COMPLIANT MEDICAL TOOL DESIGN
FOR SOFT TISSUE CUTTING

A Thesis in
Mechanical Engineering
by
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ABSTRACT

Needles are some of the most commonly used medical devices in minimally invasive procedures. Examples of these procedures include drug delivery, biopsies and brachytherapy cancer treatment. The efficacy of these procedures is highly dependent on the ability for precise placement of the needle. However, high insertion forces can lead to complications resulting in poor needle placement and possibly a poor outcome for the patient. Ongoing concerns for physician include the needle diverging from the target path and deformation or rotation of the soft tissue, both the result of high insertion forces. Several studies have been carried out to reduce the insertion force in the needle insertion procedure. These include geometry studies, the speed of insertion, the use of vibrational insertion, and more recently the use of a novel compliant hinge to act as a cutting mechanism, however the work with the novel compliant hinge was limited to the exploration of the concept. This research advances knowledge in tissue cutting using a novel compliant needle design. The properties of the compliant needle design are empirically derived and used to determine the cutting motion of a dynamically inserted needle. By understanding how the compliant mechanism aids in transverse cutting motion an improved needle can be achieved.

Finite element analysis is used to determine empirical compliant equations for varying hinge geometries. Polynomial regression equations are generated to create the empirical equations for each directional stiffness. The limitation to the empirical equations are determined to be a function of the distance between the hinges and the thickness of the hinge. It was found that if the distance between the hinges is less than 1.68 times the thickness than the cross hinge is a primary flexible member. After this, its contribution to the compliance quickly diminishes. To verify the compliant equations the models are compared to FEA results. The percent error for a distance ratio of 1.04, 1.57, 3.15, and 4.72 was 4.7, 23, 164, and 438 percent respectively. This research allows for the design of the compliant mechanisms to be simplified for these ranges. Outside of
these ranges it is possible to use compliant equations developed by Lobontiu et al.[1] found in Appendix A.
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To my Wife.
Chapter 1

Introduction

Motivation

Needles are among the mostly widely used medical devices. Some minimally invasive procedures include tissue biopsy, blood sampling, drug delivery, and the cancer treatment of interstitial brachytherapy [2]. The ability to accurately place the needle with respect to the location of the target is paramount for the procedures to be successful. For example, a poor needle placement during tissue biopsy can lead to a false negative diagnosis or unwanted tissue damage and result in a poor prognosis. Another example is that of internal radiation therapy, also known as brachytherapy treatment, a common treatment plan for many types of cancer. During interstitial brachytherapy radioactive seeds are placed inside of the body in a precise pattern near or inside of the tumor. It can be seen then that if the placement is poor, the outcome may not be optimal.

One such type of cancer commonly treated with interstitial brachytherapy is that of prostate cancer. Prostate cancer is the most frequently diagnosed cancer in men with an estimated 238,590 new cases of prostate cancer to arise in the United States in 2013 [3]. In permanent brachytherapy 50-70 radiation seeds, roughly the size of a grain of rice, are placed by hand using thin gauged needles into and around the tumor using ultrasound guidance [4, 5]. The accuracy of the needle placement is typically between 2 to 3 mm inside the tissue [6]. The level of accuracy is constrained due to the chance of edema from multiple needle repositioning. An edema may further lead to complications such as shifting of anatomy or the seed position after the procedure [7]. With this knowledge, physicians will commonly implant more seeds than necessary to
compensate [8]. This type of over seeding will lead to a larger dose of radiation and can lead to complications and a lower quality of life for the patient [4, 9]. A needle that can improve the accuracy during these minimally invasive procedures is needed to improve patient outcomes.

**Needle Cutting Background**

In many of these minimally invasive procedures, small diameter needles are chosen in order to lessen pain and reduce recovery time [10]. However, with a thinner needle, there is an increased chance of deflection from insertion forces [11, 12]. This type of needle deflection can lead to inaccurate needle placement as seen in Figure 1-1(a). Additionally, the soft tissue target is free to rotate and deflect as seen in Figure 1-1(b). This soft tissue deflection is hard to predict due to the heterogeneous nature of soft tissue. Prostate tissue is especially prone to tissue deflection. One study showed that the prostate is capable of rotations up to a maximum of 13.8° [13]. This high degree of rotation would greatly affect the dosage of radiation in brachytherapy [14].

Figure 1-1. Common Complications from (a) needle deflection and (b) soft tissue deflection.
Previous studies have investigated ways to improve the needle insertion process. Some of these methods included a predictive and correction method that predicts the deformation and needle trajectories by modeling the needle-tissue interaction [15, 16], measuring needle insertion forces [17-20], and the use of robotic-assisted tools [21-24]. However, these types of predictive methods are limited in use due to the large variability found in soft tissue.

The cause of poor needle placement can be attributed to the high tissue cutting force that acts parallel to the direction of needle insertion as seen in Figure 1-2. The reduction of high tissue cutting force that acts parallel to the needle insertion could allow for increased accuracy which would greatly improve the outcomes of medical procedures.

![Fracture Component Forces for Needle Insertion](image)

Figure 1-2. Fracture Component Forces for Needle Insertion.

Previous work has investigated how to reduce these insertion forces. One method was to look at the effects of needle tip geometry on insertion forces. Studies have shown that by changing the needle tip geometry the needle can cut with less force [25, 26]. There has been work on optimizing the standard hypodermic grind angles of the three plane bevel tip [27, 28]. Additionally, a five plane bevel-tipped needle has been shown to cut with less force [29-31].

Further investigation in reducing insertion force include reducing the diameter of the needle. Reducing the gauge of a needle from 30 gauge to 20 gauge have shown to reduce patient pain as well as a reduction in insertion force by 20% [10]. A large body of work has been investigating microneedles. Microneedles have sub-micron diameters and have shown to
significantly reduce insertion force as well as pain [32-35]. However, microneedles are limited in that they cannot allow for larger particulates to be passed through the sub-micron diameters [36]. This would exclude them from many minimally invasive procedures including brachytherapy seeding.

Insertion speed and vibratory insertion have been another area of research to reduce insertion forces. A study by Heverly et al. [37] showed when the use of high insertion speed is used there is a reduction in insertion force needed to puncture soft tissue. This has also been demonstrated in porcine liver [38] and turkey breast tissue as well [39]. Conversely, Koelsman et al. [40] showed an increase in insertion force of 12% when increasing the insertion speed into skin like simulant, a tougher tissue type.

Vibrational cutting has been adopted in the medical field as well. For example harmonic scalpels have been used to cut tissue and bone. These devices use high frequency vibration to improve the safety, precision, and accuracy of the cutting operation [41]. Vibrational cutting with needles has been shown to decrease insertion forces in excised animal skin [42], reduce tissue deformation in chicken breast [43] as well as silicone gel [21]. One study by Huang et al. showed that ultrasonic vibrations reduce needle insertion force by 28% [44]. Other work has shown a reduction of insertion force of up to 70% with the use of microneedles and axial vibrations [45]. However, none of these studies investigates vibration using a needle designed to generate cutting motions. It was shown by Barnet et al. [46] that cutting force is reduced when a compliant needle is coupled with vibration. This type of design allows for transverse cutting, however, it was left to future work to fully explore how compliance affects the cutting motion.
Role of Compliant Mechanism

As mentioned in traditional needle insertion the cutting force acts parallel to the insertion direction. A novel compliant hinge initially developed by Barnet et al. [46] can be seen in Figure 1-3(a). This compliant needle design will allow for the high cutting parallel force to be redirected to a more perpendicular direction relative to the target tissue and the insertion direction as seen in Figure 1-3(b). This cutting motion will reduce the forces acting on the needle, thus reducing the chance of the needle to deflect, as well as the forces pushing on the soft body target and in turn reducing inaccurate placement of the needle with respect to the target. In all other ways the needle geometry, grind angles and gauge, can remain the same as a traditional needle allowing for this design to be available for a large number of applications.

Figure 1-3. (a) Novel compliant hinge design and (b) the transverse cutting motion perpendicular to insertion direction.
The use of compliance in needle design is not solely novel. There have been several studies that have investigated the use of built in compliance to allow for steerable needle insertions and path prediction. Chen and Chen [47] used a multi-hinged design to generate an algorithm to predict the path direction of a beveled needle during a traditional needle insertion procedure. Swaney et. al [48] used a flexural hinge coupled with an internal actuator to reduce tissue damage but maintain steerability of a needle compared to the more traditional kinked bevel-tip needle. Again, there is little research on how a compliant mechanism could be used to generate transverse cutting motion and improve the cutting mechanics. The work presented here investigates the compliance for several asymmetric designs and will allow for the cutting motion to be optimized using standard mechanism design practices.

**Research Outline**

The purpose of this research is to study the utility of a compliant hinge coupled with dynamic insertion to reduce cutting forces. In order to design the needle, it is necessary to know the compliance of the hinge. In Chapter 2 background on compliant mechanisms is provided. In Chapter 3 a geometric design study is carried out using finite element analysis to generate empirical compliant equations for a lumped asymmetric flexure hinge. The empirical equations are then compared to FEA results to determine the error. Further, the range of when it is appropriate to use the empirical model vs other models is explored. Chapter 4 provides a summary of the findings as well as limitations and potential future works.
Chapter 2

Flexure Hinge Design Theory

Compliant Mechanisms

The general definition of a mechanism is a mechanical device that is used to transfer energy, force or motion [49, 50]. This can further be broken down into the more traditional rigid-body mechanism, and compliant mechanisms. Where the rigid-body mechanism is one that is made of rigid links and kinematic pairs, such as revolute joints, a compliant mechanism uses flexible members to transfer input forces, energy or displacements using stored strain energy in the flexible members of the mechanism [51].

Compliant mechanisms have gained favor recently over their rigid-body mechanisms counterpart due to their cost reduction, increased performance, and reduction in parts, wear, weight and maintenance [51]. For these reasons, compliant mechanisms are widely used in microscale mechanisms, such as Micro-Electro-Mechanical Systems (MEMS). An example of how a compliant mechanism can replace a traditional joint pair is shown in Figure 2-1.

![Figure 2-1. Rigid-link vise grip with its compliant mechanism counterpart [51].](image-url)
One commonly used compliant mechanism is the flexure hinge, typically used to replace a revolute joint. Like its rigid-body counterpart it acts as a kinematic pair that restricts movement in the translation directions, however, unlike a the revolute joint, a flexural hinge can only create a limited angular motion about its rotational axis due to the lack of freedom to fully rotate without failure [52]. For a flexural hinge to be effective it needs to provide increased compliance about its favored axis, but remain relatively rigid about the remaining axes to minimize any unwanted motion and stay within the materials yield stress [52].

Generally, flexural hinges are classified within two categories; primitive flexural hinges and complex flexural hinges. Complex flexure hinges provide multiple degrees of freedom and are typical for three-dimensional applications. Primitive flexural hinges, the more common of the two, predominantly focus on two-dimensional applications [53]. A few examples of the primitive flexural hinge, which includes the circular flexural hinge, corner filleted, and elliptical, can be seen in Figure 2-2. The monolithic design of these flexural hinges allows for high reproducible motion, reduced friction and reduced wear [51].

![Common Flexural Hinge Geometries](image)

**Figure 2-2. Common Flexural Hinge Geometries for Planer Motion.**
For any flexural hinge designs, the defining characteristic is the compliance of the hinge, which allows for the relationship between displacement and force to be determined. In Figure 2-3 a flexure hinge with a discrete spring model can be seen. If only two dimensional planer motion is assumed, common in primitive flexure hinges, then this relationship is defined as:

\[
\begin{bmatrix}
  u_x \\
  u_y \\
  u_\theta
\end{bmatrix} = [C]
\begin{bmatrix}
  F_x \\
  F_y \\
  M_\theta
\end{bmatrix}
\]

(2.1)

where \(u_x, u_y, u_\theta\) are the displacements in the axial, transverse and rotational direction, \(F_x, F_y, F_\theta\) are the applied forces in the axial, transverse or rotational direction. The compliance, \(C\) is defined as:

\[
C = \begin{bmatrix}
  C_{x-F_x} & 0 & 0 \\
  0 & C_{y-F_y} & C_{y-M_z} \\
  0 & C_{\theta-F_\theta} & C_{\theta-M_z}
\end{bmatrix}
\]

(2.2)

Here, the axial, transverse, and rotational compliances are denoted by the \(C_x, C_y, C_\theta\) respectively. It is common to report either the compliance or the stiffness of the hinge. Here the stiffness will be used to describe the hinge where stiffness \(K\) is defined as:

\[
K = C^{-1}
\]

(2.3)

Figure 2-3. (a) Flexible hinge connecting two rigid links and (b) its equivalent discrete spring model counterpart [1]
There has been considerable research on analyzing and designing compliant mechanisms. In 1965 Paros and Weisbord [54] published the first work on a compliance-based approach to flexure hinges. They provided the compliance equations for a symmetric circular and right circular flexure hinge. Smith et al. [53] used similar methods to provide the closed form compliance equations for the elliptical flexure hinge. Compliant equations were further provided by Lobontiu et al. who, using the reciprocity principle and Castigliano’s displacement theorem, formulated closed-form equations for the symmetric corner-filleted [55], parabolic and hyperbolic [56], and the symmetric circular [57] hinge types. Lobontiu’s compliance equations for a symmetric circular hinge as well as the transversely asymmetric hinge, Figure 2-4, are based on the capacity of rotation about its sensitive axis and is a function of the thickness of the hinge. For the compliant needle design either of these equations could be used if \( d \) is zero, fully symmetric hinge, or far enough apart, transversely asymmetric, that there would be no coupling effects between the two hinges. As previously stated, the purpose of this study is to determine a compliance model for this transition phase as seen in Figure 2-5.

![Figure 2-4](image)

Figure 2-4. (a) Circular Flexure Hinge and (b) transversely asymmetric hinge [1].
Figure 2.5. Transition of the (a) circular symmetric hinge to a (b) transversely asymmetric hinge.

**Pseudo-Rigid Body Method**

The Pseudo-Rigid-Body Method (PRBM) was developed to simplify the design and analysis of compliant mechanisms. Previously a common method to analyze the deflection for flexible mechanisms was to use elliptic integrals. However, this can be exceedingly difficult for anything but simple geometries and not provide intuitive results [51]. The PRBM creates a parametric approximation model of the deflection of flexible members using rigid-body components that have equivalent force-deflection characteristics [51]. Additionally, the PRBM can be used to compute the forces required for a given deflection and works for linear as well as non-linear applications. A straight cantilever beam PRBM with a vertical end load developed by Howell et al. [58] can be seen in Figure 2-6 and has been provided to give context to the theory.
Figure 2-6. (a) A flexible cantilever beam with end load, and (b) along with its PRBM counterpart [51]

Figure 2-6 (b) shows the PRBM of a simple cantilever beam with and end load. The PRBM consists of two rigid links, connected by a characteristic pivot point, $\gamma$. The characteristic pivot is the fraction of the beam length where the path of the rigid beam matches closely to the path of the compliant beam. Once the location of the characteristic pivot is determined, the deflection path may be described in terms of the pseudo-rigid-body angle, $\Theta$, as described by equations (2.6) – (2.8). The distance from the end of the beam to the pivot point is known as the
characteristic radius, \( \gamma l \). The angle of rotation, \( \Theta_0 \), of the compliant beam can also be determined using equation (2.9), where \( C_\theta \) is the angle coefficient equal to 1.2385 [51, 59]. The PRBM has been shown to be able to predict the deflection of the end of the beam within 0.5% of the closed-form elliptic integral solutions for quite large deflections by using a \( \gamma \) of 0.8517 allowing a maximum \( \Theta \) of 64.3 degrees[59]. A limitation of this method is that the accuracy decreases as the relative length of the hinge increases [51].

\[
\frac{x_0}{l} = 1 - \gamma (1 - \cos \Theta) \tag{2.6}
\]

\[
\frac{y_0}{l} = \gamma \sin \Theta \tag{2.7}
\]

\[
\Theta = \tan^{-1}\left(\frac{y_0}{x_0 - l(1 - \gamma)}\right) \tag{2.8}
\]

\[
\Theta_0 = C_\theta \Theta \tag{2.9}
\]

The PRBM also incorporates a torsional spring located at the characteristic pivot location that is used to model the stiffness, \( K \), in the beam. From Figure 2.6(b) the initial force causing the beam to bend tangential component can be described by equation 2.10.

\[
F_t = F \sin(\phi - \Theta) = \frac{K\Theta}{\gamma l} \tag{2.10}
\]

This transverse force creates the torque, \( T \), at the characteristic pivot point.

\[
T = F_t \gamma l \tag{2.11}
\]

Substituting equation (2.11) into (2.12) it is then possible to write the equation for torque as a function of the spring stiffness and pseudo rigid body angle.

\[
T = K\Theta \tag{2.12}
\]

Howell presented a simplified way for calculating the consent spring stiffness, \( K \). Using the beam geometry, \( l/l \), and Young’s Modulus, \( E \), as well as the PRBM constants \( \gamma \) and \( K_\theta \) the spring
stiffness can be calculated using equation (2.13). Howell et al. [59] presented an average $K_0$ of 2.61 for load angles of $11.3^\circ < \phi < 174.3^\circ$.

$$K = \gamma K_0 \frac{EI}{l}$$

(2.13)

Finally the equation (2.13) can be used to accurately predict the forces.

$$F_t = \frac{EIK_0 \Theta}{l^2}$$

(2.14)

**Finite Element Method**

The use of numerical methods such as the finite element method (FEM) have been utilized more due to the inherent difficulties of using the previously mentioned analytical methods. Where analytical methods can typically only be used for simple geometries and require complicated derivations, FEM allows for more general compliant mechanisms to be modeled more efficiently. There have been several research groups that have used FEM to derive empirical compliant equations and compared the results to known analytical solutions. Yong et al. used FEM to create empirical stiffness equations for a circular flexural hinge where the ratio of the thickness of the hinge with respect to the radius of the hinge was iterated through [60]. They then compared analytically derived equations of the rotational and translational compliance to the FEM predictions. They showed the percentage error of the empirical equations was less than 3%.

Meng et al. [61] used FEM to explore the existing stiffness equations for corner filleted flexural hinges. They created three empirical stiffness equations for corner-filleted flexural hinges based on finite element analysis by iterating over the thickness with respect to the length of the hinge. In this study, they found that the empirical models were a closer approximation, 6% error, than the
analytical methods developed by other research groups. A similar approach is used to formulate the empirical compliant equations for the lumped asymmetric flexure hinge design.
Chapter 3

Empirical Compliant Equations for Asymmetric Transition

It is necessary to design an asymmetric flexure hinge so when an axially vibration is applied a transverse cutting motion will be induced. This type of hinge can be seen in Figure 3-1. As previously mentioned in order to design and analyze a compliant mechanism the compliance of the flexure hinge is required. For a simple longitudinal asymmetric hinge we could potentially use an equation similar to the one described by Lobontiu et al. [1], however, it is necessary to fully understand the interactions of the two hinges when they are in close proximity.

A geometric study is performed using the finite analysis method. The resulting empirical stiffness equations are then presented. It is shown that there is a transition phase between when a lumped model is appropriate and the more traditional compliant equations can be used. To better determine this transition period the elastic strain energy of the mechanisms is investigated.

Figure 3-1. Geometric parameters for the asymmetric hinge design.
Variable Geometric Study

Three design parameters were chosen to explore due to their physical relationship to beam bending and can be seen in Figure 3-1 and described in Table 3-1. The first design explored was how the distance between the two hinges affected the compliance of the lumped hinge mechanism. This was done by iterating the distance between the hinges such that the ratio was $0.05 \leq d/t \leq 8$, where $d$ is the distance between the hinges and, $t$ is the thickness of the hinges. The second design explored was how the thickness, $t$, with respect to the length, $h$, of the hinge affected the compliance of the hinge mechanism. This was done by iterating the thickness and the length of the hinge between $0.1 \leq h/t \leq 0.5$ with a fixed $d$ of $1/6$ of the total length, $L$. For the third study, the effect of the thickness of the hinge, $t$ with respect to the thickness of the rod, $w$ was explored. This was done by iterating the thickness of the hinge from $0.1 \leq t/w \leq 0.9$.

Table 3-1. Design Parameters for Geometric Study.

<table>
<thead>
<tr>
<th>Study</th>
<th>$d$(mm)</th>
<th>$t$(mm)</th>
<th>$L$(mm)</th>
<th>$h$(µm)</th>
<th>$w$(mm)</th>
<th>Number of iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d/t$</td>
<td>0.032 – 5.08</td>
<td>0.635</td>
<td>5-10.08</td>
<td>90</td>
<td>1.27</td>
<td>25</td>
</tr>
<tr>
<td>$h/t$</td>
<td>0.8</td>
<td>0.635</td>
<td>5</td>
<td>63.5- 318</td>
<td>1.27</td>
<td>25</td>
</tr>
<tr>
<td>$t/w$</td>
<td>0.8</td>
<td>0.127 - 1.14</td>
<td>5</td>
<td>90</td>
<td>1.27</td>
<td>25</td>
</tr>
</tbody>
</table>

Finite Element Analysis Setup

ABAQUS software was used to model the hinge designs. To calculate the stiffness for the parametric studies, the hinges were modeled as three-dimensional cantilever beams fully fixed on the left end and free on the right end as seen in Figure 3-2. To exclude any deformation caused by either end of the beam, both the areas left and right of the hinge were modeled as rigid bodies.
For the loading conditions either an end axial load, $F_x$, of 1N, or end moment load, $M_\theta$, of 1N*D was applied at the reference point, RP1 seen in Figure 3-2. For transverse loading, $F_y$, a load of 1N was applied perpendicular to the beam at reference point RP2 in the negative direction. To isolate the stiffness in the transverse direction a negative moment was applied to the non-loaded hinge equal to:

$$M_{fy} = -F_y d$$  \hspace{1cm} (3.1)

The displacement in the axial, transverse and rotational directions were recorded from RP2 and used to calculate the stiffness in the respective direction using equation 2.1. The material properties chosen for the study were that of AISI 304 stainless steel with a Modulus of Elasticity of 200 GPa, density of 8000 kg/m3 and a Poisson’s ratio of 0.29.

![Free Body Diagram of Simulation Setup](image)

Figure 3-2. Free Body Diagram of Simulation Setup.

The mesh chosen for this study were quadratic hexahedral elements (C3D8R) and an example of the mesh can be seen in Figure 3-3. Since the right and the left end of the geometry are modeled as rigid bodies, it is not critical to have a fine mesh. However, it is critical that the mesh for the hinge is adequate to capture an accurate approximation of the material response. A mesh convergence study was performed to determine the optimal seeding for the geometry. It was determined that after a mesh density of approximately 700 elements/mm$^3$ there is no appreciable improvement, so this seeding was chosen for all hinges design models.
Results

For all design studies, the recorded displacements were used to calculate the stiffness for each design iteration. This stiffness was then plotted versus the design ratio. A polynomial regression line was then fit to determine the function of stiffness, \( R^2 > 0.9 \). It was found that the rotational stiffness was the dominating factor of the three principle directions, as expected. The rotational stiffness plots from each study can be seen in Figures 3-4, 3-6 and 3-7. Similar graphs were produced for the stiffness’s in each of the remaining direction to determine the polynomial function. The form of the empirical stiffness equations are presented in Equations 3.2(a)-(d):

\[
\frac{K_x}{Ew} = \sum_{i=0}^{n} a_{n-i}(\alpha)^{n-i} \quad (3.2a)
\]

\[
\frac{K_{\theta}}{Ew} = \sum_{i=0}^{n} b_{n-i}(\alpha)^{n-i} \quad (3.2b)
\]

\[
\frac{K_y}{Ew} = \sum_{i=0}^{n} c_{n-i}(\alpha)^{n-i} \quad (3.2c)
\]

\[
\frac{K_{\theta-fy}}{Ew} = \sum_{i=0}^{n} d_{n-i}(\alpha)^{n-i} \quad (3.2d)
\]
Where the Young’s modulus ($E$), and the diameter ($w$) of the rod are divided out in order to create a non-dimensional function. The design ratio, $\alpha$, is multiplied by its corresponding polynomial coefficients that can be found in Tables 3-2 to - 3-4.

**FEA Empirical results for $d/t$ study**

Table 3-2 and Figure 3-4 show the empirical stiffness equations for the study that explores how the hinge stiffness changes with respect to the length between the hinges. It can be seen that Figure 3-4 is broken into two distinct phases. In Phase A, the distance between the two hinges is very small with respect to the hinge thickness and the loading conditions on both hinges are nearly equivalent. The stiffness of these hinge configurations are a product of the hinge thickness, $t$ and the cross hinge thickness, $T_c$, between the two hinges as seen in Figure 3-5. As $d$ is increased further, the cross hinge thickness increases until the mechanism becomes two distinct asymmetric hinges with a short center section that can be considered a short rod element. For some distance this rod elements contribution to stiffness can be ignored. This can be seen in Euler-Bernoulli’s beam equation for constant cross section beams:

$$\theta = \frac{Ml}{EI} \quad (3.3)$$

It is then easy to see that as the length goes to zero, the rotational displacement for the rod section will go to zero. However, the cross thickness $T_c$ is still equivalent to the hinge thickness, $t$ during this distance. In this way it is possible to consider the hinges as a lumped mechanism. In Phase B, the distance between the two hinges starts to have a more dominating effect on the overall stiffness. Therefore, in Phase B the previous assumption is not valid and the section of the rod in-between the hinges must be considered using a different function.
Table 3-2. Coefficients for Empirical Stiffness Equations for $d/t$ ratio.

<table>
<thead>
<tr>
<th>n</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1.032E-04</td>
<td>---</td>
<td>6.331E-05</td>
<td>---</td>
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<tr>
<td>6</td>
<td>-0.0031</td>
<td>6.139E-12</td>
<td>-0.002</td>
<td>9.829E-10</td>
</tr>
<tr>
<td>5</td>
<td>0.0386</td>
<td>-1.365E-10</td>
<td>0.0253</td>
<td>-2.098E-08</td>
</tr>
<tr>
<td>4</td>
<td>-0.2451</td>
<td>1.036E-09</td>
<td>-0.1679</td>
<td>1.450E-07</td>
</tr>
<tr>
<td>3</td>
<td>0.8491</td>
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<td>0.6163</td>
<td>-2.099E-07</td>
</tr>
<tr>
<td>2</td>
<td>-1.5479</td>
<td>-4.694E-09</td>
<td>-1.2128</td>
<td>-1.517E-06</td>
</tr>
<tr>
<td>1</td>
<td>1.2451</td>
<td>2.590E-08</td>
<td>1.1034</td>
<td>5.376E-06</td>
</tr>
<tr>
<td>0</td>
<td>0.0140</td>
<td>-4.092E-09</td>
<td>-0.2597</td>
<td>-8.696E-07</td>
</tr>
</tbody>
</table>

Figure 3-4. Rotational Stiffness Empirical Model for $d/t$ study.
Figure 3-5. Rotation of the symmetric hinge into two asymmetric hinges with short center element.

**FEA Empirical Results for h/t study**

In Figure 3-6 the rotational stiffness is shown to be nearly linear, $R^2$ of 0.96, with the change in length of the hinge. Table 3-3 provides the coefficients for a polynomial regression with an $R^2$ of 0.99. As the hinge length increases the rotational stiffness reduces. However, it should be noted that the scale of the rotational stiffness does not change as much as any of the other studies. It has a maximum stiffness of 5.6 Nm/rad and a minimum stiffness of roughly 4.8 Nm/rad. It can be seen that the length of the hinge has little to no effect on the overall stiffness of the lumped hinge design. The reason for this can be seen in Castigliano’s theorem for bending:

\[
\Delta = \int_0^l \frac{M}{EI} \left( \frac{\partial M}{\partial Q} \right) \, dx
\]  

(3.4)

It can be seen that for a constant flexural rigidity, EI, a constant moment, M, for any arbitrary load, Q, the deflection, $\Delta$, will not increase sizably due to the short length and the even smaller
increment. This would obviously not be the case for much larger lengths, but due to design
restrictions is outside the scope of this study due to long hinges having a potential of causing
unwanted tissue damage.

Table 3-3. Coefficients for Empirical Stiffness Equations for $h/t$ ratio.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>-776.197</td>
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<td>---</td>
<td>---</td>
</tr>
<tr>
<td>6</td>
<td>1.995E03</td>
<td>---</td>
<td>-1.147E03</td>
<td>---</td>
</tr>
<tr>
<td>5</td>
<td>-2.053E03</td>
<td>-4.436E-08</td>
<td>2.0087E03</td>
<td>---</td>
</tr>
<tr>
<td>4</td>
<td>1.0918E03</td>
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<td>-1.383E03</td>
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<tr>
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<tr>
<td>2</td>
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<tr>
<td>1</td>
<td>-4.4513</td>
<td>6.2121E-09</td>
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<td>-4.8345E-07</td>
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<tr>
<td>0</td>
<td>0.4168</td>
<td>2.2069E-08</td>
<td>-0.1837</td>
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</tr>
</tbody>
</table>

Figure 3-6. Rotational Stiffness Empirical Model for $h/t$ study.
FEA Empirical Results for t/w Study

Rotational stiffness is shown to greatly increase as the hinge becomes thicker as seen in Figure 3-7. There is an increase of over 500 times as the thickness increases from 10% up to 90% of the total diameter of the rod. Again, referring back to Castigliano’s theorem it is apparent that the deflection is dependent on the second moment of area, \( I \):

\[
I = \int_0^R \rho^2 dA
\]  
(3.5)

Where \( \rho \) is the distance from the origin of the axis to the differential area. It is then readily apparent that as the cross-sectional area becomes smaller, the deflection will increase. However, for very thin hinges the stress experienced would cause failure in the part so care must be taken. Table 3-4 provides the coefficients for the polynomial regression fits for Equations 3.2(a)-(d). All fits had an \( R^2 \) greater than 0.97.

Table 3-4. Coefficients for Empirical Stiffness Equations for t/w ratio

<table>
<thead>
<tr>
<th>n</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
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<td>---</td>
<td>-8.649E03</td>
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<td>---</td>
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<tr>
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<td>-3.651E03</td>
<td>-6.480E-05</td>
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<td>7.241E-07</td>
<td>627.645</td>
<td>1.335E-04</td>
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<tr>
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<td>11.9023</td>
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<td>-13.8646</td>
<td>1.445E-05</td>
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<tr>
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<td>0.1325</td>
<td>-5.397E-09</td>
<td>0.9536</td>
<td>-1.0214E-06</td>
</tr>
</tbody>
</table>
Figure 3-7. Rotational Stiffness Model for $t/w$ study

**Validation of FEA Based Empirical Equations**

To validate the empirical compliant equations the end displacement and rotation angle of several needle designs were calculated using the empirical equations. The results were then compared to FEA results. Figure 3-8 shows the design parameters and loading condition for the simulation. The values of parameters for each needle, N1-N4, can be found in Table 3-5. For validation, only the distance with respect to hinge thickness is looked at. AISI 304 stainless steel with a Modulus of Elasticity of 200 GPa, density of 8000 kg/m3 and a Poisson’s ratio of 0.29 was used for the FEA simulation. For loading conditions an end moment, $M$, was incrementally increased by 5 Nmm from 0 to 100 Nmm at the end of the needle. The left end of the needle was fixed in all directions. The end displacement, as well as the angle of rotation at the right end of the needle, were recorded.
Table 3-5. Needle Geometric Parameters for Verification.

<table>
<thead>
<tr>
<th></th>
<th>D1 (mm)</th>
<th>D2 (mm)</th>
<th>r (mm)</th>
<th>t (mm)</th>
<th>h (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>1</td>
<td>0.33</td>
<td>0.3175</td>
<td>0.3175</td>
<td>90</td>
</tr>
<tr>
<td>N2</td>
<td>1</td>
<td>0.5</td>
<td>0.3175</td>
<td>0.3175</td>
<td>90</td>
</tr>
<tr>
<td>N3</td>
<td>1</td>
<td>1.0</td>
<td>0.3175</td>
<td>0.3175</td>
<td>90</td>
</tr>
<tr>
<td>N4</td>
<td>1</td>
<td>1.5</td>
<td>0.3175</td>
<td>0.3175</td>
<td>90</td>
</tr>
</tbody>
</table>

Figure 3-8. Verification Simulation Set Parameters.

It can be seen in Figures 3-9 to 3-12 that the lumped model starts to deviate from the FEA results as the distance between the hinges increases as well as when the load is increased. As the distance, D2, is increased the empirical model over predicts the stiffness of the flexure hinge. This is to be expected, due to the added deflection of the rod section in between the hinges starts to become the dominating contributor to the overall rotational deflection. Table 3-6 shows the end rotation angle percent error increases from an average of 4.7% for a ratio of 1.04 to an average of 438% for the ratio of 4.72. This increasing error highlights that there is a limited range that the
empirical compliance model can be used for. It is necessary to know when the transition from Phase A to Phase B occurs in order to know when it is appropriate to use the empirical model.

Figure 3-9. Rotation Angle at End of Beam for d/t of 1.04.

Figure 3-10. Rotation Angle at End of Beam for d/t of 1.57.
Figure 3-11. Rotation Angle at End of Beam for $d/t$ of 3.15.

Figure 3-12. Rotation Angle at End of Beam for $d/t$ of 4.72.
Table 3-6. Verification Results. Rotation from $M = -50$ Nmm load

<table>
<thead>
<tr>
<th></th>
<th>FEA – $\theta$ (rad)</th>
<th>Empirical – $\theta$ (rad)</th>
<th>Average Percent Error(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>-0.126703</td>
<td>-0.12077</td>
<td>4.97</td>
</tr>
<tr>
<td>N2</td>
<td>-0.097389</td>
<td>-0.10869</td>
<td>23.97</td>
</tr>
<tr>
<td>N3</td>
<td>-0.083813</td>
<td>-0.02878</td>
<td>164.6</td>
</tr>
<tr>
<td>N4</td>
<td>-0.121683</td>
<td>-0.02077</td>
<td>438</td>
</tr>
</tbody>
</table>

**Limitation of Lumped Hinge Model**

To determine when the transition from Phase A and Phase B occurs elastic strain energy can be used. This allows the mechanism to be decomposed into its individual components then the individual strain energy values can be compared to the total strain energy of the system. As previously described, if the mechanism is considered to be a lumped model as the distance between the centers of the hinges is increased, the new configuration can be considered a single hinge that is rotating and increasing in thickness, $T_c$. At some distance, $d$, the lumped hinge is no longer valid since the distance, $t$, will be much less than the $T_c$, and it will be more appropriate to model it as two separate hinges. This is due to the fact that at some point, the solid rod sections ability to store elastic strain energy will outpace that of the lumped hinge model and cannot be neglected. By determining the strain energy in the total mechanism while the degree of asymmetry is increased then comparing it to numerically determined strain energy of a single hinge model versus two hinges and a rod model it will be possible to determine the transition.

To determine the total elastic strain energy of the mechanism an FEA simulation was performed using ABAQUS software. To simplify the numerical calculations the hinges were loaded under a pure end moment of 1 Nmm. As before quadratic hexahedral elements (C3D8R)
were used to mesh the hinge geometry. The left end of the hinge was fixed in all directions while the moment end was applied to the right end of the hinge. As for the design iteration, the degree of asymmetry was increased by from $0.02 \leq d/t \leq 10$. After the simulation completed the total strain energy was recorded and can be seen in Figure 3-13. The phases can clearly be seen, with Phase A transitioning to Phase B approximately around $d/t$ of two.

![Figure 3-13. AStrain Energy of compliant mechanism from FEA.](image)

For the numerical calculations, the only loading condition for this set up is a moment about the z axis. To calculate the strain energy for both models the strain from pure bending was used.

$$U_{bending} = \int_{L} M^2 \frac{ds}{EI}$$

(3.6)

As for the two hinges with a rod it is numerically solved as two constant hinges with an increasing rod length starting from $0 \leq L < 8$ mm. Then using equation 3.6 the individual strain
energy was calculated. For the cross hinge design a function for the second moment of area (I) was generated by fitting a polynomial to several calculated geometry’s and can be seen in equation 3.7.

\[ I = -3E - 09t(x)^4 + 6E - 09t(x)^3 - 6E - 09t(x)^2 ... + 3E - 09t(x) - 4E - 10 \]  

(3.7)

Where \( t(x) \) is defined as:

\[ t(x) = t + r - \sqrt{x(2r - x)} \]  

(3.8)

Figure 3-14. Analytical Strain Energy calculation to determine transition period.

The elastic strain energy for both models can be seen in Figure 3-14. It can be seen that at a distance ratio of 1.68 the strain energy of a rod model starts to quickly outpace the lumped hinge model. This is also supported by the FEA determined total strain energy seen previously in Figure 3-13. It stands to reason then that when the distance between the hinges is less than 1.68 the cross hinge is a primary flexible member. Within this range, the empirical compliant models allow for a simplified way to determine the transverse motion of the needle tip. For ranges greater than the 1.68 a rod section, seen in Figure 3-5, can no longer be ignored or modeled as a rigid
element since its contribution to the overall elastic strain energy quickly outpaces that of a cross hinge configuration. After a ratio of five, there is a difference of an order of magnitude between the two models. At this point, it is best to model the hinge mechanism as a multilink-hinge mechanism using previously mention compliant equations.
Chapter 4

Conclusions

Empirical compliance models for an asymmetric flexural hinge were presented in this paper. It was shown that the asymmetric hinge design, modeled as a single lumped element, has a limited scope. Since the length between the hinges will eventually become a dominating factor in the overall stiffness. However, can greatly simplify the design of a compliant needle within a given range. It was also shown that the cross-sectional area of the hinge, as well as the distance between the hinges, affect the stiffness of the hinges greater than the length of the hinge. It is recommended that if the distance between the flexure hinges is less than 1.68 times the thickness of the hinge that the empirical compliance model can be used for the mechanism design. However, if the distance is larger it should be modeled using non-symmetric compliance equations, such as the ones developed by Lobontiu et al. [1], and a multilink approach.

Limitations and Future Work

This work presented an empirical compliance model for a hinge to be coupled with ultrasonic axial vibration. Previous work demonstrated the feasibility of this type of hinge to generate transverse cutting motion. With the current compliance models it will be possible to optimize the needle design. This future work will have to incorporate stress considerations into the optimization, due to the compliance model not taking into account fatigue failure from the cyclic loading conditions. Further work should be done to improve the models prediction around the transition between Phase A and Phase B.
A PRB kinematic model can be used to analyze the motion of the cutting path of the needle tip. It can be seen in Figure 4-1 that the needle can be modeled as a double link manipulator with two torsion springs with a stiffness $K_\theta$ connecting two rigid links or if link 2 is ignored a single rigid link with a torsion spring. If a single lumped mechanism is used the stiffness of the torsion springs can be determined by the empirical equations presented in chapter 3. The driving torque, $M(t)$, can be applied to point 1 and should be a sinusoidal function of time to mimic the loading conditions of the vibratory needle.

Figure 4-1. (a) Flexible hinge connecting two rigid links and (b) its equivalent discrete spring model counterpart.
References


P. J. Swaney, Jessica Burgner, Hunter B. Gilbert, and Robert J. Webster, "A Flexure-Based Steerable Needle: High Curvature With


Appendix

Selected Compliant Equations

Lobontui et al. compliance equations

In-plane compliance for symmetric right-circular flexure hinge:

\[
C_{x - Fx} = \frac{1}{Ew} \left[ \frac{2(2r + t)}{\sqrt{t(4r + t)}} \arctan \left( \frac{1 + \frac{4r}{t} - x}{2} \right) \right]
\]

\[
C_{y - Fy} = \frac{3}{4Ew(2r + t)} \left\{ \frac{2(2 + \pi)r + \pi + \frac{8r^3(44r^2 + 28rt + 5t^2)}{t^2(4r + t)^2}}{\sqrt{t^2(4r + t)^5}} \right\}
\]

\[
= \frac{3}{4Ew(2r + t)} \left\{ \frac{2(2 + \pi)r + \pi + \frac{8r^3(44r^2 + 28rt + 5t^2)}{t^2(4r + t)^2}}{\sqrt{t^2(4r + t)^5}} \right\}
\]

\[
= \frac{3}{4Ew(2r + t)} \left\{ \frac{2(2 + \pi)r + \pi + \frac{8r^3(44r^2 + 28rt + 5t^2)}{t^2(4r + t)^2}}{\sqrt{t^2(4r + t)^5}} \right\}
\]

\[
C_{y - Mz} = \frac{24r^2}{Ewt^3(2r + t)(4r + t)^3} \left[ \frac{t(4r + t)(6r^2 + 4rt + t^2)}{\sqrt{t^2(4r + t)^5}} \arctan \left( \frac{1 + \frac{4r}{t}}{2} \right) \right]
\]

\[
C_{\theta - Mz} = \frac{C_{y - Mz}}{r}
\]
Lobontui et al. compliance equations

In-plane compliance for nonsymmetric circular flexure hinge:

\[
C_x - F_x = \frac{1}{Ew} \left\{ \frac{2(2r + t)}{\sqrt{t(2r + t)}} \arctan \left[ \frac{r}{\sqrt{t(2r + t)}} \right] + \pi \left[ \frac{r + t}{\sqrt{t(2r + t)}} - 1 \right] \right\}
\]

\[
C_y - F_y = \frac{12}{Ew} \left\{ \frac{3r^3(r + t)}{t^2(2r + t)^2} + \frac{2r}{r + t} + \pi - \frac{(r + t)^3(-3r^2 + 4rt + 2t^2)}{\sqrt{t(2r + t)}} \arctan \left[ \frac{r}{\sqrt{t(2r + t)}} + \frac{\pi}{2} \right] \right\}
\]

\[
C_y - M_z = \frac{12r^2}{Ew(r + t)\sqrt{t^5(2r + t)^5}} \left\{ \frac{(3r^2 + 4rt + 2t^2)}{\sqrt{t(2r + t)}} \right\} + 3r(r + t)^2 \arctan \left[ \frac{r}{\sqrt{t(2r + t)}} + \frac{\pi}{2} \right]
\]

\[
C_\theta - M_z = \frac{C_y - M_z}{r}
\]