PROPAGATION AND CLUTTER CONSIDERATIONS FOR LONG RANGE RADAR SURVEILLANCE USING NOISE WAVEFORMS

A Thesis in
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by
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Abstract

The use of noise waveforms is investigated for long range radar surveillance. In addition to the noise signal, a chirp waveform was also simulated for the various scenarios to act as a direct comparison of traditional radar signals. The correlation and relative ratio of received to transmitted power was found for the two waveforms after reflecting from simple targets and terrain. For the simple shapes, the correlation of the two signals were similar in value and pattern with respect to incidence angle. The reflection from terrain gave smaller correlations for the noise waveform indicating that it may be less susceptible to false alarms when terrain is considered clutter. Advanced simulations were then run with a realistic hummer target and terrain clutter model. Accounting for the atmospheric propagation loss, system gains, and receiver noise, the probability of detection and false alarm were found to create receiver operating characteristic curves. It was found that the noise waveform performs as well as the chirp for cases of strong clutter response, and much better for cases of weak clutter response. Next, modeling radar propagation as a series of cascaded two-port devices was explored. This allowed different sections of propagation, such as through air or rain, to be computed separately for a particular wavelength and then combined together to form a set of system parameters. The first radar that was modeled was forward-looking which examined an air-rain-air-target scenario. The system parameters for this case were computed for various rain rates and rain path lengths then applied to the noise and chirp waveforms. It was found the both the noise and chirp signals resulted in similar correlations with respect to path length and rain rate. The final radar that was modeled was down-looking where the waveforms were reflected and transmitted through different layers of soil with various moisture content. The correlation of both waveforms were similar in that they varied with path length due to the phase introduced by the system $S_{11}$ parameter. However, the noise signal correlation was consistently lower than the chirp’s. This again indicates that the noise waveform
may be a better alternative for reducing clutter false alarms. Finally, the use of double spectral processing for determining target ranges was investigated in comparison to the use of cross-correlation. It was found that while the double spectral processing method efficiently determines target range, it produces echo responses in the case of multiple targets which may cause false alarms. Additionally, this method has lower peak-to-average responses than the correlation for noisy return signals, again increasing the false alarm rate.
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Chapter 1

Introduction

1.1 Noise Radar

Noise radar was first introduced in the 1940s and has been developed for many short range systems in the years since [1]. One such example is a ground penetrating radar developed by the University of Nebraska using Gaussian noise [2]. This system employed a variable delay line allowing the correlation of returns from different depths to be investigated. Further developments by this group using a heterodyne correlator allowed for the conservation of phase. By retaining the phase the detection of Doppler frequencies were then possible. This was demonstrated for both short and intermediate ranges up to 200 meters.

Another application is SAR imaging of short range areas using low transmit power [3]. Using an X-band noise source, ground based SAR systems with both linear and circular polarization were demonstrated for imaging areas near buildings. In the results, the location of tree, building, and car targets closely matched the actual layouts.

The detection of targets using through-wall radar with noise signals was also demonstrated [4], [5]. Using the cross-correlation of the received and delayed transmitted noise waveforms, along with the background subtracted, the detection of human targets within buildings was shown to be possible. Another technique for localization utilized the slow varying amplitude in the quadrature cross-correlation to spot breathing and heartbeats or the fast variation to notice quicker human movements.
1.2 Thesis Objectives

While many noise radar systems have been developed for short and intermediate range applications, long range noise radar has not been thoroughly investigated. In this thesis, the target, clutter, and propagation effects on noise waveforms was investigated for long range radar.

In addition to examining these effects on noise signals, chirp waveforms were also applied to the same conditions to act as a direct comparison to traditional radar waveforms. The purpose of this was to understand how noise signals compare in radar cases. That is if they have similar, or possibly better, performance to established waveforms.

The correlation between a received and transmitted ultra-wideband (UWB) noise waveform is directly related to the probability of detection [1]. As such, the reflections of the signal off of different targets and clutter were examined. Simple target shapes offered responses for constant, linear, and squared radar cross section (RCS) dependencies with respect to frequency. Simple clutter normalized radar cross sections (NRCSs) statistics offered the response from objects other than the target of interest. These objects were used to obtain a response from waveforms which may trigger a false alarm.

While the simple models offer clear responses, more complex targets and clutter was also examined to give a more realistic radar response. Typically to compare radar systems, receiver operating characteristic (ROC) curves are used. These plot the probability of detection and false alarm against one another. To directly compare chirp and noise waveforms using more than just correlation, ROC curves were examined for complex target and clutter models.

An important aspect that was examined in this thesis was the impact of propagation itself on noise waveforms. This explored the attenuation, phase shift, and reflections from air and rain as well as from wet soils. Because long range applications were explore here, the frequency dependent attenuation and phase introduced can be severe. Understanding the impact of the propagation on noise waveforms is therefore essential for this investigation.

Finally, the use of double spectral processing (DSP) for determining the time delay, or range, to a target was investigated. Traditionally in noise radar cross-correlation is used to find this information, however, the use of the DSP method is
given here to offer a full account of all options available for radar systems. In particular the range resolution and the ability to detect targets in noisy environments was examined.

1.3 Thesis Overview

The body of this work is composed of chapters dealing with the objectives outlined above. Background information on the generation of waveforms as well as the use of two-port devices is given first. This explains how the different radar cases were modeled and simulated. Following this, the correlation investigation for simple target and clutter objects is given. This naturally extends to the next chapter which examines more complex targets and clutter as well as including propagation and receiver noise in the models. Work detailing the development of propagation through air, rain, and soil in terms of two-port networks is then given. In all three of these sections, the development of the theory, simulations, and the results are given so that the reader takes away a clear understanding of that particular work. Following the two-port analysis of propagation, a comparison of the DSP and cross-correlation methods is given for noise waveforms. Finally, concluding remarks are made highlighting key points from each chapter as well as offering suggestions for the continuation of long range noise radar research.
Background

2.1 Noise Waveform

The noise waveform in this work was represented using a Fourier series. This method allows for the direct approximation of band-limited Gaussian white noise. In traditional representations of white noise, a delta function autocorrelation and infinite power is often assumed. Since these signals do not exist, a more accurate representation is found using the Fourier series which gives a real shaped autocorrelation and a finite power [6], [7]. This section discusses the definition of the noise waveform used in this work as well as the method of generating the signal for MATLAB simulations.

2.1.1 Definition

The primary reasons that the Fourier approximation of a white noise signal was used was its accuracy in representing the signal as well as the rigorous mathematical definition. It can be shown that the Fourier approximation’s amplitudes approach a Gaussian distribution and its spectrum becomes flat as the time period being approximated approaches infinity [6]. An alternative way of approaching this limit is by approximating multiple periods of the noise signal. By using a Poisson distribution to select a random number of frequencies for each time period, different length time periods can be approximated. From the central limit theorem, it can be shown that with a large number of samples the approximations approach Gaussian band-limited noise [7].
The Fourier series approximation of the noise signal, $x(t)$, is given in (2.1) where the step frequency size, $\Delta f$, is found by the relationship $T\Delta f = 1$. The random phase shift, $\varphi_n$, introduced in each sinusoid is uniformly generated within the range of $[0 - 2\pi]$ where $N$ is the total number of frequency steps and $n \in (1, 2, \ldots N)$. The coefficient $A_0$ is the total average power that would be dissipated over the bandwidth through an impedance of 1 Ω.

$$x(t) = \frac{\sqrt{2}A_0}{\sqrt{N}} \sum_{n=1}^{N} \cos \left( \frac{2\pi nt}{T} - \varphi_n \right)$$ (2.1)

The Fourier transform of the noise signal can be easily derived using the properties of the transform as well as assumptions about the signal. Since the noise waveform in the time domain is simply a summation, the linearity of the transform entails that in the frequency domain the signal is simply a summation of the individual cosine transforms. The transform of a cosine is a set of delta functions at the positive and negative frequency of that particular cosine.

In this approximation the cosines are truncated in the time domain using rectangular windows. These windows are multiplied on in the time domain correspond to sinc functions convolved with the delta functions in the frequency domain. Since the sinc functions are convolved with delta functions, the convolution only causes the sinc functions to become centered on the frequencies of the delta functions. The final Fourier transform of the noise signal can be shown to be (2.2).

$$X(f) = \left( \frac{\sqrt{2}A_0}{\sqrt{N}} \right) \left( \frac{T e^{-j\pi f T}}{2} \right) \sum_{n=1}^{N} \left\{ \text{sinc} \left[ (f - f_n) T \right] e^{j\pi f_n T} e^{j\varphi_n} + \text{sinc} \left[ (f + f_n) T \right] e^{-j\pi f_n T} e^{-j\varphi_n} \right\}$$ (2.2)

This derivation closely follows that given in [7] with only slight differences caused by the fact that the representation in (2.1) used a cosine while the approximation in [7] used a sine. The derivation of (2.2) is given in Appendix A.

From the Fourier transform of the signal, the useful cross power spectral density (CPSD) can be found. In a similar manner to the power spectral density of a single signal, the CPSD shows the frequencies where the two waveforms share power. In (2.3) the CPSD of two noise waveforms, defined as (2.1) is given. This was found so that the relationship between an original transmit signal and attenuated received signal could be seen. The derivation, given in Appendix B, closely follows [8] in which the author derives the spectral density of an un-attenuated Fourier...
approximation of noise.

\[ G_{XY}(T, f) = \frac{A_T}{2Nf_{\text{max}}} \sum_{i=1}^{N} Q_1(f_i) Q_2^*(f_i) \int_{0}^{f_{\text{max}}} \left\{ \text{sinc} \left[ (f - f_i) T \right] + \text{sinc} \left[ (f + f_i) T \right] \right\} df_i \]  

(2.3)

In (2.3) the variables \( Q_1 \) and \( Q_2 \) describe the specific attenuation coefficients for the transmit and receive signals respectively at a particular frequency, \( f_i \), of the Fourier series approximation. While the integral in (2.3) is independent of the summation and can be found analytically, the process is nontrivial as explained in Appendix B. As such, the form shown above is kept and evaluated numerically. The attenuation coefficients are dependent on the filter parameters, environmental parameters such as weather, and overall applications.

With the CPSD found for a particular situation, the power shared by the two signals can then be determined by integrating it across all frequencies. From the Wiener-Khintchine theorem, this power is the same as the cross-correlation of the two signals evaluated with a time delay of zero [9]. By normalizing this value with the square root of the product of the individual signal powers, the correlation coefficient between -1 and 1 can be found.

2.1.2 Generation

To generate the noise waveform, the pulse length of the signal is the first aspect that needs to be determined. The frequency step size can be found from its reciprocal relationship with the pulse length. This relationship, given above in 2.1.1, has important implications for utilizing the signal in MATLAB. By increasing the pulse length being estimated, the frequency step size decreases giving a more accurate representation of the signal. However, based upon the Nyquist frequency chosen, the number of frequency points, \( N \), may quickly become very large. The pulse length and Nyquist frequency must therefore be balanced to give an accurate representation of the signal as well as reasonable number of points that will not limit simulations.

After determining the number of frequency steps from the maximum frequency and step size, an equal number of random phase values are needed. These were
generated by drawing from a uniform random distribution between 0 and 1 in MATLAB and scaling these values to between 0 and $2\pi$ radians. With these aspects determined, the power of the pulse is the final required variable before the noise waveform can then be generated.

### 2.2 Chirp Waveform

Throughout this work, a more traditional waveform was also simulated as a direct comparison to the noise signal. For this role, a chirp waveform, or more generally a linear frequency modulated (LMF) signal, was chosen. This simple signal is frequently used in radar applications and so offers insight to how real-world systems would respond to these simulated cases. Additionally, the relationship between the instantaneous frequency and time step allows the frequency dependent targets to be easily applied to the signal. This section briefly discusses the definition of the chirp waveform followed by the method of generating the signal for MATLAB simulations.

#### 2.2.1 Definition

The chirp waveform is defined by (2.4) where $\theta_i(t)$ is the instantaneous quadratic phase of the signal defined in (2.5). In chirp signals, the time-bandwidth product should be kept large so that the signal’s frequency spectrum is accurately represented [10], [11]. For the purpose of the correlation work discussed in Chapter 3 the time-bandwidth product was selected to be 50. This value offered a good tradeoff between the spectrum accuracy and the number of samples needed. The false alarm probabilities discussed in Chapter 4 depended directly upon a signal’s pulse width. To ensure adequate spectrum representation as well as keep the sampling frequency, pulse width, and frequency range of interest the same as the noise signal, the time-bandwidth product for that work was chosen to be 100.

$$f(t) = \cos (2\pi \theta_i(t)) \quad (2.4)$$

$$\theta_i(t) = \frac{kt^2}{2} + f_L t \quad (2.5)$$
where

\[ k = \frac{f_H - f_L}{T} \quad (2.6) \]

and \( f_H \) and \( f_L \) are the highest and lowest frequencies of the chirp, respectively, and \( T \) is the pulse width.

### 2.2.2 Generation

The chirp waveform used in simulations was often implemented following the noise signal. For this reason, the noise parameters could easily be applied to the chirp to ensure that the results of the waveforms were comparable. The first aspect that is needed in generating the chirp signal is the frequency range of interest. From (2.5) and (2.6), it is clear that this bandwidth heavily influences the instantaneous phase. With this known, the sampling frequency is typically the next aspect that is needed. To have comparable results, the sampling frequency of the noise and chirp waveforms were kept the same.

Now, with the sampling frequency known, the sampling time period is also known. This plays an important role in determining the pulse length. As mentioned above, to get an accurate representation of the chirp’s spectrum the time-bandwidth product should be kept large. However, given that the frequency bandwidth is known, the time-bandwidth product and number of samples needs to be balanced. Depending on the simulation, the pulse width was varied to maximize the spectrum smoothness while limiting the number of signal samples. With the pulse length and sampling frequency chosen, a time vector is generated and used to compute the chirp waveform.

### 2.3 Filtering and Attenuating Waveforms

While the method of generating the noise waveform as outlined above does give a band-limited signal, it requires further filtering around the frequency range of interest. A requirement for filtering these representations is that the maximum frequency must be much larger than the bandwidth of interest [7]. For the work detailed in Chapters 3 and 4, the bandwidth of interest was 2.5-3.5 GHz, and so the Nyquist frequency was chosen to be 15 GHz. This met the necessary criteria.
of being much larger than the bandwidth being investigated. Additionally, 15 GHz was small enough that MATLAB simulations could be run efficiently without an excess number of frequency points. For the work detailed in Chapter 5 the bandwidth of interest was 3.5-4.5 GHz. This bandwidth remained small enough that the 15-GHz Nyquist frequency could still be used.

To filter the noise in Chapters 3 and 4, a simple FIR filter using a Hamming window was designed with stop frequencies at 2.25 and 3.75 GHz. The filter coefficients were then computed in MATLAB and used for generating the magnitude and phase response. The accuracy of the filter was tested by inspecting both the fast Fourier transform (FFT) and the power spectrum of the filtered signals. Additionally, the total power of the filtered signal was found by integrating the spectrum over the frequency range. In the ideal case, the filtered signal would have $\frac{1}{15th}$ of the unfiltered signal power, $A_0$, because $(BW/F_{max}) = 1/15$ in these cases. In practice, the actual signal gave a slightly larger average power due to the filter roll-off. An example of the final filtered signal and its respective power spectrum is shown in Figure 2.1. For the work in Chapter 5, a similar filter was created using a Hamming window with stop frequencies at 3.25 and 4.75 GHz. The same method as detailed above for generation and checking the window was also followed.

![Figure 2.1: Band-pass filtered Fourier approximation of white noise. (top) Time domain signal. (bottom) Power spectrum.](image-url)
The work in Chapter 6 was concerned with the DSP and cross-correlation methods’ ability to determine target range. As such, except for the bandwidth, the particulars of the transmitted and received noise waveforms were not of importance. A simple noise signal, rather than the Fourier series summation, was therefore used. A MATLAB randomly generated signal was chosen which had a sampling frequency of 5 GHz and a center frequency of 1.5 GHz. To investigate the impact of bandwidth on the DSP and correlation results, the signal was filtered around different frequency ranges. A built-in MATLAB function was used to design these band-pass filters in order to create a new filter for each of the required bandwidths quickly.

To ensure similar results were obtained, the chirp waveform was also filtered using the same Hamming filters as the noise signal. While an ideal chirp waveform only sweeps from the lower to upper frequency of interest, in practice there are artifacts outside the bandwidth of interest. These can be mitigated by increasing the time-bandwidth product, however, due to limitations, some will remain. By filtering the chirp, these outside frequency components can therefore be reduced. Furthermore, the filters used for the noise waveform have a phase shift which introduces a time delay. This can be seen in Figure 2.1 towards the beginning of the time domain signal. To have comparable results, the chirp waveform also has this phase introduced through filtering.

The use of the Fourier summation representation of a noise signal is very convenient for simulations because of the fact that it is simply a summation of cosines. To modify the signal’s amplitude or phase, it is regenerated using the original amplitude and random phases but with the frequency dependent modifications made to their respective cosines. This is useful for applying specific target modifications. After generating this new signal with modifications, it is then filtered using the same band-pass filter as before. This essentially represents the modified version of the transmit waveform after being reflected from a frequency dependent target. Since both the filtering and the modifications are linear, the order in which they are applied to the signal does not impact the final result.

For the chirp waveform, this method is similarly applied. Because the instantaneous frequency of the chirp varies with time, the different signal samples correspond to the different frequencies. The frequency response of targets can then be simply applied to a newly generated chirp signal at specific times. Similar to
the noise waveform, this new signal is then filtered so that it represents a modified version of the transmitted. While this method was adequate for the work in Chapters 3 and 4, a more advanced method of attenuating and modifying the waveforms was used for the work of Chapter 5.

In Chapter 5, the modifications made to a signal were applied based upon the propagation of waves though different media. As such, a phase shift was introduced which was proportional to the two-way path length. For a radar simulation, this path length is very large corresponding to a large phase shift. Since the phase introduced is linear with frequency it introduces a large time shift to the waveforms causing them to move within their MATLAB vectors. To avoid the signals being truncated by the vectors the waveforms were zero-padded after the transmit pulse allowing room for time shifts. This amount of zero padding corresponded to the propagation time of the signal causing a large number of time points.

To apply the wave modifications, the transmitted signal was first taken into the frequency domain and the frequency dependent changes were applied as a filter. The positive frequency components of the transmit were multiplied by the modification to give the received signal’s positive frequency components. By the properties of the Fourier transformation, the negative frequency components of the received signal were found by flipping and conjugating the positive spectrum. The total received signal in the frequency domain was found by then combining the positive and negative aspects together. Finally, the received signal in the time domain was computed by taking the inverse fast Fourier transformation. This received signal was then a copy of the attenuated transmit shifted in time.

For the work in Chapter 6, again the particulars of the noise signal were not of significant importance. As such, the propagation and system modifications, introduced by the two-way radar equation, were applied based upon the center frequency of the noise signal alone. This was similarly done for the rain attenuation which was computed for the center frequency and applied to the entire noise signal.
2.4 Two-Port Networks

2.4.1 Definitions

Two-port networks are simple representations of systems that can be used to find an output based upon a known input if there is sufficient knowledge of the system. The symbolic representation of a two-port network is shown in Figure 2.2 where the variables $a$ and $b$ represent input and output signals respectively. Networks are often described in terms of scattering parameters (S-parameters) which define the output wave at a particular port based upon the input at a particular port given a set of conditions. For a two-port system, four parameters are needed to completely describe the network. In these S-parameters, the first subscript number indicates where the output is defined while the second indicates where the input is defined.

![Two-port network representation.](image)

The voltages of a system are often normalized with respect to a given impedance resulting in the terms $a$ and $b$ as defined above. The relationship between the voltages, normalization impedance, and normalized parameters are given in (2.7) and (2.8) [12]. The negative and positive superscripts on the voltages indicate the output and input voltage waves respectively. For coax and circuit analysis, the reference impedance is often chosen to be 50 Ω. For this application, however, the S-parameters are used to describe radar propagation. As such, since the transmit and receive antenna are in air, the normalization impedance chosen was that of air which is often approximated as the impedance of free space, $Z_0 \approx 120\pi$ Ω. For
networks that are not air, such as rain or soil, their impedance is a function of the relative dielectric constant and conductivity [13] causing their S-parameters to be inherently normalized to that impedance value.

\[ a_n = \frac{V_n^+}{\sqrt{\Re \{Z_0\}}} \]  
\[ b_n = \frac{V_n^-}{\sqrt{\Re \{Z_0\}}} \]  

With the normalized waves defined, the S-parameters themselves can be written in terms of either the voltage waves or the normalized values, (2.9)-(2.12) [12]. These definitions are based upon a strict condition that there is only one input to the system at a time from one particular port. This is achieved in a two-port system by attaching a matched load, with respect to the network’s impedance, to the output port. For a network with more than two ports, all ports except the input should be attached to a matched load. Another important aspect to recall is that S-parameters are voltage ratios. As such, they are complex values. Additionally, when defining their decibel values a factor of 20 should be multiplied against the logarithm.

\[ S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} = \frac{b_1}{a_1} \bigg|_{a_2=0} \]  
\[ S_{22} = \left. \frac{V_2^-}{V_2^+} \right|_{V_1^+=0} = \frac{b_2}{a_2} \bigg|_{a_1=0} \]  
\[ S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0} = \frac{b_2}{a_1} \bigg|_{a_2=0} \]  
\[ S_{12} = \left. \frac{V_1^-}{V_2^+} \right|_{V_1^+=0} = \frac{b_1}{a_2} \bigg|_{a_1=0} \]  

Now given the formal definitions of the S-parameters, the two-port network values can be represented as a matrix relating the output values to the scaled input values. This representation is given in (2.13) using the normalized input and output parameters [12]. Note the order of the parameters with respect to the port numbers.
2.4.2 Change of Reference

Since the S-parameters for a particular network are defined with respect to that network’s impedance, their values change when the reference impedance changes. This difference becomes significant for cascaded networks that need to be combined together to form an overall set of system values. Before they can be combined, all of the S-parameters need to be defined with respect to a single reference. In [12] a simple set of formulas, based upon the reflection coefficient between the two impedances, defines the relationship between S-parameters of two different reference values. This relationship is used throughout this work for modifying parameters of a particular network to values that are normalized to an overall system impedance.

2.4.3 Cascaded Networks

For this work, S-parameters are solely used to represent radar propagation. One reason why this representation was chose was because of its ability to easily combine cascaded networks. Cascading representation is ideal for radar because a waveform moves from one section to another, such as from air into rain. By finding the individual S-parameters for each section they can then be combined together to give a picture of the overall propagation.

A cascaded network is defined as where the output of one network directly feeds into the input of another. Because of this, the output of port two for the first network is the input of port one for the second. Also, the output of port one for the second network is the input to port two for the first. This can be seen in Figure 2.3 and the relationship between the variables from the first network’s second port and from the second network’s first port can be written as: $a_2^{(A)} = b_1^{(B)}$ and $b_2^{(A)} = a_1^{(B)}$ [12].

For a cascaded system, it is convenient to write the normalized wave values in terms of one another so that the values at port one are a scaled version of the values at port two. The scaling factors used here are defined as the transfer parameters

\[
\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = S \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (2.13)
\]
Figure 2.3: Two networks connected in cascade.

(T-parameters) and their relationship with the input and output waves can be written in a matrix such as (2.14) [12]. The order of the $a$ and $b$ variables in (2.14) is important because other definitions are commonly used. The definition given here is followed throughout this thesis work.

\[
\begin{bmatrix}
a_1 \\
b_1
\end{bmatrix} =
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
b_2 \\
a_2
\end{bmatrix}
= T
\begin{bmatrix}
b_2 \\
a_2
\end{bmatrix}
\tag{2.14}
\]

The T-parameters are particularly useful for cascaded networks because they specifically define the port one variables in terms of the port two variables. Because the values between the two ports, such as between System A and B in Figure 2.3, are relatable, the port one variables of the first system can be related to the port two variables of the second. This equates to the T-parameters for the two systems being multiplied together as in (2.15) [12].

\[
\begin{bmatrix}
a_1^{(A)} \\
b_1^{(A)}
\end{bmatrix} = T_A T_B
\begin{bmatrix}
b_2^{(B)} \\
a_2^{(B)}
\end{bmatrix}
\tag{2.15}
\]

This can be extended to any number of cascaded networks allowing the very first system’s port one variables to be related to the last system’s port two variables. The relationship between the T-parameters and S-parameters can be found by simply writing out the definition equations in (2.14) and connecting them to S-parameters. A fully derived set of equations going from one to the other is given in [12]. It should be noted that the S-parameters for the different networks should first all be put in reference to the same impedance before they are converted to T-parameters and combined together.
Terrain and Simple Target Shape Correlations

In noise radar systems, the ability to correlate the received signal with a delayed copy of the transmitted is essential for target detection. The time delay to the peaks of the cross-correlation indicates the range of the targets. Furthermore, the correlation coefficients, or normalized values of these peaks, have been shown to be directly related to the probability of detection for UWB noise radars [1]. Because of this, the correlation of noise waveforms reflected off of various targets is of importance for the application of long range surveillance.

This chapter focuses on the practicality of utilizing noise radar for long range applications by computing the correlation of a reflected signal from various clutter terrains and targets. As a direct comparison, the reflections from a chirp waveform were also found for the same set of conditions. In order to understand the impact of the targets alone, path loss, weather attenuations, and other losses were neglected from the simulations with only the targets impacting the correlation. This work was originally detailed in [14] and is included here as part of the overall investigation of long range noise radar.

3.1 Correlation and Power Ratios

The two factors that were examined in this work were the correlation coefficient and the ratio between the transmitted and received signals’ power. As mentioned,
a received signal is correlated with a delayed copy of the original transmitted and the lag at which the peaks occur indicate the time delays to the target. The relationship between different targets and the signals’ correlation is therefore very important.

The correlation of a transmitted and received signal only examines their similarity. As such, the relative power of a reflected signal over the transmitted was also computed to see the impact of different targets. This relative power ratio is found for the reflections only, neglecting path and other attenuation losses, and can be compared between the noise and chirp signals to ensure that an equal amount of power is being reflected back to the radar. If for instance the noise waveform values were lower than the chirp’s, it may indicate that it is more susceptible to system noise.

For each modification case, the correlation coefficient between the transmitted and reflected signals was found. This was done neglecting other attenuations so that only the reflection itself was examined. The coefficient was found by determining the cross-correlation evaluated at its maximum. This occurs when the time delay, $\tau$, between the received and transmitted is zero. The peak value is then normalized by the square root of the product of the time average power for the two signals [1], [15]. The correlation coefficient ranges between -1 and 1 with zero indicating completely orthogonal signals and 1 indicating identical signals.

Another method of finding the cross-correlation was through the use of the CPSD. From the Wiener-Khinchin theorem, the cross-correlation and CPSD of two signals are known to be Fourier pairs [9]. The cross-correlation with no time delay can therefore be found by integrating the CPSD across all frequencies. This relationship occurs because the exponential in the Fourier transform becomes unity when the time delay is zero. A simplified version of the CPSD, $G_{XY}$, for two dependent signals can be found using a straightforward relationship which relates the Fourier transforms and the joint probability of the two signals. The resulting value of this integration can then be normalized in a similar manner to the cross-correlation, giving the correlation coefficient for the two signals.

Initially, the cross-correlation was found using both the correlation and CPSD methods. However, because of the averaging that was required to find the CPSD for each set of conditions, the computation time of simulations increased significantly. As such, finding the correlation from the CPSD was not utilized after it was
confirmed that a simulation was working properly. Implementing the correlation relationship in MATLAB was made easy by the function `xcorr` which finds the cross-correlation of two vectors. Because the phase shifts in the band-pass filter and terrain modifications introduced time delays in the signals, the `xcorr` function was needed to find the maximum value. This value was then normalized using the product of the Euclidean normalizations of the two signals. The final coefficient value used was found by taking an average of 50 iterations for each target type, waveform, and polarization.

As mentioned above the ratio of the reflected, or modified signals’ power, to the original transmitted power was another factor investigated in this work. While correlation of a reflected and transmit signal are important for signal detection, high correlation values can be obtained for extremely small amplitudes in these simulations since system noise was neglected. These ratios were found for the different targets when the incidence angles were set to zero degrees and the phase shifts introduced were zero degrees. By doing this, a standard set of values could be found for the reflected noise and chirp signals which allowed the relative estimation of their resistance to thermal noise and other variations.

### 3.2 Terrain Reflections

The reflection of the noise and chirp signals off various different ground clutter was examined to understand how the noise compared to traditional waveforms in dealing with unwanted targets. The various terrain were examined for different angles of incidence, polarization, and assumed phase shifts.

#### 3.2.1 Generating Terrain Reflections

In order to accurately represent different terrain, values from *Handbook of Radar Scattering Statistics for Terrain* were utilized [16]. In this reference, statistics for different terrain, frequency of operation, incidence angle, and polarization were given for the NRCS in decibels (dBs). The use of NRCS values eliminates the need for the antenna spot size on the different terrain, allowing the results to be independent of individual radar systems. The modifications to the signal are therefore only dependent on the terrain itself, angle of incidence, polarization, and
assumed phase shifts.

The data were provided in dBs because the distribution of the NRCS values approach a log-normal with large sample sizes \cite{16}, \cite{17}. Another benefit is that utilizing the log-normal distribution is convenient because the mean and standard deviation values are easily interpreted. This is not the case for the statistics of the linear NRCS distribution which is asymmetric. Finally, the log-normal distribution is useful for NRCS values because its average corresponds to the geometric average in linear units. Using the geometric average is advantageous because it is less dependent on outliers which frequently occur in terrain reflections and small data sets \cite{17}.

From the data set, the mean and standard deviation of different terrains in the S-band were utilized to generate voltage amplitude modifications. To get a practical understanding of how the incidence angle impacted the results, data at 20-degree intervals were utilized when available. Using a normal random number generator, a set of NRCS values in dBs were created for a given standard deviation and mean. The NRCS set was converted to linear values and then to voltage amplitude modifications by taking the square root of the linear set. An example of the dB, linear, and voltage terrain modification histograms is shown in Figure 3.1.

The simulated modifications made by the terrain also included a phase shift to the signal. This was done using both a random and linear, with respect to frequency, phase range centered on a shift of zero degrees. The reasoning for a center phase shift kept at zero degrees was that a constant phase modification can be attributed to the range between the radar system and clutter. Small movements of the radar towards or away from the target can cause a dramatic phase difference. For the purpose of this work, the center phase shift was assumed to have been either averaged out from reflections over a large swath of land or accounted for in an exponential term which can be multiplied on at the end.

In both phase modification types, random and linear, the range was varied from \( \pm 0^\circ \) to \( \pm 90^\circ \). For the random case, a vector the length of the number of discrete frequencies used was created from a random uniform number generator. The random numbers were then scaled to the phase range bounded by the upper and lower limits. In the linear phase shift case, the lower bound of the phase range was assigned to the lower stop frequency of the bandwidth and the upper bound
Figure 3.1: Histograms for wet snow at an incidence angle of zero degrees. (top) NRCS in dB. (middle) NRCS in linear units. (bottom) Voltage amplitude modifications.

assigned to the upper stop frequency. The frequencies between these two were given a phase shift which fell linearly between the bounds. For frequencies outside the band of interest, the phase associated with the band-pass filter being used was assigned.

3.2.2 Application of Terrain Reflections

The application of the amplitude and phase modifications were made in the time domain for both the noise and chirp signals as discussed in Section 2.3. For the noise cases, a new signal was generated using the same random phase values used for the transmitted. The frequency dependent terrain modifications were then applied using the particular frequency of the Fourier series as a reference. To modify the chirp signal, a similar method was utilized. The modifications were applied by re-generating the chirp with the frequency dependent modifications assigned to their corresponding frequency samples.
3.2.3 Terrain Results

In these simulations, both the noise and chirp signals showed similar trends for their correlation values but had slight variations at some incidence angles. The power ratios of the different signals were likewise similar for the two, however, there were certain terrain cases where the noise waveform had slightly larger values of around 0.5 to 1.5 dB over the chirp. For these cases, the results held across the different polarization types.

The first aspect that was examined from the simulation results was the general trends of the correlation values. For both the chirp and noise signals, the different terrain responses followed nearly identical patterns for their correlation values with respect to the angle of incidence. This is a direct result of the statistics which were used. As the standard deviation of the terrain backscatter increased or decreased, the correlation of the signals decreased or increased respectively. This is due to the fact that the correlation only examines the similarity between signals with no regard to their relative power. Regardless of whether the backscatter mean is very low, if the standard deviation remains small then all of the signal samples are essentially scaled by the same factor.

Another similarity between the signals was that the correlation dropped as the phase range was increased. This held for both the linear and random phase shifts and makes intuitive sense. A larger phase shift introduced to the signal will cause a greater difference between the transmitted and received signals and thus decrease their correlation. Though the linear phase shifts in general do have slightly larger correlations than the random phase shifts, these are small differences.

While the correlation patterns may have been similar between the two signals, the exact correlation values were sometimes different. This was particularly apparent for low incidence angles where the correlation of the chirp signals can range from 0.1 to 0.2 larger than that of the noise waveform. These differences can be attributed to larger terrain standard deviations and the random nature of the noise waveform. Since the correlation values given are an average of 50 iterations, variation of the noise signals and terrain backscatter may have averaged together to decrease the correlation. The chirp signal, however, was deterministic, meaning only the terrain variations caused the changes in correlation. It is interesting to note that this lower correlation for the noise waveform may be considered a ben-
efit as there will be less false alarms caused by ground clutter. The correlation coefficients of noise and chirp signals for grass terrain are shown in Figure 3.2.

Figure 3.2: Correlation coefficient verses incidence angle for chirp and noise signals; grass terrain.

The final aspect that was investigated for terrain reflections was the relative power of the reflected to the transmitted signals. These values were also found by taking an average of 50 iterations for each terrain type and polarization. The actual numerical values of these power reflections are not of much significance because path loss, antenna patterns, and weather attenuations are all excluded, but they do offer a direct comparison of the signals. For most terrains, the two signals have reasonably similar power ratios. In some cases such as the soil and rock, grasses, shrubs, vegetation, and snow, the noise waveform has a slightly larger ratio of reflected power on the order of 0.5 to 1.5 dB. The ratios for all cases can be seen in Table 3.1.

### 3.3 Simple Shape Target Reflections

To act as a direct comparison to terrain reflections, three simple targets were chosen which had frequency dependent RCSs. These shapes, a metal sphere, metal cylinder, and metal plate, all have well defined equations that describe their RCS
Table 3.1: Power ratios for noise and chirp signals reflected from terrain

<table>
<thead>
<tr>
<th></th>
<th>Noise</th>
<th>Chirp</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HH-Pol. (dB)</td>
<td>HV-Pol. (dB)</td>
</tr>
<tr>
<td>Soil and Rock</td>
<td>9.19</td>
<td>-11.15</td>
</tr>
<tr>
<td>Trees</td>
<td>-10.68</td>
<td>-18.5</td>
</tr>
<tr>
<td>Grasses</td>
<td>10.35</td>
<td>-10.68</td>
</tr>
<tr>
<td>Shrubs</td>
<td>2.44</td>
<td>-17.13</td>
</tr>
<tr>
<td>Short Vegetation</td>
<td>5.05</td>
<td>-14.47</td>
</tr>
<tr>
<td>Dry Snow</td>
<td>8.57</td>
<td>-8.64</td>
</tr>
</tbody>
</table>

with respect to the angle of incidence. A voltage amplitude modification was obtained by taking the square root of these RCS values after being normalized, allowing the transmitted signal to be modified.

3.3.1 Generating Target Reflections

As mentioned above, the targets used for this work were a metal sphere, cylinder, and flat plate which were chosen because they respectively have a constant, linear, and squared RCS dependency with frequency. These dependencies offered a comparison to terrain which had independent backscatter values with respect to frequency. The RCS formulas used for these simple targets are given in [18] and [13]. It should be noted that for the circular cylinder, the RCS value given at an angle of 90 degrees corresponds to a incidence wave along the cylinder side, not the circular end. In plotting and comparing the results for this target, the angles are flipped so that the coordinate system corresponds with the other shapes. Another point of note is that the angular dependence of the flat plate’s RCS contains a frequency component from a $\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$ term. If the incidence angle is zero, the angular component evaluates to one and this does not impact the RCS. For all other angles however, the $\beta$ term introduces a larger RCS difference with respect to frequency which generally corresponds to a lower correlation.

Using these equations, RCS vectors were generated with the same length as the number of chirp samples or number of samples for the noise between the two stop frequencies. These values were then all normalized to the sphere RCS so that only the relative difference between shapes, frequency, and incidence angle could be compared. The voltage amplitude modifications were found by taking the square root of the normalized RCS vectors.
Since these were metal targets, no phase shift range was introduced to the reflections. This was done because the phase shift from metal target reflection is 180 degrees. Similar to the center phase shift, it was assumed that this could be absorbed into the term accounting for the phase caused by propagation.

### 3.3.2 Application of Target Reflections

The application of the target modifications to the signal was identical to the method used for terrain. For both the noise and the chirp, the modified signal was created by re-generating the waveforms with the same original phase and amplitude. The target amplitude modifications were then multiplied onto the corresponding frequency component.

### 3.3.3 Simple Shape Reflection Results

For all three target shapes, the chirp and noise signals had similar correlation values. The noise waveforms had correlation responses which varied slightly. This variation was caused by the random nature of the noise signal itself which varied from iteration to iteration. The chirp results were smooth with respect to angle and had little variation because the signal was deterministic, giving identical correlations each iteration.

For the metal sphere, both waveforms had a correlation of nearly one across all angles. This was expected since the amplitude modification was constant with both frequency and incidence angle. Though the noise correlations were slightly lower than the chirp waveform’s, this was attributed to the fact that the noise signal had higher and lower frequencies generated outside the range of interest which were attenuated by the band-pass filter.

The metal cylinder also had a constant correlation of around 0.99 across all incidence angles. This large correlation is caused by the fact that the cylinder RCS is linear with frequency, meaning the voltage amplitude modification has a square root with frequency response. A square root dependency offers little change in the amplitude between the frequency limits of the signals and so only a small degradation of the correlation is seen. The correlation is also constant with angle because the angular dependence lowers the magnitude of the amplitude modification for all frequencies equally, which does not impact correlation.
The flat plate offers the only correlations which decrease with increasing incidence angle. The $\beta$ component of the RCS equation causes the decrease and becomes important when the incidence angle is not zero. In these cases the amplitude modification has a significant difference between the upper and lower frequencies of the signal causing the varying correlation values. Another important aspect is that the $\beta$ component also appears inside a sinusoidal argument in the RCS equation alongside the incidence angle. This is the cause of the sinusoid pattern in the correlation as the angle is increased. The correlation with respect to angle for the flat plate is shown in Figure 3.3.

![Correlation Coefficient versus Angle of Incidence](image)

**Figure 3.3:** Correlation coefficient verses incidence angle for flat plate.

The final aspect that was investigated for these targets was the relative power ratios of the modified and transmitted signals. Because the RCS values of the targets were all normalized to the sphere, all of the amplitude modifications were less than or equal to one. As with for the terrain, however, the absolute values are not significant but rather the relative difference between the two signals. From the results in Table 3.2, it is clear that both the noise and chirp waveforms have similar power ratios with the noise having only slightly larger values.
Table 3.2: Power ratios for noise and chirp signals reflected from simple targets

<table>
<thead>
<tr>
<th></th>
<th>Noise</th>
<th>Chirp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>0 dB</td>
<td>0 dB</td>
</tr>
<tr>
<td>Cylinder</td>
<td>-0.69 dB</td>
<td>-0.78 dB</td>
</tr>
<tr>
<td>Flat Plate</td>
<td>-1.34 dB</td>
<td>-1.54 dB</td>
</tr>
</tbody>
</table>

### 3.4 Conclusions

In this chapter, it was found that the correlation patterns for the two waveforms were similar for the various terrain and heavily influenced by the standard deviation of the NRCS. The noise had slightly lower correlation values than the chirp at small incidence angles which indicates that it may offer immunity to certain types of ground clutter. Additionally, the ratio of reflected to transmitted power was similar for both signal types. In the case of simple target shapes, the correlation values held similar patterns and were nearly identical for all angles of incidence. The power ratios were also nearly identical for the different shapes.

These initial results indicate that a noise waveform may be a useful alternative for long range applications. To further expand upon this, more detailed simulations involving path loss, real target responses, and receiver noise were generated. This is detailed in the following chapter where ROC curves are generated for a variety of different scenarios.
Chapter 4

Noise and Chirp Waveform ROC Curves

With the correlation and reflected power levels found to be similar for noise and chirp waveforms, additional comprehensive simulations were run. This was done to further understand the difference between the signals for a set of more complex targets than those examined in Chapter 3. Because previous results showed that noise waveforms have decreased correlation for certain terrain, the probability of false alarm may also be decreased in certain scenarios.

To further investigate this, the probability of detection and false alarm were found for the noise and chirp signals under numerous conditions. In order to find the probabilities, the return voltage from reflections off of a target of interest and various terrains were found. These probabilities were used to generate ROC curves to directly compare the different target scenarios as well as the different waveforms.

4.1 FDTD Simulation of Hummer Target

In this investigation, a hummer vehicle was chosen to represent the target of interest. Because of the limited nature of this work, actual target RCS values were unavailable for use. As an alternative, values were computed using a finite difference time domain (FDTD) software produced by Remcom [19]. This software was able to compute the RCS based upon the frequency of operation, incidence angle, and polarization.
4.1.1 FDTD Simulation

The FDTD software works by solving Maxwell’s equations in the time domain for discrete cubic cells. These cells are smaller than the wavelength in question and this process is completed for an entire geometry which composes the target. These equations are solved for the different cells at a particular time instance. The time is then stepped forward in intervals equal to the amount of time it takes an electromagnetic wave to travel from one cell to another. After a large number of steps a steady state is reached which terminates the simulation [20], [21].

The hummer target examined in this work was obtained from a database of commercially available computer-aided designs [22]. With the overall design created, material properties were assigned to the body and tires to give realistic scattering. The body was assigned to be a perfect electric conductor (PEC) and the tires were given rubber properties with a relative dielectric constant of 3 and a conductivity of $10^{-15}$ S/m [13]. The hummer model in the FDTD simulation space can be seen in Figure 4.1. An elevation angle of 15 degrees was chosen to represent a radar system looking down on the target from an observation plane. To characterize the variation of the RCS, simulations were run for all azimuth angles in five degree increments. Additionally, the VV, HH, and cross-polarization data were collected for each angle.

![Figure 4.1: Hummer model in FDTD simulation space.](image)
In this work, the frequency range of interest chosen was 2.5-3.5 GHz which offered a wide bandwidth for range resolution as well as suitable frequencies to be utilized for long range surveillance. A broadband waveform was selected as the excitation source for the simulation which experienced a three decibel drop in the amplitude between the lower and upper frequency. Because the maximum cell size allowed for FDTD simulations is one tenth the smallest wavelength [20], the grid size was set to 8.5 mm which is slightly smaller than the wavelength corresponding to 3.5 GHz. This was done to limit the required memory and expected run time.

With these parameters selected, two gigabytes memory was necessary for each polarization and angle combination. Because of the significant requirements for the target alone, a ground surface was not included in this model. An empirical model of ground reflections, discussed in Section 4.2, was utilized separately to give a clutter response. Each individual simulation took an average of 1.5 hours to run on a large personal computer giving an overall computational time of 216 hours for all combinations.

4.1.2 FDTD Simulation Results

With the simulations done, the RCS data for the hummer target were exported to MATLAB for processing. In post-processing, the RCS values were plotted in two dimensional maps to show the variation with respect to frequency and with respect to the azimuth angle. The results of these plots can be seen in Figure 4.2. In these plots, the color layout suggested by [23] was used to ensure grayscale images maintained the same relative magnitude between levels.

From these plots, it is clear that the VV and HH-polarization cases have very similar patterns with peak values occurring at 0, 90, 180, and 270 degrees. These angles make intuitive sense since at them the incidence wave was perpendicular to the target’s flat surface resulting in maximum backscatter. At other angles the hummer body acted as a flat plate with specular reflections. The cross-polarization case shows a more uniform distribution of RCS with respect to angle without the peaks at 0, 90, 180, 270 degrees. This again was due to the planar surfaces at these angles which reflect mainly co-polarized signals back. The 180° angle has some of the lowest values because it was the flattest surface for the hummer.

For all three polarizations, the results are consistent with respect to frequency.
A plot of the average values with respect to frequency can be seen in Figure 4.3. The cross-polarization is clearly lower than the two co-polarization cases as expected and there are only minor differences, on the order of one or two dB, between the VV and HH cases.

### 4.2 Clutter RCS

To directly contrast the hummer target, a model of ground clutter reflections was also implemented in this study. Returns from the ground could be considered a false alarm to the system if they exceeded a set threshold. This is particularly
4.2.1 NRCS Model

To determine the NRCS values for bare soil, an empirical model developed in [24] was used. This model was developed by taking data over different soil conditions, polarizations, frequencies, and angles of incidence and fitting curves to the results. One thing to note is that the data used for these models were taken at incidence angles from zero to 70°. In this work the model is extended to an incidence angle of 75° to correspond to the RCS results from the hummer target. The use of the model for this angle was justified based upon the strong fit of the curves to the data and the fact that it only extends outside the original range by five degrees.
With this model, NRCS values can be computed for a variety of different scenarios. Firstly, different values must be computed for each individual frequency of the respective signals. In addition, different combinations of soil roughness and moisture were examined. These play a role in the equations directly in the case of the RMS soil height, and indirectly through the Fresnel reflectivity for the soil moisture.

Four roughness values were examined which were considered; Very Smooth with a soil RMS $S$ of 0.32 cm, Smooth with $S$ being 0.4 cm, Average with $S$ being 1.12 cm, and Very Rough with $S$ being 3.02 cm [24]. In addition three different soil moisture values were examined which were given for a wavelength of 0.03 m. These included; Good Soil (wet) with $\varepsilon_r = 13$ and $\sigma = 3$ (S/m), Average Soil with $\varepsilon_r = 7$ and $\sigma = 1$ (S/m), and Poor Soil with $\varepsilon_r = 3.5$ and $\sigma = 0.3$ (S/m) [25].

The moisture content plays a direct role in both the real and imaginary parts of the dielectric constant which in turn impacts the Fresnel reflectivity. For three different moisture levels, the real part of the dielectric constant and the conductivity of the soil were given in [25]. The conductivity is used because it is directly related to the imaginary component of the dielectric.

### 4.2.2 Clutter RCS Values

With the NRCS values found, the next step was to use these values to find the RCS for a particular set of system conditions. To accomplish this, the antenna spot size was computed so that it could be multiplied by the constant value. The antenna spot size, given in (4.1), was computed for three different ranges in this work [18]. In this equation, $R$ is the range to the target, $\theta_{3dB}$ is the 3-dB beamwidth of the antenna system, assumed to be one degree, $\tau$ is the pulse width of the signal, and $\psi$ is the grazing angle of the system. All angles are given in radians.

$$ A = R\theta_{3dB}\frac{c\tau}{2}\sec\psi \quad (4.1) $$

The area of the antenna spot size can be directly multiplied to the linear NRCS value to give an RCS for a particular set of system and soil conditions. In this work, three different ranges of 7.5, 10, and 15 km were examined to see their impact. To ensure a direct comparison, the pulse width of the noise and traditional waveforms were kept the same to ensure identical antenna spot sizes.
4.3 Detection and False Alarm Probabilities

To directly compare the results of different waveforms, as well as different system parameters, the detection and false alarm probability was found. By finding the probabilities using identical threshold values, they can be plotted against one another to form ROC curves. To find these probabilities, a modified version of the formula used to find the probability for a single voltage was utilized. This modified version accounts for the variation of return voltage from a target by including the probability density function (PDF) of the return voltages.

4.3.1 Modified Probability Formula

For a single received voltage at a noisy receiver, the PDF is well known [26]. Given a particular SNR, integrating this function from a threshold value to infinity gives the probability that a voltage exceeds the threshold and indicates a detection. While this formula is useful for a particular return voltage and hence SNR, it neglects the variation of return voltage caused by a changing RCS from a moving target, a fluctuating clutter RCS, or change of power distribution with respect to frequency for a noise waveform. These variations cause differing SNR values from pulse to pulse giving different probability values.

To account for changing returns in this work, the original formula was modified by multiplying it against the return voltage PDF for a target, changing the SNR for all possible return voltages, and integrating over all possible return values [27]. This equation is correct because the SNR varies for the range of all possible return voltages. This modified formula therefore uses a possible return voltage to compute the SNR of the signal. The double integral formula works by cycling through all possible return voltages, scaling the probability of particular cases by the return voltage PDF, and finds the overall probability that the threshold is exceeded.

This is valid because both PDFs must integrate to one by definition and so the overall value can itself only be a maximum of one. To compute the probability of detection, the return voltage PDF for the hummer target was used. For the probability of false alarm, the return voltage of the clutter target was used.

The modified formula is given in (4.2) where $r$ is the receiver output normalized to the thermal noise variance, $r_b$ is a particular normalized threshold, $S_r$ is the SNR
of the particular returned voltage, $I_0$ is the modified Bessel function of the first kind order zero, and $\rho(v)$ is the PDF of the returned voltage.

$$P_D = \int_0^\infty \int_0^\infty r \exp \left( -\frac{r^2 + S_r}{2} \right) I_0 \left( r \sqrt{S_r} \right) \rho(v) dr dv$$

(4.2)

### 4.3.2 Generating Return Voltage PDFs

The return voltage PDFs introduced in the previous section were not only dependent on the target or clutter response, but also upon the range to the target, antenna gain, and other system parameters. To keep matters consist, the overall parameters were kept the same and three ranges as well as three receiver noise bandwidths were examined. These parameters are given in Table 4.1.

<table>
<thead>
<tr>
<th>Radar System Parameters</th>
<th>7.5, 10, 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range (km)</td>
<td>7.5, 10, 15</td>
</tr>
<tr>
<td>Transmit Pulse Power (W)</td>
<td>200</td>
</tr>
<tr>
<td>$T_x/R_x$ Antenna Gain (dB)</td>
<td>30</td>
</tr>
<tr>
<td>Receiver Temperature (K)</td>
<td>300</td>
</tr>
<tr>
<td>Noise Bandwidth (MHz)</td>
<td>25, 50, 1000</td>
</tr>
</tbody>
</table>

Since the noise and chirp signal are voltage waveforms, a modified version of the radar range equation was used. Firstly, the reflected signals from different targets were found by multiplying the original transmit by the square root of the frequency dependent RCS. The square root was taken because the RCS is a power quantity. Next, the other components of the two-way radar equation needed to be taken into account. That included aspects such as the antenna gains and the range to the target which account for the overall frequency dependent path loss of the signal. Again, these values are all used for finding the received power so the square root of these terms were found before being applied to the voltage signal.

With these modifications made, the signal then represented what would be returned after reflecting from a target. In order to create the PDF of returned voltages the average values of the returned pulses were found. This was done by generating the envelope of the signal and then finding the mean of that envelope. Multiple returns were collected to generate histograms and then PDFs.
To get an overall response for different targets, multiple iterations were run for each RCS set. Since there were 72 different angles for the hummer target, each of these cases were used to generate responses. In addition, the noise signal generated by the simulation varied each iteration causing slight power differences between frequencies. To account for these variations, as well as generate a full histogram, 50 iterations were run for both the noise and chirp waveforms at each of the 72 hummer angles. In the case of the clutter targets, there was only one set of RCS data for each combination. As such, 150 iterations were used for both the noise and chirp waveforms to give a balanced histogram.

The histograms were then used to create PDFs for the individual target and range conditions. First, a best-fit line was generated for a particular histogram. This was then normalized to the number of points in the histogram and the bin width. A low-pass filter was applied to these lines to smooth the overall shape. Finally, the curves were tested by ensuring they integrated to one over all possible return voltages. Only then were they used in (4.2) to modify the probability of detection or false alarm.

4.3.3 ROC Results

After generating a set of return voltage PDFs, the probability of detection and false alarm were found using the same threshold values. This was done so that the two probabilities could be directly plotted against one another to form ROC curves. The threshold values were also kept the same for the different soil backscatter combinations as well as different ranges so the results could all be directly compared to one another.

As mentioned previously, the clutter targets were used to find the probability of false alarm. This is important to note because for the long range cases the false alarm rate is much greater than detection at particular thresholds. This causes ROC curves to be pulled in the false alarm direction much more than would be typical if thermal noise alone was used for determining false alarm. By varying system parameters such as the range and half power beamwidth, the spot size of the illuminated clutter will change and hence the false alarm values. For this work, these were kept constant so a variation of range alone could be compared.

Plotting the different conditions together revealed overall patterns which were
consistent across the ranges. The average and very rough soil combinations had stronger returns causing an increased false alarm probability for a given threshold. On the other hand, the smooth and very smooth soils had weaker returns leading to less severe probabilities of false alarm.

Combining the soil roughness levels with different moisture values gave more pronounced effects. Using the wet soil along with the average and very rough soil types led to stronger probability of false alarm results than drier soils. This was caused by the larger conductivity of the wet soil giving rise to larger reflections. The alternative also held with the smooth roughness and dry soil combination. In these cases, the probability of false alarm was the lowest out of all the combinations for a particular range. The dry soil’s lower conductivity as well as the smooth surface led to less reflections compared to the other cases.

Another common trend that was consistent across different ranges was the performance of different thermal noise bandwidths. In this work, three receiver noise bandwidths were examined to see how their respective SNR values impacted the ROC curves. In cases where the clutter response was overwhelming, such as the very rough surfaces, the 1 GHz noise bandwidth signals fared the best. Often these ROC curves were on top of, if not approaching, the 50/50 line between false alarm and detection. The smaller bandwidths of 25 and 50 MHz fell below the line with the false alarm probability dominating. This counterintuitive result was caused by the fact that the noise in the larger bandwidth cases dominate over the clutter returns. In the more limited bandwidths, the clutter response was much larger pulling the overall curves below the 50/50 line.

As the range to the target was decreased from 15 km down to 10 and 7.5 km, the spot size of the antenna beam decreased. This in turn acted to reduce the clutter RCS values. However, even with a decreased RCS the significant path length reduction led to larger return voltages in general. In the cases of average and very rough soils these increased voltage returns from the clutter caused more significant false alarm triggers than the larger spot sizes at further ranges.

In the cases of smooth roughness values and wet soil, the results varied between polarizations. The VV-polarization generally was worse than the cross and HH-polarization because of its larger clutter return. In the cases of drier and very smooth soils, the smaller RCS allowed for a decrease in the false alarm rate for all polarizations. As such the ROC curves for the shorter ranges were much better.
than the 15 km case with the curves tending towards high detection at low false alarm rates.

An example of poor ROC curves can be seen in Figure 4.4 which shows the results of wet very rough soil at a range of 15 km. In this combination, the curves for both the noise and chirp waveforms favored the clutter response leading to a larger false alarm probability. As previously mentioned, Figure 4.4 shows the 1-GHz noise bandwidth case faring the best and the 25 MHz case the worst. This is consistent for all three polarizations and waveforms. An interesting note is the cross-polarization curves fare better than the co-polarization because of their lower clutter return. For the two shorter ranges the ROC plots for the wet very rough soil became worse giving a false alarm probability of near one at low detection values. For this reason the plots are more difficult to visualize and are not shown.

The results for the very smooth and dry soil case at a range of 7.5 km are given in Figure 4.5 to contrast the 15 km rough and wet soil case. In this case, the clutter returns are significantly smaller giving a decreased false alarm rate. The low false alarm rate indicates that a hummer target on this type of soil should be more easily distinguished. In addition, the different noise bandwidth cases fall in line as expected with the 25 MHz case giving the best results and the 1 GHz case the worst. Another interesting note is that the co-polarization cases show better results than the cross-polarization because of the much stronger hummer returns.

After examining the various range and clutter combinations, a pattern between the results of the noise and chirp waveforms emerged. For cases where the clutter dominates, the noise and chirp results are comparable while the noise did fare slightly better. This can be seen in Figure 4.4 where noise signal results for the 25- and 50-MHz thermal noise bandwidth cases have clearly higher detection probabilities. In smoother and drier cases, where the ROC curves tend to favor detection, the noise waveform dramatically outperforms the chirp giving much larger detection probabilities at low false alarm rates. This can clearly be seen in Figure 4.5.

In particular areas, the chirp waveform does experience slightly better results for the smooth and dry soil case at a range of 7.5 km. As the threshold voltages are decreased, and both the detection and false alarm probabilities increased, the noise waveform detection results flatline, slowly increasing with respect to the chirp’s. This is clearly visible in the VV and HH-Polarizations of Figure 4.5 for the 25-
Figure 4.4: Receiver operating characteristic curves for a very rough and wet soil surface at a range of 15 km. (a) VV-Polarization. (b) HH-Polarization. (c) VH/HV-Polarization.
Figure 4.5: Receiver operating characteristic curves for a very smooth and dry soil surface at a range of 7.5 km. (a) VV-Polarization. (b) HH-Polarization. (c) VH/HV-Polarization.
and 50-MHz receiver noise bandwidth cases. While the noise waveform does get surpassed by the chirp, it is only in certain instances where the false alarm rate is very high. For the most part, the noise waveform outperforms the chirp signal in these simulations.

The cause of the better ROC curves for the noise waveform is attributed to the random nature of the signal itself. Since it was randomly generated, the noise waveform shifted power to different frequencies from iteration to iteration. This may have helped to bring the average clutter return down for the noise signal. The chirp waveform on the other hand was deterministic and distributed power to different frequencies identically each pulse. Because the noise waveform had comparable, if not better, results than the chirp signal for the various range and clutter combinations, it indicates that it may be a useful alternative for long range radar systems.

4.4 Conclusions

This section directly examined the use of noise and chirp waveforms for long range radar surveillance. This was done by comparing the probability of detection and false alarm for the signals under various conditions. To find the probability of detection, signals were reflected from a hummer target. The RCS values for the hummer were found using FDTD simulations for 72 azimuth angles and a set incidence angle. The false alarm probabilities were found by reflecting the signals from various terrain conditions. Using different soil roughness and moisture values within a clutter model allowed for different comparisons. These target and clutter RCS values were used to find the return voltage of the noise and chirp waveforms and with them the probability of detection and false alarm.

From the results, clear patterns emerged. Rough and wet soil combinations gave large returns which in turn increased the probability of false alarm. Drier and smoother soils on the other hand gave smaller false alarm probabilities improving the ROC curves. The results for the different conditions indicated that the noise waveforms perform as well as, if not better than, the chirp. The noise signals may therefore be useful as an alternative waveform for surveillance purposes.
This chapter discusses the modeling of radar propagation and scattering using two-port network analysis. By choosing this method of analysis, different sections of the propagation, such as through rain or air, can be individually tailored before being combined together. The overall combination of the two-port parameters can then be utilized in determining the final output of a system given a specific input.

To analyze noise radar, two propagation scenarios were examined; one looking forward and one looking downward to the ground.

The two-port network, or scattering parameter (S-parameter) analysis, has unique properties that make it ideal for modeling radar propagation. As briefly mentioned, the S-parameters for cascaded networks can easily be combined together to form an overall set of system values. This is useful since a transmitted signal moves from one area to the next in a cascaded manner. By setting up a simulation properly, the different input variables such as path length or rain rate can easily be modified for a given section. The overall parameters could then be changed based upon those modifications allowing for simpler system analysis.

Another useful feature is that S-parameters are frequency dependent. Based upon the frequency, the phase or attenuation caused by a certain section of propagation may change. This allows for easy modification of the transmitted waveforms and is very useful for the analysis of noise signals.
5.1 Forward-Looking Model

The first radar scenario that was examined was a forward-looking radar. This was chosen first because of its simplicity in design with only three sections involved; a section of air, a rain column, and a final section of air. A representation of this radar can be seen in Figure 5.1. Here, the three networks are connected in a cascaded form as discussed previously.

![Figure 5.1: Simple two-port representation of a forward-looking radar system.](image)

5.1.1 Air S-Parameters

The first network analyzed was the air section. To derive some parameters, approximations were made. In the air, it was assumed that there was no attenuation of a propagating wave so that the network could be represented as a lossless transmission line. Using [12], the S-parameters for a transmission line attached to a matched load are easily computed. In the transmission line cases, there is a phase component introduced to the voltage signals. This is the same phase introduced to a propagating plane wave and for the $S_{21}$ parameter can be represented simply as, $\exp(-j\beta l)$, where $l$ is the path length through the air and $\beta = 2\pi\sqrt{\varepsilon_r}/\lambda$. For air, $\varepsilon_r \approx 1$ and so, $\beta \approx 2\pi/\lambda$. Since the networks are considered constant in terms of attenuation and phase with respect to direction, the $S_{12}$ parameter is equal to the $S_{21}$.

For the $S_{11}$ and $S_{22}$ parameters, the reflection coefficient for particular port were found assuming the other port was attached to a matched load which is a requirement for deriving S-parameters. In the first air section, the reflection coefficient of port one is zero. This is because it is assumed that a wave entering port one was previously traveling from air and so the impedances are matched.

For the $S_{22}$ parameter, the impedance of rain impacts the reflection at the
second port. Since the impedance of rain is very close to air, as will be discussed in Section 5.1.2, the reflection at port two is considered to be zero. A similar method was used to find the S-parameters of the other air network with the assumptions from the first air section reversed for the respective ports. With this in mind, the S-parameters for the air networks can be written as (5.1).

\[
S_{air} = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} = \begin{bmatrix}
0 & e^{-j\beta l} \\
e^{-j\beta l} & 0
\end{bmatrix}
\] (5.1)

5.1.2 Rain S-Parameters

The next section evaluated was the rain which required different techniques from those of the air. This is because the rain acts to attenuate the waveform, introduce a phase shift dependent on the rain rate, and reflects power back to the source. The first parameter examined is the \(S_{11}\) which is derived based upon the reflected power from the transmitted waveform. For this case, the phase introduced is neglected and simply a magnitude response is found. The reason for only examining the magnitude is that the phase response of the different rain drops average out. The connection between the \(S_{11}\) parameter and the reflected power is given below in (5.4).

Given;

\[
S_{11} \triangleq \frac{V_1^-}{V_1^+} \] (5.2)

\[
P_x^n = \left| \frac{V_{x^n}}{2Z_0} \right|^2 \rightarrow \frac{P_x^-}{P_1^+} = \left| \frac{V_{x^n}^+}{2Z_0} \right|^2 \rightarrow \sqrt{\frac{P_x^-}{P_1^+}} = \left| \frac{V_{x^n}^-}{V_1^+} \right| \] (5.3)

therefore;

\[
|S_{11}| = \sqrt{\frac{P_{1^-}}{P_1^+}} = \sqrt{\frac{P_r}{P_t}}
\] (5.4)

To find the reflected power from rain, a modified version of the derivation given in [28] was used. This modification accounts for two different ranges to the rain.
One range was used to determine the path loss of a transmitted signal. In order to give an average response from the column of rain, half the length of the column was used. This would account for the path loss given by transmitting to different areas of the rain. The second range that was used was for determining the volume illuminated by the radar. This range included the path length of the first air section as well as the half distance used in the rain formulas. By including the air path length, the volume of the illuminated section was significantly larger. The complete steps for the derivation are given in [28] but are briefly discussed here to illustrate the final result.

The derivation begins with the two-way radar equation, with equal transmit and receive antenna gains, relating the received power to the transmitted and a single target’s RCS. The average received power from a group of independent scatterers can then be written as the two-way radar equation with the scatterers’ RCS values simply summed together. This is a valid assumption because the objects’ position varies from pulse to pulse which causes the phases of the shared voltage returns from multiple objects to average out. The final average return power is therefore only proportional to the individual scatterer returns. This summation can then be normalized to a volume which is illuminated by the radar beam. For this work, a simple Gaussian beam was assumed as well as that the $\theta$ and $\phi$ half power beamwidths were both equal to one degree.

The summation of the individual RCS values normalized to a volume can then be directly related to the summation of rain drop diameter when the Rayleigh approximation criteria are met. This is applicable only for frequencies below 10 GHz in typical rain. The drop diameter can then be related to the radar reflectivity factor, $Z$, which can in turn be found from the rain rate, $R$, based upon multiple relationships [28], [29]. The final average power received can then be written as shown in (5.5) where $r$ is equal to half the path length of the rain, $r_T$ is equal to the path length of the air plus half the rain, and $|K|^2$ is the magnitude squared of the complex index of refraction.

$$\bar{P}_r = \frac{P_t G^2 r_T^2 \theta \phi c \tau \pi^3 |K|^2}{1024 \ln (2) \lambda^2 r^4} Z$$

(5.5)

From (5.4) and (5.5) the magnitude of the $S_{11}$ parameter can therefore be written as (5.6). Because of the reciprocal nature of rain, this $S_{22}$ parameter is
equal to the $S_{11}$ parameter.

$$|S_{11}| = \sqrt{\frac{P_r}{P_t}} = \sqrt{\frac{G^2r_T^2\theta\phi e T\pi^3 |K|^2}{1024\ln(2)\lambda^2\tau^4} Z} \quad (5.6)$$

For the rain’s transmission parameters, both the attenuation and phase shift needs to be accounted for. The attenuation caused by rain is frequency dependent and values for four specific wavelengths are given in [28]. Using these values an interpolated line was created to act as an approximation of the attenuation across the bandwidth of interest. The attenuation values are given in dB/km which requires first that the path length is multiplied on and then the dB values are returned to voltage linear units.

The phase shift introduced by the rain is similar to the air sections with the $S_{12}$ and $S_{21}$ values being equal due to the independent nature of rain with respect to direction. In the rain however, the relative permittivity differs from air causing a difference in the wave number and hence phase shift. To find the dielectric constant of rain, the Maxwell-Garnett mixing formula was used which assumes spherical scatterers embedded in a background medium [30]. To use this formula, the volumetric fraction of the water to air needs to be first computed. This can be found using simple relationships between the liquid-water content of rain and the rain rate [28]. In nearly all cases except for extremely heavy rain, the proportion of water to air is very small leading to an effective dielectric constant very similar to air.

With the effective dielectric constant found, the phase shift for the rain section can be found. Essentially, the only aspect that this changes is the wave number in the phase of the $S_{21}$ and $S_{12}$ parameters. The wave number can be computed as, $\beta = 2\pi\sqrt{\varepsilon_{eff}}/\lambda$. Finally, the $S_{21}$ parameter of rain can be written as, $S_{21} = \alpha e^{-j\beta l}$ with $\alpha$ being the attenuation. The overall parameters for the rain can then be written together in (5.7) where $S_{11}$ is defined in (5.6).

$$S_{\text{rain}} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} S_{11} & \alpha e^{-j\beta l} \\ \alpha e^{-j\beta l} & S_{11} \end{bmatrix} \quad (5.7)$$

In addition, the dielectric constant allows the rain network’s impedance to be computed. Since the rain water contributes very little to the overall volume fraction of rain, the conductivity is neglected and the impedance is found using the formula,
\[ Z_{\text{rain}} = \frac{Z_0}{\sqrt{\varepsilon_{eff}}} \] [13]. This value is the inherent normalization impedance of the rain network’s S-parameters.

### 5.1.3 System Parameters

With the individual network S-parameters found, they can be combined together to form an overall set of parameters for the radar. This can be done by first converting the rain parameters to values normalized by the air’s characteristic impedance. Following that, the different networks can be combined based upon the transformation given in Section 2.4.3.

Since the parameters are frequency dependent, the S-parameters for particular frequencies were examined. Increasing the rain rate or rain path length increases the overall system attenuation seen in the forward and backward transmission coefficients as expected. In addition, the system parameters as a whole have significant phase shifts for the transmission and reflection parameters which are related to the path length of the overall system. These phase shifts add together when computing the final received signal which propagates forward and back through as system resulting in significant time shifts.

### 5.1.4 Forward-Looking Radar Correlations

With the system parameters found, they were then applied to waveforms to examine the impact of various different rain and path length conditions. This was done by finding the input reflection coefficient for the system which is dependent on the system parameters as well as the reflection coefficient from a target [12]. The target reflection coefficient is given in (5.8) where \( Z_L \) is the impedance of the target and \( Z_0 \) is the system impedance which is equal to that of air. Using this value, the input reflection coefficient can be found from (5.9) where the S-parameters given here are for the total system.

\[
\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (5.8)
\]

\[
\Gamma_{in} = S_{11} + \frac{\Gamma_L S_{12} S_{21}}{1 - \Gamma_L S_{22}} \quad (5.9)
\]

For each frequency component of a signal, the S-parameters can be found and
then used to compute the input reflection coefficient. Because of the large number of points needed for zero-padding, a significant number of S-parameters, and hence reflection coefficients, are computed for each waveform’s frequency range of interest. Applying this coefficient is done to give a returned signal in a sense which is reflected off of a particular target impedance. By adding an additional multiplicative term to each frequency, a true return waveform can be found. This term is the square root of the two-way radar equation so that path loss, antenna gains, and a target RCS can be accounted for. The square root is taken because the radar equation relates to power while the desired return waveform is a voltage constant.

To get a general understanding on how the different rain rates, path lengths, and overall system parameters impact waveforms, this method was applied to both a noise and chirp signal. In both of these waveforms the frequencies are known allowing the particular S-parameters and input reflection coefficients to be computed. As briefly mentioned in Section 2.3, the frequency band of the waveforms has been shifted up from the previous chapters to 3.5-4.5 GHz. This was done so that the frequencies of interest would remain above 3 GHz which is the bottom frequency limit for rain models.

To get a comparison of systems, different target impedance values were used. Values of 0, 50, $50 + j50$, $50 - j50$, $120\pi$, and $10 \times 10^9 \, \Omega$ were all examined to represent a perfect conductor, variations of a 50 Ω impedance, air impedance, and infinite impedance. In addition, rain rates of 0, 10, 25, 50, and 100 mm/hr were examined to sweep from none to very heavy rain. The US National Weather Service default model for relating the rain rate and radar reflectivity factor was used for all simulations [29]. Finally, while the air path lengths were kept constant at 1500 meters each, the rain path length was varied from 5 to 1000 meters in 5 meter increments. A summary of the system conditions used for this investigation is given in Table 5.1.

With the attenuated received signals computed, the actual signal pulse was found within the zero-padding. Using an envelope detector, the index values for the rise and fall of the pulse were noted and used to remove the signal. Thermal noise was then added to represent the noise experienced at a receiver. This was done using the simple formula relating the thermal noise to the temperature and bandwidth of the receiver. Because of the way MATLAB’s additive noise function
Table 5.1: Forward-looking radar model’s system parameters for generating return waveforms

<table>
<thead>
<tr>
<th>Radar System Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Range (m)</td>
<td>3005-4000</td>
</tr>
<tr>
<td>Transmit Pulse Power (W)</td>
<td>500</td>
</tr>
<tr>
<td>$T_x/R_x$ Antenna Gain (dB)</td>
<td>30</td>
</tr>
<tr>
<td>Target RCS (dBsm)</td>
<td>5</td>
</tr>
<tr>
<td>Frequencies of Interest (GHz)</td>
<td>3.5-4.5</td>
</tr>
<tr>
<td>Receiver Temperature (K)</td>
<td>300</td>
</tr>
<tr>
<td>Assumed Noise Figure (dB)</td>
<td>3</td>
</tr>
<tr>
<td>Receiver Noise Bandwidth (GHz)</td>
<td>1</td>
</tr>
</tbody>
</table>

$awgn$ works, noise with a power proportional to 15-GHz was first added. This spread the noise power from zero to $F_s/2$ Hz. Band-pass filtering the new signal then resulted in noise with 1-GHz proportional power being added to the 1-GHz frequency band of interest. The correlation between the received signals and their respective transmitted could then be found.

As the rain rate and rain path length is increased, the correlation for the two signals decreases as expected. This trend holds for all target impedance cases except for the air. In that case, the correlation drop is caused by the decreased received power. Since there is no target reflection, the received signal is only proportional to the reflection from the rain itself. As such, the received power is very little and dominated by the receiver noise. When the target impedance is set to air, the correlation for all rain rates and ranges centered on 0.15.

The results for the target impedance of 0 $\Omega$ can be seen in Figure 5.2. In this case, as in the others, the noise and chirp waveform correlations follow the same pattern with respect to rain path length. Depending on the rain rate, the correlation drops 0.15-0.2 between the shortest and longest path lengths. The chirp signal, however, seems to have a slightly larger correlation for all rain rates and rain path lengths. To allow for easier comparison between the noise and chirp waveforms, their correlations for a rain rate of 25 mm/hr are plotted together in Figure 5.3 where the small increase of the chirp over the noise signal’s results is clearly visible.

For the case of an impedance of $10 \times 10^9$ $\Omega$ the correlations can be seen in Figure 5.4. In this case the correlation values are nearly identical for the noise...
and chirp waveforms for all rain rates at low path lengths. Only as the path length is increased, and hence there is more attenuation, does the rain and additive receiver noise cause differences between cases. Clearly from these results there is a pattern of varying correlation with path length. This is caused by the S-parameter’s transmission phases adding together and varying in $\pi$ increments for the input reflection coefficient. From (5.9), it is clear that the transmission S-parameters are the dominant factors of the input reflection coefficient when a target reflection coefficient is near 1 and the $S_{11}$ and $S_{22}$ parameters are low.

The results for the chirp and noise signals in the target impedance case of $10 \times 10^9$ $\Omega$ are also given for a single rain rate of 25 mm/hr in Figure 5.5. This was
done to show the varying pattern of the correlation with respect to path length more clearly as well as give a direct comparison of the two waveforms.

When the $10 \times 10^9$ and the 0 $\Omega$ cases are considered together, it appears that the correlation may be heavily influenced by the phase of the target reflection coefficient. For the 0 $\Omega$ case, the target reflection coefficient is -1 giving a phase of 180 degrees. This may offer a significant change in the phase of the input reflection coefficient which in turn directly impacts the received signal. If, like the 0 $\Omega$ case, there is no dependency of the reflection coefficient on the S-parameter’s phase then there will be no varying correlation with respect to the path length. On the other hand, in the very large impedance case which is meant to represent an infinite impedance, the target reflection coefficient is near 1. This introduces no target phase shift and correlation varies with phase and hence path length.

The results for the 50 $\Omega$ variation cases also fit this pattern. In these cases, the target impedances are all less than that of the system impedance, $120\pi$ $\Omega$, leading to negative target reflection coefficients. In addition, the complex target loads will result in complex reflection coefficients. These phases differences are introduced to the input reflection coefficient giving correlations which are not dependent on the path length.

For the different cases the chirp waveform seems to perform better than the noise signal but only slightly, with generally comparable correlations. The 50 $\Omega$ variations all have lower correlation values than the previous two impedances, especially at larger path lengths. This is caused by the decreased magnitude of
the target reflection coefficient and so decreased amount of reflected signal. For larger path lengths this becomes apparent and noticeable because of the increased attenuation. These results are consistent for all 50 $\Omega$ impedance variations and as such, only the results for the purely real 50 $\Omega$ case are shown here in Figure 5.6.

Again, to directly compare the noise and chirp signal results, the correlations for the case of a rain rate of 25 mm/hr is shown in Figure 5.7. Here it is clear that the 50 $\Omega$ target impedance does not induce a varying correlation like the $10\times10^9$ $\Omega$ case. Additionally, the differences between the chirp and noise waveform results are more easily seen.

An interesting note is that this type of phase may be introduced to a signal
regardless of a target’s impedance. Multiple target reflections, such as off a corner or from a rough surface, will introduce a phase shift to a waveform. However, this may be an advantage for examining certain targets. If a phase shift is introduced by a target, then there less chance that phase cancelation will occur between the transmitted and received signals and the correlation will be higher. If not, then a dependency on the path length will be evident in the correlation. A clear pattern that can be seen may offer additional information about the additive phase and hence the target itself.

5.2 Down-Looking Model

The second radar scenario that was examined was a down-looking radar. This was investigated to act as a complement to the forward-looking model and to allow for the representation of soil in terms of two-port networks. For this work, a radar looking directly down on soil was modeled with multiple layers varying in volumetric soil moisture, $m_v$. In Figure 5.8, the different areas of propagation are modeled with rough surfaces to represent a realistic interface.

This can be represented as a cascaded system of two-port networks which are sized based upon the antenna spot size on the ground. The cascaded connection of the networks is utilized because there are reflections back towards the radar at the different soil layers as well as transmission into the layers based upon their individual characteristic impedance values. These impedance values are based upon
Figure 5.6: Correlation of transmitted and received signals for a target impedance of 50 Ω. (a) Noise Waveform. (b) Chirp Waveform.

the moisture content of the layers which indicate the different amounts of water and dry soil being mixed together. The two-port representation of the networks is given in Figure 5.9 where air is the first network because it is the location of the transmitter and receiver.

5.2.1 Soil Permittivity

In this model, the reflection, transmission, and attenuation of the waveforms from different networks are all directly related to the permittivity of the soil layers. Based upon the relative permittivity of the layers the characteristic impedance varies which in turn impacts the reflection and transmission. Additionally, the
Rain Rate = 25 mm/hr, Target Impedance of 50 Ω, \( P_t = 500 \) W, NF = 3 dB
Tx/Rx Antenna Gain = 30 dB, Target RCS = 5 dBsm, \( R_{air1} = R_{air2} = 1500 \) m

Figure 5.7: Correlation of transmitted and received signals for a target impedance of 50 Ω and a rain rate of 25 mm/hr.

Air

Soil 1 – \( m_{v1} \)

Soil 2 – \( m_{v2} \)

\vdots

Soil \( N \) – \( m_{vN} \)

Figure 5.8: Radar representation of a down-looking radar system with multiple layers of varying soil moisture.

Figure 5.9: Two-port network representation of a down-looking radar system with multiple layers of varying soil moisture.
complex part of the permittivity is directly related to the conductivity of the soil layer determining the attenuation of the waveform.

To compute the complex permittivity for the soil layers, an empirical model developed in [31] and [32] based upon data from multiple locations presented in [33], was used. This model is dependent upon the frequency, volumetric soil moisture $m_v$, bulk density of the soil $\rho_b$, specific density of the soil $\rho_s$, the percent of sand in the soil, and the percent of clay in the soil. Throughout this work, the soil characteristics of Field 2 from [31] which contained 41.96% sand, 8.53% clay, had a specific density of 2.7 g/cm³, and an assumed bulk density of 1.1 g/cm³ were utilized.

Based upon the different soil layers’ moisture content, their complex relative dielectric constants were computed for the range of frequencies of interest. Since the soil model is valid for 1.4-18 GHz, the frequency range of 3.5-4.5 GHz was chosen for this work. Using the complex component of the dielectric constant, the equivalent conductivity was found which combines the static and alternating conductivity. For wet soil, this can be written as (5.10) [13].

$$\sigma_e = \sigma_s + \sigma_a = 10^{-3} + \omega\varepsilon_0\varepsilon''$$  \hspace{1cm} (5.10)

With the conductivity found, the attenuation constant, phase constant, and complex impedance for the particular layers were computed based upon the exact formulas given in [13]. These were then used for deriving the S-parameters of the different networks represented in this model.

### 5.2.2 Air S-Parameters

The first network analyzed for the down-looking model was the air section. This derivation was similar to that of Section 5.1.1 except that there are reflections when a waveform enters port two. This impacts the $S_{12}$ and $S_{22}$ parameters. The $S_{11}$ parameter is zero however, because it is assumed that a waveform entering the air section from port one enters from another air section. There is therefore no reflection and the transmission coefficient is equal to one allowing the full wave to pass into the network. For the $S_{21}$ parameter then, there is simply a phase shift dependent on the air’s phase constant and path length. The parameter can be written as, $\exp(-j\beta l)$, where $l$ is the path length through the air and $\beta = 2\pi\sqrt{\varepsilon_r}/\lambda$.  

55
For air, $\varepsilon_r \approx 1$ and so, $\beta \approx 2\pi/\lambda$.

In the case of the port two parameters, a waveform must travel from the top soil level into the air network. Since there is a significant difference between the air and soil impedance values, there is a large reflection coefficient which is directly related to the $S_{22}$ parameter. For a perfectly smooth surface, the reflection coefficient at a normal incidence angle is well known. However, the soil interfaces are rough which reduces the overall reflection coefficient. This is captured by (5.11) where $s$ is the standard deviation of surface roughness, $\beta$ is the phase constant of the network in which the waveform is originating from, and $\theta$ is the angle of incidence which is zero for this model [34]. For the S-parameter derivation and simulations, a standard deviation of one cm was chosen for the soil roughness.

$$\Gamma_{\text{effective}} = \Gamma \exp \left[ (-s\beta \cos \theta)^2 \right]$$

(5.11)

Using this equation, the effective reflection coefficient from a rough interface can be found. The $S_{22}$ parameter of the air network is equal to the reflection coefficient of a wave traveling from the top soil level into the air. This is given in (5.12) where the air parameters are denoted with subscripts $\text{air}$ and the top soil layer parameters with subscripts $\text{soil}_1$.

$$S_{22\text{air}} = \left( \frac{Z_{\text{air}} - Z_{\text{soil}_1}}{Z_{\text{air}} + Z_{\text{soil}_1}} \right) \exp \left[ (-s\beta_{\text{soil}_1})^2 \right]$$

(5.12)

The magnitude of the air network’s $S_{12}$ is equal to the transmission coefficient of a waveform traveling from the top soil level into the air because there is a difference in the two networks’ impedance values. This differs from the $S_{21}$ parameter which assumes the waveform enters the air network from another air section. Since the modified reflection coefficient scatters the incident wave in directions other than towards the receiver, the transmission coefficient is unaffected by the soil roughness. That is, it is given for a smooth surface while the rough soil reflection coefficient only is used to describe the reflection back towards the source in this normal incidence case. The transmission coefficient is given in [13] and is well known for two media of different impedances. The phase component of the air’s $S_{12}$ parameter is identical to that $S_{21}$ value, only dependent on the phase constant and path length. Using this information, the $S_{12}$ parameter can be written as in (5.13).
\[ S_{12\text{air}} = \left( \frac{2Z_{\text{air}}}{Z_{\text{air}} + Z_{\text{soil1}}} \right) \exp \left[ -j\beta_{\text{air}} l_{\text{air}} \right] \] (5.13)

With the different parameters computed for each port, the air network’s S-parameters can be written together in matrix form such as (5.14).

\[
S_{\text{air}} = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} = \begin{bmatrix}
0 & \left( \frac{2Z_{\text{air}}}{Z_{\text{air}} + Z_{\text{soil1}}} \right) e^{-j\beta_{\text{air}} l_{\text{air}}} \\
e^{-j\beta_{\text{air}} l_{\text{air}}} & \left( \frac{Z_{\text{air}} - Z_{\text{soil1}}}{Z_{\text{air}} + Z_{\text{soil1}}} \right) e^{-s\beta_{\text{soil1}}}^2
\end{bmatrix} \] (5.14)

### 5.2.3 Soil S-Parameters

The S-parameters for the different soil layers are very similar. The only instances where there are slight difference is the top layer, where there is an air dependency, and the bottom layer where there is no further soils attached. Because the parameters are so similar for the different layers, the subscripts \(s_n\) are used to identify the particular layer numbers.

The first soil network has S-parameters that are dependent not only on its own impedance, but the air’s and next soil layer’s as well. This air dependency is given in the reflection and transmission coefficients for port one which impact the \(S_{11}\) and \(S_{21}\) parameters. Port two reflection and transmission coefficients are dependent upon the following soil layer’s impedance which in turn impact the \(S_{22}\) and \(S_{12}\) parameters.

While a signal propagates through the soil layer, not only is there a phase shift introduced but attenuation also occurs. The attenuation amount is dependent upon the path length in the soil layer as well as the moisture content. The moisture content was increased as the layers went deeper giving the top layer the least amount of moisture and hence the least amount of attenuation. The S-parameters for the top soil layer can be written as (5.15) where \(\alpha\) is the attenuation coefficient computed based upon the frequency and moisture level.
\[
S_{s_n} = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\]

\[
S_{s_1} = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
= \begin{bmatrix}
\left(\frac{Z_{s_1}-Z_{\text{air}}}{Z_{s_1}+Z_{\text{air}}}\right) e^{-s \beta_{s_1}} & \left(\frac{2Z_{s_1}}{Z_{s_1}+Z_{s_2}}\right) e^{-(\alpha_{s_1} + j \beta_{s_1})} \\
\left(\frac{2Z_{s_1}}{Z_{s_1}+Z_{\text{air}}}\right) e^{-(\alpha_{s_1} + j \beta_{s_1})} & \left(\frac{Z_{s_1}-Z_{s_2}}{Z_{s_1}+Z_{s_2}}\right) e^{-(s \beta_{s_1})^2}
\end{bmatrix}
\]

(5.15)

In the middle layers of soil, the parameters are dependent upon the impedance values of the soils above and below the layer of interest. By using the notation of \(s_n\) for the current layer of interest, the layer above can be denoted as \(s_{n-1}\) and the layer below as \(s_{n+1}\). This allows the middle soil layers to be written generally as in (5.16).

\[
S_{s_n} = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
= \begin{bmatrix}
\left(\frac{Z_{s_n}-Z_{s_{n-1}}}{Z_{s_n}+Z_{s_{n-1}}}\right) e^{-s \beta_{s_{n-1}}} & \left(\frac{2Z_{s_n}}{Z_{s_n}+Z_{s_{n+1}}}\right) e^{-(\alpha_{s_n} + j \beta_{s_n})} \\
\left(\frac{2Z_{s_n}}{Z_{s_n}+Z_{s_{n-1}}}\right) e^{-(\alpha_{s_n} + j \beta_{s_n})} & \left(\frac{Z_{s_n}-Z_{s_{n+1}}}{Z_{s_n}+Z_{s_{n+1}}}\right) e^{-(s \beta_{s_{n+1}})^2}
\end{bmatrix}
\]

(5.16)

The final soil network has parameters different than the middle layers. This is because it is considered that there is no network attached to the final layer. Since there is no layer attached after this network, any waveform entering port two will not be reflected. This is under the assumption that the waveform is entering from a medium of the same impedance as the final network. As such, the \(S_{22}\) parameter is equal to zero and the transmission coefficient magnitude at port two is equal to one. Using the subscript \(N\) for the final layer, the S-parameters of the network can be written as (5.17).

\[
S_{s_N} = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
= \begin{bmatrix}
\left(\frac{Z_{s_N}-Z_{s_{N-1}}}{Z_{s_N}+Z_{s_{N-1}}}\right) e^{-(s \beta_{s_{N-1}})^2} & e^{-(\alpha_{s_N} + j \beta_{s_N})} \\
\left(\frac{2Z_{s_N}}{Z_{s_N}+Z_{s_{N-1}}}\right) e^{-(\alpha_{s_N} + j \beta_{s_N})} & 0
\end{bmatrix}
\]

(5.17)
5.2.4 System Parameters

With the S-parameters of the different networks derived, the system parameters for the down-looking radar model were simulated. The first aspect that was required to combine the different network parameters together was the re-normalization of the soil S-parameters. This was done so that they were given with reference to air’s impedance which is the chosen system impedance.

After re-normalization, the parameters of the different networks were transformed into T-parameters allowing the overall system T-parameters to be found. The system S-parameters were then computed based upon these values. During this work, an upper limit to the volumetric soil moisture was required. This was due to the nature of the system T and S-parameter conversion and the non-reciprocal nature of the individual networks’ parameters.

Since the attenuation experienced by the networks was very large, hence giving small $S_{12}$ and $S_{21}$ values, the T-parameters for the networks were very large due to their $1/S$ dependencies. However, the S-parameters are not reciprocal leading to very large but different T values. This difference becomes apparent for large volumetric soil moisture levels giving simulation errors in the system $S_{12}$ magnitude. To avoid these errors the moisture levels were limited.

To understand the impact of different soil moisture values on the system parameters, several simulations were run for different values and number of networks. For all of these simulations the frequency range was 3.5-4.5 GHz, the air network path length was kept to 1000 meters, and the different soil path lengths were assigned to be the range resolution in free space which is 14.99 cm in this case. Different step sizes, in terms of moisture content, and maximum moisture levels were then investigated.

The first simulation ran for volumetric soil moisture values ranging from 0.025 to 0.2 g/cm$^3$ in 0.025 increments. In this case, the system $S_{12}$ and $S_{21}$ parameters experienced exponential decay with very low values. These magnitudes were not equal, however, since the system is not reciprocal. The $S_{21}$ experiences more attenuation because the attenuation caused by lower levels are able to contribute more to the system values. The two transmission S-parameters also had a large phase introduced based upon the large air path length.

The $S_{11}$ parameter experienced a varying magnitude with respect to frequency
due to the larger attenuation of high frequency components in the soil. These attenuated frequency components were then reflected back towards the transmitter contributing to the system $S_{11}$. This parameter also experienced a large phase shift because the signal first propagates through air and then soil levels.

The $S_{22}$ parameter for this simulation, and the others as well, experienced a different response in that the reflection was constant across the range of frequencies. The phase introduced across the range was also very small compared to the other parameters. The reasoning for these behaviors is that a signal entering port two of the overall system must first propagate through the more moist soil levels. As such, the contributions from the other soils and air network are severely attenuated resulting in the system $S_{22}$ parameter being mainly dependent on the reflection between the second to last and final soil layers.

The second simulation was run with soil moisture values ranging from 0.05 to 0.25 g/cm$^3$ in 0.05 increments. The first thing to note is that the upper limit of the moisture level has increased. This is due to the fact that there are less soil networks, since the step size is larger, and hence the simulation error does not occur until a larger value. In this simulation the phase component of the different S-parameters was the same as the previous results. The magnitude of the values did change however. For the transmission parameters, their magnitude increased indicating lower overall attenuation. This makes intuitive sense as there are less soil levels, and hence less attenuation even as the moisture values are larger.

The $S_{11}$ magnitude decreased for this simulation. This is caused by the increased attenuation of the signal from the upper soil levels. Previously, the soil levels and step sizes were small allowing the lower soils to contribute to the $S_{11}$. In this simulation, the higher moisture values attenuate the signal more in the lower levels leading to a smaller contribution from them. The $S_{22}$ parameter, however, increased because of the larger difference between the final two layer’s moisture and hence impedance.

The volumetric moisture distribution used in this simulation was also used in work to find the correlation from reflected waveforms, discussed in the following section. As such, the magnitude response of the system S-parameters are given in Figure 5.10.

The final simulation that was run was for moisture values ranging from 0.1 to 0.3 g/cm$^3$ in 0.1 step sizes. The results from this case were similar to the second
Air Path Length = 1000 m, Soil Layer's Depth = 14.99 cm
Soil Layer's $m_v = 0.05, 0.1, 0.15, 0.2, 0.25$ g/cm$^3$

Figure 5.10: System S-parameter magnitude response for down-looking radar. (a) $S_{11}$ and $S_{22}$. (b) $S_{12}$ and $S_{21}$.

Simulation. The transmission S-parameters increased because of the fewer number of soil networks. The $S_{22}$ increased due to the larger step sizes for moisture. Finally, the $S_{11}$ decreased due to the larger attenuation and decreasing contribution from lower levels of soil.

By examining these different cases, a clear pattern emerges for the system parameters. While the overall system attenuation may decrease due to a decreased number of soil layers, the attenuation of each network increases. Furthermore, the decrease of the system $S_{11}$ become relevant for investigating the correlation response from soil. Because of the direct dependency of the input reflection coefficient on the $S_{11}$ parameter (see (5.9)), as the step size of the moisture content
increases the waveform’s return voltage decreases. This directly impacts the correlation of the transmitted and received signals.

5.2.5 Down-Looking Radar Correlation

With the S-parameters derived and simulated for the down-looking radar, they were used to create a received signal reflecting from the ground clutter. This was used to find the correlation between a transmitted and received signal for both the noise and chirp waveforms. These simulations were completed to give a direct comparison of the waveforms’ results and understand their respective responses to a clutter source.

To complete these simulation, many of the techniques used for the forward-looking model were applied here. The most important aspect was how the modifications to the waveforms were applied. As in the forward-looking model, the large path lengths of the air section creates large phase shifts for the S-parameters, and hence waveforms. These phase shifts introduce large time delays to the signals. To avoid truncation, the transmit waveforms were zero-padded with enough time samples to account for the maximum round trip time and the modifications were applied in the frequency domain. The modified signals were then taken back to the time domain where receiver noise was added to the signals.

The basic system parameters were similar to the previous simulation as well. These are given in Table 5.2 and kept consistent for the two waveforms. The electrical properties of the field were taken to be the same as those given in Section 5.2.1.

To compute the received waveform the input reflection coefficient of the system, given in (5.9), and the square root of the two-way radar equation were used to modify the transmitted signal. To compute the RCS of the ground clutter, the spot size of the antenna and the clutter NRCS statistics were used. Since this model looks directly down at the ground, the spot size of the antenna was computed based upon the assumption of an elliptical shape. The area of the spot size was then multiplied against the average linear NRCS value of soil taken from [16].

Using this general technique, the correlation of the noise and chirp waveforms were found for a range of air path lengths and for two different target loads. The first target impedance was 0 Ω and used to represent a metal target buried under
Table 5.2: Down-looking radar model’s system parameters for generating return waveforms

<table>
<thead>
<tr>
<th>Radar System Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Path Length (m)</td>
<td>500-1000</td>
</tr>
<tr>
<td>Soil Path Lengths (cm)</td>
<td>14.99</td>
</tr>
<tr>
<td>Transmit Pulse Power (W)</td>
<td>500</td>
</tr>
<tr>
<td>$T_x/R_x$ Antenna Gain (dB)</td>
<td>30</td>
</tr>
<tr>
<td>3 dB Beamwidth (deg.)</td>
<td>1</td>
</tr>
<tr>
<td>Clutter NRCS (dB)</td>
<td>2.5</td>
</tr>
<tr>
<td>Number of Soil Layers</td>
<td>5</td>
</tr>
<tr>
<td>Soil Moisture Range (g/cm$^3$)</td>
<td>0.05-0.25</td>
</tr>
<tr>
<td>Standard Dev. of Surface Roughness (cm)</td>
<td>1</td>
</tr>
<tr>
<td>Frequencies of Interest (GHz)</td>
<td>3.5-4.5</td>
</tr>
<tr>
<td>Receiver Temperature (K)</td>
<td>300</td>
</tr>
<tr>
<td>Assumed Noise Figure (dB)</td>
<td>3</td>
</tr>
<tr>
<td>Receiver Noise Bandwidth (GHz)</td>
<td>1</td>
</tr>
</tbody>
</table>

the soil. This was done to give a target reflection coefficient of -1 and hence include all of the system S-parameters in the computation of the input reflection coefficient. For both waveforms, the $S_{11}$ parameter was much larger than the remaining sections of (5.9). As such, the correlation was directly dependent on the phase introduced by the parameter and varied sinusoidally with path length. These dips are introduced because at certain path lengths the phase is a multiple of $\pi$ radians and hence the transmitted and received signals are out of phase.

Apart from the sinusoidal pattern, the correlation of both waveforms was constant with increasing range. This was a result of the antenna spot size, and hence clutter RCS, growing larger with range negating the added path loss.

The second target load chosen was that of the final soil layer. This was done to represent the final soil layer extending to infinity and hence having no reflection from a target. The correlation results for this case were very similar to the PEC simulation because the input reflection coefficient was strongly dependent upon the $S_{11}$ parameter. The results of the two waveforms for this target impedance are shown plotted together in Figure 5.11 for direct comparison.

As mentioned, the correlation is very similar to the previous target load with the average value remaining constant with range and a sinusoidal pattern shown. The results are also the same for the two waveforms as expected since the same range of
frequencies were examined. The noise waveform, however, had a peak correlation around 0.025 less than the chirp signal which may be significant for clutter targets. A decreased correlation indicates that there would be fewer false alarms triggered for the system when a radar was looking downward for a target. This shows that the noise waveform may be better suited for this type of surveillance than the chirp waveform depending upon the correlation values needed.

5.3 Conclusions

The modeling of radar propagation and reflections as two-port devices has significant benefit. The parameters for each network can be computed individually without regard to the waveform type. These network’s S-parameters can then be re-normalized to air impedance since this is a model for radar propagation. Combining the networks together allows for a set of system parameters that can be directly applied to the waveforms.

The forward-looking radar model allowed for the comparison of different rain rates and path lengths on the correlation of waveforms. For both the chirp and noise signals, the correlation decreased with increasing range and rain rate as expected. The correlation values were also very similar between the signals indicating that the rain and air propagation does not prefer one waveform over another.

In the forward-looking model, an interesting pattern occurred for the case of a
very large target impedance. In that case, the target reflection coefficient was near one and so the input reflection coefficient was directly dependent upon the system S-parameters. As such, the phase of these parameters caused a sinusoidal pattern with path length due to multiples of $\pi$ radians introduced to the received signal.

The down-looking radar model was a direct extension of the forward case. Instead of inspecting a target response, the responses for moist soils were examined. The moisture content of different soil layers greatly impacts the electrical properties and hence the attenuation, phase, and reflection of a waveform. Depending upon the exact moisture values chosen, the S-parameters can be computed for individual networks and then the overall system. In all cases, however, the attenuation caused the transmission S-parameters to be extremely small. This in turn caused the $S_{11}$ parameter to be the main factor of the input reflection coefficient.

This direct relationship with the $S_{11}$ parameter caused a sinusoidal pattern in the correlation with respect to path length as in the forward-looking model. This occurred for both waveforms where it was clear that the chirp signal had a consistently larger correlation than the noise waveform. This may indicate that noise signals are beneficial for down-looking radar as a smaller correlation may result in fewer false alarms from clutter responses.
Double Spectral Processing

In this chapter, the use of double spectral processing for determining target ranges is investigated. Primarily, the capabilities of DSP is directly compared to the cross-correlation method of determining target locations. Previous work has been done in applying DSP to pseudorandom signals and it was found that there is some use compared to traditional correlation [35]. It was found that while the correlation method offers higher correlation coefficients in the presence of noise, the DSP algorithm is much less complex. In general, the algorithm for the DSP method can be shown to be more computationally efficient than the cross-correlation. As such the DSP process is considered more efficient, even with two iterations of FFT, and can be a useful alternative for real-time processing and long-time signals.

This work looks to expand upon this brief introduction by investigating how the DSP and cross-correlation methods compare in the application of noise radar. A general overview of the method is first given followed by the investigation of the two method’s ability to resolve target ranges based upon the transmitted signal’s bandwidth. Finally, the performance of both methods were examined for a given set of system conditions, varying weather attenuations, and different SNR values.

6.1 DSP Background

The DSP technique is a clever method of finding the target ranges, that takes advantage of the wait time between a transmitted pulse and received signal to modulate the overall power spectrum. This section details the math behind the
DSP method, gives an intuitive explanation, discusses the method of implementing this method in MATLAB, and finally outlines some issues that can occur using this method.

6.1.1 DSP Mathematics

The DSP method works by utilizing the time gap between the transmitted and received signals. A detailed derivation of this method is given in [36] while a brief overview based upon that work is given here. The DSP method begins by working on a summation signal, \( s(t) \), which is composed of the transmitted waveform, \( x(t) \), and a delayed and modified version of this waveform, \( y(t) \). This signal is essentially the transmitted pulse followed by silence and then the received signal pulses. It can be written as (6.1) where \( T_0 \) is the time delay to a target and \( E \) is a thermal noise waveform added to the received signal.

\[
\begin{aligned}
    s(t) &= x(t) + y(t) = x(t) + A_0 x(t - T_0) + E(t) \\
\end{aligned}
\]

(6.1)

The autocorrelation of this waveform can be shown to be written as (6.2). From the relationship between the autocorrelation and power spectrum density then, the power spectrum can be written as (6.3) where \( \theta \) is the angle of \( A_0 \).

\[
\begin{aligned}
    R_{ss}(\tau) &= \left(1 + |A_0|^2\right) R_{xx}(\tau) + A_0^* R_{xx}(\tau + T_0) + A_0 R_{xx}(\tau - T_0) + R_{EE}(\tau) \\
\end{aligned}
\]

(6.2)

\[
\begin{aligned}
    S_{ss}(\omega) &= \mathcal{F}\{R_{ss}(\tau)\} = S_{xx}(\omega)|A_0|\cos(\omega T_0 - \theta) + S_{xx}(\omega)(1 + |A_0|^2) + S_{EE}(\omega) \\
\end{aligned}
\]

(6.3)

In (6.3) the modulation of the power spectrum caused by the time delay is clearly seen in the cosine. Assuming the power spectrum of the signal pulses and thermal noise is approximately constant, the Fourier transformation of the power spectrum then reveals the modulation frequency. This is shown in (6.4) where \( C_0 \) and \( C_1 \) are constants and the delta functions correspond to peaks at zero, for the signal pulses and thermal noise components, and at the positive and negative time instances, \( T_0 \), for the modulation frequency.
\[
F \{ S_{ss}(\omega) \} \approx C_0 \delta(u) + C_1 \delta(u - T_0) + C_1 \delta(u + T_0)
\]  

(6.4)

### 6.1.2 Intuitive DSP Understanding

While the mathematics of the DSP method is important, it is also informative to examine the method from an intuition point of view. For the case of a single target, the signal \( s(t) \) is essentially two peaks in time caused by the transmitted and received pulses. For \( T_0 \) seconds between these pulses, the receiver does not detect any signal. As such, this separation of pulses corresponds to a frequency \( f_0 = 1/T_0 \) Hz. The empty time and flat pulses correspond to DC or low frequencies.

By taking this signal into the frequency domain then, the frequency spectrum of the transmitted and received noise pulses are given, but modulated by the frequency \( f_0 \). Taking the power spectrum density from this transform gives the magnitude of the frequency response allowing this modulation to be more easily seen.

To recover the frequency \( f_0 \), the Fourier transform is again taken to separate the power spectrum into its low frequency components, corresponding to the noise signals, and the high frequency components, corresponding to the modulation frequency. Because the Fourier transform is taken again, the modulation frequency appears at both positive and negative time instances similar to how a signal has both positive and negative frequency spectrums. Additionally, the transformation cuts the maximum time range in half giving a range of \( \pm T_{\text{max}}/2 \) similar to a frequency range of \( \pm F_s/2 \) for one iteration of the transformation.

### 6.1.3 DSP Simulation Implementation

With a general understanding of the DSP process, different conditions were then tested in MATLAB simulations. This was done to understand how the various conditions, such as signal bandwidth and attenuations, impact the detection capabilities. While in these simulations the summation signal is generated using the transmitted and received pulses, in practice only \( s(t) \) is available. As such, all simulations approximated the power spectrum using the FFT rather than using the precise definition available in the mathematical formula.
To run the DSP simulations, a similar set of general steps were followed regardless of the particular system parameters. Firstly, a noise signal was generated and then band-pass filtered around the frequency range of interest. This was used to generate a transmit signal with the pulse first, followed by the appropriate amount of zero-padding corresponding to the target delay time plus extra time after the received signal. A received signal was also generated by attenuating the noise pulse as required, zero-padding before the pulse, zero-padding after the pulse, and adding the required thermal noise to achieve a particular SNR. These two signals were then summed together to form \( s(t) \).

The power spectrum of the summation signal was then found using the FFT. Following this, the spectrum was high-pass filtered to attenuate the final DC component that is present due to the noise waveforms. A second FFT is then taken to return the signal back to the time domain. Finally, only the positive time values are taken with the peaks corresponding to the time delay of the targets.

An example of the DSP method locating a target range is given in the top plot of Figure 6.1 for a target of 500 meters. As will be discussed in the following section, DSP is difficult to normalize. An example of the correlation method for determining the same target range is also given in Figure 6.1, in the bottom plot. Besides the normalization issue for the DSP method, the other obvious difference between these two methods is the range that is shown. Because of the FFT used in the DSP method, only time delays up to half the maximum value can be seen. The correlation, however, shows targets up to the full time delay which the receiver records.

### 6.1.4 DSP Issues

While the above method works for nearly all cases, there are some issues that occur in certain simulations which are the result of the DSP mathematics itself. The first issue that commonly occurs, if care is not taken, is that the time delay of the target is incorrect. This is the result of insufficient zero-padding after the received signal. Because the DSP ends with a time range of \( T_{\text{max}}/2 \), the receive pulse must be received before half the total time vector. If this criterion is not met, the peak is aliased back to an incorrect time delay.

The second issue that occurs is when multiple targets at different ranges are
Figure 6.1: Target range detection. (a) Double spectral processing method. (b) Cross-correlation method.

used. In this case, the targets themselves introduce additional modulating frequencies which are the result of the time gap between the return pulses. As such, the time signal at the end of the DSP method has additional peaks at these time delay differences. This is clearly seen in the top plot of Figure 6.2 where targets at 500 and 800 meters result in an extra peak at a range of 300 meters where no target exists.

There is a simple method to remove this extra peak, however, it requires that the DSP method be run again which is a major drawback to the DSP technique’s main strength, processing speed. To remove the peak, the DSP method is run again except for with the received signals only, excluding the transmitted pulse in
this case. The received pulses will result in peaks occurring only at the difference ranges. This set of peaks can then be subtracted from the original DSP results giving a set of ranges which corresponds to the targets only. This subtraction method is demonstrated on the case of targets at 500 and 800 meters giving the final results shown in the middle plot of Figure 6.2.

Of course, the correlation method of finding the target ranges does not experience this extra peak phenomenon. As such, the correlation only needs to be run once and so may be considered faster than the DSP method even though it requires more computations. The correlation results for the same targets at 500 and 800 meters can be seen in the bottom plot of Figure 6.2 to act as a direct comparison to the DSP method. Again, the correlation also gives a further set of ranges of which it can detect targets. This is also seen in the plot.

Finally, a general issue with DSP is the difficulty in normalizing the end result. Compared to the cross-correlation, it is awkward to normalize the target returns so that they correspond to the amount of received power at different ranges and through different attenuations.

One possible technique that could be used is the normalization of the DSP results to the maximum value. However, this will result in the peak value always being one, no matter the amount of attenuation experienced by the received pulse. As such, these normalized results would not have much significance compared to the correlation coefficient. The correlation coefficient, for example, describes the similarity between the received and transmitted signals and does decrease as the attenuation is increased or the SNR increased. This creates a problem in comparing the DSP to the correlation method for determining target locations. An alternative method of comparing the techniques is described below when the different attenuation and SNR cases are examined.

6.2 Peak Resolution

With a general understanding of the DSP method and how it compares to the cross-correlation, the peak resolution of the two methods was then compared. This was done to see how accurate the two methods were in determining the range, or time delay, to the target. The comparison of the peak width, measured in meters, was done for the two techniques at various noise signal bandwidths and for two SNR
Figure 6.2: Two target detection at 500 and 800 meters. (a) DSP with extra peak. (b) DSP with extra peak removed. (c) Cross-correlation.
values. It was expected and confirmed that the resolutions increases with increased bandwidth which is a well known radar concept. This section first describes the method of determining the signal’s peak width followed by the results for the various cases.

6.2.1 Determining Peak Resolution

In order to compare the two methods, the peak width must be found for the same relative magnitude in each bandwidth case. For this work, the range between the first and last points which are -10 dB from the maximum value was used as the peak width. This allowed for a similar comparison to the two methods since both could simply be normalized to the maximum value.

In the MATLAB simulation, the overall system conditions for the various bandwidth ranges and SNR cases were kept the same. The sampling frequency of the noise signal was 5 GHz and the bandwidth was varied from 100 to 1000 MHz centered on a frequency of 1.5 GHz. Also the target range was kept at a constant 500 meters throughout the different cases.

Two different SNR values were also examined for this work. The first, an infinite dB SNR, corresponded to no thermal noise being added to the received signal. This was the most ideal case and allowed the bandwidth effects alone to be examined. The second cases was for a SNR of 0 dB which introduced significant thermal noise to the received signal.

6.2.2 Peak Resolution Results

In general, both the DSP and cross-correlation methods resulted in decreasing peak widths as the bandwidth increased. While there were slight variations with respect to bandwidth, these were the result of simulation rounding errors in determining the points corresponding to -10 dB. Additionally, both the DSP and the correlation methods gave very similar results regardless of the SNR or bandwidth.

For the case of no thermal noise added, both the DSP and correlation methods gave very similar results with their peak widths being nearly exactly identical at large bandwidths. This is can be seen in the top plot of Figure 6.3 where the two method’s peak widths converge at 700 MHz. Since there was no additive noise, these similar peak widths were expected for large bandwidths. Overall, the
peak widths varied from around 80-90 meters down to near 20-30 meters at large bandwidths.

For the case of the 0 dB SNR, the two methods again gave the same general pattern. However, compared to the infinite SNR case, there were greater variations between the two methods and their peak widths. They did not converge at the large bandwidths and their peak widths did not decrease as significantly as the bandwidth increased. These results can be seen in the bottom plot of Figure 6.3. The larger peak widths, for larger bandwidths, were caused by the thermal noise which exceeded the -10 dB threshold. Additionally, the thermal noise was responsible for the variation between the two methods. Overall, the peak widths varied from 80-90 meters down to near 30-40 meters at the large bandwidths.

### 6.3 Varying SNR and Attenuation

The final aspect that was investigated for the DSP method was how it compared to the cross-correlation for varying SNR values and attenuation. This examined the ability of the two methods to identify the target peaks from the background noise. This section first describes how the two methods were compared and then describes the results for various SNR and attenuation cases.

#### 6.3.1 Method of Comparison

Since the DSP method does not normalize to a correlation coefficient simply, a different relationship was required to compare the two techniques. To accomplish this, the peak-to-average ratio was computed for each specific case. This measured the average return of the two methods across all ranges and used that value to normalize the maximum peak. The logarithm of this value was then taken since it was typically a large number.

This peak-to-average ratio was found for a variety of different cases and so the attenuation caused by the two-way radar propagation was kept constant for all of these situations. This was computed based upon a transmit and receive antenna gain of 30 dB, a sampling frequency of 5 GHz, a center frequency of 1.5 GHz, a bandwidth of 1 GHz, a signal power of 0 dBW before bandpass filtering, a range to the target of 1 km, and a target RCS of 5 dBsm. With these parameters kept
constant, the square root of the two-way radar equation was computed and applied to the received voltage signal.

To understand how thermal noise alone impacts the two methods, additive noise was included with SNR values varying from -10 dB to 50 dB. In addition, the impact of rain was also examined with its attenuation computed for the center frequency of 1.5 GHz [37]. This was done for a rain rate of 25 mm/hr, or moderate rain, and a rate of 100 mm/hr, which is very heavy rain. Assuming rain for the entire length to the target, the rain attenuation was computed for twice the path length, or a two-way trip.

Figure 6.3: Measured peak width with respect to signal bandwidth. (a) SNR of infinite dB. (b) SNR of 0 dB.
6.3.2 Comparison Results

The case of no rain allows for the direct comparison of the two methods. It was found both the DSP and cross-correlation methods have increasing peak-to-average ratios with increasing SNR as expected. At a SNR value of 20 dB, both method’s peak-to-average ratio level out. This indicates that for a SNR of 20 or above the thermal noise no longer is a major factor in detecting the target peak. The main issue then becomes the artifacts surrounding the peak which typically are the largest contributors to the mean value.

Across the range of SNR values, however, the cross-correlation method outperforms the DSP. This is consistent with the findings of [35] which found that the correlation offers a more robust solution in the presence of thermal noise. At very low SNR values, the two methods begin to converge coming within a few dB of one another. This is to be expected as the thermal noise acts to increase the average values for both methods. The top plot of Figure 6.4 shows the results for the two methods with no rain attenuation.

The impact of rain on the two methods was also examined. The results indicate that the rain has very little impact on the peak-to-average ratio. This is true even for the case of the heavy rain, shown in the bottom plot of Figure 6.4. At the low SNR values for the heavy rain, the two methods seem to converge slightly more than the no rain case, however, it is not very significant given the rain rate. The probable reason for this is the fact that the rain attenuation is very low for a center frequency of 1.5 GHz. Additionally, the two-way path length of 2 km is not long enough to allow for significant degradation to occur.

6.4 Conclusion

The DSP method is a clever method of finding radar target’s ranges. However, in comparing the DSP to cross-correlation, it is found that the correlation offers an overall simpler and more effective approach. The main selling point for the DSP method is the decreased amount of computation that is required allowing for easier real-time processing. In the case of multiple targets, however, the DSP method needs to be performed twice to reduce any artifacts introduced by the technique’s mathematics. This essentially eliminates the DSP’s advantage of being better
Figure 6.4: Peak-to-average ratio for DSP and cross-correlation methods. (a) No rain attenuation. (b) Rain attenuation for rain rate = 100 mm/hr.

This work also examined the range resolution of the two methods with respect to signal bandwidth. It was found that both method’s resolution increases with increasing bandwidth. The ability of the two methods to detect a target peak was also examined for various SNR values and attenuations. It was found that the cross-correlation significantly outperformed the DSP method in all cases. Overall, while the DSP method is an ingenious method of determining target ranges, the cross-correlation outperforms it in nearly all radar applications.
Conclusions

The goals of this research were to investigate the use of noise waveforms for long range radar surveillance. This included not only looking at the ability of noise signals to detect targets and discriminate against clutter, but to also examine the impact of propagation itself on the waveforms. To accomplish these research aims, multiple different radar cases were derived and simulated. In addition, in each situation not only was the noise waveform response examined, but a chirp waveform as well to act as a direct comparison for traditional radar signals.

Three main radar cases were examined in this thesis. These included comparing the impact of simple target and clutter reflections, more complex radar simulations including path loss and real-world targets, and finally modeling radar propagation using two-port networks. In addition to these radar case examinations, a comparison of the DSP and cross-correlation methods’ ability to detect target ranges was also performed.

The examination of simple target and clutter reflections was done to understand the impact of random clutter responses as well as frequency dependent targets on the correlation of transmitted and received waveforms. It was found that the noise and chirp signals had nearly identical correlations and relative power ratios for the different targets examined. In certain clutter cases, however, the chirp waveform had higher correlations. This may be beneficial for the noise signal since a large correlation from a clutter target may cause false alarms.

More advanced radar cases were then examined for the noise and chirp waveforms. This was done as a natural extension of the previous work to include complex target RCS values for a hummer vehicle, intricate NRCS values for the
clutter, as well as propagation effects, and thermal noise. The return signals from both the target and clutter were used to determine the probability of detection and false alarm respectively so that ROC curves could be generated.

The direct comparison of the noise and chirp waveforms in this manner indicate that the two signals are roughly equal in performance for the cases of large clutter returns. For weaker clutter returns, the noise waveform outperforms the chirp in giving high detection probabilities at low false alarms. These results conform to the previous simple radar case were the clutter correlation was lower for noise signals. The random distribution of signal power with respect to frequency may offer the best explanation of why the noise waveforms are less impacted by the clutter targets.

The next aspect of this work involved modeling forward and down-looking radar as two-port networks. This allowed for the propagation effects of areas such as air and rain to be computed individually and then combined together to form overall system parameters. By looking at propagation as a network, the received signal can be simply computed based upon the radar equation and the input reflection coefficient of the network.

In the case of forward-looking model, which examined reflections off of targets, the correlation of both the noise and chirp waveforms were nearly identical with respect to path length and rain rate. This indicates that the performance of these two waveforms is similar for target detection. The down-looking radar model gave results which agree well with the previous work. While the correlation pattern of the two signals are similar, the chirp waveform’s was consistently higher than the noise indicating a higher probability of false alarm.

Throughout this work, the noise waveform was compared to the chirp signal to offer insight on how traditional waveforms respond under the same situations. It was found that not only do the two waveforms have comparable correlations and detection probabilities for targets of interest, but the noise signal may be better suited for clutter suppression. Depending upon the situational needs, a noise waveform may therefore be a better choice than traditional signals for long rang radar surveillance.

The final aspect of this thesis involved examining the DSP and cross-correlation method for determining target range. This was done by comparing the peak resolution of both methods for various bandwidths. It was found that both techniques
gave nearly identical resolutions which increased with increasing bandwidth. In addition, the ability to detect a target peak was also examined under various SNR and attenuation values. It was found that the correlation method greatly outperformed the DSP technique for all cases.

While this work offers a beginning to the investigation of noise signals for long range radar, there are many opportunities for future work. Using different targets of interest, ROC curves can be generated to directly compare to the hummer. Additionally, the use of real target RCS and clutter NRCS data would allow for the direct comparison of real to simulated ROC curves.

The modeling of radar propagation as two-port networks can also be expanded for other types of weather such as snow or anisotropic rain, where the rain rate is not uniform throughout the rain column. The modeling of ground clutter as networks can also be expanded. Using the correlation length of the soil as a diameter, sub-spots within the overall antenna spot size can be individually computed as two-port networks. These can then be combined together to give a more comprehensive representation of the soil’s random nature. Finally, ground clutter could be looked at for incidence angles other than normal if a monostatic (or backscatter) reflection coefficient can be determined for a rough surface.
Appendix A

Derivation of Fourier Transform for Band-limited Gaussian White Noise

This appendix shows the derivation of the Fourier transform for a Fourier series approximation of Gaussian white noise. A simplified derivation is shown for a single component of the Fourier series summation. This transformation can then be modified for each respective cosine and its parameters. From the linearity property, these individual transformations can then be summed together to form the overall transform of the noise signal. In this derivation, [15] is frequently cited for the various transform properties.

The derivation begins with (2.1) where it is noted that the cosines are truncated using a rectangular window of length $T$. This window is multiplied onto the cosine in the time domain which corresponds to the convolution of the cosine and window’s respective transformations in the frequency domain. A single cosine of this summation can be expanded as in (A.1) where $c_n$ represents the constant amplitude multiplied onto the signal.

\[
x_k(t) = c_n \cos(\omega_n t - \varphi_n) \left( u[t] - u[t-T] \right) \\
= c_n \left[ \cos(\omega_n t) \cos(-\varphi_n) + \sin(\omega_n t) \sin(-\varphi_n) \right] \text{rect} \left[ \frac{t}{T} - \frac{1}{2} \right] \\
= c_n \cos(\varphi_n) \left[ e^{j\omega_n t} \text{rect} \left[ \frac{t}{T} - \frac{1}{2} \right] + e^{-j\omega_n t} \text{rect} \left[ \frac{t}{T} - \frac{1}{2} \right] \right] \\
- \frac{c_n \sin(\varphi_n)}{2j} \left[ e^{j\omega_n t} \text{rect} \left[ \frac{t}{T} - \frac{1}{2} \right] + e^{-j\omega_n t} \text{rect} \left[ \frac{t}{T} - \frac{1}{2} \right] \right] \\
\]

\[(A.1)\]

Now, each of these components has a constant which can simply be applied
to its respective transform. Additionally, each of these four components has an exponential multiplied by a rectangular function. The respective transforms of the exponential and rectangular functions therefore are convolved in the frequency domain.

The Fourier transformation of an exponential is simply a delta function centered on the frequency of that exponential. A generic rectangle function centered on zero has a transformation which corresponds to a sinc function. From the time shifting property of the Fourier transform, the shifted rectangle function scales its sinc function transformation by an exponential. This exponential has an argument corresponding to the time shift amount.

The convolution of these two functions corresponds to the sinc function shifted to the frequency of the original exponential. This is shown in (A.2) where the sinc function is defined as sinc \( x = \frac{\sin (\pi x)}{\pi x} \).

\[
\mathcal{F} \left\{ e^{j\omega_n' \text{rect} \left[ \frac{T}{2} - \frac{1}{2} \right]} \right\} = \delta (f - f_n) \ast T \text{sinc} (fT) e^{-j\pi fT} \\
= T \text{sinc} ((f - f_n) T) e^{-j\pi fT} e^{j\varphi_n T}  \tag{A.2}
\]

Using (A.2), the Fourier transformation of (A.1) can be written as shown in (A.3).

\[
\mathcal{F} \{ x_k (t) \} = \frac{c_n \cos (\varphi_n)}{2} \left[ T \text{sinc} ((f - f_n) T) e^{-j\pi fT} e^{j\varphi_n T} \right] \\
+ \frac{c_n \cos (\varphi_n)}{2j} \left[ T \text{sinc} ((f + f_n) T) e^{-j\pi fT} e^{-j\varphi_n T} \right] \\
- \frac{c_n \sin (\varphi_n)}{2j} \left[ T \text{sinc} ((f - f_n) T) e^{-j\pi fT} e^{-j\varphi_n T} \right] \\
+ \frac{c_n \sin (\varphi_n)}{2j} \left[ T \text{sinc} ((f + f_n) T) e^{-j\pi fT} e^{-j\varphi_n T} \right]  \tag{A.3}
\]

By expanding the cosine and sine constants in (A.3) into exponentials, the like terms can be grouped together and cancellations can be made. This leads to the final form of the transform in (A.4).

\[
\mathcal{F} \{ x_k (t) \} = \frac{T e^{-j\pi fT} c_n}{2} \left\{ \text{sinc} ((f - f_n) T) e^{j\varphi_n T} + \text{sinc} ((f + f_n) T) e^{-j\varphi_n T} \right\}  \tag{A.4}
\]

This final form can then be used to give the overall Fourier transform of the Fourier series approximation of Gaussian white noise. This is given in (A.5).
\[
\mathcal{F}\{x(t)\} = \mathcal{F}\left\{ x(t) = \frac{\sqrt{2A_0}}{\sqrt{N}} \sum_{n=1}^{N} \cos \left( \frac{2\pi nt}{T} - \varphi_n \right) \right\} = X(f) = \frac{\sqrt{2A_0}}{\sqrt{N}} \frac{T e^{-j\pi f T}}{2}
\]
\[
\cdot \sum_{n=1}^{N} \left\{ \text{sinc} \left( (f - f_n) T \right) e^{j\pi f_n T} e^{j\varphi_n} + \text{sinc} \left( (f + f_n) T \right) e^{-j\pi f_n T} e^{-j\varphi_n} \right\}
\]
Appendix B

Derivation of Cross Power Spectral Density for Band-limited Gaussian White Noise

This appendix shows the derivation of the CPSD for a transmitted and received copy of the Fourier series approximation of noise. The derivation closely matches that given in [8] where the spectral density of an un-attenuated Fourier approximation of noise was found.

Firstly, a noise signal using this representation has the Fourier transform given in (2.2) with \( \text{sinc}(x) = \sin(\pi x) / \pi x \), \( A_0 \) being the average power dissipated across the entire bandwidth through an impedance of one Ohm, and \( \varphi_n \) a randomly selected phase from 0 to \( 2\pi \).

To represent a band-pass filtered signal, attenuation coefficients \( Q_1(f_n) \) can be multiplied onto the Fourier transform of the transmitted signal. The coefficients \( Q_1(f_n) \) are complex numbers and frequency dependent. As such, each are included within the signal’s summation so that the individual frequency components are attenuated by their respective amount.

The received noise signal is a copy of the transmitted signal with further weather, reflections, and path loss attenuations included. These are all also frequency dependent and can be applied using the simple coefficients \( Q_2(f_n) \). Since the received signal is a copy of the transmitted signal, it implies that \( Q_2(f_n) \) contains the band-pass coefficients \( Q_1 \) within itself.
Now, from [38] the CPSD, $G_{XY}(T, f)$, can be found by integrating the product of the two signal’s Fourier transforms and the probability density functions of their variables. This is shown in (B.1) where the individual frequency and randomly selected phase variables are represented by $\lambda$. These variables have probability densities represented by $f(\lambda)$.

$$G_{XY}(T, f) = \frac{1}{T} \int_{-\infty}^{\infty} X(\lambda, T, f) Y^*(\lambda, T, f) f(\lambda) \, d\lambda \quad (B.1)$$

This equation can be expanded to include each frequency and phase integral and probability density as in (B.2).

$$G_{XY}(T, f) = \frac{1}{T} \int_{0}^{f_{max}} \cdots \int_{0}^{f_{max}} 2\pi \cdots \int_{0}^{2\pi} X((\phi_1, f_1), \ldots, (\phi_N, f_N), T, f)$$

$$\cdot Y^*((\phi_1, f_1), \ldots, (\phi_N, f_N), T, f)$$

$$\cdot f_{\phi_1}(\phi_1) \cdots f_{\phi_N}(\phi_N) f_{F_1}(f_1) \cdots f_{F_N}(f_N) \, d\phi_1 \cdots d\phi_N \, df_1 \cdots df_N \quad (B.2)$$

The density functions for the phase and frequencies can be written as $f_{\phi}(\phi) = 1/2\pi$ and $f_{F}(f) = 1/f_{max}$. This can be substituted into the previous equation to give (B.3).

$$G_{XY}(T, f) = \frac{1}{T} \frac{1}{2\pi} \frac{1}{(f_{max})^N}$$

$$\cdot \int_{0}^{f_{max}} \cdots \int_{0}^{f_{max}} 2\pi \cdots \int_{0}^{2\pi} X((\phi_1, f_1), \ldots, (\phi_N, f_N), T, f)$$

$$\cdot Y^*((\phi_1, f_1), \ldots, (\phi_N, f_N), T, f) \, d\phi_1 \cdots d\phi_N \, df_1 \cdots df_N \quad (B.3)$$

Now, from the Fourier transform of the noise waveform, the multiplication of the two signals $X$ and $Y$ for a particular frequency $k$ of the summation can be written as show in (B.4).

$$X_k(T, f) Y_k^*(T, f)$$

$$= \left( \frac{\sqrt{2\pi N}}{\sqrt{\pi}} \right) \left( \frac{Te^{-j\pi T}}{2} \right) Q_1(f_k)$$

$$\{ \text{sinc} [(f - f_k) T] e^{j\pi f_k T} e^{j\phi_k} + \text{sinc} [(f + f_k) T] e^{-j\pi f_k T} e^{-j\phi_k} \}$$

$$\left( \frac{\sqrt{2\pi N}}{\sqrt{\pi}} \right) \left( \frac{Te^{j\pi T}}{2} \right) Q_2^*(f_k)$$

$$\{ \text{sinc} [(f - f_k) T] e^{-j\pi f_k T} e^{-j\phi_k} + \text{sinc} [(f + f_k) T] e^{j\pi f_k T} e^{j\phi_k} \} \quad (B.4)$$
This can be reduced as shown in (B.5).

\[
X_k (T, f) Y_k^* (T, f) = \left( \frac{2\Lambda_0}{N} \right) \left( \frac{T^2}{4} \right) Q_1 (f_k) Q_2^* (f_k) \\
\left\{ \text{sinc} [(f - f_k) T]^2 + \text{sinc} [(f + f_k) T]^2 \right. \\
+ \text{sinc} [f - f_k) T] \text{sinc} [(f + f_k) T] e^{j2\pi f_k T} e^{j2\varphi_k} \\
\left. + \text{sinc} [(f + f_k) T] \text{sinc} [(f - f_k) T] e^{-j2\pi f_k T} e^{-j2\varphi_k} \right\} \tag{B.5}
\]

From this multiplication and the fact that the noise waveforms are summations, the CPSD can be written as shown in (B.6).

\[
G_{XY} (T, f) = \frac{1}{(2\pi)^N} \frac{1}{(f_{\text{max}})^N} \frac{1}{f_{\text{max}}^2} \\
\int_0^{f_{\text{max}}} \cdots \int_0^{f_{\text{max}}} \frac{A_0 T^2}{2\pi N} \sum_{i=1}^N \sum_{k=1}^N Q_1 (f_i) Q_2^* (f_k) \\
\cdot \left\{ \text{sinc} [(f - f_i) T]^2 + \text{sinc} [(f + f_i) T]^2 \right. \\
+ \text{sinc} [(f - f_i) T] \text{sinc} [(f + f_i) T] e^{j2\pi f_i T} e^{j2\varphi_i} \\
\left. + \text{sinc} [(f + f_i) T] \text{sinc} [(f - f_i) T] e^{-j2\pi f_i T} e^{-j2\varphi_i} \right\} \\
\cdot d\phi_1 \ldots d\phi_N df_1 \ldots df_N \tag{B.6}
\]

Changing the order of the integrals and summations, as well as splitting the summations into their diagonal and non-diagonal terms allows the CPSD to be re-written as (B.7).

\[
G_{XY} (T, f) = \frac{A_0 T}{2\pi N} \frac{1}{(f_{\text{max}})^N} \frac{1}{f_{\text{max}}^2} \\
\sum_{i=1}^N \sum_{k=1}^N \frac{1}{f_{\text{max}}} \frac{1}{f_{\text{max}}^2} \\
\int_0^{f_{\text{max}}} \cdots \int_0^{f_{\text{max}}} \int_0^{f_{\text{max}}} \int_0^{f_{\text{max}}} Q_1 (f_i) Q_2^* (f_k) \\
\cdot \left\{ \text{sinc} [(f - f_i) T]^2 + \text{sinc} [(f + f_i) T]^2 \right. \\
+ \text{sinc} [(f - f_i) T] \text{sinc} [(f + f_i) T] e^{j2\pi f_i T} e^{j2\varphi_i} \\
\left. + \text{sinc} [(f + f_i) T] \text{sinc} [(f - f_i) T] e^{-j2\pi f_i T} e^{-j2\varphi_i} \right\} \\
\cdot d\phi_1 \ldots d\phi_N df_1 \ldots df_N \tag{B.7}
\]
Now, the remaining aspects of the double summation as well as the sections of the diagonal summation that includes \( \varphi \) exponents can all be disregarded. This is because the integration of these components from 0 to \( 2\pi \) evaluates to zero. The remaining section of the diagonal summation is independent of the random phase. As such, the \( \varphi \) integrals surrounding it evaluate to \( 2\pi \). The CPSD can then be written as (B.8).

\[
G_{XY}(T, f) = A_0 \frac{T}{2N} \frac{1}{(f_{\text{max}})^N} \sum_{i=1}^{N} f_{\text{max}} \cdots f_{\text{max}} Q_1(f_i) Q_2^*(f_i) \left\{ \text{sinc} \left( (f - f_i) T \right)^2 + \text{sinc} \left( (f + f_i) T \right)^2 \right\} df_1 \cdots df_N \tag{B.8}
\]

Now, the attenuation and sinc functions are all only dependent on their respective frequencies, \( f_i \). They can therefore be pulled from the other integrals which then all evaluate to \( f_{\text{max}} \) and cancel with the denominator of the leading coefficient. Additionally, the attenuation coefficients are constant across the entire remaining integral with respect to the dummy frequency \( f_i \). They can therefore be pulled from the integral leaving the final form of the CPSD as (B.9).

\[
G_{XY}(T, f) = A_0 \frac{T}{2N f_{\text{max}}} \sum_{i=1}^{N} \left[ Q_1(f_i) Q_2^*(f_i) \int_{0}^{f_{\text{max}}} \left\{ \text{sinc} \left( (f - f_i) T \right)^2 + \text{sinc} \left( (f + f_i) T \right)^2 \right\} df_i \right] \tag{B.9}
\]

While the integral in (B.9) is independent of the summation and can removed and evaluated analytically, the process is nontrivial and requires the use of advanced functions such as the sine integral [8]. As such, the form shown in (B.9) is kept and evaluated numerically. The attenuation coefficients are dependent on the filter parameters, weather, and overall applications. This generic form of the CPSD therefore offers the most robust solution.
Bibliography


