The Pennsylvania State University
The Graduate School

Department of Economics

ESSAYS IN THE ECONOMIC THEORY OF SOCIAL NETWORKS

A Dissertation in
Economics

by
Wang Xiang

Submitted in Partial Fulfillment
of the Requirements
for the Degree of
Doctor of Philosophy

August 2017

The dissertation of Wang Xiang was reviewed and approved* by the following:

Kalyan Chatterjee
Distinguished Professor of Economics and Management Science
Dissertation Advisor
Chair of Committee

Vijay Krishna
Distinguished Professor of Economics

James Jordan
Liberal Arts Research Professor of Economics

Réka Albert
Distinguished Professor of Physics and Biology

Barry W. Ickes
Professor of Economics
Head of the Department of Economics

*Signatures are on file in the Graduate School

ABSTRACT

In this dissertation, I provide three social network models addressing different aspects of the economy.

First, I consider the issue of social network formation. Meeting with strangers and meeting with friends' friends are two common patterns when observing the formation of social networks and thus are widely adopted in many statistical network formation settings. I establish a network formation model trying to find the individual level incentive behind the two patterns. Each agent enters the network sequentially choosing between forming global random links or connecting with friends' friends, knowing that his payoff depends on the distance between the player with valuable information and himself. I find that the distance between players in the resulting network asymptotically follows a Weibull distribution and the link formation decision depends on how easily information can be transmitted under the network. When the transmission is relatively easy and information can be received from far away, players prefer to meet with strangers. Otherwise they prefer to connect with friends' friends.

The second model considers a moral hazard scenario in which a monitor must detect deviations so as to provide proper incentives to attain an efficient outcome. What if the monitor himself were to deviate after being bribed by his monitored subject? I explore a multi-agent public-good provision game in which each player prefers shirking to working in the absence of exogenous enforcement and can bribe those assigned to monitor him. I find that when players agree to cooperate in choosing an optimally designed monitoring network, a core-periphery monitoring network results, with a small group of heavily monitored players who monitor all others. Under this network, a perfect Bayesian equilibrium (PBE) is supported in which shirking is prevented, bribing among players ceases and total monitoring costs are minimized. Further, the efficiency of this core-periphery structure is robust under various settings, including the dynamic spread of bribing, and under different punishments and monitoring schedules.

Lastly, I consider a phenomenon of innovation adoption and diffusion. Innovations are crucial to the long run economic growth; however, not all new ideas are adopted by the majority of the society even though similar ones which appear later turn out to be commercial successes. I establish a model in which multiple potential entrants each can bring a new technology into a market with consumers connected under a social network. Technology has the character of network externality and entrants must rely on the word-of-mouth communication under the social network to promote their products. I find that high quality innovations are not guaranteed to perform better than low quality ones under the scenario and timing is a crucial determinant of their commercial performances. Low quality innovation firms are likely to enter the market whenever possible but high quality ones might strategically wait for a proper time to acquire a better market share.

TABLE OF CONTENTS

LIST OF FIGURES

ACKNOWLEDGEMENTS

I am indebted to my adviser, Prof. Kalyan Chatterjee, for his help and guidance. Many thanks to Prof. Edward Green, Prof. Vijay Krishna, Prof. James Jordan, Prof. Réka Albert, Prof. Syed Nageeb Ali, Prof. Ron Siegel, Prof. Nima Haghpanah, Prof. Rohit Lamba, Prof. Marc Henry, Prof. Navin Kartik, Prof. Benjamin Golub, Prof. Brendan Daley, Prof. Shengwu Li, Prof. Utku Unver, Prof. Mohammad Akbarpour, Prof. Geoffroy de Clippel, Prof. Francis Bloch, Prof. Maximilian Mihm, Prof. Ignacio Esponda, Prof. Alessandra Casella, Prof. Bruno Strulovici, Prof. Emir Kamenica, Prof. Erik Eyster, Prof. Nicole Immorlica, Prof. Giancarlo Spagnolo for useful comments and discussions. I also thank Chaohai Shen, Xun Li, Yiming Guo, Wenjing Ruan and Jianpeng Deng for useful discussions. The remaining errors are my own.

Chapter 1 Introduction

1.1 Network as a framework to study social interactions

Interactions among individuals are important topics in Economics. And the conceptual framework of network plays as an important tool in studying those topics because it can help better understand a number of different dimensions involved in those topics. The Economics of network provides insights about the way that individuals form relationships with others, games played under a given set of restrictions determining who could interact with whom, and the mechanism behind the social relationship formation and the corresponding social interaction. In this thesis I provide three social network models addressing different aspects of the economy.

First, I consider the issue of social network formation. Meeting with strangers and meeting with friends' friends are two common patterns when observing the formation of social networks and thus are widely adopted in many statistical network formation settings. In chapter 2 I establish a network formation model trying to find the individual level incentive behind the two patterns. Each agent enters the network sequentially choosing between forming global random links or connecting with friends' friends, knowing that his payoff depends on the distance between the player with valuable information and himself. I find that the distance between players in the resulting network asymptotically follows a Weibull distribution and the link formation decision depends on how easily information can be transmitted under the network. When the transmission is relatively easy and information can be received from far away, players prefer to meet with strangers. Otherwise they prefer to connect with friends' friends.

The second model considers a moral hazard scenario in which a monitor must detect deviations so as to provide proper incentives to attain an efficient outcome. What if the monitor himself were to deviate after being bribed by his monitored subject? In chapter 3 I explore a multi-agent public-good provision game in which each player prefers shirking to working in the absence of exogenous enforcement and can bribe those assigned to monitor him. I find that when players agree to cooperate in choosing an optimally designed monitoring network, a core-periphery monitoring network results, with a small group of heavily monitored players who monitor all others. Under this network, a perfect Bayesian equilibrium (PBE) is supported in which shirking is prevented, bribing among players ceases and total monitoring costs are minimized. Further, the efficiency of this core-periphery structure is robust under various settings, including the dynamic spread of bribing, and under different punishments and monitoring schedules.

Lastly, I consider a phenomenon of innovation adoption and diffusion. Innovations are crucial to the long run economic growth; however, not all new ideas are adopted by the majority of the society even though similar ones which appear later turn out to be commercial successes. In chapter 4 I establish a model in which multiple potential entrants each can bring a new technology into a market with consumers connected under a social network. Technology has the character of network externality and entrants must rely on the word-of-mouth communication under the social network to promote their products. I find that high quality innovations are not guaranteed to perform better than low quality ones under the scenario and timing is a crucial determinant of their commercial performances. Low quality innovation firms are likely to enter the market whenever possible but high quality ones might strategically wait for a proper time to acquire a better market share.

1.2 Terms and notations

Before going into details of my thesis, in this section I will provide some terms and notations used through this work to make the remaining sections clearer.

Networks are modeled as graphs in my thesis. Given a finite number of players $N = \{1, 2, ..., n\}$, a directed network G is described as a $N \times N$ matrix in which the entry g_{ij} represents the link from agent i to agent j. When the network is non-weighted, a common notation is that $g_{ij} = 1$ represents the existence of a link while $g_{ij} = 0$ represents the absence. For a weighted graph, on the other hand, $g_{ij} > 0$ indicates the existence of a link and the value represents the corresponding weight. I further define $\forall i \in N(g), g_{ii} = 0$ for convenience. In later sections, I will use the notation g_{ij} directly to represent the directed link from agent i to j henceforth as long as no confusion may occur. For any agent $i \in N$, I define $M_i^+ = \{j \in N | g_{ij} > 0\}$ the out-neighborhood of agent i, indicating the set of agents with direct links from agent i. I define $m_i^+ = |M_i^+|$ the out-degree of agent i. Similarly I can define the in-neighborhood and in-degree as $M_i^- = \{j \in N | g_{ji} > 0\}$ and $m_i^- = |M_i^-|$ representing those agents with direct links to agent i.

A network is non-directed if $g_{ij} = g_{ji}, \forall i, j \in N$, denoting as g. Given a directed graph G, if $\nexists i, j \in N$ such that $g_{ij} > 0, g_{ji} > 0$ and $g_{ij} \neq g_{ji}$, the corresponding non-directed graph g is constructed by the following process. For each $g_{ij} > 0$ in G, let $g_{ji} = g_{ij}$ in $g$and let $g_{ij} = 0$ in g if $g_{ij} = g_{ji} = 0$ in G. Similar to the definition of in-degree and out-degree, for any agent $i \in N$, I define $M_i = \{j \in N | g_{ij} > 0\}$ the neighborhood of agent i, representing the set of agents that have links with agent i in a non-directed network g. I further define $m_i = |M_i|$ the degree of agent i.

For a non-directed network g, I denote $N(g)$ to be all involved players in g and $E(g)$ to be all involved links in g. A graph $\tilde{g}$ is a sub-graph of g if $N(\tilde{g}) \subseteq N(g)$ and $E(\tilde{g}) \subseteq E(g)$. A (u, v)-walk is a sub-graph of g with agents $u, k_1, k_2...k_\kappa, v \in N(g)$ such that $g_{uk_1}, g_{k_1k_2}, ...g_{k_\kappa v} > 0$[1]. The length of the walk is the number of links in it. When the walk involves no duplicated agents, it is called a (u, v)-path and the minimum length of all (u, v)-paths is called the distance between u and v, denoting as $d_g(u, v)$. For convenience I define $d_g(u, u) = 0, \forall u \in N(g)$ and define $d_g(u, v) = +\infty$ if there is no path between u and v. When $d_g(u, v) < +\infty$, I call the two agents u and v are connected with each other under the network g. A network g is called connected if $\forall i, j \in N(g), d_g(i, j) < +\infty$.

An indicator function $\mathbb{I}_E$ is a function with $\mathbb{I}_E = 1$ if the event E occurs. Its value equals zero otherwise.

Without further clarification, a distribution function F of a random variable $X \in \mathbb{R}$ represents the cumulative distribution function of the random variable satisfying $F(x) = Pr(X \leq x)$.

[1] They are not necessarily distinct.

Chapter 2 Strong ties or structural holes? A network formation game

2.1 Background

2.1.1 Motivation

Relationship between individuals plays a crucial role in both social and economic issues because it guides the way people interact with each other. Why people sometimes want to enter a new environment meeting strangers, in which case the individual plays the role of a bridge between two otherwise far away groups, while other times want to meet friends' friends and thus form close relationships with a group of people via strong ties? Answering this question is helpful for us to better understand the structure of social networks and economic interactions between individuals.

However, the analysis of the social network formation is not a easy task because in many scenarios the social network is formed in a dynamic way. And as a result, people who participate in the network formation process do not necessarily know everyone else involved in the network. Consider the network of alumni of an university. Ignoring the potential member loss, the network grows with more and more graduates enters the group annually. An old graduate, when managing his relationship with other members in the group, might potentially know several future entrants. But it is never possible for him to know every future entrants into the group, needless to say the network structure among them. Therefore a dynamic network formation framework with incomplete information is needed when we want to understand the structure of those types of social networks.

People may still argue that without complete information about other participants, a farsighted member in the network might be able to form a good expectation of members he does not know as well as the relationships among them. However, one argument is that network structures in reality are usually too complicated for a single agent involved in it to form a detailed expectation. Assuming in the alumni case people become friends if and only if they used to attend a same class. Thus a single graduate in the network might potentially know how many other friends one of his friends has. He might even have some ideas about how many other friends one of his friends' friends has. But how about the relationship among those friends' friends' friends? It is very unlikely that when the individual manages his own social relationships, he takes all those detailed expectations into consideration. As a result, it is reasonable to analyze a social network formation process in which each individual's information and action sets are limited. And people's decision making only relies on some aggregate level information.

In this chapter I establish a network formation model. Each agent enters the network sequentially choosing between global random meetings and meetings with friends' friends to form relationships with other existing agents. After that, agents start sharing valuable private information under the network formed. However, information can only be shared within some distances and thus each agent's payoff depends on the number of others close enough to him under the network. I find that the distance between players in the resulting network asymptotically follows a Weibull distribution. And the easier the information can be passed through, the more likely agents will choose to meet strangers during the social network formation process.

This chapter unfolds as follows. The remaining part of section 2.1 briefly discusses the relationship between my work and the previous literature. The basic model is provided in section 2.2. In section 2.3, I formally present the main results of the paper by analyzing the asymptotic distance distribution of the resulting network and individual's incentive accordingly. Section 2.4 presents the conclusion and some potential extensions of the work.

2.1.2 Related literature

This work is related with researches in the field of social network formation. For long, sociologists discuss the advantages of strong ties in a social network which lead to closely linked communities and weak ties between communities acting as bridges[2]. Starting from Myerson (1977) and followed by Jackson and Wolinsky (1996) and several others, many attempts are made by economists to analyze the network formation process from individuals' incentives. Those models, commonly known as strategic models, begin with each individual's choice given his utility function, mainly depending on the final structure of the network, and his cost function, usually depending on the number of links formed. Interestingly, albeit the rich economic intuitions behind, the equilibria induced by those models sometimes do not fit empirical data, especially networks with enormous number of nodes, well enough[3]. One potential reason why those incentive-driven models do not work well is that most of them treat all possible ways of link formations the same, ignoring the fact that in realty individuals can not always meet and form relationship with another particular agent. On the other hand, another branch of models, usually called statistical models, are developed first by mathematicians, physicians and computer scientists and followed by economists looking at the topology and evolution

[2] See, for example, Lin, Cook and Burt (2001) for a review.

[3] For instance, Bala and Goyal (2000) establish a model where each agent gets payoff from all others connected with him and pays costs towards links directly formed by him. They conclude that the stable converging non-empty networks can be either circles or stars depending on exogenous parameters. However, those minimally connected networks are rarely found in real world social networks with large scales.

of real networks [4]. Comparing with strategic models, those frameworks usually do not specify the incentives that drive agents making choices but focus on the resulting structures for some given mechanisms of network formations, trying to explain how some empirical stylized patterns can be reached via specific procedures. Thus, those models fit empirical data better at a cost of the lack of economic intuitions and interpretations.

One trial to provide economic intuitions into statistical models is the hybrid growth mechanism. One classical hybrid growth model, which is a combination of the random growth graph model (a growing variation of Erdős and Rényi, 1959) and the preferential attachment mechanism (Barabási and Albert, 1999), is first discussed by Jackson and Rogers (2007) although the idea that local rules can lead to the scale free degree distribution is first raised by Barabási and Albert (1999) and concluded by Alexei Vázquez (2003). Jackson and Rogers show that many empirical patterns including the small world property, the scale free degree distribution and the high clustering of links are natural consequences of this framework and also provide the relationship between "the random/network-based meeting ratio" and the resulting degree distribution of the network. They further conclude that the social welfare under the case of concave utility functions with respect to degree is determined by the ratio of the two kinds of meetings due to the resulting stochastic dominance of degree distributions. However, their model still lack the incentive of single player in the sense that the ratio is exogenously given without economic reasons. In other words, we know the combination of random relationship formation and meeting friends' friends can lead to the desired empirical pattern but still do not know the driving force behind each individual when he makes the decision between the two ways of relationship formations. Unlike their model, the network formation model established in this chapter mainly focuses on the incentive of a single agent, considering how the network structure may influence the payoff of an individual and how he may react accordingly to change the local network structure around him. Another study needs to mention is by Kleinberg et al (2008) in which each agent competes to be the bridge between groups of agents in order to charge rents from information transmitted across the bridge. Different from this setup, agents in my model do not have the direct incentive to become bridges. Instead, I will show that even when the potential externality from the network formation stage can not be endogenized by rents, it can be resolved simply by the improvement of information transmission technology.

There are other works focusing on the issue of information transmission under some given networks. Callaway et al (2000) and Newman et al (2001) provide a probability generating function method to analysis the percolation issue on a given random graph with fixed node occupation rate,

[4] See Newman (2003) for a review.

which is applied in Campbell (2013) where information about new commodities is transmitted under an Erdős-Rényi random network of potential buyers. Unfortunately, as far as I know there is no analytical way to deal with the percolation problem under a general non-random network with strictly positive average clustering coefficient. On the other hand, when consider a more general case in which the return from a given link is endogenously determined, Brueckner (2006) build a model considering the scenario where the return is decided by the effort payment and the closeness of the network, which is exogenously given. Since the utility function used in Brueckner's model is additive, one may assert that the denser the network the higher the return. My model does not focus on the network game and adopts a simplified payoff function where the information transmission is purely determined by the distance between the source and the receiver. More generalized ways of modeling will be discussion later in the conclusion section.

2.2 Model setup

Throughout this chapter all networks are non-weighted. There are in total N agents in the model, where N is large enough but finite, playing a network formation game. Each agent i chooses an action a_i in $A = \{Local, Global\}$ at time 0 and the network is formed in the following way.

N_0 agents are chosen uniformly at random from all agents and form an initial network G^0 [5], regardless of their actions. The initial network can be arbitrary with only two restrictions that each agent has at least 1 neighbor in the corresponding g^0 and $N_0 \ll N$[6][7]. Other agents enter the network sequentially with one agent being picked uniformly randomly without replacement from the remaining agents each time. Each agent i who chooses the action $a_i = Global$ forms 2 links uniformly at random to the existing agents when entering. Each agent i who chooses the action $a_i = Local$, on the other hand, first forms 1 link uniformly at random to the existing agents when entering and, after the random match, the agent forms 1 link uniformly at random within the neighbors of his neighbor. Denoting the directed network when agent i finishes his random meeting with all existing agents as G^i and the corresponding non-directed network as g^i, those who choose *Local* form their second link randomly within $M_j(g^i)$ in which $g^i_{ij} = 1$. The final network after all agents enter the network is denoted as G and g.

The first way of link formation can be interpreted as a global random meeting[8] while the second

[5] And the corresponding non-directed network can be denoted as g^0.
[6] The initial network is not necessarily connected.
[7] This can also be modeled as an exogenously arbitrarily given initial network whose nodes are not contained in N in which case all n agents in N enter the network sequentially.
[8] I will call it global meeting henceforth.

is a local meeting process [9], that is, meeting with friends of friends. Many real world networks follow this pattern of formation. Examples include individuals enter new environments like social network websites, schools, firms or other organizations where the global meeting represent the forming of relationships with people they newly recognize and the local meeting is more likely to happen when new friends are introduced by existing friends[10].

One difference from the model of Jackson and Rogers (2007) that needs to be remarked is that in the local meeting process an agent i can random meet among the neighborhood of his neighbor under the non-directed network g^i instead of the directed one G^i. In other words this process implicitly means that a link becomes non-directed once it is formed regardless of who starts it, which is the case in a friendship network because a friendship is usually non-directed regardless of the starter. This difference has a huge influence on an individual's strategy because, unlike in Jackson and Rogers where a local meeting of agent i from his existing neighbor j always ensures him a meeting with an agent who enters the network earlier than the existing friend j, forming a link by local meeting from a friend j in this model can not guarantee an agent i to meet people who has a first mover advantage comparing with his existing friend j[11].

Also need to mention is that I require a large enough N to ensure that the limiting structure of the network to be reached approximately and the initial network g^0 not to affect the limiting structure of the network. However, N is still fixed and finite to ensure the game to be well defined.

Each agent owns a unique valuable information worth V that can be shared in the resulting network. To share information with others, the distance between the source and the receiver should be no greater than D. That is to say, each agent i can receive the information from an agent j if and only if $d_g(i,j) \leq D$. The threshold distance D in my model is exogenous. In reality it can be interpreted as a measure of the information transmission technology. A higher threshold indicates a better technology and thus people far away from each other can still efficiently exchange information.

I assume that the whole game is common knowledge for all agents although when making the decision in A, agents don't know their entry order.

Finally, each agent i's payoff is given by $u_i = V \cdot \sum_{j \in N} \mathbb{I}_{(d_g(i,j) \leq D)}$, in which $\mathbb{I}_E$ is the indicator

[9] I will call it local meeting henceforth.

[10] Note agents are assumed to be homogeneous with fixed number of total links allocating between global meetings and local meetings. This can easily be achieved by assuming a suitable convex cost function of link formation and solving the payoff maximization problem. Adding uncertainty or heteroskedasticity among agents in the model can largely increase the complexity but bring little insight, at least not focusing on the issue I are looking for in this chapter.

[11] Also note in my model an agent must first form the global meeting link and then search for neighbors, while in Jackson and Rogers one can search globally for agents and search for neighbors directly without forming the global meeting link. All those lead to the fact that the best strategy of an agent in Jackson and Rogers framework is to search globally but not form any global meeting link. Instead, all links should be formed via local meetings resulting in a preferential attachment model.

function.

To conclude, the model includes a triple $(N, A, \{u_i\}_{i \in N})$. This model describes a dynamic network formation process in which each individual's purpose of participating is to receive information from others. Each agent needs to decide using which way of meeting to form links, knowing that the distances between him and others in the resulting network eventually determines his payoff.

2.3 Optimal formation strategy

The equilibrium concept applied here is a Perfect Bayesian Equilibrium (henceforth PBE) in which the strategy of agent i is a mapping $\sigma_i : B_i \to \Delta(M)$ satisfies sequential rationality, that is, σ_i maximizes the expected payoff of player i under his belief B_i about all other players' actions and the resulting network structure, while the network structure coincides with each individual i's belief B_i. Ideally, I would like to get the accurate relationship between the network structure and actions of all agents. However, the network structure is too complicated since the potential value of $m_i(g)$ for each agent i ranges from 2 to $N-1$ while for each given value the potential number of structures among agent i's neighborhood is $2^{\frac{m_i(g)\cdot(m_i(g)-1)}{2}}$ because each two of agent i's neighbors my have a link in between. Therefore, instead of providing a close form belief system, I follows two steps to give a discussion of the equilibrium solution.

I first provide a description of the distance distribution of the network given agents' choices in A. Then I discuss the circumstance faced by a single agent, focusing on the relationship between the distance distribution from other agents towards him and his own action. And thus I can give some descriptions of the equilibrium outcome of the game. But before I start the detailed analysis, there are two examples to illustrate some basic intuitions behind the argument.

Example 1. Low information transmission technology

Consider the case where the information transmission is given by $D = 2$. Note the information transmission is such that each agent i can only receive information from his friends $M_i(g)$ and friends' friends $\cup_{j \in M_i(g)} M_j(g)$. To see how this environment can influence agents' choices between *Local* and *Global*, we first consider the mean-field approximation of the degree distribution of the network[12]. Notice that the out-degrees of all agents are given by $m_i^+(G) \equiv m$ and thus is uniformly

[12]The mean-field approximation is a commonly used method when analyzing the behavior of a large and complex stochastic system. Instead of considering the system itself, a simple version is analyzed in which all components act in the same way as if they are the average player.

distributed. Therefore, if one randomly picks an agent via the agent's out-link, it is equivalent as if the agent is uniformly randomly picked from all agents.

Now consider the in-degrees of agents. An agent i with $m_i^-(G^t)$ has a probability $\frac{2-r/2}{t} + (\frac{(r/2)\cdot m_i^-(G^t)}{2t})$ of getting a new in-link in G^{t+1}, where r represent the proportion of agents choosing *Local*. The first term represents the chance that the newly added agent forms a directed link with this agent i via a global meeting or the new link is formed via a local meeting but the newly entered agent randomly picks his neighbor's neighbor via an out-link. The second term shows the chance that the newly added agent links with agent i via a local meeting and randomly picks his neighbor's neighbor via an in-link. Therefore an approximation of the in-degree distribution can be found by solving the differential equation:

$$\frac{dm_i^-(G^t)}{dt} = \frac{2-r/2}{t} + \frac{(r/2)\cdot m_i^-(G^t)}{2t} \tag{1}$$

Which gives:

$$F_t(m^-) = \begin{cases} 1 - exp(-\frac{m^-}{2}), & r = 0 \\ 1 - (\frac{8-2r}{r\cdot m^- + 8 - 2r})^{4/r} & \text{otherwise} \end{cases}, \forall t \text{ for } m^- \geq 0, \text{ where } r = \frac{\#Local}{N} \tag{2}$$

Together with the uniform out-degree, we have the following result:

Lemma 1. *The asymptotic degree distribution of the resulting network is given by:*

$$F(m) = \begin{cases} 1 - exp(-\frac{m-2}{2}), & r = 0 \\ 1 - (\frac{8-2r}{r\cdot m + 8 - 4r})^{4/r} & otherwise \end{cases}, m \geq 2, r = \frac{\#Local}{N} \tag{3}$$

A simulation with 10,000 agents are done with all agents choosing the same action. Figure 1 shows results from the simulation, in which the left graph illustrates the degree distribution when all agents choose *Local* and the right graph illustrates the degree distribution when all agents choose *Global*. Both graphs are log-linearized.

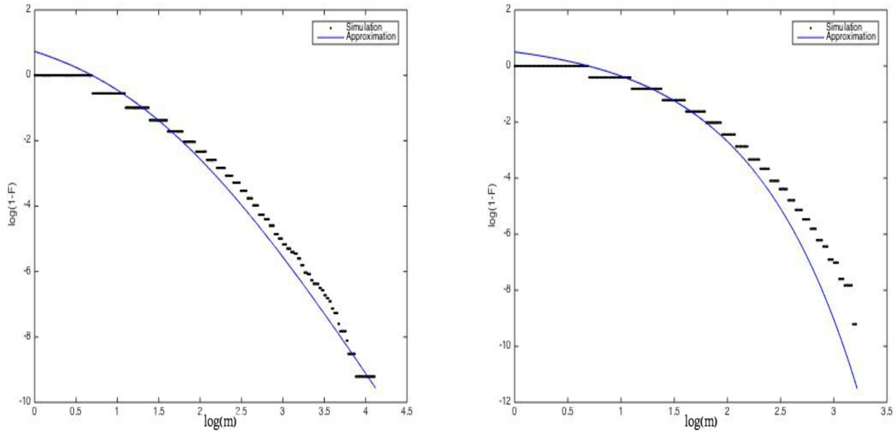

Figure 1: Simulated degree distribution

Given the degree distribution of the network[13], one can start to consider the best action of a single agent i. Since $E(m_i(g)) = 4$ regardless of his action and all agents who enter the network later than agent i do not influence agent i's best action, the problem faced by agent i is $max_{a_i \in \{Local, Global\}} E(| \cup_{j \in M_i(g^i)} M_j(g^i)|)$. Note that by global meeting agent i can randomly pick a neighbor whose expected number of neighbors is 4 while a local meeting gives him half the chance of linking with an agent with 4 expected neighbors and half the chance of linking with an agent with $\sum m \cdot \frac{m \cdot k_m}{\sum m \cdot k_m}$ expected number of neighbors, in which $k_m = F(m+1) - F(m)$. Given the degree distribution in lemma 1 we find that $(\sum m \cdot \frac{m \cdot k_m}{\sum m \cdot k_m} - 1) + (4 - 1) > 8$[14] indicating that *Local* is a dominating strategy under this circumstance.

Given agent i is selected arbitrarily, we can assert that under the case that only agents within distance 2 can share information, the final outcome is that all agents choose to have local meetings instead of global ones. Intuitively, this example shows that the local meeting acts as a preferential attachment mechanism such that one is more likely to find a person with a high degree. I conclude this example with the following lemma:

Lemma 2. *With $D = 2$, the unique equilibrium is that each player i chooses $a_i = Local$.*

[13]This result is a reasonable approximation only when n is large enough. For a discussion of the finiteness of n one can refer to Krapivsky and Redner (2002).

[14]This is because one expected number of degree is used for each neighbor if agent i chooses the local meeting.

Example 2. High information transmission technology

We now turn to another example where the information transmission is given by $D = N$. Note the total number of players in the game is N and thus the distance between any two connected players can be no greater than $N - 1$. Therefore, the information transmission is such that each agent can receive the information from any other agent as long as they are connected. Because choosing *Global* can weakly increase the probability that the whole network is connected, each agent i will choose *Global* as his dominating strategy[15], which can be concluded as following:

Lemma 3. *With $D \geq N$, the unique equilibrium is that each player i chooses $a_i = Global$.*

Together with example 1, one might guess that the determinant of players' actions is the easiness of information transmission. The easier the information can be transmitted the more likely each individual will choose to form global links. Now I start to show that the guess is close to the truth. I first derive a mean-field approximation of the distance distribution of the network. The main result is given by:

Proposition 1. *(a) The distance between two randomly selected agents in g asymptotically follows a Weibull distribution $F^{dis}(x) \approx 1 - e^{-(\frac{x}{\lambda})^{\kappa}}$ with $\kappa > 1$;*

(b) Denoting $r = \frac{\#Local}{N}$ as before, the distance distribution satisfies $d\kappa/dr < 0$ and $d\lambda/dr > 0$.

The intuition behind the result is that the distance between two randomly selected agents in the network can be considered as the minima of all paths between the two nodes. And thus the extreme value theorem can be applied. Further, when players form more local links, the number of pairs with short distances increases but the average distance of all pairs increases. This is intuitive because global links usually play the role of bridges to connect two otherwise far away parts of the network. Local links, on the other hand, make the network denser and thus there will be more close friends. Figure 2 shows the simulation results with 10,000 agents, in which the left graph illustrates

[15]This is because the initial network g^0 is not necessarily connected.

the probability density function of distance distribution when all agents choose *Local* and the right illustrates the one when all agents choose *Global*. One can check the approximation is quite accurate. Unfortunately, the explicit expression of parameters with respect to agents' action (represented by r) is not clear yet and further study is needed.

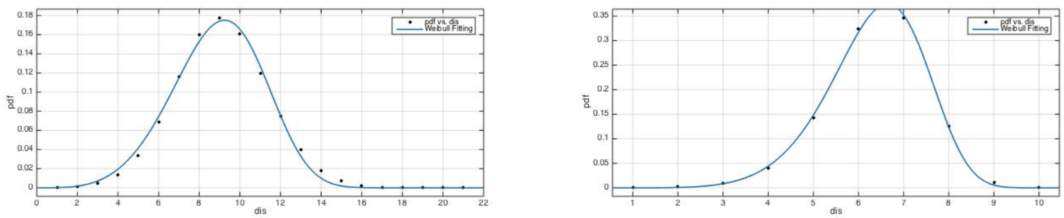

Figure 2: Simulated probability density function of distance distribution

Now we turn to the problem faced by an individual i. Similar to the argument in the first example, the agent does not care about actions of people entering the network after him. Therefore, without loss of generality I can simply assume that the agent, when making the decision in A, believes that he is the last agent entering the network facing a distance distribution follows Weibull distribution F^{dis} given in proposition 1.

The two actions can lead to different distance distribution from other agents and agent i. To be more precise, his first link uniformly randomly picks an agent j in the network whose distance towards all other agents $N \backslash \{i, j\}$ also follows a Weibull distribution F^{dis}. Thus agent i's distance x towards all other agents $N \backslash \{i, j\}$ approximately follows a Weibull distribution $F^{dis}(x-1)$. Further from the first example we also know that the expected degree of agent j is 4. Now if the second link of agent i is a local meeting, 1 out of the 4 neighbors of agent j will be linked. For every other agent $k \in N \backslash \{i, j\}$ there will be at least a chance of $\frac{1}{4}$ that the distance between agent k and agent i to be one step closer due to the new link. Therefore, we have the boundary for the distance distribution from agent i towards all other agents as:

$$\frac{3}{4}F^{dis}(x-1) + \frac{1}{4}F^{dis}(x) \leq F^{dis|local}(x) \leq F^{dis}(x) \tag{4}$$

On the other hand, if agent i chooses to have the second link as a global meeting, he again randomly selects an agent other than j and the distance from all other agents to agent i can be described as:

$$F^{dis|global}(x) = 1 - (1 - F^{dis}(x-1))^2 \tag{5}$$

With the analysis above one can get the following result:

Proposition 2. *For any given number of players N, there exist $2 < d_l^\star \leq d_u^\star$ such that: (a) for any $D \leq d_l^\star$, there is a unique equilibrium under which all players choose to form local links; and (b) for any $D \geq d_u^\star$, there is a unique equilibrium under which all players choose to form global links.*

The result shows that when the transmission is relatively easy and information can be received from far away, players prefer to meet with strangers. Intuitively, this is because when the information transmission is relatively easy, people are more likely to link with two different areas of the whole network in order to receive benefits from both areas. When the information is not easy to transmit, on the other hand, people prefer to strengthen their connections within an area in order to find more important people (in the sense of degree) within that area. One example in reality is the relationship between the communication technology and the social interaction of individuals. In areas where the communication technology is poor, people usually only contact with their close neighbors and thus the resulting social network is dense. While in areas where the technology is well developed, individuals usually rely less on meeting with friends' friends and the resulting social network turns to have more "bridges". Figure 3 shows a simulation result of the threshold with $N = 10,000$. The upper left graph compares $F^{dis|local}$ and $F^{dis|global}$ under the case of $r = 1$ and the upper right one compares the two distribution under the case of $r = 0$. A more clear view is from the bottom left graph which shows the difference $F^{dis|global} - F^{dis|local}$ under the case of $r = 1$. And the bottom right one illustrates the difference $F^{dis|global} - F^{dis|local}$ under the case of $r = 0$.

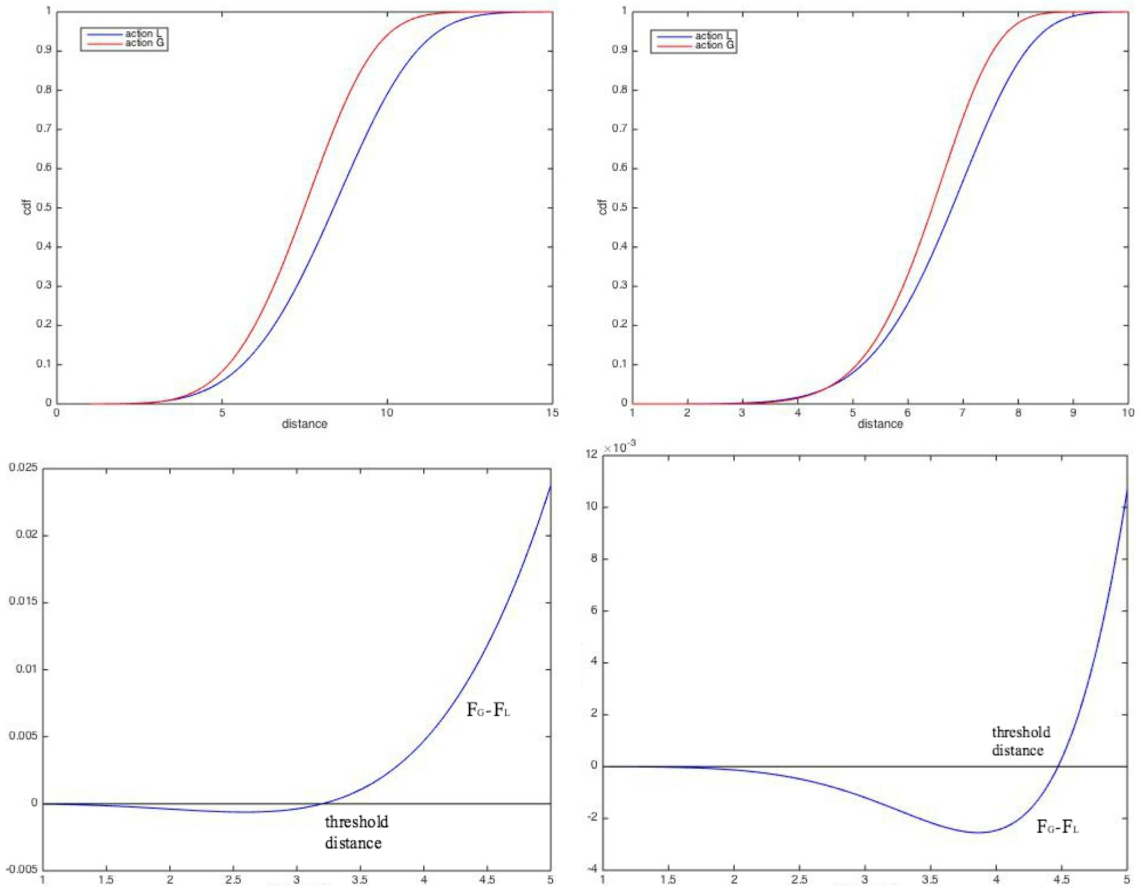

Figure 3: Simulated threshold distance

One remark is that when agent i making the choice in A, he only cares about the influence of distances between other agents and himself. Clearly his action will also affect the distance between two other agents. Specifically, when he chooses *Global*, he is more likely to be the bridge and helps shorter the distances between other agents. On the other hand, choosing *Local* will not shorten the distance between other agents. However, this kind of positive externality is ignored by agent i and thus it is possible that the network structure is suboptimal. However, the externality can be endogenized even without the help of a rent mechanism. When the information transmission technology is improved, people are likely to choose *Global* to maximize their own payoffs.

Another remark of the result is that when the threshold distance D lies in between $d_l^\star$ and $d_u^\star$, the equilibrium is not clear yet. In fact, the number of local links formed by others have two opposite effects on each individual's best response. With a higher r the shape parameter of the Weibull distribution decreases, which drives a single individual to choose *Global* as a complementary action. However, it also increases the scale parameter, which drives the individual to choose *Local* because the benefit from linking with two distinct areas decreases. Further researches are needed to better understand the mechanism behind.

2.4 Conclusion

The driving force of social network formation decisions is discussed in many different backgrounds. This paper considers a model focusing on two classical patterns: meeting with strangers and meeting with friends' friends. When each individual's action is limited in picking one of the two formation method and the payoff depends on the information sharing under the resulting network, I show that the information transmission threshold determines each individual's choice. When information can be received from relatively far away, individuals are more likely to act as structural holes by forming global links and the resulting network will have a shorter average distance. On the other hand, people prefer strong ties when the information transmission technology is poor.

I conclude by discussing some limitations of the result and potential opportunities of future research. As mentioned before, my work provides a complementary piece in between the strategic network formation models and the statistic network formation models. However, the individual action set in my model is simplified to a binary choice with no cost concerns. As a result, the best response of each individual is likely to be the combination of a cost-benefit comparison specific to my payoff function and the effect the network structure. A more realistic approach should also include the cost of different links in order to better distinguish the effect of the network structure on each individual's best strategy.

Second, my work adopts a benchmark payoff function purely relying on the distance among players. This simplification has at least three implicit assumptions. The valuable information transmitted is not memoryless, in other words, only each agent i's own information can benefit others and the fact that one agent i's information improves due to the reception of the information from another agent j will not further benefit other agents. Also, the model underestimates the benefit from strong tie by applying a deterministic information transmission threshold. And the threshold is exogenous and thus all players are assumed to be homogenous during the information transmission, regardless of their heterogeneity of network locations. Introducing a generalized payoff function or even endogenizing the transmission threshold by introducing a second stage effort payment game are necessary for more insights into the network formation problem.

Finally, my work provides an approximation of the distance distribution of the resulting network but the explicit expression of parameters is still unknown. Further studies are needed, especially the extreme value theorem in discrete scenarios, from both the theoretical and the empirical perspectives.

Chapter 3 Who will watch the watchers? On optimal monitoring networks

3.1 Background

3.1.1 Motivation

In many scenarios, a monitor is needed to ensure that economic agents have the appropriate incentives to perform the tasks which they have been contracted to execute. However, the monitor himself might also deviate after being bribed by the individuals he is monitoring. For instance, in the Enron scandal, an audit firm (Arthur Andersen) should have, in principle, executed its monitoring duty carefully but seemed unable to uncover and counter the various financial devices used by Enron to conceal its actual position. It was not until the SEC began its independent investigation of Enron that this was finally revealed to the public, resulting in a total loss to all stakeholders of more than USD 40 billion and a dramatic shock to the entire market.

The problem that arises in the setting above is essentially how to monitor a monitor. Some research on incentives suggests that the monitor should be made into a residual claimant, or more generally, that the payoff of each player should depend on the final outcome[16]. However, in the Enron example, both the CEO of Enron and the accounting firm are indeed residual claimants: they will also suffer from the market shock once the true position of the company becomes apparent. Unfortunately, compared with the potential gain, the residual rights shared by each are insufficient to prevent misbehavior. Further, it is not always possible to write contingent contracts that cover all possible outcomes. What makes my problem even more complicated is that the auditor not only might shirk his responsibilities alone but also might accept bribes from and engage in collusion with the company.

As a result, the remaining solution for the public is to hire more independent experts to monitor the company in the hope that at least some of them will not collude with the monitored subject because bribing multiple monitors is intuitively more expensive. Perhaps a superior monitor should be brought in to monitor the auditor and punish him if he is found taking bribes. In fact, these ideas were indeed adopted after the scandal. To prevent similar scenarios in the future, the monitoring system of public companies was reformed and a committee was established to provide independent

[16] See, e.g., Alchian and Demsetz's (1972), Holmstrom (1982) and Ma, Moore and Turnbull (1988).

oversight of public accounting firms and their audit services[17]. Although the solution is helpful in preventing potential collusion between the monitor and the company, it increases total monitoring costs and introduces more players whose potential misbehavior must also be accounted for. In light of these concerns, the public will attempt to determine which is the most efficient monitoring arrangement among all the players involved.

In this chapter, I explore a stylized multi-agent model in which people can either work properly, by devoting effort to the production of public goods and conducting the assigned monitoring duties if there are any, or shirk in both tasks. Each player prefers shirking to working in the absence of exogenous enforcement and can bribe others who are assigned to monitor him. Bribery helps agents collude with their monitors and shirk safely henceforth; however, it will also harm the social welfare. Therefore, I search for an efficient monitoring arrangement, including both the monitoring relationship among players and the corresponding monitoring intensity to each of the monitored subjects, that prevents individuals from shirking their responsibilities and discourages bribery among players while not resulting in unnecessary monitoring costs. I first report my main result informally[18]:

Theorem. *Among all monitoring networks that result in no shirking and no bribing, the ones that minimize the total monitoring costs have a core-periphery structure — a small group of heavily monitored players monitor all others.*

Beside the Enron scandal, this type of problem is pervasive in many other economic realms. Another example is the management of poverty reduction programs. Low-income families in rural areas are typically a burden to their neighbors because they constantly require financial assistance. A plan in China thus includes subsidies to such low-income families for both factors of production and corresponding training. Ideally, the farmers who are involved will work properly and efficiently raise their incomes. However, many of these subsidized farmers instead choose to shirk their responsibilities because it is typically not possible for the representative assigned by the central government to properly monitor all the local participants and to punish all misbehavior. As a result, the program now requires a group of local participants to monitor both one another and the remainder of the subsidy receivers[19]. Based on my results, both the reformation of the auditing system and the reorganization of the poverty reduction program not only work, but are also efficient in the sense of

[17] Public Company Accounting Oversight Board, established based on the Sarbanes–Oxley Act of 2002 (Law: Pub.L. 107–204, 116 Stat. 745).

[18] See section 3.3.1 for a formal statement of the result and an example to show the intuition behind.

[19] The new program is called the 8-7 National Poverty Reduction Plan. Wang et al. (2004) has a detailed description of the policy.

collusion control[20].

This chapter unfolds as follows. The remaining part of section 3.1 briefly discusses the relationship between my work and the previous literature. The basic model is provided in section 3.2. In section 3.3, I formally present the main results of the paper by constructing an optimal monitoring system and providing some of its properties. Some potential extensions of this study are discussed in section 3.4. Section 3.5 presents the conclusion.

3.1.2 Related literature

My work is related to several research areas. It contributes to the extensive past discussion of incentives to monitor and for monitors. Alchian and Demsetz's (1972) seminal paper first raises the idea that making a monitor the residual claimant can provide him enough incentive to avoid misconduct. Holmstrom (1982), on the other hand, suggests that an output dependent mechanism of rewards or punishments can help reaching the desired outcome. Ma, Moore and Turnbull (1988) further extend the idea by introducing private monitoring among agents to eliminate suboptimal equilibria that may occur in Holmstrom's (1982) case. However, as suggested by Rahman (2012), an output dependent mechanism might fail once publicly verifiable output is missing[21]. Instead, he considers an innovative mechanism in which agents are asked to occasionally deviate according to a predetermined schedule designed by the principal. The monitor's action thus becomes detectable because he is asked to find out the schedule. My work has a similar background to Rahman's (2012) in the sense that the punishment must be given before the realization of the output and monitoring is costly. However, I consider bribing as another possible deviation of involved players and thus Rahman's mechanism fails due to the potential collusion between agents and their monitors[22]. As a result, extra enforcement is needed and I focus on the efficient monitoring arrangement among all players involved. A few other papers discuss the issue of bribing and monitoring together; one representative example is Basu, Bhattacharya and Mishra (1992) in which a recursive model of bribing is established. Under a linear monitoring relationship, the authors find that the required monitoring intensity to each player must increase to stop bribing. Unlike the assumption of the exogenous monitoring relationship in Basu et al. (1992) with either an infinite number of monitors

[20] Although in those complex real world examples, there are debates concerning the efficiency of aspects other than collusion control. For the Enron scandal and the Sarbanes–Oxley Act, see Shakespeare (2008) for a review. For the 8-7 National Poverty Reduction Plan, see the debate between Meng (2013) and Lv (2015).

[21] See also, Prendergast (1999), MacLeod (2003), Levin (2003) and Fuchs (2007).

[22] In section 1 of his paper, Rahman mentioned that "a problem with this contract is that it is not robust to collusion: both agents could avoid effort if Friday simply told Robinson his recommendation."

or a set of elite supervisors who can not be bribed, in my model the total number of homogenous players is fixed and there is no bribeproof player. Bribery is stopped by the endogenously chosen structure of the monitoring network.

This work is also related to the literature on the cooperation under networks. Among the research surveyed in Goyal (2007), Jackson (2008) and Nava (2016), the two closest works are done by Wolitzky (2013) and Ali and Miller (2013). Wolitzky (2013) studies repeated network games with a fixed discount factor, local monitoring, global cooperation and exogenous network structures. He concludes that the maximal level of cooperation can be achieved via a grim trigger strategy. Ali and Miller (2013) consider repeated network games with a fixed discount factor, local monitoring, local cooperation and endogenously determined costly networks. Given all interactions are at the local level, a maximal level of cooperation relies on third party punishments. Ali and Miller (2013) thus concludes that the optimal network structure contains isolated cliques[23]. My model, however, adopts a non-repeated setting with local monitoring, global cooperation and endogenously determined costly monitoring networks. Instead of determining the maximal level of cooperation, I focus on the minimal cost network structure that can support a given level of cooperation. Other studies also consider issues like the communication protocol under monitoring networks[24], the Folk Theorem[25] and the costly punishment issue[26], which are not the main concerns of my paper.

Finally, my work is a complement of the literature on organizational structure. Starting from Knight (1921), the strand of literature on hierarchies discusses various driving forces behind the structure including the knowledge-based specialization[27], the communication-based specialization[28]and the decentralized decision making procedure[29]. Most of those can be placed under the category of team optimization in which there is no conflict of interest between each individual and the whole community. One exception is Baccara and Bar-Isaac (2008) in which the conflict of interest is involved and an external threat acts both as the cost of the organizational structure and the punishment to support the desired cooperation within the community[30]. In my work, however, the conflict of interests is solved by costly monitoring from within. Mishra (2006) analyzes a scenario similar

[23] Lippert and Spagnolo (2011), on the other hand, follow the argument of Wolitzky (2013) and Ali and Miller (2013) and further emphasize the importance of information transmission speeds in determining the network structure and the corresponding strategy to sustain local cooperations.

[24] Examples includes Laclau (2014) and Wolitzky (2015).

[25] The classical example is given by Ben-Porath and Kahneman (2003) and further extended by many others.

[26] Acemoglu and Wolitzky (2015) provide a model with a specialized punishment provider. The renegotiation and forgiveness issue discussed by Ali, Miller and Yang (2016) can also be considered as a cost of punishments.

[27] A stylized model is established by Garicano (2000) and further extended by Garicano and Rossi-Hansberg (2012).

[28] See, for example, Crémer, Garicano and Prat (2007).

[29] Examples include Crémer (1980) and Geanakoplos and Milgrom (1991).

[30] Recently, Ferrali (2015) has a similar model in which the boundary between the organization and the outsider is endogenous.

to mine where individuals can bribe their monitors to achieve their own goals[31]. He compares the bribing decision under a few hierarchy structures and concludes that for an exogenously given monitoring intensity, ex-post bribing ceases either by a suitable punishment or a bribeproof supervisor. I, on the other hand, search for the optimal monitoring structure and the corresponding monitoring intensity to stop ex-ante bribing under an exogenously determined punishment.

3.2 Model setup

I first provide a brief description of the model, and further details will be provided in the following subsections. Consider the scenario in which a group of homogenous risk-neutral players $N = \{1, 2, ...n\}$ attempt to cooperate with one another to produce some public good. The non-repeated game contains two main phases, and each player has a transferable endowment of 1 at the beginning of the game that can be used either as an input of production or as a value transfer to others[32]. The first phase is a bribing phase in which players can privately bribe their monitors to safely shirk later. The second phase is a public good provision process in which each player privately decides whether he wants to work.

A social planner is involved to design a monitoring system[33], and the participants play the public good game under the given monitoring system, in which they are assumed to play a Perfect Bayesian Equilibrium (PBE) of the ensuing sequential game. However, the social planner faces the problem of maximizing expected total social welfare by choosing the optimal monitoring system. Because reallocating value through bribery does not increase the total welfare of the community, a socially optimal outcome should be that all players work instead of bribing and shirking.

Therefore, the main goal for the social planner is to design an efficient monitoring system in terms of minimizing total monitoring costs while still supporting an outcome of cooperation. Figure 4 shows the process of the game. I will begin by describing the game in detail.

[31] See also, Bac (1996a, 1996b), Carrillo (2000) and Mishra (2002).

[32] Non-repeatedness is crucial in my model for two main reasons. First, with sufficiently patient players under a repeated game setting, monitoring costs can always be ignored by decreasing monitoring frequency and increasing the severity of the corresponding punishment. However, my model focuses on the case in which the Folk Theorem cannot be applied and monitoring costs are not negligible. Second, as discussed above, even if the modification is such that the discount factor is fixed, grim strategies can be adopted because the historical payoff can be observed to infer others' past actions. In my case, however, the punishment must be provided before the outcome is revealed.

[33] Although it might be argued that the existence of such a risk-neutral social planner (who is not involved in the public good provision game directly but who obtains utility from the total social welfare) is questionable in reality, an equivalent decentralized method can easily be found. One example is that all the players discuss voting for a monitoring system that can maximize the expected total welfare ex ante and use a random mechanism to determine each player's role under the selected system. Because decentralization is not the main focus of this model, I will follow the social planner's problem from this point forward.

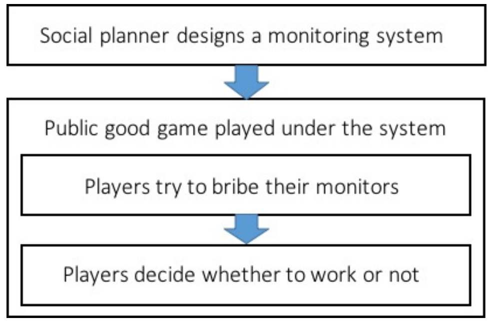

Figure 4: Process of the game

3.2.1 Monitoring system

Before the game begins, the social planner determines a monitoring system with the goal of preventing both free riding and bribing while minimizing the system's total costs. A monitoring system should contain information about both who is in charge of monitoring whom and how heavily each player is monitored. Therefore, in my model the system is described as a triple of $\{N, E, P\} = \{N, (e_{ij})_{i,j \in N \times N}, (p_i)_{i \in N}\}$ where $e_{ij} \in \{0, 1\}, p_i \in (0, 1], \forall i, j \in N$. N is the set of players who are involved, and each $e_{ji} = 1$ indicates that player j is appointed as a monitor of player i. The monitoring intensity towards each player i is described by $p_i \in [0, 1]$, which will thus be called the monitoring level of player i, and I assume that this is the probability by which player i's deviation can be caught by each of his monitors once the monitoring is realized[34]. Observations of player i by each of his monitors are assumed to be perfectly correlated[35]. However, the monitoring levels of distinct players are not necessarily equal, and the corresponding monitoring realizations are independent. Because each agent can always fully observe his own actions without the help of the monitoring system, I define $e_{ii} = 0, \forall i \in N$. Figure 5(a) shows an example of a monitoring system[36].

[34] The realization is stated more clearly later.

[35] The assumption of perfectly correlated observations ensures the tractability of the model, and relaxing this assumption does not significantly change the main result but will substantially increase the difficulty of the calculation, which is discussed in section 3.4.

[36] 5(a): An example of a monitoring system with $e_{14} = e_{15} = e_{23} = e_{24} = e_{25} = e_{35} = 1$, $e_{ij} = 0$ otherwise and $p_1 = 0.1, p_2 = 0.2, p_3 = 0.3, p_4 = 0.4, p_5 = 0.5$. 5(b): Rewrite the monitoring system as a directed weighted graph where $g_{23} = p_3 = 0.3$, $g_{14} = g_{24} = p_4 = 0.4$, $g_{15} = g_{25} = g_{35} = p_5 = 0.5$ and $g_{ij} = 0$ otherwise. Note that in this case no one monitors players 1 and 2, and thus the corresponding monitoring levels can be ignored.

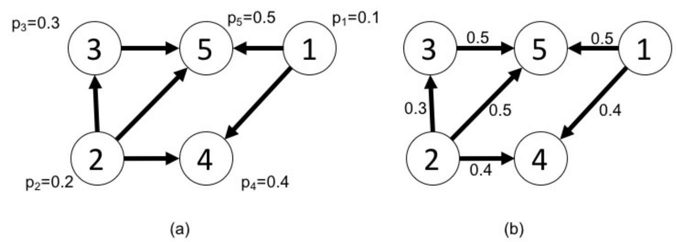

Figure 5: An example of a monitoring system

Clearly, the triple can be rewritten as a weighted directed network $G = \{(g_{ij})_{i,j \in N \times N}\}$ where $\forall i, j \in N, g_{ji} = e_{ji} \cdot p_i$. In other words, when $g_{ji} > 0$, agent j is appointed as one of the monitors of agent i, and the monitoring level of agent i is given by $g_{ji} = p_i$. Figure 5(b) shows the transformation, and for the simplicity of notation I will henceforth use G to represent a monitoring system and interchangeably refer to it as a monitoring system or as a monitoring network.

The total cost of the monitoring system is given by:

$$Cost_G = \sum_{i \in N} (\sum_{j \in N} e_{ji} \cdot p_i \cdot \beta) \tag{6}$$

The cost is paid before the game begins as a sunk cost of cooperation. To interpret this, considering a player i with a monitoring level p_i, a cost of βp_i is generated for each player in the group who is assigned to monitor him[37]. I further assume that parameter β is small enough but not negligible. In other words, a social planner prefers to have lower monitoring costs as long as players can still cooperate under it, and the participation constraints for all the players are non-binding.

To better illustrate the structure of the monitoring system, I provide some additional notations here[38]. For each player i, the set of players who should monitor him is denoted by $M_i^- = \{j \in N | g_{ji} > 0\}$), and the set of agents who are to be monitored by him is given by $M_i^+ = \{k \in N | g_{ik} > 0\}$. I denote $m_i^- = |M_i^-|$ to represent the cardinality of M_i^- and $m_i^+ = |M_i^+|$, accordingly. I further define $\mathbb{M} = \{i \in N | M_i^+ \neq \emptyset\}$, which is the set of all players with monitoring duties under the system. With those notations, I can also rearrange the cost function as follows:

$$Cost_G = \beta \cdot \sum_{i \in N} \sum_{j \in N} g_{ji} = \beta \cdot \sum_{i \in N} p_i \cdot m_i^- \tag{7}$$

Finally, the entire structure of the monitoring system is common knowledge to all players when

[37] One can interpret this cost as the effort payment to establish the monitoring devices or other protocols, such as the auditing mechanism in which each firm is required to provide its financial statement under a special format and all auditors must pay in effort to read it. A detailed statement represents a high monitoring level and therefore requires more effort payment in reading.

[38] For simplicity, all notations ignore the subscript G.

they play the public good game under it.

3.2.2 The public good game under the monitoring system

Once it has been established, the participants begin to play the public good game under the monitoring system. The game has the following schedule: 1) The players privately offer bribes to their monitors to collude with them; 2) The players privately determine whether to work; 3) The monitoring is realized before the reveal of the final production outcome, and misbehaviors are punished if the perpetrators are caught; and 4) Finally, the outcome of the public good production is realized and players obtain their payoffs. Next I illustrate the entire procedure in detail.

At the beginning of the game, each player has a transferable endowment of value 1. The endowment can either be used as bribes for other players or as inputs into the production. Before choosing public good production, players privately offer bribe plans to their monitors to engage in collusion and to avoid being monitored. Each player i independently and simultaneously chooses a bribe plan $B_i^+ = (b_{ij})_{j \in M_i^-}, \sum_{j \in M_i^-} b_{ij} \leq 1$ in which $b_{ij} \in \mathbb{R}$ is the value transfer from player i to player j. More precisely, if $b_{ij} > 0$ player i offers a positive amount of bribe to player j. Meanwhile, $b_{ij} < 0$ indicates that player i asks for a positive amount of bribe from player j[39][40].

After all the offers are made, each player i observes all $B_i^- = (b_{ji})_{j \in M_i^+}$ and simultaneously decides his response to each bribe offer, $R_i(B_i^-) = (r_{ij})_{j \in M_i^+}$ where $r_{ij} \in \{accept, reject\}$. In other words, each player i can only directly observe the bribe plan that is offered to him. Additionally, for each offer he receives, the player must decide whether to accept it. Figure 6(a) provides an example of the bribe offering and responding process[41].

[39]The situation in which a player asks for a bribe from his monitor is never acceptable under an optimal monitoring network; however, for a general model setup I still allow b_{ij} to be negative.

[40]In addition, only the player being monitored can offer bribe plans to his monitors, and a monitor cannot directly ask for value transfers from the players he monitors. In other words, collusion in this paper is modeled as bribing instead of soliciting for bribes.

[41]6(a): Player 3 offers no bribe $b_{32} = 0$, player 4 offers $b_{41} = b_{42} = 0.5$ and player 5 offers $b_{51} = -0.5, b_{52} = 0.8, b_{53} = 0.7$. Player 1 observes $b_{41} = 0.5, b_{51} = -0.5$ and rejects both, player 2 observes $b_{32} = 0, b_{42} = 0.5, b_{52} = 0.8$ and accepts all, player 3 observes $b_{53} = 0.7$ and accepts it. 6(b): Because player 3 does not offer a bribe, there is no value transfer between player 2 and 3 regardless of player 2's response. Because player 4's offer is rejected by player 1, only the transfer from player 4 to player 2 is executed. Finally, the negative part of player 5's offer is rejected by player 1, and player 5's entire plan is thus abandoned due to his budget constraints.

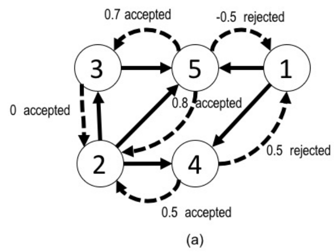

Figure 6: An example of bribing

All bribe offers that are accepted by receivers are executed, and all rejected offers that result in a zero transfer at the end of the bribing phase are abandoned with only the following exception. In a case in which player i's bribe plan B_i^+ contains some $b_{ij} < 0$ and the negative offer is rejected by player j [42], the entire plan B_i^+ is abandoned because – as a general rule – player i may face budgetary constraints [43] and might be unable to clear the remainder of his plan without the approval of a negative bribe offer [44]. In other words, when player i only offers non-negative bribes in his plan, value transfers are undertaken separately between player i and each of his monitors, depending on the responses. However, if player i offers some negative bribes in his plan, the execution of the entire plan will also depend on whether those negative bribes are accepted.

For simplicity of notation, I will assume that a player i who decides not to offer a bribe to player j simply chooses $b_{ij} = 0$ and $r_{ji}(0) = reject, \forall j \in M_i^-$, although in such a case the response is trivial because both an acceptance and a rejection lead to the same no-bribe result, which is also represented by $b_{ij} = 0$ as well. The only response that matters is the response to a non-zero bribe offer[45]. Figure 6(b) shows the execution of the bribe plans that are given in 6(a).

Before describing the monitoring realization in detail, note that the bribe plan is private between the briber and the monitor because each monitor can only observe a bribe that is offered to him. However, the bribe plan is directly observable to the corresponding monitor, which means that the monitor can observe the bribe offer without the help of the monitoring network. From the perspective of the monitoring realization, this direct observation can also be modeled as perfect monitoring. Finally, both accepting and giving a non-zero bribe plan are considered misbehaviors in cooperation, and such misbehaviors will be punished if reported. As a result, private bribe offer and acceptance is a credible collusion contract among those players[46].

[42]E.g. $r_{ji}(b_{ij}) = reject$.
[43]E.g. $\sum_{j \in M_i^-} b_{ij} \leq 1$.
[44]Allowing the execution of the plan once a sufficient amount of negative bribe offers are accepted is another way to model this same problem.
[45]E.g. $r_{ji}(b_{ij} \neq 0)$.
[46]The case in which only the briber or the receiver can be punished is discussed in section 3.4 below.

When the bribing process ends and all the bribe plans are executed, each player simultaneously privately chooses an action $a_i \in \{work, shirk\}$. Each player who shirks generates neither a public good nor a cost. Further, the shirking player neglects his monitoring duty, which means that he can obtain no information from the monitoring system even if information is available[47]. However, each player who decides to work can produce α unit(s) of public goods that can be shared with all the players at the end of the game and thus induces a cost of 1. In addition, if the working player is a monitor under the system, he can observe the information that is realized from the system accordingly. I will assume $\alpha \in (0, \frac{1}{2})$ to mean that each player prefers to shirk rather than work in the absence of other enforcements and that it is possible for each player to bribe other players[48]. I will also assume $n > N_l = sup\{x \in \mathbb{N} | \alpha x < 1\}$ to mean that it is socially optimal for each player to work instead of shirk.

After the cooperation choice, monitoring begins, and each player i receives a set of signals $S_i = \{s_{ji}\}_{j \in M_i^+}$ with $s_{ji} \in \{good, bad\}$ that indicate the actions of players who are monitored by him accordingly. The default signal is good, and bad signals can be considered a type of solid evidence that can be used to convincingly prove existing misbehaviors. Only those monitors who pay in effort by working during the cooperation phase can observe bad signals from the network, and those who shirk neglect their duties, which results in a lack of evidence to report misbehaviors, if there are any.

For each player i who tries to bribe his monitor j, his misbehavior will certainly be caught by player j if player j eventually works because the non-zero bribe plan is a direct reveal of deviation, and the monitor j who works may thus receive a bad signal with probability 1. In the case in which player i does not bribe his monitor j directly, if player i accepts a non-zero bribe plan from other players or shirks in the cooperation choice, a bad signal will be received by his monitor j with probability p_i. Thus, under the monitoring system, accepting non-zero bribes or shirking can be observed[49]. Although bribe-giving behavior cannot be observed by players other than the one who

[47]The combination of production action and monitoring action is intended for the simplicity of the model and is without loss of generality. Given that both deviations can be caught by the monitoring network, each player either chooses to do – or to neglect to do – both.

[48]The case in which $\alpha \geq \frac{1}{2}$ is trivial because the smallest collusion must contain two players and is thus not profitable by itself. In other words, in the case of $\alpha \geq \frac{1}{2}$, naturally, no player has incentive to bribe others.

[49]The foregoing is true in many real world examples in which people are found to "hold a huge amount of property

directly receives it, each player faces a budget constraint. Thus, giving out positive bribes directly leads to shirking and can be caught under the system.

Denoting $\mathbb{I}_E$ to be the indicator function with respect to event E, receiving a bad signal can be represented by equation (3) in which the first term represents the case of offering a non-zero bribe and being rejected, and the second term represents the case of deviating and being caught with probability p_i.

$$Pr(s_{ij} = bad)_{i \in M_j^+} = \mathbb{I}_{(a_j=work)\cap(b_{ij}\neq 0)} + p_i \mathbb{I}_{(a_j=work)\cap(b_{ij}=0)\cap[(a_i=shirk)\cup(R_i(B_i^- \neq 0)=accept)]} \tag{8}$$

Players whose bad signals are observed are reported, and they are therefore punished[50]. However, the punishment is bounded, and I assume that the only possible punishment for one player is to ostracize him from the community, such that the player can no longer share the payoff from the public goods that are produced[51]. However, because all bribes are executed before the punishment stage, those who already accept bribes can still retain them even if they are excluded from sharing public goods.

Finally, the production of public goods is revealed, and each player i's payoff is given by the net value transfers and the gain from public goods if he is not ostracized. The number of players who work and are not ostracized is denoted as N_w, while the payoff for each player i can be written as follows:

$$u_i = \sum_{j \in M_i^+} b_{ji} \cdot \mathbb{I}_{(r_{ij}=accept)} - \sum_{j \in M_i^-} b_{ij} \cdot \mathbb{I}_{(r_{ji}=accept)} + \alpha N_w (1 - \mathbb{I}_{(\exists k, s_{ik}=bad)}) - \mathbb{I}_{(a_i=work)} \tag{9}$$

This finishes the model. To summarize, the public good game begins with a commonly known monitoring system under which each player privately bribes his monitors before the private cooperation decision. The monitoring results are released after the private cooperation decision and before

with unidentified sources." One special case is when a player i offers a negative bribe, which will be treated as a type of bribe receiving instead of giving, and thus the player can still be caught by his monitors with probability p_i.

[50] Because solid evidence is collected, even a single reporter can convince the community and determine the punishment.

[51] Additionally, his production will not be shared, although it is never optimal for him to produce any public good but is instead optimal to deviate by giving or accepting bribes.

the final outcome realization, which allows those who are reported to be punished by ostracism. For any given monitoring system, players make choices about bribing and production, which results in a PBE. In addition, the main goal for the social planner is to design a monitoring network that maximizes total social welfare.

3.3 Optimal monitoring system

This section will focus on the main question of the paper. From the social planner's perspective, what is the optimal monitoring system that minimizes monitoring costs while incentivizing the players to work together under the network? Unfortunately, when the public good game is played under a network, it will generally end up with multiple PBEs. To be more precise, there is always a trivial equilibrium under which no one works, as is provided in the following result whose proof is omitted.

Proposition 3. *There is always a PBE under which all players shirk and nobody offers any non-zero bribe plan, regardless of the monitoring network.*

However, given that players are willing to pay the sunk cost of monitoring the network establishment, it is reasonable to assume that they are optimistic regarding others' willingness to cooperate and are thus likely to adopt a cooperation equilibrium whenever possible. In other words, a social planner should design a monitoring network such that at least some types of equilibria in which players work together can be supported. The following subsection will discuss the main results of the paper in which the social planner's target is to support one cooperation equilibrium without further restrictions. The refinements of the equilibrium are discussed in section 3.3.2, and a proof sketch of the main result is provided at the end of this section by analyzing some of the properties of the optimal monitoring network.

3.3.1 Optimal network

Before providing the main results of the paper, one example can help illustrate the importance of the structure of the monitoring network.

Example 3. Bribe-proof monitoring network

Consider the game where $N = \{1, 2, 3, 4, 5, 6\}$ and $\alpha = 0.4$. Four different monitoring structures are given in which, for the simplicity of the example, each player is exactly monitored by one another: (a) Monitoring pairs $g_{12}, g_{21}, g_{34}, g_{43}, g_{56}, g_{65} > 0$, in which three pairs are formed, and players within each pair monitor one another; (b) A cycle structure where $g_{12}, g_{23}, g_{34}, g_{45}, g_{56}, g_{61} > 0$, which means that each player monitors the next player, and the last player monitors the first player; (c) A hierarchy structure where $g_{12}, g_{23}, g_{31}, g_{14}, g_{45}, g_{46} > 0$, which means that the first three players form a committee to monitor one another and player 4, who further monitors the remaining two players; (d) A core-periphery structure where $g_{12}, g_{23}, g_{31}, g_{14}, g_{25}, g_{36} > 0$, which means that the first three players form a cycle, and each of these players monitors one of the remaining players. Figure 7 shows the three different monitoring networks.

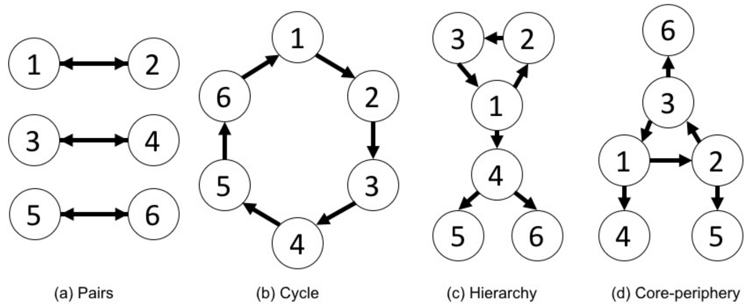

Figure 7: Four monitoring networks

I first claim that if the players cannot bribe other players during the game, all the monitoring networks can support an equilibrium under which all the players work equally efficiently with $p_i = 0.3, \forall i \in N$ because, with no bribing phase, each player i only makes a simultaneous decision of a_i. Given that all the others will work, player i can ensure a payoff of $\alpha n - 1 = 1.4$ when he works and a payoff of $(1 - p_i) \cdot \alpha(n - 1) = 1.4$ when he shirks, which indicates that $a_i = work$ is indeed a weakly dominating strategy for each player i and that all the monitoring networks support the equilibrium. Given that all three monitoring systems have the same number of links and that the monitoring level for each player is the same, all the networks have the same level of efficiency. It can also easily be checked that for any monitoring level $p_i < 0.3$, those monitoring networks can no longer support the equilibrium.

However, when the bribing phase is added, the equivalence no longer holds. Without receiving or offering any non-zero bribe plan, each player i continues to believe that all the others are working. However, he should believe that the one, if any, who offers him a positive bribe will shirk. What about when player i himself offers positive bribe plans to his monitor? Clearly, if his monitor rejects the plan, player i will be reported and punished. However, if the bribe offer is large enough, his monitor will accept it and shirk[52]. Further, player i can shirk safely because no one will receive a bad signal from him.

Thus, I claim that the monitoring network with pairs can never support a PBE in which all the players work – regardless of each player's monitoring level – because each player i's expected payoff in the case of not receiving any bribe is $\alpha n - 1 = 1.4$ if he does not deviate. However, if he receives any positive bribe plan $B_i^- > 0$[53], he can infer that his partner is deviating. Further, his partner will deviate and shirk even if the offer is rejected because the bad signal is already sent. Given that his partner is also the only one who monitors him, player i's best response is to accept the bribe and shirk, which will result in an expected payoff of $\alpha(n-2) + B_i^- > \alpha(n-1) - 1$. Finally, knowing that any positive bribe will be accepted, the cooperative strategy without deviation is dominated by the following strategy for each player i: accepting any bribe offers, offering a small positive amount of bribe to his monitor and shirking. Therefore, collusion within each pair ruins the possibility of global cooperation.

Unlike the network with pairs, the other three monitoring systems can support a PBE, but the minimal requirements of the monitoring levels are different. Using an argument similar to that derived above, in all three cases each player i will accept a non-zero bribe if and only if the strategy of accepting the bribe and shirking is at least as good as the action of rejecting the bribe and working. Thus, I present $(1-p_i)\alpha(n-2) + B_i^- \geq \alpha(n-1) - 1$, which can be rewritten as $p_i \geq \frac{1-\alpha+B_i^-}{\alpha(n-2)}$. However, each player i will offer a positive bribe plan if and only if the highest bribe that he can offer will be accepted by his monitor, where the highest bribe is such that player i's payoff under offering the bribe and shirking is the same as that under not deviating. Therefore, I present $\alpha(n-2) - B_i^+ \geq \alpha n - 1$, which yields $B_i^+ \leq 1 - 2\alpha$. Because player i is arbitrary, $B_i^- \leq 1 - 2\alpha$ also holds. Together it can be confirmed that the following monitoring level is the minimal requirement for all cases: $p_i = \frac{2-3\alpha}{\alpha(n-2)} = 0.5$ for $M_i^+ \neq \emptyset$ and $p_i = 0.3$ otherwise. Because all the players have monitoring duties in the cycle structure, while in the hierarchy structure only the first four players are in charge of monitoring, the hierarchy structure is more efficient than the

[52]Note that the acceptance of a non-zero bribe plan that includes working is strictly dominated by the acceptance of a non-zero bribe and shirking.

[53]Because in all cases each player is only monitored by one other, I can simply write B_i^- here without generating any confusion.

cycle structure. Further, the core-periphery monitoring network is even more efficient because only three players act as monitors.

The example above shows that the social planner's problem is not trivial. An arbitrary monitoring system cannot prevent bribing in general, and among monitoring networks that can support cooperation, the total costs that are generated are typically different. As shown in the example, the structure with small pairs cannot prevent collusion within the pairs because the bribe transfer helps to build credible collusion between a player and his monitor.

Secondly, the required monitoring level for each player should not only ensure that the player will not shirk alone but should also ensure that the player will not collude with the briber after receiving a bribe offer. Because only monitors have the opportunity to receive bribe offers, it is typically the case that monitors are more heavily monitored than those who do not have any monitoring duty. Therefore, one might reason that it is better to let a small group of heavily monitored players monitor all others and to have many other players that do not monitor at all, which results in a core-periphery monitoring network. In fact, the structure in figure 7(d) is indeed an optimal monitoring network under the scenario above. In addition, more generally, among all the monitoring networks that can support a PBE in which all the players work, a core-periphery network has the lowest cost, as suggested by the following result.

Given the construction:

- For any set of parameters $\{n, \alpha\}$, construct a set $\mathbb{K} = \{k \in \mathbb{N} | 0 \leq \frac{1-(k+1)\alpha}{k} \leq (n-1)\alpha - 1\}$. One can check that $\mathbb{K} \neq \emptyset$ and $inf(\mathbb{K}) \leq N_l$. If $inf(\mathbb{K}) = 1$, I will call it case A, otherwise it is case B.

- Under case A, let $g_{12}, g_{23}, g_{31} > 0$ and each player other than the first three, if any, is monitored by one of the first three players. Monitoring levels are given by $p_i = \frac{2-3\alpha}{(n-2)\alpha}$ for $\forall M_i^+ \neq \emptyset$ and $p_i = \frac{1-\alpha}{(n-1)\alpha}$ otherwise.

- Under case B, pick the first $inf(\mathbb{K}) + 1$ players to form a set $\mathbb{C}_b = \{1, 2...inf(\mathbb{K}) + 1\}$. Each player $i \in \mathbb{C}_b$ is monitored by all the other players in $\mathbb{C}_b$. Each player not in set $\mathbb{C}_b$, if any, is monitored by $inf(\mathbb{K})$ distinct players in set $\mathbb{C}_b$. The monitoring level is given by $p_i = \frac{(inf(\mathbb{K})+1)-(2inf(\mathbb{K})+1)\alpha}{inf(\mathbb{K})(n-2)\alpha}$ for $\forall M_i^+ \neq \emptyset$ and $p_i = \frac{1-\alpha}{(n-1)\alpha}$ otherwise.

I present the following result whose proof is in the Appendix:

Theorem 1. *Among all monitoring networks that can support a PBE under which all players work and nobody bribes, the core-periphery network that is constructed above has the lowest total cost.*

What PBE can be supported under the optimal monitoring network? Note that whether an equilibrium of cooperation can be supported by a given monitoring network in general also depends on the off-path belief of each player. The more optimistic each player is when he is faced with an unexpected deviation, the easier it is that cooperation can be sustained because if a player who receives an unexpected bribe plan expects that there are still a sizeable number of players working, he should expect a higher payoff from rejecting the plan and working[54], and it is thus more likely that he will reject the offer. Therefore, the cooperation PBE that can be supported under the optimal monitoring network should be one in which each player is holding an optimistic off-path belief and expecting a large number of working players even if deviations are observed.

How optimistic can each player be when he observes an unexpected deviation? The benchmark is the passive belief assumption (McAfee and Schwartz, 1994 and Segal, 1999) in which each player believes that the deviations observed are the only deviations and that other players are not deviating from the equilibrium strategy. To be more precise, the PBE under which all players work with passive beliefs is given by (henceforth, a passive cooperation PBE):

- Each player i does not offer bribe plans and will accept any bribe offer and shirk if and only if the expected payoff is at least the same as that from rejecting the offer and working. Otherwise player i will reject bribe offers and work.
- Each player i believes that all others will play the same strategy if no bribe plan is observed. Otherwise, he believes that no bribe plan is offered other than those observed by him, that all others except bribers still play the equilibrium strategy and that bribers will shirk.

This is the PBE that can be supported under the optimal monitoring network, as is shown in the following result whose proof is already contained in theorem 1:

Corollary 1. *The passive cooperation PBE can be supported under the optimal monitoring network.*

[54] Or, in the other direction, he should expect a more severe punishment if he deviates and is caught.

Notably, the optimal monitoring network is not unique. However, the lack of uniqueness is not an essential issue in considering the network structure. It can be ascertained that the optimal network has a core-periphery structure, but the monitoring duty can still be rearranged among the core players while maintaining efficiency. For instance, consider the optimal structure in example 1. Instead of having each of the three core players monitor one periphery player as in figure 7(d), the monitoring efficiency remains the same if player 1 monitors all three periphery players, as shown in figure 8. Notably, although the two networks are not identical, they both satisfy the core-periphery structure that is constructed above.

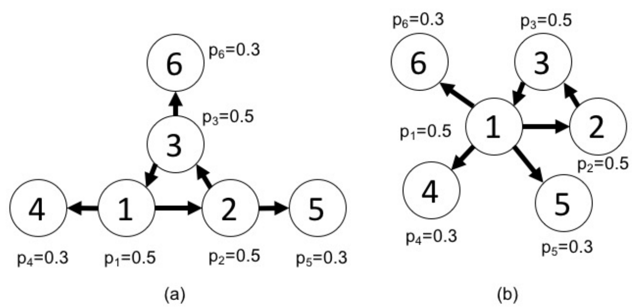

Figure 8: Two core-periphery networks

Additionally, if one can further assume that players are optimistic regarding others' willingness to cooperate, i.e., when possible players work together instead of picking an equilibrium without cooperating, the core-periphery structure indeed solves the problem of maximizing total welfare. When the monitoring network is formed, the passive cooperation PBE can be supported, and thus players can choose the equilibrium and work together.

Finally, a core-periphery structure can be treated as a special case of hierarchies with only two ranks. A generalized hierarchy structure can also be optimal under some further assumptions, which is discussed in section 3.4.

3.3.2 Optimal network with equilibrium refinement

Although players with a passive belief are indeed the most optimistic when they are faced with bribe offers, in general it is questionable whether such a belief is suitable when the observed briber is monitored by more than one player. The following example illustrates this concern.

Example 4. More than one monitor

Consider the game with $N = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $\alpha = 0.2$. As shown in figure 9, two network structures are involved: (a) The first three players act as monitors and all players are monitored by two of the three players; and (b) The first four players act as monitors and all players are monitored by two of the four players. Monitoring levels in network (a) are given by $p_i = \frac{5}{6}, \forall i = 1, 2, 3$ and $p_i = \frac{2}{7}$ otherwise. On the other hand, monitoring levels in network (b) are given by $p_i = 1, \forall i = 1, 2, 3, 4$ and $p_i = \frac{2}{7}$ otherwise.

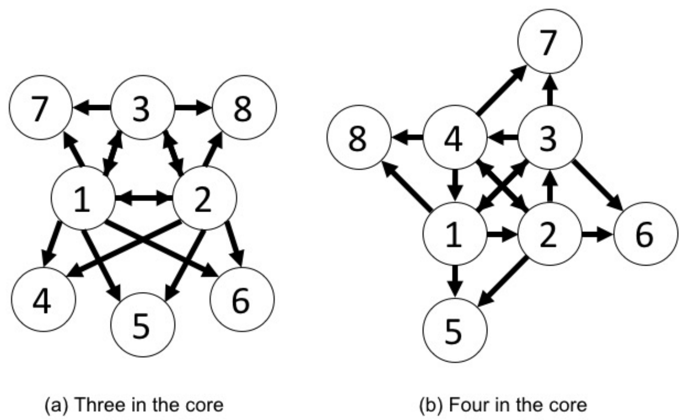

Figure 9: Two core-periphery structures with different core

Clearly, both networks can support the passive cooperation PBE. Further, based on theorem 1, the first structure is optimal because of minimized total cost. However, it might need to be reconsidered, given the core under network (a). If player 1 indeed receives a bribe offer from player 2, why should player 1 believe that player 2 is bribing only him instead of both him and player 3? Given that player 2 is monitored by both and that their observations are perfectly correlated, only bribing player 1 is costly to player 2; however, he cannot help avoid being monitored. Therefore, only bribing player 1 is never an optimal deviation from the perspective of player 2. On the other hand, if player 2 bribes both players 1 and 3, both should be happy to accept the offer because no one else can monitor them, which results in a shirking core under network (a) and a collapse of cooperation.

However, network (b) is not challenged by the concern that is described above. Even if player 1 receives a bribe offer from player 2 and believes that player 4 also receives the offer, he is still monitored by player 3. Given that player 2's offer to player 1 is less than $\frac{1-3\alpha}{2} = 0.2$ by symmetry, player 1 still prefers the cooperation payoff in lieu of taking the bribe and being punished. Thus player 1 will reject the plan, which in turn will prevent the bribing behavior of player 2.

The problem with the passive belief in the example above is essentially a problem of rationality. From the perspective of a potential deviator who wants to offer non-zero bribe plans, he must either gain directly from the plan itself or gain from the fact that he can shirk safely because all his monitors are bribed and will no longer work. Therefore, when faced with more than one monitor, a potential briber will never offer bribes only to some monitors because the strategy is a dominated one regardless of the assigned monitoring levels. This in turn indicates that a bribe receiver should not believe that the observed briber is doing so.

Figure 10 shows the argument more precisely. When under 10(a), player 1 receives a bribe offer from player 2. Given that player 1 is the only monitor of player 2, it is clear that player 1 already observes the entire bribe plan of player 2, and there is little that player 1 can believe other than the passive belief. When player 2 is also monitored by player 3, one possibility is that player 2 offers a negative plan $b_{21} < -\alpha$ to player 1 such as that which is shown in figure 10(b); this means that he asks player 1 to give him a large bribe. Because it is profitable for player 2 to only receive the bribe from player 1 if he is not caught during the monitoring realization, it is possible that player 2's bribe plan will involve only player 1[55]. However, when player 1 observes a plan of $b_{21} \geq -\alpha$ such as that shown in figure 10(c), he probably should not believe that he is the only person who is involved in the plan because if player 2 tries to bribe only player 1 with the plan $b_{21} \geq -\alpha$, he will either be rejected and caught or, even if the offer is accepted by player 1, he should expect that player 1 will shirk after accepting the plan, which will decrease his payoff by α. Further, player 2 still cannot shirk safely because he should expect that player 3 is still working, and thus his chance of being caught by player 3 does not decrease after the bribing phase. Therefore, the passive belief is generally not rationalizable[56] and player 1 should instead expect that player 2's plan also contains an acceptable bribe offer to player 3.

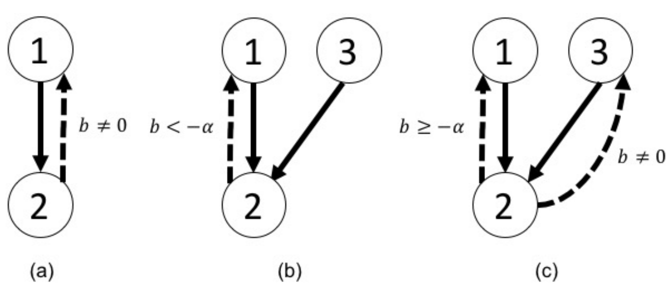

Figure 10: Passive belief is not rationalizable in general

[55]However, this situation turns out to be trivial because player 1 can never accept a plan that requires him to offer such a huge sum of money.

[56]There is no monitoring level assignment to make it rationalizable.

Unfortunately, given that the off-path belief by itself is in the not rationalizable domain in which the observed deviator plays a weakly dominated strategy, one might argue that the concern above is not sufficiently solid. However, the problem of the passive belief is also similar to the properness concern (Myerson, 1978). The main argument of properness is that an action with a lower payoff will be played with a significantly lower probability than an action with a relatively higher payoff. In other words, if a player is found not to play the best strategy by mistake, it is significantly more likely that he made a small mistake rather than a large one, as measured by the final payoff. Applied to example 4, to minimize the total monitoring cost, the monitoring levels must decrease until each player's payoff under the equilibrium strategy is the same as the payoff from bribing all of his monitors and shirking. As a result, the equilibrium strategy of working and not bribing is a weakly dominating strategy, bribing all the monitors is the "second best" action and only bribing some of the monitors is a strictly dominated action, which should be reflected in the off-path belief.

Incorporating both of the above concerns, a modified version of theorem 1 is given with the following construction:

- For any set of parameters $\{n, \alpha\}$, construct a set $\mathbb{K}' = \{k \in \mathbb{N} | 0 \leq \frac{1-(k+1)\alpha}{k} \leq (n-k)\alpha - 1\}$. If $\mathbb{K}' = \emptyset$, it will be called case A', and otherwise it is case B'.
- Under case A', pick the first $N_l + 1$ players to form a set $\mathbb{C}'_a = \{1, 2, ...N_l + 1\}$, in which $N_l = sup\{x \in \mathbb{N} | \alpha x < 1\}$ as defined above. Each player in set $\mathbb{C}'_a$ is monitored by all the other players in the set. Each player not in set $\mathbb{C}'_a$, if any, is monitored by N_l distinct players in set $\mathbb{C}'_a$. The monitoring levels for all the players are the same given by $p_i = \frac{1-\alpha}{(n-1)\alpha}, \forall i \in N$.
- Under case B', the number $K_l = inf(\mathbb{K}')$ is well defined and $1 \leq K_l < N_l < n$[57]. Pick the first $K_l + 2$ players to form a set $\mathbb{C}'_b = \{1, 2...K_l + 2\}$. Each player $i \in \mathbb{C}'_b, i \neq K_l + 2$ is monitored by all players in $\mathbb{C}'_b$ other than player $i + 1$, and player $K_l + 2$ is monitored by all players in $\mathbb{C}'_b$ other than player 1. Each player not in set $\mathbb{C}'_b$, if any, is monitored by K_l distinct players in set $\mathbb{C}'_b$. The monitoring level is given by $p_i = \frac{K_l+1-(2K_l+1)\alpha}{K_l(n-K_l-1)\alpha}$ for $\forall M_i^+ \neq \emptyset$ and $p_i = \frac{1-\alpha}{(n-1)\alpha}$ otherwise.

In both cases, only those players in the set $\mathbb{C}'_a$ or $\mathbb{C}'_b$ have monitoring duties under the network,

[57] This is true because N_l and all numbers greater than N_l are not in $\mathbb{K}$.

while all the other players are monitored by those central players, which forms another core-periphery structure. Employing the construction above, I can give the next result whose proof will be provided briefly in the next subsection and in detail in the appendix.

Theorem 2. *Among all monitoring systems that can support a proper PBE under which all players work and nobody bribes, the core-periphery system that is constructed above has the lowest total cost.*

Notably, the core-periphery network that is constructed above coincides with that in theorem 1 when $K_l = inf(\mathbb{K}) = 1$, which is intuitive because when each player is only monitored by one other player, the passive belief is indeed proper.

In addition, the refinement of properness is first defined under a finite strategic form game and later under a finite extensive form game. There is no generalized definition regarding an arbitrary infinite game although many trials are made (e.g., Simon and Stinchcombe, 1995; a comprehensive review can be found in van Damme, 2002). In my model, however, the properness can be checked by requiring that each player's off-path belief puts probability 1 on the deviator's action, which gives him the highest payoff conditional on the observed part.

3.3.3 Proof sketch

In this subsection, I provide a proof sketch of theorem 2 above. Several properties of the optimal monitoring network are examined that can help to eliminate suboptimal structures. Detailed proofs can be found in the appendix. The proof of theorem 1 has similar steps and is also found in the appendix. Throughout the entire subsection, the social planner's problem is to support a proper PBE under which all players work.

Proposition 4. *An optimal monitoring network satisfies $m_i^- = N_l, \forall i \in N$ if $\mathbb{K}' = \emptyset$ and $K_l \leq m_i^- \leq N_l$ otherwise.*

The value N_l gives the maximum collusion size such that those who collude can potentially obtain a higher payoff than under the cooperative outcome that is conditional on those that are not reported. Therefore, once the total number of monitors for one player is N_l, the player who is being monitored will never try to offer a non-zero bribe plan. Otherwise, the collusion size will be $N_l + 1$, which is not profitable by itself. Adding more monitors to one player after reaching N_l is not optimal because N_l monitors can already fully prevent any potential bribing. Therefore, N_l is the upper bound of the number of monitors for each player.

Once the number of monitors is less than N_l, the player who is being monitored can offer a non-negative bribe plan to his monitors to collude with them. The set $\mathbb{K}'$ provides the possible number of monitors for each player i such that the largest total amount of bribes that are offered by player i is not greater than the total cooperative gains that are received by all of his monitors. The only possible number of monitors for each player i must fall within the set $\mathbb{K}' \cup \{N_l\}$ to prevent all the monitors of player i from being bribed regardless of their monitoring levels. Therefore, when the set $\mathbb{K}$ is non-empty, the value $K_l = inf(\mathbb{K}')$ yields the lower bound of the number of monitors for each player.

One special case is when $\mathbb{K}' = \emptyset$, which means that as long as the number of monitors is less than N_l, the player who is being monitored can offer a large enough bribe plan to drive all the monitors to deviate. Therefore, to prevent collusion, each player must be monitored by at least N_l others under this scenario, which, together with the upper bound argument, leads to the following corollary.

Corollary 2. *If $\mathbb{K}' = \emptyset$, every monitoring network such that $m_i^- = N_l$ and $p_i = \frac{1-\alpha}{(n-1)\alpha}, \forall i \in N$ is optimal.*

Because the only possible monitoring network under case A' is that everyone is monitored by at least N_l others, a non-zero bribe plan can never be offered. As a result, each player i only must be monitored such that he does not deviate alone, which leads to the monitoring level of $p_i = \frac{1-\alpha}{(n-1)\alpha}, \forall i \in N$. This corollary directly proves the theorem under case A'. Below, I focus on case B' in which each player has the possibility of being monitored by fewer than N_l others. With the possibility of a non-zero bribe offering, the monitoring level for each player i should both ensure that he will not deviate alone and that he will not deviate after receiving bribes.

Proposition 5. *If $\mathbb{K}' \neq \emptyset$, the optimal monitoring network satisfies $m_i^- \geq m_j^-, \forall i \in M_j^-$.*

In other words, this proposition states that the number of monitors for each player j is no greater than the number of monitors for any of player j's monitors, which can be shown by reductio ad absurdum. Excluding the several exceptions that are discussed in detail in the appendix, if there is a monitoring network that can support a proper PBE under which all players work, and there is a player j who is monitored by more players than one of his monitors i, one can rearrange the monitoring duty as follows. Let all players who initially are in charge of monitoring player i monitor player j, and let all players who initially are in charge of monitoring player j stop monitoring player j. Keeping all the monitoring levels fixed, this rearrangement can still support the PBE but strictly improve efficiency. Intuitively, this argument posits that if the monitor can be monitored by fewer people, why can't the one who is being monitored?

With proposition 4, I can order players based on the number of their monitors. Assuming an optimal network is found, re-index players such that $m_1^- \geq m_2^- \geq ... \geq m_n^-$. The value $K_u = sup\{m_i^- | i \in N\}$ and the set $\mathbb{K}_u = \{i \in N | m_i^- = K_u\}$ are well defined and obtain the following property.

Corollary 3. *The set $\mathbb{K}_u$ that is defined above satisfies $\forall i \in \mathbb{K}_u, \forall j \in M_i^-, j \in \mathbb{K}_u$ under an optimal monitoring network.*

This corollary is a direct deduction from proposition 5. With those definitions, I present proposition 6 as follows.

Proposition 6. *If $\mathbb{K}' \neq \emptyset$, the optimal monitoring network satisfies $K_u = K_l$ and thus $m_i^- = K_l, \forall i \in N$.*

Proposition 6 shows that the optimal monitoring network is such that each player is monitored by exactly the minimum necessary number of players that is given in proposition 4. This result can also be shown by reductio ad absurdum. As opposed to the proof of proposition 5, if $K_u > K_l$ is under an optimal monitoring network, one can remove some of the monitoring duty among the players in the set $\mathbb{K}_u$ and increase the monitoring level accordingly. The underlying intuition is that the monitoring level for each player i depends on both the maximum potential bribe that will be offered to him and the cooperative payoff that he can receive. For each player $j \in M_i^+$, increasing m_j^- lowers the minimal requirement of the monitoring level for player i because the amount of bribe b_{ji} can be offered to player i decreases. However, the cooperative payoff for player i also decreases once player j offers him a non-zero bribe plan because player i might expect the remaining monitors of player j to deviate, given that his belief is proper. The overall effect of the two driving forces leads to the result that the monitoring level for each monitor of player j decreases, but the total cost increases. Thus, for purposes of efficiency, the monitoring network should maintain a minimum number of necessary monitors for each player, which is given by K_l.

The last step to the core-periphery structure is the following proposition, which calculates the total number of monitors.

Proposition 7. *If $\mathbb{K}' \neq \emptyset$, the optimal monitoring network has $|\mathbb{M}| = K_l + 2$.*

Remember $\mathbb{M} = \{i \in N | M_i^+ \neq \emptyset\}$. Proposition 7 shows that in the optimal monitoring network, there are only $K_l + 2$ players who have monitoring duty. To show this result, consider the set of all the monitors in the optimal monitoring network. With proposition 6, each player i in this set must be monitored by K_l other players. Further, there should be at least one more player $j \notin M_i^- \cup \{i\}$ who can monitor some of those players in M_i^-. Otherwise, player i and his monitors are the only monitors in the network, which indicates that any non-zero bribe offer from player i can always be accepted because none of them are monitored by players outside of this collusion. Therefore, the size of the set $\mathbb{M}$ is at least $K_l + 2$. However, the monitoring level for each player with monitoring duty is higher than those without monitoring duty because of the potential bribe that is offered to him. To minimize the total monitoring cost, the number of players without monitoring duty should be as large as possible. Given that the construction in theorem 2 can support a proper PBE under which all the players work, and it also shrinks the size of $\mathbb{M}$ to the lower bound. In the optimal monitoring network the size of $\mathbb{M}$ should be exactly $K_l + 2$, which finishes the proof of the proposition.

Finally, the propositions that are presented above show that the only remaining network structure is indeed the core-periphery structure. Together with a simple calculation of the minimally required monitoring levels, the proof of the theorem is complete.

3.4 Discussion

This section discusses some of the justifications of my model and its potential extensions. I will first extend my model to a dynamic scenario. Settings of the monitoring and punishment mechanisms will be discussed afterwards.

3.4.1 Spread of bribes

Ideally, my model should be extended in such a way that players can have multiple trials of bribing and offer their plans that are conditional on what they receive from others because bribes can be spread over time in reality, and the size of the collusion grows accordingly. For instance, in my model, the maximum bribe that each player can offer is bounded above by each individual's budget constraints. However, it is entirely possible that, after receiving a positive number of bribes from other players, a player will decide to deviate from the cooperative strategy by offering his monitor an amount that is greater than what he initially had. However, describing possible strategies that can be adopted by each player is extremely difficult, and it is not clear what type of belief each player should hold about the existing collusion size. One example is given below.

Example 5. Bribe spreading

Consider the scenario with $N = \{1, 2, 3, 4, 5, 6\}$, $\alpha = 0.3$ and a cycle monitoring system with $g_{12}, g_{23}, g_{34}, g_{45}, g_{56}, g_{61} > 0$. Bribes can spread such that players can offer their plans based on what they receive. Now, player 1 receives a positive bribe plan of $b_{21} = 0.2$ during the bribing phase. In addition to his own monitoring level, player 1 should also have an expectation about how many players are already involved in the deviation. However, it is generally impossible for him to infer the number of existing deviators merely from the amount of the bribe offered.

Clearly it is possible that player 2 is the only player who bribes, as is shown in figure 11(a). Expecting player 1 to accept the plan and shirk, player 2 can safely shirk and obtain a payoff of $4\alpha - b_{21} = 1$, which is strictly better than the cooperative payoff of $6\alpha - 1 = 0.8$. However, the bribe that is received by player 1 can also potentially come from a chain of bribing that began with player 3, such as that which is illustrated in figure 11(b). If player 3 perfectly predicts the spread, he might want to offer player 2 a bribe plan of $b_{32} = 0.1$ and shirk. He ends up with a payoff of $3\alpha - b_{32} = 0.8$, which is as good as his cooperative payoff. Further, player 2 now has a bigger budget and can thus offer player 1 a plan of $b_{21} = 0.2$. He ends up with a payoff of $3\alpha + b_{32} - b_{21} = 0.8$ once his plan is accepted by player 1, which is also as good as his cooperative payoff. Unfortunately, player 1 is unable to distinguish the two situations merely from the amount of bribe that is received.

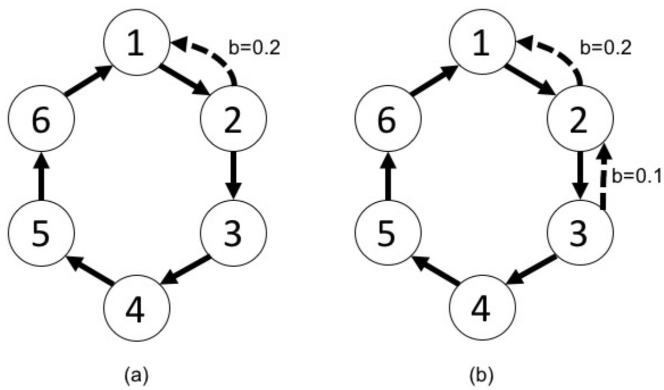

Figure 11: Bribe spreading

Once the bribe receiver does not perfectly know the number of remaining working players, it is not possible for him to determine whether accepting the plan is profitable. The situation is more complicated when the receiver takes the option of accepting the bribe and offers plans to others to consider because other than information regarding the existing collusion, he also must have some type of expectations about future bribe spreading. As a result, in this paper, I will only briefly discuss a benchmark case of bribe spreading. The design of a more general model is an opportunity for future research.

The bribing phase is modified as follows. Players are assumed to have a perfect forecast, and bribe plans are assumed to be perfectly informative. In other words, each player can perfectly expect the final outcome when he wants to bribe others. Further, once receiving a bribe offer, each player knows the existing deviators, both without and with his participation. The bribing phase contains n periods, and at the beginning of period one, each player i simultaneously chooses a bribe plan $\tilde{B}_{i1}^{+} = (b_{ij1})_{j\in M_i^-}$. In each following period, each player simultaneously chooses a contingent bribe plan $\tilde{B}_{it}^{+}(\tilde{B}_{it-1}^{-}) = \{(r_{ijt})_{j\in M_i^+}, (b_{ijt})_{j\in M_i^-}\}$.

Notably, the total number of bribing periods is fixed. From the perspective of a potential briber, this constraint is not binding because no spread requires more than n periods given that the total number of players is n. However, from the perspective of a working player who never receives a bribe, he can infer nothing from the timing of the game. This constraint mainly represents the idea that in reality, players who are involved in bribe spreading can hardly know anything about others' bribing activities. The other remark is that the bribe is assumed to be informative and thus the belief of a bribe receiver is simply assumed to coincide with what has already occurred. However, each player's contingent plan is extremely complicated, including the responses to all types of bribes spreading as long as the player himself is involved and his belief is updated accordingly. The final remark is that with the new bribing phase, the corresponding monitoring realization should also be modified slightly to ensure that only the player who is directly caught by his monitor will be punished. However, because this modification is only necessary when a plan is rejected by some players, which by itself is not rationalizable, for purposes of simplicity, I will not restate the whole model.

Similar to section 3, the social planner must design the monitoring network such that the following PBE can be supported:

- Each player's contingent plan is such that he will not bribe others if he does not receive a bribe. He will work if he is not involved in any bribe spreading.
- Each player believes that all others adopt the equilibrium strategy if he does not receive a bribe plan.
- When receiving a bribe plan, each player updates his belief accordingly.

At this point, with the bribe spreading modification, a weaker but similar result can be given whose proof is in the appendix.

Proposition 8. *When the bribing phase is modified as above, and the number of players is large enough, an optimal monitoring system that can support the PBE above is such that there are players without monitoring duties (e.g.* $\exists i \in N$ *such that* $M_i^+ = \emptyset$*) and the total number of monitors is no less than* $N_l + 1$ *(e.g.* $|\mathbb{M}| \geq N_l + 1$*).*

Although the detailed monitoring structure of players with monitoring duties remains unclear, proposition 8 shows that the optimal monitoring network continues to be close to a core-periphery structure when the social planner must prevent more general collusions in which bribes can spread over time. The underlying intuition is similar to that in the basic model. A monitor is more likely to be involved in collusion and thus must be monitored more heavily. Therefore, it is better to have a small group of heavily monitored players to monitor all the others who, as a result, do not require high monitoring levels.

3.4.2 Other punishments

In my paper, the punishment is modeled such that both the player who offers a bribe plan and the player who accepts it are considered deviators, who can be punished if they are caught. This setting is essential because it is the foundation of the credibility of collusions. From the perspective of a bribe receiver, if someone offers him a non-zero bribe plan, he knows that the provider must promise collusion because the behavior of bribe giving already explicitly ensures deviation. However, from the perspective of a bribe giver, he can also trust the monitor who accepts his offer because once it has been accepted, the monitor must deviate and shirk, which results in safe shirking for the bribe giver.

Given the importance of the punishment mechanism, one discussion might address what will occur if only one party is punishable within the bribing scheme. Notably, the result depends on which party is considered a deviator. When the acceptance of bribes is not punishable, the credibility of collusion disappears, and it is thus highly likely that no one will try to offer bribes because from his perspective, the potential bribe giver cannot shirk safely because none of his monitors promise to collude with him and shirk even if his plan is accepted. In fact, a monitor can always accept a positive bribe plan but still work given that the acceptance of a bribe is not punishable. Only when a single deviator can render the cooperation not profitable (e.g. $n = N_l + 1$) can players collude credibly by bribe giving. Therefore, forgiving the misbehavior of bribe taking actually prevents collusion[58].

As opposed to the case above, when the offering of bribe plans is not punishable, preventing collusion will become extremely difficult. First, this change notably does not ruin the credibility of collusions. Upon receiving an acceptable bribe plan, a monitor can still believe in the bribe giver's deviation based on the strategy of offering bribes, but continuing to work is always a dominated strategy regardless of whether the offering of bribe plans is punishable. Further, the change persuades players to offer bribe plans because even if their plans are rejected, unlike the basic model, they can still revert to a cooperative strategy without being punished. In other words, the change makes the cooperative strategy a side option to all potential bribers, and it is costless for them to at least offer a plan regardless of its acceptability. As a result, the players no longer independently offer bribe plans. Instead, they should expect the possibility that many players simultaneously offer bribes to a single monitor, which greatly increases the chance that the monitor will eventually deviate.

[58] Basu (2011) has a similar argument in which he considers the issue of soliciting for bribes (Basu calls it "harassment bribes") and suggests that the bribe giver should be treated as legal. In my case it is the briber actively offers his plan to his monitors, and thus the bribe receiver, who is the passive side, should be treated as legal. However, this argument faces other credibility problems. Dufwenberg and Spagnolo (2015) have a detailed discussion on this topic.

Another aspect of the punishment in my paper is that the payoff is eventually assigned to those deviators. In my model, only those deviators who are directly caught can be ostracized, and the value that is already transferred is irreversible. It can easily be conceived that it is easier to prevent collusion if the community can recover the bribes or extort confessions from deviators and thus further catch bribe givers. This result is intuitive because both changes make potential collusion less profitable. However, both changes will not largely affect the optimal monitoring structure because the key driving force toward the core-periphery network remains unchanged. Monitors remain more likely to be bribed and thus must be more heavily monitored.

Alternatively, the model might be much more complicated if the community could renegotiate with deviators. Unfortunately, this modification is beyond the scope of my paper because by adopting a non-repeated setting, my model focuses on the structure of the optimal monitoring system instead of on the sustainable level of cooperation with renegotiations. Further studies are thus needed in this regard.

3.4.3 Monitoring mechanisms

Another crucial assumption in my model is that the cost of monitoring is small enough but not negligible. However, how small of a cost is small enough? One obvious sufficient condition is $\beta N_l \leq \alpha n.$). In other words, the monitoring cost is small enough if it is still socially optimal to obtain cooperation even when all players must be fully monitored by a sufficiently large number of others. In fact, after constructing the optimal system, one can provide the necessary condition as follows. The monitoring cost is small enough if it is socially optimal to have cooperation under an optimal monitoring network. In other words, as long as the sunk cost of an optimal monitoring network is less than the gain from the cooperative public good provision, the monitoring cost is low enough. A further observation is that no matter whether one adopts the sufficient condition or the necessary condition, the per-unit monitoring cost parameter, β, is positively correlated with the number of players, n. This result is intuitive because the more players who are involved in the provision of the public good, the higher the gain from cooperation. Thus, it is more likely that the monitoring cost will be low enough, and the cooperation easier to achieve. The heterogeneous monitoring cost might also be considered. For instance, if the costs of monitoring different players are different (e.g., β_i is different for each player i), and it is extremely costly to monitor certain players, it is possible that a social planner should allow those players to shirk because enforcing their production of public goods is no longer socially optimal[59].

What if the cost of monitoring grows? For example, the marginal cost of monitoring one additional player may continue to increase. Even more extremely, each player may have a capacity constraint such that the number of players that he can monitor is bounded as above. One intuitive conjecture is that the total number of players with monitoring duties increases in both scenarios because to cooperate efficiently each player should now not monitor too many others. Luckily, both scenarios will not greatly affect the optimal monitoring structure because monitors are still more likely to be bribed and thus must be more heavily monitored. Taking the extreme case of the bounded capacity of $C > K_l$ for example[60], an optimal network now might have more players in the core due to the constraint, but the core-periphery structure should not be affected. This is true because propositions 4, 5 and 6 still hold, and the constraint only affects the total number of monitors needed. Now, the social planner can let $max\{\lceil n/C \rceil, K_l + 2\}$ players form the core to monitor all the others, and each player is still monitored by the minimal number of monitors, as given in proposition 6. Clearly, there might be other structures that are also optimal under the scenario. For example, a multi-rank hierarchy structure (e.g. $K_l + 2$ players act as top rank monitors who monitor $(K_l + 2)C - (K_l + 2)K_l$ secondary rank monitors, who further monitor others, and so on) can be imagined, which is more common in reality and might also reach the minimum total monitoring cost, meaning that it is thus also optimal. Figure 12 illustrates this idea[61].

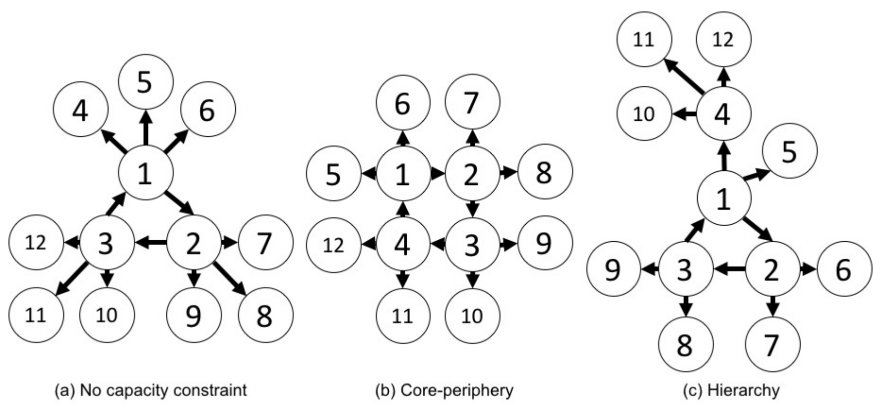

Figure 12: Optimal networks

And I summarize the idea to present the following result whose proof is omitted:

[59] Haag and Lagunoff (2006) has a similar argument when discussing local cooperations.

[60] If the capacity is such that $C \leq K_l$, no system can support the cooperative PBE because the total number of monitors is lower than the minimal number that is required

[61] All networks are with $n = 12$ and $K_l = 1$. (a) Without the monitoring capacity, players 1, 2 and 3 act as core players monitoring others. (b) Each player can monitor 3 others at most, and the core is thus enlarged to include player 4. (c) A hierarchy structure can also reach the same monitoring cost in which player 4 acts as a secondary monitor.

Proposition 9. *When there is a monitoring capacity $C > K_l$ such that each player can monitor at most C others, both a monitoring network with a core-periphery structure and one with a multi-rank hierarchy structure are optimal.*

Finally, the assumption that observations from all the monitors of a player are perfectly correlated can also be relaxed. When the monitors have independent observations of a single player but there is still a restriction that the chance of capturing the misbehavior of that player is the same across monitors, the optimality of the core-periphery structure will not be significantly influenced. Intuitively, the argument comes from the following facts. At this juncture, each player i's chance of being caught by at least one monitor is given by $p_i^\star = 1 - (1 - p_i)^{m_i^-}$, which is concave with respect to m_i^-. However, the cost of monitoring player i is given by $Cost_i = m_i^-(1 - (1 - p_i^\star)^{1/m_i^-})$, which is convex with respect to $p_i^\star$. Therefore, it is still optimal to evenly allocate the monitoring levels for the players in M_i^-. Further, from the perspective of a potential briber, the marginal gain from bribing one additional monitor increases. The potential briber will again either shirk alone or bribe all of his monitors. Now with independent monitoring realizations, propositions 4 and 5 still hold, which means that under the optimal monitoring network there is still a group of players who each have the highest number of monitors. Thus, they can act as the monitors of all the remaining players. The last step is to find the optimal number of monitors. Given that the marginal cost of monitoring player i increases with respect to m_i^-, it is again not optimal to have many monitors for a single player, which concludes the argument. Unfortunately, it is not clear what the optimal network is if the restriction of equal capturing chance is to be relaxed, which complicates the problem to an unsolvable level.

3.5 Conclusion

When a group of people want to produce public goods, they may face the problem that private incentives are insufficient to ensure cooperation and thus a monitoring mechanism is required. Unfortunately, people not only shirk alone but also sometimes bribe and collude with their monitors. As a result, the monitoring mechanism should not only prevent individual deviations but should also exclude potential collusion. One efficient way to achieve both goals is to design and establish a core-periphery monitoring system in which a small set of people are in charge of monitoring all the others. The main intuition behind this result is that monitors are more likely to be tempted by bribes and thus must be more heavily monitored. Further, the efficiency of the core-periphery structure is robust under various settings, including scenarios in which bribes can be spread over time and in which there are different punishment rules and monitoring devices.

I conclude by discussing some limitations of the result and potential opportunities of future research. As mentioned before, my work provides a complementary piece of the extensive discussion of cooperations by analyzing the monitoring arrangement that minimizes the total costs while still achieving a given level of cooperation. A comprehensive question unsolved is to generalize the cooperation decision to a richer set and to find the socially optimal solution among pairs of the cooperation level and the corresponding costly monitoring network.

Second, my work describes the ex-ante bribing behavior and the prevention mechanism, accordingly. In reality bribing can occur in various ways including ex-post bribing when people try to collude with their monitors after being caught, soliciting for bribes when the monitors extort money from an innocent agent for not framing him. Although many of those variations have a similar mechanism as that of the ex-ante bribing behavior, some might bring significant complications. For example, instead of focusing on the PBE concept, when the potential bribers are allowed to communicate before offering the bribe plans, cheap talk and other coordination issues arise. This is also true for a generalized spread of bribing with incomplete information. It would be challenging – but surely pretty interesting – to consider the problem of efficiently stopping multi-agent bribing.

Also, the assumption of perfect correlated monitoring realizations is very strong. Even the relaxed version of independent observations with equal probabilities cannot catch many real world scenarios. Further, it is possible that monitors have different abilities such that, naturally, they might execute different level of monitoring even if their observations are identical. For example, even when facing a same financial report from a public company, a professional auditor might have entirely different inferences than a regular investor. Introducing heterogeneity of individuals' abilities into the model can lead to more insights into monitoring systems.

Finally, my work provides an alternative explanation of the core-periphery (or more general, the hierarchy) organizational structures widely observed in reality. Both reformations of the auditing system for public companies in the US and the local monitoring system for the poverty reduction program in China turn out to be efficient in the sense of collusion control. However, the driving force of those two hierarchies and others, like the banking network (Fricke and Lux, 2015), might be a combination of the traditional team optimization and the monitoring concern. Hence, extending the model to allow for a richer relationship between players seems to be an interesting problem, from both the theoretical and the empirical perspectives.

Chapter 4 Network Externality and Innovation Adoption

4.1 Background

4.1.1 Motivation

Innovations support the long run growth of the economy. And the process of diffusion and adoption of an innovation is an interesting and extensively discussed topic. Within it, one phenomenon that draws considerable attention is why some new ideas fail in being adopted by the majority of the society while similar ones end up with commercial success. One example is the different performances of Couchsurfing and Airbnb. Couchsurfing is a hospitality service and social networking website founded in 2003 whose main function is to help members to stay at some others' house during traveling, at a cost or for free. Although the website is a pioneer in the field of sharing economy and the main idea behind it is proved to be a good one, Couchsurfing did not receive any venture capital funding until 2011. Sharing a similar idea, Airbnb was founded in 2008 and soon in 2010 it received the financial support from Greylock Partners and Sequoia Capital. Nowadays, Airbnb has over 3 million lodging listings across 191 countries and a total number of users over 150 million. Comparing with the performance of Airbnb, the current market share of Couchsurfing is negligible.

A common argument towards innovation firms like Couchsurfing is that a good idea might not always becomes a commercial success especially if the operation is poor. However, even with reliable management, there are still good ideas not receiving promising feedbacks from the market at the first time they appear. Another example is the Google Ride Finder, a project operated by Google starting in 2005. Attempting to provide services similar to Uber, Google's project only survived for 4 years and discontinued in 2009, the year during which Uber just started its business. With those observations in reality, one might want to ask whether other channels can also help explaining the various performances of innovation diffusion and adoption.

One observation from the two examples above is that those innovations have the property of network externality, meaning that consumers in the market not only cares about the gain from switching to those new products but also about whether others also adopt the same product. In this paper I explore an entry model in which multiple potential entrants each can bring a new technology into the market. Technology has the character of network externality, while entrants also rely on the word-of-mouth communication under the social network among consumers to promote their products. The two properties naturally generate an entry barrier in the market. And to overcome the obstacle, each entrant not only need to consider the pricing strategy of its new product but also account the timing of its entry. Early trials are likely to fail, but gradually weaken the existing network externality, which in turn helps later entrants. With this model, I find that timing is a crucial determinant of the commercial performance of an innovation. Under some conditions, bad innovation firms will always enter the market whenever possible but good innovation firms might strategically wait for a proper time in order to acquire a better market share.

The paper unfolds as follows. The remaining part of section 1 briefly discusses the relationship between my work and the previous literature. The basic model is provided in section 2. In section 3, I formally present the main results of the paper. Section 4 presents the conclusion. All detailed proofs are provided in the appendix.

4.1.2 Related literature

My work is related to several research areas. It provides an alternative view to the issue of innovation adoption, which is mainly studied in growth theory. Starting from Schumpeter (1934), many models are established to describe the relationship between technological change and business dynamics[62]. Firms adopting new technology bring creative destructions of incumbent firms and resources are reallocated to those more efficient producers. Finally, firms sticking in old technology exit the market, resulting in the phenomenon of technology switching in the market level. In those models, learning by doing[63] and R&D competition[64] are usually used to explain individual firm's technology choice, especially the reason why some firms fail in switching to a new technology. My work, on the other hand, considers a different mechanism behind the innovation adoption failure. The deterrence force in my model comes from the social preference of existing products and the word-of-mouth communication process of information transmission. As a result, the timing can explain different performances of innovations. Similar technologies entering the market in different time might perform significantly differently.

This work is also related to the literature on the network externality issue in industrial organization. In their seminal paper, Katz and Shapiro (1985) first addresses the character of network externality in many products and analyzes the competition behavior of producers under this scenario. Later researches focus on issues including the entry license[65], the dynamic pricing[66] and

[62]See also, Aghion and Howitt (1992) and Klette and Kortum (2004).
[63]See, for example, Jovanovic and Nyarko (1994).
[64]See, for example, Aghion et al. (2001) and Acemoglu and Akcigit (2012).
[65]See, for example, Economides (1996).
[66]See, for example, Dhebar and Oren (1986), Bensaid and Lesne (1996) and Cabral et al. (1999).

consumer loyalty[67]. Unlike those studies in which the network externality of products is global, in my work a local externality under some given social network structures is considered. Therefore, consumers in my model are not required to be fully informed about all others' choices or to have rational beliefs, which in turn eliminates the existence of multiple equilibria. Further, the diffusion of new product can be better described and consumers with identical preference might adopt different products according to their status under the social network.

Finally, my work is an application of percolation theory in Economics. In Physics and Mathematics, many researches are done regarding the percolation process under different network structures[68]. Among them, Callaway et al (2000) and Newman et al (2001) provide a probability generating function method to analysis the percolation issue on a given random graph. However, few researches have applied the percolation theory in Economics. One exception is Campbell (2013) in which the author discusses the word-of-mouth communication as the promotion method for a single new product. Different from Campbell (2013), I focus on the market dynamic of multiple products. The network externality of products provides natural entry barriers for incumbent products while potential entrants not only need to care about their pricing strategy but more importantly need to strategically determine the time to enter the market.

4.2 Model setup

I first provide a brief description of the model, and further details will be provided in the following subsections. Consider the scenario in which a group of innovation firms attempt to enter a market. Consumers in the market is connected by a social network which describes the friendship among them. Each consumer can and only can observe their friends' choices of products. And his welfare comes from both an individual level utility directly obtained from the product he adopts and a social preference depending on the number of friends using the same product. Each firm in the group holds a unique technology which can be sold as an update of existing products and provide heterogenous additive utilities to consumers. In other words, if a consumer chooses to purchase the update and thus switch to the new product, he will keeps the previous individual level utility and an additional utility coming from the latest update.

Time is discrete and at the beginning of each period, a random firm is picked to make an entry decision. If the firm chooses to enter the market, it will sell its technology by determining a uniform price. And when the new technology is introduced into the market, only an exogenously

[67] See, for example, Zhou and Lu (2011).
[68] See Grimmett (1999) for a review.

determined small fraction of consumers are informed about it at the beginning. Each other consumer is uninformed and thus will not consider to make the update until he directly observes some of his friends doing it. The update is irreversible and each consumer will only make the update if his net welfare improvement from both the individual level utility and the social preference is greater than the price of the update. The period ends when no more consumers are willing to purchase the update.

At the end of the period, the entrant quits the market and the technology is no longer on sell. A new innovation firm will be added into the potential entrant group to replace it. On the other hand, the picked firm can also choose not to enter the market in the current period, in which case another randomly selected firm will make the entry decision. No new potential entrant will be added if no firms decide to enter the market in the current period. However, waiting is costly and each firm maximizes its discounted profit when making the entry decision. The main question the paper want to answer is: when will a firm choose not to enter the market? Furthermore, given potential entrants can choose not to enter the market, will all firms find the optimal time to introduce their products and thus be adopted by a non-negligible fraction of consumers? I now provide details of my model.

4.2.1 Market components

The market involved in my model contains a population of consumers which is denoted by $N = \{1, 2, ...n\}$ where $n \gg 0$. The population is connected by a social network which is denoted by a non-weighted graph $G = (g_{ij})_{i \neq j \in N}$ where $g_{ij} = 1$ indicates that consumer i is a friend of consumer j and $g_{ij} = 0$ otherwise. Here the friendship is undirected and thus $g_{ij} = g_{ji}$. I assume the social network is described by a random graph with some arbitrary degree distribution $\{p_k\}$ where $\sum_k p_k = 1$. That is to say, for any randomly picked consumer under this network, the probability that he has k friends is given by p_k. The average degree of the social network is denoted as $\langle k \rangle$, which is given by $\langle k \rangle = \sum_k p_k \cdot k$. I further assume that the social network is connected.

A set of innovation firms, denoting as F attempt to enter the market. The cardinality of the set F is given by $m^f = |F|$. Each firm $f \in F$ is endowed with a unique technology $Tech^f$ which can be introduced to the market as an update of the existing product at a marginal cost of 0. The update is additive. In other word, denoting the technology adopted by consumer i to be $Tech_i$, the consumer who switch from the product $Tech_i^{f'}$ he initially adopted to the new product $Tech_i^f$ will keep the initial individual level utility and gain an additional individual level utility from the new technology $Tech^f$. Technologies have two types. A high type technology can provide an individual level utility to each consumer drawn i.i.d. from a uniform distribution of $U[0, \theta_h]$ and a low type technology brings utilities drawn from $U[0, \theta_l]$, in which $0 < \theta_l < \theta_h$. For all firms in the set F, γm^f hold a high type technology and $(1-\gamma)m^f$ hold a low type technology, in which both numbers are non-zero integers.

Before the game starts, consumers in the market adopt no technology and their individual level utilities are normalized to 0. Each consumer i also has a social preference measured by $|\{j|g_{ij} = 1, Tech_j = Tech_i\}| \cdot \theta_s$. That is to say, for each of his friends who adopts the same product as consumer i does, consumer i will gain an additional utility of θ_s. Clearly at the beginning of the game no technology is adopted and thus the social preference of each consumer is also normalized to 0. To ensure the utility from the social preference is not too large, I further assume $0 \leq \theta_s < \theta_h$.

4.2.2 Entry game

Time is discrete. At the beginning of each period t, a firm f from the set F is selected uniformly randomly. The firm can observe current market shares at the beginning of the period, which is denoted as $\Delta_t = \{\Delta_{tk}|0 < k < t\}$ where Δ_{tk} indicates the market share of firm k at the beginning of period t and $\sum_{k=1}^{t} \Delta_{tk} \leq 1$. With this information, the firm decides whether it wants to enter the market in the current period or not. If the firm chooses not to enter the market, another firm from the remaining pool is selected in the same way. However, each firm does not know whether others are picked before it and reject to enter. The process ends either when a firm decides to enter or all firms in F do not want to enter. If a firm enters the market, it is removed from the set F and a new firm with same type of technology is added to replace it. Otherwise, no new firm is added to the set F.

The firm f who chooses to enter the market at period t determines a unit price P^f for its product $Tech^f$. Consumers must become informed about the product before they can make the purchasing decision. A small fraction $\epsilon = O(\frac{1}{n})$ of consumers will be exogenously informed about the newly released product once it enters the market and all others are uninformed initially. Each consumer i keeps observing his friends' actions and thus when one of his friends adopts the new product, consumer i becomes informed. In reality, the small fraction who is informed directly can be considered as consumers who find the advertisement of the new product via some exogenous channels. While the rest consumers must be informed via learning under the network, which is similar to the classical word-of-mouth communication. An informed consumer i will purchase the new product and thus update his technology if and only if the price is no greater than the net benefit the new product will bring to him. That is to say, consumer i who initially adopts a product $Tech_i^{f'}$ will make the update and switch to the product $Tech_i^f$ if the following inequality holds:

$$P^f \leq \theta_i|Tech_i^f + \theta_s \cdot (|\{j|g_{ij} = 1, Tech_j^f = Tech_i^f\}| - |\{j|g_{ij} = 1, Tech_j^f = Tech_i^{f'}\}|) \tag{10}$$

Note the term $\theta_i|Tech_i^f$ is the individual level utility consumer i can get from the new product, which is drew from $U[0, \theta_h]$ or $U[0, \theta_l]$ according to the type of the technology. Finally, the period ends when no more consumer is willing to switch to the new product. The firm quit the market forever and the product is no longer on sell. The firm receives a discounted revenue with a common discount factor of $\delta < 1$. This finishes the model.

Note the setting captures the idea that innovations usually have network externalities. Consumers prefer to adopt the same product as their friends, as modeled by the social preference θ_s. Examples in the motivation section all share the property. For the case of Airbnb and Couchsurfing, consumers are more likely to register if residents in places where they want to travel to are members of those websites. While for the case of Uber and Google Ride Finder, on the other hand, people are willing to adopt if many drivers nearby have joined the system. Other examples include the social network websites like Facebook and Twitter, and also standards like Blu-ray and HD DVD. Although network externalities in reality have various forms, a random graph performs well as a benchmark scenario.

Also, the way consumers become informed in my model follows the standard word-of-mouth model, which is a special case of a more general class of percolation model. Consumers keep observing their friends under the social network and adopting a new technology if either a lot of others already adopt it or the individual level utility from the innovation is large enough. This illustrates the diffusion process of innovations. Further, given each period in my model ends only when no new consumers are willing to switch, I allow for a complete diffusion process within each period.

4.3 Market evolution

In this section I will analyze the behavior of potential entrants in the market modeled above. To be more precise, given the information each potential entrant has, a Markov perfect equilibrium under which each potential entrant's strategy only depends on its own type and the state of the market should be considered. Note each potential entrant needs to choose both a timing strategy which indicates when to enter the market and a pricing strategy in order to maximize its total revenue. In other words, each potential entrant f's strategy should be written as $A_f(\Delta_t, \sigma) = \{a_t^f, P_t^f\}$ where $\sigma \in \{l, h\}$ indicates the type of the firm, $a_t^f \in \{wait, enter\}$ indicates whether the firm want to enter the market in period t if picked and P_t^f is the price the firm will choose if it enters the market. Unfortunately, it is in general not possible to give an explicit description of the pricing strategy of potential entrants because the state space Δ_t has infinite dimensions as t becomes large. Instead, in the following discussion I will first focus on a benchmark case of the pricing issue which only relies on one dimension of the whole state space. And the benchmark case can provide enough knowledge to further analyze the main focus of this paper: the timing strategy of potential entrants.

4.3.1 Pricing issue and market share

Since the pricing decision is made after a firm enters the market, in this subsection the discussion will all be conditional on that the potential entrant decides to enter the market in the current period. First note when $\theta_s = 0$, my model is a special case of Campbell (2013). Without the network externality, inequality (1) is simplified to $P^f \leq \theta_i | Tech_i^f$. As a result, there will be no interaction among firms and each entrant just acts as if it enters an unoccupied market. This scenario is already carefully discussed and I will restate one result whose proof can be found in Campbell (2013):

Lemma 4. *(Campbell, 2013) If $\theta_s = 0$, for a type σ firm f, there exists a price $P_\sigma^\star \in (0, \theta_\sigma]$ such that only a negligible fraction $n \cdot O(\frac{1}{n})$ of consumers will buy the product $Tech^f$ when $P^f \geq P_\sigma^\star$ and a non-negligible fraction $n \cdot O(1)$ of consumers will buy the product when $P^f < P_\sigma^\star$.*

This result shows that for each entrant, there exists a threshold price such that the firm can only sell its product to a majority of consumers if the price is lower than the threshold. The result mainly comes from the percolation theory and the idea is that when the price is too high, the chance that an informed consumer to purchase the product is too low. Therefore, it is unlikely that an initially uninformed consumer to observe any of his friends using the new product.

Another remark of this result is that without the network externality, each firm can find a price low enough to ensure its product being purchased by a non-negligible fraction of consumers. It is true because each consumer's outside option is simply not purchasing the good and receiving a utility of zero. It is no longer held when $\theta_s > 0$ and especially when the social preference is strong. Consider the case when a non-negligible fraction of consumers in the market adopts a same old technology when the new product is introduced. It is possible that even if the firm charges a zero price, few consumers will purchase the product because now the outside option of an informed consumer is to keep adopting the same old technology as his friends and receive a positive utility. Denoting $\eta_t = max_k(\Delta_t)$ to be the fraction of consumers adopting the most widely adopted technology at the beginning of period t, the following lemma shows the scenario described above:

Lemma 5. $\exists \bar{\theta}_s > \underline{\theta}_s > 0$ *such that for a type* σ *firm* f*:*

(a) If $\theta_s \in (0, \underline{\theta}_s), \forall \eta_t \in [0,1], \exists P^\star_\sigma(\Delta_t) \in (0, \theta_\sigma]$ *such that a non-negligible fraction* $n \cdot O(1)$ *of consumers will buy the product if and only if* $P^f < P^\star_\sigma(\Delta_t)$*.*

(b) If $\theta_s \in [\underline{\theta}_s, \bar{\theta}_s), \exists \underline{\eta}(\theta_s) \in [0,1]$ *such that only a negligible fraction* $n \cdot O(\frac{1}{n})$ *of consumers will buy the product* $Tech^f$ *regardless of* P^f *when* $\sigma = l$ *and* $\eta_t \geq \underline{\eta}$*. And for* $\sigma = h$*,* $\forall \eta_t \in [0,1], \exists P^\star_h(\Delta_t) \in (0, \theta_h]$ *such that a non-negligible fraction* $n \cdot O(1)$ *of consumers will buy the product if and only if* $P^f < P^\star_h(\Delta_t)$*.*

(c) If $\theta_s \geq \bar{\theta}_s, \exists \underline{\eta}(\theta_s) < \bar{\eta}(\theta_s) \in [0,1]$ *such that only a negligible fraction* $n \cdot O(\frac{1}{n})$ *of consumers will buy the product* $Tech^f$ *regardless of* P^f *when* $\sigma = l$ *and* $\eta_t \geq \underline{\eta}$ *or when* $\eta_t \geq \bar{\eta}$*.*

Lemma 5 says that when both the social preference θ_s and the fraction η_t are large, it is possible that a new entrant can never sell its product to a majority of consumers regardless of its pricing strategy. The intuition behind is that when the social preference is too strong, the individual level utility from a new product is overwhelmed by the willingness of remaining the same as friends. And thus those who get informed about the new product early are less likely to switch, resulting in a majority of consumers keep uninformed. One remark of this result is that the social preference, which acts as the outside option of consumers, is now strictly positive. One can also consider this scenario in a similar way as that of $\theta_s = 0$ but with the adjustment that each consumer's utility from the new product is deducted by his outside option from the social preference. As a result, the threshold price might be lower than zero when the social preference is large, which just coincides with the scenario in which no price can make the new product be purchased by a majority of consumers. Note when considering outside options, only the highest one matters. Therefore, the threshold market state only depends on the biggest fraction η_t, which together with θ_s determines the highest outside option of consumers. In other words, although to know the optimal price the firm needs to use all information in Δ_t, it only requires the information η_t to check whether it has the possibility of diffusing to a non-negligible fraction of the market. Therefore, lemma 5 provides a good benchmark for this problem and thus allows us to further analyze the entry behavior of firms.

Another remark of this result is that the existence of the social preference creates an entry barrier in the market. New entrants now face a lower threshold price in order to sell their products to a majority of consumers, comparing with scenarios without the social preference. However, this is only one side of the coin. It is true that the social preference makes the threshold price lower, but when the new entrant indeed take a market share comparable with the old market overlord, the social preference has a positive effect on the new product, as shown in the following lemma:

Lemma 6. *For $\forall\theta_s, \exists\tilde{\eta} > 0$, such that if $\eta_t < \tilde{\eta}, \exists P^f$ for a high type firm f which can lead to a total demand $D(P^f) \geq \frac{\theta_h - P^f}{\theta_h} \cdot n$.*

Lemma 6 shows that when the new entrant indeed picks a price lower than the threshold, it is possible that the total sales will be greater than the case with full information and no social preference. A similar result can be given for a low type firm. The intuition behind is that once the threshold price requirement is met, the new product will be widely adopted and thus even consumers whose individual level utility from the new product is low will purchase it in order to follow their friends who switch to the new product. Mathematically, the second term in the right side of inequality (10) can become greater than zero and thus the social preference has a positive effect on the new product. Comparing with the case of no social preference, in which the total demand is bounded above by $\frac{\theta_\sigma - P^f}{\theta_\sigma} \cdot n$, the existence of the social preference stimulates total sales.

Although lemma 6 shows that the existence of the social preference is likely to lead to a large market share for successful entrants, without further information about the social network structure $\{p_k\}$ and the parameters $\{\theta_s, \theta_l, \theta_h\}$ one can not provide a further quantitative comparison of the market share for an new entrant under an optimal pricing strategy and the market share for the old market overlord, the one with the highest market share at the beginning of the period. Instead, for the remaining part of the paper I will consider a special scenario described as following:

Assumption. *(Sufficient gap, SG) The SG condition holds if the demand of a high type technology under its optimal price satisfies $max\{\eta_t \cdot n - D(P^f(\Delta_t)), D(P^f(\Delta_t))\} \geq \underline{\eta} \cdot n, \forall\Delta_t$.*

The idea behind the SG condition is that the high type technology is significantly better than the low type technology. When the market state is close to the threshold state for the low type, it is good enough for the high type in such a way that a high type firm can at least perform as good as the current market overlord. Note with the idea from lemma 3, the condition is likely held once the gap between the two thresholds $\bar{\eta}$ and $\underline{\eta}$ is large. In fact, one special case under which the SG condition is satisfied is when $\bar{\eta} \geq 2\underline{\eta}$. Because both thresholds only depends on the social network structure $\{p_k\}$ and the exogenous parameters $\{\theta_s, \theta_l, \theta_h\}$, and with a fixed social network the gap is larger when the difference between θ_l and θ_h is bigger, the condition actually assumes that the individual level utility from a high type technology is sufficiently greater than the individual level utility from a low type technology. In the remaining discussion of the paper I will assume the SG condition holds without further clarification. I will also discuss the scenario when the condition fails at the end of the paper.

To summarize, the network externality caused by the social preference both acts as an entry barrier because of the lower threshold price and also stimulates sales when the new product becomes widely adopted. With those characters in mind, the following discussion will focus on the timing issue of potential entrants.

4.3.2 Timing strategy

From lemma 1, the interaction among potential entrants only comes from the existence of the social preference. Also, the interaction is via the transformation of η_t according to lemma 2. The following result further describes the effect of entrants on the fraction of consumers adopting the most widely adopted technology:

Lemma 7. *When $\eta_{t_0} > \bar{\eta}$ for some t_0, if the total number of entrants goes to infinity with t increasing, $\exists T$ such that $\eta_{t_0+T} \leq \bar{\eta}$.*

Lemma 7 shows that the majority of consumers will switch to the new product eventually even if the social preference is strong. The result is obvious because even when each previous entrant only sells its product to a small fraction of consumers $O(\frac{1}{n}) \cdot n$, the remaining consumers adopting a same old product keeps shrinking as T becomes large. The idea behind this lemma is that although early entrants are likely to fail, they still break the existing network of consumers adopting the most popular old technology into pieces, which in turn helps later entrants to occupy the market. To interpret this lemma in another way, waiting is beneficial in the way that later entrant are more likely to face a fragile market and thus are able to sell to a non-negligible fraction of consumers by choosing the correct price. The remaining question is who at what time will choose to wait instead of entering the market directly when picked? The following lemma partly answers the question.

Lemma 8. *A firm with low type technology will always enter the market when picked.*

Because of the cost of waiting, under equilibrium there must be at least one type of firms accepts to enter the market in each period. Lemma 8 shows that a low type firm will never choose to wait, and thus the requirement in lemma 7 holds. Note a firm will choose to wait only when it faces a high η_t. Under this circumstance, the firm either accepts a small total revenue or waits, hoping that others will enter before it and break the existing strong network externality. However, if the latter is profitable to a low type firm, it is also profitable to a high type firm but not vise versa. Therefore, if a low type firm's best strategy under some state of the market condition is to wait, it should also be the best strategy of a high type firm and thus all potential entrants in the set F. However, in this case no firm will ever enter the market and firms will need to wait forever, which is clearly not an equilibrium. The lemma thus shows half of the picture: in each period, at least a low type firm will enter the market. Together with the previous lemma one could expect that there will be switches of products involving non-negligible fractions of consumers periodically. Once the best strategy of high type firms is figured out, the whole picture is complete, which is done in the following result.

Proposition 10. *When $\theta_s \geq \bar{\theta}_s$, $\exists \bar{\eta}_h^\star \in (\bar{\eta}, 1]$ such that a high type firm will choose to wait when picked if $\eta_t \in [\bar{\eta}, \bar{\eta}_h^\star]$.*

Proposition 10 shows that when the market contains a widely adopted old technology whose market share is close to the threshold market share $\bar{\eta}$, the high type firm would prefer to wait. As a result, low type products will be introduced into the market in a smooth frequency. High type products, on the other hand, will be introduced in a periodical order.

One remark of this result is that the strategy involving waiting does not guarantee a high type firm to sale its product to a non-negligible fraction $n \cdot O(1)$ of consumers. This is because when the current market is densely occupied by some old products, the firm will need to wait for too long before the market is ready for a takeover. Given the cost of waiting measured by the discounting factor δ, the high type firm might rather enter the market immediately instead of waiting. Further, even a high type firm who chooses to wait is not guaranteed to sale its product to a non-negligible fraction of consumers in the future. In fact, it is possible that before the firm is re-picked, another high type firm enters and occupies the market.

Another observation is that a strategy involving waiting does not always exist. There are two scenarios in which high type firms will also enter the market whenever picked. It is quite clear that when the social preference is not strong, e.g. $\theta_s < \bar{\theta}_s$, a high type firm can always ensure a non-negligible fraction of consumers and thus does not necessarily have the incentive to wait. Another scenario is relatively subtle. When the social preference is indeed strong, it is possible that the optimal pricing of high type firms leads to a low market occupation rate, which results in a low η_{t+1} for the forthcoming high type firms. In other words, the social preference parameter θ_s along does not guarantee an entry barrier, it is only when the optimal market share of entrants is greater than the threshold market state $\bar{\eta}$ will the timing strategy of high type firms contains a contingent waiting.

Thirdly, when the social preference is strong as that in case (c), a high type firm will only choose to wait when the current market state η_t is close to the threshold level $\bar{\eta}$. This result is mainly due to the fact that the sales of the entrant under an optimal pricing strategy will experience a sharp increase around the threshold, which offsets and overwhelms the waiting cost of the firm. When the current market state η_t is too high, as discussed above, the exponential discounting from waiting will dominate. While on the other hand, when the current market state η_t is lower, the gain from waiting is less significant because the firm can already occupy a non-negligible fraction $n \cdot O(1)$ of consumers.

Finally, this result explains why in reality we do observe new technologies failing in spreading in the market. Imagine an innovation is introduced into the market with some market state greater than the threshold $\bar{\eta}$, by lemma 5 the innovation can never be sold to a non-negligible fraction of consumers. However, as time goes by the market state will decrease and once it falls under the threshold, a similar innovation can make a commercial success. And the interval $[\bar{\eta}, \bar{\eta}_h^\star]$ can be considered as an indicator of how often a high type firm will strategically postpone its entrance into the market. In the following part I will further discuss the determinants of the interval.

4.3.3 Discussion

Since firms are picked uniformly randomly at the beginning of each period, the interval $[\bar{\eta}, \bar{\eta}_h^\star]$ can be used to measure the frequency of strategic waiting of high type firms. One obvious result about the interval is given as following:

Lemma 9. *(Patient firms) With δ increasing, $\bar{\eta}_h^\star$* will weakly increase.

The result above is obvious because when those potential entrants are more patient, they are likely to wait longer. An extreme case is when δ is close to 1, in which firms are sufficiently patient and thus high type firms will only choose to enter the market when they can sale to a non-negligible fraction of consumers.

Beside the patience of potential entrants, another crucial determinant of the waiting interval is the intensity of competition among firms, as shown in the following result:

Lemma 10. *(Competition intensity) With the increase of m^f or γ, $\bar{\eta}_h^\star$* will weakly decrease.

In words, each high type firm is less likely to wait when either the total size of the potential entrant set increases or the total number of high type firms increases. The former is more intuitive because when the number of firms increases, the chance that each firm will be picked again when the market state becomes attractive decreases. Therefore, waiting becomes more risky to high type firms, resulting in a shrinking waiting interval. The case when only the total number of high type firms increases is more subtle. Note when the total number of high type firms increases, even though the probability each potential entrant to be picked remains unchanged, it is more likely that another high type firm is picked before the waiting firm being re-picked. As a result, high type firms again are reluctant to wait. The consequential result of this lemma is that under a more competitive circumstance, more high type technology will enter the market with a negligible adoption rate. In reality this might mean that more resources are wasted in those failing trials. However, those unlucky high type firms might help breaking the existing entry barrier at a faster speed. An open question remaining unsolved is whether the fierce competition helps or hurts the total welfare of the society.

4.4 Conclusion

When innovations have network externalities and rely on the word-of-mouth communication under the social network to promote sales, potential entrants are not necessarily able to get a non-negligible demand regardless of the price. As a result, high type entrants might strategically wait for a better market state instead of entering the market immediately. However, the waiting phenomenon can only be observed when the market state is close to a threshold state and thus allowing for waiting does not guarantee a high type innovation to commercially succeed. Innovations might successfully diffuse in the market if they happened to enter the market at a good time even if similar ones have failed before.

I conclude by discussing some assumptions and limitations of the result and potential opportunities of future research. The two main assumption needed to reach my result are the existence of network externality, as described by the social preference, and the word-of-mouth communication. The social preference assumption is necessary as discussed in the beginning of section 4.3. Without the network externality, there is no interaction among potential entrants and the entry barrier which leads to the waiting strategy disappears. On the other hand, the word-of-mouth communication mainly captures the idea of innovation diffusion. If firms no longer rely on the word-of-mouth communication and all consumers are informed about each new product, firms can always at least sale their products to a non-negligible fraction of consumers. The time of entry still matters because the market state still determines the market share a new entrant can obtain, but potential entrants might no longer wait, depending on how they discount the potential high profit in the future and compare it with the current small but still non-negligible demand. One challenging but likely more realistic modification of the model is to allow firms to both rely on the word-of-mouth communication and also strategically post advertisements to inform consumers.

Secondly, the assumption of a random graph structure of the social network makes it possible to explicitly express the threshold market state. Releasing the assumption will not significantly change the main result that potential entrants might strategically wait, as long as the network structure under consideration still has a non-zero percolation threshold. However, changing the network structure can significantly change the pricing strategy, as discussed in Campbell (2013). Unfortunately, with the complication of the market state, it is not easy to get an explicit expression of the optimal price regardless of the network structure.

Finally, the SG condition leads to the result that low type firms will never wait. When the difference between a high type technology and a low type technology is not significant and the SG condition fails, one can imagine that low type firms will also adopt a similar strategy with waiting. However, proposition 10 still holds because when the current market power is strong, waiting is still not profitable for a low type firm and thus a high type one can still benefit from those low type firms entering the market. One might extend the model to include more types and an intuitive conjecture is that each type that is not significantly worse than the type higher than it can choose to strategically wait while those which are far worse will always enter the market whenever picked. A further generalization is to consider a continuous type space, which seems to be an interesting future topic from both the theoretical and the empirical perspectives.

Appendix Omitted proofs

Proof of proposition 1

First show the asymptotic distance distribution follows a Weibull distribution. To reach the goal we first consider an equivalent network formation procedure. Firstly each agent i enters the network sequentially only forming the number of global meetings he should form based on his action a_i. We call the network when all global meetings are formed g^G which can be considered as a random network with exponential degree distribution, as shown in example 1. After the formation of g^G, each agent i forms his local meeting if he indeed chooses *Local* in the same order as the entrance with one more requirement that the local meeting can only link with an agent whose order is earlier than i. The final network is g^{GL} and one can easily find that the procedure introduced here is indeed equivalent to the one in the model, e.g. $g^{GL} = g$.

Now that we can adopt the method used in Bauckhage, Kersting and Rastegarpanah (2013) to get the distribution of walk length between two randomly selected nodes[69] in g^G. Let $F^G(x)$ be the distribution function of the length of walk between two randomly selected agents in g^G. Notice if there is no walk with length smaller or equal to x between the two nodes, there is no walk with length smaller or equal to $x-1$ between the first node and any neighbor of the second node. Thus we can get the mean-field approximation of the distribution $F^G(x)$ in the recursive way of $1 - F^G(x) = [1 - F^G(x-1)]^m$ where m is the expected degree of the network. Together with the initial value $F^G(0) = \frac{1}{N} \approx 1 - e^{-\frac{1}{N}}$ and $m = 2 \times (2 - r)$ we have:

$$F^G(x) \approx 1 - e^{-\frac{1}{N}(4-2r)^x}, x \geq 0 \tag{11}$$

Since the case $x = 0$ is trivial, we might normalize the equation above to $F^G(x) \approx 1 - A \cdot e^{-\frac{1}{N}(4-2r)^x}$ for $x \geq 1$ and some constant $A = e^{\frac{1}{N}}$. And one can get the density for the second node to have a walk with a length exactly x from the first one by $f^G(x) = F^G(x) - F^G(x-1)$, where $F^G(0) = 0$ if the normalization is done. When considering the continuous approximation one can get $f^G(x) = \frac{d}{dx}F^G(x)$.

Now consider the network g^{GL}. Comparing with g^G, the local meetings added in g^{GL} lead to the result that for any randomly selected pair of nodes with distance 2 in g^G, there is a chance $q = \frac{r}{m}$ that the two nodes are neighbors in g^{GL} because one of them might link with the other via a local

[69]They are not necessarily distinct.

meeting. More general, for each path in g^G with length $x+1$, there is a corresponding path in g^{GL}[70] which is a random variable following $x+1-Bi(x,q)$ where $Bi(\cdot,\cdot)$ is the binomial distribution[71]. One can apply Ilienko (2013) to get the following function as a continuous approximation of the binomial distribution $Bi(d,q)$:

$$\tilde{F}_{x,q}(y) = 1(0 < y \leq x+1) \cdot \frac{B(y, x+1-y, q)}{B(y, x+1-y, 0)} + 1(y > x+1) \tag{12}$$

Where $B(\cdot,\cdot,\cdot)$ is the incomplete Euler B-function:

$$B(x, y, q) = \int_q^1 t^{x-1}(1-t)^{y-1} dt \tag{13}$$

Now combining equations above we have the following approximation for the distribution of paths between two randomly selected nodes in g^{GL}:

$$F^{GL}(x) = \int_0^{+\infty} \tilde{F}_{d-1,q}(d-x) dF^G(d) \tag{14}$$

Since the distance between the two nodes is the minima of all realizations, by Gnedenko (1943) a Weibull distribution can be considered as an approximation of the extreme if the following two conditions are satisfied:

$$x_l = inf(x|1 - F^{GL}(x) > 0) < +\infty \tag{15}$$

$$lim_{\alpha \to 0^+} \frac{1 - F^{GL}(\alpha x - x_l)}{1 - F^{GL}(\alpha - x_l)} = x^\beta \text{ for some } \beta > 0 \tag{16}$$

The condition one is obvious given the finiteness of the network. For condition two, applying the L'Hôpital's rule gives the result[72].

The remaining task is to show the relationship among the parameters of the Weibull distribution and the action of players, r.

[70] It is not necessarily the distance between the two nodes but just the corresponding path of the randomly chosen path after all local links are formed.

[71] This is true for all paths in g^G instead of walks but the function F^G represents the distribution of walks between two randomly selected nodes. However, since the graph is pretty sparse given all those global random links, the distribution approximately represents the distribution of paths. Also, since the result is not true only when dealing with walks that are not path and lengths of those walks can never be the distances between nodes, the approximation will not affect the following proof. See Bauckhage, Kersting and Rastegarpanah (2013) for details.

[72] For a continuous extreme distribution, de Haan (1976) shows that the parameter β in condition two just represents the shape parameter of the Weibull distribution but since the path distribution in my paper is only a continuous approximation of the discrete variable, the relationship is not clear.

The shape parameter $\kappa > 1$ due to the definition of the distance between nodes.

As for the scale parameter λ, Bollobás and Riordan (2004) shows that the diameter, and thus the average distance, of the network g^G is negatively correlated with the number of global links each agent forms. Fitting the result in my model, the average distance of g^G shirks from $O(log(N))$ to $O(log(log(N)))$ as r goes to zero. Note the local links in g^{GL} only affect the average distance proportionally, it will not affect the relationship. Therefore, one can conclude that $d\lambda/dr > 0$.

Finally, given the fact that $d\lambda/dr > 0$ and the result in example 1, the shape parameter satisfies $d\kappa/dr < 0$, which finishes the proof. □

Proof of proposition 2

For any given N and r, each individual's best response is L if and only if $F^{dis|local}(D) - F^{dis|global}(D) \geq 0$. Now given the boundary of $F^{dis|local}(d) \geq \frac{3}{4}F^{dis}(d-1) + \frac{1}{4}F^{dis}(d)$ and the fact that the parameter of the Weibull distribution located in a compact set $[\kappa(1), \kappa(0)] \times [\lambda(0), \lambda(1)]$, there exists $d_l^\star$ such that for all $D \leq d_l^\star$ the inequality holds.

Similarly, using the boundary $F^{dis|local}(d) \leq F^{dis}(d)$ one can show the existence of $d_u^\star$, which finishes the proof. □

Proof of theorem 1

First, note that with a passive belief, even if monitor j receives a bribe plan from player i who is also monitored by others, player j believes that the remaining players receive no bribe plan and will thus continue to work. As a result, the following lemma holds:

Lemma 11. *If a monitoring network can support a PBE under which all the players work, it can also support the passive cooperation PBE.*

Proof: This result is obvious because a passive cooperation PBE has the most optimistic off-path belief. Thus, if a player with some beliefs rejects an offer under a monitoring network, he will also reject the same offer with a passive belief. □

Notably, lemma 12 indicates that to prove theorem 1 and corollary 1 is equivalent to proving that among all the monitoring networks that can support a passive cooperation PBE, the core-periphery network that is constructed above has the lowest total cost. Therefore, the remaining part of the proof will focus on the passive cooperation PBE.

Lemma 12. *A negative bribe offer is never acceptable under passive belief.*

Proof: Without loss of generality consider the case of $i \in M_j^-$. From the perspective of player i, he either has $M_i^- = \{j\}$ or $M_i^- \neq \{j\}$. In the first case, a negative offer $b_{ji} < 0$ is not acceptable because player i believe that player j will shirk no matter what his response is, and thus player i can safely shirk without giving player j the required transfer. However, in the second case, player i still cannot shirk safely and will thus still not accept the offer. □

From the perspective of a potential briber:

Lemma 13. *The total amount of bribes that player i can offer satisfies $\sum_{j \in M_i^-} b_{ij} \leq 1 - (m_i^- + 1)\alpha$.*

Proof: If player i offers a non-zero bribe plan, he should bribe all his monitors and believe that his plan is acceptable such that all his monitors collude with him and shirk. Therefore, he should expect that a total of $m_i^- + 1$ players deviate from working. To make the bribe plan profitable for him, I find: $n\alpha - 1 \leq (n - m_i^- - 1)\alpha - \sum_{j \in M_i^-} b_{ij}$. Rearranging the inequality leads to obtaining the lemma. □

With the lemma above, the following condition for a network to support a passive cooperation PBE can also be obtained:

Lemma 14. *A monitoring network can support a passive cooperation PBE if and only if $M_i^- \neq \emptyset$, $p_i \geq \frac{1-\alpha}{(n-1)\alpha}$ and $\sum_{j \in M_i^-, M_j^- \neq \{i\}} p_j \geq \frac{1+m_i^- - (2m_i^- + 1)\alpha}{(n-2)\alpha}, \forall i \in N$.*

Proof: The requirements $p_i \geq \frac{1-\alpha}{(n-1)\alpha}$ and $M_i^- \neq \emptyset$ ensure that each player will not shirk alone. The requirement $\sum_{j \in M_i^-} p_j \geq \frac{1+m_i^- - (2m_i^- + 1)\alpha}{(n-2)\alpha}, \forall i \in N$ ensures that player i cannot offer a bribe to each of his monitors that is greater than the monitor's gain from cooperation because player i can offer $\sum_{j\in M_i^-} b_{ij} \leq 1-(m_i^-+1)\alpha$ to make his payoff no less than that from the equilibrium strategy. For each of his monitors j, the offer is not acceptable if and only if $M_j^- \neq \{i\}$ and $(1-p_j)(n-2)\alpha + b_{ij} \leq (n-1)\alpha - 1$. Rearrange the offer, and the required result can be obtained. □

Note $p_i \leq 1, \forall i \in N$. To satisfy the last condition in lemma 4, the number of monitors for each player i should be located in the following range:

Lemma 15. *An optimal monitoring network must have* $m_i^- \in \mathbb{K} \cup \{N_l\}$.

Proof: First show that under an optimal monitoring network, $m_i^- \leq N_l, \forall i \in N$. This formula is derived from lemma 14, if $m_i^- = N_l$, player i will never bribe his monitors. Any number that is greater than N_l only increases total monitoring costs and is thus not efficient.

In the case of $m_i^- < N_l$, player i can potentially offer positive bribes to his monitors. Now put $p_x \leq 1, \forall x \in N$ into the condition $\sum_{x\in M_i^-, M_x^- \neq \{i\}} p_x \geq \frac{1+m_i^- -(2m_i^-+1)\alpha}{(n-2)\alpha}$ in lemma 4, and the necessary condition $m_i^- \in \mathbb{K}$ can be obtained, which concludes the proof. □

In the following proof, I will focus on the case of $inf(\mathbb{K}) < N_l$ because when $inf(\mathbb{K}) = N_l$, the result is trivial.

Under $K_u' = sup\{m_i^- | i \in N, m_i^+ \neq 0\}$, I claim that under the optimal network there must be at least one player i such that $m_i^- = K_u'$ and $p_i \geq \frac{1+K_u' -(2K_u'+1)\alpha}{(n-2)\alpha K_u'}$ because I can always find a player j with $m_j^- = K_u'$ and $h \in M_j^+$ with $m_h^- \leq K_u'$. By the necessary and sufficient condition above, $\sum_{x\in M_h^-} p_x \geq \frac{1+m_h^- -(2m_h^-+1)\alpha}{(n-2)\alpha}$. Assuming $p_j < \frac{1+m_h^- -(2m_h^-+1)\alpha}{(n-2)\alpha m_h^-}$, it must be the case that there is a player $j' \in M_h^-$ such that $p_{j'} > \frac{1+m_h^- -(2m_h^-+1)\alpha}{(n-2)\alpha m_h^-} \geq \frac{1+K_u' -(2K_u'+1)\alpha}{(n-2)\alpha K_u'}$. Now if $m_{j'}^- = K_u'$, player j' is the target player i in my claim. Otherwise, it must be the case that $m_{j'}^- < K_u'$. However, I can thus rearrange the monitoring duties to allow all the players in $M_{j'}^-$ to monitor player j without changing any monitoring level[73], which can still support the passive cooperation PBE, but has strictly lower costs than the initial network, which violates the requirement that the initial network is optimal. Therefore, if $m_{j'}^- < K_u'$, it must be that $p_j \geq \frac{1+m_h^- -(2m_h^-+1)\alpha}{(n-2)\alpha m_h^-}$, which indicates that player j is the target player i in my claim.

Now, I find a player i with $m_i^- = K_u'$ and $p_i \geq \frac{1+K_u' -(2K_u'+1)\alpha}{(n-2)\alpha K_u'}$. By the definition of K_u', it must be

[73] If $j \in M_{j'}^-$, include j' in the new M_j^- instead.

the case that $m_x^- \leq K'_u, \forall x \in M_i^-$. By lemma 15, $\exists x^\star \in M_i^-$ such that $\sum_{x \in M_i^-, M_x^- \neq \{i\}, x \neq x^\star} p_x \geq \frac{1+K'_u-(2K'_u+1)\alpha}{(n-2)\alpha} \cdot \frac{K'_u-1}{K'_u}$. Now I claim $K'_u = inf(\mathbb{K})$. If not, it must be that $K'_u > inf(\mathbb{K})$. The monitoring duty $g_{x^\star i}$ can be eliminated by increasing the monitoring levels of the players that remain in M_i^- such that $\sum_{x \in M_i^-, M_x^- \neq \{i\}, x \neq x^\star} p_x = \frac{K'_u-(2K'_u-1)\alpha}{(n-2)\alpha}$ and does not decrease any of them. Together with $m_x^- \leq K'_u, \forall x \in M_i^-$ and $p_i \geq \frac{1+K'_u-(2K'_u+1)\alpha}{(n-2)\alpha K'_u}$, the rearrangement can still support the passive cooperation PBE, but it has a strictly lower total cost, which violates the requirement that the initial network is optimal. Therefore, $K'_u = inf(\mathbb{K})$.

For each remaining player i' with $M_{i'}^+ = \emptyset$, the monitoring level is given by $p_{i'} = \frac{1-\alpha}{(n-1)\alpha}$, and the set of players M_i^- can sufficiently monitor him. Any further monitors for player i' costs more and is thus not optimal.

Finally, it can be ascertained that under case A, the minimal number of necessary monitors is 3, and it is $inf(\mathbb{K})+1$ under case B, which concludes the proof. □

Proof of theorem 2

First, consider an equilibrium candidate as follows, which will be referred to as the targeted PBE throughout the proof:

- Each player i does not offer non-zero bribe plans and will accept any bribe offer and shirk if and only if the expected payoff is at least the same as that from rejecting the offer and working. Otherwise, he will reject bribe offers and work.
- Player i believes that all the others will adopt the same strategy if no bribe plan is observed.
- When some bribe plans are observed, player i believes that he observes all the bribers. Further, for each briber j who is observed with either $M_j^- = \{i\}$ or $b_{ji} < -\alpha$, player i believes that player j only offers the bribe to himself. Otherwise player i believes that player j also offers acceptable bribe plans to all his other monitors.

Note that the off-path belief mainly derives from the passive belief with the only modification under the scenario of $M_j^- \neq \{i\}$ and $b_{ji} \geq -\alpha$, which coincides with the rationality argument in section 3.3.2. It can easily be checked that the targeted PBE can be supported under the monitoring network that is constructed in theorem 2. For the following proof, I will show that the network structure that is given in theorem 2 is the optimal structure to support the targeted PBE. It can easily check that a proper PBE under which all players' work can be supported under the resulting network. At the end of the proof, I will argue that no other network with a lower total monitoring cost can support a proper PBE in which all the players work.

I begin the proof with the following lemmas:

Lemma 16. *Under any network that can support a PBE under which all the players work,* $M_i^- \neq \emptyset, p_i \geq \frac{1-\alpha}{(n-1)\alpha}, \forall i \in N$.

Proof: The result is a weak version of lemma 15. □

Lemma 17. *Any bribe offer* $b_{ij} \leq -\alpha$ *is not acceptable with the off-path belief in the targeted PBE.*

Proof: The result is a weak version of lemma 13. □

Lemma 18 shows that once a non-zero bribe plan B_i^+ is offered, it must be the case that $B_i^+ \geq -\alpha$. Because the only possible profitable deviation for the potential briber is still to bribe all his monitors, the largest bribe that each player i can offer remains $1 - (m_i^- + 1)\alpha$, given lemma 14.

Now, by the definition of N_l, it can be seen that when $m_i^- \geq N_l$, $\sum_{j \in M_i^-} b_{ij} \leq 1 - (m_i^- + 1)\alpha < 0$. Therefore, offering a non-zero bribe plan to his monitor is never profitable for player i if $m_i^- \geq N_l$. However, if for some monitoring network that can support the targeted PBE there is a player i with $m_i^- > N_l$, the monitoring duty can be rearranged in the following manner. Removing a player j in M_i^- such that $m_i^- = N_l$ and maintaining the monitoring levels of all the players unchanged, the new monitoring network can still support the PBE because for each player j who is removed from M_i^-, the required monitoring level p_j is non-increasing. However, because player i still has no profitable bribe plan, the required monitoring levels for the players who remain in M_i^- are unchanged. Therefore, after rearranging, the new monitoring network can still support the PBE and has a strictly lower total cost than the initial one, which indicates $m_i^- \leq N_l$ under an optimal monitoring network.

With regard to the case in which $1-(m_i^-+1)\alpha \geq 0$, player i can potentially offer a non-zero bribe plan to his monitors. For each player $j \in M_i^-$, he should expect both player i and all the other monitors of player i to deviate from the cooperative strategy if he observes $b_{ij} \geq -\alpha$. Player j's payoff from rejecting the plan and working in this manner is given by $\alpha(n-m_i^-)-1$, and the payoff is at least $(1-p_j)(n-m_i^- -1)\alpha + b_{ij}$ if he accepts the plan and shirks. Because $p_j \leq 1$, it is necessary that $b_{ij} \leq \alpha(n-m_i^-)-1$ or player j will surely accept the bribe and shirk. Applying the argument to all the monitors in M_i^-, a necessary condition for player i not able to bribe all of his monitors is given by $1-(m_i^-+1)\alpha \leq m_i^-(\alpha(n-m_i^-)-1)$. Together with the condition $1-(m_i^-+1)\alpha \geq 0$, the necessary condition $0 \leq \frac{1-(m_i^-+1)\alpha}{m_i^-} \leq (n-m_i^-)\alpha - 1$ can be obtained.

Notably, the necessary condition above is simply the definition of the set $\mathbb{K}'$, and the necessary condition for each m_i^- under an optimal monitoring network is $m_i^- \in \mathbb{K}' \cup \{N_l\}$ can easily be checked, which concludes the proof of proposition 2. □

Given $m_i^- \in \mathbb{K}' \cup \{N_l\}, \forall i \in N$ under an optimal monitoring network, and $\mathbb{K}' = \emptyset$, the only remaining possibility is $m_i^- = N_l$.

Further, with $m_i^- = N_l$, a non-zero bribe plan is not profitable for all players. As a result, the required monitoring level p_i for each player i must only make him indifferent between the equilibrium strategy and shirking alone, which is given by $p_i = \frac{1-\alpha}{(n-1)\alpha}, \forall i \in N$, based on lemma 17. □

Now, I will focus on the case in which $\mathbb{K}' \neq \emptyset$. To prove proposition 5, I require the following lemmas.

Lemma 18. *Under an optimal monitoring network, it is not possible to have $m_i^- < N_l$ and $\exists j \in M_i^-, M_j^- \subseteq M_i^- \cup \{i\}$.*

Proof: Suppose that it is possible that such an optimal monitoring network satisfies the two conditions. Now I claim that by removing player j from M_i^- and keeping all monitoring levels unchanged, the new monitoring system can still support the targeted PBE and cost strictly less than the initial system.

The decrease in cost is obvious because the monitoring duty g_{ji} is removed.

To show that the new network can still support the PBE, note that for any non-zero bribe plan that player i can offer under the new network, he can make exactly the same offer in the initial network together with $b_{ij} = -\alpha$. For player j under the initial network, he believes that all the other players in $M_i^- \cup \{i\}$ will shirk if he accepts the plan, in which case he can shirk safely because $M_j^- \subsetneq M_i^- \cup \{i\}$. However, player i will still shirk, but other players in M_i^- will not if player j rejects the plan. Together, player j's payoff from accepting the plan is given by $(n - m_i^- - 2)\alpha$ and the payoff is $(n-1)\alpha - 1$ under a rejection. Given $m_i^- < N_l$, $(n-1)\alpha - 1 < (n - m_i^- - 2)\alpha$ and thus player j will accept the plan.

Now, given that the initial network can support the targeted PBE, it must be the case that none of those offers can be accepted by all the players in $M_i^- \backslash \{j\}$, which indicates that the new network can also support the PBE.

Therefore, I find a new network that is strictly better than the initial one, which violates the assumption that the initial network is optimal. This concludes the proof. □

Note further that this network structure intuitively can never be optimal because it actually encourages rather than prevents bribery. Players i and j have the opportunity to combine their funds to bribe their common monitors. Further, together with lemma 18 and proposition 4, it can easily be confirmed that I only need to discuss a non-negative bribe plan when considering the optimal monitoring network because no negative bribe plan can be accepted under an optimal monitoring network, which leads to the following result, whose proof is omitted:

Lemma 19. *Under an optimal monitoring network, negative bribes are not acceptable.*

Lemma 20. *Under an optimal monitoring network, it is not possible to have* $i \in M_j^-, j \in M_i^-$ *and* $m_i^- \neq m_j^-$.

Proof: Suppose it is possible that there is such an optimal monitoring network that satisfies the two conditions. Without loss of generality, consider $K_l \leq m_i^- < m_j^- \leq N_l$. By lemma 19 $M_i^- \nsubseteq M_j^-$, which means that $\exists k \in M_i^-, k \notin M_j^-$. In addition, $\exists h \in M_j^-, h \notin M_i^-$ is clear.

Now for the total monitoring level for the players in M_i^-, I have $\sum_{x \in M_i^-} p_x \geq \frac{m_i^-(1-\alpha)+\sum_{x \in M_i^-} b_{ix}}{(n-m_i^- -1)\alpha}$ and similarly $\sum_{x \in M_j^-} p_x \geq \frac{m_j^-(1-\alpha)+\sum_{x \in M_j^-} b_{jx}}{(n-m_j^- -1)\alpha}$. Together with the individual monitoring level requirement of $p_x \geq \frac{1-\alpha}{(n-1)\alpha}, \forall x \in N$, the maximum possible bribe plan $\sum_{x \in M_i^-} b_{ix} = 1-(m_i^- +1)\alpha$ and the condition $m_x^- \geq K_l$ from proposition 4, it can be confirmed that the total cost of the monitoring network is strictly greater than the total cost of the network that is constructed in theorem 2, which violates the assumption that the network is optimal. □

Note that the proof of lemma 21 also provides the condition for the required monitoring level, which can be summarized as follows:

Lemma 21. *The minimal required monitoring levels should satisfy* $\sum_{x \in M_i^-} p_x \geq \frac{1+m_i^- -(2m_i^- +1)\alpha}{(n-m_i^- -1)\alpha}, \forall i \in N$.

Lemma 22. *Under an optimal monitoring network, it is not possible to have* $i \in M_j^-$, $j \notin M_i^-$, $m_j^- > m_i^-$ *and* $\exists k \in M_i^-, M_k^- \subseteq M_i^- \cup \{j\}$.

Proof: Suppose it is possible that there is such an optimal monitoring network that satisfies those conditions. By lemma 19 and $M_k^- \subseteq M_i^- \cup \{j\}$, it must be such that $j \in M_k^-$. In addition, by lemma 8, $\exists h \in M_i^-, h \notin M_j^-$. Now a similar calculation can be followed as that in the proof of lemma 21 to show that the total cost of this monitoring network is strictly greater than that of the network that is constructed in theorem 2, which again violates the assumption that the network is optimal. □

Suppose under an optimal monitoring network that there is $i, j \in N$ such that $i \in M_j^-$ and $m_j^- > m_i^-$. By lemma 21, $j \notin M_i^-$ and by lemma 23, $M_k^- \nsubseteq M_i^- \cup \{j\}, \forall k \in M_i^-$. Now remove all the players in M_j^-, let $g_{kj} > 0, \forall k \in M_i^-$ and maintain all the monitoring levels unchanged. I claim that after rearranging the monitoring duty, the new monitoring network can still support the targeted PBE and costs that are strictly less than the initial one.

First, show that the new network can still support the PBE. The new network does not violate the requirement in lemma 19. Initially the players in M_j^- bear less monitoring duty, and the required monitoring level is thus non-increasing. For those players in M_i^-, because player i does not initially have a profitable non-zero bribe offer, player j cannot have a profitable non-zero bribe plan in the rearranged network, given that the monitoring levels remain unchanged. The monitoring duty and the monitoring level for all the remaining players remain unchanged, and the network can thus still support the PBE.

Next, show that the cost falls in the new network, which is true simply because $m_j^- > m_i^-$ in the initial network, and the monitoring level p_j remains the same. Therefore, the new network can still support the PBE, and it has strictly lower cost than the initial network, which violates the assumption that the initial monitoring network is optimal. □

$\forall i \in \mathbb{K}_u, m_i^- = K_u = sup\{m_i^- | i \in N\}$. Now consider $j \in M_i^-$, by proposition 4, one can obtain $m_j^- \geq m_i^-$. However, by the definition of K_u one can also obtain $m_i^- \geq m_j^-, \forall j \in N$. Together I obtain $m_i^- = m_j^- = K_u$, which indicates $j \in \mathbb{K}_u$. □

$K_u \geq K_l$. Suppose under an optimal monitoring network $K_u > K_l$. I will show a contradiction for the case of $K_u < N_l$, and a similar argument can be made when $K_u = N_l$, with tiny differences in detailed numbers.

First, pick a player $i \in \mathbb{K}_u$, by the definition of $\mathbb{K}_u$, I have $m_i^- = K_u > K_l$. Consider the monitoring levels of the players in M_i^-. By lemmas 14 and 22, there is at least one player $j \in M_i^-$, such that $p_j \geq \frac{K_u+1-(2K_u+1)\alpha}{(n-K_u-1)\alpha K_u}$.

Next, consider the players in M_j^-. By corollary 3, $j \in \mathbb{K}_u$ and thus $m_j^- = K_u$ as well. Find the player $x^\star \in M_j^-$ with the lowest monitoring level $p_{x^\star} = inf\{p_x | x \in M_j^-\}$. By similar argument as above, I obtain $\sum_{x \in M_j^-, x \neq x^\star} p_x \geq \frac{K_u+1-(2K_u+1)\alpha}{(n-K_u-1)\alpha} \cdot \frac{K_u-1}{K_u}$.

Now consider the following rearrangement. Remove player $x^\star$ from M_j^-, increase the total monitoring level of the remaining players in M_j^- such that $\sum_{x \in M_j^-, x \neq x^\star} p_x = \frac{K_u-(2K_u-1)\alpha}{(n-K_u)\alpha}$ and do not decrease any of them, while keeping all the rest of the network unchanged. I claim that the rearrangement is well defined and that the new network can still support the targeted PBE and costs strictly less than the initial network.

The rearrangement is well defined because $K_u > K_l$, and thus $K_u - 1 \geq K_l$, which indicates $\frac{K_u-(2K_u-1)\alpha}{(n-K_u)\alpha} \leq K_u - 1$, which does not violate the upper bound of the monitoring levels.

The new network can still support the PBE because the players other than those in $M_j^- \cup \{j\}$ remain unchanged. The monitoring levels of the players in M_j^- do not decrease, such that those players will not shirk alone. Finally, player j continues to not have a profitable and acceptable non-zero bribe plan under the new monitoring network due to the increase in the monitoring levels of the players in M_j^- other than player $x^\star$, and player j will not shirk alone because the monitoring level p_j remains unchanged. Therefore, player j will not deviate from the equilibrium strategy.

Given $j \in \mathbb{K}_u$, I obtain $M_j^- \subseteq \mathbb{K}_u$ and thus $m_h^- = K_u, \forall h \in M_j^-$. The difference in the total costs before and after the rearrangement can thus be calculated to show that the new monitoring network indeed costs less. This violates the assumption that the initial monitoring network is optimal, and it thus concludes the proof. □

Applying lemma 19 to any player i with $M_i^+ \neq \emptyset$, $|\mathbb{M}| \geq K_l + 2$ can be obtained. For players outside of the set $\mathbb{M}$, the optimal monitoring level is $p_i = \frac{1-\alpha}{(n-1)\alpha}, \forall i \notin \mathbb{M}$ because they will not receive any bribe plan and thus the only requirement for monitoring levels is to ensure that they do not shirk alone. Applying lemma 20, it can be confirm that increasing $|\mathbb{M}|$ strictly increases the total cost of the monitoring network. Finally, the construction that is given in theorem 2 is such that $|\mathbb{M}| = K_l + 2$, which concludes the proof. □

With propositions 4 to 7, the network structure that is given in theorem 2 is the optimal one to support the targeted PBE. Further, it can easily be confirmed that the network can support a proper PBE in which all the players work. The remaining task is to show that no other network with a lower total monitoring cost can support a proper PBE in which all the players work. However, this is obvious because of the following argument. Except for those with off-path beliefs and assuming a briber is observed who does not bribe all of his monitors, no other PBE in which all the players work can be supported under a network with a lower total monitoring cost. Given those exceptions, the only off-path belief that can potentially be proper is a passive belief. However, to make the passive belief proper, it must be the case that the bribing of all the monitors is a strictly dominated strategy, which indicates a higher monitoring cost than that in theorem 2. Therefore, the network in theorem 2 is indeed the network with the lowest cost, which concludes the proof of theorem 2. □

Proof of proposition 8

During the proof I consider the case in which n is large enough with α fixed. Cases in which α becomes smaller with an increase in n can be proven in a similar manner. The following is first claimed.

Lemma 23. *With bribe spreading, a monitoring network can support the PBE in which all the players work if and only if no player can offer a profitable and acceptable bribe plan in the first bribing period.*

Proof: This lemma is obvious because for any contingent bribe plan each player i can offer at period t, which is conditional on $\tilde{B}^{-}_{it'} = 0, \forall t' < t$, and the player can also offer the same plan at period $t-1$. □

Given lemma 24, I focus on the prevention of deviations in the first bribing period to find the optimal monitoring network. Now I define the following.

Definition. A full bribe plan of player i is defined as the combination of a set of players $\mathbb{B} \subseteq N$ and a set of bribes $\{b_{jk} \neq 0\}_{j,k \in \mathbb{B}}$ such that: (1) $i \in \mathbb{B}$; and (2) $\forall j \in \mathbb{B}, \exists k_1, k_2, ..., k_x \in \mathbb{B}$ with $b_{ik_1}, b_{k_1 k_2}, ..., b_{k_x j} \neq 0$.

Lemma 24. *With bribe spreading, a monitoring network can support the PBE in which all the players work if and only if no player can offer a profitable and acceptable full bribe plan in the first bribing period.*

Proof: Notably, if a player chooses to accept certain bribe offers, it must be the case that he can find a profitable non-zero contingent bribe plan that is based on the amount of the bribe that he receives. However, his deviation must also be profitable to the initial briber because the initial briber has a perfect forecast. Thus the initial briber should be able to directly plan the entire spread, which is indeed the centralized full plan that is defined above. □

Lemma 25. *With bribe spreading, a monitoring network can support the PBE in which all the players work if and only if for any full plan* $\sum_{j\in\mathbb{B},M_j^-\not\subseteq\mathbb{B}} p_j \geq \frac{|\mathbb{B}|-|\mathbb{B}|^2\alpha}{(n-|\mathbb{B}|)\alpha}$.

Proof: Rearranging the condition the inequality $(n\alpha-1)|\mathbb{B}| \geq (n-|\mathbb{B}|)\alpha(|\mathbb{B}| - \sum_{j\in\mathbb{B},M_j^-\not\subseteq\mathbb{B}} p_j)$ can be obtained. The left hand side is the total cooperative payoff of the players in $\mathbb{B}$, and the right hand side is the expected total payoff if they all accept the plan and shirk. Therefore, the condition that is given above ensures that there is no profitable full plan, which is indeed the necessary and sufficient condition that is required. □

Lemma 26 is a natural extension of lemmas 15 and 22. In the basic model, a player will not deviate if he can neither profitably deviate alone nor successfully collude with his monitors. However, with bribe spreading, the monitoring system requires further assurances that the player cannot raise a profitable and acceptable full bribe plan. The lemma can also help deduce the following result, whose proof is omitted.

Lemma 26. *With bribe spreading, the following monitoring system can support the PBE in which all the players work: the players in* $\{1,2,...,N_l+1\}$ *form a complete network, the remaining players are all monitored by player 1, and the monitoring levels are otherwise given by* $p_i = \begin{cases} \frac{2-3\alpha}{\alpha(n-2)} & i\in\{1,2,...N_l+1\} \\ \frac{1-\alpha}{(n-1)\alpha} & otherwise \end{cases}$.

Lemma 27. *A monitoring system that can support the PBE in which all the players work under the scenario of bribe spreading can also support the targeted PBE under the basic model.*

Proof: Lemma 28 is obvious because under the scenario of bribe spreading, the set of bribe plans each player i can offer is strictly richer than the set of bribe plans that each player i can offer under the basic model. □

Notably, lemma 28 indirectly provides a lower bound of total cost $Cost_G$, which can be summarized as the following result whose proof is omitted:

Lemma 28. *The total cost of an optimal monitoring system under the scenario of bribe spreading is no less than the total cost of an optimal monitoring system under the basic model.*

With all results above I can present the proof of proposition 8. The second part of proposition 8 is obvious. If the total number of monitors is less than $N_l + 1$, the condition in lemma 26 fails.

Assuming with bribe spreading, an optimal monitoring system must have $M_i^+ \neq \emptyset, \forall i \in N$. By lemmas 28 and 29, it must be the case that the total cost of the monitoring system is no less than the total cost of an optimal monitoring network under the basic model with the additional requirement of $M_i^+ \neq \emptyset, \forall i \in N$.

When $n \to +\infty$, clearly $\mathbb{K} \neq \emptyset$, and $K_l = 1$. Under the basic model with the additional requirement of $M_i^+ \neq \emptyset, \forall i \in N$, proposition 6 still holds and thus the following network is an optimal monitoring system: $g_{12}, g_{23}, ...g_{n-1n}, g_{n1} > 0$ and $p_i = \frac{2-3\alpha}{\alpha(n-2)}, \forall i \in N$. And the corresponding total monitoring cost is given by $Cost_G = n \cdot p_i = \frac{n(2-3\alpha)}{\alpha(n-2)}$. Together with $n \to +\infty$, I obtain $Cost_G \to \frac{2-3\alpha}{\alpha}$.

Now, consider the scenario of bribe spreading. The monitoring network given in lemma 27 has a total monitoring cost of $Cost_m = N_l(N_l + 1)\frac{2-3\alpha}{\alpha(n-2)} + (n - N_l - 1)\frac{1-\alpha}{(n-1)\alpha}$. Together with $n \to +\infty$, I obtain $Cost_m \to \frac{1-\alpha}{\alpha} < \frac{2-3\alpha}{\alpha}$, which violates lemma 29 and thus concludes the proof. □

Proof of proposition 10

Proof of lemma 5

The case with threshold price is similar to the proof of lemma 4 which is an application of percolation theory. Now I will focus on the case in which no threshold price can be found.

Denote $q_k = \frac{p_k \cdot k}{\langle k \rangle}$ which is the probability that a consumer who is found via a friendship has k friends. Without loss of generality consider a firm σ entering the market at time $t$with a price $P^\sigma = 0$, for a consumer with k friends, the expected number of friends who are first informed by consumer i and then purchase the update $Tech^\sigma$ is given by $max\{0, \frac{\sum_{\eta \in \Delta} \eta(\theta_\sigma - \eta(\sum_k q_k \cdot k - 2)\theta_s)}{\theta_\sigma}\} \cdot (k-1)$. Now take the expectation over the expected number of friends that a consumer found via a random friendship has, I have $\sum_k q_k \cdot max\{0, \frac{\sum_{\eta \in \Delta} \eta(\theta_\sigma - \eta(\sum_k q_k \cdot k - 2)\theta_s)}{\theta_\sigma}\} \cdot (k-1)$. The product can be purchased by a non-negligible fraction of consumers if and only if the value is greater than 1.

Since $\eta_t = max_\eta(\Delta)$, the value $\sum_{\eta \in \Delta} \eta(\theta_\sigma - \eta(\sum_k q_k \cdot k - 2)\theta_s) = \theta_\sigma - (\sum_k q_k \cdot k - 2)\theta_s$ if $\eta_t = 1$. Also note the value $\sum_k q_k \cdot k$ is greater than $\langle k \rangle + 1$, which by the assumption of connectedness of the consumer network is greater than 3. Now one can easily check that the value $\sum_k q_k \cdot max\{0, \frac{\sum_{\eta \in \Delta} \eta(\theta_\sigma - \eta(\sum_k q_k \cdot k - 2)\theta_s)}{\theta_\sigma}\} \cdot (k-1)$ is strictly smaller than 1 if $\eta_t = 1$ and $\theta_s = \theta_\sigma$ and it is strictly greater than 1 if $\eta_t = 0$ and $\theta_s = 0$. Further, the value is continuous and monotonic with respect to Δ and θ_s, which indicates the existence of the threshold $\bar{\theta_s}$ and $\underline{\theta}_s$. Finally, given $\theta_h > \theta_l$ one can easily get $\bar{\theta_s} > \underline{\theta}_s$, which finishes the proof. □

Proof of lemma 8

A low type firm can never gain by waiting a high type firm to enter.

When the low type firm is picked at time t with $\eta_t \geq \bar{\eta}$, waiting for a high type firm to enter can lower η_t. However, since the high type firm also can not obtain a non-negligible fraction of consumers, if the low type firm is picked again at time $t+1$, it still faces $\eta_{t+1} > \underline{\eta}$ and thus its product still can not be purchased by a non-negligible fraction of consumers according to lemma 5.

When the low type firm is picked at time $t$with $\eta_t < \bar{\eta}$, by assumption SG waiting for a high type firm to enter first will end up with $\eta_{t+1} > \underline{\eta}$ and thus the similar argument applies, which finishes the proof. $\square$

Now consider proposition 10. By lemma 8 low type firms will always enter the market when picked. And thus if a high type firm i chooses to wait when picked, it should expect that a low type firm is picked and enters the market at current period. By lemma 7 the market power measured by η_t will decrease if $\eta_t \geq \bar{\eta} > \underline{\eta}$. And thus $\exists \bar{\eta}_h^\star > \bar{\eta}$ such that $\eta_{t+T} < \bar{\eta}$ and $\eta_{t+T-1} \geq \bar{\eta}$ if $\eta_t \leq \bar{\eta}_h^\star$ for some T satisfying $D(P^i(\Delta t)) \leq \frac{\delta^T}{m^f} D(P^i(\Delta t + T))$. The last inequality holds because $D(P^i(\Delta t + T)) = O(1) \cdot n$ but $D(P^i(\Delta t)) = o(1) \cdot n$ according to lemma 5.

Note $\frac{\delta^T}{m^f} D(P^i(\Delta t + T))$ is a lower bound of the benefit from waiting because the high type firm i can either be picked at time $t + T$ with probability $\frac{1}{m^f}$, which already leads to the benefit, or it can be picked after period $t + T$ conditional on no other high type firm is picked before it. Together with the argument above, waiting is indeed profitable for the high type firm, which finishes the proof of proposition 10.

The argument above can also help to show lemma 9 and 10. Lemma 9 is obvious. As for lemma 10, note the increase in γ will lower the probability that firm i is picked after period $t + T$ and no other high type firm is picked before it. $\square$

References

[1] Acemoglu, D. and Akcigit, U. (2012) Intellectual property rights policy, competition and innovation. Journal of the European Economic Association, 10(1), 1-42.

[2] Acemoglu, D. and Wolitzky, A. (2015) Sustaining cooperation: Community enforcement vs. specialized enforcement. National Bureau of Economic Research, No. w21457

[3] Aghion, P. and Howitt P. (1992) A model of growth through creative destruction. Econometrica, 60(2), 323-351.

[4] Aghion, P., Harris, C., Howitt, P. and Vickers, J. (2001) Competition, imitation and growth with step-by-step innovation. The Review of Economic Studies, 68(3), 467-492.

[5] Alchian, A. and Demsetz, H. (1972) Production, information costs, and economic organization. American Economic Review, 62(5), 777-795.

[6] Ali, S. and Miller, D. (2013) Enforcing cooperation in networked societies. Unpublished paper.

[7] Ali, S., Miller, D. and Yang, D. (2016) Renegotiation-Proof Multilateral Enforcement. Working paper.

[8] Bac, M. (1996a) Corruption and supervision costs in hierarchies. Journal of Comparative Economics, 22(2), 99-118.

[9] Bac, M. (1996b) Corruption, supervision, and the structure of hierarchies. Journal of Law, Economics, and Organization, 12(2), 277-298.

[10] Baccara, M. and Bar-Isaac, H. (2008) How to organize crime. The Review of Economic Studies, 75(4), 1039-1067.

[11] Bala, V. and Goyal, S. (2000) A noncooperative model of network formation. Econometrica, 68(5), 1181-1229.

[12] Barabási, A. and Albert, R. (1999) Emergence of scaling in random networks. Science, 286(5439), 509-512.

[13] Basu, K. (2011) Why, for a class of bribes, the act of giving a bribe should be treated as legal. Unpublished paper.

[14] Basu, K., Bhattacharya, S. and Mishra, A. (1992) Notes on Bribery and the Control of Corruption. Journal of Public Economics, 48(3), 349-359.

[15] Bauckhage, C., Kersting, K. and Rastegarpanah, B. (2013) The Weibull as a model of shortest path distributions in random networks. In Proc. Int. Workshop on Mining and Learning with Graphs, Chicago.

[16] Ben-Porath, E. and Kahneman, M. (2003) Communication in repeated games with costly monitoring. Games and Economic Behavior, 44(2), 227-250.

[17] Bensaid, B. and Lesne, J. P. (1996) Dynamic monopoly pricing with network externalities. International Journal of Industrial Organization, 14(6), 837-855.

[18] Bollobás, B. and Riordan, O. (2004) The diameter of a scale-free random graph. Combinatorica, 24(1), 5-34.

[19] Brueckner, J. (2006) Friendship networks. Journal of Regional Science, 46(5), 847-865.

[20] Cabral, L., Salant, D. and Woroch, G. (1999) Monopoly pricing with network externalities. International Journal of Industrial Organization, 17(2), 199-214.

[21] Callaway, D., Newman, M., Strogatz, S. and Watts, D. (2000) Network robustness and fragility: Percolation on random graphs. Physical review letters, 85(25), 5468.

[22] Campbell, A. (2013) Word-of-mouth communication and percolation in social networks. American Economic Review, 103(6), 2466-2498.

[23] Carrillo, J. (2000) Corruption in hierarchies. Annales d'Economie et de Statistique, 59, 37-61.

[24] Cremer, J. (1980) A partial theory of the optimal organization of a bureaucracy. Bell Journal of Economics, 11(2), 683-693.

[25] Cremer, J., Garicano, L. and Prat, A. (2007) Language and the Theory of the Firm. The Quarterly Journal of Economics, 373-407.

[26] Dhebar, A. and Oren, S. (1986) Dynamic nonlinear pricing in networks with interdependent demand. Operations Research, 34(3), 384-394.

[27] Dufwenberg, M. and Spagnolo, G. (2015) Legalizing bribe giving. Economic Inquiry, 53(2), 836-853.

[28] Economides, N. (1996) The economics of networks. International Journal of Industrial Organization, 14(6), 673-699.

[29] Erdös, P., and Rényi, A. (1959) On random graphs. Publicationes Mathematicae (Debrecen), 6, 290-297.

[30] Ferrali, R. (2015) Corruption as organized crime: a game-theoretic approach. Working paper.

[31] Fricke, D. and Lux, T. (2015) Core–periphery structure in the overnight money market: Evidence from the e-mid trading platform. Computational Economics, 45(3), 359-395.

[32] Fuchs, W. (2007) Contracting with Repeated Moral Hazard and Private Evaluations. American Economic Review, 97(4), 1432-1448.

[33] Garicano, L. (2000) Hierarchies and the Organization of Knowledge in Production. Journal of Political Economy, 108(5), 874-904.

[34] Garicano, L. and Rossi-Hansberg, E. (2012) Organizing growth. Journal of Economic Theory, 147(2), 623-656.

[35] Geanakoplos, J. and Milgrom, P. (1991) A theory of hierarchies based on limited managerial attention. Journal of the Japanese and International Economies, 5(3), 205-225.

[36] Gnedenko, B. (1943) Sur la distribution limite du terme maximum d'une serie aleatoire. Annals of Mathematics, 423-453.

[37] Grimmett, G. (1999) Percolation. Springer Berlin Heidelberg.

[38] Goyal, S. (2007) Connections: An introduction to the economics of networks. Princeton: Princeton University Press.

[39] Haag, M., and Lagunoff, R. (2006) Social norms, local interaction, and neighborhood planning. International Economic Review, 47(1), 265-296.

[40] Haan, L. D. (1976) Sample extremes: An elementary introduction. Statistica Neerlandica, 30(4), 161-172.

[41] Holmstrom, B. (1982) Moral hazard in teams. The Bell Journal of Economics, 324-340.

[42] Ilienko, A. (2013) Continuous counterparts of Poisson and binomial distributions and their properties. Working paper.

[43] Jackson, M. (2008) Social and economic networks. Princeton: Princeton University Press.

[44] Jackson, M. and Rogers, B. (2007) Meeting strangers and friends of friends: How random are social networks? American Economic Review, 97(3), 890-915.

[45] Jackson, M. and Wolinsky, A. (1996) A strategic model of social and economic networks. Journal of Economic Theory, 71(1), 44-74.

[46] Jovanovic, B. and Nyarko, Y. (1994) The Bayesian foundations of learning by doing. National Bureau of Economic Research.

[47] Katz, M. and Shapiro, C. (1985) Network externalities, competition, and compatibility. American Economic Review, 75(3), 424-440.

[48] Kleinberg, J., Suri, S., Tardos, É. and Wexler, T. (2008) Strategic network formation with structural holes. In Proceedings of the 9th ACM Conference on Electronic Commerce (pp. 284-293). ACM.

[49] Klette, T. and Kortum, S. (2004) Innovating firms and aggregate innovation. Journal of Political Economy, 112(5), 986-1018.

[50] Knight, F. (1921) Risk, uncertainty and prot. New York: Hart, Schaffner and Marx.

[51] Krapivsky, P. and Redner, S. (2002) Finiteness and fluctuations in growing networks. Journal of Physics A: Mathematical and General, 35(45), 9517.

[52] Laclau, M. (2014) Communication in repeated network games with imperfect monitoring. Games and Economic Behavior, 87, 136-160.

[53] Levin, J. (2003) Relational Incentive Contracts. American Economic Review, 93(3), 835-847.

[54] Lin, N., Cook, K. and Burt, R. (2001) Social capital: Theory and research. Transaction Publishers.

[55] Lippert, S. and Spagnolo, G. (2011) Networks of relations and word-of-mouth communication. Games and Economic Behavior, 72 (1), 202-217.

[56] Lv, X. (2015) Intergovernmental transfers and local education provision: Evaluating China's 8-7 national plan for poverty reduction. China Economic Review, 33, 200-211.

[57] Ma, C., Moore, J. and Turnbull, S. (1988) Stopping agents from "cheating". Journal of Economic Theory, 46(2), 355-372.

[58] MacLeod, B. (2003) Optimal Contracting with Subjective Evaluation. American Economic Review, 93(1), 216-240.

[59] McAfee, R. and Schwartz, M. (1994) Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity. American Economic Review, 210-30.

[60] Meng, L. (2013) Evaluating China's poverty alleviation program: A regression discontinuity approach. Journal of Public Economics, 101, 1-11.

[61] Mishra, A. (2002) Hierarchies, incentives and collusion in a model of enforcement. Journal of Economic Behavior & Organization, 47(2), 165-178.

[62] Mishra, A. (2006) Corruption, hierarchies and bureaucratic structure. International Handbook on the Economics of Corruption, 189-216.

[63] Myerson, R. (1977) Graphs and cooperation in games. Mathematics of operations research, 2(3), 225-229.

[64] Myerson, R. (1978) Refinements of the Nash equilibrium concept. International Journal of Game Theory, 7(2), 73-80.

[65] Nava, F. (2016) Repeated games on networks, Oxford Handbook on Economics of Networks, Oxford University Press.

[66] Newman, M. (2003) The structure and function of complex networks. SIAM Review, 45(2), 167-256.

[67] Newman, M., Strogatz, S. and Watts, D. (2001) Random graphs with arbitrary degree distributions and their applications. Physical Review E, 64(2), 026118.

[68] Prendergast, C. (1999) The provision of incentives in firms. Journal of Economic Literature, 37(1), 7-63.

[69] Rahman, D. (2012) But who will monitor the monitor? American Economic Review, 102(6), 2767-2797.

[70] Schumpeter, J. (1934) The theory of economic development: An inquiry into profits, capital, credit, interest, and the business cycle.Transaction publishers.

[71] Segal, I. (1999) Contracting with externalities. Quarterly Journal of Economics, 337-388.

[72] Shakespeare, C. (2008) Sarbanes-Oxley Act of 2002 five years on: What have we learned. Journal of Business and Technology Law, 3, 333.

[73] Simon, L. and Stinchcombe, M. (1995) Equilibrium refinement for infinite normal-form games. Econometrica: Journal of the Econometric Society, 1421-1443.

[74] Van Damme, E. (2002) Strategic equilibrium. Handbook of Game Theory with Economic Applications, 3, 1521-1596.

[75] Vázquez, A. (2003) Growing network with local rules: Preferential attachment, clustering hierarchy, and degree correlations. Physical Review E, 67(5), 056104.

[76] Wang, S., Li, Z., Ren, Y. and Liu, F. (2004) The 8-7 national poverty reduction program in China: The national strategy and its impact. In ponencia en la Shanghai Poverty Conference, Shanghai, Vol. 25.

[77] Wolitzky, A. (2013) Cooperation with network monitoring. The Review of Economic Studies, 80(1), 395-427.

[78] Wolitzky, A. (2015) Communication with tokens in repeated games on networks. Theoretical Economics, 10(1), 67-101.

[79] Zhou, T. and Lu, Y. (2011) Examining mobile instant messaging user loyalty from the perspectives of network externalities and flow experience. Computers in Human Behavior, 27(2), 883-889.

Wang Xiang

Department of Economics
The Pennsylvania State University
State College, PA 16802 USA
http://wxiang.weebly.com
https://www.linkedin.com/in/wxiangpsu
Email: wxx108@psu.edu, Mobile: +1 (814) 321-2125

Education:

The Pennsylvania State University, 2012 to present
Ph.D. Candidate in Economics
Thesis Title: "Essays in the economic theory of social networks"
Expected Completion Date: Summer 2017

University of British Columbia, 2011 to 2012
M.A. in Economics

Tsinghua University, 2007 to 2011
B.A. in Economics and Finance

References:

Professor Kalyan Chatterjee (Advisor)
Distinguished Professor of Economics and Management Science, Penn State
(814) 865-6050, kchatterjee@psu.edu

Professor Vijay Krishna
Job Market Placement Director
Distinguished Professor of Economics, Penn State
(814) 863-8543, vkrishna@psu.edu

Professor James Jordan
Liberal Arts Research Professor of Economics, Penn State
(814) 865-2201, jxj13@psu.edu

Research Fields:

Primary field: Microeconomic theory
Secondary field: Applied Micro-theory, Economics of Networks

Experiences:

2016	Research Assistant to Prof. V. Krishna, Penn State
2016, 14	Instructor for Introductory Macroeconomic Analysis and Policy, Penn State
2015	Instructor for Money and Banking, Penn State
2016	Teaching Assistant for Prof. D. Cai, Penn State (Labor Economics and Labor Markets)
2013-15	Teaching Assistant for Dr. A. Kearney, Penn State (Money and Banking, Introductory Macroeconomic Analysis and Policy)
2012-13	Teaching Assistant for Dr. W. Goffe, Penn State (Introductory Macroeconomic Analysis and Policy)
2011-12	Teaching Assistant for Dr. D. Whistler, University of British Columbia (Introduction to Empirical Economics, Methods of Empirical Research)

Honors, Scholarships, and Fellowships:

2012-17	Graduate Assistantship, Dept. of Economics, Penn State
Fall 2015	Rosenberg Liberal Arts Centennial Graduate Endowment in Economics, Penn State
2011-12	Department of Economics Fellowship, University of British Columbia

Other Information:

Languages: Chinese (Native), English (Fluent)
Skills: MS office, LaTeX, Matlab, Stata, Python