A STUDY OF THE ACOUSTICAL PROPERTIES OF THE
CLARINET IN ORDER TO PREDICT PLAYING FREQUENCIES

A Dissertation in
Acoustics
by
Whitney L. Coyle

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Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Doctor of Philosophy

August 2016
The dissertation of Whitney L. Coyle was reviewed and approved* by the following:

Daniel A. Russell  
Professor of Acoustics, Director of Distance Education for the Graduate Program in Acoustics  
Co-Thesis Advisor, Co-Chair of Committee

Jean Kergomard  
Director of Research, Laboratoire de mecanique et d’acoustique, Marseille, France  
Co-Thesis Advisor, Special Member

Victor W. Sparrow  
Professor of Acoustics, Director of the Graduate Program in Acoustics

Stephen A. Thompson  
Professor of Acoustics

Michelle Vigeant  
Assistant Professor of Acoustics and Architectural Engineering

Anthony Costa  
Professor of Music, Clarinet

*Signatures are on file in the Graduate School.
The modern clarinet has been in use for nearly 100 years, however, there remains much concerning the physics of the instrument that researchers have yet to understand. One of the more important aspects of the instrument that concerns musicians, is the quality of the instrument. There are many aspects of this study of instrument quality that could be subjective. This dissertation takes a step towards an objective acoustical quality marker of the clarinet by focusing on analytical predictions of playing frequencies given certain variable inputs such as blowing pressure and reed opening at rest. This research offers a study of this acoustical property by means of simplified mathematical formulas, numerical simulations, and experimental methods in order to create tuning maps of the clarinet over a normal range of playing parameters.

The dissertation begins by offering an introduction to the concept of impedance as perhaps the most important and applicable concept in musical instrument acoustics. In this work, the resonance frequencies of the instrument provide a basis for all playing frequency predictions and these resonance frequencies are found by analyzing the measured input impedance. The dissertation then describes the analytical methods for predicting playing frequencies by considering changes from the instrument’s resonance frequencies due to a number of fairly complicated effects including: reed induced flow, reed dynamics, instrument inharmonicity, and a player introduced temperature gradient. Numerical prediction methods are also described in order to provide a benchmark for the newly defined analytical formulas. Next, experimental validation methods and preliminary data are detailed in which an artificial mouth and instrumented mouthpiece were used to create tuning maps in order to further test the bounds of the analytical formulas. Finally, tuning maps are created using the simplified analytical formulas. These tuning maps can show, for all combinations of possible blowing pressures and reed openings, the resulting clarinet playing frequencies. These tuning maps could eventually give musicians
and instrument makers fast, reliable, and perhaps more importantly, objective information about the quality of their clarinets.

This material is based upon work supported by the National Science Foundation Graduate Research Fellowship under Grant No. DGE1255832. Any opinion, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.
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ACKNOWLEDGMENTS

First and foremost to my advisors (Dr. Victor Sparrow, Dr. Daniel Russell, Jean Kergomard) and committee members (Dr. Michelle Vigeant, Dr. Anthony Costa, Dr. Stephen Thompson) past and present: as difficult a road as a Ph.D. is, it would be absolutely impossible without the investment that each one of you has made in me, my work, and my future. I thank you from the bottom of my heart for believing in my and pushing me to be the best that I can possibly be.

To the professors and teachers that came before (in particular Dr. McCarthy, Dr. Fister, Dr. Locke, Dr. Cottingham, Mr. McClosky, Mr. McAllister): your guidance, patience, confidence in my abilities and obvious interest in my future pushed me forward and helped make me what I am today. I think of all of you often and what roles you played in my success. I appreciate each and every one of you and hope that as a teacher I will have the same caliber of impact for my future students.

To my dear, dear friend, Karen Thal: what would I do without you? You have continuously gone above and beyond for me. Thank you, thank you, thank you! Honestly, I have more thank yous to send to your way than I could ever actually have time express. And obviously words aren’t enough ...

To my colleagues, labmates and friends: I could not possibly list everyone (after
six years in and out of Pennsylvania/Marseille/Paris/Kentucky) and so I will blanket the statement to everyone that picked me up when I fell down, metaphorically slapped some sense into me when I needed it, took me away from study/research to detox every once in a while, and overall made my non-academic life so much more rich. Thank you for helping me stay sane (for the most part).

To my amazing family: you are such a big part of my heart. I never wanted to burden you much with my thoughts and feelings when it came to getting through my education but you were always there if I needed a shoulder to cry on (in person or virtually). You are all magnificent, beautiful, smart, and full of love for everyone around you. I am lucky to know you all. Thank you for everything!

Of course I must acknowledge the ARCS foundation and specifically the Bennet-Coppersmith-Palmer award as well as the National Science Foundation for their generous support of my education throughout my time at Penn State and abroad. The opportunity for me to perform research in the field of musical acoustics and collaborate with world-renowned researchers abroad would not have been possible without the people within these wonderful foundations believing in me and my potential.

A la France : vous m’avez transformée de mille manières différentes. Merci.

Enfin, à mon mari, Adrien : tu mérites toute une dissertation pour tout ce que tu as fait pour moi. Tu m’as supportée quand j’ai râlé, délié, pleuré et souri. Tu m’as soutenue pendant tout ce temps, et ne m’as jamais doutée, même pas pendant une seconde. Ta patience et ton soutien m’ont permise de survivre et de prospérer pendant qu’on affranchit, ensemble, cette prochaine étape de notre vie. T’es mon cœur - mille fois merci !
“You can do the best research and be making the strongest intellectual argument, but if readers don’t get past the third paragraph you’ve wasted your energy and valuable ink.” -Carl Hiaasen
PLAYING A CLARINET IN TUNE? IMPOSSIBLE!

“Clarinets, like all other wood-wind instruments, are considerably out of tune, perhaps played out of tune, but most certainly made out of tune [91].”

“No matter how good the instrument is that is in your hands, it is impossible to manufacture any wind instrument, brass or wood or plastic, that is perfectly in-tune with the tempered scale. If we were to produce such an instrument, the holes and the keys would be in such places that no human fingers could reach them [Alfred Reed in Casey, 1993, p. 318].”

“There is a trade-off between excellence in sound and excellence in intonation...”

1.1 Brief History of the Clarinet

The clarinet itself was invented in the late 1600s but began as an instrument known as the chalumeau (see Fig. 1.1), an instrument resembling the present clarinet with far fewer keys. The clarinet is basically a cylindrical resonator with holes and a small flared bell at the end and is acoustically characterized as a single-reed woodwind instrument since it is the vibrations of the reed that drive the clarinet system as seen as Fig. 1.2.

The first mention of the clarinet was by a German instrument maker Johann Christoph Denner [87]. Born in Leipzig, Denner is credited with the addition of the register key to the chalumeau which increased the range and allowed pitches a
12th above the lower register to be produced \[4\]. In 1812 Iwan Muller invented a 13-key clarinet with improved intonation due to better placement of the tone holes. This 13-key clarinet could play certain accidentals, though these added keys were not meant to allow the instrument to play in all key signatures but to allow the player to elongate the finger, cover larger holes (with pads) and to play trills. Two of the keys added by Muller were operated by the right thumb, but at this point Muller’s clarinet still does not have any rings around the tone holes. Muller further refined his instrument by adding holes and keys that replaced certain awkward cross-fingerings. In 1844 the Boehm fingering system was patented, though Boehm himself wasn’t involved. The Boehm fingering system for the clarinet is based on the Boehm flute fingering system that was introduced in 1832. This system utilized ring keys in order to cover holes that were bigger than the finger. This allowed for cross fingerings and multiple alternate fingerings found extremely useful, if not vital, by clarinetists today. There have been many smaller changes which have led to the instrument used today. However, the Boehm system is still the most popular
Figure 1.3: Fingering chart for the comfortable playing range of B♭ soprano clarinet [125].

The terminology clarinetists use to describe each part of the instrument can be found in Appendix A.
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1.2 Introduction

Manufacturers and musicians alike have been designing and building musical instruments in various ways throughout the span of the instrument’s history. Some instruments were conceived as they are for historical or religious purposes and are not necessarily meant for the ears of humans but to please a perhaps higher power; yet there is an intrinsic need for beauty in sound. The sounds created that are not considered beautiful are considered noise, unwelcome, unnecessary. It is for this reason that in our recent history researchers, musicians, hobbyists, and engineers have begun using physics and the understanding of science to aid in the creation of these more refined instruments.

Although musicians are able to create a suitable sound from instruments such as the modern clarinet, without multiple difficult adjustments there could be interest in finding ways to make the instrument as easily playable as possible. This can be seen in the many tuning accessories available for wind instruments. The more minute adjustments players must make to each instrument as he/she plays, the less room there is for expressivity, interpretation or more generally individual musicianship. Musicians struggle from a young age to understand what it is that makes their instrument good, bad or simply playable. Questions arise such as, “why must we buy the expensively made instrument to sound like a professional; why do these reeds hurt or help me, this mouthpiece, thumb rest, etc. Why are so many accessories made to aid in making a better sound?” Would it not be more efficient to just understand what it is about the instrument that is making it more or less playable and fix it at the “source”? What does it mean for an instrument to be playable? And how can one characterize a playable instrument as a good
A broad leading question for the research described in this dissertation was - what characteristics make one musical instrument more playable and preferable compared to another? More specifically, what acoustical properties or quality markers can characterize an ideal clarinet? For this dissertation research, the purpose is to explore the specific quality marker of intonation, or tuning homogeneity, for any given instrument both qualitatively, through musician interaction, and quantitatively, through numerical, analytical and experimental techniques.

One hypothesis is that a musician would prefer an instrument that has a high degree of tuning homogeneity. This “...trade-off between excellence in sound and excellence in intonation” [87] should not be the case. Perhaps the assumption that musicians just need to be accustomed to an instrument playing out of tune can change. One significance of this study is that it will be a multifaceted work with analytical, physical equations as well as numerical simulations and experimental results for validation. The author collaborated with researchers, musicians and industry clarinet makers to address these questions and the hope is that the results can be used by all involved with a future goal to improve the instrument making process in order for musicians to have the best possible clarinet with which to perform.

1.3 Statement of the Problem

“The student should be taught early to listen to himself as a dual personality, one a Dr. Jekyll projecting himself across the room where he critically listens to his other playing self, Mr. Hyde...A serious instrumentalist must devote much time and energy in learning to play in tune.” [116]

1.3.1 Tuning concerns for a clarinetist

Playing in tune is important. For many many decades the tuning system of choice for the piano has been the equal tempered scale. In the United States the tuning note given in an orchestral setting is generally a frequency of 440 Hz. This is a convention set after many years of orchestras playing with different tuning systems.
The equal tempered scale is based on the idea that the 12 tones in any scale will have the same frequency spacing between each of them (thus, equal). However, musicians are not generally concerned with remaining within the bounds of the equal tempered scale as it stands. Ensembles allow themselves to tune to one another instead, a very flexible, free-form type of tuning. Nevertheless, the equal-tempered scale is a safe place to begin the study tuning.

So, what exactly does it mean to play “in-tune”? Unfortunately, finding a perceptual definition that is agreeable to all musicians, instrument makers and researchers is well beyond the scope of this work. However, this work assumes that a musician wishes to approach the corresponding frequencies of the equal-tempered scale.

In the Art of Clarinet Playing, Stein points out that the equal temperament tuning system works wonderfully for keyboard instruments (e.g., a highly soloistic instrument) but that the system “plays havoc” on wind instruments – instruments that rely on over-blowing in the harmonic series for tones in their upper register. Tuning wind instruments at any extreme of their range to equal temperament results in extremely out of tune octaves (or 12ths in the case of the clarinet). Clarinet manufacturers continue to attempt matching this equal tempered tuning system, and actually do well in this goal. However, as Stein reiterates, the complexities of modern instruments lead to out of tune tendencies, despite the best efforts. This fact leaves a great burden on the instrumentalist to attend to these discrepancies within a scale, between registers on their own instruments and when attempting to play in tune with other players. Because of this considerable responsibility and difficulty, Stein stresses the absolute need to use every possible outlet for help, “scientific or otherwise” in order to approach a more reasonable remedy to these tuning difficulties.

For musicians, intonation is largely determined by embouchure formation. Small adjustments can be made according to the musician’s knowledge of and comfort with the equal tempered scale. Due to the fact that the average ear can tolerate a quarter tone (or 25 cents) sharp (though players tend to be much less tolerant of flat pitches) many musicians play with listener perception and use being slightly out of tune to their advantage, adding to tone color and originality in their sound.

The clarinet tends to be sharp in the lower register and flat in the higher registers. However, if played softly, the higher registers will also be sharp. Larry Guy points
out that the fundamental notes are generally flat with their corresponding 12ths being flat (depressing the register key to play register two)\textsuperscript{[71]}. Guy also points out a few general rules for clarinet tuning tendencies: hard reeds will play sharp, as will the highest notes on the instrument. The lowest notes (utilizing the bell) and the extremes of registers will play flat, and finally the throat tones will be unpredictable. The throat tone tendencies change with reed and dynamic change but are often very flat when played at a loud dynamic. Another interesting tendency understood and addressed by professionals is that the instrument will play sharper after a breath due to the composition of air entering the clarinet\textsuperscript{[87]}.

1.3.2 The acoustical problem

It is now obvious that tuning homogeneity is a problem for even professional level musicians due to the tendencies of the clarinet. The issue of tuning is of great concern for musicians both when choosing an instrument and when performing. This dissertation addresses the problem by providing tools to predict the tuning homogeneity of the clarinet and to identify problems in this regard quickly and accurately for manufacturer and musician knowledge. The main goals of this thesis are to provide an accurate and fast prediction method and to produce visual tuning maps for a given instrument. These tuning maps give playing frequencies based on a combination of two playing parameters: blowing pressure and reed opening. Clarinetists could use these maps to quickly identify the combination of playing parameters necessary to play each note on their instrument in tune. This method offers a scientifically accurate and subjective method for musicians to comprehend the tuning tendencies of their instrument. Manufacturers could use the maps to remedy these tuning discrepancies from note to note. From there, manufacturers could make small changes to the instrument and see the effect in these tuning maps for the entire range of the instrument.

1.3.3 Research plan

To solve this problem, simplified mathematical formulas that describe the sound production for the clarinet will be used to create a fast and accurate prediction method to find the playing frequencies for a clarinet. In order to validate the use of this simplified analytical model, numerical simulations will be used. These
numerical simulations were written by another researcher in musical acoustics (Philippe Guillemain, [68]) and were modified for use in the current research. These numerical simulations also have the capability to create tuning maps, although the computation time is much longer. Another validation technique is the use of experimental measurements with actual clarinets. By measuring the playing frequency of an instrument being played by an artificial mouth and using an instrumented mouthpiece played by musicians, further comparisons can be made as to the validity of the analytical formula predictions. The artificial mouth and instrumented mouthpiece measure the blowing pressure and reed opening in different ways, but are capable of offering the same data as the output of the analytical formulas and numerical simulations, making these key comparisons for this dissertation. Using the numerical simulations and measurements from the experimental apparatus, the analytical formulas will be validated and therefore can be used confidently by researchers, musicians and manufacturers alike to predict the playing frequencies and tuning tendencies across the range of the instrument, quickly and accurately.

1.4 Outline of Dissertation

Chapters 2, 3, and 4 will offer the background information necessary for readers. Chapter 2 will include a detailed literature review of musical and clarinet acoustics. Chapter 3 will detail how sound is created in the clarinet using the body of the instrument as the resonator. And Chapter 4 will describe the reed and mouthpiece system of the instrument as the nonlinear generator for this instrument.

Chapters 5, 6, and 7 describe three different approaches to understanding the problem of tuning inhomogeneity using analytical, numerical, and experimental methods. Chapter 5 will describe the mathematical background that is used to create a playing frequency prediction method with analytical formulas. Chapter 6 will present the numerical simulation code used to validate the analytical formulas. Finally, in Chapter 7 the experimental approaches used in this research, including input impedance measurements and artificial mouth measurements, will be presented.

The dissertation will conclude with Chapters 8 and 9 in which the results and comparisons will be presented and discussed. Chapter 9 will detail the validation
of the analytical formulas and include a detailed discussion of the resultant tuning maps. Chapter [10] will conclude the dissertation and offer possibilities for future work on the topic of clarinet acoustics and the prediction of quality of musical instruments in regards to tuning.
2.1 Introduction

There is a wide gap between the literature written for researchers, scientists and physicists who study musical instruments and that directed towards practitioners, the musicians and manufacturers making the instruments. This chapter will address the historical context of clarinet acoustics research, summarize current work on the more focused view of tuning homogeneity and predictive models, and will provide a detailed discussion of the results and findings from previously published work in the field of musical acoustics.

2.2 Historical Context

The clarinet is a self-sustained oscillator resulting from the coupling of a linear resonator and a nonlinear generator \[30\]. There are many physically complicated parts of the instrument that should be studied in depth individually in order to understand and model the functioning of the instrument in its entirety.

Early research on musical instrument functioning was done by Helmholtz in the early 1860s \[72\] when he laid the groundwork for studying the frequency and timbre of musical instruments based on acoustical principles. Some of the first major clarinet acoustics research was done by Arthur Benade in the mid 20th century. For
over three decades Benade was the authority on clarinet acoustics. Unfortunately much of his work was not published in journals. However, an archive of his writings is housed at Stanford [3].

In 1959 Benade discussed the importance of the woodwind bore and the effect of the finger holes on the overall playing behavior of the instrument [16]. This early publication, which seems to have sparked many other studies by Benade himself, discussed how larger bore diameter caused the quality factor of the resonance peaks to decrease throughout the spectrum of the instrument. He mentioned the possibility of future work concerning the effects of the wall material (clarinet body material) on the resulting tone quality of the instrument. Benade pointed out that the reed-pipe combination emits a sound that is of lower frequency than a closed-end frequency (as is usually the first prediction for a clarinet-like system).

Figure 2.1 shows the normal mode frequencies for a simple cylindrical pipe, closed at one end, as a function of length (solid lines) compared with a pipe “closed” at one end by a reed (dotted lines). There are four curves present on the plot, each representing the numbered resonance frequency and the effect on each as length increases. Of course, as the tube length increases the frequencies decrease. However, this figure also shows that adding the reed causes each resonance to lower in frequency, an effect which is more noticeable for shorter tubes. As the tube length decreases, the two lines (closed pipe and pipe closed with reed) begin to separate. There is also an increasing significance in this difference between the two curves for increasing mode number (n). Each of the dotted curves is limited by the “limiting frequency”, which happens to be the reed resonance frequency. This limiting frequency is present regardless of pipe length.

Even at this early stage, Benade mentioned the difficulties in making predictions above the value of blowing pressure at which the reed begins to beat against the mouthpiece since the reed mass, stiffness, type of reed table, blowing pressure and lip “tension” need to be taken into account. From Fig. 2.1 Benade became, perhaps, the first author to stress the importance that the reed resonance, in playing conditions, needs to lie far above the first two or three modal resonance frequencies of the instrument in order to have as small of an effect on the resonances of the tube as possible and that if this condition were not met, the instrument would not play.

In 1960, Benade published a follow-up paper that began the discussion on
Figure 2.1: Normal mode frequencies for a simple cylindrical pipe, closed at one end, as a function of length compared with a pipe “closed” at one end by a reed. From [16].

the importance of tone hole placement and design [17]. The primary interest of instrument makers had been to place the holes in such a way as to achieve the desired note in a given musical scale. Benade was the first to apply impedance theory to the analysis of musical instruments. Benade explained that the effect of any individual hole is that of a length correction, since the hole creates a small cavity at some point along the bore. The cavity acts as a compliance which will lower the resonance of the instrument and more generally perturb the intonation. He discussed the effects of different types of holes (closed or open: large vs. small holes, side holes, cross fingering creating missed or skipped open holes) as each one individually will create a different effect [17]. Benade recognized that understanding the coupling between reed, bore, and holes was necessary for understanding instrument design. In 1973 Benade returned to the more classical look at musical instrument acoustics and addressed the register hole design [18]. Benade observed that opening the register hole accomplishes two (perhaps expected) main effects for the instrument: the open register hole reduces the amplitude of the first peak (allowing the second peak to become the tallest in the spectrum) and this open register hole shifts the first peak.
upward slightly in frequency.

John Backus was performing research concerning clarinet acoustics at the same time as Benade. In a 1963 paper, Backus stated “musical instruments have developed into their present forms through empirical processes in which acoustical theory has played a negligible role” and that by applying acoustical theory, the designs and performance of musical instruments could be greatly improved [77]. Backus was the first to propose a relationship between flow rate and pressure and in his 1963 paper, he used an artificial mouth system to test the frequency shifts for different reed openings and qualitative reed damping. The playing frequency for a clarinet was seen to be below the resonance frequency of the clarinet by just a small amount. This playing frequency was also found to vary linearly with the reed opening and reed damping. This was the first mention that the threshold blowing pressure is proportional only to the reed opening and stiffness but does not vary with the reed damping. This means that the threshold blowing pressure should not change from player to player because of lip differences between players.

In 1967, Backus took a more focused look at the resonances of the clarinet due to the approximate cylindrical form of the bore not being as simple as it seems [13]. Backus insisted that the clarinet is “considerably” different than a simple cylinder in that the mouthpiece is a wedge-like shape at the entrance, there are covered tone holes jetting out from the bore, and the bell varies from a simple straight cylindrical geometry. These derivations from a simple cylinder mean that the resonances will not be simple odd integer multiples of one another.

In 1974 Backus further studied the resonance curves of reed woodwind instruments by studying their input impedences [14]. His paper posed the question, based on information found in the 1974 study, whether the stretching of the resonances for the bassoon and oboe are relative to the compression for the clarinet and if the suppression of this compression/stretching would improve the playability of the instrument or not. This question, though yet to be answered, is partially addressed (for the clarinet) in this dissertation and by Chaigne and Kergomard in Chapter 9 of [30] when focusing on the inharmonicity of the instrument.

Wilson and Beavers, around 1973, studied the effect of damping of reeds on the operating regimes of the clarinet [126]. Figure 2.2 from their paper, shows the reed resonance limiting the frequency on the resonances of the tube. Their analytical models and experiments used show that for every value of damping
parameter chosen, the pipe will either be excited near the resonance of the reed (where \( f/f_0 = 1 \) on the top plot) or near the resonance of the tube, as shown by the chain-dotted lines. An important result from these plots is that as long as the reed is heavily damped (in practice by the lip), the lowest operating frequency possible for the given tube (clarinet) can only be excited at the lowest blowing pressure, regardless of tube length. This paper gave the first analytical nonlinear characteristic equation for the flow as a function of pressure difference, when reed dynamics are ignored.

![Figure 2.2: Figure 2 from [126]. Resonances that can be excited by a particular reed with given damping, theory (lines) and measurement (circles). Top plot \( f/f_0 \) represents the tube resonance divided by the reed resonance.](image)

The ever important nonlinear characteristic flow curve describing the flow as a function of pressure difference for the clarinet was perhaps introduced by Backus in the early 60s and Nederven continued the discussion in the 1969 first edition of his woodwind acoustics book [99]. However, an important paper in the history of woodwind acoustics was written by McIntyre, Schumacher and Woodhouse in 1983 [92]. Their paper further developed this now well-known nonlinear characteristic flow curve for flow \( u(\Delta p) \) as a function of pressure difference.
\( \Delta p \) (inside the mouth and inside the mouthpiece) and is the treatment still used in current research.

Stewart and Strong presented the first version of a numerical model of clarinet using lumped element approximations in order to obtain the radiated pressures and subsequent playing frequencies comparable to those of a real version of a simplified clarinet [118]. Figure 2.3 represents several nonlinear characteristic flow curves \( u(\Delta p) \) for different reed openings. The figure shows that for different initial reed openings, the volume velocity into the instrument first increases as the pressure difference between the mouth and mouthpiece increases but as the pressure difference continues to increase, this pressure difference pushes the reed closer to the reed table and the flow then decreases. Finally the pressure difference is large enough the reed will be pushed against the lay and the flow stops all together. The dotted line represents the threshold blowing pressure where the reed will begin to oscillate. Before this point, the reed is stationary.

![Nonlinear characteristic curves for different reed openings](image)

*Figure 2.3: Nonlinear characteristic curves for different reed openings from [118]. Different curves show nonlinear characteristic for different openings. Bottom curve to top curve represents the most closed reed to most open reed respectively.*

The volume velocity into the clarinet as a function of pressure difference across the reed is an important characteristic measurement for a wind instrument. Figure 2.3 also shows that the threshold blowing pressure (indicated by the dotted line
going up the plot) should increase with increasing reed opening at rest. It also shows that the closing pressure is much higher for an open reed than for a closed reed.

2.3 Resonator: instrument body

In order to model the clarinet it is necessary to understand each individual piece and how it functions. The clarinet is comprised of a linear resonator and a nonlinear excitor (or generator). Benade [21], Worman [128], Grand et al. [65], Kergomard et al. [84], studied the sound production and timbre of the instrument in an analytical fashion using the classical characteristic equations describing each piece of the instrument - the nonlinear generator, the passive linear resonator and the bell as a radiator. Schumacher et al. [113] and Gilbert [62] used numerical approaches in their research to achieve this same goal. The following sections will offer a literature review for the research that has been done concerning each part of the clarinet.

The body of the clarinet is the resonator - the part of the instrument that facilitates the creation of standing wave resonances in the instrument. Chapter 3 will discuss the clarinet acoustics that show how the clarinet body can be modeled to a first approximation as a closed-open cylinder. However, this model and prediction of the resonance frequencies of the instrument needs to be modified when the loss mechanisms are considered. In 1964 Backus studied the effects of wall materials on the tone quality of instruments [12]. Backus mentioned that the clarinet body is not perfectly rigid and that, depending on the amount of elasticity in the walls, the air-column resonances could be altered, changing the harmonic structure and eventually affecting the tone of the instrument.

According to Keefe, in 1868 Kirchhoff discussed the viscothermal losses that are an important consideration for the clarinet bore [81]. Keefe used Kirchhoff’s work to find approximate equations that describe the characteristic impedance and the propagation wave number for linear acoustic transmission through a particular medium in a cylindrical duct. Keefe presented expressions for these bore/wall losses for the isothermal (no transfer of heat) and nonisothermal cases (where heat will be transferred to the walls of the duct). Keefe’s work continues to be cited today as the best method of approximation for the thermoviscous boundary layer losses in cylindrical ducts.
More recently researchers have been turning to modeling the loss phenomena numerically. Van Walstjin et al. developed a discrete-time model of the woodwind bore and were able to represent the frequency dependent wall losses by approximated digital filters (essentially a filter that decreases the output pressure by a scaling factor) [123]. Their general model is not stable when including losses. This limitation does not have a significant consequence for the relatively simple continuously cylindrical bore for the clarinet but could have implications for a piece-wise conical bore as is the case for the saxophone.

The tone holes act to effectively shorten the length of the instrument in order to play each note of the equal-tempered scale by shortening the wavelength and therefore increasing the frequency. The inclusion of these breaks in the continuity of the clarinet bore also introduce complications including unwanted turbulence and a lattice of small chimneys creating multiple volumes which will further alter the resonances of the instrument. Benade studied the tonehole lattice as early as 1973 [18] and a more complete description of the problem of adding a tonehole lattice can be found in Benade’s book [19]. Benade considered the resulting clarinet spectrum from a theoretical and experimental standpoint by treating the clarinet as a cylindrical tube which is terminated by a uniform lattice of tone holes and found that the turbulence loss due to the toneholes is a second order effect and can generally be ignored for the purpose of studying the spectrum. This paper shows the formula for the input impedance that describes the resulting spectrum of the instrument for the clarinet when a tonehole lattice is included and shows that the tone hole lattice creates a cutoff frequency of around 1500 Hz for a clarinet-like duct. In 1982 Keefe published two papers which looked into the theoretical and experimental aspects of a single woodwind tonehole [79][80]. The tone hole he included was added to the resonator impedance in series and provided a negative inertance (the series impedance for both open and closed holes is a negative inertance). The closed hole can be modeled as an additional compliance related to the closed-hole volume. The open hole contribution is written as an effective length addition and written in terms of the dimensions of the tone-hole. Keefe’s experimental work in the second of these papers validated the use of this simple acoustic transmission line equation with equivalent volumes and length corrections. Though the paper did not offer additional information about the amplitude dependence for losses involving the toneholes (as the paper was written
with a small amplitude approximations in mind), it did provide significant evidence that the losses will depend strongly on the dimensions of these toneholes (diameter and chimney length/height).

Benade introduced a new design for a clarinet that would address some of the issues that clarinetists face, such as the tonehole lattice creating a particular cutoff frequency for each instrument, the effects of the placement of the register hole and how to address the “un-clarinetlike” throat tones [82]. In 1990, Feng and Strong alluded to an optimization procedure for tone hole position, diameter and height in an effort to mitigate the tuning problems created by current tonehole placement [52]. Unfortunately, the results have not been published.

Moers and Kergomard studied the trade-off between geometrical versus acoustic regularity in the tone-hole lattice [95]. Their paper asks an important question that has yet to be answered: “why do makers provide an increase of the spacing between holes together with an increase of their radius?” It seems that instrument makers do this in search of correct tuning. But, moving and shaping the holes could change the characteristic cutoff frequency and therefore alter the tone color of the instrument as well. Optimization of tonehole geometry and placement in regards to tuning homogeneity and musician ergonomy has been explored recently by Noreland et al [103]. This paper uses numerical optimization techniques with a first target of matching the exact tuning compared to the equal tempered scale in order to create the perfectly placed tone hole lattice for the instrument. Initial constraints on the optimization included the use of only one register hole and an instrument with a fixed length and bore diameter. This research is ongoing and the researchers now hope to create a more clarinet-like instrument with keys and the inclusion of a second register key to facilitate more accurate tuning of the 12ths.

Another important aspect of the current design of the clarinet resonator is its effect on tuning. In 2004 Debut wrote that although the tuning of the first to second register has been optimized as much as possible (by hand tuning and optimization), the inharmonicity between the first register notes and their second register counterparts (reached by pressing the small register key by the left thumb) near the mouthpiece is still large [46]. Debut et al. offer an interesting method of taking the effective length of the instrument (assuming a closed-open cylinder) and interpreting all changes from this cylindrical shape as effective length corrections (addition or subtraction) to the original effective length (Fig. 2.4). Since the authors
stated all perturbations are small, these then small length corrections can simply be added together to find a new effective length and subsequently new playing frequency (since the frequency is inversely proportional to the length). Debut’s work was focused on the tuning between the 12th intervals achieved when pressing this register key and stated that the 12ths corresponding to the 4 lowest tones are slightly too large by 20 to 30 cents.

Another important point, discussed by Coltman et al. [34,35], and more recently revisited by Noreland [102], is that the temperature gradient inside the resonator of the instrument is important for predicting the playing frequencies and for correctly modeling the functioning of the instrument. This was also pointed out by Cramer [39] and for the flute by Coltman who said that the inclusion of the amount of CO$_2$ in the instrumentalists air column is not necessarily negligible. Recently, Fuks [59] and Noreland [103], in separate experiments, have taken measurements that show that the temperature gradient within the clarinet linearly decreases along the length of the instrument.

### 2.4 Generator: reed-mouthpiece combination

For the clarinet, the study of the excitor/generator includes the vibrating reed and incoming flow that causes this vibration. In order to study the clarinet model and the complicated mathematics that describe the instrument, it is necessary to make simplifications to large parts of the model in order to focus on one particular complicated piece. For example, the functioning of the reed is modeled as a single
degree-of-freedom oscillator (damped or undamped depending on the complexity of
the model) \[126,128\], even though the reed actually behaves like a damped free beam
with non uniform thickness exhibiting many bending and torsional modes \[119\].

The nonlinear characteristic of the generator/excitor was measured empirically
by Backus in 1963 and the means of excitation was studied notably by Wilson and
Beavers \[77,126\]. The nonlinear characteristic of the reed-mouthpiece combination
is used to describe the flow as a function of the pressure difference between the
mouth and mouthpiece, usually written \(u(p)\) or \(u(\Delta p)\) (see Fig. \[2,3\]). Fletcher
described the nonlinear generators of air driven woodwind (flute), lip generators
(brass) and reed generator (clarinets). Fletcher reviewed different generators he
showed that the clarinet system is relatively simple to analyze. Since in regular
playing conditions only the lowest “cantilever” mode of the reed is excited, first
order analysis can be done to show that the displacement of the reed is nearly
proportional to the pressure difference in the mouth and mouthpiece with very
little phase shift \[54\]. His discussion of nonlinear reed generators mentioned that
the slope of the nonlinear characteristic curve is like a resistance. At low pressures,
where the slope is positive (positive resistance) there is flow but with a loss of
energy. As the pressure increases (past the threshold of oscillation) to the negative
sloped portion of this curve (see Fig. \[2,3\]), the resistance is negative —this is where
the generator is providing energy to the resonator. Reed vibrations at the entrance
of the instrument in this oscillating regime are in phase with the standing pressure
waves in the resonator.

This idea of considering the coupling of the generator to the resonator was
acknowledged by Thompson in 1979 \[121\] who studied the significant effect of
shifting reed resonance frequencies. He pointed out that players often intuitively
shift the reed resonance to match a harmonic component of the playing frequency
and in doing so can stabilize the loudness, timbre and sounding pitch of the intended
note. Thompson treated the reed as a simple harmonic oscillator, driven below
its resonance by the pressure difference created across the reed. Nonlinearity
is introduced the the beating effect when the reed is driven at medium to high
pressures. Thompson added the impedance of the reed and the input impedance
of the air column in parallel in order to find the total input impedance of the air
column. The implications of tuning the reed resonance to add energy to higher
peaks of the clarinet’s spectrum are perhaps of interest to musicians. Thompson
mentioned that the reason clarinetists are able to play the second register of the instrument without opening a key is due to the fact that they are doubling the reed resonance to match a multiple of the playing frequency of interest. In this case, the lower regime is still preferred, but the extra energy in the higher harmonics thanks to the reed will help the musician make that jump.

There are considerable complications when studying the amount of volume control possible by the reed, turbulence surrounding the reed threshold, and the resistance of nonlinear wave propagation in musical instruments. The 1994 paper by Hirschberg et al. justified the work by Wilson and Beavers [126] concerning the aero-acoustics of the clarinet (volume flow control by the reed, the Bernoulli force on the reed, turbulence, vortex shedding and non-linear wave propagation) [73]. The vibration of the reed creates extra flow, reed-induced flow, into the instrument that must be modeled. Modeling of the reed-induced flow has been carried out by many, the first of which was likely Worman in the early 1970s [128]. Others include Dalmont et al. [43,45], Nederveen and Dalmont [100] and Coyle et al. [37]. Dalmont et al., Nederveen et al. and Coyle et al. all chose to model the extra flow entering the instrument as a type of length correction to the resonator, as the extra flow could be seen as a volume compliance, effectively lengthening the resonator and lowering the resulting playing frequency [37,45,100].

Using the correct reed resonance is of vital importance since the resonance and damping of a reed will absolutely alter the playing frequency in the instrument. The resonance frequency of the reed, as well as its damping are difficult to measure. Recent research by Chatziioannou et al. have focused on finding a way to model the reed parameters inversely [31]. Chatziioannou sought to estimate reed parameters in a lumped element fashion. The reed in his model used a constant stiffness and a reed-lay interaction was considered as a conditional repelling force. Taillard [120] measured the low and higher order mode shapes of reeds using holography techniques. Pinard [106] and Picart [107] used similar holography techniques to measure reed resonances, and observed that a fundamental flexural mode resonance frequency for (dry) reeds is around 1800 - 2200 Hz. However, Pinard states that “very good” reeds had a first flexural mode resonance frequency higher than this, closer to 2300 Hz. However, each of the methods used to find this reed resonance frequency was done without the use of a lower lip which would affect the exact value measured. Bouillon studied the stiffness of saxophone reeds using the reactive power approach [27].
and concluded that modeling the reed as a linear spring is sufficient. He used experimental methods to measure the stiffness of a reed indirectly while the reed was mounted on a mouthpiece and played by a musician. Boutillon’s measurements showed that the playing frequency was always slightly higher when a stiffer reed was used. He attributed this to the reactive power of the pipe, since the reactive power of the pipe is positive if driven below resonance, zero if driven just at an impedance peak (or at resonance) and, negative if driven above. Boutillon showed that any increase in frequency is in turn followed by a decrease in reactive power and if the stiffness of the reed increases, the reactive power decreases and the playing frequency will increase.

One of the more difficult reed parameters to measure is the vibrating surface area of the reed while the reed is vibrating. Many models allow for an input of the surface area of the reed, but the value generally chosen by convention and is not a validated number. Nederveen and Dalmont studied the effect of a small change in reed surface area by using two separate values and showing this effect on the resulting playing frequencies \[100\]. In one particular test, the authors showed that adjusting the reed vibrating surface area from 100 to 200 mm\(^2\) resulted in a decrease in playing frequency at the threshold of oscillation of about \(-30\) cents. They suggested a reasonable value for the reed surface area to be around 100 mm\(^2\), which is the value assumed for this dissertation. It would be useful in the future to have a more accurate estimation or measurement of this particular parameter, as mentioned in Dalmont et al. \[45\].

Recently there has been an interest in not only measuring, but also modeling the flow coming into the instrument and perhaps the actual blowing pressure exhibited by musicians. Fuks states that most of the measurements reported in literature were for sustained notes in a nonmusical context. Fuks asked musicians with various woodwind instruments to play a particular scale or short musical piece which helps measure the actual blowing pressures \[58\].

It is often interesting to measure this nonlinear characteristic curve for a given instrument in order to characterize the reed opening without needing to measure such a small opening. Dalmont et al. and Doc et al. use the same technique to measure the static nonlinear characteristic \[43,48\]. Both authors create enough damping on the reed (clarinet and saxophone respectively) in order that when the pressure is increased to just below threshold, the reed will not vibrate but that the
flow will begin to decrease so that the initial portion of the nonlinear characteristic can be measured. Figure 2.5 shows this nonlinear characteristic curve measurement for a saxophone. Since the closing pressure for the reed is an estimated 3 times the maximum pressure shown on this curve [30], there is no need to take a measurement past this pressure value as the authors will merely take this point in pressure where there is maximum flow and multiply by three to get the closing pressure that can be seen as represented by the small square in Fig. 2.6.

Nevertheless, a measurement of this full curve is possible and can be found in Dalmont et al. 2005 [43]. Figure 2.6 shows the nonlinear characteristic curve for a clarinet reed. The vertical shows the flow entering the instrument and the horizontal axis shows the pressure difference across the reed. The circle corresponds to the threshold of oscillation (in pressure), the square shows at what value of pressure the reed will begin to beat against the mouthpiece lay and the diamond represents the saturation threshold (the maximum amplitude of the oscillating regime). This figure shows that in practice (the thin dotted lines), measuring this characteristic curve completely is difficult and generally can only reach the reed closing pressure.
Figure 2.6: Nonlinear characteristic flow curve for a clarinet, from Dalmont et al. [43]. The circle corresponds to the threshold of oscillation (in pressure), the square shows at what value of pressure the reed will begin to beat against the mouthpiece lay and the diamond represents the saturation threshold (where the reed will cease to oscillate). Thick line is the model and the thin line is from experiment.

2.5 Experimentation methods

2.5.1 Impedance Measurements

The research explored in this dissertation is built on a need for accurate measurements of the pressure and volume velocity at the input of the clarinet. Knowing pressure and volume velocity provides the input impedance curve for the instrument; the impedance curve may be used to characterize the instrument. The input impedance curve gives the information necessary to understand the resonance frequencies of the instrument and is vital to ensure correct predictions of the final playing frequencies.

In 1987 Benade offered a survey of techniques and calibration methods for measuring impedance. Most of these techniques are now obsolete as measurement equipment has improved. More recent and relevant techniques are described in review papers by Dalmont published in 2001. The first [40] gave a review of the methods readily available and the second [41] addressed a new calibration method for input impedance sensors. Dalmont mentions that the easy measurement is that of the pressure at the input and is usually done with a microphone and that
impedance sensors are generally classified by the way they measure the volume velocity. Since the measurement of interest for musical acoustics is the input impedance, the pressure and volume velocity need to be measured at the same location. The choice of excitation signal at the input will depend on the signal-to-noise ratio, frequency resolution and time available for measurement. Examples include a slowly swept sine wave, white noise and chirps. Dalmont insisted that calibration is necessary for each measurement. Knowing the resonance frequencies of a closed open tube (no holes, flares, etc.), the calibration can be quite simple and the microphone and source parameters can be adjusted accordingly. Dalmont’s new calibration method is based on using two different closed tubes. The method is shown to be simple and accurate and helpful in reducing the number of calibration parameters.

In 2006 Dickens et al. wrote a review and comparison of different techniques to measure the input impedance of musical instruments [47]. In order to optimize the output signal for the input impedance measurement source, one should use white noise, a swept sine measurement takes much longer. Dickens demonstrated that a three microphone technique using three nonresonant calibrations yielded the best results but that for a smaller frequency range (2-3 octave range), two microphones would be sufficient.

### 2.5.2 Artificial Mouth Measurements

In order to, at least partially, rid measurements of player variability and subjectivity, many researchers turn to the use of an artificial mouth to “play” the clarinet. Most systems consist of an air source attached to a cavity that houses the mouthpiece, reed and ligature. The pressure driven air source pushes air into the mouthpiece. Given sufficient damping on the reed by an artificial lip, the reed will oscillate as necessary for the clarinet system to reach its oscillating regime and produce sound.

As early as 1941 an artificial blowing machine was being used to take measurements of wind instruments (notably the clarinet) by McGinnis et al. [90]. Worman and Thompson also used this tool to have experimental data to validate models in literature at the time [121,128]. More recently, researchers, such as Dalmont, have been using an artificial clarinet mouth [42–45]. Wolfe’s research team at University of New South Wales has created an artificial mouth and even built a robot player
to pilot the entire system \cite{127}. Ferrand et al. published a paper in 2010 which introduces a feed-back system to pilot the pressure input to the system instead of hand regulating the increase and decreases usually used to measure playing characteristics. The artificial mouth described in Ferrand et al. is the one used for the work in this dissertation. Finally, Almeida et al. used an artificial mouth to create the experimental (subjective) tuning and sound pressure level maps for a clarinet \cite{10}.

### 2.5.3 Instrumented Mouthpiece Measurements

The only documented use of an instrumented mouthpiece is by Munoz et al. for their study of reed behavior comparisons in real and artificial playing conditions \cite{98}. Munoz uses a CAD modeled, and then 3D printed (in Nylon) clarinet mouthpiece equipped with sensors to measure the playing parameters for the instrument. His mouthpiece included phototransistors inside the mouthpiece to measure the reed displacement, and microphones were used to measure the static pressure in the mouth and the dynamic, oscillating pressure inside the mouthpiece. The instrumented mouthpiece used by Munoz resembles the instrumented mouthpiece which was used to take measurements for this dissertation.

### 2.6 Numerical Simulations

As technology and numerical capabilities progress there have been improvements to the basic research and inquiries made by the initial researchers in the field (Benade, Backus, etc.). For example, in the sixties Benade was making his spectrum graphs by hand and now we have the numerical capability to model an entire system as well as simulate the sounds that the instrument creates. Guillemain has demonstrated the power of the numerical simulations by recreating a clarinet sound spectrum in real-time \cite{69}. His numerical model is based upon the characteristic equations found in \cite{30} which describe the nonlinear coupling between the generator and resonator through the $u(p)$ Bernoulli flow equation. The model describes each important portion of the clarinet resonator, generator and generator numerically including the losses and reed functioning. The numerical model can take the input impedance information (giving resonance frequencies and modal parameters) of a
given clarinet-like instrument and solve the characteristic equations numerically in order to produce an external pressure or resulting sound through synthesis. The numerical model by Guillemain et al. can be broadened to include conical bored instruments as well. This dissertation makes use of this numerical model in order to validate the use of the even more computationally efficient analytical formulas.

Several authors have since cited Guillemain’s numerical model in their own research. In 2008, Sterling et al. used the numerical code to synthesize solo clarinet sounds [117]. They took digital recordings and used numerical methods to extract playing parameters from these recordings for use in the synthesis code to create an even more realistic resulting sound. In 2009, Bilbao began using this numerical model to create direct simulations of reed woodwind instruments [25]. Bilbao uses a digital waveguide model to represent the resonator of the clarinet as well as the impedance-type descriptions of the clarinet system as a whole, as did Guillemain. Silva et al. used this model to focus on understanding the physical interaction of the reed and acoustic resonator numerically [115].

Mignot et al. used the digital waveguide approach to study numerical sound propagation in tubes [93]. The authors reference the work by Guillemain but do not expressly state that the research is meant for clarinets alone, but for any acoustic resonator based on tubes. Their time-domain digital model can take into account discontinuities in the tube (in radius, slope and curvature). They applied the model to propagation in a trombone compared their theory with measurements. Unfortunately, Mignot et al. only used linear and static models. Nevertheless, to include the nonlinearity of real musical instruments a modular numerical model such as theirs would make the inclusion of these elements easily integrateable (tone holes, valves, lips, reeds, etc.).

In 2012, Karkar also studied the clarinet numerically in order to better model and predict the oscillation threshold of a clarinet [78]. More recently, Bergeot has used the numerical models similar to those formulated in Guillemain, Karkar, and Silva to predict the oscillation and extinction thresholds [24]. As computation power increases with technology, more powerful tools become available to study the physics of these complicated instruments.
2.7 Map Making and Instrument Playability

A newer idea, and one that continues to inspire the work included in this dissertation, is the idea of creating maps that show the tuning, sound production, or dynamic range of any given instrument. The first publication concerning instrument cartography was written in 1978 by Schelleg et al. [112]. This paper involved showing, in a type of three-dimensional map form, effects of different parameters on a string instrument’s ability to create sound. Their parameters were force applied to the string, bow position, and velocity of bow movement.

Almeida, et al. have offered a look into the creation of these tuning maps from an experimental standpoint for a clarinet [9], [10]. Almeida et al. showed that through the use of an artificial mouth, experimental tuning and sound pressure level maps could be created. Figure 2.7 shows the schematic used in their work, with increasing blowing pressure on the horizontal and increasing lip force on the vertical axis. Within certain combinations of these two parameters (P and F), the clarinet will play more or less in tune (to the equal tempered scale) and more or less loudly. There are regions where the clarinet will not be able to play a pitched sound (when the oscillation threshold in pressure has not been met or the lip force is not sufficient). There are also small regions where, for that fingering, the higher regimes will be reached for those combinations of playing parameters.

![Figure 2.7: Fig. 1 from [10] showing the mapping capabilities from experiments of Almeida et al.](image)

Figure 2.8 shows a map of the playing frequency for a clarinet with increasing blowing pressure for different discrete choices of lip force (reed opening). Higher
lip force creates a smaller reed opening. This figure shows that for the highest
values of lip force, the range of pressures possible to play that note decreases. It
also shows that as the lip force increases (reed opening decreases), the note will be
played more and more sharp, i.e., at a higher frequency.

![Figure 2.8](image)

*Figure 2.8: Playing frequency (Hz) versus increasing blowing pressure (kPa) on the
horizontal axis for discrete values of lip force (N). Figure 3a from [10].*

Finally, Fig. 2.9 shows one of Almeida’s experimental tuning maps for Note 16
on the clarinet, an open G. The horizontal axis is increasing blowing pressure and
the vertical axis is the lip force. The shaded regions (greyscale color bar) represents
the sound pressure level (in dB) for that particular combination of lip force and
blowing pressure. The bottom right region shows that, as expected, a note will
play louder when increasing the blowing pressure. However, if the intention is to
play in tune, the black lines can be followed on this plot. If the player wants to
maintain the playing frequency of about 344 Hz, for example, there would need to
be a nearly linear decrease in lip force as blowing pressure increases.

Almeida et al. made the following conclusions about such maps: the tuning
lines (lines of equal frequency depending on P and F) will have a negative slope.
If the lip force is constant in the lower range of F for weak reeds, the frequency
decreases first when the mouth pressure increases and then will increase until the
extinction (the dotted line on the rightmost portion of the plot).

Missoum et al. studied yet another way to create playability maps that outline
boundaries of sound or no sound based on the input parameters chosen [94]. The
Figure 2.9: Figure 4a from [10]. Horizontal axis: increasing blowing pressure, vertical axis: lip force. Shaded regions (greyscale color bar) represents the sound pressure level (in dB) for that particular combination of lip force and blowing pressure.

Map boundaries are obtained through the use of a support vector machine (SVM) classifier and an adaptive sampling scheme. Such maps show promise in advancing the understanding of parameter interaction in numerical models in order to optimize instrument design.

These experimental maps provide another way for instrumentalists and instrument makers to quickly and easily analyze and assess their clarinets. This is a goal of the present research as well, to create similar maps, but in a numerical and analytical way so as to be useful for all clarinet models and configurations.

As a part of this current research, Coyle et al. began to study simplified analytical formulas that could be used to create such tuning maps [36]. These analytical formulas create a computationally quick and accurate way to represent the tuning tendencies of a given clarinet. This approach to maps will be a significant focus of this dissertation.

### 2.8 Other related topics

The clarinet design, as a cohesive instrument, has been relatively unchanged for many years but there are numerous incremental advancements being made due to research and better understanding of the physical model behind the instrument
and each of its pieces and parts. Nevertheless, there is still much to explore and discover.

2.8.1 Instrument Making and Modifications

The research concerning the functioning and modeling of the clarinet has sparked several ideas for modifications, some more feasible than others. Most of the modifications to the fabrication of clarinets are made in small increments. There are often changes to the undercutting of tone holes, new barrel designs or reed offerings. Some new research concerning comparably revolutionary new designs for clarinets, or ideas of modifications can be found in [46, 82, 103].

The effect of the pads, holes and keys on the clarinet is of great interest since modification of these smaller aspects would not alter the musicians comfort with the instrument but could possibly greatly improve intonation and ergonomics. The paper entitled, *The Physics of a New Clarinet Design* [82], written by Benade and Keefe but published after Benade’s passing, discusses several factors of the clarinet which influence the tuning of the instrument and how these aspects can be altered to help the homogeneity of the entire range. Benade suggested many alterations to tone hole configurations as well as some ideas for cutting the tone holes in a different way. More recently, Eveno studied the effect of resonators on pads of saxophones [50]. Although the saxophone pads are different than clarinet pads, this should be an important consideration as changing the pad’s surface area and material can significantly change the resulting playing frequency or timbral characteristics for a wind instrument.

In 2015 Guilloteau et al. studied the power transmitted by open holes of certain geometries in an attempt to seek an optimized method of choosing each hole chimney height and diameter [70]. A further consideration is the placement and size of the register hole at the top of the instrument. A new design for the clarinet, a logical clarinet [103], shows that there could be a different placement for the tone holes and perhaps a different configuration of the bell. As the physical modeling becomes more refined for the clarinet, the more important the details of the instrument become. Research is being performed concerning the tone holes of the instrument and their interaction with the playing frequency, and perhaps even the functioning of the instrument [32, 78, 100].
2.8.2 Vocal Tract Influence

The impact of the vocal tract is a difficult aspect to study when it is coupled to a musical instrument. Due to advances in numerical simulations and three-dimensional modeling from scans there is work done on this topic \[32\], \[33\]. Apart from its obvious function to regulate the blowing pressure coming from the lungs, the vocal tract can actually act as another resonator in series with the clarinet’s body \[20\]. Recent research stresses the importance of including the vocal tract in the modeling of clarinet functioning \[56\], \[111\], \[68\] in order to shift the instrument resonances to fit the playing style whereas other prominent researchers once believed this not to be the case \[11\]. The research in this dissertation does not include this effect, but interested readers are referred to the Ph.D. dissertation of Fritz \[56\] for a more detailed bibliography.

2.8.3 Player Perception

There is another aspect not always directly tied to acoustics, but that is vastly important and drives all musical acoustics research, this being player-instrument interaction and subjective perception. This is heavily tied to the pure acoustical research since the interaction between the instrumentalists playing style (fingers, force on the reed, vocal tract volume changes, etc.) is important. But it is also the instrumentalist’s perception of an instrument and its sound which leads to changes in the playing style or their preference of instrument to begin with. This is, perhaps, more closely linked to psychological or physiological acoustics. The player’s perception of a good instrument is the underlying motivation of this research but it is very difficult to measure \[15\].

In 1968 Backus devised an experimental set-up to measure the resonances of the instrument and all of its complicated components, and showed multiple resonance plots of the resonance peaks and amplitudes for the higher peaks beyond the fundamental. Backus mentioned in that he hoped to compare the resonance plots to some subjective judgments of clarinets by musicians so that the resonance curves could serve as a means of objective evaluation. Unfortunately, he reported that the results from this experiment were disappointing since the resonance curves for the clarinets tested were quite similar. However, the seventh harmonic showed the greatest difference and this point was worth further investigation \[13\].
Fritz et al. are working on the perception of a good violin, searching for an exhaustive database of quality-marker vocabulary as perceived by professional musicians [57]. As noted in their work, the subjective aspect of judging musical instrument quality is difficult to quantify but their work aims to bring a quantifiable aspect to these markers through having musicians describe sounds with this database and therefore be able to match descriptors with sound through acoustical analysis.

In a 2016 paper, Gazengel et al. began to search for a way to link the objective characteristics and subjective opinions of $B\flat$ clarinet reeds [61]. Due to variability in the mechanized manufacturing process, there are many discrepancies per box of 10 reeds bought by clarinetists. The authors conclude that the static and dynamic compliance are well correlated with the musician description of a reed that is “easy to play”. However, there are not yet good correlations between acoustical measurements of reeds and musician defined descriptors such as roundness or brightness.
3.1 What is Impedance?

“Acoustic impedance is an important quantity in musical acoustics because acoustic impedance has the advantage of being a physical property of the instrument alone – it can be measured (or calculated) for the instrument without a player. It is a spectrum, because it has different values for different frequencies – one can think of it as the acoustic response of the instrument for all possible frequencies. For instance, we measure it at the embouchure of an instrument because it tells us a lot about the way the player’s lips, reed or air jet from the mouth will interact with the instrument itself. So it tells us about the acoustic performance of the instrument, in an objective way that is independent of who might play it, and it allows us to compare subtle differences between instruments." [127]

3.1.1 Standard Definitions for Impedance

There are many different types of impedance depending on the system being studied. If the system under investigation is a mechanical system, then mechanical impedance would be the quantity of interest. If the system is acoustical, with an input acoustic pressure, then an acoustical impedance would be useful.

Impedance can be defined as a particular system’s frequency response function
and gives a relationship between an input and output (for acoustics this is often a relationship between pressure and velocity). The ANSI standard definition states that impedance is, at a specified frequency, the quotient of a dynamic acoustic variable (e.g., force, sound pressure) by a kinematic acoustic variable (e.g., vibration velocity, particle velocity) \[8\]. The units of this value will depend on the system.

### 3.1.1.1 Acoustic impedance

The acoustic impedance is the complex ratio of acoustic pressure \(p\) (oscillating pressure) to the resulting volume velocity \(U\) in the direction of the wave propagation and has units of \(\text{Pa} \cdot \text{s}^3\) \[8\],

\[
Z_{\text{acs}} = \frac{p}{U}. \tag{3.1}
\]

Another important measure, concerning acoustic impedance is the specific acoustic impedance and similarly the characteristic impedance. The specific acoustic impedance is the ratio of the acoustic pressure to the resulting particle velocity in \(\text{Pa} \cdot \text{s}/\text{m}\):

\[
Z_{sp} = \frac{p}{u}. \tag{3.2}
\]

This quantity removes the effect of the system geometry and focuses on the medium. The characteristic acoustic impedance of the medium has the same units as the specific acoustic impedance, the characteristic acoustic impedance of the medium is:

\[
z_0 = \rho c. \tag{3.3}
\]

The characteristic impedance for air at 20°C is about 413.3 \(\text{Pa} \cdot \text{m}^{-1}\text{s}\). There is also a different characteristic impedance, the characteristic acoustic impedance is generally said to be:

\[
Z_0 = \frac{\rho c}{S}, \tag{3.4}
\]

which is here actually the characteristic impedance of a pipe with cross-sectional area \(S\).
### 3.1.1.2 Mechanical impedance

Mechanical impedance is the ratio of the force $F$ at a point to the resulting velocity $u$ in the direction of the force at that same point,

$$Z_m = \frac{F}{u}. \quad (3.5)$$

The units for the mechanical impedance are $\frac{N \cdot s}{m}$.

### 3.1.1.3 Impedance is a complex quantity

Impedance is, in general, a complex quantity:

$$Z = R + jX. \quad (3.6)$$

The imaginary part of the impedance ($X$, the reactance) describes the energy storage by the elastic and inertial properties found in the system. The real part ($R$, resistance) describes the energy loss in the system. In musical acoustics it is good to have some loss in the system since this is usually how the sound exits the instrument.

### 3.1.1.4 Going between systems

It is useful to state relationships that will help transfer between systems. Considering the different quantities discussed for impedance, acoustic impedance, specific acoustic impedance and mechanical impedance, these values are related to one another by a factor of $S$, the cross-sectional area,

$$Z = \frac{P}{U} = \frac{P}{u \cdot S} = \frac{F/S}{u}, \quad (3.7)$$

and therefore,

$$Z_{acs} = \frac{Z_{sp}}{S} = \frac{Z_m}{S^2}. \quad (3.8)$$
3.2 The Simple Harmonic Oscillator (SHO)

3.2.1 A Mechanical System

A simple harmonic oscillator is the simplest mechanical system of interest in acoustics and vibrations. The system is made of a mass $m$ (inertial element), spring $s$ (an elastic element) and a damper $R$ (the resistive element). Figure 3.1 shows a simple harmonic oscillator being driven by a force $F$ and undergoing a displacement $x$.

![Figure 3.1: Diagram of a simple harmonic oscillator](image)

The equation of motion for the system represented in Fig. 3.1 is:

$$m\ddot{x} + R\dot{x} + sx = F(t), \quad (3.9)$$

where $F(t)$ is a periodic forcing function as a function of time. Written in a more familiar way,

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2x = \frac{F(t)}{m}, \quad (3.10)$$

where $\beta = \frac{R}{2m}$ is the damping rate in rad/s and $\omega_0 = \sqrt{\frac{s}{m}}$ is the undamped natural frequency (rad/s). There will also be an important frequency at which the damping is taken into account called the damped natural frequency:

$$\omega_d = \sqrt{\omega_0^2 - \beta^2} \quad (3.11)$$

$$= \sqrt{\frac{s}{m} - \frac{R^2}{4m^2}}. \quad (3.12)$$

The damped natural frequency is important for all damped harmonic oscillations including those in musical acoustics such as the clarinet reed where the system behavior relies heavily on the damping in the system. Compared to the natural
frequency of the system, the frequency at which the system would vibrate forever
until stopped without the inclusion of damping, the damped natural frequency
represents the effect of the damper.

3.2.2 Resonance: displacement, velocity, acceleration

Resonance is defined as a phenomena which exists for linear systems in harmonic
forced oscillation when any change in the excitation frequency results in a decrease
in the response of the system. Resonance occurs at the frequency where there is
maximum response of a system to a given input. There is often confusion when
describing the resonance of a system since technically there can be a resonance
for any variable in the system. It is important to be careful when considering a
particular resonance since it depends on what quantity is of interest. A system can
have a number of different resonances: natural resonance, damped resonance, a
resonance in displacement, velocity or even acceleration [8].

Figure 3.2 shows the amplitude response versus frequency for the displacement
(black line), velocity (blue line) and acceleration (red line) of a simple harmonic
oscillator. Each of these variables has an associated resonance frequency, which are
not all three the same and are not necessarily $\omega_0$. Notice that at low frequencies
and high frequencies (far from resonance) each variable reacts differently. The
steady-state displacement of the mass in a system, as defined in Fig. 3.1 and Eq.
3.10 is,

$$|x| = \frac{F/m}{\sqrt{(2\beta\omega)^2 + (\omega^2 - \omega_0^2)^2}},$$

(3.13)

and will reach a maximum value when $x'(\omega) = 0$, at $\omega = \sqrt{\omega_0^2 - 2\beta^2}$. At low
frequencies the displacement $x$ in Fig. 3.2 tends to an amplitude value of $F/s$
(static displacement) whereas at high frequencies, the displacement will be zero.
The velocity in the system will be

$$|u| = \omega \frac{F/m}{\sqrt{(2\beta\omega)^2 + (\omega^2 - \omega_0^2)^2}},$$

(3.14)

and will reach a maximum value when $u'(\omega) = 0$, at $\omega = \omega_0$. For the velocity, at
low frequencies it tends to zero (since static displacement will create no oscillation)
and will approach an asymptotic value at high frequencies. Most texts refer to the
resonance frequency, $\omega_0$ as the velocity resonance frequency. The acceleration of the system is

$$|a| = \omega^2 \frac{F}{m} \sqrt{(2\beta \omega)^2 + (\omega^2 - \omega_0^2)^2},$$  

(3.15)

and will reach a maximum value when $a'(\omega) = 0$, at $\omega = \sqrt{\omega_0^2 - 2\beta^2}$. The acceleration, in Fig. 3.2, the low frequency value will be zero and high frequencies will actually tend to $F/m$. A close look at Fig. 3.2 will show that the resonance frequency $\omega_0$ for each of these variables is slightly different in the case represented, which reinforces the need to be extremely clear when discussing resonance of any system.

### 3.2.3 Impedance of a Simple Harmonic Oscillator

In order to utilize the idea of impedance, one must consider a force at the input (force on the mass) of the mechanical system (the simple harmonic oscillator in Fig. 3.1). In this way there will be a resulting displacement, $x$, and therefore system velocity $u$, and a resulting mechanical impedance can then be calculated (or measured). Understanding how the mechanical input impedance works for a simple harmonic oscillator will aid in building a model for more complicated systems, such
as a clarinet, in future sections. The impedance of a simple harmonic oscillator is given by

\[ Z(\omega) = R + j \left( \omega m - \frac{s}{\omega} \right). \]  

(3.16)

In Eq. (3.16), the real part of the impedance is represented by \( R \), the damping in the mass-spring system. The mass element \( m \) and stiffness (spring) element \( s \) provide the energy storage in the system, represented by the imaginary part of the system impedance. Figure 3.3 shows the behavior of the impedance for a simple harmonic oscillator. The red curve represents the magnitude of the impedance, the brown curve is the real part (the resistive element, not frequency dependent) and the green curve is the imaginary part of Eq. (3.16).

![Figure 3.3: Particular representations of the amplitude of the impedance of a simple harmonic oscillator as a function of frequency: Red - magnitude of \( Z \), green - imaginary part of \( Z \) (the reactance), brown - real part of \( Z \) (the resistance) (\( \beta = 0.1, s = 0.1, m = 0.1 \)).](image)

Important observations can be made from Fig. 3.3. Resonance indicates a large response to a unit input. For \( Z = F/u \), a maximum velocity \( u \), and given force \( F \), the impedance \( Z \) will tend to \( R \). This means that a resonance can be found at the minimum of the impedance. This coincides with the frequency at which the imaginary part of the impedance is zero, where the impedance is purely real. (Note: This system behaves similarly to the open-open pipe later in this chapter where the resonances are occurring where the impedance is minimized. See Fig. 3.7a - 3.7c.)
3.2.4 Impedance of a different type of oscillator

A similar system which behaves differently is an oscillator that is forced at the base of the system as represented in Fig. 3.4.

![Diagram of an oscillator with mass m, stiffness s, and a resistive element R which is forced at the base of the system in order to achieve a displacement x.](image)

The impedance associated with this type of system is,

\[
Z(\omega) = \frac{2\beta \omega^2}{(2\beta \omega)^2 + (\omega^2 - \frac{s}{m})^2} + \frac{j \left(4\beta^2 \omega^3 - \frac{\omega^2 s}{m} + \omega \left(\frac{s}{m}\right)^2\right)}{(2\beta \omega)^2 + (\omega^2 - \frac{s}{m})^2}.
\]  

(3.17)

![Normalized Frequency \(\Omega/\Omega_0\), Re\(\{Z\}\), Im\(\{Z\}\)](image)

**Figure 3.5:** Particular representations of the amplitude of the impedance of a base driven simple harmonic oscillator as a function of frequency: Red - magnitude of \(Z\), green - imaginary part of \(Z\) (the reactance), brown - real part of \(Z\) (the resistance) \((\beta = 0.1, s = 0.1, m = 0.1)\)
In Fig. 3.5 the minimum of the imaginary part of the impedance occurs around \( \omega = 1 \) and coincides with the peak of the magnitude and real part of the impedance. For this oscillating system, resonance occurs at the peak, or maximum of the magnitude of the impedance.

Important observations can be made from this figure, as in Fig. 3.3. From the definition of resonance that there is a minimum response to a given force at the input. For \( Z = F/u \), a minimum velocity \( u \) and maximum force \( F \), \( Z \) will be a maximum. This means that a resonance can be found at the maximum of the impedance. In this situation, this coincides with the point where the imaginary part of the impedance is zero (as was the case in Fig. 3.3), where the impedance is purely real. (Note: This system behaves similarly to the closed-open pipe later in this chapter where the resonances are occurring where the impedance is a maximum. See Fig. 3.8a - 3.8c).

### 3.2.5 Operating regimes for the SHO

The displacement of a simple harmonic oscillator is dependent on the input force \( F \), the mass \( m \) and stiffness \( s \) in the system, the damping present in the system \( \beta \), and the driving frequency \( \omega \). The equation for displacement \( x \) of the system shown in Fig. 3.1 can be written as stated before in Eq. (3.13),

\[
|x| = \frac{F/m}{\sqrt{(2\beta \omega)^2 + (\omega^2 - \omega_0^2)^2}} = \frac{F/m}{\sqrt{(2R \omega)^2 + (\omega^2 - \frac{s}{m})^2}}.
\]

Having the displacement written in this way can help describe the behavior of the system in different frequency regions as is represented in Fig. 3.6.

- Low frequency: If the system is being driven at a low value of \( \omega \) (below resonance, where \( \omega < \omega_0 \)) the system is said to be stiffness dominated since changing the mass or damping ends up having little effect on the displacement response of the system in this frequency range, but changing the stiffness has a large effect.

- At/around resonance \( \omega_0 \): The system is resistance (damping) dominated since slight changes in the damping factor \( \beta \) will cause large changes in displacement in the frequency region surrounding \( \omega_0 \).
Figure 3.6: The displacement response vs. frequency for a simple harmonic oscillator. The regimes are labeled as follows: low frequency, the stiffness dominates the oscillation, around resonance the damping dominates and at high frequencies, the mass dominates.

- High frequency: In this region (above resonance, where \( \omega > \omega_0 \)) the displacement response drops to zero (shown in the high frequency region of the plot for the displacement, Fig. 3.6). This region is said to be mass dominated since in this region changes in mass have the greatest effect.

A few notes about phase in SHO: when the system is being driven below the system's resonance frequency, the system displacement is in phase with the force. When driven above resonance the displacement is 180 degrees out of phase with the driving force and when the system is driven at resonance the displacement lags the force by 90 degrees. Secondly, in regards to the frequency response to different values of stiffness, mass, and damping: the damped natural frequency of the system can demonstrate what will happen if certain values vary in the system. The damped natural frequency \( \omega_d \), as previously presented in this section, differs from the natural frequency of a system \( \omega_0 \) and is a natural frequency of the damped oscillator without a driving force.

\[
\omega_d = \sqrt{\omega_0^2 - \beta^2} \tag{3.18}
\]

\[
= \sqrt{\frac{s}{m} - \frac{R^2}{4m^2}} \tag{3.19}
\]
If in Eq. (3.18) all other variables are held constant and stiffness is increased, the resonance (max displacement) frequency increases. If only the mass increases, the resonance frequency decreases. If only the damping is increased, the resonance frequency decreases (and eventually the peak is less sharp as well).

3.3 Impedance of a Cylindrical Tube: calculations and implications

The clarinet is a much more complicated system than a simple harmonic oscillator. However, the theory behind calculating the input impedance of a SHO can help in determining the input impedance for a clarinet. As a first approximation, the clarinet is simply a pipe with a cylindrical cross section and a particular length. Each of these things will be included in the input impedance value. The cylinder will be subject to some pressure value at the input and a resulting particle velocity could be measured. In practice, the acoustic impedance is measured, and not the specific acoustic impedance since we have a cross-sectional area $S$.

3.3.1 Boundary Conditions (free oscillations)

A convenient place to begin is with two simple but useful configurations in musical acoustics, considering a pipe open at both ends and a pipe that is closed at one end and open at the other. This section will explore the mode shapes (pressure or volume velocity) possible inside pipes with these conditions imposed on the end boundaries.

3.3.1.1 Standing waves in an open-open pipe

An open-open pipe could be used as a simple model of the resonator of a flute. In order to determine which mode shapes are allowed in a pipe that is open at both ends there must first be an equation describing the particular acoustic variable of interest. Within the pipe, in the sinusoidal regime, there will be a pressure wave traveling in the positive $x$ direction and a wave that will travel in the negative $x$ direction:

$$p(x, t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}. \quad (3.20)$$
Equation (3.20) shows that there are two traveling pressure waves (traveling in opposite directions), each with a particular amplitude ($A$ and $B$). At $x = 0$ in the pipe, the open end, the pressure will be zero since this open end acts as a pressure-release condition. If the boundary condition at $x = 0$ is imposed then this equation becomes:

$$p(0, t) = Ae^{j\omega t} + Be^{j\omega t} \rightarrow 0 = Ae^{j\omega t} + Be^{j\omega t} \rightarrow B = -A.$$ (3.21)

This shows that at an open end, the pressure wave will invert upon reflection. Knowing that the amplitudes are related in this way, Eq. (3.20) can be rewritten as:

$$p(x, t) = -2jAe^{j\omega t} \sin kx.$$ (3.22)

This equation shows that after imposing this boundary condition, the pressure waves are no longer traveling, but standing waves. A standing wave being a wave that is dependent on time and space, the superposition of two traveling waves of same amplitude and opposite direction. In order to know what mode shapes are allowed in this configuration, the second boundary condition at $x = L$ must be taken into account. Since there is another open end at $x = L$:

$$p(L, t) = 0 = -2jAe^{j\omega t} \sin kL$$ (3.23)

$$0 = \sin kL,$$ (3.24)

which will be true if $kL = n\pi$ for all $n$. This leads to an expression for the natural or allowed resonance frequencies for a cylinder of this open-open configuration:

$$\omega_n = ck_n \quad \rightarrow \quad f_n = \omega_n/(2\pi) \quad \rightarrow \quad f_n = \frac{ck_n}{2\pi} = \frac{cn}{2L}.$$ (3.25)

Figure 3.7a shows the first pressure mode shape for an open-open cylinder ($n = 1$). Figure 3.7b shows the second pressure mode shape, and Fig. 3.7c shows the fifth pressure mode shape for this configuration. Notice that the pressure must always be zero at the open ends and that all integer multiple half wavelengths are allowed. This also means that at the open ends the volume velocity $U$ will be a maximum based on Euler's equation [85]. Considering Eq. (3.1), the equation for acoustic impedance at the input, there will be a maximum volume velocity and
minimum acoustic pressure at the input (zero), making \( Z \) a minimum (zero). This leads to the idea that an open-open cylindrical pipe, such as a simple model of a flute, will operate at the minima of the impedance spectrum which is analogous to the SHO represented in Figs. 3.1 and 3.3.

### 3.3.1.2 Standing waves in a closed-open pipe

The formulation of the input impedance for a pipe that is closed at one end \( (x = 0) \) and open at the other is similar to that of the previous section. Beginning with the expression for a right and left going pressure wave:

\[
p(x, t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}.
\]

(3.26)

Imposing a rigid boundary, or closed end, to a cylinder will cause the pressure to be a maximum, this also means that the derivative of the pressure will be zero (first derivative rule in calculus). At \( x = 0 \) (the closed end) the pressure is going to be a maximum and the boundary condition results in:

\[
\left. \frac{\partial p}{\partial x} \right|_{x=0} = -Ajke^{j\omega t} + Bkje^{j\omega t} \rightarrow 0 = -Ae^{j\omega t} + Be^{j\omega t} \rightarrow A = B.
\]

(3.27)

This means that at the closed end the pressure will not invert. Knowing that the amplitudes are related in this way, Eq. (3.26) can be rewritten as:

\[
p(x, t) = 2Ae^{j\omega t} \cos kx.
\]

(3.28)

This equation shows that after imposing this boundary condition, the pressure waves are no longer traveling, but standing waves. In order to know what mode
shapes are allowed in this configuration, the second boundary condition at $x = L$ must be taken into account. Since there is an open end at $x = L$ the pressure must be zero:

$$p(L, t) = 0 = 2Ae^{j\omega t} \cos kL$$  \hspace{1cm} (3.29)

$$0 = \cos kL,$$  \hspace{1cm} (3.30)

this will be true if $kL = m\pi/2$ for odd $m$. This leads us to an expression for the resonance frequencies for a cylinder of this open-closed configuration,

$$\omega_m = c \frac{k_m}{k_m} \quad \rightarrow \quad f_m = \frac{\omega_m}{(2\pi)}$$

$$= \frac{c k_m}{2\pi} \quad \rightarrow \quad \frac{c m}{4L} = \frac{(2n + 1) \cdot c}{4L},$$  \hspace{1cm} (3.31)

where $m$ is a positive odd integer and $n$ is any positive integer.

Figure 3.8a shows the first pressure mode shape for an closed-open cylinder (n=1). Figure 3.8b shows the second pressure mode shape, and Fig. 3.8c shows the fifth pressure mode shape for this configuration. Notice that the pressure must always be zero at the open end and pressure will be a maximum at the closed end. With these boundary conditions only odd integer multiple quarter wavelengths are allowed. Equation (3.1), the equation for acoustic impedance at the input (the closed end), results in a minimum volume velocity and maximum acoustic pressure at the input, making $Z$ a maximum (infinite). This leads to the idea that an closed-open cylindrical pipe driven at the closed end, such as a simple model of a clarinet, will operate at the impedance maxima. This is analogous to the oscillator represented in Figs. 3.4 and 3.5. The system operates at the maxima of the impedance spectrum.
3.4 Input Impedance (or the impedance looking into a cylinder, forced oscillations)

When attempting to classify a system, such as a cylinder, it is often useful and interesting to determine its input impedance. In reality instruments are not passive resonators like the cylinders described in the previous section, but complicated coupled systems. Input impedance is a measure of the ratio of acoustic force at the input (acoustic pressure) to the resulting particle velocity (or volume velocity also at the input) and is a spectrum of values representing the response of the system for a range of frequencies.

3.4.1 Assumption of Plane Waves

An important assumption, in order for the following math to work out, is that only plane waves will propagate down the cylinder. This means that the only frequencies that will be allowed to propagate must be below the cylinder’s cutoff frequency for the cavity mode above the plane wave mode. This frequency for a cylinder will be based on considering the pressure mode shapes in a cylindrical wave guide [85] and is:

\[ \omega = \frac{\beta_{m,n}c}{a} \rightarrow f_{m,n} = \frac{\beta_{m,n}c}{2\pi a} \]  (3.33)

where \( a \) is the cylinder radius, and \( \beta_{m,n} \) is the \( nth \) root of the \( J'_m \) Bessel function of the 1st kind [2]. The plane wave cutoff frequency (more appropriately considered the cut-on frequency, the frequency at which this particular mode will propagate) is 0 Hz therefore the plane wave will always propagate. However, in order to insure
that only this mode will propagate, it is necessary to find the next highest mode’s cut-on frequency. For a cylindrical pipe this mode is the (1,0) mode (the mode with one nodal diameter line and zero nodal circles). The first zero of the \( J_1 \) is 1.84. Therefore the clarinet must operate below \( f_{1,0} \approx 6,000 \text{ Hz} \) in order to remain in the plane wave regime. The fundamental frequency for the lowest note fingered on the clarinet is about 147 Hz and the highest is around 2,093 Hz so this qualification is met.

Another way to consider the idea of a plane wave approximation is to consider the small diameter (or small radius \( a \)), low frequency or small \( ka \) approximations (where \( k \) is the wave number). There is a lot of math involved in the following sections, much of which is left out except the mention that \( ka \) will be small (so that terms with values like \((ka)^2\) will be very small). For a clarinet with diameter \( \approx 15 \text{ mm} \) the range of \( ka \) values (which correspond to the range of possible playing frequencies) is as follows: \( 0.02 \leq ka \leq 0.27 \). Both of the extremes are less than 1, therefore this assumption is justified.

### 3.4.2 Transmission Line Equation

Consider a system composed of a cylinder with cross sectional area \( S \) and a given length \( L \), driven at one end \( x = 0 \) by a mechanical piston with a mechanical impedance termination \( Z_{mL} \) at the other end, \( x = L \).

Beginning with the pressure in the cylinder, as before:

\[
p(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}.
\]

When finding the allowed mode shapes, the actual, numerical values of \( A \) and \( B \) will come from initial conditions in the system. According to Euler’s equation for conservation of momentum, taking the derivative of this equation will result in the particle velocity \( u(x,t) \):

\[
u(x,t) = \frac{1}{\rho_0 c} \left[ Ae^{j(\omega t - kx)} - Bke^{j(\omega t + kx)} \right]
\]

The termination impedance \( Z_{mL} \) is determined by the pressure at this point and
particle velocity at $x = L$. Therefore the equation for $Z_{mL}$ will become,

$$Z_{mL} = \rho_0 c S \left[ \frac{A e^{-jkL} + B e^{jkL}}{A k e^{-jkL} - B e^{jkL}} \right]. \quad (3.35)$$

Two important quantities result from this calculation: the power reflection coefficient and the power transmission coefficient. The first will give a value, between zero and one, describing the fraction of the initial power that will get reflected at the boundary with an impedance of $Z_{mL}$. The second, the transmission coefficient, will characterize how much power will be transmitted across this boundary, beyond the termination. The ratio of the reflected amplitude to the initial amplitude will be given by the ratio of amplitude $B$ to amplitude $A$,

$$\frac{B}{A} = e^{-2jkL} \left( \frac{Z_{mL} - \rho_0 c S}{Z_{mL} + \rho_0 c S} \right). \quad (3.36)$$

In order to find the reflected power ratio, the reflected amplitude ratio is squared (since power is proportional to pressure squared):

$$R_{II} = \left| \frac{B}{A} \right|^2 = \left| \frac{Z_{mL} - \rho_0 c S}{Z_{mL} + \rho_0 c S} \right|^2. \quad (3.37)$$

This equation demonstrates that there is no reflection if $Z_{mL} = \rho_0 c S$. This basically says that there is no reflection if the pipe is infinite, if $Z_{mL} = Z_{pipe}$. Otherwise the reflection could be complete, $R_{II} = 1$, if $Z_{mL} = \infty, 0$. If $Z_{mL}$ is some other value (radiation impedance from an open end, for example) then some sound is reflected back into the pipe and some sound is transmitted beyond the end.

The value of greater interest is the input impedance looking into the pipe at $x = 0$,

$$Z_{in} = \frac{p(0,t)S}{u(0,t)} = \rho_0 c S \left[ \frac{A + B}{A - B} \right]. \quad (3.38)$$

Equations (3.38) and (3.35) can now be combined to eliminate $A$ and $B$ and solve for $Z_{in}$:

$$Z_{in} = \rho_0 c S \left[ \frac{Z_{mL} + j \rho_0 c S \tan kL}{\rho_0 c S + j Z_{mL} \tan kL} \right]. \quad (3.39)$$

Equation (3.39) is called the acoustic transmission line equation and is a powerful tool in musical acoustics since, as was stated by the quote at the beginning of this
chapter, the input impedance is a measure of the system alone and describes the acoustic performance of the instrument in an objective way [127].

### 3.4.3 Resonance of a cylindrical pipe

The concept of resonance must be carefully defined here. As was stated in Section 3.2 resonance is a condition where any change in excitation frequency will result in a decrease in the response in the system. In order to determine the resonance frequencies of a clarinet-like instrument (a cylindrical pipe) an understanding of the boundary conditions at the ends is necessary. At one end, the open end at $x = L$ there is the boundary condition that pressure is a minimum and the volume velocity is a maximum. At $x = L$, a first guess would be that $Z_{mL} = 0$ and the expression for $Z_{\text{in}}$ (Eq. (3.38)) reduces to:

$$Z_{\text{in}} = j \tan kL. \quad (3.40)$$

Applying the second boundary condition, that volume velocity $U = 0$ at $x = 0$ means that $Z_{\text{in}}$ in the previous equation must equal infinity. This will occur when $kL = m\pi$ for odd integer $m$, at frequencies equal to $f_n = (2n + 1)c/4L$ for all integer $n$.

Figure 3.9a (and similarly Fig. 3.9b) shows the lossless case of resonance frequencies found in a cylinder, forced at the input $x = 0$ and open at the end $x = L$. In order to know which frequencies will be allowed by the system a boundary at the input (the force location) is considered. For a clarinet-like system, that is closed at the input, the impedance at the input will be a maximum since the...
pressure at the input is maximum and so, the resonance frequencies for the clarinet-like instrument will occur at the maxima of this impedance curve. For a flute-like, open-open system, the impedance at the input will be a minimum since the pressure at the input is a minimum and the volume velocity is a maximum. Therefore the resonance frequencies for the flute-like instrument would be the minima of this impedance curve shown in Fig. 3.9a.

The calculations of the input impedance and allowed frequencies (resonance frequencies) in this section are found in the extremest of ideal cases. The previous sections considered a system with perfectly rigid boundaries, lossless walls and ignored the idea of radiation impedance, hole and bell effects as well, which will all be necessary additions in order to calculate accurate resonance frequencies of the instrument.

3.4.4 Radiation loss

The bell end of a clarinet is not a perfect termination; there must be a sound radiated at the open end so some energy can escape the resonator. Musical instruments are considered unflanged terminations or unbaffled since some of the sound would theoretically be able to wrap around the small bell. Considering the acoustic transmission line equation stated in Eq. (3.39), if it is normalized to resemble an acoustic input impedance it would look like this:

$$Z_{in} = \frac{Z_{mL} \rho_0 c S}{\rho_0 c S + j \tan kL}$$

(3.41)

Now the $Z_{mL}$ termination at the open end of the clarinet will not merely be $Z_{mL} = 0$ (for zero pressure at the open end) but will be a more complicated expression for the radiation impedance at an open end of a pipe. In order to keep the expression workable, the radiation impedance expression used will be for that of a plane piston in an infinitely baffled or flanged opening [30]:

$$Z_{rad} = \pi a^2 \rho c \left( 1 - \frac{2J_1(2ka)}{2ka} \right) + \frac{j\pi a^2 \rho c (2H_1(2ka))}{2ka}$$

(3.42)

where $J_1$ is the Bessel function of the first kind and $H_1$ is the Struve function of the first kind [2]. Figure 3.10 shows the difference between the simplified baffled vs.
unbaffled case. There are slight differences in the peak height (a baffle doubles the radiated power), but slightly more noticeable are the shifts down (in frequency) in the baffled case’s impedance peaks at higher frequencies. Overall, it is just important to know what effects the simplifications chosen will have on the end results.

3.4.5 Wall losses

Another loss mechanism is found at the walls of the cylinder. Until now the assumption was that there were no viscous or thermal losses at the walls. In reality, the air moving along the pipe walls will behave differently in the middle of the pipe (away from the walls). There are two distances that are important in determining the wall loss effects: the viscous and thermal penetration depths, a distance from the pipe wall where the viscous and/or thermal effects will be important. The viscous penetration depth is a function of frequency, the dynamic viscosity $\mu$, and the air density $\rho$:

$$\delta_{\nu} = \sqrt{\frac{2\mu}{\omega \rho}}. \quad (3.43)$$

The thermal penetration depth is a function of frequency, the thermal conductivity $\kappa$, the pipe density $\rho$, and the specific heat at constant pressure $C_p$,

$$\delta_{t} = \sqrt{\frac{2\kappa}{\omega \rho C_p}}. \quad (3.44)$$
In order to account for wall losses it is common practice to include the losses in a complex wavenumber $k$,

$$ k = \frac{\omega}{\nu} + j\alpha. $$

The two variables $\nu$ (the phase speed) and $\alpha$ (attenuation coefficient) are a function of both the viscous and thermal penetration depths (units of meters), as

$$ \alpha = \frac{\omega}{2c} \left( \frac{1}{a} \sqrt{\frac{2\mu}{\omega \rho}} + \frac{1}{a} (\gamma - 1) \sqrt{\frac{2\kappa}{\omega \rho C_p}} \right), $$

$$ \nu = c \left( 1 - \frac{1}{2a} \sqrt{\frac{2\mu}{\omega \rho}} - \frac{(\gamma - 1)}{2a} \sqrt{\frac{2\kappa}{\omega \rho C_p}} \right). $$

If these values are included in the calculation of the complex wavenumber $k$ and then used in Eq. (3.42) the graphical result for the input impedance is the green curve shown in Fig. 3.11. In this figure there is also a black curve which is the input impedance for a cylinder without wall losses. This figure shows that the wall losses will further reduce the amplitude of the input impedance peaks, slightly broaden them and shift peaks to lower frequencies, as would be expected when adding damping to the system.
3.4.6 Tone Hole Lattice

A more complicated phenomenon that will alter the resonance frequencies of the clarinet is the inclusion of a tone hole lattice. The addition of a tone hole will effectively shorten the length of the tube. However, the tone holes also add a sort of filtering effect to the spectrum of possible (passable) frequencies.

Baroque instruments, following many historical instruments, were diatonic, playing in only one key. The addition of a change in tone hole closing to allow for fork fingerings or cross fingerings would offer the possibility of using semi-tones, and finally the use of key-changes throughout classical music (as opposed to using one instrument per given key, often C).

The low frequency approximation for the radiation of sound from an unflanged open end of a cylinder is [85],

$$Z_L = Z_0 \left( \left( \frac{k a}{2} \right)^2 + j 0.61 a \right).$$  \hspace{1cm} (3.48)
And the acoustic impedance at the input of the cylinder is then,

\[ Z_{in} = Z_0 \left( \frac{Z_L + jZ_0 \tan kL}{Z_0 + jZ_L \tan kL} \right). \]  

(3.49)

(where \( Z_0 \) is the characteristic impedance of the medium in this Acoustic Transmission Line Equation). If wall losses are ignored this equation gives an impedance spectrum that resembles that in Fig. 3.11.

Adding holes could be considered as adding a side branch on the tube, the theory of which is well understood. Consider a pipe that resembles Fig. 3.13. For this situation the following is true: There will be continuity of pressure at this boundary:

\[ P_i + P_r = P_1 = P_2, \]  

(3.50)

and there will be a continuity of volume velocity at the boundary,

\[ U_i + U_r = U_1 + U_2. \]  

(3.51)

If we divide these two equations then,

\[ \frac{(U_i + U_r)}{P_i + P_r} = \frac{(U_1 + U_2)}{P_1} = \frac{U_1}{P_1} + \frac{U_2}{P_2}. \]  

(3.52)

This shows that the impedances add in parallel,

\[ \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}, \]  

(3.53)

and the admittances, \( Y = 1/Z \) add in series,

\[ Y = Y_1 + Y_2. \]  

(3.54)

First, in treating the case of a single open tone hole for a cylindrical instrument in terms of admittance, the acoustic transmission line equation becomes,

\[ Y_{in} = Y_0 \left( \frac{Y_L + jY_0 \tan kL}{Y_0 + jY_L \tan kL} \right). \]  

(3.55)

Figure 3.14 shows a variation of the pipe with a side branch, appropriate for a first step toward a tone hole in a pipe.
Beginning with the radiation from the far right side of this instrument,

\[ Y_L = \frac{Y_0}{25(ka)^2 + j0.61ka}. \]  
(3.56)

Then the admittance at the input of the junction (red dot) is,

\[ Y_{\ell_1} = Y_0 \frac{Y_L + jY_0 \tan k\ell_1}{Y_0 + jY_L \tan k\ell_1}. \]  
(3.57)

The radiation from the side branch/hole of radius \( b \) is now,

\[ Y_{b,rad} = \frac{Y_{0b}}{(\frac{kb}{2})^2 + j0.61kb}. \]  
(3.58)

Finally, the admittance at the junction (red dot) looking into the side branch/hole is,

\[ Y_b = Y_0 \left( \frac{Y_{b,rad} + jY_{0b} \tan kh}{Y_{0b} + jY_{b,rad} \tan kh} \right). \]  
(3.59)
From the previous discussion, the admittances of the cylindrical resonator and the hole will combine in series,

\[ Y_1 = Y_{\ell 1} + Y_b. \]  

(3.60)

Figure 3.16 shows the input impedance plot for a duct of length 0.5 m and one hole of diameter 6 mm located 0.1 m from the end. The plot includes the end effects that have been discussed to this point (radiation losses). Figure 3.15 shows the input impedance plot for a duct of length 0.5 m and no tone hole for comparison. Notice that the resonance frequencies are integer multiples of one another and the maxima and minima are indicated with the red dashed lines. In Fig. 3.16 the red lines remain in their original "no hole" locations. The immediate visible effect of a small open hole near the termination of the duct on the input impedance and resulting resonance frequencies of the tube is the increase the fundamental frequency of the tube by effectively shortening the tube. Figure 3.17 shows the input impedance plot for a duct of length 0.5 m and one hole of diameter 6 mm located 0.2 m from the end. The plot includes all other effects that have been discussed to this point (losses). In this case, when the hole is near the middle of the duct there are much more interesting changes occurring for the input impedance plot. Since the hole is close to the middle of the tube it seems that the fundamental frequency is nearly doubled. However, this relationship is much more complicated after the first peak. The first minimum and second peak shown are oddly very close to one another and the remaining peaks (and valleys) shown are very unusual in comparison to the original duct shown in Fig. 3.15. In practice, of course, the clarinet (or other cylindrical instrument) would actually be played by opening a lattice of holes, and not just one in the middle, in order to double the playing frequency (or have effectively half of the length of tube).

Adding one hole to the instrument, even if the hole is closed by the finger, will effectively lengthen the resonator (due to the added equivalent volume due to the hole), pulling all of the resonances to lower frequencies. This length change and severity of change will depend on the size (radius) of the open hole. If the tone holes are closed, there is still a small volume where the hole protrudes from the instrument body. This volume will create its own individual frequency shift (down).
Figure 3.15: Input impedance plot for a duct of length 0.5 m, open at both ends. The red dotted lines indicate the resonance frequencies of such a duct.

Figure 3.16: Input impedance plot for a duct of length 0.5 m, open at both ends but with a hole of radius 0.003 m a distance of 0.1 m from the end of the duct (furthest from the entrance).
Figure 3.17: Input impedance plot for a duct of length 0.5 m, open at both ends but with a hole of radius 0.003 m a distance of 0.2 m from the end of the duct (furthest from the entrance).
The next step would be to add a row of tone holes to make this resonator resemble more of a clarinet-like instrument. The holes could be regularly spaced with equal radii (simpler) or the spacing could change and the holes could change radius as well (reality and more difficult). In adding these holes, the instrument can be effectively shortened by lifting a finger to create open tone-hole and a new effective length of the instrument. Adding this row of tone holes, or tone hole lattice as this is often called, creates what is called a cut-off frequency. The lattice of open holes acts (at low frequencies) as a mass-like impedance at the open end (the row of open holes acts like a completely open end).

Benade lists a formula for the cutoff frequency, or the highest frequency that will propagate considering this tone hole lattice for a clarinet [19],

\[ f_{\text{cutoff}} = \frac{1}{2\pi\sqrt{2}} \frac{b}{a} \frac{c}{\sqrt{s \ t_{\text{eff}}}}, \]  

(3.61)

where \( b \) is the bore radius, \( a \) is the tone hole radius, \( c \) is the speed of sound, \( s \) is the half separation between subsequent holes, and \( t_{\text{eff}} \) is the effective tone hole length including end effects. The cutoff frequency for the clarinet is around 1500 Hz. The cutoff frequency creates a situation where the higher frequencies above cutoff will not have the chance to set up strong standing wave resonances in the instrument and therefore will not influence the timbre of the resulting sound. Figure 3.18 shows the effect of tone holes on the clarinet’s impedance spectrum. The bottom plot shows the acoustic impedance for an open-open cylinder, the middle plot is for a flute and the top plot is for a clarinet. It is most interesting to compare the open-open tube without a tone hole lattice and the clarinet with a tone hole lattice. Notice that for the clarinet, after approximately the cutoff frequency of 1500 Hz, the higher modes are rapidly decreasing in amplitude due to the lattice [33].
3.4.7 The Register hole

There were, early-on, register holes included for instruments giving the opportunity to use the same resonator, operating in different modes to play higher or lower in its playing range. The clarinet has one register hole which is opened to reach the second register and the third register is reached through use of cross fingerings with the register key open. These cross fingerings for the third register are generally performed by “venting” or opening the hole that is usually closed by the left index finger. Interestingly, the saxophone uses two register holes and the bassoon has four. The clarinet (and other instruments) can use this particular register hole for notes in the scale as well, specifically, the B♭ clarinet will use this key along with another in the throat tone region to play a B♭ (note 19), the note just before the break (or second register of the instrument [75]). The register key on the clarinet opens the resonator about 1/3 of the way from the mouthpiece. Putting a small hole here creates a node in pressure at that location and disrupts the fundamental,
therefore making it possible to jump to the second mode (for the clarinet, a 12th). Obviously, the location of this hole would ideally be changed for each note since the location of that pressure node would need to be in a different place depending on the wavelengths (choice in note/fingering). However, this is quite impossible with today’s instruments and so there has been a compromise for location. This compromise can lead to tuning issues when playing 12ths but is in general rectified by the musician understanding the tendencies of their individual instrument [46]. The third register of the clarinet (around note 34 and higher) is actually reached in another way, by opening a key usually covered by the left hand, about an inch below the usual register key but on the front of the clarinet. This hole is much larger and because of this some players half-open the hole while playing in the third register to help with stability and tuning.

3.4.8 Bell

One effect of the bell on the instrument is obviously the elongation of the instrument. Acoustically the bell also allows for a less abrupt change in impedance. Figure 3.19 shows the input impedance plot for a simple cylinder versus the impedance spectrum for a cylinder with a bell attached. Notice from this figure that there is a change in overall shape in the impedance spectrum when comparing a clarinet with a bell and one without. Each of the resonance peaks are now lower in amplitude. The bell is actually acting as an impedance matcher and will help the sound waves radiate from the end of the instrument. The bell allows more sound to be radiated and therefore less sound reflected within the bore, causing the impedance spectrum to show a decrease in strength of standing waves. Notice, however, that the lengths of the two resonators represented in this figure are different lengths (due to the additional length added with the bell) and therefore the frequency shift for the peaks is due to this change in length.

This bell effect does not weaken the lower frequency peaks, as is seen in the figure, the peak amplitudes are nearly identical. This is because the bell is much smaller than the wavelength of those low frequency waves and therefore would not be an effective radiator for sounds at these frequencies. The bell can then be considered as a high pass filter: high frequencies can radiate out of the instrument and are not able to set up strong standing waves that contribute to the sound that
Figure 3.19: Two impedance spectra: tube with (red curve) and without bell (black curve) from [127], no holes. The bell is like a high pass filter, diminishing the high frequencies shown in the impedance spectrum in allowing them to propagate out of the cylinder. The bell also adds effective length to the instrument, lowering the frequency value for all peaks.

is eventually heard. Because of this, the bell somewhat acts as a cutoff, as did the tone hole lattice. In a way, the bell acts as the tone hole lattice for the lowest few notes of each register (where none or only a few holes are open) so that these low notes in each register act and sound more similar in timbre as notes higher up, which are more affected by the tone hole lattice.

3.5 Impedance measurements

In order to determine the actual impedance spectrum and therefore resonances for a real system (such as a clarinet of interest) two quantities must be measured, $p$, the acoustic pressure at the input and $U$, the volume velocity at the input. This is not always an easy task to achieve. The literature offers many options for measuring these quantities. Chapter 7 will describe a few different methods, including the one used in this research. The input impedance and resulting resonance frequencies measured for various clarinets will be used in the numerical and analytical prediction
methods to determine the playing frequencies of the instruments, to be described in Chapters 5 and 6.
4.1 Introduction

The generator is the portion of the instrument that will begin the oscillations necessary to create sound (working together with the resonator and radiator). The generator for the clarinet is the air-reed-mouthpiece combination. The blowing pressure provided by the musician can be considered a separate mechanism, the power supply, but for now the generator will be modeled to contain this information as well. The reed, attached to the mouthpiece, offers two important functions: First, the reed is driven by the pressure variations inside the mouthpiece and oscillates as a simple harmonic oscillator linearly at the entrance of the resonator and has a measurable resonance frequency. Secondly, it is the nonlinear control value, allowing a particular amount of air to enter into the system, at a particular velocity and in a particular direction. This nonlinear valve function can add energy to the system.

Figure 4.1 shows a simplified sketch of the clarinet mouthpiece-reed combination in a playing condition. The reed has a vibrating surface area and is attached to the mouthpiece by the ligature. In Fig. 4.1 the portion partially enclosed at the left of the top sketch represents the mouth cavity of the musician. Notice that the teeth (in grey) will sit on the top of the mouthpiece and on the bottom, touching the reed, the bottom teeth covered by the bottom lip. Having the softer surface...
of the lip touching the reed allows the player to regulate the damping on the reed depending on the applied force. The vibrating portion length of the reed is called $L$ and is difficult to define, whether it is between the tip and lip or tip and ligature or some length in between. However, Taillard et al. mentioned that past the ligature, the reed’s vibrations can perhaps play a significant role in the overall behavior of the instrument [119]. The vibrating width of the reed (the entire width of the reed) is $w$. At rest, the reed will also be separated, towards the tip, by a certain distance $H$ shown in the top sketch. The ligature, an adjustable material, usually metal or leather, attaches the reed to the mouthpiece. Appendix A offers photographs of a real clarinet, mouth piece and body, to understand how this simplified sketch compares.

Figure 4.2 shows a picture of the clarinet mouthpiece without the ligature and reed present. Notice how the opening increases as the distance from the reed-lay increases all the way to the tip of the mouthpiece, where the tip of the reed will hit at high blowing pressures (when $H = 0$).

The two main playing parameters of interest throughout this research represent the reed opening at rest and the pressure difference across the reed. These two parameters are often nondimensionalized and will be described in greater detail in Chapters 5 and 6.
Figure 4.1: Set-up of the clarinet mouthpiece-reed combination showing important playing variables which are adjustable by the player: $H$ the reed opening at rest, $L$ the vibrating length of the reed, $w$ the vibrating width of the reed. This figure also shows the positioning of the lips and teeth on the mouthpiece and reed as well as the positioning of the ligature, the adjustable mechanism that attaches the reed to the mouthpiece.

Figure 4.2: Picture of clarinet mouthpiece alone showing the opening when the reed and ligature are not present [97].

### 4.2 The reed as a flow controller

The clarinet is a single-reed woodwind instrument. The reed is attached to the mouthpiece and is considered, in this combination, to be the instrument’s generator. The clarinet reed is inward striking in that the reed will be forced shut by an over-pressure in the mouth [109]. The pressure difference and the reed will move...
in-phase with one another (high pressure closes reed, low pressure causes reed to open allowing for more flow to enter). Figure 4.3 shows the simple spring analogy that a positive pressure difference in this system, like that for the clarinet, will cause the reed to close against the mouthpiece lay. Here $p_m$ is the pressure in the mouth and $p$ is the pressure in the mouthpiece. The pressure difference $\Delta p = (p_m - p)$ will provide the force that causes the reed to oscillate. Figure 4.4 shows the comparison of the different types of reeds. A clarinet, being an inward striking reed follows the left column whereas brass (or lip-reed) instruments follow the right column.

Figure 4.3: Simple mass-spring model of the clarinet reed-mouthpiece system [64]. This shows that a positive pressure difference will cause the reed to close as for the clarinet. The harder a player blows, the closer the reed comes to closing against the reed table. Here the vertical plate will move toward the aperature and shut the door with a sufficient pressure difference.

Figure 4.4 illustrates the static response of the clarinet system where the clarinet exhibits Bernoulli flow due to the player input of blowing pressure across the reed. Bernoulli pressure comes into effect because, within any region of streamlined flow with a velocity $v$ and initial pressure $P$, the quantity $P + \frac{1}{2} \rho v^2$ will remain constant [64]. In order to satisfy this condition, a pressure drop must occur across the reed. After passing through the relatively narrow reed opening the air will emerge (into the mouthpiece cavity) as a jet and turn and twirl around inside the voluminous mouthpiece cavity on the downstream side of the reed. This turbulence will allow mixing of the air and create a flow that is no longer streamlined. The pressure that is now inside the mouthpiece will remain low, never quite recovering to the initial pressure ($p_m$, the mouth pressure).

Figure 4.4 also shows the behavior of an outward striking / outward swinging reed. For the inward striking (like the clarinet) case the (−,+ ) refers to the fact that a negative pressure difference will open the valve. For the outward striking case (as for brass instruments) the (+,−) refers to the fact that a negative pressure difference will close the valve. For the clarinet, the flow rate first increases for a
given pressure difference across the reed, but then decreases as the difference in pressures tends to close the valve, leading to the reed closing after a certain pressure difference is achieved. For an outward striking reed, such as the brass player’s lips, the steady-state flow velocity always increases with increasing pressure across the reed (lips).

4.2.1 The nonlinear characteristic flow curve

Figure 4.5 shows a simplified nonlinear characteristic curve for a clarinet reed-resonator system. The vertical axis shows the volume flow $u$ and the horizontal axis is the pressure difference $\Delta p$ across the reed ($p_m$ being the static mouth pressure and $p$ the mouthpiece pressure).

The nonlinear flow characteristic of the clarinet follows the Bernoulli equation formulation (discussed further in Chapter 5). The Bernoulli equation creates a nonlinear characteristic static flow curve $u(p)$.

This nonlinear characteristic curve will have a different slope and point where it meets the zero on the horizontal axis depending on the choice of blowing pressure and reed opening. In Fig. 4.5, at low pressure differences (below A) the slope
Figure 4.5: Nonlinear Characteristic Curve, from Fletcher [54]. Different characteristic values represented by A (before oscillation), O (operating regime), C (closing). The solid and dashed lines are for two different reed openings.

is sharp and nearly linearly increasing. Point A corresponds to the transient, and during the transient the acoustic quantities are rapidly increasing. Just after the peak of this curve (just after A) is what is considered the acoustic pressure threshold of oscillation; the volume flow (vertical axis) increased as the pressure difference increased until this point where the reed begins to oscillate. The amount of static pressure introduced into the system will determine where on this curve the instrument will function. At very high pressures (around C), the reed actually begins; to close against the mouthpiece (shown in the fact that there is no more volume flow allowed at maximum pressure differences). The development in Chapter 5 will reveal that the Bernoulli flow equation shows the shape of this curve relies heavily on the initial reed opening and width. If the reed is more open, H is larger and it will take a greater pressure difference for the oscillations to occur (the threshold of oscillation will occur later for this particular reed opening). This also means that there will be more flow allowed into the instrument in general during the oscillations. The Figures represented in Fig. 4.6 show this flow curve (each with the same axes) for three different reed openings. Notice when comparing Figures 4.6a and 4.6c, respectively for small H and large H, the first will have a steeper slope and reach its threshold of oscillation sooner than the reed that is more opened in the second figure. For a musician this is intuitive: a reed which allows a larger
Figure 4.6: Nonlinear characteristic flow curve for three different initial reed openings a) small $H$, b) mid-range $H$, c) large $H$.

Initial opening will take more blowing pressure to make oscillate than a reed that is initially more closed. This initial opening can also be affected when choosing a mouthpiece as manufacturers can change this value to make the instrument more or less responsive depending on the needs of the musician.

Figure 4.7 shows a theoretical nonlinear characteristic flow curve compared to a measured one using the same parameters (from [30] and [42]).

The increasing portion of the curve resembles a square root of the pressure difference (Bernoulli) since the motion of the reed is very small. The decreasing portion follows the progressive closure of the reed. Notice that the measurement
curve does not show the beating of the reed (where the flow is zero for high pressure differences), because in reality for many systems, the flow does not completely vanish for high pressure differences. A measurement of this type using an artificial mouth would offer the information necessary for two important parameters (see Chapter 7), the reed closure pressure and the oscillation threshold.

4.2.2 Notes about oscillation threshold for an inward striking reed

One important point on this nonlinear characteristic flow curve, discussed in the previous section, is the point where the oscillation actually begins, at what point, in pressure difference over the reed, the sound can be expected to begin. In Fig. 4.5 the threshold of oscillation occurred just after point A and the closing pressure would have occurred past point C. The derivation of the equation representing this point will be discussed further in Chapter 5 but for now the idea will be treated qualitatively. Chapter 5 will show that the value for the pressure oscillation threshold is around one-third of the reed closing pressure. There are actually two very different reed effects that alter the actual point on the static flow curve where oscillation will begin, the value of the oscillation threshold. These two effects are (i) the reed dynamics, or damping of the reed and its stiffness causing a spring-like behavior and (ii) the flow created by the motion of the reed. The difference is subtle but important in understanding the complete functioning of the system.
Research on this topic began with Weber in 1830 and then Helmholtz joined (and was followed by many authors). Both were interested in understanding the reed induced flow in order to have a solid comparison between the clarinet and other reed instruments such as a reed organ pipe [72]. For reed instruments like the clarinet, a parameter such as the ratio of the pipe’s resonance frequency to the reed resonance frequency is small and therefore the choice of reed resonance frequency has only a small effect of the playing frequency of the instrument of this type. For the reed organ pipe however, this ratio of the pipe’s resonance frequency to the reed resonance frequency is large and therefore the reed resonance will have a great effect of the playing frequency. The study of this effect was done using linear theory, using the assumption that the playing frequency will turn out to be simply the resonance frequency of the composite air-column and reed system. In this case, the reed damping/dynamics are also ignored and so the reed is considered to be acting as a simple spring. Thinking of the system in this way highly simplifies the equations and easily shows how the playing frequency will shift from the pipe resonance to that of the reed resonance when the resonance frequency of the pipe approaches the reed resonance.

However, in order to prove that when the damping on the reed is ignored the playing frequency will be that of the composite resonator, the input admittance of this composite generator-resonator should be considered. If the nonlinear equation representing the excitor were to be linearized it will represent the flow used for the input impedance thus giving both the playing frequency and the mouth pressure at the threshold of oscillation. For the nonlinear model used in this research the mouth pressure for the oscillation threshold is found to be equal to one third of the closing pressure of the reed $p_M$ for any combination of pipe resonance frequency and reed resonance frequency.

The next group of researchers to refine these ideas were Wilson and Beavers [126]; they were the first to propose a model for the clarinet based on the Bernoulli equation for flow. They attempted to understand the effect of reed damping in the case of a single degree of freedom simple harmonic oscillator, but in this case they were obliged to ignore the effect of the reed induced flow. The dependence of the playing frequency on the ratio of pipe resonance to reed resonance remains the same as when ignoring reed damping. However, the resulting deviation of playing frequency to pipe resonance is much smaller than when only including the flow
effect. Interestingly, if this is taken further, if both effects are taken into account, Silva et al. [114] showed that the reed flow effect is dominant when considering the resulting playing frequency and the effects of each of these factors to it. On the other hand, the effect of including reed damping and the resulting oscillation threshold is much greater than that of the reed flow. For a strongly damped reed (considered to be the case for the clarinet, where the reed is highly damped by a player’s lip) the oscillation threshold remains close to the value of $p_M/3$ whereas for the lightly damped reed (a reed pipe organ reed) this pressure threshold for oscillation is lowered drastically. This is why, for the clarinet, a squeak is obtained if the player does not damp the reed enough before beginning to blow (clamp the mouthpiece/reed combination with teeth on top and lip on bottom). When the reed is not damped enough in this configuration, the reed’s resonances (harmonics) are more prevalent (stronger than the peaks of the pipe resonances) than those of the sound emitted. This also means that a harder reed, a reed with a higher resonance frequency that is less easily damped by the player’s lips, will be more likely to squeak than a soft reed which has a lower resonance frequency and is more easily damped by the player.

4.2.3 Putting it all together

When taking a look at the relationship between the volume flow that can enter the instrument ($U$) as a function of the pressure difference between the pressure in the mouth $p_m$ and the pressure inside the mouthpiece $p_{ins}$, the flow rate $U$ initially increases rapidly for a small pressure difference $\Delta p$ but then it decreases as $\Delta p$ tends to close the reed against the mouthpiece (no more flow allowed through the reed/mouthpiece). This increase in $\Delta p$ will lead to complete closure once it reaches a pressure difference called $p_{\max}$. All of these behaviors put together create a positive feedback loop - once the reed starts oscillating due to this $\Delta p$ it works with the resonator to continue the loop of pressure fluctuations into sound, as long as there is pressure offered to the system by the musician (this is considered a self sustained oscillation) [113]. The system will be able to excite resonances of the clarinet-like cylinder if the feedback is strong enough to fight against any losses present (viscous, thermal and radiation).

This process is more easily seen if offered in list form:
• Air supply: the lungs supply the constant blowing pressure which results in the steady flow of air $U$

• $P_{\text{mouth}}$: the lungs, supplying that steady flow of air, maintain a steady pressure inside the mouth in order to play a particular note.

• Pass through: the neat, and tidy, streamlined air flows towards the narrow reed opening (where the tip of the reed and mouthpiece are) and will pass over this threshold with a velocity $v$.

• Pressure drop: because the initial pressure was streamlined, once the air passes over the reed into the mouthpiece, it will suffer a pressure drop by $\frac{1}{2} \rho v^2$ according to Bernoulli’s principle.

• Jet Formed: after the threshold point when the air passes over the reed an outward going jet will be formed and quickly breaks up into much less tidy turbulent flow.

• Oscillation: since the pressure is continuously offered to the system by the player this process will continuously loop. This process is what causes the reed to oscillate and offer the conditions necessary for sound waves to propagate in the resonator and be heard through the resonator. The reed is what causes the steady pressure to be converted into acoustic pressure (fluctuations $\rightarrow$ oscillations $\rightarrow$ sound)

4.2.4 Implications of the functioning generator

It is important to stress that a linear model does not give sufficient information to create peaks other than the fundamental. The nonlinear behavior is necessary in order to create musical sounds (multi-peak resonator); linear theory cannot account for amplitude and spectrum stability of musical oscillations. The flow for a clarinet is a nonlinear function of reed tip opening and pressure difference across the reed. If the model were linear, a sinusoidal mouthpiece pressure (or reed displacement) would imply a sinusoidal excitation. But the excitation is continuous (DC, static mouth pressure) and this sound is produced by self-sustained oscillation, the coupled reed-resonator system. Therefore, in order to transform a DC, continuous input into a sinusoidal signal, the nonlinearity is necessary.
Figure 4.8: The flow wave forms that result from playing in different parts of the nonlinear characteristic curve, from [127]. Top are the nonlinear characteristic curves - left: soft dynamic, middle: medium, right: high pressure values or loud dynamic. The bottom plots show the corresponding flow curves entering the instrument. These curves are sketches, conceptually presenting the behavior and may not be drawn to exact scale [127]. It should be noted that the actual pressure ranges are not necessarily in those exact places along the nonlinear characteristic curve and have been shown to oscillate around the threshold of oscillation on either side after the onset [55].

Figure 4.8 shows that at small amplitudes (low blowing pressures / soft dynamic, past the threshold of oscillation) where the range of pressure difference is small, oscillations in the instrument due to reed vibrations will be nearly sinusoidal and nearly only the self sustained frequency is present and the pressure wave is nearly sinusoidal (with that one, fundamental frequency present). This is a very weak sound, unlike a real musical instrument, or the clarinet timbre one is accustomed to hearing. As blowing pressure is increased (and subsequently the pressure difference is greater) the playing regime is further right on the static nonlinear characteristic flow curve and the oscillations become more and more like square waves. As the player continues to crescendo and increase the blowing pressure, this nonlinear system will give more energy to the high frequency peaks; the amount of energy at each harmonic of the playing frequency is determined by the amplitude of the input impedance at each harmonic. At higher blowing pressures a greater part of the vibrational energy is spread over an increasing spectral range, despite the low pipe impedance offered to harmonics above a certain point (above the tone hole cutoff frequency). These curves are sketches, conceptually presenting the behavior and may not be drawn to exact scale [127]. It should be noted that the actual pressure ranges are not necessarily in those exact places along the nonlinear characteristic...
curve and have been shown to oscillate around the threshold of oscillation on either side after the onset \cite{55}.

4.2.5 Nonlinearity and harmonics

Inputting high blowing pressures into the clarinet will give no feedback since the reed will close. The clarinet system functions as it does due to positive feedback: since the reed motion and the pressure fluctuations depend on each other and are in phase with one another, the more energy added to the system, the higher the reed displacement amplitude \cite{121}. The pressure moves the reed until a certain point (the oscillation threshold) where the reed begins to oscillate based on its variable reed properties. Past that point the system will contain pressure fluctuations between two values and be controlled by the opening and closing of the reed, which is controlled by the level of pressure input in the system. The air-mouthpiece-reed and pipe resonator are highly coupled.

The periodic sound produced by a nonlinear system of the type under discussion will, in principle, include all harmonics of the fundamental frequency, depending on the boundary conditions imposed. This content, based on the boundary conditions and the effects of the generator coupling to the resonator, leads to a definite tone color characteristic of the instrument. It also allows the trading of energy between various harmonics in the spectrum. If the system was exactly harmonic then oscillations started at a few of the normal modes might grow and other normal mode oscillations would die out (non-harmonic ones). The harmonics that would normally continue to grow in a linear system will tend to die out in a non-linear system in order to help maintain the lower peaks. This drain on the system as blowing pressure increases leads to an absolute amplitude stability because an increase in amplitude will add energy to the high harmonics. This leads to an even more increased drain of energy on the system (less energy available to for the fundamental). A feedback oscillation can be set up in which the oscillatory air flow through the reed creates an oscillatory pressure in the mouthpiece. This will cause an oscillatory motion of the reed which will then maintain this oscillatory flow (self sustained oscillation). The incoming flow adds energy to the air column oscillation whose amplitude increases until the rate of energy loss due to radiation and damping in the air column just balances the rate of energy input.
The reed can be treated as an undamped simple harmonic oscillator,

\[ m \cdot \frac{d^2x}{dt^2} + k \cdot x = 0 \quad \rightarrow \quad \frac{d^2x}{dt^2} + \frac{k}{m} \cdot x = 0. \tag{4.1} \]

This equation shows all necessary pieces for oscillation: inertia = mass term \((m)\), and elasticity (restoring force) = spring \((k)\). The frequency at which this system will oscillate (if displaced from its equilibrium) is dependent on the ratio of the restoring force to the inertial force. For the clarinet, reed resonance frequencies are on the order of \(\approx 2400\) Hz. This is well above the normal playing frequencies for the clarinet, which is good (and necessary).

There are many factors that will alter the functioning of the reed system as a linear oscillator: high blowing pressures (playing in the fortissimo range or higher), player dependent lip placement and force on the reed (adding or subtracting damping to the system). However, for simple calculations and for most research situations, this approximation will suffice.

The differential equation that describes the motion of the reed based on reed characteristics and constant blowing pressure is:

\[ \frac{d^2y}{dt^2} + q_r \omega_r \frac{dy}{dt} + \omega_r^2 y = -H \omega_r^2 \frac{\Delta p}{p_M}. \tag{4.2} \]

This equation shows that the important reed characteristics necessary for proper modeling are the reed quality factor \(q_r\), initial opening \(H\) and resonance frequency \(\omega_r\). Furthermore, the reed vibrations will be controlled by these characteristics as well as the values of pressure that are considered as inputs for this system. The solution will be dependent on the amplitude of the input. In other words, the fundamental resonance frequency of the reed, \(\omega_r\), will be dependent on the square root of the blowing pressure amplitude \(30\). Due to this amplitude dependence and because the spectrum of resonance frequencies in the resonator is dependent on the fundamental resonance frequency of the generator, the strength or amplitude of the resulting resonances in the tube will depend on this amplitude in blowing pressure as well, causing nonlinear behavior based on blowing pressure amplitude \(60\).
4.3.1 Further Implications

Quite early on in musical acoustics research Helmholtz presented the linear theory of self-sustained oscillations and mentioned the reed as an undamped simple harmonic oscillator [72]. His research led to equations directly implying that this coupled SHO would lower the playing frequencies from the natural frequency of the pipe (though he did not say this directly). An undamped harmonic oscillator driven below resonance runs in phase with driving frequency. At low frequencies compared to the reed resonance, flow is greatest when pressure in the mouthpiece is highest (the reed is mostly open). In the SHO equation for the reed (see Chapter 5), the negative sign for pressure is present due to the fact that higher pressure difference closes the reed. In a single reed woodwind instrument, using a stiff reed at the input of the instrument will allow sustained oscillations at frequencies far below the reed’s natural frequency and so the resonator can only operate at these low frequencies (in relation to the reed resonance frequency $\omega_r$).

4.3.2 The reed resonance frequency

It is an important point to reinforce that the playing frequencies in a reed instrument will be well below the resonance frequency of the reed. In this case, for excitation frequencies (playing frequencies) much lower than the reed resonance frequency, the excitation is happening in the reed’s low frequency regime, meaning the SHO is excited in the stiffness regime, where damping and mass play a negligible role. Analytical formulas simplify greatly and mass and damping in the SHO equations are ignored. There is trouble with this when the playing frequencies either begin to approach the choice of reed resonance frequency and perhaps if the reed resonance matches a modal component of the note being played. Thompson says this can actually help at times [121]. Playing frequencies end up being just below the frequency at which the air column impedance is a maximum, near natural tube resonances, but will be a little less since the system operates well below reed resonance and therefore it can be modeled as a spring or compliance (volume) leading to a length correction, lowering the playing frequency.
4.4 Considerations and Conclusions

The reed plays a dual role in this model: the linear oscillator at the "closed" end of the air column driven by pressure variations in mouthpiece and (2) it serves as a nonlinear flow control valve which adds to the oscillation of air column. In the bore, at low frequencies (compared to the reed resonance), the wave is strongly reflected at the first hole the wave encounters. At frequencies where these reflections return to the mouthpiece in phase with exciting flow, mouthpiece pressure amplitude, $Z_{in}$ becomes very large. For frequencies above cutoff, reflection no longer occurs. Following previous work, Thompson restates that far below reed resonance, the natural frequency of the air column are slightly lowered due to the elastic termination of the air column provided by the reed, the reed provides solely a stiffness and can be considered a compliance or extra volume at the end of the clarinet. This translates into a length correction, which in turn lowers the playing frequency below resonance of the tube [121], [30].

Based on Benade’s text (and others), the presence of a reed system will lower the resonance frequency of the instrument and the system will not be able to produce frequencies higher than the plucked resonance frequency of the reed [19]. A reed will only be able to ideally sustain stable oscillations in an air column if (1) the reed resonance is much higher than the intended playing frequencies of the system and (2) the player’s embouchure is set in such a way that the operating point is in a good position on the nonlinear characteristic curve (the Bernoulli flow curve). Ideally the operating point would be somewhere in the middle of the downward slope [128], after the threshold of oscillation of course but lower in blowing pressure than the maximum (closing) pressure [19].

Sound may be defined as (1) the disturbance in pressure that propagates through a compressible medium [96]. This chapter has offered an encounter with the pressure “disturber”, the reed-mouthpiece set-up (aka the generator) and led to the information necessary to understand the possibility of this system creating sound when attached to the resonator discussed in Chapter 3.
5.1 Introduction

A perhaps obvious, yet very important aspect, of clarinet acoustics is the research on sound production, regimes of oscillation and the conditions that make the particular sounds and regimes obtainable. The basic acoustics of the clarinet were presented in Chapters 3 and 4, but the clarinet is much more complicated than this. There are multiple aspects of the instrument that are not perfectly taken into account when using the simplistic model of taking the first peak of the input impedance as the playing frequency. The theoretical model presented in Chapter 5 on impedance does not take into account the following three effects: reed induced flow, reed damping / dynamics, and inharmonicity of the resonator.

These three effects have been researched, developed, used, and discussed in literature (30, 45, 42). However, they have yet to be included in a cohesive analytical model to predict the playing frequencies of the clarinet. If the results of these three effects can be found and their direct influence on the playing frequency realized, these analytical methods would be a simple, fast way to predict the playing frequencies of any instrument, given its input impedance measurement. This means that a full tuning chart, or map of sorts, could be made available to each musician at the time of purchase of a new clarinet. If the clarinetist were testing multiple instruments, they could see, visually and quickly, what the tuning tendencies of
each instrument would be, and this would inform their choice of instrument.

The results and explanations from the analytical formulas described within this chapter were used to write two conference proceedings and a manuscript that has been published in the Journal of the Acoustical Society of America [37, 38, 36].

5.2 Reed mechanics (restatements and additions)

In Chapter 3 the resonance of the reed was discussed. For this analytical model the reed motion can, at first approximation, be considered to follow one mass and spring simple harmonic oscillator system,

\[ M_r \frac{d^2 y}{dt^2} + R \frac{dy}{dt} + K_r y = F, \tag{5.1} \]

or more specifically in our case,

\[ \frac{d^2 y}{dt^2} + q_r \omega_r \frac{dy}{dt} + \omega_r^2 y = -\Delta p M_r, \tag{5.2} \]

where \( \omega_r \) is the first reed resonance frequency, \( y \) is the reed displacement, \( -\Delta p \) is the difference in force of acoustic pressure on each side of the reed. \( \Delta p = p_m - p \) where \( p_m \) is the pressure inside the mouth. In Eq. 5.2 the negative sign in front of \( \Delta p \) indicates that as the pressure difference rises the reed will close. \( M_r \) is the mass per unit area where \( K_r \) is the spring constant per unit area for the reed (N/m³).

One statement made in Chapter 4 is that the fundamental frequency, the playing frequency of the instrument, initially inferred from the first peak of the input impedance shown in Chapter 3, is much lower than the fundamental frequency of the reed. Based on measurements in previous research the first reed resonance frequency is around 2400 Hz (31, 9, 10).

The problem is simplified if reed dynamics are ignored, so that \( q_r = 0 \) and \( \omega_r = \infty \), then \( K_r y = -\Delta p \). The static case would allow for a measurement of the static spring constant \( K_r \). The static case also allows us to define the pressure that would completely block the flow of air into the mouthpiece, so the reed would close against the mouthpiece,

\[ p_M = K_r H. \tag{5.3} \]

Figure 5.1 shows the orientation used for reed displacement the positive direction for
$y$ is to increase the opening. The choice of orientation is arbitrary but complements choices made in past literature [30]. The clarinet mouthpiece imposes the condition that $y > -H$ since the reed cannot go past the lay of the mouthpiece itself.

Figure 5.2 shows the relationship between blowing pressure and reed displacement for low blowing pressures and Fig. 5.3 shows this relationship when blowing pressure is high and the reed pushes against the mouthpiece, blocking incoming pressure. In the second configuration, at higher blowing pressures, the reed is said to be beating. When the reed ‘beats’ the opening is closed ($y = -H$). The pressure difference is negative, and the flow velocity of the air through the mouthpiece is zero.

![Diagram of clarinet mouthpiece coordinates](image)

*Figure 5.1: Clarinet mouthpiece coordinates. Reed is at rest when $y = 0$. Displacement $y$ is positive when $y$ moves downward, increasing the reed opening.*
Figure 5.2: Pressure (black line) vs. reed displacement (grey line) for a low blowing pressure, amplitude over time. Notice that the reed follows the trend of the pressure, they are in phase. From [105]
Figure 5.3: Pressure (black line) vs. reed displacement (grey line) for a high blowing pressure, amplitude over time. Notice that the reed follows the trend of the pressure, they are in phase. However, at this high blowing pressure the displacement goes to zero when pressure is negative. This is the point where the reed is beating against the mouthpiece. From [105]
5.2.1 Fluid dynamics around the reed: total flow

There are two sources of flow into the instrument, the flow from the jet of air entering the instrument and a source of flow that is created by the vibrating reed motion. Consider the velocity of the jet of air entering the instrument,

\[ v_j = \sqrt{\frac{2\Delta p}{\rho}}. \]  

(5.4)

This is the velocity of a jet described by Bernoulli flow. The volume velocity \( u_b \), can be characterized by

\[ u_b = v_j S_j, \]  

(5.5)

where \( S_j = (y + H)w \) is the relevant surface area of the incoming jet and \( w \) is the width of the reed where the jet will be in contact. The volume velocity that is created by the reed motion is

\[ u_r = -S_r \frac{dy}{dt}, \]  

(5.6)

where \( S_r \) is the vibrating surface area of the reed and a reasonable value for \( S_r \) for the clarinet is 0.0001 m\(^2\). When \( y \) gets smaller (reed moves towards the mouthpiece) there will be a positive flow (due to reed motion) entering the instrument. If \( y \) is getting larger (reed moves away from the mouthpiece) then the flow due to the reed motion will decrease.

Now, the total flow entering the instrument can be described as \( u_{tot} \),

\[ u_{tot} = u_b + u_r. \]  

(5.7)

If the assumption stands that there are plane waves entering the instrument then the pressure at the entrance of the instrument can be related to the input impedance,

\[ Z(\omega) = \frac{P(\omega)}{U(\omega)}, \]  

(5.8)

which can be rewritten as,

\[ U(\omega) = Y(\omega)P(\omega), \]  

(5.9)

where \( Y = 1/Z \) is the admittance function. The inverse Fourier transform into the time domain is the convolution of the impulse response of the reed \( h(t) \) (the signal
produced by the reed when the input to the reed system is a delta function) and the input pressure signal \( p(t) \),

\[
    u_{tot}(t) = h(t) * p(t). 
\]  

(5.10)

Then the equation for the total flow relates the Bernoulli flow and the flow created by the motion of the reed,

\[
    u_b = u_{tot} - u_r = u_{tot} + S_r \frac{dy}{dt}. 
\]  

(5.11)

This equation shows that the reed flow, the second term, is responsible for an additional flow term in parallel with the Bernoulli flow entering the instrument. In the frequency domain,

\[
    U_b(\omega) = U_{tot} - U_r = Y(\omega)P(\omega) - S_r j\omega Y(\omega),
\]  

(5.12)

and admittances are added this way when in parallel.

If the playing frequencies of interest are much lower than the resonance frequency of the reed \( \omega_r \) then the system can be considered to operate in the spring/stiffness regime (see Chapter [3]). If the reed behavior acts as a spring at the beginning of the instrument, it could also be seen as an acoustic compliance (reciprocal of stiffness) at the entrance of the instrument.

Starting with

\[
    K_r y = -\Delta p 
\]  

(5.13)

one can take the time derivative and rearrange,

\[
    \frac{dy}{dt} = \frac{1}{K_r} \frac{dp}{dt}, 
\]  

(5.14)

since \( \Delta p = p_m - p \) and \( \frac{dp_m}{dt} = 0 \) (static pressure).

\[
    \frac{dy}{dt} = \frac{1}{K_r} \frac{dp}{dt} = \frac{H}{p_M} \frac{dp}{dt}, 
\]  

(5.15)

where \( K_r = \frac{\nu_M}{H} \) and \( y = H \) is the location of the reed when it is against the
mouthpiece. This leads to,

$$u_r = -S_r \frac{dy}{dt} = -S_r \left[ \frac{H \ dp}{p_M \ dt} \right].$$  \hspace{1cm} (5.16)

We can define an equivalent volume as,

$$V_{eq} = \frac{\rho c^2}{p_M} S_r H,$$  \hspace{1cm} (5.17)

so that,

$$u_r = \frac{-V_{eq} dp}{\rho c^2 \ dt}.$$  \hspace{1cm} (5.18)

The acoustical compliance of a volume is,

$$C_{acs} = \frac{V}{\rho c^2},$$  \hspace{1cm} (5.19)

$$V = \frac{\rho c^2 S_r}{K_r},$$  \hspace{1cm} (5.20)

$$V = \frac{\rho c^2 S_r H}{p_M}.$$  \hspace{1cm} (5.21)

This shows why the mouthpiece can be considered as an acoustic compliance with volume of $V_{eq}$ at the entrance of the instrument.

Now,

$$u_b = u_{total} + \frac{V_{eq} dp}{\rho c^2 \ dt}.$$  \hspace{1cm} (5.22)

In the case of a cylinder whose length is much longer than its diameter and considering this portion as a lumped element model, if this small acoustic compliance, or equivalent volume, is added due to the reed motion, the frequency of the first impedance peak (and resulting playing frequency), would decrease. This effect could be treated as if there were a small correction in length to the original cylinder. This length correction, $\Delta \ell$ due to the volume compliance can be defined,

$$\Delta \ell = \frac{V_{eq}}{S}.$$  \hspace{1cm} (5.23)

Then,

$$\Delta \ell = \frac{\rho c^2 S_r H}{S_{pM}},$$  \hspace{1cm} (5.24)

where $S$ is the surface area of the entrance of the clarinet that was used in the input.
impedance measurements (different than $S_r$ which is the surface area of the reed). Measurements [42] show that this small length correction due to the addition of the reed compliance (equivalent volume) is close to 10 mm.

5.3 Three characteristic equations governing reed motion and flow into the instrument

This chapter has thus far given two of the three coupled equations (5.22) and (5.2) that can be solved in order to describe the functioning of the reed and resulting behavior of the instrument in the presence of an initial incoming pressure and known playing parameters and reed characteristics.

The Bernoulli flow through the reed opening becomes,

$$u_b = u_{tot} - u_r = u_{tot} + \frac{S\Delta_\ell}{H} \frac{p_M}{\rho c^2} \frac{dy}{dt}, \quad (5.25)$$

with the closing pressure $p_M = K_r H$. The equation for the reed as a simple harmonic oscillator becomes,

$$\frac{d^2 y}{dt^2} + q_r \omega_r \frac{dy}{dt} + \omega_r^2 y = -\frac{\Delta p}{M_r} = -\frac{\Delta p \omega_r^2}{K_r} = -\Delta p \omega_r^2 \frac{H}{p_M}. \quad (5.26)$$

Since the linear dependence of the reed-mouthpiece opening is assumed (the reed will not bend to meet the mouthpiece), using equation (5.4) and (5.5),

$$u_b = v_j S_j = \sqrt{\frac{2\Delta p}{\rho}} (y + H) w \quad (5.29)$$

$$= w H \left(1 + \frac{u}{H}\right) \sqrt{\frac{2}{\rho} p_M \sqrt{\frac{\Delta p}{p_M}}} \text{sgn}(\Delta p) \quad (5.30)$$

The sgn($\Delta p$) function is included to ensure a positive number under our square
root but to allow for negative flow if \( \Delta p \) is negative. The function will introduce a negative sign into the equation if the value of \( \Delta p \) is negative. Finally,

\[
\begin{align*}
  u_b &= \begin{cases} 
    w \sqrt{2 \rho p_M \left( 1 + \frac{y}{H} \right)} \sqrt{\frac{\Delta p}{p_M}} \text{sgn}(\Delta p) & \text{if } y > -H \\ 
    0 & \text{if } y < -H.
  \end{cases} 
\end{align*}
\]

(5.32)

(5.33)

The second case defines the beating reed situation.

Equation (5.29) could be rewritten, if \( y = -\frac{\Delta p}{K_r} \) is substituted in,

\[
  u_b = w H \left( 1 + \frac{\Delta p}{K_r H} \right) \sqrt{\frac{2 \rho}{\rho p_M}} \sqrt{\frac{\Delta p}{p_M}} \text{sgn}(\Delta p)
\]

(5.34)

and through some rearrangement,

\[
  u_b = \sqrt{\frac{2 \rho p_M}{\rho}} w \frac{p_M}{K_r} \left( 1 - \frac{\Delta p}{p_M} \right) \sqrt{\frac{\Delta p}{p_M}}
\]

(5.35)

Equation (5.35) could be used to plot the nonlinear characteristic curve that is often mentioned throughout this thesis. Figure 5.4 shows this nonlinear relationship between \( u \) and \( \Delta p \). All other variables in Eq. (5.35) are constants.

---

![Figure 5.4: Nonlinear characteristic flow curve \( U(\Delta P) \).](image)

Equations (5.25), (5.27) and (5.35) form a system of three equations and contains...
three unknowns: the vertical displacement of the reed \( y \), the pressure at the entrance of the resonator \( \Delta p \) and the Bernoulli flow at the entrance of the resonator \( u_b \). The first two are linear and the third is nonlinear, a necessary condition for self sustained oscillation like that in a clarinet.

### 5.4 Towards the analytical formulas (with dimensions)

The playing frequency can be considered as a perturbation from the resonance frequency of the instrument. In this section, three effects or perturbations from this resonance frequency will be considered: reed-induced flow, reed damping or dynamics, and the inharmonicity of the resonator. Each of these derivations will begin with the previously stated characteristic equations that describe the instrument.

#### 5.4.1 Reed flow effect

In order to find simplified analytical formulas that describe the effect of the individual mechanisms which convert the resonance frequency into the playing frequency, we begin with equations (5.25), (5.27) and (5.31). First, with Eq. (5.31), the Bernoulli equation, where \( HK_s = p_M \), and the reed compliance, \( C_s = 1/K_s \). This is the case if \( y > -H \) based on the orientation in Fig. 5.1 corresponding to the non-beating reed regime, before the reed hits the reed table (lay) at \( y = -H \). The case where the reed is beating will be treated later.

For the resonator with natural resonance frequencies \( \omega_n \), the input impedance may be decomposed into a sum of its modal components,

\[
Z_{\text{tot}}(\omega) = \frac{\rho c}{S} j \omega \sum_n \frac{F_n}{\omega_n^2 - \omega^2 + j \omega Q_n},
\]

where, if \( \omega = \omega_n \) the impedance has a maximum value,

\[
Z(\omega_n) = \frac{\rho c}{S} \frac{F_n Q_n}{\omega_n^2}.
\]

The value of \( F_n \), a modal factor described for a maximum value of impedance at a
resonance $\omega_n$ is,

$$F_n = Z_{\text{max}_n} \frac{S}{\rho c} \omega_n,$$  \hspace{1cm} (5.38)

where $Z_{\text{max}_n}$ is the maximum value of the $n^{th}$ impedance peak. For a perfect cylinder this value should be around $2c\ell$, and is independent of $n$.

Restating the reed equation, Eq. (5.2),

$$M_r \frac{d^2 y}{dt^2} + R \frac{dy}{dt} + K_s y = -\Delta p$$  \hspace{1cm} (5.39)

Rearranging (5.16) in terms of a compliance,

$$\frac{dp}{dt} = \frac{1}{C_s} \frac{dy}{dt},$$  \hspace{1cm} (5.40)

the reed flow rate then becomes,

$$u_r = -S_r C_s \frac{dp}{dt}.$$  \hspace{1cm} (5.41)

Coming back to $u_{tot}$ as the total flow calculated by adding the Bernoulli flow term $u_b$ and the reed flow term $u_r$, as was shown in Eq. (5.7),

$$u_b = u_{tot} - u_r.$$  \hspace{1cm} (5.42)

In the frequency domain the total volume flow could be represented as,

$$U_{tot}(\omega) = Y(\omega)P(\omega),$$  \hspace{1cm} (5.43)

where $Y(\omega)$ is the admittance (inverse of impedance). Then the frequency domain version of the Bernoulli flow from Eq. (5.7) becomes,

$$U_b(\omega) = U_{tot}(\omega) - U_r(\omega),$$  \hspace{1cm} (5.44)

which may be expanded as,

$$U_b(\omega) = Y(\omega)P(\omega) + j\omega \frac{V_{eq}}{\rho c^2} P(\omega).$$  \hspace{1cm} (5.45)
Finally,

\[ U_0(\omega) = P(\omega) \left[ Y(\omega) + j\omega \frac{V_{eq}}{\rho c^2} \right] \]  \hspace{1cm} (5.46)

Writing the volume as the product of a length correction and the cross-sectional area of the tube, \( V_{eq} = \Delta \ell_0 S \), with \( \Delta \ell_0 \) represented in Eq. (5.24),

\[ V_{eq} = \Delta \ell_0 S_{tube} \quad \text{and} \quad \Delta \ell_0 = \frac{V_{eq} S}{\rho c^2} \]

(5.47)

(where here the subscript 0 was represented to \( \Delta \ell \) as to not confuse with the other length corrections in the forthcoming subsections). Then the Bernoulli flow term becomes, in the frequency domain,

\[ U_b = \left[ Y(\omega) - j\omega \Delta \ell_0 \frac{S}{\rho c^2} \right] \]  \hspace{1cm} (5.48)

The first playing frequency \( \omega_p \) may be expressed as a small variation from the first resonance frequency \( \omega_1 \): \( \omega_p = \omega_1 + \epsilon \). The resonances can be found by taking the imaginary part of the admittance from Eq. (5.36) and setting this equal to zero. The total admittance is,

\[ Y_{tot}(\omega_p) = \frac{S}{\rho c} \frac{1}{j\omega_p} \frac{\omega_1^2 - \omega_p^2 + j\omega_p \omega_1/Q_1}{F_1} \]  \hspace{1cm} (5.49)

and taking the imaginary part yields,

\[ \text{Im} \left[ Y_{tot}(\omega_p) \right] = \frac{S}{\rho c} \frac{1}{\omega_p F_1} \left[ \omega_p^2 - \omega_1^2 \right] \]  \hspace{1cm} (5.50)

Substituting \( \omega_p = \omega_1 + \epsilon \), and rearranging,

\[ \text{Im} \left[ Y_{tot} \right] = \frac{S}{\rho c \omega_1 F_1} \left[ (\omega_1 + \epsilon)^2 - \omega_1^2 \right] \]  \hspace{1cm} (5.51)

\[ = \frac{S}{\rho c \omega_1 (1 + \epsilon/\omega_1) F_1} \left[ 2\epsilon \omega_1 + \epsilon^2 \right] \]  \hspace{1cm} (5.52)

\[ = \frac{S}{\rho c \omega_1 (1 + \epsilon/\omega_1) F_1} \left[ 1 + \epsilon/2\omega_1 \right] \]  \hspace{1cm} (5.53)

Considering only the first order terms in \( \epsilon/\omega_1 \) (considering that \( \epsilon/\omega_1 \ll 1 \) for
small perturbations),

\[ \text{Im}[Y_{\text{tot}}(\omega_p)] = \frac{S}{\rho c} \frac{2\varepsilon}{F_1} = \frac{S}{\rho c} \frac{2(\omega_p - \omega_1)}{F_1} \]  

(5.54)

Normalizing the admittance,

\[ \text{Im}[Y_{\text{tot}}] \frac{\rho c}{S} = \frac{2}{F_1} (\omega_p - \omega_1). \]  

(5.55)

Now the criteria for auto-oscillations states that \( \text{Im}[Y] = \text{Im}[\frac{U_b}{F}] \) must be zero. In other words, the playing frequency is given when \( \text{Im}[Y_b(\omega)] = 0 \).

Solving the equations using the criteria that \( \text{Im}(Y_{\text{tot}}) = 0 \) will offer the playing frequencies if they are assumed to be close to the resonances \( \omega_n \). Thus,

\[ \text{Im}(Y_b) = \text{Im}(Y_{\text{tot}}) - \text{Im}(Y_r) = 0 \]  

(5.56)

\[ \frac{\omega_p - \omega_1}{F_1} \frac{S}{\rho c} + \frac{\omega_p \Delta \ell_0 S}{\rho c^2} = 0, \]  

(5.57)

and in rearranging,

\[ -\frac{\omega_1 + \omega_p}{F_1} + \frac{\omega_p \Delta \ell_0}{c} = 0. \]  

(5.58)

This leaves the value for \( \omega_1 \) in terms of the modal factor \( F_1 \), the length correction term \( \Delta \ell_0 \) and the playing frequency \( \omega_p \). In order to understand this small perturbation from resonance to playing frequency, a ratio \( \Delta f/f \) can be found in order to see the frequency difference in cents. In this case,

\[ \frac{\Delta f}{f} = \frac{\omega_p - \omega_1}{\omega_1}, \]  

(5.59)

and

\[ \omega_1 = \omega_p \left[ 1 + \frac{F_1 \Delta \ell_0}{c} \right]. \]  

(5.60)

This gives a final value for the playing frequency in terms of the resonance, modal

\[ ^1 \text{This is because } u_b \text{ is a nonlinear function of } p = F_0 + Ap + BP_2 + Cp^3 \text{ of which the first harmonic approximation is } P_1^2 = \frac{1}{2} \frac{Y_{12} A}{c}. \text{ Considering that } \text{Im}[Y_b] = 0 \text{ will offer the resonance frequencies since there are only solutions to this when } Y_b \text{ is real).} \]
factor and length correction due to reed flow information,

\[ \omega_p = \frac{\omega_1}{1 + \frac{F_1 \Delta \ell_{0}}{c}}. \]  

(5.61)

This last equation offers the possibility to predict the playing frequency based on

5.4.2 Reed dynamics effect

The study of the influence of the reed dynamics on the oscillation threshold

(5.62)

First, the continuous and variable parts of this equation should be separated. The

continuous part is when \( p = 0 \) and \( y = -p_m/K_r = -Hp_m/p_M \). For small \( p \), when \( \Delta p > 0 \) and if the reed displacement is small \( y + Hp_m/p_M \) can be replaced by

variable \( x \),

\[ \frac{y}{H} = \frac{x - p_m}{p_M}. \]  

(5.63)

Substitution into (5.31) and performing a binomial expansion on \( \sqrt{\Delta p} \) yields,

\[ u_b = wH \sqrt{\frac{2}{\rho}} \left( 1 + \frac{x}{H} - \frac{p_m}{p_M} \right) \sqrt{p_m} \left( 1 - \frac{p}{2p_m} \right). \]  

(5.64)

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When \( p = 0 \) and \( x \) is very small (approaching 0) the continuous part will be,

\[
    u_{b0} = wH \sqrt{\frac{2}{\rho}} \left( 1 - \frac{p_m}{p_M} \right) \sqrt{p_m}.
\]  

(5.65)

The variable portion, in the frequency domain, to the first order of \( x \) and \( p \) is then,

\[
    U_b = wH \sqrt{\frac{2}{\rho}} \sqrt{p_m} \left( \frac{X}{H} - \frac{P}{2p_m} \left( 1 - \frac{p_m}{p_M} \right) \right). \quad (5.66)
\]

The relationship between reed displacement and pressure is given by Eq. (5.39) in the time domain. If a new variable is defined \( D \),

\[
    D = \frac{X/H}{P/p_M}
\]  

(5.67)

then the admittance is,

\[
    Y_b = \frac{U_b}{P} = wH \sqrt{\frac{2}{\rho \rho p_M}} \sqrt{\frac{p_m}{p_M}} \left[ D - \frac{1}{2p_m} (p_M - p_m) \right].
\]  

(5.68)

This equation is shown in dimensionless form in [30] (Chaigne and Kergomard’s Eq. 9.124), but here it is shown with dimensions and without considering the reed induced flow since here the effects are considered separately.

To find the playing frequencies at the oscillation threshold (see Chapter 4) the losses are disregarded which means that \( Y \) is purely imaginary so this equation resolves to,

\[
    Y_b = \frac{U_b}{P} = j wH \sqrt{\frac{2}{\rho \rho p_M}} \sqrt{\frac{p_m}{p_M}} \text{Im}(D). \quad (5.69)
\]

At low frequencies (when the playing frequency is much lower than the reed resonance frequency) the imaginary part of \( D \) becomes,

\[
    \text{Im}(D) = \frac{-\omega \omega_r q_r}{\omega_r^2} = \frac{\omega q_r}{\omega_r}. \quad (5.70)
\]

Based on previous studies [24][30] the threshold of oscillation can be estimated to occur when \( p_m/p_M \) is about 1/3 (since the effect of the reed dynamics is assumed
to be a small amplitude perturbation). Then,

\[ Y_{\text{threshold}} = -jwH\sqrt{\frac{2}{\rho pM}}\sqrt{\frac{1}{3}}\frac{\omega q_r}{\omega_r}. \quad (5.71) \]

Let \( Z_c = \rho c/S \) be the characteristic impedance. Then if,

\[ \text{Im}(Y) \cdot Z_c = \frac{2}{F_1}(\omega_p - \omega_1), \quad (5.72) \]

then the characteristic equation may be written,

\[ \omega_p - \omega_1 = \frac{-\omega_p F_1 q_r}{2\sqrt{3}\omega_r} \left( wHZ_c\sqrt{\frac{2}{\rho pM}} \right), \quad (5.73) \]

then,

\[ \omega_p = \frac{\omega_1}{1 + \frac{F_1 q_r}{2\sqrt{3}\omega_r} \left( wHZ_c\sqrt{\frac{2}{\rho pM}} \right)}, \quad (5.74) \]

and,

\[ \omega_p = \omega_1 \left[ 1 - \frac{F_1 q_r}{2\sqrt{3}\omega_r} \left( wHZ_c\sqrt{\frac{2}{\rho pM}} \right) \right]. \quad (5.75) \]

So, for a cylinder where \( F_1 = 2c/\ell \),

\[ \omega_p = \omega_1 \left[ 1 - \frac{c}{\ell\sqrt{3}\omega_r} \left( wHZ_c\sqrt{\frac{2}{\rho pM}} \right) \right]. \quad (5.76) \]

This result provides a way to estimate the effect on the playing frequency \( \omega_p \) due to variations in the reed dynamics \( (q_r/\omega_r) \). Finally, if a length correction were sought based on this frequency shift due to the reed dynamics it would be,

\[ \frac{\Delta \ell}{\ell} = -\frac{\Delta \omega}{\omega} = \left[ \frac{c}{\ell\sqrt{3}}wHZ_c\sqrt{\frac{2}{\rho pM}} \frac{q_r}{\omega_r} \right]. \quad (5.77) \]

This is similar to the result at the oscillation threshold that was given in the form of a length correction by Silva \cite{114} Eq. 31: \( \Delta \ell = q_r Z_c w\sqrt{2y_0/(K_r\rho)}/((\omega_r/c)\sqrt{3}) \), where \( y_0 \) is the initial reed opening. The method of Silva \cite{114} leads to the following
result for playing frequency, if the tube is a one-mode oscillator,

$$\omega_p = \omega_1 \left[ 1 - \frac{Z_r w \sqrt{2 y_0 / (K_r r) F_1 q_r}}{2 \sqrt{3} \omega_r} \right]. \quad (5.78)$$

It should be noted that Nederveen tried to describe this effect, but he considered a reed with high damping and an infinite natural frequency \[99\], in his Equation (25.23), the length correction is denoted $\Delta \ell_r$, and increases with the reed opening at rest, in accordance with the above result. Further comparison with his result is difficult, because a different nonlinear characteristic was used.

### 5.4.3 Resonator Inharmonicity effect

If the resonance frequencies are not exactly harmonic, due an imperfect cylindrical instrument or the presence of losses, then the playing frequency changes with the level of excitation \[19\]. In order to find the changes in playing frequency due to inharmonicity a method is considered which is valid for any given shape of the nonlinear characteristic for both non-beating and beating reed cases. In this case the reed dynamics are ignored for simplicity. Then a static characteristic exists in Eq. (5.25) since each effect is considered independently.

The level of inharmonicity will be named $\eta_n$,

$$\omega_n = n \omega_1 (1 + \eta_n) \quad (5.79)$$

where $\omega_n$ are the natural resonance frequencies. Clarinet-like instruments will exhibit weak inharmonicity and the “reactive power rule” found by Boutillon for bowed instruments \[26\] can be limited to odd harmonics. Seeking the playing frequency in the form,

$$\omega_p = \omega_1 (1 + \varepsilon), \quad (5.80)$$

with $\varepsilon$ being very small, only the first order terms of the spectrum will be kept in consideration. If the nonlinear characteristic $u = F(p)$ is considered in the frequency domain as $U(\omega) = Y(\omega) P(\omega)$, using the harmonic balance technique found in \[30\, Section 4.1\], this leads to the following,

$$\sum_{n>0} |P_n|^2 n \text{Im}[Y(n \omega_p)] = 0, \quad (5.81)$$
where \( P_n \) are the pressure components in the spectrum for each \( n \) peak. Rearranging,

\[
\sum_{n>0} |P_n|^2 n \frac{2}{F_n} (n\omega_p - \omega_n) = 0.
\] (5.82)

Replacing the playing frequency with Eq. (5.80) and \( \omega_n \) from Eq. (5.79),

\[
\sum_{n>0} |P_n|^2 2n \frac{2}{F_n} (n\omega_1(1 + \varepsilon) - (n\omega_1(1 + \eta_n))) = \sum_{n>0} |P_n|^2 n^2 \frac{2}{F_n} \omega_1(\varepsilon - \eta_n) = 0
\] (5.83)

Finally,

\[
\sum_{n>0} |P_n|^2 n^2 \varepsilon - \eta_n F_n = 0.
\] (5.84)

If \( \eta_1 = 0 \) (the difference between the first impedance peak and the resonance frequency should be zero) and \( F_n = 2c/\ell \) (nearly cylindrical instrument),

\[
\sum_{n>0} |P_n|^2 n^2 (\varepsilon - \eta_n) = 0.
\] (5.85)

Then the first harmonic can be separated from the others:

\[
\sum_{n>1} |P_n|^2 n^2 (\varepsilon - \eta_n) = -|P_1|^2 \varepsilon.
\] (5.86)

Solving for \( \varepsilon \) while ignoring the even harmonics in the mouthpiece pressure spectrum of a clarinet,

\[
\varepsilon = \frac{\sum_{n \text{ odd} \geq 3} \eta_n d_n}{1 + \sum_{n \text{ odd} \geq 3} d_n}
\] (5.87)

with

\[
d_n = n^2 \left| \frac{P_n}{P_1} \right|^2.
\] (5.88)

Note that for a square wave signal, \( d_n = 1 \) for every odd \( n \). If the dependence of the spectrum on the excitation pressure is known, it is possible to deduce the variation in the playing frequency. Approximate formulas for clarinet-like instruments were given by Kergomard et al. [84]. For a clarinet, the decrease of the higher harmonics is always faster than it is for a square wave signal, therefore it is reasonable to
search for an approximate formula by limiting the series to the third harmonic only.

The ratio of the amplitude $P_3/P_1$ is given by applying the harmonic balance method and can be deduced from Eq. (21b) of [84].

\[ \frac{P_3}{P_1} = -\frac{1}{3} \frac{1}{1 + z}, \]  \hspace{1cm} (5.89)

\[ z = \frac{Y_3 - Y_1}{A - Y_1}. \]  \hspace{1cm} (5.90)

Here, $Y_n$ is the admittance value of the $n^{th}$ peak. Here, $A$ is dimensionless and is both a function of blowing pressure and reed opening.

\[ A = \frac{2}{p_m \rho} \frac{\sqrt{2 \left( \frac{p_m}{p_M} \right) - 1}}{2 \sqrt{\frac{p_m}{p_M}}}. \]  \hspace{1cm} (5.91)

Experimentally, there have been measurements of $P_3/P_1$ smaller than $1/3$, but this formula is a good approximation for both non-beating and beating reeds. Finally, from Eq. (5.87), a first order approximation is found:

\[ \varepsilon = \frac{\eta_3}{1 + |1 + z|^2}, \]  \hspace{1cm} (5.92)

where $\eta_3 = (\omega_3 - 3\omega_1)/3\omega_1$. At the oscillation threshold $A = Y_1$, $\varepsilon = 0$ (the signal is sinusoidal), and $Y_1$ is real. For large excitation pressure, $z$ tends to zero, and $\varepsilon$ to $\eta_3/2$. Notice that because the reasoning is based upon a perturbation at the first order, the values of $Y_3$ and $Y_1$ can be determined without the effect of inharmonicity being present, i.e. the values for both are real.

Refinements to Eq. (5.92) are quite difficult; the formula for the 5th harmonic is very complicated, [84, Eq. (21b)]. Nevertheless, this formula offers the extent of the variation of the effect on inharmonicity of the second peak (3rd harmonic) and doesn’t necessarily warrant calculation beyond this as the higher order effects would be quite a bit smaller.
5.5 Dimensionless Equations

The formulation of the numerical simulation code that will be discussed in Chapter 6 use normalized (dimensionless) variables for blowing pressure and reed opening. The numerical simulations results will be used to validate the analytical formula predictions. Therefore, in order to make comparisons between the analytical formulas and numerical simulations, it is necessary to make all variables dimensionless. All pressures will be divided by the closing pressure $p_M$, flow terms will be divided by $p_M/Z_c$ where $Z_c = \rho c/S$, and all displacements will be divided by $H$, the opening at rest. For example, dimensionless displacement, pressure, and velocity terms will become,

$$\tilde{y} = \frac{y}{H},$$ (5.93)

$$\tilde{p} = \frac{p}{p_M},$$ (5.94)

$$\tilde{u} = \frac{uZ_c}{p_M}.$$ (5.95)

Two dimensionless parameters are very important in the remainder of this dissertation. The dimensionless blowing pressure $\gamma$ is the pressure inside the mouth normalized by the closing pressure,

$$\gamma = \frac{p_m}{p_M},$$ (5.96)

and the dimensionless reed opening $\zeta$, depending on the characteristic impedance and amount of flow allowed into the mouthpiece is,

$$\zeta = Z_c \frac{u}{p_M} = Z_c wH \sqrt{\frac{2}{\rho p_M}}.$$ (5.97)

In the case of the clarinet the dimensionless reed opening $\zeta$ will be most interesting in the lower bound of 0 (a reed very closed) and in this work, an upper bound of 1 (a reed mostly open). The values for $\gamma$ will range between 0 and 1 (from blowing pressure beginning at zero ranging to the upper limit of the static closing pressure $p_M$). The blowing pressure is not limited to this value in the general model. However, beyond $\gamma = 1$ the static and oscillating regime could both be stable.
Blowing pressures above $\gamma = 1$ would not be of interest in musical acoustics research since only blowing pressures within the oscillating regime ($\gamma_{th} \geq \gamma \leq 1$) will produce the sounds deemed usable in a musical context. Throughout this work, the values for each of these parameters will be limited between 0 and 1. This range of $\gamma$ values makes it easier to visualize and map the three different effects (reed induced flow, reed damping/dynamics and inharmonicity) for each note of the instrument.

5.5.1 Dimensionless forms of the important equations

The three characteristic equations, (5.25), (5.26), and (5.35) can be made dimensionless by using the techniques from the previous section. The closing pressure is negative so that a dimensionless quantity,

$$x = \tilde{y} - \gamma,$$  \hspace{1cm} (5.98)

can be introduced. Then Eqs. (5.25), (5.26), and (5.35) can be replaced respectively with the following,

$$\tilde{u} = \left[ h \ast \tilde{\rho} \right](t) + \frac{\Delta \ell}{c} \frac{dx}{dt}$$ \hspace{1cm} (5.99)

$$\frac{1}{\omega_r^2} \frac{d^2 x}{dt^2} + \frac{q_r}{\omega_r} \frac{dx}{dt} + x = -\tilde{p}$$ \hspace{1cm} (5.100)

$$\tilde{u}_b = \zeta (1 + x + \gamma) \sqrt{\gamma - \tilde{p}}$$ \hspace{1cm} (5.101)

For the remainder of the chapter, the tilde will, in general, be omitted from these dimensionless variables and when necessary, a subscript ‘d’ will be included to signify a variable with dimensions.

5.5.2 Threshold of Oscillation

To understand at what point in blowing pressure the reed will begin to oscillate, the most basic form of the nonlinear characteristic equation should be considered.

The series expansion for the nonlinear characteristic equation for total flow $u_{tot}(p)$ is $u(p) = F_0 + Ap + Bp^2 + Cpq^3$. $F_0$, $A$, $B$, and $C$ are the zeroth, first, second and third order series expansion coefficients respectively. In this representation, Chaigne and Kergomard [30] show that the coefficients have the following values,
when made dimensionless,

\[ F_0 = \zeta(1 - \gamma)\sqrt{\gamma}, \quad A = \zeta \frac{3\gamma - 1}{2\sqrt{\gamma}}, \quad B = -\zeta \frac{3\gamma + 1}{8\gamma^{3/2}}, \quad C = -\zeta \frac{\gamma + 1}{16\gamma^{5/2}}. \] (5.102)

Linearizing the series expansion of \( u(p) \) gives \( u(p) = F_0 + Ap \) and since \( A = \frac{\zeta(3\gamma - 1)}{2\sqrt{\gamma}} \) Chaigne and Kergomard \[30\] were able to show that at the pressure threshold of oscillation \( A = 1/3 \). Therefore a dimensionless parameter, \( \gamma_{th} \) can be found,

\[ \gamma_{th} \simeq \frac{1}{3} + \frac{2\text{Re}[Y(\omega_1)]}{3\sqrt{3}\zeta}. \] (5.103)

For more information on the expansion of the nonlinear characteristic and the series representation of the pressure function, the reader is referred to Chaigne and Kergomard’s Chapter 9 \[30\].

### 5.6 Towards dimensionless individual effects on playing frequency

The impedance measurement may be used to find the playing frequency for a clarinet. The first peak of the impedance spectrum can serve as an estimate of the playing frequency, but the input impedance measurement itself characterizes the resonator and does not include the external factors such as the player nor reed effects. As was discussed in the introduction of this chapter, there are known formulas that describe the effects of the reed induced flow, reed damping/dynamics, and inharmonicity of the resonator.

The research represented in this dissertation is the first of its kind to include these three effects into simplified analytical formulas in order to predict the playing frequency for a clarinet. Each of these effects can be formulated as small corrections to the total length of the clarinet which will either raise or lower the resonance frequency. All of the effects will create a small correction, and thus they can be added all together. The resulting frequency is called the playing frequency. It is assumed that ideally this would be close to the tempered scale frequency so that the clarinet is able to be played in tune in an orchestral setting. Therefore comparisons will be made with 12-tone equal tempered scale frequencies as the benchmark.
5.6.1 Flow rate due to the reed movement

The effect of reed induced flow into the instrument is a relatively new phenomenon to be studied and added to the model. It is important to understand the behavior of this effect in the two different reed regimes. For a reed in the beating reed regime, the reed displacement is limited by the mouthpiece lay, and therefore the flow rate is limited as well. When the flow rate is considered, the condition \( \frac{dx}{dt} = 0 \) if \( x < -1 \) replaces the classical condition \( u_b = 0 \) if \( x < -1 \) [74]. The following discussion of the beating reed regime follows the approach of Dalmont et al. [45], who published a satisfactory comparison between a simple model and experiment. For oscillations in the beating reed regime, the signal of the mouthpiece pressure is not far from a square wave signal, which is the exact shape for a perfect cylinder when losses, radiation and reed dynamics are ignored. With these assumptions, the pressure takes the following values: \(-\gamma\) when the reed beats, and \(+\gamma\) when the reed is open. For the first harmonic of frequency \( \omega_1 \), the amplitude of the mouthpiece pressure is therefore \( 2\gamma/\pi \). The reed displacement is expected to vary between \(-1\) and 0. Thus its first harmonic has an amplitude of \( 1/\pi \), and \( P(\omega_1) = 2\gamma X(\omega_1) \) in the frequency domain. This means,

\[
\frac{dp}{dt} \simeq 2\gamma \frac{dx}{dt},
\]

in the time domain. This result is in agreement with the case of the non-beating reed regime (before the reed touches the facing): for the simplest case of a lossless resonator, ignoring reed dynamics, the beating-read threshold (the point, in pressure when the reed first touches the facing) is given by \( \gamma = 1/2 \). Finally, both cases of the non-beating and beating reed regime are considered by using the equation,

\[
U_b(\omega) = \left[ Y(\omega) + j\frac{\omega \Delta\ell_{eq}}{c} \right] P(\omega),
\]

where \( \Delta\ell_{eq} = \frac{\Delta\ell_0}{G(\gamma)} \) with \( \Delta\ell_0 = \frac{pc^2}{p_m} \frac{s_e}{S} H \) and where \( G(\gamma) = 1 \) when \( \gamma < 0.5 \) and \( G(\gamma) = 2\gamma \) when \( \gamma \geq 0.5 \) (see [122]). The function \( G \) is here given for an abrupt stop of the reed when closing the reed channel. A more sophisticated function \( G \) could be considered in order to take into account a progressive reduction of the moving surface, as observed in practice for high blowing pressures [45].
5.6.2 Extracted modal parameters

The input impedance has been measured using a device which was built in Le Mans, France [88] and will be discussed in Chapter 7. The source is a piezoelectric transducer and the pressure in the back cavity of the sound source is measured by a microphone, which gives an estimation of the flow rate. The method to extract the modal parameters from the measured impedance plots is based upon a local optimization procedure (nonlinear least-square algorithm), and is not discussed here. For each mode the modal frequency $\omega_n$, the quality factor $Q_n$, and the modal factor $F_n$ are extracted. The factor $F_n$ is roughly proportional to the fundamental frequency of the notes and is nearly independent of register (the values are almost equal for $n = 2$ and $n = 1$); this behavior is very similar to that of a perfect cylinder (please see Chapter 6 for a further discussion about the modal parameters).

$$Z_n = F_n Q_n / \omega_n$$

is the value of the impedance at $\omega = \omega_n = 2\pi f_n$, when ignoring the effect of the other modes in the series. Moreover, the inharmonicity $\eta_n$ between Mode $n$ and Mode 1 is given by the solution of,

$$\eta_n = \frac{\omega_n - n\omega_1}{n\omega_1}.$$  (5.106)

5.7 Dimensionless Analytical Formulas for the Playing Frequency

5.7.1 Playing frequency in the ideal case

The ideal case could be considered where the effects of the reed flow rate and reed dynamics are ignored: ($\Delta\ell_{eq} = 0$, $p = x + \gamma$). For this ideal case, the playing frequency at the oscillation threshold is given by,

$$A = Y(\omega) \text{ with } A = \frac{\zeta(3\gamma - 1)}{2\sqrt{\gamma}}.$$  (5.107)

$A$ is the coefficient for the linear term of Kergomard’s polynomial representation of the nonlinear characteristic $u_b(p)$ in Eq. (5.35), when expanding the function with respect to $p$ around the static regime $p = 0$ (if $Z(0)$ is assumed to vanish). Because
A is real,

\[ \text{Im} \{ Y(\omega) \} = 0, \]  

thus for the first register \( \omega = \omega_1 \), and \( A = \text{Re} \{ Y(\omega_1) \} = A_{th} \), the threshold value. For small \( \text{Re} \{ Y(\omega_1) \} \), Equation (5.107) yields the value of the pressure threshold \( \gamma_{th} \):

\[ \gamma_{th} \simeq \frac{1}{3} + \frac{2\text{Re} \{ Y(\omega_1) \}}{3\sqrt{3\zeta}}. \]  

(5.109)

Above the oscillation threshold, if the resonance frequencies are also harmonically related, the playing frequency \( \omega_p \) remains the frequency found at the oscillation threshold, i.e. \( \omega_1 \). Also, when the reed dynamics are ignored, there is a static nonlinear characteristic which links the two variables of pressure \( p \) and flow rate \( u \). This brings us back to the “reactive power rule” found by Boutillon for bowed instruments \(^{[26]}\):

\[ \sum_n |P_n|^2 n\text{Im}[Y(n\omega_p)] = 0. \]

This equation, where \( P_n = P(n\omega_p) \) is the amplitude of the \( n^{th} \) harmonic of the pressure, is one of the harmonic balance system of equations. If all the resonance frequencies are harmonically related to the first one, this equation is satisfied for \( \omega_p = \omega_1 \) regardless of the spectrum (or equivalently, the excitation conditions). Thus the playing frequency does not change with the excitation level. The previous explanation seems to be trivial, but this is useful when studying the non-ideal case, treated as a perturbation of the ideal one.

### 5.7.2 Approximations for the playing frequency in the non-ideal case

With Eqs. (5.25) - (5.109) from the model, it is now possible to deduce approximations for the difference between natural frequencies and the playing frequencies \( \omega_p \) of the clarinet. The frequency difference \( \Delta \omega \) is sought:

\[ \Delta \omega = \omega_p - \omega_1. \]  

(5.110)
If this value is small enough, the relative difference can be expressed in cents, as follows:

\[ N_{cents} = 1200 \log_2(\omega_p/\omega_1) \approx \frac{100}{0.06} \frac{\Delta \omega}{\omega_1}. \]  

(5.111)

This is because one semitone (100 cents) corresponds to a ratio of \( f_2 = 1.06 \times f_1 \) and because we can approximate the logarithm after the difference as \( \frac{100}{0.06} \), for a cylinder of length \( \ell \), a length correction can be defined as:

\[ \frac{\Delta \ell}{\ell} = -\frac{\Delta \omega}{\omega_1}. \]  

(5.112)

In what follows, the three effects are considered separately, assuming that the frequency shifts (or the length corrections) can simply be added.

### 5.7.2.1 Flow rate due to the reed movement

If both the reed dynamics and the influence of higher order harmonics are ignored, the playing frequency \( \omega_p \) is given by its value at the oscillation threshold [65].

Because \( A \) is real in Eq. (5.107), Eq. (5.105) yields a generalization of Eq. (5.108),

\[ \text{Im} \left[ Y(\omega_p) + j\omega_p \Delta \ell_{eq} \right] = 0. \]  

(5.113)

Given that the quality factor of a chosen impedance peak is high enough, only this peak is kept in the modal decomposition around the resonance frequency \( \omega_n \) and the following approximation is valid,

\[ \text{Im} [Y(\omega_p)] = \frac{2}{F_n} (\omega_p - \omega_n). \]  

(5.114)

Therefore, for the first register, the solution of Eq. (5.113) is:

\[ \omega_p = \frac{\omega_1}{1 + \frac{\Delta \ell_{eq} F_1}{2c}}. \]  

(5.115)

For a perfect cylinder, if the length correction is small enough that \( \omega \Delta \ell/c \approx \tan (\omega \Delta \ell/c) \), the same result is found immediately:

\[ \omega_p = \frac{\omega_1}{1 + \frac{\Delta \ell_{eq}}{\ell}}. \]  

(5.116)
with \( \omega_1 = \frac{\pi c}{2\ell} \). For this particular case, the effect of the flow rate can be viewed as a simple length correction. Eqs. (5.105) and (5.115) are those proposed for this first effect, for both a beating reed and a non-beating reed. Finally, it is written:

\[
N_{cents_{flow}} = -\frac{100}{0.06} \frac{F_1 \Delta \ell_0}{2G(\gamma)c}, \tag{5.117}
\]

where \( \Delta \ell_0 = \frac{\rho c^2 s}{p M S} H \) and where \( G(\gamma) = 1 \) when \( \gamma < 0.5 \) and \( G(\gamma) = 2\gamma \) when \( \gamma \geq 0.5 \).

### 5.7.2.2 Reed dynamics

When reed dynamics are considered \( A \neq Y(\omega_p) \) at the threshold of oscillation. The study of the influence of the reed dynamics on the oscillation threshold (frequency and mouth pressure) has been done by Wilson and Beavers \[126\], and extended by Silva et al. \[114\] by adding the effect of the reed flow rate. The results are valid for the case of strong reeds (e.g., organ reeds, with small reed damping), and weak reeds (e.g., woodwind reeds, with high damping by the lips). This method involves the linearization of Eq. (5.29), and solving the characteristic equation. Here the case of high damping, i.e., large \( q_r \) is considered. For a cylinder, the result at the oscillation threshold was given in the form of a length correction in \[114\, Eq. (31)\]:

\[
\Delta \ell = q_r \zeta/(k_r \sqrt{3}),
\]

where \( k_r = \omega_r/c \). The method of Ref. \[114\] leads to the following result if the tube is a one-mode oscillator:

\[
\omega_p = \omega_1 \left[ 1 - \frac{\zeta F_1 q_r}{2\sqrt{3} \omega_r} \right]. \tag{5.118}
\]

Nederveen tried to describe this effect, but he considered a reed with high damping and an infinite natural frequency \[99\]. In his Eq. (25.23), the length correction is denoted \( \Delta \ell_r \), and increases with the reed opening at rest, which is in accordance with the result in Eq. (5.118). Further comparison with his result is difficult because a different nonlinear characteristic was used.

Equation (5.78) is valid at the threshold of oscillation, for a non-beating reed and a very small excitation pressure \( \gamma \). Nevertheless, Kergomard and Gilbert \[83\], using the harmonic balance method analytically (limited to the first harmonic),
found the following dependence on the excitation pressure:

\[
\omega_p = \omega_1 \left[ 1 - \frac{\zeta F_1}{2\sqrt{3} \omega_r} \left[ 1 + \frac{3}{4}(\gamma - \gamma_{th}) \right] \right].
\]  
(5.119)

This is the beginning of a series expansion above the oscillation threshold \(\gamma_{th}\), consequently the formula is not necessarily valid at high values of \(\gamma\), i.e. Eq. (5.119) is obtained for the non-beating reed regime only. When losses in the pipe are ignored, the value of the oscillation threshold \(\gamma_{th}\) is given by Silva et al. [114]. However, the current work uses small perturbation reasoning and therefore an approximate version of Silva’s threshold of oscillation can be used, as in Eq. (5.109).

This effect can be transformed into a simple change in frequency, in cents, as well and is represented as follows:

\[
N_{\text{cents, dynamics}} = -100 \cdot \frac{\zeta F_1}{0.06} \frac{q_r}{2\sqrt{3} \omega_r} \left[ 1 + \frac{3}{4}(\gamma - \gamma_{th}) \right].
\]  
(5.120)

### 5.7.2.3 Effect of the inharmonicity of the resonator

If the resonance frequencies are not exactly harmonic, the playing frequency changes with the level of excitation [19]. The method is valid for any given shape of the nonlinear characteristic \(u = F(p)\), for both non-beating and beating reed regimes. In this case, because reed dynamics are ignored, this (static) characteristic exists. For clarinet-like instruments with weak inharmonicity (small \(\eta_n\)), the summation in Eq. (5.87) can be limited to odd harmonics. It is possible to use Eq. (5.114) near every resonance frequency, seeking the playing frequency in the form:

\[
\omega_p = \omega_1 (1 + \varepsilon).
\]  
(5.121)

At the first order in \(\eta_n\) and \(\varepsilon\), Eq. (5.87) yields:

\[
\sum_{n \text{ odd}} n^2 |P_n|^2 \varepsilon - \frac{\eta_n}{F_n} = 0.
\]  
(5.122)
For cylindrical instruments, because the modal factor $F_n$ is nearly independent of $n$ (and equal to $2c/\ell$) and because $\eta_1 = 0$, the final result for the playing frequency is Eq. (5.80), with:

\[
\varepsilon = \sum_{n \text{ odd} \geq 3} \frac{\eta_n d_n}{1 + \sum_{n \text{ odd} \geq 3} d_n} \text{ with } d_n = n^2 \left| \frac{P_n}{P_1} \right|^2 .
\]  

(5.123)

(note that for a square wave signal $d_n = 1$ for every odd $n$). If the dependence of the spectrum with respect to the excitation pressure is known, it is possible to deduce the variation in the playing frequency. Approximate formulas for clarinet-like instruments were given by Kergomard et al. [84]. The decrease of the higher harmonics is always faster than in the case of the square wave signal, therefore it is reasonable to search for an approximate formula by limiting the series to the third harmonic only. The ratio of the amplitude $P_3/P_1$ is given by Eq. (21b) of [84]:

\[
\frac{P_3}{P_1} = -\frac{1}{3} \frac{1}{1 + z}
\]  

(5.124)

where,

\[
z = \frac{Y_3 - Y_1}{A - Y_1} \text{ and } Y_n = \frac{1}{Z_n} \text{ and } A = \frac{\zeta (3\gamma - 1)}{2\sqrt{\gamma}}
\]

Experimentally, there have been measurements of $P_3/P_1$ smaller than 1/3, but this formula is a good approximation for both the non-beating and beating reed regimes. Finally, a first order approximation is found:

\[
\varepsilon = \frac{\eta_3}{1 + |1 + z|^2}.
\]  

(5.125)

At the threshold $A = Y_1$, $\varepsilon = 0$ (the signal is sinusoidal), and $Y_1$ is real. For large excitation pressure, $z$ tends to zero, and $\varepsilon$ to $\eta_3/2$. Notice that because the reasoning is based upon a perturbation at the first order, the values of $Y_3$ and $Y_1$ can be determined without inharmonicity, i.e. they are real.

Refinements to this formula would be quite intricate (as an example the formula for the 5th harmonic is very complicated, see [84, Eq. (21b)]). Nevertheless, this formula exhibits the sense of variation of the effect on inharmonicity of the second peak and doesn’t necessarily warrant calculation beyond this. As before it is helpful
to transform the frequency changes into a simpler value, in cents:

\[ N_{cents_{inharmonicity}} = \frac{100}{0.06} \cdot \frac{\eta_3}{1 + |1 + z|^2} \]  

(5.126)

5.8 The temperature gradient effect

Up to this point, the calculations for playing frequencies have been based on the resonance frequencies measured at 20°C, for both numerical and analytical calculations. It is intuitive that the air closest to the mouthpiece, having just left the air-column of the instrumentalist, would have a higher temperature than the air that will reach the open end of the clarinet. In general, an average temperature over the instrument was used in predictions of playing frequencies since it was considered to not vary greatly with a change in note \[46\]. Previous research stated that the temperature could be seen as an average over the instrument since it does not vary greatly with a change in note \[63,124\]. However, recent measurements by Noreland \[102\] show that these temperature differences can be as much as a 9°C difference from the top of the barrel (\(T_0 = 31°C\)) to the bottom of the bell (22°C). He also found that the temperature profile is linear and nearly independent of note (i.e. the number of open holes). Thus the consideration of this effect is simple. In order to take the temperature value and gradient into account, the frequency shift due to a temperature change of 11°C (from 22°C to 31°C), is first computed, then the effect of the temperature decrease inside the instrument. A correction to the final results is simply added which is equivalent to the effect of a \(\Delta T = 11°C\) change in temperature, nearly 33 cents.

The calculation for the effect of the gradient is similar to that for flute-like instruments done by Coltman \[35\], where he considered a change in temperature over a length \(dx\) to be an additional acoustic mass \(\rho LA\), since the temperature affects the density. Since here we consider a closed tube at the entrance, Coltman’s approximations are modified by replacing the cosine function by the sine function:

\[ \Delta \ell = \int_0^\ell \frac{\delta \rho}{\rho} \sin^2(kx)dx, \]  

(5.127)
where $\delta \rho$ is the variation of density and $\ell$ is the equivalent length of the instrument at the particular note. Using the measured linear temperature gradient from Eq. (5.127):

$$T(x) = T_0 \left[ 1 - \kappa \frac{x}{L} \right], \quad (5.128)$$

since a change in density $\rho$ can be considered comparable to a change in temperature,

$$\frac{\delta \rho}{\rho} = - \frac{\delta T}{T} = \kappa \frac{x}{L}. \quad (5.129)$$

Here, $L$ is the total length of the instrument, $\kappa$ is an empirical constant depending on the temperature difference between mouthpiece and bell, and $x$ here is the distance from the top of the barrel, a useful analytical formula is found from Eq. (5.127). The equivalent length correction for the temperature gradient inside the clarinet is found to be:

$$\frac{\Delta \ell}{\ell} = \left( \frac{1}{4} + \frac{1}{\pi^2} \right) \kappa \frac{\ell}{L}, \quad (5.130)$$

where $\kappa = 9/(T_0 + 273)$ (based on measurements in [102]). In this calculation, the formula $k \ell = \frac{\pi}{2}$ is assumed since in this work only the first register was studied. The specific effect of the CO$_2$ content and the percent humidity are not included in these calculations since they are small values and can be assumed to have a very small effect [34, 35, 58, 59]. In a 1997 study, Fuks states that the effect of gas changes can cause as much as a 20 cent decrease in fundamental frequency [58]. However this was for a long sustained note whereas this work is interested in a small portion of the playing frequency in the steady-state regime and not on the evolution of pitch throughout a sustained note.

The final result, in cents without dimensions, for this effect is:

$$N_{\text{cents, temperature}} = \frac{100}{0.06} \left( - \left( \frac{1}{4} + \frac{1}{\pi^2} \right) \kappa \frac{\ell}{L} + \frac{1}{2} \frac{\Delta T}{(T_0 + 273)} \right). \quad (5.131)$$
5.9 Example results

Now that expressions have been obtained which can describe the frequency shifts due to each individual effect of reed flow, reed dynamics, inharmonicity, and temperature, the equations can be used to predict the playing frequencies for a given note of the clarinet. Figures in this section will show each individual effect as well as the combination of the first three in order to show the magnitude of each individual effect per note and obtain an accurate prediction of the final playing frequency.

Each note in the following subsections was run once with $\zeta = 0.2$ and once with $\zeta = 0.45$. Both plots for each note were made using the following parameters:

- reed quality factor: $q_r = 0.2$
- reed resonance frequency: $\omega_r = 2\pi \cdot 2400$ Hz
- vibrating width of reed: $w_r = 10 \times 10^{-3}$ m
- vibrating length of reed: $l_r = 6.5 \times 10^{-3}$ m
- $c = 343$ m/s
- mouth temperature: $26.5^\circ$ C
- impedance measurement room temperature: $20^\circ$ C
- $\rho = 1.2$ kg/m$^3$

Each of the calculated playing frequencies are then compared to the resonance frequency of the clarinet which was determined from the input impedance measurements (see Chapter 3 and 7). These values can also be compared to the tempered scale frequencies, these will be discussed more in Chapter 9 when comparing to other prediction methods and measurements. For each plot the color scheme is the same:

- RED line: reed dynamics effect
- TEAL line: inharmonicity effect
- GREEN line: reed induced flow effect
• **BLUE** line: total of above three effects

• **MAGENTA** line: temperature gradient effect

Notice that, for the moment, the blue line (representing the total of all effects) does not include the temperature variation, this is because the method used in creating these results for the total difference will eventually be compared to the numerical simulations where, this additional length correction due to a temperature gradient is not possible.

Each plot shows the same important trends:

• reed dynamics: this effect causes a very small shift downward in frequency for each of the notes shown here. This effect is nearly linear, showing that it is not heavily dependent on the choice of blowing pressure $\gamma$.

• inharmonicity: at the threshold of oscillation (the beginning of the plot on the horizontal axis) this effect is zero. This effect is quite dependent on the reed opening parameter $\zeta$.

• reed induced flow: the effect is constant until the beating reed, at $\gamma = 0.5$.

• temperature: the effect is independent of reed opening $\zeta$ and blowing pressure $\gamma$ but highly dependent on note (effective total length depending on which holes are open).

There are three example notes shown in the following subsections, notes 1, 12 and 17. These notes represent a full resonator, half resonator, and short resonator respectively. For more detailed plots, discussions and comparisons, refer to Chapter 9.

### 5.9.1 Note 1

Figures 5.5a and 5.5b show the analytical formula predictions for Note 1 of the clarinet using $\zeta$ value of 0.2 and 0.45 respectively, for comparison. The literature states that a generally a value of about 0.3 is used when playing the clarinet. Notice that for a higher value of $\zeta$ the reed will begin to vibrate at a lower value of blowing pressure $\gamma$. Note as well that the vertical axis values in cents are different for the two values of $\zeta$. The higher $\zeta$ value affects the threshold of oscillation and most
significantly the reed dynamics effect in red. A high value of $\zeta$ means a more open reed therefore more margin for the reed dynamics to have an effect. For each value of $\zeta$ there are similar trends in the overall behavior of the individual effects such as the fact that the inharmonicity and the reed dynamics effects will increase (become more flat) with increasing blowing pressure. In contrast, the reed induced flow effect will decrease (have a greater slope) with increasing blowing pressure. The dominant effect for each note and for each $\zeta$ is the reed induced flow effect which seems to largely influence the shape of the total trendline (in blue).

5.9.2 Note 12

Figures 5.6a and 5.6b show the analytical formulas predictions for Note 12 of the clarinet using a $\zeta$ value of 0.2 and 0.45 respectively, for comparison. Comparing Figs. 5.6a and 5.6b with 5.5a and 5.5b respectively it is apparent that with increasing note the inharmonicity effect decreases and the temperature and reed induced flow effects increase.

5.9.3 Note 17

Figures 5.7a and 5.7b show the analytical formulas predictions for Note 17 of the clarinet using a $\zeta$ value of 0.2 and 0.45 respectively, for comparison. As the notes continue to increase the intensity of the effect of the reed induced flow drastically increases. For Note 17, in Figs. 5.7a and 5.7b the effect is nearing -50 cents flat, almost a semitone. This seems to follow a trend that the closer the first open hole is to the mouthpiece, the reed effects will have more of an impact and the resonator effect (inharmonicity) will have a lesser impact.
(a) Variation in playing frequency for Note 1 as a function of increasing blowing pressure \( \gamma \) and a fixed reed opening of \( \zeta = 0.2 \).

(b) Variation in playing frequency for Note 1 as a function of increasing blowing pressure \( \gamma \) and a fixed reed opening of \( \zeta = 0.45 \).

Figure 5.5: Analytical formula results - Note 1 (lowest note on the clarinet): reed dynamics, inharmonicity, reed induced flow, total of above three effects, temperature gradient effect.
(a) Variation in playing frequency for Note 12 as a function of increasing blowing pressure $\gamma$ and a fixed reed opening of $\zeta = 0.2$.

(b) Variation in playing frequency for Note 12 as a function of increasing blowing pressure $\gamma$ and a fixed reed opening of $\zeta = 0.45$.

Figure 5.6: Analytical formula results - Note 12: reed dynamics, inharmonicity, reed induced flow, total of above three effects, temperature gradient effect.
(a) Variation in playing frequency for Note 17 as a function of increasing blowing pressure $\gamma$ and a fixed reed opening of $\zeta = 0.2$

(b) Variation in playing frequency for Note 17 as a function of increasing blowing pressure $\gamma$ and a fixed reed opening of $\zeta = 0.45$

Figure 5.7: Analytical formula results - Note 17: reed dynamics, inharmonicity, reed induced flow, total of above three effects, temperature gradient effect
5.10 Analytical Tuning Maps

This method of predicting playing frequencies of a clarinet could be very useful to musicians. Using the analytical formulas and taking a loop over each of the playing parameters can give a full picture of the tuning tendencies described by the analytical formulas.

For a very small grid of 1000x1000 in $\gamma$ and $\zeta$, tuning maps were run for each note in under 4 hours. The limits for all color plots were placed at no more or less than 100 cents different seeing as how this would mean the clarinet was predicted to be playing a different note entirely (a semitone or half-step different).

Figure 5.8 shows the analytical formulas tuning maps for the first register of the clarinet. For each plot the horizontal axis is $\gamma$ increasing from 0 to 1 and the vertical axis is $\zeta$ from 0 to 1. The black portion at the beginning of each plot represents the threshold of oscillation for each note, before which no pitch is detectable. The color bars also match for each plot, ranging from -30 cents flat to 75 cents sharp (compared to, in this case, the resonance frequencies of the clarinet). The values for tuning on these maps correspond to the figures where only one value of $\zeta$ was chosen in Section 5.9.

However, for a player the more interesting comparison is to the equal tempered scale frequencies. Figure 5.9 shows the tuning maps compared to the equal tempered scale frequencies for all notes in the first register. The maps represent some interesting, yet not surprising data for the upper throat tones of the instrument (notes 17-19). These notes get increasingly further away from the equal tempered scale, especially for high values of $\zeta$, and most significantly for note 19. Although it could be a coincidence, this note is known to be a problem note, one that is nearly impossible to tune well without the help of “tuning fingerings”, which cover up some of the lower holes of the instrument to help with pitch without interfering with the fundamental itself.

An important trend is apparent from these maps, one that holds with musician intuition: for a fixed value of $\zeta$, the increasing pressure will cause the pitch to first drop and then increase when approaching $\gamma = 1$. If the value of blowing pressure was fixed and the reed opening $\zeta$ increased from 0 to 1 the pitch will begin sharp (small reed opening) and grow increasingly flatter as the reed reaches its maximum opening.

120
Figure 5.8: Analytical formulas tuning map - playing frequency compared with resonance frequencies of the instrument for each note. The colorbar axis (same color scheme for each plot) is $\Delta f$ in cents, horizontal axis is normalized blowing pressure $\gamma$ and the vertical axis is normalized reed opening $\zeta$. 
Figure 5.9 also shows that there is a marked change around note 10 or 11. At these notes, the tuning begins to shift, leaving more regions of light blue again at the top (near in tune), somewhat like the notes at the bottom of the register. Note 10 is the first note to have an open tonehole in the top joint of the clarinet so it would make sense that the behavior of this note would be different than those before. The more red areas (very sharp) seem to come back when the clarinet reaches the throat tones and continues to take over the lower values of $\zeta$ until note 19, which as mentioned before, does not seem to follow the same data trend as the other notes in this register.

If a player wanted to tune one note only, based on a particular value of blowing pressure and reed opening (or here $\zeta$ and $\gamma$), a map such as Figure 5.10 could be created. In Fig. 5.10, the only difference is that a tuning line of 20 cents sharp has been inserted to show the effect of one adjustment to the other notes in the first register. Meaning that, if the player wanted to play to that tuning line (20 cents sharper than the equal tempered scale) they would need to follow that line when changing their blowing pressure and reed opening playing parameters. The player could also just choose one note and dynamic and tune from there. 20 cents sharp actually corresponds to Note 1 where $\gamma \approx 0.45$ and $\zeta \approx 0.3$ a normal opening and mezzo forte dynamic just below beating. If the player was concerned with this particular note, and pulled out the barrel to tune, or chose a different reed to compensate, these maps quickly show the effect on the other notes. For example, if a player wanted to increase the frequency of a note, say Note 1, by pushing the barrel of the clarinet in, perhaps to result in a 20 cent increase in playing frequency, Fig. 5.10 shows how the other notes in the first register would fare. Note 8, for example, would not have any combination of $\zeta$ and $\gamma$ that would result in playing the note in tune (compared to the chosen tuning for Note 1).
Figure 5.9: Analytical formulas tuning map comparing the playing frequency with equal tempered scale frequencies for each note. The colorbar axis (same color scheme for each plot) is $\Delta f$ in cents, horizontal axis is normalized blowing pressure $\gamma$ and the vertical axis is normalized reed opening $\zeta$. 
Figure 5.10: Analytical formulas tuning map. A tuning line is added to follow the 20 cents line for each note. The tuning is compared with equal tempered scale frequencies.
5.11 Conclusions

This chapter has explained the work necessary to take the three important characteristic equations for the functioning of the clarinet and create three separate length corrections due to the important frequency shifting effects of reed induced flow, reed dynamics and the resonator inharmonicity. Some example results were shown in the previous sections but a more in depth look at how these results compare to the validated and previously established numerical simulations will be described in Chapter 6. Chapter 7 will continue by describing the experimental procedures followed in order to take data from an actual clarinet for comparison with the analytical formulas. Chapter 8 will offer preliminary data from the artificial mouthpiece and instrumented mouthpiece experiments. Finally, Chapter 9 will offer a full comparison and discussion of the results obtained for each method.
CHAPTER

SIX

NUMERICAL SIMULATIONS

6.1 Introduction

The purpose of this chapter is to briefly introduce and outline the functioning of
the numerical code (written in MATLAB) used as a benchmark for this research.
Because of its well documented success in the literature [7,100,114], this numerical
model and its results will be used to validate the results found from using analytical
formulas from Chapter 5 and comparisons between the two will be offered in
Chapter 9. The numerical code was written initially by Philippe Guillemain in [69]
to simulate sounds that would be produced by a clarinet and not specifically to
predict merely the playing frequencies and many of the unique capabilities remain
unpublished. This research represents the first time the code has been used to
generate tuning maps.

This chapter will offer an overview of the basic mathematics within the numerical
code, taken from two separate references [37,69], a flowchart giving a work-flow
overview, and finally some possible results will be discussed. For further information
on the physical model upon which the numerical code is based, the reader is referred
to [69]. The numerical code synthesizes wind instrument sounds using a computer
coded classical physical model (which was the base for the analytical formulas
as well) and solves the three characteristic equations for pressure ($p$), reed
displacement ($x$, in this chapter, $y$ in Ch. 5) and total flow ($u$). The advantage
of using the numerical code lies in the fact that the code is able to replace the
linear parts of the mathematical model of the instrument with more complex bore geometries and reed behavior. The physical model on which the numerical code is based successfully describes the complicated nonlinear coupling between the resonator and generator through the Bernoulli flow equation (see Ch. 5, Eq. (5.4)). The numerical code uses sampled versions of the dimensionless physical variables throughout the synthesis process and offers a digital transposition of each part of the physical model [69]. As was the case for the analytical formulas, the algorithms are expressed analytically as functions of the physical variables and control variables ($\zeta$ and $\gamma$). Although this numerical code was not written directly for this research, Guillemain was in direct collaboration throughout this project and offered advice and guidance in the research process. Numerical sound synthesis (the intended purpose for the numerical code) is a complicated subject itself, and this chapter will merely offer a simplified explanation of the use and results of this numerical code and not an in depth review of its inner workings, nor the details of the mathematics behind it.

6.1.1 Theory and its comparison to analytical formulas

The numerical model follows the physical model presented in Section 5.3 of Chapter 5 and provides a straightforward digital transposition of each part of this physical model. The three characteristic equations for flow $u$, pressure $p$ and reed displacement $x$ are solved for their corresponding dimensionless variables, each as a function of the dimensionless control parameters for blowing pressure $\gamma$ and reed opening $\zeta$.

As was the case for the analytical formulas, the reed displacement is considered a single degree of freedom oscillator driven by the pressure difference between the mouthpiece and the mouth. Equation (5.2) in Chapter 5 shows this for the analytical model. For the numerical model can be written in dimensionless form as,

$$\frac{1}{\omega_r^2} \frac{d^2 x(t)}{dt^2} + \frac{q_r}{\omega_r} \frac{dx(t)}{dt} + x(t) = p_e(t),$$

with

$$p_e(t) = p(t) - \gamma(t).$$

The nonlinear “shock condition” should be added which corresponds to the value of blowing pressure at which the reed begins to beat against the mouthpiece
lay. This can be written such that the velocity of the reed is zero if the reed is touching the mouthpiece lay,

$$\frac{dx(t)}{dt} = 0 \text{ if } x < -1.$$  \hfill (6.3)

Introducing the shock condition ensures that the reed will not pass the barrier of the mouthpiece lay. This point is intended to mark the transition from the non-beating to beating reed regimes.

The resonator can be described through a modal decomposition of its input impedance described by the equation, \( P(\omega) = Z(\omega)U_{tot}(\omega) \), as was explained in Chapter 5. Modal decomposition is a mathematical tool that uses the information in the resonator’s input impedance spectrum to extract certain characteristics about each individual impedance peak and resulting resonance frequency [30]. The numerical modal decomposition leads to the extracted values of the modal parameters \( \omega_n \) (the resonance frequencies of the clarinet), \( F_n \) (the modal factor, a frequency value that is nearly equal for all peaks of the spectrum) and \( Q_n \) (the quality factor of the peaks), where \( n \) represents the mode number. Each one of these extracted parameters is used in the numerical simulations.

The dimensionless impedance \( Z(\omega) = 1/Y(\omega) \) can be written as a modal expansion,

$$Z(\omega) = \sum_n Z_n(\omega) = j\omega \sum_n \frac{F_n}{\omega_n^2 - \omega^2 + j\omega Q_n}, \hfill (6.4)$$

where the \( \omega_n \) are the resonance frequencies, and \( Q_n \) are the quality factors of mode \( n \). \( F_n \) are the “modal factors” obtained from the modal shapes calculated, at the input (The dimension of \( F_n \) is that of a frequency. For a perfect cylinder of length \( \ell \), \( F_n \) is equal to \( 2c/\ell \), and is independent of the mode number \( n \)).

The flow rate entering the reed channel \( u_b(t) \) is given by,

$$u_b(t) = \zeta(1 + x(t))\text{sgn}(\gamma - p(t))\sqrt{|\gamma - p(t)|}, \hfill (6.5)$$

$$u_r(t) = \frac{\lambda dx(t)}{dt}, \hfill (6.6)$$

$$u_b(t) = u_{tot}(t) - u_r(t). \hfill (6.7)$$

Where the parameter \( \lambda \) is a dimensionless coefficient that will be discussed shortly. Equations (6.5) and (6.1) are discretized according to [69] and for the
resonator, at each time sample, each impedance mode (Eq. (6.4)) is discretized after an estimation of modal parameters.

\[ p_k(n) = b_{k0}u_{tot}(n) + V_{k\text{known}}(p_k(n) - y), u_{tot}(n - y), y > 0 \]  

(6.8)

This yields:

\[ p(n) = \sum_{k=1}^{K} p_k(n) = b_{M0}u_{tot}(n) + V_{\text{known}} \]  

(6.9)

where

\[ b_{M0} = \sum_{k=1}^{K} b_{k0} \text{ and } V_{\text{known}} = \sum_{k=1}^{K} V_{k\text{known}}. \]  

(6.10)

This leads to the definition of the three discretized characteristic equations to be solved:

\[ \frac{1}{\omega_r^2} \frac{d^2 x(t)}{dt^2} + \frac{q_r}{\omega_r} \frac{dx(t)}{dt} + x(t) = p_e(t), \]  

(6.11)

\[ P_e(\omega) = Z_e(\omega)U_e(\omega), \]  

(6.12)

\[ u_e = \Theta(1 - \gamma + x(t))\zeta(1 - \gamma + x(t))\text{sgn}(\gamma - p_e(t))\sqrt{|\gamma - p_e(t)|}, \]  

(6.13)

where \( \Theta \) is the Heaviside function, the role of which is to keep the opening of the reed positive. These equations match those offered in Chapter 5 by Eqs. (5.99) and (5.105). Here, the subscript \( e \) represents the respective nondimensionalized variables of interest in order to match the formulation in [69].

### 6.1.2 Expressing the Input Impedance

The description of the underlying physical model matches that of the analytical formulas. The more important points from Chapter 5 will be discussed and compared here, the portion that describes the transformation from the physical model to the numerically coded synthesis model. This begins with a difference in impedance relation formulations. For a cylindrical bore the nondimensional impedance for an open end, ignoring radiation impedance contribution, is defined by,
\( Z_e(\omega) = i \tan(k(\omega)L) = i \frac{\sin(k(\omega)L)}{\cos(k(\omega)L)} \)

\[ = \frac{\exp(ik(\omega)L) - \exp(-ik(\omega)L)}{\exp(ik(\omega)L) + \exp(-ik(\omega)L)}. \quad (6.15) \]

\[ (6.16) \]

The final expression for the impedance can be written as

\[ Z_e(\omega) = \frac{1}{1 + \exp(-2ik(\omega)L)} - \frac{\exp(-2ik(\omega)L)}{1 + \exp(-2ik(\omega)L)} = \frac{1 - e^{-2ikL}}{1 + e^{-2ikL}}. \quad (6.17) \]

In the case of a perfectly cylindrical bore, Equation (6.17) is not the most efficient way to describe the resonator because in this expression the resonator is represented by the wave variables; [69] cites that the digital waveguide model would be more efficient. However most musical instruments, and especially the clarinet, are not perfectly cylindrical throughout the length of the bore and so the numerical simulations must be capable of including flares and varying cross-sections. Following [69], the acoustic pressure and flow can be decomposed into wave variables and there can be a useful definition of the reflection coefficient, \( R(\omega) \), which can model the forward and backward propagation of the pressure waves in the cylinder,

\[ R(\omega) = \frac{Z_e(\omega) - 1}{Z_e(\omega) + 1} = -\exp(-2ik(\omega)L). \quad (6.18) \]

As a first step, the reed displacement discretization scheme (see [69]) is:

\[ x(n) = F_{\text{known}}(x(n - y), p(n - y), y > 0); \]

\[ \text{if } 1 + x(n) < 0, x(n) = x(n - 1); \]

\[ u_r(n) = \lambda(x(n) - x(n - 1)); \]

\[ W = \zeta(1 + x(n)). \]

As a second step, since \( W \) and \( u_r \) are known at time sample \( n \), this leads to a set of two equations with two unknowns. Omitting the subscripts for time sample
\[ p = b_{M0}(u_b + u_r) + V; \]  
\[ u_b = W \text{sgn}(\gamma - p)\sqrt{|\gamma - p|} \]  
(6.23, 6.24)

As a third step, these equations are then transformed into two second order polynomial equations in \( u \) corresponding to either positive or negative values of \( u_b \), yielding the final solution:

\[ u_{tot} = \frac{\text{sgn}(\gamma - V - b_{M0}u_r)}{2} (-b_{M0}W^2 + ...) \]
\[ W \sqrt{(b_{M0}W)^2 + 4|\gamma - V - b_{M0}u_r|} + u_r; \]  
(6.25)

\[ p_k = b_{k0}u_{tot} + V_k; \]  
\[ p = \sum_{k=1}^{K} p_k; \]  
(6.26, 6.27)

The two physical equations Eq. (6.11) and (6.13) and any impedance model corresponding to Eq. (6.12) will lead to the introduction of nonlinearity due to the reed movement. This will link the output \( p_e \) to the input flow \( u_e \). The output of the model, after solving the three physical equations, will be the three coupled variables \( p_e, u_e \) and \( x \) (pressure, flow, and reed displacement). The model depends on the length of the resonator, \( L \) and two nondimensional variables slowly varying in time \( \gamma(t) \), the blowing pressure parameter and \( \zeta(t) \) the reed channel opening parameter, as it was in the analytical equations discussed before.

### 6.1.3 Discretizing the continuous system

In order to form the complete synthesis model, the authors of [69] used a discrete, time-domain formulation for the impedance and reed-displacement models. In general, most references approximate the loop filter by using a one-pole filter which can be modified according to the geometry of the resonator using the variables of length and radius of the resonator [37,114]. This can lead to an expression for the impedance of a cylindrical bore as a function of two filter coefficients and can then be expressed as a difference equation for \( p_e \) (see Section IV from Coyle et
al. [37] for details of the filter coefficients and difference equation). This difference equation allows a model to take into account additional losses such as losses from tone hole radiation and bell effects which aren’t taken into account in the physical model. These effects have a greater presence at high frequencies. The loss effect is estimated by the difference equation method by having the higher order modes decay faster.

6.1.4 Reed displacement

The continuous impulse response of the reed is modeled as an exponentially damped sine function which satisfies \( x(0) = 0 \) (the initial reed displacement is zero).

A digital filter can be built and the digital transfer function of the reed is

\[
\frac{X(z)}{P_e(z)} = \frac{\omega_r^2}{\omega_r^2 + f_e^2(z - 2 + z^{-1}) + \frac{L_c}{q_r}(z - z^{-1})q_r\omega_r} \tag{6.28}
\]

This produces the difference equation for \( x \) as well for the reed displacement due to an impulse pressure \( p_e(t) = \delta(t) \) is given by [37],

\[
x(t) = \frac{2\omega_r \exp\left(-\frac{1}{2}q_r\omega_r t\right)}{\sqrt{4 - q_r^2}} \sin \left(\frac{1}{2} \sqrt{4 - q_r^2} \omega_r t\right). \tag{6.29}
\]

The response of the reed to an impulse can be considered non-instantaneous \( (x(0) = 0) \) and since the convolution of \( x(t) \) and \( p_e(t) \) is the resulting total flow \( u_{tot} \) (as discussed in Chapter [5]), this property will be true for any excitation \( p_e(t) \).

6.1.5 Reed Flow

The article by Guillemain et al. [69] did not include the reed flow effect. This effect was, however, included in the more recent article by Coyle et al. [37]. In order to describe the beating reed condition, a point where the velocity is zero (for a given pressure) is created; this is where the reed hits the table for the first time. The two sources of flow are then added together as was done in the analytical formulas, \( u_{tot}(n) = u_b(n) + u_r(n) \). In the discrete time domain, Eqs. (6.3) and (6.6) are written as:

\[
\text{if } 1 + x(n) \leq 0 \text{ then } x(n) = x(n - 1). \tag{6.30}
\]
\[ u_r(n) = -f_e \frac{\Delta \ell_0}{c} [x(n) - x(n - 1)], \]  

(6.31)

where \( f_e \) is the sampling frequency (with dimensions). This leads to the dimensionless coefficient \( \lambda = -f_e \Delta \ell_0 / c \). In order to compare the simulations and analytical formulas (which will be described later), \( \lambda = -0.7 \) has been chosen as an initial input: this value corresponds to \( \Delta \ell_0 = 5.5 \text{ mm} \), as explained in [45] and discussed in Section 9.3 of Chapter 9 of this dissertation.

Here, unlike as was seen for the analytical formulas, the reed flow effect is valid above the beating reed regime because within the simulations there are a few added stipulations, such as the inclusion of the ability for the reed to touch the table of the mouthpiece and at that point have a velocity of zero as well as the ability to have negative flow \( (1 + x(t) < 0) \).
6.2 Flow chart and code usage

Figure 6.1 shows a simplified flow chart of the work order detailing tasks and computations achieved by the numerical simulations.

6.2.1 First block: Input User Variables

The very first step in the numerical simulation is to have the user enter the various parameters that will be held constant. These parameters include the reed characteristics, such as resonance frequency and quality factor ($\omega_r$ and $q_r$).
respectively), sampling frequency $f_e$ (i.e. 44100 Hz), speed of sound $c$ (assuming constant temperature throughout the instrument), and the density of air at this temperature.

The most important input is the choice of the note of interest. For the clarinet there are 38 notes in the first two registers, 19 in the first (before crossing the break and using the register key) and 19 above. Notes past 36 are sometimes considered to be the third register, though there is no second register key for reaching these notes. They are reached through the use of different though equivalent, sometimes creative, fingerings.

In certain cases, the value of the reed opening $\zeta$ is also set as a constant parameter in order to study the change in playing frequency with only the blowing pressure increasing. This is useful when making comparisons to the artificial mouth (see Chapter 7). However, there is the option of varying this parameter and creating a loop over these values, as is done for $\gamma$.

### 6.2.2 Second Block: Extract Modal Variables

The second block is quite special to this code. Here is where the code will extract (in a sense) the modal variables needed for computation. This portion of the code is the numerically simulated modal decomposition described in [30], briefly in [37] and in Section 6.1.1 of this chapter. These modal variables are the modal frequencies (the resonance frequencies of the instrument, taken from the impedance spectrum measurement), a second value, sometimes called the modal factor $F_n$, which as mentioned in Chapter 5 for a cylinder is simply $2c/\ell$, and finally the quality factor of the impedance peaks $Q_n$. In order to find these values the numerical simulations call for either a simulated input impedance spectrum or a measured spectrum from a clarinet. In this research the input impedance spectra were measured as described in Chapter 7.

### 6.2.3 Third Block: Solve Characteristic Equations

This block is where the numerical simulations take place. The subject of the mathematical theory described earlier in regards to discretizing the system and expressing the physical variables, using the results from the previous step, etc. takes place in this block. This is the computationally expensive portion of the code.
This portion of the code numerically solves the characteristic equations and, based on all inputs, will give a resulting internal pressure (in the mouthpiece, past the reed) and external pressure (that listeners would hear).

### 6.2.4 Fourth Block: Results

The main outputs from running this code are the internal pressure in the mouthpiece and the external pressure (external to the instrument) as a function of time. These values can be used to find the playing frequency as was done with the analytical formulas. From here, other values can be found as well such as current playing regime (first vs. second, amplitude, energy, transient vs. steady state, etc.).

### 6.2.5 Loops

At this point the code can be rerun and looped over a number of variables. The main playing parameters of interest are $\gamma$ and $\zeta$, blowing pressure and reed opening. The code can also be looped over the note number so that at some point 3D maps can be made as a function of note, blowing pressure and reed opening.

### 6.3 Separating the effects in the numerical simulations

The main contribution of the current research is altering the simulations mentioned above by adding an important feature, the ability to separate the effects in order to eventually compare the individual effects of reed induced flow, reed dynamics, and inharmonicity to the analytical formulas presented in the Chapter 5. The task of separating the effects from one another in the numerical simulations was not trivial. The easiest effect to suppress was the inharmonicty effect. The peak frequencies from the input impedance spectrum were simply forced to be harmonic multiples ($\omega_3 = 3 \ast \omega_1$). The reed dynamics effect was suppressed by forcing the quality factor to be very small and the resonance frequency to be very high (the quality factor could not be zero since this would cause a division by zero in the simulations) hence minimizing the terms including $1/\omega_r^2$ and $q_r/\omega_r$ in Eq. (6.1). Finally, to suppress the reed induced flow effect, the equivalent value for the vibrating surface area of
the reed was set to be zero (affecting only this effect, not others, and specifically the value of \( \lambda = 0 \)).

To find the difference in playing frequency due to only one of the three effects, assumed to be independent of each other, the playing frequency can be said to be equal to the resonance frequency \( f_1 \) plus the three different frequency shifts, calculated from the length corrections discussed in Chapter \[\Delta f_{flow}, \Delta f_{dynamics}, \text{and} \Delta f_{inharmonicity}\]. So the playing frequency \( f_p \) is essentially,

\[
f_p = f_1 + \Delta f_{flow} + \Delta f_{dynamics} + \Delta f_{inharmonicity}.
\] (6.32)

Writing the playing frequency as a function of the three effects,

\[
f_p = f(\lambda, q_r/\omega_r, \eta_3),
\] (6.33)

where \( \lambda = fe\Delta l_0/c \) represents the reed induced flow effect, \( q_r \) is the reed quality factor, \( \omega_r \) is the reed resonance frequency and the ratio represents the reed dynamics effect, and \( \eta_3 = (\omega_3 - 3\omega_1)/3\omega_1 \) represents the resonator inharmonicity.

One way to separate the three effects is to run the simulations multiple times to create the following. First, the change in frequency due to reed induced flow,

\[
\Delta f_{flow} = f(\lambda, 0, 0) - f(0, 0, 0)
\] (6.34)

\[
= f(\lambda, q_r \approx 0/\omega_r(\text{large}), \eta_3 = 1) - f(0, q_r \approx 0/\omega_r(\text{large}), \eta_3 = 1).\] (6.35)

This equation shows for each of the effects, what variables are set to zero (or very small). For example, \( f(\lambda, 0, 0) \) means that \( \lambda \) is set to its initial (normal) value of around 0.7 to match what is used in the analytical formulas for \( \Delta \ell_0, q_r/\omega_r \) should be set close to zero (very small) and \( \eta_3 \) (the inharmonicity) should be set to zero (the third peak frequency would be exactly three times the first). In reality, \( q_r \) will be set close to, but not equal to zero (e.g. .001), \( \omega_r \) will be set to a very high value (e.g. \( f_r \approx 2400 \text{ Hz} \)) and to set inharmonity equal to zero is equivalent to \( \eta_3 = 1 \). In this way, the effects can be subtracted safely from and compared to the total playing frequency \( f_p \).

Next, for reed dynamics,

\[
\Delta f_{dynamics} = f(0, q_r/\omega_r, 0) - f(0, q_r \approx 0/\omega_r(\text{large}), \eta_3 = 1).
\] (6.36)
Then, for the inharmonicity effect,

\[ \Delta f_{\text{inharmonicity}} = f(0, 0, \eta_3) - f(0, q_r \approx 0/\omega_r(\text{large}), \eta_3 = 1). \] (6.37)

And finally, the difference in cents is calculated as follows,

\[ -\frac{f_p + f_{\text{effect}}}{f_p} \] (6.38)

or

\[ \frac{\Delta f_{\text{effect}}}{f_p}, \] (6.39)

where \( f \) is the output playing frequency considering all three effects.

### 6.4 Discussion of Results

The primary focus of this thesis is on the first register of the clarinet. This section offers three separate output figures from the numerical simulations, for notes 1, 12 and 17, notes representing a full resonator, half resonator and short resonator respectively. Figures \[6.2a\], \[6.2b\] and \[6.2c\] show the numerical simulation results for notes 1, 12 and 17 respectively for a particular set of input parameters. Each curve represents the numerical simulation results for the separate effects (and the total of all three effects is shown in a bold dashed blue line on each plot (bottom of plot). This value, in cents, represents the total adjustment to the resonance frequency of the instrument in order to find the playing frequency for each note.

The input parameters for each figure are as follows:

- **Reed information:**
  - Notes 1, 12: \( q_r = 0.2, f_r = 2400 \text{ Hz} \).
  - Note 17: \( q_r = 0.2, f_r = 3000 \text{ Hz} \).

- **Reed opening:** \( \zeta = 0.3 \)

- **Atmosphere:** \( c = 343 \text{ m/s}, \rho = 1.21 \text{ kg/m}^3 \)

For each of these figures,

- The effect of inharmonicity should be near zero at the threshold of oscillation (the beginning of the plot shown).
Figure 6.2: Numerical simulations results. Cyan - inharmonicity effect, Red - reed dynamics effect, Green - reed induced flow effect and the Blue dotted line - total of all three effects.
• The reed dynamics effect has the least effect on the overall playing frequency difference.

• The reed flow effect will initially be stable at a particular value (before the beating reed regime, near $\gamma = 0.5$) then increase with increasing blowing pressure.

Figure 6.2a represents the results for Note 1 on the clarinet. In general, the lower notes behaved as expected compared to the higher notes. This could be because for the input impedance spectra provided, more peaks were valid (there are more peaks present in the impedance spectrum below the cutoff frequency, for the clarinet $\approx 1500$ Hz). Figure 6.2a shows a relatively well-behaved and expected behavior with an interesting feature around the beating reed regime, near $\gamma = 0.45$ for the reed dynamics effect (in Red) there is an abrupt change in the effect’s behavior yet this feature still has quite a small affect on the total difference. Figure 6.2b shows similar behavior as Note 1 with the exception of the inharmonicity effect. This effect has a much less dramatic decrease as $\gamma$ increases. Figure 6.2c shows similar behavior as the other two notes but notice that as the note increases, the total of all three effects is greater and greater even though the reed dynamics and inharmonicity effects (Red and Cyan respectively) remain small. This shows that the closer the playing note is to the mouthpiece, the reed induced flow becomes more prevalent and important in the calculation of the playing frequency.

6.4.1 Constraints, Run-time

One of the great interests in using the analytical formulas in this research instead of the numerical simulations is that the computational expense (CPU time) for the numerical simulations is extremely costly. One important application of the simulations (analytical or numerical) would be to show musicians, in real-time, their instrument’s tuning characteristics for example. With the current computing power of a typical laptop computer (2010 Macbook Pro with a 2.4 GHz Intel Core 2 Duo processor, with 8 GB of 1067 MHz DDR3 memory using MatLab 2010a) the run time for a full set of computations (36 notes, $\zeta$ and $\gamma$ from 0 to 1 with a resolution of 65 points in $\gamma$ and 16 points in $\zeta$) requires around 15 hours (which corresponds to about 15 hours of sound!). In contrast, for the analytical formulas, this same computation takes less than 30 minutes.
Because the numerical simulations do not make quite as many simplifications as do the analytical formulas which makes computation time longer, the numerical simulations could possibly lead to better predictions of the playing frequencies. Though the computation time is the main constraint when using this simulation scheme, the simulation results offer more information about the sound than do the analytical formulas. The numerical simulations can reveal the natural regime played for a set of reed characteristic parameters and playing parameters - this is impossible to predict with the analytical formulas.

To better understand the behavior of the simulations, several values of input parameters were attempted even with unrealistic values of reed parameters \(f_r \approx 1200\) Hz, or much lower than necessary based on assumptions) and the resulting playing frequency for low values of \(\gamma\) were unreasonable as well, giving playing frequencies in the second register when this was not expected for higher values of \(\gamma\) in the first register. Figure 6.3a illustrates this issue for Note 12. Notice that between \(\gamma = 0.3 - 0.43\) the results for the simulations (Blue - total and Red - dynamics effect) are not consistent and shoot up significantly for the reed dynamic effect (red).

The interaction between the reed resonance frequency and the input impedance peaks of these higher notes in the first register is important to comment on. If the reed resonance frequency was lower than the first or second peak in the impedance spectrum, the playing frequency would not sound. Throughout the theory it is assumed that the reed generator can be modeled as a massless spring or single degree of freedom simple harmonic oscillator and since the playing frequencies of the clarinet are much lower than the resonance of the reed, it operates in this stiffness regime. If the reed resonance frequency happens to coincide with one of the lower impedance peaks for a particular note (in general this could happen for higher notes in the first register) then it is possible that the note in question would actually jump into the second register for certain values of \(\gamma\). Once the reed resonance frequency was shifted up to around 2400 Hz for most notes of the first register and perhaps even a bit higher for the throat tones of the clarinet, this problem disappeared. An example, for comparison with Fig. 6.3b is Figure 6.2c where there is a much more reasonable, continuous output for the numerical simulations for all considered values of \(\gamma\).
(a) Note 12 simulations showing the effect of using a lower value of reed resonance frequency. When trying to run the numerical simulations for higher notes in the first register we were having trouble getting the correct note to be played. After some investigation it was decided that the value for the reed resonance frequency was too low (simulations were using $f_r = 1200$ Hz vs. $f_r = 2400$ Hz as shown in Fig. 6.2b). This low resonance frequency was too low to include all information from the spectrum and could have perhaps been interacting with the harmonics of the spectrum.

(b) Note 17 simulations showing the effect of using a lower value of reed resonance frequency. When trying to run the numerical simulations for higher notes in the first register we were having trouble getting the correct note to be played. After some investigation it was decided that the value for the reed resonance frequency was too low (simulations were using $f_r = 2400$ Hz vs. $f_r = 3000$ Hz as shown in Fig. 6.2c). This low resonance frequency was too low to include all information from the spectrum and could have perhaps been interacting with the harmonics of the spectrum.

Figure 6.3: Numerical simulations results for specific situations.
6.4.2 Tuning Maps

One of the end goals in this thesis is the capability to use the analytical formulas to quickly create visually useful maps for use by musicians given their particular instrument’s input impedance measurement. These maps can be made with either the analytical formulas or the numerical simulations to create tuning maps over the range of playing parameters $\zeta$ and $\gamma$. Figures 6.4 and 6.5 show example output maps from the numerical simulations where the former shows comparison with the instrument’s resonance frequencies and the latter with the equal tempered scale frequencies. Each color map represents the tuning in cents different from the tempered scale frequency (Fig. 6.4) or to the resonance frequency (Fig. 6.5) for each of the 19 notes in the first register for the clarinet. Each run of the numerical code was performed for all values of $\gamma$ (beginning at each note’s oscillation threshold $\gamma = 0.3$) and $\zeta$ (from 0 to 1).

Each of the maps displayed shows the same basic trends: in the permanent regime (at the left side of the plot), the frequency decreases with increasing blowing pressure until a certain point where the frequencies begin to rise as the blowing pressure approaches the closing pressure. There is less possibility of playing a sound, or playing in tune, when the values of $\zeta$ are very high. A completely open reed for $\zeta = 1$ will not produce sound for a clarinet and the reed resonance will more likely be played. As the notes continue higher, there are increasingly fewer playing parameter combinations that will allow the player to play in tune. This could point to the fact that there are many other playing parameters that are not being modeled here at the moment that are extremely important in playing notes in tune in the correct register even, for wind instrument players (for example, changing the vibrating surface area of the reed as the note being played increases). These other playing parameters effects on the playing frequency would require a three-dimensional, or more, representation in map form (see, for example [94]).
Figure 6.4: Example tuning map to show comparison of playing frequencies to the equal tempered scale frequencies for a particular clarinet. The color scheme is as follows: Darker blue is further out of tune (further away from the intended frequency, in this case the tempered scale frequencies) and as the color approaches red (in the MATLAB color scheme Jet) the instrument is playing more closely in tune. The horizontal axis is $\gamma$ increasing from the threshold of oscillation for that particular note to 1 (reed closing pressure) and the vertical axis is $\zeta$ ranging from 0 (most closed) to 1 (most open).
Figure 6.5: Example tuning map to show comparison of playing frequency to resonance frequencies for a particular clarinet. The color scheme is as follows: Darker blue is further out of tune (further away from the intended frequency, in this case the resonances of the instrument) and as the color approaches red (in the MATLAB color scheme Jet) the instrument is playing more closely in tune. The horizontal axis is $\gamma$ increasing from the threshold of oscillation for that particular note to 1 (reed closing pressure) and the vertical axis is $\zeta$ ranging from 0 (most closed) to 1 (most open).
Each tuning map figure (Figs. 6.4 and 6.5) has a similar color scheme but have identical axis labels. The horizontal axis is $\gamma$ increasing from the threshold of oscillation for that particular note to 1 (reed closing pressure) and the vertical axis is $\zeta$ ranging from 0 (most closed) to 1 (most open), hence the said changing reed parameters. For Fig. 6.4 the colorbar ranges from -100 cents to 50 cents and for Fig. 6.5 has a colorbar ranging from -100 cents to 0 cents.

In each figure, if there are black portions this means that the frequency of the note being played is lower than a given threshold, and the grey portions mean the frequency of the note being played is higher than a given threshold. The threshold here was -100 to 100 cents different since this would mean that a semi-tone lower or higher would be played, respectively and would therefore no longer be an issue of tuning but of playing a different note all together. This threshold is in place regardless of the colorbar thresholds in the right hand portion of each plot.

The hope is that, as the analytical formulas discussed in the previous chapter are refined, the closer the predictions made by the formulas to the actual playing frequencies will become. If the analytical formula predictions become very accurate and reliable then manufacturers and musicians alike could use this tool to predict the tuning tendencies of a particular instrument before sale (or repair), saving time and money on each side of the sale.

### 6.5 Conclusion

The model described in this chapter used numerical methods to describe and solve the characteristic equations discussed in Chapter 5. The numerical formulation allows for the addition of the necessary nonlinear modeling of the interaction between the pressure, reed displacement and flow to the linear parts of the model. The system, in its discretized version (in time) is utilized throughout. An advantage lies in the numerical simulation’s ability to replace the linear parts within the model with more complex bore geometries as well as different models for the reed. With this model, the coefficients of the digital filters and the control parameters are explicitly expressed in terms of physical parameters. This code will be used to validate the analytical equations from Chapter 5. These comparisons and validation will be offered in Chapter 9.
CHAPTER

SEVEN

EXPERIMENTATION

7.1 Introduction

The research in this dissertation utilized three different experimental techniques: input impedance measurements, measurements with an artificial mouth and an instrumented mouthpiece for use with musicians. The input impedance measurements gathered the initial resonance frequency information necessary to use in the analytical formulas and numerical simulations. The artificial mouth measurements were an attempt at taking data objectively and repeatably to compare to the analytical formula results. Finally, using the instrumented mouthpiece with musicians was an attempt at placing the instrument in actual playing conditions and comparing to the objective measurements of the artificial mouth and the results from the analytical formulas.

This chapter will detail the method that each of the experimental techniques employed and will present a brief set of results from each method.

7.2 Input impedance measurements

The analytical and numerical models described in Chapters 5 and 6 require an accurate measurement of the input impedance of a clarinet in order to predict the playing frequency. This measurement offers the information necessary to deduce the resonance frequencies which depend on the cylindrical geometry of the instrument.
and the possible changes in assumed resonances due to the tone hole lattice and undercut tone holes.

The input impedance measurement does not take into account: reed/mouthpiece set up, the musician’s vocal tract, or the characteristics of the mouth. The resonance frequency of the instrument, discerned from this measurement is not equivalent to the playing frequency emitted by the instrument. The effects of the musician and musician-mouthpiece interaction can highly affect the playing frequency and would be complicated additions to a playing frequency prediction method. These effects are not included in this work.

The resonance frequency of the clarinet is found from analysis of the first peak of the input impedance measurement. The analytical formulas and numerical simulations use this frequency as a base value for the playing frequency and add several different effects due to musician contribution and realistic formation of the clarinet in order to calculate a final, realistic playing frequency which could then be compared to data from actual sound recordings of this clarinet being played either live or by an artificial mouth.

7.2.1 Method and apparatus used for input impedance measurements

In order to measure the input impedance of the clarinet, the methods and instruments used are described in the documentation for the apparatus created in Le Mans, France by the Centre de transfert de technologie (Center for Transfer of Technology) [29]. There are methods described in the literature for measuring the input impedance, but the most popular is in using two pairs of microphones in order to determine the volume velocity at the input of the instrument. Figure 7.1 shows the measurement apparatus sketch. There are the two cavities separated by a piezo-electric pressure transducer (labeled “buzzer”). Each cavity has a microphone placed at equal known distances from rigid boundaries. The far right side is where the instrument will be placed in order to measure the input impedance. For the apparatus created by the CTTM, one microphone measures the pressure $p_2$ in the second cavity, behind a source. This microphone will allow an estimated measurement of the volume velocity of the source. The clarinet (or other instrument of interest) is connected in front of the source after another small cavity in which
another microphone sits, measuring $p_1$.

Figure 7.2a shows the full apparatus that is used in these measurements. The small gold cylinder (Fig. 7.2b) is what the instrument will be connected to and where the cavities and the speakers are housed.

Figure 7.1: A sketch of the impedance measurement apparatus, from [88]. The instrument is inserted perpendicular into the right side of the apparatus (at $d_2$) not to go further than the reference plane and the other end, at $d_1$, is sealed. The piezo-electric source is the source. The $\phi$ symbol refers the the ends being closed (sealed).
7.2.2 Basic Theory and Operation

The impedance apparatus used employs, as a source, a piezo-electric patch closed at the back by a cavity connected to the front, airtight, to the instrument to be measured. The pressure at the entrance of the instrument $p_1$ is measured by mic 2 and mic 1 measures the pressure $p_2$ inside the other cavity, proportional to the flow pulsed by the source.

The input impedance is $Z_{11} = p_1/u_1$, both pressure and volume flow being measured at the input of the instrument. The flow, $u_1$ is created by the piezo-electric source. Benade states in his 1987 paper that the acoustical input impedance of some unknown air column may be calculated in terms of pressure $p_1$ measured at the driving point (close to the microphone on the right side of the apparatus) if the injected flow amplitude $u_1$ produced by the excitory piston in known \[22\]. This $u_1$ is indirectly measured by our second microphone in the second cavity as being the same as the flow in the front cavity but 180 degrees out of phase.

For this configuration, at low frequencies, the back cavity at the left in Fig. 7.1 acts as an acoustic compliance for which the impedance is,

$$Z(\omega) = \frac{1}{-j\omega C'},$$  \hspace{1cm} (7.1)

where $C' = V/\rho c$. The input impedance of interest is $Z_1 = p_1/u_1 = -p_1/u_2$.

Figure 7.2: (a) Impedance Box created by LAUM and CTTM in Le Mans, France. The whole box contains necessary cables, an National Instruments data acquisition box and should be used in conjunction with their provided program on a PC. (b) Picture of gold box that houses the apparatus to take impedance measurements.
since the volume velocity should be the same on each side of this source but 180 degrees out of phase (thus the negative sign). The impedance of the left cavity is \( p_2/u_2 = Z(\omega) = 1/ -j\omega C \) therefore \( u_2 = p_2(j\omega C) \) and finally,

\[
Z_{in}(\omega) = -\frac{p_1}{u_2} = -\frac{p_1}{p_2(j\omega C)}.
\] (7.2)

In terms of the transfer function between microphones,

\[
\frac{p_1}{p_2} = -j\omega C Z_{in}(\omega).
\] (7.3)

This means that the input impedance can be measured directly by simply using the pressure signals from microphones 1 and 2 in the separate cavities.

### 7.2.3 Calibration

The calibration of the impedance head could be done in three ways: (1) measuring the sensor output when the cavity is open, (2) when the cavity is blocked by a rigid plate, and (3) by taking the input impedance measurement of a long tube, closed at one end, with known dimensions. This way, the measurements taken with the system can be compared with a theoretical calculation of the input impedance spectrum for an infinite end, closed volume cavity and a long closed-closed or closed-open cylinder of known length. For this section the calibration with the tube will be discussed as this method is the most applicable to the current research.

Calibration of the apparatus in Marseille was performed with a tube made of PVC piping with a length of 1 m and a diameter of 14 mm. A 10 second log sweep from 0 Hz to 4 kHz was sent through the tube 10 times after which an average was taken and the results were shown. There is an automatic program that treated the data and compared it to the theoretical input impedance curves for a tube of the same dimensions. There was a small tolerance (in frequency) given on the upper and lower bounds of each impedance peak and if the measurements were within these bounds then the preceding measurements could be kept, with confidence. Figure 7.3 shows a sample screenshot of a calibration measurement. This figure shows the frequency range of interest (10 Hz - 1000 Hz), a sampling frequency (25000 Hz), the duration of sweep (1s), and the number of averages taken (10). The three plots within Fig. 7.3 are top: the modulus of the signal (absolute value of a complex
signal), middle: the coherence in the calibration measurement from one average to the next, bottom: the signal sent into the resonator. If the measurement was sufficiently coherent (coherence close to 1, an inherently subjective choice), the peaks could be user verified after the measurement was saved in order to ensure that the apparatus measured the expected resonance frequencies for this tube of known dimensions and boundary conditions (i.e. all integer harmonics for a closed-closed tube).

![Figure 7.3: Screenshot of calibration of program to take impedance measurements.](image)

### 7.2.4 Example Measurements

For each note of the instrument (and for each particular fingering) an impedance measurement was taken in the same manner, by choosing the time for the sine sweep and the number of averages the program should take (how many times the sweep was sent). If the coherence between each subsequent measurement, and the final result was sufficiently good, then the measurement was saved for further analysis by the program. The coherence tolerance was a subjective choice, between 85 and 90% coherent was the generally accepted benchmark. The user sees the coherence of the measurement after each average and ideally the coherence would be exactly 1, meaning that the results of the measurements from one to the next had not changed. However, many factors could lead to poor coherence, such as a noise
source beginning in a nearby room. Because of the subjectivity, three independent runs for each note were done and the measurement with the best coherence was kept.

Figure 7.4 shows an example measurement set-up that was in place at IRCAM in early 2015. The gold impedance measurement apparatus is in place so that a clarinet would be inserted (barrel up) with the bell pointed down in a somewhat natural playing position (as if the gold cylinder were the mouth) so that the person taking the measurements could easily cover and uncover holes as necessary for each subsequent measurement. This could also be done by adding modeling clay to the open tone holes while being careful to not insert the modeling clay into the holes during the measurement. Both methods showed good agreement and one was not preferred over the other except that having a user cover and uncover holes was a much faster method.

Figure 7.4: Set-up of an input impedance measurement at IRCAM in an anechoic chamber.

### 7.2.5 Experimental Input Impedance Data

The input impedance data for a particular professional level B♭ soprano clarinet were measured and are recorded in Table 7.1. The table shows the equal tempered
scale frequency (Temp. Freq.) for the notes in the first register of the clarinet (Notes 1 - 19), the first and third harmonic peak frequencies (Res 1, Res 2), the quality factor of the peaks and the modal factor $F_n$. These are the values for resonance frequencies used throughout this research for two purposes: (1) as the input for the numerical simulations and analytical formulas in order for each method to predict playing frequencies and (2) as a reference for which to compare the predicted playing frequencies calculated with the analytical formulas or numerical simulations, and playing frequencies measured with the artificial mouth and instrumented mouthpiece.

Table 7.1: Table of resonance frequencies and modal information gathered from input impedance measurement and corresponding modal parameter extraction based on the work in [69] for each note in the first register. From left to right: Note number, tempered frequency reference for that note, frequency of first peak of the input impedance, second peak (third harmonic), quality factor of the first peak, quality factor of the second peak, modal factor of the first and second peaks.

<table>
<thead>
<tr>
<th>Note #</th>
<th>Temp. Freq.</th>
<th>Res. 1</th>
<th>Res. 2</th>
<th>Qn 1</th>
<th>Qn 2</th>
<th>Fn 1</th>
<th>Fn 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>146.8</td>
<td>149.45</td>
<td>439.3</td>
<td>27.3</td>
<td>40.1</td>
<td>1245.8</td>
<td>1231.8</td>
</tr>
<tr>
<td>2</td>
<td>155.56</td>
<td>157.46</td>
<td>468.6</td>
<td>24.1</td>
<td>38.4</td>
<td>1411.8</td>
<td>1315.4</td>
</tr>
<tr>
<td>3</td>
<td>164.8</td>
<td>166.97</td>
<td>499.2</td>
<td>24.1</td>
<td>39.3</td>
<td>1467.1</td>
<td>1411.6</td>
</tr>
<tr>
<td>4</td>
<td>174.6</td>
<td>178.14</td>
<td>531.7</td>
<td>24.3</td>
<td>38.7</td>
<td>1591.3</td>
<td>1524.3</td>
</tr>
<tr>
<td>5</td>
<td>184.99</td>
<td>189.6</td>
<td>564.4</td>
<td>25.7</td>
<td>38.3</td>
<td>1679.1</td>
<td>1643.1</td>
</tr>
<tr>
<td>6</td>
<td>195.99</td>
<td>200.9</td>
<td>600.5</td>
<td>25.3</td>
<td>39.8</td>
<td>1802.1</td>
<td>1718.3</td>
</tr>
<tr>
<td>7</td>
<td>207.65</td>
<td>212.8</td>
<td>635.7</td>
<td>26.6</td>
<td>42.3</td>
<td>1871.2</td>
<td>1794.9</td>
</tr>
<tr>
<td>8</td>
<td>220</td>
<td>226.5</td>
<td>674.7</td>
<td>27.8</td>
<td>44.5</td>
<td>1965.1</td>
<td>1892.2</td>
</tr>
<tr>
<td>9</td>
<td>233.08</td>
<td>239.3</td>
<td>716.3</td>
<td>27.3</td>
<td>45.7</td>
<td>2100.8</td>
<td>2016.8</td>
</tr>
<tr>
<td>10</td>
<td>246.9</td>
<td>253.9</td>
<td>759.8</td>
<td>25.8</td>
<td>40.8</td>
<td>2305.8</td>
<td>2191.3</td>
</tr>
<tr>
<td>11</td>
<td>261.6</td>
<td>268.7</td>
<td>801.9</td>
<td>27.6</td>
<td>48.1</td>
<td>2404.1</td>
<td>2214.8</td>
</tr>
<tr>
<td>12</td>
<td>277.18</td>
<td>283.1</td>
<td>846.3</td>
<td>27.8</td>
<td>45.6</td>
<td>2490.7</td>
<td>2387.3</td>
</tr>
<tr>
<td>13</td>
<td>293.66</td>
<td>302.2</td>
<td>900.7</td>
<td>28.1</td>
<td>48.1</td>
<td>2663.4</td>
<td>2493.1</td>
</tr>
<tr>
<td>14</td>
<td>311.1</td>
<td>321.2</td>
<td>951.2</td>
<td>26.8</td>
<td>47.4</td>
<td>2830.5</td>
<td>2633.1</td>
</tr>
<tr>
<td>15</td>
<td>329.6</td>
<td>342.5</td>
<td>1008.5</td>
<td>28.6</td>
<td>46.6</td>
<td>3018.4</td>
<td>2748.9</td>
</tr>
<tr>
<td>16</td>
<td>349.2</td>
<td>362.4</td>
<td>1077.1</td>
<td>28.4</td>
<td>46.8</td>
<td>3203.9</td>
<td>3232.8</td>
</tr>
<tr>
<td>17</td>
<td>369.99</td>
<td>385.9</td>
<td>1141.4</td>
<td>29.4</td>
<td>45.9</td>
<td>3337.9</td>
<td>3168.6</td>
</tr>
<tr>
<td>18</td>
<td>391.99</td>
<td>409.7</td>
<td>1208.3</td>
<td>29.5</td>
<td>44.5</td>
<td>3562.9</td>
<td>3246.8</td>
</tr>
<tr>
<td>19</td>
<td>415.3</td>
<td>435.4</td>
<td>1234.7</td>
<td>25.9</td>
<td>40.7</td>
<td>3745.8</td>
<td>3036.3</td>
</tr>
</tbody>
</table>

Figures 7.5, 7.6 and 7.7 show the resonance frequencies (the first resonance versus the tempered scale) from measurement, the modal factor and quality factor
of the peaks respectively.

Figure 7.5 shows that the resonance frequencies of the instrument found by measuring the input impedance of the instrument start out very close to the equal tempered values, with only a 2.65 Hz difference between the two. However, the difference becomes increasingly more significant as the note increases (playing closer and closer to the mouthpiece). Note 19 has a difference between the resonance frequency and the equal tempered scale frequency of 20 Hz. This could mean that manufacturers are capable of matching quite closely the lower range of the instrument to the equal tempered scale but as less of the resonator is used (more open holes), the task becomes increasingly more difficult.

Figure 7.6 shows the modal factors for each note of the instrument. This value would be \( F_n \approx \frac{2c}{\ell} \) for a perfect cylinder where \( \ell \) is the effective length of the resonator. \( F_n \) is independent of the peak number, i.e. the first and second peak in the input impedance spectrum should have the same \( F_n \). This assumption matches quite well for the first ten notes in the first register and then begins to exhibit a larger difference between the two registers. Note 10 is where the notes begin to be operated by the left hand, played in the upper body of the instrument. This change in behavior (the values for \( F_n \) getting farther apart) could be caused by the abrupt change in bore. Although manufacturers work to make this transition smooth, there is a break in the clarinet (in order for simplified storage and repairs) and the approximation of a perfect cylinder is again perturbed. The values diverge further around Notes 17, 18 and 19. These notes are throat tones, played with the small trill keys at the top of the instrument. Note 19 in particular is generally a problem note for clarinetists in tuning and tone quality [71] therefore it is not surprising to see odd behavior in the modal characteristics.

Figure 7.7 plots the quality factor for each of the first two impedance peaks for a professional level B\( \flat \) clarinet. The quality factor is a measure of damping for the input impedance peaks. The damping seems fairly constant for notes 2 through 7 and is markedly different for the first and last note of the first register. The second peak follows this trend for the first and last note and notes 2 through 7. However there are differences for the rest of the notes as the damping for the second peak seems to increase except for note 10. As note 10 marks this transition into the upper joint, there seems to be a slight different in this note’s behavior for each of the quality factors. Note 10 has a minor decrease for the first register peak and a
large decrease for the second register peak.

![Figure 7.5: Plot of the resonance frequency (red solid line) and tempered scale frequencies (blue dashed line) for the first register of a professional level B♭ clarinet.](image)

Figures 7.8 and 7.9 show two example input impedance measurement data for three different clarinets: Pro+ model (a top professional model), Pro model (intermediate professional model), and a student model clarinet. Although past literature claims that the input impedance data cannot be used as the sole data with which to judge clarinet quality or playability, it is interesting to see that for each of these figures the impedance data for the two top tier clarinets behave very similarly, and the student model is markedly different (a significant shift down for the second frequency peak).
Figure 7.6: Plot of $F_1$ (blue) and $F_2$ (red) values, the modal factor for the first register of the a professional level B♭ clarinet (for a cylinder $\approx 2c/\ell$, independent of rank).

Figure 7.7: Plot of $Q_1$ (blue) and $Q_2$ (red) values (nondimensional), the quality factor of the first and second impedance peak for the first register of a professional level B♭ clarinet.
Figure 7.8: Measured input impedance for Note 1, three different clarinets: blue - Pro+, red - Pro, yellow - Student, and red dotted line shows the equal tempered scale frequency.

Figure 7.9: Measured input impedance for Note 16, three different clarinets: blue - Pro+, red - Pro, yellow - Student, and red dotted line shows the equal tempered scale frequency.
7.3 Artificial Mouth Measurements

It is of great interest for researchers in all fields dealing with variability imposed by human/experiment interaction, to find a way to limit or eliminate human variability. This research uses an artificial mouth to play the clarinet. This offers the ability to maintain a constant and measurable (while still adjustable) blowing pressure and reed opening. Musicians would find that holding these two variables constant would be impossible. The motivation in using this apparatus in research is that measurements from the artificial mouth compared to measurements taken from actual musicians would offer insight into the musician’s effect on the playing frequency directly. This also offers a reliable real-life comparison for the analytical formulas which, in general include a fixed value for blowing pressure or reed opening.

![Figure 7.10: Top-down sketch of artificial mouth set up.](image)

7.3.1 The artificial mouth components

The artificial mouth apparatus is sketched in Fig. 7.10. The air balloon source at top right is connected to the flow meter and is piloted by the Simulink program and a servo-valve. The air flows past the flow meter and into the artificial mouth where the instrument is attached (mouthpiece enclosed in plexiglass box labeled AM). There are two endevco microphones, one in the mouth (inside AM) and one
in the mouthpiece (outside of the AM). These are connected to a conditioning box that allows for calibrated measurements of blowing pressure. This conditioning box is attached to the acquisition system and finally to the PC where the data is saved. Each of the components are stated here and explained in detail in the following sections:

- **Air source**: The air to be input into the system is contained in a 120 Liter balloon at the beginning of the artificial mouth. The static pressure in the artificial mouth is adjustable through the real-time feedback loop.

- **Flow measurement**: at the first step, the air will flow past a flow meter. It can detect up to 80 L/mn.

- **Input**: after the flow meter there is another small tube that leads into a small Plexiglass encasing (with an inner volume of 15 cm$^3$) that surrounds the clarinet mouthpiece and creates a tight air seal. It is necessary to have a small removable part of the Plexiglas in the front for adjustments (and insertion of the incoming flow tube) and the back must be open until the addition of the clarinet.

- **Lips**: just after the Plexiglas box begins sits an artificial bottom lip. This lip is made from latex and can be made more stiff or loose (while maintaining
relatively realistic properties of a human lip) by the measurable addition and subtraction of water from the lip using an attached syringe. Rising the pressure of water increases the bearing force of the lip on the reed, which results in a decreased reed channel opening. The artificial lip is placed at around 1 cm from the tip of the reed to simulate real musician playing conditions. The latex lip surrounds a small metal cylinder that is fixed inside the Plexiglass box. The clarinet is inserted into the box and remains stationary. The lip is filled and presses on the reed to close it from rest. There always remains enough water in the lip to prevent the clarinet mouthpiece and reed combination from ever touching this small cylinder inside the lip.

- **Clarinet**: The instrument itself is added to the Plexiglas box and a thick piece of silicone is placed around the reed/mouthpiece/ligature combination and attached to the box. This creates the, at times, an extremely difficult to achieve airtight seal.

- **Measurements**: The measurements possible with the artificial mouth are: static flow rate, static pressure in the mouth, acoustic pressure in the mouthpiece and a resulting reed opening based on the previous three values. The static pressure inside the artificial mouth as well as the pressure inside the mouthpiece are measured with Endevco pressure sensors (8507C-5 model, a high sensitivity piezoresistive pressure transducer). The National Instrument data acquisition system (NI9215 model) digitizes these experimental signals with a sampling frequency of 25600 Hz and the signals are then sent to the computer to be analyzed in MATLAB.

### 7.3.2 Feedback loop

The air pressure allowed in the system is piloted by a program created in Simulink and is carried out through a closed-loop. A real-time dSpace controller drives a servo-valve (placed upstream an air tank) according to the static pressure measured in the box (the artificial mouth). The program allows for a static pressure or a ramp up or down in pressure as desired. Specific developments have been necessary to achieve a precise control through time of the pressure in the artificial mouth [53].

Figure 7.12 shows the feedback loop sketch used in the artificial mouth set-up. Beginning at the left side is the computer and included dSpace software. The
compressed air is sent though a pressure reducer and servo valve that is piloted by the computer and dSpace program (Figs. 7.17b and 7.17a) then sent into the artificial mouth and instrument. These values that are measured from the artificial mouth system are sent back to the computer and the system is re-regulated based on this feedback loop [53]. The user can input a target pressure, either stable or ramped. The air balloon has the capability of inputting up to about 6 bars of compressed air based on the capability of the servo-valve. This pressure reducer and servo valve is piloted by the computer program. The computer reads in the pressure value at $P_1$ in the diagram and adjusts to meet the target pressure and the servo-valve allows that pressure through. There is then another pressure measurement, $P_2$ at the artificial mouth to ensure that the target is actually being met. The pressure is then sent into the artificial mouth and the pressure in the mouth is measured by the pressure sensor there and this information is sent back and read into the computer.

Figure 7.13 shows the top view of the artificial mouth set-up showing the inserted Endevco microphone and one of the two possible supports for the clarinet. This particular support, in black, was 3D printed to offer more flexibility in how the instrument was inserted (angle, etc.). However, this support was not as sturdy and caused unwelcome vibrations as the pressure levels were increased so it was not used for the final measurements.

To further describe the apparatus, several figures showing smaller sections of the artificial mouth are included here. Figure 7.14a shows an inside view for the artificial mouth, this is near the location where the flow would enter the system. Figure 7.14b is a side view of the artificial mouth showing the syringe that is used to fill artificial lip. This figure also shows the location of the inserted clarinet. Figure 7.14c shows in greater detail the airtight seal around the mouthpiece and the location for the microphone that is inserted into mouthpiece (between ligature and modeling clay). This airtight seal ensures that the values taken for flow into the mouth (though taken outside of the mouth) are actually what is entering the instrument and that air flow is not lost between the flow meter and the mouth/mouthpiece/instrument.

Figure 7.15a shows the flow meter used with the artificial mouth, capable of measuring 80 L/mn, hooked up between the balloon and artificial mouth to measure flow into the instrument. Figure 7.15b details the air balloon that contains the air that will be supplied to the artificial mouth. The air input can be regulated in the
computer system with a piloting program. Figure 7.15c shows the placement of the flow meter between the balloon and artificial mouth.

Figure 7.16a shows a close-up of the syringe used to inflate and deflate the artificial mouth latex lip. Figure 7.16b shows the clarinet used in all measurements (artificial mouth, instrumented mouthpiece and input impedance measurements). Here it is shown with some modeling clay covering holes for a particular measurement. The clay was placed over open tone holes while ensuring it would not enter far into the chimney of the tone hole (which could alter results, taking away this added volume from the tone hole chimney). Figure 7.16c shows the long tube used during nonlinear characteristic measurements (discussed in next section). The tube is inserted into the clarinet in order to suppress reed vibrations, to maintain a static measurement situation.

Figure 7.17a shows an example screen shot of the control desk piloting program used to run the artificial mouth. The user can select the target pressure (for a linearly increasing then decreasing ramp) initial pressure (as a constant) as well as view, in real time, the output from the Endevco pressure sensors (mouth and mouthpiece pressure) and flow value (from flow meter). Figure 7.17b shows the Simulink piloting program for the artificial mouth measurements. The program
Figure 7.13: Top view of the artificial mouth set-up showing the inserted Endevco microphone into the mouthpiece and one of the two possible supports for the clarinet.

allows individual input of calibration factors as well as the intended pressure targets and ramp length (max and min pressure values for beginning and end of ramp as well as the ramp duration). Figure 7.17c shows the National Instruments acquisition box used during all artificial mouth measurements (as well as instrumented mouthpiece measurements).

Finally, Fig. 7.18 shows an example measurement session with the artificial mouth. The left laptop computer is showing the real-time signal being acquired by the NI system - the two pressure signals and one flow signal. The right computer screen shows the ControlDesk piloting program output including the ramp in blowing pressure (in red) and the measured pressure in the mouthpiece (in blue).
Figure 7.14: Photos from the Artificial Mouth: (a) Inside view of the artificial mouth, (b) Side view of the artificial mouth, (c) Airtight seal around mouthpiece.
Figure 7.15: Photos from the Artificial Mouth: (a) Flow Meter, (b) Air balloon, (c) Balloon, flow meter, and artificial mouth.
Figure 7.16: Photos from the Artificial Mouth: (a) Syringe for lip, (b) The clarinet shown with modeling clay covering holes, (c) The long tube used during nonlinear characteristic measurements.
Figure 7.17: The measurement system components: (a) Control desk piloting program, (b) Simulink piloting program, (c) National Instruments acquisition box.
7.3.3 Nonlinear Characteristic Measurement

One of the more difficult measurements is of the reed opening based on the lip force on the reed and blowing pressure. This information will be used to calculate the nondimensional playing parameter $\zeta$ for comparing the artificial mouth data with the analytical formulas. Therefore it is very important to have this measurement be as accurate as possible. Within the artificial mouth, the static reed opening is controlled by the addition or subtraction of water from a syringe into the artificial latex lip. While the addition of water can be done with relative precision using the syringe and ml markers, the effect on the reed opening can change from one measurement day to the next. The most accurate way to determine the static reed opening is to do a measurement of the static nonlinear characteristic of the reed opening and calculate the corresponding value of $\zeta$ based on the pressure input and resulting flow allowed into the instrument. The method of measuring this parameter follows the methods described in the recent article by Doc and Vergez [48].

Using the concept that the clarinet reed will react to a pressure in accordance with Bernoulli flow, the expectation is that the flow will increase with an increasing pressure difference until a point where the reed begins to vibrate. The pressure difference continues to increase but the flow decreases as the reed vibrations begin. As the pressure difference continues to increase, flow decreases and the reed reaches a closure point on the mouthpiece at a pressure $p_M$. The pressure threshold of oscillation can be estimated to be around $P_{max} \approx P_M/3$ and the corresponding point on the vertical axis of flow is $U_{max}$. The reed opening, in the nondimensionalized
parameter $\zeta$ can then be found by using an experimental estimate of this reed closure pressure, $P_M = 3P_{\text{max}}$ and $U_{\text{max}}$. This offers the information necessary to calculate a value of $\zeta$ using the formula from [30],

$$\zeta = Z_c S \sqrt{\frac{2}{\rho P_M}} = \frac{\sqrt{3} U_{\text{max}}}{2 P_{\text{max}}} Z_c. \quad (7.4)$$

Knowing the value of $P_M$ will allow for the determination of $\gamma$ as well since,

$$\gamma = \frac{P_m}{P_M}. \quad (7.5)$$

Figure 7.19 shows an experimental trial to find the values of $P_{\text{max}}$ and $U_{\text{max}}$. The idea is to eliminate the possibility for the reed to vibrate so that, within this experiment, the pressure and flow values will reach their maximums, just before the threshold of oscillation for the reed and allow for the experiment to show decline in flow before the pressure ramp descends back to zero. To do this, a small piece of foam is placed inside the plexiglass “mouth” of the apparatus and at times, even a piece of tape may be added to the reed to prevent vibration. Another way to help suppress reed vibrations in order to measure this static nonlinear characteristic curve is to insert a long tube with diameter smaller than that of the clarinet. In essence this alters the resonator and inhibits the reed from vibrating by offering an extra resistance inside the original resonator without affecting the reed opening. Figure 7.19 is an example of what this measurement is supposed to show. Ideally, the measurement of flow velocity would show (on the top plot) a nearly vertical ascent in the beginning of the plot and at the height of the pressure ramp, a slight decrease in flow. This allows for easy identification of the pair $U_{\text{max}}$ and $P_{\text{max}}$ (the pressure when the maximum flow is achieved) in order to calculate $\zeta$ and $\gamma$.

For each experimental set-up and data acquisition there will be a different setting of the reed opening. Therefore it is necessary to remeasure the nonlinear characteristic curve (value of $\zeta$) each time the set-up changes to obtain an accurate representation of the parameters used, and the resulting playing frequencies obtained. This process is long and oftentimes difficult. The possible combinations of $\zeta$ and $\gamma$ values which produce useful experimental data are extremely limited compared to the results obtained from the analytical and numerical simulations. There is, however, a large enough range to validate the results obtained in the aforementioned
Figure 7.19: The pressure ramp and resulting flow used to capture the nonlinear characteristic curve that will offer the reed opening.

Figure 7.20: A measured nonlinear characteristic curve for a particular reed opening, from [48].

Figure 7.21: A measured nonlinear characteristic curve for a particular reed opening.
analytical and numerical predictions, which is the point of the experimentation.

Figure 7.20 shows an example measurement of the nonlinear characteristic curve \([48]\). This measurement (in green) is nearly perfectly matching the theory (in red), but this is rarely the case due to the complicated nature of the measurement. The dotted red line is the theoretical curve based on the initial ramp location and the maximum flow location (in pressure difference). The curve begins with a steep increase in flow as the pressure difference increases, this continues until a point in maximum flow is reached. This point corresponds to just before the reed would begin to oscillate. Since this measurement was done with the intention of suppressing the reed vibrations, the pressure is only allowed to increase until just past this point, where the flow begins to decrease. If the measured flow curve (green) follows closely this dotted red line then the measurement is kept and used to determine the value of \(\zeta\) for this particular reed opening.

Figure 7.21 shows actual data for one nonlinear characteristic curve measurement from the artificial mouth for one of the clarinets used in this study. The initial ramp, although not as tidy as the one in Fig. 7.20 shows the expected vertical slope at the beginning and follows quite closely the dotted line expected curve. The red circle shows the maximum flow value and position in flow for this reed opening. Those values are used in determining the parameter \(\zeta\).

### 7.3.4 Example Measurements and Preliminary Data

After finding the necessary playing parameters, \(\zeta\) through measurement of the nonlinear characteristic curve and \(\gamma\) through dividing the blowing pressure by the closing pressure, the artificial mouth was also used to measure the actual sounds produced when the clarinet is blown past it’s oscillation threshold to create sound. A discussion about the conversion of data can be found in Appendix \([3]\). This section offers one such measurement with the graphical output shown in Fig. 7.22. This figure represents the the lowest note on the B\(\flat\) clarinet, note 1 (\(\approx 145\) Hz) being played with a linearly increasing ramp in blowing pressure. The data shown here is the pressure in the mouth (top), pressure in the mouthpiece (bottom plot) and the flow entering the instrument (middle plot). The static mouth pressure shown in the top plot is contrasted by the large oscillating pressure inside the mouthpiece (measured after the flow passed the reed opening). In the plot for volume flow,
notice the decrease in overall flow at the point (in blowing pressure) where the reed begins to oscillate and the instrument is emitting sound. When the reed begins to oscillate, less flow is able to enter the instrument as the reed begins to be blown closed with increasing blowing pressure.

### 7.4 Instrumented Mouthpiece Measurements

It is extremely important to have some concrete way to gauge the realism in the measurements taken by the artificial mouth. Although well and easily regulated, the artificial mouth does not give measurements in a real playing setting. One goal within this work is to understand the playability from a musician’s standpoint. The idea behind the instrumented mouthpiece is just that; place a measurement apparatus within the mouthpiece in order to measure all quantities of interest while the musician plays. This measurement apparatus should be as unencumbering as possible in order to obtain accurate measurements. This instrumented mouthpiece was developed in France through partnerships between the labs in Paris (IRCAM)
and Marseille (CNRS-LMA). In using this instrumented mouthpiece the following parameters can be measured: pressure in the mouth, pressure in the mouthpiece, musician lip force on the reed, and reed displacement. Using the instrumented mouthpiece is a necessary step in order to validate the analytical formulas for use in predicting playing frequencies for clarinets because these instruments will be played by human musicians. This fact carries with it all of the variability and difficulties of human-instrument interaction. The data from the instrumented mouthpiece could, in the future, help to further improve the prediction capabilities of the analytical formulas.

This instrumented mouthpiece is attached to a clarinet, played by a musician and is able to capture the following signals:

- **Pressure inside the mouthpiece:** this is done through a small hole drilled in the side of the mouthpiece where a small tube is inserted. This tube then leads to a pressure sensor (Honeywell) at the exterior of the instrument as to limit condensation getting to the sensors.

- **Pressure inside the musician’s mouth:** the same sized tube as was used to take the pressure inside the mouthpiece is inlay at the side of the clarinet mouthpiece and is inserted a small distance into the corner of the musician’s mouth in such a way as to not inhibit comfortable playing technique. The tube again leads to the sensor at the exterior of the instrument to limit condensation affecting the sensors.

- **Reed displacement:** is measured by two small optical sensors that are embedded into the top of the mouthpiece parallel to the reed at rest.

- **Force on the reed:** an FSR sensor can be placed (and easily removed) onto the reed and will be pressed by the bottom lip during play. The FSR is there to measure the amplitude of force a player is putting on the reed at any given time and will change based on the note and blowing pressure. This measurement of force and the reed displacement are two pieces of data that can help to understand the reed opening in different playing situations.

Using this method of testing clarinets, by having real-time scientific feedback about the functioning of the clarinet, could be very useful to manufacturers seeing as how they could keep their professional testers in order to have the “best case
Figure 7.23: An example measurement instance showing the real-time data acquisition on the laptop computer and the musician holding the apparatus.

scenario” for their results while being able to rely on the measured data and analysis as opposed to one tester’s opinion.

Figure 7.23 shows the instrumented mouthpiece in action, capturing the pressure in the mouth, in the mouthpiece and the reed displacement (as well as the force on the reed) on the computer shown in the figure.

Figure 7.24a shows the real, fully formed instrumented mouthpiece and Fig. 7.24b shows the 3D rendering of the instrumented mouthpiece as designed at CNRS-LMA in Marseille, France. Notice the tubes coming from the parallel to the mouthpiece and perpendicular. The latter captures the pressure inside the mouthpiece and the former captures the pressure in the mouth. The pink cords coming down from the mouthpiece represent the cables attached to the optical sensor that will measure the reed displacement, this sensor is not shown in this sketch.

The electronics card shown in Fig. 7.24c holds the two Honeywell pressure sensors to measure pressure in the mouth and inside the mouthpiece, optical sensor electronics and FSR sensor electronics. The card has the capability of being run on a 9 V battery (or by plugging in) and the musician can wear the card around the neck while playing for more easy and comfort.

One major drawback to the current design, is the difficulty in calibration. The optical sensor, which measures the reed displacement is relatively easy to calibrate
Figure 7.24: Instrumented Mouthpiece photos: (a) Close-up view of instrumented mouthpiece, (b) A second close-up picture of the instrumented mouthpiece, (c) Small electronics card housing the two Honeywell pressure sensors and electronics necessary to measure the reed displacement from optical sensor and player Lip force from FSR sensor.
by closing the reed completely and regulating the output voltage to zero for this position. The pressure in the mouthpiece and in the mouth are very difficult to calibrate since this would require inserting the Endevco microphones into the mouthpiece, but this is not necessarily ideal since this would put the expensive Endevco microphones in contact with condensation which could be detrimental to their future functioning. For the moment, the pressure values that are taken from these Honeywell microphones in the instrumented mouthpiece’s acquisition card are taken for qualitative comparisons only. Having pressure thresholds from the artificial mouth for similar playing situations, a user could estimate the pressure values for each measurement based on the reed opening measured with the optical sensor and subsequent value of non-dimensional reed opening $\zeta$. The optical sensor is the only sensor that can, for the moment, be confidently calibrated since the completely open and completely closed positions of the reed can be taken for any particular set-up.

7.4.1 Example Measurements and Preliminary Data

Figure 7.26 shows an example measurement taken at IRCAM with the instrumented mouthpiece in Paris played on a professional level B♭ clarinet, reed strength 2.5 Vandoren. The top plot in this figure is the audio signal taken about 0.5 m away from the bell, the second plot from the top is the pressure taken inside the mouthpiece, the third from the top is the pressure inside the mouth, the second from the bottom is the force signal taken from the change in force on the reed (exerted by the player) and the bottom most plot is the optical sensor measurement of the reed oscillation (reed displacement with 0 V representing the closed reed against the mouthpiece).

7.4.1.1 Chromatic Scales

To collect the data shown in the next few plots the musician was asked to play a chromatic scale (beginning at the lowest note and playing 1 second on each half step of the instrument). Figure 7.26 shows the Instrumented Mouthpiece (IM) recordings for the first register chromatic scale (notes 1 - 19). Figure 7.25 shows similar recordings except that the musician was asked to separate the notes in some way (i.e. playing the notes detached from one another by tonguing, etc.).
Figure 7.25: Instrumented mouthpiece measurement showing the musician playing the first register chromatic scale. The musician was asked to play the notes articulated or detached. The subplots from top to bottom represent: Audio file, Mouthpiece Pressure (Pa), Mouth Pressure (Pa), Lip Force (measured by FSR) (V), and Reed Displacement (scaled from 0 to 1).
Figure 7.26 shows the instrumented mouthpiece data for the first register chromatic scale, played slurred. There are 19 distinct notes played here, with a pause after note 11 for the player to breathe. This figure shows an interesting trend in that the mouth pressure, lip ‘force’ and reed displacement (envelope) are nearly constant, these variables are being held impressively uniform without the musician doing this deliberately. The mouthpiece pressure shows that as the player begins, more air is being supplied to the instrument. This could point to the responsiveness of the instrument itself and could be a measurement to compare the quality of different clarinets. If the pressure in the mouthpiece is high this could mean that the instrument is more responsive at that note than at others. Note 9, for example has a high amplitude in the audio file, a louder sound. This note is just near the break between the lower joint notes and the upper joint and this increase in loudness could be due to that change. Further, in regards to responsiveness, the break in the file (after Note 11) is from the player taking a breath, when the notes resume (at note 12) the mouthpiece pressure is much less than it was before. This could allude to the fact that these notes are easier to play and do not require as much blowing pressure to sound as the lower register notes. This is not the main focus of this current research but could be further studied at a later time.
Figure 7.26: Instrumented mouthpiece file showing the musician playing the first register chromatic scale. The musician was asked to play the notes slurred (attached). The subplots from top to bottom represent: Audio file, Mouthpiece Pressure (Pa), Mouth Pressure (Pa), Lip Force (measured by FSR) (V), and nondimensional Reed Displacement (scaled from 0 to 1).
Figure 7.25 shows, from top to bottom: Audio recording file (what corresponds to the .wav file), the pressure in the mouthpiece (Pa), pressure in the mouth (Pa), Lip ‘Force’ (in volts) and Reed Displacement from equilibrium (in volts). The musician was not asked to play in any particular awareness of tuning but to play naturally as possible with a similar blowing pressure over the range of the clarinet.

Figure 7.27 shows the playing frequencies of the first register chromatic scale (black) compared with the equal tempered scale frequencies (red). For the entire first register of this clarinet, for this player, the playing frequencies are lower than the equal tempered scale, increasingly more so as the musician approaches the throat tones (notes at the top of the first register).

Figure 7.28 shows a closer view of three notes in particular, Notes 1, 12, and 17. Note 1, in Fig. 7.28a and Note 12 in Fig. 7.28b show a similar trend in that, while the playing frequencies both remain under the equal tempered scale, both of these notes tends to begin very close to the equal tempered scale frequency and then decrease rapidly (Fig. 7.28a drops 18 cents, Fig. 7.28b drops 15 cents). Since the player was not given any device to indicate their tuning, it could be that the musician had an inaccurate intuition of their tuning for the entire clarinet (throughout this measurement as they were not given a beginning reference nor a tuner during the measurement and was not their instrument.). This could also be attributed to the fact that the notes were all slurred. In contrast to this trend Fig. 7.29 shows that when each note was tongued most notes showed an opposite trend, beginning flat and increasing in frequency.
Figure 7.27: Extracted playing frequencies vs. equal tempered scale frequencies from first register chromatic scale.
(a) Played frequency vs. equal tempered scale frequency for Note 1 (lowest note of clarinet, closest to the bell).

(b) Played frequency vs. equal tempered scale frequency for Note 12.

(c) Played frequency vs. equal tempered scale frequency for Note 17 (throat tones, near mouthpiece).

Figure 7.28
Figure 7.25 shows the measured AM data for a musician playing a chromatic scale in the first register, articulating each note. The notes shown here were played for about 1.5 seconds before changing to the next. For this measurement, notice that the musician releases the reed for each note, seen in the reed displacement and lip force curves. However, the minimum mouth pressure never goes to zero throughout the measurement except for a few notes (after notes 12, 15, 16, 17, and 18). This shows that the musician needed to do something extra to keep the same dynamic and tuning before each of these notes. It would be interesting to research this further to see if this were something all musicians need to do for these particular notes or not.

One interesting comparison can be made between the pressure in the mouthpiece and the audio recording for certain notes. The location of the microphone did not change for each of the notes but it is interesting to see that although the mouthpiece pressure (the input by the musician) seems to decrease as the note increases, around note 8 and 9 there is a greater output in the audio signal than before. This could mean that those notes are more responsive on this particular clarinet. This is generally the case for clarinets since for note 9 all right hand holes are open. This tends to be a very free playing note. Seeing that this ease of playing a particular note does not necessarily depend on the particular note allowing for more pressure into the instrument (actually, as shown, there is less here) would lead to another worthwhile study in the ease of play idea, though that question is not in the scope of the present work.

Looking closely at the frequency difference from the equal tempered scale frequencies offers a much different picture than when the notes were slurred. Figure 7.29 shows the playing frequencies of the first register chromatic scale (black) compared with the equal tempered scale frequencies (red) while being played detached (articulated when the note changes). In contrast to Fig. 7.27 when the notes were slurred, when the notes are articulated, as in Fig. 7.29, the clarinet tends to play above the tempered scale frequencies for the lower notes in the first register but play below the equal tempered scale for notes 9 and above.

Figure 7.30 shows a closer view of three notes in particular, Notes 1, 12, and 17. There is no apparent trend over this first register for articulated notes. Note 1 in Fig. 7.30a shows that at the bottom of the first register, when the note is articulated it is generally played sharp. Note 12 in Fig. 7.30b shows that in the middle of
Figure 7.29: Extracted playing frequencies vs. equal tempered scale frequencies from first register detached chromatic scale.
(a) Played frequency vs. equal tempered scale frequency for Note 1 (lowest note of clarinet, closest to the bell).

(b) Played frequency vs. equal tempered scale frequency for Note 12

(c) Played frequency vs. equal tempered scale frequency for Note 17

Figure 7.30
the register beginning the note tongued will play flat and finally, note 17 (a throat
tone) shows an odd mix of the two, beginning flat and ending sharp. A hypothesis
could be made here that when the tongue is involved in beginning or ending a note
it will complicate the expected results due to the changes in vocal tract and mouth
cavity shape. When slurring the notes the tongue generally stays stationary (as
much as the musician possibly can) and will not alter the tuning throughout the
held note.

7.4.1.2 One note: constant reed opening H, varying blowing
pressure P

Figure 7.31 shows the instrumented mouthpiece measurement for the case where
the musician was asked to keep the reed opening as constant as possible while
varying the blowing pressure. This task is not an easy one for musicians. Changing
either blowing pressure $\gamma$ or reed opening $\zeta$ will cause a frequency shift and in
musical settings this is avoided so it is difficult habit to change. Notice in the third
plot from the top, the mouth pressure and mouthpiece pressure were basically held
constant though not perfectly so and the lip force was decreased then increased
again as time increased.

7.4.1.3 One note: constant blowing pressure P, varying reed
opening H

Figure 7.32 shows the instrumented mouthpiece measurement for the case where
the musician was asked to keep the blowing pressure (pressure in the mouthpiece)
constant. The figure shows, in the third plot from the top, that overall pressure in
the mouth was fairly constant during this task. This task is also very difficult to do
as musicians inherently will compensate for higher blowing pressures by opening
the reed and closing when playing softly.
Figure 7.31: Instrumented mouthpiece measurement (Whitney Coyle as clarinetist). Constant reed opening ($H$) and ramp in mouth pressure.
Figure 7.32: Instrumented mouthpiece measurement (Whitney Coyle as clarinetist). Constant pressure in the mouth and a ramp up and down in reed opening (shown in the FSR measurement of Lip Force).
7.5 Conclusions

The purpose of all experimental techniques described in this chapter were to give more options for real world comparisons for the analytical formulas and numerical simulations, to create a data set that could validate using these fast and accurate prediction methods alone. Chapter 9 will offer further results and comparisons between the experiments using each apparatus and the analytical formulas.
8.1 Introduction

The next step in validating the analytical formulas would be to see how the results compare to those found when a clarinet is actually played. As a first step toward that goal, the clarinet was placed in a controlled environment and played with an artificial mouth, in order to vary one parameter at a time. Then similar experimental results were attempted with the instrumented mouthpiece. This chapter will offer the preliminary data along those lines and briefly discuss future possibilities with these techniques.

8.2 Just-Noticeable Frequency Difference as a Benchmark

It is important to address the threshold and standard to which the comparisons between any of the discussed techniques are held in order to be considered “close enough”. Psycho-acoustic literature generally describes this difficult distinction by assuming pure tones, played and separated by a pause. In this scenario a 10 cent difference is perceptible [51]. In fact, there are many articles that claim a discrepancy of about 10 cents is the general rule for perceived difference in
frequency [57,67,110]. However, many assumptions go into using this rule and this value changes based on a number of factors: the listener’s musical training, sound level, sound source type and location, duration of tones being played, complexity of the sound (pure tones, sine waves, square waves, a clarinet vs. piano, etc.), and whether the tone is played alone or together with another tone.

The main ideas about this threshold of tuning perception come from research involving a just noticeable difference (JND). Psycho-acousticians study JND in amplitude, frequency, etc. In the frequency case, when the source output is a sine wave, the total variation is actually $2\Delta f$, where $\Delta f$ is the smallest detectable difference in frequency (Hz) between two tones being played (either in succession or simultaneously). This $\Delta f$ is highly frequency dependent, especially in the higher frequency range. According to [51] at low frequencies, below 500 Hz, $\Delta f$ is nearly constant and is about 1.8 Hz. Above 500 Hz this value of $\Delta f$ increases proportionally at approximately $0.007 \cdot f$. At low frequencies human hearing is rather insensitive to these changes and below 50 Hz humans are capable of discerning no more than a semi-tone (100 cents).

Another intricacy is whether or not the two tones are played successively or simultaneously. There is actually a great deal of research investigating the perception and performance of intonation in the instrumental performance of individuals using each case ([6,23,28,49,66,101,129]). There is still no agreement as to what the appropriate JND for $\Delta f$ would be over the range of a musical instrument.

The JND in frequency is also highly subject dependent. The listener’s musical background and personal hearing thresholds will affect their ability to discern between two frequencies [51]. Nikjeh et al. studied the difference limens for frequency (DLFs) [101]. The DLF is similar to the JND and their results show that musicians had a DLF of 23 cents. In contrast the non-musicians will only be able to hear a difference of 53 cents, which is greater than a quartertone.

Fastl et al. mention an important point, however, that musical tones are rarely simply sinusoidal and therefore this deserves a more complex look at “how close is close enough” [51]. There is not a consensus in regard to the difference limens for frequency (DLF) or just noticeable difference (JND). Nikjeh et al. say 23 cents [101]. Madsen et al. revealed an average of 10 cents for wind players with a wide variety of years of experience in a single context [89]; Rodman reported a minimum of 15 cents for young instrumental music students and approximately 6 cents for adult
musicians when distinguishing between pairs of notes \[108\]; and Clark found that college and high school wind players were best able to detect differences between 10 and 20 cents in a variety of presented situations \[28\].

Throughout the research for this thesis, the only players considered are those playing professional grade instruments who have thorough musical training, giving them a trained ear. The frequency range of interest, the fundamental frequency range, is from 146 Hz to around 1100 Hz and the experimental procedures call for a mix of sustained tones and broken passages. Therefore, due to the general rules presented above and the frequency range and subject base of interest, the goal for the work presented attempts to reach agreement between the numerical simulations and the analytical formulas within 10 cents, for each individual effect and more importantly, for the total of the combined effects.

8.3 Comparing Analytical Formulas with Artificial Mouth Data

As was shown in Chapter \[7\] the artificial mouth (AM) is used in order to ensure that the reed opening $\zeta$ remains constant while varying the blowing pressure. Most experiments run with the aim to mimic the analytical formulas, i.e. a constant $\zeta$ and increasing $\gamma$ from $\gamma_{th}$ up to a maximum. In order to match the analytical formulas, the range of $\gamma$ would ideally be from $\gamma_{th}$ to $\gamma = 1$. Unfortunately, it was discovered that for most tests run, the values of $\gamma$ were not pushed far enough past what is considered the beating reed regime, or $\gamma = 0.5$. This should not be a big issue since the analytical formulas are known to be accurate up to the beating reed regime and merely approximate after this. Therefore, the data should still offer enough information for comparison.

8.3.1 Uses of Artificial Mouth in Literature

In this section, the functionality of the Artificial Mouth (AM) will be discussed in greater detail to explain how the feedback loops must function. The artificial mouth used for these experiments was described in Chapter \[7\] and is detailed in \[53\]. Ferrand et al. discuss the technological details of the AM and coded feedback loop that essentially pilots the AM. Figure 8.1 shows an example of the ramps.
that can be produced with this artificial mouth. This data from Ferrand et al. shows pressures only reaching about 3-3.5 kPa. This is likely due to the Endevco microphone chosen, with a limit near 5 kPa. In the future, the measurements discussed in this chapter will need to be repeated using a much higher amplitude microphone. Unfortunately, this limitation was not discovered until after the experiment was completed at which point the author no longer had access to experimental equipment.

Figure 8.1: Figure 16 from Ferrand et al. [53] showing the limits of their artificial mouth set-up. The blue was the target pressure (reference) in kPa. The pink and red curves show two separate measurements of static pressure in the artificial mouth reaching around 3.5 kPa.

Many researchers are beginning to use an artificial mouth/mouthpiece apparatus in order to understand better how musical instruments function given varied input parameters. Before discussing the artificial mouth data for the current work, it may be useful to summarize how the AM has appeared in literature and the limitations of its operation. Most recently, Alemeida et al. worked with an artificial mouth in order to create playing maps (in frequency and loudness) for clarinets [10]. Figure 8.2 shows that their artificial mouth was able to reach pressures between 3 and 7
kPa for a large reed opening, before the reed closure pressure. For very open reeds the closing pressure is higher (7 kPa) and for more closed reed (small H, large Lip Force F, small ζ) the closing pressure is much lower (about 3 kPa). For a somewhat below average ζ or lip force in Fig. 8.2 the closing pressure is between 4.5 and 6 kPa.

![Graph showing blowing pressure limits](https://example.com/graph.png)

**Figure 8.2:** Figure from Almeida et al. [10] showing the blowing pressure limits in their artificial mouth set-up. For a large opening of reed (low lip force), a blowing pressure of nearly 7 kPa was reached.

Figure 8.3 from [45] shows how Dalmont et al. created tuning lines (fixed ζ and increasing γ) for a clarinet. The pressures possible with their set-up were between 3 and 8 kPa for Note 1 of the clarinet. This figure shows (with blue circles) the measured playing frequency for a particular note. For this note, the pressure in the mouth reaches up to 90 hPa = 9 kPa.
Dalmont and Nederveen showed the functioning of the artificial mouth and resulting playing frequencies for different playing parameters (including reed parameters of vibrating surface area, $q_r$ and $\omega_r$, the reed resonance frequency) \cite{100}. Figure 8.4 shows that with their artificial mouth pressures reached about 5.5 kPa. Their threshold is nearly the same as for the artificial mouth and clarinet used for this research, but the beating reed regime seems to begin much earlier (judging by the decrease leading up this point and increase in frequency afterwards), closer to about 3 kPa.
Figure 8.4: Artificial mouth data from Dalmont and Nederveen showing the playing frequency for six different combinations of reed parameters and reed openings with increasing blowing pressure [100].

Recently the artificial mouth used in this dissertation was used for the work of Bergeot et al. [24]. Bergeot studied the response of the artificially blown clarinet to different pressure ramps and transients. Figure 8.5 shows a simple pressure ramp (top black line) able to reach nearly 8 kPa with their chosen pressure sensor. The artificial mouth used in Bergeot’s work is the exact apparatus used for this project. However, the exact make and model for the pressure microphone they used was not listed in the reference.
8.3.2 Discussion of Artificial Mouth Data

In each of the previous cases the blowing pressure reached up to at least 8 kPa, whereas with the artificial mouth in use for this project (as measured with Endevco pressure microphones) did not reach higher than 5 kPa, with most ramps being below the threshold of oscillation, starting around 2 kPa and ending at or around 5 kPa. This upper limit could also be due to when subjective listening of the artificial mouth playing the instrument. Since a musician was running the apparatus, there may have been a bias considering what sound was expected from this particular instrument. Had a non-clarinetist piloted the artificial mouth ramps, perhaps this limit could have been overcome.

The artificial mouth used for this research is still a work in progress and was not continuously available during this research for taking measurements. Due to the need for sharing the apparatus with other research groups, as well as other unforeseeable circumstances throughout the project, the data gathered for this project spanned perhaps two weeks of testing and during those perhaps only four days worth of somewhat reliable data was gathered. The artificial mouth was extremely temperamental, working wonderfully one day and not at all the next. Any change made at the end of a measurement day (taking the clarinet from the apparatus in order to store it safely for the night) would affect the data gathered.
the following day. The method of closing the reed with the water-filled latex lip was also less than ideal in that the addition of water was not done with any precision and the reed opening was not able to be closed in a smooth ramp meaning that all values of $\zeta$ were not even possible. In future work it will be necessary to spend time to design and build a better artificial mouth in order to have more reliable and repeatable data to compare with the analytical formulas. However, in spite of the problems and limitations encountered with the AM, some potentially useful data may be discussed.

For each experimental situation the nonlinear characteristic curve was measured as per Chapter 7 in order to determine the particular value of $\zeta$ or reed opening depending on how much the latex lip was inflated with water. This was a very tedious measurement since the clarinet would not play all notes for every value of $\zeta$. In reality, the musician would adjust their embouchure (lip force and position) for the note they want to play. But here, the idea was to keep a constant reed opening for the entire first register.

The input impedance measurements for a clarinet were necessary to input for the analytical formulas. These input impedance measurements were taken at a particular temperature (22°C) and the artificial mouth measurements, not taken on the same day, were taken at another (26°C). This temperature difference was taken into account in the analytical formulas as a fixed length correction for each note in order to account for the effect of the 4°C difference.

The main notes of interest for the first register of the clarinet were: Note 1 (fingered E, $f_{\text{tempered}} \approx 146$ Hz), Note 4 (fingered G), Note 9 (fingered C), Note 12 (fingered E♭), Note 14 (fingered F), Note 16 (fingered open G), and Note 17 (fingered A♭). Some notes were more responsive than others; this could be reed or clarinet dependent and could not always be remedied.

The data for each note is presented in two sets of figures. The first plot shows the measurements that the artificial mouth allows, pressure in the mouth (Pa), pressure in the mouthpiece (Pa) and flow into the instrument (L/mn). Notice that for each of these plots as the input pressure (pressure in the mouth) increases the flow is decreasing. With a high enough blowing pressure ($\gamma = 1$), the flow should theoretically tend towards zero. The second set of figures shows (i) the playing frequency as a function of blowing pressure $\gamma$, (ii) the difference between measured playing frequency and the equal tempered scale, and (iii) the analytical formula.
prediction of the difference between playing frequency and the equal tempered
frequency.

Figures 8.6, 8.7a, 8.7b and 8.7c are the data for Note 4 and $\zeta = 0.37$. This
particular note has a threshold of oscillation around $\gamma = 0.35$, begins about 10
cents sharp (higher than the tempered scale frequency), drops quickly to meet the
tempered scale frequency and between $\gamma = 0.4 - 0.5$ seems to level out a bit. The
analytical formulas predict that after the beating reed threshold of $\gamma = 0.5$ the
frequency should increase again. This particular measurement did not show data
past this point so it is difficult to say if this would happen or not. Lower notes on
the clarinet tend to be flat [116] so it is not surprising that for very low blowing
pressures into the normal playing range the playing frequency would be well under
the tempered scale frequency. Comparing this to the analytical formula predictions
in Fig. 8.7c shows that there is similar trend between the prediction method and
the artificial mouth measurements. The analytical formulas predict that below
the beating reed threshold the frequency will first decrease then level out until
its increase due to the reed closure. The formulas do not, however, predict the
extreme of the frequency drop as the pressure increases; formulas predict a drop of
only about -13 cents whereas the experiment shows a drop to -40 cents below the
tempered scale frequency. Note 4 is towards the lower range of the instrument; in
general, these notes would begin to be affected by a temperature gradient when
a musician is playing. However, for the artificial mouth, the temperature was
measured during playing and the temperature out of the balloon and into the
mouth was nearly identical to the temperature measured at the bell. Therefore,
one should not include a temperature gradient in the predictions of the playing
frequency of the clarinet played by the artificial mouth. For note 4, Fig. 8.7c the
temperature effect was left out; ignoring the temperature effect does bring the
playing frequency prediction into the sharp region for the note, which is where
the artificial mouth played, but it is predicting a higher sharpness than actually
occurred. In other words, the playing frequency begins sharp by about 15 cents
matching the measured data at $\gamma = 0.35$, but does not drop flat for higher blowing
pressure $\gamma$. 
Figure 8.6: Artificial mouth measurements, Note 4, $\zeta = 0.37$. 
(a) Playing frequency of the AM. Red line is equal tempered scale frequency.

(b) Difference in cents between AM measurement and equal tempered scale frequency.

(c) Analytical formulas results for the total difference between predicted playing frequency and the equal tempered scale, in cents.

*Figure 8.7: Note 4 - $\zeta = 0.37$*
Figures 8.8, 8.9a, 8.9b, and 8.9c are the four output plots for Note 14 and $\zeta = 0.32$. Although these figures use the same reed opening value as the previous two, the trend and overall playing frequency found from the artificial mouth measurements are much closer to what is predicted by the analytical formulas. Figure 8.9b shows that the artificial mouth in this configuration gave a difference between -15 cents and 2 cents and that the trend is similar to what was expected with a decrease in playing frequency until about the beating reed regime, here being around $\gamma = 0.48$. The analytical formula prediction shown in Fig. 8.9c shows this decreasing trend with a difference between 28 to 18 cents. This difference in prediction and experimental results, nearly 25 cents different at the threshold of oscillation is not within the JND and does not give validation to the use of the analytical formulas just yet (under the assumption that the data is accurate and reliable).

The third subplot in Fig. 8.8 shows a good example of the flow trend during this increasing blowing pressure. At the beginning, the flow decreases as pressure increases and around 8 ms begins to increase slightly again. It is perhaps at that point, the point of increase, that the beating reed is actually beginning, earlier than predicted.
Figure 8.8: Artificial mouth measurements, Note 14 $\zeta = 0.32$. 
(a) Playing frequency of the AM. Red line represents the equal tempered scale frequency.

(b) Difference in cents between the playing frequency of the AM and the equal tempered scale frequency.

(c) Analytical formulas results for the total difference between predicted playing frequency and the equal tempered scale, in cents.

*Figure 8.9: Note 14 - $\zeta = 0.32$*
Figures 8.10, 8.11a, 8.11b, and 8.11c are the data plots for Note 16, $\zeta = 0.17$. This is for a much lower value of reed opening and the predictions show a noticeable increase in the playing frequency for such a small reed opening. This is seen in Fig. 8.11b for example, in that the trend still seems familiar, the decrease with increasing blowing pressure until $\gamma = 0.5$, but this time the playing frequency for all values is around 20 cents higher. The analytical formula predictions in Fig. 8.11c shows a similar sharpness in playing frequency (whereas in previous figures the playing frequency tended to be below the equal tempered scale frequency). The artificial mouth data shows a slight increase in playing frequency towards the end of the data but not nearly as marked as the analytical formulas predict. The analytical formulas adequately predict (correct range of cents different) the range of playing frequencies that would result for this note and reed opening despite the difference in trend between the predictions and experiment.

8.3.3 Artificial Mouth Conclusions

For the moment there is not a satisfactory explanation for the larger discrepancies presented in this section other than perhaps misrepresentation of the following within the analytical formulas: reed parameters not input as closely as they could be (reed vibrating surface area $S_r$, reed resonance $\omega_r$ and reed quality factor $q_r$), the predictions only using two impedance peaks in the predictions, etc. This hypothesis can be tested both analytically (by taking a very close look at the comparison of results for a fine grid of input parameters) and experimentally, by carefully measuring the reed characteristics and temperature information within the artificial mouth. Of greater importance would be to repeat artificial mouth measurements with several systematic experiments. There is a need to precisely adjust the lip force on the reed and subsequently the static reed tip opening. Testing the reed stiffness and resonance frequency before measurements in order to have exact inputs would also help to create a more accurate prediction.
8.4 Possible Preliminary Comparisons with Instrumented Mouthpiece Data

For many decades clarinet manufacturers have used musicians to test their instruments before they leave the factory intended for sale. The clarinet testers at Buffet typically come in for 3-5 hours at a time and test nearly one hundred instruments, noting their flaws, aesthetic and mechanical, tuning variability and overall timbre. These testers try to be objective, but it is expected that each individual would have a personal preference for sound and feel of an instrument. This testing process is exhausting for the musicians and not completely reliable for the manufacturers. The approach discussed in this section is meant to be a step away from relying on one person’s subjective opinion of the clarinet and more towards a scientifically objective evaluation of a great instrument. The use of the artificial mouth, discussed in the previous section, was able to approximately validate the trend in playing frequency, though not completely matching the analytical formulas.

Another method of validation is to use an instrumented mouthpiece, described in Chapter 7, which can measure the pressure in the mouthpiece, pressure in the mouth, reed displacement and lip force on the reed.

There is not a great wealth of data using the instrumented mouthpiece just yet. For each individual type of clarinet studied there needed to be a mouthpiece made. There were many issues in calibrating each of the sensors and the FSR currently used is not sensitive enough to get accurate results for large reed openings (little force on the reed). Although experiments with this apparatus are still in the beginning stages there are several preliminary tests that show interesting results, and those will be presented here. In each of the experiments presented, the musicians were not watching a real-time view of their playing data. So, when requested to hold blowing pressure constant, they were using their judgment and not the feedback necessary to make the measurement as constant as it could perhaps be.

Chapter 7 described a few capabilities of the instrumented mouthpiece, including the chromatic scales in each register and holding a constant (static) reed opening, as seen in the measurements of the ‘lip force’ and ramping the blowing pressure. The musician could also attempt to hold the blowing pressure constant while ramping the ‘lip force’. The following subsections will detail these experiments and offer the
sample data that was taken with the instrumented mouthpiece.
Figure 8.10: Artificial mouth measurements, Note $16 \zeta = 0.17$. 

\[ \zeta = 0.17. \]
(a) Playing frequency of the AM. Red line represents the equal tempered scale frequency.

(b) Difference in cents between the playing frequency of the AM and the equal tempered scale frequency.

(c) Analytical formula results for the total difference between predicted playing frequency and the equal tempered scale, in cents.

Figure 8.11: Note 16 - $\zeta = 0.17$
8.4.1 Individual Notes: Constant P, Ramp H

When studying individual notes with the instrumented mouthpiece the two main playing parameters used were: blowing pressure $\gamma$ and reed opening $\zeta$. The instrumented mouthpiece equivalent parameters for $\zeta$, the reed displacement or the lip ‘Force’. For this test, the musician was asked to keep the blowing pressure as constant as possible while varying the reed opening. For musicians, this is very difficult to do as changing either blowing pressure $\gamma$ or reed opening $\zeta$ will cause a frequency shift. In musical settings this is avoided so it is difficult to change this habit.

Figure 8.12 shows the instrumented mouthpiece file for Note 1 played with a constant blowing pressure and the reed opening ramped from closed to open. This can be seen in the ‘lip force’ measurement. The reed opening is controlled in play by pressing the reed harder or softer. A ramp in reed opening from closed to open would be large force (nearly closing the reed) to small force (just enough to have the reed still oscillate in the correct manner). The measurement was considered successful if the mouth pressure (subplot three from the top in Fig. 8.12) was a nearly straight line starting at onset. The FSR measuring the lip force on the reed went quickly to zero as is seen from the thin straight line in this figure around time $t = 5.5$ seconds. This was an ongoing problem throughout the measurements and has yet to be remedied. This represents zero force on the FSR and subsequently the reed. But this was not the case; the FSR was not sensitive enough to pick up these small forces placed on the reed. Nevertheless, plenty of information can still be gathered from these measurements.

Figures 8.13a and 8.13b show the frequency variation for this measurement. While the musician played with a normal mezzo forte dynamic and a closed embouchure at the beginning of the measurement the frequency matched well the equal tempered scale. Towards the end of this measurement the difference was nearly 30 cents flat. The ability of the musician to have the instrument play with this flexible of a range in tuning for one note is quite impressive since it was achieved with only the use of opening and closing the embouchure (decreasing and increasing the force exerted on the reed by the bottom lip and teeth).

Figure 8.14 shows the complete instrumented mouthpiece measurement for Note 4. For this measurement the mouth pressure was much more constant, dipping
Figure 8.12: Instrumented mouthpiece measurement for Note 1 (low E) for an attempted constant mouth pressure (blowing pressure) and ramp in reed opening (closed to open to closed again - Lip force is high, then low, then high). The subplots from top to bottom represent: Audio file, Mouthpiece Pressure (Pa), Mouth Pressure (Pa), Lip Force (measured by FSR) (V), and Reed Displacement (scaled from 0 to 1).
Figure 8.13: Instrumented mouthpiece measurement of constant blowing pressure and ramp in reed opening (closed to open) for Note 1.
slightly in the middle of the measurement but only slightly. The lip force shows again the limitations of the FSR (as it drops to zero) but does show that the reed much more closed at the beginning than it was for Note 1. Note the difference in vertical axis scale here versus Fig. 8.12. For Note 1 the reed begins at a voltage of about 1.5 whereas Note 4 begins closer to 2.5. Since the reed began much more closed one would expect to see a higher frequency throughout the measurement (compared to the equal tempered scale), which seems to occur with the onset of the note beginning 15 cents sharp and dropping to about 25 cents flat in the middle of the measurement. Figures 8.15a and 8.15b show the frequency variation for this measurement compared to the equal tempered scale frequency for this note. The results are similar to those from Note 1 where the frequency varies between 15 cents sharp to -22 cents flat.

Figure 8.16 shows the complete instrumented mouthpiece measurement for Note 12. The measurements shows a similar trend that the mouth pressure is relatively constant but dips slightly in the middle. This is at the same place that the reed is at its most open. Notice too that at this point the optical sensor measuring the reed displacement has the highest output which makes sense here because the reed is able to oscillate more freely. Considering the frequency change due to these embouchure adjustments in Figs. 8.17a and 8.17b the same trend follows as before. But this time an impressive drop from 10 cents sharp to 40 cents flat is achieved.
Figure 8.14: Instrumented mouthpiece measurement for Note 4 (low G) for an attempted constant mouth pressure (blowing pressure) and ramp in reed opening (closed to open to closed again - Lip force is high, then low, then high). The subplots from top to bottom represent: Audio file, Mouthpiece Pressure (Pa), Mouth Pressure (Pa), Lip Force (measured by FSR) (V), and Reed Displacement (scaled from 0 to 1).
Figure 8.15: Instrumented mouthpiece measurement of constant blowing pressure and ramp in reed opening (closed to open) for Note 4.
Figure 8.16: Instrumented mouthpiece measurement for Note 12 (low E♭) for an attempted constant mouth pressure (blowing pressure) and ramp in reed opening (closed to open to closed again - Lip force is high, then low, then high). The subplots from top to bottom represent: Audio file, Mouthpiece Pressure (Pa), Mouth Pressure (Pa), Lip Force (measured by FSR) (V), and Reed Displacement (scaled from 0 to 1).
Figure 8.17: Instrumented mouthpiece measurement of constant blowing pressure and ramp in reed opening (closed to open) for Note 12.
8.4.2 Individual Notes: Constant H, Ramp P

In this test the musician was asked to keep the reed opening as constant as possible while varying the blowing pressure. Again, this task is not an easy one for musicians. Changing either blowing pressure $\gamma$ or reed opening $\zeta$ will cause a frequency shift, and in musical settings this is avoided so it is difficult habit to change.

Figure 8.18 shows the results from the instrumented mouthpiece measurements for Note 1 of the clarinet. The musician was asked to hold the reed opening constant (referencing to a constant lip force) and increase then decrease the blowing pressure. For the measurement here, as for each task in this section, the musician did not have immediate feedback as to their success in this task but was allowed to view the results just after and re-do the task if necessary. Notice that in this figure, for the Lip Force (subplot four) the FSR sensor decreased to zero around 10 seconds in this instance. This does not mean that the musician was no longer pressing on the reed. As in the previous experiment, this was a downside of using this particular force sensor. This ongoing problem with this particular build of instrumented mouthpiece is evident throughout this section of measurements.

Figures 8.19a and 8.19b offer a frequency analysis of the data in the previous figure in playing frequency and cents different from the equal tempered scale respectively. The red line in Fig. 8.19a shows the equal tempered scale frequency for this fingering. These figures show that the musician actually begins to play quite close to ‘in-tune’ and quickly drops below the intended note while increasing the blowing pressure and returns to the intended frequency by decreasing the blowing pressure. With the idea of JND in mind, the 7 cents sharp is within the range of ‘in-tune’ for this research. The range in tuning is between 10 cents sharp and -15 cents flat by varying the blowing pressure for this note, the lowest of the instrument. If the musician was able to play this note, at the extreme of the instrument, nearly in tune for a low dynamic, it is expected that for a note higher than that will be quite sharp in comparison (for the same blowing pressure level) [71].

Figure 8.20 shows the results from the instrumented mouthpiece measurements for Note 4 of the clarinet. This figure shows that the musician was better able to control both the blowing pressure as well as the reed opening since the ‘lip force’ line was nearly straight and constant around 2.5 V and did not drop at or below the zero (limit of FSR) as the other measurements had been. On the other hand,
Figure 8.18: Instrumented mouthpiece measurement for Note 1 for an attempted constant $H$ (horizontal ‘lip force’ measurement) and ramp up then down of mouth pressure (blowing pressure). The subplots from top to bottom represent: Audio file, Mouthpiece Pressure (Pa), Mouth Pressure (Pa), Lip Force (measured by FSR) (V), and Reed Displacement (scaled from 0 to 1).
Figure 8.19: Instrumented mouthpiece measurement of constant reed opening and ramp in blowing pressure for Note 1.
Figure 8.20: Instrumented mouthpiece measurement for Note 4 for an attempted constant $H$ (horizontal ‘lip force’ measurement) and ramp up then down of mouth pressure (blowing pressure). The subplots from top to bottom represent: Audio file, Mouthpiece Pressure (Pa), Mouth Pressure (Pa), Lip Force (measured by FSR) (V), and Reed Displacement (scaled from 0 to 1).
Figure 8.21: Instrumented mouthpiece measurement of constant reed opening and ramp in blowing pressure for Note 4.
the musician was not able to reach the same blowing pressure level as for Note 1. Many musicians believe (correctly) that the lowest note of the instrument is the most free and are able to play very loudly, whereas for notes in the mid-low range of the instrument are less responsive. Notice also that for all instrumented mouth measurements in this section the crescendo is steady and long and the decrescendo is shorter. This could be attributed to the decrease in air capacity of the player, but this effect is evident in all measurements in this section.

Figures 8.21a and 8.21b offer a frequency analysis of the data for Note 4 in playing frequency and cents different from the equal tempered scale respectively. These figures follow a very similar trend to that of Note 1, beginning in-tune and dropping below following the increase in blowing pressure. Note 4 here is capable of playing nearly 10 cents sharp and 15 cents flat compared to the equal tempered scale.

Figure 8.22 shows the results from the instrumented mouthpiece measurements for Note 12 of the clarinet. As with the previous notes the crescendo or increase in blowing pressure lasts about 5 seconds whereas the decrescendo or return to below the oscillation threshold lasts only 3 seconds. Notice also that the maximum mouthpiece pressure is 1000 Pa and the mouth pressure 4000 Pa compared to 3000 Pa and 4000 Pa respectively for Note 1 in Fig. 8.18. There is noticeably more resistance when playing this note as the note was played with a fingering utilizing the side keys (sometimes called trill keys) which are played with the right hand. These holes are much smaller than the other keys played by the left hand (controlling the upper joint of the clarinet) and sometimes create an airy timbre, especially at the extremes of the dynamic range.

Figures 8.23a and 8.23b show the frequency variation with blowing pressure for Note 12. This note shows a frequency variation between -5 cents flat to -25 cents flat, a much larger dip below the intended equal tempered scale frequency than the previous notes.

Figure 8.24 shows the results from the instrumented mouthpiece measurements for Note 16 of the clarinet - the open G in the first register where no keys are being depressed and the clarinet is supported only by the mouth and the thumb rest hanging on the right thumb. This note has a tendency to be quite flat but has a great flexibility in playing frequency [71].

Figures 8.25a and 8.25b show the frequency variation for this note. Notice the
Figure 8.22: Instrumented mouthpiece measurement for Note 12 for an attempted constant H (horizontal ‘lip force’ measurement) and ramp up then down of mouth pressure (blowing pressure). The subplots from top to bottom represent: Audio file, Mouthpiece Pressure (Pa), Mouth Pressure (Pa), Lip Force (measured by FSR) (V), and Reed Displacement (scaled from 0 to 1).
(a) Note 12 - Frequency vs equal tempered scale frequency (red)

(b) Note 12 - Difference in cents

Figure 8.23: Instrumented mouthpiece measurement of constant reed opening and ramp in blowing pressure for Note 12.
Figure 8.24: Instrumented mouthpiece measurement for Note 16 for an attempted constant H (horizontal ‘lip force’ measurement) and ramp up then down of mouth pressure (blowing pressure). The subplots from top to bottom represent: Audio file, Mouthpiece Pressure (Pa), Mouth Pressure (Pa), Lip Force (measured by FSR) (V), and Reed Displacement (scaled from 0 to 1).
Figure 8.25: Instrumented mouthpiece measurement of constant reed opening and ramp in blowing pressure for Note 16.
smoothness of the drop and lift in frequency for this measurement. This really shows the ease at which the musician is able to manipulate the tuning of this note by beginning the note about 10 cents sharp and dropping it to 25 cents flat with an increase in peak blowing pressure of about only 1 kPa (mouthpiece (oscillating) pressure).

Figure 8.26 shows the results from the instrumented mouthpiece measurements for Note 17 of the clarinet - the side-key, A♭ - at the top of the first register. This note is considered to be in the throat tones and does not always behave as the others notes presented in this section thus far [116]. Figures 8.27a and 8.27b show a similar trend and a great flexibility in frequency as was seen for the open G in the previous figures. It does seem that this note begins much more sharp than the others (25 cents sharp). All notes presented in this section were played sequentially from lowest to highest in frequency, so there is a possibility of musician fatigue as the measurements went on.
Figure 8.26: Insrtumented mouthpiece measurement for Note 17 for an attempted constant H (horizontal ‘lip force’ measurement) and ramp up then down of mouth pressure (blowing pressure). The subplots from top to bottom represent: Audio file, Mouthpiece Pressure (Pa), Mouth Pressure (Pa), Lip Force (measured by FSR) (V), and Reed Displacement (scaled from 0 to 1).
Figure 8.27: Instrumented mouthpiece measurement of constant reed opening and ramp in blowing pressure for Note 17.
8.4.3 Comparing the two measurements

The tuning flexibility was measured in the previous sections in two different ways. First, by holding the blowing pressure (mouth pressure) constant and varying the reed opening (lip force) and secondly, by holding the reed opening constant and varying the blowing pressure. Comparing, for example, Figure 8.17a to 8.23a shows that a greater range for the shift in playing frequency is achieved by varying the reed opening or lip force. This is not a definitive test in order to determine this shift specifically since many other playing parameters are varied when using real musicians (for example as the reed is closed by adding force from the bottom lip and teeth, the reed’s vibrating surface area is smaller).

8.4.4 Instrumented Mouthpiece Conclusions

From these measurements it is clear that the musician is quite successful at maintaining the mouth pressure and varying the lip force. However, it seems that the opposite (constant opening, varied pressure) is much more difficult to accomplish. Regardless, in order to actually make these possible comparisons of the experiment with the instrumented mouthpiece and the analytical formulas the change in lip force will be quite complicated to model accurately. This is because the lip force is actually changing several parameters of the model at once: reed opening at rest, reed damping, the reed resonance frequency and therefore the reed compliance and reed flow rate. Here, two of the three frequency shift effects are concerned (reed dynamics and reed-induced flow) when the player changes force on the reed. For this reason, future comparisons with theoretical results will be difficult.

Nevertheless, the use of the instrumented mouthpiece to extract playing parameters will be interesting in future work. The analytical formulas predict with dimensionless playing parameters, many of which are based on user input of the reed characteristics including the vibrating surface area and rest position. Ideally, these measurements taken from use of the instrumented mouthpiece can be used as a third validation (after the numerical simulations and artificial mouth) and perhaps even a way to improve the analytical formula predictions through the use of empirical fits to data trends in regions that are difficult to understand through simplified analytical formulas, such as after the beating reed regime.
9.1 Introduction

Research on any given topic begins with a hypothesis, deciding the best way to test this hypothesis, determining the possible outcomes the research could present, and finally assessing the implications of these results. The main focus of this research was to combine the simplified analytical formulas found in the literature and use MATLAB to compute the resulting playing frequencies in order to have a computationally fast and simple way to make these predictions for a given clarinet. A number of validation techniques were sought in order to justify the use of the analytical formulas as a sole predictor of the resulting playing frequency for a clarinet. The hypothesis is that combining the formulas that model the individual effects (inharmonicity, reed-induced flow, reed dynamics, temperature gradient) to predict a resulting playing frequency will yield a realistic representation of the playing frequencies that one would measure when testing a given clarinet with a musician and tuner. Using these analytical formulas to make these determinations, as opposed to musician testing, will completely control the input variables based on user choice. However it is important to validate their functioning against models in literature, such as the numerical model described in Chapter 6, as well as to verify
against an actual clarinet, played either by an artificial mouth (blowing machine) or a musician using an instrumented mouthpiece (all introduced in Chapter 7). Using numerical and experimental techniques in this research provides multiple ways to confirm the validity of the analytical formulas introduced in this research.

The numerical simulations use the same physical model as the analytical formulas and so it would be expected that the two would match very closely. However, as was discussed in Chapter 5, there are many simplifications in the analytical formulas and it would not be surprising if the results were to vary in certain cases. One of these cases is for the playing regime above the beating reed ($\gamma$ values above 0.5). Due to the simplifications and the empirical fit that the analytical formulas take above the beating reed threshold it is likely that the agreement will be less than satisfactory (based on the criteria discussed in Section 8.2). Results in this playing range will require further investigation that is beyond the scope of the current work due to the current model.

This chapter will begin by establishing a benchmark, what level of agreement between the analytical formula results and other validation techniques (numerical simulations and experimental measurements) will be acceptable. This will include a discussion about the idea of just noticeable difference. Following this will be a brief presentation of the analytical formulas’ output for various notes in the first register of the clarinet for a number of reed openings. Finally, a presentation and discussion of results from each of the different validation techniques will be offered: comparison with the numerical simulations for individual notes considering one value of reed opening $\zeta$ and finally the tuning maps for all values of $\gamma$ and $\zeta$.

## 9.2 Analytical Formulas

The most important result from this dissertation is to show that the analytical formulas developed (as described in Chapter 5) are capable of being used, on their own, to predict the playing frequencies for a B♭ clarinet. In order to do this, a number of different validation methods have been called upon. These include comparisons with the numerical simulations (Chapter 6) and the preliminary comparisons with measurements from the artificial mouth and the sensor equipped mouthpiece (discussed in Chapter 8). In this section the results from the analytical formulas will be presented on their own. The formulas were programmed easily
into MATLAB and are overall a function of the two playing parameters ($\gamma$ and $\zeta$), fixed mouth temperature and fixed reed parameters (reed resonance frequency, quality factor, and the vibrating surface area).

Figure 9.1 shows four separate plots of the analytical results. In each of the analytical formulas plots in this section the color scheme is the same: red is the reed dynamics effect, cyan is the inharmonicity effect, green is the reed induced flow effect, blue is the total of those three effects and magenta is the temperature effect. Each of the plots has the same horizontal and vertical axis. The horizontal axis shows $\gamma$ increasing from around the threshold of oscillation (different depending on $\zeta$). The vertical axis is the difference from the resonance frequency of the clarinet, in cents. Figure 9.1 includes four individual subfigures, all sharing the same axes (cents different as a function of increasing $\gamma$) but each subplot represents the output of the analytical formulas for a different choice of reed opening $\zeta$. From top left, $\zeta$ ranges from 0.2 to 0.35, all reasonable choices for a playable reed opening, fairly closed. For comparison, $\zeta$ for saxophone mouthpiece-reed combination is closer to 0.4-0.5, a much larger opening. This follows intuition in playing that a clarinet utilizes a much firmer embouchure [48].
Figure 9.1: Output for analytical formulas showing the difference, in cents, between the playing frequency and resonance frequency as a function of blowing pressure $\gamma$ for Note 1 using four different values of $\zeta$: 0.2, 0.25, 0.3, 0.35. Each effect is represented on the plots: Cyan: inharmonicity, Red: reed dynamics, Green: reed-induced flow, Magenta: temperature, Blue: total of the first three effects. The horizontal axis is a range of $\gamma$ values from $\gamma_{th}$: the threshold of oscillation to $\gamma = 1$, the closing pressure.
Figure 9.1 represents a trend that is expected, as a function of $\zeta$, that as the reed opening increases the pitch will drop. This is evidenced by the fact that the total difference (in blue) is lowered in each successive plot. Notice that one of the variables affected quite a bit by this change in $\zeta$ is the reed dynamics (in red). This makes sense because as the reed is closed physically there is less reed to vibrate; the musician is damping the reed more by the lips and teeth as the reed closes. In the equations themselves this is happening due to their dependence on $\zeta$ (refer to Eq. 5.7.2.2). The inharmonicity effect is not changed by increasing $\zeta$ since this formula is not a function of that variable (see Eq. 5.7.2.3). The same is true of the temperature effect (see Eq. 5.8) and the reed-induced flow effect (see Eq. 5.7.2.1). The main changes occurring for the effects of reed-induced flow and inharmonicity are for increasing $\gamma$, the blowing pressure. This is the case because for the reed-induced flow effect, the higher the blowing pressure, the more the reed is oscillating and this is when the reed is forcing flow into the instrument. For the inharmonicity effect, the harder a musician blows into the instrument, the more pronounced this effect will be as higher blowing pressures will cause the higher harmonics to have greater amplitude [30].

In comparing Note 1 (Fig. 9.1) to a mid-range first register note, Note 12 (Fig. 9.2) and an upper-range first register note, Note 17 (Fig. 9.3) the changes due to location in the resonator (where the standing waves “see” the end of the instrument) are evident. Note that the vertical axis for each of the figures above are not the same. This choice was made so that each curve would be as clear as possible. Notice also that the horizontal axis for each of these figures begins at $\gamma = 0.38$. The inharmonicity curve will be zero at the threshold of oscillation and the threshold of oscillation will increase as the value of $\zeta$ increases. For the best comparison, the limits on the horizontal and vertical axis were held constant. Moving from Note 1 to Note 12 to Note 17 the following trends are observed:

- Inharmonicity: this effect is decreased as the note increases. Lower notes are more affected by inharmonicity than higher notes in the first register.

- Reed dynamics: this effect is nearly unaffected by the increase in note. This effect is localized at the reed and is therefore not as frequency dependent. For example, Note 1 shows a reed dynamics effect of -3 cents, Note 12 of -5 cents and Note 17 of -8 cents. This is not a noticeable decrease from note to note.
- Reed-induced flow: this effect is highly frequency dependent. The frequency shift increases rapidly as the note increases. For $\zeta = 0.2$ for Note 1 at the threshold of oscillation $N_{\text{cents}} \approx 18$ and for Note 17 $N_{\text{cents}} \approx 45$.

- Temperature: the temperature effect is highly frequency dependent since the linear gradient is based on location in the resonator. So the difference in temperature between notes occurring towards the beginning of the resonator (upper-range first register, Note 17) will be affected more by a temperature gradient (about 25 cents sharp) than the notes at the end of the resonator, far from the mouth (lower-range first register notes, Note 1, 12 cents sharp).
Figure 9.2: Output for analytical formulas showing the difference, in cents, between the playing frequency and resonance frequency as a function of blowing pressure $\gamma$ for Note 12 using four different values of $\zeta$: 0.2, 0.25, 0.3, 0.35. Each effect is represented on the plots: Cyan: inharmonicity, Red: reed dynamics, Green: reed-induced flow, Magenta: temperature, Blue: total of the first three effects. The horizontal axis is a range of $\gamma$ values from $\gamma_{th}$: the threshold of oscillation to $\gamma = 1$, the closing pressure.
Figure 9.3: Output for analytical formulas showing the difference, in cents, between the playing frequency and resonance frequency as a function of blowing pressure $\gamma$ for Note 17 using four different values of $\zeta$: 0.2, 0.25, 0.3, 0.35. Each effect is represented on the plots: Cyan: inharmonicity, Red: reed dynamics, Green: reed-induced flow, Magenta: temperature, Blue: total of the first three effects. The horizontal axis is a range of $\gamma$ values from $\gamma_{th}$: the threshold of oscillation to $\gamma = 1$, the closing pressure.
9.3 Comparisons between Analytical Formulas and Numerical Simulations

Results from the author’s published article in JASA [37], which offered comparisons between the numerical simulations and analytical formulas, will be given in this section. Since the numerical simulations have already been validated [69], making this comparison offers confidence in using the much faster analytical formulas to predict playing frequencies for a clarinet. Ideally, these comparisons will yield a great match between the trends for each individual effect as well as the total frequency difference from resonance. The JND of 10 cents is used for a benchmark in comparisons.

9.3.1 Comparison of the Individual Effects

Figures 9.4, 9.5 and 9.6 show the comparisons between the analytical formulas (solid lines) and numerical simulations (dotted lines) for Notes 1, 12 and 17 respectively. The figures plot the frequency difference $N_{\text{cents}}$ as a function of $\gamma$. There are four individual subplots in each figure to represent (beginning at the top of the figure): The inharmonicity effect (cyan), the reed dynamics effect (red), the reed induced flow effect (green), and the total of all three effects (blue). Solid lines represent the analytical results, while dotted lines represent the numerical results. The horizontal axis is the same for all four plots and is an increasing blowing pressure $\gamma$. The vertical axis is different for each plot, ranging anywhere from 0 to -40 cents. For each figure $\zeta = 0.3$ (chosen as a normal reed opening).

Each figure shows close agreement between the total difference from resonance as well as for the individual effects. When looking at Note 1, Fig. 9.4 one should notice the following:

- Inharmonicity: At the threshold of oscillation (far left point in $\gamma$) the inharmonicity effect is zero for both the analytical formulas and numerical simulations as is required. The difference between the two prediction techniques increases with $\gamma$, however this difference never exceeds our JND of 10 cents.

- Reed Dynamics: This effect is more difficult to model with the numerical
Figure 9.4: The frequency difference $N_{\text{cents}}$ between the 1st impedance peak frequency (resonance frequency) and the playing frequency for Note 1 (fingering for E) of the clarinet. Note 1 values: $\zeta = 0.3$, $\eta_3 = -0.0201$, $F_1 = 1243$ Hz and $f_1 = 146$ Hz. Solid lines represent the analytical results, while dotted lines represent the numerical results.
Figure 9.5: The frequency difference $N_{\text{cents}}$ for Note 12 (fingering for $E\flat$) of the clarinet. Note 12 values: $\zeta = 0.3$, $\eta_3 = -0.0036$, $F_1 = 2490$ Hz, $f_1 = 277$ Hz. Solid lines represent the analytical results, while dotted lines represent the numerical results.
Figure 9.6: The frequency difference $N_{\text{cents}}$ for Note 17 (fingering for $A\flat$) of the clarinet. Note 17 values: $\zeta = 0.3$, $\eta_3 = -0.0142$, $F_1 = 3338$ Hz, $f_1 = 369$ Hz. Solid lines represent the analytical results, while dotted lines represent the numerical results.
simulations and was not always stable depending on the value of reed resonance and $q_r$. However, up until the beating reed regime (until $\approx \gamma = 0.5$) the two techniques give nearly identical results. After this point, the difference increases. However, the difference between the analytical and numerical results is only 2 cents, and this is very small and much less than the JND.

- **Reed-induced flow**: for this note, the analytical formulas results are nearly identical to the numerical simulations, being less than 2 cents different over the entire range of $\gamma$.

- **Total**: the difference between the total of these three effects remains under the required JND of 10 cents.

Figure 9.5 shows the comparison for Note 12:

- **Inharmonicity**: At the threshold of oscillation (far left point in $\gamma$) the inharmonicity effect is zero for both the analytical formulas and numerical simulations as is required. The difference between the two prediction techniques increases with $\gamma$, however this difference remained very small, not exceeding 1 cent.

- **Reed Dynamics**: the difference between the methods showed similar results here as for Note 1 in Fig. 9.4. Until just before the assumed beating reed regime the two results are identical and after this the difference increases with increasing $\gamma$. At the upper range of blowing pressure $\gamma$ the difference is around 4 cents.

- **Reed-induced flow**: this effect is again very well reproduced by both prediction methods. The numerical simulations do not show the marked change between the non-beating reed and beating reed regime as in Fig. 9.4 but instead shows a relatively steady, nearly linear increase with increasing $\gamma$. The difference between method never exceeds 1 cent here.

- **Total**: due to the excellent agreement in each of the individual effects, the total frequency shift in blue shows much less than a 10 cent difference between analytical formula and numerical simulations predictions.

Figure 9.6 shows the comparison results for Note 17:
• Inharmonicity: the inharmonicity for notes in the upper-range of the first register has much less of an effect on the difference from resonance. The effect is zero at the threshold, as required and as $\gamma$ increases the difference remains nearly constant at about 3 cents.

• Reed Dynamics: the numerical simulations seem to be somewhat unstable for this note. This could be perhaps attributed to the reed resonance being near one of the harmonics ($f_1 = 369$ Hz, $f_r = 2400$ Hz) but further investigation would be needed to verify this. Despite the irregular behavior of the numerical simulations for this effect, the difference between the analytical formulas and the numerical simulations is, at its greatest about 4 cents, much less than the JND.

• Reed-induced flow: as was the case for Notes 1 and 12 in Figs. 9.4 and 9.5 this effect is very well modeled by the analytical formulas in comparison with the numerical simulations. The numerical simulations again show a somewhat linear increase whereas the analytical formulas have a more marked change before and after the beating reed (as expected). The difference between these two prediction methods for the reed-induced flow never exceeds 5 cents.

• Total: though the trend for the numerical simulations seems to be more linear than the analytical formulas, which shows the marked decrease leading up to the beating reed and the increase following that point, the difference between the two is in general less than our benchmark JND of 10 cents. The one area where there are discrepancies that should be further studied is at the beating reed ($\gamma = 0.5$). Around this point, the difference is closer to 15 cents, greater than the intended threshold.

This discussion points out an important results – for these three example notes (Notes 1, 12, and 17 in the first register of the clarinet), the analytical formulas are able to approach the numerical simulation’s results within the intended just noticeable difference of 10 cents.

### 9.3.2 Frequency Shifts as Length Corrections

Due to the fact that these frequency shifts (in cents) were first regarded as a simple length correction it is interesting to see that total frequency shift transformed back
into a length correction as a function of frequency, or note, in the first register. Figures 9.7 and 9.8 show the length correction, in mm, for each note of the first register. Figure 9.7 shows this for a fixed value of $\gamma = \gamma_{th}$ at the threshold of oscillation and $\zeta = 0.3$, as with previous results. Figure 9.8 shows the same but using $\gamma = 0.65$ and $\zeta$ remains 0.3.

In Fig. 9.7, notice that for $\gamma = \gamma_{th}$ the curve for the inharmonicity is zero for all notes since the inharmonicity (black thin line) of the resonator has no effect at the threshold of oscillation. For the reed induced flow effect (dark grey squares), a value near 5.5 mm is found. This, as well as the total magnitude of the length corrections for these notes, corresponds well to the work done by Dalmont et al. in [42] which showed length corrections totaling 7 mm to 10 mm. It is noted that Dalmont’s work was based on experiments using an artificial mouth and does not consider the effect of a temperature gradient.

In Fig. 9.8 the effect of inharmonicity shape and trend is similar for each note, creating a length correction between about 1 mm over the first register. However, for Note 1 with all holes closed, and for Note 19 with all holes open, the effect of inharmonicity is more pronounced.

These figures show that the reed induced flow effect is stable and offers the largest frequency shift of the three effects. This was to be expected based on the work in [45] and should show the change in behavior above the beating reed regime. For the case of reed dynamics, the effect is small for every note and nearly linear as a function of $\gamma$. It is obvious from these figures that the effects are very much dependent on note (length of resonator). This is intuitive to a clarinetist since, as the effective length of the instrument is shortened there are more factors influencing the playing frequency and therefore more compensation is needed in order to play “in-tune”.
Figure 9.7: Analytical length corrections ($\Delta L$) values, in mm, as a function of note number, in the first register, at the threshold of oscillation. Black thin line: inharmonicity effect, light grey, thin dashed line: reed dynamics, Dark grey thick solid line: temperature effect, Dark grey squares: reed flow effect and finally, the thick black solid line is the total of all four effects. In this figure, $\gamma = \gamma_{th}$ (a value that changes depending on Note number, generally around 0.33).
Figure 9.8: Analytical length corrections ($\Delta L$) values, in mm, as a function of note number, in the first register. Black thin line: inharmonicity effect, light grey, thin dashed line: reed dynamics, Dark grey thick solid line: temperature effect, Dark grey squares: reed flow effect and finally, the thick black solid line is the total of all four effects. The data are represented for $\gamma = 0.65$. 
9.3.3 Summery and Overall Observations

A few general comments can be made about the small discrepancies between analytical and numerical results, for each of the three effects (and resulting total difference):

- For the inharmonicity effect, the general shape and trend is satisfactory, especially for higher notes where the comparison between analytical and numerical is quite close. For Note 1, Fig. 9.4, the order of magnitude of the discrepancy remains less than 10 cents, but the variation and trend of the line with the excitation pressure $\gamma$ was well predicted. The discrepancy here for lower notes is found to be the most important and explains the larger total discrepancy between analytical formulas and numerical simulations for Note 1. This is likely due to the analytical formulas only using the first and third harmonic peaks in the impedance spectrum. Chaingne and Kergomard [30, p. 441] showed that if the most basic approximation for this effect is considered, the order of magnitude should be near $\varepsilon = \eta/2$ (see Chapter 5, which is in fact the case for the three specific notes studied here).

- For the reed flow, the hypothesis leading to the analytical formula for the beating reed regime [45] is validated due to the difference from numerical values being limited to 2 or 3 cents. The decrease of the length correction with increasing $\gamma$ is also well predicted. However, the agreement between analytical and numerical results is not as close for Note 17 (Fig. 9.6).

- For the reed dynamics, the effect is rather small in each case as is the difference between analytical formulas output and numerical simulation. The slight frequency decrease just above the threshold is correctly predicted [83]. Surprisingly however, for the lower notes, the discrepancy between methods remains small past the beating reed threshold ($\gamma \approx 0.5$) since the analytical formula was not derived for this case (the formula for the non-beating reed regime is simply continued past the known point).

Overall, for the notes in the first register of the clarinet, the comparison of the analytical formula and the simulations is good. There remain two points worth studying further: the inharmonicity effect for lower notes and the reed dynamics.
effect for higher notes. These topics are of great interest in future work and will be further discussed in the following chapter, Chapter 10.

In Ref. [100] Nederveen and Dalmont compared experimental data with a numerical computation for 3 fingerings of a clarinet. The 3 fingerings give the same note, but the inharmonicity values used in their work for those three fingerings were different (between -0.04 and -0.007). Their model is similar to that of used here in this thesis, but a comparison between their models and the solving methods is out of the scope of the current research. However, using some of the parameters from their computations, the main trends of the analytical results shown in Figs. 9.4, 9.5, and 9.6 are in qualitative agreement with the computation performed in Nederveen and Dalmont [100]. In particular, their results show that the reed dynamic effect is weak, except for fingerings at the top of the first register. For fingering 17, between the oscillation and the beating-reed thresholds, both reed dynamics and inharmonicity contribute the same order of magnitude to the decrease of the playing frequency. As observed in [100], the inertia of the reed increases with frequency, and it is likely that its influence is even more important in the second register, leading to an increase of the length correction along the second register.

In general, the results of the analytical formulas prediction of the playing frequencies show a decrease in playing frequency above the oscillation threshold. This seems to be mainly due to inharmonicity, while the increase above the beating reed threshold is due to the decrease of the flow rate effect. For high mouth pressures, the predicted frequency increase is exaggerated by the analytical formulas. Again, the reason for this is probably due to not taking the impedance peaks above the second one into consideration in Eq. (5.7.2.3), as well as perhaps underestimating the interaction between the three effects.

### 9.4 Analytical Formulas, Map-Making

A final useful tool for musicians and manufacturers alike would be the ability to see, quickly, the tuning characteristics of a particular clarinet. This could be done with the use of tuning maps made by using the analytical formulas with a full range of reed opening ($\zeta$) and blowing pressure ($\gamma$). Section 5.10 showed the tuning maps for the full first register of the clarinet and Section 6.4.2 offered the same but made by the numerical simulations. This section will present individual notes as a
comparison between the numerical simulations data and analytical formulas.

\textbf{9.4.1 Tuning Maps}

Figure 9.9 shows one of many analytical tuning maps as introduced in Chapter 5 Section 5.10. These figures are basically a 3D type view of the tuning tendencies for a particular clarinet as a function of increasing blowing pressure (horizontal axis) and increasing reed opening (vertical axis). The color represents the third “axis” as it shows the difference in cents from the equal tempered scale of the predicted playing frequencies. On Fig. 9.9 the white line shows the chosen value of $\zeta$ that would produce the blue curve in the graph inset, a graph that matches those that have come before. By running the code for all values of $\zeta$ instead of just one, we can see the overall tuning differences in all possible regions of blowing pressure ($\gamma$) and reed opening ($\zeta$, reminder: low $\zeta$ means a smaller reed opening, high $\zeta$ means a larger reed opening).

The two figures shown in Fig. 9.10 represent the tuning maps made using the analytical formulas and the numerical simulations respectively.

For any tuning map the trend was expected to be as follows:

\begin{itemize}
  \item there will be no sound (black region) before around $\gamma = 0.3$ as this is the estimated threshold of oscillation
  \item the note should play more flat for large $\zeta$
  \item the playing frequency should decrease with increasing $\gamma$ until $\gamma = 0.5$ (the beating reed threshold) and then increase slightly again
  \item After the threshold of oscillation, if the numerical simulation’s output map shows a black region, this means that this particular combination of $\zeta$ and $\gamma$ will cause the clarinet to play in another regime (it is further than 100 cents flat)
  \item If there is a grey region, the note being played is higher than the highest tuning placed on the color bar to the right of the plot
\end{itemize}

For the two figures shown in Fig. 9.10 the first visibly noticeable difference between the two maps is the resolution. The top plot, the analytical formulas
Figure 9.9: Example of an analytical formula tuning map with blowing pressure ($\gamma$) on the horizontal axis and reed opening ($\zeta$) on the vertical axis. The white inset graph shows the analytical formula prediction for the total difference from the equal tempered scale for a fixed value of $\zeta = 0.3$ and increasing $\gamma$. The color plot has a white line at $\zeta = 0.3$ showing the location on this tuning map that corresponds to the graph inset.
output was run on a much finer grid than the numerical simulations on the bottom. The numerical simulations, as discussed before in Chapter 6, are much more computationally expensive and cannot be run at the same level in the same amount of time. The next difference between the figures is the color bar scales. Figure 9.10a showing the analytical output shows the tuning between 5 and 40 cents sharp compared to the equal tempered scale whereas the numerical simulations show nearly 100 cents flat (nearly a semi-tone below the intended frequency) to 30 cents sharp. This difference perhaps makes the side-by-side comparison more difficult but it is a necessary difference due to the vast discrepancy in color bar range. When the two figures were plotted with the same ranges it led to large black regions for the analytical formula plot and large white regions on the numerical simulation plot since the two methods created different extremes at the upper and lower ends of $\gamma$ and $\zeta$.

Although the results seem much different, when focusing on a normal reed opening, $\zeta = 0.3$ for example shows that both of the prediction methods offer about the same value, between 13 and 28 cents sharp depending on the value of $\gamma$. Seeing as how the comparison and validation with the numerical simulations was done for a $\zeta$ value of 0.3, this is not surprising. It may be necessary to study the analytical formula predictions for high and low values of reed opening to ensure its validity for all values of $\zeta$ in order to confidently use the analytical tuning maps.

### 9.4.2 Other Notes

Some other maps are shown for Notes 12 (Fig. 9.11), 17 (Fig. 9.12) (to match earlier represented results) and for Note 19 (Fig. 9.13), as its behavior was much different than the other studied notes.

Figure 9.11 represents the tuning maps for Note 12 of the clarinet made by the analytical formulas and the numerical simulations respectively. Note 12 is near the middle of the first register. As was the case for the Note 1 maps, the first major noticeable difference is in the resolution due to the much longer computation time for the numerical method. This trend will continue for the remaining maps as the numerical maps were very long to run. The color bars were not set to the same scale. This is because the numerical simulations maps tended to drop well below the analytical formula predictions for high $\zeta$ values. However, for low $\zeta$ values,
Figure 9.10: Note 1 difference of playing frequency from equal tempered scale frequency, in cents as a function of $\gamma$ (horizontal axis) and $\zeta$ (vertical axis): Analytical Formula tuning map (a), Numerical Simulation tuning map (b). The color bar ranges are not the same due to the extremes at the upper and lower values of $\zeta$. 
Figure 9.11: Note 12 difference of playing frequency from equal tempered scale frequency, in cents as a function of $\gamma$ (horizontal axis) and $\zeta$ (vertical axis): Analytical Formula tuning map (a), Numerical Simulation tuning map (b). The color bar ranges are not the same due to the extremes at the upper and lower values of $\zeta$. 
between 0 and nearly 0.3, the results are similar and show an average difference in frequency around 0 and 40 cents sharp. This corresponds well to the results found in the plots for comparison where a single realistic value of $\zeta$ was chosen (0.3).

Figure 9.12 shows the two figures representing the tuning maps for Note 17 made by the analytical formulas and the numerical simulations respectively. This note lies in the upper part of the first register, in the throat tones. The resulting comparison that can be made here is quite similar to that for Note 12; for low values of $\zeta$, the predictions are similar and show between 40 and 70 cents sharp.

Figure 9.13 shows the tuning maps for Note 19 of the clarinet. This is the final note of the first register, an unusual note in the throat tones that is notorious for being out of tune and generally unpredictable [71]. Note 19 had much different and fairly bizarre behavior compared to the other notes shown. For Note 19, the difference in cents was around 80 cents sharp for low $\zeta$ values but at high $\zeta$ values. However, for the numerical simulations, a different note would be played, more than 100 cents flat would refer to playing at least half step below for that combination of parameters. For high values of $\zeta$ for the analytical formulas, the results show a difference of nearly 50 cents flat, corresponding to a quarter-tone difference. This difference is quite noticeable and significantly greater than any other note studied.

The tuning maps have the ability to represent the tuning tendencies of an instrument quickly based on the chosen prediction method. Though the extreme values of $\zeta$ seem less reliable than the value chosen for the direct comparison ($\zeta = 0.3$), plenty of information can be gained from a quick glance at the map, including relative comparisons from tuning to a particular value of $\zeta$ and $\gamma$. This particular use could be perhaps more important and practical for musicians, since they all exhibit different individual subjective playing characteristics that are not included in the prediction models and further change and complicate the possible predictions being made.

9.5 Conclusions

The analytical formulas have been shown to offer an acceptable match to the numerical simulations and can now be used with confidence to predict the playing frequencies of the clarinet. Many model modifications could be further studied in order to create an even better comparison and therefore better predictions. For
Figure 9.12: Note 17 difference of playing frequency from equal tempered scale frequency, in cents as a function of $\gamma$ (horizontal axis) and $\zeta$ (vertical axis): Analytical Formula tuning map (a), Numerical Simulation tuning map (b). The color bar ranges are not the same due to the extremes at the upper and lower values of $\zeta$. 
Figure 9.13: Note 19 difference of playing frequency from equal tempered scale frequency, in cents as a function of $\gamma$ (horizontal axis) and $\zeta$ (vertical axis): (a) Analytical Formula tuning map, (b) Numerical Simulation tuning map. The color bar ranges are not the same due to the extremes at the upper and lower values of $\zeta$. 
example, improving the measured input parameters such as the vibrating reed surface area. Studying more carefully the behavior of the model after the beating reed regime will be an important next step as well.

The artificial mouth comparisons in Chapter 8 offered a promising possibility for an additional validation technique for the analytical formulas. The use of the artificial mouth will be vital in continued research as the ability to hold certain playing parameters constant proved to be difficult for musicians when using the instrumented mouthpiece. The artificial mouth can be further used to investigate the reed parameters as well. With better model parameter inputs will come better, more reliable results.
10.1 Research Summary

The thesis began in Chapter 1 with the purpose and introduction to the current work. Chapter [2] continued with a survey of the relevant literature concerning clarinet acoustics. This literature review focused especially on (a) historical context of clarinet acoustics, (b) research concerning the two important separate parts of the instrument (resonator, generator), (c) experimental methods, and (d) important research focusing on the concept of instrument playability.

Chapter [3] offered an in depth explanation of the concept of input impedance with the intention of beginning at the simplest model (a simple harmonic oscillator) and building in complexity until the instrument is complete: full resonator with bore losses, holes, bell, etc.

Next, Chapter [4] gave the conceptual explanation of how the nonlinear generator functions for the clarinet and introduced the extremely important nonlinear characteristic equation $u(p)$ and the concept of Bernoulli flow that describes the blown closed clarinet reed and how it will interact with the linear cylindrical resonator of the clarinet.

Chapter [5] stepped through the derivation of the analytical formulas that were used to predict the playing frequencies of a clarinet. The three main characteristic equations (in simplest terms: one for the reed, one for the resonator, and the nonlinear characteristic equation that couples the two). The equations were first
represented in their physical form and then made dimensionless for use in comparison with the numerical simulations (Chapter 6). The analytical formulas become basically perturbations (frequency corrections) to the resonance frequency that can be found by measuring the input impedance of the instrument. The analytical formulas allowed for the prediction of playing frequencies for each note in the first register and even included the effect of a linear temperature gradient within the instrument. These analytical formulas were then used to create tuning maps for a range of reed opening, $\zeta$, and blowing pressure, $\gamma$, for each note of the first register of the clarinet.

Chapter 6 gave an overview of the numerical simulations, written by Guillemain [37,69], and showed how this numerical code could be used as a tool to test the validity of the analytical formula results.

Chapter 7 gave an overview of the experimental methods performed in this work. These included (1) input impedance measurements, (2) use of the artificial mouth, and (3) the instrumented mouthpiece and its possible applications.

The preliminary data from the instrumented mouthpiece was shown and discussed in Chapter 8. And finally, in Chapter 9 the validation of the analytical formulas and their possible implications were detailed. This work offered the first time that the equations and code have been used in this way and the analytical formulas matched well with the numerical simulations. After this, tuning maps were shown to be a useful tool that could, in future work, give researchers a quick glance at tuning tendencies of any clarinet being studied.

10.2 Introduction to Discussion and Future Work

A broad leading question for the research described in this dissertation was - what characteristics make one musical instrument more playable and preferable compared to another? More specifically, what acoustical properties or quality markers can characterize an ideal clarinet? The purpose of this dissertation was to explore the specific quality marker of intonation or tuning homogeneity for any given clarinet both qualitatively, through musician interaction, and quantitatively, through numerical, analytical and experimental techniques.

The hypothesis that a musician would prefer the instrument that has a high degree of tuning homogeneity can only be tested subjectively through interviews
and interaction with musicians. Such interviews and subsequent statistical analysis is left for future work; Appendix C contains possible interview questions and topics of interest. However, this thesis has shown that analytical prediction tools, as well as numerical simulations and experimental results for validation are a useful first step in creating a computationally fast and valid playing frequency prediction method. The steps to be taken in future work to reach this goal and further improve the instrument making process using these prediction techniques are discussed in this chapter.

10.3 The Validation of Analytical Formulas

The simplified analytical methods formulated for use in this dissertation to predict the playing frequencies of a clarinet have been validated by comparison with the numerical simulations. The comparison between the two was satisfactory, well below the threshold of the JND of 10 cents, up until the beating reed regime (until $\gamma = 0.5$) as reported in Coyle et al. [37]. There was some discrepancy above this point, due to the empirical nature of the reed induced flow formula. Some areas for future work include measuring and modeling a more precise temperature gradient to use for the predictions, studying the behavior of the reed induced flow beyond the beating reed threshold, and measuring the vibrating surface area of the reed $S_r$ as this variable heavily influences the reed induced flow impact of the playing frequency (see Eq. 5.7.2.1). Modal analysis of the reed in playing conditions could be done in order to determine the behavior of the reed vibrations at higher blowing pressures. Based on current literature, it is still unclear whether or not the reed has partial closure at some point during play based on possible curling of the reed onto the mouthpiece lay. This could be tested in the future using the artificial mouth and instrumented mouthpiece discussed below.

One area of concern to be considered in the future is the resolution of the input impedance measurements and the algorithm that is used to find the peaks automatically (in MATLAB) for use in the analytical formulas. Figure 10.1 shows, for Note 1 of the clarinet, the resolution available for an input impedance measurement. The figure shows a detailed view of the first peak (first resonance). The resolution in this figure is 0.1 Hz. However, it is difficult to say which peak the algorithm would or should choose (the highest one? a smoothed or re-sampled version of
these peaks?, etc.).

![Graph showing impedance vs frequency]

**Figure 10.1:** Note 1 measured input impedance as a function of frequency. This specifically shows a zoomed-in version of peak number 1 (first resonance frequency) and the resolution from the measurement (0.1 Hz steps).

With this in mind, as a preliminary investigation, the table below shows the effect of changing the resonance frequency by 0.1 Hz on either side (of what on the total difference between the playing frequency predicted by the analytical formulas).

**Table 10.1: Effect of input resonance frequency peaks due to resolution in measurement of input impedance**

<table>
<thead>
<tr>
<th>Resonance (peak) Frequency (Hz)</th>
<th>Total difference predicted by analytical formulas at threshold, in cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>149.3</td>
<td>14.96</td>
</tr>
<tr>
<td>149.4 (original)</td>
<td>16.11</td>
</tr>
<tr>
<td>149.5</td>
<td>17.27</td>
</tr>
</tbody>
</table>

Notice from Table 10.1 that choosing a 'peak' 0.1 Hz below or above will affect the total difference. Shifting the peak from the original to 0.1 Hz down or up changes the total difference (up or down) about 1.15 cents (compared with results from Chapter 5). Though this is not noticeable (based on JND) it could be important
for certain notes and in the future it would be interesting to study further the sensitivity of the model to certain other input variables in order to ensure that the discrepancy between the analytical formulas results and the numerical simulations would be within or outside of the uncertainty in these user input variables ($S_r$ (vibrating surface area of the reed), and other reed characteristics ($q_r$ and $\omega_r$)).

10.4 Artificial Mouth Measurements

The artificial mouth is another tool that could help improve the analytical formulas. The hope for this dissertation research was to have similar data to the analytical formulas predictions of the playing frequency when a real clarinet was being played by this artificial mouth. While the preliminary artificial mouth data was promising and showed similar trends as the analytical formulas predicted, there is much room for improvement.

In the future experimental data could be retaken using an instrumented mouthpiece (discussed below) in conjunction with an artificial mouth in order to have consistent, reliable and repeatable measurements to use for comparison.

The literature showed that there are models of artificial mouths that are capable of reaching much higher blowing pressures than those reached in this research. One goal in future work would be to attempt to reach these higher pressures to get a better picture of the behavior of the reed and clarinet response at high blowing pressures.

10.5 Potential Topics for Exploration with an Instrumented Mouthpiece

The instrumented mouthpiece remains in its early development stages. It offers the researcher and musician alike the ability to see in real-time the acoustical effects that their playing gestures produce. The instrumented mouthpiece currently lacks a reliable calibration method in order to provide confidence in the values that are measured with the FSR (force on the reed) and optical sensors (reed displacement). A better force sensor (FSR) that is much less cumbersome to the musician while playing would also be a welcome addition to future models.
From a musician’s standpoint, the instrumented mouthpiece could be used to study the more interesting phenomenon created using extended or non-classical techniques in playing. For example, in one particular measurement session the musician was asked to perform a number of interesting tasks. Figure 10.2 shows the task where the musician first plays one note normally (here it is Note 1, fingered E or the lowest note of the clarinet) and then changed whatever was necessary in order to produce multiphonics (or multiple notes at one time). This technique is difficult for beginning and intermediate players and it is often difficult for the player to describe what exactly they are doing (in a repeatable or pedagogical way) to create these effects.

Having the instrumented mouthpiece would be an excellent tool to discover the small changes in reed opening, displacement as well as the mouth and mouthpiece pressure when multiphonics are present. Due to the fact that the clarinet is an open-closed acoustic resonator it is expected that the musician could produce the harmonic series including all odd multiples of the fundamental. If the fundamental was played in-tune with the equal tempered scale this fundamental would be 146.83 Hz. The third and fifth harmonics possible here then would be: 440.49 Hz, and 734.15 Hz. These would represent notes in the second register of the clarinet.

Figure 10.3 shows a side-by-side comparison of the beginning of the multiphonics recording and the end (with the least to the most harmonics being added to the spectrum) with a wider frequency range in order to study the higher harmonics behavior. Notice that between the two, Fig. 10.3b shows a much richer upper harmonic content (above the range studied in the previous figures) which give the sound a very harsh quality. Many composers use this growling sound in contemporary compositions.
Figure 10.2: Instrumented Mouthpiece measurement showing a player using techniques to play multiphonics, multiple tones played at the same time. Played on Note 1 (fingered E). The subplots from top to bottom represent: Audio file, Mouthpiece Pressure (Pa), Mouth Pressure (Pa), Lip Force (measured by FSR) (V), and Reed Displacement (scaled from 0 to 1).
Figure 10.3: Comparison of spectrum 1 and 6: (a) Spectrum 1 - no attempt at multiphonics, purely fundamental driven timbre., (b) Spectrum 6 - significant multiphonics present, growling sound, fundamental nearly lost in tone.
Figure 10.4 shows the IM data for the musician playing the opening measures from Gershwin’s Rhapsody in Blue. This is a very popular excerpt for clarinetists to learn and is quite difficult, requiring the musician to use techniques to smoothly glide or glissando from a very low note to an upper register note seamlessly. Notice that in the second portion of this expert the mouthpiece pressure gets very low and the lip force is increased continuously yet the mouth pressure stays high throughout. In other words, the pressure drops in the mouthpiece but the reed opening begins to close as the lip force increases - the flow through the reed should get smaller as well. This is an example of how this tool could be used pedagogically. If a student were able to see and attempt to mimic a professor’s gestures using this instrumented mouthpiece they could perhaps better understand what is expected in technique (both in reed opening and in controlling the breath).
Figure 10.4: Instrumented Mouthpiece measurement showing a player using advanced techniques to play the opening to Gershwin’s Rhapsody in Blue. Begins with a low trill and uses techniques to glissando to the upper second register smoothly. The subplots from top to bottom represent: Audio file, Mouthpiece Pressure (Pa), Mouth Pressure (Pa), Lip Force (measured by FSR) (V), and Reed Displacement (scaled from 0 to 1).
10.6 Evolution of Musical instruments...past and present

An initial area of interest in this project was the study of what actually makes a clarinet easy to play, “in tune” or otherwise. Although the modern clarinet has been in use for nearly 100 years, its predecessors were around long before this. A study of historical clarinets as well as other models, such as the German model, is currently being conducted in order to understand what acoustical characteristics musicians are searching for in a “perfect” instrument. Studying instruments from the past and how they evolved over time can offer clues as to what specific acoustical effects instrument makers are modifying and how each one affects player perception of the instrument.

10.6.1 The CAGIMA project and Historical Clarinets, Future Work

The French National Research Agency (ANR) funded project CAGIMA is a multi-year, multi-lab collaboration to understand how one could produce a better reed instrument. The project CAGIMA (Conception Acoustique Globale d’Instruments de Musique a Anche justes et homogenes) translates to global acoustic conceptualization of in-tune and homogeneous reed musical instruments. That is a lengthy way to say that this project seeks to understand what good instruments have in common, searching for these acoustical quality markers in historical instruments, and instruments from present day, as well as the possibility of creating a new design that combines all of the best characteristics through numerical optimization and studies about ergonomy.

Although this dissertation research was not directly part of CAGIMA (as this project began before this thesis), the research was very closely linked and shared many of the same objectives. With that in mind, one goal for this thesis was to compare what has been gained from studying the modern clarinets with the quality markers of the past. The CAGIMA project included working with Buffet-Crampon, the world renowned clarinet manufacturers in France in order to study a corpus of historical instruments from their museum collections. As with an study of the present and future it is always helpful to learn from our past - whether it be mistakes or triumphs.
10.6.2 Historical Clarinets

While the author of this thesis was studying in France she was granted access to a small corpus of historical clarinets and offer a comparison of the modern instruments with their past counterparts. Thanks to the cooperation of Buffet-Crampon the following clarinets were available for use in this project:

- 1780s reconstruction of a Viennese 5-key clarinet
- 1835 13-key clarinet
- 1845 Boehm-like system clarinet
- 1935 Full Boehm system clarinet
- 2010 Tosca Clarinet
- 2012 Modern German style clarinet

Measurements of these historical clarinets were limited to the input impedance and instrumented mouthpiece experiments. At the moment these instruments are unable to be transported from Paris and therefore could not yet be measured using the artificial mouth in Marseille. The input impedance measurements were already a useful tool since these older instruments were not made in the same way as modern instruments and seeing that they were still exhibiting similar characteristics is perhaps surprising. In the following subsections the input impedance measurements for a few different historical clarinets are offered and compared to a modern clarinet. The instrumented mouthpiece data is not yet available as many of the historical clarinets were played with special mouthpieces and so artisans specializing in historical instrument reproductions were contacted and at the time of writing this dissertation had not yet completed the mouthpieces including the modifications for including sensors (optical/pressure tube holes, etc.).

Plots of the input impedance for a few notes in the first register of one of the historical clarinets, the 5-key system, are shown in Figures [10.5] and [10.6] and [10.7] inlaid with the input impedance plots from a few modern clarinets. Major differences in the input impedance plots show that although the instruments have the same length and general clarinet-like cylinder characteristics, their resonance frequencies are quite different. The historical 5-key system seems to have much
lower resonances than its modern counterparts. As the note increases, as seen for Note 16 in Fig. 10.7, the student model has less space between the first and second peak.

The inharmonicities between the first and second peak for each of the four clarinets can be found in Table 10.3. From Table 10.2 the difference in cents between the second peak and three times the first peak is found. These values show that the second peak in all cases for Notes 1 and 4 are lower than three times the fundamental. This can be seen visually in the aforementioned figures as well. This table also shows that the top professional model (Pro+) indeed has lower values for this difference than the other clarinets and the student model has the highest differences of the modern clarinets, as expected.

Table 10.2: The first and second peak frequencies for the listed clarinets.

<table>
<thead>
<tr>
<th>Note</th>
<th>First two peak resonance frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Pro+</td>
<td>149.4</td>
</tr>
<tr>
<td>Pro</td>
<td>149.6</td>
</tr>
<tr>
<td>Student</td>
<td>150.1</td>
</tr>
<tr>
<td>5-key</td>
<td>117.5</td>
</tr>
</tbody>
</table>

Table 10.3: The difference in cents between the second peak and three times the first peak from Table 10.2.

<table>
<thead>
<tr>
<th>Note</th>
<th>1</th>
<th>4</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pro+</td>
<td>-17.5</td>
<td>-3.3</td>
<td>-7</td>
</tr>
<tr>
<td>Pro</td>
<td>-17.8</td>
<td>-6.7</td>
<td>-13</td>
</tr>
<tr>
<td>Student</td>
<td>-26.5</td>
<td>-12</td>
<td>-50</td>
</tr>
<tr>
<td>5-Key</td>
<td>-9</td>
<td>-11</td>
<td>+70</td>
</tr>
</tbody>
</table>
Figure 10.5: Input impedance plots for four clarinets: Top pro model, mid-pro model, student model and the 5-key viennese model for Note 1.
Figure 10.6: Input impedance plots for four clarinets: Top pro model, mid-pro model, student model and the 5-key vienese model for Note 4.
Figure 10.7: Input impedance plots for four clarinets: Top pro model, mid-pro model, student model and the 5-key vienese model for Note 16.
Looking forward to the future of the clarinet, the CAGIMA project is an interesting combination of working towards the future of reed instruments and taking an educated look back at past advances which came about slowly through the ages. The next step in the CAGIMA project was to model and build prototypes of what is being termed a Logical Clarinet. This instrument is meant to be brand new while using the good quality markers of a modern clarinet. This instrument is modeled to play as in tune as possible over the range of the instrument and have a greater facility in emission over the entire range. More about this instrument can be found by reading the following references (103, 70).

10.7 Tuning Maps

Although the mapping process is not quite ready for use as a pedagogical tool, it has the capability to aid researchers in understanding what musicians are doing during play that help them achieve certain effects. Musicians know intuitively that when they crescendo (increase blowing pressure) they cannot maintain the same force on the reed (static reed opening) if they wish to play in tune. It is a combination of the two effects changing that will allow this. The tuning maps show which combination will help the musician achieve this for any given clarinet.

The maps could be used to see difference between models quickly in order to make conclusions about the instruments playability in comparison with another. Input impedance data was measured for four different clarinets throughout this research and could be used to make tuning maps to compare tuning tendencies of these instruments.

One drawback to the current formulation is the extensive use of dimensionless variables. This makes it less likely that a musician could use this as a tool. Though musicians do not think about their blowing pressure in Pascals nor their reed opening in millimeters, perhaps packaging the variables in a more intuitive way for the player would make this an easy to use tool and diagnostic for a given clarinet (with measured resonance frequencies) and reed and mouthpiece characteristics (mouthpiece: open, closed lay. reed: soft, hard, humid, dry, etc.).
10.8 Final Words

The goal for this research was to create a fast and simple way to show musicians and instrument makers important acoustical information about the clarinet through the use of these quality marker maps. This dissertation focused on one quality marker - tuning tendencies of clarinets. However, in the future this work could grow to include the creation of multiple acoustical quality marker maps to display acoustical characteristics of musical instruments. The hope is that the results can be used by all involved with a future goal to better the instrument making process in order for musicians to have the best possible clarinet with which to perform. As clarinetists could use these maps to quickly identify the combination of playing parameters necessary to play each note on their instrument in tune (in the correct regime, correct dynamic, etc.), this method offers a scientifically accurate and objective way for musicians to comprehend the tuning tendencies (and other characteristics) of their instrument. Further, manufacturers could use the maps to remedy these tuning discrepancies from note to note. From there, manufacturers can make small changes to the instrument and see the effect in these tuning maps for the entire range of the instrument.

The clarinet is already a beautiful and impressive feat of artistry both in its design and sound brought about after years of improvements and adjustments, many made from the heart of the creators involved. Nevertheless, there is always room for improvement and advancement; this dissertation has offered a few steps in that very direction.
A.1 Terminology

The entirety of the clarinet is made of many smaller, sometimes intricate pieces. Each of these pieces has a specific function that is worth mentioning here, for the understanding of all readers. See Fig. A.1 for references to individual parts.

The resonator, The Body

The body of the clarinet has been made from a variety of different materials: wood, plastic, hard rubber, metal, resin and even ivory. Generally student model clarinets, the least expensive, are made from plastic and the professional models are made from grenadilla wood (African M’pingo wood).

The Reed

The reed on the clarinet is generally made of cane from the strong grass called Arundo donax L. Musicians generally buy them pre-cut and many advanced players will file the reeds down using some sort of abrasive material such as a fine grain sand paper in order for the reed to exhibit the desired properties and response. The reeds can be bought pre-cut to a range of hardness values (as set by the manufacturers) from 1.5 - 5+. Reeds may also be bought from synthetic materials which may tend to last longer but have a different timbre than the cane reeds 104.
The ligature
The ligature is a type of fastening mechanism that will keep the reed laying flat against the reed table while not inhibiting the desired vibrations. The ligature can be metal, leather, string, etc. The placement of the ligature is essential to finding the correct length of reed vibration and the placement can affect the ease of playing and even playing frequency.

The Mouthpiece
The mouthpiece is where the ligature fastens the reed. The reed lays mostly flat against the mouthpiece however there are a variety of different mouthpieces with different 'lays'. The lay describes how much the tip of the mouthpiece will slant away from the 'straight' position. An important term to note here is the table of the mouthpiece, this is the part that the reed touches when it is completely flat against the mouthpiece. This will often be referred to for the two separate modeling regimes, the beating reed regime and the non-beating regime (when the pressure difference inside the mouth and inside the mouthpiece is great enough to cause the reed to close against the table, after which would be the beating regime.

The Barrel
The barrel is the portion of the clarinet that is just below the mouthpiece. It has two sides which the top joint fits into with it’s corked joint and the top part where the mouthpiece will fit into with its corked joint as well. The barrel can come in a variety of bore profiles including different diameters that will help in tuning. The barrel is made so that the player can either pull out or push in depending on the tuning needs. Changing the position of this piece will greatly affect the notes that are closest to this position in fingering, the top of the first and second registers.

The Embouchure
The formation of the lips and mouth cavity around the mouthpiece and reed.

The top joint
This is the portion just below the barrel. The left hand is responsible for fingering these notes (as well as the register key). Throat tones: we may refer to the throat tones often. These are the notes that are affected most by the
register key and barrel. These are usually the F4, F♯4, G, G♯4, A, B♭4, The left hand plays these notes.

The **bottom joint**
Below that is the bottom joint. The right hand is involved here and will hold some of the weight of the clarinet on the right thumb. There are a few notes played with the right hand on the top joint, these are played with the side of the index finger and the keys are called trill keys.

The **Bell**
The flared portion at the end of the clarinet.

**Tone holes**
The holes, drilled into the clarinet, at specific points, in order to create an effectively shorten or elongate the instrument depending on the note desired.

**Register**
The clarinet has three main operating registers. Lowest to highest: chalumeau, clarion, and altissimo. The first two registers are separated by using the register key with the left thumb, or not. The third register is reached through alternate fingerings (see A.7), usually first lifting the left index finger.
A.1.1 Note naming

There are a variety of ways we can discuss the notes played on the clarinet. The clarinet is a transposing instrument. This means that the note that is written for the B♭ clarinet is not what is heard. If the clarinetist sees and plays the fingering for middle C what will actually be heard is the frequency for a concert B♭. This is why the soprano clarinet is considered in the key of B♭. Therefore it is sometimes complicated and confusing to describe the note that we are focusing on in literature. For this thesis we will always discuss the notes on the instrument as follows: When talking about the clarinet scale we begin with the lowest note, the fingered E3 (concert D3), the highest note on the clarinet that we will consider is the high D7 (concert C7).

A.1.2 Fingering system

The clarinet used by the majority of players in the United States and France exhibit the Boehm system. The current Boehm key system consists of generally 6 rings,
on the thumb, 1st, 2nd, 4th, 5th and 6th holes, a register key just above the thumb hole, easily accessible with the thumb. Above the 1st hole, there is a key that lifts two covers creating the note A in the throat register (high part of low register) of the clarinet. A key at the side of the instrument at the same height as the A key lifts only one of the two covers, producing G♯ a semitone lower. The A key can be used in conjunction solely with the register key to produce A♯/B♭

A.1.3 The basic acoustical pieces of the clarinet

Now that the terminology well known to the musician has been explained there are some specifics concerning clarinet acoustics that must be mentioned before describing the mathematics and physics behind the instrument.

The clarinet is composed of three main acoustical parts to consider in modeling. There is an energy source, a nonlinear element at the input and a passive linear resonator. These parts for the clarinet are, respectively, the input air, the reed/mouthpiece and the body of the instrument.

For the simplest models, we can consider that the nonlinear element, which is necessary in transforming the continuous source pressure into an oscillating one, is localized, here, at the entrance of the resonator - the mouthpiece, Fig. A.2.
Figure A.3: Clarinet barrel - serving as a link between the mouthpiece and body of the clarinet, this portion of the instrument is easily changed for different bore shapes and sizes to aid in tuning the instrument.

Attaching the mouthpiece to the body of the instrument is the barrel, shown in Fig. A.3. This accessory is available in a variety of bore profiles including different diameters that will help in tuning and is easily interchangeable. The barrel offers an option for musicians to quickly change the tuning tendencies for certain notes of the instrument by lengthening or shortening the instrument by small amounts.

Further down on the instrument is the top-joint of the instrument. The left hand plays the notes here. The register key (to jump the 12th) lies at the thumb of the left hand and all of the throat tone notes are played by the thumb and index finger of this hand, Fig. A.4.

Further down on the instrument is the bottom joint which is played by the right hand. This is also where the instrument is balanced on the thumb of the right hand. This can be an issue for some and for younger students it is helpful to use a neck-strap or stand to help support the weight of the instrument as to not interfere with playing and proper embouchure, Fig. A.5.

Attached to the bottom joint is the bell of the instrument. The bell generally serves to bridge the gap between the abrupt ending to the cylindrical tube (lower joint) and the outside acoustic environment. It also aids in tuning the instrument for these lower notes.

Figure A.7 shows a fingering chart for registers 1, 2 and 3 of the B♭ soprano clarinet. Holes that are shaded for a particular note represent closed keys and white holes represent open, uncovered holes.
Figure A.4: The top joint of the clarinet - these notes are played by the left hand. The register key (to play 12ths, into the second register) is played with the thumb of the left hand. [5].
Figure A.5: The bottom joint of the clarinet - these notes are played by the right hand. The thumb rest lies at the back of the bottom joint and rests on the right thumb, holding the weight of the instrument. [5].

Figure A.6: The bell of the clarinet. [5].
Figure A.7: Clarinet fingering chart for registers 1, 2, 3. From: http://rlmf.org/
CONVERSION OF DATA FROM ARTIFICIAL MOUTH IN ORDER TO COMPARE TO ANALYTICAL FORMULAS

Chapter 5, where the theoretical development of the analytical formulas was presented, the parameters of interest (pressure and flow) were expressed in terms of dimensionless variables $\gamma$ and $\zeta$. However, the data obtained from the artificial mouth as shown in Fig. 7.21 is expressed in terms of flow $U$ and pressure $p$ which both have their respective dimensions (units). In order to facilitate the comparison of theoretical and experimental data, both results need to be expressed in the same manner. It is far easier to represent the experimental results in terms of the dimensionless parameters, but several steps are required.

Within MATLAB multiple steps had to take place:

- The artificial mouth data was taken, already converted and the portion of the file to be considered was isolated. In most of the artificial mouth data taken there was a symmetric slow ramp up in pressure and then back down again, beginning just before the threshold of oscillation (a manually estimated $\gamma_{th}$). Choosing the up-ramp portion of the file facilitated comparisons to the current form of the analytical formulas. The file was manually cut just past the apparent onset transient since the steady-state oscillations were of interest for this research. Once the pressure data was cut, the file was converted to a .wav in MATLAB in order to use a powerful (free) signal processing toolbox.
named MirTools. MirTools allows for finding the frequency and variations throughout the file and compare this to a chosen reference, in this case the tempered scale frequencies.

- The MirToolbox article reads: “MIRToolbox is a MATLAB toolbox dedicated to the extraction of musically-related features from audio recordings. It has been designed in particular with the objective of enabling the computation of a large range of features from databases of audio files, that can be applied to statistical analyses.”

- An FFT was taken over the chosen data and many features of the file were calculated, such as: basic statistics of the spectrum giving several timbral characteristics (spectral centroid, roll-off, brightness, flatness, etc), an estimation of roughness, or sensory dissonance can be assessed by adding the beating provoked by each couple of energy peaks in the spectrum. Finally, tonality was estimated based on the frequency spectrum.

- Once done, there was a conversion of the axis data from pressure values (in Pa) to $\gamma = p_m/P_M$ values (from 0 to 1). The information necessary for this is the closing pressure $P_M$. This value comes from the nonlinear characteristic curve studied earlier. The value for $\zeta$ came from this nonlinear characteristic curve but also a value of $p_{max}$, this is the maximum pressure reached in the nonlinear characteristic curve measurement and corresponds to the maximum pressure reached by the clarinet before oscillation begins. The closing pressure necessary to nondimensionalize the mouth pressure values, $P_M$, is said to be nearly equal to $p_{max} \times 3$.

- There was then the possibility of showing the frequency difference between the measurements and the tempered scale frequencies (in cents or Hz) and were compared to the analytical formulas. The other necessary input information was temperature during the measurement (mouth temperature = outside temperature for the artificial mouth). This information was placed on the x-axis with the corresponding values of $\gamma$.

Figure 3.2 shows the full file that is taken from the artificial mouth data. The file is read in in Volts (y-axis) and time (x-axis). From there the file was cut into the parts to be compared against the analytical formulas. The file was first cut
from the beginning to the top of the pressure ramp, about halfway through the file, shown in Fig. B.3. From there, the transient was cut in order to consider only the steady state portion, shown in Fig. B.4.

Figure B.1 shows a flowchart from [86]. The interesting part of this flowchart is the portion showing how the program user can go about finding pitch using the MirToolbox. The analysis code uses mainly the following functions found in this Toolbox since the playing frequency (pitch) is the main value of interest: mirgetdata (to read in the .wav file created), mirsegment (to segment the file), mirpitch (to find the pitch of the segment of the file of interest).

Figure B.5 shows one potential output plot after the data analysis in MirToolbox. The data is shown in frequency evolution with increasing blowing pressure (x-axis). The red straight line is the frequency reference point for this particular note (note 1 of the clarinet), the reference being the equal tempered scale frequency for that fingering. Figure B.6 shows another potential output file after the data analysis from MirToolbox. The data is represented as an evolution in playing frequency, in cents different from the tempered scale frequency as the pressure is increased on the x-axis.
Figure B.1: Work flow possibilities with MirToolBox. This research focuses mainly on the pitch tracer capabilities. (from MirToolBox Documentation).
Figure B.2: This is full file taken from a pressure measurement in the mouthpiece of the artificial mouth. The blowing (mouth) pressure began just below the threshold of oscillation and increased for 20 seconds and decreased for 20 seconds. The increase in pressure would go up to a blowing pressure just a bit below an manual estimation of the reed closing pressure.

Figure B.3: This is a portion of the measurement file shown in Fig. B.2 where only the up-ramp is being considered in order to compare to the analytical formulas which are a function of increasing $\gamma$ or nondimensionalized blowing pressure.
Figure B.4: An even smaller portion of Fig. B.2 where the data has been cut to include only the up-ramp portion shown in Fig. B.3 and has been cut even further to not include the transient portion of the file since the portion of interest in calculating the playing frequency is, for now, in the steady state regime.

Figure B.5: Analysis from MirToolbox. The playing frequency calculated from the measurement by the toolbox function mirpitch in blue and in red, the tempered scale frequency for note 1 of clarinet.
Figure B.6: Playing frequency analysis from the MirToolbox representing the playing frequency for this particular note in cents different from the tempered scale frequency.
C.1 Initial Survey

In order to ensure that the current research could be of use to the clarinetists and instrument makers, it is important to gather some sort of subjective data in order to attempt correlations with the physical modeling that was the focus of this dissertation. Below are the initial survey questions in the format that was offered, in email form, to respondents. Unfortunately, due to its depth and breadth, there were zero responses offered from those contacted.

- What are the important characteristics (specifically related to the sound (emission and quality)) when choosing a new clarinet?

- Which coveted characteristics are difficult to find in clarinets? Define a great clarinet.

- After you purchase a clarinet, how do you go about adjusting the tuning issues to best suit your needs?

- What is your most important tuning mechanism? Where do small adjustments happen? –reed (strength, position in mouth), mouthpiece (model), bore (lengthening or shortening), player (embouchure, alternate fingerings, vocal tract modifications)?

- How are your tuning rituals different in:
1. Ensemble vs. solo playing
2. Long vs. short duration notes
3. Crescendos vs. decrescendos
4. Large vs. small performance spaces
5. Melodic vs. harmonic playing

- What qualities do you look for in a new clarinet in regards to how the clarinet will "break-in" (after 1 month, 1 year, 20 years)? Is this something a clarinetist can generally predict?

- Which permanent modifications do you make on your own instrument or on the instruments of students in order to make the instrument easier to play, more in-tune, have a better tone quality, etc.?

- What can be improved in the general clarinet design?

- Can a plastic beginner clarinet sound great? What does a professional level clarinet offer the musician?

- Which notes do you find the most 'out of tune'?

- Please give your best definition for any/all of the following commonly used terms to describe instrument playability (based on work by Fritz et al.: Violin timbre and acoustic properties):
  1. Full
  2. Mellow
  3. Sweet
  4. Unbalanced
  5. Weak
  6. Clean
  7. Dull
  8. Resonant
  9. Singing
10. Thin
11. Dark
12. Muffled
13. Open
14. Responsive
15. Bright
16. Heavy
17. Muted
18. Quiet
19. Rich
20. Warm

- Feel free to add any information here that you may find helpful in our research regarding the "ease-of-play" for the clarinet.

**C.2 Changes after comprehensive exam**

Committee member Dr. Michelle Vigeant suggested multiple ways to improve the above survey in order to gather reliable and valid data for future use. The sections that follow are the comprehensive exam responses that helped shape the new survey aim and scope.

**C.2.1 Considerations in creating rating scale for survey**

The key things to consider when writing a scale would be:

- Include a large number of relevant items
  Offer enough items in order to make the measurement worth the time of the respondents and researchers.

- Be redundant
  It’s okay to ask the same question in multiple ways, multiple times. As the paper mentions, redundancy with respect to content is an asset.
• Don’t include package-deal questions
  Be careful not to address two separate constructs (or variables) in one question. Questions should not be written so that a respondent would have to choose to say yes to one part of the question while wanting to say no to the other.

• Include both positively and negatively worded (grammatically correct) questions
  Writing questions with the construct of interest included (positively worded) or not included (negatively worded) can both be of interest. The addition of the grammatically correct parenthesis is necessary to avoid so-called “sources of ambiguity”. Be clear in each case so that the respondent comprehends what they are answering.

• Decide on a scale and choose the number of response categories:
  Thurstone Scaling, Guttman Scaling, Scale with equally weighted items, etc.

• Decide on response formats
  – Scales with specific choices offered (circle 1,2,3,4...) or Scales with visual analog (place your choice along scale...).
  – Numerical response (scale of numbers, binary response).

### C.2.2 Types of scales used in surveys

#### C.2.2.0.1 Thurstone Scaling:
  a scale where the items correspond with different intensities of response, spaced at equal intervals and given the option of agree or disagree.

#### C.2.2.0.2 Guttman Scaling:
  a series of questions that will be asked at progressively higher levels. This could help gauge the respondent’s level comfort or perhaps their conviction for the particular response.

#### C.2.2.0.3 Likert Scale:
  a scale in which an item is presented as a declaration and the responses that follow indicate varying degrees of agreement with the declaration. Usually using responses like: strongly disagree, mildly agree, agree, strongly agree, etc.
C.2.2.0.4 Semantic Differential: here, a target stimulus is declared and a spectrum of adjectives is placed, usually two being on opposite ends (happy — sad). The respondent adds their response on the continuum between the adjectives.

C.2.2.0.5 Visual Analog: this is similar to the semantic differential in that the respondent is given a continuum in which to place their response. Unlike the semantic differential, this scale is continuous. Good for measuring phenomena before and after an event perhaps but due to their sensitivity (and variability) they are not suited for every situation. However, this helps in situations where test could become biased based on repeating the experiment. Respondents would be less likely to memorize their answers.

C.2.2.0.6 Numerical Response Formats: there are situations where a numbered response is useful. The paper suggests that the mental number line helps some people think in spatial terms, based on these numbers presented visually
### C.2.3 New constructs and questions

<table>
<thead>
<tr>
<th>Construct</th>
<th>Quest. Num.</th>
<th>Original Question(s) Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality of current clarinet design and manufacture (Sound quality)</td>
<td>1</td>
<td>What are the important characteristics (specifically related to the sound (emission and quality)) when choosing a new clarinet?</td>
</tr>
<tr>
<td>Quality of current clarinet design and manufacture (Sound quality)</td>
<td>9</td>
<td>Can a plastic beginner clarinet sound great? What does a professional level clarinet offer the musician?</td>
</tr>
<tr>
<td>Quality of current clarinet design and manufacture (Personal definition)</td>
<td>2b</td>
<td>Which coveted characteristics are difficult to find in clarinets? Define a great clarinet.</td>
</tr>
<tr>
<td>Quality of current clarinet design and manufacture (Personal definition)</td>
<td>12</td>
<td>Feel free to add any information here that you may find helpful in our research regarding the “ease-of-play” for the clarinet.</td>
</tr>
<tr>
<td>Quality of current clarinet design and manufacture (faults, adjustments)</td>
<td>3</td>
<td>After you purchase a clarinet, how do you go about adjusting the tuning issues to best suit your needs?</td>
</tr>
<tr>
<td>Quality of current clarinet design and manufacture (faults, adjustments)</td>
<td>7</td>
<td>Which permanent modifications do you make on your own instrument or on the instruments of students in order to make the instrument easier to play, more in-tune, have a better tone quality, etc.?</td>
</tr>
<tr>
<td>Quality of current clarinet design and manufacture (faults)</td>
<td>6</td>
<td>What qualities do you look for in a new clarinet in regards to how the clarinet will “break-in” (after 1 month, 1 year, 20 years)? Is this something a clarinetist can generally predict?</td>
</tr>
<tr>
<td>Quality of current clarinet design and manufacture (faults)</td>
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<td>What can be improved in the general clarinet design?</td>
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<td>Quality of current clarinet design and manufacture (faults)</td>
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<td>Which notes do you find the most “out of tune”?</td>
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<tr>
<td>Quality of current clarinet design and manufacture (faults)</td>
<td>2a</td>
<td>Which coveted characteristics are difficult to find in clarinets? Define a great clarinet.</td>
</tr>
<tr>
<td>Tuning (faults)</td>
<td>10</td>
<td>Which notes do you find the most “out of tune”?</td>
</tr>
<tr>
<td>Tuning (personal adjustments)</td>
<td>5</td>
<td>How are your tuning rituals different in: Ensemble vs. solo playing, Long vs. short duration notes, Crescendos vs. decrescendos, Large vs. small performance spaces, Melodic vs. harmonic playing?</td>
</tr>
<tr>
<td>Tuning (personal adjustments)</td>
<td>7b</td>
<td>Which permanent modifications do you make on your own instrument or on the instruments of students in order to make the instrument easier to play, more in-tune, have a better tone quality, etc.?</td>
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<tr>
<td>Tuning (personal adjustments)</td>
<td>4</td>
<td>What is your most important tuning mechanism? Where do small adjustments happen? reed (strength, position in mouth), mouthpiece (model), bore (lengthening or shortening), player (embouchure, alternate fingerings, vocal tract modifications)?</td>
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*Table C.1: Table 1 for Question 6: Identifying constructs within the survey questions*
<table>
<thead>
<tr>
<th>Construct</th>
<th>Term(s) from Question 11</th>
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<tbody>
<tr>
<td>Transient, Dynamics, Timbre</td>
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<td>Muffled</td>
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<tr>
<td>Timbre, Dynamics</td>
<td>Mellow</td>
</tr>
<tr>
<td>Transient, Dynamics</td>
<td>Resonant</td>
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<tr>
<td>Transient, Timbre</td>
<td>Unbalanced</td>
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<td>Transient, Timbre</td>
<td>Clean</td>
</tr>
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<td>Transient, Timbre</td>
<td>Dull</td>
</tr>
<tr>
<td>Transient, Timbre</td>
<td>Singing</td>
</tr>
<tr>
<td>Transient, Timbre</td>
<td>Thin</td>
</tr>
<tr>
<td>Timbre</td>
<td>Dark</td>
</tr>
<tr>
<td>Timbre</td>
<td>Sweet</td>
</tr>
<tr>
<td>Timbre</td>
<td>Open</td>
</tr>
<tr>
<td>Timbre</td>
<td>Bright</td>
</tr>
<tr>
<td>Timbre</td>
<td>Heavy</td>
</tr>
<tr>
<td>Timbre</td>
<td>Muted</td>
</tr>
<tr>
<td>Timbre</td>
<td>Rich</td>
</tr>
<tr>
<td>Timbre</td>
<td>Warm</td>
</tr>
<tr>
<td>Dynamics</td>
<td>Quiet</td>
</tr>
<tr>
<td>Transient</td>
<td>Responsive</td>
</tr>
</tbody>
</table>

*Table C.2: Table 2 for Question 6: Terms used in question 11 from survey grouped by Construct: Transient (the quickness in response of the system), Dynamics (loudness of the emitted sound) and Timbre (tone color). The terms can be used to describe one or more of the constructs mentioned.*
## C.2.4 Further refinements to constructs, defining importance

<table>
<thead>
<tr>
<th></th>
<th>Construct</th>
<th>Quest. Num.</th>
<th>Why is it important?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sound Quality</td>
<td>1, 9</td>
<td>This construct is hinting at playability, asking the musician to offer their opinion on whether or not their ideal instrument (what they look for) and what is offered to them currently (in a professional level instrument) coincide.</td>
</tr>
<tr>
<td>2</td>
<td>Instrument faults</td>
<td>2a, 6, 8, 10</td>
<td>This construct is asking the musician to admit where there are specific faults in current clarinet design and manufacture. The answers to this question would help give my experiments direction when taking data to show these faults (and subsequently using the analytical formulas to prove these faults would show up in theoretical work (or not)).</td>
</tr>
<tr>
<td>3</td>
<td>Adjustments to instrument faults</td>
<td>3, 7</td>
<td>This construct is useful since it gives us an idea of what we could do to the instrument to fix the faults mentioned above. I could use the results from this question and try them out. If the adjustments (whether permanent or temporary) work it would be proven scientific data to offer a manufacturer, other than a personal opinion.</td>
</tr>
<tr>
<td>Page</td>
<td>Concept</td>
<td>References</td>
<td>Description</td>
</tr>
<tr>
<td>------</td>
<td>----------------------------------</td>
<td>------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>4</td>
<td>Tuning faults</td>
<td>5, 10</td>
<td>In the same way as the instrument faults construct above, I would be asking here what current tuning trends show up during mass manufacture. Having the artificial mouth, I would be able to test these hypothesis formed from this question. I could also test a wide range of clarinets to be sure that this hypothesis is a general one and not model dependent (Buffet Professional Clarinet vs. Selmer vs. Yamaha, etc). Since tuning homogeneity is a major theme in my current research, having these player opinions of where the faults lie will greatly help direct my attention and hopefully allow me the tools to begin describing the faults more completely with theory and further validate the theory with the playing experiments.</td>
</tr>
<tr>
<td>5</td>
<td>Adjustments to tuning faults</td>
<td>4, 5, 7b</td>
<td>In the same way as the adjustments to instrument faults construct was addressed, having a good data set that tells us what the professional musicians are doing to overcome the tuning faults in their own instruments we could try these out and come up with acoustical data to back up their personal adjustments. We could then show the musician what it is they are actually doing and offer the data to manufactures to perhaps make a broad alteration to the current design of an instrument.</td>
</tr>
</tbody>
</table>
Terminology used to describe the timbre of the instrument often coincides with the tuning homogeneity of the instrument. If a note is flexible on an instrument, this can mean that the peaks of the impedance spectrum are wide and the player can play around the actual peak. Perhaps if a note is resonant, the impedance peaks will be high. If we could zero in on how the musician terminology matches up with the acoustical measures, this would help us understand what instrumentalists really want acoustically and therefore give the manufacturers not only an instrumentalist’s desires but the mathematical theory and experimental data to back it all up.

**Table C.3: Table for Question 7 representing the selection of important constructs for my survey**

<table>
<thead>
<tr>
<th></th>
<th>Timbre descriptive terms</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I did not want to completely stray from an open-ended type question. The majority of the questions I was asking are relatively leading, they are based on my own opinion of the function of the instrument from a performers perspective and giving the musician a chance to freely respond will help to guide any follow-up surveys I might have in the future.</td>
<td>12</td>
</tr>
</tbody>
</table>
C.2.5 Sample questions that fit constructs

C.2.5.1 Construct 1: Sound Quality

C.2.5.1.1 Question and possible answers:

1. The sound quality produced by a professional level clarinet is superior to that which could be produced on a student model.
   - Agree
   - Disagree

2. The sound quality produced by a clarinet is the most important aspect in choosing a new clarinet.
   - Strongly Agree
   - Somewhat Agree
   - Equally Agree and Disagree
   - Somewhat Disagree
   - Strongly Disagree

3. ...

C.2.5.1.2 Rating scale type used: Likert Scaling

C.2.5.1.3 Why we use this scale type: In the Likert scale I am able to place strong statements while allowing a range of agreement (disagreement) from the respondent. This is great for questions regarding the overall sound quality of the clarinet. The feelings that musicians have about the sound quality of their instrument versus others is highly varied and the Likert scale handles this well. I have added the option in the second question to equally agree and disagree since, as the book mentions, this will imply strong but equal attraction. For me, I know that sound quality is important but perhaps I am pulled in the direction that I believe another aspect is more important (tuning homogeneity perhaps??) and so this gives me the option to be neutral, but still not apathetically disinterested.
C.2.5.2 Construct 2: Instrument Faults

C.2.5.2.1 Question and possible answers: When I purchase a clarinet, I am surprised by the manufacturing discrepancies and differences from one clarinet to another

- Strongly Agree
- Somewhat Agree
- Neither Agree nor Disagree
- Somewhat Disagree
- Strongly Disagree

There are many improvements that could be added to the clarinet in order to be my opinion of ideal.

- Strongly Agree
- Somewhat Agree
- Equally Agree and Disagree
- Somewhat Disagree
- Strongly Disagree

The quality of my clarinet was better after many years of playing

- Strongly Agree
- Somewhat Agree
- Neither Agree nor Disagree
- Somewhat Disagree
- Strongly Disagree

C.2.5.2.2 Rating scale type used: Likert Scaling
C.2.5.2.3 Why we use this scale type: It’s necessary for this construct to get a range of feelings about current clarinet design and production from the musician. This is a pretty general question to be answered, that there are still improvements that can be made before we can consider the modern clarinet a perfect (ideal) instrument. Allowing the respondents a range of answers on general “instrument quality” questions could help in understanding what aspects are important. Also, being a bit vague on what we mean by “quality” and “ideal” will keep the questions from leading the musician in any biased direction.

C.2.5.3 Construct 3: Adjustments to instrument faults

C.2.5.3.1 Question and possible answers: After I purchase a clarinet I tend to make _________ adjustments in order for the instrument to play exactly how I want it to.

- zero
- very few
- some
- many
- numerous

C.2.5.3.2 Rating scale type used: Likert Scaling

C.2.5.3.3 Why we use this scale type: This gives a quick intensity check on how the musicians feel about the initial tuning of their instrument.

C.2.5.4 Construct 4: Tuning faults

C.2.5.4.1 Question and possible answers: Please place on the scale provided to indicate a level of ease/difficulty tuning your instrument in the following situations:

- Ensemble Playing
• Solo Playing
• Long duration notes
• Short duration notes
• Crescendos
• Decrescendos
• Large performance space
• Small performance space
• Melodic passages
• Harmonic passages

Example Scale: Difficult __________________________ Easy

C.2.5.4.2 Rating scale type used: Visual Analog

C.2.5.4.3 Why we use this scale type: I think this would be useful since it would give the musicians to indicate in which situations it is more difficult for them to play in tune. This information tells us that perhaps they are having difficulty playing in tune as the note crescendos, as the blowing pressure increases ... this would have important pointing questions and implications for my research. It means that the tuning of the clarinet is decided at a particular level of blowing pressure and if that is the case, in other playing conditions the accuracy would deteriorate. It would probably be useful to offer these options (questions) not in the order I have presented, but in a random order so as not to create a bias. For example, if the musician writes their mark on the far right DIFFICULT for solo playing then they would be likely to write their mark on the far left for EASY when asked about ensemble playing if they are listed one after the other. This might not be the case if they were to think of the situations independent of each other.
C.2.5.5 Construct 5: Adjustments to tuning faults

C.2.5.5.1 Question and possible answers: I have found that, in general, with my clarinet, I am able to make necessary tuning adjustments:

- simply using my mouth (reed opening and/or vocal tract)
  - Agree
  - Disagree

- using my mouth, and pulling out at the clarinet barrel or mid-joint
  - Agree
  - Disagree

- using my mouth, pulling our at barrel or mid-joint AND choosing different accessories as necessary (barrel with different length or bore, different mouthpiece face, new clarinet, etc.)
  - Agree
  - Disagree

- by changing everything about my instrument and playing style based on the context
  - Agree
  - Disagree

- ... I don’t. The tuning on my instrument is impossible. I’ve basically given up.
  - Agree
  - Disagree

C.2.5.5.2 Rating scale type used: Thurstone Scale
C.2.5.5.3 Why we use this scale type: I liked the idea of this scale and I am not sure if it would be correct with this construct exactly but I think it is good for the question of tuning. It gives me as the researcher an idea of, firstly, how important tuning issues are to the musician and secondly, it tells me at what “lengths” the musicians have to go to make the current clarinets play in tune. This question would need some guidance from actual professional musicians (who have much more experience than myself) in order to offer enough options for the question to really come across as having increasing intensity.

C.2.5.6 Construct 6

C.2.5.6.1 Question and possible answers: Please place a mark on this scale that would indicate the importance of a particular timbre descriptor when choosing to purchase instrument.

- between dark and bright
- between thin and full
- between mellow and sweet
- between open and muted
- between clean and unbalanced
- between rich and dark
- between warm and dark
- ...

C.2.5.6.2 Rating scale type used: Semantic Differential

C.2.5.6.3 Why we use this scale type: I think this would be an interesting scale to use here because we could start out asking for their range of importance between actual opposites (concerning dictionary definitions, i.e. dark vs. bright) but as the question continues the questions would lead the musicians to choose which timbre descriptor would be more important to them (choosing between a
dark vs. warm sound). This type of scale wouldn’t, however, necessarily help us gain understanding of the acoustical meaning of the words. In order to have that extra piece of (very important and useful) information, we could offer sound files after this survey and another type of scale where the musicians place their perception of the sound they hear on a spectrum of terms used in this question.

C.3 Short Pilot Survey

In order to test the initial success of the redefined question types, a small survey was created that could be delivered via an online response gathering site called SurveyMonkey. The number of responses increased with this streamlined approach. The survey will be further modified and made available to the public of clarinet enthusiasts as this research continues.
Figure C.2: Survey Page 3

3. The sound quality produced by a professional level clarinet is superior to that which could be produced on a student model.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Neither Agree nor Disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comment on response:

4. The sound quality produced by a clarinet is the most important aspect in choosing a new clarinet.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Neither agree nor disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What affects sound quality of a particular instrument?

Figure C.3: Survey Page 4

5. When I purchase a clarinet, I am surprised by the manufacturing discrepancies and differences from one clarinet to another.

<table>
<thead>
<tr>
<th>Disagree</th>
<th>Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comment on response:

6. There are many improvements that could be added to the clarinet design and manufacture that would make the instrument more ideal.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Somewhat Disagree</th>
<th>Equally Agree and Disagree</th>
<th>Somewhat Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Comment on response:

7. The quality of my clarinet has improved after many years of playing.

<table>
<thead>
<tr>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Neither Agree nor Disagree</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Comment on response:
8. After I purchase a clarinet, I tend to make _____ adjustments in order for the instrument to play exactly how I want it to.

- [ ] zero
- [ ] very few
- [ ] some
- [ ] many
- [ ] numerous

For example:

9. On a scale from 1 (easy) to 10 (difficult), rank the level of ease / difficulty in tuning your instrument in the following situations:

<table>
<thead>
<tr>
<th></th>
<th>1 (EASY)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10 (DIFFICULT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ensemble playing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Solo playing</td>
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<td></td>
<td></td>
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<tr>
<td>Long duration notes</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Short duration notes</td>
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<tr>
<td>Crossed</td>
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<tr>
<td>Descending</td>
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<td></td>
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<td></td>
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<tr>
<td>Descending</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Large performance passages</td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small performance passages</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Melodic passages</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Harmonic passages</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Other opinion specify:
BIBLIOGRAPHY


Whitney L. Coyle was born in Louisville, Kentucky on December 11, 1986. Her interest in music (clarinet) and mathematics was sparked early in middle school and was fostered by many great teachers and mentors. As a student at South Oldham High School in Crestwood, Kentucky Whitney was chosen as a Governor’s Scholar, was a two time Kentucky All-State Bass Clarinetist, competed in Kentucky’s High School State doubles tennis tournament, and graduated with the highest honors in 2005. In an effort to continue studying both music and math in college, Whitney attended Murray State University in Murray, Kentucky on full scholarship as a Music Education and Mathematics major. While at Murray State, Whitney participated in two NSF sponsored Research Experiences for Undergraduates (REUs) first at Rice University in Statistics and finally at Coe College studying musical acoustics. Graduating from Murray State *Summa Cum Laude* with both degrees, Whitney decided to attend Penn State to study acoustics.

At Penn State Whitney was an ARCS scholar and accepted a research assistantship for her masters work on outdoor sound propagation modeling with Dr. Victor Sparrow. After this Whitney accepted a National Science Foundation Graduate Research Fellowship (NSF GRFP) to fund her Ph.D. research in musical acoustics working with Dr. Jean Kergomard and Dr. Daniel A. Russell. This work was done in collaboration with the Laboratoire de mechanique et d’acoustique (LMA) in Marseille, France. At the completion of this work Whitney accepted a position of Assistant Professor of Physics at Rollins College in Winter Park, Florida where she will continue to perform research in musical acoustics with a strong emphasis on undergraduate participation.