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AN EVOLUTIONARY GAME MODEL OF SELF-DECEPTION

AND THE EFFECT OF BELIEF ON PERFORMANCE

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ABSTRACT

Self-deception at first thought may appear to be counterintuitive as it pertains to evolutionary strategies. It would seem that in order to best adapt, it is first necessary to make an accurate assessment of one's state. Yet, phenomena such as the placebo effect continue to suggest that there is some benefit to self-deception when it comes in the form of optimistic belief.

In previous work, Byrne and Kurland demonstrated that self-deception could be fitness enhancing if it enables one to better deceive an opponent into not competing for a resource. However, their model did not consider any effect of self-deception on one's actual performance if the opponent competes. This thesis is a natural extension of Byrne and Kurland's work. In this work, the relationship between beliefs and performance in fitness competition is examined.

The present work first assumes that belief in victory enhances one's performance and subsequently one's probability of victory. It further assumes that one's capacity to believe in victory can be limited by past experiences of defeat. Based on these assumptions, an evolutionary game model is used to analyze the relationship between a player's belief in victory and the final outcome of a competitive encounter. Simulation is employed to provide a bridge between Byrne & Kurland's prior work based on probability distributions and future studies in which discrete player histories must be tracked.

The first step is to study a model where belief in victory enhances performance. Next, the trends of a model where belief does not affect performance are examined. The evolutionarily stable strategies resulting from the simulation runs are presented and interpreted. Comparing the two models, conclusions are made about the relationship between beliefs and performance.

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Chapter 1

INTRODUCTION

Cognitive scientist Marvin Minsky developed the concept of a modular mind as a hierarchy of independent agents each seeking a unique interest, [Minsky 1986]. Should conflict arise between equally strong agents, Minsky's model passes control to a completely different agent. However, this shift in control could prove detrimental to the host's survival if the conflict occurs within fitness-enhancing agents. As an example, consider "fear" and "hunger" as subordinate agents of a parent "survival" agent. In Minsky's model, control of the host would pass away from the survival agent if the fear and hunger agents were in conflict with one another. Ultimately this shift in control would mean the demise of the host.

Byrne and Kurland introduce the concept of self-deception as a conflict resolution technique, evolved to preserve the life of the host [Byrne & Kurland 2001]. By temporarily suppressing one of the fitness-enhancing agents in conflict, control of the host remains with the survival agent. Ramachandran argued that any potential benefits of self-deception would be overshadowed by its cost, [Ramachandran 1996, Ramachandran & Blakeslee 1998]. Convincing oneself of fearlessness, for example could cause harm. Trivers, however, suggested that self-deception could be beneficial if it enabled one to better deceive an opponent [Trivers 1976, 1985].

The hypotheses of Ramachandran and Trivers were tested in the study performed by Byrne and Kurland. Their model extended evolutionary biology's widely known hawk-dove model. For a player of size 0.5, considered to be an average size player, the authors showed non self-deceiving players to be eliminated from the population in the evolutionarily stable strategies (ESS). This conclusion supported Trivers' hypothesis by concluding the benefit of enabling the self-deceived players to better deceive their opponents outweighed the cost of sometimes fighting a much larger opponent. Whether a player's belief in victory has any impact on the outcome of a competition was deferred to future work.

Section 1.1 Research Objectives and Expected Contributions

This thesis is an extension of the study performed by Byrne and Kurland. The purpose of this extension is to develop and analyze a game theoretic model of the effect of belief on performance. In doing so, two trends are identified: 1) Any deviation in the ESS from those reported in the original paper and 2) Any sensitivity of these strategies to player sizes. Specifically, does self-deception remain fitness-enhancing as previously indicated, and are distinct strategies preferable for players of a particular size? Finally, ESS mixtures are compared between a model that is not influenced by belief and one in which belief affects performance. Conclusions are made about the effect of belief on performance in fitness competitions.

The results of this study may offer valuable insight for the fields of mental and emotional rehabilitation. By understanding the effect of belief on performance, enhanced assistance can be provided to those in recovery. Furthermore, this research is one of the first of its kind in its use of mathematics to model sociological behavior. Researchers in this field may use this work as a foundation for similar studies.

Section 1.2 Thesis Overview

Chapter 2 provides a brief review of the literature most relevant to this study. It begins with a summary of the original paper by Byrne and Kurland, highlighting the aspects of their model that are essential to understanding this extension. An additional work is presented which is necessary for understanding the simulation that serves as the medium through which this work is performed. Lastly, some open questions resulting from these two studies are presented and this thesis is properly motivated.

Chapter 3 presents the formal problem statement. A few examples are used in support of the theory that belief affects performance. The model assumptions are listed and explained. The chapter concludes with the two major questions to be answered.

Next, Chapter 4 details the methodology utilized to answer the questions posed in the previous chapter. A model of the effect of belief on performance is first developed. Then the simulation used to perform the experimentation is described. Finally, the specifications of the experimental design are provided.

The results of the experimentation are provided in Chapter 5. The first results examine the average size player's ESS for any divergence from those of the original paper. These results are then analyzed for sensitivity to player sizes by finding the evolutionarily stable strategies of the entire population. The ESS values are also determined for the population of a model which does not allow belief to impact performance.

Chapter 6 concludes this thesis. This chapter begins with a discussion of the results given in Chapter 5. An overall summary of the background, purpose, and results of this study is ultimately provided. Closing remarks raise a few additional questions intended for further study.

Chapter 2

BACKGROUND

This thesis builds upon the substantial work published by previous authors concerning the nature of self-deception. Of particular note for this thesis, Byrne and Kurland explored the usefulness of self-deception in an evolutionary setting [Byrne & Kurland, 2001]. Their research provides the foundation for what follows. A thorough review of their paper is necessary to facilitate the discussion of its extension. Following is a presentation of that review.

Section 2.1 Self-Deception in an Evolutionary Game

Intuitively, self-deception may seem to be disadvantageous in a competitive encounter. One might think that in order to gain any possible advantage, it is first necessary to make an accurate assessment of the environment and one's capabilities. Cognitive scientist Ramachandran additionally argued that any benefit of self-deception would be overshadowed by its cost [Ramachandran 1996, Ramachandran & Blakeslee 1998]. In a competitive encounter for a resource, for example, a player might devalue the resource in order to avoid conflict. The player avoids the conflict, but also loses the resource which was perhaps essential for survival.

Trivers conversely argued that self-deception could be beneficial if it enabled one to better deceive an opponent [Trivers 1976, 1985]. Consider that the same player was to deceive herself into believing a resource is worthless, perhaps she could trick an opponent into believing the same. The opponent might then walk away from the encounter. Thus, the player would be left to enjoy the true value of the resource.

This leads to Byrne and Kurland's first conjecture, "episodes of self-deception need not be permanent," [Byrne & Kurland, 2001]. In the second example, the opponent forfeiting the resource effectively terminates the encounter. If at that point the player ends her self-deception, she is able to take advantage of the available resource.

The authors test the hypotheses of Ramachandran and Trivers utilizing Minsky's modular model of the mind as a basis for the definition of self-deception. Minsky developed the concept of the mind as a hierarchy of independent agents each seeking a unique interest, referred to by the authors as *daemons* [Minsky 1986]. A host's actions are determined at any given time by the most dominant daemon. Conflicts occurring between subordinate daemons lessen the parent daemon's control over the host's actions. As a result, Minsky suggests that control will pass away from the parent daemon.

Minsky's example of a child at play with blocks illustrates this process. The child builds towers and then knocks them down. As shown in Figure 2-1, the *play*, *eat*, and *sleep* daemons are positioned at the same level in the hierarchy. If *wrecker* causes the child to knock down blocks before *builder* has finished a tower, the two daemons would come into conflict. To resolve this conflict, control would pass to the stronger of the remaining two daemons at the level of *play*. The child might go to sleep, for example, if the conflict occurs just before bedtime.

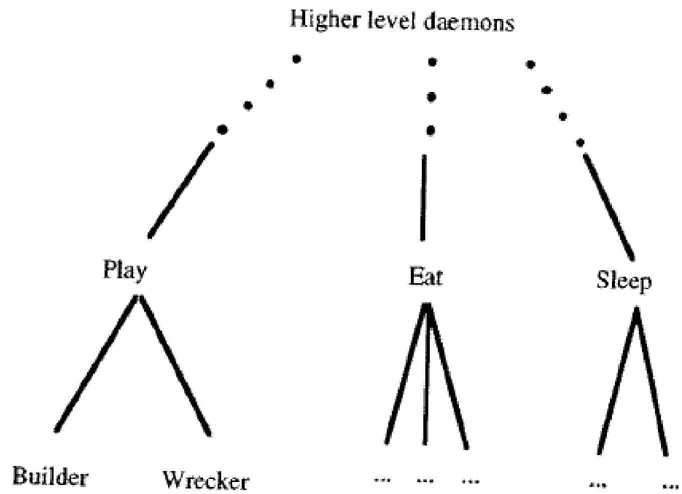


Figure **2-1**: Modified image of Minsky's model of the mind as a hierarchy of "daemons," [Byrne & Kurland, 2001].

Although this solution may be suitable for subordinate *play* daemons, this transfer of control could prove to be detrimental for the host if the conflict occurs between fitness-enhancing daemons. Figure 2-2 presents *fear* and *hunger* as subordinate survival daemons. Should these two daemons enter conflict and cause control to be transferred away from survival, it would mean the demise of the host.

Figure **2-2**: *Survival* daemon with subordinates *Fear* and *Hunger*. Transferring control away from survival could be detrimental to the host.

Alternatively, one of the conflicting daemons could be temporarily suppressed. This would preserve the life of the host by granting dominance to one of the subordinate daemons, and preventing control from being transferred away from *survival*. Byrne and Kurland hypothesize that this resolution of inter-daemonic conflict constitutes self-deception. The authors employ the multi-dimensional dynamic character (MDDC) modeling approach to create a game theoretic structure of their cognitive self-deception model [Byrne 1995, 1996]. This approach allows a two-way interaction between the game and the psychology of the players:

> "MDDC defines a meta-structure for models that maps…daemons, into player information and strategies, enabling cognitive models to determine actions in a game-theory model. Reciprocally, the meta-model maps parameters and outcomes of game-theory models into states or state changes of cognitive models, enabling the game circumstances to affect the cognitive states of the players, [Byrne and Kurland 2001]."

Evolutionary game theory specifically is used to structure the model. In each generation of the evolution model, a multi-period game is played between members of the population. The game of each period is an asymmetric hawk-dove game [Maynard Smith & Price, 1973]. The encounter is such that two players simultaneously discover a valuable resource. A player applying the hawk strategy will fight for the resource if the other player is unwilling to forfeit it. The dove strategy begins with an attempt to intimidate the opponent, yet will always yield as opposed to engaging in an actual fight.

During the encounter, the two players are each controlled by a *survival* daemon with subordinate *fear* and *hunger* daemons. The *fear* daemon is responsible for maintaining the physical safety of its host, the player, and accordingly influences the player to play dove. The *hunger* daemon ensures adequate nourishment is provided for its host and thus influences the player to play hawk.

As previously noted, Byrne and Kurland characterize self-deception as a mechanism for resolving inter-daemonic conflict without jeopardizing the fitness of the host. They distinguish three types of players according to their variations on this mechanism. In response to inter-daemonic conflict, the *fear* daemon is suppressed in a type SDF player. This player's fear is rendered "unconscious," and the now *hunger*-dominated daemon will play hawk. Conversely, a type SDH player suppresses the *hunger* daemon. This player's hunger is thus rendered "unconscious" and the *fear*-dominated player plays dove. A third player type, NSD, does not enter self-deception. For this player, the inter-daemonic conflict persists and the player plays either hawk or dove with some positive probability. The authors show that the final results are not dependent on the exact value of this probability.

The interaction between the two players engaged in an encounter consists of a series of assessments and belief updates. At the start of the encounter, the *hunger* and *fear* daemons of each player form respective initial assessments about the value of the resource and the cost of a fight with the opponent. Ambivalence, or conflict between the two daemons, occurs when the difference between the value of the resource and the cost of the fight does not exceed some ε - referred to as the cognitive resolution parameter. A self-deceiving player experiencing ambivalence will enter self-deception and remain in that state for the duration of the encounter.

During the display phase the players signal their beliefs to each other. If a player has entered self-deception, the new beliefs are signaled. Otherwise a player signals the initial assessments. After receiving the signaled beliefs of the opponent, a player updates his assessments based on how sensitive he is to the opponent's signal. This sensitivity is governed by λ, the susceptibility parameter. Here, players have another prospect of entering self-deception, if their updated assessments cause the difference between the resource's value and the fight's cost to fall below ε. Once beliefs are updated, the players' actions are determined by daemon dominance as influenced by their player type.

In each generation, the players engage in numerous two-player games, each time matched with a different member of this three-type population and a different resource value. Beta distributions on the resource value and the expected cost of the fight are used as continuous approximations of these "real life" encounters. As standard in an evolutionary model, the composition of the population in the next generation depends on the individual success of the respective types.

The entire encounter space of individual hawk-dove encounters are analyzed for a player of average stature, size 0.5, through the use of beta distributions modeling the likelihood of encountering an opponent inflicting a given cost (C_1 on player 1) and a resource of a given value, V.

> "That is, we assume that in a reproductive lifetime, a player encounters the game a large number of times, with expected V and C_1 determined by the probability distributions. We integrate the payoff function of V and C_1, against the probability distributions of V and C_1, over the encounter space, to compute a 3x3 type vs. type expected lifetime fitness matrix, [Byrne & Kurland 2001]."

This integrand is the basic asymmetric hawk-dove payoff function used in the standard replicator dynamic, which computes the subsequent population dynamics based on the expected lifetime fitness matrix. The expected lifetime fitness matrix provides the expected payoff in the given generation for each player type. The average expected payoff in a given generation is computed by summing the products of the proportion of players of each type and the expected payoff for that type. In the next generation, the proportion of players of a given type will be equal to the product of the proportion of players of that type and the expected payoff for that type, divided by the average expected payoff for that generation. The equations from the paper are given below.

$P_{\sigma,\tau}$ = proportion of type σ players in the population in generation τ (1a)

$$P_\tau = (P_{NSD,\tau}, P_{SDF,\tau}, P_{SDH,\tau}) \qquad (1b)$$

$\pi_\sigma(P_\tau)$ = the payoff to type σ in generation t (1c)

$$\pi_{ave}(P_\tau) = P_{NSD,\tau}\pi_{NSD}(P_\tau) + P_{SDF,\tau}\pi_{SDF}(P_\tau) + P_{SDH,\tau}\pi_{SDH}(P_\tau) \qquad (1d)$$

$$P_{\sigma,t+1} = P_{\sigma,t}\pi_\sigma(P_\tau) / \pi_{ave}(P_\tau) \qquad (1e)$$

Byrne and Kurland proved that for all values of ε and λ tested, the NSD type is eliminated from the population in the ESS. Furthermore, the resulting SDF-SDH population has a unique mixed-strategy ESS. The ESS is purely SDF when the ratio of λ/ε is very high. Figure 2-3 displays the sensitivity of the ESS values to ε and λ.

In conclusion, the results support Trivers' hypothesis. Self-deception can be fitness-enhancing when it enables a player to better deceive his opponent.

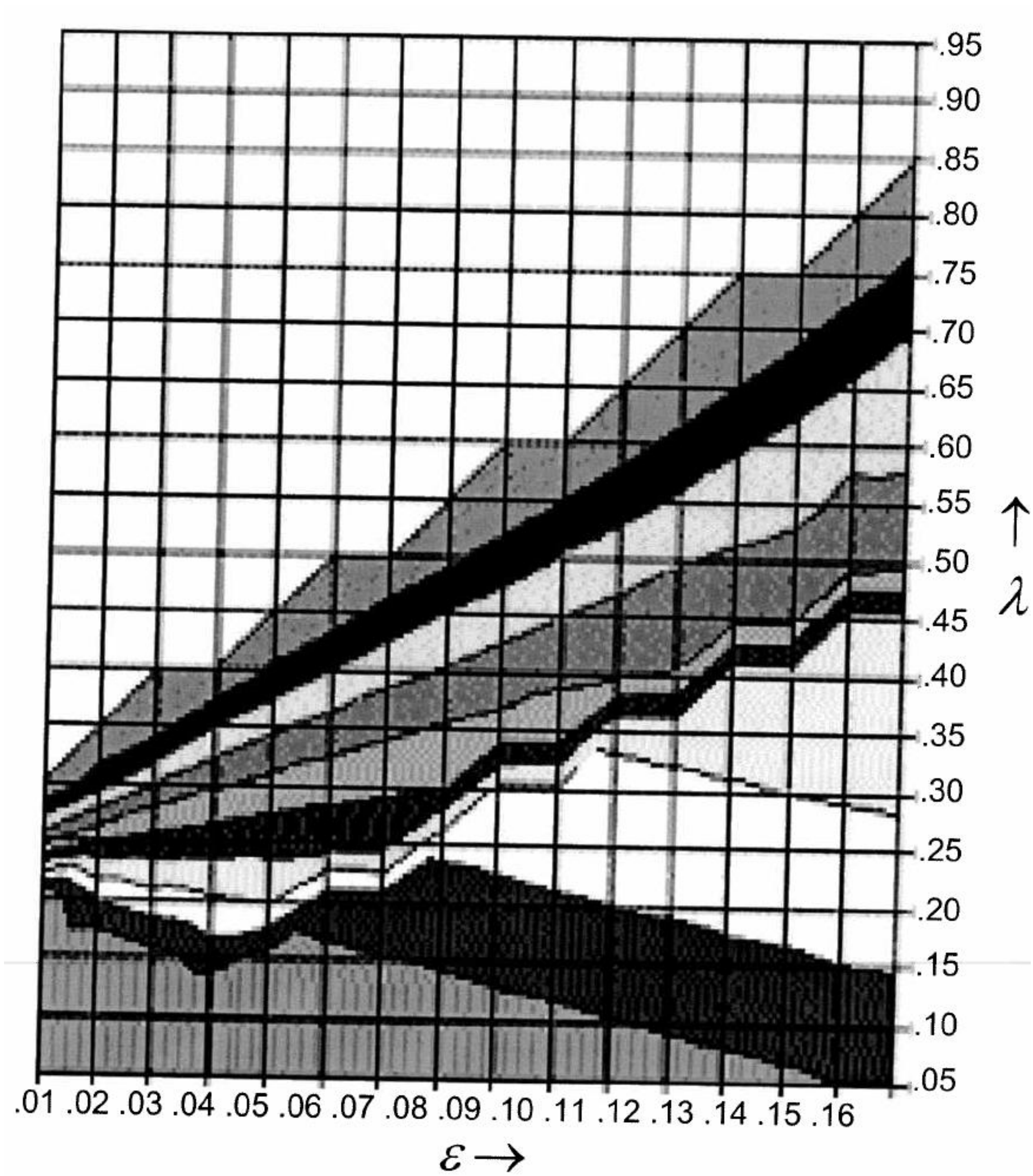


Figure **2-3**: The sensitivity of the ESS to ε and λ. The shades of gray represent 10% increments in the SDF-content of the ESS SDF-SDH population, [Byrne & Kurland 2001].

Section 2.2 Additional Work

One additional reference is noteworthy for the purposes of this thesis. While the theoretical and numerical analysis provided by Byrne and Kurland was insightful, a computer model is needed in order to provide the flexibility to further explore the implications of their results. Specifically, a finite population is needed to examine belief capacity dynamics in order to track individual encounter histories. Haney developed a computer simulation model that reproduces the results of Byrne and Kurland, [Haney 2007]. This model can be easily modified to analyze the results of variations applied to the original model, variations in the model parameters for example.

In Haney's Matlab simulation program, the population of players is initialized with a sampled beta distribution. Information about the players is stored in a matrix, with each row containing the information for a single player. The columns of the matrix contain the player's size, the player's type, the value of ε, and the value of λ. The same values for ε and λ are shared by the entire population. In this model there is a finite approximation of the beta distribution that is a continuous approximation of reality. Whereas the original paper integrated the entire encounter space, here the expected cumulative fitness values of the players are calculated by summing the sampled points weighted by the value of the beta distribution at that point.

As in the original paper, player one, the prototype player, is set to be the average size player at 0.5 [Byrne & Kurland 2001]. The sampled points of opponent player sizes range linearly from 0.01 to 0.99. The cost to player one is simply the size of the opponent. In every encounter, the value of the resource is set to equal the cost to player one. Haney follows the original model and equates the opponent size with the cost to player one [Haney 2007]. As a result, the opponent size and the value of the resource in an encounter both follow beta distributions. This characteristic is important for comparison with the original paper.

For each player type, the prototype player engages every representative player of the sampled beta distribution in a hawk-dove game. Each of these games is repeated for all representative resource values, and for all possible types of the opponent. Expressions (2a) and (2b) present the cumulative fitness value calculations for Byrne & Kurland's and Haney's models respectively.

$$\int_v \int_{c_1} \pi(p_1, p_2, v)\, dc_1 dv \qquad (2a)$$

$$\sum_{V=0.01}^{0.99} \sum_{c_1=0.01}^{0.99} \pi(p_1, p_2, v)\beta(v)\beta(c_1) \qquad (2b)$$

In the expressions, the player characteristics $p_1 = (0.5,\sigma)$ and $p_2 = (c_1,\sigma)$, where σ represents the player's self-deception type. Moreover, π is the payoff of each encounter, and in (2b) the β distribution is distinctly called for the value of the resource and the cost of the fight. Results of each game contribute to the cumulative lifetime fitness value of the prototype player for each player type. After playing an entire population, the overall fitness score for the prototype player is stored in a matrix. The final result is a 3x3 matrix of payoff values for each combination of types. Standard replicator dynamics initialize the next population following each generation.

The results for the prototype players are compared against those of the original paper for validation. The graph of ESS values from Haney's model is given in Figure 2-4. It shows the percentage of SDF players in the evolutionarily stable strategies. This figure should be compared to Figure 2-3 which shows the ESS values of the original paper.

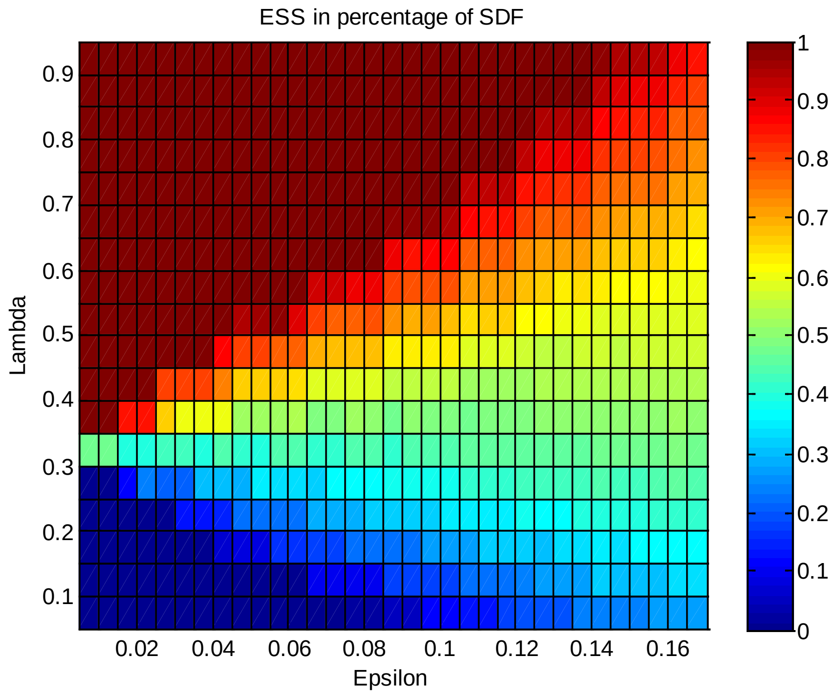


Figure **2-4**: Haney's ESS Results [Haney 2007].

These results are found to closely match those of Byrne and Kurland, with deviations minor enough to attribute to the difference in methodology (i.e. sampling versus integration). Haney successfully reproduces the results of the original paper employing a computer model [Haney 2007].

Section 2.3 Motivation

With validated results, Haney's computer model can be modified to answer some of the questions raised by Byrne and Kurland at the end of their paper, while maintaining an analytical bridge to their results. As mentioned, self-deception was found to be fitness enhancing when it enables one to better deceive an opponent into not competing for a resource. However, the model of Byrne and Kurland does not consider any effect of self-deception on one's actual performance if the opponent competes. An examination of the effect of beliefs on performance is the first possible extension of their work.

Furthermore, the original paper considers only the encounter space of an average size player against a sampled beta distribution of opponents. For the average size player, the NSD type is eliminated from the ESS, with an implicit assumption of the independence of type and size. A worthwhile exercise would be to analyze the encounters of a sampled distribution of player sizes against opponents from the same distribution. Perhaps some noteworthy trends would arise.

This thesis is an extension of Byrne and Kurland's original work designed to address these two issues.

Chapter 3

PROBLEM STATEMENT

Athletes often observe strict rituals before playing in a game. They may always wear the same undershirt, carry a special trinket, or mismatch their socks. These athletes believe that following these rituals will place the odds of winning the game in their favor. Patients participating in medical experiments may be issued a placebo rather than the actual treatment being tested. Nevertheless, these patients will often report an improvement in their condition. Authors of self-help books generally stress the importance of positive thinking. They suggest that what one believes to happen plays a major role in determining what actually does happen. All of these scenarios seem to support the same conclusion: belief affects performance.

Consider a competitive game between two players in which the size of a player directly relates to his or her chance of winning the game. A larger player is more likely to win and a smaller player is more likely to lose. In boxing and wrestling, for example, matches are coordinated based on one's weight class. A match pairing two players of considerable size difference is deemed unfair in that the larger player is likely to dominate the smaller one.

However, suppose that size is not the only characteristic that impacts the players' chances of winning. Suppose the players' beliefs about their performances in the game will have an impact on the outcome as well. The smaller player might be convinced that she is stronger than her size would suggest, and that she therefore has a 50-50 chance of beating her opponent. The larger player might be rebounding from a series of losses and doubting his abilities at the moment. Is the larger player still more likely to emerge as the winner?

The fundamental problem addressed in this thesis relates to the study of the relationship between self-deception and the outcome of a competitive encounter. Particularly, the effect of a player's belief in victory on his or her actual performance is examined.

Two assumptions serve as the foundation for this study. The assumptions made are:

1) Belief in victory enhances one's actual performance.

2) One's capacity for belief in victory is influenced by personal history.

The first assumption expresses the view that belief indeed affects performance. This assertion holds for both the affirmative and negative cases. A player who believes he will win a competitive encounter is more likely to do so. Similarly, a player who believes she will lose a competitive encounter increases her chances of defeat.

However, if it were that simple people would just maintain an infinite belief in victory. The second assumption is that one's ability to believe in victory is affected by past experiences; i.e. previous experiences have the potential to both expand and restrict one's capacity for belief.

Based on these assumptions, the objective is to learn more about the effect of belief on performance. In order to study this problem the following questions are answered:

1) Is self-deception still fitness-enhancing as indicated by the original paper?

2) Do the results display size sensitivity?

Recall that Byrne and Kurland demonstrated self-deception could be fitness enhancing if it enables one to better deceive an opponent into not competing for a resource. However, in this thesis the case is considered where self-deception has the potential to both positively and negatively affect one's performance. The first question addresses whether this additional aspect of self-deception changes the results of the original paper. Perhaps now in the ESS for the average size player NSD types are no longer eliminated from the population.

Secondly, trends in the results are discussed as it pertains to size. This step is necessary to obtain an accurate image of the evolutionarily stable strategies. It may be that it is more beneficial for smaller players to enter self-deception. This would be consistent with phenomena such as the "Napoleon complex" and small dog behavior. The Napoleon complex describes the theory that smaller men are more aggressive in an effort to dominate larger men. Small dogs often noticeably bark in encounters more frequently and for longer periods of time than their larger counterparts. As another example of size dependence perhaps large players stand to gain more, or at least lose less, by being realistic. Yet, large players risk less by inflating their beliefs.

Finally, the ESS results of this model are compared to a model that does not allow belief to affect performance. Addressing all of these issues will enable a full appreciation of the relationship between belief and performance. What is the effect of belief on performance in fitness competitions? The subsequent chapter details our methodology for discovering the answer.

Chapter 4

METHODOLOGY

At the foundation of this work are the assumptions that belief in victory enhances performance, and that one's ability to believe in victory is limited by past experiences of defeat. Based on these assumptions, an evolutionary game model of the effect of belief on performance is formulated. With this model as the basis, the simulation used to produce the player fitness values for analysis is described. Finally, the technical details of the experimental runs are provided.

Section 4.1 Model Formulation

Player sizes range exclusively from 0 to 1, $s \in (0, 1)$, with 0 considered to be a player of small stature and 1 considered to be a player of large stature. The baseline probability of victory, $\alpha(s_1, s_2)$, is a function of the two player sizes. Consistent with the work of the original paper, the players' probabilities of losing a fight are equal to the cost, C, they incur in the fight [Byrne & Kurland, 2001].

In the present model, however, the fight probability and costs are a function of the two player sizes. This allows the consideration of payoffs to any size player. The cost of a fight is equal to the difference in player sizes, normalized to (0, 1) to maintain a probability.

A player's belief in victory ranges from [0, 1]. This is a belief about his or her baseline probability of victory. A belief close to zero is considered to be weak and a belief close to one is considered to be strong. The two players' beliefs, β_1 and β_2, need not sum to unity. Their beliefs are based on a number of factors, including their baseline probabilities of victory, their opponent's signal, and/or their self-deception type. Furthermore, a player's belief in victory is affected by his or her current capacity (upper limit) for belief, χ. The capacity for belief in victory is fixed in this model. However, it will be dynamic in the future. Note that in Byrne and Kurland, capacity was implicitly 1, as SDF players had complete belief in victory when in self-deception.

In the present model of the effect of belief on performance, beliefs lead to an adjusted probability of victory, γ, for each player throughout their competitive encounters. The weighted average of each player's believed and baseline probabilities of victory are normalized to calculate this adjusted probability. The normalization accounts for the opponent's belief based advantage or disadvantage. The parameter f is utilized to denote the weight placed on belief in victory. All variables and parameters along with the final model of the adjusted probability of victory are listed below. It may be seen that $C_1 + C_2 = 1$, and $\gamma_1 + \gamma_2 = 1$.

$\mathbf{s_1}$ = player 1's size (3a)

$\mathbf{s_2}$ = player 2's size (3b)

$\alpha_1(\mathbf{s_1}, \mathbf{s_2}) = \frac{1}{2} + \frac{(s_1 - s_2)}{2}$ = baseline probability of victory for player 1 (4a)

$\alpha_2(\mathbf{s_1}, \mathbf{s_2}) = \frac{1}{2} + \frac{(s_2 - s_1)}{2}$ = baseline probability of victory for player 2 (4b)

β_1 = player 1's belief in victory (5a)

β_2 = player 2's belief in victory (5b)

χ_1 = player 1's capacity (upper limit) for belief in victory (6a)

χ_2 = player 2's capacity (upper limit) for belief in victory (6b)

f = weight placed on belief in victory (7)

$\mathbf{C_1} = \alpha_2(s_1, s_2)$ (8a)

$\mathbf{C_2} = \alpha_1(s_1, s_2)$ (8b)

$$\gamma_1 = \frac{f\beta_1 + (1 - f)\alpha_1}{f\beta_1 + (1 - f)\alpha_1 + f\beta_2 + (1 - f)\alpha_2} \tag{9a}$$

$$\gamma_2 = \frac{f\beta_2 + (1 - f)\alpha_2}{f\beta_1 + (1 - f)\alpha_1 + f\beta_2 + (1 - f)\alpha_2} \tag{9b}$$

Section 4.2 Simulation

Both the original and present models are deterministic and can be largely analyzed utilizing numerical methods. However, anticipated additional features of future models will shift the model to a dynamic one that requires a more sophisticated analysis. Therefore, Matlab is used to simulate the present model with the collaboration of Haney. His previous work in creating a computer model that accurately reproduces the results of Byrne and Kurland serves as the foundation for the present model.

The simulation engages two players in a hawk-dove encounter. Encounters are characterized by the two players, the baseline cost of the fight, the value of the resource, and the weight that is placed on the beliefs of the players. Each player has five attributes: size, self-deception type, cognitive resolution, susceptibility, and belief capacity. Both the cognitive resolution and susceptibility parameters are shared by the entire population. This could be examined for further extensions.

As in the work of Byrne and Kurland, a player who is not in ambivalence will play hawk if the value of the resource exceeds the cost of a fight. Such a player will play dove if the cost of a fight is greater than the value of the resource. Otherwise, the player will enter ambivalence when the difference between the value of the resource and the cost of the fight is less than the player’s cognitive resolution. This ambivalence is defined in the inequality given below.

Ambivalence => $|V - C| \leq \varepsilon$ (10)

Upon encountering a resource and an opponent, each player makes an initial assessment about the value of the resource and the cost of a fight with the opponent. The present model defines this cost to be C_{1a}, the maximum between C_1 and 1 - χ_1. In this way, the model reflects the fact that a player's capacity for belief in victory (or lack thereof) has the potential to increase the actual cost incurred by the player. As noted above, belief capacity is implicitly equal to 1 in Byrne and Kurland's model. In this model the belief capacity of each player is fixed. Further note that when belief capacity is introduced to the model, all players are affected regardless of self-deception type. The fixed capacity of the present model is an interim step toward analyzing dynamic capacities.

Following the initial assessment, the players signal their beliefs to each other. Players of the non self-deceiving type or not in ambivalence accurately signal a value of V for the resource and of C_{1a} for the cost of the fight. The signal of players in ambivalence will vary based on the assessments resulting from their self-deception types. A list of self-deception types and corresponding assessments is provided in Table 4-1.

Table **4-1**: Self-Deception Types and Corresponding Assessments.

Self-Deception Type	Hunger Assessment	Fear Assessment
SDF	V	$1-\chi_1$
SDH	0	C_{1a}

Based upon their individual susceptibilities, the players then update their assessments in accordance with the signals received from their opponents. The updates are calculated with a simple convex sum, weighting the opponent's signal by the susceptibility parameter λ. This period of updated assessments provides another opportunity for players to enter ambivalence. Since ambivalence once entered lasts the duration of an encounter, ambivalent players from the initial assessment remain ambivalent throughout the update phase of play. Ambivalence occurs in the update phase if the difference between the updated hunger and fear beliefs is less than ε. The players entering ambivalence during this phase also share the assessment values listed in Table 4-1.

Players not in ambivalence choose to play hawk when their hunger exceeds their fear and play dove otherwise. An ambivalent player facing a non-ambivalent opponent will play hawk if he is an SDF player and dove if he is SDH. Ambivalent NSD players play either hawk or dove with equal probability. The various strategies of two ambivalent players are listed in Table 4-2. In each entry, the first strategy refers to player 1 and the second strategy refers to player 2.

Table **4-2**: Ambivalent Player Strategies.

		Opponent		
		NSD	SDF	SDH
Player 1	NSD	0.25*(Hawk, Hawk) + 0.25*(Hawk, Dove) + 0.25*(Dove, Hawk) + 0.25*(Dove, Dove)	Dove, Hawk	Dove, Dove
	SDF	Hawk, Dove	Hawk, Hawk	Hawk, Dove
	SDH	Dove, Dove	Dove, Hawk	Dove, Dove

The final fear assessments of the update phase of play are used in the calculation of the players' beliefs. For each player, belief is set to equal one minus the final fear assessment. These belief values are then entered in the adjusted probability of victory equations (9a) and (9b). Lastly, the adjusted probability of victory is used to determine the fitness value of each player for a single generation of play. These fitness values are broken out by a player's performance against various opponent sizes. The next section describes the computation of these fitness values, preceded by a discussion of the variable and parameter levels employed in this study.

Section 4.3 Experimental Design

Experiment One

Recall that in the present model, belief has the potential to both positively and negatively affect one's performance. The first question addressed pertains to the ESS of the type(s) of average size players. Does this additional aspect of the model prevent the NSD player type from being eliminated from the population in the ESS as was the case in the original Byrne and Kurland model?

To this end, an average size player of each player type engages opponents of each size of a 99-point sampled beta distribution. The value of the resource in these encounters is also taken from a 99-point sampled beta distribution. An initial weight of $f = 0.5$ is given to the players' beliefs in victory. Setting $f = 0.5$ results, before normalization to account for the opponent, in a player's adjusted probability of victory being a simple average of the players' believed and baseline probabilities of victory.

Epsilon is held constant at a value of 0.1. With reasonable epsilon values ranging from 0.01 to 0.16, this value is about average for the cognitive resolution. Yet, it is high enough to frequently force players into ambivalence.

Referring back to Figure 2-3, there is more sensitivity in the ESS to the variation of the lambda parameter than to the variation of epsilon. For this reason the encounters are repeated at the lambda values of 0.2, 0.45, and 0.7. These values sufficiently sample the lambda range of 0.05 to 0.95.

Finally, the belief capacity of each player is tested at 0, 0.25, 0.5, 0.75, and 1. The experiments test every possible combination of belief capacity for player 1 and player 2. The output of these experiments is 25 3x3 matrices for each level of lambda. The matrices contain fitness values for player 1 resulting from the play of each player type against an opponent of each player type. An example of the output from the experiments is provided in Table 4-3. The lower case letters are constants representing the fitness values for player 1 resulting from each encounter. Player 1 is the row player and the opponent (player 2) is the column player.

Table **4-3**: Example Output of the Average Size Player's Encounter Payoffs.

	NSD	SDF	SDH
NSD	a	b	c
SDF	d	e	f
SDH	g	h	i

The fitness values are computed by summing the payoff functions of V and ($1-\gamma 1$) against the sampled probability distributions of V and ($1-\gamma 1$). Consider the case where both players are ambivalent. In this case NSD players choose to play either hawk or dove with equal probability. For a hawk-hawk encounter, the payoff is the value of the resource times the adjusted probability of victory, minus the adjusted cost. The hawk receives the value of the resource at no cost in a hawk-dove encounter. In a dove-dove encounter, the payoff is the value of the resource times the adjusted probability of victory, again at no cost. These equations may be seen in Table 4-4.

Table **4-4**: Payoff Matrix for two Ambivalent Players.

	NSD	SDF	SDH
NSD	0.25*[γ1*V-(1-γ1)] + .25*V + .25*(0) + .25*(γ1*V)	0	γ1*V
SDF	V	γ1*V - (1-γ1)	V
SDH	γ1*V	0	γ1*V

Using these fitness results, the equilibrium values are calculated by solving for the proportions of each type in the population that render the fitness of each type equal, the types being the three varieties of self-deception: SDF, SDH, and NSD. The system of equations for Table 4-3 is provided below. Define x_1, x_2, and x_3 to be the NSD, SDF, and SDH strategies respectively. As the strategies are the probabilities of playing each type, their values must sum to unity.

$$ax_1 + bx_2 + cx_3 = dx_1 + ex_2 + fx3 = gx_1 + hx_2 + ix_3 \quad (11a)$$

$$x_1 + x_2 + x_3 = 1 \quad (11b)$$

Similarly, a system of two equations and two unknowns are solved to calculate a two-type ESS whenever one of the strategies is strictly dominated. A pure strategy ESS results whenever one of the strategies strictly dominates the other two.

Experiment Two

The second study addresses the question of size sensitivity in the results. Do players benefit more from particular strategies depending on their sizes? These experiments include two additional player sizes to consider a population of small, medium, and large players, each of whom can exhibit any of the three types of self-deception. For fitness calculations, the full sampled beta distribution of opponent sizes is preserved and the distribution is divided in thirds such that opponents greater than 0.66 in size are considered large, those less than 0.33 in size are considered small, and the rest are considered medium. This allows for direct comparison of the results of the three sample size players to the ESS results of the simulated version of Byrne and Kurland's model.

Originally, 11 different player sizes were tested, resulting in 33 size-type pairs. These sizes ranged from 0.05 to 0.94 in increments of 0.09. Preliminary testing showed the resulting trends to be the same for three sets of players: smaller, average, and larger players. A sample of three player sizes was found to be sufficient to describe the trends found in the entire space while reducing the complexity of the model. Avoiding the endpoints, the player 1 sizes of 0.14, 0.5, and 0.85 were chosen for examination.

For comparison with that model, the ESS values of the population are examined with belief capacity set to $\chi_1 = \chi_2 = 1$. Having observed the affect of lambda variation in the first experiment, lambda is held constant in this experiment at the middle value of 0.45. All other levels for the model variables and parameters remain the same as in experiment one.

The key to analyzing size dependence of the ESS results is to allow self-deception types to evolve separately for each size, while holding the distribution of sizes constant and continuing to match each player (of any size and type) with the full distribution of opponent sizes and types. The three possible types for each size bin result in 27 extreme points of the convex set of mixed populations. For example, a population in which all small players have type NSD, all medium players have type SDF and all large players have type SDF is one extreme. Another would be type SDH small players, type SDH medium and type SDF large. A mixture these extremes would feature a mix of NSD and SDH in the small players, a mix of SDF and SDH in the medium players, and SDF in the large players. Any population featuring a mix of types for each player size is in the convex hull of the 27 extreme points, which will be used to search for any ESS of the population. It should be noted that not every mixed population has a unique representation as a convex combination of the extreme points, which becomes evident in the analysis section.

By analogy, there is a "payoff" to each extreme point (triple) of the population paired with any other triple, resulting in a 27 x 27 matrix of payoffs to which standard evolutionary game methodology can be applied. The payoff of one triple matched with another is computed as the sum of the payoffs to each size-type combination represented by the pair of triples. This models each size player having to play the entire population (all sizes of players) in his or her lifetime. Therefore, nine payoff values are summed to calculate the total payoff to each triple versus each other triple. Table 4-5 provides a blank example of the triple payoff matrix that is summed to calculate a single entry in the total triple payoff matrix. When all player size, opponent size and type combinations have been exhausted, 729 such matrices are produced.

Table **4-5**: Blank Triple Payoff Matrix.

		Opponent		
		Small, Type	Medium, Type	Large, Type
Player 1	S1 = 0.14, Type			
	S1 = 0.5, Type			
	S1 = 0.85, Type			

The total triple payoff matrix is extensively analyzed to identify the evolutionarily stable strategies. Payoffs are first evaluated for pure strategies and then to identify all mixed-strategy equilibria. The matrix is evaluated for pure ESS values by inspecting the payoffs of each triple versus itself to determine whether they are the maximum payoff in their columns. A maximum triple versus itself payoff effectively indicates a pure ESS. Otherwise the population will gravitate towards the higher paying strategy.

Mixed strategy equilibria are found by exhaustively checking all possible combinations of types and solving the system of linear equations that equate the payoffs of every type in the combination. To narrow the search, any strategy that is strictly dominated in the total triple payoff matrix is eliminated because it will not be part of an ESS. Considering this to be a one population model of players versus other players, only the diagonal entries are checked for equilibria. On the diagonal, the row player takes on the role of each pure strategy and the column player represents the entire population.

Identified equilibria are tested for stability, leading to the discovery of the ESS. Stability is proven by showing that the disruption of an equilibrium in any direction within a given neighborhood leads to a return to the equilibrium. ESS values are observed across the full range of parameter values to confirm local or global attraction.

We repeat this process for a model where beliefs have no influence over performance by setting the belief parameter to f=0. The results of these experiments are provided in the following chapter.

Chapter 5

RESULTS

This chapter provides the results of experiment one and experiment two. Experiment one examines whether non self-deceiving players are eliminated from the population in the evolutionarily stable strategies. The second experiment finds identifies any sensitivity in the results to distinct player sizes. Experiment two also compares the ESS results of the present model to a model where belief has no impact on the performance of a player.

Section 5.1 Experiment One

Utilizing the simulation, an average size player competed in hawk-dove encounters against opponents from a 99-point sampled beta distribution. The value of the resource in each encounter was also taken from a 99-point sampled beta distribution. While ε was held constant at 0.1, the encounters were repeated for λ values of 0.2, 0.45, and 0.7. The beliefs of both players varied across all combinations of the values 0, 0.25, 0.5, 0.75, and 1. Tables 5-1 (a), (b), and (c) list the resulting fitness and ESS values of these encounters at the belief level $\chi_1 = 1$, for the three λ values. This belief value is used to facilitate the comparison of this model to the work of Byrne and Kurland in which all self-deceiving players have a belief level of one [Byrne and Kurland 2001]. As in the paper, no size sensitivity is assumed in the ESS results. Fitness and ESS values for all belief levels of both players are provided in Appendix B.

Table **5-1(a)**: Fitness (in thousands) and ESS Values for the 0.5 Player at λ = 0.2.

	X2 = 1			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	0.81	0.04	1.2962	
SDF	1.34	0.00	2.00	0.64
SDH	1.04	0.33	1.42	0.36

Table **5-1(b)**: Fitness (in thousands) and ESS Values for the 0.5 Player at λ = 0.45.

	X2 = 1			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	0.76	0.15	1.11	
SDF	1.69	0.19	2.22	0.78
SDH	1.13	0.44	1.37	0.22

Table **5-1(c)**: Fitness (in thousands) and ESS Values for the 0.5 Player at λ = 0.7.

	X2 = 1			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	0.77	0.26	1.03	
SDF	1.92	0.81	2.13	1
SDH	1.22	0.61	1.37	

Figures 5-1 (a), (b), and (c) show the type dominance for these encounters. These figures display the type dominance for a player 1 belief level of χ_1 = 1 as the belief level of player 2 varies across all five values.

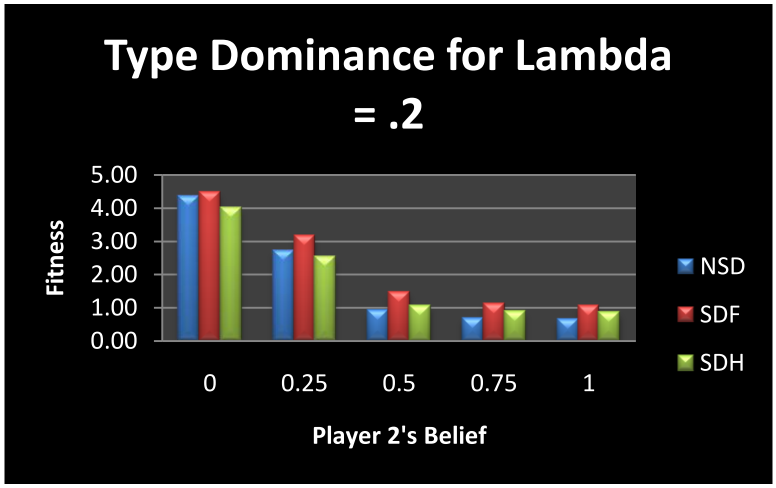


Figure **5-1(a)**: Player 1 Type Dominance for λ = 0.2.

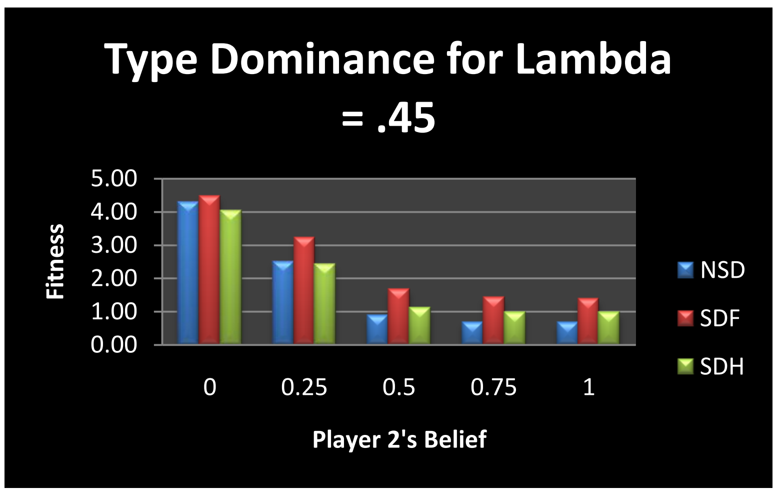


Figure **5-1(b)**: Player 1 Type Dominance for λ = 0.45.

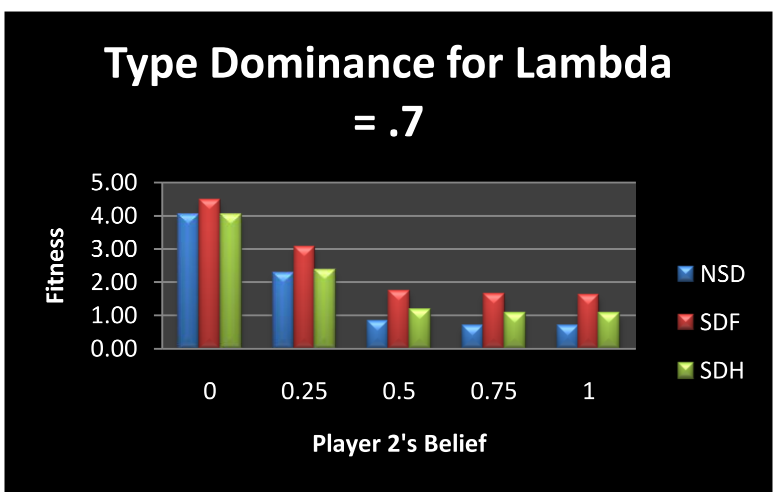


Figure **5-1(c)**: Player 1 Type Dominance for λ = 0.7.

Section 5.2 Experiment Two

In experiment two, the distribution of opponent sizes is divided into thirds to manage complexity while analyzing size dependence in the results of the evolutionarily stable strategies. The opponents are classified as small, medium and large respectively with the thirds in ascending order, (i.e. sizes 0.01 to 0.33, 0.34 to 0.66, and 0.67 to 0.99). Having now understood the effect of λ variation, the susceptibility parameter is held constant at 0.45 in this experiment.

Original experimentation was performed with 11 different player one sizes. These preliminary results showed the same general player type trends for three categories of players: smaller, average, and larger. That being the case, the player one sizes of 0.14, 0.5, and 0.85 serve as a sufficient sample of the encounter space. Graphs of the preliminary results are provided in Appendix C.

The total triple payoff matrix for the present model did not produce a pure strategy ESS. Before exhaustively searching the matrix for mixed equilibria, dominated strategies were eliminated. Following this elimination, five triple size and type strategies remained. These five strategies and their payoffs are listed in Table 5-2. In Tables 5-2 and 5-3, F refers to the SDF strategy, H to the SDH strategy, and N to the NSD strategy. The first letter is the strategy of the small player, the second is that of the medium player, and the final letter is the large player's strategy.

Table **5-2**: Non-dominated Strategies.

	FFF	FHN	FHF	HHF	HFF
FFF	1541.36	5095.52	4311.23	5755.18	2985.31
FHN	1713.55	4157.67	3868.21	5114.68	2960.02
FHF	1787.60	4551.42	3997.06	5243.13	3033.67
HHF	2154.17	4360.51	3807.16	4722.82	3069.83
HFF	1907.93	4904.60	4121.32	5234.87	3021.47

The five strategies were examined for mixed equilibria by checking all possible combinations of types and solving the system of linear equations that equates the payoffs of every type in the combination. Table 5-3 provides the equilibria identified as a result of this search. These equilibria were calculated utilizing Mathematica software. Appendix D contains the calculations.

Table **5-3**: Equilibrium Strategies.

Mix A

FFF	FHN
0.84	0.15

Mix D

FHN	HHF
0.66	0.34

Mix G

FFF	HHF	HFF
0.09	0.12	0.79

Mix B

FFF	FHF
0.56	0.44

Mix E

FHF	HFF
0.09	0.91

Mix H

FHF	HHF	HFF
0.09	0.03	0.88

Mix C

FFF	HHF
0.63	0.37

Mix F

HHF	HFF
0.09	0.91

Mix I

FFF	FHF	HHF	HFF
0.09	0.00	0.12	0.79

In search of stability, the member strategies of identified equilibria were first examined for maximum payoff among all the strategies. An equilibrium in which the positively weighted pure strategies do not have the maximum payoffs among all strategies is not stable. The population will eventually move from such an equilibrium toward the strategies with better payoffs. This initial criterion eliminated all but Mix G, H, and I. Furthermore, mixes G, H and I are actually all the same mixed population; that is, G, H and I are merely different representations of the same underlying population as convex combinations of extreme points. The equilibrium population is 9% SDF and 91% SDH for the small player, 88% SDF and 12% SDH for the medium player, and 100% SDF for the large player.

To confirm the stability of this population mix, it must be shown that there exists a neighborhood within which no matter how the strategies are perturbed, the population always returns to the mixed strategy of interest.

The large player employs a pure strategy SDF in all of the aforementioned mixed strategies. This pure strategy SDF strictly dominates SDH and NSD and is therefore stable. Furthermore, SDF and SDH strictly dominate NSD for the small and medium players. Consequently, only the perturbation of the mix of SDF and SDH are considered for the small and medium player strategies.

Figures 5-2 (a), (b), and (c) chronicle the examination of such perturbations for Mix G. The first two figures graph the difference between the payoffs of the SDF and SDH strategies for the small and medium players respectively. It may be seen that the lines in each graph cross the x-axis at the equilibrium. Where the lines are in the positive region, the SDF strategy will continue to increase. Once it surpasses the equilibrium, it enters the negative region and will return to the equilibrium. Thus the mixed strategy remains stable as the small or medium player changes strategy given the other player is held constant.

The third figure displays a graph of the equilibria for both the small and medium player. With this figure one may examine the effect of simultaneously perturbing the strategies of these two players. In the figure the blue line represents the small player and the medium player is represented in red. The arrows in the graph illustrate the direction in which the mixed strategies will move when they are in various combinations of small and medium player strategies. One may observe that the mixed strategies will ultimately return to the equlibrium point.

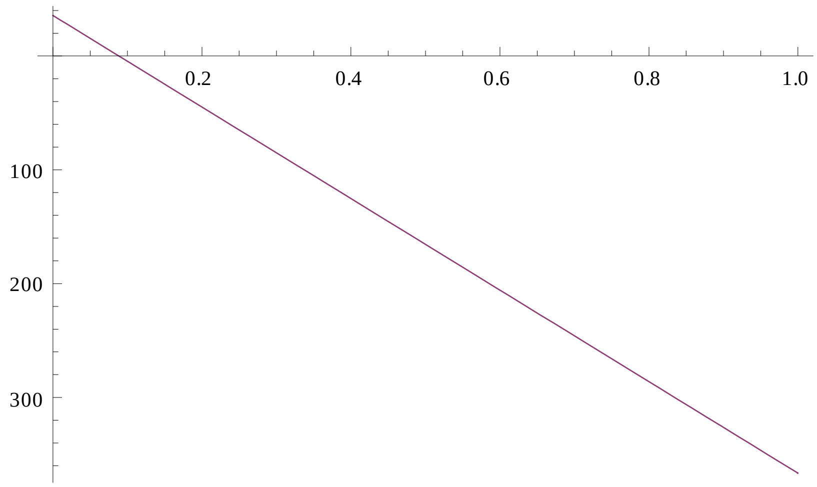


Figure **5-2(a)**: Difference in Payoffs between Small SDF and SDH Players in Mix G.

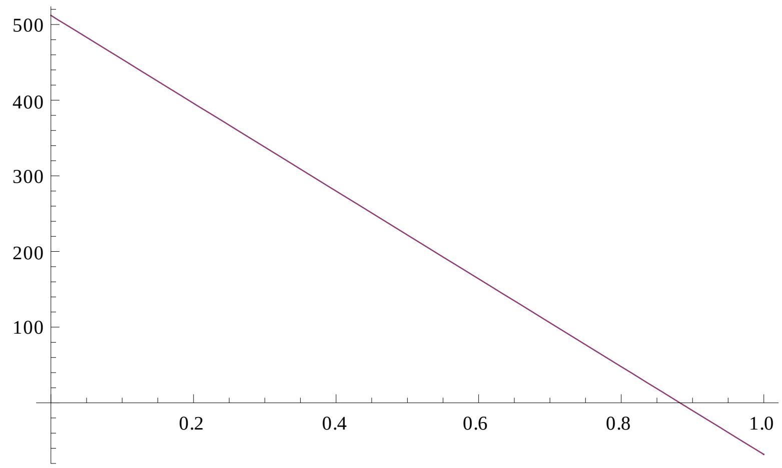


Figure **5-2(b)**: Difference in Payoffs between Medium SDF and SDH Players in Mix G.

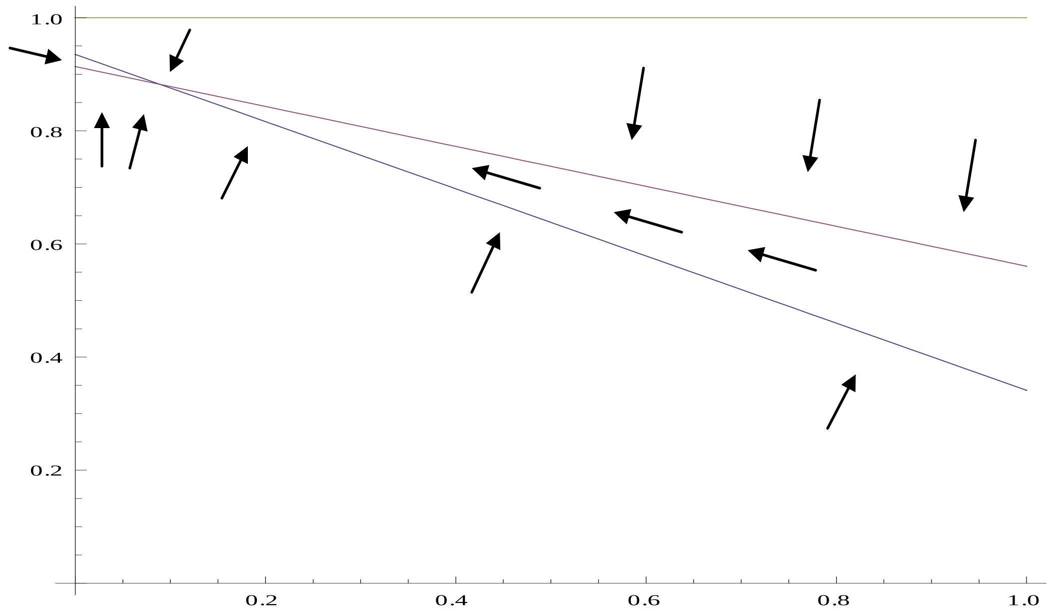


Figure **5-2(c)**: Equilibria of the Small and Medium Players in Mix G.

This process was repeated for a model that does not allow a player's belief to impact his or her performance, referred to as the original model. Again there was not a pure ESS. Table 5-4 provides the payoffs for the three strategies which were non-dominated in the total triple payoff matrix. These strategies were evaluated for equilibria – the Mathematica calculations may be reviewed in Appendix E.

Table **5-4**: Original Model Non-dominated Strategies.

	FFF	HHF	HFF
FFF	1529.14	5273.71	2639.97
HHF	3134.98	4741.41	3372.62
HFF	2451.98	5140.57	3211.05

Table 5-5 lists the resulting equilibria. After inspecting the payoffs for stability, Mix B is determined to be the only possible stable strategy. In this model, both small and large players have pure strategies. The small player is pure SDH and the large player is pure SDF. Therefore only the mixed strategy of the medium player is examined for stability. Figure 5-3 shows that perturbations of the mixed strategy will lead to a return to the equilibrium.

Table **5-5**: Original Model Equilibrium Strategies.

Mix A	FFF	HHF
	0.2490	0.7510

Mix B	HHF	HFF
	0.2881	0.7119

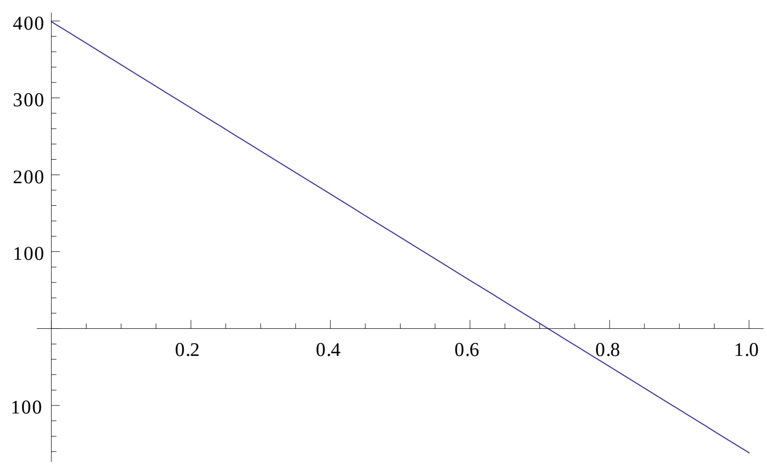


Figure **5-3**: Original Model Difference in Payoffs between Medium SDF and SDH Players.

The evolutionarily stable strategy for this comparative model is 100% SDH for the small player, 71% SDF and 29% SDH for the medium player, and 100% SDF for the large player. Since both models are observed over the full range of parameter values, both of these ESS results are global attractors. This fact will be very instrumental in making predictions about the population composition.

In the next and final chapter these results are discussed, along with their implications. This study is summarized, and a few issues are highlighted which remain open for future work.

Chapter 6

CONCLUSION

Section 6.1 Discussion

The tables for experiment one in Appendix B show that for all belief levels and all values of λ, the fitness values of non self-deceiving players are strictly dominated. This means that non self-deceiving players again will be eliminated from the evolutionarily stable strategies. The figures for this experiment also show that the NSD type is always dominated, in these cases by the SDF type.

A few additional trends are evident in the fitness and ESS values provided in Appendix B. As λ increases, the SDF strategy becomes more dominant. This trend is true for all belief levels. Furthermore, as one might expect, player 1's overall fitness increases as his or her belief level increases. Conversely, player 1's overall fitness decreases as the opponent's belief level increases. These trends occur almost monotonically across all parameters, with the exception of a few irregularities at $\lambda = 0.75$. At this high level of susceptibility, the irregularities may be the result of a player either engaging in or restraining from fights which otherwise should have been surrendered or engaged respectively. Finally, there is very little change in the fitness of NSD and SDH players as the belief levels (of either player) increase from 0.75 to 1. This may hint at an approaching ceiling to the effect belief can have on these two player types.

In experiment two, the equilibria in both the belief influenced and belief neutral models were found to be stable and were confirmed as the evolutionarily stable strategies. From these two models follows a discussion of the effect of belief on performance. Whereas without beliefs affecting performance small players would all be SDH, 9% of small players become SDF with belief effects. This phenomenon provides evidence in support of the effectiveness of the Napoleon complex and small dog behavior. Medium players also experience an increase in SDF types in the model influenced by belief. Large players, however, do not appear to be much affected by the inclusion or exclusion of belief in the self-deception model.

Of final note, Byrne and Kurland's implicit denial of interdependence between type and size is not necessarily valid. The results indicate that the evolutionarily stable strategies are sensitive to the size of the player. Large players are comprised entirely of SDF types in the ESS while small and medium players have a mixed ESS of SDF and SDH types.

Section 6.2 Summary

Byrne and Kurland demonstrated that self-deception could be fitness enhancing if it enables one to better deceive an opponent into not competing for a resource [Byrne and Kurland 2001]. This thesis extension of that work examines the relationship between beliefs and performance. It specifically studies the effect of belief on one's actual performance in competition.

Two major assumptions form the foundation of the model. The first states that belief in victory enhances one's performance and subsequently one's probability of victory. The second assumption states that one's capacity to believe in victory is limited by past experiences of defeat. Based on these assumptions, evolutionary game theory is utilized to model and analyze the relationship between a player's belief in victory and the final outcome of a competitive encounter. Given its inherent ability to adapt to chaotic dynamics necessary for future study, simulation is employed to connect between Byrne and Kurland's continuous model with future discrete models.

This study sought first to answer whether self-deception remained fitness-enhancing as indicated in the original work of Byrne and Kurland, as self-deception now has the potential to both positively and negatively affect one's performance. It showed that notwithstanding the additional aspects of the present model, non self-deceiving players are still eliminated from the population in the equilibrium strategies. Thus, self-deception remains fitness-enhancing.

Secondly, this study addressed the issue of whether there would be size sensitivity in the results. To this end the evolutionarily stable strategies of players small, medium, and large in stature were examined. The results showed that there is indeed size sensitivity when considering the distribution of opponents from each player size. Small players will be comprised of 91% SDH types and 9% SDF types. The majority of medium size players will be SDF types, at 88%, and 12% SDH types. Large players are comprised entirely of SDF types, making it their pure ESS.

Finally, in order to formulate a conclusion about the effect of belief on performance, these results are compared to the evolutionarily stable strategies of a model in which belief has no influence on performance. For this model, both the small and large players have a pure ESS. The small players will be made up completely of SDH types, and the large players again are comprised of all SDF types. The medium players have a decreased SDF strategy of 71% and an increased SDH strategy of 29%.

In summary, the incorporation of the effect of belief in a self-deception model effectively strengthens the strategy of SDF. One will be more successful in fitness competitions if he or she believes in victory. The SDF strategy is the beneficiary of belief as the present model considers performance. An analogous scenario for the SDH strategy would be whether a devalued resource is actually worth less if it is received.

Section 6.3 Future Work

Recall that the first major assumption of this study is that one's capacity for belief in victory is influenced by past experiences. If a player loses a fight he thought he would win, he is more likely to be skeptical about his chances of victory in the future. Conversely, winning a fight one thought she would lose may encourage her to believe more in the future. The present model fixes the players' belief capacities at static levels. Furthermore, it is memory less in that there is no mechanism for tracking the encounters of the players.

Future work will examine the nature of the limit that experience places on belief. Two adjustments will be made to the present model in order to facilitate this study. First, a history-dependent model will be developed. From such a model an observation of the effect of past experiences on a player's belief will be possible. Secondly the model will be dynamic, enabling the belief capacities of the players to increase and decrease as they experience victory and defeat respectively.

Perhaps several factors determine the effect personal history will have on a player's belief. Primarily, belief is effected by the accuracy of one's past predictions of victory or defeat. Secondly, belief is effected by the magnitude of past wins or losses. A player might accurately predict a loss, but not lose as badly as he thought he would. Or, perhaps a player accurately predicted a win, but won barely. Both scenarios would influence belief level. As a final note, it is presumed that early and recent experiences form lasting impressions upon a player. First impressions are commonly hard to forget. Additionally, what occurred last week for the most part is more prominent than what occurred last year. All of these factors of past experiences affect one's capacity to believe in victory.

Employing these assumptions, several issues will be addressed. It would seem that below a certain belief level a player with a low capacity for belief in victory would cease to engage in competitive encounters.

Considering past experiences of defeat, it would also seem that there is an upper limit to a player's ability to believe in victory. The first issue addresses whether there is ultimately an upper and/or lower bound to a player's capacity for belief in victory. A belief level may be reached where additional losses no longer decrease a player's capacity for belief. Additionally, the possibility of an optimal level of overconfidence is examined. Pessimism is classified as negative overconfidence, realism as zero overconfidence, and optimism as positive overconfidence. Finally, given a history of persistent defeat, what is required for one to return to any optimal overconfidence level? This final question searches for an optimal path of victorious experiences which will lead a player to recovery from a low belief capacity.

Appendix A

VARIABLES AND PARAMETERS

$P_{\sigma,\tau}$ = proportion of type σ players in the population in generation τ (1a)

P_{τ} = ($P_{NSD,\tau}$, $P_{SDF,\tau}$, $P_{SDH,\tau}$) (1b)

$\pi_{\sigma}(P_{\tau})$ = the payoff to type s in generation t (1c)

$\pi_{ave}(P_{\tau}) = P_{NSD,\tau}\pi_{NSD}(P_{\tau}) + P_{SDF,\tau}\pi_{SDF}(P_{\tau}) + P_{SDH,\tau}\pi_{SDH}(P_{\tau})$ (1d)

$P_{\sigma,t+1} = P_{\sigma,t}\pi_{\sigma}(P_{\tau}) / \pi_{ave}(P_{\tau})$ (1e)

$\mathbf{s_1}$ = player 1's size (3a)

$\mathbf{s_2}$ = player 2's size (3b)

α_1 = $\mathbf{C_2}$ = baseline probability of victory for player 1 (4a)

α_2 = $\mathbf{C_1}$ = baseline probability of victory for player 2 (4b)

β_1 = player 1's belief in victory (5a)

β_2 = player 2's belief in victory (5b)

χ_1 = current belief capacity of player 1 (6a)

χ_2 = current belief capacity of player 2 (6b)

f = weight placed on belief (7)

$$\mathbf{C_1} = \frac{1}{2} + \frac{(s_2 - s_1)}{2} \quad (8a)$$

$$\mathbf{C_2} = \frac{1}{2} + \frac{(s_1 - s_2)}{2} \quad (8b)$$

$$\gamma_1 = \frac{f\beta_1 + (1 - f)\alpha_1}{f\beta_1 + (1 - f)\alpha_1 + f\beta_2 + (1 - f)\alpha_2} \quad (9a)$$

$$\gamma_2 = \frac{f\beta_2 + (1 - f)\alpha_2}{f\beta_1 + (1 - f)\alpha_1 + f\beta_2 + (1 - f)\alpha_2} \quad (9b)$$

$\mathbf{C_{1a}}$ **=** max(C_1, 1- χ_1)

ε = cognitive resolution parameter

λ = susceptibility parameter

Appendix B

FITNESS VALUE AND ESS RESULTS

Lambda = 0.2 ***Epsilon = 0.1*** ***S_1 = 0.5***

	X2 = 0			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	2.35	2.35	2.57	
SDF	2.49	2.35	2.64	0.93
SDH	2.43	2.36	2.50	0.07

	X2 = 0.25			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	1.02	1.06	1.75	
SDF	0.86	1.05	1.73	
SDH	1.21	1.11	1.79	1

	X2 = 0			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	3.26	2.78	3.78	
SDF	3.74	3.64	3.89	1
SDH	3.05	2.96	3.21	

	X2 = 0.25			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	1.57	1.24	2.55	
SDF	2.37	1.06	3.07	0.74
SDH	1.88	1.36	2.22	0.26

	X2 = 0			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	4.31	4.18	4.43	
SDF	4.44	4.32	4.54	1
SDH	3.89	3.78	4.00	

	X2 = 0.25			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	2.50	1.74	3.52	
SDF	2.94	2.16	3.95	1
SDH	2.29	1.67	3.02	

	X2 = 0			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	4.38	4.25	4.49	
SDF	4.49	4.38	4.59	1
SDH	4.02	3.90	4.12	

	X2 = 0.25			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	2.67	1.87	3.65	
SDF	3.10	2.35	4.03	1
SDH	2.52	1.88	3.25	

	X2 = 0			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	4.38	4.25	4.49	
SDF	4.49	4.38	4.59	1
SDH	4.02	3.90	4.12	

	X2 = 0.25			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	2.67	1.87	3.65	
SDF	3.11	2.42	4.03	1
SDH	2.52	1.88	3.25	

Appendix B

FITNESS VALUE AND ESS RESULTS

	X2 = 0.5			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	0.52	0.40	0.95	
SDF	0.48	0.37	0.92	
SDH	0.57	0.46	1.00	1

	X2 = 0.75			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	445.13	347.59	822.84	
SDF	412.07	316.00	791.25	
SDH	506.38	407.36	882.61	1

	X2 = 0.5			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	0.42	0.20	0.92	
SDF	0.88	0.00	1.66	0.28
SDH	0.96	0.53	1.45	0.72

	X2 = 0.75			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	0.50	0.29	0.94	
SDF	0.78	0.00	1.33	
SDH	1.02	0.64	1.42	1

	X2 = 0.5			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	0.78	0.21	1.41	
SDF	1.45	0.00	2.26	0.65
SDH	1.07	0.45	1.42	0.35

	X2 = 0.75			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	0.93	0.31	1.45	
SDF	1.39	0.00	2.12	0.47
SDH	1.22	0.65	1.54	0.53

	X2 = 0.5			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	0.94	0.32	1.54	
SDF	1.58	0.28	2.34	0.79
SDH	1.17	0.49	1.58	0.21

	X2 = 0.75			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	0.81	0.12	1.30	
SDF	1.26	0.00	1.96	0.59
SDH	1.04	0.37	1.42	0.41

	X2 = 0.5			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	0.94	0.32	1.54	
SDF	1.63	0.45	2.35	0.95
SDH	1.17	0.49	1.58	0.05

	X2 = 0.75			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	0.76	0.07	1.25	
SDF	1.29	0.12	1.95	0.75
SDH	1.00	0.32	1.37	0.25

Appendix B

FITNESS VALUE AND ESS RESULTS

	X2 = 1			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	445.09	347.57	822.773	
SDF	412.05	316.00	791.2071	
SDH	506.36	407.36	882.5659	1

	X2 = 1			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	0.57	0.35	1.0082	
SDF	0.85	0.00	1.3938	
SDH	1.09	0.71	1.4915	1

	X2 = 1			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	1.10	0.42	1.6167	
SDF	1.56	0.00	2.29	0.42
SDH	1.39	0.81	1.70	0.58

	X2 = 1			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	0.98	0.21	1.467	
SDF	1.43	0.00	2.13	0.52
SDH	1.21	0.50	1.59	0.48

	X2 = 1			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	0.81	0.04	1.2962	
SDF	1.34	0.00	2.00	0.64
SDH	1.04	0.33	1.42	0.36

Appendix B

FITNESS VALUE AND ESS RESULTS

Lambda = 0.45 *Epsilon = 0.1* *S_1 = 0.5*

	X2 = 0			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	2.35	2.35	2.57	
SDF	2.49	2.35	2.64	0.93
SDH	2.43	2.36	2.50	0.07

	X2 = 0.25			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	0.57	1.04	1.72	
SDF	0.50	1.04	2.04	0.73
SDH	0.91	1.11	1.86	0.27

	X2 = 0			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	2.55	2.01	3.26	
SDF	3.65	3.65	3.89	1
SDH	2.97	2.92	3.14	

	X2 = 0.25			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	1.06	1.29	2.36	
SDF	2.73	1.06	3.07	0.74
SDH	2.07	1.36	2.22	0.26

	X2 = 0			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	4.17	3.97	4.53	
SDF	4.34	4.31	4.54	1
SDH	3.80	3.76	4.00	

	X2 = 0.25			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	2.00	1.17	3.45	
SDF	2.78	2.21	3.95	1
SDH	2.09	1.67	2.65	

	X2 = 0			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	4.27	4.05	4.58	
SDF	4.43	4.36	4.59	1
SDH	3.99	3.92	4.15	

	X2 = 0.25			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	2.37	1.56	3.60	
SDF	3.00	2.46	4.03	1
SDH	2.41	1.88	2.94	

	X2 = 0			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	4.27	4.05	4.58	
SDF	4.43	4.36	4.59	1
SDH	3.99	3.92	4.15	

	X2 = 0.25			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	2.37	1.56	3.60	
SDF	3.00	2.60	4.03	1
SDH	2.41	1.88	2.94	

Appendix B

FITNESS VALUE AND ESS RESULTS

	X2 = 0.5			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	0.29	0.35	0.89	
SDF	0.27	0.39	0.95	
SDH	0.46	0.46	1.00	1

	X2 = 0.75			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	218.13	291.64	729.73	
SDF	194.37	329.57	787.32	
SDH	408.73	407.36	845.86	1

	X2 = 0.5			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	0.03	0.16	0.86	
SDF	1.05	0.00	2.71	0.59
SDH	1.07	0.57	1.88	0.41

	X2 = 0.75			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	0.27	0.31	0.91	
SDF	1.23	0.00	2.50	0.47
SDH	1.18	0.74	1.84	0.53

	X2 = 0.5			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	0.57	0.17	1.19	
SDF	1.81	0.08	2.29	0.71
SDH	1.12	0.45	1.37	0.29

	X2 = 0.75			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	0.61	0.13	1.08	
SDF	1.66	0.00	2.26	0.64
SDH	1.09	0.50	1.38	0.36

	X2 = 0.5			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	0.87	0.39	1.37	
SDF	1.94	0.43	2.39	0.87
SDH	1.32	0.56	1.51	0.13

	X2 = 0.75			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	0.76	0.18	1.11	
SDF	1.65	0.19	2.22	0.77
SDH	1.13	0.45	1.37	0.23

	X2 = 0.5			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	0.87	0.39	1.37	
SDF	1.97	0.60	2.39	1
SDH	1.32	0.56	1.51	

	X2 = 0.75			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	0.76	0.18	1.11	
SDF	1.69	0.36	2.22	0.91
SDH	1.13	0.45	1.37	0.09

Appendix B

FITNESS VALUE AND ESS RESULTS

	X2 = 1			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	217.37	290.98	727.46	
SDF	194.28	327.97	785.72	
SDH	408.63	407.36	844.26	1

	X2 = 1			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	0.41	0.45	1.05	
SDF	1.37	0.00	2.64	0.43
SDH	1.32	0.89	1.98	0.57

	X2 = 1			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	0.78	0.27	1.25	
SDF	1.83	0.00	2.43	0.57
SDH	1.26	0.67	1.55	0.43

	X2 = 1			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	0.76	0.15	1.11	
SDF	1.65	0.03	2.22	0.67
SDH	1.13	0.44	1.37	0.33

	X2 = 1			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	0.76	0.15	1.11	
SDF	1.69	0.19	2.22	0.78
SDH	1.13	0.44	1.37	0.22

Appendix B

FITNESS VALUE AND ESS RESULTS

Lambda = 0.7 ***Epsilon = 0.1*** ***S_1 = 0.5***

	X2 = 0			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	2.35	2.35	2.57	
SDF	2.49	2.35	2.64	0.93
SDH	2.43	2.36	2.50	0.07

	X2 = 0.25			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	0.33	1.04	1.72	
SDF	0.53	1.04	2.08	0.71
SDH	0.46	1.11	1.91	0.29

	X2 = 0			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	2.04	2.00	2.22	
SDF	3.65	3.65	3.89	1
SDH	2.97	2.92	3.09	

	X2 = 0.25			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	1.06	1.33	2.22	
SDF	2.98	1.06	3.07	0.74
SDH	2.22	1.36	2.22	0.26

	X2 = 0			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	3.77	3.34	4.30	
SDF	4.34	4.33	4.54	1
SDH	3.80	3.73	3.97	

	X2 = 0.25			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	1.54	0.87	3.12	
SDF	2.60	2.22	3.55	1
SDH	1.90	1.67	2.65	

	X2 = 0			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	4.00	3.68	4.39	
SDF	4.42	4.40	4.59	1
SDH	4.00	3.91	4.14	

	X2 = 0.25			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	2.11	1.43	3.30	
SDF	2.91	2.45	3.77	1
SDH	2.34	1.86	2.91	

	X2 = 0			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	4.02	3.71	4.39	
SDF	4.43	4.42	4.59	1
SDH	4.03	3.93	4.16	

	X2 = 0.25			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	2.11	1.43	3.30	
SDF	2.91	2.55	3.77	1
SDH	2.34	1.86	2.91	

Appendix B

FITNESS VALUE AND ESS RESULTS

	X2 = 0.5			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	0.09	0.45	0.99	
SDF	0.00	0.46	1.32	0.66
SDH	0.50	0.56	1.13	0.34

	X2 = 0.75			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	0.02	0.32	0.75	
SDF	0.00	0.35	1.04	0.54
SDH	0.37	0.46	0.91	0.46

	X2 = 0.5			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	0.00	0.38	1.07	
SDF	1.57	1.39	3.17	1
SDH	1.49	1.28	2.19	

	X2 = 0.75			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	0.00	0.26	0.83	
SDF	1.69	1.02	2.97	0.96
SDH	1.30	1.06	1.92	0.04

	X2 = 0.5			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	0.40	0.11	1.1147	
SDF	1.91	0.50	2.18	0.93
SDH	1.11	0.56	1.37	0.07

	X2 = 0.75			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	0.42	0.00	0.92	
SDF	1.80	0.57	2.13	1
SDH	1.01	0.55	1.28	

	X2 = 0.5			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	0.81	0.42	1.2772	
SDF	2.09	0.79	2.238	1
SDH	1.38	0.67	1.5105	

	X2 = 0.75			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	0.77	0.26	1.03	
SDF	1.92	0.81	2.13	1
SDH	1.22	0.61	1.37	

	X2 = 0.5			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	0.81	0.42	1.28	
SDF	2.11	0.89	2.24	1
SDH	1.38	0.67	1.51	

	X2 = 0.75			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	0.77	0.26	1.03	
SDF	1.92	0.88	2.13	1
SDH	1.22	0.61	1.37	

Appendix B

FITNESS VALUE AND ESS RESULTS

	X2 = 1			
X1 = 0	NSD	SDF	SDH	**ESS**
NSD	0.02	0.34	0.74	
SDF	0.00	0.35	1.05	0.5
SDH	0.39	0.49	0.91	0.5

	X2 = 1			
X1 = 0.25	NSD	SDF	SDH	**ESS**
NSD	0.00	0.26	0.83	
SDF	1.69	0.92	2.97	0.88
SDH	1.30	1.06	1.92	0.12

	X2 = 1			
X1 = 0.5	NSD	SDF	SDH	**ESS**
NSD	0.43	0.00	0.94	
SDF	1.82	0.48	2.14	0.91
SDH	1.02	0.57	1.30	0.09

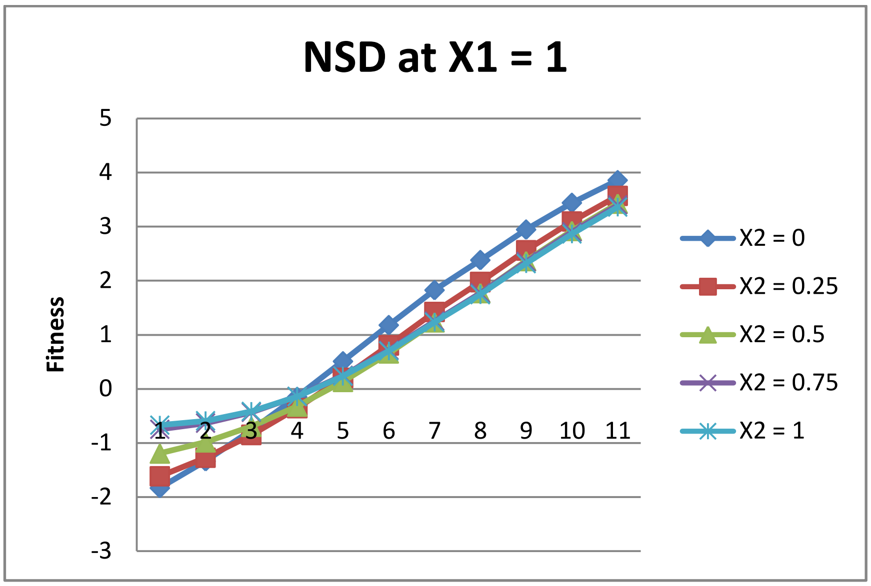
	X2 = 1			
X1 = 0.75	NSD	SDF	SDH	**ESS**
NSD	0.77	0.26	1.03	
SDF	1.92	0.73	2.13	1
SDH	1.22	0.61	1.37	

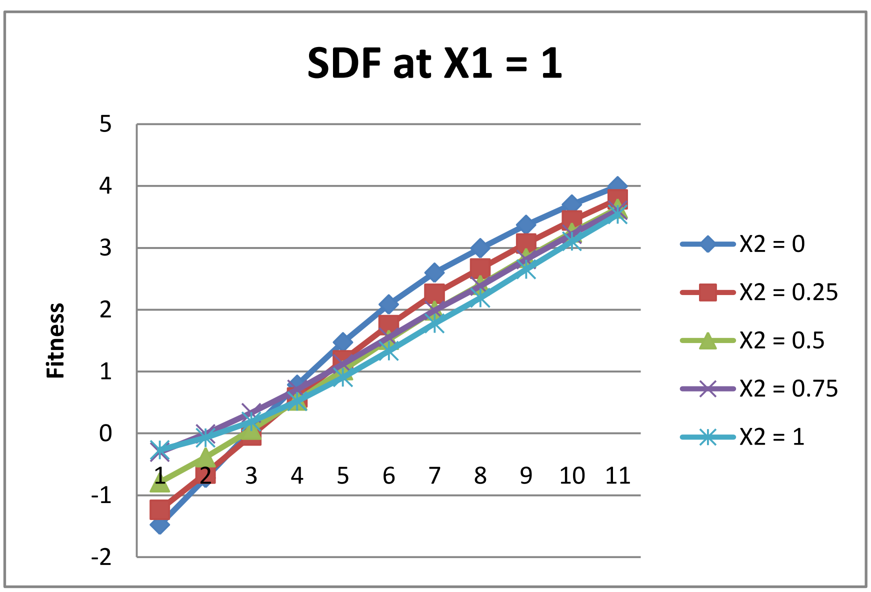
	X2 = 1			
X1 = 1	NSD	SDF	SDH	**ESS**
NSD	0.77	0.26	1.03	
SDF	1.92	0.81	2.13	1
SDH	1.22	0.61	1.37	

Appendix C

PRELIMINARY SIZE RESULTS

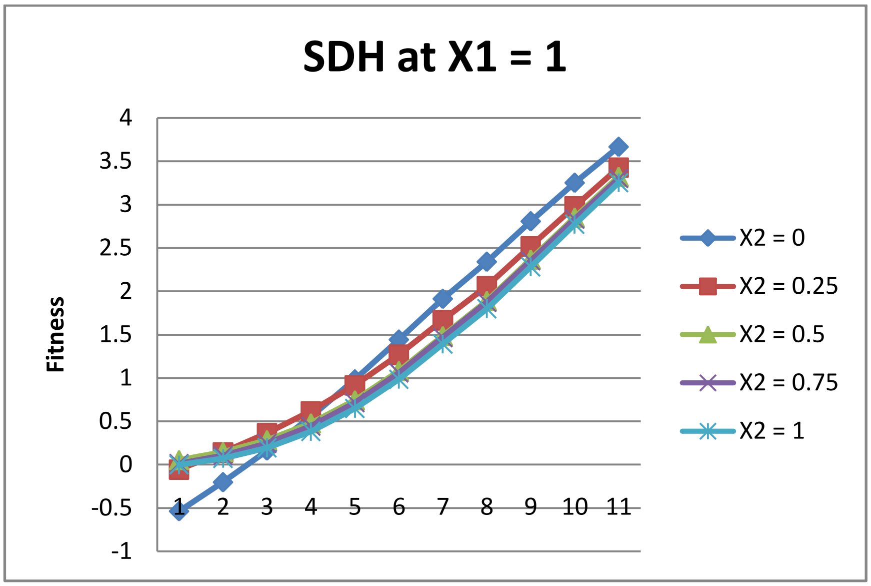
These graphs display fitness values on the y-axis and the eleven original player one sizes on the x-axis, which ranged from 0.05 to 0.94 in increments of 0.09. Note that for the most part the trends are the same for three categories of players: smaller, average, and larger.





Appendix C

PRELIMINARY SIZE RESULTS



Appendix D

MATHEMATICA EQUILIBRIA CALCULATIONS

```
p={{1541.36, 5095.52, 4311.23, 5755.18, 2985.31},
   {1713.55, 4157.67, 3868.21, 5114.68, 2960.02},
   {1787.60, 4551.42, 3997.06, 5243.13, 3033.67},
   {2154.17, 4360.51, 3807.16, 4722.82, 3069.83},
   {1907.93, 4904.60, 4121.32, 5234.87, 3021.47}};

MatrixForm[p]
( {
  {1541.36, 5095.52, 4311.23, 5755.18, 2985.31},
  {1713.55, 4157.67, 3868.21, 5114.68, 2960.02},
  {1787.6, 4551.42, 3997.06, 5243.13, 3033.67},
  {2154.17, 4360.51, 3807.16, 4722.82, 3069.83},
  {1907.93, 4904.6, 4121.32, 5234.87, 3021.47}
 } )

(* mixA *)
Solve[x1 p[[1,1]]+x2 p[[1,2]]==x1 p[[2,1]]+x2 p[[2,2]]&&x1+x2□1]
{{x1→0.844879,x2→0.155121}}

(* mixB *)
Solve[x1 p[[1,1]]+x3 p[[1,3]]==x1 p[[3,1]]+x3 p[[3,3]]&&x1+x3□1]
{{x1→0.560607,x3→0.439393}}

(* mixC *)
Solve[x1 p[[1,1]]+x4 p[[1,4]]==x1 p[[4,1]]+x4 p[[4,4]]&&x1+x4□1]
{{x1→0.62751,x4→0.37249}}

(* mixD *)
Solve[x2 p[[2,2]]+x4 p[[2,4]]==x2 p[[4,2]]+x4 p[[4,4]]&&x2+x4□1]
{{x2→0.65892,x4→0.34108}}

(* mixE *)
Solve[x3 p[[3,3]]+x5 p[[3,5]]==x3 p[[5,3]]+x5 p[[5,5]]&&x3+x5□1]
{{x3→0.0894035,x5→0.910597}}

(* mixF *)
Solve[x4 p[[4,4]]+x5 p[[4,5]]==x4 p[[5,4]]+x5 p[[5,5]]&&x4+x5□1]
{{x4→0.086294,x5→0.913706}}
```

```
(* mix123 *)
Solve[x1 p[[1,1]]+x2 p[[1,2]]+x3 p[[1,3]]==x1 p[[2,1]]+x2 p[[2,2]]+x3 p[[2,3]]==x1
p[[3,1]]+x2 p[[3,2]]+x3 p[[3,3]]&&x1+x2+x3□1]
{{x1→0.394525,x2→-0.404794,x3→1.01027}}

(* mix124 *)
Solve[x1 p[[1,1]]+x2 p[[1,2]]+x4 p[[1,4]]==x1 p[[2,1]]+x2 p[[2,2]]+x4 p[[2,4]]==x1
p[[4,1]]+x2 p[[4,2]]+x4 p[[4,4]]&&x1+x2+x4□1]
{{x1→0.680616,x2→-0.293828,x4→0.613212}}

(* mix125 *)
Solve[x1 p[[1,1]]+x2 p[[1,2]]+x5 p[[1,5]]==x1 p[[2,1]]+x2 p[[2,2]]+x5 p[[2,5]]==x1
p[[5,1]]+x2 p[[5,2]]+x5 p[[5,5]]&&x1+x2+x5□1]
{{x1→-0.150934,x2→-0.0603757,x5→1.21131}}

(* mix134 *)
Solve[x1 p[[1,1]]+x4 p[[1,4]]+x3 p[[1,3]]==x1 p[[4,1]]+x4 p[[4,4]]+x3 p[[4,3]]==x1
p[[3,1]]+x4 p[[3,4]]+x3 p[[3,3]]&&x1+x4+x3□1]
{{x1→0.882436,x3→-0.793876,x4→0.91144}}

(* mix135 *)
Solve[x1 p[[1,1]]+x5 p[[1,5]]+x3 p[[1,3]]==x1 p[[5,1]]+x5 p[[5,5]]+x3 p[[5,3]]==x1
p[[3,1]]+x5 p[[3,5]]+x3 p[[3,3]]&&x1+x5+x3□1]
{{x1→-0.028999,x3→0.117567,x5→0.911432}}

(* mixG *)
Solve[x1 p[[1,1]]+x4 p[[1,4]]+x5 p[[1,5]]==x1 p[[4,1]]+x4 p[[4,4]]+x5 p[[4,5]]==x1
p[[5,1]]+x4 p[[5,4]]+x5 p[[5,5]]&&x1+x4+x5□1]
{{x1→0.0885595,x4→0.117564,x5→0.793876}}

(* mix235 *)
Solve[x5 p[[5,5]]+x2 p[[5,3]]+x3 p[[5,3]]==x5 p[[2,5]]+x2 p[[2,2]]+x3 p[[2,3]]==x5
p[[3,5]]+x2 p[[3,2]]+x3 p[[3,3]]&&x5+x2+x3□1]
{{x2→-0.160658,x3→-0.4026,x5→1.56326}}

(* mix245 *)
Solve[x4 p[[4,4]]+x2 p[[4,2]]+x5 p[[4,5]]==x4 p[[2,4]]+x2 p[[2,2]]+x5 p[[2,5]]==x4
p[[5,4]]+x2 p[[5,2]]+x5 p[[5,5]]&&x4+x2+x5□1]
{{x2→-0.106707,x4→0.199101,x5→0.907605}}

(* mixH *)
Solve[x4 p[[4,4]]+x5 p[[4,5]]+x3 p[[4,3]]==x4 p[[5,4]]+x5 p[[5,5]]+x3 p[[5,3]]==x4
p[[3,4]]+x5 p[[3,5]]+x3 p[[3,3]]&&x4+x5+x3□1]
{{x3→0.0885661,x4→0.029002,x5→0.882432}}
```

(* mix1234 *)
Solve[x1 p[[1,1]]+x2 p[[1,2]]+x3 p[[1,3]]+x4 p[[1,4]]==x1 p[[2,1]]+x2 p[[2,2]]+x3 p[[2,3]]+x4 p[[2,4]]==x1 p[[3,1]]+x2 p[[3,2]]+x3 p[[3,3]]+x4 p[[3,4]]□x1 p[[4,1]]+x2 p[[4,2]]+x3 p[[4,3]]+x4 p[[4,4]]&&x1+x2+x3+x4□1]
{{x1→0.495173,x2→-0.38358,x3→0.628014,x4→0.260393}}

(* mix 1235 *)
Solve[x1 p[[1,1]]+x2 p[[1,2]]+x3 p[[1,3]]+x5 p[[1,5]]==x1 p[[2,1]]+x2 p[[2,2]]+x3 p[[2,3]]+x5 p[[2,5]]==x1 p[[3,1]]+x2 p[[3,2]]+x3 p[[3,3]]+x5 p[[3,5]]□x1 p[[5,1]]+x2 p[[5,2]]+x3 p[[5,3]]+x5 p[[5,5]]&&x1+x2+x3+x5□1]
{{x1→0.234807,x2→-0.383584,x3→0.888427,x5→0.260349}}

(* mix1245 *)
Solve[x1 p[[1,1]]+x2 p[[1,2]]+x5 p[[1,5]]+x4 p[[1,4]]==x1 p[[2,1]]+x2 p[[2,2]]+x5 p[[2,5]]+x4 p[[2,4]]==x1 p[[5,1]]+x2 p[[5,2]]+x5 p[[5,5]]+x4 p[[5,4]]□x1 p[[4,1]]+x2 p[[4,2]]+x5 p[[4,5]]+x4 p[[4,4]]&&x1+x2+x5+x4□1]
{{x1→1.12306,x2→-0.383554,x4→0.888329,x5→-0.627835}}

(* mixI *)
Solve[x1 p[[1,1]]+x5 p[[1,5]]+x3 p[[1,3]]+x4 p[[1,4]]==x1 p[[5,1]]+x5 p[[5,5]]+x3 p[[5,3]]+x4 p[[5,4]]==x1 p[[3,1]]+x5 p[[3,5]]+x3 p[[3,3]]+x4 p[[3,4]]□x1 p[[4,1]]+x5 p[[4,5]]+x3 p[[4,3]]+x4 p[[4,4]]&&x1+x5+x3+x4□1]
{{x1→0.0885595,x3→2.0305×10^{-23},x4→0.117564,x5→0.793876}}

(* mix2345 *)
Solve[x5 p[[5,5]]+x2 p[[5,2]]+x3 p[[5,3]]+x4 p[[5,4]]==x5 p[[2,5]]+x2 p[[2,2]]+x3 p[[2,3]]+x4 p[[2,4]]==x5 p[[3,5]]+x2 p[[3,2]]+x3 p[[3,3]]+x4 p[[3,4]]□x5 p[[4,5]]+x2 p[[4,2]]+x3 p[[4,3]]+x4 p[[4,4]]&&x5+x2+x3+x4□1]
{{x2→-0.383589,x3→1.12327,x4→-0.234808,x5→0.495132}}

(* mix12345 *)
Solve[x1 p[[1,1]]+x2 p[[1,2]]+x3 p[[1,3]]+x4 p[[1,4]]+x5 p[[1,5]]==x1 p[[2,1]]+x2 p[[2,2]]+x3 p[[2,3]]+x4 p[[2,4]]+x5 p[[2,5]]==x1 p[[3,1]]+x2 p[[3,2]]+x3 p[[3,3]]+x4 p[[3,4]]+x5 p[[3,5]]□x1 p[[4,1]]+x2 p[[4,2]]+x3 p[[4,3]]+x4 p[[4,4]]+x5 p[[4,5]]□x1 p[[5,1]]+x2 p[[5,2]]+x3 p[[5,3]]+x4 p[[5,4]]+x5 p[[5,5]]&&x1+x2+x3+x4+x5□1]
{{x1→0.739607,x2→-0.38358,x3→0.38358,x4→0.504827,x5→-0.244434}}

Appendix E

MATHEMATICA ORIGINAL MODEL EQUILIBRIA CALCULATIONS

```
p={{1529.14, 5273.71, 2639.97},
  {3134.98, 4741.41, 3372.62},
  {2451.98, 5140.57, 3211.05}};

MatrixForm[p]
( {
 {1529.14, 5273.71, 2639.97},
 {3134.98, 4741.41, 3372.62},
 {2451.98, 5140.57, 3211.05}
} )

(* mixA *)
Solve[x1 p[[1,1]]+x2 p[[1,2]]==x1 p[[2,1]]+x2 p[[2,2]]&&x1+x2□1]
{{x1→0.248955,x2→0.751045}}

(* mixB *)
Solve[x2 p[[2,2]]+x3 p[[2,3]]==x2 p[[3,2]]+x3 p[[3,3]]&&x2+x3□1]
{{x2→0.288142,x3→0.711858}}

(* mix123 *)
Solve[x1 p[[1,1]]+x2 p[[1,2]]+x3 p[[1,3]]==x1 p[[2,1]]+x2 p[[2,2]]+x3 p[[2,3]]==x1
p[[3,1]]+x2 p[[3,2]]+x3 p[[3,3]]&&x1+x2+x3□1]
                    {{x1→1.21465,x2→1.41766,x3→-1.63231}}
```

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