The Pennsylvania State University
The Graduate School
College of Engineering

A NEW OPTIMIZED DOUBLE STACKED TURNSTILE ANTENNA DESIGN

A Thesis in
Electrical Engineering
by
Mohamed Alkhatib

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Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

August 2016
The thesis of Mohamed Alkhatib was reviewed and approved* by the following:

James K. Breakall  
Professor of Electrical Engineering  
Thesis Advisor

Julio V. Urbina  
Associate Professor of Electrical Engineering

Victor Pasko  
Professor of Electrical Engineering  
Graduate Program Coordinator

*Signatures are on file in the Graduate School
ABSTRACT

The Turnstile Antenna is one of the many types of antennas that have been developed to be primarily used for omnidirectional very high frequency (VHF) communication. The basic turnstile consists of two horizontal half-wave antennas (half-wave dipoles) mounted at right angles of each other on the same plane. When these dipoles are excited with equal currents that are 90 degrees out of phase, the typical figure-eight radiation pattern of the two dipoles are merged into an almost circular radiation pattern. Typically, however, the gain of such an antenna is not very high in the horizontal direction, thus to increase the gain in the horizontal and to eliminate the gain in vertical direction, pairs of the same dipole antenna are stacked vertically and are separated by a distance, thus creating the Stacked Turnstile Antenna. In the original stacked turnstile antenna design, both 50 ohm and 75 ohm cables are used to connect the dipoles, using quarter wave impedance matching transformers, which leads to inaccurate radiation patterns and a higher VSWR. In this paper, we are addressing this issue by introducing a new design using only 75 ohm cables to achieve a more proper impedance matching and omnidirectional pattern. By using this approach, we are introducing a double stacked turnstile antenna that has a more circular radiation pattern, and a low VSWR at the frequencies of desired operation. Rigorous computer modeling and simulations codes were used for designing and evaluating the antenna. Furthermore, five models of this antenna were made, three computer models and two experimental models: an aluminum tubing model at 50.1 MHz which was done by Dr. James K. Breakall, a wire model at 700 MHz, and a cylindrical model at 700 MHz modeled by the author. All of these models were optimized using the Nelder-Mead Simplex method that FEKO provides. Two experimental models were also built, a scaled down antenna operating at 700 MHz (Mohamed’s Antenna) and a regular antenna operating at 50.1 MHz (Dr. James’s Antenna), where in both cases, the results agreed with the simulated models.
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ACKNOWLEDGEMENTS

I would like to take this opportunity to thank my parents and family for their indefinite support. If it was not for them, I would not have the strength and the conviction to stand by my decision to live abroad for five years each day. I would also like to thank my country, the United Arab Emirates for providing me with the means to continue my studies by covering all my expenses. I would also like to send a special thanks His Highness Sheikh Khalifa Bin Zayed AlNahyan, His Highness Sheikh Mohamed Bin Zayed AlNahyan, His Highness Sheikh Mohamed Bin Rashid AlMaktoum for setting up funds and scholarships and providing opportunities for people like me to pursue their dreams.

I would also like to thank Dr. James Breakall for accepting me as a graduate student and taking me under his guidance. Working under the guidance of Dr. Breakall has changed the way I think about the world and my entire view of it. Dr. Breakall is the best professor I have had the pleasure to study under at Penn State. Not only is Dr. Breakall an established figure in the field of antennas, he is also an outstanding teacher, because of his clarity in thoughts, which has enhanced my understanding of the subject. He shall forever serve as someone whom I inspire to be like in the future. I also extend my appreciation to Dr. Julio Urbina for taking time out of his schedule and agreeing to be a member of my committee on such short notice.
Chapter 1

Introduction

1.1 Summary of the Turnstile Antenna

The turnstile antenna [1] is a radio antenna that comprises a pair of dipole antennas aligned in such a way that they are at right angles to one another. The antenna becomes fed by currents supplied to the dipoles at an angle of 90° out of phase [2]. The derivation of the name of this type of antenna comes from the fact that it looks like a turnstile when someone mounts it horizontally. Typically, the gain of this antenna is almost 3 dB less than that of a single dipole in its direction of maximum radiation. This is due to the fact that each element receives only half of the transmitter power. The antenna becomes omnidirectional to the plane on the horizontal surface when horizontally mounted. The mode, in this case, is called the normal mode. On the other hand, it becomes circularly polarized and directional at 90° to its plane when mounted vertically [3]. That mode is called the Axial Mode.

1.1.1 Normal Mode

The turnstile antenna is in the normal mode when the axis of the antenna is vertical. Thus, it radiates polarized radio waves in a linear direction. When superposing the two dipole patterns to create a radiation pattern, it becomes omnidirectional [4]. The dipoles form a cloverleaf-shaped pattern having four maxima at the ends of each element. However, the patterns do not form an exact circle in the horizontal direction, there is an error margin of about +5% [5]. The operator
can easily increase the horizontal directions by stacking several turnstile antennas in a vertical manner, and fed in phase. Although this causes an increase in the gain in directions that are horizontal, it partially cancels the radiation that is in the vertical direction. The turnstile antennas stacked in the normal mode are useful for transmitting radio waves at frequencies of UHF and VHF used for television and FM broadcasting [6]. A typical stacked turnstile antenna is shown in Figure 1-1. The turnstile antenna operated in the normal mode is usually referred to as the George Brown turnstile antenna because of the person who invented it in 1935. [1]

![Image of the Double Turnstile Antenna](image-url)

Figure 1-1. Image of the Double Turnstile Antenna. [2]

1.1.2 Axial Mode

This arrangement involves placing the antenna in such a way that its axis is at a right angle to the plane of reference. Thus, it radiates circularly polarized radio waves. One of the radiated waves in one end is left hand circularly polarized while the other one is right hand circularly polarized [7]. The feed connections phasing usually determines the end that produces
which mode of circular polarization. A flat surface that acts as a conductor such as a metal screen reflector can be added since the directional antenna requires only one beam.

The waves become reflected back in that direction at 180° causes a reversal of the polarization, and hence these reflected waves strengthen radiation towards the forward direction [6]. The other method of increasing the radiation in the axial mode is through the replacement of every dipole with a Yagi array [5]. The turnstile antenna in the axial mode is useful for making missile and satellite antennas. It is because the gain of circularly polarized waves does is not easily affected by the antenna’s elements orientation.

1.2 Feeding the Antenna

For a turnstile to function in the appropriate way, the basic requirement is that the currents in each of the dipoles should have magnitudes that are equal and in phase quadrature [8]. One can use feed-line methods to carry out this process. This process is possible by the addition of the reactance in a way that the dipoles should be in series with each other. Another feed-line technique is the quadrature feed. It involves splitting the signal that comes from the transmission line into two signals that are equal. The operator usually does this using a two-way splitter, and then delay one of the signals by an additional transmission line of electrical length of 90°, after that, each phase gets applied to each of the dipoles. A basic outline of the turnstile setup is given in Figure 1-2.
Figure 1-2. The Basic Turnstile Setup.[3]
1.3 The Stacked Turnstile Antenna

![Diagram of double turnstile antenna]

Figure 1-3. The basic double turnstile setup. [9]

The stacked turnstile antenna is the basic turnstile antenna with dipoles stacked above each other vertically. It is commonly used for broadcasting TV due to its omnidirectional characteristics [9]. As an example of the stacked turnstile antenna, there is the double stacked turnstile, as shown in Figure 1-3, which is in this case used for Ham Radio purposes. Another
type of the stacked turnstile is the batwing antenna or super-turnstile antenna. With the advancements of the HDTV, the super-turnstile antennas are becoming more popular for UHF broadcasters who must share antennas with several transmitters simultaneously. Figure 1-4 below shows a typical batwing antenna.

![Image of the super-turnstile antenna](image.png)

Figure 1-4. Image of the super-turnstile antenna[^10].

The advantages of this antenna over the regular turnstile is the fact that the stacking allows for a strong for strong directivity and increases the gain horizontally. This antenna is a huge improvement over the original George brown Turnstile due to the stacking. The stacked turnstile is used often for many applications these days. Note that in the design in Figure 1-3, both 50 ohm and 75 ohm cables are being used.
1.4 Previous Work

The turnstile antenna in the normal mode, as previously mentioned, is mostly used for TV broadcasting. While a lot of work has been done in improving the antenna, the radiation patterns tend to be more circular for the single turnstile due to the amount of dipoles being fewer, where there are only 2 dipoles, as shown in Figure 1-2.

![Radiation Pattern of a typical single turnstile antenna](image.png)

Figure 1-5. Radiation Pattern of a typical single turnstile antenna \[^{[11]}\].
Looking at both patterns in Figures 1-5 and 1-6, respectively, it is clear that the radiation pattern is not circular. Moreover, if more stacks were added to the model, it is safe to assume that the radiation pattern will get even worse.
Chapter 2
Antennas and Electromagnetic Modeling

2.1 Antenna Theory

Whenever one tries to design an antenna, the input impedance and radiation patterns are of great concern to the antenna engineer. These parameters are calculated typically from both the electric field and the magnetic field. The far-field radiation can be determined through a multi-step process that starts with the vector potential $\vec{A}$ expressed in Equation 2.1:

$$\vec{A} = \iiint_V \mu \vec{J} e^{-j\beta \vec{R}} \frac{dV}{4\pi \vec{R}}$$

(2.1)

Where:

$\vec{J}$ is the source current density

$\vec{R}$ is the vector distance from the source point $r'$ to the observation point $r_p$.

Both the electric field $\vec{E}$ and the magnetic field $\vec{H}$ can be calculated from the vector potential by using Equations 2.2 and 2.3:

$$\vec{H} = \frac{1}{\mu} \nabla \times \vec{A}$$

(2.2)

$$\vec{E} = \frac{1}{j\omega \varepsilon} (\nabla \times \vec{H} \times \vec{J})$$

(2.3)
If the electric field is computed away from the current source region, then the current distribution $\mathbf{J}$ is zero, which simplifies the equation even further.

After finding the electric field and magnetic field, the total radiated power and input resistance can be calculated. The power radiated is expressed in Equation 2.4, which is done by integrating the real part of the complex Poynting vector over a surface located in the far-field.

$$P_{rad} = \frac{1}{2} Re \iint_S \mathbf{E} \times \mathbf{H}^* \, dS$$

(2.4)

The power delivered to an antenna is the sum of the radiated power plus power dissipated due to ohmic losses as shown in equation 2.5. The dissipative impedance $R_A$ of the antenna is made of radiation resistance $R_r$ and ohmic losses $R_{Ohmic}$. Energy is also stored in the near field, which is represented by the reactance $jX_A$ which represents the reactive part of the antenna input impedance. The combination of impedances in 2.6 yields the total input impedance of the antenna, which is represented as $Z_A$.

$$P_{in} = P_r + P_{ohmic} = \frac{1}{2} R_A |I_A|^2 = \frac{1}{2} R_r |I_A|^2 + \frac{1}{2} R_{Ohmic} |I_A|^2$$

(2.5)

$$Z_A = R_r + R_{Ohmic} + jX_A$$

(2.6)

The power that is not radiated away, is then stored in the antennae near field. The time average stored power $P_{st}$ as shown in Equation 2.7. $P_{st}$ depends on the stored electric and magnetic energies.

$$P_{st} = j2\omega (W_{mAverae} - W_{eAverage})$$
The time average stored electric and magnetic field energies are shown in the Equations 2.8 and 2.9:

\[ W_{e,\text{Average}} = \frac{1}{2} \iiint_{V} \frac{1}{2} \varepsilon |\vec{E}|^2 \, dV \] \hspace{1cm} (2.8)

\[ W_{m,\text{Average}} = \frac{1}{2} \iiint_{V} \frac{1}{2} \mu |\vec{H}|^2 \, dV \] \hspace{1cm} (2.9)

Once the currents have been determined, the antenna parameters are easily extracted. For example, the current distribution of a simple \( \frac{1}{2} \lambda \) dipole antenna can be approximated by a sinusoidal waveform. If the dipole is infinitesimally short, then the current can be simplified even further by assuming that it is constant across the dipole. However, with complicated geometries, the current distribution tends to be unpredictable and has to be solved for. This is done using computational electromagnetics which involves computers.

### 2.2 Numerical Solutions

The problems that are typically solved using computational electromagnetics are classified into two groups: Numerical Methods and asymptotic methods [13]. Depending on the problem size, the appropriate method is then chosen to solve the problem. As a rule of thumb, the transition between using asymptotic methods of electromagnetics or numerical methods falls on the order of \( 10\lambda \). To clarify, for a problem that fits within a sphere diameter of \( 10\lambda \) or smaller, then numerical methods are the methods of choice. When the problem size is higher than \( 10\lambda \), asymptotic methods are used. The computational power and solution time are also a major factor.
when choosing the appropriate method for solving a problem, where in some cases the numerical methods would be solved faster than the asymptotic methods and vice versa.

With the current problem at hand, when factoring in the required results, and the available computational power, the numerical based integral solver was chosen as the best method for solving the present problem.

Several integral equations are available to determine the unknown current distributions in a particular model. The general form of the equation is shown in Equation 2.10 for a conductor position parallel to the z-axis with length $l$.

$$\int_{-l/2}^{l/2} \tilde{I}(z')\overline{K}(z,z')dz' = \overline{E}^i(z)$$  \hspace{1cm} (2.10)

This particular equation is arranged in a way that the unknown current $\tilde{I}(z')$ is part of the integrand, i.e. the unknown is inside of the integral. The kernel, $\overline{K}(z,z')$ depends on the selected integral equation. $\overline{E}^i(z)$ is the applied electric field.

### 2.2.1 General solution technique

An approach for solving this type of equation involves approximating it with a system of linear equations [14]. The Method of Moments is a common procedure for solving such set of linear equations. A summary of the method of moments will be given below.

$$L(f) = g$$  \hspace{1cm} (2.11)
Starting with an inhomogeneous equation as in Equation 2.11, the unknown, is \( f \), \( L \) is a linear operator and \( g \) is a known value. The solution process is then started by letting the unknown, \( f \), be represented by a series expansion functions \( f_n \) weighted by unknown constants \( \alpha_n \) as shown below in equation 2.12

\[
f = \sum_n \alpha_n f_n
\] (2.12)

In this case, an infinite set of basis functions \( f_n \) can be determined, usually through an infinite summation and an exact solution to the problem can be then found; however, approximate solutions of the equation 2.12 occur when the summation is not infinite, i.e. finite. Equation 2.13 is found by substituting 2.12 back into 2.11 and thus we get the following inhomogeneous equation as a series:

\[
\sum_n \alpha_n L(f_n) = g
\] (2.13)

Now, a transformation from an equation to a set of equations has to be done, thus a set of weighting functions \( w \) are defined within the range of \( L \) for taking the inner product with Equation 2.13, which results in Equation 2.14:

\[
\sum_n \alpha_n \langle w_m, Lf_n \rangle = \langle w_m, g \rangle
\] (2.14)

To translate into matrix notation:

\[
[l_{mn}][\alpha_n] = [g_m]
\] (2.15)
Where:

\[
[l_{mn}] = \begin{pmatrix}
\langle w_1, l f_1 \rangle & \langle w_1, l f_2 \rangle & \cdots \\
\langle w_2, l f_1 \rangle & \langle w_2, l f_2 \rangle & \cdots \\
\vdots & \vdots & \ddots
\end{pmatrix}
\]

(2.16)

\[
[\alpha_n] = \begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots
\end{bmatrix}
\]

(2.17)

\[
[g_m] = \begin{bmatrix}
\langle w_1, g \rangle \\
\langle w_2, g \rangle \\
\vdots
\end{bmatrix}
\]

(2.18)

Then by inverting the \( I \) matrix, and multiplying it by the known function \( g \), the unknown constants for the expansion functions can now be found by using the following Equation 2.19:

\[
[\alpha_n] = [g_m][l_{mn}]^{-1}
\]

(2.19)

Having properly dealt with the constants, the original unknown of the function can now be solved for appropriately.

### 2.2.2 Applying the Method of Moments to the Integral Equation

If the method of moments is to be applied to the integral equation, several approaches are then available for solving for the unknown currents. Three well known kernel \( \mathbf{K}(z, z') \) can be used with or without the weighting functions. Kernels of choice will affect the problem size and the accuracy, while on the other hand, the weighting functions determine how the boundary conditions are being enforced. If we neglect the use of the weighting functions, we are left with the simplest approach for solving for the unknown currents and understanding the solution method, and the approach is point matching.
Point matching is a method where the only boundary condition that is being enforced is that the tangential electric field on the conductor surface is zero at discrete points. In between these points however, the condition may not be satisfied. Nevertheless, with point matching, it is assumed that the boundary condition is not violated tremendously to the point where the solution is extremely wrong. This error can also be reduced at the expense of time, where more segments can be added and the boundary condition is applied to more sections of the conductor, however, the more segments that are added, the more time it will take for the computer to solve the problem.

For point matching of the line current $\vec{I}(z')$ as in Equation 2.10; it is represented as a series of unknown functions as shown in Equation 2.20 below.

$$\vec{I}(z') = \sum_{n=1}^{N} \vec{I}_n \vec{F}_n(x', y')$$  \hspace{1cm} (2.20)

Where $F_n$ are the chosen expansion functions.

For surface currents in the x-y plane, the expansion becomes

$$\vec{j}(x', y') = \sum_{n=1}^{N} \vec{I}_n \vec{F}_n(x', y')$$  \hspace{1cm} (2.21)

Substituting everything back into Equation 2.10 yields the following

$$- \int_{-\frac{L}{2}}^{\frac{L}{2}} \sum_{n=1}^{N} \vec{I}_n \vec{F}_n(z') \vec{K}(z_m, z') dz' \approx \vec{E}_z(z_m)$$  \hspace{1cm} (2.22)

Note that adding the subscript m on $z_m$ indicates the boundary condition is being enforced on the segment m. The following equation can now also be represented in matrix notation as it was done earlier for equation 2.14.
\[ [Z_{mn}] | I_n] = [V_m] \]  \hspace{1cm} (2.23)

Where in this case, the complex expansion coefficients \( I_n \) are unknown and

\[ Z_{mn} = - \int_{\Delta x_n} \tilde{P}_n(z') \tilde{R}(z_m, z') dz' \]  \hspace{1cm} (2.24)

\[ V_m = \tilde{E}_z^i(z_m) \]  \hspace{1cm} (2.25)

In this case, it is observed that Equation 2.23 holds similarities to Kirchhoff’s network equations.

Now, to solve this problem using weighted functions, the boundary condition is enforced on a weighted average over the entire surface. The actual method of moments technique is used when the weighted residuals are used with conjunction of the inner product to minimize the deviation from the boundary condition, where the actual deviation of the true boundary condition from the match points boundary condition is defined as the residual in Equation 2.26 below [2].

\[ \Delta \tilde{E}_{Tan} = \tilde{E}_{Tan}^{inc} + \tilde{E}_{Tan}^{ref} \neq 0 \]  \hspace{1cm} (2.26)

Using then, the pulse expansion function to represent the scattered field, the residual for a wire along the \( z \)-axis becomes

\[ R(z) = - \sum_{n=1}^{N} \tilde{I}_n \int_{\Delta x_n} \tilde{R}(z, z'_n) dz' + \tilde{E}_{z}^{inc}(z) \]  \hspace{1cm} (2.27)

By this, the residual is defined as the difference between the solved scattered field and the incident field. When the weighted residuals are used in conjunction with the inner product, the series of expansion functions now becomes

\[ \sum_{n=1}^{N} \tilde{I}_n \langle \tilde{W}_m, \tilde{P}_n(z') \rangle = \langle \tilde{W}_m, \tilde{I}(z') \rangle \]  \hspace{1cm} (2.28)

This leads to the integral in Equation 2.10 that becomes the following
\[
- \int_{-L/2}^{L/2} \tilde{W}_m(z) \sum_{n=1}^{N} \tilde{I}_n \tilde{F}_n(z') \tilde{K}(z_m, z') dz' + \int_{-L/2}^{L/2} \tilde{W}_m(z) E_z^i(z_m) dz = 0
\]
The matrix notations remain the same; however, \( Z_{mn} \) and \( V_m \) are defined as follows

\[
Z_{mn} = - \int_{\Delta z_m} \int_{\Delta z_n} \vec{F}_n \vec{K} \vec{R}(z_m, z')dz
\]  

\[
V_m = \int_{\Delta z_m} \vec{E}_z^i(z_m)dz
\]

Using the weighted residuals to get the current solutions will enforce a boundary condition on average over the wire as opposed to when neglecting the weighted residuals. Having the boundary condition enforced on an overall basis gives a better representation of the currents along the wire. The greatest factors to consider when representing the unknown currents are the choice of expansion functions, the kernel and finally the weighting residuals. One method is to pick the expansion and weighting functions such that they closely represent the actual current distribution, in this circumstance, when the same function is used both for expansion functions and the weighting functions, this is called the Galerkin’s Method. Finally, the kernel choice is influenced mostly by the complexity of the scattering structure and the ability to write the proper computer code. The final solution of the problem, after all, depends greatly on these three factors, the expansion functions, the weighting functions and the kernel.
2.2.3 Other Integral Equations

Another famous integral equation that makes use of the kernels is the Pocklington’s equation for an infinitely conducting wire with length L as shown in Equation 2.32. With the assumption of a highly conductive wire, the unknown currents can be taken to be on the surface of the wire instead of being within the volume. Due to that, the volume current is replaced with a surface current which in result allows for the use of the vector potential in the electric field integral equation. Furthermore, if we assume that the wire is electrically thin and the currents are symmetric with respect to the wire cross sectional area, additional simplifications can be made.

\[
\vec{E}_z = \frac{1}{j\omega \varepsilon_0} \oint \int \left[ \frac{\partial^2 \psi(z,z')}{\partial z^2} + \beta^2 \psi(z,z') \right] \vec{J}_s dz' d\phi
\]

(2.32)

The derivation of the potential form of the electric field integral equation where both vector and scalar potentials are being utilized in Equation 2.33 was done by Harrington [14]. The use of this form of equation, depending on the problem, can provide a faster convergence than Pocklington’s integral equation.

\[
\vec{E}_z^i = \int_{-L/2}^{L/2} \left[ j\omega \mu_0 \vec{I}(z') - \frac{1}{j\omega \varepsilon_0} \frac{\partial \vec{I}(z')}{\partial z'} \frac{\partial}{\partial z'} \right] e^{-j\beta \vec{R}} d\vec{z}'
\]

(2.33)

Finally, Hallen’s integral Equation, which is the simplest to work with. It is shown in Equation 2.34. The extra unknown constant C₁ requires one extra equation than the number of unknowns. In this case, the excitation field is represented in the form of voltage, where it is assumed that Vₐ is placed at the antenna’s feed point.
\[
\int_{-\ell/2}^{\ell/2} \tilde{I}(z') e^{\frac{-j \beta R}{4\pi R}} dz' = -\frac{j}{\eta} \left( C_1 \cos \beta z + \frac{V_A}{2} \sin \beta |z| \right)
\]

(2.34)

### 2.3 FEKO

FEKO is one of the many electromagnetics commercialized antenna modeling and simulation software packages. The problem at hand was modeled, optimized and simulated through FEKO. FEKO utilizes the method of moments when solving for the unknown current distributions through the wire. Further details on how FEKO utilizes the weighting and expansion functions will be discussed in section 2.3.1. Moreover, FEKO has four optimization tools, Grid search, Simplex Nelder-Mead, Genetic Algorithm and Particle Swarm. Each of the following tools of optimization has its own unique and distinct features. Each of these tools allows the user to optimize the parameters to his desired specification. There are five total goals that the user can choose from, Impedance goal, Near-Field goal, Far-Field goal, S-parameter goal, and SAR goal. With the impedance goal, the user is allowed to control the following: Input impedance, input admittance, Reflection Coefficient, Transmission coefficient, VSWR and return losses. The Near-Field goal allows the user to control the E-field, H-field, Directional component and coordinate system. The Far-Field goal allows the user to specify the E-field (in the far-field), the gain, the directivity and RCS. Finally, the S-parameter goal allows the user to control and specify the coupling, reflection and transmission coefficients, VSWR and return losses.
2.3.1 Expansion Functions, Weighting Functions and FEKO

The choice of expansion functions results in different approximations for the current distributions on the structure. Faster convergence of the solution is very much possible if the expansion function is chosen to very closely represent the actual current. Just as the model in any electromagnetics modeling software is divided into sections, the unknown currents are also similarly expressed in subdomain functions that are defined over each individual segment. This division of the expansion functions allows it to represent the current distribution without knowledge of the actual current distribution. By overlapping subdomain functions, the actual distribution is very closely approximated. However, due to the expansion functions not having smooth properties as opposed to the actual currents distribution, this could cause the approximation to miss sharp changes in the actual distribution, thus care should be taken.

The simplest expansion function is the piecewise constant that represents the current distributions with pulses as shown in Equation 2.35.

\[ F_n(z) = \begin{cases} 1 & z_{n-1} \leq z \leq z_n \\ 0 & Otherwise \end{cases} \]  

(2.35)

For smoother representation of the current distributions, overlapping sets of triangles or sinusoidal functions can be used. FEKO uses triangle functions or piecewise linear equations to represent the currents on wires as shown in Equation 2.36

\[ F_n(z) = \begin{cases} \frac{z - z_{n-1}}{z_n - z_{n-1}} & z_{n-1} \leq z \leq z_n \\ \frac{z_n - z_{n-1}}{z_{n+1} - z_n} & z_{n-1} \leq z \leq z_{n+1} \end{cases} \]  

(2.36)

Other subdomain functions include the following:
piecewise sinusoidal as shown in Equation 2.37

\[ F_n(z) = \begin{cases} \frac{\sin[\beta(z - z_{n-1})]}{\sin[\beta(z_n - z_{n-1})]} & z_{n-1} \leq z \leq z_n \\ \frac{\sin[\beta(z_{n+1} - z)]}{\sin[\beta(z_n+1 - z_n)]} & z_{n-1} \leq z \leq z_{n+1} \end{cases} \tag{2.37} \]

Truncated Cosine Equation as in Equation 2.38

\[ F_n(z) = \begin{cases} \cos \left( \frac{\beta(z - z_{n-1})}{2} \right) & z_{n-1} \leq z \leq z_n \\ 0 & Otherwise \end{cases} \tag{2.38} \]

Sinusoidal interpolation as in Equation 2.39

\[ F_n(z) = \begin{cases} A_n + B_n(z - z_n) + C_n \cos \beta(z - z_n) & z_{n-1} \leq z \leq z_n \\ 0 & Otherwise \end{cases} \tag{2.39} \]

Equations 2.37, 2.38, and 2.39 all deal with current representations on a line. For approximation of surface currents, other representations are required. Dr. Sadasiva M. Rao, was the first to initially use triangular mesh with Roof Top functions to represent surface currents. The details are in his Thesis [15]. In FEKO, objects that are constructed with triangular meshing use Roof Top functions for estimating the currents [16]. The Roof Top functions are applicable to both triangular and square meshes [17]. For a square mesh with edge centers located at \((x_n, y_n)\) and cell dimensions of \(A\) by \(B\) in the \(x-y\) system, the Roof Top is accordingly shown in Equation 2.40

\[ \begin{align*} B_{xn}(x, y) &= t(x; x_n - a, x_n, x_n + a)p \left( y; y_n - \frac{1}{2}b, y_n + \frac{1}{2}b \right) \\ B_{yn}(x, y) &= t(y; y_n - b, y_n, y_n + b)p \left( x; x_n - \frac{1}{2}a, x_n + \frac{1}{2}a \right) \end{align*} \tag{2.40} \]

Where Roof Top representation for triangles is as follows:

\[ B_i = \frac{\omega_i}{4A^2} \left\{ \begin{array}{l} \hat{x}\left[ a_j c_k - a_k c_j + (b_j c_k - b_k c_j)x \right] \\ \hat{y}\left[ a_j b_k - a_k b_j + (c_j b_k - c_k b_j)y \right] \end{array} \right\} \tag{2.41} \]
This is but a short list of functions that can be choices for the representation of currents. Many others do exist and have a dependency on many factors; for example, wire to wire and plate to wire junctions can have functions of their own, higher number of conductors joined at a node can also require special functions. Functions greatly affect the convergence and the accuracy of the results.

2.4 Method of Optimization

While there are many tools of optimization around, FEKO offers four tools; grid search, Nelder-Mead Simplex Method, Genetic Algorithm and Particle Swarm. To determine the best course of optimization for the current problem, a tool of optimization had to be picked out of the mentioned four.

Looking at each tool of optimization individually, and according to FEKO [18], grid search is not strictly a tool of optimization where it is just a technique that linearly scans the solution space, and determines the optimum value upon completion. This method is also computationally very expensive, and due to that, it is not the recommended tool of optimization for a solution space with more than two variables. Considering that the current problem has more than two variables to optimize, grid search was deemed unfit for optimizing the turnstile antenna. On the other hand, while the Genetic Algorithm and Particle Swarm methods are very accurate, they are also computationally extremely expensive, where it would take over 12 hours to optimize a problem such as the Turnstile Antenna, thus due to that limitation, they too, were also out of consideration to be the choice in optimizing the Turnstile. Finally, we look at the Nelder-Mead Simplex method, where the optimization results are greatly influenced by the user’s initial values and limits. In the case of the turnstile, this method is ideal, due to the fact that all the variables that are going to be optimized have been already calculated with basic antenna equations.
2.4.1 Nelder-Mead Method

The Nelder-Mead Method [19] is a very widely used method of optimization, where it is used to optimize unconstrained problems without the use of derivatives. Due to the fact that this method does not use derivatives, it is very suitable for problems with non-smooth functions or discontinuous functions. It is a very widely used method of optimization for real life small dimensional problems.

Due to the dimensions of the turnstile antenna being rather small, this allows the use of the N-M simplex method when optimizing the antenna.

Dr. Fuchang Gao and Dr. Lixing Han have done an excellent introduction and an outline in their paper about the Nelder-Mead Method which is given below. Further details about the topic can be found in their paper. [20]

Consider the following:
\[
\min f(x),
\]
\[(2.42)\]

Where \( f: \mathbb{R}^n \to \mathbb{R} \) is called the objective function and \( n \) is the dimension and denoting a simplex with vertices \( x_1, x_2, \ldots, x_{n+1} \) by \( \Delta \).

Simplex is a geometric figure in \( n \) dimensions that is the convex hull of \( n+1 \) vertices.

The N-M method then generates a sequence of simplexes to approximate the optimal point of \( \min f(x) \). With each sequence, the vertices of the simplex are ordered according to the functions values.
\[
f(x_1) \leq f(x_2) \leq \cdots \leq f(x_{n+1})
\]
\[(2.43)\]
Where in this case, \( x_1 \) is referred as the optimum vertex, and \( x_{n+1} \) is the worst vertex; however, several vertices might hold the same objective value, in that case, tie breaking rules should be set for this method to properly work.

The tie breaking algorithm uses four possible operations: expansion, contraction, reflection and shrinkage. Each of the following operations are associated with the following parameters: \( \alpha, \beta, \gamma, \) and \( \delta, \)

Where:

\( \alpha \) is reflection
\( \beta \) is expansion
\( \gamma \) is contraction

and \( \delta \) is shrinkage.

In the general N-M simplex method, the following parameters are chosen:

\[
\{ \alpha, \beta, \gamma, \delta \} = \left\{ 1, 2, \frac{1}{2}, \frac{1}{2} \right\}
\] (2.44)

We denote \( \bar{x} \) to be the centroid of the best \( n \) vertices, thus we end up with the following equation 2.45:

\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i
\] (2.45)

One iteration of the Nelder-Mead Algorithm starts with the following steps:

1. **Sort**: Evaluate \( f \) at \( n+1 \) vertices of \( \Delta \) and sort the vertices so that Equation 2.43 holds true.

2. **Reflection**: Compute the reflection point \( x_r \), from
\[ x_r = \bar{x} + \alpha (\bar{x} - x_{n+1}) \]

and evaluate \( f_r = f(x_r) \); if \( f_1 \leq f_r \leq f_n \), replace \( x_{n+1} \) with \( x_r \)

3. **Expansion:** If \( f_i < f_i \) then compute expansion point \( x_e \) from

\[ x_e = \bar{x} + \beta (x_r - \bar{x}) \]

and evaluate \( f_e = f(x_e) \). if \( f_e < f_r \), replace \( x_{n+1} \) with \( x_e \); otherwise replace \( x_{n+1} \) with \( x_r \)

4. **Outside Contraction:** if \( f_n \leq f_r \leq f_{n+1} \) compute the outside contraction point \( x_{oc} \) from

\[ x_{oc} = \bar{x} + \gamma (x_r - \bar{x}) \]

and evaluate \( f_{oc} = f(x_{oc}) \). if \( f_{oc} \leq f_r \), replace \( x_{n+1} \) with \( x_{oc} \); otherwise go to step 6

5. **Inside Contraction:** if \( f_r \geq f_{n+1} \), compute the inside contraction point \( x_{ic} \) from

\[ x_{ic} = \bar{x} - \gamma (x_r - \bar{x}) \]

and evaluate \( f_{ic} = f(x_{ic}) \). if \( f_{ic} < f_{n+1} \), replace \( x_{n+1} \) with \( x_{ic} \); otherwise go to step 6

6. **Shrinkage:** for \( 2 \geq i \geq n+1 \), define

\[ x_i = x_1 + \delta (x_i - x_1) \]

These six steps summarize and outline how the Nelder-Mead method iteration works, where it would go through this process over and over until the optimum values are found. While this method might not be the optimal choice for large dimensional structures when optimizing, in which case, the Particle Swarm or the Genetic Algorithm tools of optimization would be more...
ideal, the simplex method have worked perfectly fine for a small dimensional antenna like the
turnstile.
3.1 Antenna Model

After thoroughly studying the original double turnstile antenna, shown in Figure 1-4, and trying to optimize it using the regular 75 and 50 ohm cables, it was deduced that there is a quarter wave impedance matching problem, where the cable impedances were not matching properly and in result, causing the VSWR to be higher than ideal and the radiation pattern to sway away from being circular. From the previous design as shown in Chapter 1, Figures 1-5 and 1-6, the radiation patterns are not circular, in fact, as you increase the number of stacks, the pattern tends to sway away from being circular, due to the number of cables increasing and resulting in impedance matching becoming more difficult. Figure 1-4 shows the original design of the double turnstile, where it exclusively mentions that the voltage standing wave ratio (VSWR) of that design is 1.4. Figure 1-4 is also the configuration that our antenna was modeled after.

The first solution that comes to mind is to select all the cables to have the same impedances, however, the antenna does not respond very well if all cables are matched to 50 ohms. Thus, going to the next common cable of 75 ohms is tried. The next step is to optimize the antenna. However, in order to optimize the antenna, the parameters that are needed to be optimized must first be figured out. Looking at Figure 1.4 again, we can clearly deduce that the parameters that hold the most effect over the radiation pattern of this particular antenna are summarized in Table 3-1.
Table 3-1. List of Optimized Variables.

<table>
<thead>
<tr>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the Dipoles</td>
</tr>
<tr>
<td>Spacing Between the Stacks</td>
</tr>
<tr>
<td>Length of Transmission Line 1</td>
</tr>
<tr>
<td>Length of Transmission Line 2</td>
</tr>
<tr>
<td>Length Transmission Line 3</td>
</tr>
<tr>
<td>Length Transmission Line 4</td>
</tr>
</tbody>
</table>

With the parameters determined, the next step is to set the goals of optimization; however, in order to optimize the antenna to get a fully circular pattern, certain goals had to be given to the optimizer, as there is no option to exactly set the goal of radiation pattern to be circular. A smart way around that issue is to set two different goals, one to maximize the minimum gain, and another is to minimize the maximum gain, thus that way the optimizer would work in such a way that once it finds the maximum gain, it minimizes it, and vice versa with the minimum gain. A final goal was also set, which is to minimize the voltage standing wave ratio (VSWR).

3.2 The Double Turnstile at 50.1 MHz

Before determining the re-design with the antenna, a model of Figure 1-4 had to be done, with everything taken into consideration, a wire model of the double turnstile was deemed
enough of an approximation to the real double turnstile antenna at a frequency of 50.1 MHz.

Figure 3-1 shows the FEKO model of the antenna.

Figure 3-1. Image of the FEKO Wire Model.

Figure 3-2 shows the network schematic of antenna.

Figure 3-2. Image of the Wire Model Network Schematic.
With the model done, the next step is to determine the limits of optimization and the initial values. Considering this model is done at 50.1 MHz, and the fact that this model represents an actual antenna above ground, not in free space, the effect of the ground plane has to be taken into account as well, thus a new parameter is introduced that describes the height of the antenna above ground.

However, due to the original design using of 50 ohms and 75 ohm cables, the calculated lengths of dipoles and transmission lines were not appropriate for this new design. With that in mind, a technique around this is to use the optimizer a multiple of times and to increase or decrease the limits to give room for the optimizer to get the best possible outcome on each session. Once the optimum values are determined, they then serve as initial values in the next optimization session with the limits of optimization decreasing or increasing accordingly. This was done numerous of times in order to get the best possible results out of the optimizer that fit our design criteria. This method of multiple trials was only possible due to the choice of optimization tool, the Nelder-Mead Simplex method, where the initial specifications greatly influence the optimum value; this unique and distinct feature of the N-M simplex method made optimizing the antenna to our new design much easier. The “final” initial values and limits of optimization are given in Table 3-2 below.

Table 3-2. Parameters Initial Values and Limits

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
<th>Initial Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the Dipoles</td>
<td>1.2 m</td>
<td>1.7 m</td>
<td>1.4 m</td>
</tr>
<tr>
<td>Spacing Between the Stacks</td>
<td>3 m</td>
<td>5 m</td>
<td>3.2 m</td>
</tr>
<tr>
<td>Length of Transmission Line 1</td>
<td>5 m</td>
<td>8 m</td>
<td>6.8 m</td>
</tr>
<tr>
<td>Length of Transmission Line 2</td>
<td>5 m</td>
<td>8 m</td>
<td>6.4 m</td>
</tr>
<tr>
<td>Length Transmission Line 3</td>
<td>0.1 m</td>
<td>4 m</td>
<td>3 m</td>
</tr>
<tr>
<td>Length Transmission Line 4</td>
<td>2 m</td>
<td>6 m</td>
<td>4.2 m</td>
</tr>
<tr>
<td>Height Above ground</td>
<td>2 m</td>
<td>6 m</td>
<td>3 m</td>
</tr>
</tbody>
</table>
3.2.1 Wire Model Results

With the optimization done, Table 3-3 summarizes the optimum value of each of the optimized parameters.

Table 3-3. Parameters Optimum Values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Optimum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the Dipoles</td>
<td>3.176 m</td>
</tr>
<tr>
<td>Spacing Between the Stacks</td>
<td>3.742 m</td>
</tr>
<tr>
<td>Length of Transmission Line 1</td>
<td>7.170 m</td>
</tr>
<tr>
<td>Length of Transmission Line 2</td>
<td>6.753 m</td>
</tr>
<tr>
<td>Length Transmission Line 3</td>
<td>1.765 m</td>
</tr>
<tr>
<td>Length Transmission Line 4</td>
<td>3.874 m</td>
</tr>
<tr>
<td>Height Above ground</td>
<td>3.991 m</td>
</tr>
</tbody>
</table>

The VSWR of the antenna dropped significantly when using these specifications along with the proposed design, as shown in Figure 3-3.

Figure 3-3. VSWR of the wire model.
While it is not very clear from the figure, the VSWR has dropped from 1.4 in the original design to 1.1 at 50.1 MHz. The bandwidth of the antenna is wide as well.

Since this antenna was optimized at a frequency of 50.1 MHz, the radiation patterns have been taken at different frequencies to show the difference as we get closer to the frequency which we have optimized at. Figures 3-4 through 3-8 shows the radiation pattern in ascending order of frequencies from 49 MHz to 52 MHz.

Figure 3-4. Wire Model radiation pattern at 49 MHz
Figure 3-5. Wire Model radiation pattern at 49.5 MHz.
Figure 3-6. Wire Model radiation pattern at 50.1 MHz.
Figure 3-7. Wire Model radiation pattern at 51 MHz.

Figure 3-8. Wire Model radiation pattern at 52 MHz.

Table 3-4 summarizes the results of each radiation pattern as shown.

Table 3-4. Results of the wire model radiation patterns

<table>
<thead>
<tr>
<th>Frequency of Operation</th>
<th>Minimum Gain</th>
<th>Maximum Gain</th>
<th>Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>49 MHz</td>
<td>4.488 dBi</td>
<td>9.86 dBi</td>
<td>5.37 dB</td>
</tr>
<tr>
<td>49.5 MHz</td>
<td>5.88 dBi</td>
<td>9.75 dBi</td>
<td>3.88 dB</td>
</tr>
<tr>
<td>50.1 MHz</td>
<td>6.90 dBi</td>
<td>9.22 dBi</td>
<td>2.33 dB</td>
</tr>
<tr>
<td>51 MHz</td>
<td>5.53 dBi</td>
<td>9.91 dBi</td>
<td>4.39 dB</td>
</tr>
<tr>
<td>52 MHz</td>
<td>2.29 dBi</td>
<td>11.11 dBi</td>
<td>8.82 dB</td>
</tr>
</tbody>
</table>

Where Δ is the difference between the maximum gain and the minimum gain.
It is clear from Table 3-4 and the radiation patterns, and the summarized results that the antenna has been significantly improved with this new design. It is also shown that as you get closer to the frequency of optimization, \( \Delta \) becomes smaller as expected.

This model worked perfectly fine at 50.1 MHz, where the thin wires can be used to approximate dipoles, however, when going to a higher frequency, in the next case, 700 MHz, the thin wire approximation no longer holds true due to the radius of the dipoles becoming too large.

Regardless of the FEKO’s issues when it comes to meshing the wire model at higher frequencies, at 50.1 MHz, the VSWR and radiation pattern of this antenna did yield results that were more promising than the original double turnstile results. However, these results have to be confirmed by an actual experimental model.

### 3.2.2 Experimental Antenna Results at 50.1 MHz

To confirm the results of the wire model at 50.1 MHz, a prototype antenna at has been built according to the parameters that were obtained from the wire model. Figure 3-9 shows the Antenna that was built according to the results obtained. Figure 3-10 shows the wiring of the Double Turnstile Antenna.
When building the antenna two things were taken into consideration, approximating the optimum parameters are close as possible, and taking into account the effect of velocity of
propagation of the transmission lines. Velocity of propagation indicates how the wave propagates inside a transmission line, which means that the wavelength is different in the transmission line that it is in air.

RG59 is the cable of choice for the transmission lines, those cables typically have a velocity of propagation of 66%, which means that the transmission lines will be smaller (66%) for the same wavelength that the line was for 100%.

Ferrite beads were used to suppress high frequency noises on the RG59 cable. In this case, since the antenna is to be operated at a frequency of 50.1 MHz, a material 31 ferrite bead has been used on the cable, with 4 beads covering each end (at the feed point) of the cable. Note that the length of the ferrite bead used in this case is about 0.531 inches, which allows for more beads to be stacked at each end, as opposed to the 1.125 inches long beads.

Looking at table 3-3, it is also clear that some approximation had to be done in order to properly cut the pieces of the antenna, it is very hard to actually cut everything to that precision without the use of special machinery. Table 3-5 shows the approximated values with the velocity of propagation taken into account.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Ft and Inches</th>
<th>Meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the Dipoles</td>
<td>10’ 5”</td>
<td>3.18</td>
</tr>
<tr>
<td>Spacing Between the Stacks</td>
<td>12’ 3”</td>
<td>3.73</td>
</tr>
<tr>
<td>Length of Transmission Line 1</td>
<td>15’ 6 1/4 ”</td>
<td>4.73</td>
</tr>
<tr>
<td>Length of Transmission Line 2</td>
<td>14’ 7 1/2 ”</td>
<td>4.46</td>
</tr>
<tr>
<td>Length of Transmission Line 3</td>
<td>3’ 9 7/8 ”</td>
<td>1.17</td>
</tr>
<tr>
<td>Length of Transmission Line 4</td>
<td>8’ 4 5/8 ”</td>
<td>2.26</td>
</tr>
</tbody>
</table>
By building the antenna according Table 3-5, a VSWR measurement has been taken and is compared with the model VSWR as shown in Figure 3-11.

![VSWR Over Frequency](image)

Figure 3-11. VSWR Vs Frequency

Looking at Figure 3-11 it is clear that the measured VSWR agrees with the model VSWR. The results are almost identical, where the lowest region of VSWR is between 49 and 50.5, and it picks up right after that. The lowest that was recorded for this antenna was at 50 MHz, which the VSWR came out to be around 1.1.

However, this antenna is too large to be put into the Anechoic Chamber at Penn State, where the total height of the antenna is about 3.7 meters, or 12 Ft. An accurate radiation pattern
cannot be measured. Due to that, a scaled down model of the antenna had to be simulated and built to show a representational radiation pattern.

3.3 The Double Turnstile at 700 MHz

Due to the fact that a radiation pattern from the actual antenna cannot be accurately measured, a scaled down model (at 700 MHz) of the turnstile has been made for the purpose of getting a representative radiation pattern of the antenna. Moreover, the frequency of 700 MHz was specifically chosen so that the antenna would fit inside the Anechoic Chamber which is located at Penn State, Electrical East Building, without having to worry about dimensions of the antenna.

Modeling the turnstile at a higher frequency caused an issue when using the thin wire approximation, since it no longer holds true. The radius of the dipoles was getting too large, and in FEKO, if the radius of the wires is too large, it cannot be meshed properly and is not broken into proper segments, and thus the radiation pattern tends to have some errors.

In this case, each dipole was only broken down into 18 segments, which led to errors in the radiation pattern and impedance. Thus a wire model is not sufficient enough to accurately build and experimentally measure a scaled down antenna with these specifications. A cylindrical surface model of the antenna has been constructed to combat this issue.

To further highlight this issue, a comparison between the cylindrical model and the wire model at 700 MHz is done.
3.3.1 Wire Model at 700 MHz

Taking into consideration the original antenna given in Figure 1-4 again, the model in 3-11 is based off that model. However, notice that there is no ground plane anymore as opposed to the 50.1 MHz model. This is due to the fact that this antenna is made to be tested inside the Anechoic Chamber and not outdoors, thus there is no need for the ground plane.

![Image of the FEKO Wire Model.]

Again, as it was done for determining the parameters for the 50.1 MHz model, the same rigorous method was used. Table 3-6 summarizes the “final” initial values and limits of optimization.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
<th>Initial Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the Dipoles</td>
<td>85.7 mm</td>
<td>121 mm</td>
<td>112 mm</td>
</tr>
<tr>
<td>Spacing Between the Stacks</td>
<td>214 mm</td>
<td>357 mm</td>
<td>255 mm</td>
</tr>
<tr>
<td>Length of Transmission Line 1</td>
<td>357 mm</td>
<td>571 mm</td>
<td>512 mm</td>
</tr>
<tr>
<td>Length of Transmission Line 2</td>
<td>357 mm</td>
<td>571 mm</td>
<td>482 mm</td>
</tr>
<tr>
<td>Length Transmission Line 3</td>
<td>7.14 mm</td>
<td>285 mm</td>
<td>126 mm</td>
</tr>
<tr>
<td>Length Transmission Line 4</td>
<td>142 mm</td>
<td>428 mm</td>
<td>277 mm</td>
</tr>
</tbody>
</table>
With the optimization done, Table 3-7 summarizes the optimum value of each of the optimized parameters.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Optimum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the Dipoles</td>
<td>112 mm</td>
</tr>
<tr>
<td>Spacing Between the Stacks</td>
<td>256 mm</td>
</tr>
<tr>
<td>Length of Transmission Line 1</td>
<td>525 mm</td>
</tr>
<tr>
<td>Length of Transmission Line 2</td>
<td>491 mm</td>
</tr>
<tr>
<td>Length Transmission Line 3</td>
<td>124 mm</td>
</tr>
<tr>
<td>Length Transmission Line 4</td>
<td>263 mm</td>
</tr>
</tbody>
</table>

Since this antenna was optimized at a frequency of 700 MHz, the radiation patterns have been taken at different frequencies to show the difference as we get closer to the frequency which we have optimized it at. Figures 3-4 through 3-8 shows the radiation patterns in ascending order of frequencies from 695 MHz to 710 MHz.
Figure 3-13. Wire Model radiation pattern at 690 MHz.
Figure 3-14. Wire Model radiation pattern at 695 MHz
Figure 3-15. Wire Model radiation pattern at 700 MHz
Figure 3-16. Wire Model radiation pattern at 705 MHz
Table 3-8 summarizes the results of each radiation pattern as shown.

Table 3-8. Results of the wire model radiation patterns

<table>
<thead>
<tr>
<th>Frequency of Operation</th>
<th>Minimum Gain</th>
<th>Maximum Gain</th>
<th>Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>690 MHz</td>
<td>0.49 dBi</td>
<td>3.98 dBi</td>
<td>3.48 dB</td>
</tr>
<tr>
<td>695 MHz</td>
<td>0.91 dBi</td>
<td>4.01 dBi</td>
<td>3.11 dB</td>
</tr>
<tr>
<td>700 MHz</td>
<td>1.22 dBi</td>
<td>4.16 dBi</td>
<td>2.93 dB</td>
</tr>
<tr>
<td>705 MHz</td>
<td>1.37 dBi</td>
<td>4.39 dBi</td>
<td>3.03 dB</td>
</tr>
<tr>
<td>710 MHz</td>
<td>1.29 dBi</td>
<td>4.37 dBi</td>
<td>3.38 dB</td>
</tr>
</tbody>
</table>

As expected, due to the thin wire approximation failing at higher frequencies, the radiation patterns of this specific model were not correct. As shown in Figures 3-13 through 3-17, there is barely any change in the radiation pattern as the frequency increases. It is clear that the radiation pattern of this model is being incorrectly computed.
3.3.2 Cylindrical Surface Model at 700 MHz

The main difference between the two models is the way they get meshed. Since FEKO uses Method of Moments to solve for the current distributions, from chapter 2, it was concluded that FEKO treats structures and wires differently, where objects like squares, cylinders etc. are meshed using triangles, while wires are broken into segments. Due to that, limitations are set upon wires as the radius increases, since wires are broken into segments when they are assumed to be electrically thin. As the radius increases, the wires will become thicker, in which case they resemble cylinders instead of thin wires, at which point FEKO fails to break the wires properly into segments. This results into current distributions being miscalculated which ultimately leads to the radiation patterns to be inaccurate.

Modeling dipoles as cylinders as opposed to wires is a little bit more difficult due to the fact that there are more variables that have to be incorporated. For example, with the wire model, worrying about spacing at the feed-point between the dipoles is not really an issue because the port can just be placed in the middle and that would separate the wires. On the other hand, when modeling a cylinder, a simple dipole must consist of two cylinders and a thin wire in between to properly connect them. In the case of the turnstile, the major concern was how much space must be left between each cylinder to properly model a dipole, having too wide of a gap between the dipoles would cause matching issues, in other words, the VSWR and radiation patterns are very sensitive to the gap between the cylinders. Thus, the goal has been set to minimize the gap as much as it is possible physically, while keeping in mind that this antenna has to be built in real life for more extensive testing.

Due to the sensitivity of the VSWR to the gap between the cylinders, an extra variable had to be added to the optimizer as opposed to the wire model. In this case, this variable is the spacing between cylinders. Table 3-9 summarizes the new parameters that need to be optimized.
Table 3-9. List of the new optimized variables.

<table>
<thead>
<tr>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the Dipoles</td>
</tr>
<tr>
<td>Spacing Between the Stacks</td>
</tr>
<tr>
<td>Length of Transmission Line 1</td>
</tr>
<tr>
<td>Length of Transmission Line 2</td>
</tr>
<tr>
<td>Length Transmission Line 3</td>
</tr>
<tr>
<td>Length Transmission Line 4</td>
</tr>
</tbody>
</table>

In this case, proper spacing between cylinders had to be approximated. Physically speaking, it is almost impossible to solder one half of a dipole together with its mirror half with a gap below 1 centimeter, it is also not possible that the feed-point spacing (or the gap) between the two halves could exceed 3 centimeters without affecting the results. After some more testing with FEKO, it turns out that if the gap is set to 2 centimeters, everything else will fall into place.

Moreover, under the assumption that the antenna is going to be physically soldered together, the radius of the thin wire used in the model to connect the two halves of the dipoles together is set to 0.29 millimeters, which is the radius of a typical RG-59 inner conductor.

Figure 3-18 clearly shows the feed-point spacing and the thin wire used to connect the two halves of the dipoles together.
Figure 3-18. Image of the Cylindrical Surface Model feed-point spacing.

Every other variable limit has been left the same as the wire model. The reasoning behind that is the fact that even if the spacing between the cylinders effects the length of the dipoles, it should theoretically still be within the limits that were initially set for the other variables, thus the optimizer had to be run again to get the ideal results. Table 3-10 summarizes the optimum results achieved through the optimizer.

Table 3-10. List of Optimized Variables.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Optimum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the Dipoles</td>
<td>198.93 mm</td>
</tr>
<tr>
<td>Spacing Between the Stacks</td>
<td>224.38 mm</td>
</tr>
<tr>
<td>Length of Transmission Line 1</td>
<td>520.03 mm</td>
</tr>
<tr>
<td>Length of Transmission Line 2</td>
<td>484.16 mm</td>
</tr>
<tr>
<td>Length Transmission Line 3</td>
<td>130.49 mm</td>
</tr>
<tr>
<td>Length Transmission Line 4</td>
<td>263.59 mm</td>
</tr>
</tbody>
</table>
Comparing Tables 3-7 and 3-10, respectively, there is clearly a difference in the lengths of all the variables, which indicates that the gap plays somewhat of a role in determining the accuracy of the double turnstile antenna. This is expected due to the fact that the gap between the dipoles majorly affects the reflection coefficient, which in turn affects the VSWR of the antenna, and one of the optimization goals is to minimize the VSWR. Setting the gap to about 20 millimeters or 2 centimeters, has made building the scaled down turnstile feasible.

For the purposes of comparison between the cylindrical and the wire model at 700 MHz, the same testing method that was done on the wire model has been carried out on the cylindrical model, where it was tested over a range of frequencies from 690 MHz to 710 MHz (instead of 49 MHz to 51.7 MHz) to determine the radiation patterns at different frequencies. The radiation pattern at each frequency starting from 690 MHz with a frequency increment of 5 MHz is also provided.
Figure 3-19. Cylindrical Model Radiation pattern at 690 MHz

Figure 3-20. Cylindrical Model Radiation pattern at 695 MHz
Figure 3-21. Cylindrical Model Radiation pattern at 700 MHz
Figure 3-22. Cylindrical Model Radiation pattern at 705 MHz
Figure 3-23. Cylindrical Model Radiation pattern at 710 MHz

The results of radiation patterns with regards to its frequency of operation are given below in table 3-11.

Table 3-11. Results of the Cylindrical mode radiation patterns

<table>
<thead>
<tr>
<th>Frequency of Operation</th>
<th>Minimum Gain</th>
<th>Maximum Gain</th>
<th>∆</th>
</tr>
</thead>
<tbody>
<tr>
<td>690 MHz</td>
<td>1.11 dBi</td>
<td>3.47 dBi</td>
<td>2.36 dB</td>
</tr>
<tr>
<td>695 MHz</td>
<td>1.54 dBi</td>
<td>3.34 dBi</td>
<td>1.80 dB</td>
</tr>
<tr>
<td>700 MHz</td>
<td>1.76 dBi</td>
<td>3.13 dBi</td>
<td>1.37 dB</td>
</tr>
<tr>
<td>705 MHz</td>
<td>1.45 dBi</td>
<td>3.42 dBi</td>
<td>1.97 dB</td>
</tr>
<tr>
<td>710 MHz</td>
<td>1.10 dBi</td>
<td>3.82 dBi</td>
<td>2.72 dB</td>
</tr>
</tbody>
</table>

It is clear from the radiation patterns in Figures 3-18 through 3-22 that the patterns are more accurate when compared to the wire model.

To put things into perspective, Table 3-12 summarizes the complete difference between the two models.
Looking at Table 3-12, and comparing the $\Delta$ at 700 MHz, which is the frequency of optimization, it is clear that the radiation pattern of the cylindrical model is more circular than the wire model, where $\Delta$ of the cylindrical model is only 1.21 dB while the $\Delta$ of the wire model is 2.93 dB. In terms of the difference between the maximum and the minimum, the cylindrical model is clearly more circular. As a result, a prototype experimental antenna model is going to be based of the cylindrical model.

While both of these models have shown promising results, the wire model compared to the cylindrical model is far less accurate due to the meshing of the wire model just being inferior at higher frequencies.

### 3.3.3 Prototype Antenna at 700 MHz

This prototype, as shown in Figure 3-24, was purely built for the purposes of getting a representational radiation pattern for the model that was done at 50.1 MHz. However, due to this antenna, the issue of the wire model versus the cylindrical model has risen, which leads to some
pretty interesting comparisons between the two models, which was greatly emphasized in the previous section, 3.3.2. Figure 3-25 also shows the wiring of the scaled down double turnstile.

Figure 3-24. Prototype of the Double Turnstile at 700 MHz
Again, taking into consideration the fact that even with the optimum values found, without special machinery, it is very hard to get the cables and dipoles cut to that precision. All the values have been rounded according to the closest physically possible length.

Since RG59 cables are also used for the scaled down model, the effect of the velocity of propagation through the transmission line has been taken into consideration which leads to the lengths of the transmission lines to be changed accordingly. Similarly, 4 ferrite beads were used to cover each end of the cables (at the feed point of the dipoles shown in Figure 3-25), to suppress
high frequency noise. However, in this case, material 61 ferrite beads are used due to the fact that this antenna is to be operated at a frequency of 700 MHz.

To elaborate further, the difference between material 61 and material 31 ferrite beads is typically the range of frequency that the material could suppress. In the case of material 31 ferrite, the beads would suppress frequency noises from 1 MHz up to 300 MHz, which made it ideal for the experimental model that is operated at a frequency of 50.1 MHz. On the other hand, material 61 ferrite are used to suppress frequency noises from 200 MHz up to 1 GHz, which made it ideal for the scaled down turnstile antenna which is set to operate at a frequency of 700 MHz.

It is also notable that when actually building the antenna, the length of the feed-point spacing becomes part of the total dipole length. This was also taken into consideration, and the results are summarized in Table 3-13 below.

Table 3-13. Difference Between Cylindrical and Wire models

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model Parameters (mm)</th>
<th>Model Parameters (inches)</th>
<th>Actual Physical Parameters (Inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the Dipole</td>
<td>218</td>
<td>8.58”</td>
<td>8.6”</td>
</tr>
<tr>
<td>Spacing Between the Stacks</td>
<td>224</td>
<td>8.82”</td>
<td>9”</td>
</tr>
<tr>
<td>Length of Transmission Line 1</td>
<td>358</td>
<td>14.1”</td>
<td>14.1”</td>
</tr>
<tr>
<td>Length of Transmission Line 2</td>
<td>333</td>
<td>13.11”</td>
<td>13.1”</td>
</tr>
<tr>
<td>Length Transmission Line 3</td>
<td>89.9</td>
<td>3.54”</td>
<td>3.6”</td>
</tr>
<tr>
<td>Length Transmission Line 4</td>
<td>182</td>
<td>7.16”</td>
<td>7.2”</td>
</tr>
</tbody>
</table>

Looking at Table 3-13, it is clear that some approximation has been done in order to be able to properly build the antenna.
Figure 3-26. Representational Radiation Pattern of the double turnstile Prototype

The purpose of building this antenna is to purely show a representational radiation pattern that shows that the new antenna design should in theory give a more circular radiation pattern that has been done with previous designs. Figure 3-26 serves that purpose and shows that the antenna is properly working even at higher frequencies. At 50.1 MHz, the agreement between the VSWR curves of the FEKO model and the actual antenna is remarkable. At 700 MHz, the radiation patterns do somewhat agree, where the error in the radiation from the prototype antenna is faulted at the dimensions of the antenna itself. Due to its small size, soldering the parts properly together was extremely challenging. Another factor is the fact that the feed point spacing between the dipoles was also difficult to get to be 2 centimeters due to again, the dimensions of the double turnstile being extremely difficult to work with. Despite the latter, the representational radiation pattern was very close to that of FEKO.
Chapter 4
Conclusion

The New Double Turnstile Antenna that was proposed in this paper is a complete improvement over the old turnstile design. Every aspect of the antenna has been improved, the VSWR, the radiation pattern and in turn the gain. Even with the “improved” 50 ohm and 75-ohm combination between the cables, the VSWR still suffered and no radiation pattern was given for that particular design. With the current design, where only 75 ohm cables are used, being tested at both 50.1 MHz and 700 MHz, it has proved that this particular design works despite the changes in the frequency of operation, as long as the transmission lines and dipoles lengths are accounted for and are adjusted appropriately. Then, the antenna should give the desired results.

Chapter 1 gave a complete and comprehensive review of the turnstile antenna, where numerous types of Turnstile antennas were introduced. The old design of the double Turnstile antenna was also discussed, where the VSWR of that antenna was around 1.4, and no radiation pattern of that antenna was ever provided. However, radiation patterns of a single turnstile antenna were also shown to further illustrate the purpose of this thesis.

Next, some basic background antenna theory was given, which led to the discussion of the Method of Moments. With the introduction of the method of moments done, the coding software that was used throughout the thesis was introduced, FEKO. FEKO is an electromagnetic modeling software package that was used to simulate and optimize the turnstile antenna to the desired goals. Finally, the method of optimization that was explicitly used for this thesis is introduced and explored towards the end of the 2nd chapter.
Finally, the models and the actual antenna results are provided in Chapter 3, where the proposed design was put to the test, where the results of the computer model and the actual models were compared. Moreover, in the final chapter, an issue with the modeling software FEKO was realized, which led to the development of the cylindrical model of the scaled down turnstile. This led to some interesting conclusions on how FEKO treats thick wires modeled as just lines versus how they are treated when they are modeled as cylinders. It was finally deduced that when the wire becomes too thick, FEKO fails to appropriately break down the line into segments which results in inaccurate radiation patterns.

To conclude, we demonstrated in this thesis that the new method of using 75 ohm cables to connect the Turnstile instead of the 50 and 75-ohm combination is clearly far more superior. The VSWR has significantly dropped as opposed to the original design, while the radiation pattern became more circular in the Omni-direction. It is also evident that there are limitations to the Simplex optimization method that was used to optimize this antenna. Further studies can be done by using different optimization methods to optimize the antenna, which could lead to even better results. Improvements to the design that was proposed is still an area of interest. Another area of interest would be using the proposed approach and applying it to numerous different types of Turnstile antennas and comparing the results with those as well.
Bibliography


Appendix A

Antenna Modeling

Figure A-1. List of Defined Variables in FEKO
Appendix B
Antenna Modeling

Figure B-1. Transmission Line Modification
Appendix C

Antenna Optimization

Figure C-1. Method of Optimization Choice
Appendix D

Antenna Optimization

Figure D-1. Goals of Optimization