FIELD AND ANALYTICAL INVESTIGATION OF
A 3D DYNAMIC TRAIN-TRACK
INTERACTION MODEL AT CRITICAL SPEEDS

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ABSTRACT

Railroad transportation, especially high-speed passenger rail, has been developing at sensational speed, creating the needs to evaluate the track safety and predict the potential hazards for railroad operation. As train speed increases, the responses of the train and track substructure present larger dynamic behavior, which raises problems in passenger comfort, operational safety and track structures. With the boom of modeling and computing techniques, numerical simulation becomes a more feasible, safe and effective tool to identify the problems in railroad engineering. In this dissertation, a self-developed three-dimensional train-track interaction model is formulated, validated and implemented to predict the potential risks and explore the mechanisms of rail track problems.

The developed 3D dynamic track-subgrade interaction model includes a train model, 2D discrete support track model and 3D computation-efficient finite element (FE) soil subgrade model. The rail beam is modeled as a Euler-Bernoulli Beam. The 2D track model discretizes the tie and ballast as rigid bodies with designated spacing. The discretely supported system enables the track model to take the track longitudinal variations into consideration. The subgrade model is simulated by finite element method with quadrilateral elements. Since the subgrade of a tangent track is considered as homogeneous in the moving direction, the formulation of the subgrade is derived in a computing-efficient way by expanding the longitudinal direction in the frequency domain. Therefore, the computing time is significantly reduced. The entire model is a self-developed program, which is coded in MATLAB.

Field data collected in x, y, z directions were used to validate the accuracy of the dynamic track-subgrade interaction model. Testing sites were located on Amtrak’s highest speed (240 km/hr) line near Kingston, Rhode Island, on the Northeast Corridor in United States. Track deflections and the ground wave motion measured in the field were compared with those predicted by the 3D dynamic track-subgrade interaction model.

Some applications of the 3D dynamic track-subgrade interaction model are included in this dissertation, as follow.

Critical speed study: Amtrak requires more frequent maintenance for the North East Corridor (NEC) at Kingston, Rhode Island (known as the Great Swamp area). It was suspected that a condition, known as critical speed, might exist at this particular location. The conventional definition of critical speed of a railroad system is the speed at which vibrations propagate within
the track structure and subgrade at a speed close to the Rayleigh wave velocity of the subgrade soil. As trains travel at speeds approaching the critical speed, all track components are expected to present significantly increased vibrations. However, no historical data indicated such dramatic increment in track responses at the “Great Swamp” area. Therefore, the 3D dynamic track-subgrade interaction model was used to evaluate the track performance under the current range of operation speeds and predicted the track responses at train speeds higher than the track speed limits. According to the results and analysis, the critical speed effect at the Kingston site could not be comprehensively explained by the conventional definition of the critical speed. Therefore, a two-level explanation is used to address the phenomenon at this site.

Hanging tie detection: the hanging tie condition is that in which voids have developed beneath the ties, causing tie-ballast gaps. The existence of the tie-ballast gaps can result in larger peak accelerations at the unsupported or poorly supported ties. This will increase the dynamic contact force between tie and ballast, and then further deteriorate the track structure. However, currently the railroad industry has difficulties in identifying hanging ties in the field since the problem is buried beneath the track. Therefore, the previously developed model was used to propose a fast, nondestructive screening method to identify the hanging tie problem. The method utilized a dynamic track model to characterize the track's “Moving Deflection Spectrum (MDS)” under different tie-supporting scenarios. The MDSs of tracks without a hanging tie problem have a clear peak at the “Tie Spacing Frequency.” However, tracks have a hanging tie problem if the MDSs present a significantly increased peak in the low frequency region (the frequency below the “Tie Spacing Frequency”). The modeling results were further validated by the field investigations conducted on three metro lines in Boston and St. Louis. Accelerometers were mounted on a high-rail vehicle to measure the acceleration of the moving wheels. The MDSs were then calculated to predict the potential locations of hanging ties. The locations of hanging ties predicted by the model matched the field observations.

Tie movements characterization: modeling techniques were used to predict the movements of railroad ties under moving train passage in this study. The ballast and other track substructures performances are largely dependent on the tie-ballast contact. Therefore, characterizing the tie movements could potentially bring new concepts to the conventional laboratory tests and practical maintenance for railroad engineering. In order to obtain the tie movements numerically, two steps were made by combining a commercial vehicle dynamics model with a track model established in the commercial software ABAQUS. The modeling results showed that the motion of ties not only
contains translational movements, but also rotations. The field tests to validate this finding were conducted on an Amtrak high-speed passenger line and a freight railroad short line. The measuring units were mounted on ties to record the accelerations and the changes in Euler angles of the ties in three orthogonal directions. The measurements of tie displacements and rotations in the field tests had good agreement with the modeling results. Moreover, field tests indicated that the tie-ballast gaps may cause higher accelerations and angular velocities of ties. Then the effect of tie rotation was investigated by discrete element modeling. The modeling results showed that the acceleration of individual ballast particles and ballast contact forces could have significant increase when the tie rotation is considered.

Overall, the 3D train-track interaction model is capable of simulating the railroad tracks of various conditions by integrating the discrete supports and 3D subgrade model. The model can be a reliable and effective tool for railroad industry. The versatility of the model makes it have great potential for future research and references.
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Introduction and Motivation

Passenger transport becomes a significant portion of rail transport as the train speed increases. Due to its environmentally friendly technologies and time saving in short-distance travel, high speed rail (HSR) has been proven to be a powerful technological advance in the transportation industry. Over ten thousand miles of HSR rail have been built in the world. Without exception, as many other countries, United States also has a long-term goal of the HSR program. However, one of the most critical problems of the HSR development in United States is to evaluate and then upgrade the current railway lines for high-speed trains. Unlike some other countries build new railway lines for HSR, United States is prone to upgrading existing railway tracks for most HSR lines for economical reason. But, increasing speed brings many challenges to a conventional railway system, which include rail system control, passenger safety and comfort, noise and vibration hazards. Among those challenges, ground-borne vibration induced by HSR is the focus in this research. Those vibrations could bring potential problems of operational safety by accelerating the deterioration of the conventional railway tracks and affect passenger comfort. However, it is a complex problem to measure the effect of vibrations on vehicle, track and human comfort. Therefore, a comprehensive and effective modeling method integrating the entire railway structures will be an appropriate tool to investigate the railway performance.

According to previous research, high speed trains on soft ground could induce a significant increase in vibration level when trains move at critical speeds. The critical speed is that at which resonance occurs when the speed of a moving train is close to or above the speed of the Rayleigh wave of subgrade soil. Many high speed lines\textsuperscript{1-5} have been suspected to have the critical speed effect. In the United States, Amtrak requires more frequent maintenance for the North East Corridor (NEC) at Kingston, Rhode Island (known as the Great Swamp area), and so identified the need for investigating the site. It was suspected that the critical speed effect might exist at this particular location as well. However, it is expensive and perilous to run a train at critical speed to test the vibration of the subgrade soil. Therefore, it would be applicable and effective to develop and
validate a 3D dynamic train-track interaction model, and then use it to predict the track and ground vibrations and substructure performance under trains moving at critical speeds. The dynamic train-track interaction model should be a complex system with train, track and soil subgrade. The model needs to have the capabilities to simulate any track profiles, different train speeds and wave propagation generated by HSR.

Field validation is an important step that is made to validate the developed numerical model which inherently has many assumptions and simplifications. Once it is validated, the model can be implemented with confidence to solve specific problems under certain conditions. For this reason, the validation of the model was conducted at the Kingston site. The following tasks were planned: 1) comprehensive field investigation of the problem site, including both laboratory testing and on-site characterization; 2) development of an instrumentation plan designed to quantitatively evaluate track vibration and its change under different train speeds; 3) validation of the 3D dynamic train-track interaction model to study the track performance and critical speed phenomenon.

The validated model has the versatility to solve many problems in an effective and efficient way. The model can be appropriately modified and coupled with other benchmarked programs to extend the capabilities of the current model. If the right assumptions are made, the modified model can still have its own ability and take full advantages of other programs at the same time. Then, it will be an efficient and effective way to solve some specific problems. Hanging tie detection and characterization of tie movements are two examples discussed in this dissertation. The commonality of these two cases is that currently there are no effective tools or inspection techniques that can explore the mechanism and predict potential hazards of both issues. The developed model is then modified and coupled with other numerical programs to solve the problems. Therefore, the developed model is not just a method to settle the specific problems in this dissertation, but also has great potential to be extended to solve many other railway problems.

Objectives

The overarching goal of this dissertation is to formulate and validate an effective and comprehensive train-track interaction model to investigate the mechanism of various railway problems. The interaction of railway system becomes more and more essential as the heavy axle load and high speed trains develop rapidly. The dissertation combines a comprehensive literature review (Chapter 2) with a series of manuscripts to fulfill the following specific objectives:

- Formulate a 3D dynamic train-track interaction model. Make sure it is capable of predicting the performance of vehicle, track, and subgrade; (Chapter 3)
- Validate the developed model by conducting field tests on the rail deflection and ground wave motion on selected railway lines; (Chapter 3)
- Use the model to investigate the critical speed effect at the Kingston site and predict the track performance that could not be measured directly; (Chapter 4)
- Use the model to propose a fast, nondestructive screening method to identify the hanging tie problem; (Chapter 5)
- Combine the model and other commercial programs to characterize the tie movements under repeated train loading. (Chapter 6)

**Scope and Methodology**

In this dissertation, a semi-analytical model was formulated which is capable of simulating the wheel-rail contact scenario, movements of tie, discrete support condition of the track and soil subgrade response to moving trains. The wheel forces were coupled with the rail using Hertzian contact theory. The ties were discretized along the longitudinal direction with specific tie spacing. The ties were modeled as constant masses interacting with rail and ballast by springs and dampers. In addition, the subgrade was modeled as 3D finite element method with plane-stress quadrilateral finite elements, which makes the calculation time-saving but needs to assume the subgrade as a homogeneous domain in the longitudinal direction. The entire model is self-coded in MATLAB.

For the field validation, a $3 \times 3$ array of piezoelectric, triaxial accelerometers were temporarily installed along the track and right-of-way with a general longitudinal spacing of 25 tie spaces (15 m) and a general lateral spacing of 7.5 m. The Dynamic Cone Penetrometer Test (DCP) was conducted to construct the cross-sectional data input for the dynamic track model. In addition, traffic information, such as vehicle speed and train types, was also collected during the field validation tests. The acceleration data was calibrated by Multi-depth Deflectometer (MDD) and then double integrated to get the rail deflections.

For the hanging tie detection, the track's “Moving Deflection Spectrums (MDS)” under different tie-supporting scenarios were investigated and characterized by the dynamic track model. The track model included a moving dynamic load and a track with discrete supports. For describing the tie movements under repeated train loading, a vehicle dynamics model coupled with a three-dimensional finite element track model was used.
Expected Contribution

The ultimate contributions of this dissertation to the railroad community will be:

- An effective and reliable modeling methodology to predict track performance in terms of rail deflection, ground surface wave motion and vertical compressive stress in cross sections at different train speeds, train loadings and many other conditions.
- Applications by using the model to substantially improve the current method of maintenance, which is high-cost and time-consuming.

Engineering Significance

Significance of the dissertation research can be communicated best in terms of the need for the aforementioned contributions. The developed model is able to simulate the track performance and explore the mechanism of track problems in an efficient and effective way. The following explanations address the significance of this dissertation in addressing the limitations of current analysis approaches.

- **A 3D dynamic train-track interaction model at critical speeds**

  The conventional definition of critical speed of a railroad system is the speed at which vibrations propagate within the track structure and subgrade at a speed close to the Rayleigh wave velocity of the subgrade soil. One of crucial issues of increasing speed is that high speed trains on soft ground where Rayleigh waves travel slower can generate a significant increase in vibration level when train speeds approach the critical speed of the system. In another word, when a train operates at its critical speed, it could cause high-level of track vibration, excessive rail deflection and even derailment. It has been considered as an important factor of operational safety when high-speed rails run on soft subgrade. A track section on the North East Corridor (NEC) at Kingston, Rhode Island (known as the Great Swamp area) was suspected that the critical speed effect might exist at this particular location because Amtrak requires more frequent maintenance for this site. The modeling and field study indicated that the critical speed effect does not match the conventional criteria and could be defined in two levels for the Kingston site: 1) the speed causing significant increase in the cross section stress intensity, at which more frequent ballast maintenance becomes a concern; 2) the speed causing significant increase in rail deflection, at which derailment becomes a concern.
Study of the mechanism and detection of the hanging tie problem

A hanging tie is a form of railroad track distress that occurs when voids have developed beneath the ties due to uneven ballast settlement and improper maintenance practices. It will lead to an increase in dynamic impact loading on the top of the ballast and therefore further deteriorate the track structure. The study proposed a fast, nondestructive screening method to identify the hanging tie problem by using the self-developed model. The method utilized the dynamic track model to characterize the track's “Moving Deflection Spectrum (MDS)” under different tie-supporting scenarios. The field experiments were also conducted to validate findings from numerical modeling. The finding of this research is significant because hanging tie problem requires a long maintenance period and high cost at current state. The quick identification method could improve the maintenance efficiency in railroad industry and provide a safer rail service.

Characterization of the movements of railroad ties

The ballast performance, degradation, breakage and deformation largely depends on the tie-ballast contact condition. Previous research on ties or ballast assumes that the movements of a railroad tie under the repeated train loading are along the vertical direction. A unidirectional actuator which was used for these laboratory tests was perpendicular to the ballast surface. The setup results in tie-ballast contact forces being applied in the vertical direction. However, field instrumentations installed at Kingston indicated that the tie movements in real world were not only translational but also rotational. Therefore, an accurate description of the tie-ballast contact interaction is an essential and imperative study and has the potential to improve the laboratory research and practical maintenance.
Chapter 2

BACKGROUND AND LITERATURE REVIEW

A Brief Introduction to Railway Engineering

In 1825, the world’s first railroad\(^6\) (George Stephenson Locomotion No. 1, as shown in Figure 2-1) was operated from Darlington to Stockton in Great Britain. It was capable of pulling 90 tons of coal at 15 miles per hour. Also, conical wheels began to be introduced. The conicity (wheel tread taper) is the slope of the wheel tread or running surface relative to the axis of the wheelset. Wheel tread taper may change with respect to the lateral location of the rolling contact point as the wheelset shifts laterally from its centered position. Tread taper is generally expressed as a ratio of the unit rise per lateral distance; for example, 1:20 or 0.05. Both taper and conicity represent the same quantities. The existence of the tapered wheels makes the moving vehicle tend to move back to the centerline of the track.

Figure 2-1. Locomotion No. 1\(^6\)

In the late 19th century, some countries, like Japan (1872) and China (1876), set out to construct their first railway lines. Since 1825, more than a million kilometers of railroads have been built. The list of leading countries of current rail transport network size is shown in Table 2-1.\(^7\)

Table 2-1. List of countries by rail transport network size.\(^7\)
<table>
<thead>
<tr>
<th>Rank</th>
<th>Country</th>
<th>Length (km)</th>
<th>Date of Info.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>United States</td>
<td>224,792</td>
<td>2014</td>
</tr>
<tr>
<td>2</td>
<td>China</td>
<td>112,000</td>
<td>2014</td>
</tr>
<tr>
<td>3</td>
<td>Russia</td>
<td>86,000</td>
<td>2013</td>
</tr>
<tr>
<td>4</td>
<td>India</td>
<td>65,808</td>
<td>2014</td>
</tr>
<tr>
<td>5</td>
<td>Canada</td>
<td>46,552</td>
<td>2008</td>
</tr>
<tr>
<td>6</td>
<td>Germany</td>
<td>38,445</td>
<td>2008</td>
</tr>
</tbody>
</table>

The history of railroad development could be divided into three stages. The first stage happened during the Second Industrial Revolution (1840-1913); more than 20,000 km of railway were built in the Western World every year. The second stage was stagnation due to World War I and II; the starting point of the third stage was around the 1960s. Since the 1960s, except that U.S. and Canada railway are still based on combustion traction, most countries have turned to the development of electrified lines. The overall trend of development of the world’s railroads is faster, heavier and cleaner.

According to the European Union Directive 96/48/EC, Annex 1, high speed rail is defined as the trains operating at minimum speed of 250 km/h (155 mph) on new lines or 200 km/h (124 mph) on existing lines that have been specially upgraded. In 1964, the Shinkansen, the world’s first high-speed line, began to serve from Tokyo to Osaka, shortening the travel time from 7 hours to 4 hours. Since then, the wind of the high-speed rail (HSR) has swept across the world. As of 2014, 19,369 km of high-speed lines have been built in China, which is more than fifty percent of the high-speed railway lines in the world. The world speed record for a train on a national railway system is 584.8 km/h in France in 2007 on a conventional rail.

The heavy haul rail (HHR) also has been developing quickly in the most recent two decades. The common haul tonnage has increased to 6,000 tons, and some trains can carry more than 10,000 tons of freight. In 2001, a rolling stock with 682 cars in Australia, hauled by eight AC6000 diesel electric locomotives, set a world record of HHR with a total weight of 99,734 tons.

In the United States, the railroad industry of HHR is in the leading position, while HSR is underdeveloped. As of 2010, U.S. freight railroads operated over 140,000 miles of track with standard gauge. The revenue of freight rail keeps increasing; at the same time, the number of employees per ton-mile is reduced substantially. However, for high-speed railway, the United States possesses less than 1,600 km of electrified lines and only 362 km of high-speed railway lines. Higher jet fuel prices, congested highways, and annoying airport security rules have now combined to make HSR become a more attractive option for passengers. The HSR program has been one of the goals of the Obama administration which came into office in January 2009. However, building
a national network of high speed rail is a long-term plan. President Obama’s vision is giving 80 percent of Americans access to high-speed rail within the next 25 years.\textsuperscript{8}

In North America, the freight rail network (Figure 2-2) has seven class I railroad companies (revenue > 400 million/yr), plus several hundreds of class II and III of regional or short lines.\textsuperscript{8}

![Class I Railroads of North America](image)

Figure 2-2. Class I railroads in the United States (2006)\textsuperscript{9}

Based on the data from "National Rail Plan Progress Report,"\textsuperscript{10} September 2010, the rail network accounts for approximately 40 percent of U.S. freight moves by ton-miles (the length freight travels) and 16 percent by tons (the weight of freight moved). The freight transport by railway in U.S. is promising and poised to face the challenge by offering a more efficient and reliable freight railroads network to meet customer needs. The rail industry’s planned future map of freight transport is shown in Figure 2-3.
Passenger transport is operated by the National Railroad Passenger Corporation of the USA, usually called Amtrak (from the words “America” and “track”). With the boom of the highway network in the 1970s, the railroad passenger companies were declining gradually. In order to protect the railroad industry, Congress exempted all the railroad companies and founded Amtrak. Amtrak is the only unique railroad company in the U.S. that provides passenger transport service. The passenger transport lines in United States are shown in Figure 2-4.\textsuperscript{8}
In the United States, there is only one actual high-speed rail – the “Acela” Express, which operates from Boston to Washington, DC. It is the busiest line in the United States. The highest operational speed can be 240km/h. However, the average speed is significantly reduced due to sharing the line with other trains. The average speed is only 127 km/h from New York City (NYC) to DC and as low as 101 km/h from NYC to Boston.

However, since 1965, United States has foreseen the development of HSR and committed to the development of HSR. During the 1960s, the federal government passed The High Speed Ground Transportation Act of 1965 and started to put in effort to develop HSR technologies. From 1970 to 1979, FRA built the foundation of an HSR system by successful improvements of the Northeast Corridor (NEC). From 1980 to 1991, FRA explored the potential of an HSR network across the U.S. In the next 20 years, The Intermodal Surface Transportation Efficiency Act (ISTEA) of 1991 planned five high-speed rail corridors and six more in 1998. In April 2009, FRA published the High-Speed Rail Strategic Plan and launched the High Speed Intercity Passenger Rail (HSIPR) Program in June 2009. Figure 2-5 presents the planned future high-speed railway network in the U.S.  

Figure 2-5. The planned United States high-speed rail map[^5]
To build a nation-wide HSR railway network, many challenges, such as collecting funds, construction, and operation safety, have to be overcome. Among them, operation safety is one of the most important issues in running high speed rail, which contains signals, train control, track maintenance, etc. As the operation speed increases, the vehicle and track will show more dynamic behavior which could cause problems that never occurred at lower speeds. Therefore, accurate railway system monitoring and performance prediction is on demand and imperative. Field measurements and monitoring have been used to monitor the behavior of track and vehicle under high speed. However, the field tests are expensive, time-consuming and site-specific. Thus, modeling techniques have become a prevailing tool to simulate the railway system. Commercial vehicle dynamics programs, such as NUCARS, SAMSRAIL, SIMPACK, and VAMPIRE, have proven to be effective to investigate the vehicle responses and wheel-rail contact at different train speeds. In addition, many advancements on track modeling from research organizations and industries around the world have been achieved. The following sections of this chapter will introduce the fundamentals of railway vehicle/track structure modeling and the vibrations induced by moving trains.

**Railway Vehicle**

From the early design to the modern design, many types and structures of rolling stocks have been developed. Generally, a rail vehicle consists of a car body, primary and secondary suspension systems, bogies, wheelsets and traction/braking system. The function of the car body is to accommodate passengers or freight. The car body is supported by two bogies with a suspension system which is used to enhance the riding quality by attenuating the outside excitations. Bogie, as an important component of rolling stock, is located between the wheel and car body. It can distribute the upper load evenly to each wheelbase and ease the high-level vibrations from lower structures. The suspension system is also installed in the bogies. The wheels are typically pressed onto an axle and mounted directly on a rail car or locomotive or indirectly on a bogie.\(^\text{11}\) As stated in the previous section, rail wheels are tapered slightly. The conical section of a railroad wheel is often tapered with a 1-in-20 slope.
The wheel-rail contact is one of the most important issues in railway engineering. It is related to many railway problems, such as vehicle stability, passenger comfort, rail deterioration, rail climbing and even derailments. Figure 2-8 shows the tread and flange contact points of elastic and rigid contact scenarios. Due to the clearance between the rail and the flange of the wheels, the wheelset actually moves left and right between the rails as the truck moves forward. This phenomenon is called truck hunting oscillation (shown in Figure 2-9). Figure 2-10 shows the only high speed rail in the United States running on the Northeast Corridor (NEC) which is also the major test site in this dissertation.
Vehicle Modeling

Rail vehicles usually consist of a car body, primary and secondary suspension systems, bogies and wheelsets. Therefore, the vehicle is a multi-degree-of-freedom system. It is essential to understand the relationship between forces and displacements in the solving process.

The vehicle modeling in the vertical direction was introduced first in the railway modeling history. According to different suspension systems, the bogies can be categorized to single-stage suspension bogie and double-stage suspension bogie, as shown in Figure 2-11. The single-stage suspension bogie has lower manufacturing costs and is easy to maintain. This kind of bogie is mostly used on freight cars, which do not have high demand on riding quality. However, the double-
stage suspension bogie will experience twice the vibration attenuation so that the riding quality is largely increased and is used for passenger cars.

![Diagram of single-stage and double-stage suspension bogies](image)

**Figure 2-11.** Classic vehicle dynamics models: (a) the single-stage suspension bogie; (b) double-stage suspension bogie

As can be seen from Figure 2-11, the car body, bogies and wheels can be simplified as rigid bodies with constant masses. The suspension system is described by a series of springs and dampers. According to the force diagram, one can construct the vibration equation for each component. Then the vehicle system equation can be depicted by the following equation:

\[
[M]\ddot{z} + [C]\dot{z} + [K](z) = (P) \tag{2-1}
\]

Where, \([M]\), \([C]\), \([K]\), \((P)\) are the mass matrix, damping matrix, stiffness matrix and external force vector, respectively; \(\{z\}\) is the displacement vector.

The vehicle body motion usually contains six types of motions and can be formulated by the Cartesian coordinates shown in Figure 2-12: the displacement in the longitudinal direction (X); the displacement in the lateral direction (Y); the displacement in the vertical direction (Z); three rotation angles of the body \(\psi\) (yaw), \(\phi\) (roll), and \(\theta\) (pitch) that define the orientation of the body. These angles can be written in a vector form as \(\theta = [\psi \ \phi \ \theta]^T\).
The wheel-rail contact scenario is another critical issue in the railway vehicle dynamics modeling. It combines the knowledge of contact theory, creepage, kinematic oscillation, wheel-rail profile and wheel/rail damages.

In 1855, Redtenbacher introduced the equilibrium rolling line in his book. It pointed out that the wheelset will not be centered when the vehicle is on the curve. Later, in 1883, Klingel derived the famous Klingel’s formula for the kinematic oscillation. The equation shows that the truck hunting oscillation is independent of train speed and has a fixed wavelength.
Classical contact mechanics is most notably associated with Heinrich Hertz. In 1880, Heinrich Hertz developed the Hertz contact theory of contact in 1880 during the Christmas holiday at the age of 23. Hertzian contact stress (shown in Figure 2-14) refers to the localized stresses that develop as two curved surfaces come in contact and deform slightly under the imposed loads. This amount of deformation is dependent on the modulus of elasticity of the material in contact. It gives the contact stress as a function of the normal contact force, the radii of curvature of both bodies and the modulus of elasticity of both bodies. The Hertzian contact theory is widely used for describing the wheel-rail contact.

Later, it was found by Nadal and Carter that the wheel-rail forces are not those of pure rolling. The friction between wheel and rail should be included due to slippage at the interface. It was demonstrated that the wheel-rail contact forces could not be entirely explained by rolling or sliding, but could be explained by creep. This leads to a conservative lateral/vertical ratio for rail climbing or derailment. In 1967, Kalker developed a complete analytical formulation for wheel/rail forces due to creep, which includes the spin component. Even though researchers later introduced improvement to Kalker’s work or even different models, Kalker’s model is considered to be the basis of all modern wheel-rail contact analytical creep models.

Recently, along with the thriving computing technologies, the field of vehicle dynamics has also been developing rapidly. Many vehicle dynamics simulation software, such as NUCARS (Figure 2-15), SIMPACK, VAMPIRE, and ADAMS, enable researchers to calculate the vehicle
that even takes up hundreds of degrees of freedom. However, most commercial vehicle dynamics programs (except NUCARS) do not possess a track model. The vehicles rest on the rails on a rigid foundation or a Winkler’s foundation. The effect of the track and subgrade is not comprehensively presented in the calculation process.

![Vehicle model in NUCARS](image)

**Figure 2-15. Vehicle model in NUCARS**

**Railway Track**

Track, as the foundation of running trains, is an important structure in the railway lines. Track guides the vehicle, carries and distributes the loads from the vehicles. The track structure needs to guarantee enough strength, smooth and reasonable maintenance period for running trains under required carrying loads and speeds. Conventional ballast track structure shown in Figure 2-16 contains rails, rail pads, ties, subballast, ballast and subgrade.

![Typical cross section of ballast track structure](image)

**Figure 2-16. Typical cross section of ballast track structure**
Rails

Rail is an important longitudinal steel track component put on the top of the rail track and is used to support and guide the vehicle by providing smooth running surfaces. It can accommodate the wheel loads and distributes these loads over the ties or supports. The horizontal transverse forces on the rail head can be transferred to the ties and lower track components. Also, the rail enables the vehicle to move in a stable direction.

There are many types of rail with regards to its profile, including flat-bottom rail, non-standard profile, grooved rail, block rail and crane rail, which are shown in Figure 2-17. Flat-bottom rail is the standard profile used as a general rule in conventional track, and it is also introduced in this dissertation. Also, in the United States, flat-bottom rail can be divided into several categories according to different self-weights per linear yard. In addition, each rail is connected together by joint bars.

![Figure 2-17. Rail profile types](image)

Rail Pad and Tie Plate

The rail tie plate is also called rail base plate. Tie plates are used not only to support the rails, but also to fix the entire rail fastening systems. The rail plate is shown in Figure 2-18.

The rail pad is put on top of the tie plate. The function of the rail pad is to provide an absorbing component between the steel rail and ties through transferring the rail load to ties and screening out the high frequency force. Also, the rail pad can make a more stable track and significantly lengthen the life of wood ties. In addition, rail pads are embedded under rails acting as electrical insulation.
Figure 2-18. Rail plate\textsuperscript{26}

Rail Tie

The ties rest on the transverse direction of the moving direction of the vehicle. The function is to maintain track gauge and fasten the rails to be aligned. Also, it can be considered as electrical insulation for the rails. Ties transmit the train loading to the lower track structure. The available materials for ties can be wood, steel and concrete. Timber and concrete ties are widely used and steel ties are in limited use.

Ballast

Ballast is a layer that is formed by crushed granular material and placed on the top of the subgrade. Ballast bed can absorb considerable compressive stresses, but not tensile stresses. Also, it has a large bearing strength in the vertical direction, but it is reduced in the lateral direction. The thickness of the ballast bed is typically about 50 cm from the top of the ballast or 25 to 30 cm from the lower side of the tie. The main functions of ballast are to: 1) distribute the stresses transmitted by ties; 2) drain rainwater; 3) resist transverse and longitudinal shifting of track; and 4) attenuate train vibration significantly.

Track Modeling

The function of rail track models is to interrelate each component in the track structure in order to simulate the integrated responses of track under the moving train load. Rail track models that include all the track components enable us to predict the track performance more effectively and precisely, but could be more time consuming. A track model needs to be selected appropriately to solve track problems efficiently.

As an important track component, rail is used to support and guide the vehicle by providing smooth running surfaces. In track modeling, the rails are usually simplified as two mathematical
models: Euler-Bernoulli Beam (E-B beam) and Rayleigh-Timoshenko Beam (R-T beam). E-B beam only considers bending behavior of rails. R-T beam theory includes not only bending, but also shear deformation of the beam. Train-induced vibrations with frequency less than 500 Hz carry higher energy. As a result, the vibration dissipates at a lower rate and its impact is felt farther from the source. Also, it was found that when the frequencies of train loading are less than 500 Hz, shear deformation of the rail can be neglected. Hence, E-B beam theory is sufficient to simulate the ground vibration induced by HSR. Railpads are absorbing components between the steel rail and ties. Pairs of springs and dashpots are introduced to simulate the railpads. For the railroad ties, they are usually simplified as a constant mass per unit track length. For the ballast, it denotes a layer of crushed stone of uniform size. It is usually simulated as a deformed continuous body or discrete rigid masses when the individual particle movement is not of concern. If the individual ballast motion is the focus, discrete element method (DEM) can be an effective tool.

Many researchers have been working on track models. Two-dimensional models are suitable for study of vertical track performance, but they ignore the transverse cross section of the track. With the boom of computing power, three-dimensional models are rapidly being developed because three dimension can provide more detailed performance of the track and responses from all directions. However, 3D models are still very time-consuming compared to 2D models. As a compromise, a 2.5D finite element method is introduced. The elements are in 2D shapes but have three dimensional movements by expanding the responses in the wavenumber domain.

**Two-Dimensional Track Models**

Many early models are based on a beam-on-elastic-foundation (BOEF) formulation. In this system, rails, simulated as continuous Euler-Bernoulli beams, are placed on elastic spring supports. Thus, the rail reactions in the longitudinal direction are proportional to their deflections. This model, which is shown in Figure 2-19, has long been accepted and used in modeling rail track, and is the backbone of the subsequent models.

```
<table>
<thead>
<tr>
<th>EI</th>
<th>Rail beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td></td>
</tr>
</tbody>
</table>
```

Figure 2-19. Beam (bending stiffness EI) on elastic foundation (bed modulus k)^12
The governing equation for the beam is in the following equation:

$$EI \frac{d^4w(x)}{dx^4} + p(x) = q(x) \quad (2-2)$$

Where, the \( w(x) \) describes the deflection of the beam in the vertical direction at some position \( x \); \( p(x) \) is the spring forces; \( q(x) \) is the distributed load on beam; \( E \) is the elastic modulus; \( I \) is the second moment of area of the beam's cross section.

Kaynia\(^2\) developed a 2D model, which evolved from the basic BOEF. Besides the bending rigidity and mass per unit length of embankment, hysteretic damping ratio is also added. The whole track system is bonded to the half-space at discrete points (nodes) along the embankment. The train loads that applied to the nodes with time shifts are combined into one concentrated load at the centerline of the bogie. The track model is illustrated in Figure 2-20.

![Figure 2-20. Schematic representation of embankment-ground interaction model by Kaynia\(^2\)](image)

Madshus\(^2\) simulated and analyzed the response of the track-ground system from high-speed train passage by using a computer program, Vibtrain. The ground is modeled as layered half-space by Green’s functions. And the rail/ballast system is taken as a beam by finite elements. Track and ground are related by enforcing vertical displacement and stress. Train loads are applied to the system through delaying the loads point by point according to different train speeds.

Kalker\(^2\) introduced a discretely supported beam model, which is shown in Figure 2-21. In his research, the rail is modeled as a Euler beam. The most salient feature is the irregular discrete support of the ties. The vertical displacement of a railway rail due to a traveling vertical point load of variable intensity is calculated.
The governing equation for the Kalker’s model is as follows:

$$EI \frac{d^4w(x,t)}{dx^4} + \rho \frac{d^2w(x,t)}{dt^2} = p(x,t)$$ (2-3)

Where, the $w$ describes the deflection of the beam in the vertical direction; $p$ is the distributed load on beam; $\rho$ is the tie mass per unit length.

In order to obtain the ballast effect, Huang\textsuperscript{30} introduced a 2D track model, called the Sandwich track model, which is shown in Figure 2-22. The ballast is modeled as discrete masses that are connected to ties and the ground with spring/dashpot. Rail, in this Sandwich model, is also modeled as a Euler beam. The rail pad, tie and ballast are all represented by mass and spring/dashpot.

The governing equation for the Sandwich track model is:
EI \frac{\partial^4 u(x,t)}{\partial x^4} + T \frac{\partial^2 u(x,t)}{\partial x^2} + \rho \frac{\partial^2 u(x,t)}{\partial t^2} + c \frac{\partial u(x,t)}{\partial t} = f(t) \delta(x - vt) - \sum m a_m(t) \delta(x - x_m) \tag{2-4}

where EI is the bending stiffness of rail; u (x, t) is the rail deflection as a function of time and position; \rho is the unit mass of rail; \varepsilon is damping of rail itself (which will be set to zero for convenience); T is the rail axial force caused by temperature increase; f (t) is the wheel load function; \delta is the delta function; x_m is the location of the m^{th} tie; and v is the wheel speed.

However, 2D models are insufficient to simulate the ground vibrations vertical to the track, thus the Mach radiation effect of soil cannot be detected. Also, most early researchers’ work on ground-borne vibrations was based on plane strain assumptions, and the beam on elastic foundation system. Hence, strictly speaking, only a more sophisticated 3D model is appropriate to investigate the wave propagation in the ground.

**Three-Dimensional Track Models**

Due to the limitations of 2D models, many researchers proposed 3D models to simulate the integrated track system to study the ground vibration generated by moving trains.

The track system of Takemiya shown in Figure 2-23, including rails, ties and ballast bed, is also modeled as a Euler-Bernoulli beam on elastic foundation. The Fourier Transform that changes the time-domain problem to a frequency domain problem is applied to obtain the solution for a moving load. The railway track is modeled as a Euler beam resting on ballast, then on the ground. However, the effect of ties is not considered in this model.

![Figure 2-23. The 3D track model by Takemiya](image)

Cai\textsuperscript{31} introduced a 3D model with coupling spring/damper elements, representing the mechanism of railpads, ties and ballast bed. The rail can be either modeled as Euler-Bernoulli Beam (E-B beam) or Rayleigh-Timoshenko Beam (R-T beam) to describe the rails which are discretely coupled with ties. Another beam element with mass is placed to model the ties. The tie rests on another spring-damper system, as shown in Figure 2-24.

![Figure 2-24. The model with discrete support by Cai\textsuperscript{31}](image)

Sheng\textsuperscript{32} proposed a 3D model that can calculate vibrational response of a layered ground subject to a fixed-position harmonic load acting via a railway track structure, or acting directly on the ground surface. As can be seen in Figure 2-25, the train is simulated as a harmonic loading and the track structure is continuous. Also, continuous springs are modeled to connect the rail beam and embankment.
For the rail, the governing equation is given as follows:

\[
EI \frac{\partial^4 w_1}{\partial x^4} + m_R \frac{\partial^2 w_1}{\partial t^2} + F_1 = P_0 e^{i\omega t} \delta(x)
\]  

(2-5)

where EI is the bending stiffness of rail; \( w_1 \) is the rail deflection; \( m_R \) is the unit mass of rail; \( P_0 \) is the wheel load function; \( \delta \) is the delta function; \( F_1 \) is the force between rail and ties.

The previous types of rail track models do not include the effects of real ballast behavior, such as ballast deformation. Ballast is formed by granular material, which cannot be modeled the same as subgrade soil. To fulfill this task, discrete ballast masses need to be used to mimic the mechanical responses of a granular material. Furthermore, when more and more details of the track such as geometry and/or material properties have to be considered, numerical solving techniques are employed. 3D finite element model (3D FEM) is one of the most widely used models as it is able to cover almost all geometry considerations and is commercially available. In order to investigate the train speed effect, XiTRACK Limited proposed a 3D FEM model incorporating all track components, including rail, ties, ballast, and subgrade. Train load is modeled as a sequence of constant load running at a constant speed over the rails. Detailed information about the model is given by Banimahd.\(^{33}\)
2.5-Dimensional Track Models

Although 3D FEM can always serve as a benchmark program to calibrate different track models when field measured data is not available, it is time consuming and is usually utilized only for particular cases. Hence, realizing the limitations of two-dimensional models and the unfavorable time efficiency associated with three-dimensional models, researchers\textsuperscript{34-36} proposed an innovative model called 2.5D FEM by assuming the track property remains uniform along the direction of train movement; only a profile of half-space vertical to the direction of load movement is considered. Also, the 2.5D approach is suitable for tunnels due to its assumption of uniform material properties along the movement direction. The accuracy of this approach is verified via comparison of results obtained from analytical solutions.\textsuperscript{36}

Bian\textsuperscript{34} modeled track-ground interaction by moving load using 2.5D FEM. The material properties and geometry are assumed consistent along the movement direction. The ground is modeled by isoparametric quadrilateral elements to condense the 3D issue to a plane strain problem. The 2.5D BOEF model is shown in Figure 2-26.

For 3D soil, the 2.5D FEM technique is employed. Fourier Transform was only performed in the direction of train movement, and the transverse and vertical directions were discretized by plane stress quadrilateral finite elements. A typical 2.5D element is illustrated in Figure 2-27.\textsuperscript{34}
In this formulation, the governing equation of rail is given as follows:

$$EI \frac{\partial^4 u(x,t)}{\partial x^4} + m \frac{\partial^2 u(x,t)}{\partial t^2} = f(x,t) - \sum_{n} p_n(t) \delta(x - ct) \quad (2-6)$$

where $EI$ is the bending stiffness of rail; $u(x,t)$ is the rail deflection as a function of time and position; $m$ is the unit mass of rail; $p_n(t)$ is the wheel load function; $c$ is the wheel speed.

**Train-Track Interaction Modeling**

**Wheel-Rail Contact and Track Irregularities**

In train-track interaction modeling, two major problems should be well-treated: the wheel-rail contact and track irregularities.

The moving, hauling and braking of trains are driven by the interaction between wheel and rail. The wheel-rail contact is a complex problem that combines mechanical theories, law of motion, geometry and material types, wheel/rail abrasion, etc. Carter$^{21}$ first introduced the concept of creep in the field rolling contact. He proposed the adhesive region and sliding region at the contact area, which means the wheel moving on the rail is not a pure rolling motion, but contains a motion of sliding. Kalker$^{22}$ then made huge contributions to the rolling contact theory. His study not only analyzes the adhesion and creep mechanism in the contact region, but also provides the solution to calculate creep rate.

The problem of track irregularities contains certain and uncertain factors which come from both vehicle and track. Track irregularities that result from vehicles can be flat wheel, wheel abrasion, and geometric eccentricity of wheel. As to the track, the connection of rails, rail surface unevenness, turnouts, and variation of track stiffness could be the sources of track irregularities.
Track irregularities have huge influence on the vehicle and track dynamic performance and thus have effect on maintenance and operational safety.

**Train-Track Interaction Model**

The train-track interaction model has consistently not been valued in previous research. However, due to the track irregularities and vehicle dynamic properties, a train-track interaction model is essential. A 3D train-track system allows dynamic loads to transmit to the track structure and subgrade through the wheel-rail contact. Over the past decade, research on train-track interaction models had been rapidly developing, with computer technology facilitating the possibility of analyzing large and sophisticated dynamic train-track coupled models.\(^{37,38}\)

In Sheng’s model\(^{37}\), the train model is simplified as sequence of loads. For the track structure, two rails are taken as a single Euler beam with mass per unit length and bending rigidity. The lower beam, also with a mass per unit length but no bending rigidity, represents ties. The continuous springs between two beams have a complex stiffness by considering spring stiffness and damping loss factor. The ballast has an infinite length and mass; vertical complex stiffness is modeled as a viscoelastic layer. The ground is simply modeled as a layered elastic medium of infinite extent. This model is shown in Figure 2-28.

![Figure 2-28. Model for track-ground system with multiple moving loads by Sheng\(^{37}\)](image_url)

The governing equation for the train-track model is:
where EI is the bending stiffness of rail; w (x, t) is the rail deflection; m is the unit mass of rail; δ is the delta function; c is the wheel speed; kp is the stiffness of rail pad.

However, the validity of the assumption for the soil part (i.e. half infinite space) is still limited. Also, this model becomes insufficient in cases that tracks are laid on slopes, or inside tunnels, where cross section geometry characteristics of the track and foundation are of great importance.

By considering coupling effect of train-track interaction, Zhai38 proposed a model with train and track substructure interacting. The model is shown in Figure 2-29. In this model, the ballastless slab track is taken into consideration, due to its wide application in Chinese high-speed railways. The vehicle model is double-stage suspension system and has 6 degrees of freedom. The track structure consists of rail, railpad, slab, cement and asphalt mortar and concrete foundation.

Figure 2-29. A complete model with vehicle and track structure for slab track by Zhai38

Ground Vibration Due to Moving Loads

Introduction

The ground vibration induced by moving trains is a complicated dynamic problem. The surface irregularities of wheels and rails, and the discrete tie support under rail can generate the train and track vibrations. The vibration will not only have influence on passenger comfort and
operational safety, but also can transport through the track structure, propagate through the soil medium, eventually reach the buildings to discomfort the residents and break down high-precision instruments in the buildings nearby. Three major phases can be identified for the mechanism of vibrations induced by the moving train: (1) generation; (2) transmission; and (3) reception. Various factors may affect the levels of vibrations, such as the train type, train speed, track design, embankment design, ground condition, etc. In this dissertation, the major focus is the modeling of generation of the train-induced vibrations at the site of interest.

The governing equations for a homogenous isotropic elastic half-space can be written in terms of displacements $u$:

$$\nabla \cdot (\lambda + \mu \nabla) u + \mu \nabla^2 u + \rho f = \rho \ddot{u}$$  \hspace{1cm} (2-8)

where $\lambda$ and $\mu$ are Lame’s constants, $u$ and $f$ denote the displacement and body force components, respectively, and $\rho$ is the mass density of the elastic solid. Lame’s constants can be expressed in terms of other elastic constants, such as Young’s modulus $E$, Poisson’s ratio $\nu$, and the bulk modulus $K$.

The speeds of the compressional and shear wave are defined as the following equations:

Compressional wave: $C_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}$ or $\sqrt{\frac{2\mu(1-\nu)}{\rho(1-2\nu)}}$

Shear wave: $C_s = \sqrt{\frac{\mu}{\rho}}$

The waves mentioned above are two major forms of waves propagating through the interior of an elastic solid. However, when an elastic wave travels to a free boundary, energy is reflected from the boundary causing a third type of waves which are confined closely to the surface. The third type of waves is called Rayleigh waves whose effect decreases rapidly with depth and their velocity of propagation is slightly less than that of shear waves. The distribution of the three wave was studied by Woods, and the Rayleigh wave contains most of the total energy.
Figure 2-30. Distribution of displacement waves from a circular footing on a homogenous, isotropic, elastic half-space

The Rayleigh waves are surface waves, commonly produced by earthquakes but also caused by heavy, fast-moving loads such as high speed trains. Rayleigh waves typically take two-thirds of the energy in a vibrating soil system, and also attenuate at a much slower rate than P waves and S waves. Consequently, Rayleigh waves are so important because they are able to cause considerable structural vibrations when trains reach critical speed. As can be seen in Figure 2-31, the velocity of a Rayleigh wave is slightly less than the velocity of a shear wave, particularly for higher Poisson’s ratio. Also how the Rayleigh wave propagates on the surface of the ground is shown in Figure 2-32. The relationship between the velocities of the two waves is:

$$V_R \approx \frac{0.87 + 1.12\nu}{1+\nu} V_S$$

(2-9)

Where, \(\nu\) is the Poisson’s ratio; \(V_R\) is the velocity of Rayleigh wave; \(V_S\) is the velocity of shear wave.
Figure 2-31. Variations of Rayleigh wave and Body wave propagation velocities with Poisson’s ratio

Figure 2-32. Motion of Rayleigh wave

Critical Speed Effect

With the continuous increase in the operation speed of passenger trains, the effect of speed has been a critical issue in the railway dynamics. One of the major concerns on the train speed effect is that the moving train could encounter high-level vibrations as the train passes through some elastic barriers. A similar phenomenon has been found when an airplane passes through the sound barrier; the Mach radiation of shock waves will occur. A moving train could suffer the shock waves by the same mechanism when it surpasses the characteristic speed of the waves of the soil medium. Therefore, the speed effect on trains, especially for high-speed rails, has to be taken into account when to study the wave propagation for the railway system. The effect of train speed is illustrated in Figure 2-33.
The following explanation for critical speed is cited from *CRITICAL SPEED OF SHAFTS*, written by Krueger:

In solid mechanics, in the field of rotordynamics, the critical speed is the theoretical angular velocity that excites the natural frequency of a rotating object, such as a shaft, propeller, leadscrew, or gear. As the speed of rotation approaches the object's natural frequency, the object begins to resonate, which dramatically increases system vibration. The resulting resonance occurs regardless of orientation. When the rotational speed is equal to the numerical value of the natural vibration, the speed is referred to as critical speed.

In railway engineering, the theory is similar. If train speeds reach critical velocity and the vibrations resonate with the natural frequency of the subgrade soil or a building nearby, it can cause increase in ground vibration, internal noise in building, and even structural damage in buildings up
to 250m from the track. This critical velocity is relative to the speed of propagation of Rayleigh waves on the ground, which depends on geotechnical properties.

The research on critical speed can be traced back to 1927. Theoretical modeling of a rail as a beam supported by track structure and ground reveals that the dynamic amplification will occur when train reaches a certain speed. However, with the knowledge of normally assumed soil properties, this critical speed is around 500 m/s, which is highly above the realistic HSR speed, because only pressure wave velocity is considered in Timoshenko’s theory. Hence, for a long period, the train loads have been assumed as quasi-static moving loads. Krylov considered that a train will encounter the ‘sound barrier’ when reaching the velocity of Rayleigh surface waves propagating in the ground. The velocity of Rayleigh waves traveling in soft sandy soils is 90 to 130 m/s, which is already reachable by today’s high speed trains. Furthermore, for peat, marine clays, and other soft clays, the characteristic wave velocities could be reduced to as low as 30-40 m/s. Rayleigh waves have a speed slightly less than shear waves, depending on the soil properties. The ground medium which has low shear-wave velocity needs much attention because it is susceptible to the increasing vibrations generated by HSR.

Therefore, according to previous research, high speed trains on soft ground are able to reach the critical speed and induce a significant increase in vibration level. The critical speed is defined as that at which resonance occurs when the speed of moving train is close to or above the speed of the Rayleigh wave of subgrade soil. The soft subgrade which has low Rayleigh wave velocity is susceptible to the increasing vibrations generated by high speed trains. Two major characteristics of the critical speed effect are high vibration of track and cone-shaped wave motion of ground surface which is analogous to a boat marching in the water. The resulting high level of vibration imposes limits on train speed. For an example, X-2000 (Swedish HSR), in 1997, was running at the maximum speed on the Goteborg-Malmo line and generated excessive vibration in the soil and overhead contact line support poles. Furthermore, at Ledsgard, Sweden, a new railway line with design speed of 200 km/h encountered high vibrations on soft cohesive soil. The speed of trains had been decreased from 200 km/h to 160 km/h, then to 130 km/h. The reason for the high vibrations turned out to be the approaching of critical speed. Also, the similar phenomenon happened at Northern Ireland Railways where the subgrade is constructed on peaty soils. Similar to Ledsgard, the site at Kingston is located on The Great Swamp Management Area, Rhode Island Section of the Northeast Corridor, where it is underlain by a soft organic silt (peaty) deposit. The high-speed line there has been suffering highly frequent maintenance. It is suspected that the critical
speed effect exists at this particular site. This is one of the major motivations of this research from the industry perspective.

In the past decade, critical speed effect has been considered as one of the most important factors affecting high speed rail safety and precluding higher speed operations. The critical speed effect is the combination of train suspension system, train speed, rail surface smoothness, track structure, and most importantly the ground soil or subgrade. It has been usually investigated case-by-case and there is a lack of general solutions. One possible solution is to develop a sophisticated computer model to identify any potentially critical train-track combinations. Through the computer model proposed in this research, the critical speed effect can be predicted which will greatly decrease the risk of the operation of railway traffic.

**Summary**

In summary, railway modeling is a complex topic combining the train, track and subgrade together with the interaction among those. Many train/track/train-track models have been developed using numerical and analytical methods. With the boom of modern computing technology, the experimental test is no longer the unique method to investigate the railway system. Modeling techniques have proven to be an effective tool to provide solutions to real problems. Three components, train, track and subgrade, make up a railway system as a whole. A train load can be modeled as a stationary static load, a stationary dynamic load, a moving static load or a moving dynamic load. Also, train model can be simplified as a point load, a series of train loads, several degrees of freedom system or even hundreds of degrees of freedom system depending on the type of problem. The track model can be one-layer to multi-layer: a continuous or discrete support system. The subgrade of the track can be two-dimensional, three-dimensional, even 2.5-dimensional. As said before, which of the models will be selected relates to the specific problem of focus. For examples, if the interaction between individual tie and ballast is of interest, the discrete model must be selected; if the passenger comfort or train ride quality is the focus, the acceleration and other responses of a vehicle need to be fully investigated. Therefore, a detailed vehicle model has to be used.

Some of the previous work on the modeling in railway engineering is briefly summarized in Table 2-2.
Table 2-2. Summary of railway modeling.

<table>
<thead>
<tr>
<th>Author</th>
<th>Dim.</th>
<th>Modeling Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaynia^27</td>
<td>2D</td>
<td>The quasi-static loads are used in the model. The ground is modeled as a layered viscoelastic half-space. The railway embankment is only modeled as a viscoelastic beam excited by the moving loads.</td>
</tr>
<tr>
<td>Kalker^29</td>
<td>2D</td>
<td>The track model considers the periodicity of ties and a moving dynamic load.</td>
</tr>
<tr>
<td>Krylov^46</td>
<td>3D</td>
<td>A train moving with speed v on welded track with sleeper periodicity d. The quasi-static pressure mechanism of excitation results from load forces applied to the track from each wheel axle.</td>
</tr>
<tr>
<td>Mashus^28</td>
<td>3D</td>
<td>The rail/embankment system behaves as a beam in dynamic interaction with the ground.</td>
</tr>
<tr>
<td>Cai^31</td>
<td>3D</td>
<td>3D dynamic load for a conventional rail track subjected to arbitrary loading. The rail track is idealized as periodic elastically coupled beam system resting on a Winkler foundation</td>
</tr>
<tr>
<td>Galvin^47</td>
<td>3D</td>
<td>A fully three-dimensional analysis of high-speed train-track-soil-structure dynamic interaction in the time domain was developed using a coupled FE/BE method.</td>
</tr>
<tr>
<td>Kourossis^48</td>
<td>3D</td>
<td>The complete vehicle-track-soil model is developed according to an uncoupled approach.</td>
</tr>
<tr>
<td>Connolly^49</td>
<td>3D</td>
<td>The model is a time domain explicit, dynamic finite element model capable of simulating non-linear excitation mechanisms. To increase boundary absorption performance, the soil structure is modeled using an elongated spherical geometry.</td>
</tr>
<tr>
<td>Sheng^32</td>
<td>3D</td>
<td>A continuously supported multi-layer model. A method of calculation of the vibrational response of a layered ground subject to a fixed-position harmonic load acting via a railway track structure, or acting directly on the ground surface, has been produced.</td>
</tr>
<tr>
<td>Sheng^37</td>
<td>3D</td>
<td>A continuously supported multi-layer model. Multiple wheel/rail contact points, to models representing a number of carriages. A method of calculation of the vibrational response of a layered ground subject to a fixed-position harmonic load acting via a railway track structure, or acting directly on the ground surface, has been produced.</td>
</tr>
<tr>
<td>Kacimi^50</td>
<td>3D</td>
<td>3D finite element coupled train-track model by using the same approach but using a half rectangular ground model and the influence of the soil material damping was investigated.</td>
</tr>
<tr>
<td>Metrikine^51</td>
<td>3D</td>
<td>This paper presents a theoretical study of the stability of a two-mass oscillator that moves along a beam on a viscoelastic half-space. The oscillator and the beam on the half-space are employed to model a bogie of a train and a railway track, respectively. Using Laplace and Fourier integral transforms, expressions for the dynamic stiffness of the beam are derived in the point of contact with the oscillator.</td>
</tr>
<tr>
<td>Dieterman^52</td>
<td>3D</td>
<td>An analytical model of a harmonic load moving uniformly along an elastic layer is introduced.</td>
</tr>
<tr>
<td>Vostroukhov^53</td>
<td>3D</td>
<td>This paper presents a theoretical study of the steady state dynamic response of a railway track to a moving train. The model for the railway</td>
</tr>
<tr>
<td>Author</td>
<td>Dim.</td>
<td>Modeling Details</td>
</tr>
<tr>
<td>-----------------</td>
<td>------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Zhai</td>
<td>3D</td>
<td>The analysis based on FEM is performed to determine the 3D dynamic response of the ground induced by the high-speed train.</td>
</tr>
<tr>
<td>Degrande</td>
<td>3D</td>
<td>The three-dimensional dynamic tunnel–soil interaction problem is solved with a subdomain formulation, using a finite element formulation for the tunnel and a boundary element method for the soil. A periodic coupled finite element–boundary element formulation is used to study the dynamic interaction between a tunnel and a layered soil due to a harmonic excitation on the tunnel invert.</td>
</tr>
<tr>
<td>Anderson</td>
<td>2.5D</td>
<td>A coupled finite element and boundary element scheme is applied in both two and three dimensions.</td>
</tr>
<tr>
<td>Bian</td>
<td>2.5D</td>
<td>A continuously supported model. An efficient 2.5D finite element numerical modeling approach was developed to simulate wave motions generated in ground by high-speed train passages.</td>
</tr>
<tr>
<td>Lombaert</td>
<td>2.5D</td>
<td>A continuously supported model.</td>
</tr>
<tr>
<td>Lombaert</td>
<td>2.5D</td>
<td>A continuously supported model.</td>
</tr>
<tr>
<td>Yang</td>
<td>2.5D</td>
<td>In this paper, the transmissibility of soils for vibrations induced by trains moving at different speeds is studied. The 2.5 D finite/infinite element approach adopted herein allows us to consider the load-moving effect of the train in the direction normal to the two-dimensional profile of the soils considered, and, therefore, to obtain three-dimensional responses for the soils using only plane elements. However, the model has simplified track model and moving loads have no self-oscillation.</td>
</tr>
<tr>
<td>Pedro</td>
<td>2.5D</td>
<td>A study on the critical speed of railway tracks. Detailed and simplified Approaches: 1) 2.5D model; 2) simplified: obtain an accurate assessment of the critical speed by an insightful analysis of the dispersion relationships.</td>
</tr>
<tr>
<td>Sheng</td>
<td>2.5D</td>
<td>In this paper, the wavenumber-based modeling approach is outlined and then the applicability of the method to surface vibration and tunnel vibration analyses is demonstrated.</td>
</tr>
</tbody>
</table>
Chapter 3

MODEL FORMULATION AND FIELD VALIDATION OF A THREE-DIMENSIONAL DYNAMIC TRAIN-TRACK INTERACTION MODEL

Overview

An efficient 3D dynamic track-subgrade interaction model has been formulated and then validated by field investigations at various field and traffic conditions including the effect of different train speeds and types of trains. The model includes a train model, 2D discrete support track model and 3D computation-efficient finite element (FE) soil subgrade model. In the 2D track model, the rail beam is modeled as a Euler-Bernoulli Beam. The 2D track model discretizes the tie and ballast as rigid bodies with designated spacing. The 3D FE subgrade model is simulated by plane-stress quadrilateral finite elements. The longitudinal direction of the subgrade model is assumed to be homogeneous which makes the subgrade model possible to be expanded in the wave-number domain. Therefore, the computing time is largely reduced. A moving dynamic loading is applied on top of the rail. The model is capable of taking train speed variations and the profile change of the cross section into consideration. Multiple field instrumentation tests covering a wide range of cross sections and train speeds were then conducted to verify the accuracy of the dynamic track-subgrade interaction model. Testing site was located on the Amtrak’s highest speed (240 km/hr) line near Kingston, Rhode Island on the Northeast Corridor in United States. Track deflections measured in the field were compared with those predicted by the 3D dynamic track-subgrade interaction model. It is concluded that this model can predict track performance accurately for the Kingston site.

Introduction

The simulation of train-induced vibration is one of the most important topics in railway modeling. Many models have been utilized to investigate the mechanism of the vibrations and predict the track performance under train loading. The vibrations are generated by a variety of excitation mechanisms: the parametric excitation due to discrete supports of the track (such as variations in the tie/ballast stiffness), the “supersonic” motion (critical velocity: a train moves faster
than the wave speed in the track-subgrade system), the excitation due to wheel flats, rail gaps and other surface irregularities.

Early developed models\(^2^7,\ 2^8,\ 3^1,\ 4^6\) are based on a beam-on-elastic-foundation (BOEF) formulation where rails were simulated as Euler-Bernoulli beams placed on continuous elastic spring supports. The excitation by discrete supports of track is weakened due to the simplified railway tracks in the models. Krylov\(^4^5\) developed a prediction model with discrete tie support, but only a quasi-static loading is considered. The model is valid and effective for the high speed train running close to critical velocity of the track-subgrade system because the “supersonic” motion is dominant in that case.\(^4^7\)

More modern models adopt three-dimensional numerical or analytical (semi-analytical) approaches to take more detailed information of the track and subgrade into consideration. Train-induced vibrations, particularly at the train speed around the critical velocity, are studied by many researchers as high speed trains can reach over 500 km/hr\(^4^6\) nowadays. Numerical approaches are widely used to construct three dimensional models, especially finite element method (FEM). Kouroussis et al.\(^4^8\) applied three-dimensional FEM software ABAQUS with C++ user subroutine to investigate ground-borne vibrations. The complete vehicle-track-soil model is developed using an uncoupled approach. Connolly et al.\(^4^9\) adopted ABAQUS and Fortran user subroutine to develop a time domain explicit, dynamic finite element model capable of simulating non-linear excitation mechanisms. Boundary element modeling\(^4^7,\ 5^4,\ 5^5\) is often applied in the FE models to increase the boundary absorption performance. Kacimi et al.\(^5^0\) have applied a damping model based on Rayleigh damping approach in the time-domain FE model to investigate train-induced vibrations. Analytical and semi-analytical models are also developed to understand the mechanism and physical effects of train-induced vibrations. Sheng et al.\(^3^2,\ 3^7\) have proposed a continuous support model with multi-layer ground to study the train-induced ground vibration propagation. Vostroukhov et al.\(^5^3\) have presented a periodically positioned support model with a viscoelastic 3D layer to obtain the steady state dynamic response of a high-speed railway track. Zhai et al.\(^3^8\) proposed a train-track-ground system coupled model to determine the 3D dynamic response of the ground induced by the Chinese high-speed system which is constructed on slab (ballastless) tracks.

Fully three-dimensional analysis is usually time-consuming. Researchers later proposed innovative models using 2.5D approach to reduce the computing time. The 2.5D approach assumes that the track property remains uniform along the direction of train movement. Lombaert et al.\(^5^6,\ 5^7\) was inspired by the formulation derived by Metrikine et al.\(^5^1,\ 6^1\) and Dieterman et al.\(^6^2\) to obtain train-induced vibrations using a 2.5D FE/BE formulation in the frequency domain. The model has
a continuously-supported track and a half-space subgrade. Sheng et al.\textsuperscript{37, 59} have outlined a wave number-based modeling approach to investigate the surface vibration and tunnel vibration under moving vehicles. Bian\textsuperscript{34} and Yang\textsuperscript{36} used 2.5D finite element approach to simulate the soil ground. The previous models using 2.5D approach do not have a detailed track model for periodical supports. Takemiya et al.\textsuperscript{63} presented a multi-layered model in the frequency–wave-number domain by applying the Fourier transform procedure. The train loading is formulated by a series of moving axle loads without self-oscillation.

In order to fully explore the train-induced vibrations, a model that can take track cross-sectional irregularities, moving dynamic load and 3D soil subgrade into consideration is needed.

\textit{Track cross-sectional irregularities.} To account for the detailed track structure, Huang, et al.,\textsuperscript{30} primarily inspired by Kalker’s work as well as others,\textsuperscript{28, 29} introduced a discrete support “Sandwich” track model which will be used in the following formulation of this research. Track irregularities such as the spatial variation of the support stiffness in the longitudinal direction could generate dynamic track responses. It cannot be ignored in the dynamic train-track-subgrade interaction modeling.

\textit{Moving dynamic load.} The train speed effect on track performance has been considered as a critical factor causing track and subgrade vibrations, especially trains running at the critical velocity. Therefore, the moving load at various train speeds is required to model the track-subgrade performance. In addition, the dynamic load is necessary for a comprehensive simulation since the quasi-static load is accurate only when the train speed is close to the critical velocity of the track-subgrade system.\textsuperscript{47}

\textit{3D soil subgrade.} A three-dimensional subgrade model is needed to capture the critical velocity effect. The critical velocity is defined as a train speed that is close to or higher than the wave velocity of the track-subgrade system. This phenomenon mostly occurs on the railway tracks that have soft subgrade underlay. A cone-shaped ground wave motion is generated and moves along with the train. The critical velocity effect is well studied and explained in many researchers’ work.\textsuperscript{28, 37, 52, 58, 64}

In this chapter, model formulation and field validation of the model are presented. In the first section, a computing-efficient dynamic track-subgrade interaction model is formulated with discrete supports, moving dynamic load, and 3D subgrade. The track model is two-dimensional including rail as a Euler-Bernoulli beam, ties and ballast positioned discretely along the train moving direction. The 3D subgrade model is formulated by 2.5D FEM with boundary absorption elements. The subgrade can be partitioned into several regions with different geometries and
material properties. The Green's function is used to couple the track model with the subgrade model. Secondly, the field investigation and validation which are conducted at various field and traffic conditions including the effect of different train speeds and types of trains at the Amtrak Kingston site, Rhode Island, United States. According to the comparison of the field measurements and the model results, the model is validated, which means it is capable of predicting the track vibrations at different speeds for the site.

3D Dynamic Train-Track Interaction Model

Figure 3-1 shows the 3D dynamic train-track interaction model proposed for this dissertation. The model has several parts: a train model, discrete supports (including pad, tie, and ballast), rail beam, and 3D finite element domain for subgrade.

![Figure 3-1. Conceptual dynamic track-subgrade interaction model for U.S. high speed rail](image)

The conceptual model shown in Figure 3-1 is mathematically described in Figure 3-2. Key parameters in the model are listed in Table 3-1. The model couples a 2D track model with the finite element formulation of subgrade. The rail is modeled by a Euler beam. The track structure includes discrete ties with designated spacing and ballast masses. Springs and dampers are used to describe the contacts of rail/tie and tie/ballast. The subgrade is modeled by finite element method with plane-stress quadrilateral elements.
Three steps need to be taken to calculate the dynamic responses of the moving train in the proposed model. The first step is to generate the numerical stiffness and damping of the subgrade (Ks and Ds) by finite element method. The second step is to input the properties of subgrade into the track model to get the responses of the track components. The last step is to extract the contact forces of ballast and soil interface from the track model as input loads for the FEM model to get the subgrade responses. The entire model is a self-developed program which is coded in MATLAB. The detailed formulation and coupling scheme will be presented in the following sections.

![Mathematical description of the 3D dynamic track-subgrade interaction model](image)

**Figure 3-2. Mathematical description of the 3D dynamic track-subgrade interaction model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train Load (f)</td>
<td>(kg)</td>
</tr>
<tr>
<td>Speed (v)</td>
<td>(km/hr)</td>
</tr>
<tr>
<td>Rail Mass per Unit Length (Mr)</td>
<td>(kg m(^{-1}))</td>
</tr>
<tr>
<td>Moment of Inertia of Rail (I)</td>
<td>(m(^4))</td>
</tr>
<tr>
<td>Tie Mass (Mt)</td>
<td>(kg)</td>
</tr>
<tr>
<td>Pad Stiffness (Kp)</td>
<td>(MN m(^{-1}))</td>
</tr>
<tr>
<td>Pad Damping (Dp)</td>
<td>(MN sec m(^{-1}))</td>
</tr>
<tr>
<td>Ballast Stiffness (Kb)</td>
<td>(MN m(^{-1}))</td>
</tr>
<tr>
<td>Ballast Damping (Db)</td>
<td>(MN sec m(^{-1}))</td>
</tr>
<tr>
<td>Subgrade Density</td>
<td>(kg m(^{-3}))</td>
</tr>
<tr>
<td>Subgrade Modulus</td>
<td>(MN m(^2))</td>
</tr>
</tbody>
</table>
Train Model

A simplified vehicle with both primary and secondary suspensions was employed (Figure 3-3). It has 10 degrees of freedom and the displacements of the vehicle can be expressed in frequency domain after the following procedures.

![Typical ten-degree-of-freedom vehicle model](image)

Figure 3-3. Typical ten-degree-of-freedom vehicle model

The governing equation for the train model can be expressed as:

$$[K - \omega^2M]\{dV(\omega)\} = \{f(\omega)\} \tag{3-1}$$

Where: “K” and “M” are the stiffness matrix and mass (including mass moment of inertia) matrix of the car, respectively. “{dV(\omega)}” is the nodal displacement vector. “{f(\omega)}” is the nodal external force vector.

Since \(\{f(\omega)\} = \begin{bmatrix} 0 \\ I \end{bmatrix}\{P(\omega)\}\) where “\{P(\omega)\}” is the nodal wheel force vector and \(\{dW(\omega)\} = \begin{bmatrix} 0 & I \end{bmatrix}\{dV(\omega)\}\) Equation (2) can be rewritten as:

$$\{dW(\omega)\} = [GV]\{P(\omega)\} \tag{3-2}$$

Where: \([GV] =\begin{bmatrix} 0 & I \end{bmatrix}[K - \omega^2M]^{-1}\begin{bmatrix} 0 & I \end{bmatrix}^T\) is called “Green Function of the Vehicle” in this research. \([0 I]\) is a matrix with null matrix \([0]\) and unit matrix \([I]\).

Equation (3-2) is the relationship between the wheel displacements and the wheel rail contact forces in frequency domain. It can be explained as: applying excitation forces \(\{P\}\) on those wheels at frequency “\(\omega\)”, those wheels will vibrate with those magnitudes of “\(dW\)".
Track Structure Formulation

For the track system, the point load \( f(x,t) \) is applied on top of the beam. For the point load, we have

\[
f(x,t) = f(t)\delta(x-vt)
\]  

(3-3)

Where, \( x \): load position;  
\( t \): time;  
\( v \): train speed.

The rail beam is modeled beginning with the governing equation given by Euler Beam Theory:

\[
EI\frac{\dddot{u}}{\ddot{x}^2} + M_r \frac{\dddot{u}}{\dot{x}^2} = f_{ext}(x,t) = \sum_{m=1}^{n} a_m(t)\delta(x-x_m) + f(t)\delta(x-vt)
\]  

(3-4)

Where, \( a_m \): tie force at \( x_m \) acting on the rail beam;  
\( f_{ext} \): external load distribution;  
\( E \): the modulus of elasticity;  
\( I \): Bending moment of inertia;  
\( M_r \): Mass of rail per unit length;  
\( u \): rail beam deflection;  
\( v \): train speed.

Equation (3-4) is rewritten using force equilibrium and Newton’s 2nd law to give the tie force \( a_m \) on rail and ballast force \( b_m \) on the subgrade.

\[
a_m = (U_r-U_t)K_p + (\dot{U}_r-\dot{U}_t)D_p
\]  

(3-5)

\[
b_m = U_bK_s + \dot{U}_bD_s
\]  

(3-6)

\[
M_i\dddot{U}_t = a_m - [(U_i-U_{t})K_p + (\dot{U}_i-\dot{U}_t)D_p]
\]  

(3-7)

\[
M_b\dddot{U}_b = (U_t-U_b)K_p + (\dot{U}_t-\dot{U}_b)D_p - (U_bK_s + \dot{U}_bD_s)
\]  

(3-8)

Where, \( U_t \): rail deflection; \( U_t \): tie deflection; \( U_b \): ballast deflection;  
\( K_p \), \( K_b \), \( K_s \): stiffness of rail pad, ballast and subgrade;  
\( D_p \), \( D_b \), \( D_s \): damping of rail pad, ballast and subgrade;  
\( M_t \): mass of tie per unit length;  
\( M_b \): mass of ballast per unit length.
A Complex Fourier transform is used to remove the time differentiation from Equations (3-5) through (3-8) and are given in Equations (3-9) through (3-12). The subscript ‘w’ means the calculation is transformed to frequency domain and ‘i’ symbols are imaginary numbers.

\[ a_m(w) = [U_{r(w)} - U_{t(w)}](K_p + iwD_p) \]  \hspace{1cm} (3-9)

\[ b_m(w) = U_{b(w)}(K_s + iwD_s) \]  \hspace{1cm} (3-10)

\[ [U_{r(w)} - U_{t(w)}](K_p + iwD_p) - [U_{t(w)} - U_{b(w)}](K_b + iwD_b) = -w^2U_{t(w)}M_t \]  \hspace{1cm} (3-11)

\[ [U_{t(w)} - U_{b(w)}](K_b + iwD_b) - U_{b(w)}(K_s + iwD_s) = -w^2U_{b(w)}M_b \]  \hspace{1cm} (3-12)

Where, \( w \): wave number

The wave number is determined by the minimum wave length of the train induced vibration.

Equations (3-11) and (3-12) are rewritten in order to solve \( U_t \) and \( U_b \) with respect to \( U_r \),

\[ (K_p + iwD_p)U_{r(w)} - [w^2M_t - (K_p + iwD_p + K_b + iwD_b)]U_{t(w)} + U_{b(w)}(K_s + iwD_s) = 0 \]  \hspace{1cm} (3-13)

\[ (K_b + iwD_b)U_{t(w)} + [w^2M_b - (K_b + iwD_b + K_s + iwD_s)]U_{b(w)} = 0 \]  \hspace{1cm} (3-14)

In order to make Equations (3-13) and (3-14) simple and concise in the following expression, the two equations are rewritten below:

\[ (DK_p)U_{r(w)} - (MT)U_{t(w)} + U_{b(w)}(DK_s) = 0 \]  \hspace{1cm} (3-15)

\[ (DK_b)U_{t(w)} + (MB)U_{b(w)} = 0 \]  \hspace{1cm} (3-16)

Where, we set

\[ DK_p = K_p + iwD_p; \quad DK_b = K_b + iwD_b; \quad DK_s = K_s + iwD_s \]

\[ MT = w^2M_t - DK_p - DK_b; \quad MB = w^2M_b - DK_b - DK_s \]

By solving \( U_t \) and \( U_b \) with respect to \( U_r \), the tie force \( a_m \) on rail and ballast force \( b_m \) on the subgrade are written in the form below:

\[ a_{m(w)} = DK_p (1 + DK_p / \Delta)U_{r(w)} \]  \hspace{1cm} (3-17)
\[ b_{m(w)} = (DK_s^*DK_b^*DK_p^*)U_{r(w)}/(MT*MB-DK_b^2) \]

(3-18)

Where, \( \Delta = (MT-DK_b^2/MB) \)

Equations (3-17) and (3-18) are used in the model to represent the forces between the tie and ballast, and the ballast and soil, respectively.

As can be seen from Equations (3-17) and (3-18), the tie force \( a_m \) on rail and ballast force \( b_m \) are the functions of \( U_r \). In order to solve \( U_r \), the rail beam equation is recalled and transformed with respect to \( x \) and \( t \),

\[ (Elk^4-Mr^2+ikw)U_{r(k,w)} = \sum_{m=1}^{n} a_m(w)e^{-ikw} + f(w)(vk+w) \]

(3-19)

To solve the Equation (3-19), the process is addressed in Huang et al.\(^{30, 64}\) Once \( U_r \) is determined, the tie and ballast contact force and deflection can be calculated after using Equations (3-17) and (3-18).

**The Green’s Function for Subgrade**

The subgrade is modeled by 3D FEM. The Green’s function\(^{65, 66}\) is used to describe the subgrade. The model has two major advantages that enable it to simulate different tracks accurately. The first advantage is that the model can take the profile change of the cross section into consideration. Therefore, it is possible to simulate any landforms, such as a tunnel or slope as shown in Figure 3-4. The cross section is modeled by 2D elements with 3D motions. Then, in the longitudinal direction, the domain is expanded by wavenumber through Fourier transform as the track cross section change is considered in the model.
The second advantage is that the 3D subgrade formulation takes the speed effect into consideration. As the speed increases, the soil deflection will increase in a non-linear fashion. Therefore, the soil stiffness considered here will be less as the train speed increases. In this way, the stiffness and damping of the soil have to be obtained by a numerical method to account for the train speed, unlike the values of $K_p$, $D_p$, $K_b$, and $D_b$ which are constant. The method of achieving this is to apply a unit load on top of the subgrade and obtain the numerical stiffness and damping of the subgrade at different speeds. The values of the numerical stiffness and damping of the subgrade is shown as a complex modulus (“$H_s$”) for the subgrade at different train speeds. “$H_s$” varies as the speed of train changes. This is the Green’s function of subgrade used to account for the train speed effect. The real part of “$H_s$” is the value of stiffness ($K_s$) of the subgrade; the imaginary part of “$H_s$” is the damping ratio ($D_s$) of the subgrade.

Figure 3-5 shows a unit point load running directly on top of the soil at a given speed. Because the subgrade shows different dynamic properties at different train speeds, soil stiffness should have different values under different speed conditions. At a given speed, the reciprocal of soil deflection under a unit load is given by the Green function of subgrade for that speed. The Fourier Transform is performed in the travel direction of the train, while the transverse and vertical directions were discretized by Plane Stress Quadrilateral finite elements. The depth and width of the subgrade cross section used in this research is 20 m and 160 m, respectively. In the longitudinal direction, the subgrade domain has a length of 200 m. The sensitivity analysis of the domain size can be found in Gao’s publication. The domain size used in the model is most suitable one because the train-induced ground vibrations can be fully investigated. The absorbing boundary is constructed by dashpots to prevent the wave reflection.
According to the stain-displacement relationship, the strain matrix is given by Equation (3-20) and the stress-displacement relationship gives the elastic matrix in Equation (3-21).

\[
[B] = \begin{bmatrix}
-i\lambda & 0 & 0 \\
0 & \frac{\partial}{\partial \xi} & 0 \\
0 & 0 & \frac{\partial}{\partial \eta} \\
\frac{\partial}{\partial \xi} & -i\lambda & 0 \\
0 & \frac{\partial}{\partial \eta} & \frac{\partial}{\partial \xi} \\
\frac{\partial}{\partial \eta} & 0 & -i\lambda
\end{bmatrix}
\]  (3-20)

\[
[C] = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{22} & C_{23} & 0 & 0 & 0 \\
C_{33} & 0 & 0 & 0 & 0 \\
C_{44} & 0 & 0 & 0 & 0 \\
C_{55} & 0 & 0 & 0 & 0
\end{bmatrix}
\]  (3-21)

Where,

\[
C_{11} = E_1 (1 - \nu_{23} \nu_{32}) \gamma \\
C_{22} = E_2 (1 - \nu_{13} \nu_{31}) \gamma \\
C_{33} = E_3 (1 - \nu_{12} \nu_{21}) \gamma \\
C_{12} = E_1 (\nu_{23} - \nu_{32} \nu_{31}) \gamma = E_2 (\nu_{13} - \nu_{32} \nu_{13}) \gamma \\
C_{13} = E_1 (\nu_{31} - \nu_{21} \nu_{32}) \gamma = E_3 (\nu_{13} - \nu_{12} \nu_{23}) \gamma
\]
\[ C_{23} = E_2 \left( V_{32} - V_{12} V_{31} \right) \gamma = E_3 \left( V_{23} - V_{12} V_{13} \right) \gamma \]
\[ C_{44} = \mu_{23}, \quad C_{55} = \mu_{13}, \quad C_{66} = \mu_{12} \]
\[ \gamma = \frac{1}{1 - V_{12} V_{21} - V_{23} V_{32} - V_{31} V_{13} - 2 V_{21} V_{32} V_{13}} \]

Where, \( \nu \) is the Poisson’s ratio and \( \mu \) is the shear modulus of subgrade. The adopted shape function is presented:

\[
[N] = \begin{bmatrix}
N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 & 0 \\
0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4 & 0 \\
0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & N_3 & 0 & 0 & N_4
\end{bmatrix}
\tag{3-22}
\]

Where, \( N_1, N_2, N_3, N_4 \) are parameters based on quadrilateral elements described by Equation (3-23).

\[ N_i (\xi, \eta) = \frac{1}{4} (1 + \xi \xi)(1 + \eta \eta) \]

\[ N_i = \frac{1}{4} (1 - \xi)(1 - \eta) \]
\[ N_2 = \frac{1}{4} (1 + \xi)(1 - \eta) \]
\[ N_3 = \frac{1}{4} (1 + \xi)(1 + \eta) \]
\[ N_4 = \frac{1}{4} (1 - \xi)(1 + \eta) \]

Equations (3-20) and (3-21) combine with Equations (3-22) and (3-23) to provide the mass matrix (M) and stiffness matrix (K) as shown in Equations (3-24) and (3-25).

Mass matrix:
\[
[M] = \sum \rho \int_{\xi_1}^{\xi_2} \int_{\eta_1}^{\eta_2} N^T N |J| d\xi d\eta
\tag{3-24}
\]

Stiffness matrix:
\[
[K] = \sum \int_{\xi_1}^{\xi_2} \int_{\eta_1}^{\eta_2} (B^T N)^T C (B N) |J| d\xi d\eta
\tag{3-25}
\]

Where, ‘e’ represents element-wise integration.

The equivalent nodal force vector is given in Equation (3-26):
\[
[F^e] = \sum \int_{\xi_1}^{\xi_2} \int_{\eta_1}^{\eta_2} N^T f |J| d\eta d\xi
\tag{3-26}
\]
And J is the Jacobian matrix:

\[
[J] = \begin{bmatrix}
\sum_{i=1}^{4} \frac{\partial N_i}{\partial \xi} y_i & \sum_{i=1}^{4} \frac{\partial N_i}{\partial \xi} z_i \\
\sum_{i=1}^{4} \frac{\partial N_i}{\partial \eta} y_i & \sum_{i=1}^{4} \frac{\partial N_i}{\partial \eta} z_i
\end{bmatrix}
\]

(3-27)

|J| is the corresponding determinant, |J|=det[J].

Equation (3-28) is obtained from the derived equations above, and is used to calculate the ground surface displacement.

\[
([K] - (\omega - \lambda v)^2 \times [M]) \times [d\bar{S}] = [\bar{F}]
\]

(3-28)

From the equation above, one can solve the soil deflections under unit moving point load with any excitation frequency, which is the numerical soil Green’s Function, the matrix [GS]. By inserting this Green’s Function back into the model, a fully coupled model is constructed. The complete construction and solving process were coded and solved in MATLAB.

**Train-Track-Soil Coupling**

Figure 3-6 shows the wheel rail coupling scheme. Equation (3-29) is the mathematical expression of this contact coupling scheme:

\[
-\{dW(\omega)\} - \{dR(\omega)\} - \{ds(\omega)\} = \{P(\omega)\}/[HK]
\]

(3-29)

Where: “\{dR(\omega)\}” is the downwards rail displacement in frequency domain; “\{ds(\omega)\}” is the combination of rail surface roughness and train speed which is usually obtained by field measurements and is in fact the cause of the vehicle vibration.
Figure 3-6. Wheel-rail coupling scheme

It is worth noticing that similar to Equation (3-30), track deflection and wheel rail contact force can also be expressed as: \( \{dR(\omega)\} = [GT]\{P(\omega)\} \) where “[GT]” is the “Green Function of the Track”. Therefore, wheel rail contact force can be expressed by using rail surface roughness and train speed:

\[
\{P(\omega)\} = -([HK][GV] + [HK][GT] + [I])^{-1}\{ds(\omega)\}
\]  (3-30)

In order to couple the subgrade formulation with the track formulation, the soil stiffness \( Ks(\omega) \) is considered as a frequency dependent value. A unit load is applied on top the soil at each excitation frequency. Then, at each frequency, the subgrade domain could generate a frequency-dependent subgrade stiffness that will be input into the track formulation. This means that the subgrade stiffness is a changing value for different frequencies. In this way, the speed effect on the entire track-subgrade system can be fully taken into consideration.

**Field Validation**

Field validation is necessary to ensure the accuracy of the developed dynamic track model. To verify the capability of the model at different speed conditions, it requires a variety of speeds ranging from lower speed to high speed. In this section, a field case study on the Northeast Corridor at Kingston, Rhode Island is presented to validate the model. The Kingston site has both low speed conditions (regional trains: 90-140 km/hr) and high-speed conditions provided by Amtrak’s Acela Express Train which travels to 240 km/hr.

NEC is a rail line owned primarily by Amtrak, which runs 731 km from Boston, Massachusetts to Washington, D.C. This line has sections of Class 8 Track allowing speeds of 240 km/hr. This line is not restricted to high-speed trains (the Acela), but also serves regional passenger trains and limited commuter and freight trains.

The subdivision of the NEC that is of interest to this study is a straight section of track that is currently classified for speeds up to 240 km/hr. A straight section of the NEC runs northeast/southwest through the northern section of the Great Swamp, Rhode Island, just to the west of South Kingston, Rhode Island. Currently, this section of the NEC is a double track section that is used for both HSR trains and regional passenger trains. The ballasted track consists of UIC60 Rail (\( I = 3.04 \times 10^{-5} \text{ m}^4; \text{Mass/Length} = 60 \text{ kg m}^{-1} \)) is supported by precast, pre-stressed concrete ties.
(Length = 2.62 m; Width = 0.28 m; Height = 0.25 m; Mass = 340.2 kg) spaced at 0.61 m. Some geotechnical and geophysical tests were adopted to determine the track condition and site geological information.

**Instrumentation Plan**

A $3 \times 3$ array of piezoelectric, triaxial accelerometers were temporarily installed along the track and right-of-way with a general longitudinal spacing of 25 tie spaces (15 m) and a general lateral spacing of 7.5 m. Steel mounting plates were machined and attached to the concrete ties using an epoxy adhesive (Figure 3-7). The mounting plates were machined with a bevel to accommodate the geometry of the tie, allowing for a horizontal surface so that the accelerometers could be level. The six accelerometers installed in the soil adjacent to the track were mounted on insertion spikes, and pushed into the embankment and swamp to a depth of six inches. The installation methods were designed to permit rapid installation and removal so that the accelerometer network could be installed at different sites along the track.

Each accelerometer recorded the accelerations in three directions: vertical, longitudinal, and lateral. The measurement range of the accelerometers are $\pm 490 \text{ m/s}^2$ and the frequency range is 0.8 to 8000 Hz for 3 dB. According to the manufacturer’s specifications, the accelerometers are capable of measuring the vibrations generated by moving trains. The specifications of the accelerometer are attached in the Appendix D. The sampling was conducted at a rate of 1000 Hz. The accelerometer data were recorded using conventional amplification, signal conditioning, and data acquisition using a notebook computer. 500 Hz low-pass passive filters were used to eliminate aliasing effects. Figure 3-8 shows the general plan and cross-sectional view of instrumentation. More details of the instrumentation are provided in Appendix B.

Figure 3-7. Installation of accelerometers at the Kingston site: (a) concrete tie; (b) swamp deposit
Figure 3-8. Layout of instrumentation at the Kingston site: (a) plan view; (b) cross section

**Model Parameter Characterization**

Site characterization provides the material properties and site geometry for model validation. Two different locations (called site 1 and site 2 in the following context) were selected at the Kingston site to conduct the site characterization. Accurate topography was available from
Light Detection and Ranging (Lidar) surveying. Layer configuration at the site was determined using GPR. The Dynamic Cone Penetrometer Test (DCP) was conducted to construct the cross-sectional data input for the dynamic track model. In addition, the traffic information, such as vehicle speed, train types, was recorded and listed in the Appendix A. The configurations of different types of trains were offered at the courtesy of Amtrak.

*Ground penetrating radar (GPR) and light detection and ranging (Lidar)*

Ground penetrating radar was collected at the sites by HyGround Engineering in June 2011. The Kingston site has a two-way track and the track instrumented is called Track 1 in this chapter. A hi-rail truck with antennas mounted on the left, center, and right sides of Track 1 was used to survey the track. The data were processed to estimate the fouling indices, layer depths, and moisture profiles as shown in Figure 3-9. Geometry and Lidar data were also shown with the GPR data. Lidar data were provided for the track and right-of-way which were used to develop accurate ground surface topography.

Window (a) shows the Fouling Index based on GPR with the left on top, then center, and right on the bottom. The color bar shows that it is typically around 10, but comes close to 20 where light green and 25 where yellow; Window (b) shows the moisture profile of left center and right. The blue dots represent moisture; dark blue is very wet while anything that is red is not wet. The approximate layer depths are drawn on these plots; Window (c) shows the Roughness with mid chord offset (MCO) of 19 m. These data are values above and below zero, so we square the data over a window of 60 feet to give an absolute value of the roughness in a spot. The y-axis is a number representing the months since January 2010, from early 2011 to January 2015; Window (d) shows the space curve data taken from the time that the survey was done. This is the relative vertical position of the rail over a 120-meter window, and the units are in inches; Window (e) shows the Lidar data, zeroed at Track 1; Window (f) shows the surficial geology provided from maps from the state of Rhode Island. Red is organics; Orange is fill over till.
Dynamic cone penetrometer (DCP)

The Dynamic Cone Penetrometer test was conducted at the track centerline, shoulder and swamp area of Track 1 at both sites. The right side of Figure 3-10 (Track 2), which is omitted from the figure, was assumed to have subgrade stiffness properties that were symmetrical to the track shown (Track 1). The DCP results indicated the in-place relative density of the soil, which was used to estimate the subgrade stiffness. The DCP device uses an 8-kg hammer that drops 575 mm driving a 60° cone tip with 20 mm base diameter into the ground. Tests were performed at both sites whenever possible, though testing in the gage of the track was only possible during overnight work. Samples were taken from the embankment at sites for visual identification and are shown on Figure 3-10. Figure 3-11 shows the DCP device being used along the edge of the embankment closest to the track. More DCP data is included in Appendix C.
Figure 3-10. Cross sections of site 1 and site 2 with soil description and layers

Figure 3-11. Dynamic cone penetrometer (DCP) device
Figure 3-12 shows the DCP Results of site 1 and site 2 collected in 2013. DCP tests were also conducted in the years of 2014 and 2015. All the DCP data was analyzed and summarized in the figure. For each site, three locations were chosen to be tested: center of the track, embankment and swamp. Test locations were marked in the plot and the number of blows per inch were recorded and shown in the Figure 3-12.

The stiffness of the ballast and subgrade are calculated using the DCP data. The stiffness ranges at each depth are listed in Table 3-2. The values have a wide range because the data were collected in different seasons and conditions. The modulus of the soil can be estimated using the following equation \(68\). Equation (3-31) is for standard DCP equipment only (drop height of 575 mm and a hammer mass of 8 kg).

\[
E_{DPI} = 10^{3.04758 \cdot [1.06166 \log(DPI)]} \tag{3-31}
\]
Where: $E_{DPI} = \text{modulus (MPa)}$

$\text{DPI} = \text{DCP penetration index [mm/drop]}$

Table 3-2. Track and subgrade stiffness of site 1 and site 2.

<table>
<thead>
<tr>
<th>Site 1</th>
<th>Centerline</th>
<th>Embankment</th>
<th>Swamp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (m)</td>
<td>E (MPa)</td>
<td>Depth (m)</td>
<td>E (MPa)</td>
</tr>
<tr>
<td>0-0.6</td>
<td>150-350</td>
<td>0-0.6</td>
<td>30-90</td>
</tr>
<tr>
<td>0.6-1.3</td>
<td>30-100</td>
<td>0.6-2.7</td>
<td>10-30</td>
</tr>
<tr>
<td>1.3-2.6</td>
<td>40-120</td>
<td>2.7-3.6</td>
<td>100-120</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Site 2</th>
<th>Centerline</th>
<th>Embankment</th>
<th>Swamp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (m)</td>
<td>E (MPa)</td>
<td>Depth (m)</td>
<td>E (MPa)</td>
</tr>
<tr>
<td>0-1.1</td>
<td>200-350</td>
<td>0-2.0</td>
<td>30-90</td>
</tr>
<tr>
<td>1.1-2.8</td>
<td>150</td>
<td>2.0-2.5</td>
<td>150</td>
</tr>
<tr>
<td>2.8-3.5</td>
<td>70-120</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Traffic Information

Figure 3-13 shows the Acela Express and the Regional trainset configuration. The Acela Express has one power car in each end of the train and six passenger cars in between. The Regional train has only one locomotive which is followed by seven passenger cars. Information regarding the axle loads for the Acela and Regional train set is listed in Table 3-3. The train speed is an important factor in evaluating track performance, which has great influence on track performance. A high-speed video camera and a photo-electric sensor were used to record the passing train so that an accurate measure of train speed could be obtained and the exact location of the train could be reconciled with the acceleration time histories. A complete train passage log can be found in Appendix A.
Figure 3-13. Acela express and regional trainset configuration (dimensions in meters, courtesy of Amtrak)\(^{69}\)

Table 3-3. Acela express and regional train set information.\(^{69}\)

<table>
<thead>
<tr>
<th>Acela Train</th>
<th>No. of Cars</th>
<th>Axles/Car</th>
<th>Car Length (m)</th>
<th>Inter-Bogie Spacing (m)</th>
<th>Inter-Axle Spacing (m)</th>
<th>Mass/Axle (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Locomotive</strong></td>
<td>2</td>
<td>4</td>
<td>21.219</td>
<td>10.744</td>
<td>2.849</td>
<td>23,175</td>
</tr>
<tr>
<td><strong>First Class</strong></td>
<td>2</td>
<td>4</td>
<td>26.645</td>
<td>18.136</td>
<td>2.997</td>
<td>16,150</td>
</tr>
<tr>
<td><strong>Business Class</strong></td>
<td>3</td>
<td>4</td>
<td>26.645</td>
<td>18.136</td>
<td>2.997</td>
<td>15,775</td>
</tr>
<tr>
<td><strong>Regional Train</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Locomotive</strong></td>
<td>1</td>
<td>4</td>
<td>15.692</td>
<td>7.799</td>
<td>2.943</td>
<td>22,908</td>
</tr>
<tr>
<td><strong>Passenger</strong></td>
<td>7 to 9</td>
<td>4</td>
<td>26.01</td>
<td>18.136</td>
<td>2.591</td>
<td>13,155</td>
</tr>
<tr>
<td><strong>Café</strong></td>
<td>1</td>
<td>4</td>
<td>26.01</td>
<td>18.136</td>
<td>2.591</td>
<td>12,475</td>
</tr>
</tbody>
</table>
The Acceleration Data at Kingston Calibrated by Multi-depth Deflectometers (MDDs)

The acceleration measured by accelerometers could have drift when being double integrated to obtain displacements. In this section, a method is proposed to perform double integration on accelerometer data combining spectrum analysis and the Multi-depth Deflectometers (MDDs). The MDD measurements were used as ground truth data to calibrate the accelerometer data in the field. The instrumentation of MDDs was conducted by the researchers at the University of Illinois and the test site was also on the NEC but at different locations. The installation location of MDDs was in Chester, PA, to monitor the deflection of individual track substructure layers. In this section, all of the field data from MDDs and accelerometers were collected at the Chester site and will be used to calibrate the data from the Kingston site. Both MDDs and accelerometers were installed at the site for this calibration and comparison. Figure 3-14 shows the schematic of MDDs installed in the track substructure. The basic idea of MDDs is to measure the deflection of each layer with respect to a fixed spot deep in the ground by installing several linear variable differential transformers (LVDTs) vertically.

The test was conducted by the researchers from University of Illinois. The displacement transducers (LVDTs) used in the test were based on the inductive half-bridge principle and an HBM MGCPLUS signal conditioner amplifier was used as the power supply. These particular types of LVDTs were selected because they are capable of providing noise-free precise measurements for extended periods of time. The displacement measurement range of the LVDTs was up to ±4 mm with an accuracy of < 1% (often < 0.5%) over their full range. For the LVDT at the top of MDD, where the highest deformations are expected, a specially manufactured LVDT capable of measuring ±10 mm was used. Therefore, the error in the measurements is assumed to be of negligible significance for the overall track deflection. The MDD-measured deflections of tracks will be considered as ground truth in this study. The MDD installation and data recording were conducted by the researchers at University of Illinois at Urbana-Champaign. Initial calibrations of the LVDTs were conducted by using the HBM MGCPLUS signal conditioner and repeating the calibration process with two different cable types. Each LVDT was calibrated with its own ferrite core. Each LVDT module was connected to the signal conditioner amplifier and then were inserted into the hole which was pre-drilled on the track. The details of the instrumentation plan and data acquisition can be found in the related University of Illinois publications.
Figure 3-15 shows a perfect plot of the rail deflection measured by the MDDs for a typical Acela train. It can be clearly detected in the graph that two locomotives are at the two ends and six passenger cars are in between. The MDD data is considered as ground truth to calibrate the acceleration measured by accelerometers.

However, the displacements double integrated from the accelerometer data could not be able to catch the train responses since the level of noise and drift is sufficiently high to interfere the
displacement data. Therefore, authors start to analyze the raw acceleration data from the perspective of frequency domain. The displacement measured by MDDs are differentiate twice to get the acceleration. Double differentiation will not introduce any error so the data can still be considered as ground truth. Figure 3-16 shows the comparison of the acceleration measured by MDDs and accelerometers in the frequency domain. As can be seen, the two spectrums overlap at most of the major peaks with comparable magnitudes. It means the two measurements (MDDS and accelerometers) actually are able to catch the responses of tracks under moving trains. The reason is that each peak in the spectrum indicates one characteristics of the train type associated with the train speed (train set information is shown in Table 3-4). For example, the train speed was 177 km/h (49 m/s). The first peak in the spectrum is around 1.83 Hz. The train speed combined with the characteristic frequency results in a wavelength of 49/1.83 = 26.8 m which is very close the car length. In other words, the peaks at those frequencies in the spectrum reflect the modes of vibrations of the track induced by a specific type of train at a specific velocity. Those responses between the major peaks will not have significant effect on the magnitude and shape of the displacement contour. Through this finding, one can extract the useful acceleration from the raw data to obtain the correct displacement. The procedure will be illustrated in the following paragraphs.

Table 3-4. The configuration of the Acela train.

<table>
<thead>
<tr>
<th>Acela Train</th>
<th>No. of Cars</th>
<th>Axles/Car</th>
<th>Car Length (m)</th>
<th>Inter-Bogie Spacing (m)</th>
<th>Inter-Axle Spacing (m)</th>
<th>Mass/Axle (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Locomotive</td>
<td>2</td>
<td>4</td>
<td>21.219</td>
<td>10.744</td>
<td>2.849</td>
<td>23,175</td>
</tr>
<tr>
<td>First Class</td>
<td>2</td>
<td>4</td>
<td>26.645</td>
<td>18.136</td>
<td>2.997</td>
<td>16,150</td>
</tr>
<tr>
<td>Business Class</td>
<td>3</td>
<td>4</td>
<td>26.645</td>
<td>18.136</td>
<td>2.997</td>
<td>15,775</td>
</tr>
</tbody>
</table>
As stated before, the acceleration data in the frequency domain have different peaks at different frequencies which represent the characteristics of the train type and speeds. However, the magnitude between those peaks do not have significant effect on results. Therefore, the responses could be filtered into different intervals in the frequency domain in order to find out which interval contributes most of the noise that causes the drift. The authors found that for this specific train type and speed, the noise from the responses of 0-4 Hz interferes the results of double integration most. As long as 0-4 Hz is filtered from the data, the displacement obtained from the double integration could have similar shape as the MDD data. Figure 3-17 represents the two measurements of the deflection of the rail from 4 Hz above: 1) obtained directly from MDDs; 2) double integrated from accelerometer. As can be seen, the displacement fields have quite similar responses when the train passes. Therefore, the acceleration data over 4 Hz can be double integrated without significant interference for this specific condition.
Figure 3-17. The comparison of the displacements obtained from the data over 4 Hz for the Chester site

Then, the rest of the work becomes how to obtain the correct responses for the interval of 0-4 Hz. As can be seen in Figure 3-18, the spectrum from 0 to 4 Hz of MDDs data only has two peaks at 1.83 Hz and 3.66 Hz which correspond to the car length and half of the car length. But no significant responses are detected other than these two peaks. However, the spectrum of accelerometer data contains much more noise and even barely can detect the response at 1.83 Hz. This also helps explain the acceleration data of 0-4 Hz could not be integrated correctly. In order to obtain the acceleration data without interference, the responses at 1.83 Hz and 3.66 Hz are extracted to reproduce the responses in 0-4 Hz by a simple equation:

\[ Y = A \cos(2\pi f_1 x) + B \cos(2\pi f_2 x + 90) \]

Where, \( f_1 \) and \( f_2 \) are the two frequencies in the 0-4 Hz; \( A \) and \( B \) are the magnitude in the spectrum of the two peaks.

Figure 3-18. The spectrum from 0 to 4 Hz of MDDs and accelerometer data at the Chester site
Then, the responses from 0-4 Hz is added to the responses from 4 Hz above as shown in Figure 3-19. The total deflection is shown in Figure 3-20 and matches the MDDs data in terms of the maximum rail deflection at each wheel. However, the match between each bogie is not good enough and some uplifts can be found in the accelerometer data. From the spectrum of the rail deflection (Figure 3-21), the accelerometer data and MDDs results match at most peaks. The difference in the spectrum could be caused by the double integration and abilities of accelerometers. However, compared to the results directly integrated by accelerometers (not shown here because drifting is huge and results are not even able to be comparable with MDDs), the match has a great improvement. Therefore, we can conclude that this method can be an effective way to calibrate the acceleration data. Also, there are some limitations on this method: 1) the method only works for specific train type. If the configuration of the train changes, the method may not be useful; 2) MDDs has to be installed for this specific train, otherwise there is no correct data for comparison and validation. Even though this method has those limitations, for this specific train type (The Acela train and regional train), this method can be considered as valid and effective to obtain the displacements form accelerometers.

![Graph showing rail vertical deflection](image)

Figure 3-19. The reproduced deflection (0-4 Hz) and accelerometer measured deflection (over 4 Hz) at the Chester site.
Figure 3-20. The comparison of rail vertical deflection between MDDs and accelerometers at the Chester site

Figure 3-21. Spectrum analysis for rail deflection measured by MDD and accelerometer at the Chester site

In summary of this method, three steps are made to obtain the rail deflection:

1) Use a high-pass filter to get the acceleration data from 4 Hz above and obtain the displacements from the filtered acceleration by double integration;

2) Multi-depth Deflectometer (MDD) was used to obtain the displacement directly. MDD has been considered as a ground truth measurement for the displacement. 0-4 Hz of the MDD data has been bandwidth filtered and then been conducted second derivative to get the acceleration. In the frequency domain, the MDD data of 0-4 Hz (ground truth data) was compared with the accelerometer data of 0-4 Hz. The peak responses from 0 to 4 Hz were selected and extracted to
reproduce the acceleration in 0-4 Hz in order to get rid of the noise and interference signals. Then the accelerometer data of 0-4 Hz could be double integrated to get the displacement;

3) Add up the displacement from accelerometers in step 1 and 2 to obtain the total rail vertical displacement.

Model Prediction

Measurements of the track and ground vibration due to passing trains in the Great Swamp Area were conducted at different vehicle speeds. Figure 3-22 presents the calculated displacement versus time. Each downward peak presents a wheel load of the end locomotives and six passenger cars. The downward displacement for locomotives are larger than passenger cars since the locomotives are heavier; the bogie spacing for locomotive is smaller than for passenger cars.

Figure 3-22. Vertical displacement of tie for an Acela traveling at 193 km/hr, recorded on 3-18-14 at 12:49 pm at the Kingston site 1

Figure 3-23 shows the match of rail deflection between the field tests and simulation for three different Acela Trains at different speeds similar to Figure 3-22, but with the model shown. The rail deflection was obtained by filtering and double integration. In the simulation, the maximum track deflections with train speeds of 193 km/hr, 222 km/hr and 240 km/hr were 2.3 mm, 2.4 mm and 2.8 mm, respectively. The maximum track deflections in field test were 2.2 mm, 2.3 mm and 2.7 mm respectively. The data from accelerometers 1A, 2A and 3A were used to perform the calculation. The match for all three measurement points were only shown for the speed at 193 km/hr. 1A measurement point is shown for all speeds. The results for other measurement points can be provided upon request. As can be seen from Figure 3-23, the maximum rail deflection from the model can match the field measurements. The signatures between wheel axles from field
measurements have uplifts that the model results do not possess. However, MDD measurements of the rail deflection do not have uplifts, indicating that the uplifts could be outcomes caused by numerical processing of the data. The uplifts may come from the double integration and the method of processing the acceleration data.
Figure 3-23. Field measurement and simulation of deflection of rail at different Acela train speeds (3-18-14, the Kingston site 1): (a) $v = 193$ km/hr at 1A; (b) $v = 193$ km/hr at 2A; (c) $v = 193$ km/hr at 3A; (d) $v = 222$ km/hr at 1A; (e) $v = 240$ km/hr at 1A

Figure 3-24 shows the match for Regional train. The train speed was 110 km/hr and the maximum track deflection is 2.0 mm for both simulation and field measurements. The responses from locomotive and passenger cars can be clearly detected in the plot.

Figure 3-24. Field measurement and simulation of deflection of rail for regional trains (3-18-14, the Kingston site 1)
Figure 3-25 shows the vertical track displacement match of site 2 at Kingston for Acela and Regional trains. By site investigation, site 2 had larger subgrade stiffness than that of site 1. This also has been reflected in the results of simulation and field measurement of vertical track displacement. The Acela train and Regional train had the maximum track displacement around 2.1 mm and 2.0 mm respectively. The Acela train was running 237 km/hr that is close to the train speed in Figure 3-24 (e) in which the maximum track displacement was 2.7 mm. Therefore, it shows the applicability of the model to different sites and demonstrates the validity of the model at least within the range of the speed tested (125 km/hr-240 km/hr) at Kingston.

In order to further investigate the difference between the field measurement and model results, the rail deflections in the Figure 3-23 (a), (d) were transformed into frequency domain for comparison. The Acela train speeds were 193 k/hr and 222 km/hr respectively. Only the frequency range 0 to 30 Hz were presented for the comparison because the deflection profile is only affected by the frequency under 30 Hz or even lower. As can be seen, the spectrum of field measurements and simulation have peak responses at most frequencies. It means that the model and field
measurements both caught the major responses generated by the moving trains. At most peaks, the magnitudes were close except for the frequencies around 10 Hz and 17 Hz. At these two frequencies, the field measurements have much lower magnitude than the model have. The attenuation in the field measurements may be induced by the track structure, subgrade or other uncertainties which may be hard to be identified. Despite the magnitude at these two frequencies, the model is capable of predicting the field results at most frequencies as shown in the spectrum.

Figure 3-26. Spectrum analysis for field measurement and simulation of rail deflection (3-18-14, the Kingston site 1): (a) $v = 193$ km/hr at 1A; (b) $v = 222$ km/hr at 1A
Lac La Biche Field Verification

The field verification at Lac la Biche is used as a backup validation for the model. This site is very similar to the Kingston site, though is used for the transportation of freight at lower speeds. Four locations within 1000 m were selected to test the model because they were close to an area where samples had been collected. Data were available for the properties of the soil and track that were needed to use the model. This includes samples collected in the track using an Automatic Ballast Sampler (ABS), soil layers from GPR, Cross sections from terrestrial laser scanning, and surficial geology. The rail size which was much smaller than the Kingston site (41kg per m), and has inertia values twice as small.

The soil modulus was back calculated using GEOTRACK with the loading conditions of the Falling Weight Deflectometer (FWD) test. This was done by forcing the layer deflections to match the results of FWD tests as close as possible with realistic modulus values. The layers were decided based on the layers from GPR and the samples collected. Figure 3-27 shows all of this information for one of the four sites. Once the results of the individual layer deflections were satisfactory, the sites were analyzed under MRail conditions to ensure that the properties of the ballast, fastener stiffness, and other track properties matched the deflections available from the MRail tests. Figure 3-28 shows a plot of the GPR, ground surface, geometry, MRail, and FWD data of the LLB Site.

Figure 3-27. Subgrade Properties of the Lac la Biche Site and GEOTRACK Results at 157+654
The MRail system has been developed over years of research developed by Shane Farritor at the University of Nebraska–Lincoln. It is an autonomous system that is capable of measuring vertical rail deflection from a rail car by using a laser. GEOTRACK was used with the loading conditions of the MRail test as shown in Figure 3-29. This provides the rail deflections under 10 ties so interpolations of the deflections in this figure give the equivalent MRail values. Once the results were satisfactory, the modulus values shown in Table 3-5 were trusted and the original model was used to predict the deflection to further verify it. Figure 3-30 shows the automatic ballast sampling (ABS) data that was used.
Figure 3-29 MRail measurement setup
Table 3-5 Lac La Biche track substructure properties.

<table>
<thead>
<tr>
<th>Milepost</th>
<th>157+654</th>
<th>157+969</th>
<th>157+3246</th>
<th>157+3476</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Surficial Geology</strong></td>
<td><strong>Moraine</strong></td>
<td><strong>Moraine</strong></td>
<td><strong>Organics</strong></td>
<td><strong>Organics</strong></td>
</tr>
<tr>
<td></td>
<td>L Dept h From TOT (m)</td>
<td>C Dept h From TOT (m)</td>
<td>R Dept h From TOT (m)</td>
<td>L Dept h From TOT (m)</td>
</tr>
<tr>
<td>Ballast</td>
<td>0.32</td>
<td>0.30</td>
<td>0.27</td>
<td>0.40</td>
</tr>
<tr>
<td>SubBallast</td>
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<td>0.60</td>
<td>0.55</td>
<td>0.63</td>
</tr>
<tr>
<td>Embankment</td>
<td>1.16</td>
<td>1.14</td>
<td>1.10</td>
<td>1.34</td>
</tr>
</tbody>
</table>

Table 3-6 shows the results of the prediction of the rail deflection, comparing the maximum displacement for field and simulation results. From the table, the model prediction for LLB site is

![Figure 3-30 ABS data used for developing subgrade layers](image_url)
Table 3-6 Lac La Biche rail deflection predictions.

<table>
<thead>
<tr>
<th></th>
<th>157+654</th>
<th>157+969</th>
<th>157+3246</th>
<th>157+3476</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRail</td>
<td>3.30 mm</td>
<td>4.82 mm</td>
<td>3.30 mm</td>
<td>3.30 mm</td>
</tr>
<tr>
<td>Model Prediction</td>
<td>4.06 mm</td>
<td>5.08 mm</td>
<td>4.06 mm</td>
<td>4.06 mm</td>
</tr>
</tbody>
</table>

Conclusions

In this chapter, a 3D dynamic train-track interaction model is formulated to investigate and predict the track, and soil dynamic responses under different train speeds. It was then validated at a range of train speeds at different sites for different types of trains. The model simulated the track structure with discrete ties with designated spacing and ballast masses. The ballast/soil interface was described by a complex Green’s function which was derived from a 3D FE model considering speed effects on the subgrade. Also, the cross section variation was considered in this model which enables the model to simulate any profiles.

The field investigation was performed at the Great Swamp in Kingston Rhode Island. The site investigation included Dynamic Cone Penetrometer (DCP), Ground Penetrating Radar (GPR), Light Detection and Ranging (Lidar). For the Kingston site, the ballast and soil stiffness were obtained at different times of a year. The traffic information was also collected and analysed to provide the inputs for the numerical model.

Field validation was conducted to validate the accuracy of the 3D dynamic track model. This was done by using a 3*3 array of accelerometers were mounted on one side of the track to record the track performance at Kingston site. The rail deflection was calculated by double integrating the accelerations for tests performed on trains running at several speeds. Based on the comparison of the track performance at different train speeds, the maximum rail deflections of the model results at the track centerline match the field measurements. This shows the applicability of the model to different sites and demonstrates that the model is capable of predicting the track performance at different train speeds, at least within the range of the speed tested (125 km/hr-240 km/hr) at Kingston.
Chapter 4
HIGH SPEED RAILWAY TRACK DYNAMIC BEHAVIOR NEAR CRITICAL SPEED

Overview

Field observations and measurements had indicated a critical speed at which moving trains will cause substantial amount of track vibrations both horizontally and vertically. The critical speed of a High Speed Rail (HSR) system is the speed at which vibrations propagate within the track structure and subgrade at a speed close to the Rayleigh wave velocity of the subgrade soil. This study was performed on the Amtrak Northeast Corridor (NEC) at Kingston, Rhode Island, in an area known as the Great Swamp where high speed trains (Acela Express) and regular trains (Regional) operate at fast speeds within an area with relatively thick deposits of soft organic soil (swamp) that makes it a favorable site for investigation of the critical speed phenomena. The objective of this research is to investigate and evaluate the track performance at the Kingston site, predict the critical speed effect through a numerical model and determine if critical speed effects exist at the Kingston site. The track performance was investigated by a three-by-three (3x3) array of accelerometers. Site investigations were carried out to characterize the site and provide input data for modeling. The previously developed and validated 3D dynamic track-subgrade interaction model (Chapter 2) was used to predict the track and ground vibrations and substructure performance. Field measurements and model results of rail deflection match for the maximum rail deflection at certain speed range; ground surface wave propagation has the similar cone-shaped mode. A slight critical speed effect was detected at Kingston site, but did not cause significant track deflection. In addition, model simulation pointed out that the stress contour in the subgrade would encounter a significant increase when trains running at predicted critical speed. Due to the complicated tie-ballast-soil system and the geometry of the cross section, the critical speed is defined in two levels: 1) the speed causing significant increase in rail deflection, at which derailment becomes a concern; 2) the speed causing significant increase in the cross section stress intensity, at which more frequent ballast maintenance becomes a concern.
Introduction

Increasing speed brings new challenges to conventional railway engineering, which include rail system control, passenger safety and comfort, and noise and vibration hazards. Ground-borne vibration induced by high speed rail is the focus of this research. According to previous research, high speed trains on soft ground can induce a significant increase in vibration level when trains move at the critical speed. Theoretical modeling of a rail as a beam supported by track structure and ground reveals that the dynamic amplification will occur when train reaches a certain speed, this has been studied since 1927. For typical soil properties, this previous modeling suggests dynamic amplification around 1800 km/hr, which is greatly above the realistic HSR speed, because only pressure wave velocity is considered in Timoshenko’s theory. Hence, for a long period, the train loads have been assumed to be quasi-static moving loads. However, Krylov considered that a train will encounter something equivalent to the ‘sound barrier’ when reaching the velocity of Rayleigh surface waves propagating in the ground. This phenomenon is somewhat analogous to the sonic boom of supersonic jets. The velocity of Rayleigh waves traveling in soft sandy soils is 320 to 470 km/hr, which is already reachable by today’s high speed trains outside of The United States. Furthermore, for peat, marine clays, and other soft clays, the Rayleigh wave velocities could be as low as 110-140 km/hr.

The critical speed is the speed at which vibrations propagate within the track structure and subgrade at a speed close to the Rayleigh wave velocity of the subgrade soil. Rayleigh waves typically have a speed slightly less than shear waves. Thus, soil with low shear-wave velocity needs much attention because it is susceptible to increased vibrations when the Rayleigh Wave speeds are approached. As a consequence, the high-level of vibration imposes limits on train speed. Another phenomenon of the critical speed effect is the cone-shaped wave motion of ground surface which is analogous to a boat moving through water. As can be seen in Figure 4-1, the displacement fields of the ground surface become a cone-shaped wave motion as the train speed reaches the critical speed and the magnitude of the displacement increases at the same time. For an example, X-2000 (Swedish HSR), in 1997, was running at the maximum speed on the Goteborg-Malmo line and generated excessive vibration in the soil and overhead contact line support poles. Furthermore, at Ledsgard, Sweden, a new railway line with design speed of 200 km/h encountered high vibrations on soft cohesive soil. The speed of trains had been decreased from 200 km/h to 160 km/h, then to 130 km/h. The reason for the high vibrations turned out to be the approaching of critical speed.
Also, the similar phenomenon happened at Northern Ireland Railways where the subgrade is constructed on peaty soils.\textsuperscript{75}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image}
\caption{Vertical displacement fields of the ground surface when train running at different speeds: low speed (left) and critical speed (right)\textsuperscript{64}}
\end{figure}

In this study, a previously developed and validated 3D dynamic track model\textsuperscript{30,64} is used to predict the track and ground vibrations and substructure performance under trains moving at critical speeds. The dynamic track model is a complex system with train, track and soil subgrade. The vehicle model is a detailed system with car body, primary and secondary suspension system. The ballasts-soil interface is numerically modeled by the complex Green’s function \textsuperscript{76} to account for different speeds and subgrade stiffnesses. The subgrade is modeled by 3D finite element method so that the motion of wave propagation can be completely simulated.

To investigate the critical speed effect, the site was chosen to be a section of the Northeast Corridor located within the Great Swamp Management Area, Kingston, Rhode Island, where the track is underlain by a soft organic silt (peaty) deposit. This is most favorable for this research since the velocity of Rayleigh wave may be low enough to observe the critical speed effect resulting from both high speed trains and conventional passenger trains. A 3x3 array of piezoelectric accelerometers was used for the measurement of near and far field track and ground response. A series of methods, such as Ground Penetrating Radar (GPR), Seismic Wave Velocity Testing (SWVT) and Dynamic Cone Penetrometer (DCP), were used to investigate the site.
Site Investigation

Some geotechnical and geophysical tests were adopted to determine the track condition and site geological information. The Kingston test site is shown in Figure 4-2 and the two instrumentation sites were named site 1 and site 2 shown in Figure 4-3. Track 1 (Southbound) was used for all tests. The Dynamic Cone Penetrometer (DCP) was performed by the authors to determine the soil modulus and changes in the layers. Ground penetrating Radar (GPR) was performed by HyGround Engineering and the data were used to obtain the layer depths, fouling index, and moisture. Light Detection and Ranging (Lidar) data were provided and used to determine the cross sections. Seismic Wave Velocity Testing (SWVT) tests were performed to calculate the shear wave velocity profiles. Laboratory tests were performed on samples taken from the swamp at Site 1 such as bender elements were used to estimate the shear wave velocity. Shear vane and fall cone tests were performed giving the shear strength and modulus. Water content and density of shallow samples in the swamp were also determined.

Figure 4-2. The Kingston site location in southern Rhode Island (RI)
Amtrak Northeast Corridor (NEC) Study Area

The Northeast Corridor (NEC) is a rail line owned primarily by Amtrak, which runs 731 km from Boston, Massachusetts to Washington, D.C. This line has sections of Class 8 Track allowing speeds of 250 km/hr. This line is not restricted to HSR trains (the Acela), but also serves regional passenger trains with some commuter and freight train traffic.

The subdivision of the NEC that is of interest to the critical speed study is a straight section of track that is currently classified for speeds up to 250 km/hr. A straight section of the NEC runs northeast/southwest through the northern section of the Great Swamp, Rhode Island, just to the west of South Kingston, Rhode Island. Currently, this section of the NEC is a double track section that is used for both HSR trains and regional passenger trains. The ties are concrete and the rail weight is 60 kg m$^{-1}$.

The surficial geology in the general region where the Great Swamp is located is comprised entirely of various glacial deposits (moraines, kames, outwash, etc.) or swamp deposits. In the area of the Great Swamp, the soil deposits are dominated by ground moraine, subglacial till, undifferentiated ice contact deposits, and an organic swamp soil deposit. The glacial ground
moraine is a light colored deposit consisting of uniform fine sand. The ground moraine is a competent material for foundations as it is generally strong. The subglacial till and undifferentiated ice contact deposits have many of the same engineering characteristics of the ground moraine. The swamp deposits are completely underlain by this ground moraine. As a result of the uneven glacial melting, the ground moraine has a hummocky topography that resulted in a variation in elevation and swamp deposit thickness. The swamp deposits consist of normally consolidated dark organic clays and silts which are generally soft and deformable. The thickness of the swamp deposits varies from 2 m to 7.5 m, with most of the deposits having the lower range of thickness. It would not be uncommon to find peaty soil with high water content to have Rayleigh wave velocities as low as 110 km/hr to 140 km/hr. Figure 4-4 presents a surficial Geology map with the mileposts and sites marked.
Ground Penetrating Radar (GPR) and Light Detection and Ranging (Lidar)

Ground Penetrating Radar was collected at the sites by HyGround Engineering in June 2011. A hi-rail truck with antennas mounted on the left, center, and right sides of Track 1 was used to survey the track. The data were processed to estimate the fouling indices, layer depths, and moisture profiles as shown in Figure 4-5. Geometry and Lidar data are also shown with the GPR data. Light Detection and Ranging data were provided for the track and right-of-way which were used to develop accurate ground surface topography.

Window (a) shows the Fouling Index based on GPR with the left on top, then center, and right on the bottom. The color bar shows that it is typically around 10, but comes close to 20 where light green and 25 where yellow; Windows (b) shows the moisture profile of left center and right. The blue dots represent moisture; dark blue is very wet while anything that is red is not wet. The approximate layer depths are drawn on these plots; Window (c) shows the Roughness with mid chord offset (MCO) of 19 m. These data are values above and below zero, so the data is squared over a window of 60 feet to give an absolute value of the roughness in a spot. The y-axis is a number representing the months since January 2010, from early 2011 to January 2015; Window (d) shows the space-curve data taken from the time that the survey was done. This is the relative vertical position of the rail over a 120-meter window, and the units are in inches; Window (e) shows the Lidar data, zeroed at Track 1; Window (f) shows the surficial geology provided from maps from the state of Rhode Island. Red is organics; Orange is fill over till.
Site 1 has a spot of slightly fouled ballast, roughness and moisture near the area where measurements were taken. Moisture is present at the top layer of soil on the right (towards Track 2). Site 2 has mildly fouled ballast all around the site and moisture down to the second layer of both the left and right side of Track 1. There is a spot of very mild roughness too. Both sites are in a fill, over several hundred feet on each longitudinal side, which made these locations an excellent place to perform field measurements.

**Seismic Wave Velocity Testing (SWVT)**

The SWVT method is an in-situ seismic method for determining shear wave velocity profiles. Testing is performed on the ground surface, allowing for less costly measurements than with traditional borehole methods. A dynamic impact is used to generate surface waves which are monitored by two or more receivers at known offsets. Figure 4-6 shows an example of the SWVT test. It plots the wave traveling time versus surface displacement. Different curves represent the signals received at different locations. The strongest response (usually the first downward peak) is the time point when the surface wave arrives. The surface wave velocity is determined by the time lag and length of offset, providing the modulus of soil.
Laboratory Tests

Several samples from the top 0.3 m of the swamp were collected and brought to the laboratory for tests. Torvane and fall cone tests were performed to calculate the shear strength, and water contents were measured. Bender elements were used to measure the shear wave velocity, which gives an estimate of the modulus of elasticity and the shear modulus. Table 4-2 includes the results of these tests.

Table 4-1 Subgrade stiffness by seismic wave velocity testing at the Kingston site.

<table>
<thead>
<tr>
<th>Location</th>
<th>Elastic Modulus (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Site 1</td>
</tr>
<tr>
<td>Embankment</td>
<td>42</td>
</tr>
<tr>
<td>Swamp</td>
<td>2.9</td>
</tr>
</tbody>
</table>

Table 4-2 Laboratory tests results at the Kingston site.
### Field Measurements and Speed Effect Characterization

Field measurements were collected at a class 8 (240 km/hr top operating speed) test section of the Northeast Corridor (the Great Swamp Management Area, Amtrak NEC MP155-156). Further, the study on train speed effect at the Kingston site was carried out.

#### Field Measurement

A 3 × 3 array of piezoelectric, triaxial accelerometers were temporarily installed along the track and right-of-way with a longitudinal spacing of 25 tie spaces (15 m) and a general lateral spacing of 7.5 m. Steel mounting plates were machined and attached to the concrete ties using an epoxy adhesive (Figure 4-7). The mounting plates were machined with a bevel to accommodate the geometry of the tie, allowing for a horizontal surface so that the accelerometers could be level. The six accelerometers installed in the soil adjacent to the track were mounted on insertion spikes, and pushed into the embankment and swamp to a depth of six inches. The installation methods were designed to permit rapid installation and removal so that the accelerometer network could be installed at multiple sites along the track.

Each accelerometer recorded the accelerations in three directions: vertical, longitudinal, and lateral. The measurement range of the accelerometers are ±490 m/s² and the frequency range is 0.8 to 8000 Hz for 3 dB. According to the manufacturer’s specifications, the accelerometers are capable of measuring the vibrations generated by moving trains. The specifications for the accelerometer are attached in the Appendix D. The sampling was conducted at a rate of 1000 Hz. The accelerometer data were recorded using conventional amplification, signal conditioning, and

<table>
<thead>
<tr>
<th>Tube No.</th>
<th>Sample Depth</th>
<th>ρ (g cm⁻³)</th>
<th>v₀ (m·s⁻¹)</th>
<th>Gmax (kPa)</th>
<th>E (kPa)</th>
<th>w (%)</th>
<th>Sᵤ, TV (kPa)</th>
<th>Sᵤ, FC (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-)</td>
<td>(m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-3</td>
<td>0.05-0.13</td>
<td>1.20</td>
<td>29.4</td>
<td>1037</td>
<td>3008</td>
<td>384</td>
<td>4.0</td>
<td>5.6</td>
</tr>
<tr>
<td>1-3</td>
<td>0.13-0.21</td>
<td>1.13</td>
<td>28.2</td>
<td>899</td>
<td>2606</td>
<td>204</td>
<td>5.7</td>
<td>6.2</td>
</tr>
<tr>
<td>1-3</td>
<td>0.21-0.29</td>
<td>1.02</td>
<td>21.5</td>
<td>471</td>
<td>1367</td>
<td>215</td>
<td>6.1</td>
<td>13.9</td>
</tr>
<tr>
<td>1-4</td>
<td>0.23-0.30</td>
<td>1.07</td>
<td>24.2</td>
<td>627</td>
<td>1817</td>
<td>405</td>
<td>4.2</td>
<td>7.1</td>
</tr>
<tr>
<td>1-5</td>
<td>0.08-0.15</td>
<td>1.42</td>
<td>27.7</td>
<td>1090</td>
<td>3160</td>
<td>70</td>
<td>5.6</td>
<td>9.2</td>
</tr>
<tr>
<td>1-5</td>
<td>0.18-0.25</td>
<td>1.30</td>
<td>24.4</td>
<td>774</td>
<td>2245</td>
<td>161</td>
<td>4.9</td>
<td>4.8</td>
</tr>
<tr>
<td>1-6</td>
<td>0.08-0.15</td>
<td>1.23</td>
<td>31.7</td>
<td>1236</td>
<td>3584</td>
<td>131</td>
<td>5.7</td>
<td>14.5</td>
</tr>
<tr>
<td>1-6</td>
<td>0.15-0.23</td>
<td>1.06</td>
<td>25.1</td>
<td>668</td>
<td>1937</td>
<td>376</td>
<td>5.2</td>
<td>11.4</td>
</tr>
<tr>
<td>1-6</td>
<td>0.23-0.30</td>
<td>1.06</td>
<td>17.1</td>
<td>310</td>
<td>899</td>
<td>487</td>
<td>4.4</td>
<td>11.9</td>
</tr>
</tbody>
</table>
data acquisition using a notebook computer. 500 Hz low-pass passive filters were used to eliminate aliasing effects. Figure 4-8 and Figure 4-9 show the general plan and cross-sectional view of instrumentation. The setup in the field changed a bit due to the different topography. The detailed information for each test can be found in the Appendix B.

Figure 4-7. Installation of accelerometers on concrete tie (left) and in the swamp deposit (right) at the Kingston site

Figure 4-8. Plan view of setup of accelerometers at the Kingston site
Figure 4-9. Cross-sectional view of setup of accelerometers at the Kingston site

Measurements of the track and ground vibrations due to passing trains in the Great Swamp Area were conducted at different vehicle speeds at different times of the year. Figure 4-10 presents the vertical acceleration versus time since the start of the test of the tie-mounted and ground-pinned accelerometers and for an Acela train traveling at 193 km/hr. The train was traveling in the southwest direction on Track 1, as is indicated by the delay in wave motion arrival times.
Figure 4-10. Vertical acceleration time histories for the passage of an Acela Express traveling at 193 km/hr, recorded on 11-18-13 at 12:49 pm at the Kingston site.

Speed Effect Characterization

One of the objectives of this research is to determine if the critical speed effect exists at Kingston site. Generally, critical speed effect will cause high vibration and generate cone-shaped ground wave motion. Therefore, to characterize the speed effect at the Kingston site, the vertical deflection of the rail and ground surface wave motion were plotted in this section.

In conventional railroad engineering, dynamic speed effect is actually considered in calculation of rail deflection. However, the effect is just described by a linear equation. Unlike conventional dynamic calculation, the critical speed effect is non-linear and is generated from resonance between vehicle and soft subgrade. The vibration level can be significantly increased when trains are running at critical and super-critical speed. In this section, the comparison between conventionally calculated and field measured rail deflection is performed to better understand the different mechanisms.

The conventional method presented by Talbot has suggested that the static load is increased by 1% over 5 mph as shown in Eq. (1):
\[ P_v = P + 0.01P (V-5) \]  
(4-1)

where, \( P_v \) = dynamic load in pounds at train speed of \( v \);  
\( V \) = speed in mile per hour;  
\( P \) = static load in pounds.

The maximum deflection can be calculated by Eq. (2):

\[ Y_o = \frac{P_v}{(64EIu^3)^{1/4}} \]  
(4-2)

where, \( Y_o \) = maximum deflection;  
\( EI \) = bending rigidity of rail beam;  
\( u \) = modulus of elasticity of track support;  
\( P_v \) is calculated by Eq. 1.

As can be seen in Figure 4-11, the rail vertical deflection calculated by the conventional method is a linear response. However, the results by field measurement have an exponential increase when the train speed increases. But perhaps because of the heavy rail and other factors, the change in track deflection is not large. The track deflection only changes from 2.2 mm to 2.7 mm when train speed increases from 193 km/hr to 240 km/hr.

Figure 4-11. Comparison of the downward vertical maximum rail deflection for field measurement at the Kingston site 1 and conventional method
In addition, Figure 4-12 shows the contour that represents the wave front arrival times in seconds relative to the arrival of the first axle at Accelerometer 1A for Acela train at 193 km/hr. The forward bend in the wave propagation near the track could result from that the shear wave velocity of the embankment soil exceeding the train speed. However, when the wave propagates to the area about 5m away from the track centerline to the swamp, the shear wave velocity significantly reduces and wave movement lags behind the train. The cone-shaped wave motion is seen, so the test site at Kingston may have a slight critical speed effect in this test. Figure 4-13 shows a figure of Regional train at 125 km/hr. As can be seen, the cone-shaped ground wave motion was attenuated largely at lower speed.

Figure 4-12. Ground surface wave front arrival times (seconds) of Acela at 193 km/hr at the Kingston site 1
Even though the train speeds at the Kingston site did not cause significant rail deflection at current operational speed, other potential track problems need to be further explored. Modeling is an ideal way to predict the track performance for current operational speed and higher speeds. The previously developed and validated 3D dynamic track-subgrade interaction model (Chapter 2) was used. Railroad track components, including rail, tie, ballast, and subgrade soil, are all considered in this model. More importantly, in order to take speed effect and cross-section change into consideration, ballast soil interface is modeled by a complex Green’s function which is derived from a 3D FEM model of subgrade. The detailed formulation can be found in the authors’ other publications. The modeling of rail vertical deflection and ground surface wave motion was carried out to compare with the field measurement. In addition, the prediction of compressive stress distribution in ballast and subgrade was performed by the model. The key parameters of track components and subgrade in the model are listed in Table 4-3.

3D Dynamic Track-Subgrade Interaction Model Prediction
### Table 4-3 Key parameters considered in the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train Load (f)</td>
<td>(kg)</td>
</tr>
<tr>
<td>Speed (v)</td>
<td>(km/hr)</td>
</tr>
<tr>
<td>Rail Mass per Unit Length (Mr)</td>
<td>(kg m(^{-1}))</td>
</tr>
<tr>
<td>Moment of Inertia of Rail (I)</td>
<td>(m(^4))</td>
</tr>
<tr>
<td>Tie Mass (Mt)</td>
<td>(kg)</td>
</tr>
<tr>
<td>Pad Stiffness (Kp)</td>
<td>(MN m(^{-1}))</td>
</tr>
<tr>
<td>Pad Damping (Dp)</td>
<td>(MN sec m(^{-1}))</td>
</tr>
<tr>
<td>Ballast Stiffness (Kb)</td>
<td>(MN m(^{-1}))</td>
</tr>
<tr>
<td>Ballast Damping (Db)</td>
<td>(MN sec m(^{-1}))</td>
</tr>
<tr>
<td>Subgrade Density</td>
<td>(kg m(^{-3}))</td>
</tr>
<tr>
<td>Subgrade Modulus</td>
<td>(MN m(^{-2}))</td>
</tr>
<tr>
<td>Subgrade Poisson’s Ratio</td>
<td>N/A</td>
</tr>
</tbody>
</table>

#### Model Inputs and Site Characterization

The Dynamic Cone Penetrometer test was conducted at the track centerline, shoulder and swamp area of Track 1 at both sites. The DCP device uses an 8-kg hammer that drops 575 mm driving a 60° cone tip with 20 mm base diameter into the ground. The DCP device was used along the edge of the embankment closest to the track. The DCP tests were performed from 2013 to 2015. Figure 4-14 shows the DCP Results in 2013. The test results of other years can be provided if needed. Table 4-4 presents the variations of track and subgrade stiffness at different depths and different test time.
The stiffness of the ballast and subgrade are calculated using the DCP data. The stiffness ranges at each depth are listed in Table 4-4. The values have a wide range because the data were collected in different seasons during three years and ballast/subgrade stiffness have variable conditions. The model considered stiffness values from the DCP values collected on that date, or closest to it.

The modulus of the soil can be estimated using Eq. (3)\textsuperscript{[68]}:

$$E_{DPI} = 10^{3.0475 - [1.06166 \log(DPI)]}$$  (4-3)

where: $E_{DPI} =$ modulus (MPa)

DPI = DCP penetration index [mm per drop]

Table 4-4 Track and subgrade stiffness.
<table>
<thead>
<tr>
<th>Site 1</th>
<th>Centerline</th>
<th>Embankment</th>
<th>Swamp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (m)</td>
<td>E (MPa)</td>
<td>Depth (m)</td>
<td>E (MPa)</td>
</tr>
<tr>
<td>0-0.6</td>
<td>150-350</td>
<td>0-0.6</td>
<td>30-90</td>
</tr>
<tr>
<td>0.6-1.3</td>
<td>30-100</td>
<td>0.6-2.7</td>
<td>10-30</td>
</tr>
<tr>
<td>1.3-2.6</td>
<td>40-120</td>
<td>2.7-3.6</td>
<td>100-120</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Site 2</th>
<th>Centerline</th>
<th>Embankment</th>
<th>Swamp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (m)</td>
<td>E (MPa)</td>
<td>Depth (m)</td>
<td>E (MPa)</td>
</tr>
<tr>
<td>0-1.1</td>
<td>200-350</td>
<td>0-2.0</td>
<td>30-90</td>
</tr>
<tr>
<td>1.1-2.8</td>
<td>150</td>
<td>2.0-2.5</td>
<td>150</td>
</tr>
<tr>
<td>2.8-3.5</td>
<td>70-120</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**The Maximum Vertical Displacement of Rail**

Figure 4-15 shows the rail vertical downward deflection by conventional method, field measurement, and model prediction. The field measurement only has a certain range since the train speeds recorded were only within that range. As stated in the previous section, the dynamic speed effect is only described by a linear portion in conventional railroad engineering. The linear equation is not sufficient to describe the dynamic responses generated by the moving train. As can be seen in the figure, the model predicted vertical maximum rail deflection presented exponential behavior and could better describe the trend of field measurements than the Talbot’s equation does. However, despite of the exponential behavior of the rail deflection, the absolute value of the rail deflection change was still small. This indicates that the critical speed effect has not fully developed.
From previous analysis, the current train speed has not reached the critical speed of the site. Therefore, by increasing the speed in the proposed model, Figure 4-16 shows the model results of track maximum vertical deflection up to 324 km/hr. As can be seen, at the train speed being increased to 306 km/hr, the track maximum deflection reached the highest value which presents a fully developed critical speed condition and the track deflection is substantially increased to 4.4 mm.
Ground Surface Wave Motion

The ground wave motion at the ground surface was calculated by the data from the 3 x 3 array of accelerometers. Figure 4-17(a) shows the contour that represents the wave front arrival times in seconds relative to the arrival of the first axle of the Acela train. The shear wave velocity estimated by the arrival time contours (72 km/hr) for the swamp soil is very similar to the bender element testing by Gnatek⁷⁹ (89 km/hr). The ground wave motion by simulation is plotted in Figure 4-17(b) to compare with the field results. The white elliptical circles represent where the wheels are. As can be seen, both field measurement and model simulation had cone-shaped ground wave motion at the speed of 193 km/hr. This indicates that the subgrade started to encounter the critical speed effect; however, the effect was slight at the current state.
Figure 4-17. Arrival time of shear wave front at 193 km/hr (Acela): (a) field measurement; (b) simulation (values in contour are ‘seconds’)

Figure 4-18 shows the ground wave motion by Regional train which operates at a lower speed (125 km/hr). The cone-shaped motion still can be detected but was largely attenuated.

Figure 4-18. Arrival time of shear wave front at 125 km/hr (Regional): field measurement (left); simulation (right, values in contour are ‘seconds’)

Figure 4-19 shows the ground wave motion predicted by the developed model ranging from low speed up to the critical speed (306 km/hr). It is clearly shown that the surface wave motion
starts to grow even within the operational speed of the Regional train and becomes more intense with increasing speeds.

Figure 4-19. The ground surface wave motion of subgrade at different speed conditions for the Kingston site 1

**Compressive Stress Contour in Ballast and Subgrade**

Figure 4-20 shows the compressive stress contour of cross section of subgrade at different speed conditions. The plots were generated by the model. The plotted regions are half domains, which mean that just half of the cross section is presented here. From the left to right, the compressive stress contour represents the train speed condition from 18 km/hr up to 306 km/hr, respectively. Figure 4-20(a) shows the train running at low speed (18 km/hr in this case) which is much below the critical speed; Figure 4-20(b) presents the train speed when the cone-shaped ground wave motion starts to grow; Figure 4-20(c) has a larger area of influence that describes the highest speed condition of Acela at the Kingston site; Figure 4-20(d) presents the predicted critical condition which is train running at 306 km/hr, which shows the critical speed condition.

As can be seen in the plot, the influence region and magnitude of stress increase slightly from (a) to (b), but increase significantly when the Acela train reaches 240 km/hr which is approaching the current highest operational speed. The rail deflection in the prediction changes from 2.3 mm to 2.8 mm without significant increase when the train speed increases from 125 km/hr to 240 km/hr. However, the prediction shows that the stress distribution in the subgrade could be substantially increased. This is important since the high stress could bring problems in ballast and subgrade, such as ballast breakage and high-rate deformation of the subgrade. It may be the reason causing the higher maintenance requirements at this site. This could be the next step of this research, which is to investigate the subgrade performance. In addition, from modeling results, the deformed region is substantially enlarged when a train is running at critical speed (306 km/hr) in this area.
The compressive stress in the ballast layer could accelerate the ballast breakage and affect the track stability. Once the ballast breaks, it could make the small-size particles even easier to be broken, and eventually the fine aggregate could lower the drainage ability of the track. The ballast issues, such as pumping, track instabilities, could affect the operational safety. In practice, if the ballast particle distribution does not satisfy the standards, new ballast should be used to replace the in-service ballast. Therefore, the high-stress sections are expected to have more frequent maintenance, which could cause higher maintenance costs and more traffic delays.

**Discussion of Critical Speed at Kingston Test Site**

This section analyzes the critical speed effect at the Kingston test site based on the field measurements and model prediction. The Kingston site is known as the Great Swamp area and requires more frequent track maintenance. It was suspected that a condition so called critical speed might exist at this particular location. The conventional definition of the critical speed is the speed at which vibrations propagate within the track structure and subgrade at a speed close to the Rayleigh wave velocity of the subgrade soil. As trains travel at speeds approaching the critical speed, all track components are expected to present cone-shaped surface wave motion and significantly increased vibrations. According to the field tests at Kingston, the cone-shaped surface wave motion started at the speed as low as 90-120 km/hr. However, no data indicates such dramatic increment in rail/track deflection. This indicates that the cone-shaped surface wave motion and the increase of the maximum rail deflection did not occur at the same time. Also, neither the surface wave motion nor the slightly increased rail deflection under the current operational speed can well
explain the extra track maintenance required for the site. Therefore, the effect caused by the critical speed cannot be interpreted the same as the conventional definition, and a more appropriate and accurate interpretation will be provided for the Kingston site.

As shown in Figure 4-17 and Figure 4-18, the cone-shaped surface wave pattern started as early as the train speed reaches the soil shear wave velocity. It can be explained by the train resonating with the swamp part of the subgrade since train speed hits the Rayleigh wave velocity of that area. However, it is not the critical condition because the resonance did not spread in the entire subgrade and the rail vertical displacement was recorded as safe condition. The two signs which are conventionally used to indicate potential critical speed condition: cone-shaped surface wave and increased rail deflection, actually did not occur at the same time at Kingston site. As the train speed increases to 240 km/hr, which was the highest recorded train speed at Kingston, the rail displacement still was not significantly increased which may be due to the heavy rail and thick ballast layer. However, the compressive stress contour presented an evident increase at this speed. This might explain that Amtrak has no problem with safety in Rhode Island but may have issues with ballast maintenance. In addition, the critical condition predicted by the model is the train at the speed over 300 km/hr. The stress level and rail displacement will be substantially increased. The summary of the speed effect on track performance is shown in Table 4-5.

<table>
<thead>
<tr>
<th>Description</th>
<th>18 km/hr</th>
<th>125 km/hr</th>
<th>240 km/hr</th>
<th>306 km/hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cone-shaped Surface Wave</td>
<td></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Rail Vertical Displacement</td>
<td>Safe</td>
<td>Safe</td>
<td>Safe</td>
<td>Significantly increased</td>
</tr>
<tr>
<td>Stress Level in Subgrade</td>
<td></td>
<td>Slightly increased</td>
<td>May affect track maintenance</td>
<td>May affect track maintenance</td>
</tr>
</tbody>
</table>

* RWV: Rayleigh wave velocity

Therefore, according to the analysis, the track performance at Kingston site experienced three stages as the speed increases: the appearance of the cone-shaped surface wave motion (around 90-120 km/hr); the compressive stress increase in the subgrade (around 240 km/hr); the rail deflection increase (by modeling, over 300 km/hr). The high-level stress can explain the more frequent track maintenance and the predicted larger rail deflection could affect the operational
safety. The main reasons causing the divided critical speed characters could be the complicated tie-ballast-soil system and the geometry of the cross section. If a train is directly running on soft and homogeneous subgrade, increase in deflection and stress intensity will occur at the same time. Therefore, authors would define the critical speed in two levels for the Kingston site:

1. The speed causing significant increase in the cross section stress intensity, at which more frequent ballast maintenance becomes a concern;

2. The speed causing significant increase in rail deflection, at which derailment becomes a concern.

**Conclusions**

In this research, track and ground vibration and substructure performance under trains moving at high speeds over soft ground are investigated through the field tests and the simulation by a 3D dynamic track-subgrade interaction model. The field investigation was performed on Amtrak’s Northeast Corridor at the Great Swamp site in Kingston Rhode Island. The site investigation included several geotechnical and geophysical field tests, such as Laboratory tests, Dynamic Cone Penetrometer (DCP), and Seismic Wave Velocity Testing (SWVT). Ground Penetrating Radar (GPR) and Light Detection and Ranging (Lidar) had previously been done at this site so those data were also used. The investigation indicated that the subgrade was soft with stiffness as low as 1 to 3 MPa within the Great Swamp.

For rail vertical deflection, the simulation results could fit the trend of field measurements better than the conventional method for the range of recorded speeds (190 km/hr to 240 km/hr). Both simulation and field measurements show that the increase in rail deflection is non-linear with the train speed as the train approaches critical speed. The conventional method (Talbot’s Equation) is not able to thoroughly predict the track behavior at critical speed. Therefore, the track deflection non-linear increase with speed is not only caused by the linear dynamic effect described by the conventional method, but also by some other mechanisms. In addition, the modeling results show that the rail deflection at the current operational speed will not have significant critical speed effect, but will encounter a significant increase when trains run at higher speeds.

Also, the ground deflection contours show that the cone-shaped surface wave motion is clearly detected in the field and modeling, even without having any significant increase in rail deflection. The cone-shaped wave motion starts to grow at a much lower speed than the speed causing large rail deflection.
Further, by increasing the train speed in model prediction, the influence region and magnitude of compressive stress increase significantly when the train reaches critical speed, even without significant rail deflection change. This finding indicates that the critical speed effect will not only affect the rail vibration, but also have an influence on track subgrade. Higher-speed-induced stress in the subgrade can result in greater progressive deformation of the subgrade over time and high deterioration rate of ballast. Therefore, for the Kingston site, authors would define the critical speed in two levels: 1) The speed causing significant increase in rail deflection, at which derailment becomes a concern; 2) The speed causing significant increase in the cross section stress intensity, at which more frequent ballast maintenance becomes a concern. Further study is needed to verify the model prediction of stress.
Chapter 5

HANGING TIE STUDY USING THE “MOVING DEFLECTION SPECTRUM”

Overview

A hanging tie is a form of railroad track distress that occurs when voids have developed beneath the ties due to uneven ballast settlement and improper maintenance practices. It will lead to an increase in dynamic impact loading on the top of the ballast and therefore further deteriorate the track structure. In this chapter, a fast, nondestructive screening method to identify the hanging tie problem is proposed. The method utilized a dynamic track model to characterize the track's “Moving Deflection Spectrum (MDS)” under different tie-supporting scenarios. The model includes a moving dynamic load and ties with discrete supports. The MDS shows dynamic responses of the “Track Moving Deflection (TMD)” in the frequency domain. The modeling results indicate that a significant discrepancy exists in the MDS depending on whether or not there is a hanging tie condition. To validate this method, preliminary field tests were carried out on both Boston Metro and St. Louis Metro lines. Then the MDS generated by the model was compared with the MDS in the field tests as measured by the accelerometers installed on a high-rail vehicle. Results showed that the method is effective in identifying the hanging tie problem and has great potential to be employed by the rail industry in the future.
Introduction

The hanging tie condition is illustrated in Figure 5-1, showing that the voids have developed beneath the ties, causing tie-ballast gaps. The existence of the tie-ballast gaps can result in larger peak accelerations at the unsupported or poorly supported ties. This will increase the dynamic contact force between tie and ballast, and then further deteriorate the track structure. Moreover, the hanging tie condition can increase the rail deflection and cause differential movements of rail. Therefore, it could impose limits on increasing train speed because high-speed rails are more sensitive to changes in track longitudinal geometry.

The hanging tie condition is developed gradually and involves three steps that determine its severity. At the initial condition, due to the variations of ballast density and compactness in the longitudinal direction, the ballast layer will have differential settlement. That results in an uneven surface of the ballast layer, causing the ties to be supported by varying ballast stiffness in the longitudinal direction of tracks. Next, under repeated train loading, the regions with lower ballast stiffness will settle more than the adjacent regions until the tie-ballast gap fully develops. Once a tie is “floating” or “hanging” on the ballast, the problem can infect surrounding ties by introducing larger dynamic loads in this region. Eventually, consecutive hanging ties will be developed if no maintenance is performed. However, current maintenance practices could not identify the existence of hanging ties quickly and effectively. Therefore, it would be beneficial to develop a fast and nondestructive method to identify the hanging tie(s) problem.

The objective of this research is to introduce and validate a method to identify tracks with a hanging tie problem by characterizing the track's “Moving Deflection Spectrum” (MDS). Efforts have been made to relate hanging tie identification to the MDS. The MDS is the frequency spectrum.
of the “Track Moving Deflection” (TMD), which is defined as the vertical movement of the wheel rail contact point when the wheel moves forward along the rail. In other words, the “Track Moving Deflection” can illustrate the trajectory of the wheel rail contact point as the wheel goes forward. It is worth noting that the TMD is different from the track deflection which is normally measured at a fixed location on the track structure. The “Track Moving Deflection” should be a perfectly horizontal line if the track is longitudinally homogeneous (as shown in Figure 5-2). However, in reality, due to rail surface roughness, tie spacing, and other variations in the longitudinal direction of the track, the “Track Moving Deflection” will always present fluctuations. In this research, the focus is to characterize the fluctuations in the frequency domain in order to identify the hanging tie problem.

![Diagram of track moving deflection](image)

**Figure 5-2. Schematic plot of the track moving deflection of a longitudinally homogenous track.**

To characterize the differences in the track responses between a “healthy” track and a track with hanging tie(s), a dynamic track model which is capable of generating the TMD was formulated. Both tie spacing and individual tie support were considered in the formulation. The corresponding MDS can be generated through the Fourier Transform. To validate the modeling results, the “Track Moving Deflection” was measured on Boston Metro and St. Louis Metro lines for comparison. Two methods are normally used to measure the TMD. The first method is called MRail, as shown in Figure 5-3(a). It is an autonomous system that is capable of measuring vertical rail deflection from a rail car moving at full track speed. By measuring the width of the two laser beams projected on the rail, the moving deflection can be calculated. The other option, shown in Figure 5-3(b), is to install accelerometers on the moving wheels. The “Track Moving Deflection” can be calculated by double integration of the measured accelerations.
Numerical Modeling

Figure 5-4 shows the dynamic track model for this research. The arrow denotes the wheel load moving along the rail. The rail was modeled as a Euler beam. The rail pad, tie, and ballast layer were represented by a system of mass, spring, and damper, with designated spacing. Since the focus is on characterizing the rail-tie-ballast performance, the soil subgrade was simplified as another spring and damper system. The moving load, track and subgrade soil were coupled to obtain the responses of the entire system.

Two conditions of tracks (those with and without the hanging tie) were constructed in the numerical model by changing the value of $K_b$, which represents the tie-ballast contact stiffness. The TMDs were calculated by the model for both conditions, as well as their corresponding MDS. Figure 5-5 demonstrates the tracks simulated in the model with and without hanging tie conditions.
In the model, rails were placed on 50 ties with a typical tie spacing of 0.508m. The speed of the load was 2 m/s, which was close to that of the testing vehicle.

![Figure 5-5. The track with and without hanging tie](image)

**Characterization of the “Moving Deflection Spectrum (MDS)”**

Figure 5-6 presents the “Track Moving Deflections” without hanging ties. The periodic fluctuation originates from the changes in tie support since the rail is discretely supported by ties. The wave troughs of the responses indicate that the wheel load is at the middle of two adjacent ties, and the wave crests denote the condition when the wheel load is on the top of a tie.

![Running Time (s)](image)

Figure 5-6. The “Track Moving Deflections” without hanging ties by modeling

Figure 5-7 shows the MDS plot corresponding to the TMD in Figure 5-6. It can be seen in the spectrum that there is a clear peak occurring at a frequency of around 4 Hz. This frequency will
be called the “Tie Spacing Frequency” in the following context because the response is excited by the tie spacing and train speed. A simple equation can explain the relationship: Tie Spacing Frequency = train speed / tie spacing = 2(m/s) / 0.508 (m) = 3.9 (Hz). The other peaks are related to the vehicle or track physical properties, such as the wheelbase, car length, etc. The magnitude at the “Tie Spacing Frequency” has the largest value, indicating that the responses excited by the tie spacing are dominant as compared to other excitations.

![Graph](image)

Figure 5-7. The “Moving Deflection Spectrum” without hanging ties by modeling

With the definition of hanging ties, the ties are "floating" on the ballast, resulting in less or even no support from the ballast layer. Therefore, the hanging tie scenario was modeled by setting the tie-ballast contact stiffness to relatively low values, in this case: 7 kN/m. In Figure 5-8, it shows the TMD with the hanging tie compared to without the hanging tie situation. According to the graph, although the fluctuations which were excited by the tie spacing still exist, a significant hump near the hanging tie location was detected in the “Track Moving Deflection” when the hanging tie situation occurs. In these cases, the shape and mode of the TMD have been changed significantly near the location of the hanging tie(s).
Figure 5-8. The “Track Moving Deflections” with hanging ties by modeling

As can be seen, the two lines overlap each other in the range over the “Tie Spacing Frequency” meaning that the two tracks have the same characteristics in many aspects. However, the significant difference in the spectrum for both conditions is that the track with the hanging tie possesses a much larger peak in the low frequency region (the frequency range below the “Tie Spacing Frequency”). The magnitude of the peak is about three times larger than the magnitude at the “Tie Spacing Frequency” in this case. This observation is the key to distinguish the track with hanging ties from those without hanging ties. Clearly, the largest peak in the spectrum was excited by the hanging tie. Therefore, the magnitude of the peak may indicate the severity of the hanging tie problem.
Figure 5-9. The “Moving Deflection Spectrum” with hanging ties by modeling

Therefore, with the aid of the dynamic track model, some useful observations for hanging tie identification are summarized: 1) the track might have a hanging tie(s) problem if the MDS presents a dramatically increased peak in the low frequency region (the frequency below the Tie Spacing Frequency); and 2) the magnitude of the peak in the low frequency region may indicate the degree of deterioration of the hanging tie problem.

Field Investigations and Discussions

Field investigations were carried out at three different sites in Boston and St. Louis: Scott Air Force Base Westbound (SAFBW), Cross-country Westbound (CCW), and Redline Inbound (RI) to validate the predictions of the numerical modeling. Accelerometers were mounted on the axle of steel wheels of a high-rail vehicle (as shown in Figure 5-10). This vehicle is ideal for this research because the weight of the vehicle is heavy enough to generate track deflections, but not too heavy to close the tie-ballast gaps. If the gaps are closed, the deterioration level of the hanging ties cannot be tested accurately.
Figure 5-11 shows the measurement of the “Track Moving Deflection” and the corresponding “Moving Deflection Spectrum” at the SAFBW site. In Figure 5-11(a), the total running time is 900 seconds and the vertical axis presents the vertical track deflection. As can be seen, the vertical track deflection has significant fluctuations due to the changing track stiffness in the longitudinal direction. Figure 5-11(b) shows the “Track Moving Deflections” in the frequency domain, which is the “Moving Deflection Spectrum.” The low-frequency responses were strong because the speed of the testing vehicle was only in the range of 4 and 8 mph.
The measurements from the SAFBW site were analyzed in detail to examine how to identify hanging ties using the MDS. The total testing time for SAFBW was 900 seconds. In order to accurately determine the locations of hanging ties on the track, the field measurements need to be deconstructed into smaller segments for the analysis. Therefore, the total testing time was divided into nine segments and each segment has 100 seconds of testing time.

In Figure 5-12, two segments (Segment 2 and Segment 4) are chosen to make comparisons to explicate how to use the MDS to identify the hanging tie problem. As can be seen in Figure 5-12(a), the second segment has clear peaks at around 7 Hz which is the “Tie Spacing Frequency” in this case. This implies that the testing vehicle was moving at a speed of around $0.508 \times 7$ (Hz) = 3.6 m/s=8 mph (where 0.508 m is the tie spacing). The calculated speed matched the speed of the high-rail vehicle in the field test. As concluded from the modeling results, hanging tie conditions have a significant response in the low frequency region, while those without hanging tie conditions do not. In Figure 5-12, significant responses were detected in the low frequency region for both segments. However, the authors believe that the second segment had no hanging tie condition because the peaks in the low frequency region were caused by the long-wave-length track vertical profile which is normally seen in field tests. For the field measured MDS, as long as the peaks excited at the Tie Spacing Frequency can still be seen in the MDS plots, the track can be regarded as not having a hanging tie or a very minor hanging tie problem.

In contrast, as shown Figure 5-12(b), when the testing vehicle reached the fourth segment, the magnitude of the peaks in the low frequency region increased significantly, which caused the peaks at the Tie Spacing Frequency to be barely observed. This is consistent with the signs of a
hanging tie as described in the numerical modeling. High disturbances and noises generated by hanging ties can explain why the peaks at the Tie Spacing Frequency attenuated. This phenomenon suggests that the fourth segment might have a hanging tie problem. This prediction based on the MDS is consistent with a visual inspection at the site. Further, it is worth noting that the magnitude of the peaks at the low frequency region may have a correlation with the vibration level of the ties. The larger the magnitude, the more severe the hanging tie problem. But the field data are not enough to conduct a statistical analysis to verify the surmise. Future work will have to be performed to draw more quantitative conclusions for determining the severity of the hanging ties.

Then, the MDS's of the sites at Cross-country Westbound (CCW) and Redline Inbound (RI) were analyzed by an analogous method. The MDSs of these two sites can be provided upon request. For all three tested sites, the modeling predictions match the field observations very well. This could show that the MDS may be an effective and efficient way to fast screen for the hanging tie in the field. The segments that could have potential hanging ties are summarized in Table 5-1.

Table 5-1. Potential positions of hanging ties.
In order to more accurately locate the hanging ties, segment four of the SAFBW site which may have a potential hanging tie problem was analyzed by further dividing it into ten pieces. Therefore, each piece has ten seconds of running time, reducing the tested track interval to 36 m. This length of track is practical to conduct visual inspection and is applicable for track maintenance. The MDS of each piece of the third segment is plotted in Figure 5-13. With the same analytical procedure, piece number 6, 7, 8 and 9 were considered to have potential hanging tie problems. Hence, the locations of hanging ties can be quickly narrowed down to a 36-meter-long track which is manageable and convenient for field inspection.
Conclusions

In this chapter, a fast, nondestructive screening method to identify a hanging tie problem is introduced and validated. The method used a dynamic track model which is capable of simulating a moving load and discrete tie supports. The model results characterized the “Track Moving Deflection (TMD)” and the corresponding “Moving Deflection Spectrum (MDS)” to identify the hanging tie problem. It is found that the MDS's of tracks without a hanging tie problem have a clear peak at the “Tie Spacing Frequency.” However, tracks have a hanging tie problem if the MDSs present a significantly increased peak in the low frequency region (the frequency below the “Tie
Spacing Frequency”). To validate the finding, field investigations were conducted on three metro lines in Boston and St. Louis. Accelerometers were mounted on a high-rail vehicle to measure the acceleration of the moving wheels. The TMD and corresponding MDS were then calculated to predict the potential locations of hanging ties. The locations of hanging ties predicted by the model matched the field observations. Therefore, it is concluded that the “Moving Deflection Spectrum (MDS)” can be an applicable and effective tool to identify the hanging tie problem.
Chapter 6

CHARACTERIZATION OF THE MOVEMENTS OF RAILROAD TIES UNDER TRAIN PASSAGE ON STRAIGHT LINES

Overview

In this chapter, the movements of railroad ties under moving train loads was characterized by numerical modeling and field investigation. To numerically simulate the tie movements, a vehicle dynamics model (commercial software) coupled with a three-dimensional finite element track model (commercial software) was utilized. To obtain the real tie movements under train passage, field measurements were conducted on an Amtrak high-speed passenger line and a freight railroad short line. The measuring units were mounted on top of ties to record the accelerations and the changes in Euler angles of the ties in vertical, lateral and longitudinal directions. The tie displacements obtained by field investigation and modeling results showed that the ties could have translational movements in the three directions rather than just in the vertical direction. The rotation of railroad ties represented by the changes in Euler angles were in a range of 0.2 to 0.75 degree. In addition, angular velocity and angular acceleration measured in the field tests indicated that the rotation could cause extra moment up to 5000 N-m on top of the ballast layer. Further, the effect of the tie rotation on ballast was investigated by discrete element modeling (DEM). The results of DEM showed that the tie rotation would increase the acceleration of individual ballast particles and contact forces between ballast particles.

Introduction

A tie, resting on the transverse direction of railroad tracks, transmits the moving train loads to the lower track structure and maintains track gauge as well as fastens the two rails to be aligned. In the United States, 140,000 rail miles are being operated by Class I, plus regional and local railroads. As a consequence, hundreds of millions of railroad ties are under maintenance by the railroad industry. Tie problems, such as rail seat abrasion and concrete tie cracking, need to be well addressed. Also, ballast issues, such as fouling, particle degradation and migration, are related to the tie-ballast contact interaction forces transmitted from the bottom of the ties. Therefore,
obtaining an accurate description of the tie-ballast contact interaction is a meaningful and imperative study and has the potential to improve the laboratory research and practical maintenance. Previous laboratory research on ties or ballast assumed that the movements of a railroad tie under the repeated train loading are exclusively along the vertical direction.\textsuperscript{4, 88, 89} A unidirectional actuator perpendicular to the ballast surface was used for these laboratory tests. The setup results in tie-ballast contact forces being applied in the vertical direction. In order to account for the lateral movements, the lateral loads were also applied in some laboratory tests on railroad ties recently.\textsuperscript{90, 91} However, field instrumentations installed at Kingston, Rhode Island, United States, indicate that the tie could have translational movements as well as rotational movements\textsuperscript{92}.

In order to obtain the tie movements, a modeling method that couples a vehicle dynamics model (VSD: VAMPIRE) with a 3D finite element (FE) track model (ABAQUS) was proposed. This integrated approach can fully use the advantages of each model by taking the vehicle system and track system into consideration. The simulation contains two steps: (1) The vehicle dynamics model was used to obtain the wheel-rail contact forces in the lateral, vertical and longitudinal directions. The vehicle model allows users to build a multi-body vehicle of any typical train configurations and takes rail and wheel profiles into consideration. Therefore, the wheel-rail contact forces can be more accurately calculated; (2) The wheel-rail contact forces were extracted from the VSD model and then input into the 3D FE track model to obtain the movements of ties. The FE track model is a multilayer system, including rail, ballast layer and soil subgrade.

Field investigation was conducted at two sites, including a high-speed line with concrete ties and a short line with wood ties. The high-speed line site is located on the Northeast Corridor (NEC) at Kingston, Rhode Island where the tracks are well-maintained. The short line is located at Hollidaysburg, Pennsylvania, and is owned by the Everett Railroad. To measure the tie movements, a specially-designed device was mounted on top of ties to measure the accelerations, angle changes and angular velocities of ties. The measuring unit has a tri-axial gyroscope, a tri-axial accelerometer, and a tri-axial magnetometer integrated together. It is a 9-degree-of-freedom motion/vibration sensor which records translation, orientation, and rotation. The gyroscope and magnetometer work simultaneously to measure the angle change of ties. The translational movements can be obtained by double integration of the acceleration data. Figure 6-1 presents the field instrumentation plan and the definitions of the directions of tie motion used in this paper.
Further, the effect of rotational movements of ties was investigated by a ballast modeling software (BLOKS3D) based on the Discrete Element Method (DEM). The software is developed by the University of Illinois and can predict individual particle accelerations and contact forces between ballast particles within the ballast layer for given loading profiles. The principle of the DEM software is to discretize the ballast layer and analyze the performance of individual particles. Accordingly, through the integrated approach of field investigation and numerical modeling, the characterization of tie movements and the effect of rotations of railroad ties on ballast layer can be studied.

**Train-Track Interaction Modeling**

**Vehicle Dynamics Model**

The vehicle dynamics model was created in a commercial program (VAMPIRE), which can simulate rail vehicles traveling over tracks. It can calculate the wheel/rail contact forces by accurately simulating how vehicles and tracks interact. The vehicle model used in this research has a four-axle car and rails on rigid foundation (shown in Figure 6-2). The car model also possesses vertical and lateral suspension systems. Moreover, besides a detailed vehicle model, the program is capable of simulating the track irregularities and hunting oscillation. The hunting oscillation (also called truck hunting, shown in Figure 6-3) is a swaying motion of a railway vehicle caused by the coning action of the wheels. The wheel-rail forces, especially the lateral forces, are more accurate by considering the truck hunting. The wheel-rail contact forces in the vertical, lateral and longitudinal directions are able to be recorded through the output channels.
Figure 6-2. Vehicle dynamics model used in the simulation

Figure 6-3. Truck hunting\(^\text{12}\) (also known as the Klingel’s Movement)

**Track Model**

A three-dimensional finite element track model (established in ABAQUS), as shown in Figure 6-4, is used to study the tie movements under moving train loads. The model consists of rails, ties and track substructure (ballast and subgrade). All the components were simulated by solid elements. Springs and dampers were used for simulating the rail-tie contact and the tie-ballast contact. The length and width of the subgrade domain were 200 m and 160 m. The detailed model information can be found in the publications.\(^\text{64,67,94}\) The moving train loads were simplified as point loads, which contain three components in three orthogonal directions. The magnitude of the loads as a function of time were obtained in the vehicle dynamics model.
Modeling Results

Two steps were made to obtain the tie movements: (1) extract the wheel-rail contact forces in the vertical, lateral and longitudinal directions from the vehicle dynamics model; (2) apply the wheel-rail contact forces to the track model to obtain the tie movements. Separating the modeling process into two steps could take full advantage of each model and reduce the computing time.

Figure 6-5 shows the wheel-rail contact forces in the three directions. The forces are periodic due to the Klingel’s Movement and track irregularities. The tie movements obtained by the model are shown in the following figures. The definitions of directions and Euler angles are described in Figure 6-1.
Figure 6-5. The vertical, lateral and longitudinal forces extracted from VAMPIRE

Figure 6-6 presents the translational movements of a railroad tie in the FE model. It can be seen from the graphs that the tie does not only have up-and-down motion, but also has lateral and longitudinal movements. The magnitudes of the movements in the lateral and longitudinal directions are relatively small compared to the vertical movements.
Euler angles represent the rotations in degree along the axis of longitudinal, lateral and vertical directions, respectively. Figure 6-7 presents the pitch of a tie, and the magnitude is ±0.2 degree.

**Figure 6-7. Rotational movements by modeling along the lateral direction**

**Field Investigation and Analysis**

**Field Instrumentation and Site Characterization**

The two sites picked to perform the tests were a high-speed passenger line with concrete ties and a Class III common carrier short railroad line with wood ties. The high speed line is on the Northeast Corridor (NEC) at Kingston, Rhode Island. The short line is located at Hollidaysburg, Pennsylvania, and is owned by the Everett Railroad Company. NEC is a rail line owned by Amtrak, which runs 731 km from Boston, Massachusetts to Washington, D.C. This line has sections of Class 8 Track allowing speeds of 240 km/hr. This line has high-speed trains (the Acela Express passenger
trains), but also serves regional passenger trains and limited commuter and freight trains. The segment of the NEC near Kingston runs northeast/southwest and comprises two tracks for the high-speed Acela operating at 240 km/hr. Compared to the high-speed passenger track at the Kingston site, the track at the Hollidaysburg site has lower maintenance standard, resulting relatively lower track quality. Some track defects, such as hanging tie and missing fasteners can be found by visual inspection. The track servers the local community with carload freight service and the top train speed at the Hollidaysburg site is around 32 km/hr. The train information and track condition of the two sites are summarized in Table 6-1.

<table>
<thead>
<tr>
<th>The Kingston Site</th>
<th>The Hollidaysburg Site</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Train Speed (km/h)</strong></td>
<td><strong>Axle Load (kg)</strong></td>
</tr>
<tr>
<td>240</td>
<td>16,150</td>
</tr>
</tbody>
</table>

The instrumentation used at both sites was a customized measuring unit which was developed at the Pennsylvania State University. The measuring unit is a 9-degree-of-freedom motion/vibration sensor consisting of a tri-axial gyroscope, a tri-axial accelerometer, and a tri-axial magnetometer, which records rotation, translation, and orientation, respectively. The sensor was mounted on top of a tie and the data was transferred to laptops through Bluetooth. Figure 6-8 shows the instrumentation method at two sites. Due to the one-side-only access allowed by Amtrak at the Kingston site, the measuring unit was only installed on the right edge of a tie with respect to the train moving direction. A mounting plate was machined with a bevel to accommodate the geometry of the tie, allowing for a horizontal surface so that the measuring unit could be level. For the Hollidaysburg site, three measuring units were installed to the middle and two edges of the ties, as shown in Figure 6-8.
Figure 6-8. Field instrumentations at the Kingston and Hollidaysburg site (red circles are the locations of the measuring devices)

The Kingston Site

Detailed site investigation had previously been performed at Kingston, and the results can be found in Chapter 3 and Chapter 4. Sufficient site information is available for conducting the simulation. Therefore, the field test results will be compared with the modeling results only for the Kingston site. The test at Kingston was on September 27th, 2014 and all the five tests were conducted on the same day with one type of train (the Acela). The results of the first test were shown in this section. The results from the other tests were consistent and can be provided upon request. Figure 6-9 (a), (b) and (c) present the tie translational movements by field measurements and modeling in the vertical, lateral and longitudinal directions respectively. The displacement was obtained by double integration of the acceleration. As can be seen, the vertical displacement has the largest maximum magnitude and is the primary motion of all three translational movements since the train loading was applied in the same direction.
In Figure 6-10, the rotational movements of the tie along the lateral axis were plotted. Both simulation and field results show the existence of rotational movements. The changes in pitch were within ±0.2 degrees and the values fluctuate significantly around 0 degrees, indicating that the tie was rocking with respect to its original position. The rotations were described by the changes in Euler angles. The maximum angle change with respect to original position were 0.3 degree for pitch. The values fluctuate significantly near 0 degree, indicating that the tie was rocking with respect to its original position. Therefore, the field measurements indicate that the tie movements under train loading contain both translational and rotational motions.
The Hollidaysburg Site

The instrumentation method at the Hollidaysburg site differed from the Kingston site. Three sensors at the middle and two edges of a tie were installed. This setup allows the test to study the tie movements at different locations of a tie. The track has a train operating on Monday, Wednesday, and Friday, and only has one train passage for each day at this site. A total of three tests were conducted during February 29 to March 11, 2016. Test 1 was conducted on a well-supported tie; Test 2 was conducted on the same tie as Test 1, but exchanged the positions of the measuring unit on the right and the middle one, in order to test the repeatability of the measuring unit; Test 3 had the same configuration as Test 1 but was conducted on another tie with a tie-ballast gap. As stated before, the track quality at the Hollidaysburg site is relatively low compared to the track at Kingston. Therefore, the study can be extended to characterize the tie responses with different ballast supporting conditions. The tie in Test 1 and Test 2 had a good supporting condition, while a clear tie-ballast gap can be found in Test 3.

Figure 6-11 (a), (b) and (c) show the rotational movements of the tie along the vertical, lateral and longitudinal directions, respectively, in Test 1. The maximum angle changes with respect to original position were around 0.375 degree for pitch and roll, but only 0.03 degree for yaw. As shown in the plots, the maximum value of roll and pitch increased 0.075 and 0.175 degree, respectively, compared to the measurements at Kingston. The increase could come from the lower track quality and type of ties, etc., which needs to be further investigated. Test 2 has comparable measurements showing that the test setup and measuring unit is repeatable.
The vertical accelerations obtained from the left edge and right edge are presented in Figure 6-12. The peak accelerations caused by wheel loads from the right edge under each wheel pass were larger than those from the left edge. The magnitude ranges from 1 g to 2 g. This may be caused by the different ballast support stiffness on the two sides of the track.

Figure 6-13 shows the rotational movements of the tie of Test 3 at the Hollidaysburg site. Test 3 was conducted on the tie with a clear tie-ballast gap by visual inspection. Compared to Test 1 and Test 2, the measurements showed that the maximum changes in Euler angles were increased to 0.75 degree in roll, 0.6 degree in pitch and about 0.4 degree in yaw. The angle changes nearly doubled the measurements compared to the well-supported tie in Test 1. This indicates that the tie with a tie-ballast gap could increase the tie rotations.
The previous results show that the tie movements not only have translational movements but also have rotational movements. However, how the tie rotations affect the track structure could not be directly measured. Therefore, an estimate on the forces on top of ballast generated by tie rotations is presented here. Figure 6-14 (a) and (b) display the angular velocities of a tie at the Hollidaysburg site for Test 1 and Test 3, respectively. As can be seen in the figure, the maximum magnitudes of the angular velocities were 47 degrees per second in Test 1 and over 100 degrees per second in Test 3. Further by differentiating the angular velocity, the angular acceleration can be obtained in Figure 6-14 (c). The angular acceleration with respect to the lateral direction could reach 15,000 rad/s² (Test 3). According to the basics of engineering dynamics, moment can be calculated by the equation $M = I \alpha$ (moment of inertia) * $\alpha$ (angular acceleration). This will generate an extra moment around 5,000 kg·m²/s² (equivalent to N-m) along with the lateral direction. The extra moments may not cause significant track failure, but may accelerate ballast degradation and breakage.
Figure 6-14. The rotation in the lateral, vertical and longitudinal directions: (a) angular velocity of test 1; (b) angular velocity of test 3; (c) angular acceleration of test 3
Figure 6-15 shows the comparison of tie accelerations in Test 1 and Test 3. As can be seen, the accelerations in the longitudinal and vertical directions significantly increased if tie-ballast gap existed. Therefore, the existence of the tie-ballast gap could increase the impact load between railroad ties and ballast layer, and eventually may affect the ballast deterioration and maintenance.

![Graph showing acceleration comparison](image)

(a) Longitudinal direction; (b) Vertical direction.

According to the field investigations at the Kingston and Hollidaysburg site, the movements of a railroad tie not only contain translational motion, but also rotational motion. The NEC is classified as Class 8 Track, which is frequently maintained and high-quality. This helps explain that the tie rotational and translational movements were not significant. However, the measurements at the Hollidaysburg site, which has a lower-quality track, showed larger movements of ties. The tie which has a tie-ballast gap doubled the rotational movements compared to the results of a well-supported tie.

**Effect of Tie Rotation on Ballast Performance by Discrete Element Modeling (DEM)**

In the previous sections, the field results have shown that the tie movements in the “real world” consist of not only translations but also rotations. In this section, the effect of the tie rotation on ballast performance (ballast particle acceleration and contact forces) is studied. A discrete element program (BLOKS3D) developed at the University of Illinois was used in this study to predict the ballast particle acceleration levels and contact forces between particles with and without
tie rotation considered. A total of 6695 ballast particles were modeled in the simulation. Figure 6-16\textsuperscript{70} shows the dimensions of the half-track model generated by the BLOKS3D program. A half tie resting on a half ballast track was modeled. Figure 6-17\textsuperscript{96} shows a set of typical vertical train loads that were applied in the modeling. Tie rotations were simulated by applying periodic angle changes with respect to the vertical, lateral and longitudinal directions. The detailed inputs are listed in Table 2. Condition 1 represents a well-supported tie and condition 2 represents a tie with tie-ballast gap.

![Figure 6-16. The dimensions of the half-track model used in the BLOKS3D program\textsuperscript{96}](image)

<table>
<thead>
<tr>
<th>Amplitude (degree)</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
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<th>Y</th>
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<tbody>
<tr>
<td>Direction</td>
<td>X</td>
<td>Y</td>
<td>Z</td>
<td>X</td>
<td>Y</td>
<td>Z</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>20</td>
<td>35</td>
<td>20</td>
<td>20</td>
<td>35</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 2. Tie rotation information in the discrete element modeling (BLOKS3D).
Figure 6-18 shows the vertical acceleration of a chosen ballast particle under different tie rotation conditions. In Figure 6-18 (a), the tie without rotation only has a maximum value of 12 m/s² of the vertical acceleration. However, the maximum vertical accelerations in condition 1 and condition 2 are increased to 70 m/s² and 80 m/s², respectively. This indicates that the ballast particles will suffer larger accelerations if the tie rotation is considered.

![Graph showing vertical acceleration vs sample number](image)

Figure 6-18. Acceleration of ballast particle # 162 under different tie rotation conditions by BLOKS3D: (a) no tie rotation; (b) tie rotation condition 1; (c) tie rotation condition 2

Figure 6-19 shows the vector plot of ballast particle contact forces of the track cross section. The arrows are in the vertical direction and the length of the vector indicates the magnitude of the contact forces. Figure 6-19 (a) and (b) represent the conditions of the modeling without and with tie rotation, respectively. It can be seen from the figure that the magnitude of the ballast inter-particle forces, especially in the area beneath the tie, would have a significant increase if the rotational movement of the tie is considered.
Conclusions and Future Work

In this chapter, the tie movements under train passage were characterized by numerical modeling and field tests conducted on a high-speed passenger line and a Class III freight line. The modeling results and field measurements show that the movements of a railroad tie not only have translational movements, but also rotational movements. Further, discrete element modeling (DEM) was used to predict the effect of the tie rotation on the ballast performance, such as individual particle acceleration and ballast inter-particle forces. The following list summarizes the main findings of the study:

- Field study reveals that the motion of a railroad tie under train loading could have both translational movements and rotational movements.
The magnitude of the movements depends on the track quality, such as fasteners and ballast support. If tie-ballast gap exists under a tie, the tie rotation and acceleration of the tie could have a significant increase which may accelerate the deterioration of ballast.

By installing multiple measuring units on a single tie, the motion of a single tie at different positions was measured. The results show that the left and right edge of a single tie could have different magnitudes of accelerations.

The angular velocities and angular acceleration of the tie are obtained by the first and second derivative of the Euler angle. The angular acceleration of the tie with tie-ballast gap could generate extra moment up to 5,000 N-m. It may be large enough to increase the rate of ballast breakage and fouling, thus affect the service life of track.

The interaction of individual ballast particle was investigated by the discrete element modeling. The vertical acceleration of ballast particle and inter-particle contact forces would have significant increases if the rotation of ties is taken into consideration.

Future research will investigate how the tie rotation with respect to each axis individually affects the ballast performance and other track components. Laboratory tests will also be conducted to validate the findings of modeling in this paper and evaluate the ballast performance with tie rotation.
Chapter 7

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

This dissertation is a combination of five manuscripts aimed at investigating the critical speed effect and other railroad track problems by using a 3D train-track interaction model. The formulation and validation of the model are presented. The critical speed effect, hanging tie detection and characterization of tie movements are studied with the aid of the models.

In Chapter 3, the model derivation coupled a 2D discrete support track model with a 3D computationally-efficient FE soil subgrade model. In the 2D track model, the rail beam was modeled as a Euler -Bernoulli Beam. The 2D track model discretized the tie and ballast as rigid bodies with designated spacing. The 3D FE subgrade model was simulated by plane-stress quadrilateral finite elements. The longitudinal direction of the subgrade model was expanded in the frequency domain and is assumed to be homogeneous. Therefore, the computing time could be largely reduced. A moving dynamic loading was applied on top of the rail. The model is capable of taking train speed variations and the profile change of the cross section into consideration. The limitation of the current model is that by assuming the subgrade homogeneity in the longitudinal direction, the model cannot take the variations of the subgrade in moving direction. The entire model was self-coded in MATLAB.

Multiple field instrumentation tests covering a wide range of cross sections and train speeds were then conducted to verify the accuracy of the dynamic track-subgrade interaction model. The main testing site was located on Amtrak’s highest speed (240 km/hr) line near Kingston, Rhode Island, on the Northeast Corridor in United States. Track deflections measured in the field were compared with those predicted by the 3D dynamic track-subgrade interaction model. It is concluded that this model can predict track performance accurately for the Kingston site.

In Chapter 4, the previously developed and validated 3D dynamic track-subgrade interaction model was used to predict the track and ground vibrations and substructure performance. Field measurements and model results of maximum rail deflection matched well at the selected speed range; the ground surface wave propagation had the similar cone-shaped mode. A slight critical speed effect was detected at Kingston site, but did not cause significant track deflection. In addition, model simulation pointed out that the stress contour in the subgrade would encounter a significant increase when trains were at the predicted critical speed. Due to the
complicated tie-ballast-soil system and the geometry of the cross section, the critical speed is defined in two levels for the Kingston site: 1) the speed causing significant increase in rail deflection, at which derailment becomes a concern; and 2) the speed causing significant increase in the cross section stress intensity, at which more frequent ballast maintenance becomes a concern.

For hanging tie detection, the dissertation proposed a fast, nondestructive screening method to identify the hanging tie problem. The method utilized the proposed model in Chapter 3 to characterize the track's “Moving Deflection Spectrum (MDS)” under different tie-support scenarios. The model includes a moving dynamic load and ties with discrete supports. The MDS shows dynamic responses of the “Track Moving Deflection (TMD)” in the frequency domain. The modeling results indicate that the MDSs present a significantly increased peak in the low frequency region if there is hanging tie problem existing. To validate this method, preliminary field tests were carried out on both Boston Metro and St. Louis Metro lines. The TMDs in the field tests were measured by the accelerometers installed on a high-rail vehicle. Then the TMDs were divided into many segments with equal length in order to locate the hanging tie(s). If the corresponding MDSs show a clear peak at the Tie Spacing Frequency, the track is considered as not having a hanging tie or a very minor hanging tie problem. The results match the field visual observation, and therefore the method can identify the hanging tie problem and has great potential to be employed by the rail industry in the future.

In Chapter 6, the characterization of tie movements is conducted by a numerical model which couples two commercial software: a vehicle dynamics model (VAMPIRE) and a three-dimensional finite element track model (ABAQUS). The modeling results showed that the motion of ties not only contains translational movements, but also rotations. The field tests to validate this finding were conducted on an Amtrak high-speed passenger line and a freight railroad short line. The measuring units were mounted on ties to record the accelerations and the changes in Euler angles of the ties in three orthogonal directions. The measurements of tie displacements and rotations in the field tests had good agreement with the modeling results. Moreover, field tests indicate that the tie-ballast gaps may cause higher accelerations and angular velocities of ties. It is also found that different positions on a single tie could have different accelerations. The effect of the tie rotation was investigated by the discrete element modeling. The vertical acceleration of ballast particle and inter-particle contact forces would have significant increases if the rotation of ties is taken into consideration.
Overall, the 3D train-track interaction model is capable of simulating the railroad tracks in various conditions by integrating the discrete supports and 3D subgrade model. It is a calculation-efficient, effective and reliable (validated) tool to predict and solve the train and track problems.

**Recommendations**

The current track model contains a 2D track substructure and a 3D subgrade domain. The 2D track substructure only provides the information of the track in the plane of longitudinal and vertical directions. The lateral direction of the track is ignored in the formulation. The main purpose of the track lateral setup is to provide the lateral stability and rail alignment. The friction between ties and ballast layer should be sufficient to balance the centrifugal forces when trains are on a curve. Meanwhile, the friction largely depends on the vertical forces applied on top of the rail. Therefore, the current formulation combined with the formulation of the track in the lateral makes the model capable of investigating the lateral stability of the track. The lateral formulation of the track is recommended to be included in the future modeling. The lateral stability of the track highly relies on the loading condition in the vertical direction; therefore, the lateral stability can only be evaluated after the vertical loading is provided or the vertical formulation provides the contact forces for the layer of interest.

Temperature change will build up the longitudinal forces in the rail. At the rail neutral temperature, the longitudinal forces are close to zero. However, temperature increase will cause significant change in compressive load in the rail, which could lead to rail buckling. The governing equation of the rail in the current model has included the longitudinal forces due to the temperature change. The potential risk of the rail buckling can be evaluated by combining the effects of temperature change, moving train load, speed condition and rail type (bending rigidity).

The formulation of the track model is based on the assumptions of the linearity, which means that the rail-tie and tie-ballast interactions are described by linear equations. However, the layer interactions, especially the tie-ballast interaction, more realistically involves nonlinear behavior. Therefore, in future research, the nonlinearity can be incorporated into the formulation in order to simulate the track system more accurately and realistically.
References

23. Transportation Technology Center I. NUCARS tutorial.


84. Selig ETAW, John M. *Track geotechnology and substructure management*.


Appendix A

Train Passage Log at Kingston

A high-speed video camera and a photo-electric sensor were used to record the passing train so that an accurate measure of train speed could be obtained and the exact location of the train could be reconciled with the acceleration time histories. The information of each train passage is listed in the table.

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<th>Date</th>
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<th>Location</th>
<th>Velocity (km/hr)</th>
<th>Type</th>
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Appendix B

Field Equipment at Kingston
11-18-13 MP 156-23 (Site 1) Accelerometer Array
11-18-13 MP 155-41 (Site 2) Accelerometer Array
12-19-13 MP 156-23 (Site 1) Accelerometer Array
3-18-14 MP 156-23 (Site 1) Accelerometer Array
4-9-14 MP 156-23 (Site 1) Accelerometer Array
4-10-14 MP 155-41 (Site 2) Accelerometer Array
Appendix C

DCP Analysis at Kingston

The DCP results are provided by HyGround Engineering and University of Massachusetts Amherst
DCP 1
MP 155-41 (Site 2)
Centerline

![Graphs showing cumulative blows, DCP index, and CBR values against depth.](image-url)
DCP 2
MP 155-41 (Site 2)
Embankment

Cumulative Blows

DCP Index (mm/blow)

CBR (%)
DCP 3
MP 155-41 (Site 2)
Swamp

![Graphs showing cumulative blows, DCP index (mm/blow), and CBR (%)]
DCP 4
MP 156-23 (Site 1)
Centerline

![Cumulative Blows](chart1.png)

![DCP Index (mm/blow)](chart2.png)

![CBR (%)](chart3.png)
DCP 5
MP 156-23 (Site 1)
Embankment

![Graph of Cumulative Blows](image)

![Graph of DCP Index (mm/blow)](image)

![Graph of CBR (%)](image)
DCP 6
MP 156-23 (Site 1)
Swamp

**Cumulative Blows**

**DCP Index (mm/blow)**

**CBR (%)**
DCP 7
MP 156-23 (Site 1)
Embankment
DCP 9
MP 155-41 (Site 2)
Embankment

Cumulative Blows

DCP Index (mm/blow)

CBR (%)
DCP 10
MP 155-41 (Site 2)
Swamp

Cumulative Blows

DCP Index (mm/blow)

CBR (%)
DCP 11
MP 155-41 (Site 2)
Swamp

Cumulative Blows

Depth (m)

DCP Index (mm/blow)

Depth (m)

CBR (%)
Appendix D

Accelerometer Specifications

The model of the accelerometers used in the field validation is 629A61 from PCB Piezotronics. The specifications are from the website of the manufacture.


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<td>Measurement Range</td>
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<td>Frequency Range (±5 %)</td>
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<tr>
<td>Frequency Range (±10 %)</td>
<td>1.7 to 5000 Hz</td>
</tr>
<tr>
<td>Frequency Range (±3 dB)</td>
<td>0.8 to 8000 Hz</td>
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<tr>
<td>Resonant Frequency</td>
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<tr>
<td>Broadband Resolution (1 to 10000 Hz)</td>
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<td>Transverse Sensitivity</td>
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<td>Frequency Range (± 5 %)</td>
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<tr>
<td>Frequency Range (± 10 %)</td>
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</tr>
<tr>
<td>Frequency Range (± 3 dB)</td>
<td>0.8 to 8000 Hz</td>
</tr>
</tbody>
</table>

Environmental

| Overload Limit (Shock)               | 49050 m/s² pk        |
| Temperature Range                    | -54 to +121 °C       |
| Temperature Response                 | See Graph %/°F       |

Electrical

| Settling Time (within 1% of bias)    | ≤3.0 sec             |
| Discharge Time Constant             | ≥0.2 sec             |
| Excitation Voltage                  | 18 to 28 VDC         |
| Constant Current Excitation         | 2 to 20 mA           |
| Output Impedance                    | <100 Ohm             |
| Output Bias Voltage                 | 8 to 12 VDC          |
| Spectral Noise (10 Hz)              | 68.7 (µm/sec²)/√Hz   |
| Spectral Noise (100 Hz)             | 27.5 (µm/sec²)/√Hz   |
| Spectral Noise (1 kHz)              | 9.8 (µm/sec²)/√Hz    |
| Electrical Protection               | RFI/ESD              |
| Electrical Isolation (Case)         | >1000000000 Ohm      |

Physical

<p>| Size - Length                        | 38.1 mm              |
| Size - Width                         | 38.1 mm              |
| Size - Height                        | 20.8 mm              |
| Weight (without cable)               | 139 gm               |
| Mounting Thread                      | Not Applicable       |
| Mounting Torque                      | 2.7 to 6.8 Nm        |
| Sensing Element                      | Ceramic              |
| Sensing Geometry                     | Shear                |
| Housing Material                     | Stainless Steel      |</p>
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<tr>
<td>Cable Type</td>
<td>Polyurethane</td>
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</table>

![Typical Sensitivity Deviation vs Temperature](image)
VITA: Yin Gao

Yin Gao was born on August 17th, 1989 in Nanjing, China. He completed his undergraduate studies in Civil Engineering at the Southeast University in 2011. In the summer of 2011, he started his master studies at the Pennsylvania State University (PSU) majoring in Civil Engineering, with specialty of Geotechnical Engineering. In August 2013, he was conferred a degree of Master of Science in Civil at PSU.

Afterwards, he started his PhD work at Penn State in the fall of 2013. He worked as a graduate research assistant in the Civil Infrastructure Testing and Evaluation Laboratory (CITEL) at Penn State. His research topic is the train-track interaction modeling and its applications. He received V. Terrey Hawthorne Memorial Scholarship Fund (A scholarship for ASME RTD graduate student) in 2014 and 2016. He also worked as a laboratory assistant for CE 337 (Civil Engineering Materials Laboratory) in the spring semester of 2015.

During the summer of 2014, he interned at the Transportation Technology Center which is a world-renowned research center in railroad engineering. He worked on the failure mechanism of the track section with soft subgrade, the train-track interaction modeling, the track gauge and rail profile data collection, etc.

Continuing his interest in the train-track interaction modeling which was developed during his master’s studies, Yin formulated and validated a model with discrete supports of track for his PhD dissertation. As a research assistant, he contributed to projects for the Federal Railroad Administration, Amtrak. On May 26th 2016, Yin successfully defended his PhD dissertation in Civil Engineering.