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ESSAYS ON INCOME DISTRIBUTION AND TRADE VOLUMES

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ABSTRACT

CHAPTER 1: Income Distribution, Market Structure, and Individual Welfare

This essay proposes a new insight on how income distribution influences market structure and affects the economic well-being of different groups. It shows that inequality may be good for the poor via a trickle-down effect operating through entry. I consider a general equilibrium model of monopolistic competition with free entry, heterogenous firms and consumers that share identical but non-homothetic preferences. The general model is formulated. The case of two types of consumers, which are different in terms of efficiency units of labor they are endowed with, is considered in detail. I show that higher income inequality in the economy can benefit the poor. An increase in personal income of the rich raises welfare of the poor, while an increase in the fraction of the rich has an ambiguous impact on the poor: welfare of the poor has an inverted U shape as a function of the fraction of the rich. In addition, an increase in the personal income of the rich together with a decrease in the fraction of the rich, keeping the aggregate income in the economy fixed, raises the well-being of the poor. I also analyze the effect of changes in market size and entry cost. I show that the rich gain more from an increase in market size and lose more from an increase in the cost of entry than the poor.

CHAPTER 2: Globalization: Intensive versus Extensive Margins

There is empirical evidence that globalization leads to higher income inequality within a country. However, in the economic literature not much attention is paid to the fact that globalization may influence inequality through the consumption channel. In particular, if different groups of consumers consume different sets of goods and in different amounts, then globalization can change consumption patterns and increase or decrease welfare inequality among the groups. In this essay, I look at two margins of globalization, namely trade liberalization (the intensive margin) and a rise in the number of trading partners (the extensive margin), and explore their impact on the economic well-being of different population groups through the consumption channel. I extend the model formulated in Chapter 1 to a world with many symmetric countries. I show that the impact of globalization on the relative welfare of the rich with respect to the poor depends on the margin of globalization considered. In particular, the relative welfare of the rich is first increasing and then decreasing as transportation costs fall. As for a rise in the number of trading partners, the rich always gain more than the poor. Moreover, in some cases the rich can even be worse off from trade liberalization, while welfare of the poor and aggregate welfare both increase.

CHAPTER 3: Per Capita Income, Market Access Costs, and Trade Volumes

In this essay, I document and analyze several phenomena of trade data. First, countries with higher per capita income tend to have greater trade volumes even after controlling for total income. Second, many country pairs in the world do not trade with each other in one or both directions. Finally, there are substantial costs of access to foreign markets. I construct and estimate a general equilibrium model of trade in an asymmetric world with many countries that squares the above data features. There are two novelties in the essay. First, in my model I introduce a relationship between the costs of access to foreign markets and exporter development level. I show that this relationship can account for the effect of per capita income on trade volumes and explain the many zeros in bilateral trade data. Second, I develop an estimation procedure, which allows me to identify separate effects of variable and fixed costs of trade on trade volumes. The model performs well in fitting the data. The trade elasticities with respect to aggregate and per capita incomes predicted by the model are close to those in the data. I find that the aggregate spending on access to foreign markets constitutes on average around the half of the total export profits.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

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Chapter 1. Income Distribution, Market Structure, and Individual Welfare

1.1 Introduction

What are the possible consequences of income redistribution for market structure, consumption allocation, and welfare? As Atkinson and Bourguignon (2000) argue, "it is difficult to think of economic issues without distributive consequences and it is equally difficult to imagine distributive problems without some allocational dimension." There is a large empirical and theoretical literature that relates income distribution and inequality to a number of social and economic outcomes[1]. Alesina and Rodrik (1994) show that an increase in income inequality has a negative impact on economic growth (see also Persson and Tabellini (1994)). Waldmann (1992) argues that the level of inequality is positively correlated with infant mortality. Glaeser, Scheinkman and Shleifer (2003) suggest that high inequality can negatively affect social and economic progress through the subversion of institutions in the economy. This paper develops another insight into the interaction between income distribution and economic outcomes, which has not been explored extensively. I examine how income distribution affects market structure, pricing, and the welfare of heterogenous agents. In particular, I show that higher income inequality in the economy may benefit the poor via a trickle-down effect operating through entry.

I consider a general equilibrium model of monopolistic competition with heterogenous firms and consumers. In traditional models of monopolistic competition, income distribution plays no role. This rests on two standard preference assumptions. First, if preferences are identical and homothetic, it is well understood that the distribution of income does not affect equilibrium: only aggregate income matters. Second, when preferences are quasi-linear, the presence of a numeraire good eliminates the influence of income distribution on equilibrium outcomes. In this paper, I assume that all consumers share identical but non-homothetic preferences. I introduce income heterogeneity in the model by assuming that consumers differ in the efficiency units of labor they are endowed with. In models with identical homothetic preferences, any price changes have the same impact on all consumers regardless of whether consumers are identical or not. Non-homothetic preferences and income heterogeneity imply that changes in prices may

[1] See Atkinson and Bourguignon (2000) for more substantial literature review.

affect different groups differently. In the model, the presence of market power induces variable markups across firms, which are in turn affected by income distribution. Hence, changes in income distribution may have different consequences for different groups of agents.

I adopt the preference structure from Murphy, Shleifer and Vishny (1989) and Matsuyama (2000). The basic idea is that goods are indivisible and potential consumers want to buy only one unit of each good. This implies that given prices, goods can be arranged so that consumers can be seen as moving down some list in choosing what to buy. For instance, in developing countries, consumers first buy food, then clothing, then move up the chain of durables from kerosene stoves to refrigerators, to cars. Notice that the consumer utility can only be increased by the consumption of a greater number of goods. Moreover, higher income consumers consume the same set of goods as lower income consumers plus some others. This structure of consumer preferences has enough flexibility to be applied as to the whole economy as to a certain industry where goods differ in quality. On the one hand, each good can be interpreted as a distinct good sold in the market. In this case, the structure describes the whole economy. On the other hand, we might think that firms sell not distinct goods but some characteristics of a good produced in a certain industry. For instance, consider a car industry. Each good can be treated as some characteristic of a car. The poor purchase main characteristics associated with a car, while the rich buy the same characteristics as the poor plus some additional luxury characteristics. That is, both groups of consumers buy the same good but of different quality.

Goods differ in terms of the valuations consumers attach to them. By the valuation of a certain good, I mean the utility delivered to consumers from the consumption of one unit of this good. That is, there are goods that are more essential in consumption (necessities) and goods that are less essential (luxuries). There is free entry in the market. To enter the market, ex ante identical firms have to make costly investments that are sunk. Once firms enter, they learn about the valuations attached to their goods. The only source of firm ex-post heterogeneity is the difference in the valuations placed on their goods[2]. Depending on the valuations drawn, firms choose whether to stay or to leave the market. Firms that decide to stay engage in price competition with other firms. This and the preference structure lead to the endogenous distribution of markups, which is influenced not only by market size, but also

[2] To simplify the analysis, I assume that the marginal cost of production is identical across all firms.

by the distribution of income in the economy. Hence, the model incorporates two key features: imperfect competition and non-homothetic preferences, which allow analyzing the consequences of changes in the income distribution on pricing in the equilibrium, the market structure and, thereby, welfare of different groups of consumers.

While the general model is established and solved, the heart of the paper focuses on the case of two types of consumers: rich and poor[3]. Depending on the valuations attached to the goods they produce, firms are endogenously divided into three groups in the equilibrium. Firms with high valuations choose to serve all consumers. Firms with medium valuations decide to sell only to the rich. Finally, firms with low valuations leave the market. Therefore, the poor consume on average more essential goods than the rich. This is in line with common intuition that the rich spend higher share of their income on luxury goods, which are less essential in consumption[4].

As sources of income inequality, I consider changes in the income and the fraction of the rich consumers. An increase in the income level of the rich has two effects: redistribution of firms across the groups and the higher number of firms entering the market, which results in tougher competition. The former effect is negative for the poor, while the latter one is positive. I show that due to additional entry, the poor gain from an increase in the income of the rich. This is reminiscent of the trickle-down effect in Aghion and Bolton (1997), who show that in the presence of imperfect capital markets, the accumulation of wealth by the rich may be good for the poor. The intuition, which is behind these results, may also work in traditional models with homothetic preferences. In Melitz (2003), higher income of some consumers results in higher entry, tougher competition, and, thereby, higher welfare of all consumers. However, there are some differences. In the short run, when the mass of firms is unchanged, there is only a negative impact on the poor resulting in welfare losses. In traditional models, if we fix the mass of firms then higher income of one part of consumers does not affect welfare of the other part. Moreover, in the present model higher income of the rich raises the markups of firms selling only to the rich and decreases the markups of firms serving all consumers. In traditional models, there is the same or no impact on firms' markups.

[3]Recall that in the model, income distribution is exogenous. I deliberately leave out of the scope the situation when income distribution is endogenous, since my main goal is to understand the effects of changes in income distribution not in some other parameters.

[4]Notice that the present model is not a model about quality differences. Agents do not choose between high quality and low quality potatoes. They choose between potatoes, TV's, refrigerators, and so on.

Another intriguing issue is to compare welfare of the poor in economies with different fractions of the rich. What is better for the poor: tiny minority or vast majority of the rich? Keeping the same personal incomes and the mass of the consumers, an increase in the fraction of the rich has two opposite implications for the poor. First, some firms that served all consumers choose to sell only to the rich. Second, the larger fraction of the rich results in more firms entering the market. The former effect hurts the poor and the latter one benefits them. I show that if the fraction of the rich is small then the positive effect prevails, while if the fraction of the rich is sufficiently high the opposite happens. Hence, we might expect that welfare of the poor has an inverted U shape as a function of the fraction of the rich. The fact that firms endogenously choose the type of consumers they wish to serve makes the results regarding changes in fraction of the rich different from that ones in traditional models with homothetic preferences. In Melitz (2003), higher fraction of the rich always leads to higher welfare of the poor in the long run and has no impact in the short run. In the present model, we observe an ambiguous impact in the long run and a negative impact in the short run. There is a common feature of both comparative statics considered above. An increase in the personal income of the rich as well as an increase in the fraction of the rich raises the aggregate income in the economy. In the view of policy implications, we need to explore the effects of changes in income distribution keeping aggregate income constant. To capture a pure redistribution effect, I consider an increase in the personal income of the rich together with a decrease in the fraction of the rich keeping aggregate income in the economy fixed. In models with homothetic preferences, these changes in income distribution do not affect entry, prices, and welfare of the poor. In the present model, I show that they result in higher entry in the market. This in turn leads to higher welfare of the poor.

The effects of changes in entry cost or market size on consumer welfare are similar to those in traditional literature. However, consumers do not equally gain or lose from these changes as we might expect in models with homothetic preferences. Who gains more: the rich or the poor? I show that given some plausible assumption about the distribution function of valuation draws, the rich gain more from a rise in market size and lose more from a rise in entry cost than the poor.

The related literature in this area can be divided into three strands. First, there are papers that consider monopolistic competition models with firm heterogeneity assuming homothetic or quasi-linear preferences. Melitz (2003) develops a general equilibrium model with firm heterogeneity and Dixit-Stiglitz preferences, which imply constant markups. Melitz and Ottaviano (2005) examine a similar framework, but incorporate variable markups considering a linear demand system. However, in both these papers, the distribution of income does not play any role. In contrast, the model presented here includes all the key features of the papers mentioned while also establishing a connection between income distribution and the market structure. The second group of papers, for instance Flam and Helpman (1987), Stokey (1991), and Matsuyama (2000), explores the implications of non-homothetic preferences in a perfectly competitive environment for open economies. These papers mainly analyze the interaction between income distribution and trade patterns. There is a set of papers written by Krishna and Yavas[5], in which the role of indivisibilities and market distortions is investigated. However, the impact of income distribution on market structure is not considered in these papers. Finally, the third group of papers deals with both monopolistic competition and non-homothetic preferences. Markusen (1986) extends the Krugman type model of trade with monopolistic competition by adding non-homothetic demand. He examines the role of income per capita in interindustry and intra-industry trade. Mitra and Trindade (2005) also consider a model of monopolistic competition with non-homothetic preferences. However, the way they introduce nonhomotheticity has a shortcoming: the share of consumer income spent on a certain type of goods is exogenous and depends on the income.

Closer to this paper is the work of Foellmi and Zweimueller (2004) that develops a general equilibrium model with an exogenous mass of identical firms. In contrast, I consider heterogenous firms and free entry in the market, which in turn implies endogeneity of the mass of potential producers in equilibrium. Moreover, Foellmi and Zweimueller (2004) do not address welfare issues. They show that, depending on the parameters of the model, an increase in income inequality has either no impact on firm markups or increases them. The present paper suggests that this is not necessarily the case; in fact, an increase in income inequality affects different firms differently. Due to free entry, greater income inequality may raise markups for firms that sell their goods only to the rich and reduce markups for firms that sell their goods to all consumers. Murphy, Shleifer and Vishny (1989) study how income inequality affects the adoption of modern

[5] See Krishna and Yavas (2001), Krishna and Yavas (2004), and Krishna and Yavas (2005).

technologies. In their model, prices and markups are exogenous. In fact, Murphy, Shleifer and Vishny (1989) leave the questions of competition, markups, and welfare outside their analysis. In the paper closest to the present work, Foellmi and Zweimueller (2006) examine a dynamic variation of Murphy, Shleifer, and Vishny (1989). Assuming learning by R&D, they focus their analysis on the link between possible growth and inequality. In contrast, I do not consider the learning by R&D spillover and explore the impact of income distribution and inequality on the level of competition, markups, and individual welfare.

The rest of the paper is organized as follows. Section 2 introduces the basic concepts of the general model. Section 3 develops a special case with two types of consumers, rich and poor, and establishes existence and uniqueness of equilibrium for this case. It also derives the implications of the distribution of income on market structure and individual welfare. Section 4 extends the analysis to the general case with N types of consumers, and Section 5 concludes.

1.2 The Model

I consider a general equilibrium model of monopolistic competition with heterogenous firms and consumers. The preference structure is adopted from Murphy, Shleifer and Vishny (1989) and Matsuyama (2000).

1.2.1 Production

The timing of the model is as follows. There is free entry in the market. To enter the market, firms have to make sunk investments f_e. If a firm incurs the cost of entry, it obtains a draw b of the valuation of its good from a common distribution $G(b)$ on $[0, A]$. This is meant to capture the idea that before they enter, firms do not know how well they will end up doing, as they do not know how highly consumers will value their products. I assume that $G'(b) = g(b)$ exists. The valuation b is interpreted as the utility delivered to consumers from the consumption of one unit of the good. Depending on the valuation they draw, firms choose to leave the market or to stay. Firms that decide to stay compete in price with other firms. The only factor of production is labor. I assume that marginal costs of production are the same for all firms and equal to c, i.e., it takes c effective units of labor (which are paid a wage of unity[6]) to produce a unit of any

[6]In the model, wage is taken as numeraire.

good[7].

Consumers differ in the number of efficiency units of labor they are endowed with[8]. I assume that there are N types of consumers indexed by n. A consumer of type n is endowed with I_n efficiency units of labor. I choose indices so that $I_n > I_{n-1}$. Let α_n be the fraction of type n consumers in the aggregate mass L of consumers. Then, the total labor supply in the economy in efficiency units is $L\sum_{i=1}^{N}\alpha_i I_i$.

1.2.2 Consumption

All consumers have the same non-homothetic preferences given by utility function

$$U = \int_{\omega\in\Omega} b(\omega)x(\omega)d\omega,$$

where Ω is the set of available goods in the economy, $b(\omega)$ is the valuation of good ω, and $x(\omega) \in \{0, 1\}$ is the consumption of good ω. Each consumer owns a balanced portfolio of shares of all firms. Due to free entry, the total profits of all firms are equal to zero in the equilibrium. This implies that the value of any balanced portfolio is equal to zero. Thus, all consumers have the same wealth, while their incomes vary with their productivity. To simplify the notation, I assume that consumers have equal shares of all firms. Let π be the total profit of all firms in the economy. Given prices of available goods, a type n consumer maximizes

$$\int_{\omega\in\Omega} b(\omega)x(\omega)d\omega$$

subject to the budget constraint

$$\int_{\omega\in\Omega} p(\omega)x(\omega)d\omega \leq I_n + \frac{\pi}{L},$$

where $p(\omega)$ is the price of good ω. The utility maximization merely involves moving down the list of products ordered by their valuation to price ratios, $\frac{b(\omega)}{p(\omega)}$, until all income is exhausted.

The analysis of the general case with N types is rather complicated. Therefore, I focus the analysis on the simpler case when consumers have one of two possible labor productivities. In the

[7] The assumption that marginal costs of production are the same across firms simplifies analytical derivations in the model and does not change qualitative results. In general, we can assume that marginal costs are also drawn from some common distribution.

[8] Throughout the paper, I use terms, endowments of efficiency units of labor and labor productivities, interchangeably.

next section, I show the existence and uniqueness of the equilibrium and analyze its properties. In section 4, I prove the existence and uniqueness of the equilibrium in the general case and briefly discuss the case when the distribution of labor productivities is continuous.

1.3 A Special Case: Two Types of Consumers

There are two types of consumers: a high income (high productivity) type and a low income type. The productivity of the high income type is defined by I_H, the productivity of the low income type is I_L. Given the preferences, all goods consumed by less productive consumers are also consumed by more productive. Thus, goods in the economy can be divided into three groups: the "common" group includes goods that are consumed by all consumers; the "exclusive" group includes goods that are only consumed by high income consumers; finally, there is the group of goods that are consumed by no one.

A firm that produces a good ω obtains the profit of $(p(\omega) - c)Q(\omega)$, where $Q(\omega)$ is demand for good ω. If all consumers buy the good then the demand is L. If only the rich buy it, the demand is $\alpha_H L$, where α_H is the fraction of a high income type. Hence, $Q(\omega) \in \{L, \alpha_H L, 0\}$. Each firm takes the valuation to price ratio of all other firms as given and maximizes its profit. The following proposition holds.

Proposition 1 *Even though all goods have different valuation to marginal cost ratios, goods from the same group have the same valuation to price ratio in the equilibrium.*

Proof. Suppose not. In this case, there exists some group, in which there are at least two goods with different $\frac{b(\omega)}{p(\omega)}$ ratios. Since both goods belong to the same group, the firm that produces its good with higher $\frac{b(\omega)}{p(\omega)}$ can raise its $p(\omega)$ without affecting the demand. This in turn would increase its profit. ■

Define V_C as the valuation to price ratio of goods from the "common" group and V_E as valuation to price ratio of goods from the "exclusive" group in the equilibrium. Here V_C and V_E are endogenous parameters and V_C is strictly greater than V_E[9]. Thus, if a firm with valuation

[9] Notice that by definition, $V_C \geq V_E$. If $V_C = V_E$ then all available goods have the same valuation to price ratios. In this case, the equilibrium concept implies that the high income consumers buy all goods, while the poor buy only some part (for instance, this part can be randomly determined). This means that expected demand for a certain good is strictly less than L. Thus, firms can increase their profits by slightly decreasing their prices and acquiring greater demand share. Therefore, if $V_C = V_E$, equilibrium does not exist.

$b(\omega)$ sells to all consumers then its price is equal to $\frac{b(\omega)}{V_C}$ and its profit is given by

$$(p(\omega) - c)\, L = \left(\frac{b(\omega)}{V_C} - c\right) L,$$

while if the firm sells only to the rich, its profit is given by

$$(p(\omega) - c)\alpha_H L = \left(\frac{b(\omega)}{V_E} - c\right) \alpha_H L.$$

As $V_C > V_E$, the firm chooses between selling to more people at a lower price and selling to fewer of them but at a higher price. Hence, the firm chooses $p(\omega) \in \{\frac{b(\omega)}{V_C}, \frac{b(\omega)}{V_E}\}$ to maximize its profit, taking V_C and V_E as given. In the equilibrium, the price of good ω only depends on $b(\omega)$. Therefore, hereafter I omit the notation of ω and consider prices as a function of b.

Let b_M be the unique solution of the equation

$$\left(\frac{b}{V_C} - c\right) L = \left(\frac{b}{V_E} - c\right) \alpha_H L. \tag{1}$$

In the equilibrium, the condition $\frac{\alpha_H}{V_E} < \frac{1}{V_C}$ is satisfied. Otherwise, for any $b \geq 0$, $\left(\frac{b}{V_E} - c\right) \alpha_H L > \left(\frac{b}{V_C} - c\right) L$ and all firms would choose to sell only to high income consumers. However, this is impossible in the equilibrium. This condition guarantees that

$$\begin{array}{lcll} \left(\frac{b}{V_C} - c\right) L & \geq & \left(\frac{b}{V_E} - c\right) \alpha_H L & \text{if } b \geq b_M, \\ \left(\frac{b}{V_C} - c\right) L & < & \left(\frac{b}{V_E} - c\right) \alpha_H L & \text{otherwise.} \end{array}$$

This means that if a firm draws $b \geq b_M$ then in the equilibrium, it sells to both types of consumers, otherwise it sells only to the rich or exits. A firm with valuation b_M of its good is indifferent between selling to all consumers and selling only to the rich (see *Figure 1*). Hence, even in the presence of market power, products have a natural hierarchy: consumers first buy goods with higher b, i.e., goods that are more essential in consumption. This result is supportive of the common intuition that the poor mostly spend their incomes on necessities, which are more essential in consumption, while the rich can afford to buy not only necessities but also luxuries. Without loss of generality, I assume that a firm with valuation b_M sells to both types of consumers. Let a function $V(b)$ be defined by $\frac{b}{p(b)}$. In the equilibrium, $V(b)$ looks as in *Figure 2*, where $b_L \geq 0$ is a cutoff level such that firms drawn $b < b_L$ exit.

Figure 1: The Profit Function

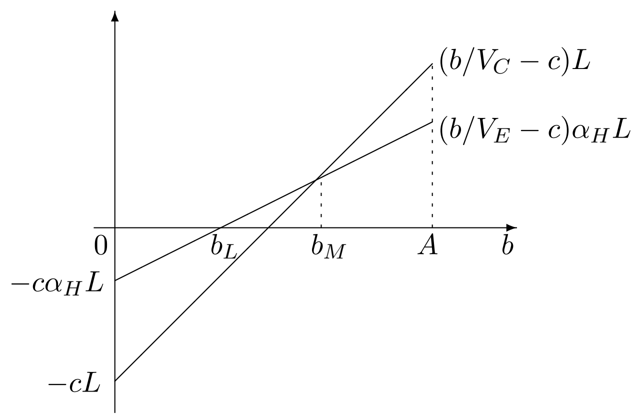
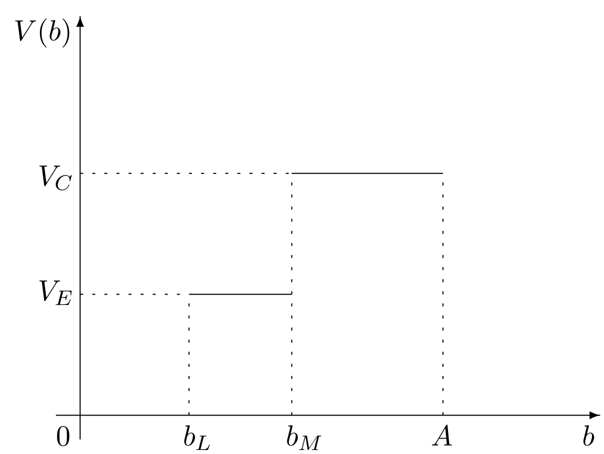


Figure 2: The Valuation to Price Function: A Special Case



1.3.1 The Equilibrium

Let M_e be the mass of firms that enter the market. One can think of M_e as that there are $M_e g(b)$ different firms with a particular valuation b. In the equilibrium, several conditions should be satisfied. First, as there is free entry in the market, the ex ante expected profits of firms have to be equal to zero. Second, the goods market clears. Since the poor consume only goods from the "common" group, the aggregate cost of the bundle of goods from the "common" group should be equal to the income of a poor consumer. Similarly, the aggregate cost of the bundle of all available goods in the economy should be equal to the income of a rich consumer.

Definition 1 *The equilibrium of the model is defined by the price function $p(b)$ on $b \geq b_L$, the cutoff level $b_L \geq 0$, b_M, M_e, and the valuation to price ratios V_C and V_E such that*

1) The ex ante expected profits of firms are equal to zero.

2) The goods market clears.

Further, I derive equations that satisfy the conditions mentioned above and prove that equilibrium in the model always exists and is unique. Let $\pi(b)$ be the variable profit of a firm with valuation b. To find the equilibrium, I express $\pi(b)$ and $p(b)$ as functions of b, b_L, b_M and exogenous parameters. As b_L is the cutoff level, firms with valuation b_L have zero profits. This implies that $\left(\frac{b_L}{V_E} - c\right)\alpha_H L = 0$ or $V_E = \frac{b_L}{c}$. From (1), we can express V_C as a function of b_L and b_M. As a result, the following lemma holds.

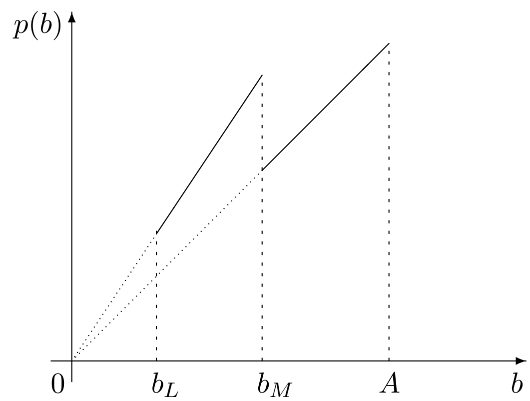
Lemma 1 *In the equilibrium,*

$$p(b) = \begin{cases} \frac{b}{V_C} = cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) & \text{if } b \geq b_M, \\ \frac{b}{V_E} = cb\frac{1}{b_L} & \text{if } b \in [b_L, b_M), \end{cases}$$

$$\pi(b) = \begin{cases} \left(cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) - c\right)L & \text{if } b \geq b_M, \\ \left(cb\frac{1}{b_L} - c\right)\alpha_H L & \text{if } b \in [b_L, b_M). \end{cases}$$

Since firms with valuation b_M have the same profits from selling to all consumers as from selling only to the rich, the price function has a jump at b_M; i.e., to compensate for lower demand, firms raise their prices (see *Figure 3*). This results in the nonmonotonicity of the price

Figure 3: The Price Function



function.

Due to free entry in the market, the ex ante expected profits of firms are equal to zero in the equilibrium. Using the results from *Lemma 1* and taking into account that firms with $b < b_L$ exit, I obtain

$$\begin{aligned} f_e &= (G(b_M) - G(b_L))E(\pi(b)|b_L \leq b < b_M) + (1 - G(b_M))E(\pi(b)|b \geq b_M) \iff \\ \frac{f_e}{cL} + 1 &= \alpha_H H(b_L) + (1 - \alpha_H)H(b_M), \end{aligned} \tag{2}$$

where $H(x) = G(x) + \frac{\int_x^A xdG(x)}{x}$. The goods market clearing condition implies that

$$\begin{cases} I_L = M_e \int_{b_M}^A p(t)dG(t) \\ I_H = M_e \int_{b_L}^A p(t)dG(t) \end{cases}. \tag{3}$$

The aggregate cost of the bundle of goods from the "common" group is equal to the income of a poor consumer, while the aggregate cost of the bundle of all available goods in the economy is equal to the income of a rich consumer. Dividing the second line in (3) by the first one and using *Lemma 1*, I obtain

$$\frac{\int_{b_L}^{b_M} tdG(t)}{\int_{b_M}^A tdG(t)} = \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1 - \alpha_H)}{b_M}\right).$$

Hence, given the exogenous parameters I_H, I_L, α_H, f_e, c, L, and the distribution of draws $G(\cdot)$, we can find endogenous b_M and b_L from the system of equations, which is given by

$$\begin{cases} \frac{\int_{b_L}^{b_M} tdG(t)}{\int_{b_M}^A tdG(t)} = \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right) \\ \frac{f_e}{cL} + 1 = \alpha_H H(b_L) + (1 - \alpha_H)H(b_M) \end{cases}. \tag{4}$$

The following lemma states the existence and uniqueness of the equilibrium.

Lemma 2 *The system of equations* (4) *has a unique solution.*

Proof. In Appendix A. ■

Once b_M and b_L are found, V_C and V_E can be derived using the results in *Lemma 1.* Finally, the mass of firms can be found from (3).

1.3.2 Income Inequality and Welfare

Before analyzing the effects of income inequality on market structure and welfare, I examine how consumer welfare and income inequality are determined in the model.

Welfare Given the preference structure, welfare of a certain consumer is equal to the sum of valuations of goods she consumes. In this way, welfare of a poor consumer is equal to $M_e \int_{b_M}^{A} t dG(t)$. From (3), $M_e = \frac{I_L}{\int_{b_M}^{A} p(t) dG(t)}$. This implies that

$$W_p = I_L V_C.$$

Welfare of a poor consumer naturally rises with an increase in either her income or the valuation to price ratio of goods she consumes. Similarly, welfare of a rich consumer is given by

$$W_r = I_L V_C + (I_H - I_L) V_E.$$

As the rich consume the same bundle of goods as the poor plus some others, welfare of the rich is equal to welfare of the poor plus additional welfare from the consumption of "exclusive" goods, which is in turn equal to income spent on these goods multiplied by their valuation to price ratio.

Notice that all changes in individual welfare are divided into two components: an income effect and a price effect. The price effect is determined by changes in V_C and V_E, which implicitly depend on the incomes (I_H and I_L) and the level of competition inside the groups of goods. The income effect is explicitly determined by changes in exogenous I_L and I_H.

Income Inequality As income inequality in the economy, I consider the variance of the income distribution[10], which is given by

$$VAR = \alpha_H(1-\alpha_H)\,(I_H - I_L)^2\,. \tag{5}$$

Income inequality is increasing in the income difference $I_H - I_L$ and has an inverted U shape as a function of α_H. Since one of the comparative statics I analyze deals with fixed aggregate income per capita, I express the variance in terms of aggregate income per capita, the fraction of the rich, and the income of the poor. Aggregate income per capita in the economy is given by

$$AG = \alpha_H I_H + (1-\alpha_H) I_L.$$

Then, the variance can be rewritten as follows

$$VAR = (\frac{1}{\alpha_H} - 1)\,(AG - I_L)^2\,. \tag{6}$$

The expression in (6) implies that keeping AG fixed, an increase in I_H together with a decrease in α_H raise income inequality in the economy.

In the next sections, I examine the impact of income inequality on the market structure and individual welfare. Since one of my main goals is to consider the effects on welfare of the poor consumers, in the subsequent analysis, I keep the income of the poor fixed and only consider changes in α_H and I_H. Recall that while changes in α_H affect consumer welfare only through the price effect, changes in I_H affect welfare through both the price and the income effects.

Changes in the Income of the Rich If the rich get even richer, do the poor gain or lose? What is the impact on prices? In this section, I consider an increase in the income of the rich I_H. Higher I_H has an impact on the poor only through changes in the prices of the "common" goods. Two opposite effects influence these prices. First, since I_H increases, some firms that used to sell their goods to all consumers find it more profitable to sell only to the rich. This

[10] Another possible way to describe income inequality in the model is to use the Gini coefficient. However, in the case of the income distribution considered in the paper, the Gini coefficient is highly correlated with the variance. Changes in the parameters of the distribution, which increase the Gini coefficient, usually increase the variance. The exception is changes in α_H. In some cases, higher α_H decreases the Gini coefficient but increases the variance. As my main goal is to analyze the qualitative implications of changes in income distribution, without loss of generality, I consider the variance of the distribution as the measure of income inequality.

reduces competition among firms serving all consumers and, therefore, raises the prices of the "common" goods. Second, higher income of the rich results in higher expected profits of firms, this in turn implies that more firms enter the market inducing tougher competition and reducing the prices. I show that the latter effect prevails over the former one. As a result, higher I_H positively affects V_C increasing welfare of the poor. The following proposition summarizes the results above.

Proposition 2 *An increase in income of the rich reduces the prices of the "common" goods increasing welfare of the poor.*

Proof. In Appendix *A*. ■

In contrast, an increase in I_H affects the rich through both the price and the income effects. Higher income of the rich allows firms that sell only to the rich to raise their prices. In spite of higher entry in the market, the prices of the "exclusive" goods rise and as a result, V_E falls. However, the income effect is stronger than the effect of changes in prices of the "exclusive" goods and the rich gain from higher I_H. The following proposition holds[11].

Proposition 3 *An increase in income of the rich raises the prices of the "exclusive" goods and increases welfare of the rich.*

Proof. In Appendix *A*. ■

The intuition, which is behind the results above, may also work in traditional models with homothetic preferences. In Melitz (2003), higher income of some consumers results in higher entry, tougher competition, and, thereby, higher welfare of all consumers. However, there are some differences. In the present model, higher income of the rich raises the markups of firms selling only to the rich and decreases the markups of firms serving all consumers. In traditional models, there is the same or no impact on firms' markups. Moreover, assume for a moment that the mass of firms does not change in the model[12]. In this case, higher income of the rich raises prices of all goods and the poor are worse off. In traditional models, if we fix the mass of firms then higher income of one part of consumers does not affect welfare of the other part.

[11] Similar intuition works if we consider changes in I_L. An increase in I_L raises the prices of the "common" goods and decreases the prices of "exclusive" goods. The poor and the rich are better off (see details in the Appendix).

[12] In some sense, this case can be interpreted as a short run version of the model.

Changes in the Fraction of the Rich In the previous section, I have established that higher income of the rich always benefits the poor. What is the impact of an increase in the fraction of the rich? What is better for the poor: tiny minority or vast majority of the rich? In this section, I analyze how changes in α_H affect the poor consumers. As above, a rise in α_H affects the poor through the price effect. Because of higher α_H, profits from selling to the rich become higher. This implies that some firms switch from serving all consumers to serving only the rich. This reduces competition among firms selling the "common" goods and, consequently, raises the prices of the "common" goods. At the same time, higher fraction of the rich results in higher ex ante expected profits and this in turn increases entry in the market inducing tougher competition and lower prices of all goods. In the previous section, the negative effect on the prices of "common" goods always dominates the positive one. In this case, it is not necessarily true. I show that in a neighborhood around $\alpha_H = 0$, a rise in α_H increases welfare of the poor. While in a neighborhood around $\alpha_H = 1$, higher fraction of the rich decreases welfare of the poor consumers. The following proposition holds.

Proposition 4 *If α_H is close to zero (close to one), a rise in α_H decreases (increases) the prices of the "common" goods increasing (decreasing) welfare of the poor.*

Proof. In Appendix *A*. ■

The last proposition suggests that welfare of the poor has an inverted U shape as a function of α_H. Because of mathematical difficulties arising in the analysis, I cannot strictly prove this conjecture. Instead, I make a number of numerical exercises where I consider welfare of the poor as a function of α_H. The results are supportive of the claim that welfare of the poor has an inverted U shape as a function of α_H[13].

The fact that firms endogenously choose the type of consumers they wish to serve makes the results regarding changes in α_H different from that ones in traditional models with homothetic

[13] It appears that the sign of $(W_p)'_{\alpha_H}$ is the same as the sign of $K(\alpha_H) - \frac{\alpha_H}{1-\alpha_H}$, where $K(\alpha_H) > 0$ for any $\alpha_H \in [0,1]$ and $K(1) < \infty$. This implies that in the neighborhood of $\alpha_H = 0$ ($\alpha_H = 1$), $(W_p)'_{\alpha_H} > 0$ ($(W_p)'_{\alpha_H} < 0$). If $K(\alpha_H)$ is well behaved, i.e., the equation $K(\alpha_H) = \frac{\alpha_H}{1-\alpha_H}$ has a unique solution, then W_p has an inverted U as a function of α_H. Unfortunately, the analysis of the behavior of $K(\alpha_H)$ on $[0,1]$ is rather complicated. We cannot exclude the possibility that the equation $K(\alpha_H) = \frac{\alpha_H}{1-\alpha_H}$ has multiple solutions (see details in the Appendix). In the numerical examples I consider, I take the power distribution $G(b) = \left(\frac{b}{A}\right)^k$ with $k > 0$ as the distribution of draws. For a number of different sets of the exogenous parameters, I find the solution of $K(\alpha_H) = \frac{\alpha_H}{1-\alpha_H}$. In all cases, the solution is unique.

preferences. In Melitz (2003), higher fraction of the rich always leads to higher welfare of the poor in the long run and has no impact in the short run (when the mass of firms is fixed). In this model, we observe an ambiguous impact of α_H on the poor in the long run and a negative impact in the short run[14].

Changes in the Income and the Fraction of the Rich Keeping Aggregate Income Fixed There is a common feature for both comparative statics mentioned above. An increase in I_H as well as an increase in α_H raises aggregate income in the economy. To capture a pure redistribution effect, I consider an increase in the personal income of the rich together with a decrease in the fraction of the rich keeping aggregate income in the economy fixed[15]. In models with homothetic preferences, these changes in income distribution do not affect entry, prices, and welfare of the poor. In the present model, I show that they result in higher entry in the market and, therefore, higher welfare of the poor.

For better understanding of the intuition behind, I first consider the short run implications of the changes in income distribution. From the previous sections we know that in the short run, higher I_H decreases welfare of the poor, while lower α_H increases it. Thus, two effects work in opposite directions. However, it appears that the impact of I_H is always stronger than that of α_H. Here the assumption that aggregate income is unchanged plays a key role[16]. This

[14] While a rise in α_H has an ambiguous impact on the poor, the rich always benefit from it. Higher α_H raises competition among firms selling the "exclusive" goods and, thereby, reduces the prices of these goods. Recall that welfare of the rich is equal to $W_r = I_L V_C + (I_H - I_L)V_E$. Even though in some cases V_E falls, due to a rise in V_C, W_r increases. As a result, the rich gain from higher α_H. See details in the Appendix.

[15] Another comparative static that holds aggregate income constant is changes in incomes of the rich and the poor keeping the fraction of the rich fixed. Since the main goal of this paper is to explore the effects on welfare of the poor, I do not pay a lot of attention on this comparative static. If we consider a rise in I_H and a decrease in I_L holding AG constant then we might expect that the income effect prevails over the price effect. That is, the rich gain and the poor lose. Notice that the poor consume on average more valuable goods than the rich. At the same time, the changes in the incomes substitute the consumption of more valuable goods for the consumption of less valuable goods. Therefore, given that $bg(b)$ is increasing in b, total welfare in the economy may decrease.

[16] Recall that welfare of a poor consumer is given by $W_p = M_e \int_{b_M}^{A} t dG(t)$. Since in the short run M_e is fixed, we only need to examine the effects on b_M. Notice that b_M solves $\left(\frac{b}{V_C} - c\right) L = \left(\frac{b}{V_E} - c\right) \alpha_H L$. This implies that $b_M \left(\frac{1}{V_C} - \frac{\alpha_H}{V_E}\right) = c(1-\alpha_H)$. From the goods market clearing condition, $I_L = \frac{M_e}{V_C} \int_{b_M}^{A} t dG(t)$ and $I_H - I_L = \frac{M_e}{V_E} \int_{b_L}^{b_M} t dG(t)$. This results in $\frac{b_M}{M_e} \left(\frac{I_L}{\int_{b_M}^{A} t dG(t)} - \frac{\alpha_H (I_H - I_L)}{\int_{b_L}^{b_M} t dG(t)} \right) = c(1-\alpha_H)$. Notice that in the long run, both an increase in I_H and a decrease in a_H drive the prices of the "exclusive" goods up (see the previous sections). This implies that in the long run, $b_L = cV_E$ falls. In the short run, only firms that were active before may operate in the market. Therefore, in the short run, b_L is unchanged. That is, firms that produced before the changes

implies that in the short run, the poor are worse off from the changes in the income distribution considered.

What is about the long run? On the one hand, higher income of the rich allows firms to impose higher prices of their goods and, consequently, leads to higher entry in the market. On the other hand, lower fraction of the rich reduces the demand for the "exclusive" goods making ex ante expected profits lower. This results in lower entry in the market. As in the previous section, I focus the analysis on two extreme cases: $\alpha_H \approx 0$ and $\alpha_H \approx 1$. I show that in these cases, the impact of I_H prevails over that of α_H leading to higher entry in the market. Moreover, I show that the positive effect on welfare of the poor from higher entry is stronger than the negative short run effect. These results are derived for neighborhoods of $\alpha_H = 0$ and $\alpha_H = 1$ and an arbitrary distribution function $G(b)$. However, if we limit the analysis to the cases when $bg(b)$ is increasing in b then the results hold for any $\alpha_H \in [0,1]$[17]. The next proposition summarizes these findings.

Proposition 5 *If α_H is in neighborhoods of $\alpha_H = 0$ and $\alpha_H = 1$ or $G(b)$ is such that $bg(b)$ is increasing in b, an increase in I_H together with a decrease α_H keeping aggregate income fixed raise welfare of the poor and the number of firms entering the market.*

Proof. In Appendix *A*. ■

The assumption that $bg(b)$ is increasing in b has a strong economic interpretation. It implies that $g(b)$ does not decrease too fast; i.e., the probability of getting higher values of b does not decrease too fast with b. Moreover, in some sense, utility from the consumption of all goods with a particular valuation b is equal to $M_e bg(b)$. Hence, this assumption also guarantees that this utility increases with b.

1.3.3 Entry Cost, Market Size, and Welfare

The impact of higher entry cost and market size on consumer welfare is the same as in traditional models. However, the present model implies that changes in market size or the cost of entry

in I_H and α_H find it profitable to produce after. Moreover, as aggregate income in the economy is unchanged, $\alpha_H(I_H - I_L)$ does not change too. This implies that only b_M changes in $\frac{b_M}{M_e}\left(\frac{I_L}{\int_{b_M}^{A} t dG(t)} - \frac{\alpha_H(I_H - I_L)}{\int_{b_L}^{b_M} t dG(t)}\right)$. As a result, an increase in $c(1-\alpha_H)$ leads to a rise in b_M.

[17] For instance, the set of power distributions satisfies this condition.

have different impacts on different types of consumers. In this section, I briefly describe the effects of changes in f_e and L on individual welfare and focus my analysis on the effects on the relative welfare of the rich with respect to the poor.

An increase in the cost of entry f_e reduces the ex ante expected profits of firms. This in turn decreases the number of firms entering the market and reduces competitive pressure. As a result, prices of goods from both groups rise and welfare of all consumers falls. An increase in L results in higher ex ante expected profits of firms. This leads to the higher number of firms entering the market and tougher competition. Prices of goods from both groups fall and consumers of both types are better off. Finally, any changes in f_e and L such that the ratio $\frac{f_e}{L}$ remains the same do not change prices and individual welfare. Two opposite effects completely compensate each other (see (4)). The following proposition holds.

Proposition 6 *Larger countries and countries with lower entry cost have higher individual welfare: a rise in* $\frac{f_e}{L}$ *reduces welfare of all individuals.*

Proof. In Appendix A. ■

In the next section, I examine the effect of $\frac{f_e}{L}$ on relative welfare of the rich with respect to the poor.

Relative Welfare Relative welfare of the rich with respect to the poor is given by

$$\frac{W_r}{W_p} = 1 + \frac{I_H - I_L}{I_L}\frac{V_E}{V_C}.$$

Notice that welfare inequality is divided into two components: income inequality and consumption inequality. The income inequality is determined by the ratio $\frac{I_H - I_L}{I_L}$, while the consumption inequality $\frac{V_E}{V_C}$ depends on relative prices of the "exclusive" goods with respect to the "common" goods. Changes in the exogenous parameters of the model may affect either type of inequality or both. For instance, higher income of the rich raises income inequality but decreases consumption inequality.

The relative welfare can be rewritten as follows

$$\frac{W_r}{W_p} = 1 + \frac{I_H - I_L}{I_L}\left(\alpha_H + (1 - \alpha_H)\frac{b_L}{b_M}\right). \tag{7}$$

From (7), changes in $\frac{f_e}{L}$ affect $\frac{W_r}{W_p}$ only through the ratio $\frac{b_L}{b_M}$. Moreover, changes in $\frac{f_e}{L}$ have no direct impact on the goods market equilibrium condition

$$\frac{\int_{b_L}^{b_M} tdG(t)}{\int_{b_M}^{A} tdG(t)} - \frac{I_H - I_L}{I_L}\left(\alpha_H + (1-\alpha_H)\frac{b_L}{b_M}\right) = 0. \tag{8}$$

From (8), we can find an implicit dependence of b_M on b_L: $b_M = b_M(b_L)$. Notice that $\frac{f_e}{L}$ is negatively correlated with the cutoff $b_L = cV_E$. Hence, exploring the impact of $\frac{f_e}{L}$ on relative welfare, we need to analyze the sign of $\left(\frac{b_L}{b_M(b_L)}\right)'_{b_L}$. In the Appendix, I show that to determine the sign of $\left(\frac{b_L}{b_M(b_L)}\right)'_{b_L}$, we need to know the sign of $\left(\frac{b^2 g(b)}{\int_b^A tdG(t)}\right)'_b$. If $\left(\frac{b^2 g(b)}{\int_b^A tdG(t)}\right)'_b$ is always greater than zero then $\left(\frac{b_L}{b_M(b_L)}\right)'_{b_L}$ is always positive. Otherwise, depending on the exogenous parameters of the model, the sign of $\left(\frac{b_L}{b_M(b_L)}\right)'_{b_L}$ might be either. The following proposition formalizes the findings above.

Proposition 7 *If* $\left(\frac{b^2 g(b)}{\int_b^A tdG(t)}\right)'_b > 0$ *for any* $b \in [0, A]$, *then the rich gain more from an increase in market size and lose more from an increase in the cost of entry than the poor.*

Proof. In Appendix A. ■

Limiting the analysis to the cases when $\left(\frac{b^2 g(b)}{\int_b^A tdG(t)}\right)'_b$ is always positive, we derive that the rich lose more from an increase in $\frac{f_e}{L}$ than the poor. To understand the intuition, I separately consider two markets. The first market is the market for goods from the "common" group, while the second one is the market for the "exclusive" goods. I divide the effect of higher $\frac{f_e}{L}$ into two steps. First, given an increase in $\frac{f_e}{L}$, fewer firms enter the both markets decreasing b_L and b_M. Second, due to less competitive pressure, some firms that sold their goods only to the rich switch to selling to all consumers. This effect decreases b_M even more and in turn reduces competition in the second market allowing firms with low valuations to survive. As a result, the cutoff b_L falls. Since firms that switched from the second market to the first one have relatively high valuations of their goods compared with firms that "survived", the prices of these goods were relatively high. This implies that b_L has to fall by more than b_M to compensate for the difference in the prices.

1.4 A General Model

To complete the model, I consider the general case with N types of consumers. I show the existence and uniqueness of the equilibrium and discuss some issues related to the case when the distribution of efficiency units of labor among consumers is continuous.

In the general case, consumers differ in the number of efficiency units of labor they are endowed with. A consumer of type n is endowed with I_n efficiency units of labor. I choose indices so that $I_n > I_{n-1}$. Here α_n is the fraction of consumers of type n in the aggregate mass L of consumers. The equilibrium in the general model is similar to the equilibrium in the simple case considered before. All goods that are consumed by a certain type of consumers are also consumed by more productive consumers. Thus, goods in the economy are divided into $N+1$ groups. Goods belong to group $k = 1..N$ if they are only consumed by consumers whose type is greater or equal to k. Goods belong to group $N+1$ if they are consumed by nobody. In the equilibrium, goods from the same group have the same valuation to price ratio. Let V_k be the valuation to price ratio of goods from group k. Then, in the equilibrium, $V(b)$ looks as in *Figure 4,* where b_k is such that firms with b_k are indifferent between selling to consumers with types greater or equal to k and selling to consumers with types greater or equal to $k+1$. For instance, firms with b_1 are indifferent between selling to all consumers and selling to everyone except the poorest. Firms with $b < b_N$ leave the market. Without loss of generality, I assume that firms with b_k choose to sell to consumers with types greater or equal to k. As before, let M_e be the mass of firms that enter the market and draw valuation of their goods.

Definition 2 *The equilibrium of the model is defined by the price function $p(b)$ on $b \geq b_N$, M_e, the sequences $\{V_k\}_{k=1..N}$ and $\{b_k\}_{k=1..N}$ such that*

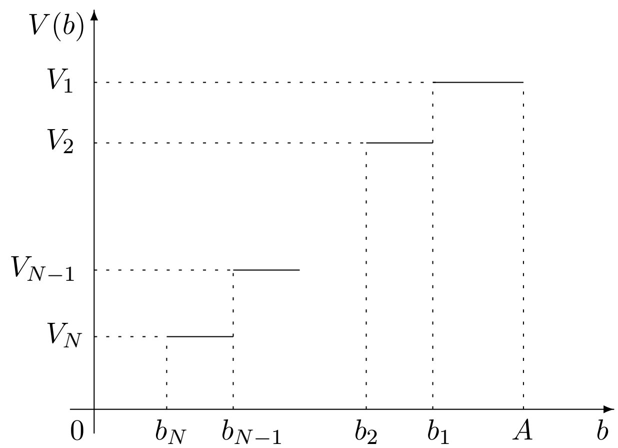
1) The ex ante expected profits of firms are equal to zero.

2) The goods market clears.

Let $\pi_k(b)$ and $p_k(b)$ be the profit and the price of a firm with valuation $b \in [b_k, b_{k-1})$, respectively[18]. Then, the following lemma holds.

[18] $b_0 = A$.

Figure 4: The Valuation to Price Function: A General Model



Lemma 3 *In the equilibrium,*

$$p_k(b) = \frac{b}{V_k} = bc\frac{\sum_{i=k}^{N} \frac{\alpha_i}{b_i}}{\sum_{i=k}^{N} \alpha_i},$$

$$\pi_k(b) = cL \sum_{i=k}^{N} \frac{\alpha_i(b - b_i)}{b_i}.$$

Proof. In Appendix A. ■

In the equilibrium, the expected profits of firms are equal to zero. This implies that

$$\begin{aligned} f_e &= \sum_{k=1}^{N} (G(b_{k-1}) - G(b_k)) E(\pi_k(b) | b \in [b_k, b_{k-1})) \iff \\ \frac{f_e}{cL} + 1 &= \sum_{k=1}^{N} \alpha_k H(b_k). \end{aligned}$$

In addition, the goods market clearing condition should be satisfied. This implies that the aggregate cost of the bundle of goods from group k should be equal to income of a consumer of type k. In this way, I obtain

$$I_k = M_e \int_{b_k}^{A} p(t) dG(t) \quad k = 1..N.$$

Hence, there is the system of $N + 1$ equations

$$\begin{cases} I_k = M_e \int_{b_k}^{A} p(t) dG(t) \quad k = 1..N \\ \frac{f_e}{cL} + 1 = \sum_{k=1}^{N} \alpha_k H(b_k) \end{cases} \tag{9}$$

with $N + 1$ unknowns: $\{b_k\}_{k=1..N}$ and M_e.

Proposition 8 *The equilibrium in the general model always exists and is unique.*

Proof. The proof is based on the fact that the system of equations (9) has a unique solution. Details are in Appendix A. ■

Assume that the distribution of consumer productivities is continuous. Notice that any continuous distribution can be approximated by the sequence of discrete distributions. Therefore, we can interpret equilibrium in the continuous model as the limit of equilibria in the discrete models. In this case, the function $V(b)$ is continuous, increasing on $[b^c_L, b^c_M)$, and flat on $[b^c_M, A]$, where $0 \geq b^c_L > b^c_M \geq A$. The parameter b^c_L represents the cutoff level: firms with $b < b^c_L$ leave the market. While b^c_M is determined by the support of the productivity distribution. Namely, goods with $b \in [b^c_M, A]$ are consumed by everybody in the equilibrium. This implies that $b^c_M < A$ if and only if the lower bound of the distribution support is strictly greater than zero; i.e., the minimum income in the economy is greater than zero.

Due to mathematical difficulties, it is hard to solve the continuous model for an arbitrary distribution of productivities[19]. To solve the problem explicitly, I need to make a simplifying assumption about the distribution of efficiency units of labor. I assume that this distribution has a constant hazard rate. That is, I consider the set of exponential distributions on $[s, \infty)$, where $s \geq 0$ is the minimum endowment of efficiency units of labor. Since the upper bound of the support is infinity, the maximum income in the economy is also equal to infinity. This implies that the cutoff b^c_L equals to zero in the equilibrium. I show that in a neighborhood $b = 0$, the price function $p(b)$ is decreasing in b and $p(0) = \infty$. Hence, this model gives us a simple straightforward explanation of why some luxury goods with relatively low valuation (or quality) to price ratios are so expensive: the rich are ready to pay such high prices for these goods.

1.5 Conclusion

In this paper, I consider a general equilibrium model of monopolistic competition with heterogenous firms and consumers. The model incorporates two key features: imperfect competition and non-homothetic preferences, which allow us to analyze the consequences of changes in income distribution on pricing, market structure and, thereby, welfare of different groups of consumers in equilibrium. The general model is constructed and solved. Due to technical difficulties in

[19]See details in the Appendix.

exploring comparative statics in the general case, I focus on the case of two types of consumers: rich and poor.

This framework leads to interesting theoretical results that help to understand the impact of income inequality on individual well-being. In particular, I analyze how income inequality influences welfare of the poor. I show that higher income inequality in the economy may benefit the poor via a trickle-down effect operating through entry. This model also allows us to analyze the effects of changes in market size and entry cost. An increase in market size leads to tougher competition. Therefore, markups of all firms fall and welfare of all consumers rises. Similarly, an increase in entry cost induces lower competition, raises markups, and, thereby, decreases welfare of all consumers. Moreover, I show that the rich may gain more from an increase in market size and lose more from an increase in entry cost compare to the poor.

There are a number of plausible extensions of this model. For instance, it would be interesting to consider an open economy version of the model. In this case, the paper can be modified in two ways. First, one can explore a model of trade between two countries with different income distributions and examine how this difference affects the trade pattern. Second, it would be interesting to consider the case when income distribution is endogenous and, for instance, affected by the level of openness. I leave these issues for future work.

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Chapter 2. Globalization: Intensive versus Extensive Margins

2.1 Introduction

It is well known that different groups of consumers (for instance, the rich or the middle class) consume different bundles of goods and in different amounts: while the rich can afford to buy Ferrari, the poor are constrained by buying Nissan or Toyota. Thus, gains or losses from globalization are not equally distributed across consumers. Some consumers gain or lose more, some of them less. During the nineties in Russia, the government imposed high tariffs on imported cars to protect the domestic industry. While rich consumers continued buying cars at higher prices, some of poorer consumers had to do without any car at all. Whereas the rich lost from higher prices, the poor lost from incapability to buy a car. A natural question arises: who lost more? In this paper, I analyze the effects of globalization on relative welfare of the rich with respect to the poor.

There is a large empirical and theoretical literature that examines the impact of world globalization on income inequality within a country[20]. However, in economic literature there is not much attention paid to the fact that globalization may influence inequality through consumption: globalization can change consumption patterns and increase or decrease welfare inequality among the groups. To my best knowledge, there is only one study that examines the implications of globalization on welfare inequality not only through the income channel, but also through the consumption channel. In his paper, Porto (2006) empirically explores the impact of Argentinean trade reform, namely entry into Mercosur, on the distribution of welfare. He examines two possible effects caused by the trade reform: price changes and income changes. Given the structure of Mercosur, the prices of such goods as food and beverages increased, while the prices of nontradable goods such as health and education decreased. Food and beverages have a larger share in the consumption of the poor than that of the rich; health and education are mostly consumed by the rich. Thus, one of the effects of Mercosur is an increase in inequality through consumption. Even though Porto (2006) argues that the income channel dominates over the consumption channel, the consumption channel seems to play an important role in the analysis of globalization and inequality and needs to be explored further.

[20] See Goldberg and Pavcnik (2007) for a literature review.

The goal of this paper is to construct a model that establishes a link between globalization and inequality through the consumption channel. I develop a general equilibrium trade model of monopolistic competition with heterogenous firms and consumers. Consumer heterogeneity in the model is introduced by assuming that consumers differ according to the efficiency units of labor they are endowed with. I assume that all consumers share identical but non-homothetic preferences. In traditional models of monopolistic competition and trade, there are two standard preference assumptions: consumer preferences are identical and homothetic, or identical and quasi-linear. In both cases, it is well understood that the presence of consumer heterogeneity does not affect equilibrium. In the case of homothetic preferences, only aggregate income matters; while in the case of quasi-linear preferences, the presence of a numeraire good eliminates the influence of consumer heterogeneity on equilibrium outcomes. In these models, any price change has the same (not relative to income) impact on all consumers. In the present model, non-homothetic preferences and income heterogeneity imply that price changes (caused by globalization) affect different population groups differently.

I adopt the preference structure in Murphy, Shleifer and Vishny (1989) and Matsuyama (2000). The basic idea is that goods are indivisible, and potential consumers want to buy only a single unit of each good. This implies that given prices, goods can be arranged so that consumers choose what to buy by moving down a certain list. For example, in developing countries, consumers first buy food, then clothing, then move up the chain of durables from kerosene stoves to refrigerators, to cars. Notice that the consumer utility can only be increased by the consumption of a greater number of goods. Moreover, consumers with high income consume the same set of goods as consumers with low income, plus some others. Goods differ in terms of the valuations that consumers attach to them. By the valuation of a good, I mean the utility derived by consumers from the consumption of one unit of this good. Such differences between goods generate ex-post heterogeneity across firms, as firms enter the market before the valuations placed on their goods are realized[21]. There is free entry in the market. To enter the

[21] This structure of consumer preferences has enough flexibility to be applied as to the whole economy as to a certain industry where goods differ in quality. On the one hand, each good can be interpreted as a distinct good sold in the market. In this case, the structure describes the whole economy. On the other hand, one can think that firms sell not distinct goods but some characteristics of a good produced by a certain industry. For instance, consider a car industry. Each good can be treated as some characteristic of a car. The poor purchase some main characteristics associated with a car, while the rich buy the same characteristics as the poor plus some additional

market, firms have to make costly sunk investments. Once firms enter, they learn about the valuations attached to their goods. Depending on the valuation drawn, firms choose whether to exit or to stay. Firms that decide to stay engage in price competition with other firms. Hence, the model incorporates two key features: imperfect competition and non-homothetic preferences, which permit to analyze the impact of globalization on mark-ups charged on different goods and on the welfare of different groups of consumers.

The heart of the paper focuses on the case of two types of consumers: rich and poor, which are distinguished only by income (associated with productivity). Given the preferences, all goods consumed by the poor are also consumed by the rich. In equilibrium, domestic and importing firms choose between selling to everybody at a lower price and selling only to the rich at a higher price. As a result, firms with relatively high valuations decide to sell to everybody, while firms with relatively low valuations choose to sell only to the rich. Thus, potentially available goods in each country can be divided into three groups: the "common" group includes goods that are consumed by both types of consumers; the "exclusive" group includes goods that are consumed by the rich only; finally, there is the group of goods that are consumed by no one.

In the analysis, I focus on the two components of globalization: trade liberalization and a rise in the number of trading partners[22]. I show that the effect of globalization on welfare inequality depends on the component of globalization considered. While the rich always gain more from a rise in the number of trading partners than the poor, the impact of trade liberalization on relative welfare depends on transportation costs. If transportation costs are low enough then the poor compared to the rich gain more out of trade liberalization; otherwise, the opposite is true.

The intuition behind these results is as follows. Consider separately two submarkets: one for the "common" goods and one for the "exclusive" goods. Since the rich consume the same set of goods as the poor plus goods from the "exclusive" group, then changes in the relative welfare of the rich are determined by changes in the relative prices of "exclusive" goods with

luxury characteristics. That is, both groups of consumers buy the same good but of different quality.

[22] While lower transportation costs are the main consequence of globalization, the connection of globalization with a rise in the number of trading partners is not so straightforward. One can think that globalization leads to changes in the set of goods produced in a country: firms start producing goods that have potential to be sold in the world market. Thus, the number of countries trading with each other increases. Another example is related to trade blocs: a rise in the number of trading partners can be interpreted as the expansion of a trade bloc.

respect to "common" goods. If the transportation costs are sufficiently low then both types of consumers buy imported goods. In this case, an increase in transportation costs leads to exit of some importing firms from both submarkets: firms that used to sell their goods to the rich stop exporting; firms sold to everybody start selling only to the rich. This limits the scope of competition in the submarkets and drives up prices. However, as firms that switched from the submarket for "common" goods to the submarket for "exclusive" goods induce more competition, prices in this submarket rise by less than prices in the submarket for "common" goods. This implies that the rich lose less from an increase in transportation costs than the poor. Moreover, I show that depending on the exogenous parameters of the model, the rich may be better off from an increase in transportation costs, while welfare of the poor and aggregate welfare both fall. If the transportation costs are high enough then the prices of imported goods are relatively high, and only the rich can afford to buy them. In this case, it becomes that the rich lose more from an increase in transportation costs than the poor. The results regarding the expansion of the number of trading partners are based on the similar logic. If transportation costs are high enough then only the rich purchase imported goods. In this case, they gain more from a rise in the number of trading partners than the poor. If transportation costs are such that both types of consumers buy imported goods then a rise in the number of trading partners leads to higher competition in both submarkets forcing some domestic and importing firms to exit. At the same time, firms that leave the submarket for the "common" goods enter the submarket for the "exclusive" goods inducing even higher competition and lower prices in this submarket. Hence, the rich gain more from a rise in the number of trading partners than the poor irrespective of the transportation costs.

The related literature in this area can be divided into three strands. First, there are papers considering monopolistic competition and trade models with firm heterogeneity, but assuming homothetic or quasilinear preferences. Melitz (2003) develops a general equilibrium model with firm heterogeneity and Dixit-Stiglitz preferences, which imply constant mark-ups. Melitz and Ottaviano (2005) examine a similar framework, but incorporate variable mark-ups by considering a linear demand system. In both studies, the impact of globalization is the same for all consumers. In contrast, the model presented here includes all the key features of the papers mentioned, while analyzing in addition the impact of globalization on different population groups.

The second group of papers explores the implications of non-homothetic preferences in a perfectly competitive environment for open economies. Flam and Helpman (1987), Stokey (1991), and Matsuyama (2000) develop a Ricardian model of North-South trade with non-homothetic preferences. They examine the impact of technical progress, population growth, and redistribution policy on the patterns of specialization and welfare. Stibora and Vaal (2005) extend the model in Matsuyama (2000) by studying the effects of trade liberalization. They show that South loses in terms of trade from unilateral trade liberalization, while North may gain by liberalizing its trade. Krishna and Yavas (2005) investigate the impact of trade in the presence of labor market distortions and indivisibilities in consumption. However, due to their perfectly competitive framework, some important economic mechanisms (such as entry and exit of firms) related to pricing, market structure, and welfare are beyond the scope of these papers. Fieler (2007) modifies Eaton and Kortum (2002) by introducing non-homothetic preferences and technology distribution across sectors. This modification allows separating the effects of income per capita and country size on trade patterns.

The third group of papers deals with both monopolistic competition and non-homothetic preferences. Markusen (1986) extends the Krugman type model of trade with monopolistic competition by adding non-homothetic demand. He examines the role of income per capita in interindustry and intra-industry trade. Mitra and Trindade (2005) focus on the implications of asset inequality on trade flows and patterns. While they consider a model of monopolistic competition with non-homothetic preferences, the way they introduce non-homotheticity has the shortcoming that the share of income spent on a particular good is exogenous and depends on personal income. There are two papers written by Foellmi and Zweimueller that are similar to the closed economy case of the model presented in this paper. Foellmi and Zweimueller (2004) develop a general equilibrium model with an exogenous mass of identical firms. They show that, depending on the parameters of the model, an increase in income inequality has either no impact on firm mark-ups or increases them. In contrast, I consider heterogenous firms and free entry in the market, which in turn implies endogeneity of the mass of potential producers in equilibrium. Foellmi and Zweimueller (2006) examine a dynamic variation of Murphy, Shleifer, and Vishny (1989). Assuming learning by R&D, they focus their analysis on the link between possible growth and inequality. Finally, Tarasov (2007) considers the closed economy case of the model in the paper and studies the impact of income distribution on individual welfare, in particular, welfare of the poor.

The rest of the paper is organized as follows. Section 2 introduces the basic concepts for the closed economy case of the model. Section 3 extends the analysis to the open economy case with $N+1$ identical countries and derives the implications of globalization on prices, market structure, and consumer welfare. Section 4 concludes.

2.2 Closed Economy

In this section, I briefly describe the closed economy version of the model. The structure of the model is adopted from Tarasov (2007).

2.2.1 Production

The timing of the model is as follows: firms choose whether to incur sunk costs f_e or not. If a firm incurs the costs then it obtains a draw b of the valuation of its good from the distribution $G(b)$ on $[0, A]$. This captures the idea that before entry, firms do not know how well they will end up doing due to uncertainty in valuations of their products. I assume that $G'(b) = g(b)$ exists. The valuation b can be interpreted as the utility derived by consumers from the consumption of one unit of the good. Such differences between goods generate ex-post heterogeneity across firms. Depending on the valuation drawn, firms choose whether to leave the market or to stay. Firms that decide to stay engage in price competition with other firms. The only factor of production is labor. I assume that marginal cost of production is the same for all firms and equals to c, i.e., it takes c effective units of labor (which are paid a wage of unity) to produce a unit of any good.

Consumers differ in the number of efficiency units of labor they are endowed with. I assume that there are two types of consumers, indexed by L and H. A consumer of type $i \in \{L, H\}$ is endowed with I_i efficiency units of labor, and $I_H > I_L$. Let α_H be the fraction of type H consumers in the aggregate mass L of consumers. Thus, the total labor supply in the economy in efficiency units is $L(\alpha_H I_H + (1-\alpha_H) I_L)$.

2.2.2 Consumption

All consumers have the same non-homothetic preferences given by utility function

$$U = \int_{\omega\in\Omega} b(\omega)x(\omega)d\omega, \tag{10}$$

where Ω is the set of available goods in the economy, $b(\omega)$ is the valuation of good ω, and $x(\omega) \in \{0, 1\}$ is the consumption of good ω. Each consumer owns a balanced portfolio of shares of all firms. To simplify the notation, I assume that consumers have equal shares of all firms[23]. This means that all consumers have the same wealth, while their labor incomes vary with their productivity. Let π be the total profits of all firms in the economy. For given prices, a type i consumer maximizes (10) subject to the budget constraint

$$\int_{\omega\in\Omega} p(\omega)x(\omega)d\omega \leq I_i + \frac{\pi}{L},$$

where $p(\omega)$ is the price of good ω. It is clear that utility maximization merely involves moving down the list of products ordered by their valuation to price ratio, $\frac{b(\omega)}{p(\omega)}$, until all income is exhausted.

2.2.3 Equilibrium

Given preferences, all goods consumed by less productive consumers are also consumed by more productive ones. Thus, goods in the economy can be divided into three groups: the "common" group includes goods that are consumed by both types of consumers; the "exclusive" group includes goods that are consumed by more productive type only; finally, there is the group of goods that are never consumed. Hereafter, by the rich and the poor, I mean consumers of type H and L, respectively.

A firm that produces a good ω obtains a profit of $(p(\omega) - c)Q(\omega)$, where $Q(\omega)$ is the demand for good ω. If all consumers buy the good then the demand is L. If only the rich buy it, the demand is $\alpha_H L$. Thus, $Q(\omega) \in \{L, \alpha_H L, 0\}$.

Each firm takes the valuation to price ratios of all other firms as given and maximizes its profit. The following proposition holds.

[23] Due to free entry, the total profits of all firms are equal to zero in the equilibrium. This implies that the value of any balanced portfolio is equal to zero.

Proposition 1 *Even though all goods have different valuation to marginal cost ratios, goods from the same group have the same valuation to price ratio in the equilibrium.*

Proof. Suppose the opposite is true. Then, there exists some group, in which there are at least two goods with different $\frac{b(\omega)}{p(\omega)}$ ratios. Since both goods belong to the same group, the firm that produces its good with higher $\frac{b(\omega)}{p(\omega)}$ can raise its $p(\omega)$ without affecting the demand. This in turn would increase its profit. ∎

Define V_C as the valuation to price ratio of goods from the "common" group and V_E as valuation to price ratio of goods from the "exclusive" group in the equilibrium. Here V_C and V_E are endogenous parameters and $V_C > V_E$ in the equilibrium[24]. Thus, if a firm with valuation $b(\omega)$ sells to all consumers then its price is equal to $\frac{b(\omega)}{V_C}$ and its profit is given by

$$(p(\omega) - c)\,L = \left(\frac{b(\omega)}{V_C} - c\right) L,$$

while if the firm sells only to the rich, its profit is given by

$$(p(\omega) - c)\alpha_H L = \left(\frac{b(\omega)}{V_E} - c\right) \alpha_H L.$$

As $V_C > V_E$, the firm chooses between selling to more people at a lower price and selling to fewer of them, but at a higher price. Hence, the firm chooses $p(\omega) \in \{\frac{b(\omega)}{V_C}, \frac{b(\omega)}{V_E}\}$ to maximize its profit, taking V_C and V_E as given. Note that in the equilibrium the price of good ω depends only on $b(\omega)$. Therefore, hereafter I omit the notation of ω and consider prices as a function of b.

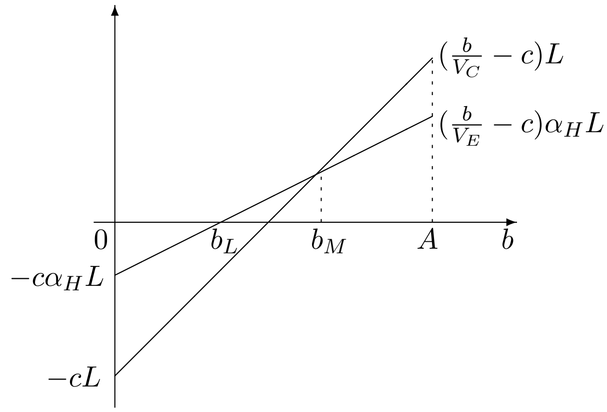
Let b_M be the unique solution of the equation

$$\left(\frac{b}{V_C} - c\right) L = \left(\frac{b}{V_E} - c\right) \alpha_H L. \tag{11}$$

In the equilibrium, the condition $\frac{\alpha_H}{V_E} < \frac{1}{V_C}$ is satisfied (otherwise $\left(\frac{b}{V_E} - c\right) \alpha_H L > \left(\frac{b}{V_C} - c\right) L$ for any $b \geq 0$ and all firms would choose to sell only to high income consumers, which is

[24] Notice that by definition, $V_C \geq V_E$. If $V_C = V_E$ then all available goods have the same valuation to price ratios. In this case, the equilibrium concept implies that the high income consumers buy all goods, while the poor buy only some part (for instance, this part can be randomly determined). This means that expected demand for a certain good is strictly less than L. Thus, firms can increase their profits by slightly decreasing their prices and acquiring greater demand share. Therefore, if $V_C = V_E$, equilibrium does not exist.

Figure 5: Profit Functions



impossible in the equilibrium). This condition guarantees that

$$\begin{aligned} \left(\frac{b}{V_C} - c\right) L &\geq \left(\frac{b}{V_E} - c\right) \alpha_H L, \quad \text{if } b \geq b_M, \\ \left(\frac{b}{V_C} - c\right) L &< \left(\frac{b}{V_E} - c\right) \alpha_H L, \quad \text{otherwise.} \end{aligned}$$

Thus, if a firm draws $b \geq b_M$ then in the equilibrium it sells to both types of consumers, otherwise it sells only to the rich or exits. A firm with valuation b_M of its good is indifferent between selling to all consumers or only to the rich (see *Figure 5*). Thus, even in the presence of market power, products have a natural hierarchy: consumers first buy goods with higher b. Notice that firms with valuations $b < b_L \equiv cV_E$ have to exit, otherwise they would have negative profits.

Let M_e be the mass of firms that enter the market[25]. In the equilibrium, several conditions should be satisfied. First, due to free entry, the expected profits of firms have to be equal to zero. Second, the goods market clears. Since the poor consume only goods from the "common" group, the aggregate cost of the bundle of goods from the "common" group should be equal to the income of a poor consumer. Similarly, the aggregate cost of the bundle of all available goods in the economy should be equal to the income of a rich consumer.

Definition 3 *The equilibrium in the model is defined by the price function $p(b)$ on $b \geq b_L$, the cutoff level b_L, b_M, M_e, and the valuation to price ratios V_C and V_E such that*

[25] One can think of M_e as that there are $M_e g(b)$ different firms with a particular valuation b.

1) The expected profits of firms are equal to zero;

2) The goods market clears.

Now I derive the equations that satisfy conditions mentioned above and prove that the equilibrium in the model always exists and is unique. Let $\pi(b)$ be the variable profit of a firm with valuation b. To find the equilibrium, I rewrite $\pi(b)$ and $p(b)$ as functions of b, b_L, b_M, and exogenous parameters. Recall that firms with valuation b_L have zero profits, i.e., $b_L = cV_E$ or $V_E = \frac{b_L}{c}$. From (11) one can easily find V_C as a function of b_L and b_M. Thus, the following lemma holds.

Lemma 1 *In equilibrium*

$$p(b) = \begin{cases} \frac{b}{V_C} = cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right), & \text{if } b \geq b_M, \\ \frac{b}{V_E} = cb\frac{1}{b_L}, & \text{if } b \in [b_L, b_M), \end{cases}$$

$$\pi(b) = \begin{cases} \left(cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) - c\right)L, & \text{if } b \geq b_M, \\ \left(cb\frac{1}{b_L} - c\right)\alpha_H L, & \text{if } b \in [b_L, b_M). \end{cases}$$

The ex-ante profits of firms are equal to zero in the equilibrium. Using the results from *Lemma 1* and taking into account that firms with $b < b_L$ exit, I obtain

$$\begin{aligned} f_e &= (1-G(b_M))\left(\int_{b_M}^{A}\left(ct\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) - c\right)LdG_1(t)\right) \\ &\quad + (G(b_M) - G(b_L))\left(\int_{b_L}^{b_M}\left(ct\frac{1}{b_L} - c\right)\alpha_H L dG_2(t)\right), \end{aligned}$$

where $G_1(t) = \frac{G(t)}{1-G(b_M)}$ and $G_2(t) = \frac{G(t)}{G(b_M)-G(b_L)}$. This equation can be rewritten as follows:

$$\frac{f_e}{cL} + 1 = \alpha_H H(b_L) + (1-\alpha_H)H(b_M), \tag{12}$$

where $H(x) = G(x) + \frac{\int_x^A tdG(t)}{x}$. The goods market clearing condition implies that

$$\begin{cases} I_L + \frac{\pi}{L} = M_e \int_{b_M}^{A} p(t)dG(t), \\ I_H + \frac{\pi}{L} = M_e \int_{b_L}^{A} p(t)dG(t). \end{cases} \tag{13}$$

At the same time, free entry in the market means that $\pi = 0$. Thus, dividing the second line by the first one and using *Lemma 1*, I obtain

$$\frac{\int_{b_L}^{b_M} tdG(t)}{\int_{b_M}^{A} tdG(t)} = \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right).$$

Hence, given the exogenous parameters I_H, I_L, α_H, f_e, c, L, and the distribution of draws $G(\cdot)$, one can find endogenous b_M and b_L from the system of equations:

$$\begin{cases} \frac{\int_{b_L}^{b_M} tdG(t)}{\int_{b_M}^{A} tdG(t)} = \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right), \\ \frac{f_e}{cL} + 1 = \alpha_H H(b_L) + (1-\alpha_H)H(b_M). \end{cases} \tag{14}$$

The detailed analysis of the existence and uniqueness of the equilibrium and comparative statics is presented in Tarasov (2007).

2.3 Open Economy

In this section, I analyze the open economy extension of the model. I assume that there are $N+1$ symmetric countries, i.e., every country has $N \geq 1$ trade partners. The symmetry assumption leads to the same wage across countries, which is normalized to unity. I assume that markets are segmented. There is an iceberg transportation costs $\tau \geq 1$ of serving a foreign market[26]. The presence of transportation costs implies that there are firms that serve only their domestic market. Thus, given the valuation b of its good, a firm has three options: to exit, to serve only its domestic market, or to serve all existing markets. To simplify the model, I assume that there are no fixed costs of exporting.

2.3.1 Equilibrium

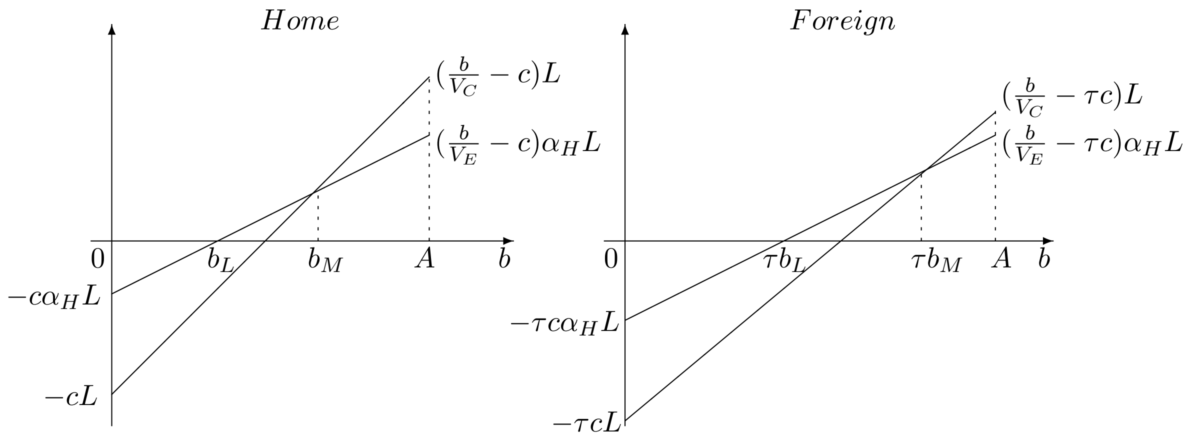
Let $\pi_D(b) \geq 0$ and $\pi_F(b) \geq 0$ be the variable profits of a firm with valuation b from selling at home and at a foreign country, respectively. Then, as there are N trade partners,

$$\pi(b) = \begin{cases} 0, & \text{if the firm exits,} \\ \pi_D(b), & \text{if the firm serves only domestic market,} \\ \pi_D(b) + N\pi_F(b), & \text{if the firm serves all existing markets.} \end{cases}$$

In every country the existing goods are divided into three groups: the "common" group, the "exclusive" group, and the "no one" group. Because the markets are segmented, valuation to price ratios of goods from the same group are the same. Define V_C^i and V_E^i as valuation to price ratios of goods from the "common" and "exclusive" groups in country i, respectively. As all countries are identical, $V_C^i = V_C$ and $V_E^i = V_E$ for all $i = 1..N+1$. From *Figure 6*, one can see

[26] In the analysis below, to avoid the possibility of arbitrage opportunities that can arise due to market segmentation, I assume that τ is sufficiently high.

Figure 6: Profit Functions: Open Economy



that

$$\pi_D(b) = \begin{cases} \left(\frac{b}{V_C} - c\right)L, & \text{if } b \geq b_M, \\ \left(\frac{b}{V_E} - c\right)\alpha_H L, & \text{if } b \in [b_L, b_M), \end{cases}$$

where b_M solves $\frac{b}{V_C} - c = \left(\frac{b}{V_E} - c\right)\alpha_H$ and $b_L = cV_E$. Similarly, since the markets are segmented,

$$\pi_F(b) = \begin{cases} \left(\frac{b}{V_C} - \tau c\right)L, & \text{if } b \geq b^*_M, \\ \left(\frac{b}{V_E} - \tau c\right)\alpha_H L, & \text{if } b \in [b^*_L, b^*_M), \end{cases}$$

where b^*_M solves $\frac{b}{V_C} - \tau c = \left(\frac{b}{V_E} - \tau c\right)\alpha_H$ and $b^*_L = \tau c V_E$. Obviously, $b^*_M = \tau b_M$ and $b^*_L = \tau b_L$. Thus, firms with $b < b_L$ exit, firms with $b \in [b_L, \tau b_L)$ serve only domestic market, and firms with $b \geq \tau b_L$ sell to all existing markets. Due to transportation costs, there are goods in every country, which are available to consumers of type i at home but not available to consumers of the same type abroad. For instance, goods with valuations $b \in [b_M, \tau b_M)$ are sold to everybody at home but exported only to the rich at the foreign countries. Hence, the model provides a clear intuition why some imported goods are available to the rich and not available to the poor.

If transportation costs τ are such that $\tau b_M \geq A$ in the equilibrium then in every country only the rich consume imported goods[27]. Moreover, given sufficiently high transportation costs[28], there is no trade among the countries. Hereafter, I assume that all consumers can afford to buy imported goods, i.e., τ is not so high and $\tau b_M < A$ in the equilibrium.

[27] I will show later that this case ($\tau b_M \geq A$) is possible in general.
[28] τ is such that $\tau b_L > A$.

Let M_e^i be the mass of firms entering the market in country i. Due to the symmetry assumption, $M_e^i = M_e$ for any $i = 1..N+1$. Let $p_D(b)$ and $p_F(b)$ be the prices of goods with valuation b charged to the consumers at home and abroad, respectively. Notice that $p_D(b)$ is not necessarily equal to $p_F(b)$, since the markets are segmented.

Definition 4 *The equilibrium in the model is defined by the price functions $p_D(b)$ and $p_F(b)$ on $b \geq b_L$ and $b \geq \tau b_L$, respectively, the cutoff level b_L, b_M, M_e, and the valuation to price ratios V_C and V_E such that*

1) The expected profits of firms in every country are equal to zero;

2) The goods markets clear in every country.

As in the closed economy case, to find the equilibrium, I rewrite $\pi_D(b)$, $\pi_F(b)$, $p_D(b)$, and $p_F(b)$ as functions of b, b_L, b_M and exogenous parameters. The following lemma holds.

Lemma 2 *In the equilibrium*

$$p_D(b) = \begin{cases} \frac{b}{V_C} = cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right), & \text{if } b \geq b_M, \\ \frac{b}{V_E} = cb\frac{1}{b_L}, & \text{if } b \in [b_L, b_M), \end{cases}$$

$$p_F(b) = \begin{cases} \frac{b}{V_C} = cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right), & \text{if } b \geq \tau b_M, \\ \frac{b}{V_E} = cb\frac{1}{b_L}, & \text{if } b \in [\tau b_L, \tau b_M), \end{cases}$$

$$\pi_D(b) = \begin{cases} \left(cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) - c\right)L, & \text{if } b \geq b_M, \\ \left(cb\frac{1}{b_L} - c\right)\alpha_H L, & \text{if } b \in [b_L, b_M), \end{cases}$$

$$\pi_F(b) = \begin{cases} \left(cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) - \tau c\right)L, & \text{if } b \geq \tau b_M, \\ \left(cb\frac{1}{b_L} - \tau c\right)\alpha_H L, & \text{if } b \in [\tau b_L, \tau b_M). \end{cases}$$

Lemma 2 implies that the prices of goods with relatively high and low valuations[29] are the same at home and abroad, i.e., $p_D(b) = p_F(b)$. Hence, firms that produce these goods receive the same revenues from exporting to a foreign country as from selling at home, but due to the presence of transportation costs the profits are lower[30]. Firms that produce goods with valuation

[29] That is, goods with $b \in [\tau b_L, b_M) \cup [\tau b_M, A]$.

[30] Here the symmetry assumption matters. In general, V_C^i and V_E^i can differ across countries. This in turn may result in the different home and foreign prices of any particular good.

$b \in [b_M, \tau b_M)$ sell to all consumers at home, but only to the rich at the foreign countries. For these goods,

$$\begin{aligned} p_D(b) &= cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right), \\ p_F(b) &= cb\frac{1}{b_L}, \end{aligned}$$

i.e., $p_F(b) > p_D(b)$. However, due to differences in quantities demanded and transportation costs, these firms have higher revenues from selling at home than from exporting to a foreign country.

What about arbitrage opportunities? No arbitrage condition means that it is not profitable for the third party to buy a good in one country and resell it in another one, i.e., $p_D(b)$ should be less or equal than $\tau p_F(b)$ and greater or equal than $\frac{p_F(b)}{\tau}$. In our case, no arbitrage condition holds for goods with $b \in [\tau b_L, b_M) \cup [\tau b_M, A]$ and does not necessarily hold for goods with $b \in [b_M, \tau b_M)$, since for these goods, $p_F(b) > p_D(b)$ for any $\tau > 1$. No arbitrage condition for the goods with $b \in [b_M, \tau b_M)$ is given by $\frac{p_F(b)}{p_D(b)} \leq \tau$. Recall that these goods are sold to all consumers at home, but only to the rich abroad. Given the assumption about market segmentation, $\frac{p_F(b)}{p_D(b)}$ positively depends on the relative income of the rich with respect to the poor $\frac{I_H}{I_L}$ (see details in Tarasov (2007)). Thus, for some values of $\frac{I_H}{I_L}$ and τ, it might be the case that $\frac{p_F(b)}{p_D(b)} > \tau$ implying arbitrage opportunities for the set of goods considered above. In detail,

$$\frac{p_F(b)}{p_D(b)} \leq \tau \iff \tau\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) \geq \frac{1}{b_L} \iff \frac{b_L}{b_M} \geq \frac{1-\alpha_H\tau}{(1-\alpha_H)\,\tau}. \tag{15}$$

Recall that $\frac{b_L}{b_M}$ is always strictly less than one. Thus, if transportation costs are sufficiently low: τ is close to one, then $\frac{b_L}{b_M} < \frac{1-\alpha_H\tau}{(1-\alpha_H)\tau}$. At the same time, $\frac{1-\alpha_H\tau}{(1-\alpha_H)\tau}$ is decreasing in τ and equal to zero at $\tau = \frac{1}{\alpha_H}$, and as I will show later, $\left(\frac{b_L}{b_M}\right)'_\tau > 0$. This implies that there exists τ^* such that for $\tau \geq \tau^*$, inequality (15) holds. In the subsequent analysis, I assume that $\tau \geq \tau^*$ and $\tau^* b_M(\tau^*) < A$[31].

In the next section, I derive equilibrium conditions and show the existence and uniqueness of the equilibrium in the model.

[31] Notice that τ^* is strictly less than $\frac{1}{\alpha_H}$. To give a reader an idea about the possible values of τ^*, I consider the following numerical exercise. I take $\frac{L}{f_e} = c = 1$, $\alpha_H = 0.45$, $\frac{I_H}{I_L} = 3$, $N = 10$, and $G(b) = b$ on $[0, 1]$. As a range for τ, I take an interval $(1, \frac{1}{\alpha_H})$ with step 0.03. That is, $\tau = 1.03$, 1.06 and so on. For any τ from the range and the other parameters, I find the equilibrium values of b_L and b_M. Recall that τ^* can be found from $\frac{b_L(\tau^*)}{b_M(\tau^*)} = \frac{1-\alpha_H\tau^*}{(1-\alpha_H)\tau^*}$. Given the values of the exogenous parameters considered above, $\tau^* \approx 1.24$. Moreover, changes in $\frac{L}{f_e}$, c, and N do not significantly alter τ^*. Thus, the assumption that $\tau \geq \tau^*$ is not so strong.

Equilibrium Conditions The ex-ante profits of firms is equal to zero in the equilibrium. This can be written as follows:

$$f_e = E\left(\pi_D(b)|b \geq b_L\right) + NE\left(\pi_F(b)|b \geq \tau b_L\right).$$

Using the results from *Lemma 2*, I obtain

$$\begin{aligned} f_e &= (1-G(b_M))\left(\int_{b_M}^{A}\left(ct\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)-c\right)LdG_1(t)\right) \\ &\quad +(G(b_M)-G(b_L))\left(\int_{b_L}^{b_M}\left(ct\frac{1}{b_L}-c\right)\alpha_H LdG_2(t)\right) \\ &\quad +N\left(1-G(\tau b_M)\right)\left(\int_{\tau b_M}^{A}\left(ct\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)-\tau c\right)LdG_3(t)\right) \\ &\quad +N\left(G(\tau b_M)-G(\tau b_L)\right)\left(\int_{\tau b_L}^{\tau b_M}\left(ct\frac{1}{b_L}-\tau c\right)\alpha_H LdG_4(t)\right), \end{aligned}$$

where $G_1(t)=\frac{G(t)}{1-G(b_M)}$, $G_2(t)=\frac{G(t)}{G(b_M)-G(b_L)}$, $G_3(t)=\frac{G(t)}{1-G(\tau b_M)}$, and $G_4(t)=\frac{G(t)}{G(\tau b_M)-G(\tau b_L)}$. Simple algebra shows that this equation can be rewritten as follows:

$$\frac{f_e}{cL}+1+N\tau=\alpha_H\left(H(b_L)+N\tau H(\tau b_L)\right)+(1-\alpha_H)\left(H(b_M)+N\tau H(\tau b_M)\right),$$

where $H(x)=G(x)+\frac{\int_x^A tdG(t)}{x}$.

The goods market clearing condition implies that

$$\begin{cases} I_L+\frac{\pi}{L}=M_e\left(\int_{b_M}^{A}p_D(t)dG(t)+N\int_{\tau b_M}^{A}p_F(t)dG(t)\right), \\ I_H+\frac{\pi}{L}=M_e\left(\int_{b_L}^{A}p_D(t)dG(t)+N\int_{\tau b_L}^{A}p_F(t)dG(t)\right). \end{cases} \tag{16}$$

Recall that free entry in the market means that $\pi=0$. Thus, dividing the second line by the first one and using *Lemma 2*, I obtain

$$\frac{\int_{b_L}^{b_M}tdG(t)+N\int_{\tau b_L}^{\tau b_M}tdG(t)}{\int_{b_M}^{A}tdG(t)+N\int_{\tau b_M}^{A}tdG(t)}=\left(\frac{I_H}{I_L}-1\right)\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right).$$

Hence, given the exogenous parameters I_H, I_L, α_H, f_e, c, L, τ, N, and the distribution of draws $G(\cdot)$, one can find endogenous b_M and b_L from the following system of equations:

$$\begin{cases} \frac{\int_{b_L}^{b_M}tdG(t)+N\int_{\tau b_L}^{\tau b_M}tdG(t)}{\int_{b_M}^{A}tdG(t)+N\int_{\tau b_M}^{A}tdG(t)}=\left(\frac{I_H}{I_L}-1\right)\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right), \\ \frac{f_e}{cL}+1+N\tau=\alpha_H\left(H(b_L)+N\tau H(\tau b_L)\right)+(1-\alpha_H)\left(H(b_M)+N\tau H(\tau b_M)\right). \end{cases} \tag{17}$$

Using the same technique as in Tarasov (2007), it is possible to prove the existence and uniqueness of the equilibrium. Once b_M and b_L are found, V_C and V_E can be derived from *Lemma 2.* Finally, the mass of firms entering the market, M_e, can be found from (16).

Before describing the comparative statics of the model, I look at consumer welfare.

2.3.2 Welfare

Welfare of a poor consumer is equal to $M_e\left(\int_{b_M}^{A} tdG(t) + N\int_{\tau b_M}^{A} tdG(t)\right)$. At the same time, from (16) $M_e = \frac{I_L}{\int_{b_M}^{A} p_D(t)dG(t)+N\int_{\tau b_M}^{A} p_F(t)dG(t)}$. This implies that

$$W_p = I_L V_C.$$

Welfare of a poor consumer naturally rises with an increase in either her income or the valuation to price ratio of goods she consumes. Similarly, welfare of a rich consumer is given by

$$W_r = I_L V_C + (I_H - I_L)V_E.$$

As the rich consume the same bundle of goods as the poor plus some others, welfare of the rich is equal to welfare of the poor plus additional welfare from consumption of goods from the "exclusive" group, which is in turn equal to income spent on these goods multiplied by their valuation to price ratio. Aggregate welfare per capita, W_a, is equal to $(1-\alpha_H)W_p + \alpha_H W_r$, which is equivalent to

$$W_a = I_L V_C + \alpha_H (I_H - I_L)V_E.$$

Finally, relative welfare of the rich with respect to the poor is given by

$$\frac{W_r}{W_p} = 1 + \left(\frac{I_H}{I_L} - 1\right)\frac{V_E}{V_C} = 1 + \frac{I_H - I_L}{I_L}\left(\alpha_H + (1-\alpha_H)\frac{b_L}{b_M}\right).$$

Given changes in τ or N, $\frac{W_r}{W_p}$ moves in the same direction as $\frac{b_L}{b_M}$ does.

Notice that all changes in individual welfare can be decomposed into two components: an income effect and a price effect. The price effect is determined by changes in V_C and V_E, which depend on the level of competition within the groups of goods. The income effect is determined by changes in exogenous I_C and I_R.

In the next section, I investigate the effects of the changes in the level of globalization, which are caused by changes in τ and N, on the market structure, trade patterns, and individual welfare in a country.

2.3.3 Comparative Statics

One of the main goals of this section is to analyze the relationship between globalization and relative welfare of the rich with respect to the poor $\frac{W_r}{W_p}$. Recall that any changes in τ and N

affect $\frac{W_r}{W_p}$ through $\frac{b_L}{b_M}$. Hence, to examine the impact of changes in τ and N on $\frac{W_r}{W_p}$, it is sufficient to determine the signs of $\left(\frac{b_L}{b_M}\right)'_\tau$ and $\left(\frac{b_L}{b_M}\right)'_N$. In general, the signs of $\left(\frac{b_L}{b_M}\right)'_\tau$ and $\left(\frac{b_L}{b_M}\right)'_N$ depend on the exogenous parameters of the model and can go in either direction. To avoid the ambiguity in the signs, I limit the analysis of the comparative statics to the case when the distribution of draws $G(b)$ is such that $b^2g(b)$ is increasing and convex in b[32]. The last condition has a strong economic interpretation. It implies that $g(b)$ does not decrease too fast, i.e., the probability of getting higher values of b does not decrease too fast with b. Moreover, the aggregate utility from the consumption of goods with a particular valuation b is equal to $M_e g(b)b$. Thus, the condition also guarantees that the aggregate utility from the consumption of more valuable goods does not decrease too fast. The convexity of $b^2g(b)$ is rather a technical condition: it helps to simplify some proofs.

To better understand the intuition, which is behind the changes in τ and N, I separately consider two submarkets: the submarket for goods from the "common" group and the submarket for goods from the "exclusive" group. Each of these submarkets is characterized by its equilibrium valuation to price ratio: V_C and V_E. Since the poor consume only goods from the "common" group, changes in τ and N affect their welfare through changes in V_C, whereas welfare of the rich is affected by changes in both V_C and V_E, as the rich consume all available goods.

One can think of V_C and V_E as some measures of the level of competition in the submarkets. The higher the level of competition is in the submarket for "common" goods or for "exclusive" goods, the higher is V_C or V_E. Changes in τ and N induce some firms to exit and some firms to enter a certain submarket. If firms exit then the level of competition decreases, while if firms enter the level of competition rises. Thus, the impact of changes in τ and N can be divided into two effects: "exiting" and "entering" effects. In the next subsections, the intuition is built on these two effects.

Changes in Transportation Costs First, I discuss the consequences of changes in transportation costs. Consider the impact of higher transportation costs τ on the submarket for goods from the "common" group. Higher transportation costs lead to exit of some importing firms from the submarket (the "exiting" effect). As a result, τb_M increases. Due to lower competition,

[32]For instance, the set of power distributions with $G(b) = \left(\frac{b}{A}\right)^k$, $k > 0$, satisfies this assumption.

firms that stay increase their prices. Some domestic firms that used to sell their goods only to rich consumers now find it more profitable to sell to all consumers (the "entering" effect). This leads to a decrease in b_M. In the short run, the "exiting" effect dominates the "entering" one[33]. To analyze the long run effect, one needs to take into account changes in M_e. On the one hand, an increase in τ reduces the profits from exporting. On the other hand, an increase in τ can raise the profits from selling domestically due to lower competition. The overall effect on the expected profits and, therefore, on M_e is ambiguous. I show that, whatever changes in M_e are, τb_M rises, b_M falls, and the "exiting" effect still dominates the "entering" effect in the long run. Thus, an increase in τ results in lower level of competition in the submarket for goods from the "common" group.

A quite different situation is observed in the submarket for goods from the "exclusive" group. Again, higher transportation costs lead to the exit of some importers from the submarket: τb_L increases. Moreover, some domestic firms move to the submarket for the goods from the "common" group. These two effects result in lower competition. At the same time, higher transportation costs imply that some importing firms that sold their goods to all consumers now find it more profitable to sell only to the rich. These firms in turn induce tougher competition. In this case, it is unclear which effect ("exiting" or "entering") is stronger. Thus, the impact of an increase in τ on b_L and, hence, on the price level in the submarket might be ambiguous both in short and long run cases. The following lemma summarizes the findings above.

Lemma 3 *In the long run, higher transportation costs raise τb_L and τb_M, decrease b_M, and have an ambiguous impact on b_L.*

Proof. The proof is in Part 1 of Appendix B. ■

Recall that $\frac{W_r}{W_p} = 1 + \left(\frac{I_H}{I_L} - 1\right)\frac{V_E}{V_C}$. Since the "exiting" effect is stronger than the "entering" effect in the submarket for "common" goods, the prices in the submarket rise, reducing V_C. At the same time, the impact of an increase in τ on b_L and, thereby, on V_E is ambiguous[34]. In the Appendix, I show that depending on the exogenous parameters, V_E can either increase or decrease. I prove that an increase in τ raises the ratio $\frac{V_E}{V_C}$, i.e., the rich lose less from higher

[33] By the short run, I mean the case when the number of entrants M_e is fixed and not affected by the changes in transportation costs, while in the long run, M_e changes.

[34] Recall that $V_E = \frac{b_L}{c}$.

transportation costs than the poor (or the poor gain more from trade liberalization than the rich)[35].

Remember that I consider the case when $\tau b_M < A$, that is, transportation costs are not significantly high. However, if $\tau b_M \geq A$ in the equilibrium then the rich gain more than the poor from trade liberalization. The intuition is quite straightforward. If $\tau b_M \geq A$ then the poor do not consume imported goods at all. Therefore, trade liberalization does not have a direct impact on their welfare. The following proposition summarizes the above results.

Proposition 2 *If transportation costs are low enough (high enough) then the poor gain more (less) from trade liberalization than the rich.*

Proof. The proof is in Part 1 and 2 of Appendix B. ■

Thus, relative welfare of the rich with respect to the poor has an inverted U shape as a function of transportation costs.

Changes in the Number of Trading Partners In this section, I turn to analyzing the effects of changes in the number of trading partners. An increase in the number of trading partners N leads to higher competition in both submarkets. As a result, some domestic and

[35] Since the "exiting" effect dominates the "entering" effect in the submarket for goods from the "common" group, welfare of the poor falls. As the rich consume goods from both groups and the impact of higher transportation cost on the submarket for "exclusive" goods is ambiguous, the impact of an increase in τ on welfare of the rich is ambiguous: the rich may gain from an increase in τ. Thus, trade liberalization results in higher welfare of the poor and may result in either higher or lower welfare of the rich. I also show that trade liberalization positively affects aggregate welfare per capita in the economy.

In what cases does welfare of the rich decrease or increase? In Part 1 of Appendix B, I demonstrate that if the fraction of the rich is high enough or the income of the rich is close to that of the poor then given an increase in τ, b_L is likely to fall and welfare of the rich decreases. While if the fraction of the rich is close to zero and the income difference between the rich and the poor is high enough, then b_L is more likely to rise and welfare of the rich may increase. The intuition is as follows. Recall that $\frac{V_E}{V_C} = \alpha_H + \frac{b_L(1-\alpha_H)}{b_M}$. Thus, if α_H or $\frac{I_H}{I_L}$ is close to one, then $\frac{V_E}{V_C}$ is close to one. That is, there is no much difference between selling only to the rich and selling to everybody. In this case, the "exiting" effect dominates the "entering" effect and welfare of the rich decreases. While if α_H is close to zero and $\frac{I_H}{I_L}$ is high enough, then the difference between selling only to the rich and selling to everybody is significant and the "entering" effect prevails the "exiting" one. Welfare of the rich may increase. See the proofs in Part A of the Appendix.

In traditional literature, for instance, in Melitz (2003), bilateral trade liberalization is beneficial for all consumers. In this paper, I show that it is not necessarily the case. While the whole economy benefits from lower transportation costs, the rich consumers may be worse off from trade liberalization. Moreover, lower transportation costs have different impacts on firm mark-ups. After bilateral trade liberalization mark-ups of some firms may rise, while in traditional literature mark-ups of all firms stay the same or fall.

importing firms leave the submarkets, and in the short run, b_L and b_M rise. Firms that leave the submarket for the "common" goods enter the submarket for the "exclusive" goods. This additional "entering" effect induces even higher competition in this submarket raising b_L. Thus, one can expect that in the short run, b_L increases by more than b_M, that is, $\frac{b_L}{b_M}$ rises. In the long run, the impact of an increase in N on b_L and b_M also depends on changes in M_e: lower entry causes b_L and b_M to fall. On the one hand, higher N implies that firms import their goods to more markets than before; this raises the profits of importing firms and results in higher M_e. On the other hand, higher N intensifies competition in each market and, consequently, reduces the profits. This in turn results in lower entry rate. A number of simulations I conduct for a wide range of parameters shows that M_e falls, while NM_e rises[36]. I demonstrate that in the long run, b_L keeps increasing, while b_M may either rise or fall. A possible fall in b_M is caused by a decrease in M_e. Finally, $\frac{b_L}{b_M}$ also rises in the long run. The following lemma summarizes the results above.

Lemma 4 *In the long run, a higher number of trading partners raises b_L and $\frac{b_L}{b_M}$, and has an ambiguous impact on b_M.*

Proof. The proof is in Part 1 of Appendix B. ■

Remember that $\frac{W_r}{W_p} = 1 + \frac{I_H - I_L}{I_L}\left(\alpha_H + (1-\alpha_H)\frac{b_L}{b_M}\right)$. Given an increase in N, $\frac{b_L}{b_M}$ and, thereby, $\frac{W_r}{W_p}$ increase. Thus, if $\tau b_M < A$ in the equilibrium then the rich gain more from a rise in the number of trading partners than the poor. The main reason for this result is that firms that leave the submarket for the "common" goods enter the submarket for the "exclusive" goods inducing tougher competition and lower prices. If transportation costs are high enough and $\tau b_M \geq A$ in the equilibrium then again, the rich gain more than the poor. The intuition is exactly the same as in the case with an increase in τ. The following proposition formulates these findings.

Proposition 3 *For any transportation costs the rich gain more than the poor from an increase in the number of trading partners.*

Proof. The part of the proof follows from *Lemma 4* and the rest of the proof is in Part 2 of Appendix B. ■

[36] It is rather complicated to show analytically that M_e falls and NM_e rises.

2.4 Conclusion

In the present paper, I construct a general equilibrium trade model of monopolistic competition with heterogenous firms and consumers, which establishes a link between different components of globalization and welfare inequality through the consumption channel. The model is based on two key assumptions: imperfect competition and non-homothetic preferences. Given these assumptions, the model allows to analyze the impact of globalization on mark-ups charged on different goods and on the welfare of different groups of consumers. The main analysis is focused on the case with two types of consumers: rich and poor.

I argue that the impact of globalization on relative welfare of the rich with respect to the poor depends on the component of globalization considered. Changes in relative welfare of the rich are determined by changes in the relative prices of "exclusive" goods (consumed only by the rich) with respect to "common" goods (consumed by everybody). These relative prices in turn depend on the level of competition inside the submarkets for "exclusive" and "common" goods. Due to non-homothetic preferences, greater globalization affects the submarkets differently, therefore, changing relative welfare. I show that while the rich always gain more from a rise in the number of trading partners than the poor, the impact of trade liberalization on relative welfare depends on transportation costs. If transportation costs are low enough then the poor compared to the rich gain more out of trade liberalization; otherwise, the opposite is true. I also show that depending on the exogenous parameters of the model, the rich may be even worse off from trade liberalization, while welfare of the poor and the aggregate welfare rise.

I would like to mention two plausible ways, which allow extending the present model. First, it would be interesting to consider a similar model of trade between two countries with different income distributions and examine the way this difference affects the distribution of gains from greater globalization across the countries. Second, it might be sensible to develop a model where income distribution is endogenous and, for instance, influenced by the level of openness. These issues are left for future research.

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Chapter 3. Per Capita Income, Market Access Costs, and Trade Volumes

3.1 Introduction

What do we learn from trade data? First, richer countries (with higher per capita income) have greater trade volumes even after controlling for aggregate income. For instance, Hummels and Klenow (2002) show that conditional on total income, richer countries export and import more than poorer countries do. Second, there are many zeros in bilateral trade flows in the data. A considerable number of country pairs in the world do not trade with each other in one or both directions. In 1995, 14 percent of bilateral trade flows among the hundred largest countries in terms of GDP are zero trade flows. Third, there are significant fixed costs of trade, which play an important role in determining trade volumes.[37] For example, Anderson and Wincoop (2004) argue that the presence of fixed costs can explain a number of zeros in bilateral trade flows. Meanwhile, no existing quantitative general equilibrium model of trade captures these features of the data simultaneously.

In this paper, I construct and estimate a general equilibrium model of trade with many countries built on Melitz (2003) that squares all the data features discussed above. There are two novelties in the paper. First, in my model I introduce an association between the costs of access to foreign markets (fixed costs of trade) and countries' development levels. This helps to explain the effect of per capita income on trade volumes and the many zeros in bilateral trade data. Second, I develop an estimation procedure, which allows me to identify separate effects of variable and fixed costs of trade on trade volumes. I use *restricted* non-linear least squares to estimate the model. Specifically, I minimize the difference between the actual and the simulated bilateral trade flows subject to the constraint that the number of zeros generated by the model is the same as that in the data.

In the model, each country is characterized by its population size and development level. As in Melitz (2003), there is an unbounded set of potential entrants into an industry in each country.[38] To enter the industry, ex ante identical firms have to make an initial investment

[37] Eaton, Kortum and Kramarz (2008) argue that to explain firm-level trade patterns, we need both variable and fixed costs of trade. Das, Roberts and Tybout (2007) estimate fixed costs of trade for Colombian exporters using a dynamic partial equilibrium model of trade. Their estimates lie between \$300.000 and \$500.000.

[38] Since the analysis of the multi-industry specification of the model is beyond the scope of this paper, I assume

associated with the creation of a new variety. Ex post, firms vary according to their productivity, which is defined as the product of a firm-specific productivity and the country development level. Exporting firms face variable and fixed costs of trade. As in Helpman, Melitz and Rubinstein (2008), the model is able to predict exports zeros observed in the data. It is possible that there are no firms in country i that are productive enough to find it profitable to export to country j. This results in zero exports from i to j.

I assume that fixed costs of trade may depend on countries' development levels. Depending on parameters of the model, firms in more developed countries may face either lower or higher costs of access to foreign markets. In the paper, I show that all else equal, countries with lower fixed costs of exporting *relative* to the other costs (the costs of entry into the industry and fixed costs of selling domestically) tend to have greater trade volumes in equilibrium. What follows from this finding is that if more developed and, therefore, richer countries have lower relative costs of access to foreign markets, then other things equal, they export and import more.[39]

The association between fixed costs of trade and country development level is a natural way to explain the phenomenon that richer countries trade more even after controlling for aggregate income. For instance, exporting firms may be required to meet certain product standards and quality requirements imposed by a destination country. The international management literature emphasizes that one of the key reasons for obtaining quality management certification (ISO 9000) is the requirements of international customers. Weston (1995) and Anderson et al. (1999) report that export considerations are one of the primary motivations for American exporting firms to seek quality management certification. Verhoogen (2008) finds that exporters in Mexico are more likely to have ISO 9000 certification than nonexporters. Potoski and Prakash (2009) find that ISO 9000 certification levels are associated with greater bilateral exports. They argue that ISO certification is a signal about the quality of a product, which is especially important for developing and less developed countries, as consumers often relate the quality of products to their countries of origin. Meanwhile, the process of certification is costly. It includes both the costs of development and implementation of new production processes satisfying the standards and the

that production in each country is represented by some average industry.

[39] In the model, per capita income is an endogenous variable. It can be shown that more developed countries have higher per capita incomes in equilibrium.

costs of certification itself (e.g. the costs of application and documentation review, registrar's visits, etc.). Mersha (1997) documents that achieving the quality management certification is especially complicated in less developed countries (in particular, he considers the countries of Sub-Saharan Africa). This is because of a shortage of skilled workers, poor infrastructure, low level of technology, and other factors. This suggests that less developed countries face relatively high costs of access to foreign markets compared, for instance, with costs of selling domestically.

To understand the gains from the imposed relationship between fixed costs of exporting and development level, I estimate the key parameters of the model using the data for 1995 on bilateral trade flows of the 100 largest countries in terms of total income. In particular, I minimize the sum of squared differences between the actual bilateral trade flows and those generated by the model subject to the constraint that the number of zero bilateral trade flows predicted by the model is the same as that in the data. This estimation procedure has several advantages. First, it allows to estimate the effects of both variable and fixed costs of trade on trade volumes. If we drop the constraint on the zeros, the effects of variable and fixed costs are not separately identifiable from the bilateral trade data. Second, the procedure accounts for the general equilibrium features of the model including the effects of free entry into the industry. Finally, it assumes away endogeneity issues that arise in the case of reduced form estimation.

The estimated parameters reveal a strong correlation between countries' development levels and fixed costs of exporting. More developed countries tend to have lower fixed costs of exporting relative to the other costs. I find that the elasticities of trade with respect to aggregate and per capita incomes generated by the model are close to those in the data. In the data, doubling a country's income per capita (keeping the total income unaltered) on average leads to a 19% increase in trade.[40] The model predicts an increase in trade of 22%. The estimation procedure I use implies that the number of zero bilateral trade flows predicted by the model is the same as that in the data. However, mismatch is possible. The model can predict some zeros that are not actually in the data and vice versa. I find that the estimated association between countries' development levels and fixed costs of trade helps to explain many zero trade flows in the data. In particular, it significantly reduces the mismatch between the zeros predicted by the model and those in the data compared to the benchmark model when fixed costs of trade do not depend

[40] By trade, I mean the average between exports and imports.

on the development level. The model also allows us to determine the magnitude of fixed costs of trade. I find that the aggregate spending on access to foreign markets constitutes on average around the half of the total export profits. This result is very similar to that in Eaton, Kortum and Kramarz (2008), who find that "fixed costs eat up a little more than half of gross profits". In particular, in my model richer countries have the lower share of fixed costs of exporting in the total export profits, while countries with larger population have the higher share.

This paper contributes to two strands of literature: the literature that analyzes the role of fixed costs of trade in explaining trade volumes and patterns and the literature on quantitative general equilibrium models of trade. The analysis of the role of fixed costs was first developed in Melitz (2003), who considered a general equilibrium model of trade between symmetric countries.[41] Building on the Melitz model, Helpman, Melitz and Rubinstein (2008) develop a two-stage reduced form estimation procedure that uses the information contained in zero bilateral trade flows to improve on the gravity regression. While there is a clear link between my paper and Helpman, Melitz and Rubinstein (2008) (both papers estimate the gravity regression using the Melitz type framework), my paper addresses different questions and focuses on the implications of a general equilibrium model of trade applying structural estimation, which is especially important for counterfactual analysis.

This paper is not the first one, which establishes the relationship between country's trade costs and per capita income. Waugh (2009) considers a general equilibrium model of trade developed in Eaton and Kortum (2002). Estimating the model, he assumes that variable trade costs are a function of symmetric relationships (e.g. distance, etc.) and an exporter fixed effect. He finds a negative correlation between exporter per capita income and the fixed effect, implying that poor countries face higher variable trade costs than rich countries. There are two points worth noting about the linkage of the present paper to the work of Waugh. First, while his finding is able to explain the relationship between per capita income and trade volumes, it does not explain why we observe a substantial amount of trade zeros in the data. In his model, zero trade in either direction between two countries occurs only if variable trade costs are infinitely high. Second, in the present paper the association between country's *variable costs of trade* and

[41] Chaney (2008) considers an extension of Melitz (2003) with asymmetric countries to analyze the impact of the elasticity of substitution on the intensive and extensive margins of trade.

development level can also account for greater trade volumes of richer countries and zero trade flows in the data. However, by applying the same estimation procedure I find that in this case the model performs much worse in matching the data. For instance, the mismatch between simulated zeros and zeros in the data is much greater.

The remainder of the paper is organized as follows. Section 2 introduces the basic concepts of the model and describes the equilibrium. In Section 3, I present the main theoretical findings of the model and derive their implications for trade patterns. In Section 4, I estimate the model and compare the quantitative implications with the data. Section 5 conducts counterfactual analysis. In Section 6, I examine the quantitative implications of alternative specifications of the model. Section 7 concludes.

3.2 Theory

I consider a variation of the Melitz model extended to a world with N asymmetric countries. Each country is characterized by its population size and development level. The only factor of production is labor, which is inelastically supplied by agents endowed with one unit of labor each. There is a continuum of monopolistically competitive heterogenous firms producing different varieties of a differentiated good. Without loss of generality, I assume that agents own equal shares of all firms.[42] Hence, consumers in country j have identical incomes (which can vary across countries) consisting of labor income w_j and the share of firms' profits π_j.

3.2.1 Consumption

I assume that consumers have identical homothetic preferences that take the constant elasticity of substitution (CES) form. In particular, a representative consumer in country j maximizes

$$Q_j = \left(\int_{\omega \in \Omega_j} q_j^{\frac{\sigma-1}{\sigma}}(\omega) d\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{18}$$

subject to

$$\int_{\omega \in \Omega_j} p_j(\omega) q_j(\omega) d\omega = (w_j + \pi_j) L_j, \tag{19}$$

[42] The more general assumption is that each agent owns a balanced portfolio of shares of all firms. However, due to free entry, the total profits of all firms are equal to zero in the equilibrium. This implies that the value of any balanced portfolio is equal to zero. Therefore, to simplify the notation, I assume that agents have equal shares of all firms.

where Ω_j is the set of available varieties in country j, $q_j(\omega)$ is quantity consumed, $p_j(\omega)$ is the price of variety ω in country j, L_j is the population size of country j, and $\sigma > 1$ is the elasticity of substitution between varieties. This maximization problem yields that

$$q_j(\omega) = \left(\frac{p_j(\omega)}{P_j}\right)^{-\sigma} \frac{(w_j + \pi_j)\, L_j}{P_j},$$

where $P_j = \left(\int_{\omega \in \Omega_j} p_j^{1-\sigma}(\omega) d\omega\right)^{\frac{1}{1-\sigma}}$ is the CES price index: i.e., $P_j Q_j = (w_j + \pi_j)\, L_j$.

3.2.2 Production

Production in each country is represented by an average industry and there is free entry into the industry. To enter the industry in country i, ex-ante identical firms have to make sunk investments f_{ei} associated with the creation of a new variety. Once a firm incurs the costs of entry, it obtains a draw θ of its firm-specific productivity from a distribution $G(\theta)$ on $[\theta_L, \theta_H]$. This distribution is common for all firms in all countries. Ex post, firms vary by their productivities, which are the product of a firm-specific productivity θ and the country development level Z_i. Hence, both population size L_i and development level Z_i could affect equilibrium outcomes for country i.

The price of variety ω sold in country j, $p_j(\omega)$, is determined by the productivity of the firm producing this variety, its country of origin, and the destination market. Therefore, hereafter I omit the notation of ω and use $p_{ij}(\theta)$ instead of $p_j(\omega)$. We define $\pi_{ij}(\theta)$ as the variable profits from exporting to country j of the firm, which produces in country i with firm-specific productivity θ. Then,

$$\pi_{ij}(\theta) = \left(p_{ij}(\theta) - \frac{w_i}{Z_i}\frac{\tau_{ij}}{\theta}\right)\left(\frac{p_{ij}(\theta)}{P_j}\right)^{-\sigma} \frac{(w_j + \pi_j)\, L_j}{P_j}, \tag{20}$$

where $p_{ij}(\theta)$ solves the following maximization problem:

$$\max_{p \geq 0} \left(p - \frac{w_i}{Z_i}\frac{\tau_{ij}}{\theta}\right) p^{-\sigma}. \tag{21}$$

Here τ_{ij} stands for variable trade costs between country i and j, which take Samuelson's iceberg form. I set τ_{ii} to unity and assume that the triangle inequality holds for any τ_{ij}: i.e., $\tau_{ij} \leq \tau_{ik}\tau_{kj}$ for any i, j, and k.[43]

[43] The triangle inequality guarantees that it is cheaper to deliver goods from country i directly to country j, rather than to use another country as an intermediary.

The pricing rule maximizing (21) is as follows:

$$p_{ij}(\theta) = \frac{w_i}{Z_i}\frac{\tau_{ij}}{\theta}\frac{\sigma}{(\sigma-1)}. \tag{22}$$

Consequently, the variable profit $\pi_{ij}(\theta)$ is given by

$$\pi_{ij}(\theta) = C\left(\frac{Z_i}{w_i\tau_{ij}}\right)^{\sigma-1}\frac{(w_j+\pi_j)L_j}{P_j^{1-\sigma}}\theta^{\sigma-1}, \tag{23}$$

where $C = \frac{1}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}$.

To export from country i to country j, firms have to pay fixed costs f_{ij} representing the costs of serving market j.[44] The presence of fixed costs implies that not all firms find it profitable to export or sell at home. Firms with relatively low productivities exit because of negative potential profits. In particular, firms located in country i with $\theta < \theta_{ij}$ decide not to export to country j, where the cutoff θ_{ij} is determined by

$$\pi_{ij}(\theta_{ij}) = f_{ij}.$$

The last expression implies that the cutoff θ_{ij} is given by

$$\theta_{ij} = \frac{w_i}{Z_i}\tau_{ij}\left(\frac{1}{C}\frac{P_j^{1-\sigma}}{(w_j+\pi_j)L_j}\right)^{\frac{1}{\sigma-1}} f_{ij}^{\frac{1}{\sigma-1}}. \tag{24}$$

A higher θ_{ij} means that fewer firms based in country i find it profitable to export to country j. In particular, if $\theta_{ij} > \theta_H$, then no firm exports from country i to country j resulting in zero exports from i to j.

We define $r_{ij}(\theta)$ as the revenues received from exporting to country j by a firm with θ located in country i. Then,

$$r_{ij}(\theta) = \sigma C\left(\frac{Z_i}{w_i\tau_{ij}}\right)^{\sigma-1}\frac{(w_j+\pi_j)L_j}{P_j^{1-\sigma}}\theta^{\sigma-1}. \tag{25}$$

As a result, the total value of exports from country i to country j, X_{ij}, is given by

$$X_{ij} = M_{ei}\int_{\theta_{ij}}^{\theta_H} r_{ij}(\theta)dG(\theta), \tag{26}$$

where M_{ei} is the mass of entrants into the industry.[45] Since there are $M_{ei}dG(\theta)$ firms with productivity θ in country i, the measure of available varieties in country j is equal to:

$$\mu(\Omega_j) = \textstyle\sum_{i=1}^{N} M_{ei}\left(1 - G(\theta_{ij})\right).$$

[44] The fixed costs of selling at home are f_{ii}.

[45] Note that the mass of firms based in country i and serving market j is equal to $M_{ij} = M_{ei}(1 - G(\theta_{ij}))$. In

3.2.3 Market Access Costs and Costs of Entry

I assume that the fixed costs of serving any market are subdivided into two parts: costs directly associated with serving a market (for instance, the construction of facilities) and costs associated with access to the market (for instance, satisfying product standards and quality requirements of the destination country). It is assumed that domestic firms pay only the former, while foreign firms pay both. Hence, the functional form for the fixed costs of exporting is as follows:

$$f_{ij} = \begin{cases} w_i f_d + w_i \frac{f_x}{Z_i^\delta}, & \text{if } j \neq i, \\ w_i f_d, & \text{otherwise,} \end{cases} \tag{27}$$

where f_d and f_x are common for all countries. The parameter δ describes how the country development level Z_i affects the fixed costs of exporting. If δ is greater (less) than zero, then more developed countries use fewer (more) units of labor to access foreign markets.

This way of representing fixed costs of trade is one of key points in the paper. In Melitz (2003), countries are symmetric, so fixed costs are the same for all countries. Chaney (2008) considers the Melitz framework with many asymmetric countries. However, he does not impose any particular relationship or structure on fixed costs of trade. In the present paper, it is assumed that fixed costs of exporting depend only on exporter characteristics. Meanwhile, Arkolakis (2008) and Eaton, Kortum and Kramarz (2008) argue that the costs of access to foreign markets depend on the characteristics of the destination market as well. For instance, Arkolakis (2008) relates the fixed costs of exporting to product advertising requiring labor services from both source and destination countries. In Section 6, I consider the alternative specifications of the model including the dependence of fixed costs on the importer development level.

Finally, I assume that the costs of entry into the industry are given by

$$f_{ei} = w_i f_e \text{ for all } i, \tag{28}$$

where f_e is common for all countries.

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this manner, the expression (26) can be rewritten as

$$X_{ij} = M_{ij} \int_{\theta_{ij}}^{\theta_H} r_{ij}(\theta) d \frac{G(\theta)}{1 - G(\theta_{ij})},$$

where $\frac{G(\theta)}{1-G(\theta_{ij})}$ is the distribution of firm-specific productivities conditional on $\theta \geq \theta_{ij}$.

3.2.4 Equilibrium

Given the set of parameters $\{\, f_d,\, f_x,\, \delta,\, f_e,\, \tau_{ij},\, \sigma,\, G(\cdot),\, Z_i,\, L_i\}_{i,j=1..N}$, the equilibrium in the model is defined by $\{p_{ij}(\theta),\, P_i,\, M_{ei},\, \theta_{ij},\, w_i\}_{i,j=1..N}$ such that

1) $\{p_{ij}(\theta)\}_{i,j=1..N}$ are determined by the firm maximization problem (see (22)).

2) $\{P_i\}_{i=1..N}$ satisfy the following equation:

$$P_i = \left(\int_{\omega \in \Omega_i} p_i^{1-\sigma}(\omega) d\omega \right)^{\frac{1}{1-\sigma}},$$

which is equivalent to

$$P_i^{1-\sigma} = \textstyle\sum_{j=1}^{N} M_{ej} \displaystyle\int_{\theta_{ji}}^{\theta_H} p_{ji}^{1-\sigma}(\theta) dG(\theta).$$

3) Expected profits of a given firm are equal to zero, meaning that

$$f_{ei} = \sum_{j=1}^{N} \Pr\left(\theta \geq \theta_{ij}\right) E\left(\left(\pi_{ij}(\theta) - f_{ij}\right) | \theta \geq \theta_{ij}\right).$$

4) $\{\theta_{ij}\}_{i,j=1..N}$ satisfy the zero profit condition (see (24)).

5) Trade is balanced, implying that

$$\sum_{j=1}^{N} M_{ei} \int_{\theta_{ij}}^{\theta_H} r_{ij}(\theta) dG(\theta) = \sum_{j=1}^{N} M_{ej} \int_{\theta_{ji}}^{\theta_H} r_{ji}(\theta) dG(\theta).$$

Note that the set $\{w_i,\, P_i,\, M_{ei}\}_{i=1..N}$ is sufficient to determine all other endogenous variables in the model such as $p_{ij}(\theta)$, $\pi_{ij}(\theta)$, $r_{ij}(\theta)$, and θ_{ij}. This implies that to find the equilibrium in the model, we need to find the set $\{w_i,\, P_i,\, M_{ei}\}_{i=1..N}$, which satisfies the following system of equations:

$$\begin{cases} P_i^{1-\sigma} = \sum_{j=1}^{N} M_{ej} \int_{\theta_{ji}}^{\theta_H} p_{ji}^{1-\sigma}(\theta) dG(\theta), \\ f_{ei} = \sum_{j=1}^{N} \Pr\left(\theta \geq \theta_{ij}\right) E\left(\left(\pi_{ij}(\theta) - f_{ij}\right) | \theta \geq \theta_{ij}\right), \\ \sum_{j=1}^{N} M_{ei} \int_{\theta_{ij}}^{\theta_H} r_{ij}(\theta) dG(\theta) = \sum_{j=1}^{N} M_{ej} \int_{\theta_{ji}}^{\theta_H} r_{ji}(\theta) dG(\theta), \end{cases} \tag{29}$$

where $p_{ij}(\theta)$, $\pi_{ij}(\theta)$, $r_{ij}(\theta)$, and θ_{ij} are expressed in terms of $\{w_i,\, P_i,\, M_{ei}\}_{i=1..N}$ and the parameters of the model. Thus, we have the system of $3N$ equations with $3N$ unknowns, $\{w_i,\, P_i,\, M_{ei}\}_{i=1..N}$. Consequently, taking w_N as numeraire, we can solve the system and find the endogenous variables for any given set of the parameters of the model.

3.3 Per Capita Income and Trade Volumes

In the equilibrium, the total income of country i is given by $w_i L_i$, where w_i is a function of both Z_i and L_i (and the other parameters of the model).[46] It is straightforward to show that all else equal, more developed countries (with higher Z_i) tend to have higher total income. This in turn means that there is a positive correlation between per capita income w_i and development level Z_i.

In this section, I compare trade volumes of two countries with identical total incomes but different components, per capita income and population size, *within* a given equilibrium. This way of doing comparative statics in the model corresponds to cross-country comparison in the data. In particular, I consider such equilibrium (defined by (29)) that there are two countries, countries 1 and 2, which are identical in every way except for Z_i and L_i. I assume that Z_1, Z_2, L_1, and L_2 are such that $Z_1 > Z_2$, $L_1 < L_2$, and $w_1 L_1 = w_2 L_2$ in the equilibrium.[47] In this way, I restrict countries 1 and 2 to have the same economy sizes (GDP) but different per capita incomes: country 1 is richer and smaller, while country 2 is poorer and larger. To capture only the effects of Z_i and L_i on trade volumes, I assume that the countries are geographically symmetric and have identical trading partners. This means that $\tau_{1j} = \tau_{2j}$ and $\tau_{j1} = \tau_{j2}$ for all $j > 2$, and countries 1 and 2 do not trade with each other in the equilibrium. For instance, we can think that country 1 is located at the North Pole, country 2 is located at the South Pole, while the rest of the world is located along the equator. Note that this approach of analyzing the effects of Z_i and L_i is equivalent to a standard comparative statics exercise (where we compare equilibrium outcomes before and after a change in a parameter) applied to a small open economy.

I show that if δ is greater than zero, then the richer country has greater trade volumes in the equilibrium. The intuition behind this result goes as follows. As $Z_1 > Z_2$, $\frac{f_{1j}}{f_{e1}} < \frac{f_{2j}}{f_{e2}}$ for all $j > 2$, meaning that country 1 has lower fixed costs of trade relative to the costs of entry into the industry than country 2 (it is relatively less expensive to export than to create a new variety in country 1). This leads to lower mass of entrants into the industry, but to a higher fraction of them finding it profitable to export to a given market compared to country 2. In

[46] Recall that free entry into the industry leads to zero total profits: i.e., $\pi_i = 0$ for all $i = 1..N$. This means that the total income in country i equals to $w_i L_i$.

[47] Note that since higher Z_i implies higher w_i in equilibrium, we can always find such values of Z_1 and Z_2 that $w_1 L_1 = w_2 L_2$ in the equilibrium.

addition, as $\frac{f_{1j}}{f_{11}} < \frac{f_{2j}}{f_{22}}$ for all $j > 2$, country 1 has lower fixed costs of trade relative to fixed costs of selling domestically. As a result, country 1 has a higher fraction of firms (in the total mass of entrants) serving foreign markets *relative* to a fraction of firms serving the domestic market. This in turn leads to greater trade volumes in country 1. That is, the richer country trades more after controlling for total income. Notice that as usual in trade theory, relative terms matter. It might be the case that a country faces lower fixed costs of trade in absolute terms but trades less, as fixed costs of trade are higher relative to other costs.

While the intuition is straightforward, the proof is quite complex. This is due to the difficulty of obtaining analytical results in the presence of asymmetries in Melitz type models. To prove this claim, I make two assumptions.

Assumption 1: $\frac{\theta^{\sigma} g(\theta)}{\int_{\theta}^{\theta_H} t^{\sigma-1} dG(t)}$ *is weakly increasing in* θ*, where* $g(\theta)$ *is the density function associated with* $G(\theta)$*.*

This assumption has a natural interpretation. It implies that $g(\theta)$ does not decrease too fast; i.e., the probability of getting higher values of θ does not decrease too fast with θ. For instance, a truncated Pareto distribution or a Power distribution satisfies this condition.

Assumption 2: *Trade costs are so high that* $\theta_{1j} \geq \theta_{11}$ *and* $\theta_{2j} \geq \theta_{22}$ *for all* $j > 2$*.*

This is a standard assumption in trade literature, saying that exporting firms serve the home market as well. It is consistent with empirical evidence: only a small fraction of firms export and those that export sell also domestically.

Proposition 1 *If* $\delta > 0$ *and Assumptions* 1 *and* 2 *hold, then country* 1 *has greater trade volumes in the equilibrium.*

Proof. A sketch of the proof is as follows. Suppose that country 2 trades more in the equilibrium. Since trade is balanced, country 2 imports more than country 1. That is,

$$\textstyle\sum_{j>2}^{N} X_{j2} \geq \sum_{j>2}^{N} X_{j1}. \tag{30}$$

As total incomes in the countries are the same ($w_1 L_1 = w_2 L_2$), (30) immediately implies that the price index in country 2 is higher than that in country 1: $P_2 \geq P_1$. Next, I show that if

$P_2 \geq P_1$ and the assumptions of the proposition hold, then country 1 exports more than country 2. This constitutes a contradiction, implying that country 1 trades strictly more than country 2 in the equilibrium. This stage of the proof has two steps.

Step 1: The free entry condition in (29) states that

$$f_{ei} = \sum_{j=1}^{N} \Pr\left(\theta \geq \theta_{ij}\right) E\left(\left(\pi_{ij}(\theta) - f_{ij}\right) | \theta \geq \theta_{ij}\right).$$

Using the expression for $\pi_{ij}(\theta)$ in (23) and dividing $\pi_{ij}(\theta)$ by $\pi_{ij}(\theta_{ij})$, we obtain that

$$\begin{aligned} \frac{\pi_{ij}(\theta)}{\pi_{ij}(\theta_{ij})} &= \left(\frac{\theta}{\theta_{ij}}\right)^{\sigma-1} \Longleftrightarrow \\ \pi_{ij}(\theta) &= \pi_{ij}(\theta_{ij})\left(\frac{\theta}{\theta_{ij}}\right)^{\sigma-1}. \end{aligned}$$

Taking into account that $\pi_{ij}(\theta_{ij}) = f_{ij}$, the free entry condition for country i can be rewritten as follows:

$$\sum_{j=1}^{N} \frac{f_{ij}}{f_{ei}} \int_{\theta_{ij}}^{\theta_H} \left(\left(\frac{\theta}{\theta_{ij}}\right)^{\sigma-1} - 1\right) dG(\theta) = 1. \tag{31}$$

Hence, condition (31) implies that for countries 1 and 2 (recall that countries 1 and 2 do not trade with each other):

$$\sum_{j:j\neq 2}^{N} \frac{f_{1j}}{f_{e1}} \int_{\theta_{1j}}^{\theta_H} \left(\left(\frac{\theta}{\theta_{1j}}\right)^{\sigma-1} - 1\right) dG(\theta) = \sum_{j:j\neq 1}^{N} \frac{f_{2j}}{f_{e2}} \int_{\theta_{2j}}^{\theta_H} \left(\left(\frac{\theta}{\theta_{2j}}\right)^{\sigma-1} - 1\right) dG(\theta). \tag{32}$$

Note that $\frac{f_{1j}}{f_{e1}}$ is strictly less than or equal to $\frac{f_{2j}}{f_{e2}}$ for all $j = 1..N$. This means that in order for (32) holds, θ_{1j} must be strictly less than θ_{2j} at least for some j.[48] I show that in fact, $\theta_{1j} < \theta_{2j}$ for all $j > 2$ (see Appendix C for details). That is, country 1 has a higher fraction of firms in the total mass of entrants exporting to country j for any $j > 2$.

Step 2: Next, I show that country 1 has higher exports to domestic sales ratio than country 2. This means that country 1 exports more than country 2, as total incomes of the countries are the same. In Appendix C, I show that the result of the previous step and the fact that $\frac{f_{1j}}{f_{11}}$ is strictly less than $\frac{f_{2j}}{f_{22}}$ for all $j > 2$ imply that

$$\frac{\int_{\theta_{1j}}^{\theta_H} \theta^{\sigma-1} dG(\theta)}{\int_{\theta_{11}}^{\theta_H} \theta^{\sigma-1} dG(\theta)} > \frac{\int_{\theta_{2j}}^{\theta_H} \theta^{\sigma-1} dG(\theta)}{\int_{\theta_{22}}^{\theta_H} \theta^{\sigma-1} dG(\theta)} \text{ for all } j > 2. \tag{33}$$

[48] Since $\frac{f_{1j}}{f_{e1}}$ is strictly less than or equal to $\frac{f_{2j}}{f_{e2}}$ for all $j = 1..N$, $\int_{\theta_{1j}}^{\theta_H} \left(\left(\frac{\theta}{\theta_{1j}}\right)^{\sigma-1} - 1\right) dG(\theta)$ must be strictly greater than $\int_{\theta_{2j}}^{\theta_H} \left(\left(\frac{\theta}{\theta_{2j}}\right)^{\sigma-1} - 1\right) dG(\theta)$ at least for some j. Otherwise, (32) does not hold. This implies that $\theta_{1j} < \theta_{2j}$ for some j.

As we assume that $\tau_{1j} = \tau_{2j}$ for all $j > 2$, (33) implies that

$$\frac{\sum_{j>2}^{N} \left(\frac{1}{\tau_{1j}}\right)^{\sigma-1} \frac{w_j L_j}{P_j^{1-\sigma}} \int_{\theta_{1j}}^{\theta_H} \theta^{\sigma-1} dG(\theta)}{\int_{\theta_{11}}^{\theta_H} \theta^{\sigma-1} dG(\theta)} > \frac{\sum_{j>2}^{N} \left(\frac{1}{\tau_{2j}}\right)^{\sigma-1} \frac{w_j L_j}{P_j^{1-\sigma}} \int_{\theta_{2j}}^{\theta_H} \theta^{\sigma-1} dG(\theta)}{\int_{\theta_{22}}^{\theta_H} \theta^{\sigma-1} dG(\theta)}.$$

Note that $P_2 \geq P_1$ implies that $\frac{w_1 L_1}{P_1^{1-\sigma}} \leq \frac{w_2 L_2}{P_2^{1-\sigma}}$. This in turn means that

$$\frac{\sum_{j>2}^{N} \left(\frac{1}{\tau_{1j}}\right)^{\sigma-1} \frac{w_j L_j}{P_j^{1-\sigma}} \int_{\theta_{1j}}^{\theta_H} \theta^{\sigma-1} dG(\theta)}{\frac{w_1 L_1}{P_1^{1-\sigma}} \int_{\theta_{11}}^{\theta_H} \theta^{\sigma-1} dG(\theta)} > \frac{\sum_{j>2}^{N} \left(\frac{1}{\tau_{2j}}\right)^{\sigma-1} \frac{w_j L_j}{P_j^{1-\sigma}} \int_{\theta_{2j}}^{\theta_H} \theta^{\sigma-1} dG(\theta)}{\frac{w_2 L_2}{P_2^{1-\sigma}} \int_{\theta_{22}}^{\theta_H} \theta^{\sigma-1} dG(\theta)},$$

which is equivalent to

$$\frac{\sum_{j>2}^{N} X_{1j}}{X_{11}} > \frac{\sum_{j>2}^{N} X_{2j}}{X_{22}}.$$

Q.E.D. ■

Notice that in general, we might assume that the costs of entry into the industry and selling at home also depend on the country development level. This would not change the main findings above (except the condition on δ), as we need differences in relative terms. In the paper, I do not introduce this dependence, as it is not identifiable from the trade data.

3.4 Estimation

To understand the contribution of the imposed association between market access costs and development level in explaining the data, I estimate the key parameters of the model. In the estimation procedure, I use data on total income, population, bilateral trade flows, and cultural and geographical barriers between country pairs for 1995. I consider the sample of the hundred largest countries in terms of GDP, for which the data sets are complete. These countries account for 91.6% of world trade in 1995. I assume that the other countries do not exist (these hundred countries constitute the entire world). Exports to non-existent countries are considered as domestic sales.

Data on total income and population are taken from the World Bank (2007). *Table 11* reports the list of the countries in the sample arranged by the size of GDP. Data on bilateral trade flows comes from the United Nation Statistics Division (2007).[49] In constructing bilateral

[49] An alternative source for data on trade flows is the NBER-UN data set constructed by Feenstra et al. (2005). However, this data set includes only trade flows in a certain category with values greater than $100.000 per year. When aggregating, this may potentially lead to underestimation of aggregate exports and imports and overestimation of the number of zero trade flows.

trade flows, I follow Feenstra et al. (2005). As a measure of trade volumes between countries, I use trade values reported by the importing country, as they tend to be more precise than those reported by the exporter. However, if an importer report is not available, I use the corresponding exporter report instead. There are 1399 export zeros in the sample, which constitutes 14% of the total number of bilateral trade flows.

As potential trade barriers, I consider distance, the effects of common border and language, and the impact of membership in free trade areas.[50] Hence, for each country pair we need data on whether these countries have a common language or share a common border plus data on distance between them.[51] I take these data sets from the Centre d'Etudes Prospectives et d'Informations Internationales (2005). In addition, I use the data on whether the pair of countries belongs to the North American Trade Agreement (NAFTA) or the European Union (EU).

Figure 11 displays the relationship in the data between the log of trade volumes (the average between exports and imports) and the log of GDP. As is evident from the figure, there is a strong positive correlation between countries' trade volumes and total income. This is in line with the previous empirical studies on the gravity equation. *Figure 12* depicts the relationship between the residuals and countries' logs of income per capita and population size. As it is inferred from the figure, there is a significant positive correlation between the residuals and income per capita. This suggests that conditional on GDP, richer countries trade more.

3.4.1 Estimation Strategy

To estimate the model, we need to parametrize the distribution of firm-specific productivity draws $G(\theta)$ and variable trade costs τ_{ij}. In parametrizing $G(\theta)$, I follow a number of studies using a truncated Pareto distribution to describe the distribution of firm productivities.[52] In

[50] Many other variables (for instance, religion or colonial origin) can be used as additional measures of trade barriers between countries. However, to reduce the number of parameters I need to estimate, I consider only language, border, distance, and membership in free trade areas.

[51] By distance between two countries, I mean the distance between the main cities in the countries. Usually, the main city is the capital. However, in some cases, the capital is not populated enough to serve a role of the economic center of the country. In these cases, the most populated city represents the country.

[52] See, for instance, Helpman, Melitz, and Rubinstein (2007) or Johnson (2007).

particular, I assume that

$$G(\theta) = \frac{\frac{1}{\theta_L^k} - \frac{1}{\theta^k}}{\frac{1}{\theta_L^k} - \frac{1}{\theta_H^k}} \text{ on } [\theta_L, \theta_H], \tag{34}$$

where $\infty \geq \theta_H > \theta_L > 0$ and $k > \sigma - 1$. The last condition guarantees that in the case when $\theta_H = \infty$, the integral $\int_{\theta_{ij}}^{\theta_H} \theta^{\sigma-1} dG(\theta)$ exists.

I assume the following functional form for variable trade costs:

$$\tau_{ij} = 1 + \gamma_0 D_{ij}^{\gamma_1} \gamma_2^{B_{ij}} \gamma_3^{LNG_{ij}} \gamma_4^{NAF_{ij}} \gamma_5^{EU_{ij}} \quad \text{for } j \neq i, \tag{35}$$

where $\{\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$ is the set of parameters describing variable costs of trade. D_{ij} is the distance between countries i and j, B_{ij} and LNG_{ij} are dummy variables for common border and language, and NAF_{ij} and EU_{ij} are dummy variables for whether countries i and j are members of NAFTA or EU, respectively. For instance, if γ_2 is less (greater) than one, then sharing a common border reduces (increases) the costs of trade between countries.

Hence, the set of parameters of the model is given by

$$\{\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, f_x, \delta, f_e, f_d, \sigma, k, \theta_L, \theta_H\}. \tag{36}$$

Note that a number of parameters in (36), namely $\{f_e, f_d, \sigma, k, \theta_L, \theta_H\}$, are not identifiable from the trade data. Therefore, to achieve identification I fix these parameters at certain values. In choosing the parameter values, I mostly follow the previous literature.

In fixing σ, I follow the results in the previous studies estimating the elasticity of substitution. Bernard et al. (2003) argue that σ equal to 3.8 captures the export behavior of the U.S. plants best. Broda and Weinstein (2006) estimate the elasticity of substitution for different aggregation levels. In period 1990-2001 for SITC-3 aggregation level, the estimates vary from 1.2 (thermionic, cold cathode, photocathode valves, etc.) to 22.1 (crude oil from petroleum or bituminous minerals) with the mean equal to 4. The number obtained in Bernard et al. (2003) is close to the mean of the estimates in Broda and Weinstein (2006). Following their results, I set σ equal to 3.8.

The distribution of productivity draws in (34) is characterized by parameters θ_L, θ_H, and k. I normalize θ_L to unity. In many studies, to simplify analytical derivations θ_H is set to infinity.[53] However, setting θ_H to infinity implies that there always exist some relatively productive firms

[53] See, for instance, Chaney (2008).

Table 1: The Values Assigned to the Unidentifiable Parameters

Parameter	Value
Elasticity of substitution, σ	3.8
Shape parameter, k	3.4
Lower support, θ_L	1
Upper support, θ_H	20
Fixed costs of selling at home, f_d	0
Costs of entry, f_e	1

finding it profitable to export to any country. This means that the model does not generate export zeros, which is at odds with the data. In the paper, I set θ_H equal to 20. On the one hand, finite θ_H allows for zero bilateral flows. On the other hand, there is not much difference in terms of the statistics of the productivity distribution (such as average, variance, etc.) between $\theta_H = 20$ and $\theta_H = \infty$. That is, the choice of θ_H is mainly consistent with the previous studies and allows for export zeros. The shape parameter k determines the behavior of the tail of the firm-specific productivity distribution. Following Ghironi and Melitz (2004) and Bernard, Redding and Schott (2007), I set k equal to 3.4.

Following Helpman, Melitz and Rubinstein (2008), I set f_d equal to zero. So that there are no fixed costs of selling domestically implying that all entering firms serve the home market. Finally, as changes in the parameter f_e only rescale the mass of entrants into the industry and have no impact on trade volumes, I normalize f_e to unity.[54]

Hereafter, I assume that the set of parameters $\{f_e, f_d, \sigma, k, \theta_L, \theta_H\}$ is fixed at the values reported in *Table 1*. In Appendix *C*, I do several robustness checks by trying some other parameter values. I find that changes in the values do not significantly alter the quantitative implications of the model.

[54] The condition (31) implies that any changes in f_{ij} and f_{ei} keeping $\frac{f_{ij}}{f_{ei}}$ fixed for all j do not affect the cutoffs θ_{ij} and, therefore, trade volumes.

Estimation Procedure The rest of the parameters of the model is given by

$$\Theta = \{\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, f_x, \delta\}.$$

To estimate these parameters, I use restricted non-linear least squares procedure. For given Θ and $\{Z_i, L_i\}_{i=1..N}$, we can solve the system of equations (29) and find the equilibrium values of $\{w_i,\ P_i,\ M_{ei}\}_{i=1..N}$ (notice that if we know Θ, we can construct $\{\tau_{ij}\}_{i,j=1..N}$ using (35)). Let $X_{ij}(Z, L, \Theta)$ denote the value of exports from country i to j generated by the model conditional on Θ, $\{Z_i, L_i\}_{i=1..N}$, and the corresponding equilibrium values of $\{w_i,\ P_i,\ M_{ei}\}_{i=1..N}$ (here $Z = \{Z_i\}_{i=1..N}$ and $L = \{L_i\}_{i=1..N}$). In words, if we know the parameters of the model and the exogenous variables Z and L, we can solve for the equilibrium and construct the corresponding bilateral trade flows. To estimate Θ, I solve the following minimization problem:

$$\min_{\Theta} \sum_{i,j:i \neq j} \left(X_{ij}^o - X_{ij}(Z, L, \Theta)\right)^2 \tag{37}$$

subject to

$$\Psi\left(Z, L, \Theta\right) = 0, \tag{38}$$

where X_{ij}^o is the value of exports from i to j observed in the data. $\Psi\left(Z, L, \Theta\right)$ stands for the difference between the number of zeros predicted by the model (given Θ and $\{Z_i, L_i\}_{i=1..N}$) and the actual number of zero bilateral trade flows in the data. The constraint $\Psi\left(Z, L, \Theta\right)$ is imposed for identification purposes. In particular, the variable and fixed costs of trade are not separately identifiable from just bilateral trade data. Any changes in f_x can be offset by proper changes in $\{\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \delta\}$ without affecting the value of the objective function in (37). This estimation technique also allows to account for the general equilibrium features of the model (including the effects of free entry into the industry) and to use the information contained in export zeros.

The estimation procedure discussed above is based on the fact that we know the values of $\{Z_i, L_i\}_{i=1..N}$. While for $\{L_i\}_{i=1..N}$ we can use the data on population size in the countries, $\{Z_i\}_{i=1..N}$ are not observable. To resolve this problem, I use the data on per capita income levels to reconstruct $\{Z_i\}_{i=1..N}$. Specifically, from the equilibrium conditions (29) we have that $w = w(Z, L, \Theta)$, where $w = \{w_i\}_{i=1..N}$. This implies that we can express Z in terms of w, L, and the parameters Θ. That is, we can invert the function $w(Z, L, \Theta)$ and obtain $Z = Z(w, L, \Theta)$. In the model, w_i is equal to per capita income in country i, which is observed in the data. Hence, using the data for $\{w_i\}_{i=1..N}$ we can reconstruct $\{Z_i\}_{i=1..N}$. In this case, the minimization problem can be rewritten as follows:

$$\min_{\Theta} \sum_{i,j:i\neq j} \left(X_{ij}^o - X_{ij}(Z(w,L,\Theta),L,\Theta)\right)^2 \tag{39}$$

subject to

$$\Psi\left(Z(w,L,\Theta),L,\Theta\right) = 0. \tag{40}$$

Note that the structure of the equations in (29) is nonlinear. This implies that the function $w(Z,L,\Theta)$ is not necessarily one-to-one. For instance, several different values of Z may lead to the same value of w. However, no such examples occur in the numerical analysis I conduct in the paper.

In Appendix C, as a robustness check I consider alternative estimation procedures such as restricted non-linear least deviations and restricted non-linear weighted least squares. I find that these estimation procedures yield similar predictions as the procedure used in the paper.

3.4.2 Results

As a measure of the explanatory power of the model, I use

$$R^2 = 1 - \frac{\sum_{i,j:i\neq j}\left(X_{ij}^o - X_{ij}(Z(w,L,\Theta),L,\Theta)\right)^2}{\sum_{i,j:i\neq j}\left(X_{ij}^o\right)^2}.$$

The explanatory power is 100%, if we are able to fit perfectly all bilateral trade flows: i.e., $\sum_{i,j:i\neq j}\left(X_{ij}^o - X_{ij}(Z(w,L,\Theta),L,\Theta)\right)^2 = 0$.

Table 2 reports the results obtained from solving (39) and (40). The explanatory power of the model is 81%. Like in traditional estimates of the gravity equation, the results show that country i and j trade more if they are closer to each other, have a common border, share a common language, or belong to the same regional trade agreement (NAFTA or EU). The estimated value of δ is 0.67, implying a strong correlation between country development level and market access costs. More developed countries tend to have lower fixed costs of exporting relative to the other costs.

To compare the quantitative implications of the model with the data, I run the following regression (robust standard errors in parentheses):

$$\ln \frac{T_i}{T_i(\hat{\Theta})} = \underset{(0.09)}{-1.23} + \underset{(0.03)}{0.16} \ln GDP_i - \underset{(0.04)}{0.04} \ln \frac{GDP_i}{L_i}, \tag{41}$$

where T_i is the actual trade volumes of country i and $T_i(\hat{\Theta})$ is the volumes of trade generated by the model given the estimated values of the parameters (see *Table 2*). As we can see from (41), the model captures the effect of per capita income on trade volumes (conditional on total income) quite well. The corresponding coefficient is not significantly different from zero. Meanwhile, the estimates in (41) suggest that the model somewhat underestimates trade volumes of large population countries. *Table 3* reports the elasticities of trade with respect to total and per capita incomes observed in the data and generated by the model (the first and second columns, respectively). In the data, doubling a country income per capita (keeping the total income unaltered) leads on average to an increase in trade volumes of 19% and doubling a country population size raises trade volumes by 85%. The model predicts a rise in trade volumes of 22% and 69%, respectively.

The restriction (40) in the estimation procedure implies that the number of zero bilateral trade flows generated by the model is the same as that in the data. However, the model fits just the number of zeros. It can generate zeros that are not actually observed in the data and vice versa. I find that the model explains 35% of "true" zeros. That is, 35% of zeros generated by the model are zeros actually observed in the data, while the rest is mismatch. The key point is that the model underestimates trade volumes of large population countries. As a result, it generates a number of "false" zeros (which are not observed in the data) among countries with large population and does not predict many zeros in the data among small population countries. This in turn constitutes the mismatch of 65%. Notice that the estimated association between country's development level and fixed costs of trade helps to explain many "true" zeros in the data. In the next subsection, I estimate a variation of my model when δ is equal to zero: i.e., fixed costs of trade (in terms of labor units) are identical across the countries. I find that in this case, the mismatch constitutes 91%.

The estimated values of the parameters also allow us to determine the magnitude of fixed costs of trade. For each country I construct the ratio of the aggregate fixed costs of exporting

Table 2: Parameter Estimates

γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	f_x	δ	R^2
0.49	0.14	0.76	0.97	0.78	0.94	2.15	**0.67**	81%

to the aggregate export profits, which is given by

$$FC_{Ri} = \frac{M_{ei}\sum_{j\neq i}^{N}\left(1-G(\theta_{ij})\right)f_{ij}}{M_{ei}\sum_{j\neq i}^{N}\int_{\theta_{ij}}^{\theta_H}\pi_{ij}(\theta)dG(\theta)}.$$

The ratios vary from 0.32 (for Iceland) to 0.64 (for India) with the mean equal to 0.45. That is, the total costs of access to foreign markets constitute on average around the half of the total export profits. The similar result is obtained in Eaton, Kortum and Kramarz (2008), who find that the share of fixed costs in the gross profits is a little more than half. By regressing the log of FC_{Ri} on the logs of GDP and GDP per capita, I find that richer countries have the lower share of fixed costs of exporting in the total export profits, while countries with larger population have the higher share. Namely,

$$\ln FC_{Ri} = \underset{(0.01)}{-0.84} + \underset{(0.003)}{0.04}\ln GDP_i - \underset{(0.002)}{0.14}\ln\frac{GDP_i}{L_i}. \tag{42}$$

In the next subsection, I estimate the benchmark model when δ is equal to zero (implying no differences in fixed costs of exporting across the countries) and compare the quantitative implications of the benchmark model with those obtained above.

Comparison with the Benchmark Model In the benchmark model, I assume that δ is equal to zero. That is,

$$f_{ij} = w_i f_x \text{ for } i \neq j.$$

This implies identical fixed costs of trade (in terms of labor) across the countries. I estimate the model using the same estimation procedure and compare the quantitative implications with those obtained above in the previous section. Since I fix δ at zero, the set of parameters estimated is $\{\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, f_x\}$.

Table 4 reports the estimated values of the parameters. The explanatory power of the model falls from 81% to 73%. Hence, differences in fixed costs of trade established in the model

Table 3: Trade Elasticities and Zeros

Estimator:	Data (OLS)	Model (OLS, $\delta > 0$)	Model (OLS, $\delta = 0$)
Dependent variable:	$\ln T_i$	$\ln T_i(\hat{\Theta})$	$\ln T_i(\hat{\Theta})$
Log of *GDP*	0.85**	0.69**	0.76**
	(0.03)	(0.03)	(0.03)
Log of *GDP* per capita	0.19**	0.22**	0.04**
	(0.04)	(0.02)	(0.02)
The percentage of "true" zeros:	100%	35%	9%
Observations:	100	100	100

explain 8% of the variation in the bilateral trade flows. The third column of *Table 3* shows the trade elasticities generated by the benchmark model. As it can be seen from the table, the benchmark model does slightly better in predicting trade volumes of large population countries. Doubling a country population size raises trade volumes by 76% compared to an increase of 85% in the data. However, the effect of per capita income on trade volumes predicted by the benchmark is substantially lower than that in the data. Conditional on the total income, doubling a country income per capita leads on average to an increase in trade volumes of 4%, while the effect observed in the data is 19%.[55] Finally, the percentage of "true" zeros predicted by the benchmark model is 9%. This constitutes the mismatch of 91%, which is significantly higher than the mismatch obtained in the case when fixed costs of exporting depend on country development level.

3.5 Counterfactuals

The estimation procedure developed in the previous section enables us to perform some counterfactual analysis. In particular, I analyze how the elimination of asymmetries in fixed costs of trade affects consumer welfare in the countries. Although the model is stylized and one should

[55] The positive and significant (at the 5% significance level) effect of per capita income on trade volumes in the benchmark model is explained by some correlation between the estimated trade barriers (distance, border, etc.) and per capita income levels. That is, some small part of the effect of per capita income can be explained by the geographical location of a country, its membership in the trade unions, or its cultural characteristics such as language.

Table 4: Parameter Estimates (δ=0)

γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	f_x	δ	R^2
0.04	0.31	0.75	0.82	0.72	1.06	122.7	**0**	73%

not consider the obtained magnitudes of effects as the actual ones, the counterfactuals give us a clear intuition about the mechanisms in the model.

In the model, consumer welfare in country i (denoted as W_i) is equal to the real wage in that country. Namely, for $i = 1..N$

$$W_i \equiv \frac{Q_i}{L_i} = \frac{w_i}{P_i}. \tag{43}$$

Hence, given the parameters of the model, we can solve (29) for $\{w_i, P_i, M_{ei}\}_{i=1..N}$ and then using (43), find the equilibrium value of consumer welfare in country i.

I examine the effects of the elimination of asymmetries in market access costs. Specifically, I assume that δ is equal to zero. This removes the relationship between market access costs and development levels, implying that

$$f_{ij} = w_i f_x.$$

The other exogenous parameters (including $\{Z_i\}_{i=1..N}$) are fixed at the values obtained in the previous section.

Let us denote $\frac{\triangle W_i}{W_i}$ as the percentage change in welfare in country i given the changes in the parameters of the model. That is,

$$\frac{\triangle W_i}{W_i} = \frac{W_i^{after}}{W_i^{before}} - 1,$$

where W_i^{before} is the equilibrium value of welfare when the parameters take the values obtained in the previous section and W_i^{after} is welfare computed after the changes in δ. I find that all countries gain from the elimination of asymmetries in market access costs with the average percentage change in welfare being equal to 18%. The next regression illustrates how the gains depend on the country characteristics. In particular, I regress $\frac{\triangle W_i}{W_i}$ on the logs of GDP_i and $\frac{GDP_i}{L_i}$:

$$\frac{\triangle W_i}{W_i} = \underset{(0.01)}{0.35} - \underset{(0.005)}{0.03} \ln GDP_i - \underset{(0.005)}{0.05} \ln \frac{GDP_i}{L_i}. \tag{44}$$

As it can be seen, doubling a country population size on average reduces the welfare gains by 3%, while doubling a country per capita income (controlling for the total income of that country) reduces the gains by 5%. The former effect is explained by the fact that setting δ to zero enhances trade in all countries. As countries with larger population tend to have a lower trade to GDP ratio, those countries gain less compared to small population countries. The latter effect is based on the feature of the model that fixed costs of trade depend on country development level: i.e., firms in less developed countries face higher market access costs. Since fixing δ at zero eliminates this relationship, the changes in welfare are more substantial for less developed countries. I also find that the number of zeros predicted by the model substantially falls. The model predicts only 4 zeros if the relationship between fixed costs of trade and country development levels is removed.

3.6 Alternative Specifications

In this section, I consider two alternative specifications of the model. First, I assume that fixed costs of trade depend on both exporter's and importer's development levels. Second, I examine the case when variable costs of trade depend on exporter's development level, while fixed costs of trade are identical across the countries.

In the paper, fixed costs of trade depend only on the characteristics of an exporting country. Meanwhile, Arkolakis (2008) emphasizes that to serve a foreign market firms may need labor services from both the source and the destination countries. Eaton, Kortum and Kramarz (2008) assume that market access costs depend only on importer characteristics. To account for the importer effect on fixed costs, I assume that

$$f_{ij} = \frac{w_i}{Z_i^{\delta}} Z_j^{\alpha} f_x \quad \text{for } i \neq j.$$

That is, fixed costs of exporting depend on importer's development level as well. I then estimate the model applying the same estimation procedure as before. The set of the parameters I estimate is given by $\{\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, f_x, \delta, \alpha\}$.

The second column in *Table 5* shows the estimates of the parameters (the first column reports the estimates obtained before). As it can be inferred, there is a negative correlation between the importer development level and fixed costs of trade. The estimate of α is -0.09 implying that it

Table 5: Alternative Specifications: Parameter Estimates

Specifications:	(1)	(2)	(3)
γ_0	0.49	0.52	0.28
γ_1	0.14	0.14	0.16
γ_2	0.76	0.77	0.72
γ_3	0.97	0.97	0.99
γ_4	0.78	0.78	0.72
γ_5	0.94	0.94	0.94
f_x	2.15	1.67	15.88
δ	**0.67**	**0.62**	–
α	–	**−0.09**	–
β	–	–	**0.22**
R^2	81%	81%	79%

is relatively easier to export to more developed countries. However, the impact of the importer development level on fixed costs is substantially lower than that of the exporter development level. The presence of the importer effect does not considerably affect the estimates of the other parameters and does not improve the explanatory power of the model. This suggests that the importer effect does not contribute a lot in explaining bilateral trade flows. The third column in *Table 6* reports the trade elasticities and the percentage of "true" zeros predicted by the model. As it can be seen, the presence of the importer effect does not significantly change the trade elasticities and slightly improves the ability of the model to match the zeros in the data.

In his paper, Waugh (2009) assumes that variable trade costs are a function of symmetric relationships and an exporter fixed effect. He finds a negative correlation between exporter per capita income and the fixed effect, implying that poor countries face higher variable trade costs than rich countries. Following Waugh (2009), I examine a variation of my model when fixed costs of trade are identical across the countries, while variable trade costs depend on the

Table 6: Alternative Specifications: Trade Elasticities and Zeros

	Data	(1)	(2)	(3)
Dependent variable:	$\ln T_i$	$\ln T_i(\hat{\Theta})$	$\ln T_i(\hat{\Theta})$	$\ln T_i(\hat{\Theta})$
Log of GDP	0.85**	0.69**	0.69**	0.68**
	(0.03)	(0.03)	(0.03)	(0.03)
Log of GDP per capita	0.19**	0.22**	0.24**	0.49**
	(0.04)	(0.02)	(0.03)	(0.03)
The percentage of "true" zeros:	100%	35%	36%	27%
Observations:	100	100	100	100

exporter development level. That is, for $i \neq j$

$$\begin{aligned} f_{ij} &= w_i f_x \quad \text{and} \\ \tau_{ij} &= 1 + \frac{\gamma_0}{Z_i^{\beta}} D_{ij}^{\gamma_1} \gamma_2^{B_{ij}} \gamma_3^{LNG_{ij}} \gamma_4^{NAF_{ij}} \gamma_5^{EU_{ij}}. \end{aligned}$$

Hence, if β is greater than zero, then other things equal, more developed countries tend to have lower variable costs of trade. I then estimate the parameters $\{\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, f_x, \beta\}$.

The third column in *Table 5* reports the parameter estimates. The estimate of β is 0.22 meaning a strong negative correlation between variable costs of trade and country development level, which is consistent with the findings in Waugh (2009). The explanatory power slightly falls from 81% to 79%. However, this variation of the model considerably overestimates the impact of per capita income on trade volumes (see the fourth column in *Table 6*). The model predicts that controlling for the total income, doubling a country income per increases trade volumes of that country by 49% (compared to 19% in the data). Moreover, the percentage of "true" zeros falls from 35% to 27% constituting the larger mismatch between the zeros predicted by the model and in the data.

Hence, while the relationship between variable costs of trade and development level can also account for greater trade volumes of richer countries and zero trade flows in the data, the model in this case performs much worse in matching the trade elasticities and zeros in the data.

3.7 Concluding Remarks

This paper contributes to a rapidly growing literature analyzing the role of fixed costs of trade in explaining trade volumes. I show that the relationship between fixed costs and exporter development level can account for the impact of per capita income on trade volumes observed in the data and explain a number of zeros among bilateral trade flows. One of the novelties of this paper is an estimation procedure that distinguishes between the effects of variable and fixed costs of trade on trade volumes. The point is that variable and fixed costs of trade cannot be separately estimated from just bilateral trade flows. Therefore, to estimate the model I use information containing in export zeros applying *restricted* non-linear least squares. I find that market access costs constitute around the half of export profits. This finding is similar to that in Eaton, Kortum and Kramarz (2008), who estimate market access costs using the data on *firm-level* trade.

There are two natural extensions of the present model. First, in the model firms in more developed countries face relatively easier access to foreign markets, implying that richer countries have greater trade volumes even after controlling for total income. An alternative explanation of the phenomenon that richer countries trade more is based on nonhomotheticity of consumer preferences. A notable example of this literature is Fieler (2009), who extends the Ricardian model of trade in Eaton and Kortum (2002) by allowing for non-homothetic preferences and cross-sector differences in production technologies. However, there is evidence that seems inconsistent with the hypothesis about the role of nonhomothetic preferences. Anderson and Marcouiller (2002) find that the presence of institutional quality indices in the gravity equation leads to a negative and significant relationship between per capita income and the share of total expenditure devoted to traded goods. This calls into question the role of preferences in explaining the effect of per capita income on trade volumes. Hence, it would be interesting to introduce nonhomothetic preferences in my model. This would enable us to capture simultaneously the effects of both consumer preferences and fixed costs of trade on trade volumes in a general equilibrium framework.

Second, in my model I consider an environment where countries trade only in a differentiated good. This framework is more applicable to the case of trade among rich countries. In particular, the setup of the model assumes away the possibility that trade flows might be generated by differences in factor endowments. To describe better trade between countries with different factor endowments (specifically, trade between rich and poor countries), we can extend the model by incorporating the Heckscher-Ohlin trade theory. This would allow us to analyze both intra-industry and inter-industry trade and, thereby, improve the fit of the model. I leave these issues for future work.

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Appendix A: Proofs to Chapter 1

Proof of Lemma 2

Consider

$$\frac{\int_{b_L}^{b_M} t dG(t)}{\int_{b_M}^{A} t dG(t)} = \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right). \tag{45}$$

Let $b_M = F_1(b_L)$ be an implicit solution of (45). $F_1(b_L)$ is strictly increasing in b_L and $A \geq F_1(b_L) \geq b_L$. This implies that $F_1(A) = A$. Now, consider

$$\frac{f_e}{cL} + 1 = \alpha_H H(b_L) + (1-\alpha_H)H(b_M). \tag{46}$$

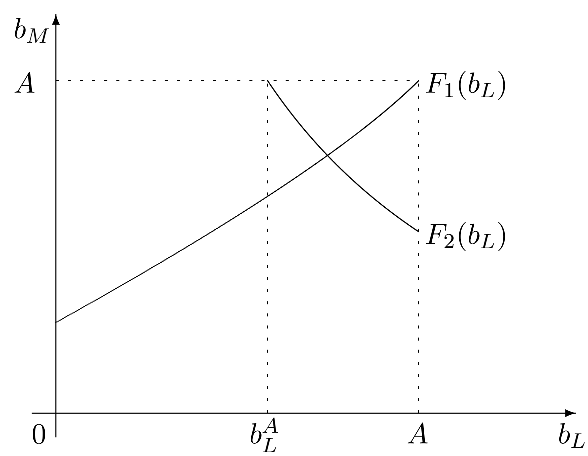
By analogy, let $b_M = F_2(b_L)$ be an implicit solution of (46) . As $H(\cdot)$ is strictly decreasing, $F_2(b_L)$ is also strictly decreasing in b_L. Since $H(A) = 1$, $H(F_2(A)) = \frac{f_e}{cL(1-\alpha_H)} + 1 > 1$. This implies that $F_2(A) < A$. Let b_L^A be such that $F_2(b_L^A) = A$. Then, $H(b_L^A) = \frac{f_e}{cL\alpha_H} + 1 > 1$, i.e., $b_L^A < A$. Hence, the solution of (4) exists and is unique (see *Figure 9*).

Proof of Lemma 3

Demand for goods from group k is equal to $L\sum_{i=k}^{N}\alpha_i$. From the definition of the sequence $\{b_k\}_{k=1..N}$, $\left(\frac{b_k}{V_k} - c\right)\sum_{i=k}^{N}\alpha_i = \left(\frac{b_k}{V_{k+1}} - c\right)\sum_{i=k+1}^{N}\alpha_i$. By induction,

$$\frac{\sum_{i=k}^{N}\alpha_i}{V_k} = \frac{1}{V_1} - c\sum_{i=1}^{k-1}\frac{\alpha_i}{b_i}. \tag{47}$$

Figure 7: The Equilibrium



From (47), $\pi_N(b) = \left(\frac{b}{V_N} - c\right)\alpha_N L = \frac{bL}{V_1} - cbL\sum_{i=1}^{N-1}\frac{\alpha_i}{b_i} - c\alpha_N L$. Recall that $\pi_N(b_N) = 0$. This implies that $\frac{1}{V_1} = c\sum_{i=1}^{N}\frac{\alpha_i}{b_i}$. From (47), $\frac{1}{V_k} = \frac{c\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}$ $k = 1..N$. Therefore,

$$\begin{aligned} p_k(b) &= bc\frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}, \\ \pi_k(b) &= cL\sum_{i=k}^{N}\frac{\alpha_i(b-b_i)}{b_i}. \end{aligned}$$

Proof of Proposition 8

Using *Lemma 3*, the system of equations (9) can be rewritten as follows[56]

$$\begin{cases} \frac{f_e}{cL} + 1 = \sum_{k=1}^{N}\alpha_k H(b_k), \\ \frac{I_k - I_{k-1}}{cM_e} = \frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}\int_{b_k}^{b_{k-1}} tdG(t) \quad k = 1..N \end{cases}. \tag{48}$$

Consider $k = N$. Then,

$$\frac{I_N - I_{N-1}}{cM_e} = \frac{1}{b_N}\int_{b_N}^{b_{N-1}} tdG(t). \tag{49}$$

Given M_e and b_{N-1}, there exists a unique solution $b_N(b_{N-1}, M_e)$ of the equation (49). The function $b_N(b_{N-1}, M_e)$ is strictly increasing in M_e and b_{N-1}. Given b_{N-1}, $\frac{M_e}{b_N(b_{N-1},M_e)} = \frac{I_N - I_{N-1}}{c\int_{b_N}^{b_{N-1}} tdG(t)}$ is strictly increasing in M_e.

Consider $k = N-1$. Then,

$$\frac{I_{N-1} - I_{N-2}}{cM_e} = \frac{\frac{\alpha_N}{b_N} + \frac{\alpha_{N-1}}{b_{N-1}}}{\alpha_N + \alpha_{N-1}}\int_{b_{N-1}}^{b_{N-2}} tdG(t). \tag{50}$$

Given M_e and b_{N-2}, there exists a unique solution $b_{N-1}(b_{N-2}, M_e)$ of the equation (50). The function $b_{N-1}(b_{N-2}, M_e)$ is strictly increasing in b_{N-2}. Since $\frac{M_e}{b_N(b_{N-1},M_e)}$ is strictly increasing in M_e, $b_{N-1}(b_{N-2}, M_e)$ is also strictly increasing in M_e. Finally, $\frac{\left(\frac{\alpha_N}{b_N} + \frac{\alpha_{N-1}}{b_{N-1}}\right)M_e}{\alpha_N + \alpha_{N-1}} = \frac{I_{N-1} - I_{N-2}}{c\int_{b_{N-1}}^{b_{N-2}} tdG(t)}$ is strictly increasing in M_e.

Using the backward induction, it can be proved that for any $k = 1..N$, there exists a unique solution $b_k(b_{k-1}, M_e)$ of the equation $\frac{I_k - I_{k-1}}{cM_e} = \frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}\int_{b_k}^{b_{k-1}} tdG(t)$ such that $b_k(b_{k-1}, M_e)$ is strictly increasing in b_{k-1} and M_e. This implies that for any M_e, there exists a unique solution $\{b_k(M_e)\}_{k=1..N}$ of the system of equations $\frac{I_k - I_{k-1}}{cM_e} = \frac{\sum_{i=k}^{N}\frac{\alpha_i}{b_i}}{\sum_{i=k}^{N}\alpha_i}\int_{b_k}^{b_{k-1}} tdG(t)$ $k = 1..N$. And for any $k = 1..N$, $b_k(M_e)$ is strictly increasing in M_e. Hence, (48) is equivalent to

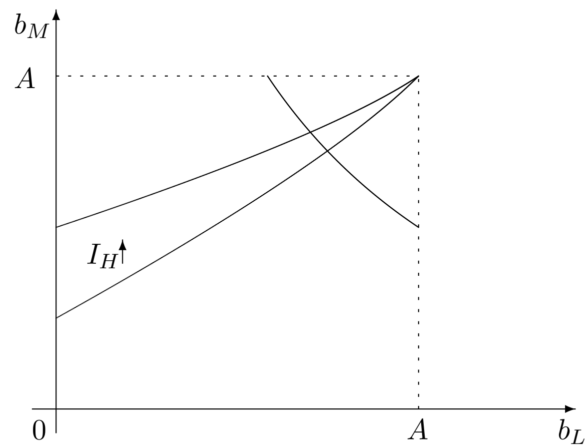
$$\begin{cases} \frac{f_e}{cL} + 1 = \sum_{k=1}^{N}\alpha_k H(b_k(M_e)), \\ b_k = b_k(M_e) \qquad k = 1..N. \end{cases} \tag{51}$$

Consider $D(M_e) = \sum_{k=1}^{N}\alpha_k H(b_k(M_e))$. As $H(x)$ is a strictly decreasing function, $D(M_e)$ is strictly decreasing in M_e. If M_e is close to zero then $b_N(M_e)$ is close to zero and, thereby, $D(M_e)$ is high enough[57]. If M_e is sufficiently high then for any $k = 1..N$, $b_k(M_e)$ is close to A and $D(M_e) \approx \sum_{k=1}^{N}\alpha_k H(A) = 1 < \frac{f_e}{cL} + 1$. This implies that there exists a unique solution M_e of (51). Therefore, there exists a unique solution of (9).

[56] $I_0 = 0$.

[57] Recall that $H(0) = \infty$.

Figure 8: An Increase in I_H



Comparative Statics

In this section, I use a simplifying notation: $\int_x^y$ means $\int_x^y tdG(t)$.

Proof of Proposition 2

An increase in I_H shifts the curve $F_1(b_L)$ up, while the curve $F_2(b_L)$ is unchanged. As a result, b_L falls and b_M rises (see *Figure 10*). The impact on welfare of the poor is not so straightforward. Rewrite (45) and (46) as follows

$$\begin{cases} J_1 \equiv (1-\alpha_H)cLH\left(b_M\right) + \alpha_H cLH\left(b_L\right) - f_e - cL = 0 \\ J_2 \equiv I_L \int_{b_L}^{b_M} - \left(I_H - I_L\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)\int_{b_M}^{A} = 0 \end{cases}. \tag{52}$$

Notice that equilibrium values of b_L and b_M solve (52). Using implicit differentiation, I obtain

$$\frac{\partial b_M}{\partial I_H} = \frac{\frac{\partial J_2}{\partial I_H}\frac{\partial J_1}{\partial b_L}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} > 0 \tag{53}$$

$$\frac{\partial b_L}{\partial I_H} = \frac{-\frac{\partial J_2}{\partial I_H}\frac{\partial J_1}{\partial b_M}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}} < 0. \tag{54}$$

Consider $\frac{1}{V_C} = \frac{\alpha_H c}{b_L} + \frac{(1-\alpha_H)c}{b_M}$. $\left(\frac{\alpha_H c}{b_L} + \frac{(1-\alpha_H)c}{b_M}\right)'_{I_H}$ is equal to $\frac{-\alpha_H c}{(b_L)^2}\frac{\partial b_L}{\partial I_H} - \frac{(1-\alpha_H)c}{(b_M)^2}\frac{\partial b_M}{\partial I_H}$. From (53) and (54),

$$\frac{-\alpha_H c}{\left(b_L\right)^2}\frac{\partial b_L}{\partial I_H} - \frac{(1-\alpha_H)c}{\left(b_M\right)^2}\frac{\partial b_M}{\partial I_H} = \frac{c^2 L\alpha_H(1-\alpha_H)\frac{\partial J_2}{\partial I_H}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}\left(\frac{H'\left(b_M\right)}{\left(b_L\right)^2} - \frac{H'\left(b_L\right)}{\left(b_M\right)^2}\right).$$

Recall that $H'(x) = -\frac{\int_x^A tdG(t)}{x^2} < 0$. Then,

$$\frac{-\alpha_H c}{\left(b_L\right)^2}\frac{\partial b_L}{\partial I_H} - \frac{(1-\alpha_H)c}{\left(b_M\right)^2}\frac{\partial b_M}{\partial I_H} = \frac{c^2 L\alpha_H(1-\alpha_H)\frac{\partial J_2}{\partial I_H}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}\frac{\int_{b_L}^{A} - \int_{b_M}^{A}}{\left(b_L\right)^2\left(b_M\right)^2}.$$

Since $\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L}-\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}>0$ and $\frac{\partial J_2}{\partial I_H}<0$, $\left(\frac{\alpha_H c}{b_L}+\frac{(1-\alpha_H)c}{b_M}\right)'_{I_H}<0$. Therefore, $(V_C)'_{I_H}>0$. This implies that an increase in I_H leads to the lower prices of the "common" goods and higher welfare of the poor, which is equal to $I_L V_C$.

Proof of Proposition 3

From the previous proof, we know that higher I_H results in lower b_L. That is, the prices of the "exclusive" goods rise. As $W_p = M_e\int_{b_M}^{A}$ and b_M increases, an increase in I_H raises M_e and, therefore, $W_r = M_e\int_{b_L}^{A}$.

Changes in I_L

Similarly, an increase in I_L shifts the curve $F_1(b_L)$ down, while the curve $F_2(b_L)$ is unchanged. Hence, b_L rises and b_M falls. To analyze the impact on consumer welfare, I use the same technique as in the previous proofs. As $W_r = M_e\int_{b_L}^{A}$ and $I_H = M_e\int_{b_L}^{A} p(t)dG(t)$, $W_r = \frac{I_H\int_{b_L}^{A}}{\int_{b_L}^{A} p(t)dG(t)}$. The sign of $(W_r)'_{I_L}$ is the same as the sign of $\left(\int_{b_L}^{A}\right)'_{I_L}\int_{b_L}^{A} p(t)dG(t)-\left(\int_{b_L}^{A} p(t)dG(t)\right)'_{I_L}\int_{b_L}^{A}$. Algebra shows that

$$\left(\int_{b_L}^{A}\right)'_{I_L}\int_{b_L}^{A} p(t)dG(t)-\left(\int_{b_L}^{A} p(t)dG(t)\right)'_{I_L}\int_{b_L}^{A} =$$

$$= \frac{c^2L(1-\alpha_H)^2\frac{\partial J_2}{\partial I_L}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L}-\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}\frac{\int_{b_L}^{b_M}\int_{b_L}^{A}\int_{b_M}^{A}}{(b_L)^2(b_M)^2}+c(1-\alpha_H)(b_M-b_L)\left\{\frac{\partial b_L}{\partial I_L}\frac{g(b_L)}{b_M}\int_{b_M}^{A}-\frac{\partial b_M}{\partial I_L}\frac{g(b_M)}{b_L}\int_{b_L}^{A}\right\}.$$

From (52), $\frac{\partial b_L}{\partial I_L}>0$, $\frac{\partial b_M}{\partial I_L}<0$, and $\frac{\partial J_2}{\partial I_L}>0$. This implies that $(W_r)'_{I_L}>0$. As $M_e = \frac{W_r}{\int_{b_L}^{A}}$, an increase in I_L raises M_e and, thereby, $W_p = M_e\int_{b_M}^{A}$.

Proof of Proposition 4

An increase in α_H shifts the curve $F_1(b_L)$ up and the curve $F_2(b_L)$ to the right around 45° degree line (see *Figure 11*). In this case, b_M rises. The impact on b_L is not so straightforward. There are two opposite effects. The upward shift of $F_1(b_L)$ decreases b_L, while the shift of the $F_2(b_L)$ increases b_L. I show that $\frac{\partial b_L}{\partial \alpha_H}>0$.

From (52),

$$\frac{\partial b_M}{\partial \alpha_H} = \frac{-\frac{\partial J_1}{\partial \alpha_H}\frac{\partial J_2}{\partial b_L}+\frac{\partial J_2}{\partial \alpha_H}\frac{\partial J_1}{\partial b_L}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L}-\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}>0$$

$$\frac{\partial b_L}{\partial \alpha_H} = \frac{-\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial \alpha_H}+\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial \alpha_H}}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L}-\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}.$$

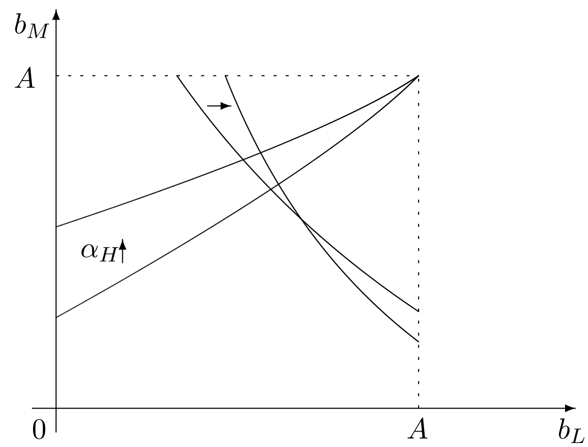
To determine the sign of $\frac{\partial b_L}{\partial \alpha_H}$, I examine

$$-\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial \alpha_H}+\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial \alpha_H}=cL\left(\left(H\left(b_L\right)-H\left(b_M\right)\right)\frac{\partial J_2}{\partial b_M}-\frac{(1-\alpha_H)}{(b_M)^2}(I_H-I_L)\left(1-\frac{b_L}{b_M}\right)\left(\int_{b_M}^{A}\right)^2\right).$$

The partial derivative of J_2 with respect to b_M can be written as follows

$$\frac{\partial J_2}{\partial b_M}=(I_H-I_L)\left(\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)b_M g\left(b_M\right)\frac{\int_{b_L}^{A}}{\int_{b_L}^{b_M}}+\frac{b_L(1-\alpha_H)}{(b_M)^2}\int_{b_M}^{A}\right). \tag{55}$$

Figure 9: An Increase in α_H



Then,

$$\begin{aligned}\frac{-\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial \alpha_H}+\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial \alpha_H}}{cL\left(I_H-I_L\right)} &= \left(H\left(b_L\right)-H\left(b_M\right)\right)\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)b_M g\left(b_M\right)\frac{\int_{b_L}^{A}}{\int_{b_L}^{b_M}}+ \\ &\quad +\frac{(1-\alpha_H)}{\left(b_M\right)^2}\int_{b_M}^{A}\left(G(b_L)b_L-G(b_M)b_L+\int_{b_L}^{A}-\int_{b_M}^{A}\right)>0,\end{aligned}$$

as $b_M>b_L$ and $G(b_L)b_L-G(b_M)b_L+\int_{b_L}^{A}-\int_{b_M}^{A}$ is increasing in b_M and equal to zero when $b_M=b_L$.

Welfare of the poor is given by $W_p=\frac{I_L}{c\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)}$. To determine the sign of $\left(W_p\right)'_{\alpha_H}$, we need to examine the sign of $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}=\frac{1}{b_L}-\frac{1}{b_M}-\left(\frac{\alpha_H}{(b_L)^2}\frac{\partial b_L}{\partial \alpha_H}+\frac{(1-\alpha_H)}{(b_M)^2}\frac{\partial b_M}{\partial \alpha_H}\right)$. The derivative of J_2 with respect to b_L can be expressed as

$$\frac{\partial J_2}{\partial b_L}=-\left(I_H-I_L\right)\left(\frac{b_L g\left(b_L\right)\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)\int_{b_M}^{A}}{\int_{b_L}^{b_M}}+\frac{(1-\alpha_H)}{b_M}\int_{b_M}^{A}\right). \tag{56}$$

Using the expressions (55) and (56), I show that

$$\frac{\alpha_H}{\left(b_L\right)^2}\frac{\partial b_L}{\partial \alpha_H}+\frac{(1-\alpha_H)}{\left(b_M\right)^2}\frac{\partial b_M}{\partial \alpha_H}=\frac{cL\left(I_H-I_L\right)\left(\left(H\left(b_L\right)-H\left(b_M\right)\right)\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)P_1+\left(1-\frac{b_L}{b_M}\right)P_2\right)}{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L}-\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}},$$

where $P_1=\left(\frac{\alpha_H b_M g(b_M)\int_{b_L}^{A}}{(b_L)^2\int_{b_L}^{b_M}}+\frac{(1-\alpha_H)b_L g(b_L)\int_{b_M}^{A}}{(b_M)^2\int_{b_L}^{b_M}}+\frac{(1-\alpha_H)}{b_L(b_M)^2}\int_{b_M}^{A}\right)$ and $P_2=\frac{(1-\alpha_H)\alpha_H}{(b_M)^2(b_L)^2}\int_{b_L}^{b_M}\int_{b_M}^{A}$. In addition,

$$\frac{\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L}-\frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}}{cL\left(I_H-I_L\right)}=\frac{\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}}P_3+\frac{(1-\alpha_H)\int_{b_M}^{A}tdG(t)}{\left(b_M\right)^2}P_4,$$

where $P_3 = \frac{(1-\alpha_H) b_L g(b_L) \left(\int_{b_M}^{A}\right)^2}{(b_M)^2} + \frac{\alpha_H b_M g(b_M) \left(\int_{b_L}^{A}\right)^2}{(b_L)^2}$ and $P_4 = \frac{(1-\alpha_H) \int_{b_M}^{A}}{b_M} + \frac{\alpha_H \int_{b_L}^{A}}{b_L}$. Therefore,

$$\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} = \frac{1}{b_L} - \frac{1}{b_M} - \frac{\left(H\left(b_L\right) - H\left(b_M\right)\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right) P_1 + \left(1 - \frac{b_L}{b_M}\right) P_2}{\frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}} P_3 + \frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2} P_4}.$$

After some simplifications,

$$\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} = \frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}} P_3 + \frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2} P_4} P_5,$$

where

$$\begin{aligned} P_5 &= \frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2}\left(\frac{1}{b_L} + \frac{b_L g\left(b_L\right)}{\int_{b_L}^{b_M}}\right)\left(G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_L}\right) \\ &\quad + \frac{\alpha_H \int_{b_L}^{A}}{(b_L)^2} \frac{b_M g\left(b_M\right)}{\int_{b_L}^{b_M}}\left(G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_M}\right). \end{aligned}$$

Hence, the sign of $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}$ is the same as the sign of P_5. As $b_M > b_L$, $G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_L} < 0$ and $G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_M} > 0$. Hence, if α_H is close enough to zero then $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} < 0$; that is, $(W_p)'_{\alpha_H} > 0$. However, if α_H is close enough to one then $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} > 0$. This implies that $(W_p)'_{\alpha_H} < 0$. It is much more complicated to determine the sign of P_5 for all values of $\alpha_H \in [0, 1]$.

The Effect of Higher α_H on the Rich

From the previous section, we know that $\frac{\partial b_L}{\partial \alpha_H} > 0$. This means that higher α_H decreases the prices of the "exclusive" goods. Welfare of the rich is given by $\frac{1}{c}\left(\frac{I_L}{\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)} + (I_H - I_L) b_L\right)$. This implies

$$c\left(W_r\right)'_{\alpha_H} = \frac{\left(I_H - I_L\right)\frac{\partial b_L}{\partial \alpha_H}\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}}{\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)^2}.$$

To determine the sign of $(W_r)'_{\alpha_H}$, we need to examine the sign of $\left(I_H - I_L\right)\frac{\partial b_L}{\partial \alpha_H}\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}$. Using the previous results,

$$\begin{aligned} &\left(I_H - I_L\right)\frac{\partial b_L}{\partial \alpha_H}\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} = \\ &= \left(I_H - I_L\right)\frac{\partial b_L}{\partial \alpha_H}\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)^2 - I_L\frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M}} P_3 + \frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2} P_4} P_5. \end{aligned}$$

After some simplifications, it appears that to prove that $(W_r)'_{\alpha_H} > 0$, it is enough to prove that

$$\begin{aligned}(I_H - I_L)\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)(H(b_L) - H(b_M)) - \frac{I_L}{b_L}\left(G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_M}\right) &> 0 \iff \\ \frac{I_L \int_{b_L}^{A}}{b_L}\left(\frac{H(b_L) - H(b_M)}{\int_{b_M}^{A}} - \frac{1}{b_L} + \frac{1}{b_M}\right) &> 0.\end{aligned}$$

For any $b_L < b_M$, $\frac{H(b_L)-H(b_M)}{\int_{b_M}^{A}} - \frac{1}{b_L} + \frac{1}{b_M} > 0$ resulting in that $(W_r)'_{\alpha_H}$ is always greater than zero. Since $(W_r)'_{\alpha_H} > 0$, $(b_L)'_{\alpha_H} > 0$ and $W_r = M_e \int_{b_L}^{A}$; the mass of firms entering the market rises, i.e., $(M_e)'_{\alpha_H} > 0$.

Proof of Proposition 5

Aggregate income per capita AG is given by $\alpha_H I_H + (1-\alpha_H) I_L$. This implies that $\alpha_H (I_H - I_L) = AG - I_L$. In this way, I rewrite (52) as follows

$$\begin{cases} J_1 \equiv (1-\alpha_H) cLH(b_M) + \alpha_H cLH(b_L) - f_e - cL = 0 \\ J_2 \equiv I_L \int_{b_L}^{b_M} - (AG - I_L)\left(1 + \frac{b_L(1-\alpha_H)}{\alpha_H b_M}\right)\int_{b_M}^{A} = 0 \end{cases}. \tag{57}$$

Hence, it is necessary to explore the impact of a decrease in α_H on welfare of the poor given new equilibrium equations (57). Using the same technique as in the proof of *Proposition 4*, I obtain

$$\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} = \frac{\left(\frac{\alpha_H}{b_L} + \frac{b_L(1-\alpha_H)}{b_M}\right)\left(\frac{(1-\alpha_H)\int_{b_M}^{A}}{b_M^2}(G(b_M) - G(b_L)) + P_6\right)}{\frac{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}{\int_{b_L}^{b_M} t dG(t)} P_3 + \frac{(1-\alpha_H)\int_{b_M}^{A}}{(b_M)^2} P_4},$$

where $P_6 = \frac{(1-\alpha_H)\int_{b_M}^{A}}{b_M^2} \frac{b_L^2 g(b_L)}{\int_{b_L}^{b_M}}\left(G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_L}\right) + \frac{\alpha_H \int_{b_L}^{A}}{b_L} \frac{b_M g(b_M)}{\int_{b_L}^{b_M}}\left(G(b_M) - G(b_L) - \frac{\int_{b_L}^{b_M}}{b_M}\right)$.

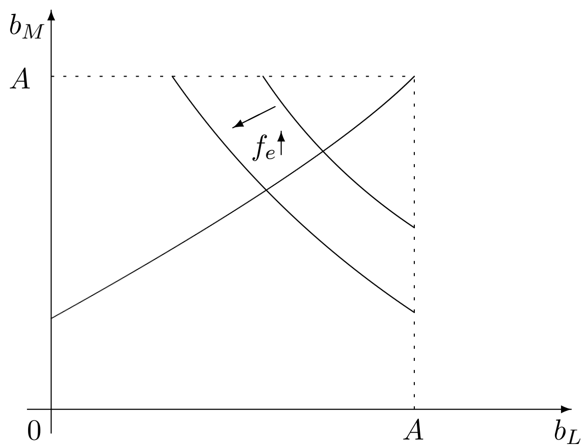
If α_H is close to one then $P_6 > 0$ and $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H} > 0$. That is, welfare of the poor rises with a decrease in α_H. This result is also supported by the fact that given sufficiently high α_H, both an increase in I_H and a decrease in α_H have a positive impact on welfare of the poor.

Consider $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H}$ when α_H is close to zero. From (57), $\lim_{\alpha_H \to 0} b_L(\alpha_H) = 0$ and $\lim_{\alpha_H \to 0} \frac{b_L(\alpha_H)}{\alpha_H}$ is a positive constant. As for any density function $g(\cdot)$, $\lim_{x \to 0} x g(x) = 0$; $\lim_{\alpha_H \to 0} P_6 > 0$. This implies that $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_{\alpha_H = 0} > 0$. Finally, it can be shown that if $bg(b)$ is increasing in b, $\frac{(1-\alpha_H)\int_{b_M}^{A}}{b_M^2}(G(b_M) - G(b_L)) + P_6 > 0$ for any $\alpha_H \in [0, 1]$.

Proof of Proposition 6

A rise in f_e shifts the curve $F_2(b_L)$ to the left, while the curve $F_1(b_L)$ is unchanged. As a result, b_L and b_M fall (see *Figure 12)*. Since $W_p = \frac{I_L}{\frac{\alpha_H c}{b_L} + \frac{(1-\alpha_H)c}{b_M}}$ and $W_r = W_p + \frac{I_H - I_L}{c} b_L$, W_p and W_r decrease. M_e, which is equal to $\frac{W_p}{\int_{b_M}^{A}}$, falls too. In the same way, a rise in L raises M_e, W_p, and W_r. Finally, any changes in f_e and L such that $\frac{f_e}{L}$ remains unchanged do not affect $F_2(b_L)$ and $F_1(b_L)$.

Figure 10: An Increase in f_e



Proof of Proposition 7

I need to show that given $\left(\frac{b^2 g(b)}{\int_b^A}\right)_b' > 0$, $\left(\frac{b_L}{b_M(b_L)}\right)_{b_L} > 0$ where $b_M(b_L)$ is an implicit solution of

$$\int_{b_L}^{b_M} - \left(\frac{I_H}{I_L} - 1\right)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)\int_{b_M}^{A} = 0.$$

Notice that the sign of$\left(\frac{b_L}{b_M(b_L)}\right)_{b_L}$ is the same as the sign of $b_M - \frac{\partial b_M}{\partial b_L} b_L$. Algebra shows that

$$b_M - \frac{\partial b_M}{\partial b_L} b_L \quad > \quad 0 \iff$$

$$\frac{b_L g(b_L) + \frac{\int_{b_L}^{b_M}}{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)} \frac{(1-\alpha_H)}{b_M}}{\frac{\int_{b_L}^{A}}{\int_{b_M}^{A}} b_M g(b_M) + \frac{b_L(1-\alpha_H)}{(b_M)^2} \frac{\int_{b_L}^{b_M}}{\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)}} \frac{b_L}{b_M} \quad < \quad 1 \iff \frac{(b_L)^2 g(b_L)}{\int_{b_L}^{A}} < \frac{(b_M)^2 g(b_M)}{\int_{b_M}^{A}}.$$

The Continuous Distribution of Efficiency Units of Labor

I assume that there is a distribution $F(\cdot)$ on $[s, S]$ (with a density function $f(\cdot)$) of efficiency units of labor. That is, given the mass L of consumers, there are $F(x)L$ consumers with income less or equal to x. Define $V(b) = \frac{b}{p(b)}$. From the main body of the paper, $V(b)$ is increasing on $[b_L^c, b_M^c)$ and flat on $[b_M^c, A]$ (see *Section 4*). I assume that $V(b)$ is differentiable on $[b_L^c, b_M^c)$. To simplify the notation, I also assume that $L = 1$.

Consider a particular firm with valuation b. If $b \in [b_M^c, A]$ then demand for this good is equal to one and $p(b) = \frac{b}{V(b_M^c)}$. Suppose $b \in [b_L^c, b_M^c)$ and the firm imposes price p of its good. Then, given $V(b)$ in the equilibrium, $s + \int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t) dG(t)$ is total spending on goods, which are bought before the good considered: goods that have higher valuation to price ratios. This implies that demand for this good is equal to $1 - F\left(s + \int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t) dG(t)\right)$. Hence, in the equilibrium, firms with $b \in [b_L^c, b_M^c)$ solve the following maximization problem

$$\max_p (p-c)\left(1-F\left(s+\int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t)dG(t)\right)\right).$$

The first order condition implies that

$$\frac{1-F\left(s+\int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t)dG(t)\right)}{f\left(s+\int_{V^{-1}\left(\frac{b}{p}\right)}^{b_M^c} M_e p(t)dG(t)\right)} = (p-c)\frac{bM_e p\left(V^{-1}\left(\frac{b}{p}\right)\right)g\left(V^{-1}\left(\frac{b}{p}\right)\right)}{p^2 V'\left(V^{-1}\left(\frac{b}{p}\right)\right)}.$$

This equation should be satisfied for any $b \in [b_L^c, b_M^c)$. That is, the price function $p(b)$ on $[b_L^c, b_M^c)$ solves the following differential equation

$$\frac{1-F\left(s+\int_b^{b_M^c} M_e p(t)dG(t)\right)}{f\left(s+\int_b^{b_M^c} M_e p(t)dG(t)\right)} = (p(b)-c)\frac{bM_e g(b)}{p(b)V'(b)} \tag{58}$$

where $V(b) = \frac{b}{p(b)}$. Using the solution of (58), free entry condition, and the goods market equilibrium, we can find b_L^c, b_M^c, and M_e.

In general, it is rather complicated to find the solution of (58). To simplify the problem, I assume that $F(x) = 1-e^{-\alpha(x-s)}$ on $[s,\infty)$. This implies that $\frac{1-F\left(s+\int_b^{b_M^c} M_e p(t)dG(t)\right)}{f\left(s+\int_b^{b_M^c} M_e p(t)dG(t)\right)} = \frac{1}{\alpha}$.Thus, (58) is equivalent to

$$V'(b) = \alpha M_e (b - cV(b))\, g(b). \tag{59}$$

As the maximum endowment of efficiency unit of labor is infinity, there is no exit and $b_L^c = 0$. Using the initial condition $V(0) = 0$ and (59), we have

$$\begin{aligned} V(b) &= \frac{1}{c}\left(b - e^{-\alpha M_e cG(b)}\int_0^b e^{\alpha M_e cG(t)}dt\right) \\ p(b) &= \frac{cb}{b - e^{-\alpha M_e cG(b)}\int_0^b e^{\alpha M_e cG(t)}dt}. \end{aligned}$$

From the goods market clearing condition, we obtain that $s = \frac{M_e}{V(b_M^c)}\int_{b_M^c}^A t dG(t)$. Using this equation and free entry condition, we can find M_e and b_M^c[58]. Notice that $\lim_{b\to 0} p(b) = \infty$. This means that goods with the lowest valuations have the highest prices.

[58] In the simplest case when $s = 0$, $b_M^c = A$ and M_e can be found from $f_e = \int_0^A (p(b)-c)\, e^{-\alpha M_e \int_b^A p(t)dG(t)} dG(b)$.

Appendix B: Proofs to Chapter 2

In this Appendix, I determine the effects of changes in τ and N on the endogenous variables of the model in Chapter 2 such as b_L, b_M, $\frac{b_L}{b_M}$, and consumer welfare. Therefore, the algebra in the Appendix is mainly based on the differentiation of implicit functions. As the intuition of this exercise (the differentiation of implicit functions) is straightforward, I present only the most important details and omit unnecessary ones.

In this section, I introduce a simplifying notation: $\int_x^y$ means $\int_x^y tdG(t)$.

Part 1

Let

$$J_1 = \alpha_H \left(H(b_L) + N\tau H(\tau b_L)\right) + (1-\alpha_H)\left(H(b_M) + N\tau H(\tau b_M)\right) - \frac{f_e}{cL} - 1 - N\tau \tag{60}$$

$$J_2 = I_L\left(\int_{b_L}^{b_M} + N\int_{\tau b_L}^{\tau b_M}\right) - (I_H - I_L)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)\left(\int_{b_M}^{A} + N\int_{\tau b_M}^{A}\right). \tag{61}$$

Then, simple algebra shows[59]

$$\begin{aligned}
\frac{\partial J_1}{\partial b_M} &= -\frac{(1-\alpha_H)}{b_M^2}\left(\int_{b_M}^{A} + N\int_{\tau b_M}^{A}\right) < 0,\ \frac{\partial J_1}{\partial b_L} = -\frac{\alpha_H}{b_L^2}\left(\int_{b_L}^{A} + N\int_{\tau b_L}^{A}\right) < 0,\\
\frac{\partial J_1}{\partial \tau} &= N\left(\alpha_H G(\tau b_L) + (1-\alpha_H)G(\tau b_M) - 1\right) < 0,\ \frac{\partial J_1}{\partial N} = \tau\left(\alpha_H H(\tau b_L) + (1-\alpha_H)H(\tau b_M) - 1\right) > 0,\\
\frac{\partial J_2}{\partial b_M} &= I_L b_M\left(g(b_M) + N\tau^2 g(\tau b_M)\right)\left(\frac{\int_{b_L}^{A} + N\int_{\tau b_L}^{A}}{\int_{b_M}^{A} + N\int_{\tau b_M}^{A}}\right) + I_L\frac{b_L(1-\alpha_H)}{b_M^2}\frac{\int_{b_L}^{b_M} + N\int_{\tau b_L}^{\tau b_M}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}} > 0,\\
\frac{\partial J_2}{\partial b_L} &= -I_L b_L\left(g(b_L) + N\tau^2 g(\tau b_L)\right) - I_L\frac{(1-\alpha_H)}{b_M}\frac{\int_{b_L}^{b_M} + N\int_{\tau b_L}^{\tau b_M}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}} < 0,\\
\frac{\partial J_2}{\partial \tau} &= I_L N\tau\left(b_M^2 g(\tau b_M)\frac{\int_{b_L}^{A} + N\int_{\tau b_L}^{A}}{\int_{b_M}^{A} + N\int_{\tau b_M}^{A}} - b_L^2 g(\tau b_L)\right) > 0,\ \frac{\partial J_2}{\partial N} = I_L\left(\frac{\int_{\tau b_L}^{\tau b_M}\int_{b_M}^{A} - \int_{b_L}^{b_M}\int_{\tau b_M}^{A}}{\int_{b_M}^{A} + N\int_{\tau b_M}^{A}}\right) > 0.
\end{aligned}$$

Changes in Transportation Costs

From (60) and (61),

$$\frac{\partial J_1}{\partial b_M}\frac{\partial b_M}{\partial \tau} + \frac{\partial J_1}{\partial b_L}\frac{\partial b_L}{\partial \tau} = -\frac{\partial J_1}{\partial \tau} \text{ and } \frac{\partial J_2}{\partial b_M}\frac{\partial b_M}{\partial \tau} + \frac{\partial J_2}{\partial b_L}\frac{\partial b_L}{\partial \tau} = -\frac{\partial J_2}{\partial \tau}.$$

Define D as $\frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L}$. Notice that $D > 0$. Then,

$$\frac{\partial b_M}{\partial \tau} = \frac{-\frac{\partial J_1}{\partial \tau}\frac{\partial J_2}{\partial b_L} + \frac{\partial J_2}{\partial \tau}\frac{\partial J_1}{\partial b_L}}{D} \text{ and } \frac{\partial b_L}{\partial \tau} = \frac{-\frac{\partial J_2}{\partial \tau}\frac{\partial J_1}{\partial b_M} + \frac{\partial J_1}{\partial \tau}\frac{\partial J_2}{\partial b_M}}{D}.$$

[59] Remember that $b^2 g(b)$ is increasing in b.

Proof of Lemma 3 Consider $(\tau b_M)'_\tau = b_M + \tau \frac{\partial b_M}{\partial \tau}$. This can be rewritten as

$$(\tau b_M)'_\tau = \frac{b_M D + \tau\left(-\frac{\partial J_1}{\partial \tau}\frac{\partial J_2}{\partial b_L} + \frac{\partial J_2}{\partial \tau}\frac{\partial J_1}{\partial b_L}\right)}{D}.$$

Since $D > 0$, one needs to determine the sign of the numerator:

$$b_M D + \tau\left(-\frac{\partial J_1}{\partial \tau}\frac{\partial J_2}{\partial b_L} + \frac{\partial J_2}{\partial \tau}\frac{\partial J_1}{\partial b_L}\right) = \frac{\partial J_2}{\partial b_L}\left(b_M\frac{\partial J_1}{\partial b_M} - \tau\frac{\partial J_1}{\partial \tau}\right) + \frac{\partial J_1}{\partial b_L}\left(\tau\frac{\partial J_2}{\partial \tau} - b_M\frac{\partial J_2}{\partial b_M}\right).$$

Using the expressions for $\frac{\partial J_i}{\partial b_L}$, $\frac{\partial J_i}{\partial b_M}$, and $\frac{\partial J_i}{\partial \tau}$, it is possible to show

$$\begin{aligned}
b_M\frac{\partial J_1}{\partial b_M} - \tau\frac{\partial J_1}{\partial \tau} &= -(1-\alpha_H)\left(N\tau H(\tau b_M) + \frac{\int_{b_M}^A}{b_M}\right) + N\tau\left(1 - \alpha_H G(\tau b_L)\right),\\
\tau\frac{\partial J_2}{\partial \tau} - b_M\frac{\partial J_2}{\partial b_M} &= -I_L\left(N\tau^2 b_L^2 g(\tau b_L) + b_M^2 g(b_M)\left(\frac{\int_{b_L}^A + N\int_{\tau b_L}^A}{\int_{b_M}^A + N\int_{\tau b_M}^A}\right) + \frac{b_L(1-\alpha_H)}{b_M}\frac{\int_{b_L}^{b_M} + N\int_{\tau b_L}^{\tau b_M}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}}\right).
\end{aligned}$$

Plugging this into the numerator and using the expressions for $\frac{\partial J_1}{\partial b_L}$ and $\frac{\partial J_2}{\partial b_L}$, I obtain that the numerator is equal to

$$\begin{aligned}
&I_L\frac{(1-\alpha_H)}{b_M}\frac{\int_{b_L}^{b_M} + N\int_{\tau b_L}^{\tau b_M}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}}\left(\frac{\alpha_H\int_{b_L}^A}{b_L} + \frac{(1-\alpha_H)\int_{b_M}^A}{b_M} + N\tau\left(\alpha_H H(\tau b_L) + (1-\alpha_H)H(\tau b_M) - 1\right)\right)\\
&+I_L N\tau^2 b_L g(\tau b_L)\left(\frac{\alpha_H\int_{b_L}^A}{b_L} + \frac{(1-\alpha_H)\int_{b_M}^A}{b_M} + N\tau\left(\alpha_H H(\tau b_L) + (1-\alpha_H)H(\tau b_M) - 1\right)\right)\\
&+I_L\frac{\alpha_H b_M^2 g(b_M)}{b_L^2}\frac{\left(\int_{b_L}^A + N\int_{\tau b_L}^A\right)^2}{\int_{b_M}^A + N\int_{\tau b_M}^A} - I_L b_L g(b_L)\left(-(1-\alpha_H)\left(N\tau H(\tau b_M) + \frac{\int_{b_M}^A}{b_M}\right) + N\tau\left(1-\alpha_H G(\tau b_L)\right)\right).
\end{aligned}$$

Remember that $H(x) = G(x) + \frac{\int_x^A}{x} \geq 1$ for any $x \in [0, A]$. This means that $\alpha_H H(\tau b_L) + (1-\alpha_H)H(\tau b_M) - 1 > 0$. Recall that $b_L < b_M$. Hence, the assumption that $b^2 g(b)$ is increasing in b implies that $b_M^2 g(b_M) > b_L^2 g(b_L)$. Moreover, $\frac{\int_{b_L}^A + N\int_{\tau b_L}^A}{\int_{b_M}^A + N\int_{\tau b_M}^A} > 1$. As a result,

$$\begin{aligned}
&I_L\frac{\alpha_H b_M^2 g(b_M)}{b_L^2}\frac{\left(\int_{b_L}^A + N\int_{\tau b_L}^A\right)^2}{\int_{b_M}^A + N\int_{\tau b_M}^A} - I_L b_L g(b_L)\left(-(1-\alpha_H)\left(N\tau H(\tau b_M) + \frac{\int_{b_M}^A}{b_M}\right) + N\tau\left(1-\alpha_H G(\tau b_L)\right)\right)\\
> \ &I_L g(b_L) b_L\left(\frac{\alpha_H\left(\int_{b_L}^A + N\int_{\tau b_L}^A\right)}{b_L} + (1-\alpha_H)\left(N\tau H(\tau b_M) + \frac{\int_{b_M}^A}{b_M}\right) - N\tau\left(1-\alpha_H G(\tau b_L)\right)\right) > 0.
\end{aligned}$$

Hence, $b_M D + \tau\left(-\frac{\partial J_1}{\partial \tau}\frac{\partial J_2}{\partial b_L} + \frac{\partial J_2}{\partial \tau}\frac{\partial J_1}{\partial b_L}\right) > 0$, which in turn implies that $(\tau b_M)'_\tau > 0$.

Similarly,

$$(\tau b_L)'_\tau = \frac{b_L D + \tau\left(-\frac{\partial J_2}{\partial \tau}\frac{\partial J_1}{\partial b_M} + \frac{\partial J_1}{\partial \tau}\frac{\partial J_2}{\partial b_M}\right)}{D}.$$

Again, it is necessary to determine the sign of

$$b_L D + \tau\left(-\frac{\partial J_2}{\partial \tau}\frac{\partial J_1}{\partial b_M} + \frac{\partial J_1}{\partial \tau}\frac{\partial J_2}{\partial b_M}\right) = \frac{\partial J_1}{\partial b_M}\left(b_L\frac{\partial J_2}{\partial b_L} - \tau\frac{\partial J_2}{\partial \tau}\right) + \frac{\partial J_2}{\partial b_M}\left(\tau\frac{\partial J_1}{\partial \tau} - b_L\frac{\partial J_1}{\partial b_L}\right).$$

Using the same technique as before, one can show that $b_L D+\tau\left(-\frac{\partial J_2}{\partial\tau}\frac{\partial J_1}{\partial b_M}+\frac{\partial J_1}{\partial\tau}\frac{\partial J_2}{\partial b_M}\right)$ equals to

$$
\begin{aligned}
& I_L\frac{b_L(1-\alpha_H)}{b_M^2}\frac{\int_{b_L}^{b_M}+N\int_{\tau b_L}^{\tau b_M}}{\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}}\left(\frac{(1-\alpha_H)\int_{b_M}^A}{b_M}+\alpha_H\frac{\int_{b_L}^A}{b_L}+N\tau\left((1-\alpha_H)H(\tau b_M)+\alpha_H H(\tau b_L)-1\right)\right)\\
&+I_L N\tau^2 b_M g(\tau b_M)\frac{\int_{b_L}^A+N\int_{\tau b_L}^A}{\int_{b_M}^A+N\int_{\tau b_M}^A}\left(\frac{(1-\alpha_H)\int_{b_M}^A}{b_M}+\alpha_H\frac{\int_{b_L}^A}{b_L}+N\tau\left((1-\alpha_H)H(\tau b_M)+\alpha_H H(\tau b_L)-1\right)\right)\\
&+\frac{I_L(1-\alpha_H)\left(\int_{b_M}^A+N\int_{\tau b_M}^A\right)}{b_M^2}b_L^2 g(b_L)\\
&+I_L b_M g(b_M)\frac{\int_{b_L}^A+N\int_{\tau b_L}^A}{\int_{b_M}^A+N\int_{\tau b_M}^A}\left(\alpha_H N\tau H(\tau b_L)+\alpha_H\frac{\int_{b_L}^A}{b_L}-N\tau\left(1-(1-\alpha_H)G(\tau b_M)\right)\right).
\end{aligned}
$$

Taking into account that $(\tau b_M)^2 g(\tau b_M)\geq b_M^2 g(b_M)$, one can derive

$$
\begin{aligned}
& I_L N\tau^2 b_M g(\tau b_M)\frac{\int_{b_L}^A+N\int_{\tau b_L}^A}{\int_{b_M}^A+N\int_{\tau b_M}^A}\left(\frac{(1-\alpha_H)\int_{b_M}^A}{b_M}+\alpha_H\frac{\int_{b_L}^A}{b_L}+N\tau\left((1-\alpha_H)H(\tau b_M)+\alpha_H H(\tau b_L)-1\right)\right)\\
&+I_L b_M g(b_M)\frac{\int_{b_L}^A+N\int_{\tau b_L}^A}{\int_{b_M}^A+N\int_{\tau b_M}^A}\left(\alpha_H N\tau H(\tau b_L)+\alpha_H\frac{\int_{b_L}^A}{b_L}-N\tau\left(1-(1-\alpha_H)G(\tau b_M)\right)\right)\\
\geq\ & I_L N b_M g(b_M)\frac{\int_{b_L}^A+N\int_{\tau b_L}^A}{\int_{b_M}^A+N\int_{\tau b_M}^A}\left(\frac{(1-\alpha_H)\int_{b_M}^A}{b_M}+\alpha_H\frac{\int_{b_L}^A}{b_L}+N\tau\left((1-\alpha_H)H(\tau b_M)+\alpha_H H(\tau b_L)-1\right)\right)\\
&+I_L b_M g(b_M)\frac{\int_{b_L}^A+N\int_{\tau b_L}^A}{\int_{b_M}^A+N\int_{\tau b_M}^A}\left(\alpha_H N\tau H(\tau b_L)+\alpha_H\frac{\int_{b_L}^A}{b_L}-N\tau\left(1-(1-\alpha_H)G(\tau b_M)\right)\right).
\end{aligned}
$$

Note that

$$
\begin{aligned}
0 &< N\left(\frac{(1-\alpha_H)\int_{b_M}^A}{b_M}+\alpha_H\frac{\int_{b_L}^A}{b_L}+N\tau\left((1-\alpha_H)H(\tau b_M)+\alpha_H H(\tau b_L)-1\right)\right)\\
&\quad+\alpha_H N\tau H(\tau b_L)+\alpha_H\frac{\int_{b_L}^A}{b_L}-N\tau\left(1-(1-\alpha_H)G(\tau b_M)\right).
\end{aligned}
$$

Thus, I show that $b_L D+\tau\left(-\frac{\partial J_2}{\partial\tau}\frac{\partial J_1}{\partial b_M}+\frac{\partial J_1}{\partial\tau}\frac{\partial J_2}{\partial b_M}\right)>0$.

Recall that $\frac{\partial b_M}{\partial\tau}=\frac{-\frac{\partial J_1}{\partial\tau}\frac{\partial J_2}{\partial b_L}+\frac{\partial J_2}{\partial\tau}\frac{\partial J_1}{\partial b_L}}{D}$. From the expressions for $\frac{\partial J_i}{\partial b_L}$ and $\frac{\partial J_i}{\partial\tau}$, $\frac{\partial J_1}{\partial\tau}<0$, $\frac{\partial J_2}{\partial b_L}<0$, $\frac{\partial J_2}{\partial\tau}>0$, and $\frac{\partial J_1}{\partial b_L}<0$. This results in the negative sign of $\frac{\partial b_M}{\partial\tau}$, i.e., given an increase in τ, b_M falls.

At the same time, the impact of an increase in τ on b_L is ambiguous. Remember that $\frac{\partial b_L}{\partial\tau}=\frac{-\frac{\partial J_2}{\partial\tau}\frac{\partial J_1}{\partial b_M}+\frac{\partial J_1}{\partial\tau}\frac{\partial J_2}{\partial b_M}}{D}$. Consider the numerator of the expression:

$$
\begin{aligned}
&-\frac{\partial J_2}{\partial\tau}\frac{\partial J_1}{\partial b_M}+\frac{\partial J_1}{\partial\tau}\frac{\partial J_2}{\partial b_M}\\
=\ & I_L N\frac{\tau(1-\alpha_H)}{b_M^2}\left(b_M^2 g(\tau b_M)\left(\int_{b_L}^A+N\int_{\tau b_L}^A\right)-b_L^2 g(\tau b_L)\left(\int_{b_M}^A+N\int_{\tau b_M}^A\right)\right)\\
&-I_L N\frac{\int_{b_L}^A+N\int_{\tau b_L}^A}{\int_{b_M}^A+N\int_{\tau b_M}^A}\left(1-\alpha_H G(\tau b_L)-(1-\alpha_H)G(\tau b_M)\right)\left(b_M g(b_M)+N\tau^2 b_M g(\tau b_M)\right)\\
&-I_L N\frac{b_L(1-\alpha_H)}{b_M^2}\frac{\int_{b_L}^{b_M}+N\int_{\tau b_L}^{\tau b_M}}{\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}}\left(1-\alpha_H G(\tau b_L)-(1-\alpha_H)G(\tau b_M)\right).
\end{aligned}
$$

In general, $-\frac{\partial J_2}{\partial \tau}\frac{\partial J_1}{\partial b_M}+\frac{\partial J_1}{\partial \tau}\frac{\partial J_2}{\partial b_M}$ can be positive or negative. If α_H is close to one then $-\frac{\partial J_2}{\partial \tau}\frac{\partial J_1}{\partial b_M}+\frac{\partial J_1}{\partial \tau}\frac{\partial J_2}{\partial b_M}$ is close to

$$-I_L N\frac{\int_{b_L}^{A}+N\int_{\tau b_L}^{A}}{\int_{b_M}^{A}+N\int_{\tau b_M}^{A}}\left(1-G(\tau b_L)\right)\left(b_M g(b_M)+N\tau^2 b_M g(\tau b_M)\right)<0.$$

If α_H is close to zero then $-\frac{\partial J_2}{\partial \tau}\frac{\partial J_1}{\partial b_M}+\frac{\partial J_1}{\partial \tau}\frac{\partial J_2}{\partial b_M}$ is close to

$$\begin{aligned}&I_L N\frac{\tau}{b_M^2}\left(b_M^2 g(\tau b_M)\left(\int_{b_L}^{A}+N\int_{\tau b_L}^{A}\right)-b_L^2 g(\tau b_L)\left(\int_{b_M}^{A}+N\int_{\tau b_M}^{A}\right)\right)\\&-I_L N\left(1-G(\tau b_M)\right)\left(\left(b_M g(b_M)+N\tau^2 b_M g(\tau b_M)\right)\frac{\int_{b_L}^{A}+N\int_{\tau b_L}^{A}}{\int_{b_M}^{A}+N\int_{\tau b_M}^{A}}+\frac{\int_{b_L}^{b_M}+N\int_{\tau b_L}^{\tau b_M}}{b_M}\right).\end{aligned}$$

The sign of last expression is not determined in general. For example, if τ is high enough (τb_M is close to A)then it is positive, while if τ is close to one then it can be negative. Thus, given sufficiently low α_H, b_L has a U shape or increasing as a function of τ, while given sufficiently high α_H, b_L is always decreasing. A number of simulations I conduct for a wide range of parameters confirm the findings above.

Proof of Proposition 2 To examine the sign of $\left(\frac{W_r}{W_p}\right)'_\tau$, one needs to analyze the sign of $\left(\frac{b_L}{b_M}\right)'_\tau=\frac{\frac{\partial b_L}{\partial \tau}b_M-\frac{\partial b_M}{\partial \tau}b_L}{b_M^2}$. Simple algebra shows that the sign of $\frac{\partial b_L}{\partial \tau}b_M-\frac{\partial b_M}{\partial \tau}b_L$ is the same as the sign of $\frac{\partial J_1}{\partial \tau}\left(\frac{\partial J_2}{\partial b_M}b_M+\frac{\partial J_2}{\partial b_L}b_L\right)-\frac{\partial J_2}{\partial \tau}\left(\frac{\partial J_1}{\partial b_M}b_M+\frac{\partial J_1}{\partial b_L}b_L\right)$. Using the expressions for $\frac{\partial J_i}{\partial \tau}$, $\frac{\partial J_i}{\partial b_M}$, and $\frac{\partial J_i}{\partial b_L}$, I derive

$$\begin{aligned}\frac{\partial J_2}{\partial b_M}b_M+\frac{\partial J_2}{\partial b_L}b_L &= I_L\left(\left(b_M^2 g(b_M)+\tau^2 b_M^2 g(\tau b_M)\right)\frac{\int_{b_L}^{A}+N\int_{\tau b_L}^{A}}{\int_{b_M}^{A}+N\int_{\tau b_M}^{A}}-b_L^2 g(b_L)-\tau^2 b_L^2 g(\tau b_L)\right)\\ \frac{\partial J_1}{\partial b_M}b_M+\frac{\partial J_1}{\partial b_L}b_L &= -\frac{(1-\alpha_H)\left(\int_{b_M}^{A}+N\int_{\tau b_M}^{A}\right)}{b_M}-\frac{\alpha_H\left(\int_{b_L}^{A}+N\int_{\tau b_L}^{A}\right)}{b_L}.\end{aligned}$$

Thus, one needs to examine the sign of

$$\begin{aligned}&I_L N\left(\tau b_M^2 g(\tau b_M)\frac{\int_{b_L}^{A}+N\int_{\tau b_L}^{A}}{\int_{b_M}^{A}+N\int_{\tau b_M}^{A}}-\tau b_L^2 g(\tau b_L)\right)\tau\left(\alpha_H G(\tau b_L)+(1-\alpha_H)G(\tau b_M)-1\right)\\&+I_L N\left(\tau b_M^2 g(\tau b_M)\frac{\int_{b_L}^{A}+N\int_{\tau b_L}^{A}}{\int_{b_M}^{A}+N\int_{\tau b_M}^{A}}-\tau b_L^2 g(\tau b_L)\right)\left(\frac{(1-\alpha_H)\left(\int_{b_M}^{A}+N\int_{\tau b_M}^{A}\right)}{b_M}+\frac{\alpha_H\left(\int_{b_L}^{A}+N\int_{\tau b_L}^{A}\right)}{b_L}\right)\\&-I_L N\left(1-\alpha_H G(\tau b_L)-(1-\alpha_H)G(\tau b_M)\right)\left(b_M^2 g(b_M)\frac{\int_{b_L}^{A}+N\int_{\tau b_L}^{A}}{\int_{b_M}^{A}+N\int_{\tau b_M}^{A}}-b_L^2 g(b_L)\right).\end{aligned}$$

Remember that $b^2 g(b)$ is increasing and convex in b. As $\tau b_M^2 g(\tau b_M)-b_M^2 g(b_M)>0$,

$$\begin{aligned}&\frac{\int_{b_L}^{A}+N\int_{\tau b_L}^{A}}{\int_{b_M}^{A}+N\int_{\tau b_M}^{A}}\left(\tau b_M^2 g(\tau b_M)-b_M^2 g(b_M)\right)-\left(\tau b_L^2 g(\tau b_L)-b_L^2 g(b_L)\right)\\ > &\left(\tau b_M^2 g(\tau b_M)-b_M^2 g(b_M)\right)-\left(\tau b_L^2 g(\tau b_L)-b_L^2 g(b_L)\right)>0.\end{aligned}$$

Since $N \geq 1$,

$$\begin{aligned}
& \tau\left(\alpha_H G(\tau b_L)+(1-\alpha_H)G(\tau b_M)-1\right)+\frac{(1-\alpha_H)\left(\int_{b_M}^{A}+N\int_{\tau b_M}^{A}\right)}{b_M}+\frac{\alpha_H\left(\int_{b_L}^{A}+N\int_{\tau b_L}^{A}\right)}{b_L} \\
> \; & \tau\left(\alpha_H H(\tau b_L)+(1-\alpha_H)H(\tau b_M)-1\right)+\frac{(1-\alpha_H)\int_{b_M}^{A}}{b_M}+\frac{\alpha_H\int_{b_L}^{A}}{b_L} \\
> \; & \frac{(1-\alpha_H)\int_{b_M}^{A}}{b_M}+\frac{\alpha_H\int_{b_L}^{A}}{b_L} > 1-\alpha_H G(\tau b_L)-(1-\alpha_H)G(\tau b_M).
\end{aligned}$$

Thus, $\left(\frac{b_L}{b_M}\right)'_\tau > 0$.

Welfare Issues (the proofs for footnote 35) Consider the following expression: $\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}$. I prove that $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_\tau > 0$. This in turn implies that $(W_p)'_\tau < 0$.

$$\begin{aligned}
\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_\tau &= -\frac{\alpha_H}{b_L^2}\frac{\partial b_L}{\partial \tau}-\frac{(1-\alpha_H)}{b_M^2}\frac{\partial b_M}{\partial \tau} \\
&= \frac{\frac{\partial J_1}{\partial \tau}\left(\frac{(1-\alpha_H)}{b_M^2}\frac{\partial J_2}{\partial b_L}-\frac{\alpha_H}{b_L^2}\frac{\partial J_2}{\partial b_M}\right)+\frac{\partial J_2}{\partial \tau}\left(\frac{\alpha_H}{b_L^2}\frac{\partial J_1}{\partial b_M}-\frac{(1-\alpha_H)}{b_M^2}\frac{\partial J_1}{\partial b_L}\right)}{D}.
\end{aligned}$$

Note that $\frac{\alpha_H}{b_L^2}\frac{\partial J_1}{\partial b_M}-\frac{(1-\alpha_H)}{b_M^2}\frac{\partial J_1}{\partial b_L} > 0$. Since $\frac{\partial J_1}{\partial \tau} < 0$, $\frac{\partial J_2}{\partial \tau} > 0$, $\frac{\partial J_2}{\partial b_L} < 0$, and $\frac{\partial J_2}{\partial b_M} > 0$, $\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)'_\tau > 0$. Thus, $(W_p)'_\tau < 0$.

Aggregate welfare per capita in a country is given by $W_a = \alpha_H W_r + (1-\alpha_H)W_p = W_p + \alpha_H (I_H - I_L) V_E$. This can be rewritten as

$$W_a = \frac{1}{c}\left(\frac{I_L}{\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}}+\alpha_H (I_H-I_L) b_L\right), \text{ and}$$

$$(W_a)'_\tau = \frac{I_L\left(\frac{\partial J_1}{\partial \tau}\left(P_1\frac{\alpha_H}{b_L^2}\frac{\partial J_2}{\partial b_M}-\frac{(1-\alpha_H)}{b_M^2}\frac{\partial J_2}{\partial b_L}\right)+\frac{\partial J_2}{\partial \tau}\left(\frac{(1-\alpha_H)}{b_M^2}\frac{\partial J_1}{\partial b_L}-P_1\frac{\alpha_H}{b_L^2}\frac{\partial J_1}{\partial b_M}\right)\right)}{cD\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)^2},$$

where $P_1 = 1+\frac{\int_{b_L}^{b_M}+N\int_{\tau b_L}^{\tau b_M}}{\int_{b_M}^{A}+N\int_{\tau b_M}^{A}}\left(\alpha_H+\frac{(1-\alpha_H)b_L}{b_M}\right)$. I show that $\frac{(1-\alpha_H)}{b_M^2}\frac{\partial J_1}{\partial b_L}-P_1\frac{\alpha_H}{b_L^2}\frac{\partial J_1}{\partial b_M} < 0$. This implies that $(W_a)'_\tau < 0$. Aggregate welfare per capita falls with higher τ.

What about the welfare of the rich? $W_r = W_p + (I_H - I_L)V_E = \frac{1}{c}\left(\frac{I_L}{\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}}+(I_H-I_L)b_L\right)$, and

$$(W_r)'_\tau = \frac{I_L\left(\frac{\partial J_1}{\partial \tau}\left(P_2\frac{\alpha_H}{b_L^2}\frac{\partial J_2}{\partial b_M}-\frac{(1-\alpha_H)}{b_M^2}\frac{\partial J_2}{\partial b_L}\right)+\frac{\partial J_2}{\partial \tau}\left(\frac{(1-\alpha_H)}{b_M^2}\frac{\partial J_1}{\partial b_L}-P_2\frac{\alpha_H}{b_L^2}\frac{\partial J_1}{\partial b_M}\right)\right)}{cD\left(\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)^2}, \tag{62}$$

where $P_2 = 1+\frac{1}{\alpha_H}\frac{\int_{b_L}^{b_M}+N\int_{\tau b_L}^{\tau b_M}}{\int_{b_M}^{A}+N\int_{\tau b_M}^{A}}\left(\alpha_H+\frac{(1-\alpha_H)b_L}{b_M}\right)$. The impact of an increase in τ on W_r is ambiguous. To show this, I plug the expressions for $\frac{\partial J_i}{\partial \tau}$, $\frac{\partial J_i}{\partial b_M}$, and $\frac{\partial J_i}{\partial b_L}$ into (62). After some simplifications, I show that the sign of the numerator in (62) is the same as the sign of the following expression:

$$\begin{aligned}
&(\alpha_H G(\tau b_L)+(1-\alpha_H)G(\tau b_M)-1)\left(b_M g(b_M)+N\tau^2 b_M g(\tau b_M)\right)P_2\frac{\alpha_H}{b_L^2}\frac{\int_{b_L}^A+N\int_{\tau b_L}^A}{\int_{b_M}^A+N\int_{\tau b_M}^A}\\
&+(\alpha_H G(\tau b_L)+(1-\alpha_H)G(\tau b_M)-1)\left(b_L g(b_L)+N\tau^2 b_L g(\tau b_L)\right)\frac{(1-\alpha_H)}{b_M^2}\\
&+(\alpha_H G(\tau b_L)+(1-\alpha_H)G(\tau b_M)-1)\frac{(1-\alpha_H)}{b_M^2}\frac{\int_{b_L}^{b_M}+N\int_{\tau b_L}^{\tau b_M}}{\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}}\left(P_2\frac{\alpha_H}{b_L}+\frac{(1-\alpha_H)}{b_M}\right)\\
&+\left(\tau b_M^2 g(\tau b_M)\frac{\int_{b_L}^A+N\int_{\tau b_L}^A}{\int_{b_M}^A+N\int_{\tau b_M}^A}-\tau b_L^2 g(\tau b_L)\right)\frac{(1-\alpha_H)^2\left(\int_{b_L}^{b_M}+N\int_{\tau b_L}^{\tau b_M}\right)}{b_M^3 b_L}.
\end{aligned}$$

In general, the sign of this expression can be either positive or negative. If α_H is close to one or incomes of the poor and the rich are close to each other then the sign is negative[60]. However, if α_H is close to zero and the difference between incomes of the poor and the rich is significant enough then the sign can be positive[61]. A number of simulations I conduct for a wide range of parameters support these results.

Proof of Lemma 4 Similarly to the previous analysis,

$$\frac{\partial b_M}{\partial N}=\frac{-\frac{\partial J_1}{\partial N}\frac{\partial J_2}{\partial b_L}+\frac{\partial J_2}{\partial N}\frac{\partial J_1}{\partial b_L}}{D}\text{ and }\frac{\partial b_L}{\partial N}=\frac{-\frac{\partial J_2}{\partial N}\frac{\partial J_1}{\partial b_M}+\frac{\partial J_1}{\partial N}\frac{\partial J_2}{\partial b_M}}{D}.$$

Note that $\frac{\partial b_L}{\partial N}>0$. Consider the sign of $-\frac{\partial J_1}{\partial N}\frac{\partial J_2}{\partial b_L}+\frac{\partial J_2}{\partial N}\frac{\partial J_1}{\partial b_L}$, which is the same as the sign of

$$\begin{aligned}
&I_L\tau\left(\alpha_H H(\tau b_L)+(1-\alpha_H)H(\tau b_M)-1\right)\left(b_L g(b_L)+N\tau^2 b_L g(\tau b_L)+\frac{(1-\alpha_H)}{b_M}\frac{\int_{b_L}^{b_M}+N\int_{\tau b_L}^{\tau b_M}}{\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}}\right)\\
&-I_L\left(\frac{\int_{\tau b_L}^{\tau b_M}\int_{b_M}^A-\int_{b_L}^{b_M}\int_{\tau b_M}^A}{\int_{b_M}^A+N\int_{\tau b_M}^A}\right)\frac{\alpha_H}{b_L^2}\left(\int_{b_L}^A+N\int_{\tau b_L}^A\right).
\end{aligned}$$

If α_H is close to zero then $\frac{\partial b_M}{\partial N}$ is likely to be greater than zero. However, if α_H is close to one then it is possible that $\frac{\partial b_M}{\partial N}<0$. For instance, if b_L is close to zero and α_H is close to one then $\frac{\partial b_M}{\partial N}<0$.

Finally, the sign of $\left(\frac{b_L}{b_M}\right)'_N$ is the same as the sign of $\frac{\partial J_1}{\partial N}\left(\frac{\partial J_2}{\partial b_M}b_M+\frac{\partial J_2}{\partial b_L}b_L\right)-\frac{\partial J_2}{\partial N}\left(\frac{\partial J_1}{\partial b_M}b_M+\frac{\partial J_1}{\partial b_L}b_L\right)$. Since $\frac{\partial J_i}{\partial N}>0$, $\frac{\partial J_2}{\partial b_M}b_M+\frac{\partial J_2}{\partial b_L}b_L>0$, and $\frac{\partial J_1}{\partial b_M}b_M+\frac{\partial J_1}{\partial b_L}b_L<0$, $\left(\frac{b_L}{b_M}\right)'_N>0$.

Part 2

Here I consider the case when $\tau b_M\geq A$ in the equilibrium. Then,

$$\begin{cases}\frac{\int_{b_L}^{b_M}tdG(t)+N\int_{\tau b_L}^A tdG(t)}{\int_{b_M}^A tdG(t))}=\left(\frac{I_H}{I_L}-1\right)\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right),\\ \frac{f_e}{cL}+1+\alpha_H N\tau=\alpha_H\left(H(b_L)+N\tau H(\tau b_L)\right)+(1-\alpha_H)H(b_M).\end{cases}$$

[60] This implies that b_L is close to b_M.

[61] This condition guarantees that $\frac{I_H}{I_L}$ is high enough. This implies that τb_M is close to one (see Tarasov (2007) for details).

Again, let

$$
\begin{aligned}
J_1 &= \alpha_H \left(H(b_L) + N\tau H(\tau b_L) \right) + (1-\alpha_H) H(b_M) - \frac{f_e}{cL} - 1 - \alpha_H N\tau, \\
J_2 &= I_L \left(\int_{b_L}^{b_M} + N \int_{\tau b_L}^{A} \right) - (I_H - I_L) \left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M} \right) \int_{b_M}^{A}.
\end{aligned}
$$

Then, algebra shows

$$
\begin{aligned}
\frac{\partial J_1}{\partial b_M} &= -\frac{(1-\alpha_H)}{b_M^2} \int_{b_M}^{A} < 0,\ \frac{\partial J_1}{\partial b_L} = -\frac{\alpha_H}{b_L^2} \left(\int_{b_L}^{A} + N \int_{\tau b_L}^{A} \right) < 0, \\
\frac{\partial J_1}{\partial \tau} &= N\alpha_H \left(G(\tau b_L) - 1 \right) < 0,\ \frac{\partial J_1}{\partial N} = \tau \alpha_H \left(H(\tau b_L) - 1 \right) > 0, \\
\frac{\partial J_2}{\partial b_M} &= I_L b_M g(b_M) \frac{\int_{b_L}^{A} + N \int_{\tau b_L}^{A}}{\int_{b_M}^{A}} + I_L \frac{b_L(1-\alpha_H)}{b_M^2} \frac{\int_{b_L}^{b_M} + N \int_{\tau b_L}^{A}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}} > 0 \\
\frac{\partial J_2}{\partial b_L} &= -I_L b_L \left(g(b_L) + N\tau^2 g(\tau b_L) \right) - I_L \frac{(1-\alpha_H)}{b_M} \frac{\int_{b_L}^{b_M} + N \int_{\tau b_L}^{A}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}} < 0 \\
\frac{\partial J_2}{\partial \tau} &= -I_L N \tau b_L^2 g(\tau b_L) < 0,\ \frac{\partial J_2}{\partial N} = I_L \int_{\tau b_L}^{A} > 0.
\end{aligned}
$$

3.7.1 Proof of Proposition 2

From Part 1 of the Appendix the sign of $\left(\frac{W_r}{W_p}\right)'_\tau$ is the same as that of $\frac{\partial J_1}{\partial \tau}\left(\frac{\partial J_2}{\partial b_M} b_M + \frac{\partial J_2}{\partial b_L} b_L\right) - \frac{\partial J_2}{\partial \tau}\left(\frac{\partial J_1}{\partial b_M} b_M + \frac{\partial J_1}{\partial b_L} b_L\right)$. Using the expressions for $\frac{\partial J_i}{\partial \tau}$, $\frac{\partial J_i}{\partial b_M}$, and $\frac{\partial J_i}{\partial b_L}$,

$$
\begin{aligned}
\frac{\partial J_2}{\partial b_M} b_M + \frac{\partial J_2}{\partial b_L} b_L &= I_L \left(b_M^2 g(b_M) \frac{\int_{b_L}^{A} + N \int_{\tau b_L}^{A}}{\int_{b_M}^{A}} - b_L^2 g(b_L) - N\tau^2 b_L^2 g(\tau b_L) \right), \\
\frac{\partial J_1}{\partial b_M} b_M + \frac{\partial J_1}{\partial b_L} b_L &= -\frac{(1-\alpha_H)\int_{b_M}^{A}}{b_M} - \frac{\alpha_H \left(\int_{b_L}^{A} + N \int_{\tau b_L}^{A} \right)}{b_L}.
\end{aligned}
$$

Thus, one needs to examine the sign of

$$
\begin{aligned}
& I_L \left(b_M^2 g(b_M) \frac{\int_{b_L}^{A} + N \int_{\tau b_L}^{A}}{\int_{b_M}^{A}} - b_L^2 g(b_L) - N\tau^2 b_L^2 g(\tau b_L) \right) N\alpha_H \left(G(\tau b_L) - 1 \right) \\
& -I_L N \tau b_L^2 g(\tau b_L) \left(\frac{(1-\alpha_H)\int_{b_M}^{A}}{b_M} + \frac{\alpha_H \left(\int_{b_L}^{A} + N \int_{\tau b_L}^{A} \right)}{b_L} \right) \\
= \; & I_L N \alpha_H \left(b_M^2 g(b_M) \frac{\int_{b_L}^{A} + N \int_{\tau b_L}^{A}}{\int_{b_M}^{A}} - b_L^2 g(b_L) \right) \left(G(\tau b_L) - 1 \right) \\
& + I_L N \tau b_L^2 g(\tau b_L) \left(\alpha_H N\tau \left(1 - H(\tau b_L) \right) - \frac{(1-\alpha_H)\int_{b_M}^{A}}{b_M} - \frac{\alpha_H \int_{b_L}^{A}}{b_L} \right) < 0.
\end{aligned}
$$

Thus, $\left(\frac{W_r}{W_p}\right)'_\tau < 0$, and the rich lose more than the poor from higher τ.

3.7.2 Proof of Proposition 3

As in Part 1 of the Appendix, the sign of $\left(\frac{W_r}{W_p}\right)'_N$ is the same as the sign of $\frac{\partial J_1}{\partial N}\left(\frac{\partial J_2}{\partial b_M}b_M + \frac{\partial J_2}{\partial b_L}b_L\right) - \frac{\partial J_2}{\partial N}\left(\frac{\partial J_1}{\partial b_M}b_M + \frac{\partial J_1}{\partial b_L}b_L\right)$. Using the expressions for $\frac{\partial J_i}{\partial N}$, $\frac{\partial J_2}{\partial b_M}b_M + \frac{\partial J_2}{\partial b_L}b_L$, and $\frac{\partial J_1}{\partial b_M}b_M + \frac{\partial J_1}{\partial b_L}b_L$, one needs to examine the sign of

$$
\begin{aligned}
& I_L\tau\alpha_H\left(H(\tau b_L)-1\right)\left(b_M^2 g(b_M)\frac{\int_{b_L}^A + N\int_{\tau b_L}^A}{\int_{b_M}^A} - b_L^2 g(b_L) - N\tau^2 b_L^2 g(\tau b_L)\right) \\
& + I_L\int_{\tau b_L}^A\left(\frac{(1-\alpha_H)\int_{b_M}^A}{b_M} + \frac{\alpha_H\left(\int_{b_L}^A + N\int_{\tau b_L}^A\right)}{b_L}\right) \\
= \; & I_L\tau\alpha_H\left(H(\tau b_L)-1\right)\left(b_M^2 g(b_M)\frac{\int_{b_L}^A + N\int_{\tau b_L}^A}{\int_{b_M}^A} - b_L^2 g(b_L)\right) + I_L\int_{\tau b_L}^A\left(\frac{(1-\alpha_H)\int_{b_M}^A}{b_M} + \frac{\alpha_H\int_{b_L}^A}{b_L}\right) \\
& + I_L\alpha_H N\tau\left(\frac{\left(\int_{\tau b_L}^A\right)^2}{\tau b_L} - \left(H(\tau b_L)-1\right)\tau^2 b_L^2 g(\tau b_L)\right).
\end{aligned}
$$

Consider the function $K(x) = \frac{\left(\int_x^A\right)^2}{x} - (H(x)-1)\,x^2 g(x)$. I show that

$$K'(x) = \frac{-x^2 g(x)\int_x^A - \left(\int_x^A\right)^2}{x^2} - (H(x)-1)\left(2xg(x) + x^2 g'(x)\right).$$

Since $x^2 g(x)$ is increasing, $K'(x) < 0$. Moreover, $K(A) = 0$, so that $K(x) \geq 0$ for any $x \in [0, A]$. Thus, the sign of $\left(\frac{W_r}{W_p}\right)'_N$ is positive. The rich gain more than the poor from an increase in N.

Appendix C: Proofs and Robustness Check of the Results in Chapter 3

Proof of Step 1 in *Proposition 1*

At this stage of the proof, I show that $\theta_{1j} < \theta_{2j}$ for all $j > 2$. Specifically, I first show that if $\theta_{1j^*} < \theta_{2j^*}$ for some $j^* > 2$, then $\theta_{1j} < \theta_{2j}$ for all $j > 2$. Then, I prove that the equality (32) implies that $\theta_{1j} < \theta_{2j}$ at least for some $j > 2$. This finishes the proof.

From the zero profit condition (24),

$$\theta_{ij} = \frac{w_i}{Z_i} \tau_{ij} \left(\frac{1}{C} \frac{P_j^{1-\sigma}}{w_j L_j} \right)^{\frac{1}{\sigma-1}} f_{ij}^{\frac{1}{\sigma-1}}.$$

Substituting for f_{ij}, the cutoffs are given by

$$\theta_{11} = \frac{w_1}{Z_1} (w_1 f_d)^{\frac{1}{\sigma-1}} \left(\frac{1}{C} \frac{P_1^{1-\sigma}}{w_1 L_1} \right)^{\frac{1}{\sigma-1}}, \tag{63}$$

$$\theta_{22} = \frac{w_2}{Z_2} (w_2 f_d)^{\frac{1}{\sigma-1}} \left(\frac{1}{C} \frac{P_2^{1-\sigma}}{w_2 L_2} \right)^{\frac{1}{\sigma-1}}, \tag{64}$$

$$\theta_{ij} = \frac{w_i}{Z_i} (w_i f_d)^{\frac{1}{\sigma-1}} \left(1 + \frac{f_x}{f_d Z_i^\delta} \right)^{\frac{1}{\sigma-1}} \tau_{ij} \left(\frac{1}{C} \frac{P_j^{1-\sigma}}{w_j L_j} \right)^{\frac{1}{\sigma-1}} \quad \text{for } j > 2 \text{ and } i = 1, 2. \tag{65}$$

Given the expressions for the cutoffs, it is straightforward to show that if $\theta_{1j^*} < \theta_{2j^*}$ for some $j^* > 2$, then

$$\frac{w_1}{Z_1} (w_1 f_d)^{\frac{1}{\sigma-1}} \left(1 + \frac{f_x}{f_d Z_1^\delta} \right)^{\frac{1}{\sigma-1}} \tau_{1j} \left(\frac{1}{C} \frac{P_j^{1-\sigma}}{w_j L_j} \right)^{\frac{1}{\sigma-1}} < \frac{w_2}{Z_2} (w_2 f_d)^{\frac{1}{\sigma-1}} \left(1 + \frac{f_x}{f_d Z_2^\delta} \right)^{\frac{1}{\sigma-1}} \tau_{2j} \left(\frac{1}{C} \frac{P_j^{1-\sigma}}{w_j L_j} \right)^{\frac{1}{\sigma-1}}.$$

As we assume that $\tau_{1j} = \tau_{2j}$ for all $j > 2$, the last inequality implies that

$$\frac{w_1}{Z_1} (w_1 f_d)^{\frac{1}{\sigma-1}} \left(1 + \frac{f_x}{f_d Z_1^\delta} \right)^{\frac{1}{\sigma-1}} < \frac{w_2}{Z_2} (w_2 f_d)^{\frac{1}{\sigma-1}} \left(1 + \frac{f_x}{f_d Z_2^\delta} \right)^{\frac{1}{\sigma-1}}.$$

This in turn means that $\theta_{1j} < \theta_{2j}$ for all $j > 2$.

To reduce the notation in this subsection, I denote $\int_{\theta_{ij}}^{\theta_H} \left(\left(\frac{\theta}{\theta_{ij}} \right)^{\sigma-1} - 1 \right) dG(\theta)$ as $H(\theta_{ij})$. Notice that $H'(\theta_{ij}) < 0$. Given the new notation, the equality (32) can be rewritten as follows (recall that country 1 and 2 do not trade with each other):

$$\sum_{j>2}^{N} \frac{f_{1j}}{f_{e1}} H(\theta_{1j}) + \frac{f_{11}}{f_{e1}} H(\theta_{11}) = \sum_{j>2}^{N} \frac{f_{2j}}{f_{e2}} H(\theta_{2j}) + \frac{f_{22}}{f_{e2}} H(\theta_{22}).$$

Substituting for f_{e1}, f_{e2}, f_{1j}, f_{2j}, f_{11}, and f_{22} (see (28) and (27)), we have

$$\left(\frac{f_d}{f_e} + \frac{f_x}{Z_1^\delta} \right) \sum_{j>2}^{N} H(\theta_{1j}) + \frac{f_d}{f_e} H(\theta_{11}) = \left(\frac{f_d}{f_e} + \frac{f_x}{Z_2^\delta} \right) \sum_{j>2}^{N} H(\theta_{2j}) + \frac{f_d}{f_e} H(\theta_{22}). \tag{66}$$

To finish the proof, I consider $\frac{w_1}{Z_1} w_1^{\frac{1}{\sigma-1}}$ and $\frac{w_2}{Z_2} w_2^{\frac{1}{\sigma-1}}$. In general, we do not know whether $\frac{w_1}{Z_1} w_1^{\frac{1}{\sigma-1}}$ is greater or less than $\frac{w_2}{Z_2} w_2^{\frac{1}{\sigma-1}}$ in the equilibrium. Therefore, we need to examine two cases. First, if

$\frac{w_1}{Z_1}w_1^{\frac{1}{\sigma-1}} < \frac{w_2}{Z_2}w_2^{\frac{1}{\sigma-1}}$, then (as $Z_1 > Z_2$),

$$\frac{w_1}{Z_1}(w_1 f_d)^{\frac{1}{\sigma-1}}\left(1+\frac{f_x}{f_d Z_1^{\delta}}\right)^{\frac{1}{\sigma-1}} < \frac{w_2}{Z_2}(w_2 f_d)^{\frac{1}{\sigma-1}}\left(1+\frac{f_x}{f_d Z_2^{\delta}}\right)^{\frac{1}{\sigma-1}}.$$

From (65) and the fact that $\tau_{1j} = \tau_{2j}$, the last inequality results in that $\theta_{1j} < \theta_{2j}$ for all $j > 2$.

Second, if $\frac{w_1}{Z_1}w_1^{\frac{1}{\sigma-1}} \geq \frac{w_2}{Z_2}w_2^{\frac{1}{\sigma-1}}$, then (remember that $w_1 L_1 = w_2 L_2$, $P_2 \geq P_1$, and $\sigma > 1$)

$$\theta_{11} = \frac{w_1}{Z_1}(w_1 f_d)^{\frac{1}{\sigma-1}}\left(\frac{1}{C}\frac{P_1^{1-\sigma}}{w_1 L_1}\right)^{\frac{1}{\sigma-1}} \geq \frac{w_2}{Z_2}(w_2 f_d)^{\frac{1}{\sigma-1}}\left(\frac{1}{C}\frac{P_2^{1-\sigma}}{w_2 L_2}\right)^{\frac{1}{\sigma-1}} = \theta_{22}$$

Since $H'(\cdot) < 0$, we obtain that

$$\frac{f_d}{f_e}H(\theta_{11}) \leq \frac{f_d}{f_e}H(\theta_{22}).$$

This means that in order for (66) holds, θ_{1j} should be strictly less than θ_{2j} for some $j > 2$, implying that $\theta_{1j} < \theta_{2j}$ for all $j > 2$ (see the considerations before).

Proof of Step 2 in *Proposition 1*

In this subsection, I show that in the equilibrium,

$$\frac{\int_{\theta_{1j}}^{\theta_H}\theta^{\sigma-1}dG(\theta)}{\int_{\theta_{11}}^{\theta_H}\theta^{\sigma-1}dG(\theta)} > \frac{\int_{\theta_{2j}}^{\theta_H}\theta^{\sigma-1}dG(\theta)}{\int_{\theta_{22}}^{\theta_H}\theta^{\sigma-1}dG(\theta)} \text{ for all } j > 2. \tag{67}$$

To prove this result, I need the following technical lemma.

Lemma 1 *For any positive numbers A and B such that $A \geq B$, a function $J(x) = \frac{\int_{Ax}^{\theta_H}\theta^{\sigma-1}dG(\theta)}{\int_{Bx}^{\theta_H}\theta^{\sigma-1}dG(\theta)}$ is weakly decreasing in x.*

Proof. The proof directly follows from differentiating $J(x)$. Namely, $J'(x) \leq 0$ if and only if $\frac{x^{\sigma}g(x)}{\int_x^{\theta_H}t^{\sigma-1}dG(t)}$ is weakly increasing in x, which is assumed in the condition of the proposition. ■

Next, I use this lemma to show that (67) holds. To do so, we need to construct several additional variables. Specifically, we define

$$\begin{aligned} x_1 &= \frac{w_1}{Z_1}(w_1 f_d)^{\frac{1}{\sigma-1}}\left(1+\frac{f_x}{f_d Z_1^{\delta}}\right)^{\frac{1}{\sigma-1}} \quad \text{and} \\ x_2 &= \frac{w_2}{Z_2}(w_2 f_d)^{\frac{1}{\sigma-1}}\left(1+\frac{f_x}{f_d Z_2^{\delta}}\right)^{\frac{1}{\sigma-1}}. \end{aligned}$$

Note that the proof of Step 1 implies that $x_1 < x_2$ in the equilibrium (as $\theta_{1j} < \theta_{2j}$ for all $j > 2$). In addition, for arbitrary $j > 2$ we define $A = \tau_{1j}\left(\frac{1}{C}\frac{P_j^{1-\sigma}}{w_j L_j}\right)^{\frac{1}{\sigma-1}}$ and $B = \frac{\left(\frac{1}{C}\frac{P_1^{1-\sigma}}{w_1 L_1}\right)^{\frac{1}{\sigma-1}}}{\left(1+\frac{f_x}{f_d Z_1^{\delta}}\right)^{\frac{1}{\sigma-1}}}$. Recall that *Assumption 2* says that $\theta_{1j} \geq \theta_{11}$ for all $j > 2$. From (63) and (65), $\theta_{1j} \geq \theta_{11}$ is equivalent to

$$\begin{aligned} \frac{w_1}{Z_1}(w_1 f_d)^{\frac{1}{\sigma-1}}\left(1+\frac{f_x}{f_d Z_1^{\delta}}\right)^{\frac{1}{\sigma-1}}\tau_{1j}\left(\frac{1}{C}\frac{P_j^{1-\sigma}}{w_j L_j}\right)^{\frac{1}{\sigma-1}} &\geq \frac{w_1}{Z_1}(w_1 f_d)^{\frac{1}{\sigma-1}}\left(\frac{1}{C}\frac{P_1^{1-\sigma}}{w_1 L_1}\right)^{\frac{1}{\sigma-1}} \iff \\ \tau_{1j}\left(\frac{1}{C}\frac{P_j^{1-\sigma}}{w_j L_j}\right)^{\frac{1}{\sigma-1}} &\geq \frac{\left(\frac{1}{C}\frac{P_1^{1-\sigma}}{w_1 L_1}\right)^{\frac{1}{\sigma-1}}}{\left(1+\frac{f_x}{f_d Z_1^{\delta}}\right)^{\frac{1}{\sigma-1}}}. \end{aligned}$$

The last inequality implies that $A \geq B$.

Next, I apply *Lemma 1* for x_1, x_2, A, and B defined above. As $x_1 < x_2$, *Lemma 1* implies that $J(x_1) \geq J(x_2)$ or

$$\frac{\int_{Ax_1}^{\theta_H} \theta^{\sigma-1} dG(\theta)}{\int_{Bx_1}^{\theta_H} \theta^{\sigma-1} dG(\theta)} \geq \frac{\int_{Ax_2}^{\theta_H} \theta^{\sigma-1} dG(\theta)}{\int_{Bx_2}^{\theta_H} \theta^{\sigma-1} dG(\theta)}. \tag{68}$$

Note that from (63) and (65),

$$\begin{aligned} Ax_1 &= \frac{w_1}{Z_1} (w_1 f_d)^{\frac{1}{\sigma-1}} \left(1 + \frac{f_x}{f_d Z_1^\delta}\right)^{\frac{1}{\sigma-1}} \tau_{1j} \left(\frac{1}{C} \frac{P_j^{1-\sigma}}{w_j L_j}\right)^{\frac{1}{\sigma-1}} = \theta_{1j}, \\ Bx_1 &= \frac{\left(\frac{1}{C}\frac{P_1^{1-\sigma}}{w_1 L_1}\right)^{\frac{1}{\sigma-1}}}{\left(1 + \frac{f_x}{f_d Z_1^\delta}\right)^{\frac{1}{\sigma-1}}} \frac{w_1}{Z_1} (w_1 f_d)^{\frac{1}{\sigma-1}} \left(1 + \frac{f_x}{f_d Z_1^\delta}\right)^{\frac{1}{\sigma-1}} = \frac{w_1}{Z_1} (w_1 f_d)^{\frac{1}{\sigma-1}} \left(\frac{1}{C} \frac{P_1^{1-\sigma}}{w_1 L_1}\right)^{\frac{1}{\sigma-1}} = \theta_{11}. \end{aligned}$$

Moreover, from (65) and the assumption that $\tau_{1j} = \tau_{2j}$,

$$\begin{aligned} Ax_2 &= \tau_{1j} \left(\frac{1}{C} \frac{P_j^{1-\sigma}}{w_j L_j}\right)^{\frac{1}{\sigma-1}} \frac{w_2}{Z_2} (w_2 f_d)^{\frac{1}{\sigma-1}} \left(1 + \frac{f_x}{f_d Z_2^\delta}\right)^{\frac{1}{\sigma-1}} \\ &= \tau_{2j} \left(\frac{1}{C} \frac{P_j^{1-\sigma}}{w_j L_j}\right)^{\frac{1}{\sigma-1}} \frac{w_2}{Z_2} (w_2 f_d)^{\frac{1}{\sigma-1}} \left(1 + \frac{f_x}{f_d Z_2^\delta}\right)^{\frac{1}{\sigma-1}} \\ &= \theta_{2j}. \end{aligned}$$

Summarizing the findings above, we can rewrite (68) as follows:

$$\begin{aligned} \frac{\int_{Ax_1}^{\theta_H} \theta^{\sigma-1} dG(\theta)}{\int_{Bx_1}^{\theta_H} \theta^{\sigma-1} dG(\theta)} &\geq \frac{\int_{Ax_2}^{\theta_H} \theta^{\sigma-1} dG(\theta)}{\int_{Bx_2}^{\theta_H} \theta^{\sigma-1} dG(\theta)} \iff \\ \frac{\int_{\theta_{1j}}^{\theta_H} \theta^{\sigma-1} dG(\theta)}{\int_{\theta_{11}}^{\theta_H} \theta^{\sigma-1} dG(\theta)} &\geq \frac{\int_{\theta_{2j}}^{\theta_H} \theta^{\sigma-1} dG(\theta)}{\int_{Bx_2}^{\theta_H} \theta^{\sigma-1} dG(\theta)}. \end{aligned} \tag{69}$$

Finally, since $\frac{P_1^{1-\sigma}}{w_1 L_1} \geq \frac{P_2^{1-\sigma}}{w_2 L_2}$ and $1 + \frac{f_x}{f_d Z_1^\delta} < 1 + \frac{f_x}{f_d Z_2^\delta}$ (this is implied by $P_2 \geq P_1$ and $Z_1 > Z_2$),

$$Bx_2 = \frac{\left(\frac{1}{C}\frac{P_1^{1-\sigma}}{w_1 L_1}\right)^{\frac{1}{\sigma-1}}}{\left(1 + \frac{f_x}{f_d Z_1^\delta}\right)^{\frac{1}{\sigma-1}}} x_2 > \frac{\left(\frac{1}{C}\frac{P_2^{1-\sigma}}{w_2 L_2}\right)^{\frac{1}{\sigma-1}}}{\left(1 + \frac{f_x}{f_d Z_2^\delta}\right)^{\frac{1}{\sigma-1}}} x_2.$$

From (64),

$$\begin{aligned} \frac{\left(\frac{1}{C}\frac{P_2^{1-\sigma}}{w_2 L_2}\right)^{\frac{1}{\sigma-1}}}{\left(1 + \frac{f_x}{f_d Z_2^\delta}\right)^{\frac{1}{\sigma-1}}} x_2 &= \frac{\left(\frac{1}{C}\frac{P_2^{1-\sigma}}{w_2 L_2}\right)^{\frac{1}{\sigma-1}}}{\left(1 + \frac{f_x}{f_d Z_2^\delta}\right)^{\frac{1}{\sigma-1}}} \frac{w_2}{Z_2} (w_2 f_d)^{\frac{1}{\sigma-1}} \left(1 + \frac{f_x}{f_d Z_2^\delta}\right)^{\frac{1}{\sigma-1}} \\ &= \theta_{22}. \end{aligned}$$

That is,

$$Bx_2 > \theta_{22}.$$

Table 7: Robustness Checks: Parameter Estimates

	Main Model	Model: $k = 4$	Model: $\sigma = 3$
γ_0	0.49	0.20	0.40
γ_1	0.14	0.18	0.15
γ_2	0.76	0.67	0.69
γ_3	0.97	0.95	0.96
γ_4	0.78	0.69	0.67
γ_5	0.94	0.90	0.85
f_x	2.15	9.67	2.38
δ	0.67	0.65	0.49
R^2	81%	81%	81%

The last inequality and (69) immediately imply that

$$\frac{\int_{\theta_{1j}}^{\theta_H} \theta^{\sigma-1} dG(\theta)}{\int_{\theta_{11}}^{\theta_H} \theta^{\sigma-1} dG(\theta)} > \frac{\int_{\theta_{2j}}^{\theta_H} \theta^{\sigma-1} dG(\theta)}{\int_{\theta_{22}}^{\theta_H} \theta^{\sigma-1} dG(\theta)}.$$

Since the choice of j was arbitrary, the last inequality holds for all $j > 2$.

Robustness Checks

In this part of the Appendix, I check the robustness of the quantitative implications of the model to changes in the values of the unidentifiable parameters. Namely, I reestimate the model fixing the parameters $\{\sigma, k\}$ at different values. I do not report the effects of changes in $\{\theta_L, \theta_H, f_e, f_d\}$, as they do not entirely affect the explanatory power and quantitative implications of the model (the generated trade elasticities and "true" zeros are the same as those predicted by the main model).

I consider one by one changes in $\{\sigma, k\}$. That is, I change the value of one parameter fixing the others at the values set in the main body of the paper. In particular, I examine the following changes: σ falls from 3.8 to 3 and k rises from 3.4 to 4. *Table 7* reports the parameter estimates. As it can be seen, the changes in $\{\sigma, k\}$ lead to the different estimates of the parameters. However, the explanatory power remains the same. *Table 8* shows the trade elasticities and the percentage of "true" zeros predicted by the model. Given the new values of $\{\sigma, k\}$, the model predicts the same effect of population size on trade volumes as the main model does, but somewhat amplifies the impact of per capita income. Doubling a country income per capita (keeping the total income unaltered) leads to an increase in trade volumes of 31% ($\sigma = 3$) and of 27% ($k = 4$), while the main model predicts a rise in trade volumes of 22%.

Alternative Estimation Procedures

Another robustness check is to apply an alternative estimation procedure. The point is that non-linear least squares (NLLS) attach greater weights to observations with higher values. Therefore, if high value

Table 8: Robustness Checks: Trade Elasticities and Zeros

	Data	Main Model	Model: $\sigma = 3$	Model: $k = 4$
Dependent variable:	$\ln T_i$	$\ln T_i(\hat{\Theta})$	$\ln T_i(\hat{\Theta})$	$\ln T_i(\hat{\Theta})$
Log of GDP	0.85**	0.69**	0.68**	0.68**
	(0.03)	(0.03)	(0.03)	(0.03)
Log of GDP per capita	0.19**	0.22**	0.31**	0.27**
	(0.04)	(0.02)	(0.03)	(0.03)
The percentage of "true" zeros:	100%	35%	34%	35%
Observations:	100	100	100	100

observations have larger variance, then non-linear least squares may lead to inefficient estimates. I examine two alternative estimation procedures: restricted non-linear least deviations (NLLD) and restricted weighted non-linear least squares (WNLLS). Both of these procedures attach lower weights to high value observations compared to non-linear least squares.

In the case of non-linear least deviations we solve the following minimization problem:

$$\min_{\Theta} \sum_{i,j:i\neq j} \left| X_{ij}^o - X_{ij}(Z(w,L,\Theta),L,\Theta) \right|$$

subject to

$$\Psi\left(Z(w,L,\Theta),L,\Theta\right) = 0.$$

While weighted non-linear least squares imply that the estimate of Θ solves

$$\min_{\Theta} \sum_{i,j:i\neq j} W_{ij} \left(X_{ij}^o - X_{ij}(Z,L,\Theta) \right)^2 \tag{70}$$

subject to

$$\Psi\left(Z,L,\Theta\right) = 0, \tag{71}$$

where W_{ij} is a weight attached to the observation X_{ij}^o. In particular, I use the following weights:

$$W_{ij} = \frac{1}{(GDP_i GDP_j)^{0.68} \left(\frac{GDP_i}{L_i} \frac{GDP_j}{L_j} \right)^{0.17}}.$$

This choice of the weights is due to the findings in Silva and Tenreyro (2006). The ideal weights in (70) are the inverted variances of X_{ij}^o, which are not observable. In Silva and Tenreyro (2006), the authors test the hypothesis that the variances of X_{ij}^o are proportional to their means: i.e., $V(X_{ij}^o) \sim E(X_{ij}^o)$, and find that this hypothesis is not rejected. Hence, a natural approach is to use the inverted means of X_{ij}^o as the weights in (70). Making one step further, I assume that $E(X_{ij}^o)$ is proportional to X_{ij}^o, which is in turn proportional to $(GDP_i GDP_j)^{\kappa_1} \left(\frac{GDP_i}{L_i} \frac{GDP_j}{L_j} \right)^{\kappa_2}$. To estimate κ_1 and κ_2, I use the data on country aggregate trade volumes and the Poisson pseudo-maximum-likelihood estimator developed in Silva and Tenreyro (2006). This estimation procedure results in $\kappa_1 = 0.68$ and $\kappa_2 = 0.17$.[62]

Table 9: Alternative Estimation Procedures: Parameter Estimates

Estimation procedure:	*NLLS*	*NLLD*	*WNLLS*
γ_0	0.49	0.60	0.50
γ_1	0.14	0.17	0.19
γ_2	0.76	0.73	0.75
γ_3	0.97	0.86	0.85
γ_4	0.78	0.63	0.78
γ_5	0.94	0.82	0.89
f_x	2.15	1.35	1.21
δ	**0.67**	**0.62**	**0.69**
R^2	81%	44%	57%

Table 10: Alternative Estimation Procedures: Trade Elasticities and Zeros

	Data	NLLS	NLLD	WNLLS
Dependent variable:	$\ln T_i$	$\ln T_i(\hat{\Theta})$	$\ln T_i(\hat{\Theta})$	$\ln T_i(\hat{\Theta})$
Log of GDP	0.85**	0.69**	0.63**	0.64**
	(0.03)	(0.03)	(0.03)	(0.03)
Log of GDP per capita	0.19**	0.22**	0.33**	0.30**
	(0.04)	(0.02)	(0.03)	(0.03)
The percentage of "true" zeros:	100%	35%	34%	35%
Observations:	100	100	100	100

[62] An alternative way of estimating κ_1 and κ_2 is to convert to the log-linear specification and to apply the usual ordinary least squares (OLS). However, Silva and Tenreyro (2006) argue that this way of estimating κ_1 and κ_2 leads to biased estimates, as the mean of the logarithm of a random variable is not necessary equal to the logarithm of the mean. This implies that the conditional mean of the errors in the log-linearized specification is not zero resulting in biased estimates of κ_1 and κ_2. Indeed, OLS applied to the logarithms of trade volumes, total income, and income per capita give us $\kappa_1 = 0.85$ and $\kappa_2 = 0.18$. In this case, there is a significant negative correlation between the ratio $\frac{T_i}{(GDP_i)^{0.85}\left(\frac{GDP_i}{L_i}\right)^{0.18}}$ (where T_i is the aggregate trade volumes of country i) and GDP_i. This means that OLS generates upward bias in estimation. In contrast, there is no significant correlation between $\frac{T_i}{(GDP_i)^{0.68}\left(\frac{GDP_i}{L_i}\right)^{0.17}}$ and GDP_i.

Table 9 reports the parameter estimates obtained by applying *NLLD* and *WNLLS* (the first column of the table shows the non-linear least squares estimates). The results of the both procedures imply a negative correlation between country development level and market access costs. Moreover, the estimates of δ are close to that obtained by applying *NLLS*. The estimates of δ vary from 0.62 (*NLLD*) to 0.69 (*WNLLS*), while *NLLS* result in δ equal to 0.67. The estimates of the other parameters do not considerably vary as well. *Table 10* shows the comparison of the simulated trade elasticities obtained by applying different estimation procedures. As it can be inferred, the model estimated by *NLLD* and *WNLLS* slightly reduces the effect of population size on trade volumes and amplifies the impact of per capita income compared to the model estimated by *NLLS.*

Figure 11: Trade vs GDP in 1995 (the 100 largest countries in terms of GDP)

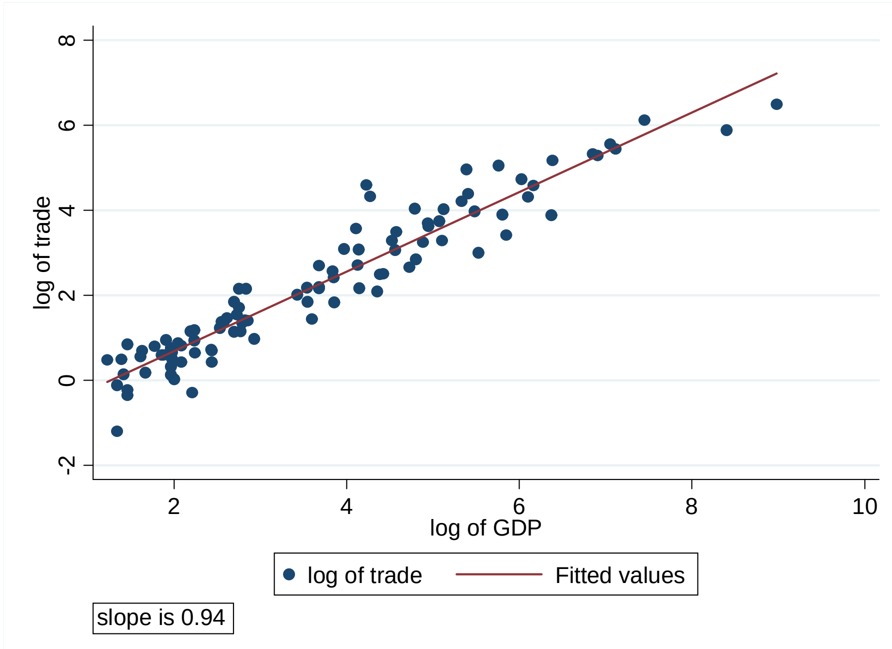


Figure 12: Residuals vs GDP per capita and population size

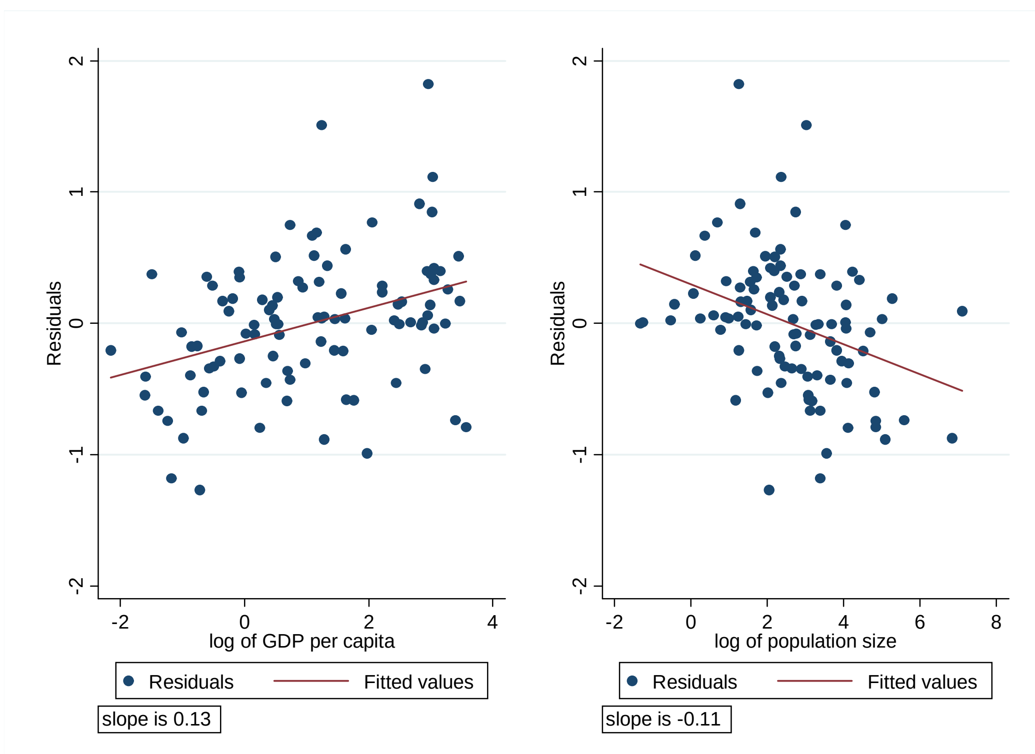


Table 11: List of Countries

Country name	**GDP per capita (2000US$)**	**GDP (2000US$ billion)**
United States	29942	7972.80
Japan	35439	4445.37
Germany	21073	1720.46
United Kingdom	21160	1232.58
France	19990	1156.29
Italy	17565	998.50
China, Hong Kong	777	941.32
Canada	20170	592.06
Brazil	3613	583.91
Spain	12056	474.85
Mexico	4892	445.85
Korea Republic	9159	413.01
India	372	346.57
Australia	18267	330.12
Netherlands	20427	315.81
Argentina	7184	250.26
Russian Federation	1618	239.71
Switzerland	31614	222.60
Belgium, Luxemburg	20672	218.01
Sweden	23374	206.42
Austria	21088	167.72
Turkey	2666	164.60
Indonesia	827	159.38
Norway	32214	140.45
Denmark	26599	139.06
Poland	3411	131.63
Greece	11445	121.71
Thailand	2086	120.01
Venezuela	5120	112.85
Finland	18899	96.54
Israel	17246	95.63
Portugal	9196	92.21
Iran	1409	83.07
Colombia	2092	80.04

Table 11: List of Countries (continued)

Country name	GDP per capita (2000US$)	GDP (2000US$ billion)
Egypt	1278	77.51
Malaysia	3471	71.47
Singapore	19359	68.23
Pakistan	515	63.00
Philippines	913	62.59
Chile	4295	61.82
Ireland	16799	60.62
Czech Republic	5100	52.69
Peru	1976	47.13
Algeria	1662	46.96
NewZealand	12635	46.41
Nigeria	363	39.54
Romania	1742	39.50
Hungary	3812	39.37
Bangladesh	289	36.54
Ukraine	672	34.60
Kuwait	19048	34.33
Morocco	1160	30.66
Uruguay	5786	18.62
Syrian Arab Republic	1181	17.26
Slovak Republic	3174	17.03
Oman	7749	16.83
Kazakhstan	1023	16.18
Guatemala	1589	15.89
Croatia	3337	15.58
Slovenia	7828	15.58
Ecuador	1335	15.21
Lebanon	4245	14.82
Tunisia	1651	14.79
Dominican Republic	1694	13.58
Bulgaria	1564	13.14
SriLanka	704	12.77
Costa Rica	3608	12.54

Table 11: List of Countries (continued)

Country name	GDP per capita (2000US$)	GDP (2000US$ billion)
Kenya	417	11.42
Uzbekistan	500	11.39
ElSalvador	2006	11.30
Belarus	920	9.38
Lithuania	2561	9.30
Panama	3470	9.27
Sudan	307	9.06
Coted'Ivoire	596	8.93
Jamaica	3241	8.04
Cameroon	568	7.99
Cyprus	11870	7.73
Tanzania	249	7.43
Jordan	1723	7.23
Yemen Rep.	465	7.22
Zimbabwe	606	7.15
Paraguay	1488	7.14
Bolivia	948	7.09
Iceland	25489	6.83
Angola	544	6.70
Bahrain	11170	6.45
Trinidad and Tobago	5037	6.40
Latvia	2364	5.95
Dem.Rep. Congo	116	5.26
Honduras	921	5.13
Gabon	4737	5.00
Nepal	200	4.34
Uganda	203	4.32
Estonia	2986	4.29
Bahamas	14477	4.06
Ghana	225	4.03
Senegal	424	3.84
Azerbaijan	488	3.75
Mauritius	3064	3.44

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