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ABSTRACT

Since its inception in 2003, the Red Bull Air Racing World Championships has sparked interest in airplane racing. While the Edge 540 and Extra 300 were flown during first years of the competition, several pilots moved to the MXS-R as the competition progressed. The 2016 season opened with two pilots flying the MXS-R and twelve pilots flying two versions of the Edge 540. However, all of these airplanes are being used for an off-design purpose, allowing for improvements to be made. Several teams had made attempts to improve the aerodynamic efficiency of their airplane, as both of the MXS-R and two of the Edge 540 airplanes have been equipped with winglets.

As previous design work on the MXS-R has been conducted and is available for reference, this airplane serves as the base for this body of work. Several notable design changes on the MXS-R are investigated for their effect on flight performance. These alterations include: the addition of winglets to the airframe and changing the wing airfoil to an NACA 6-series laminar flow airfoil. For the former, the CG location is also examined for potential gains.

The design changes are analyzed for aerodynamic efficiency, quantified with the airplanes’ lift-to-drag ratio. Winglets contribute to a significant drag reduction over the production MXS-R, as the maximum lift-to-drag ratio increases by 7.66%. By changing to a laminar flow airfoil, further gains can be made in drag reduction, particularly at high speeds. At the entry speed of 200 knots, the NACA 63(3)-012 and NACA 64A-012 yield increases of 9.91% and 9.04% in lift-to-drag ratio, respectively. Additionally, the aft-most CG location leads to the greatest gains for the MXS-R.

An optimized flight path is found to give the quickest flight time through a course for each airplane configuration, allowing for a quantitative analysis of the designs. The Dallas - Fort Worth course from the 2014 Championship season is used for this purpose. The production MXS-R posts a time of 56.67 seconds, the slowest when compared to the altered designs. The MXS-R with winglets is slightly faster with a time of 56.25 seconds, while the NACA 63(3)-012 and NACA 64A-012 airfoils stop the
clock at 55.32 seconds and 55.44 seconds, respectively. The trajectory analysis shows that the laminar flow airfoils better allow the airplane to accelerate out of high g turns, thus recovering more airspeed than the other design changes.
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LIST OF SYMBOLS

$AR, AR_{HT}$ = aspect ratio of the wing and tail, respectively

$B$ = span

$C_{D_i}$ = induced drag coefficient

$C_{D_{HT}}$ = induced drag coefficient of the horizontal tail

$C_{D_{wing}}$ = induced drag coefficient of the wing

$C_{D_{trim}}$ = trim drag coefficient

$CG$ = center of gravity

$C_{HT}$ = horizontal tail volume coefficient

$C_L$ = three-dimensional lift coefficient

$C_{L_{HT}}$ = horizontal tail lift coefficient

$C_{L_{wing}}$ = wing lift coefficient

$C_{M_{fuselage}}$ = pitching moment coefficient of the fuselage

$C_{M_{prop}}$ = pitching moment coefficient of the propulsion system

$C_{M_{0,wing}}$ = zero-lift pitching moment coefficient of the wing

$c$ = local chord length

$c̅$ = mean aerodynamic chord

$C_{d}$ = two-dimensional profile drag coefficient

$C_{l}$ = two-dimensional lift coefficient

$D_i$ = induced drag

$D_{i_{HT}}$ = induced drag of the horizontal tail

$D_p$ = profile drag

$Δb$ = increment in span
\( e, e_{HT} \) = span efficiency value for the wing and tail, respectively

\( i \) = spanwise summation index

\( L \) = lift

\( L_t \) = lift of tail

\( L/D \) = lift-to-drag ratio

\( l_{HT} \) = length from CG to tail aerodynamic center

\( M_{CG} \) = pitching moment about the CG

\( M_{wing} \) = pitching moment of the wing

\( M_{fuselage} \) = pitching moment of the fuselage

\( M_{propulsion} \) = pitching moment of the propulsion system

\( N \) = number of elements on the wing half-span

\( \rho \) = air density

\( Re \) = Reynolds number

\( S, S_{HT} \) = area of the wing and horizontal tail, respectively

\( V \) = velocity

\( V_{stall} \) = stall speed

\( W \) = weight

\( X_{CG} \) = longitudinal CG location

\( X_{AC,wing} \) = longitudinal wing aerodynamic center location
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Chapter 1

Introduction

1.1 Motivation

The Red Bull Air Race World Championship has raised public awareness and excitement for airplane racing. The Air Races present a unique sporting platform that merges piloting skill and precision, aircraft design, and flight path planning. They have grown in popularity, with individual races often seeing hundreds of thousands of spectators. In more than a decade of the Red Bull Air Races, many changes have morphed the competition into what it is today. From altering the pylon design to standardizing the engine and propeller of the airplanes, the Air Races are ever-changing and evolving in nature, and are becoming safer for both pilots and spectators.

The airplanes flown must be of the highest performance to be competitive, and any reductions in drag are of utmost importance. It is for this reason that teams focus on the aerodynamic efficiency of their airplane. In the first few years of the Air Races, the Edge 540 and the Extra 300 were flown, but it quickly became apparent that the Edge was the superior racer and all competitors moved to this airplane. The Edge 540 is shown in Fig. 1.1.

![The Edge 540 flown in the Red Bull Air Races](image1)

Fig. 1.1 – The Edge 540 flown in the Red Bull Air Races [1].
There are several pilots in the Air Races that fly the MXS-R, which was designed to be a direct competitor to the Edge 540. The MXS-R was designed using state-of-the-art technology. The fuselage was computer-optimized for aerodynamic efficiency, and the wings of the base variant MXS were adapted to include wingtip extensions for racing. It is made entirely from carbon fiber, whereas the Edge 540’s fuselage is of a steel tube design. The MXS-R is shown in Fig. 1.2.

![Fig. 1.2 - The MXS-R flown in the Red Bull Air Races [1].](image)

Between the 2009 and 2010 racing seasons, pilot Nigel Lamb had winglets designed and installed on his MXS-R. After the three year hiatus imposed by Red Bull on the Air Races to address safety concerns, Lamb went on to win the 2014 Championship. Lamb had an aerodynamic advantage in that the winglets reduced the induced drag of his airplane. In the following year, the other MXS-R pilot, Matt Hall, began racing with another winglet design. Although the geometry of the winglet designs is different, both have the potential to reduce the induced drag of the respective airplanes. The two winglet designs are shown in Fig. 1.3.
The winglets on Lamb’s MXS-R were designed by Dr. Mark Maughmer, professor of Aerospace Engineering at The Pennsylvania State University and advisor to this thesis. In designing the winglets, questions regarding the performance of the MXS-R arose, including: how the center-of-gravity location affects performance and what further gains in top speed could be achieved by taking additional drag reduction measures such as using a laminar flow airfoil on the wing.

The Red Bull Air Races limit the airplanes to a minimum flight weight of 698 kilograms [4]. Most teams leave a little room for error, and manage to keep the weight of the airplane at 700 kilograms. Some of that weight comes in the form of tail ballast, which is used to meet the minimum weight requirement and to move the center-of-gravity aft. However, the winglets on Lamb’s MXS-R weigh five kilograms, which limits the amount of tail ballast that the airplane can carry while remaining at the 700 kilogram weight. Thus, the furthest aft center-of-gravity that Lamb’s MXS-R can achieve is limited. This causes extra drag in the form of trim drag, which works against the benefit of the induced drag reduction from the winglets. It is of interest to determine the optimal condition for this trade-off, as the trim drag has the potential to offset the gain from the winglets.

Additionally, it is well known that laminar flow airfoils can achieve lower drag coefficients than their non-laminar flow counterparts over some range of lift coefficients. When properly designed, an airplane using laminar flow airfoils on the wing should have less profile drag within a given design.
operating range. However, the nature of the Red Bull Air Races causes the airplanes to fly to the bounds of the flight envelope such that the optimum operating range of a laminar flow airfoil might not be wide enough. Thus, while a laminar flow airfoil may have lower drag coefficients over some range of lift coefficients, it is not entirely clear that it will result in better racing performance.

The parameter to minimize with any design change on the MSX-R is the lap time for a Red Bull Air Race course. For example, while winglet designs work to decrease induced drag, they are accompanied by a trade-off of increasing profile and trim drag. The winglets may be beneficial at low speeds, but may become a hindrance to top speed. Similarly, a laminar flow airfoil may contribute less drag in some phases of flight when compared an aerobatic airfoil, but could contribute more drag when the airplane is operating outside of the drag bucket. Thus, such designs must be assessed for their potential improvement on a particular race course. The research presented herein provides a preliminary investigation of the aerodynamic performance of such designs, as well as a trajectory analysis of flights for these design.

1.2 Research Objective

The goal of the presented research herein is to investigate the effect of center-of-gravity location, the addition of winglets, and the use of a laminar flow airfoil on the performance of the MXS-R in the Red Bull Air Races. This investigation incorporates applied aerodynamics, aircraft design theory, and aircraft simulation to explore configurations of the MXS-R. The research makes use of existing tools when possible, as well as developing tools specifically for this purpose.

The objective is to improve the lap times of the MXS-R in a Red Bull Air Race venue. First, the study examines the potential aerodynamic benefits of adding winglets to the airplane and compares this to the stock version. Next, the wing airfoil of the MXS-R is changed to a laminar flow airfoil and the result is compared to the airplane flown by Lamb with the production airfoil. To keep the weight and structural
properties constant, the laminar flow airfoil is restricted to have the same thickness ratio as the production airfoil. Finally, the potential lap times of the various MXS-R configurations are found using a simulation tool developed specifically for this study.
Chapter 2

Background Review

Air Racing is a sport in which pilots compete in airplanes, generally over a fixed course. The winner of the competition is the pilot who returns the shortest time, the most points, or finishes with the time closest to an estimated time. Competitions vary with distance, number of pilots, and class of airplane flown, and have greatly changed since the early days of flight.

2.1 Air Racing History

2.1.1 Early Days of Air Racing

The first airplane race was held on May 23, 1909, at the Port Aviation, south of Paris, France. At this time, airplane technology was in its infancy; the Wright brothers had made the first controlled flight less than a decade prior, and many designers were still struggling to get their craft into the air. The task of this race was to fly ten laps of a 1.2 kilometer course – a total of just under seven and a half miles. Four pilots entered, yet none were able to complete the entirety of the race. Léon Delagrange was able to fly just over half of the course, and was declared the winner by virtue of flying the furthest distance. Delagrange’s Voisin biplane is shown in Fig. 2.1.
The first major international event to be held was the *Grand Semaine d’Aviation de la Champagne* in Reims, France, in August 1909. The event featured many pioneers of aviation competing in a week full of events. The premier event was the Gordon Bennett Trophy competition, a time trial competition where the pilots flew two laps of a 10 kilometer course. In this event, pilots took to the air one-at-a-time and recorded an individual time. Glenn Curtiss, the founder of Curtiss Aeroplane and Motor Company, won the Gordon Bennett Trophy as well as another prize for the fastest three laps about the course. Louis Blériot, the first to fly across the English Channel, won the prize for the fastest single lap with an average time of 47.8 miles per hour [6]. Fig. 2.2 shows Curtiss in his No. 2 biplane preparing for flight.

Fig. 2.1 – Delagrange’s Voisin biplane that was flown to victory at the first air race [5].
2.1.2 Days of Rapid Aircraft Improvement

In the first half of the 20th century, air racing provided a means to test and advance aircraft designs. From 1906 to 1934, the world record for top speed increased from 25.65 mph to 440.5 mph [8, 9].

Following Curtiss’s win of the 1909 Gordon Bennett Trophy, the event became an annual competition. Each country was able to field a team of three pilots, and the previous winning country would host the following year’s competition. In such a setting, advancement in aircraft technology is easily observed in the average speed of the winning airplanes. From 1909 to 1913, the average winning speed increased from 46.77 mph to 124.8 mph. The competition returned after the First World War, with the winner posting an average speed of 168.73 mph [10]. The winning airplane was a former WWI French fighter, the Nieuport 29, shown in Fig. 2.3.
Similarly, the Schneider Trophy was awarded annually to the winner of a race of seaplanes and is a prime example of the rapid improvement of racing airplanes. Like the Gordon Bennett Trophy, it was a time trial event in which pilots flew one-at-a-time for the fastest time.

In 1913, the first winner of the Schneider Trophy flew a SPAD Deperdussin Monocoque with an average speed of 45.71 mph; the following year, a Sopwith Tabloid cruised to victory with an average speed of 86.83 mph. From 1920 to 1931, the average speed of the winning airplane increased from 107.2 mph to 340.08 mph. The Supermarine S.6B seaplane that won the 1931 competition went on to set the world speed record just days later with a speed of 407.5 mph [12].

Many of the airplanes that were successful in competing for the Schneider Trophy were further developed into speed record holders and World War II fighters. In 1934, the Macchi M.C.72, an Italian seaplane designed to compete for the Trophy, broke the world record with a speed of 440.5 mph [12]. Shown in Fig. 2.4, the Supermarine S.6B design was heavily incorporated into the Supermarine Spitfire, Britain’s iconic and successful fighter of WWII.

Fig. 2.3 – Nieuport 29, flown to victory for the 1920 Gordon Bennett Trophy [11].
Fig. 2.4 – The Supermarine S.6B, S1596 - winner of the 1931 Schneider Trophy \cite{12}.

To highlight the advances in airplane design during this time, Fig. 2.5 shows the average speed of the winning airplanes for each year that the Gordon Bennett Trophy and Schneider Trophy were held.

Fig. 2.5 – Average speed of the winners of the Gordon Bennett Trophy and the Schneider Trophy.
These races show that with improvements in design and technology, performance gains can be made. In 1909, the Gordon Bennett Trophy was won with a biplane that had the characteristic wire bracing of the time. It is now known that wires contribute an enormous amount of drag to an airplane, having a drag coefficient on the order of 10 times that of an airfoil with the same thickness [13]. Furthermore, the biplane design became less favored as researchers made progress on airfoil design; once it was discovered that a thicker airfoil capable of carrying an internal spar could operate with low drag coefficients, monoplanes became increasing popular.

It can also be observed that the fuselages of the airplanes become increasingly ‘clean’, meaning that the lines of the body were streamlined to reduce drag. By the late 1920’s to 1930’s, extra cross sectional area and sharp edges were largely eliminated. This can be seen in the S.6B, as every surface was made with the purpose of reducing drag.

The S.6B also showcases the development of the wing planform. Rather than using a rectangular planform, the S.6B rounds the tips of the wing to reduce induced drag. This was further developed into the iconic elliptical wing planform of the Spitfire.

2.1.3 Red Bull Air Race World Championships

The Red Bull Air Race World Championships was established in 2003 as an international competition. Elite pilots race approximately eight courses in a season, earning points at each competition based on their finish positions. Each race course consists of a series of pylons, which the pilots must navigate through. Pilots fly one-at-a-time, and are scored by the time it takes to traverse the course. The top eight pilots at each race score points, and the pilot with the most points at the end of the season wins the championship [14].
Penalties are applied for various infractions that pilots make throughout the course. Penalties will either add time, lead to a did-not-finish result, or automatically disqualify the pilot if severe. A full list of penalties is shown in Table 2.1.

Table 2.1 - List of Penalties [15].

<table>
<thead>
<tr>
<th>Penalty Level</th>
<th>Penalties</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Second Penalty</td>
<td>• Insufficient or no smoke</td>
</tr>
</tbody>
</table>
| 2 Second Penalty | • Flying too high, through or over an Air Gate  
                  | • Incorrect bank angle when crossing through an Air Gate |
| 3 Second Penalty | • First time hitting an Air Gate  
                  | • Second time hitting an Air Gate |
| Did Not Finish | • Deviating from the course  
                 | • Exceeding 200 knots through the Start Gate  
                 | • Airplane weighs less than 698 pounds after the race  
                 | • Exceeding the maximum load factor of 10g’s  
                 | • Third time hitting an Air Gate |
| Disqualification | • Uncontrolled movements or flight  
                   | • Close to ground pull-up from descent  
                   | • Crossing the safety line  
                   | • Pulling a negative g turn around and Air Gate  
                   | • Flying below 15 meters between Air Gates  
                   | • Flying into clouds  
                   | • Flying through the start gate at an angle greater than 45 degrees  
                   | • Ignoring Race Director’s commands |

The 2010 season also saw the introduction of winglets to the competition, with pilot Nigel Lamb adding winglets to his MXS-R. Following the three-year hiatus, Lamb continued using winglets in 2014 and went on to win the Championship. Another pilot, Matt Hall, fitted winglets to his MXS-R for the 2015 season and finished second in the Championship. For the 2016 Championship, 4 pilots in total were using winglets on their airplanes.
The 2014 season marked the initiation of standardized engines and propellers for the Air Races. Each airplane is required to use a Lycoming engine that produces 300 horsepower and a Hartzell 3-blade propeller [16, 17]. This rule change effectively leveled the power output that the teams can expect; improvements to the airplanes must therefore come in the form of advances in aerodynamic performance.

The Red Bull Air Races push the airplanes to their limits; the races require precise and rapid flight changes that stress the airframes up to the allowable 10g’s and occur within the blink of an eye. Courses typically are about six kilometers in total length, with pilots flying two laps over the course. The start gate must be entered at less than 200 knots, after which the pilots are free to open the throttle as they maneuver throughout the course. Each venue is unique and requires different maneuvers to fly between and around pylons. Sets of double pylons require that the airplane be in level flight when passing in-between, while single pylons are set in a chicane pattern that requires tight knife-edged turns. Additionally, courses feature a vertical or near-vertical maneuver, where the pilots must fly a Half-Cuban Eight, or variant thereof.

Due to the aerobatic-based nature of the competitions, the pilots fly high-performance airplanes that are typically used in aerobatic competitions, such as MX Aircraft’s MXS-R and Zivko Aeronautics’ Edge 540. They have high power-to-weight ratios, high roll and pitch rates, and can handle load factors in excess of 10g’s. While there is an aerobatic component to the Red Bull Air Races, these aircraft are not designed or intended to be used as ‘racing aircraft’, and speed is not the driving design requirement. Though these airplanes are capable of the highest aerobatic performance for any production airplane on the market today, there are design changes that could yield significant gains in speed. Such changes are explored herein for the MXS-R.
2.2 Airplane Design

The design of the airplanes used in the Red Bull Air Races has been typically aimed at improving aerobatic performance. Many manufactures have focused on the ability of their airplane to execute precision maneuvers, with top speed being an afterthought. Flight characteristics such as roll rate, climb rate, and stall speed are emphasized. The Red Bull Air Races, on the other hand, require the airplanes to have a high top speed, high roll rate and high pitch rate.

To take an airplane that is intended for aerobatic competitions and fly it in the Red Bull Air Races is to take the airplane out of its design space. The mission that is the Red Bull Air Races requires the airplanes to fly through multiple air gates at precise bank angles, to perform a vertical maneuver while pulling less than 10 g’s, and to navigate a chicane, or slalom, pattern with rapid knife-edge turns – all while minimizing the time on the course. It is a mission that requires a fast airplane that is highly maneuverable.

The Edge 540 and the MXS-R are currently the only airplanes flown in the Master Class of the Red Bull Air Races. The Edge 540 was designed in the early 1990’s as one of the first aerobatic airplanes to be constructed with composite materials. The primary considerations for its design were a high roll rate, low stall speed, and a high-strength structure. The original composite wing produced by Zivko Aeronautics Inc. was tested to 20 g’s without failure, and the subsequent airframe was constructed with a steel tube fuselage for rigidity, strength, and ease of repair [18]. Refinements over the later versions, the Edge 540 V2 and V3, improved the aerodynamic characteristics of the airplane, making it more akin to racing and the requirements of the Red Bull Air Races.

The MXS was designed to compete with the Edge 540 in Unlimited Class aerobatic competitions, and its’ racing variant, the MXS-R, built upon the design with race-inspired specifications. For example, the MXS-R further tilts the pilot’s seat back to 45 degrees creating a lower canopy profile and smoother fuselage lines. The MXS-R also has factory installed wingtips that extend the span and mitigate the
aerodynamic losses of the original tips, reducing induced drag [19]. Both pilots flying the MXS-R have further refined the wings of their airplanes to include winglets.

Relevant baseline parameters for the Edge 540 and the MXS-R are listed in Table 2.2.

Table 2.2 – Baseline Aircraft [20, 21].

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Edge 540</th>
<th>MXS-R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, ft</td>
<td>20.7</td>
<td>21.4</td>
</tr>
<tr>
<td>Wingspan, ft</td>
<td>24.3</td>
<td>24.0</td>
</tr>
<tr>
<td>Wing Area, ft²</td>
<td>98</td>
<td>102.0</td>
</tr>
<tr>
<td>Empty Weight, lbs, kg</td>
<td>1155, 524</td>
<td>1260, 572</td>
</tr>
<tr>
<td>Max Weight, lbs, kg</td>
<td>1800, 816</td>
<td>1840, 835</td>
</tr>
<tr>
<td>Aerobatic Weight, lbs, kg</td>
<td>1550, 703</td>
<td>1600, 726</td>
</tr>
<tr>
<td>Ultimate Load Factor</td>
<td>+/- 12</td>
<td>+/- 14</td>
</tr>
<tr>
<td>Engine, HP</td>
<td>300</td>
<td>300</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Performance</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Speed, knots</td>
<td>230</td>
<td>230</td>
</tr>
<tr>
<td>Stall Speed, knots</td>
<td>58</td>
<td>58</td>
</tr>
<tr>
<td>Roll Rate, deg/sec</td>
<td>420</td>
<td>420</td>
</tr>
<tr>
<td>Max Climb Rate, ft/min</td>
<td>3700</td>
<td>3500</td>
</tr>
</tbody>
</table>

2.2.1 Winglet Design

A winglet’s purpose is to improve performance of the airplane through the reduction of drag. Typically, this involves a trade-off between a reduction in induced drag and an increase in profile drag.
Profile drag has two contributions: skin friction and pressure drag about a lifting surface. Skin friction results from the viscosity of the air moving along an aerodynamic surface, while pressure drag results from the nonzero integration of static pressure about the surface. It is a function of the amount of the surface exposed to the air (wetted area), shape of the surface, angle of attack, air density, and square of the velocity [22].

Induced drag is a consequence of producing lift. Present for a finite wing, it is the product of spanwise airflow affecting the local angle of attack along the wing. A wing that produces lift has lower pressure on the top surface than the lower surface, resulting in a pressure difference. To relieve this difference, high pressure air moves toward low pressure by flowing around the tips of the wing. This effect is minimal near the root of the wing, but is increasingly present moving outward toward the tips, as shown in Fig. 2.6.

Fig. 2.6 – Spanwise flow on a finite wing – solid lines, upper surface; dashed lines, lower surface.
As can be observed in Fig. 2.6, the streamlines begin to converge within a few chord lengths downstream and ultimately roll up into two tip vortices. This can be visually represented via an idealized horseshoe vortex system, in which each tip vortex carries the same circulation as the wing, as shown in Fig. 2.7.

The horseshoe vortices – and more generally the shed vorticity – affect the flowfield everywhere. As the lift varies along the span of the wing, the local circulation varies accordingly and leads to downwash. When the downwash is superimposed with the freestream velocity, a resulting induced angle of attack is present. This leads to a vector component from the resultant aerodynamic force that is parallel to the freestream, which is the induced drag. Naturally, less downwash yields less induced drag, which can be minimized but not eliminated.

It is well known that non-planar tip devices have the ability to reduce the spanwise flow that leads to higher induced drag. Early attempts to include such devices mainly employed end plates that attempted to halt the flow and prevent it from moving around the tip; however, they came with an increase in parasitic drag that offset any gains in induced drag [24]. This is where a winglet comes into play - by
carrying an aerodynamic load, a winglet can alleviate spanwise flow by imposing a velocity field that opposes that of the main wing \[22\]. This effect is shown in Fig. 2.8.

![Fig. 2.8 Experimentally determined flowfield crossflow velocity vectors behind model with and without winglets [25].](image)

It can be observed in Fig. 2.8 that the spanwise flow has been greatly reduced in the presence of the winglet. Effectively, the winglet is diffusing the influence of the tip vortex, reducing the downwash and hence the induced drag. The winglet is doing so by carrying an aerodynamic load, which allows it to have the same effectiveness as an endplate but with less wetted area and less profile drag. When properly designed, a winglet can yield a net decrease in overall drag and thus an increase in lift-to-drag ratio. An example of this improvement can be seen in Fig. 2.9 for the Discus 2 sailplane with winglets designed by Maughmer [26].
While the increase in maximum lift-to-drag ratio is only 2 – 3%, it is significant because it comes with no drag penalty at higher speeds.

Work done by Kody, Bramesfeld, and Schmitz [27] shows that winglets may be designed for both high speed cruise and for low speed thermalling of a sailplane. Via a multi-objective optimization, they show that gains can be made at both ends of the speed range with a well-designed winglet.

2.3 Trajectory Optimization

The goal of a time-optimal trajectory problem is to minimize the time required to fly a specified course while satisfying all operational constraints. Specifically, it requires the flight path to be integrated through a governing set of state equations and altered until the fastest time is found. Such simulations
require either a point-mass model to approximate the dynamics of the airplane, or a six degree-of-freedom (6-DOF) model to solve the rotational and translational dynamics.

Methods for solving trajectory optimization problems can generally be classified as either indirect or direct methods. Indirect methods introduce adjoint variables to augment the governing system of equations, while direct methods use control variables as parameters for the optimization. Of the two, the direct method requires less computation time and is able to use the parameters directly to satisfy constraint equations.

Moon and Kim [28] study the problem of an aircraft flying through fixed waypoints with the direct method of optimization. In their study, Euler angles are used in the simulation with angle of attack and bank angle being controlled through time. They introduce auxiliary variables to deal with the unknown times that the airplane passes through each waypoint, allowing the optimizer to find these times. They show that this method is viable, as the path generated in their study converges to the waypoints, shown sequentially in Fig. 2.10.

![Fig. 2.10 – Convergence sequence of aircraft passing through waypoints [28].](image)
Visser and van der Plas [29] compare several optimization strategies for an air race. They examine a course broken up into twelve segments, and compare the number of legs needed to be simultaneously optimized to obtain an accurate result. They examine three optimization scenarios: a path obtained from optimizing one leg at a time progressing sequentially through the course, a path resulting from a full twelve-phase optimization, and a path pieced together from a Receding-Horizon Multi-phase (RHM) approach.

The single-phase approach optimizes one leg at a time by considering the current segment with no look-ahead to the next. The process is begun by initializing the entry point with the velocity and orientation of the airplane, and the first leg of the course is optimized. Once this leg is found, the process moves on to the second leg by setting the initial state of the airplane equal to the final state of the airplane at the end of the first leg. This is repeated until the airplane has traversed the course.

The twelve-phase optimization attempts to solve for a path by considering all legs at once. In comparison to the single-phase approach, this passes twelve times as many variables to the optimizer, which takes significantly more computation time. Due to the enormity of the problem, convergence is not guaranteed and it is more likely to find a local optimum than a global optimum.

The RHM approach provides a compromise of the single-phase and twelve-phase approaches by considering several legs at once. Visser and van der Plas used three consecutive legs in their RHM analysis. The strategy begins by optimizing the first three legs of the course and saving the first leg as the optimal. The optimization then progresses to the second, third, and fourth legs and optimizes this sequence. The second leg is then saved as the optimal, and the method progress in this manner until the course is completed. The RHM approach allows for a limited look-ahead of the course, allowing the optimizer to consider future flight but not be overloaded with too many variables. A graphical representation of the RHM approach is presented in Fig. 2.11 that shows how the first and second legs of the course are optimized.
Using the three optimization approaches, Visser and van der Plas conclude that a near-optimal trajectory can be found using the RHM approach. In their analysis, they found that the flight path obtained from the three-phase RHM approach is nearly identical to the complete twelve-phase solution;
however, there is much variation when compared to the single-phase solution. Figs. 2.12 – 2.13 show the differences in the paths obtained from the three approaches.

Fig 2.12 – Comparison of a three-phase Receding-Horizon solution to a twelve-phase solution [29].

Fig 2.13 – Comparison of a three-phase Receding-Horizon solution to a single-phase solution [29].
The paths shown in Fig. 2.12 are nearly indistinguishable, indicating the similarity of the two solutions. The only perceptible difference occurs between the first and second gates where the RHM approach yields a trajectory with a lower altitude than the twelve-phase solution.

On the other hand, Fig. 2.13 shows significant variations in the paths. The single-phase solution begins by cutting the corner to the first gate, which comes at the expense of the future flight. The RHM approach rounds this first turn more, providing better entry to the next leg of the course. The sharp corner shown in the bottom left of this figure is due to a vertical maneuver, where the airplane’s heading is discontinuous at a pitch of 90°.

To further understand the differences in the three optimization approaches, Table 2.3 lists the times of each leg of the course. The single-phase approach differs in part because it does not take into account any future flight. Indeed, when examining the first two legs it becomes clear that the single-phase approach is flawed. Without examining any future flight, the approach does not account for the transition from one leg to the next. By minimizing the time of the first leg and leaving the airplane in a poor state, the second leg must recover to keep the airplane on path. Although the single-phase allows the airplane to fly the first leg 0.358 seconds faster than the RHM approach, the second leg lags behind by 0.445 seconds. Over the length of the course, this adds up to a total time that is 1.337 seconds slower than the RHM method.
Table. 2.3 – Comparison of leg times for the three optimization approaches [29].

<table>
<thead>
<tr>
<th>Phase</th>
<th>Trajectory leg</th>
<th>Transit time (sec) 12-phase sol.</th>
<th>Transit time (sec) 3-phase RHM sol.</th>
<th>Transit time (sec) Single-phase sol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gate 1 - Gate 2</td>
<td>6.543</td>
<td>6.321</td>
<td>5.963</td>
</tr>
<tr>
<td>2</td>
<td>Gate 2 - Gate 3</td>
<td>4.521</td>
<td>4.622</td>
<td>5.067</td>
</tr>
<tr>
<td>3</td>
<td>Gate 3 - Gate 4</td>
<td>7.984</td>
<td>8.075</td>
<td>8.406</td>
</tr>
<tr>
<td>4</td>
<td>Gate 4 – Vertical 4</td>
<td>1.792</td>
<td>1.778</td>
<td>1.836</td>
</tr>
<tr>
<td>5</td>
<td>Vertical 4 - Gate 5</td>
<td>6.973</td>
<td>6.955</td>
<td>6.893</td>
</tr>
<tr>
<td>6</td>
<td>Gate 5 - Gate 6</td>
<td>3.491</td>
<td>3.494</td>
<td>3.687</td>
</tr>
<tr>
<td>7</td>
<td>Gate 6 - Gate 7</td>
<td>8.245</td>
<td>8.236</td>
<td>8.517</td>
</tr>
<tr>
<td>8</td>
<td>Gate 7 – Vertical 7</td>
<td>1.815</td>
<td>1.822</td>
<td>1.890</td>
</tr>
<tr>
<td>9</td>
<td>Vertical 7 - Chalk Line</td>
<td>8.250</td>
<td>8.261</td>
<td>8.347</td>
</tr>
<tr>
<td>10</td>
<td>Chalk Line - Gate 8</td>
<td>6.237</td>
<td>6.240</td>
<td>6.297</td>
</tr>
<tr>
<td>11</td>
<td>Gate 8 - Gate 9</td>
<td>10.934</td>
<td>10.93</td>
<td>10.907</td>
</tr>
<tr>
<td>12</td>
<td>Gate 9 - Gate 10</td>
<td>4.521</td>
<td>4.518</td>
<td>4.781</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>71.306</td>
<td>71.254</td>
<td>72.591</td>
</tr>
</tbody>
</table>

The RHM approach results in the fastest time through the course. Although the twelve-phase method should have found the global optimum and the fastest time, the optimizer was not able to overcome the sheer size of the problem and resulted in a local optimum.

The work done by Moon and Kim [28], as well as Visser and van der Plas [29], demonstrates the feasibility of a trajectory optimization for the Red Bull Air Races. Following these efforts, a direct method optimization will be used, and an RHM method will be applied to find the optimal trajectory.
3.1 Airplane Drag

The drag that an airplane produces directly affects its performance. Drag not only limits top speed, but also determines the behavior of the airplane throughout its flight envelope. It is of paramount importance that drag be accurately calculated to determine the flight performance.

The total drag of an airplane can be broken down into multiple parts, including: wing profile drag, induced drag, fuselage drag, trim drag, and tailplane profile drag. Polar Generation Software (PGEN) is an in-house drag calculation routine that is used for this study [30]. PGEN has been used in the winglet design process for numerous competition sailplanes and for Lamb’s MXS-R.

3.1.1 Induced Drag

Induced drag is a consequence of generating lift [13]. By producing lift, a wing effectively adds a downward component to incoming velocity vector which can be accounted for by using an induced angle of attack. The lift vector remains perpendicular to the total velocity vector and is therefore rotated back by the induced angle of attack. Relative to the freestream velocity vector, the tilted lift vector can be decomposed to yield the induced drag, as shown in Fig. 3.1.
Induced drag is always present for a finite lifting surface. A wing creates lift by producing circulation, which may be discretized into vortices in the spanwise direction. At every location where the circulation changes in the spanwise direction, vorticity is shed downstream. This shed vorticity induces velocity in the flowfield, which results in the downwash on the wing.

Since the distribution of lift is not constant in the spanwise direction, multiple vortices with different circulation must be placed along the span such that the wing is replaced by a system of vortex structures. In the case of a vortex lattice method, the wing is broken into panels that each contain a vortex structure. The vortex structures are given constant circulation in the spanwise (bound vortex – lifting) and chordwise (trailing vortex – non-lifting) directions, yielding a horseshoe vortex. A disadvantage of the vortex lattice methods is that they require a large number of vortices along the span to sufficiently obtain the wing lift, thus increasing computational expenses. Furthermore, these methods also predict induced drag quite inaccurately due to the presence of discrete trailing vortices.

PGEN uses Horstmann’s multiple lifting-line method to calculate the induced drag [31]. Like vortex-lattice methods, the multiple lifting-line method creates panels on a lifting surface, each containing a distribution of circulation. Horstmann’s method allows the spanwise component of the circulation to

Fig. 3.1 - A wing section under the influence of the freestream velocity and the downwash.
vary in a quadratic manner, which is akin to the near-elliptic shape. Because the bound circulation is second order, the vorticity shed into the wake, $\frac{d\Gamma}{dy}$, is a first-order continuous (linear) distribution in the spanwise direction. The continuous distribution of shed vorticity leads to a more accurate calculation of induced drag than vortex lattice methods because the singularities resulting from trailing vortices are not present.

The piecewise quadratic distribution of spanwise circulation results in Horstmann’s multiple lifting-line method being of a higher order than single lifting-line and vortex-lattice methods. Horstmann’s method is shown graphically in Fig. 3.2.

![Fig. 3.2 - Multiple lifting-line method [32].](image)

To solve for the unique solution of the quadratic distribution, three boundary conditions must be satisfied. The first and second conditions are simply that the magnitude and slope of the circulation must be continuous between neighboring panels. The third boundary condition enforces the fact that no flow may pass through the surface of the body by requiring flow tangency at fixed control points. These conditions are sufficient to solve for a unique quadratic distribution of circulation.
Horstmann’s method uses a fixed wake, meaning that the effects of wake roll-up are not captured. The method is accurate nevertheless, and it allows for a fast calculation of coefficients of lift, pitching moment, and induced drag.

Once induced drag is calculated, it is expressed as a nondimensional coefficient, $C_{Di}$, which varies with the square of the lift coefficient. The induced drag coefficient is then given by

$$C_{Di} = \frac{C_L^2}{\pi e AR}$$  

After the induced drag coefficient is determined via Horstmann’s method for a given lift coefficient, $e$ is calculated such that the above relation is true. This methodology allows the span efficiency factor to be used to determine the induced drag coefficient at other lift coefficients.

### 3.1.2 Profile Drag

Profile drag results from viscous flow about a two-dimensional lifting body. It is composed of the skin friction drag and the pressure drag acting on the body. Skin friction results from shearing effects present in a viscous flow and pressure drag results from the nonzero integration of static pressure normal body acting in the incoming flow direction.

Profile drag coefficients for each airfoil are calculated using XFOIL [33] at various Reynolds numbers over the operational $c_l$ range. Figure 3.3 shows the drag polars of the Meyer MX2 airfoil (used for the wing of the MXS-R) at various Reynolds numbers.
These data are then compiled into a single input file for PGEN to read. PGEN uses the calculated lift distribution to determine the local profile drag coefficient at a number of spanwise locations, and then performs a summation over the wingspan to obtain the total wing profile drag

\[
D_p = \frac{1}{2} \rho V^2 \left( 2 \sum_{i=1}^{N} \Delta b(i) \ c(i) \ c_d(i) \right)
\]

(2)

### 3.1.3 Trim Drag

Trim drag is the consequence of maintaining trimmed flight in an airplane and is a consequence of having a lifting tail. The tail produces induced drag when lifting and it also changes the amount of lift the wing is required to produce, thus altering the wing induced and profile drags. Trim drag is a function of center of gravity (CG) location, and is affected by the wing, fuselage, and propulsion system pitching moments [13].
In trimmed flight, the tail lift coefficient is calculated via a moment summation about the CG.

$$\sum M_{CG} = M_{wing} + M_{fuselage} + M_{propulsion} + W(X_{CG} - X_{AC,wing}) - l_{HT} L_t \tag{3}$$

To trim, the summation must equate to zero. Imposing this condition and non-dimensionalizing the moment and force terms of Eq. 3, we obtain

$$0 = \frac{1}{2} \rho V^2 S \bar{c} C_{M,0,wing} + \frac{1}{2} \rho V^2 S \bar{c} C_{M,fuselage} + \frac{1}{2} \rho V^2 S \bar{c} C_{M,prop} + \frac{1}{2} \rho V^2 S C_L (X_{CG} - X_{AC,wing}) - l_{HT} \frac{1}{2} \rho V^2 S HT C_{L,HT} \tag{4}$$

where $C_L$ is the lift coefficient based on weight and wing area.

The dynamic pressure may be canceled out, as it is a common to all terms. Rearranging the constants in Eq. 4 yields

$$0 = C_{M,0,wing} + C_{M,fuselage} + C_{M,prop} + C_L \left( \frac{X_{CG} - X_{AC,wing}}{\bar{c}} \right) - l_{HT} \frac{S_{HT}}{S \bar{c}} C_{L,HT} \tag{5}$$

Equation 5 may be simplified for the MXS-R due to the symmetric airfoil that is used for the wing. A symmetric airfoil has zero pitching moment coefficient at its aerodynamic center for all angles of attack, thus the wing zero-lift pitching moment coefficient may be neglected. Also noting that the term $\frac{l_{HT} S_{HT}}{S \bar{c}}$ preceding the tail lift coefficient is by definition the horizontal tail volume coefficient, $C_{HT}$, the tail lift coefficient may be written as

$$C_{L,HT} = \frac{1}{C_{HT}} \left[ C_{M,fuselage} + C_{M,prop} + C_L \left( \frac{X_{CG} - X_{AC,wing}}{\bar{c}} \right) \right] \tag{6}$$

The trim drag may now be expressed as the sum of the tail induced drag and the change in wing induced drag due to the additional lift that the wing must produce to offset the tail lift.
Tail induced drag is calculated in a similar manner to the wing induced drag in that it is a function of the square of the tail lift coefficient.

\[
D_{iHT} = \frac{1}{2} \rho V^2 S_{HT} \frac{C_{L_{HT}}^2}{\pi e_{HT} AR_{HT}}
\]  

(7)

Alternatively expressed in terms of wing area and tail induced drag coefficient, the tail induced drag may be written as

\[
D_{iHT} = \frac{1}{2} \rho V^2 S C_{D_{iHT}}
\]  

(8)

The tail induced drag coefficient may now be calculated by setting the right-hand sides of Eqs. 7 and 8 equal to one another.

\[
\frac{1}{2} \rho V^2 S_{HT} \frac{C_{L_{HT}}^2}{\pi e_{HT} AR_{HT}} = \frac{1}{2} \rho V^2 S C_{D_{iHT}}
\]  

(9)

Simplifying and solving for the tail induced drag coefficient yields

\[
C_{D_{iHT}} = \frac{S_{HT}}{S} \frac{C_{L}^2}{\pi e_{HT} AR_{HT}}
\]  

(10)

For convenience in future equations, Eq. 10 is multiplied by \(\frac{C_{L}^2 AR e}{C_{L}^2 AR e}\) and rearranged to obtain

\[
C_{D_{iHT}} = \frac{S_{HT}}{S} \frac{C_{L}^2}{\pi AR e} \left(\frac{C_{L_{HT}}}{C_{L}}\right)^2 \frac{AR e}{AR_{HT} e_{HT}}
\]  

(11)

The fraction \(\frac{C_{L}^2}{\pi AR e}\) is the airplane induced drag coefficient, as calculated previously. Thus, the tail induced drag coefficient may be simplified to
The second component to the trim drag coefficient is the change in wing induced drag. Because a nonzero tail force is required, the wing lift changes such that total lift remains equal to weight

\[ L = W - L_{HT} \]  

(13)

Non-dimensionalizing Eq. 13 yields

\[ C_{L_{wing}} = C_L - \frac{S_{HT}}{S} C_{L_{HT}} \]  

(14)

The wing induced drag is calculated from the wing lift coefficient, \( C_{L_{wing}} \), as

\[ C_{D_{iwing}} = \frac{C_{L_{wing}}^2}{\pi e AR} \]  

(15)

By squaring both sides of Eq. 14, a substitution may be made in Eq. 15 for the square of the wing lift coefficient

\[ C_{D_{iwing}} = \frac{C_L^2 - 2C_L C_{L_{HT}} \frac{S_{HT}}{S} + \left( C_{L_{HT}} \frac{S_{HT}}{S} \right)^2}{\pi e AR} \]  

(16)

The last term in the numerator, \( \left( C_{L_{HT}} \frac{S_{HT}}{S} \right)^2 \), may be neglected as a lower order term in comparison to the first two terms. Multiplying the second term in the numerator, \( 2C_L C_{L_{HT}} \frac{S_{HT}}{S} \), by the ratio \( \frac{C_L}{C_L} \) and rearranging yields

\[ C_{D_{iwing}} = \frac{C_L^2}{\pi AR e} - \frac{2C_L^2 \frac{C_{L_{HT}} S_{HT}}{C_L S}}{\pi AR e} \]  

(17)
The first term of Eq. 17 is equal to the induced drag coefficient of the airplane; thus the induced drag coefficient of the wing is equal to the sum of the previously computed induced drag coefficient and an additional term that exists due to the tail lift. The increment in the induced drag coefficient, shown as the second term in Eq. 17, is the second component of the trim drag coefficient. It is further simplified and expressed as

\[ \Delta C_{D_i} = -2C_{D_i} \frac{C_{L_{HT}} S_{HT}}{C_L S} \]  

(18)

Adding both components of the trim drag coefficient from Eqs. 12 and 18 yields

\[ C_{D_{trim}} = C_{D_{iHT}} + \Delta C_{D_i} \]  

(19)

which is expressed as

\[ C_{D_{trim}} = \frac{S_{HT}}{S} C_{D_i} \left( \frac{C_{L_{HT}}}{C_L} \right)^2 \left( \frac{AR e}{AR_{HT} e_{HT}} - 2 \right) C_{D_i} \]  

(20)

Collecting like terms in Eq. 20, the trim drag coefficient may be simplified to its final form [13]

\[ C_{D_{trim}} = \left( \frac{S_{HT}}{S} \frac{C_{L_{HT}}}{C_L} \right) \left( \frac{C_{L_{HT}} AR e}{C_L AR_{HT} e_{HT}} - 2 \right) C_{D_i} \]  

(21)

If the horizontal tail were not required to produce lift, there would be no additional trim drag penalty. However, as the CG location is changed, the tail lift coefficient changes according to Eq. 6, resulting in an additional trim drag.

3.2 PGEN Software

PGEN is an in-house tool that is used to compute total drag of an airplane, and is used for the aerodynamic performance calculations performed herein. PGEN calculates drag through analysis of the airplane geometry, gross weight, airfoil drag polars, operational flight conditions, and CG location.
3.2.1 Software Layout

PGEN uses a set of input files to define the wing geometry, the aircraft geometry, and the airfoil data. The wing geometry file, winggeom.dat, breaks the wing into multiple panels, allowing the user to change wing properties from root to tip. Each panel represents a constant taper section of the wing, where the user specifies the inboard and outboard chord lengths as well as the incidence angles about the quarter chord. This information is needed for every panel of the wing and must be non-dimensionalized by the wing half-span.

The aircraft geometry file, acgeom.dat, contains information about the airplane as a whole. It specifies the wing airfoil locations, the wingspan, flap deployment, CG location, aircraft gross weight, empennage size, tail location, and fuselage equivalent flat-plate area and pitching moment.

The airfoil input file is comprised of tables of data for various Reynolds numbers. Each table contains values of $c_d$ and $c_m$ over a range of $c_l$ values. If the aircraft uses different flap deployments, the aerodynamic data for each flap setting is contained within this file.

PGEN uses the combination of these three input files to compute the drag over a user-defined speed range. The user has the option of minimizing wing induced drag, wing profile drag, or total aircraft drag. For this study, total aircraft drag was the parameter of interest.

3.2.2 Drag Build-Up

The baseline drag build-up results for the MXS-R are given as the lift-to-drag ratio vs. airspeed in Fig. 3.4. All calculations are shown for the CG location at the mid-point of the allowable range.
Fig. 3.4 - Lift-to-drag ratio of the baseline MXS-R.

The MXS-R has a predicted maximum lift-to-drag ratio of 12.37 at a velocity of 76 knots. Its predicted maximum speed is 230 knots, which matches the performance specifications given by Red Bull [5].

The MXS-R operates over a wide range of lift coefficients and load factors. Because of this, changes to the wing will impact the design space in a variety of ways, as will be explored in the following chapter.

3.3 Airplane Simulation

To quantify the improvement to design changes on the MXS-R, an analysis of a typical flight in the Red Bull Air Races is performed.
3.3.1 Equations of Motion

The flight of an airplane can be described with a set of equations which account for the translation and rotation of the body. Derived in Stevens and Lewis [34] and shown below, Eqs. 22-33 model the dynamics of an airplane with six degrees-of-freedom.

\[
\dot{U} = RV - QW - g\sin(\theta) + (X_{Aero} + X_{Thrust}) \left( \frac{1}{m} \right)
\]

(22)

\[
\dot{V} = -RU + PW + g\sin(\varphi)\cos(\theta) + (Y_{Aero} + Y_{Thrust}) \left( \frac{1}{m} \right)
\]

(23)

\[
\dot{W} = QU - PV + g\cos(\varphi)\cos(\theta) + (Z_{Aero} + Z_{Thrust}) \left( \frac{1}{m} \right)
\]

(24)

\[
\dot{\varphi} = P + \tan(\theta) \left( Q\sin(\varphi) + R\cos(\varphi) \right)
\]

(25)

\[
\dot{\theta} = Q\cos(\varphi) - R\sin(\varphi)
\]

(26)

\[
\dot{\psi} = \frac{1}{\cos(\theta)} \left( Q\sin(\varphi) + R\cos(\varphi) \right)
\]

(27)

\[
P = \frac{1}{I_x} \left[ J_{xz}(J_x - J_y + J_z)PQ - \left( J_z(J_z - J_y) + J_{xy}^2 \right)QR + J_zl + J_{xz}n \right]
\]

(28)

\[
\dot{Q} = \frac{1}{I_y} \left[ (J_z - J_x)PR - J_{xy}(P^2 - R^2) + m \right]
\]

(29)

\[
\dot{R} = \frac{1}{I_z} \left[ PQ(J_x(J_x - J_y) + J_{xz}^2) - J_{xz}(J_x - J_y + J_z)QR + J_{xz}l + J_xn \right]
\]

(30)

\[
p_n' = U\cos(\theta)\cos(\psi) + V \left[ -\cos(\varphi)\sin(\psi) + \sin(\varphi)\sin(\theta) \right] n \cos(\psi)
\]

(31)
\[ \dot{p}_e = U \cos(\theta) \sin(\psi) + V [\cos(\varphi) \cos(\psi) + \sin(\varphi) \sin(\theta) \sin(\psi)] \\
+ W [-\sin(\varphi) \cos(\psi) + \cos(\varphi) \sin(\theta) \sin(\psi)] \]

(32)

\[ \dot{h} = U \sin(\theta) - V \sin(\varphi) \cos(\theta) - W \cos(\varphi) \cos(\theta) \]

(33)

This 6-DOF system represents the rotational and translational motion of an aircraft. The first set of three equations, Eqs. 22-24, describe the change in velocity along each of the three principle axes of the aircraft. The second set of three equations, Eqs. 25-27, describes the rate of change of the Euler angles describing the orientation of the airplane. The third set of three equations, Eqs. 28-30, describes the angular rates about the three principle axes. The fourth set of three equations, Eqs. 31-33, describe the change in position of the aircraft in a north-east-down (NED) coordinate system with \( h \) being defined as positive in the upward direction. Control inputs are seen in the \( l, m, \) and \( n \) terms, i.e. the rolling, pitching, and yawing moments, which are primarily controlled via the ailerons, elevator, and rudder, respectively. The aerodynamic terms, \( X_{Aero}, Y_{Aero} \), and \( Z_{Aero} \), are functions of the lift and drag of the airplane in the direction of the principle axes. The thrust terms, \( X_{Thrust}, Y_{Thrust}, \) and \( Z_{Thrust} \), are functions of the throttle setting and the thrust angle of the propulsion system in the direction of the principle axes.

These equations may be simplified if the airplane is assumed to be in coordinated flight; that is, there is zero lateral acceleration about the CG. This condition may be satisfied by finding the appropriate rudder input that yields zero lateral force; however, the downside is that it is inherently an iterative process to find this input at every time step. Alternatively, Stevens and Lewis [34] derive an equation for the time rate of change of heading that produces coordinated flight, which is valid for small sideslip angles.

\[ \dot{\psi} = \frac{g}{V_{total}} \tan(\varphi) \cos(\theta) \]

(34)

Which accounts for the bank and pitch Euler angles, as well as the total velocity of the airplane.
The corresponding yaw rate $R$ of the airplane may then be found as

$$R = \dot{\psi} \cos(\varphi) \cos(\theta)$$  \hspace{1cm} (35)

Recognizing that with yaw rate thus constrained and assuming that aileron deflection dominates the roll rate of the airplane, rudder deflection ceases to be an input to the state equations. This leaves aileron and elevator deflection as the two inputs. As these inputs only present themselves in the roll and pitch rates of the aircraft, it is equivalent to state that control of the roll and pitch rates are the necessary inputs to the system.

Thus, a simplified set of state equations may be written as the collection of Eqs. 22-26, 31-34. With this set of equations, as well as Eq. 35 serving as a constraint on the yaw rate, the airplane may be simulated entirely by controlling the roll and pitch rates as functions of time.

### 3.3.2 Aircraft Constraints

The equations of motions described in Eqs. 22-26, 31-34 require knowledge of the airplane’s mass, thrust, and aerodynamic forces. As previously stated, the MXS-R flown by Lamb serves as the baseline for the analysis herein, with alterations being made to this airframe.

### 3.3.3 Flight Course

The Red Bull Air Races feature a unique course at each venue. Air gate placement often depends on natural or man-made landmarks such as trees or buildings. Figures 3.5 – 3.7 show the layouts of the Dallas – Fort Worth, Putrajaya, and Rovinj courses, respectively.
Fig. 3.5 - Dallas - Fort Worth course layout [35].
Fig. 3.6 - Putrajaya, Malaysia course layout [36].
These courses are shown because they are representative of the variation in layout from one race to the next. All locations were venues in the 2014 Championship. Dallas - Fort Worth features two identical laps that follow a figure-8 pattern; Putrajaya features two laps that double back on themselves in such a way that the pilots fly the entire length of the course four times; Rovinj requires the pilots to fly the first lap of the course in a counter-clockwise manner, and then reverse directions to fly the second lap in the clockwise manner. These venues are illustrative of the variation in the course layouts. Some are designed for tight-turning flight, while others are better suited to all-out top speed.

Lamb, who won the 2014 Championship, did well at Putrajaya and Dallas - Fort Worth, taking first and second place, respectively, but finished eighth at Rovinj. The differences in the courses can play not only to a given pilots’ strengths and weaknesses, but also favor certain airplane designs. For example, the Dallas - Fort Worth course entails a significant amount of turning flight, whereas Putrajaya involves
more straight-line flight. The fact that Lamb performed so well at these two very different venues shows both his skill as a pilot, and brings to light the design alterations of his airplane and how they affect performance.

3.3.4 Gate Constraints

The flight course can be modeled as a series of waypoints that represent the air gates. Some of the air gates are set in pairs of pylons, while others stand alone. The airplane must pass through the double sets of pylons in level flight, but is free to bank in a knife-edge orientation around the single pylons. The waypoints are placed at each single gate or in the center of each pair of gates, and the airplane is required to pass directly through the waypoint with an appropriate bank angle.

The Dallas - Fort Worth venue will be used in the simulation model. The locations of the gates are approximated to correspond to this course, shown in Fig. 3.8.

Fig. 3.8 - Simulated course modeling the Dallas - Fort Worth venue from the 2014 Championship.

The airplanes must enter the course by flying through the start gate at no greater than 200 knots with zero bank. After flying through all the gates in consecutive order, the airplane again passes through
the start gate at zero bank, completing one lap. The Dallas - Fort Worth venue featured two laps, the first of which will be simulated herein to demonstrate the differences in lap times for the various design changes.

3.3.5 Optimization Strategy

The optimization uses a direct method and RHM strategy as described in Chapter 2.3 to determine the fastest path over the course. Via the direct method, the roll and pitch rates of the aircraft as a function of time are taken as input parameters and are used to simulate a sequence of legs of the course. There are an infinite combination of inputs that result in the airplane passing through the finish line, but only one set will yield a path with minimum time.

For the trajectory optimization posed herein, the cost function used is equal to the flight time on the course.
Chapter 4

Design Studies with the MXS-R

The MXS-R was designed as an unlimited class aerobatic airplane, meaning that it is meant to be flown in the highest level of aerobatic competitions. These competitions feature precision flying; the pilots are judged based on the skill with which they complete a series of maneuvers. The unlimited class is the most challenging of the aerobatic classes in terms of difficulty of maneuvers and skill of the pilots.

The Red Bull Air Races have strict requirements on the airplanes that are flown in the races. These requirements are very similar to the specifications of the MXS-R, making the airplane a prime contender. However, the nature of the Red Bull Air Races takes the MSR-R out of its design space, making alterations a necessity to gain improved performance. For example, the MXS-R has a flight envelope of +/- 14 g’s and routinely flies inverted; yet the Red Bull Air Races limit the airplanes to a + 10 g maximum load factor and prohibit the airplanes from pulling negative g’s in turns.

There are a number of ways in which the MXS-R could be improved for the Red Bull Air Races. This study examines several such design changes including: the addition of winglets to the airframe and the advantages of changing the wing airfoil to a NACA 6-series laminar flow airfoil. For the former, the CG location and reduction in induced drag are examined for potential gains, while the reduction in profile drag is examined for the latter.

4.1 Case I: Winglets vs. Tail Ballast

The production MXS-R is a carbon fiber low wing monoplane with conventional landing gear and full-span ailerons. It is equipped with a Lycoming engine producing 300 horsepower, to which is fitted a 3-bladed Hartzell propeller.
Due to the requirements of the Red Bull Air Races, all airplanes must weigh over 698 kg at the finish of each race. This leads the teams to make weight savings a priority for the 571 kg empty airframe. With a pilot and fuel, the aerobatic flying weight as given by MX Aircraft is 725 kg. Despite this, teams are able to trim excess weight from the airframe, and anticipate a 700 kg gross weight at race end.

In the 2010 season, pilot Nigel Lamb fitted winglets to his MXS-R airplane, shown in Fig. 4.1. These winglets were designed by Dr. Mark Maughmer of The Pennsylvania State University using the PGEN code for performance calculations.

![Fig. 4.1 - MSX-R equipped with winglets [2].](image)

Non-planar wing geometries, such as winglets, are known to produce greater span efficiency values than a planar wing of the same wingspan. When properly designed, winglets yield an induced drag reduction that can offset the added profile drag of the additional lifting area. However, winglets come at a cost in the weight that they add to the airframe. The MXS-R winglets add approximately 5 kg to the airframe, requiring weight to be removed elsewhere to keep the desired 700 kg final weight. For Lamb’s MXS-R, this resulted in the center of gravity being moved forward.

As can be observed from Eqs. 6 and 21, as the CG is moved forward, the tail lift and trim drag coefficients increase, resulting in greater total drag. In most cases, tail ballast can be added to the airplane to maintain the same value of trim drag, but this is not an option when the weight of the MXS-R is to be...
held at 700 kg. By taking 5 kg of tail ballast from an aft-loaded MSX-R and instead placing 5 kg at the wingtips, the CG moves from the rear limit to approximately 83% of the allowable range. This leads to an increase in trim drag and hence a trade-off of which is more beneficial to airplane performance: the aft CG location or the winglets. Also of interest is whether adding 5 kg of ballast the tail of MXS-R with winglets would result in improved performance in the Red Bull Air Races.

4.1.2 Aerodynamic Analysis

A study on how the CG location affects the drag of the MXS-R was performed with PGEN. Figure 4.2 shows the lift-to-drag ratio of the MXS-R over its speed envelope for various CG locations.

![Lift-to-drag ratio of an MXS-R without winglets for various CG locations.](image)

Fig. 4.2 - Lift-to-drag ratio of an MXS-R *without winglets* for various CG locations.

As can be observed, the aft CG location yields the highest lift-to-drag ratio for the MXS-R and shows no adverse effects throughout the speed range.

The MXS-R equipped with winglets was also analyzed with various CG locations. Figure 4.3 shows the results of this study, which again shows that the aft CG location results in the highest lift-to-drag ratio.
Due to the added 5 kg of weight for the winglets, the CG can only be located at 83% of the allowable range for the MXS-R to maintain a gross weight of 700 kg. Figure 4.4 compares the drag of MXS-R without winglets and the CG at the aft limit to the MXS-R with winglets and the CG at the 83% limit, both with a gross weight of 700 kg.
Fig. 4.4 - Comparison of drag for an MXS-R with and without winglets. The CG is located as far aft as possible for each case while maintaining a gross weight of 700 kg.

The winglets increase the maximum lift-to-drag of the MXS-R from 13.05 to 14.05, an increase of 7.66%. At low speeds, the winglets benefit the airplane by reducing the induced drag, while at high speeds the two configurations exhibit similar drag characteristics because profile drag becomes the dominant component.

When 5 kg of weight is added to the tail of the MXS-R with winglets to move the CG back to the aft limit, the gross weight increases to 705 kg. Figure 4.5 compares the drag of the MXS-R with winglets, the CG at the aft limit, and a gross weight of 705 kg to the MXS-R with winglets, the CG at 83% of the allowable range, and a gross weight of 700 kg.
Fig. 4.5 - Comparison of drag for an MXS-R with winglets and two different CG locations and gross weights.

The aft CG increases the maximum lift-to-drag ratio of the MXS-R with winglets to 14.28, an additional 1.64% above the 83% CG configuration. This is mainly due to the reduction in trim drag at low airspeed. Drag throughout the rest of the speed range is nearly unchanged, showing the induced and profile drag are less influenced by the change. Also of note is that the stall speed of this configuration is slightly lower than the 83% CG configuration, meaning that it has a higher $C_{L_{max}}$.

4.1.1 Trajectory Analysis

While the MXS-R with winglets shows a better lift-to-drag ratio across the speed range - despite having a further forward CG location - the improvement in lap time must be investigated. Integrating the state equations shown in Eqs. 22-26, 31-34, as well imposing the gate constraints of the Dallas - Fort Worth venue, the first lap of the race is simulated and optimized.
The flight path of the MXS-R with winglets and a gross weight of 700 kg is shown in Fig. 4.6. A corresponding colorbar shows the airspeed of the MXS-R over the course in knots, and the black track shows the ground path. This configuration of the MXS-R is able to complete the first lap with a time of 56.25 seconds, while the baseline MXS-R finishes with a time of 56.67 seconds. While four tenths of a second may not be a significant amount of time, it is often more than the difference between first place and second place in the Red Bull Air Races.

Next, the MXS-R with winglets, the CG at the aft limit, and a gross weight of 705 kg is simulated on the course. The optimal flight path for this configuration is shown in Fig. 4.7.
Fig. 4.7 - Flight path of the MXS-R with winglets, CG at 100% of the allowable range, and gross weight of 705 kg.

This configuration of the MXS-R is able to complete the first lap with a time of 55.73 seconds, representing another significant gain.

To understand the difference in the two cases with winglets, consider the velocity over the course, as shown in Fig. 4.8.
Fig. 4.8 - Velocity vs. time for the Case I CG comparison.

The drop in airspeed near the 20 second mark is due to the airplane completing the vertical maneuver, where each configuration approaches stall speed.

Examining the difference in velocity shows further insights into the flight of the two cases. Figure 4.9 shows difference in velocity of the aft CG configuration and the 83% CG configuration.
Fig. 4.9 – Velocity difference between the MXS-R with gross weight of 705 kg and MXS-R with gross weight of 700 kg.

The MXS-R with the CG at the aft limit and a gross weight of 705 kg has lower drag at high lift coefficients and a higher $C_{l_{max}}$, but is more massive. The low drag allows it to maintain more speed through the vertical maneuver than the other configuration, which is evident in the Fig. 9 near the 30 second mark. After the vertical maneuver, the airplane accelerates into the next leg of the course. Having less mass, the MXS-R configuration with a gross weight of 700 kg is better able to handle this part of the course. Examining the velocities in Fig. 4.8 between 30 seconds and 43 seconds shows that the
lighter MXS-R is gaining speed on the heavier configuration. These differences are significant on this course, and allow the MXS-R configuration with the CG at the aft limit to post a faster time.

4.2 Case II: Production Airfoil vs. Laminar Flow Airfoil

The MXS-R is designed with the Meyer MX2 airfoil, which is symmetric with the location of maximum thickness relatively far forward. The drag polars for this airfoil are shown in Fig. 3.3. These data show that the airfoil has a $c_{l_{\text{max}}}$ value that approaches 1.2 for high Reynolds numbers. The airfoil also has a relatively wide region of low drag coefficients at high Reynolds numbers. For Reynolds numbers greater than six million, the profile drag coefficient is less than 80 counts up to a $c_l = 0.8$.

4.2.1 Aerodynamic Analysis

The NACA 63(3)-012 and the NACA 64A-012 airfoils were chosen for comparison because they match the 12% thickness of the Meyer MX2 airfoil. The NACA 6-series airfoils were designed for low drag; their shape is such that long runs of laminar flow are maintained, leading to the well-known drag bucket [38]. Figure 4.10 shows the airfoils overlaid on the same axes, revealing the differences in their thickness distributions.
Each of the 6-series airfoils has the location of maximum thickness much farther aft than the Meyer MX2. This is a characteristic trait of laminar flow airfoils, as it promotes a favorable pressure gradient over much of the airfoil surface, thus delaying transition. The location of maximum thickness for the NACA 63(3)-012 and the NACA 64A-012 are 34% and 40% of the chord, respectively, compared to 17.5% for the Meyer MX2.

The drag polars for the NACA 63(3)-012 and the NACA 64A-012 airfoils are shown for various Reynolds numbers in Fig. 4.11.
Fig. 4.11 - Drag polars for the NACA 63(3)-012 (top) and NACA 64A-012 (bottom) for various Reynolds numbers.
The NACA airfoils exhibit very similar aerodynamic characteristics. Both have low drag at low lift coefficients, and have $c_{t_{\text{max}}}$ values approaching 1.5 for high Reynolds numbers. The drag bucket can be distinguished for both airfoils in this figure as the near-constant drag region for low lift coefficients.

When the NACA airfoils are compared directly against the Meyer MX2 airfoil, the drag reduction becomes apparent. Figure 4.12 shows the drag polars of the three airfoils for Reynolds numbers of six million and nine million, which is the typical operating range of the MXS-R in the Red Bull Air Races.
Fig. 4.12 - Drag polars for the Meyer MX2, NACA 63(3)-012, and NACA 64A-012 for Reynolds numbers of six million (top) and nine million (bottom).
These polars show that the two NACA airfoils exhibit lower drag up to a lift coefficient of approximately $c_l = 0.7$. Above this lift coefficient, the Meyer MX2 airfoil has slightly less drag, but quickly converges to values similar to the NACA airfoils. Both of the NACA airfoils have higher maximum lift coefficients, approaching $c_{l_{max}} = 1.5$.

These airfoils lead to extremely different performance on the MXS-R. To determine the effects of altering the airfoils, PGEN was used to find the aerodynamic performance of the MXS-R with winglets and a CG location of 83% of the allowable range. These data are compared to the baseline MXS-R without winglets and the CG at the aft limit. These CG locations are consistent with the proper tail ballast to achieve a gross weight of 700 kg. The resulting lift-to-drag ratios and drag are shown in Fig. 4.13 and Fig. 4.14, respectively.

Fig. 4.13 - Comparison of lift-to-drag ratio of the MXS-R with different wing airfoils.
Fig. 4.14 - Comparison of drag vs. velocity of the MXS-R with different wing airfoils.

The MXS-R equipped with the NACA 63(3)-012 and the NACA 64A-012 wing airfoils both result in a maximum lift to drag value that is very close to that achieved with the Meyer MX2 airfoil. The maximum lift-to-drag ratio for the airplane using the NACA 63(3)-012 is 13.97, while it is 14.12 for the NACA 64A-012. This is a slight decrease of 0.57% for the former, and an increase of 0.50% for the latter.

The drag at high speed is much less when an NACA airfoil is used. At the entry speed of 200 knots, the NACA 63(3)-012 and NACA 64A-012 result in increases of 9.91% and 9.04% in lift-to-drag ratio compared to the production MX2 airfoil. This is also seen in Fig. 4.11, where the total drag at high speeds is much less when an NACA airfoil is used.

Furthermore, the NACA 63(3)-012 airfoil also allows for the MXS-R to have a much lower stall speed of 57.7 knots, compared to 63.7 knots with the production MX2 airfoil. The NACA 64A-012 airfoil also yields a lower stall speed, but it is not as significant with a speed of 62.5 knots.
4.2.2 Trajectory Analysis

The trajectory analysis is now repeated for the alternative laminar flow airfoils. It has already been seen that the MXS-R with winglets is capable of completing a lap in a faster time than the baseline MXS-R, and this result will now be compared to flight time with the NACA 63(3)-012 and the NACA 64A-012 airfoils.

Changing the wing airfoil to the NACA 63(3)-012 allows the MXS-R to complete the flight in a time of 55.32 seconds. This is a significant improvement over the production MXS-R with winglet time of 56.25 seconds. The flight path for the MXS-R with the NACA 63(3)-012 is shown in Fig. 4.15.

![Flight path of the MXS-R with the NACA 63(3)-012 airfoil.](image)

With the NACA 64A-012 airfoil, the MXS-R also shows an improvement over the MXS-R with winglets. With this airfoil, the MXS-R is able to complete the flight in a time of 55.44 seconds. This is faster that the MXS-R with the MX2 by 0.81 seconds, but slower than the NACA 63(3)-012 equipped MXS-R by 0.12 seconds. The flight path for the MXS-R with the NACA 64A-012 is shown in Fig. 4.16.
The fact that the MXS-R equipped with the NACA 63(3)-012 airfoil posts the fastest time is not surprising; examining the drag polars presented in Figs. 4.11 and 4.12, one sees that for a range of $c_l$ values, the NACA 63(3)-012 yields lower drag coefficients than the other configurations, which results in a faster airplane.

To illustrate the differences in the flights, the velocity over the course is plotted against time in Fig. 4.17 for the MXS-R with the NACA 63(3)-012, the NACA 64A-012, and the production MX2 airfoil.
Fig. 4.17 – Velocity vs. time for the Case II airfoil comparison.

The drop in airspeed near the 20 second mark is again due to the airplane completing the vertical maneuver. Interestingly, there is little variation in the flight speed prior to this point. In particular, the two cases with laminar flow airfoils have nearly the same velocity and path leading up to the vertical maneuver, while the case with the production airfoil carries a slightly lower velocity.
Figure 4.18 shows the difference in velocity of the laminar flow airfoil cases and the case with the production MX2 airfoil.

A stark difference in velocity can be seen after the vertical maneuver, when the airspeed begins to recover. The MSX-R with the NACA 63(3)-012 airfoil is able to accelerate out of the maneuver to a faster airspeed than with the NACA 64A-012 airfoil. It is in this part of the flight
that the NACA 63(3)-012 outshines the competition and ultimately leads to the fastest time through the course.

Also of interest following the vertical maneuver is that the two laminar flow airfoils clearly have an edge on the production MX2 airfoil. On this part of the course, the NACA 63(3)-012 and the NACA 64A-012 airfoils allow the MXS-R to fly approximately 8.5 knots and 6 knots faster than the production MX2 airfoil. These differences lead to the faster times posted by both laminar flow airfoils.

4.3 Summary of Results

Table 4.1 summarizes the results of the two design cases. The lift-to-drag ratio at a velocity of 200 knots is additionally recorded for the various wing airfoils, as this is the maximum entry speed of the MXS-R in the Red Bull Air Races.
Table 4.1 - Summary of MXS-R performance with different wing airfoils.

<table>
<thead>
<tr>
<th>Winglet Configuration</th>
<th>No Winglets, Meyer MX2, 83% CG, 700 kg</th>
<th>Winglets, Meyer MX2, 83% CG, 700 kg</th>
<th>Winglets, Meyer MX2, 100% CG, 705 kg</th>
<th>Winglets, NACA 63(3)-012, 83% CG, 700 kg</th>
<th>Winglets, NACA 64A-012, 83% CG, 700 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{stall}$ (knots)</td>
<td>66.6</td>
<td>63.7</td>
<td>62.7</td>
<td>57.7</td>
<td>62.5</td>
</tr>
<tr>
<td>$L/D_{max}$</td>
<td>13.05</td>
<td>14.05</td>
<td>14.28</td>
<td>13.97</td>
<td>14.12</td>
</tr>
<tr>
<td>$(L/D)_{V=200\text{ knots}}$</td>
<td>3.42</td>
<td>3.43</td>
<td>3.46</td>
<td>3.77</td>
<td>3.74</td>
</tr>
<tr>
<td>Flight Time (s)</td>
<td>56.67</td>
<td>56.25</td>
<td>55.73</td>
<td>55.32</td>
<td>55.44</td>
</tr>
</tbody>
</table>

These results show that an MXS-R equipped with winglets and a laminar flow airfoil can achieve considerable performance improvements. The additions of winglets and an NACA 6-series airfoil can improve the aerodynamic efficiency of the MXS-R over the production airplane, and each of these design changes can lower the flight time at the Dallas - Fort Worth venue.

With each of the design changes, the maximum lift-to-drag ratio of the MXS-R is changed only slightly. However, the 6-series airfoils are able to improve the high speed performance of the airplane significantly and can drastically decrease the stall speed. With the NACA 63(3)-012, the MXS-R is able to improve the flight time by the most significant amount, lowering the time from 56.67 seconds to 55.32 seconds.
Chapter 5

Conclusion

The goal of this study was to assess design changes to the MXS-R being used in the Red Bull Air Race World Championships. Winglets were first introduced to the MXS-R in 2010, which went on to win the Championship in the following racing season. However, questions arose to other possible changes and what influence they would have on the MXS-R.

The addition of winglets to the MXS-R causes the CG to move forward from the aft limit to 83% of the allowable range, increasing the trim drag. This leads to a trade-off between the induced drag benefit of the winglets and the added profile and trim drag. A study of the MXS-R shows that the winglets yield an increase in maximum lift-to-drag ratio with little adverse effects at high speed. Furthermore, keeping the CG as far aft as possible results in the lowest drag, and it can result in faster lap times even if the gross weight slightly exceeds 700 kg.

As the MXS aircraft is designed for competition aerobatics, low drag is not necessarily a design driver and is, in fact, somewhat undesirable. Thus, the use of an airfoil that does not achieve long runs of laminar flow is of little consequence; however, for the Red Bull Air Races, some gains are achievable by making use of laminar flow airfoils. Such airfoils have been researched for over 60 years, and have been shown to operate at incredibly low drag coefficients within the drag bucket. A study of the MXS-R using the NACA 64A-012 and the NACA 63(3)-012 airfoils in place of the production MX2 airfoil shows considerable performance improvements, particularly at high speed. When equipped with the NACA 63(3)-012, the lift-to-drag ratio improves by nearly 10% at an airspeed of 200 knots, resulting in a decrease in flight time from 56.67 seconds with the production MX2 airfoil to 55.32 seconds.

The performance gains shown with the NACA 6-series airfoils could be further improved upon with a custom-designed airfoil. The Red Bull Air Races present a unique mission for aircraft design, and as such would gain the most benefit from a design specific to this application.
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