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OUTSIDE AND INSIDE MONEY: A MECHANISM DESIGN APPROACH

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by

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Abstract

This thesis applies mechanism design to explore the roles of outside and inside money in achieving optimal allocations. Two chapters use settings that are related to random-matching models. The third uses an overlapping generations setting. In all three, a critical role is played by assumptions about whether people can commit to future actions and about how much, if any, of their histories is publicly known.

A MODEL IN WHICH OUTSIDE AND INSIDE MONEY ARE ESSENTIAL

I present a model in which both outside money and inside money are essential. The environment is a random-matching model of money where some people have publicly-observed histories and so can be monitored, while others have private histories and cannot be monitored. Those who are monitored issue inside money. The issuers of inside money can only be monitored imperfectly, however, because histories are observed only after a lag. I show via a mechanism design approach that for some parameters, there exist allocations that are achievable only when both outside and inside money are used.

THE SIZE OF MEETINGS AND THE ROLE OF MONEY

Double-coincidence problems are necessary for money to be essential. I amend a random-matching model with no commitment and no publicly- observable histories by making the number of people in a random meeting a parameter, denoted N. This is one way to parameterize the extent of the double-coincidence problem. I prove that as $N \to \infty$, the beneficial role of money disappears.

MECHANISM DESIGN IN FREEMAN'S MODEL OF PAYMENTS

Freeman (1996a) claims to have an environment in which outside money is essential to repay debt. I dispute that claim by providing an alternative trading mechanism that does not use money to repay debt, but achieves optimal allocations. Moreover, the use of this alternative makes irrelevant the liquidity-providing institutions - a central bank or a private clearinghouse - whose existence Freeman claims to justify.

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Chapter 1

A Model in which Outside and Inside Money are Essential

1.1 Introduction

Monetary economists have debated for a long time the relative desirability of outside (or public) money and inside (or private) money as alternative means of payment. The arguments, such as those between the so-called currency and free-banking schools, have primarily centered on the exclusive use of one or the other. Observations in history, however, show that both types of money have simultaneously circulated as means of payment. Moreover, improved information technology and the removal of legal impediments to the private issue of inside money in the United States suggest that studying their coexistence is relevant today. Surprisingly, the literature has given little attention to the study of the underlying features of an environment such that both outside and inside money are essential as means of payment. This paper takes a step in that direction.

To take this step I generalize the model in Cavalcanti-Wallace (1999a). Theirs is a random-matching model of money[1] in which some people (bankers) can be perfectly monitored via a record-keeping technology while others (nonbankers) cannot be monitored at all because their trading histories are private information. They compare the

[1] See Kiyotaki-Wright (1989), Trejos-Wright (1995), and Shi (1995). These models all assume that every agent's trading history is private information.

exclusive use of outside money to the exclusive use of inside money and show that implementable outside-money allocations are a strict subset of implementable inside-money allocations because bankers can consume independent of their recent trades whenever inside money is used; a banker may be able to issue a new note to a nonbanker in exchange for consumption.

In this paper, I assume that bankers can be monitored only imperfectly because of a lag in the updating of their histories, a random version of which is used in Kocherlakota-Wallace (1998)[2]. I compare three sets of implementable allocations: those that use only outside money, those that use only inside money, and those that use both. I show via numerical examples that there is no strict subset ordering of implementable allocations for some parameters. Indeed, for updating lags that are neither too short nor too long, there exist allocations that are only implementable when both inside and outside money are used. Inside money retains the advantage that it has in Cavalcanti-Wallace (1999a). However, inside money has a potential disadvantage in that there are tighter and additional incentive constraints arising from the fact that recent banker histories are private information. The implied trade-off is what gives rise to the possibility that both outside and inside money are essential.

The virtues of this analysis are that I look at the role trading histories and reputations play in the ability to issue inside money and that I apply mechanism design, although in a small class of allocations. At the core of the debate over outside and

[2]In their model, everyone's history ($B = 1$) is updated with probability ρ at each date, which produces an average updating lag of $\frac{1}{\rho}$. There, that led to a slightly simpler formulation of the payoff from defecting than a deterministic lag. Here, the more straightforward deterministic lag is simpler.

inside money is the fear that private issuers of inside money have incentives to overissue. The threat of overissue was one reason Friedman (1959) favored 100% reserve requirements, essentially eliminating inside money. Klein (1974) stressed the importance of reputations for private bankers to issue inside money. With this model, I can rigorously address the threat of overissue and the role of reputation by characterizing a set of incentive constraints. Bankers measure the short-run gains from issuing notes when they should not or not redeeming notes when they should with the long-term costs of a blemished reputation.

I present three different examples where each one is an element of only one of the sets of implementable allocations. These examples are numerical because the updating lag of banker histories must be neither too short nor too long. This is because inside money is superior to outside money in the absence of a lag, as shown in Cavalcanti-Wallace (1999a), and outside money is superior to inside money when the public record of histories is never updated. The latter is due to the fact that bankers cannot be monitored and so cannot be rewarded or punished in the future for actions they take currently.

Related work that looks at the role of histories and reputations play in the ability to issue inside money include Cavalcanti-Wallace (1999b) who look only at inside money in an environment where bankers are perfectly monitored, and Cavalcanti-Erosa-Temzelides (1999) who assume a related but different record-keeping technology[3]. Recent work on the coexistence of outside and inside money has not focused on the role of history and reputation. Williamson (1999) models banks as institutions where agents can invest in projects and receive claims to the dividends of that investment in the future. These claims circulate as inside money. Azariadis-Bullard-Smith (2001) and Bullard-Smith (2000) look at the simultaneous use of outside and inside money in a model of spatial separation and limited communication as in Townsend (1987, 1989). In their model, redemption is costlessly enforceable. They then explore the welfare implications for outside and inside money, as well as address the assertion that the use of inside money gives rise to excess volatility.

This rest of the chapter is organized as follows. Section 1.2 presents the environment. Section 1.3 describes the restricted class of mechanisms I study and the conditions for implementability. Section 1.4 presents examples demonstrating that there is no subset ordering of implementable allocations, while section 1.5 offers concluding remarks.

1.2 The Economic Environment

Time is discrete and the horizon is infinite. There are S distinct, divisible, and perishable types of goods at each date and there is a $[0, 1]$ continuum of each of S specialization types of agents, where $S > 2$. An agent whose specialization type is s

[3]Specifically, there exists a clearinghouse that keeps records of banker balance sheets that monitor the redemption behavior of bankers. They do not formally introduce outside money in their model, and it is unclear that the coexistence of both is essential, because they do not apply mechanism design in their framework.

consumes only good s and produces only good $s+1$ (modulo S), for $s = 1, 2, \ldots, S$. Each agent maximizes expected discounted utility with a discount factor $\beta \in (0, 1)$. Utility in a period is given by $u(c) - y$ where c is the amount consumed and y is the amount produced. The function u is defined on $[0, \infty)$, is increasing, twice differentiable, and satisfies $u(0) = 0$, $u'' < 0$, and $u'(0) = \infty$. Moreover, there exists $y^{\max} > 0$ such that $u(y^{\max}) = y^{\max}$.

In each period, agents are randomly matched in pairs. There are two kinds of meetings. A single-coincidence meeting is a meeting that contains a type s agent (the producer) and a type $s + 1$ agent (the consumer) for some s. A no-coincidence meeting is a meeting in which neither agent produces what the other consumes. Because $S > 2$, there are no meetings in which there is a double-coincidence of wants.

It is assumed that agents cannot commit to future actions. This implies that those agents who produce must expect to receive a future reward for doing so.

The society is able to keep a public record of the actions of a fraction B of each specialization type of agent, where $B \in [0, 1]$. Agents whose histories are a part of the public record are called *bankers*. The fact that banker histories are part of a public record implies that it is possible to monitor their behavior. As we shall see, this implies that bankers do not need to receive something tangible to induce them to produce in a single-coincidence meeting; bankers can be rewarded and punished in the future for actions they take currently. Society has no public record for the remaining fraction $1 - B$ of each specialization type, the *nonbankers*. The fact that nonbankers are anonymous implies that the society cannot monitor their behavior. The implication is that nonbankers must receive something tangible in order to produce. We can think of B as society's capacity for keeping track of individual trading histories.

Public information about banker histories is not perfect because the public record is not updated immediately after every action. Specifically, there is a deterministic lag of T periods, where $T \geq 0$. At the beginning of each date $t > T$, the bankers' trading histories are known up through what they did until the beginning of date $t-T$. For $t \leq T$, banker histories are unknown. Thus, if $T = 0$, then banker histories are completely publicly known and society can perfectly monitor their actions. If T is sufficiently large, then banker histories are effectively unknown and society cannot monitor their actions at all. We can think of T as society's ability to update records.

There are two distinct assets. These assets are indivisible and agents can carry at most one unit of one asset across dates. It is not possible for an agent to simultaneously hold a unit of both assets. These assets could be two outside monies, two inside monies, or one outside money and one inside money. Why I consider the possibility of two outside monies and two inside monies will become clear in the next section.

Outside money is neither produced nor consumed. It is indivisible and perfectly durable. If both assets in the environment are outside monies, then the two are distinguishable in some way.

Each agent has a technology that permits that agent to create two distinct, indivisible and perfectly durable objects, called notes. Because these notes are a type of credit instrument that is issued by private individuals and may circulate as a means of payment, they may serve as *inside money.* The notes issued by a single agent are distinguishable from those issued by another agent so that counterfeiting is not a problem.

When two agents meet, the following is common knowledge: each trading partner's specialization type, asset holdings, information type (banker or nonbanker) and the past actions of the bankers in the meeting that occurred up to $t - T$ periods ago.

1.3 Mechanisms

I compare three separate classes of monetary mechanisms: an outside-money mechanism (two outside monies), an inside-money mechanism (two inside monies), and a mixed mechanism (one outside money and one inside money). Both the outside-money mechanism and the inside-money mechanism use two assets so as to provide comparable alternatives to the mixed mechanism. Given the assumed indivisibility of assets, it is possible that two distinct assets are better than one[4].

When two agents meet, a mechanism takes as inputs each agent's specialization and information types, each agent's asset-holdings, and, for bankers, the known histories up to $t-T$ periods ago, and announced histories from $t-T+1$ to $t-1$. The outcome of a mechanism is current actions. All monetary mechanisms are restricted to be stationary, symmetric, and deterministic. A mechanism is stationary in the sense that there is a steady-state distribution of assets among agents. A mechanism is symmetric in the sense that current actions are independent of an individual's specialization type and his relative position within that type. Further, notes issued by bankers are treated symmetrically by

[4] See Aiyagari-Wallace-Wright (1996). The idea is this. Consider a single-coincidence meeting between two nonbankers in which both the producer and the consumer have an asset. If the assets are identical, then the producer does not want to produce for the asset because he already holds it. If the assets are distinct and the producer has the lower valued asset, then he may be willing to produce and give up his asset in exchange for the more valuable asset. Thus, distinct assets can increase the frequency of trades.

the agents in the economy and those issued by nonbankers are ignored[5]. If both assets in the economy are inside monies, asset 1 issued by bankers is treated symmetrically and asset 2 issued by bankers is treated symmetrically, although asset 1 and asset 2 may be treated differently. A mechanism is deterministic in the sense that it does not introduce additional randomness and there are no mixed strategies.

One final restriction that is placed on all mechanisms is that they use banker histories in a limited way. Banker histories can be used for two things. First, they can be used to reveal a banker defection in that part of history which is publicly known. This enables the mechanism to punish those bankers by proposing an alternative, less desirable outcome. Second, for bankers whose histories do not reveal a defection, their outcomes could depend on their entire histories – the part that is known and the part that is announced – in the spirit of Green (1987). It is in this sense in which histories are limited. Here, histories and asset-holdings are summarized by states, which are members of a three-element set $A \equiv \{0, 1, 2\}$. This set has the minimum number of elements such that each element could adequately represent asset-holdings, where state 0 indicates no asset-holdings, state 1 indicates that an agent holds a unit of asset 1, and state 2 indicates that he holds a unit of asset 2. One can think of an implicit function that

[5] One could think of alternative ways in which inside money could be used such that bankers would want to hold other bankers' inside money. This could be the case if bankers receive more consumption by exchanging another's note than issuing their own. In this context, redemption becomes an important issue. Specifically, since there is zero probability that any one agent meets the banker that issued the note in his possession, and all other bankers would prefer to use the note as a means of payment, then inside money is never redeemed. If it is never redeemed, steady-state requirements imply that it cannot be issued. If inside money is never issued nor redeemed, then it is missing the essential features that distinguish it from outside money.

maps asset-holdings and observed and announced histories into states. The restriction is placed on the mechanisms to keep the model tractable.[6]

Because nonbankers have unobservable histories, their states can only represent asset-holdings. As a result, the state of a nonbanker is always observable in meetings because asset-holdings are common knowledge. The state interpretation for bankers, however, is contingent on the mechanism being considered. In the outside-money mechanism, a banker's state describes asset-holdings. Because each element of A is needed to represent asset-holdings, the states do not represent a banker's history. Like the nonbanker states, banker states are completely observable in the outside-money mechanism.

Due to the symmetry imposed on mechanisms, the states cannot represent asset-holdings of bankers for the inside-money mechanism. With the outside-money mechanism, banker asset-holdings may determine whether a banker produces or consumers. For example, a banker with a unit of outside money (in state 1 or 2) may be able to consume in certain meetings while a banker without a unit of outside money (in state 0) may be induced to produce in certain meetings. In order to see if the inside-money mechanism can duplicate such an allocation, I must be able to distinguish the meetings in which bankers should produce and those in which they should consume. This requires some history dependence under the inside-money mechanism according to a set of states that is at least as large as A.[7] With the inside-money mechanism, a banker's state is private information whenever $T > 0$.

[6] The state space here is richer than that in Cavalcanti-Wallace (1999a), where histories and asset-holdings are mapped into one of two states.

[7] For example, if observed and announced histories mapped into state 0, then to duplicate what is proposed by the aforemention outside-money mechanism, the inside-money mechanism must propose that this banker produce.

For the mixed mechanism, without loss of generality, refer to asset 1 as the inside money and asset 2 as the outside money. If a banker is in state 2, that implies that she has a unit of outside money and her state is observable. If the banker is in either state 0 or state 1, then that implies she does not have a unit of outside money. Whether a banker is in state 0 or state 1 depends on her history and so is private information whenever $T > 0$.

The state interpretations above enable me to use the same notation for each of the three classes of monetary mechanisms considered. Additionally, instead of announcing histories, agents can announce states. Given the limited use of banker histories, when two agents meet, a mechanism now takes as inputs each agent's specialization and information types, each agent's announced current state, and for bankers, an indication about whether they have been discovered as defecting in the past. The outcome of a mechanism consists of each agent's next-period state, and, in single-coincidence meetings, output.

When there is no discovered defection in a meeting, the mechanism suggests outcomes according to four functions. The first three pertain to the outcomes associated with a single-coincidence meeting. The first maps the announced states of two trading partners into output. The next two map the announced states of the two trading partners into next period's state. Let the set of information types be $\{b, n\}$, where b indicates that an agent is a banker and n indicates that he is a nonbanker. Let $y_{ij}^{kl} \in \Re_+$ be output when a producer of information type k announces state i and a consumer of information type l announces state j. Formally, for all $(k, l) \in \{b, n\} \times \{b, n\}$, let

$$y_{\cdot,\cdot}^{kl} : A \times A \to \Re_+.$$

Similarly, let

$$p^{kl}_{\cdot,\cdot} : A \times A \rightarrow A$$

be the state transition for the producer in the meeting and

$$q^{kl}_{\cdot,\cdot} : A \times A \rightarrow A$$

be that for the consumer in the meeting. Thus, the outcome of a single-coincidence meeting is $(y^{kl}_{ij}, p^{kl}_{ij}, q^{kl}_{ij})$. The number of elements in $\{b, n\}^2 \times A^2$ is 36, representing the number of potentially distinct single-coincidence meetings for which the mechanism makes suggestions.

For no-coincidence meetings, let r^{kl}_{ij} be the state-transition function for the agent of information type k who announces state i in a no-coincidence meeting with an agent of information type l who announces state j. Formally, for all $(k, l) \in \{b, n\} \times \{b, n\}$, let

$$r^{kl}_{\cdot,\cdot} : A \times A \rightarrow A.$$

Thus, the outcome of a no-coincidence meeting is $(r^{kl}_{ij}, r^{lk}_{ji})$, the end-of-period states for both trading partners. There are also 36 potentially distinct no-coincidence meetings.

I now describe the game associated with any monetary mechanism. There are two stages within a meeting. At the first stage agents simultaneously announce states. Such an announcement is superfluous for nonbankers because their states always represent asset-holdings which are common knowledge in a meeting. The announcement for bankers is contingent on the class of mechanism. Formally, let $\widetilde{A} \subset A$ be the set of states over which a banker can misrepresent. For the outside-money mechanism $\widetilde{A} = \emptyset$ because states represent asset-holdings and are common knowledge in a meeting. For the inside-money mechanism, however, $\widetilde{A} = A$; bankers have complete freedom to misrepresent their state because it is private information. For the mixed mechanism, $\widetilde{A} = \{0, 1\}$. A banker can report a state in $\widetilde{A}$ only when her true state is in $\widetilde{A}$. A banker cannot misrepresent at all when her true state is 2, because it represents outside money-holdings in the mixed mechanism.

Given the information types of the agents and their announced states from the first stage, a mechanism suggests an outcome according to the functions described above if there has not been a discovered defection by either agent in the meeting. If there has been a discovered defection by either agent in the meeting, then the mechanism always suggests that there is no trade. In other words, the mechanism permanently punishes a discovered defecting banker with autarky. At the second stage, agents simultaneously decide whether to agree or disagree to the suggestions. If both agree, the suggested outcome is carried out. If at least one agent disagrees, then both leave the meeting without trading.

If a banker either misrepresents her state in the first stage, or chooses not to participate in the suggested outcome in the second stage, then that banker is a defector. A nonbanker does not receive such a label because a defection by him would never be discovered. A defection by a banker is discovered T periods from the date it occurred (say date t); a banker's true state and announced state at date t are publicly revealed at date $t + T$, as well as whether or not a banker agreed to the suggested outcome in the second stage. For the $T - 1$ periods that follow an initial defection, a banker is an undiscovered defector. An undiscovered defector can costlessly choose to either truthfully participate in trade or to defect again. From period $t + T$ on, a defecting banker is a discovered defector and is punished with autarky.

Let x_i^k denote the fraction of each specialization type with information type k in state i. An *allocation* is a list of fractions of agents in states, outcomes in single-coincidence meetings, and outcomes in no-coincidence meetings: $(x_i^k, y_{ij}^{kl}, p_{ij}^{kl}, q_{ij}^{kl}, r_{ij}^{kl})$. An allocation is *feasible* via a monetary mechanism if it satisfies certain constraints imposed on a mechanism's suggested state transitions. An allocation is *implementable* via a monetary mechanism if it satisfies some free disposal conditions, and induces truth-telling by bankers in the first stage and participation by both bankers and nonbankers in the second stage. The three sets of monetary allocations differ in two respects: the number of feasibility constraints and the number of incentive constraints that each mechanism must satisfy.

1.3.1 Steady-State and Feasibility Requirements

I look at stationary mechanisms in the sense that the distribution of agents over states is a steady-state distribution, given the state transitions. I now describe the requirements imposed on state transitions so that they follow a steady-state distribution. In doing so, I anticipate the satisfaction of one of the incentive constraints presented in the next subsection: no free disposal of assets.

Because each person must be in one of the states, the fractions of each specialization type in each state must satisfy:

$$\sum_i x_i^b = B \text{ and } \sum_i x_i^n = 1 - B. \tag{1.1}$$

A steady-state distribution of agents over states requires that the fraction of bankers in each state and the fraction of nonbankers in each state be constant. This can be expressed by equating the inflow and outflow of each state for nonbankers and bankers. For nonbankers, an inflow into a state i occurs whenever a nonbanker in state $h \neq i$ in a meeting with a banker leaves the meeting in state i. An outflow from state i occurs whenever a nonbanker in state i in a meeting with a banker leaves the meeting in state $h \neq i$. For bankers, the inflows and outflows have a similar interpretation in meetings with nonbankers. Formally, for $h \in A$, let the indicator variable $p_{ij}^{kl}(h) = 1$ if $p_{ij}^{kl} = h$ and $p_{ij}^{kl}(h) = 0$ otherwise. Define $q_{ij}^{kl}(h)$ and $r_{ij}^{kl}(h)$ similarly. Then the inflow-equal-outflow conditions are as follows. For each $i \in A$,

$$\sum_{h \neq i} x_h^n \sum_j x_j^b [p_{hj}^{nb}(i) + q_{jh}^{bn}(i) + (S-2) r_{hj}^{nb}(i)] = x_i^n \sum_j x_j^b [\sum_{h \neq i} p_{ij}^{nb}(h) + q_{ji}^{bn}(h) + (S-2) r_{ij}^{nb}(h)] \tag{1.2}$$

and

$$\sum_{h \neq i} x_h^b \sum_j x_j^n [p_{hj}^{bn}(i) + q_{jh}^{nb}(i) + (S-2) r_{hj}^{bn}(i)] = x_i^b \sum_j x_j^n [\sum_{h \neq i} p_{ij}^{bn}(h) + q_{ji}^{nb}(h) + (S-2) r_{ij}^{bn}(h)] \tag{1.3}$$

where (1.2) relates to nonbankers and (1.3) to bankers. An allocation is stationary if it satisfies equations (1.1)-(1.3).

In addition to the steady-state restrictions imposed on state transitions, there are also feasibility constraints implied by the preservation of asset-holdings in meetings. These constraints reflect the difference between outside and inside money; outside money is not issued by private agents, and inside money is issued only by bankers. Additionally, the free disposal conditions anticipated here imply that no one wants to dispose of outside money, and nonbankers do not want to dispose of inside money. As a result, the outside-money mechanism has the largest number of feasibility restrictions because outside-money holdings must be preserved in all meetings while the inside-money mechanism has the fewest number of feasibility constraints because inside-money holdings must be preserved only in meetings between nonbankers. For the mixed mechanism, the preservation of outside-money holdings must be satisfied in meetings regardless of information types, but this pertains only to asset 2. Asset 1, the inside money, must be preserved only in meetings between nonbankers. The following definitions formally express the feasibility constraints as they pertain to each mechanism.

Definition 1. *A* ***stationary and feasible outside-money allocation*** *is* $(x_i^k,\ y_{ij}^{kl},\ p_{ij}^{kl},\ q_{ij}^{kl},\ r_{ij}^{kl})$ *for* $i, j \in A$ *and* $k, l \in \{b, n\}$ *that satisfies (1.1) - (1.3) and*

$$p_{ii}^{kl} = q_{ii}^{kl} = r_{ii}^{kl} = i \tag{1.4}$$

$$
\begin{aligned}
p_{ij}^{kl} &= j \textit{ if and only if } q_{ij}^{kl} = i \\
r_{ij}^{kl} &= j \textit{ if and only if } r_{ji}^{lk} = i
\end{aligned}
\tag{1.5}
$$

for $i, j \in A$ *and* $k, l \in \{b, n\}$.

Condition (1.4) says that if both agents have the same asset-holdings, then they leave with the same asset-holdings. Condition (1.5) says that if agent A's next-period state is agent B's current state, then B's next-period state is agent A's current state (they swap asset-holdings). Conditions (1.4) and (1.5) imply that outside money is never issued nor redeemed.

Definition 2. *A* ***stationary and feasible inside-money allocation*** *is* $(x_i^k, y_{ij}^{kl}, p_{ij}^{kl}, q_{ij}^{kl}, r_{ij}^{kl})$ *for* $i, j \in A$ *and* $k, l \in \{b, n\}$ *that satisfies (1.1) - (1.3), and*

$$
p_{ii}^{nn} = q_{ii}^{nn} = r_{ii}^{nn} = i \tag{1.6}
$$

$$
\begin{aligned}
p_{ij}^{nn} &= j \textit{ if and only if } q_{ij}^{nn} = i \\
r_{ij}^{nn} &= j \textit{ if and only if } r_{ji}^{nn} = i.
\end{aligned}
\tag{1.7}
$$

With the inside-money mechanism, conditions (1.6) and (1.7) are weaker than (1.4) and (1.5) in the sense that they only pertain to meetings between nonbankers.

Note that (1.6) and (1.7) allow for both note issue and note redemption in meetings between bankers and nonbankers. Note issue occurs whenever a nonbanker without a note leaves such a meeting with a note. Note redemption occurs whenever a nonbanker with a note leaves such a meeting without one. Because a banker does not wish to hold notes, she always destroys them (they are redeemed).

Definition 3. *A* ***stationary and feasible mixed allocation*** *is* (x_i^k, y_{ij}^{kl}, p_{ij}^{kl}, q_{ij}^{kl}, r_{ij}^{kl}) *for* $i, j \in A$ *and* $k, l \in \{b, n\}$ *that satisfies (1.1) - (1.3) for* $i \in A$ *and* $k \in \{b, n\}$,

$$
\begin{aligned}
p_{ii}^{nn} &= q_{ii}^{nn} = r_{ii}^{nn} = i \\
p_{22}^{bl} &= q_{22}^{bl} = r_{22}^{bl} = 2 \\
p_{22}^{lb} &= q_{22}^{lb} = r_{22}^{lb} = 2
\end{aligned} \tag{1.8}
$$

$$
\begin{aligned}
p_{ij}^{nn} &= j \text{ if and only if } q_{ij}^{nn} = i \\
r_{ij}^{nn} &= j \text{ if and only if } r_{ji}^{nn} = i.
\end{aligned} \tag{1.9}
$$

$$
\begin{aligned}
p_{i2}^{lb} &= 2 \text{ if and only if } q_{i2}^{lb} \neq 2 \\
p_{i2}^{bl} &= 2 \text{ if and only if } q_{i2}^{bl} \neq 2 \\
q_{2i}^{bl} &= 2 \text{ if and only if } p_{2i}^{bl} \neq 2 \\
q_{2i}^{lb} &= 2 \text{ if and only if } p_{2i}^{lb} \neq 2 \\
r_{i2}^{lb} &= 2 \text{ if and only if } r_{2i}^{bl} \neq 2 \\
r_{i2}^{bl} &= 2 \text{ if and only if } r_{i2}^{bl} \neq 2
\end{aligned} \tag{1.10}
$$

for $i, j \in A$ *and* $l \in \{b, n\}$.

Condition (1.10) states that whenever someone acquires a unit of outside money, the trading partner must give it up. This is weaker than requiring the agents to swap states. It is easy to see that condition (1.8) is less restrictive than (1.4) (it applies to strictly fewer meetings) but is more restrictive than (1.7) (it pertains to some meetings involving bankers). For similar reasons, conditions (1.9) and (1.10) are less restrictive than (1.5) but more restrictive than (1.7). Conditions (1.8)-(1.10) allow for the issue and redemption of asset 1, the inside money, but not asset 2, the outside money.

1.3.2 Incentive Constraints

This subsection describes a general set of participation, truth-telling, and free-disposal constraints. The participation constraints reflect the fact that agents can always choose to reject the suggested outcome in the second stage of a meeting. The truth-telling constraints represent the fact that bankers can misrepresent their states whenever they are private information. Free-disposal constraints reflect the fact that an agent is not required to hold an asset.

I begin by describing the expected discounted utility for nondefecting bankers and nonbankers, and for undiscovered defecting bankers. These are all expressed given that no one else defects.

Let v_i^k denote the no-defection, expected discounted utility of an agent of information type k who is in state i at the start of a period. Suppose that everyone else follows the suggested outcome. For a single-coincidence meeting in which the producer is of information type k in state i and the consumer is of information type l in state j, let P_{ij}^{kl} and Q_{ij}^{kl} be producer and consumer payoffs, respectively, from following the suggested outcome in the second stage of a meeting. Then

$$P_{ij}^{kl} \equiv -y_{ij}^{kl} + \beta \sum_h p_{ij}^{kl}(h) v_h^k \tag{1.11}$$

and

$$Q_{ij}^{kl} \equiv u(y_{ij}^{kl}) + \beta \sum_h q_{ij}^{kl}(h) v_h^k. \tag{1.12}$$

For a no-coincidence meeting in which an agent is of information type k in state i and his partner in the meeting is of information type l in state j, let R_{ij}^{kl} be the payoff to the agent of information type k in state i from following the suggested outcome in the second stage of a meeting. Then

$$R_{ij}^{kl} \equiv \beta \sum_h r_{ij}^{kl}(h) v_h^k. \tag{1.13}$$

Given these definitions, v_i^k satisfies

$$v_i^k = \sum_j \frac{x_j^n}{S}[P_{ij}^{kn} + Q_{ji}^{nk} + (S-2)R_{ij}^{kn}] + \sum_j \frac{x_j^b}{S}[P_{ij}^{kb} + Q_{ji}^{bk} + (S-2)R_{ij}^{kb}]. \tag{1.14}$$

It is easy to show, via a contraction mapping argument, that there is a unique solution for v_i^k given an allocation.

I calculate recursively the initial-defector expected discounted utility, given that no one else defects. The defecting banker's payoff must include the option of misrepresenting herself in the first stage whenever her state is private information and she must also have the option of disagreeing to the suggested outcome in the second stage of a meeting. It must also reflect the fact that she knows with certainty that her first defection will be discovered T periods after it occurs and she will be punished with autarky from that date on. Let $\widetilde{v}^{b}_{i\tau}$ denote the expected discounted utility of a defecting banker who enters the period in state i and who first defected τ periods ago.

I now describe producer and consumer payoffs in single-coincidence meetings and payoffs from no-coincidence meetings for undiscovered defecting bankers who agree to the suggested outcome in the second stage. Suppose that everyone else follows the suggested outcome. Let $\widetilde{P}^{bl}_{mj\tau}$ and $\widetilde{Q}^{lb}_{jm\tau}$ be producer and consumer payoffs, respectively, to an undiscovered defecting banker who first defected τ periods ago and announced state m in the first stage of a single-coincidence meeting with a trading partner of information type l in state j from following the suggested outcome in the second stage. Then

$$\widetilde{P}^{bl}_{mj\tau} = -y^{bl}_{mj} + \beta \sum_{h} p^{bl}_{mj}(h) \widetilde{v}^{b}_{h,\tau+1} \tag{1.15}$$

and

$$\widetilde{Q}^{lb}_{jm\tau} = u(y^{lb}_{jm}) + \beta \sum_{h} q^{lb}_{jm}(h) \widetilde{v}^{b}_{h,\tau+1}. \tag{1.16}$$

Similarly, let $\widetilde{R}^{bl}_{mj\tau}$ be the payoff to an undiscovered defecting banker who first defected τ periods ago and announced state m in the first stage of a no-coincidence meeting with an agent of information type l who is in state j from following the suggested outcome in the second stage. Then

$$\widetilde{R}^{bl}_{mj\tau} \equiv \beta \sum_{h} r^{bl}_{mj}(h) \widetilde{v}^{b}_{h,\tau+1}. \tag{1.17}$$

The difference between (1.11)-(1.13) and (1.15)-(1.17) is that v_h^b is replaced with $\widetilde{v}_{h,\tau+1}^b$. The presence of τ reflects the fact that the continuation payoff of a defecting banker is dependent on the time left before discovery.

Now consider a banker in the first stage of a meeting whose true state is i. The announced state of an undiscovered defecting banker depends on the ability of that banker to misrepresent her state, what is known about the state of her trading partner, whether she is in a single-coincidence or no-coincidence meeting, and in the case of a single-coincidence meeting, whether she is a producer or consumer. Recall that $\widetilde{A} \subset A$ is the set over which a banker can misrepresent. Let I_i be an indicator variable that equals 1 if $i \in \widetilde{A}$ and is 0 otherwise. Thus, if $I_i = 1$, then a banker can announce a state in $\widetilde{A}$, while if $I_i = 0$, then she is constrained to report truthfully. Similarly, let J_j^l be an indicator variable that equals 1 if $j \in \widetilde{A}$ and is 0 otherwise, where j is the true state of the banker's trading partner. J_j^l not only depends on the true state of the trading partner, but also on his information type[8].

Let $\mu \in \{p, q, r\}$ denote the type of meeting a defecting banker is in. Thus, $\mu = p$ indicates that the defecting banker is a producer in a single-coincidence meeting, $\mu = q$ indicates that she is a consumer in a single-coincidence meeting and $\mu = r$ indicates that she is in a no-coincidence meeting. Then define $m_{ij}^{\mu l}(I_i, J_j^l)$ to be the optimal message of a defecting banker who is in meeting type μ, whose true state is i and whose trading partner is of information type l in state j.

---

[8] For example, $J_j^n = 0$ for all classes of mechanisms while J_j^b depends on the specific mechanism.

There are four types of optimal messages for a banker in a meeting of type μ: $m^{\mu l}_{ij}(0,0)$, $m^{\mu l}_{ij}(0,1)$, $m^{\mu l}_{ij}(1,0)$, and $m^{\mu l}_{ij}(1,1)$. Let $\widetilde{M} = \widetilde{P}^{bl}_{mj\tau}$ if $\mu = p$, $\widetilde{M} = \widetilde{Q}^{lb}_{jm\tau}$ if $\mu = q$, and $\widetilde{M} = \widetilde{R}^{bl}_{mj\tau}$ if $\mu = r$. Then the optimal messages are

$$\begin{aligned} m^{\mu l}_{ij}(0,0) &= i \\ m^{\mu l}_{ij}(0,1) &= i \\ m^{\mu l}_{ij}(1,0) &= \arg\max_{m} \widetilde{M} \\ m^{\mu l}_{ij}(1,1) &= \arg\max_{m} \sum_{i \in \widetilde{A}} \frac{x^l_j}{\sum_{j \in \widetilde{A}} x^l_j} \widetilde{M}. \end{aligned} \tag{1.18}$$

The first two messages reflect the fact that if a banker cannot misrepresent her type ($I_i = 0$) then she reports truthfully. The final two messages say that when given the freedom to misrepresent, a defecting banker chooses the state that gives the highest expected discounted utility; $m^{\mu l}_{ij}(1,0)$ indicates that the state of the trading partner is known with certainty while $m^{\mu l}_{ij}(1,1)$ indicates that what is known is that the state of the trading partner is an element of $\widetilde{A}$.

Finally, consider a defecting banker in the second stage. The announcements concerning states have been revealed and now a defecting banker chooses whether to agree to the suggested outcome. Let $\widehat{P}^{bl}_{\tau}(m^{pl}_{ij}(I_i, J^l_j), j) \equiv \widetilde{P}^{bl}_{mj\tau}$ such that $m = m^{pl}_{ij}(I_i, J^l_j)$. Similarly define $\widehat{Q}^{lb}_{\tau}(j, m^{ql}_{ij}(I_i, J^l_j))$ and $\widehat{R}^{bl}_{\tau}(m^{rl}_{ij}(I_i, J^l_j), j)$.

Then for $i \in A$ and $\tau \in \{1, 2, \ldots, T-1\}$, the expected discounted utility of an undiscovered defecting banker is

$$\begin{aligned}
\widetilde{v}^b_{i\tau} &= \sum_j \frac{x^n_j}{S} \max[\widehat{P}^{bn}_\tau(m^{pl}_{ij}(I_i, 0), j), \beta \widetilde{v}^b_{i,\tau+1}] \\
&+ \sum_j \frac{x^n_j}{S} \max[\widehat{Q}^{nb}_\tau(j, m^{ql}_{ij}(I_i, 0)), \beta \widetilde{v}^b_{i,\tau+1}] \\
&+ (S-2) \sum_j \frac{x^n_j}{S} \max[\widehat{R}^{bn}_\tau(m^{rl}_{ij}(I_i, 0), j), \beta \widetilde{v}^b_{i,\tau+1}] \\
&+ \sum_j \frac{x^b_j}{S} \max[\widehat{P}^{bb}_\tau(m^{pl}_{ij}(I_i, J^b_j), j), \beta \widetilde{v}^b_{i,\tau+1}] \\
&+ \sum_j \frac{x^b_j}{S} \max[\widehat{Q}^{bb}_\tau(j, m^{ql}_{ij}(I_i, J^b_j)), \beta \widetilde{v}^b_{i,\tau+1}] \\
&+ (S-2) \sum_j \frac{x^b_j}{S} \max[\widehat{R}^{bb}_\tau(m^{rl}_{ij}(I_i, J^b_j), j), \beta \widetilde{v}^b_{i,\tau+1}] \qquad (1.19)
\end{aligned}$$

with the terminal condition that

$$\widetilde{v}^b_{iT} = 0 \qquad (1.20)$$

for all $i \in A$.

Equation (1.19) takes into account the two decisions a defecting banker makes. At the first stage, a banker assumes that she will agree to the suggested outcome in the second stage, because the payoff to defecting at the second stage is autarky in the meeting and so is not dependent on the announcement in the first stage. The first three terms on the right-hand side of (1.19) pertain to meetings with nonbankers. Consider the first expression. The sum over $\frac{x^n_j}{S}$ pertains to the probability that a banker meets a nonbanker-consumer in state j. A defecting banker knows the state of a nonbanker and chooses $m_{ij}^{pl}(I_i, 0)$ in the first stage to maximize the payoff she would receive if she agrees in the second stage. At the second stage, given that she chooses $m_{ij}^{pl}(I_i, 0)$, she then compares the payoff to agreeing at the second stage, given that her trading partner is agreeing, $\widehat{P}_{\tau}^{bn}(m_{ij}^{pl}(I_i, 0), j)$, with the payoff to disagreeing, $\beta\widetilde{v}_{i,\tau+1}^{b}$. She chooses so that she receives the higher payoff. The second and third expressions on the right-hand side of (1.19) reflect the similar decision when a defecting banker is the consumer in a single coincidence meeting with a nonbanker and is in a no-coincidence meeting with a nonbanker, respectively. All three incorporate the fact that $J_j^n = 0$ in all possible meetings.

The final three terms on the right-hand side pertain to meetings with bankers. In this situation, J_j^b could be either 0 or 1, depending on the mechanism. In the case where $J_j^b = 1$, the banker chooses an announcement in the first stage that maximizes the *expected* payoff she receives if she agrees in the second stage. That is the only significant difference between the first three terms and the final three terms.

The expected discounted utility for a banker from an initial defection given that no one else defects, $\beta\widetilde{v}_{i1}^{b}$, is what is relevant for the incentive constraints. This is obtained by solving $\widetilde{v}_{i\tau}^{b}$ recursively from the terminal condition $\widetilde{v}_{iT}^{b} \equiv 0$. The terminal condition incorporates the fact that once discovered, a defecting banker is punished with autarky forever. It is easy to establish that $\widetilde{v}_{i1}^{b}$ is finite and unique.

Now consider the constraints that are relevant for implementation. There are three sets of constraints: participation, truth-telling, and free-disposal. Participation constraints require that agents are ex-post sequentially rational. This is equivalent to the requirement that they receive non-negative gains from trade. For nonbankers, the participation constraints are

$$\min\{P_{ij}^{nl}, Q_{ji}^{ln}, R_{ij}^{nl}\} \geq \beta v_i^n \tag{1.21}$$

for all $i, j \in A$ and $l \in \{b, n\}$. The right-hand side of (1.21) is due to the fact that defecting nonbankers can only be punished with no trade at that date because they will never be discovered.

For bankers, the participation constraints are

$$\min\{P_{ij}^{bl}, Q_{ji}^{lb}, R_{ij}^{bl}\} \geq \beta \widetilde{v}_{i1}^b \tag{1.22}$$

for all $i, j \in A$ and $l \in \{b, n\}$. Notice that for bankers that the right-hand side of (1.22) reflects the fact that if a banker disagrees to the suggested outcome, she does not trade at that date and becomes an initial defector.

Now consider truth-telling constraints on bankers whose states are not known. All that is required here is that bankers report their true state in the first stage of a meeting for all possible meetings:

$$m_{ij}^{\mu l}(I_i, J_j^l) = i \tag{1.23}$$

for all $i, j \in A$, $l \in \{b, n\}$ and $\mu \in \{p, q, r\}$.

Notice from (1.18) that $m_{ij}^{\mu l}(0, J_j^l) = i$ so that truth-telling constraints are necessarily satisfied and that $m_{ij}^{\mu l}(1, J_j^l)$ is the result of optimizing behavior and truth-telling constraints must be considered. Thus, with the outside-money mechanism, there are no truth-telling constraints because $\widetilde{A} = \emptyset$ and for defecting bankers $I_i = 0$ for all i[9]. For the inside-money mechanism, $\widetilde{A} = A$ and $I_i = 1$ for all i so that there are truth-telling constraints for all bankers. For the mixed mechanism, $\widetilde{A} = \{0, 1\}$ and so $I_i = 1$ when $i \in \widetilde{A}$ and truth-telling constraints must be satisfied for those meetings. When $i = 2$, $I_i = 0$, and no truth-telling constraints are needed.

Finally, the free-disposal constraints for nonbankers and nondefecting bankers are:

$$v_i^k \geq v_0^k \tag{1.24}$$

For undiscovered defecting bankers, the free-disposal constraints are:

$$\tilde{v}_{i\tau}^b \geq \tilde{v}_{0\tau}^b \tag{1.25}$$

for all $\tau \in \{1, 2, \ldots, T-1\}$.

The following definitions formalize the conditions necessary for an allocation to be implementable via each mechanism.

Definition 4. *An* ***implementable outside-money allocation*** *is* $(x_i^k,\ y_{ij}^{kl},\ p_{ij}^{kl},\ q_{ij}^{kl},\ r_{ij}^{kl})$ *for* $i, j \in A,\ k, l \in \{b, n\},\ \mu \in \{p, q, r\},\ \tau \in \{1, 2, ..., T\}$ *that satisfies Definition 1 and (1.11)-(1.25) where* $I_i = J_j^b = 0$ *for all* $i, j \in A$.

[9] One could drop the assumption that asset-holdings are common knowledge, so that there are truth-telling constraints when outside money is used in the above situation. The use of outside money would still have fewer truth-telling constraints than the use of inside money because it is not possible for individuals to claim to have a unit of an asset when they do not. Thus, the outside money mechanism would still have the fewest incentive constraints and retain the trade-off between feasibility constraints and incentive constraints.

Definition 5. *An **implementable inside-money allocation** is* $(x_i^k,\ y_{ij}^{kl},\ p_{ij}^{kl},\ q_{ij}^{kl},\ r_{ij}^{kl})$ *for* $i,j \in A,\ k,l \in \{b,n\},\ \mu \in \{p,q,r\},\ \tau \in \{1,2,...,T\}$ *that satisfies Definition 2, (1.11)-(1.23) where* $I_i = J_j^b = 1$ *for all* $i,j \in A$*, and (1.24) only for* $k = n$ *with* $i \in A$*.*

Definition 6. *An **implementable mixed allocation** is* $(x_i^k, y_{ij}^{kl}, p_{ij}^{kl}, q_{ij}^{kl}, r_{ij}^{kl})$ *for* $i,j \in A,\ k,l \in \{b,n\},\ \mu \in \{p,q,r\},\ \tau \in \{1,2,...,T\}$ *that satisfies Definition 3, (1.11)-(1.23) where* $I_2 = J_2^b = 0$ *and* $I_i = J_j^b = 1$ *for* $i,j \in \{0,1\}$*, and (1.24)-(1.25) only for* $k = n$ *with* $i \in A$ *and for* $k = b$ *with* $i = 2$*.*

The incentive constraints are most restrictive for the inside-money mechanism, and least restrictive for the outside-money mechanism. This is reflected by the statements regarding I_i and J_j^b in Definitions 4-6, which determine the relevant terms of (1.18) and affect both the value of (1.19) and the relevance of (1.23)[10]. There are two reasons for the difference. The first is that the outside-money mechanism has, in effect, no truth-telling constraints, the mixed mechanism has truth-telling constraints only for some bankers, and the inside-money mechanism has truth-telling constraints for every banker.

The second reason is that, because the different mechanisms have different numbers of truth-telling constraints, the value to a banker from making an initial defection, $\widetilde{v}_{i1}^b$, may also vary with the mechanisms. This leads to stricter participation constraints for bankers in mechanisms where $\widetilde{v}_{i1}^b$ is higher. In the outside-money mechanism, there are no truth-telling constraints so future defections can only occur at the second stage of a meeting. In both the inside-money and mixed mechanisms, a defecting banker can defect in both the first and second stage. In the inside-money mechanism, a banker can

[10]The conditions in Defintions 1-3 imply that the relevant free-disposal constraints in (1.24) and (1.25) are satisfied.

defect in the first stage regardless of her true state and she can announce any state in A. Thus, the expected discounted utility for a banker from an initial defection is highest in the inside money-mechanism and lowest in the outside-money mechanism.

1.4 Examples of Implementable Allocations

As mentioned in the introduction and demonstrated in Cavalcanti-Wallace (1999b), if the public record of histories is updated immediately ($T = 0$) so that bankers can be monitored perfectly, inside money has an advantage over outside money in that banker consumption does not have to be tied down to recent histories. Not only are the feasibility constraints less stringent with its use, but the incentive constraints are identical to those with outside money because there are effectively no truth-telling constraints. At the other extreme is where the public record is never updated. This leads to a value of defection that exceeds the value to being honest because histories are never updated. Bankers have an incentive to always issue notes for consumption and to never produce to redeem notes; they never have an incentive to tell the truth about their recent histories. Given that, no one would accept notes issued by bankers. Outside money is superior to inside money because it caries with it a recent trading history that cannot be misrepresented[11]; its use is not tied to banker reputation.

For both types of money to be essential, therefore, the updating lag must be neither too short nor too long. With an intermediate lag, bankers can be monitored

[11] Actually, the agents in the economy could use the initial steady-state stock of banknotes and not accept notes newly issued by bankers. Such an allocation can have trade, but the inside money is just playing the role of outside money; no new notes are ever issued and no existing notes are ever redeemed.

only imperfectly. Inside money has an advantage over outside money in that bankers can consume more frequently. However, there are more and tighter incentive constraints whenever inside money is used. Outside money has an advantage over inside money in that it is not tied to banker histories that can be misrepresented.

I now present three examples of allocations and compares their implementability via the three classes of monetary mechanisms. Each example takes as given the same parameters: $\{T, S, B, \beta, u(x)\}$, which include the length of the updating lag, the number of specialization types in the economy, the fraction of each specialization type that are bankers, the discount factor, and the specification of the period utility function. The parameter of most interest for this analysis is T, the length of the updating lag. Because of the need for an intermediate lag to find an allocation where the use of both outside and inside money are essential, I use numerical examples to establish the result.

In Example 1, both assets trade for more output in single-coincidence meetings than can be traded with inside money. This causes both truth-telling constraints and banker participation constraints to fail in the inside-money and mixed mechanisms. Example 2 cannot be duplicated by either the outside-money mechanism or the mixed mechanism because it does not satisfy the stricter feasibility constraints in Definitions 1 and 3. Example 3 cannot be duplicated due to a combination of the reasons given above. The outside-money mechanism cannot duplicate it because the example does not satisfy the stricter feasibility constraints in Definition 1. The inside-money mechanism cannot because the outside money in the mixed mechanism trades for more output than can be traded with inside money. This leads to the violation of several banker participation constraints. I use the three numerical examples to prove the following proposition.

Proposition 1.1. *There exist parameters, $\{T, S, B, \beta, u(x)\}$, such that there is no subset ordering of implementable allocations.*

For each example, the parameters are:

$$\{T, S, B, \beta, u(x)\} = \{65, 3, .1, .99, x^{\frac{1}{2}}\}.$$

Motivations for these choices are as follows. The choices of S, B, and β are somewhat arbitrary. The only importance given to the number of specialization types is that there are enough to eliminate the possibility of a double-coincidence of wants in a meeting. The minimum number of specialization types that accomplishes this is $S = 3$. The choice of a small fraction of each specialization type being bankers, B, is motivated by the use of a small B in Cavalcanti-Wallace (1999a). The choice of a relatively high discount factor, β, while important in determining the values of the expected utilities and T, is somewhat arbitrary.

The explicit utility function for the calculations is $u(x) = x^{\frac{1}{2}}$. Such a utility function has a very simple form that satisfies all of the assumptions made in Section 1.2. Note $u(x) = x^{\frac{1}{2}}$ implies that the solution to $\max u(y) - y$ is $y = 0.25$. I shall focus on attempting to implement this level of output, because it has been tied to several characterizations of welfare in these models[12].

Most important is the choice of T, the length of the updating lag. To arrive at the choice of 65, I first used a version of an implementable inside-money allocation

[12] See Cavalcanti-Wallace (1999a,b) and Kocherlakota-Wallace (1998).

presented in Cavalcanti-Wallace (1999a)[13]. Such an allocation provides a level of output consistent with $y = 0.25$. Then I searched for the minimum lag such that banker-producer participation constraints would be violated, given the value of the other parameters. This lag was $T = 65$. This gives me an idea of how much output is too much to be traded for inside money.

Now let us consider the specific examples. Each example will stipulate that no gifts be given in no-coincidence meetings ($r_{ij}^{kl} = i$). Example 1 has the feature that asset 2 is more valuable than asset 1. It trades for a higher output, and in single-coincidence meetings between nonbankers, consumers who hold a unit of asset 2 can swap assets with producers who hold a unit of asset 1 and receive some positive consumption. In single-coincidence meetings between bankers, there is positive output. In most of those meetings, banker-producers give gifts to banker-consumers and do not switch states, but in certain meetings the bankers do switch states. In the outside-money mechanism, this corresponds to a banker-consumer with a unit of outside money trading the unit to the banker-producer who does not have a unit of outside money in exchange for consumption.

Example 1. *Production in single-coincidence meetings is as follows:*

$$\begin{aligned}
y_{01}^{bb} &= y_{01}^{nn} = y_{01}^{bn} = y_{01}^{nb} = 0.25 \\
y_{02}^{bb} &= y_{02}^{nn} = y_{02}^{bn} = y_{02}^{nb} = 0.3 \\
y_{00}^{bb} &= y_{1j}^{bb} = y_{2j}^{bb} = 0.2 \\
y_{12}^{nn} &= 0.1
\end{aligned}$$

[13] See their Example 2 allocation.

and $y_{ij}^{kl} = 0$ *for all other meetings for* $i \in A$. *The following are the state transitions whenever they stipulate a new state for the agent:*

$$\begin{aligned} q_{01}^{bb} &= q_{01}^{bn} = q_{01}^{nb} = q_{01}^{nn} = q_{02}^{bb} = q_{02}^{bn} = q_{02}^{nb} = q_{02}^{nn} = 0 \\ p_{01}^{bb} &= p_{01}^{bn} = p_{01}^{nb} = p_{01}^{nn} = q_{12}^{nn} = 1 \\ p_{02}^{bb} &= p_{02}^{bn} = p_{02}^{nb} = p_{02}^{nn} = p_{12}^{nn} = 2 \end{aligned}$$

All other $p_{ij}^{kl} = i$ *and* $q_{ij}^{kl} = j$. *For all no-coincidence meetings,* $r_{ij}^{kl} = i$. *The fractions of each specialization type in each state are*

$$\begin{aligned} x_0^n &= 0.45; x_1^n = x_2^n = 0.225 \\ x_0^b &= 0.05; x_1^b = x_2^b = 0.025. \end{aligned}$$

It is easy to verify that Example 1 satisfies the feasibility restrictions in Definition 1, and, as a direct consequence, those in Definitions 2 and 3. To check the incentive constraints, we need to know the expected discounted utility for nonbankers, bankers, and defecting bankers who defect for the first time. Table 1.1 reproduces those values for the above example. Recall that the expected discounted utility to initial defectors is contingent on the type of mechanism used. Let $\widetilde{v}_{i1}^b(O)$ be the expected discounted value to a banker of making an initial defection when the outside-money mechanism is used. Similarly define $\widetilde{v}_{i1}^b(I)$ for the inside-money mechanism and $\widetilde{v}_{i1}^b(M)$ for the mixed mechanism.

Table 1.1. Value Functions for Example 1

State	v_i^n	v_i^b	$\widetilde{v}_{i1}^b(O)$	$\widetilde{v}_{i1}^b(I)$	$\widetilde{v}_{i1}^b(M)$
0	2.272	2.512	1.361	4.684	4.359
1	2.626	2.849	1.708	4.684	4.359
2	2.748	2.895	1.754	4.684	4.398

Given the expected discounted utilities in Table 1.1, I can verify that all of the participation constraints are satisfied for the outside-money mechanism and so Definition 4 is satisfied. Thus, Example 1 is implementable via the outside-money mechanism. Such is not the case for the inside-money mechanism and the mixed mechanism as Table 1.1 suggests. Under both of those mechanisms, $\widetilde{v}_{i1}^b > v_i^b$ for any i and all banker participation constraints and all relevant truth-telling constraints are violated. It is obvious that (1.23) is violated whenever the mechanism calls for production by a banker because $\widetilde{v}_{i1}^b(I)$ is independent of state, and a defecting banker can announce any state in A. It turns out that participation constraints for banker producers are violated as well. Banker truth-telling constraints are violated because banker consumers in different states receive significantly different outcomes, and banker producers in different states receive significantly different outcomes. This creates incentives to misrepresent their state because they can defer punishment well enough into the future. Thus, Definitions 5 and 6 are violated for Example 1, and it is only implementable via the outside-money mechanism.

Example 2, like Example 1 has the second asset more valuable than the first. In meetings between nonbankers, the second asset trades for more output than the first, and consumers who hold a unit of asset 2 can swap assets with producers who hold a unit of asset 1 and receive some positive consumption. In contrast with Example 1, the states of the bankers never change in any type of meeting. Banker-producers give gifts to banker-consumers, and produce for nonbankers who have a unit of either asset. In the inside-money mechanism, the latter is equivalent to bankers redeeming notes from nonbankers (the bankers then destroy the notes). Nonbanker-producers produce output for banker-consumers, regardless of state. In the inside-money mechanism, this is equivalent to bankers, regardless of state, issuing a note to a nonbanker for output.

Example 2. *Production in single-coincidence meetings is as follows:*

$$
\begin{aligned}
y_{ij}^{bb} &= y_{01}^{nn} = y_{i2}^{bn} = y_{0i}^{nb} = 0.2 \\
y_{02}^{nn} &= 0.25 \\
y_{i1}^{bn} &= 0.15 \\
y_{12}^{nn} &= 0.1
\end{aligned}
$$

and $y_{ij}^{kl} = 0$ *for all other meetings for* $i \in A$ *. The following are the state transitions whenever they stipulate a new state for the agent:*

$$
\begin{aligned}
q_{01}^{nn} &= q_{02}^{nn} = q_{i1}^{bn} = q_{i2}^{bn} = 0 \\
p_{01}^{nn} &= p_{00}^{nb} = p_{01}^{nb} = q_{12}^{nn} = 1 \\
p_{02}^{nn} &= p_{02}^{nb} = p_{12}^{nn} = 2
\end{aligned}
$$

All other $p_{ij}^{kl} = i$ *and* $q_{ij}^{kl} = j$*. For all no-coincidence meetings,* $r_{ij}^{kl} = i$*. The fractions of each specialization type in each state are*

$$\begin{aligned} x_0^n &= 0.45; x_1^n = x_2^n = 0.225 \\ x_0^b &= x_1^b = 0.025; x_2^b = 0.05. \end{aligned}$$

It is easy to verify that an Example 2 allocation satisfies Definition 2, but violates Definitions 1 and 3. The fact that bankers remain in the same state in which they entered in single-coincidence meetings with nonbankers that leave their original state violates the feasibility constraints whenever outside money is used. For example, $p_{02}^{nb} = 2$ and $q_{02}^{nb} = 2$ violates (1.5) for the outside-money mechanism and (1.10) for the inside-money mechanism when outside money is used.

Finally, I must demonstrate that an Example 2 allocation is implementable via the inside-money mechanism. Table 1.2 reports the expected discounted utilities for nonbankers, bankers, and initial banker defectors.

Table 1.2. Value Functions for Example 2

State	v_i^n	v_i^b	$\widetilde{v}_{i1}^b(O)$	$\widetilde{v}_{i1}^b(I)$	$\widetilde{v}_{i1}^b(M)$
0	2.453	4.907	3.890	3.890	3.890
1	2.751	4.907	3.890	3.890	3.890
2	2.871	4.907	3.890	3.890	3.890

From these values we can verify that all of the participation and truth-telling constraints are satisfied and therefore, Definition 5 is satisfied. I conclude that Example 2 is implementable only in the inside-money mechanism.

In Example 3, as in the other two, asset 2 is more valuable than asset 1 and banker-producers always produce for banker-consumers. Like Example 1, most banker-producers give gifts of output to banker-consumers, but in some meetings between bankers, they leave their states. In the mixed mechanism, this is a banker-consumer with a unit of outside money giving it to a banker-producer in exchange for output. Like Example 2, banker-producers produce for nonbankers who have a unit of either asset and nonbanker-producers produce output for banker-consumers, regardless of state. In the mixed mechanism, the former is equivalent to bankers redeeming notes (if the nonbanker state is 1) or acquiring outside money (if the nonbanker state is 2). The latter is equivalent to bankers issuing a note (if the banker is not in state 2) or surrendering a unit of outside money (if the banker is in state 2). Thus, banker states are important here in the sense that outside money exchanges for more output than inside money.

Example 3. *Production in single-coincidence meetings is as follows:*

$$
\begin{aligned}
y_{00}^{bb} &= y_{01}^{bb} = y_{10}^{bb} = y_{11}^{bb} = y_{2j}^{bb} = y_{01}^{nn} = y_{i1}^{bn} = y_{00}^{nb} = y_{01}^{nb} = 0.2 \\
y_{02}^{bb} &= y_{12}^{bb} = y_{02}^{nn} = y_{02}^{bn} = y_{12}^{bn} = y_{02}^{nb} = 0.25 \\
y_{12}^{nn} &= 0.1
\end{aligned}
$$

and $y_{ij}^{kl} = 0$ *for all other meetings for* $i \in A$. *The following are the state transitions whenever they stipulate a new state for the agent:*

$$\begin{aligned} q_{02}^{bb} &= q_{12}^{bb} = q_{01}^{nn} = q_{02}^{nn} = q_{i1}^{bn} = q_{02}^{bn} = q_{12}^{bn} = q_{02}^{nb} = 0 \\ p_{01}^{nn} &= p_{01}^{bn} = p_{00}^{nb} = p_{01}^{nb} = q_{12}^{nn} = 1 \\ p_{02}^{bb} &= p_{12}^{bb} = p_{02}^{nn} = p_{12}^{nn} = p_{02}^{bn} = p_{12}^{bn} = p_{02}^{nb} = 2 \end{aligned}$$

All other $p_{ij}^{kl} = i$ *and* $q_{ij}^{kl} = j$. *For all no-coincidence meetings* $r_{ij}^{kl} = i$. *The fractions of each specialization type in each state are*

$$\begin{aligned} x_0^n &= 0.36; x_1^n = 0.18; x_2^n = 0.36 \\ x_0^b &= 0.0333; x_1^b = 0.0167; x_2^b = 0.05. \end{aligned}$$

Inspection of Example 3 shows that it satisfies the conditions in Definition 3, and therefore, Definition 2, because there are fewer feasibility constraints for the inside-money mechanism than for the mixed mechanism. By a similar argument, it is easy to verify that Definition 1 is not satisfied for the Example 3 allocation. There are more feasibility constraints for the outside-money mechanism than for the mixed mechanism. In the Example 3 allocation there exist single-coincidence meetings in which bankers and nonbankers do not swap states, even though nonbankers leave their original state. For example, $p_{01}^{nb} = 1$ and $q_{01}^{nb} = 1$ violates (1.5) for the outside-money mechanism. Such an allocation, therefore, cannot be implemented via the outside-money mechanism.

Table 1.3 provides the relevant expected discounted utilities for the Example 3 allocation. These values can be used to verify that the Example 3 allocation satisfies

Table 1.3. Value Functions for Example 3

State	v_i^n	v_i^b	$\widetilde{v}_{i1}^b(O)$	$\widetilde{v}_{i1}^b(I)$	$\widetilde{v}_{i1}^b(M)$
0	2.432	3.742	3.253	3.595	3.253
1	2.712	3.742	3.253	3.595	3.253
2	2.821	3.877	3.303	3.595	3.303

the participation constraints and relevant truth-telling constraints that are required by Definition 6, and so the allocation is implementable via the use of one outside money and one inside money. Finally, we can use Table 1.3 to verify that the inside-money mechanism cannot implement Example 3. In fact, certain banker-producer participation constraints are violated. Specifically, in meetings with nonbanker-consumers who enter with a unit of asset 1, bankers in states 0 and 1 are not willing to produce the amount called for by the allocation. Also, in meetings with banker-consumers, bankers in states 0 and 1 are not willing to produce for other bankers in state 0 or 1.

1.5 Conclusion

This paper has presented a framework for understanding the features of an environment that make both outside money and inside money essential as means of payment. Both must coexist for specific parameters to implement allocations that reflect a trade-off between inside money's ability to increase the frequency of banker consumption (because bankers can be monitored) and outside money's ability to avoid incentive problems that lower the amount of consumption that bankers can receive when issuing inside money (because monitoring bankers is imperfect). Additionally, for those specific parameters, there are allocations that can only be implemented via outside money, and those that can only be implemented via inside money.

In deriving the results, I make use of the assumption that agents can only hold one unit of one asset at a time. I conjecture that this assumption is not crucial. This is because what makes both types of money essential is the trade-off between outside and inside money. This trade-off would still exist if the assumption about the unit upper bound on money-holdings were dropped. The issue and redemption of inside money would still be subject to the imperfect monitoring of bankers, implying that the value of inside money is less than that of outside money. Nonetheless, the issuance of inside money would still provide bankers with liquidity that permits them to consume more frequently.

Given that there is no subset ordering of the three monetary allocations, the logical next step in this analysis is to compare the welfare attainable under all three mechanisms. To strengthen the claim that both outside and inside money are essential as means of payment would then require, for specific parameters, that there exist mixed allocations that achieve a level of welfare that is higher than the highest level of welfare achieved by using outside money only and by using inside money only. The surmise is that there exist parameters, specifically with a lag that is neither too short nor too long, for which the trade-off between inside money and outside money makes the use of both desirable in terms of this welfare criterion. The difficulty is finding, for given parameters, the highest level of welfare attainable via outside money alone, and that attainable via inside money alone because there are 36 potentially distinct single-coincidence meetings, and adjusting the level of output in one such meeting affects the incentive constraints.

This framework is useful for further study of outside and inside money. Its main advantage is that it links the possiblity of overissue of inside money to what is known about the histories and reputations of the issuers. Further study might look at alternative complications in the ability to monitor banker reputations. For example, bankers could possess idiosyncrasies, or there could be some aggregate uncertainty concerning production or consumption shocks. Additionally, one could explore regulatory design in terms of alternative trigger strategies than the one proposed here for discovered defections. One could then study how banker incentives change under different circumstances, and how that impacts the relative desirability of outside and inside money.

Chapter 2

The Size of Meetings and the Role of Money

2.1 Introduction

Double-coincidence problems are necessary for money to be essential. Random-matching models such as Kiyotaki-Wright (1989), Trejos-Wright (1995) and Shi (1995) depict such situations by (i) having people meet randomly in pairs, (ii) having them be specialized in both production and consumption, and (iii) having goods be perishable. Alternatively, if (i) were replaced by a meeting of everyone at each date, then there would be no double-coincidence problem at all and money would not be needed. This paper fills the gap between these two extremes by making the number of people in a random meeting, denoted N, a parameter. I prove that as $N \to \infty$, the benefit of money disappears.

The competitive allocation for an economy in which everyone is together at each date provides an upper bound on expected utility under random meetings. I give an example of a nonmonetary allocation that is immune to group defections and show that as N gets large, the expected lifetime utility attained via this allocation approaches the upper bound. The makeup of meetings is determined randomly via the multinomial distribution. My proof uses a result on the distribution of the minimum number of a type for the multinomial distribution.

The chapter is organized as follows. Section 2.2 describes the environment and an upper bound on utility. Section 2.3 presents the main result. Section 2.4 concludes.

2.2 The Environment

Time is discrete and infinite. There are K divisible and perishable types of goods at each date and a [0,1] continuum of each of K specialization types of people, where $K \geq 3$.

A type k person consumes good k and produces good $k+1$ (addition is modulo K). Each person maximizes expected discounted utility with a discount factor $\beta \in (0,1)$. Utility in a period for a type k person is $u(c) - v(y)$, where c is the amount of good k that a type k agent consumes and y is the amount of good $k+1$ that a type k agent produces. The function u is twice differentiable and satisfies $u' > 0$, $u'' < 0$, $u(0) = 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. The function v is twice differentiable and satisfies $v' > 0$, $v'' > 0, v(0) = 0$, $v'(0) = 0$, and there exists $y^{\max}$ such that $v(y) \to \infty$, as $y \to y^{\max}$.

Agents cannot commit to future actions and the trading histories of agents are private information. This rules out any possibility for credit arrangements to exist. Because credit arrangements are a substitute for money, ruling them out gives money the best chance to be essential in this environment.

At each date, people are randomly matched into a group of N individuals, where $N \geq 2$. Each agent is in only one such meeting per date. The specialization type (henceforth type) of each agent is common knowledge in a meeting.

Formally, define $I \subset \Re_+^K$ as the set of *meeting profiles* of size N. That is,

$$I = \{(i_1, i_2, \ldots, i_K) : i_k \geq 0 \text{ and } \sum i_k = N\} \tag{2.1}$$

where i_k denotes the number of agents of type k in the meeting. The probability that a particular meeting profile, $\bar{i} \in I$, occurs is given by the multinomial distribution with equal probabilities:

$$\Pr(i = \bar{i}) = \frac{N!(\frac{1}{K})^N}{\prod_{k=1}^{K} i_k(\bar{i})!}. \tag{2.2}$$

The standard random-matching model of money is the special case of the above model where $N = 2$ and there is a complete lack of double coincidences. For $N > 2$, however, it is possible that some agents in a meeting have a double-coincidence of wants. Those agents can receive current consumption to induce them to produce. Thus, larger meeting sizes, which increase the likelihood that agents have a double-coincidence, can alleviate the problem that money is intended to alleviate.

To prove the limit result, I use the following result, derived by Johnson and Young (1960), concerning the distribution of the extreme values, $\frac{i_{\min}}{N}$ and $\frac{i_{\max}}{N}$, where $i_{\min} = \min\{i_1, i_2, \ldots, i_K\}$, and $i_{\max} = \max\{i_1, i_2, \ldots, i_K\}$ for a meeting profile.

Lemma 2.1. *Consider the multinomial distribution with K equally-probable types and sample size N. Let $\upsilon_1, \ldots, \upsilon_K$ be a sample of K independent unit normal random variables with sample mean $\overline{\upsilon}$. Define $\upsilon_{\min} = \min\{\upsilon_1, \ldots, \upsilon_K\} - \overline{\upsilon}$. For any $\alpha \in (0,1)$,* *define* V_α *such that* $\Pr(\upsilon_{\min} < -V_\alpha) = \alpha$. *Then,*

$$\Pr(\frac{i_{\min}}{N} < \frac{1}{K} - (\frac{V_\alpha}{(NK)^{\frac{1}{2}}} + \frac{1}{2N})) = \alpha. \tag{2.3}$$

For my result, I need a high probability that meetings contain roughly the same number of agents of each specialization type. In other words, I want to get sufficiently close to the scenario where every meeting profile at every date is balanced (has an equal number of each type), so that there is no double-coincidence problem and money is not needed. I want to find a sufficiently large N that simultaneously makes α and $\frac{V_\alpha}{(NK)^{\frac{1}{2}}} + \frac{1}{2N}$ arbitrarily small. This can be done because, by Lemma **??**, V_α is determined only by α and K, and so is independent of N. Because K is a parameter of the model, the choice of α determines V_α. This makes it possible to find an N associated with an arbitrarily small α such that $\frac{V_\alpha}{(NK)^{\frac{1}{2}}} + \frac{1}{2N}$ is also arbitrarily small. Because I shall refer to it later, let $f(\alpha, N) \equiv \frac{V_\alpha}{(NK)^{\frac{1}{2}}} + \frac{1}{2N}$.

Money is inessential if it cannot achieve allocations that are more desirable than what is achievable via nonmonetary allocations. To prove that money is inessential, I first characterize an upper bound on the symmetric ex-ante expected discounted utility of an arbitrary agent. The symmetry imposed is that all agents receive the same ex-ante expected discounted utility regardless of both their type and identity within that type. Because nothing is more desirable than this upper bound, money is inessential if there exists an implementable nonmonetary allocation that provides symmetric ex-ante expected discounted utility that is arbitrarily close to this upper bound. An allocation is implementable if it is immune to group defections.

Random meetings are a friction of the environment that limits what can be done in meetings. This is because what is feasible in a meeting depends on the particular meeting profile. An upper bound on what can be achieved in a certain meeting is what can be attained if the entire economy were in one meeting. Given the symmetry imposed, the best that can be achieved with everyone together is that everyone produce and consume y^* at every date, where y^* is the unique solution to $u'(y^*) = v'(y^*)$.[1] Therefore, an upper bound on ex-ante expected discounted utility is

$$W^* = \frac{w(y^*)}{1-\beta}, \tag{2.4}$$

where $w(y^*) \equiv u(y^*) - v(y^*)$. For my purpose, it is sufficient to show that an agent's expected period utility is arbitrarily close to $w(y^*)$ at every date when N is sufficiently large.

2.3 The Limit Result

I propose a nonmonetary allocation, called the *Walrasian-autarky allocation,* which is the static Walrasian allocation for each meeting profile with at least one agent of every type, and is autarky otherwise. As is well-known, such an allocation is indeed immune to group defections; that is, it is in the meeting-specific core. I then prove that as $N \to \infty$, the expected period utility of any agent approaches $w(y^*)$. This is done via two propositions. The first proposition shows that for meetings that are sufficiently balanced, the Walrasian equilibrium outcome is such that period utility is close to $w(y^*)$.

[1] The outcome that everyone produce and consume y^* is a Walrasian equilibrium allocation of a K-good economy in which everyone is together at every date. See Proposition 2.1.

The second proposition then shows that as $N \to \infty$, expected period utility approaches $w(y^*)$ because as N gets large, the probability that a meeting is sufficiently balanced approaches unity.

For my purposes, I need the existence of a Walrasian equilibrium only for meeting profiles in which $i_{\min}(i) > 0$. I use properties of the excess demand function for such meeting profiles to prove the existence of Walrasian equilibria. For a meeting profile i, let $c_k(i), y_k(i) \in \Re_+$, and $p_k(i) \in \Re_{++}$ be the consumption of good k by an agent of type k, the production of good k by an agent of type $k-1$, and the price of good k, respectively, for $k = 1, 2, ..., K$. The maximization problem for an agent of type k in the meeting is:

$$\max_{c_k, y_{k+1}} u(c_k) - v(y_{k+1})$$

subject to:

$$p_k c_k \le p_{k+1} y_{k+1}$$

Because u is concave, v is convex, and the constraint set is linear, I can invoke the Kuhn-Tucker conditions as necessary and sufficient for a maximum so that the first order condition is

$$u'(c_k) - \frac{p_k}{p_{k+1}} v'(\frac{p_k}{p_{k+1}} c_k) = 0. \tag{2.5}$$

The solution to (2.5) represents the demand function of an agent of type k for good k. Let $c_k = \phi(p_k, p_{k+1})$ denote this demand function. Because u and v are both C^2, ϕ is C^1. Totally differentiating (2.5) and using the Implicit Function Theorem, I find that

$$\phi_1 = \frac{\frac{1}{p_{k+1}}v' + \frac{p_k}{p_{k+1}^2}v''c_k}{u'' - \frac{p_k^2}{p_{k+1}^2}v''} < 0$$

$$\phi_2 = \frac{-[\frac{p_k}{p_{k+1}^2}v' + \frac{p_k^2}{p_{k+1}^3}v''c_k]}{u'' - \frac{p_k^2}{p_{k+1}^2}v''} > 0 \tag{2.6}$$

where ϕ_i is the derivative of ϕ with respect to its ith argument. From the budget constraint $y_{k+1} = \frac{p_k}{p_{k+1}}\phi(p_k, p_{k+1})$. Therefore, the per capita excess demand for good k is

$$z_k(\mathbf{p}) = \frac{i_k}{N}\phi(p_k, p_{k+1}) - \frac{i_{k-1}}{N}\frac{p_{k-1}}{p_k}\phi(p_{k-1}, p_k) \tag{2.7}$$

where $\mathbf{p} = (p_1, p_2, ..., p_K)$. Let $z(\mathbf{p}) = (z_1(\mathbf{p}), z_2(\mathbf{p}), ..., z_K(\mathbf{p}))$.

The following lemma shows that a Walrasian equilibrium exists for meeting profiles where every type is represented.

Lemma 2.2. *If i is such that $i_{\min}(i) > 0$, then there exists a Walrasian equilibrium.*

Proof. I appeal to an existence result found in Mas-Colell, Whinston and Green (1995)[2]. It entails verifying the following properties of the excess demand function.

1. z is continuous

2. z is homogeneous of degree zero

[2]See pages 585-587.

3. $\mathbf{p} \cdot z(\mathbf{p}) = 0$ for all $\mathbf{p}$

4. There is an $s > 0$ such that $z_k(\mathbf{p}) > -s$ for every commodity k and all $\mathbf{p}$.

5. If $\mathbf{p}^n \to \mathbf{p}$, where $\mathbf{p} \neq 0$ and $p_k = 0$ for some k, then

$$\max\{z_1(\mathbf{p}^n), \ldots, z_K(\mathbf{p}^n)\} \to \infty.$$

Continuity follows directly from the continuity of ϕ. Homogeneity of degree zero follows from individual demand functions being homogeneous of degree zero. Therefore, $\mathbf{p} \cdot z(\mathbf{p}) = 0$, also holds. The property that the aggregate excess demand function is bounded below comes from the fact that production is bounded from above at $y^{\max}$. The final property stems from the fact that because there is at least one agent of every type present, there is always someone who will demand a good whose price is near zero. Suppose $p_k = 0$ and the price vector $\mathbf{p} \neq 0$. Then there exists a $p_k = 0$ and a $p_{k+1} \neq 0$ such that, by inspection of (2.5), as $p_k^n \to 0$, agent k's demand function is arbitrarily close to $u'(c_k) = 0$ and $c_k = \phi(p_k, p_{k+1}) = \infty$. Because $y^{\max}$ is the most that an agent of type $k - 1$ would produce, $z_k(\mathbf{p}^n)$ will tend towards infinity. ■

Given that a Walrasian equilibrium exists from meeting profiles in which $i_{\min}(i) > 0$ and that Walrasian allocations are core allocations, the Walrasian-autarky allocation is immune to group defections if autarky is a core allocation for meeting profiles in which $i_{\min}(i) = 0$. This is fairly trivial. It amounts to arguing that there is no coalition that can give at least one of its members positive period utility in the meeting without someone else receiving negative period utility. To see this, suppose that k is the type that is not represented in the meeting. Then there is no good $k+1$ for a person of type $k+1$ to consume. So the type $k+1$ agents do not produce type $k+2$ goods, etc.

I now turn to the limit result. Proposition 2.1 shows that if a meeting profile is sufficiently balanced, then the corresponding Walrasian equilibrium outcome is such that period utility is arbitrarily close to $w(y^*)$.

PROPOSITION 2.1. *For any $\varepsilon > 0$, there exists $\delta > 0$ such that if $\left|\frac{i_{\min}(i)}{N} - \frac{1}{K}\right| \leq \delta$ for meeting profile i, then a corresponding Walrasian equilibrium outcome is such that $u(c_k(i)) - v(y_{k+1}(i)) \geq w(y^*) - \varepsilon$ for all k.*

Proof. The first part of the proof shows that $\mathbf{p}^* = (p^*, p^*, ..., p^*)$ is an equilibrium price vector for a balanced meeting profile that gives period utility of $w(y^*)$ to everyone in the meeting. Because inspection of (2.5) reveals that $\phi(p^*, p^*) = y^*$ and so period utility is $w(y^*)$ for every agent in the meeting, I just need to verify that $z(\mathbf{p}^*) = 0$ where z is the per capita aggregate excess demand function for a balanced meeting profile. Using the fact that $i_k = i_{k-1}$ for all k for a balanced meeting profile, the per capita excess demand for any good k is

$$\frac{1}{K}\phi(p^*, p^*) = \frac{1}{K}\frac{p^*}{p^*}\phi(p^*, p^*) \tag{2.8}$$

and so $z(p^*) = 0$.

The next part of the proof establishes the link between $\frac{i_{\min}(i)}{N} \to \frac{1}{K}$ and $\mathbf{p}^i \to \mathbf{p}^*$, where $\mathbf{p}^i$ is the equilibrium price vector for the meeting profile i. If $\mathbf{p}^i \to \mathbf{p}^*$ then $u(c_k(i)) - v(y_{k+1}(i)) \to w(y^*)$ for all k. I show this by appealing to a result in Proposition 5.4.3 in Mas-Colell (1985), which gives conditions for when this is the case. It turns out that I only need to show that z, the excess demand function for a balanced meeting profile, is C^1 and rank $\partial z(\mathbf{p}^*) = K - 1$.

The proof that z is C^1 is trivial since u and v are both assumed to be C^2. For the proof that the rank $\partial_p z(\mathbf{p}^*) = K - 1$, note that for any k, z_k is a function of only three prices, p_{k-1}, p_k, and p_{k+1}. From this, the following are partial derivatives of z_k, evaluated at $\mathbf{p}^*$:

$$\begin{aligned}
\frac{\partial z_k}{\partial p_{k-1}}|_{\mathbf{p}^*} &= -\frac{1}{K}[y^* + \phi_1] \\
\frac{\partial z_k}{\partial p_k}|_{\mathbf{p}^*} &= \frac{1}{K}[y^* + \phi_1 - \phi_2] \\
\frac{\partial z_k}{\partial p_{k+1}}|_{\mathbf{p}^*} &= \frac{1}{K}\phi_2
\end{aligned}$$

Note from (2.6) that, evaluated at $\mathbf{p}^*$,

$$\begin{aligned}
\phi_1 &= \frac{\frac{1}{p^*}[v'(y^*) + v''(y^*)y^*]}{u''(y^*) - v''(y^*)} < 0 \\
\phi_2 &= \frac{-\frac{1}{p^*}[v'(y^*) + v''(y^*)y^*]}{u''(y^*) - v''(y^*)} > 0
\end{aligned}$$

Notice that $\phi_1 = -\phi_2$ and that I can simplify the partial derivatives of z_k by:

$$\begin{aligned}
\frac{\partial z_k}{\partial p_{k-1}}|_{\mathbf{p}^*} &= -\frac{1}{K}[y^* - \phi_2] \\
\frac{\partial z_k}{\partial p_k}|_{\mathbf{p}^*} &= \frac{1}{K}[y^* - 2\phi_2] \\
\frac{\partial z_k}{\partial p_{k+1}}|_{\mathbf{p}^*} &= \frac{1}{K}\phi_2
\end{aligned}$$

It is easy to show that $y^* - 2\phi_2$ and hence $y^* - \phi_2$ are positive[3]. Note that from this $\frac{\partial z_k}{\partial p_{k-1}}|_{\mathbf{p}^*} < 0$, $\frac{\partial z_k}{\partial p_k}|_{\mathbf{p}^*} > 0$, and $\frac{\partial z_k}{\partial p_{k+1}}|_{\mathbf{p}^*} > 0$. Then $\partial_p z(\mathbf{p}^*)$ for $\mathbf{p}^* \in \Re^K_{++} =$

$$\frac{1}{K}\begin{bmatrix} y^*-2\phi_2 & \phi_2 & 0 & \cdots & 0 & 0 & -(y^*-\phi_2) \\ -(y^*-\phi_2) & y^*-2\phi_2 & \phi_2 & \cdots & 0 & 0 & 0 \\ 0 & -(y^*-\phi_2) & y^*-2\phi_2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & y^*-2\phi_2 & \phi_2 & 0 \\ 0 & 0 & 0 & \cdots & -(y^*-\phi_2) & y^*-2\phi_2 & \phi_2 \\ \phi_2 & 0 & 0 & \cdots & 0 & -(y^*-\phi_2) & y^*-2\phi_2 \end{bmatrix}$$

By normalizing $p^*_K \equiv 1$, I can focus attention on $\mathbf{p}^* \in \Re^{K-1}_{++}$ by removing both the last row and last column of the matrix above. Then $\partial_p z(\mathbf{p}^*)$ for $\mathbf{p}^* \in \Re^{K-1}_{++} =$

$$\frac{1}{K}\begin{bmatrix} y^*-2\phi_2 & \phi_2 & 0 & \cdots & 0 & 0 & 0 \\ -(y^*-\phi_2) & y^*-2\phi_2 & \phi_2 & \cdots & 0 & 0 & 0 \\ 0 & -(y^*-\phi_2) & y^*-2\phi_2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & y^*-2\phi_2 & \phi_2 & 0 \\ 0 & 0 & 0 & \cdots & -(y^*-\phi_2) & y^*-2\phi_2 & \phi_2 \\ 0 & 0 & 0 & \cdots & 0 & -(y^*-\phi_2) & y^*-2\phi_2 \end{bmatrix}$$

[3]The requirement that $y^* > \frac{2(v'+v''y^*)}{p^*(u''-v'')}$ reduces to $y^* > \frac{2v'}{p^*u''-(p^*+2)v''}$, a negative number. Obviously y^* must be positive and so the requirment is satisfied.

To show that rank $\partial_p z(\mathbf{p}^*) = K - 1$, I need only show that this matrix is nonsingular. I argue by induction. Throughout the argument, I ignore the $\frac{1}{K}$ term because it has no influence on the signs of the determinants. First note that the following 2×2 matrix is nonsingular:

$$\begin{bmatrix} y^* - 2\phi_2 & \phi_2 \\ -y^* + \phi_2 & y^* - 2\phi_2 \end{bmatrix}$$

This can be shown via the determinant, which reduces to:

$$(y^* - 2\phi_2)^2 + (y^* - \phi_2)\phi_2 > 0$$

The 2×2 matrix is, therefore, nonsingular, because $(y^* - \phi_2) > 0$. Now consider the following 3×3 matrix:

$$\begin{bmatrix} y^* - 2\phi_2 & \phi_2 & 0 \\ -(y^* - \phi_2) & y^* - 2\phi_2 & \phi_2 \\ 0 & -(y^* - \phi_2) & y^* - 2\phi_2 \end{bmatrix}$$

The determinant of this matrix is simply $y^* - 2\phi_2$ times the determinant of the 2×2, a positive value, minus:

$$\phi_2 \cdot \begin{vmatrix} -(y^* - \phi_2) & \phi_2 \\ 0 & y^* - 2\phi_2 \end{vmatrix}$$

which is clearly negative. So the determinant of the 3×3 matrix is positive.

Now consider the $(K+1) \times (K+1)$ matrix that is composed of both a $K \times K$ matrix and a $(K-1) \times (K-1)$ matrix, each with a positive determinant. I now show that the determinant of the $(K+1) \times (K+1)$ matrix is also positive. The determinant of the $(K+1) \times (K+1)$ matrix takes the following form:

$$(y^* - 2\phi_2) \cdot |K \times K| - \phi_2 \begin{vmatrix} -(y^* - \phi_2) & \phi_2 & \mathbf{0} \\ \mathbf{0} & (K-1) \times (K-1) & \end{vmatrix}$$

This reduces to

$$(y^* - 2\phi_2) \cdot |K \times K| + \phi_2(y^* - \phi_2) \cdot |(K-1) \times (K-1)|$$

which is clearly positive since the determinant of both the $K \times K$ matrix and the $(K-1) \times (K-1)$ matrix are positive. Thus, $\partial_p z(\mathbf{p}^*) \neq 0$ for any K. ■

The proof of Proposition 2.1 essentially verifies that a Walrasian equilibrium of a balanced meeting profile provides period utility $w(y^*)$, and that the Walrasian correspondence is continuous around a balanced meeting profile.

The next proposition states the actual limit result. It shows that as $N \to \infty$, the probability that an agent is in a sufficiently balanced meeting at every date approaches unity and so expected period utility approaches $w(y^*)$.

PROPOSITION 2.2. *For any $\eta > 0$, there exists an N_η such that for all $N > N_\eta$, the expected period utility of an arbitrary agent exceeds $w(y^*) - \eta$.*

Proof. Let $\varepsilon > 0$ be given and let δ_ε be as in Proposition 2.1. Let $\widetilde{I} \subset I$ be the set of meeting profiles such that $\frac{i_{\min}(i)}{N} \geq \frac{1}{K} - f(\alpha, N)$ for a given α and N where $f(\alpha, N) \equiv \frac{V_\alpha}{(NK)^{\frac{1}{2}}} + \frac{1}{2N}$ as defined in Lemma **??**. For any $\alpha \in (0,1]$, let N_α be such that for all $N > N_\alpha$, $f(\alpha, N) \leq \delta_\varepsilon$. Under these conditions, the expected period utility of an arbitrary agent is at least as large as $(1-\alpha)[w(y^*)-\varepsilon]$, where $1-\alpha$ is the probability of being in a meeting in $\widetilde{I}$. Let $\eta > 0$ be given. Then I want to show

$$(1-\alpha)[w(y^*) - \varepsilon] \geq w(y^*) - \eta. \tag{2.9}$$

This is equivalent to showing

$$\alpha w(y^*) + \varepsilon \leq \eta. \tag{2.10}$$

Choose $\varepsilon = \frac{\eta}{2}$. Then (2.10) becomes

$$\alpha w(y^*) \leq \frac{\eta}{2}. \tag{2.11}$$

Finally, let $\alpha = \frac{\eta}{2w(y^*)}$, which satisfies (2.11). ■

2.4 Conclusion

This paper presents a limit result establishing that money is inessential when the size of meetings is sufficiently large. This leads to the surmise that the role of money decreases as the size of meetings increases. Establishing such a result is difficult because measuring the role of money requires identifying all of the meetings where some double-coincidence problems occur and finding out how money can improve upon allocations arising from nonmonetary trading mechanisms. This task is extremely difficult even for relatively small meeting sizes.

Chapter 3

Mechanism Design in Freeman's Model of Payments

3.1 Introduction

This chapter is a comment on a body of work based on Freeman (1996a). Freeman provides a model of the payments system with the following features observed in actual economies: (i) agents make some purchases with debt and other purchases with fiat money; (ii) debts are repaid with fiat money; and, (iii) there is an active second-hand market for outstanding debt.

The first feature in Freeman's payments system is not new to the literature. The coexistence of money and debt has been explored in a variety of ways. The second feature, however, does seem to be new. In most models in which debt and money coexist, debt is repaid with goods, which seems different from what is observed in actual payments systems. Freeman offers an environment where it is possible for money to be used for the repayment of debt. He is able to characterize a competitive equilibrium of a trading mechanism in which debt is repaid with money. In fact, Freeman goes one step further by claiming that money is *essential* to repay debts in his environment: "Money serves not only as a medium of exchange, but also as the means by which debts are cleared. *Money is essential in this model for the clearing of debts* and the existence of a credit market; without valued money, equilibrium debt equals zero (1996a, page 1129; emphasis added)."

The use of money to clear debts is important for most of the results found in the body of work initiated by Freeman (1996a). It is used to justify an institutional role for either a central bank (Freeman, 1996a,b) or a private clearinghouse (Freeman 1996a, Green 1999) to provide liquidity whenever outstanding debt sells at a discount for money. Freeman (1999) adds aggregate default risk to the environment and explores how alternative liquidity-providing institutions react to such risk. McAndrews and Roberds (1999) use the model to analyze the general equilibrium of check float, with the interpretation that outstanding debt is the check float.

To effectively evaluate whether Freeman's environment provides an essential role for money as a means by which debts are cleared requires a mechanism design approach. I take such an approach and find that money is not essential for the clearing of debts. Specifically, I show that there exists an alternative trading mechanism that implements Freeman's notion of socially desirable outcomes. This mechanism involves debt-issues serving as inside money and has debts repaid with goods, not money. Since Freeman's scheme by which money is used to repay debt can do no better from a social point of view, his essentiality claim is unjustified. As a direct consequence, I show that there is no need for a central bank or a private clearinghouse in such an environment.

The chapter is organized as follows. Section 3.2 summarizes the important features of Freeman's model. Section 3.3 derives a notion of socially desirable allocations. Section 3.4 discusses how Freeman's trading mechanism with money as the means by which debt is cleared achieves those allocations unless certain additional difficulties are present. Section 3.5 contains the main result which provides an inside money mechanism and disproves Freeman's claim that money is essential to clear debt. Section 3.6

offers a discussion of the implications when there is aggregate default risk and Section 3.7 concludes.

3.2 The Environment

The model is that presented in Freeman (1996a), although described in a somewhat simpler way. It is an endowment model of two-period-lived overlapping generations with two goods at each date, *good 1* and *good 2*. The economy starts at date $t = 1$. There is a $[0, 1]$ continuum of each of two types of agents born at every date. These two types are distinguished by their endowments and preferences. One type are *creditors* and the other type are *debtors*.

Each creditor is endowed with y units of good 1 when young and nothing when old. Each debtor is endowed with x units of good 2 when young and nothing when old.

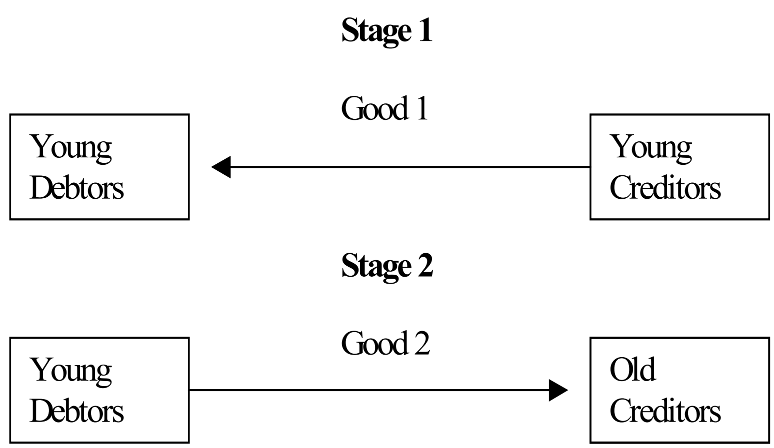
Let $c^t_{zt'}$ denote consumption of good $z \in \{1, 2\}$ at date t' by a creditor of generation t. The utility of a creditor is $u(c^t_{1t}, c^t_{2,t+1})$ where $u : \Re^2_+ \to \Re$. Notice that a creditor wishes to consume good 1 when young and good 2 when old. The function u is strictly increasing and concave in each argument, is C^1, and $u'(0) = \infty$ and $u'(\infty) = 0$.

Let $d^t_{zt'}$ denote consumption of good $z \in \{1, 2\}$ at date t' by a debtor of generation t. The utility of a debtor born at date t is $v(d^t_{1t}, d^t_{2t})$ where $v : \Re^2_+ \to \Re$. Hence, a debtor wishes to consume both good 1 and good 2 when young. A debtor does not wish to consume either good when old. The function v is strictly increasing and concave in each argument, is C^1, and $v'(0) = \infty$ and $v'(\infty) = 0$.

At date $t = 1$, there is a [0,1] continuum of initial old creditors. These creditors are each endowed with M divisible units of outside money. By outside money I mean that money is not produced by agents within the economy, but rather, is determined exogenously.

Trading opportunities are limited by a specific sequence of events that occur at each date (see Figure 3.1).

Fig. 3.1. Trading Patterns at Each Date



At the first stage, only young debtors and young creditors are together. This is the only opportunity for young debtors to acquire good 1. At the second stage, only young debtors and old creditors are together. This is the only opportunity for old creditors to acquire good 2. There are two additional assumptions that are implicit in Freeman's environment. Both are important in the sense that they make trade difficult given the sequence of events. One is that agents cannot commit to future actions. The other is that the past trading histories of agents are private information to those agents (no public memory).

One difficulty is that young debtors acquire good 1 before they surrender any of their endowment. Since there is no commitment to future actions, once they obtain some of good 1 for consumption in the first stage, young debtors have no incentive to surrender some of their endowment of good 2 at the second stage because they do not wish to consume anything in the future. To overcome this difficulty, there is a final stage within a period called *enforcement*[1]. At this stage, Freeman assumes that there exists an exogenous enforcement authority that has the power to levy punishment when agents fail to show some type of evidence of "appropriate behavior" in their past actions. Since the no commitment problem is particular to young debtors, the enforcement authority verifies whether debtors "paid" for their acquisition of good 1. If debtors fail to show evidence of having paid, the enforcement authority has the ability to punish them. The enforcement is limited by the fact that the authority can only punish agents currently at the enforcement stage.[2]

The second difficulty is that young creditors must be induced to surrender some of their endowment before they have an opportunity to consume. This is a difficulty

[1] This enforcement stage occurs at a separate calendar date in Freeman's original model, but this is unnecessary because no consumption occurs at the enforcement stage at that separate calendar date.

[2] Freeman implicitly assumes no commitment by the mere presence of an enforcement authority that has only limited enforcement. Otherwise, with full enforcement, the authority oversees agents at every stage and could impose certain actions regardless of whether it is individually rational for them to do so. This would be just another way of implying that agents could commit to future actions.

because there is no memory.[3] Young creditors must somehow be assured that they will be rewarded with consumption of some of good 2 when old if they sacrifice some of good 1 when young. Without public memory, these creditors have no way of communicating how much of their endowment they surrendered in the past.

To overcome this difficulty, Freeman first assumes that all agents have a technology which enables them to issue notes or IOUs. These notes are completely verifiable in the sense that they correctly identify the issuer, including his type and generation. Thus, young debtors can issue notes or IOUs to young creditors in exchange for some of good 1.

Old creditors need to bring something tangible with them when they trade with young debtors at the second stage of a period. It should be the case that creditors acquired these tangible items only by sacrificing some of their endowment when young. Then these items can signal that creditors shared their endowment in the past. It is also important that debtors wish to obtain these tangible items and are willing to exchange some of good 2 to acquire them. They will do so if they need the items to show evidence that they have repaid their debt. In the absence of memory, therefore, tangible items are necessary to serve as a medium of exchange at the second stage of each date. The tangible items proposed by Freeman are the divisible units of outside money with which initial old creditors are endowed. In this paper, I propose that the tangible items need only be the notes issued by the young debtors to young creditors at the previous date. This implies that the notes issued by young debtors serve as inside money.

[3] No memory is implicitly assumed by Freeman through the assumption that agents are spatially separated as in Townsend (1980, 1989). Spatial separation of agents is one way to model the absence of memory. See Kocherlakota (1998) and Huggett and Krasa (1996).

3.3 The Optimal Consumption Levels

To explore the validity of Freeman's claim that money is essential to repay debt in such an environment, I need a sense in which an outcome is socially desirable. I follow Freeman by using his notion of optimality. I look for outcomes that maximize a weighted average of the expected steady-state utility of debtors and creditors. Denote the steady state levels of consumption for a debtor of good z by d_z and that for a creditor by c_z, $z \in \{1, 2\}$. The problem is then to

$$\max_{d_1, d_2, c_1, c_2} \mu v(d_1, d_2) + (1 - \mu) u(c_1, c_2) \tag{3.1}$$

subject to:

$$y \geq d_1 + c_1 \tag{3.2}$$

$$x \geq d_2 + c_2 \tag{3.3}$$

where μ is the weight given to debtor utility.

Define v_z as the partial derivative of debtor utility with respect to good z and u_z as the partial derivative of creditor utility with respect to good z for $z \in \{1, 2\}$. The first order conditions, which satisfy the Kuhn-Tucker conditions for necessity and sufficiency, simplify to:

$$\frac{v_1}{v_2} = \frac{u_1}{u_2}. \tag{3.4}$$

These are simply "golden rule" allocations or allocations along the Pareto frontier. Denote the set of golden rule allocations $(d_1^*, d_2^*, c_1^*, c_2^*) \in \Re_+^4$ as the steady-state allocations (d_1, d_2, c_1, c_2) that satisfy (3.4).

3.4 Freeman's Outside Money Trading Mechanism

Freeman demonstrates that competitive trade with outside money serving both as a medium of exchange and a means by which debt is cleared satisfies (3.4) when the only complications are no commitment and no memory; that is, when all young debtors and all young creditors are simultaneously present at the enforcement stage.

The first stage of date t provides trading opportunities between young debtors and young creditors only. In exchange for some of good 1, young debtors issue notes that are promises to repay outside money to the young creditors when they are reunited at the enforcement stage[4]. Young debtors then meet old creditors in the second stage of date t. Each of these old creditors has M units of outside money with which they purchase some of good 2. Finally, young debtors and young creditors arrive at the enforcement stage. It is at this stage when debts are cleared with fiat money. The young debtors take the money acquired from the old creditors to the enforcement stage and pay the creditors from whom they borrowed. The enforcement authority ensures that the debtors pay the creditors with money. Otherwise, the enforcement authority punishes the debtors. Freeman is vague as to how the enforcement authority inflicts punishment on defaulting

[4]Note that Freeman requires that all young debtors and all young creditors arrive at the enforcement stage.

debtors. He assumes that the punishment is severe enough so that debtors always pay creditors.

Without further complications, Freeman's environment with competitive trade is one in which (i) agents make some purchases with debt and others with money; and, (ii) debts are repaid with money. In order to observe the third feature of actual payments systems, a second-hand market for outstanding debt, Freeman models a potential lack of liquidity in his environment. This lack of liquidity not only leads to a second-hand market for outstanding debt, but may also lead to a potentially inefficient outcome since the competitive equilibrium outcome may not satisfy (3.4). Thus, the feature that money is used to repay debts can be a source of inefficiency when there is a lack of liquidity.

Freeman models the lack of liquidity in the following way. At the beginning of the enforcement stage all young creditors and an exogenous subset (λ) of young debtors arrive. Then an exogenous subset of creditors $(1-\alpha)$ leave the enforcement stage before the remaining $(1-\lambda)$ debtors arrive. When the number of early-leaving creditors exceeds the number of late-arriving debtors, $(1-\alpha) > \lambda$, some creditors leave before they can be fully repaid. A second-hand market for outstanding debt arises at the enforcement stage where creditors who leave early sell their outstanding claims to creditors who remain until the last subset of debtors arrive.

The outstanding debt sells at a discount to the benefit of late-leaving creditors and to the detriment of early-leaving creditors. That is, late-leaving creditors leave the enforcement stage with more outside money than early-leaving creditors even though they sacrificed the same amount of their endowment of good 1 at the first stage of their youth. Late-leaving creditors, therefore, can purchase more of good 2 when old than can early-leaving creditors. Thus, the competitive equilibrium outcome is not a golden-rule allocation since creditors who leave the enforcement stage at different times but are otherwise identical consume different levels of good 2.

To correct this problem, Freeman introduces a monetary authority that can provide liquidity at the enforcement stage by printing additional units of outside money to fully compensate early-leaving creditors. This has the effect of removing the discount on outstanding debt in the second-hand market. Then, when the final subset of debtors arrives at the enforcement stage, the monetary authority collects money from them as repayment of debt and destroys as much money as it produced when some of the creditors left. This elastic supply of currency implements steady-state allocations that satisfy (3.4) by virtually eliminating the liquidity problem. Thus, the feature that money is used to repay debt combined with a lack of liquidity gives a justification for the presence of a central monetary authority.

Both Freeman (1996b) and Green (1999) discuss the essentiality of a central bank in the model. In Freeman (1996b), the author shows that the liquidity problem can be alleviated with banknotes provided by a distinct set of agents who form a private clearinghouse. Green (1999) shows that a subset of late-leaving creditors can issue *novation securities* to early-leaving creditors that serve as perfect substitutes for outside money. What is common in each of these works is that they offer a role for a liquidity-providing institution *taking the essentiality of money to clear debt as given.* Testing for the essentiality of money to clear debt in this work can shed some light on the need for such liquidity-providing institutions in this environment.

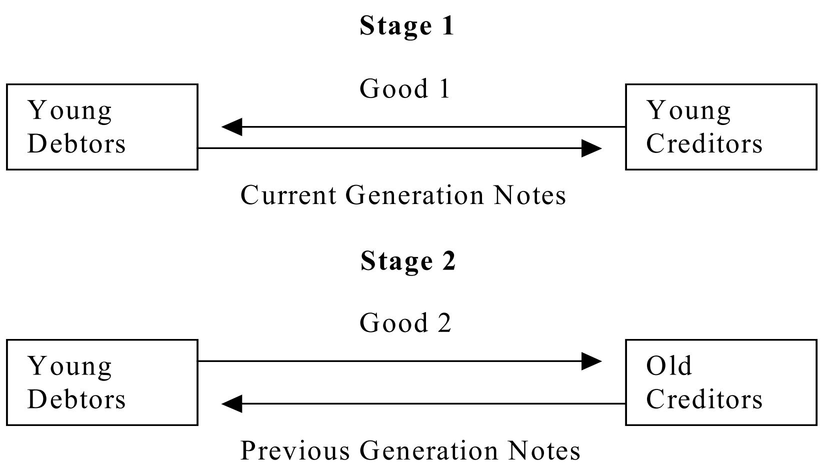
3.5 An Inside Money Mechanism

I now ask whether there exists a trading mechanism that implements a golden-rule outcome $(d_1^*, d_2^*, c_1^*, c_2^*)$ without the feature that money is used to repay debt. In order to do this, I shall employ a different equilibrium concept than that employed by Freeman. My notion of an implementable outcome is one which is the outcome of a sub-game perfect equilibrium for some game that respects the timing of events within a period in Freeman's environment and the absence of both commitment and memory. This is a rather weak notion of implementability, since it does not require that the equilibrium outcome be unique.

I now propose the following game that is played at each date $t \geq 2$. A slightly modified version of the game will be presented below for date 1. I shall refer to the entire game as the *inside money game* or *inside money mechanism*. At the first stage of date t, young creditors and young debtors simultaneously choose to either agree or disagree to a proposed consumption level of good 1 for an arbitrary debtor, $\overline{d_1}$. If all agents agree, then $d_1^t = \overline{d_1}$, $c_1^t = y - \overline{d_1}$, and young debtors issue notes to the young creditors. If at least one agent disagrees, then there is autarky; $d_1^t = 0$, $c_1^t = y$, and no notes are exchanged.

At the second stage of date t, the old creditors and the young debtors will choose to either accept or reject a proposed consumption level of good 2 for an arbitrary old creditor, $\overline{c_2}$. If all agents agree, then $c_2^{t-1} = \overline{c_2}$, $d_2^t = x - \overline{c_2}$, and old creditors surrender notes issued to them by the previous generation's debtors to the current generation's young debtors. If at least one agent disagrees or if old creditors do not enter the second stage with notes issued by their generation's debtors, then autarky results; $c_2^{t-1} = 0$, $d_2^t = x$, and no notes are exchanged. (See Figure 3.2)

Fig. 3.2. Trading Patterns with Inside Money



Finally, at the third stage of date t, if young debtors have a note issued by a debtor from the previous generation, then the enforcement authority destroys the note and the young debtor avoids punishment. If a young debtor does not have a note issued by a debtor from the previous generation, then that debtor faces punishment.

Recall that Freeman was not explicit about how the punishment is administered at the enforcement stage. Rather, he assumed that it would be severe enough to encourage debtors to repay loans. Here, I shall be more precise about punishment. I assume that agents do not consume until the end of each period. Thus, if a debtor does not have a note issued by a debtor from the previous generation, then the enforcement authority confiscates the amount of good 1 with which he enters the enforcement stage.

The game is virtually the same for date 1. The only difference occurs at the second stage, at which time the initial old creditors enter trade with outside money instead of debtor-issued notes. The following allocation is proposed for the second stage of date 1. Old creditors surrender all of their M units of outside money in exchange for $c_2^0 = \overline{c_2}$. Again, if all agents agree, $c_2^0 = \overline{c_2}$, $d_2^1 = x - \overline{c_2}$ and creditors surrender all of their outside money to debtors. If at least one agent disagrees, then autarky results; $c_2^0 = 0$, $d_2^1 = x$, and no notes are exchanged. At the enforcement stage, if the young debtors come with M units of outside money, they are destroyed and the young debtor avoids punishment. Otherwise, the young debtors face the same punishment as in any other period[5].

A couple of comments are worth noting regarding the aforementioned game. First, note that the feasibility constraints (3.2) and (3.3) hold at equality. Second, even though outside money is used at date 1, its only role is as a medium of exchange. It is never used as a means by which debt is cleared. The primary media of exchange are the debtor-issued notes or IOUs since they are what circulate for dates $t \geq 2$.

The inside money game alleviates the problems of no commitment and no memory. No commitment is trivially taken care of by the external enforcement authority as it is in Freeman (1996a). This is consistent with the role that the enforcement stage played

[5] If a young debtor comes to the enforcement stage with $M' \neq M$ units of outside money, the authority inflicts punishment and confiscates and destroys those M' units.

in Freeman's trading scheme[6]. The notes issued by debtors also alleviate the problem of no memory. By trading away some of their endowment when young, creditors can now acquire something tangible that will be demanded by young debtors when they are old. Since the notes issued by debtors provide a correct signal about the past trading history of the old creditors, they can be induced to trade when young.

I now ask what types of steady-state outcomes can be implemented via the inside money game. My first result demonstrates that the set of implementable outcomes corresponds with a couple of participation constraints, one for debtors and one for creditors.

PROPOSITION 3.1. *A steady-state outcome* $(\overline{d_1}, \overline{d_2}, \overline{c_1}, \overline{c_2})$ *is implementable (via the inside money game) if and only if it satisfies the following participation constraints:*

$$v[\overline{d_1}, x - \overline{c_2}] \geq v[0, x] \tag{3.5}$$

for debtors and

$$u[y - \overline{d_1}, \overline{c_2}] \geq u[y, 0] \tag{3.6}$$

for creditors.

Proof. First, I show that if an outcome satisfies (3.5) and (3.6), then it is implementable. Recall that an outcome is implementable if it is the sub-game perfect equilibrium of the inside money game. I solve for a sub-game perfect equilibrium by backwards induction. Since the game is stationary, I can solve for an arbitrary date t.

[6] A notable distinction is that all that is required under the inside money mechanism is that debtors arrive at the enforcement stage. It is not necessry for creditors to arrive as well.

Let us begin with the second stage at date t. There are two possible states that a generation t debtor could be in at this stage. Define the state of a generation t debtor at the second stage as $\delta \in \{0, 1\}$. When $\delta = 0$ a generation t debtor enters the second stage with none of good 1; that is, at least one person disagreed at stage one of date t. When $\delta = 1$, then a generation t debtor enters the second stage with $\overline{d_1}$ of good 1 acquired in the first stage of date t. The state of generation t debtors is private information to them.

Suppose that all other agents will agree in the second stage. An arbitrary generation t debtor will also agree if:

$$v[\delta\overline{d_1}, x - \overline{c_2}] \geq v[0, x]. \tag{3.7}$$

The right hand side of the expression takes into account the fact that if a debtor disagrees, he will not receive a note and will then be punished with autarky at the enforcement stage. Equation (3.7) is clearly not satisfied when $\delta = 0$. That is, if a generation t debtor did not acquire any of good 1 at the first stage of date t, then he will not want to offer any of his endowment of good 2 in the second stage.

When $\delta = 1$, (3.7) is equivalent to (3.5) which is satisfied by assumption. Thus, a strategy for a generation t debtor at the second stage of date t is to agree if $\delta = 1$ and to disagree if $\delta = 0$. I need $\delta = 1$ to be an equilibrium state for debtors.

There are also two possible states for a generation $t - 1$ creditor at the second stage of date t. Define the state of a generation $t - 1$ creditor at the second stage as $\kappa \in \{0, 1\}$. Creditor states are private information to the generation $t - 1$ creditors. When $\kappa = 0$ a generation $t - 1$ creditor enters the second stage without a note; that is, at least one person disagreed at stage one of date $t-1$. If this is the case, then the generation $t-1$ creditors have no notes to offer in exchange. Thus, the generation $t-1$ creditor receives autarkic utility, $u[y,0]$, which incorporates the fact that if a generation $t-1$ creditor has no note, then she did not share any of her endowment when young.

When $\kappa=1$ a generation $t-1$ creditor enters the second stage with a note. Now, suppose that all other agents will agree in the second stage. An arbitrary old creditor will also agree if:

$$u[y-\overline{d_1},\overline{c_2}] \geq u[y-\overline{d_1},0]. \tag{3.8}$$

Clearly, an old creditor will agree at the second stage when she has a note. Thus, I need $\kappa=1$ to be an equilibrium state for creditors.

Now let us consider an arbitrary generation t debtor at the first stage of date t. It remains to be shown that an arbitrary generation t debtor has no incentive to deviate from agreeing in the first stage of date t given that every other agent agrees. If the debtor disagrees, he enters the second stage in state $\delta=0$ and will disagree in the second stage as well. Thus, he will receive only autarkic utility which is the right hand side of (3.5). If the debtor agrees, he enters the second stage in state $\delta=1$. He knows that if generation $t-1$ creditors enter the second stage with notes, he will agree in the second stage, receive a note issued by a generation $t-1$ debtor and receive $v[\overline{d_1},x-\overline{c_2}]$, which is at least as large as autarky given that (3.5) is satisfied. If the generation $t-1$ creditors do not enter the second stage with a note, then generation t debtors are assured autarky. Thus, it is a weakly dominant strategy to agree at the first stage if every other agent agrees at that stage.

Finally, consider an arbitrary generation $t-1$ creditor at the first stage of date $t-1$. It remains to be shown that an arbitrary generation $t-1$ creditor has no incentive to deviate from agreeing in the first stage of date $t-1$ given that every other agent agrees. If the generation $t-1$ creditor disagrees at the first stage when young, she enters the second stage when old in state $\kappa = 0$ and receives autarkic utility, $u[y, 0]$. She knows that generation t debtors will agree in the first stage of date t and thus agree in the second stage of date t since it is a weakly dominant strategy for them to do so. Thus, if she agrees at the first stage of date $t-1$, she will enter the second stage of date t in state $\kappa = 1$ and agree so that she receives $u[y - \overline{d_1}, \overline{c_2}]$. This is at least as large as the utility of autarky since (3.6) holds. As a result, generation $t-1$ creditors agree in the first stage of date $t-1$.

Nothing substantive changes for the first date, when outside money is exchanged at the second stage. The initial old creditors enter in only one possible state, $\kappa = 1$, which implies that the initial old creditors enter the second stage of date 1 with M units of outside money. Thus, the initial old creditors have no incentive to deviate from agreeing in the second stage of date 1. Generation 1 debtors will not deviate from agreeing in the second stage of date 1 so long as they enter in state $\delta = 1$, which they do for the reasons given above for an arbitrary date t.

To prove the second part of Proposition 3.1 suppose that the outcome $(\overline{d_1}, \overline{d_2}, \overline{c_1}, \overline{c_2})$ is implementable. Since autarky is always an option, it must be the case that (3.5) and (3.6) hold. ■

Proposition 3.1 provides two simple conditions to check to ensure that an outcome is implementable via the inside money mechanism. Thus, to disprove Freeman's claim that money is essential for the repayment of debts in his environment is to show that the golden rule allocations satisfy the two participation constraints, (3.5) and (3.6). But this is rather easy to show, as the next result demonstrates.

COROLLARY 3.1. *Any golden rule outcome* $(d_1^*, d_2^*, c_1^*, c_2^*)$ *is sub-game perfect implementable at date t (via the inside money mechanism) for all t.*

Proof. By Proposition 3.1, all that is required is to show that the golden rule allocations satisfy equations (3.5) and (3.6). As Freeman shows, any outcome that satisfies (3.4) can arise from a competitive equilibrium. Since a competitive equilibrium outcome implies that trade is voluntary, the golden rule outcomes must do at least as well as autarky and thus, satisfy (3.5) and (3.6). ■

Freeman's essentiality claim is, therefore, unjustified. The inside money mechanism does as well as Freeman's trading mechanism which has debts being repaid with money. Both alleviate the problems of no commitment and no memory to achieve the golden rule allocations. Moreover, because only debtors need to be threatened in the inside-money mechanism, the liquidity issue does not arise. That is, because the difficulties of debt-clearing do not arise with the inside money mechanism, there is no second-hand market for outstanding debt, and the asserted role of a central bank or private clearinghouse does not arise. Thus, a liquidity-providing institution is also inessential in Freeman's environment. In particular, there is no need for either a central monetary authority or a private clearinghouse when the inside money mechanism is used in this environment instead of Freeman's trading mechanism.

3.6 Adding Aggregate Default Risk

In Freeman (1999), the author adds yet another friction, aggregate default risk, and explores its various consequences in terms of risk sharing under alternative institutional payments systems. He does this by assuming that there is a probability that a subset of debtors may avoid the enforcement stage entirely and go unpunished if they do not repay their debt. No one, including the eventual defaulting debtors, knows of such a default until the enforcement stage.

The addition of aggregate default risk in such an environment makes it difficult for even a central bank or private clearinghouse to aid in achieving the golden-rule allocations via Freeman's trading mechanism. Thus, one might expect that the presence of aggregate default risk may have important consequences for whether the inside money mechanism can implement the golden rule allocations.

Formally, suppose as in Freeman (1999) that at any date θ is the probability that there is a subset of debtors with measure η who escape the enforcement stage. With probability $1-\theta$ all debtors will be required to go to the enforcement stage. Agents do not learn of the presence of a default shock until the final subset $(1-\lambda)$ of debtors arrive at the enforcement stage[7]. Thus, an arbitrary debtor does not go to the enforcement stage with probability $\theta\eta$.

[7] It is a maintained assumption that there is still a lack of liquidity $((1-\alpha) > \lambda)$.

The presence of aggregate default risk directly affects the second stage debtor participation constraint. It now becomes

$$v[\delta\overline{d_1}, x - \overline{c_2}] \geq (1 - \theta\eta)v[0, x] + \theta\eta v[\delta\overline{d_1}, x]. \tag{3.9}$$

The right hand side is now the expected utility to disagreeing in the second stage. With probability $(1 - \theta\eta)$ a debtor arrives at the enforcement stage and is punished with autarky. With probability $\theta\eta$ a debtor avoids punishment.

Since socially optimal outcomes must now satisfy (3.9), it is not obvious that any golden rule outcomes are implementable via the inside money mechanism. If the inequality (3.9) holds, then all debtors will agree to the allocations proposed in the second stage and there will be no aggregate default risk, even with the possibility that some debtors could avoid punishment. The fact that they do not know they can get away with it a priori, prevents them from defaulting. Thus, the inside money mechanism is unaffected by the presence of aggregate default risk if (3.9) holds.

If, on the other hand, inequality (3.9) does not hold, then all debtors will take the risk and default. Thus, the mere possibility of aggregate default causes every debtor to default with certainty. Of course, knowing this in advance, creditors will not accept notes in exchange for good 1 at the first stage. Autarky results whenever (3.9) does not hold.

One simple way around the potential problem that (3.9) creates is for the enforcement authority to increase the punishment. Specifically, as noted above, Freeman is vague as to how the authority may punish debtors who are caught defaulting. It is implicitly assumed that the penalty is severe enough that they do not intentionally default. If I were to assume that the authority inflicts enough punishment so that if debtors defaulted at the second stage, their utility would be negatively infinite if caught, then (3.9) is trivially satisfied and the inside money mechanism still implements the golden-rule allocations.

3.7 Conclusion

Freeman proposes a model of a payments system with the features that debt and outside money are used to purchase goods, outside money is used to repay debt, and there exists a secondary market for outstanding debt. In particular, he claims that money is essential as a means by which debt is repaid. Because of this role for money, when there is a lack of liquidity, there is a need for an institution such as a central bank or a private clearinghouse to achieve socially optimal outcomes.

I have shown that Freeman's model does not provide an essential role for money as a means by which debt is repaid. In particular, an inside money mechanism which uses the notes issued by debtors as a medium of exchange can implement socially optimal outcomes without the feature that money is used to repay debt. In addition, a potential lack of liquidity does not prevent the inside money mechanism from implementing the socially desirable outcomes. As a result, the asserted role for a liquidity-providing institution, such as a central bank or a private clearinghouse, does not arise.

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