The Pennsylvania State University

The Graduate School

College of Engineering

# BI-OBJECTIVE INTEGRATED SUPPLY CHAIN NETWORK DESIGN UNDER SUPPLY AND DEMAND UNCERTAINTIES

A Thesis in

Industrial Engineering and Operations Research

by

Boyi Wu



Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

August 2016

The thesis of Boyi Wu was reviewed and approved* by the following:

Jeya Chandra

Professor of Industrial Engineering

Thesis Adviser

Saurabh Bansal

Assistant Professor of Supply Chain Management

Reader

Janis P. Terpenny

Professor of Industrial Engineering

Head of the Department of Industrial and Manufacturing Engineering

*Signatures are on file in the Graduate School

# ABSTRACT

A supply chain has a network structure and its design involves production-distribution decisions on the location of production, distribution facilities, capacities and transportation quantities. A multi-echelon supply chain involving multiple suppliers and multiple retailers for multiple products among multiple periods is considered in this thesis, which is required to be modeled as a multi-objective problem coping with uncertainties during the process. The uncertainties considered in this thesis include demand uncertainties and suppliers' disruptions. The end retailer's demand is assumed to be uncertain following a certain distribution. Supplier's disruptions are represented by a set of discrete scenarios with given probabilities of occurrence of shrink in storage capacity, which can cause insufficient supplies from suppliers. Hence, the model in this thesis is different from traditional supply chain network models under a deterministic case or the models with uncertainties in demand only.

In order to study the effects of the various uncertainties involved in the chain on the optimal decisions, multiple methods are tested using a multi-objective mixed-integer nonlinear model. After comparison of computational performance, the min-max method is applied to obtain detailed mathematical solution. Numerical examples are conducted to illustrate the developed model with scenario analysis and sensitivity analysis is also conducted.

# TABLE OF CONTENTS

# LIST OF FIGURES

# LIST OF TABLES

# ACKNOWLEDGEMENTS

I would like to take this opportunity to express my thanks to my advisor, Dr. Jeya Chandra for his patience, guidance and persistent support throughout this research and the writing of this thesis.

I would additionally like to thank my thesis committee members, Dr. Saurabh Bansal, and Dr. Janis Terpenny for their guidance and feedback helping me complete this thesis through the hurdles.

I am grateful to all my colleagues in the Department of Industrial Engineering who have assisted me in the course, and shared their graduate school life with me.

Last but not least, I would like to thank the commitment, sacrifice and support of my parents, who have always motivated me.

# Chapter 1

# INTRODUCTION AND PROBLEM STATEMENT

*In this chapter the background related to the subject of this research is presented. The problem definitions are given followed by the purpose of thesis; in addition a focus and limitation are provided in order to define the scope of the research and outline the study.*

## 1.1 Introduction

Supply chain has been considered as one of the most important coordinated set of activities in majority of organizations and firms. It has a network structure and its design involves production-distribution decisions on the location of production, distribution facilities, capacities and transportation quantities. Efficient supply chain network is essential to competitive current world. It not only has to cope with rapidly changing conditions to fulfill the needs from customers but also has to make more profits. The aim of supply chain network design is to provide an optimal platform for efficient and effective supply chain management, which involves making strategic and tactical decisions on the production, location, transportation and coordination of each facility. It is not easy to coordinate physically distinct geographically separated stages to do their jobs effectively and make sure that the network runs as anticipated. Many companies in every stage are currently facing unanticipated issues disrupting the normal products flow through procurement, production and distribution process though they are always pursuing for robust supply chain network to work with. The main focus of this thesis is to study common supply chain network design issue under various uncertainties and propose mathematical modeling framework to optimize supply chain problems with multiple objectives. Multiple stages with multiple firms lead to large scale of dataset. Numerical examples are presented and analyzed with proposed methodologies to show optimized result when facing various uncertainties.

## 1.2 Problem Statement

Over the past decade, there has been great increase in focusing on supply chain network and integrated logistics because of the emphasis on productivity and customer satisfaction. An effective, efficient and robust logistics network becomes a sustainable competitive advantage for companies and organizations to help them cope with increasing environmental turbulence and more intense competitive pressures. These pressures have led to unplanned and unanticipated events affecting normal flow of materials and products within a supply chain. Hence, the presence of uncertainty affects the real-time decision-making.

The uncertainties discussed in supply chain network can be classified into three categories, based on supply stage, retailer stage and middle stage. The uncertainties affect the materials supply, production resources, the operating parameters (lead times, shortage, spare value, holding stock management, etc.) and the demand-side scenarios, which are related to the market (prices, delivery requirements, etc.). Especially when accidental events significantly affect the performance of supply chain, the uncertainties relates closely to disruption. Natural disasters like earthquake, SARS and terrorism attack damage each stage of supply chain and disrupt the network. That is, unintended, unwanted situations result in a negative supply chain performance.

Considering uncertainties with disruptions helps companies to supply a better service level to their customers. The essential problem being analyzed in the thesis is the model and method to solve common supply chain network under various uncertainties. Solving the problem requires an in-depth understanding of uncertainties and disruption from a supply chain perspective. This research is based on mathematical modeling to describe such uncertainties in supply chains.

The available literature has mainly paid attention to demand uncertainty while a few kinds of literature also take supply side uncertainty into consideration. Nevertheless, supply side disruption may have a negative impact on procurement and consequently on the availability of materials and products for distribution. Besides, very limited research has focused on supply chains with multiple stages and multiple periods. Focus on multiple objectives has also been neglected even though it is necessary to consider tradeoffs among multiple objectives. During last few decades, only single objective case has been widely studied. From the current view of supply chain risk management, analyzing and handling both supply and demand uncertainties enable supply chain network to respond quickly to the market.

Optimization is a mathematical procedure to determine the optimal distribution and allocation to limited resources. In this thesis, a multi-echelon supply chain involving multiple suppliers and multiple retailers for multiple products among multi-period is considered. The problem has two conflicting objectives to optimize. The first objective is to minimize the total cost of the supply chain which includes the cost of running manufacturing plants and distribution centers, the cost of buying and transporting raw materials, the cost of transporting products from plants to distribution centers and from distribution centers to retailers, and cost of holding products at distribution centers. The second objective is to minimize disruption cost, which occurs because of uncertain demand and supply disruptions. These two objectives require a trade-off acquiring materials from a single supplier or multiple suppliers in network with supply and demand uncertainties. The problem is formulated into a bi-objective mixed integer nonlinear programming model. Then multiple methods are employed to solve the problem using the MATLAB platform.

This thesis includes analytical review of theoretical base of bi-objective supply chain optimization approaches and focuses on further understanding and utilization of it to solve a certain model. The academic contribution of this thesis is the in-depth scenario analysis of supplier and demand uncertainty based on large-scale data related to supply chain Network Performance and its modification according to results of analysis. The paper provides an extension to Taha's work[1]. Model developed in this thesis contains more parameters, some of which vary with period and product.

## 1.3 Thesis Outline

In the following chapter 2, relevant current studies in the literature are discussed. In chapter 3, the mathematical model is developed. Chapter 4 proposes solution method to the problem and describes specific numerical example on the bi-objective supply chain network design applied using proposed methodologies. We analyze the detailed data presented with forms and charts. Finally, chapter 5 summarizes the conclusions drawn from the thesis and identifies future research directions.

# Chapter 2

# LITERATURE REVIEW

*In this chapter the study of literature related to the subject of this research is presented. The reviews of literature from seven criteria are introduced and originality of this work when compared to the existing literature is explained.*

A review of literature related to supply chain network design area specified in allocation and distribution problem under risks is presented in this section. There has been a growing interest in supply chain network design criteria which has drawn great attention of a number of researchers to propose advanced models and develop efficient methods. Traditionally, procurement, manufacturing, distribution and planning processes are studied individually. However, with risk management in demand and supply side, there has been an increasing attention placed on the integration of each process as a network design problem. We conducted a search of references through library's database using criteria key words, which are "supply chain network", "distribution-allocation", "risk management ", "uncertainty", "stochastic demand" and "supplier disruption". Appropriate mathematical programming methods and algorithms possibly used to solve problems in this criterion is reviewed in this section. A large body of references shows that mixed-integer programming models are commonly used in supply chain network design under conditions of uncertainty.

## 2.1 Supply chain network

In recent business world, supply chain is an important part of many businesses. The supply chain concept arose from a number of changes in the manufacturing environment, including the rising costs of manufacturing, the shrinking resources of manufacturing bases, shortened product life cycles and the globalization of market economies [2]. The growing trend of globalization creates opportunities for firms to conduct business operations in diverse conditions. However the current global market is filled with highly demanding customers pursuing customized products, lead times are short and product life cycles are also short. As the result of this, supply chain is becoming more uncertain, complex and disordered. A typical supply chain is a network of suppliers, production sites(manufacturers), storage facilities(distribution centers) and retailers. Each stage facility and distribution performs the functions of acquiring materials, converting of these materials into intermediate and finished products, and distribution of these products to customers.

Beamon [2] defines supply chain as an integrated process wherein a number of various business entities (i.e., suppliers, manufacturers, distributors, and retailers) work together in an effort to (1) acquire raw materials and parts (2) transform these raw materials and parts into finished products (3) add value to these products (4) distribute and promote these products to either retailers or customers and (5) facilitate information exchange among various business entities involving suppliers, manufacturers, distributors, third-party logistics, and retailers.

Supply chain network design problem has been gaining importance due to increasing competitiveness introduced by the market globalization [3] and it is one of the most comprehensive strategic decision problems that need to be optimized for long term efficient operation of the whole supply chain. Operation of efficient supply chain network usually requires many integrated processes including (1) the production assignment that deals with specific quantities of products produced to meet each retailer's demand (2) distribution planning and logistic process that determines optimal distribution channel and quantity and (3) inventory control process that affects stock in distribution centers dealing with storage. These processes need to be jointly optimized to fulfill retailers' demand and requirement and also to minimize the total cost of the supply chain network while maximizing the final profitability to fertilize each stage (facilities).

Efficient and robust design and operation of supply chain network can help firms to be sustainable enough to cope with uncertain situations and survive in increasing competitive environment filled with risks and pressures. The models and approaches in many literatures are based on deterministic cases; however, only few examples in our life are under ideal conditions, and the real supply chain network problems are characterized by uncertainties in various process. It is also not realistic to assume that all parameters are known with certainty, such as demand and storage. For instance, either products have to be delivered earlier to the distribution centers or late to retailers sometimes, incurring holding or shortage costs in the process.

Although organizations for each stage have uncertainties in the process and their own objectives might conflict with each other, they are corporately pursuing the profitability of the entire network. The key driver of the overall productivity and profitability of a supply chain is its distribution network. Efficient distribution network can be used to achieve a variety of the supply chain objectives ranging from total network cost to network responsiveness.

## 2.2 Allocation distribution problem

In wide range of supply chain network problems, the solution to allocation-distribution problem suggests the strategic decision of the whole network, such as distribution flows, number, location and capacity of entities in each stage of the supply chain network. Tsiakis[4] is one of the few to consider flexible production facilities in which a number of products are produced, making use of shared resources. The performance of a supply chain depends mostly on the effective and balanced allocation from factory stage to retailer stage. Gebennini[5] states that the network structure of the chain strongly influences the later operational decisions of flow management throughout the chain. Hence, in addition to strategic locating and capacity setting costs, the resulting operational inventory holding and transportation costs should also be considered at the network design stage. As the result, how products are retrieved and transported from the warehouse to retailers through this network is initially considered in allocation distribution problem. For any supply chain, logistics and distribution play a key role in its success, corresponding allocation distribution plan not only achieves multiple objectives in the chain but also enhances the total profitability and production of the whole network. The objective is to allocate demand so as to minimize total cost of the whole network system including operation, manufacturing and distribution at each period when subjected to inventory, shortage and capacity constraints. Ivan [6] proposes that when both variable cost and fixed cost are involved in transportation cost, a non-linear model is formulated in a multi-period perspective. Analyzing location issues is also considered one of the important decision making in supply chain network design. Appropriate facility location has positive impact on economic profit, responsiveness and retailer's satisfaction, even though it might not be possible to satisfy all the requests on time. Hence, these processes interact with one another to produce an integrated supply chain network. As Beamon[2] addresses, the design and management of these integrated processes determine to what extent can supply chain meet the required performance objectives.

Although the literature on allocation distribution problem is large, many of them are single-stage supply chain, which only consists of a set of suppliers and a set of retailers. Allocation-distribution problems cover wide range of formulations ranging from simple single product to complex multiple products, and from linear deterministic models to complex non-linear or mixed-integer stochastic models. Flow of products between suppliers and retailers passes through several stages consisting of many facilities. Hence, there are only a small number of works devoted to studying multi-echelon multi-period supply chain network. As the result, the supply chain network design literature is extended by considering multi-echelon multi-product multi-period case including uncertainty.

## 2.3 Uncertainty/Disruption in supply chain network

Uncertainty is the main source that leads to supply chain risk and disruption; therefore, it is one of the inherent attributes of the supply chain which affects supply chain performance and real-time decision making [7]. If these problems are not handled well, supply chain network will be imbalanced, which will result in poor responsiveness of each stage and firms will suffer huge loss, as well as loss of reliability. Considering uncertainties and disruptions helps firms to deliver a better service level to customers. Uncertainty is early defined as the difference between the amount of information required to execute a task and the information that is available [8]. In supply chain network design decision processes, uncertainty is a main factor that can influence the effectiveness of the configuration and coordination of supply chains and tends to propagate up and down along the supply chain, appreciably affecting its performance [9]. Santoso [10] states that unless the supply chain is designed to be robust with respect to the uncertain operating conditions, the impact of operational inefficiencies such as delays and disruptions will be larger than necessary. As the result, product distribution flow cannot be run well as expected and holding costs with backorder costs rise in intermediate stages. Different level of uncertainties can affect market, development of technology and even instability among international trade. Disruption is typical in uncertainty. Supply chain disruptions are unanticipated incidents that disrupt the normal flow of raw materials and products within a chain network, which expose firms within the supply chain to operational and financial risks [11]. Therefore considering both demand-side uncertainty and supply-side disruption are necessary in the supply chain network design stage. Supply-side disruption (storage disruption & delivery delay), process disruption (manufacturing failure and reliability of transportation) and retailer's demand side disruption (uncertain demand quantity and bullwhip effect) are three groups of disruptions in a supply chain. With only a small number of work being devoted to consider production and supply-side disruption, developing recovery models can really help handle situation and make supply chain more resilient. Paul [12] introduces a supply disruption management model in three-tier supply chain network considering recovery/revised plan to minimize disruption.

There are also several ways to deal with uncertainty problems. In a number of literatures, some models simply ignore the uncertain parameters and solve the problem taking "average" values. However, some others demonstrate uncertainties by fuzzy theory and also distribution function in stochastic models [13]. Scenario and probabilistic approaches are mainly used to model uncertainty problem. The scenario approach sets up different scenarios to represent all combinations of uncertain parameters in discrete cases, while probabilistic approach set up probability distributions for uncertain parameters. This description is leading to conclude that dynamic mathematical formulations approach uncertainty in two ways[14].(1) scenario or multi-period approaches often including discretization applied to a continuous uncertain parameter space;(2) probabilistic approaches using two stage stochastic programming.

Both demand and supply uncertainties are considered in this thesis work. Demand uncertainty will be modeled with a demand following uniform distribution and supply uncertainty will be modeled in terms of the probability of the occurrence of disruption scenarios in supplier capacity. The following related literature study could be classified into two groups. The first group studies the disruption on supply-side; the second group studies the uncertain demand.

## 2.4 Supplier disruption

Supplier (supply-side) uncertainty is the variability in network brought by how the supplier operates and usually modeled as disruption events because unpredictable disaster or accident results in disturbance and loss of supply. A supply disruption can be defined as any form of interruption in the raw material supply that may be caused due to delay, unavailability, or any other form of disturbance [15]. Disruption events are haphazard and unexpected but they can significantly affect the performance of supply chain, lead to extra inventory and lost sales in retailing outlets, such as natural disasters like earthquake, SARS epidemic and human–caused disruption like labor strike, terrorism attack, financial crisis damage each stage of supply chain and disrupt the network, that is, unwanted situations resulting in a negative supply chain performance. As mentioned by Norrman and Jansson [16], a fire at one of the major suppliers of the Ericsson Company resulted in several serious problems for this company and the shutdown of its manufacturing plants for several days. Lin's work [17] discusses manufacturing postponement with downward substitution mitigation and early differentiation of products which increases the fulfillment uncertainty affecting the supply-side as well. Random yield, also as one of supplier disruption, affects the procurement and production process because of mechanical failure, material quality problem, labor strike and other incidents. Giri's work [18] takes random yield in to account that randomness in production yield is very likely to occur whereas the incident of disruption is very less probable, although the later has very high effect on the supply chain's performance, the result of which will lead to loss in sales for retailer and extra inventory in stock of intermediate stage facilities. It is necessary to specify the effective factors on suppliers' reliability and the importance weight of each factor in the reliability level of a supplier since supplier reliability is a parameter which, in an uncertain way, influences the ability of supply chain to satisfy demand [19]. According to Giri[18], to fulfill supply agreement, the production companies often set the production level higher than the desired level which again results in excessive production and holding cost. It is also possible to have redundant suppliers, increased storage capability, increased flexibility, increased responsiveness, or more retailers to manage supply-side disruption. There are several strategies to cope with these problems and robust optimization is widely used. The body of literature related to this criteria is extensive [20][21][22].

To solve network design problem with the supply disruption, the major task is to simultaneously determine an integrated stochastic procurement and deterministic manufacturing operations plan. This usually requires an additional decomposition approach. Tomlin [23] discusses three general strategies for coping with supply disruptions: inventory control, sourcing and acceptance. Describing supplier disruption through various scenarios and associated probability of occurrence is considered, which is to extend Pan's work [24] to show the scenario approach accessible to find a robust solution. This ensures that the solution is "close" to the optimum in response to changing input data, considering all specified scenarios.

## 2.5 Demand uncertainty

Nowadays, retailers' demands have been more various and uncertain. Gupta [25] proposes that the demand variety can be recognized as one of the important sources of uncertainty in a supply chain. Xu [7] notes that there exists relationship between uncertain demand and fluctuation of price. Demand uncertainty can cause “Bullwhip effect” which results in affecting inventory stock, changing the manufacturing operations and consequently the procurement plan of production facilities. “Bullwhip effect ” is caused by lack of communication between stages and eventually increases variability at successive upstream stages of a supply chain and the negative impact of demand uncertainty has been amplified.

But what causes demand uncertainty? Xu’s work[7] shows that the uncertainties of demands are determined by the uncertainties of prices. If suppliers have more demand information from retailers or downstream stages, they might be able to operate better and cope with rapid change in demand in current dynamic environment. The fact that demand information only belongs to retailers determines that other stages cannot obtain accurate information from retailers. However, Cardona-Valdés[26] states that meeting customer demand is what mainly drives most supply chain initiatives. This motivated to study the problem considering that the demand is a random variable whose value is not known at the time of designing the network. Risk-pooling strategy can be adopted to deal with variability of retailers’ demands, inventory and product holding in distribution centers. As Tavakkoli-Moghaddam[27] indicates that the proposed risk-pooling strategy and centralizing the inventory at distribution centers are considered as one of the effective ways to manage such a demand uncertainty to achieve appropriate service levels to customers. As the result, two stochastic models with risk pooling have been developed (i) probabilistic approaches (ii) scenario approaches. Supply chain models under demand uncertainty have received significant attention in the literature. In Chen’s model[28], the demand uncertainty is modeled as discrete scenarios with given probabilities for different expected outcomes, and the uncertain product prices are described as fuzzy variables. Goh[29] also utilizes stochastic programming to design an algorithm under the scenario of a variety of risks. Listes and Dekker [30] propose a stochastic mixed integer programming (SMIP) model in a sand recycling network to maximize the total profit. They develop their model for different situations regarding several scenarios. Salema[31] develops a stochastic model for multi-product networks under demand uncertainty using stochastic mixed integer programming. Yin[32] demonstrates the impact of demand uncertainty on profit for the average case and worst case and conducts comparative experiments by analyzing the deviation of demand of finished products.

In this work, stochastic demand with specific uniform distribution is used to describe demand uncertainty, which is formulated into the framework of a constrained bi-objective mixed integer linear programming model to solve a multi-echelon supply chain.

## 2.6 Mathematical Programming-Based Methodology

The review in this part focuses on model-based quantitative methodologies based on mathematical models dealing with design and operation of supply chain network. The present work formulates the supply chain network design problem as a multi-objective stochastic mixed-integer programming problem. It is recognized that multi-objective optimization problem involves simultaneous optimization of problems with at least two objective functions which are conflicting in nature. In multi-objective optimization problem there is no single optimum solution, but a number of them exist that are optimal called fronts[33]. Except multi-objective optimization techniques that provide Pareto frontiers, there exist a lot of methods for quantitative description of supply chain uncertainty modeled in multi-objective optimization problem. Such approaches include Fuzzy optimization, stochastic programming, genetic algorithm, goal programming and robust optimization.

Fuzzy optimization has been traditionally used to solve multi-objective optimization problem. Xu[7] uses fuzzy sets theory and the method of robust linear programming based on interval analysis to construct multi-objective fuzzy programming model. Chen[28]proposes fuzzy decision-making method, as one can see in the case study, demonstrates that the method can provide a compensatory solution for the multiple conflict objectives and the fuzzy product prices problem in a supply chain network with demand uncertainties. But they can only handle limited cases when uncertainty exists.

Stochastic programming is also one of the major approaches dealing with uncertainty in supply chain especially in mixed-integer programming problem. It is assumed that the uncertain parameter,which are mainly uncertain demand, follows a multidimensional integral involving the joint probability distribution which is known or can be predicted from historical data. Georgiadis's study [34] shows that in the two-stage stochastic programming approach, the decision variables of a model under uncertainty are partitioned into two sets. First stage variables are those which have to be decided before the actual realization of the uncertain parameters. Second stage variables is used for a recourse decision which compensates for any bad effects as a result of the realization of uncertain parameters. The optimal policy from such a model is a single first-stage policy and a collection of recourse decisions (a decision rule) defining which second-stage action should be taken in response to each random outcome. Its objective function consists of the sum of the first-stage performance measure and the expected second-stage performance involving uncertain parameters. Santoso[9] develops stochastic programming methodology which integrates a recently proposed sampling strategy, the sample average approximation scheme, with an accelerated Benders decomposition algorithm to solve supply chain design problems with continuous distributions for the uncertain parameters, and hence an infinite number of scenarios.

Besides above, Genetic Algorithm is one of the widely implemented approaches many papers are working on and is very effective to solve allocation-production problem. GA is a stochastic search technique whose solution process mimics natural evolutionary phenomena: genetic inheritance and Darwinian strife for survival[35]. Altiparmak[36] develops a new solution methodology based on GA for multi-objective optimization of supply chain networks to obtain all Pareto-optimal solutions considering two different weight approaches for the supply chain network design problem and enable the decision maker for evaluating a greater number of alternative solutions. But particular GA can take long time searching for optimal solutions when facing large-scale quantity, and there exists obstacle solving multi-objective function when it converges towards a limited region of Pareto fronts ignoring solutions. Farahani[37] succeeds in finding the Pareto fronts for the large-size problem instances of this multi-objective mixed-integer linear programming problem. This helps to develop a new model for a distribution network in a three-echelon supply chain better in representing the real-world situations, which not only minimizes the total cost, but also follows certain distribution purposes.

Goal programming approach has fewer variables to work with and will be computationally faster since it is only an one-stage method compared to other multi-stage methodology. In order to reduce logistics cost and enhance product quality simultaneously, Patia [38] proposes a mixed integer goal programming (MIGP) model to assist in the proper design of a multi-product paper recycling supply chain distribution network and finally obtain environmental benefits.

Unlike stochastic programming, Robust optimization does not assume that the uncertain parameters are random variables with known distributions; rather it models uncertainty in parameters using deterministic uncertainty sets in which all possible values of these parameters reside [39]. Robust optimization attempts to compute feasible solutions for a whole range of scenarios of the uncertain parameters, while optimizing an objective function in a controlled and balanced way with respect to the uncertainty in the parameters [40]. In order to enhance computational efficiency, Bertsimas and Thiele [41] proposes a general methodology based on robust optimization to address the problem of optimally controlling a supply chain subject to stochastic demand in discrete time. Yu [42] presents a robust optimization model to integrate classical goal programming techniques with a scenario-based data set to assist a manager in solving stochastic logistic problems wisely.

## 2.7 Work in this thesis

Most of the studies in supply chain network design problem only consider single stage, single echelon or single level parameters and they only considers in single objective. They also do not usually count uncertainties in the model they studies. Hence, it is not easy to find literatures that simultaneously focus on production, distribution and inventory task involved to optimize supply chain planning with uncertainties, especially for the case of more general operation networks and their mathematical model. This usually leads to a mixed-integer linear programming, the solution of which determines the optimal values for the mentioned variables. Few are listed below: Lei [43] investigates the interaction between disruption risk management in an one-supplier and one-retailer supply chain. Pishvaee[44] offers an efficient stochastic MILP model for single period, single product, multi-stage integrated supply chain network design. Amiri [45] develops a MILP model for a multi-stage forward network and also considers multiple capacity levels for each facility. The present work formulates the supply chain network design problem as a multi-objective stochastic mixed-integer programming problem, which is solved by using the goal programming approach. This formulation takes into account not only supply chain network total cost, but also the uncertainties reflected by stochastic demand and the disruption cost of suppliers. A number of supplier disruption scenarios with assigned probabilities shown in matrix are used as discrete stochastic supplier capacity. All scenarios are simultaneously taken into account in the supply chain network design. In order to find the best supply chain network allocation and production plan, it is necessary to consider trade offs among multiple objectives transformed as a set of Pareto-optimal solutions which are to be used by the decision-maker.

# Chapter 3

# MATHEMATICAL MODEL

*In this chapter, the mathematical model to the problem is presented. The problem description is provided and detailed formulations related to the model are explained part by part.*

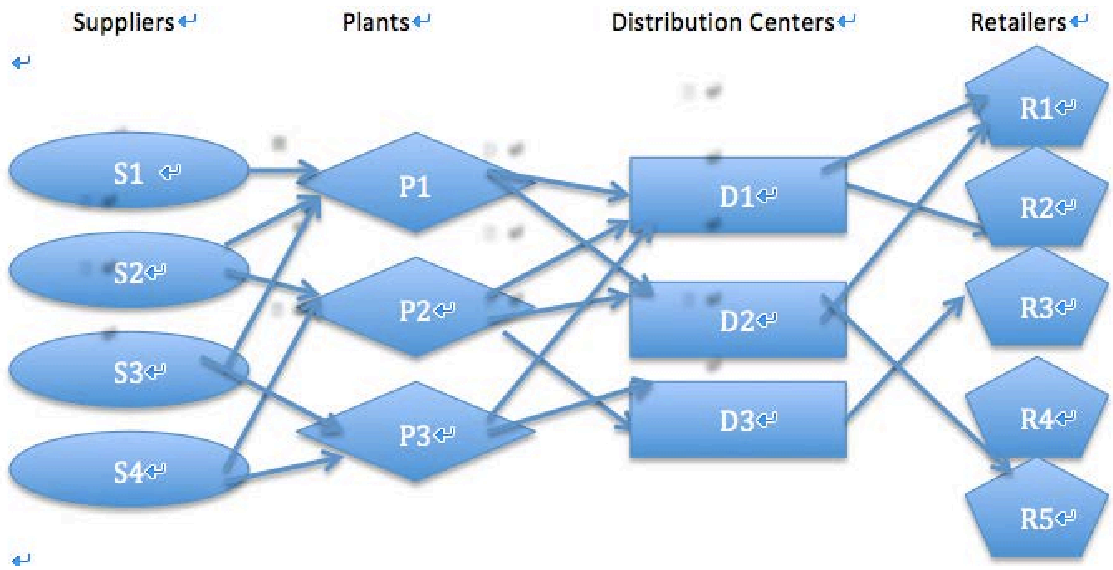
## 3.1 Problem description

The aim for this thesis is to determine the supply network configuration and associated production and distribution plans in an integrated manner in order to achieve the best use of available resources along entire planning horizon so that all retailer demands are met at minimum cost. So we model the problem based on designing a multi-echelon multi-product multi-period supply chain network system. The supply chain can accommodate for the production of more than one product with different production rates. Decisions to be made include strategic decisions like determining the subsets of plants and distribution centers to be opened, and a distribution network strategy that satisfy all capacities and retailer's demands in such a way that total cost is minimized. The structure of the supply chain system includes the following elements:

- A set of suppliers where raw materials are purchased prior to be sent to the plants;$\{S_1, S_2 \ldots S_S\}$
- A set of plants where products are manufactured prior to be sent to the warehouse; $\{P_1, P_2 \ldots P_P\}$
- A set of distribution centers where products are distributed and stored before being transported to the retailers; $\{D_1, D_2 \ldots D_D\}$
- A set of retailers where products are available to markets. $\{R_1, R_2 \ldots R_R\}$

The structure of the supply chain system with the four levels is depicted in Figure.3.1.

Figure 3.1 Basic structure of supply chain network model




The following functions are carried out: (1) acquiring raw materials, (2) manufacturing these raw materials and parts into finished products, (3) distributing and promoting these products to retailers, (4) making orders, gaining products and making them to market. Ideally we would like to deliver the products at the right time the retailers have requested and in the right amount as demanded. This helps to save holding cost and maximize retailer services. So we require that every supplier must deliver the right amount of products at the right time and to the right place. Two directions of this supply chain system are order flow and product flow. For order flow, retailers are always under uncertain demand with probability density function. Then distribution centers integrate the received orders from retailers and pass them to plants with limited capacity and finally orders reach suppliers. For product flow, plants acquire raw materials from suppliers and manufacture them to products, which are then delivered to retailers through distribution centers. Distribution centers then supply the items to independent identical retailers. When the demand occurs at the retailer, it is satisfied from the retailer's available stock. Otherwise, the demand is backordered. It is assumed that the retailers' demands are stochastic with a known probability density function for each product at specific time. The supply chain allows for building up of inventory at the distribution centers for optimization purposes, but due to capacity constraints of the distribution centers and plants, it might not be possible to satisfy all the requests on time hence we either have to deliver earlier or late sometimes, incurring shortage and holding costs in the process. Therefore, investment cost occurs at distribution centers and plants, production cost occurs at plants, raw material cost occurs at suppliers, while the product is transferred through the entire system, incurring costs like inventory holding costs, backorder cost, etc. All the costs take into account time value of money and interest rate. There exists disruption risk in suppliers' storage and uncertainty in retailers' demands. Suppliers' disruption is represented by a set of discrete scenarios with given probabilities of occurrence of shrinkage in storage capacity. This shrinkage can be due to the terrible weathers or disruptive accidents. The stochastic demand is assumed to be a uniform distribution U($a_{rpt}$, $b_{rpt}$). In the model two objective functions are simultaneously optimized. One objective function is to minimize total cost of the supply chain which includes the cost of running manufacturing plants and distribution centers, cost of buying and transporting raw materials, cost of transporting products from plants to DCs and from DCs to retailers, and cost of holding products at DCs. The other objective function is to minimize disruption cost, which occurs because of uncertain demand and supply disruption.

The proposed model aims to find:

1.Whether to run or close plants & DCs
2.Quantities (raw materials and products) transferred between each echelon in the network.

## 3.2 Model description

### 3.2.1 Notation

S:number of suppliers, s=[1,2,…,S]
f:number of plants, f=[1,2,…,F]
d:number of Distribution Centers, d=[1,2,…,D]
r:number of retailers, r=[1,2,…,R]

t:number of periods, t=[1,2,…,T]
p:number of products, p=[1,2,…,P]
ω:number of scenarios, $\omega = [1,2, \dots, \beta]$

### 3.2.2 Parameters

i: interest rate in each period
$\mathrm{rm_s}$:unit cost of raw material from supplier s
$\mathrm{pc_{pf}}$: production cost per unit for product p in plant f
$\mathrm{h_t}$:holding cost per unit in period t at the distribution center
$\mathrm{F_{ij}}$:failure rate from stage i to stage j
$\mathrm{u_{pt}}$:production rate of raw material in production for each unit of product p in period t
$\alpha_{\mathrm{rpt}}$: service level for product p by retailer r in period t
$Fcf_f$: Fixed cost of operating plant per period
$\mathrm{Fcd_d}$: Fixed cost of operating DC per period

$\mathrm{MCS_{sp}}$: maximum capacity of supplier s for product p
$\mathrm{Cs_{spt}}$(ω): capacity of supplier s for product p in scenario ω at period t
$\mathrm{Cf_{fp}}$: capacity of plant f for product p
$\mathrm{Cd_{dp}}$: capacity of DC d for product p

Psc: supplier losing capacity penalty loss per unit because of disruption
Pbc: backorder(shortage) penalty loss per unit in demand
Poc: spare value per unit in overproduction

$\mathrm{Tf_{psf}}$: transportation cost of raw material for product p per unit from supplier s to plant f
$\mathrm{Td_{pfd}}$: transportation cost of product p per unit from plant f to DC d
$\mathrm{Tr_{pdr}}$: transportation cost of product p per unit from DC d to retailer r
Capacity disruption parameter vector
$\Omega = [\Omega_1, \Omega_2, \Omega_3] = [1,0.5,0]$, $\Omega_\mathrm{i}$ means disruption status i, here 1 means supplier runs in full capacity, 0.5 means supplier capacity shrinks, 0 means supplier capacity totally

disrupts.

Disruption probability matrix

$\gamma=[\gamma_1, \gamma_2, \gamma_3]$ is the probability matrix corresponding to $\Omega$ matrix, which means the probability of supplier runs in full capacity is $\gamma_1$, the probability of supplier capacity shrinks is $\gamma_2$ and the probability of supplier capacity totally disrupts is $\gamma_3$

$\tau$= number of column in matrix $\Omega$ = 3

S= number of suppliers

$\beta = \text{Total scenarios} = (\text{number of } \Omega)^s = \tau^s = 3^s$ , each scenario corresponds to a combination of different supplier working under specific capacity

Capacity scenario matrix M

$$M = M_{S*\beta} = M_{S*\tau^s} = M_{S*3^s} = \begin{bmatrix} \Omega_1 & \Omega_1 & \cdots & \Omega_2 & \Omega_3 \\ \Omega_1 & \Omega_1 & \cdots & \Omega_3 & \Omega_3 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \Omega_1 & \Omega_1 & \cdots & \Omega_3 & \Omega_3 \\ \Omega_1 & \Omega_2 & \cdots & \Omega_3 & \Omega_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 0.5 & 0 \\ 1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 0 & 0 \\ 1 & 0.5 & \cdots & 0 & 0 \end{bmatrix} \;;$$

the number of rows corresponds to the number of suppliers and the number of columns corresponds to the number of scenarios. The value of $M_{s\omega}$ corresponds to the status of capacity for supplier s under scenario $\omega$.

Suppose we have two suppliers (s =2), which means we have 9 scenarios for this case ($3^s = 3^2 = 9$ ). Then matrix $M_{S*\tau^s}$ is a 2 x $3^2$ matrix, which is $M_{2*3^2} =$ $\begin{bmatrix} 1 & 1 & 1 & 0.5 & 0.5 & 0.5 & 0 & 0 & 0 \\ 1 & 0.5 & 0 & 1 & 0.5 & 0 & 1 & 0.5 & 0 \end{bmatrix}$. In this matrix, column 4(scenario 4) means supplier 1 (row 1)runs in shrunk capacity while supplier 2 (row 2)runs in full capacity. The value of $M_{24}$ corresponds to the status of capacity for supplier 2 under scenario 4.

$$P = \begin{bmatrix} \gamma_1{}^{s} \\ \gamma_1{}^{s-1} * \gamma_2{}^{1} \\ \vdots \\ \gamma_1{}^{s-k-1} * \gamma_2{}^{k} * \gamma_3{}^{1} \\ \vdots \\ \gamma_2{}^{1} * \gamma_3{}^{s-1} \\ \gamma_3{}^{s} \end{bmatrix}$$ is the probability column vector (β x 1 matrix) corresponding to matrix M (S x β matrix); this vector gives the probability of each scenario. Suppose we have three suppliers (s =3), which means we have 27 scenarios for this case ($3^{s} = 3^{3} = 27$ ). Then vector

P=

$[\gamma_1\gamma_1\gamma_1 \quad \cdots \quad \gamma_1\gamma_1\gamma_3 \quad \gamma_1\gamma_2\gamma_1 \quad \cdots \quad \gamma_1\gamma_2\gamma_3 \quad \cdots \quad \gamma_1\gamma_3\gamma_2 \quad \cdots \quad \gamma_2\gamma_3\gamma_1 \quad \cdots \quad \gamma_3\gamma_3\gamma_2 \quad \gamma_3\gamma_3\gamma_3]^{T}$

In this vector, row 4 gives the probability of scenario 4 (supplier1 and supplier 3 run in full capacity while supplier 2 runs in shrink capacity), which is $P_4 = \gamma_1{}^{2} * \gamma_2{}^{1}$.

Calculation of actual capacity

$CS_{sp}(\omega) = MCS_{sp} * M_{s\omega}$

$Cs_{sp}(\omega)$ is the capacity of supplier s for product p in scenario ω at period t, $MCS_{sp}$ is the maximum capacity of supplier s for product p, $M_{s\omega}$ is the element for $s^{th}$row $\omega^{th}$ column value in Matrix M. The value of $M_{s\omega}$ corresponds to the status of capacity for supplier s under scenario ω.

Stochastic Demand

$Dem_{rpt}$: stochastic demand at retailer r for product p in period t

$f_{rpt}(Dem_{rpt})$: demand's probability density function at retailer r for product p in period t

$a_{rpt}$: demand's lower bound at retailer r for product p in period t

$b_{rpt}$: demand's upper bound at retailer r for product p in period t

### 3.2.3 Decision Variables

$Q^{12}_{psft\omega}$: quantity of raw material for product p transported from supplier s to plant f in period t under scenario $\omega$

$Q_{pfdt}^{23}$: quantity of product p transported from plant f to DC d in period t

$Q_{pdrt}^{34}$: quantity of product p transported from DC d to retailer r in period t

$Q_{pdt}^{33}$: quantity of product p stored in DC d in period t

$A_{ft}$: binary variable to indicate whether plant p is open or closed in period t

$B_{dt}$: binary variable to indicate whether DC d is open or closed in period t

### 3.2.4 Assumptions

1. This supply chain network includes four echelons: supplier, plant, DC and retailer.
2. Shrinkage in supply capacity performs as source of disruption.
3. Backorder (shortage) and inventory holding exist and are allowed.
4. Lead time is assumed to be zero.
5. Links between each echelon has a different defect rate.
6. Number of suppliers, plants, DCs and retailers are known.
7. Capacity of plants and DCs are known, maximum capacities of suppliers are known.
8. Probabilities related to disruption scenarios are known, which affect supplier capacity.
9. Holding costs, utilization rates in production and demand are different for different period of time.
10. Each DC receives products from several plants and supplies several retailers.
11. A stochastic demand with a known probability density function for each product is assumed, which is uniform distributed.

### 3.2.5 Formulation

$$Investment\ Cost = \sum_{t=1}^{T}\sum_{d=1}^{D}\frac{Fcd_d * B_{dt}}{(1+i)^t} + \sum_{t=1}^{T}\sum_{f=1}^{F}\frac{Fcf_f * A_{ft}}{(1+i)^t}$$

The investment cost includes the fixed cost of operating plant and fixed cost of operating DC, while interests occur in each period.

Transportation Cost

$$= \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{f=1}^{F}\frac{\mathrm{T_{psf}} * \mathrm{Q_{psft\omega}^{12}}}{(1+\mathrm{i})^{\mathrm{t}}} + \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{f=1}^{F}\sum_{d=1}^{D}\frac{\mathrm{T_{pfd}} * \mathrm{Q_{pfdt}^{23}}}{(1+\mathrm{i})^{\mathrm{t}}}$$

$$+ \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{d=1}^{D}\sum_{r=1}^{R}\frac{\mathrm{T_{pdr}} * \mathrm{Q_{pdrt}^{34}}}{(1+\mathrm{i})^{\mathrm{t}}}$$

The transportation cost includes the transportation cost of raw materials from suppliers to plants, transportation cost of products from plants to DCs and transportation cost of products from DCs to retailers, while interests occur in each period.

$$\text{Holding Cost} = \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{d=1}^{D}\frac{\mathrm{h_t} * \mathrm{Q_{pdt}^{33}}}{(1+\mathrm{i})^{\mathrm{t}}}$$

The holding cost occurs when inventories exists in distribution centers, while interests occur in each period.

Raw material & production Cost

$$= \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{f=1}^{F}\frac{\mathrm{rm_s} * \mathrm{Q_{psft\omega}^{12}}}{(1+\mathrm{i})^{\mathrm{t}}} + \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{d=1}^{D}\frac{\mathrm{pc_{pf}} * \mathrm{Q_{pfdt}^{23}}}{(1+\mathrm{i})^{\mathrm{t}}}$$

The raw material and production cost include cost for plants purchasing raw materials from suppliers and cost for plants producing products from raw materials, while interests occur in each period.

$$\text{Shortage Cost} = \left[\sum_{r=1}^{R}\sum_{p=1}^{P}\sum_{t=1}^{T}\mathrm{Pbc} * \int_{\sum_{d=1}^{D}\mathrm{Q_{pdrt}^{34}} * \mathrm{F_{34}}}^{\mathrm{b_{rpt}}}\left(\mathrm{Dem_{rpt}} - \sum_{d=1}^{D}\mathrm{Q_{pdrt}^{34}} * \mathrm{F_{34}}\right)\right.$$

$$\left. * \, \mathrm{f_{rpt}}(\mathrm{Dem_{rpt}})\mathrm{dDem_{rpt}}\right] / (1+\mathrm{i})^{\mathrm{t}}$$

Shortage cost equals to shortage quantity of products times per unit penalty cost. The term $\mathrm{Dem}_{rpt} - \sum_{d=1}^{D} \mathrm{Q}_{pdrt}^{34} * \mathrm{F}_{34}$ ) is the shortage quantity of products experienced by retailer r. The random variable is $\mathrm{Dem}_{rpt}$ which follows uniform distributed density function $\mathrm{f}_{rpt}(\mathrm{Dem}_{rpt})$. The range of this integration is from $\sum_{d=1}^{D} \mathrm{Q}_{pdrt}^{34} * \mathrm{F}_{34}$ which is the amount supplied to retailer r to maximum demand which is $\mathrm{b}_{rpt}$.

Value of surplus supplied to retailer r

$$= \left[\sum_{r=1}^{R}\sum_{p=1}^{P}\sum_{t=1}^{T} \mathrm{Poc} * \int_{\mathrm{a}_{rpt}}^{\sum_{d=1}^{D} \mathrm{Q}_{pdrt}^{34} * \mathrm{F}_{34}} \left(\sum_{d=1}^{D} \mathrm{Q}_{pdrt}^{34} * \mathrm{F}_{34} - \mathrm{Dem}_{rpt}\right) * \mathrm{f}_{rpt}(\mathrm{Dem}_{rpt})\,\mathrm{dDem}_{rpt}\right] / (1+\mathrm{i})^{\mathrm{t}}$$

The term $(\sum_{d=1}^{D} \mathrm{Q}_{pdrt}^{34} * \mathrm{F}_{34} - \mathrm{Dem}_{rpt})$ means surplus quantity of products sent to retailer in excess of the current retailer's demand. The random variable is $\mathrm{Dem}_{rpt}$ which follows uniform distributed density function $\mathrm{f}_{rpt}(\mathrm{Dem}_{rpt})$. The range of this integration is from minimum demand which is $\mathrm{a}_{rpt}$ to $\sum_{d=1}^{D} \mathrm{Q}_{pdrt}^{34} * \mathrm{F}_{34}$ which is the amount supplied to retailer r and equal to the demand.

$$\text{Supplier disruption Cost} = \left[\left(\mathrm{MCS}_s - \sum_{\omega=1}^{\beta} \mathrm{CS}_{sp}(\omega)\right) * \mathrm{Psc}\right] / (1+\mathrm{i})^{\mathrm{t}}$$

The term $\left(\mathrm{MCS}_s - \sum_{\omega=1}^{\beta} \mathrm{CS}_{sp}(\omega)\right)$ is the disrupted quantity and equal to the maximum capacity minus current supplier capacity under certain scenario.

### 3.2.6 Objective function

1. Min f1 = Total Cost = Inventory Cost + Transportation Cost + Holding Cost + Raw material& production Cost = $\sum_{t=1}^{T}\sum_{d=1}^{D}\frac{Fcd_d*B_{dt}}{(1+i)^t}+\sum_{t=1}^{T}\sum_{f=1}^{F}\frac{Fcf_f*A_{ft}}{(1+i)^t}+$

$\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{f=1}^{F}\frac{\mathrm{T_{psf}}*\mathrm{Q_{psft\omega}^{12}}}{(1+\mathrm{i})^{\mathrm{t}}}+\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{f=1}^{F}\sum_{d=1}^{D}\frac{\mathrm{T_{pfd}}*\mathrm{Q_{pfdt}^{23}}}{(1+\mathrm{i})^{\mathrm{t}}}+$

$\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{d=1}^{D}\sum_{r=1}^{R}\frac{\mathrm{T_{pdr}}*\mathrm{Q_{pdrt}^{34}}}{(1+\mathrm{i})^{\mathrm{t}}}+\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{d=1}^{D}\frac{\mathrm{h_t}*\mathrm{Q_{pdt}^{33}}}{(1+\mathrm{i})^{\mathrm{t}}}+$

$\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{f=1}^{F}\frac{\mathrm{rm_s}*\mathrm{Q_{psft\omega}^{12}}}{(1+\mathrm{i})^{\mathrm{t}}}+\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{d=1}^{D}\frac{\mathrm{pc_{pf}}*\mathrm{Q_{pfdt}^{23}}}{(1+\mathrm{i})^{\mathrm{t}}}$

(1.1)

2.Min f2 = Disruption Cost = Shortage Cost − Value of surplus supplied to retailer + Supplier disruption Cost =

$\Big[\sum_{r=1}^{R}\sum_{p=1}^{P}\sum_{t=1}^{T}\mathrm{Pbc}*\int_{\sum_{d=1}^{D}\mathrm{Q_{pdrt}^{34}}*\mathrm{F_{34}}}^{\mathrm{b_{rpt}}}(\mathrm{Dem_{rpt}}-\sum_{d=1}^{D}\mathrm{Q_{pdrt}^{34}}*\mathrm{F_{34}})*$

$\mathrm{f_{rpt}}(\mathrm{Dem_{rpt}})\mathrm{dDem_{rpt}}-\sum_{r=1}^{R}\sum_{p=1}^{P}\sum_{t=1}^{T}\mathrm{Poc}*\int_{\mathrm{a_{rpt}}}^{\sum_{d=1}^{D}\mathrm{Q_{pdrt}^{34}}*\mathrm{F_{34}}}(\sum_{d=1}^{D}\mathrm{Q_{pdrt}^{34}}*$

$\mathrm{F_{34}}-\mathrm{Dem_{rpt}})*\mathrm{f_{rpt}}(\mathrm{Dem_{rpt}})\mathrm{dDem_{rpt}}+\left(\mathrm{MCS_s}-\sum_{\omega=1}^{\beta}\mathrm{CS_{sp}}(\omega)\right)*\mathrm{Psc}\Big]/(1+\mathrm{i})^{\mathrm{t}}.$

(1.2)

Failure rate in transportation from distribution center stage to retailer stage is denoted by $\mathrm{F_{34}}$)

### 3.2.7 Constraints

Service Level Constraints

$\int_{\mathrm{a_{rpt}}}^{\sum_{d=1}^{D}\mathrm{Q_{pdrt}^{34}}*\mathrm{F_{34}}}*\mathrm{f_{rpt}}(\mathrm{Dem_{rpt}})\,\mathrm{dDem_{rpt}}\geq\alpha_{\mathrm{rpt}}.$ (2.1)

This guarantees that the probability when the demand is smaller than quantities of products sent to retailer r is greater than or equal to the service level $\alpha_{\mathrm{rpt}}$

Capacity Constraints

Supplier Capacity:

For supplier s at period t, the total flow of products sent from supplier s to all plants f, noted as $\sum_{f=1}^{F} Q_{psft\omega}^{12}$ is constrained by the actual capacity of supplier s for product p in scenario ω,noted as $CS_{sp}(\omega)$

$$\sum_{f=1}^{F} Q_{psft\omega}^{12} \leq CS_{sp}(\omega), \text{ for t=1,2…T, p=1,2...P,s=1,2…S, ω=1,2,… β} \quad (2.2)$$

Plant Capacity:

For plant f at period t, the total flow of materials sent from all suppliers to plant f, noted as $\sum_{s=1}^{S} Q_{psft\omega}^{12}$, multiplied by the probability of failure during transportation from stage 1 (supplier stage) to stage 2 (plant stage) is constrained by the capacity of plant f for product p, noted as $Cf_{fp}$. Also $A_{ft}$ is the binary variable to indicate whether plant f is open or closed in period t

$$\sum_{s=1}^{S} Q_{psft\omega}^{12} * F_{12} \leq Cf_{fp} * A_{ft}, \text{ for t=1,2…T, p=1,2...P, f=1,2…F} \quad (2.3)$$

DC Capacity:

The quantity of products in distribution center d should be less than or equal to the capacity of distribution center d, noted as $Cd_{dp}$,. At period t, the quantity of products in distribution center d includes the total flow of products sent from all plants to distribution center d, noted as $\sum_{f=1}^{F} Q_{pfdt}^{23}$ multiplied by probabilities of failure during transportation from stage 2 (plant stage) to stage 3 (distribution center stage) , the inventory held in distribution center, noted as $Q_{pdt}^{33}$ and total flow of products sent from distribution center d to all retailers, noted as$\sum_{r=1}^{R} Q_{pdrt}^{34}$. Also $B_{dt}$ is the binary variable to indicate whether distribution center d is open or closed in period t.

$$\sum_{f=1}^{F} Q_{pfdt}^{23} * F_{23} + Q_{pdt}^{33} - \sum_{r=1}^{R} Q_{pdrt}^{34} \leq Cd_{dp} * B_{dt}, \text{ for t=1,2…T, p=1,2...P, d=1,2…D}$$

(2.4)

Balance Constraints

Plant Balance:

For plant f at period t, the total quantity of raw materials sent from all suppliers to plant f, noted as the input flow$\sum_{s=1}^{S} \mathrm{Q}_{\mathrm{psft\omega}}^{12}$, be equal to total quantity of products sent from plant f to all distribution centers, noted as the output$\sum_{d=1}^{D} \mathrm{Q}_{\mathrm{pfdt}}^{23}$. The input quantities multiplied by production rate $\mathrm{u}_{\mathrm{pt}}$ should always be greater than or equal to output quantities.

$$\sum_{s=1}^{S} \mathrm{Q}_{\mathrm{psft\omega}}^{12} * \mathrm{u}_{\mathrm{pt}} \geq \sum_{d=1}^{D} \mathrm{Q}_{\mathrm{pfdt}}^{23}, \quad \text{for t=1,2…T, p=1,2...P, f=1,2…F, for each } \omega \qquad (2.5)$$

DC Balance:

For distribution center d, the sum of the total quantity of inventory held at period t-1 for product p in distribution center d, noted as $\mathrm{Q}_{\mathrm{pd(t-1)}}^{33}$, and total quantity of products sent from all plants to distribution center d, noted as input flow $\sum_{f=1}^{F} \mathrm{Q}_{\mathrm{pfdt}}^{23}$ is equal to the sum of the total quantity of inventory held at period t for product p in distribution center d, noted as $\mathrm{Q}_{\mathrm{pdt}}^{33}$,and total quantity of products sent from distribution center d to all retailers, noted as output flow $\sum_{r=1}^{R} \mathrm{Q}_{\mathrm{pdrt}}^{34}$

$$\mathrm{Q}_{\mathrm{pd(t-1)}}^{33} + \sum_{f=1}^{F} \mathrm{Q}_{\mathrm{pfdt}}^{23} = \mathrm{Q}_{\mathrm{pdt}}^{33} + \sum_{r=1}^{R} \mathrm{Q}_{\mathrm{pdrt}}^{34}, \quad \text{for t=1,2…T, p=1,2...P, d=1,2…D} \qquad (2.6)$$

Demand Constraints

upper and lower demand limits under uncertainty

$$\mathrm{a}_{\mathrm{rpt}} \leq \sum_{d=1}^{D} \mathrm{Q}_{\mathrm{pdrt}}^{34} \leq \mathrm{b}_{\mathrm{rpt}} \qquad (2.7)$$

Facility operation Constraints

$$\sum_{f=1}^{F} \mathrm{A}_{\mathrm{ft}} \geq 1 \qquad (2.8)$$

This guarantees that at least one plant is operating. $\mathrm{A}_{\mathrm{ft}}$ is the binary variable to indicate whether plant f is open or closed in period t.

$$\sum_{d=1}^{D} \mathrm{B}_{\mathrm{dt}} \geq 1 \qquad (2.9)$$

This guarantees that at least one distribution center is operating. $\mathrm{B}_{\mathrm{dt}}$ is the binary variable to indicate whether distribution center d is open or closed in period t.

Non-negativity Constraints

$$\mathrm{Q}^{12}_{\mathrm{psft}\omega}, \mathrm{Q}^{23}_{\mathrm{pfdt}}, \mathrm{Q}^{34}_{\mathrm{pdrt}}, \mathrm{Q}^{33}_{\mathrm{pdt}}, \geq 0 \qquad (2.10)$$

Binary Constraints

$$\mathrm{A}_{\mathrm{ft}}, \mathrm{B}_{\mathrm{dt}} = \{0,1\} \qquad (2.11)$$

### 3.2.8 Summary of formulation

Decision Variables: $Q^{12}_{psft\omega}$, $Q^{23}_{pfdt}$, $Q^{34}_{pdrt}$, $Q^{33}_{pdt}$, $A_{ft}$, $B_{dt}$

Objective functions: Min f1=$\sum_{t=1}^{T}\sum_{d=1}^{D}\frac{Fcd_d*B_{dt}}{(1+i)^t}+\sum_{t=1}^{T}\sum_{f=1}^{F}\frac{Fcf_f*A_{ft}}{(1+i)^t}+$
$\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{f=1}^{F}\frac{T_{psf}*Q^{12}_{psft\omega}}{(1+i)^t}+\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{f=1}^{F}\sum_{d=1}^{D}\frac{T_{pfd}*Q^{23}_{pfdt}}{(1+i)^t}+$
$\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{d=1}^{D}\sum_{r=1}^{R}\frac{T_{pdr}*Q^{34}_{pdrt}}{(1+i)^t}+\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{d=1}^{D}\frac{h_t*Q^{33}_{pdt}}{(1+i)^t}+\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{f=1}^{F}\frac{rm_s*Q^{12}_{psft\omega}}{(1+i)^t}+$
$\sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{s=1}^{S}\sum_{d=1}^{D}\frac{pc_{pf}*Q^{23}_{pfdt}}{(1+i)^t}$ (1.1)

Min f2 = $\Big[\sum_{r=1}^{R}\sum_{p=1}^{P}\sum_{t=1}^{T} Pbc * \int_{\sum_{d=1}^{D}Q^{34}_{pdrt}*F_{34}}^{b_{rpt}}(Dem_{rpt}-\sum_{d=1}^{D}Q^{34}_{pdrt}*F_{34})$
$* f_{rpt}(Dem_{rpt})dDem_{rpt}-\sum_{r=1}^{R}\sum_{p=1}^{P}\sum_{t=1}^{T} Poc * \int_{a_{rpt}}^{\sum_{d=1}^{D}Q^{34}_{pdrt}*F_{34}}(\sum_{d=1}^{D}Q^{34}_{pdrt} *$
$F_{34}-Dem_{rpt}) * f_{rpt}(Dem_{rpt})dDem_{rpt}+\left(MCS_s-\sum_{\omega=1}^{\beta}CS_{sp}(\omega)\right)*Psc\Big]/(1+i)^t.$
(1.2)

Constraints(subject to)

$\int_{a_{rpt}}^{\sum_{d=1}^{D}Q^{34}_{pdrt}*F_{34}} * f_{rpt}(Dem_{rpt})\, dDem_{rpt} \geq \alpha_{rpt}.$ (2.1)

$\sum_{f=1}^{F}Q^{12}_{psft\omega} \leq CS_{sp}(\omega)$, for t=1,2…T, p=1,2...P,s=1,2…S, ω=1,2,… β (2.2)

$\sum_{s=1}^{S}Q^{12}_{psft\omega} * F_{12} \leq Cf_{fp} * A_{ft}$, for t=1,2…T, p=1,2...P, f=1,2…F (2.3)

$\sum_{f=1}^{F}Q^{23}_{pfdt} * F_{23} + Q^{33}_{pdt} - \sum_{r=1}^{R}Q^{34}_{pdrt} \leq Cd_{dp} * B_{dt}$, for t=1,2…T, p=1,2...P, d=1,2…D (2.4)

$\sum_{s=1}^{S}Q^{12}_{psft\omega} * u_{pt} \geq \sum_{d=1}^{D}Q^{23}_{pfdt}$, for t=1,2…T, p=1,2...P, f=1,2…F, for each ω (2.5)

$Q^{33}_{pd(t-1)} + \sum_{f=1}^{F}Q^{23}_{pfdt} = Q^{33}_{pdt} + \sum_{r=1}^{R}Q^{34}_{pdrt}$, for t=1,2…T, p=1,2...P, d=1,2…D (2.6)

$a_{rpt} \leq \sum_{d=1}^{D}Q^{34}_{pdrt} \leq b_{rpt}$ (2.7)

$\sum_{f=1}^{F}A_{ft} \geq 1$ (2.8)

$\sum_{d=1}^{D}B_{dt} \geq 1$ (2.9)

$Q^{12}_{psft\omega}$, $Q^{23}_{pfdt}$, $Q^{34}_{pdrt}$, $Q^{33}_{pdt}$, $\geq 0$ (2.10)

$A_{ft}$, $B_{dt}$ ={0,1} (2.11)

# Chapter 4

# METHODOLOGIES

*In this chapter the detailed mathematical methodologies are presented prior to numerical example study in order to understand the problem-solving methods in use and theoretical optimization process.*

In order to demonstrate the application of proposed model dealing with the problem of supply side uncertainty and demand uncertainty, methodologies in solving numerical example and the computing study analysis are provided in following chapters.

The model developed in Chapter 3 is a constrained bi-objective mixed-integer nonlinear programming for the supply chain network design problem, including $f_1$ as a linear objective and $f_2$ as a nonlinear objective composed of a set of quadratic terms. The general form of mathematical models with w objective functions and p variables in the vector X=$[x_1, x_2, x_3 \ldots x_p]$ is

$$Min[f_1(x), f_2(x) \ldots f_w(x)]$$

$$s.t.:$$

$$g_d(x) \leq b_d \text{ ; } d = 1,2 \ldots u$$

$$x_c \geq 0 \text{ ; } c = 1,2, \ldots p$$

where u and p represent the number of constraints and variables. The ideal solutions to the model not only satisfy all the constraints but also optimize the set of objective functions simultaneously. However there usually exists trade-off between these objectives. To some extent, many problems in real-life contain conflicting objectives and feasible solutions do not usually optimize all objective simultaneously. Hence, it is critical to make fine decision to find an effective solution.

## 4.1 Basic method: Pareto frontiers

The solution of multi-objective problems contains of a set of Pareto frontiers in supply chain network, solved by the $\varepsilon$-constraint method. Based on Mavrotas's work [46], the $\varepsilon$-constraint comes up with idea of keeping only one of the objectives and restricting the rest of the objectives within some specified value. Specifically, the $\varepsilon$-constraint method is based on optimizing one of the objective functions and considering others as constraints bounded by some allowable level $\varepsilon_0$. The entire Pareto optimal set will be generated to solve mixed integer problem by adjusting the different levels of $\varepsilon_0$ value. The following is a basic model for the $\varepsilon$-constraint method[47].

$$\min[f_1(x), f_2(x),..., f_n(x)]$$

$$x \in S,$$

where n > 1 and S is the set of constraints defined above. The objective space is the space that objective vector belongs to, and the corresponding feasible set under F is called the attained set$[f_1(x), f_2(x),..., f_n(x)]$. Such a set will be denoted in the following with

$$C=\{y\in R^n :y= f(x),x\in S\}$$

Essentially, a vector $x^* \in$ S is said to be Pareto optimal for a multi-objective problem, if all other vectors x $\in$ S have a higher value for at least one of the objective functions $f_i$, with i = 1, 2⋯, n, or have the same value for all the objective functions$[f_1(x), f_2(x),..., f_n(x)]$. The space containing the efficient solutions lied in is called Pareto curve or surface formed by sets of Pareto fronts. The trade-off which exists in the different objective functions can be determined by the shape of such Pareto surface. Since the model in this work stands for a mixed-integer problem, the image is presented as a set of discrete points instead of a continuous curve. Pareto surface cannot be computed efficiently in many cases. Hence, except multi-objective optimization techniques that provide Pareto frontiers, various methods are available to solve multi-objective programming models.

## 4.2 Individual optimization

Individual optimization method is a simple and direct way to find optimal values. It is supposed that the optimal solution for each objective solution is an effective solution to multiple objectives problem. If the effective solutions obtained from the first objective problem satisfy the second or more objective problems constraints, then ideal solution can be obtained [48]. In the model, the first objective function is a linear function containing 108 decision variables and the second objective function is a nonlinear function composed of a set of quadratic terms. Each quadratic term contains two decision variables. In MATLAB, the function linprog () is applied to solve linear objective function and the function quadprog() is usually applied to solve quadratic function with a set of Hessian matrixes required.

## 4.3 Goal attainment method

Goal attainment method is also an effective method to solve the multi-objective problem. In this method, the optimal value varies with weight vectors and goal vectors given by the decision-maker, which is the same as the goal programming technique. Specifically, this method finds the solution that minimizes the highest weighed deviation between the individual and overall objective function values. In the model, goal vectors are gained from individual optimization method to make preferred solution close to each objective function value. In addition, goal attainment method is computationally faster than some interactive multi-objective techniques such as complex genetic algorithm [49]. It has fewer variables to work with and is an one-stage method. Hence, it is one of the efficient methods to solve large-scale mixed-integer nonlinear program. In MATLAB, the function fgoalattain() function is applied and the sets of weight vectors are tested in the model to find the best solution to the problem.

## 4.4 Sequential quadratic programming

Sequential quadratic programming, known as the function fmincon() in MATLAB, solves the multi-objective nonlinear problem as sets of quadratic programming subproblem in a few iterations. The function fmincon() performs line searches using a merit function similar to the function proposed by Powell [50]. The quadratic programming subproblem is solved using an active set strategy similar to that described in Gill's work[51]. It is efficient that it updates an estimate of the Hessian of the Lagrangian quickly at each iteration using the dense quasi-Newton approximation formula.

## 4.5 Min-max method

Min-max method aims to find a solution that minimizes the maximum difference between model output and design specification [52]. In MATLAB, the function fminimax() is a special case of approximated Hessian(quasi-Newton SQP) algorithm that uses the soft line search method. It uses fewer iterations than the function fmincon() while performing generally well. In the numerical results, min-max methods stands out on overall performance when compared to other accessible methods.

## 4.6 NSGA-II

NSGA-II is a non-dominated sorting genetic algorithm solving multi-objective problem. This method has strong applicability to find a much better spread of solutions and better convergence near the true Pareto-optimal front for most problems. It adapts a suitable automatic mechanics based on the crowding distance in order to guarantee diversity and spread of solutions[53]. More importantly, NSGA-II has its corresponding solver in MATLB represented by the function gamultiobj(). This function in NSGA-II requires to set the data for Pareto fraction, population size on Pareto frontier, time limit of stall generation and function tolerance to create a set of Pareto optima for a multi-objective minimization.

## 4.7 Scenario analysis

In the model, supplier disruption is described by scenario analysis. A scenario is a description of a future situation and the course of events that enables one to progress from the original situation to the future situation [54]. Scenario analysis attempts to capture uncertainty by using moderate number of discrete realizations of the stochastic quantities, which constitutes distinct scenarios. The aim for adopting scenario analysis is that problem-solving methods mentioned above should be robust enough to perform well under both optimistic and pessimistic situations. In the model, the supplier capacity can be totally disrupted or shrunk to some extent caused by some disruption events. Hence, each scenario represents a supplier capacity situation. The number of scenarios varies with the number of suppliers. Different optimal values can be obtained by computing each problem-solving method under every scenario, even though there are some non-available data. Such non-available data shows that it is infeasible to satisfy all constraints under certain scenario. The weighted average of objective value among all scenarios for each method is calculated to determine the best solving method.

These above problem-solving methods are all first employed to solve the proposed model to obtain the optimal value for both objective functions. The CPU running time for each corresponding program is also observed. Then the displaced ideal solution method is used to evaluate these outputs to judge the overall performance for each method and the best method is selected to obtain the optimal solution and further sensitivity analysis.

# Chapter 5

# NUMRICAL EXAMPLES

*In this chapter, the practical use of proposed model is presented by studying numerical example. The numerical case study is given followed by the detailed data analysis.*

In order to demonstrate the application of proposed model dealing with the problem of supply side uncertainty and demand uncertainty, the resulting study analysis is provided.

## 5.1 Numerical results

A set of experiments are designed and conducted to study the effectiveness of the proposed model. In this example, the network structure and allocation-distribution plans for a three-period three-echelon planning horizon are provided. The input parameters in this supply chain network design model are given in Table 5.1.

Table 5.1 : Data set for numerical example

| Parameters | Value | Parameter | Value |
|---|---|---|---|
| No. of suppliers (s) | 2 | No. of plants (f) | 2 |
| No. of DCs (d) | 2 | No. of retailers (r) | 3 |
| No. of periods (t) | 3 | No. of products (p) | 2 |
| No. of scenarios ($\omega$) | 9 | Interest rate (i) | 10% |
| Raw material unit cost for supplier s ($rm_s$) | $rm_1$=24, $rm_2$=20 | Production cost per unit for product p in plant f ($pc_{pf}$) | $pc_{11}$ =7 , $pc_{12}$ =8, $pc_{21}$=6, $pc_{22}$=7 |
| Holding cost per unit in period t at DC ($h_t$) | $h_1$=7,$h_2$=9,$h_3$=7 | Quality rate from stage i to stage j ($F_{ij}$) | $F_{12}$ =0.85, $F_{23}$ =0.9, $F_{34}$=0.88 |
| Production rate of raw material in production ($u_{pt}$) | $u_{11}$ =70%, $u_{12}$ =75%, $u_{13}$ =60%, $u_{21}$ =70%, $u_{22}$=65%,$u_{23}$=70% | Maximum capacity of supplier s for product p ($MCS_{sp}$) | $MCS_{11}$ = $MCS_{12}$ =3000, $MCS_{21}$=$MCS_{22}$=2500 |
| Service level for product p by retailer r in period t($\alpha_{rpt}$) | $\alpha_{111}$ =90%, $\alpha_{112}$ =92%, $\alpha_{113}$=90%,$\alpha_{121}$=95%, $\alpha_{122}$=95%,$\alpha_{123}$=90%, $\alpha_{211}$=85%,$\alpha_{212}$=88%, $\alpha_{213}$=92%,$\alpha_{221}$=88%, $\alpha_{222}$=90%,$\alpha_{223}$=90%, $\alpha_{311}$=85%,$\alpha_{312}$=85%, $\alpha_{313}$=88%,$\alpha_{321}$=88%, $\alpha_{322}$=93%,$\alpha_{323}$=91% | Fixed cost of operating plant per period ($Fcf_f$) | $Fcf_1$ =8500, $Fcf_2$=9500 |
| | | Fixed cost of operating DC per period ($Fcd_d$) | $Fcd_1$ =6000, $Fcd_2$=7000 |
| Capacity of plant f for product p ($Cf_{fp}$) | $Cf_{11}$ = $Cf_{12}$ =1700 $Cf_{21}$=$Cf_{22}$=1500 | Capacity of DC d for product p ($Cd_{dp}$) | $Cd_{11}$ =600, $Cd_{12}$ =580, $Cd_{21}$=570,$Cd_{22}$=730 |
| Penalty loss per unit in supplier stage (Psc) | 15 | Penalty loss per unit for shortage (Pbc) | 20 |
| Spare value per unit for overproduction (Poc) | 5 | | |
| Transportation cost per unit from plant f to DC d ($Td_{pfd}$) | $Td_{111}$ =10, $Td_{112}$ =9, $Td_{121}$=10,$Td_{122}$=8, $Td_{211}$=8,$Td_{212}$=12, $Td_{221}$=10,$Td_{222}$=8 | Transportation cost per unit from supplier s to plant f ($Tf_{psf}$) | $Tf_{111}$=10,$Tf_{112}$=12, $Tf_{121}$=12,$Tf_{122}$=10, $Tf_{211}$=8,$Tf_{212}$=10, $Tf_{221}$=10,$Tf_{222}$=10 |
| Transportation cost per unit from DC d to retailer r ($Tr_{pdr}$) | $Tr_{111}$ =7, $Tr_{112}$ =9, $Tr_{113}$ =8, $Tr_{121}$ =10, $Tr_{122}$=7,$Tr_{123}$=7, $Tr_{211}$=12,$Tr_{212}$=10, $Tr_{213}$=8,$Tr_{221}$=12, $Tr_{222}$=12,$Tr_{223}$=9 | | |

As mentioned earlier, retailers are always facing uncertain demand, which is assumed to follow uniform distribution. Hence, lower bound and upper bound for each demand at certain circumstance of this example are given in Table 5.2 .

Table 5.2: Stochastic demand for at retailer r for product p in period t ($a_{rpt} \leqq Dem_{rpt} \leqq b_{rpt}$)

| Parameters | t=1 | t=2 | t=3 |
|---|---|---|---|
| $Dem_{11t}$ | ~Unif[400,500] | ~Unif [450,520] | ~Unif [380,420] |
| $Dem_{12t}$ | ~Unif [600,800] | ~Unif [580,720] | ~Unif [600,700] |
| $Dem_{21t}$ | ~Unif [300,350] | ~Unif [380,400] | ~Unif [400,480] |
| $Dem_{22t}$ | ~Unif [480,600] | ~Unif [520,600] | ~Unif [500,650] |
| $Dem_{31t}$ | ~Unif [500,600] | ~Unif [480,620] | ~Unif [420,480] |
| $Dem_{32t}$ | ~Unif [560,600] | ~Unif [500,550] | ~Unif [520,540] |

There also exist disruption risk in suppliers' storage and uncertainty in retailers' demands. Suppliers' disruption is represented by a set of discrete scenarios and probabilities are associated with scenarios. A solution is sought against every possible scenario. It should be noted that the individual solution varies with scenario. The probabilities assigned to scenarios represent the importance weight of each scenario when calculating weighed average of optimal value. These probabilities are concluded in the probability column vector P (9 x 1 matrix) corresponding to capacity scenario matrix M (2 x 9 matrix). Capacity scenario matrix M is also related to capacity disruption parameter vector $\Omega = [\Omega_1, \Omega_2, \Omega_3] = [1,0.5,0]$. Both sets of values for vector P and Ω are tested in sensitivity analysis to find the effect of changes in probability and scale of supplier disruption. Vector P and matrix M are listed below.

$$P = [0.7225 \quad 0.085 \quad 0.0425 \quad 0.085 \quad 0.01 \quad 0.005 \quad 0.00425 \quad 0.005 \quad 0.0025]^T$$

$$M = \begin{bmatrix} 1 & 1 & 1 & 0.5 & 0.5 & 0.5 & 0 & 0 & 0 \\ 1 & 0.5 & 0 & 1 & 0.5 & 0 & 1 & 0.5 & 0 \end{bmatrix}$$

Weighed average of objective value plays a key role in the method selection section. Actual capacity data with probability of corresponding scenario are given in Table 5.3 .

Table 5.3: Actual capacity

| Scenario ω | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $Cs_{1p}$(ω), p=1,2 | 3000 | 3000 | 3000 | 1500 | 1500 | 1500 | 0 | 0 | 0 |
| $Cs_{2p}$(ω), p=1,2 | 2500 | 1250 | 0 | 2500 | 1250 | 0 | 2500 | 1250 | 0 |
| Probability | 0.7225 | 0.085 | 0.0425 | 0.085 | 0.01 | 0.005 | 0.0425 | 0.005 | 0.0025 |

With changing supplier capacity, the solutions and optimal values are assessed under each scenario by allowing models to update their decision variables on quantity of flows between network facilities under each scenario.

Using the above numerical data, the mixed-integer nonlinear programming problem modeled in Chapter 3 is solved with different methods using MATLAB software platform on Core i5 2.8GHz processor and 4GB memory. The detailed mathematical formulations for this example are attached in Appendix A.

### 5.1.1 The experiment on selecting best set of weight for goal attainment method

Goal attainment method is widely used in solving multi-objective problem. However, this method cannot perform well until some parameters are set appropriately, such as weight vectors and goal vectors. Considering that the result of using goal attainment method varies with different weights, the aim of this experiment is to study the effect of applying different sets of weight values in the model and find the best set for solving the model. Goal vectors are obtained from independent optimized values for each objective function. Result for each weight set applied to goal attainment method is given in Table 5.4 .

Table 5.4: Goal Attainment method with different weights

| Weight(f1,f2) | Scenarios | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | WeightedAvg |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.5,0.5 | Total Cost | 570649.8 | 573494.9 | 587435.8 | 570649.8 | 573494.9 | N/A | 570649.8 | N/A | N/A | 574395.83 |
| | Disruption Cost | 29063.27 | 125185.93 | 223198.64 | 140963.27 | 237085.93 | N/A | 252863.27 | N/A | N/A | 168061.72 |
| 0.6,0.4 | Total Cost | 576280.5 | 580001.8 | 588914.7 | 576280.5 | 580001.8 | N/A | 576280.5 | N/A | N/A | 579626.63 |
| | Disruption Cost | 27222.83 | 122946.77 | 222174.36 | 139122.83 | 234846.77 | N/A | 251022.83 | N/A | N/A | 166222.73 |
| 0.4,0.6 | Total Cost | 566074.3 | 568342.7 | 587435.8 | 566074.3 | 568342.7 | N/A | 566074.3 | N/A | N/A | 570390.68 |
| | Disruption Cost | 30688.79 | 127341.58 | 223198.64 | 142588.79 | 239241.58 | N/A | 254488.79 | N/A | N/A | 169591.36 |
| 0.8,0.2 | Total Cost | 593208.8 | 599485.9 | 612735.3 | 593208.8 | 599485.9 | N/A | 593208.8 | N/A | N/A | 598555.58 |
| | Disruption Cost | 22030.22 | 116870.81 | 213421.09 | 133930.22 | 228770.81 | N/A | 245830.22 | N/A | N/A | 160142.23 |
| 0.2,0.8 | Total Cost | 559077.8 | 564075.8 | 587435.8 | 559077.8 | 564075.8 | N/A | 559077.8 | N/A | N/A | 565470.13 |
| | Disruption Cost | 33434.8 | 129948.64 | 223198.64 | 145334.8 | 241848.64 | N/A | 257234.8 | N/A | N/A | 171833.39 |

Data under scenario 6,8 and 9 stays vacant, which indicates that the disruption of suppliers’ storage capacity causes insufficient transporting of products to satisfy the demand. In these scenarios there are no feasible solution to satisfy all constraints listed. Based on the results obtained, it can be easily seen that none of the weight sets can optimize both objective functions simultaneously. For instance, weight set (0.2,0.8) performs best in optimizing total cost; however, it has the worst performance in optimizing disruption cost. Hence, the displaced ideal solution method is used here to select the best solution set [55]. In this selection method, Fi is a vector of the solution for each method and in this part it is obtained based on two criteria, which are the weighted average of objective 1 value and the weighted average of objective 2 value. The results of weight selection are given below, in Table 5.5 .

Table 5.5: Data to determine the best weight for goal attainment method

| Weight(f1,f2) | (0.5,0.5) | (0.6,0.4) | (0.4,0.6) | (0.8,0.2) | (0.2,0.8) |
|---|---|---|---|---|---|
| Avg(object1) | 574395.8333 | 579626.6333 | 570390.6833 | 598555.5833 | 565470.1333 |
| Avg(object2) | 168061.7183 | 166222.7317 | 169591.3617 | 160142.2283 | 171833.3867 |
| Distance1 | 0.0158 | 0.0250 | 0.0087 | 0.0585 | 0 |
| Distance2 | 0.0495 | 0.0380 | 0.0590 | 0 | 0.0730 |
| Total Distance | 0.0653 | 0.0630 | 0.0677 | 0.0585 | 0.0730 |

Weighted average here means the sum of probability of a scenario occurrence times the objective value under that scenario. Weighted average values of objective 1(total cost) and weighted average values of objective 2 (disruption cost) for each weight set obtained from table 5.4 are listed in the second and the third rows in table 5.5. The ideal solution is first specified by determining the minimum value in the five columns for each row in table 5.5, which are 565470.1333 for weighted average value of objective 1 and 160142.2283 for weighted average value of objective 2. Then, the values in Fi are normalized and the direct distance of each is computed. Here, the direct distance means the deviation of current value from optimal value in criteria divided by optimal value.

$$Distance = \frac{Fi - F^*}{F^*}$$

Lastly, a solution with a minimum total distance is selected. It is obvious that fifth column with weight set (0.8,0.2) gets the minimum total distance for 0.0585. This set of weight values performs well especially on optimizing disruption cost. Hence, the weight set(0.8,0.2) is selected as the best fit for goal attainment method when compared to the result of other sets.

### 5.1.2 The experiment on selecting best method for solving the model

After the best weight set for goal attainment method is selected, various methods of solving multi-objective problem are compared for their optimization performances. Individual optimized values are calculated first to set the goal for each method. Min-max method, sequential quadratic programming method, goal attainment method and NSGA-II method are tested. The aim of this experiment is to select the best method to solve the model as the final output. The tables show that different methods have different values for each objective function and different running time. Results for each method are given in Table 5.6 .

Table 5.6: Method comparison

| Methods | Scenarios | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | WeightedAvg | WeightAvgCPUtime |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Individual optimal value | Total Cost | 553751.7 | 564076.6 | 587435.8 | 553751.7 | 564076.6 | N/A | 553751.7 | N/A | N/A | 556194.7 | 2.06 seconds |
| | Disruption Cost | 12197.77 | 105447.77 | 198697.77 | 124097.77 | 217347.77 | N/A | 235997.77 | N/A | N/A | 49592.20 | 6.65 seconds |
| fminimax | Total Cost | 553751.7 | 564076.6 | 587435.8 | 553751.7 | 564076.6 | N/A | 553751.7 | N/A | N/A | 556194.7 | 4.39 seconds |
| | Disruption Cost | 36698.64 | 129948.64 | 223198.64 | 148598.64 | 241848.64 | N/A | 260498.64 | N/A | N/A | 74093.07 | |
| fmincon | Total Cost | 561750.3 | 566652.5 | 587435.8 | 561750.3 | 566652.5 | N/A | 561750.3 | N/A | N/A | 563327.4 | 5.09 seconds |
| | Disruption Cost | 32273.16 | 127741.47 | 223198.64 | 144173.16 | 240141.47 | N/A | 256073.16 | N/A | N/A | 70076.52 | |
| goal attainment (weight 0.8,0.2) | Total Cost | 593208.8 | 599485.9 | 612735.3 | 593208.8 | 599485.9 | N/A | 593208.8 | N/A | N/A | 594653.1 | 7.35 seconds |
| | Disruption Cost | 22030.22 | 116870.81 | 213421.09 | 133930.22 | 228770.81 | N/A | 245830.22 | N/A | N/A | 59788.16 | |
| NSGA-II | Total Cost | 568413.6 | 570196.5 | 587435.8 | 553751.7 | 564076.6 | N/A | 553751.7 | N/A | N/A | 567448.8 | 65.81 seconds |
| | Disruption Cost | 30583.57 | 126384.53 | 223198.64 | 142483.57 | 238284.53 | N/A | 254383.57 | N/A | N/A | 68486.59 | |

Based on the results obtained, it can be easily seen that none of the problem-solving methods can optimize both objective functions simultaneously. For instance, goal attainment method performs best in optimizing disruption cost; however it has the worst performance in optimizing the total cost. It is also a time-consuming method when compared to other methods, except the NSGA-II method. Displaced ideal solution method is again applied to select best solution method. In this experiment, the selection is obtained based on three criteria, which are the weighted average of objective 1 value (total cost), the weighted average of objective 2 value (disruption cost), and the mean average of CPU. The results of method selection are given in Table 5.7 .

Table 5.7: Data to determine the best weight for the best solution method

| Methods | fminimax | fmincon | goalattain | NSGA-II |
|---|---|---|---|---|
| Avg(object1) | 556194.6770 | 563327.3559 | 594653.0539 | 567448.7673 |
| Avg(object2) | 74093.0703 | 70076.5244 | 59788.1623 | 68486.5894 |
| Avg(CPUtime) | 4.39 | 5.09 | 7.35 | 65.81 |
| Distance1 | 0 | 0.0128 | 0.0691 | 0.0202 |
| Distance2 | 0.2392 | 0.1721 | 0 | 0.1455 |
| Distance3 | 0 | 0.1595 | 0.6743 | 13.9909 |
| Total Distance | 0.2392 | 0.3444 | 0.7434 | 14.1566 |

The result shows that the min-max method in second column has the minimum total distance for 0.2392. It is obvious that the min-max method has the best performance both on optimizing total cost and CPU running time. The optimal CPU time is not even close for other methods. Goal attainment method and NSGA-II method seem to have impressive performances on the first two criteria, while CPU running time costs too much to drag them behind. It is critical to take CPU running time into account, especially when solving large-scale problems that require not only effective but also efficient way dealing with data. Therefore, min-max method is applied to solve the model and conduct sensitivity analysis on key parameters. It is worth noting that numerical results include optimal values under each scenario, flow tables of the products during transportation between facilities and facility operation plan table (tables 5.8, 5.9, 5.10 and 5.11). The detailed computational results for applying goal attainment method are shown in table 5.12.

Table 5.8: Flow of the raw materials from suppliers to plants(scenario 2) $Q_{psft2}^{12}$

| Products | Product1 | | | | Product2 | | | |
|---|---|---|---|---|---|---|---|---|
| From | Supplier1 | | Supplier2 | | Supplier1 | | Supplier2 | |
| To | Plant1 | Plant2 | Plant1 | Plant2 | Plant1 | Plant2 | Plant1 | Plant2 |
| T=1 | 464 | 0 | 107 | 1143 | 1093 | 0 | 107 | 1143 |
| T=2 | 497 | 0 | 103 | 1147 | 1212 | 0 | 103 | 1147 |
| T=3 | 542 | 0 | 0 | 1250 | 1064 | 0 | 0 | 1250 |

Table 5.9: Flow of the products from plants to distribution centers(scenario 2) $Q_{pfdt}^{23}$

| Products | Product1 | | | | Product2 | | | |
|---|---|---|---|---|---|---|---|---|
| From | Plant1 | | Plant2 | | Plant1 | | Plant2 | |
| To | DC1 | DC2 | DC1 | DC2 | DC1 | DC2 | DC1 | DC2 |
| T=1 | 400 | 0 | 0 | 800 | 840 | 0 | 0 | 800 |
| T=2 | 450 | 0 | 0 | 860 | 855 | 0 | 0 | 745 |
| T=3 | 325 | 0 | 55 | 820 | 745 | 0 | 0 | 875 |

Table 5.10: Flow of the products from distribution centers to retailers(scenario 2) $Q_{pdrt}^{34}$

| Products | Product1 | | | | | | Product2 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| From | DC1 | | | DC2 | | | DC1 | | | DC2 | | |
| To | R1 | R2 | R3 | R1 | R2 | R3 | R1 | R2 | R3 | R1 | R2 | R3 |
| T=1 | 400 | 0 | 0 | 0 | 300 | 500 | 0 | 480 | 360 | 600 | 0 | 200 |
| T=2 | 450 | 0 | 0 | 0 | 380 | 480 | 0 | 520 | 335 | 580 | 0 | 165 |
| T=3 | 380 | 0 | 0 | 0 | 400 | 420 | 0 | 500 | 245 | 600 | 0 | 275 |

Table 5.11: Facility operation plan(scenario 2) $A_{ft}$, $B_{dt}$

| Facilities | Plant1 ($A_{1t}$) | Plant2($A_{2t}$) | DC1($B_{1t}$) | DC2($B_{2t}$) |
|---|---|---|---|---|
| T=1 | 1 | 1 | 1 | 1 |
| T=2 | 1 | 1 | 1 | 1 |
| T=3 | 1 | 1 | 1 | 1 |

Scenario 2 is arbitrarily selected for result study. Tables 5.8- 5.11 present the optimal chain configuration and transportation flow breakdown of the network. These tables indicate that flow of products differs with time, products and also different facilities. Operation plan represented by the binary variables determines whether to run or close facilities. The optimal values of binary variables show that the stochastic model determines the number and locations of facilities in a way that they can satisfy under certain scenario. For scenario 2 at each period, both plants and both distribution centers are operated normally.

Table 5.12: Optimized result (Using fminimax method)

| Scenarios | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Total Cost | 553751.7 | 564076.6 | 587435.8 | 553751.7 | 564076.6 | N/A | 553751.7 | N/A | N/A |
| Disruption Cost | 36698.64 | 129948.64 | 223198.64 | 148598.64 | 241848.64 | N/A | 260498.64 | N/A | N/A |
| CPU time | 4.44s | 4.26s | 3.89s | 4.32s | 4.38s | N/A | 4.53s | N/A | N/A |

Table 5.12 depicts the two objective values which vary with scenarios. Disruption cost increases with steeper gradient when disruption level escalates; the maximum disruption cost is reached for scenario 7 when supplier 1 is totally disrupted. By contrast, the total cost slightly increases, which means min-max method is robust in optimizing the total cost. Hence, disruption cost is more sensitive to disruption scale. The CPU running time remains stable.

This numerical example indicates that after comparison of performances on optimizing multiple objectives, min-max method can lead to the optimal solution that is robust under supplier disruption and uncertain demand. This could be new to the area of stochastic programming and decision-making under uncertainty. Detailed computational results are provided to understand network configuration and real-time distribution flow. Sensitivity analysis is conducted in the following part to show key parameters' influence to the model.

## 5.2 Sensitivity analysis

The experiments are run in cases to study the sensitivity of parameters on supplier disruption, demand uncertainty and problem-solving method selection. These parameters are critical because their changes affect the network configuration of this chain.

### 5.2.1 The effect of variation in disruption scale in supplier capacity

All the parameters are fixed and scale value of capacity disruption parameter vector $\Omega = [\Omega_1, \Omega_2, \Omega_3]$ changes with different sets. Initial set Ω = [1,0.5,0] means corresponding case has three possibilities: $\Omega_1 = 1$ means that supplier runs in full capacity at best, $\Omega_2$=0.5 means supplier capacity shrinks to half of capacity and $\Omega_3$=0 means that supplier capacity disrupts and the supplier has no ability to store any products. In another example, Ω = [1,0.8,0.3] means corresponding case has three possibilities: $\Omega_1 = 1$ means that supplier runs in full capacity at best, $\Omega_2$=0.8 means that supplier capacity shrinks by 20% of the full capacity and $\Omega_3$=0.3 means that supplier capacity disrupts and has only 30% of capacity to store products. Different values for scale of supplier capacity disruption have been tested and illustrative results are presented in following table.

Table 5.13: Comparison of different capacity disruption scale matrix Ω (Using fminimax method )

| $\Omega = [\Omega_1, \Omega_2, \Omega_3]$ | Scenarios | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Weighted Avg | Weighted Avg CPUtime |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| [1,0.5,0] | Total Cost | 553751.7 | 564076.6 | 587435.8 | 553751.7 | 564076.6 | N/A | 553751.7 | N/A | N/A | 556194.677 | 4.30 seconds |
| | Disruption Cost | 36698.64 | 129948.64 | 223198.64 | 148598.64 | 241848.64 | N/A | 260498.64 | N/A | N/A | 74093.07 | |
| [1,0.8,0.5] | Total Cost | 553751.7 | 555577.6 | 564076.6 | 553751.7 | 555577.6 | 564076.6 | 553751.7 | 555577.6 | 564076.6 | 554450.5 | 4.82 seconds |
| | Disruption Cost | 36698.64 | 73998.64 | 129948.64 | 81458.64 | 118758.64 | 174708.64 | 148598.64 | 185898.64 | 241848.64 | 55162.14 | |
| [1,0.8,0.3] | Total Cost | 553751.7 | 555577.6 | 572126.8 | 553751.7 | 555577.6 | 572126.8 | 553751.7 | 555577.6 | N/A | 554809.8 | 4.84 seconds |
| | Disruption Cost | 36698.64 | 73998.64 | 167248.64 | 81458.64 | 118758.64 | 212008.64 | 193358.64 | 230658.64 | N/A | 58601.87 | |
| [1,0.8,0] | Total Cost | 553751.7 | 555577.6 | 587435.8 | 553751.7 | 555577.6 | 587759.5 | 553751.7 | N/A | N/A | 555540.2 | 4.62 seconds |
| | Disruption Cost | 36698.64 | 73998.64 | 223198.64 | 81458.64 | 118758.64 | 267958.64 | 260498.64 | N/A | N/A | 63287.81 | |

It is worth noting that total cost decreases with higher $\Omega_i$ value and disruption cost decreases as well. This observation can be explained by the impact of scale of disruption. It can also be concluded that disruption cost is more sensitive to disruption scale compared to total cost. Mild scale of disruption enables chain to be more robust to lower values of both disruption cost and total cost. In addition, more scenarios are satisfied to obtain objective value with mild scale of disruption because some capacity constraints can be satisfied and optimal value can be found. In the table, initial set $\Omega = [1,0.5,0]$ obviously has the worst performance in optimizing both objective values, and data in some scenarios remains non-available. Set $\Omega = [1,0.8,0.5]$ has the best performance and robustness for the chain. The CPU running time remains stable for each set.

### 5.2.2 The effect of variation in uncertain demand

Uncertain demand is another critical part affecting configuration and objective values in the chain. To evaluate the effect of variance of uncertainty ranges, the experiments with different values for stochastic demand variability have been tested. These experiments vary from deterministic case ($\sigma^2$=0) to case with variance of 200 and illustrative results are presented in following table.

Table 5.14: Comparison of different demand uncertainties (Using fminimax method)

| Case | Scenarios | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Weighted Avg | WeightedAvg CPUtime |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| General case (Avg($\sigma^2$ )= 85.55 ) | Total Cost | 553751.7 | 564076.6 | 587435.8 | 553751.7 | 564076.6 | N/A | 553751.7 | N/A | N/A | 556194.7 | 4.80 seconds |
| | Disruption Cost | 36698.64 | 129948.64 | 223198.64 | 148598.64 | 241848.64 | N/A | 260498.64 | N/A | N/A | 74093.07 | |
| $\sigma^2$ =0 | Total Cost | 598616.5 | 610586.1 | 635640.5 | 598616.5 | 610599.3 | N/A | N/A | N/A | N/A | 601485.0 | 4.07 seconds |
| | Disruption Cost | 0 | 93250 | 186500 | 111900 | 205150 | N/A | N/A | N/A | N/A | 29011.11 | |
| $\sigma^2$ =50 | Total Cost | 572590.0 | 583713.9 | 607761.8 | 572591.5 | 583742.8 | N/A | N/A | N/A | N/A | 575290.5 | 4.56 seconds |
| | Disruption Cost | 31832.43 | 125082.43 | 218332.43 | 143732.43 | 236982.43 | N/A | N/A | N/A | N/A | 60843.54 | |
| $\sigma^2$ =100 | Total Cost | 547373.0 | 557041.4 | 580355.7 | 547379.0 | 557026.5 | N/A | 547373.0 | N/A | N/A | 549723.0 | 4.73 seconds |
| | Disruption Cost | 44042.00 | 137292 | 230542.00 | 155942.00 | 249192.00 | N/A | 267842.00 | N/A | N/A | 81436.43 | |
| $\sigma^2$ =200 | Total Cost | 497187.0 | 503617.5 | 525676.4 | 497187.0 | 503616.6 | N/A | 497187.0 | N/A | N/A | 499031.7 | 6.12 seconds |
| | Disruption Cost | 74405.27 | 167655.27 | 260905.27 | 186305.27 | 279555.27 | N/A | 298205.27 | N/A | N/A | 111799.70 | |

The results show that the total cost decreases while disruption cost increases, if the variance is increased. It is indicated that if the demand from retailer becomes unstable, the supply chain network can have chance of saving total cost; however it will face more risk of the loss of disruption cost. Compared to deterministic demand($\sigma^2$ =0) , demand with variability can generate more scenarios. CPU running times increases with increased variance.

### 5.2.3 The effect of changes in disruption probability in method selection

The probabilities assigned to scenarios represent the weights of each scenario when calculating weighed average of optimal value. Hence, such probability vector affects the method selection by making a difference in displaced ideal solution method. Different values for disruption probability sets have been tested and illustrative results are presented in the following table.

Table 5.15: Comparison of different disruption probabilities

| Total distance | fminimax | fmincon | goalattain | NSGA-II |
|---|---|---|---|---|
| [0.9,0.1,0] | 0.2880 | 0.3772 | 0.7244 | 14.2142 |
| [0.85,0.1,0.05] | 0.2392 | 0.3444 | 0.7434 | 14.1566 |
| [0.7,0.2,0.1] | 0.1526 | 0.2829 | 0.7223 | 14.1022 |
| [0.6,0.35,0.05] | 0.1313 | 0.257 | 0.7389 | 14.0874 |
| [0.6,0.3,0.1] | 0.1276 | 0.2588 | 0.7218 | 14.0851 |
| [0.5,0.4,0.1] | 0.1087 | 0.2333 | 0.7177 | 14.0716 |
| [0.4,0.3,0.3] | 0.0895 | 0.2274 | 0.6553 | 14.0587 |
| [0.2,0.4,0.4] | 0.0727 | 0.1831 | 0.6368 | 14.0453 |

The results show that min-max method performs consistently well even though sets with wide range have been tested. Results also show that with the increase of disruption probability, the total distance of every method decreases, which means that the performance of each method is getting closer to the overall optimal value. Hence, conclusion can be drawn that overall performance is sensitive to disruption probability.

The above sensitivity analysis shows that computational performances of both objective values are sensitive to all three key parameters. There exist some trends between objective values and tested parameters, which helps to realize the critical influence of supplier disruption and uncertain demand on network chain. Min-max method is applied in all three experiments.

# Chapter 6

# CONCLUSIONS AND FUTURE STUDY

*This final chapter presents the conclusions from this thesis study. Future study possibilities are suggested based on the findings and limitations experienced in this research effort.*

The motivation of this work is to develop an effective, efficient and robust supply chain network model coping with increasing uncertain environment and competitive pressures. Over the past decades, there has been a great increase in focusing on supply chain network and integrated logistics because of the emphasis on productivity and customer satisfaction.

This work first reviews the literature related to supply chain network design criteria. Most of the former work studies either only a single stage, single echelon or consider each parameter at a single level. The models in former work are usually single objective and uncertainties are not taken into account. Hence, there has been an increasing attention placed on the integration of each process as a network design problem. Appropriate mathematical programming methods and algorithms possibly used to solve problems in this criterion are also a big part of this literature review. Large body of references shows that mixed-integer programming models are commonly used in supply chain network design under conditions of uncertainty.

Then this work presents detailed mathematical formulations for the problem of designing supply chain network. The supply chain network considered in this work is a multi-period, multi-product, multi-echelon supply chain with supplier disruption and uncertain demand. In the model, two objective functions are simultaneously optimized. One objective function is to minimize the total cost of the supply chain which includes the cost of running manufacturing plants and distribution centers, cost of buying and transporting raw materials, cost of transporting products from plants to DCs and from DCs to retailers, and cost of holding products at DCs. The other objective function is to minimize the disruption cost, which occurs because of uncertain demand and supply disruption. Disruption, as the one typical type of uncertainty, is captured in terms of a number of likely scenarios which may affect supplier capacity that influence the network configuration. Uncertain demand follows uniform distribution. The problem is formulated as a multi-objective mixed integer non-linear programming model and solved to global optimality. A numerical example has been used to illustrate the applicability of the developed model. Multiple methods are applied to compare the computational performance because there exists trade-off between the objectives, shown as Pareto curve. After comparison, min-max method is selected as the best method and its detailed computational results are presented. The computational cost related to model, shown as CPU running time, has been found to be relatively low (only takes around 4.5 seconds), thus making the model favorable in solving large-scale problems.

The proposed multi-objective mixed integer non-linear programming model aims to assist operations management to take strategic and tactical decisions under both supplier disruption and uncertain demand conditions. The model takes production allocation, product flow distribution and network configuration into account. Further investigation has been conducted to study sensitivity of the solutions following changes to key parameters of the problem, such as variation in disruption scale, variation in uncertain demand and changes in disruption probability. Sensitivity study part has shown how changing these parameters can affect the network configuration. This also verifies robustness and correctness of the proposed model because it reacts reasonably well to the key parameter changes.

The amount of literature of supply chain network design considering uncertainty both in demand and supply side is still short and this work is one of the few works solving multi-objective function with both uncertainties considered. Many possible future research directions can be defined in this area. For example, computational complexity of scenario analysis approach will increase as the number of scenarios increases. The number of scenarios, which relates to the number of suppliers, determines the number of experiments to run in the program. Hence, with the increase of scenarios, whether this model can be applied to solve large-scale problem remains to be studied. The way in which scenario changes with time-period and affect network design is also an interesting part to study and it can be useful to cope with more dynamic network. Last but not least, extending the proposed model to the case of supplier uncertainty outside capacity disruption and process uncertainty between stages can make the model more robust to cope with more situations.

# REFERENCES

[1] Taha R and Abdallah (2015). Multi-Objective Supply Chain Network Design Under Demand Uncertainity Using Robust Goal Programming Approach. International Conference on Operations Excellence and Service Engineering.

[2] Beamon B. M (1998). Supply chain design and analysis: Models and methods. International Journal of Production Economics, 55(3), 281–294.

[3] Thomas D. J and Griffin P. M (1996). Coordinated supply chain management. European Journal of Operational Research, 94, 1–115.

[4] Tsiakis P, Shah N and Pantelides C.C (2001). Design of multiechelon supply chain networks under demand uncertainty. Industrial and Engineering Chemistry Research, 40, 3585–3604.

[5] Gebennini E, Gamberini R and Manzini R (2009). An integrated production–distribution model for the dynamic location and allocation problem with safety stock optimization. International Journal of Production Economics, 122, 286–304.

[6] Ivan A (2015). Modelling and Analysis of Multi-period Distribution-Allocation Problem in a Two-Stage Supply Chain. International Journal of Industrial Engineering and Management Science, Vol. 5, No. 4.

[7] Xu N and Nozick L (2009). Modeling supplier selection and the use of option contracts for global supply chain design. Computers and Operations Research, 36, 2786-2800.

[8] Galbraith J (1973). Designing complex organizations. Addison-Wesley, Hardcover ISBN: 020-1025-590.

[9] Peidro D, Mula J and Poler R (2009). Quantitative models for supply chain planning under uncertainty: a review. International Journal of Advanced Manufacturing Technology, 43, 400–420.

[10] Santoso T, Ahmed S, Goetschalckx G and Shapiro A(2005). A stochastic programming approach for supply chain network design under uncertainty. European Journal of Operational Research, 167, 96–115.

[11] Craighead C.W, Blackhurst J and Rungtusanatham M.J(2007). The severity of supply chain disruptions: design characteristics and mitigation capabilities. Decision Sciences 38 (1), 131–156.

[12] Paul S. K, Sarker R. A and  Essam D. L (2014). Managing supply disruption in a three-tier supply chain with multiple suppliers and retailers. IEEE International Conference on Industrial Engineering and Engineering Management, Bandar Sunway, 194-198.

[13] Baghalian A, Rezapour S and Farahani R.Z (2013). Robust supply chain network design with service level against disruptions and demand uncertainties: A real-life case. European Journal of Operational Research, 227, 199–215.

[14] Ierapetritou M.G, Pistikopoulos E.N and Floudas C.A(1996). Operational planning under uncertainty. Computers and Chemical Engineering, 20(12), 1499–1516.

[15] Xiao T.J, Qi X and Yu G (2007). Coordination of supply chain after demand disruptions when retailers compete. International Journal of Production Economics, 109 (1), 162–179.

[16] Norrman A and Jansson U (2004). Ericsson's proactive supply chain risk management approach after a serious sub-supplier accident. International Journal of Physical Distribution & Logistics Management, 34 (5), 434–456.

[17] Lin C and Wang T (2011). Build-to-order supply chain network design under supply and demand uncertainties. Transportation Research Part B, 45, 1162–1176.

[18] Giri B and Bardhan S (2015). Coordinating a supply chain under uncertain demand and random yield in presence of supply disruption. International Journal of Production Research, Vol. 53, No. 16, 5070–5084.

[19] Mahnam M, Yadollahpour M. R, Famil-Dardashti V and Hejazi S. R (2009). Supply chain modelling in uncertain environment with bi-objective approach. Computers and Industrial Engineering, 56, 1535–1544.

[20] Pishvaee M, Rabbani M and Torabi S.A (2011) A robust optimization approach to closed-loop supply chain network design under uncertainty. Applied Mathematical Modelling, 35, 637–649.

[21] Yu C and Li H (2000) A robust optimization model for stochastic logistic problems. International Journal of Production Economics, 64 (1–3), 385–397.

[22] Ben-Tal A, Golany B, Nemirovski A and Vial J.-Ph (2005). Retailer–supplier flexible commitments contracts: A robust optimization. Manufacturing and Service Operations Management Approach, 7, 248–271.

[23] B. Tomlin (2006). On the value of mitigation and contingency strategies for managing supply chain disruption risks. Management Science, 52(5), 639–657.

[24] Pan F and Nagi R (2010). Robust supply chain design under uncertain demand in agile manufacturing. Computers and Operations Research, 37(4), 668–683.

[25] Gupta A and Maranas C. D (2000). A two-stage modelling and solution framework for multisite midterm planning under demand uncertainty. Industrial & Engineering Chemistry Research, 39, 3799–3813.

[26] Cardona-Valdés Y, Álvarez A and Ozdemir D (2011). A bi-objective supply chain design problem with uncertainty. Transportation Research: Part C, 19, 821–832.

[27] Tavakkoli-Moghaddam R, Forouzanfar F and Ebrahimnejad S (2013) Incorporating location, routing, and inventory decisions in a bi-objective supply chain design problem with risk-pooling. Journal of Industrial Engineering International, 9(1), 1–6.

[28] Chen C and Lee W (2004). Multi-objective optimization of multi-echelon supply chain networks with uncertain product demands and prices. Computers and Chemical Engineering, 28, 1131–1144.

[29] Goh M, Lim J and Meng F.A (2007). Stochastic model for risk management in global supply chain networks. European Journal of Operational Research, 182, 164–173.

[30] Listes O and Dekker R (2005). A stochastic approach to a case study for product recovery network design. European Journal of Operational Research, 160, 268–287.

[31] Salema MIG, Barbosa-Povoa AP and Novais AQ (2007). An optimization model for the design of a capacitated multi-product reverse logistics network with uncertainty. European Journal of Operational Research, 179, 1063–1077.

[32] Yin S and Nishi T (2014). A Supply Chain Planning Model with Supplier Selection under Uncertain Demands and Asymmetric Information. Procedia CIRP 17, 639 – 644.

[33] Dzupire N and Nkansah-Gyekye Y (2014). A Multi-Stage Supply Chain Network Optimization Using Genetic Algorithms. Mathematical Theory and Modeling, Vol.4, No.8.

[34] Georgiadis M. C, Tsiakis P, Longinidis P and Sofioglou M (2011). Optimal design of supply chain networks under uncertain transient demand variations. Omega, 39, 254–272.

[35] Gen M and Cheng R (2000). Genetic algorithms and engineering optimization. New York: John Wiley and Sons, Hardcover ISBN: 978-0-471-31531-5.

[36] Altiparmak F, Gen M, Lin L and Karaoglan I (2009). A steady-state genetic algorithm for multi-product supply chain network design. Computers & Industrial Engineering, 56, 521–537.

[37] Farahani R.Z and Elahipanah M(2008). A genetic algorithm to optimize the total cost and service level for just-in-time distribution in a supply chain. International Journal of Production Economics, 111, 229-243.

[38] Patia RK, Vratb P and Kumarc P (2008). A goal programming model for paper recycling system. Omega, 36, 405–417.

[39] Bienstock D and Ozbay N (2008), Computing robust basestock levels, Discrete Optimization, 5 (2), 389–414.

[40] Akbari A.A and Karimi B (2015). A new robust optimization approach for integrated multi-echelon, multi-product, multi-period supply chain network design under process uncertainty. International Journal of Advanced Manufacturing Technology, Volume 79, Issue 1, 229-244.

[41] Bertsimas D and Thiele A (2006). A robust optimization approach to inventory theory. Operations Research, 54(150), 168.

[42] Yu C and Li H (2000). A robust optimization model for stochastic logistic problems. International Journal of Production Economics, 64, 385–397.

[43] Lei D, Li J and Liu Z (2002). Supply chain contracts under demand and cost disruptions with asymmetric information. International Journal of Production Economics, 139, 116-126.

[44] Pishvaee M, Jolai F and Razmi J (2009). A stochastic optimization model for integrated forward/reverse logistics network design. Journal of Manufacturing Systems, 28, 107–114.

[45] Amiri A (2006). Designing a distribution network in a supply chain system: formulation and efficient solution procedure. European Journal of Operational Research, 171, 567–576.

[46] Mavrotas G (2009). Effective implementation of the e-constraint method in Multi-Objective Mathematical Programming problems. Applied Mathematics and Computation, 213, 455–465.

[47]Caramia M and Dell'Olmo P (2008) Multi-objective Management in Freight Logistics Increasing Capacity, Service Level and Safety with Optimization Algorithms. Springer, Hardcover ISBN: 978-1-84800-381-1.

[48] Pasandideh S and Niaki S (2015) Optimizing a bi-objective multi-product multi-period three echelon supply chain network with warehouse reliability. Expert Systems with Applications, 42, 2615–2623.

[49] Marker R and Arora J (2004) Survey of multi-objective optimization methods for engineering. Structural and Multidisciplinary Optimization, Volume 26, Number 6, April, 369-395(27)

[50] Powell M (1978) The Convergence of Variable Metric Methods For Nonlinearly Constrained Optimization Calculations. Nonlinear Programming, 3, 27-63.

[51] Gill P, Murray W and Wright M (1981) Practical Optimization. Elsevier Academic Press, Hardcover ISBN: 978-0-12286-952-8.

[52] Coello C (1998) Using the Min-Max Method to Solve Multiobjective Optimization Problems with Genetic Algorithms. IBERAMIA '98, 303–313.

[53] Deb K (2002) A Fast and Elitist Multiobjective Genetic Algorithm:NSGA-II. IEEE Transactions on Evolutionary Computation, VOL. 6, NO. 2.

[54] Godet M (2000) The Art of Scenarios and Strategic Planning: Tools and Pitfalls. Technological Forecasting & Social Change, 65 (1), 3-22.

[55] Hwang C and Masud A(1979). Multiple objective decision making, methods and applications: A state-of-the-art survey. Springer, Hardcover ISBN: 978-3-54009-111-0.

# APPENDIX A

## Detailed ma

## thematical formulations

Decision Variables: $Q^{12}_{psft\omega},\ Q^{23}_{pfdt},\ Q^{34}_{pdrt}, Q^{33}_{pdt},\ A_{ft}, B_{dt}$

Objective functions:

$$Min\ f1 = 5454.55 * B_{11} + 4958.68 * B_{12} + 4507.89 * B_{13} + 6363.64 * B_{21} + 5785.12 * B_{22} + 5259.20 * B_{23} + 7727.27 * A_{11} + 7024.79 * A_{12} + 6386.18 * A_{13} + 8636.36 * A_{21} + 7851.24 * A_{22} + 7137.49 * A_{23} + 30.91 * Q^{12}_{1111\omega} + 32.73 * Q^{12}_{1121\omega} + 29.09 * Q^{12}_{1211\omega} + 27.27 * Q^{12}_{1221\omega} + 29.09 * Q^{12}_{2111\omega} + 30.91 * Q^{12}_{2121\omega} + 27.27 * Q^{12}_{2211\omega} + 27.27 * Q^{12}_{2221\omega} + 28.10 * Q^{12}_{1112\omega} + 29.75 * Q^{12}_{1122\omega} + 26.45 * Q^{12}_{1212\omega} + 24.79 * Q^{12}_{1222\omega} + 26.45 * Q^{12}_{2112\omega} + 28.10 * Q^{12}_{2122\omega} + 24.79 * Q^{12}_{2212\omega} + 24.79 * Q^{12}_{2222\omega} + 25.54 * Q^{12}_{1113\omega} + 27.05 * Q^{12}_{1123\omega} + 24.04 * Q^{12}_{1213\omega} + 22.54 * Q^{12}_{1223\omega} + 24.04 * Q^{12}_{2113\omega} + 25.54 * Q^{12}_{2123\omega} + 22.54 * Q^{12}_{2213\omega} + 22.54 * Q^{12}_{2223\omega} + 15.45 * Q^{23}_{1111} + 14.55 * Q^{23}_{1121} + 16.36 * Q^{23}_{1211} + 14.55 * Q^{23}_{1221} + 12.73 * Q^{23}_{2111} + 16.36 * Q^{23}_{2121} + 15.45 * Q^{23}_{2211} + 13.64 * Q^{23}_{2221} + 14.05 * Q^{23}_{1112} + 13.22 * Q^{23}_{1122} + 14.88 * Q^{23}_{1212} + 13.22 * Q^{23}_{1222} + 11.57 * Q^{23}_{2112} + 14.88 * Q^{23}_{2122} + 14.05 * Q^{23}_{2212} + 12.40 * Q^{23}_{2222} + 12.77 * Q^{23}_{1113} + 12.02 * Q^{23}_{1123} + 13.52 * Q^{23}_{1213} + 12.02 * Q^{23}_{1223} + 10.52 * Q^{23}_{2113} + 13.52 * Q^{23}_{2123} + 12.77 * Q^{23}_{2213} + 11.27 * Q^{23}_{2223} + 6.36 * Q^{34}_{1111} + 8.18 * Q^{34}_{1121} + 7.27 * Q^{34}_{1131} + 9.09 * Q^{34}_{1211} + 6.36 * Q^{34}_{1221} + 6.36 * Q^{34}_{1231} + 10.91 * Q^{34}_{2111} + 9.09 * Q^{34}_{2121} + 7.27 * Q^{34}_{2131} + 10.91 * Q^{34}_{2211} + 10.91 * Q^{34}_{2221} + 8.18 * Q^{34}_{2231} + 5.79 * Q^{34}_{1112} + 7.44 * Q^{34}_{1122} + 6.61 * Q^{34}_{1132} + 8.26 * Q^{34}_{1212} + 5.79 * Q^{34}_{1222} + 5.79 * Q^{34}_{1232} + 9.92 * Q^{34}_{2112} + 8.26 * Q^{34}_{2122} + 6.61 * Q^{34}_{2132} + 9.92 * Q^{34}_{2212} + 9.92 * Q^{34}_{2222} + 7.44 * Q^{34}_{2232} + 5.26 * Q^{34}_{1113} + 6.76 * Q^{34}_{1123} + 6.01 * Q^{34}_{1133} + 7.51 * Q^{34}_{1213} + 5.26 * Q^{34}_{1223} + 5.26 * Q^{34}_{1233} + 9.02 * Q^{34}_{2113} + 7.51 * Q^{34}_{2123} + 6.01 * Q^{34}_{2133} + 9.02 * Q^{34}_{2213} + 9.02 * Q^{34}_{2223} + 6.76 * Q^{34}_{2233} + 6.36 * Q^{33}_{111} + 6.36 * Q^{33}_{121} + 6.36 * Q^{33}_{211} + 6.36 * Q^{33}_{221} + 7.44 * Q^{33}_{112} + 7.44 * Q^{33}_{122} + 7.44 * Q^{33}_{212} + 7.44 * Q^{33}_{222} + 5.26 * Q^{33}_{113} + 5.26 * Q^{33}_{123} + +5.26 * Q^{33}_{213} + 5.26 * Q^{33}_{223} \quad (1.1)$$

$$Min\ f2 = 0.0528 * (Q^{34}_{1111} + Q^{34}_{1211})^2 - 64 * (Q^{34}_{1111} + Q^{34}_{1211}) + 19090.91 + 0.0686 * (Q^{34}_{1112} + Q^{34}_{1212})^2 - 84.68 * (Q^{34}_{1112} + Q^{34}_{1212}) + 25947.46 + 0.2182 * (Q^{34}_{1113} + Q^{34}_{1213})^2 - 201.65 * (Q^{34}_{1113} + Q^{34}_{1213}) + 46543.95 + 0.0264 * (Q^{34}_{2111} + Q^{34}_{2211})^2 - 52 * (Q^{34}_{2111} + Q^{34}_{2211}) + 25000 + 0.0343 * (Q^{34}_{2112} + Q^{34}_{2212})^2 - 59.74 * (Q^{34}_{2112} + Q^{34}_{2212}) + 25637.54 + 0.0436 * (Q^{34}_{2113} + Q^{34}_{2213})^2 - 72.73 * (Q^{34}_{2113} + Q^{34}_{2213}) + 30052.59 + 0.1056 * (Q^{34}_{1121} + Q^{34}_{1221})^2 - 88 * (Q^{34}_{1121} + Q^{34}_{1221}) + 18181.82 + 0.24 * (Q^{34}_{1122} + Q^{34}_{1222})^2 - 221.82 * (Q^{34}_{1122} + Q^{34}_{1222}) + 51198.35 + 0.0545 * (Q^{34}_{1123} + Q^{34}_{1223})^2 - 62.81 * (Q^{34}_{1123} + Q^{34}_{1223}) + 17881.29 + 0.044 * (Q^{34}_{2121} + Q^{34}_{2221})^2 - 64 * (Q^{34}_{2121} + Q^{34}_{2221}) + 22909.1 + 0.06 * (Q^{34}_{2122} + Q^{34}_{2222})^2 - 85.45 * (Q^{34}_{2122} + Q^{34}_{2222}) + 30206.61 + 0.029 * (Q^{34}_{2123} + Q^{34}_{2223})^2 - 46.28 * (Q^{34}_{2123} + Q^{34}_{2223}) + 18031.56 + 0.0528 * (Q^{34}_{1131} + Q^{34}_{1231})^2 - 76 * (Q^{34}_{1131} + Q^{34}_{1231}) + 27045.45 + 0.0343 * (Q^{34}_{1132} + Q^{34}_{1232})^2 - 51.95 * (Q^{34}_{1132} + Q^{34}_{1232}) + 19291.62 + 0.073 * (Q^{34}_{1133} + Q^{34}_{1233})^2 - 82.64 * (Q^{34}_{1133} + Q^{34}_{1233}) + 23328.32 + 0.132 * (Q^{34}_{2131} + Q^{34}_{2231})^2 - 184 * (Q^{34}_{2131} + Q^{34}_{2231}) + 64000 + 0.096 * (Q^{34}_{2132} + Q^{34}_{2232})^2 - 123.64 * (Q^{34}_{2132} + Q^{34}_{2232}) + 39669.42 + 0.22 * (Q^{34}_{2133} + Q^{34}_{2233})^2 - 271.1 * (Q^{34}_{2133} + Q^{34}_{2233}) + 84147.26 + \left[11000 - \left(CS_{11}(\omega) + CS_{12}(\omega) + CS_{21}(\omega) + CS_{22}(\omega)\right)\right] * 37.3 \quad (1.2)$$

Constraints(subject to)

(2.1 Service level constraints)

$0.0088 * (Q^{34}_{1111} + Q^{34}_{1211}) \geq 0.9$
$0.0125 * (Q^{34}_{1112} + Q^{34}_{1212}) \geq 0.92$
$0.044 * (Q^{34}_{1113} + Q^{34}_{1213}) \geq 0.9$
$0.0044 * (Q^{34}_{2111} + Q^{34}_{2211}) \geq 0.95$
$0.0063 * (Q^{34}_{2112} + Q^{34}_{2212}) \geq 0.95$
$0.0088 * (Q^{34}_{2113} + Q^{34}_{2213}) \geq 0.9$
$0.0176 * (Q^{34}_{1121} + Q^{34}_{1221}) \geq 0.85$
$0.044 * (Q^{34}_{1122} + Q^{34}_{1222}) \geq 0.88$
$0.011 * (Q^{34}_{1123} + Q^{34}_{1223}) \geq 0.92$
$0.0073 * (Q^{34}_{2121} + Q^{34}_{2221}) \geq 0.88$
$0.011 * (Q^{34}_{2122} + Q^{34}_{2222}) \geq 0.9$
$0.0059 * (Q^{34}_{2123} + Q^{34}_{2223}) \geq 0.9$
$0.0088 * (Q^{34}_{1131} + Q^{34}_{1231}) \geq 0.85$
$0.0063 * (Q^{34}_{1132} + Q^{34}_{1232}) \geq 0.85$
$0.0147 * (Q^{34}_{1133} + Q^{34}_{1233}) \geq 0.88$
$0.022 * (Q^{34}_{2131} + Q^{34}_{2231}) \geq 0.88$
$0.0176 * (Q^{34}_{2132} + Q^{34}_{2232}) \geq 0.93$
$0.044 * (Q^{34}_{2133} + Q^{34}_{2233}) \geq 0.91$

(2.2 Supplier capacity constraints)

$Q^{12}_{1111\omega} + Q^{12}_{1121\omega} \leq CS_{11}(\omega)$
$Q^{12}_{1112\omega} + Q^{12}_{1122\omega} \leq CS_{11}(\omega)$
$Q^{12}_{1113\omega} + Q^{12}_{1123\omega} \leq CS_{11}(\omega)$
$Q^{12}_{2111\omega} + Q^{12}_{2121\omega} \leq CS_{12}(\omega)$
$Q^{12}_{2112\omega} + Q^{12}_{2122\omega} \leq CS_{12}(\omega)$
$Q^{12}_{2113\omega} + Q^{12}_{2123\omega} \leq CS_{12}(\omega)$
$Q^{12}_{1211\omega} + Q^{12}_{1221\omega} \leq CS_{21}(\omega)$
$Q^{12}_{1212\omega} + Q^{12}_{1222\omega} \leq CS_{21}(\omega)$
$Q^{12}_{1213\omega} + Q^{12}_{1223\omega} \leq CS_{21}(\omega)$
$Q^{12}_{2211\omega} + Q^{12}_{2221\omega} \leq CS_{22}(\omega)$
$Q^{12}_{2212\omega} + Q^{12}_{2222\omega} \leq CS_{22}(\omega)$
$Q^{12}_{2213\omega} + Q^{12}_{2223\omega} \leq CS_{22}(\omega)$

(2.3 Plant capacity constraints)

$0.85 * (Q^{12}_{1111\omega} + Q^{12}_{1211\omega}) \leq 700 * A_{11}$
$0.85 * (Q^{12}_{1112\omega} + Q^{12}_{1212\omega}) \leq 700 * A_{12}$
$0.85 * (Q^{12}_{1113\omega} + Q^{12}_{1213\omega}) \leq 700 * A_{13}$
$0.85 * (Q^{12}_{2111\omega} + Q^{12}_{2211\omega}) \leq 680 * A_{11}$
$0.85 * (Q^{12}_{2112\omega} + Q^{12}_{2212\omega}) \leq 680 * A_{12}$
$0.85 * (Q^{12}_{2113\omega} + Q^{12}_{2213\omega}) \leq 680 * A_{13}$
$0.85 * (Q^{12}_{1121\omega} + Q^{12}_{1221\omega}) \leq 720 * A_{21}$

$0.85 * (Q^{12}_{1122\omega} + Q^{12}_{1222\omega}) \leq 720 * A_{22}$
$0.85 * (Q^{12}_{1123\omega} + Q^{12}_{1223\omega}) \leq 720 * A_{23}$
$0.85 * (Q^{12}_{2121\omega} + Q^{12}_{2221\omega}) \leq 670 * A_{21}$
$0.85 * (Q^{12}_{2122\omega} + Q^{12}_{2222\omega}) \leq 670 * A_{22}$
$0.85 * (Q^{12}_{2123\omega} + Q^{12}_{2223\omega}) \leq 670 * A_{23}$

(2.4 DC capacity constraints)

$0.9 * (Q^{23}_{1111} + Q^{23}_{1211}) + Q^{33}_{111} - (Q^{34}_{1111} + Q^{34}_{1121} + Q^{34}_{1131}) \leq 600 * B_{11}$
$0.9 * (Q^{23}_{1112} + Q^{23}_{1212}) + Q^{33}_{112} - (Q^{34}_{1112} + Q^{34}_{1122} + Q^{34}_{1132}) \leq 600 * B_{12}$
$0.9 * (Q^{23}_{1113} + Q^{23}_{1213}) + Q^{33}_{113} - (Q^{34}_{1113} + Q^{34}_{1123} + Q^{34}_{1133}) \leq 600 * B_{13}$
$0.9 * (Q^{23}_{2111} + Q^{23}_{2211}) + Q^{33}_{211} - (Q^{34}_{2111} + Q^{34}_{2121} + Q^{34}_{2131}) \leq 580 * B_{11}$
$0.9 * (Q^{23}_{2112} + Q^{23}_{2212}) + Q^{33}_{212} - (Q^{34}_{2112} + Q^{34}_{2122} + Q^{34}_{2132}) \leq 580 * B_{12}$
$0.9 * (Q^{23}_{2113} + Q^{23}_{2213}) + Q^{33}_{213} - (Q^{34}_{2113} + Q^{34}_{2123} + Q^{34}_{2133}) \leq 580 * B_{13}$
$0.9 * (Q^{23}_{1121} + Q^{23}_{1221}) + Q^{33}_{121} - (Q^{34}_{1211} + Q^{34}_{1221} + Q^{34}_{1231}) \leq 570 * B_{21}$
$0.9 * (Q^{23}_{1122} + Q^{23}_{1222}) + Q^{33}_{122} - (Q^{34}_{1212} + Q^{34}_{1222} + Q^{34}_{1232}) \leq 570 * B_{22}$
$0.9 * (Q^{23}_{1123} + Q^{23}_{1223}) + Q^{33}_{123} - (Q^{34}_{1213} + Q^{34}_{1223} + Q^{34}_{1233}) \leq 570 * B_{23}$
$0.9 * (Q^{23}_{2121} + Q^{23}_{2221}) + Q^{33}_{221} - (Q^{34}_{2211} + Q^{34}_{2221} + Q^{34}_{2231}) \leq 730 * B_{21}$
$0.9 * (Q^{23}_{2122} + Q^{23}_{2222}) + Q^{33}_{222} - (Q^{34}_{2212} + Q^{34}_{2222} + Q^{34}_{2232}) \leq 730 * B_{22}$
$0.9 * (Q^{23}_{2123} + Q^{23}_{2223}) + Q^{33}_{223} - (Q^{34}_{2213} + Q^{34}_{2223} + Q^{34}_{2233}) \leq 730 * B_{23}$

(2.5 Plant balance constraints)

$0.7 * (Q^{12}_{1111\omega} + Q^{12}_{1211\omega}) \geq (Q^{23}_{1111} + Q^{23}_{1121})$
$0.75 * (Q^{12}_{1112\omega} + Q^{12}_{1212\omega}) \geq (Q^{23}_{1112} + Q^{23}_{1122})$
$0.6 * (Q^{12}_{1113\omega} + Q^{12}_{1213\omega}) \geq (Q^{23}_{1113} + Q^{23}_{1123})$
$0.7 * (Q^{12}_{2111\omega} + Q^{12}_{2211\omega}) \geq (Q^{23}_{2111} + Q^{23}_{2121})$
$0.65 * (Q^{12}_{2112\omega} + Q^{12}_{2212\omega}) \geq (Q^{23}_{2112} + Q^{23}_{2122})$
$0.7 * (Q^{12}_{2113\omega} + Q^{12}_{2213\omega}) \geq (Q^{23}_{2113} + Q^{23}_{2123})$
$0.7 * (Q^{12}_{1121\omega} + Q^{12}_{1221\omega}) \geq (Q^{23}_{1211} + Q^{23}_{1221})$
$0.75 * (Q^{12}_{1122\omega} + Q^{12}_{1222\omega}) \geq (Q^{23}_{1212} + Q^{23}_{1222})$
$0.7 * (Q^{12}_{1123\omega} + Q^{12}_{1223\omega}) \geq (Q^{23}_{1213} + Q^{23}_{1223})$
$0.7 * (Q^{12}_{2121\omega} + Q^{12}_{2221\omega}) \geq (Q^{23}_{2211} + Q^{23}_{2221})$
$0.65 * (Q^{12}_{2122\omega} + Q^{12}_{2222\omega}) \geq (Q^{23}_{2212} + Q^{23}_{2222})$
$0.7 * (Q^{12}_{2123\omega} + Q^{12}_{2223\omega}) \geq (Q^{23}_{2213} + Q^{23}_{2223})$

(2.6 DC balance constraints)

$(Q^{23}_{1111} + Q^{23}_{1211}) = Q^{33}_{111} + (Q^{34}_{1111} + Q^{34}_{1121} + Q^{34}_{1131})$
$Q^{33}_{111} + (Q^{23}_{1112} + Q^{23}_{1212}) = Q^{33}_{112} + (Q^{34}_{1112} + Q^{34}_{1122} + Q^{34}_{1132})$
$Q^{33}_{112} + (Q^{23}_{1113} + Q^{23}_{1213}) = Q^{33}_{113} + (Q^{34}_{1113} + Q^{34}_{1123} + Q^{34}_{1133})$
$(Q^{23}_{2111} + Q^{23}_{2211}) = Q^{33}_{211} + (Q^{34}_{2111} + Q^{34}_{2121} + Q^{34}_{2131})$
$Q^{33}_{211} + (Q^{23}_{2112} + Q^{23}_{2212}) = Q^{33}_{212} + (Q^{34}_{2112} + Q^{34}_{2122} + Q^{34}_{2132})$
$Q^{33}_{212} + (Q^{23}_{2113} + Q^{23}_{2213}) = Q^{33}_{213} + (Q^{34}_{2113} + Q^{34}_{2123} + Q^{34}_{2133})$
$(Q^{23}_{1121} + Q^{23}_{1221}) = Q^{33}_{121} + (Q^{34}_{1211} + Q^{34}_{1221} + Q^{34}_{1231})$
$Q^{33}_{121} + (Q^{23}_{1122} + Q^{23}_{1222}) = Q^{33}_{122} + (Q^{34}_{1212} + Q^{34}_{1222} + Q^{34}_{1232})$
$Q^{33}_{122} + (Q^{23}_{1123} + Q^{23}_{1223}) = Q^{33}_{123} + (Q^{34}_{1213} + Q^{34}_{1223} + Q^{34}_{1233})$
$(Q^{23}_{2121} + Q^{23}_{2221}) = Q^{33}_{221} + (Q^{34}_{2211} + Q^{34}_{2221} + Q^{34}_{2231})$
$Q^{33}_{221} + (Q^{23}_{2122} + Q^{23}_{2222}) = Q^{33}_{222} + (Q^{34}_{2212} + Q^{34}_{2222} + Q^{34}_{2232})$
$Q^{33}_{222} + (Q^{23}_{2123} + Q^{23}_{2223}) = Q^{33}_{223} + (Q^{34}_{2213} + Q^{34}_{2223} + Q^{34}_{2233})$

(2.7 Demand constraints)

$400 \leq Q_{1111}^{34} + Q_{1211}^{34} \leq 500$
$450 \leq Q_{1112}^{34} + Q_{1212}^{34} \leq 520$
$380 \leq Q_{1113}^{34} + Q_{1213}^{34} \leq 400$
$600 \leq Q_{2111}^{34} + Q_{2211}^{34} \leq 800$
$580 \leq Q_{2112}^{34} + Q_{2212}^{34} \leq 720$
$600 \leq Q_{2113}^{34} + Q_{2213}^{34} \leq 700$
$300 \leq Q_{1121}^{34} + Q_{1221}^{34} \leq 350$
$380 \leq Q_{1122}^{34} + Q_{1222}^{34} \leq 400$
$400 \leq Q_{1123}^{34} + Q_{1223}^{34} \leq 480$
$480 \leq Q_{2121}^{34} + Q_{2221}^{34} \leq 600$
$520 \leq Q_{2122}^{34} + Q_{2222}^{34} \leq 600$
$500 \leq Q_{2123}^{34} + Q_{2223}^{34} \leq 650$
$500 \leq Q_{1131}^{34} + Q_{1231}^{34} \leq 600$
$480 \leq Q_{1132}^{34} + Q_{1232}^{34} \leq 620$
$420 \leq Q_{1133}^{34} + Q_{1233}^{34} \leq 480$
$560 \leq Q_{2131}^{34} + Q_{2231}^{34} \leq 600$
$500 \leq Q_{2132}^{34} + Q_{2232}^{34} \leq 550$
$520 \leq Q_{2133}^{34} + Q_{2233}^{34} \leq 540$

(2.8 Plant operation constraints )

$A_{11} + A_{21} \geq 1$
$A_{12} + A_{22} \geq 1$
$A_{13} + A_{23} \geq 1$

(2.9 Distribution center operation constraints)

$B_{11} + B_{21} \geq 1$
$B_{12} + B_{22} \geq 1$
$B_{13} + B_{23} \geq 1$

(2.10 Non-negative constraints)

$Q_{psft\omega}^{12}$ ,$Q_{pfdt}^{23}$, $Q_{pdrt}^{34}$, $Q_{pdt}^{33}$, ≥0 for t=1,2,3; p=1,2; f=1,2; s=1,2; d=1,2;r=1,2,3 for each $\omega$ and they are all integers.

(2.11 Binary constraints)

$A_{ft}$, $B_{dt}$ ={0,1} for t=1,2,3; f=1,2; d=1,2

# APPENDIX B

# CODE

Funexselect.m

```
%case example
clear
clc
tic
CoIL=xlsread('C:\Thesis\Coefficient.xlsx','left inequ','A2:DD217');
CoIR=xlsread('C:\Thesis\Coefficient.xlsx','right inequ','A2:A217');
CoEL=xlsread('C:\Thesis\Coefficient.xlsx','left equ','A2:DD13');
CoER=xlsread('C:\Thesis\Coefficient.xlsx','right equ','A2:A13');
x0=xlsread('C:\Thesis\Coefficient.xlsx','x0','A2:DD2');
ub=[inf(96,1);ones(12,1)];
lb=zeros(108,1);
op = optimset('fmincon');
op = optimset(op,'MaxFunEvals',1000000,'MaxIter',1000000);


%fminmax
[x,fval]=fminimax('Funselect',x0,CoIL,CoIR,CoEL,CoER,lb,ub);
x2=round(x);
fval2=Fun(x2);

%fmincon
% [x,fval]=fmincon('Funselect',x0,CoIL,CoIR,CoEL,CoER,lb,ub,[],op);
  x2=round(x);
  [~,F]=Fun(x2);

%goalattain
  g=[553751.7;12197.77];
  w=[1;1]

[x,fval]=fgoalattain('Funselect',x0,g,w,CoIL,CoIR,CoEL,CoER,lb,ub,[],op
);
  x2=round(x);
  fval2=Fun(x2);

% NSGA-II
  fitness=@Funselect;

options=gaoptimset('paretoFraction',0.3,'populationsize',100,'generatio
ns',200,'stallGenLimit',200,'TolFun',100,'PlotFcns',@gaplotpareto);
```

```
  [x,fval]=gamultiobj(fitness,96,CoIL,CoIR,CoEL,CoER,lb,ub,options);
   x2=round(x);
  fval2=Fun(x2);

toc
```

Funselect.m

```
%case 1 function
function F =Funselect(x)
% function [A,F] =Fun(x)

% F(1)=xlsread('C:\Thesis\Coefficient.xlsx','min f1 co','A2:CR2');
F(1)=30.91*x(1)+32.73*x(2)+29.1*x(3)+27.27*x(4)+29.1*x(5)+30.91*x(6) ..
.,

+27.27*x(7)+27.27*x(8)+28.1*x(9)+29.75*x(10)+26.45*x(11)+24.8*x(12)  ...,

+26.45*x(13)+28.1*x(14)+24.8*x(15)+24.8*x(16)+25.54*x(17)+27*x(18) ...,

+24*x(19)+22.54*x(20)+24*x(21)+25.54*x(22)+22.54*x(23)+22.54*x(24) ...,

+15.45*x(25)+14.55*x(26)+16.36*x(27)+14.55*x(28)+12.73*x(29)+16.36*x(30
) ...,

+15.45*x(31)+13.64*x(32)+14.05*x(33)+13.22*x(34)+14.88*x(35)+13.22*x(36
) ...,

+11.57*x(37)+14.88*x(38)+14.05*x(39)+12.4*x(40)+12.77*x(41)+12*x(42) ..
.,

+13.52*x(43)+12*x(44)+10.52*x(45)+13.52*x(46)+12.77*x(47)+11.27*x(48) .
..,

+6.36*x(49)+8.18*x(50)+7.27*x(51)+9.09*x(52)+6.36*x(53)+6.36*x(54) ...,

+10.91*x(55)+9.1*x(56)+7.27*x(57)+10.91*x(58)+10.91*x(59)+8.18*x(60) ..
.,

+5.785*x(61)+7.44*x(62)+6.61*x(63)+8.26*x(64)+5.785*x(65)+5.785*x(66)..
.,

+9.92*x(67)+8.26*x(68)+6.61*x(69)+9.92*x(70)+9.92*x(71)+7.44*x(72) ...,
    +5.26*x(73)+6.76*x(74)+6*x(75)+7.51*x(76)+5.26*x(77)+5.26*x(78)  ...,
    +9*x(79)+7.51*x(80)+6*x(81)+9*x(82)+9*x(83)+6.76*x(84) ...,

+6.36*x(85)+6.36*x(86)+6.36*x(87)+6.36*x(88)+7.44*x(89)+7.44*x(90) ...,

+7.44*x(91)+7.44*x(92)+5.26*x(93)+5.26*x(94)+5.26*x(95)+5.26*x(96) ...,

+7727.27*x(97)+7024.79*x(98)+6386.18*x(99)+8636.36*x(100)+7851.24*x(101
)...,

+7137.49*x(102)+5454.55*x(103)+4958.68*x(104)+4507.89*x(105)+6363.64*x(
106)...,
    +5785.12*x(107)+5259.2*x(108);
```

```
F(2)=0.0528*((x(49)+x(52))^2)-64*(x(49)+x(52))+19090.91 ...,
 +0.0686*((x(61)+x(64))^2)-84.6753*(x(61)+x(64))+25947.46 ...,
 +0.2182*((x(73)+x(76))^2)-201.653*(x(73)+x(76))+46543.95 ...,
 +0.0264*((x(55)+x(58))^2)-52*(x(55)+x(58))+25000 ...,
 +0.0343*((x(67)+x(70))^2)-59.74*(x(67)+x(70))+25637.54 ...,
 +0.0436*((x(79)+x(82))^2)-72.73*(x(79)+x(82))+30052.59 ...,
 +0.1056*((x(50)+x(53))^2)-88*(x(50)+x(53))+18181.82 ...,
 +0.24*((x(62)+x(65))^2)-221.82*(x(62)+x(65))+51198.35 ...,
 +0.0545*((x(74)+x(77))^2)-62.81*(x(74)+x(77))+17881.29 ...,
 +0.044*((x(56)+x(59))^2)-64*(x(56)+x(59))+22909.1 ...,
 +0.06*((x(68)+x(71))^2)-85.45*(x(68)+x(71))+30206.61 ...,
 +0.029*((x(80)+x(83))^2)-46.28*(x(80)+x(83))+18031.56 ...,
 +0.0528*((x(51)+x(54))^2)-76*(x(51)+x(54))+27045.45 ...,
 +0.0343*((x(63)+x(66))^2)-51.95*(x(63)+x(66))+19291.62 ...,
 +0.073*((x(75)+x(78))^2)-82.64*(x(75)+x(78))+23328.32 ...,
 +0.132*((x(57)+x(60))^2)-184*(x(57)+x(60))+64000 ...,
 +0.096*((x(69)+x(72))^2)-123.64*(x(69)+x(72))+39669.42 ...,
 +0.22*((x(81)+x(84))^2)-271.1*(x(81)+x(84))+84147.46;
 A=((F(1)-553725.68)^2+(F(2)-12197.77)^2)^0.5;
```