USING A SELF-MANAGED INTERVENTION TO BUILD COMPUTATIONAL 
AND PROCEDURAL FLUENCY WITH COMPLEX COMPUTATION 

A Dissertation in 
Special Education 
by 
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ABSTRACT

Behavioral fluency synthesizes accuracy and speed and enables students to operate successfully in the learning environment. The current investigation tested a self-managed frequency building intervention designed to increase behavioral fluency in four, seventh grade students with three critical pre-algebra skills: order of operations, adding and subtracting fractions with uncommon denominators, and long division with and without a remainder. Elements of the frequency building intervention comprised of cue cards, timed practice, and feedback. The experimenter applied an alternating treatments design with the first experimental condition consisting of three, one-minute frequency building trials with 30 seconds of feedback provided immediately after each one-minute trial. The second condition consisted of one, three-minute frequency building trial immediately followed by 90 seconds of feedback. A baseline condition had the students participate in one-minute timed trial with no feedback. Findings indicate that all four students successfully self-managed and benefited from frequency building, especially with long division and adding and subtracting fractions.

Keywords: frequency building; mathematics fluency; pre-algebra fluency; complex computation; behavioral fluency; feedback; self-managed interventions
TABLE OF CONTENTS

List of Tables........................................................................................................... v
List of Figures......................................................................................................... vi

Chapter 1. INTRODUCTION...................................................................................... 1

Chapter 2. METHODS............................................................................................ 7

Chapter 3. RESULTS.............................................................................................. 17

Chapter 4. DISCUSSION......................................................................................... 25

Bibliography............................................................................................................. 36

Appendices
  Appendix A: Figures............................................................................................ 47
  Appendix B: Student Materials............................................................................ 50
  Appendix C: Experimenter Materials................................................................. 53
  Appendix D: Review of the Literature................................................................. 58
LIST OF TABLES

Table 1: Intervention Assignments.......................................................... 11
Table 2: Intervention Schedule............................................................ 53
Table 3: Student Performance Indicators.............................................. 19
Table 4: Level and Celeration Comparison Analysis............................ 21
Table 5: Results from Simple Computation Probes.............................. 22
Table 6: Student Examples: Order of Operations............................... 34
LIST OF FIGURES

Figure 1: Correct Digits/Symbols and Incorrect Digits/Symbols…………….47
Figure 2: Celeration Lines and Level Lines…………………………………48
Figure 3: Weekly Median Performance………………………………………49
Figure 4: Order of Operations………………………………………………..50
Figure 5: Fractions with Uncommon Denominators………………………….51
Figure 6: Long Division with and without Remainders………………………52
Figure 7: Procedural Integrity Checklist………………………………………55
Figure 8: Instructions for Students…………………………………………...56
Chapter 1

Introduction

Algebraic proficiency has become the benchmark for access to higher education, employability, and competitive wages – all components associated with a higher quality of life (Adelman, 2006; National Mathematics Advisory Panel, NMAP, 2008). Over the next decade, 62% of American jobs will require advanced mathematics skills (Hanushek, Peterson, & Woessmann, 2010) yet the United States has difficulty producing workers in key sectors of the workforce that necessitate science, technology, engineering, and mathematics (STEM) competency (Xue & Larson, 2015). Consequently, the growth and viability of a competitive economy will continue to hinge on schools graduating students ready to meet the mathematical demands of a global workforce.

On a personal level, successful daily living has traditionally required the fluent execution of basic mathematics skills such as counting money, telling time, estimation, balancing a checkbook, and other basic problem solving. However, successful living in the 21st century requires an even higher level of mathematical proficiency to adapt to new technologies and related applications as well as process increasingly complex information in matters such as finance, healthcare, and the environment (Price & Ansari, 2013). Citizens who lack essential skills in mathematics, science, and technology will have difficulty contributing to and benefiting from a rapidly evolving knowledge-based society (Atkinson & Mayo, 2010).

Despite the importance of mathematics, students in the United States who enter school with deficits in mathematics often continue to struggle or fail to reach the standards necessary to participate in high school algebra (Duncan et al., 2007). The
The sharpest decline in mathematics occurs at the middle school level. For instance, on the latest National Assessment of Educational Progress (2015), only 40% of grade four students performed at or above proficient. Forty-two percent scored at basic and 18% below basic. In grade eight, only 33% scored at or above proficient, 38% basic and 29% below basic. The performance of students with disabilities warrant further concern – by grade eight 38% scored basic and 45% below basic. Regrettably, students who fail to meet mathematics standards have a greater likelihood to fail courses, endure retention, and possibly dropout (Calhoon, Emerson, Flores, & Houchins, 2007; Duncan et al., 2007; NMAP, 2008).

The Role of Behavioral Fluency in Mathematics Education

*Fluency* refers to a behavior performed to a high level of accuracy plus speed reflected in a competent performance (Binder, 1996). Fluency evokes descriptors such as smooth, rhythmic, effortless, fluid, well-practiced, automatic, and masterful (Johnson & Layng, 1996). Fluency, or more accurately behavioral fluency, denotes the “attainment of a performance standard or fluency aim and the subsequent associated critical learning outcomes (Kubina & Yurich, 2012, p. 318).” Three examples of critical learning outcomes and descriptions include *retention* or the ability to perform the behavior after significant periods of time without practice; *endurance* or the ability maintain a behavior at a given standard over a period of time; *application* or the ability to combine two or more element behaviors to a compound behavior (Johnson & Layng, 1996; Kubina & Morrison, 2000; Kubina & Yurich, 2012; Binder, 1996). A number of fluency studies have reported the relationship between reaching a performance standard and critical learning outcomes in math (Brady & Kubina, 2010; Beverly, Hughes, & Hastings, 2009;
Bullara, Kimball, & Cooper, 1993; Chiesa & Robertson, 2000; Miller, Hall, & Heward, 1995; Stromgren, Berg-Mortensen, & Tangen, 2014).

However, some researchers subscribe to a narrower view of fluency that associates mathematical fluency to computational proficiency without considering the interconnectedness between conceptual understanding, procedural adeptness, and the automatic recall of facts (Biancarosa & Shanley, 2016; Clarke, Nelson, & Shanley, 2016). For instance, research has long indicated that computational fluency frees up the limited cognitive resources to engage in more complex mathematical tasks (Gersten, Jordan, & Flojo, 2005; LeFevre, DeStefano, Coleman, 2005; Raghubar, Barnes, & Hecht, 2010). Furthermore, students who struggle to retrieve math facts fluently also tend to work more slowly and prone to make additional errors when completing more complex tasks (Geary, 2004; Lin & Kubina, 2005; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). As a result, students who lack fluency with basic facts experience difficulty comprehending fundamental mathematical concepts or accessing problem-solving approaches (Gersten & Chard, 1999) leading to problems with procedural fluency and generalization (Fuchs, Seethaler, Powell, Fuchs, Hamlett, & Fletcher, 2008).

**Behavioral Fluency in Mathematics Education**

To learn a new concept or skill students first concentrate on acquisition and conceptual understanding, then the quality and accuracy of the response (Archer & Hughes, 2011; Ardoin & Daly, 2007; Binder, 2003; Haring & Eaton, 1978; National Council of Teachers of Mathematics, 2014). Students then subsequently engage in *frequency building* – systematic practice to build fluency – with the goal of *behavioral fluency*: the ability to apply the element behavior to a new compound behavior.
*Frequency building* does not suggest sacrificing conceptual understanding; *frequency building* operates in tandem with acquisition and conceptual understanding to reach *behavioral fluency* which allow students to successfully transition to more complex topics (Biancarosa & Shanley, 2016; Partnership for Assessment of Readiness for College and Careers, 2014, PARCC). Unfortunately, educators often move prematurely to new compound skills before performing element skills fluently (Binder, 2003). And despite the importance of providing ample opportunities to develop behavioral fluency, the quality and quantity of practice as well as the available materials within the American classroom do not effectively support fluency instruction (Daly, Martens, Barnett, Witt, & Olson, 2007; Witzel & Riccomini, 2007).

**Methods Used to Increase Mathematics Fluency**

The majority of fluency research in computation focuses on math facts (Foegen, Olson, & Impecoven-Lind, 2008; Geary, Hoard, Nugent, & Byrd-Craven, 2007). In a review of simple computation fluency interventions, Codding, Burns, and Lukito (2011) discovered interventions that incorporate practice with modeling, drill, and three or more treatment components yielded the largest effect sizes. Self-managed or student directed interventions also yielded significant effect sizes. Components of the effective interventions allowed students to review the problem and solution, receive immediate feedback, and participate in an error correcting procedure that reinforced correct responding versus recurring errors (Burns, VanDerHeyden, & Boice, 2008, Daly et al. 2007; Fuchs et al., 2008; Rivera & Bryant, 1992). The self-managed interventions that included the effectual components showed improved focus, incentive, and responsibility over their own learning with minimal teacher mediation (Hughes, Korinek, & Gorman,
1991; Mace, Belfiore, & Hutchinson, 2001; McDougall & Brady, 1998; Reid, Trout, & Schartz, 2005). Conversely, practice without a modeling component generated the smallest effect size which indicates that more intense treatment packages provide added stimulus control or sensitivity to individual responses (Codding et al. 2011).

Explicit timing procedures function as the primary tool to measure and evaluate computational and procedural fluency. Researchers and teachers typically quantify responses via digits correct per minute (DCPM) or correct problems per minute (CPPM) to assign a frequency to each performance. Frequency denotes a count over a specified time of observation yielding a detailed representation of student behavior (Johnston & Pennypacker, 2009). With behavioral fluency, frequency building signifies timed repetition of a target behavior followed by performance feedback (Kubina & Yurich, 2012). When a student reaches an established fluency criterion of an element skill through frequency building, both the student and educator can have more confidence progressing onto a compound skill (Kubina & Yurich, 2012).

Middle school students rely on the fluent execution of element skills (e.g. math facts, multi-digit addition, fractions, decimals) learned in prior grades to solve higher-level compound problems. As students move through the hierarchical structure of the mathematics curriculum, a continuous recycling occurs where element skill(s) fit into a new compound skill and in turn, the new compound can become an element within another compound skill. Students who successfully transition between element and compound skills have a distinct advantage over non-fluent students later in the high school algebra curriculum (NMAP, 2008). Order of operations, multi-digit multiplication computation, and adding and subtracting fractions with unlike denominators serve as but
a few examples of complex computation that require fluency. Still, a lack of applied research exists on interventions designed to increase behavioral fluency with complex computation.

To examine the effects of frequency building on complex computation, the experimenter posed the following questions: What effects does frequency building with a self-managed intervention have on student performance with order of operations, long division with and without remainders, and adding and subtracting fractions with unlike denominators? Also, what performance differences occur in students between a three, one-minute frequency building intervention, a one, three-minute frequency building intervention, and a baseline condition?
Chapter 2

Methods

Participants and Setting

The seventh grade mathematics teacher nominated four students experiencing mathematics difficulties and solicited parental consent. Two female students (Cara and Poppy) and two male students (John and Jono) participated in the study. All four students have received instruction in the skills examined in the present investigation. Located in a central Pennsylvania charter school, the intervention took place in a separate room next to the main office where small group instruction and staff meetings occur. The room had a long conference table where the four students and experimenter sat in the same seats for the 15 days of intervention.

Materials

Student materials (Appendix B) consisted of (a) practice sheets, (b) answer keys for feedback, (c) cue cards that outline steps to solve the corresponding skill, and (d) assessments. Experimenter materials (Appendix C) included (a) instructions, (b) procedural integrity checklists, and (c) an intervention schedule. Ancillary materials included a stopwatch, an application (Chartlytics, 2014) for recording, displaying, analyzing data, and a camera for recording student performance.

Three exclusive sets of practice sheets, corresponding answer keys, and assessments focused on either order of operations, long division with and without remainders, or adding or subtracting fractions with unlike denominators. Each practice sheet and assessment included nine problems. Below lists the decision rules for each complex computation skill to balance level of difficulty between assessments.
Order of Operations:
- 18 sets of parentheses total, 2 per problem;
- 9 exponents total, 1 per problem with products of 27 or less;
- 5-8 multiplication facts per assessment, no more than 2 per problem;
- 5-8 division facts per assessment, no more than 1 per problem;
- 5-8 addition facts per assessment, no more than 2 per problem;
- 5-8 subtraction facts per assessment, no more than 2 per problem

Long Division w/ and w/o Remainders
- All 9 problems have one digit divisors, 2-9 randomly assigned
- 1 problem with 2 digit dividend
- 3-4 problems with 3 digit dividend
- 3-4 problems with 4 digit dividend
- 4-5 problems with remainders counterbalanced

Adding or Subtracting w/ Unlike Denominators
- Common denominators occur between 4 and 81
- 5 problems have denominators with products up to 35.
- 4 problems have denominators with products up to 81
- Addition and subtraction of fractions counterbalanced
- 4 problems counterbalanced reducing and/or converting improper fractions to mixed numbers

Response Measurement and Accuracy

Dependent variables. The dependent variables include the number of correct digits and symbols per minute (CDSM) and the number of incorrect digits and symbols per minute (IDSM) a student made during each practice trial and assessment. A CDSM signifies an accurate written presentation of a number. An IDSM denotes (a) an illegible written digit or symbol, (b) an inaccurate numerical presentation, or (c) a digit or symbol separate from the procedure of reaching the solution. Each assessment contained nine problems with more digits than a student could complete. Examples of the dependent variable appear in Appendix B. Students completed three, one-minute assessments of the dependent variable per day over a period of 15 days for Baseline, a three, one-minute
frequency building intervention, and a one, three-minute frequency building intervention (described below in the Independent Variable section).

**Accuracy.** Accuracy signifies the quality to which experimental values deliver a precise account of behavior that transpired during an experiment. Accuracy delivers more information than inter-observer agreement by calculating the exact values of experimental data (Johnston & Pennypacker, 2009; Kostewicz, King, Datchuk, Brennan & Casey, 2016). In the present experiment, the lead investigator created an answer key for the assessments. The answer key verified by the lead investigator and research assistant served as the true value. The lead investigator and research assistant corrected written student responses against the answer key.

**Independent Variable and Procedural Integrity**

**Independent variable.** The independent variable included two experimental conditions that contained the same self-managed intervention but differed in the allocation of timed frequency building trials. The first intervention included three, one-minute frequency building trials. After each one-minute trial the students self-corrected their problems and solutions using an answer key for 30 seconds (i.e., a total of 90 seconds). The second intervention had one, three-minute frequency building trial. After the three-minutes of practice, students self-corrected problems and solutions using an answer key for 90 seconds. During the both frequency building interventions the students had access to a cue card that outlined the steps of the corresponding algorithm.

**Procedural integrity.** The experimenter used a procedural integrity checklist to ensure accuracy and consistency in implementation of the intervention (Appendix C). The checklist also confirmed the readiness of practice sheets, assessments, answer keys,
checklists, and proper sequence for reading instructions prior to the start of the day’s intervention. On four separate days a research assistant checked procedural integrity. Before the first day of the intervention, the research assistant received instructions to appropriately execute procedural integrity. The research assistant reviewed the materials, participated in a simulation of the intervention, and conducted a simulated procedural integrity check. Computing procedural integrity consisted of dividing the number of steps correctly executed over the total number of possible steps, then multiplying by 100 (Gast, 2010). The mean procedural integrity came to 100%.

**Experimental Design**

The study employed an alternating treatments design (Cooper, Heron, & Heward, 2007; Johnston & Pennypacker, 2009; Kazdin, 2011; Sindelar, Rosenberg, & Wilson, 1985) to evaluate and compare the effects of the two frequency building conditions and the baseline condition. The three, one-minute frequency building trial intervention, three-minute frequency building trial intervention, and Baseline Condition systematically alternated in order each day to isolate the influence of the independent variable assigned to the different conditions (Kazdin, 2011). Alternating treatments design originates from single case design research. Single case design research denotes an experimental design where each participant functions as his or her own control. Single case research designs often test ideas, hypotheses, and sources for developing new techniques in education (Kazdin, 2011).

In order to alternate interventions in the current study, the experimenter (a) randomly assigned three discreet skills to each student (see Table 1) and (b) counterbalanced the order in which the students received the three conditions (see Table
2). By randomly assigning three separate skills, the design eliminates confounds that could occur when students share the same skill in each condition. For instance, counterbalancing attempts to control the unwanted effects of a static order that may favor one condition.

Table 1. Intervention Assignments

<table>
<thead>
<tr>
<th>Student</th>
<th>Baseline</th>
<th>Intervention #1</th>
<th>Intervention #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cara</td>
<td>Order of Operations</td>
<td>Add/Sub Fractions</td>
<td>Long Division</td>
</tr>
<tr>
<td>John</td>
<td>Long Division</td>
<td>Order of Operations</td>
<td>Add/Sub Fractions</td>
</tr>
<tr>
<td>Jono</td>
<td>Add/Sub Fractions</td>
<td>Order of Operations</td>
<td>Long Division</td>
</tr>
<tr>
<td>Poppy</td>
<td>Add/Sub Fractions</td>
<td>Long Division</td>
<td>Order of Operations</td>
</tr>
</tbody>
</table>

Baseline: no practice; Intervention 1: Three, one-minute practice trials; Intervention 2: One, three-minute practice trial

Procedure

The experimenter arranged four stapled packages of intervention materials (e.g. assessments, practice sheets with answer keys, and cue cards) on a long rectangular table. Students chose permanent seats for the duration of the experiment and listened to the first of four sets of instructions corresponding with the intervention schedule. The instructions requested the students to (a) show all their work, (b) work left to right across the page starting with problem number one, (b) not skip problems, and (d) complete the task as rapidly as possible. The instructions also requested students to (a) calculate the remainder to the tenths or one decimal place and (b) remember to simply fractions and/or convert to a mixed number. During the assessment the experimenter prompted a student to “please continue working” when he or she paused for more than five seconds, had a question, or caused a disruption before the timer expired.

On the first day students started with the one-minute Baseline condition. The students then tore off the fastened sheet and handed in the Baseline assessment and proceeded to listen to instructions. The students then attended to the first of three, one-minute practice sheets. Following the first timed frequency building exercise, the
students tore off the practice sheet and evaluated their work from an answer key (the next page) for 30 seconds. The students then tore off the answer key, handed in the first practice sheet, turned over the answer key, and then repeated the same process two more times. The three, one-minute frequency building trials produced a total three minutes of frequency building and 90 seconds of self-assessment and feedback. Next, the students completed a one-minute assessment for the dependent variable without the cue card.

For the next intervention the students practiced for three minutes and with a cue card and then self-evaluated their work for 90 seconds with the answer key. After frequency building, the student took another one-minute assessment for the dependent variable. Afterwards the experimenter thanked the students for their participation and hard work. Anecdotal notes based on observation occurred throughout the intervention. Notes included absences, questions students asked, and observable behavior (e.g. student not showing work, student unfocused). The experimenter then promptly collected, scored, and inputted the data into a digital application for evaluation. Each student participated a total of 15 intervention days. On the last day students completed a questionnaire for social validity.

**Data Display**

The experimenter recorded, evaluated, and visually displayed CDSM and IDSM data for each condition on three separate panels from segments of Standard Celeration Charts (SCC; Graf and Lindsley 2002; Lindsley 2005; Pennypacker, Guitierrez, & Lindsley, 2003). Separate panels, side by side, allow for a clear comparison of multiple measures presented on semi-logarithmic charts. The SCC segments display changes in
behavior proportionally in calendar time and supported by precise measures: Level, Celeration (i.e., slope on a ratio graph), and the Improvement Index (I.I.).

**Level.** Level signifies central tendency or the mean for both CDSM and IDSM. The geometric mean determined Level in the present study (Kubina, Kostewicz, & Al-Shammari, 2016). The geometric mean provides a measure that normalizes the set of numbers calculated without weighing or favoring a certain subset(s) of numbers over another subset(s) of numbers. In addition, the geometric mean also eases the impact of outliers that can skew data (Clark-Carter, 2005).

**Level Comparison Analysis.** The present investigation applies a Level Comparison to quantify the difference in levels of CDSM to CDSM and IDSM to IDSM between baseline and the individual experimental condition. The calculation involves dividing the larger value by the smaller value. The quotient then incorporates a multiply or divide sign signifying the greater initial value of the two compared levels. In the following example, a student produces a level of 12 CDSM during baseline and a level of 18 IDSM during an experimental condition. The Level Comparison, or change in performance between the student’s baseline and experimental condition performance equals a ÷1.5 (33%) difference in IDSM (i.e., 18 ÷ 12 = 1.5; apply the ÷ sign because from baseline to intervention the errors show a reduction).

**Celeration.** Celeration refers to a standard unit of measurement that quantifies a change in frequency or rate of performance over time (Johnston & Pennypacker, 2009). For instance, a student who solves 40 CDSM on Monday’s assessment and then accelerates to 60 IDSM on the following Monday’s assessment, will produce a celeration value of x1.5 – a 50% weekly growth rate. Another student in the same class who
accelerates from 40 CDSM to 80 CDSM will double his or her performance thus producing a celeration value of $x \times 2.0$ or a $100\%$ weekly growth rate. Similarly, the SCC also assigns celeration values quantifying a decrease in frequency of performance. A student who produces 10 CDSM on Friday’s assessment and decelerates to 5 CDSM on next Friday’s assessment will record a celeration value of $\div 1.5$ or a $33\%$ decay rate for errors.

**Celeration Comparison Analysis.** Similar to Level Comparison, the Celeration Comparison Analysis quantifies the differences between CDSM to CDSM, and IDSM to IDSM in baseline and the experimental conditions. However, the Celeration Comparison Analysis finds the differences in speed of change (celeration) versus the differences in the geometric mean (level). A student with a celeration of $x1.1$ for CDSM in baseline, for instance, has a celeration of $x2.2$ during the experimental condition. The Celeration Comparison Analysis measure must find not only speed differences, but also the trends of the celerations. Thus, the subsequent rules apply: if both values have the same sign (i.e., both $x$ or both $\div$) divide the larger value by the smaller value and employ the sign denoting the comparison of the change (i.e., if the resulting change from baseline to intervention sped up, a $x$ sign would appear; for instances where the speed decayed the value has a $\div$ sign). However, for celerations with different signs ($x$ to $\div$ or $\div$ to $x$) the rule states to multiply the values together and use the sign representing the speed difference ($x$ for accelerating speed difference and $\div$ for decelerating speed difference). Therefore, a speed comparison of $x1.1$ in baseline and $x2.2$ in an intervention yields $2.2 \div 1.1 = 2$ with a $x$ sign for $x2$ stating the speed comparison of the intervention value occurred twice as fast, or $x2$ faster, when compared to baseline.
**Improvement Index.** The Improvement Index (I.I.) measures the degree of progress improvement. To find I.I., one finds the ratio of the two coexisting celerations. The greater the I.I. value, the more improvement or student progress has occurred (Pennypacker et al. 2003). For instance, an I.I. of x1.5 indicates progress has advanced by 50% over the span of the experimental condition. Conversely, an I.I. of ÷ 1.5 indicates that progress worsened by 33% over the span of the experimental condition. I.I. provides educators a sensible, quantitative summary that measures the magnitude of progress change in learning due to the effects of the intervention or prescribed instructional activity (Kubina & Yurich, 2012).

**Improvement Index Comparison Analysis.** The Improvement Index Comparison Analysis compares two Index measures: baseline and an intervention. The resulting ratio of ratios offers a value quantifying progress change from baseline to intervention. The Improvement Index Comparison applies the same algorithm as the Celeration Change Comparison; when both values have like signs (i.e., x and x, or ÷ and ÷) divide the larger value by the smaller value and apply the sign indicating the comparison change (i.e., growth from baseline to intervention requires a x whereas decline would have a ÷ sign). When I.I. values have unlike signs, multiply the values and use the sign that represents the progress difference. As previously noted, apply a multiply sign (i.e., x) for an improvement progress difference and a ÷ sign for a worsening progress difference.

**Additional Measures**

**Weekly Median for Correct Problems.** A secondary measure included correct problems per minute (CPPM) after the experimenter and research assistant calculated
CDSM and IDSM. The investigation incorporated CPPM to reflect a performance measure typically used by classroom teachers to evaluate student progress. By eliminating the two highest and two lowest numbers the experimenter sought to temper the impact of possible outliers in performance.

**Retention Measures.** Retention signifies the relationship between performance frequencies measured at two points where learners have not had the opportunity to perform the behavior (Fabrizio & Moors, 2003; Kubina & Yurich, 2012). One month after the intervention the students completed three one-minute assessments for retention. The retention measures mirrored the dependent variable for each skill (i.e. number of problems, difficulty).
Chapter 3

Results

Table 3 shows the results for the initial and exit probes measuring frequency in digits correct per minute (DCPM) on multiplication and division math facts probes. Table 4 provides individual performance outcomes for Level Corrects, Celeration Corrects, Level Incorrects, Celeration Incorrects, and Improvement Index values. Table 5 includes a Celeration and Level Comparison Analysis between Baseline and the three, one-minute frequency building trial intervention and Baseline versus the three-minute frequency building trial intervention.

Figure 1 displays data on Standard Celeration Chart Segments representing frequencies for number of correct digits/symbols per minute (CDSM) and incorrect digits/symbols per minute (IDSM). Dots signify CDSM and x’s for IDSM. Figure 2 displays the level and celeration lines representing the data from Figure 1. The black lines (celeration lines) provide a visual representation of the growth or decay of performance frequencies. The lines followed by the dot signify CDSM whereas the x’s depict IDSM. The grey lines denote the level (geometric mean) of CDSM and IDSM for each student and corresponding condition. Dots and x’s also depict CDSM and IDSM. Level and celeration lines shown together provide a visual reference for the mean performance and trend for the individual conditions from the alternating treatments design. Figure 3 provides a visual display of the weekly median of correct problems per minute for each student on bar charts.

Student Performance Outcomes

During the Baseline condition (order of operations) Cara produced a level of 54
digits and symbols correct (CDSM) and 1 digits and symbols incorrect (IDSM). She accelerated digits and symbols correct per minute (CDSM) by x1.1 (10% weekly growth) and incorrect digits and symbols per minute (IDSM) by x1.3 (30% weekly growth) to yield a ÷1.2 (or a 20% decrease in progress improvement) Improvement Index (I.I.). Within the three, one-minute frequency building trial intervention (fractions), Cara produced levels of 67 CDSM and 1 IDSM. She accelerated CDSM by x1.2 (20% weekly growth) and IDSM by ÷1.2 (20% weekly decay) to yield a x1.4 (40% progress improvement) I.I. The three-minute frequency building trial intervention (division) saw Cara produce levels of 41 CDSM and 1 IDSM. She accelerated CDSM by x1.2 (20% weekly growth) and decreased IDSM by ÷ 1.2 (17% weekly decay) to yield a x1.4 (40% progress improvement) for I.I.

When compared to Baseline Levels (i.e., Level Change), Cara produced x1.2 (20%) more CDSM and had no change with IDSM (a x1.0 Level Change) during the Intervention #1 condition. For Celeration Comparison, her CDSM changed by x1.1 (10% faster) and IDSM by ÷1.4 (29% slower) for the Intervention #1 condition. Cara’s Improvement Index Change came to an overall x1.5 or 50% progress difference favoring three, one minute timings over Baseline. For Intervention #2, she produced a Level Change of ÷1.3 for CDSM and ÷1.2 for IDSM when compared to baseline. In the Celeration Comparison, Cara produced similar differences in CDSM (x1.1) and IDSM (÷1.4) for a x1.5 (50%) progress difference also favoring one, three-minute timing over Baseline.

John produced a level of 24 CDSM and 1 IDSM during the Baseline condition (long division). He decelerated digits and symbols correct per minute (CDSM) by ÷1.1
(10% weekly decay) and incorrect digits and symbols per minute (IDSM) by ÷1.2 to yield a ÷1.1 (or a 10% decrease in progress improvement) Improvement Index (I.I.).

Within the three, one-minute frequency building trials intervention (order of operations), John produced levels of 39 CDSM and 1 IDSM. He accelerated CDSM by x1.2 (20% weekly growth) and IDSM by x1.1 (10% weekly growth) to yield a x1.1 (10% progress improvement) I.I. The three-minute frequency building trial intervention (fractions) saw John produce levels of 36 CDSM and 0 IDSM. He accelerated CDSM by x1.3 (30% weekly growth) and did not have any IDSMs to yield a x1.3 (30% progress improvement) I.I.

Table 3. Student Performance Indicators

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Correct Digits/Symbols</th>
<th>Incorrect Digits/Symbols</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Condition</td>
<td>Skill</td>
<td>Level</td>
</tr>
<tr>
<td>Cara</td>
<td>Baseline</td>
<td>Order of Operations</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>Intervention #1*</td>
<td>Fractions</td>
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<td></td>
<td>Intervention #2**</td>
<td>Division</td>
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<td>Baseline</td>
<td>Division</td>
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<td></td>
<td>Intervention #1</td>
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<td></td>
<td>Intervention #2</td>
<td>Division</td>
<td>33</td>
</tr>
<tr>
<td>Poppy</td>
<td>Baseline</td>
<td>Fractions</td>
<td>52</td>
</tr>
<tr>
<td></td>
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<td>Division</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>Intervention #2</td>
<td>Order of Operations</td>
<td>57</td>
</tr>
</tbody>
</table>

*three, one minute practice trials  **one, three minute practice trial

The comparison analysis shows that John produced a level of x1.6 (60%) more CDSM and had no change for IDSM, x1.0 (0%), during the Intervention #1 Condition versus Baseline. For Celeration Comparison, his CDSM changed by x1.3 (30% faster) and IDSM by x1.4 (40% faster) for the Intervention #1 condition. John’s I.I. did not change between the two conditions. In other words, no progress change occurred between the three, one minute trials (order of operations) and baseline according to the Improvement Index Change metric. For Intervention #2, John had a Level Change of
x1.5 (50%) for CDSM and x1.2 (20%) for IDSM when compared to baseline. For the Celeration Comparison, John produced differences in CDSM (x1.4) and IDSM (x1.2) for a x1.2 (20%) progress difference favoring one, three minute timing when compared to Baseline.

In the Baseline condition (fractions), Jono produced a level of 40 CDSM and 1 IDSM. He accelerated digits and symbols correct per minute CDSM by x1.4 (40% weekly growth) and incorrect digits and symbols per minute IDSM by x1.5 to yield a ÷1.1 (or a 10% decrease in progress improvement) Improvement Index (I.I.). Within the three, one-minute frequency building trial intervention (order of operations), Jono produced levels of 35 CDSM and 1 IDSM. He accelerated CDSM by x1.2 (20% weekly growth) and IDSM by x1.3 (30% weekly growth) to yield a ÷1.1 (10% decrease in progress improvement) I.I.. The three-minute frequency building trial intervention (division) saw Jono produce levels of 33 CDSM and 1 IDSM. He accelerated CDSM by x1.4 (40% weekly growth) and decreased IDSM by ÷ 1.3 (23% weekly decay) to yield a x1.8 (80% progress improvement) for I.I.

For the Level Change Comparison, Jono had ÷1.1 (10%) less CDSM and did not change IDSM (a x1.0 Level Change) for the Intervention #1 condition versus baseline. For the celeration comparison, his CDSM slowed by ÷1.3 (23%) and by ÷1.2 (17%) for IDSM for the Intervention #1 condition comparison. Jono’s Improvement Index Change yielded a similar x1.0 or 0% progress difference between the three, one minute timings and Baseline. For Intervention #2, Jono had a Level Change of ÷1.2 (17%) less CDSM and ÷2.4 (58%) less IDSM when compared to Baseline. Consequently, Jono increased his overall progress by x2.2 (120%) favoring the one, three-minute timing to the Baseline I.I.
Poppy yielded levels of 52 CDSM and 1 IDSM during the Baseline condition (fractions). She accelerated digits and symbols correct per minute (CDSM) by $x1.1$ (10% weekly growth) and incorrect digits and symbols per minute (IDSM) by $x1.0$ which computed to a $x1.1$ (or a 10% increase in progress improvement) Improvement Index (I.I.). Within the three, one-minute frequency building trials intervention (division), Poppy produced levels of 69 CDSM and 1 IDSM. She accelerated CDSM by $x1.3$ (30% weekly growth) and IDSM by $x1.5$ (33% weekly decay) to yield a $x2.0$ (100% progress improvement) I.I.. The one, three-minute frequency building trial intervention (order of operations) saw Poppy produce levels of 57 CDSM and 1 IDSM. She accelerated CDSM by $x1.3$ (20% weekly growth) and accelerated IDSM by $x1.4$ to yield a $x2.0$ (27% decrease in progress) I.I.

**Table 4. Level and Celeration Comparison Analysis**

<table>
<thead>
<tr>
<th></th>
<th>Skill</th>
<th>Level Comparison</th>
<th>Celeration Comparison</th>
<th>Level Improvement</th>
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<tbody>
<tr>
<td><strong>Cara</strong></td>
<td>Baseline</td>
<td>Order of Op.</td>
<td>54</td>
<td>x1.1</td>
</tr>
<tr>
<td></td>
<td>Intervention #1* Fractions</td>
<td>x1.2</td>
<td>x1.1</td>
<td>x1.0</td>
</tr>
<tr>
<td></td>
<td>Intervention #2** Division</td>
<td>$\div1.3$</td>
<td>x1.1</td>
<td>x1.0</td>
</tr>
<tr>
<td><strong>John</strong></td>
<td>Baseline</td>
<td>Division</td>
<td>24</td>
<td>$\div1.1$</td>
</tr>
<tr>
<td></td>
<td>Intervention #1 Order of Op.</td>
<td>x1.6</td>
<td>x1.3</td>
<td>x1.0</td>
</tr>
<tr>
<td></td>
<td>Intervention #2 Fractions</td>
<td>x1.5</td>
<td>x1.4</td>
<td>-</td>
</tr>
<tr>
<td><strong>Jono</strong></td>
<td>Baseline</td>
<td>Fractions</td>
<td>40</td>
<td>x1.5</td>
</tr>
<tr>
<td></td>
<td>Intervention #1 Order of Op.</td>
<td>$\div1.1$</td>
<td>$\div1.3$</td>
<td>x1.0</td>
</tr>
<tr>
<td></td>
<td>Intervention #2 Division</td>
<td>$\div1.2$</td>
<td>$\div1.2$</td>
<td>x1.0</td>
</tr>
<tr>
<td><strong>Poppy</strong></td>
<td>Baseline</td>
<td>Fractions</td>
<td>52</td>
<td>x1.1</td>
</tr>
<tr>
<td></td>
<td>Intervention #1 Division</td>
<td>x1.3</td>
<td>x1.0</td>
<td>x1.0</td>
</tr>
<tr>
<td></td>
<td>Intervention #2 Order of Op.</td>
<td>x1.1</td>
<td>x1.1</td>
<td>x1.0</td>
</tr>
</tbody>
</table>

**Bold** indicates original Baseline measure; *three, one minute practice trials; **one, three minute practice trial.

When compared to Baseline Levels (i.e., Level Change), Poppy produced $x1.3$ (30%) more CDSM and did not change her IDSM (a $x1.0$ Level Change) during the Intervention #1 condition. The Celeration Comparison, showed her CDSM did not change in speed ($x1.0$ or 0%) and her IDSM slowed by $\div1.6$ (38% slower) for the Intervention #1 condition. Poppy’s I.I. came to an overall $x1.4$ or 40% progress.
difference favoring three, one minute timings over Baseline. For Intervention #2, she had a level change of x1.1 for CDSM and x1.4 for IDSM when compared to baseline. In the celeration comparison, Poppy had speed changes for CDSM x1.1 and IDSM x1.4. for a ÷1.2 (or 17% decrease) I.I. favoring Baseline over the one, three-minute timing.

**Initial and Exit Assessments.** Before the start of intervention, the students completed two, one-minute simple simple computation assessments for multiplication and division. The students then completed two more one-minute assessments the day after intervention ended to evaluate performance change (see Table 3 below). Three of the four students increased performance on the simple computation multiplication probes. Jono and Poppy increased 12 DCPM and 26 DCPM, respectively. John had a 3 DCPM gain whereas Cara decreased by 3 DCPM. All four students increased performance on the simple computation division probes. Again, Poppy showed a gain of 13 DCPM. John, Cara, and Jono increased by 6 DCPM, 5 DCPM, and 3 DCPM, respectively.

<table>
<thead>
<tr>
<th>Name</th>
<th>Operation</th>
<th>Initial Probe</th>
<th>Exit Probe</th>
<th>± DCPM Change</th>
</tr>
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<tbody>
<tr>
<td>Cara</td>
<td>Multiplication</td>
<td>42 DCPM</td>
<td>39 DCPM</td>
<td>−3 DCPM</td>
</tr>
<tr>
<td>Division</td>
<td>13 DCPM</td>
<td>18 DCPM</td>
<td>+5 DCPM</td>
<td></td>
</tr>
<tr>
<td>John</td>
<td>Multiplication</td>
<td>21 DCPM</td>
<td>24 DCPM</td>
<td>+3 DCPM</td>
</tr>
<tr>
<td>Division</td>
<td>12 DCPM</td>
<td>18 DCPM</td>
<td>+6 DCPM</td>
<td></td>
</tr>
<tr>
<td>Jono</td>
<td>Multiplication</td>
<td>23 DCPM</td>
<td>35 DCPM</td>
<td>+12 DCPM</td>
</tr>
<tr>
<td>Division</td>
<td>16 DCPM</td>
<td>19 DCPM</td>
<td>+3 DCPM</td>
<td></td>
</tr>
<tr>
<td>Poppy</td>
<td>Multiplication</td>
<td>23 DCPM</td>
<td>49 DCPM</td>
<td>+26 DCPM</td>
</tr>
<tr>
<td>Division</td>
<td>17 DCPM</td>
<td>30 DCPM</td>
<td>+13 DCPM</td>
<td></td>
</tr>
</tbody>
</table>

**Weekly Median for Correct Problems.** During the first week, Jono produced a median of one correct problem per minute in all three conditions. By the end of week three, he produced two correct problems during Baseline (fractions), one correct problem in the three, one minute trials condition (order of operations), and four correct problems
in the one, three-minute trial condition (long division). Cara had a median of one correct problem per minute during the first week of Baseline (order of operations) and completed the third week with a median of two correct problems per minute. During the first week of three, one-minute trials (fractions), Cara produced a weekly median of three correct problems per minute and for week three increased to a median six correct problems per minute. For the first week of the one, three-minute trial (long division), she produced a weekly median of two correct problems per minute and for week three increased to four correct problems per minute.

John produced a median of one correct problem per minute through the first week of Baseline (long division) and completed the third week with a median of two correct problems per minute. For the first week of the three, one minute trials (order of operations) he posted a median of two correct problems per minute and for week three a weekly median of three correct problems. During the first week of the one, three-minute trial (fractions), John produced a weekly median of one correct problem per minute and for week three yielded a weekly median of three correct problems.

Poppy yielded a median of three correct problems per minute through the first week of Baseline (fractions) and decreased during week three to a median of two correct problems. For the first week of three, one minute trials (long division) she produced a median of three correct problems per minute and for week three increased her weekly median to four correct problems per minute. Through week one of the one, three-minute trial (order of operations), Poppy produced a median of two correct problems per minute and for week three produced a weekly median of four correct problems per minute.
Retention Measures. Cara produced 66 CDSM and 0 IDSM for order of operations (Baseline skill). She yielded 3 CPPM. For adding and subtracting fractions (three, one-minute frequency building skill), Cara produced 72 CDSM and 2 IDSM. She yielded 4 CPPM. For division (one, three-minute frequency building skill), Cara produced 48 CDSM and 0 IDSM. She yielded 5 CPPM.

John produced 39 CDSM and 1 IDSM for division (Baseline skill). He yielded 3 CPPM. For order of operations (three, one-minute frequency building skill), John produced 25 CDSM and 0 IDSM. He completed 1 CPPM. For adding and subtracting fractions (three, one-minute frequency building skill), John produced 54 CDSM and 0 IDSM. He yielded 4 CPPM.

Jono produced 41 CDSM and 1 IDSM for adding and subtracting fractions (Baseline skill). He yielded 1 CPPM. For order of operations (three, one-minute frequency building skill), Jono produced 49 CDSM and 0 IDSM. He yielded 3 CPPM. For division (one, three minute frequency building skill), Jono produced 51 CDSM and 0 IDSM. He yielded 4 CPPM.

Poppy produced 70 CDSM and 0 IDSM for fractions (Baseline skill). She yielded 4 CPPM. For long division (three, one-minute frequency building skill), Poppy produced 63 CDSM and 1 IDSM. She yielded 3 CPPM. For order of operations (one, three-minute frequency building skill), Poppy produced 72 CDSM and 0 IDSM. She yielded 4 CPPM.
Chapter 4

Discussion

Researchers have successfully established that self-managed practice interventions can build fluency with simple computation (e.g. Hulac, Wickerd, & Vining, 2013; Poncy, McCallum, & Schmitt, 2007; Skinner, Turco, Beatty, & Rasavage, 1989). The present investigation set out to extend the research base by testing whether students experiencing mathematics difficulties could self-manage a frequency building intervention with complex computation. The experiment evaluated the differences in performance outcomes between (a) baseline, (b) three, one-minute frequency building (i.e., systematic practice) trials with three, 30 second opportunities for feedback, and (c) one, three-minute frequency building trial with one, 90 second opportunity for feedback. Visual and quantitative analysis obtained from the Standard Celeration Chart (SCC) segments served as tools to measure and evaluate student performance.

The experimenter counterbalanced baseline and experimental conditions (i.e. frequency building) using an alternating treatments design in order to separate the effects of the intervention from baseline as well as to compare the two approaches to frequency building (Kazdin, 2011). Unlike multiple baseline or a withdrawal design, the application of an alternating treatments design allows for the simultaneous comparison of different instructional approaches (Kazdin, 2011; Sindelar, Rosenberg, & Wilson, 1985) and promptly discriminates the features of the intervention that affects student performance (Cooper, Heron, & Heward, 2007; Kazdin, 2011). Specifically, the alternating treatments design in the present investigation demonstrated the comparative difference of two self-managed frequency building interventions against baseline.
After 15 days of intervention, the data suggests frequency building functioned as an efficient and effective method to build behavioral fluency with long division and adding and subtracting fractions. The critical features of frequency building, timed repetition of behavior and feedback given after each trial, led to enhanced speed and accuracy and the quality of the response (Binder, 1996; Hughes, Beverley & Whitehead, 2007; Kubina & Yurich, 2012, Stocker & Kubina, manuscript submitted for publication). Timed trials provided the students with an uninterrupted number of opportunities to respond executed at a brisk pace which translated to increased momentum and response speed over the span of the intervention (Binder, 1996; Lee, 2006). For instance, all four students significantly accelerated CDSM (≥ x1.2 or 20% weekly growth) in the intervention conditions except for Poppy (x1.1) in the three, one-minute frequency building condition (order of operations). Without the benefit of frequency building, three students (Cara, John, and Poppy) produced an insignificant speed change in CDSM of ≤ x1.1 in the baseline condition.

Self-correction allows students to develop error detecting skills (Hattie & Timperley, 2007) and implement improvements without the mediation of a peer or teacher (Skinner et al., 1997). For long division, Jono and Poppy frequently made component skill errors (Stein, Kinder, Silbert, & Carnine, 2005) when computing remainders during the first and second weeks of intervention. After two weeks of frequency building, both students calculated remainders accurately which led to a robust decay in IDSM of ÷1.5 and ÷1.6 prompting an upturn in CDSM during the last week of intervention. For fractions, Cara converted improper fractions to mixed numbers with more consistency and fluidity stimulating a steady acceleration in CDSM over the second
half of the investigation. The consistent performance that occurred after successful self-correction combined with improved speed and accuracy on subsequent assessments suggests that frequency building increased the future probability of correct responding versus the reinforcement of errors (Binder, 1996; Burns, VanDerHeyden & Boice, 2008; Daly et al., 2007; Rivera & Bryant, 1992).

Prior research in mathematics shows that fluency with element skills can affect the fluent execution of compound skills (Lin & Kubina, 2005; Lin, Kubina, & Shimamune, 2011; McDowell & Keenan, 2002). While all four students in the present investigation exhibited element skill deficits, error analysis from assessments suggest the students experienced more difficulty increasing speed and accuracy with order of operations due to the larger number of different element skills (i.e., positive and negative numbers, exponents, decimals) presented in random sequences. In a recent investigation, Stocker and Kubina (manuscript submitted for publication) applied the same frequency building intervention to build fluency with order of operations in sixth grade students. The students who exhibited similar element skill deficits also presented similar error patterns. Persistent error patterns that occur in compound problem-solving practice activities that do not respond to feedback often necessitate separate fluency building activities such as frequency building to remedy element skill deficits (Beverley et al., 2009).

Testing different iterations of interventions (e.g. timings, feedback, assessment) provide opportunities for researchers to determine which iteration best fits a learner (e.g. Brady & Kubina, 2010; Poncy, Skinner, & Jaspers, 2007; Skinner, Bamberg, Smith, & Powell, 1993). Unlike the results from Brady and Kubina (2010) where students
performed best in three, twenty second practice trials versus a one, one-minute practice trial, the present experiment could not determine an advantage of using three, one-minute practice trials versus one, three-minute practice. The results of the present experiment yielded a similar outcome from the initial Stocker and Kubina (manuscript submitted for publication) investigation where students entered the intervention at different levels of proficiency with element skills which affected the performance outcomes regardless of the allocation of time in the experimental conditions.

Still, elevating the number of feedback opportunities in the three, one-minute practice trials can allow more instances to self-evaluate performance which in turn can increase the probability of detecting errors and establishing strategies for future problem solving (Burns et al., 2008; Hattie & Timperley, 2007). For example, in the one, three-minute frequency building trial intervention (order of operations), Poppy produced levels of 57 CDSM and 1 IDSM, accelerated CDSM by x1.3 (30% weekly growth), but also accelerated IDSM by x1.4 to yield a ÷1.4 (27% decrease in progress) for the Improvement Index. Based on visual analysis of the Standard Celeration Chart and error analysis, she made only one IDSM over the first eight days of intervention. Poppy then made sporadic errors over four of the last seven days mostly confusing operations (e.g. adding or subtracting) or computing positive and negative numbers. In Poppy’s scenario, she could have plausibly benefitted from receiving more frequent feedback delivered within the three, one-minute trials (Brady & Kubina, 2010; Burns et al., 2008).

Conversely, Cara produced Levels of 67 CDSM and 1 IDSM, accelerated CDSM by x1.2 (20% weekly growth), and decelerated IDSM by ÷1.2 (20% weekly decay) to yield a x1.4 (40% progress improvement) Improvement Index computing fractions in the
three, one-minute practice trials condition. Furthermore, on her last three assessments she did not commit an error. Perhaps Cara would benefit switching to the one, three-minute practice trial to build endurance in order to extend speed and accuracy over an extended period of time as well as increase resistance to competing distractions (Brady & Kubina, 2010; McTiernan, Holloway, Healy, and Hogan, 2016; Stromgren, Berg-Mortensen, and Tangen, 2014).

**Weekly Median for Correct Problems Per Minute**

Although the experiment did not test for a direct causal relationship between frequency building and weekly median for correct problems per minute, the data suggests an upward trend occurred in correct problems per minute from the first week through the third week of intervention. The increase in speed, accuracy, and endurance that resulted from the intervention provides initial evidence that a relationship plausibly exists between frequency building that accelerates CDSM and decelerates IDSM and the completion of more correct problems per minute.

**Effects of Performance with Simple Computation**

Non-fluent performance with simple computation and subsequent element skills effects the speed and accuracy of the compound skill (Binder, 1996; Datchuk, 2015; Kubina, Young & Kilwein, 2004; Kubina & Yurich, 2012, NMAP 2008). For instance, the four students showed deficits in simple computation from the initial probes. They frequently had more difficulty with multiplication facts that generate products between 36 and 81 and their division complements. Nevertheless, Poppy and Cara who entered the intervention with the highest frequencies of DCPM on the initial probes proceeded to yield the highest levels (geometric mean) of CDSM and number of correct problems
when compared to John and Jono. By the end of the study all the students generally increased in compound skill performance, but the results from the exit probes showed that John and Jono still functioned at the instructional level for multiplication facts (< 40 DCPM) while all four students continued to function within the instructional or frustration level (< 20 DCPM) for division facts according to fluency criteria set by Deno & Mirkin (1977).

Still, within 15 days three out of the four students did surpass the realistic growth rate of approximately .50 digit per week for simple computation as indicated in the progress monitoring literature (Fuchs, Fuchs, Hamlett, Walz, & Germann, 1993). The increase in probe scores provides initial evidence that using frequency building to increase fluency with complex computation can plausibly maintain and improve performance with simple computation.

**Retention Measure**

The retention measure demonstrated that frequency building can lead to sustained gains in performance. All four students emitted comparable levels of CDSM and IDSM. Similar error patterns occurred such as making component skill errors calculating remainders as well as fact errors with positive and negative numbers. The students also maintained a greater number of correct problems per minute versus the first week of intervention. Although retention occurred, the data suggests that without continued systematic practice to build fluency the students will conceivably experience difficulty making gains to perform at grade level.

**Social Validity**

All four students preferred the one, three-minute time allocation for frequency
building. The students did not quit or exhibit frustration over the span of the investigation. The students liked the opportunity to check their work with the answer key but wanted the opportunity to complete the practice sheet. Working for three minutes without pause on appropriately matched materials suggests that the intervention encouraged endurance - a key feature of behavioral fluency in which a student exhibits the capacity to stay on task at a certain speed and level of accuracy over an extended period of time (Kubina & Yurich, 2012; Fabrizio & Moors, 2003). Comments made by students included, “it helps me in math (class)”, “I like getting better with practice”, “this can help me get better grades”, and “I can solve problems faster in class”. All the students could see themselves committing to ten minutes of frequency building per day.

Limitations

In the current investigation, dexterity with element skills such as simple computation can influence outcomes between conditions as does prior knowledge of compound skills. As shown in the Level Comparison Analysis, Cara and Poppy tended to have a stronger grasp of element skills when compared to Jono and John. Coupled with the small sample size and difficulty students experienced with order of operations, the experiment could not determine whether one iteration of the intervention worked more effectively.

Implications for Practice

As already indicated, error patterns that occur in compound problem-solving practice activities that do not respond to feedback often require independent fluency building activities to remedy deficits (Beverley et al., 2009). Separate, systematic practice such as frequency building with simple computation and subsequent element
skills can support students before embarking on, or in conjunction with building fluency on compound skills. If students do not receive intensive support, they may continue to work slower than their peers, experience difficulty keeping up with the pace of instruction, and fall behind in the hierarchical configuration of the mathematics curriculum (Biancorosa & Shanley, 2016).

**Future Directions for Research**

A paucity of research exists for developing fluency interventions with complex computation. The present study serves as the second investigation examining frequency building for complex computation. Replication studies can provide more evidence as to whether frequency building qualifies as an evidence-based practice for complex computation. Another direction for research may include conducting comparison investigations of students who have reached a level of fluency with element skills versus students who have not reached a level of fluency to evaluate performance differences. Performance differences can lead to establishing numerical markers (i.e., fluency criteria) that distinguishes a proficient performance with a compound skill (Binder, 1993; Johnson & Layng, 1996; Kubina & Morrison, 2000).

Element skills extend beyond simple computation, encompassing subsequent concepts such as converting improper fractions to mixed numbers, decimals, and positive and negative numbers. Therefore, future research to develop fluency criteria for element skills can yield critical information to support teacher decision-making, especially when deciding to introduce frequency building to new compound skills. Additional studies that align fluency criteria to the mathematics curriculum will also allow for norm-referenced comparisons of students at the same or different ages or grade levels.
Future research that matches the appropriate fluency criteria on a component skill may also lead to better matching instructional materials to the performance level of a student. Matching instructional materials to a student's skill level can help maximize gains in performance (VanDerHeyden & Burns, 2008; Kubina & Morrison, 2000; Daly et al., 2007). In order to successfully implement a mathematics program that supports fluency instruction, teachers must have access to a substantial reserve of leveled materials designed to meet the need for repeated practice opportunities over extended periods of time (Daly et al. 2007; Witzel & Riccomini, 2007). Unfortunately, instructional materials by math textbooks often do not meet the needs of students who struggle with mathematics (Cawley et al., 2001; NMAP, 2008; Witzel & Riccomini, 2007). Future research can plausibly outline a continuum for critical element-compound skills. The continuum would benefit from including fluency criteria or numerical markers representing each skill in conjunction with the development of curricular materials.

Pre-assessment allows for more precise matching of materials to a student’s present level of performance (i.e., order of operations problems). Future investigations would benefit from pre-assessing students on a larger number of element skills such as calculating positive and negative numbers, decimals, and fractions. In addition to assessing element skills, achievement tests can also assist in selecting students for intervention who exhibit more similarities in mathematics performance. Students who share common performance indicators prior to intervention can enhance comparisons between subjects using single case design (Kazdin, 2011). Furthermore, conducting post-assessments with the aforementioned element skills and achievement tests provide another dimension in which to measure gains in performance.
The time allocation difference did not favor performance in one condition over
the other. Unlike three of the four students in Stocker & Kubina (manuscript submitted
for publication) who preferred the three-one minute timings, the four students in the
present study preferred the one, three-minute timing. Yet, students who exhibit
mathematics difficulties typically require more opportunities for feedback (Archer &
Hughes, 2011). Future studies can explore the application of matching theory or choice
theory in order to predict and increase the desired response (Martens, Lochner, & Kelly,
1992; Skinner, Robinson, Johns, Logan, & Belfiore, 1996). Increasing the likelihood of
students selecting preferred academic tasks has shown to increase learning rates and
decrease inappropriate behavior (McCurdy, Skinner, Grantham, Watson, & Hindman,
2001).

When the students began to work faster, they continued to increase accuracy,
speed, and operate more efficiently when problem solving. For instance, Cara and Poppy
who exhibited higher levels of fluency with simple computation performed best with
order of operations regardless of condition. Consequently, they learned new strategies to
solve problems (Hattie & Timperley, 2007) more efficiently by reducing the number of
written steps suggesting more efficient use of cognitive resources (see Table 6 below).

<table>
<thead>
<tr>
<th></th>
<th>Student #1</th>
<th>Student #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ (-4 + 2^2 \cdot -2) - (2 \cdot 3^2 - 10) = ]</td>
<td>[ (-4 + 2^2 \cdot -2) - (2 \cdot 3^2 - 10) = ]</td>
<td></td>
</tr>
<tr>
<td>[ (-4 + 4 \cdot -2) - (2 \cdot 3^2 - 10) = ]</td>
<td>[ (-4 + -8) - (18 - 10) = ]</td>
<td></td>
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<tr>
<td>[ (-4 + -8) - (2 \cdot 9 - 10) = ]</td>
<td>[ (-12 - 8) = ]</td>
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<tr>
<td>-12</td>
<td>[ (-12 - 10) = ]</td>
<td>-20</td>
</tr>
<tr>
<td>-12</td>
<td>[ 8 = ]</td>
<td></td>
</tr>
<tr>
<td>-20</td>
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</tbody>
</table>

Perhaps pre- and post-intervention measures in replication studies can conclude that
frequency building with complex computation assists students in other areas of
mathematics or daily routines such as counting money, telling time, and estimating.
Bibliography


Brady, K. K., & Kubina, R. M. (2010). Endurance of multiplication fact fluency for students with attention deficit hyperactivity disorder. *Behavior modification, 34*, 79-93.


National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring


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Appendix A

Figures

Figure 1. Correct Digits/Symbols and Incorrect Digits/Symbols
Figure 2. Celeration Lines and Level Lines
Figure 3. Weekly Median Performance
Appendix B

Student Materials

Figure 4. Order of Operations

Name ___________________________  Date ___________________________

\[
(1.2 + 2.5) + (4^2 - 3) = \quad (-5 \times -3) + (2^2 \times 2) = \quad (4 \times 5 - 10) - (2 + 2^2) = \\
\]

\[
(-3 + 2 \times 2) - (4 \times 2^2 - 10) = \quad (3 \times 4) \div (18 \div 3^2) = \quad (-5 \times -3) + (6 \times 4) = \\
\]

\[
(4^2 \div 2) + (7 \times 7) = \quad (5^2 - 10) + (4 \times 4 - 12) = \quad (-5 - 3 - 4) + (2^2 \times 7) = \\
\]
Figure 5. Fractions with Uncommon Denominators

\[
\begin{align*}
\frac{1}{3} + \frac{1}{2} &= \\
\frac{4}{5} - \frac{3}{4} &= \\
\frac{1}{5} + \frac{3}{4} &= \\
\frac{3}{5} - \frac{1}{4} &= \\
\frac{2}{3} + \frac{5}{9} &= \\
\frac{6}{7} + \frac{2}{5} &= \\
\frac{5}{8} + \frac{1}{3} &= \\
\frac{3}{4} + \frac{1}{2} &= \\
\frac{5}{6} - \frac{5}{8} &= 
\end{align*}
\]
Figure 6. Long Division with and without Remainders

\[ \begin{array}{ccc}
5 & \overline{675} & 3 & \overline{524} & 4 & \overline{1826} \\
9 & \overline{927} & 7 & \overline{2821} & 6 & \overline{434} \\
2 & \overline{423} & 8 & \overline{644} & 9 & \overline{544} \\
\end{array} \]
### Appendix C

#### Experimenter Materials

Table 2. Intervention Schedule

<table>
<thead>
<tr>
<th>Day</th>
<th>Cara</th>
<th>Intervention #1</th>
<th>Intervention #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>Order of Operations</td>
<td>Fractions</td>
<td>Long Division</td>
</tr>
<tr>
<td></td>
<td>Long Division</td>
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<td>Fractions</td>
</tr>
<tr>
<td></td>
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<td>Fractions</td>
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</tr>
<tr>
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</tr>
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<td>Order of Operations</td>
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<tr>
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<td>Order of Operations</td>
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<td>Long Division</td>
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<td>Fractions</td>
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<td>Order of Operations</td>
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<td>Order of Operations</td>
<td>Long Division</td>
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<td>Order of Operations</td>
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<td>Order of Operations</td>
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<td>Day</td>
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<td>Topic</td>
<td>Topic</td>
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<td>#11</td>
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<td></td>
<td>John</td>
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<td>Fractions</td>
</tr>
<tr>
<td></td>
<td>Jono</td>
<td>Order of Operations</td>
<td>Long Division</td>
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<td>Order of Operations</td>
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<td>John</td>
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<td>Long Division</td>
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<tr>
<td></td>
<td>Jono</td>
<td>Long Division</td>
<td>Fractions</td>
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<td>Fractions</td>
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<td>Order of Operations</td>
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<td>Fractions</td>
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<td>Fractions</td>
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<td>John</td>
<td>Fractions</td>
<td>Long Division</td>
</tr>
<tr>
<td></td>
<td>Jono</td>
<td>Long Division</td>
<td>Fractions</td>
</tr>
</tbody>
</table>
Figure 7. Procedural Integrity Checklist

**Procedural Integrity Checklist**

_____ Appropriate number of probes/DV and answer keys developed in each category for each student

_____ Cue cards created and placed above probe during the Intervention 1 and Intervention 2 tasks

_____ Stopwatch for timings

_____ Instructions for the independent variable (IV) prepared and read to students before baseline

_____ Instructions for the independent variable (IV) prepared and read to students before intervention 1

_____ Answer key for intervention 1

_____ Instructions for the independent variable (IV) prepared and read to students before intervention 2

_____ Answer key for intervention 2

_____ Timings match between experimenter and research assistant
**Instructions for Students**

**Baseline**
Say to the students:

“We’re going to take a 1-minute math probe. I want you to show your work and write your answers to each of the problems. Start with the first problem and then continue to the next problem. Do not skip problems.”

“Work as quickly as you can on each problem. At the end of the probe we will take very short break.”

“When I say ‘BEGIN’ you can work ACROSS the page. Then go to the next row.”

“Are there any questions? (Pause)”

“Begin”

****If a student stops working before the test is done, say to the student: “Keep doing the best work you can.”

**Intervention 1**
Say to the students:

“We’re going to take (3) 1-minute math probes. I want you to show your work and write your answers to each of the problems. Start with the first problem and then continue to the next problem. Do not skip problems.”

“You have a cue card placed in front of you to help you if you forget how to solve the problem.”

“On long division problems with remainders, go out only one decimal place.”

“On fractions, please make sure to reduce fraction and/or change improper fraction to a mixed number.”

“Work as quickly as you can on each problem.”

“At the end of each probe you will have 30 seconds to compare your answers to the answer key”

“When I say ‘BEGIN’ you can work ACROSS the page. Then go onto the next row.”

“Are there any questions? (Pause)”

“Begin”

****If a student stops working before the test is done, say to the student: “Keep doing the best work you can.”
**Intervention 2**

Say to the students:

“We’re going to take (1) 3-minute math probe. I want you to show your work and write your answers to each of the order of operations problems. Start with the first problem and then continue to the next problem.

“You have a cue card placed in front of you to help you if you forget how to solve the problem.”

“On long division problems with remainders, go out only one decimal place.”

“On fractions, please make sure to reduce fraction and/or change improper fraction to a mixed number.”

“Work as quickly as you can on each problem”

“At the end of each probe you will have 90 seconds, a minute and a half, to compare your answers to the answer key”

“When I say ‘BEGIN’ you can work ACROSS the page. Then go onto the next row.”

“Are there any questions? (Pause)"

“Begin”

****If a student stops working before the test is done, say to the student: “Keep doing the best work you can.”
Appendix D

Review of Relevant Literature

Abstract

Behavioral fluency denotes a relationship between the achievement of performance standards, or frequency ranges of behavior, and associated critical learning outcomes. Over the years Precision Teaching and related behavioral intervention research have contributed a number of studies designed to increase behavioral fluency. The subsequent review examined the Precision Teaching literature and practice interventions to examine whether empirical studies support behavioral fluency in mathematics. The review identified 16 experimental studies that demonstrate a relationship between critical learning outcomes and performance standards. Results suggest practice interventions and frequency building interventions can successfully increase behavioral fluency and yield critical learning outcomes.
The Evidence Base for Behavioral Fluency and Mathematics Education: A literature Review of Performance Standards and Critical Learning Outcomes

Mathematics permeates many aspects of successful 21st century living. The inextricable link between mathematics and science fuels progress – citizens who can reason and communicate mathematically have greater opportunities to understand and contribute to the social, political, and economic structures of modern society (Geary, 2013; Gross, Hudson, & Price, 2009). While conventional skills such as counting money, telling time, estimation, and basic problem solving remain necessary for successful daily living, managing healthcare and finances in the new economy as well as acclimating to new technologies and related applications require advanced mathematical skillsets (Price & Ansari, 2013). Moreover, in order to meet the demands of a burgeoning scientific and technological workforce, current students must acquire key competencies in mathematics. Nearly two-thirds of entry-level jobs in the coming decade will require at least some advanced knowledge in algebra, statistics, data interpretation, and geometry to support gainful employment (Friedman, 2007; Hanushek, Peterson, & Woessmann, 2010; Lockard & Wolf, 2012).

National initiatives deem fluency a key component to mathematical competency (National Mathematics Advisory Panel, NMAP, 2008; National Council of Teachers of Mathematics, NCTM, 2014). Both NCTM (2014) and NMAP (2008) emphasize the importance of accuracy, automatic execution, and flexible use of whole number operations and standard algorithms, as well as extending fluent skills to problem solving. In response, the National Common Core State Standards for Mathematics (National
Governors Association Center for Best Practices, & Council of Chief State School Officers, 2010, CCSS-M) crafted fluency standards originating in kindergarten that extend through middle school (see Table 1 below).

<table>
<thead>
<tr>
<th>Gr.</th>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>CCSS.MATH.CONTENT.K.OA.A.5</td>
<td>Fluently add and subtract within 5.</td>
</tr>
<tr>
<td>1</td>
<td>CCSS.MATH.CONTENT.1.OA.C.6</td>
<td>Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.</td>
</tr>
<tr>
<td></td>
<td>CCSS.MATH.CONTENT.1.NBT.C.5</td>
<td>Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.</td>
</tr>
<tr>
<td>2</td>
<td>CCSS.MATH.CONTENT.2.OA.B.2</td>
<td>Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.</td>
</tr>
<tr>
<td></td>
<td>CCSS.MATH.CONTENT.2.NBT.B.5</td>
<td>Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</td>
</tr>
<tr>
<td>3</td>
<td>CCSS.MATH.CONTENT.3.OA.C.7</td>
<td>Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.</td>
</tr>
<tr>
<td></td>
<td>CCSS.MATH.CONTENT.3.NBT.A.2</td>
<td>Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</td>
</tr>
<tr>
<td>4</td>
<td>CCSS.MATH.CONTENT.4.NBT.B.4</td>
<td>Fluently add and subtract multi-digit whole numbers using the standard algorithm.</td>
</tr>
<tr>
<td>5</td>
<td>CCSS.MATH.CONTENT.5.NBT.B.5</td>
<td>Fluently multiply multi-digit whole numbers using the standard algorithm.</td>
</tr>
<tr>
<td>6</td>
<td>CCSS.MATH.CONTENT.6.NS.B.2</td>
<td>Fluently divide multi-digit numbers using the standard algorithm.</td>
</tr>
<tr>
<td></td>
<td>CCSS.MATH.CONTENT.6.NS.B.3</td>
<td>Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.</td>
</tr>
<tr>
<td>7</td>
<td>CCSS.MATH.CONTENT.7.EE.B.4.A</td>
<td>Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where $p$, $q$, and $r$ are specific rational numbers. Solve equations of these forms fluently.</td>
</tr>
</tbody>
</table>

In order to prepare students for Algebra, NMAP (2008) established three areas of focus termed the Critical Foundations of Algebra: *Fluency with Whole Numbers*, *Fluency with Fractions*, and *Particular Aspects of Geometry and Measurement*. CCSS-M (2010) did not set explicit numerical standards for fluency for simple or complex computation, but recommended students focus on developing fluency in writing, interpreting, and translating a variety of linear equations and inequalities, in addition to
application in the problem solving process. The Partnership for Assessment of Readiness for College and Careers Model Content Frameworks for Mathematics (PARCC, 2014) provides additional detailed information and recommendations regarding fluency in order for students to:

…get past the need to manage computational details so that they can manage computational details so that they can observe structure and patterns in problems. Such fluency can also allow for smooth progress beyond the college and career readiness threshold toward readiness for further study/careers in science, technology engineering, and mathematics (STEM) fields. (pp. 41-42)

Although initiatives, standards, and prior researchers have established the significance of mathematical fluency, the quality and quantity of practice that occurs in the classroom often fails to promote fluency (NMAP, 2008). In a prior study completed for NMAP (2008), a sample of Algebra teachers (n=748) conveyed students need more basic skills preparation in areas such as fractions and decimals, order of operations, and positive and negative integers (Hoffer, Venkataraman, Hedberg, & Shagle, 2007). The teachers also preferred students show less reliance on calculators. The survey concluded that “careful attention to pre-algebra curriculum and instruction in the elementary grades is needed” to remedy skill deficits (p. 35).

Several reasons plausibly explain the discrepancy. Ginsburg et al. (2005) suggest U.S. textbooks present a wide range of topics with less in-depth study versus world leaders in mathematics achievement such as Singapore who spend more time on a smaller number of topics in order to attain mastery. Hence, schools do not produce mathematics proficiency because of the pressure teachers have to move students to new complex, or
compound, skills before they can execute the element skills fluently (Binder, 1996, 2003; Daly, Martens, Barnett, Witt, & Olson, 2007).

Other researchers have asserted that textbooks and curricular materials do not appropriately support fluency instruction (Witzel and Riccomini, 2007). Commercial resources frequently contain extraneous materials designed to expand market share and as a consequence, do not contain the appropriate stimuli where students can make effective discriminations; examples may also contain too much information in order for students to systematically and successfully work through the steps to solve a problem (Daly et al., 2007).

**Behavioral Fluency and Critical Learning Outcomes**

Descriptors of fluency include words such as smooth, rhythmic, effortless, fluid, well-practiced, automatic, and masterful (Binder, 2003, Johnson & Layng, 1996; Kubina & Yurich, 2012). However, a highly accurate and well-paced (i.e., proper speed) performance typically serves as a common portrayal of fluency (Binder, 1996). Computational fluency, procedural fluency, and conceptual understanding require efficient and effective practice so that students can successfully navigate through the mathematics curriculum (NMAP, 2008; PARCC, 2014). Yet, some researchers and educators associate practice and fluency with rote memorization missing the mutually reinforcing benefits between conceptual understanding and arithmetical dexterity (Baroody, 2006; Biancarosa & Shanley, 2016; Clarke, Nelson, & Shanley, 2016; NCTM, 2014).

Despite narrow interpretations of fluency, a database has steadily emerged detailing critical learning outcomes – retention, endurance, application, maintenance, and
stability (see terms and descriptions below in Table 2) – that occur after students reach a “performance standard” through systematic practice (Binder, 1996). The term behavioral fluency marks the evolution and further refinement of fluency (Kubina & Yurich, 2012). Students who reach behavioral fluency through precise and well-defined quantitative benchmark(s), or the performance standard, often experience easier transitions from simple to more complex skills within the hierarchical configuration of the mathematics curriculum. As a result, students who exhibit high levels of behavioral fluency tend to function efficiently and effectively in their natural environments (Binder, 1996).

<table>
<thead>
<tr>
<th>CriticalOutcome</th>
<th>Definition</th>
<th>Source of Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retention</td>
<td>The relationship between behavior frequencies measured at two points in when a learner has not had the opportunity to perform the behavior</td>
<td>Fabrizio &amp; Moors (2003)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kubina &amp; Yurich (2012)</td>
</tr>
<tr>
<td>Endurance</td>
<td>The capacity for behavior to occur at a certain speed and accuracy level over extended periods of time; also increases resistance to distraction</td>
<td>Binder, Haughton, &amp; Van Eyk (1996)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kubina &amp; Yurich (2012)</td>
</tr>
<tr>
<td>Application</td>
<td>The capacity to easily apply the skill as a prerequisite or component of a more complex performance</td>
<td>Johnson &amp; Layng (1996)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kubina &amp; Morrison (2000)</td>
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<td></td>
<td></td>
<td>Kubina &amp; Yurich (2012)</td>
</tr>
<tr>
<td>Maintenance</td>
<td>The relationship between a behavior’s frequencies maintained across multiple points in time</td>
<td>Binder (1996)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kubina &amp; Yurich (2012)</td>
</tr>
<tr>
<td>Stability</td>
<td>The predictability of performance over multiple points in time; maintenance of performance in the presence of distractors</td>
<td>Binder (1996)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Johnson &amp; Layng (1992)</td>
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<tr>
<td></td>
<td></td>
<td>Lindsley (1995)</td>
</tr>
</tbody>
</table>

**Behavioral Fluency and the Learning Process**

Educators regularly plan instruction where students first concentrate on acquisition and conceptual understanding, then the quality and accuracy of the response (Binder, 2003; Archer & Hughes, 2011; Ardoin & Daly, 2007; NCTM, 2014). Practice interventions designed to build fluency (e.g. cover, copy, compare; incremental rehearsal) usually include rehearsal with immediate feedback followed by a timed assessment to
track progress. In the Precision Teaching literature, *frequency building* refers to the application of timed rehearsal followed by immediate feedback; students then take an additional timed assessment to track progress (Kubina & Yurich, 2012).

The metric used to calculate frequency of a timed performance involves count over the recorded time of an observed behavior (Johnston & Pennypacker, 2009). In mathematics behavioral intervention literature, researchers typically assign digits correct per minute or correct problems per minute to measure and record the frequency of a performance. When learners achieve certain frequencies of accurate performance they seem, to retain and maintain what they have learned, endure or persist for sufficient periods of time, even in the face of distraction (Binder, 1996). And as students apply newly minted fluent element skill(s) to a new compound skill, a recurring cycle of learning emerges starting with essential fluent element skill(s) merging into a compound skill. As the cycle moves forward the compound skill may become an element skill for a subsequent compound skill.

As noted above, behavioral fluency signifies a relationship between reaching performance standards, or frequency ranges of behavior, and associated critical learning outcomes. The critical learning outcomes that occur through frequency building support conceptual understanding and further achievement as students move through the hierarchical configuration of the mathematics curriculum. The aim of the present review involves locating empirical studies that provide evidence of critical learning outcomes after a student has reached a fluency performance standard in mathematics. In order to conduct the review, the researcher posed the following questions:
(1) What critical learning outcomes occur from students after attaining a performance standard in mathematics?

(2) Does the attainment of a performance standard or fluency aim have associated critical learning outcome(s) from the effects of mathematics interventions?

Method

The following criteria led to the inclusion of an article: (1) assessment of a particular skill for behavioral fluency, (2) an identified performance standard, (3) examination of a critical learning outcome of fluency (i.e., retention, endurance, application, maintenance, stability), (4) report an empirical investigation, and (5) appear in a peer-refereed journal. Search procedures included a computerized search of PsycINFO, ERIC, and ProQuest databases as well as Google Scholar from 1980 to 2016. Descriptors for the studies included the following word combinations: fluency and mathematics, fluency and computation, mathematics fluency and retention, mathematics fluency and endurance, mathematics fluency and application, mathematics fluency and maintenance, mathematics fluency and stability. A hand search examined articles contained in the Journal of Precision Teaching. Additionally, an ancestral search of reference lists from the articles determined by the above processes and related review articles (Codding et al, 2007; Codding, Burns, & Lukito, 2011) identified additional studies.

The initial search of on-line databases yielded six articles which met the inclusion criteria. An ancestral search of the six articles and two pertinent literature reviews generated three additional articles and an additional eight articles resulted from a hand search of the Journal of Precision Teaching. The qualifying 16 articles, noted with an
asterisk in the reference section meet the review criteria.

**Results**

The results consist of four sections – retention, application, maintenance, and multiple critical outcomes. Table 3 provides a summary of the studies conducted including critical learning outcome(s), and referenced performance standards for further review.

**Retention**

VanDerHeyden and Burns (2009) applied a year-long intervention designed to increase fluency with simple computation for students in grades two through five to establish grade-level fluency criteria leading to long-term retention. Students received the practice intervention via class wide peer tutoring four days per week with a changing criterion to gradually reach pre-established computational skills goals suggested by Deno and Mirkin (1977). Each week the students completed two weekly probes: one assessing the present simple computation skill and the other a mixed skill probe assessing retention of prior weekly skills. Students also took a monthly progress mixed-skill probe to monitor for year-end goals. Results from the investigation showed that students who exhibited retention required significantly higher fluency frequencies than recommended by Deno and Mirkin (1977). VanDerdHeyden and Burns (2009) recommended 60 DCPM for fourth and fifth grade students may better support retention.

In an effort to increase addition fact fluency performance of a student receiving special education services at a tutoring clinic, Bullara, Kimball, and Cooper (1993) applied frequency building with flash cards and practice sheets. Intervention components included assessment for accuracy, review of math rules, sprints (15, 30, or 45 seconds),
and at least two 30 second timings for the dependent variable. The participant entered performing at 20 digits correctly in 30 seconds, however, could not maintain that pace for a full minute. After attaining the performance standard of 70-90 (74) correct written responses in one minute the school recessed for summer break. Three months later in the fall, the researchers conducted a retention check. The participant exhibited a significant degree of retention by writing 56 correct digits in one minute.

In a large number of prior studies, the practice intervention Cover, Copy, and Compare (CCC) has shown to increase accuracy and fluency in multiple subjects such as reading, spelling, geography, and mathematics. Skinner, Bamberg, Smith, and Powell (1993) conducted a study that modified CCC by withdrawing the written component and substituting it with a sub-vocal component. The participants had to reach a performance standard of 40 digits correct per minute with no errors on three separate sets containing 12 division math facts in each. Participants had to meet the goal on the first set before moving onto the second set. All three participants met the performance standard during the intervention phase as well as maintained the standard eight months later on retention assessments.

**Application**

Five studies focused on application of element skill(s) to a compound skill. Chiesa and Robertson (2000) conducted a study in a fifth grade classroom with five learning support students. The balance of the class (20 students) served as the control. Participants practiced component skills (1-5 times tables) including (a) multiplication sheets emphasizing a particular multiplier, (b) finding the missing factor, (c) reciting facts with a peer, and (d) number writing. Students completed a one-minute assessment for
each component skill. The researchers set the performance standard of 40-50 correct responses (which included one and two digit responses) per minute. Participants took pretests and posttests of simple (2 digit by 1 digit without remainders) and more complex (2 digit by 1 digit with remainders) division problems with divisors ranging from ÷ 1 to ÷ 5. After 12 weeks, the results showed that the fluency training group outperformed the control group with the exception of one student. Only 50% of the control group exhibited any gain in performance with 35% scoring lower on the final assessment.

Successful entrance and completion of post-secondary education often depends on prospective undergraduates having a firm grasp of algebra skills (Adelman, 2006). Fitzgerald and Garcia (2006) led a study to improve the performance of five undergraduate students in a remedial algebra course that functioned as a pre-requisite for college level algebra coursework. Seven undergraduates in the same course functioned as the control. Presented in 12 sessions over a time period of six to nine weeks, the interventions consisted two to eight one-minute timings designed to increase math fact fluency in all four operations. Although only one participant met the performance objective of 80-100 correct responses in one operation and the group only averaging an increase of 6.14 digits correct per minute, the implementation of a fluency building intervention proved successful as the experimental group outperformed the matched controls by a mean of 15% in quizzes, exams, and course grades.

In an investigation to examine the relationship between accuracy and fluency in component to composite skills with simple to complex computation, Lin and Kubina (2005) screened 157 fifth grade students using explicit timing in a (3) one-minute assessment sequence. The researchers measured (a) writing speed, (b) accuracy and
fluency with multiplication facts, and (c) accuracy and fluency with multi-digit
multiplication problems. Performance standards included 80-120 digits correct per
minute performance standard for multiplication facts (element skill) and 40-60 digits
correct per minute for multi-digit multiplication (compound skill). Only 14% met the
standard for simple computation and 3.2% met the standard for complex computation.
The study reported a mean accuracy for component and composite skills at 98.12% and
88.40%, respectively. From the results of the investigation which included a high
correlation between component and composite skill fluency, Lin and Kubina (2005)
determined the speed and accuracy students exhibited would not support transition and
mastery of new and more complex calculation skills.

McDowell and Keenan (2002b) designed a study to evaluate the outcome of
frequency building with element skills prior to the compound skill versus frequency
building with the compound skill prior to the first element then the second element skills.
Researchers assigned participants to either +2, x2, and x4 written or pointing to keywords
as compound skills. Depending on the participant, element skills included (a) written
answers to +1, +2, and +4 problems (criteria of 70-90 DCPM), (b) written dots on a
number line to skip count by +2 (criteria of 80-100 dots per minute) or +4 (criteria of 60-
80 dots per minute), and/or (c) reciting numbers (60-80 numbers per minute), letter
sounds (60-80 sounds per minute). The study reported that the participants made the
most gains across all skills from fluency training starting with component skills versus
fluency training starting with only the composite skill.
### Table 3. Critical Learning Outcomes and Performance Indicators

<table>
<thead>
<tr>
<th>Reference</th>
<th>Reference Information</th>
<th>Performance Indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berens, Boyce, Berens, Doney, &amp; Kenzer (2003)</td>
<td>x x x x</td>
<td>R,M- 65 CPPM; E- 100 DCPM; A- CPPM</td>
</tr>
<tr>
<td>Beverley, Hughes, &amp; Hastings (2009)</td>
<td>x x</td>
<td>40-60 CPPM with up to 2 errors</td>
</tr>
<tr>
<td>Bullara, Kimball, &amp; Cooper (1993)</td>
<td>x</td>
<td>70-90 DCPM</td>
</tr>
<tr>
<td>Chiesa, M., &amp; Robertson, A. (2000)</td>
<td>x</td>
<td>40-50 correct responses per minute</td>
</tr>
<tr>
<td>Coddington, Eckert, Fanning, Shiyo, &amp; Solomon (2006)</td>
<td>x x</td>
<td>40 DCPM (Deno &amp; Mirkin, 1977)</td>
</tr>
<tr>
<td>Fitzgerald &amp; Garcia (2006)</td>
<td>x</td>
<td>80-100 Correct Responses per minute</td>
</tr>
<tr>
<td>Lin, F. Y., &amp; Kubina, R. M. (2005)</td>
<td>x</td>
<td>80-120 DCPM element; 40-60 DCPM compound</td>
</tr>
<tr>
<td>McDowell, C., &amp; Keenan, M. (2002)</td>
<td>x</td>
<td>70-90 DCPM; 80-100 dots per minute 60-80 dots per minute</td>
</tr>
<tr>
<td>McTiernan, Holloway, Healy, &amp; Hogan (2015)</td>
<td>x x x</td>
<td>50-70 CPPM</td>
</tr>
<tr>
<td>MacDonald, Wilder, &amp; Binder (2006)</td>
<td>x x</td>
<td>60-90 CPPM</td>
</tr>
<tr>
<td>Mong &amp; Mong (2010)</td>
<td>x x</td>
<td>32 DCPM</td>
</tr>
<tr>
<td>Nelson, Burns, Kanive, Ysseldyke (2013)</td>
<td>x x</td>
<td>40 correct problems within 2 minutes</td>
</tr>
<tr>
<td>Singer-Dudek &amp; Greer (2005)</td>
<td>x</td>
<td>100 DCPM</td>
</tr>
<tr>
<td>Skinner, Bamberg, Smith, &amp; Powell (1993)</td>
<td>x</td>
<td>40 DCPM/100% accuracy</td>
</tr>
<tr>
<td>Stromgren, Berg-Mortensen, &amp; Tangen (2014)</td>
<td>x x</td>
<td>70 CPPM</td>
</tr>
<tr>
<td>VanDerHeyden &amp; Burns (2009)</td>
<td>x x</td>
<td>Grades 2 and 3: 20 DCPM Grades 4 and 5: 40 DCPM</td>
</tr>
</tbody>
</table>

CRPM- correct responses per minute, DCPM=digits correct per minute, R=retention, E=endurance, A=application, M=maintenance, S=stability

### Maintenance

In order to increase multiplication facts fluency in adults with schizophrenia, MacDonald, Wilder, and Binder (2006) incorporated four phases within the study – baseline, accuracy, fluency, and follow-up (maintenance). During baseline phase, the researchers assigned participants one of ten worksheets that had 80 problems. After exhibiting stability, the accuracy phase began where participants had to reach 100% accuracy (successfully responding within 5 seconds of introducing the fact) on 10 sets of facts. The participants then started the fluency practice phase. Practice comprised of goal setting, prompting, verbal feedback, and one minute timings for two larger combined sets of facts. For the dependent variable, each participant completed a 15
second assessment at the end of each session. To attain a level of fluency the participants had to answer correctly 60-90 written problems transformed from a 15 second timing. Both reached the fluency criteria. The follow-up phase tested for maintenance occurring once a week for four weeks. Results showed that both participants maintained the fluency criteria for the duration of the study.

Mong and Mong (2010) compared the performance of two practice interventions: Cover, Copy, and Compare (a self-managed intervention) and Math to Mastery (teacher mediated) to evaluate which intervention best increased the fluent performance of three second graders with addition and subtraction math facts. Both interventions included the following components—modeling, practice, immediate feedback, and reinforcement. The researchers used an alternating treatments design counterbalancing the interventions on a daily basis. A fluency criteria of 32 digits correct per minute indicated mastery for second grade students (adapted from Deno & Mirkin, 1977).

After practicing with Cover, Copy, and Compare the participants completed a 2-minute CBM probe. Conversely, after practicing with Math to Mastery, the participants completed one minute timings until they reached the fluency criteria and then completed a 2-minute CBM probe. Results indicate that Math to Mastery increased student performance more rapidly than Cover, Copy, and Compare. Follow-up data to test for maintenance occurred six days and 18 days after the last day of the intervention. Results for maintenance showed that the participants maintained the fluency criterion.

In two separate experiments, Singer-Dudek and Greer (2005) focused on the maintenance of compound skills. The researchers assigned participants to either a mastery learning condition where participants practiced to 100% accuracy or a fluency
instruction condition where the participants practiced to a fluency aim of 100 DCPM on math facts while learning complex computation. In the first experiment, the participants practiced simple computation and the multiplication of two-digit by two-digit numbers. In the second experiment, the participants practiced simple computation to the division of three digits numbers evenly by either the numbers two or three. Results from both experiments demonstrated that participants learned the compound skills in a similar measure of time, however, students who practiced to fluency maintained the compound skill after a two-month period.

**Multiple Critical Outcomes**

**Endurance, Stability, and Application.** McTiernan, Holloway, Healy, and Hogan (2016) conducted a random control experiment to evaluate the effects of fluency building on five critical learning outcomes. Prior to intervention both groups of students completed the WIAT-II (Wechsler, 2005), a norm-referenced standardized test of mathematical achievement. Both groups also participated in pre-test and post-test measures designed to evaluate fluency, endurance, stability, and application. During intervention, students had to orally recite individual sets of fact families within four to six seconds each to meet the fluency building criteria. On separate written assessments, fluency aims ranged from 50-70 correct written responses on 26 target tasks.

Seventy-one percent of participants reached all 26 fluency aims. The balance of the participants met between 13-24 fluency aims. A statistically significant difference occurred between the two groups on the WIAT-II (Wechsler, 2005) mathematical reasoning subtest with the experimental group scoring higher \( t(13) = 2.52, p = .025, n_p^2 = .333 \) versus the control group exhibiting no significant difference \( t(13) = 3.43, p = \)
.211) from pre-test to post-test. Students in the experimental group also exhibited significant improvement in stability and endurance measures versus the control group. No statistically significant difference occurred between the experimental and control group in relation to application although the mean score of students in the experimental group yielded 13.8 correct responses on complex computation problems versus 11.1 correct responses for the students in the control group on a two-minute assessment.

**Retention, Endurance, and Application.** Berens et al. (2003) conducted three separate experiments that concentrated on retention, endurance, and application. The first study focused on retention from practice with simple computation using flash cards. Results suggest that the participants who emitted vocal responses closer to the fluency aim of 65 correct responses performed better on the retention probe assigned one month later. For the study of endurance, the participants vocally identified Arabic numbers in a specific place value. As with retention, the closer the participant performed to the fluency aim (100 digits per minute), the better the participant performed on the critical outcome—a five-minute test of endurance. In the third study that tested for application, the participants vocally identified place values starting at a lower level of difficulty (e.g. ones to tens) and then applied the skills to a higher level of difficulty (e.g. ones up to millions). During training of the target skill, the students also completed application probes at the next level of performance. The results showed that as the participants came closer to the fluency aim of 90 correct responses per minute for the target skill, the participants likewise showed improved performance on higher-level application probes that they had yet to receive instruction.
Retention and Application. Beverley, Hughes, and Hastings (2009) conducted a study of 55 psychology undergraduates exhibiting difficulty with statistics coursework. Drawing on the results from a pre-test, 24 students participated a Precision Teaching flashcards intervention (i.e. SAFMEDS, Say All Fast Minute Everyday Shuffled) condition and 31 students received treatment as usual in the control condition. The SAFMEDS cards consisted of key statistical terms from the textbook such as bimodal (as related to distribution) and square root (as a related to standard deviation and variance). Participants attended bi-weekly meetings to review charted progress and receive instructional guidance. Results indicated that students in the experimental condition scored higher score than the control condition on all of the statistics tests throughout the semester. The students in the experimental condition also achieved a statistically significant gain in post-test performance when compared to the control group (F(1, 53) = 5.23, p = .026, d = 0.62).

Nelson, Burns, Kanive, and Ysseldike (2013) compared the effects of a mnemonic strategy versus fluency rehearsal in the application of math facts to word problems. In the fluency rehearsal condition, the participants practiced using a computer software program. During a 5-15 minute practice session the software provided feedback for errors by having the participant repeat the problem and solution. The performance criteria established within the software consisted of 40 correct problems in two minutes before moving onto the next level. The interventions and data collection lasted six days allowing the participants to complete one level. The participants then completed an application assessment with 18 word problems. Although fluency practice showed
students had highest fluency scores, the results indicated no statistical effect to the application of fluency practice to the word problems.

**Application and Maintenance.** Codding et al. (2007) evaluated the effects of applying simple computation to complex computation with multiplication and the maintenance of the assigned math facts using three different iterations of the cover, copy, and compare practice intervention: (a) cover, copy, and compare, (b) cover, copy, and compare plus performance feedback using digits correct per minute, and (c) cover, copy, and compare plus feedback using digits incorrect per minute. Assessments for simple computation did not necessarily match the same problems practiced during the intervention.

Participants all met the noted performance standard of 40 DCPM (Deno & Mirkin, 1977) with participants reported reaching as high as 70 DCPM, 57 DCPM, and 43 DCPM. Two participants applied multiplication math facts to three-digit by one-digit with regrouping multiplication problems and one participant applied addition facts to three-digit by two-digit with regrouping addition problems. Codding et al. (2007) assessed application through a pre-test and post-test of complex computation. Two participants completed maintenance checks four days and 12 days after the intervention whereas one participant completed one maintenance check four days after the intervention. Results showed that all three participants maintained basic computation skills, however, the interventions only led to small increases in complex computation fluency.

**Endurance and Stability.** In an eight-week study of fifth, sixth, and seventh grade students, Stromgren, Berg-Mortensen, and Tangen (2014) tested for endurance and
stability with math facts using fluency criteria of 70 correct responses per minute. Participants in the experimental group received a Precision Teaching frequency building intervention. Practice included 30 second timings with feedback for a maximum of 10 timings per day. Endurance tests lasted 90 seconds. Stability tests lasted 30 seconds while students listened to music in headphones as a distractor. Although most students only reached between 55-65 in the experimental condition, Stromgren et al. (2014) reported participants who scored above 50 correct responses per minute in the experimental condition maintained scores reliably on the stability and endurance assessments.

**Discussion**

The present review examined frequency building and practice interventions used to increase behavioral fluency in mathematics. Behavioral fluency denotes a student’s ability to (a) retain information for later retrieval, (b) endure and complete tasks, (c) apply fluent skills to new learning scenarios, (d) maintain a level of fluent performance after achieving a standard or benchmark, and (e) exhibit a stable or predictable performance in the face of distraction – critical learning outcomes that support successful participation in the mathematics curriculum. Studies that qualified for the review had to include a pre-determined fluency criteria and a measurement of one or more of the associated critical learning outcomes. Findings indicate frequency building and practice interventions can successfully build speed and accuracy which in turn encourages behavioral fluency and the aforementioned associated critical learning outcomes (Binder 1996; Johnson & Lanyg, 1992; Kubina & Yurich, 2012). The majority of the students in the studies reached or came close to the fluency criteria within weeks or months versus
years of less structured and inconsistent practice (Kubina & Yurich, 2012; NMAP, 2008).

In the six studies that focused on retention, students exhibited comparable levels of performance on the retention measure when compared to the last day of intervention. Retention or the capacity to automatically recall an element skill over an extended period of time without practice supports students as they engage in compound tasks that require the skill. Conversely, students who hesitate (e.g. finger count, tally marks) in recalling number combinations and/or executing standard algorithms have shown to struggle following new and more complex material introduced by the teacher as well as keep up with the pace of instruction (Clarke, Nelson, & Chandley, 2016; NMAP, 2008). The studies applied a wide range of fluency criteria for the retention of single digit computation starting at 20-40 DCPM to upwards of 60-120 DCPM and 40 correct problems in two minutes. Despite the increase in student performance, an empirical question persists as to the criteria necessary to make the “retention useful or capable of being emitted, reinforced, and thereby maintained in its natural environment is the same as the frequency that will ensure retention of the behavior after a period of time in which it has not occurred (Binder, 1996, p.24).” Unfortunately, limited evidence exists to pinpoint grade-level or developmentally appropriate numerical standards for computational/behavioral fluency.

Maintenance entails measuring student performance on a set number of occasions (e.g. weekly, bi weekly) within closer proximity to the last day of intervention versus retention measures that typically occur after an extended period of time without intervention (e.g. one month). Maintenance provides educators and researchers with short-term numerical markers indicating whether students have sustained performance
gains made during frequency building or systematic practice. Maintenance also functions as an effective ‘in-between’ measure to discern whether the intervention worked and students will likely retain skills. Similar to the retention outcomes measures and fluency criteria, the five studies that measured maintenance reported students sustained levels of performance.

As students increase in frequency of responding, they typically build endurance and stability – the capacity to stay on task for an extended period of time and not engaging in competing stimuli that previously delivered higher levels of reinforcement. Theorists have attributed the increase in frequency or the repetitive, continual movement and associated reinforcement analogous to building momentum necessary to complete the practice sequence in the face of competing stimuli or “distraction” (Banda, Matuszny, & Therrien, 2008; Lee, 2006; Maag, 2007). When frequency of accurate responses increases, so does rate of reinforcement providing added benefits such as assignment completion and enhanced academic performance (Banda & Kubina, 2009; Lee, 2006). The process of building endurance and stability may also alleviate fatigue and propel students to “not give-up” or quit on element and compound skills as well as show more interest in the activity (Binder, Haughton, and Van Eyk, 1990; Brady & Kubina, 2010).

Although criteria and assessments varied between the three studies that measured endurance and/or stability, two researchers indicated students perform better when emitting more than 50 correct responses per minute on 90 second written endurance and stability assessments (Stromgren et al., 2014) and 100 oral responses per minute on five-minute endurance assessments (Berens et al., 2003). McTiernan et al. (2015) did not provide a specific numerical recommendation. Yet, the mean response of the treatment
group which included 29% of students who did not reach full criteria averaged approximately 40 DCPM over the five-minute endurance measure and 45.5 DCPM on the three-minute stability measure.

Fluent completion of complex compound skills typically entails the successful application of a number of discreet, element skills (Kubina & Yurich, 2012). Students who have reached a level of behavioral fluency have the ability to quickly recall elements as they build procedural fluency practicing a compound skill. For instance, within an order of operations problem a student frequently have to compute a combination of fractions, decimals, and integers then execute with procedural dexterity. The same persists in word problems where reading fluently plays an important role. Hence, a student who has a deficit in one or more element skillset(s) may experience limited gains in overall performance. Conversely, when a student has reached a level of fluency with element skills, the new compound skill can plausibly come together more seamlessly after acquisition.

Within the included studies that measured for application, five out of nine studies showed a positive outcome. Results suggest that students who performed at 70-100 DCPM with simple computation demonstrated superior outcomes on complex computation application measures than students in studies that incorporated fluency criteria below 50 DCPM. Still, when reviewing application studies, outcomes can prove difficult to interpret without comprehensive diagnostic fluency data across all the required element skills or when students have not reached a formidable level of fluency in the pertinent element skills prior to measuring application outcomes. Moreover, the outcomes researchers seek may occur more expeditiously when students apply the fluent
element skills to the systematic practice of the compound skill versus expecting an immediate effect or impact.

**Recommendations for Future Research**

Inconsistencies exist in the literature base that (a) link numerical fluency indicators to grade-level performance. The discrepancy in numerical indicators for fluency plausibly exists due to two separate reference points. The first originating from the Precision Teaching literature where recommendations begin with a minimum of 60 DCPM to 90+ DCPM for simple computation. The second originating from Deno and Mirkin (1977) and subsequent researchers (e.g., Shapiro, 1996; Shinn, 1989) who typically follow 40 DCPM. For practical interpretation, 40 DCPM allows students to emit responses above one second. Conversely, 60+ DCPM ensures students emit a digit in one second or less.

Within the included studies, VanDerHeyden and Burns (2009) suggests useful retention likely occurs closer to 60 DCPM versus Deno and Mirkin’s (1977) recommendation of 40 DCPM; Berens et al. (2003) suggests that optimal student performance occurs closer to 100 DCPM. Perhaps national initiatives such as NMAP (2008), NCTM (2000), and CCCSS (2010) have not recommended numerical standards due to a lack of research to provide empirical evidence. Therefore, replications of VanDerHeyden and Burns (2009) study can lend additional validity and reliability to pinpoint grade-level criteria. The Lin and Kubina (2005) investigation can also serve as a reference to further evaluate numerical benchmarks that link simple to complex computation (application). From appropriate grade-level criteria further studies can test differentiated curricular materials that meet the needs of the learner.
Another plausible direction for research includes conducting comparisons between frequency building versus practice interventions to discern whether one methodology works more efficiently and effectively in increasing behavioral fluency. Frequency building typically presents more opportunities to respond with feedback. Simply stated, if an individual student requires a specified number of practice opportunities to reach fluency on simple computation (Burns, Ysseldyke, Nelson, & Kanive, 2015) then the student should have the most efficient intervention designed to build accuracy and speed. Furthermore, efficiency that leads to behavioral fluency on precursor element skills within the hierarchical configuration of mathematics allows students to engage in compound skills then have the capacity to complete ‘more reps’ on later skills.

Unfortunately, the majority of the studies focus on simple and complex computation. Future studies that incorporate vocabulary, word problems, estimation, and computation (e.g. order of operations, solving for $x$) related to element and compound skills can extend the research base in mathematics.

**Conclusion**

Computational and procedural fluency supports conceptual understanding (NMAP, 2008). Research demonstrates that a fluent performance with math facts leads to more accurate complex computation and word problem solving (Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008; Fuchs et al., 2006; Hecht, Close, & Santisi, 2003) as well as stronger estimation skills (Dowker, 2003). Students who have a grasp on conceptual understanding of elementary skills but lack fluency can plausibly experience difficulty at different points in time keeping up with the pace of instruction especially
when conducting complex Algebraic concepts (Clarke et al., 2016). The studies included in the present review provide insight into interventions that lead to behavioral fluency in mathematics. The review also highlights the discrepancy in fluency criteria within the research literature of what constitutes mastery in simple computation. As suggested, further research to determine grade level criteria and the most effective fluency interventions can help students expeditiously increase behavioral fluency that yield critical learning outcomes.
Bibliography


Shapiro, E. S. (1996). Academic skills problems: Direct assessment and intervention (2nd


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Manuscripts Submitted for Publication