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SUPPLY CHAIN INVENTORY MODELS CONSIDERING SUPPLIER SELECTION AND PRICE SENSITIVE DEMAND

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Abstract

The purpose of this research is to develop supply chain inventory models that simultaneously coordinate supplier selection and pricing decisions for a range of retailing situations. The selection of appropriate suppliers plays an important role in improving companies' purchasing performance. Many researchers and firms have studied the supplier selection problem without taking into account the price-sensitive nature of the demand for certain products. A product's selling price has a significant impact not only on a company's ability to attract consumers, but also on its strategic decisions on matters such as supplier selection that are taken at the upstream stages of the supply chain.

In this dissertation, we start with the development of a new mathematical model for the supplier selection problem that refines and generalizes some of the existing models in the literature. We propose a mixed integer nonlinear programming (MINLP) model to find the optimal inventory replenishment policy for a particular type of raw material in a supply chain defined by a single manufacturer and multiple suppliers. Each supplier offers an all-unit quantity discount as an incentive mechanism. Multiple orders can be submitted to the selected suppliers within a repeating order cycle. We initially assume the demand rate to be constant. The model provides the optimal number of orders and corresponding order quantities for the selected suppliers such that the replenishment and inventory cost per time unit is minimized under suppliers' capacity and quality constraints. Then, we extend the model to simultaneously find the optimal selling price and replenishment policy for a particular type of product in a supply chain defined by a single retailer and multiple potential suppliers. Hence, we replace the manufacturer with a retailer subject to a demand rate considered to be dependent on the selling price. We propose an MINLP model to find

the optimal order frequency and corresponding order quantity allocated to each selected supplier, and the optimal demand rate and selling price such that the profit per time unit is maximized taking into consideration suppliers' limitation on capacity and quality. In addition, we provide sufficient conditions under which there exists an optimal solution where the retailer only orders from one supplier. We also apply the Karush–Kuhn–Tucker conditions to investigate the impact of the supplier's capacity on the optimal sourcing strategy. The results show that there may exist a range of capacity values for the dominating supplier, where the retailer's optimal sourcing strategy is to order from multiple suppliers without fully utilizing the dominating supplier's capacity.

Next, we study the integrated pricing and supplier selection problem in a two-stage supply chain that comprises a manufacturer stage followed by a retailer stage, both controlled by a single decision-maker. The manufacturer can procure the required raw material from a list of potential suppliers, each of which has constraints in regard to capacity and quality. In this model, the manufacturer periodically replenishes the retailer's inventory, the demand for which is proving to be price-sensitive. We propose an MINLP model designed to determine the optimal replenishment policy for the raw material, the optimal amount of inventory replenished at each stage, and the optimal final product's selling price at which the profit per time unit is maximized. Additionally, we provide upper and lower bounds for the optimal selling price and for the manufacturer's lot size multiplicative factor, which result in a tight feasible search space.

Next, we propose an MINLP model to extend the prior model by considering a serial supply chain controlled by a decision-maker responsible for maximizing the profit per time unit by determining the following: the optimal amount of raw material to order from the selected suppliers, the optimal amount of product to transfer between consecutive stages in order to avoid any inventory shortage, and the optimal final product's selling price. Coordinating all these decisions simultaneously is a topic that has been neglected in literature. In addition, our model requires the order quantity received from each selected supplier to be an integer multiple of the order quantity delivered to the following stage, which means that a different multiplicative factor can be assigned to each supplier. This coordination mechanism shows an improvement in the objective function compared to those of existing models that assign the same multiplicative factor to each selected supplier. Moreover, we develop a heuristic algorithm that generates near-optimal solutions. A numerical example is presented to illustrate the proposed model and the heuristic algorithm.

Table of Contents

List of Figures ix

List of Tables x

Acknowledgements xii

Chapter 1: Introduction and Overview 1

1.1. Introduction 1

1.2. Research Contributions 5

1.3. Structure of the Dissertation 6

Chapter 2: Literature Review 7

2.1. Supplier Selection under Constant Demand Rate 7

2.2. Pricing and Supplier Selection Problem 11

2.3. Pricing and Supplier Selection Problem in a Two-Stage Supply Chain 14

2.4. Pricing and Supplier Selection Problem in a Multi-Stage Serial Supply Chain 17

Chapter 3: Quantity Discount Decisions considering Multiple Suppliers with Capacity and Quality Restriction 20

3.1. Problem Description and Model Formulation 20

3.2. Numerical Example 25

3.3. Conclusions 33

Chapter 4: Determining the Retailer's Replenishment Policy considering Multiple Capacitated Suppliers and Price-Sensitive Demand 34

4.1. Problem Description and Model Development 34

4.2. Model Analysis 38

4.2.1. Uncapacitated Dominating Supplier 38

4.2.2. Capacity Analysis for Dominating Supplier 42

4.3. Numerical Example 49

4.4. Conclusions 55

Chapter 5: Integrated Pricing and Supplier Selection Problem in a Two-Stage Supply Chain 57

5.1. Problem Description and Model Development 57

5.2. Model Analysis 62

5.2.1. Finding the Dominating Supplier 62

5.2.2. Developing Lower and Upper Bounds for the Selling Price. 66

5.2.3. Feasible Region for the Multiplicative Factor. 66
5.2.4. Improving the Multiplicative Factor's Bounds 72
5.3. Numerical Examples 78
5.4. Conclusions 84
Chapter 6: Integrated Pricing and Lot-sizing Decisions in a Serial Supply Chain 86
6.1. Problem Description and Model Development 86
6.2. Heuristic Algorithm using the Power-of-Two (POT) Policy 95
6.3. Illustrative Examples 102
6.4. Conclusions 110
Chapter 7: Research Summary and Future Directions 111
7.1. Research Summary 111
7.2. Future Directions 113
Appendix 115
References 116

List of Figures

Figure 3.1. Orders submitted to each supplier within a repeating order cycle. 22

Figure 3.2. Behavior of Model (*M1*) over different values of *m*. 27

Figure 3.3. Cost comparison between Models (*M1*) and (*M2*). 29

Figure 3.4. Model (*M1*)'s behavior with respect to the inventory holding cost rate 32

Figure 4.1. Supply chain with a single retailer and multiple suppliers...34

Figure 4.2. Illustration of a solution for Model (*M1*). 39

Figure 4.3. Total monthly profit and cycle time for *m* = 1,…, 20. 50

Figure 4.4. Model (*M1*) vs. Model (*M2*)... 52

Figure 4.5. The impact of supplier 1's capacity on the sourcing strategy when *m* = 2. 54

Figure 5.1. Two-stage supply chain system...57

Figure 6.1. Pricing and supplier selection in a serial system...86

Figure 6.2. Unsynchronized inventory levels in a two-stage supply chain. 90

Figure 6.3. Synchronized inventory levels in a two-stage supply chain...................................... 91

List of Tables

Table 3.1. Suppliers' all unit quantity discounts. 26

Table 3.2. Detailed solutions of Model (*M1*) over different values of m. 28

Table 3.3. Model (*M2*)'s detailed solutions for different values of m. 30

Table 3.4. Cost comparison between Models (*M1*) and (*M2*) for m = 7. 31

Table 4.1. Suppliers' parameters. ………………………..…………………………………49

Table 4.2. Model (*M1*)'s detailed solutions for $m = 1, \dots, 20$………………………………..51

Table 4.3. Model (*M2*)'s detailed solutions for $m = 1, \dots, 20$…………………………..……...52

Table 4.4. Results obtained from selecting each supplier separately……………………………53

Table 5.1. Suppliers' parameter information……………………..……………………………80

Table 5.2. Parameter information for each stage………………………………………………...80

Table 5.3. Optimal solution when each supplier is selected separately………………………….81

Table 5.4. CPU time comparison………………………………………………………………...81

Table 5.5. Calculation to improve the upper bound for the multiplicative factor……………….82

Table 5.6. CPU time comparison………………………………………………………………...83

Table 5.7. Suppliers' parameter information…………………………………………………….83

Table 5.8. Optimal solution when each supplier is selected separately………………………….83

Table 5.9. CPU time comparison………………………………………………………………...84

Table 6.1. Suppliers' parameters information…………………………………………………..103

Table 6.2. Cost parameters of each stage……………………………………………………….103

Table 6.3. Optimal solution for Model (*M1*)…………………………………………………...104

Table 6.4. Several values for the selling price……………………………………………….105

Table 6.5. Initial order quantities for each stage………………………………………………..105

Table 6.6. First iteration of Procedure 1……………………………..………………………106

Table 6.7. Second iteration of Procedure 1…………………………………………………..106

Table 6.8. Third iteration of Procedure 1……………………………..……………………107

Table 6.9. Suppliers' parameters information………………………………………………..108

Table 6.10. Cost parameters for each stage…………………………………………………108

Table 6.11. Optimal solution for Model (*M1*)…………………………………………………109

Table 6.12. Optimal solution for Model (*M2*)…………………………………………………109

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Chapter 1: Introduction and Overview

1.1. Introduction

In today's competitive environment, companies tend to leverage their profits and reduce the cost of the final product by acquiring some of the parts and/or services needed for the final product from outside suppliers. On this point, it should be noted that the purchasing cost of raw materials and component parts may represent a significant portion of the cost of a product. For instance, Weber et al. (1991) stated that purchased services and parts in the high-tech industry represent up to 80% of a product's total cost. In addition, Wadhwa and Ravindran (2007) showed that in the automotive industry the purchased components and parts exceed 50% of the total sales. Therefore, efficient management of purchasing processes is needed for a company to remain competitive and reduce costs. For these reasons, researchers studied companies' purchasing policies and found that the selection of appropriate suppliers is a key strategic decision in enhancing companies' purchasing performance (Ravindran and Warsing, 2012).

Supplier selection is a multi-criteria problem that aims is to select a group of preferred suppliers from a large set of potential suppliers based on the buyer's qualitative and quantitative criteria. Price, delivery lag, quality, production capacity, and location are the most common criteria influencing the supplier selection process discussed in the literature. Hence, the multi-criteria nature of the supplier selection problem increases the level of complexity, since different contradictions will take place when a trade-off between qualitative and quantitative factors is performed to select the best supplier. For example, the supplier offering the lowest unit price may not have the best quality or the supplier with the best quality may not be able to offer timely product delivery.

The majority of supplier selection problems in practice take place in a multiple source purchasing environment in which there is no single supplier who is capable of satisfying the buyer's demand due to suppliers' constraints on matters such as capacity, quality, delivery lag, and price. A central goal in supply chain management, therefore, is that of maintaining long-term relationships with suppliers that provide competitive prices and reliable service. Thus, it is crucial to use a pre-selection decision-making approach to screen suppliers and on that basis generate a manageable list of effective suppliers. Then, a mathematical model can be formulated to select the most appropriate suppliers and determine the corresponding order quantities considering mostly quantitative criteria. Comprehensive surveys on the supplier selection problem include those by Minner (2003), Aissaoui et al. (2007), Ho et al. (2010), Agarwal et al. (2011), and Chai et al. (2013).

The majority of the existing supply chain models available in the literature consider placing a single order to each selected supplier within a repeating order cycle. However, Mendoza and Ventura (2008) showed that the cost per time unit can be reduced by submitting multiple orders with different frequencies to the selected suppliers within an order cycle. This effective approach is well suited when the most efficient supplier is unable to satisfy the demand due to capacity limitations. For instance, assume that supplier 1, the most efficient supplier, can satisfy only 10% of the demand whereas supplier 2, a less efficient supplier, can satisfy all the demand. Then, allowing at most one order to each supplier per order cycle would either result in using both suppliers at the expense of having a very high holding cost due to the large order quantity submitted to supplier 2 or it would result in ruling out supplier 1 and satisfying all the demand requirements from supplier 2. Clearly, suboptimal solutions will be obtained in both cases, since the cost could be reduced further by allowing multiple orders to be submitted to supplier 2 for each order placed to supplier 1 in a repeating order cycle.

Most of the research on the supplier selection problem is founded on the unrealistic assumption that product demand is deterministic and constant. In the contemporary environment, however, supply chain inventory management faces challenges presented by the price-sensitive nature of demand for certain products. In practice, it is common for product demand to vary with selling price, given that low prices in most markets play a significant role in attracting consumers. Considering the impact of price on sales, many researchers and firms have focused on developing mathematical models to optimize pricing decisions alone. However, pricing decisions can affect other aspects of the supply chain, such as production and distribution decisions. Therefore, it is essential to coordinate all these decisions simultaneously. Such environment setting is unlike the classical economic order quantity (EOQ) model that unrealistically assumes the demand rate to be fixed and independent of the selling price. Hence, product demand must be considered as a decision variable, as it is strongly influenced by the selling price. It can also be argued that in retail contexts, more profit can be obtained when pricing and lot-sizing decisions are jointly determined.

Another unrealistic assumption found in the literature on the supplier selection problem is that most of the models optimize supplier selection decisions in regard to a specific member of the supply chain. Inventory decisions at any given stage can have an effect on key decisions in the entire supply chain. In addition, due to the challenges facing today's supply chains, such as the increase in manufacturing, transportation, and holding costs, there is a clear need to consider the supply chain as a whole. A supply chain is defined as "a coordinated set of activities concerned with the procurement of raw materials, production of intermediate and finished products, and the distribution of these products to customers within and external to the chain" (Ravindran and

Warsing, 2013). This integrated process is one in which several activities are simultaneously coordinated among the various members of a supply chain: suppliers, manufacturers, warehouses, distribution centers, retailers, and customers. These activities can include designing the supply chain network, selecting appropriate suppliers, designing transportation channels, allocating order quantities, and distributing finished goods to customers. Comprehensive studies on supply chain management include those by Thomas and Griffin (1996), Vidal and Goetschalckx (1997), Beamon (1998), Chandra and Kumar (2000), Min and Zhou (2002), Meixell and Gargeya (2005), and Badole et al. (2012).

Many scholars have shown that each member of a supply chain is better off when all members work together to determine a joint economic inventory policy compared to when each member determines its own inventory policy independently. The challenging aspect of a multi-stage supply chain is that the inventory at any given stage is used to replenish the inventory for the next stage. Therefore, the principal critical decision in a multi-stage supply chain system pertains to coordinating the flow of products from one stage to the next stage such that inventory shortages are avoided. Researchers have shown that inventory shortages are avoided when two conditions hold: (1) replenishment orders are placed only when the inventory level drops to zero and (2) when an order is placed at any stage in the supply chain, orders are placed at all the downstream stages as well. These assumptions require the order quantity placed at any stage to be an integer multiple of the order quantity placed at the downstream stage. This ordering policy is known as the zero-nested inventory ordering (Love, 1972; Schwarz, 1973; Schwarz and Schrage, 1975; Maxwell and Muckstadt, 1985). Following this line of thought, Roundy (1985) proved that a near-optimal solution can be obtained if the vendor's integer multiplicative factor is a power-of-two.

1.2. Research Contributions

The main contributions of this research are as follows:

1. A mixed integer nonlinear programming (MINLP) model is developed to solve a supplier selection problem for a manufacturer that procures a particular type of raw material from a set of potential suppliers, across which ordering and purchasing costs, production capacity, and quality limitations all vary. Multiple orders can be placed to the selected suppliers within a repeating order cycle. Further, the model is realistic and practical in nature given that the suppliers offer all-unit quantity discounts as an incentive mechanism to increase the order quantity placed, thereby reducing the average replenishment cost.
2. An MINLP model is developed to solve the integrated pricing and supplier selection problem for a retailer that procures a particular type of product from a set of potential suppliers and faces price-sensitive demand. In addition, an investigation of the retailer's sourcing strategy when the dominating supplier is capacitated is also provided. Results show that the dominating supplier can be selected without fully utilizing its capacity.
3. An MINLP model is developed to solve the integrated pricing and supplier selection problem in a two-stage supply chain. Moreover, a procedure to compute tight bounds for the profit per time unit and for the main decision variables, such as the selling price and integer multiplicative factors is presented. These bounds help in obtaining a reduced feasible region and solve the problem in a timely manner.
4. An MINLP model is developed to solve the integrated pricing and supplier selection problem in a serial supply chain. In addition, a heuristic algorithm that provides a solution within 2% of the optimal solution is presented. Furthermore, the model allows a different multiplicative factor to be applied to the order quantity placed for each selected supplier.

This effective tool yields a higher profit per time unit compared to those of models that use the same multiplicative factor.

1.3. Structure of the Dissertation

This dissertation is organized as follows. Chapter 2 presents a literature review of the most recent and significant related work on pricing and the supplier selection problem and on multi-stage inventory systems. Chapter 3 proposes a model for a supply chain inventory problem for a specific type of raw material with multiple suppliers, where the final product's demand is assumed to be constant and known in advance. Then, in Chapter 4, we simultaneously study pricing and inventory replenishment decisions by extending the model proposed in Chapter 3 to the case where the demand rate is a decreasing function of the selling price. Chapter 5 presents an inventory replenishment model with supplier selection and pricing decisions in a two-stage supply chain comprising a manufacturer stage followed by a retailer stage. Then, Chapter 6 presents a study of the integrated pricing and supplier selection problem in a serial supply. Chapter 7 summarizes the dissertation research, and suggest some directions for future work.

Chapter 2: Literature Review

2.1. Supplier Selection under Constant Demand Rate

The supplier selection problem has been studied widely for the past three decades due to the critical role it plays in the purchasing decision processes. Some good comprehensive surveys were done in reviewing and analyzing previous published work. For instance, Weber et al. (1991) classified 74 articles based on the specific purchasing situations and the decision tools used in supplier selection. Degraeve et al. (2000) presented rating models to evaluate and compare different supplier selection approaches based on the total cost of ownership which considers the purchasing price and other related costs. De Boer et al. (2001) considered the entire supplier selection process. Thus, they did not just review the final supplier selection models, but also considered all the decision making steps involved in the supplier selection process. Ho et al. (2010) and Agarwal et al. (2011) reviewed and evaluated the multi-criteria decision making methods in supplier selection. More recently, Chai et al. (2013) reviewed 123 journal articles from year 2008 to 2012 based on decision-making techniques in supplier selection. The rest of this section provides a brief discussion on the most significant and recent published articles related to all unit quantity discounts and supplier selection problem which have been considered in the development of our proposed general model in Chapter 3.

Researchers considered two purchasing situations in the supplier selection problem. Firstly, in the single sourcing purchasing situation, any supplier can satisfy the buyer's demand without any capacity constraint. Benton (1991) developed nonlinear programming models using the concept of economic order quantity to find the best supplier among all the candidate suppliers who offered all-unit quantity discounts and under the conditions of multiple items and suppliers' constraints. Instead, Lee et al. (2001) applied a multi-criteria approach for selecting the preferred supplier. They used the analytical hierarchy process (AHP) to compare suppliers based on a predetermined set of attributes. Conversely, Jayaraman et al. (1999) concluded in their work that single sourcing is not necessarily the most efficient approach. When a large number of products need to be ordered, it may not be practical to drop some suppliers just to reduce the total ordering cost.

Secondly, in a multiple sourcing purchasing situation, there may not be one individual supplier who can satisfy the buyer’s demand due to suppliers’ constraints on quality, capacity, lead time, etc. Rosenblatt et al. (1998) developed a single item EOQ model considering multiple capacitated suppliers and a constant demand rate. Ghodsypour and O’Brien (2001) developed an MINLP model for a supply chain inventory problem to determine the optimal allocation of products assigned to suppliers while minimizing the total annual cost of purchasing, ordering, and holding under capacity, budget, and quality constraints. Later, Mendoza and Ventura (2008) proposed a two-phase approach, where in the first phase the subset of preferred suppliers was selected within a large set of suppliers, and then in the second phase an MINLP model was formulated to find the optimal order quantity allocation to suppliers. It was found that the optimal solution obtained by Ghodsypour and O’Brien (2001) could be improved by allowing multiple number of orders to be placed to the selected suppliers within a repeating order cycle. Moreover, Mendoza and Ventura (2011) compared two MINLP models for supplier selection, where the first one allows the submission of different order quantities to each supplier while in the second one all the order quantities are restricted to have the same size. Recently, Mohammaditabar and Ghodsypour (2016) developed a supplier selection model to determine the joint replenishment inventory decision for multiple items. They showed that the ordering cost decreases when ordering multiple items from the same supplier. The cost components that they considered are: ordering, inventory holding, and purchasing cost.

However, in none of the aforementioned supplier selection models, suppliers are allowed to adopt incentive mechanisms such as quantity discounts. All-unit quantity discounts are commonly used as incentive tools offered by suppliers to motivate buyers to place larger order quantities by reducing the unit ordering cost and, as a result, the setup cost per time unit. Monahan (1984) analyzed the quantity discount problem with respect to the supplier point of view. Later, Lee and Rosenblatt (1986) generalized the model and discussed the ordering and price discount problem. Similarly, Rubin and Benton (2003) developed a generalized framework for quantity discount pricing schedules to increase the supplier's profits. In addition, Abad (1994) formulated the problem of coordination between a vendor and a buyer as a two-person fixed threat bargaining game, and proposed two pricing scheduled for a vendor who is supplying several buyers. One of them is based on profit sharing and the other one resembles the all-unit quantity discount schedule. Recently, Ke and Bookbinder (2012) developed a model to find the optimal all-unit quantity discounts that should be offered by a supplier for the non-cooperative (Stackelberg equilibrium) and cooperative (Pareto efficient solution) cases. It was found that, if the quantity discount is determined cooperatively, the supply chain efficiency can be enhanced.

Thereafter, scholars have incorporated all-unit quantity discounts with the supplier selection problem. For instance, in Chaudhry et al. (1993), suppliers offer all-unit (cumulative) and incremental (non-cumulative) quantity discounts to motivate the buyer to increase the order quantity. They developed linear and mixed binary integer programming models to find the best suppliers under capacity, quality, and delivery performance constraints. Tempelemeier (2002) considered supplier selection and purchase order quantity under time sensitive demand and quantity discounts. He developed a mixed integer linear programming model and proposed an efficient heuristic algorithm. However, neither Chaudhry et al. (1993) nor Tempelemeier (2002) consider the concept of economic order quantity (EOQ) in their procedures. Moon et al. (2008) developed a hybrid genetic algorithm to solve the joint replenishment problem when multiple items need to be ordered from a number of suppliers offering different quantity discounts schedules. Lee et al. (2013) developed a mixed integer programming model to solve an integrated supplier selection and lot-sizing problem considering multiple suppliers, multiple periods, and quantity discounts. They also proposed a genetic algorithm to determine the order quantity in each time period. Kamali et al. (2011) applied a multi-criteria approach by developing an MINLP model considering qualitative and quantitative objectives for a supply chain inventory model to achieve buyer-suppliers coordination under all-unit quantity discounts; however, they also restrict the buyer to place only one order to each selected supplier within a repeating order cycle.

In Chapter 3, we propose a new model for the supplier selection problem that refines and generalizes some existing models in which the goal is to minimize the average replenishment and inventory cost under suppliers' limitations on capacity and quality. Furthermore, to ensure a more realistic and practical situation, the suppliers in our model are offering all-unit quantity discounts as an incentive mechanism to increase the placed order quantities, and hence reduce the average replenishment cost. Under all-unit quantity discounts, every unit in the order is discounted if the ordered quantity is above certain order level. In addition, multiple orders to the selected suppliers are allowed within a repeating order cycle. Computational results show that the average replenishment and inventory cost is reduced in comparison to the models that allow at most one order to each supplier within an order cycle. Moreover, two versions of our model are considered based on the type of order quantity: the first one considers independent order quantities where different order quantities can be placed to the selected suppliers, while the second one considers equal-size order quantities where all the order quantities submitted to all suppliers have the same size. Thus, the proposed model determines the optimal set of selected suppliers, number of orders to each supplier per order cycle, and the corresponding order quantities to take advantage of possible discounts.

2.2. Pricing and Supplier Selection Problem

Whitin (1955) was the first to incorporate the concept of linking inventory theory and economic price theory in which he considered the demand rate to be linearly dependent on the selling price. Later, Kunreuther and Richard (1971) studied the interrelationship between pricing and inventory decisions, and determined the retailer's optimal pricing and ordering decisions. Thereafter, Abad (1988) found the optimal selling price and lot size when the supplier offers all-unit quantity discounts considering two types of price varying demands, namely linear and negative power functions of price. He also considered the problem when the supplier offers incremental quantity discounts. In this direction, researchers have considered various settings and provided extensions for the joint pricing and inventory problem proposed by Abad (1988). For instance, Kim and Lee (1998) jointly determined the optimal price and lot size for a capacitated manufacturing firm facing a price-sensitive demand, and provided managerial insights on the firm's optimal capacity decisions. Deng and Yano (2006) also considered the case of a capacitated manufacturer facing a price sensitive demand for which they studied the optimal prices and production quantities for a constant and time-varying capacity.

In the context of incorporating coordination mechanisms such as quantity discounts the following scholars: Weng and Wong (1993), Weng (1995), and Viswanathan and Wang (2003), have addressed the effectiveness of quantity discounts and their managerial insights when price sensitive demand is considered. And, Qin et al. (2007), and Lin and Ho (2011) proposed an integrated inventory model with quantity discounts and price sensitive demand to find the optimal pricing and ordering strategies. Other scholars included transportation cost in determining the retailer's optimal pricing and lot-sizing decisions, e.g. Burwell et al. (1997), Abad and Aggrawal (2005), Yildirmaz et al. (2009), and Hua et al. (2012). While others extended the Abad (1988) model by considering a set of geographically dispersed retailers, where each has a price-sensitive demand e.g. Boyaci and Gallego (2002), Wang and Wang (2005), Mokhlesian and Zegordi (2014), and Taleizadeh et al. (2015).

Although, several studies have been developed in coordinating pricing and lot sizing decisions for different retailing settings, all are restricted to a single reliable supplier. However, it's a common practice that retailers would consider multiple potential suppliers to seek the best supplier(s) based on certain qualitative and quantitative criteria. Therefore, in Chapter 4 we consider supplier selection decisions, which have often been neglected in related literature of joint pricing and lot-sizing problem. Research on coordinating pricing and inventory replenishment decisions considering multiple suppliers include Qi (2007) who developed heuristic and dynamic programming algorithms to find the optimal selling price for a manufacturer who faces a price sensitive demand and procures a single product from multiple capacitated suppliers. The proposed model is considered as a fractional knapsack model, where the manufacturer needs to split the demand (i.e., the total number of products) among a set of capacitated suppliers. Thus, he was able to prove that there exists an optimal solution to the proposed problem where at most one of the selected suppliers gets a less than full-capacity order. The same result was obtained earlier by Rosenblatt et al. (1998) who developed a single item EOQ model considering multiple capacitated suppliers and a constant demand. Although the model we present in Chapter 4 can be considered as an extension to the model proposed by Rosenblatt et al. (1988), we show in an illustrative example that the aforementioned result does not hold true and more than one supplier can be selected without fully utilizing their capacities.

More recently, Huang et al. (2011) coordinated pricing, and inventory decisions in a supply chain that consists of multiple suppliers, a single manufacturer, and multiple retailers. The problem was modeled as a three level dynamic non-cooperative game. However, they assumed that only a single sourcing strategy can be considered between the manufacturer and suppliers. Later, Rezaei and Davoodi (2012) proposed a multi-objective nonlinear programming model and a robust genetic algorithm for selecting suppliers and finding Pareto near-optimal selling prices and lot-sizes of multiple products in multiple periods considering budget, storage, and supplier capacity limitations. Qian (2014) studied supplier selection problem under a linear attribute-dependent demand function. The considered the demand to be a function of serval product attributes such as price, delivery time, service level, and quality. However, similar to the majority of the supplier-selection models found in literature, inventory management was ignored in Rezaei and Davoodi (2012), Qian (2014) models. In Chapter 4, we consider a single item EOQ model with multiple capacitated suppliers. Each supplier offers an all-unit quantity discount to motivate the retailer to place larger orders for a lower unit price. The retailer faces a price-sensitive demand which is modeled as a negative power function of the selling price. In addition, the retailer can place multiple orders to the selected suppliers in a cycle. Thus, the goal of the proposed supplier model is to simultaneously find the optimal number of orders per cycle and the corresponding order quantities for the selected suppliers, and the optimal selling price that maximize the retailer's profit per time unit under suppliers' limitations on capacity and quality. We also propose a model that only considers submitting equal-size order quantities to the selected suppliers. Furthermore, in Chapter 4, we provide sufficient conditions under which there exists an optimal solution where the retailer only orders from a single supplier. Moreover, we study the impact of the dominating supplier's capacity on the retailer's sourcing strategies. Thus, we apply Karush–Kuhn–Tucker (KKT) conditions to monitor the change in the retailer's sourcing strategy as the dominating supplier's capacity decreases, and to check whether the supplier's capacity is fully utilized or not (i.e., active or inactive capacity constraint).

2.3. Pricing and Supplier Selection Problem in a Two-Stage Supply Chain

Research on the joint economic lot sizing (JELS) problem has typically focused on coordinating inventory replenishment decisions for a single vendor and a single buyer in a centralized decision-making process. Schwarz (1973) was the first to develop an integrated inventory model to address supply chain inventory problems faced by a single warehouse and a single retailer for a particular product type. Goyal (1977) showed that when supply chain members work together to determine their joint economic inventory policy, each can achieve significant savings compared to the case in which each party determines its own inventory policy independently. Banerjee (1986) developed a joint economic lot-size model for the case of a single vendor with a finite production rate and a single buyer. However, in his model, the vendor is obliged to follow a lot-for-lot policy; i.e., the vendor procures only the quantity of a given product that is required by the buyer. Goyal (1988) extended this research by showing that if the vendor produces an integer multiple of the buyer's order quantity, the obtained cost in Banerjee's (1986) model can be reduced. This ordering policy is defined in the literature as the nested ordering policy (Love, 1972; Schwarz, 1973; Schwarz and Schrage, 1975; Maxwell and Muckstadt, 1985). Following this line of thought, Roundy (1985) proved that a near optimal solution can be obtained if the vendor's integer multiplicative factor is a power-of-two. In the following decades, various aspects of the joint economic lot-sizing problem were addressed, including most recently in comprehensive studies such as those by Ben-Daya et al. (2008) and Glock (2012).

Most of the research on the JELS problem makes the unrealistic assumption that the product demand is deterministic and constant, although in practice the demand is a function of the selling price. The first to address the importance of linking inventory policies with pricing decisions were Whitin (1955), and Kunreuther and Richard (1971), who attempted to determine the optimal selling price and the optimal order quantity for a single retailer. Scholars who studied the JELS problem in conditions of price-sensitive demand include Abad (1994), who determined the optimal policy for a centralized vendor–buyer channel, and characterized the Pareto efficient and Nash bargaining solutions for a decentralized vendor–buyer channel. Viswanathan and Wang (2003) published a similar study considering the coordination mechanisms between two supply chain members in which quantity and volume discounts were considered. Sajadieh and Jokar (2009) analyzed a two-stage supply chain that consists of a vendor with a certain production rate and a buyer who is facing a price-sensitive demand. They proposed a solution algorithm to find the optimal ordering, shipment, and pricing policies that maximize the joint profit of the supply chain members. They also showed that, it is beneficial for supply chain members to cooperate in high competitive environments where customers can easily shift to other less expensive suppliers. Wang et al. (2015) studied the same problem and they proposed two sequential algorithms to find the optimal order quantity, the selling price, and the multiplicative factor. They also investigated coordination mechanisms for cases in which the supply chain is decentralized. Mokhlesian and Zegordi (2014) developed a nonlinear multidivisional bi-level programming model to coordinate pricing and inventory decisions in a multiproduct two-stage supply chain that consists of a single manufacturer and multiple retailers. Pal et al. (2015) considered a two-stage supply chain that is defined by a single manufacturer and a single retailer. They assumed the demand to be not only sensitive to the selling price, but also to the product's quality, and retailer's promotional effort. Taleizadeh et al. (2015) developed vendor managed inventory model for a two-stage supply that is defined by a single vendor and multiple retailers. They considered a certain production rate for the vendor and a different price-sensitive demand function for each retailer. However, they developed their model based on Stackelberg approach in which the vendor is the leader and the retailers are the followers. Recently, Mohabbatdara et al. (2016) studied the ordering and pricing problem for a supply chain that consists of a manufacturer who deliver the final product with an imperfect quality to the retailer. The retailer on the other hand receives the products and determines the optimal selling price.

Another unrealistic assumption found in the literature on the JELS problem is that the vendor procures all the required raw material to produce a good from a single supplier even though in practice many companies use multiple suppliers. On the other hand, many scholars have studied the advantages to companies for selecting more than one supplier. Researches that addressed the coordination of supplier selection and pricing decisions as presented in Section 2.2 are limited to Qi (2007), Rezaei and Davoodi (2012), and Qian (2014). All these studies limited their investigations of the pricing and supplier selection problem to a single stage supply chain. Although Huang, Huang and Newman (2011) coordinated supplier selection, pricing, and inventory in a three-stage supply chain, they considered a non-cooperative game between supply chain members. Moreover, the manufacturer in their model is restricted to a single sourcing strategy. Therefore, in Chapter 5, we formulate the integrated inventory problem for a single manufacturer and a single retailer. The retailer is responsible for deciding the selling price and the size of the order placed to the manufacturer. The manufacturer must make decisions on the integer multiplicative factor in order to determine the number of orders per order cycle and the amount of raw material to order from the selected suppliers, where setup and purchasing costs, production capacity, and quality limitations all vary across suppliers. We develop an MINLP model considering that the two supply chain stages are vertically integrated (i.e., centralized control system) in order to find the number of orders placed to the selected suppliers per order cycle and the corresponding order quantity, the optimal manufacturer's integer multiplicative factor, and the optimal selling price whereby the joint profit per time unit for the manufacturer and retailer is maximized. Moreover, we develop upper and lower bounds on the optimal selling price and the multiplicative factor to obtain a tight feasible region.

2.4. Pricing and Supplier Selection Problem in a Multi-Stage Serial Supply Chain

Inventory policies for a multi-stage supply chain were first studied by Clark and Scarf (1960) and Hadley and Whitin (1963). The challenging aspect of a multi-stage supply chain is that the inventory at any given stage is used to replenish the inventory for the next stage. Therefore, the principal critical decision in a multi-stage supply chain management pertains to coordinating the flow of products from one to stage to another such that inventory shortages are avoided. Researchers have shown that inventory shortages are avoided when two conditions hold: replenishment orders are only placed when the inventory level drops to zero and, when any stage in the supply chain orders, all the downstream stages order as well. This requires the order quantity placed at any stage to be an integer multiple of the order quantity placed at the downstream stage. This ordering policy is known as the zero-nested inventory ordering (Love, 1972; Schwarz, 1973; Schwarz and Schrage, 1975; Maxwell and Muckstadt, 1985). In this direction, Roundy (1985) considered a one-warehouse multi-retailer system and showed that if the integer multiplicative factor is of a power of two, then a near-optimal solution can be obtained for the multi-stage supply chain system. He proved that, when the base cycle time is fixed and known in advance, then the power-of-two (POT) policy solution is within 6% of the optimal solution. And, when the base cycle time is treated as a decision variable, the obtained solution is even closer to the optimal solution, i.e., within 2%. Then, Roundy (1986) generalized his work to consider a multi-product, multi-stage production/inventory system. Further, Muckstadt and Roundy (1993) contributed to the literature by providing a comprehensive analysis of multi-stage production systems. In addition, Li and Wang (2007) provided a review of supply chain coordination mechanisms. Khouja (2003) studied three different inventory coordination mechanisms in a three-stage supply chain. The first mechanism is to use the same cycle time for each member in the supply chain. The second mechanism is to make sure that the cycle time for any stage is an integer multiple of the cycle time of the downstream stage. And, in the third mechanism the integer multiplier is considered to be of a power of two. Reviews of supply chain coordination mechanisms include those by Li and Wang (2007), and Bahinipati et al. (2009).

Studies of supplier selection decisions in a serial supply chain include one by Jaber and Goyal (2008), who studied inventory coordination decisions in a supply chain that consists of multiple suppliers, a single manufacturer, and multiple buyers. Mendoza and Ventura (2010), who developed an MINLP to determine supplier selection decisions and order quantity allocation in a serial supply chain. They also proposed a heuristic algorithm that obtains near-optimal solutions in a timely manner. Later, Ventura et al. (2013) considered the same problem, but for a multi-period inventory lot-sizing model. They developed an MINLP model to determine the optimal inventory policy for each stage in each period. And, most recently, Pazhani et al. (2015) developed an MINLP model to simultaneously determine inventory replenishment and supplier selection decisions for a serial supply chain in which transportation costs are accounted for.

Research on pricing has recently become the subject of considerable scholarly work, although as shown in Section 2.2 only few studies have been published in the last decade that address supplier selection and price-sensitive demand. However, in all these studies, the model accounts for only a single-stage supply chain. Taleizadeh and Noori-daryan (2016) considered a three-stage supply chain that consist of multi-supplier, a single manufacturer, and multi-retailer. They assumed the demand to be price sensitive, yet they established Stackelberg game among the supply chain members. Kumar et al. (2016) studied a three stage supply chain under linear price sensitive demand. They considered three cost components which are the ordering, transportation, and holding costs. They developed an inventory system for coordinated and non-coordinated supply chain. However, they unrealistically assumed that there is only a single supplier to procure the required raw material from. Given this gap in the literature, in Chapter 6, we study pricing and supplier selection decisions in a serial supply chain. Hence, we develop an MINLP model to simultaneously determine the number of orders and corresponding order quantities to submit to the selected suppliers, lot-size decisions between consecutive stages, and the selling price such that the long-run average profit is maximized. And, in order to coordinate inventory decisions to avoid inventory shortages at any stage of the supply chain, the zero-nested ordering policy is considered. We also consider different multiplicative factors to apply to order quantities allocated to the selected suppliers. This approach results in an increase in the average profit in comparison with the average profit generated by models that consider the same multiplicative factor, e.g., Mendoza and Ventura (2010). In addition, we develop a heuristic algorithm through which a near-optimal solution is obtained in a timely manner.

Chapter 3: Quantity Discount Decisions considering Multiple Suppliers with Capacity and Quality Restriction

3.1. Problem Description and Model Formulation

This section presents an MINLP formulation to model a supply chain inventory problem considering a single type of item and multiple (n) potential suppliers. The manufacturer's demand rate d is constant and can be satisfied by allocating multiple orders to the selected suppliers in a repeating order cycle. Also, the manufacturer's unit purchase price is defined by the following all-unit quantity discounts mechanism offered by supplier i,

$$p(Q_i) = \left\{ \begin{array}{lll} 0 & if & Q_i = 0 \\ p_{i1} & if & 0 < Q_i < u_{i1} \\ p_{i2} & if & u_{i1} \leq Q_i < u_{i2} \\ \vdots & if & \vdots \\ p_{i,a_i-1} & if & u_{i,a_i-2} \leq Q_i < u_{i,a_i-1} \\ p_{i,a_i} & if & u_{i,a_i-1} \leq Q_i < \infty \end{array} \right\}, i = 1,2, \dots n,$$

where Q_i is the purchased lot size from supplier i, $u_{i0} = 0 < u_{i1} < \cdots < u_{i,a_i-1} < u_{i,a_i} = \infty$ are the sequence of quantities at which the unit price changes, and a_i is the number of quantity discount intervals offered by supplier i. For instance, the purchasing cost for a lot size Q_i is $p_{ij}Q_i$, if $u_{i,j-1} \leq Q_i < u_{ij}$, where u_{ij} is supplier i's strict upper bound of discount interval j and p_{ij} is the unit price, $j = 1,2, \dots a_i$, and $p_{i1} > \cdots > p_{i,a_i-1} > p_{i,a_i} > 0$.

In addition, each supplier has a certain capacity (or production) rate c_i and quality level q_i (i.e. percentage of acceptable units), and also the minimum acceptable quality level for the buyer (or manufacturer) is q_a. Therefore, the goal of the proposed model is to minimize the replenishment and inventory cost per time unit by finding the optimal number of orders and the corresponding order quantity Q_i for each supplier i where $u_{i,j-1} \leq Q_i < u_{ij}$.

Additional Parameters

r Inventory holding cost rate.

k_i Setup cost for supplier i, where $i = 1,2, \ldots n$.

m Maximum number of orders that can be placed to the selected suppliers in a repeating order cycle.

Additional Decision Variables

Q Total order quantity from all suppliers per order cycle.

Y_{ij} Binary variable; equals one if discount interval j is selected for supplier i, and zero otherwise, where $i = 1,2, \ldots n$, $j = 1,2, \ldots a_i$.

J_{ij} Number of orders submitted to supplier i in interval j per a repeating order cycle, where $i = 1,2, \ldots n$, $j = 1,2, \ldots a_i$.

T_i Time interval to consume the ordered quantity Q_i, where $i = 1,2, \ldots n$.

T_c Repeating order cycle time.

The goal of the objective function is to minimize the replenishment and inventory cost per time unit, which consists of setup cost, holding cost, and purchasing cost. Note that in our model, multiple orders can be allocated to the selected suppliers within an order cycle. For instance, as shown in Figure 3.1, six orders are submitted in an order cycle of length T_c. Two orders are submitted to supplier 1 ($J_{1j} = 2$), three orders to supplier 2 ($J_{2j} = 3$), and one order to supplier 3 ($J_{3j} = 1$). Also note that the number of orders submitted to each supplier and the corresponding order quantities will be repeated in each cycle.

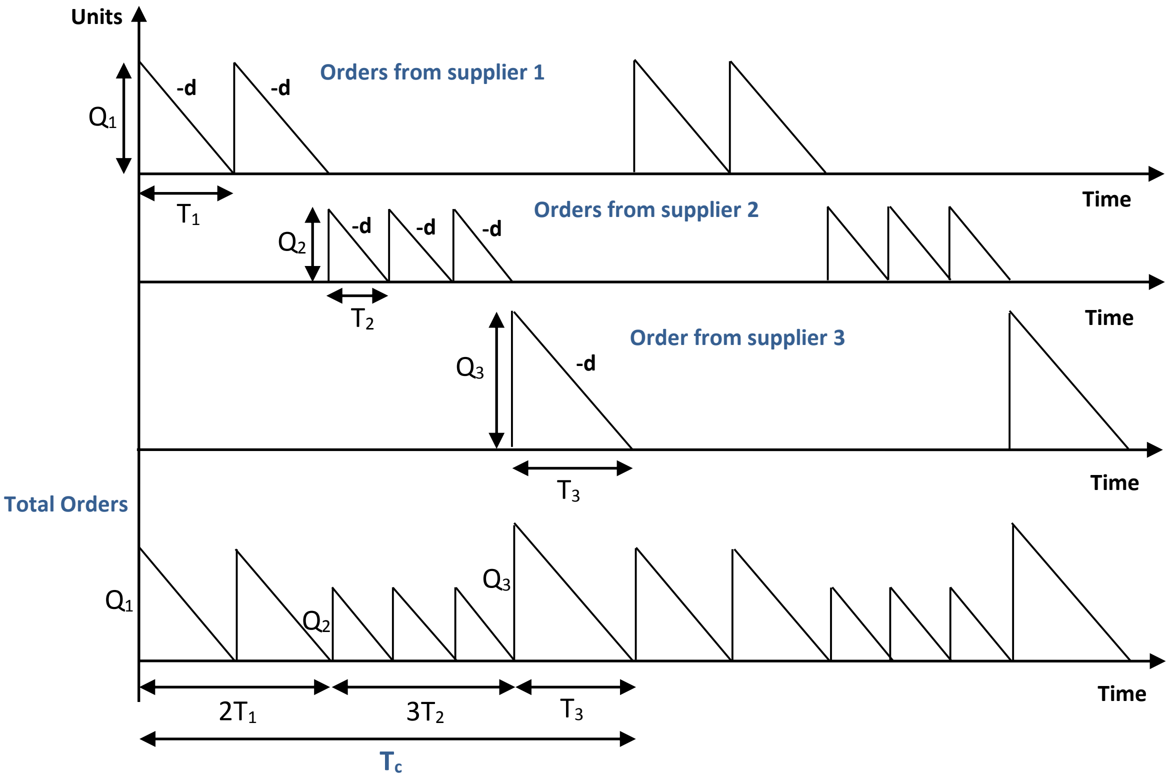

Figure 3.1. Orders submitted to each supplier within a repeating order cycle.

The time interval to consume the ordered quantity Q_i under a constant demand is equal to $T_i = Q_i/d$. Since it is allowed to place J_{ij} orders to supplier i in interval j, then the total time to consume all the units ordered from supplier i is equal to $T_i \sum_{j=1}^{a_i} J_{ij} = (Q_i/d\,) \sum_{j=1}^{a_i} J_{ij}$. Thus the total repeating order cycle time $T_c = \sum_{i=1}^{n} T_i \sum_{j=1}^{a_i} J_{ij} = \sum_{i=1}^{n} (Q_i/d) \sum_{j=1}^{a_i} J_{ij} = Q/d$, where $Q = \sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij}$.

Now, the three objective function components are explained as follows. The first component is the setup cost per time unit which is equal to the total setup cost per cycle $\sum_{i=1}^{n} k_i \sum_{j=1}^{a_i} J_{ij}$ divided by the cycle time T_c. Hence,

$$\text{Setup Cost per Time Unit} = \frac{\sum_{i=1}^{n} k_i \sum_{j=1}^{a_i} J_{ij}}{T_c} = \frac{d \sum_{i=1}^{n} k_i \sum_{j=1}^{a_i} J_{ij}}{Q}.$$

The second component is the holding cost per time unit. Note that, the unit holding cost depends on the unit price, since every supplier is offering different unit price based on the order quantity. Now, for instance the holding cost per time unit due to supplier i is the product of the average inventory per time unit for the purchased units that are received from supplier i in interval j , $(Q_i/2)\left(T_i J_{ij}\right)/T_c$ and the corresponding unit holding cost for that supplier rp_{ij}. Thus,

$$\text{Holding Cost per Time Unit} = \frac{\sum_{i=1}^{n}(Q_i/2)\ \left((Q_i/d)r\sum_{j=1}^{a_i} J_{ij}p_{ij}\right)}{T_c}$$

$$= \frac{(1/2)r\sum_{i=1}^{n} Q_i^2 \sum_{j=1}^{a_i} J_{ij}\, p_{ij}}{Q}.$$

Finally, the third component is the purchasing cost per time unit which is equal to the purchasing cost per cycle $\sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij}p_{ij}$ divided by the cycle time T_c. Hence,

$$\text{Purchasing Cost per Time Unit} = \frac{\sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij}p_{ij}}{T_c} = \frac{d\sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij}p_{ij}}{Q}.$$

Therefore, the MINLP model ($M1$) can be formulated as follows:

$$Min\ \ Z = \frac{1}{Q}\left[d\sum_{i=1}^{n} k_i \sum_{j=1}^{a_i} J_{ij} + (1/2)r\sum_{i=1}^{n} Q_i^2 \sum_{j=1}^{a_i} J_{ij}\, p_{ij} + d\sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij}p_{ij}\right],$$

subject to

$$Q = \sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij}, \tag{1}$$

$$dQ_i \sum_{j=1}^{a_i} J_{ij} \leq Qc_i\,, \qquad i = 1, \dots, n\,, \tag{2}$$

$$\sum_{i=1}^{n} Q_i(q_i - q_a) \sum_{j=1}^{a_i} J_{ij} \geq\ \ 0\,, \tag{3}$$

$$\sum_{j=1}^{a_i} Y_{ij} \leq 1\,, \qquad i = 1, \dots, n, \tag{4}$$

$$Q_i \leq \sum_{j=1}^{a_i} u_{ij} Y_{ij}, \qquad i = 1,..,n \text{ , and } j = 1, \dots, a_i, \qquad (5)$$

$$Q_i \geq \sum_{j=1}^{a_i} u_{i,j-1} Y_{ij}, \qquad i = 1,..,n \text{ , and } j = 1, \dots, a_i, \qquad (6)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{a_i} J_{ij} \leq m, \qquad (7)$$

$$J_{ij} \leq m Y_{ij}, \qquad i = 1,..,n \text{ , and } j = 1, \dots, a_i, \qquad (8)$$

$$J_{ij} \geq 0, \qquad integer, \; i = 1,..,n \text{ ,and } j = 1, \dots, a_i, \qquad (9)$$

$$Q_i \geq 0, \qquad i = 1,..,n, \qquad (10)$$

$$Y_{ij} \in (0,1), \qquad i = 1,..,n, \text{ and } j = 1, \dots, a_i, \qquad (11)$$

In this model, constraint (1) represents the total order quantity from all suppliers per order cycle. Constraint (2) represents the suppliers' capacity restrictions, where the total purchased quantity from a certain supplier over the order cycle time should be less than or equal to the supplier's capacity rate c_i. Constraint (3) represents the suppliers' quality restriction in which the average quality level offered by all suppliers should be greater than or equal to the minimum acceptable quality level q_a. The following three constraints, (4) to (6), are related to the quantity discount intervals; for instance, constraint (4) guarantees that at most one of the supplier's quantity discount intervals is selected, and constraints (5) and (6) make sure the purchased quantity is within the supplier's quantity discount interval. Note that, when $Q_i = u_{ij}$, both Y_{ij} and $Y_{i,j+1}$ can be set to 1; however, in this cost model $Y_{i,j+1}$ will be set to 1 because of the lower unit price. Moreover, constraint (7) represents the restriction on the total number of orders that can be placed to the suppliers within an order cycle. This constraint allows controlling the length of the cycle time. In addition, constraints (8) and (9) guarantee that the total number of orders placed to each supplier is integer and less than or equal to m. Finally, non-negativity and binary conditions are represented by constraints (10) and (11), respectively.

Note that the previous model considers independent order quantities, meaning that a different order quantity can be placed by each selected supplier. However, as shown in Munson and Rosenblatt (2001), coordinating the channel of supply chain can be achieved by ensuring that the placed order quantities at each stage in the supply chain is an integer multiple of the order quantities at the downstream stage. Hence, to facilitate this coordination mechanism, the second version of our Model $(M2)$ allows only equal-size of order quantities to the selected suppliers. Thus, the independent order quantities Q_i in Model $(M1)$ are replaced by an equal-size order quantity Q_c in Model $(M2)$.

3.2. Numerical Example

Suppose that a supplier selection problem consists of three suppliers and a manufacturer. The manufacturer's demand per time unit is 500 units per month, and he/she is allowed to place m orders to supplier(s) within a repeating order cycle. The manufacturer wants to select the best set of suppliers to purchase from, given that the manufacturer's minimum acceptable quality level is 0.95, while the quality levels for the three suppliers are 0.92, 0.95, and 0.98, respectively. Moreover, each supplier has a production capacity of 300, 350, and 250 units per month, respectively. Also, the suppliers are offering the manufacturer all-unit quantity discounts with the discount intervals and prices shown in Table 3.1. The manufacturer's inventory holding cost rate is 0.3 per month. Furthermore, the manufacturer's fixed ordering cost from each supplier is \$500, \$250, and \$450 per order, respectively. Note that the transportation cost is considered to be fixed and part of the ordering cost. The goal of the manufacturer is to determine the optimal order quantities that need to be placed to the selected suppliers and how often they need to be placed during an order cycle time in order to minimize the replenishment and inventory cost per time unit under supplier's capacity and quality constraints.

Table 3.1. Suppliers' all unit quantity discounts.

Suppliers	j	Lower bound (u_{j-1})	Upper bound (u_j)	Unit price ($)
Supplier 1	**1**	0	50	9
	2	50	100	8.9
	3	100	150	8.8
	4	150	200	8.7
	5	200	∞	8.6
Supplier 2	**1**	0	75	9.8
	2	75	150	9.6
	3	150	225	9.4
	4	225	∞	9.2
Supplier 3	**1**	0	100	10.5
	2	100	200	10.4
	3	200	∞	10.3

This problem was formulated and solved using LINGO 13.0 with global optimizer on a PC with INTEL(R) Core (TM) 2 Duo Processor at 2.10 GHz and 4.0 GB RAM. In order to determine the absolute minimum cost, m in constraint (7) is set to a very large value and it has been found that the absolute minimum cost is obtained at $m = 117$ ($5566.21/month). However, it corresponds to an impractical cycle time of 76.45 months. Observe that in Figure 3.2, as m increases, the order cycle time increases. Thus, decision makers should select a reasonable value for m that achieves a low average monthly cost and reaches a reasonably small order cycle. Consequently, constraint (7) is changed to equality to observe model ($M1$)'s behavior concerning the optimal solution for different values of m. Table 3.2 shows the detailed solutions for $m = 2, \ldots, 20$. Also, Figure 3.2 depicts the change in average monthly cost and order cycle time with respect to m. In this case, the first and second lowest average monthly cost occurs at $m = 17$($5567.44/month) and $m = 8, 16$ ($5567.16/month), respectively. Notice that, the average monthly cost is almost the same; hence the key decision factor is the cycle time. Accordingly, $m = 8$ is selected since the cycle time is reduced to 5.27 months which can justify the small increment in cost of $0.022%/month comparing to the absolute minimum cost value.

Model $(M1')$ can be seen as a combination of the model proposed by Ghodsypour and O'Brien (2001), where only one order can be placed to each selected supplier within an order cycle, and Model $(M1)$, where suppliers offer all-unit quantity discounts. As shown in Figure 3.2, the average monthly cost of Model $(M1)$ when $m = 3$ is \$5717.15/month ($J_{24} = 2, J_{33} = 1$,and $Q_{24} = 349.21$ units, $Q_{33} = 299.32$ units), which is lower than the \$5741.04/month ($J_{15} = J_{24} = J_{33} = 1$,and $Q_{15} = 358.39$ units, $Q_{24} = 413.42$ units, $Q_{33} = 358.39$ units) obtained by Model$(M1')$. Accordingly, it can be concluded that, by restricting the manufacturer to submit at most one order to each supplier within an order cycle may result in a suboptimal solution.

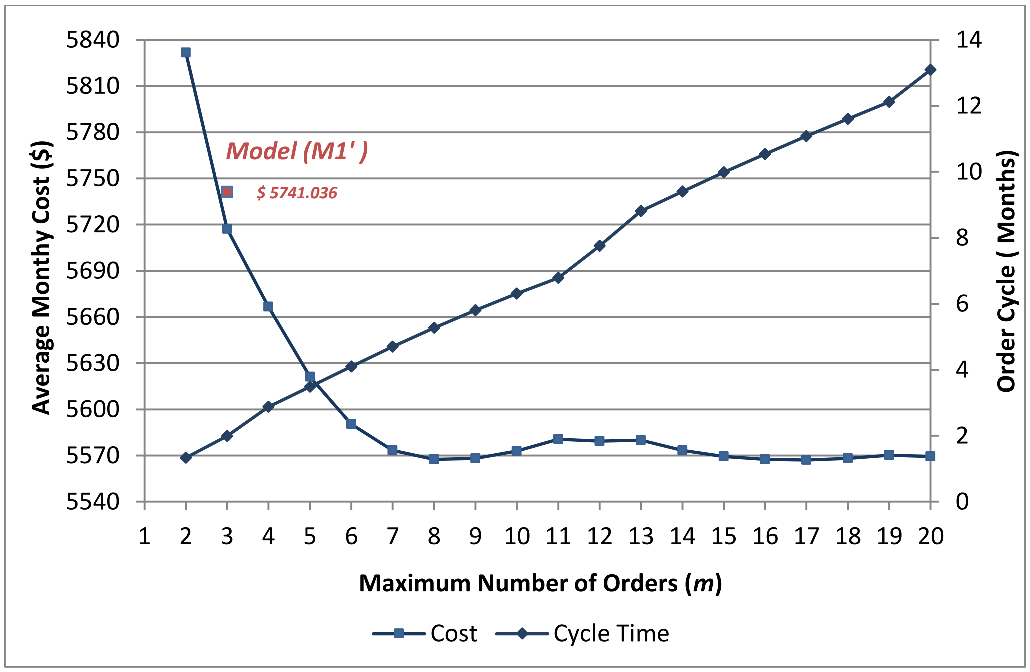

Figure 3.2. Behavior of Model (*M1*) over different values of *m*.

Table 3.2. Detailed solutions of Model (*M1*) over different values of m.

Number of Orders (m)	Supplier 1			Supplier 2			Supplier3			Cost (*\$/month*)	Cycle Time (*months*)
	(j)	(J_{1j})	(Q_1) (*units*)	(j)	(J_{2j})	(Q_2) (*units*)	(j)	(J_{3j})	(Q_3) (*units*)		
2	0	0	0	4	1	446.25	3	1	220.60	5831.65	1.33
3	0	0	0	4	2	349.21	3	1	299.32	5717.15	2.00
4	5	1	332.15	4	2	386.47	3	1	332.15	5666.65	2.87
5	5	1	316.11	4	3	369.98	3	1	316.11	5621.16	3.48
6	5	1	306.95	4	4	358.11	3	1	306.95	5590.43	4.09
7	5	1	352.64	4	5	329.13	3	1	352.64	5573.30	4.70
8	5	1	395.19	4	6	307.37	3	1	395.19	5567.44	5.27
9	5	1	435.18	4	7	290.12	3	1	435.18	5568.14	5.80
10	5	1	473.04	4	8	275.94	3	1	473.04	5572.94	6.31
11	5	1	509.06	4	9	263.96	3	1	509.06	5580.57	6.79
12	5	1	581.25	4	9	301.39	3	2	290.63	5579.43	7.75
13	5	2	330.24	4	9	342.47	3	2	330.24	5580.01	8.81
14	5	2	352.64	4	10	329.13	3	2	352.64	5573.30	9.40
15	5	2	374.26	4	11	317.56	3	2	374.26	5569.33	9.98
16	5	2	395.19	4	12	307.37	3	2	395.19	5567.44	10.54
17	5	2	415.48	4	13	298.29	3	2	415.48	5567.16	11.08
18	5	2	435.18	4	14	290.12	3	2	435.18	5568.14	11.60
19	5	2	454.36	4	15	282.71	3	2	454.36	5570.13	12.12
20	5	2	490.94	4	15	305.48	3	3	327.30	5569.38	13.09

Moreover, based on the problem structure, it is important to mention that, the average monthly cost for any value of m should be less than or equal to the resulting cost when m is set to any of its factors. In addition, if the number of orders placed to the selected suppliers has a greatest common factor k, such that $k > 1$, then an alternative solution can be generated by dividing the number of orders placed to each selected supplier by k; this solution will have the same average monthly cost, but the cycle time will be reduced by $100 \times (k-1)/k\%$. For example, the average monthly cost for $m = 18$ should be less than or equal to the average monthly cost for$m = 2, 3, 6, 9$; Note that, when $m = 1$, the problem is infeasible due to the supplier capacities. In addition, given that $k = 2$ for $m = 18$, the average monthly cost for $m = 18$ and $m = 9$ is the same. Also, the cycle time for $m = 9$ is 50% less than the corresponding cycle time at $m = 18$.

The second version of Model ($M2$) places an equal-size order quantity to the selected suppliers to allow coordination among the various stages of a supply chain. Table 3.3 shows Model ($M2$)'s detailed solutions over different values of m. Also, Figure 3.3 depicts the increase in the average monthly cost from Model ($M1$) to Model ($M2$).

Notice that, when $m = 3$ in Model ($M2$), the average monthly cost is equal to \$5736.66/month ($J_{24} = 2, J_{33} = 1$and $Q_c = 332.17$ units), which is less than the average monthly cost obtained from Model ($M2'$), which is \$5743.52/month ($J_{15} = J_{24} = J_{33} = 1$and $Q_c = 377.29$ units), where the manufacturer only places one order to the selected suppliers during an order cycle. Consequently, as shown in Figure 3.3, it can be concluded that, by allowing the manufacturer to place multiple orders to the selected suppliers per order cycle, the average monthly cost can be reduced.

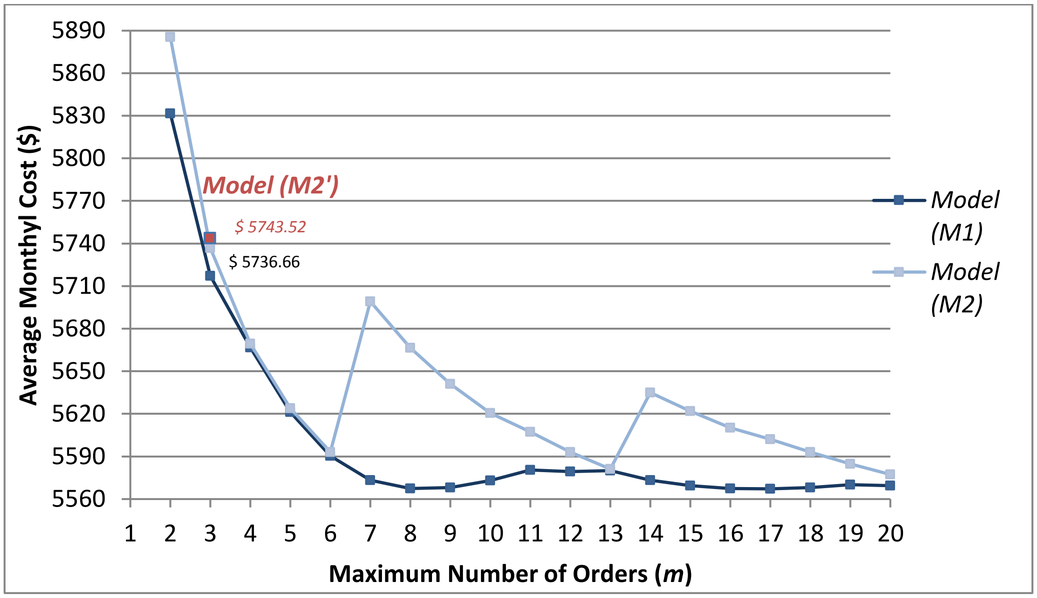

Figure 3.3. Cost comparison between Models (*M1*) and (*M2*).

Table 3.3. Model (*M2*)'s detailed solutions for different values of m.

Number of Orders (m)	Supplier 1 (j)	Supplier 1 (J_{1j})	Supplier 2 (j)	Supplier 2 (J_{2j})	Supplier 3 (j)	Supplier 3 (J_{3j})	(Q_c) (*units*)	Cost (*\$/month*)	Cycle time (*months*)
2	5	1	0	0	0	1	409.33	5885.44	1.64
3	0	0	4	2	3	1	332.17	5736.66	1.99
4	5	1	4	2	3	1	359.97	5669.52	2.88
5	5	1	4	3	3	1	349.09	5623.96	3.49
6	5	1	4	4	3	1	341.61	5593.05	4.10
7	5	1	4	4	3	2	348.16	5699.07	4.87
8	5	1	4	5	3	2	342.73	5666.50	5.48
9	5	1	4	6	3	2	338.43	5641.00	6.09
10	5	1	4	7	3	2	334.93	5620.49	6.70
11	5	2	4	7	3	2	345.03	5607.16	7.59
12	5	2	4	8	3	2	341.61	5593.05	8.20
13	5	2	4	9	3	2	338.68	5581.03	8.81
14	5	2	4	9	3	3	342.25	5635.02	9.58
15	5	2	4	10	3	3	339.70	5621.83	10.19
16	5	2	4	11	3	3	337.44	5610.24	10.80
17	5	3	4	11	3	3	343.83	5602.19	11.69
18	5	3	4	12	3	3	341.61	5593.05	12.30
19	5	3	4	13	3	3	339.61	5584.84	12.91
20	5	3	4	14	3	3	337.80	5577.42	13.51

In addition, by implementing this coordination mechanism, the average monthly cost also increases, compared to Model $(M1)$, due to the changes in the manufacturer's order allocations. For instance, when $m = 4$, both models have the same order allocations; however, the optimal order quantities submitted to each supplier in Model $(M1)$, $Q_{15} = 332.15$ units, $Q_{24} = 386.46$ units, and $Q_{33} = 332.15$ units, are changed to a single order quantity $Q_c = 359.97$ units in Model $(M2)$. This change in order quantities leads to a slight increase in the purchasing cost and hence an increase in the overall average monthly cost. Note that, since the order allocations have not been changed, the average monthly cost does not increase significantly and that explains the good behavior of Model $(M2)$ compared to Model $(M1)$ for $m = 4, 5, 6$ and 13, as shown in Figure 3.3. However, when the order allocations change in Model $(M2)$ compared to those in Model $(M1)$, the average monthly cost may increase significantly. For instance, let us consider $m = 7$ in both models. In this case, it can be noticed in Table 3.2 that in Model ($M2$) one additional order is placed to supplier 3 (i.e., in Model ($M1$), $J_{33} = 1$ and in Model ($M2$), $J_{33} = 2$). Hence, in the case of supplier 3 all the cost components increase because one more order is allocated to supplier 3 as a result of the reduction in supplier 3 order quantity from Model ($M1$) to Model ($M2$). Table 3.4 shows the change of each cost component for each supplier when $m = 7$. Even though all the cost components decrease in the case of suppliers 1 and 2, this reduction is not enough to reduce the average monthly cost because all the cost components for supplier 3 increase.

Table 3.4. Cost comparison between Models (*M1*) and (*M2*) for *m* = 7.

$\boldsymbol{m} = 7$	**Ordering Policy**		**Setup cost**		**Holding cost**		**Purchasing cost**	
	$M1$	$M2$	$M1$	$M2$	$M1$	$M2$	$M1$	$M2$
Supplier 1	$J_{15} = 1$ $Q_1 = 352.64$	$J_{15} = 1$ $Q_c = 348.16$	106.34	102.58	68.23	64.15	645	614.28
Supplier 2	$J_{24} = 5$ $Q_2 = 329.13$	$J_{24} = 4$ $Q_c = 348.16$	265.85	205.16	317.93	274.54	3220	2628.57
Supplier 3	$J_{33} = 1$ $Q_3 = 352.64$	$J_{33} = 2$ $Q_c = 348.16$	95.706	184.64	81.72	153.68	772.5	1471.42
Total			467.9	492.39	467.9	492.39	4637.5	4714.29

Now, sensitivity analysis for the inventory holding cost rate r is performed for its significant influence on the behavior of model ($M1$). Thus, for $m = 8$, different values for the inventory holding cost rate r are considered, keeping the values of the remaining parameters unchanged. Figure 3.4 shows the order quantities for the three suppliers and the average monthly cost for different values of r.

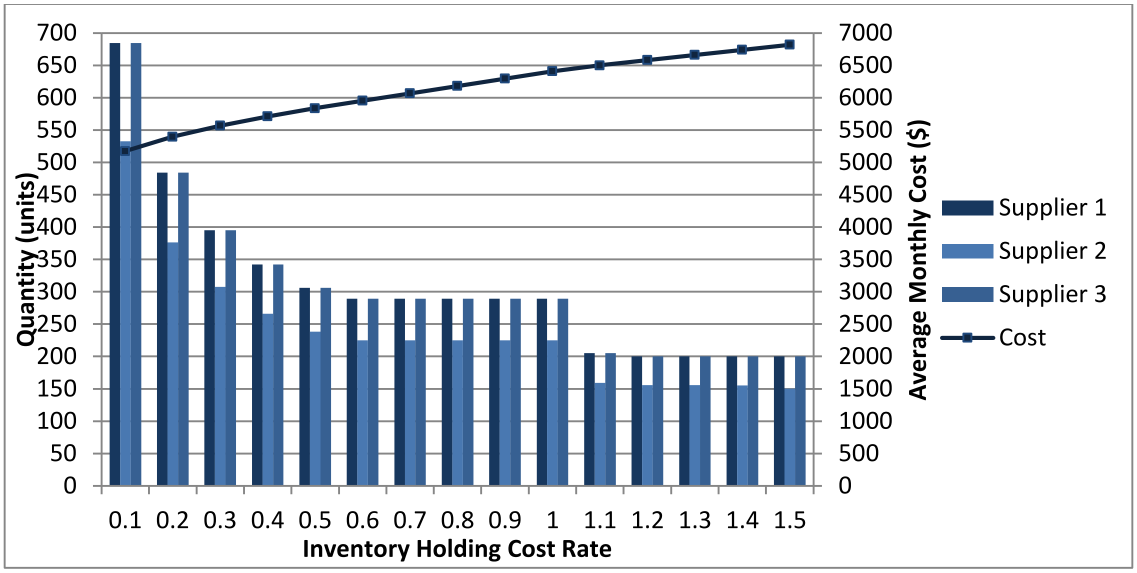

Figure 3.4. Model (*M1*)'s behavior with respect to the inventory holding cost rate.

It can be argued that, as the holding cost rate increases, the optimal order quantities will either keep decreasing or remain the same. For instance, as shown in Figure 3.4, the optimal order quantities keep decreasing until the inventory holding cost rate reached the value of 0.6. Then, for all $0.6 \leq r \leq 1.0$, the optimal order quantities remained unchanged. In some cases, it is more efficient to keep the optimal order quantities unchanged, even though that would result in increasing the holding cost, but it would avoid incurring some additional setup and purchasing costs. Thus, over some ranges of the holding cost rate, the optimal order quantities remain the same until the holding cost rate becomes really high, at which point it becomes more efficient to decrease the optimal order quantities instead of incurring in a very high holding cost. In this example, the optimal order quantity for supplier 2 at $r = 0.1$ is $Q_2 = 532.38$ units and it starts decreasing until it reaches $u_{23} = 225$ units at $r = 0.6$. Note that the average monthly cost would increase significantly if the optimal order quantity (Q_2) was decreased to a value lower than u_{23} , because that would result in ordering at a higher unit price in a lower quantity discount interval. Therefore the optimal order quantities remained unchanged for $0.6 \leq r \leq 1.0$.

3.3. Conclusions

An MINLP model has been developed for a supplier selection problem with n suppliers. The proposed model combines two efficient approaches to incur the least possible average replenishment and inventory cost associated to the supplier selection process: the first one allows submitting multiple orders to each selected supplier during a repeating order cycle, and the second one allows suppliers to offer all-unit quantity discounts to encourage the increase of the order quantities and reduce the ordering cost per time unit. Moreover, two versions of the proposed model have been considered based on the type of order quantities: the first one considers independent order quantities for the selected suppliers and the second one considers equal-size order quantities. The proposed model minimizes the replenishment and inventory cost per time unit under supplier's capacity and quality constraints. The model determines the appropriate suppliers to order from, the number of orders placed to the selected suppliers, the optimal order quantities, and the suppliers' prices based on the corresponding quantity discount intervals.

Chapter 4: Determining the Retailer's Replenishment Policy considering Multiple Capacitated Suppliers and Price-Sensitive Demand

4.1. Problem Description and Model Development

In this section, we develop a joint pricing and inventory replenishment model in a supply chain that consists of a single retailer and n potential suppliers, as shown in Figure 4.1. Suppliers can deliver a particular type of product to the retailer. Let c_i denote the capacity or production rate, k_i be the setup cost, and q_i denote the quality level, which represents the percentage of acceptable units, for supplier i. In addition, let q_a denote the retailer's minimum acceptable quality level on the average quality level obtained from all suppliers, and let the inventory holding cost rate be r.

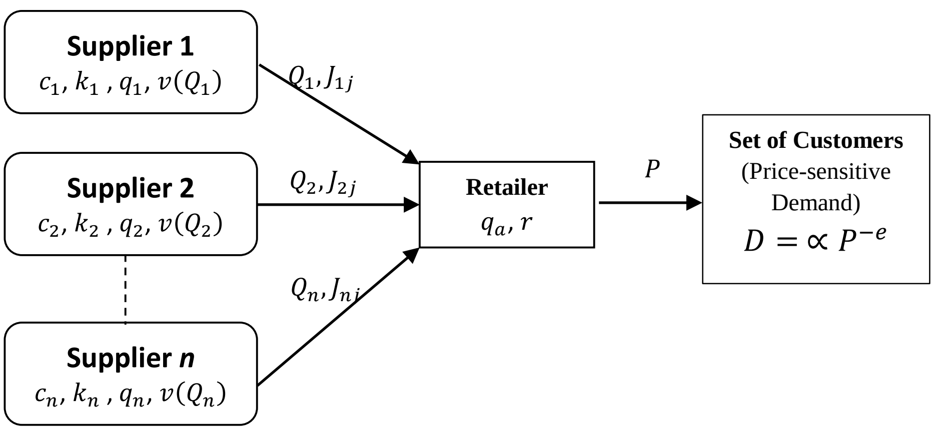

Figure 4.1. Supply chain with a single retailer and multiple suppliers.

Also, the retailer's unit purchasing price depends on the following all-unit quantity discount mechanism that is offered by the suppliers:

$$v(Q_i) = \left\{ \begin{array}{lll} 0 & if & Q_i = 0 \\ v_{i1} & if & 0 < Q_i < u_{i1} \\ v_{i2} & if & u_{i1} \leq Q_i < u_{i2} \\ \vdots & if & \vdots \\ v_{i,a_i-1} & if & u_{i,a_i-2} \leq Q_i < u_{i,a_i-1} \\ v_{i,a_i} & if & u_{i,a_i-1} \leq Q_i < \infty \end{array} \right\}, i = 1,2, \dots n,$$

where Q_i is the purchased lot size from supplier i, $u_{i0} = 0 < u_{i1} < \cdots < u_{i,a_i-1} < u_{i,a_i} = \infty$ provide the sequence of order quantities at which the unit price changes, and a_i is the number of quantity discount intervals offered by supplier i. For instance, the purchasing price for a lot size Q_i is $v_{ij}Q_i$, if $u_{i,j-1} \leq Q_i < u_{ij}$, where u_{ij} is supplier i's strict upper bound of discount interval j and v_{ij} is the unit price, $j = 1,2, \ldots a_i$, such that $v_{i1} > \cdots > v_{i,a_i-1} > v_{i,a_i} > 0$. Let Y_{ij} be a binary variable that equals one if discount interval j for supplier i is selected, and equals zero otherwise.

The retailer's demand rate is a decreasing function of the selling price P given by a constant price elasticity function, $D = \propto P^{-e}$, where $\propto$ and e are the scaling factor and the price elasticity index, respectively. We consider the contribution made by Mendoza and Ventura (2008) who recommend allowing multiple orders to be submitted to the selected suppliers during a repeating order cycle. Thus, let J_{ij} denote the number of orders submitted to supplier i in interval j per order cycle, and m denote the maximum number of orders that can be placed to the selected suppliers in a cycle. Therefore, the goal is to find the optimal number of orders per cycle and the corresponding order quantity for the selected suppliers, and the optimal selling price that maximize the retailer's profit per time unit subject to quality and capacity constraints.

In this section, we propose two mixed integer nonlinear programming formulations depending on the type of order quantities submitted to the selected supplies, which can be supplier-dependent order quantities and equal-size order quantities. In the first case, when the retailer receives an order quantity of size Q_i from supplier i, let T_i be the time to consume that order under price-elastic demand; thus, $T_i = Q_i/D = Q_i/(\propto P^{-e})$. Hence, the total time to consume all the units ordered from supplier i in an order cycle is equal to $T_i \sum_{j=1}^{a_i} J_{ij} = Q_i/(\propto P^{-e}) \sum_{j=1}^{a_i} J_{ij}$. Given that the repeating order cycle time T_c is the time where all the units from the selected suppliers are consumed, $T_c = \sum_{i=1}^{n} T_i \sum_{j=1}^{a_i} J_{ij} = \sum_{i=1}^{n} Q_i/(\propto P^{-e}) \sum_{j=1}^{a_i} J_{ij} = Q/(\propto P^{-e})$, where Q denotes the sum of the order quantities received from all the suppliers within an order cycle (i.e., $Q = \sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij}$). Note that the number of orders submitted to each supplier and the corresponding order quantities will be repeated in each cycle. Therefore, the mixed integer nonlinear programming Model ($M1$) is as follows:

$$Max\ \ Z = \propto P^{1-e} - \frac{1}{Q}\left[\propto P^{-e}\sum_{i=1}^{n} k_i \sum_{j=1}^{a_i} J_{ij} + (1/2)r\sum_{i=1}^{n} Q_i^2 \sum_{j=1}^{a_i} J_{ij}\, v_{ij} + \propto P^{-e}\sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij} v_{ij}\right],$$

subject to

$$Q = \sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij}\ , \tag{1}$$

$$\propto P^{-e} Q_i \sum_{j=1}^{a_i} J_{ij} \le Q c_i\ , \qquad i = 1, \dots, n, \tag{2}$$

$$\sum_{i=1}^{n} Q_i (q_i - q_a) \sum_{j=1}^{a_i} J_{ij} \ge 0\ , \tag{3}$$

$$Q_i \le \sum_{j=1}^{a_i} u_{ij} Y_{ij}\ , \qquad i = 1, \dots, n, and\ \ j = 1, \dots, a_i, \tag{4}$$

$$Q_i \ge \sum_{j=1}^{a_i} u_{i,j-1} Y_{ij}\ , \qquad i = 1, \dots, n, and\ \ j = 1, \dots, a_i, \tag{5}$$

$$\sum_{j=1}^{a_i} Y_{ij} \le 1\ , \qquad i = 1, \dots, n, \tag{6}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{a_i} J_{ij} \le m\ , \tag{7}$$

$$J_{ij} \le m Y_{ij}\ , \qquad i = 1, \dots, n, and\ \ j = 1, \dots, a_i, \tag{8}$$

$$J_{ij} \ge 0, \qquad integer\ , i = 1, \dots, n, and\ \ j = 1, \dots, a_i, \tag{9}$$

$$P \ge 0\ , \tag{10}$$

$$Q_i \ge 0\ , \qquad i = 1, \dots, n, \tag{11}$$

$$Y_{ij} \in (0,1)\ , \qquad i = 1, \dots, n\ , and\ \ j = 1, \dots, a_i. \tag{12}$$

In this formulation, the objective function maximizes the retailer's profit per time unit, which is equal to the sales revenue per time unit, $\propto P^{1-e}$, minus the cost per time unit; the latter includes the setup cost per time unit, holding cost per time unit, and purchasing cost per time unit. The setup cost per time unit is obtained by dividing the total setup cost per cycle $\sum_{i=1}^{n} k_i \sum_{j=1}^{a_i} J_{ij}$ by the repeating order cycle time T_c; this cost component becomes $\left(\propto P^{-e} \sum_{i=1}^{n} k_i \sum_{j=1}^{a_i} J_{ij}\right)/Q$. The holding cost per time unit corresponding to the units purchased from supplier i is obtained by multiplying the unit holding cost rv_{ij} by the average inventory level $(Q_i/2)\left(T_i \sum_{j=1}^{a_i} J_{ij} /T_c\right)$. Hence, the holding cost per time unit is equal to $\left((1/2)r \sum_{i=1}^{n} Q_i^2 \sum_{j=1}^{a_i} J_{ij}\, v_{ij}\right)/Q$. Lastly, the purchasing cost per time unit is obtained by dividing the total purchasing cost per cycle $\sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij} v_{ij}$ by the repeating order cycle time T_c; thus, the purchasing cost per time unit becomes $\left(\propto P^{-e} \sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij} v_{ij}\right)/Q$.

Model ($M1$) is subject to a number of constraints. Constraint (1) represents the sum of the order quantities submitted to all suppliers within an order cycle. Constraint set (2) insures that, on average, the number of units ordered from each supplier (i.e., the supplier's demand share) does not exceed the supplier's capacity. Constraint (3) guarantees that the average quality level computed from all suppliers, $\sum_{i=1}^{n} Q_i q_i \sum_{j=1}^{a_i} J_{ij} / \sum_{i=1}^{n} Q_i \sum_{j=1}^{a_i} J_{ij}$, is greater than or equal to the retailer's minimum acceptable quality level q_a. Constraint sets (4) and (5) make sure that the ordered quantity for a particular supplier is within the proper quantity discount interval, and constraint set (6) assures that at most one of the supplier's quantity discount intervals is selected. Constraint (7) restricts the maximum number of orders that can be placed to the selected suppliers within a repeating order cycle. Note that, this constraint helps in controlling the length of the cycle time as it will be shown in Section 4.3. Constraint sets (8) and (9) make sure that the number of orders allocated to a certain supplier is integer and less than or equal to m. Finally, constraint sets (10) and (11) impose non-negativity conditions, and constraint set (12) enforces binary conditions.

The second model, denoted as $(M2)$, allows only order quantities of equal size to the selected suppliers. The independent order quantities Q_i in Model $(M1)$ are replaced by an equal-size order quantity Q_c. In addition, constraint set (11) is replaced by $Q_c \geq 0$. Model $(M2)$ is useful because it can be extended to a multi-stage supply chain system, where inventory coordination between consecutive stages is essential to facilitate the inventory planning process and eliminate the possibility of having undesirable shortages. Coordination occurs when the equal-size order quantity employed at a given stage is an integer multiple of the equal-size order quantity employed at the downstream stage (Ravindran and Warsing, 2012).

4.2. Model Analysis

4.2.1. Uncapacitated Dominating Supplier

The general solution for Model $(M1)$ can be represented by $(P, J_1, \dots, J_n, Q_1, \dots, Q_n)$, where $J_i = \sum_{j=1}^{a_i} J_{ij}$, $i = 1, \dots, n$. For instance, Figure 4.2 illustrates a solution for Model $(M1)$; it shows that, for a given price P, the same supplier can be selected multiple times within an order cycle, i.e., $J_1 = 2$, $J_2 = 3$, and $J_3 = 2$. In Theorem 1 below, we provide sufficient conditions under which there exists an optimal solution where the retailer orders from a single supplier.

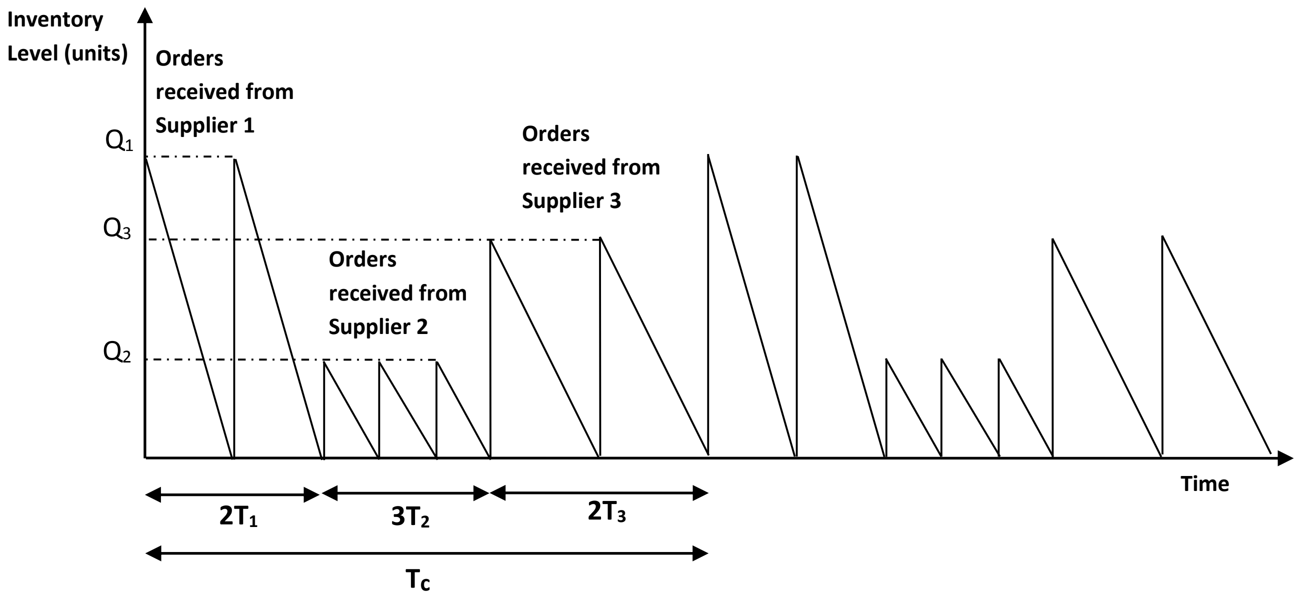

Figure 4.2. Illustration of a solution for Model (*M1*).

Theorem 1 Consider a special case of Model $(M1)$ where the retailer's demand rate is constant (i.e., $\propto P^{-e}$ is replaced by D). In addition, without loss of generality, assume that supplier 1 is such that $c_1 \geq D,\ q_1 \geq q_a$, and $TC'_1 \leq TC'_i,\ i = 2, \dots, n$, where TC'_i is the cost per time unit incurred by only selecting supplier i without considering capacity and quality constraints: $TC'_i = D\frac{k_i}{Q'_i} + (1/2)\, rQ'_i v_{ij^*} + Dv_{ij^*}, i = 1, .., n$ and $j^* \in \{1, \dots, a_i\}$ (the Appendix shows the procedure to calculate the optimal order quantity Q'_i and identify the supplier's best quantity discount interval j^*). Then, there exists an optimal solution where the retailer only orders from supplier 1, i.e., $J'_1 = 1, J'_2 = \cdots = J'_n = 0,\ Q'_1 > 0, Q'_2 = \cdots = Q'_n = 0$, where $J'_1 = \sum_{j=1}^{a_i} J'_{1j} = J'_{1j^*} = 1$.

Proof (By contradiction):

Since the demand rate is assumed to be constant, the selling price P is also constant. Consequently, the revenue component $\propto P^{1-e}$ in the objective function for Model $(M1)$ is the same for any ordering strategy. Thus, we only need to consider the cost component in the proof.

Now, assume that the solution stated in Theorem 1, say **Solution I**, is not optimal. Instead, there exists an optimal solution, say **Solution II**, $(J_1'', \dots, J_n'',\ Q_1'', \dots, Q_n'')$, where $J_i'' = \sum_{j=1}^{a_i} J_{ij}'' = J_{ij^*}''$, $i = 1, \dots, n$ where j^* is the quantity discount interval used when supplier i is selected. Also, in **Solution II**, at least one supplier different from supplier 1 is selected, i.e., $\sum_{i=2}^{n} J_i'' > 0$ and $\sum_{i=2}^{n} Q_i'' > 0$. Hence, the cost per time unit for **Solution II** is

$$TC'' = \frac{1}{\sum_{i=1}^{n} Q_i'' J_i''} \left[D \sum_{i=1}^{n} k_i J_i'' + \left(\frac{1}{2}\right) r \sum_{i=1}^{n} Q_i''^2 J_i'' \, v_{ij^*} + D \sum_{i=1}^{n} Q_i'' J_i'' v_{ij^*} \right]$$

$$= \sum_{i=1}^{n} \frac{Q_i'' J_i''}{\sum_{i=1}^{n} Q_i'' J_i''} \left[D \frac{k_i}{Q_i''} + (1/2) r Q_i'' v_{ij^*} + D v_{ij^*} \right].$$

Let $\beta_i = Q_i'' J_i'' / \sum_{i=1}^{n} Q_i'' J_i''$ represent the proportion of cycle time associated to supplier i, where $\sum_{i=1}^{n} \beta_i = 1$; hence,

$$TC'' = \sum_{i=1}^{n} \beta_i \left[D \frac{k_i}{Q_i''} + (1/2) r Q_i'' v_{ij^*} + D v_{ij^*} \right]. \tag{13}$$

Note that $\frac{k_i}{Q_i''} + (1/2) r Q_i'' v_{ij^*} + D v_{ij^*} \geq TC_i'$, because in **Solution II** the determination of the order quantity for a certain supplier is influenced by the order quantities placed to the other suppliers. In addition, the suppliers in **Solution II** may have some quality and capacity limitations. Thus,

$$TC'' \geq \sum_{i=1}^{n} \beta_i \, TC_i' \, . \tag{14}$$

By assumption, $TC_1' \leq TC_i'$, $i = 2, \dots, n$. Therefore, $\sum_{i=1}^{n} \beta_i \, TC_i' \geq \sum_{i=1}^{n} \beta_i \, TC_1'$, and accordingly,

$$TC'' \geq \sum_{i=1}^{n} \beta_i TC_1' = TC_1' \sum_{i=1}^{n} \beta_i = TC_1' . \quad (15)$$

This shows that **Solution I**, when the retailer only orders from supplier 1, is at least as good as **Solution II**. Therefore, this is a contradiction, because it implies that **Solution I** must also be optimal. ■

Corollary 1.1 In Theorem 1, let the cost per time unit incurred from selecting supplier 1 only be strictly less than the cost per time incurred from selecting any other supplier (i.e., $TC_1' < TC_i', \ i = 2, \dots, n$), then ordering from supplier 1 alone is the retailer's unique optimal solution.

Proof: This proof is a variation of the proof for Theorem 1. Since, by assumption, $TC_1' < TC_i', \ i = 2, \dots, n$, then $D\frac{k_i}{Q_i''} + (1/2)rQ_i''v_{ij^*} + Dv_{ij^*}$ becomes strictly greater than TC_1'. Hence, (15) is rewritten as follows:

$$TC'' > \sum_{i=1}^{n} \beta_i TC_1' = TC_1' \sum_{i=1}^{n} \beta_i = TC_1' .$$

Therefore, **Solution II** is not optimal and the only optimal solution is to only order from supplier 1. ■

Note that Corollary 1.1 proves the following property that was identified by Rosenblatt et al. (1998) for their model: "if no capacity constraints exist, then only one supplier with the minimum cost per time unit is selected". However, if quality constraints are considered, then this single sourcing strategy might be infeasible.

4.2.2. Capacity Analysis for Dominating Supplier

In this subsection, we study the retailer's optimal sourcing strategy as the capacity of the dominating supplier changes. As a matter of fact, supplier's capacity (i.e., production capability) may vary over time; it can majorly be affected by several factors, such as a natural disaster, business stability, and economic crisis. Any reduction in suppliers' capacity may ultimately disrupt the retailer's supply chain, and hence, may require the retailer to reevaluate sourcing decisions based on new supplier new capacities.

For instance, if the dominating supplier (i.e., retailer's sole source) experiences a capacity shortage, then the retailer would generally tend to either keep sourcing from the dominating supplier, switch to other supplier(s), or consider a multiple sourcing strategy with fully utilizing the available capacity of the dominating supplier. In this subsection, however, we show that for some capacity values, the retailer's optimal sourcing strategy is to consider a multiple sourcing strategy but without fully utilizing the dominating supplier's capacity.

In order to address all the above possible retailer's sourcing strategies, let us consider a special case of Model $(M1)$, detonated as $(M1^{*})$, where there are only two potential suppliers (i.e., $n = 2$) such that $q_i \geq q_a$, $i = 1,2$. Assume also that there are no quantity discounts (i.e., J_{ij} and v_{ij} are replaced by J_i, and v_i, respectively). Moreover, without loss of generality, assume that supplier 1 is the dominating supplier, i.e., $TP_1 > TP_2$, where TP_i is the profit per time unit obtained by only selecting supplier i without considering capacity and quality constraints: $TP_i = \propto P^{1-e} - \left[\propto P^{-e} \frac{k_i}{Q_i} + (1/2) r Q_i v_i + \propto P^{-e} v_i\right]$, $i = 1,2$. Let also D_i represent the optimal demand rate that is obtained by only selecting supplier i without considering capacity and quality constraints, where $i = 1, 2$. In addition, assume that supplier 1's capacity c_1 is strictly less than D_1, and supplier 2 has no capacity limitations. Consequently, Model $(M1^*)$ can be written as follows:

$$Max\ \ TP = \propto P^{1-e} - \frac{1}{\sum_{i=1}^{2} Q_i J_i}\left[\propto P^{-e}\sum_{i=1}^{2} k_i J_i + (1/2)r\sum_{i=1}^{2} Q_i^2 J_i v_i + \propto P^{-e}\sum_{i=1}^{2} Q_i J_i v_i\right],$$

subject to

$$\frac{\propto P^{-e} Q_1 J_1}{\sum_{i=1}^{2} Q_i J_i} \le c_1\ , \tag{16}$$

$$\sum_{i=1}^{2} J_i \le m\ , \tag{17}$$

$$J_i \ge 0\ , \qquad integer, and\ i = 1,2\ , \tag{18}$$

$$P \ge 0\ , \tag{19}$$

$$Q_i \ge 0\ , \qquad i = 1,2\ . \tag{20}$$

Now, let the integer variables J_i, and m be known in advance and assumed to be constant. Under these assumptions, we can generate the corresponding KKT conditions for Model $(M1^*)$ to address the retailer's possible optimal sourcing strategies, and analyze supplier 1's capacity constraint to find out whether it is fully utilized (i.e., active capacity constraint) or underutilized (i.e., inactive capacity constraint) for each sourcing strategy. Therefore, let μ_1 and γ be the nonnegative Lagrangian multiplier associated with constraints (16) and (19), respectively. Let also λ_1 and λ_2 be the nonnegative Lagrangian multipliers for the two variables in constraint set (20). Then the Lagrangian function is written as follows:

$$\mathcal{L}(\mu_1, \gamma, \lambda_1, \lambda_2, Q_1, Q_2, P) =$$

$$\propto P^{1-e} - \frac{1}{\sum_{i=1}^{2} Q_i J_i}\left[(\propto P^{-e})\sum_{i=1}^{2} k_i J_i + (1/2) r \sum_{i=1}^{2} Q_i^2 J_i v_i + (\propto P^{-e})\sum_{i=1}^{2} Q_i J_i v_i\right]$$
$$+ \mu_1\left(c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e}) Q_1 J_1\right) + \gamma(P) + \lambda_1(Q_1) + \lambda_2 (Q_2)\,.$$

Note that, the objective function of Model ($M1^*$) has a singularity at $P = 0$ (i.e., the objective function is not defined at $P = 0$). In addition, it can be also noticed that there is a singularity when Q_1 and Q_2 are both equal to zero. Therefore, for any sourcing strategy, $P > 0$ which implies that $\gamma = 0$ (i.e., $\gamma > 0$ never happens) also at least one of $Q_i > 0$, where $i = 1, 2$ and that implies that at least one of $\lambda_i = 0$, where $i = 1, 2$ (i.e., both $\lambda_1 > 0$ and $\lambda_2 > 0$ never happen).

Hence, the necessary KKT conditions are:

$$\frac{(\propto P^{-e}) Q_1 J_1}{\sum_{i=1}^{2} Q_i J_i} \leq c_1\,, \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial P} = \frac{\propto}{P^e} + \frac{\propto e}{P^{e+1}}\left(-P + \frac{\sum_{i=1}^{2} k_i J_i}{\sum_{i=1}^{2} Q_i J_i} + \frac{\sum_{i=1}^{2} Q_i J_i v_i}{\sum_{i=1}^{2} Q_i J_i}\right) + \mu_1 Q_1 J_1 \frac{\propto e}{P^{e+1}} = 0\,, \tag{22}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial Q_1} &= \lambda_1 + \mu_1\left(J_1 c_1 - J_1 \frac{\propto}{P^e}\right) + \frac{\propto J_1}{P^e \sum_{i=1}^{2} Q_i J_i}\left(\frac{\sum_{i=1}^{2} Q_i J_i v_i}{\sum_{i=1}^{2} Q_i J_i} - v_1 + \frac{\sum_{i=1}^{2} J_i k_i}{\sum_{i=1}^{2} Q_i J_i}\right)\\ &\quad + \frac{J_1 r \sum_{i=1}^{2} Q_i^2 J_i v_i}{2(\sum_{i=1}^{2} Q_i J_i)^2} - \frac{J_1 Q_1 r v_1}{\sum_{i=1}^{2} Q_i J_i} = 0\,,\end{aligned} \tag{23}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial Q_2} &= \lambda_2 + \mu_1 J_2 c_1 + + \frac{\propto J_2}{P^e \sum_{i=1}^{2} Q_i J_i}\left(\frac{\sum_{i=1}^{2} Q_i J_i v_i}{\sum_{i=1}^{2} Q_i J_i} - v_2 + \frac{\sum_{i=1}^{2} J_i k_i}{\sum_{i=1}^{2} Q_i J_i}\right)\\ &\quad + \frac{J_2 r \sum_{i=1}^{2} Q_i^2 J_i v_i}{2(\sum_{i=1}^{2} Q_i J_i)^2} - \frac{J_2 Q_2 r v_2}{\sum_{i=1}^{2} Q_i J_i} = 0\,,\end{aligned} \tag{24}$$

$$\mu_1\left(c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e}) Q_1 J_1\right) = 0\,, \tag{25}$$

$$\lambda_1 Q_1 = 0\,, \tag{26}$$

$$\lambda_2 Q_2 = 0\,, \tag{27}$$

$$\mu_1, \lambda_1, \lambda_2 \geq 0\,,\ and\ \ P, Q_1, Q_2 \geq 0\,. \tag{28}$$

The capacity analysis for supplier 1 can be performed by considering the following cases:

Case 1: $\lambda_1 = 0$, $\lambda_2 > 0$, and $\mu_1 > 0$. Consequently, $Q_1 \geq 0$, $Q_2 = 0$, and from constraint (25), $c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e}) Q_1 J_1 = 0$.

Case 1.1: If $Q_1 > 0$, the retailer selects only supplier 1 and utilizes its full capacity. Based on constraint (25), $c_1 = \propto P^{-e}$.

Case 1.2: When $Q_1 = 0$, both Q_1 and Q_2 end up being equal to zero. Hence, this case is excluded because at least one Q_i must be positive.

Case 2: $\lambda_1 = 0$, $\lambda_2 > 0$, and $\mu_1 = 0$. Consequently, $Q_1 \geq 0$, $Q_2 = 0$ and, from constraint (25), $c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e}) Q_1 J_1 \geq 0$.

Case 2.1: If $Q_1 > 0$ and $c_1 \sum_{i=1}^{2} Q_i\, J_i - (\propto P^{-e})\, Q_1\, J_1 = 0$, the retailer selects only supplier 1 and fully utilizes its capacity, i.e., constraint (25) implies that $c_1 = \propto P^{-e}$.

Case 2.2: If $Q_1 > 0$ and $c_1 \sum_{i=1}^{2} Q_i\, J_i - (\propto P^{-e})\, Q_1\, J_1 > 0$, the retailer selects only supplier 1 without fully utilizing supplier 1's capacity, i.e., using constraint (25), $c_1 > \propto P^{-e}$. This case is valid when supplier 1 has a high value for the capacity rate c_1. However, Model $(M1^*)$ assumes that $c_1 < D_1$ (recall that D_1 is the maximum demand rate that is obtained when only supplier 1 is selected without considering capacity and quality constraints). Thus, this case results in a suboptimal solution because additional supplier 1's capacity can still be utilized to maximize the retailer's profit per time unit.

Case 2.3: If $Q_1 = 0$ and $c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e})\, Q_1 J_1 \geq 0$, then both Q_1 and Q_2 are equal to zero and this case can be ignored.

Case 3: $\lambda_1 > 0$, $\lambda_2 = 0$, and $\mu_1 \geq 0$. Note that we can ignore again the cases at which $Q_2 = 0$ since $Q_1 = 0$ (i.e., $\lambda_1 > 0$). Hence, this case implies that the retailer selects only supplier 2, i.e., $Q_1 = 0$ and $Q_2 > 0$. In addition, constraint (25) is not considered because supplier 1 is not selected.

Case 4: $\lambda_1 = 0$, $\lambda_2 = 0$, and $\mu_1 > 0$. Consequently, $Q_1 \geq 0$, $Q_2 \geq 0$, and from constraint (25), $c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e})\, Q_1 J_1 = 0$.

Case 4.1: If $Q_1 > 0$ and $Q_2 > 0$, the retailer selects both suppliers, where supplier 1's capacity is fully utilized, i.e., using constraint (25), $c_1 = \frac{\propto P^{-e} Q_1 J_1}{\sum_{i=1}^{2} Q_i J_i}$.

Case 4.2: If $Q_1 > 0$ and $Q_2 = 0$, the retailer selects only supplier 1 and fully utilizes its capacity. Considering constraint (25), $c_1 = \propto P^{-e}$.

Case 4.3: If $Q_1 = 0$ and $Q_2 > 0$, the retailer selects only supplier 2. Note that, constraint (25) is not considered because supplier 1 is not selected.

Case 4.4: We exclude the case $Q_1 = 0$ and $Q_2 = 0$ because at least one Q_i must be positive.

Case 5: $\lambda_1 = 0$, $\lambda_2 = 0$, and $\mu_1 = 0$. Consequently, $Q_1 \geq 0$, $Q_2 \geq 0$, and from constraint (25), $c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e})\, Q_1 J_1 \geq 0$.

Case 5.1: If $Q_1 > 0$, $Q_2 > 0$, and $c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e})\, Q_1 J_1 \geq 0$, the retailer selects both suppliers.

Case 5.1.1: When $c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e})\, Q_1 J_1 = 0$, supplier 1's capacity is fully utilized i.e., $c_1 = \frac{\propto P^{-e} Q_1 J_1}{\sum_{i=1}^{2} Q_i J_i}$.

Case 5.1.2: When $c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e})\, Q_1 J_1 > 0$, supplier 1's capacity is not fully utilized, i.e., $c_1 > \frac{\propto P^{-e} Q_1 J_1}{\sum_{i=1}^{2} Q_i J_i}$. Therefore, over a range of c_1 values, there exists an optimal solution to the supplier selection problem in which the dominating supplier's capacity is not fully used.

Since in this case supplier 1's capacity is not fully utilized, let $d_1 = \frac{\propto P^{-e} Q_1 J_1}{\sum_{i=1}^{2} Q_i J_i}$ be the component of the demand rate assigned to supplier 1, where $c_1 > d_1$. Note that, if c_1 decreases and becomes less than or equal to d_1, then the optimal sourcing strategy might change or supplier 1's capacity might become fully utilized.

Case 5.2: If $Q_1 > 0$, $Q_2 = 0$, and $c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e})\, Q_1 J_1 \geq 0$, the retailer selects only supplier 1.

Case 5.2.1: When $c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e})\, Q_1 J_1 = 0$, supplier 1's capacity is fully utilized, i.e., $c_1 = \propto P^{-e}$.

Case 5.2.2: When $c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e})\, Q_1 J_1 > 0$, supplier 1's capacity is not fully utilized, i.e., using constraint (25), $c_1 > \propto P^{-e}$. This case results in a suboptimal solution, as discussed in Case 2.2.

Case 5.3: If $Q_1 = 0$, $Q_2 > 0$, and $c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e})\, Q_1 J_1 \geq 0$, the retailer selects only supplier 2. Note that constraint (25) is not analyzed because supplier 1 is not selected.

Case 5.4: If $Q_1 = 0$, $Q_2 = 0$, and $c_1 \sum_{i=1}^{2} Q_i J_i - (\propto P^{-e})\, Q_1 J_1 \geq 0$, then this case is excluded because at least one Q_i should be positive.

The model proposed in this subsection can be considered as an extension to the model developed by Rosenblatt et al. (1998) who studied the problem of multiple capacitated suppliers under a constant demand rate. Since they do not use the J_i variables and constraint (17), their optimal solutions may be difficult to implement since they are not guaranteed to be stationary, i.e., some solutions may not produce finite repeating cycle times. Nevertheless, they introduced the following optimality property for their model: "there is at most one supplier whose capacity is not fully utilized". This property, however, does not hold true in our case. Note that Case 5.1.2 shows that two suppliers can be selected without fully utilizing their corresponding capacities. For instance, let us assume that the dominating supplier is only able to satisfy the majority of demand due to some capacity limitations, and hence the retailer will need to satisfy the remaining small portion of the demand by placing a very small order quantity to the second supplier. Now, since both suppliers will be used, the retailer will incur an increase in the ordering cost per time unit because a very small order quantity will be placed to the other supplier. Then, in order to lighten this increase in the ordering cost, the order quantities submitted to both suppliers is altered to ensure that both of them are relatively large. As a result, both suppliers will be used without fully utilizing their capacity.

Rosenblatt et al. (1998) also provided an optimal greedy algorithm that ranks the suppliers based on their effective unit cost and orders the maximum allowed quantity from each supplier starting with the lowest cost supplier until satisfying all the demand. However, in our model, we show that in Case 3, Case 4.3, and Case 5.3, the dominating supplier is not selected at all, and instead the retailer switches to the second supplier. These cases occur when the dominating supplier's capacity is reduced significantly, and hence the retailer can only place a very small order quantity to the dominating supplier, which makes the use the dominating supplier inefficient due to the increase in the ordering cost per time unit.

The numerical example shown in Section 4.3 illustrates the retailer's optimal sourcing strategy and the related cases of the KKT conditions as the dominating supplier's capacity decreases.

4.3. Numerical Example

In this section we consider a numerical example in which the retailer's monthly demand rate is a decreasing function of the selling price: $D = \propto P^{-e}$, where $\propto = 3{,}375{,}000$ and $e = 3$. The retailer's inventory holding cost rate is 0.3 per month, and the retailer's minimum acceptable quality level is 0.95. Table 4.1 shows the suppliers' parameters. The retailer wants to determine the optimal number of orders per cycle and the corresponding order quantity for the selected suppliers, and the optimal selling price that maximize the profit per time unit.

Table 4.1. Suppliers' parameters.

Suppliers	j	**Lower Bound** (u_{j-1})	**Upper Bound** (u_j)	**Unit Price** ($)	**Quality Level** (q_i)	**Capacity** (Units/month)	**Ordering Cost** ($/order)
Supplier 1	**1**	0	50	9	0.92	300	500
	2	50	100	8.9			
	3	100	150	8.8			
	4	150	200	8.7			
	5	200	∞	8.6			
Supplier 2	**1**	0	75	9.8	0.95	350	250
	2	75	150	9.6			
	3	150	225	9.4			
	4	225	∞	9.2			
Supplier 3	**1**	0	100	10.5	0.98	250	450
	2	100	200	10.4			
	3	200	∞	10.3			

This problem is solved using the global solver in LINGO 14.0 on a PC with INTEL(R) Core (TM) 2 Duo Processor at 2.10 GHz and 4.0 GB RAM. We first set m in constraint (7) to a very large value to find out the optimal value of m that obtains the absolute maximum profit per time unit. In this example, the optimal value of m is 29 ($4180.98/month). However, this value of m results in a very large cycle time of 15.98 months. Therefore, a reasonable small value of m has to be selected to reduce the cycle time. For this purpose, constraint (7) is changed to equality to evaluate the monthly profit over different values of m, and hence, select a reasonable small value of m with a profit close to the absolute maximum. Table 4.2 shows Model ($M1$)'s detailed solutions for $m = 1, \ldots, 20$, and Figure 4.3 illustrates the total monthly profit and cycle time versus the total number of orders allowed per order cycle m. In this example, $m = 4$ is selected because it corresponds to a short cycle time of just 2.17 months and results in a small decrease in profit of only 0.06% compared to the absolute maximum profit per time unit obtained at $m = 29$.

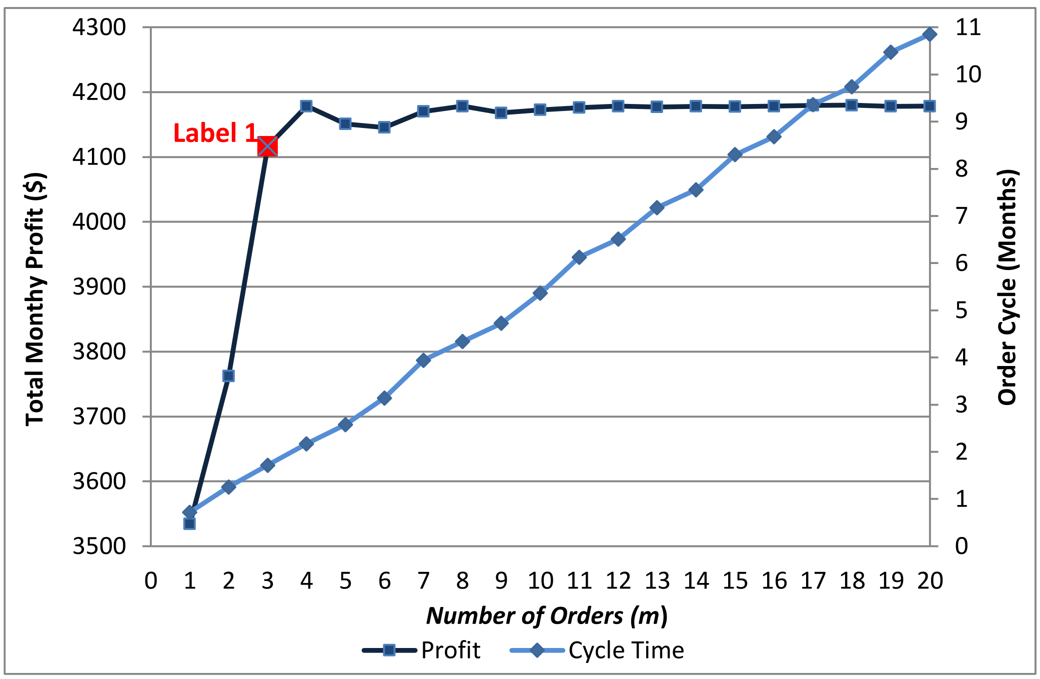

Figure 4.3. Total monthly profit and cycle time for m = 1,…, 20.

Table 4.2. Model ($M1$)'s detailed solutions for $m = 1, \ldots, 20$.

Number of Orders (m)	Supplier 1 (j)	(J_{1j})	(Q_1) (units)	Supplier 2 (j)	(J_{2j})	(Q_2) (units)	Supplier3 (j)	(J_{3j})	(Q_3) (units)	Price (\$)	Profit (\$/month)	Cycle Time (months)
1	0	0	0	4	1	251.81	0	0	0	21.29	3534.68	0.72
2	0	0	0	4	1	440.12	3	1	314.37	17.78	3762.34	1.26
3	5	1	429.10	4	1	600.73	3	1	429.10	15.84	4116.46	1.72
4	5	1	542.52	4	2	379.77	3	1	542.53	15.84	4178.42	2.17
5	5	1	595.82	4	3	300.74	3	1	595.82	16.08	4151.03	2.58
6	5	1	785.20	4	3	366.43	3	2	392.60	15.84	4145.69	3.14
7	5	2	492.77	4	3	459.92	3	2	492.77	15.84	4170.31	3.94
8	5	2	542.53	4	4	379.77	3	2	542.53	15.84	4178.42	4.34
9	5	2	576.81	4	5	330.68	3	2	576.81	15.91	4168.04	4.72
10	5	2	670.63	4	5	375.55	3	3	447.09	15.84	4172.74	5.37
11	5	3	510.48	4	5	428.80	3	3	510.48	15.84	4176.14	6.13
12	5	3	542.52	4	6	379.77	3	3	542.53	15.84	4178.42	6.51
13	5	3	598.15	4	6	418.71	3	4	448.61	15.84	4177.31	7.18
14	5	3	629.74	4	7	377.85	3	4	472.31	15.84	4178.24	7.56
15	5	4	518.87	4	7	415.10	3	4	518.87	15.84	4177.72	8.30
16	5	4	542.53	4	8	379.77	3	4	542.53	15.84	4178.42	8.68
17	5	4	585.19	4	8	409.63	3	5	468.15	15.84	4179.71	9.36
18	5	4	608.68	4	9	378.74	3	5	486.95	15.84	4179.91	9.74
19	5	5	523.78	4	9	407.38	3	5	523.78	15.84	4178.31	10.48
20	5	5	542.52	4	10	379.77	3	5	542.53	15.84	4178.42	10.85

In Figure 4.3, Label 1 represents the solution of Model ($M1'$) in which the retailer is restricted to place at most one order to each supplier per order cycle. As shown in Figure 4.3, the proposed Model ($M1$) and Model ($M1'$) have the same monthly profit when $m = 3$. Accordingly, it can be concluded that restricting the retailer to place at most one order to each supplier results in a suboptimal solution.

To help coordinate inventory between consecutive stages of a supply chain, the placed order quantities at each stage in the supply chain need to be an integer multiple of the placed order quantities at the downstream stage. Thus, as stated in Section 4.1, Model ($M2$) facilitates this coordination mechanism by placing equal-size order quantities to the selected suppliers. Table 4.3 shows Model ($M2$)'s detailed solutions for $m = 1, \ldots, 20$. Also, Figure 4.4 compares Models

$(M1)$ and $(M2)$ with respect to the total monthly profit. Note that, the decrease in the total monthly profit from Model $(M1)$ to Model $(M2)$ is due to the changes in the retailer's order allocations.

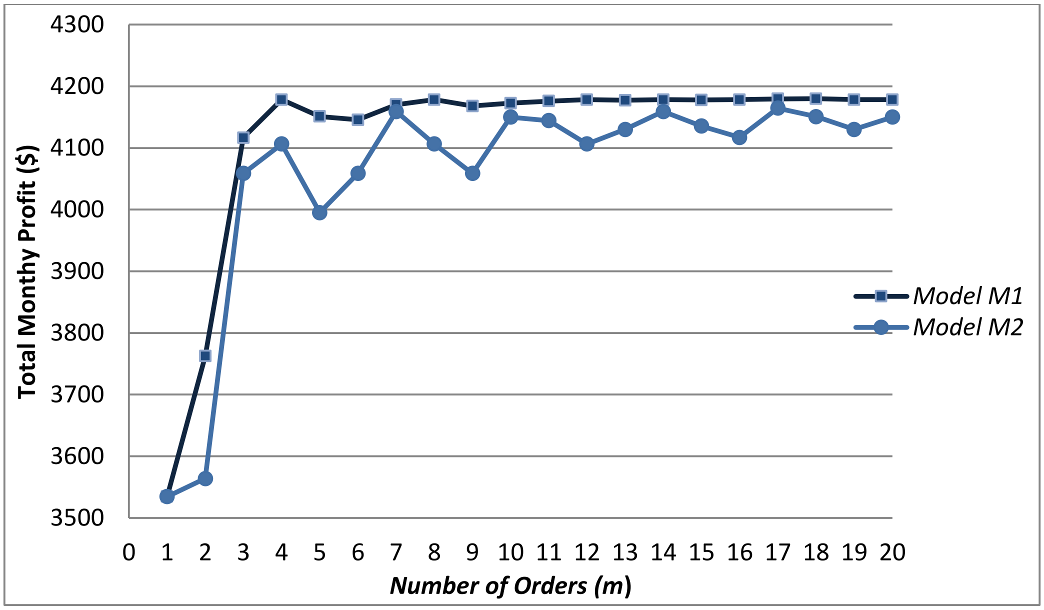

Figure 4.4. Model (*M1*) vs. Model (*M2*).

Table 4.3. Model (*M2*)'s detailed solutions for $m = 1, \ldots, 20$.

Number of Orders (m)	**Supplier 1** (j)	(J_{1j})	**Supplier 2** (j)	(J_{2j})	**Supplier 3** (j)	(J_{3j})	(Q_c) (*units*)	**Price** (\$)	**Profit** (*\$/month*)	**Cycle Time** (*months*)
1	0	0	4	1	0	0	251.81	21.29	3534.68	0.72
2	5	1	0	0	3	1	409.33	18.90	3563.97	1.64
3	5	1	4	1	3	1	462.09	16.51	4058.77	1.85
4	5	1	4	2	3	1	425.92	16.89	4106.58	2.43
5	5	1	4	3	3	1	377.06	17.95	3995.15	3.23
6	5	2	4	2	3	2	462.09	16.51	4058.77	3.70
7	5	2	4	3	3	2	469.69	16.05	4159.05	4.03
8	5	2	4	4	3	2	425.93	16.89	4106.58	4.87
9	5	3	4	3	3	3	462.09	16.51	4058.77	5.55
10	5	3	4	4	3	3	478.29	15.94	4149.97	5.74
11	5	3	4	5	3	3	452.70	16.37	4144.44	6.47
12	5	3	4	6	3	3	425.92	16.89	4106.58	7.30
13	5	4	4	5	3	4	474.29	16.08	4129.98	7.58
14	5	4	4	6	3	4	469.69	16.05	4159.05	8.05
15	5	4	4	7	3	4	445.21	16.51	4135.63	8.91
16	5	5	4	6	3	5	471.88	16.16	4117.11	9.44
17	5	5	4	7	3	5	481.47	15.84	4164.69	9.63
18	5	5	4	8	3	5	459.14	16.24	4150.86	12.24
19	5	5	4	9	3	5	441.00	16.59	4130.06	11.34
20	5	6	4	8	3	6	478.29	15.94	4149.97	11.48

Now, to study the impact of the dominating supplier's capacity on the sourcing strategy, let us consider only suppliers 1 and 2 from the previous numerical example, and assume that both suppliers have no quality restrictions (i.e., $q_i \geq q_a$, $i = 1,2$). In addition, assume that none of the suppliers offer all-unit quantity discounts, and the fixed unit purchase prices for suppliers 1 and 2 are \$8.6 and \$9.2, respectively. Let us first obtain the optimal order quantity, and the optimal selling price and demand rate that maximize the profit per time unit when each supplier is selected separately without considering capacity and quality constraints. A summary of the results are shown in Table 4.4. Notice that, supplier 1 is the dominating supplier (i.e., $TP_1 > TP_2$).

Table 4.4. Results obtained from selecting each supplier separately.

	Order Quantity, $\boldsymbol{Q_i}$ (*Units*)	**Selling Price, *P*** (*\$*)	**Demand, $\boldsymbol{D_i}$** (*Units/month*)	**Profit, $\boldsymbol{TP_i}$** (*\$/month*)
Supplier 1	691.61	13.98	1234.10	4860.41
Supplier 2	440.95	14.65	1073.30	4632.94

Let us assume supplier 2 has no capacity limitation. Then, as long as supplier 1's capacity is greater than the optimal demand rate (i.e., $c_1 > D_1 = 1234.10$), the retailer's optimal sourcing strategy will always be to select supplier 1 alone. Once supplier 1 experiences some capacity shortages, the retailer's sourcing strategy might change. Figure 4.5 is developed to illustrate the change in the retailer's sourcing strategy as c_1 decreases. Note that, the total number of orders allowed per order cycle is two (i.e., $m = 2$).

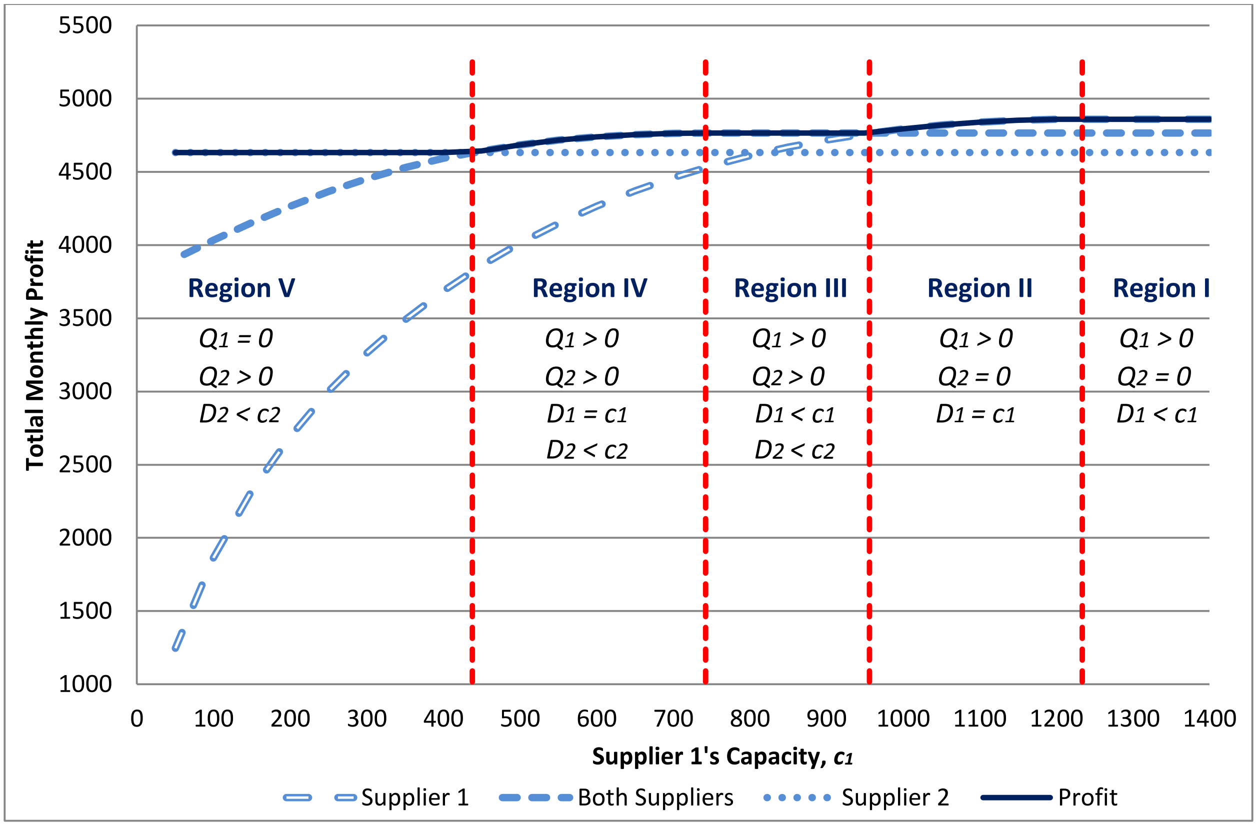

Figure 4.5. The impact of supplier 1's capacity on the sourcing strategy when m = 2.

Figure 4.5 is divided into five regions, where each region represents a different sourcing strategy. Now, starting form Region I where supplier 1's capacity is very high (i.e., $c_1 > D_1 = 1234.10$), the optimal sourcing strategy is to use supplier 1 alone without fully utilizing the corresponding capacity. This region is equivalent to solving the problem for the case of uncapacitated suppliers. Then, as c_1 decreases, the optimal sourcing strategy remains almost unchanged ,as shown in Region II ($956 < c_1 \leq 1234.10$); however, supplier 1's capacity in Region II is fully utilized which corresponds to Cases 1.1, 2.1, 4.2, and 5.2.1. Note that, the reduction in supplier 1's capacity in Region II is not significant enough to consider the use of supplier 2.

Next, as c_1 keeps decreasing, both suppliers are selected without utilizing any of them at full capacity, as shown in Region III ($742.15 < c_1 \leq 956$), which corresponds to Case 5.1.2. In this region, selecting supplier 1 alone is not enough to satisfy all the demand. Hence, instead of ordering a small order quantity from supplier 2 to satisfy the remaining portion of the demand and incurring in a significant increase in the ordering cost per time unit, the order quantity submitted to each supplier is relatively large and, as a result, both suppliers are used without fully utilizing their capacities. For instance, the demand rate fulfilled by supplier 1 is less than all the capacity values in Region III (i.e., $d_1 = 742.15 < c_1$). Notice that, d_1 becomes equal to the lower bound of Region III. Subsequently, in Region IV ($438 < c_1 \leq 742.15$), the optimal sourcing strategy remains to order from both suppliers but fully utilizing supplier 1's capacity. This region corresponds to Cases 4.1 and 5.1.1.

Lastly, once supplier 1's capacity is reduced significantly, the optimal sourcing strategy is to only order from supplier 2, as shown in Region V ($c_1 \leq 438$) that corresponds to Cases 3, 4.3, and 5.3. In this region, supplier 1 is not selected because it can only provide a small order quantity due to capacity limitations. Thus, it is not efficient for the retailer to use supplier 1 and increase the ordering cost per time unit.

4.4. Conclusions

In this chapter, we have proposed a mixed integer nonlinear programming model for a supplier selection problem in which the goal is to maximize the retailer's profit per time unit under suppliers' limitations on capacity and quality. In addition, to ensure a more realistic and practical situation, the suppliers in our model are adopting all-unit quantity discounts as an incentive mechanism for the retailer. Also, we assume that the demand is price-sensitive and multiple orders are allowed to be submitted to the selected suppliers during a repeating order cycle. The proposed model simultaneously finds the optimal number of orders and the corresponding order quantities for the selected suppliers, and the selling price that maximize the retailer's profit per time unit.

Moreover, we have considered two versions of our model based on the type of order quantity: the first one allows a different order quantity for each selected supplier and the second one considers equal-size order quantities. Furthermore, we have developed sufficient conditions under which there exists an optimal solution where the retailer only orders from one supplier. We have also investigated the impact of the dominating supplier's capacity on the retailer's sourcing strategies.

Chapter 5: Integrated Pricing and Supplier Selection Problem in a Two-Stage Supply Chain

5.1. Problem Description and Model Development

Consider an integrated company that controls its entire production process from purchasing raw material to selling the final product to a set of customers (i.e., a centralized decision-making process). We assume that this company can be modeled as a two-stage supply chain system, i.e., $l = 2$, as shown in Figure 5.1.

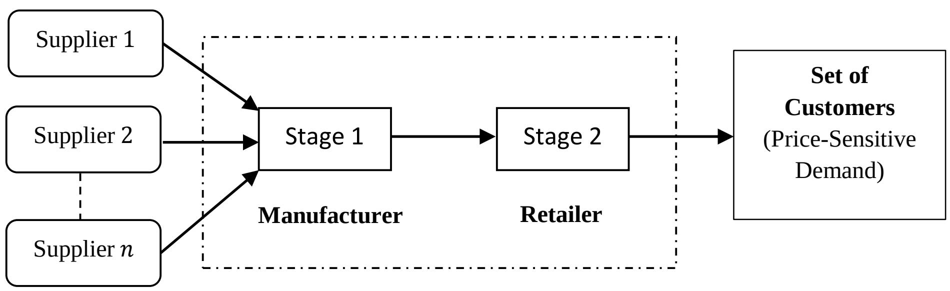

Figure 5.1. Two-stage supply chain system.

The first stage in the supply chain is represented by a manufacturer who can procure the needed amount of raw material from n potential suppliers. The suppliers differ from each other in terms of quality level (i.e., percentage of acceptable units), production capacity, unit purchasing cost, and unit setup cost. We assume that the manufacturer at the first stage has a minimum acceptable quality rate q_a and can place multiple orders to the selected suppliers in a repeating order cycle. Mendoza and Ventura (2008) showed that placing at most one order per order cycle per supplier results in a suboptimal solution. Hence, in the model proposed in this chapter, supplier-selection decisions at the first stage include finding the number of orders placed to the selected suppliers per order cycle and the corresponding order quantities taking into account capacity and quality restrictions. Thereafter, the manufacturer produces the finished product for which the retailer is facing a price-sensitive demand. Therefore, the problem is to simultaneously carry out supplier-selection decisions at the first stage, coordinate inventory-replenishment decisions across supply chain stages, and handle the retailer's pricing decisions in order to maximize the company's profit per time unit.

Now, we introduce the notation and assumptions underlying our model. Subsequently, we describe the proposed mixed integer nonlinear programming model.

Indices

s	Stage, where $s = 1,2$.
i	Potential supplier, where $i = 1, \dots, n$.

Parameters

c_i	Production capacity rate of supplier i, where $i = 1, \dots, n$.
q_i	Quality level (i.e., portion of acceptable units) of supplier i, $0 < q_i \leq 1$, where $i = 1, \dots, n$. This level is defined as a positive rational number.
q_a	Minimum acceptable quality level for the manufacturer at stage 1, $0 < q_a \leq 1$. This level is defined as a positive rational number.
v_{1i}	Purchasing cost of one unit from supplier i at stage 1, where $i = 1, \dots, n$. Note that $v_1$is the weighted average unit purchasing cost for the selected suppliers.
k_s	Setup cost for placing one order in stage s, where $s = 1,2$.
k_{1i}	Setup cost for placing one order to supplier i at stage 1, where $i = 1, \dots, n$. Note that k_1 is the weighted average unit setup cost for the selected suppliers.
h_s	Unit inventory holding cost at stage s, where $s = 1,2$.
e_s	Unit echelon inventory holding cost at stage s, i.e., $e_1 = h_1$, and $e_2 = h_2 - h_1$.

$\propto$ Scaling factor for price-sensitive demand.

δ Price elasticity index for price-sensitive demand.

Decision variables

J_i Number of orders submitted to supplier i per a repeating order cycle, where $i = 1, \dots, n$.

Q_s Order quantity at stage s, where $s = 1,2$.

X_1 Multiplicative factor for the order quantity received from stage 2.

P Retailer's optimal selling price.

D Retailer's price-sensitive demand rate.

D_i Maximum demand rate met by supplier i when quality and capacity constraints are not considered, where $i = 1, \dots, n$.

The following assumptions are considered:

1. The time horizon is infinite.
2. The supply chain produces a single type of product.
3. The lead times are constant and can be assumed to be zero.
4. The manufacturer has an infinite production rate and storage capacity.
5. To avoid inventory shortages at any of the supply chain stages, inventory replenishment decisions must follow the zero-nested inventory ordering policy. Love (1972), and Schwarz (1973) proved that the zero-nested inventory ordering policy is optimal for a serial inventory system. The zero-inventory ordering policy implies that the inventory at each stage is replenished by the immediate upstream stage only when the inventory level drops to zero. A policy is nested if, when any stage in the supply chain orders, all the downstream stages order as well. This requires the order quantity placed at any stage to be an integer multiple of the order quantity

placed at the downstream stage. For instance, in this chapter, the manufacturer's order quantity is an integer multiple of the retailer's order quantity, i.e., $Q_1 = X_1 Q_2$.

6. The product gains more value upon the arrival to the retailer stage. Thus, the echelon holding cost of the first stage is strictly less than that of the second stage (i.e., $h_1 < h_2$).
7. The stages are treated separately if and only if $\frac{k_1}{e_1} > \frac{k_2}{e_2}$; otherwise, they are combined into a single stage with a setup cost of $k_1 + k_2$ and an echelon holding cost of $e_1 + e_2$.
8. The demand rate is considered to be a power function of the selling price, i.e., $D(P) = \propto P^{-\delta}$, where $\propto$, and δ are the scaling factor and the price elasticity index for the price-sensitive demand, respectively.

Now, the following mixed integer nonlinear programming Model $(M1)$ is developed, where the goal is to maximize the profit per time unit such that supplier-selection, inventory-replenishment, and pricing decisions are coordinated simultaneously.

$$\begin{aligned} Max.\ TP_1 = \propto P^{1-\delta} - \left(\propto P^{-\delta} \frac{\sum_{i=1}^{n} J_i\, k_{1i}}{Q_1 \sum_{i=1}^{n} J_i} + \frac{e_1}{2} Q_1 \ + \propto P^{-\delta} \frac{\sum_{i=1}^{n} J_i\, v_{1i}}{\sum_{i=1}^{n} J_i} \right) \\ - \left(\propto P^{-\delta} \frac{k_2}{Q_2} + \frac{e_2}{2} Q_2 \right), \end{aligned} \tag{1}$$

subject to

$$\propto P^{-\delta} J_i \leq \ c_i \sum_{i=1}^{n} J_i \ , \qquad i = 1, \dots, n\ , \tag{2}$$

$$\sum_{i=1}^{n} J_i\, q_i \geq \ q_a \ \sum_{i=1}^{n} J_i \ , \tag{3}$$

$$Q_1 = X_1 Q_2\ , \tag{4}$$

$$X_1 \geq 1\ , integer\ , \tag{5}$$

$$\sum_{i=1}^{n} J_i \geq 1 \,, \tag{6}$$

$$J_i \geq 0 \,, integer, \qquad\qquad i = 1, \ldots, n \,, \tag{7}$$

$$P, Q_2 > 0 \,. \tag{8}$$

The first component in the objective function represents the revenue per time unit. The second component includes the replenishment cost (i.e., setup cost and purchasing cost per time unit), and holding cost per time unit at the first stage. And the third component accounts for the setup and holding cost per time unit for the retailer stage.

The model is subject to a set of constraints. Constraint set (2) represents the suppliers' capacity limitations. Constraint (3) imposes the manufacturer's restriction on the minimum acceptable quality level. Constraints (4) and (5) ensure that the order quantity placed at the first stage is an integer multiple of the order quantity received from stage 2. Constraint (6) makes sure that at least one supplier is selected to be able to satisfy the retailer's demand. In addition, as discussed in Mendoza and Ventura (2008), the total number of orders submitted to the selected suppliers can be fixed to a small integer value to obtain a reasonable small cycle time. Constraint (7) imposes the integrality condition on the number of order submitted to each supplier. Finally, Constraint set (8) avoids the division by zero. Hence, a small positive number can be set as a lower bound for these non-zero variables.

In the following section, we provide some analysis regarding the determination of the supplier, who under ideal conditions, would generate the highest profit per time unit for the company. Throughout this chapter, we refer to this supplier as the dominating supplier. In addition, we identify upper and lower bounds for the profit per time unit that corresponds to multiple sourcing strategy, and hence, we develop bounds on the selling price. Moreover, we identify the feasible region for the optimal multiplicative factor, and propose an algorithm to improve the bounds when the suppliers have no capacity limitations. And then, we find a tight feasible region for the manufacturer's multiplicative factor for the special case when there are only two potential suppliers to order from and the suppliers have capacity and quality restrictions.

5.2. Model Analysis

5.2.1. Finding the Dominating Supplier

In this subsection, we identify the supplier with whom the company can obtain the highest profit per time unit when capacity and quality restrictions are not considered. Thus, we need to determine the profit per time unit when each supplier is selected individually without accounting for the supplier's limitations on capacity and quality. This requires finding the optimal order quantity placed by the manufacturer, Q_{1i}^*, order quantity placed by the retailer, Q_{2i}^*, multiplicative factor, X_{1i}^* , and selling price, P_i^* , when supplier i is selected individually without considering capacity and quality constraints, $i = 1, \dots, n$. Therefore, the following mixed integer nonlinear programming Model $(M1_i')$ is developed for supplier $i, i = 1, \dots, n$:

$$Max.\ TP_i' = \propto P_i^{1-\delta} - \left(\propto P_i^{-\delta} \frac{k_{1i}}{Q_{1i}} + \frac{e_1}{2} Q_{1i} + \propto P_i^{-\delta} v_{1i}\right) - \left(\propto P_i^{-\delta} \frac{k_2}{Q_{2i}} + \frac{e_2}{2} Q_{2i}\right), \tag{9}$$

subject to

$$Q_{1i} = X_{1i} Q_{2i}\,, \tag{10}$$

$$X_{1i} \geq 1, integer, \tag{11}$$

$$P_i, Q_{2i} > 0. \tag{12}$$

The objective function of Model $(M1_i')$ represents the profit per time unit which is equal to the gross revenue per time unit minus the replenishment cost and inventory holding cost per time unit occurred at the first stage, and the setup cost and inventory holding cost per time unit for the second stage. Model $(M1_i')$ is subject to Constraint (10), which ensures that the quantity ordered at the first stage is an integer multiple of the order quantity placed at the second stage. Constraint (11) guarantees that the multiplicative factor is a positive integer. Finally, Constraint set (12) avoids the division by zero.

Now, let us first replace Q_{1i} in Equation (9) by $X_{1i}Q_{2i}$, then TP_i' is a concave function in Q_{2i} for given values of P_i and X_{1i}. Hence, the optimal EOQ policy is established as follows:

$$Q_{2i}^* = \sqrt{\frac{2\propto P_i^{-\delta}\left(\frac{k_{1i}}{X_{1i}}+k_2\right)}{(X_{1i}e_1+e_2)}}\ , i = 1, \dots, n. \tag{13}$$

By substituting Q_{2i}^* in Equation (9), we obtain a profit function that is only a function of the multiplicative factor for a given selling price, i.e.,

$$TP_i' = \propto P_i^{-\delta}(P_i - v_{1i}) - \sqrt{2 \propto P_i^{-\delta}\left(\frac{k_{1i}}{X_{1i}} + k_2\right)(X_{1i}e_1 + e_2)}\ . \tag{14}$$

The next step is to find the optimal multiplicative factor that maximizes the profit function shown in Equation (14). Note that, this is equivalent to minimizing the expression under the square root, i.e.,

$$\overline{TP_i'} = 2 \propto P_i^{-\delta}\left(\frac{k_{1i}}{X_{1i}} + k_2\right)(X_{1i}e_1 + e_2)\ . \tag{15}$$

Hence, by relaxing the integrality of X_{1i}, and taking the first partial derivative of $\overline{TP_i'}$ with respect to X_{1i}, we get

$$\frac{\partial \overline{TP_i'}}{\partial X_{1i}} = 2 \propto P_i^{-\delta}\left(e_1 k_2 - e_2 \frac{k_{1i}}{X_{1i}^2}\right). \tag{16}$$

Note that $\overline{TP_i'}$is a convex function of X_{1i}, i.e., $\frac{\partial^2 \overline{TP_i'}}{\partial X_{1i}^2} = \frac{4 \propto P_i^{-\delta} e_2 k_{1i}}{X_{1i}^3} > 0$. Thus, by setting Equation (16) to zero and solving for X_{1i}, we get a closed-form solution for the multiplier that is independent on Q_{2i} and P_i, i.e.,

$$\hat{X}_{1i} = \sqrt{\frac{k_{1i} e_2}{e_1 k_2}} \ , i = 1, \dots, n. \tag{17}$$

Clearly, $\hat{X}_{1i}$ can be a real number. Thus, we need to round it either to the nearest lower or upper integer. The optimal $\hat{X}_{1i}$ must satisfy $\overline{TP_i'}(\hat{X}_{1i}) \le \overline{TP_i'}(\hat{X}_{1i} + 1)$ and $\overline{TP_i'}(\hat{X}_{1i}) \le \overline{TP_i'}(\hat{X}_{1i} - 1)$. Therefore, by simplifying the inequalities and using the quadratic formula, lower and upper bounds can be derived as follows:

$$\frac{-1 + \sqrt{1 + 4\frac{k_{1i} e_2}{k_2 e_1}}}{2} \le \hat{X}_{1i} \le \frac{1 + \sqrt{1 + 4\frac{k_{1i} e_2}{k_2 e_1}}}{2} \quad , i = 1, \dots, n. \tag{18}$$

Let us denote the lower and upper bound in Relation (18) by $\hat{X}_{1i,lo}$, and $\hat{X}_{1i,up}$, respectively. The difference between the upper bound and the lower bound is 1. Therefore, if the bounds are not integers, then the optimal integer multiplicative factor X_{1i}^* is either equal to the nearest upper integer of the lower bound, i.e., $\lceil \hat{X}_{1i,lo} \rceil$ or to the nearest lower integer of the upper bound, i.e., $\lfloor \hat{X}_{1i,up} \rfloor$. And, when the bounds are integer, then the lower and upper bounds are the two possible solutions for X_{1i}^*. Hence, the optimal integer multiplicative factor can be written:

$$X_{1i}^* = \begin{Bmatrix} \lfloor \hat{X}_{1i,up} \rfloor & if & \hat{X}_{1i,up} \in \mathbb{R} \\ \lceil \hat{X}_{1i,lo} \rceil, \lfloor \hat{X}_{1i,up} \rfloor & if & \hat{X}_{1i,up} \in \mathbb{N} \end{Bmatrix}, i = 1, \dots, n. \tag{19}$$

Note that, Munson and Rosenblatt (2001) consider only one solution for the integer multiplicative factor that is equal to $\lfloor \hat{X}_{1i,up} \rfloor$, when in fact, two solutions can be obtained if the bounds in Relation (18) are integer.

Now, for given values of Q_{2i} and X_{1i}, we obtain the first partial derivative of the profit function with respect to P_i, i.e.,

$$\frac{\partial TP_i'}{\partial P_i} = \propto P_i^{-\delta} + \frac{(-\delta) \propto P_i^{-\delta}}{P_i}\left(P_i - v_{1i} - \frac{\left(\frac{k_{1i}}{X_{1i}} + k_2\right)}{Q_{2i}}\right). \tag{20}$$

Note that TP_i' is a concave function of P_i, i.e., for $\delta > 1$, $\frac{\partial^2 TP_i'}{\partial P_i^2} < 0$ at the stationary point. Hence, by setting Equation (20) to zero and solving for P_i, we obtain the optimal selling price:

$$P_i^* = \frac{\delta}{\delta - 1}\left(v_{1i} + \frac{\frac{k_{1i}}{X_{1i}} + k_2}{Q_{2i}}\right), i = 1, \dots, n. \tag{21}$$

Therefore, **Algorithm I** below can be used to find the values of P_i^*, Q_{2i}^*, and X_{1i}^* that maximize the company's profit per time unit when each supplier is selected when capacity and quality constraints are not taking into consideration. **Algorithm I** is an extension to the algorithm proposed by Abad (1988) who studied pricing and lot sizing decisions for a single stage supply chain. He proved that, if the optimal selling price is known, then the profit function is a convex-concave function of the order quantity, thus by starting with a large value for the order quantity and iteratively solving for the selling price and order quantity, the algorithm keeps getting closer to the optimal order quantity until it converges at a desired accuracy threshold ε.

Algorithm I – Finding the unrestricted dominating supplier

Step 1. For each supplier i, $i = 1, \dots, n$.

Step 1.1 Let $k = 0$, $Q_{2i,k=0} = \infty$, and calculate X_{1i}^* using Equation (19).

Step 1.2 Calculate $P_{i,k}^*$ using Equation (21). Calculate $Q_{2i,k+1}^*$ using Equation (13).

If $\left|Q_{2i,k+1}^* - Q_{2i,k}^*\right| < \varepsilon$, stop and go to Step 1.3. Otherwise, let $k = k + 1$ and repeat Step 1.2.

Step 1.3 Return the values of X_{1i}^* $Q_{2i,k}^*$, and $P_{i,k}^*$. Calculate TP_i' using Equation (9), and determine $D_i^* = \propto P_{i,k}^{*-\delta}$.

Step 2. If $i < n$, then let $i = i + 1$ and go back to Step 1.1. Otherwise, find the optimal supplier i^*, $i^* \in arg\,max\ \{TP_i': i = 1, \dots, n\}$.

Note that **Algorithm I** can be extended to the case where one supplier needs to be selected from a set of potential capacitated suppliers. This can be done by checking if the respective supplier's capacity is enough to cover the maximum demand rate that the supplier can achieve when is selected individually and there is no capacity or quality limitations. Hence, Step 1.3 can be replaced by, if $\propto P_{i,k}^{*-\delta} \leq c_i$, then return the values of X_{1i}^* $Q_{2i,k}^*$, and $P_{i,k}^*$, and calculate TP_i' using Equation (9). Otherwise, set $P_{i,k}^* = (c_i/\propto)^{-1/\delta}$, update $Q_{2i,k}^*$ using Equation (13), and calculate TP_i' using Equation (9).

5.2.2. Developing Lower and Upper Bounds for the Selling Price.

The goal of this subsection is to determine the lower and upper bounds for the optimal selling price when suppliers' limitations on capacity and/or quality requirements are considered. Let us first present **Lemma 1**, which shows the lower and upper bounds for the profit per time unit when the suppliers have no capacity limitations. Then, we use the result from **Lemma 1** to develop the optimal selling price's bounds.

Lemma 1. Consider a special case of Model $(M1)$ where the suppliers are such that $c_i \geq \propto P_i^{*-\delta}, i = 1, \ldots, n$. And, without loss of generality, assume that supplier 1 is the dominating supplier; if there are multiple dominating suppliers, then supplier 1 is the dominating supplier with the highest quality level among all the dominating suppliers. In addition, assume that there is a supplier k such that $q_k \geq q_a$. Therefore, the obtained profit per time unit is $TP'' \in [\ min\{TP_i'; i = 2, \ldots, n\},\ TP_1'\ ]$, where TP_i' is the profit per time unit obtained by selecting only supplier i without considering capacity and quality constraints: $TP_i' = \propto P^{1-\delta} - \left(\propto P^{-\delta} \frac{k_{1i}}{Q_1} + \frac{e_1}{2} Q_1 \ + \propto P^{-\delta} v_{1i}\right) - \left(\propto P^{-\delta} \frac{k_2}{Q_2} + \frac{e_2}{2} Q_2\right), i = 1, \ldots, n.$

Proof.

The two possible sourcing strategies are either the single or multiple sourcing strategy. Now, in case of a single sourcing strategy, it is clear that the obtained profit per time unit is $TP'' \in [\ min\{TP_i'; i = 2, \ldots, n\},\ TP_1'\ ]$. Now, the same bounds can also be considered for the case of a multiple sourcing strategy. To prove this, assume that multiple sourcing strategy is the optimal sourcing strategy and let us analyze the corresponding profit per time unit equation. We know that

$$TP'' = \propto P^{1-\delta} - \left(\propto P^{-\delta} \frac{\sum_{i=1}^{n} J_i\, k_{1i}}{Q_1 \sum_{i=1}^{n} J_i} + \frac{e_1}{2} Q_1 \ + \propto P^{-\delta} \frac{\sum_{i=1}^{n} J_i\, v_{1i}}{\sum_{i=1}^{n} J_i}\right) - \left(\propto P^{-\delta} \frac{k_2}{Q_2} + \frac{e_2}{2} Q_2\right). \quad (22)$$

By rewriting Equation (22), we obtain

$$TP'' = \sum_{i=1}^{n} \frac{J_i}{\sum_{i=1}^{n} J_i} \left(\propto P^{1-\delta} - \left(\propto P^{-\delta} \frac{k_{1i}}{Q_1} + \frac{e_1}{2} Q_1 \ + \propto P^{-\delta} v_{1i} \right) - \left(\propto P^{-\delta} \frac{k_2}{Q_2} + \frac{e_2}{2} Q_2 \right) \right). \quad (23)$$

Let $\beta_i = \frac{J_i}{\sum_{i=1}^{n} J_i}$ represent the proportion of orders submitted to supplier i, where $\sum_{i=1}^{n} \beta_i = 1$. Hence,

$$TP'' = \sum_{i=1}^{n} \beta_i \, TP_i'', \quad (24)$$

where TP_i'' is the profit per time unit obtained from supplier i when a single sourcing strategy is considered given that the optimal order quantities and selling price are obtained from the multiple sourcing strategy. Note that $TP_i'' \leq TP_i'$ because in a multiple sourcing strategy the determination of the order quantities and selling price are functions of the selected suppliers' parameters. In addition, the suppliers may have quality limitations. Thus,

$$TP'' \leq \sum_{i=1}^{n} \beta_i \, TP_i'. \quad (25)$$

By assumption, supplier 1 is the dominating supplier, i.e., $TP_1' \geq TP_i', i = 2, \ldots, n$. Hence, $TP'' \leq \sum_{i=1}^{n} \beta_i \, TP_1' = TP_1'$. This ends the first part of the proof regarding the upper bound, i.e., $TP'' \leq TP_1'$.

The goal of the remaining part of the proof is to show that $TP'' \geq min \{TP_i' : i = 2, \ldots, n\}$. This part of the proof can be done by contradiction. Thus, let us assume that the optimal multiple sourcing strategy corresponds to a profit per time unit that is lower than the minimum profit per time unit obtained with a single sourcing strategy, i.e.,

$$TP'' < min\ \{TP_i' : i = 2 \ldots, n\}. \tag{26}$$

By assumption, there is a supplier k such that $q_k \geq q_a$. Then, supplier k can be selected individually without violating the quality constraint. Accordingly, the profit per time unit obtained from selecting supplier k is greater than or equal to the minimum profit per time unit obtained in the case of a single sourcing strategy, i.e., $TP_k' \geq min\ \{TP_i' : i = 2 \ldots, n\}$. Therefore, Relation (26) can be written as follows:

$$TP'' < min\ \{TP_i' : i = 2 \ldots, n\} \leq TP_k' \ . \tag{27}$$

Equation (27) thereby shows in regard to profit per time unit that selecting supplier k individually produces results at least as good as those obtained with the multiple sourcing strategy. Therefore, this is a contradiction, because it violates the assumption that multiple sourcing strategy is the optimal sourcing strategy. ■

Note that **Lemma 2** below shows that for the case in which suppliers have no capacity limitations, there is no need to consider all the non-dominating suppliers given that their quality level is less than or equal to the quality level of supplier 1.

Lemma 2. In **Lemma 1,** supplier r such that $TP_r'' < TP_1''$ and $q_r \leq q_1 < q_a$ will not be selected, i.e., $\beta_r = 0, r = 2, \ldots, n$,

Proof.

Let us address all the possible sourcing strategies to show that supplier r will not be selected. Based on the suppliers' quality level, the following three sourcing strategies are considered:

A. If $q_1 \geq q_a > 0$, then only the dominating suppliers can be used because there is at least one dominating supplier with a good quality level that can be selected and generates profit per time unit greater than the one obtained from any sourcing strategy that includes supplier r.

B. If $0 < q_1 < q_a$ and all the non-dominating suppliers are such that $q_k \leq q_a$, but at least one of the non-dominating suppliers has a quality level of q_a, then neither the dominating suppliers nor supplier r are selected because if they are selected the minimum average quality level cannot be achieved. Note that the problem becomes infeasible if all the non-dominating suppliers have also a quality level that is strictly less than q_a.

C. If $0 < q_1 < q_a$ and there is a supplier k such that $q_k > q_a$ and $TP_k'' < TP_1''$, then multiple sourcing strategy is the optimal sourcing strategy where supplier r is not selected. This can be proved by contradiction. Let us assume that there exists an optimal solution where S is the optimal set of suppliers selected in the case of the multiple sourcing strategy, in which $\beta_r > 0, \beta_1 > 0$, and $\beta_k > 0$. Hence, $TP'' = \beta_r TP_r'' + \beta_1 TP_1'' + \beta_k TP_k'' + \sum_{i \in S-\{r\}-\{1\}-\{k\}} \beta_i TP_i''$.

Now, if we replace supplier r by the dominating supplier (i.e., supplier 1), and allocate the proportion of orders submitted to supplier r to supplier 1, then, $TP'' < (\beta_r + \beta_1) TP_1'' + \beta_k TP_k'' + \sum_{i \in S-\{r\}-\{1\}-\{k\}} \beta_i TP_i''$ because $\beta_r > 0$ and $TP_r'' < TP_1''$. This shows that TP'' can be increased by replacing supplier r by supplier 1. Therefore, this is a contradiction, because it violates the assumption that supplier $r \in S$.

Note that the quality constraint is not violated when supplier r is replaced by supplier 1 because $q_r \leq q_1$, i.e., $q_a \leq \beta_r q_r + \beta_1 q_1 + \beta_k q_k + \sum_{i \in S-\{r\}-\{1\}-\{k\}} \beta_i q_i \leq (\beta_r + \beta_1) q_1 + \beta_k q_k + \sum_{i \in S-\{r\}-\{1\}-\{k\}} \beta_i q_i$. ■

Theorem 1. Consider a special case of Model $(M1)$ where the suppliers have no capacity problems, i.e., $c_i \geq P_i^{*-\delta}, i = 1, \ldots, n$. Then, the selling price is $P \in [P_{lo}, P_{up}]$, where P_{lo} is the minimum optimal selling price in case of a single sourcing strategy, i.e., $P_{lo} = min\ \{P_i^*: i = 1, \ldots, n\}$, and P_{up} is the maximum optimal selling price in case of a single sourcing strategy, i.e., $P_{up} = max\ \{P_i^*: i = 1, \ldots, n\}$.

Proof.

Lemma 1 shows that $TP'' \in [\ min\ \{TP_i'; i = 2, \ldots, n\},\ TP_1'\ ]$, where TP'' is a weighted sum function as shown in Equation (24). Thus, as β_i approaches value 1, the optimal solution gets closer to the case in which supplier i is the only selected supplier, $i = 1 \ldots, n$. Therefore, when the suppliers are such that $c_i \geq \propto P_i^{*-\delta}, i = 1 \ldots, n$, the obtained selling price is $P \in [P_{lo}, P_{up}]$. ■

The remaining part of this subsection discusses the calculation of the lower and upper bounds for the selling price when there is a possibility that the suppliers also have capacity restrictions, i.e., $c_i < \propto P_i^{*-\delta}, i = 1, \ldots, n$. Now, the lower bound of the selling price P_{lo} remains the same because it is the lowest selling price calculated when capacity and quality constraints are dropped. And, in order to calculate the upper bound for the selling price, we need to account for the sourcing strategy that obtains the highest selling price (i.e., lowest demand rate). Note that similar to **Lemma 1**, it can be shown that, the profit per time unit when capacity and quality constraints are considered is $TP'' \in \left[min\left\{TP_i'|_{P_i=(c_i/\propto)^{-1/\delta}} :\ i = 2, \ldots, n\right\}, TP_1' \right]$ given that supplier 1 is the dominating supplier. Therefore, the upper bound shown in **Theorem 1** can be rewritten as follows:

$$P_{up} = max\ \left\{P_i^*,\ (c_i/\propto)^{-1/\delta} : i = 1, \ldots, n\right\}. \tag{28}$$

Note that the second component in parentheses, i.e., $(c_i/\propto)^{-1/\delta}$, is added so that we can consider the case in which a single supplier is selected at full capacity.

5.2.3. Feasible Region for the Multiplicative Factor.

In this subsection, we assume that a multiple sourcing strategy is implemented due to the dominating supplier's limitations in regard to capacity and/or quality requirements. Therefore, by carrying the same derivations shown in Subsection 3.1 for given values of $J_i, i = 1, \dots, n$, we obtain the optimal multiplicative factor $\hat{X}_1 = \sqrt{k_1 e_2/e_1 k_2}$, where k_1 is the weighted average unit setup cost, i.e., $k_1 = \sum_{i=1}^{n} J_i\, k_{1i}/\sum_{i=1}^{n} J_i$. Recall that $\hat{X}_1$ can be a real number; thus, the optimal integer multiplicative factor can be determined as follows:

$$X_1^* = \begin{Bmatrix} \lfloor \hat{X}_{1,up} \rfloor & if \quad \hat{X}_{1,up} \in \mathbb{R} \\ \lceil \hat{X}_{1,lo} \rceil, \lfloor \hat{X}_{1,up} \rfloor & if \quad \hat{X}_{1,up} \in \mathbb{N} \end{Bmatrix}, \; i = 1, \dots, n \,, \tag{29}$$

where $\hat{X}_{1,lo} = -\,0.5 + 0.5\sqrt{1 + 4(k_1 e_2/e_1 k_2)}$, and $\hat{X}_{1,up} = 0.5 + 0.5\sqrt{1 + 4(k_1 e_2/e_1 k_2)}$. The challenging aspect here is the inability to determine in advance the values of $J_i, i = 1, \dots, n$, and as a result a specific value for X_1^* cannot be calculated. For instance, if multiple suppliers are selected because the dominating supplier has limits on capacity, then it is difficult to determine in advance the extent to which the suppliers' capacity is utilized, it was shown in Adeinat and Ventura (2015) that in some cases multiple suppliers can be selected without fully utilizing their capacity. However, **Lemma 3** shows that lower and upper bounds can be obtained for the multiplicative factor.

Lemma 3. The optimal multiplicative factor is $\hat{X}_1 \in [X_{1,lo}, X_{1,up}]$, where $X_{1,lo}$ is the multiplicative factor obtained using the lowest unit setup cost, i.e., $k_{1,min} = min\,\{k_{1i}: i = 1, \dots, n\}$, and $X_{1,up}$

is the multiplicative factor obtained using the highest unit setup cost, i.e., $k_{1,max} = max\ \{k_{1i}: i = 1, \dots, n\}$.

Proof.

In the case of a multiple sourcing strategy, the optimal multiplicative factor is a function of the weighted average unit setup cost, i.e.,

$$\hat{X}_1 = \sqrt{\frac{k_1 e_2}{e_1 k_2}} = \sqrt{\frac{\frac{\sum_{i=1}^{n} J_i\, k_{1i}}{\sum_{i=1}^{n} J_i} e_2}{e_1 k_2}}. \tag{30}$$

Note that $\frac{\sum_{i=1}^{n} J_i k_{1i}}{\sum_{i=1}^{n} J_i} \leq \frac{\sum_{i=1}^{n} J_i\, k_{1,max}}{\sum_{i=1}^{n} J_i} = k_{1,max}$. Similarly, $\frac{\sum_{i=1}^{n} J_i k_{1i}}{\sum_{i=1}^{n} J_i} \geq \frac{\sum_{i=1}^{n} J_i\, k_{1,min}}{\sum_{i=1}^{n} J_i} = k_{1,min}$. Thus,

$$\sqrt{\frac{k_{1,min} e_2}{e_1 k_2}} \leq \hat{X}_1 \leq \sqrt{\frac{k_{1,max} e_2}{e_1 k_2}}. \tag{31}$$

$$X_{1,lo} \leq \hat{X}_1 \leq X_{1,up}\ . \blacksquare \tag{32}$$

In the following subsection, we show that the bounds described in **Lemma 3** can be improved.

5.2.4. Improving the Multiplicative Factor's Bounds

Assume that the suppliers have no capacity limitations, i.e., $c_i \geq \propto P_i^{*-\delta}, i = 1, \dots, n$. Also, assume that the suppliers are re-indexed as

$$X_1^{(i=1)} \leq X_1^{(2)} \leq \cdots \leq X_1^{(n-1)} \leq X_1^{(i=n)}\ , \tag{33}$$

where $X_1^{(i)}$ is the corresponding multiplicative factor obtained when supplier i is selected individually when capacity and quality constraints are not considered, $i = 1, \dots, n$. The superscripts in this subsection represent the selected supplier(s).

Now, in order to improve the bounds, we need to check the corresponding quality level for supplier 1 and supplier n. For instance, if supplier 1 is such that $q_1 \geq q_a$, then the lower bound shown in Relation (33) remains the same because this supplier can be selected individually without violating the quality constraint. Now, if supplier 1 is such that $q_1 < q_a$, then this supplier will not be selected unless supplier i such that $q_1 < q_a < q_i$ is also selected, $i = 2, \dots, n$. Hence, the corresponding multiplicative factor when supplier 1 and supplier i are selected, i.e., $X_1^{(1,i)}$, might improve the lower bound shown in Relation (33). Consequently, the lower bound can be improved by selecting the minimum between $X_1^{(1,i)}$ and $X_1^{(2)}$, i.e.,

$$X_{1,lo} = min\left\{X_1^{(1,i)}, X_1^{(2)}\right\}, \ q_1 < q_a < q_i \text{ and } i = 2, \dots, n. \tag{34}$$

Similarly, if supplier n is such that $q_n \geq q_a$, then the upper bound shown in Relation (33) remains the same because this supplier can be selected individually without violating the quality constraint. And, if supplier n is such that $q_n < q_a$, then there is a possibility of improving the multiplicative factor's upper bound shown in Relation (33). This is because if supplier n needs to be selected, then supplier i such that $q_n < q_a < q_i$, $i = 1, \dots, n-1$ must be selected to achieve the quality requirement. Hence, the corresponding multiplicative factor, i.e., $X_1^{(n,i)}$, might improve the upper bound as follows:

$$X_{1,up} = max\left\{X_1^{(n,i)}, X_1^{(n-1)}\right\}, \ q_n < q_a < q_i \text{ and } i = 1, \dots, n-1. \tag{35}$$

Now, the challenge is to find the supplier who if selected with supplier 1 obtains the minimum multiplicative factor, and the supplier who if selected with supplier n obtains the maximum multiplicative factor. In **Theorem 2**, we show a property that helps in finding the potential pair of suppliers that need to be considered when improving the multiplicative factor's bounds shown in **Lemma 3**.

Theorem 2. Assume that $c_i \geq \propto P_i^{*-\delta}, i = 1, \ldots, n$. Also, without loss of generality, assume that the lower and upper bounds shown in **Lemma 3** correspond to supplier 1 and supplier n, respectively. In addition, assume that supplier 1 is such that $q_1 < q_a$. Thus, the minimum multiplicative factor is obtained when supplier 1 and supplier i are selected, given that supplier i is such that $0 < q_a < q_j \leq q_i$ and $0 < k_{11} \leq k_{1i} \leq k_{1j}, i, j = 2, \ldots, n$ and $i \neq j$.

Similarly, if supplier n is such that $q_n < q_a$, then the maximum multiplicative factor is obtained when supplier n and supplier i are selected, given that supplier i is such that $0 < q_a < q_j \leq q_i$ and $0 < k_{11} \leq k_{1j} \leq k_{1i}, i, j = 1, \ldots, n-1$ and $i \neq j$.

Proof. (By contradiction).

The goal of the first part of the proof (i.e., when $q_1 < q_a$) is to show that $X_1^{(1,i)} \leq X_1^{(1,j)}$, $i, j = 2, \ldots, n$ and $i \neq j$. This implies that the weighted average unit setup cost is such that $k_1^{(1,i)} \leq k_1^{(1,j)}$. Instead, let us assume that $k_1^{(1,i)} > k_1^{(1,j)}$, i.e.,

$$\frac{J_1^{(1,i)} k_{11} + J_i^{(1,i)} k_{1i}}{M^{(1,i)}} > \frac{J_1^{(1,j)} k_{11} + J_j^{(1,j)} k_{1j}}{M^{(1,j)}}, \qquad i, j = 2, \ldots, n \text{ and } i \neq j. \tag{36}$$

Replacing $J_1^{(1,i)}$ by $M^{(1,i)} - J_i^{(1,i)}$ and $J_1^{(1,j)}$ by $M^{(1,j)} - J_j^{(1,j)}$, we obtain

$$k_{11} + \frac{J_i^{(1,i)}}{M^{(1,i)}}(k_{1i} - k_{11}) > k_{11} + \frac{J_j^{(1,j)}}{M^{(1,j)}}(k_{1j} - k_{11}), \qquad i, j = 2, \ldots, n \text{ and } i \neq j. \tag{37}$$

By canceling out k_{11} from both sides and then by adding a positive number ∇ to the lower bound in order to change the inequality into an equality, we obtain

$$\nabla = \frac{J_i^{(1,i)}}{M^{(1,i)}}(k_{1i} - k_{11}) - \frac{J_j^{(1,j)}}{M^{(1,j)}}\left(k_{1j} - k_{11}\right), \qquad i, j = 2, \dots, n \text{ and } i \neq j \,. \tag{38}$$

Since, by assumption, $0 < k_{11} \leq k_{1i} \leq k_{1j}$, then $(k_{1i} - k_{11}) \leq \left(k_{1j} - k_{11}\right)$. Also, since by assumption, $0 < q_a < q_j \leq q_i$, then $\frac{J_i^{(1,i)}}{M^{(1,i)}} \leq \frac{J_j^{(1,j)}}{M^{(1,j)}}$. Thus, a negative value is obtained for ∇, which is a contradiction because ∇ is assumed to be a positive number. Therefore, the minimum multiplicative factor is obtained when supplier 1 and the supplier with the highest quality level and lowest unit setup cost are selected.

The second part of the theorem can be proved in the same way to show that the maximum multiplicative factor is obtained when supplier n and the supplier with the highest quality level and highest unit setup cost are selected. ∎

Theorem 2 implies that supplier j such that $q_j > q_a$ and $k_{1j} > k_{1i}$, $i, j = 2, \dots, n$ and $i \neq j$, must be excluded when we attempt to improve the lower bound in **Lemma 3** because a lower multiplicative factor can be obtained by selecting supplier 1 and supplier i such that $0 < q_a < q_j \leq q_i$ and $0 < k_{11} \leq k_{1i} \leq k_{1j}, i, j = 2, \dots, n$ and $i \neq j$. Hence, **Algorithm II** is developed using **Theorem 2** to show the steps needed to improve the lower bound shown in **Lemma 3**.

Algorithm II – Improving the lower bound

Step 1. Re-index the suppliers such that $X_1^{(i=1)} \leq X_1^{(2)} \leq \cdots \leq X_1^{(n-1)} \leq X_1^{(i=n)}$.

Step 2. If $q_1 \geq q_a$, then $X_{1,lo} = X_1^{(1)}$. Otherwise, find supplier i such that $q_a < q_j \leq q_i$, $i, j = 2, \dots, n$, and $i \neq j$.

Step 2.1 If $k_{1i} \leq k_{1j}$. Then, $X_{1,lo} = min\left\{ X_1^{(1,i)}, X_1^{(2)} \right\}$. Otherwise, go to Step 2.2.

Step 2.2 Consider only supplier j such that $k_{1j} < k_{1i}$ and $q_j > q_a$, $j = 2, \dots, n$ and $i \neq j$.

Hence, $X_{1,lo} = min\left\{ X_1^{(1,i)}, min\left\{ X_1^{(1,j)}: j = 2, \dots, n, \ \& \ i \neq j \right\}, X_1^{(2)} \right\}$.

Similarly, **Theorem 2** implies that supplier j such that $q_j > q_a$ and $k_{1j} < k_{1i}$, $i, j = 2, \dots, n$ and $i \neq j$, must be excluded when we attempt to improve the upper bound in **Lemma 3** because a higher multiplicative factor can be obtained by selecting supplier n and supplier i such that $0 < q_a < q_j \leq q_i$ and $0 < k_{11} \leq k_{1j} \leq k_{1i}$, $i, j = 1, \dots, n-1$ and $i \neq j$. Hence, **Algorithm III** is developed to improve the multiplicative factor's upper bound shown in **Lemma 3**.

Algorithm II – Improving the upper bound

Step 1. Re-index the suppliers such that $X_1^{(i=1)} \leq X_1^{(2)} \leq \cdots \leq X_1^{(n-1)} \leq X_1^{(i=n)}$.

Step 2. If $q_n \geq q_a$, then $X_{1,up} = X_1^{(n)}$. Otherwise, find supplier i such that $q_a < q_j \leq q_i$, $i, j = 1, \dots, n-1$ and $i \neq j$.

Step 2.1 If $k_{1i} \geq k_{1j}$. Then, $X_{1,up} = max\left\{ X_1^{(n,i)}, X_1^{(n-1)} \right\}$. Otherwise, go to Step 2.2.

Step 2.2 Consider only supplier j such that $k_{1j} > k_{1i}$ and $q_j > q_a$, $j = 2, \dots, n$ and $i \neq j$.

Hence, $X_{1,up} = max\left\{ X_1^{(n,i)}, max\left\{ X_1^{(n,j)}: j = 1, \dots, n-1, \& \ i \neq j \right\}, X_1^{(\mathrm{n}-1)} \right\}$.

Now, in order to calculate the multiplicative factor when two suppliers are selected. Let us assume that supplier r and supplier k are selected. Hence, we first need to determine the minimum number of orders to be submitted to each supplier in a cycle time such that the lower bound of the quality constraint is satisfied, i.e.,

$$\frac{J_r q_r + J_k q_k}{J_r + J_k} = q_a \, . \tag{39}$$

Let M represents the total number of orders allocated to supplier r and supplier k during a cycle time. Thus, by substituting $M - J_r,$ for J_k we obtain

$$J_r = M \left(\frac{q_k - q_a}{q_k - q_r}\right). \tag{40}$$

Recall that the quality levels for the suppliers and the manufacturer are defined as positive rational numbers. Therefore, after the ratio $\left(\frac{q_k - q_a}{q_k - q_r}\right)$ shown in Equation (40) is turned into a fraction in the simplest form (i.e., the numerator and dominator are relative prime numbers), the minimum integer value for M that guarantees the integrality of J_r is equal to the value of the dominator, and the number of orders placed to supplier r per order cycle is equal to the numerator's value. Now, after determining the number of orders submitted to supplier r and supplier k per order cycle, we can obtain the weighted average unit setup cost needed to calculate the multiplicative factor.

Let us now discuss the determination of the multiplicative factor's feasible region when there are only two potential suppliers to order from, i.e., $n = 2$, and with taken into consideration suppliers' limitations in regard to capacity and quality requirements. Now, if $0 < k_1 < k_2$, then based on **Lemma 3** the optimal multiplicative factor is $X_1^* \in \left[X_1^{(1)}, X_1^{(2)}\right]$. Consequently, if $q_1 < q_a$

and $q_2 > q_a$, then the lower bound can be improved, i.e., $X_1^* \in \left[X_1^{(1,2)}, X_1^{(2)}\right]$, where $X_1^{(1,2)}$ is the multiplicative factor obtained when both suppliers are selected considering only the quality constraint. Note that, as $X_1^{(1,2)}$ is calculated such that the lower bound of the quality constraint is satisfied, hence, the obtained average quality level is equal to q_a. Therefore, any ordering policy that would result in a multiplicative factor that belongs to the range of $\left[X_1^{(1)},\ X_1^{(1,2)}\right)$ is not feasible, because the proportion of orders placed to supplier 1 in this range is more than the one placed to supplier 1 when $X_1^{(1,2)}$ is calculated. Hence, the corresponding average quality level will be less than q_a since supplier 1 has the lower quality level of the two suppliers. Similarly, if the both suppliers are such that $q_1 > q_a$ and $q_2 < q_a$, then the range of $\left(X_1^{(1,2)}, X_1^{(2)}\right]$ is not feasible, and the optimal multiplicative factor is $X_1^* \in \left[X_1^{(1)},\ X_1^{(1,2)}\right]$.

5.3. Numerical Examples

Two numerical examples are addressed in this section. In the first example, we develop upper and lower bounds on the selling price and the multiplicative factor when capacity and quality constraints are considered. Then, in this first example, we show how the obtained bounds are changed when we consider only the quality constraint. Next, in the second example, we develop a tight feasible region for the case in which there are only two potential suppliers and in doing so we take the suppliers' limitations in regard to capacity and quality into consideration.

Example 1. Consider a two-stage serial supply chain in which a manufacturer is located at the first stage and a retailer is at the second stage. The manufacturer can replenish inventory from six potential suppliers. Table 5.1 shows the parameter data of the six suppliers. Also, the manufacturer's minimum acceptable quality level is $q_a = 0.95$. The retailer is facing price-sensitive demand, i.e., $D = \propto P^{-\delta}$, where $\propto = 4.00\text{E}+08$ and $\delta = 2$.

Table 5.1. Suppliers' parameter information.

Supplier i	**Quality level** q_i	**Capacity** c_i (units/month)	**Unit setup cost** k_i ($/order)	**Unit purchasing cost** v_i ($/unit)
Supplier 1	0.94	9,000	6,100	45
Supplier 2	0.99	9,500	1,900	54
Supplier 3	0.94	8,000	14,500	49
Supplier 4	0.98	9,000	8,500	51
Supplier 5	0.96	32,000	30,000	50
Supplier 6	0.94	15,000	40,000	46

Table 5.2 shows the unit setup cost and the unit echelon cost for each stage. Note that the unit setup cost at the first stage is equal to the weighted average unit setup cost that depends on the number of orders submitted to the selected suppliers per order cycle.

Table 5.2. Parameter information for each stage.

Stages	**Unit setup cost** k_s ($/order)	**Unit holding cost** h_s ($/unit/month)	**Unit echelon cost** e_s ($/unit/month)
Stage 1	-	4	4
Stage 2	1,000	34	30

Next, using **Algorithm I**, we need to find the dominating supplier(s) (i.e., the supplier who generates the highest profit per time unit for the company when capacity and quality constraints are not considered). Table 5.3 shows the result when each supplier is selected separately and when there are neither capacity nor quality requirements.

Table 5.3. Optimal solution when each supplier is selected separately.

Supplier i	**Order quantity** Q_1(units)	**Multiplicative factor** X_1	**Order quantity** Q_2(units)	**Selling price** P ($/unit)	**Demand rate** D(units/month)	**Profit rate** TP ($/month)
1	12,204.95	7	1,743.56	92.15	47,108.65	2,119,889
2	5,822.70	4	1,455.68	110.03	33,039.83	1,784,263
3	16,574.69	10	1,657.47	100.96	39,245.79	1,923,043
4	12,471.34	8	1,558.92	104.65	36,527.01	1,862,878
5	23,594.90	15	1,572.99	103.81	37,114.61	1,855,731
6	29,331.61	17	1,725.39	95.89	43,505.51	2,001,253

Now, we need to find the lower and upper bounds for the selling price and the manufacturer's multiplicative factor when both capacity and quality constraints are considered. Thus, based on **Theorem 1**, the lower bound for the selling price is $P_{lo} = min\ \{P_i^*: i = 1, \dots, 6\} = \$\ 92.15$ per unit. And, when Equation (28) is used, the upper bound for the selling price is $P_{up} = max\left\{P_i^*, (c_i/\propto)^{-1/\delta} : i = 1, \dots, 6\right\} = \$\ 210.82$ per unit. In addition, based on **Lemma 3**, the optimal multiplicative factor is $X_1^* \in [4,17]$. This problem is solved using the global solver in LINGO15.0 on a PC with an INTEL(R) Core (TM) 2 Duo Processor at 2.10 GHz and 4.0 gigabytes RAM. Table 5.4 shows the reduction in computational time when the selling price and multiplicative factor bounds are considered.

Table 5.4. CPU time comparison.

	Original problem without bounds	**Original problem with bounds**
X_1	$X_1 \geq 1$	$4 \leq X_1 \leq 17$
P	$P \geq 0.1$	$92.15 \leq P \leq 210.82$
CPU time	17 seconds	2 seconds

The optimal solution obtains a profit per time unit of 1,970,010 $/month, i.e., $J_1 = 21, J_2 = 1, J_3 = 11, J_4 = 21, J_5 = 0, J_6 = 35$, $Q_1 = 20{,}274.68$ units, $X_1 = 13$, $Q_2 = 1{,}559.59$ units, and $P = \$102.41$ per unit.

Next, let us assume that the suppliers have no capacity limits, i.e., $c_i \geq \propto P_i^{*-\delta}, i = 1, \ldots, n$. Then, based on **Theorem 1**, the upper bound for the selling price is equal to the maximum selling price obtained when each supplier is selected individually and when capacity and quality restrictions are not considered. Thus, from Table 5.3, $P_{up} = max\{P_i^*: i = 1, \ldots, 6\} = \$\ 110.03$ per unit. Note that the lower bound for the selling price remains unchanged, i.e., $P_{lo} = min\{P_i^*: i = 1, \ldots, 6\} = \$\ 92.15$ per unit.

And, in regard to the multiplicative factor, $X_1^* \in [4,17]$. However, the upper bound value is obtained when supplier 6, who has quality limitations, is selected. Therefore, the upper bound can be improved based on **Algorithm III**. Consequently, we need to find the supplier who has the maximum quality level, which in this case is supplier 2. However, supplier 2 does not have the highest unit setup cost. Therefore, we also need to calculate the multiplicative factor when supplier 6 is selected with supplier j such that $k_j > k_2$ and $q_j > q_a, j = 1, \ldots, 5$, and $j \neq 2$. Table 5.5 shows the number of orders submitted to each supplier per order cycle and the corresponding multiplicative factor.

Table 5.5. Calculation to improve the upper bound for the multiplicative factor.

Supplier i	$\boldsymbol{J_j}$	$\boldsymbol{J_6}$	$\boldsymbol{X_1^{(j,6)}}$	$\boldsymbol{max\left\{X_1^{(j,6)}\right\}}$	**Feasible region**
Supplier 2	1	4	16		
Supplier 4	1	3	16	16	[4,16]
Supplier 5	1	1	16		

Note that the number of orders shown in Table 5.5 are calculated such that the lower bound of the quality constraint is satisfied, i.e., $J_6 = M\left(\frac{q_j - q_a}{q_j - q_6}\right)$. Thus, after the ratio $\left(\frac{q_j - q_a}{q_j - q_6}\right)$ is turned into a fraction in the simplest form, the value for M that guarantees the integrality of J_6 is equal to the value of the dominator, and J_6 is equal to the numerator's value. Table 5.6 shows some reduction in the CPU time after we impose the proposed bounds.

Table 5.6. CPU time comparison.

	Original problem without bounds	**Original problem with bounds**
X_1	$X_1 \geq 1$	$4 \leq X_1 \leq 16$
P	$P \geq 0.1$	$92.15 \leq P \leq 110.03$
CPU time	4 seconds	<1 second

The optimal solution obtains a profit per time unit of 2,049,290 \$/month, i.e., $J_1 = 3, J_2 = 0,\ J_3 = 0\ , J_4 = 1, J_5 = 0,\ J_6 = 0$, $Q_1 = 12{,}072.18$ units, $X_1 = 7$, $Q_2 = 1{,}724.60$ units, and $P = \$\ 95.27$ per unit.

Example 2. Consider the same two-stage serial supply chain presented in Example 1. The only difference between this example and the first example is that the manufacturer can replenish inventory from only two potential suppliers. Table 5.7 shows the parameter data of the two suppliers.

Table 5.7. Suppliers' parameter information.

Supplier i	**Quality level** q_i	**Capacity** c_i (units/month)	**Ordering cost** k_i (\$/order)	**Unit purchasing cost** v_i (\$/unit)
Supplier 1	0.92	5,000	7,500	25
Supplier 2	0.962	30,000	35,000	50

The goal of this example is to find a tight feasible region for the multiplicative factor when the suppliers' capacity and quality are considered. Thus, as a first step, we use **Algorithm 1** to find the dominating supplier. Table 5.8 shows the results when each supplier is selected separately in a case in which there are neither capacity nor quality constraints.

Table 5.8. Optimal solution when each supplier is selected separately.

Supplier i	**Order quantity** Q_1 (units)	**Multiplicative factor** X_1	**Order quantity** Q_2 (units)	**Selling price** P (\$/unit)	**Demand rate** D (units/month)	**Profit rate** TP (\$/month)
Supplier 1	24,678.22	8	3,084.78	51.25	152,253.7	3,806,341
Supplier 2	25,332.76	16	1,583.30	104.03	36,963.48	1,848,174

Let us initially assume that both suppliers will be selected to satisfy the quality requirement. Hence, we need to find the number of orders submitted to each supplier during a cycle time such that the quality requirement is satisfied. Accordingly, $M = 7, J_1 = 2$, and $J_2 = 5$. Therefore, by using Equation (29), the integer multiplicative factor when supplier 1 and 2 are selected with only the quality constraint taken into consideration is $X_1^{(1,2)} = 14$. And, given that $k_1 < k_2$ and $q_1 < q_a$, then the optimal multiplicative factor, $X_1^* \in \left[X_1^{(1,2)}, X_1^{(2)}\right] = [14{,}16]$. Now, we need to find the selling price bounds when both capacity and quality constraints are considered. Thus, by using **Theorem 1** and Equation (28), $P^* \in [\$\ 51.25, \$\ 282.84]$. Table 5.9 shows the difference between the computational time when the bounds are imposed and when they are not imposed.

Table 5.9. CPU time comparison.

	Original problem without bounds	Original problem with bounds
X_1	$X_1 \geq 1$	$14 \leq X_1 \leq 16$
P	$P \geq 0.1$	$51.25 \leq P \leq 282.84$
CPU time	44 seconds	20 seconds

The optimal solution obtains a profit per time unit of 1,977,553 $/month (i.e., $J_1 = 1, J_2 = 6, Q_1 = 23{,}184.05$ units, $X_1 = 15$, $Q_2 = 1{,}545.60$ units, and $P = \$106.90$ per unit.

5.4. Conclusions

In this chapter, we have considered the integrated pricing and supplier selection problem in a two-stage supply chain. We have developed an MINLP model to find the number of orders placed to the selected suppliers per order cycle and the corresponding order quantity, the inventory lot size for the second stage, and the retailer's selling price such that the profit per time unit is maximized. Moreover, we have proposed an algorithm to find the supplier who, if selected, would yield the highest profit per time unit when capacity and quality limitations are not considered. Then, we show how the algorithm can be edited to find the best supplier when the capacity of each potential supplier's capacity is taken into consideration. In addition, we have identified lower and upper bounds for the optimal selling price. We have also identified the feasible region for the multiplicative integer factor and shown a tighter feasible region when there are only two potential suppliers. Further, we have presented two numerical examples to explain the proposed model and the calculation of the lower and upper bounds for the selling price and the integer multiplicative factor.

Chapter 6: Integrated Pricing and Lot-sizing Decisions in a Serial Supply Chain

6.1. Problem Description and Model Development

In the present chapter, we consider a company that controls a series of supply chain stages through which a single product is produced. Figure 6.1 describes the serial supply chain that we consider in this chapter. The manufacturer in the first stage facility determines the required amount of raw material and how often it should be ordered from the selected suppliers in a cycle time taken into account suppliers' limitations in regard to capacity and quality. Then after the first stage, the products made from the raw materials go through a series of stages that can represent additional manufacturing facilities or warehouses until reaching the last stage, i.e., the distribution center. Finally, the distribution center identifies the optimal selling price, as we assume that demand is price-sensitive. Therefore, the goal is to simultaneously coordinate supplier selection, inventory replenishment, and pricing decisions such that the profit per time unit is maximized.

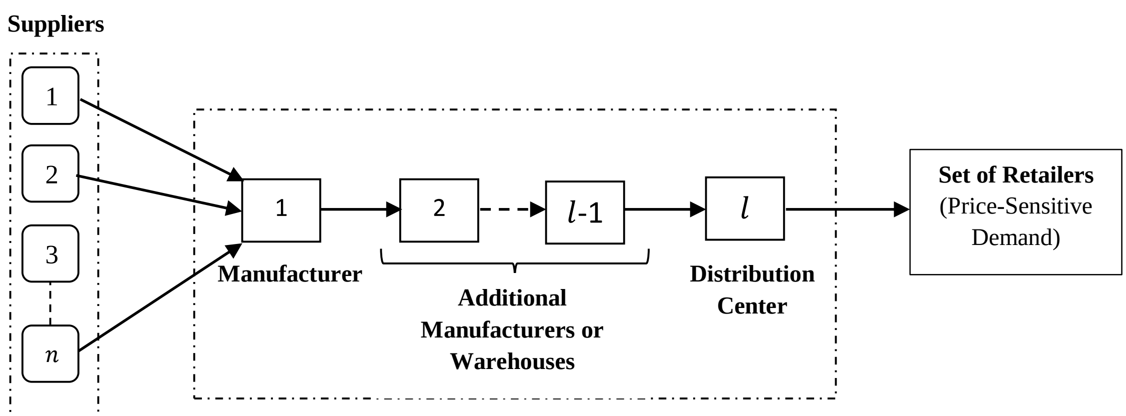

Figure 6.1. Pricing and supplier selection in a serial system.

The challenging aspect of the serial system shown in Figure 6.1 is that the inventory at any given stage is used to replenish the inventory for the next stage. For instance, the manufacturer at the first stage uses its inventory to periodically replenish the inventory at the second stage. Hence, once the inventory level at stage 2 drops to zero, then immediately stage 1 uses its inventory to replenish the inventory at stage 2 by providing a batch of the required order quantity. Accordingly, the manufacturer should receive the order quantities placed to the selected suppliers in time to avoid any inventory shortages. Note that, the lead time is fixed and or it can be assumed to be zero because each stage would order just at the right time such that the order quantity is received when inventory level drops to zero.

Let us now introduce the notation and the assumptions underlying our model. Then, we show the development of the proposed MINLP model. Listed below are the model's indices, parameters, and decision variables.

Indices

s Stage, where $s = 1, \ldots, l$.

i Potential supplier, where $i = 1, \ldots, n$.

Parameters

c_i Production capacity rate of supplier i, where $i = 1, \ldots, n$.

q_i Quality level (i.e., portion of acceptable units) of supplier i, $0 < q_i \leq 1$, where $i = 1, \ldots, n$.

q_a Minimum acceptable quality level for the manufacturer at stage 1, $0 < q_a \leq 1$.

v_{1i} Purchasing cost of one unit from supplier i at stage 1, where $i = 1, \ldots, n$. Note that v_1 is the weighted average unit purchasing cost for the selected suppliers.

k_s Setup cost for placing one order at stage s, where $s = 1, \ldots, l$.

k_{1i} Setup cost for placing one order to supplier i at stage 1, where $i = 1, \ldots, n$. Note that k_1 is the weighted average unit setup cost for the selected suppliers.

h_s Unit inventory holding cost at stage s, where $s = 1, \ldots, l$.

e_s Unit echelon inventory holding cost at stage s, i.e., $e_1 = h_1$, and $e_s = h_s - h_{s-1}$, where $s = 2, \dots, l$.

$\propto$ Scaling factor for price-sensitive demand.

δ Price elasticity index for price-sensitive demand.

m Maximum number of orders that can be placed to the selected suppliers in an order cycle.

Decision variables

J_i Number of orders submitted to supplier i per order cycle.

X_{1i} Multiplicative factor for order quantity submitted to supplier i, where $i = 1, \dots, n$.

X_s Multiplicative factor for order quantity received from stage $s + 1$, where $s = 2, \dots, l - 1$.

Q_s Order quantity at stage s, i.e., $Q_s = X_s Q_{s+1}$, where $s = 2, \dots, l - 1$.

Q_{1i} Order quantity placed to supplier i, i.e., $Q_{1i} = X_{1i} Q_2$, where $i = 1, \dots, n$.

P Distribution center's optimal selling price.

D Distribution center's price-sensitive demand rate, i.e., $D = \propto P^{-\delta}$.

The assumptions are summarized as follows:

1. The time horizon is infinite.
2. Backordering is not allowed.
3. The supply chain produces a single type of product.
4. The lead times are constant and can be assumed to be zero.
5. All the supply chain stages have an infinite production rate and storage capacity.
6. The variable cost components for each stage are mainly the setup and holding costs, with the exception of the first stage in which purchasing cost is also considered.

7. An inventory replenishment decision must follow the zero-nested inventory ordering policy. Love (1972) and Schwarz (1973) proved that the zero-nested inventory policy is optimal for a serial inventory system.

8. The product gains more value as it moves down to the downstream stages. Thus, the unit holding cost increases, as the product gets closer to the final consumer, i.e., $h_1 < h_2 < \cdots < h_l$. In addition, we consider the unit echelon inventory holding cost for each stage, i.e., $e_1 = h_1$, and $e_s = h_s - h_{s-1}$, $s = 2, \ldots, l$.

9. Two consecutive stages are treated separately if and only if $\frac{k_s}{e_s} > \frac{k_{s+1}}{e_{s+1}}$, $s = 1, \ldots, l-1$. Otherwise, they are combined into one stage with a unit setup cost of $k_s + k_{s+1}$ and a unit echelon holding cost of $e_s + e_{s+1}$. Schwarz and Schrage (1975), and Muckstadt and Roundy (1993) proved the optimality of applying this rule for combining consecutive stages in a serial supply chain.

10. Retailers' demand rate D occurs at stage l and is a decreasing power function of the selling price P, i.e., $D(P) = \propto P^{-\delta}$, where $\propto$ and δ are the scaling factor and price elasticity index, respectively. And, to guarantee that the selling price has a significant impact on the demand, we assume that $\delta > 1$ (Kim and Lee, 1998).

Note that, Assumption (7) indicates that an optimal policy in a serial inventory system must be nested and inventory replenished only when inventory level is zero. The zero-inventory ordering policy implies that the inventory at each stage is replenished by the immediate upstream stage only when the inventory level drops to zero and it is time to replenish the inventory at the immediate downstream stage. The nested ordering policy implies that when any stage in the supply chain orders, all the downstream stages order as well, where the quantity ordered at each stage is an integer multiple of the order quantity at the immediate downstream stage, i.e., $Q_{1i} = X_{1i}Q_2$, $i =$

$1, \dots, n$, and $Q_s = X_s Q_{s+1}$, $s = 2, \dots, l-1$. Figure 6.2 provides an example of a two-stage supply chain with inventory shortages because the order quantity placed at stage 1 is not an integer multiple of the order quantity placed at stage 2. The example assumes that the demand rate is $D = 1$ unit per time unit, the order quantity placed at the first stage is $Q_1 = 3$ units, and the order quantity placed at the second stage is $Q_2 = 2$ units.

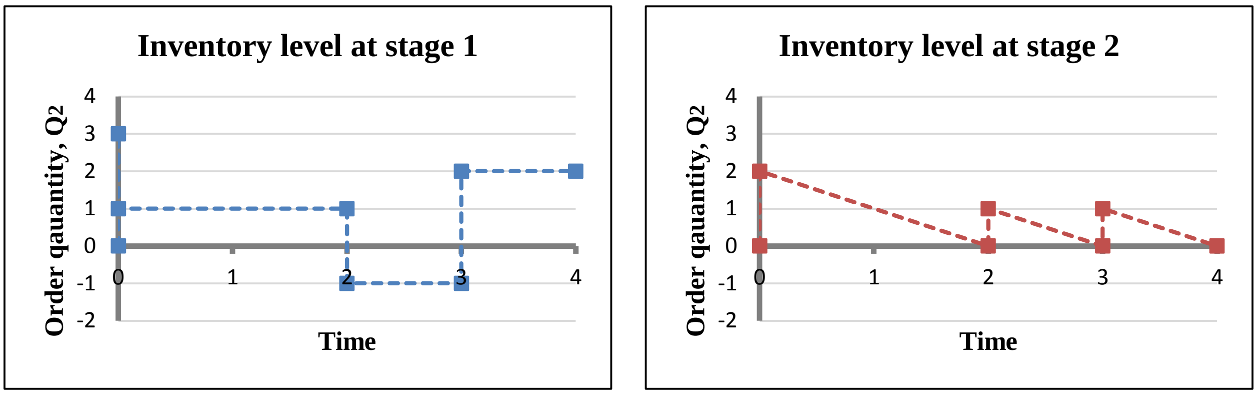

Figure 6.2. Unsynchronized inventory levels in a two-stage supply chain.

As shown in Figure 6.2, at the beginning of the cycle time, stage 1 uses its inventory of 3 units to deliver 2 units to the second stage. Hence, the inventory level at stage 1 drops to 1 unit. Then after two time units, the inventory level at stage 2 drops to zero. Hence, stage 2 immediately places an order of 2 units to stage 1. However, stage 1 does not have enough inventory to cover the required order quantity, thus inventory shortages occur at the first stage and only 1 unit is delivered to the second stage. The second unit has to be delivered at time unit 3. Note that, if Q_1/Q_2 is not rational, then the ordering policy would also result in a nonstationary inventory policy where the order cycle is not repeatable.

On the other hand, if the inventory levels are synchronized such that the order quantity placed at stage 1 is an integer multiple of the order quantity placed at stage 2. Then an optimal policy can be obtained and inventory shortages are voided. Figure 6.3 shows an example of coordinated inventory levels in a two-stage supply chain. The example assumes that the demand rate is $D = 1$ unit per time unit, the order quantity placed at the first stage is $Q_1 = 4$ units, and the order quantity placed at the second stage is $Q_2 = 2$ units.

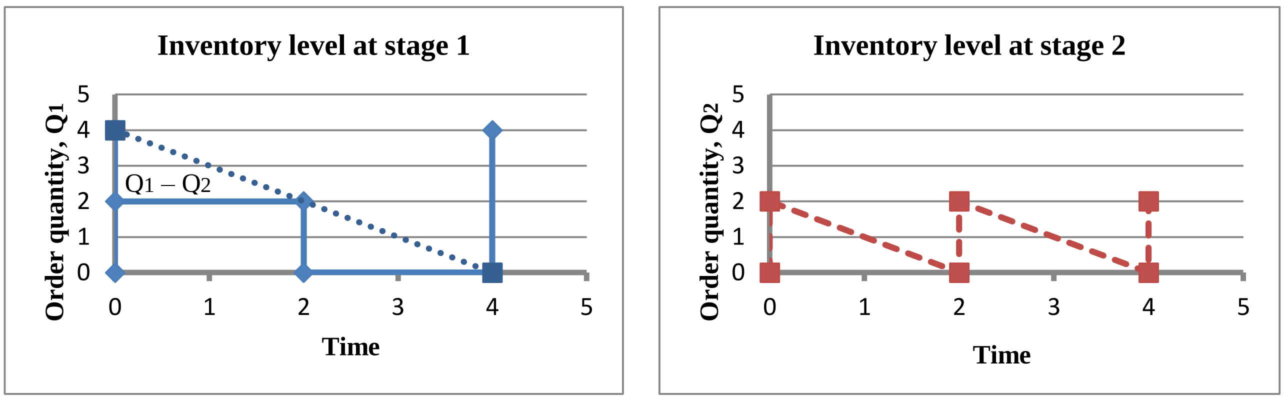

Figure 6.3. Synchronized inventory levels in a two-stage supply chain.

A shown in Figure 6.3, inventory shortages are avoided because the order quantity at the first stage is an integer multiple of the order quantity placed at the second stage. Note that, stage 1 replenishes its inventory only when the inventory level drops to zero and it is the time to replenish the inventory of the second stage in order to avoid inventory leftovers and shortages.

Assumption (8) is made to simplify the calculation of the holding cost per time unit (Muckstadt and Roundy, 1993). Recall that for a single-stage system, the on-hand inventory follows the well-known saw-toothed curve. Now, if we consider a two-stage serial system in which the zero-nested inventory policy is applied, then the on-hand inventory for stage 2 follows the saw-toothed curve, but the on-hand inventory for stage 1 does not take this form, because over a certain time range the on-hand inventory level for stage 1 remains constant until stage 2 places an order. Thus, if we add the amount of on-hand inventory at stage 1 and the amount of on-hand inventory at stage 2, we obtain what is known by the echelon inventory for stage 1 which follows the saw-toothed curve. Therefore, it is easier to consider the echelon inventory instead of the on-hand inventory because the average on-hand inventory for stage 1 is a function of the order quantity ordered at stage 1 and the order quantity ordered at stage 2. Note that the holding cost per time unit that is calculated using the unit echelon holding cost is the same as that obtained using the unit inventory holding cost. For instance, the holding cost per time unit for the two-stage supply chain shown in Figure 6.3 is $TH = \frac{1}{2}(Q_1 - Q_2)h_1 + \frac{1}{2}Q_2h_2$, where $Q_1 = X_1Q_2$, and X_1 is a positive integer multiplicative factor. Hence, TH can be written as $TH = \frac{Q_2}{2}\big((X_1 - 1)h_1 + h_2\big) = \frac{Q_2}{2}(X_1e_1 + e_2)$, where $e_1 = h_1$, and $e_2 = h_2 - h_1$.

In addition, Assumption (9) ensures that the condition for any feasible solution is satisfied, i.e., the order quantity at any stage must be greater than or at least equal to that of the immediate downstream stage. Thus, if two consecutive stages are such that $Q_s < Q_{s+1}$, $s = 1, \dots, l - 1$, then the order quantities at these consecutive stages are set to be equal to each other in order not to violate the feasibility condition. This is achieved by combining the two stages into one stage with a unit setup cost of $k_s + k_{s+1}$ and a unit echelon holding cost of $e_s + e_{s+1}$. Similarly, if two consecutive stages are such that $Q_s = Q_{s+1}$, $s = 1, \dots, l - 1$, then these two stages can also be treated as a single combined stage. Hence, by using the economic order quantity (EOQ) formula, any two consecutive stages are combined into one stage if $\frac{k_s}{e_s} \leq \frac{k_{s+1}}{e_{s+1}}$, $s = 1, \dots, l - 1$.

Based on these assumptions, the following MINLP model $(M1)$ is formulated:

$$Max.\ TP_1 = \propto P^{1-\delta} - \frac{1}{Q_1}\left(\propto P^{-\delta}\sum_{i=1}^{n} J_i\, k_{1i} + \frac{e_1}{2}\sum_{i=1}^{n} J_i Q_{1i}^2 + \propto P^{-\delta}\sum_{i=1}^{n} J_i\, Q_{1i} v_{1i}\right)$$

$$-\left(\propto P^{-\delta}\sum_{s=2}^{l}\frac{k_s}{Q_s} + \frac{1}{2}\sum_{s=2}^{l} Q_s e_s\right),$$

subject to

$$Q_1 = \sum_{i=1}^{n} J_i\, Q_{1i}\,, \qquad (1)$$

$$\propto P^{-\delta} J_i Q_{1i} \leq Q_1 c_i\,, \qquad i = 1, \dots, n\,, \qquad (2)$$

$$\sum_{i=1}^{n} J_i\, Q_{1i} q_i \geq Q_1 q_a\,, \qquad (3)$$

$$Q_{1i} = X_{1i} Q_2\,, \qquad i = 1, \dots, n\,, \qquad (4)$$

$$X_{1i} \geq 1\,, \qquad integer, i = 1, \dots, n\,, \qquad (5)$$

$$Q_s = X_s Q_{s+1}\,, \qquad s = 2\,, \dots, l-1\,, \qquad (6)$$

$$X_s \geq 1\,, \qquad integer, s = 2\,, \dots, l-1\,, \qquad (7)$$

$$\sum_{i=1}^{n} J_i \leq m\,, \qquad (8)$$

$$J_i \geq 0\,, \qquad integer,\ i = 1, \dots, n\,, \qquad (9)$$

$$Q_{1i} \geq 0\,, \qquad i = 1, \dots, n\,, \qquad (10)$$

$$Q_s > 0\,, \qquad s = 2\,, \dots, l\,, \qquad (11)$$

$$P > 0\,. \qquad (12)$$

The objective function consists of three components: the first one is the revenue per time unit $\propto P^{1-\delta}$; the second component includes the replenishment cost (i.e., setup cost and purchasing cost per time unit) and holding cost per time unit at the first stage; and, lastly, the third component is the setup and holding costs per time unit for stages 2 to l. The model is subject to a number of constraints. Constraint (1) represents the total quantity ordered from all the selected suppliers during a cycle time. Constraint set (2) requires that a supplier's replenishment rate does not exceed the capacity rate. Constraint (3) imposes the manufacturer's restriction on the minimum acceptable quality level. Constraint sets (4) and (5) guarantee that the purchased order quantity from each supplier is an integer multiple of the quantity ordered at stage 2, where a different multiplicative factor can be applied to each supplier. Constraint sets (6) and (7) ensure that the order quantity at each stage is an integer multiple of the order quantity at the immediate downstream stage. Constraint (8) imposes an upper bound on the total number of orders submitted to suppliers in a repeating order cycle. This constraint is important in regard to controlling the length of the cycle time, as discussed by Mendoza and Ventura (2008). Constraint set (9) imposes the integrality condition on the number of orders placed to the selected suppliers during an order cycle. Constraint set (10) represents the non-negativity conditions. Constraint sets (11) and (12) ensure avoiding the division by zero. Note that the objective function of Model $(M1)$ has singularities at $P = 0$ and $Q_s = 0$, $s = 2, \dots, l$.

Note that Model $(M1)$ makes it possible to order different quantities from the selected suppliers because each order quantity is multiplied by a different integer multiplicative factor. Model $(M1)$ is compared with models that consider only the same multiplicative factor for the quantity of each order submitted to the selected supplier, e.g., Mendoza and Ventura (2010). For this purpose, Model $(M2)$ is developed by allowing only an equal-size order quantity to be submitted to the selected suppliers. Hence, Model $(M1)$ is edited so that $Q_{1i} = Q_1$, and $X_{1i} = X_1$, $i = 1, \dots, n$. In Section 6.3, we show that Model $(M1)$ obtains a higher profit per time unit than Model $(M2)$.

6.2. Heuristic Algorithm using the Power-of-Two (POT) Policy

As the number of suppliers and stages increases, the problem becomes more complex and, consequently, more computational time is needed to solve it. Therefore, there is an increasing need for a heuristic algorithm that is capable of reducing the feasible region to solve the problem in a timely manner, and obtaining near-optimal solutions. One possible way to reduce the size of the feasible region is to consider the POT nested policy, which restricts the integer multiplicative factors to be powers of two, i.e., $X_{1i} = 2^{r_{1i}}$, $i = 1, \dots, n$, and $X_s = 2^{r_s}$, $s = 2, \dots, l$, where r_{1i} and r_s are non-negative integers. Roundy (1986) proved that this policy provides a solution that is within 6% of the model's optimal nested solution if the base cycle time is fixed and within 2% of the model's optimal nested solution if the base cycle time is treated as a decision variable. In addition, POT policies haven been considered by many scholars to determine inventory policies in a multi-stage supply chain (Maxwell and Muckstadt 1985; Roundy 1989; Hahm and Yano, 1995; Ouenniche and Boctor, 2001; Mendoza and Ventura 2010), and were found to provide a practical approach in determining supply chain inventory policies as they are simple to implement and computationally efficient. In addition, Khouja (2003) showed that implementing POT policies yield to significant cost reductions. Therefore, we propose a heuristic algorithm to solve the pricing and supplier selection problem in the case of a serial supply chain using the POT inventory policy.

Let us first turn our attention to developing an upper bound for Model ($M1$). An upper bound for the problem can be obtained by relaxing the coordination mechanism from Model ($M1$), which if it remained would ensure that the order quantity for any given stage in the supply chain would be an integer multiple of the order quantity of the immediate downstream stage. Therefore, by replacing Constraints (4–7) in Model $(M1)$ with $Q_{1i} \geq Q_2, i = 1, \dots, n,$ and $Q_s \geq Q_{s+1},\ s = 2, \dots, l-1$, the relaxed model that provides an upper bound, denoted as $(\overline{M1})$, is obtained:

$$Max.\ \overline{TP}_1 = \propto P^{1-\delta} - \frac{1}{Q_1}\left(\propto P^{-\delta} \sum_{i=1}^{n} J_i\, k_{1i} + \frac{e_1}{2} \sum_{i=1}^{n} J_i Q_{1i}^2 + \propto P^{-\delta} \sum_{i=1}^{n} J_i\, Q_{1i} v_{1i}\right)$$

$$- \left(\propto P^{-\delta} \sum_{s=2}^{l} \frac{k_s}{Q_s} + \frac{1}{2} \sum_{s=2}^{l} Q_s e_s\right),$$

subject to

$$Q_1 = \sum_{i=1}^{n} J_i\, Q_{1i}\,, \tag{13}$$

$$\propto P^{-\delta} J_i Q_{1i} \leq Q_1 c_i\,, \qquad i = 1, \dots, n\,, \tag{14}$$

$$\sum_{i=1}^{n} J_i\, Q_{1i} q_i \geq Q_1 q_a\,, \tag{15}$$

$$Q_{1i} \geq Q_2\,, \qquad i = 1, \dots, n\,, \tag{16}$$

$$Q_s \geq Q_{s+1}\,, \qquad s = 2\,, \dots, l-1\,, \tag{17}$$

$$\sum_{i=1}^{n} J_i \leq m\,, \tag{18}$$

$$J_i \geq 0\,, \qquad integer,\ i = 1, \dots, n\,, \tag{19}$$

$$Q_{1i} \geq 0\,, \qquad i = 1, \dots, n\,, \tag{20}$$

$$Q_s > 0\,, \qquad s = 2\,, \dots, l\,, \tag{21}$$

$$P > 0\,. \tag{22}$$

Even though, Model $(\overline{M1})$ provides an upper bound on the profit of the optimal solution, the resulting solution might not satisfy the zero-nested inventory policy because the coordination mechanism is dropped. Hence, inventory shortages might occur, and the inventory policy becomes non-stationary.

Note that, solving Model ($\overline{M1}$) for a given selling price is equivalent to optimizing the inventory decisions at each stage separately without taking into consideration the inventory policy of other stages. Hence, one way to obtain an upper bound for a given selling price is to consider one stage at a time and optimize its inventory decisions alone. Thus, the supplier selection problem at the first stage needs to be solved, and the optimal order quantities for stages$s$, $s = 2, \dots, l$ need to be determined by applying the EOQ formula at each stage using the unit echelon holding cost. If the order quantity at any stage is greater than or equal the order quantity of the downstream stage, then the obtained solution is optimal for Model ($\overline{M1}$) and the corresponding profit is an upper bound for Model ($M1$).

The heuristic algorithm consists of four main steps. In the first step, initial values for the order quantities are obtained for each stage. For this purpose, an initial value is set for the selling price, i.e., $P = P_o$. Then, the EOQ formula is used to compute the initial order quantities at each stage. Let Q_{1i}^R donate the initial order quantity placed to supplier i, i.e., $Q_{1i}^R = \sqrt{2 \propto P^{-\delta} k_{1i}/e_1}$, $i = 1, \dots, n$. And, let Q_s^R donate the initial order quantity placed at stage s, i.e., $Q_s^R = \sqrt{2 \propto P^{-\delta} k_s/e_s}$, $s = 2, \dots, l$. This step is similar to solving Model ($\overline{M1}$) for a given selling price and without taking into consideration suppliers' limitation in regard to capacity and quality constraints. In addition, the resulting solution might not satisfy the zero-nested inventory policy because the coordination mechanism is dropped.

Nevertheless, the obtained initial order quantities are going to be altered to ensure that the order quantities between consecutive stages are coordinated with multiplicative factors that are powers of two, i.e., $Q_{1i} = X_{1i} Q_2$, $X_{1i} = 2^{r_{1i}}$, $i = 1, \dots, n$, and $Q_s = X_s Q_{s+1}$, $X_s = 2^{r_s}$, $s = 2, \dots, l-1$, where r_{1i} and r_s are non-negative integers. Hence, as a second step, we generalize the POT procedure proposed by Roundy (1986) in order to determine a POT solution using the initial values of the order quantities. Procedure 1 shown below presents a generalized POT procedure that can be used for a multi-supplier serial supply chain system in which a different multiplicative factor is assigned to the order quantities placed to the selected suppliers.

Procedure 1 below shows the detailed steps that must be followed in altering the initial order quantities. Let Q_{1i}^{POT}, $i = 1, \dots, n$, and Q_s^{POT}, $s = 2, \dots, l$, be the POT solution that is obtained from implementing Procedure 1. The procedure starts with using the POT solution for that last stage, i.e., Q_l^{POT} in order to determine the POT solution for the immediate upstream stage, i.e., Q_{l-1}^{POT}. Not that, Q_l^{POT} is not known initially, hence Q_l^R can instead be used as an estimation of Q_l^{POT}. Then, Q_{l-1}^{POT} is used to obtain the POT solution for stage $l - 2$, i.e., Q_{l-2}^{POT}. Similarly, the procedure continues until the POT solution is obtained for stage 2. Then, as we are allowing different order quantities to be placed to the selected suppliers, each order quantity placed to the selected supplier must be an integer multiple of the order quantity for stage 2. Thus, the POT solution for stage 2 is used to determine the POT solution for the order quantity placed at stage 1 to supplier i, i.e., Q_{1i}^{POT}, $i = 1, \dots, n$.

Procedure 1. A Generalized Power-of-Two Procedure

Step I. If Q_l^{POT} is known, then go to Step II. Otherwise, set Q_l^{POT} to Q_l^R.

Step II. For $s = l-1, \dots, 2$, find the positive integer r, such that $2^r Q_{s+1}^{POT} \leq Q_s^R \leq 2^{r+1} Q_{s+1}^{POT}$.

If $\frac{Q_s^R}{2^r Q_{s+1}^{POT}} \leq \frac{2^{r+1} Q_{s+1}^{POT}}{Q_s^R}$, set $X_s = 2^r$ and $Q_s^{POT} = X_s\, Q_{s+1}^{POT}$.

Otherwise, set $X_s = 2^{r+1}$ and $Q_s^{POT} = X_s\, Q_{s+1}^{POT}$.

Step III. For $s = 1$, find the positive integer r, such that $2^r Q_2^{POT} \leq Q_{1i}^R \leq 2^{r+1} Q_2^{POT}$, $i = 1, \dots, n$.

If $\frac{Q_{1i}^R}{2^r Q_2^{POT}} \leq \frac{2^{r+1} Q_2^{POT}}{Q_{1i}^R}$, set $X_{1i} = 2^r$ and $Q_{1i}^{POT} = X_{1i} Q_2^{POT}$.

Otherwise, set $X_{1i} = 2^{r+1}$ and $Q_{1i}^{POT} = X_{1i} Q_2^{POT}$.

Once the POT solutions are determined, we need to update the value of the selling price and find the number of orders that must be placed to the selected suppliers during a cycle time to ensure that the model's constraints are not violated based on the order quantities obtained from Procedure 1. Therefore, as a third step, the supplier selection problem at the first stage must be solved given that the order quantities submitted to each supplier, i.e., Q_{1i}, are equal to $Q_{1i}^{POT}, i = 1, \dots, n$. Thus, Model $(M1')$ below is developed to solve the supplier selection problem at the first stage, where Q_{1i} need to be treated as input parameters:

$$Max.\ TP_1' = \propto P^{1-\delta} - \frac{1}{Q_1}\left(\propto P^{-\delta} \sum_{i=1}^{n} J_i\, k_{1i} + \frac{e_1}{2} \sum_{i=1}^{n} J_i Q_{1i}^2 + \propto P^{-\delta} \sum_{i=1}^{n} J_i\, Q_{1i} v_{1i}\right),$$

subject to

(1), (2), (3), (8), (9), (10), and (12).

Now, after determining the X_{1i},X_s, J_i, and P values, as a fourth step, we need to refine the value of the order quantity at the last stage Q_l in order to try obtaining an optimal POT solution for Model ($M1$). Consequently, to obtain a closed form solution that refines the value of Q_l, let us first rewrite the objective function in Model ($M1$) in terms of Q_l, i.e.,

$$TP_1 = \propto P^{1-\delta} - \frac{1}{Q_l \sum_{i=1}^{n} J_i Y_{1i}} \left(\propto P^{-\delta} \sum_{i=1}^{n} J_i\, k_{1i} + \frac{e_1 Q_l^2}{2} \sum_{i=1}^{n} J_i Y_{1i}^2 + \propto P^{-\delta} Q_l \sum_{i=1}^{n} J_i\, Y_{1i} v_{1i} \right) - \left(\frac{\propto P^{-\delta}}{Q_l} \sum_{s=2}^{l} \frac{k_s}{Y_s} + \frac{Q_l}{2} \sum_{s=2}^{l} Y_s e_s \right) \tag{23}$$

where $Y_{1i} = X_{1i}X_2X_3 \dots X_l$, $i = 1, \dots, n$ and $Y_s = X_s X_{s+1} \dots X_l$, $s = 2, \dots, l$. Note that the capacity and quality constraints are not functions of Q_l Also, note that TP_1 is a function of only Q_l because the selling price P and the number of orders per supplier per cycle J_i, $i = 1, \dots, n$ are determined by solving Model ($M1'$). And, the multiplicative factors X_{1i}, $i = 1, \dots, n$ and X_s, $s = 2, \dots, l$, are obtained from Procedure 1. Thus, we take the first partial derivative of $TP_1$with respect to Q_l:

$$\frac{\partial TP_1}{\partial Q_l} = \frac{\propto P^{-\delta}}{Q_l^2} \left(\frac{\sum_{i=1}^{n} J_i k_{1i}}{\sum_{i=1}^{n} J_i Y_{1i}} + \sum_{s=2}^{l} \frac{k_s}{Y_s} \right) - \frac{1}{2} \left(\frac{e_1 \sum_{i=1}^{n} J_i Y_{1i}^2}{\sum_{i=1}^{n} J_i Y_{1i}} + \sum_{s=2}^{l} Y_s e_s \right). \tag{24}$$

Note that TP_1 is a concave function of Q_l , i.e., $\frac{\partial^2 TP_1}{\partial {Q_l}^2} = -\frac{2\propto P^{-\delta}}{Q_l^3} \left(\frac{\sum_{i=1}^{n} J_i k_{1i}}{\sum_{i=1}^{n} J_i Y_{1i}} + \sum_{s=2}^{l} \frac{k_s}{Y_s} \right) < 0$. Thus, by setting Equation (24) to zero and solving for Q_l, we establish the optimal EOQ policy as follows:

$$Q_l^* = \sqrt{\frac{2\propto P^{-\delta} \left(\frac{\sum_{i=1}^{n} J_i k_{1i}}{\sum_{i=1}^{n} J_i Y_{1i}} + \sum_{s=2}^{l} \frac{k_s}{Y_s} \right)}{\frac{e_1 \sum_{i=1}^{n} J_i Y_{1i}^2}{\sum_{i=1}^{n} J_i Y_{1i}} + \sum_{s=2}^{l} Y_s e_s}}, \tag{25}$$

where $Y_{1i} = X_{1i}X_2X_3 \dots X_l$, $i = 1, \dots, n$ and $Y_s = X_s X_{s+1} \dots X_l$, $s = 2, \dots, l$.

Now, we use the refined value of Q_l as an input to Procedure 1 to make sure all the order quantities placed at the upstream stages are power-of-two integer multiples of the refined Q_l value. Hence, we repeat Procedure 1 by first updating the initial order quantities using the selling price P obtained from solving Model $(M1')$. And then the remaining steps of the algorithm are followed. If the multiplicative factors and the number of orders per supplier per cycle remain unchanged, then we can terminate the algorithm and report the resulting solution. The heuristic algorithm is summarized as follows:

Heuristic Algorithm. Solving the Pricing and Supplier Selection Problem in a Serial System

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Let $t = 0$, and set $P = P_0$, where P_0 is an initial value for the selling price.

Step 1. ***Obtain initial values for the order quantities.***

Compute $Q_{1i}^R = \sqrt{2 \propto P^{-\delta} k_{1i}/e_1}$, $i = 1, \dots, n$.

Compute $Q_s^R = \sqrt{2 \propto P^{-\delta} k_s/e_s}$, $s = 2, \dots, l$.

Step 2. ***Modify the initial order quantities.***

Use Procedure 1 to find Q_{1i}^{POT} and X_{1i}, $i = 1, \dots, n$, and to find Q_s^{POT} and X_s, $s = 2, \dots, l$.

Step 3. ***Update the selling price and find the number of orders per supplier per cycle.***

Solve Model $(M1')$ given Q_{1i}^{POT}, $i = 1, \dots, n$, to obtain P and J_i, $i = 1, \dots, n$.

Step 4. ***Refine the order quantity placed at the last stage.***

Use Equation (25) to find a new value for Q_l.

Step 5. ***Check for termination.***

1. If $t = 0$, let $t = t + 1$ and go to Step 1.
2. If the number of orders placed to the selected suppliers during a cycle time and the multiplicative factors at iteration t are the same as those obtained in iteration $t - 1$, then stop. Otherwise, let $t = t + 1$ and go to Step 1.

Now, in order to obtain an initial value for the selling price P_0, we first need to determine several values for the selling price, then these values can be used to estimate an initial value. For instance, the mean/median can be calculated to be set as an initial value for the selling price. One possible way to obtain several values for the selling price is by considering the supplier selection problem at the first stage where one supplier can be selected. Thus, we need to determine the selling price obtained when each supplier is selected individually without considering capacity and quality constraints. Note that, Abad (1988) proposed an algorithm to determine the optimal selling price and order quantity for a retailer purchasing a product from a single supplier. Therefore, we can use Abad's algorithm to determine the optimal selling price when each supplier is selected individually without considering capacity and quality constraints. Let P_i^* denote the optimal selling price when supplier i is selected individually without taking into consideration supplier's limitations in regard to capacity and quality, $i = 1, \dots, n$. Now, if the respective supplier's capacity is not enough to cover the maximum demand rate that the supplier can achieve when is selected individually and there is no capacity or quality limitations, i.e., $c_i < \propto P_i^{*-\delta}$, $i = 1, \dots, n$, then more values for the selling price can be obtained by considering the case where the supplier is selected at a full capacity, hence $P_i = (c_i/\propto)^{-1/\delta}$, $i = 1, \dots, n$.

6.3. Illustrative Examples

In this section, two numerical examples are analyzed. The first one illustrates the proposed algorithm and shows that it converges to a near-optimal solution in a timely manner. The second one provides a comparison between Model $(M1)$, and Model $(M2)$.

Example 1. Consider a five-stage serial supply chain in which a manufacturer is located at the first stage and a distribution center is at the last stage. And, the intermediate stages can represent additional manufacturers and warehouses. The manufacturer at the first stage can replenish raw material inventory from four potential suppliers. Table 6.1 shows the parameter data of the four suppliers. Also, the manufacturer's minimum acceptable quality level is $q_a = 0.95$. The retailers are facing price-sensitive demand, i.e., $D = \propto P^{-\delta}$, where $\propto = 4.0E + 08$ and $\delta = 2$.

Table 6.1. Suppliers' parameters information.

Supplier i	**Quality level** q_i	**Unit setup cost** k_i (\$/order)	**Unit purchasing cost** v_{1i} (\$/unit)	**Capacity rate** c_i (units/month)
1	0.94	5000	180	700
2	0.92	1500	150	1200
3	0.96	4500	240	900
4	0.98	3500	300	600

Table 6.2 shows the unit setup cost and unit echelon cost for each stage. Note that the setup cost at the first stage is determined once the number of orders placed to the selected suppliers during a cycle time is determined, then it equals the weighted average unit setup cost.

Table 6.2. Cost parameters of each stage.

Stage s	**Unit holding cost** h_s (\$/unit/month)	**Unit echelon cost** e_s (\$/unit/month)	**Unit setup cost** k_s (\$/order)
1	5	5	-
2	20	15	200
3	50	30	150
4	95	45	100
5	145	50	50

The problem is solved using the global solver in LINGO15.0 on a PC with an INTEL(R) Core (TM) 2 Duo Processor at 2.10 GHz and 4.0 gigabytes RAM. Table 6.3 shows the optimal values for the decision variables in each stage such that the profit per time unit is maximized. Note that, for the serial supply chain inventory system, researchers have found closed form solutions for the decision variables to obtain the optimal policy for the case of two-stage supply chain. However, as shown by Roundy (1986), as the number of stages increase, the problem becomes more complex due to the integer variables and the nonlinear terms in the constraints that result in a non-convex feasible space. Hence, he developed a heuristic algorithm using the POT policy to obtain a near optimal solution, and he showed that the average cost of a POT policy is a convex function since the objective function can be represented as a sum of convex functions. Therefore, to obtain the optimal solution for Model $(M1)$, we use the global solver in LINGO15.0 that converts the problem into several convex, linear subproblems and uses a branch and bound technique to perform exhaustively search over these subproblems until a global solution is found.

Table 6.3. Optimal solution for Model (*M1*).

Stage 1				Stage 2		Stage 3		Stage 4		Stage 5		
i	J_i	Q_{1i} (units)	X_{1i}	Q_2 (units)	X_2	Q_3 (units)	X_3	Q_4 (units)	X_4	Q_5 (units)	X_5	P ($/unit)
1	112	2036.52	9	226.28	2	113.14	1	113.14	2	56.57	1	461.20
2	50	1131.4	5									
3	162	1810.24	8									
4	22	1583.96	7									
Profit per time unit								**$ 443,379 per month**				

As shown in Table 6.3, the order quantities placed to the selected suppliers vary among the suppliers. This is the case because each supplier has a different multiplicative factor, which results in different order quantities.

Next, the algorithm that is proposed by Abad (1988) can be used to find the optimal selling price P_i^* when each supplier is selected individually without considering capacity and quality constraints. In addition, if the maximum demand rate met by supplier i when quality and capacity constraints are not considered $\propto P_i^{*-\delta}$ is less than or equal to the supplier's capacity rate c_i , then additional values for the selling price can be obtained by computing the selling price when supplier i is selected at full capacity. Hence, $P_i = (c_i/\propto)^{-1/\delta}$ because the demand rate obtained when supplier i is selected at full capacity is equal to the supplier's capacity rate c_i since $\propto P_i^{*-\delta} \leq$

$c_i, i = 1, \ldots, n$. Table 6.4 presents several values for the selling price that can be used to obtain an initial value for the selling price.

Table 6.4. Several values for the selling price.

Supplier i	P_i^*	$P_i = (c_i/\propto)^{-1/\delta}$
Supplier 1	364.07	755.93
Supplier 2	301.84	577.40
Supplier 3	485.15	666.67
Supplier 4	605.67	816.50

Now, from Table 6.4, let us set the initial the selling price to be the average selling price values shown in Table 6.4 without considering the highest and lowest values, i.e., $P_0 = \$575.80$ per unit. Consequently, as shown in Table 6.5, the initial order quantities for each stage are obtained using the EOQ formula considering the unit echelon holding cost of each stage.

Table 6.5. Initial order quantities for each stage.

	Stage 1	**Stage 2**	**Stage 3**	**Stage 4**	**Stage 5**
$\boldsymbol{i}$	**Order quantity** Q_{1i}(units)	**Order quantity** Q_2(units)	**Order quantity** Q_3(units)	**Order quantity** Q_4(units)	**Order quantity** Q_5(units)
1	1553.35	179.37	109.84	73.23	49.12
2	850.81				
3	1473.64				
4	1299.63				

The initial order quantities shown in Table 6.5 are adjusted such that the order quantity of each stage is an integer multiple of the immediate downstream stage, where the integer multiplicative factor is an integer of the power of two. Table 6.6 shows the order quantities obtained after implementing the first iteration of Procedure 1. Note that the value of Q_5 in Table 6.6 is the same as to the one shown in Table 6.5.

Table 6.6. First iteration of Procedure 1.

	Stage 1		Stage 2		Stage 3		Stage 4		Stage 5	
i	Q_{1i}^{POT} (units)	X_{1i}	Q_2^{POT} (units)	X_2	Q_3^{POT} (units)	X_3	Q_4^{POT} (units)	X_4	Q_5^{POT} (units)	X_5
1	1571.84	8	196.48	2	98.24	1	98.24	2	49.12	1
2	785.92	4								
3	1571.84	8								
4	1571.84	8								

As new order quantities have to be placed to the selected suppliers, we need to update the selling price and find the number of orders to be placed to the selected suppliers during a cycle time such that the capacity and quality constraints are not violated. Therefore, Model $(M1')$ is solved in which $Q_{1i} = Q_{1i}^{POT}, i = 1, \dots, n$. The obtained number of orders submitted to each of the selected suppliers per order cycle are as follows: $J_{11} = 84$, $J_{12} = 50, J_{13} = 108$, and $J_{14} = 17$, and the updated selling price is $\$452.91$ per unit. Consequently, the order quantity at the last stage can now be refined by using Equation (25), i.e., $Q_l = 58.73$ units. Accordingly, the algorithm continues until the number of orders submitted to the selected suppliers during a cycle time and the integer multiplicative factors remain unchanged. Table 6.7 shows the order quantities after implementing the second iteration of Procedure 1.

Table 6.7. Second iteration of Procedure 1.

	Stage 1		Stage 2		Stage 3		Stage 4		Stage 5	
i	Q_{1i}^{POT} (units)	X_{1i}	Q_2^{POT} (units)	X_2	Q_3^{POT} (units)	X_3	Q_4^{POT} (units)	X_4	Q_5^{POT} (units)	X_5
1	1879.36	8	234.92	2	117.46	1	117.46	2	58.73	1
2	939.68	4								
3	1879.36	8								
4	1879.36	8								

Similarly, given Q_{1i}^{POT} from Table 6.7, $i = 1, \dots, n$, we need to solve Model $(M1')$ to check the new number of orders submitted to each supplier in a repeating order cycle time and also to update the selling price. As a result, the number of orders submitted to each supplier in a cycle are $J_{11} =$

63, $J_{12} = 38$, $J_{13} = 81$, $J_{14} = 13$, and the selling price is $452.27 per unit. Therefore, since the number of orders allocated to the suppliers in the cycle time got changed. Then, we need to refine the order quantity at the last stage and start a new iteration. Hence, by using Equation (25), i.e., $Q_l = 58.82$ units. Table 6.8 shows the order quantities after implementing the third iteration of Procedure 1.

Table 6.8. Third iteration of Procedure 1.

	Stage 1		Stage 2		Stage 3		Stage 4		Stage 5	
$\boldsymbol{i}$	$\boldsymbol{Q_{1i}^{POT}}$ (units)	$\boldsymbol{X_{1i}}$	$\boldsymbol{Q_2^{POT}}$ (units)	$\boldsymbol{X_2}$	$\boldsymbol{Q_3^{POT}}$ (units)	$\boldsymbol{X_3}$	$\boldsymbol{Q_4^{POT}}$ (units)	$\boldsymbol{X_4}$	$\boldsymbol{Q_5^{POT}}$ (units)	$\boldsymbol{X_5}$
1	1882.24	8	235.28	2	117.64	1	117.64	2	58.82	1
2	941.12	4								
3	1882.24	8								
4	1882.24	8								

Similarly, by solving Model $(M1')$ where the order quantity placed to the selected suppliers are given in Table 6.8, we obtain $J_{11} = 63$, $J_{12} = 38$, $J_{13} = 81$, $J_{14} = 13$, and the selling price is $452.27 per unit. Therefore, since the number of orders allocated to the suppliers in the cycle time and the multiplicative factors in Tables 6.7 and 6.8 remain unchanged, then we terminate the algorithm and compute the profit per time unit, which is equal to $443,238.4 per month (i.e., 0.031% less than the optimal profit per time unit shown in Table 6.3, and 0.07 % less than the upper bound that is $443,567.2 per month obtained by solving Model ($\overline{M1}$)). Note that the CPU time needed to solve the problem using the heuristic algorithm is about 10 seconds, whereas the time needed to solve the problem optimally is 104 seconds.

Example 2. Consider a five-stage serial supply chain in which a manufacturer is located at the first stage and a distribution center is at the last stage, and the intermediate stages can represent additional manufacturing facilities or warehouses. The manufacturer at the first stage can replenish raw material inventory from four potential suppliers. Table 6.9 shows the parameter data of the four suppliers. Also, the manufacturer's minimum acceptable quality level is $q_a = 0.95$. The retailers are facing price-sensitive demand, i.e., $D = \propto P^{-\delta}$, where $\propto = 3{,}375{,}000$ and $\delta = 2$.

Table 6.9. Suppliers' parameters information.

Supplier i	**Quality level** q_i	**Unit setup cost** k_i ($/order)	**Unit purchasing cost** v_{1i} ($/unit)	**Capacity rate** c_i (units/month)
1	0.94	16000	5	1000
2	0.92	1500	4	1400
3	0.96	4000	6	1300
4	0.98	1000	7	1250

Table 6.10 shows the unit setup cost and unit echelon cost for each stage. Note that the unit setup cost at the first stage is determined once the number of orders placed to the selected suppliers during a cycle time is determined, then it equals the weighted average unit setup cost.

Table 6.10. Cost parameters for each stage.

Stage s	**Unit holding cost** h_s ($/unit/month)	**Unit echelon cost** e_s ($/unit/month)	**Unit setup cost** k_s ($/order)
1	5	5	-
2	20	15	200
3	50	30	150
4	95	45	100
5	145	50	50

The goal of this example is to show that when each supplier has a different multiplicative factor, the optimal solution can be improved in comparison with the optimal solution for models in which the same multiplicative factor is assigned to each supplier. The optimal solutions for Model $(M1)$ and Model $(M2)$ are presented in Table 6.11 and Table 6.12, respectively.

Table 6.11. Optimal solution for Model (*M1*).

Stage 1				Stage 2		Stage 3		Stage 4		Stage 5		
i	J_i	Q_{1i} (units)	X_{1i}	Q_2 (units)	X_2	Q_3 (units)	X_3	Q_4 (units)	X_4	Q_5 (units)	X_5	P ($/unit)
1	570	5498.1	15	366.5	2	183.3	1	183.3	2	91.63	1	26.11
2	3003	1832.7	5									
3	1743	2932.3	8									
4	3352	1466.16	4									
Profit per time unit							**$ 64,430.5 per month**					

Table 6.12. Optimal solution for Model (*M2*).

Stage 1				Stage 2		Stage 3		Stage 4		Stage 5		
i	J_i	Q_1 (units)	X_1	Q_2 (units)	X_2	Q_3 (units)	X_3	Q_4 (units)	X_4	Q_5 (units)	X_5	P ($/unit)
1	0	1942.8	6	323.8	2	161.9	1	161.9	2	80.9	1	29.2
2	28											
3	26											
4	25											
Profit per time unit							**$ 62,550.7 per month**					

As shown in Tables 6.11 and Table 6.12, when we apply different multiplicative factors to the order quantities submitted to the selected suppliers, i.e., Model ($M1$), the profit per time unit has increased by 3% in comparison with the case in which the same multiplicative factor is considered, i.e., Model ($M2$). Note that supplier 1 in Model ($M2$) is not selected because a relatively small order quantity will be placed to supplier 1 if selected. Hence, this small order quantity is not enough to justify the increase in the setup cost per time unit as supplier 1 has the highest unit setup cost. Hence, when we allow different multiplicative factors as shown in Model($M1$), supplier 1 is selected with a relatively large order quantity that justifies selecting supplier 1 who has the highest unit setup cost.

A similar analysis is performed for the model proposed by Mendoza and Ventura (2010) who considered the same multiplicative factor for each order quantity placed to the selected suppliers.

However, we found that the cost per time unit can be reduced by 1.5% if each order quantity placed to the selected suppliers has a different multiplicative factor.

6.4. Conclusions

Coordinating pricing and inventory decisions in retail and manufacturing industries has recently gained a considerable attention in the literature and practice. Many firms including Dell, Amazon, J.C. Penney, and Grainger change their selling prices frequently based on several factors such as inventory levels and demand variations (Elmaghraby and Keskinocak, 2003; Chan et al., 2005). Therefore, it is important to develop models that simultaneously coordinate pricing decisions with other key decisions in the supply chain, such as supplier selection, and inventory replenishment decisions. In this chapter, we have considered the integrated pricing and inventory replenishment problem in a serial supply chain. We have developed an MINLP model to find the number of orders placed to the selected suppliers during a cycle time and corresponding order quantities, inventory lot sizes between consecutive stages, and the final product's selling price, such that the profit per time unit is maximized. Moreover, we have proposed an algorithm capable of finding near-optimal solutions within 2% of the optimal solution, as verified in the numerical analysis. In addition, we have shown that models that use different multiplicative factors for the selected suppliers obtains an increase in the average profit in comparison with the average profit generated by models that consider the same multiplicative factor.

Chapter 7: Research Summary and Future Directions

7.1. Research Summary

In this dissertation, we have addressed the importance of coordinating pricing decisions with other aspects of the supply chain such as inventory replenishment decisions. The simultaneous coordination of these decisions has recently become a focus of attention by many scholars, as it leads to optimizing the entire supply chain rather than to optimizing decision-making around specific members of it. Therefore, this dissertation has focused on integrating pricing decisions with other aspects of the supply chain, particularly in regard to supplier selection and inventory coordination in a serial supply chain.

Supplier selection is a key strategic decision in improving companies' purchasing performance. Studies have shown that the cost of acquiring a product's parts from outside suppliers represents a significant portion of the final product's cost. Therefore, it is crucial to develop supplier selection models that reflect real-life situations so that companies can use them to improve their purchasing performance. Accordingly, we have developed a new model for the supplier selection problem that refines and generalizes some of the existing models in the literature. The suppliers in the model offer all-unit quantity discounts to ensure a more realistic and practical situation. In addition, the model accounts for placing multiple orders to the selected suppliers during a cycle time. Previous research in the area showed that this ordering strategy reduces the cost per time unit compared to the cost shown by models that allow at most one order to each supplier during a cycle time. Thereafter, we have developed a more general model for the supplier selection problem in which the pricing problem is also considered. Many researchers and firms have studied these two problems separately or unrealistically assume that the demand rate is constant and known in advance. In real retailing environments, it can be argued that the selling price plays a significant role not only in attracting more consumers, but also in determining the appropriate suppliers. Hence, we have studied the supplier selection problem under the assumption that the demand rate is not constant but is, instead, dependent on the selling price. In addition, we applied KKT conditions to monitor any changes in the retailer's sourcing strategy as the dominating supplier's capacity decreases, and to determine whether the supplier's capacity is fully utilized or not (i.e., active or inactive capacity constraint). Researchers showed that if an optimal solution exists, then at most one of the selected suppliers receives a less than full-capacity order whereas the dominating supplier's capacity is fully utilized. However, we have shown that this result does not hold true, such that more than one supplier can be selected without fully utilizing the capacity of any of them. In some cases, the retailer considers a multiple-sourcing strategy whereby the dominating supplier's capacity is not fully utilized.

Next, we have studied the integrated pricing and supplier selection problem in a two-stage supply chain. The two stages are considered to be vertically integrated (i.e., a centralized-making process). Thus, the goal becomes to simultaneously determine pricing, supplier selection, and inventory coordination decisions whereby the joint profit per time unit is maximized. In addition, we have obtained lower and upper bounds on the profit per time unit obtained when more than one supplier is selected. In addition, we have developed lower and upper bounds on the optimal selling price and the multiplicative factor in order to obtain a tight feasible region and solve the problem in a timely manner. We have also identified a tighter feasible region for the multiplicative factor when there are only two potential suppliers. Then, we have studied the integrated pricing and supplier selection problem in a serial supply chain. As the number of suppliers and stages increases, the problem becomes more complex and, consequently, more computational time is needed to solve it. Therefore, we have proposed an algorithm capable of finding near-optimal solutions as verified in our numerical analysis, within 2% of the optimal solution. In addition, we have showed that models using the same multiplicative factor for the selected suppliers obtains an increase in the average profit in comparison with the average profit generated by models that consider the same multiplicative factor.

7.2. Future Directions

In regard to future research, it would be worthwhile to investigate algorithms that eliminate some of the suppliers before solving the mathematical model taking into consideration a price-sensitive demand and suppliers' limitations on capacity and quality. For instance, a supplier with a very low capacity rate and a low quality level might not be selected; therefore, this supplier can be eliminated in advance. However, price-sensitive demand and the multiple orders that can be placed to selected suppliers per order cycle make the problem more interesting and challenging.

In addition, the MINLP model developed in Chapter 6 to solve the integrated pricing and supplier selection problem in a serial supply chain can be extended to consider distribution decisions as well. The distribution center in the proposed model can be considered to serve a number of independent markets, where each has a different price-sensitive demand function. Hence, the proposed heuristic algorithm can be generalized to solve this integrated pricing, supplier selection, and distribution problem in a serial supply chain. In addition, researchers can also consider the case in which the company negotiates with its suppliers such that the latter lower their unit prices and provide discounts on raw material so that the selling price can be decreased. By reducing the selling price, the demand will increase, which will result in improve profits for the company and selected suppliers.

Furthermore, pricing and inventory replenishment decisions are highly influenced by the price-sensitive demand function that may change over time. Therefore, researchers can formulate a robust optimization model to deal with demand uncertainty in the integrated pricing and lot-sizing problem.

Appendix

We suggest the procedure discussed in Hopp and Spearman **(**2001) to determine the retailer's optimal order quantity Q_i' when supplier i offers all unit quantity discounts, for $i = 1, \dots, n$. Let TC_{ij} denote the cost per time unit incurred form ordering Q_{ij}^* units from discount interval j. Then,

$TC_{ij} = D \frac{k_i}{Q_{ij}^*} + (1/2)r\, Q_{ij}^* v_{ij} + Dv_{ij}$, $i = 1, .., n$ & $j = 1, \dots, a_i$,

where Q_{ij}^* is the order quantity that minimizes the cost per time unit in interval j. The steps of the procedure are provided below.

Step 1. For each quantity discount interval j, where $j = 1, \dots, a_i$, use the corresponding v_{ij} to compute the economic order quantity Q_{ij}, i.e., $Q_{ij} = \sqrt{2k_i D/(rv_{ij})}$, $i = 1, \dots, n,$ & $j = 1, \dots, a_i$.

Step 2. Find out the largest quantity discount interval index j^o such that $u_{i,j^o-1} \le Q_{ij^o} < u_{ij^o}$ (i.e., Q_{ij^o} is the largest realizable EOQ that is within the correct discount interval) and then calculate the corresponding cost per time unit TC_{ij^o}, where $Q_{ij}^* = Q_{ij^o} = \sqrt{2k_i D/(rv_{ij^o})}$; hence, $TC_{ij^o} = \sqrt{2k_i D r v_{ij^o}} + Dv_{ij^o}$.

Step 3. Ignore all the discount intervals that are less than j^o, and for each $j > j^o$, use the corresponding v_{ij} to calculate the cost per time unit donated by TC_{ij}^r, where $Q_{ij}^* = u_{i,j-1}$. Thus,

$$TC_{ij}^r = D\frac{k_i}{u_{i,j-1}} + (1/2)ru_{i,j-1}v_{ij} + Dv_{ij}, \quad i = 1, \dots, n, \&\ j > j^o .$$

Step 4. Select the order quantity Q_{ij}^* and the index of the supplier's best quantity discount interval j^* corresponding to the minimum cost per time unit: $TC_i' = min\left\{TC_{ij^o}, TC_{i,j^o+1}^r, \dots, TC_{i,a_i}^r\right\}$. Then, the retailer's optimal order quantity is $Q_i' = Q_{ij^*}^*$.

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