ANALYTICAL PROCESS MODELING AND NONLINEAR CONTROL OF MELT-POOL HEIGHT AND TEMPERATURE IN DIRECTED ENERGY DEPOSITION

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by

Jianyi Li

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The dissertation of Jianyi Li was reviewed and approved* by the following:

Qian Wang
Professor of Mechanical Engineering
Dissertation Advisor
Chair of Committee

Panagiotis Michaleris
Adjunct Professor of Mechanical Engineering

Bo Cheng
Assistant Professor of Mechanical Engineering

Edward W. (Ted) Reutzel
Adjunct Professor of Engineering Science & Mechanics Department

Karen Thole
Distinguished Professor of Mechanical Engineering
Department Head of Mechanical Engineering

* Signatures are on file in the Graduate School
Abstract

Additive manufacturing is a cutting-edge manufacturing technology which can be applied to many areas, for example bio-fabrication, automotive and aerospace. However, this high-potential manufacturing technology hasn’t been widely applied in industry due to several severe drawbacks, such as the poor accuracy of part geometry and inferior metallurgical properties, which can be addressed by utilizing feedback control system. This dissertation focuses on developing control-oriented model and model-based control system design for melt-pool height and temperature control.

In this dissertation, we first construct a reduced-order physics-based nonlinear model of the melt-pool height and temperature for 1-D case of directed energy deposition process, in which only single straight track is deposited on the substrate by LENS system. One main contribution of our proposed model compared with existing literature lies in a novel parameterization of the entire material transfer rate as a function of the major process parameter in manufacturing process. This novel parameterization is derived from the perspective of physics and can improve the accuracy of our model prediction of the steady-state melt-pool geometry. Then system identification are conducted to calibrate the unknown parameters in our 1-D model using experimental data and FEA prediction collected from deposition of Ti-6AL-4V and Inconel 718, followed by validation of 1-D model with calibrated parameters. A LQR controller and nonlinear MPC controller are designed based on this model to control the melt-pool height and temperature by regulating the laser power and scanning speed applied in the manufacturing process.

The dynamics of additive manufacturing process varies inter-bead or inter-layer, which motivates
the modification of 1-D model to multi-dimensional one. The 1-D model can be extended to multi-dimensional case by incorporating the thermal history in the existing part, which can be characterized approximately by Rosenthal’s solution. Our extended multi-dimensional model is then validated by experimental data obtained from three multi-dimensional examples: thin-wall structure, parallel patch build and L-shape structure. Next a feedback linearization controller is developed based on this multi-dimensional model to track the melt-pool height reference trajectory using laser power.
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Chapter 1  Introduction

The Directed Energy Deposition (DED) process is a kind of additive manufacturing (AM) technology where metal parts are fabricated based on a three-dimensional computer-aided design (CAD) model with various material, for example Inconel 718 and Ti64. One of the most distinct features of AM is that it builds metal parts in a layer-by-layer fashion, where metal material is deposited either on the substrate or the previous layer. This is essentially different from traditional manufacturing technology where parts are fabricated by subtraction of material. AM begins with constructing a 3-D CAD model either by computer software or by scanning of an existing artifact, which is then sliced into a bunch of layers for defining the deposition path. The number of layers depends on the process characteristics. Then the path of power source, usually laser or electron beam in the DED process, can be planned within these layers. Next, a melt-pool is formed during the fabrication of one layer by heating up the substrate and adding metal in wire or powder form. The molten material solidifies after the heating source is moved away, and each layer is formed gradually as the heating source moves along the prescribed path within the layer. This process will be repeated for many layers to manufacture parts with more complicated geometry. At last, post-processing may be required, which usually includes removal of the support structure and polish of the part surface in DED process. AM technology can be applied to many areas, for example fabrication of personal product, bio-fabrication, automotive, electronics, aerospace and even food industry. One unique application of DED technology compared with other AM technologies is to repair worn metal parts.
1.1 History of AM Technology

The appearance of early AM technology can be traced back to 1980s. In 1981, two AM manufacturing techniques to produce a three-dimensional plastic model with a photo-hardening polymer were developed by Hideo Kodama of Nagoya Municipal Industrial Research Institute [1]. Then Chuck Hull of 3D Systems Corporation developed a prototype system to add layers by curing photopolymers with ultraviolet light lasers, which is similar to Kodama’s idea [1]. The file format STL (StereoLithography), which was also invented by Chuck Hull, is widely used in rapid prototyping, 3D printing and computer-aided manufacturing (CAM) [2]. AM processes with metal started to appear in the 1980s and 1990s, which impacted the traditional manufacturing fashion of removing metal from raw material. By the mid 1990s, new AM technologies were developed at Stanford and Carnegie Mello University, for example micro-casting [1]. Nowadays, AM has already been commercialized and various types of AM equipment can be purchased in the market, for example the Optomec® LENS MR-7 System. In addition, the terms 3D printing and additive manufacturing have become more and more interchangeable, although the former is usually used to refer to polymer deposition and the latter is usually for metal deposition.

1.2 Categories of AM Technology

There are multiple ways to categorize the AM technology based on different criterions. AM techniques were classified into three categories by Kruth: liquid-based, powder-based and solid-based system with respect to the form of material used in deposition [3]. In addition, other ways of classification of AM technology has been created by Helsinki University of Technology & German
production process standard and Christopher B. Williams from Virginia Polytechnic Institute and State University [3]. In 2013, ASTM international (American Society for Testing and Materials) proposed an innovative classification of AM technology into seven categories, which has been widely accepted by the public: (1) Material Extrusion, e.g. Fused Deposition Modeling (FDM) and Contour Crafting, (2) Powder Bed Fusion, e.g. Selective Laser Sintering (SLS), Direct Metal Laser Sintering (DMLS), Selective Laser Melting (SLM) and Electron Beam Melting (EBM), (3) Vat Photopolymerization, e.g. Stereolithography (SLA), (4) Material Jetting, e.g. Polyjet/Inkjet Printing, (5) Binder Jetting, e.g. Indirect Inkjet Printing (Binder 3DP), (6) Sheet Lamination, e.g. Laminated Object Manufacturing (LOM) and (7) Directed Energy Deposition, e.g. Laser Engineered Net Shaping (LENS) and Electronic Beam Welding(EBW) [3]. Different materials and power sources are involved in these seven categories. Specifically, directed energy deposition technology is explored in my research, where the “ink” for printing is molten and re-solidified metal powder and the power source is laser beam.

1.3 Development of AM in Academia and Industry

Additive manufacturing has gained significant attention and investment in both academia and industrial application due to its ability to manufacture parts with complicated geometry while saving material. From the perspective of academia, the challenges and promising directions of AM lie in four areas [3]: (1) geometrical description for AM, (2) material development for AM process, (3) development of computational instruments & interfaces, (4) fabricating instruments & processes development, for example computer simulation of AM process. In this proposal, an effort has been
made within area (4), specifically for AM process model development and feedback control design. From the perspective of industrial application, as we mentioned before, AM has gained broad usage in industries such as automotive, aerospace, engineering, medicine, biological systems, and food supply chains in recent years. Many big companies consider AM as a significant revolution in industry due to its essential difference with traditional manufacturing. For example, GE is using AM technology to build parts and products which can be cost prohibitive using traditional manufacturing technology. A model of the GEnx jet engine was produced by GE engineers with direct metal laser melting technique [4]. However, the spread of AM in industrial application has been restricted by several issues, including precision of the part geometry, mechanical and material properties of the fabricated part, and surface quality, etc [5].

### 1.4 Literature Review for Modeling and Controller Design for AM Processes

To solve these issues, advanced sensing, modeling and control technology are highly desired for the AM process. Continuing effort has been made to develop models of the AM process, including physical, computational and empirical models. Numerous FEA models have been developed and validated by experimental measurement [6], [7], [8], [9], [10]. They are found helpful in the study of microstructure, material properties, residual stress, and distortion of the final part [6]. However, FEA models are inappropriate for control system design, since the involved partial differential equations are too complex for real-time control. Thus reduced-order lumped-parameter models of AM process are highly desired for control system design. Several control-oriented models can be discovered from
existing literature. In Devesse’s work [11], a high-order physical model for laser cladding is presented. This model is derived from the heat conduction equation in the workpiece and formulated in state space form with the states equal to a set of isotherm positions and input equal to laser power. Then the model can be used to regulate the melt pool width in the various initial thermal conditions by adjusting the power of the laser beam, provided a one dimensional mesh of temperature measurements over time as feedback. In Jun’s work [12], a controlled autoregressive moving average model is applied to simulate the GMAW-LAM (layer additive manufacturing using gas metal arc welding) process, in which the input is the scanning speed of deposition and the output is the nozzle to top surface distance (NTSD). A recursive least squares algorithm is implemented to learn unknown parameters of the model from input-output data generated by random scanning speed within 4-8 mm/s. A strong linear relationship has been observed between the melt pool width and power of laser beam in Miyagi’s work [13]. Then this empirically linear model is deployed to regulate the melt pool width by changing the laser power. A knowledge-based (semi-empirical) model consisting of a linear dynamic and a nonlinear memoryless block is constructed in Fathi’s work [14] and [15], with the scanning speed as the control input and the melt pool height as the output. The unknown parameters in the model are estimated offline with experimental data by recursive least-square (RLS) method. In Song’s paper [16] and [17], an empirical state-space model is developed for relating the laser power to the melt pool temperature. The matrices in the state-space model are identified experimentally using the randomly modulated laser power and the corresponding melt pool temperature by subspace method. In Kruth’s paper [18], a first-order transfer function between the melt pool area and laser power is used, where the steady-state gain and time constant are determined from step response. In Heralić’s work [19], the process is described through two first-order transfer function based on the
assumption that the melt pool width depends only on the laser power and the melt pool height depends only on the material feed rate. The unknown parameters in this empirical model are investigated through a set of step response experiments. In Tang’s paper [20], a first-order transfer function is proposed with three inputs laser power, scanning speed, powder flow rate and single output melt pool temperature. A set of experiments with step input are conducted to estimate the parameters in the transfer function using least-squares method. In Doumanidis’ work [21], the MIMO system is approximated by first-order transfer function matrix. The scanning speed and the powder flow rate are selected as the control input and the melt pool width and height are chosen as the output. The unknown parameters are identified off-line with the experimental data generated by step inputs. An important lumped-parameter model was derived by Doumanidis and Kwak in [22]. A dynamic model relating the three inputs laser power, scanning speed, powder flow rate to the outputs melt pool width, height, length and temperature are constructed based on governing equations about mass, momentum and energy balance. The model from [22] is also frequently adopted and modified for controller design in [23], [24], [25], [26], [27], [28], [29] by Sammons, Tang, Cao and Landers. While most of the control-oriented models in existing literature ignore the dependency of system dynamics on other dimension, such as height dimension, some take this into account. In [23], a height-dependent model for laser metal deposition process is constructed. The height-dependence of this model is achieved through the solidification rate term in the mass balance equation. This solidification rate is correlated with the temperature gradient along scanning direction in the solid region evaluated at the phase-change interface, and this temperature gradient is a function of the part height. In Sammon’s work [30], [31] and [32], the author presents a process control oriented model in repetitive form for laser metal deposition process, which includes both in-layer and layer-to-layer dynamics.
There are limited studies on the design and implementation of closed-loop controls for laser-based AM processes. Most of the controllers in the literature are classical controllers, e.g. PID controller. In [13], [18], [21], [22] by Miyagi, Kruth and Doumanidis, either PID controller or PI controller is implemented on the plant for tracking melt pool geometry or temperature with certain reference. In [11] by Devesse, a linear state feedback controller designed using pole placement and combined PI control law is adopted for regulating the melt pool size by modulating the laser power. A PID controller combined with feedforward controller is developed to adjust the melt pool height with scanning speed in [14] by Fathi. In Fathi’s work [15], a sliding mode controller (SMC) plus conventional PID controller is designed to regulating the melt pool height with scanning speed. In [16] and [17] by Song, the melt pool height is controlled by a classic rule-based controller and the melt pool temperature is regulated by a generalized predictive controller (GPC). In [19] by Heralić, a PI controller is developed to control the melt pool width and feedforward compensator is used to regulate the melt pool height. A general tracking controller using the internal model principle is implemented to do online temperature control of the melt pool in [20] by Tang. In [28] by Landers, a feedback linearization controller is adopted in the closed-loop system, where the melt pool width and temperature are regulated by material feed rate and laser power, respectively. In the multi-layer case, iterative learning control (ILC) strategy, which is widely applied in repeated process, is used for either melt pool geometry or temperature tracking in [12], [25], [26] by Jun and Tang. A repetitive process MPC controller was designed in [31] by Sammons for melt-pool height control by using the spatial powder flow rate as the control input. In [33] and [34] by Jun, a single-neuron self-learning controller was employed to control the width of a wall structure using the scanning speed as the control input, which was designed based on a nonlinear Hammerstein model.
1.5 Thesis Outline

The remainder of this proposal is organized as follows. A reduced-order physics-based MIMO model of one-dimensional DED process is constructed in chapter 2. The contribution of this reduced-order analytical model lies in a novel parameterization of the material transfer rate in the deposition as a function of the process operating parameter. In chapter 3, a LQR controller and a MPC controller are designed using the model proposed in chapter 2 to regulate the melt pool height, or the melt pool height and temperature at the same time, by adjusting the laser power and scanning speed during the deposition process. In chapter 4, the one-dimensional model of DED process proposed in chapter 2 is extended to multi-dimensional case by incorporating the thermal history of the process part. Based on the extended multi-dimensional model in chapter 4, then feedback linearization controller is designed for melt-pool height tracking in multi-dimensional case of DED process in chapter 5. Chapter 6 briefly concludes the achievements in this thesis and discusses potential directions of future research.
Chapter 2  Analytical Model of Melt-Pool Geometry and Temperature in 1-D Directed Energy Deposition Process

2.1 Introduction

As we mentioned in chapter 1, although continuing effort has been made to develop multiple types of AM model and the performance of these AM models has been validated by experimental measurement, they are considered inappropriate for control system design. In this chapter, our objective is to develop a lumped-parameter physics-based reduced-order model, which is severely lacking in existing literature and suitable for control system design. Empirical models are not preferred here because their applicability to dynamic process other than their training situation is limited by its statistical nature, while physics-based model can provide a better understanding and insight of the nature of the process [22].

The remainder of this chapter is organized as follows. In section 2.2, the classic lumped-parameter physics-based model proposed by Doumanidis and Kwak in [22] is introduced. This model describes the material and thermal transfer in DED process with three (mass, momentum and energy) balance equations. Next, our model is created in section 2.3 following Doumanidis and Kwak’s model. The contribution of our model primarily lies in a novel parameterization of the entire material transfer rate during deposition as a function of the process operating parameter such as laser power and scanning speed in the mass balance equation. Such parameterization improves the accuracy of steady-state melt pool geometry prediction compared with other existing lumped-parameter analytical models [5]. This parameterization in section 2.3 is inspired by Eagar and Tsai’s work in [35]. In section 2.4, our
proposed model in section 2.3 is calibrated and validated by a set of experiments of conducted on LENS system using material Ti-6AL-4V and Inconel 718. Our model prediction of melt-pool geometry is validated through width and height measurement, while our model prediction of melt-pool temperature is validated by predictions from FEA method. A brief summary of this chapter is given in section 2.5.

2.2 A Lumped-Parameter Analytical Model of DED Process Derived by Doumanidis and Kwak

DED is a complex process involving both material and thermal transfer. Among multiple process parameters in DED process, three of them, i.e. material transfer rate, laser power and the scanning speed, are considered as major process parameters. As we mentioned before, a simplified model is more desirable than FEA model for the purpose of control, due to its computational efficiency. In this thesis, our work follows the lumped-parameter physics-based model proposed by Doumanidis and Kwak in [22], which consists of mass, momentum, and energy balance equations. Doumanidis and Kwak’s model is constructed based on an assumption of the geometrical configuration of the melt-pool in DED process, which is stated in assumption 2.2.1. This assumption is also adopted in our model since our model is derived based on Doumanidis and Kwak’s.
Assumption 2.2.1  In DED process, the melt-pool geometry can be approximated by a three-dimensional ellipsoid segmented into two halves by the substrate surface. These two halves are assumed to be symmetric to the substrate surface, although the bottom half is observed to have a larger geometry than the top half in experiments. Since the bottom half of the melt-pool can be considered as pure melting of the substrate having no direct material exchange with the adding material and uniform temperature equal to the melting temperature, the object characterized in the model is the top half of the melt-pool, which is the shaded geometry in Fig. 2.2.1. Three variables can be used here to describe the geometrical information of this half 3-D ellipsoid: melt pool width $w$ (measured on the substrate surface and perpendicular to the velocity direction), melt pool height $h$ (measured on the maximum cross section and vertical to the substrate surface), and melt pool length $l$ (measured on the substrate surface along the velocity direction) [22].

Based on assumption 2.2.1, now we are ready to introduce Doumanidis and Kwak’s model in [22].
The first equation in their model is called mass balance equation, which can be written as

\[
\frac{d(\rho V(t))}{dt} = -\rho A(t)v(t) + \mu_m f(t),
\]

(2.2.1)

where \( \rho \) is the material density \( (kg/m^3) \), \( V(t) \) is the volume of the melt pool \( (m^3) \), \( A(t) \) is the cross-sectional area formed by \( w \) and \( h \) \( (m^2) \), \( v(t) \) is the travelling speed of the substrate in the direction of deposition \( (m/s) \), \( f(t) \) is the powder flow rate/material transfer rate \( (kg/s) \), \( \mu_m \) is the material catchment efficiency. Because of assumption 2.2.1, the volume and cross-sectional area of the melt-pool, which is approximated by a half 3D ellipsoid, can be written as

\[
V(t) = \frac{\pi}{6} w(t)h(t)l(t),
\]

(2.2.2)

\[
A(t) = \frac{\pi}{4} w(t)h(t),
\]

(2.2.3)

where \( w(t), h(t) \) and \( l(t) \) are respectively, the width, height and length of the melt pool. The physical meaning of (2.2.1) is quite straight forward: the first term denotes time rate of melt-pool mass, the second term denotes the loss rate of melt-pool mass due to solidification, while the last term denotes the adding material transfer rate.

The second equation in Doumanidis and Kwak’s model is a momentum balance equation,

\[
\frac{d(\rho V(t)v(t))}{dt} = -\rho A(t)v(t)[-v(t)] + \alpha w(t),
\]

(2.2.4)

where the parameter \( \alpha \) is given by

\[
\alpha = [1 - \cos(\theta)][\gamma_{GL} - \gamma_{SL}],
\]

(2.2.5)

where \( \theta \) is the wetting angle \( (rad) \), \( \gamma_{GL} \) is the gas to liquid surface tension coefficient \( (N/m) \), and \( \gamma_{SL} \) is the solid to liquid surface tension coefficient \( (N/m) \). \( \alpha \) is negative since \( \gamma_{GL} < \gamma_{SL} \).

The third equation in the model is the energy balance equation,
\[
\frac{d(\rho V(t)e)}{dt} = -\rho A(t)\nu(t)c_i(T_m - T_0) + \eta Q(t) - A_s\alpha_s(T(t) - T_m) \\
- A_g \times [\alpha_g(T(t) - T_0) + \varepsilon\sigma(T^4(t) - T_0^4)]
\]

(2.2.6)

where \( e \) denotes the specific internal energy of the melt-pool per unit mass with respect to the ambient temperature \( T_0(\text{K}) \),

\[
e = c_i(T_m - T_0) + h_{sl} + c_i(T(t) - T_m)
\]

(2.2.7)

\( T(t) \) is the average melt pool temperature (K), \( c_i \) is the solid material specific heat (J/(kg K)), \( T_m \) is the melting temperature (K), \( h_{sl} \) is the specific latent heat of fusion-solidification (J/kg), \( c_i \) is the molten material specific heat (J/(kg K)), \( \eta \) is the laser transfer efficiency, \( Q(t) \) is the laser power (W), \( \alpha_s \) is the convection coefficient (W/(m² K)), \( \alpha_g \) is the heat transfer coefficient (W/(m² K)), \( \varepsilon \) is the surface emissivity, and \( \sigma \) is the Stefan-Boltzmann constant (W/(m² K⁴)). \( A_s \) and \( A_g \) denote the areas of melt-pool interface to the substrate and to the free surface, respectively. Based on assumption 2.2.1, we have

\[
A_s = \frac{\pi}{4} w(t)l(t)
\]

(2.2.8)

\[
A_g = \frac{\pi}{\sqrt{2}} \left[ w(t)h(t)l(t) \right]^{2/3}
\]

(2.2.9)

It should be clarified that (2.2.9) is an approximated solution of half of the surface area of a 3D ellipsoid, which is adopted by Doumanidis and Kwak for the simplicity of their model.

The physical meaning of the energy balance equation (2.2.6) is quite straightforward as well. The first term denotes the rate of melt-pool internal energy, the second term denotes the loss rate of melt-pool internal energy due to solidification, the third term is the laser power absorbed by the melt-pool, the forth term denotes the heat convection rate from the melt-pool to the substrate, and the last term denotes the convection and radiation rate from the melt-pool to the free surface.
The last equation involved in the model is

\[ \frac{l}{X} = 1 + \frac{1}{4} \left( \frac{w}{X} \right)^2 \]  \hspace{1cm} (2.2.10)

where

\[ X = \max \left[ \frac{w}{2}, \frac{\eta Q}{2\pi k(T - T_0)} \right] \]  \hspace{1cm} (2.2.11)

(2.2.10) and (2.2.11) provide a thermal relationship between melt-pool length and width.

The model proposed by Doumanidis and Kwak is in simple form (ordinary differential equation and algebraic equation) and has been shown to capture the dynamic of DED process quite well in [22] and [25]. However, there still exist several issues that need to be fixed in this model:

1. The material catchment efficiency \( \mu_m \) in the mass balance equation (2.2.1) is assumed to be constant during the entire deposition process in [22] and [25]. However, this coefficient \( \mu_m \) is time-varying and dependent on the real-time working condition. For example, if the laser power is not high enough to melt the metal powder, then the material catchment efficiency \( \mu_m \) is equal to zero, which equivalently says that the metal powder distributed by the powder nozzle is not captured at all. Besides this, if the laser power applied is increased, the melt-pool geometry will increase correspondingly, which will cause more metal powder captured by the melt-pool. Thus it’s unreasonable to assume the material catchment efficiency \( \mu_m \) to be a constant independent of the working condition during the entire manufacturing process.

2. The steady-state melt-pool area can be solved from the mass balance equation (2.2.1) by setting
the left hand side of the equation to be 0.

\[ A_{ss} = \frac{\mu_s f}{\rho v} \]  

(2.2.12)

In addition, the steady-state melt-pool width can be solved from the momentum balance equation (2.2.4) by setting the left hand side of the equation to be 0,

\[ w_{ss} = -\frac{\mu_s f v}{\alpha} \]  

(2.2.13)

The steady-state melt-pool height can be solved by combining (2.2.12) and (2.2.13),

\[ h_{ss} = -\frac{4\alpha}{\rho \pi v^2} \]  

(2.2.14)

It can be seen from (2.2.12)-(2.2.14) that the steady-state melt-pool geometry \( w_{ss}, h_{ss} \) and \( A_{ss} \) are independent of the laser power \( Q \) applied, which conflicts our observation from the experiment.

3. The material properties gas to liquid surface tension coefficient \( \gamma_{GL} \) (the surface tension coefficient on the interface between air and melt pool) and solid to liquid surface tension coefficient \( \gamma_{SL} \) (the surface tension coefficient on the interface between substrate/previous layer and melt pool) in (2.2.5) are difficult to be measured or looked up in other reference. The values of \( \gamma_{GL} \) and \( \gamma_{SL} \) of H13 tool steel used by simulations in Ref. [25] are actually estimated by fitting the experimental data of melt-pool height.

Our model of melt-pool geometry and temperature, which will be introduced in section 2.3, will follow Doumanidis and Kwak’s work. However, some modifications have to occur to the model.
proposed by Doumanidis and Kwak to resolve these issues analyzed above. The modification to the mass balance equation (2.2.1), which is inspired by Eagar and Tsai’s work in Ref. [35], will be focused on in section 2.3, since it is one of the main contributions of this thesis on the modeling part.

2.3 Reduced-Order Multivariable Modeling of DED Process

As we stated in section 2.2, several issues exist in the Doumanidis and Kwak’s model, such as the assumption of constant material catchment efficiency independency of steady-state melt-pool geometry on the laser power applied and the difficulty of measuring some coefficients in the model. In this section, our model of melt-pool geometry and temperature will be constructed following Doumanidis and Kwak’s idea with modifications. One thing should be noted about our model is that assumption 2.2.1 is also adopted in our model.

(1) Improved mass balance equation

The first equation in our model is called improved mass balance equation, which is a modified version of the original mass balance equation (2.2.1). To solve the issue caused by assumed constant material catchment efficiency \( \mu_m \) in (2.2.1), a novel parameterization of the entire material transfer rate \( \mu_m f \) as a function of the process parameters is proposed in this subsection, instead of estimating or iteratively learning the material catchment efficiency \( \mu_m \) in the deposition process. This is equivalently to say that a function \( F \) is to be sought such that \( \mu_m f = F(Q,v) \). This novel parameterization of \( \mu_m f \) is inspired by Eagar and Tsai’s work in Ref. [35], which will be introduced first. Generally speaking, Ref. [35] provides a relationship between the operating parameter (such as laser power and
scanning speed) and the steady-state melt pool area, which can be utilized in the parameterization of the material transfer rate $\mu_{m,f}$ as a function of the process parameters.

The work in Ref. [35] is an extension and application of the classical Rosenthal’s solution (2.3.1), which is an analytical solution of the temperature field generated by heat source travelling on plate.

\[ T - T_{\text{initial}} = \frac{Q}{2\pi kR} e^{-\frac{(v+R)^2}{2a}}, \quad (2.3.1) \]

where $T$ is the temperature of point of interest (K), $T_{\text{initial}}$ is the initial temperature of the point of interest (K), $Q$ and $v$ denote the power and moving speed of heat source respectively, $k$ is the thermal conductivity (W/m·K), $a$ denotes the thermal diffusivity (m$^2$/s), $w$ denotes the distance from the point of interest to heat source along the moving direction (m), $R$ is the distance from point of interest to heat source (m). More details of definition of $w$ and $R$ can be found in Ref. [35].

**Remark 2.3.1:** The notation of initial temperature of point of interest is changed from $T_0$ in Ref. [35] to $T_{\text{initial}}$ in (2.3.1), since $T_0$ has been used to denote the ambient temperature in the energy balance equation (2.2.6).

The Rosenthal’s solution is solved from the heat conduction equation, which is a partial derivative equation, with several assumptions for simplifying. The simplifying assumptions include the ideal point form of the heat source, absence of convective or radiative heat transfer, invariance of material properties with respect to temperature and a quasi-steady state semi-infinite plate. It should be emphasized that work in Ref. [35] adopted all these simplifying assumptions except the ideal point form of the heat source.

In Ref. [35], the point heat source assumed in Rosenthal’s solution is extended to a Gaussian
distributed heat source. The extended scenario of Gaussian distributed heat source in Ref. [35] is illustrated by Fig. 2.3.1 below.

Here we note that $\sigma$ in Fig. 2.3.1, which is an important distribution parameter of Gaussian heat source, denotes the standard deviation of the Gaussian distribution. In other words, $\sigma$ determines the shape of Gaussian heat source, e.g. $\sigma = 0$ for a point heat source and $\sigma$ gets larger for a “fatter” distributed heat source.

The motivation of the extension from ideal point heat source to Gaussian distributed heat source is backed up by Nestor and Schoeck’s work in Ref. [36], [37]. The heat distributions of arcs on water-cooled copperanodes were measured by Nestor an Schoeck in [36], [37]. Although the measured
distributions are not exact Gaussians, an equivalent (in terms of total heat input) Gaussian heat source can be constructed by a least square regression of their data to fit a Gaussian distribution, which supports the extension mentioned in Ref. [35].

The solution of temperature field generated by Gaussian distributed heat source is provided in Ref. [35] by equation (2.3.2). Details of obtaining solution (2.3.2) by solving heat conduction equation together with Gaussian distributed heat source can be found in Ref. [35]. Additionally, the solution (2.3.2) has been transformed in a dimensionless form, i.e. the variables in (2.3.2) have no units.

\[
\theta = \frac{n}{\sqrt{2\pi}} \int_0^{vT} \frac{e^{-\frac{x^2}{2n^2}}}{\sqrt{2\pi}} dx = \frac{\eta Q v}{4\pi a^2 \rho c_i (T_m - T_{\text{initial}})}
\]

where \( \theta = \frac{T - T_{\text{initial}}}{T_m - T_{\text{initial}}} \) is the dimensionless temperature, \( n = \frac{\eta Q v}{4\pi a^2 \rho c_i (T_m - T_{\text{initial}})} \) denotes the operating parameter associated with laser power \( Q \) and scanning speed \( v \), which plays a critical role in our analysis. \( u = \frac{v\sigma}{2a} \) is the dimensionless distribution parameter of Gaussian distributed heat source,

\[
\xi = \frac{v\xi}{2a}, \psi = \frac{v\psi}{2a}, \zeta = \frac{v\zeta}{2a}
\]

are dimensionless form of \( w, y \) and \( z \), respectively. More details of parameters in (2.3.2) can be found in Ref. [35].

**Remark 2.3.2:** The definition of the dimensionless temperature \( \theta \) and the operating parameter \( n \) in Ref. [35] is \( \theta = \frac{T - T_{\text{initial}}}{T_c - T_{\text{initial}}} \) and \( n = \frac{\eta Q v}{4\pi a^2 \rho c_i (T_c - T_{\text{initial}})} \) respectively, where \( T_c \) denotes the critical temperature. Since we are interested in the geometry of the melt-pool, which is characterized by its boundary and associated with the melting temperature, we select the critical temperature to be the melting temperature, i.e. \( T_c = T_m \).
An important application of (2.3.2) is that the dimensionless boundary of the melt-pool can be characterized by setting $\theta = \frac{T - T_{\text{initial}}}{T_m - T_{\text{initial}}} = 1$. Further the dimensionless melt pool width, height and length can be solved separately in a numerical way by setting the coordinates on the other two directions equal to 0. It’s straightforward to tell from (2.3.2) that the dimensionless melt-pool width, height and length are all dependent on the operating parameter $n$ and dimensionless distribution parameter $u$. Thus the dimensionless melt-pool area can be formulated as a function of operating parameter $n$ and dimensionless distribution parameter $u$, which is shown in Fig. 2.3.2 [35]. More details of obtaining the result in Fig. 2.3.2 by numerical integration can be found in Ref. [35].

![Graph showing dimensionless steady-state melt pool area as a function of parameters $n$ and $u$.][1]

Fig. 2.3.2 Dimensionless steady-state melt pool area as a function of parameters $n$ and $u$ [35]

In Fig. 2.3.2, the X-axis denotes the operating parameter $n$ defined in (2.3.2) while Y-axis denotes the dimensionless melt-pool area,

$$A_d = \frac{v^2}{4\alpha^2} A_{ss}$$  

(2.3.3)
One thing should be noted in (2.3.3) is that $A_d$ is associated with the melt-pool area $A_{ss}$ in steady-state form, which is backed by the assumption of quasi steady-state.

For simplicity, assumption 2.3.1 is introduced below.

**Assumption 2.3.1:** The dimensionless distribution parameter $u = \frac{v\sigma}{2a}$ is a constant during deposition.

Now we are ready to introduce our novel parameterization of the entire material transfer rate $\mu_m f$ as a function of the process parameters, utilizing Eagar and Tsai’s achievement in [35].

Based on Fig. 2.3.2 and assumption 2.3.1, the dimensionless melt pool area can be written as a function of operating parameter $n$, i.e.

$$A_d = \Gamma(n) = \Gamma\left(\frac{\eta Q_v}{4\pi a^2 \rho c_i (T_m - T_{initial})}\right),$$

(2.3.4)

where the dimensionless melt pool area $A_d$ is defined as (2.3.3).

Remark: The function $\Gamma(\cdot)$ in (2.3.4) could be different forms for different types of material [5]. For example, it can be approximated by a piecewise-linear function as Fig. 2.3.2, i.e.

$$\frac{v^2}{4a^2} A_{ss} = A_d = \Gamma(n) = \Gamma\left(\frac{\eta Q_v}{4\pi a^2 \rho c_i (T_m - T_{initial})}\right) = \begin{cases} 0 & (Q \leq Q^c) \\ \beta \frac{\eta(Q - Q^c)v}{4\pi a^2 \rho c_i (T_m - T_{initial})} & (Q > Q^c) \end{cases}$$

(2.3.5)

$$\Rightarrow A_{ss} = \begin{cases} 0 & (Q \leq Q^c) \\ \beta \frac{\eta(Q - Q^c)}{4\pi c_i (T_m - T_{initial})v} & (Q > Q^c) \end{cases}$$

where $Q^c$ is the critical laser power that metal powder starts to get melted by the laser beam ($Q^c$ corresponds to the crossover value of $n$ where $A_d$ starts to become nonzero in Fig. 2.3.2), $\beta$ is the slope of the linear approximation. The piece-wise linear approximation (2.3.5) is also supported by our
experimental data collected from Ti-6Al-4V data. One thing needs to clarify is that both $Q^c$ and $\beta$ are unknown parameters and can be calibrated by experimental data. In the remainder of this subsection, we use this piecewise-linear approximation as example to introduce our novel parameterization of $\mu_m f$.

Consider the steady-state form of the mass balance equation (2.2.1) by setting the left hand side 0, we can obtain

$$\mu_m f = \rho A_{ss} v,$$  \hspace{1cm} (2.3.6)

which implies us the possibility of characterizing $\mu_m f$ through the steady-state melt-pool area $A_{ss}$.

By plugging (2.3.5) into (2.3.6), then the entire material transfer rate $\mu_m f$ can be rewritten as

$$\mu_m f = \rho A_{ss} v = \begin{cases} 0 & (Q \leq Q^c) \\ \beta \frac{\eta(Q - Q^c)}{\pi c_i (T_m - T_{\text{initial}})} & (Q > Q^c) \end{cases}$$  \hspace{1cm} (2.3.7)

It can be seen clearly in (2.3.7) that the entire material transfer rate $\mu_m f$ has already been parameterized as a function of the process parameters, i.e. $\mu_m f = F(Q, v)$ has been construction in (2.3.7).

Then we put (2.3.7) back into the right hand side of the original mass balance equation (2.2.1). As a result, our improved mass balance equation takes the following form.

$$\frac{d(\rho V(t))}{dt} + \rho A(t) v(t) = \begin{cases} 0 & (Q \leq Q^c) \\ \beta \frac{\eta(Q - Q^c)}{\pi c_i (T_m - T_{\text{initial}})} & (Q > Q^c) \end{cases}$$  \hspace{1cm} (2.3.8)

The melt-pool volume $V(t)$ and cross-sectional area are defined in (2.2.2) and (2.2.3), respectively.

The powder catchment efficiency $\mu_m$ is eliminated in (2.3.8). In addition, the dependency of the melt-pool geometry on the process parameters can be seen easily in (2.3.8) as well. This is the main
contribution of our model for 1-D directed energy deposition process.

(2) Relationships between melt-pool width, height and length

The improved mass balance equation (2.3.8) describes the dynamics of melt-pool area and volume depending on the process parameters. However, (2.3.8) lacks of information about how the melt-pool area and volume are distributed on three dimensions: melt-pool width, height and length.

It is not uncommon to assume a fixed ratio between the melt-pool width and height in existing literature [5],

\[ r = \frac{w}{h} \]  \hspace{1cm} (2.3.9)

It needs to be clarified here this fixed ratio \( r \) is also unknown and can be calibrated from experimental data.

Apart from (2.3.9), we assume an approximated equality between melt-pool width and length obtained by simplifying (2.2.10)-(2.2.11). The detail of this simplification can be found in Ref. [25].

\[ l(t) = w(t), \]  \hspace{1cm} (2.3.10)

So far it’s complete to characterize the melt-pool geometrical information by a combination of assumption 2.2.1 and (2.3.8)-(2.3.10).

(3) Energy balance equation

The last equation in our model is the energy balance equation (2.2.6), which is adopted from Doumanids and Kwak’s model in [22] without any modification. The energy balance equation is used to describe the dynamics of the melt-pool average temperature.
To summarize, our model follows Doumanidis and Kwak’s model with modification to resolve issues analyzed in section 2.2. The main contribution in our 1-D model lies in a novel parameterization of the entire material transfer rate as a function of the process parameter in (2.3.7). Our model is derived based on assumption 2.2.1, and consists of (2.2.2), (2.2.3), (2.2.6) and (2.3.8)-(2.3.10).

Our model can be transformed into state-space form easily by simple algebra, which is included in the appendix. The state-space model is written as (2.3.11) and (2.3.12):

\[
\dot{h} = \frac{\beta \eta (Q - Q^c)}{\pi c_i [T_m - T_{\text{initial}}]} - \rho \frac{\pi}{4} r h^2 \nu \frac{\pi}{2} \rho r^2 h^2,
\]

\[
\dot{T} = \left\{ \left( \rho \frac{\pi}{4} r h^2 \nu - \frac{\beta \eta (Q - Q^c)}{\pi c_i [T_m - T_{\text{initial}}]} \right) \left[ c_i (T_m - T_0) + h_{sl} + c_i (T - T_m) \right] \right. \\
- \rho \frac{\pi}{4} r h^2 c_i (T_m - T_0) + \eta Q - \rho \frac{\pi}{4} r h^2 \alpha_i (T - T_m) - \frac{\pi}{2} \left( r^2 h^3 \right)^{3/2} \left. + \alpha_i (T - T_0) + \varepsilon \sigma (T^4 - T_0^4) \right\} / \left( \frac{\pi}{6} r^2 h^3 \rho c_i \right)
\]

The state vector is selected as \( x(t) = [h(t) \quad T(t)]^T \) and the inputs are \( u(t) = [Q(t) \quad v(t)]^T \). Three unknown parameters \( Q^c, \beta, r \) are supposed to be calibrated from experimental data. The melt-pool width and height are characterized with respect to melt-pool height by (2.3.9) and (2.3.10). It should be noted that the state-space model (2.3.11)-(2.3.12) is a nonlinear one and the order of the model is reduced.

In next subsection, a set of experiments are conducted with Ti-6Al-4V and Inconel 718 to calibrate the unknown parameters and validate the reduced-order model (2.3.11)-(2.3.12).
2.4 Model Calibration and Validation

In this subsection, the model proposed in section 2.3 will be calibrated and validated by a set of experiments conducted on laser engineered netting shaping (LENS) system using different materials. Fig. 2.4.1 is a picture of the Optomec® LENS MR-7 machine operated by CIMP 3D at Applied Research Laboratory (ARL) of Pennsylvania State University.

![Optomec® LENS MR-7 Laser-Based DDM System](image)

Fig. 2.4.1 Optomec® LENS MR-7 Laser-Based DDM System, picture courtesy of ARL at The Pennsylvania State University
Fig. 2.4.2 shows a schematic plot of the LENS system used in the experiments for model calibration and validation. The general procedure of DED process described in Fig. 2.4.2 has been introduced briefly at the beginning of chapter 1. The system will be operated in manually-programmed mode, which allows direct interfacing with its motion controller. This operating mode is appropriate for communication with external data collection devices as well as dynamic control of processing parameters [5].

In our model proposed in section 2.3, the major process parameter laser power and scanning speed are selected as the inputs, thus the experiment is designed to investigate the system response with respect to these two variables. In each run of the experiment, a single track is deposited on a substrate by the LENS system, which is a typical situation of 1-D directed energy deposition process. The
Experiment setup is separated into two subcases:

Case 1: Time-varying laser power is applied following a multiple step-input while the scanning speed is fixed.

Case 2: Time-varying scanning speed is applied following a multiple step-input while the laser power is fixed.

The length of a deposited track is around 13 cm and the substrate is at ambient temperature for both cases. The multi-step input trajectory is evenly split into seven intervals. The reason for selecting step-input is that the system response can reach a steady-state correspondingly if the system is well-behaved. In addition, the step response of certain types of system has been explored a lot by researchers, which can provide us some insight on the behavior of our model. As we mentioned earlier, the width and height information of the deposited tracks are collected by processing the 3D spatial optical profilometry data.

For the two subcases of experiment setup, case 1 (varying power fixed speed case) is utilized to calibrate the unknown parameters \((Q, \beta, r)\) in our model. Then our model with calibrated parameters is validated through the experimental data collected from case 2. The procedure and methodology of calibration of unknown parameters \((Q, \beta, r)\) using measurements from case 1 is explained below:

Step 1: Calibrate the width to height ratio \(r = \frac{w}{h}\) using measurements of melt pool width and height collected from case 1,
\[ r = \frac{1}{N} \sum_{i=1}^{N} \frac{w_i^m}{h_i^m}, \]  

(2.4.1)

where \( N \) is the number of measurements, \( i \) is the index of measurement, \( w_i^m \) and \( h_i^m \) are the measured melt pool width and height with same index.

Step 2: The values of \( Q^c \) and \( \beta \) can be calibrated by solving the optimization problem (2.4.2).

Basically, the optimization problem (2.4.2) minimizes the summed square error between our model prediction and measurement of steady-state melt-pool area.

\[ \min_{Q^c, \beta} \sum_{i=1}^{7} (A_i^m - \hat{A}_i^{ss})^2, \]  

(2.4.2)

where \( i \) denotes the index of interval of multi-step input trajectory \((i = 1, \ldots, 7)\), \( A_i^m \) is the mean value of melt-pool area in interval \( i \), which can be calculated based on measurements of melt-pool width and height as (2.4.3).

\[ A_i^m = \frac{1}{N_i} \sum_{j} \left( \frac{\pi}{4} w_i^j h_i^j \right) \]  

(2.4.3)

where \( N_i \) is the number of measurement in interval \( i \), \( j \) is the index of melt-pool width and height measurement within interval \( i \).

\( \hat{A}_i^{ss} \) in (2.4.2) is our model prediction of steady-state melt-pool area, which can be solved by setting \( \frac{d(\rho V(t))}{dt} = 0 \) for \( Q > Q^c \) case of (2.3.8).

\[ \hat{A}_i^{ss} = \beta \frac{\eta(Q_i - Q^c)}{\rho \pi c_i (T_m - T_{\text{inumal}}) v_i}, \]  

(2.4.4)

where \( Q_i \) and \( v_i \) denote the laser power and scanning speed applied in interval \( i \), respectively (\( Q_i \) is a constant in case 2 while \( v_i \) is a constant in case 1).

For better validation of our 1-D model, two materials Ti-6AL-4V and Inconel 718 are utilized in the
deposition of single tracks on LENS system. The experimental work of this thesis was supported by Office of Naval Research and these two materials possess some properties desired for military application, for example their resistance of corrosion and capability of serving in extreme environment.

(1) Parameter calibration and model validation with Ti-6AL-4V experiment

For Ti-6AL-4V experiment, the varying laser power applied in case 1 follows Fig. 2.4.4 with fixed scanning speed 25 ipm, while the varying scanning speed applied in case 2 follows Fig. 2.4.5 with fixed laser power 450 W. The powder flow rate is 3 g/min for both cases. The material properties of Ti-6Al-4V used are listed in Table 2.4.1. The calibrated parameters $Q'$, $\beta$, $r$ are listed in Table 2.4.2, which are identified by the methodology introduced in (2.4.1)-(2.4.4).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
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<td>Density ($kg/m^3$)</td>
<td>$\rho$</td>
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<td>Solid material specific heat (J/(kg K))</td>
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<td>Initial temperature (K)</td>
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<td>Specific latent heat of fusion-solidification (J/kg)</td>
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<td>Molten material specific heat (J/(kg K))</td>
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</tr>
<tr>
<td>Thermal diffusivity ($m^2/s$)</td>
<td>$a$</td>
<td>2.4794e-6</td>
</tr>
<tr>
<td>Convection coefficient ($W/m^2K$) in case 1</td>
<td>$\alpha_s$</td>
<td>5.7e$^{-5}$-9.7e$^{-5}$</td>
</tr>
<tr>
<td>Convection coefficient ($W/m^2K$) in case 2</td>
<td>$\alpha_s$</td>
<td>4.13e$^{-5}$-5.8e$^{-5}$</td>
</tr>
<tr>
<td>Laser transmission efficiency</td>
<td>$\eta$</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table 2.4.2 Parameters calibrated using measurements from case 1 of Ti-6Al-4V

<table>
<thead>
<tr>
<th>$r = w/h$</th>
<th>$Q^c$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/0.1298</td>
<td>111.72W</td>
<td>0.30256</td>
</tr>
</tbody>
</table>

The curve fitting result of calibration of $Q^c, \beta, r$ using measurements collected from case 1 of Ti-6Al-4V is shown in Fig. 2.4.3. The measurement of melt-pool area (blue curve) in Fig 2.4.3 is calculated by $A'' = \frac{\pi}{4} w'' h''$, while the red curve (Our model prediction of steady-state melt-pool area with calibrated parameters in table 2.4.2) is calculated by (2.4.4). It should be noted that the red curve doesn’t provide any information of the transient behavior of our model, since (2.4.4) is a purely steady-state solution.
Fig. 2.4.3 Curve fitting results of optimization problem (2.4.2): Our model prediction of steady-state melt-pool area vs experimental data from case 1 of Ti-6AL-4V

Then we do simulation for case 1 of Ti-6AL-4V with state-space model (2.3.11) and (2.3.12), our model prediction of melt pool width and height is compared with experimental data. Since the melt pool temperature was not measured in the experiments, average melt-pool temperatures were computed using FEA method as the reference for comparison. The value of $\alpha_G$ in Table 2.4.1 is estimated in Ref. [6] and the values of $\alpha_s$ are estimated through the FEA heat flux computation. More details on the FEA estimation of average temperature and $\alpha_s$ can be found in Ref. [42]. The simulation result is shown in Fig. 2.4.4.
Fig. 2.4.4 Ti-6Al-4V: Our model prediction vs. experimental measurements of melt-pool geometry and FEA prediction of melt-pool temperature, subjected to a multiple step-input trajectory of laser power Q and a fixed scanning speed $v = 10.58\text{mm/s}$.

It is shown in Fig. 2.4.4 that our model prediction of melt pool width matches the experimental data quite well. And both the model prediction and experimental measurement have displayed clear step pattern, which is due to the step changes of laser power. However, the experimental data of melt pool height does not show this step pattern clearly. Especially, a “balling” effect can be observed from distance intervals $2 \times 10^4 - 4 \times 10^4 \mu m$ and $8 \times 10^4 - 10 \times 10^4 \mu m$. For LENS system, the “balling” effect is mainly caused by the variation of distribution of powder size, since metal powder in larger size is more difficult to be melted completely, which will result in a rougher surface. For the temperature profile, our model prediction of average melt-pool temperature follows the FEA
prediction of average melt pool temperature closely.

Next, our model with calibrated parameters listed in Table 2.4.2 is validated through experimental data collected from case 2 of Ti-6AL-4V, where the laser power is fixed at $Q = 450W$ and the scanning speed follows a step-input trajectory displayed at the bottom of Fig. 2.4.5. The reference of our model prediction of melt pool geometry and temperature are generated in similar way as Fig. 2.4.4. The simulation result is shown in Fig. 2.4.5.

![Fig. 2.4.5 Ti-6Al-4V: Our model prediction vs. experimental measurements of melt-pool geometry and FEA prediction of melt-pool temperature, subjected to a multiple step-input trajectory of laser scanning speed $v$ and a fixed laser power $Q = 450W$](image)

It is shown in Fig. 2.4.5 that our model prediction of melt pool width and height matches the experimental measurement quite well except for the first and last intervals. For the temperature profile,
our model prediction of average melt pool temperature follows the FEA prediction of average melt pool temperature closely.

(2) Parameter calibration and model validation with Inconel 718 experiment

To further validate our model, similar analysis will be conducted with Inconel 718 as (1). The varying laser power applied in case 1 follows Fig. 2.4.8 with fixed scanning speed 25\text{ipm}, while the varying scanning speed applied in case 2 follows Fig. 2.4.10 with fixed laser power 350\text{W}. The powder flow rate is 6.5\text{g/min} for both cases. As stated before, case 1 of Inconel 718 is used to calibrate the unknown parameters $Q^r, \beta, r$ in our model and case 2 of Inconel 718 is used for validation. One significant difference of Inconel 718 from Ti-6Al-4V is that a square root form is assumed for function $\Gamma(\cdot)$ here, i.e.

$$A_d = \Gamma(n) = \beta \sqrt{\frac{\eta(Q - Q^r)\nu}{4\pi a^2 \rho_c (T_m - T_{initial})}} \text{ for } Q > Q^r, \quad (2.4.5)$$

Correspondingly, the expression of $\hat{A}_{ss}^{i}$ in optimization problem (2.4.2) is rewritten as

$$\hat{A}_{ss}^{i} = \frac{4a^2}{\nu_i^2} \beta \sqrt{\frac{\eta(Q^r_i - Q^r)\nu_i}{4\pi a^2 \rho_c (T_m - T_{initial})}}. \quad (2.4.6)$$

Remark: The linear form of function $\Gamma(\cdot)$ was also tried for Inconel 718 in the parameter calibration as well as model validation for comparison. The comparison result (Fig. 2.4.6 and Fig. 2.4.7) shows that the square root form of function $\Gamma(\cdot)$ serves as a better option in terms of the summed square prediction error. This switch of function $\Gamma(\cdot)$ is purely for better curve fitting result, so far we don’t have any physical explanation for it.
Fig. 2. 4. 6 Curve fitting results of optimization problem (2.4.2): Our model prediction of steady-state melt-pool area (linear & square root form) vs experimental data from case 1 of Inconel 718

$\text{SSE}_{\text{linear}} = 1.2008 \times 10^4 \mu \text{m}^4$

$\text{SSE}_{\text{square root}} = 6.9265 \times 10^3 \mu \text{m}^4$
Fig. 2. 4. 7  Our model prediction of steady-state melt-pool area (linear&square root form) vs measurements from case 2 of Inconel 718

The material properties of Inconel 718 used are listed in Table 2.4.3 and the calibrated parameters are listed in Table 2.4.4.

### Table 2. 4. 3  Material properties of Inconel 718

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
<td>ρ</td>
<td>8145</td>
</tr>
<tr>
<td>Solid material specific heat (J/(kg K))</td>
<td>c_s</td>
<td>652</td>
</tr>
<tr>
<td>Melting temperature (K)</td>
<td>T_m</td>
<td>1570</td>
</tr>
<tr>
<td>Ambient temperature (K)</td>
<td>T_0</td>
<td>292</td>
</tr>
<tr>
<td>Parameter</td>
<td>Symbol</td>
<td>Value</td>
</tr>
<tr>
<td>---------------------------------------------------------------------------</td>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>Initial temperature (K)</td>
<td>$T_{initial}$</td>
<td>$T_0$</td>
</tr>
<tr>
<td>Specific latent heat of fusion-solidification (J/kg)</td>
<td>$h_{SL}$</td>
<td>2.5e5</td>
</tr>
<tr>
<td>Molten material specific heat (J/(kg K))</td>
<td>$c_i$</td>
<td>778</td>
</tr>
<tr>
<td>Heat transfer coefficient ($W/m^2K$)</td>
<td>$\alpha_G$</td>
<td>75</td>
</tr>
<tr>
<td>Surface emissivity</td>
<td>$\varepsilon$</td>
<td>0.53</td>
</tr>
<tr>
<td>Stefan-Boltzann constant ($W/m^2K^4$)</td>
<td>$\sigma$</td>
<td>5.67e-8</td>
</tr>
<tr>
<td>Thermal conductivity constant (W/m K)</td>
<td>$k$</td>
<td>31.3</td>
</tr>
<tr>
<td>Thermal diffusivity ($m^2/s$)</td>
<td>$a$</td>
<td>5.8939e-6</td>
</tr>
<tr>
<td>Convection coefficient ($W/m^2K$) in case 1</td>
<td>$\alpha_s$</td>
<td>10.8e5-13.7e5</td>
</tr>
<tr>
<td>Convection coefficient ($W/m^2K$) in case 2</td>
<td>$\alpha_s$</td>
<td>8.25e5-11.75e5</td>
</tr>
<tr>
<td>Laser transmission efficiency</td>
<td>$\eta$</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table 2.4.4 Parameters calibrated by measurements from case 1 of Inconel 718

<table>
<thead>
<tr>
<th>$r=w/h$</th>
<th>$Q^c$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/0.2039</td>
<td>132.63W</td>
<td>0.33223</td>
</tr>
</tbody>
</table>

Then we do simulation for case 1 of Inconel 718 with state-space model (2.3.11) and (2.3.12). Our model prediction of melt-pool width and height is compared with experimental data, while our model prediction of average melt-pool temperature is compared with FEA prediction. The simulation result is shown in Fig. 2.4.8.
Fig. 2.4.8  Inconel 718: Our model prediction vs. experimental measurements of melt-pool geometry and FEA prediction of melt-pool temperature, subjected to a multiple step-input trajectory of laser power $Q$ and a fixed scanning speed $v = 10.58\text{mm/s}$

It is shown in Fig. 2.4.8 that our model prediction of melt pool width matches the experimental data quite well. In addition, clear pattern caused by the step changes of laser power is clearly displayed in both our model prediction and experimental measurement of melt pool width. However, a “balling” effect can be observed in the melt pool height profile within distance intervals $3 \times 10^4 - 3.5 \times 10^4 \mu m$ and $8.6 \times 10^4 - 9.3 \times 10^4 \mu m$, which occurred in Ti-6AL-4V experiment too. For the temperature profile, our model prediction of average melt pool temperature follows the FEA prediction of average
melt pool temperature closely.

Next, our model with calibrated parameters listed in Table 2.4.4 is validated through experimental data collected from case 2 of Inconel 718, where the laser power is fixed at $Q = 350W$ and the scanning speed follows a step-input trajectory displayed at the bottom of Fig. 2.4.10. Our model prediction of melt pool geometry is compared with experimental data, while the average melt-pool temperatures were computed using FEA method as the reference. The simulation result for melt-pool area in case 2 of Inconel 718 is shown in Fig. 2.4.9.

![Fig. 2.4.9 Simulation vs. experimental data of melt-pool area from case 2 of Inconel 718, subjected to a multiple step-input trajectory of scanning speed $v$ and a fixed laser power $Q = 350W$](image)

Fig. 2.4.9 shows that our model with calibrated parameters captures the dynamic process of the melt-pool area quite well except for the last interval (distance between $10.6 \times 10^4$ – $12.4 \times 10^4 \mu m$). In
Fig. 2.4.10, the simulation result of melt-pool width and height are compared vs. experimental data of melt-pool width and height, while the simulation result of melt-pool average temperature is compared vs FEA prediction of average melt-pool temperature.

![Graph showing comparison between simulation and experimental data](image)

Fig. 2.4.10 Inconel 718: Our model prediction vs. experimental measurements of melt-pool geometry and FEA prediction of melt-pool average temperature, subjected to a multiple step-input trajectory of scanning speed $v$ and a fixed laser power $Q = 350W$.

The dynamics of melt-pool width and height are not predicted well by our model in Fig. 2.4.10, while the dynamics of the melt-pool area is captured well by our model prediction in Fig. 2.4.9. This observation can be explained by the variation of the width to height ratio $r$ along the track in experimental data of case 2 of Inconel 718. In Fig. 2.4.11, the value of ratio $r$ along the entire track,
which is calculated based on melt-pool width and height measurements of Inconel 718 in case 2, is plotted vs. the calibrated constant $r$ used in simulation.

Fig. 2.4.11 Width to height ratio $r$ calculated from measurement of Inconel 718 in case 2 vs calibrated constant value used in simulation

Fig. 2.4.11 reveals that a constant calibrated $r$ is not a good approximation of the real $r$ value along the entire track. However, the value of $r$ used in simulation has no effect on our model prediction of steady-state melt-pool area $\hat{A}_\mu$, which is backed up by equation (2.4.6). This fact explains the consistency between our model prediction of steady-state melt-pool area simulated with a constant $r$ and the measurements of melt-pool area in Fig. 2.4.9, although using a constant $r$ in simulation is disproved by Fig. 2.4.11.
2.5 Summary

In this chapter, a reduced-order multivariable physics-based model of melt-pool geometry and temperature in 1-D DED process is developed following Doumanidis and Kwak’s work in [22]. The main contribution of our 1-D model lies in a novel parameterization of the entire material transfer rate as a function of the operating parameter, which can be used to update the mass balance equation in [22]. This novel parameterization is inspired by Eagar and Tsai’s work in Ref. [35]. Then a set of experiments are conducted on LENS system using material Ti-6AL-4V and Inconel 718 for unknown parameters calibration and validation of our proposed model. Our model prediction of melt-pool geometry is validated by the experimental data, while our model prediction of the melt-pool temperature is validated by FEA prediction.
Chapter 3 Melt-Pool Height and Temperature Controller Design for 1-D DED Process

3.1 Introduction

As stated in the previous chapters, there exist several issues that restrict the widespread use of AM technology in industrial application, although AM possesses beneficial features that can’t be realized by traditional manufacturing technology such as milling, turning and cutting. Existing issues regarding AM technology include but not limited to the accuracy of part geometry and metallurgical properties of the processed part. The first issue is governed by the melt-pool geometry and the second issue is directly related to the melt-pool temperature [20]. Consequently, we are highly motivated to design a control system to regulate the melt-pool height and temperature for 1-D case of DED process, which is also the objective of this chapter.

Several controllers for 1-D AM process in the existing literature are reviewed in chapter 1. Most of the existing controllers are restricted to classical controllers, for example model-free PID controller based on pure sensing information. In addition, most of the models of AM process used in controller design in existing literature are usually empirical ones, whose applicability to dynamic process other than their training scenario is limited by its statistical nature. Due to the severe lacking of advanced, physical model based controllers in the existing literature, two types of controller, an LQR (linear quadratic regulator) and an MPC (model predictive control), are developed in this chapter based on our model proposed in chapter 3.

The remainder of this chapter is organized as follows. The introduction, design methodology and a
A numerical example of LQR controller design are introduced in subsection 3.2. The introduction, design methodology and a numerical example of MPC controller design are included in subsection 3.3. Subsection 3.4 serves as a brief summary of this chapter.

3.2 Introduction, Design Methodology and a Numerical Example of LQR Controller Design

3.2.1 Introduction of LQR controller

Linear Quadratic Regulator is an optimal control strategy designed for linear systems subjected to a quadratic cost function. In this subsection we will focus on infinite horizon continuous-time LQR, whose standard form can be formulated as an optimization problem:

\[
\min_u J(u) = \int_0^\infty (x(t)^T Q x(t) + u(t)^T R u(t)) dt
\]

subject to \( \dot{x}(t) = Ax(t) + Bu(t) \)

where matrix \( Q \geq 0 \) and matrix \( R > 0 \) are weighting matrices on the state \( x(t) \) and input \( u(t) \) respectively, equation (3.2.1.2) is the dynamic equation of a LTI system. Due to the penalty imposed on state \( x(t) \), cost function (3.2.1.1) can drive the state to the origin eventually.

The solution of the optimization problem (3.2.1.1)-(3.2.1.2) is a state-feedback control law

\[
u(t) = -K x(t)
\]

where matrix \( K \) can be obtained from the following equation,

\[
K = R^{-1} P B^T
\]

where matrix \( P \geq 0 \) can be solved from the algebraic Riccati equation (ARE) as follows
\[ A^T P + PA - PBR_p^{-1}B^T P + Q_p = 0, \quad (3.2.1.5) \]

More details of derivation of solution (3.2.1.3)-(3.2.1.5) can be referred to Ref. [38].

3.2.2 LQR controller design for nonzero target height and temperature tracking

In subsection 3.2, our objective is to design a LQR controller so that both the melt-pool height \( h(t) \) and average temperature \( T(t) \) can track nonzero reference trajectories by regulating laser power \( Q \) and scanning speed \( v \) at the same time. Our dynamic model of 1-D DED process proposed in chapter 2 is used here

\[ \dot{x} = f(x, u) \leftrightarrow \begin{bmatrix} \dot{h} \\ \dot{T} \end{bmatrix} = \begin{bmatrix} f_1(h, Q, v) \\ f_2(h, T, Q, v) \end{bmatrix} \quad (3.2.2.1) \]

where \( u = [Q \quad v]^T \) is the control input of the system, the nonlinear vector field \( f = [f_1 \quad f_2]^T \) takes the following form:

\[ f_1 = \frac{\beta}{\pi c_s[T_m - T_{\text{initial}}]} - \rho \frac{\pi}{4} rh^2 v \]

\[ f_2 = \frac{\rho \frac{\pi}{4} rh^2 v}{\pi c_s[T_m - T_{\text{initial}}]} \left[ c_s(T_m - T_0) + h_{\text{sl}} + c_l(T - T_m) \right] \]

\[ -\rho \frac{\pi}{4} rh^2 c_s(T_m - T_0) + \eta Q - \frac{\pi}{4} r^2 h^2 \alpha_s(T - T_m) - \frac{\pi}{\sqrt{2}} (r^2 h^3)^{2/3} \]

\[ \times \left\{ \alpha_G(T - T_0) + \varepsilon\sigma(T^4 - T_0^4) \right\} [\frac{\pi}{6} r^2 h^3 \rho c_i] \quad (3.2.2.2) \]

where \( \beta, Q^*, r \) are unknown parameters that can be calibrated from experimental data.

Note that (3.2.2.1) is a nonlinear system, which is inappropriate for LQR controller design. Thus linearization is applied to (3.2.2.1) prior to LQR controller design. Assume that the nonlinear system (3.2.2.1) is linearized around certain operating point \( [\bar{x} \quad \bar{u}]^T = [\bar{h} \quad \bar{T} \quad \bar{Q} \quad \bar{V}]^T \), the linearized
system can be written as

\[ \Delta \dot{x} = A_L \Delta x + B_L \Delta u \]  

(3.2.2.4)

where \( \Delta x = x - \bar{x} = [h - \bar{h} \quad T - \bar{T}]^T \), \( \Delta u = u - \bar{u} = [Q - \bar{Q} \quad v - \bar{v}]^T \), \( A_L \) and \( B_L \) are the Jacobian matrices.

\[ A_L = \begin{bmatrix} \frac{\partial f_1}{\partial h} & \frac{\partial f_1}{\partial T} \\ \frac{\partial f_2}{\partial h} & \frac{\partial f_2}{\partial T} \end{bmatrix} |(\bar{x}, \bar{u}) \]  

(3.2.2.5)

\[ B_L = \begin{bmatrix} \frac{\partial f_1}{\partial Q} & \frac{\partial f_1}{\partial v} \\ \frac{\partial f_2}{\partial Q} & \frac{\partial f_2}{\partial v} \end{bmatrix} |(\bar{x}, \bar{u}) \]  

(3.2.2.6)

One thing needs to be emphasized here is that solution (3.2.1.3)-(3.2.1.5) can’t be directly employed for LQR controller design for original nonlinear system (3.2.2.1) and its linearized system (3.2.2.4). This is because the objective of the optimal control input solved by (3.2.1.3)-(3.2.1.5) is to drive the states to zero, while our objective is to let the states track non-zero trajectories.

This non-zero target tracking can be realized by decomposing the control input \( u \) in (3.2.2.1) into three parts \( u = \bar{u} + \Delta u^* + \Delta u_d \), where \( \Delta u^* \) denotes the increment needed to change the trim point \( \bar{x} \) to the target value \( x_t \) in the linearized system (3.2.2.4), \( \Delta u_d \) is to ensure stability and shape the response.

Since \( \Delta u^* \) can change the trim point \( \bar{x} \) to the target value \( x_t \) in the linearized system, equation (3.2.2.4) should hold with LHS equal to 0 if we plug \( \Delta u^* \) in it, i.e.

\[ 0 = A_L \Delta x^* + B_L \Delta u^* \]  

(3.2.2.7)

where \( \Delta x^* = x_t - \bar{x} \). Here we assume the target value \( x_t \) is a constant trajectory, or a piecewise-constant trajectory which can be considered as separated constant trajectories. If matrix \( B_L \) is invertible, the solution of \( \Delta u^* \) is
\[ \Delta u^* = -B_L^*A_L(x_r - x) , \quad (3.2.8) \]

The last component \( \Delta u_d \) in \( u \) will be generated by the standard LQR introduced in section 3.2.1. If we plug \( \Delta u = u - \bar{u} = \Delta u^* + \Delta u_d \) into the linearized equation \( (3.2.2.4) \), we have

\[
\Delta \dot{x} = A_L \Delta x + B_L (\Delta u^* + \Delta u_d) = A_L \Delta x + B_L \Delta u^* + B_L \Delta u_d, \quad (3.2.2.7)
\]

\[
= A_L \Delta x - A_L \Delta x^* + B_L \Delta u_d = A_L (\Delta x - \Delta x^*) + B_L \Delta u_d, \quad (3.2.2.9)
\]

We define state variables \( z_1 = \Delta x - \Delta x^* \), \( z_2 = \int_0^t (\Delta x - \Delta x^*)d\tau \), so we have the state-space model

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} =
\begin{bmatrix}
A_L & 0 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2
\end{bmatrix} +
\begin{bmatrix}
B_L
\end{bmatrix}
\Delta u_d, \quad (3.2.2.10)
\]

Note that \( z_1 = \Delta x - \Delta x^* = x - x_r \) can be considered as the tracking error, \( z_2 = \int_0^t (\Delta x - \Delta x^*)d\tau = \int_0^t (x - x_r)d\tau \) can be considered as the integral of the tracking error from 0 to time \( t \). The integral of tracking error \( z_2 \) is introduced as a state in \((3.2.2.10)\) to improve the steady-state performance of the closed-loop system. LQR introduced in section 3.2.1 will be designed w.r.t. equation \( (3.2.2.10) \) so that \( z = [z_1 \ z_2]^T \) can be driven to zero by \( \Delta u_d \). The weighting matrices in cost function \( (3.2.2.1) \) can be tuned based on the requirements of the designed LQR. The control law can be written as a state feedback

\[
\Delta u_d = -Kz = -K \left[ x - x_r \right]^T \int_0^t (x - x_r)d\tau, \quad (3.2.2.11)
\]

where matrix \( K \) can be solved by ARE. It can be observed that \( (3.2.2.11) \) is actually in the form of a PI (proportional-integral) controller, which is optimal in the sense of minimization of the cost function \( (3.2.2.1) \). More details regarding LQR design for non-zero reference tracking of nonlinear system can be referred to Ref. [38].
3.2.3 Numerical example

In this subsection, a numerical example will be provided to test the performance of LQR controller introduced in section 3.2.2. To test the performance, a single track deposition will be simulated using material Ti-6AL-4V. The material properties and calibrated parameters ($\beta, Q^c, r$) involved in this numerical example are listed in Table 2.4.1 and Table 2.4.2 of section 2.4.

Fig. 3.2.3.1 Diagram of nonlinear MIMO control of melt-pool height and temperature using LQR

Fig. 3.2.3.1 is a diagram of the nonlinear MIMO control of melt-pool height and temperature with LQR. The operating point used for linearization is $\begin{bmatrix} h & T & Q & v \end{bmatrix}^T = [163.8 \mu m \ 2032 K \ 337.5 W \ 25 ipm]^T$ since $[Q \ v] = [337.5 W \ 25 ipm]$ is the middle point of the operating range employed in the experimental work involved in section 2.4. Then a LQR controller will be designed with respect to the linearized model with the methodology introduced in section 3.2.2. The weighting matrices in cost function of LQR are prescribed as
\[
Q_p = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0.5 & 0 & 0 \\
0 & 0 & 0 & 10^{-11}
\end{bmatrix}
\]  
(3.2.3.1)

\[
R_p = \begin{bmatrix}
\frac{1}{Q^2} & 0 \\
0 & \frac{1}{v^2}
\end{bmatrix}
\]  
(3.2.3.2)

(3.2.3.1) and (3.2.3.2) are selected so that tracking performance is guaranteed and the corresponding control input is within operating range of LENS system. At last the designed LQR controller will be implemented with the original nonlinear plant model for performance test.

Fig. 3.2.3.2 shows the simulation results of applying LQR controller to track the reference melt-pool height and temperature, where the blue solid curve is the system response with LQR implemented while the red dash line represents the reference trajectories.

Fig. 3.2.3.2  Simulation of nonlinear MIMO control of melt-pool height and temperature using LQR
Fig. 3.2.3.2 shows that our closed-loop system can track its reference trajectory quite well in both melt-pool height and temperature profiles, although overshoot occurs at the step change of melt-pool height reference trajectory. For the melt-pool height, the reference trajectory is selected as it in Fig. 3.2.3.2 since these three values (150µm, 175µm and 200µm) were realized in Ti-6AL-4V experiment in Fig. 2.4.4; for the melt-pool temperature, the reference is selected as a constant value 2000K which is slightly above the melting temperature of Ti-6AL-4V so that the metallurgic properties can be uniform over the entire processed part and optimized, for example the porosity and dilution can be reduced.

Regarding the control inputs \( Q \) and \( v \), we can observe that \( Q \) keeps increasing to drive melt-pool height \( h \) from 150µm to 200µm, which is quite intuitive; meanwhile, the scanning speed \( v \) also keeps increasing to maintain the melt-pool temperature at a constant level. This can be explained by the energy balance equation (2.2.6), in which \( v \) can be increased to increase the heat loss rate due to solidification \( \rho \frac{\pi}{4} w(t)h(t)v(t)c_s(T_m - T_0) \).

3.3 NMPC Control for Melt-Pool Height in 1-D DED Process

3.3.1 Introduction of model predictive control (MPC)

Model predictive control (MPC) is an advanced control technology which is widely used in chemical process. MPC is a model-based control strategy since the calculation of the optimal control input is related with the prediction of the future states and outputs over certain prediction horizon,
which is based on the model of the process and the measurement of current state. MPC has a sense of feedforward from the perspective of prediction of future states as well as a sense of feedback due to the measurement of current state.

The MPC control strategy is a repetitive process of solving the optimal control problem. The optimal control problem can be formulated as equations below:

\[
\begin{align*}
\min_u J(x,u) &= \sum_{i=0}^{N-1} l(x(k+i),u(k+i)) + p(x(k+N)) \\
\text{subject to} \quad &u(k+i) \in U, x(k+i) \in X, x(k+N) \in \Omega
\end{align*}
\]

where \(k\) denotes the present index of the state vector \(x\), \(N\) is the prediction horizon of MPC, (3.3.1.1) is the cost function imposed on both states and inputs, certain constraints are given to inputs and states in (3.3.1.2). In addition, the system dynamic equation is considered as equality constraint in (3.3.1.2).

The method and complexity of optimizing (3.3.1.1) subjected to (3.3.1.2) depends on the type of this optimization problem: for example, if the cost function is (3.3.1.1) is in the form of linear or quadratic function, and the constraint (3.3.1.2) is linear, then this optimization problem is a simple linear or quadratic programming problem whose close-form solution can be achieved; if the optimization problem is more complicated, then sequential quadratic programming and interior point algorithms are commonly used to find a suboptimal solution of the optimization problem locally while the stability may be guaranteed by the suboptimal control input [39].

After the optimization problem (3.3.1.1 – 3.3.1.2) is solved,

\[
u^* = \{u^*(k), u^*(k+1), \ldots, u^*(k+N-1)\},
\]

usually only the first element \(u^*(k)\) will be applied to the plant. Then the current state \(x(k)\) will be
updated by the new measurement, which is for the formulation of the next optimal control problem analogous to (3.3.1.1 – 3.3.1.2).

In this section, our objective is to design a MPC controller so that the melt-pool height can track its reference trajectory by adjusting laser power and scanning speed. The melt-pool temperature is not selected as control output in MPC controller design due to the difficulty in discretization of melt-pool temperature dynamic equation (2.3.12). A big overshoot occurs in the melt-pool temperature profile as the power of the laser beam increases, which constrains the discretization of dynamic equation of the melt-pool temperature. In addition, our biggest concern about the melt-pool temperature is to prevent the occurrence of high melt-pool temperature, which can be realized by a simple on/off controller if needed.

3.3.2 Numerical example of NMPC design for melt-pool height control

In this subsection, a numerical example of NMPC controller design is given based on the 1-D physics-based model proposed in chapter 2. A single track deposition will be simulated using Ti-6AL-4V. The length of the track is approximately 10mm. Prior to the formulation of the NMPC problem, our model proposed in chapter 2 can be transformed from time-domain to space-domain with a change of variable as follows:

\[
\frac{dh}{dt} = \frac{dh}{ds} \frac{ds}{dt} = \frac{dh}{ds} \times v \Rightarrow \\
\frac{dh}{ds} = \frac{1}{v} \frac{dh}{dt} = \frac{\beta \eta (Q(s) - \dot{Q})}{\pi c_i [T_m - T_{\text{initial}}]} - \rho \frac{\pi}{4} rh(s)^2 v(s) - \rho \frac{\pi}{2} rh(s)^2 v(s)
\]

where \(s\) denotes the variable in the spatial S-coordinate and \(ds\) is defined as the infinitesimal in this
coordinate similar to \( dt \) in the time domain. The purpose of this transformation is to reformulate melt-pool height \( h \) as a function of distance \( s \) instead of time \( t \), since the reference trajectory is usually provided in the space-domain. Then discretization using Euler method can be applied to this space-domain model (3.3.2.1) with ZOH (zero order hold) condition. The discretized dynamic equation of melt-pool height \( h \) in space-domain model is shown in (3.3.2.2).

\[
\begin{align*}
    h_{k+1} &= h_k + \frac{\beta \eta (Q_k - Q')}{\pi c_i [T_m - T_{init}]} - \frac{\rho}{4} \frac{r h_k^2 v_k}{\pi} + \frac{\pi}{2} \rho r^2 h_k^2 v_k \times D_s
\end{align*}
\]  

(3.3.2.2)

where \( D_s \) denotes the sampling distance. If the value of \( D_s \) is too large, the discretized model (3.3.2.2) may not be a good approximation of the originally continuous-time model (3.3.2.1); if the value of \( D_s \) is too small, the corresponding computational efficiency will be hindered. Thus the selection of sampling distance \( D_s \) is a trade-off between the performance of discretization and corresponding computational load. Sampling distance \( D_s = 0.25 \text{mm} \) is selected in this numerical example.

The repeated optimal control problem in MPC design is formulated as follows:

\[
\begin{align*}
    \min_{u} J(h,u) &= \sum_{i=1}^{N} \left\{ (h_{k_i} - h_f)^T Q_p (h_{k_i} - h_f) + \{Q(k+i-1); v(k+i-1)\}^T R_p \{Q(k+i-1); v(k+i-1)\} \right\} \\
    \text{subject to} & \quad h_{k+i+1} = h_{k+i} + \frac{\beta \eta (Q(k+i) - Q')}{\pi c_i [T_m - T_{init}]} - \frac{\rho}{4} \frac{r h_k^2 v(k+i)}{\pi} + \frac{\pi}{2} \rho r^2 h_k^2 v(k+i) \times D_s, \\
    & \quad Q_{\min} \leq Q(k+i) \leq Q_{\max}, v_{\min} \leq v(k+i) \leq v_{\max}, i = 0, \ldots, N-1
\end{align*}
\]  

(3.3.2.3)

where \( N \) is the MPC horizon, matrix \( Q_p \) and \( R_p \) are the weighting matrices for tracking error and control input respectively, the cost function is in quadratic form of the tracking error and control input. \( Q_{\min} \) and \( Q_{\max} \) are the lower and upper bound for the laser power, \( v_{\min} \) and \( v_{\max} \) are the lower and upper
bound for the scanning speed. Note that the dynamic equation of melt-pool height (3.3.2.2) is considered as an equality constraint in (3.3.2.3). Because of the nonlinearity of the imposed constraints, the optimization problem (3.3.2.3) becomes a NMPC (nonlinear model predictive control) problem.

The material properties and calibrated parameters of Ti-6Al-4V used in simulation are listed in Table 2.4.1 and Table 2.4.2 of section 2.4. The parameters of NMPC design are listed in Table 3.3.2.1, where “diag” refers to diagonal matrix.

Table 3.3.2.1 Parameters of NMPC design

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_p$</td>
<td>10</td>
<td>$Q_{\text{min}}$</td>
<td>225W</td>
</tr>
<tr>
<td>$R_p$</td>
<td>diag ([4.94e-6, 396.8])</td>
<td>$Q_{\text{max}}$</td>
<td>450W</td>
</tr>
<tr>
<td>$N$</td>
<td>8</td>
<td>$v_{\text{min}}$</td>
<td>12.5ipm (5.2917mm/s)</td>
</tr>
<tr>
<td>$D_s$</td>
<td>0.25mm</td>
<td>$v_{\text{max}}$</td>
<td>37.5ipm (15.875mm/s)</td>
</tr>
</tbody>
</table>

The lower and upper bounds of laser power $Q$ and scanning speed $v$ are referred to the experimental setup in section 2.4. As we can observe in Fig. 3.3.2.1, the distance for transient behavior of melt-pool height is approximately 1mm which can be covered by our MPC horizon $N = 8$, while additional computational load can be caused by longer horizon $N$. The weighting matrices $Q_p$ and $R_p$ are tuned empirically to guarantee the tracking performance and smoothness of melt-pool height at the same time.

Fig. 3.3.2.1 shows the simulation results on applying MPC controller to track the reference
melt-pool height, where the red dash line represents the reference trajectory. This simulation is conducted in MATLAB with toolbox Yalmip, which is used to formulate the repetitive optimization problem. The nonlinear optimization problem (3.3.2.3) is solved using “fmincon” solver with interior point algorithm.

Fig. 3.2.3.3  Simulation of NMPC of melt pool height by regulating laser power and scanning speed

It is shown in Fig. 3.3.2.1 that the melt-pool height of the closed-loop system can track the reference trajectory quite well, which is desirable for the accuracy of part geometry. In the temperature profile, the melt-pool temperature stays around 2000K, which is just a little bit above the melting temperature 1923K. Although the experimental data of melt-pool temperature is lacking in this proposal, this predicted value stays within the usual melt-pool temperature range we observe in experiments. Regarding the control inputs, the laser power is quite high and the scanning speed is quite low at the start so that the melt pool height can be increased to its target value quickly. Then the
laser power is lowered and the scanning speed is increased generally as the melt-pool height approaches its target value.

3.4 Summary

In this chapter, both a LQR controller and a NMPC controller are designed based on the model proposed in chapter 2. The LQR controller is designed to track the reference trajectories of melt-pool height and temperature at the same time by regulating the laser power and scanning speed. This is a case of MIMO control, which can be considered as an advantage of our LQR control over our NMPC control. It should be noted that NMPC is capable of realizing MIMO control, but MISO NMPC was designed in my proposal. However, no constraints on control inputs are involved in the formulation of LQR controller design. Apart from this, the control input generated by the LQR controller is a continuous-time control law, which needs to be discretized for implementation in experiment. The NMPC controller is designed to track the reference trajectory of melt-pool height by adjusting the laser power and scanning speed, which is a MISO case as stated before. The control of melt-pool temperature is not included in NMPC controller design due to the difficulty in discretization of dynamic equation of the melt-pool temperature. An advantage of NMPC over LQR is that constraints can be imposed on the control inputs, e.g. laser power $Q \leq Q_{\text{max}}$, which is for the practicality of operations. In addition, the control inputs generated by NMPC are already in discrete form, which can be applied to the plant directly in experiment. Another significant difference between these two types of controller is that LQR is an off-line control strategy while MPC has to be solved online. Numerical examples are conducted in section 3.2.3 and section 3.3.2 to test the performance of these two
controllers. It is shown that both controllers can work well for tracking their reference trajectories.
Chapter 4 Control-Oriented Nonlinear Model of Melt-Pool Geometry for Multi-Dimensional DED Process

4.1 Introduction

In chapter 2, we proposed a second-order nonlinear state-space model of the melt-pool geometry and temperature, which is displayed by equation (2.3.11) and (2.3.12). This model is proposed based on physical insights of the directed energy deposition process with several unknown parameters that can be tuned for the purpose of calibration. Then this proposed model is calibrated and validated by the experimental data and finite element analysis prediction of single-bead deposition of two materials, Ti-6AL-4V and Inconel 718. So far, our research about process model of directed energy deposition in chapter two is limited within the one-dimensional (1-D) case where only single-bead deposition occurs, while the variation of process model between different beads or layers in the same building hasn’t been taken into account yet. However, the achievements in several references clearly support the existence of this variation: In [12], a 21-layer thin-wall structure is deposited with constant process parameters, and it is observed in the experiment that the layer height profile changes with respective to the layer number; In [23], a height-dependent model for the laser metal deposition process is constructed, in which the height-dependence is achieved through the solidification rate term in the mass balance equation. By Stefan equation, this solidification rate term is correlated with the temperature gradient along the scanning direction in the solid region evaluated at the phase-change interface, and this temperature gradient can be formulated as a function of the part height by the finite element method; In [30], [31] and [32], the author presents a control-oriented process model in
repetitive form for laser metal deposition process, which includes both in-layer and layer-to-layer dynamics; In [33], a ten-layer single-bead wall is deposited by wire and arc additive manufacturing technology. The scanning speed is changed from layer to layer in order to maintain a constant layer width, which reversely implies that this process is layer-dependent; In [40], the author conducts experiment and FEA simulation for a nine-layer wall structure built by laser-assisted additive manufacturing technology with constant process parameters. The melt-pool geometry can be observed to increase with the layer index in both the experimental data and FEA simulation result, which strongly backups the layer-to-layer variation of the dynamics of additive manufacturing process; In [34], gas metal arc welding technology is applied to build an eleven-layer wall structure with constant process parameters. The average measured bead width varies with respect to layer index, which is another solid evidence of the layer-dependency of additive manufacturing technology. These achievements in reference mentioned above support our intention to explore more about our proposed process model so that it can be applied to multi-dimensional case of DED process, where structure composed of multiple beads and layers will be deposited.

The extension of our directed energy deposition process model from 1-D case to multi-dimensional case is very beneficial for the process development. For example, it is very common to build a single-bead wall by the directed energy deposition technique, during which constant process parameters are usually applied to produce constant layer geometry. However, it has been observed that the layer geometry will increase as the layer index goes up, which will induce accumulated error in wall height. This accumulated error of wall height will result in defocus of the powder stream and laser beam on the top surface of the wall, which will jeopardize the stability of the manufacturing process in turn. This example demonstrates one of the uses of multi-dimensional process model in the
process development.

The rest of this chapter will be organized as follows: Section 4.2 introduces the methodology for our model extension, which is realized by taking into account the thermal history of the build. Section 4.3 validates this extended model by three builds having multi-dimensional geometry: a thin-wall structure, a patch build and a L-shape structure. At the end of this chapter, section 4.4 gives a brief summary of this chapter.

4.2 Extension of proposed model from one-dimensional case to multi-dimensional case

We proposed a reduced-order nonlinear model of melt-pool geometry and temperature during one-dimensional DED process in chapter two. This proposed model consists of two ordinary differential equations (ODEs) and two algebraic equations, which are listed as (4.2.1)-(4.2.4).

\[ \dot{h} = \frac{\beta \eta (Q - Q^*)}{\pi c_f [T_m - T_{initial}]} - \frac{\rho \pi}{2} r^2 h^2 \]  

\[ \dot{T} = \left( \frac{\pi}{4} rh^2 v - \frac{\beta \eta (Q - Q^*)}{\pi c_f [T_m - T_{initial}]} \right) [c_s (T_m - T_0) + h_{sl} + c_i (T - T_m)] \]

\[ -\rho \frac{\pi}{4} r h^2 v c_j (T_m - T_0) + \eta Q - \frac{\pi}{4} r^2 h^2 \alpha_i (T - T_m) - \frac{\pi}{\sqrt{2}} (r^3 h^3)^{\gamma/3} \]

\[ \alpha_o (T - T_0) + \varepsilon \sigma (T^4 - T_0^4)] / (\frac{\pi}{6} r^2 h^3 \rho c_i) \]

\[ \frac{w}{h} = r \]  

\[ w = l \]  

The state variables are \( x(t) = [w(t) \ h(t) \ l(t) \ T(t)]^T \), where \( w(t), h(t), l(t) \) and \( T(t) \) denote the
melt-pool width, height, length and average temperature, respectively. The inputs of the model are
\[ u(t) = [Q(t), v(t)]^T, \]
where \( Q(t) \) and \( v(t) \) denote power and scanning speed of the laser beam, respectively. In addition, there are three unknown parameters \( Q^*, \beta, r \) in (4.2.1)-(4.2.4), whose values can be calibrated by experimental data. The calibration process is introduced using material Ti-6AL-4V and Inconel 718 as example in section 2.4, and the corresponding calibrated values are listed in Table 2.4.2 and Table 2.4.4. Parameter \( T_0 \) is the ambient temperature and parameter \( T_{\text{initial}} \) in (4.2.1) is the initial temperature/preheated temperature of the substrate in 1-D case. All the other parameters involved in (4.2.1)-(4.2.4) are material properties, whose physical meanings are listed in Table 2.4.1 and Table 2.4.3.

Remark: As we mentioned in section 2.3, the function \( \Gamma(\cdot) \) in equation (2.3.4) can have different forms for different materials, which will in turn affect the form of (4.2.1) and (4.2.2). For example, (4.2.1) and (4.2.2) are the result derived by linear approximation of function \( \Gamma(\cdot) \), which has been validated by experiments conducted with material Ti-6AL-4V; the function \( \Gamma(\cdot) \) is selected as square root form for material Inconel718 and validated by measurements collected from Inconel718 experiment. Since material Ti-6AL-4V will be used as example in the remaining part of this thesis, we will keep function \( \Gamma(\cdot) \) approximated by linear form and the corresponding dynamic equation of melt-pool height \( h(t) \) and average temperature \( T(t) \) will be in the form of (4.2.1) and (4.2.2).

The DED process is a thermal process since a laser source is applied to melt the metal powder distributed by the nozzle. Thus heat accumulation will happen during the deposition process especially for the multi-dimensional case, which will cause the beads to be built preheated by the heat.
accumulated in the existing part. Based on our observation in experiments of 1-D case, the geometry of the melt-pool will be increased by a preheated temperature of the substrate. Consequently, it is reasonable to model the variation between the 1-D process model and multi-dimensional process model by incorporating the thermal information occurring during the deposition process. Note in (4.2.1) the temperature $T_{\text{initial}}$ denotes the initial/preheated temperature, which can be used to involve the in-process thermal information. Then the remaining challenge is to find a way to characterize the preheated/initial temperature $T_{\text{initial}}$ caused by heat accumulation in the existing part, and this way should be applicable to our control-oriented model (2.4.1) and (2.4.2). In this thesis, we propose an idea to characterize $T_{\text{initial}}$ by the Rosenthal’s solution.

The Rosenthal’s solution is an analytical solution of steady-state temperature field generated by point heat source travelling on semi-infinite medium, which is described by (2.3.1) and has been briefly introduced in section 2.3 [35]. The Rosenthal’s solution was solved from the heat conduction equation, which is a partial derivative equation, with several assumptions. These assumptions involved in Rosenthal’s solution are: the ideal point form of the heat source, absence of convective or radiative heat transfer, invariance of material properties with respect to temperature and a quasi-steady state semi-infinite plate. These assumptions were introduced to simplify the solving process of the heat conduction equation. Our method of calculating preheated temperature $T_{\text{initial}}$ in multi-dimensional model utilizes the Rosenthal’s solution. However, an assumption needs to be introduced first before we explain the methodology of calculating $T_{\text{initial}}$.

**Assumption 4.2.1** The simplifying assumptions employed in the Rosenthal’s solution are also adopted in this thesis for characterization of the initial/preheated temperature $T_{\text{initial}}$ by the Rosenthal’s
solution.

In the Rosenthal’s solution, the point heat source is assumed to be travelling in a continuous way, which is the situation of 1-D DED process. Thus the temperature field generated in 1-D case (single bead deposition) of DED process can be approximated by applying the Rosenthal’s solution once. However, the temperature field in the multi-dimensional case is generated as a combination of multiple heat sources, with each heat source corresponding to a single bead. Assumption 4.2.2 is introduced to deal with these multiple heat source.

**Assumption 4.2.2** The combined heating effect from multiple heat sources can be calculated by the superposition principle, with each heat source corresponding to one single bead.

Another thing should be emphasized is that the transient behavior of the temperature field is left out in the characterization of $T_{\text{initial}}$ with Rosenthal’s solution, which is due to the quasi-steady state nature of the Rosenthal’s solution. Apart from the quasi steady-state nature, another reason that the transient behavior of the temperature field can be left out here is that our process model is a dynamic one, which can capture the transient behavior of the system instead.

Based on Assumption 4.2.1 and Assumption 4.2.2, now we are ready to explain our methodology to calculate the approximated preheated temperature $T_{\text{initial}}$ in multi-dimensional case of DED process. We will use the single-bead wall as an example, which is schematically plotted in Fig. 4.2.1.
The global coordinate system $XOZ$ is defined as in Fig. 4.2.1, the $Y$ axis is left out here since the single-bead wall can be considered as a 2-D case with $y = 0$. The layer index $j$ starts from 1 at the bottom layer of the wall and increases as the layer goes up. The magnitude of the scanning speed $v$ is assumed to be constant within each layer while the direction of the scanning speed switches every layer, for example the odd layers are deposited from left to right but the even layers are deposited from right to left. The length of the wall is denoted by $L$. In addition, there may be dwell-time at the completion of each layer, which can be denoted as $\Delta t$.

The initial/preheated $T_{\text{initial}}$ temperature of the pre-deposition point can be computed as follows:

1) $j = 1$. Suppose we are now working on the first layer $j = 1$. The parameter $T_{\text{initial}}$ in equation (4.2.1) should be a constant equal to the ambient temperature if the substrate is under room-temperature, i.e.

$$T_{\text{initial}} = T_0 \quad \text{(4.2.5)}$$

In Fig. 4.2.2, we define a moving local coordinate system $(x_1, z_1 = 0)$ associated with heat source 1. When the first layer is finished and the laser beam reaches the end of layer 1, the real-time coordinate of heat source 1 is $(x_1, z_1) = (L, 0)$, which is the situation in Fig. 4.2.2. At this time instant, based on
assumption 4.2.1, the temperature field generated by heat source 1 can be characterized by the Rosenthal’s solution easily as below [35]:

\[ T_1(x, z) = T_0 + \frac{Q_1}{2\pi k R_1} e^{-\frac{(v_1 R_1 + R_1)^2}{2a}}, \]  

(4.2.6)

where \( T_1(K) \) is the temperature of target point \((x, y = 0, z)\). \( T_0(K) \) is the ambient temperature. The last term of (4.2.5) denotes the temperature increment caused by heat source 1, \( Q_1 \) and \( v_1 \) denote the power and scanning speed of heat source 1 in Fig. 4.2.2 respectively, \( k \) is the thermal conductivity of the deposition material (\( W/m \cdot K \)), \( a \) denotes the thermal diffusivity of the deposition material (\( m^2/s \)). \( w_1(m) = x - x_1 = x - L \) denotes the \( x \) coordinate of the target point \((x, y = 0, z)\) in local coordinate \( x_1, z_1 \), \( R_1(m) = \sqrt{(x-x_1)^2 + (z-z_1)^2} = \sqrt{(x-L)^2 + z^2} \) is the distance from target point \((x, y = 0, z)\) to heat source 1 \((x_1, z_1) = (L, 0)\). More details about calculating \( w_1 \) and \( R_1 \) with respective to local coordinate system \( x_1, z_1 \) can be found in Ref. [35].

**Fig. 4.2.2  Schematic plot at the completion of layer 1**

As we mentioned above, dwell time \( \Delta t \) may occur at the end of layer 1, during which the temperature field generated by heat source 1 will cool down and affect the initial/preheated temperature \( T_{\text{initial}} \) of layer 2 in some way. Thus a way to describe this cooling off effect of \( T_{\text{initial}} \) induced by heat source 1 needs to be figured out. Obviously, the initial/preheated temperature \( T_{\text{initial}} \) of layer 2 is no longer the ambient temperature \( T_0 \) since it is deposited on top of layer 1 and subjected to the preheated effect.
induced by layer 1. The initial/preheated temperature $T_{\text{initial}}$ of layer 2 should own these two properties:

1. The preheated temperature $T_{\text{initial}}$ of layer 2 varies with respect to the location: the zone close to the heat source 1 should have a higher $T_{\text{initial}}$ than the zone far away from heat source 1.

2. The preheated temperature $T_{\text{initial}}$ will cool off eventually during the deposition of layer 2 since the preheated effect induced by heat source 1 will diminish eventually.

The first property has already been characterized by the Rosenthal’s solution. In order to characterize the cooling-off effect mentioned in 2, we assume heat source 1 will change into its “virtual version” as soon as it is shut down at the end of layer 1, with its power and speed inherited from the original heat source 1. This assumed “virtual version” of heat source 1 will be called “virtual heat source 1” in this thesis. Figure 4.2.3 schematically plots the situation at the end of dwell-time $\Delta t$ of layer 1, where the real-time coordinate of “virtual heat source 1” is $(x_1, z_1) = (L + v_1 \Delta t, 0)$. The temperature field of Fig. 4.2.3 can be calculated easily by replacing $x_1 = L$ with $x_1 = L + v_1 \Delta t$ in (4.2.6).

2) $j = 2$. Next layer 2 will be deposited on top of layer 1, with the moving local coordinate defined as $(x_2, z_2 = \bar{h}_1)$ in Fig. 4.2.4, where $\bar{h}_1$ denotes the average height of layer 1. As we explained in the situation of layer 1, the “virtual heat source 1” will continue its movement during the deposition of layer 2, with laser power and scanning speed equal to $Q_1$ and $v_1$, respectively. The initial/preheated
temperature $T_{\text{initial}}$ of the pre-deposition point in Fig. 4.2.4 can be calculated as (4.2.7)

$$T_{\text{initial}}(x_2,z_2) = T_0 + \frac{Q}{2\pi k R_{12}} e^{-\frac{y_0(w_2 + R_{12})}{2a}}$$

(4.2.7)

where $w_{12} = x_2 - (L + v_1(\Delta t + t_2))$ is the x-coordinate of the pre-deposition $(x_2, z_2 = \bar{h}_1)$ in the local coordinate system $(x_1, z_1) = (L + v_1(\Delta t + t_2), 0)$ and $R_{12} = \sqrt{w_{12}^2 + (z_2 - z_1)^2} = \sqrt{w_{12}^2 + \bar{h}_1^2}$ is the distance between the pre-deposition point and the “virtual heat source 1. So far we introduced how to approximately calculate the real-time initial/preheated temperature $T_{\text{initial}}$ of layer 2 by applying Rosenthal’s solution to “virtual heat source 1”. This real-time preheated temperature $T_{\text{initial}}$ can be plugged in (4.2.1) for simulation of melt-pool height of layer 2. The laser source will become the “virtual heat source 2” after layer 2 is finished, with power $Q_2$ and scanning speed $v_2$ inherited from laser source of layer 2.

3) $j = 3$. Next we are going to deposit layer 3 after the first two layers are finished. The heat source of layer 3 is travelling on top of layer 2 from left to right while the virtual heat sources 1 and 2 keep moving away from the single-bead wall. An instant of layer 3 is plotted schematically in Fig. 4.2.5.
In Fig. 4.2.5, the local coordinate system associated with laser source in layer 3 is denoted as 
\((x_3, z_3 = \overline{h_1} + \overline{h_2})\), where \(\overline{h_1}\) and \(\overline{h_2}\) denotes the average height of layer 1 and layer 2, respectively. The real-time initial/preheated temperature \(T_{initial}\) at \((x_3, z_3)\) of layer 3 is generated by virtual heat source 1 and 2 together, which can be calculated by applying Rosenthal’s solution and superposition principle to virtual heat source 1 and 2, as (4.2.8).

\[
T_{initial}(x_3, z_3) = T_0 + \frac{Q_1}{2\pi kR_{13}} e^{-\frac{R_{13}}{2a}} + \frac{Q_2}{2\pi kR_{23}} e^{-\frac{R_{23}}{2a}}
\]

where \(w_{13} = x_3 - x_1 = v_3 t_3 - L - v_1 (2\Delta t + t_2 + t_3)\) denotes the x-coordinate of the pre-deposition point \((x_3 = v_3 t_3, z_3 = \overline{h_1} + \overline{h_2})\) in the local coordinate system \((x_1, z_1)\), 
\(R_{13} = \sqrt{w_{13}^2 + (z_3 - z_1)^2} = \sqrt{w_{13}^2 + (\overline{h_1} + \overline{h_2})^2}\) denotes the distance between the pre-deposition point and the “virtual heat source 1”. \(w_{23} = x_2 - x_3 = -v_2 (\Delta t + t_3) - v_2 t_3\) denotes the x-coordinate of the pre-deposition point in the local coordinate system \((x_2, z_2)\), \(R_{23} = \sqrt{w_{23}^2 + (z_3 - z_2)^2} = \sqrt{w_{23}^2 + \overline{h_2}^2}\) denotes the distance between the pre-deposition point and the “virtual heat source 2”. By (4.2.8) we introduced how to calculate the real-time initial/preheated temperature \(T_{initial}\) of layer 3 based on the virtual heat source 1 and 2. This real-time preheated temperature \(T_{initial}\) can be plugged in (4.2.1) for
simulation of deposition of layer 3.

In the following layers \((j = 4, 5, 6...)\) of the single-bead wall, the preheated temperature \(T_{\text{initial}}\) in (4.2.1) can be calculated in a similar fashion, in which superposition of the temperature fields generated by past heat sources happens, with one virtual heat source corresponding to one past bead. A pseudo-code of the algorithm for computing \(T_{\text{initial}}\) iteratively using a single-bead wall as the example is given in Fig. 4.2.6.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Iterative Computation of (T_{\text{initial}}(x_j, z_j)) for single-bead wall</th>
</tr>
</thead>
</table>
| **Input** | \(j\) : index of current layer  
(x\(j\), z\(j\)) : position of deposition point of layer \(j\) in global coordinate system \((x, z)\)  
\(T_0\) : initial temperature of the substrate  
\(L\) : wall length  
\(\Delta t\) : dwell time between layers (assumed constant for all layers)  
\(Q_i(1 \leq i \leq j-1)\) : power of laser source in layer \(i\)  
v\(i\)(1 \(\leq i \leq j-1\)) : speed of laser source in layer \(i\)  
t\(i\)(1 \(\leq i \leq j-1\)) : total deposition time of layer \(i\)  
\(h_i(1 \leq i \leq j-1)\) : average layer height of layer \(i\)  
t\(j\)(\(j\)) : deposition time from the start of layer \(j\) to deposition point \((x_j, z_j)\) |
| **Output** | \(T_{\text{initial}}(x_j, z_j)\) : initial temperature at \((x_j, z_j)\) before deposition |
| 1: | if \(j\) equal to 1 then |
| 2: | \(T_{\text{initial}} \leftarrow T_0\) |
| 3: | else |
| 4: | \(T_{\text{initial}} \leftarrow T_0\) |
| 5: | for \(i \leftarrow 1\) to \(j-1\) do |
| 6: | if \(i\) is an odd number then |
7: \[ x_i \leftarrow L + v_i[\Delta t (j-i) + \sum_{k=i+1}^{j-1} t_k + t_d'] \]

8: \[ w_i \leftarrow x_j - x_i \]

9: \[ \text{else} \]

10: \[ x_i \leftarrow -v_i[\Delta t (j-i) + \sum_{k=i+1}^{j-1} t_k + t_d'] \]

11: \[ w_i \leftarrow x_i - x_d \]

12: \[ \text{end if} \]

13: \[ z_i \leftarrow \sum_{k=i}^{j-1} \bar{b}_k \]

14: \[ R_i \leftarrow \sqrt{w_i^2 + (z_j - z_i)^2} \]

15: \[ \Delta T_i \leftarrow \frac{Q_i}{2\pi k R_i} e^{-\frac{v_i(w_i+R_i)}{2a}} \]

16: \[ T_{\text{initial}} \leftarrow T_{\text{initial}} + \Delta T_i \]

17: \[ \text{end for} \]

18: \[ \text{return } T_{\text{initial}} \]

---

Fig. 4. 2. 6  Pseudo-code of computing $T_{\text{initial}}$ iteratively for single-bead wall

In addition, even though 2-D single-bead wall is used here as the example, this introduced methodology for calculating preheated temperature $T_{\text{initial}}$ can be applied to 3-D case. So far, we finished the introduction of our methodology for extending our model from one-dimensional case to multi-dimensional case by incorporating the thermal history of the existing build. In next subsection 4.3, our extended multi-dimensional model will be validated by simulation and comparison with three experiments conducted on LENS system with Ti-6Al-4V.
4.3 Validation of multi-dimensional model of DED process

In subsection 4.2, we proposed a methodology to extend our process model of melt-pool geometry in DED process from one-dimensional case to multi-dimensional case by incorporating the thermal information of the existing part of the build. The thermal information of the existing build lies in the parameter $T_{\text{initial}}$ in (4.2.1) denoting the initial or preheated temperature of the current bead. In subsection 4.2, we introduced a method of predicting $T_{\text{initial}}$ by superposition of the Rosenthal’s solution of temperature fields generated by past virtual heat sources, with each virtual heat source corresponding to one past bead. In this subsection, three multi-dimensional structures fabricated by DED technology using Ti-6Al-4V will be utilized as examples to validate our proposed model in subsection 4.2. These three multi-dimensional structures are the single-bead wall, the parallel patch build and the L-shape structure.

4.3.1 Model validation with the single-bead wall

In this subsection, our extended multi-dimensional model will be validated by two single-bead walls deposited in [6], which is a common example of 2-D case. The image of these two single-bead walls is shown in Fig. 4.3.1.1. In [6], only one bead is deposited in each layer of a single-bead wall and 62 layers in total are deposited for each wall. The deposition direction switches every single layer, which means the odd layers are fabricated from left to right and even layers are fabricated from right to left. The single-bead wall is deposited by Optomec® LENS MR-7 system with a 500W IPG Photonics fiber laser. The deposition happens in an environment filled with argon atmosphere so that the laser optics and melt-pool are under protection. The material used in deposition is Ti-6Al-4V, which is in the form of
powder and distributed by four equally spaced nozzles with a rate of 3.0g/min. The material properties of Ti-6AL-4V used in our model simulation are listed in Table 2.4.1 in section 2.4. The single-bead wall are deposited on a 76.2mm long, 25.4mm wide and 6.4mm thick Ti-6AL-4V substrate with one side clamped. The desired dimension of the single-bead wall is 38.1mm long, 12.7mm tall and 3mm wide. The primary experiment conditions and geometry measurements of these two single-bead walls are listed in the Table 4.3.1.1. The main difference between the working condition of these two subcases lies in the dwell time at the completion of each layer. More details about deposition of these two single-bead walls can be found in [6].

(a) Single-bead wall with no dwell between layers (case 1)  
(b) Single-bead wall with 20s dwell between layers (case 2)

Fig. 4. 3. 1. 1 Images of two subcases of single-bead wall [6]
Table 4.3.1.1 Experiment conditions and geometry measurements of two single-bead walls [6]

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured laser power (W)</td>
<td>410</td>
<td>415</td>
</tr>
<tr>
<td>Scanning speed (mm/s)</td>
<td>8.5</td>
<td>8.5</td>
</tr>
<tr>
<td>Additional dwell between layers(s)</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>Total wall height (mm)</td>
<td>11.2</td>
<td>10.7</td>
</tr>
<tr>
<td>Measured wall length (mm)</td>
<td>39.2</td>
<td>37.2</td>
</tr>
<tr>
<td>Measured wall width (mm)</td>
<td>3.0</td>
<td>2.2</td>
</tr>
</tbody>
</table>

As we introduced in chapter 2, there are three unknown parameters $Q^c$, $\beta$ and $r$ in our model, whose values for material Ti-6AL-4V have been calibrated in section 2.4 with experimental data collected from 1-D case/single bead deposition. The calibrated values of $Q^c$, $\beta$ and $r$ for Ti-6AL-4V are listed in Table 2.4.2. The calibrated $Q^c$ and $\beta$ listed in Table 2.4.2 will re-used in our simulation of the single-bead wall here, while the width to height ratio $r$ will be re-calibrated according to the measured wall width and total height provided in Table 4.3.1.1. The values of parameters $Q^c$, $\beta$ and $r$ used in our simulation of the single-bead wall are displayed in Table 4.3.1.2.

Table 4.3.1.2 Calibrated parameters of Ti-6AL-4V used in simulation of thin-wall structure

<table>
<thead>
<tr>
<th>$r = w/h$</th>
<th>$Q^c$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2/(10.7/62)</td>
<td>111.72W</td>
<td>0.30256</td>
</tr>
</tbody>
</table>

Fig. 4.3.1.2 is the plot of parameter $T_{initial}$ for wall 1 and wall 2 calculated by our approach proposed
in section 4.2. In (a), $T_{\text{initial}}$ of the first layer is fixed as constant equal to the ambient temperature; the deposition of the second layer starts from the right-hand side of the wall, whose parameter $T_{\text{initial}}$ is higher than the rest part of this bead. This is a result of the fact that the right end of layer 2 has just been heated up by laser source of layer 1; the deposition of the third layer begins from the left-hand side, whose parameter $T_{\text{initial}}$ is higher than the rest of this bead. This is also because this part has just been heated up by laser source in the second layer. In (c), the curve shape of $T_{\text{initial}}$ for wall 2 is consistent with the result in (a). In addition, it can be seen that $T_{\text{initial}}$ of wall 2 is much cooler than $T_{\text{initial}}$ of wall 1, which is due to the additional 20s dwell time at the end of each layer in wall 2.

Fig. 4.3.1.2 Initial temperature $T_{\text{initial}}$ calculated by our approach in terms of wall distance and layer number. (a) $T_{\text{initial}}$ of first three layers of wall 1; (b) $T_{\text{initial}}$ of all 62 layers of wall 1; (c) $T_{\text{initial}}$ of first three layers of wall 2; (d) $T_{\text{initial}}$ of all 62 layers of wall 2.

Our model prediction of layer height profile of several layers in case 1 are plotted in Fig. 4.3.1.3. The layer height profiles of even layers are flipped over horizontally so that they also start from the
LHS, which is convenient for comparison with layer height profiles of the odd layers. In Fig. 4.3.1.3, a bump can be observed near the start of each layer except for the first layer. This observation can be explained by the shape of our simulated $T_{\text{initial}}$ in Fig. 4.3.1.2, in which we can see that the start of each layer has a higher initial/preheated temperature $T_{\text{initial}}$ than the rest of this layer, except for the first layer. In addition, the layer height profile increases in the bottom layers as the layer index goes up due to the heat accumulation effect, while the layer height profiles of the top layers almost overlap each other since the heat accumulation effect has already reached its steady-state. Overall, these observations of the simulated layer height profiles in Fig. 4.3.1.3 make a lot of sense according to our physical insight of DED process.

Fig. 4. 3. 1. 3  
Layer height profile of several layers in case 1
The total height of 62 layers of case 1 is shown in Fig. 4.3.1.4, which is obtained by simply adding up our simulated layer heights of all 62 layers. Our model prediction and measurement of total wall height are plotted by blue solid curve and red dash line, respectively. As shown in Fig. 4.3.1.4, our model prediction of the total wall height is a good approximation of the measurement. The prediction error is within 0.8% - 2.0% between wall distance 6.5mm – 33mm. In addition, bump occurs on both edges of the wall, which is consistent with our analysis in Fig. 4.3.1.3. In Fig. 4.3.1.1 (a), bumps can also be seen on both edges of the deposited wall, which supports the shape of our model prediction curve strongly.

![Fig. 4.3.1.4 Total wall height profile of case 1](image)

Apart from melt-pool geometry, our extended model can also be used to predict the average
melt-pool temperature in the multi-dimensional case. The simulated melt-pool temperature profiles for several layers are displayed in Fig. 4.3.1.5. The temperature profiles of even layers are also flipped over horizontally for the convenience of comparison. In Fig. 4.3.1.5, we can observe that the melt-pool temperature has an overshoot at the start of these layers and will drop to its steady-state value eventually. It can be observed that the steady-state melt-pool temperature is just a little bit above the melting temperature of material Ti-6AL-4V (1923K), which matches our experience from experiments. In addition, the melt-pool temperature profile of the first layer is slightly higher than that of other layers, which can be explained by the smaller melt-pool geometry in the first layer. However, there exists several points to be discussed about our melt-pool temperature prediction: First, it should be mentioned that a constant convection coefficient $\alpha_c = 664597 \frac{W}{m^2 K}$ is used in simulation of all 62 layers, which may not be true in reality. In future, FEA method can be employed to produce rough estimations about this convection coefficient for different beads; Second, the accuracy of our model prediction of melt-pool temperature is still questionable due to the lack of experimental data. But this numerical simulation demonstrates the capability of our extended model to produce temperature prediction.
Next let’s proceed to case 2 of the single-bead wall in [6], where idling time occurs at the completion of each layer. The layer height profiles of several layers in case 2 are plotted in Fig. 4.3.1.6. The profile of the even layers is also flipped over horizontally for the convenience of comparison. We can observe that the layer height profile increases as the layer index is increased, which also occurs in case 1. However, the increment between different layer height profiles in case 2 is not as obvious as it in case 1, which is due to the additional 20s dwell at the completion of each layer in case 2. In addition, the bump of layer height profile observed in Fig. 4.3.1.3 disappears in Fig. 4.3.1.6, which is also a result of the additional 20s dwell time used in case 2.
The profile of total wall height in case 2 is displayed in Fig. 4.3.1.7, which is easily calculated by adding up all 62 simulated layer height profiles. Our model prediction and the measurement are plotted by blue solid curve and red dash line, respectively. It can be seen that our model predicts the experimental data quite well with a prediction error around 3% within 10mm − 27mm of the wall. In addition, the simulated wall height profile in case 2 looks flatter than it in case 1, which is the result of disappearance of bump in simulated layer height profile of case 2. This disappearance of the bump in case 2 is also supported by Fig. 4.3.1.1 (b), which is an image of deposited single-bead wall of case 2.
Fig. 4. 3. 1. 7  Total wall height profile in case 2

Fig. 4.3.1.8 compares our model prediction of wall 2 height profile and scanned image of wall 2, which is generated by scatter plot with color as a function of the wall height. Based on this comparison, we can also tell our model servers as a good approximation of the height profile of wall 2.
Fig. 4.3.1.8  Our model prediction of wall 2 height profile versus scanned image

(MATLAB scatter plot with color as a function of height).

The melt-pool temperature profile of several layers in case 2 is shown in Fig. 4.3.1.9. The profile of even layers is flipped over horizontally as well. The profiles of different layers almost overlap each other in this figure, since the melt-pool geometry of different layers in case 2 are very close to each other and same laser power is applied to every layer. Except for this, all the other observations in Fig. 4.3.1.9 are consistent with our analysis of Fig. 4.3.1.5. The points to be discussed mentioned in Fig. 4.3.1.5 also exist for Fig. 4.3.1.9 here.
After analyzing our model prediction of case 1 and case 2 separately, they can be plotted together for comparison. The wall height profile of several representative layers in case 1 and case 2 are shown together in Fig. 4.3.1.10. As expected, it can be observed that the wall height of case 1 at a given layer is higher than it of case 2 with same layer index, which can be explained by the additional 20s dwell time in case 2.
Based on the simulation result and our analysis, our extended model of multi-dimensional DED process can predict the geometry of the single-bead walls from [6] quite well, which is a common example of 2-D case. Our extended multi-dimensional model can also be used to predict the average melt-pool temperature for the single-bead walls, which needs further investigation by FEA method or experimental data.
4.3.2 Model validation with the patch build

In section 4.3.1, our extended multi-dimensional model is validated by single-bead walls deposited in [6], which is a common two-dimensional example. In this section 4.3.2, a more complex case in [41] – parallel patch build will be employed to validate our model for three-dimensional case. The parallel patch build is deposited on Optomec® LENS MR-7 with directed energy deposition technique using material Ti-6AL-4V. The dimension of the parallel patch build and the substrate is illustrated in Fig. 4.3.2.1.

![Parallel Patch and Substrate Diagram](image)

**Fig. 4.3.2.1** Dimension of parallel patch build and substrate [41]

The target height of the parallel patch is 0.76mm, which is designed to be achieved by deposition of three layers. The pattern of this three-layer parallel patch build is illustrated in Fig. 4.3.2.2.
The hatch patterns of these three layers are parallel to each other, while the deposition direction switches every single bead. It is designed to have 36 beads in each layer. Fig. 4.3.2.3 are images of the parallel patch sample from different views. The patch sample in Fig. 4.3.2.3 was cut into halves to study the microstructure of the sample.

(a) Top view
Fig. 4. 3. 2. 3 Images of the parallel patch sample from different views, picture courtesy of ARL at The Pennsylvania State University
After the patch sample is finished, we cut four cross-sections down from it as illustrated in Fig. 4.3.2.4. Fig. 4.3.2.4 (a) shows the top view of the half sample, in which the origin of the coordinate system denotes the starting point of the deposition of the first layer, and the first bead in the first layer is deposited along the $+X$ direction. The $x$-coordinate of A, B, C and D four cross-sections are 18mm, 9mm, 4.5mm and 0.4mm, respectively. Figure 4.3.2.4 (b) is the scanned image of the half patch sample, which is generated by the MATLAB scatter plot based on the image data. In the scatter plot, the color is associated with the height of the sample, and the segments $A_1A_2$, $B_1B_2$, $C_1C_2$ and $D_1D_2$ are corresponding to cross-sections A, B, C and D in (a), respectively. These four segments will be scanned and the scanned images will be compared with prediction from our process model. In fact, these four segments are thin-band segments rather than line segments, which is to ensure enough data in the scanned image: for example, the width of segments $A_1A_2$, $B_1B_2$ and $C_1C_2$ is 0.8mm and the width of segment $D_1D_2$ is 0.2mm.
Fig. 4.3.2.4 (a): Photo of the half patch sample. (b): Scanned image of the half sample.

Photo and scanned image courtesy of ARL at The Pennsylvania State University

The experiment conditions of the patch build used in my simulation are listed in Table 4.3.2.1 below. More details about experimental setup can be found in [41].

<table>
<thead>
<tr>
<th>Laser Power (W)</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scanning Speed (mm/s)</td>
<td>10.58</td>
</tr>
<tr>
<td>Hatch Spacing (mm)</td>
<td>0.71</td>
</tr>
<tr>
<td>Layer Height (mm)</td>
<td>0.25</td>
</tr>
</tbody>
</table>

The material properties of Ti-6AL-4V used in simulation are listed in Table 2.4.1 in section 2.4.

The calibrated values of three unknown parameters $Q^c$, $\beta$ and $r$ of Ti-6AL-4V are listed in Table
4.3.2.2 below. The values of $Q^c$ and $\beta$ are the ones from section 2.4, while the melt-pool width to height ratio $r$ is re-calibrate here: the measured melt-pool width for the parallel patch is around 0.892mm and the designed melt-pool height is equal to the layer height 0.25mm.

<table>
<thead>
<tr>
<th>$r = w/h$</th>
<th>$Q^c$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.892/0.25</td>
<td>111.72W</td>
<td>0.30256</td>
</tr>
</tbody>
</table>

Table 4.3.2.2 Calibrated parameters of Ti-6AL-4V used in simulation of parallel patch build

Fig. 4.3.2.5 shows the melt pool height profiles of the first and last (42nd) bead in each layer. The height curves of the first beads in these three layers are plotted in solid lines, and the height curves of the last beads in these three layers are plotted in dash lines. As expected, the last (42nd) bead of each layer has a larger height than the first bead in the same layer due to the heat accumulated from deposition of previous beads. In addition, the three beads on the front side of the patch (1st bead of layer 1, 42nd bead of layer 2, and 1st bead of layer 3) have increasing height profile with the increase of layer number, which is also the case of three beads on the back side of the patch. These observations are both caused by the heat accumulation during the deposition process. Besides, the layer height profile of the last bead in each layer has a bump near its start, which is caused by the higher preheated temperature $T_{initial}$ of this region. However, no bumps can be observed on the height profiles of the first bead in each layer, which is due to the idling time consumed by the z-movement of the nozzle before the start of each layer. The observations from Fig. 4.3.2.5 are consistent with our analysis of single-bead wall in subsection 4.3.1.
Fig. 4.3.2.5  Simulation of melt-pool height profile of different layers in parallel patch

Fig. 4.3.2.6 shows a comparison between our model prediction of patch height profile and scanned image at cross-section A, B, C and D in Fig. 4.3.2.4. The relative error between the average height from our model prediction and the average height measured from the actual patch sample is less than 15%, which implies that our model is a good approximation of the experimental work here.
The melt-pool temperature profiles of all beads are plotted in Fig. 4.3.2.7 below. The blue, red and green curves denote the melt-pool temperature profile occurring in first, second and third layer, respectively. The melt-pool temperature profiles are plotted in the same direction for the convenience of comparison. In general, an overshoot happens at the start of each bead then the temperature profile drops to its steady-state value around 2200K, which is very close to the melting temperature of
Ti-6AL-4V. The average melt-pool temperature within each layer is calculated, it can be observed that this value decreases as the layer index increases, which can be explained by the larger melt-pool geometry in higher layer. However, the points to be discussed mentioned w.r.t. Fig. 4.3.1.5 still exist for Fig. 4.3.2.7 here.

![Simulation of melt-pool temperature profile of different layers](image)

Fig. 4.3.2.7 Simulation of melt-pool temperature profile of different layers

The 3-D surface plot of our simulated three-layer parallel patch build is displayed in Fig. 4.3.2.8. It occurs that the top surface of the patch is unflat: the two edges of the patch are higher than the middle part. This unflatness can be observed in the image of patch sample Fig. 4.3.2.3(c) and is consistent with our analysis of the single-bead walls in section 4.3.1. The average height of our simulated parallel patch is 0.7689mm, which is very close to the designed patch height 0.76mm and the prediction error is 1.17%.
Based on the simulation result and our analysis, our extended model of multi-dimensional DED process can predict the geometry of parallel patch build quite well, which is a relatively simple example of 3-D case. In addition, our extended multi-dimensional model also can be used to predict the average melt-pool temperature for the parallel patch build, which needs further investigation by FEA method or experimental data.

4.3.3 Model validation with the L-shape structure

In section 4.3.2, our multi-dimensional model of DED process is validated by parallel patch build, which is a relative simple 3-D case since all the beads are deposited in parallel direction. In this section, a more complicated 3-D case, the L-shape structure from [42] will be employed to further
validate our multi-dimensional model.

Two L-shape structures are directed energy deposited on Optomec® LENS MR-7 system using material Ti-6AL-4V, with a 7.62cm long, 7.62cm wide and 0.635cm thick substrate [42]. The maximum laser power of the system used for deposition is 500W. The L-shape structure is designed to consist of a one-bead leg and a three-bead leg, with a length of 2.54cm and a height of 5.08cm for each leg. The target height 5.08cm is designed to be achieved through deposition of 286 layers with an adjustment of laser power applied to one-bead leg and three-bead leg. The entire deposition process happens in a sealed environment filled with argon to protect the laser optics and avoid the occurrence of oxidation [42]. The deposition path of the L-shape structure is illustrated in Fig. 4.3.3.1.

![Deposition path of L-shape structure](image)

Hatches 1-4 and 5-8 denote the deposition path in odd and even layer, respectively. Hatch spacing exists between hatches 2-4, and hatches 5-7, during which transition time occurs with laser beam turned off. The difference between these two L-shape structures from [42] lies in the dwell time between layers, which happens at the completion of hatch 4 and hatch 8. The images of these two
cases of L-shape structure are displayed by Fig. 4.3.3.2.

![Image of L-shape structure](image1)

(a)

![Image of L-shape structure](image2)

(b)

Fig. 4.3.3.2 Images of L-shape structure (a) 0s dwell (b) 4s dwell [42]

The experimental conditions used in simulation of two cases of L-shape structure are listed in Table 4.3.3.1 below. The value of calibrated parameters $Q^*$, $\beta$ and $r$ are listed in Table 4.3.1.2. The material properties of Ti-6AL-4V are listed in Table 2.4.1. More details about the experimental setup can be referred to [42].
Table 4. 3. 3. 1 Experimental conditions used in simulation of L-shape structure

<table>
<thead>
<tr>
<th>Laser Power (W)</th>
<th>450W for 1-bead leg and 350W for 3-bead leg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scanning Speed (cm/min)</td>
<td>63.5</td>
</tr>
<tr>
<td>Hatch Spacing (mm)</td>
<td>0.81</td>
</tr>
<tr>
<td>Transition Time Between Hatches (ms)</td>
<td>97</td>
</tr>
<tr>
<td>Dwell Between Layers (s)</td>
<td>0s for case 1 and 4s for case 2</td>
</tr>
</tbody>
</table>

The 3-D plot of our simulated L-shape structure in case 1 (no dwell) is displayed in Fig. 4.3.3.3. We can see that bump occurs on both sides of every leg, which is a result of the higher initial/preheated temperature $T_{initial}$ at that region. This observation of bumps happening on the sides is consistent to our analysis for the single-bead walls and the parallel patch build, and supported by the images of the samples. In addition, we can see that the corner of our simulated L-shape structure is higher than the rest of the structure, which makes sense since more heat is accumulated at the corner part. The average heights of the one-bead leg and the three-bead leg are 52.9mm and 48.9mm, respectively, which are relatively close to the target height 50.8mm. The one-bead leg has a higher average height due to the higher laser power applied to it.
Fig. 4. 3. 3. 4 3-D plot of simulated L-shape structure in case 1

Fig. 4.3.3.4 shows a comparison between the height profile simulated by our model and the scanned image of case 1. The scanned image is generated by MATLAB Scatter plot, whose color is associated with the height. In (a), the average height of our model predicted curve is 52.9mm and the average height from the scanned image is 52.17mm, which implies that our model prediction is around 2% higher than the measurement. In (b), the average height from our model prediction is 48.9mm, which is 6% lower than the average height from the scanned image (51.83mm). There are three height profiles simulated by our model in (b), the overall height profile of the three-bead leg is achieved by taking the maximum of these three curves corresponding to the same location. In terms of the prediction error of the average height of the L-shape structure, our model serves as a good approximation of the experimental work.
Our model prediction of height profile versus scanned image (MATLAB scatter plot) of case 1. (a) One-bead leg in case 1, in which the right-hand side corresponds to the intersection of one-bead leg and three-bead leg. (b) Three-bead leg in case 1, in which the left-hand side corresponds to the intersection of one-bead leg and three-bead leg.

The melt-pool temperature profile of all layers in case 1 is plotted in Fig. 4.3.3.5 below, with each subplot corresponding to one wall which is indexed as Fig. 4.3.3.1. The melt-pool temperature profiles are plotted in the same direction for the convenience of comparison. It can be seen that overshoot occurs at the start of each bead in wall 3 and wall 4, while only beads in odd layers of wall 1 and even layers of wall 2. This is because that hatch 1 and 2 in odd layers are simulated in a continuous fashion, as well as hatch 7 and 8 in even layers. Then the temperature profile eventually decreases to its steady-state value, whose value is just slightly above the melting temperature 1923K of Ti-6Al-4V. In addition, the average melt-pool temperature of wall 2, 3 and 4 are very close to each other, and slightly lower than that of wall 1, which is caused by the higher laser power applied to wall 1. The points to be discussed mentioned in Fig. 4.3.1.5 still apply to Fig. 4.3.3.5, for example the use
of a constant convection coefficient for all the beads, which is questionable and can be further investigated by FEA method.

The 3-D plot of our simulated L-shape structure for case 2 (with dwell) is displayed in Fig. 4.3.3.6. We can see that bump also occurs on both sides of every leg, which is consistent with our previous analysis and backup by Fig. 4.3.3.2 (b). In addition, we can see that the corner of the L-shape structure is higher than the other parts, which is also consistent with our observation and analysis in Fig. 4.3.3.3. The average heights of the one-bead leg and three-bead leg are 50mm and 45.8mm, respectively, which are lower than those numbers in case 1 due to the dwell happening at the completion of each layer.
Fig. 4. 3. 3. 6 3-D plot of simulated L-shape structure in case 2

Fig. 4.3.3.7 compares our model prediction of height profile and scanned image of case 2, where the scanned image is also obtained by MATLAB scatter plot. In (a), the average height of our model predicted curve is 50mm and the average height from the scanned image is 51.3mm, which implies that our model prediction is around 3% lower than the measurement. In (b), the average height from our model prediction is 45.8mm, which is 7% lower than the average height from the scanned image (49.1mm). We also take the maximum of the three curves in (b) as the overall height profile of the three-bead-leg. In terms of the prediction error of the average height of the L-shape structure, our model serves as a good approximation of the experimental work.
Our model prediction of height profile versus scanned image (MATLAB scatter plot) of case 2. (a) One-bead leg in case 2, in which the right-hand side corresponds to the intersection of one-bead leg and three-bead leg. (b) Three-bead leg in case 2, in which the left-hand side corresponds to the intersection of one-bead leg and three-bead leg.

The melt-pool temperature profiles of all the layers of case 2 are displayed in Fig. 4.3.3.8, with each subplot corresponding to one wall, which are indexed as Fig. 4.3.3.1. The melt-pool temperature profiles are plotted in the same direction for the convenience of comparison. It is very similar to Fig. 4.3.3.5 that overshoot occurs at the start of each bead in wall 3 and wall 4, while only beads in odd layers of wall 1 and even layers of wall 2. This is also because that hatch 1 and 2 in odd layers are simulated in a continuous fashion, as well as hatch 7 and 8 in even layers. Then the temperature profile drops to its steady-state value which is just slightly above the melting temperature 1923K of Ti-6Al-4V. The average melt-pool temperature of wall 2, 3 and 4 are very close to each other, and slightly lower than that of wall 1, which is due to the higher laser power applied to wall 1. In addition,
the average melt-pool temperatures of case 2 are a little bit higher than those of case 1, which can be explained by the smaller melt-pool geometry of case 2. However, these questionable points mentioned in Fig. 4.3.1.5 still apply to Fig. 4.3.3.8, for instance the use of a constant convection coefficient for all the beads, which can be further investigated by FEA method.

![Melt-pool temperature profile of simulated L-shape structure in case 2](image)

**Fig. 4.3.3.8** Melt-pool temperature profile of simulated L-shape structure in case 2

Based on the simulation result and our analysis, our extended model of multi-dimensional DED process can predict the geometry of two L-shape structures in [42] quite well, which is a relatively complex example of 3-D case. Our extended multi-dimensional model can also be used to predict the average melt-pool temperature for the L-shape structure, which needs further investigation by FEA method or experimental data.
4.4 Summary

In this chapter, we propose the methodology for extending our 1-D model of melt-pool geometry and temperature of DED process to a multi-dimensional model. The main novelty of our multi-dimensional model lies in its incorporation of the thermal history of the existing part, which can be captured by the parameter $T_{\text{initial}}$ in (4.2.1) and (4.2.2). In the multi-dimensional case, this parameter $T_{\text{initial}}$ can be explained as the initial/preheated temperature of the pre-deposition point, which can be calculated by superposition of Rosenthal’s solutions of heat sources in existing part of the build, with one heat source associated with one past bead. After proposing the method for our model extension, three types of multi-dimensional builds from existing literature are utilized to validate our extended model, including the single-bead wall (2-D), the parallel patch build (3-D) and the L-shape structure (3-D). The simulation result and corresponding analysis show that our extended model can provide a good prediction of the geometry of the deposited part, while the prediction of the average melt-pool temperature needs further investigation by FEA method or experimental data.
Chapter 5 Melt-Pool Height Control for Multi-Dimensional DED Process

5.1 Introduction

As we mentioned in chapter 1, additive manufacturing technology is a cutting-edge technology which can bring us a lot of benefits, such as the freedom to design parts with complex geometry. However, the application of this technology is still very limited in industry due to several issues. One of the biggest issues of AM technology is the poor accuracy of its part geometry, which may result in instability of the manufacturing process as our analysis at the end of section 4.1. In chapter 3, the possibility of improving geometry accuracy of the one-dimensional DED process has been explored by designing advanced model-based control algorithms, whose effectiveness has been demonstrated in simulation environment. In the multi-dimensional DED process, the accuracy of part geometry can be further impaired due to the heat accumulated from the entire deposition process. The three examples (single-bead wall with no dwell, parallel patch, L-shape structure with no dwell) employed in section 4.3 has demonstrated this conclusion, where constant working conditions are applied to all layers and the layer height increases as the layer index goes up. In addition, some other defects of part geometry in multi-dimensional DED process can be observed from these three examples, for instance the unflatness of the top surface of the part. Our extended multi-dimensional model proposed in section 4.2 has been validated to capture these dynamics quite well by these three examples.

These imperfections of the part geometry produced in multi-dimensional DED case provide us the
motivation for advanced control system design. However, there exists very limited studies on design and implementation of closed-loop control system for multi-dimensional DED process in existing literature. In [24], an optimal iterative learning controller was developed for melt-pool width, height and temperature control by regulating the laser power, scanning speed and powder flow rate at the same time, based on a height-dependent dynamic model. However, the height dependence of this multi-dimensional model relies on calculation through FEA method and interpolation, which is a relatively complicated way compared with our model. A repetitive process MPC controller was designed in [31] for melt-pool height control by using the spatial powder flow rate as the control input. However, the multi-dimensional model used in control system design lacks dependency of part geometry on laser power, which is a deficiency of the model. In [33] and [34], a single-neuron self-learning controller was employed to control the width of a wall structure using the scanning speed as the control input, which was designed based on a nonlinear Hammerstein model. The model used in [33] and [34] is a fixed one even in a multi-dimensional build, which is questionable based on our analysis in chapter 4. In addition, the nature of the single-neuron self-learning controller in [33] and [34] is just a discrete form PID controller, which can be applied even without a process model.

The remaining part of this chapter will be organized as follows. The methodology of designing a feedback linearization controller based on our multi-dimensional model for melt-pool height reference tracking will be introduced in section 5.2. In section 5.3, the L-shape structure introduced in section 4.3.3 will be used as a numerical example to test the performance of our feedback linearization controller design.
5.2 Introduction and design methodology of feedback linearization controller

5.2.1 Introduction of feedback linearization control

Feedback linearization is an ordinary method where the original nonlinear system can be equivalently converted into a linear system through a change of variables and defining new suitable control input. To apply feedback linearization controller, usually we will consider a class of nonlinear systems in the following form [43].

\[ \dot{x} = f(x) + G(x)u \]  
\[ y = h(x) \]

(5.2.1.1)  
(5.2.1.2)

where \( f : D \rightarrow \mathbb{R}^n \), \( G : D \rightarrow \mathbb{R}^{m \times n} \), and \( h : D \rightarrow \mathbb{R}^l \) are sufficiently smooth on a domain \( D \subseteq \mathbb{R}^n \). \( m \) and \( l \) denotes the order of the system, number of inputs and number of outputs, respectively.

There are two types of feedback linearization control, the first type is called full-state linearization, where the state equation (5.2.1.1) is transformed to an equivalent linear form completely, but nonlinearity may still stay in the output equation (5.2.1.2). The other type is called input-output linearization, where the input-output relation is equivalently converted into linear form, while the state equation may be only partially linearized. Since the state variable and system output are both the melt-pool height in our control system design, these two types of feedback linearization are equivalent concepts in our case. Thus only full-state linearization will be considered in the remaining part of this chapter.

A sufficient condition for cancelling the nonlinearity in the state equation (5.2.1.1) is that the state equation (5.2.1.1) has the form of (5.2.1.3).
\[
\dot{x} = Ax + B\gamma(x)(u - \alpha(x)) \quad (5.2.1.3)
\]

where \( A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{m \times m} \) and the pair \((A, B)\) is controllable, functions \( \alpha : \mathbb{R}^n \to \mathbb{R}^m \) and \( \gamma : \mathbb{R}^n \to \mathbb{R}^{m \times m} \) are defined on domain \( D \). In addition, the non-singularity of function \( \gamma(\cdot) \) is assumed for any \( x \in D \). Then the state equation in the form of (5.2.1.3) can be converted into an equivalent linear system easily via defining a new control input and a relationship between the original and the new control input which depends on the state variable \( x \).

\[
u = \alpha(x) + \beta(x)\nu \quad (5.2.1.4)
\]

where \( \nu \) is the new control input and \( \beta(x) = \gamma(x)^{-1} \), whose existence is guaranteed by our assumption about the non-singularity of function \( \gamma(\cdot) \). If we plug (5.2.1.4) into (5.2.1.3), the state equation becomes a linear equation (5.2.1.5)

\[
\dot{x} = Ax + B\nu \quad (5.2.1.5)
\]

Then state variables \( x \) can be driven to arbitrary value by new control input \( \nu \) since the pair \((A, B)\) is controllable. And the corresponding original control signal \( u \) can be calculated via (5.2.1.4), which requires information of state variables \( x \).

However, the absence of form (5.2.1.3) for state equation (5.2.1.1) doesn’t necessarily imply that the state equation (5.2.1.1) can’t be feedback linearized. As we know, there exists more than one state-space model of a system, which depends on the selection of the state variables. Even though the state equation is not in the form of (5.2.1.3), this form may be achieved through a change of variables \( z = T(x) \), which means the state equation has structure (5.2.1.3) with \( z \) as the state variables. For the transformation \( T(\cdot) \), its range should include the origin and be invertible. In addition, we require the transformation \( T(\cdot) \) to be a diffeomorphism, which means both \( T(\cdot) \) and \( T^{-1}(\cdot) \) are continuously differentiable. If there exist such a transformation \( T(\cdot) \) so that the state equation of the system can be
converted into the form (5.2.1.3), this nonlinear system (5.2.1.1) is said to be feedback linearizable. More details of feedback linearization, such as a sufficient and necessary condition for the system to be feedback linearizable, can be found in Ref. [43].

5.2.2 Design of feedback linearization controller for melt-pool height control in multi-dimensional DED process

As our analysis in previous chapters, we are highly motivated to design a melt-pool height controller to improve the accuracy of part geometry, which is beneficial for the stability of the manufacturing process, especially for the multi-dimensional case. The dynamic process model of melt-pool height has been extended and validated for multi-dimensional case of DED process in chapter 4, which in turn will be used in the model-based control system design in this subsection.

The objective of this controller is to realize melt-pool height reference tracking for each layer so that the accuracy of the part geometry and the stability of the manufacturing process can be both improved. Since the melt-pool height is the only variable to be controlled, it is enough to employ only one variable as the control input. In this thesis, the laser power $Q$ is selected as the control input and the melt-pool height is the output to be controlled, which makes it a single-input-single-output (SISO) case of control. The laser power $Q$ is chosen as the control input here since it is easy to be regulated and has been investigated a lot by researchers for melt-pool geometry control. Since our model of melt-pool height (4.2.1) is in the form of (5.2.1.1), a feedback linearization controller can be an intuitive choice for controlling the melt-pool height.

The dynamic equation of melt-pool height for multi-dimensional case of DED process is constructed in chapter 4 as (5.2.2.1).
\[ \dot{h} = \frac{2\beta \eta}{\rho \pi^2 r^2 c_i (T_m - T_{\text{initial}}) h^2} (Q - Q^*) - \frac{v}{2r} \tag{5.2.2.1} \]

(5.2.2.1) is expressed in time-domain, which is preferred to be converted to an equivalent space-domain form since the reference trajectory of the melt-pool height is usually expressed in space-domain. This transformation between time-domain and space-domain can be completed easily via a change of variable (5.2.2.2).

\[
\frac{dh}{ds} \times \frac{ds}{dt} = \frac{dh}{dt} \Rightarrow \frac{dh}{ds} = \frac{1}{v} \times \frac{dh}{dt} \tag{5.2.2.2}
\]

With the help of (5.2.2.2), the dynamic equation of melt-pool height in space-domain can be expressed as (5.2.2.3) below.

\[
\frac{dh(s)}{ds} = \frac{2\beta \eta}{\rho \pi^2 r^2 c_i v(T_m - T_{\text{initial}}(s)) h(s)^2} (Q(s) - Q^*) - \frac{1}{2r} \tag{5.2.2.3}
\]

One thing that should be noted in (5.2.2.3) is that the preheated/initial temperature \(T_{\text{initial}}(s)\) of the current track is a priori, which means \(T_{\text{initial}}(s)\) is completely known before the current track is deposited. The method of calculating \(T_{\text{initial}}(s)\) based on information of previous beads has been introduced in section 4.2. If we define

\[
f(s, h(s)) = \frac{2\beta \eta}{\rho \pi^2 r^2 c_i v(T_m - T_{\text{initial}}(s)) h(s)^2} \tag{5.2.2.4}
\]

\[
\tilde{Q}(s) = Q(s) - Q^* \tag{5.2.2.5}
\]

Then (5.2.2.3) becomes

\[
\frac{dh(s)}{ds} = f(s, h(s)) \tilde{Q}(s) - \frac{1}{2r} \tag{5.2.2.6}
\]

Since the objective of our controller design is to track the reference trajectory, which is equivalent to drive the tracking error to zero, we consider the dynamics of the tracking error defined as

\[
e(s) = h(s) - h_r(s) \tag{5.2.2.7}
\]

where \(h_r(s)\) denotes the reference trajectory of melt-pool height in space-domain. Then the dynamic
equation of tracking error can be derived as
\[
\frac{de(s)}{ds} = \frac{dh(s)}{ds} - \frac{dh_r(s)}{ds} = f(s, h(s))\tilde{Q}(s) - \frac{1}{2r} - \frac{dh_r(s)}{ds}
\]  
(5.2.2.8)

The second equality holds due to (5.2.2.6). One thing that should be emphasized in (5.2.2.8) is that the term \(\frac{dh_r(s)}{ds}\) is known since \(h_r(s)\) is provided by the specification of the part to be manufactured.

It’s straightforward to tell that (5.2.2.8) is an instance of (5.2.1.3) with \(A = 0, B = 1, \gamma(\cdot) = f(\cdot)\) and \(\alpha(\cdot) = \frac{1}{f(\cdot)} \left( \frac{1}{2r} + \frac{dh_r(s)}{ds} \right)\), which is a sufficient condition for feedback linearizing the state equation of tracking error (5.2.2.8) in space-domain.

If we define a new control input \(\tilde{Q}(s)\) such that
\[
\tilde{Q}(s) = \frac{1}{f(s, h(s))} (\dot{\tilde{Q}}(s) + \frac{1}{2r} + \frac{dh_r(s)}{ds})
\]  
(5.2.2.9)
then (5.2.2.8) becomes
\[
\frac{de(s)}{ds} = \dot{\tilde{Q}}(s)
\]  
(5.2.2.10)

Equation (5.2.2.10) is in the form of an integrator and controllable. A simple proportional controller can be designed with respect to \(\tilde{Q}(s)\) to drive the tracking error \(e(s)\) to zero.

\[
\dot{\tilde{Q}}(s) = -K_p e(s)
\]  
(5.2.2.11)

where \(K_p > 0\) denotes the proportional gain. The close-loop system of the tracking error becomes (5.2.2.12) with the proportional control law (5.2.2.11) plugged in
\[
\frac{de(s)}{ds} = -K_p e(s)
\]  
(5.2.2.12)

Obviously, the dynamics of the tracking error (5.2.2.12) is a first-order system with solution as (5.2.2.13), which will decrease to zero as \(s \to \infty\).

\[
e(s) = e(s_0) \times e^{-K_p(s - s_0)}
\]  
(5.2.2.13)

The complete form of the corresponding control input laser power \(Q\) can be obtained by combining...
(5.2.2.5), (5.2.2.7), (5.2.2.9) and (5.2.2.11).

\[
\begin{align*}
Q(s) &= \tilde{Q}(s) + Q^c \\
\tilde{Q}(s) &= \frac{1}{f(s, h(s))} \left( \dot{Q}(s) + \frac{1}{2r} + \frac{dh(s)}{ds} \right) \\
\dot{Q}(s) &= -K_p e(s) \\
e(s) &= h(s) - h_1(s)
\end{align*}
\]

\[Q(s) = \frac{1}{f(s, h(s))} \left[ -K_p (h(s) - h_1(s)) + \frac{1}{2r} + \frac{dh(s)}{ds} \right] + Q^c \tag{5.2.2.14}
\]

where \( f(s, h(s)) \) is defined as (5.2.2.4). Obviously, the control law (5.2.2.14) depends on real-time information of the melt-pool height \( h(s) \), which requires assistance from in-processing monitoring equipment, such as CCD camera.

In addition, there may be other constraints applied to the control law (5.2.2.14), for example the maximum laser power of the LENS system, which can be considered as a hard constraint.

To summarize, we introduce the basic concept and a sufficient condition for applying feedback linearization to nonlinear system in subsection 5.2.1, and the methodology of designing feedback linearization controller based on our extended model for melt-pool height control in multi-dimensional DED process is explained in subsection 5.2.2. In subsection 5.3, a numerical example will be utilized to demonstrate the effectiveness of the proposed feedback linearization controller for part height control.

### 5.3 Numerical example of feedback linearization controller design for part height control

In this subsection, we will use a numerical example to illustrate the effectiveness of our feedback linearization controller proposed in subsection 5.2 for melt-pool height control. The L-shape structure
used in section 4.3.3 to validate our multi-dimensional DED process model will be utilized here as the example due to the complexity of its geometry and thermal history. The deposition path for L-shape structure is shown in Fig. 5.3.1.

![Deposition path of L-shape structure](image)

**Fig. 5.3.1 Deposition path of L-shape structure [42]**

As our analysis in subsection 5.2, the laser power $Q$ is selected as the control input since it is easy to be regulated and has been investigated a lot by researchers for control of melt-pool or part geometry. The material used in this numerical example is Ti-6AL-4V, whose material properties are listed in Table 2.4.1. The calibrated parameters $Q^c$, $\beta$ and $r$ used in this numerical example are listed in Table 4.3.1.2. In this numerical example, we also assume the equipment used for deposition of the L-shape structure is the Optomec® LENS MR-7 system, which is the case of Ref. [42]. The process parameters of the L-shape structure is displayed in Table 5.3.1.
In Table 5.3.1, the idling time at the completion of each layer is assumed to be 0s since more heat can be accumulated in this case, which can better serve as the example to prove the effectiveness our proposed controller. The feedback linearization control law (5.2.2.14) designed in section 5.2.2 will be iteratively employed in a bead-to-bead fashion to realize melt-pool height reference tracking for every single bead in the L-shape structure. In (5.2.2.14), as we mentioned in section 5.2.2, the preheated temperature parameter $T_{\text{initial}}(s)$ in $f(s, h(s))$ is a priori to the current track, which can be calculated using the laser power applied to the previous beads with the methodology introduced in section 4.2.

The value of the proportional gain $K_p$ is selected as 5000 in this simulation so that the tracking error $e(s)$ can decay to zero relatively fast. The remaining question in control law (5.2.2.14) is to figure out the reference trajectory of the melt-pool height $h_i(s)$ for every single bead.

Since AM technology builds a mechanical part in a layer-by-layer fashion, the reference trajectory of the melt-pool height depends on the slicing strategy of the 3D CAD model of the mechanical part.

### Table 5.3.1 Process parameters employed in numerical example of L-shape structure [42]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target Part Height $H_r$ (cm)</td>
<td>5.08</td>
</tr>
<tr>
<td>Layer Number $N$</td>
<td>286</td>
</tr>
<tr>
<td>Leg Length $L$ (cm)</td>
<td>2.54</td>
</tr>
<tr>
<td>Scanning Speed $v$ (cm/min)</td>
<td>63.5</td>
</tr>
<tr>
<td>Hatch Spacing $H_s$ (mm)</td>
<td>0.81</td>
</tr>
<tr>
<td>Transition Time Between Hatches $\Delta t_s$ (ms)</td>
<td>97</td>
</tr>
<tr>
<td>Dwell Between Layers $\Delta t_l$ (s)</td>
<td>0</td>
</tr>
<tr>
<td>Maximum Laser Power of LENS MR-7 System (W)</td>
<td>500</td>
</tr>
</tbody>
</table>
It is very common to slice the CAD model into layers with different thickness according to the variation of the curvature [3]. Since the L-shape structure here has a uniform curvature along its height direction, we assume the 3D CAD model will be sliced with a constant layer thickness \( \bar{h} = H/N = 177.6224 \mu m \). Thus the ideal situation of deposition of this L-shape structure has a constant melt-pool height reference trajectory equal to \( \bar{h} \) for every single bead. However, the melt-pool height profile in the first several layers may be lower than its reference value \( \bar{h} \) even though the maximum laser power of the Optomec® LENS MR-7 system is applied. This can be made up in the deposition of the following layers due to the heat accumulated in the deposition process. To summarize, the reference height of the melt-pool for every bead is defined by (5.3.1) below,

\[
 h_{i,j} = \begin{cases} 
 \bar{h} & (i=1) \\
 i \times \bar{h} - \sum_{k=1}^{i-1} h_{j,k} & (i=2,...,N)
\end{cases}
\]

(5.3.1)

where \( i \) is the layer index \((i = 1,2,...,N)\) and \( j \) is the wall index \((j = 1,2,3,4)\) as defined in Fig. 5.3.1. \( \sum_{k=1}^{i-1} h_{j,k} \) in case \( i = 2,...,N \) denotes the height profile of the existing part of wall \( j \), which is assumed to be simply calculated by adding up the melt-pool height profiles from layer 1 to layer \( i-1 \) of wall \( j \).

Another thing that should be noted in the feedback linearization control law (5.2.2.14) is that the corresponding control input \( \dot{Q} \) is upper bounded by the maximum laser power of LENS system (500W), which means 500W will be applied if the control input generated by (5.2.2.14) is larger than 500W.

Before we display the simulation result of our closed-loop system, the direction of simulation for all these four walls will be defined in the first place. Fig. 5.3.2 will be utilized to illustrate the defined simulation direction of the L-shape structure.
In Fig. 5.3.2, assume you’re standing at the position marked by the red star to view the simulation results of the L-shape structure, which will lead to the simulation direction listed in Table 5.3.2.

<table>
<thead>
<tr>
<th></th>
<th>Odd Layer</th>
<th>Even Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall 1</td>
<td>From D to U</td>
<td>From U to D</td>
</tr>
<tr>
<td>Wall 2</td>
<td>From L to R</td>
<td>From R to L</td>
</tr>
<tr>
<td>Wall 3</td>
<td>From R to L</td>
<td>From L to R</td>
</tr>
<tr>
<td>Wall 4</td>
<td>From L to R</td>
<td>From R to L</td>
</tr>
</tbody>
</table>

The simulated height profiles of all the four walls are displayed in Fig. 5.3.3-5.3.6, where several typical layers occurring in the deposition process are plotted out. In these figures, the blue solid line
denotes the closed-loop system with our feedback linearization controller, while the red dash line denotes the reference height of the part. It’s clear that the closed-loop system can track its reference very well at these typical layers in all the four walls except for the edge, which corresponds to the transient behavior in the open-loop system. In Fig. 5.3.3, no transient behavior happens on the right edge of wall 1 since track 7 and track 8 in even layers are simulated in a continuous fashion. Similarly, no transient behavior occurs on the left edge of wall 2 since track 1 and track 2 in even layers are also simulated in a continuous fashion. In Fig. 5.3.5 and Fig. 5.3.6, transient behavior appears on both sides of the wall since the deposition direction reverses every layer.

![Wall 1 of L-Shape Structure](image)

**Fig. 5.3.3** Simulated height profile of wall 1 in L-shape structure
Fig. 5.3.4  Simulated height profile of wall 2 in L-shape structure

Fig. 5.3.5  Simulated height profile of wall 3 in L-shape structure
Although the closed-loop system works really well with respect to the part height at these typical layers in simulation environment, it is still worthwhile to be more careful by checking the simulated melt-pool height profile for the first several individual beads of the L-shape structure. The simulated melt-pool height profiles and the corresponding control inputs of four beads in the first layer are displayed in Fig. 5.3.7 to Fig. 5.3.10, while those of the second layer are displayed in Fig. 5.3.11 to Fig. 5.3.14.

The simulated height profile and corresponding laser power applied to bead 1 in layer 1 are displayed in Fig. 5.3.7. The blue solid line denotes the response of the closed-loop system while the red dash line is the reference trajectory. Obviously, the melt-pool height profile is below its reference trajectory even though the maximum laser power $500W$ is applied, which is caused by the relatively cool preheated/initial temperature $T_{\text{initial}}$ of this bead.
The simulated melt-pool height profile and control input of bead 2 in layer 1 are displayed in Fig. 5.3.8. Since this bead is simulated with bead 1 of layer 1 in a continuous way, the initial melt-pool height is equal to the final value of bead 1 in layer 1. We can still observe that the melt-pool height profile is below its reference even though the maximum laser power of the system (500W) is applied.
The simulated height profile and control input of bead 3 in layer 1 are shown in Fig. 5.3.9. Since the preheated temperature $T_{\text{initial}}$ at the right edge of this bead is higher than the rest of itself, we can observe that the simulated melt-pool height profile is higher at its right edge. In addition, the corresponding laser power profile is slightly below its maximum value 500W near distance 20mm, which is also caused by its relatively higher local preheated temperature $T_{\text{initial}}$. 

Fig. 5.3.8  Height profile and control input of bead 2 in layer 1
The simulated melt-pool height profile and laser power profile are plotted in Fig. 5.3.10. The simulation result for bead 4 in layer 1 is consistent with our analysis for Fig. 5.3.9. The simulated melt-pool height profile is higher at its left edge and the corresponding laser power is smaller than the maximum laser power (500W) within interval 2.5mm-10mm due to the high local preheated temperature $T_{\text{initial}}$. 
The simulated melt-pool height profile and the corresponding laser power of bead 5 in layer 2 are displayed in Fig. 5.3.11. The reference trajectory is raised at its left edge, which is for making up the transient behavior happened at the left edge of Fig. 5.3.10.
The simulated melt-pool height profile and control input of bead 6 in layer 2 is plotted in Fig. 5.3.12. The closed-loop system can track its reference quite well except for both edges: the gap at the left edge is the result of the transient behavior of the dynamic process, while the restriction from the maximum laser power leads to the gap at the right edge. The corresponding laser power is lower than 500W within interval 3mm to 11mm, since the left-hand side of this bead is heated up by the left-hand side of bead 5.
The simulation result and corresponding control inputs of bead 7 and bead 8 in layer 2 are plotted in Fig. 5.3.13 and Fig. 5.3.14, respectively. The observations in these two figures are consistent with our previous analysis.
Next the control input profile of several layers of the L-shape structure will be plotted for
comparison in Fig 5.3.15-5.3.18. Generally speaking, the corresponding laser power applied to the system decreases as the layer index increases since more heat is accumulated as the layer index goes up.

In Fig. 5.3.15, the simulation direction is from right to left for even layers in wall 1. The laser power applied to the right-hand side of the bead is lower than that of the left hand side, since the preheated temperature $T_{\text{initial}}$ is higher on the right hand side of wall 1 in even layers. The laser power reaches its maximum value 500W on the left edge of wall 1 to make up the transient behavior occurred in wall 1, which can be observed in Fig. 5.3.3.

![Fig. 5.3.15 Control input profile for several layers in wall 1](image)

In Fig. 5.3.16, the simulation direction is from right to left for even layers in wall 2. The laser power reaches its maximum value 500W at the start of the bead to make up the transient behavior occurred at the right edge of wall 2, which can be observed in Fig. 5.3.4. Since the preheated temperature $T_{\text{initial}}$ is higher near the right edge of wall 2 than its left edge, the laser power keeps
increasing after the transient behavior of wall 2 while the deposition moves towards left.

In Fig. 5.3.17, the simulation direction is from left to right for even layers in wall 3. The laser power reaches its maximum value 500W at both edges of wall 3 to make up the transient behavior happened at both edges of wall 3, which can be observed in Fig. 5.3.5. Since the preheated temperature $T_{\text{initial}}$ is higher near the left edge of wall 3 than its right edge, the laser power keeps increasing after the transient behavior of wall 3 while the deposition moves towards right.
In Fig. 5.3.18, the simulation direction is from right to left for even layers in wall 4. The laser power reaches its maximum value 500W at both edges of wall 4 to make up the transient behavior happened at both edges of wall 4. Since the preheated temperature $T_{\text{init}}$ is higher at the right edge of wall 4 than its left edge, the laser power increases slightly after the transient behavior while the deposition moves towards left.
The 3-D plot and top surface plot of the simulated L-shape structure with feedback linearization controller implemented are displayed in Fig. 5.3.19 and Fig. 5.3.20, respectively. It can be observed clearly that the top surface of the simulated part is flat and matches its designed height 5.08 cm except for the edges, which is due to the transient behavior of the dynamic process.

So far, this numerical simulation can show the effectiveness of our feedback linearization controller design for part height control via regulating the laser power in multi-dimensional DED process. In addition, the details extracted from the simulation results match our physical insight about the DED process, which also supports our design of height control. However, there are still several issues that may jeopardize the effectiveness of our designed control algorithm in experimental environment, for instance the accuracy of our process model based on which our feedback linearization controller is designed, and the accuracy and efficiency of measuring the real-time melt-pool height with the in-processing monitoring equipment.
Fig. 5.3.19 3D-plot of L-shape structure with feedback linearization controller

Fig. 5.3.20 Top surface plot of L-shape structure with feedback linearization controller
5.4 Summary

The objective of this chapter is to control the melt-pool and part height in multi-dimensional DED process based on the model we proposed in chapter 4. In section 5.1, the motivation and a brief review of the height controller in multi-dimensional DED process is provided. Then we give a concise introduction about the concept and design methodology of feedback linearization controller in section 5.2, in which the laser power is selected as the control input and the melt-pool height is the output to be controlled. The L-shape structure mentioned in section 4.3.3 is used as a numerical example in section 5.3. The numerical example shows that the closed-loop system can track its reference height effectively in simulation environment. However, several issues that may jeopardize the effectiveness of our designed control algorithm need to be investigated more in experimental environment.
Chapter 6  Conclusion and Future Work

In this thesis, we first developed a physics-based reduced-order process model, in the form of nonlinear state-space equations, of the melt-pool geometry and average temperature in one-dimensional directed energy deposition process in chapter 2. This one-dimensional model follows the work of Doumanidis and Kwak in [22] with modification on flaws in it. Our model was calibrated and validated by collected experimental data and FEA prediction of single bead deposition on LENS system using two types of material Ti-6AL-4V and Inconel 718. In chapter 3, based on our validated model, then a LQR controller was designed to regulate the melt-pool height and temperature simultaneously using both the laser power and scanning speed as the control inputs, and a MPC controller was developed to regulate the melt-pool height with laser power and scanning speed. The designed two types of controller are both showed to be effective in simulation environment for the objective of reference tracking. Some experimental work and conclusions in literature about multi-dimensional DED process proves the existence of variation between 1-D and multi-dimensional DED process, which is also investigated in this thesis. In chapter 4, our one-dimensional process model of the melt-pool geometry and temperature in DED process was extended to the multi-dimensional case by incorporating the thermal information of the finished portion of the part, which can be used to update the initial/preheated temperature $T_0$ in the 1-D model. Our proposed methodology of calculating $T_0$ is also introduced in chapter 4, which is realized by superposition of Rosenthal’s solution, with each heat source corresponding to one past bead. This extended multi-dimensional model was validated by three multi-dimensional samples in existing literature,
which are all deposited on LENS system using Ti-6AL-4V: the single-bead wall structure, the parallel patch build and the L-shape structure. Our extended multi-dimensional model was shown to be a good prediction of the part height profile by simulation, while its effectiveness of predicting melt-pool temperature profile can’t be validated due to lack of experimental data and estimation of convection coefficient in the entire DED process. In chapter 5, a feedback linearization controller is developed based the validated model in chapter 4 for reference part height profile tracking with the laser power as the control input. The L-shape structure introduced in chapter 4 with no dwell at the end of each layer is utilized as the numerical example to test the performance of our feedback linearization controller. It can be observed in simulation result that the L-shape structure with feedback linearization controller implemented has a flat top surface with height equal to its target value, except for the transient behavior on both edges of the build.

In future work, first we plan to validate our methodology of calculating preheated/initial temperature $T_0$ in multi-dimensional DED with FEA method. Secondly, our model prediction of the melt-pool temperature in multi-dimensional DED process also needs to be validated either by the FEA prediction or experimental data, with the convection coefficient employed in our model estimated by FEA method. The last step is to test the effectiveness of our proposed control system in experiment environment.
Bibliography


Appendix

Derivation of reduced-order nonlinear state-space model (2.3.11) and (2.3.12)

In chapter two, we proposed our model of melt-pool geometry and temperature in one-dimensional DED process, which consists of the improved mass balance equation (A.1), the energy balance equation (A.2) and two algebraic equations (A.3)&(A.4),

\[
\frac{d(\rho V(t))}{dt} + \rho A(t)v(t) = \beta \frac{\eta(Q - Q^c)}{\pi c_i(T_m - T_{\text{initial}})}, Q > Q^c
\]  

(A.1)

\[
\frac{d(\rho V(t)e)}{dt} = -\rho A(t)v(t)c_s(T_m - T_0) + \eta Q(t) - A_s \alpha_s(T(t) - T_m) - A_c \times [\alpha_c(T(t) - T_0) + \varepsilon \sigma(T^4(t) - T_0^4)]
\]  

(A.2)

\[r = \frac{w}{h}
\]  

(A.3)

\[l = w
\]  

(A.4)

Since the material density \( \rho \) is assumed to be constant, then we have

\[(A.1) \Rightarrow \rho \dot{V} + \rho Av = \beta \frac{\eta(Q - Q^c)}{\pi c_i(T_m - T_{\text{initial}})}\]  

(A.5)

By plugging \( V(t) = \frac{\pi}{6} w(t)h(t)l(t) \), \( A(t) = \frac{\pi}{4} w(t)h(t) \), (A.3) and (A.4) into (A.5), we can obtain the dynamic equation of the melt-pool height,

\[
\dot{h} = \frac{\beta \eta(Q - Q^c)}{\pi c_i[T_m - T_{\text{initial}}]} - \rho \frac{\pi}{4} \frac{rh^2v}{r^2h^2}
\]  

(A.6)
If we plug $e = c_s(T_m - T_0) + h_{sl} + c_i(T(t) - T_m)$ into (A.2), then we can have

$$\frac{d(\rho Ve)}{dt} = \rho V e + \rho \dot{V} e = \rho V [c_s(T_m - T_0) + h_{sl} + c_i(T - T_m)] + \rho V c_i \dot{T}$$

$$= -\rho Avc_s(T_m - T_0) + \eta Q - A_i \alpha_s(T - T_m) - A_G \times \left[ \alpha_G(T - T_0) + \varepsilon \sigma(T^4 - T_0^4) \right]$$  \hspace{1cm} (A.7)

By using (A.5), (A.7) becomes

$$\frac{\beta \eta (Q - Q^c)}{\pi c_i (T_m - T_{initial})} - \rho A v \left[ c_s(T_m - T_0) + h_{sl} + c_i(T - T_m) \right] + \rho V c_i \dot{T}$$

$$= -\rho Avc_s(T_m - T_0) + \eta Q - A_i \alpha_s(T - T_m) - A_G \times \left[ \alpha_G(T - T_0) + \varepsilon \sigma(T^4 - T_0^4) \right]$$  \hspace{1cm} (A.8)

By plugging $V(t) = \frac{\pi}{6} w(t) h(t) l(t)$, $A(t) = \frac{\pi}{4} w(t) h(t)$, (A.3), (A.4), $A_3 = \frac{\pi}{4} w(t) l(t)$ and $A_G = \frac{\pi}{\sqrt{2}} \left[ w(t) h(t) l(t) \right]^{2/3}$ into (A.8), the dynamic equation of the average melt-pool temperature can be obtained,

$$\dot{T} = \{ (\rho \frac{\pi}{4} rh^2 V - \frac{\beta \eta (Q - Q^c)}{\pi c_i (T_m - T_{initial})}) [c_s(T_m - T_0) + h_{sl} + c_i(T - T_m)]$$

$$- \rho \frac{\pi}{4} rh^2 vc_s(T_m - T_0) + \eta Q - \frac{\pi}{4} r^2 h^2 \alpha_s(T - T_m) - \left[ \frac{\pi}{\sqrt{2}} (r^2 h^3)^{2/3} \right]$$

$$\times \left[ \alpha_G(T - T_0) + \varepsilon \sigma(T^4 - T_0^4) \right] \} \left( \frac{\pi}{6} r^2 h^3 \rho c_i \right)$$  \hspace{1cm} (A.9)
Vita

Jianyi Li

Jianyi Li was born in Shenyang, China in the year of 1990. And he spent 12 years receiving education in this hometown before leaving for college education. In 2012, Jianyi Li obtained his bachelor of science degree in Department of Mechanical Engineering and Automation from Zhejiang University, which is located in Hangzhou, China and considered as one of the top universities in China. Then Jianyi started his Ph. D. study in department of Mechanical and Nuclear Engineering at the Pennsylvania State University since September 2012. Meanwhile, he joined Dr. Qian Wang’s group “Information and Controls Lab” as a graduate research assistant. During Jianyi’s Ph. D. study and graduate research assistant appointment, his research area mainly focuses on process modeling and control of laser-based metal additive manufacturing technology. In addition, Jianyi has two internship experiences during his Ph. D. study: one is in Manufacturing Process Lab at GE Global Research Center (Shanghai, China) and the other one is in Process Data Technology Group at Air Products and Chemicals R&D (Allentown, PA).