The Pennsylvania State University
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ZEBRA FINCH VOCAL DEVELOPMENT
THROUGH REINFORCEMENT OF THE
ANTERIOR FOREBRAIN PATHWAY

A Dissertation in
Physics
by
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Abstract

This dissertation explores reinforcement learning in the context of zebra finch song development. In the first chapter, we explore why we believe reinforcement learning is present in songbirds. We argue that similarities in vocal learning and neurophysiology between humans and songbirds offer compelling evidence that the two develop vocalization under similar constraints. A general constraint of motor learning tasks in animals is the requirement of the neurotransmitter dopamine. In mammals dopamine has been shown to encode reward information that is subsequently used to reinforce the motor activity.

The second chapter presents our reinforcement learning model in the context of song development during the sensorimotor phase of zebra finch. We use simplified binary neurons and synaptic plasticity rules to model activity in critical nuclei that are involved in zebra finch song learning. The model generates exploration in the anterior forebrain pathway (AFP) to guide the song trajectory to a stored tutor song. More specifically, random activity in the lateral magnocellular nucleus of the anterior nidopallium (LMAN) drives random exploration of HVC (proper name) projections to area X. When the model’s juvenile song moves towards the memorized tutor template a reward is generated. The reward is represented by activity in the ventral tegmental area (VTA) which globally projects dopamine to area X. The reinforcement of area X activity is permanently mapped
onto the premotor projection from HVC to the robust nucleus of the arcopallium (RA). The reward activity is delayed by 100 ms for biological reasons creating a temporal difference problem between activity and its corresponding reward. We resolve this issue using sustained area X activity and a plastic excitatory projection from area X to the VTA. The model is able to guide song development to the tutor song template by using biologically reasonable connectivity and synaptic learning rules.

Following the presentation of our birdsong model, a brief summary of a computational neuron model previously developed is presented in chapter 3. We look at the important aspects of neurons contained in the medial nucleus of the dorsolateral thalamus (DLM). The DLM neurons show unique activity transmission within the AFP and its role in song learning is unknown. The computational model is a single compartment, conductance based neuron with several ion channels. The properties of the ion channels were derived from a neuron model of mammalian thalamic relay neurons. The model reproduces the electrophysiological properties experimentally reported for the DLM neuron that are critical to the timing reported in the AFP.
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I would feel more optimistic about a bright future for man if he spent less time proving that he can outwit Nature and more time tasting her sweetness and respecting her seniority.

– E. B. White (1899 - 1985)
Chapter 1

Reinforcement Learning and Songbirds

1.1 Reinforcement Learning

In life we often struggle to advance tools we use to solve every day problems. From assembly line machinations that build the cars of the future to the way communication lines interact with each other to transfer data efficiently. Time after time we see the very same problems being solved in nature albeit more efficiently and in a more elegant fashion. Reinforcement learning is a prime example of this parallel. From learning to hit a puck with a hockey stick to bears learning to catch fish swimming upstream, reinforcement learning is believed to model extensive animal behaviour. With its beginnings in the psychology of animal learning it has been applied more recently to research involving machine learning (113).

This dissertation explores reinforcement learning in the context of zebra finch learning song. The following chapter hopes to explain why reinforcement learning represents an ideal paradigm to be applied to songbird development. We begin by introducing reinforcement learning, and provide an example in mammalian motor learning tasks. We next explore human and songbird vocalizations which is an interesting motor learning task and reinforcement learning problem.
The motor learning tasks of mammals is related to human and songbird vocalization development through similar electrophysiological organization.

Reinforcement learning is characterized by three basic features. First there exists an agent, actor or controller that must chose a course of action. The agent’s actions must be capable of changing the environment or system that the problem is defined in. Finally, the change in system generates a reward or response that the actor can use to determine future actions needed to solve the problem. This reward usually comes in the form of a simple scalar value and is often delayed. A distinguishing characteristic of the reward is that it is a positive reinforcement signal that does not communicate negative or incorrect evaluations.

The agent of the system often does not know the results of actions taken in the beginning. Without knowing how to solve the problem the agent must attempt to maximize the reward received from the sequence of actions generated. By maximizing the reward generated the agent is hopefully maximizing the chances of reaching or getting as close to their goal as possible. Therefore as the system evolves the agent must balance exploration of the system with exploiting actions that are known generators of reward. A book published by two pioneers in the field, Sutton and Barto (147) explores what strategies can be used to balance these two requirements.

The generic conditions of reinforcement learning lead to a large array of examples both computational and biological. This can include problem solving
strategies for systems such as chess and backgammon to artificial intelligence and motor skill learning in humanoid robotic. All of these tasks require trial and error and an ability to balance long term goals with short term gains. Our focus begins with motor learning tasks in mammals where reinforcement learning is well established (86; 128).

1.2 The Dopamine System

In a classic motor task experiment, monkeys were trained to press a lever at the sight of a light cue to receive a reward (juice) (86). The reward is delivered at a delayed time so that the monkey must learn to associate pressing the lever with the light cue and reward juice. The brain is an excellent example of where reinforcement learning is used to obtain results that are beneficial to the animal. Understanding how neuronal structure encodes the action selection, reward representation and overall goal orientation should provide valuable insight into how nature and neural dynamics can solve complex problems using reinforcement learning.

One area in the brain identified as being involved in the motor learning experiment is the midbrain dopaminergic nucleus or VTA. Dopamine is a neurotransmitter released by the VTA that has been shown to encode reward information during the task oriented learning experiment (86; 128). The experiment showed that individual VTA neurons respond in phasic bursts to positive stimulus such as
fruit juice (86). The dopaminergic neurons in the brain project to areas known to be involved in goal oriented activities such as the striatum and frontal cortex.

Initially when the monkey has not been trained to pull the lever, the VTA bursts when the reward or fruit juice is delivered (Fig. 1.1 top). After the monkey has learned to push the lever at the sight of the light, the VTA neurons fire brief bursts during the light cue. The VTA no longer bursts during the presentation of the food reward (Fig. 1.1 middle). Finally if the food is not delivered, the VTA neurons show a depression in activity at the exact time that the reward was expected (Fig. 1.1 bottom). The experiment shows that VTA neurons readily code for the prediction and receiving of reward. Few dopamine neurons show coding of tasks that produce unpleasant outcomes.

It is desirable to understand how one species codes for learning so that it can be adapted to similar systems. In the motor learning task involving the monkey, firm evidence for reward encoding was established in the VTA. Understanding how the VTA is used with the other areas of the brain to accomplish the actual learned motor task is the next critical step. There are several computational and theoretical models attempting to explain how dopamine is used in motor learning.
Fig. 1.1 Dopamine neurons learn to predict reward. Top The dopamine neurons are active in the presence of an unpredicted reward (fruit juice). Middle After the correlation between the conditioned stimulus (light cue) and the reward (fruit juice), the dopamine neurons only fire during the predicted reward or the sensory cue. Bottom Reward (fruit juice) is absent. Even in the absence of reward, the dopamine neurons still code for predicted reward. A depression in activity occurs at the time of reward. Adapted with permissions from Shultz, Dayan and Montague (1997) (128).
1.2.1 Brown Model

In actor/critic paradigms, the critic evaluates the outcome of the actor’s performance and produces positive or negative feedback depending on if the performance has improved or deteriorated respectively. There are many neuronal actor/critic architectures that model dopamine behaviour. The models are distinguished by the connectivity between the neuronal areas representing the actor (cortex/striatum) and the critic (SNc/VTA). A review of several dopamine models and their properties can be found in Daw (2003) (19). A model proposed by Brown, Bullock and Grossberg (1999) uses divergent reciprocal architecture between the corresponding actor/critic neuronal areas (10). The model reproduces dopamine activity using biologically realistic connectivity.

The Brown model consists of an indirect excitatory pathway and a direct inhibitory pathway originating at the actor or striatum and ending at the critic or dopamine neurons. The indirect excitatory pathway consists of the ventral striatum, ventral pallidum and the pedunculopontine tegmental nucleus (PPTN) (Fig. 1.2 left side). The ventral striatum has an inhibitory projection to the ventral pallidum which in turn has an inhibitory projection to the PPTN. The striatum inhibits the tonically active pallidum neurons releasing the PPTN from inhibition and thus generating excitation. The PPTN projects an excitatory signal to the dopamine neurons of the SNc/VTA. With primary reward projecting to the PPTN, the nucleus serves as relay of excitation for both the stimulus and reward
to dopamine neurons. The SNc reciprocally projects back to the striatum. The
direct inhibitory projection originates from the striosome neurons of the striatum
(52; 11). The cortex sends an excitatory projection to both the ventral striatum
and the striosome. The Brown model connectivity is illustrated in Fig. 1.2.

The cortical input to the striatum learns to drive subsequent PPTN activity
at the time of stimulus cues. The experimental light or conditioned stimulus gen-
erates sustained activity that represents a long lasting memory of the event. This
long lasting activity comes in the form of input from the cortex. The cortex ($I_i$)
projects an excitatory current to the ventral striatum and striosome neurons. After
an appropriate time delay, a reward signal caused by the unconditioned stimulus
(the food reward) drives a dopamine burst in the VTA. The dopamine facilitates
synaptic potentiation between the working memory site (cortex) and the ventral
striatum (108). Once this correlation is learned further presentation of the con-
ditioned stimulus will drive the striatum and generate a dopamine burst through
the PPTN.

As the cortex and striatum learn to drive the dopamine neurons at the sight
of the stimulus cue the striosome are learning to inhibit the VTA. The synaptic
weight between the cortical input and the striosome is potentiated during the time
of unconditioned stimulus (juice). Using a spectrum of delays, calcium spikes in
the striosome drive the learning at the precise time of reward delivery. The delays
do not begin until shortly after the cortical input begins excitation. Otherwise the
Fig. 1.2 → - Excitation. —○- Inhibition. The conditioned stimulus drives cortical input $I_i$. Correlation of cortical input and DA reward from SNc drives synaptic potentiation (W) along the corticostrialal pathway between the cortex and ventral striatum (S). The striatum disinhibits the PPTN through the ventral pallidum (VP) generating activity proportional to W. Once PPTN activity is above threshold $\Gamma$, the SNc releases dopamine during the conditioned stimulus. Meanwhile striosome activity learns to depress SNc dopamine activity at the time of primary reward ($I_R$). S - Ventral Striatum, VP - Ventral Pallidum, PPTN - Pedunculopontine Tegmental Nucleus, SNc - Substantia Nigra pars Compacta, W - Plastic Projection, $\Gamma$ - Cortical Driven Activity Threshold.
The phasic dopamine activity in single recordings is also observed on the population level in the motor learning experiment (128). The Brown, Bullock and Grossberg model would show similar population level activity if all cortical input was identical (10). How the model deals with large populations of striatal neurons with different cortical input is unclear. This situation is important because many learned motor tasks such as vocalization involve sequential movements to achieve the desired result. Dopamine projects diffusely to the striatum but the model delivers reward from the activity of the single neuron. How a continuous reward signal would distinguish between the multitudes of sustained cortical activity projecting to the striatum is unknown.

The global projection of dopamine neurons, coupled with their encoding of reward in experiments, indicates dopamine must play a vital role in learning sequential motor control. Any reinforcement learning model of the dopamine experiment requires a representation of stimulus that bridges the temporal gap between conditioned stimulus and reward. The Brown model used a continuous level of activity in the cortex during the entire time of learning to bridge the gap. How humans and songbirds represent the sensory stimulus in the cortex will dictate how
reward encoded dopamine neurons are used to reinforce learning in vocalization development.

1.3 Similarities in Vocal Development Between Humans and Songbirds

The development of vocalization in humans presents an interesting learning paradigm that should be explored. Unfortunately studying the neurophysiology of humans during active learning is difficult. An alternative strategy to direct experimentation on humans is to study similar biological systems found in other animals. Nonhuman primates, our closest evolutionary neighbours, have many qualitative and quantitative similarities to human learning and behaviour. However studies of nonhuman primates show no obvious similarities with the vocal development observed in humans (130). Little evidence exists of learned vocal behaviour in nonhuman primates and it is postulated that their vocal abilities are innate (70; 130). There are other mammals that show vocal development such as the cetaceans and some bats (98; 8; 30). A group of species that share in many of the behavioural characteristics in human vocal development are songbirds. An extensive review on the common themes and mechanisms between human and songbird vocalizations is presented in Doupe and Kuhl (1999) (30). Experimentation on songbirds is significantly easier than on humans, making them an ideal subject to study the underlining neurophysiology of vocal development.
Songbirds generate complex vocalization with acoustic structure similar to speech in humans. The smallest component of a song is considered a note which is strung together to form a syllable (16). In humans, phonetic units are pieced together to form words. The syllables are put together with brief moments of silence in between to form what is referred to as a motif. Motifs typically last several hundred milliseconds to a few seconds depending on the species of songbird. Thus songbirds string together syllables into a motif in a similar way to humans piecing together words in a sentence. In contrast, syllable order does not imply that song syntax is analogous to grammar used by humans in language. In zebra finch for example, the syllable order is held fixed in motifs and the song remains static in structure during adulthood. The song is believed to have some very basic uses such as courtship, defense and identification (129).

Humans and songbirds develop vocalization in a similar sequence of behavioural stages. The early stage of vocal development in both humans and songbirds is marked by a silent period of sensory acquisition and memorization of the exposed acoustic features from the language or song (142; 150). The next stage of development involves the practice of rudimentary vocalization (63). There is greater vocal plasticity during the early stages of development for both humans and songbirds (142; 126). In contrast to juveniles, adults show more stereotyped acoustic structure in their speech or song.
Humans and songbirds require auditory feedback to develop normal vocalization. Without the ability to hear the sounds produced by others, speech and song develop abnormally. Human speech develops deficiencies when there is insufficient social interaction or speech exposure as an infant (43; 143). Songbirds raised without a tutor or kept in isolation develop very abnormal song (76; 104). The song developed in isolation is known as isolate song (150). Speech and song are impaired if the ability to hear one’s own vocalizations is damaged or removed. If deafened during vocal development children show substantial degradation in speech (115). Deafening in developing songbirds during the sensorimotor phase after adequate exposure to tutor song also report abnormal song afterwards (77; 126). The behavioural similarities of humans and songbirds during vocal development indicate that neurophysiological similarities might exist as well.

1.4 Neurophysiological Similarities Between Mammals and Songbirds

Songbirds and humans have many homologous brain structures important for vocal development (34; 30). Electrophysiological similarities between species are often attributed to common evolutionary ancestors. The stem amniotes are believed to represent the most recent ancestors of both humans and birds (34; 35). It is believed the common ancestry is the reason there is similar brain connectivity and structure (65).
The cerebrum or telencephalon is the brain area primarily involved in the development of human and songbird vocalization. The telencephalon is organized into three distinct domains that are homologous in fish, amphibians, reptiles, birds and mammals: pallial, striatal and pallidal. We next explore how each of these domains are involved in vocal development.

The avian pallium processes information in a similar manner to mammalian sensory and motor cortices (34). In zebra finch, this includes HVC and RA nuclei, believed to be involved in syllable order and structure (55; 156). The neuronal areas of human and songbird palliums do show some differences in structural format. Humans tend to have a more folded or layered organization in the higher forebrain area, whereas songbirds have a more nuclear neuronal organization (121; 122). The connectivity between the folds in humans and connections of nuclei in songbirds are paralleled.

The organizational differences between corresponding brain regions is believed to be correlated more with brain size than behavioural complexity (34). Cognitive abilities that require these areas such as tool use are observed in both humans and birds (62). Therefore vocal learning is probably more dependent on the connectivity and functional dynamics of the brain’s subdivisions rather than brain size, or a nuclear versus folded paradigm. General principles of learning strategies could be determined from the connectivity and functional activity of the nuclei in songbirds.
The basal ganglia are composite of striatal and pallidal areas of the brain and are crucial for vocal learning and maintenance in mammals and songbirds (39). These areas of the mammalian brain include the striatum, globus pallidus, substantia nigra (pars reticulata (SNr) and pars compacta (SNc)) and subthalamic nucleus. The songbird equivalent of the basal ganglia includes area X, the medial nucleus of the dorsolateral thalamus (DLM), and the lateral magnocellular nucleus of the anterior nidopallium (LMAN) (122). The three nuclei form a closed loop known as the anterior forebrain pathway (AFP) that projects onto the premotor control nuclei of the avian pallium. The similarities between the two basal ganglia structures are illustrated in Fig. 1.3. The basal ganglia in mammals is crucial for learning of sequential movement (71; 51). The anterior forebrain pathway is essential for learning song; a highly specialized sequential movement of the vocal organs in songbirds.

The mammalian ventral tegmental area (VTA) is found in the substantia nigra and is critical for motor learning (86). The VTA diffusely projects dopamine to the mammalian striatum and avian area X (100; 33). Dopamine is known to excite membrane potentials and necessary for synaptic potentiation in the basal ganglia (108). Dopamine affects the songbird basal ganglia in a similar way (27; 26; 45). With the similar connectivity between humans and songbirds, the avian VTA and dopamine likely plays a critical role in motor learning tasks.
Fig. 1.3 The connectivity and electrophysiological properties of the basal ganglia and subsequent interaction with the cortex in mammalian brains has striking similarities to songbirds. The striatum and globlus pallidus of the mammalian basal ganglia are almost identical to the spiny and aspiny neurons respectively which are condensed into the single AFP nucleus area X. The thalamus is believed to process and relay sensory information to the cortex, while the role of the DLM in song learning is largely unknown. The DLM projection to LMAN appears to be similar to the thalamocortical projection found in mammals.
We began by looking at reinforcement learning and explored it in the context of motor learning tasks in monkeys. Another possible example of reinforcement learning is human and songbird vocalization development. The homologous brain structures used in human and songbird vocalization development are also used in mammalian motor task experiments. Mammalian motor learning involving dopamine has been extensively studied and modeled (161; 19). Therefore mammalian dopamine learning models could lead to important inferences for the songbird learning system.
Chapter 2

An AFP Model of Birdsong Learning

2.1 Background Information

2.1.1 Song Development

Zebra finch song develops in two major stages: the sensory and the sensorimotor phase (63). The sensory phase consists of the juvenile finch receiving auditory information from a tutor. The sensory phase typically begins at 25 days post hatch (dph) and ends around 60 dph. The sensorimotor phase starts when the juvenile initiates rudimentary vocalization or subsong. The juvenile’s singing is characterized by highly variable plastic song. As the juvenile’s singing matures, it begins to take on the characteristics of the tutor song heard during the sensory phase. This sensorimotor process begins around 35 dph and the song becomes stable around 90 dph when exploration ends (Fig. 2.1). Past 90 dph the zebra finch’s song varies little and is referred to as crystallized. The crystallized song is usually very close to the acoustic features of the tutor song.

During the sensorimotor phase two distinct patterns of development have been observed in zebra finch (14). The first strategy involves repeating a syllable until it is sufficiently learned before introducing more syllables into the motif.
Subsequent syllables introduced into the motif occur in the same order as the tutor vocalized song. The second strategy involves the zebra finch singing all of the syllables from the tutor motif during the sensorimotor phase. Exploration and learning occurs across the entire motif as the songbird develops. Thus zebra finch can focus in a syllable specific way or on the entire motif structure.

Song variability during development makes it difficult to observe when syllable structure learning occurs. Recent computational work in syllable recognition has made it possible to map the trajectory of syllable development (148; 22). Syllable trajectories are identifiable even in highly variable juvenile song because similar acoustic features such as pitch are still recognizable across renditions. Figure 2.2 depicts the gradual evolution of a syllable and the corresponding pitch error (149). The gradual trajectory indicates that zebra finches are capable of incremental adaptation.

Syllables can be introduced into the motif with acoustic structure that is very similar to the corresponding tutor features (22). Figure 2.3 shows a starting syllable with very similar end features. Although there are notable windows of time
Fig. 2.2 (Left) Spectral derivative of a developing harmonic stack. The syllable starts out very close to the tutor syllable and collapses relatively quickly. (Right) The pitch error (compared to tutor syllable) evolves gradually over the course of days. Adapted from Tchernichovski, Mitra, Lints and Nottebohm (2001) (149).

on the order of days when large changes can occur, the final pitch profile is very similar to the beginning pitch (22). Understanding how these gradual trajectories form and how this learning translates on the neuronal activity level is critical.

2.1.2 Song Related Nuclei and Neuronal Activity

Nottebohm, Stokes and Canard (1976) identified two distinct groups of nuclei involved in song learning (105). The first group is the premotor nuclei HVC (proper name) and the robust nucleus of the arcopallium (RA) which control the motor system in sound production. RA projects to the motor production system, which involves the motor neurons of the hypoglossal nucleus, (nXIIIts), which in turn innervates the syrinx (146; 30; 144). The anterior forebrain pathway (AFP) is a secondary group of nuclei necessary for learning during the sensorimotor phase
Fig. 2.3 (Left) Spectral derivative of initial syllable. (Middle) The dotted box represents where 80% of the variable change occurs. The fundamental pitch appears to change very little, and the trajectory over the course of development is gradual. (Right) Spectral derivative of final syllable. Adapted from Deregnaucourt et al. (2004) (22)

The AFP includes Area X of the striatum, the medial nucleus of the dorsolateral thalamus (DLM), and the lateral magnocellular nucleus of the anterior nidopallium (LMAN). Together they form an analogous group found in mammals known as the basal ganglia (31). Another important area possibly involved in song learning is the midbrain dopaminergic nuclei. These nuclei project diffusely throughout the telencephalon including HVC, RA, LMAN and densely to Area X (85; 133; 25; 27; 26). The primary nucleus projecting to area X is the midbrain ventral tegmental area or VTA (45; 46). Figure 2.4 depicts the connectivity between the premotor, AFP and dopaminergic nuclei.
Fig. 2.4 The green nuclei depict the premotor areas (HVC and RA) primarily involved in control of motor output with projections ending at the motor nuclei directly involved in motor production (light blue nXIIIts). There is also an HVC projection to the AFP (area X). The AFP is primarily involved in learning during the sensorimotor phase and is depicted in red. Area X also receives projections from the dopaminergic nucleus VTA. Not depicted is the recently discovered projection from Area X to the ventral pallidum (VP) which then projects to the VTA.
2.1.2.1 Premotor Nuclei

The HVC contains three distinct neuronal types, two of which are projection neurons (101). The projection neuron that targets the RA nucleus, HVC(RA), has diffuse synaptic targets (102; 42). HVC(RA) neurons fire in precise temporal sequences associated with the production of song and motif structure (36; 55; 163). The inhibitory interneuron HVC(I) are involved in the suppression of auditory responses during singing (127). A raster plot of the HVC(RA) and HVC(I) neurons reported by Hahnloser, Kozhevnikov, and Fee (2002) (55) is depicted in Fig. 2.5. The last HVC neuronal type, HVC(X), projects diffusely to the AFP nucleus area X.

The HVC(RA) neurons do not initially drive the RA neurons alone. A large source of variability in RA is caused by LMAN projection neurons (107; 2). At the end of the sensorimotor phase, the RA activity is largely driven by the HVC as lesion studies have indicated (36). The RA activity in adult zebra finch is characterized by temporally precise burst firing that aligns with the structure of the song similar to the HVC(RA) neurons (83). The temporally precise burst firing is depicted in figure 2.6. Thus during the sensorimotor phase the RA activity transitions from highly variable HVC(RA) and LMAN driven activity to very precise song specific activity driven by HVC(RA) neurons. Therefore RA can be considered a map of motor commands that generate song output.
Fig. 2.5 (Top) Sonogram of a motif. (Middle) The song profile is depicted at slot 1 of the rasterplot. A raster plot of HVC(RA) neuronal activity showing precise alignment with acoustic features of the motif. (Bottom) The HVC interneurons are associated with inhibiting auditory feedback responses in HVC(X) neurons (127). Obtained with permissions from Hahnloser, Kozhevnikov and Fee (2002) (55).
Fig. 2.6 (Top) Sonogram of a motif with syllable identification on top. (Bottom) Raster plot recordings of RA neurons aligned with the sonogram located at the top. Coloring is used to distinguish neuron identification. The RA neurons have temporally precise bursting characteristics that are similar to the HVC(RA) projection neurons. Two RA interneurons are included in the shaded box at the bottom. Obtained with permissions from Leonardo and Fee (2005) (83)
2.1.2.2 Anterior Forebrain Pathway

The transition from juvenile subsong to a full song resembling the original tutor template requires the activity of the anterior forebrain pathway (126). The AFP consists of three main nuclei of which area X excites the DLM, the DLM subsequently drives LMAN, and LMAN has an excitatory projection to both RA and area X (Fig. 2.4). The nuclei form closed topographic projections and are segregated into multiple compartments, possibly controlling individual vocal muscles (87; 64). Experiment has shown that LMAN lesions immediately cease acoustic variability in zebra finch (126). In the absence of LMAN input to RA song development ceases towards the tutor song in juveniles. Lesions in area X do not disrupt song variability, but arrest song development toward the tutor song (126). This evidence on the surface seems to indicate that area X is necessary for guiding learning to tutor song, and LMAN is required to produce variability or exploration.

When examined more closely, area X seems to contain several electrophysiologically similar neuronal types found in mammalian basal ganglia. Spread across two distinct structures in mammals, these neuronal types are tightly compacted together in the area X of songbirds (120; 39). Three major neuronal types include inhibitory interneurons, spiny and aspiny neurons, all of which release GABA, an inhibitory neurotransmitter. The spiny neurons, $X_{SN}$, are inhibitory and exhibit electrophysiologically properties similar to the spiny neurons in the striatum of mammalian basal ganglia (39). The spiny neurons receive excitatory projections
from HVC(X) neurons and LMAN. The striatal spiny neurons respond to current injection with membrane potentials that resemble plateaus (37). This response suggests a bistable membrane capacity similar to the UP/DOWN states observed in mammals (141; 13; 39).

The spiny neurons inhibit the aspiny neurons, \( X_{AF} \), within area X. The aspiny neurons are fast-firing, tonically active and are similar to the pallidal neurons in the globus pallidus of mammals (39). In summary, area X has the makings of a compact basal ganglia unit that contain striatal and pallidal mammalian neurons connected in a similar fashion. Roughly half of the aspiny neurons form GABAergic projections to the DLM nucleus (89; 88). Evidence for feedback and lateral projections between spiny and aspiny neurons in area X is detailed in Farries, Ding and Perkel (2005) (38). Clearly there is still much work to be done to explore and understand the true nature of Area X and its exact role in the learning of song.

The projection from area X to the DLM is GABAergic, yet evidence indicates signal transmission through the AFP occurs (29). The striatal spiny neurons inhibit the pallidal aspiny neurons, which are otherwise tonically active and inhibiting the DLM. Once the inhibited aspiny neurons cease their inhibition of DLM neurons, the DLM is released from inhibition. This release from inhibition produces an action potential even in the absence of excitatory current input. The neuron produces what is called a post-inhibitory rebound spike (PIR), or burst of spikes (111). Specific ionic channels exist that become more sensitive to firing
when a neuron is being inhibited (58). This channel mechanism facilitates the rebound spikes recorded in DLM neurons. Each DLM neuron receives only one to two axonal projections from area X neurons (87). The DLM forms an excitatory connection with LMAN projection and interneurons.

LMAN projections to RA are initially diffuse during the sensory phase. Before the onset of the sensorimotor phase the projections become topographic, conforming to the rest of the AFP topology (64). Individual LMAN neurons project to both area X and RA through axonal splitting (154; 87). Thus a closed loop forms between the three nuclei of the AFP, with incoming projections from HVC and VTA to area X and outgoing projections from LMAN to RA (Fig. 2.4). Experiment has shown LMAN neurons project NMDA to RA which facilitates synaptic potentiation between HVC(RA) and RA neurons (99; 107).

LMAN is critical during the sensorimotor phase for normal song development. Lesion studies indicate that without LMAN the juvenile’s song actually becomes stereotyped and does not collapse to the tutor template (126). Normal development indicates that as the sensorimotor phase ends, RA variability due to LMAN activity diminishes, and song becomes stereotypic (126). LMAN neurons during the learning and experimental phase appear to fire randomly in a range of 2 to 25 Hz, (136; 82). Highly variable activity in LMAN neurons is depicted in Fig. 2.7.
Fig. 2.7 Raster plot of RA-projecting LMAN neurons aligned with song. LMAN activity in adult zebra finch is highly variable. Differences in correlation distributions suggest it may not be completely random with respect to song. Figure obtained with permissions from Olveczky, Andalman and Fee (2005) (107).

Although mean LMAN activity shows correlation with song structure in adults, the neuronal activity is highly variable compared to song structure from rendition to rendition (57). There is some experimental evidence that this variability may not exist for all LMAN neurons. Figure 2.8 obtained from Leonardo (2004) displays small high frequency correlations in LMAN with temporally scaled syllables (82). This stereotypic activity is also observed in LMAN during directed song (57). The total fraction of LMAN neurons that displays this precise high frequency activity that correlates to song is largely unknown.

After the song has crystallized the AFP is no longer necessary to produce song (36). The AFP still remains crucial for song maintenance in adult zebra finch
Fig. 2.8 (Top) Top 3 graphs are sonograms temporally scaled to align. The middle sonogram has distorted auditory feedback and is marked by the gray shading in the 2nd raster plot. (Middle) The two raster plots are LMAN activity of a single neuron scaled to the same motif alignment. (Bottom) Firing rate plot showing high frequency activity that is temporally aligned and stable across motif renditions. The asterisk marks where LMAN activity is synchronized to the song’s acoustic features regardless of auditory distortion. Adapted with permissions from Leonardo (2004) (82).
RA activity is highly variable early in the sensorimotor phase due to LMAN input (2). In adult zebra finch RA activity is temporally precise and HVC(RA) neurons drive activity (55). How RA transitions from LMAN to HVC driven activity is a necessary component of understanding learning in zebra finch (2).

### 2.1.2.3 VTA and Dopamine

Dopamine is extremely important in mammalian motor learning (128; 161; 10; 19). Therefore exploring the role dopaminergic nuclei have in zebra finch song learning, an intrinsically motor control learning task, is critical. The VTA projects dopamine diffusely to large areas of the telencephalon, and densely innervates the striatal or spiny neurons of area X (85; 46; 25). Dopamine facilitates synaptic potentiation in area X spiny neurons when high frequency input is coupled to postsynaptic firing (27; 26; 45; 46). It is likely that the dopamine system within zebra finch behaves in a similar way to that of mammals and hence the VTA is a strong possible destination or representation of a reward signal for song performance (31).

In vivo recordings presented by Yangihara and Hessler (2006) showed VTA activity can align with song structure as depicted in Fig. 2.9 (162). In adult male zebra finch it is believed dopamine is used to distinguish between directed and undirected singing, regulating attention and overall variability (124; 162).
Whether dopamine is used in a reinforcement learning context in juvenile zebra finch is an open question.

2.1.3 Auditory Processing

In the context of reinforcement learning a goal must be established to define the critic and guide the actor. Experiment indicates that juvenile zebra use a memorized template of the tutor’s song to develop their vocalizations (44). The exact location and nature of the tutor template is currently unknown. Recent experimental evidence has shown that memory formation does exist in nuclei shown to be required for auditory processing in songbirds (114). The caudal medial nidopallium (NCM), a auditory processing center of the caudal telencephalon, has shown the capacity to distinguish a difference between novel and tutor stimuli (155; 114). Neuronal recordings indicate a quicker attenuation of its activity over multiple trials in response to the tutor song over novel stimuli and birds own song (BOS) (114). The tutor and novel stimuli responses of the NCM are depicted in Fig. 2.10.

2.1.3.1 HVC(X) Projection

How auditory feedback impacts the AFP to affect song learning is one of the main unsolved problems confronting the songbird community. It is largely believed that auditory feedback travels through the HVC and is used in either some form of an efference copy or error signal (151; 152; 137; 135). Evidence for
Fig. 2.9 In vivo VTA Activity. (Top) Song profile of directed and undirected song. (Middle) VTA raster plot aligned with song recordings displayed at the top. The firing rate and the colored bars depicted on the right side differentiate between directed and undirected song. (Bottom) This firing rate plot indicates a mean increase in firing rate at a specific time in relation to song structure. The increase in activity appears to occur approximately 60 ms after the syllable. Although speculative, this activity could represent delayed reward. Adapted with permission from Yanagihara and Hessler (2006) (162).
Fig. 2.10 RMS response amplitude of repeated presentation of a novel song stimulus (♦) and tutor stimulus (□). The habituation rate is different for the two stimuli allowing for the distinction between BOS (birds own song) and tutor song. Adapted with permissions from Phan, Pytte and Vicario (2006) (114).

The auditory selective activity in HVC and AFP was largely recorded in asleep or anesthetized zebra finch (137; 136). The auditory activity was not observed in the HVC(X) neurons of awake singing zebra finch (127). In the presence of distorted auditory feedback HVC(X) neuronal responses were not altered (80). The distorted auditory feedback experiment was presented by Kozhevnikov and Fee (2007)(80) and is illustrated in Fig. 2.11.

Zebra finch require auditory feedback for song learning and maintenance (126; 103). Studies of song development indicate that the majority of learning occurs during singing (149; 148). Experimental evidence indicates zebra finch require area X to guide development towards the tutor song (126). Thus experiment sheds doubt on the idea that auditory feedback and AFP activity interact to learn the tutor song through the HVC projection to area X. If it is assumed that the
Fig. 2.11 Raster plots of HVC(X) neurons in the presence of distorted auditory feedback. The two sonograms and HVC(X) raster activity are aligned with the song depicted at the top. The second sonogram depicts the distorted auditory feedback recorded on top of singing. The green indicates time of distorted auditory feedback. The red and blue indicate HVC(X) responses during singing that show no difference prior to (red) and post (blue) auditory feedback distortion. Adapted with permissions from Kozhevnikov and Fee (2007) (80).
AFP requires the auditory feedback information to guide song development then an alternative pathway is required.

2.1.3.2 Alternate Auditory Pathway to AFP

Recent evidence suggests a pallium-midbrain pathway could carry auditory information to the AFP during singing. An area of the arcopallium surrounding RA projects to the VTA (47; 110). The RA cup known to be involved in auditory processing is part of the same arcopallium area projecting to the VTA (155). It is reasonable to assume that at least one way auditory feedback could affect learning is through the projection of dopamine to the AFP via the arcopallium VTA projection.

The study also reports aspiny area X neurons project to the ventral pallidum, a nuclei which has synaptic terminals in the VTA (110). Thus strong evidence would indicate the ability of the anterior forebrain pathway to affect its own activity with auditory information through the release of dopamine. How the VTA is used in juveniles during singing is largely dependent on the nature of these projection interactions. The ability to explore these and other possible learning paradigms in a computational model are an important step in determining future experimental design.
2.2 Previous Computational Models of Zebra Finch Song Learning

The trajectory of a juvenile zebra finch song converges to what was learned early in development during tutor exposure. How this exposure to a tutor song guides subsequent learning is largely unknown. A stored representation of the tutor song referred to as a template has been the assumption guiding experiment and theory within the field for several years (44). Currently concrete evidence does not exist of a highly localized static representation of this tutor template. There are many theories using the idea of a template to guide learning in juvenile zebra finch.

Two main conjectures involving a tutor template are an error driven learning and the well established reinforcement learning (151; 32). In error driven learning, a signal is created that represents the difference between the tutor template and the juvenile song. This error signal is then used to guide the song development trajectory. With reinforcement learning, the dynamics are adjusted to maximize a generic reward signal that is sent to the system. The signal is maximized when the juvenile song matches the tutor template and is usually delivered in a diffuse global way. How these two paradigms could be used in conjunction or individually has been the purpose of several past models.
2.2.1 Doya and Sejnowski Model

The model of Doya and Sejnowski proposed using the AFP as an adaptive critic to simulate how songbirds learn (32). This form of actor/critic used in reinforcement learning has its origins in Sutton and Barto (147). Exploration occurs by allowing LMAN noise projecting to RA to perturb the synaptic weights between HVC and RA. The model uses reinforcement learning by assuming that specific area X neurons exist that are tutor song selective to modulate LMAN output (28; 137). They state primary reinforcement is delivered to area X by dopamine projecting VTA neurons. A vector representation of the tutor and synthesized syllable is created and their correlation defines the VTA reward signal. The correlated activity between the VTA reward and the auditory input from HVC are used as generators of the reinforcement in area X.

A critique of the model voiced by the authors involved the auditory response in HVC neurons. The HVC neurons that contain both auditory and motor activity have their auditory responses suppressed during singing (127). If this behaviour is consistent in juveniles, then correlated activity between HVC auditory responses and reward input is no longer a plausible learning mechanism. Another issue noted by the authors is the lack of a temporal delay in auditory processing. The signal is assumed to represent an expected level of performance and hence does not contain any time delay in the auditory information. Overall the model is well constructed, has many insights into the possible dynamics involved in song learning and is
mindful of the biologically homologous structures in mammals. With the striking similarities between the zebra finch AFP and mammalian basal ganglia, the role the VTA plays in zebra finch learning should be as essential as it is in mammals.

2.2.2 Troyer and Doupe Model

The role that auditory selective HVC(X) neurons play and their interaction with the delayed reward is an important strategy in tackling the learning problem if the suppression of auditory responses are not complete (80; 127). The model proposed by Troyer and Doupe (2000) uses an efference copy of the song generated between the HVC(RA) and HVC(X) projecting neurons of the HVC (151; 152). The efference copy is learned in HVC by associating the premotor activity in HVC(RA) neurons with the resulting auditory feedback that is projected to HVC(X) neurons (28). This association then represents a prediction of the auditory feedback at the time of premotor activity.

The AFP does not directly evaluate the auditory feedback but guides the development of motor output in RA based on the comparison between the efference copy and the tutor template. This comparison represents the error signal that is used to then guide subsequent activity towards the tutor song. The predicted feedback or efference copy within HVC(X) occurs at a significantly earlier time relative to normal auditory feedback and therefore addresses the issue of evaluative delay.
There is evidence that during song production the activity of the HVC(X) projecting neurons are actually indifferent to auditory feedback, placing some doubt on a possible efference copy (127; 80). Given the auditory feedback is on the order of 50-65 ms (97; 96; 93), and assuming that in juveniles the AFP guides or even drives activity which is a further 45 ms behind motor output (29), addressing how this 100 ms delay is handled is a difficult standing issue and central to songbird models. This issue aside, the Troyer Doupe model is a novel and interesting approach to the delay problem. The model treats each syllable as a neuronal ensemble potentially obscuring how the detailed structure of the syllable can form. The syllable structure is contained in the pre-grouped ensembles and the syllable sequence is learned through the HVC to RA connections. The model does represent one of the first comprehensive attempts at understanding song learning in zebra finch.

2.2.3 Margoliash Sleeping Paradigm

An early proponent of the importance of sleep in zebra finch learning was Daniel Margoliash (18; 92). Margoliash asserts that sleep is used in the consolidation of memory during periods of intense procedural learning, similar to hippocampal studies in mammals (18; 41; 160). The efference copy paradigm was used to explain the role of sleep in zebra finch learning by Dave and Margoliash (2000) (18). The efference copy is assumed to travel through the AFP during singing and
project back onto area X via LMAN. Using the long temporal delay generated by DLM activity (111), the efference copy becomes temporally in sync with the actual auditory feedback associated with the original premotor activity. The coincident inputs to area X are then reinforced while LMAN is assumed to have little role during singing because it is considered out of sync with premotor activity. During sleep it is believed that premotor activity is exactly the same as activity recorded in awake zebra finch. The predicted auditory feedback of actual singing contained in LMAN is projected onto the artificial premotor command signal from HVC to RA (18). The coincidence of these two signals then modulates RA activity.

The model was proposed prior to evidence reporting deterioration of song caused by sleep (23). How this learning mechanism could be modified to address for the degrading of song is unknown. The model is the first put forward to incorporate sleep as a necessary process in zebra finch song learning. The model would also need to account for evidence that learning can occur during singing (149). Although the proposed model was not implemented it represents an excellent suggestion of how the role of sleep in learning can be explored.

2.2.4 Fiete Model

A recent model proposed by Fiete, Fee and Seung (2007) uses local synaptic learning rules, coupled with a global reinforcement signal to guide song learning
A projection from LMAN to RA perturbs activity and represents exploration. The direction of exploration is compared to a tutor template and a reinforcement signal is then generated. When the RA neuron’s membrane is perturbed via glutamatergic synaptic input from LMAN, an eligibility trace is generated. The correlation between this eligibility trace and the delayed scalar reinforcement signal, or the lack thereof, determines whether long term potentiation (LTP), or long term depression (LTD) occurs. The reinforcement signal itself is defined as a binary scalar signal that is nonzero only when the song amplitude or pitch moves, on average, in the direction of the tutor song. The location and exact nature of this evaluative signal is not addressed, but they suggest acetylcholine (131) or norepinephrine (100; 134) inputs as possible candidates.

The model depicts the LMAN nuclei as a set of completely independent noisy neurons projecting to RA. The role the rest of the AFP plays is not addressed or incorporated into the model. Given the evidence that area X lesions cause juvenile song to halt its trajectory towards the tutor song, it is unlikely LMAN is a solely independent entity guiding all learning (126). They assume that temporal correlations exhibited by LMAN with ongoing song, reported by Leonardo (2004) is negligible when compared with the Poisson nature of activity in their model (82). The gradient decent formulation of the learning rule and diffuse global reward signal does provide a powerful mechanism for fast, efficient learning. This learning
mechanism shows how local synaptic learning rules can use a global reinforcement input to collectively guide learning.

The previous computational models of zebra finch learning have proven to be valuable tools exploring the numerous aspects of song development. Although new perspectives have been provided, important issues and challenges must still be confronted. Currently no model incorporating the AFP addresses the possibility that auditory responses are not used in HVC(X) neurons during singing. Without the use of an efference copy, or an eligibility trace, it is unknown how the correlation might form between the premotor activity in area X and the substantially delayed auditory feedback. Investigating the crucial role that area X plays in song learning must be a central component in any realistic paradigm. A model that can address these major issues and incorporate their solutions into a coherent framework would provide new insight into zebra finch song learning.

2.3 Model Overview

In zebra finch, both area X and auditory feedback are required for song learning (126). We assume auditory feedback is compared with the tutor song memorized during the sensory phase (44) to generate a reward to reinforce AFP activity (27; 26). The area X activity that affects song and the corresponding auditory feedback are separated by a substantial time delay (97; 96; 93; 29; 151).
Determining how the temporal difference between desired activity and its subsequent reward is correlated presents an interesting challenge. The purpose of our model is to solve this credit assignment problem and show how zebra finches learn to sing.

The HVC generates high frequency burst activity in area X to provide a temporally precise signal through the AFP to guide learning in RA. Juvenile song requires contributions from both HVC(RA) and LMAN initially (15; 80; 2). We therefore assume that initially HVC(X) population activity does not dominate the premotor signal in area X. With a combination of high frequency input and background dopamine levels, random synaptic potentiation is possible between HVC(X) and area X spiny neurons (27; 26). Through these synaptic fluctuations, our model assumes that HVC(X) neurons drive area X activity with its characteristic precise bursting. The synaptic fluctuations are assumed to be slow, lasting several bouts (48).

The HVC to AFP projection represents syllable structure exploration. The subsequent AFP activity is then projected onto RA. When these explorations change the syllable structure towards the tutor template the system generates reward in the VTA (Fig. 2.12 A). Through the use of sustained activity in area X and a plastic projection from area X to the VTA, we correlate the neurons involved in exploration with VTA reward activity (Fig. 2.12 B). The area X to VTA plastic projection is excitatory. Therefore through correlation this projection
drives the VTA at the time of premotor area X activity (Fig. 2.12 B-C). With the VTA activity transferred to the time of premotor activity in area X, the HVC to AFP projection is potentiated with dopamine (Fig. 2.12 C). The learned AFP activity is held for several motifs and subsequently transferred to the HVC to RA pathway (Fig. 2.12 B-D). These interactions lead to local improvement in syllable structure. Ultimately the juvenile’s plastic song develops into an acoustic structure that agrees with the tutor template.

In the following sections we will detail the important assumptions behind our model. Auditory feedback is discussed to provide motivation for its interaction with area X in songbird learning. Our model assumes the principle mechanism in which area X activity is affected by auditory feedback is through dopamine release from the VTA (47; 110). Dopamine activity has been shown to encode reward information in the motor learning of mammals and solves the temporal difference and credit assignment problem (128). Therefore we analyze a dopamine learning model from mammalian basal ganglia to better understand the role dopamine plays in area X (10).

We investigate mammalian striatum to extrapolate what area X activity might be like. After describing our assumptions of area X neuronal activity, we explore the interaction it has with the VTA. In our model area X affects its own development through excitation of the VTA. The balancing of VTA excitation is modeled by an inhibitory projection from the medial striatum based on the
Fig. 2.12 Model learning dynamics. A - Slow HVC(X) synaptic fluctuations generate explorations in the AFP and hence RA. Reward (red arrow from VTA) is generated if explorations are desired. B - Correlations between sustained area X activity and reward driven VTA activity occur along an excitatory pathway (XPT). C - With excitatory pathway (XPT) potentiated HVC(X) driven area X activity drives VTA. By driving VTA area X reinforces the initial HVC(X) projections that generated the explorations and therefore the subsequent VTA reward activity. HVC(X) timing is transferred to RA through AFP. The VTA activity is inhibited afterwards depressing the excitatory pathway (XPT) prevent over excitation of the VTA. D - The HVC(RA) projection representing the exploration is locked in and new explorations in HVC(X) proceed.
dopamine learning model in mammals (Fig. 2.12 D). Using these critical assumptions motivated by songbird and mammalian data, we model syllable development in zebra finch.

2.3.1 Auditory Feedback

Determining the critical factors in a complex problem is the most important step in modeling any solution. Understanding how auditory feedback is used and temporally represented during learning is the most critical unanswered question in the songbird field. Zebra finch require auditory feedback for song development during the sensorimotor phase (78). Area X is also needed in juveniles to develop their plastic vocalizations into the memorized tutor song (126). Therefore it is highly probable that area X either processes or receives auditory feedback information in order to drive learning. Addressing how auditory information is delivered to area X is a crucial component in a realistic model solution. There exist two main candidates for the delivery of auditory feedback to area X: the HVC and VTA nuclei.

The HVC is the most modeled pathway for auditory transmission in learning models but it may not be the most probable candidate. Anesthetized zebra finch show auditory selective activity in HVC(X) and area X during song playback (127; 137). Previous computational models use the auditory selective neurons during sleep to justify the HVC to area X pathway for transmission of auditory
feedback during singing (32; 151; 152). It is important to address auditory processing in singing zebra finch because experiments indicate this is the time that learning occurs (148; 22). In contrast, recordings in HVC(X) during singing indicate that auditory responses are suppressed by the HVC inhibitory interneurons (127). Therefore finding another pathway that carries auditory information during singing may provide important information and a unique solution to learning.

The arcopallium projection to VTA is a strong candidate for the delivery of auditory feedback to area X. As previously detailed, the arcopallium surrounding RA projects to the VTA (47; 110). This arcopallium area includes the RA cup known to be involved in auditory processing (155). The VTA heavily innervates area X by releasing the neurotransmitter dopamine (45; 25). With dopamine required for synaptic potentiation in area X (27; 26), a mechanism exists for auditory feedback to affect network activity. In our model, we assume the arcopallium VTA pathway is used by zebra finch to transmit auditory feedback to the AFP to affect song learning.

The temporal delay between activity in area X affecting song production and subsequent auditory feedback associated with that area X activity is significant. First, the transmission time of a signal travelling between HVC(X) and RA through area X and the rest of the AFP must be accounted for. Allison Doupe reported significant correlation between HVC(X) and RA neuronal activity with a temporal separation of approximately 40 milliseconds (29; 74; 151). The activity
from the premotor nucleus RA is then transmitted through the motor control system. The delay between HVC(RA) activity and song output is typically around 45 milliseconds (97; 96). As the songbird sings, acoustic information is absorbed and processed. The time delay from song output to its associated auditory responses in HVC(X) is approximately 15 milliseconds (93). We assume for simplicity that the delay to area X through the arcopallium VTA pathway is of the same order. With all of the delays combined, the separation between area X activity and the communication of information from auditory feedback would be roughly 100 milliseconds. This process is summarized in Fig. 2.13. If the interaction between area X activity and auditory feedback is needed for learning, a mechanism to link these temporally disjoint events is critical.

Fig. 2.13 The time between area X activity and any subsequent reward is separated by a 100 ms delay. This temporal difference is larger than what most synaptic plasticity ranges can achieve. The delay of transmission from HVC(X) through area X to RA is estimated as 40 ms. The time from RA activity to correlated song output is approximately 45 ms. From the time of song to observed auditory responses is approximately 15 ms.
2.3.2 Dopamine and the Brown Model

With a possible pathway linking auditory feedback and VTA activity established the release of dopamine from VTA during singing becomes important \((47; 110)\). Unfortunately there does not exist extensive experimental literature detailing dopamine activity in juvenile zebra finch \((124; 162)\). With large similarities in the development of vocalization in humans and songbirds, looking to mammalian experiments in dopamine learning is a plausible alternative strategy. In mammalian motor learning experiments, dopamine activity is shown to phasically increase initially at the time of primary reward \((86)\). Dopamine release transitions to the early time of motor activity or sensory cue (light) associated with the primary reward (juice). The phasic transition is depicted for a monkey performing the motor control task in Fig. 2.14. The transition addresses the temporal difference between the sensory cue (light) and the reward (juice). Studying mammalian models that bridge the temporal difference between action and reward activity could provide insight for songbird learning.

Brown, Bullock and Grossberg introduced a mammalian model that offers several plausible mechanisms to solve temporal difference learning in avian systems (Fig. 1.2, \((10)\)). Our model uses three main mechanisms adapted from the Brown model. The three mechanisms are sustained cortical input to the striatum, an excitatory pathway from the striatum to the SNc and an inhibitory projection to the SNc (Section 1.2.1). The use of sustained cortical input to the striatum
Fig. 2.14 Dopamine neurons learn to predict reward. Top The dopamine neurons are activity in the presence of an unpredicted reward of fruit juice. Middle After the correlation between the conditioned stimulus (light cue) and the reward (fruit juice), the dopamine neurons only fire during the predicted reward or the sensory cue. Bottom Reward (fruit juice) is absent. Even in the absence of reward, the dopamine neurons still code for predicted reward. A depression in activity occurs at the time of reward. Adapted with permissions from Shultz, Dayan and Montague (1997) (128).
correlates the motor control signal to the delayed dopamine projection from the SNc. This allows synaptic potentiation along the corticostriatal projection to occur. The model assumes an excitatory pathway from the striatum to the SNc to drive dopamine activity at the time of conditioned stimulus or motor control activity. Inhibition of the SNc is a component needed in reproducing dopamine behaviour during motor control learning experiments (Fig. 2.14, (86)). The model uses striosome neurons to project inhibition to the SNc (52; 11). Adapting the model’s learning mechanisms in zebra finch could provide important elements of how songbirds solve the temporal difference problem.

The sustained cortical input to the striatum used in the Brown model is a useful mechanism. The Brown model uses synaptic plasticity between the cortex and striatum to drive the dopamine in SNc at the appropriate time. The cortical input is active during the duration of learning which allows activity correlation between the SNc reward (dopamine), and the corticostriatal projection. The constant level of activity between conditioned and unconditioned stimulus was a necessary component in the Brown Model.

An avian equivalent of the corticostriatal pathway is the HVC(X) projection to spiny neurons in area X. In zebra finch, HVC(X) neurons fire in short high frequency bursts, much shorter than the associated 100 ms time delay of auditory feedback (80). Although HVC(X) neuronal activity can play the role of conditioned stimulus (CS) it does not have a constant level of activity to correlate with the
delayed reward input. Currently no evidence exists that a 100 ms eligibility trace or other biological mechanism can correlate activity between HVC and area X while ignoring all intermediate activity. Another candidate for sustained activity correlating premotor activity in the AFP and the delayed reward delivered from the VTA is required.

The combination of LMAN input and membrane bistability of area X spiny neurons could be a plausible neuronal mechanism acting as sustained cortical input. The spiny neuron membrane bistability is characterized by an UP and a DOWN state (37; 39). The DOWN state is recognizable by the hyperpolarized membrane potential of the neuron and the lack of activity. The UP state is distinguished by an elevated membrane potential, approximately 4 to 5 mV below the neuron’s firing threshold. The spiny neuron’s membrane plateau (UP state) can be maintained for between 150 and 400 milliseconds (37; 39). With LMAN projections providing excitatory input, sustained spiny neuron activity can be maintained long enough for delayed auditory feedback to arrive. Therefore we model the UP state of the area X spiny neuron with excitatory LMAN input as the bridge between premotor AFP activity and the associated delayed auditory feedback. Before we can model the learning involved with auditory feedback in area X, we must first understand the complex striatal dynamics observed in mammals and zebra finch.
2.3.3 Area X Neuronal Activity

The modeling of area X spiny neurons is motivated by properties observed in mammalian striatal neurons. Area X spiny neurons exhibit electrophysiological properties similar to the mammalian striatum neurons (120; 37). There are three mammalian striatal properties that we assume are used to generate the area X dynamics: population level correlated UP states, lateral inhibitory projections and fast firing inhibitory interneurons (13; 158; 116). The last two assumptions are combined for the possibility of correlated activity that is temporally aligned.

Evidence that populations of neurons in UP and DOWN states are correlated has recently been shown in mammals (13; 141). Specifically one study indicated that through the injection of NMDA, there were subpopulations of neurons that were in UP states and DOWN states in distinctly windowed time segments (13). The neurons formed distinct UP state groups that were able to transition to other distinct UP state groups across several seconds. The correlations formed through the injection of NMDA or stimulation of cortical input.

Motor control sequences could be represented using UP and DOWN states correlated at the population level. The UP states last approximately 400 ms, which would easily cover the duration of the typical syllable in a zebra finch song. We therefore assume that during learning a specific set of neurons are in the UP state. The required cortical input for correlated UP states is assumed to originate from
LMAN and HVC input. The rest of the spiny neurons in area X are considered quiescent in an inhibited UP state or hyperpolarized in the DOWN state.

Mammalian striatal neurons have connectivity and electrophysiological properties that could cause the correlated UP states to be sparse. Strong lateral inhibition among striatal neurons is present in the mammalian striatum (153). Strong short term synaptic plasticity induced by high frequency activity between striatal neurons is also present (17; 116). The lateral inhibition coupled with the short term plasticity could generate a local winner-take-all effect among UP state neurons (145; 158; 116). With a local winner-take-all effect the correlated UP state activity induced by NMDA would be sparsely distributed. The sequence of activity that could induce strong lateral inhibition and a winner-take-all effect is depicted in Fig. 2.15. Determining if the spiking activity of neurons in UP states is correlated through some winner-take-all mechanism is currently unknown and still debated (3; 158). Our model uses this possibility to assume the correlated UP states are sparse relative to population density.

Fast firing inhibitory interneurons reported in mammalian and avian striatum could generate precise firing among correlated UP states (79; 120). When the inhibitory interneuron is itself inhibited, all spiny neuron targets have a high probability to fire. The longer or more intense the inhibition of the interneuron the higher the probability that neuronal activity would occur. If a spiny neuron or HVC(X) projection inhibited the interneuron through a high frequency burst,
Fig. 2.15 Lateral inhibitory projection initially has a low value between spiny neurons 1 and 2. After coupling of high frequency activity the synaptic weight is potentiated. With a stronger synaptic weight spiny neuron 1 dominates during the next trial or song rendition inhibiting neuron 2.

Spiny neurons that receive projections from the interneuron could increase activity for the duration of the burst. Therefore high frequency activity from UP state spiny neurons could bias activity of other UP state spiny neurons that are connected to the same interneuron. The interaction described is depicted in Fig. 2.16. The HVC(X) activity could be a possible candidate for the high frequency input that is needed for the mechanism to work.

The modeled area X dynamics coupled with sparse HVC(X) input generates correlated activity across multiple song renditions and is used to drive song learning. The model assumes that HVC(X) input does not dominate area X activity in the same way that HVC(RA) activity does not dominate RA activity in juvenile zebra finch. The HVC(X) projections that have stronger than average synapses will drive spiny neurons in short high frequency bursts. Evidence indicates in adult zebra finch fluctuations in syllable structure occur over durations on the order of
Fig. 2.16 Possible mechanism to generate temporally precise activity in multiple UP states. Lateral inhibitory projections can generate large short term plasticity (> 200%) to allow local winner-take-all dynamics (116). Using this strong plasticity between spiny neurons and the global inhibitory interneurons may allow the HVC high frequency activity driving spiny neurons to be inhibited in the interneurons. By reducing the global inhibition with high frequency precise bursting, other spiny neurons receiving projections from the interneuron could fire at this precise time.
500-1000 ms (48). Thus our model assumes random HVC(X) driven activity in X is projected through the AFP to RA for some set number of trials or bouts.

The slow HVC(X) fluctuations combined with correlated UP states introduce stereotypic activity that allow for correlations to form across song renditions (13; 48). Our model tracks the correlations across song bouts and generates reward if the precise HVC(X) activity improves the juvenile’s song when compared to the tutor template. The reward is modeled as excitation from the arcopallium area involving auditory processing being projected to the VTA (47; 110). The sparse UP state and weak HVC(X) projection dynamics reduces the interference of uncorrelated activity with the delayed reward. The correlation between the dominant UP states of X and the incoming reward at the VTA is a necessary, but not sufficient condition in our solution of the time delay problem.

2.3.4 XPT Pathway

A plastic excitatory pathway from area X to the VTA is assumed in our model. Recently reported by Gale, Person and Perkel, a pathway exists that projects from area X to the ventral pallidum (VP) and from the VP to the VTA (47). The area X-VP-VTA or XPT pathway projects GABA along each projection but evidence shows excitation of area X leads to an increase in dopamine concentration (45). How the XPT pathway is a net excitation is explored in the
For simplicity our model assumes the XPT pathway is a direct excitatory projection from the aspiny neurons in area X to the VTA.

The XPT pathway is used in our model to accomplish two objectives. The model first uses the plastic XPT pathway to correlate the sustained activity of the grouped UP states in area X to the reward generated from delayed auditory feedback in the VTA (Fig. 2.17 A). The second purpose the projection fulfills is to drive the VTA at the time of the premotor activity which caused the delayed reward (Fig. 2.17 C). This pathway is able to drive the VTA once the correlation between reward and UP state activity is strong enough. Therefore the XPT pathway is a critical component in solving the temporal difference problem for our model.

The XPT pathway represents the best candidate for correlating the sustained area X activity to subsequent delayed reward in the VTA. The correlation between the spiny neurons and the VTA occurs during the time of delayed auditory feedback and not during the time of premotor X activity (Fig. 2.17 A). Using mammalian data, spike time dependent plasticity (STDP) was shown to operate on short time scales for single spikes (108). It is reasonable to assume zebra finch spiny neurons operate on a similar time scale given the similar electrophysiological properties with mammalian striatum neurons (38; 31; 39). Therefore short HVC(X) burst activity undermines the role STDP can have as an effective mechanism for correlating area X activity with the 100 millisecond delayed reward. The short time scale issue is addressed by random LMAN input that prolongs spiny
neuron activity while in the UP state. The correlation of UP states maintained by LMAN input with the delayed VTA dopamine reward (via XPT pathway) is used to associate reward with the HVC(X) and UP state explorations.

For synaptic plasticity to develop between HVC(X) and area X neurons, two requirements need to be met: relatively high frequency concurrent pre and postsynaptic activity, and the presence of dopamine (27; 26; 108). Therefore concurrent HVC and area X activity with high concentrations of dopamine drive potentiation more than concurrent events with low or baseline levels of dopamine. The most direct way to achieve high concentrations of dopamine in area X is for the desired HVC(X) driven activity in X to drive the VTA dopamine release. The XPT pathway between the UP state area X neurons and the VTA must first be potentiated through the correlation of X activity with delayed reward (Fig. 2.17 B). If we assumed that area X could drive the VTA initially then any random activity would be potentiated. After the XPT pathway is potentiated sufficiently, the established UP states and precise HVC(X) driven firing in area X drives the VTA’s dopamine release. With increased dopamine concentrations during the time of primary premotor activity in area X the correct HVC(X) to X projection is potentiated. The potentiated XPT projection now represents the excitatory pathway mechanism adopted from the Brown model.

In summary, with sustained UP state X activity, and a plastic excitatory projection from area X to VTA, we have been able to adapt two mechanisms
used from the Brown model to address the temporal difference problem. The projection from the VTA to X is initially responsible for the delivery of dopamine as primary reward based on the delayed auditory feedback. The reciprocal XPT pathway to the VTA acting as a closed gate opens if correlation between X UP state activity and the VTA occurs. If the XPT pathway is above a synaptic threshold, high frequency activity can drive the VTA and allow for long term potentiation between HVC and X (38; 31; 39). The proposed mechanism creates a stereotypical drive through the AFP to a small number of LMAN neurons driving RA across a brief succession of bouts. The bias pushes the HVC(RA) projection above a synaptic threshold where it is considered supersynaptic or permanently learned. This learning sequence transitions RA activity from the noisy LMAN driven juvenile state to the precisely timed HVC driven state observed in adults. The learning paradigm and relative order of synaptic potentiation is described graphically in Fig. 2.17.

2.3.5 VTA Inhibition

The proposed framework is not complete and is insufficient because it generates excessive excitation of the VTA and precision in LMAN activity. Under the current proposal, as strong connections between HVC(X) and X develop, more area X neurons are capable of driving the VTA. As the accumulation of excitation from area X to the VTA develops, the VTA would saturate area X with dopamine.
Fig. 2.17 Event sequence order generating a correctly learned connection. A - #1. HVC connection weight randomly fluctuates above firing threshold driving $X_{SN}$ reliably for several trials (indicated by the dashed line from HVC to $X_{SN}$). #2. A correct sequence in RA due to HVC(X) activity generates a delayed reward in VTA (bottom of A with purple arrow). B - #3. Correlation between reward and the sustained activity of the UP state of $X_{SN}$ develops, potentiating $W_{X-VTA}$. C #4 - When $W_{X-VTA}$ is above a synaptic threshold area X drives the VTA at the time of #1 HVC(X) activity (indicated by black arrow along XPT pathway). This occurs several bouts after the initial reward delivery. #5. - With $X_{SN}$ driving activity of VTA at the time of HVC(X) activity, dopamine causes potentiated between HVC(X) and X. D #6. - During this learning AFP activity is learned between HVC(RA) and RA(indicated in green). Also reward generated by exploration attenuates explained in song evaluation section. E #7 - Learning the timing of stereotyped VTA activity caused by X, striosome from the medial striatum (MSt) inhibits the connection and the synaptic weights decay. F After the XPT pathway has been weakened new exploration between HVC and area X occurs.
The lack of correlated activity between dopamine concentrations and desired corrections in juvenile song would prevent song development. Another criticism is as more connections are learned and maintained in X, LMAN would become substantially correlated with the syllable profile. Therefore an adaptation or inhibitory mechanism is needed to prevent the VTA from drowning the system in dopamine.

The third and final mechanism inspired by the Brown, Bullock, and Grossberg model and mammalian data, is an inhibitory projection to the VTA (10). First reported by Ann Graybeil, inhibitory neurons in the medial striatum of mammals project GABA to the VTA and SNc and are referred to as striosomes (52; 11). The basal ganglia of zebra finch slightly differs from mammals, but the overall connectivity and functionality of the striatum is believed to remain largely intact (35; 121). The medial striatum of songbirds contain GABAergic spiny neurons that project to the VTA and substantia nigra pars compacta (SNc) (33; 121; 110). Therefore the most probable location of neurons resembling striosome in zebra finch is the medial striatum (MSt). The neurons are assumed to receive excitatory input that generates a complete timing spectrum during learning (10). The striosome of the avian medial striatum are innervated by the diffusely projecting VTA (110). This allows the striosome to learn the timing of the VTA.

Long term synaptic potentiation and depression are related to concentrations of dopamine and dopamine receptors in mammalian and avian striatum
Depression of the VTA could drive dopamine concentrations below baseline values in area X. With low concentrations of dopamine, long term depression can occur between HVC(X) and spiny neurons in area X. If the VTA activity is depressed then correlations between area X and VTA cease. Without correlations between these two nuclei the XPT projection can decay or is weakened (Fig. 2.17 E). Therefore the saturation of dopamine through accumulative excitation along the XPT pathway is avoided. The striosome inhibition of the VTA’s activity allows learning to remain controlled and dopamine concentrations unsaturated.

Figure 2.18 is a schematic of the connectivity proposed in our model, and labels the connections that are assumed to be plastic and necessary for learning. The following sections detail the neuronal modeling, synaptic learning rules, and auditory processing implementations of the proposed neural network.

2.4 Model Implementation

Our learning model is a complex sequence of interactions involving multiple nuclei and pathways. To reduce the complexity of the modeling, we use simple representations of neuronal activity and connectivity. Experimental evidence indicates the AFP forms a loop that is considered microscopically closed (87; 64). Through small tracer injections Luo, Ding and Perkel (2001) indicate that area X neurons projecting to DLM receive input from LMAN neurons that are targeted by
Fig. 2.18 The double inhibitory connection from $X_{SN}$ to DLM is actually considered net excitation via DLM PIR spikes. Net excitation is also assumed for the projection from X through the VP to the VTA labeled $W_{X-VTA}$. The order in which the connections become dominate in the learning are $W_{XV}, W_{HX}, W_{HR}$, and $W_{HS}$. $W_{HR}$ is potentiated during the whole process. LMAN activity is assumed to be random accept for the precisely driven activity originating in area X.
the very same DLM neurons (87). The authors suggest that these sets of neurons that form closed loops within the AFP might act as computational units. These units may be used to control muscles given the myotopical organization of RA (156).

Our model assumes that functionally we can approximate connectivity within the AFP as one to one. Luo and Perkel (1999) report area X axonal projections target only one to two DLM neurons (89; 88). Individual RA neurons receive approximately 50 synaptic connections from LMAN (56; 40) and approximately 1000 synapses from HVC (75; 40). With the reduced number of HVC projections in our model (approximately 6) it is a reasonable approximation that RA receives one projection from LMAN.

Single unit recordings of area X aspiny neurons in response to BOS in anthesitized adult zebra finch display short bursts of quiescents (approximately 16 ms) in the axonal projections prior to their targeted DLM neuron firing (112). The most probable explanation for these short bursts of inhibition are direct HVC input to spiny neurons in area X. Their hypothesis is supported by an increase in axonal firing rate prior to the quiescent because of evidence of a direct projection of excitatory HVC input to pallidal neurons reported in Farries et al. (2005) (38). If pallidal neurons received diffuse projections from spiny neurons then the responses to BOS would show many bouts of quiescent due to the diffuse HVC projections.
to spiny neurons. We therefore conclude that spiny neurons projections can be approximated as one to one with respect to aspiny neurons with area X.

2.4.1 AFP Neuronal Models

We begin by treating neurons as simple binary units, where the firing state $R(t)$ is one if the neuron is spiking and zero otherwise. We also use a discrete time representation with a one millisecond time step. The firing state of a neuron with input $x$ is defined as:

$$R(t) = \varphi(x)$$ \hspace{1cm} (2.1)

The $\varphi$ is the step function which is 1 only if the input $x$ is greater than zero. The step function is defined as:

$$\varphi(x) = \begin{cases} 
1 & \text{if } x \geq 0 \\
0 & \text{if } x < 0
\end{cases} \hspace{1cm} (2.2)$$

We assume a signal transmission time of one millisecond between all connected neuron pairs. If the presynaptic neuron is firing at time $t$, then we assign a characteristic time delay, $T_F$, for the postsynaptic neuron to fire. This time to fire, $T_F$, is used to simulate the time required for the incoming current to build up the neuron’s membrane potential to spike.
The neurons that have static synaptic connections are assumed to generate perfect transmission. Projections that are plastic must surpass a synaptic strength threshold for the presynaptic neuron to drive the postsynaptic neuron. Let us suppose neuron i of nucleus A receives a perfect transmission from neuron k of nucleus B. Let neuron i also receive a plastic projection from neuron j of nucleus C with firing threshold $\gamma_{AC}$. Combining the two projections the firing state for neuron i of nucleus A is defined as:

$$R_{A,i}(t + T_{F,A}) = \varphi \left[ R_{B,k}(t - 1) + W_{ij}^{AC} R_{C,j}(t - 1) - \gamma_{AC} \right]$$

(2.3)

The $t-1$ argument for the projecting neuron’s firing state function represents the assumed signal transmission delay of 1 ms. The unit step function, $\varphi$, is used to represent a threshold mechanism to separate learned $W_{AC}$ connections from silent or weak synapses. The threshold $\gamma_{AC}$ is assumed less than one so that $R_B$ can generate activity in the absence of nuclei C input.

2.4.1.1 HVC

The two projection neuron types in HVC are assumed to generate static high frequency burst spiking patterns. The HVC activity is believed to represent a spatiotemporal pattern known as a synfire chain (66; 67). Synfire chains have been shown to develop relatively quickly suggesting that the HVC temporal precision
observed in adults is already present in juveniles (69). Supporting this conjecture is
evidence that both RA and X projecting HVC neurons fire precisely during singing
in juveniles (55). We set each HVC neuron to fire 5 times in a 10 millisecond
window. The HVC firing times are assumed to be static across learning.

The HVC projects diffusely to both the RA nucleus and the spiny neurons of
area X, $X_{SN}$. Each RA and $X_{SN}$ neuron is modeled to receive $N_h$ projections from
the HVC(RA) and HVC(X) neurons respectively. The HVC neurons are selected
such that the time of bursts cover the length of the syllable to allow for a complete
timing representation. The timing spectrum is justified by RA neurons that show
the capacity to fire over the course of the entire syllable as illustrated in Fig. 2.6.

The interval between adjacent bursts of HVC neurons that project to a
RA neuron was set to 20 ms. There are two reasons why we modeled a 20 ms
time interval between adjacent bursts. The first reason is the separation generates
sparse enough activity that the timing of HVC projections to individual neurons
does not overlap. The second motivation is that syllable duration can be covered
by RA activity with a relatively small number of HVC projections and RA neurons,
thus reducing simulation time.

2.4.1.2 RA

The RA nucleus receives a static projection from LMAN neurons and a
plastic projection from HVC(RA) neurons. Experimental evidence shows LMAN
as a primary driving force in juvenile RA and area X activity (2; 15). We model LMAN to generate noise and bias RA through the precise bursting from HVC(X). We therefore assume that there is perfect transmission from LMAN to RA and LMAN to area X spiny neurons. The HVC activity develops into the dominating driving force in RA as the song crystallizes and RA activity becomes more stereotyped (55). In juveniles, the dominance of the HVC projection is determined by its synaptic strength. We model this by using a synaptic weight threshold, $\gamma_{hr}$ to separate the weak and strong HVC to RA connections. The time to fire, $T_{F,RA}$, was set to a conservative 6 ms given the 10 ms delay between LMAN and RA correlation peaks (74). The firing state of the $i^{th}$ RA neuron is defined as

$$R_{RA,i}(t + T_{F,RA}) = \varphi \left[ R_{LMAN,i}(t-1) + \sum_{j=1}^{N_h} W_{HR}^{ij} R_{HVC(RA),j}(t-1) - \gamma_{hr} \right]$$

(2.4)

2.4.1.3 Area X

The firing dynamics of area X spiny neurons is modeled in a similar fashion to RA dynamics. Area X receives diffuse projections from HVC(X) and topographic projections during the sensorimotor phase from LMAN. LMAN noise drives activity and a firing threshold mechanism separates weak and strong HVC(X) projections. Area X spiny neurons have a time to fire of 6 ms (37).

The probability is small that X activity is correlated to the same LMAN neuron it has projected to through the DLM given that this would represent a
transmission of activity through the AFP a second time when only one delayed correlation peak is observed in RA (74). We therefore assumed an independent source of noise for input to $X_{SN}$ neurons. We simulated noise as a Poisson spike train at a frequency of $f_x$. With $\eta_{X,i}(t)$ defined as the independent noise source for the $i^{th}$ striatal neuron in X. The variable input from LMAN is assumed to exist only if the spiny neuron is in the UP state as low frequency input is unlikely to drive the spiny neuron into an UP state (37). Therefore the kronecker delta function $\delta_{\text{State},UP}$ is added as a factor to $\eta_X$. The spiny neuron firing state is defined by:

$$R_{X_{SN},i}(t + T_{f,X_{SN}}) = \varphi \left[ \eta_{X,i}(t-1)\delta_{\text{State},UP} + \sum_{j=1}^{N_e} W_{Hx}^{ij} R_{HVC(X),j}(t-1) - \gamma_{hx} \right]$$

(2.5)

The UP state dynamics and precise high frequency activity detailed in the previous section is modeled through the use of a connection matrix. For a set number of trials or bouts, $T_w$, a group of X spiny neurons are randomly selected to be in the UP state. Of these spiny neurons a random number between 1 and $N_e = 10$ are selected to have a HVC projection above firing threshold. Neurons can transition to UP states only through this randomly selected correlated group state activity or if an HVC projection is super-synaptic ($W_{Hx} > \gamma_{hx}$). This selection mechanism simplifies the complex interaction with spiny neuron UP states and
cortical input (13) and assumes that the neurons involved in exploring HVC(X) projections are in UP states.

The HVC(X) projections are selected so that the group of spiny neurons receive the high frequency burst input in the same narrow time window representing an individual note variation in the syllable (48). This temporally narrow window is argued by the inhibition dynamics of the spiny neurons previously discussed. Relaxing this mechanism to variations across the entire syllable is explored in the discussion. After the set number of trials, $T_w$, the connections are assumed to decay below firing threshold again unless through the learning dynamic of our model the synaptic weight $W_{HX}$ was potentiated above the threshold $\gamma_{hx}$ as is defined by equation 2.5.

The spiny neurons inhibit the tonically firing $X_{AF}$ neurons which respond immediately with the 1 ms transmission delay. The immediate response is used to model how quickly the neuron can be inhibited and prevent baseline firing from occurring. The spiny neurons are assumed to project to aspiny neurons in an one to one fashion (38). We keep track of aspiny activity by stating if the spiny neuron is on, then the aspiny neuron is off one millisecond later. With a simplified pallidal neuron in area X, only the times the neuron is inhibited are important. We therefore only keep track of inhibited activity, denoted by $\tilde{R}(t)$. At all other times the aspiny neuron is tonically active, inhibiting the corresponding DLM neuron.
Therefore the time of inhibition of the aspiny neuron $X_{AF}$ is defined as:

$$\tilde{R}_{X_{AF},i}(t) = R_{X_{SN},i}(t - 1)$$

(2.6)

2.4.1.4 DLM

The nature of transmission through DLM neurons is unique within the AFP. The post inhibitory rebound spikes require a sustained amount of inhibition and then a release from that inhibition in order for spikes to occur (90; 111; 112). The minimum time of release from inhibition required before the onset of a rebound spike in vivo is not known. In vitro evidence suggests that it may be as low as 20 milliseconds (111).

The DLM requires high frequency input from area X to provide the necessary release from inhibition to fire. Low frequency noise is unlikely to release the neuron from inhibition long enough for a rebound spike to form. The necessary and sufficient condition in our paradigm is that DLM should be capable of transmitting HVC(X) activity through the AFP to RA (29; 74). Therefore the burst activity observed in HVC(X) neurons which drives activity in $X_{SN}$ neurons and the subsequent release from inhibition of aspiny pallidal neurons must be sufficient to drive DLM activity. We assumed the HVC(X) fires a 5 spike burst in 10 milliseconds (55), and use this high frequency activity as the requirement before allowing the onset of a DLM rebound spike. The inhibitory spike threshold, $\gamma_{zd} = 4$, defines the
necessary inhibition of aspiny X neurons and is set to 4 spikes over a 10 millisecond window.

\[ R_{DLM,i}(t + \vec{T}_F, DLM) = \varphi(\sum_{j=0}^{10} \tilde{R}_{X_{AF},i}(t - j - 1) - \gamma_{sd}) \]  

(2.7)

The vector form of the time to fire for the DLM, denoted \( \vec{T}_F, DLM \), is used to indicate that the DLM can generate multiple sodium spikes on top of a calcium rebound spike (90). The firing time predicted by equation 2.8 is then followed by 2 spikes with an ISI of 2 ms to simulate the rebound burst in equation 2.7.

The actual time to fire or rebound latency of DLM neurons is not a constant but a function of the length of inhibition prior to its release. The DLM time to fire, \( T_F, DLM \), follows the post-inhibitory rebounds described in Person and Perkel (2005) (111). \( T_F, DLM \) is given by the following equation that was a best fit estimate:

\[ T_F,dlm(\tau_{Inh}) = Ae^{-B\tau_{Inh}} + C \]  

(2.8)

where,

\[ A = 36.15 \quad B = -0.0078 \quad C = 23.75 \]

\( \tau_{Inh} \) is the time of inhibition since the last DLM spike minus a refractory period of 10 milliseconds. We do not address the effect of firing of the \( X_{AF} \) pallidal neurons has on the DLM firing time while the DLM is in the process of rebounding. If a DLM rebound occurs an excitatory signal is transmitted to the corresponding LMAN neuron.
2.4.1.5 LMAN

LMAN neurons are treated as noise generators for area X and RA. Most studies indicate that LMAN is a noise generator, and is incapable of guiding syllable and song development alone (126; 107). In vivo recordings have shown that LMAN is capable of high frequency bursts that align with temporally scaled syllables sung by zebra finch (82). We therefore associated the temporally precise, high frequency burst activity in LMAN with the DLM burst input generated by HVC(X) driven area X dynamics. We assume the principle importance of LMAN noise is to cause the synaptic weights between HVC(X) projecting neurons and area X spiny neurons to fluctuate. The synaptic weight fluctuates are idealized by the random connection selection between HVC(X) and spiny neurons detailed previously.

We model the noise generator for LMAN neurons using $\eta_{L,i}(t)$ as a Poisson train with frequency $f_{Lman}$. The time to fire for LMAN neurons was set to 6 ms (7). The firing state of the LMAN neuron is thus defined as:

$$R_{LMAN,i}(t + T_{F,LMAN}) = \varphi\left[\eta_{L,i}(t) + \sum_{j} R_{DLM,i}(t - 1 - j)\right]$$  \hspace{1cm} (2.9)

The sum over $j$ previous firing states of DLM defines the width of the burst generated in LMAN due to the post inhibitory rebound bursts observed in DLM. The number of LMAN spikes caused by the DLM rebound burst is set to 5 spikes in a 10 millisecond window.
2.4.1.6 Striosome

The mammalian striosome represent the template for our model’s inhibitory neurons that project to the VTA (52; 11). In mammals, the striosome are located throughout the dorsal and ventral striatum and produce an inhibitory GABAergic projection to the VTA and SNc. Mammalian striatums receive input from the SNc/VTA, amgydala and medial limbic cortex (95). In zebra finch, the dorsal aspect of the caudal lateral nidopallium (dNCL) residing near the HVC, projects to the medial striatum (64). In our model we assume the striosome, $X_{St}$, project to the VTA neuron and receive input from the dNCL. With the dNCL’s close proximity to HVC, we assume a similar temporal representation to HVC activity. We use $N_{X_{St}} = 1$ for simplicity because our model only uses one VTA neuron. Either strong input from the dNCL or the VTA will excite the striosome neurons:

$$R_{X_{St}}(t + 1) = \varphi [R_{VTA}(t - 1) + \sum_{j = 1}^{N_{HS}} W_{HS,j} R_{HS,j}(t - 1) - \gamma_{st}] \Delta_{fst}$$  \hspace{1cm} (2.10)

$\Delta_{fst}$ is used to restrict the striosome activity so that it is more stereotypical. If the nidopallium and VTA input overlap then the neurons would otherwise be over saturated and the synaptic strengths would increase relatively quickly. Our learning paradigm requires that striosome synaptic dynamics be relatively slow, therefore we restrict the interspike interval (ISI) and have a saturation factor. The ISI is restricted to a minimum of 2 ms (factor \([1 - R_{X_{St}}(t - 1)]\)). The second factor
maintains that the striosome fire a stereotypic 5 spike burst ($\gamma_{fst} = 5$) once the VTA or nidopallium input is sufficiently active. These restrictions are combined and are defined in $\Delta_{fst}$ as:

$$\Delta_{fst} = [1 - R_{X_{St}}(t - 1)]\varphi(\gamma_{fst} - \sum_{j=1}^{20} R_{X_{St}}(t - j))$$ (2.11)

Although artificial, these two mechanisms keep the learning dynamics described within an optimal time frame necessary for learning. A summary of the firing properties of neurons used in the premotor and AFP nuclei are described in table 2.1 below.

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>Firing Time, $T_F$</th>
<th>Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA</td>
<td>6 ms</td>
<td>HVC(RA) and LMAN</td>
</tr>
<tr>
<td>$X_{SN}$</td>
<td>6 ms</td>
<td>HVC(X) and LMAN</td>
</tr>
<tr>
<td>$X_{AF}$</td>
<td>1 ms</td>
<td>$X_{SN}$ of Area X</td>
</tr>
<tr>
<td>DLM</td>
<td>$T_{F,dlm}(\tau_{Inh})$</td>
<td>Area X aspiny neurons $X_{AF}$</td>
</tr>
<tr>
<td>LMAN</td>
<td>6 ms</td>
<td>DLM and random noise at $f_{lman}$ Hz</td>
</tr>
<tr>
<td>$X_{St}$</td>
<td>1 ms</td>
<td>Nidopallium Projection and VTA</td>
</tr>
</tbody>
</table>

Table 2.1 Firing Times of AFP Neurons

2.4.2 VTA and the Dopamine System

Neurons that require dopamine (DA) for synaptic potentiation express several DA receptors. There are two main receptors that have been shown to be necessary for typical synaptic potentiation in area X spiny neurons. D1 neural receptors activate long term synaptic potentiation (LTP) and D2 receptors have
been shown necessary for long term depression (LTD) (27; 26; 108). Dopamine concentration fluctuations relative to baseline levels and relative receptor densities determine if potentiation occurs. In mammalian striatum the DA fluctuations from baseline can be used as a reinforcement signal (159; 10). Therefore our model distinguishes LTP and LTD in area X as positive and negative fluctuations of DA activity from baseline respectively.

Given the complex nature of dopamine activity in mammals and songbirds, using a single binary VTA neuron may not allow for all observed VTA dynamics. In simple motor learning tasks in mammals VTA/SNc neuronal activity acting as a reinforcement signal can be correlated at the population level (86; 128). A firing rate threshold mechanism for a single neuron may not be sufficient to model the distinction between the LTP and LTD dopamine concentrations. If the VTA population is depressed it may still be difficult to distinguish in a sparsely active VTA neuron. Our model uses the excitatory and inhibitory input to the VTA neuron to define a state of overall dopamine concentration in the striatum. Therefore the firing state of the VTA neuron can be thought of as the activity level of the entire VTA.

The net value of excitatory and inhibitory input into the model VTA neuron determines three types of activity exhibited by mammalian VTA neurons (Fig. 2.14). There is phasic burst activity during the time of conditioned and unconditioned reward, normal baseline activity, and inhibited activity below baseline when
expected reward does not occur (128). With our model there are two mechanisms that can cause the phasic bursting: the reinforcement of correct exploration in song through auditory feedback, and the excitation of the VTA by area X via the XPT pathway. Inhibition is modeled using the GABAergic projection of the striosome neurons in the medial striatum.

The excitatory input to the VTA is modeled from two sources: the synaptically strengthened XPT pathway, and a reinforcement learning signal. The reward generated by improvement in song performance is labeled $R(t)$ and defined further below in the reward section. Therefore the combined excitatory input to the VTA is:

$$I_+(t) = \varphi(R(t) + \sum_{i=1}^{N_{SN}} I_{XV, i}(t - 1))$$

(2.12)

The summation over $I_{XV}$ includes all striatal driving input. Experimentally the convergence of projections to a single VTA neuron is not observed, but the VTA projects diffusely to area X and populations of VTA neurons can behave in a collective manner (86). Therefore it is reasonable to assume that any individual area X neuron can modulate the dopamine concentration it is affected by.

The principal mechanism that the XPT pathway uses to excite the VTA is through the inhibition of the aspiny X neurons. The inhibition of the fast firing aspiny neurons must be sufficiently long for the VTA to generate a noticeable increase in dopamine concentration. We assume that at minimum the aspiny
neuron’s inhibition generated from spiny neurons driven by HVC(X) activity is sufficient to excite the VTA. Therefore we introduce a firing threshold of 3 spikes over 10 milliseconds from a single aspiny neuron to excite the VTA neuron. The synaptic connection strength must also be above threshold $\gamma_{xv}$ in order to generate enough activity. The XPT excitatory pathway input is defined as:

$$I_{XV,i}(t) = \varphi \left[ \sum_{j=0}^{10} R_{XAF,i}(t - 1 - j) - 3 \right] \varphi(W_{X-{VTA}} - \gamma_{xv})$$

(2.13)

The inhibition current is nonzero if the striosome neuron’s synaptic weight has reached a threshold, $\gamma_{st}$. There is no way for $X_{St}$ to fire except through NCL or VTA activity. If there is excitatory input concurrently with inhibition, then VTA activity is considered baseline. This is to mimic the dynamics introduced by Brown, Bullock and Grossberg (10) and experimental results depicted in Fig. 2.14 (128). The depression is maintained for 10 ms or what is necessary to inhibit the phasic activity generated by the excitatory input. The inhibitory current to the VTA neuron is defined as:

$$I_-(t) = -\varphi \left( \sum_{j=1}^{N_{HS}} W_{HS,j}^j R_{HS,j}(t - 1) - \gamma_{st} \right)$$

(2.14)
VTA population level activity is defined in terms of the combined excitation and inhibition of equations 2.12 and 2.14:

\[ S_{VTA}(t) = I_-(t) + I_+(t) \]  

(2.15)

With the inhibitory and excitatory input defined, the conditionality used to define the three states of VTA activity observed is described below.

\[ S_{VTA}(t) = 0 : \text{When both } I_+ = 1 \text{ and } I_- = -1, \text{ or when both equal 0}, \]

the VTA activity is at baseline levels.

\[ S_{VTA}(t) = -1 : \text{When } I_+ = 0 \text{ and } I_- = -1, \text{ the VTA is depressed and can cause long term depression, LTD}. \]

\[ S_{VTA}(t) = 1 : \text{When } I_+ = 1 \text{ and } I_- = 0, \text{ the VTA is active, above baseline and allows for long term potentiation, LTP}. \]

The baseline noise activity, \( \eta_{VTA}(t) \), is modeled as a Poisson train with frequency \( f_{vta} \). Combining the noise and projected input, the firing state of our model VTA neuron is thus defined:

\[ R_{VTA}(t) = \varphi(S_{VTA}(t) + \eta_{VTA}(t)) \]  

(2.16)

Once the XPT projection is learned, the VTA neurons begin to fire at the time of premotor activity in area X. The reward signal from the auditory
feedback diminishes across motif renditions detailed in the reward section. The XPT driven VTA activity is inhibited through striosome activity. Equation 2.15 in combination with our network dynamics is sufficient to reproduce the basic experimental findings in the mammalian dopamine nuclei based on reinforcement learning experiments (86; 128).

2.5 Synaptic Weight Dynamics

Spike timing dependent plasticity (STDP) is a well established framework for synaptic plasticity that has been observed in several areas of the brain (94; 21; 108). In zebra finch synaptic plasticity is largely present in a majority of the song nuclei (27; 26; 123; 99; 4; 133). STDP is observed in mammalian corticostriatal projections supporting our assumption of plasticity in area X (108). Although the XPT pathway consists of three inhibitory projections, STDP has been observed along inhibitory projections experimentally (54). We adopt the model introduced by Song, Miller and Abbott (2000) to capture the synaptic dynamics of the HVC projections to RA and area X, nidopallium projections to the striosome and the XPT pathway.

Synaptic eligibility traces for pre and postsynaptic neuronal activity are used to update synaptic weights based on activity. We define $\Delta t = t_{\text{post}} - t_{\text{pre}}$, where $t_{\text{post}}$ represents the spike time of the postsynaptic neuron, and $t_{\text{pre}}$ represents the firing time of the presynaptic neuron. The magnitude of synaptic modification
is obtained from (138):

\[
F_{uw}(\Delta t) = \begin{cases} 
  A_+ e^{-\Delta t/\tau} & \text{if } \Delta t > 0 \\
  -A_- e^{\Delta t/\tau} & \text{if } \Delta t \leq 0 
\end{cases}
\]

Following Song, Miller, and Abbott (2000) (138) we define two synaptic input variables to model pre and postsynaptic activity. The presynaptic activity trace is defined for projecting neuron \( j \) as \( P_j(t) \). When a presynaptic neuron fires the trace is updated:

\[
P_j(t) = P_j(t) + A_+ 
\]  

(2.17)

If the \( i^{th} \) postsynaptic neuron fires, a similar function, \( M_i(t) \) is updated:

\[
M_i(t) = M_i(t) + A_- 
\]  

(2.18)

When no there is no activity the two eligibility traces decay,

\[
\tau_- \frac{dM_i}{dt} = -M_i \quad \text{and} \quad \tau_+ \frac{dP_j}{dt} = -P_j 
\]  

(2.19)

It is assumed the decay times \( \tau_+ \), and \( \tau_- \) are equal. The decay times are labeled \( \tau_i \) and specified in appendix A for each nucleus.

The modification of the synaptic weight occurs when either a presynaptic or postsynaptic neuron fires. Therefore if the \( i^{th} \) neuron receives presynaptic activity
at time \( t \) from neuron \( j \), the weight is augmented accordingly:

\[
W_{ij}^{\prime} \rightarrow W_{ij}^{\prime} + W M_i(t) \quad (2.20)
\]

and if the \( i^{th} \) postsynaptic neuron fires, the synaptic connection between it and the \( j^{th} \) projection neuron is modulated by:

\[
W_{ij}^{\prime} \rightarrow W_{ij}^{\prime} + WP_j(t) \quad (2.21)
\]

The synaptic weight \( W_{ij} \) is bound between 0 and 1. The scale factor \( W \) is used as a gating mechanism when dynamic conditions are not conducive to long term potentiation or the need for long term depression arises. The conditions for LTP/LTD are projection specific in our model, therefore \( W \) is defined uniquely for each projection type. The decay time of the synaptic weight is controlled by parameters \( \alpha_i \), and are considered to be on a very slow time scale and hence are only computed at the end of a trial or bout. Thus for nucleus A projecting to nucleus B,

\[
\frac{dW_{ij}^{AB}}{dT} = -\alpha_B W_{ij}^{AB} \quad (2.22)
\]

Where \( T \) stands for trial, and this equation is approximated as:

\[
W_{ij}^{AB} \rightarrow W_{ij}^{AB}(1 - \alpha_B) \quad (2.23)
\]
Synaptic spiketime dependent plasticity (STDP), alone is not sufficient to model how the synaptic weights change in the critical areas of the AFP. STDP does not account for the dependence of dopamine in area X and the medial striatum (108; 27; 26). We augment the STDP framework by adjusting the parameters of the synaptic learning rules to model the observed dependencies for learning in the AFP and RA. We introduce these dependences by modulating the synaptic weight coefficient $W$ accordingly.

2.5.1 HVC(RA) to RA : $W_{HR}$

The correlation between burst activity observed in LMAN and HVC is used for the development of synaptic potentiation between HVC and RA connections. LMAN activity facilitates synaptic potentiation between HVC(RA) and RA neurons by activating NMDA receptors in RA (99; 107). The AMPA receptors are activated by the precise burst activity observed in HVC. The AMPA couples with the NMDA to generate the synaptic potentiation. The stereotyped high frequency burst activity from LMAN will couple with the HVC input to RA to generate LTP.

If the synaptic weight $W_{HR}$ has been above threshold $\gamma_{hr}$ longer than the normal fluctuations assumed in HVC(X) ($T_w$ trials) then the connection is considered supersynaptic and is maintained for the remainder of exploration. The synaptic weight decays at rate $\alpha_{hr}$. The scale factor determining potentiation for
the HVC to RA projection is defined as:

\[ W = \beta_{hr} \]  \hspace{2cm} (2.24)

### 2.5.2 HVC(X) to Spiny neurons: \( W_{HX} \)

Experimental evidence indicates that LTP can occur between HVC(X) and \( X_{SN} \) neurons if there is concurrent high frequency activity between the two neurons and dopamine present (26). We use the population state of the VTA, \( S_{VTA}(t) \), to distinguish between bursting reward activity and random sparse firing. Therefore the synaptic scaling factor, \( \overline{W} \), between HVC(X) projection and area X spiny neurons is defined as:

\[ \overline{W} = \beta_{hx}S_{VTA}(t) \]  \hspace{2cm} (2.25)

If the VTA is being inhibited, the VTA nuclei state is defined as \( S_{VTA} = -1 \) and LTD occurs. To prevent potentiation from presynaptic activity we apply \( \overline{W} \) only when the postsynaptic spiny neuron fires. Therefore for the \( i^{th} \) spiny neuron during depressed dopamine concentrations:

\[ \overline{W} = \beta_{hx}S_{VTA}(t)R_{X_{SN}, i}(t) \]  \hspace{2cm} (2.26)
2.5.3 Area X to VTA: $W_{X-VTA}$

The XPT pathway contains three inhibitory projections from $X_{SN}$ to $X_{AF}$ neurons, $X_{AF}$ to VP and finally from VP to the VTA (47; 110). We assume there is a net excitatory projection along this pathway, therefore when spiny neurons are more active, VTA activity increases. The high frequency bursts from HVC and LMAN noise driving UP state spiny neuron activity represents the inhibitory input of aspiny X neurons. Therefore the inhibition of aspiny neurons, $\tilde{R}(t)$, is modeled as the excitatory presynaptic input to the VTA.

For significant potentiation to occur, we assume VTA firing rates or the equivalent dopamine concentrations, must meet an activity threshold. If the VTA’s frequency is comparable to either HVC driven activity via $X_{SN}$ or reward driven activity, then we allow for potentiation to occur. The VTA spike count threshold, $R_{XV}$, is set at 3 in a time window of 4 ms.

$$W = \beta_{XV}\varphi\left(\sum_{k=0}^{4} R_{VTA}(t-k) - R_{XV}\right) \quad (2.27)$$

Without long term potentiation, the weight will decay at rate $\alpha_{XV}$, to below VTA driving threshold. When the synaptic weight $W_{X-VTA}^i$ for the $i^{th}$ presynaptic aspiny neuron is above threshold $\gamma_{XV}$, the inhibited $X_{AF}$ neuron can generate VTA activity through the XPT pathway. This excitation mechanism is defined in equation 2.13.
2.5.4 Striosome to VTA: $W_{HS}$

The striosome synaptic potentiation develops between the NCL and medial striatum spiny neurons that project to the VTA. The striosome’s purpose in our model is to inhibit the VTA, in a similar way proposed by Brown, Bolluck and Grossberg (1999) (10). The time scale of synaptic potentiation is assumed slow to allow for learning in area X, therefore the scale factor $\beta_{hs}$ is set to 0.05. We simplify our model by using one striosome neuron projecting to the single VTA neuron.

The medial striatum neurons in zebra finch receive large diffuse projections of dopamine from the SNc and VTA (47; 110). Therefore activity and synaptic potentiation is probably sensitive to dopamine input. Our model imposes a high frequency VTA or high dopamine concentration requirement for potentiation to occur. The firing threshold was dependent on the overall dopamine concentration function $S_{VTA}(t)$:

$$\overline{W} = \beta_{hs} \varphi(S_{VTA}(t))$$  \hspace{1cm} (2.28)

The decay rate of the NCL to striosome projection must be low during learning to mimic mammalian experiments but large after AFP learning to facilitate our simulations. In mammalian experiments, after several trials the dopamine activity returns to baseline levels at the time of primary reward. When the time of unconditioned stimulus is changed or no reward occurs, the SNc/VTA activity
levels are depressed. This inhibitory dynamic is depicted in Fig. 2.14. In order to mimic the mammalian dopamine depression at the time of unconditioned stimulus, the striosome projection must be maintained when no VTA activity is present. Therefore we need a small decay rate when the system is exploring so that the projection behaves biologically and is maintained in the absence of VTA activity.

Our model assumes one striosome neuron projects to the one VTA neuron to speed simulation. The synaptic decay rate must be large in between HVC(X) explorations otherwise the single striosome neuron would learn all firing times of the VTA preventing dopamine delivery. This is an artificial problem because we do not have multiple striosome neurons projecting to multiple VTA neurons that would otherwise spread the inhibition out and allow VTA to fire. When exploration is over (zebra finch are not singing) we introduce a strong synaptic decay rate to solve this issue.

We set the striosome decay rate $\alpha_{hs} = 0$ when the inhibitory connection is above firing threshold $\gamma_{hs}$ and the system is learning. This threshold is met only when correct VTA learning occurs during AFP exploration. Otherwise the system is either not learning or can be potentiated beyond the decay rate through STDP and $\alpha_{hs} = 0.01$. This is summarized below:
\[
\alpha_{hs} = \begin{cases} 
0 & \text{If } W_{HS} > \gamma_{hs} \text{ during exploration} \\
0.07 & \text{If } W_{HS} < \gamma_{hs}
\end{cases}
\]

When the synaptic weight is above threshold, \(\gamma_{hs}\), the weight does not decay even though the VTA is depressed or fires at baseline levels due to the striosome activity (Middle of Fig. 2.14). This allows our model to maintain the single striosome neuron synaptic weight without VTA activity. Once exploration ends, the weight decays due to uncorrelated activity of the learning circuit. If connections were maintained, eventually all VTA activity would be inhibited. We next explore the motor system and how we model sound production and processing.

### 2.6 Motor Control System

Song production is directly modulated by the RA nucleus and indirectly by AFP activity in juvenile zebra finch. The avian vocal organ syrinx is located near the intersection of the trachea and the bronchi (9). Air flow in the bronchi is coincident with the use of the dorsal syrinx muscles. The activity of the ventral syrinx muscles have been linked to the production of sound oscillating at the fundamental frequency (50). The syrinx, is innervated by the motor neurons located in the hypoglossal nucleus (XIIIts) (144). Finally the premotor nucleus RA directly innervates the syringeal part of the hypoglossal motor nucleus (157). The RA nucleus activity is heavily influenced by NMDA projecting LMAN during the sensorimotor
phase of juveniles (72; 2). Therefore juvenile song is indirectly influenced by the
development of AFP activity.

We adopt the motor neuron model of Fiete, Fee and Seung (2007) which
models the motor output control derived from RA activity (40). These motor
neurons are used to define pulse excitation mimicking the affect the motor neurons
have on the syrinx. The sound generated from the syrinx then passes through the
vocal tract and beak further augmenting sound output. The model uses the pulse
excitation as input to a filter that models the affects that the vocal tract and beak
have on the air flow. The output from the filter is the final song output. This
process is illustrated in Fig. 2.19.

Fig. 2.19 RA neurons project myotopically to the motor system. We use two motor
neurons to determine pulse period \( m_1(t) \) and pulse amplitude \( m_2(t) \). The pulse
input is used in the LPC filter to generate the corresponding song. The coefficients
for the filter are determined from real zebra finch song data donated by Alexay
Kozhevnikov.

The motor neuron’s output is used to define the song’s pitch or fundamental
frequency \( m_1(t) \), and the other neuron controls song amplitude \( m_2(t) \) (72). The
neurons are nonspiking and obey the following equation:
\[
\tau_m \frac{dm_i(t)}{dt} + m_i(t) = \sum_j A_{ij} s_j^{RA}(t) + b_i
\]  \quad (2.29)

The equations for \( m_1(t) \) and \( m_2(t) \) are discretized and evolved on a 1 millisecond time step using runge kutta 4 (117). The motor neurons sum RA input based on two factors; amplitude coefficients \( A_{ij} \) and synaptic activity function \( s_j^{RA} \). The motor amplitude coefficients are equal in magnitude for a given motor neuron with \( |A_{1j}| = 3.2 \) and \( |A_{2j}| = 25.0 \). There is an equal number of positive and negative coefficients so that RA can control pitch and amplitude in both directions. An equal number of RA neurons project to each motor neuron so that the projections are myotopically organized and the projections are assumed to be static (156). The \( b_i \) coefficient is the baseline motor value and set to 60 and 40 for \( m_1 \) and \( m_2 \) respectively. The value of the steady state and range for the motor functions are set to correspond to the value of the pitch period sampled at 40 kHz and excitation amplitude required to generate song using the filter.

The synaptic function \( s_j^{RA} \) is used to track RA activity, and is updated by 1 every time the \( j^{th} \) RA neuron fires:

\[
s_j(t) \rightarrow s_j(t) + R_{RA,j}(t)
\]  \quad (2.30)
The synaptic eligibility then decays exponentially with time constant $\tau_s$.

$$\tau_s \frac{ds_j}{dt} = -s_j$$  \hfill (2.31)

We next detail how the motor neuron output is used to model the syrinx and vocal tract output to generate song.

2.7 Sound Generation

2.7.1 Impulse Train

The motor neurons are combined to define a pulse train used for sound generation. This pulse train is passed through a digital filter to model song production. The sampling frequency of the song generated by the digital filter is set at 40 kHz. Therefore the motor neuron output is linearly interpolated to match the same sampling frequency of the digital filter. We adopt the same notation from Fiete, Fee and Seung (2007) for the new interpolated motor functions as $m_i(t)$ (40). The filter maps the intrinsic zebra finch acoustic features onto the pulse train.

The timing of the pulse is defined using motor neuron 1 as a counter function. When the summation of $1/m_1(t)$ passes the threshold of 1 a pulse is generated the subsequent time step. After the pulse is generated the summation is reset. The amplitude of the pulse is equal to the scaled magnitude of motor neuron 2, $m_2(t_n) \times 10^{-3}$, where $t_n$ is the time of the pulse. To successfully model song using
the excitation signal characterized by the motor neurons, we use the LPC filter common to speech synthesis (119; 91; 40).

### 2.7.2 LPC Filter

The speech synthesis model detailed in Rabiner argues that speech can be modeled as a linear system excited by quasi-periodic pulses (119). The vocal tract’s resonant properties and beak features filter the sound generated initially by the vocal organs (106). Therefore the impulses represent the syrinx activity and the effects of the vocal tract and beak can be modeled as a linear combination of the previous speech samples and syrinx input. We define the impulse as \( u(t) \) and speech as \( s(t) \) in a discrete time representation so that \( u(t_n) \) and \( s(t_n) \) are the \( n^{th} \) sample point and are referenced as \( u(n) \) and \( s(n) \) from this point on. Therefore the \( n^{th} \) speech sample point is defined as:

\[
s(n) = \sum_{k=1}^{p} a_k s(n - k) + Gu(n)
\]

(2.32)

The coefficients \( a_k \) represent the vocal tract characteristics and the \( G \) is the gain of the impulse train previously defined.

In linear predictive analysis a speech sample is approximated as a linear combination of previous speech samples (91; 119; 40). The linear predictor \( \tilde{s} \) is
defined as an estimate using a linear sum of previous song samples:

\[ \tilde{s}(n) = \sum_{k=1}^{p} \alpha_k s(n - k) \]  

(2.33)

The prediction error, \( e(n) \), is defined by Rabiner (119) as the difference between the actual song sample and the linearly predicted sample:

\[ e(n) = s(n) - \tilde{s}(n) = s(n) - \sum_{k=1}^{p} \alpha_k s(n - k) \]  

(2.34)

Looking at equation 2.34, we see that it conforms to the digital filter for song synthesis if \( e(n) = Gu(n) \) and \( \alpha_k = a_k \). The overall purpose of the linear predictive method is to reproduce the song as accurately as possible. Therefore by minimizing the error in the estimate we should generate the desired acoustic features. This is equivalent to finding the minimum impulse input required to generate the speech sample using a linear combination of previous inputs.

Following the procedure outlined in (91), the method of least squares is applied to the error \( e(n) \):

\[ E = \sum_{n} e^2(n) = \sum_{n} (s(n) - \sum_{k=1}^{p} \alpha_k s(n - k))^2 \]  

(2.35)
We minimizing $E$ with respect to the coefficients $\alpha_i$:

\[
\frac{\partial E}{\partial \alpha_i} = 0, \quad 1 \leq i \leq p \tag{2.36}
\]

This leads to the following equations for minimization,

\[
\sum_{k=1}^{p} \alpha_k \sum_{n} s(n-k)s(n-i) = \sum_{n} s(n)s(n-i), \quad 1 \leq i \leq p \tag{2.37}
\]

This set of equations can be solved using the Levinson-Durbin recursion method to obtain the coefficients $\alpha_i$ (84). The method can be used incrementally if the filter changes in time, but we assume a static filter for the course of learning (40). The coefficients $\alpha_i$ were determined to order $p = 10$ using zebra finch song data donated by Alexay Kozhevnikov.

2.8 Auditory Processing

The AFP can directly modulate acoustic features such as fundamental frequency and song amplitude that are controlled by zebra finch during learning (72; 149). Our model uses these two acoustic features as parameters in song evaluation during auditory feedback processing. The parameters are extracted from the song generated using the LPC filter described in the previous section. The auditory feedback parameters are compared to the tutor song acoustic features to generate a reinforcement signal. The amplitude extraction was modeled by Fiete,
Fee and Seung (2007) (40). Pitch period extraction was motivated by Fiete, Fee and Seung (2007) but the YIN algorithm for pitch period extraction was used instead (40; 20).

2.8.1 Pitch Extraction

There are numerous methods of extracting the fundamental frequency of a song (119; 20; 12; 132). The accuracy of a pitch extraction method is typically measured in gross % error so variability can be an issue. We found that the YIN algorithm provided accurate results with minimal computational cost (20). The method is summarized below.

If a sound sample has a period T, and is perfectly periodic, then the following should be true:

$$\sum_{n=0}^{T} (S(n) - S(n + T)) = 0$$

(2.38)

Fortunately zebra finch songs are not that simplistic. The YIN algorithm attempts to minimize the error in the extraction of the fundamental pitch period by modifying Eq. 2.38 to account for the complexity in song and speech. They begin by defining the function $d_t$:

$$d_t(\tau) = \sum_{n=t+1}^{t+N_{samp}} (S(n) - S(n + \tau))^2$$

(2.39)
The $\tau$ that minimizes equation 2.39 for the specific sample is considered the pitch period. In order to minimize contributions from smaller $\tau$ which are almost on top of the original sample, they reduce Eq. 2.39 by the scaled sum of previous summations:

$$d'_t(\tau) = \frac{d_t(\tau)}{\frac{1}{\tau} \sum_{j=1}^{\tau} d_t(j)}$$

(2.40)

The minima of equation 2.40 are interpolated using a parabolic fit. The smallest $\tau$ in the minima set below threshold $\gamma$ is selected. If $d'_t$ does not have a minimum below the threshold then the global minimum is found. When the extracted pitch is drastically different from the previous song sample, the local minimum of equation 2.40 is found within a 5 ms window centered at the time of the poorly estimated pitch. This minimum across the windowed time steps is then substituted as the new pitch period. This procedure is defined as step 6 in the YIN algorithm (20).

Equation 2.40 is calculated for lag $\tau$ between 12 and 80 time steps. The restricted range is determined by the zebra finch’s ability to generate pitch. The time representation is the same as the LPC song generation which is set at 40 kHz. Therefore 40 time steps is equivalent to 1 millisecond. The sample window is $N_{samp}$ time steps in length (at 40 kHz sampling rate) and the middle $n_{ps}$ points are set to the extracted pitch. The window then slides $n_{ps}$ points down and the process
is reiterated. The extracted pitch is fit to a Savitzky-Golay smoothing filter with sample width 11 (125; 117).

2.8.2 Amplitude Extraction

A non-overlapping window of width $N_{\text{amp}}$ is used to sample the song profile for amplitude extraction. The maximum of the absolute value of the segment is found and multiplied by 0.3 to scale it to the pulse train filter. The extracted value is defined as the amplitude for the $N_{\text{amp}}$ points over the entire window.

It is important to ensure the methods described work effectively and that the LPC coefficients capture the song generation dynamics sufficiently. We applied the pitch and amplitude extraction methods to a real syllable sample, and then used the extracted values to generate a pulse train. This pulse train was then passed through the LPC filter to generate an artificial syllable. The real syllable and LPC filter generated syllable are shown in Fig. 2.20. The spectrogram is a useful measure for understanding and visualizing the frequency component of song as it changes with respect to time. The sonograms of the real and LPC generated syllables are depicted in Fig. 2.21. In both figures the general characteristics of the real syllable are captured by the LPC generated syllable.
Fig. 2.20 The syllable was obtained from data given by Alexay Kozhevnikov. The lpc method was generated from a pulse train that used the extracted pitch and amplitude from the real syllable data. The lpc coefficients $\alpha_i$ are defined in appendix A.
Fig. 2.21 Vertical axis is frequency with a range of 20 kHz. Horizontal axis is time. *Left Syllable* - Original Tutor, *Right Syllable* - LPC Generated Tutor. The frequency profile as a function of time represented by the lpc method is not exact, but is a close match to the real syllable sonogram. The method does capture the harmonic stack features.

### 2.9 Song Evaluation

Where the tutor template maybe represented and how it is used in song evaluation is still unknown (1). Recordings reported by Phan, Pytte and Vicario (2006) (114) show experimental evidence that a long lasting representation of the tutor template is possible. A novel stimulus generated distinguishably different rates of activity attenuation from the tutor stimulus across repeated playback. This rate of habituation was recorded in the NCM and is depicted in Fig. 2.10. Therefore over several trial presentations, there were distinct differences between the activity responses of a novel stimulus and that of the tutor song. This neuronal mechanism could provide the evaluation required between the bird’s own song and the tutor song to drive learning.
We begin by assuming that the model's acoustic features extracted from song are stored across trial renditions. The acoustic features are the same proposed by Fiete, Fee and Seung (2007) which are song amplitude and pitch period (40). The evaluation of the feature trajectories must be designated such that it can guide the direction of the song parameters whether in a positive or negative direction towards the tutor song amplitude and pitch period. The comparison must also be sensitive enough to small changes so juvenile song can come to a very close match with the tutor template.

HVC projections to area X produce high frequency activity that is sustained across several motifs. This slow synaptic modulation represents the elementary unit of learning and exploration in our zebra finch model. The LMAN drives variability in the HVC(X) projections and carries the burst signal from area X (via DLM) through to the RA. The high frequency burst activity in LMAN can be seen in figures 2.7 and 2.8. In order for the network to recognize such a small incremental change, compared to the level of variability that can be caused by all LMAN noise driving RA, we use averages across renditions to define change.

To determine the direction of the juvenile’s syllable trajectory, we use the difference of two sliding averages of trial width $T_w$ that are separated by a constant trial distance of $T_w/2$. The two windowed averages represent a recent and older version of the same song features that are combined to determine the direction across trials. The current average of song amplitude and pitch period is denoted
by $A(t)$ and $P(t)$:

$$\bar{A}_{T_n}(t) = \frac{1}{T_w} \sum_{i=0}^{T_w-1} A_{T_n-i}(t) \quad \text{and} \quad \bar{P}_{T_n}(t) = \frac{1}{T_w} \sum_{i=0}^{T_w-1} P_{T_n-i}(t) \quad (2.41)$$

Where the subscripts extracted pitch and amplitude is trial number and $T_n$ denotes the current trial. The delayed or older averaged parameter values $A_L(t)$ and $P_L(t)$ are defined as:

$$A_{L,T_n}(t) = \frac{1}{T_w} \sum_{i=\frac{1}{2}T_w}^{\frac{3}{2}T_w-1} A_{T_n-i}(t) \quad \text{and} \quad P_{L,T_n}(t) = \frac{1}{T_w} \sum_{i=\frac{1}{2}T_w}^{\frac{3}{2}T_w-1} P_{T_n-i}(t) \quad (2.42)$$

The difference between the current and older average representations provides the measure of direction for the overall activity across bouts. The older average $A_L$ could represent the song amplitude prior to a bout of motifs, and $\bar{A}$ could represent what the juvenile considers the current song. The difference between the two will approach zero if the song does not change. This would be equivalent to the diminishment of the level of activity with repeated presentations of the same stimulus observed in the NCM (114).

The direction the song travels in relation to the tutor song is computed and combined with the song trajectory to give an overall evaluation. The comparison to the tutor song amplitude $A_T(t)$ is defined as a constant coefficient that changes sign depending on what direction the song parameters need to change. Dropping
the subscript $T_n$ denoting trial number, the directional component $A^*$ is defined as

$$A^*(t) = \begin{cases} A_c & \text{if } (A_T(t) - \overline{A}(t)) > 0 \\ -A_c & \text{if } (A_T(t) - \overline{A}(t)) < 0 \end{cases}$$

and the corresponding comparison to the tutor pitch defines $P^*$:

$$P^*(t) = \begin{cases} P_c & \text{if } (P_T(t) - \overline{P}(t)) > 0 \\ -P_c & \text{if } (P_T(t) - \overline{P}(t)) < 0 \end{cases}$$

The magnitudes $A_c$ and $P_c$ are used as scale factors so the two song properties evaluated give equal responses to neural dynamics. The evaluation of song direction is the combination of the direction the song needs to go ($A^*$) and the direction it is currently going ($\overline{A}(t) - A_L(t)$):

$$E_A(t + \Delta \tau) = A^*(t)\left(\overline{A}(t) - A_L(t)\right) \quad (2.43)$$

and,

$$E_P(t + \Delta \tau) = P^*(t)\left(\overline{P}(t) - P_L(t)\right) \quad (2.44)$$

The auditory feedback and processing outlined above reaches the VTA at a delayed time, $\Delta \tau$, and is set to 65 milliseconds (97; 96; 93). With the evidence of declined activity for all novel stimuli in the NCM, (114), we believe an evaluation of song improvement using trial averages is a plausible assumption. When the
tutor song is determined to be unvoiced, \( P^*(t) \) is set to 0, this is defined when \( A_T(t) \leq 0.01 \).

Our model takes the evaluated directions of the amplitude and pitch, and combines them to ultimately form the reinforcement signal sent to the VTA. In order to generate an excitatory signal to project to the VTA, the evaluated activity must accumulate and pass some threshold. This accumulation is defined as \( E_T \):

\[
E_T(t) = \sum_{k=0}^{7} \left[ E_A(t-k) + E_P(t-k) \right]
\]

(2.45)

If the evaluation of the juvenile song has moved in the correct direction over a small time window (7 ms), enough to pass some sensitivity threshold, \( E_\gamma \), reward is generated. The threshold is useful in ignoring random fluctuations in the song trajectory that occur at low values while focusing on systematic changes that are explored over several song bouts. The reward, \( R \), used in equation 2.12 is defined as:

\[
R(t) = \varphi \left( E_T(t) - E_\gamma \right) \varphi (10 - \sum_{k=0}^{20} R(t-k))
\]

(2.46)

The first factor can generate constant reward for a long period of time depending on the nature of fluctuations, evaluation threshold and learning increment. The second factor is used as a saturation mechanism for the VTA neurons to
address any prolonged reward signal. The saturation factor facilitates parameter-izations of the synaptic weights in our simulations by generating more stereotypic reward input. This description generates the necessary reinforcement reward to properly characterize improvement in the juvenile song with respect to the tutor template.

Improvement in song is also measured as positive when incorrect exploration away from the tutor template ends. At the end of a series of bouts when the zebra finch are known to temporarily rest, reward would be generated if the session overall had directed song in a poor direction. There is no singing during this time therefore no incorrect activity would be learned. After \( T_w \) trials of exploration or after a UP state group has been learned, the model is simulated with no singing to allow striosome synaptic weights to decay, and to avoid this erroneous reward generation. Reseting amplitude and pitch averages could also avoid this issue. Simulation results reported only include the singing sessions or trials.

2.10 Results

2.10.1 Learning Single Note Variations

How a single HVC to area X projection uses reinforcement to improve juvenile song is the basis for correct song crystallization. Detailed in Fig. 2.22 is a raster of neuronal activity involved in learning the correct projection. We find that randomly selected area X UP states (Fig. 2.22 A) that have above threshold
HVC(X) projections can drive reward at the same time and thus reinforce the initially random exploration (Fig. 2.22 C). The sustained area X activity (across syllable repetitions or trials) provides the AFP with consistent output that would not be present otherwise. The consistent AFP output translates into precise RA activity (Fig. 2.22 B). Through striosome inhibition of the VTA, dopamine oversaturation is prevented (Fig. 2.22 D). Once learning is complete the X neuron goes back to a DOWN state (Fig. 2.22 A) but RA has learned to maintain its HVC projection (Fig. 2.22 B). Thus our model illustrates how a single note variation or HVC(X) projection (48) can use reward encoded dopamine activity to guide syllable development to the tutor template.

The sequence of activity illustrated in Fig. 2.22 is dependent on the order of synaptic potentiation of the plastic projections in our model. We explore the synaptic plasticity dynamics in Fig. 2.23. The first synaptic event is the HVC(X) projection going above firing threshold when the spiny neuron is in its UP state (Fig. 2.22 A). The firing threshold transition of the HVC projection is not depicted in Fig. 2.23 A because the projection is artificially selected. We assume the synaptic strength difference between baseline and above firing threshold does not affect the temporal dynamics of learning.

The HVC(X) driven precise firing in area X generates correct RA activity allowing reward to drive the VTA (Early trials of Fig. 2.22 C). The correlation of the reward with UP state spiny neuron activity drives the XPT pathway ($W_{X-V_{TA}}$)
Fig. 2.22 Time is located on the horizontal axis and trial number increases up the vertical axis. Plots C and D have fewer trials displayed to focus on the learning dynamics. A - The area X spiny neuron is initially in a DOWN state. Once in UP state, activity is driven by noisy LMAN and a precise HVC(X). The neuron is later reinforced by dopamine allowing the HVC connection to be maintained longer. After learning of HVC to RA projection, X is assumed to be in DOWN state again (top of raster plot). B - An RA neuron driven by precise firing in LMAN via the spiny neuron in area X (A). If the X neuron is maintained longer than normal fluctuations then the HVC(RA) connection is learned permanently. C - Initial VTA trials display reward reinforcement signal. Once the XPT pathway is learned \( W_{X-VTA} > \gamma_{xv} \) the VTA neuron fires at the precise area X timing as well (A). The reward timing activity ends when the averaged parameters approach each other late in the exploration (Eq. 2.43 and 2.44). The last few trials the VTA goes to baseline once striosome learn to inhibit at the area X driven time. D - Striosome activity is modeled to resemble VTA activity to aid in learning. The last 2 striosome bursts are when \( W_{HS} > \gamma_{hs} \) and the VTA is inhibited. Once inhibition of the VTA occurs the XPT pathway and \( W_{HX} \) connections are pushed below threshold.
potentiation (Fig. 2.23 C). When the XPT projection is above the synaptic threshold $\gamma_{xv}$, the spiny neurons can drive the VTA dopamine neurons (top half of Fig. 2.22 C). Now dopamine is present in high concentrations during the precise bursting in area X. The HVC(X) projection is quickly potentiated to maximum synaptic value (Fig. 2.23 A). During the entire process that spiny neurons are transmitting precise firing through the AFP the HVC to RA projection is learned at the correct time (Fig. 2.23 B).

The process of inhibition and decline of the connections learned in the AFP is illustrated in late trials of Fig. 2.23. Initially striosome synaptic potentiation occurs at the time of primary reward (dashed line in Fig. 2.23 D). The primary reward generated is attenuated when the song parameters $(A_L, \overline{A})$ used in evaluation (Eq. 2.43 and 2.44) become equal. The time that striosome neurons learn in our model is the time of precise firing in area X which drives the VTA (solid line in Fig. 2.23 D). Therefore the function of striosome is to control the VTA activity potentiating premotor activity in area X and not the primary reward. Thus our model focuses striosome activity at a different time than the Brown, Bullock and Grossberg model (10).

The inhibition of the VTA at the time of precise area X activity, and the attenuation of VTA activity at the primary reward time leaves the VTA firing at baseline levels (top trials of Fig. 2.22 C). Without correlated activity between spiny neuron UP states and the VTA, the XPT projection decays below VTA driving
threshold (Fig. 2.23 C). Without the excitatory current from area X, the VTA only receives inhibition from the striosome \(S_{VTA} = -1\) from Eq. 2.15. When the VTA is depressed we assume a similar LTD dynamic along the corticostriatal pathway of the Brown model. Thus the HVC(X) to X synaptic weight, \(W_{HX}\), is depressed (Fig. 2.23 A). Without consistent VTA activity to correlate striosome activity the projection decays away (Fig. 2.23 D) leaving only the permanent HVC to RA projection left (Fig. 2.23 B).

### 2.10.2 Parameter Exploration

Three pathways and their respective parameters largely determine the rate of learning in our model. The three model dynamics are the striosome inhibition of the VTA, synaptic potentiation between HVC and area X spiny neurons \(X_{SN}\), and potentiation between the \(X_{AF}\) and VTA neurons along the XPT pathway. We first looked at the striosome projection because of the required slowness. The striosome synaptic weight is incrementally potentiated by \(\beta_{hs}\) and the synaptic weight decays at rate \(\alpha_{hs}\). These two parameters largely determine the projections effectiveness. Figure 2.24 explores how dependent the model is on the striosome parameters.

The parameter search of Fig. 2.24 looked at learning a single connection for a given parameter set with a HVC(X) projection that was predetermined to be correct. After \(T_w\) trials if the HVC(RA) connection was learned then the simulation was considered a success. The simulation was repeated several times to account
Fig. 2.23 A - When $W_{X-VTA} > \gamma_{xv}$, area X drives VTA during the precise burst activity. With dopamine concentrations higher at that time, the synaptic weight $W_{HX}$ is potentiated (trials 58-62). If LTP did not occur the HVC(X) connection would fluctuation back below firing threshold. When the striosome inhibit the VTA, synaptic depression occurs and $W_{HX}$ decays (> trial 79). B - When HVC(RA) activity is correlated to the RA bursting for a longer duration of trials than the characteristic HVC(X) fluctuations, a permanent connection is formed. The trial axis is shortened to focus on the trials that include learning. C - $W_{X-VTA}$ is the first to be potentiated if the UP states of the neuronal group in area X are still firing when reward arrives from the VTA (trials 43-58). Once $W_{X-VTA} > \gamma_{xv}$ the area X UP state group can drive the VTA (plot A). When the VTA is inhibited, $W_{X-VTA}$ begins to decay because there is no correlation between reward and UP states any longer (trials > 78). D - Striosome synaptic potentiation is assumed much slower than other connections to allow for learning in plots A-C to occur first. The dashed curve in plot D is the $W_{HS}$ synaptic weight learning the primary reward time that occurs first in the VTA (Fig. 2.22 C-D). $W_{HS}$ reaches threshold and inhibits the VTA during the precise burst activity in X at trial 78. Once the VTA has been inhibited and the relevant connections unlearned, there is no activity to maintain the striosome connection and it decays (> trials 79).
for random noise, and song fluctuations then the fraction of successful trials was calculated and plotted. When $\beta_{hs}$ is large the striosome learn to inhibit the VTA quickly. Therefore the primary reward from a correct exploration is not presented long enough to potentiate the XPT pathway or HVC(X) projection. The model was fairly robust for $\alpha_{hs}$ but the parameter search did not include values above 0.1. The search was restricted because larger values would be involve unrealistic decay rates and are not necessary.

Formation of the HVC to area X projection is highly dependent on the learning rate of the XPT pathway projection. The synaptic potentiation along the XPT pathway cannot take longer than the trial duration of the fluctuating HVC(X) projection that is considered above firing threshold. If the XPT pathway learns too slowly then the HVC(X) projection fluctuates back below firing threshold or the UP state correlation ends, and the process must start over. The dependence of the HVC(X) synaptic weight increment $\beta_{hx}$ on the synaptic increment $\beta_{xv}$ is explored in Fig. 2.25.

In our model, small $\beta_{xv}$ represents a slow synaptic learning for the XPT pathway. Our parameter search verifies that when the XPT pathway learning is slow (small $\beta_{xv}$), the model’s learning rate is low (bottom section of Fig. 2.25). The XPT pathway is unable to learn before the UP states become uncorrelated or the HVC(X) projections fluctuate below firing threshold. The HVC(X) projection can also disrupt learning if the synaptic potentiation occurs slower (small $\beta_{hx}$)
Fig. 2.24 Fractional success rate of learning connections. Red is 1 or 100% learning rate and blue is 0 learning rate. Simulations were run for 100 trials, and repeated 10 times. $\alpha_{hs}$ was varied by 0.01 and $\beta_{hs}$ by 0.05 creating a 200 point grid. The learning rate increases substantially for low $\beta_{hs}$ values and appears robust across the $\alpha_{hs}$ parameter.
than the duration of the correlated UP states. If the projection is not strong enough to induce an UP state in the spiny neuron by the time the group becomes uncorrelated then the learning does not occur.

2.10.3 Model Parameters

Model parameters were explored to understand their properties in an attempt to optimize the learning rate of the model. The $\beta$ and $\alpha$ parameters were explored in the previous section and were defined to generate a robust learning rate given a trial exploration window of $T_w = 20$. The striosome parameters explored were set to $\alpha_{hs} = 0.01, \beta_{hs} = 0.05$. Although not depicted in Fig. 2.25, $\beta_{xv} = 1.0$ and $\beta_{hx} = 0.7$ were used and produced robust learning. The other $\alpha$ and $\beta$ parameters were selected at reasonable values and verified that the model learned at a good rate.

The evaluative parameters $A_c$ and $P_c$ of equations 2.43 and 2.44 were explored through individual simulations to determine their affect on the learning rate. If the pitch and amplitude coefficients ($A_c, P_c$) are too small then the song performance evaluation function $E_T$ (Eq. 2.45) cannot surpass the reward generating threshold $E_\gamma$. If the pitch and amplitude coefficients are too large, the VTA is over excited and all exploration is reinforced. Therefore values between these two extremes were used: $A_c = 17.544$ and $P_c = 0.25$. 
Fig. 2.25 Fractional success rate of learning connections as a function of $\beta_{hx}$ and $\beta_{xv}$ was explored. Red is 1 or 100% learning rate and blue is 0 learning rate. Simulations were run for 100 trials, and repeated 20 times. $\beta_{hx}$ and $\beta_{xv}$ were varied by 0.05 creating a 400 point grid. For low values of $\beta_{xv}$ the XPT pathway does not learn to drive the VTA before the HVC(X) projection fluctuates back below firing threshold or the UP state correlation ends. For low values of $\beta_{hx}$ no learning occurs because the synaptic connection is not potentiated before the spiny neuron UP states become uncorrelated. Large values of $\beta_{xv}$ and $\beta_{hx}$ produce high learning rates.
Finally the magnitude of the motor neuron coefficients $A_{ij}$ and motor time constants $\tau_m$ were explored through individual full-length simulations. Coefficient $A_{ij}$ determines how large the $i^{th}$ motor neuron is affected by the $j^{th}$ RA neuron with activity trace $s_j$. The time constant $\tau_m$ determines the time scale that the RA activity contributes. Both parameters are defined by the motor neuron equation 2.29. The larger the coefficients, the larger the effect that a single RA spike has on the motor neurons. The motor neurons directly control the songs amplitude and pitch period. Therefore the coefficient's affect on the motor neurons determines how many RA spikes are required to go from juvenile subsong to the tutor template. The affect of a RA spike on the motor neurons also has a direct relationship with the level of fluctuations and accuracy the model can obtain. A compromise between fast learning with fewer required RA neurons using large coefficients and final accuracy of the acoustic features using smaller coefficients was determined. We defined the amplitude motor neuron coefficient $|A_{1j}| = 25.0$ and the pitch period motor neuron coefficient $|A_{2j}| = 3.2$. The motor neuron time constants were set to $\tau_{m_1} = \tau_{m_2} = 1\text{ms}$. The remaining model parameters either have a passive effect such as time duration of a trial ($T = 500\text{ms}$) or have similar effects as the parameters explored here ($\tau_{xx}$, etc.). These remaining parameter values are listed in appendix A.
2.10.4 Song Development

Our model was simulated for several thousand trials to determine if an unstructured syllable could be evolved into a tutor syllable template. The first parameters evaluated were the very acoustic features used in song trajectory evaluation. The juvenile amplitude and pitch period extracted from early and final developmental stages are compared to the tutor template. This comparison is depicted in figures 2.26 and 2.27. We average the extracted amplitude and pitch period to remove any remaining variability and generate the song profile. Both the extracted amplitude (Fig. 2.26) and pitch period (Fig. 2.27) are in good agreement with their respective tutor template.

Two main factors determine the limits on the final accuracy of the extracted pitch and amplitude. The first factor is the magnitude of the motor coefficients $A_{ij}$ already discussed; they determine the overall contribution the RA burst has on the song acoustics and therefore determine how small an increment that can be learned. The second factor is an extension of the first: the HVC driven activity limits RA improvements to discrete bursts. Our model uses binary neurons and does not have a continuous current input determined by the relative strength of the synaptic weight. The model uses a firing threshold for HVC driven RA activity which produces an all or nothing effect (and a one to one correspondence between an HVC spike and an RA spike). The discrete HVC driven RA activity is learned
and not the individual RA spikes caused by noise. Therefore the discrete increment learning of RA burst activity limits the model’s accuracy.

Our model is capable of shaping an initially unstructured plastic song into a highly ordered song that models the acoustic features of the tutor syllable template. The song profile is depicted in Fig. 2.28. In comparison, the juvenile song captures the overall song envelope fairly well. There is some disagreement at the edges which is correlated with amplitude error (illustrated in Fig. 2.26). An evolution of the juvenile plastic song into the crystallized adult song and how they compare to the tutor syllable template is illustrated with the sonograms of Fig. 2.29. The initially unstructured plastic syllable clearly evolves into a harmonic stack that models the tutors.

The error trajectory is a useful measure for depicting the time scale of syllable development. The amplitude error was calculated based upon:

\[
\delta E_A(trial) = \sqrt{\frac{\sum_{t=t_s}^{t_e} \left( A_{model}(t) - A_{tutor}(t) \right)^2}{\sum_{t=t_s}^{t_e} A_{tutor}(t)}}
\]  \hspace{1cm} (2.47)

Where \(t_s\) and \(t_e\) are defined as the limits of the timing spectrum created by the HVC neurons projecting to RA. A similar equation for extracted pitch period is defined:

\[
\delta E_P(trial) = \sqrt{\frac{\sum_{t=t_s}^{t_e} \left( P_{model}(t) - P_{tutor}(t) \right)^2}{\sum_{t=t_s}^{t_e} P_{tutor}(t)}}
\]  \hspace{1cm} (2.48)
Fig. 2.26 Black Diamond - Juvenile Amplitude, Blue Square - Adult Amplitude, Red Solid Line - Tutor Amplitude. The final adult amplitude profile (blue square) conforms well to the tutor amplitude extracted from the original tutor syllable. There is some variance from the tutor template at 240 ms and 275 ms. This is visible in the song profile plotted in Fig. 2.28.
Fig. 2.27 Black Diamond - Juvenile Pitch Period, Blue Square - Adult Pitch Period, Red Solid Line - Tutor Pitch Period. The juvenile syllable is highly variable but evolves into the harmonic stack of the adult song. The tutor and adult pitch period show a fairly uniform value across the majority of the syllable. This uniformity indicates a harmonic stack which is a constant fundamental frequency across the syllable.
Fig. 2.28 The model syllable is lpc generated from the extracted pitch period and amplitude. The tutor syllable is lpc generated from the extracted pitch period and amplitude of the original tutor syllable song file. The overall syllable profiles agree. There are deviations at the edges. The attenuation of the envelope on both sides mismatches the tutor. This mismatch is correlated with variation in the extracted amplitude depicted in Fig. 2.26.
Fig. 2.29 Vertical axis represents frequency range from 0 to 20 kHz. Horizontal axis is trial time from 220 ms to 290 ms. Left Sonogram - Juvenile input syllable. Middle Sonogram - Crystallized juvenile or adult syllable sonogram averaged over 40 trials. Right Sonogram - Tutor syllable generated with lpc. The trajectory of the syllable clearly forms a harmonic stack as it crystallizes and is close to the tutor template acoustic structure.

The song error for both the extracted amplitude and pitch period were calculated and plotted as a function of trial number in Fig. 2.30. The overall trajectory is towards the tutor template and eventually stabilizes to the crystallized syllable.

2.10.5 Area X Influence on Song Development

The area X dynamics involving correlated UP states and high frequency HVC input direct learning in our model. The nature of correlation of the UP states is unknown in zebra finch area X. To the knowledge of the author there are no in vivo experiments showing the existence of correlated UP states in spiny neurons of area X. If there are attractor-like groups that form as suggested by the mammalian data (13) what influence in syllable trajectory development they
Fig. 2.30 The error of the extracted juvenile amplitude is represented in blue and is scaled by the vertical axis on the left. The error of the extracted juvenile pitch is depicted in green and scaled by the right vertical axis. The overall trajectory takes several thousand trials, and is highly dependent on the parameters $T_w$, and the projection parameters involved in synaptic potentiation explored earlier.
have is unclear. Perhaps a natural bias emerges to allow for independent control of the topological projections observed in the AFP (64). Our model indicates that neuronal groups controlling specific trajectories in song parameters that push the song to the tutor values are favored. This UP state preference is depicted in Fig. 2.31.

The number of neurons in the grouped UP states determines the size of stable fluctuations and affects the model’s learning rate. In our simulations when selecting the synaptic projections that form with the HVC(X) neuron, the size was determined randomly from a scale of 1 to $N_e = 10$. The maximum neuronal number of UP states $N_e$, was selected based on the motor coefficients $A_{ij}$ to maintain reasonable fluctuations. Experimentally the natural size of the proposed correlated UP states could have a direct affect on the nature of exploration song during learning. By varying the size of the correlated UP state group we allow a greater range in exploratory signal originating in area X.

Figure 2.32 depicts the dependence of the rate of learning on the maximum size of correlated UP states in area X. Multiple simulations were run for a given maximum UP state size that received HVC(X) exploration projections. The maximum UP state size was varied and the learning rate was investigated. The error reduction was calculated for the extracted amplitude and pitch and plotted versus the UP state number. The error terms are defined in Eq. 2.47 and Eq. 2.48 and
Fig. 2.31 Directional bias of song parameters defined by the composite UP states is represented on the horizontal axes. Therefore the +3 on amplitude axis and -4 on the pitch axis represents an UP state group that has an overall effect of driving 3 RA neurons projecting to the amplitude motor neuron in the positive direction, and 4 RA neurons that drive pitch period in the negative direction. The vertical axis is number of trials maintained by the area X UP state group with those group attributes. During simulations the size and distribution of the UP state group was selected at random with a maximum of 10 neurons driven by the same HVC(X) neuron. The overall bias is generated because more trials are spent on learned UP states than on the random UP state fluctuations that do not improve song. The initial pitch period and amplitude required a more positive pitch and a reduction in amplitude as can be seen in Figs. 2.27 and 2.26. The trial bins are therefore larger for negative amplitude and positive pitch UP state groups.
T stands for trial:

\[
E_f = \frac{1}{2} \left[ \frac{\delta E_A(T_1) - \delta E_A(T_{end})}{\delta E_A(T_1)} + \frac{\delta E_P(T_1) - \delta E_P(T_{end})}{\delta E_P(T_1)} \right] \tag{2.49}
\]

The size of the correlated UP states directly affects the model’s rate of learning. As the number of correlated UP states with precise HVC(X) projections goes up, the learning rate increases.

2.11 Discussion

2.11.1 Model Learning Rate and Song Variability

The model’s learning rate is slower than previously proposed models but is reasonable for zebra finch song development. Accounting for the trials not involved in exploration, the model’s juvenile syllable typically crystallized within 50,000 trials. The Fiete, Fee and Seung model learned relatively quickly, obtaining an adult syllable comparable to the tutor template within 2000 trials (40). The Doya and Sejnowski model learned within 1000 trials (32). The Troyer Doupe model was able to learn within approximately 25000 trials (151; 152). The actual zebra finch can generate over 50,000 bouts during song development (68; 40). With a variable number of motifs ranging from one to several renditions per bout the actual number of syllable repetitions is well above the number of trials our model requires.
Fig. 2.32 Simulations were run for approximately 14000 trials as a function of the number of spiny neuron UP states that receive strong HVC(X) projections. The fraction in error reduction was calculated for both pitch and amplitude separately and then combined. The solid line represents a linear fit using the polyfit algorithm in MATLAB. The error reduction fraction increases as a function of strong HVC(X) projections to area X. There is a clear relationship indicating that larger UP state groups should increase the rate of learning.
The model’s learning rate can be improved by adjusting the time scale of variability and song evaluation. Simulations with a smaller $T_w$ were explored and results indicated that learning was possible and significantly faster. Therefore HVC(X) projections and correlated UP states in area X can be maintained for shorter trial durations. One weakness in reducing $T_w$ is that the model becomes more sensitive to LMAN noise. The ratio of high frequency area X activity to LMAN noise becomes smaller for shorter trial durations. Experiments show LMAN is capable of precise high frequency activity across multiple song renditions in adult zebra finch (82). Thus the ability of the AFP to learn precise high frequency activity in the presence of LMAN noise is possible. Perhaps a memory mechanism within NCM or another nucleus has a nonlinear method of extracting more stable fluctuations. The nonlinear extraction would allow reward evaluations for the slower temporal variations while ignoring the novel ones associated with LMAN activity. This evaluation mechanism would also improve the model’s learning rate.

Fluctuations about the mean syllable trajectory are determined by the size of the RA nucleus and LMAN variability. With a finite number of motor neurons a larger RA nucleus will contain more RA activity driven by LMAN noise thus producing more song variability. The magnitude of song variability has important implications in our song evaluation. Our model assumes that high frequency HVC activity drives area X explorations used in learning. The averaging of extracted song parameters defined in Eqs. 2.41 and 2.42 must have a signal to noise ratio
greater than one. In other words, correlated reward generation requires the average of HVC(X) generated activity in RA to be greater than the average of LMAN noise activity generated in RA:

$$\langle \sum_j A_{ij} \eta_{LMAN,j}(t) \rangle_{T_w} < \langle \sum_k A_{i,f(k)} R_{HVC(X),k}(t) \rangle_{T_w}$$ (2.50)

The first summation over j is assumed to be all LMAN neurons associated with the RA neurons projecting to motor neuron i. The second term includes a summation over the $N_e$ HVC(X) projections that generate the high frequency activity required by our model. The function f(k) maps the $k^{th}$ HVC(X) projection to its corresponding area X neuron affecting motor neuron i. When fluctuations violate Eq. 2.50 the evaluation $E_\Gamma$ of Eq. 2.45 is pushed above the reward threshold $E_\gamma$. This violation represents VTA reward activity generated by random noise fluctuations. Therefore if $T_w$ is small enough that Eq. 2.50 is violated frequently then noise rather than correlated song improvement would drive the model’s trajectory.

2.11.2 The Role of Sleep

How zebra finch incorporate sleep into song learning is currently unknown. Precisely timed burst sequences of HVC and RA neurons recorded during sleep are similar to recordings observed during previous day singing (18). It is believed that this activity in sleeping zebra finch may represent song replay or practice. The purpose of this song replay in song development is currently unknown. Analysis
of morning song has revealed an initial deterioration in the structure of the song when compared to the previous day’s performance (23). This deteriorated state recovers following an intense morning singing session. During the latter part of the morning session new learning appears to occur. How might song deterioration in the morning be correlated with song replay during sleep? A plausible explanation put forward by Deregnaucourt et al. (2005) suggests song deterioration occurs during song replay because no auditory feedback is generated (23). Without feedback there is nothing to guide development and uncorrelated activity between the noisy LMAN input and HVC driven bursts would degrade song structure.

Our model would support the proposition of Deregnaucourt et al. (2005) because without auditory feedback directing VTA activity, synaptic potentiation would be random and uncorrelated. This theory alone is not sufficient to explain another observation made by Deregnaucourt et al. (2005) concerning overall song development (23). The tutor imitation score was found to be higher in birds that showed a greater deterioration in their morning song. Investigation into the role HVC song replay has on the AFP during subsequent singing sessions will be an important step in addressing this problem.

An error signal generated during sleep could be used during song replay to improve overall imitation scores. The error signal would require HVC or auditory nuclei to manufacture auditory feedback. During song replay an efference copy could be mapped between the "motor" signal in RA and the auditory feedback
There would be a direct map correlating the motor activity to auditory song representation. An error signal could be generated if the auditory feedback is first compared to an auditory representation of the tutor template. The error signal processed in the auditory representation would be mapped onto the motor signal through the efference copy. Song degradation would still occur because of the observed increase in noise (18) and lack of feedback evaluation through the VTA.

The error signal mapped onto the motor signal could bias the degradation away from the correctly learned activity. This would explain the overall increase in imitation scores for zebra finch who show large daily song deterioration. Intense song reconstruction during early morning singing could be modeled using recent AFP activity. The previous day’s explorations could bias the projection exploration between HVC and area X during the next morning. This bias would speed up correct exploration allowing the song trajectory to retrace its previous path quickly. Experiments show new learning occurs during the late stages of morning singing (23). The gradual learning of note variations in our model likely corresponds to this phase of song learning.

### 2.11.3 XPT Pathway Dynamics

Experiment shows spiny neuron stimulation increases dopamine concentration in area X (45) but how the XPT pathway excites the VTA to accomplish
this is unknown. Until recently, detailed experiments investigating the pathway between the spiny neurons of area X and the VTA did not exist. Gale, Person and Perkel (2008) (47) report a triple inhibitory projection pathway from area X to the VTA. The spiny neurons inhibit the aspiny neurons that are contained in area X. The pallidal aspiny neurons of area X project to the ventral pallidum (VP), which in turn projects to the VTA (47; 110). The pathway from the spiny neurons in X through to VTA, are all considered GABAergic inhibitory projections. This pathway is illustrated in Fig. 2.33 A. In contrast, evidence indicates that the pathway generates net excitation in the VTA (45). Further experimentation investigating how a triple inhibitory pathway increases dopamine concentration in area X needs to be explored.

The XPT pathway would provide a net excitation if the target of VP projection neurons were the inhibitory interneurons of the VTA. Within area X, spiny neurons inhibit the pallidal aspiny projection neurons. The aspiny neurons of X are tonically active and send a GABAergic signal to the VP. An excitation of the spiny neurons in X would thus lead to a release from inhibition for VP neurons. If a release from inhibition of the VP increases activity, this would cause an increase in GABA inhibiting the VTA. Therefore the excitation of area X spiny neurons would increase inhibition of the VTA, which would seem to contradict experimental evidence (45). Postulating that VP inhibitory projection neurons target the VTA inhibitory interneurons resolves this contradiction. Inhibiting VTA interneurons
would release the dopamine projecting VTA neurons from inhibition thus increasing dopamine concentrations in area X. The suggested excitatory mechanism is illustrated in Fig. 2.33 B. This disinhibition mechanism could increase dopamine concentration globally in area X if the VTA interneurons project diffusely.

Our model proposes that the medial striatum (MSt) plays a larger role in song learning than has previously been suggested. The inhibition and regulation of dopaminergic midbrain neurons in the VTA is critical to our model dynamics. Spiny neurons of the medial striatum strongly project to the VTA and SNc (85; 35; 110). Largely located medial to area X, the neurons exhibit electrophysiological properties similar to the GABA projecting spiny neurons of area X (110). Our model requires that the area X driven VTA neurons also innervate the medial striatum so inhibition can be correlated with excitation. However it has been shown that separate areas of the VTA innervate area X and the medial striatum contradicting our requirement (110).

An alternate striosome pathway involves the spiny neurons of the medial striatum projecting to the VP. A dorsal strip at the edge of the medial striatum containing spiny neurons project to the VP (110). This alternate projection to the VP can resolve the contradiction. Instead of direct inhibition of the VTA, inhibitory projection neurons of the medial striatum could short circuit the XPT pathway at the VP juncture. By inhibiting VP activity, the medial striatum
Fig. 2.33 A - The XPT pathway consists of three GABAergic projections: spiny neurons to aspiny neurons in area X, aspiny neurons to ventral pallidum (VP), and VP to VTA. B - A possible excitatory mechanism involves the VP inhibitory neurons targeting VTA inhibitory interneurons. Spiny neuron activity in area X ultimately increases VP activity. Therefore if VP inhibits VTA interneurons, spiny neuron activity would increase inhibition of VTA interneurons, exciting VTA neurons. C - The existence of plastic along the excitatory XPT projection from area X to VTA is not known. By providing reward information to both the VP and the VTA, the arcopallium allows for either projection along the XPT pathway to represent the required plastic projection.
projection effectively inhibits the VTA. Currently there is no evidence linking the neuronal activity of the medial striatum, VP and VTA during singing.

Our model assumes the reinforcement signal processed from auditory feedback is represented by the arcopallium shelf projection to the VTA. Experiment suggests that an alternate indirect pathway from the arcopallium shelf to the VTA could exist (110). The alternate projection originates from the same VTA projecting region of the arcopallium but instead forms connections with the ventral pallidum. Therefore both area X and the arcopallium, or both premotor and auditory feedback (reward) activity could drive the VTA by exciting the VP. The two pathways represent distinct excitation mechanisms: area X releases the VP from inhibition increasing activity, and the arcopallium shelf provides direct excitation.

A consequence of the alternate reward projection to the VP is that the synaptic gate $W_{X-VTA}$ could now exist between aspiny X projecting neurons and the VP. The existence of the $W_{X-VTA}$ projection between area X and the VP is possible because the VP would now receive direct information about primary reward. A weakness in this argument is that VP neurons that receive projections from area X and arcopallium do not overlap significantly (110). The possibility of an excitatory projection within the VP linking the two separated areas has been suggested (110). Therefore the ventral pallidum would represent a nexus of dopamine control for area X (Fig. 2.33 C). Our model indicates any lesion
experiments or studies involving the disruption of VP activity would cause real
deterioration in the trajectory of zebra finch vocal development.

The gating of dopamine release downstream of area X is a critical assump-
tion in our model. Our model represents the XPT pathway as an excitatory pro-
jection with a plastic synaptic weight $W_{X-VTA}$. Whether this gating or synaptic
plasticity exists, and what role it has will depend upon the nature of the XPT
pathway. Although the existence of plastic inhibitory projections along the XPT
pathway is unknown, experiments have shown the existence of STDP between
inhibitory neurons (54). The magnitude and range of plasticity would have im-
portant implications to the model’s learning dynamics. If experiments show that
a projection with a large capacity for plasticity does exist along the XPT pathway
then our model would be supported. A sufficient condition on $W_{X-VTA}$ would be
the ability to generate distinguishable dopamine concentrations in area X using
distinct neuronal projections along the XPT pathway.

The discrete gating effect of $W_{X-VTA}$ provides a solution to the temporal
difference problem in area X. A more effective approach could include replacing the
binary threshold of $W_{X-VTA}$ with a continuous response. Area X excitation to the
VTA would be proportional to the synaptic weight $W_{X-VTA}$. The all or nothing
approach to the XPT pathway is hopefully a first step in addressing learning
in the AFP. Defining the XPT pathway modulation in a continuous way may
support increased variability of song exploration. How often a particular spiny
neuron is associated with reward would determine how much dopamine is released to reinforce the HVC projection.

The variable modulation dynamic assumes area X driven VTA activity could not potentiate the XPT projection itself. Otherwise all projections would eventually potentiate $W_{X-VTA}$ through random high frequency activity in area X. Long term depression along $W_{X-VTA}$ between spiny neurons in DOWN states or lacking high frequency activity when reward occurs could resolve the self potentiation problem. Thus UP states would eventually have stronger XPT pathways if their HVC projections generated correct firing on average. Therefore our model's requirement of sparse HVC driven activity could be relaxed and allow for more variability in trial by trial exploration.

2.11.4 Time Scale of Fluctuations and Reward Evaluation

The population level correlation of UP states observed in vitro in mammals has important implications if the same can be observed in spiny neurons of area X (13). The existence of stable attractors for UP states across bouts or from day to day would provide important clues about how action selection takes place during song exploration. The statistical stability of these UP attractor states across bouts would directly impact the HVC projection explorations that correlate with the sustained activity in area X. By transitioning to a different network attractor, the current UP states could change into DOWN states. Any HVC(X) projection
that was being learned with the previous UP state would now decay due to a lack of correlation between the newly hyperpolarized or DOWN state X neurons. Weaker synaptic decay rates or stronger potentiation between HVC and area X could address these shorter or more diffuse correlations. Further simulation and exploration of UP state dynamics across motifs is required.

A learning model must be capable of correlating exploration to subsequent reward to incorporate desired changes into the system. A global reinforcement signal is indiscriminate in the face of a temporal delay when direct association with the reward driven activity cannot form. Direct association is prevented when temporal delays do not facilitate biological mechanisms like spike time dependent plasticity. Therefore an indirect association is required that uses different projection and activity dynamics.

Our model uses slow incremental variation in single notes to develop a one to one correspondence between explorative activity and reward. The correspondence is facilitated in the model by Eq. 2.45 which uses brief neural integration to capture single note variations. This sensitivity in our model can lead to uncorrelated reinforcement. When single note variations generate global reward the model can erroneously reinforce all variations regardless of their intrinsic value to the system. Therefore for temporally disperse variations a temporally narrow integrator is insufficient.
A temporally broad neural integrator used for reward evaluation allows for reinforcement of desired fluctuations that occur across the entire syllable. The neural integrator of Eq. 2.45 with a syllable long summation would evaluate the combined effect of the syllable’s fluctuations. The system could collectively learn across the syllable provided the threshold \( E_\gamma \) generated reward when a majority of the variations were correct. A single primary reward evaluation per syllable would generate VTA activity similar to what is observed in Fig. 2.9. Recording VTA neuronal activity similar to Fig. 2.9 during the sensorimotor phase could reveal correlations between singular bursting and global improvements in the syllable. This integration mechanism may be efficient early on, but using a constant threshold for reward would prevent single note corrections during the later stages of development. If exploration variability attenuates during song development, a scaled integrator could then be used to learn single note variations to improve song.

2.11.5 Conclusions

Through the use of specifically defined projections, neuronal dynamics, and synaptic learning rules, we have proposed a framework that allows zebra finch to learn syllable structure. The model exploits HVC driven AFP explorations to improve song performance. This learning provides stereotyped activity lasting several motifs that is then adopted in the HVC to RA pathway. After the changes have become permanent the HVC to area X projections generate new exploration
starting the process anew. The proposed paradigm generates gradual learning and develops the juvenile’s plastic song into a crystallized syllable similar to its tutor.

The model emphasizes HVC(X) interactions with area X, VTA dopamine activity and VP dynamics as important for song learning. The correlation of UP states in area X across several motif renditions was an important assumption in our model that currently has no experimental support in zebra finch. How statistically stable these UP states are in area X determines the interaction with the HVC(X) projection to spiny neurons. In a similar way the stability of the HVC(X) driven activity in area X across motif renditions (48) determines the influence of high frequency activity on explorations generated by the AFP.

The duration of correlated UP states and HVC(X) synaptic fluctuations together determine the influence the VTA has on learning in the AFP. Without reinforcement of the AFP activity through dopamine release our model would not learn. The pathway linking the HVC-area X interactions to the VTA is critical to solving the temporal difference problem. Our model suggests that the VP may be a critical nexus of control and deserves further study. These model dynamics could frame future experiments that reveal important elements of song learning in zebra finch.

Our model reproduces critical observations from experimental results in song development and provides valuable interpretations of how the AFP circuit and activity could be used to drive song learning. The model assumes reward
derived from improvements in song performance is generated through the arcopallium projection to either the VP or VTA. Thus auditory feedback is treated in a novel way and provides an alternative to using auditory selective HVC neurons while the bird is singing.

The model represents a paradigm shift from the use of random LMAN variability directly to affect song learning. Our model suggests that LMAN’s noisy activity is actually used indirectly by driving variations in HVC(X) projections rather than HVC(RA) and RA activity. The variable HVC(X) projections are then used as the primary drive guiding song exploration. The temporal delay problem is solved using a novel interaction between HVC(X) projections, area X UP states and the XPT pathway. In conclusion, the model represents a novel approach that could aid experimentation in solving the complex problem of song learning in zebra finch.
Chapter 3

Conductance Based Model of DLM Neurons

3.1 Introduction

The medial portion of the dorsolateral thalamic nucleus (DLM) is an important relay nucleus necessary for anterior forebrain pathway (AFP) dynamics during songbird learning. The exact role that the DLM plays during learning and how it shapes vocal development is largely unknown. There is currently no evidence that indicates it has a direct role in information processing to guide syllable structure development in the AFP. The most immediate affect DLM has on the learning dynamics is the transmission time of AFP activity to the robust nucleus of the arcopallium (RA) (29; 111). A long time delay for transmission of activity through the AFP is believed to be largely caused by the firing properties of neurons in the DLM (111). Activity transmission through the DLM involves two inhibitory projections from area X. The firing time and characteristics are unique within the AFP. Therefore understanding the firing properties of DLM neurons is an important first step in exploring the purpose the DLM fulfills in birdsong learning.

The majority of electrophysiological properties of DLM neurons are developed prior to the sensorimotor phase but synaptic evolution continues during song
development (90). Therefore the firing characteristics shaped by the electrophysiology of the DLM neurons are already stereotyped prior to song exploration and crystallization. Luo and Perkel (2002) showed the synaptic reversal potential and decay time constants in juvenile and adult zebra finch are different (90). Boettiger and Doupe (1998) have indicated that dendritic regression of the synaptic connections between DLM and LMAN occur during the sensorimotor phase and continues into adulthood (4). Therefore the overall firing pattern or projection properties of DLM onto LMAN are not static, but what affect the changes have on learning is unknown (6). The decline of LMAN’s affect on song output during song development could be correlated to the decrease in the number of dendritic connections or silent synapses projected from the DLM (5). The possibility exists that the DLM plays a crucial role in the learning or plasticity dynamics within the AFP.

The double inhibitory projection pathway in area X generates an excitatory effect and determines the nature of the transmission delay in DLM neurons. The spiny neurons of area X project GABA which inhibits aspiny neurons (39). The spiny neuronal activity subsequently inhibits the tonically active aspiny neurons. The aspiny projection neurons of area X project GABA to the DLM neurons (89). The release from prolonged inhibition in the DLM generates an event known as a post-inhibitory rebound or PIR (111). The PIR is characterized by a single or multiple spikes which can occur when a neuron is released from inhibition,
even in the absence of positive current. The time to fire or rebound latency is directly related to the nature of inhibition that preceded the release (111). The majority of the axonal projections from area X to DLM are one to one (89). With the approximate one to one correlation in projections between area X and DLM, the activity of individual aspiny neurons can directly modulate the time delay of activity in the AFP.

This chapter presents a summary of the computational model created to reproduce the timing properties of the DLM. The theoretical framework for computational conductance based models is introduced and its use motivated. The argument is then made that with limited experimental data, using electrophysiological properties of neurons in similar systems is useful. Therefore the experimental data and conductance based model of thalamic relay neurons of the guinea pig and cat first introduced by Huguenard and McCormick (1992) is summarized and then compared to the known electrophysiological properties of the DLM neurons (61; 60). Finally the model created in this chapter is presented and the temporal properties are explored and discussed.

3.1.1 Conductance Based Neuron Model

A conductance based neuron model first proposed in 1952 by Hodgkin and Huxley is ideal for replicating the required PIR dynamics (59). The neuron model
focuses on the membrane potential of the soma and treats it as a single compartment. The change in electric potential of the soma membrane is described as:

\[
c_m \frac{dV}{dt} = I_{ext}(t) - i_m
\]  

(3.1)

Where \( I_{ext} \) is external current being injected into the neuron, \( i_m \) is the membrane current travelling through the membrane ion channels, and \( c_m \) is the membrane’s capacitance. The membrane’s current is controlled by several currents that are distinguished by specific ion flow through the neuron’s membrane. Each current has gates that open and close based on the membrane’s potential difference, \( V \). The gates are specific to the current ”channel” and only allow the specified ions to cross through the membrane. For example, the ion channels are sometimes mixed ion or cationic channels, other common channels include calcium, sodium and potassium. Therefore the membrane current \( i_m \) is defined as:

\[
i_m = \sum_{\text{Ion Channels}} i_{ion}
\]  

(3.2)

The ion channel’s current is dependent upon several characteristic features including reversal potential, membrane conductance, and inactivation and activation gates. The reversal or Nernst potential is the membrane voltage at which an equilibrium point for inward and outward flow of the particular ion is reached.
Therefore the overall current flow for that particular ion channel is zero at its reversal potential. The measure of ion flow for a given voltage gradient across an ion channel is defined as the channel’s conductance. The relative conductance values between different ion channels determine what influence each of current channels has on the overall change in membrane voltage. The activation and inactivation gates together describe the opening and closing of the ion channel as a function of the membrane’s electric potential difference. A thorough description of various ionic channels found in neurons can be found in Hille (1992) (58). Combining the described channel properties, an individual ion current \( i_{ion} \), is defined as:

\[
i_{ion} = m^i(V)h^j(V)(V - E_{rev})
\]  

(3.3)

The \( E_{rev} \) is the channel’s reversal potential. The \( m(V) \) is the activation gate and \( h(V) \) is the inactivation gate. The dynamics of these gates are voltage dependent and usually described by the following:

\[
\tau(V) \frac{dm}{dt} = m_\infty(V) - m
\]

(3.4)

The steady state values \( m_\infty \) conform to the functional forms depicted in Fig. 3.1. How quickly the gate approaches the asymptotic value \( m_\infty \) is also voltage dependent detailed by \( \tau(V) \). The amount of overlap between the two gates \( m \) and
h is a measure of how easily the gate can remain open in a particular voltage range. With a time constant $\tau(V)$ the voltage range is dynamic and path dependent.

The variability in ion channels and their corresponding voltage functionality is large. Therefore to determine what ion channels to include in the DLM model requires experiment. However, there has not been a complete study and determination of all ion channels and their characteristics for the DLM neuron experimentally. The most current experimental data of the DLM's electrophysiological properties has been reported largely by the Perkel group including Luo and Person (89; 88; 90; 111). A useful strategy is to combine existing experimental data and to draw inferences from channels of neurons in similar systems. There has been extensive experimentation in cat and rodent thalamic relay neurons by Huguenard and McCormick (1992) (61; 60). The thalamic relay neurons are similar in electrophysiological characteristics to the DLM neurons and are contained in a homologous neuronal structure. Therefore we next explore the conductance based model that was derived on the cat and rodent experimental data presented in the same papers (61; 60).

3.2 Experimental Data

3.2.1 McCormick Model

Mammalian thalamic neurons are often studied in the context of their behaviour under inhibition because they receive a tonic inhibitory projection from
Fig. 3.1 The inactivation gates become activated when the neuron is hyperpolarized while the opposite is true for activation gates. The timing mechanism for an activation gate is typically on a much faster time scale than inactivation gates. The post inhibitory rebound spikes require the inactivation gate to be large enough upon release from inhibition for a rebound to occur. Both gates have a maximum value of 1 at their respective voltage ranges.
the pallidal neurons of the basal ganglia. After release from prolonged exposure to inhibition, the thalamic neurons will typically display a broad calcium rebound spike. The calcium rebound is accompanied by a burst of sodium spikes. The Huguenard and McCormick model reproduces the experimental findings including the inhibitory responses of the thalamocortical relay neurons (61; 60). A generic post inhibitory rebound spike simulated by their model is depicted in Fig. 3.2.

Understanding what channels contribute to the dynamics of a PIR is essential if an analogous computational model for DLM neurons is to be constructed. The McCormick model details several ion channels, including multiple sodium, potassium and calcium channels and a mixed ion channel. Therefore it is important to determine what channels are involved and are proved crucial for the PIR spiking. The multiple channels modeled by McCormick (61; 60) and are summarized in table 3.1. The membrane current for the thalamic neurons was defined as:

$$i_m = I_T + I_L + I_{Na} + I_{Na(p)} + I_C + I_{K2} + I_A + I_h + I_{leak}$$

(3.5)

All of the described currents use a functional form similar to the voltage dependent gates depicted in Fig. 3.1 except for the calcium currents $I_T, I_L$ and the potassium current $I_C$. The calcium currents use the Goldman-Hodgkin-Katz relationship which models the concentration of the calcium as a dynamical parameter. The derivation for an ion current using the Nernst-Planck differential equation
Fig. 3.2 After a current injection of -0.5 nA, the neuron’s membrane potential becomes hyperpolarized. Upon release from inhibition the inactivation gate of the calcium channel is sufficiently large to initiate a calcium rebound spike with a burst of sodium spikes riding on top of it. The amount of bursting is slightly larger than shown in (61; 60) but agrees with results found in Destexhe, Contreras and Steriade (1998) (24) which uses the McCormick model in its study.

<table>
<thead>
<tr>
<th>Current</th>
<th>Ion</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_T$</td>
<td>$[Ca^{2+}]$</td>
<td>transient, low-voltage activated</td>
</tr>
<tr>
<td>$I_L$</td>
<td>$[Ca^{2+}]$</td>
<td>low voltage threshold activation</td>
</tr>
<tr>
<td>$I_{Na}$</td>
<td>$[Na^+]$</td>
<td>fast and transient</td>
</tr>
<tr>
<td>$I_{Na(p)}$</td>
<td>$[Na^+]$</td>
<td>persistent, depolarization-activated</td>
</tr>
<tr>
<td>$I_C$</td>
<td>$[K^+]$</td>
<td>$Ca^{2+}$ activated, contributes to repolarization of $Ca^{2+}$ and $Na^+$ spikes</td>
</tr>
<tr>
<td>$I_{K2}$</td>
<td>$[K^+]$</td>
<td>depolarization activated with slow inactivation</td>
</tr>
<tr>
<td>$I_A$</td>
<td>$[K^+]$</td>
<td>transient and depolarization activated, slows $Ca^{2+}$ spike timing</td>
</tr>
<tr>
<td>$I_h$</td>
<td>Cation</td>
<td>hyperpolarization activated, rhythmic spike generator during inhibition</td>
</tr>
<tr>
<td>$I_{Leak}$</td>
<td>$[Na^+],[K^+]$</td>
<td>Leak current.</td>
</tr>
</tbody>
</table>

Table 3.1 McCormick Thalamocortical Relay Current Channels
for ion diffusion is detailed in Hille (1992) (58). Applying it to calcium currents Huguenard and McCormick obtain:

\[ I_T = m^2(V)h(V)P \frac{(zF)^2V [Ca^{+2}]_i - [Ca^{+2}]_o e^{-\frac{zFV}{RT}}}{1 - e^{-\frac{zFV}{RT}}} \]  

(3.6)

Where F is faraday’s constant, R the gas constant, T is the temperature in K, P is the membrane permeability, z is the valence number of the ion, and \([Ca^{+2}]_i, o\) refer to the inside and outside calcium concentrations relative to the membrane.

The m(V) and h(V) are standard activation and inactivation gates. The interior calcium concentration is assumed to evolve based on:

\[ \frac{d[Ca^{+2}]_i}{dt} = \beta[Ca^{+2}]_i \]  

(3.7)

The L-type calcium current follows a similar equation and the potassium current has the traditional voltage dependent gates but it is proportional to the inside calcium concentration \([Ca^{+2}]\). For negative voltage ranges the current-voltage relationship behaves in an ohmic fashion (Eq. 3.3) (58).

The rebound burst of thalamic neurons is mediated by the calcium current \(I_T\) and is dependent on the voltage characteristics of its inactivation gate \(h_T(V)\). The interaction between the inactivation and activation gate during prolonged inhibition and the subsequent release from inhibition determines rebound dynamics. During inhibition, the activation gate \(m_T(V)\) is closed and therefore \(I_T\) is zero.
The inactivation gate $h_T(V)$ is open and the longer the neuron is hyperpolarized, the larger the value of the gate (to a maximum of $h_\infty(V)$ determined by the input current). The time constant for the $I_T$ inactivation gate is slow compared to the time constant of the activation gate. Therefore during release from inhibition, the activation gate can open and allow for firing before the inactivation gate can close at the high membrane potential.

Once the inactivation gate has opened up beyond some threshold during inhibition, there is sufficient nonzero overlap of $m^i_T(V)h_T(V)$ to allow for a rebound once release from inhibition occurs. The sodium spikes are sensitive to the membrane voltage near the top of the calcium spike thus generating fast spiking on top of the relatively broad calcium rebound. The described firing dynamic is depicted in the PIR spikes of Fig. 3.2.

The primary ion responsible for the repolarization of a neuron’s membrane to the resting potential is potassium. The three potassium channels involved in the McCormick model; $I_C$, $I_{K2}$, and $I_A$ all contribute to the repolarization to the neuron’s resting potential. In the McCormick study, $I_C$ was found to be primarily responsible for repolarization of the $Na^+$ spikes (61; 60). The secondary currents $I_{K2}$ and $I_A$ still contributed to the firing dynamics and repolarization. An interesting feature of the three currents was when one of the channels was removed, the other $K^+$ channels would compensate maintaining little change in the overall
$Na^+$ spiking dynamics. Ultimately the $I_C$ current was sufficient to control the repolarization of the sodium spikes.

The cation current $I_h$ generates oscillatory behaviour during inhibition and is important in the timing of PIR spikes. An important aspect of prolonged inhibition in the thalamus is the appearance of a rhythmic oscillation of activity. The spiking occurs when an increase in $I_h$ current is driven by the hyperpolarized membrane potential. The oscillation occurs at a low rate known as the delta frequency (1-4 Hz) and is believed to be important during sleep (140; 139). During shorter durations in inhibition on the order of AFP dynamics (100 - 500 ms), the membrane potential shows a depolarizing sag prior to the release from inhibition and subsequent PIR. The depolarizing sag is also attributed to $I_h$ and directly affects the rebound latency dynamics of the PIR spikes. The cation $I_h$ current is also observed in the DLM and is therefore an important ion current to model.

Regardless of the ion currents involved, any conductance based model that is capable of a PIR has a rebound latency that depends upon the duration of inhibition. There is an inverse relationship between the rebound latency and the length of inhibition and this can be seen in Fig. 3.3 and is also depicted for the DLM in Fig. 3.5. Determining what ion currents from the McCormick model are observed and important for the rebound latency in DLM neurons requires the study of current DLM data.
Fig. 3.3 The rebound latency is measured as the difference between the time of release from inhibition and the time to fire of the first sodium spike. The timing of the rebound latency is within the range of the experimental data reported for the DLM neuron dynamics.
3.2.2 Current Electrophysiological Data for DLM Neurons

An important property of signal transmission through the DLM is the average number of spikes generated in a post-inhibitory rebound. The Perkel group reports a wide range in the number of spikes during a PIR for DLM neurons (88; 90; 111). Multiple burst spiking typified in the McCormick recordings (61; 60), is also reported in DLM recordings by Luo and Perkel (1999,2002) (88; 90). The number of sodium spikes varied, but usually was considered between the range of 2 to 4 spikes. A more recent study by Person and Perkel (2005) showed a majority of recordings that contained DLM PIRs that were typified by a calcium rebound and a unitary sodium spike (111). It is uncertain whether this represents a reduction in the availability of the sodium channel, or other experimental technicality. The recordings could more closely resemble the activity seen extracellularly in the mammalian thalamic nucleus, where single spikes have been recorded (109).

Several currents were shown to exist in the DLM that are known ion channels of the mammalian thalamic relay neurons and the Huguenard and McCormick model (88; 90; 61; 60). The two specific currents are the transient low voltage calcium current responsible for the PIR, $I_T$, and the cation hyperpolarization activated current $I_h$ (88; 90). The recordings include sodium spikes that are definitely repolarized by potassium but the exact gating characteristics of the potassium channels are unknown. The $I_T$ and $I_h$ currents are the most influential in determining the timing dynamics of the rebound latency of post-inhibitory rebounds.
Using stimulation of aspiny projections to DLM neurons in vitro, Person and Perkel (2005) showed that the unitary PIR’s were capable of reliable transmission (111). The stimulation consisted of poisson trains with a mean frequency of 140 Hz. The DLM spikes were aligned to the stimulus train to determine if a correlation existed between the stimulus train and DLM activity. The aligned raster plot is depicted in Fig. 3.4. The raster plot shows that DLM post-inhibitory rebounds occur following a brief break in the inhibitory activity of area X. Thus the double inhibitory pathway in area X has been shown to be a plausible transmission mechanism of activity through the DLM.

Another feature of DLM PIRs studied by Person and Perkel (2005) was the nonlinear relationship between the rebound latency of PIRs and the duration of inhibition prior to the activity (111). A reciprocal relationship exists between the PIR rebound latency and length of inhibition for both the McCormick model (Fig. 3.3), and DLM neurons. This negative correlation in DLM neurons reported by Person and Perkel (2005) (111) is depicted in Fig. 3.5. The negative correlation is a general feature of conductance based models and can be reproduced with very few ion channels. The Hodgkin Huxley model that has a sodium and potassium channel (59) shows the negative correlation but only on a very short time scale. The variability in timing originates from the relative time scales associated with the ion channel’s time constants in equation 3.4. The relationship is best fit to a decaying
Fig. 3.4 A - The membrane potential trace of a DLM neuron in response to a stimulus train of IPSPs. The raster of the stimulus train is depicted below the voltage plot. The resting membrane potential was typically -55 mV and spike range was approximately 40 mV leaving the highest point of a PIR to be approximately -30 to -20 mV. B - The raster plot of stimulus input aligned to the time of the DLM spike. The raster represents several trials of the results in A. The DLM firing time is indicated by the black arrow, and the white arrow is the average break from inhibition prior to the DLM spike. Figure was obtained from (111) with permission from author D. J. Perkel.
exponential which was the motivation behind equation 2.8. The properties of the channel dynamics causing post-inhibitory rebound spikes is briefly explored in (73).

### 3.3 Reduced Computational Model

The goal of our computational model was to reproduce the PIR timing observed in Person and Perkel (2005) at the reported resting potential and voltage scale as depicted in Fig. 3.5 (111). The evidence supporting PIR spikes as the probable means of information transmission through the DLM points to the importance of having the timing dynamics reproduced as accurately as possible. Another important aspect of experimental and learning importance is the robustness of the PIR. Both the mammalian data and McCormick model show several sodium spikes riding on top of the calcium rebound spike. Evidence reported by Person and Perkel (2005) (111) shows only a single sodium spike and at most 2 or 3 spikes based on previous papers (88; 90). Therefore adjusting the number of sodium spikes that the DLM model produces upon rebound was another important goal. Without a full knowledge of all the gates involved, using as few channels as possible reduces the number of critical parameters required to explore.
Fig. 3.5 If exposed to enough inhibitory current long enough, upon release from inhibition DLM neurons have the ability to spike. A - Displayed are PIRs with a calcium rebound spike (wide short peak), with a sodium spike activated by the rebounding potential. The graph with multiple spikes contains membrane voltage traces that were aligned to show the spiking that occurs in the same range. B - The rebound latency is the time from release to the time of the spike (highest point in membrane potential). There was a 30 ms spread between firing times of the two inhibition duration extremes. The longer the inhibition, the smaller the change in rebound latency. Figure was obtained from (111) with permission from author D. J. Perkel.
Beginning with the McCormick model, by reducing the number of ion channels to a critical few and adjusting the remaining channel’s properties, we were able to reproduce the important physiological properties of DLM neurons. The T-type calcium current $I_T$ was crucial to reproduce the broad PIR that generates the long rebound latencies. The cation current $I_h$ was kept to reproduce the depolarization sag observed in hyperpolarized DLM neurons. The sodium current $I_{Na}$ was kept to produce the sodium spikes that occur on top of the calcium rebound. Of the three potassium channels, only the $I_C$ current was kept to repolarize the membrane during the sodium and calcium spiking activity. Although there was evidence for the $I_A$ current (88), using as few channels as possible was the main motivation. The evolution of this reduction and hand tuning of the McCormick model is depicted in Fig. 3.6.

Further adjustments to the model parameters was required to scale the resting potential and the voltage range over which the firing dynamics occur to that of the DLM neuron. By adjusting reversal potentials, slope values of the gating functions for the various ion currents and leak conductance, the resting potential was similar to the experimental data (Fig. 3.5 A). Typical calcium spikes in the DLM data occurred in lower voltage range than the McCormick model. The lower range suggests a simple ohmic equation rather than the Goldman-Hodgkin-Katz relationship could maintain timing accuracy (58).
Fig. 3.6 The major stages of adapting the McCormick Model to a reduced DLM model. A - The original McCormick model with all current channels and Gold-Katz Calcium implementation. B - Removed all currents except $I_T, I_{Na}, I_C, I_h$ and leak current. Without as many potassium channels, more sodium spikes are generated. C - Converted the Goldman-Hodgkin-Katz (GHK) calcium current, $I_T$, to a simple ohmic relation. Removed the dependency on the calcium concentration for $I_C$, and adjusted the voltage form of the gates to react as previously for the same voltage range. D - Removed the excessive sodium spiking on top of the calcium spike by adjusting the voltage range of the overlap in the sodium gates.
The important timing dynamic of the DLM neurons shown previously in Fig. 3.5 are reproduced in our reduced computational model. The rebound latency as a function of inhibition duration is simulated and depicted in Fig. 3.5. The next step in evaluating our model is exploring the robustness of the ion channel parameters in reproducing the required dynamics. The rebound latency’s dependence on various gate and conductance parameters is explored next.

Fig. 3.7 **Solid Line** - The rebound latency dependency as a function of inhibition duration for the model neuron at a resting potential of -62 mV. **Dashed Line** - The rebound latency relationship with an altered leak current reversal potential to mimic a constant current so that the resting potential is at -55 mV. The simulation and experimental results are similar in range and membrane resting potentials.
3.3.1 DLM Model Parameters

The following is a description of our DLM model neuron parameters. The membrane current $i_m$ is defined as:

$$i_m = I_T + I_{Na} + I_{C} + I_h + I_{leak}$$  \hfill (3.8)

The membrane conductance was set to $c_m = 0.29 \text{ nF}$. The reversal potential, conductance and various gate parameters for individual currents are defined below.

**Leak Current, $I_l$**

$$I_{leak} = G_l(V - e_l)$$  \hfill (3.9)

With $G_l = 0.0105 \mu S$ and $e_l = -47.5607 \text{ mV}$.

**Calcium Current, $I_T$**

$$I_T = G_T m^2 h(V - e_T)$$  \hfill (3.10)

With $G_T = 1.0 \mu S$, $e_T = 126.325 \text{ mV}$.

$I_T$ Current Activation Gate

$$m_{\infty}(V) = \frac{1}{1 + e^{-(V+48.6475)/5.549}}$$  \hfill (3.11)

$$\tau_{\infty}(V) = 0.612 + \frac{1}{e^{-(V+112.64)/14.9465} + e^{(V+5.2848)/16.2890}}$$  \hfill (3.12)
$I_T$ Current Inactivation Gate

$$h_\infty(V) = \frac{1}{1 + e^{(V+72.365)/3.58}} \quad (3.13)$$

$$\tau_\infty(V) = 28.0 + e^{-(V+16.875)/9.3975} \quad (3.14)$$

and for $V < -63.415$ mV

$$\tau_\infty(V) = e^{(V+415.15)/59.607} \quad (3.15)$$

Sodium Current, $I_{Na}$

$$I_{Na} = G_{Na} m^3 h(V - e_{Na}) \quad (3.16)$$

With $G_{Na} = 12 \mu S$, $e_{Na} = 45.775$ mV

$I_{Na}$ Current Activation Gate

$$\alpha(V) = \frac{0.1017(V + 24.2587)}{1 - e^{-(V+24.2587)/4.4750}} \quad (3.17)$$

$$\beta(V) = \frac{-0.0693(V + 24.2587)}{1 - e^{(V+24.2587)/4.4750}} \quad (3.18)$$

Where

$$m_\infty(V) = \frac{\alpha(V)}{\alpha(V) + \beta(V)} \quad (3.19)$$
and

\[ \tau _{\infty } (V) = \frac{1}{\alpha (V) + \beta (V)} \] (3.20)

**I_{Na} Current Inactivation Gate**

\[ \alpha (V) = 0.016e^{-(V+49.095)/13.425} \] (3.21)

\[ \beta (V) = \frac{2.07}{1 + e^{-(V-15.345)/18.795}} \] (3.22)

**Potassium Current, I_{C}**

\[ I_{C} = G_{C}m(V - e_{K}) \] (3.23)

Where \( G_{C} = 3.25 \mu S, \ e_{K} = -88.475 \) mV

**I_{C} Current Activation Gate**

\[ \alpha (V) = 0.125e^{(V-10.4225)/21.48} \] (3.24)

\[ \beta (V) = 0.1e^{-(V-10.4225)/21.48} \] (3.25)
Cation Current, $I_h$

\[ I_h = G_h m(V - e_h) \]  
(3.26)

With $G_h = 0.005 \mu S$, and $e_h = -32.985 \text{ mV}$.

$I_h$ Inactivation Gate

\[ h_\infty(V) = \frac{1}{1 + e^{-(V+61.625)/4.9225}} \]  
(3.27)

\[ \tau_\infty(V) = \frac{1}{e^{-(0.0961V+14.0615)} + e^{0.0783V-2.3008}} \]  
(3.28)

For easy reference the gates are also listed in appendix B.

### 3.3.2 Parameter Space Exploration

With the current channels defined, exploring the interplay between the various parameters of the conductance based model and their affect on the timing dynamics is important. For example, the relative voltage distance between the activation and inactivation gates of the sodium channel determines the voltage range that the current can generate the sodium spikes. The shift in sodium gates affects how sensitive the current is to the stereotyped calcium spike. The number of sodium spikes as a function of this voltage shift is explored in Fig. 3.8. The
exploration reveals a steady decline in the number of sodium spikes and a relatively large parameter regime of just a single or no spike.

An important factor in determining the DLM timing dynamics is the relative strengths or densities of the calcium and potassium channels. The interplay between $G_T$ and $G_C$ determines the magnitude of the calcium spike and also affects the timing of rebound spikes. Figure 3.9 depicts the parameter space evaluation of the timing of the rebound spikes relative to the data extracted from Fig. 3.5 for rebound latency. Four inhibition durations were evolved for a given set of parameters and the rebound latency of the new parameters were compared to the experimental data:

$$\left[ \sum_{i=1}^{4} (RL_{m,i} - RL_{e,i})^2 \right]^{-1}$$

(3.29)

Where $RL_m$ and $RL_e$ are the rebound latency of the model and experimental data respectively. The $i$ spans the set of inhibition durations set to $\{50, 100, 200, 500\}$ ms.

The parameter space exploration shows an increase in potassium conductance decreases the neurons ability to fire. This is seen in the transition to the blue regime of the space representing poor agreement with experiment. Conversely if there is an increase in calcium conductance then there is a increase in the neurons ability to fire. In the regime of high calcium conductance and low potassium conductance there is relatively good agreement with the experimental rebound latencies.
Fig. 3.8 A parameter space of the number of sodium spikes passing a -30 mV threshold. The transformation is very uniform, although Na spike height can be variable. The additional potassium channels are removed, but the GHK calcium dynamics are still implemented in this depiction to agree with the previous DLM data that reported multiple sodium spikes (88; 90).
Fig. 3.9 The variability in rebound timing as evaluated in equation (3.29) is plotted with red indicating relative close agreement to experimental findings and blue relative disagreement. The rebound latency changed in a smooth fashion except in the direction of increasing $G_C$ and decreasing $G_T$. Beyond a threshold of $G_T$ and $G_C$ the rebound latency did not agree with experiment. When the potassium conductance becomes too large (upper left) the neuron was unable to spike increasing the parameter distance from the experimental rebound latency. This represents a transition to a non-spiking regime.
Specific ion channels have distinct voltage gate characteristics that show strong evolutionary conservatism, but there exists small variability. Variability in ion channels can be observed across experiment, among models and possible ligand interactions across species (81). Our goal is not necessarily to have the exact voltage gate characteristics of the DLM ion channels but to reproduce the timing dynamics of the experimental results. With the variance in rebound latency timing due to changes in the conductance of the calcium and potassium channels $I_T$, and $I_C$, depicted in Fig. 3.9, shifts in their inactivation and activation gates illustrate how sensitive the dynamics are to changes. This exploration is shown in Fig. 3.10. By shifting the inactivation gate of the calcium current to a high voltage regime the neuron is able to fire during hyperpolarization. An analogy would be the $I_h$ current on steroids. The shift in potassium activation gradually shifts the rebound latencies further away from the experimental results.

The post inhibitory rebound spike requires the inactivation gate of the $I_T$ current to pass a threshold value to enact the rebound. The rebound threshold of the inactivation gate determines how small in amplitude or how short in duration an inhibitory current can be to induce firing. The parameters that determine the $I_T$ inactivation gate value at the release from inhibition is crucial in the timing of the rebound latency.
Fig. 3.10 The variability in rebound timing as defined by equation (3.29) is plotted with red indicating relative close agreement to experimental findings and blue relative disagreement. The large fluctuating structure in the lower half indicates a different state of the neuron where the calcium channel oscillates during inhibition. The black points indicate firing during inhibition. The large orange region in the middle left, centered around 0 shift in $I_C$ gate, and from -5 to 0 mV shift in the $I_T$ gate is extremely stable and close to the defined rebound timing set.
One ion channel that has a large effect on the $I_T$ inactivation gate value is the cation current $I_h$. This hyperpolarization activated current is a single inactivation gate that causes the membrane potential to increase during hyperpolarization. The shift in membrane potential causes the inactivation gate of the calcium current $I_T$ to approach a different value at the time of release from inhibition. Therefore having an idea of where the cation gate’s conductance and voltage dependence can be defined is important.

Figure 3.11 depicts exploration of the timing of the rebound latency as the conductance $G_h$ and a voltage shift of the voltage midpoint in the inactivation gate, $h_T(V)$, are varied. The variability in the performance evaluation (Eq. 3.29) was largest during shifts in $G_h$ when compared to the other parameter searches. Over the majority of parameter space the overall firing profile was largely maintained. Therefore $I_h$ is very influential in the timing of PIR rebounds in DLM neurons without adversely affecting the firing capacity.

There is sufficient insensitivity to the parameters as displayed in the explored parameter regimes of figures 3.9, 3.10, and 3.11 to show that the timing relationship is a robust feature of these ion channels during a post-inhibitory spiking. The model captures the essential firing dynamics and timing aspects of the post-inhibitory rebound of DLM neurons.
Fig. 3.11 The variability in rebound timing as defined by equation (3.29) is plotted with red indicating relative close agreement to experimental findings and blue relative disagreement. The neuron shifts from no change in membrane potential due to $I_h$ indicated in blue, to a regime with very close timing agreement and observable depolarization sag due to $I_h$. The parameter adjustments created the largest changes in rebound latency timing while still maintaining the overall firing profile.
3.4 Discussion

Given the wide range of voltage properties reported experimentally for the DLM neuron, it is difficult to capture all of them in a single parameterized conductance based model. Further experimentation of the DLM ion channels needs to be explored to further the accuracy of any model. From the wide range in the number of sodium spikes during a PIR, the variable timing of the calcium spike itself, to the voltage range over which the spikes occurred, all point to a large variability in electrophysiological properties or experimental conditions.

The McCormick model serves as a strong example of the possible channels that might exist in the actual DLM neurons. Our goals were not to mimic the DLM neurons exactly because experimentation is necessary to establish the exact gating channel voltage characteristics. The volume of variability in a conductance model with just a few channels is staggering. A slight variation in slope or shift in voltage of any gate can push the neuron into a completely different firing pattern space (Fig. 3.10). These large shifts in firing properties are asymmetrical and highly sensitive to what particular direction of conductance space you are moving, as is illustrated in Goldman, Golowasch, Marder and Abbott (49).

Fine tuning the existing conductance and channel properties of a known neuron model from a homologous structure found in mammals offers a compelling strategy for our zebra finch DLM neuron model. With only slight adjustments to the ion channels of the mammalian thalamic neuron, our DLM model reproduces
the timing and unitary sodium spike dynamics. An alternative approach would be to generate a vast conductance parameter search but exploring all the voltage gate parameter may prove difficult (118; 53). One weakness in the model is the shifted sodium and potassium channels. Robust bursting is more difficult to reproduce with the shifted sodium and potassium channels. A simple switch between unitary and multiple sodium spikes involves modeling the calcium spikes by the Goldman-Hodgkin-Katz equation. The switch in calcium equations reintroduces the voltage overlap and allows for multiple spikes.

The unique timing of PIR spikes is an interesting functional and evolutionary property, with many benefits. How this firing dynamic relates to the current unknown role that the DLM plays in learning in the AFP is a critical question. If there is cyclic correlated activity within AFP feeding back onto itself, the DLM neuron could act as a possible restoring force. The large refractory period, and inverse correlation between inhibition time and rebound latency would tend to maintain the period of a cycle of activity within the AFP. How information is processed in the DLM with respect to auditory information is still a largely unexplored problem. Knowing and understanding the electrophysiological and synaptic properties of the DLM neurons (90) is an important step in the direction of solving these questions. Overall our hand-tuned model reproduces the key electrophysiological properties of the DLM neurons that we deemed important. Our model is
a computationally fast, reduced ion channel, conductance based model that could be used in further simulations of AFP dynamics.
Appendix A

AFP Model Parameters
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<td>$\gamma_Y$</td>
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Table A.1 Motor Neuron Evolution

Linear Predictive Coding Coefficients, Order $p=10$

$$\alpha_k = \{2.4459, -2.7735, 1.5796, -0.5344, 0.0006, 0.3165, -0.4442, 0.1994, 0.0084, -0.0327\}$$
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Table A.2 Model Parameters
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Table A.3 Synaptic Weight Parameters
Appendix B

DLM Gating Functions
The following is a description of our DLM model neuron parameters. The membrane current $i_m$ is defined as:

$$i_m = I_T + I_{Na} + I_C + I_h + I_{\text{leak}} \tag{B.1}$$

The membrane conductance was set as $c_m = 0.29 \text{ nF}$. The reversal potentials, conductances and various gate parameters for individual currents are defined below.

**Leak Current, $I_l$**

$$I_{\text{leak}} = G_l (V - e_l) \tag{B.2}$$

With $G_l = 0.0105 \ \mu\text{S}$ and $e_l = -47.5607 \ \text{mV}$.

**Calcium Current, $I_T$**

$$I_T = G_T m^2 h (V - e_T) \tag{B.3}$$

With $G_T = 1.0 \ \mu\text{S}$, $e_T = 126.325 \ \text{mV}$.

**$I_T$ Current Activation Gate**

$$m_\infty(V) = \frac{1}{1 + e^{-(V+48.6475)/5.549}} \tag{B.4}$$

$$\tau_\infty(V) = 0.612 + \frac{1}{e^{-(V+112.64)/14.9465} + e^{(V+5.2848)/16.2890}} \tag{B.5}$$

**$I_T$ Current Inactivation Gate**

$$h_\infty(V) = \frac{1}{1 + e^{(V+72.365)/3.58}} \tag{B.6}$$

$$\tau_\infty(V) = 28.0 + e^{-(V+16.875)/9.3975} \tag{B.7}$$

and for $V < -63.415 \ \text{mV}$

$$\tau_\infty(V) = e^{(V+415.15)/59.607} \tag{B.8}$$

**Sodium Current, $I_{Na}$**

$$I_{Na} = G_{Na} m^3 h (V - e_{Na}) \tag{B.9}$$
With $G_{Na} = 12 \mu S$, $e_{Na} = 45.775 \text{ mV}$

$I_{Na}$ Current Activation Gate

$$\alpha(V) = \frac{0.1017(V + 24.2587)}{1 - e^{-(V+24.2587)/4.4750}} \quad (B.10)$$

$$\beta(V) = \frac{-0.0693(V + 24.2587)}{1 - e^{(V+24.2587)/4.4750}} \quad (B.11)$$

Where

$$m_{\infty}(V) = \frac{\alpha(V)}{\alpha(V) + \beta(V)} \quad (B.12)$$

and

$$\tau_{\infty}(V) = \frac{1}{\alpha(V) + \beta(V)} \quad (B.13)$$

$I_{Na}$ Current Inactivation Gate

$$\alpha(V) = 0.016e^{-(V+49.095)/13.425} \quad (B.14)$$

$$\beta(V) = \frac{2.07}{1 + e^{-(V-15.345)/18.345}} \quad (B.15)$$

Potassium Current, $I_{C}$

$$I_{C} = G_{C}m(V - e_{K}) \quad (B.16)$$

Where $G_{C} = 3.25 \mu S$, $e_{K} = -88.475 \text{ mV}$

$I_{C}$ Current Activation Gate

$$\alpha(V) = 0.125e^{(V-10.4225)/21.48} \quad (B.17)$$

$$\beta(V) = 0.1e^{-(V-10.4225)/21.48} \quad (B.18)$$
Cation Current, $I_h$

$$I_h = G_h m(V - e_h) \quad (B.19)$$

With $G_h = 0.005 \, \mu S$, and $e_h = -32.985 \, \text{mV}$.

$I_h$ Inactivation Gate

$$h_\infty(V) = \frac{1}{1 + e^{-(V+61.625)/4.9225}} \quad (B.20)$$

$$\tau_\infty(V) = \frac{1}{e^{-(0.0961V+14.0615)} + e^{0.0783V-2.3008}} \quad (B.21)$$
Bibliography


Vita
David Jeffrey Fraser

Jeff Fraser was born in New Glasgow, Nova Scotia on May 20, 1981, the son of Frederick and Patricia Fraser.

Education

Ph.D. in Physics, expected in May 2009
Area of Specialization: Biophysics

York University Toronto, Ontario 1999–2003
B.Sc. in Physics and Astronomy, Specialized Honors in Physics

Awards and Honors

Miller Donald Graduate Fellowship 2004
Roberts Scholarship 2003
York University Undergraduate Bursary 2002 - 2003
York Faculty of Science and Engineering Scholarship 1999
York University Entrance Award 1999
Governor General’s Bronze Academic Medal 1999

Research Experience

Thesis Advisor: Prof. Dezhe Jin
Research included modeling of songbird neural networks that use reinforcement learning in the motor control of song development. Models were explored using C++ and MATLAB environments.

Undergraduate Research York University 1999–2003
Research Advisor: Prof. Marko Horbatsch, Prof. Sampa Bhadra and Prof. Scott Menary
Quantum wave packet evolution using the De Broglie Bohm interpretation was explored in a MAPLE environment. Spherical harmonic basis expansions were developed to order n in a FORTRAN environment.

Teaching Experience

Teaching Assistant The Pennsylvania State University 2003-2006
Recitation and experimental labs for 212: Electricity and magnetism and experimental labs for introductory physics I and II.