DENSITY-BASED RESCALED EXPONENTIAL MODEL FOR GAS-CONDENSATE RESERVOIRS DURING BOUNDARY-DOMINATED FLOW

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Abstract

Over the last decade, the importance of natural gas as an energy resource has grown significantly. In July 2015, dry natural gas production worldwide reached an all-time high of 350 BCF/Day and it now accounts for almost a third of the world’s energy consumption, as reported by the U.S. Energy Information Administration. Accurate reserve estimation and the capability to predict production performance is an essential aspect for the development of natural gas reservoirs. Once available, analysis of production data from natural gas reservoirs can serve as one of the most powerful tools to estimate remaining reserves and provide a forecast of its future performance.

Well performance forecasting is important for field development to guide economic decisions during the life of a reservoir. For liquid reservoirs, these fundamentals are very well understood and described in literature, using a linear flow rate-pressure relationship. In contrast, traditional well performance methods for dry and wet gas wells rely on the use of single phase pseudo-pressure and pseudo-time linearizations to solve the highly non-linear gas flow equations due to the presence of strongly pressure-dependent gas properties like viscosity and compressibility. However, natural gas reservoirs often exhibit multi-phase flow of fluids resulting in simultaneous production of oil and gas, sometimes accompanied by water. Such rich gas reservoirs also undergo changes in reservoir fluid composition during their producing life due to retrograde condensate dropout within the reservoir once reservoir pressure falls below dew point pressure. Any predictive model used to analyze these reservoirs must account for the appearance of these multi-phase flow effects.

Typically, analytical models use a pressure-based approach for rate forecasting and tracking fluid property changes with reservoir pressure. This work presents rigorous mathematical models
capable of predicting non-linear flow behavior in multi-phase gas-condensate reservoirs using a density-based approach, i.e., tracking fluid property changes with variation in fluid density, applicable to boundary-dominated flow regimes under variable bottom-hole flowing pressure conditions. Here, gas-condensate systems have been modeled analogous to dry gas reservoirs through an elegant material balance approach which uses equivalent fluid molar densities in multi-phase systems, resulting in simplified analytical equations for the different flowing phases. This work shows that a rescaled exponential model is a rigorous decline solution for liquid-rich gas wells producing under variable bottom-hole pressure during boundary dominated flow, under conditions of water being present but immobile in the reservoir.

The goal of this work is to build a framework for a three-phase analytical model valid under variable bottom-hole pressure conditions. This will provide a rigorous, neat, analytical model suitable for estimating reserves and forecasting production from multi-phase gas-condensate reservoirs.
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1 **Background**

Decline curve analysis in its traditional definition is a procedure used to analyze the decline in oil and gas production rates due to loss in reservoir pressure or changing relative volumes of the produced fluids with time. It is based on the assumption that factors controlling production trends in the past will continue to do so in the future, and hence, is an important tool in forecasting production rates, aiming to predict the performance of oil and gas wells ahead of time.

1.1 **Single Phase Flow**

Arps (1945) was the first to empirically identify a set of mathematical equations to describe the declining behavior of oil and gas wells. He used the equation of the hyperbola to define three main types of equations to model decline, namely - hyperbolic, exponential and harmonic - based on his definition of the flow exponent ‘b’. It was observed that the value of ‘b’ typically ranged from 0 to 1, resulting in the exponential and hyperbolic decline types being the most widely used. Arps also suggested a way to extrapolate an exponential or hyperbolic decline to understand production from wells in the future.

A basis for Arps’ empirical observation was given by Fetkovich (1980), when he introduced the idea of applying type curve analysis by using the Arps’ decline curves for matching solution gas drive and gas reservoir depletion. He modified Arps’ equations to a dimensionless form and extended the analysis to generate type curves for the entire flow regime using his analytical solution and Arps’ empirical equations. He proposed using the parameter ‘b’ for Arps’ family of curves in the range of 0 to 1 for the boundary dominated flow regime. His method was intended as a quick way of analyzing gas well performance for wells producing under constant bottom-hole pressure. With his study, he demonstrated that a closed reservoir producing an incompressible
single phase fluid under constant bottom-hole pressure exhibits an exponentially declining production.

Most analysis was based on analyzing production rates from oil reservoirs, i.e., incompressible fluids. But the use of empirical techniques for analyzing compressible fluids, such as single phase gas reservoirs, was not always reliable due to their highly pressure-dependent fluid properties of viscosity and compressibility resulting in a non-linear flow equation. In order to linearize this flow equation for gases, the concept of pseudo-functions – pseudo-pressure and pseudo-time - was introduced. Al-Hussainy and Ramey (1966) derived a mathematical relationship between gas flow rate and average pseudo-pressure during boundary-dominated flow and were the first to analytically prove this. Agarwal (1979) introduced pseudo-time, allowing further linearization of the gas flow equation by accounting for the changing gas viscosity-compressibility product within the pseudo-time function.

To determine the decline curve behavior for gas systems, Carter (1985) presented two complimentary studies for radial and linear flow regimes for closed systems producing gas under a constant bottom-hole pressure condition. He proposed the use of type-curves using dimensionless flow rate and dimensionless time along with an approximate drawdown parameter, $\lambda$, which accounted for the variation of the gas viscosity-compressibility product over the range of average reservoir pressure during depletion. $\lambda$ was defined as the ratio of the viscosity-compressibility product at initial reservoir pressure to the product at average reservoir pressure. He also showed that this product can be expressed in terms of the difference between initial and flowing pressures and pseudo-pressures. The limiting case, where $\lambda$ takes a value of unity, represents the case of liquid flow.
Building on Agarwal’s pseudo-time concept, Fraim and Wattenbarger (1987) introduced the normalized pseudo-time, evaluated at average reservoir conditions. Using the concepts of pseudo-pressure and pseudo-time in conjunction, they were able to collapse the boundary-dominated flow regime under constant bottom-hole pressure to an exponential decline, similar to a volumetric oil reservoir depletion.

However, since a constant bottom-hole pressure is not always observed practically in the field, Blasingame and Lee (1986) and (1988) introduced the concept of the normalized pseudo-pressure and material balance pseudo-time empirically, later rigorously derived by Palacio and Blasingame (1993). They mathematically proved the validity of this approach to linearize the gas flow equation for variable rate/pressure depletion scenarios, enabled by the use of material balance pseudo-time, thus extending this analysis to a more realistic field case for single phase gas reservoirs.

For the case of dry gas wells, Ayala and Zhang (2013) and Zhang and Ayala (2014a) and (2014b) proved that gas rate decline under variable rate/pressure constraints can be successfully captured in terms of rescaled exponential models,

\[
q_{gSC} = q_{gi}^e \cdot r_p \cdot \tilde{\lambda} \cdot \exp\left(-D_i^e \cdot \tilde{\beta} t\right)
\]

Equation 1-1

where \(\tilde{\lambda}\) and \(\tilde{\beta}\) are dimensionless rescaling parameters quantifying the effect of non-linearities (i.e., gas viscosity-compressibility dependencies with pressure) in the performance equation, \(r_p\) is the density drawdown ratio, \(q_{gi}^e\) is the initial decline rate, and \(D_i^e\) is the initial decline. The authors showed that these rescaled exponential models can be rigorously derived for both constant and variable bottom-hole pressure conditions for single-phase flow without the use of pseudo-time.
Vardcharragosad and Ayala (2015) have extended the dry gas density-based rescaled exponential to account for apparent permeability and sorbed-phase effects for dry gas reservoirs.

The primary motivation behind the above simplifications is to linearize the dry gas flow equation and use well-established fundamentals of liquid flow to analyze these systems. However, with increasing interest in gas production, a number of gas-condensate systems are also encountered, wherein an oil or condensate phase drops-out from liquid-rich gases as the reservoir depletes. This leads to a multi-phase system, with gas, oil and possibly even water flowing in the reservoir. This multi-phase production with time, from gas-condensate reservoirs, is the main focus of this study.

1.2 Multiphase Flow

In order to accurately determine well production rates from a gas-condensate reservoir, it is important to consider the effect of liquid drop-out and its consequences to gas and oil production rates. For the Headlee Devonian field, O'Dell and Miller (1967) observed a high condensate build-up near the well-bore when reservoir average pressure fell within 50 psi of the dew point pressure. To account for this high condensate build-up, they proposed a method to calculate the volume of retrograde liquid drop-out and its effect on producing rates. In order to compute saturations for their formulation, they use the relation between the relative permeability ratios to the relative fluid volumes from constant composition expansion (CCE) data, and evaluate saturation, which has a monotonic relationship with the gas-oil relative permeability ratio. However, a requirement for their method is the maintenance of reservoir pressure near dew point, such that a steady state is established and the well is producing under a constant drawdown.

To simulate production from multiphase reservoirs without using black-oil properties, Roebuck et al. (1969) presented a series of studies beginning from 1-D and leading to a two dimensional three
phase compositional reservoir simulator. Fussell (1973) modified Roebuck’s 1-D radial model to study long term production from a single well in a gas-condensate reservoir and also tested the applicability of the steady state model proposed by O’Dell and Miller. Fussell’s radial model accurately predicts gas production rate for cases where average reservoir pressure falls significantly below dew point pressure, when compared to O’Dell-Miller’s steady-state model. Fussell concluded that the steady-state model may not suitably predict the distances at which high condensate saturations exist and would thereby underestimate production rates.

Jones and Raghavan (1988) investigated multiphase flow in gas-condensate reservoirs with the aim to provide a theoretical basis for analyzing well-test data. They observed that the O’Dell-Miller’s equation can be derived rigorously and from fundamental considerations. They also stated that the modified form of the O’Dell-Miller equation which accounts for gas dissolved in condensate, as used by Fussell, is accurate at all times during depletion. Their analysis uses the gas-oil relative permeability ratio calculated from relative vapour and liquid volumes obtained from CCE data and they acknowledge the fact that pressure-saturation profiles are essential to analyze multi-phase flow in reservoirs, especially under boundary-dominated flow conditions. For transient flow conditions, an approximate pressure-saturation profile can be obtained from the steady-state flow theory which is not applicable to the boundary-dominated flow regime, resulting in erroneous calculations.

Fevang and Whitson (1996) proposed a method of evaluating the much debated pseudo-pressure integral for well-deliverability using the producing gas-oil ratio. Their method accounts for near well-bore damage, vertical or horizontal well trajectory which is included in the rate equation. The pseudo-pressure integral and their integration approach can be described with the following equations:
\[ \Delta m_{tp}(p) = \int_{p_{wf}}^{p} \left( \frac{k_{rg}}{\mu_g B_g} + R_s \frac{k_{ro}}{\mu_o B_o} \right) dp \]

**Equation 1-2**

Elaborating **Equation 1-2**

\[
\Delta m_{tp}(p) = \int_{p_{wf}}^{p_1} \left( \frac{k_{rg}}{\mu_g B_g} + R_s \frac{k_{ro}}{\mu_o B_o} \right) dp + \int_{p_1}^{p_{dew}} \left( \frac{k_{rg}}{\mu_g B_g} \right) dp + k_{rg}(S_{wi}) \int_{p_{dew}}^{\bar{p}} \left( \frac{1}{\mu_g B_g} \right) dp
\]

**Equation 1-3**

Their basic setup is based on the observation of three possible regions, of which one or more may exist at a time, in the reservoir. These are the near well-bore region which experiences condensate build-up due to liquid drop-out, an intermediate region where condensate exists but remains immobile, and a farther single phase gas region where pressure is still above dew point and no condensate drop-out is observed. This method effectively captures the important changes experienced by the reservoir and is able to mimic production from fine-grid single-well simulations fairly accurately. It is based on the assumption that flowing composition in the near well-bore region is constant and equal to that seen at the surface, thus showing a constant value of gas-oil ratio in this region. This gas-oil ratio is used to evaluate the relative permeabilities to gas and oil using the following modified Evinger and Muskat (1942) approach:

\[ \frac{k_{rg}}{k_{ro}}(p) = \left( \frac{R_p - R_s}{1 - R_v R_p} \right) \frac{\mu_g B_g}{\mu_o B_o} \]

**Equation 1-4**

They also study the effect of relative permeability and show that well-deliverability impairment due to condensate blockage is dependent on relative permeabilities in the range defined by \( 1 < \)
$\frac{k_{rg}}{k_{ro}} < 50$. Other observations from their study are that critical oil saturation and interfacial tension dependence of relative permeability have little or no effect on gas-condensate well performance. This method has been adopted for the development of the model in this manuscript and a detailed explanation is presented as a part of the model description.

In other notable works, Guehria (2000) proposed a scheme to obtain inflow-performance relationship curves for a gas-condensate reservoir under boundary-dominated flow. The basic framework consisted of the gas and oil flow equations in porous media, based on their respective pseudo-pressure differentials. In order to calculate production rates, an initial guess on the value of gas-oil ratio is used until convergence. This process is repeated at every required pressure level. The model uses an empirical scheme for the initial guess of gas-oil ratio.

Mott (2003) proposed an easy-to-use spreadsheet model based on the approach presented by Fevang and Whitson, with a modification applied to the calculation of the pseudo-pressure integral in the mobile condensate region. According to the author, the assumption of a constant gas-oil ratio in the near well-bore region results in over-estimation of the volatilized oil-gas ratio, causing under-prediction of the fluids produced. Thus, he proposed a calculation based on tracking the expansion of the region with mobile condensate using pore volumes. The study indicated that this region’s volume is the most important parameter to accurately estimate the pseudo-pressure integral here.

As an improvement to Mott’s proposed method, Xiao and Al-Muraikhi (2004) modified the pseudo-pressure integral in the near well-bore region to use constant volume depletion (CVD) data to calculate relative permeability throughout the producing life of the reservoir. They provided strong evidence of the validity of their model for vertical, horizontal and hydraulically fractured
wells. They used an iterative process to calculate the pressure \( P^* \), the pressure contour forming the outer boundary of the near well-bore mobile condensate region, and the bisection method to converge on oil saturation. Their results for a rich gas being produced from a vertical well for nearly 20 years shows, under engineering accuracy, a deviation in gas-oil ratio from the proposed model when compared to reported numerical simulator values, after the first few years. This may be attributed to the assumption of constant volume depletion data being representative throughout the simulation time.

Gerami et al. (2010) proposed an iterative scheme to calculate well-deliverability in gas-condensate reservoirs using the pseudo-pressure integral approach proposed by Fevang and Whitson. Their model used the established well-deliverability equation for gas flow and an analogous equation for oil flow rate to be able to compute the gas-oil ratio from these flow rates. They used an initial guess on gas-oil ratio, a requirement for the Fevang and Whitson approach, and iterated on this until convergence. Although this is based on a sound methodology, its validation has been done for one synthetic case from where the advantages or limitations of this approach are unclear. Simulation time is not evident from the graphical results presented and there are no available comments on the physical significance of their results with observed field data. Also, the model does not compare differences that the richness of the gas may create in the results obtained.

Following this, Sureshjani and Gerami (2011) proposed an analytical model to analyze long term production data of gas-condensate reservoirs with all formulations written for the gas component, as has been done by many previous works mentioned here. They introduced the two-phase material balance pseudo-time and when used in conjunction with two-phase pseudo-pressure, they were
able to estimate average reservoir pressure and initial gas-in-place for a lean and rich gas under variable bottom-hole pressure conditions. Their two-phase pseudo-time is given by:

\[ t_{a,tp}(t) = \frac{1}{m_{tp,i}} \int_0^t \frac{dt}{\partial \bar{m}_{tp}\left(\frac{S_g(\bar{P})}{B_g(\bar{P})} + \frac{R_s(\bar{P})S_o(\bar{P})}{B_o(\bar{P})}\right)} \]

Equation 1-5

where, their definition of two-phase pseudo-pressure, \( \bar{m}_{tp} \) is given by

\[ \bar{m}_{tp} = \int_{P_{dew}}^{\bar{P}} \left( \frac{k_{rg}}{\mu_g B_g} + R_s \frac{k_{ro}}{\mu_o B_o} \right) d\bar{P} \]

Equation 1-6

Near well-bore region sees relative permeability to be a function of saturation as well as capillary number and inertial effects. They used the model proposed by Whitson et al. (1999) to include capillary number in their analytical model. They investigated the effect of non-Darcy flow by using the Forchheimer equation, and showed that it has a negative effect on relative permeability when compared to the effect of capillary pressure. Since both non-darcy factor and capillary number are functions of velocity, they determined velocity as a function of pressure, by modifying the Whitson’s model for velocity calculation. The use of their model as a forward one, to predict gas and oil flow rates, has not been shown, and an equation for oil flow rate or estimation of initial oil-in-place is also missing from their analysis of gas-condensate reservoirs.

A follow up on their previous study of 2011, Sureshjani et al. (2014) modified their material balance pseudo-time to neglect the oil phase mobility term with the aim to simplify calculations of this parameter. The new calculation used \( k_{rg_{CVD}} \) and a modified gas compressibility factor which accounted for flow properties of the fluid.
\[ t_{acr, tp} = \mu_{gi} \frac{1}{q_{gsc}(t)} \int_0^t \frac{k_{rgCVD}}{S_g \mu_g c_g} q_{gsc}(\tau) d\tau \]

**Equation 1-7**

This simplification is also seen in their pseudo-pressure calculation where producing gas-oil ratio is not required because of the assumed negligible contribution of gas dissolved in the condensate phase. Thus, a pressure-saturation relationship from CVD data is used. Their model applies to constant well-bore flowing pressure cases only, due to the simplification in material balance pseudo-time. Their results for rich gas show major deviations from the numerical simulator values.

The model may not be applicable to all cases of gas-condensate reservoirs owing to the simplifications and consequently, the use of a number of empirical constants in the process.

Arabloo et al. (2014) presented an alternative to the previous two studies. They modified the dry gas equation proposed by Palacio and Blasingame (1993) for gas-condensate reservoirs by using two-phase \( Z \)-factors for their calculations instead of dry gas \( Z \)-factors, as was the case in 1993. Their equation is described here as follows

\[ \frac{m_i^* - m_{wf}^*}{q_{gsc}} = b_{PSS}^* + \frac{1}{G_{C_{gi}}^*} t_{ca}^* \]

**Equation 1-8**

where, their definition of pseudo-pressure and material balance pseudo-time is

\[ m^* = \frac{\mu_{gi} Z_{tp,i} P_i}{P_t} \int_0^P \frac{P}{\mu_g Z_{tp}} dp \]

**Equation 1-9**
\[ t_{ca}^* = \frac{\mu_g c_g^*}{q_{gsc}(t)} \int_0^t \frac{q_{gsc}(t)}{\mu_g c_g^*} \, dt \]

Equation 1-10

and compressibility factor is calculated using the equation

\[ c_g^* = \frac{1}{P} - \frac{1}{Z_{tp}} \frac{\partial Z_{tp}}{\partial P} \]

Equation 1-11

This modified equation along with the two-phase material balance approach proposed by Hagoort (1988) enabled them to predict average pressure and initial gas-in-place for a range of lean and rich gases. Their model is limited to the constant bottom-hole pressure case and can be used only when average reservoir pressure falls below dew point.

1.3 Material Balance

An important aspect of this work is the integration of well-deliverability equations with zero-dimensional material balance to track gas and oil production with time. This is detailed out in the model description. A large part of this work is based on a direct extension of the material balance model proposed by Zhang and Ayala (2016), which related equivalent gas molar density to cumulative surface gas production in order to predict average reservoir pressure for liquid rich gas reservoirs. However, it is important to mention notable studies of zero-dimensional material balance models by Hagoort (1988), Vo et al. (1989), and Walsh et al. (1994a) and (1994b) before delving into the specifics of the model this work is based on.

J. Hagoort in 1988 introduced the concept of two-phase Z-factors and proposed a zero-dimensional material balance equation for volumetric gas-condensate reservoirs. He related average reservoir
pressure to the total moles of hydrocarbon produced using two-phase $Z$-factors which were obtained from CVD data.

\[
\frac{\bar{P}}{Z_{tp}} = \frac{P_i}{Z_{tp,i}} \left(1 - \frac{n_p}{n_i}\right)
\]

**Equation 1-12**

This could be related to cumulative surface gas production, which is a measurable quantity, using condensate-gas molar ratios $R_{MLG_i}$ and $R_{MLG_p}$. Thus, an alternate form of the material balance equation is

\[
\frac{\bar{P}}{Z_{tp}} = \frac{P_i}{Z_{tp,i}} \left(1 - \frac{G_p \left(1 + R_{MLG_p}\right)}{G_i \left(1 + R_{MLG_i}\right)}\right)
\]

**Equation 1-13**

where,

\[
R_{MLG_i} = \frac{n_{oi}}{n_{gi}} = \frac{5.615 \times R_{vl} \times G_i \times \rho_{ost}}{G_i \rho_{gsc}} = 5.615 \times \frac{\rho_{ost}}{\rho_{gsc}} \times R_{vl}
\]

**Equation 1-14**

\[
R_{MLG_p} = \frac{n_{op}}{n_{gp}} = \frac{5.615 \times N_p \times \rho_{ost}}{G_p \rho_{gsc}} = 5.615 \times \frac{\rho_{ost}}{\rho_{gsc}} \times \frac{N_p}{G_p}
\]

**Equation 1-15**

Vo et al. (1989) showed that two-phase $Z$-factors are molar weighted averages of reservoir gas and oil phase $Z$-factors. Thus, two-phase $Z$-factors can be expressed as
\[ Z_{tp} = f_{nl}Z_L + f_{nv}Z_V \]

Equation 1-16

They also showed the approximate validity of using CCE data instead of CVD data to obtain two-phase \( Z \)-factors, relating it to cumulative surface gas production as follows,

\[ \frac{\bar{P}}{Z_{tp,CCE}} = \frac{P_i}{Z_{tp,i}} \left( 1 - \frac{G_p}{G_i} \right) \]

Equation 1-17

Walsh et al. (1994a) and (1994b) derived the generalized material balance equation applicable to the full range of conventional reservoir fluid types. When simplified for a gas-condensate reservoir fluid, the material balance can be written as follows:

\[
\frac{(G_i - G_p)(B_V - B_LR_v) + (N_i - N_p)(B_L - B_VR_s)}{1 - R_sR_v} = \frac{G_i(B_{V,i} - B_{L,i}R_{vi}) + N_i(B_{L,i} - B_{V,i}R_{si})}{1 - R_{si}R_{vi}}
\]

Equation 1-18

Based on a detailed study of the available zero-dimensional material balance models applicable to liquid-rich gas reservoirs, Zhang and Ayala (2016) proposed a density based material balance equation on the basis of surface gas component species conservation. The equation is written in terms of the \textit{equivalent} gas molar density, relating directly to cumulative surface gas production and thus posing as a direct extension to the dry gas material balance equation. This equation can be related to CVD parameters, giving a very good average reservoir pressure prediction for gas-dominated systems. It does not require the use of two phase \( Z \)-factors and relates to physical quantities directly tracking the evolution of the surface gas storage term of the surface gas partial differential equation.
2 Problem Statement

As the name suggests, liquid-rich reservoirs may contain low to significant quantities of condensate in vaporized form. Hydrocarbons are typically found in vapor phase owing to high temperature and pressure conditions, resulting in a classification under the gas reservoirs family. Water may also be present in the reservoir - either immobile, or substantial enough to flow along with gas and condensate. A detailed study of the available analytical and semi-analytical models suggests potential for a rigorous three-phase analytical model to understand the flow of gas, condensate and water in liquid-rich reservoirs. Thus, creating a framework for a multi-phase analytical model with the ability to forecast production and estimate gas and oil reserves is the foundation of this work.

Based on the thermodynamics of a liquid-rich system, depletion of the reservoir results in a retrograde behavior, where liquid dropout is observed when average reservoir pressure falls the fluid’s dew point pressure. This starts at the well-bore and progresses outward to the boundaries of the reservoir. Condensate dropout near the well-bore is a cause for concern because of the loss in hydrocarbon productivity observed due to condensate banking. In some cases, water may also be produced along with hydrocarbons, further competing for a share of total productivity.

Real-time changes observed in a gas condensate reservoir are compositional. However, a modified black oil approach is used here (a) to understand how close a representation this may be and (b) to make use of all readily available data from CVD and CCE experiments to enable a straightforward analysis. Being able to understand the physics of flow explaining this behavior and liquid-rich reservoirs in general, can provide useful insights to help in decision making during field development, and thus help follow best practices for such reservoirs.
Specifically, the main objectives of this work can be summarized as follows:

a. Develop multi-phase analytical models to account for flow of gas and condensate during production. The models will be used to forecast production from the reservoir and assess hydrocarbon reserves by solving the inverse problem, using production data as input to generate reliable estimates.

b. Verify the developed analytical solutions with the results from direct numerical simulations.

c. Apply the models to real field cases and thus, implement variable rate/pressure-drop handling capability.

Few mathematical models have been developed to understand the flow phenomena of gas, condensate and water in the reservoir. Having a rigorous model will help in comprehending this less understood behavior by tying observations to fundamental physics, creating a strong foundation of concepts. This, as a basis, can supplement future work and studies on this topic, with the added advantage of minimizing empiricism.
3 Introduction

Liquid-rich gas reservoirs are important sources of hydrocarbons to supplement the world’s demand for this resource. However, due to the thermodynamics of this particular reservoir type, multi-phase flow is seen within the reservoir, unlike other members of the family of gas reservoirs, i.e., dry and wet gas reservoirs. This is illustrated by the pressure-temperature diagram below.

Figure 3-1: Pressure-Temperature Path in a Gas-Condensate Reservoir

These reservoirs exhibit temperatures above the critical temperature, and below the cricondentherm. Thus, when pressure depletes isothermally due to hydrocarbon production from the reservoir, the pressure-temperature path followed by the reservoir fluid results in retrograde condensation of heavier hydrocarbons from the gas phase, forming a secondary liquid phase during production, along with a tertiary water phase inherent in any reservoir. Owing to the rich content of hydrocarbon condensation observed and produced at the surface, these reservoirs are considered attractive for economic investment, resulting in various analyses methodology to understand flow behavior and estimate amounts of hydrocarbon produced from them.
The following model development centers on the density-based approach. Since changes in fluid density are observed with respect to time, this model helps one analyze the physical significance of pressure depletion when the reservoir is produced. This approach ties production from a gas-condensate reservoir to the physical changes in the fluid, and thus, is a rigorous method that would prove useful for a first-hand analysis of reservoir production data. This method provides useful insight about liquid-rich gas reservoir behavior, and can provide valuable information for further reservoir development and production.

The proposed model is developed and applied to three distinct reservoir and fluid types which are described in detail in the subsequent section. These case studies form the basis of the discussion and carry on throughout the implementation of the model, starting with the analysis of the material balance equation followed by the well deliverability equation. Finally, both of these equation sets culminate into the rescaled exponential model, whose behavior is shown relative to results reported by a commercial numerical simulator. In each case, this manuscript talks about the conditions where the model proves to be successful, along with associated caveats to be kept in mind while working with the rescaled exponential equations. The proposed model is then used to estimate reserves for the three cases discussed. The lean, intermediate and rich gas case studies chosen are outlined with the required input parameters below.

3.1 The Cases Studied

Three cases are studied here to establish the working of the model. These cases are representative of a gas condensate reservoir that may be present in extreme cases of being lean or rich. An intermediate liquid-rich gas case is also shown.
Real PVT data from three gas wells in a field in Texas USA are used, representing three case studies of lean, intermediate, and liquid-rich gas condensate, to validate the developed mathematical model. Reservoir parameters are shown in Table 3-1 followed by the constant volume depletion liquid dropout results for all three cases. Lean gas shows a maximum CVD liquid dropout of 1.35 % in contrast to the 40.2 % seen in the rich gas CVD experiment, while intermediate gas shows a dropout of 22%.

Table 3-1: Reservoir and fluid properties for three case studies: liquid-rich, intermediate, and lean gas.

<table>
<thead>
<tr>
<th>Property</th>
<th>Case 1: Lean Gas</th>
<th>Case 2: Inte. Gas</th>
<th>Case 3: Rich Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir temperature, $T$ (F)</td>
<td>235</td>
<td>250</td>
<td>226</td>
</tr>
<tr>
<td>Reservoir pressure, $P_l$ (psia)</td>
<td>6000</td>
<td>2790</td>
<td>4300</td>
</tr>
<tr>
<td>Dew Point pressure, $P_{dew}$ (psia)</td>
<td>3208.14</td>
<td>2788.00</td>
<td>4578.94</td>
</tr>
<tr>
<td>Bottom-hole pressure, $P_{wf}$ (psia)</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>Porosity, $\phi$ (%)</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Permeability, $k$ (mD)</td>
<td>30</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>Formation thickness, $h$ (ft)</td>
<td>300</td>
<td>50</td>
<td>300</td>
</tr>
<tr>
<td>Reservoir radius, $r_e$ (ft)</td>
<td>3000</td>
<td>500</td>
<td>3000</td>
</tr>
<tr>
<td>Wellbore radius, $r_w$ (ft)</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Oil API at SC</td>
<td>63.81</td>
<td>54.70</td>
<td>50.07</td>
</tr>
<tr>
<td>CVD liquid dropout (%)</td>
<td>1.35</td>
<td>22.00</td>
<td>40.20</td>
</tr>
<tr>
<td>Initial gas-in-place, $G_i$ (BCF)</td>
<td>773.61</td>
<td>1.92</td>
<td>547.81</td>
</tr>
<tr>
<td>Initial oil-in-place, $N_i$ (MMSTB)</td>
<td>36.17</td>
<td>0.32</td>
<td>67.12</td>
</tr>
</tbody>
</table>
The above case studies have been chosen in order to test the model for an extremely lean, an intermediate and an extremely rich gas case, operating over a wide range of pressures. The lean gas case sees an under-saturated reservoir at an initial reservoir and dew point pressures of 6000 psia and 3200 psia respectively, the intermediate gas reservoir is found at its dew point of 2788 psia and the rich gas reservoir is saturated at an initial reservoir and dew point pressures of 4300 psia, and 4500 psia, respectively.

Figure 3-2: CVD liquid dropout curves for three case studies: lean, intermediate, and liquid-rich gas.

Relative permeability tables are generated using Corey’s correlation with exponent values 4 and 2 for gas and oil respectively. Connate water \( S_{wc} \) is neglected, but can be readily incorporated, resulting in a decrease in the net porosity value observed in the reservoir and thus effecting the absolute value of the flow rates, but maintaining similar trends to the ones shown here.
Figure 3-3: Relative Permeability Curves Using Corey's Correlation

Results for all cases are summarized under their respective sections below. All results have been validated against values reported by a commercial numerical simulator (CMG-IMEX).

3.1.1 The Lean Gas Case

This case study is for a lean gas reservoir with a maximum CVD liquid drop-out of 1.35%. Related black oil PVT properties are shown in Figure 3-4 and Figure 3-5, for pressures ranging from well-bore flowing pressure to the fluid’s dew point pressure.
Figure 3-4: Lean Gas Black Oil PVT Data

Figure 3-5: Lean Gas Viscosity Data
3.1.2 The Intermediate Gas Case

The intermediate gas shows a CVD liquid dropout of 22% with the following black oil PVT properties and viscosity data ranging from well-bore flowing pressure to the fluid’s dew point pressure.

![Intermediate Gas PVT Data](image1)

**Figure 3-6: Intermediate Gas PVT Data**

![Intermediate Gas Viscosity Data](image2)

**Figure 3-7: Intermediate Gas Viscosity Data**
3.1.3 The Rich Gas Case

This case study is for a rich gas reservoir with a maximum CVD liquid drop-out of 40.2%, representative of an extreme case of liquid-rich reservoirs, in order to test the model at the limits of the problem at hand. Related black oil PVT properties are shown in Figure 3-8 and Figure 3-9.

Figure 3-8: Rich Gas Black Oil PVT Data
With the above introduction to the three cases under consideration, subsequent section outline the development of the model and documents responses of these three cases to the different aspects studied.
4  The Rescaled Exponential Model for Multiphase Boundary-dominated Flow

The development of the aforementioned rescaled exponential model for multi-phase boundary-dominated flow follows a two-step process, of which the first is to identify an appropriate material balance equation for the components under consideration and the second is to use the correct well-deliverability equation. These two equations, when combined, describe the behavior of production from the reservoir as an exponential decline. The assumptions made to develop this model are outlined below.

4.1  Assumptions

In the analytical methodology described here, the following assumptions are used in order to arrive at the proposed BDF multi-phase solution:

1. Modified black oil formulation assumptions are applicable. i.e., surface gas and surface oil are the two primary components considered for this model.
2. Rock compressibility is neglected i.e., pore volume $V_p$ remains constant
3. Connate water is immobile and incompressible (water saturation $S_w = S_{wc}$ throughout production life)
4. Capillary pressure effects are neglected.
5. Well is produced at a constant bottom-hole pressure.

Note that assumption 5 is relaxed in the subsequent sections and the model is extended to a variable bottom-hole pressure specification as well.
4.2 Model Development

This section goes through a step-by-step development of the model and explains various linked concepts. It starts with the material balance equation, followed by well-deliverability equations for the fluids under consideration. It then goes onto the final rescaled exponential model by tying the material balance to its respective well deliverability equation. This has been detailed out below.

4.2.1 Material Balance Equations

In order to develop the model, the first step is to identify the appropriate material balance equation to determine average reservoir pressure as it undergoes depletion. Commonly used material balance equations for average pressure calculation are in the form of Z-factors for single-phase fluids and two-phase Z-factors for multi-phase reservoirs.

The approach used here does not require calculation of two-phase Z-factors, and is adopted from the method proposed by Zhang and Ayala (2016), given below:

\[
\bar{\rho}_g = \rho_{g,i}^* \left(1 - \frac{G_p}{G_i}\right)
\]

Equation 4-1

where, \(\rho_g^*\) is the equivalent gas molar density given by,

\[
\rho_g^* = \rho_{gsc}^* \left(\frac{S_g}{B_g} + \frac{S_o}{B_o} R_s\right)
\]

Equation 4-2

\(\rho_{gsc}^*\) is the gas molar density at standard condition and equal to \(\frac{p_{sc}}{T_{sc} R}\). This equation, proven to work well for gas-condensate reservoirs to predict average reservoir pressure as the reservoir depletes,
led to the development of similar expressions for the surface oil component and total hydrocarbons produced at the surface.

Material balance based on an equivalent oil molar density results in the following equation, analogous to that of the surface gas component.

\[
\bar{\rho}_o^* = \rho_{o, i}^* \left(1 - \frac{N_p}{N_i}\right)
\]

**Equation 4-3**

where, \( \rho_o^* \) is the equivalent oil molar density,

\[
\rho_o^* = \rho_{osc}^* \left(\frac{S_o}{B_o} + R_v \frac{S_g}{B_g}\right)
\]

**Equation 4-4**

\( \rho_{osc}^* \) is the surface oil molar density at standard conditions, and unlike \( \rho_{gsc}^* \), depends on the composition of the fluid. It can be obtained from flash calculation at standard conditions of the original reservoir fluid, at initial reservoir pressure and temperature. A detailed derivation of **Equation 4-3** and **Equation 4-4** is provided in Appendix A.

The surface gas and surface oil components can also be lumped in the form of total hydrocarbons produced at the surface. The usefulness of this third methodology will be evident in the results of this section. Thus, following a similar development, detailed in Appendix B, a material balance done on the total hydrocarbon production at the surface results in the following,

\[
\bar{\rho}_t^* = \rho_{t, i}^* \left(1 - \frac{G_{pE}}{G_{iE}}\right)
\]

**Equation 4-5**
where, $\rho^*_t$ is the equivalent molar density of all hydrocarbons produced, given by,

$$
\rho^*_t = \rho^*_{osc} \cdot \left( \frac{S_g}{B_o} + R_v \frac{S_g}{B_g} \right) + \rho^*_{gsc} \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right)
$$

\textbf{Equation 4-6}

All three material balance equations are used in the following section to generate an average reservoir pressure prediction using the relationship of the corresponding equivalent molar density with PVT properties, thus making them a function of pressure.

\textbf{4.2.1.1 Results}

The following results compare the gas, oil and total hydrocarbon material balance equations to each other, as well as to the widely accepted Hagoort’s (1988) two-phase Z-factor equation and average pressure performance given by the numerical simulator. This comparison has been shown for the three cases of lean, intermediate and rich gases, using two sets of pressure-saturation relationships. The pressure-saturation relationship plays a significant role in the ability of the material balance equation to estimate an average reservoir pressure response.

\textit{a.} A convenient way of estimating the pressure and oil saturation relationship in the reservoir is to use the pressure-saturation data obtained from a CVD experiment. Here, the relationship obtained is between pressure, seen analogous to reservoir pressure, and saturation of condensate that has dropped out from gas. It is important to understand that in this experiment, condensate is not produced along with gas and remains in the experimental cell. This may not always hold true for field production and condensate is usually an essential by-product when producing from gas reservoirs. This CVD pressure-saturation relationship is an approximation, at best.
b. Pressure-saturation relationship can also be obtained from the numerical simulator. This will closely represent the changes in the reservoir, and thus, will act as a better estimate when used with the analytical model. However, this information is not available unless a reservoir simulation is carried out, which may not be desirable at all times.

Case 1: Lean Gas

Figure 4-1: A comparison of average reservoir pressure obtained from the proposed material balance equations, Hagoort’s material balance equation, and numerical simulation for the lean gas case (Case 1) using two pressure-saturation relations: (1) CVD test and (2) direct numerical simulation.
Case 2: Intermediate Gas

Figure 4-2: A comparison of average reservoir pressure obtained from the proposed material balance equations, Hagoort’s material balance equation, and numerical simulation for the intermediate gas case (Case 2) using two pressure-saturation relations: (1) CVD test and (2) direct numerical simulation.
Case 3: Rich Gas

Figure 4-3: A comparison of average reservoir pressure obtained from the proposed material balance equations, Hagoort’s material balance equation, and numerical simulation for the rich gas case (Case 3) using two pressure-saturation relations: (1) CVD test and (2) direct numerical simulation.

Being a gas-dominated system, both gas and total hydrocarbon material balance equations work very well for the lean, intermediate and rich gas case, giving an average reservoir pressure almost identical to that reported by the numerical simulator, which is representative of the real reservoir condition at that time. However, the oil material balance, heavily skewed by the dominance of its corresponding oil saturation in its material balance equation, results in a poor pressure estimate when a CVD pressure-saturation relationship is used. This is contrary to the case where a more realistic pressure-saturation relationship is provided by the numerical simulator.
Another observation from the results is the fact that the material balance equations give an excellent estimate for the lean and intermediate gas, and comparatively, a good approximation for a rich gas. As the fluid deviates from being a dry gas to a liquid rich gas, all principles understood and derived for a dry gas may not be able to capture reservoir non-linearities, but can still be used to serve the purpose of broadly understanding the flow phenomena seen in the reservoir.

4.2.2 Well Deliverability

In order to follow through with the development of the rescaled exponential model, identifying the appropriate well-deliverability equation applicable to boundary-dominated flow is an important step. This, for the surface gas component (Zhang et al. 2016), the surface oil component and total hydrocarbons produced at the surface, in liquid-rich gas systems respectively, corresponds to

**Surface Gas**

\[
q_{gsc} = \frac{2\pi k h \delta}{b_{D, PSS}} [m_{tg}(\bar{p}) - m_{tg}(p_{wf})] = \frac{2\pi k h \delta}{b_{D, PSS}} \int_{p_{wf}}^{\bar{p}} \left( \frac{k_{rg}}{\mu_g B_g} + R_s \frac{k_{ro}}{\mu_o B_o} \right) dp
\]

**Equation 4-7**

This considers the flow of free gas as well as gas dissolved in the condensate phase in the reservoir. A significant proportion of gas flows in the free phase as compared to the dissolved component in reservoir oil/condensate phase.

**Surface Oil**

\[
q_{osc} = \frac{2\pi k h \delta}{b_{D, PSS}} [m_{to}(\bar{p}) - m_{to}(p_{wf})] = \frac{2\pi k h \delta}{b_{D, PSS}} \int_{p_{wf}}^{\bar{p}} \left( \frac{k_{ro}}{\mu_o B_o} + R_v \frac{k_{rg}}{\mu_g B_g} \right) dp
\]

**Equation 4-8**
The well-deliverability equation for oil, analogous to that of gas, considers the free flow of condensate in the reservoir as well as its component present vaporized in the reservoir gas phase, which is significant for the case of gas-condensate reservoirs.

**Total Hydrocarbons**

\[
q_{t,sc} = \frac{2\pi k h \delta}{b_{D,PSS}} \left[ m_{tt}(\bar{p}) - m_{tt}(p_{wf}) \right]
\]

Equation 4-9

This incorporates pseudo-pressure drawdown from the gas and oil phase, as shown,

\[
q_{t,sc} = \frac{2\pi k h \delta}{b_{D,PSS}} \left[ \left( m_{tg}(\bar{p}) - m_{tg}(p_{wf}) \right) + R_{go} \left( m_{to}(\bar{p}) - m_{to}(p_{wf}) \right) \right]
\]

Equation 4-10

Elaborating on this,

\[
q_{t,sc} = \frac{2\pi k h \delta}{b_{D,PSS}} \int_{p_{wf}}^{\bar{p}} \left( \frac{k_{rg}}{\mu_g B_g} + R_s \frac{k_{ro}}{\mu_o B_o} \right) + R_{go} \left( \frac{k_{ro}}{\mu_o B_o} + R_v \frac{k_{rg}}{\mu_g B_g} \right) dp
\]

Equation 4-11

Thus, the above well-deliverability equation considers the flow of surface gas in reservoir gas and reservoir oil phases, the flow of surface oil in reservoir gas and reservoir oil phases and these two components together, with appropriate conversion factors for unit consistency, result in the total flow rate of hydrocarbons produced.

Detailed derivations of **Equation 4-7** and **Equation 4-8** are shown in **Appendix C**. **Equation 4-9** is seen as a summation of **Equation 4-7** and **Equation 4-8**.
An important aspect in this development is evaluating the pseudo-pressure integral in the well-deliverability equations described above. As mentioned earlier, this work calculates the pseudo-pressure integral based on the method proposed by Fevang and Whitson (1996). Their method has been adopted to calculating the pseudo-pressure integral for the oil and total hydrocarbon well-deliverability equations and is presented in detail below.

4.2.2.1 The Pseudo-Pressure Integral

Unlike the original pseudo-pressure proposed for dry gas reservoirs \( m(p) = 2 \int_{p_b}^{p} \frac{p}{\mu_g z} dp \), which is a sole function of pressure, two-phase pseudo-pressure \( (m_{tp}) \) is a function of both pressure and saturation. Thus, its calculation requires knowledge of a pressure-saturation relationship. In Equation 4-7, Equation 4-8 and Equation 4-9, the approach proposed by Fevang and Whitson (1996) is used to calculate the pseudo-pressure differential, \( m_{tp}(\bar{p}) - m_{tp}(p_{wf}) \). Following their approach, the integral on the right-hand-side of the equations is divided into three sections or flow regions:

**Integral for Surface Gas**

\[
m_{tg}(\bar{p}) - m_{tg}(p_{wf}) = \\
\int_{p_{wf}}^{p_1} \left( \frac{k_{rg}}{\mu_g B_g} + R_s \frac{k_{ro}}{\mu_o B_o} \right) dp + \int_{p_1}^{p_{dew}} \left( \frac{k_{rg}}{\mu_g B_g} \right) dp + k_{rg} (S_{wc}) * \int_{p_{dew}}^{\bar{p}} \left( \frac{1}{\mu_g B_g} \right) dp
\]

**Equation 4-12**

**Integral for Surface Oil**

\[
m_{to}(\bar{p}) - m_{to}(p_{wf}) =
\]
\[\int_{p_{wf}}^{p_1} \left( \frac{k_{ro}}{\mu_o B_o} + R_v \frac{k_{rg}}{\mu_g B_g} \right) dp + \int_{p_1}^{p_{dew}} \left( \frac{k_{rg}}{\mu_g B_g} - R_v \right) dp + \int_{p_{dew}}^{p} \left( R_v \frac{1}{\mu_g B_g} \right) dp\]

\textbf{Equation 4-13}

\textit{Integral for Total Hydrocarbons}

\[m_{tt}(\bar{p}) - m_{tt}(p_{wf}) = \]

\[\int_{p_{wf}}^{p_1} \left( \frac{k_{rg}}{\mu_g B_g} (1 + R_v R_{go}) + \frac{k_{ro}}{\mu_o B_o} (R_s + R_{go}) \right) dp + \int_{p_1}^{p_{dew}} \left( \frac{k_{rg}}{\mu_g B_g} (1 + R_v R_{go}) \right) dp + k_{rg}(S_{wc}) * \int_{p_{dew}}^{p} \left( \frac{1 + R_v R_{go}}{\mu_g B_g} \right) dp\]

\textbf{Equation 4-14}

\textbf{Region 1}

The first integral on the right hand side of \textbf{Equation 4-12}, \textbf{Equation 4-13} and \textbf{Equation 4-14} represents the near wellbore region where both gas and condensate are flowing. In this integral, \(p_1\) corresponds to the pressure below which the condensate becomes mobile. This pressure can be obtained from the PVT property table of the reservoir fluid as the pressure corresponding to a volatilized oil-gas ratio equal to \(R_v^1 = \frac{1}{R_p}\), where \(R_v^1\) is oil-gas ratio for the producing well stream and \(R_p\) is the known producing gas-oil ratio. Relative permeability values within this region can be related to \(R_p\), the producing gas-oil ratio, using the following relationship, modified from Evinger and Muskat (1942), expressed as \textbf{Equation 1-4} and shown here,

\[\frac{k_{rg}}{k_{ro}}(p) = \left( \frac{R_p - R_s}{1 - R_v R_p} \right) \frac{\mu_g B_g}{\mu_o B_o}\]
Using relative permeability curves ($k_{rg}$ and $k_{ro}$ versus $S_o$), this pressure-dependent relative permeability ratio $\frac{k_{rg}}{k_{ro}}$ is directly linked to saturation; and a pressure – saturation relationship is established.

**Region 2**

The second integral in Equation 4-12, Equation 4-13 and Equation 4-14 is calculated for the region where gas and condensate coexist but only gas phase flows and condensate remains immobile. In this region, $k_{rg}(S_o)$ can be approximately determined from the liquid dropout volume from a CVD test. Immobile water can be accounted for by rescaling the saturation obtained from CVD data by $(1 - S_{wc})$.

**Region 3**

For the third integral in the aforementioned equations, only single –phase gas exists and flows and can be easily calculated through gas PVT properties. Immobile water is accounted for by evaluating $k_{rg}(S_{wc})$ for this region.

**4.2.3 The Rescaled Exponential Model**

With the above background of the material balance and well-deliverability equations in place, the rescaled exponential under constant bottom-hole pressure assumption can be written for the surface gas, surface oil, and total hydrocarbons produced as follows,

**Surface Gas**

$$
q_{gsc} = q_{tg, i}^e \times r^*_{tg} \times \bar{\lambda}_{tg} \exp \left( -D_{tg, i}^e \bar{\beta}_{tg} t \right)
$$

Equation 4-15
Surface Oil

\[ q_{osc} = q_{to,i}^e \times r_{p,ro}^* \times \bar{\lambda}_{ro} \times \exp(-D_{to,i}^e \tilde{\beta}_{to} t) \]

Equation 4-16

Total Hydrocarbons

\[ q_{tsc} = q_{tt,i}^e \times r_{p,rt}^* \times \bar{\lambda}_{tt} \times \exp(-D_{tt,i}^e \tilde{\beta}_{tt} t) \]

Equation 4-17

A detailed derivation of all three equations is presented in Appendix D, E and F respectively.

The derivations show that the exponential nature of the equation stems from the fact that it is the variation in equivalent molar density with time which contributes to the exponential nature of the equations. It is also evident from the development that the respective two-phase depletion parameters play an important role in the development of the equation and take into account the non-linearities observed in the reservoir due to the presence of the gas phase. The depletion parameter is described in detail below.

4.3 Significance of \( \bar{\lambda}_t \)

In a multiphase system, \( \bar{\lambda}_{tg} \) basically tracks the average equivalent gas viscosity-compressibility ratio (slope) evaluated from the straight-line connecting bottom-hole and average conditions, as shown in Figure 4-4 below adapted from Zhang et al., (2016) — also shown by Zhang and Ayala (2014a) for the case of dry gases. The slope of two-phase pseudo-pressure (\( m_{tp} \)) vs. density (\( \rho_{g}^* \)) plot directly gives the product of viscosity-compressibility \( \mu_{g}^* c_{g}^* \), which again follows from rewriting the \( m_{tp} \) definition in terms of the equivalent gas properties \( (\rho_{g}^*, \mu_{g}^*, c_{g}^*) \).
Figure 4-4: Two-phase pseudo-pressure vs. equivalent gas density plot

(Adapted from Zhang et al., 2016)

We can re-write the two-phase pseudo-pressure integral for gas as:

\[
m_{tg}(p) = \int_{\rho_{g,wf}^{*}}^{\rho_{g}^{*}} \frac{1}{\rho_{g,sc}^{*} \cdot \mu_{g}^{*} c_{g}^{*}} \frac{1}{\rho_{g}^{*} \cdot \mu_{g}^{*} c_{g}^{*}} d\rho_{g}^{*}
\]

Equation 4-18

Differentiating the above with respect to the equivalent gas molar density:

\[
\frac{d \left( m_{tg}(\tilde{p}) \right)}{d\rho_{g}^{*}} = \frac{1}{\tilde{\mu}_{g}^{*} \tilde{c}_{g}^{*} \rho_{g,sc}^{*}}
\]

Equation 4-19
This can be obtained from the slope of the graph between two-phase pseudo pressure defined for the surface gas component and its corresponding equivalent gas molar density. Thus, this can be written as

\[
\frac{d \left( m_{tg}(\bar{p}) \right)}{d \rho_g^*} = \frac{m_{tg}(\bar{p}) - m_{tg}(p_{wf})}{\bar{p}_g^* - \rho_{g,wf}^*} = \frac{1}{\mu_g^* \bar{c}_g^* \rho_{g,sc}^*}
\]

**Equation 4-20**

which leads to the following definition of the depletion parameter

\[
\bar{\lambda}_{tg} = \frac{\mu_{gi}^* c_{gi}^*}{\mu_g^* \bar{c}_g^*} = \frac{\mu_{gi}^* c_{gi}^*}{(\bar{p}_g^* - \rho_{g,wf}^*)/\rho_{g,sc}^*}
\]

**Equation 4-21**

A similar procedure can be used to arrive at the depletion parameter used for the surface oil component and total hydrocarbons produced at surface. Their final forms are as given below by **Equation 4-22** and **Equation 4-23** respectively.

**Depletion parameter for surface oil:**

\[
\bar{\lambda}_{to} = \frac{\mu_{oi}^* c_{oi}^*}{\mu_o^* \bar{c}_o^*} = \frac{\mu_{oi}^* c_{oi}^*}{(\bar{p}_o^* - \rho_{o,wf}^*)/\rho_{o,sc}^*}
\]

**Equation 4-22**

**Depletion parameter for total hydrocarbons produced at the surface:**
\[
\bar{\lambda}_{tt} = \frac{\mu_t^* c_t^*}{\bar{\mu}_t^* \bar{c}_t^*} = \frac{\mu_t^* c_t^*}{(\bar{\rho}_t^* - \rho_{w,f}^*)/\rho_{tsc}^*}
\]

Equation 4-23

4.4 Results

Results for the aforementioned case studies implementing the rescaled exponential models for gas, oil and total equivalent gas flow rate are shown below for the lean, intermediate and rich gas cases respectively. Here, we use the developed model to generate production data for flow rates of gas and gas equivalents of total hydrocarbons, which is then used to calculate oil flow rates and hence to determine gas-oil ratio. This follows from the previous discussion of the material balance equation, which is unable to predict the correct average reservoir pressure using the oil molar density due to the difference in oil saturation observed in a CVD test as compared with observations during production or a field scale simulation.

4.4.1 Case 1: Lean Gas Reservoir

This case was simulated for 20 years, using a time-step of 1 day with reservoir average pressures being nearly equal to the well-bore pressure towards the end of the simulation, i.e., ~1000 psia. \(G_i\) and \(N_i\) for this reservoir are reported to be 773.61 BCF and 36.17 MMSTB respectively. Figure 4-5 shows a comparison of results obtained from the proposed rescaled exponential model, compared to those reported by the numerical simulator and the widely used well-deliverability equation.
Figure 4-5: Comparisons of (a) gas flow rate, (b) total hydrocarbon flow rate, (c) calculated oil flow rate, and (d) calculated gas-oil-ratio between the developed analytical model and numerical results for the lean gas case (Case 1).

Results from the proposed model are in agreement with values reported by the numerical simulator for the entire producing life of the reservoir. This shows the stability of the model even at low hydrocarbon flow rates, where many simulators run into instabilities caused by error accumulation.

4.4.2 Case 2: Intermediate Gas Reservoir

This case was simulated for 11 years with a constant pressure of 1000 psia maintained at the wellbore. $G_t$ and $N_t$ for this reservoir are reported at 1.92 BCF and 319 MSTB respectively. Results obtained for this case study on the intermediate gas reservoir are shown in Figure 4-6, where
results obtained from the proposed model are compared against those from the numerical simulator and well-deliverability equation.

Figure 4-6: Comparisons of (a) gas flow rate, (b) total hydrocarbon flow rate, (c) calculated oil flow rate, and (d) calculated gas-oil-ratio between the developed analytical model and numerical results for the intermediate-gas case (Case 2).

As seen for the lean gas case, results are in excellent agreement for the reservoir under study. Simulation for this case ends with an average reservoir pressure of 1000 psia, indicating production for the entire life of the well, at the specified bottom-hole pressure.

4.4.3 Case 3: Rich Gas Reservoir

Simulation time for this rich gas reservoir was nearly 10 years, with a time step of 1 day, constant well-bore pressure at 1000 psia attaining an average reservoir pressure of 1126 psia by the end of the simulation. $G_i$ and $N_i$ for this reservoir are reported to be 547.81 BCF and 67.12 MMSTB
respectively. **Figure 4-7** compares results from the rescaled exponential model with those of the numerical simulator and the well-deliverability equation, as was done for the previous case.

Results are in good agreement for about 7 years, beyond which, oil flow rates are lower than those reported by the numerical simulator, resulting in an increasing trend of gas-oil ratio. This mismatch is primarily due to the calculation of equivalent molar density at the well-bore ($\rho_{gwf}^*, \rho_{owf}^*$ or $\rho_{twf}^*$) which is done using CVD gas and condensate saturations. As is seen in the case of the oil material balance, this mis-match can be attributed to the fact that CVD data is not completely representative of the conditions in the reservoir and thus, for a case as rich as the one presented, this approximation may not hold good throughout the producing life of the reservoir.

![Figure 4-7: Comparisons of (a) gas flow rate, (b) total hydrocarbon flow rate, (c) calculated oil flow rate, and (d) calculated gas-oil-ratio between the developed analytical model and numerical results for the liquid-rich gas case (Case 3).](image)
At late time, most of the oil that can be produced along with gas is seen at the surface, with comparatively less oil remaining in the reservoir. However, a CVD experiment does not show any oil production, having a large saturation of residual oil in the reservoir at late time. These two scenarios do not represent the same phenomena, and we see this trend during the simulation.

However, since CVD is one way of estimating reservoir oil saturations apriori, this methodology has been adapted here. Estimates of equivalent molar densities are, at best, an approximation which may not hold good especially at late time, and as the gas gets richer in the condensate content it holds.

Overall, for the three cases presented above, the model is in good agreement with data reported by the numerical simulator under boundary-dominated flow and constant bottom-hole pressure specification. The next section extends this model to incorporate a more realistic scenario – that of variable bottom-hole pressure conditions, which is often seen during production.
5 Extension to Variable Bottom-hole Pressure Condition

The rescaled exponential model established above can also be extended to a variable bottom-hole pressure specification and thus, help study a real field case scenario where well bore pressures are rarely seen to be constant with time. Bottom-hole pressure variation with time can be seen as a superposition of numerous constant bottom-hole pressures specified for very short time durations during production, such that, when observed from day zero, a variable profile for bottom-hole pressure with time is obtained. This is illustrated with an example below.

![Figure 5-1: A cartoon illustration of the implementation of variable BHP in the model](image)

‘Pwf_1’ is the well bottom-hole pressure applied at a time interval of 200 days. On reducing this time interval to 100, 20 and finally, 1 day intervals, the bottom-hole pressure specification ‘Pwf_2’, ‘Pwf_3’ and ‘Pwf_4’ can be treated as different cases of varying bottom-hole pressure. This illustrates a change in well-bore pressure that decreases with time. A real field case may see
well-bore pressure increase or decrease with time, based on how the production rate is controlled for a particular well.

Thus, respective extensions of the gas, oil and total hydrocarbons equations to incorporate a variable bottom-hole pressure condition are outlined below:

**Surface Gas**

\[ q_{gsc} = q_{tg,i}^e \times R_{p,tg}^* \times \bar{\lambda}_{tg} \times \exp \left( -D_{tg,i}^e \int_{t_j}^{t} \bar{\lambda}_{tg} dt \right) \]

Equation 5-1

Where, \( R_{p,tg}^* = r_{p,tg}^* - \frac{p_{tg,i}^e}{q_{tg,i}^e} G_{p,j} \). In the case of a constant bottom-hole pressure specification, Equation 5-1 will reduce to Equation 4-15 with \( t_j = 0 \), \( G_{p,j} = 0 \) and \( R_{p,tg}^* = r_{p,tg}^* \). An alternate decline model for the density based approach is derived as

\[ q_{gsc} = q_{tg,i}^e \bar{\lambda}_{tg} \left( r_{p,tg}^* - \frac{D_{tg,i}^e}{q_{tg,i}^e} G_p \right) \]

Equation 5-2

Thus, Equation 5-2 can be seen as the generalized gas flow rate equation valid for any bottom-hole pressure specification.

**Surface Oil**

\[ q_{osc} = q_{to,i}^e \times R_{p,to}^* \times \bar{\lambda}_{to} \times \exp \left( -D_{to,i}^e \int_{t_j}^{t} \bar{\lambda}_{to} dt \right) \]

Equation 5-3
where $R_{p, to}^* = r_{p, to}^* - \frac{D_{to,i}^e}{q_{to,i}^e} N_{p, j}$. An alternate expression for the rescaled exponential can be written in terms of cumulative oil production,

$$q_{osc} = q_{to,i}^e \bar{\lambda}_{to} \left( r_{p, to}^* - \frac{D_{to,i}^e}{q_{to,i}^e} N_{p} \right)$$

Equation 5-4

**Total Hydrocarbons**

$$q_{t, sc} = q_{tt,i}^e \times R_{p, tt}^* \times \bar{\lambda}_{tt} \times \exp \left( -D_{tt,i}^e \int_{t_j}^{t} \bar{\lambda}_{tt} dt \right)$$

Equation 5-5

where $R_{p, tt}^* = r_{p, tt}^* - \frac{D_{tt,i}^e}{q_{tt,i}^e} G_{p_E,j}$. An alternate expression for the rescaled exponential can also be written in terms of cumulative hydrocarbon production,

$$q_{t, sc} = q_{tt,i}^e \bar{\lambda}_{tt} \left( r_{p, tt}^* - \frac{D_{tt,i}^e}{q_{tt,i}^e} G_{p_E} \right)$$

Equation 5-6

5.1 Results

Using the lean, intermediate and rich gas reservoirs outlined above under a variable bottom-hole specification, while keeping all other parameters unchanged, the following results are obtained using the rescaled exponential model.
Table 5-1: Variable bottomhole pressure schedule for the lean, intermediate and rich gas cases

<table>
<thead>
<tr>
<th>Case 1: Lean gas</th>
<th>Case 2: Intermediate gas</th>
<th>Case 3: Liquid-rich gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (Days)</td>
<td>BHP (psia)</td>
<td>Time (Days)</td>
</tr>
<tr>
<td>0 - 731</td>
<td>3000</td>
<td>0 - 365</td>
</tr>
<tr>
<td>732 - 1096</td>
<td>2000</td>
<td>366 - 730</td>
</tr>
<tr>
<td>1097 - 1461</td>
<td>1200</td>
<td>731 – 1095</td>
</tr>
<tr>
<td>1462 - 1827</td>
<td>1800</td>
<td>1096 – 1461</td>
</tr>
<tr>
<td>1828 - 3000</td>
<td>1000</td>
<td>1461 – 2191</td>
</tr>
<tr>
<td>3001 – 7300</td>
<td>1000</td>
<td>2192 – 4000</td>
</tr>
</tbody>
</table>

5.1.1 Case 1: Lean Gas Reservoir

The lean gas reservoir simulated for the constant bottom-hole pressure case was subjected to its aforementioned variable bottom-hole pressure schedule during production and simulated for 7300 days (~ 20 years). **Figure 5-2** shows results generated by the rescaled exponential model, compared to the well-deliverability equation, and the numerical simulator.
Figure 5-2: Comparisons of (a) gas flow rate, (b) total hydrocarbon flow rate, (c) calculated oil flow rate, and (d) calculated gas-oil-ratio between the developed analytical model and numerical results for the lean gas case (Case 1).

Results are in excellent agreement with each other for the entire producing life of the well, pointing to the successful extension of the model to a variable bottom-hole pressure specification at the well. The model successfully captures changes in flow rates, and the resultant gas-oil-ratio as bottom-hole pressure changes.

5.1.2 Case 2: Intermediate Gas Reservoir

The intermediate gas reservoir, simulated for 11 years, was subject to its respective variable bottom-hole pressure schedule as given in Table 5-1 with results presented in Figure 5-3.
Figure 5-3: Comparisons of (a) gas flow rate, (b) total hydrocarbon flow rate, (c) calculated oil flow rate, and (d) calculated gas-oil-ratio between the developed analytical model and numerical results for the intermediate gas case (Case 2).

A similar trend to that of the lean gas is observed for this case, with results from the proposed model being in agreement with numerical simulator and well-deliverability data. However, as we move to a richer gas, deviations in the results are observed, which is shown and explained in the next section.

5.1.3 Case 3: Rich Gas Reservoir

As done for cases 1 and 2, the rich gas reservoir was also subjected to a varying bottom-hole pressure specification, given by its respective aforementioned schedule. Figure 5-4 shows results for the gas flow rate and total equivalent gas flow rate simulated by the rescaled exponential model and compared to the well-deliverability equation as well as results from the numerical simulator.
Figure 5-4: Comparisons of (a) gas flow rate, (b) total hydrocarbon flow rate, (c) calculated oil flow rate, and (d) calculated gas-oil-ratio between the developed analytical model and numerical results for the liquid-rich gas case (Case 3).

These results show that while the rescaled exponential model is able to capture flow rate trends qualitatively for a rich gas, quantitatively it deviates from flow rate values reported by the numerical simulator. An accumulation of error with time can also be observed.

To summarize these results for the rich-gas case, Figure 4-7 shows results to be in good agreement until ~7 years under a constant bottom-hole pressure specification, beyond which, oil flow rates are significantly lower than those reported by the numerical simulator, resulting in an increasing trend of gas-oil ratio. This is primarily due to the estimation of equivalent molar density at the wellbore ($\rho_{g,\text{wf}}, \rho_{o,\text{wf}}, \rho_{t,\text{wf}}$) which is done using CVD liquid and gas saturations. For the same reasons, Figure 5-4 shows that while the rescaled exponential model is able to capture flow rate
trends qualitatively for a liquid-rich gas condensate reservoir, it quantitatively deviates from flow rate values reported by the numerical simulator under a variable bottom-hole pressure schedule, where an accumulation of error with time can also be observed. Owing to the deviations of gas and total hydrocarbon flow rates as seen in Figure 5-4 calculations of oil flow rates and gas-oil-ratio also inherit these errors and depart from their expected behavior, even qualitatively.

It is also essential to note that Fevang and Whitson’s pseudo-pressure integral approach used here possibly assumes co-existence of three flow regions described above, although not explicitly mentioned in their work. Although, \( p_i < p_{dew} \) is seldom encountered in naturally occurring gas-condensate systems, one may argue that using \( p_i < p_{dew} \) may result in the disappearance of regions 2 and 3, and thus, a potential mismatch when compared to results from the numerical simulator. This may be a plausible explanation for results observed for the rich gas reservoir, i.e., Case 3, where initial reservoir pressure is less than the fluid’s dew point pressure. A conclusive analysis of this hypothesis requires further investigation of the applicability of the pseudo-pressure integration approach. However, in this work, results calculated from the well-deliverability equation use the same approach for integration and show perfect matches with simulated data for all case studies.

This points to an important aspect of the developed analytical solution and can be explained by assumptions made during its development. As we move from a lean gas to an extremely liquid-rich gas, a quantitative match between the results from the rescaled exponential model and flow rates reported by well-deliverability and the numerical simulator progressively weakens. A CVD approximation holds good for the cases of the lean- and intermediate-gas but can only be used qualitatively when analyzing an extremely rich-gas reservoir. Since the fluid under study is an extremely rich gas with 40% CVD liquid dropout, it leads to a situation where this approximation
does not hold true for this scenario and CVD ceases to be representative of production from the reservoir. A significant amount of condensate is produced from such a rich-gas reservoir, whereas, a CVD process does not see substantial condensate amounts at the surface.

For the purpose of having prior knowledge of *equivalent* molar density \((\rho_{g, wf}^*, \rho_{o, wf}^*, \rho_{t, wf}^*)\) at the wellbore, these are derived as functions of black-oil PVT properties and CVD data, and their values correspond to pressures observed at the wellbore. This calculation introduces approximations because its exact value cannot be determined *apriori* with the tools at hand. The variation in bottom-hole pressure leads to variation in the *equivalent* molar density values, causing the approximation induced errors to accumulate over time as the number of changes to bottom-hole pressure increase. It should also be noted that using a good estimate of the *equivalent* molar densities at the wellbore can result in a quantitative match in gas and oil flow rates for a liquid-rich gas reservoir as well, similar to that observed for the lean and the intermediate gas cases. Currently, such a match would rely on using empirical values for *equivalent* molar density, and estimating these values in a rigorous manner is yet to be investigated.

To demonstrate the significance of using a representative pressure-saturation relation on the results, a pressure-saturation relation directly obtained from numerical simulation for Case 3 (see Figure 5-5) and re-calculated flowrates for the liquid-rich gas case. Figure 5-6 compares the numerical and analytical results for the same case discussed in Figure 5-4, but using the pressure-saturation relation directly obtained from numerical simulation. As expected, when the pressure-saturation data is provided by the numerical simulator (Figure 5-6), the analytical results are in close agreement with those obtained from numerical simulations. The results depicted in Figure 5-6 reveal that having a representative pressure-saturation relation is crucial in accurate prediction of flowrates, particularly for liquid-rich gas condensate reservoirs.
Figure 5-5: A comparison between the pressure-saturation relations obtained directly from numerical simulation and the CVD test for the liquid-rich gas case (Case 3).

Figure 5-6: Comparisons of (a) gas flow rate, (b) total hydrocarbon flow rate, (c) calculated oil flow rate, and (d) calculated gas-oil-ratio between the developed analytical model and numerical results using the pressure-saturation relation directly obtained from numerical simulator for the liquid-rich gas case (Case 3).
With these caveats associated with the stated assumptions, the model mimics reservoir productions for the lean and the intermediate gas cases very well when compared to a numerical simulator, whereas, a liquid-rich gas reservoir can still be qualitatively understood and assessed in order to provide insights into the dynamics of the reservoir and trends of its expected behavior. The model operates over a wide range of pressures and thus, different stages of reservoir fluid saturation, as demonstrated by the cases studied. The rigorous nature of the rescaled exponential model makes it a powerful tool to analyze reservoir behavior in a timely and efficient manner. Having known production data, it is also capable of solving the inverse problem and generating estimates of in-place hydrocarbons, the knowledge of which is paramount for understanding the extent and economics associated with any reservoir. This capability has been explored in the subsequent section.
6 Reserve Estimation

Reserves comprise of hydrocarbons that can be economically recovered from known accumulations. Thus, estimating the amount of gas and oil reserves is key to decision-making and field development for profitable production from a reservoir. A vital component in this estimation is the degree of certainty to which reserves can be accounted for. This is heavily dependent on one’s interpretation of the available geological and geophysical data at the time of estimation. It is, therefore, important to have regularly updated values of in-place hydrocarbons as more relevant data comes to surface.

The rescaled exponential model provides a method to calculate total in-place hydrocarbons by using available production data to estimate this value through a straight-line analysis method. This procedure is applied to the calculation of original gas-in-place and original total hydrocarbons-in-place, using which, original oil-in-place can be readily computed. Such a procedure has been adopted to minimize errors that may be introduced into the calculation owing to the fact that a CVD approximation to production from a gas-condensate reservoir may not be representative for direct calculations of the condensate phase throughout the reservoir’s producing life.

The sections below provide a step-by-step guide to this estimation method, followed by results obtained for the lean, intermediate and rich gas cases studied in this manuscript, which are then compared to in-place hydrocarbon values reported by the numerical simulator.

6.1 Initial Gas in Place

Equation 5-2, the density-based decline model for gas production under variable bottom-hole pressure conditions, can be re-arranged in the form of a straight line equation, of which, the reciprocal of the slope yields estimation of initial gas in place, $G_i$. 
\[ \bar{\lambda}_{tg} \frac{r_{\rho, tg}}{q_{gsc}} = \frac{1}{G_i} \bar{\lambda}_{tg} \frac{G_p}{q_{gsc}} + \frac{1}{q_{tg,i}} \]

Equation 6-1

Thus, a Cartesian plot of \( \bar{\lambda}_{tg} \frac{r_{\rho, tg}}{q_{gsc}} \) vs. \( \bar{\lambda}_{tg} \frac{G_p}{q_{gsc}} \) yields a straight line, with slope \( \frac{1}{G_i} \) and intercept \( \frac{1}{q_{tg,i}} \).

The following procedure is detailed below to be able to reach a reasonable reserve estimate for initial gas in place.

1) An initial guess, which is required to start the calculation procedure, may be obtained from the slope of the Cartesian plot of \( \bar{\lambda}_{tg} \frac{r_{\rho, tg}}{q_{gsc}} \) vs. \( \bar{\lambda}_{tg} \frac{G_p}{q_{gsc}} \), where \( G_i = \frac{1}{\text{slope}} \).

2) This first estimate can be used to go through the following calculation of average pressures and flow rates

   a. Calculate average pressure from the material balance equation, **Equation 4-2**

   b. Using this average pressure, the pseudo-pressure drawdown is calculated using **Equation 4-12** and the approach described there-after.

   c. The depletion parameter \( \bar{\lambda}_{tg} \) can then be calculated with the value of the pseudo-pressure integral calculated in step 2b.

3) Using this value of \( \bar{\lambda}_{tg} \), known values of flow rates and cumulative production can be rescaled and plotted on a Cartesian plot of \( \bar{\lambda}_{tg} \frac{r_{\rho, tg}}{q_{gsc}} \) vs. \( \bar{\lambda}_{tg} \frac{G_p}{q_{gsc}} \).

4) Inverse of the slope gives an updated estimate of initial gas in place \( G_i \).

5) This procedure is then iterated on, until a convergence of desirable tolerance on \( G_i \) is reached. A tolerance of 0.001% is used for this study.
6.2 Initial Total Hydrocarbons in Place

A similar procedure can be followed to calculate the total initial hydrocarbons in place. Equation 5-6, the density-based decline model for total hydrocarbon production, can be re-arranged in the form of a straight line equation, of which, the reciprocal of the slope yields estimation of total initial hydrocarbons in place, $G_{IE}$.

$$
\bar{\lambda}_{tt} \frac{r_{\rho_{tt}}^{*}}{q_{t,sc}} = \frac{1}{G_{IE}} \bar{\lambda}_{tt} \frac{G_{PE}}{q_{t,sc}} + \frac{1}{q_{tt,i}}
$$

Equation 6-2

Thus, a Cartesian plot of $\bar{\lambda}_{tt} \frac{r_{\rho_{tt}}^{*}}{q_{t,sc}}$ vs. $\bar{\lambda}_{tt} \frac{G_{PE}}{q_{t,sc}}$ yields a straight line, with slope $\frac{1}{G_{IE}}$ and intercept $\frac{1}{q_{tt,i}}$.

Here, two out of three ($G_i, N_i, G_{IE}$) are required to be able to calculate the third parameter. To obtain reasonable estimates, the recommended approach is to determine $N_i$ as a consequence of the values obtained for $G_i$ and $G_{IE}$. $N_i$ is calculated from $G_i$ and $G_{IE}$ as follows:

$$
N_i = \frac{G_{IE} - G_i}{R_{go}}
$$

Equation 6-3

where,

$$
R_{go} \left[ \frac{SCF}{STB} \right] = \frac{\rho_{ost}^{*} \left[ \frac{lbmol}{STB} \right]}{\rho_{gsc} \left[ \frac{lbmol}{SCF} \right]}
$$

Equation 6-4
And thus, this calculation is dependent on an accurate value of $\rho_{ost}^*$, which may be known or can be determined from the specific gravity of the condensate produced, at stock tank conditions, as follows.

$$\rho_{ost}^* \left[ \frac{lbmol}{STB} \right] = \frac{\rho_{ost} \left[ \frac{lb}{ft^3} \right]}{MW_{condensate}}$$

**Equation 6-5**

where, $MW_{condensate}$ is the molecular weight of condensate and is given by

$$MW_{condensate} = \frac{42.43 \gamma_{condensate}}{1.008 - \gamma_{condensate}}$$

**Equation 6-6**

with $\gamma_{condensate}$ being the known specific gravity of the condensate.

This procedure is recommended because the use of a pressure-saturation relationship from CVD data results in large deviations when estimating original oil/condensate in place directly. The estimates are in agreement with reported values when calculated from initial gas and total hydrocarbons in place, as detailed out in the results presented below. However, it is important to note that the commercial numerical simulator used in this study does not use **Equation 6-3** to calculate $N_t$, and instead uses the approach $N_t = R_{vl} \ast G_t$, where $R_{vl}$ is the volatalized condensate-gas ratio at initial reservoir pressure.

### 6.3 Results

The aforementioned procedure is applied to the lean, intermediate and rich gas cases being studied in this manuscript. In order to successfully estimate gas and oil reserves for these reservoirs, production data is assumed to be available and data reported by the numerical simulator under a
variable bottom-hole pressure specification is taken to be equivalent to known production data. On solving the inverse problem, the following results are obtained.

6.3.1 Case 1: Lean Gas Reservoir

Assuming production data is known for a period of ~20 years, values of initial gas-in-place and total hydrocarbons-in-place were estimated using the outlined procedure and compared to values reported by the numerical simulator. An initial guess on these two parameters is required in order to start the solution to the problem. This initial guess is obtained by plotting density drawdown ratio against cumulative production, both scaled by the respective flow rate, in order to obtain a straight line on a Cartesian plot. The inverse-slope of this straight line is then used as an initial guess for the model. This is true for both $G_i$ (also referred to as OGIP in this section) and $G_{IE}$ (also referred to as OGEIP in this section) values, as shown in Figure 6-1.

![Figure 6-1: Lean Gas: Initial guess for OGIP (left) and OGEIP (right) using a straight-line analysis approach](image)

Figure 6-1: Lean Gas: Initial guess for OGIP (left) and OGEIP (right) using a straight-line analysis approach
**Figure 6-2** shows the same plot, but now re-scaled by the respective depletion parameter, with the inverse-slope giving final converged values of OGIP and OGEIP, which are shown in the table below.

![Graph showing re-scaled values for OGIP and OGEIP](image)

**Figure 6-2: Lean Gas: Re-scaled values after the final iteration for OGIP (left) and OGEIP (right) calculation**

**Table 6-1** shows values of OGIP and OGEIP obtained from the model after 6 iterations under a convergence criteria set at 0.001% for errors in OGIP and OGEIP. On convergence to the below mentioned values, $N_i$ (also referred to as OOIP here) was then computed as a consequence of the values of OGIP and OGEIP using **Equation 6-3**. All three values were compared to their corresponding values reported by the numerical simulator, with errors of 0.032%, 0.138% and 0.03% for OGIP, OGEIP and OOIP respectively, also shown below.
Table 6-1: Lean Gas: OGIP, OGEIP and OOIP values compared to reported numerical simulator data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analytical Model</th>
<th>Numerical Results</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_i$ ($\times 10^{-9}$ SCF)</td>
<td>773.8</td>
<td>773.6</td>
<td>0.03</td>
</tr>
<tr>
<td>$G_{IE}$ ($\times 10^{-9}$ SCF)</td>
<td>803.9</td>
<td>805.1</td>
<td>0.14</td>
</tr>
<tr>
<td>$N_i$ ($\times 10^{-6}$ STB)</td>
<td>36.18</td>
<td>36.17</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Figure 6-3 shows error values computed for OGIP and OGEIP calculation at the end of each iteration. Convergence was obtained after 6 iterations, with a total simulation time of ~150 seconds.

Figure 6-3: Lean Gas: Errors computed during convergence

Thus, OGIP, OGEIP and OOIP values are successfully estimated for the lean gas reservoir under limits of engineering accuracy, with a maximum error of 0.14% observed in OGEIP when
compared to the reported numerical simulator values. OGIP and OGEIP, however, are in excellent agreement with their corresponding reported values.

6.3.2 Case 2: Intermediate Gas Reservoir

For the intermediate gas reservoir, known production data for its producing life of 11 years was available and this was used to estimate its gas and oil reserves using the aforementioned straight line analysis procedure. Convergence was achieved over 10 iterations for a tolerance of 0.001% for errors in OGIP and OGEIP.

![Figure 6-4: Intermediate Gas: Initial guess for OGIP (left) and OGEIP (right) using a straight-line analysis approach](image)

*Figure 6-4* and *Figure 6-5* show a Cartesian plot using original data for the first iteration, and corresponding re-scaled data for the last iteration, respectively. The values of OGIP and OGEIP are then evaluated as the inverse slope of their respective straight-lines.
Figure 6-5: Intermediate Gas: Re-scaled values after the final iteration for OGIP (left) and OGEIP (right) calculation

The estimated reserves as obtained from this procedure are tabulated in Table 6-2, showing errors of 0.005%, 0.19% and 1.7% when the values calculated by the proposed method are compared to those reported by the numerical simulator.

Table 6-2: Intermediate Gas: OGIP, OGEIP and OOIP reserve estimates as compared to reported numerical simulator data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analytical Model</th>
<th>Numerical Results</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_i$ ($\times 10^{-9}$ SCF)</td>
<td>1.921</td>
<td>1.922</td>
<td>0.01</td>
</tr>
<tr>
<td>$G_{IE}$ ($\times 10^{-9}$ SCF)</td>
<td>2.148</td>
<td>2.152</td>
<td>0.19</td>
</tr>
<tr>
<td>$N_i$ ($\times 10^{-3}$ STB)</td>
<td>314</td>
<td>319</td>
<td>1.69</td>
</tr>
</tbody>
</table>

Figure 6-6 and Figure 6-7 show the progression in errors during convergence, as well as their variation with values reported by the numerical simulator with subsequent iterations. The declining
nature of errors indicates a good rate of convergence and their approach to reported simulator values, pointing to a successful estimation of reserves for the intermediate gas reservoir.

Figure 6-6: Intermediate Gas: Errors computed during convergence

Figure 6-7: Intermediate Gas: Errors with respect to reported numerical simulator values
6.3.3 Case 3: Rich Gas Reservoir

A similar procedure was followed to determine initial gas-in-place, total hydrocarbons-in-place and original oil-in-place data for the rich gas reservoir using production data for a duration of ~7 years. An initial guess on OGIP and OGEIP for this case was also obtained using a straight line analysis approach, also shown in Figure 6-8 for the first iteration step.

Figure 6-8: Rich Gas: Initial guess for OGIP (left) and OGEIP (right) values

Figure 6-9 shows the same graph for the last iteration re-scaled with the depletion parameter for gas and total hydrocarbons respectively, giving a final estimate of OGIP and OGEIP on convergence.
It can be observed that re-scaling with the depletion parameter helps fit the data better to a straight line in a Cartesian plot when compared to the original data in Figure 6-8, but increase to 9.8% for OGEIP, and are at 0.22% for OOIP calculations.

Table 6-3 tabulates final estimated values for OGIP, OGEIP and OOIP, and compares this to their corresponding numbers reported by the numerical simulator. Errors are at 0.2% for OGIP calculation, but increase to 9.8% for OGEIP, and are at 0.22% for OOIP calculations.
Table 6-3: Rich Gas: Reserve estimates using CVD pressure-saturation relation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analytical Model</th>
<th>Numerical Results</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_i$ ($\times 10^{-9}$ SCF)</td>
<td>548.9</td>
<td>547.8</td>
<td>0.20</td>
</tr>
<tr>
<td>$G_{iE}$ ($\times 10^{-9}$ SCF)</td>
<td>654.9</td>
<td>596.4</td>
<td>9.80</td>
</tr>
<tr>
<td>$N_i$ ($\times 10^{-6}$ STB)</td>
<td>67.25</td>
<td>67.10</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Figure 6-10 shows error progression as convergence is achieved at 0.001\% for OGIP and OGEIP calculations over 10 iterations, with a simulation time of ~130 seconds.

Figure 6-10: Rich Gas: Error calculation during convergence when CVD pressure saturation relation is used.

Thus, an extremely rich gas sees a slight increase in errors computes for total hydrocarbon reserves, but provides an excellent estimate for in-place gas and condensate. Values from the proposed
model can, thus, serve as first estimates to understanding a range of liquid-rich gas reservoirs with efficient simulation times using a rigorous and robust approach. Careful analysis of results is important for any numerical simulator and for the proposed model, it is essential to do so when studying an extremely rich gas reservoir, owing to the underlying assumption of a constant volume depletion process representing field production from a gas-condensate reservoir. While this assumption holds good for lean to intermediate gas reservoirs, it may lead to increased errors, although marginal, when an extremely rich-gas reservoir is under study. Having known production data, the proposed model can provide excellent estimates of hydrocarbon reserves, shown for a wide range of liquid-rich gas systems.
7 Conclusions

This work proposes a tool for well performance forecasting and reserve estimation for gas-condensate reservoirs operating under a boundary-dominated flow regime and variable well-bore flowing pressure conditions, with the objective to have the model well-grounded to the physics of the problem and thereby reduce empiricism.

The set of equations describing surface gas, surface oil, and total hydrocarbons produced from liquid-rich reservoirs are rigorously developed and verified with finely gridded numerical simulation, demonstrating the power of rescaled exponential models to successfully capture multi-phase reservoir behavior. Non-linearities in fluid viscosity and compressibility are captured using the spatial average depletion parameter, \( \lambda_{tp} \), and its time-average, \( \tilde{\beta}_{tp} \), without the use of pseudo-time calculations. This, along with well-deliverability and material balance for surface gas and surface oil, forms the crux of the model, allowing accurate performance analysis and hence, enabling one to estimate available reserves by solving the inverse problem. Along with this, drawbacks of not having access to real-time pressure-saturation relationship in the reservoir are highlighted, influencing average pressure calculation via material balance, and subsequently, the results. Knowledge of this pressure-saturation relationship may not be available \textit{apriori}, and approximations, such as that of CVD used here, provide a good estimate of expectations from a given reservoir.

It must be noted that a CVD process is only a representation of production from the reservoir, and it is invoked here as an assumption to be able to provide a solution to the problem at hand, while staying true to the physics of the problem using a robust method to understand liquid-rich gas reservoir dynamics.
In light of the multifaceted nature of the problem, this work provides a set of rigorous analytical equations which can be used to assess the multi-phase reservoir under study and guide subsequent economic and field development decisions, without resorting to the much involved numerical models during the first stages of progress.
8 Future Work

The proposed model in its current form works very well for gas-condensate (multi-phase) reservoirs, but to a certain limit of gas richness when the model is being used to forecast production. Errors observed for rich gases, mainly due to the estimated value of equivalent wellbore flowing density from the P-S relation in use, exacerbate as the richness increases. Empirically changing this equivalent wellbore density value can result in significant changes in flow rate trends and also in perfect matches. However, to make it less arbitrary, this aspect requires in-depth investigation on certain parameters, with the equivalent wellbore density being an important one.

Thus, for further development of the rescaled exponential model, the following may be considered as additions to the model as a tool to estimate well performance and hydrocarbon reserves going forward.

a) An important aspect of the model is its ability to provide reliable estimates of the gas-condensate reservoir under study. However, due to the assumptions involved in the development of the proposed model, it becomes essential to analyze its results from a statistical point of view. This may be implemented in the form of a statistical analysis on essential parameters, such as equivalent molar density at the well-bore for gas, oil and total hydrocarbons, $\rho_{gwf}^*, \rho_{owf}^*, \rho_{twf}^*$, whose uncertainty has an impact on estimated results from the model. Such an analysis would provide the required results over a plausible range of values in the absence of real-time pressure-saturation relations, and thus, make room for interpretation over a number of scenarios ranging from the best possible case to the least probable one.
b) The model presented in this work is an extension to models analyzing well performance of dry gas reservoirs. When extended to liquid-rich gas cases, it is imperative to test the model at various levels of gas-richness, currently measured using maximum liquid dropout observed in a CVD analysis. Thus, testing the proposed models as well as its underlying building blocks over a wide range of maximum CVD liquid dropouts will help identify a range of gas-richness for which the model(s) hold valid and above which they may be used, albeit with caution.

c) Understanding the importance and implications of compositional changes as well as addition of water production from liquid-rich reservoirs is also essential. Potentially extending the model to be able to account for flow of water in the reservoir will help provide a comprehensive understanding fluid flow in gas-condensate reservoirs. Also, comparing a compositional analytical model to the current black oil one would help understand the extent to which composition may or may not impact the current model.

d) As mentioned above, further investigation is required into understanding the applicability of the pseudo-pressure integral approach. The hypothesis is that this integral may assume the co-existence of three flow regions and the absence of one or more of these regions may be a potential cause for result mis-matches observed, especially for Case 3. However, this is not an explicitly stated as limitation of the model of Fevang and Whitson (1996), and the well-deliverability equation, which uses the same approach, is able to mimic results from the numerical simulator fairly accurately.

e) It is also important to validate results of the model with experimental data under the assumption that such a study is experimentally feasible. This would add value to the model
and its results, and may also help provide a platform to develop correlations towards estimating a pressure-saturation relationship under certain experimental conditions.

f) As shown for the dry-gas case, this model can readily be extended to include effects of rock compressibility by modification of the depletion parameter. Impact of this and other effects, such as interfacial tension or real-time compositional changes, on production may be studied, either in isolation or coupled with a statistical analysis. This will also help gauge the reservoir response under dynamic conditions, helping one understand its behavior as the reservoir depletes.

Thus, the aforementioned suggestions may contribute to the rescaled exponential model as tool, and add to its capability of analyzing well-performance and estimating gas and oil reserves.
Nomenclature

**Letters**

$A$: Reservoir surface area, ft$^2$

$B_g$: Gas formation volume factor, RB/SCF

$B_o$: Oil formation volume factor, RB/STB

$B_y$: Vapour formation volume factor, RB/SCF

$B_l$: Liquid formation volume factor, RB/STB

$b_{PSS}$: Pseudo steady state parameter, -

$b_{D,PSS}$: Pseudo steady state component, -

$C_A$: Dietz’s reservoir shape factor, -

$c_g$: Gas compressibility, 1/psi

$\bar{c}_g$: Gas compressibility at average reservoir pressure, 1/psi

$c_g^*$: Equivalent gas compressibility, 1/psi

$c_{gi}^*$: Equivalent gas compressibility at initial reservoir pressure, 1/psi

$\bar{c}_g^*$: Equivalent gas compressibility at average reservoir pressure, 1/psi

$D_i^e$: Initial decline parameter for single-phase gas, 1/day

$G_i$: Initial gas-in-place, SCF

$G_p$: Cumulative gas produced, SCF

$G_{iE}$: Total equivalents of gas in place, SCF

$G_{pE}$: Total equivalents of gas produced, SCF

$h$: Thickness, ft

$k$: Absolute permeability, mD

$k_{rg}$: Relative permeability to gas, -

$k_{ro}$: Relative permeability to oil, -

$m_{tp}$: Two-phase pseudo-pressure, psi SCF/cP RB
\( m_{tg} \): Two-phase pseudo-pressure for gas component, psi SCF/cP RB

\( m_{to} \): Two-phase pseudo-pressure for oil/condensate component, psi SCF/cP RB

\( N_i \): Initial oil-in-place, STB

\( N_p \): Cumulative oil produced, STB

\( n_i \): Number of moles at initial condition, lb-mol

\( n_p \): Number of moles produced, lb-mol

\( n_g \): Number of moles of gas, lb-mol

\( n_o \): Number of moles of oil, lb-mol

\( n_t \): Number of moles of total hydrocarbons, lb-mol

\( P \): Pressure, psia

\( P_i \): Initial reservoir pressure, psia

\( \bar{P} \): Average reservoir pressure, psia

\( P_{ave} \): Average reservoir pressure, psia

\( P_{dew} \): Dew point pressure, psia

\( P_{wf} \): Wellbore flowing pressure, psia

\( P_1 \): Well-stream pressure below which condensate becomes mobile, psia

\( q_{Da} \): Dimensionless flow rate

\( q_g \): Gas flow rate at reservoir condition, RB/day

\( q_{gsc} \): Gas flow rate at standard condition, SCF/day

\( q_{gi} \): Initial decline rate for single-phase gas, SCF/day

\( q_{tg,i} \): Initial decline rate for gas component under two-phase flow, SCF/day

\( q_o \): Oil flow rate at reservoir condition, RB/day

\( q_{osc} \): Oil/condensate flow rate at standard condition, STB/day

\( q_{to,i} \): Initial decline rate for oil/condensate component under two-phase flow, STB/day

\( q_{tsc} \): Total flow rate at standard condition in equivalents of gas, SCF/day
$q_{tt,i}^e$: Initial decline rate for total hydrocarbons under two-phase flow in equivalents of gas, SCF/day

$R$: Universal gas constant, psi ft³/lb-mol R

$R_{go}$: Gas-oil equivalency factor, SCF/STB

$R_p$: Producing gas-oil ratio, SCF/STB

$R_s$: Solution gas-oil ratio, SCF/STB

$R_o$: Condensate oil-gas ratio, STB/SCF

$R_{MLG_i}$: Hagoort’s initial condensate-gas molar ratio, -

$R_{MLG_p}$: Hagoort’s produced condensate-gas molar ratio, -

$R_{\rho,tg}^*$: Modified equivalent gas density drawdown ratio for variable bottom-hole pressure specification

$R_{\rho,to}^*$: Modified equivalent oil/condensate density drawdown ratio for variable bottom-hole pressure specification

$R_{\rho,tt}^*$: Modified total hydrocarbon density drawdown ratio for variable bottom-hole pressure specification

$r$: Radius, ft

$r_e$: External radius of the reservoir, ft

$r_w$: Wellbore radius, ft

$r_p$: Single-phase gas density drawdown ratio, -

$r_{\rho,tg}^*$: Equivalent gas density drawdown ratio under two-phase flow, -

$r_{\rho,to}^*$: Equivalent oil/condensate density drawdown ratio under two-phase flow, -

$r_{\rho,tt}^*$: Equivalent total hydrocarbon density drawdown ratio under two-phase flow, -

$S_g$: Gas saturation, -

$S_o$: Oil/condensate saturation, -

$S_w$: Water saturation, -

$S_{wc}$: Connate water saturation, -

$\tilde{S}_w$: Water saturation at average reservoir pressure, -
\(T_{\text{sc}}\): Temperature at standard condition, R

\(t\): Time, Day

\(t_{DAd}\): Dimensionless time

\(t_{a,tp}\): Two-phase pseudo time, Day

\(t_{acr,tp}\): Two-phase material balance pseudo time, Day

\(t_{ca}^*\): Material balance pseudo time defined by Arabloo \textit{et al.} (2014), Day

\(V\): Volume, ft\(^3\)

\(V_b\): Bulk volume, ft\(^3\)

\(V_p\): Pore volume, ft\(^3\)

\(v\): Phase velocity for gas or oil flow, ft/s

\(x_i\): Mole fraction of component \(i\) in oil phase, -

\(y_i\): Mole fraction of component \(i\) in gas phase, -

\(Z\): Gas compressibility factor, -

\(Z_{tp}\): Two-phase gas compressibility factor, -

\textit{Greek symbols}

\(\pi\): Constant, 3.141592, -

\(\delta\): Unit conversion constant, 1 for SI units and 0.001127 for field units, -

\(\gamma\): Euler’s constant, 0.5772157749, -

\(\phi\): Porosity, -

\(\bar{\lambda}\): Depletion-driven parameter (viscosity-compressibility ratio) for single-phase gas, -

\(\bar{\lambda}_{tg}\): Gas component depletion-driven parameter under two-phase flow, -

\(\bar{\lambda}_{to}\): Oil/condensate component depletion-driven parameter under two-phase flow, -

\(\bar{\lambda}_{tt}\): Total hydrocarbons depletion-driven parameter under two-phase flow, -

\(\bar{\beta}\): Time averaged depletion parameter for single-phase gas, -

\(\bar{\beta}_{tg}\): Time averaged depletion parameter for gas component under two-phase flow, -

\(\bar{\beta}_{to}\): Time averaged depletion parameter for oil/condensate component under two-phase flow, -
\( \bar{\beta}_{tt} \): Time averaged depletion parameter for total hydrocarbons under two-phase flow.

\( \mu_g \): Gas viscosity, cP

\( \mu^*_g \): Equivalent gas viscosity, cP

\( \mu^*_{gi} \): Equivalent gas viscosity at initial reservoir pressure, cP

\( \bar{\mu}^*_g \): Equivalent gas viscosity at average reservoir pressure, cP

\( \bar{\mu}_g \): Gas viscosity at average reservoir pressure, cP

\( \mu_o \): Oil viscosity, cP

\( \mu^*_o \): Equivalent oil/condensate viscosity, cP

\( \mu^*_{oi} \): Equivalent oil/condensate viscosity at initial reservoir pressure, cP

\( \bar{\mu}^*_o \): Equivalent oil/condensate viscosity at average reservoir pressure, cP

\( \bar{\mu}_o \): Oil viscosity at average reservoir pressure, cP

\( \rho_{gsc} \): Gas density at standard condition, lbm/SCF

\( \rho^*_g \): Equivalent gas molar density, lb-mol/RB

\( \rho^*_{gi} \): Equivalent gas molar density at initial reservoir pressure, lb-mol/RB

\( \bar{\rho}^*_g \): Equivalent gas molar density at average reservoir pressure, lb-mol/RB

\( \rho^*_{g,wf} \): Equivalent gas molar density at wellbore flowing pressure, lb-mol/RB

\( \rho^*_{gsc} \): Equivalent gas molar density at standard condition, lb-mol/SCF

\( \rho_{ost} \): Oil density at stock tank condition, lbm/STB

\( \rho^*_o \): Equivalent oil/condensate molar density, lb-mol/RB

\( \rho^*_{oi} \): Equivalent oil/condensate molar density at initial reservoir pressure, lb-mol/RB

\( \bar{\rho}^*_o \): Equivalent oil/condensate molar density at average reservoir pressure, lb-mol/RB

\( \rho^*_{o,wf} \): Equivalent oil/condensate molar density at wellbore flowing pressure, lb-mol/RB

\( \rho^*_{osc} \): Equivalent gas molar density at standard condition, lb-mol/STB

\( \rho^*_t \): Total equivalent hydrocarbon molar density, lb-mol/RB

\( \rho^*_{ti} \): Total equivalent hydrocarbon molar density at initial reservoir pressure, lb-mol/RB

\( \bar{\rho}^*_t \): Total equivalent hydrocarbon molar density at average reservoir pressure, lb-mol/RB
**Subscripts**
c: Connate

E: Equivalents of gas
g: Gas
i: Initial condition
L: Liquid
o: Oil
p: Produced
sc: Standard condition
st: Stock tank
t: Total
tg: Gas component under two-phase flow
to: Oil/condensate component under two-phase flow
tp: Two-phase condition
(tt: Total hydrocarbons under two-phase flow
w: Water
wf: Wellbore flowing condition
V: Vapour

**Abbreviations**
BHP: Bottom-hole pressure
CCE: Constant composition expansion
CVD: Constant volume depletion
BDF: Boundary-dominated flow
PSS: Pseudo steady state
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Appendix

Appendix A: Material Balance Equation for Surface Oil Component

Molar conservation of surface oil component present in the reservoir gas \((n_o^V)\) and reservoir oil \((n_o^L)\) phase can be expressed as

\[ n_o = n_o^V + n_o^L = n_{o,i} - n_{o,p} \]  

Equation A-1

Where, \(n_{o,i}\) is the number of moles of surface oil component present initially and \(n_{o,p}\) is the number of moles produced. By defining an *equivalent oil molar density* as total moles of surface oil \((n_o = n_o^V + n_o^L)\) present at reservoir conditions per unit reservoir pore volume \(V_p\), we can write:

\[ \rho_o^* = \frac{n_o}{V_p} \]  

Equation A-2

For any average pressure, the ratio of the fluid density at an average reservoir pressure to fluid density at initial reservoir pressure can be written as,

\[ \frac{\bar{\rho}_o^*}{\rho_{o,i}^*} = \frac{n_o}{n_{o,i}} \]  

Equation A-3

Substituting from Equation A-1,
\[ \frac{\bar{\rho}_o}{\rho^*_o, i} = \frac{n_{o,i} - n_{o,p}}{n_{o,i}} = \left(1 - \frac{n_{o,p}}{n_{o,i}}\right) \]

**Equation A-4**

Using the following proportionality between moles and surface oil quantities:

\[ \bar{\rho}_o = \rho^*_o, i \cdot \left(1 - \frac{\rho_{osc}^{* \frac{lbmol}{STB}} \frac{N_p}{[STB]}}{\rho_{osc}^{* \frac{lbmol}{STB}} \frac{N_i}{[STB]}}\right) = \rho^*_o, i \cdot \left(1 - \frac{N_p}{N_i}\right) \]

**Equation A-5**

However, the constant of proportionality, \( \rho_{osc} \), is dependent on the oil produced and is not constant like \( \rho_{gsc}^* \). It can be obtained from CVD data using,

\[ \rho_{osc}^{* \frac{lbmol}{STB}} = R_{go}^{\frac{SCF}{STB}} \rho_{gsc}^{* \frac{lbmol}{SCF}} \]

**Equation A-6**

where, \( R_{go} \) is defined as the gas-oil equivalency factor at surface conditions. This value depends on the composition of the gas-condensate mixture and varies with the fluid under study. Thus, re-writing **Equation A-5**, the material balance equation for oil,

\[ \bar{\rho}_o = \rho^*_o, i \cdot \left(1 - \frac{N_p}{N_i}\right) \]

**Equation A-7**

In terms of standard PVT black-oil properties and saturation data an alternate expression for equivalent oil molar density as a function of pressure can be written as,
\[ \rho_o^* = \frac{n_o^V + n_o^L}{V_p} = \left( \frac{\rho_{osc}^* V_L}{B_L} + R_v \frac{\rho_{osc}^* V_V}{B_V} \right) \frac{V_p}{V_p} \]

**Equation A-8**

which, when re-arranged gives,

\[ \rho_o^* = \rho_{osc}^* \cdot \left( \frac{S_L}{B_L} + \frac{S_V}{B_V} R_v \right) = \rho_{osc}^* \cdot \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \]

**Equation A-9**

where, \( \rho_{osc}^* \) is the surface oil molar density at standard conditions, and \( V_V \) and \( V_L \) are the vapor phase and liquid phase volumes at the given reservoir condition, respectively.
Appendix B: Material Balance Equation for Total Hydrocarbons

Molar conservation of total hydrocarbons present in the reservoir can be expressed as

\[ n_t = n_{t,i} - n_{t,p} \]

\textbf{Equation B-1}

where, \( n_{t,i} \) is the total number of moles of hydrocarbons present initially and \( n_{t,p} \) is the total number of moles of hydrocarbons produced. As done for the surface oil case, an \textit{equivalent total hydrocarbon molar density} can be defined as the total moles of hydrocarbon present at reservoir conditions per unit reservoir pore volume \( V_p \), we can write:

\[ \rho_t^* = \frac{n_t}{V_p} \]

\textbf{Equation B-2}

Thus, for any average pressure, the ratio of the fluid density at an average reservoir pressure to fluid density at initial reservoir pressure can be written as,

\[ \frac{\bar{\rho}_t^*}{\rho_{t,i}^*} = \frac{n_t}{n_{t,i}} \]

\textbf{Equation B-3}

Substituting from \textbf{Equation B-1},

\[ \frac{\bar{\rho}_t^*}{\rho_{t,i}^*} = \frac{n_{t,i} - n_{t,p}}{n_{t,i}} = \left(1 - \frac{n_{t,p}}{n_{t,i}}\right) \]

\textbf{Equation B-4}
Using the following proportionality between total moles \( n_t \) and volume of total hydrocarbons produced \( G_{pE} \):

\[
\rho_t^* = \rho_{t,i}^* \left( 1 - \frac{\rho_{t,sc}^* \frac{lbmol}{SCF} G_{pE}[SCF]}{\rho_{t,sc}^* \frac{lbmol}{SCF} G_{iE}[SCF]} \right) = \rho_{t,i}^* \left( 1 - \frac{G_{pE}}{G_{iE}} \right)
\]

Equation B-5

\( G_{iE} \) is the total volume of hydrocarbons present initially and can alternatively be expressed as the sum of initial gas and initial oil in place,

\[
G_{iE} = G_i + R_{go} \cdot N_i
\]

Equation B-6

The constant of proportionality, \( \rho_{tsc} \), is calculated from \( \rho_{osc}^* \) in \( \frac{lbmol}{STB} \) and \( \rho_{gsc}^* \) in \( \frac{lbmol}{SCF} \) using the following relation with 5.615 \( \frac{SCF}{STB} \) being the constant of proportionality used for unit consistency,

\[
\rho_{tsc}^* = \rho_{gsc}^* + \frac{\rho_{osc}^*}{5.615}
\]

Equation B-7

In order to derive an expression for total equivalent hydrocarbon molar density, we write the total equivalent hydrocarbon molar density as the sum of the equivalent molar densities of surface oil and surface gas, provided all equivalent molar density values are expressed in consistent units. Here, units of \( lbmol/RB \) are used.

\[
\rho_t^* = \frac{n_o + n_g}{V_p} = \rho_o^* + \rho_g^*
\]

Equation B-8
In terms of standard PVT black-oil properties and saturation data, using the definition of gas and oil equivalent density,

\[
\rho^*_t = \rho^{*}_{osc} \cdot \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) + \rho^{*}_{gsc} \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right)
\]

Equation B-9

This gives a relationship of the total hydrocarbon molar density with pressure-dependent modified black-oil PVT properties and can be calculated using a pressure-saturation relationship from CVD data.
Appendix C: BDF Gas and Oil Rate-Density Equation for Liquid-Rich Gas Systems

This section shows a detailed derivation of the gas and oil rate BDF well-deliverability equation used in the development of the proposed rescaled exponential model. Zhang and Ayala (2016) discusses that the continuity equation for the flow of component (‘i’) in multiphase flow environment in a reservoir is

\[-\nabla \cdot \left( \rho_g v_g y_i + \rho_o v_o x_i \right) = \frac{\partial}{\partial t} \left[ \phi \left( \rho_g S_g y_i + \rho_o S_o x_i \right) \right]\]

Equation C-1

where \(\rho_g\) and \(\rho_o\) are the molar densities of reservoir gas and oil phase, \(v_g\) and \(v_o\) represent phase velocity vectors for gas and oil flow, and \(y_i\) and \(x_i\) are the molar fraction of component \(i\) in gas and oil phases, respectively. In black-oil fluid formulations, only two pseudo-components are allowed in hydrocarbon phases—namely, surface gas and surface oil. Compositional and volumetric effects in black-oil models are captured through the concepts of \(R_s\) (solution gas-to-oil ratio), \(R_v\) (volatilized oil-to-gas ratio), reservoir oil formation volume factor \((B_o)\) and reservoir gas formation volume factor \((B_g)\).

For the surface gas component, for example, Equation C-1 collapses to Equation C-2

\[-\nabla \cdot \left( \frac{v_g}{B_g} + \frac{v_o}{B_o} R_s \right) = \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right]\]

Equation C-2

whereas, for the surface oil component, Equation C-1 gives Equation C-3
\[-\nabla \cdot \left( \frac{\mathbf{v}_o}{B_o} + R_v \frac{\mathbf{v}_g}{B_g} \right) = \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right] \]

Equation C-3

Applying Darcy’s Law, and neglecting the effect of gravity, phase velocity terms \((\mathbf{v}_V \text{ and } \mathbf{v}_L)\) in the above equations can be written as,

\[
\mathbf{v}_g = -\frac{kk_{rg}}{\mu_g} \nabla p
\]

Equation C-4

\[
\mathbf{v}_o = -\frac{kk_{ro}}{\mu_o} \nabla p
\]

Equation C-5

Here, integration of Equation C-2 over reservoir volume \(V\), yields

\[- \iiint_V \nabla \cdot \left( \frac{\mathbf{v}_g}{B_g} + \frac{\mathbf{v}_o}{B_o} R_s \right) \, dV = \iiint_V \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right] \, dV \]

Equation C-6

and integration of Equation C-3 over the same reservoir volume \(V\), yields

\[- \iiint_V \nabla \cdot \left( \frac{\mathbf{v}_o}{B_o} + R_v \frac{\mathbf{v}_g}{B_g} \right) \, dV = \iiint_V \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right] \, dV \]

Equation C-7

Following Gauss divergence theorem, the volume integral of the divergence over the region inside the surface is equal to the outward flux of a vector through a close surface. Therefore, the left-hand-side of Equation C-6 yields gas production.
− ∫∫∫_V \nabla \cdot \left( \frac{v_g}{B_g} + \frac{v_o}{B_o} R_s \right) dV = − \left( ∫∫∫_V \nabla \cdot \frac{v_g}{B_g} dV + ∫∫∫_V \nabla \cdot \frac{v_o}{B_o} R_s dV \right) \\
= − \left( ∫_S \frac{1}{B_g} v_g \cdot \hat{n} dS + ∫_S \frac{R_s}{B_o} v_o \cdot \hat{n} dS \right) = − \left( \frac{q_g}{B_g} + \frac{q_o}{B_o} R_s \right) = −q_{gsc}

Equation C-8

Similarly, the left hand side of Equation C-7 yields surface oil production

− ∫∫∫_V \nabla \cdot \left( \frac{v_o}{B_o} + R_v \frac{v_g}{B_g} \right) dV = − \left( ∫∫∫_V \nabla \cdot \frac{v_o}{B_o} dV + ∫∫∫_V \nabla \cdot \frac{v_g}{B_g} R_v dV \right) \\
= − \left( ∫_S \frac{1}{B_o} v_o \cdot \hat{n} dS + ∫_S \frac{R_v}{B_g} v_g \cdot \hat{n} dS \right) = − \left( \frac{q_o}{B_o} + R_v \frac{q_g}{B_g} \right) = −q_{osc}

Equation C-9

Applying Leibniz’s rule, the right hand side of Equation C-6 can be integrated and written as

∫∫∫_V \frac{∂}{∂t} \left[ \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right] dV = \frac{d}{dt} ∫∫∫_V \left[ \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right] dV = \frac{d}{dt} \left( \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) V \right)

Equation C-10

And that of Equation C-7 can be written as

∫∫∫_V \frac{∂}{∂t} \left[ \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right] dV = \frac{d}{dt} ∫∫∫_V \left[ \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right] dV = \frac{d}{dt} \left( \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) V \right)

Equation C-11

For a system producing from a fixed drainage area, Equation C-6, using the substitutions from Equation C-8 and Equation C-10, becomes
\[-q_{gsc} = V_b \frac{d}{dt} \left[ \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right] \]

Equation C-12

and Equation C-7, when Equation C-9 and Equation C-11 are substituted, becomes

\[-q_{osc} = V_b \frac{d}{dt} \left[ \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right] \]

Equation C-13

where \( \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \) and is the volumetric-average of the surface gas component storage term that includes the surface gas component in both reservoir gas and oil phases and \( \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \) is its counterpart for the surface oil component in the reservoir oil and gas phases, respectively. \( V_b = Ah \) is the bulk volume of the reservoir.

For a radial-cylindrical system, Equation C-12 is also written as

\[
\frac{d}{dt} \left( \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right) = - \frac{q_{gsc}}{Ah} = - \frac{q_{gsc}}{\pi h (r_e^2 - r_w^2)}
\]

Equation C-14

and Equation C-13 as

\[
\frac{d}{dt} \left( \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right) = - \frac{q_{osc}}{Ah} = - \frac{q_{osc}}{\pi h (r_e^2 - r_w^2)}
\]

Equation C-15

Also, the introduction of Darcy’s law (Equation C-4 and Equation C-5) into Equation C-2 results in
\[
\n\nabla \cdot \left[ k \left( \frac{k_{rg}}{\mu_g B_g} + R_v \frac{k_{ro}}{\mu_o B_o} \right) \nabla p \right] = \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right]
\]

Equation C-16

A similar substitution into Equation C-3 gives

\[
\nabla \cdot \left[ k \left( \frac{k_{ro}}{\mu_o B_o} + R_v \frac{k_{rg}}{\mu_g B_g} \right) \nabla p \right] = \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right]
\]

Equation C-17

With the substitution of two-phase pseudo-pressure with respect to the gas component defined by Equation 4-12, Equation C-16 becomes

\[
\nabla^2 m_{tg}(p) = \frac{1}{k} \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right]
\]

Equation C-18

and the substitution of two-phase pseudo-pressure with respect to the oil component defined by Equation 4-13, Equation C-17 becomes

\[
\nabla^2 m_{to}(p) = \frac{1}{k} \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right]
\]

Equation C-19

For a homogeneous and isotropic system, integrating Equation C-18 over an arbitrary volume \(V\)

\[
\iiint_V \nabla^2 m_{tg}(p) \, dV = \frac{1}{k} \int_v \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right] \, dV = \frac{V}{k} \frac{d}{dt} \left[ \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right]
\]

Equation C-20

and similarly, integrating Equation C-19 over the same volume,
\[
\iiint_V \nabla^2 m_{t_0}(p) \, dV = \frac{1}{k} \iiint_V \frac{\partial}{\partial t} \left[ \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right] \, dV = \frac{V}{k} \frac{d}{dt} \left( \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right)
\]

Equation C-21

where, for a 1-D radial-cylindrical system, \( dV = r \, dr \, d\varphi \, dz \), \( \nabla^2 m_t = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial m_t}{\partial r} \right) \). In order to introduce the BDF no-flow boundary condition \( \frac{\partial m_t}{\partial r} = 0 \) at the outer boundary \( r = r_e \), the system of study for the integration of Equation C-20 and Equation C-21 is defined as the control volume defined within \( r = r_e \) and any arbitrary radius \( r \), for which \( V = \pi h (r^2 - r_e^2) \).

The left-hand-side of Equation C-20 is now

\[
\iiint_V \nabla^2 m_{tg}(p) \, dV = \int_0^h \int_0^{2\pi} \int_r^{r_e} r \frac{\partial}{\partial r} \left( r \frac{\partial m_{tg}}{\partial r} \right) \, dr \, d\varphi \, dz = -2\pi h r \frac{\partial m_{tg}}{\partial r}
\]

Equation C-22

and that of Equation C-21 is

\[
\iiint_V \nabla^2 m_{to}(p) \, dV = \int_0^h \int_0^{2\pi} \int_r^{r_e} r \frac{\partial}{\partial r} \left( r \frac{\partial m_{to}}{\partial r} \right) \, dr \, d\varphi \, dz = -2\pi h r \frac{\partial m_{to}}{\partial r}
\]

Equation C-23

Substitution of Equation C-22 in Equation C-20 yields,

\[
2\pi h r \frac{\partial m_{tg}}{\partial r} = \frac{V}{k} \frac{d}{dt} \left( \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right) = \frac{\pi h (r^2 - r_e^2)}{k} \frac{d}{dt} \left( \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right)
\]

Equation C-24

and substitution of Equation C-23 in Equation C-21 yields,
\[ 2\pi hr \frac{\partial m_{t_o}}{\partial r} = \frac{V}{k} d \left( \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right) = \frac{\pi h (r^2 - r_e^2)}{k} d \left( \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right) \]

**Equation C-25**

Integrating **Equation C-24** from \( r = r_w \) to an arbitrary radius \( r \):

\[
m_{tg} = m_{tg, wf} + \frac{1}{2k} \frac{d}{d t} \left[ \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right] \int_{r_w}^{r} \left( r - \frac{r_e^2}{r} \right) dr
\]

\[
= m_{tg, wf} + \frac{1}{2k} \frac{d}{d t} \left[ \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right] \left[ \frac{r^2 - r_w^2}{2} - r_e^2 \ln \left( \frac{r}{r_w} \right) \right]
\]

**Equation C-26**

And a similar integration on **Equation C-25** yields

\[
m_{to} = m_{to, wf} + \frac{1}{2k} \frac{d}{d t} \left[ \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right] \int_{r_w}^{r} \left( r - \frac{r_e^2}{r} \right) dr
\]

\[
= m_{to, wf} + \frac{1}{2k} \frac{d}{d t} \left[ \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right] \left[ \frac{r^2 - r_w^2}{2} - r_e^2 \ln \left( \frac{r}{r_w} \right) \right]
\]

**Equation C-27**

which, when applied to the outer boundary \( r = r_e \) to **Equation C-26** gives

\[
m_{tg, r_e} = m_{tg, wf} + \frac{1}{2k} \frac{d}{d t} \left[ \phi \left( \frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right) \right] \left[ \frac{r_e^2 - r_w^2}{2} - r_e^2 \ln \left( \frac{r_e}{r_w} \right) \right]
\]

**Equation C-28**

and when applied to **Equation C-27** gives
\[ m_{to,r_e} = m_{to,wf} + \frac{1}{2k} \frac{d}{dt} \left( \phi \left( \frac{S_o}{B_o} + R_f \frac{S_g}{B_g} \right) \right) \left[ \frac{r_e^2 - r_w^2}{2} - r_w^2 \ln \left( \frac{r_e}{r_w} \right) \right] \]

Equation C-29

Substitution of Equation C-14 in Equation C-28 results in the BDF rate equation for gas component in terms of outer boundary condition \((r_e \gg r_w)\):

\[ q_{gsc} = \frac{2\pi k h}{\ln \left( \frac{r_e}{r_w} \right) - \frac{1}{2}} \left[ m_{tg,r_e} - m_{tg,wf} \right] \]

Similarly, a substitution of Equation C-15 in Equation C-29 gives the BDF rate equation for the surface oil component,

\[ q_{osc} = \frac{2\pi k h}{\ln \left( \frac{r_e}{r_w} \right) - \frac{1}{2}} \left[ m_{to,r_e} - m_{to,wf} \right] \]

Alternatively, this gas rate BDF solution can be written in terms of average pseudo-pressure values \((\bar{m}_{tg} \text{ or } \bar{m}_{to})\) rather than pseudo-pressure at the outer boundary \((m_{t,r_e})\). For the surface gas and surface oil component, the definition of average pseudo-pressure in a volumetric reservoir \((r_e \gg r_w)\) can be written as

\[ \bar{m}_{tg} = \frac{1}{V} \iiint_{V} m_{tg} dV = \int_{0}^{r_e} \int_{0}^{2\pi} \int_{r_w}^{r} m_{tg} r dr d\varphi dz = \frac{2}{r_e^2} \int_{r_w}^{r} m_{tg} r dr \]

Equation C-30

and

\[ \bar{m}_{to} = \frac{1}{V} \iiint_{V} m_{to} dV = \int_{0}^{r_e} \int_{0}^{2\pi} \int_{r_w}^{r} m_{to} r dr d\varphi dz = \frac{2}{r_e^2} \int_{r_w}^{r} m_{to} r dr \]

Equation C-31
For the surface gas component, this yields,

\[
\bar{m}_{tg} = m_{tg,wf} + \frac{1}{2k} \frac{d}{dt} \left( \phi \left( \frac{S_g}{B_g} + \frac{R_s}{B_o} \right) \right) \int_{r_w}^{r} \left[ \frac{r^2 - r_w^2}{2} - r_e^2 \ln \left( \frac{r}{r_w} \right) \right] r dr
\]

\[
= m_{tg,wf} + q_{gsc} \left[ \frac{\ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4}}{2\pi kh} \right]
\]

Equation C-32

and for the surface oil component, we have,

\[
\bar{m}_{to} = m_{to,wf} + \frac{1}{2k} \frac{d}{dt} \left( \phi \left( \frac{S_o}{B_o} + R_v \frac{S_g}{B_g} \right) \right) \int_{r_w}^{r} \left[ \frac{r^2 - r_w^2}{2} - r_e^2 \ln \left( \frac{r}{r_w} \right) \right] r dr
\]

\[
= m_{to,wf} + q_{osc} \left[ \frac{\ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4}}{2\pi kh} \right]
\]

Equation C-33

which can be rearranged as the BDF solution in terms of average reservoir condition (\(\bar{m}_{tg}\)) for the gas component as

\[
q_{gsc} = \frac{2\pi kh}{\ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4}} \left[ \bar{m}_{tg} - m_{tg,wf} \right]
\]

Equation C-34

and \(\bar{m}_{to}\) for the oil component as

\[
q_{osc} = \frac{2\pi kh}{\ln \left( \frac{r_e}{r_w} \right) - \frac{3}{4}} \left[ \bar{m}_{to} - m_{to,wf} \right]
\]

Equation C-35
For a generic non-circular drainage area, the denominator of Equation C-34 and Equation C-35 is customarily rewritten in terms of the \( b_{D,PSS} \) component to give the surface gas flow rate,

\[
q_{gsc} = \frac{2\pi kh}{b_{D,PSS}}[\bar{m}_{tp} - m_{tp,\text{wf}}] = \frac{2\pi kh\delta}{b_{D,PSS}} \int_{p_{\text{wf}}}^{\bar{p}} \left( \frac{k_{rg}}{\mu_g B_g} + \frac{R_s}{k_{ro}} \frac{k_{rg}}{\mu_o B_o} \right) dp
\]

Equation C-36

and for the surface oil flow rate,

\[
q_{osc} = \frac{2\pi kh}{b_{D,PSS}}[\bar{m}_{to} - m_{to,\text{wf}}] = \frac{2\pi kh\delta}{b_{D,PSS}} \int_{p_{\text{wf}}}^{\bar{p}} \left( \frac{k_{ro}}{\mu_o B_o} + \frac{R_v}{\mu_g B_g} \right) dp
\]

Equation C-37

\( \delta \) is a conversion factor required to express gas flow rate in \( \text{SCF/Day} \) and oil flow rate in \( \text{STB/Day} \). For SI units, \( \delta = 1 \), whereas, for field units, \( \delta = 0.001127 \).

Equation C-36 and Equation C-37 are the gas rate and oil rate equations, respectively, for BDF conditions (no-flow outer boundary, constant drainage area) regardless of well production condition at the inner boundary. The only assumption held in both equations is that formation compressibility is negligible (i.e., porosity remains constant); thus they are general equations applicable for gas condensate reservoirs regardless of initial water and its mobility. The only difference between the conditions where in situ water is present or not will be the evaluation of relative permeabilities, where saturation vs. Relative permeability data will require a re-scaling, based on the amount of in-situ water present.
Appendix D: Rescaled Exponential for Gas

The current development starts with identifying the appropriate BDF well deliverability equation, which, for the surface gas component, in liquid-rich gas systems corresponds to

\[ q_{gsc} = \frac{2\pi kh\delta}{b_{D,PSS}} [m_{tg}(\bar{p}) - m_{tg}(p_{wf})] = \frac{2\pi kh\delta}{b_{D,PSS}} \int_{p_{wf}}^{\bar{p}} \left( \frac{k_{rg}}{\mu_g B_g} + \frac{R_s}{\mu_o B_o} \right) dp \]

Equation D-1

and the density-based material balance equation for liquid-gas system proposed by Zhang and Ayala (2016), written as follows:

\[ \bar{p}_g^* = \rho_{g,i}^* \left( 1 - \frac{G_p}{G_i} \right) \]

Equation D-2

Following Equation 4-2, the equivalent gas molar density \( \rho_g^* \) is a function of pressure and can be conveniently obtained from constant volume depletion test parameters on the right-hand-side. The differentiation of Equation D-2 with respect to time yields:

\[ \frac{d\bar{p}_g^*}{dt} = -\frac{\rho_{g,i}^*}{G_i} q_{gsc} = -\frac{\rho_{gsc}^*}{Ah\phi(1 - S_{wc})} q_{gsc} \]

Equation D-3

Equation D-3 enables Equation D-1 to be rewritten in terms of equivalent gas molar density and the multiphase phase counterpart to Zhang and Ayala (2014a)’s \( \lambda \) rescaling parameter, or \( \lambda_{tg} \),
where $\tilde{\lambda}_{tg}$ is the two-phase equivalent viscosity-compressibility ratio written for the surface gas component as shown:

$$
\tilde{\lambda}_{tg} = \frac{\mu^*_g}{\mu^*_g} = \frac{\mu^*_g c^*_g}{(\rho^*_g - \rho^*_{g, wf})/\rho^*_{gsc}}
$$

**Equation D-5**

Here, $\mu^*_g$ and $c^*_g$ are the *equivalent* viscosity and compressibility functions defined for the surface gas component present in the multiphase system as follows:

$$
\frac{1}{\mu^*_g} = k_{rg} \frac{1}{\mu_g B_g} + k_{ro} \frac{R_s}{\mu_o B_o}
$$

**Equation D-6**

$$
c^*_g = \frac{1}{\rho^*_g} \frac{d \rho^*_g}{dp}
$$

**Equation D-7**

Equivalent viscosity for the gas component ($\mu^*_g$) results from a weighted average of the reciprocals of oil and gas phase viscosities, and retains the same units as those of phase viscosity. Above definitions are consistent with the two-phase viscosity and compressibility functions proposed by Stewart (2011) for gas and oil systems. It should be noted, however, that the definitions in *Equation D-6* and *Equation D-7* are written for gas component only, which accounts for the total surface gas component existing in both the reservoir gas phase and oil phase. It is also interesting
to note that the use of equivalent gas properties \((\rho_g^*, \mu_g^*, c_g^*)\) given in \textbf{Equation D-2}, \textbf{Equation D-6} and \textbf{Equation D-7} allows recasting the multiphase governing continuity equation as follows:

\[
\nabla \cdot \left( \frac{1}{\mu_g^* c_g^*} \nabla \rho_g^* \right) = \frac{\phi}{k} \frac{\partial \rho_g^*}{\partial t}
\]

\textbf{Equation D-8}

which is fully analogous to the governing equation for dry gas systems used in Zhang and Ayala (2014a)’s original dry-gas rescaled exponential work. This strongly supports the premise that the derivation of an analogous rescaled-exponential function for multiphase systems is possible using \textit{equivalent gas} definitions and a similar development. As a result, \(\tilde{\lambda}_{tg}\) is termed the \textit{equivalent gas} viscosity-compressibility ratio.

Substitution of \textbf{Equation D-4} in \textbf{Equation D-3}, and subsequent integration on both sides, yields:

\[
\int_{\rho_g^* - \rho_{g,wf}^*}^{\tilde{\rho}_g} \frac{1}{\rho_{g,i}^* \tilde{\rho}_g^* - \rho_{g,wf}^*} d(\rho_g^* - \rho_{g,wf}^*) = -\frac{2\pi k h \delta}{A \phi (1 - S_{wc}) b_{D,PSS} \mu_g^* c_g^*} \int_0^t \tilde{\lambda}_{tg} dt
\]

\textbf{Equation D-9}

from where an expression for \((\tilde{\rho}_g^* - \rho_{g,wf}^*)\) as an exponential function of time is derived,

\[
\tilde{\rho}_g^* - \rho_{g,wf}^* = (\rho_{g,i}^* - \rho_{g,wf}^*) \exp(-D_{tg,i}^e \tilde{\lambda}_{tg} t)
\]

\textbf{Equation D-10}

which, upon substitution into \textbf{Equation D-4}, leads to the multiphase rescaled-exponential solution for liquid-rich gas reservoir systems,

\[
q_{gsc} = q_{tg,i}^e \times r_{p,tg} \times \lambda_{tg} \exp(-D_{tg,i}^e \tilde{\lambda}_{tg} t)
\]

\textbf{Equation D-11}
where, $\bar{\beta}_{tg}$ is the time-average of the depletion parameter $\bar{\lambda}_{tg}$ and is defined as

$$
\bar{\beta}_{tg} = \frac{1}{t} \int_{0}^{t} \bar{\lambda}_{tg} dt
$$

Equation D-12

$r_{\rho, tg}^*$ is the initial wellbore density drawdown ratio defined as

$$
r_{\rho, tg}^* = \frac{\rho_{g,i}^* - \rho_{g, wf}^*}{\rho_{g,i}^*}
$$

Equation D-13

with initial decline $D_{tg,i}^e$ and initial decline rate $q_{tg,i}^e$ parameters defined as

$$
D_{tg,i}^e = \frac{2\pi kh\delta}{Ah\phi(1 - S_{wc})b_{D, PLL_i}c_{g,i}^*}
$$

Equation D-14

$$
q_{tg,i}^e = \frac{2\pi kh\delta \rho_{g,i}^*}{b_{D, PLL_i}c_{g,i}^* \rho_{g, sc}^*}
$$

Equation D-15

The dimensionless form of Equation 18 can be written as:

$$
q_{Dd} = \bar{\lambda}_{tg} \exp(-\bar{\beta}_{tg} t_{DAd})
$$

Equation D-16

where $q_{Dd} = q_{gsc}/q_{tg,i}^e$ and $t_{DAd} = D_{tg,i}^e t$.

Equation D-11 is the rescaled-exponential model extended to liquid-rich gas wells producing under a constant bottom-hole pressure condition. Detailed step-by-step procedure for
implementation is provided in Appendix G. It is noted that the constant bottom-hole pressure restriction results from Equation D-9 where \( \rho_{g,\text{wf}}^* \) was assumed constant to carry out this integration. This analytical development not only proves the validity of the rescaled exponential for liquid-rich gas wells under constant bottomhole pressure but also showcases its analogy to the single-phase dry gas analysis. Initial decline \( D_{tp,i} \) and decline rate \( q_{tp,i} \) parameters are also directly linked to reserve estimations through their relationship with initial gas in place \( G_i \), as shown below:

\[
G_i = \frac{q_{tg,i}}{D_{tg,i}} = \frac{\rho_{g,i} Ah \phi (1 - S_{wc})}{\rho_{g,sc}}
\]

**Equation D-17**

**Extension to Variable Bottom-hole Pressure Specification**

To adapt the rescaled exponential model to a variable bottom-hole pressure specification, an identical procedure is followed as done for the case of constant bottom-hole pressure until the bottom-hole pressure specification is incorporated. This happens when Equation D-9 is integrated over a time interval. Here, this equation is integrated over any two arbitrary times \( t = t_j \) and \( t = t \), which yields,

\[
\int_{\rho_{g,j} \hat{\rho}_g - \rho_{g,\text{wf},j}}^{\rho_{g,t} \hat{\rho}_g - \rho_{g,\text{wf},j}} d \left( \hat{\rho}_g - \rho_{g,\text{wf},j}^* \right) = - \frac{2\pi kh \delta}{Ah \phi (1 - S_{wc}) b_{D,\text{PSS}} \mu_i c_{g,i}^*} \int_{\beta_{tg}^j t}^{\beta_{tg}^t t} d(\hat{\beta}_{tg} t)
\]

**Equation D-18**

from where an expression for the equivalent density drawdown over the specified time interval, \( (\hat{\rho}_g - \rho_{g,\text{wf},j}) \), as an exponential function of time is derived,
\[
\frac{\tilde{\rho}_g - \rho_{g,wf,j}^*}{\tilde{\rho}_{g,i} - \rho_{g,wf,i}^*} = \exp\left(-D_{tg,i}^e \int_{t_j}^{t} \tilde{\lambda}_{tg} dt\right) = \exp\left[-D_{tg,i}^e \left(\tilde{\rho}_{tg} t - (\tilde{\rho}_{tg} t)_j\right)\right]
\]

Equation D-19

This is based on the premise that bottom-hole pressure remains constant between the times \( t = t_j \) and \( t = t \) only, and it does not require bottom-hole pressure to remain constant during the entire production life of the reservoir. Based on the varying bottom-hole pressure schedule, the prevailing fluid density in the reservoir is given by \( \tilde{\rho}_{g,j}^* \) at \( t = t_j \). The above expression can be collapsed to the constant bottom-hole pressure scenario, in which case, the equation is reset to the start of production time. i.e., \( t_j = 0 \) and \( \tilde{\rho}_{g,j}^* = \rho_{g,i}^* \) to give,

\[
\frac{\tilde{\rho}_g^* - \rho_{g,wf}^*}{\rho_{g,i}^* - \rho_{g,wf}^*} = \exp[-D_{tg,i}^e \tilde{\rho}_{tg} t]
\]

Equation D-20

which is the variation of fluid density with time in the case where bottom-hole pressure remains constant throughout the producing life of the reservoir, as shown in Equation D-10. Equation D-20, upon substitution into Equation D-4 yields

\[
q_{gsc} = \frac{2\pi kh\delta}{b_{D,PSS} \mu_{g,i}^* c_{g,sc}^* \rho_{gsc}^*} \tilde{\lambda}_{tg} (\tilde{\rho}_{g,j}^* - \rho_{g,wf,j}^*) \exp\left(-D_{tg,i}^e \int_{t_j}^{t} \tilde{\lambda}_{tg} dt\right)
\]

Equation D-21

with initial decline \( D_{tg,i}^e \) and decline rate \( q_{tg,i}^e \) parameters defined as in Equation D-14 and Equation D-15 respectively. Equation D-21 can further be simplified to
\[ q_{gsc} = q_{tg,i} e \times (\tilde{p}_{g,j}^* - \rho_{g,wf,j}^*) \times \exp \left( -D_{tg,i}^e \int_{t_j}^t \tilde{\lambda}_{tg} dt \right) \]

Equation D-22

Further, at any time \( t = t_j \), the prevailing fluid density in the reservoir can be described by the material balance equation

\[
\tilde{p}_{g,j}^* = \rho_{g,i}^* \left( 1 - \frac{G_{p,j}}{G_i} \right)
\]

Equation D-23

where, \( G_{p,j} \) represents the cumulative surface gas production attained at time \( t = t_j \). Using this, we can write

\[
\tilde{p}_{g,j}^* - \rho_{g,wf,j}^* = \rho_{g,i}^* \left( 1 - \frac{G_{p,j}}{G_i} \right) - \rho_{g,wf,j}^* = \rho_{g,i}^* \left( r_{p,tg}^* - \frac{D_{tg,i}^e}{q_{tg,i}} \frac{G_{p,j}}{G_i} \right)
\]

Equation D-24

\( r_{p,tg}^* \) is the well-bore density drawdown ratio defined as:

\[
r_{p,tg}^* = \frac{\rho_{g,i}^* - \rho_{g,wf,j}^*}{\tilde{p}_{g,i}^*}
\]

Equation D-25

Thus, the rescaled exponential model for the case of a varying bottom-hole pressure specification is written as follows, using Equation D-22, Equation D-24 and Equation D-25

\[
q_{gsc} = q_{tg,i} e \times R_{p,tg}^* \times \tilde{\lambda}_{tg} \times \exp \left( -D_{tg,i}^e \int_{t_j}^t \tilde{\lambda}_{tg} dt \right)
\]

Equation D-26
where, \( R_{p,tg}^* = r_{p,tg}^* - \frac{p_{tg,i}^e}{q_{tg,i}^e} G_{p,j} \). In the case of a constant bottom-hole pressure specification, Equation D-26 will reduce to Equation D-11 with \( t_j = 0 \), \( G_{p,j} = 0 \) and \( R_{p,tg}^* = r_{p,tg}^* \).

An alternate expression for the rescaled exponential can be written in terms of cumulative gas production. In order to obtain the associated forecast for cumulative production during a variable rate/pressure drawdown scenario, Equation D-26 can be integrated, as follows:

\[
G_p - G_{p,j} = \int_{t_j}^{t} q_{gsc} dt = q_{tg,i}^e \times R_{p,tg}^* \int_{t_j}^{t} \exp(-D_{tg,i}^e \tilde{\beta}_{tg} t) d(\tilde{\beta}_{tg} t)
\]

\[
= \frac{q_{tg,i}^e}{D_{tg,i}^e} R_{p,tg}^* \left( 1 - \exp(-D_{tg,i}^e \int_{t_j}^{t} \tilde{\lambda}_{tg} dt) \right)
\]

\[
= \frac{q_{tg,i}^e}{D_{tg,i}^e} R_{p,tg}^* \left( 1 - \frac{q_{gsc}}{q_{tg,i}^e R_{p,tg,i}^* \tilde{\lambda}_{tg}} \right)
\]

Equation D-27

Using the above, an alternate decline model for the density based approach is derived as

\[
q_{gsc} = q_{tg,i}^e \tilde{\lambda}_{tg} \left( r_{p,tg}^* - \frac{D_{tg,i}^e}{q_{tg,i}^e} G_p \right)
\]

Equation D-28

Thus, Equation D-28 can be seen as the generalized gas flow rate equation valid for any bottom-hole pressure specification.
Appendix E: Rescaled Exponential for Oil

Starting with the well deliverability equation for oil, with the presence of connate water, we have,

\[
q_{osc} = \frac{2\pi kh \delta}{b_{D,PSS}} [m_{to}(\bar{p}) - m_{to}(p_{wf})] = \frac{2\pi kh \delta}{b_{D,PSS}} \int_{p_{wf}}^{\bar{p}} \left( \frac{k_{ro}}{\mu_o B_o} + R_v \frac{k_{rg}}{\mu_g B_g} \right) dp
\]

Equation E-1

For the constant bottom-hole pressure specification, this can be written in terms of density drawdown as,

\[
q_{osc} = \frac{2\pi kh \delta}{b_{D,PSS} \mu^*_o c^*_o \rho^*_{osc}} \tilde{\lambda}_{to} (\bar{\rho}_o - \rho^*_o - \rho^*_w f)
\]

Equation E-2

with the depletion parameter for the surface oil component defined as,

\[
\tilde{\lambda}_{to} = \frac{\mu^*_o c^*_o}{\mu^*_o e^*_o} = \frac{\mu^*_o c^*_o}{(\bar{\rho}_o - \rho^*_w f)/\rho^*_o} \left( \frac{m_{to}(\bar{p}) - m_{to}(p_{wf})}{m_{to}(\bar{p}) - m_{to}(p_{wf})} \right)
\]

Equation E-3

Here, \( \mu^*_o \) and \( c^*_o \) are the equivalent viscosity and compressibility functions defined for the surface oil component and given by the following expressions, which are analogous to their counterparts for the surface gas component.

\[
\frac{1}{\mu^*_o (\rho^*_o/\rho^*_o)} = k_{ro} \frac{1}{\mu_o B_o} + k_{rg} \frac{R_v}{\mu_g B_g}
\]

Equation E-4
\[ c_o^* = \frac{1}{\rho_o^*} \frac{d\rho_o^*}{dp} \]

Equation E-5

From the material balance equation derived above, the relationship between equivalent oil density and surface production of oil is written as,

\[ \bar{\rho}_o^* = \rho_{o,l}^* \left( 1 - \frac{N_p}{N_i} \right) \]

Equation E-6

Differentiation of the material balance equation, Equation E-6, with respect to time yields,

\[ \frac{d\rho_o^*}{dt} = \frac{\rho_{o,l}^*}{N_i} q_{osc} = - \frac{\rho_{o,l}^*}{Ah\phi(1 - S_{wc}) \left( \frac{\rho_{o,l}^*}{\rho_{osc}} \right)} q_{osc} = - \frac{\rho_{osc} q_{osc}}{Ah\phi(1 - S_{wc})} \]

Equation E-7

where, initial oil in place, \( N_i = \frac{Ah\phi(1 - S_{wc}) \rho_{o,l}}{\rho_{osc}} \). Substituting the surface oil flow rate expression from Equation E-1, i.e., the oil well-deliverability equation recast in terms of equivalent molar oil density drawdown, we get,

\[ \int_{\rho_{o,l}^*}^{\rho_o^*} \frac{1}{\rho_o^* - \rho_{o,wf}} d\left( \rho_o^* - \rho_{o,wf} \right) = - \frac{2\pi k h\delta}{Ah\phi(1 - S_{wc}) b_{D,PSM} \mu_{o,l}^* c_{o,l}^*} \int_0^t \bar{\lambda}_{to} dt \]

Equation E-8

This integration can be performed assuming a constant bottom-hole pressure specification, and can be written as,
\[
\ln \frac{\bar{\rho}_o^* - \rho_{o,wf}^*}{\rho_{o,i}^* - \rho_{o,wf}^*} = -\frac{2\pi kh\delta}{Ah\phi(1 - S_{wc})b_{D,PSS}\mu_{o,i}^* c_{o,i}^* - \bar{\rho}_{to} t}
\]

Equation E-9

which gives an exponential relation of density drawdown with respect to time,

\[
\bar{\rho}_o - \rho_{o,wf}^* = (\rho_{o,i}^* - \rho_{o,wf}^*) \exp(-D_{to,i}^e \bar{\rho}_{to} t)
\]

Equation E-10

The initial decline is expressed as

\[
D_{to,i}^e = \frac{2\pi kh\delta}{Ah\phi(1 - S_{wc})b_{D,PSS}\mu_{o,i}^* c_{o,i}^*}
\]

Equation E-11

By substituting the average density drawdown, Equation E-10, into the well deliverability equation, Equation E-1, we get the following rescaled exponential for oil flow rate in a gas condensate reservoir producing under constant bottom-hole pressure in the presence of immobile water,

\[
q_{osc} = \frac{2\pi kh\delta}{b_{D,PSS}\mu_{o,i}^* c_{o,i}^* \rho_{osc}} \bar{\lambda}_{to} (\rho_{o,i}^* - \rho_{o,wf}^*) \exp(-D_{to,i}^e \bar{\rho}_{to} t)
\]

Equation E-12

which can be simplified by grouping terms as follows,

\[
q_{osc} = \left(\frac{2\pi kh\delta \rho_{o,i}^*}{b_{D,PSS}\mu_{o,i}^* c_{o,i}^* \rho_{osc}}\right) \left(\frac{\rho_{o,i}^* - \rho_{o,wf}^*}{\rho_{o,i}^*}\right) \bar{\lambda}_{to} \exp(-D_{to,i}^e \bar{\rho}_{to} t)
\]

Equation E-13
\[ q_{osc} = q_{to,i}^e \times r_{\rho, to}^* \times \tilde{\lambda}_{to} \exp(-D_{to,i}^e \beta_{to,t}) \]

Equation E-14

with the initial decline rate defined as

\[ q_{to,i}^e = \frac{2\pi kh\delta \rho_{o,i}^*}{b_{D,PSS,\mu_o, l} c_{o,i}^* \rho_{osc}^*} \]

Equation E-15

and the initial well-bore density drawdown ratio defined as

\[ r_{\rho, to}^* = \left( \frac{\rho_{o,i}^* - \rho_{o,wf,j}^*}{\rho_{o,i}^*} \right) \]

Equation E-16

Also, initial oil in place, \( N_i \), can be obtained from the following:

\[ N_i = \frac{q_{to,i}^e}{D_{to,i}^e} \frac{\rho_{o,i}^* A h \phi (1 - S_{wc})}{\rho_{osc}^*} \]

Equation E-17

**Extension to Variable Bottom-Hole Pressure Specification**

With **Equation E-8** as the starting point, where bottom-hole pressure specification decides the course of the integral, following the development for surface gas component, between to arbitrary times \( t = t_j \) and \( t = t \) the integration is done as follows,

\[ \int_{\rho_{o,i}^*}^{\rho_{o,j}^*} \frac{1}{\rho_{o,i} - \rho_{o,wf,j}^*} d(\rho_{o,i}^* - \rho_{o,wf,j}^*) = -\frac{2\pi kh\delta}{A h \phi (1 - S_{wc}) b_{D,PSS,\mu_o, l} c_{o,i}^*} \int_{t_j}^{t} \tilde{\lambda}_{to} dt \]

Equation E-18
which gives an exponential relation of density drawdown with respect to time in the specified time interval,

\[ \bar{\rho}_o - \rho_{o,wf,j}^* = (\rho_{o,i}^* - \rho_{o,wf,j}^*) \exp \left[ -D_{t_0,i}^e \left( \bar{\rho}_{t_0} t - (\bar{\rho}_{t_0} t)_j \right) \right] \]

Equation E-19

This equation reduces to that of the constant bottom-hole pressure case when \( t_j = 0 \) and \( \bar{\rho}_{o,j}^* = \rho_{o,i}^* \). Substitution into Equation E-1 and further simplification results in the following rescaled exponential model under a variable bottom-hole pressure specification,

\[ q_{osc} = q_{t_0,i}^e \times \bar{\lambda}_{t_0} \times (\bar{\rho}_{o,j}^* - \rho_{o,wf,j}^*) \times \exp \left( -D_{t_0,i}^e \int_{t_j}^{t} \bar{\lambda}_{t_0} dt \right) \]

Equation E-20

Further, as shows for the surface gas component, at any time \( t = t_j \), the prevailing fluid density in the reservoir can be described by the material balance equation

\[ \bar{\rho}_{o,j}^* = \rho_{o,i}^* \left( 1 - \frac{N_{p,j}}{N_i} \right) \]

Equation E-21

where, \( N_{p,j} \) represents the cumulative surface oil production attained at time \( t = t_j \). Using this, we can write

\[ \bar{\rho}_{o,j}^* - \rho_{o,wf,j}^* = \rho_{o,i}^* \left( r_{\rho,t_0}^* - \frac{D_{t_0,i}^e}{q_{t_0,i}^e} N_{p,j} \right) \]

Equation E-22

\( r_{\rho,t_0}^* \) is the well-bore density drawdown ratio for surface oil defined as.
Thus, a rescaled exponential model for the case of a varying bottom-hole pressure specification is written analogous to its counterpart for surface gas,

\[ q_{osc} = q_{to,i}^e \times R_{p, to}^e \times \tilde{\lambda}_{to} \times \exp \left( -D_{to,i}^e \int_{t_j}^{t} \tilde{\lambda}_{to} \, dt \right) \]

Equation E-24

where, \( R_{p, to}^* = r_{p, to}^* - \frac{D_{to,i}^e}{q_{to,i}^e} N_{p,j} \).

An alternate expression for the rescaled exponential can be written in terms of cumulative oil production,

\[
N_p - N_{p,j} = \int_{t_j}^{t} q_{gsc} \, dt = q_{to,i}^e \times R_{p, to}^e \int_{t_j}^{t} \exp \left(-D_{to,i}^e \tilde{\beta}_{to} \, t \right) \, d(\tilde{\beta}_{to} \, t) \]

\[
= \frac{q_{to,i}^e}{D_{to,i}^e} R_{p, to}^e \left( 1 - \frac{q_{osc}}{q_{to,i}^e R_{p, to,j}^* \tilde{\lambda}_{to}} \right) \]

Equation E-25

Thus, an alternate decline model is derived as

\[ q_{osc} = q_{to,i}^e \tilde{\lambda}_{to} \left( r_{p, to}^* - \frac{D_{to,i}^e}{q_{to,i}^e} N_p \right) \]

Equation E-26
Appendix F: Rescaled Exponential for Total Hydrocarbon Flow Rate

The well-deliverability equation for total hydrocarbon flow rate can be derived using the well-deliverability equations for surface gas and surface oil flow rates, by considering the flow of surface gas and surface oil in both reservoir gas and reservoir oil phases. This results in the following well-deliverability equation for total hydrocarbon flow rate, which is written here in terms of a total pseudo-pressure drawdown,

\[ q_{t,sc} = \frac{2\pi kh\delta}{b_{D,PSS}} \left[ m_{tt}(\bar{p}) - m_{tt}(p_{wf}) \right] \]

Equation F-1

This, when re-cast in terms of density as has been done previously with surface gas and surface oil, can be written in terms of total hydrocarbon density drawdown as,

\[ q_{t,sc} = \frac{2\pi kh\delta}{b_{D,PSS} \mu_{t,i}^* c_{t,i}^* \rho_{t,sc}^*} \lambda_{tt}^* (\bar{\rho}_t^* - \rho_{t,wf}^*) \]

Equation F-2

with the depletion parameter defined as,

\[ \lambda_{tt}^* = \frac{\mu_{t,i}^* c_{t,i}^*}{\bar{\mu}_t^* \bar{c}_t^*} = \frac{\mu_{t,i}^* c_{t,i}^*}{(\bar{\rho}_t^* - \rho_{t,wf}^*)/\rho_{t,sc}^*} \frac{m_{tt}(\bar{p}) - m_{tt}(p_{wf})}{m_{tt}(\bar{p}) - m_{tt}(p_{wf})} \]

Equation F-3

The depletion parameter can be explained in a manner similar to that shown for surface gas, where it can be calculated from the slope of the pseudo-pressure vs. corresponding density drawdown.
plot. Also, $\mu^*_t$ and $c^*_t$ are the equivalent viscosity and compressibility functions defined for total hydrocarbons, if it were all in one single vapour phase, and given by the following expressions,

$$
\mu^*_t \left( \frac{\rho_{tsc}^*}{\rho_t^*} \right) = \frac{1}{\mu_g^* \left( \frac{\rho_{gsc}^*}{\rho_g^*} \right) + R_g \frac{1}{\mu_o^* \left( \frac{\rho_{osc}^*}{\rho_o^*} \right)}}
$$

Equation F-4

$$
c^*_t = \frac{1}{\rho_t^*} \frac{d\rho_t^*}{dp}
$$

Equation F-5

Continuing with the development of the rescaled exponential, differentiation of the material balance equation, Equation B-5, with respect to time yields,

$$
\frac{d\rho_t^*}{dt} = -\frac{\rho_{t,i}^*}{G_{IE}} q_{t,sc} = -\frac{\rho_{t,i}^*}{Ah\phi(1-S_{wc})\left(\frac{\rho_{t,i}^*}{\rho_{tsc}}\right)} q_{t,sc} = -\frac{\rho_{tsc}^* q_{t,sc}}{Ah\phi(1-S_{wc})}
$$

Equation F-6

where, initial hydrocarbons in place can also be expressed as,

$$
G_{IE} = \frac{Ah\phi(1-S_{wc})\rho_{t,i}}{\rho_{tsc}}
$$

Equation F-7

Substituting the surface total flow rate expression from Equation F-2, we get,

$$
\int_{\rho_{t,i}^*}^{\rho_t^*} \frac{1}{\rho_t^* - \rho_{t,wf}^*} d(\rho_t^* - \rho_{t,wf}^*) = -\frac{2\pi kh\delta}{Ah\phi(1-S_{wc}) b_{D,PSS} \mu_{t,i}^* c_{t,i}} \int_0^t \lambda_{l,t} dt
$$

Equation F-8
which gives an exponential relation of density drawdown with respect to time, showing that the exponential nature of decline is due to the nature of variation of fluid density with time and is expressed as,

$$\bar{\rho}_t^* - \rho_{t,wf}^* = (\rho_{t,i}^* - \rho_{t,wf}^*) \exp(-D_{tt,i}^e \bar{\beta}_tt t)$$

Equation F-9

The initial decline is similar to previous developments, given by,

$$D_{tt,i}^e = \frac{2\pi kh \delta}{Ah\phi(1 - S_{wc})b_{D_PSS} \mu_t^i c_{t,i}^*}$$

Equation F-10

By substituting the average density drawdown, Equation F-9, into the well deliverability equation, Equation F-2, we get the following rescaled exponential for total hydrocarbon flow rate in a gas-condensate reservoir producing under constant bottom-hole pressure,

$$q_{tsc} = q_{tt,i}^e \times r_{p,tt}^* \times \bar{\lambda}_{tt} \exp(-D_{tt,i}^e \bar{\beta}_tt t)$$

Equation F-11

With the initial decline rate defined as

$$q_{tt,i}^e = \frac{2\pi kh \delta \rho_{t,i}^*}{b_{D_PSS} \mu_{t,i}^* c_{t,i}^* \rho_{tsc}^*}$$

Equation F-12

And the initial well-bore density drawdown ratio defined as

$$\bar{\rho}_t = \rho_t$$
\[ r_{ρ,tt}^* = \left( \frac{ρ_{t,i}^* - ρ_{t,wf}^*}{ρ_{t,i}^*} \right) \]

**Equation F-13**

**Extension to Variable Bottom-Hole Pressure Specification**

Instead of integrating over the entire time duration, integration of **Equation F-8** is done between to arbitrary times \( t = t_j \) and \( t = t \) as follows,

\[
\int_{\rho_{t,j}^*}^{\rho_{t,i}^*} \frac{1}{\rho_{t,i}^* - ρ_{t,wf,j}^*} d(ρ_{t,i}^* - ρ_{t,wf,j}^*) = -\frac{2\pi khδ}{Ahφ(1 - S_{wc})b_{D,PSS}μ_{t,i}c_{t,i}^*} \int_{t_j}^{t} \bar{λ}_{tt} dt
\]

**Equation F-14**

which gives an exponential relation of density drawdown with respect to time, in the time interval specified,

\[
\bar{ρ}_{t}^* - ρ_{t,wf,j}^* = (ρ_{t,i}^* - ρ_{t,wf,j}^*) \exp\left[ -D_{tt,i}^e \left( \bar{λ}_{tt} t - (\bar{λ}_{tt} t) _j \right) \right]
\]

**Equation F-15**

Substitution into **Equation F-2** and further simplification results in the following rescaled exponential model under a variable bottom-hole pressure specification,

\[
q_{t,sc} = q_{t,tt}^e \times R_{ρ,tt}^* \times \bar{λ}_{tt} \times \exp \left( -D_{tt,i}^e \int_{t_j}^{t} \bar{λ}_{tt} dt \right)
\]

**Equation F-16**

where,

\[
R_{ρ,tt}^* = r_{ρ,tt}^* - \frac{D_{tt,i}^e}{q_{t,tt}^e} G_{pE,j}
\]

**Equation F-17**
which is obtained from the following equation, expressing the density drawdown at any time \( t = t_j \) in terms of the corresponding cumulative hydrocarbons produced,

\[
\bar{\rho}_{t,j} - \rho_{t,\text{wf},j} = \rho_{t,i}^* \left( r_{\rho,tt}^* \frac{D_{tt,i}^e}{q_{tt,i}^e} G_{PE,j} \right)
\]

\text{Equation F-18}

\( r_{\rho,tt} \) is the well-bore density drawdown ratio for total hydrocarbons, defined as,

\[
r_{\rho,tt}^* = \frac{\rho_{t,i}^* - \rho_{t,\text{wf},j}^*}{\rho_{t,i}^*}
\]

\text{Equation F-19}

An alternate expression for the rescaled exponential can also be written in terms of cumulative hydrocarbon production,

\[
q_{t,sc} = q_{t,i}^e \overline{\lambda}_{tt} \left( r_{\rho,tt}^* \frac{D_{tt,i}^e}{q_{tt,i}^e} G_{PE} \right)
\]

\text{Equation F-20}
Appendix G: Procedure for Gas Rate BDF Forecasting

This section contains a step-by-step guide to use the model. To implement the proposed rescaled exponential model to calculate rate of decline of a gas condensate well, the recommended procedure is as follows:

1. At time step 1, calculate initial pseudo-pressure drop \( m_{tg}(p_i) - m_{tg}(p_{wf}) \) following its definition given by Equation 4-12. Depletion-dependent parameters \( \bar{\lambda}_{tg} \) and \( \bar{\beta}_{tg} \) are calculated as

\[
\bar{\lambda}_{tg} <1> = \frac{\mu_{gi}^e c_{gi}^e}{(\rho_{gi}^e - \rho_{g,wf}^e)/\rho_{g,sc}^e} \frac{m_{tg}(p_i) - m_{tg}(p_{wf})}{m_{tg}(p_i) - m_{tg}(p_{wf})}
\]

Equation G-1

\[
\bar{\beta}_{tg} <1> = \frac{1}{t <1>} \times \bar{\lambda}_{tg} <1> \times t <1> = \bar{\lambda}_{tg} <1>
\]

Equation G-2

2. Gas rate is calculated using proposed rescaled exponential model

\[
q_{gsc} <1> = q_{tg,i}^e \times r_{p,tg}^* \times \bar{\lambda}_{tg} <1> \times \exp\left(-D_{tg,i}^e \bar{\beta}_{tg} <1> t\right)
\]

Equation G-3

3. Cumulative production at time step 1 is calculated by

\[
G_p <1> = q_{gsc} <1> \times t <1>
\]

Equation G-4

4. The average reservoir condition at time step 2 is obtained from material balance equation
\[ \tilde{\rho}_g^{<2>} = \rho_{g,i}^* \left( 1 - \frac{G_p^{<1>}}{G_i} \right) \]

Equation G-5

5. From previously-prepared table of P vs. \( \rho_g^* \), using available CVD parameters in Equation 4-2, average pressure \( \bar{P} \) at time step 2 is obtained by interpolation, and pseudo-pressure drop \( m_{tg} (\bar{P}) - m_{tg} (p_{wf}) \) can be calculated.

6. Depletion-dependent parameters at this next step \( \tilde{\lambda}_{tg} \) and \( \tilde{\beta}_{tg} \) are calculated as

\[ \tilde{\lambda}_{tg}^{<2>} = \frac{\mu_{gi} c_{gi}^*}{m_{tg} (\bar{P}^{<2>}) \rho^*_g} \left( \frac{\rho^*_g - \rho^*_{g, wf}}{\rho^*_{g, sc}} \right) \]

Equation G-6

\[ \tilde{\beta}_{tg}^{<2>} = \frac{1}{t^{<2>}} \times \left[ \tilde{\beta}_{tg}^{<1>} \sqrt{t^{<1>}} + 0.5 \left( \tilde{\lambda}_{tg}^{<2>} + \tilde{\lambda}_{tg}^{<1>} \right) \times (t^{<2>} - t^{<1>}) \right] \]

Equation G-7

From them \( q_{g, sc}^{<2>} \) can be calculated.

7. Continue the calculation for time step i:

\[ \tilde{\rho}_g^{<i>} = \rho_{g,i}^* \left( 1 - \frac{G_p^{<i>}}{G_i} \right) \]

Equation G-8

\[ \tilde{\lambda}_{tg}^{<i>} = \frac{\mu_{gi} c_{gi}^*}{m_{tg} (\bar{P}^{<2>}) \rho^*_g} \left( \frac{\rho^*_g - \rho^*_{g, wf}}{\rho^*_{g, sc}} \right) \]

Equation G-9

\[ \tilde{\beta}_{tg}^{<i>} = \frac{1}{t^{<i>}} \times \left[ \tilde{\beta}_{tg}^{<i>} \sqrt{t^{<i>}} + 0.5 \left( \tilde{\lambda}_{tg}^{<i>} + \tilde{\lambda}_{tg}^{<i-1>} \right) \times (t^{<i>} - t^{<i-1>}) \right] \]

Equation G-10
Two-phase pseudo-pressure calculation requires producing gas-oil-ratio vs. time as an input. An analogous procedure can be followed to calculate oil flow rate and total hydrocarbons produced using their respective rescaled exponential models.