stricter stability margin is enforced. In general, the EVW scenario yielded eigenvalues with larger bounce and pitch vibration frequencies than the GVW scenario, which makes sense because natural vibration frequencies are inversely proportional to the square root of mass (bounce) or rotational inertia (pitch).
Figure 3.3. Root locus plots. (a) 250 ton haul truck when empty and (b) a close-up view of the associated unstable cases. (c) 250 ton haul truck when loaded to 100% payload capacity and (d) a close-up view of the associated unstable cases. Potentially unstable cases are shown in blue and unstable cases are shown in red. The dashed lines extending radially from the origins in (a) and (c) are lines of constant damping at 5% and 2.5%.
Figure 3.3(b) magnifies the unstable region for the EVW scenario and Figure 3.3(d) magnifies the unstable region for the GVW scenario. As can be seen, there are bands of frequency where the eigenvalue group together and cross into the right half of the complex plane. These bands are primarily the result of the different combinations of front and rear tire stiffnesses tested from case to case. Unstable vibration frequencies in the EVW scenario range from approximately 1.8 Hz to 2.5 Hz and unstable frequencies in the GVW range from approximately 0.95 Hz to 1.55 Hz. The very small magnitude of the growth rates indicate that the unstable frequencies are very close to the truck’s natural frequencies—within 99.9%. These frequencies correlate well with haul truck vibration frequencies reported in the literature. Hugo et al. (2008) determined experimentally that the bounce and pitching frequencies of an unladen Komatsu 730E electric drive truck (200 ton payload) are near 1 Hz and 2 Hz, respectively. Kobayashi et al. (2016) state that “it is generally known that the pitching vibrational frequency of a heavy electric vehicle such a dump truck is 1 to 3 Hz,” where this pitching frequency is proportional to load. That the 3-DOF model predicts dynamic instability with frequencies in this range is reflects an appropriate choice of parameters.

3.2.4 Dependency of Instability on Model Parameters

To gain a better understanding of how the variable parameters affect haul truck stability, histograms were plotted to show the frequency with which unstable and potentially unstable cases occurred for given values of the variable parameters. The histograms juxtapose the unstable cases and the potentially unstable cases because their dependencies on the variable parameter values are not guaranteed to be the same. Also, because the number of potentially unstable cases was far greater than the number of unstable cases in both the EVW and GVW scenarios, the frequency distributions have all been normalized so that heights of the bars add up to one.
**Road Grade**

Figure 3.4 shows the occurrences of unstable and potentially unstable cases for the tested range of road grades. Here, we see that the 3-DOF model predicts instability not only on steep uphill inclines, but on steep downhill inclines as well. This type of dynamic instability while traveling on downhill grades has not been reported in previous power hop studies.

![Figure 3.4](image)

(a) 250 Ton Truck, Empty

(b) 250 Ton Truck, 100% Payload

**Figure 3.4.** Frequency distribution of unstable cases (green bars) and potential unstable cases (blue bars) for (a) the empty truck and (b) the loaded truck versus road grade, $\beta$.

Power hop instability normally occurs when a vehicle exerts tractive effort in the presence of a draft load. Wiley and Turner (2008) note that power hop has been observed on a 4WD tractor climbing a steep hill on loose soil without pulling any implement, where the “downslope component of the tractor weight was enough of a draft load to induce instability.” Whether it is the downslope component of a vehicle’s weight or a towed implement, the idea is that the draft load acts in the opposite direction of the tractive forces and thus the direction of forward motion. However, when a vehicle travels downhill, the downslope weight component acts in the direction of forward motion.

One possible explanation for the model’s prediction about instability on downhill grades is a reversal in the roles of the vehicle’s weight and longitudinal tire forces. A retarding force will be needed to maintain constant speed if the road grade is great enough such that downslope component of the vehicle’s weight overcomes the rolling
resistance forces. In this case, opposing longitudinal forces are still present, but with the roles of the vehicle’s weight and longitudinal tire forces reversed. The model appears to predict that instability occurs at certain levels of retarding force on downhill grades, where such a retarding force occurs in place of the tractive force in order to oppose the downslope component of the vehicle’s weight—a mirror image of the uphill case with the same end result.

When observing the other parameters that led to these unstable cases where the road grade is steeper than -12% in the EVW scenario, or steeper than -8% in the GVW scenario, the forward travel speed is also typically very high—in the high 20 mph and 30 mph range. This suggests that a combination of steep downhill grades and high speeds are conducive to dynamic instability. Regardless of whether this matches real world observations, this is a hazardous situation that is avoided in surface mining operations for obvious safety reasons. The immense inertia of these several hundred ton vehicles combined with limited retarding and braking capabilities means that downhill grades must be traversed at slower speeds, especially when loaded. However, this does not mean definitively that dynamic instability cannot occur while traveling down typical grades at safe operating speeds. It is important to note that potentially unstable cases appear over the entire range of road grades for both the empty and loaded truck.

Another important detail to point out is the appearance that instability is more likely when traveling downhill rather than uphill, especially in the GVW scenario. This is due to the vehicle power limitations imposed on the model. As noted previously, a 250 ton truck does not have the power to be able to climb a 16% incline at 35 mph, empty or loaded. On the other hand, it is technically possible for the truck to careen down a 16% incline at 35 mph. For this reason, there were fewer physical possible cases on positive grades overall.

From the above observations, it is clear that some refinement is needed in how one chooses model parameters or in how the MATLAB® program determines realistic cases. Also, it must be reiterated that the model’s prediction about power hop instability occurring on downhill grades is just that—a prediction. Obviously, work must still be done to experimentally to judge the validity of model predictions.
**Front and Rear Axle/Tire Stiffnesses**

Figure 3.5 shows a histogram of unstable and potentially unstable cases as they depend on total front axle stiffness. Figure 3.6 shows a histogram of unstable and potentially unstable cases as they depend on total rear axle stiffness.

**Figure 3.5.** Frequency distribution of unstable cases (green bars) and potential unstable cases (blue bars) for (a) the empty truck and (b) the loaded truck versus front tire stiffness. Total front axle stiffness is shown ($k_f = 2k_{tf}$).

**Figure 3.6.** Frequency distribution of unstable cases (green bars) and potential unstable cases (blue bars) for (a) the empty truck and (b) the loaded truck versus rear tire stiffness. Total rear axle stiffness is shown ($k_r = 4k_{tr}$).
According to Figure 3.5, instability is more likely to occur for larger values of front tire stiffness. The potentially unstable cases also show a general trend of occurring more frequently with larger front axle stiffnesses. Practically, increased axle stiffness would correspond to increased tire inflation pressures and loads. Of particular concern might be when a truck is traveling downhill and/or if a payload is concentrated farther forward in the body of the truck than what is recommended by the manufacturer. A truck with two tires in the front and four tires in the rear is typically designed to achieve precisely or close to a 33/67 front-to-rear weight split when loaded so that weight is evenly distributed amongst all the tires. A truck operating with a more forward-biased weight distribution could be more likely to become unstable.

It is difficult to say whether there is a clear dependence on rear axle stiffness when observing Figure 3.6. However, if one looks at the unstable cases on an individual basis, a clear pattern emerges. Rear axle stiffness was greater than or equal to front axle stiffness in nearly all unstable cases where $\beta > 0$ (uphill travel). This matches Wiley and Turner’s observations that agricultural tractors operating on firm surfaces are more likely to become unstable when stiffness is higher in the rear than it is in the front. The fact that the typical haul truck has two more tires on the rear axle than on the front, all of which are usually inflated to the same pressure, means that the rear axle will be stiffer than the front axle in most circumstances. Even so, it is worth noting that the opposite pattern was observed for cases where $\beta < 0$ (downhill travel): instability occurred exclusively for cases when front axle stiffness was greater than rear axle stiffness.
**Front and Rear Tire Damping Coefficients**

Figure 3.7 shows a histogram of unstable and potentially unstable cases as they depend on total front axle damping. Figure 3.8 shows a histogram of unstable and potentially unstable cases as they depend on total rear axle damping.

**Figure 3.7.** Frequency distribution of unstable cases (green bars) and potential unstable cases (blue bars) for (a) the empty truck and (b) the loaded truck versus front tire damping. Total front axle damping is shown ($c_f = 2c_{tf}$).

**Figure 3.8.** Frequency distribution of unstable cases (green bars) and potential unstable cases (blue bars) for (a) the empty truck and (b) the loaded truck versus rear tire damping. Total rear axle damping is shown ($c_r = 4c_{tr}$).
Figures 3.7 and 3.8 show the expected result that instability is more likely to occur when damping in the system is low. In the EVW scenario, only the minimum value of front and rear tire damping (5 kN.s/m per tire) yielded unstable cases. In the GVW scenario, relatively few unstable cases occurred with tire damping values greater than the minimum value. The occurrence of potentially unstable cases also decreased with increasing tire damping coefficient values. It is a general rule that damping levels in off-road tires are kept low to minimize heat dissipation (Goering et al., 2003), so the fact that model predicts instability in the cases where tire damping is low seems reasonable.

**Rolling Resistance Coefficient**

From Figure 3.9, the 3-DOF model predicts rolling resistance does not play a critical role in contributing to or inhibiting instability, though trends suggest it instability might be less likely to occur on sections of road with very high rolling resistance (unmaintained loose earth, loose gravel, muddy rutted materials). The main take away is that the model shows instability is possible over the entire range of tested rolling resistance values in both empty and loaded conditions.

![Figure 3.9](image)

**Figure 3.9.** Frequency distribution of unstable cases (green bars) and potential unstable cases (blue bars) for (a) the empty truck and (b) the loaded truck versus rolling resistance coefficient, $\rho$. 

50
Pitch Moment of Inertia

Figure 3.10 shows a histogram of unstable and potentially unstable cases as they depend on pitch moment of inertia. The number of potentially unstable cases generally increased with moment of inertia in both the EVW and GVW scenarios. The unstable cases do not show any clear trends. Instead, there are spikes between $1 \times 10^6$ and $2 \times 10^6$ kg.m$^2$ in Figure 3.10(a) and at $3.5 \times 10^6$ kg.m$^2$ in Figure 3.10(b) where the system was most frequently unstable.

Figure 3.10. Frequency distribution of unstable cases (green bars) and potential unstable cases (blue bars) for (a) the empty truck and (b) the loaded truck versus pitch moment of inertia, $I_{yy}$. 
CG Longitudinal Location

Figure 3.11 shows a histogram of unstable and potentially unstable cases as they depend on the longitudinal location of the CG behind the front axle. In Figure 3.11(a) for the EVW scenario, the unstable and potentially unstable cases show opposing trends. The unstable cases indicate that instability is generally more likely the closer the CG is to the front axle. Slightly more potentially unstable cases occurred when the CG was shifted farther back, but the overall variation is small, so it cannot be said that there is any definite trend. The unstable cases in Figure 3.11(b) indicate that instability is more likely the closer the CG is to the rear axle, while the variation in potentially unstable cases is again very small.

![Frequency distribution of unstable cases (green bars) and potential unstable cases (blue bars) for (a) the empty truck and (b) the loaded truck versus CG fore-aft location given by the longitudinal distance of the CG behind the front axle, \( a \).](image)

Some comparisons can again be drawn with Wiley and Turner’s model and field experiments when looking at the unstable cases on an individual basis. A tractor pulling a draft load on a level surface becomes more stable as the CG is shifted rearward. Here, the same is observed when the haul truck travels up a positive road grade, where the vehicle weight serves as an effective, rearward-acting draft load; shifting the CG rearward increases stability. The exact opposite is observed when the haul truck travels down a negative road grade; the truck was less stable for downhill cases where the CG was
farther rearward. Referring back to Figure 3.4, the percentage of unstable EVW cases that occurred on positive road grades was greater than that of unstable GVW cases that occurred on positive road grades, which might account for opposing trends between Figures 3.11(a) and 3.11(b).

**CG Height from Road Surface**

Figure 3.11 shows a histogram of unstable and potentially unstable cases as they depend on height/normal distance of the CG from the road surface. Based on the unstable cases only, the model predicts that the likelihood of instability generally increases with CG height from the road surface. This result is somewhat expected, as increasing CG would also reduce any ground vehicle’s lateral/rollover stability. Potentially unstable cases do not show any definite trends, suggesting that instability may be possible over the entire range of CG heights tested.

![Figure 3.12](image)

**Figure 3.12.** Frequency distribution of unstable cases (green bars) and potential unstable cases (blue bars) for (a) the empty truck and (b) the loaded truck versus CG height from the road surface, \( h \).
Forward Travel Speed

Figure 3.12 shows a histogram of unstable and potentially unstable cases as they depend on forward travel speed. According to both Figure 3.12(a) and 3.12(b), a haul truck is more likely to be unstable at high speeds. However, there appears to be significant potential for instability at lower speeds as well, as the most potentially unstable cases occurred at 5 mph (8 km/h). A curious result of the EVW scenario is the gaps in the green bars that indicate the unstable cases. Instability did not occur at 5 mph or at 20 mph (32.2 km/h). This is because unstable EVW cases on positive road grades only occurred at 12.5 mph, while unstable cases on negative road grades only occurred at 27.5 mph (42.3 km/h) or greater (note that \( V_0 \) was taken at increments of 7.5 mph). These gaps in speed were filled in by the GVW scenario. This suggests that loading condition plays an important role in determining whether a haul truck is stable at certain speeds. Hypothetically speaking, based on these simulations, a 250 ton truck that is stable when traveling at 20 mph while empty may all of a sudden experience hopping after it has picked up a load.

![Graphs showing frequency distribution of unstable cases](image)

**Figure 3.13.** Frequency distribution of unstable cases (green bars) and potential unstable cases (blue bars) for (a) the empty truck and (b) the loaded truck versus initial forward travel speed, \( V_0 \).
CHAPTER 4

CONCLUSIONS AND FUTURE WORK

4.1 Conclusions

In this research, a 3-DOF mathematical model was formulated to predict the onset of a pitch/bounce type dynamic instability in mining haul trucks. The 3-DOF haul truck model builds upon a model originally developed to predict power hop instability in agricultural tractors. Unlike the previous model, the 3-DOF model in this research specifically addresses circumstances where the vehicle under consideration is rear-wheel drive; operates on firm, sloped road surfaces; and, perhaps most importantly, does not tow any implements behind it, which is the primary tractive condition that leads to power hop in other OHVs. Analytical stability criteria were derived through a closed-form eigenvalue analysis of the equations of motion while neglecting all damping terms.

A program was written to carry out simulations of the 3-DOF model. There is a degree of uncertainty in many of the model parameters, whether due to a lack of available data or inherent variability with loading conditions. Because of this, the program was written to perform numerical stability analysis of the 3-DOF model using a sensitivity study wherein tens of millions of different parameter combinations were simulated.

Simulation results show that the 3-DOF does lead to the prediction of dynamic instability in a representative haul truck with a payload capacity of 250 tons (227 metric tons). More than 33 million different parameter combinations were tested for two payload scenarios: empty (EVW) and 100% payload capacity (GVW). Only a small percentage of these cases exhibited actual instability. However, by utilizing a stability margin as a measure of “near” or “potential” instability for eigenvalues that lie very close to the unstable region of the complex plane, the results show that a haul truck can exist in numerous different operating point configurations that could potentially lead to dynamic
instability. Of the 33 million cases tested for the chosen ranges of parameters, 21% were deemed potentially unstable for the empty truck and 38% were deemed potentially unstable for the loaded truck. Potentially unstable cases existed over the full ranges of all the different parameters simulated.

The analytical stability criteria derived from the undamped equations of motion did not always correlate with the numerical eigenvalue analysis of the full set of equations of motion that included all damping terms. For both payload scenarios, the numerical results and the stability criterion $\hat{H}$ were in near 100% agreement in predicting whether a specific combination of parameters led to instability. The stability criterion $D$ was in agreement with the numerical results less than two-thirds of the time. The reason for this discrepancy is likely to be that inclusion/exclusion of damping drastically changes the characteristics of the system.

There was shown to be a heavy dependence of instability on road grade, tire stiffness and damping properties, the location of the center of gravity, and forward travel speed. The model mainly predicts instability on severe road grades and at high speeds. A result not seen in previous studies was the prediction of dynamic instability when a vehicle travels downhill. Normally, power hop instability occurs when an off-highway vehicle exerts tractive effort in the presence of a draft load that acts opposite to the direction of forward motion. For a haul truck, the only “draft load” present would be the component of its own weight acting down a slope, which would act in the direction of forward motion when traveling downhill and, therefore, no longer be a draft load. In this case, the draft load that induces instability may come in the form of retarding forces generated at the tires, which are needed to prevent the vehicle from accelerating down steep slopes. Future studies should investigate this prediction by the model to determine if it is supported by real world observations. Furthermore, while instability on uphill grades primarily occurred when the rear axle is stiffer than the front axle, the opposite was observed when the truck travels downhill. Increased tire damping was shown to reduce the likelihood of instability in both loading scenarios and on all grades. Like tire stiffness, longitudinal center of gravity location was also shown to play a role in determining the likelihood of instability on positive or negative road grades.
4.2 Future Work

The 3-DOF model was shown to be capable of predicting dynamic instability for a configuration of parameters that describe a representative haul truck. However, this model merely provides a starting point for gaining insight into the nature of the dynamic instability/power hop problem encountered on mining haul trucks. The primary question at this juncture is whether it can reliably predict instability for the exact circumstances under which it is actually observed on real haul trucks. Work needs to be done to refine and verify the theory with experimental observations. Proposed in this section are some possible avenues for model refinement.

4.2.1 Inclusion of Powertrain Dynamics

The 3-DOF models fail to consider how the diesel engine in a mechanical drive truck or the traction motors in an AC electric drive truck deliver torque to the drive axle. Because the engine or motor is the source of energy that creates the tractive forces that ultimately drive dynamic instability, additional DOFs relating to the rotational dynamics of the powertrain could be incorporated in future mathematical models. Wiley and Turner (2008) showed that the onset of power hop could be predicted adequately without the need for additional DOFs. However, this may or may not be the case for the current problem due to the very different configuration of mining haul trucks in terms of sheer size, power and speed capabilities, and the mechanics of their propulsion systems. It could be possible that dynamic instability in haul trucks is more directly related to power train dynamics and certainly deserves further investigation.

A preliminary approach to including powertrain dynamics into a mathematical model can be found in Chapter 14 of Goering et al. (2013), where EOMs are developed to describe the motions of both the drive wheels and chassis of an agricultural tractor. Incorporating powertrain dynamics requires additional DOFs relating to the rotational motion of the drive wheels and corresponding EOMs describing both external and internally developed forces acting on the drive wheels and the powertrain. It would be necessary to relate the drive wheel rotational speed to the rotational speed of the
engine/traction motor, which requires knowledge of power train speed ratios and efficiencies. This would also necessarily require theoretical or empirical motor torque-speed or power-speed curves to determine operating point forces and speeds.

4.2.2 Inclusion of Suspension Systems

In the 3-DOF model of the current work, a haul truck is represented as a single rigid body system supported by two sets of parallel spring and damper combinations that reflect tires characteristics only. This simplification allows for a relatively quick and easy analysis of dynamic instability, but is likely to omit some important dynamical features. Haul trucks incorporate independent front and rear suspension systems, where part of the rear suspension is a trailing arm that constrains the solid rear axle to rotational motion about a pivot point located on the vehicle chassis. Furthermore, the hydropneumatic struts used in haul truck suspension systems exhibit nonlinear stiffness and damping characteristics (Hugo et al., 2008). It is recommend that models be developed that account for the motions of the unsuspended mass of the front wheels, the unsuspended mass of the rear axle/trailing arm, and the suspended mass of the vehicle chassis/body. Such models will necessarily have additional DOFs and kinematic constraints. Models that include suspensions will also be much more complex because many more parameters will be needed to describe the geometry of the suspension, masses and rotational inertias of the independent moving bodies, and stiffness and damping characteristics of the hydropneumatic struts. Given this increased complexity, development and analysis of such a model would be greatly aided by the use of sophisticated multibody dynamics software.

4.2.3 Alternative Traction Models

The 3-DOF model of the present work uses a traction equation by developed by Zoz and Brixius (1979) for agricultural tires on concrete. Chapter 3 showed that using this traction model does lead to the prediction of dynamic instability in mining haul
trucks, but the extent of its suitability to this problem is a lingering question. Models that describe the traction characteristics of haul truck tires on mine haul roads were not able to be found for the current research. Other traction models could be used, but it is difficult to know whether one model is more suitable than another without any experimental data on the tractive performance of haul truck tires on mining haul roads. Still, it may be worthwhile to conduct investigations into the suitability of alternative traction models. Ideally, experiments should be performed to identify appropriate tractive coefficient and rolling resistance coefficient characteristics for haul truck tires on typical mine haul roads, much like what has been done extensively for agricultural tractors (see Brixius, 1987; Wismer and Luth, 1973; Wolf et al., 1996; and Zoz and Brixius 1979). It is noted that such full-scale experiments would be entire research efforts in and of themselves, so practicality is a concern. Scaled experiments may be more worthwhile.

4.2.4 Experimental Parameter Measurement

A lack of concrete data meant that certain parameters for the 3-DOF model had to be treated as variables with ranges of likely values (see Table 3.2). However, it is possible to conduct relatively simple experiments to directly measure or estimate characteristics of certain variable parameters. Doing so could help in assessing the capability of the 3-DOF model to accurately predict instability. Having concrete, well-defined parameter data could also reduce the number of variable parameters in the 3-DOF model or any higher-order model that includes powertrain dynamics and/or suspension system effects. This would thereby help reduce computation time or allow for more granularity of other parameters.

For example, Lines and Murphy (1991) and Nguyen et al. (2009) provide experimental procedures for measuring baseline damping coefficient values of agricultural tires, which could, in theory, be extended to haul truck tires. Having baseline values of tire damping coefficients for a specific haul truck tire would eliminate front tire damping and rear tire damping as variable parameters.

As detailed in Wiley and Tuner (2008), tire stiffness characteristics can be calculated from empirical static load vs. deflection data. Off-the-road (OTR) tires for
OHVs like tractors and earthmovers typically exhibit nonlinear load-deflection characteristics that depend on inflation pressure. Tire stiffness is taken as the slope of the tire’s static load vs. deflection relationship for a given inflation pressure. Therefore, individual tire loads found via the operating point calculations could be used to calculate static deflections of the front and rear tires (refer to section 2.1.4). These static deflections could then be used to calculate the front and rear tires stiffness (and thus the total front and rear axle stiffnesses) required by the EOMs. If tire manufacturer load-deflection data cannot be obtained, Hugo et al. (2008) outlines an experiment that can be performed to characterize haul truck tire load-deflection relationships in situ on an open-air weighbridge. This would be useful when applying the 3-DOF model or any higher-order models to a specific truck. Having tire stiffness as a function of axle loads would mean that they are dependent on specific operating point conditions. As a result, tire stiffness would be eliminated as a variable parameter.

Hugo et al. (2008) also outlines techniques for characterizing hydropneumatic struts used in haul truck suspensions as well as measuring truck accelerations in situ and then using power density spectrums to identify pitching frequency. Understanding the nonlinear stiffness and damping characteristics of hydropneumatic struts would be beneficial for any model that incorporates suspension effects. Measured pitching frequencies could be used to estimate pitch mass moment of inertia, $I_{yy}$, for specific trucks under specific loading conditions.
REFERENCES


APPENDIX

FULL PITCH EQUATION

Wiley and Turner (2008) notes that a common assumption when formulating EOMs for rotational DOFs is that rotations are small enough that moment arms do not vary with the rotational DOF, but that “this assumption has the effect of omitting some linear terms that should be included for a rigorous analysis.” Therefore, the pitch EOM is formulated considering the effects of varying moment arms for the normal forces $N_f$ and $N_r$ as shown in Eq. (2.31). As presented in Chapter 2, Section 2.2.1, Eq. (2.31) does not consider varying moment arms for the gross tractive force $F_{r0}$ and the rolling resistance forces $TF_{f0}$ and $TR_{r0}$. Here, we develop the pitch equation considering all varying moment arm terms.

Consider a haul truck with the operating point geometries and forces shown in Figure A.1. The normal force $N_{f0}$ acts at the center of the front wheels at a distance $a$ from the CG. The rolling resistance force $TF_{f0}$ acts at the center of the front wheels at a distance $h_f$ from the CG. The dimensions $a$ and $h_f$ can be considered to be two sides of a right triangle with hypotenuse $g_f$ representing the distance between the front axle center and the CG. The normal force $N_{r0}$ acts at the center of the rear wheels at a distance $b$ from the CG. The rolling resistance force $TF_{r0}$ acts at the center of the rear wheels at a distance $h_r$ from the CG. The dimensions $b$ and $h_r$ can be considered to be two sides of a right triangle with hypotenuse $g_r$ representing the distance between the rear axle center and the CG. The gross tractive force $F_{r0}$ acts at the contact patch of the rear tires at a distance $h$ from the CG.
From Figure A.1, we have

\[
\begin{align*}
a &= g_f \cos \theta_f \\
h_f &= g_f \sin \theta_f \\
b &= g_r \cos \theta_r \\
h_r &= g_r \sin \theta_r
\end{align*}
\]

and the pitch equilibrium equation about the vehicle’s CG is

\[
\sum M_{CGy} = 0 = N_{f0}a - N_{r0}b + F_{r0}h - TF_{r0}h_r - TF_{f0}h_f
\]

As the vehicle is perturbed from its operating point, it undergoes positive displacements in the \( \theta \) and \( z \) directions. CCW and downward displacements are positive in Figure A.1. The moment arms \( a, b, h_f, \) and \( h_r \) vary with pitching motion, \( \theta \), and the moment arm \( h \) varies with bouncing motion, \( z \). Under perturbed motion, Eq. (A.5) becomes
\[ I_{yy} \ddot{\theta} = N_f g_f \cos(\theta_f - \theta) - N_r g_r \cos(\theta_r + \theta) + F_r (h - z) - T F_f g_f \sin(\theta_f - \theta) - T F_r g_r \sin(\theta_r + \theta) \quad (A.6) \]

where the 0 subscripts have been dropped to indicate that the forces during perturbed motion are no longer equivalent to their operating point values. For small perturbations, the small angle approximation and angle sum and difference trigonometric identities can be used to obtain the following simplified, approximate pitch EOM:

\[ I_{yy} \ddot{\theta} \cong N_f (a + h_f \theta) - N_r (b - h_r \theta) + F_r (h - z) - T F_f (h_f - a \theta) - T F_r (h_r + b \theta) \quad (A.7) \]

Eq. (A.7) resembles Eq. (2.31), but now with additional terms that describe the varying moment arms of the gross tractive force, given by \( h - z \), and the rolling resistance forces, given by \( h_f - a \theta \) and \( h_r + b \theta \). Because the moment arms vary according to displacements (not velocity or acceleration), the only changes to the EOMS, Eq. (2.44), is the appearance of additional terms in the \( k_{21} \) and \( k_{22} \) positions of the stiffness matrix \([K]\), which correspond to coefficients of the \( z \) and \( \theta \) DOFs in Eq. (A.7), respectively. The full pitch equation can be derived using the force expressions described in Chapter 2, Section 2.2.3 and the linearization method described in Chapter 2, Section 2.3.3. It can be shown that the above considerations result in the following new stiffness matrix:

\[
[K] = \begin{bmatrix}
    k_f + k_r & -k_f a + k_r b & 0 \\
    -k_f a + k_r b & k_f a^2 + k_r b^2 & -\mu h_f k_r a \\
    +\rho h_f k_f & -\rho h_f k_r a & -(\mu_0 h - \rho h_r) k_r b \\
    -\mu_0 N_f & +\mu_0 N_r & -h_f + \rho a N_f_0 \\
    +\mu_0 N_r & +\mu_0 N_r & +h_r + \rho b N_r_0 \\
    \rho k_f - (\mu_0 - \rho) k_r & -\rho k_f a - (\mu_0 - \rho) k_r b & 0 
\end{bmatrix} \quad (A.8)
\]
Compared to $[K]$ in Eq. (2.44), the $k_{21}$ sees the additional term $\mu_{r_0}N_{r_0}$ and the $k_{22}$ sees the additional terms $-\rho a N_{f_0}$ and $\rho b N_{r_0}$. As explained in Chapter 2, Section 2.2.1, these additional stiffness terms do not change the overall structure of the EOMs and their effects on overall stability are minimal. It is noted, however, that $k_{12}$ appears in the analytical stability criteria $\tilde{H}$ given by Eq. (2.57). The addition of the $\mu_{r_0}N_{r_0}$ term to $k_{12}$ further complicates this criterion because now both $\mu_{r_0}$ and $N_{r_0}$ must be determined through operating point analysis. These additional terms may be included for a more rigorous analysis.