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ABSTRACT

In this thesis, a multi-criteria multi-period supplier selection and order allocation problem is studied. The supply chain system is modeled with a single buyer and multiple suppliers with deterministic demands as a mixed integer programming problem. Five objectives are included in the multi-period and multi-sourcing problem. All-unit quantity discount policy is also incorporated in the model.

The five objectives considered are (1) procurement cost; (2) weighted average inventory; (3) weighted average shortage; (4) weighted average lead time (5) weighted average quality defect rate. Model constraints include demand balance constraints, all-unit discount policy constraints and capacity constraints of the suppliers.

Weighted objective method and Goal programming methods (both preemptive and non-preemptive) are utilized to solve the multi-criteria mathematical programming problem. Weighted objective method is used to generate several efficient solutions by applying different sets of weights assigned to the five criteria. In both Preemptive goal programming and Non-preemptive goal programming, the specified goals for the five objectives are satisfied according to the decision maker’s preference. A numerical example is presented to illustrate the Weighted objective and Goal programming methods. Twelves efficient solutions are analyzed visually through the Value Path Approach. A sensitivity analysis is conducted to study the impact of varying inventory factors and shortage factors on the optimal sourcing decisions.
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Chapter 1

Introduction

Multi-period supplier selection problems often exist in industries with different scenarios. Due to the uncertainty and fluctuation of demand over a long period, the planning horizon of a company is usually divided into multiple periods in practice. In this thesis, the supplier selection problem will be considered in multiple periods. Moreover, some of the objectives of the supplier selection problem are often conflicting, for example the tradeoff among procurement cost, product quality, and transportation lead time. Nowadays, because of fierce competition, the low-price strategy forces companies to pay more attention on their budgets. Although procurement cost, as a type of direct cost, may be the initial concern for most managers, other factors would incur potential cost in budget also, such as product quality, shortage and inventory. Therefore, one of the most efficient ways to control budgets is the optimization of the whole business process. And during the optimization, several factors can be employed to evaluate the quality of the business process. Furthermore, in many industries, cost of raw material and component parts constitutes a major percentage of total expense. For instance, in high technology firms, purchased materials and services account for up to 80% of the total product cost (Ravindran and Warsing, 2013). Normally the price given by different suppliers would be different, and a right combination of suppliers could help companies to lower budgets considerably. Hence, supplier selection problem has gradually become one of the most important issues for establishing an effective supply chain system. In recent years an unrelenting rise in the cost of raw materials already makes the companies realize that management of the right suppliers is critical for their business. Besides the price, different suppliers would employ their own transportation methods, discount policies, and production standard etc.,
so these factors should also be considered in the process of supplier selection and order allocation. The order quantity in each period would not only determine the inventory but also shortage, which increases cost and reduces customer service level.

1.1 General Supplier Selection Criteria and Process

Supplier selection is the complicated process of determining the suitable suppliers who are able to provide the buyer with the right quality products and/or services at the right price, at the right time and in the right quantities. It is also a strategic decision since it may affect multiple functional areas from purchasing of raw material and components, production planning, to inventory management and the delivery of end products.

Supplier selection process is difficult because the criteria for selecting suppliers could be conflicting. Buyers must define and measure what “best value” means for the whole supply chain system, and the best suppliers are those offering products or services that match up to, or exceed, the buyer’s requirements. In this process, several factors and criteria can be significant for buyers. Criteria for supplier selection have been studied extensively. An important review of supplier selection methods and key criteria was presented by Weber et al. (1991). In their studies, 74 related articles, which appeared since 1966, were reviewed, annotated and classified, and specific attention was given to the criteria and analytical methods used in the vendor selection process. They found 47 of the 74 articles discussed more than one criterion, which demonstrated inherently the multi-objective nature of supplier selection decisions. They also found that supplier selection criteria might change over time, although the key criteria generally included price, delivery, quality, production facilities and capacity. For example, increased concern over the geographical location of vendors was one major change brought about by the implementation of Just-In-Time (JIT) strategies. Also, it was concluded that the application of multi-objective programming techniques
was another fruitful area of research in the supplier selection process given the complexity and economic importance of supplier selection. According to Wilson (1994), it appeared that there had been a shift away from price as a primary attribute in supplier selection since 1970s. Price was ranked second in importance in early studies, while it tended to be less important in the later examination of supplier selection criteria. Probably due in part to the globalization of the marketplace and the increasingly competitive atmosphere, quality and service considerations dominated price and delivery criteria in the examination of studies conducted in 1974, 1982, and 1993 separately (Lehmann and O’Shaughnessy 1974, 1982; Evans 1982; Wilson 1993). Changes in the supplier-buyer relationships were also observed. For example, buyers and suppliers traditionally had viewed each other as adversaries, but buyers and suppliers were gradually changing the way they dealt with one another, and both were encouraged to maintain a more cooperative relationship. Recently, additional criteria, such as new technologies, flexibilities and risk management, are gaining importance.

Normally the number of candidate suppliers is very large, and the pre-qualification is a useful process to shorten the list and maintain a smaller manageable number of suppliers. During pre-qualifications, the buyer needs to make tradeoff among the conflicting criteria and rank the suppliers. There exists several multiple criteria ranking methods available (Ravindran and Warsing 2013). Earlier studies about pre-qualification were done by Holt (1998). Although his study was based on the procurement of the construction contractor, the results could be applied in other areas. In his study, contractors’ viewpoint on prequalification was solicited, in contrast to earlier works which tended to present clients’ perspectives. He found that the five factors considered by contractors in pre-qualification were price, construction experience, company reputation, financial standing, and prior business relationship. For the three highest ranking factors, levels of importance tended to decline with contractor size. Furthermore, this study also stated that the lack of cyclic
review was a big problem in pre-qualification. Contractors rarely reassessed more frequently than just annually.

In the second phase of the supplier selection, detailed quantitative data such as price, capacity, quality etc., are collected on the shortlisted suppliers and are used in a multi-objective frameworks for the actual order allocation. In the final selection step, optimization models help buyers to make two decisions at the same time, the combination of most favorable suppliers who would meet the selection criteria and the optimal order quantities from the chosen suppliers. Moore and Fearon (1973) stated that price, quantity and delivery were important criteria for supplier selection, and discussed the use of linear programming in decision making. Gaballa (1974) applied the mathematical formulation using integer and mixed-integer programming to formulate a decision making model. The study was conducted in the context of Australian Post Office (A.P.O.), and its cost allocation problem was dealt with in two sections; one is devoted to quantity discounts, the other to order value discounts. Feng et al. (2001) presented a stochastic-integer programming model for the simultaneous selection of tolerances and suppliers based on the quality loss function and process capability index. Tung and Torng (2006) presented a fuzzy decision-making approach to deal with the supplier selection problem in a supply chain system. In their work, linguistic values were used to assess the ratings and weights for various factors, and a hierarchical multiple criteria decision-making (MCDM) model, based on fuzzy set theory, was proposed. Since supplier selection problems dealt with uncertain and imprecise data, this study showed that fuzzy-set theory was adequate to deal with them. Mendoza et al (2008) put forward a three-phase multi-criteria framework to reduce the number of potential suppliers. They proposed a screening process, using a multi-criteria ranking method, in the first phase. A distance metric was used with the respect to the ideal solution, which offered an easy way to reduce a large number of potential suppliers to a manageable number. The criteria weights were computed in the second phase, and the suppliers were ranked according to the AHP method. In the third phase, a preemptive goal programming
model was implemented to allocate orders among the selected suppliers. Ravindran et al (2010) stated that singular emphasis on supply chain cost can make the supply chain brittle and more susceptible to the risk of disruptions. Considering supplier risk, they developed two different types of risk models for supply disruptions, value-at risk (VaR) and miss-the-target (MtT) risk. In their 2-phase method, pre-qualification was done in phase 1 to shortlist the suppliers by ranking them under multiple conflicting criteria. It was found that cost, quality and delivery criteria were the three most important measures. In phase 2, order quantities were allocated among the short listed suppliers using goal programming models.

1.2 Problem statement

This thesis is an extension of the master’s thesis of Ding (2014). In her thesis, a supply chain system with a single buyer and multiple suppliers for multiple items was considered. It was a technical decision making model covering multiple periods. The key decision was to determine the optimal quantities ordered from the selected suppliers, which was a supplier selection and order allocation problem. The three objectives being considered were total cost, weighted average lead time and weighted average quality defect rate. Since the three objectives were conflicting, multi-criteria optimization methods were utilized to solve the problem. The methods used in her thesis were Weighted Objective Method, Pre-emptive Goal Programming, and Non-preemptive Goal Programming. A numerical example was used to illustrate the application of the model.

In this thesis, Ding’s thesis (2014) will be extended in three ways:

1. The impact of shortage will be considered on the supplier selection, and weighted sum of shortages over the horizon would be minimized as an additional objective. It would be assumed that the shortage will negatively affect customer service, because shortage could impact
customer trust and subsequent orders;

2. The total inventory will be minimized directly rather than estimating the total inventory cost. Usually it is hard to estimate inventory holding cost accurately in real problems. Therefore, in this thesis, the weighted sum of inventory will be minimized;

3. Supplier quantity discounts would be considered. Generally, there exists two types of discount policy, all-unit discount and graduated discount. In this thesis, all-unit discount will be studied. All-unit discount is the policy where the lower price applies to all the units ordered, provided the order exceeds a certain threshold.

In this thesis, a two-stage supply chain consisting of a company and several suppliers is considered. The company faces a deterministic stream of external demands for specific parts. Supply chains are classified as either centralized or decentralized. In a centralized setting, the stages are owned by the same company and a single decision maker has the authority to make decisions for all the stages in the supply chain. In the decentralized system, each stage is owned by a different company and decisions are made at each stage independently of other stages. In this thesis, we assume there is a company, who needs to order $N$ types of products from multiple suppliers over $T$ periods. Both the company’s inventory capacity and supplier’s capacities are limited. Thus, the company will have to choose the best combination of suppliers, from a given set of potential suppliers, and will order optimal quantities in each period from the selected suppliers to meet the demands. Thus, a comprehensive model of the multi-period supplier selection problem will be established with five objectives: “procurement cost”, “weighted inventory”, “weighted shortage”, “weighted quality”, and “weighted lead time”. The constraints will include demand constraints, capacity constraints, and all-unit discount constraints. We will formulate a mixed-integer programming model, and obtain an optimal solution by utilizing multi-objective optimization methods, such as, Weighted Objected Method, Non-preemptive Goal Programming, and
Preemptive Goal Programming. Managerial insights to select the best efficient solution using a Value Path Approach will also be discussed.

1.3 Thesis outline

The thesis is organized as follows: Chapter 2 consists of a literature review in the field of supplier selection, discount policies, and order allocation models under multiple criteria. Chapter 3 illustrates the multi-criteria optimization models for order allocation among the selected suppliers over finite time periods. Chapter 4 presents a numeric example with realistic data, and a sensitive analysis is conducted to study the effects of the model parameters on the optimal policies. Managerial insights provided by the models will also be discussed. Chapter 5 consists of conclusions and further research.
Chapter 2

Literature Review

In this section, a review of literature will be presented related to methods of supplier selection and order allocation. Topics covered include single-period, multiple-period, multiple sourcing, quantity discounts and transportation alternatives. Then, the methodology of multi-criteria optimization will be reviewed as well as its application in the field of supplier selection and order allocation.

2.1 Supplier selection and order allocation models under multiple sourcing strategy

Regarding decisions associated with supplier selection, the first major decision is to decide when to use a single sourcing strategy or a multiple sourcing strategy. In single sourcing only one supplier is selected to satisfy buyer’s requirements of demand, quality, delivery, etc. Multiple sourcing uses several suppliers to satisfy the buyer’s requirement, under the constraints of suppliers’ capacity, quality, etc. Although single sourcing may reduce the cost due to quantity discounts and economics of scale in shipping, purchasing managers may worry about the dependency on a single supplier associated with several kinds of risk, for example, increasing prices and supply disruption. As a result, many buyers have taken the strategy of multiple sourcing. Moreover, simplifications in trade regulations and price differentials between suppliers in developing and developed countries offer a broader scope for supplier search and sourcing strategies compared to the situation where only domestic suppliers are considered.

In order to assess the benefits of order splitting in an economic context, it is necessary to consider a model, where the total cost for ordering (order releases, order receipts), purchase prices,
inventory holding, and stockout penalties are minimized. Ramasesh et al (1991) present such a cost minimization approach for two potential suppliers where the order quantity is evenly split between the suppliers. Demand is assumed to be constant, shortages are backordered and a penalty cost per item and time unit is incurred. Both suppliers have identical lead time distributions, either being uniformly or exponentially distributed. The final conclusion shows that a dual-sourcing technique can lead to the effective lead time with lower variability and hence savings in the inventory holding and shortage costs.

Supplier selection and order allocation under multiple sourcing strategy is a multi-criteria problem with both qualitative and quantitative factors being considered. There are several types of methods and models in this field. Ravindran and Wadhwa (2009) provide a review of methods in supplier selection. For the prequalification of suppliers, the commonly used techniques are categorical methods, Data Envelopment Analysis (DEA), cluster analysis, Case-Based Reasoning (CBR) systems and Multi-criteria Decision Making (MCMD) methods. In the final selection phase, which is to identify the most suitable suppliers and allocate orders among them, there are two cases to consider: (1) single sourcing and (2) multiple sourcing. Some of the methods used in single sourcing are linear weighted point, cost ratio, Analytic Hierarchy Process (AHP) and Total Cost of Ownership (TCO). The most appropriate method used in multiple sourcing is mathematical programming. There are two types of mathematical programming methods, multiple sourcing single objective method and multiple sourcing multi-objective method. These methods are discussed in detail in the textbook by Ravindran & Warsing (2013). We will focus on reviewing these methods in this chapter.
2.1.1 Single-period supplier selection and order allocation models

Since supplier selection is a strategic-level problem, the main concern may include purchasing price, product quality and delivery. The approaches that are commonly applied can be divided into different groups: Multi Attribute Decision Making Techniques (MADM) such as Analytic Hierarchy Process (AHP), Mathematical programming and statistical approaches. The first two categories of approaches are often integrated to figure out the final order allocation results. The most commonly used methods in supplier selection research field involve mathematical programming, which is sometimes combined with MADM techniques.

Mendoza et al (2008) proposed a three-phase multi-criteria method to solve the supplier selection and order allocation problem. The first phase was to screen suppliers on the given criteria from a list of potential suppliers. The second phase was to calculate the criteria weights and come up with the ranking of suppliers screened from the first phase. The third phase was to allocate the orders among the selected suppliers. L2 metric, AHP and Goal Programming (GP) methods were applied to the three phases, respectively. This model provided systematic guidance and specific way to the whole supplier selection process with multiple criteria being considered, including flexibility, quality, price, service and delivery.

Ravindran et al (2010) developed a two-phase model to solve the risk-adjusted supplier selection problem with multi-criteria optimization methods. In the first phase, suppliers were shortlisted by ranking them under multiple criteria using Rating, Borda Count and AHP methods. The criteria were delivery, business performance, quality, cost, information technology, long term improvement and risk. Some attributes were given for each criterion to measure the suppliers’ performance. In the second phase, only lead time, cost and risk were used as criteria to formulate a multi-criteria optimization model to solve the supplier selection problem for multiple products with incremental price discount. The authors considered two types of supply risk: value-at-risk (VaR)
and miss-the-target (MtT) risk. VaR type risks are used to model globally disruptive events at suppliers with severe impacts to buyers. Preemptive, Non-preemptive, Tchebycheff (Min-Max) and Fuzzy Goal Programming methods were used to solve the problem. The models were illustrated with an actual application.

In Baruch Keren’s (2009) study, a special form of the single period inventory problem (newsvendor problem) with a known demand and stochastic supply (yield) is studied. A general analytic solution for two types of yield risks, additive and multiplicative, is described. Numerical examples demonstrate the solutions for special cases of uniform distribution yield risks. An analysis of a two-tier supply chain of customer and producer reveals that customer may find it optimal to order more than is needed, since a larger order increases the producer’s optimal production quantity.

Ruoning and Xiaoyan Zhai (2008) developed an optimization model for single period supply chain problems with fuzzy demand. The aim of their study was to develop an optimal techniques for dealing with fuzziness aspect of demand uncertainties. Triangular fuzzy numbers were used to model external demand, and decision models in both non-coordination and coordination situations were constructed. It was shown that in the decision models there existed a unique solution that can be expressed analytically. Based on the closed form solutions for both models, the behaviors and relationships of both the manufacturer and the retailer were quantitatively analyzed, and cooperative policy for the optimization of the whole supply chain was put forward.
2.1.2 Multi-period supplier selection and order allocation models

The multi-period problem is an extension of the single-period supplier problem. Under the multi-period supplier selection and order allocation models, the lead time and inventory management have to be considered explicitly over several periods.

EOQ model is a popular method for multi-period problem with infinite horizon. EOQ model is derived for constant, continuous demand. Donaldson’s (1977) study extends the analytical method to cover cases of irregular demand, so the classical no-shortage inventory policy is examined for the case of linear trend in demand bounded by a time horizon $H$, and the methods used are a computationally simple procedure for determining the optimal times for replenishment of inventory is established. Although Donaldson’s method is mathematically convenient for obtaining a solution to some extent, Donaldson’s analysis of demand “up to a time horizon $H$” is just a device to aid the mathematical solution and not an essential feature of real life, which also complicates the calculation of the optimal replenishment policy. E. Ritchie (1984) extends the time horizon of Donaldson’s model so that it no longer influences the replenishment times and simplify the calculation of the optimal policy. Ritchie finds that this simple optimal policy is analogous to the EOQ for constant demand, though it has been derived for the case of linear increasing demand. And for the decreasing demand, Smith (1976) shows that EOQ could be used for determining lot size for declining demand with little cost penalty.

Ghodsypour and O’Brien (2001) proposed a mixed-nonlinear programming model to solve a multiple sourcing problem minimizing the sum of transportation, ordering and inventory cost with some constraints on service, quality and budget. In that model, the inventory cost was calculated using EOQ model with the assumption of constant demand rate. Mendoza and Ventura (2008) extend EOQ model by considering two types of transportation, truckload (TL) and less than
truckload (LTL) carriers, and they also introduce all-units and incremental quantity discount policy in EOQ method.

Sudip Bhattacharjee and R. Ramesh (2000) studied a multi-period profit maximizing model for retail supply chain management, where the product has a fixed life perishability for a certain number of periods. This problem has significant importance for an efficient operation of the marketing/manufacturing interface at the retail end of the supply chain. The profit maximization problem is modeled as a dynamic program, and the Wagner-Whitin dynamic programming recursions are developed for both perishable and non-perishable products. The structural properties of the model are investigated, and it is shown that the maximum profit function is continuous piecewise concave. Two efficient search heuristics are presented, and the results are compared with benchmark optimum values. The heuristics have been extensively tested and the results indicate that the proposed approach is robust, efficient and practically viable.

2.1.3 Supplier selection and order allocation models under quantity discounts

Many suppliers would doubtlessly like to see their customers increase the size of their current orders. Larger individual orders, if maintained with the same order frequency, obviously would mean higher total annual sales. Thus, price discounts should be considered carefully in the period of making purchasing cost.

According to the study of Mostafa et al (2012), the current popular discount policy includes all-unit discount, incremental price break, business volume discount, bundling and other forms. In all-unit discount, the variable price to pay for an item applies for all units that are ordered. In an incremental discount-policy the discount applies only for quantities exceeding the price break
quantities. Standard formulations of modeling both all-unit discount and incremental price discounts as integer programming problems are given by Ravindran and Warsing (2013). Business volume discount is a type of price discount that applies to the total dollar volume of business awarded to a given supplier, not the order quantity of each individual product (Xia & Wu, 2007). Bundling is a class of price discount wherein the price of an item is related to the order quantities of other items (Aissaout et al, 2007).

Li and Liu (2006) develop a model for illustrating how to use a quantity discount policy to achieve supply chain coordination. In their model, a supplier-buyer system selling one type of product with multi-period and probabilistic customer demand is considered. It is first shown that if both the buyer and supplier can find a coordination mechanism to make joint decisions, the joint profit in this situation is more than the sum of their profits in the decentralized decision situation. The quantity discount policy is shown to be a way that may be implemented to achieve coordination. Their results illustrate that there is a bound of quantity discount in which both sides can accept and the increased profit due to joint decision can be measured using this bound.

Lal and Staelin (1984) do a study of discount policy based on the perspective of sellers. The problem addressed is why and how a seller should develop a discount pricing structure even if such a pricing structure does not alter ultimate demand. A model buyer reaction to any given pricing scheme is developed to show that there exists a unified pricing policy, which motivates the buyer to increase its ordering quantity per order, thereby reducing the joint (buyer and seller) ordering and holding costs. As a result, the seller faces numerous groups of variable ordering and shipping costs and situations where the seller faces numerous groups of buyers, each having different ordering policies. Finally a case study is presented explicitly showing how the proposed pricing policy can be applied to the situation of a large seller selling to a number of different buyer groups.
2.1.4 Supplier selection and order allocation models with multiple transportation modes

Transportation modes play an important role in logistics and supply chain management for reducing cost and improving service. There is no doubt that the changing role of the corporate transportation function in the modern business environment has impacted the decision making in supply chain management. Successful managers today need a broad view of transportation management’s role and responsibilities in an integrated supply chain. Stank and Goldsby (2000) clarify the major transportation decision areas and introduce a framework that positions corporate transportation management within the overall integrated supply chain environment. The framework portrays initial transportation decision as strategic, long-term decisions that focus on the overall supply chain transportation system. Once decisions are understood at this level, the decision-making scope becomes increasingly tactical in nature, focusing on operations that implement the overall system decisions.

Liang (2007) presents a novel fuzzy multi-objective linear programming (f-MOLP) model for solving integrated production-transportation planning decision problems in supply chains in a fuzzy environment. The proposed model attempts to simultaneously minimize total production and transportation costs, total number of rejected items, and total delivery time with reference to available capacities, labor level, quota flexibility, and budget constraints at each source, as well as forecast demand and warehouse space at each destination. An industrial case demonstrates that the proposed f-MOLP model achieves an efficient compromise solution and overall decision maker satisfaction with determined goal values.

Liu and Kao (2004) develop a procedure to derive the fuzzy objective value of the fuzzy transportation problem, in that the cost coefficients and the supply and demand quantities are fuzzy numbers. The idea is based on the extension principle. A pair of mathematical programs is formulated to calculate the lower and upper bounds of the fuzzy total transportation cost at
possibility level. Two different types of the fuzzy transportation problem are discussed: one with inequality constraints and the other with equality constraints.

2.2 Multi-criteria optimization

Multi-criteria optimization methods belong to the field of multiple criteria decision making (MCDM), which are concerned with finding the best alternatives under multiple conflicting objectives. Trade-off needs to be made by Decision Maker (DM) and one or more efficient solutions may be chosen to maximize DM’s preference. Here, we focus on the MCDM problems with infinite number of alternatives. These problems are called multi-criteria mathematical programming (MCMP) problems. A review of problem definition, terminology and methods are given below.

2.2.1 Definition of MCMP

Following Shin and Ravindran (1991), a multi-criteria mathematical programming (MCMP) problem can be written as:

\[
\text{Max } F(x) = \{f_1(x), f_2(x), ..., f_k(x)\}
\]

subject to: \(g_j(x) \leq 0 \text{ for } j = 1, ..., m\)

Where:

x: n-vector of decision variables

S: feasible region in the decision space, which is defined as \(S = \{x|g_j(x) \leq 0, \forall j\}\)

Y: criteria or objective space, which is defined as \(Y = \{y|F(x) = y \forall x \in S\}\)
2.2.2 Terminology (Shin & Ravindran, 1991)

**Ideal solution**

It is defined as the vector of best values achievable for each criterion. In other word, it is the individual optima of each objective function regardless of others. In a multi-criteria optimization problem, the ideal solution is not achievable since the criteria or objectives are conflicting with each other.

**Efficient, Non-dominated or Pareto optimal solution** (Shin & Ravindran, 1991)

A solution to MCMP is said to be a superior solution if it is feasible and maximizes all the objectives simultaneously. However, in most problems, superior solutions do not exist, because objectives conflict with each other. Hence, we are interested in efficient solutions or non-dominated solution. A feasible solution $x^0$ is said to be efficient if $f_k(x) > f_k(x_0)$ for some feasible solution $x$ implies that $f_j(x) < f_j(x_0)$ for at least one other index $j$.

**Dominated solution**

A dominated solution is a feasible solution where at least one objective can be improved without losing achievements in other objectives.

**Best compromise solution**

The best compromise solution is one that can maximize the Decision Makers’ preferences. It is generally assumed that the DM’s preferences can be modeled as a real-valued function, but it is not known explicitly.
2.2.3 MCMP Methods

There are several methods to solve MCMP problems. Based on the classification made by Shin and Ravindran (1991), the approaches can be categorized by the basic assumptions associated with the DM’s preference function.

Methods with Pre-specified preference information from Decision Maker

In this category, prior articulation preferences is made by the DM before the MCMP is solved. Weighted Objective Method is one of the commonly used techniques in multi-objective problems to find the efficient solutions. The basic idea is to assign weights for each objective after it has been appropriately scaled. The multi-objective problem is then transformed to a single objective, using the weighted sum of original multiple objectives. A brief description of this method is given below (Masud & Ravindran, 2008):

The multi-criteria mathematical programming (MCMP) problem can be written as:

\[
\underset{x}{\text{Max}} \; F(x) = \{f_1(x), f_2(x), \ldots, f_k(x)\}
\]

Subject to: \( g_j(x) \leq 0 \) for \( j = 1, \ldots, m \)

*Where* \( x \) *is an* \( n \) *– vector of decision variables,* \( S = \{x \mid g_j(x) \leq 0, j = 1, \ldots, m\} \) *is the feasible region, and* \( f_i(x), i = 1, \ldots, k \) *are the k criteria functions.*

The formulation of the weighted objective problem, also known as the \( P_\lambda \) problem, is given below: (Masud & Ravindran, 2008)

\[
\text{Max} \; Z = \sum_{i=1}^{k} \lambda_i f_i(x)
\]
Subject to: \( x \in S \)

\[
\sum_{i=1}^{k} \lambda_i = 1
\]

\[
\lambda_i \geq 0
\]

Where \( \lambda_i \) are the weight assigned to objective \( i \).

Let \( \lambda_i > 0 \) for all \( i \) be specified. If \( x^0 \) is an optimal solution for the \( P_\lambda \) problem, then \( x^0 \) is an efficient solution to the MCMP problem. However, there may be efficient solutions to MCMP problem which could not be achieved as an optimal solution to the \( P_\lambda \) problem (Geoffrion, 1968).

One typical method belonging to this category is Goal Programming (GP). Goal Programming is a widely used method to deal with multi-criteria optimization problems. The basic idea is to specify a set of targets or goals for the conflicting objectives and minimize the deviations from the targets. Undesired deviations from each target should be minimized to implement the goal programming method, with the priority or weight according to each criterion’s relative importance. There exists several types of goal programming models, and the Pre-emptive Goal Programming and Non- preemptive Goal Programming are utilized in this thesis, which are more commonly used in practice.

In goal programming, DM needs to assign target levels for each objective. In addition, relative priority on achieving those target levels should also be provided by the DM. The objective of GP is to find an optimal solution that is as close as possible to the targets with specified priorities. GP can be classified as Preemptive and Non-preemptive based on the different ways priorities are determined for each goal.

In the Preemptive GP, ordinal ranking is used to assign goals to different priority levels from highest to lowest. The goal assigned to a lower priority will not be considered until satisfying the goal with a higher priority. The model becomes a sequential optimization problem. Arthur and Ravindran (1978) provide efficient algorithms for solving the preemptive linear GP problems.
Under the preemptive priorities, different priority levels are assigned to the goals. In this case, the objective function of the GP model is reformulated as:

$$\text{Minimize } Z = \sum_x P_x \sum_i (d_i^+ + d_i^-)$$

Where $P_x$ represent the priority assigned to the criteria. Notice that the target with lower priority cannot be considered without satisfying the target with higher priority. By this way, the problem turns to a sequential single objective optimization process.

In the non-preemptive GP, pre-specified weights are assigned to each goal and the objective is to minimize the weighted sum of goal deviations. In this case, the goal program can be reduced to a single-objective optimization problem. The general formulation of Non-preemptive goal programming (GP) approach shown below is given by Masud & Ravindran (2008).

$$\text{Minimize } Z = \sum_{i=1}^{k} (w_i^+ d_i^+ + w_i^- d_i^-)$$

Subject to: $f_i(x) + d_i^- - d_i^+ = b_i$ for $i = 1, \ldots, k$

$g_j(x) \leq 0$ for $j = 1, \ldots, m$

$x_j, d_i^-, d_i^+ \geq 0$ for all $i$ and $j$

Where $b_i$ represents the acceptable level of achievement for each criterion $f_i$ and a weight $w_i$ (ordinal or cardinal) is assigned to the deviation between $f_i$ and $b_i$. The variables, $d_i^-$ and $d_i^+$, are the deviational variables representing the under achievement and over achievement of the $i^{th}$ goal, respectively.

Under Non-preemptive (cardinal) weights, $w_i^-, w_i^+$ should be assigned specific values on a relative scale, representing the relative importance of the goals given by the Decision Maker (DM). The specific values of the weights can be obtained from the DM using Analytic Hierarchy Process (AHP) or other approaches. Once $w_i^-, w_i^+$ are specified, the problem reduces to a single
objective optimization problem. The drawback of this method is that the criteria value should be scaled and cardinal weights are difficult to obtain.

**Methods with no preference information available**

This approach is involved in generating all efficient solutions to the DM to choose from. Global criterion method (Hwang & Masud, 1979) and compromise programming (Zeleny, 1982) belong to this category. A detailed discussion of these 2 methods are given in Masud & Ravindran (2008).

**Interactive methods**

Methods that rely on partial preference information which is obtained from the DM progressively are called interactive methods. In Sadagopan and Ravindran’s (1986) study, several interactive procedures for solving multiple criteria nonlinear programming problems have been developed. These are based on the generalized reduced gradient method for solving single objective nonlinear programming problems. They can handle maximization problems with nonlinear concave objectives, nonlinear convex constraints and an implicit quasi-concave preference function of the decision maker. The interactive procedures work with information of varying degrees of accuracy from the decision maker, thereby extending and strengthening a number of existing methods.

The implementation of these approaches begins with finding an efficient solution. Afterwards, DM’s response to that solution is obtained. These two steps should be repeated until the DM is satisfied or until other termination criterion is met to terminate. Shin and Ravindran (1991) present a detailed survey of MCMP interactive methods including classification scheme, review of methods in each category and a rating of each category of methods. According to the authors, typical interaction styles include binary pair-wise comparison, pair-wise comparison, vector comparison and precise local tradeoff ratio.
2.3 Multi-criteria supplier selection and order allocation models

Applying the multi-criteria optimization methods in the supplier selection problem has been analyzed by many scholars. Wind and Robinson (1968) apply linear weighting method to a vendor selection problem. In their study, an evaluation function is developed, which is a linear weighted function consisting of several scaled performance criteria, such as quality/price ratio and delivery reliability, and weights assigned to each criterion. By calculating the score of each supplier’s evaluation function, the supplier can be chosen with the highest score.

Wadhwa and Ravindran (2007) applied three different methods to solve the multi-objective supplier selection model in a multiple sourcing environment. There were three conflicting objectives being minimized, which were total cost, weighted average lead time and weighted average percentage of quality rejects. Moreover, the incremental price discount was incorporated into the model to make it more realistic. The solution methods included weighted objective, goal programming and compromise programming. The Value Path Approach was utilized to compare and analyze the results obtained from different methods. Their work provided some insight into the supplier selection field on utilizing different methods and making comparisons to solve multiple criteria optimization problems.

Ziqi (2014) apply multi-criteria optimization methods to supplier selection and order allocation problem over a planning horizon under multiple sourcing strategy, and the supply chain system is modeled with a single buyer and multiple suppliers with deterministic demands as a mixed integer programming problem. Based on Ziqi’s study, in this thesis, the all-unit discount policy will be included in the mixed integer programming model. Meanwhile the weighted inventory and the weighted shortage are treated as separated objectives, and the trade-off among five objectives will be discussed, including total procurement cost, weighted inventory, weighted shortage, quantity weighted lead time and quantity weighted defect rate. Therefore, the focus of
this thesis is to build a multi-period supplier selection and order allocation model under an all-unit discount price policy using multi-criteria approaches.

Table 2-1 summarizes the preceding presentation. The first column gives the authors of the articles, the second column presents the topics of the article, and the third column shows the main content in the articles.

Table 2-1. Summary of Articles Related to this Thesis

<table>
<thead>
<tr>
<th>Article</th>
<th>Topic</th>
<th>Main content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ramasesh et al (1991)</td>
<td>Sole versus dual sourcing in stochastic lead-time (s, Q) inventory models</td>
<td>In the two-vendor model, the order quantity is split equally between the two vendors and the split orders are placed simultaneously when the inventory position reaches the reorder level.</td>
</tr>
<tr>
<td>Ravindran &amp; Wadhwa (2009)</td>
<td>Muti-criteria optimization models for supplier selection</td>
<td>Review the methods in supplier selection, including Data envelopment analysis and cluster analysis etc.</td>
</tr>
<tr>
<td>Ravindran et al (2010)</td>
<td>Risk adjusted multi-criteria supplier selection models with applications</td>
<td>Developed a multi-criteria supplier selection models incorporating supplier risk and apply them to a real company</td>
</tr>
<tr>
<td>Baruch Keren (2009)</td>
<td>Single-period inventor problem: Extension to random yield from the perspective of the supply chain</td>
<td>Described a general analytic solution for two types of yield risks, additive and multiplicative; An analysis of a two-tier supply chain of customer and producer reveals that the customer may find it optimal to order more than is needed.</td>
</tr>
<tr>
<td>Ruoning &amp; Xiaoyan Zhai (2008)</td>
<td>Optimal models for single-period supply chain problems with fuzzy demand</td>
<td>Developed an optimal technique for dealing with the fuzziness aspect of demand uncertainties. It is shown that in the decision models there exists a unique solution that can be expressed analytically.</td>
</tr>
<tr>
<td>Donaldson (1977)</td>
<td>Inventory replenishment policy for a linear trend in demand</td>
<td>Examined the classical no-shortage inventory policy for the case of a linear trend in demand; Established a computationally simple procedure for determining the optimal times for replenishment of inventory.</td>
</tr>
<tr>
<td>Author(s) (Year)</td>
<td>Description</td>
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<tr>
<td>E. Ritchie (1984)</td>
<td>The EOQ for linear increasing demand extended the time horizon of Donaldson’s model so that it no longer influences the replenishment times and simplify the calculation of the optimal policy.</td>
<td></td>
</tr>
<tr>
<td>Ghodsypour &amp; O’Brien (2001)</td>
<td>The total cost of logistics in supplier selection, under conditions of multiple sourcing, multiple criteria and capacity constraint. Presented a mixed integer non-linear programming model to solve the multiple sourcing problem, which takes into account the total cost of logistics.</td>
<td></td>
</tr>
<tr>
<td>Xia &amp; Wu (2007)</td>
<td>Supplier selection with multiple criteria in volume discount environments. Proposed an integrated approach of AHP to simultaneously determine the number of suppliers to employ and the order quantity allocated to these suppliers in the case of multiple sourcing, multiple products, with multiple criteria and with supplier’s capacity constraints.</td>
<td></td>
</tr>
<tr>
<td>Li &amp; Liu (2006)</td>
<td>Supply chain coordination with quantity discount policy. Developed a model for illustrating how to use quantity discount policy to achieve supply chain coordination.</td>
<td></td>
</tr>
<tr>
<td>Lal &amp; Staelin (1984)</td>
<td>An approach for developing an optimal discount pricing policy. Addressed the problem of why and how a seller should develop a discount pricing structure even if such a pricing structure does not alter ultimate demand.</td>
<td></td>
</tr>
<tr>
<td>Stank &amp; Goldsby (2000)</td>
<td>A framework for transportation decision making in an integrated supply chain. Clarified the major transportation decision areas and introduced a framework that positions corporate transportation management within the overall integrated supply chain environment.</td>
<td></td>
</tr>
<tr>
<td>Liu &amp; Kao (2004)</td>
<td>Solving fuzzy transportation problems based on extension principle. Developed a procedure to derive the fuzzy objective value of the fuzzy transportation problem, in that the cost coefficients and the supply and demand quantities are fuzzy numbers.</td>
<td></td>
</tr>
<tr>
<td>Authors</td>
<td>Title</td>
<td>Summary</td>
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<tr>
<td>Sadagopan &amp; Ravindran</td>
<td>Interactive algorithm for multiple criteria nonlinear programming problems</td>
<td>Developed several interactive procedures for solving multiple criteria nonlinear programming problems, which are based on the generalized reduced gradient method for solving single objective nonlinear programming problems.</td>
</tr>
<tr>
<td>Shin &amp; Ravindran</td>
<td>Interactive multiple objective optimization: Survey I—continuous case</td>
<td>Surveyed the interactive methods developed for solving continuous multiple objective optimization problems and their applications.</td>
</tr>
<tr>
<td>Wind &amp; Robinson</td>
<td>The determinants of vendor selection: the evaluation function approach</td>
<td>Applied linear weighting method to a vendor selection problem.</td>
</tr>
<tr>
<td>Wadhwa &amp; Ravindran</td>
<td>Vendor selection in outsourcing</td>
<td>Applied three different methods to solve the multi-objective supplier selection model in a multiple sourcing environment.</td>
</tr>
<tr>
<td>Ziqi</td>
<td>Multi-criteria multi-period supplier selection and order allocation models</td>
<td>Applied multi-criteria optimization methods to supplier selection and order allocation problem over a planning horizon under multiple sourcing strategy.</td>
</tr>
</tbody>
</table>
Chapter 3
Model Formulation

In this chapter, a multi-criteria mixed integer programming model will be presented to solve the multi-period supplier selection and order allocation problem. Multi-criteria optimization techniques will be utilized to minimize the procurement cost, inventory, shortage, lead time, and quality defect rate. This model is an extension to the model proposed by Ding (2014).

3.1 Model description and formulation

Consider a system of single retailer, multiple suppliers and multiple types of products with deterministic demand over multiple periods. The problem is to select the appropriate combination of suppliers among the given set of suppliers in each period and allocate orders among the selected suppliers. The four objectives are to minimize procurement cost, weighted average inventory, weighted average shortage, lead time, and quality defect rate. The constraints considered include demand constraints, capacity constraints, and quantity discount constraints.

3.1.1 Assumptions

1) The planning horizon is assumed to be $T$ periods.
2) The demand for each type of product in each period is deterministic, and a forecast of demand is given.

3) For product $i$, $i=1,...,N$, the retailer has $M_i$ suppliers to choose from.

4) The retailer places order at the beginning of each period.

5) For product $i$ supplied by supplier $j$, the lead time $L_{i,j}$ is assumed to be a constant.

6) A unit transportation cost $CT_{i,j}$ occurs when product $i$ is supplied by supplier $j$.

7) A fixed cost $CF_j$ occurs when an order is placed to supplier $j$ in any period.

8) Shortage are allowed.

9) The suppliers offer an all-unit discount model.

10) The retailer’s inventory capacity is limited.

11) Supplier’s capacity over the planning horizon is limited.

12) The price for each supplier’s part will not change during the planning horizon.

13) The initial shortage and the initial inventory for each product are given.

14) The final shortage and the final inventory for each product are assumed to be zero.

### 3.1.2 Notation

**Indices**

$i$: index of products, $(i=1,...,N)$

$j$: index of suppliers, $(j=1,...,M)$

$k$: index of quantity discount of price level

$t$: index of time periods, $(t=1,...,T)$

**Parameters**
\( N \): total number of products

\( M \): total number of suppliers

\( T \): total number of periods

\( L_{i,j} \): lead time for product \( i \) from supplier \( j \)

\( CT_{i,j} \): unit transportation cost of supplier \( j \) for product \( i \)

\( D_{i,t} \): demand for product \( i \) in period \( t \)

\( CF_j \): fixed set-up cost to order any part from supplier \( j \) in any period

\( p_{i,j,k} \): unit price of level \( k \) of product \( i \) from supplier \( j \)

\( Q_{i,j,k,t} \): break point for unit price \( p_{i,j,k} \)

\( U_{i,j} \): number of discount price levels offered by supplier \( j \) for product \( i \)

\( q_{i,j} \): quality defect rate of product \( j \) from supplier \( i \)

\( CAP_{i,j} \): capacity of supplier \( j \) for product \( i \) in each period

Decision Variables

\( Q_{i,j,k,t} \): order quantity for product \( i \) from supplier \( j \) in period \( t \) at price level \( k \). (Note: order placed in period \( t \) will arrive in period \( t+L_{i,j} \))

\( I_{i,t} \): inventory level of product \( i \) at the end of period \( t \). (Note: \( I_{i,0} \) and \( I_{i,T} \) are the initial and final inventories for product \( i \), which are specified constants)

\( S_{i,t} \): cumulative shortage of product \( i \) at the end of period \( t \). (Note: \( S_{i,0} \) is given and \( S_{i,T} \equiv 0 \))

\( \delta_{j,t} \): binary variable, \( \delta_{j,t} = 1 \) if supplier \( j \) is selected in period \( t \), and \( \delta_{j,t} = 0 \) otherwise.

\( \nu_{i,j,k,t} \): binary variable, \( \nu_{i,j,k,t} = 1 \) if \( Q_{i,j,k,t} \) is in the quantity interval \( k \), and \( \nu_{i,j,k,t} = 0 \) otherwise.
3.1.3 Objective functions

There are five objective functions in the model: total procurement cost, weighted average inventory, weighted average shortage, quantity weighted lead time, and quantity weighted quality defect rate.

1) Total procurement cost.

The total procurement cost consists of fixed ordering cost, variable cost, and shipping cost.

- The fixed ordering cost $C_f$:

  Once the product is ordered from the supplier $j$ in period $t$, there is a fixed ordering cost associated with it. The total ordering cost is given by

  $$
  C_f = \sum_{t=1}^{T} \sum_{j=1}^{M} CF_j \delta_{j,t}
  $$

- The variable cost $C_v$:

  The total purchasing cost over the planning horizon is given by

  $$
  C_v = \sum_{t=1}^{T} \sum_{j=1}^{M} \sum_{i=1}^{N} \sum_{k=1}^{U_{i,j}} \bar{p}_{t,j,k} Q_{i,j,k,t}
  $$

- The shipping cost $C_s$:

  This is the cost generated to ship the parts from suppliers to the buyer. The total shipping cost is given by
Here it is assumed that the buyer would pay the shipment cost.

- Total procurement cost:

Summing up the fixed ordering cost, variable cost, and shipping cost, the first objective is given by

\[
\min Z_1 = \sum_{t=1}^{T} \sum_{j=1}^{M} CF_{j,t} \delta_{j,t} + \sum_{t=1}^{T} \sum_{j=1}^{M} \sum_{i=1}^{N} \sum_{k=1}^{Q_{i,j,k,t}} \bar{p}_{i,j,k} Q_{i,j,k,t}
\]

\[
+ \sum_{t=1}^{T} \sum_{j=1}^{M} \sum_{i=1}^{N} \sum_{k=1}^{Q_{i,j,k,t}} CT_{i,j,k,t} Q_{i,j,k,t}
\]

(3.1)

2) Weighted average inventory

Normally, inventory cost is a significant portion of total operational cost in a company. Therefore, lowering inventory cost is an important concern in supply chain optimization. Inventory cost includes rent, taxes, insurance, and cost of capital. However, it is usually difficult to estimate the holding cost accurately. Hence, in this thesis, the weighted average inventory will be directly minimized rather than the inventory cost. The weights will reflect the importance of the different products.

The importance of the parts is different for holding inventory and the ranking or the weight can be used to reflect their priorities. In this thesis, the weight \( \omega_i \) assigned to each part will be calculated considering the value, size, and the demand for the part. Ranking methods, such as, Rating Method, Borda Count Method, and AHP can be used to calculate the weights.
\[ \text{Min } Z_2 = \sum_{t=1}^{T} \sum_{i=1}^{N} l_{i,t} \omega_i \]

(3.2)

3) **Weighted average shortage**

Not fulfilling customer demand leads to not only extra cost but poor customer service. However, most factors related to estimating the shortage cost are hard to measure, for example, goodwill loss due to shortage. Thus in this thesis the weighted average shortage will be minimized directly. The weight \( v_i \) assigned to each part will be calculated considering the influence of shortage on the whole supply chain. The weights will reflect the importance of the parts.

\[ \text{Min } Z_3 = \sum_{t=1}^{T} \sum_{i=1}^{N} s_{i,t} v_i \]

(3.3)

4) **Quantity weighted lead time.**

According to Ravindran and Warsing (2013), the quantity weighted lead time is commonly used to measure the average lead time of selected suppliers. The quantity weighted lead time is calculated by summing the product of lead time by the order quantity for all products, suppliers, periods and price levels, and then dividing it by the total order quantity. Thus, the numerator of this ratio is given by:

\[ \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{U} L_{i,j} Q_{i,j,k,t} \]

The denominator (total order quantity) is given by:
Hence, the objective of minimizing the quantity weighted lead time is given by:

\[
\min \frac{\sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} U_{i,j} \sum_{k=1}^{M} L_{i,j,k,t} Q_{i,j,k,t}}{\sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} U_{i,j} \sum_{k=1}^{M} Q_{i,j,k,t}}
\]

According to the assumptions (13) and (14), the initial inventory and shortage for each product are given, and the inventory and shortage at the end of horizon plan are zero. Hence, the total order quantity over the time horizon T will be constant, making the denominator of the objective function a constant. Thus, the fourth objective is equivalent to:

\[
\min Z_4 = \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} U_{i,j} L_{i,j,k,t} Q_{i,j,k,t}
\]  
(3.4)

5) **Quantity weighted quality defect rate.**

According to Ravindran and Warsing (2013), quantity weighted quality defect rate is commonly used to measure the combined quality of products provided by the suppliers. It is given by:

\[
\min \frac{\sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} U_{i,j} q_{i,j,k,t} Q_{i,j,k,t}}{\sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} U_{i,j} Q_{i,j,k,t}}
\]
Because the total number of orders within the planning horizon is constant, the quantity weighted quality defect rate is equivalent to:

\[
Min Z_S = \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{t=1}^{T} \sum_{k=1}^{U_{ij}} q_{t,j,k,t} Q_{i,j,k,t}
\]  

(3.5)

3.1.4 Constraints

1) Demand constraints:

For most supply chains, satisfying customer demands is very important. In an aggregate planning problem, although shortages are permitted, the total demand should be satisfied by the end of the planning horizon. Therefore, in each period \( t \), equality constraint is used for material balance. Let \( t_i^* \) denote the minimum lead time for product \( i \) among all suppliers, i.e., \( t_i^* = Min_j L_{i,j} \), and the initial shortage is assumed to be zero, namely, \( S_{i,0} = 0 \) for all \( i \).

From the period 1 to period \( t_i^* \), the demand of product \( i \) can only be satisfied by the initial inventory:

\[
S_{i,t} + I_{i,t-1} = D_{i,t} + I_{i,t} + S_{i,t-1}
\]

\[
S_{i,0} = 0 \text{ and } I_{i,0} \text{ are given}
\]

for all \( i = 1, \ldots, N \) and \( 1 \leq t \leq t_i^* \)
From period $t_i^*$ to period $T$, the demand of each product and the shortage from the previous period can be satisfied by both inventory and new orders received at each period.

$$S_{i,t} + I_{i,t} + \sum_{j=1}^{M} \sum_{k=1}^{U_{i,j}} Q_{i,j,k,t-L_{i,j}} = D_{i,t} + I_{i,t} + S_{i,t-1}$$

(3.7)

for all $i = 1 \ldots N$ and $t_i^* + 1 \leq t \leq T$

2) Capacity constraints:

Since supplier’s capacity is limited, order allocation is necessary in procurement. The following constraints make sure that order quantity from each supplier in each period cannot exceed the supplier’s capacity:

$$\sum_{k=1}^{U_{i,j}} Q_{i,j,k,t} \leq \text{CAP}_{i,j} \times \delta_{j,t}$$

(3.8)

for all $i = 1 \ldots N$, $j = 1 \ldots M$ and $t = 1 \ldots T$

The binary variable $\delta_{j,t}$ make sure that no order will be placed to supplier $j$ in the period $t$, when supplier $j$ is not selected in period $t$, i.e., $\delta_{j,t} = 0$.

3) All-unit discount constraints:

Quantity discounts refer to the practice of offering lower prices for large volume purchases. Under all-unit discount policy, the entire purchase will be charged at a lower price based on the order quantity:

$$(\bar{Q}_{i,j,k} + 1)v_{i,j,k,t} \leq Q_{i,j,k,t} \leq \bar{Q}_{i,j,k+1}v_{i,j,k,t}$$

(3.9)

for $i = 1 \ldots N$, $j = 1 \ldots M$, $k = 1 \ldots U_{i,j}$, $t = 1 \ldots T$
\[
\sum_{k=1}^{u_{i,j}} \nu_{i,j,k,t} \leq \delta_{j,t} \\
\text{for } i=1, \ldots, N, \ j=1, \ldots, M, \ t=1, \ldots, T
\] (3.10)

Note: \(\nu_{i,j,k,t}\) are binary variables

In constraint (3.9), \(\bar{Q}_{i,j,k}\) are the breakpoints in the all-unit discount case, and each set of inequalities with these breakpoints denote a discount interval, corresponding to a specific price level, \(\bar{p}_{i,j,k}\). The constraints and purchasing cost functions make sure that the order quantities are located in the right discount level and are charged by the right purchasing price for the specific product, supplier, and period. Constraint (3.10) makes sure that the orders will only be placed to the suppliers that are selected in that period, and all \(\nu_{i,j,k,t}\) will be forced to zero by constraint (3.10), if the supplier \(j\) is not selected in period \(t\), i.e., \(\delta_{j,t} = 0\). Thus, the zero values of \(\nu_{i,j,k,t}\) in constraint (3.9) make all orders placed to supplier \(j\) zero in period \(t\), i.e., \(Q_{i,j,k,t} = 0\).

4) Maximum number of suppliers:

The number of suppliers used in each period is restricted as given below:

\[
\sum_{j=1}^{M} \delta_{j,t} \leq \alpha \\
\text{for } t=1, \ldots, T
\] (3.11)

\(\alpha\) denotes the maximum number of suppliers that can be selected in each period.

5) Other constraints:
These constraints make sure that all the decision variables are non-negative and all the binary variables are well defined:

\[
Q_{i,j,k,t} \geq 0 \quad (3.12)
\]

for \( i = 1, \ldots, N; \ j = 1, \ldots, M; \ k = 1, \ldots, U_{i,j}; \ t = 1, \ldots, T \)

\[
l_{i,t} \geq 0 \text{ and } s_{i,t} \geq 0
\]

for all \( i = 1, \ldots, N; \ t = 1, \ldots, T \)

\[
l_{i,0}, l_{i,T}, s_{i,0}, \text{ and } s_{i,T} \text{ are specified}
\]

\[
\delta_{j,t} \in \{0,1\}, \nu_{i,j,k,t} \in \{0,1\}
\]

## 3.2 Solution approaches

For solving the multi-objective optimization problems, there exists several methods, such as Goal Programming, Compromise Programming, and weighted objective method (Masud & Ravindran, 2008). In this thesis, the model will be solved using both weighted objective and goal programing methods.

### 3.2.1 Weighted objective method

Under the weighted objective method, the problem is transformed into a single objective mixed integer programming problem, as shown below.
where $W_1$, $W_2$, $W_3$, $W_4$, and $W_5$ represent the relative weights assigned to the five objectives: total procurement cost, average inventory, average shortage, weighted average lead time, and average quality defect rate, respectively. The weight assigned to each objective demonstrates the relative importance of that objective. The weights and ratings can be obtained by Borda Count or AHP methods. A sensitivity analysis will also be done on the weights to demonstrate its impacts on the final solutions.

Since the magnitude of objectives can affect the solutions, appropriate scaling methods have to be used. To apply the weighted objective method, it is essential to scale each objective. There are several scaling methods, such as simple scaling, ideal value method, simple linearization and $L_p$ Norm (Vector Scaling). The different scaling methods are discussed in Ravindran and Warsing (2013). In this thesis, ideal value method is used for scaling.

To scale the objectives, let $\pi_1, \pi_2, \pi_3, \pi_4$ and $\pi_5$ be the ideal values of the objectives. Ideal value represents the best value that can be achieved for each objective, ignoring all other objectives. Then, we scale each objective by dividing the objective function by its ideal value. The scaled objectives are given below:
Once the weights are specified, we have a single objective optimization problem to solve.

\[
\text{Min} \frac{W_1}{\pi_1} \left\{ \sum_{t=1}^{T} \sum_{j=1}^{M} CF_j \delta_{j,t} + \sum_{t=1}^{T} \sum_{j=1}^{M} \sum_{i=1}^{N} \sum_{k=1}^{U_{ij}} \tilde{p}_{i,j,k} Q_{i,j,k,t} \right. \\
+ \sum_{t=1}^{T} \sum_{j=1}^{M} \sum_{i=1}^{N} \sum_{k=1}^{U_{ij}} CT_{i,j} Q_{i,j,k,t} \left. \right\} + \frac{W_2}{\pi_2} \left( \sum_{t=1}^{T} \sum_{i=1}^{N} I_{i,t} \omega_{i,t} \right) \\
+ \frac{W_3}{\pi_3} \left( \sum_{t=1}^{T} \sum_{i=1}^{N} s_{i,t} \nu_{i,t} \right) + \frac{W_4}{\pi_4} \left( \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{U_{ij}} L_{i,j} Q_{i,j,k,t} \right) \\
+ \frac{W_5}{\pi_5} \left( \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{t=1}^{T} \sum_{k=1}^{U_{ij}} q_{i,j} Q_{i,j,k,t} \right) \right.
\]

(a) \text{Pre-emptive goal programming}

Here, the relative importance of goals are specified by preemptive priorities.
Objective function

\[
\min Z = P_1 d_1^- + P_2 d_2^+ + P_3 d_3^+ + P_4 d_4^+ + P_5 d_5^+
\]

where \(P_1, P_2, P_3, P_4\) and \(P_5\) are the pre-emptive priorities assigned to each criterion. That is, goals with lower priority can only be considered after goals with higher priority are achieved. Since pre-emptive goal programming does not specify a weight to an objective, scaling of the objectives is not necessary. In this case, the five objectives are to minimize. Hence, the positive deviations from the objective targets are minimized.

Goal constraints

(1) Total procurement cost

Based on the objective function of total cost, given by Equation (3.1), the goal constraint to satisfy the cost target is given by:

\[
\sum_{t=1}^{T} \sum_{j=1}^{M} CF_j \delta_{j,t} + \sum_{t=1}^{T} \sum_{j=1}^{M} \sum_{i=1}^{N} \sum_{k=1}^{U_{i,j}} \bar{p}_{i,j,k} Q_{i,j,k,t} + \sum_{t=1}^{T} \sum_{j=1}^{M} \sum_{i=1}^{N} \sum_{k=1}^{U_{i,j}} CT_{i,j} Q_{i,j,k,t} + d_1^- = \text{Total procurement cost target}
\]

where \(d_1^-\) & \(d_1^+\) are the negative and positive deviation from the cost target.

(2) Weighted average inventory

Based on the objective function of weighted average inventory, given by Equation (3.2), the goal constraint to satisfy the weighted average inventory target is given by:
\[
\sum_{t=1}^{T} \sum_{i=1}^{N} I_{i,t}w_i + d_2^- - d_2^+ = \text{Weighted average inventory target} \quad (3.16)
\]

(3) **Weighted average inventory**

Based on the objective function of weighted average shortage, given by Equation (3.3), the goal constraint to satisfy the weighted average shortage target is given by:

\[
\sum_{t=1}^{T} \sum_{i=1}^{N} S_{i,t}v_i + d_3^- - d_3^+ = \text{Weighted average shortage target} \quad (3.17)
\]

(4) **Weighted average lead time**

Based on the objective function of total lead time, given by Equation (3.4), the goal constraint to satisfy the lead time target is given by:

\[
\sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{U_{i,j}} L_{i,j,k,t}Q_{i,j,k,t} + d_4^- - d_4^+ = \text{Weighted lead time target} \quad (3.18)
\]

(5) **Quality defect rate**

Based on the objective function of total quality defects, given by Equation (3.5), the goal constraint to satisfy the quality defects target is given by:

\[
\sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{T} \sum_{U_{i,j}} q_{i,j,k,t} Q_{i,j,k,t} + d_5^- - d_5^+ = \text{Weighted quality target} \quad (3.19)
\]

(6) **Non-negative constraints**
\[ d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+, d_5^-, d_5^+ \geq 0 \]

Other constraints are shown as the real constraints, including demand constraints, capacity constraints and non-negative and binary constraints, which are given by Equations (3.6) through (3.12).

The Preemptive goal programming model has to be solved as a sequence of optimization problems. Priority 1 objective is minimized first, before priority 2 is even considered, and so on.

(b) Non-preemptive goal programming

In Non-preemptive goal programming, relative weights are assigned to the goals. Hence the objective have to be scaled. If the criteria values are not scaled properly, the criteria with large magnitudes would simply dominate the final result, irrespective of the assigned weights. Hence, the goals will be scaled by the ideal value method so that the weights can be applied to comparable metrics. The ideal value for total cost refers to the minimum cost obtained by simply minimizing the first objective ignoring other objectives. Similarly, the ideal values for lead time, quality defects and service level are obtained by minimizing the respective objectives. Let \( \pi_1, \pi_2, \pi_3, \pi_4 \) and \( \pi_5 \) represent the ideal values for the procurement cost, weighted average inventory, weighted average shortage, lead time, and quality defects, respectively. The deviational variables in this model represent the amount by which the scaled values of the objectives are away from their targets values.

- Objective function:

\[ Min \ Z = w_1 d_1^+ + w_2 d_2^+ + w_3 d_3^+ + w_4 d_4^+ + w_5 d_5^+ \]

Here, \( w_1, w_2, w_3, w_4 \) are the non-preemptive weights assigned to each objective.
· Goal constraints:

(1) Procurement cost target

\[ \frac{Z_1 + d_1^- - d_1^+}{\pi_1} = \frac{\text{Procurement cost target}}{\pi_1} \]

(2) Inventory target

\[ \frac{Z_2 + d_2^- - d_2^+}{\pi_2} = \frac{\text{Inventory target}}{\pi_2} \]

(3) Shortage target

\[ \frac{Z_3 + d_3^- - d_3^+}{\pi_3} = \frac{\text{Shortage target}}{\pi_3} \]

(4) Lead time target

\[ \frac{Z_4 + d_4^- - d_4^+}{\pi_4} = \frac{\text{Lead time target}}{\pi_4} \]

(5) Quality target

\[ \frac{Z_5 + d_5^- - d_5^+}{\pi_5} = \frac{\text{Quality target}}{\pi_5} \]

(6) Non-negativity constraints

\[ d_1^+, d_1^-, d_2^+, d_2^-, d_3^+, d_3^-, d_4^+, d_4^-, d_5^+, d_5^- \geq 0 \]

Other constraints are shown as real constraints, including demand constraints, capacity constraints, shortage constraints and non-negativity and binary constraints, which are given by Equations (3.6) through (3.12). Under Non-preemptive goal programming, only a single objective optimization problem is solved.
Chapter 4

Illustrative Example and Analysis

4.1 Illustrative example

In this chapter, a numerical example will be utilized to illustrate the application of the model presented in Chapter 3. As stated before, this model is used to help buyer to select the most appropriate suppliers from a number of potential suppliers and to allocate orders among the selected suppliers. In this example, two types of products and three suppliers will be considered. Different multiple criteria optimization methods are implemented to illustrate the solution process.

4.1.1 Problem description

A manufacturing company needs two key parts for making product. The company has shortlisted 3 suppliers for these parts. The supplier data are given in Tables 4-1 through 4-6. Supplier 1 has the lowest price for part 1, and supplier 3 has the lowest price for part 2; but the lower price generally leads to a lower quality (higher defect rate). Supplier 1 has the longest lead time for both parts, but charges relatively low transportation cost (Table 4-5). The company wants to restrict the number of suppliers selected in each period to two so that the third supplier can be used as a back-up supplier during a supply disruption. Demands of the two parts over the planning horizon are shown in Table 4-4, and the inventory factors and shortage factors are presented in Table 4-6 which are used to calculate the weighted average inventory and the weighted average shortage.
### Table 4-1. Fixed Ordering Cost

<table>
<thead>
<tr>
<th>Supplier (j)</th>
<th>Fixed ordering cost ($C_{F_j}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1000</td>
</tr>
<tr>
<td>2</td>
<td>$2000</td>
</tr>
<tr>
<td>3</td>
<td>$800</td>
</tr>
</tbody>
</table>

### Table 4-2. All-unit Discount Price for Part 1

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Price</th>
<th>Break point $Q_{i=1,j,k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>22</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>over 150</td>
</tr>
<tr>
<td>2</td>
<td>26</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>over 150</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>22</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>over 150</td>
</tr>
</tbody>
</table>

### Table 4-3. All-unit Discount Price for Part 2

<table>
<thead>
<tr>
<th>Supplier</th>
<th>40</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>32</td>
<td>over 120</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>36</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>over 120</td>
</tr>
<tr>
<td>3</td>
<td>38</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>34</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>over 120</td>
</tr>
</tbody>
</table>

### Table 4-4. Demands for Part 1 and Part 2 ($D_{i,t}$)
<table>
<thead>
<tr>
<th>Period ( t ) (week)</th>
<th>Part 1 demand ( (D_{1,t}) )</th>
<th>Part 2 demand ( (D_{2,t}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>320</td>
<td>230</td>
</tr>
<tr>
<td>2</td>
<td>390</td>
<td>272</td>
</tr>
<tr>
<td>3</td>
<td>469</td>
<td>246</td>
</tr>
<tr>
<td>4</td>
<td>349</td>
<td>278</td>
</tr>
<tr>
<td>5</td>
<td>468</td>
<td>252</td>
</tr>
<tr>
<td>6</td>
<td>415</td>
<td>342</td>
</tr>
<tr>
<td>7</td>
<td>430</td>
<td>262</td>
</tr>
<tr>
<td>8</td>
<td>454</td>
<td>287</td>
</tr>
<tr>
<td>9</td>
<td>368</td>
<td>286</td>
</tr>
<tr>
<td>10</td>
<td>438</td>
<td>338</td>
</tr>
</tbody>
</table>

Table 4-5. Other Data Related to Suppliers and Parts

<table>
<thead>
<tr>
<th>Part (i)</th>
<th>Supplier (j)</th>
<th>Capacity ( (CAP_{i,j}) )</th>
<th>shipping cost ( (CT_{i,j}) )</th>
<th>Lead time ( (L_{i,j}) )</th>
<th>Defect rate ( (q_{i,j}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>300</td>
<td>5</td>
<td>4 weeks</td>
<td>7.00%</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>180</td>
<td>10</td>
<td>1 weeks</td>
<td>1.20%</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>200</td>
<td>6</td>
<td>3 weeks</td>
<td>5.50%</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>250</td>
<td>7</td>
<td>3 weeks</td>
<td>1.00%</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>200</td>
<td>6</td>
<td>2 weeks</td>
<td>2.30%</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>150</td>
<td>5</td>
<td>1 weeks</td>
<td>3.40%</td>
</tr>
</tbody>
</table>

Table 4-6. Inventory Factors and Shortage Factors

<table>
<thead>
<tr>
<th>Part (i)</th>
<th>inventory factor ( (\omega_i) )</th>
<th>shortage factor ( (\nu_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.35</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.65</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Assumed initial and final constraints:

Initial inventory for part 1: \( I_{1,0} = 500 \) units

Initial inventory for part 2: \( I_{2,0} = 500 \) units

Final inventory for part 1: \( I_{1,10} = 0 \) units

Final inventory for part 2: \( I_{2,10} = 0 \) units

Initial shortage for part 1: \( S_{1,0} = 0 \) units

Initial shortage for part 2: \( S_{2,0} = 0 \) units

Final shortage for part 1: \( S_{1,10} = 0 \) units
Final shortage for part 2: $S_{2,10} = 0$ units

### 4.1.2 Mathematical model

**Decision variables**

- $Q_{i,j,k,t}$: order quantity for part $i$ from supplier $j$ at price level $k$ in period $t$.
- $I_{i,t}$: inventory level of part $i$ at the end of period $t$.
- $S_{i,t}$: shortage level of part $i$ at the end of period $t$.
- $\delta_{j,t}$: binary variable, $\delta_{j,t} = 1$ if supplier $j$ is selected in period $t$, and $\delta_{j,t} = 0$ otherwise.
- $\nu_{i,j,k,t}$: binary variable, equals to 1 if $Q_{i,j,k,t}$ is in the quantity interval $k$, and 0 otherwise.

**Objective functions**

1. Minimize the total procurement cost over the planning horizon

   $$
   Min Z_1 = \sum_{t=1}^{10} (1000\delta_{1,t} + 1500\delta_{2,t} + 800\delta_{3,t} + 24Q_{1,1,1,t} + 20Q_{1,1,2,t} + 18Q_{1,1,3,t} + 26Q_{1,2,1,t} + 22Q_{1,2,2,t} + 18Q_{1,2,3,t} + 30Q_{1,3,1,t} + 22Q_{1,3,2,t} + 16Q_{1,3,3,t} + 40Q_{2,1,1,t} + 35Q_{2,1,2,t} + 32Q_{2,1,3,t} + 40Q_{2,2,1,t} + 36Q_{2,2,2,t} + 30Q_{2,2,3,t} + 38Q_{2,3,1,t} + 34Q_{2,3,2,t} + 28Q_{2,3,3,t} + 5(Q_{1,1,1,t} + Q_{1,1,2,t} + Q_{1,1,3,t}) + 8(Q_{1,2,1,t} + Q_{1,2,2,t} + Q_{1,2,3,t}) + 6(Q_{1,3,1,t} + Q_{1,3,2,t} + Q_{1,3,3,t}) + 7(Q_{2,1,1,t} + Q_{2,1,2,t} + Q_{2,1,3,t}) + 6(Q_{2,2,1,t} + Q_{2,2,2,t} + Q_{2,2,3,t}) + 5(Q_{2,3,1,t} + Q_{2,3,2,t} + Q_{2,3,3,t}) )
   $$

2. Minimize the weighted average inventory over the planning horizon
\[ Min Z_2 = 0.35 \sum_{t=1}^{10} I_{1,t} + 0.65 \sum_{t=1}^{10} I_{2,t} \]

3. Minimize the weighted average shortage over the planning horizon

\[ Min Z_3 = 0.50 \sum_{t=1}^{10} S_{1,t} + 0.50 \sum_{t=1}^{10} S_{2,t} \]

4. Minimize the weighted average lead time over the planning horizon

\[ Min Z_4 = \sum_{t=1}^{10} \left[ 4(Q_{1,1,1,t} + Q_{1,1,2,t} + Q_{1,1,3,t}) + 1(Q_{1,2,1,t} + Q_{1,2,2,t} + Q_{1,2,3,t}) \right. \\
+ 3(Q_{1,3,1,t} + Q_{1,3,2,t} + Q_{1,3,3,t}) + 3(Q_{2,1,1,t} + Q_{2,1,2,t} + Q_{2,1,3,t}) \\
+ 3(Q_{2,2,1,t} + Q_{2,2,2,t} + Q_{2,2,3,t}) + 1(Q_{2,3,1,t} + Q_{2,3,2,t} + Q_{2,3,3,t}) \]

5. Minimize the weighted average quality defect rate over the planning horizon

\[ Min Z_5 = \sum_{t=1}^{10} \left[ 0.970(Q_{1,1,1,t} + Q_{1,1,2,t} + Q_{1,1,3,t}) + 0.988(Q_{1,2,1,t} + Q_{1,2,2,t} + Q_{1,2,3,t}) \right. \\
+ 0.975(Q_{1,3,1,t} + Q_{1,3,2,t} + Q_{1,3,3,t}) + 0.990(Q_{2,1,1,t} + Q_{2,1,2,t} + Q_{2,1,3,t}) \\
+ 0.977(Q_{2,2,1,t} + Q_{2,2,2,t} + Q_{2,2,3,t}) + 0.966(Q_{2,3,1,t} + Q_{2,3,2,t} + Q_{2,3,3,t}) \]

**Constraints**

Constraints (3.6)-(3.12) discussed in Section 3.1.4 of Chapter 3 can be written as follows for this example:

**Demand constraints:**

For part 1:

For \(t=1\):

\[ S_{1,1} + 500 = 320 + I_{1,1} + 0 \]

(4.1)
For \( t=2 \): \( S_{1,2} + I_{1,1} + (Q_{1,2,1,1} + Q_{1,2,2,1} + Q_{1,2,3,1}) = 390 + I_{1,2} + S_{1,1} \)

(4.2)

For \( t=3 \): \( S_{1,3} + I_{1,2} + (Q_{1,2,1,2} + Q_{1,2,2,2} + Q_{1,2,3,2}) = 469 + I_{1,3} + S_{1,2} \)

(4.3)

For \( t=4 \): \( S_{1,4} + I_{1,3} + (Q_{1,2,1,3} + Q_{1,2,2,3} + Q_{1,2,3,3}) + (Q_{1,3,1,1} + Q_{1,3,2,1} + Q_{1,3,3,1}) = 349 + I_{1,4} + S_{1,3} \)

(4.4)

For \( t=5 \) to \( t=10 \): \( S_{1,t} + I_{1,t-1} + (Q_{1,1,1,t-4} + Q_{1,1,2,t-4} + Q_{1,1,3,t-4}) + (Q_{1,2,1,t-1} + Q_{1,2,2,t-1} + Q_{1,2,3,t-1}) + (Q_{1,3,1,t-3} + Q_{1,3,2,t-3} + Q_{1,3,3,t-3}) = D_{1,t} + I_{1,t} + S_{1,t-1} \)

(4.5)

where \( D_{1,t} \) = demand for part 1 in period \( t \) given in Table 4-4.

Note: \( I_{1,10} = 0, S_{1,10} = 0 \)

For part 2:

For \( t=1 \): \( S_{2,1} + 500 = 230 + I_{2,1} + 0 \)

(4.6)

For \( t=2 \): \( S_{2,2} + I_{2,1} + (Q_{2,2,1,1} + Q_{2,2,2,1} + Q_{2,2,3,1}) = I_{2,2} + 272 + S_{2,1} \)

(4.7)

For \( t=3 \): \( S_{2,3} + I_{2,2} + (Q_{2,2,1,1} + Q_{2,2,2,1} + Q_{2,2,3,1}) + (Q_{2,3,1,2} + Q_{2,3,2,2} + Q_{2,3,3,2}) = I_{2,3} + 246 + S_{2,2} \)

(4.8)

From \( t=4 \) to \( t=10 \): \( S_{2,t} + I_{2,t-1} + (Q_{2,1,1,t-3} + Q_{2,1,2,t-3} + Q_{2,1,3,t-3}) + (Q_{2,2,1,t-2} + Q_{2,2,2,t-2} + Q_{2,2,3,t-2}) + (Q_{2,3,1,t-1} + Q_{2,3,2,t-1} + Q_{2,3,3,t-1}) = I_{2,t} + D_{2,t} + S_{2,t-1} \)

(4.9)
where $D_{2,t}$ = demand for part 2 in period $t$ given in Table 4-4.

Note: $I_{2,10} = 0, S_{2,10} = 0$

Capacity constraints:

$$Q_{i,j,1,t} + Q_{i,j,2,t} + Q_{i,j,3,t} \leq CAP_{i,j} \times \delta_{j,t}$$  \hspace{1cm} (4.10)

for all $i=1,2; j=1,2,3; t=1,2,3,\ldots,10$

where $CAP_{i,j}$ = capacity of part $i$ for supplier $j$, given in Table 4-5

All-unit discount constraints:

For part 1:

$$(0 + 1)v_{1,j,1,t} \leq Q_{1,j,1,t} \leq 100v_{1,j,1,t}$$  \hspace{1cm} (4.11)

$$(100 + 1)v_{1,j,2,t} \leq Q_{1,j,2,t} \leq 150v_{1,j,2,t}$$

$$(150 + 1)v_{1,j,3,t} \leq Q_{1,j,3,t} \leq CAP_{1,j}v_{1,j,3,t}$$

$v_{1,j,1,t} + v_{1,j,2,t} + v_{1,j,3,t} \leq \delta_{j,t}$

where $CAP_{1,1} = 300, CAP_{1,2} = 180, CAP_{1,3} = 200$

For part 2:

$$(0 + 1)v_{2,j,1,t} \leq Q_{2,j,1,t} \leq 80v_{2,j,1,t}$$  \hspace{1cm} (4.12)

$$(80 + 1)v_{2,j,2,t} \leq Q_{2,j,2,t} \leq 120v_{2,j,2,t}$$

$$(120 + 1)v_{2,j,3,t} \leq Q_{2,j,3,t} \leq CAP_{2,j}v_{2,j,3,t}$$
\[ \nu_{2,j,1,t} + \nu_{2,j,2,t} + \nu_{2,j,3,t} \leq \delta_{j,t} \]

\[ \forall j, t \]

where \( \text{CAP}_{2,1} = 250, \text{CAP}_{2,2} = 200, \text{CAP}_{2,3} = 150 \)

**Maximum number of suppliers:**

\[ \delta_{1,t} + \delta_{2,t} + \delta_{3,t} \leq 2 \] \hspace{1cm} (4.13)

\[ \forall t \]

**Other constraints:**

\[ Q_{i,j,k,t} \geq 0 \forall i, j, k, t \] \hspace{1cm} (4.14)

\[ I_{i,t} \geq 0 \text{ and } S_{i,t} \geq 0 \forall i, t \]

\[ \delta_{j,t} \in \{0,1\}, \nu_{i,j,k,t} \in \{0,1\} \]

### 4.2 Solution Procedure

#### 4.2.1 Identifying Ideal Values

Ideal values of total procurement cost, weighted inventory, weighted shortage, weighted lead time and weighted quality defect rate are obtained by minimizing each objective separately, ignoring the other objectives. LINGO 11.0 is used to generate the mixed integer linear program and solve it. There are 435 variables (210 integer variables) and 516 constraints.

- Minimize Objective 1 (Total procurement cost) while ignoring other objectives:
The procurement cost over ten periods=186511.00
The weighted average inventory=368.50
The weighted average shortage=2707.50
The weighted average lead time=15424.00
The weighted average quality defect rate=265.33

The ideal solution for objective 1=186511.00 (Ideal 1)

- Minimize Objective 2 (Weighted average inventory) while ignoring other objectives:
  The total procurement cost over ten periods=226461.00
  The weighted average inventory=238.50
  The weighted average shortage=2634.50
  The weighted average lead time=18576.00
  The weighted average quality defect rate=271.92

The ideal solution for objective 2=238.50 (Ideal 2)

- Minimize Objective 3 (Weighted average shortage) while ignoring other objectives:
  The total procurement cost over ten periods=225532.00
  The weighted average inventory=332.75
  The weighted average shortage=1346.50
  The weighted average lead time=15563.00
  The weighted average quality defect rate=209.00

The ideal solution for objective 3=1346.50 (Ideal 3)

- Minimize Objective 4 (Weighted average lead time) while ignoring other objectives:
  The total procurement cost over ten periods=203588.00
  The weighted average inventory=238.50
  The weighted average shortage=1810.50
  The weighted average lead time=13723.00
The weighted average quality defect rate=209.64

**The ideal solution for objective 4=13723.00 (Ideal 4)**

- **Minimize Objective 5 (Weighted average quality defect rate) while ignoring other objectives:**
  
  The total procurement cost over ten periods=211663.00
  The weighted average inventory=1156.95
  The weighted average shortage=1346.50
  The weighted average lead time=15430.00
  The weighted average quality defect rate=188.35

**The ideal solution for objective 5=188.35 (Ideal 5)**

### 4.2.2 Solutions by the Weighted Objective Method

Weighted objective method is used first to solve the multi-objective problem. However, allocating weights for the objectives is critical, and an appropriate weight allocation should be consistent with the Decision Maker’s preference. AHP method is one of the most popular method in estimating Decision Maker’s preference. In this example, 10 different sets of weight will be investigated. Note that the alternative 1, 2, 3, 4, 5 represent the solutions, where most of the weight is given respectively to the procurement cost, weighted inventory, weighted shortage, weighted lead time, and weighted quality defect rate. Alternative 6 is the equally weighted solution. Alternative 7 simulates that company stressed on the customer service, where more weight is put on the objectives of shortage and quality defect. For example, the company wants to decrease the shortage and increases the quality level so that the customer loyalty can be built quickly at the initial stage of the business. Alternative 8 emphasizes the operation efficiency, where more weight is put on the objectives of procurement cost and weighted inventory. Alternative 9 represents that company almost ignoring the objectives of shortage and quality defect rate, pursuing the short-
term profit at the cost of customer loyalty. In alternative 10, the company put less weight on the objectives of the total procurement cost and the inventory and is dedicated to cultivate the customer relationships.

Table 4-7. Values of Weights used in the Weighted Objective Method

<table>
<thead>
<tr>
<th>Alternative Number</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$w_4$</th>
<th>$w_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>3.5</td>
<td>1</td>
<td>3.5</td>
</tr>
<tr>
<td>8</td>
<td>3.5</td>
<td>3.5</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>3</td>
<td>0.5</td>
<td>3</td>
<td>0.5</td>
</tr>
<tr>
<td>10</td>
<td>0.5</td>
<td>0.5</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

As noted in Chapter 3, the five objectives have different magnitudes, and scaling is necessary. In this case, simple scaling is utilized to scale the objectives and the scaled objectives are $\frac{z_i}{c_i}$ with $c_1 = 200000$, $c_2 = 200$, $c_3 = 1000$, $c_4 = 10000$, and $c_5 = 200$. The scaled weighted objective function is given below.

$$
\text{Min} Z = w_1 \frac{Z_1}{200000} + w_2 \frac{Z_2}{200} + w_3 \frac{Z_3}{1000} + w_4 \frac{Z_4}{10000} + w_5 \frac{Z_5}{200}
$$

Subject to the constraints (4.1) to (4.14).

Table 4-8 shows eight efficient points and their objective values for different sets of weights. Table 4-9 represents the objective values as the ratios of their ideal values in percentage.
Table 4-8. Objective Values with the Weighted Objective Method

<table>
<thead>
<tr>
<th>Alternative Number</th>
<th>(w_1, w_2, w_3, w_4, w_5)</th>
<th>Objective 1</th>
<th>Objective 2</th>
<th>Objective 3</th>
<th>Objective 4</th>
<th>Objective 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(6, 1, 1, 1, 1)</td>
<td>202843.00</td>
<td>272.30</td>
<td>1377.50</td>
<td>13800.00</td>
<td>207.83</td>
</tr>
<tr>
<td>2</td>
<td>(1, 6, 1, 1, 1)</td>
<td>203941.00</td>
<td>238.50</td>
<td>1346.50</td>
<td>13862.00</td>
<td>207.83</td>
</tr>
<tr>
<td>3</td>
<td>(1, 1, 6, 1, 1)</td>
<td>203941.00</td>
<td>238.50</td>
<td>1346.50</td>
<td>13862.00</td>
<td>207.83</td>
</tr>
<tr>
<td>4</td>
<td>(1, 1, 1, 6, 1)</td>
<td>203143.00</td>
<td>257.35</td>
<td>1498.50</td>
<td>13723.00</td>
<td>209.64</td>
</tr>
<tr>
<td>5</td>
<td>(1, 1, 1, 1, 6)</td>
<td>206789.00</td>
<td>367.20</td>
<td>1346.50</td>
<td>15430.00</td>
<td>188.35</td>
</tr>
<tr>
<td>6</td>
<td>(2, 2, 2, 2, 2)</td>
<td>203941.00</td>
<td>238.50</td>
<td>1346.50</td>
<td>13862.00</td>
<td>207.83</td>
</tr>
<tr>
<td>7</td>
<td>(1, 1, 3.5, 1, 3.5)</td>
<td>206789.00</td>
<td>367.20</td>
<td>1346.50</td>
<td>15430.00</td>
<td>188.35</td>
</tr>
<tr>
<td>8</td>
<td>(3.5, 3.5, 1, 1, 1)</td>
<td>203259.00</td>
<td>238.50</td>
<td>1377.50</td>
<td>13800.00</td>
<td>208.64</td>
</tr>
<tr>
<td>9</td>
<td>(3, 3, 0.5, 3, 0.5)</td>
<td>202612.00</td>
<td>238.50</td>
<td>1528.50</td>
<td>13723.00</td>
<td>209.64</td>
</tr>
<tr>
<td>10</td>
<td>(0.5, 0.5, 3, 3, 3)</td>
<td>203941.00</td>
<td>238.50</td>
<td>1346.50</td>
<td>13862.00</td>
<td>207.83</td>
</tr>
</tbody>
</table>

Table 4-9. Objective Values as Ratios of Ideal Values with the Weighted Objective Method

<table>
<thead>
<tr>
<th>Alternative Number</th>
<th>(w_1, w_2, w_3, w_4, w_5)</th>
<th>Objective 1 Procurement cost</th>
<th>Objective 2 Weighted Inventory</th>
<th>Objective 3 Weighted shortage</th>
<th>Objective 4 Weighted lead time</th>
<th>Objective 5 Weighted defect rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(6, 1, 1, 1, 1)</td>
<td>108.76%</td>
<td>114.17%</td>
<td>102.30%</td>
<td>100.56%</td>
<td>110.77%</td>
</tr>
<tr>
<td>2</td>
<td>(1, 6, 1, 1, 1)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>3</td>
<td>(1, 1, 6, 1, 1)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>4</td>
<td>(1, 1, 1, 6, 1)</td>
<td>108.92%</td>
<td>107.90%</td>
<td>111.29%</td>
<td>100.00%</td>
<td>111.30%</td>
</tr>
<tr>
<td>5</td>
<td>(1, 1, 1, 1, 6)</td>
<td>110.87%</td>
<td>153.96%</td>
<td>100.00%</td>
<td>112.44%</td>
<td>100.00%</td>
</tr>
<tr>
<td>6</td>
<td>(2, 2, 2, 2, 2)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>7</td>
<td>(1, 1, 3.5, 1, 3.5)</td>
<td>110.87%</td>
<td>153.96%</td>
<td>100.00%</td>
<td>112.44%</td>
<td>100.00%</td>
</tr>
<tr>
<td>8</td>
<td>(3.5, 3.5, 1, 1, 1)</td>
<td>108.98%</td>
<td>100.00%</td>
<td>102.30%</td>
<td>100.56%</td>
<td>110.77%</td>
</tr>
<tr>
<td>9</td>
<td>(3, 3, 0.5, 3, 0.5)</td>
<td>108.63%</td>
<td>100.00%</td>
<td>113.52%</td>
<td>100.00%</td>
<td>111.30%</td>
</tr>
<tr>
<td>10</td>
<td>(0.5, 0.5, 3, 3, 3)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
</tbody>
</table>

In Alternative 1, although most weight is assigned to Objective 1, Objective 1 does not achieve its ideal value. In Alternatives 2, 3, 4, and 5, the objectives, which are assigned with the most weight, do achieve their ideal values. Compared with Alternative 1, procurement cost and lead time in Alternative 2 and 3 increase slightly, but the average inventory and shortage decrease. Alternative 2 and 3 have the same result, so it is possible that the average inventory and the average shortage can be minimized at the same time at the expense of higher procurement cost and longer lead time. In Alternative 4, shorter average lead time is stressed, and the ideal value of the
objective 4 is achieved. In Alternative 5, objective 5 achieves its ideal value, but the average inventory increases significantly. Alternative 6 achieves the same solution as Alternatives 2 and 3. In Alternative 7, objectives 3 and 5 achieve their ideal values at the same time with the most weight assigned. In Alternative 8, procurement cost does not achieve its ideal value although much weight is allocated to it. In Alternative 9, the weighted shortage and the weighted quality defect rate increase a lot because of less weight, and the objective of inventory and lead time achieve their ideal values. In Alternative 10, the shortage objective reaches its ideal value, and the lead time objective almost obtains the ideal value as well, but the weighted quality defect rate does not decrease.

For Objective 2 and Objective 3, the correlation coefficient between these two columns is -0.26, which indicates a negative correlation between average inventory and average shortage, thus it is reasonable that keeping more inventory can help the company to control the shortage. And the correlation coefficient between the procurement cost and the quality defect rate is -0.9764, where the strong negative relationship is consistent with the idea that high quality product tends to charge a higher price.

4.2.3 Solution by Preemptive Goal Programming

Under Preemptive goal programming discussed in Chapter 3, the formulation of the objective function is as follows:

$$\text{Min } Z = P_1 d_1^+ + P_2 d_2^+ + P_3 d_3^+ + P_4 d_4^+ + P_5 d_5^+$$

$P_1, P_2, P_3, P_4,$ and $P_5$ are the priorities assigned to goals 1, 2, 3, 4, 5 respectively.

Here, we set the ideal values as goal/targets and consider 5 different scenarios for the priority structure:
\[ P_1 \gg P_2 \gg P_3 \gg P_4 \gg P_5 \]
\[ P_2 \gg P_3 \gg P_1 \gg P_4 \gg P_5 \]
\[ P_5 \gg P_2 \gg P_3 \gg P_4 \gg P_1 \]
\[ P_3 \gg P_5 \gg P_4 \gg P_2 \gg P_1 \]
\[ P_4 \gg P_2 \gg P_5 \gg P_1 \gg P_3 \]

- **Goal constraints**

(7) **Total procurement cost goal**

Based on the objective function of total cost given in section 4.12, the goal constraint to satisfy the cost target is given by:

\[
\sum_{t=1}^{10} \sum_{j=1}^{3} CF_j \delta_{j,t} + \sum_{t=1}^{10} \sum_{j=1}^{3} \sum_{k=1}^{3} \bar{p}_{i,j,k} Q_{i,j,k,t} + \sum_{t=1}^{10} \sum_{j=1}^{3} \sum_{k=1}^{3} CT_{i,j} Q_{i,j,k,t} + d^-_1 - d^+_1 = 186511.00
\]  

(4.15)

where \(d^-_1\) & \(d^+_1\) are the negative and positive deviations from the cost goal.

(8) **Weighted average inventory goal**

\[
\sum_{t=1}^{10} \sum_{i=1}^{2} l_{i,t} w_{i,t} + d^-_2 - d^+_2 = 238.50
\]  

(4.16)

(9) **Weighted average inventory goal**

\[
\sum_{t=1}^{10} \sum_{i=1}^{2} s_{i,t} v_{i,t} + d^-_3 - d^+_3 = 1346.50
\]  

(4.17)

(10) **Weighted average lead time goal**
\[ \sum_{t=1}^{10} \sum_{i=1}^{2} \sum_{j=1}^{3} \sum_{k=1}^{3} L_{i,j}Q_{i,j,k,t} + d_4^- - d_4^+ = 13723.00 \]  

(4.18)

(11) **Quality defect rate goal**

\[ \sum_{t=1}^{3} \sum_{j=1}^{2} \sum_{i=1}^{10} \sum_{k=1}^{10} q_{i,j}Q_{i,j,k,t} + d_5^- - d_5^+ = 188.35 \]  

(4.19)

(12) **Non-negative constraints**

\[ d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+, d_5^-, d_5^+ \geq 0 \]

(13) **Other constraints**

As given by constraints (4.1) to (4.14).

**Preemptive goal Programming solutions:**

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Scenario</th>
<th>Procurement cost</th>
<th>Weighted inventory</th>
<th>Weighted shortage</th>
<th>Weighted lead time</th>
<th>Weighted defect rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>( P_1 \gg P_2 \gg P_3 \gg P_4 \gg P_5 )</td>
<td>100.00%</td>
<td>138.70%</td>
<td>177.31%</td>
<td>112.82%</td>
<td>140.50%</td>
</tr>
<tr>
<td>12</td>
<td>( P_2 \gg P_3 \gg P_1 \gg P_4 \gg P_5 )</td>
<td>109.06%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>101.95%</td>
<td>109.46%</td>
</tr>
<tr>
<td>13</td>
<td>( P_5 \gg P_2 \gg P_3 \gg P_4 \gg P_1 )</td>
<td>111.50%</td>
<td>137.61%</td>
<td>100.00%</td>
<td>112.44%</td>
<td>100.00%</td>
</tr>
<tr>
<td>14</td>
<td>( P_3 \gg P_5 \gg P_4 \gg P_2 \gg P_1 )</td>
<td>111.90%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>111.43%</td>
<td>100.81%</td>
</tr>
<tr>
<td>15</td>
<td>( P_4 \gg P_2 \gg P_5 \gg P_1 \gg P_3 )</td>
<td>109.13%</td>
<td>100.00%</td>
<td>111.29%</td>
<td>100.00%</td>
<td>111.30%</td>
</tr>
</tbody>
</table>

The objective values of the preemptive goal programming solutions, expressed as the ratios of their ideal values in the five priority scenarios are given in Table 4-10. In Alternative 11, with the priority \( P_1 \gg P_2 \gg P_3 \gg P_4 \gg P_5 \), only goal 1 is achieved, and the other goals are not satisfied. Alternative 11 stresses procurement cost and weighted inventory as very important, so it could be a seller’s market or it is probable that the parts sold are low-end products. In Alternative
12, the goal 2 and goal 3 are achieved, and the values of other objectives are slightly larger than the targets. Alternative 12 represents a scenario that emphasizes lower inventory and shortage. Some automobile manufacturers, for example Toyota, hold the goals of achieving zero inventory and less shortage. In Alternative 13, only goals 3 and 5 are achieved. Although $P_2 \gg P_3$, objective 2 is far from the target. Alternative 13 illustrates the company’s preference for high quality; thus what it sells could be high-end products. For example, jewelers prioritize the quality of jewelry, and the inventory of jewelry should be kept limited due to the high value of jewelry. In Alternative 14, the inventory and shortage goals achieve their ideal values, and quality defect rate almost reaches its ideal value. Alternative 14 could occur in a market where customers are very sensitive to the shortage and quality defect. For example, in a fully competitive market, substitute goods are available and easy to be bought by customers, and any shortage and quality problem will incur customer goodwill loss to the company and drive customers to competitors. In Alternative 15, the goals for average inventory and lead time are achieved, and other goals are also kept at a reasonable level. Alternative 15 indicates a company’s preference for lead time and inventory, for example, a company selling fresh or frozen goods. Because fresh goods have the time-critical shelf lives and high transportation cost, the company would tend to shorten the lead time and avoid large inventory.

By comparing these five priority scenarios, it is observed that a slight decrease in the total procurement cost could lead to a significant increase in all the other objectives. Hence, in this case, pursing solely the minimization of the procurement cost is not appropriate for the company. Similarly, if the company can improve the tolerance of the quality defect slightly, all the other objectives decrease significantly.
4.2.4 Solution by Non-Preemptive Goal Programming

Based on the Preemptive goal programming model discussed in Chapter 3, the formulation is shown below:

\[
Min Z = w_1 d_1^+ + w_2 d_2^+ + w_3 d_3^+ + w_4 d_4^+ + w_5 d_5^+
\]

where \(w_1, w_2, w_3, w_4, w_5\) are the relative weights assigned to goal achievements. Scaling of the objectives is critical here and we use the ideal values to scale the objectives.

- Goal constraints:

(7) Procurement cost goal

\[
\frac{Z_1}{186511} + d_1^- - d_1^+ = 1
\]

(8) Inventory goal

\[
\frac{Z_2}{238.5000} + d_2^- - d_2^+ = 1
\]

(9) Shortage goal

\[
\frac{Z_3}{1346.500} + d_3^- - d_3^+ = 1
\]

(10) Lead time goal

\[
\frac{Z_4}{13723} + d_4^- - d_4^+ = 1
\]

(11) Quality goal

\[
\frac{Z_5}{188.3490} + d_5^- - d_5^+ = 1
\]

(12) Non-negativity constraints

\(d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+, d_5^-, d_5^+ \geq 0\)

(13) Other constraints

As given by Constraints (4.1) to (4.14).
Non-preemptive goal Programming solutions:

Table 4-11. Objective Values with Non-preemptive Goal Programming

<table>
<thead>
<tr>
<th>Alternative</th>
<th>((w_1, w_2, w_3, w_4, w_5))</th>
<th>Objective 1 Procurement cost</th>
<th>Objective 2 Weighted Inventory</th>
<th>Objective 3 Weighted shortage</th>
<th>Objective 4 Weighted lead time</th>
<th>Objective 5 Weighted defect rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>(4, 3, 2, 0.9, 0.1)</td>
<td>108.98%</td>
<td>100.00%</td>
<td>102.30%</td>
<td>100.56%</td>
<td>110.77%</td>
</tr>
<tr>
<td>17</td>
<td>(2, 4, 3, 0.9, 0.1)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>18</td>
<td>(0.1, 3, 2, 0.9, 4)</td>
<td>111.50%</td>
<td>137.61%</td>
<td>100.00%</td>
<td>112.44%</td>
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<td>19</td>
<td>(0.1, 4, 1, 0.9, 4)</td>
<td>111.50%</td>
<td>137.61%</td>
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<td>20</td>
<td>(0.1, 6, 1, 0.9, 2)</td>
<td>111.89%</td>
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<td>111.39%</td>
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<td>(0.1, 0.9, 4, 2, 3)</td>
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<td>22</td>
<td>(0.9, 3, 0.1, 4, 2)</td>
<td>108.63%</td>
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<td>23</td>
<td>(0.1, 3, 0.9, 4, 2)</td>
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In Alternative 16, the weights assigned to goals 1, 2, 3, 4 and 5 decrease progressively, simulating Alternative 11 in Preemptive goal programming. The 5 objectives hold the same ranking except that the relative weights are specified in Alternative 16. Although the procurement cost does not achieve its ideal value, as in Alternative 11, all other objectives in Alternative 16 achieve better values than those in Alternative 11. This implies that trade-offs could be made among the objectives. Alternative 17 allocates more weight to goals 2, 3 and 1, simulating Alternative 12 and achieving a similar solution, where weighted inventory and weighted shortage are minimized. Alternative 18 simulates the ranking in Alternative 13 with specific weights, and obtains the same solution, in which weighted shortage and weighted defect rate are minimized but weighted inventory is very large. If the company is not satisfied with the large value of weighted inventory achieved in Alternative 18, then it can transfer some weight from weighted shortage to weighted inventory as shown in Alternative 19. However, the solution in Alternative 19 still has a large weighted inventory. In this situation, the company may be willing to try Alternative 20 by increasing the weight on inventory by sacrificing quality; then a much better solution is attained in Alternative 20, where the weighted inventory and weighted shortage achieve their ideal values, while the quality defect rate increases slightly. Alternative 21 simulates the ranking in Alternative...
14, and its solution has a shorter weighted lead time but a higher weighted defect rate than those in Alternative 14. Both Alternatives 22 and 23 are used to simulate Alternative 15 by varying the weights assigned to objectives 1 and 3. Comparing Alternatives 22 and 23, it is observed that the change on the weight allocated to the objectives 1 and 3 does not affect the other objectives. But the procurement cost is lower with more weight assigned in Alternative 22, and the weighted shortage cost is lower with more weight assigned in Alternative 23.

### 4.3 Analysis of the Efficient Discount Policy and the Choice of Suppliers

**Weighted Objective Method**

In Tables 4-12 to 4-21, the choice of suppliers is presented for each of the alternatives under weighted objective method. Number “1” indicates that the first price level is chosen for the specific part, specific supplier and specific period; number “2” indicates that the second price level is chosen given for specific part, specific supplier and specific period; number “3” indicates that the third price level is chosen for the specific part, specific supplier and specific period.

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Table 4-13. Solution of Weighted Objective Method for Alternative 2 with Weight (1, 6, 1, 1, 1)

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Table 4-14. Solution of Weighted Objective Method for Alternative 3 with Weight (1, 1, 6, 1, 1)

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In Table 4-12, most weight is allocated to procurement cost, and it is not surprising that all the orders are placed at the third price level, so that the discount policy is utilized fully. In Table 4-13, the company orders part 2 at the first price level in 2nd and 4th weeks. Although the first price level will lead to more procurement cost, small order quantity makes the company avoid holding too much inventory. The solution in Table 4-14 is same as in Table 4-13, which indicates that the average inventory and average shortage can be lowered at the same time.

Table 4-15. Solution of Weighted Objective Method for Alternative 4 with Weight (1, 1, 6, 1, 1)

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Table 4-16. Solution of Weighted Objective Method for Alternative 5 with Weight (1, 1, 1, 1, 6)

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In Table 4-15, the order allocation plan for the part 1 does not change, but the company places an order of part 2 to supplier 1 at the third price level in 1st week, at the second price level in 2nd week, and at the first price level in 3rd week. In Table 4-16, with more weight for the quality defect rate, the orders of part 2 to supplier 1 increases significantly, since part 2, produced by supplier 1, has the lowest defect rate of 1%.

Table 4-17. Solution of Weighted Objective Method for Alternative 6 with Weight (1, 1, 1, 1, 1)

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<th>Part (i)</th>
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In Table 4-17, the weight is allocated evenly to each objective and, the same solution is obtained as in Tables 4-13 and 4-14, and the average inventory and average shortage reach their minimums. In Table 4-18, the weighted quality defect rate is minimized, so there is no order for part 2 placed to supplier 3, whose quality is the worst for part 2. In Table 4-19, procurement cost and weighted inventory are emphasized and it has a similar order allocation as in Table 4-17, where the weights are assigned evenly.
Table 4-18. Solution of Weighted Objective Method for Alternative 7 with Weight (1, 1, 3.5, 1, 3.5)

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Table 4-19. Solution of Weighted Objective Method for Alternative 8 with Weight (3.5, 3.5, 1, 1, 1)

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Table 4-20. Solution of Weighted Objective Method for Alternative 9 with Weight (3, 3, 0.5, 3, 0.5)

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<th>Part (i)</th>
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Table 4-21. Solution of Weighted Objective Method for Alternative 10 with Weight 

\((0.5, 0.5, 3, 3, 3)\)

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In Table 4-20, the weighted inventory and lead time are minimized. Although the discount policy is fully utilized, the procurement cost only achieves 108.63% of its ideal value. In Table 4-21, the weighted shortage, lead time, and quality defect rate are emphasized, and it obtains the same solution as in Table 4-17.

**Goal Programming Solutions**

Table 4-22 to 4-34 presents the choice of suppliers and their price levels for preemptive and non-preemptive goal programming models.

Table 4-22. Solution of Preemptive Goal Programming for Alternative 11 with Priority 

\(P_1 \gg P_2 \gg P_3 \gg P_4 \gg P_5\)

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Table 4-23. Solution of Preemptive Goal Programming for Alternative 12 with Priority $P_2 \gg P_3 \gg P_1 \gg P_4 \gg P_5$

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<th>Part (i)</th>
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In Table 4-22, the target of low procurement cost has the first priority, making the company to fully utilize the discount policy. Although the target of low inventory has the second priority, the weighted inventory in this solution is 138.70%, which probably results from large-quantity orders. According to Table 4-23, objectives 2 and 3, the weighted inventory and the weighted shortage, have achieved their ideal values; so this solution presents an efficient solution, where inventory and shortage are minimized, with all the orders placed at the third price level.

Table 4-24. Solution of Preemptive Goal Programming for Alternative 13 with Priority $P_5 \gg P_2 \gg P_3 \gg P_4 \gg P_1$

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Table 4-25. Solution of Preemptive Goal Programming for Alternative 14 with Priority $P_3 \gg P_5 \gg P_4 \gg P_2 \gg P_1$

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Table 4-26. Solution of Preemptive Goal Programming for Alternative 15 with Priority $P_4 \gg P_2 \gg P_5 \gg P_1 \gg P_3$

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Compared with the other solutions, the solution in Table 4-24 does not include orders to supplier 3, who provides the parts with the worst quality. The quality defect rates for supplier 3 are 5.50% and 3.40% for part 1 and part 2 respectively. With the first priority for shortage and the second priority for quality, Table 4-25 has a similar solution as in Table 4-24. Because the second-priority objective will not be considered until the first-priority objective is minimized, one more order is placed to supplier 3 in Table 4-25. Thus, the average quality defect rate increases slightly compared to Table 4-24. In Table 4-26, the lead time has the first priority. Hence, fewer orders are placed to supplier 1, who has the lead time of 4 weeks, and several small-quantity orders of part 2 are placed to supplier 2.
Table 4-27. Solution of Non-preemptive Goal Programming for Alternative 16 with Weight 
\((4, 3, 2, 0.9, 0.1)\)

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Table 4-28. Solution of Non-preemptive Goal Programming for Alternative 17 with Weight 
\((2, 4, 3, 0.9, 0.1)\)

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In Table 4-27, procurement cost is given the most weight; still, it does not achieve its ideal value, and the discount policy is not used efficiently. In Table 4-28, the solution is the same as in Tables 4-13 and 4-14. The average inventory and average shortage are minimized to their ideal values.

Table 4-29. Solution of Non-preemptive Goal Programming for Alternative 18 with Weight 
\((0.1, 3, 2, 0.9, 4)\)

<table>
<thead>
<tr>
<th>Part (i)</th>
<th>Supplier (j)</th>
<th>t=1</th>
<th>t=2</th>
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<th>t=4</th>
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</tbody>
</table>
Table 4-30. Solution of Non-preemptive Goal Programming for Alternative 19 with Weight (0.1, 4, 1, 0.9, 4)

<table>
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</tr>
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</table>

Table 4-31. Solution of Non-preemptive Goal Programming for Alternative 20 with Weight (0.1, 6, 1, 0.9, 2)

<table>
<thead>
<tr>
<th>Part (i)</th>
<th>Supplier (j)</th>
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<th>t=2</th>
<th>t=3</th>
<th>t=4</th>
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<th>t=6</th>
<th>t=7</th>
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</tr>
</tbody>
</table>

Table 4-29 has the same solution as in Table 4-24, with the quality defect rate and shortage achieving their ideal values. Compared with Table 4-29, the average of inventory is assigned more weight but the solution does not change in Table 4-30. In Table 4-31, much more weight is assigned to the average inventory, and some orders of part 2 are transferred from supplier 2 to supplier 3, which leads to a slight increase in the average quality defect rate.

Table 4-32. Solution of Non-preemptive Goal Programming for Alternative 21 with Weight (0.1, 0.9, 4, 2, 3)

<table>
<thead>
<tr>
<th>Part (i)</th>
<th>Supplier (j)</th>
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<th>t=3</th>
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Table 4-33. Solution of Non-preemptive Goal Programming for Alternative 22 with Weight (0.9, 3, 0.1, 4, 2)

<table>
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</tr>
</tbody>
</table>

Table 4-34. Solution of Non-preemptive Goal Programming for Alternative 23 with Weight (0.1, 3, 0.9, 4, 2)

<table>
<thead>
<tr>
<th>Part (i)</th>
<th>Supplier (j)</th>
<th>t=1</th>
<th>t=2</th>
<th>t=3</th>
<th>t=4</th>
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<th>t=7</th>
<th>t=8</th>
<th>t=9</th>
<th>t=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tbody>
</table>

Table 4-32 has the same solution as Table 4-28, and targets for average inventory and average shortage are achieved. In Table 4-33, the solution is the same as in Table 4-20 and targets for average inventory and average lead time are achieved. Compared with Table 4-33, the weights assigned to the objective 1 and 2 are switched in Table 4-34. Therefore, procurement cost in Table 4-34 is slightly larger than the one in Table 4-33, and the average shortage is slightly lower than the one in Table 4-34, and there are first-level price orders of part 2 placed to supplier 2 in 2nd and 3rd weeks.

Based on the results in the preceding tables, it is observed that most orders are placed at the third price level, which leads to the lower unit price and the price discount policy provided by suppliers are efficiently utilized. Also, most orders in the third price level results in economics of
scale in production. On the one hand, the large scale of production can provide a lower purchasing cost to the company. On the other hand, the large scale production can make the suppliers or manufacturers lower the total setup cost, contributing to a lower production cost. From an overall perspective, the utilization of the discount policy can improve the efficiency of the whole supply chain system.

In all the efficient solutions, no order is placed in the last period, but orders are always placed in the first period. Since no order is permitted to reach after the last week, the time of placing the last order is consistent with the lead time of parts. The lead time of supplier 1 for delivering part 1 is 4 weeks, so it is apparent that no order for part 1 is placed to supplier 1 after week 6. Similarly, the lead time for part 2 by supplier 2 is 2 weeks; hence the last order for part 2 to supplier 2 should be no later than week 8. Furthermore, because the initial inventory is limited, there must be orders placed in the first week to minimize the potential shortage.

4.4 Value Path Approach

In order to help the Decision Maker (DM) choose a solution from the set of solutions, the value path approach is used to display the performance of the different solutions visually.

Value Path Approach is an efficient way to visualize and compare among different solutions with conflicting criteria. It displays the performance of each solution for the different criteria, on parallel scales, one for each criterion. The Value Path Method is an effective way to demonstrate the trade-offs among the conflicting criteria. Some properties of this approach are given by Shilling et al (1983):

- If the value paths corresponding to two alternatives A and B intersect, then neither alternative is dominated by the other.
If more than two value paths intersect, then their associated points in the objective space are collinear.

If two points do not intersect, then one path must lie entirely above the other and is therefore inferior, if the objective is to minimize.

It is possible that using different methods and different sets of weights could lead to the same solution; for example, Alternatives 13, 18 and 19 present the same solution as in Table 4-24. Therefore, there are totally 12 different efficient solutions out of the 23 solutions as shown in Table 4-35.

<table>
<thead>
<tr>
<th>Number</th>
<th>Objective 1 Procurement cost</th>
<th>Objective 2 Weighted Inventory</th>
<th>Objective 3 Weighted shortage</th>
<th>Objective 4 Weighted lead time</th>
<th>Objective 5 Weighted defect rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>108.76%</td>
<td>114.17%</td>
<td>102.30%</td>
<td>100.56%</td>
<td>110.77%</td>
</tr>
<tr>
<td>2</td>
<td>111.90%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>111.43%</td>
<td>100.81%</td>
</tr>
<tr>
<td>3</td>
<td>109.35%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>4</td>
<td>108.92%</td>
<td>107.90%</td>
<td>111.29%</td>
<td>100.00%</td>
<td>111.30%</td>
</tr>
<tr>
<td>5</td>
<td>110.87%</td>
<td>153.96%</td>
<td>100.00%</td>
<td>112.44%</td>
<td>100.00%</td>
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<tr>
<td>6</td>
<td>108.98%</td>
<td>100.00%</td>
<td>102.30%</td>
<td>100.56%</td>
<td>110.77%</td>
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<tr>
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<td>108.63%</td>
<td>100.00%</td>
<td>113.52%</td>
<td>100.00%</td>
<td>111.30%</td>
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<tr>
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<td>100.00%</td>
<td>138.70%</td>
<td>177.31%</td>
<td>112.82%</td>
<td>140.50%</td>
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<tr>
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<td>109.06%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>101.95%</td>
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<td>111.50%</td>
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<td>100.00%</td>
<td>100.00%</td>
<td>111.39%</td>
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</tr>
</tbody>
</table>

Based on the scaled objectives for the 12 solutions, we plot the data with Value Path Approach to visualize and compare solutions, as shown in Figure 4-1. The 12 solutions are all non-dominated since their lines intersect with one another.
Note that there are many other efficient solutions which are not generated here. The final decision will be made by the manager who will select the best alternative from the set of available efficient alternatives.
4.5 Sensitivity Analysis

In this thesis, the second and third objectives are the weighted average inventory and the weighted average shortage. As stated before, the weight assigned to each part is decided by the preference of decision makers, based on how critical the part is. The available methods of allocating weights include Rating method, Borda count and AHP. It is expected that different sets of weights assigned to each part would affect the optimal solution. Similarly, the weights used in the calculation of the weighted average shortage would have an impact on the optimal solution.

To analyze the impact of the inventory factors, Alternative 6 with equal weights over each objective is chosen for the sensitivity analysis. We begin with the sensitivity analysis of the inventory factors for parts 1 and 2. Table 4-36 presents the ratios of the objective values with respect to the ideal values obtained under the weight of (0.35, 0.65) for the inventory factors for parts 1 and 2.
Table 4-36. Variation of Objectives with Changes in the Inventory Factors

<table>
<thead>
<tr>
<th>((\omega_1, \omega_2))</th>
<th>Objective 1 Procurement cost</th>
<th>Objective 2 Weighted Inventory</th>
<th>Objective 3 Weighted shortage</th>
<th>Objective 4 Weighted lead time</th>
<th>Objective 5 Weighted defect rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.001, 0.999)</td>
<td>109.35%</td>
<td>113.17%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.010, 0.990)</td>
<td>109.35%</td>
<td>112.83%</td>
<td>100.00%</td>
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<td>110.34%</td>
</tr>
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<td>(0.100, 0.900)</td>
<td>109.35%</td>
<td>109.43%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
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<td>107.55%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.200, 0.800)</td>
<td>109.35%</td>
<td>105.66%</td>
<td>100.00%</td>
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<td>110.34%</td>
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<td>109.35%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.400, 0.600)</td>
<td>109.35%</td>
<td>98.11%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.450, 0.550)</td>
<td>109.35%</td>
<td>96.23%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.500, 0.500)</td>
<td>109.35%</td>
<td>94.34%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.550, 0.450)</td>
<td>109.35%</td>
<td>92.45%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.570, 0.430)</td>
<td>109.35%</td>
<td>91.70%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.580, 0.420)</td>
<td>109.35%</td>
<td>91.32%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.590, 0.410)</td>
<td>109.35%</td>
<td>90.94%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.600, 0.400)</td>
<td>109.35%</td>
<td>90.69%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.650, 0.350)</td>
<td>109.35%</td>
<td>105.41%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.700, 0.300)</td>
<td>109.35%</td>
<td>101.13%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.750, 0.250)</td>
<td>109.35%</td>
<td>96.86%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.800, 0.200)</td>
<td>109.35%</td>
<td>92.58%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.900, 0.100)</td>
<td>109.35%</td>
<td>84.03%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.990, 0.010)</td>
<td>109.35%</td>
<td>76.33%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.999, 0.001)</td>
<td>109.35%</td>
<td>75.58%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
</tbody>
</table>
Figure 4-2 is the plot of Objective 2 in Table 4-36. In the x-coordinate, the inventory factor of part 1 increases from left to right. According to Table 4-36 and Figure 4-2, it is easy to note that changing the weight assigned to each part does not impact the other objectives. As we increase the weight assigned to part 1, the value of objective 2 decreases gradually, and the turning point occurs at the weight of (0.600, 0.400). At the weight of (0.600, 0.400) for parts 1 and 2, objective 2 increases to 109.69% from 90.94%, and then decreases again as the weight of part 1 increases further. The minimum value is 75.58% under the weight of (0.999, 0.001).
Table 4-37. Variation of Objectives with Changes in the Shortage Factors

<table>
<thead>
<tr>
<th>((u_1, u_2))</th>
<th>Objective 1 Procurement cost</th>
<th>Objective 2 Weighted Inventory</th>
<th>Objective 3 Weighted shortage</th>
<th>Objective 4 Weighted lead time</th>
<th>Objective 5 Weighted defect rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.001, 0.999)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>3.91%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.010, 0.990)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>5.64%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.100, 0.900)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>22.97%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.150, 0.850)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>32.60%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.200, 0.800)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>42.23%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.250, 0.750)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>51.86%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.300, 0.700)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>61.49%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.350, 0.650)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>71.11%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.400, 0.600)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>80.74%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.450, 0.550)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>90.37%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.500, 0.500)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.550, 0.450)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>111.70%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.570, 0.430)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>115.46%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.580, 0.420)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>117.34%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.590, 0.410)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>119.22%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.600, 0.400)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>121.10%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.650, 0.350)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>130.50%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.700, 0.300)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>139.90%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.750, 0.250)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>149.29%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.800, 0.200)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>158.69%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.900, 0.100)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>179.26%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.990, 0.010)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>194.63%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
<tr>
<td>(0.999, 0.001)</td>
<td>109.35%</td>
<td>100.00%</td>
<td>196.12%</td>
<td>101.01%</td>
<td>110.34%</td>
</tr>
</tbody>
</table>

Table 4-37 illustrates the sensitivity analysis for variations of the shortage factors for parts 1 & 2. In Table 4-37, the weights assigned to each objective are equal as the shortage factor varies. The first column gives the different sets of shortage factors used, and the other columns give the values of the five objectives as ratios of the ideal values.
Figure 4-3 is the plot of Objective 3 in Table 4-37. In the x-coordinate, the shortage factor of part 1 increases from left to right. The ideal value used to calculate the ratios is the one obtained under the weight of (0.500, 0.500), so at the weight (0.500, 0.500) objective 2 reaches 100%. According to Figure 4-3, objective 3 has the ascending trend as the weight assigned to the shortage of part 1 increases. When the decision maker has more emphasis on the shortage of part 2 rather than part 1, objective 3 can reach almost 3.91%. When the decision maker has more emphasis on the shortage of part 1 rather than part 2, the weighted average shortage achieves the maximum under Weighted Objective Method.

There are three main differences between Table 4-36 and Table 4-37. Firstly, the weighted average inventory and the weighted average shortage change in the opposite direction. With more inventory factor weight put on the part 1, inventory objective will decrease, but the shortage objective will increase when the shortage factor of part 2 increases. Hence, there probably exists a trade-off for managers to decide the inventory factors and shortage factors for each part. Secondly, there exists a turning point in Figure 4-2 rather on Figure 4-3. So managers should be
more careful to determine the inventory factors because more weight on part 1 does not always mean smaller weighted inventory. Thirdly, the change range in Figure 4-3 is much larger than the one in Figure 4-2, so it is concluded that the weighted shortage is more sensitive to the factor allocation for the two parts.
Chapter 5

Conclusion

5.1 Conclusion

In this research, a multi-criteria multi-period supplier selection and order allocation problem is studied considering all-unit discount policy. The problem is formulated as a multi-objective mixed integer linear programming model. The five objectives being considered are procurement cost, weighted average inventory, weighted average shortage, weighted average lead time and weighted average quality defect rate. The model is solved with two approaches: Weighted Objective Method and Goal Programming.

A numerical example is used to illustrate the application of the model. The decision of supplier selection and order allocation is made among three suppliers for two parts in ten periods. To evaluate the efficiency of the model, the ideal value for each objective are obtained first by ignoring the other objectives. Ten different sets of weights are used to express the decision maker’s preferences in the Weighted Objective Method. They can reflect the decision maker’s preference in different kinds of market candidates. Impacts of the weights on the optimal solutions are discussed. Next, Preemptive and Non-Preemptive goal Programming models are used to solve the problem. Five different priorities for the five goals are considered under Preemptive goal programming. Eight different sets of weights are used to consolidate goals under Non-preemptive goal Programming approach, and their optimal solutions are compared. There are three general points that can be summarized here. Firstly, it is observed that two different methods can lead to the same solution, and a small change in the weight may not affect the ultimate solution. Secondly, although it is not hard to achieve a low enough procurement cost, the minimization of procurement cost can only be obtained at greater expense to the other objectives; so, it is not recommended to
pursue the minimization of procurement cost as the sole objective. Thirdly, the impact of utilization of all-unit discount policy on the order allocation is studied, and it is concluded that all-unit discount policy does influence the company’s order decision, so managers should try to exploit the discount policy efficiently. Moreover, simple scaling method and ideal value scaling method are applied to eliminate the differences among the magnitudes of each objective, so that Weighted Objective Method and Non-preemptive goal Programming can be solved properly.

There exists 12 different efficient solutions generated and analyzed out of 23 total solutions from the Weighted Objective Method and Goal Programming models. Value Path Approach is then used to visually present the trade-off among the different efficient solutions. Because demand forecasts can vary a lot in reality, it is necessary for managers to check and revise the forecast frequently. Thus, it is recommended that the model should be implemented in a rolling horizon to react quickly to the change in demand. At the end of this study, a sensitivity analysis is done by changing the inventory factors and shortage factors. It is observed that the changes in inventory factors or shortage factors have impacts on only the weighted inventory or weighted shortage objectives. However, their relative impacts are very different. It is observed that the plot of shortage factor has a bigger impact than the plot of inventory factor; so the weighted shortage objective is more sensitive to different shortage factors than the weighted inventory objective.

5.2 Future scope

The following can be considered as the future research elements:
The forecasted demand in this model is given without consideration of demand uncertainty. A further study can be done under dynamic demand or demand following specific distributions.

Only all-unit discount policy is considered in this thesis. Future study can focus on the impact of graduated discount policy and compare the results between these two policies.

The model developed here assumes that suppliers only provide one transportation alternative. However, in reality suppliers would provide different kinds of transportation modes, so the impact of transportation choice can be studied in the future.
References


discounts”, IIE Transactions, 17(3), 206-211.


Appendix A

Ideal Value

!Multi-objective MIP;
MODEL:
SETS:
  obj/1:/costobj,inveobj,shorobj,leadobj,qualobj;
  product/1 2:/invf,shof,InitialInv,FinalInv,InitialSho,FinalSho;
  supplier/1 2 3:/CF;
  level/1 2 3/;
  fakelevel/1 2 3 4/;
  week/1..10/;

  ps(product,supplier):L,CT,quality,CAP;
  pw(product,week):demand,inventory,shortage;
  sw(supplier,week):bsw;
  psw(product,supplier,week):
  psl(product,supplier,level):price;
  psf(product,supplier,fakelevel):breakpoint;
  pslw(product,supplier,level,week):Q,bpslw;

ENDSETS

!Formulation;

!Total procurement cost
[totalcost]min=@sum(sw(j,t):CF(j)*bsw(j,t))+@sum(pslw(i,j,k,t):price(i,j,k)*Q(i,j,k,t))+@sum(pslw(i,j,k,t):CT(i,j)*Q(i,j,k,t));

!Weighted average inventory
[averageinventory]min=@sum(pw(i,t):inventory(i,t)*invf(i));

!Weighted average shortage
[averageshortage]min=@sum(pw(i,t):shortage(i,t)*shof(i));

!Weighted average lead time
[leadtime]min=@sum(pslw(i,j,k,t):L(i,j)*Q(i,j,k,t));

!Average quality defect;
[qualitydefect]min=@sum(pslw(i,j,k,t):quality(i,j)*Q(i,j,k,t));
costobj(1)=@sum(sw(j,t):CF(j)*bsw(j,t))+@sum(pslw(i,j,k,t):price(i,j,k)*Q(i,j,k,t))+@sum(pslw(i,j,k,t):CT(i,j)*Q(i,j,k,t));

inveobj(1)=@sum(pw(i,t):inventory(i,t)*invf(i));

shorobj(1)=@sum(pslw(i,j,k,t):shortage(i,t)*shof(i));

leadobj(1)=@sum(pslw(i,j,k,t):L(i,j)*Q(i,j,k,t));

qualobj(1)=@sum(pslw(i,j,k,t):quality(i,j)*Q(i,j,k,t));

!Demand constraint;
!For product 1;
!t=1;
shortage(1,1)+initialInv(1)=initialsho(1)+inventory(1,1)+demand(1,1);
!t=2;
shortage(1,2)+inventory(1,1)+@sum(level(k):Q(1,2,k,1))=demand(1,2)+inventory(1,2)+shortage(1,1);
!t=3;
shortage(1,3)+inventory(1,2)+@sum(level(k):Q(1,2,k,2))=demand(1,3)+inventory(1,3)+shortage(1,2);
!t=4;
shortage(1,4)+inventory(1,3)+@sum(level(k):Q(1,2,k,3))+@sum(level(k):Q(1,3,k,1))=demand(1,4)+inventory(1,4)+shortage(1,3);
!from t=5 to t=9:
@for(week(t)|t#GE#5 #and# t#LE#9: shortage(1,t)+inventory(1,t-1)+@sum(level(k):Q(1,1,k,t-3))+@sum(level(k):Q(1,2,k,t-2))+@sum(level(k):Q(1,3,k,t-1))=inventory(1,t)+demand(1,t)+shortage(1,t-1));

!t=10;
FinalSho(1)+inventory(1,9)+@sum(level(k):Q(1,1,k,6))+@sum(level(k):Q(1,2,k,9))+@sum(level(k):Q(1,3,k,7))=demand(1,10)+FinalInv(1)+shortage(1,9);

!For product 2;
!t=1;
initialInv(2)+shortage(2,1)=initialsho(2)+inventory(2,1)+demand(2,1);
!t=2;
shortage(2,2)+inventory(2,1)+@sum(level(k):Q(2,2,k,1))=inventory(2,2)+demand(2,2)+shortage(2,1);
!t=3;
shortage(2,3)+inventory(2,2)+@sum(level(k):Q(2,2,k,1))+@sum(level(k):Q(2,3,k,2))=inventory(2,3)+demand(2,3)+shortage(2,2);
!from t=4 to t=9:
@for(week(t)|t#GE#4 #and# t#LE#9: shortage(2,t)+inventory(2,t-1)+@sum(level(k):Q(2,1,k,t-3))+@sum(level(k):Q(2,2,k,t-2))+@sum(level(k):Q(2,3,k,t-1))=inventory(2,t)+demand(2,t)+shortage(2,t-1));

!t=10;
finalsho(2)+inventory(2,9)+@sum(level(k):Q(2,1,k,7))+@sum(level(k):Q(2,2,k,8))+@sum(level(k):Q(2,3,k,7))=demand(2,10)+FinalInv(2)+shortage(2,9);
k):Q(2,3,k,9) =
finalinv(2)+demand(2,10)+shortage(2,9);

!Capacity constraint;
@for(psw(i,j,t): @sum(level(k):Q(i,j,k,t)) <= CAP(i,j)*bsw(j,t));

!All-unit discount constraints:
@for(pslw(i,j,k,t): (breakpoint(i,j,k)+1)*bpslw(i,j,k,t) <= Q(i,j,k,t));
@for(pslw(i,j,k,t): breakpoint(i,j,k+1)*bpslw(i,j,k,t) >= Q(i,j,k,t));
@for(psw(i,j,t): @sum(level(k):bpslw(i,j,k,t)) <= bsw(j,t));

!Maximum number of suppliers constraint:
@for(week(t): @sum(supplier(j):bsw(j,t)) <= 2);

!Integer constraint;
!@for(pslw(i,j,k,t): @Gin(Q(i,j,k,t)));
!@for(pw(i,t): @Gin(inventory(i,t)));
!@for(pw(i,t): @Gin(shortage(i,t)));
!Binary constraint;
@for(sw(j,t): @Bin(bsw(j,t)));
@for(pslw(i,j,k,t): @Bin(bpslw(i,j,k,t)));

!Data set;

DATA:

invf=0.35, 0.65;
shof=0.50, 0.50;
InitialInv=500, 500;
FinalInv=0, 0;
InitialSho=0, 0;
FinalSho=0, 0;

CF=1000, 2000, 800;
L=4, 1, 3, 3, 2, 1;
CT=5, 10, 6, 7, 6.5;
quality=0.0700, 0.0120, 0.0550, 0.0100, 0.0230, 0.0340;
CAP=300, 180, 200, 250, 200, 150;

demand=320, 390, 469, 415, 430, 454, 468, 438, 230, 272, 246, 278, 252, 342, 262, 287, 286, 338;

price=24, 22, 20, 26, 20, 18, 30, 22, 16, 40, 35, 32, 40, 36, 30, 38, 34, 28;
breakpoint=0,100,150,1000,0,100,150,1000,0,100,150,1000,
0,80,120,1000,0,80,120,1000,0,80,120,1000;
ENDDATA
END
Appendix B

Non-preemptive Goal Programming

!Non-preemptive GP;
MODEL:
SETS:
obj/1/costobj,inveobj,shorobj,leadobj,qualobj;
product/1..2/invf,shof,InitialInv,FinalInv,InitialSho,FinalSho;
criteria/1..5/;
supplier/1..3/:CF;
level/1..3/;
fakelevel/1..3..4/;
week/1..10/;
Dev/1..2/;
Target/1..5/;

ps(product,supplier):L,CT,quality,CAP;
pw(product,week):demand,inventory,shortage;
sw(supplier,week):bsw;
psw(product,supplier,week);
psl(product,supplier,level):price;
psf(product,supplier,fakelevel):breakpoint;
pслw(product,supplier,level,week):Q,bpslw;
DevTarget(Target,Dev):d;
weight(criteria):w;

ENDSETS

!Formulation;

!Objective;
[Preemptiveobjective] min=w(1)*d(1,2)+w(2)*d(2,2)+w(3)*d(3,2)+w(4)*d(4,2)+w(5)*d(5,2);

!Demand constraint;
!For product 1:
!t=1:
shortage(1,1)+initialInv(1)=initialsho(1)+inventory(1,1)+demand(1,1);
!t=2:
shortage(1,2)+inventory(1,1)+@sum(level(k):Q(1,2,k,1))=demand(1,2)+inventory(1,2)+shortage(1,1);
!t=3:
shortage(1,3)+inventory(1,2)+@sum(level(k):Q(1,2,k,2))=demand(1,3)+inventory(1,3)+shortage(1,2);
!t=4:
shortage(1,4)+inventory(1,3)+@sum(level(k):Q(1,2,k,3))+@sum(level(k):Q(1,3,k,1))=demand(1,4)+inventory(1,4)+shortage(1,3);
!from t=5 to t=9:
@for(week(t)|t#GE#5 #and# t#LE#9: shortage(1,t)+inventory(1,t-1)+@sum(level(k):Q(1,1,k,t-4))+@sum(level(k):Q(1,2,k,t-1))+@sum(level(k):Q(1,3,k,t-3))=demand(1,t)+inventory(1,t)+shortage(1,t-1));
!t=10:
FinalSho(1)+Inventory(1,9)+@sum(level(k):Q(1,1,k,6))+@sum(level(k):Q(1,2,k,9))+@sum(level(k):Q(1,3,k,7))=demand(1,10)+FinalInv(1)+shortage(1,9);

!For product 2:
!t=1:
initialInv(2)+shortage(2,1)=initialsho(2)+inventory(2,1)+demand(2,1);
!t=2:
shortage(2,2)+inventory(2,1)+@sum(level(k):Q(2,3,k,1))=inventory(2,2)+demand(2,2)+shortage(2,1);
!t=3:
shortage(2,3)+inventory(2,2)+@sum(level(k):Q(2,2,k,1))+@sum(level(k):Q(2,3,k,2))=inventory(2,3)+demand(2,3)+shortage(2,2);
!From t=4 to t=9:
@for(week(t)|t#GE#4 #and# t#LE#9: shortage(2,t)+inventory(2,t-1)+@sum(level(k):Q(2,1,k,t-3))+@sum(level(k):Q(2,2,k,t-2))+@sum(level(k):Q(2,3,k,t-1))=inventory(2,t)+demand(2,t)+shortage(2,t-1));
!t=10:
finalsho(2)+inventory(2,9)+@sum(level(k):Q(2,1,k,7))+@sum(level(k):Q(2,2,k,8))+@sum(level(k):Q(2,3,k,9))=finalinv(2)+demand(2,10)+shortage(2,9);

!Capacity constraint;
@for(psw(i,j,t):@sum(level(k):Q(i,j,k,t))<=CAP(i,j)*bsw(j,t));

!All-unit discount constraints;
@for(pslw(i,j,k,t):(breakpoint(i,j,k)+1)*bpslw(i,j,k,t)<=Q(i,j,k,t));
@for(pslw(i,j,k,t):breakpoint(i,j,k+1)*bpslw(i,j,k,t)>=Q(i,j,k,t));
@for(psw(i,j,t): @sum(level(k):bpslw(i,j,k,t))<=bsw(j,t));

!Maximum number of suppliers constraint;
@for(week(t): @sum(supplier(j):bsw(j,t))<=2);

!Integer constraint;
@for(pslw(i,j,k,t):@Gin(Q(i,j,k,t)));
@for(pw(i,t):@Gin(inventory(i,t)));
@for(pw(i,t):@Gin(shortage(i,t)));
!Binary constraint;
@for(sw(j,t):@Bin(bsw(j,t)));
@for(pslw(i,j,k,t):@Bin(bpslw(i,j,k,t)));

!Goal constraints
!Total procurement cost;
(1/1000000)*( @sum(sw(j,t):CF(j)*bsw(j,t))+ @sum(pslw(i,j,k,t):price(i,j,k)*Q(i,j,k,t))+ @sum(pslw(i,j,k,t):CT(i,j)*Q(i,j,k,t))+d(1,1)-d(1,2)=186511/1000000);

!Weighted average inventory;
(1/50000)*@sum(pw(i,t):inventory(i,t)*invf(i))+d(2,1)-d(2,2)=238.5/500000;

!Weighted average shortage;
(1/10000)*@sum(pw(i,t):shortage(i,t)*shof(i))+d(3,1)-d(3,2)=1346.5/100000;

!Weighted average lead time;
(1/10000)*@sum(pslw(i,j,k,t):L(i,j)*Q(i,j,k,t))+d(4,1)-d(4,2)=13723/1000000;

!Average quality defect;
(1/200)*@sum(pslw(i,j,k,t):quality(i,j)*Q(i,j,k,t))+d(5,1)-d(5,2)=188.3490/200;

d(1,1)=0;
d(2,1)=0;
d(3,1)=0;
d(4,1)=0;
d(5,1)=0;

w(1)=3;
w(2)=3;
w(3)=0.5;
w(4)=3;
w(5)=0.5;

!Data set;

DATA:

invf=0.35,0.65;
shof=0.50,0.50;
InitialInv=500,500;
FinalInv=0,0;
InitialSho=0,0;
FinalSho=0,0;

CF=1000,2000,800;

L=4,1,3,3,2,1;
CT=5,10,6,7,6,5;
quality=0.0700,0.0120,0.0550,0.0100,0.0230,0.0340;
CAP=300,180,200,250,200,150;

demand=320,390,469,468,415,430,454,368,438,
230,272,246,278,252,342,262,287,286,338;

price=24,22,20,26,20,18,30,22,16,
40,35,32,40,36,30,38,34,28;

breakpoint=0,100,150,1000,0,100,150,1000,0,100,150,1000,
0,80,120,1000,0,80,120,1000,0,80,120,1000;

ENDDATA

END
Appendix C

Preemptive Goal Programming

!Preemptive GP;
MODEL:
SETS:
obj/1:costobj,inveobj,shorobj,leadobj,qualobj;
product/1 2:invf,shof,InitialInv,FinalInv,InitialSho,FinalSho;
supplier/1 2 3:CF;
level/1 2 3;
fakelevel/1 2 3 4;
week/1..10;
Dev/1 2;
Target/1..5;

ps(product,supplier):L,CT,quality,CAP;
pw(product,week):demand,inventory,shortage;
sw(supplier,week):bsw;
psw(product,supplier,week):;
psl(product,supplier,level):price;
psf(product,supplier,fakelevel):breakpoint;
pslw(product,supplier,level,week):Q,bpslw;
DevTarget(Target,Dev):d;
ENDSETS

!Formulation;

!Objective
[Preemptiveobjective]min=(50^2)*d(1,2)+(20)*d(2,2)+(20)*d(3,2)+(2)*d(4,2)+d(5,2);

!Objective
[Preemptiveobjective]min=(50^2)*d(2,2)+(20)*d(3,2)+(5)*d(1,2)+(2)*d(4,2)+d(5,2);

!Objective
[Preemptiveobjective]min=(50^2)*d(5,2)+(50)*d(2,2)+(2)*d(3,2)+(2)*d(4,2)+d(1,2);

!Objective
[Preemptiveobjective]min=(50^2)*d(3,2)+(20)*d(5,2)+(5)*d(4,2)+(2)*d(2,2)+d(1,2);

!Objective
[Preemptiveobjective]min=(50^2)*d(4,2)+(20)*d(2,2)+(5)*d(5,2)+(2)*d(1,2)+d(3,2);

costobj(1)=(1/186511)*(@sum(sw(j,t):CF(j)*bsw(j,t))+@sum(pslw(i,j,k,t):price(i,j,k)*Q(i,j,k,t)))+
@sum(pslw(i,j,k,t):CT(i,j)*Q(i,j,k,t)));
inveobj(1)=(1/238.5)*@sum(pw(i,t):inventory(i,t)*invf(i));
shorobj(1)=(1/1346.5)*@sum(pw(i,t):shortage(i,t)*shof(i));
leadobj(1)=(1/13723)*@sum(pslw(i,j,k,t):L(i,j)*Q(i,j,k,t));
qualobj(1)=(1/188.349)*\sum(pslw(i,j,k,t):quality(i,j)*Q(i,j,k,t));

!Demand constraint;
!For product 1;
!t=1;
shortage(1,1)+initialInv(1)=initialsho(1)+inventory(1,1)+demand(1,1);
!t=2;
shortage(1,2)+inventory(1,1)+\sum(level(k):Q(1,2,k,1))=demand(1,2)+inventory(1,2)+shortage(1,1);
!t=3;
shortage(1,3)+inventory(1,2)+\sum(level(k):Q(1,2,k,2))=demand(1,3)+inventory(1,3)+shortage(1,2);
!t=4;
shortage(1,4)+inventory(1,3)+\sum(level(k):Q(1,2,k,3))+\sum(level(k):Q(1,3,k,1))=demand(1,4)+inventory(1,4)+shortage(1,3);
!from t=5 to t=9:
@for(week(t)|t#GE#5 #and# t#LE#9: shortage(1,t)+inventory(1,t-1)+\sum(level(k):Q(1,1,k,t-4))+\sum(level(k):Q(1,2,k,t-1))+\sum(level(k):Q(1,3,k,t-3))=demand(1,t)+inventory(1,t)+shortage(1,t-1));
!t=10:
FinalSho(1)+Inventory(1,9)+\sum(level(k):Q(1,1,k,6))+\sum(level(k):Q(1,2,k,9))+\sum(level(k):Q(1,3,k,7))=demand(1,10)+FinalInv(1)+shortage(1,9);

!For product 2;
!t=1;
initialInv(2)+shortage(2,1)=initialsho(2)+inventory(2,1)+demand(2,1);
!t=2;
shortage(2,2)+inventory(2,1)+\sum(level(k):Q(2,3,k,1))=inventory(2,2)+demand(2,2)+shortage(2,1);
!t=3;
shortage(2,3)+inventory(2,2)+\sum(level(k):Q(2,2,k,1))+\sum(level(k):Q(2,3,k,2))=inventory(2,3)+demand(2,3)+shortage(2,2);
!from t=4 to t=9:
@for(week(t)|t#GE#4 #and# t#LE#9: shortage(2,t)+inventory(2,t-1)+\sum(level(k):Q(2,1,k,t-3))+\sum(level(k):Q(2,2,k,t-2))+\sum(level(k):Q(2,3,k,t-1))=inventory(2,t)+demand(2,t)+shortage(2,t-1));
!t=10:
finalsho(2)+inventory(2,9)+\sum(level(k):Q(2,1,k,7))+\sum(level(k):Q(2,2,k,8))+\sum(level(k):Q(2,3,k,9))=finalinv(2)+demand(2,10)+shortage(2,9);

!Capacity constraint;
@for(psw(i,j,t):\sum(level(k):Q(i,j,k,t))<=CAP(i,j)*bsw(j,t));
!All-unit discount constraints:
@for(pslw(i,j,k,t): breakpoint(i,j,k)+1)*bpslw(i,j,k,t) <= Q(i,j,k,t));
@for(pslw(i,j,k,t): breakpoint(i,j,k+1)*bpslw(i,j,k,t) >= Q(i,j,k,t));

@for(psw(i,j,t): @sum(level(k): bpslw(i,j,k,t)) <= bsw(j,t));

!Maximum number of suppliers constraint:
@for(week(t): @sum(supplier(j): bsw(j,t)) <= 2);

!Integer constraint;
@for(pslw(i,j,k,t): @Gin(Q(i,j,k,t)));
@for(pw(i,t): @Gin(inventory(i,t)));
@for(pw(i,t): @Gin(shortage(i,t)));

!Binary constraint;
@for(sw(j,t): @Bin(bsw(j,t)));
@for(pslw(i,j,k,t): @Bin(bpslw(i,j,k,t)));

!Goal constraints
!Total procurement cost:
(@sum(sw(j,t): CF(j)*bsw(j,t)) + @sum(pslw(i,j,k,t): price(i,j,k)*Q(i,j,k,t)) + @sum(pslw(i,j,k,t): CT(i,j)*Q(i,j,k,t)))/186511 + d(1,1) - d(1,2) = 1;

!Weighted average inventory:
(@sum(pw(i,t): inventory(i,t)*invf(i)))/238.5 + d(2,1) - d(2,2) = 1;

!Weighted average shortage:
(@sum(pw(i,t): shortage(i,t)*shof(i)))/1346.500 + d(3,1) - d(3,2) = 1;

!Weighted average lead time:
(@sum(pslw(i,j,k,t): L(i,j)*Q(i,j,k,t)))/13723 + d(4,1) - d(4,2) = 1;

!Average quality defect:
(@sum(pslw(i,j,k,t): quality(i,j)*Q(i,j,k,t)))/188.3490 + d(5,1) - d(5,2) = 1;

d(1,1) = 0;
d(2,1) = 0;
d(3,1) = 0;
d(4,1) = 0;
d(5,1) = 0;

!Data set;

DATA:
invf = 0.35, 0.65;
shof = 0.50, 0.50;
InitialInv = 500, 500;
FinalInv = 0, 0;
InitialSho=0,0;
FinalSho=0,0;

CF=1000,2000,800;
L=4,1,3,3,2,1;
CT=5,10,6,7,6,5;
quility=0.0700,0.0120,0.0550,0.0100,0.0230,0.0340;
CAP=300,180,200,250,200,150;
demand=320,390,469,349,454,368,438,
230,272,246,278,252,342,262,287,286,338;
price=24,22,20,26,20,18,30,22,16,
40,35,32,40,36,30,38,34,28;
breakpoint=0,100,150,1000,0,100,150,1000,0,100,150,1000,
0,80,120,1000,0,80,120,1000,0,80,120,1000;
ENDDATA

END
Appendix D

Weighted Objective Method

!Weighted Objective Method;
MODEL:
SETS:
obj/1/:costobj,inveobj,shorobj,leadobj,qualobj;
criteria /1..5/;
product/1 2/:invf,shof,InitialInv,FinalInv,InitialSho,FinalSho;
supplier/1 2 3/:CF;
level/1 2 3/;
fakelevel/1 2 3 4/;
week/1..10/;

ps(product,supplier):L,CT,quality,CAP;
pw(product,week):demand,inventory,shortage;
sw(supplier,week):bsw;
psw(product,supplier,week):;
psl(product,supplier,level):price;
psf(product,supplier,fakelevel):breakpoint;
pslw(product,supplier,level,week):Q,bpslw;
weight(criteria):w;

ENDSETS

!Formulation;

!Weighted objective;
[Weightedobjective]
min=((@sum(sw(j,t):CF(j)*bsw(j,t))+@sum(pslw(i,j,k,t):price(i,j,k)*Q(i,j,k,t))
+@sum(pslw(i,j,k,t):CT(i,j)*Q(i,j,k,t)))*w(1)/1000000+
(@sum(pw(i,t):inventory(i,t)*invf(i)))*w(2)/50000+
(@sum(pw(i,t):shortage(i,t)*shof(i)))*w(3)/10000+
(@sum(pslw(i,j,k,t):L(i,j)*Q(i,j,k,t)))*w(4)/100000+
(@sum(pslw(i,j,k,t):quality(i,j)*Q(i,j,k,t)))*w(5)/200;

costobj(1)=(1/186511.1)*(@sum(sw(j,t):CF(j)*bsw(j,t))+@sum(pslw(i,j,k,t):price(i,j,k)*Q(i,j,k,t))
+@sum(pslw(i,j,k,t):CT(i,j)*Q(i,j,k,t));
inveobj(1)=(1/238.5)*@sum(pw(i,t):inventory(i,t)*invf(i));
shorobj(1)=(1/1346.5)*@sum(pw(i,t):shortage(i,t)*shof(i));
leadobj(1)=(1/13723)*@sum(pslw(i,j,k,t):L(i,j)*Q(i,j,k,t));
qualobj(1)=(1/188.3490)*@sum(pslw(i,j,k,t):quality(i,j)*Q(i,j,k,t));
!Demand constraint;
!For product 1;
!t=1;
shortage(1,1)+initialInv(1)=initialsho(1)+inventory(1,1)+demand(1,1);
!t=2;
shortage(1,2)+inventory(1,1)+@sum(level(k):Q(1,2,k,1))=demand(1,2)+inventory(1,2)+shortage(1,1);
!t=3;
shortage(1,3)+inventory(1,2)+@sum(level(k):Q(1,2,k,2))=demand(1,3)+inventory(1,3)+shortage(1,2);
!t=4;
shortage(1,4)+inventory(1,3)+@sum(level(k):Q(1,2,k,3))+@sum(level(k):Q(1,3,k,1))=demand(1,4)+inventory(1,4)+shortage(1,3);
!from t=5 to t=9;
@for(week(t)|t#GE#5 #and# t#LE#9: shortage(1,t)+inventory(1,t-1)+@sum(level(k):Q(1,1,k,t-4))+@sum(level(k):Q(1,2,k,t-1))+@sum(level(k):Q(1,3,k,t-3))=inventory(1,t)+demand(1,t)+shortage(1,t-1));
!t=10;
FinalSho(1)+Inventory(1,9)+@sum(level(k):Q(1,1,k,6))+@sum(level(k):Q(1,2,k,9))+@sum(level(k):Q(1,3,k,7))=demand(1,10)+FinalInv(1)+shortage(1,9);

!For product 2;
!t=1;
initialInv(2)+shortage(2,1)=initialsho(2)+inventory(2,1)+demand(2,1);
!t=2;
shortage(2,2)+inventory(2,1)+@sum(level(k):Q(2,3,k,1))=inventory(2,2)+demand(2,2)+shortage(2,1);
!t=3;
shortage(2,3)+inventory(2,2)+@sum(level(k):Q(2,2,k,1))+@sum(level(k):Q(2,3,k,2))=inventory(2,3)+demand(2,3)+shortage(2,2);
!from t=4 to t=9;
@for(week(t)|t#GE#4 #and# t#LE#9: shortage(2,t)+inventory(2,t-1)+@sum(level(k):Q(2,1,k,t-3))+@sum(level(k):Q(2,2,k,t-2))+@sum(level(k):Q(2,3,k,t-1))=inventory(2,t)+demand(2,t)+shortage(2,t-1));
!t=10;
finalsho(2)+inventory(2,9)+@sum(level(k):Q(2,1,k,7))+@sum(level(k):Q(2,2,k,9))+@sum(level(k):Q(2,3,k,9))=finalinv(2)+demand(2,10)+shortage(2,9);

!Capacity constraint;
@for(psw(i,j,t):@sum(level(k):Q(i,j,k,t))<=CAP(i,j)*bsw(j,t));

!All-unit discount constraints;
@for(pslw(i,j,k,t):(breakpoint(i,j,k)+1)*bpslw(i,j,k,t)<=Q(i,j,k,t));
@for(pslw(i,j,k,t):breakpoint(i,j,k+1)*bpslw(i,j,k,t)>=Q(i,j,k,t));
@for(psw(i,j,t): @sum(level(k):bpslw(i,j,k,t))<=bsw(j,t));

!Maximum number of suppliers constraint;
@for(week(t): @sum(supplier(j):bsw(j,t))<=2);

!Integer constraint;
@for(pslw(i,j,k,t): @Gin(Q(i,j,k,t))); @for(pw(t): @Gin(inventory(i,t))); @for(pw(t): @Gin(shortage(i,t)));
!Binary constraint;
@for(sw(j,t): @Bin(bsw(j,t))); @for(pslw(i,j,k,t): @Bin(bpslw(i,j,k,t)));

!w(1)=3.5;
!w(2)=3.5;
!w(3)=1;
!w(4)=1;
!w(5)=1;

w(1)=1;
w(2)=6;
w(3)=1;
w(4)=1;
w(5)=1;

!w(1)=1/10;
!w(2)=1/10;
!w(3)=6/10;
!w(4)=1/10;
!w(5)=1/10;

!w(1)=8/10;
!w(2)=1/10;
!w(3)=1/10;
!w(4)=1/10;
!w(5)=1/10;

!w(1)=1/10;
!w(2)=1/10;
!w(3)=1/10;
!w(4)=1/10;
!w(5)=6/10;

!w(1)=2/10;
!w(2)=2/10;
!w(3)=2/10;
!w(4)=2/10;
!w(5)=2/10;

!w(1)=1/10;
!w(2)=3.5/10;
!w(3)=1/10;
!w(4)=3.5/10;
!w(5)=1/10;

!w(1)=3.5/10;
!w(2)=1/10;
!w(3)=1/10;
!w(4)=1/10;
!w(5)=3.5/10;

!Data set;

DATA:

invf=0.35,0.65;
shof=0.50,0.50;
InitialInv=500,500;
FinalInv=0,0;
InitialSho=0,0;
FinalSho=0,0;

CF=1000,2000,800;

L=4,1,3,3,2,1;
CT=5,10,6,7,6,5;
quality=0.0700,0.0120,0.0550,0.0100,0.0230,0.0340;
CAP=300,180,200,250,200,150;

demand=320,390,469,415,430,454,368,438,230,272,246,278,252,342,262,287,286,338;

price=24,22,20,26,20,18,30,22,16,40,35,32,40,36,30,38,34,28;

breakpoint=0,100,150,1000,0,100,150,1000,0,100,150,1000,0,80,120,1000,0,80,120,1000,0,80,120,1000;

ENDDATA

END
Appendix E

Sensitivity Analysis

!Sensitive analysis;
MODEL:
SETS:
obj/1/:costobj,inveobj,shorobj,leadobj,qualobj;
criteria /1..5/;
product/1 2/:invf,shof,InitialInv,FinalInv,InitialSho,FinalSho;
supplier/1 2 3/:CF;
level/1 2 3/;
fakelevel/1 2 3 4/;
week/1..10/;

ps(product,supplier):L,CT,quality,CAP;
pw(product,week):demand,inventory,shortage;
sw(supplier,week):bsw;
psw(product,supplier,week):;
psl(product,supplier,level):price;
psf(product,supplier,fakelevel):breakpoint;
pslw(product,supplier,level,week):Q,bpslw;
weight(criteria):w;

ENDSETS

!Formulation:

!Weighted objective:
[Weightedobjective]min=( @sum(sw(j,t):CF(j)*bsw(j,t))+ @sum(pslw(i,j,k,t):price(i,j,k)*Q(i,j,k,t)) )+ @sum(pslw(i,j,k,t):CT(i,j)*Q(i,j,k,t))*w(1)/1000000+
(@sum(pw(i,t):inventory(i,t)*invf(i)))*w(2)/50000+
(@sum(pw(i,t):shortage(i,t)*shof(i)))*w(3)/10000+
(@sum(pslw(i,j,k,t):L(i,j)*Q(i,j,k,t)))*w(4)/10000+
(@sum(pslw(i,j,k,t):quality(i,j)*Q(i,j,k,t)))*w(5)/200;

costobj(1)=(1/186511.1)*(@sum(sw(j,t):CF(j)*bsw(j,t))+ @sum(pslw(i,j,k,t):price(i,j,k)*Q(i,j,k,t)) )+ @sum(pslw(i,j,k,t):CT(i,j)*Q(i,j,k,t)));
inveobj(1)=(1/238.5)*@sum(pw(i,t):inventory(i,t)*invf(i));
shorobj(1)=(1/1346.5)*@sum(pw(i,t):shortage(i,t)*shof(i));
leadobj(1)=(1/13723)*@sum(pslw(i,j,k,t):L(i,j)*Q(i,j,k,t));
qualobj(1)=(1/188.3490)*@sum(pslw(i,j,k,t):quality(i,j)*Q(i,j,k,t));
!Demand constraint;
!For product 1;
!t=1;
shortage(1,1)+initialInv(1)=initialsho(1)+inventory(1,1)+demand(1,1);            
!t=2;
shortage(1,2)+inventory(1,1)+@sum(level(k):Q(1,2,k,1))=demand(1,2)+inventory(1,2)+shortage(1,1);            
!t=3;
shortage(1,3)+inventory(1,2)+@sum(level(k):Q(1,2,k,2))=demand(1,3)+inventory(1,3)+shortage(1,2);            
!t=4;
shortage(1,4)+inventory(1,3)+@sum(level(k):Q(1,2,k,3))+@sum(level(k):Q(1,3,k,1))=demand(1,4)+inventory(1,4)+shortage(1,3);
!from t=5 to t=9;
@for(week(t)|t#GE#5 #and# t#LE#9: shortage(1,t)+inventory(1,t-1)+@sum(level(k):Q(1,1,k,t-4))+@sum(level(k):Q(1,2,k,t-1))+@sum(level(k):Q(1,3,k,t-3))=demand(1,t)+inventory(1,t)+shortage(1,t-1));
!t=10;
FinalSho(1)+Inventory(1,9)+@sum(level(k):Q(1,1,k,6))+@sum(level(k):Q(1,2,k,9))+@sum(level(k):Q(1,3,k,7))=demand(1,10)+FinalInv(1)+shortage(1,9);

!For product 2;
!t=1;
initialInv(2)+shortage(2,1)=initialsho(2)+inventory(2,1)+demand(2,1);            
!t=2;
shortage(2,2)+inventory(2,1)+@sum(level(k):Q(2,2,k,1))=inventory(2,2)+demand(2,2)+shortage(2,1);            
!t=3;
shortage(2,3)+inventory(2,2)+@sum(level(k):Q(2,2,k,2))+@sum(level(k):Q(2,3,k,2))=inventory(2,3)+demand(2,3)+shortage(2,2);
!from t=4 to t=9;
@for(week(t)|t#GE#4 #and# t#LE#9: shortage(2,t)+inventory(2,t-1)+@sum(level(k):Q(2,1,k,t-3))+@sum(level(k):Q(2,2,k,t-2))+@sum(level(k):Q(2,3,k,t-1))=inventory(2,t)+demand(2,t)+shortage(2,t-1));
!t=10;
finalsho(2)+inventory(2,9)+@sum(level(k):Q(2,1,k,7))+@sum(level(k):Q(2,2,k,9))+@sum(level(k):Q(2,3,k,9))=finalinv(2)+demand(2,10)+shortage(2,9);

!Capacity constraint;
@for(psw(i,j,t): @sum(level(k):Q(i,j,k,t))<=CAP(i,j)*bsw(j,t));

!All-unit discount constraints;
\@for(pslw(i,j,k,t):(breakpoint(i,j,k)+1)*bpslw(i,j,k,t)\leq Q(i,j,k,t));
\@for(pslw(i,j,k,t):breakpoint(i,j,k+1)*bpslw(i,j,k,t)\geq Q(i,j,k,t));

\@for(psw(i,j,t):\sum(level(k):bpslw(i,j,k,t))\leq bsw(j,t));

!Maximum number of suppliers constraint;
\@for(week(t):\sum(supplier(j):bsw(j,t))\leq 2);

!Integer constraint;
\@for(pslw(i,j,k,t):@Gin(Q(i,j,k,t)));  
\@for(pw(t):@Gin(inventory(i,t)));   
\@for(pw(t):@Gin(shortage(i,t)));   

!Binary constraint;
\@for(sw(j,t):@Bin(bsw(j,t)));  
\@for(pslw(i,j,k,t):@Bin(bpslw(i,j,k,t)));  

!w(1)=3.5;
!w(2)=3.5;
!w(3)=1;    
!w(4)=1;
!w(5)=1;

w(1)=1;
w(2)=1;
w(3)=1;
w(4)=1;
w(5)=1;

!w(1)=1/10;
!w(2)=1/10;
!w(3)=6/10;
!w(4)=1/10;
!w(5)=1/10;

!w(1)=8/10;
!w(2)=1/10;
!w(3)=1/10;
!w(4)=1/10;
!w(5)=1/10;

!w(1)=1/10;
!w(2)=1/10;
!w(3)=1/10;
!w(4)=1/10;
!w(5)=6/10;

!w(1)=2/10;
!w(2)=2/10;


!w(3)=2/10;
!w(4)=2/10;
!w(5)=2/10;

!w(1)=1/10;
!w(2)=3.5/10;
!w(3)=1/10;
!w(4)=3.5/10;
!w(5)=1/10;

!w(1)=3.5/10;
!w(2)=1/10;
!w(3)=1/10;
!w(4)=1/10;
!w(5)=3.5/10;

!Data set;

DATA:

invf=0.35, 0.65;
shof=1, 0;
InitialInv=500,500;
FinalInv=0,0;
InitialSho=0,0;
FinalSho=0,0;

CF=1000,2000,800;

L=4,1,3,3,2,1;
CT=5,10,6,7,6,5;
quality=0.0700,0.0120,0.0550,0.0100,0.0230,0.0340;
CAP=300,180,200,250,200,150;

demand=320,390,469,415,430,454,368,438,230,272,246,278,287,286,338;

price=24,22,20,26,20,18,30,22,16,40,35,32,40,36,30,38,34,28;

breakpoint=0,100,150,1000,0,100,150,1000,0,100,150,1000,0,80,120,1000,0,80,120,1000;

ENDDATA

END