LASER DOPPLER VELOCIMETRY MEASUREMENTS OF A SCALE MODEL SUPersonic EXHAUST JET IMPINGING ON A GROUND PLANE

A Thesis in
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by
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Abstract

Newer generation of fighter aircraft are designed to take off in a short distance, accelerate to supersonic flight speeds, and land vertically if necessary. During a vertical landing, these Short Takeoff and Vertical Landing, STOVL, aircraft direct the high-speed, high temperature exhaust from their primary thrust nozzles towards the ground to provide half of the required lift. The operational environment surrounding these aircraft is severely impacted by the complex flow field established when these aircraft hover in close proximity to the ground. At low stand-off distances, aeroacoustic resonances give rise to high amplitude impingement tones which may cause unsteady loading on the aircraft and be a source of hearing loss for personnel near the aircraft. Of the utmost importance is the need to better understand the high temperature, high velocity flow field generated by these aircraft which gives rise to extremely hazardous conditions for personnel and equipment in the vicinity of the aircraft.

To better understand the supersonic impinging jet flow field, two-component laser Doppler velocimetry, LDV, measurements are made in the jet plume, impingement region, and outwash region of a scale model jet impinging on a simulated ground plane. The model used for this study is representative of a generic military-style STOVL aircraft in hover. Velocity measurements are made for jet stand-off distances of 6, 12, and 23 nozzle diameters above the ground plane for an ideally expanded jet exhausting out of a contoured de Laval nozzle operating at a pressure ratio of 2.93 with an exit diameter of 13.5 mm.

Mean velocity measurements in the jet plume and wall jet are compared to previous pitot rake measurements. There is good agreement with past data. Although there is a slight mismatch in measured peak velocities between the two measurement techniques this discrepancy falls within experimental error. For plate separations of 6 and 12 nozzle diameters, axially-directed velocities as high as 250 m/s are observed within half of a nozzle diameter above the ground plane. Turbulence intensities as high as 15% of the jet exit velocity are found to be in the shear layer for all stand-off distances.

Radial velocities in the wall jet as high as 25% of the jet exit velocity are found to extend radially out to 9 nozzle diameters from the jet centerline. The wall jet becomes fully-
developed by 9 nozzle diameters from the jet centerline as well. Peak velocity decay in the outwash drops off inversely to increasing radial distance. Between 8 to 12 nozzle diameters out, LDV show a higher wall jet velocity than previous pitot probe measurements. This is attributed to the smaller LDV and improved model alignment.

Turbulence spectra obtained for a plate separation of jet stand-off distance of 6 nozzle diameters show a distinct tone at several locations in the flow field that agree well the impingement tone previously observed in acoustic field measurements. The velocity fluctuations appear to be coherent throughout the jet plume and wall jet flow out to 1.4 nozzle diameters. By 2 diameters in the outwash, the tonal information is no longer present in turbulence spectra. These large-scale instabilities have been observed with flow visualizations but have been previously difficult to quantify for supersonic impinging jets.
# Table of Contents

List of Figures viii  
List of Tables xi  
List of Symbols xv  
Acknowledgments xvii

## Chapter 1  
Introduction 1  
1.1 Motivation ........................................ 1  
1.2 Background ........................................ 3  
1.2.1 Impinging Jet Flow Field ....................... 3  
1.2.1.1 Free Jet Region ............................... 3  
1.2.1.2 Impingement Zone ............................. 6  
1.2.1.3 Wall Jet/Outwash Region ................. 6  
1.2.2 Acoustic Phenomena in Supersonic Impinging Jets 7  
1.2.2.1 Turbulence-Induced Noise .................. 7  
1.2.2.2 Shock-Associated Noise ..................... 8  
1.2.2.3 Ground Plane-Nozzle Feedback Resonance .... 9  
1.3 High-Speed Free and Impinging Jet Measurements 10  
1.3.1 Hot-Wire ........................................ 11  
1.3.2 Unsteady Pressure and Acoustic Field Measurements 11  
1.3.3 Pitot Probe / Pitot Rake ....................... 12  
1.3.4 Optical Measurements .......................... 12  
1.3.4.1 Classic Particle Image Velocimetry .......... 12  
1.3.4.2 Time Resolved Particle Image Velocimetry ... 13  
1.4 Laser Doppler Velocimetry ......................... 14  
1.4.1 Previous Applications of LDV in Supersonic Jets 14  
1.4.2 Overview of LDV Measurement Technique ........ 14
1.5 Scope of Thesis ............................................. 19
1.5.1 Research Objectives .................................. 19
1.5.2 Thesis Synopsis ........................................ 20

Chapter 2
Facility Description ........................................... 21
2.1 High Speed Jet Aeroacoustics Facility Overview .......... 21
2.2 High Pressure Air Supply .................................. 22
2.3 Impinging Jet Experimental Setup .......................... 23

Chapter 3
Experimental Methods, Data Acquisition, and Processing 27
3.1 Experimental Parameters .................................. 27
  3.1.1 Impinging Jet Operational Conditions .................. 27
  3.1.2 Coordinate System ...................................... 28
  3.1.3 Measurement Locations .................................. 28
3.2 Schlieren/Shadowgraph Flow Visualization ................ 31
3.3 Acoustic Field Measurements ................................ 32
3.4 Pitot Probe/Pitot Rake Flow Field Pressure Measurements . 33
3.5 The High Speed Jet LDV System ........................... 35
  3.5.1 Laser and Optics Equipment .............................. 35
  3.5.2 Flow Seeding ............................................ 36
  3.5.3 Sending and Receiving Probe Overview .................. 37
    3.5.3.1 Mie Scattering of Light ............................ 37
    3.5.3.2 Sending and Receiving Probe Design ................ 38
    3.5.3.3 Modifications to Probe for Near-Ground Plane Survey . 39
  3.5.4 PMT Wiring and Power Supply Design .................... 41
  3.5.5 LDV Data Acquisition Software .......................... 43
  3.5.6 Probe Calibration ........................................ 44
3.6 Particle Burst Validation and Processing .................... 45
3.7 Spectral Estimation of LDV Data ............................ 48
3.8 Experimental Uncertainty ................................... 50
  3.8.1 Facility Uncertainties .................................... 51
  3.8.2 Laser Doppler Velocimetry Uncertainty ................. 51
    3.8.2.1 Mean Flow Uncertainty ............................ 51
    3.8.2.2 Turbulence Intensity Uncertainty .................. 57

Chapter 4
Experimental Results .......................................... 59
4.1 Mean and Unsteady Velocity Measurements .................. 59
### 4.1.1 Supersonic Jet Plume Measurements

- 4.1.1.1 Ground Plane Effects on Supersonic Jet Plume
- 4.1.1.2 Two-Dimensional Survey of Impinging Jet Plumes

### 4.1.2 Investigation of Impingement Zone

- 4.1.2.1 Discussion
- 4.1.2.2 Estimation of Probe-Jet Misalignment

### 4.1.3 Characterization of Outwash Flow Development

### 4.2 Turbulence Spectra

<table>
<thead>
<tr>
<th>Chapter 5</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Summary and Conclusions</strong></td>
<td>83</td>
</tr>
<tr>
<td>5.1 Summary of Goals and Objectives</td>
<td>83</td>
</tr>
<tr>
<td>5.2 Review of Primary Results</td>
<td>84</td>
</tr>
<tr>
<td>5.3 Recommendations for Future Work</td>
<td>86</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MATLAB Processing Scripts</strong></td>
<td>88</td>
</tr>
<tr>
<td>1 LDV Velocity Processing</td>
<td>88</td>
</tr>
<tr>
<td>2 Turbulence Spectra Estimation</td>
<td>99</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bibliography</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bibliography</strong></td>
<td>107</td>
</tr>
</tbody>
</table>
## List of Figures

1.1 Photograph of the *F-35B* aircraft during a vertical landing. The thrust during hover is provided predominantly by the rear core, hot supersonic, jet and the front, "cold" sonic, lift fan jet. .............................................. 2  
1.2 Schematic diagram of the flow field produced by a single supersonic jet impinging normally on a planar surface ......................................................... 3  
1.3 Schematic of flow exhausting from an axis-symmetric round nozzle. ........ 4  
1.4 Schematic diagram of the fully-developed wall jet velocity profile .......... 7  
1.5 Averaged Schlieren Image of an over-expanded jet issuing from a converging-diverging military-style nozzle. ................................................................. 9  
1.6 Schematic diagram of probe volume and projected probe volume .......... 15  
1.7 Ideal time-series signal of a particle light burst traveling through the probe volume ........................................................................................................ 17  

2.1 Top-view schematic of the Pennsylvania State University High Speed Jet Aeroacoustics Facility. .......................................................... 21  
2.2 Schematic of the High Pressure Piping Control Cabinet in the High Speed Jet Aeroacoustic Laboratory. Pure air delivery is depicted with blue lines while red lines depict helium gas delivery. .................................................. 23  
2.3 Schematic diagrams and photograph of impinging jet model with dimensions and nozzle design parameters. Note (a) and (b) are in opposite orientations 24  
2.4 Photograph of the *F-35B* in hover configuration just after landing. The rear nozzle can been seen to protrude approximately one diameter below the aircraft ................................................................. 25  
2.5 Close-up photograph of the dual impinging jet model installed in the anechoic chamber with shroud removed to show piping .................... 26  
2.6 Photograph of the anechoic chamber with full impinging jet model installed. The exhaust duct can be seen on the left of the image. The microphone boom can been seen in the background. ............................................. 26  

3.1 Coordinate system used for impinging jet LDV measurements ............. 28
3.2 Diagram of all LDV measurement locations for plate separations of 12 and 6 $D_r$. Note the different scales for each figure. Black $c'$s denote axial velocity component measurements. Filled, red dots denote radial velocity component measurements.

3.3 Schlieren/shadowgraph z-type setup used in the High Speed Jet Aeroacoustics Facility at the Pennsylvania State University.

3.4 Far-field microphone array, composed of 23 microphones, setup around the azimuth of the baseline dual impinging jet model.

3.5 Photograph and schematic diagram of the pitot probe rake used for mean velocity measurements that were used for laser Doppler velocimetry (LDV) data validation.

3.6 The PSU High Speed Jet Aeroacoustics laser Doppler velocimeter optics bench.

3.7 Directivity of scattered light for a small particle ($d_p \leq 0.1\lambda$, left), a medium particle particle ($d_p \approx 10\lambda$, center), and a large particle particle ($d_p > 10\lambda$, right).

3.8 Photographs of sending and receiving probes mounted to traversing stages.

3.9 CAD model and photograph of the LDV setup inside the anechoic chamber for impinging jet flow-field measurements.

3.10 Schematic diagram of the final photomultiplier tube power supply circuit.

3.11 Example probably distributions of particle velocity and time between particle bursts at five different radial locations for $H/D_r = 6$, $y/D_r = 0.5$.

4.1 Supersonic jet plume profiles taken at $x/D_r = 4$ for varying lift plate heights.

4.2 Digitally averaged shadowgraph visualization of dual impinging jets positioned at $H/D_r = 12$, left, and $H/D_r = 6$, right, with jet operating conditions $M_{j,f} = 1.0$, $M_{j,r} = 1.3$, and total temperature ratio, $T_0/T_{amb}$ ($TTR$)$=1.0$. A standoff shock can be seen in the image on the right.

4.3 Velocity magnitude contours for $H/D_r = 12$ and 6 with unit vectors indicating flow directionality. Open circles denote measurement locations used to generate contours. Note: contour color levels are identical for both plots.

4.4 Axial turbulence intensity contours for $H/D_r = 12$ and 6. Contours are generated using the same points as Figure 4.3.

4.5 Normalized axial velocities and axial turbulence intensities plotted against similarity parameter, $\eta*$, for all lift plate heights and all measurement heights above the ground plane.

4.6 Axial velocity profiles in the impingement zone for plate separations of 6 and 12 $D_r$. 

ix
4.7 Radial velocity profiles in the impingement zone for plate separations of 6 and 12 D_r ................................................................. 70
4.8 Axial turbulence intensity profiles in the impingement zone for plate separations of 6 and 12 D_r ................................................................. 71
4.9 Radial turbulence intensity profiles in the impingement zone for plate separations of 6 and 12 D_r ................................................................. 72
4.10 Schematic diagram of estimated LDV probe-jet misalignment ........ 73
4.11 Mean radial velocity contour and scaled radial velocity profiles in outwash flow region for H/D_r = 12 and M_j = 1.34 ........................................ 75
4.12 Wall jet peak velocity decay compared to the r^{-1} velocity decay radially outwards from a point source .................................................. 76
4.13 Radial turbulence intensity contour and scaled radial turbulence intensity profiles in outwash flow region for H/D_r = 12 and M_j = 1.34 ........ 77
4.14 Mean outwash flow LDV measurements compared to pitot rake data at similar locations for H/D_r = 12 and M_j = 1.34. The radial position of pitot rake measurement is listed in each plot in red ........................................ 78
4.15 Normalized radial velocity and normalized turbulence intensity profiles of the outwash flow for H/D_r = 12 and M_j = 1.34 ........................................ 80
4.16 Blue spectra: Turbulence spectra obtained from LDV measurements for H/D_r = 6, M_j = 1.34, NPR = 2.93, TTR = 1, characteristic frequency, f_c = 31.8 kHz. LDV measurement locations are indicated on each graph. Frequency resolution is 122Hz. Red spectra: Acoustic field measurements taken directly aft of the rear nozzle for the same plate separation and jet conditions. Frequency resolution is 74Hz ........................................ 82


## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Text Matrix for the LDV Impinging Jet Measurements, $NPR_r = 2.927$, $M_j = 1.34$, $TTR = 1.0$ for all cases</td>
<td>30</td>
</tr>
<tr>
<td>3.2</td>
<td>High Speed Jet Aeroacoustics Laser Doppler Velocimeter Characteristics</td>
<td>41</td>
</tr>
<tr>
<td>3.3</td>
<td>Uncertainties of Laboratory Gauges and Transducers</td>
<td>51</td>
</tr>
<tr>
<td>3.4</td>
<td>Tabulated Axial and Radial Velocity Uncertainties</td>
<td>56</td>
</tr>
<tr>
<td>4.1</td>
<td>LDV Measurement Points Used to Produce Nondimensionalized Velocity and Turbulence Intensity Profiles</td>
<td>66</td>
</tr>
</tbody>
</table>
# List of Symbols

- $a$: local acoustic velocity
- $c$: speed of light ($3 \times 10^8 \text{ m/s}$)
- $D$: 'average' nozzle diameter
- $d_{\text{beams}}$: diameter of laser beam
- $D_f$: forward nozzle diameter
- $D_j$: fully expanded jet diameter, or average diameter of the jet plume
- $d_p$: seeding particle diameter
- $d_{pv}$: diameter of LDV probe volume
- $D_r$: rear nozzle diameter
- $F$: local length of lens
- $f$: frequency (in Hz)
- $f_b$: beat frequency of scattered laser light, $f_b = \frac{(f_{\text{up}}-f_{\text{down}})}{2}$
- $f_c$: Characteristic frequency (in Hz) of the rear jet, $f_c = U_{j,th}/D_r$
- $f_D$: particle Doppler frequency (in Hz)
- $f_{\text{down}}$: 'downshifted' frequency of scattered laser light due to particle, $f_{\text{down}} = f - f_D$
- $f_{\text{up}}$: 'upshifted' frequency of scattered laser light due to particle, $f_{\text{up}} = f + f_D$
- $H/D_r$: ground plane to lift plate separation nondimensionalized by the rear nozzle diameter
- $I$: current (Amperes)
- $K$: number of points in single-sided power spectral density estimation, $K = \frac{N_{PSD}}{2}$
- $Kn$: seeding particle Knudsen number
- $\ell_{pv}$: length of LDV probe volume
- $M_a$: acoustic Mach number
$M_c$ convection Mach number of the jet large scale structures
$M_d$ nozzle design Mach number
$M_j$ jet Mach number
$N_{PSD}$ number of data points used for PSD calculation
$N_s$ number of static fringes in LDV probe volume
$N_v$ number of virtual, or virtual, fringes in LDV probe volume
$O$ order of magnitude
$P$ pressure
$R$ resistance (kilo-Ohms)
$Re_p$ seeding particle Reynolds number
$Re_r$ rear jet Reynolds number
$\mathcal{R}_{\text{gas}}$ gas constant
$r/D_r$ radial position, parallel to the ground plane, nondimensionalized by the rear nozzle diameter
$St$ Strouhal Number, $f/f_c$
$T$ temperature
$U$ local velocity (no specific direction)
$\bar{U}$ mean axial velocity
$U_j$ fully expanded jet velocity, or mean exhaust velocity
$u'$ fluctuating axial velocity, $u' = \bar{U} - U$
$u_{rms}$ root-mean-square velocity
$V$ local radial velocity, positive away from the jet centerline
$\bar{V}$ mean radial velocity, positive away from the jet centerline
$v'$ fluctuating radial velocity, $v' = \bar{V} - V$, positive away from the jet centerline
$x/D_r$ downstream axial location nondimensionalized by the rear nozzle diameter
$y/D_r$ height above the ground plane nondimensionalized by the rear nozzle diameter

**Greek**

$\Delta$ change in quantity
\( \delta \) fringe spacing in probe volume
\( \delta_w \) shear layer/vorticity thickness: \( \delta_w = \frac{U(r)_{max}}{(\partial U(\partial r))_{max}} \)
\( \Delta \tau \) time spacing between autocorrelation estimation bins
\( \eta^* \) Nondimensional radius: \( \eta^* = \frac{r-r_{0.5}}{\delta_w} \)
\( \Gamma \) turbulence intensity
\( \gamma \) ratio of specific heats
\( \kappa \) smoothing parameter for LDV fuzzy slotting spectra estimation
\( \lambda \) wavelength of laser light
\( \mu \) dynamic viscosity
\( \nu \) kinematic viscosity
\( k\Omega \) kilo-Ohms (resistance)
\( \psi \) laser beam intersection half-angle
\( \rho \) density
\( \sigma \) uncertainty
\( \sigma_{LDV} \) LDV probe volume uncertainty
\( \sigma_{pos} \) uncertainty of LDV measurements due to probe positioning
\( \sigma_u \) uncertainty of the measured laser Doppler particle velocity
\( \sigma_{turb} \) uncertainty of turbulence intensity
\( \sigma_{u_j} \) uncertainty of the jet velocity
\( \theta \) LDV probe-jet misalignment angle

**Subscript**

0 total or stagnation quantity
0.5 position in a velocity profile where the local velocity is equal to half the maximum velocity measured

\( amb \) ambient quantity
\( core \) quantity relative to the core jet
\( exit \) quantity relative to the nozzle exit
\( rms \) root-mean square quantity
\( f \) front jet
jet quantity

max maximum measured value

meas quantity that is experimentally measured

r rear jet

raw quantity relative to the raw signal data

th quantity that is theoretical, or analytically derived

**Acronyms**

A/D analog-to-digital converter

BBSAN broadband shock associated noise

CAD computer-aided design

CD converging-diverging

CFD computational fluid dynamics

DAQ data acquisition

DEHS di-ethyl-hexyl sebacat

DURIP Defense University Research Instrument Program

FFT fast Fourier transform

FIR finite impulse response

LDV laser Doppler velocimetry

NPR nozzle pressure ratio, $P_0/P_{amb}$

OASPL overall sound pressure level

P2P ratio peak-to-peak ratio

PDF probability density function

PIV particle image velocimetry

PMT photomultiplier tube

PSD power spectral density

RPM revolutions per minute

SNR signal-to-noise ratio
SPL  sound pressure level
STOVL short take-off and vertical landing
TRPIV time-resolved particle image velocimetry
TTR  total temperature ratio, $T_0/T_{amb}$
I want to graciously thank my advisor Dr. Dennis K. McLaughlin for giving me the opportunity to work with him. As a Mechanical Engineering undergraduate, I first met Dr. McLaughlin through a Aerospace Engineering teammate of mine on my multi-disciplinary senior capstone project. Dr. McLaughlin graciously volunteered his time to coach us on everything from careful project planning to proper experimental design. When I approached him with a want to pursue graduate studies, he was more than welcoming and readily brought me on to the team. His practical wisdom and thoughtful analysis inspire me everyday to be a better engineer.

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1 Introduction

1.1 Motivation

The newer generation of fighter aircraft, such as the F-35B, are designed to take off in a short distance, accelerate to supersonic flight speeds, and land vertically if necessary. With the F-35B, the vertical thrust is provided primarily via the high-temperature, high-speed jet exhausting from the rear nozzle and the cold, sonic lift fan exhaust. A photograph of the F-35B during a vertical landing on a carrier deck is shown in Figure 1.1, obtained from [1]. During a vertical landing, the rear nozzle swivels downward to approximately 90° from the aircraft longitudinal axis and provides thrust equal to that of the lift fan.

Lu and Terrier [2] report the approximate rear nozzle operating conditions of the F-35B at various mission points. The authors go on to state that the nozzle pressure ratio, \( P_0/P_{\text{amb}} \) (\(NPR\)), defined as the ratio of nozzle total pressure to ambient pressure, is approximately 3-4 during cruise, 6-8 during acceleration, and near 2 for hover. The exhaust temperature is also reported to be 780 K in both cruise and hover, while the temperature is up to 2,000 K during acceleration. In order to achieve propulsive efficiency at mission points other than cruise, the nozzle exit diameter is able to expand or contract via a mechanical link with the nozzle throat.

The operational environment surrounding these short take-off and vertical landing (STOVL) aircraft is severely impacted by the complex flow field established when these aircraft hover in close proximity to the ground. Loss of lift causing aircraft suck-down is induced by the lifting jets developing a recirculation region underneath the body of the aircraft resulting in low surface pressures on the aircraft frame. Near the ground, hot temperature engine exhaust may be drawn into the engine inlet adversely affecting engine performance.
At low stand-off distances, aeroacoustic resonances give rise to high amplitude, discrete acoustic tones, known as impingement tones. These tones can cause unsteady loading on the aircraft and be a source of hearing loss for personnel near the aircraft. Of the utmost importance is the need to better understand the high temperature, high velocity flow field generated by these aircraft which gives rise to extremely hazardous conditions for personnel and equipment in the vicinity of the aircraft.

During a vertical landing of an F-35B-like aircraft, the hot supersonic jet exhaust interacting with the ground and the jet produced with the forward fan as well the outwash flow make for a highly complex flow field which current modeling capabilities lack both the accuracy and efficiency to predict. Full-scale data to validate the computations are also very difficult and costly to obtain. Recent efforts led by a U.S. Navy team involved the development of techniques used to measure the full-scale outwash velocity and temperature profiles of the F-35B during a vertical shipboard landing at sea. However, McCarthy [3] reported on the difficulty of engineering the proper instrumentation and hardware support for performing such tests in the full-scale environment. Without the ability to obtain accurate full-scale data, much of the development of future STOVL aircraft will continue to rely on sub-scale model testing.
1.2 Background

1.2.1 Impinging Jet Flow Field

Alvi and Iyer [4] describe three major regions of the imping jet flow field: i) the primary jet plume, ii) the impingement zone - characterized by strong velocity gradients and a turning of the flow, and iii) the wall jet or outwash region. A schematic diagram of the single impinging jet flow field is shown in Figure 1.2. This section outlines the primary characteristics of each major region.

![Schematic diagram of the flow field produced by a single supersonic jet impinging normally on a planar surface](image)

Figure 1.2: Schematic diagram of the flow field produced by a single supersonic jet impinging normally on a planar surface

1.2.1.1 Free Jet Region

Donaldson et al. [5] proposed that the mean properties of the jet plume remain essentially unchanged upstream of any strong local interaction effects that are a result of impingement. As such, this section discusses some characteristic flow features of supersonic free-jets.

The flow issuing from a nozzle is dependent upon the internal geometry of the nozzle, including internal roughness, boundary layer thickness, and thermodynamic properties such as stagnation pressure, $P_0$, and stagnation temperature, $T_0$. Figure 1.3, obtained from Kuo [6], shows a schematic diagram of flow exhausting from an axisymmetric round nozzle. The potential core of the jet can be considered inviscid and is defined as the region where the jet velocity is 99% of the exit velocity. The growth of the shear layer, with downstream distance, corresponds to the decay of the potential core due to entrainment.
of the quiescent surrounding air. The end of the potential core marks the beginning of the transition region where the jet becomes fully turbulent. Further downstream, the jet velocity profiles become self-similar and the jet is considered to be fully developed. The supersonic core length is also shown in Figure 1.3. For supersonic jets, this region denotes where the velocities are greater than the speed of sound. Through particle image velocimetry (PIV) measurements of a $M_j = 1.1$ jet, Andre et al. [7] demonstrated that it is possible for the supersonic core length to extend past the potential core. It was shown the potential core extended to 9 nozzle diameters downstream whereas the supersonic core existed up to 10.5 diameters. Cross-correlating unsteady LDV measurements along the centerline of a supersonic jet with simultaneous microphone measurements, Panda et al., [8] showed the primary noise production region of the jet to be downstream of the end of the potential core and extended for several nozzle diameters.

For accurate acoustic measurements, Viswanathan [9] estimated a minimum Reynolds number of 400,000 must be maintained to avoid low Reynolds number effects. In a follow-on study Viswanathan and Clark [10] conducted a detailed study comparing far field acoustic measurements of three different convergent nozzle of equal exit diameters but different internal contours. Computational fluid dynamics (CFD) analysis showed significant difference in internal boundary layer thickness between the three nozzles. Ultimately it was shown internal contours and boundary layer thickness had a small effect on the noise radiated to the far field. McLaughlin et al. [11] showed that the acoustic field produced by supersonic jets issuing from university-scale nozzles compares extremely well with that produced by moderate-scale jets. As well, Viswanathan [12] in an earlier study reported the acoustic field of moderate-scale jets compares well with jets exhausting from
full-scale nozzles. Further discussion on the aeroacoustics of supersonic jets is reserved for Section 1.2.2.

The nature of large scale turbulence in jet shear layers has been studied extensively. Brown and Roshko [13] were some of the first to discover coherent structures in the shear layer, with transverse length scales on the order of the shear layer thickness. These coherent structures are generated by Kelvin-Helmholtz instability waves. Kelvin-Helmholtz instabilities are the result of a velocity shear or density imbalance between two parallel flows. Large scale disturbances initially form at the nozzle exit and grow with downstream distance; having a characteristic length scale on the order of the shear layer thickness. Tam and Hu, [14] demonstrated analytically and computationally that at high supersonic Mach numbers, the flow can support two other unique instabilities that have supersonic and subsonic phase velocities relative to the ambient gas. Above a critical Mach number, the supersonic instability waves become the dominant instability over the Kelvin-Helmholtz instability waves.

The speed at which these large scale structures convect downstream in the shear layer is classically referred to as the convection Mach number, $M_c$. If the jet is exhausting into quiescent air, the convection Mach number can be defined as the ratio of the convection velocity, $U_j$, to the ambient speed of sound, $a_{amb}$, as shown in Equation (1.1).

$$M_c = \frac{U_c}{a_{amb}}$$ (1.1)

Extensive research has been performed to determine the relationship of the convective velocity to the fully expanded jet velocity. Hot-wire measurements performed by Troutt and McLaughlin [15] showed $U_c = 0.8U_j$ for the dominant turbulence components in a cold $M_j = 2.0$ jet. Norum and Seiner [16] performed least squares fit on far-field microphone data to arrive at $U_c = 0.7U_j$. Bridges [17] used PIV space-time correlations to arrive at $U_c$ between 0.5 and 0.8 within the potential core of subsonic cold and heated jets. Thurow et al. [18] note the wide discrepancy in reported convection velocities. Discrepancies are further complicated by the fact that a wide variety of measurement techniques have been used to measure the convective velocity. The rate of entrainment and growth of the shear layer are both dependent on the convection Mach number [19,20], and it can be used as a means to determine whether or not the shear layer is affected by compressibility. Papamoschou and Roshko [19] observed that compressibility in the shear layer caused its growth in a supersonic jet to be slower than that of a subsonic jet, resulting in an increase potential core length.
1.2.1.2 Impingement Zone

The impingement region can be defined as the region in which the supersonic jet plume begins to be significantly affected by the presence of the ground plane. If the flow remains supersonic close to the ground plane, a standoff normal shock forms in the jet [21–23]. Just above the ground plane, Carling and Hunt [24] and Alvi and Iyer [4] have shown the existence of a low-pressure recirculating stagnation bubble at the center of the jet impingement. This stagnation bubble occurs downstream of the standoff shock. If the shock is sufficiently strong, the centerline pressure drops lower than the pressure towards the edge of the jet, thus trapping the bubble. Alvi and Iyer [4] note the formation of the stagnation bubble appears to be reserved for underexpanded jets beyond a certain degree of off-design operation, however the exact parameters governing its appearance are not yet fully understood.

Davis et al. [25] recently investigated the effect of ground plane separation distance on the instability modes of a jet plume through the use of unsteady pressure sensitive paint. By applying the special paint to the impingement surface, they were able to visualize the unsteady pressure characteristics associated with resonant modes originating in the jet shear layer. At a ground plane spacing of 4 \( D \), they were able to identify purely axis-symmetric modes while at a spacing of 4.5 \( D \), a dominant symmetric mode and sub-dominant helical mode was observed.

1.2.1.3 Wall Jet/Outwash Region

The final major region shown in Figure 1.2 is the wall jet or outwash flow region. For a single jet impinging normally on a surface, the wall jet spreads radially outwards from the point of impingement. Figure 1.4 presents a schematic diagram of the fully-developed wall jet velocity profile. \( V_m \) denotes the maximum velocity in the wall jet and \( y_{0.5} \) marks the vertical position above the ground plane at which the velocity is equal to half the maximum value. The wall-jet velocity profile can be thought of as a superposition of the classic flat-plate boundary layer profile and the fully-developed free-jet velocity profile.

Of practical concern to the U.S. Navy is understanding the velocity and thermal characteristics in the outwash region. The landing of STOVL in close confines such as the flight decks of carriers means personnel and equipments are in close proximity to these high-speed, high temperature flows. Personnel and equipment hazard zones, which must remain clear, are defined based off of wind speeds and temperatures that limits the position
of personnel and objects to a certain distance from the landing aircraft. The restricts a portion of the carrier deck to be unusable.

Carling and Hunt [24] studies the early wall jet using surface pressure and flow visualization measurement. They showed that the presence of the stagnation bubble has negligible effects on the early wall jet flow. Recently, Myers et al. [26] reported measurements in the jet plume and outwash flow of a supersonic jet impinging on a simulated ground plane. High speed flows, near Mach 0.2, are observed in the outwash region out to 12 nozzle diameters.

1.2.2 Acoustic Phenomena in Supersonic Impinging Jets

1.2.2.1 Turbulence-Induced Noise

An early investigation by Troutt and McLaughlin [15] showed large scale coherent structures are direct noise producers in supersonic jets. Tam et al. [27] later provided strong evidence that both large scale and fine scale turbulence were strong producers of noise in supersonic jets. In the upstream, or forward arc spectra are dominated by fine-scale mixing noise produced near the jet exit. At downstream angles, the spectra are dominated by large-scale mixing noise. Large scale structures are efficient noise producers when they convect supersonically ($M_c > 1$) downstream [28]. These large structure produce noise in the form of Mach wave radiation. The direction of the Mach wave radiation can be found by using the convective Mach number, previously defined in Equation (1.1) where relationship
between the angle of emission, $\mu$ (measured from the jet centerline), of the Mach wave radiation and the convective Mach number is given by Equation (1.2).

$$\mu = \arcsin \left( \frac{1}{M_c} \right)$$  \hspace{1cm} (1.2)

Krothapalli et al. [29] provided an excellent overview and review of Mach wave radiation. Seiner et al. [30] showed that the peak noise radiation angle aligns well with the Mach wave angle. Equation (1.2) implies that only jets with supersonic convection velocities produce Mach wave radiation. It has however been shown that jets with subsonic convection Mach numbers sometimes still exhibit Mach waves and have large scale structure noise that has been attributed to Mach wave radiation. The growth and decay of the structures causes the wave number spectrum of the turbulence to be very broad. The result is that the phase velocity of different sized structures can be individually higher at high frequencies than the ambient acoustic velocity. As shown by Veltin et al. [31] this results in some portions of the spectrum producing Mach wave radiation, thus allowing jets with an average convective Mach number that is subsonic to produce Mach wave radiation.

### 1.2.2.2 Shock-Associated Noise

Unique to imperfectly expanded jets, supersonic jets may also produce shock-associate noise. Supersonic jets issuing from conical nozzles or converging-diverging nozzles operating off their design condition contain standing shock waves within the flow. This results from differences in pressure at the nozzle exit from the ambient air which causes shocks (over expanded) or expansion (under expanded) waves at the nozzle exit. The pressure difference at the nozzle exit causes the flow to either expand or contract, and because the flow is supersonic this can only occur through shocks or expansion waves. Shock (expansion) waves propagate through the jet and are reflected from the shear layer as expansion (shock) waves. This standing shock pattern is commonly referred to as shock cells or a shock diamond pattern, and can be seen in the flow visualization image of an over expanded jet in Figure 1.5.

Shock-associated noise is generated primarily through the weak interaction of large-scale coherent structures weakly interacting with the shock cell system [32]. Because of the broad frequency range of structures convecting through the shock cells leads to noise generated over a wide range of frequencies. This is generally referred to as broadband shock associated noise ($BBSAN$)
A second component of shock associated noise is typically referred to as shock screech. Screech is a high amplitude, discrete frequency tone(s) in spectra over all observation angles. Screech tones occur due to a positive feedback loop between the shock-associated noise and the nozzle exit. Upstream traveling acoustic fluctuations excite instabilities of the same wavenumber at the nozzle exit. Seiner [33] performed measurements to better understand the screech phenomena and visualized the effect using a spark schlieren flow visualization. Shen and Tam [34] used numerical simulations to model the frequency and amplitude of the screech tones. Comparisons to experiments were performed with good agreement. These studies also did not notice a strong effect of lip thickness on screech amplitude in contrast to the results reported by Ponton and Seiner [35]. Using a large set of experimental data, Panda [36] developed a semi-empirical relationship predicting the screech tone frequency. The frequency of the first harmonic of the screech tone was shown to be related to the shock spacing and convection velocity by:

\[
f_s = \left( \frac{U_j}{D} \right) \left( \frac{0.67}{\sqrt{M_j^2 - 1}} \right) \left[ 1 + 0.7M_j \left( 1 + \frac{\gamma - 1}{2} M_j^2 \right)^{-1/2} \left( \frac{T_0}{T_{amb}} \right)^{-1/2} \right]^{-1} \tag{1.3}
\]

1.2.2.3 Ground Plane-Nozzle Feedback Resonance

Instabilities traveling in the shear layer of the jet plume may impact the ground plane and generate strong acoustic waves that propagate upstream back to the nozzle exit or lift plate further exciting instabilities of the same frequency. The closed-loop resonant phenomenon produces what is generally referred to as impingement tones. In full-scale applications, these tones may impact the hearing of personnel within the vicinity of the aircraft as well
as generated unsteady loading on the aircraft under-surface. The production of these tones has been the subject of many studies over the past three decades.

Krothapalli et al. [37] made use of high-speed shadowgraph visualization to provide evidence that upstream-propagating acoustic waves originated in the stagnation region of the jet. Acoustic waves reflected off the lift plate were seen to propagate into the quiescent air. The findings of [37] are in agreement with the earlier model of impinging jet tone production proposed by Tam and Ahuja [38]. It is largely agreed upon that impingement tones are initiated by large scale instabilities traveling in the jet shear layer. How the feedback loop is closed remains under investigation. Tam and Ahuja [38] developed a theoretical model for high subsonic, cold jets in an attempt to predict whether the loop is closed by feedback waves traveling up the jet column or acoustic fluctuations propagating through the quiescent medium. While the results were not entirely conclusive, current evidence at the time suggested the loop could be closed within the jet column. This directly contrasted the previous experimental studies by Ho and Nosseir [39] and Nosseir and Ho [40] who found the feedback path to be outside of the jet plume for high subsonic jets. These works did not offer an explanation for how the feedback loop is closed in the case of a supersonic jet.

In a more recent study by Henderson et al. [22] of supersonic impinging jets, time-resolved particle image velocimetry (TRPIV) and shadowgraphy was used to provide strong evidence of the feedback loop being closed outside of the jet column. In short, TRPIV showed large scale instabilities displacing the stand-off shock. This displacement resulted in distortions of the impingement flow and quiescent regions. These distortions propagated out to the wall jet causing it to oscillate. Phase-locked shadowgraph images showed a sound wave originating from the boundary between the oscillating wall jet and ambient medium propagating back to the nozzle lip, thus closing the feedback loop. This studied emphasized the earlier work of Powell [21] who showed standoff shock oscillations as a source of impingement tones. However, no explanation for tone production in the absence of a standoff shock was offered. From the above discussion, it is evident further investigation into the impingement tone phenomena is required.

1.3 High-Speed Free and Impinging Jet Measurements

Both mean and unsteady measurements using a wide variety of measurement techniques have been employed over the decades to study high-speed free and impinging jet flows.
With current advancements in technology, more exotic flow diagnostics, such as plenoptic PIV [41] may become more common in the future. This section reviews previous studies employing the more classical and common flow diagnostics.

### 1.3.1 Hot-Wire

Morris and Zaman [42] performed hot-wire measurements at varying positions along the centerline and lip line of a jet to compare second and fourth order correlations to determine corresponding length scale. However, this study was confined to a Mach 0.25 jet. McLaughlin et al. [43] made hot wire measurements in supersonic jets. This study was limited to low Reynolds numbers achieved by low density flows. Hot-wire measurements in most supersonic free and impinging jet flow fields is impractical in atmospheric density flows due to the fragile wire sensors.

### 1.3.2 Unsteady Pressure and Acoustic Field Measurements

Alvi and Iyer [4] focused on small ground plane separations less than 6 nozzle diameters with a single nozzle flush mounted to a circular lift plate. They found that while the lift plate does not alter the nature of the flow field, it does influence the magnitude of surface pressures. Spectral analysis of unsteady surface pressure measurements showed pressure peaks that agreed well with impingement tones measured by acoustic field microphones. However, other spectral peaks in surface pressures were not measured in the acoustic field indicating the presence of non-dominant, localized events. Outwash flow measurements indicated that self-similarity was achieved at larger radial distances from the jet centerline as the jet velocity increases. Peak wall outwash velocities were found to decrease as lift plate height increased.

Near and far field acoustic measurements of heated supersonic jets impinging in multiple configurations were performed recently by Worden et al. [44]. For isothermal jets, (298 K) impinging normally, overall sound pressure level (OASPL) in the near field were measured to be as high as 144 dB. Peak far field OASPL was observed to be 112 dB. OASPL was seen to increase on average by 5 dB when the jet temperature was increased from 298 K to 1200 K. A maximum of 11 dB OASPL increase was observed in the far field for a ground plane separation of twelve nozzle diameters.
1.3.3 Pitot Probe / Pitot Rake

A practical method of interrogating the velocity field of high speed jets is through total pressure, or pitot probe measurement. A linear, side-by-side, array of pitot probes on a single traversing arm is generally referred to as a pitot rake. Pitot probes offer a relatively quick and solution to velocity measurements in high speed flows. If the pitot tube is long enough such that the pressure transducer is far enough from the tube opening pitot probes can be well-suited for velocity measurements in high-temperature applications. However, compared to hot-wire and unsteady pressure sensors, pitot probe measurements are only able to obtain time-mean pressure measurements. Compared to hot-wire and optical techniques, pitot probes generally have a larger probe size and as such are not capable of as fine spatial resolution. In supersonic flow applications, one must remain cognizant of the shockwave that is established ahead of the pitot tube. Improper accounting of this will lead to erroneous results.

That said, pitot rake measurements have seen use in university-scale supersonic jet applications in recent years. Powers et al. [45] performed pitot rake measurements in the jet plume of an overexpanded $M_j = 1.36$ jet issuing from a 18 mm converging-diverging (CD), faceted nozzle. Measurements were also taken in the jet plume of similar nozzles, one with distributed blowing in the divergent section and the other with hardwall corrugations (first reported by Seiner et al. [46]). Both of which are designed to achieve noise reduction. Interestingly, it was discovered that both noise reduction technologies modified the cross-sectional shape of the jet and disrupted the shock cell structure. As previously mentioned, Myers et al. [26] performed a pitot rake survey of the jet plume and outwash flow of a supersonic jet impinging on a simulated ground plane.

1.3.4 Optical Measurements

1.3.4.1 Classic Particle Image Velocimetry

Krothapalli et al. [37] studied a Mach 1.5 jet flush mounted to a circular lift plate. Lift plate height above the ground plate was varied from 2.5D to 60D (where D is the nozzle exit diameter). PIV measurements showed large scales in the jet plume traversed laterally into the outwash region upon impinging with the plate without losing their coherence. For a ground plane two nozzle diameters below the lift plate, suck-down forces were measured to be as high as 60% of the primary jet thrust. A staging effect of the impingement tone
was observed in near field spectra with increasing lift plate height. For small increases in lift plate height, the observed tone decreased slightly. Further increases in plate separation caused the impingement tone to jump to a different frequency. Tones observed in the spectra were seen in the absence of a stand-off shock. Henderson and Powell [47] had suggested in an earlier study that stand-off shock oscillations caused by large scale instabilities in the jet column had an important role in the production of tones.

The findings of Krothapalli et al. [37] are in agreement with the earlier model of impinging jet tone production proposed by Tam and Ahuja [38]. It is largely agreed upon that impingement tones are initiated by large scale instabilities traveling in the jet shear layer. How the feedback loop is closed remains under investigation. Tam and Ahuja [38] developed a theoretical model for high subsonic, cold jets in an attempt to predict whether the loop is closed by feedback waves traveling up the jet column or acoustic fluctuations propagating through the quiescent medium. While the results were not entirely conclusive, current evidence at the time suggested the loop could be closed within the jet column. No explanation was offered for how the feedback loop is closed in the case of a supersonic jet.

1.3.4.2 Time Resolved Particle Image Velocimetry

TRPIV has been used extensively by many other research groups to characterize the mean velocities of jet exhausts, as described and documented in detail by Bridges and Wernet [48, 49]. Bridges and Wernet [49] utilized TRPIV to obtain mean velocity and velocity spectra in heated supersonic free jets. The benefit of TRPIV is the ability to acquire data over large portions of the flow field compared to other measurement techniques. However, due to the large amount of data throughput required for these systems, the velocity field is currently limited to sampling rates no higher than 10-25 kHz. For the moderate scale jet used in their study, this allowed the spectral velocity field to be resolved to about a Strouhal number, $St$ of 3. However, Henderson et al. [22], successfully performed TRPIV and shadowgraphy measurements to explore the production of impingement tones in supersonic impinging jets.

Single-point measurements including molecular Rayleigh scattering and LDV have the potential to obtain velocity measurements at considerably higher data rates. This is particularly important for small jet experiments typical of university research facilities. Panda et al. [8] made use of molecular Rayleigh scattering to investigate turbulence statistics. Unfortunately this technique has seen limited application because of the high
level of difficulty and time commitments to successfully apply the technique. There are also additional limitations when applied to high speed and potentially heated, jet flows.

1.4 Laser Doppler Velocimetry

1.4.1 Previous Applications of LDV in Supersonic Jets

LDV systems have been used in the past to obtain mean flow measurements and unsteady measurements in supersonic jet exhausts. The effects of Mach number on axisymmetric jet characteristics were studied by Lau et al. [50]. In a follow-up study, Lau [51] investigated the effects of both temperature and Mach number on mean flow and turbulence characteristics of an axisymmetric jet. Lau [52] and Kerherve et al. [53] performed correlation measurements in unheated jets. Recently Brooks et al. [54] presented turbulence statistics obtained from measurements made in cold and heated over-expanded supersonic jets. Using Doppler Global Velocimetry, a derivative of the LDV measurement technique Ecker [55] presented 2nd and 4th order spatiotemporal correlations in addition to 4th order statistics in perfectly expanded cold and heated jets.

1.4.2 Overview of LDV Measurement Technique

A laser Doppler velocimeter is a non-intrusive optical instrument designed to measure flow velocities. The LDV principle is based on measuring particle velocities through the probe volume. Particles must be sufficiently small to accurately follow the flow and must have neutral buoyancy to avoid gravitational effects. For LDV measurements in air the flow must be thoroughly seeded with small particles. Further discussion on flow seeding is reserve for Section 3.5.2.

A single-point, single-component LDV creates the probe volume by crossing two laser beams of equal wavelength, $\lambda$, and coherence. Ideally the probe volume should be as small as possible to achieve a high signal-to-noise ratio (SNR) and good spatial resolution. A schematic diagram of two beams crossing to form a probe volume is shown in Figure 1.6a. A particle traveling at velocity $U$ is shown entering the probe volume in the diagram as well. A projected interference patter from the LDV system used in this study is shown in Figure 1.6b. In real application, laser beams generally have a Gaussian distribution
of power through the cross-section of the beam. This leads to the probe volume actually having an ellipsoid shape as noted by the black ellipse in Figure 1.6a.

![Figure 1.6a: Schematic diagram of the LDV probe volume created by the crossing of two laser beams of equal wavelength and coherence](image1)

![Figure 1.6b: Projected interference pattern from the probe volume used in this study](image2)

Figure 1.6: Schematic diagram of probe volume and projected probe volume

Because the two beams are coherent, constructive and destructive interference of the light waves creates a fringe pattern, with spacing $\delta$ between the fringes. The existence of this fringe pattern is integral to the LDV technique as will be highlighted later in the discussion. If the two beams were not coherent, such as if they were produced by two different lasers, no fringe pattern would be established.

The probe volume is formed from two initially parallel beams focused by a lens with focal length, $F$. As the beams are of finite size, the beams themselves get focused by the lens as well. At the focal point of the lens the beams cross and the beam diameter, $d_{beams}$ reaches a minimum known as the waist diameter. Knowing the beam diameter, lens focal length, and the wavelength of the light, the waist diameter may be determined by:

$$d_{waist} = \frac{4F\lambda}{\pi Md_{beams}}$$  \hspace{1cm} (1.4)

where $M$ is the beam expander magnification factor. For a beam not going through an expander, $M = 1$. Also indicated in Figure 1.6a are the beam half-angle, $\psi$ and fringe spacing. The half-angle and theoretical fringe spacing, $\delta_{th}$, are give by:

$$\psi = \tan^{-1}\left(\frac{d_{beams}}{2F}\right)$$ \hspace{1cm} (1.5)
\[
\delta_{th} = \frac{\lambda}{2 \sin \psi}
\]  

(1.6)

Using Equation (1.4), the length, \( \ell_{pv} \), and diameter, \( d_{pv} \), of the probe volume may be readily determined from geometric relations as:

\[
\ell_{pv} = \frac{d_{waist}}{\sin \psi}
\]  

(1.7)

and

\[
d_{pv} = \frac{d_{waist}}{\cos \psi}
\]  

(1.8)

Now, the total number of static fringes in the probe volume, \( N_s \), is given by:

\[
N_s = \frac{d_{pv}}{\delta_{th}}
\]  

(1.9)

By inspection of Equations (1.7) and (1.8), the size of the probe volume is dependent on the beam waist diameter and, by extension, the focal lens of the lens. A short focal length gives the advantage of a short probe volume and waist diameter. The trade-offs being a larger volume diameter and the optical hardware will be in close proximity to the flow. In the case of a high-speed flows mixed with seeding particles, great caution must be taken to not damage the hardware or obscure the optics with condensed seeding particles. A longer focal length yields a smaller diameter probe volume as well as allows the sending hardware to be further removed from the flow field but at the sacrifice of a longer probe volume. An additional advantage of a shallower beam crossing is that away from the probe the beams are close together which allows for measurements closer to physical surfaces than if the beams crossed at a steep angle.

As the name indicates, LDV takes advantage of the Doppler effect. Each laser beam has a frequency, \( f \), related to the wavelength by \( f = c/\lambda \). For green light of wavelength 514.5 nm, this corresponds to a frequency of 583 THz (583 \( \times \) 10\(^{14}\) Hz). As a particle passes through the probe volume, it scatters light back to the photodetector, generally a photomultiplier tube (PMT), as indicated in Figure 1.6a. The light detected by photodetector is referred to as a "burst." A time-series signal of an ideal particle burst measured by a photodetector is shown in Figure 1.7. The ideal signal is a superposition of a Gaussian function and a sine wave. The Gaussian (DC-component) of the signal is due to the Gaussian distribution of power across the beam diameter. The AC component (sine wave) is generated by
Figura 1.7: Ideal time-series signal of a particle light burst traveling through the probe volume

the particle scattering light from the fringes. As a particle with speed, \( U \), normal to the fringes, traverses the probe volume, the frequency of the light bursts is proportional to the particle’s velocity [56] by:

\[ f_D = \frac{U}{\delta_{th}} \]  

(1.10)

Where \( f_D \) is referred to as the Doppler frequency and the time between bursts, \( \Delta t \), is related to \( f_D \) by \( \Delta t = f_D^{-1} \). This is referred to as the Doppler frequency is because the frequency of a light beam is altered by \( \pm f_D \) depending on whether the particle is traveling towards or away from a beam.

Referencing Figure 1.6a, the particle is moving toward "Beam 1" so the scattered light from this beam is "upshifted" by \( f_D \), where \( f_{up} = f + f_D \). The particle is moving away from "Beam 2" and therefore the scattered light from this beam is "downshifted" by \( f_D \), where \( f_{down} = f - f_D \). As mentioned before, the frequency of light (even after "downshifting") is still on the order of terahertz. This is far too fast for any practical data acquisition (DAQ) system to handle. However, compared the frequency of light, the Doppler frequency is quite small and therefore \( f_{up} \approx f_{down} \) with both have nearly identical amplitude. When these waves are superimposed, the amplitude of the resulting signal slowly rises and falls. This slow rise and fall, or modulation, is known as the beat frequency, \( f_b \). This beat frequency is given by:

\[ f_b = \frac{(f_{up} - f_{down})}{2} = \frac{(f_D + f_D)}{2} = \frac{2f_D}{2} = f_D \]  

(1.11)
and is much slower than the frequency of a light wave. In application, $f_{\text{up}}$ and $f_{\text{down}}$ are superimposed within the PMT, so $f_b$ may be easily resolved by the DAQ computer.

By inspection of Equations (1.10) and (1.11), the Doppler (beat) frequency only depends on the magnitude of the velocity but contain no information on the direction the particle is traveling. To circumvent this ambiguity, an acoustic-optical device, more commonly known as a Bragg cell, is used to "upshift" the frequency of one of the beams. In doing so, the fringe pattern moves at a speed

$$V_{\text{fringe}} = (f_{\text{Bragg}})(\delta)$$

(1.12)

in the direction of the higher frequency beam to the lower frequency beam. Now, even when there is no particle in the probe volume, the PMT consistently detects a signal generated by the Bragg cell frequency shift. If a particle moves against the moving fringe pattern with speed $+U$, the PMT detects a burst frequency of $f = f_D + f_{\text{Bragg}}$. Any detected frequency above that of the Bragg cell frequency indicates a positive velocity. In the same fashion, a particle moving with the fringes with speed $-U$, produces a burst frequency of $f = f_D - f_{\text{Bragg}}$. Therefore, any frequency less than $f_{\text{Bragg}}$ indicates a negative velocity.

With a moving fringe pattern, the number of fringes a particle sees is a function of its velocity and the Bragg cell frequency. Consider a particle with arbitrary velocity $\vec{U}$ and a moving fringe pattern with fringe velocity $\vec{V}$. The Doppler frequency shift in the scattered light is:

$$f_D = \frac{\vec{U} \cdot \vec{V}}{\delta \|\vec{V}\|}$$

(1.13)

And the total number of fringes the particle effectively encounters, or virtual fringes, $N_v$, is the number of static fringes, $N_s$ plus an additional number of fringes based on the ratio of the Bragg cell frequency to the Doppler frequency. For a particle moving though a moving fringe patter, the total number of fringes it encounters is:

$$N_v = N_s + \left(\frac{f_{\text{Bragg}}}{f_D}\right) N_s$$

(1.14)

Compare Equations (1.12) and (1.13) to the second term on the left hand side of Equation (1.14). Another interpretation of this fringe-scaling term is the ratio of the fringe speed to the particle speed in the direction of the fringe motion.
1.5 Scope of Thesis

For the past two years experiments have been underway in the High Speed Aeroacoustics Laboratory at Penn State University of model scale impinging jets. Measurements have been conducted over 10 years for both near-field and far-field acoustic investigations. In 2012, Penn State was awarded two Defense University Research Instrument Program (DURIP) grants sponsored by Office of Naval Research to 1) acquire the experimental instrumentation for supersonic impinging jet flow field and noise source measurements, and 2) to develop an LDV system to study the flow field of supersonic jet exhausts. The focus of this thesis is to qualify the high-speed LDV system for unsteady velocity measurements throughout the flow field produced by a single supersonic jet impinging on a simulated ground plane.

1.5.1 Research Objectives

The goal of this research is to develop the capability of a hybrid fiber-optic LDV system to perform time-resolved measurements in the supersonic impinging jet flow. This includes determining the two-dimensional mean velocity field in the impingement zone of the jet and the turbulence properties in the three defined regions of the flow field: (i) the free jet, (ii) the impingement zone, and (iii) the outwash flow/wall jet. The goal of this thesis will be realized through the completion of the objectives restated below:

1. Integrate the laser and fiber-optics components to develop a viable system to measure two-components of the velocity field

2. Develop a series of MATLAB codes to perform the following tasks:
   
   (a) Process raw LDV velocity data to determine mean and \( rms \) velocities and write to EXCEL file and view velocity histograms
   
   (b) Plot mean velocity and turbulence intensity profiles
   
   (c) Determine turbulence spectra using the 'fuzzy-slotting' technique

3. Perform non-simultaneous two-component mean and unsteady velocity measurements in the supersonic jet plume, impingement zone, and outwash flow. Mean velocity measurements in the jet plume and wall jet are compared to previous pitot probe measurements at the same locations.
4. Estimate turbulence spectra for both axial and radial velocity fluctuations at various locations in the flow field and compare to previous acoustic field measurements.

5. Quantify the accuracy of the mean and unsteady LDV measurements in the jet plume and wall jet.

It is the longer term goal to use the data presented in this thesis and that of future studies to guide the development of computational models being assembled by partners in the U.S Navy.

1.5.2 Thesis Synopsis

The remainder of this thesis will address the research goal and subsequent objectives as recently described. Chapter 2 will give a detailed overview of the Penn State High Speed Jet Aeroacoustics facility. An in-depth description of the dual impinging jet model will be given as well. In Chapter 3 the technical approach used to complete all stated objectives is given. That chapter begins with the nozzle operating conditions and matrix of test points. An overview of all experimental techniques used in thesis are presented with the major focus being on the LDV system and modifications made to the sending-receiving prob for near-ground measurements. Data processing methodology is reviewed and then Chapter 3 concludes with a uncertainty analysis of the LDV system.

Chapter 4 presents the major experimental results of this thesis. Mean and unsteady supersonic jet plume data are first presented flowed by Mean and unsteady measurements in the impingement zone. Following this, a discussion is held on estimated the amount of misalignment of the probe and the jet. Chapter 4 concludes with mean and time-varying measurements in the jet outwash as a presentation of turbulence spectra obtained throughout the flow field. The thesis concludes with Chapter 5. A restatement of the research goal and objectives will be made. Major highlights, which address the research objectives, will be reviewed.
2 | Facility Description

2.1 High Speed Jet Aeroacoustics Facility Overview

All experiments presented in this study were conducted in the Pennsylvania State University High Speed Jet Aeroacoustics Laboratory. This facility is a $5.02 \times 6.04 \times 2.79$ m ($16.5 \times 20 \times 9$ ft) anechoic chamber covered in fiberglass wedges with a theoretical cutoff frequency of 250 Hz. A top-view schematic diagram of the laboratory is shown in Figure 2.1.

![Figure 2.1: Top-view schematic of the Pennsylvania State University High Speed Jet Aeroacoustics Facility.](image)

A high pressure air supply plumbed into the facility. At present the facility can operate in two modes: (i) horizontal, free jet and, (ii) dual vertical impinging jet. In the horizontal jet mode, the high pressure air exhausts into the chamber via a 2 m (78.75 in) cylindrical, 11.4 cm (4.5 in) inner diameter plenum in the side wall of the chamber. For this study, the chamber was operated exclusively in the impinging jet configuration. For the vertical impinging jet configuration, the source air is fed into the chamber through the ceiling by
way of two pipes. Further discussion on how the source air interfaces with the impinging jet experimental model is reserved for Section 2.3.

The laboratory is equipped with the capability to simulate head wind and cross wind effects in both the horizontal and impinging jet configurations. The forward flight stream generates a co-flow parallel to the horizontal jet plenum to mimic adverse wind conditions a full-scale aircraft might experience. An inlet fan pulls in outside air through a muffler and acoustically treated, rectangular duct. An extension nozzle can be applied to the duct to yield an open jet wind tunnel in the facility with a $38 \times 38$ cm ($15 \times 15$ in) cross-section. The co-flow was not operated during this study. Detailed discussion of the forward-flight system can be found in [57, 58] Opposite the forward-flight duct, an exhaust collector and fan captures jet exhaust to minimize air recirculation within the chamber. During testing, the exhaust system was operated around 7% max power to provide gentle, yet effective suction to counter the build-up of extraneous seeding particles within the chamber without noticeably altering the impinging jet flow-field. More discussion on flow seeding is addressed in Section 3.5.2.

2.2 High Pressure Air Supply

Air for the facility is supplied by a CS-121 compressor and conditioned with a KAD-370 air dryer, both manufactured by Kaeser Compressors. The air is stored in two $28.3 \text{ m}^3$ ($1000 \text{ ft}^3$), tanks to 1.31 MPa (190 psi) at room temperature. The air passes through a Leslie control valve where the supply pressure is regulated down to the desired value. Downstream, the air flow is controlled using a series of pressure regulators and control valves located within a piping cabinet. The cabinet controls the supply air to the plenum and/or impinging jets within the chamber.

The facility is equipped to run simulated hot jets up to a $TTR$ of 3.0. For heat-simulation measurements three helium cylinders pressurized to approximately 14.5 MPa (2100 psi) separately feed into the pressure control cabinet. In order to simulate the flow and acoustic properties of a hot jet, helium is mixed with the air, following a procedure developed by Doty and McLaughlin [59], to alter the effective gas constant, $R_{gas,mix}$, and specific heat ratio, $\gamma_{mix}$ of the mixture such that the acoustic velocity of the mixed gas is equal to that of a heated air jet. Adjustment of a system of piping and valves while monitoring pure air and mixture pressures allow for the accurate mixing of the gases. Figure 2.2 shows a diagram of the piping control cabinet as described. Kinzie and McLaughlin [60]
demonstrated that this technique is able to capture the dominant noise characteristics of actual heated jets. This has also been demonstrated by Doty and McLaughlin [59] and Papamoschou [61]. Recent careful comparisons [11] between the measurements conducted at Penn State with the measurements performed in other facilities have shown very good agreement as well.

![Figure 2.2: Schematic of the High Pressure Piping Control Cabinet in the High Speed Jet Aeroacoustic Laboratory. Pure air delivery is depicted with blue lines while red lines depict helium gas delivery.](image)

Currently the facility is capable of exhausting pure air jets at stable flow conditions through 1.78 or 2.54 cm (0.7 or 1 in) nozzles operating at $M_j = 1.5$ continuously for 50 or 7 min, respectively. The maximum achievable jet Mach number for a 1.78 or 2.54 cm (0.7 or 1 in) exit diameter nozzle is $M_{j,max} = 2.3$ or 1.7.

### 2.3 Impinging Jet Experimental Setup

The impinging jet experimental model was designed to be representative of a generic military-style STOVL tactical aircraft, such as the F-35B, in hover. The experimental model consists of two vertical, stainless steel nozzles embedded in a rectangular lift plate of dimensions $15.2 \times 22.9 \times 0.64$ cm ($6 \times 9 \times 0.25$ in). In lieu of a complex, aircraft-specific geometry, the flat, rectangular plate is used to mimic a generic aircraft underside. Using a simple experimental geometry also facilitates the grid generation process for a CFD study being conducted in parallel by the U.S. Navy [62]. The midpoint of the nozzles is
located 1.05 \( D_r \) aft of the midpoint of the lift plate. Complete schematic diagrams of the model are shown in Figures 2.3a and 2.3b and a photograph of the underside of the model is shown in Figure 2.3c. The coordinate system for this study is formally described in Section 3.1.2. Note that no attempt was made to model the roll-post jets present on the F-35B. While these jets certainly have an effect on the flow-field, they remain outside the scope of this study.

Visible on the underside of the lift plate in Figures figs. 2.3b and 2.3c are a sparse matrix of holes. The lift plate was instrumented with 32 pressure taps. A Honeywell ASDX \( \pm 1 \) psi differential pressure transducers was connected to each tap via Tygon\textsuperscript{®} tubing. The layout of the pressure taps was designed to provide a sufficient spacial resolution so as to accurately estimate the integrated pressure profile on the underside of the lift plate as well as capture the effect of the fountain flow interaction with the lift plate. Surface pressure measurements are outside the scope of this thesis. Further details on surface pressure measurements were presented in length by Myers et al. [63].

Figure 2.3: Schematic diagrams and photograph of impinging jet model with dimensions and nozzle design parameters. Note (a) and (b) are in opposite orientations.
The forward nozzle is a contoured, convergent nozzle with an exit diameter, $D_f$ of 13.5 mm (0.53 in) flush mounted with the underside of the lift plate. The jet produced by this nozzle is designed to be representative of the exhaust produced by the lift fan on the F-35B. The reader is reminded that this nozzle was not operated during this study; only the rear nozzle was operational. The rear nozzle is a contoured de Laval nozzle with an exit diameter of $D_r$ of 11.9 mm (0.47 in) and design Mach number, $M_d$ of 1.34. The rear nozzle protrudes $1D_r$ below the surface of the lift plate. This is designed to mimic the nozzle on the F-35B, which swivels downward when the aircraft is in hover. A photograph of the F-35B in hover is shown in 2.4.

![Figure 2.4: Photograph of the F-35B in hover configuration just after landing. The rear nozzle can been seen to protrude approximately one diameter below the aircraft](image)

Both nozzles interface with the experimental model via two vertical, schedule 80, $\frac{3}{4}$ NPT black steel pipes. Each "plenum" pipe is 50.8 cm (20 in or $\sim 43D_r$) in length. Source air is delivered to the black steel pipes through two flexible hoses running to two feed lines, on the ceiling of the anechoic chamber. The forward nozzle is fed by a, schedule 40 2 NPT (inner diameter 5.25 cm (2.067 in)) steel pipe while the rear nozzle is fed by a 1.9 cm (0.75 in) rubber hose.

Operating conditions for each jet are measured with a static pressure port and a total pressure probe installed in each plenum pipe. The static pressure ports are installed approximately 22 cm (8.66 in or $17D$) upstream of the nozzle exits. Each total pressure probe is installed approximately 28 cm (11 in or $22D$) upstream of the nozzle exit. Each probe is connected to a Setra 205-2 100 psi pressure transducer. A photograph of the impinging jet plenum piping, static pressure ports, and nozzles is shown in Figure 2.5.

The three wooden ribs around the model are symmetric NASA16021 airfoil shapes. A thin, aluminum shroud skin is attached to these ribs via six wood screws per rib. This shroud is designed to shield the plenum piping and pressure port tubing in the presence of
oncoming flow from the forward flight duct. The entire model is rigidly attached to two angle iron supports at the top of the chamber.

Figure 2.5: Close-up photograph of the dual impinging jet model installed in the anechoic chamber with shroud removed to show piping

An aluminum plate, with a smooth, machined, surface, of dimensions $43 \times 61 \times 1.27 \text{ cm}$ ($17 \times 24 \times 0.5 \text{ in}$) was used to simulate a ground plane. The ground plane was attached to four T-slot 80/20® rails. The rails were attached to two angle iron supports via rail clamps. This setup allows the distance between the ground plane and lift plate to be adjusted from $6-23D_r$. A photograph of the entire impinging jet model with shroud and ground plane installed in the chamber is shown in Figure 2.6.

Figure 2.6: Photograph of the anechoic chamber with full impinging jet model installed. The exhaust duct can be seen on the left of the image. The microphone boom can be seen in the background.
3 Experimental Methods, Data Acquisition, and Processing

3.1 Experimental Parameters

All operating conditions are monitored with LabVIEW. Current atmospheric conditions (i.e. relative humidity, temperature, and pressure) are recorded prior to each set of experimental runs and input into LabVIEW to ensure accurate jet operation. More details on the LabVIEW code used are discussed in [58].

3.1.1 Impinging Jet Operational Conditions

To simplify this preliminary LDV survey of impinging jet flows, only the rear nozzle was operated; the front jet was turned off. Because of the inherent point-nature of LDV measurements, and strong 3-dimensionality of the impinging jet flow field, only a single nozzle pressure ratio was tested and measurements were only taken in the plane containing the jet axis. The rear jet was operated at its design Mach number \((M_d)\) of 1.34 corresponding to a NPR of 2.93. Only cold, \((TTR = 1.0)\) jets were run. Thus, the rear jet has an exit static temperature of 201 K (-72 °C), exit Reynolds number, \(Re_r\) of approximately 602,000, and a sonic velocity of 294 m/s (635 mph). These operational conditions yield a theoretical jet exit velocity, \(U_{j,th}\) of 380 m/s (850 mph). The characteristic frequency of the jet, based on \(U_{j,th}\) and \(D_r\) (assumed to be equal to the fully-expanded jet diameter when running 'on-design'), is 31.8 kHz. It should be noted that due to boundary layer growth within the nozzle, the nozzle is effectively underexpanded to a small degree. Operating on or very near the nozzle design Mach number keeps shock strength to a minimum thus minimizing seeding particle lag behind shocks. This will be discussed in more detail in Section 3.5.2.
Azimuthal angles for microphone measurements around the impinging jet model are measured with respect to the centerline of the lift plate, with \(0^\circ\) being aft of the rear nozzle. The vertex for all angles is coincident with the rear nozzle. This coordinate system is shown in Figure 3.4a. All LDV measurement locations are nondimensionalized by the rear nozzle diameter, \(D_r\). A two dimensional coordinate system is used, however, the datum for the coordinate system varies as will be discussed. The lift plate height above the ground plane (otherwise referred to as plate separation) is \(H/D_r\). For jet plume measurements the datum for the coordinate system is the center of the nozzle exit. Axial distance downstream of the nozzle exit is \(x/D_r\). For impingement zone and outwash flow measurements, a more appropriate datum is the point on the ground plane directly in-line with the center of the nozzle. Height above the ground plane is \(y/D_r\). In all cases, radial position is positive to the right (i.e. towards an azimuth angle of \(0^\circ\)) and is denoted as \(r/D_r\). \(r/D_r = 0\) represented the centerline of the jet issuing from the rear nozzle. In order to ease in physical interpretation of the data presented, radial velocities directed away from the jet axis are positive whereas negative radial velocities are oriented towards the jet axis. Figure 3.1 presents a diagram of the coordinate system used in this study.

![Coordinate system used for impinging jet LDV measurements](image)

Figure 3.1: Coordinate system used for impinging jet LDV measurements

LDV measurements were taken in the impinging jet flow field at plate separations of 6, 12, and 23 \(D_r\). A plate separation of 23 \(D_r\) was expected to see little to no ground plane effects and thus act as a "free jet". Single component, axial velocity measurements, at all plate separations, were taken in the supersonic jet plume where the flow is largely one-dimensional. Radial surveys of the jet plumes employed a high spacial resolution:
0.1 to 0.2 \( D_r \) between measurement points. For all plate heights, measurements were kept within a radial span of -1.2 to 2 \( D_r \). However, for \( H/D_r = 12 \), the wall jet was interrogated out to a radial distance of \( r/D_r = 12 \) in 1 \( D_r \) increments. Measurement height above the ground plane were logarithmically spaced between 0.017 to 1 \( D_r \) so as to take full advantage of the small LDV probe volume. In the wall-affected portions of the flow two-component, axial and radial velocities were measured \( H/D_r = 6 \) and 12. Only a single velocity component was measured at a time. Figure 3.2 presents a schematic diagram of measurement locations in the study for plate heights of 6 and 12 \( D_r \). Furthermore, measurement locations for all plate separations are listed in Table 3.1. The right-most column of Table 3.1 denotes which data used for producing Figure 4.5 in Section 4.1.1

![Diagram of all LDV measurement locations for plate separations of 12 and 6 \( D_r \).](image)

(a) LDV measurement locations for \( H/D_r = 12 \)  
(b) LDV measurement locations for \( H/D_r = 6 \)  

Figure 3.2: Diagram of all LDV measurement locations for plate separations of 12 and 6 \( D_r \). Note the different scales for each figure. Black o’s denote axial velocity component measurements. Filled, red dots denote radial velocity component measurements.
Table 3.1: Text Matrix for the LDV Impinging Jet Measurements, \(NPR_r = 2.927, M_j = 1.34, TTR = 1.0\) for all cases

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<th>(y/D_r)</th>
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\(^a\)Radial positions listed as Start Position:Increment:End Position. Individual measurement positions are comma separated.
3.2 Schlieren/Shadowgraph Flow Visualization

Schlieren and shadowgraph imaging was used to visualize the impinging jet flow field. Both are light refraction techniques that exploit density gradients in compressible flows as a means to visualize the flow fields. The two techniques are nearly identical in nature except that schlieren visualization utilizes a knife edge in front of the camera to block some of the incoming light as a means to achieve higher sensitivity to small density changes within the flow. Shadowgraphy is better suited for visualizing strong, abrupt density gradients such as shock waves. The light-dark contrast in schlieren imaging is proportional to the first derivative of density whereas shadowgraphy contrast is proportional to the second derivative of density. Extensive details on these techniques are described in Settles [64]. All flow visualization images presented in this thesis were captured using shadowgraph imaging.

The High Speed Jet Aeroacoustics Facility is equipped with a z-type schlieren and shadowgraph system. Figure 3.3 presents a schematic diagram of the schlieren/shadowgraph system used in this study. A strobe light is used as the light source. The strobe light is a short duration spark light with a xenon lamp, specifically Spectralite Model 900 manufactured by Spectrum Dynamics. The strobe light pulse rate is controlled using an Agilent Model 33220A signal generator. A lens and slit focus the strobe light and direct it toward a parabolic mirror. This mirror produces a cylindrical light column that passes through the desired interrogation region. A second parabolic mirror focuses the light column back to a point in the direction of the camera.

![Schlieren/shadowgraph z-type setup](image)

Figure 3.3: Schlieren/shadowgraph z-type setup used in the High Speed Jet Aeroacoustics Facility at the Pennsylvania State University.
The parabolic mirrors are 20.32 cm (8 in) in diameter with a 1.63 m (64 in) focal length. Due to the size of the mirrors it is not possible to capture the entire impinging jet flow field. As such, a focus was placed on visualizing the rear, supersonic jet. Further details on schlieren/shadowgraph image capturing and processing practices used in this facility are discussed in [58]

### 3.3 Acoustic Field Measurements

The anechoic chamber is specially designed for polar acoustic measurements of horizontally-directed free jets. The microphone infrastructure inside the facility consists of a curved microphone boom capable of being rotated. This boom holds fifteen to twenty-three 3.2 mm (1/8 in) GRAS type 40DP microphones oriented in grazing incidence to a horizontal jet. Grazing incidence is defined as the downstream jet axis being perpendicular to the microphone diaphragm. Microphones are positioned on the boom over an arc length of 100° with higher microphone densities at angles of interest (e.g. peak noise direction). A photograph of this microphone boom can be seen in the background of Figure 2.6. As well, the schematic diagram of the facility presented in Figure 2.1 shows the location of the boom within the chamber. The boom has an average radius of 1.8 m (70 in) centered at the horizontally-oriented nozzle exit.

Acoustic field measurements of the impinging jet flow field took advantage of the existing microphone infrastructure. Without any modifications to the array, the array becomes an azimuthal array in the impinging jet configuration. Again, the microphones were oriented in grazing incidence to the vertical jet plumes. In this configuration, the rear nozzle was taken as the center of the microphone array. As shown in Figure 3.4a, the first microphone was located aft of the rear nozzle at an azimuthal angle of 0° while the last microphone was just forward of the front nozzle at an azimuthal position of 100°. Because the microphone is not exactly centered on the rear nozzle, the radial distance from the rear nozzle to the microphones varied from 180 $D_r$ to 140 $D_r$. Each microphone was at an approximate axial location of $x/D_r = 8$. A photograph of the array with the lift plate at a height of $H/D_r = 24$ is shown in Figure 3.4b.

Microphone calibration is performed with a B&K acoustic calibrator, model 4231, and the microphone calibration constants are recorded to provide the conversion from the measured voltages to the equivalent pressure. The analog time-domain signals from the microphones are routed through a GRAS model 12AG power module and then amplified and filtered for
anti-aliasing, thus enabling their accurate digital conversion in the following acquisition. A high-pass filter is also set to 500 Hz, removing any undesirable low frequency noise that could contaminate the data. A PXIe-1073 National Instruments multifunction DAQ chassis (two PXIe-6358 and one PXIe-6356 DAQ card are installed) acquired the time domain data which are then stored in binary files. The sampling rate is 300 kHz. 409,600 data points are collected. The raw data are then fed into MATLAB for processing. Data is split into $N_{FFT} = 4096$ point blocks and a Hanning window is applied with 50% overlap between each window. The fast Fourier transform (FFT) is calculated in each window and an averaged value is calculated from the 199 segments. This yields the power spectral density (PSD), with a frequency bandwidth of 74 Hz, which is then converted to a decibel ($dB$) scale using the standard reference pressure of 20 $\mu$Pa. The frequency range of the spectra shown in this work is between 500 Hz and 100 kHz.

### 3.4 Pitot Probe/Pitot Rake Flow Field Pressure Measurements

Mean flow LDV measurements made in the jet plume outwash measurements are compared against mean velocities determined via previous pitot probe measurements (see Chapter 4). Pitot probe measurements were used to validate the LDV data. Pitot rake measurements
presented in this thesis were first reported by Myers et al. [26]. All data reproduced in this thesis are with the authors’ permission.

The pitot probe rake used consists of five pitot probes each separated by 1.65 mm (0.065 in). A photograph of this probe is shown in Figure 3.5a. Each probe has an outer diameter of 0.09 $D_r$. Each probe in the rake is connected to a 100 psig Setra model 205-2 differential pressure transducer. The rake was positioned in the flow using three computerized linear traversing stages controlled with LabVIEW.

When the pitot probe rake is placed within a region of subsonic flow (e.g. shear layer or downstream of a supersonic potential core), the Mach number can be directly calculated from the measured total pressure and ambient pressure using the isentropic flow relationship:

$$\frac{P_0}{P_{amb}} = \left(1 + \frac{(\gamma + 1)M_j^2}{2}\right)^{\frac{\gamma}{\gamma - 1}}$$  \hspace{1cm} (3.1)

During supersonic jet plume pressure measurements a normal shock develops ahead of the pitot probe rake. Because of this, the pitot probe physically measures the total pressure behind the shock. This total pressure is less than the pressure upstream of the shock. A diagram of this flow field is shown in Figure 3.5b.

![Figure 3.5: Photograph and schematic diagram of the pitot probe rake used for mean velocity measurements that were used for LDV data validation](image)

In order to properly determine the true Mach number of the flow, the change in total pressure across the shock must be taken into consideration. To this end, Equation (3.1) is combined with the normal shock pressure drop equation to yield the Rayleigh pitot relation:

$$\frac{P_{0,2}}{P_{amb}} = \left(\frac{(\gamma + 1)M_{j,1}^2}{2}\right)^{\frac{\gamma}{\gamma - 1}} \left(\frac{\gamma + 1}{2\gamma M_{j,1}^2 - (\gamma - 1)}\right)^{\frac{1}{\gamma - 1}}$$  \hspace{1cm} (3.2)
Here the subscripts 1 and 2 denotes a values upstream and downstream of a shock, respectively. As Equation (3.2) indicates, the static pressure is still taken to be the ambient pressure. This is a necessary assumption as it is difficult to accurately quantify the local static pressure near the tip of the pitot probe due the complex shock and expansion wave structure generated by the probe itself. However, the error caused by this assumption is assumed to be small so calculated velocities are still valuable.

3.5 The High Speed Jet LDV System

The impinging jet flow field was interrogated using the High Speed Jet Aeroacoustic Lab’s LDV system. This system was developed in-house with funding provided by a DURIP award granted in 2012. It was designed with the capability to non-intrusively measure flow velocities ranging from low-speed to supersonic at high data rates to in order to resolve both mean and time-varying velocity components. Details on the development of this LDV system are described in [65].

3.5.1 Laser and Optics Equipment

The LDV system used in this study is powered by a 5 Watt rated Coherent Innova 90C-5 Laser System. A Bragg cell is incorporated in the setup just after the laser exit to split the beam and impart a shift frequency of 40 MHz into one of the beams to allow for negative velocities to be resolved. As opposed to conventional LDV configurations which up-shift one of the beams in frequency, due to space and Bragg cell geometry constraints, one beam was down-shifted in frequency by 40 MHz. This was taken into consideration to ensure the fringe pattern still moved against the primary flow direction being measured. The Bragg shifted beams are then passed through a double prism to split the light into separate pairs of blue and green wavelength light (488 and 514.5 nm wavelengths, respectively). The collimated laser light pairs are each focused into transmitting fiber optic cables and sent to the transmitting probe within the anechoic chamber through a small hole in the wall of the facility. All optical equipment is positioned on a heavy, vibration-resistant optics bench just outside the anechoic chamber. This allows for convenient assembly and operation of these components during set-up and operations. A photograph of the optics bench and schematic are shown in Figure 3.6.
3.5.2 Flow Seeding

Flow seeding is provided with a LaVision Aerosol Generator using di-ethyl-hexyl sebacat (DEHS) particles. Typical particle sizes are approximately 0.3 \( \mu m \) to 0.5 \( \mu m \), giving them an average Reynolds number, \( Re_p \), based on jet exit condition velocity and average particle diameter, of 0.20. Such a low \( Re_p \) gives suggests that the particles trace the flow. Following Davis and Kumar [66], the theoretical jet exit velocity, \( U_j \) and rear nozzle diameter (\( D_r = 11.9 \text{mm} \)) produce a characteristic flow time scale of 31.4 \( \mu s \). The corresponding seeding particle relaxation (or response) time is given by [67] as:

\[
\tau_0 = \frac{\rho_p d_p^2}{18 \mu_{\text{air}}} \tag{3.3}
\]

where \( \rho_p \) and \( d_p \) are the average density and diameter, respectively, of the DEHS particles. [68] lists the density of DEHS as 912 \( \text{kg/m}^3 \). This yields an average particle response time of 61 picoseconds. The ratio of particle response time to the characteristic flow time scale is the Stokes number [69]. For a Stokes number < 1, particles trace the flow with errors less than 1% [70]. In this case the Stokes number for the seeding particles is \( 1.94 \times 10^{-6} \).

As was mentioned previously (see Section 3.1.1), the rear nozzle was operated on or very near the nozzle design Mach number in order to minimize shock strength thus minimizing seeding particle lag behind shocks. Rapid, nearly instantaneous flow deceleration through a shockwave causes particle slip in which the seeding particle velocity lags that of the fluid.
for some distance downstream of the shock [70]. Melling [71] discussed the appropriate particle sizes for PIV measurements and provided an extensive literature review. Ragni et al. [72] employed PIV to quantify particle response times behind an oblique shock in a Mach 2.0 flow. Tedeschi et al. [73] analyzed different effects on spherical particle motion in high speed flows. Important flow properties investigated were relative particle Reynolds number (based on fluid properties, particle diameter, and relative particle velocity), compressibility, and large velocity gradients. Importantly, it was pointed out that small tracer particles experience non-continuum effects as well. As such, the classic Stokes’ expression for drag becomes of very limited use. A new expression for drag on spherical particles is presented. More recently, Ecker et al. [74] discuss the development of a novel LDV probe to quantify seeding particle accelerations in shock-containing flows.

Seeding particles for the core flow to the rear nozzle are introduced 56 $D_r$ upstream of the nozzle exit. The feed line for the nozzle is split to feed seeding particles to three holes in the lift plate surrounding the rear nozzle. A valve is attached to this line to control the seeding rate of the entrainment flow independently of the core flow seeding. This provides entrainment flow seeding so as to prevent bias towards higher velocity regions of the flow. Thurow et al. [18] discuss the importance of uniform flow seeding for particle-based flow measurements in order to accurately determine convection velocities of large-scale turbulence structures. Seeding rates were optimized to achieve peak data rates before each series of tests.

### 3.5.3 Sending and Receiving Probe Overview

As the seeding particles pass through the probe volume, the scattered light is collected by the receiving probe and focused on to the PMT.

#### 3.5.3.1 Mie Scattering of Light

Light scattered by particles or molecules whose diameter is less than the wavelength of the light is known as Rayleigh scattering. In the Rayleigh scattering regime, the intensity of the scattered light is strongly depended on particle size and light wavelength. This phenomena has been exploited to non-intrusively study supersonic jets. However, because the particles or molecules are small, the intensity of the scattered light can be low [75, 76].

As the particle diameter increases beyond the wavelength of the light, the scattering of light enters the Mie regime. The intensity of Mie scattered light is highly dependent upon
observation angle. A majority of the light is scattered in the forward direction (i.e. in the same direction as the incoming light source) although scattering intensity in the backwards direction (i.e. light scattered back towards the source) is appreciable as well [70]. The scattering directivity for different size particles is shown in Figure 3.7. The ratio of $DEHS$ particle diameter to the wavelength of green light is approximately 8.

![Figure 3.7: Directivity of scattered light for a small particle ($d_p \leq 0.1\lambda$, left), a medium particle particle ($d_p \approx 10\lambda$, center), and a large particle particle ($d_p > 10\lambda$, right). Image taken from [77].](image)

3.5.3.2 Sending and Receiving Probe Design

Understanding the directivity of the scattered light was critical to the design and positioning of the transmitting (sending) and receiving probes. The sending and receiving probes were designed and assembled in-house.

The transmitting probe is 13.34 cm (5.25 in) in length overall with an interchangeable 7 cm (2.75 in) diameter achromatic doublet lens with a focal length of 30 cm (11.81 in). The back end of the send probe is designed to accept two pair of beam-carrying fiber optic cables. The transmitting probe focuses the coherent laser beam pairs exiting the fiber optic cables into an ellipsoid optical probe volume within the jet flow. The two laser wavelengths (514.5 nm and 488 nm) allow for simultaneous, two velocity component, single point measurements. Although for this study, the two velocity components were not measured simultaneously.

The receiving probe was aligned with the probe volume and focuses the scattered light onto a PMT. The receiving optics are placed next to the transmitting probe in a 30° backscatter configuration. While a forward scatter configuration would ultimately have a higher incoming signal amplitude, a backscatter orientation was chose for two primary reasons. The first being that a forward scatter receiving probe would require a separate traversing system that would have to move in tandem with that of the sending optics. This added complexity was determined to be an unworthy trade-off. The second reason
for the off-angle backscatter configuration is that it reduces the effective probe volume length because the viewing optics "cut-through" the elliptic probe volume at an angle. From the center graphic in Figure 3.7, it can be seen that the intensity of the reflected light at 30° is significant. The placement of a 200 \( \mu m \) pinhole just ahead of the PMT further reduces the effective size of the probe volume and prevents extraneous light from reaching the PMT. With this arrangement it was estimated that using both the pinhole and the 30° off axis receiver reduces the probe volume length by about a factor of 5 [65].

The sending and receiving optics are mounted on the same aluminum plate. The plate is mounted on a manually controlled, high resolution scissor jack to allow for a total of 10.16 cm (4 in) vertical travel. The height of the probe volume above the plate is measured with calipers. The scissor jack is mounted to a computer-controlled linear traverse. The traverse is a Velmex model number: \textit{MA6012W1-S6}, 6 in wide traversing stage with 12 in of travel. The direction of motion of the traverse is parallel to the centerline of the lift plate. Figure 3.8 shows photographs of the sending and receiving optics mounted to the traversing stages.

![Photographs of sending and receiving optics mounted to traversing stages](image)

(a) Front view of sending and receiving optics mounted to horizontal and vertical traverses  
(b) Rear view of sending and receiving optics mounted to horizontal and vertical traverses

Figure 3.8: Photographs of sending and receiving probes mounted to traversing stages

3.5.3.3 Modifications to Probe for Near-Ground Plane Survey

A lens with a focal length of 50 cm (19.68 in) was selected to replace the original 30 cm on the sending probe so that the optical equipment could be moved back sufficiently far away from the high speed impinging jet outwash flow during near-ground measurements. As a secondary precaution, aluminum flow deflectors were placed in front of the optics to divert
the outwash flow. This prevents the optics from being contaminated by seeding particles or any extraneous debris that might be in the flow.

To achieve near-ground plane measurements, the aluminum plate holding the receiving optics is attached to the scissor jack via two 15° angle mounts so the optical equipment is "looking down" at the ground plane. Without these mounts, the bottom half of the lens on the receiving probe would be below the ground plane. If this were the case, the ground plane would block a significant portion of the light scattered by the seeding particles significantly impairing SNR. Orienting the probes in the fashion inherently introduces some error in vertical velocity measurements proportional to the cosine of the angle of inclination. For an inclination angle of 15°, this translates to a 3.4% error in axial velocity. This is deemed to be a very reasonable trade-off for improved SNR for near-ground measurements. No attempt is made to "correct" velocity measurements for this error, however, it is taken into consideration in the uncertainty analysis in Section 3.8.2.

A computer-aided design (CAD) model and photograph of the LDV system set up in the 15° orientation are shown in Figures 3.9a and 3.9b, respectively. Pertinent characteristics of the LDV system are listed in Table 3.2.

![Figure 3.9: CAD model and photograph of the LDV setup inside the anechoic chamber for impinging jet flow-field measurements.](image)
Table 3.2: High Speed Jet Aeroacoustics Laser Doppler Velocimeter Characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Green Laser Value</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Separation</td>
<td>39.8 mm</td>
<td></td>
</tr>
<tr>
<td>Transmitting Focal Length</td>
<td>500 mm</td>
<td></td>
</tr>
<tr>
<td>Beam Half Angle</td>
<td>2.28 deg</td>
<td>Equations (1.5) and (3.11)</td>
</tr>
<tr>
<td>Probe Volume Length</td>
<td>5.40 mm</td>
<td>Equation (1.7)</td>
</tr>
<tr>
<td>Probe Volume Diameter</td>
<td>0.215 mm</td>
<td>Equation (1.7)</td>
</tr>
<tr>
<td>Effective Probe Volume Length</td>
<td>0.5 mm</td>
<td></td>
</tr>
<tr>
<td>Fringe Spacing</td>
<td>6.47 µm</td>
<td>Equations (1.6) and (3.10)</td>
</tr>
<tr>
<td>Number of Static Fringes</td>
<td>33.3</td>
<td>Equation (1.9)</td>
</tr>
<tr>
<td>Pinhole Diameter</td>
<td>200 µm</td>
<td></td>
</tr>
</tbody>
</table>

3.5.4 PMT Wiring and Power Supply Design

Light scattered from the probe volume is focused on to a *Hamamatsu H10721 Series* PMT. This model requires a constant +5 VDC input to provide necessary operational power. A secondary, control voltage is required to adjust the sensitivity of the PMT to incoming light. A higher sensitivity is required in low-light situations, such as a dim probe volume (i.e. low laser output from fiber optic cables). The amplitude of the signal output by the PMT is proportional to the intensity of the incoming light. Therefore, if the light has low power, a high SNR can be achieved through increasing the sensitivity. The control voltage can be adjusted from +0.5 to +1.1 VDC. Maximum sensitivity (+1.1 VDC) corresponds to an output signal gain of approximately 30 dB over the minimum gain of the output signal ($V_{sensitivity} \leq +0.5$ VDC). A control voltage any higher than the maximum value risks overloading the PMT.

With these power requirements and restrictions in mind, a custom PMT power supply was designed and fabricated. The power supply is designed to run off of simple wall power. The incoming ±120 VAC power is split and sent to two *Acopian* AC-DC voltage converters (models 1.5EB50 and 5EB50). The 5EB50 model converts wall power to +5 VDC and is solely used to provide the operational power for the PMT. The 1.5EB50 model provides a nominal output voltage of +1.5 VDC used for the control voltage.

Originally the system was designed with a 5 kΩ resistor to step down the voltage to +1 VDC. The sensitivity voltage would then be controlled via a 10 kΩ potentiometer and monitored with a voltmeter wired in parallel with the PMT. The PMT and voltmeter were assumed to have effectively infinite resistance. Unfortunately, during a breadboard mock-up of the circuit it was discovered that the voltmeter had a resistance of only 3.07 kΩ.
Thus it was drawing current resulting in an unexpected voltage drop in the system. A redesign of the system modeled the actual voltmeter as an ideal voltmeter in parallel with a 3.07 kΩ resistor. A complete schematic diagram of the final PMT power supply circuit is shown in Figure 3.10.

Figure 3.10: Schematic diagram of the final photomultiplier tube power supply circuit

Knowing the output voltage from the 1.5EB50 converter and the resistances of the voltmeter and potentiometer, the necessary resistance to step down the output voltage from the converter so as to not overload the PMT can be determined as follows:

1. The average voltage drop across $R_1$ must be $0.4 \leq \Delta V_{R_1} \leq 0.5$ and will therefore be taken as the average required drop: $\Delta V_{R_1,avg} = 0.45V$ and must be equal to:

$$\Delta V_{R_1,avg} = 0.45V = IR_1 \quad (3.4)$$

2. Since the potentiometer and voltmeter are wired in parallel and are 'downstream' of $R_1$, they may be theoretically combined into a single equivalent resistor in series with $R_1$. The equivalent resistance of this resistor is:

$$R_{eq} = \left( \frac{1}{R_{pot}} + \frac{1}{R_{voltmeter}} \right)^{-1} \quad (3.5)$$

$$R_{eq} = \left( \frac{1}{10} + \frac{1}{3.07} \right)^{-1} = 2.35k\Omega$$

3. Then, assuming zero current flow through the PMT and ideal voltmeter, the total current in the system is:

$$I = \frac{1.5V_{DC}}{R_{eq} + R_1} \quad (3.6)$$
4. Finally, Equations (3.4) and (3.6) may be combined and rearranged to solve for $R_1$:

$$R_1 = \frac{\Delta V_{R_1,\text{avg}} R_{eq}}{1.5 V_{DC} - \Delta V_{R_1,\text{avg}}} \quad (3.7)$$

This yields a required resistance of $R_1 = 1 \text{k}\Omega$ and is a relatively common resistor.

The sensitivity of the PMT is set prior to each set of run to achieve the highest possible data rates and SNR. The output signal from the PMT is further gained by 32 dB using a Sonoma Instrument® 310 Broadband amplifier before being passed along to the DAQ computer.

### 3.5.5 LDV Data Acquisition Software

The DAQ computer is located outside of the anechoic chamber. The data acquisition, analog-to-digital converter (A/D) conversion, particle light burst detection, and initial velocity processing is done using the Studio Software Suite developed by AUR Inc. The computer digitizes the analog signal output by the PMT data at a sampling-rate of 1-2 GHz, depending on expected magnitude of the particle velocity. It should be noted at this point that some a priori knowledge of the flow field must be had prior to data acquisition. In particular the primary direction of the flow and the expected range of velocities must be known. One crucial input to the AUR software is the expect range of Doppler frequencies (proportional to particle velocity, see Equation (1.13)) to look for when analyzing the incoming signal. Other pertinent inputs into the software are the Bragg cell frequency and the fringe spacing within the probe volume (critical to converting frequencies to velocities). More details on all inputs for the AUR software are given in Appendix A. in Karns [65].

Proprietary burst detection algorithms are used to identify and record particle bursts and their subsequent properties. Specifics on the algorithms and additional details can be found in [78]. Output data from each run include the particle burst time (computer internal clock time), particle velocity, Doppler frequency, burst duration (temporal length of particle light burst), SNR, and burst peak-to-peak ratio (P2P ratio). To avoid biasing towards any particular measurement, $100,000$ nominal valid bursts, as determined by the processing, were acquired. The entire dataset output by AUR Studio was then post-processed using MATLAB. Details on particle burst validation and processing are discussed at length in Section 3.6.
3.5.6 Probe Calibration

As was discussed in Section 3.5.5, the fringe spacing must be known in order to properly determine particle velocities. Fringe spacing is determined through the calibration process of the transmitting and receiving probes. The system is calibrated against a known linear velocity. In this case, the reference velocity is provided by a *Thorlabs model MC2000* optical chopper system. This is a precision instrument that can spin at operator-chosen blade-passage frequencies with negligible change in revolutions per minute (RPM) during operation. From Equation (1.6), it can be seen that the fringe spacing is a function of the beam intersection half angle. The calibration process outlined here is derived from that described by Park *et al.* [79] in which the fringe spacing and beam half angle were indirectly measured with a sandpaper wheel.

The calibration process is as follows:

1. Using Equations (1.5) and (1.6), the theoretical fringe spacing for an ideal system is determined. The measured wheel Doppler frequency is determined by:

\[
f_{D,\text{wheel}} = \frac{\bar{U}_{\text{meas,\text{wheel}}}}{\delta_{th}} \tag{3.8}\]

where \(\bar{U}_{\text{meas,\text{wheel}}}\) is the mean linear velocity of the wheel as measured by the LDV system and \(\delta_{th}\) is the theoretical fringe spacing determined by Equation (1.6).

2. Knowing the blade passage frequency, \(f_b\) and number of blades, \(b\) on the chopper wheel, the theoretical mean linear velocity of the wheel at the LDV measurement location is given by:

\[
\bar{U}_{th,\text{wheel}} = 2\pi r \left(\frac{f_b}{b}\right) \tag{3.9}\]

where \(r\) is the distance from the center of the wheel to the measurement location.

3. Combining Equations (3.8) and (3.9) the new, corrected fringe spacing is calculated as:

\[
\delta_{corr} = \frac{\bar{U}_{th,\text{wheel}}}{f_{D,\text{wheel}}} = \left(\frac{\bar{U}_{th,\text{wheel}}}{\bar{U}_{\text{meas,\text{wheel}}}n}\right) \delta_{th} \tag{3.10}\]

4. Rearranging Equation (1.6), and substituting in the corrected fringe spacing as determined in Equation (3.10), the corrected beam half-angle is:

\[
\psi_{corr} = \sin^{-1}\left(\frac{\lambda}{2\delta_{corr}}\right) \tag{3.11}\]
5. This process is repeated over a range of wheel rotational frequencies (on the order of fifteen to twenty) to calculate a more accurate, average corrected beam half-angle. This corrected half-angle value is then input into the AUR data acquisition software prior to experimental measurements.

This calibration process was first outlined by Karns [65].

3.6 Particle Burst Validation and Processing

As was previously discussed in Section 3.5.5, data is processed by the AUR software. Output data from each run include the particle burst time, velocity, Doppler frequency, burst duration, SNR, and burst P2P ratio. Included also in the output files is a column of 1’s and 0’s denoting which particle bursts are 'valid' and which are not. Because the software’s exact particle burst validity criteria are unknown, all raw data sets are post-processed in MATLAB to determine valid bursts.

Bursts are validated based on SNR and P2P ratio. Additionally, the burst must be within a certain number standard deviations of the mean value for the data set. While each data set is ultimately processed independently of other data sets, data sets are processed in groups to limit variation in exact validation criteria. Before the exact validation methodology is discussed, a brief definition of burst P2P ratio is warranted.

For a given particle velocity, \( U \), the Doppler frequency created as the particle traverses the probe volume is given by Equation (1.10). Ideally, this will yield a single peak in the frequency domain. However due to noise, electronic or otherwise, this is never exactly the case. As a signal is being read, the software analyzes the signal in the frequency domain. The P2P ratio is the amplitude ratio of secondary peaks in the spectrum to the peak determined to be the primary Doppler frequency of the particle. For burst validation, this ratio should be as small as possible i.e. the Doppler frequency dominates the power spectrum. Aside from low SNR \( (A_{f\delta} = \mathcal{O}(\text{noise peaks})) \), high-amplitude secondary spectral peaks may be present if the particle is undergoing significant acceleration through the probe volume, as is the case near shockwaves. Strong accelerations cause the software to detect at least two, strong secondary velocity peaks in the power spectrum again resulting in a high P2P ratio.

Valid particle bursts are determined via two sets of validation criteria: i) rigorous (tight) validation criteria for an accurate first estimation of both the mean and \( \text{rms} \) velocities
followed by, \( ii \) more liberal criteria to achieve high data rates and updated mean and \( rms \) velocities, \( u_{rms} \).

In the first validation step, the raw data is processed using tight validation criteria (e.g. Min. SNR of burst = 18 and Max. P2P ratio of burst = 0.2). Because occasionally particle bursts come in at extremely high rates, the large influx of data leads to processor 'dead times' in which it is processing/writing the data. This even is known as "binning" because the time history of the data has several (\( \sim 10 \)) artificially long time delays (approx. 0.2 s) between valid particle bursts. To combat this, the raw data file is split into data blocks at these binning events. For each block, the mean velocity and \( rms \) velocity are estimated.

Acquired LDV data is unevenly sampled in time as data is only acquired when a particle is present within the probe volume and produces a validated light burst. What’s more, in turbulent flows, LDV velocity measurements are subject to a bias error caused by the unequal particle arrival times, as originally identified by McLaughlin and Tiederman [80]. Assuming the entire flow field is uniformly seeded, in a given span of time, a greater number of higher velocity particles will pass through the measurement volume than slower velocity particles. To account for this, Kerherve at al. [53] suggested a particle interarrival time weighting to calculate the mean and \( rms \) velocities as:

\[
\bar{U} = \frac{\sum_{i=2}^{N} U_i w_i}{\sum_{i=2}^{N} w_i} \tag{3.12}
\]

where

\[
w_i = t_i - t_{i-1} \tag{3.13}
\]

and,

\[
u_{rms} = \sqrt{\frac{\sum_{i=2}^{N} (U_i - \bar{U})^2 w_i}{\sum_{i=2}^{N} w_i}} = \sqrt{\frac{\sum_{i=2}^{N} u_i^2 w_i}{\sum_{i=2}^{N} w_i}} \tag{3.14}
\]

This bias correction is essentially equivalent to that originally suggested by McLaughlin and Tiederman [80], although it might be more convenient to implement. At this point it is appropriate to note that for all further discussions turbulence intensities are defined on a percentage-wise basis as:

Axial Turbulence Intensity, \( \Gamma_u = \frac{u_{rms}}{U_j} \times 100\% \)

Radial Turbulence Intensity, \( \Gamma_v = \frac{v_{rms}}{U_j} \times 100\% \) \tag{3.15}

where \( U_j \) is the fully expanded jet velocity and is taken to be 380 m/s.
The second step of validation is designed to capture only valid particle bursts but with a significantly more liberal set of validation criteria so as many bursts are validated as possible. The raw data is processed using looser validation criteria (e.g. Min. SNR of burst = 12 and Max. P2P ratio of burst = 0.5). Velocities only within $\pm 3u_{rms}$ of the original estimated mean are retained. The raw data is then split into blocks and the new mean and $rms$ velocities are determined using Equations (3.12) and (3.14). This two-step process can yield mean data rates as high as 40kHz. The MATLAB scripts used for data processing are included in Appendix A.

Prior to calculating mean velocities in each step, a notch filter is in place to allow the user to manually remove any unphysical velocities which seldom appear in the data. After processing the data, the velocity probability density function (PDF)'s are inspected to ensure no anomalies appear in the data (e.g. unphysical velocity peaks, PDF's not tapering to zero). Example of PDF's for a "good, clean" set of data points taken at $H/D_r = 6$, $y/D_r = 0.5$ is shown in Figure 3.11a. Figure 3.11 shows the probability distribution for time between particle bursts for the same sets of data. This gives the user an idea of how far out the frequency content of the velocity fluctuations may be resolved. Because particle interarrival times are as low as $10^{-5}$s or less, this suggests spectra may be resolved out to approximately 50kHz. Spectral processing of unevenly sampled LDV data is discussed in Section 3.7.

![Velocity PDF](image1.png)

(a) Velocity PDF for $H/D_r = 6$, $y/D_r = 0.5$

![Particle Interarrival Time PDF](image2.png)

(b) Particle arrival separation times PDF for $H/D_r = 6$, $y/D_r = 0.5$

Figure 3.11: Example probably distributions of particle velocity and time between particle bursts at five different radial locations for $H/D_r = 6$, $y/D_r = 0.5$
3.7 Spectral Estimation of LDV Data

In order to perform meaningful spectral analysis on the unsteady velocity measurements, the data sets must effectively be converted into an equally-spaced (in time) data set. Sample-and-hold reconstruction of the signal is attractive in its simplicity, however, it involves signal contamination due to the addition of imprecise information. An alternative approach is a technique known as 'fuzzy slotting' first introduced by Nobach et al. [81]. Following this, van Maanen et al. [82] combined the work of Nobach et al. [81] with the earlier work of van Maanen and Tummers [83]. van Maanen and Tummers [83] employed local normalization to reduce the variance of correlation coefficients and improved the estimation of turbulence PSD’s. Excellent overviews of the fuzzy slotting technique for LDV spectral estimation, its history, and implementation can be found in Benedict et al. [84] and Nobach [85].

The fuzzy slotting autocorrelation estimation technique involves calculating all possible velocity cross-products. For each cross-product, the associated time lag is determined as well. The velocity cross-products are then organized into $K$ number of bins (slots) equally spaced in time by $\Delta \tau$. $K$ is also the total number of points in the autocorrelation, where $K = \frac{N_{PSD}}{2}$. The time lag for each bin is defined as:

$\text{Time lag} = k\Delta \tau$ for $k=0$ to $K-1$

For this study, $\Delta \tau = 4 \mu s$ and $K = 2048$ resulting in an autocorrelation that extends out to nearly 8.2 ms. It is important to note at this point that the autocorrelation is only calculated for positive time delays.

Within each bin the cross-products are averaged resulting in equally spaced correlation coefficients in the correlation domain. The term "fuzzy slotting" is derived from the fact that the bins are triangular shaped in the correlation domain with the central peak (correlation value=1) at the associated time lag ($k\Delta \tau$). At the 'base' of these bins (correlation value=0), bins overlap each other by 50%. Products with time lags closer to the centers of bins contribute more heavily to the correlation estimate for that time lag (bin) than those towards the slot edges. Cross-products residing in regions where two bins overlap contribute to the correlation estimate for both bins. Similarly, cross-products in between bins, contribute to neither bin and get discounted. Kerherve et al. [53] provide an excellent schematic of the 'fuzzy slots.'

To calculate the velocity PSD, the data set are split in to blocks. Calculating the PSD for each block and averaging them together reduces the variance of the PSD for the entire
data set. For this study, data sets are split into 10 to 20 blocks, depending on the total length of the data set, with each block containing on the order of 5000 valid velocity measurements which typically span an average time of 0.5 s. The mean velocity and rms velocity for each block is calculated using Equations (3.12) and (3.14), respectively. The autocorrelation for each block is then calculated as:

\[
\hat{R}_k = \frac{u_{\text{rms}}^2 A}{\sqrt{BC}} \quad \text{for } k = 0 \text{ to } K \tag{3.16}
\]

with

\[
A = \sum_{i=2}^{N-1} \sum_{j=i+1}^{N} u_i u_j w_i w_j b_k(t_j - t_i) \tag{3.17}
\]

\[
B = \sum_{i=2}^{N-1} \sum_{j=i+1}^{N} u_i^2 w_i w_j b_k(t_j - t_i) \tag{3.18}
\]

\[
C = \sum_{i=2}^{N-1} \sum_{j=i+1}^{N} u_j^2 w_i w_j b_k(t_j - t_i) \tag{3.19}
\]

where \( w_i \) is previously defined in Equation (3.13) and

\[
w_j = t_{j+1} - t_j \tag{3.20}
\]

The triangular fuzzy masking function, \( b_k(t_j - t_i) \) is defined as:

\[
b_k(t_j - t_i) = \begin{cases} 
1 - \left| \frac{\Delta t}{\Delta \tau} - k \right| & \text{for } \left| \frac{\Delta t}{\Delta \tau} - k \right| < 1 \\
0 & \text{otherwise}
\end{cases} \tag{3.21}
\]

Once the autocorrelation is calculated (Equation (3.16)), the scaled, single-sided PSD is determined using a discrete cosine transform. The unscaled PSD is calculated as follows:

\[
\hat{G}(f) = 2\Delta \tau \left( \hat{R}_0 + 2 \sum_{k=1}^{N_{PSD}} d_k(f) \hat{R}_k \cos(2\pi f k \Delta \tau) \right) \tag{3.22}
\]

The term \( d_k(f) \) on the right-hand side of Equation (3.22) is a frequency-dependent variable window originally recommended by Tummers and Passchier [86] to reduce the variance of
the PSD. It is formally defined as:

\[
d_k(f) = \begin{cases} 
\frac{1}{2} + \frac{1}{2} \cos \left( \frac{\pi f \Delta \tau}{\kappa} \right) & \text{for } |\frac{\pi f \Delta \tau}{\kappa}| < \kappa \\
0 & \text{otherwise}
\end{cases} \tag{3.23}
\]

The parameter \( \kappa \) in Equation (3.23) is a 'smoothing parameter' whose value can be chosen arbitrarily. A larger \( \kappa \) reduces the amount of 'smoothing' the window applies to the PSD. The fuzzy slotting literature suggests \( \kappa = 6 \). However, for the data collected in this study, \( \kappa = 6 \) was found to provide far too much smoothing. As such, a value of \( \kappa = 50 \) is used for all spectra presented in this thesis.

After calculating the PSD using Equations (3.22) and (3.23), the PSD is rescaled such that its integral returns the mean square velocity, \( u_{rms}^2 \). A 3-point Savitzky-Golay finite impulse response (FIR) smoothing filter is applied to the rescaled PSD to reduce the variance further. Because LDV does not sample regularly in time, incoming data rates may be as high as 200kHz thus allowing spectra in this study to be resolved out to 100kHz with a frequency resolution of 122Hz.

### 3.8 Experimental Uncertainty

The reliability and repeatability of experiments is strongly dependent on the accuracy of experiments. Truly meaningful analysis and interpretation of experimental results can only be accomplished once there is a thorough understanding of the associate uncertainties with those measurements. Such is especially true when attempting to develop and validate the LDV for supersonic impinging jet measurements. It is envisioned that this LDV system will be used to gain further understanding of the flow fields produced by single and dual supersonic impinging jets and validate on going CFD efforts at partnering facilities such Naval Air Station Patuxent River. General facility uncertainties for instruments crucial to flow metering are described in Section 3.8.1. Uncertainties specific to the laser Doppler system are discussed in Section 3.8.2. Specifically, a detailed analysis on the uncertainties in mean velocities and turbulence intensities is provided. The following uncertainty analysis is restricted to only the primary velocity component in each region of the flow. A separate discussion on uncertainties associated with jet-LDV probe misalignment is reserved for Section 4.1.2.2.
Table 3.3: Uncertainties of Laboratory Gauges and Transducers

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<thead>
<tr>
<th>Measurement</th>
<th>Uncertainty (σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient Temperature</td>
<td>±1 K (0.5 °F)</td>
</tr>
<tr>
<td>Ambient Pressure</td>
<td>±100 Pa (0.015 psi)</td>
</tr>
<tr>
<td>Relative Humidity</td>
<td>±1 %rh</td>
</tr>
<tr>
<td>Pressure Transducer</td>
<td>±690 Pa (0.1 psi)</td>
</tr>
</tbody>
</table>

3.8.1 Facility Uncertainties

This section discusses general facility uncertainties associated with all gauges and transducers in the laboratory pertinent to high-speed jet operation. Atmospheric pressure, temperature, and relative humidity are read off of analog gauges mounted within the anechoic chamber. Uncertainty stems from the ability of the operator to distinguish between two values. The uncertainty in nozzle operating pressure, \( P_0 \), is given by the manufacturer of the pressure transducer (Setra). Facility uncertainties are tabulated in Table 3.3

3.8.2 Laser Doppler Velocimetry Uncertainty

This section presents the uncertainties associated with mean jet velocities, \( \sigma_{u_j} \), and turbulence intensities, \( \sigma_{turb} \), measured by the LDV system. First the uncertainty in jet velocity measurements is discussed in Section 3.8.2.1, followed by an analysis of the uncertainty of turbulence measurements in Section 3.8.2.2.

3.8.2.1 Mean Flow Uncertainty

The uncertainty of any jet velocity LDV measurement presented in this thesis, \( \sigma_{u_j} \), will be assumed to be dependent on three primary sources: 1) uncertainty in the source jet, \( \sigma_u \), 2) uncertainty of the LDV probe volume (\( \sigma_{LDV} \)), and 3) positioning error of the probe within the flow (\( \sigma_{pos} \)). This is summarized in Equation (3.24).

\[
\sigma_{U_j} = \sqrt{\sigma_U^2 + \sigma_{LDV}^2 + \sigma_{pos}^2} \tag{3.24}
\]
The fully expanded jet velocity at any operating condition, $T_0$ and $NPR$, is given by:

$$U = \sqrt{\frac{2\gamma}{\gamma - 1} \Re T_0 \left(1 - NPR^{\frac{1}{\gamma}}\right)}$$  \hspace{1cm} (3.25)$$

The uncertainty in the jet velocity can be calculated using the data reduction equation as described in Coleman and Steele [87]. In this way, the uncertainty of the jet velocity is dependent upon the uncertainty of each term within Equation (3.25). Taking the uncertainties in specific heat ratio and gas constant to be negligible with respect to other uncertainties, the jet velocity uncertainty, $\sigma_u$, is:

$$\sigma_u^2 = \left(\frac{\partial U}{\partial T_0}\right)^2 \sigma_{T_0}^2 + \left(\frac{\partial U}{\partial NPR}\right)^2 \sigma_{NPR}^2$$  \hspace{1cm} (3.26)$$

The partial derivatives in Equation (3.26) are evaluated analytically, where:

$$\left(\frac{\partial U}{\partial T_0}\right)^2 = U^2 \frac{1}{4T_0^2}$$  \hspace{1cm} (3.27)$$

and

$$\left(\frac{\partial U}{\partial NPR}\right)^2 = U^2 \left(\frac{1 - \gamma)^2}{4\gamma^2}\right) \left(\frac{NPR^{\frac{2-\gamma}{\gamma}}}{1 - NPR^{\frac{1}{\gamma}}}\right)$$  \hspace{1cm} (3.28)$$

Because the supply air tanks and connecting lines are unheated and exposed to ambient conditions, then to a good approximation the total temperature of the jet can be assumed to be equal to ambient temperature. Therefore, the total temperature uncertainty in Equation (3.26) is equal to the uncertainty in atmospheric temperature listed in Table 3.3.

However, $\sigma_{NPR}$ in Equation (3.26) needs to be calculated. Given the definition of $NPR$:

$$NPR = \frac{P_0}{P_{\text{amb}}}$$  \hspace{1cm} (3.29)$$

it is readily shown:

$$\sigma_{NPR}^2 = \left(\frac{\partial NPR}{\partial P_0}\right)^2 \sigma_{P_0}^2 + \left(\frac{\partial NPR}{\partial P_{\text{amb}}}\right)^2 \sigma_{P_{\text{amb}}}^2$$

$$= \left(\frac{1}{P_{\text{amb}}^2}\right) \sigma_{P_0}^2 + \left(-\frac{P_0}{P_{\text{amb}}^2}\right)^2 \sigma_{P_{\text{amb}}}^2$$

$$= \left(\frac{NPR}{P_0}\right)^2 \sigma_{P_0}^2 + \left(\frac{NPR^2}{P_0}\right)^2 \sigma_{P_{\text{amb}}}^2$$  \hspace{1cm} (3.30)$$
where $\sigma_{P_0}$ and $\sigma_{P_{amb}}$ are the uncertainties associated with the pressure transducer and ambient pressure gauge, respectively, and are tabulated in Table 3.3.

The velocity uncertainty associated with the LDV probe itself, $\sigma_{LDV}$, is partially due to the wheel calibration method outlined in Section 3.5.6 as well as the signal processing method. Lowe [78] describes the method used by the AUR Inc. system and shows that the uncertainty is proportional to the time duration of the particle burst, which can then be determined to be proportional to the minimum number of fringes encountered by the particle.

The relative uncertainty of the LDV particle velocity due to calibration errors is explained as follows. During the calibration process, the theoretical linear velocity of the wheel is dependent on the distance of the probe volume from the center of rotation. Given the finite diameter of the probe volume, the linear velocity of the wheel measured at the "bottom" of the probe volume is less than the linear velocity of the wheel measured at the "top" of the probe volume. This leads to uncertainty bars directly proportional to the diameter of the probe volume. Additional error may be introduced if the tangential velocity of the wheel blades is not perpendicular to the fringes. Therefore uncertainty in wheel velocity is dependent on the uncertainty of the disk rotation frequency ($f$) and the radial distance from the probe volume to the center of the wheel ($r$). Uncertainties in linear wheel velocity will propagate into value of the corrected beam half angle. From Equations (1.6) and (1.10), the beam half angle is a critical parameter for determining particle velocities during measurements.

The uncertainty of the Doppler frequency calculation can be shown to be dependent on the particle time in the probe volume and the number of fringes ($N_f$) encountered by the particle (e.g. [78]). Uncertainty in the Doppler frequency measurement is shown in Equation (3.31):

$$
\sigma_{LDV}^2 = U^2 \left[ \left( \frac{\sigma_f}{f} \right)^2 + \left( \frac{\sigma_R}{R} \right)^2 + \left( \frac{1}{10N_f} \right)^2 \right] \quad (3.31)
$$

Where $\sigma_f$ and $\sigma_R$ are the uncertainties associated with the wheel rotation frequency and radius to the center of rotation, respectively. In this case, $U$ is the linear velocity of the disk at radius, $r$. For the calibration performed the dominant uncertainty is due to the non-zero probe volume size and subsequently, uncertainty in radius.

The remaining source of error comes from the uncertainty of the probe's position within the flow and includes errors associate with probe orientation as well. This discussion will
draw conclusions based off of mean velocities presented in Myers and McLaughlin [26] and will make reference to the results of the jet-probe misalignment analysis presented in Section 4.1.2.2 as well as results in Section 4.1.3 of this thesis. Although it is not ideal to refer to data before they are formally presented, it is felt that the reader will strongly benefit from having a complete understanding of the experimental uncertainty prior to reading the results, especially when attempting to draw conclusions from comparisons to previous studies.

Jet velocity is a function of position \((x,r)\) within the flow, assuming an axisymmetric jet. Therefore, any uncertainty in position translates to an uncertainty in velocity. Moreover, any errors due probe-jet to misalignment, \(\sigma_{\theta}\), also contribute to uncertainties in measured particle velocities. As discussed in Section 3.5.3.3, most measurements were performed with the LDV probe inclined \(15^\circ\) to the horizontal. From simple geometry, it can be shown that this introduces an error of 3.4\% in axial (vertical) velocities only, such that:

\[
\sigma_{15^\circ}^2 = (0.034U_{ax})^2 \approx 1.16 \times 10^{-3}U_{ax}^2
\]  
(3.32)

Then, the total positioning uncertainty is given by:

\[
\sigma_{pos}^2 = (\frac{\partial U}{\partial x})^2 \sigma_x^2 + (\frac{\partial U}{\partial r})^2 \sigma_r^2 + \sigma_\theta^2 + \sigma_{15^\circ}^2
\]  
(3.33)

For axial velocity measurements in the jet plume, to a good approximation, the gradient of velocity in the axial direction can be considered negligible compared the radial gradient in velocity via a simple order of magnitude analysis. From the results of the jet-probe misalignment analysis (Section 4.1.2.2), the jet appears to be angled \(3.5^\circ\) to the vertical and shifted \(0.1D_r\) \((\Delta r)\) to the left of the assumed zero position. Therefore, \(\sigma_{\theta}\) for axial velocity measurements can be approximated as:

\[
\sigma_{\theta,U}^2 = \left(\frac{\partial U}{\partial r}\right)^2 \Delta r^2 + \left((1 - \cos(3.5^\circ))U\right)^2 \approx (0.002U)^2 = 4 \times 10^{-6}U^2
\]  
(3.34)

From the above argument, the axial velocity uncertainty due to probe positioning and misalignment errors is:

\[
\sigma_{pos,U}^2 = \left(\frac{\partial U}{\partial r}\right)^2 \sigma_r^2 + \left(\frac{\partial U}{\partial r}\right)^2 \Delta r^2 + 4 \times 10^{-6}U^2 + 1.16 \times 10^{-3}U^2
\]
\[ \approx U^2 \left[ \left( \frac{\partial U}{\partial r} \right)^2 \left( \frac{\sigma_r^2 + \Delta r^2}{U^2} \right) + (1.2 \times 10^{-3}) \right] \] (3.35)

From geometry, there is negligible effect of LDV probe inclination on radial velocity measurements. Similar to the argument made for the axial measurements in the jet plume, with respect to radial velocity measurements in the outwash flow, velocity gradients in the radial direction are negligible compared to those in the direction normal to the ground plane. As result of this argument, uncertainty due to jet-probe misalignment is now only due to the angular misalignment; the radial offset \((\Delta r)\) may be considered negligible. That is:

\[ \sigma^2_{\theta,V} = (0.061V)^2 \approx 3.7 \times 10^{-3}V^2 \] (3.36)

Now, the radial velocity uncertainty due to probe positioning and misalignment errors may be approximated as:

\[ \sigma^2_{pos,V} \approx V^2 \left[ \left( \frac{\partial V}{\partial y} \right)^2 \frac{\sigma_y^2}{V^2} + \left( 3.7 \times 10^{-3} \right) \right] \] (3.37)

An attempt will now be made to quantify the error in axial and radial velocity measurements. In the above equations there are 6 unknowns which must first be quantified: \(\sigma_R, \sigma_y, \sigma_r, \sigma_f, \frac{\partial u}{\partial r}, \) and \(\frac{\partial v}{\partial y}\). Because both the radial distance to the center of the wheel and the height of the probe volume above the ground plane are determined with analog calipers, their relative uncertainties, \(\sigma_R\) and \(\sigma_y\) are equal. These uncertainties are:

\[ \sigma_R = \sigma_y \approx 0.0005in \ (12.7\mu m) \]

Where it is assumed calipers can be reliably read to within half of the smallest tick spacing. The radial positioning of the probe, as previously discussed in Section 3.5.3.2, is controlled via a computer-operated traversing stage. The manufacturer, Velmex, lists the accuracy of this stage, \(\sigma_r\), to be \(0.0001in \ (2.54\mu m)\). However, because the mounted sending-receiving probes on this traverse are heavy, the uncertainty of this traverse will be taken to be \(\sigma_r = 12.7\mu m\).

Myers and McLaughlin [26] reported free-jet, axial velocity measurements for a jet exhausting from the same nozzle and \(NPR\) as in this study. Using the velocity profile data provided by the authors at an axial station 4 \(D_r\) downstream of the nozzle exit, the radial gradient in velocity can be simply estimated using a linear fit. Note: these data are reproduced in Figure 4.1a in Section 4.1.1. To be conservative, this uncertainty analysis
Table 3.4: Tabulated Axial and Radial Velocity Uncertainties

<table>
<thead>
<tr>
<th>Uncertainty Component</th>
<th>Jet Plume</th>
<th>Wall Jet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center of wheel rotation, $\sigma_R$</td>
<td>12.7$\mu m$</td>
<td>12.7$\mu m$</td>
</tr>
<tr>
<td>Height above ground plane, $\sigma_y$</td>
<td>12.7$\mu m$</td>
<td>12.7$\mu m$</td>
</tr>
<tr>
<td>Linear traversing stage, $\sigma_r$</td>
<td>12.7$\mu m$</td>
<td>12.7$\mu m$</td>
</tr>
<tr>
<td>Wheel rotation frequency, $\sigma_f$</td>
<td>$4 \times 10^{-4}Hz$</td>
<td>$4 \times 10^{-4}Hz$</td>
</tr>
<tr>
<td>Probe position in flow, $\sigma_{pos}$</td>
<td>51 m/s (14%)</td>
<td>9.7 m/s (6.1%)</td>
</tr>
<tr>
<td>LDV prove (500 Hz, 1 in), $\sigma_{LDV}$</td>
<td>1.1 m/s (0.3%)</td>
<td>0.5 m/s (0.3%)</td>
</tr>
<tr>
<td>Jet velocity, $\sigma_U$</td>
<td>26 m/s (7.2%)</td>
<td>11.6 m/s (7.25%)</td>
</tr>
</tbody>
</table>

**Total Jet Uncertainty, $\sigma_{U_{j}}$** | 57 m/s (16%) | 15 m/s (9.4%) |

will use the most extreme velocity gradient in the data. By inspection, the axial velocity drops from $370 \text{ m/s}$ to $0 \text{ m/s}$ over a span of $0.75 D_r$. This results in a velocity gradient of:

$$\frac{\partial U}{\partial r} \approx \frac{370 \text{ m/s}}{(0.75 \times 0.47 \text{ in} \times 0.0254 \text{ m/in})} \approx 41325 \text{ s}^{-1}$$

In a similar fashion, the vertical gradient of radial velocity in the outwash flow regime can be estimated. To do so, outwash data presented in Section 4.1.3 at $r/D_r = 4$ are used to approximate $(\partial V/\partial y)$. A radial station of $r/D_r = 4$ was chosen to remain consistent with the estimation of $(\partial U/\partial r)$. Radial velocity is seen to drop from $160 \text{ m/s}$ to $0 \text{ m/s}$ over a vertical distance of $0.5 D_r$, so:

$$\frac{\partial V}{\partial y} \approx \frac{160 \text{ m/s}}{(0.50 \times 0.47 \text{ in} \times 0.0254 \text{ m/in})} \approx 26800 \text{ s}^{-1}$$

The uncertainty of the wheel rotation frequency is provided by *Thorlabs*: $\sigma_f = 4 \times 10^{-4} \text{ Hz}$. Uncertainties for an axial jet velocity of $360 \text{ m/s}$ and a radial wall jet velocity of $160 \text{ m/s}$ for $M_j = 1.34$, $NPR = 2.93$ are tabulated in Table 3.4. It is clear the major source of uncertainty in axial jelocity measurements stems from error the positioning of the LDV probe volume within the flow. For an axial velocity of $360 \text{ m/s}$, the velocity uncertainty is 16% of the mean flow speed. However, it must be kept in mind this is a very conservative estimate of uncertainty based on the maximum radial gradient of axial velocity in the flow. In actuality an axial velocity of $360 \text{ m/s}$ would occur closer to the jet centerline where gradients are comparatively small. The accuracy of radial velocity measurements in the outwash flow appear to be nearly equally dependent on probe positioning (including relative alignment with the jet) and source jet velocity. It is imperative to understand that
this uncertainty analysis breaks down near shockwaves as they would introduce massive
gradients in the primary flow direction as well as particle lag effects.

3.8.2.2 Turbulence Intensity Uncertainty

Uncertainty in reported turbulence intensities, $\sigma_{turb}$, will be shown to be entirely a
consequence of the uncertainty in the LDV velocity measurements, $\sigma_{uj}$. Applying the data
reduction equation outlined by Coleman and Steele [87] to the definition of turbulence
intensity, Equation (3.15):

$$
\sigma^2_{turb} = \left( \frac{\partial \Gamma_u}{\partial u_{rms}} \right)^2 \sigma^2_{u_{rms}} + \left( \frac{\partial \Gamma_u}{\partial U_j} \right)^2 \sigma^2_{u_j} \quad (3.38)
$$

Evaluating the partial derivatives analytically:

$$
\left( \frac{\partial \Gamma_u}{\partial u_{rms}} \right)^2 = \frac{1}{U_j^2} \quad (3.39)
$$

$$
\left( \frac{\partial \Gamma_u}{\partial U_j} \right)^2 = \frac{\Gamma^2}{U_j^2} \quad (3.40)
$$

where $\sigma_{u_j}$ was determined in Section 3.8.2.1. Then the only unknown is the error associated
with the $rms$ velocity, $\sigma_{u_{rms}}$. This term can be calculated via:

$$
\sigma^2_{u_{rms}} = \left( \frac{\partial u_{rms}}{\partial U_i} \right)^2 \sigma^2_{u_j} + \left( \frac{\partial u_{rms}}{\partial \bar{U}} \right)^2 \sigma^2_{\bar{U}} \quad (3.41)
$$

Determining the two partial derivatives in Equation (3.41):

$$
\left( \frac{\partial u_{rms}}{\partial U_i} \right) = \frac{1}{u_{rms}} \left( \frac{\sum (U_i - \bar{U}) w_i}{\sum w_i} \right)
$$

$$
\left( \frac{\partial u_{rms}}{\partial \bar{U}} \right) = -\frac{1}{u_{rms}} \left( \frac{\sum (U_i - \bar{U}) w_i}{\sum w_i} \right)
$$

$$
\left( \frac{\partial u_{rms}}{\partial \bar{U}} \right) = - \left( \frac{\partial u_{rms}}{\partial U} \right)
$$

(3.42)

and from the definition of $\bar{U}$,

$$
\bar{U} = \frac{\sum_{i=2}^N U_i w_i}{\sum_{i=2}^N w_i} \implies \sigma_{\bar{U}} = \left( \frac{\partial \bar{U}}{\partial U_j} \right) \sigma_{u_j} \implies \sigma_{\bar{U}} = \sigma_{u_j} \quad (3.43)
$$
Because \( \left( \frac{\partial \overline{u}_{\text{rms}}}{\partial \bar{U}} \right) = - \left( \frac{\partial \overline{u}_{\text{rms}}}{\partial \bar{U}} \right) \) and \( \sigma_{\bar{U}} = \sigma_{U_j} \), then by inspection of Equation (3.41), it can be seen \( \sigma_{\overline{u}_{\text{rms}}} = 0 \). Therefore, the turbulence intensity uncertainty, given by Equation (3.38), reduces to:

\[
\sigma_{\text{turb}} = \frac{\partial \Gamma_u}{\partial U_j} \sigma_{U_j} = \frac{\Gamma_u}{U_j} \sigma_{U_j}
\]  

(3.44)

For an axial turbulence intensity of 16\% and fully-expanded jet velocity \( U_j = 380 \text{ m/s} \), the uncertainty in turbulence intensity is 2.5\%. In being a function of the uncertainty of the LDV measurements, \( \sigma_{U_j} \), this analysis has accounted for the effects positioning error, probe volume size, and errors in jet operational velocity. What this analysis does not account for is bimodal histograms of velocity measurements. Due to finite probe volume size, in the vicinity of a shockwave, a portion of the probe volume reside on either side of the shock. As such, the LDV software would detect two distinct velocities. The same phenomenon was reported by Panda [88] who performed LDV measurements in underexpanded jets. This bimodality would yield an artificially high \( u_{\text{rms}} \) as the mean velocity would lay somewhere in between the two histogram peaks.
4 Experimental Results

4.1 Mean and Unsteady Velocity Measurements

This section presents the major findings from mean flow and turbulence intensity measurements in the supersonic jet plume, impingement zone and outwash flow produced by the rear jet. Jet operating conditions for these measurements were $M_{j,f} = 0$, $NPR_r = 2.93$, $M_{j,r} = 1.34$, and $TTR = 0$ (Section 3.1.1).

4.1.1 Supersonic Jet Plume Measurements

In order to better understand what effects the ground plane has on the mean flow features and turbulence characteristics of the supersonic jet plume, mean velocity and turbulence intensity profiles $4D_r$ downstream of the nozzle exit ($x/D_r = 4$) were measured. Lift plate to ground plane separations were $H/D_r = 23$, 12, and 6 with $H/D_r = 23$ sufficiently far away so as to be approximated as a 'free jet.' Mean flow measurements are compared to free-jet pitot-rake measurements at the same location and jet conditions originally presented in Myers and McLaughlin [26].

4.1.1.1 Ground Plane Effects on Supersonic Jet Plume

Mean velocity measurements are presented in Figure 4.1a. Mean flow pitot rake measurements taken at $x/D_r = 4$ without the ground plane installed are shown as the solid black line with black triangles. Corresponding turbulence intensities are shown in Figure 4.1b. Mean velocity profiles for $H/D_r = 23$ and 12 agree well with each other and with free jet pitot rake measurements for $r/D_r \geq 0$. Free jet pitot rake measurements experience a peak velocity approximately 3.5% higher than that observed for $H/D_r = 23$ and 5%
(a) Mean axial velocity profiles at $x/D_r = 4$ for $H/D_r = 22.87, 12$, and $6$ compared to free jet pitot rake measurements.

(b) Axial turbulence intensity profiles at $x/D_r = 4$ for $H/D_r = 22.87, 12$, and $6$.

Figure 4.1: Supersonic jet plume profiles taken at $x/D_r = 4$ for varying lift plate heights.

higher than that observed at $H/D_r = 12$. Therefore, at these standoff distances, the jet Mach numbers ($M_j$) throughout the potential core is approximately 1.3: very close to the design Mach number ($M_d$) of 1.34. At $r/D_r \geq 0.4$ the velocity decay in the shear layer is identical between the free jet and the lift plate 23 Dr above the ground plane. At a plate separation of $H/D_r = 12$ the velocity decay rate in the shear layer appears more rapid. In general, the ground plane appears to exert minimal influence on the jet potential core.
at there standoff differences. While there is noticeable disagreement in the shapes and shear layer velocity decay rates for $r/D_r \leq 0$, this is expected to be a misalignment error between the probe and the jet plume. This will be investigated further in Section 4.1.2.2.

At the closest plate separation, $H/D_r = 6$, the velocity profile has noticeably broadened. The peak velocity has dropped significantly to $330 \text{ m/s}$: 14\% from the peak velocity measured by the pitot rake. However, near the jet centerline the flow remains supersonic ($M_j = 1.17$). The ground plane appears to have a substantial effect on the jet plume $4D_r$ downstream of the nozzle exit at this plate height. Although, it much be kept in mind that because the nozzle protrudes $1D_r$ below the surface of the lift plate, the cross-jet measurements for $H/D_r = 4$ are taken only $1D_r$ above the surface of the ground plane. Digitally averaged shadowgraphs shown in Figure 4.2 indicate this may be within the impingement zone of the jet, where flow turning and deceleration are significant.

Along the jet centerline, for the largest standoff distances, turbulence intensities are low, $\leq 3\%$. For $H/D_r = 6$, the turbulence intensity is between 6-9\% in the jet core. It appears that the presence of the ground plane causes increased mixing in the jet. As well, the radial decay of turbulence outside of the shear layer is slowed as well. For all plate separations, the intensities increase rapidly in the shear layer to a peak value of 15-17\% at the point of maximum velocity gradient. Here shear stresses in the flow are expected to be largest.

Figure 4.2: Digitally averaged shadowgraph visualization of dual impinging jets positioned at $H/D_r = 12$, left, and $H/D_r = 6$, right, with jet operating conditions $M_{j,f} = 1.0$, $M_{j,r} = 1.3$, and $TTR=1.0$. A standoff shock can be seen in the image on the right.

In the shadowgraph images presented in Figure 4.2, the front nozzle was operating at $M_{j,f} = 1.0$ and the rear nozzle at $M_{j,r} = 1.3$. Because of the large separation between
nozzles, the forward, sonic/subsonic jet is not expected to exert any tangible influence on
the supersonic jet plume or immediate impingement region produced by the aft, supersonic
jet. In the left image, the ground plane is 12 Dr below the lift plate while the right image
shows the ground plane 6 Dr below the lift plate. A standoff shock can be seen just
above the ground plane in the right image. Careful inspection places the location of this
shock approximately 0.5 Dr above the surface of the ground plane. This appears to be in
agreement with Krothapalli et al. [37] who presented shadowgraph images for a moderately
underexpanded $M_j = 1.5$ jet impinging on a flat plate six diameters downstream of the
nozzle exit. An important flow feature of supersonic impinging jets hinted at in the
shadowgraphs is that even for moderate to large plate separations, the turning radius of
the flow is small. The impingement zone appears to be restricted to a region very near the
surface of the ground (within 1 $D_r$).

4.1.1.2 Two-Dimensional Survey of Impinging Jet Plumes

Vector fields and contours of velocity were generated for the impinging jet plumes. Fig-
ures 4.3a and 4.3b show the vector fields and contours for lift plate standoff distances
of 12 $D_r$ and 6 $D_r$, respectively. Open circles mark the locations of the data points used
to generate the contours. With the exception of vertical-component-only measurements
taken at $y/D_r \geq 4$, velocity vectors and magnitudes are calculated from both axial and
radial components. Vector arrows have been converted to unit vectors and as such are only
meant to indicate flow directionality. Contours are generated by interpolating between
measurement locations. Due the relatively spare matrix of measurement locations, contours
meant to yield a qualitative representation of the flow field.
Figure 4.3: Velocity magnitude contours for $H/D_r = 12$ and $6$ with unit vectors indicating flow directionality. Open circles denote measurement locations used to generate contours. Note: contour color levels are identical for both plots.
Within 0.5 $D_r$ of the ground plane, there remains a strong axially directed component of velocity on the order of 250 m/s for both jets. However, significant turning can start to be seen at radial locations greater than 1 $D_r$. This is consistent with the previous observation that impingement phenomena occur very near the surface of the ground plane.

For $H/D_r = 6$, the flow can be seen to be slowing down by $y/D_r = 0.25$. By 2 $D_r$ radially out from the centerline the flow can be seen to be flowing horizontal or upward indicating the start of the outwash region. In both contours, the flow directionality near the ground appears to have a slight bias towards the negative radial direction. This is further evidence
that there is some slight degree of misalignment between the jet and the LDV probe. An attempt is made to quantify this misalignment in Section 4.1.2.2.

Axial turbulence intensity contours for $H/D_r = 12$ and 6 are shown in Figures 4.4a and 4.4b, respectively. Turbulence intensity contours are generated using the same points as those shown in Figure 4.4. For a plate separation of $12D_r$ the strongest mixing appears to be in the jet shear layer greater than $5D_r$ above the surface of the ground plane. For both plate separations, turbulence intensities greater than 14% exist throughout the jet $0.5D_r$ above the plate. This is especially pronounced for $H/D_r = 6$. As previously mentioned in Section 4.1.1.2, inspection of the right shadowgraph photograph in Figure 4.2 places the stand-off shock at approximately this location. Oscillations in the location of the stand-off shock could result in the higher turbulence intensities observed as oscillations would produce strong fluctuations in the local velocity. While this phenomena was not observed directly in this work, Henderson et al. [22,23,47] has shown that upstream jet instabilities cause significant stand-off shock oscillations and thus strong oscillations in the surrounding flow.

In an attempt to compare all jet profiles measured and more clearly see the effects of an impingement surface in a supersonic jet, axial velocity data are plotted on nondimensional coordinates. It was expected that the presence of a ground plane would cause the data to deviate from the well-known two-dimensional shear layer similarity profile. Following Brown and Roshko [13] and Samimy and Elliot [20], the similarity parameter is defined as:

$$\eta^* = \frac{r - r_{0.5}}{\delta_w}$$

where the shear layer thickness, $\delta_w$, is defined as:

$$\delta_w = \frac{\bar{U}(r)_{\text{max}}}{\left( \frac{\partial \bar{U}(r)}{\partial r} \right)_{\text{max}}}$$

This may also be interpreted as the vorticity thickness, as discussed by Brown and Roshko [89]. For a given height above the ground plane, $r_{0.5}$ is the radial position at which the local axial velocity is equal to half the maximum measured velocity; that is, $r_{0.5} = r \left( \frac{1}{2} \bar{U}(r)_{\text{max}} \right)$. Peak velocity gradient (denominator of Equation (4.2)) and $r_{0.5}$ are found by applying a 5th order polynomial fit to the velocity profiles. $r_{0.5}$ is determined from where the curve fit has a value equal to half the maximum measured velocity. Maximum velocity gradient is determined by where the derivative of the polynomial fit reaches a maximum value.
For the present analysis, only half of the jet measurements, between \( 0 \leq r/D_r \leq 2 \), were normalized. \( \bar{U}(r)_{\text{max}} \) is the peak velocity measured in this region for a given height above the ground plane. The exact measurement locations are explicitly listed in Table 4.1. As well, the right-most column of Table 3.1 denotes which sets of data were used. Roman numerals denote the sets of axial velocity measurements used.

Table 4.1: LDV Measurement Points Used to Produce Nondimensionalized Velocity and Turbulence Intensity Profiles

<table>
<thead>
<tr>
<th>Measurement Location</th>
<th>( H/D_r )</th>
<th>( x/D_r )</th>
<th>( y/D_r )</th>
<th>( r/D_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>22.87</td>
<td>4</td>
<td>17.87</td>
<td>0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8</td>
</tr>
<tr>
<td>II</td>
<td>12</td>
<td>10.5</td>
<td>0.5</td>
<td>0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0</td>
</tr>
<tr>
<td>III</td>
<td>12</td>
<td>10</td>
<td>1</td>
<td>0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0</td>
</tr>
<tr>
<td>IV</td>
<td>12</td>
<td>7</td>
<td>4</td>
<td>0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2</td>
</tr>
<tr>
<td>V</td>
<td>12</td>
<td>4</td>
<td>7</td>
<td>0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8</td>
</tr>
<tr>
<td>VI</td>
<td>6</td>
<td>4.75</td>
<td>0.25</td>
<td>0, 0.4, 0.8, 1.2</td>
</tr>
<tr>
<td>VII</td>
<td>6</td>
<td>4.5</td>
<td>0.5</td>
<td>0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0</td>
</tr>
<tr>
<td>VIII</td>
<td>6</td>
<td>4</td>
<td>1</td>
<td>0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0</td>
</tr>
</tbody>
</table>

Normalized axial velocities are shown in Figure 4.5a. Between \(-1.5 \leq \eta^* \leq 0.5\) there is excellent collapse of the data for all plate separations and measurement heights, \( y/D_r \). Because of the good collapse, this provides evidence that free-jet behavior is still observed close to the ground plane even for small jet stand-off distances (e.g. Location VI: \( H/D_r = 6 \), \( y/D_r = 0.25 \)). There is some scatter in the data for \( \eta^* > 0.5 \) however, depending on measurement height, these data are either in the wall jet, where the flow is no longer a part of the jet plume, or the quiescent region, where flow speeds are very low. All data compare well with Gortler's error-function profile. The equation for this profile is:

\[
U/U_{\text{max}} = \frac{1}{2}\left[1 - \text{erf}(1.5\eta^*)\right] \tag{4.3}
\]

where the 1.5 inside Equation (4.3) is typically referred to as the 'spreading rate parameter' as discussed in Samimy and Elliot [20] and Lau et al [50] and is determined experimentally. For this study, a value of 1.5 was chosen as it produced a curve-fit that best matched the data. There was found to be equally good collapse of the data when jet exit velocity \( (U_j = 380 \, \text{m/s}) \) is used in place of \( \bar{U}(r)_{\text{max}} \), as discussed in Lau et al [50] and Kerherve et al. [53]. For brevity however, the results of this analysis are not presented.
(a) Normalized axial velocities plotted against the similarity parameter. Locations of velocity measurements used are between $0 \leq r/D_r \leq 2$ for $H/D_r = 22.9, 12, \text{ and } 6$. Error function equation is: $U/U_{\text{max}} = 1/2(1 - \text{erf}(1.5\eta^*))$

(b) Axial turbulence intensities plotted against similarity parameter for $y/D_r \geq 1$

(c) Axial turbulence intensities plotted against similarity parameter for $y/D_r < 1$

Figure 4.5: Normalized axial velocities and axial turbulence intensities plotted against similarity parameter, $\eta^*$, for all lift plate heights and all measurement heights above the ground plane
Figures 4.5b and 4.5c present the turbulence intensities plotted against nondimensionalized radius. Figure 4.5b shows the turbulence intensities for all measurements at $y/D_r \geq 1$ whereas Figure 4.5c shows the turbulence intensities for all measurements at $y/D_r < 1$. For $y/D_r \geq 1$ there is a generally good collapse of the turbulence intensities on to a single curve. Although $H/D_r = 12$, $y/D_r = 4$ (Roman numeral VI) shows some discrepancies. All turbulence intensities reach a maximum between $-0.25 \leq \eta^* \leq 0$. Physically, this suggests that for a given height above the ground plane, the peak turbulence in the impinging jet plume peaks where the local axial velocity is approximately equal to half the maximum velocity. Closer to the ground plane (Figure 4.5c, $y/D_r < 1$), near/in the impingement zone where ground affects on the flow become significant, the trends observed in Figure 4.5b begin to break down. The data no longer collapse to a single curve and position of peak turbulence intensity occurs anywhere between $-1.25 \leq \eta^* \leq 0.25$.

4.1.2 Investigation of Impingement Zone

Axial and radial components of velocity and turbulence intensity are compared to each other for two plate separations, $6D_r$ and $12D_r$ in order to better understand the physics of the impingement region and the effect of surface proximity to the jet source.

4.1.2.1 Discussion

Mean axial velocity measurements for $H/D_r = 12$ and $6$, $y/D_r \leq 1$ are presented in Figure 4.6. At $1D_r$ above the ground plane, the peak velocity measured for $H/D_r = 6$ is approximately 10% larger than that measured for $H/D_r = 12$. Axial velocities are observed to decay faster with increasing $r/D_r$ for $H/D_r = 6$ than $H/D_r = 12$. This is expected as the jet plume for $H/D_r = 12$ has more time to spread and mix than the jet plume for $H/D_r = 6$. Although the axial profiles are within $5D_r$ of the nozzle exit for $H/D_r = 6$, they no longer exhibit a classic "top-hat" profile; the profiles have a bell-curve shape typical of a fully-developed turbulent jet. For closer plate separations, the presence of the ground plane appears to disrupt or shorten the potential core of the jet.

Peak velocities for both plate separations decline by about 10% between $1 \geq y/D_r \geq 0.5$. This suggests that the axial velocity rapidly decays and get redirected close to the ground plane ($y/D_r \leq 0.5$) regardless of lift plate height. Indeed, in Figure 4.6b, between $0.5 \geq y/D_r \geq 0.25$, peak axial velocity decays by nearly 20%: the velocity decays twice as much in half distance compared to the drop observed between $1 \geq y/D_r \geq 0.5$. 
(a) Axial velocities in the impingement region, \(y/D_r \leq 0.5\), for \(H/D_r = 12\)

(b) Axial velocities in the impingement region, \(y/D_r \leq 0.5\), for \(H/D_r = 6\)

Figure 4.6: Axial velocity profiles in the impingement zone for plate separations of 6 and 12 \(D_r\)

Mean radial velocity measurements for \(H/D_r = 12\) and 6, \(y/D_r \leq 1\) are presented in Figure 4.7. Again, as discussed in Section 3.1.2, positive radial velocities are directed away from the jet centerline. The observed magnitudes of peak radial velocity are between 6% to 33% of corresponding peak axial velocities. Peak radial velocity rapidly increases with decreasing height above the ground plane. For \(H/D_r = 12\), peak radial velocity increases by 200% between \(1 \geq y/D_r \geq 0.5\). A 200% increase is also observed for \(H/D_r = 12\) between \(0.5 \geq y/D_r \geq 0.25\). Interestingly, from Figure 4.6, axial velocities in the jet shear layer for \(y/D_r = 1\) and 0.5 are virtually unchanged. It therefore appears that it is primarily the energy lost in the flow along the jet centerline that contributes to the increase in radial velocities.

In the impingement zone, radial velocities reach a maximum approximately \(1 D_r\) away from the jet centerline, then decay farther outwards. Negative radial velocities observed at larger radii are the result of the quiescent air being entrained by the jet. The velocity vector fields in Figure 4.3 support this observation. The antisymmetry in the radial profiles and relatively large negative radial velocities along the centerline is due to jet-LDV probe
(a) Radial velocities in the impingement region, \( y/D_r \leq 0.5 \), for \( H/D_r = 12 \)

(b) Radial velocities in the impingement region, \( y/D_r \leq 0.5 \), for \( H/D_r = 6 \)

Figure 4.7: Radial velocity profiles in the impingement zone for plate separations of 6 and 12 \( D_r \)

misalignment. An estimation of this misalignment based off of the mean axial and radial velocity profiles is discussed in Section 4.1.2.2.

Figure 4.8 presents axial turbulence intensities for \( H/D_r = 12 \) and 6. Where a minimum axial turbulence intensity of 3\% – 6\% is observed in the supersonic jet plume along the jet centerline (see Figures 4.1b and 4.4), in the impingement region, axial turbulence intensities are over 12\% throughout the jet indicating that this is a region of strong velocity fluctuations and intense mixing. The trough in turbulence intensity in the core of the jet is almost nonexistent for \( H/D_r = 12 \). Turbulence intensity appears to become more uniform throughout the jet with decreasing height above the plate. For \( y/D_r = 6 \), \( y/D_r = 1 \) and \( 0.5 \), a noticeable decrease in axial turbulence intensity is still observed in the core of the jet. This may be attributable to the fact that the measurements are still within 5 \( D_r \) of the supersonic nozzle exit. Although, between \( y/D_r = 1 \) and \( 0.5 \) a significant increase in axial turbulence intensities in the jet core increase by approximately 250\%. At \( y/D_r = 0.25 \) axial turbulence intensity is nearly uniform throughout the jet. It therefore appears that the trends observed for \( H/D_r = 12 \) may appear closer the surface of the ground plane for
$y/D_r = 6$ although measurements closer to the ground plane must be made to confirm this hypothesis.

![Graph](image)

(a) Axial turbulence intensities in the impingement region, $y/D_r \leq 0.5$, for $H/D_r = 12$

(b) Axial turbulence intensities in the impingement region, $y/D_r \leq 0.5$, for $H/D_r = 6$

Figure 4.8: Axial turbulence intensity profiles in the impingement zone for plate separations of 6 and 12 $D_r$

Radial turbulence intensities are presented in Figures 4.9a and 4.9b for plate separations of 12 $D_r$ and 6 $D_r$, respectively. Radial turbulence intensities exhibit a maximum value along the jet centerline for a lift plate to ground plane separation of 12 $D_r$. Radial turbulence intensities throughout the jet appear to increase with decreasing height above the ground plane. This is not unexpected as the magnitudes of radial velocities increase closer to the ground plane. However, an opposite trend in radial turbulence intensities is observed for a plate separation of 6 $D_r$. Radial turbulence intensities reach a minimum value along the jet centerline. The magnitude of centerline of centerline turbulence intensities does increase with decreasing $y/D_r$. Peak radial turbulence intensities are measured at $|r/D_r| \geq 0.5$ and appear to move out away from the jet centerline with decreasing $y/D_r$. Whether the trends observed for $H/D_r = 12$ occur closer to the ground plane for $H/D_r = 6$ or there is some physical mechanism is responsible for altering the radial turbulence characteristics has yet to be determined.
4.1.2.2 Estimation of Probe-Jet Misalignment

As has been mentioned in Sections 4.1.1.2 and 4.1.2.1, it is apparent there is some misalignment between the LDV sending/receiving optics and the impinging jet. In this section an attempt is made to quantify the extent of the misalignment.

Given the skewed nature of the radial velocity profiles presented in Figure 4.7, there is some degree of angularity between the probe and the jet. The true centerline of the jet is taken as the radial location where the axial velocity for \( H/D_r = 6 \), \( y/D_r = 1 \) (Figure 4.6b) reaches a maximum value. This particular velocity profile location will be used to estimate the jet-probe angularity because it is closest profile to the nozzle exit in which both velocity components were measured. Ground plane effects such as increased radial velocities should be at minimum compared to other velocity profiles. In this profile, axial velocity is seen to reach a peak value of \( \bar{U} \approx 330 \text{ m/s} \) at \( r/D_r = 0.2 \). The corresponding magnitude of radial velocity at this radial position, determined from Figure 4.7b, is \( \bar{V} \approx 20 \text{ m/s} \). Therefore, the
probe-jet misalignment angle may be determined by:

\[ \theta = \tan^{-1} \left( \frac{\bar{V}}{\bar{U}} \right) = \tan^{-1} \left( \frac{20}{330} \right) \tag{4.4} \]

and the misalignment angle is found to be \( \theta \approx 3.5^\circ \) towards the negative \( r/D_r \) direction.

The negative radial velocities to the left and right of \( r/D_r = 0 \) in Figure 4.7 can be easily explained by a radial misalignment, \( \Delta r/D_r \) between the probe and the jet. If \( \Delta r/D_r \) is negative (i.e. the jet is to the left of what is considered to be \( r/D_r = 0 \)), as the jet spreads radially outwards, the radial velocity would be directed towards \( r/D_r = 0 \) and would therefore be interpreted as negative. Again, radial velocities directed towards \( r/D_r = 0 \) are taken as negative (See: Section 3.1.2). At positive \( r/D_r \)'s, the entrainment flow is directed towards \( r/D_r = 0 \) and opposing the outward spreading of the jet. This accounts for the lower magnitude negative radial velocities observed at positive \( r/D_r \)'s. Further from the jet centerline, the large, positive radial velocities are the jet outwash flow. If indeed the jet is rotated and linearly displaced with respect to \( r/D_r = 0 \) the velocity vector with zero radial velocity component would occur at an \( r/D_r \) position off the centerline and independent of height above the ground plane. In fact, this is observed in both Figures 4.7a and 4.7b. All radial velocity profiles intersect 0 on the ordinate at \( r/D_r = -0.1 \). This confirms a negative radial misalignment of \( \Delta r/D_r = -0.1 \). A schematic diagram of the LDV probe-jet misalignment is given in Figure 4.10.

![Figure 4.10: Schematic diagram of estimated LDV probe-jet misalignment](image)
4.1.3 Characterization of Outwash Flow Development

Unsteady radial-component velocity measurements were performed in the wall jet, or outwash flow region, of the flow field for a lift plate height of 12 diameters. Measurements were taken from two to twelve diameters radially outward from the centerline of the rear jet in $1D_r$ increments. Logarithmic vertical spacing of measurements above the ground plane allow for good resolution of the wall jet velocity profile both near the wall and in the entrainment shear layer. These data supplement the steady outwash flow measurements previously performed by Myers and McLaughlin [26] using a 5-probe pitot rake. Because of the relatively small probe volume size, the LDV technique is able to measure within 0.02 $D_r$ of the ground plane whereas the pitot probe is physically limited to measurements no closer than $y/D_r = 0.1$ for the nozzle used in this study.

Figure 4.11a presents a contour plot of the mean radial velocity in the wall jet. Velocities as high as $125 \text{ m/s}$ are observed out as far as $6.5D_r$ from the jet centerline. As well, velocities as much as 25%$U_j$ persist out to $9D_r$. The wall jet is largely confined close to the ground plane, $y/D_r \leq 0.25$. Significant upward spreading of the outwash flow appears to begin where the peak wall jet velocity rapidly decays, around $r/D_r = 6$.

Scaled outwash flow velocity profiles are shown in Figure 4.11b in black. Each profile is zeroed to its corresponding $r/D_r$ location on the abscissa. The spacing of major vertical ticks corresponds to $\bar{V} = 0.80\%U_j$. Also shown are the 95th percentile velocity (blue) and 99th percentile velocity (red) velocity profiles. These profiles give an indication of the maximum jet speeds that could be experienced in the wall jet due to turbulent velocity fluctuations. Within the first five diameters from the jet centerline, mean velocities are about 50% of the jet exit velocity but with turbulent fluctuations they may be as high 80%. Maximum velocities are initially largest near the ground plane. However, at larger radial distances away, velocity fluctuations are largest higher above the plate. Interestingly, for $6 \leq r/D_r \leq 8$, peak velocities due to turbulent fluctuations occurs farther above the ground plane ($y/D_r \approx 0.25$) than position of maximum mean velocity ($y/D_r \approx 0.125$).

Maximum wall jet velocity is plotted against radial position in Figure 4.12a. Peak velocity declines rapidly with increasing distance from the jet centerline. For $r/D_r \geq 4$, max velocity in the wall jet is inversely proportional to radial position, in the form:

$$V_{r,max} \approx \left( \frac{A}{r/D_r} \right)$$  \hspace{1cm} (4.5)
(a) Outwash flow mean radial velocity magnitude contour for $H/D_r = 12$

(b) Scaled outwash flow radial velocity profiles for: mean velocity (black), 95th percentile velocity (blue), and 99th percentile velocity (red) for $H/D_r = 12$

Figure 4.11: Mean radial velocity contour and scaled radial velocity profiles in outwash flow region for $H/D_r = 12$ and $M_j = 1.34$
Excellent agreement with the data was found for $A = 759$. Equation (4.5) and the velocity decay in Figure 4.12a shows good agreement with the 2-dimensional velocity field generated by a point source of strength $Q$ from potential flow theory, Figure 4.12b. For $r/D_r \geq 4$, the rear jet can reasonably be modeled as a point source as a first approximation to estimate peak velocities. The constant, $A$ in Equation (4.5) is equal to the volume flow rate of the jet, $0.054 \text{ m}^3/\text{s}$, multiplied by an empirically-determined scale factor of 14100 to match the data.

![Graph](image)

(a) Peak outwash flow velocity decay with increasing distance from jet centerline for $H/D_r = 12$

(b) Top view schematic of jet impinging on a ground plane (dotted circle) modeled as a point source of strength $Q$

Figure 4.12: Wall jet peak velocity decay compared to the $r^{-1}$ velocity decay radially outwards from a point source

Figure 4.13a presents a contour plot of the radial turbulence intensity throughout the wall jet. Turbulence intensities as much as 12% persist in the wall jet out to $7D_r$. There is a steady increase in the position of peak turbulence intensity above the plate with increasing radial distance for the $r/D_r \leq 7$.

Scaled outwash flow velocity profiles are shown in Figure 4.13b. Each profile is zeroed to its corresponding $r/D_r$ location on the abscissa. The spacing of major lateral ticks corresponds to 10% turbulence intensity. Peak radial turbulence intensity increases up to $r/D_r = 4$ then decreases with increasing $r/D_r$ as the wall jet spreads and loses momentum to the quiescent air. Turbulence intensities become more uniformly distributed throughout the height of the wall jet as the boundary layer and entrainment shear layer converge. When the two shear layers converge, the outwash flow may be considered 'fully-developed'.

76
Figure 4.13: Radial turbulence intensity contour and scaled radial turbulence intensity profiles in outwash flow region for $H/D_r = 12$ and $M_j = 1.34$.
Outwash flow measurements between $8 \leq r/D_r \leq 12$ are compared to pitot rake data in Figure 4.14. Pitot rake measurements in the outwash flow of was previously reported by Myers and McLaughlin [26]. The exact radial position of pitot rake measurements is indicated on each graph in red, italicized font. There is good agreement between the LDV and pitot probe measurements in the outwash region, particularly above $y/D_r = 0.5$. However, at nearly all positions, peak velocities measured using LDV are higher than those observed in the pitot rake measurements. This is likely due to spatial averaging in the pitot probes near the point maximum velocity. Because the pitot-probe tip is larger than the size of the LDV probe volume, pitot probe measurements are averaging over a larger vertical section of the wall jet. As such, the pitot probes may not be able to accurately resolve the peak velocity in a wall jet of this scale.

It is important to note that the authors of [26] believe the impinging jet apparatus was not oriented perfectly vertical at the time pitot rake measurements were taken. This would introduce angularity in the resulting outwash flow such that there would be a non-negligible velocity component tangential to the pitot rake that could not be measured.

Normalized radial outwash velocities and radial turbulence intensities are presented in Figure 4.15. Wall jet velocities are normalized by the maximum velocity observed at the corresponding radial position, $\bar{V}_{max}(r/D_r)$. Similarly, turbulence intensities are normalized by the peak turbulence intensity at each radial station, $v'_{max}$. The vertical distance is nondimensionalized by $y_{0.5}$: the height above the plate at which the local velocity is equal to half the peak velocity for a given radial position. That is, $y_{0.5} = y(\frac{1}{2} \bar{V}_{max})$. A collapse of

Figure 4.14: Mean outwash flow LDV measurements compared to pitot rake data at similar locations for $H/D_r = 12$ and $M_j = 1.34$. The radial position of pitot rake measurement is listed in each plot in red.
the data using these parameters has been interpreted by Miller and Wilson [90] to indicate that the wall jet has become fully developed.

The left-hand column of Figure 4.15 (Figures 4.15a, 4.15c and 4.15e) presents the normalized velocities. It is clear that the outwash flow velocities between \(2 \leq r/D_r \leq 12\) collapse on to three distinct curves. This is indicative of three regions in the wall jet: 1) the early and 2) intermediate wall jet (Figures 4.15a and 4.15c), and 3) the fully-developed wall jet (Figure 4.15e). The early wall jet, \(2 \leq r/D_r \leq 5\) is characterized by a "concave-up" shape with a maximum velocity occurring near \(0.1y_{0.5}\). Moving outwards to the intermediate (or developing) wall jet region, \(6 \leq r/D_r \leq 8\), the profile has a largely linear shape above the point of peak velocity. Interestingly, this intermediate region appears to have two distinct heights at which the flow speed reaches a maximum: \(0.1y_{0.5}\) and \(0.4y_{0.5}\).

Above a radial distance of \(9D_r\), the outwash flow is fully developed. This is in agreement with Myers and McLaughlin [26] who observed fully-developed flow from \(r/D_r = 8.3\) to 24. The velocity profile takes on the classic wall jet boundary layer shape with a "bending back" of the data above and below the position of peak velocity. Maximum velocity in the wall jet occurs at \(0.3y_{0.5}\) above the ground plane. This is nearly the same as the height of the second velocity peak in the intermediate wall jet region.

The right-hand column of Figure 4.15 presents the normalized radial turbulence intensities. In the early wall jet, Figure 4.15b, there is a very rough collapse of the data. Although, this is an aerodynamically complex region region of the flow with significant turning and strong velocity gradients; a collapse of turbulence intensities is not necessarily expected. Figure 4.15d shows the normalized radial intensities for \(r/D_r \geq 6\). There is an excellent collapse of the data onto a single curve. It appears that outwash flow turbulence becomes fully-developed (i.e normalized velocity fluctuations become independent of radial position) more quickly than the mean flow velocities. Comparison with Figures 4.15c and 4.15e, shows that peak turbulence intensity occurs near or just above the position of peak velocity i.e. in the entrainment shear layer. There is a distinct local maxima in turbulence intensity near \(y/D_r = 0.1\) in the boundary layer as well.
Figure 4.15: Normalized radial velocity and normalized turbulence intensity profiles of the outwash flow for $H/D_r = 12$ and $M_j = 1.34$
4.2 Turbulence Spectra

Turbulence spectra have been estimated at a series of points along the jet axis, jet shear layer, and wall jet for a lift plate height of 6 $D_r$ using the fuzzy-slotting technique discussed in Section 3.7. This height was chosen because Myers et al. [63] observed a strong tone, greater than 20 dB in relation to surrounding amplitudes, at this lift plate height. LDV spectra obtained at five positions throughout the flow field are presented in Figure 4.16 in solid blue. Also included on these graphs is the spectrum obtained from acoustic field measurements of a single microphone located aft of the rear nozzle in red. The scale for the acoustic measurements is on the right ordinate. The top row of Figure 4.16 presents turbulence spectra determined from axial velocity fluctuations while the bottom three figures present those determined from radial velocity fluctuations. The exact coordinates of where spectra were obtained within the jet are indicated in the lower-left corner of each graph.

There is significant energy, over $10^{-1} \frac{u'_{rms}}{Hz}$ in the low frequency, large-scale structures for both axial and radial velocity components. The only position where is not observed is for radial velocity fluctuations at $y/D_r = 0.25, r/D_r = 0.8$ (Figure 4.16c). However, this is in the early portion of the wall jet, where it is thinnest so the largest scale structures will possess higher frequencies. At this station, the turbulence spectrum appears flat over a wide range of frequencies, $1 - 10$ kHz. This is not observed in the other spectra. In Figures 4.16a, 4.16b, 4.16d and 4.16e, above 8 kHz, the decline in turbulence spectra intensity parallels the decline in acoustic field intensity.

All graphs in Figure 4.16, with the exception of Figure 4.16e, show a distinct tone present in the turbulence spectra. This tone has excellent agreement with previous acoustic measurements. Both measurements indicate a tone frequency of 12.7 kHz. This tone is observed in the jet shear layer, Figure 4.16a, and near the jet centerline near the stand-off shock, Figure 4.16b. The tone increases in amplitude closer to the impingement region. In Figure 4.16c it appears strongest (relative to the local mean) in the impingement zone near the outside of the jet shear layer, $y/D_r = 0.25, r/D_r = 0.8$. Moving outwards radially, the tone decreases in magnitude. By 2 $D_r$ away from the jet centerline the tone was no longer observed in the turbulence spectra. These findings are in agreement with Henderson et al. [23] who proposed that the source of the impingement tone, in the presence of a stand-off shock, is the result of velocity fluctuations in the early wall jet.
(a) $\bar{U} = 76.5\text{m/s, } u_{\text{rms}} = 57.3\text{m/s}$, Axial velocity

(b) $\bar{U} = 258\text{m/s, } u_{\text{rms}} = 46.7\text{m/s}$, Axial velocity

(c) $\bar{V} = 41.3\text{m/s, } v_{\text{rms}} = 43\text{m/s}$, Radial velocity

(d) $\bar{V} = 73.4\text{m/s, } v_{\text{rms}} = 73\text{m/s}$, Radial velocity

(e) $\bar{V} = 53.5\text{m/s, } v_{\text{rms}} = 55\text{m/s}$, Radial velocity

Figure 4.16: Blue spectra: Turbulence spectra obtained from LDV measurements for $H/D_r = 6$, $M_j = 1.34$, $NPR = 2.93$, $TTR = 1$, characteristic frequency, $f_c = 31.8\text{kHz}$. LDV measurement locations are indicated on each graph. Frequency resolution is 122Hz. Red spectra: Acoustic field measurements taken directly aft of the rear nozzle for the same plate separation and jet conditions. Frequency resolution is 74Hz.
5 | Summary and Conclusions

5.1 Summary of Goals and Objectives

The main focus of this thesis is to qualify the high-speed LDV system for unsteady velocity measurements throughout the flow field produced by supersonic exhaust jets impinging on a simulated ground plane. This included determining the two-dimensional mean velocity field in the impingement zone of the jet and the turbulence properties in the three defined regions of the flow field: (i) the free jet, (ii) the impingement zone, and (iii) the outwash flow/wall jet. The goal of this thesis was realized through the completion of the objectives restated below:

1. Integrate the laser and fiber-optics components to develop a viable system to measure two-components of the velocity field

2. Develop a series of MATLAB codes to perform the following tasks:
   (a) Process raw LDV velocity data to determine mean and rms velocities and write to EXCEL file and view velocity histograms
   (b) Plot mean velocity and turbulence intensity profiles
   (c) Determine turbulence spectra using the 'fuzzy-slotting' technique

3. Perform non-simultaneous two-component mean and unsteady velocity measurements in the supersonic jet plume, impingement zone, and outwash flow. Mean velocity measurements in the jet plume and wall jet are compared to previous pitot probe measurements at the same locations.

4. Estimate turbulence spectra for both axial and radial velocity fluctuations at various locations in the flow field and compare to previous acoustic field measurements.
5. Quantify the accuracy of the mean and unsteady LDV measurements in the jet plume and wall jet.

The first objective was met and described in detail in Sections 3.6 and 3.7. These sections describe in detail the exact methodology used for data processing and spectral analysis. Using the equations for mean and \( \text{rms} \) velocity discussed in Section 3.6 as well as the equation for fully expanded jet velocity, the second objective was met in Section 3.8, in which a semi-empirical uncertainty analysis of the mean and unsteady velocity measurements is outlined. Using a large matrix of measurement locations for three plate heights, the third objective objective is completed. Mean velocity measurements in the supersonic jet plume and wall jet are shown to compare well with previous pitot probe measurements. In performing spectral analysis on the unsteady measurements obtained during the completion of the third objective, the final objective was met. Impingement tone frequency in both turbulence and acoustic field spectra show excellent agreement.

### 5.2 Review of Primary Results

Two-component time-mean and unsteady laser Doppler velocimetry measurements are performed in the supersonic jet plume, impingement region, and outwash flow of a jet exhausting from a nozzle operating at its design Mach number of 1.34. The geometry of the experimental model used in this study is representative of the underside of a generic STOVL aircraft in hover. Measurements were made with lift plate heights of 23 \( D_r \), 12 \( D_r \), and 6 \( D_r \). A major purpose of these experiments is to help guide the development of computational models for vertical landing scenarios of powered lift STOVL aircraft.

Mean flow measurements in the jet plume show good agreement with previous free-jet pitot probe rake measurements for lift plate heights of 23 \( D_r \) and 12 \( D_r \) 4 jet diameters downstream of the nozzle exit. Turbulence measurements made in the same locations show a peak intensity of 17% in the jet shear layer. At a lift plate height of 6 \( D_r \) and \( x/D_r=4 \), the presence of the ground plane has had significant effects of the velocity profile: The flow has been strongly decelerated and the shear layer has been broadened. Turbulence intensities remain between 6-9% throughout the core flow. As well, the radial decay in turbulence appears to be less rapid compared to the larger jet stand off distances.

For lift plate height of 12 \( D_r \) and 6 \( D_r \), axial velocities as high as 250 m/s are observed 0.5 \( D_r \) above the ground plane. Normalized half-jet velocity profiles show jet self-similarity is preserved down to 0.25 \( D_r \) above the ground plane: well within the ground-affected
region of the flow. Plotted against the same similarity parameter, turbulence intensities measured above $y/D_r=1$ show a peak very near the radial position where the local axial velocity is equal to half of the centerline value.

Inspection of the axial and radial velocity profiles in the jet plume appear to indicate a jet-probe misalignment of $\Delta r/D_r = -0.1$ and a jet orientation angle of $3.5^\circ$ with respect to vertical. This misalignment is taken into consideration in the uncertainty analysis of the velocity measurements.

Near the ground plane, significant redirecting of the flow is observed at radial positions greater than one nozzle exit diameter outwards from the jet centerline. Radial-component measurements in the wall jet are made out to $12D_r$ with a logarithmic vertical spacing of measurement locations above the ground plane. A clear development of the wall jet is observed. Wall jet velocity profiles appear to collapse on to three distinct curves indicating early, transitional, and fully-developed regions within this region of the flow. The fully-developed wall jet appears to begin 8-9 $D_r$ away from from the jet centerline. Mean velocities as large as 25% of the theoretical jet exit velocity are seen to persist in the wall jet out to 7 diameters. As well, peak outwash velocities show a clear $1/r$ drop-off at for radial positions greater than 4 diameters.

Turbulence intensities in the wall jet remain as high as 15% of the jet exit velocity out to $r/D_r = 4$. With turbulent fluctuations, wall jet air speeds may reach as high as $80\% U_j$ within out to 5 jet diameters. Normalized turbulence intensities appear to become radially independent by $r/D_r = 6$; nearly $3D_r$ prior to the beginning of the fully-developed wall jet.

Turbulence spectra for axial and radial velocity components are estimated at five positions throughout the flow field for $H/D_r = 6$. A distinct tone of 12.7kHz is observed in spectra along the jet centerline, jet shear layer near the stand-off shock, and the early outwash flow ($1.4D_r$). The frequency of this tone shows excellent agreement with previous acoustic field measurements at the same operating conditions. There does not appear to be a tone in spectra for locations farther out in the wall jet. This is consistent with the findings of with Henderson et al. [23] who proposed the production of the impingement tone occurs early in the wall jet.
5.3 Recommendations for Future Work

Development of the laser Doppler velocimeter in the High Speed jet Aeroacoustics facility is ongoing. Future experiments are planned with an increased focus on near-ground measurements in the impingement zone of the jet. While the current study provided a detailed velocity map of the flow field of a single supersonic jet impinging on a flat plate, it is clear the impingement region was not fully resolved. A second focus should be on interrogating the flow field produced by both the forward, sonic jet, and rear supersonic jet operating simultaneously. The interaction of these jets produce a more complex flow field that would include an upward-directed fountain flow and recirculation zones. It is important to collect a large database of measurements to better understand these complex flows and to guide the development of computational models.

Work has already been completed on upgrading the experimental model used for the work presented in this thesis to a larger, more accurate model with the ability to incorporate military-style faceted nozzles. The increased accuracy in model design and construction is expected to mitigate the misalignment issues discovered during this investigation. As well, for a fixed LDV probe volume size, a larger model allows for measurements closer to the ground plane with respect to nozzle exit diameter.

To gain a better understanding of the flow field, more advanced LDV measurements should be performed in the form of simultaneous two-point measurements and simultaneous two-component measurements. Unsteady simultaneous single-point two-component velocity measurements and simultaneous two-point, single component velocity measurements are critical to the development of accurate CFD models of these flows. As the flow field is comprised of three very different regimes: supersonic jet, impinging flow, and subsonic wall jet, the turbulence characteristics of each of these regions is different. Advanced measurements would yield insight into the components of the Reynolds Stress tensor. Insight gained will be used to ensure computational models are capturing the proper physics. As well, single-point two-component velocity measurements would allow for the determination of the convective velocities of the large-scale structures present in the flow. Recall, Mach wave radiation is a primary noise-production mechanism in flows with supersonic convective velocities. It is critical to understand how flow instabilities radiate acoustically to the far field in this environment to protect the hearing of military personnel in the vicinity of these aircraft.
Hopefully the current study has laid the ground work for more advanced laser Doppler velocimetry measurements in supersonics jets issuing from military-style nozzles. This highly-capable tool would be well-suited to perform measurements in jets in practical configurations relevant full-scale military applications. This includes, but is not at all limited to dual impinging jets, jets over simulated aft decks and carrier jets, and jets exhausting from rectangular nozzles.
Appendix  |  
MATLAB Processing Scripts  

1 LDV Velocity Processing

This script processes the raw LDV data output by the AUR Studios software to determine valid velocity measurements based on SNR, P2P ratio, and standard deviation. The user also has an option to exclude any velocities above or below a certain value or within a defined band. This allows for the exclusion of non-physical velocities which may be present in the data due to system noise. The code outputs time-mean velocity and \textit{rms} velocity to an \textsc{Excel} file.

The script also displays two PDF’s of: 1) all valid velocity measurements, and 2) particle inter-arrival times. The velocity PDF allows for quick monitoring of the data to make sure there aren’t any non-physical velocities in the data due to system noise. The particle inter-arrival time PDF gives an indication of the highest velocity fluctuation frequencies that may be resolved when estimating turbulence spectra.

As well, two 2-dimensional histogram contours of SNR and P2P ratio. The abscissa is velocity and the ordinates are SNR and P2P ratio. These give a qualitative indication of the processing criteria to assess whether the settings are too tight or too loose. High occurrence is denoted by yellow-red contour levels and low occurrence is marked by purple and blue contour levels. Valid measurements appear as a 2-dimensional Gaussian distribution of velocities centered near the mean. Moreover, contours allow for easy visualization of non-physical velocities present in the data as they show up as a distinct, higher-occurrence band at a single velocity.
clc
close all
clearvars -except numeric_entries text_entries full_spreadsheet_name

%%% I. Laser Info Probe Volume Info
% Calculation of Probe Volume Characteristics

% Focal length of sending probe lens (m)
fSend = 500e-3;

% beam half angle.
kappa = 2.3485;

% wavelength of light to be used (m)
lamda = 488e-9;

% beam diameter input in (m)
beam_diameter = 1.04e-3;

% beam waster diameter at focal point for guassian beam
beam_waist_diameter = 4*fSend*lamda/(pi*beam_diameter);

% diameter of probe volume
probe_vol_diameter = beam_waist_diameter/cosd(kappa);
II. Filtering parameters to determine valid bursts

%=========================================================================%
% Tight Validation Criteria
SNR_cutoff_tight = 18;
P2P_cutoff_tight = 0.2;

% Minimum distance particle traveled through probe volume as a percent of
% the probe volume diameter
particle_travel = 0;

%=========================================================================%
% Loose Validation Criteria
SNR_cutoff_loose = 14;
P2P_cutoff_loose = 0.5;

%=========================================================================%
% Use a velocity cutoff filter (y/n)?
velocity_filter='n';

% If yes, choose the cutoff velocity in m/s
cutoff_velocity=[259];

% Choose the filter style (highpass, lowpass, notch)
filter_style={'notch'};

notch_width=1.5;

%=========================================================================%
III. Processing Parameters

% User has the option to choose whether or not to write the processed data
% to a text file and/or write the mean and RMS velocity and data rate to
% the Excel spreadsheet (y/n)
write_processed_data_to_txt='n';
write_processed_data_to_xl='n';

% Minimum number of bursts per block
min_block_size=30;

% Number of points in autocorrelation
Nk=2048;

% Time separation between fuzzy slot peaks
del_tau=1E-5;

% Data block length (in time)
window_length_factor=50;
block_length=window_length_factor*Nk*del_tau;

% IV. Select Files to Process
%
% Prompt the user to load in the files
[dummy_filenames, file_path] = uigetfile('*txt','Select files to process','MultiSelect','on');

if iscell(dummy_filenames)
    number_of_files=length(dummy_filenames);
    filenames=dummy_filenames;
else
    number_of_files=1;
    filenames{1}=dummy_filenames;
end

clear dummy_filenames

% V. Determine valid bursts
%
try
    Excel = actxserver ('Excel.Application');
    invoke(Excel.Workbooks,'Open',full_spreadsheet_name);
    for counter=1:number_of_files
        counter
        % Find the location of the current file in the spreadsheet
        match=strcmp(filenames{counter}(1:end-4),text_entries);
    end
end
[match_row,~]=find(match==1);

% Create the full file name including directory and file type
current_file_full_name=strcat(file_path,filenames{counter});

% Load in the data file
ldv_data=load(current_file_full_name);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Tight Validation criteria
% Find all bursts in the data set that meet the validation % criteria
valid_ldv_indices=find(((ldv_data(:,2).*abs(ldv_data(:,4))/probe_vol_diameter
*100) > particle_travel)...
.*(ldv_data(:,6) > SNR_cutoff_tight).*(ldv_data(:,7) <
P2P_cutoff_tight).*(ldv_data(:,3) < 1800));

% Store the valid data in a new variable
tight_validity_ldv_data=ldv_data(valid_ldv_indices,:);

%-----------------------------------------------------------------%
% Filter to use if nonphysical velocities are in data
for counter2=1:length(cutoff_velocity)
  if strcmp(velocity_filter,'y')&&!strcmp(filer_style{counter2},'lowpass')
    tight_validity_ldv_data=tight_validity_ldv_data...
    (tight_validity_ldv_data(:,4)<=cutoff_velocity(counter2),:);
  elseif strcmp(velocity_filter,'y')&&!strcmp(filer_style{counter2},'highpass')
    tight_validity_ldv_data=tight_validity_ldv_data...
    (tight_validity_ldv_data(:,4)>=cutoff_velocity(counter2),:);
  elseif strcmp(velocity_filter,'y')&&
    strcmp(filer_style{counter2},'notch')
    notch_indices_a=find((tight_validity_ldv_data(:,4)<=
    cutoff_velocity(counter2)-notch_width));
    notch_indices_b=find((tight_validity_ldv_data(:,4)>=
    cutoff_velocity(counter2)+notch_width));
    notch_indices=[notch_indices_a;notch_indices_b];
    clear notch_indices_a notch_indices_b
    tight_validity_ldv_data=tight_validity_ldv_data(notch_indices,:);
% Split the data into blocks to remove binning effects
%         \[\text{block\_times, block\_velocities, mean\_data\_rate, mean\_velocity,}
% vel\_std\_dev, std\_devOrig, vMeanOrig]\ldots
%             = RWP\_BlockSplitting(tight\_validity\_ldv\_data(:,4),
tight\_validity\_ldv\_data(:,1), block\_length, min\_block\_size);

[block\_times, block\_velocities, mean\_data\_rate, mean\_velocity, 
vel\_std\_dev, std\_devOrig, vMeanOrig, num\_blocks, peak\_vel]...
= SMH\_BlockSplitting(tight\_validity\_ldv\_data(:,4),
tight\_validity\_ldv\_data(:,1), block\_length);

% "Resample" the valid bursts and only keep bursts within +/- 3
% standard deviations of the mean velocity
revalidated\_ldv\_indices=find((tight\_validity\_ldv\_data(:,4)>=mean\_velocity-
(3*vel\_std\_dev))... 
.*(tight\_validity\_ldv\_data(:,4)<=mean\_velocity+(3*vel\_std\_dev)));

tight\_validity\_ldv\_data=tight\_validity\_ldv\_data(revalidated\_ldv\_indices,:);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Loose Validation criteria
% Find all bursts in the data set that meet the validation
% criteria
clear valid\_ldv\_indices
valid\_ldv\_indices=find(((ldv\_data(:,2).*abs(ldv\_data(:,4))/probe\_vol\_diameter 
*100) > particle\_travel)... 
.*(ldv\_data(:,6) > SNR\_cutoff\_loose).* (ldv\_data(:,7) < 
P2P\_cutoff\_loose).*(ldv\_data(:,3) < 1800));

% Store the valid data in a new variable
loose\_validity\_ldv\_data=ldv\_data(valid\_ldv\_indices,:);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% vel\_filt\_type=''
% Filter to use if nonphysical velocities are in data
if strcmp(velocity\_filter,'y')
   for counter2=1:length(cutoff\_velocity)
      switch filer\_style{counter2}
      case 'lowpass'

loose\_validity\_ldv\_data=loose\_validity\_ldv\_data...
(loose_validity_ldv_data(:,4)<=cutoff_velocity(counter2),:);
    vel_filt=1;
    vel_filt_type_dummy=1;

    case 'highpass'

    loose_validity_ldv_data=loose_validity_ldv_data...
    (loose_validity_ldv_data(:,4)>=cutoff_velocity(counter2),:);
    vel_filt=1;
    vel_filt_type_dummy=2;

    case 'notch'

    notch_indices_a=find((loose_validity_ldv_data(:,4)...
        <=cutoff_velocity(counter2)-notch_width));
    notch_indices_b=find((loose_validity_ldv_data(:,4)...
        >=cutoff_velocity(counter2)+notch_width));
    notch_indices=[notch_indices_a;notch_indices_b];
    clear notch_indices_a notch_indices_b

    loose_validity_ldv_data=loose_validity_ldv_data(notch_indices,:);
    vel_filt=1;
    vel_filt_type_dummy=3;

    end

    vel_filt_type=strcat(filer_style{counter2},',',vel_filt_type);
end

else

    vel_filt=0;
    vel_filt_type='none';

end

if strcmp(vel_filt_type(end),',');
    vel_filt_type=vel_filt_type(1:end-1);
end

% "Resample" the valid bursts and only keep bursts within +/- 3
% standard deviations of the mean velocity

revalidated_ldv_indices=find((loose_validity_ldv_data(:,4)>=mean_velocity-
(3*vel_std_dev))... 
.*(loose_validity_ldv_data(:,4)<=mean_velocity+(3*vel_std_dev)));

loose_validity_ldv_data=loose_validity_ldv_data(revalidated_ldv_indices,:);
% Split the data into blocks to remove binning effects  
% (block_velocities is the fluctuating component of velocity)  
[block_times, block_velocities, mean_data_rate, mean_velocity,...  
  vel_std_dev, std_devOrig, vMeanOrig, num_blocks, peak_vel]...  
=  
SMH_BlockSplitting(loose_validity_ldv_data(:,4),loose_validity_ldv_data(:,1),  
block_length);  
num_blocks=num_blocks-1;  
% 95th Percentile Velocity  
ninetyfifth_vel=mean_velocity+(2*vel_std_dev);  
% 99th Percentile Velocity  
ninetyninth_vel=mean_velocity+(3*vel_std_dev);  
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
SNR(counter)=loose_validity_ldv_data(:,6);  
peak_to_peak(counter)=loose_validity_ldv_data(:,7);  
velocity_measurements{counter}=loose_validity_ldv_data(:,4);  

% time_differences{counter}=loose_validity_ldv_data(2:end,1)-loose_validity_ldv_data(1:end-1,1);  
[f2{counter}, x2{counter}] = hist(time_differences{counter},10.^(-  
8.5:0.05:-0.5));

% Write valid data to text file and put file in the same folder  
% as the parent file. If the file already exists it will be  
% overwritten. The mean velocity and the variance for each file will  
% also be written to the parent spreadsheet for quick access  

if strcmp(write_processed_data_to_txt,'y')  
  % Write data blocks to individual text files  
  for counter2=1:length(block_times)  
    data2write=horzcat(block_times{counter2},...  
      block_velocities{counter2},block_velocities{counter2},...  
      block_velocities{counter2});  

    data2write=data2write';  

    processed_file_name=strcat(file_path,filenames{counter}(1:end-  
4),'_processed_',num2str(counter2),'.txt');  

    % Open the test file  
    fileID = fopen(processed_file_name,'w');  

    % Write data to test file  
    fprintf(fileID,'%10.8e %10.5f %10.5f %10.5f\n',data2write);  

    % Close text file and clear unneeded variables  
  end  
end
fclose(fileID);

clear data2write
end

% Write the mean and RMS velocity, and mean data rate to % Excel spreadsheet
if strcmp(write_processed_data_to_xl,'y')

    % Define the range in the spreadsheet to write the data
    sheet_range=strcat('S',num2str(match_row),':AJ',num2str(match_row));

    if (length(cutoff_velocity)>=1)&&(strcmp(velocity_filter,'y'))
        cutoff_velocity_w='';
        for counter2=1:length(cutoff_velocity)
            cutoff_velocity_w=strcat(num2str(cutoff_velocity(counter2)),',',cutoff_velocity_w);
        end
    else
        cutoff_velocity_w='n/a';
    end

    if strcmp(cutoff_velocity_w(end),',');
        cutoff_velocity_w=cutoff_velocity_w(1:end-1);
    end

    % Compile all the data to write into a linear array
    excel_data={mean_velocity,vel_std_dev,peak_vel,ninetyfifth_vel,...
                ninetyninth_vel,mean_data_rate,SNR_cutoff_tight,P2P_cutoff_tight,...
                SNR_cutoff_loose,P2P_cutoff_loose,num_blocks,...
                window_length_factor,Nk,del_tau,vel_filt,cutoff_velocity_w,...
                vel_filt_type,particle_travel};

    % Write the data to the spreadsheet
    xlswrite1(full_spreadsheet_name,excel_data,'Sheet1',sheet_range)
end

end

% Close the Excel workbook
invoke(Excel.ActiveWorkbook,'Save');
Excel.Quit
Excel.delete
clear Excel

% Catch any errors, close the Excel workbook if it's open, and rethrow the
catch err
    if exist('Excel','var');
        invoke(Excel.ActiveWorkbook,'Save');
        Excel.Quit
        Excel.delete
        clear Excel
    end
    rethrow(err);
end

% VI.    Plot the data
%=========================================================================%

close all
clc

color=[1,0,0; 0.87,0.49,0; 0.75,0.75,0; 0,0.5,0; 0,0,1; 1,0,1;
      0.48,0.06,0.89; 0.5,0.5,0.5];

lines=['- '-'--' '-.'];

velocity_prob_density_fig = figure;
AX1 = axes('Parent',velocity_prob_density_fig);
xlabel('Velocity (m/s)');
ylabel('Probability Density');
grid on;

arrival_time_prob_density_fig = figure;
AX2 = axes('Parent',arrival_time_prob_density_fig,'XScale','log');
xlabel('Particle Arrival Separation Time (m/s)');
ylabel('Probability Density');
grid on;

color_counter=0;
line_counter=1;

for counter = 1:number_of_files
    color_counter=color_counter+1;
    if color_counter>length(color)
        color_counter=1;
        line_counter=line_counter+1;
    end

    figure(velocity_prob_density_fig);
    hold on
    Unorm = (velocity_measurements{counter});
    [f, x] = hist(Unorm,50);
    plot(x,f/trapz(x,f),'Color',color(color_counter,:),
         'LineStyle',lines(line_counter,:),'LineWidth',2);
    hold off
figure(arrival_time_prob_density_fig);
    hold on
    plot(x2{counter}(2:end),f2{counter}(2:end)/sum(f2{counter})*100,...
    'Color',color(color_counter,:), 'LineStyle', lines(line_counter,:), 'LineWidth',
2);
    xlim([1e-8 1]);
    hold off
    figs = figure;
    AX = axes('Parent',figs,'FontWeight','bold','FontSize',12.15);
    ndhist(velocity_measurements{counter},SNR{counter}, 'binsx',4,'binsy',2.25);
    xlabel('Particle Velocity (m/s)', 'FontWeight','bold');
    ylabel('Signal to Noise Ratio','FontWeight','bold');
    figs = figure;
    AX = axes('Parent',figs,'FontWeight','bold','FontSize',12.15);
    ndhist(velocity_measurements{counter},peak_to_peak{counter}, 'bins',4);
    xlabel('Particle Velocity (m/s)', 'FontWeight','bold');
    ylabel('Peak to Peak Ratio','FontWeight','bold');
    end

legend(AX1,filenames);
title(AX1,strcat('P2P = ',num2str(P2P_cutoff_tight),',',num2str(P2P_cutoff_loose),...
    ' and SNR = ',num2str(SNR_cutoff_tight),',',num2str(SNR_cutoff_loose)));
legend(AX2,filenames);
title(AX2,strcat('P2P = ',num2str(P2P_cutoff_tight),',',num2str(P2P_cutoff_loose),...
    ' and SNR = ',num2str(SNR_cutoff_tight),',',num2str(SNR_cutoff_loose)));

%=========================================================================%
%% VII.     End of script
%=========================================================================%
2 Turbulence Spectra Estimation

This script calculates the estimated turbulence spectrum using text files of autocorrelation values output by a separate executable program provided by Dr. K. Todd Lowe of Virginia Tech. The executable performs the autocorrelation calculation given by Equation (3.16).

For each autocorrelation file, the MATLAB script applies a frequency-dependent window to the data and calculates the single-sided PSD’s using the discrete cosine transform defined in Equation (3.22). The final turbulence spectrum is the ensemble-averaged PSD’s. This averaged spectrum is scaled such that the sum of the values returns the square of the \textit{rms} velocity. A 3-point Savitzky-Golay filter is applied to further reduce variance in the final turbulence spectrum.
% Author: Scott Hromisin
% Name: LDV_spectra_plotting.m
% Date Started: March 12 2015
% Date Completed: March 12 2015

% About this script
This script calculates the PSD and variance for a set (of any length) of autocorrelation files. First, it calculates the PSD and variance using the FFT function in MATLAB. Then it calculates the PSD and variance using the Tukey-Hanning window. The PSD output by the Tukey-Hanning window is then scaled such that the variance is equal to that output by the FFT function.

% Date Modified:
% Description of Modification:

format compact
clc
clear all
close all

%=========================================================================%
%% I. Define Spectral Processing Parameters
%=========================================================================%

% Define the time step
delTAU=1E-5;

% Define the number of points in the autocorrelation
Nk=2048;

% Kappa value for Tukey-Hanning Window
kappa = 75;

%=========================================================================%
%% II. Import the autocorrelation text files to calculate PSDs
%=========================================================================%

% Prompt the user to load in the autocorrelation file
[autocorr_filenames,autocorr_path] = uigetfile('*\.txt','Multiselect','on');
if iscell(autocorr_filenames)
    for counter=1:length(autocorr_filenames);
        % Create the full file name including path
        autocorr_full_name{counter}=strcat(autocorr_path,autocorr_filenames{counter});
    end
    % Import the autocorrelation text files into MATLAB workspace
    autocorr_data{counter}=load(autocorr_full_name{counter},'-ascii');
end
% Resort the autocorrelation into the MATLAB-expected format that
% goes from Tau = 0 to Tau Nk-1*del_tau instead of -(Nk/2)*dTau :
% dTau : Nk/2*dTau

autocorr_vals_arranged{counter}=[autocorr_data{counter}((Nk/2+1:Nk),2);...
(autocorr_data{counter}((1:Nk/2),2))];

% Unsorted autocorrelation data for Tukey-Hanning Window
autocorr_vals_unarranged{counter}=autocorr_data{counter}(:,2);
end
else

% Create the full file name including path
autocorr_full_name{1}=strcat(autocorr_path,autocorr_filenames);

% Import the text files into MATLAB workspace
autocorr_data{1}=load(autocorr_full_name{1},'
ascii');

% Resort the autocorrelation into the MATLAB-expected format that
% goes from Tau = 0 to Tau Nk-1*del_tau instead of -(Nk/2)*dTau :
% dTau : Nk/2*dTau
autocorr_vals_arranged{1}=[autocorr_data{1}((Nk/2+1:Nk),2);...
(autocorr_data{1}((1:Nk/2),2))];

% Unsorted autocorrelation data for Tukey-Hanning Window
autocorr_vals_unarranged{1}=autocorr_data{1}(:,2);
end

%=========================================================================%
%% III.     Prepare to import the input files to the autocorrelation code
%=========================================================================%
% This is section is written in such a way to ensure the files imported are
% only those that correlate to those selected in the previous section.

% Prompt the user to select the directory containing the files
autocorr_inputfile_path=uigetdir('','Select the folder containing the
autocorrlation input files');
autocorr_inputfile_path=strcat(autocorr_inputfile_path,'\');

% Import the files
if iscell(autocorr_filenames)

    for counter=1:length(autocorr_filenames);

        % Get the last 1 or 2 numbers in the file name corresponding to the
        % number of the data block
        spectrum_number_a=str2num(autocorr_filenames{1,counter}(end-5));
spectrum_number_b=str2num(autocorr_filenames{1,counter}(end-4));

        % Create the name of the input file for the autocorrelation code
        autocorr_inputfile_name{counter}=strcat(autocorr_filenames{1,counter}(1:18),
..
'processed_’,num2str(spectrum_number_a),num2str(spectrum_number_b),'.txt');

% Create the full name of the input file for the autocorrelation
% code including path name

autocorr_inputfile_fullname{counter}=strcat(autocorr_inputfile_path,autocorr_inputfile_name{counter});
end
else
% Get the last 1 or 2 numbers in the file name
spectrum_number_a=str2num(autocorr_filenames(end-5));
spectrum_number_b=str2num(autocorr_filenames(end-4));

% Create the name of the input file for the autocorrelation code
autocorr_inputfile_name{1}=strcat(autocorr_filenames(1:18),'

'processed_’,num2str(spectrum_number_a),num2str(spectrum_number_b),'.txt');

% Create the full name of the input file for the autocorrelation
% code including path name

autocorr_inputfile_fullname{1}=strcat(autocorr_inputfile_path,autocorr_inputfile_name{1});

end

%=========================================================================%
%% IV. Import the pre-autocorrelation files and calculate variance
%=========================================================================%

% Load in the text files
for counter=1:length(autocorr_inputfile_fullname)
    % Import the text file into MATLAB workspace
    autocorr_input_data{counter}=load(autocorr_inputfile_fullname{counter},'-ascii');

    % Extract the velocity data from each data block
    valid_ldv_velocities{counter}=autocorr_input_data{counter}(:,4);

    % Calculate all time differences for each data block
    time_differences{counter}=(autocorr_input_data{counter}(2:end,1)-autocorr_input_data{counter}(1:end-1,1));

    % Calculate the mean square value for each data block
    variance(counter)=sum((valid_ldv_velocities{counter}(2:end).^2).*...time_differences{counter}))/sum(time_differences{counter});
end

%=========================================================================%
%% V. Calculate the PSD and variance based on the PSD

102
% Define the average PSD to be 0
Sxx_avg=0;

% Define the average autocorrelation to be 0
autocorr_vals_avg=0;

for counter=1:length(autocorr_vals_arranged)

    % Scale the autocorrelation for each data block by the variance of the
    % data block
    autocorr_vals_scaled{counter}=autocorr_vals_arranged{counter}.*variance(counter);

    % Scale the autocorrelation for each data block by the variance of the
    % data block
    autocorr_vals_scaled_b{counter}=autocorr_vals_unarranged{counter}.*variance(counter);

    % Calculate the PSD for each data block
    Sxx{counter}=fft(autocorr_vals_scaled{counter}).*delTAU;

    % Integrate the PSD to get the approx. variance
    integrated_psd_sxx(counter) = sum(((Sxx{counter})))./(Nk*delTAU);

    % Check accuracy of PSD esimate
    % integrated_psd{2,counter}=abs(integrated_psd{1,counter}-variance(counter))*100/variance(counter);

    % Calculate the average PSD using a running average
    Sxx_avg=Sxx_avg.*((counter-1)/counter)+(Sxx{counter}./counter);

    % Calculate the average autocorrelation using a running average
    autocorr_vals_avg=autocorr_vals_avg.*((counter-1)/counter)+(autocorr_vals_scaled{counter}./counter);
end

net_mean_sq_sxx=sum(Sxx_avg)/(Nk*delTAU);
std_dev_sxx=sqrt(net_mean_sq_sxx);

%===============================================
%%%%%%%%%%%%%%%%%%%%%
%% VI A.      Calculate the PSD using the Tukey-Hanning Window and rescale
%               PSD determined from averging the PSDs of each data block
%========================================================================% 
% Define the frequencies for the fuzzy slots.
% frequencies_fuzzy=0:length(1:(Nk))-1;
% frequencies_fuzzy=frequencies_fuzzy*1./(Nk*delTAU);
% GxxTukey_avg=0;
% Calculate the PSD for each data block
% for counter=1:length(autocorr_vals_arranged);
%     
% Calculate the PSD value at each frequency
% for counter2=1:Nk/2+1;
%     
%     win_xformed_acf_sum=0;
%     
% Calculate the frequency dependent Tukey-Hanning Window and apply
% to PSD
% for k=1:Nk
%     if abs(frequencies_fuzzy(counter2)*(k-1)*delTAU)<kappa
%         dk=0.5*(1+cos(pi*frequencies_fuzzy(counter2)*(k-1)*delTAU/kappa));
%     else
%         dk=0;
%     end
%     
%     if (k-1)*delTAU<kappa
%         win_xformed_acf_sum=win_xformed_acf_sum +
%             dk*autocorr_vals_scaled{counter}(k)*(
%                 cos(2*pi*frequencies_fuzzy(counter2)*(k-1)*delTAU)+
%                 1i*sin(2*pi*frequencies_fuzzy(counter2)*(k-1)*delTAU));
%     end
%     
% Single-sided PSD using Tukey-Hanning Window
%     GxxTukey{counter}(counter2)=2*delTAU*win_xformed_acf_sum;
% end
%     
% GxxTukey{counter}(1) = GxxTukey{counter}(1)/2;
% GxxTukey{counter}(Nk/2+1)=GxxTukey{counter}(Nk/2+1)/2;
%     
% Calculate the power in the spectrum
%     integrated_psd_gxx(counter) = sum(GxxTukey{counter})/(Nk*delTAU);
%     
% Determine the ratio of the power in the true PSD to that in the PSD
% with the Tukey-Hanning window
%     scaling_factor(counter)=integrated_psd_sxx(counter)/integrated_psd_gxx(counter);
%     
% Scale the PSD
%     GxxTukey_scaled{counter}=GxxTukey{counter}.*scaling_factor(counter);
%     
% Calculate the average PSD using a running average
%     GxxTukey_avg=GxxTukey_avg.*((counter-1)/counter)+(GxxTukey_scaled{counter})./counter;
% end
%     
% Apply a 3 point smoothing filter to the spectrum
% GxxTukey_smooth=filter(ones(3,1)/3,1,GxxTukey_avg);
% net_mean_sq_gxx=sum(GxxTukey_avg)/(Nk*delTAU);
% std_dev_gxx=sqrt(net_mean_sq_gxx);

%=========================================================================%
%% VI B. Calculate the PSD using the Tukey-Hanning Window and rescale
%                    PSD Determined from Averaged Autocorrelation
%=========================================================================%

% Define the frequencies for the fuzzy slots.
frequencies_fuzzy=0:length(1:(Nk))-1;
frequencies_fuzzy=frequencies_fuzzy*1./(Nk*delTAU);

% Calculate the PSD value at each frequency
for counter=1:Nk/2+1;
    win_xformed_acf_sum_b=0;

    % Calculate the frequency dependent Tukey-Hanning Window and apply
    % to PSD
    for k=1:Nk
        if abs(frequencies_fuzzy(counter)*(k-1)*delTAU)<kappa
dk=0.5*(1+cos(pi*frequencies_fuzzy(counter)*(k-1)*delTAU/kappa));
        else
dk=0;
        end

        % Calculate PSD with window applied
        win_xformed_acf_sum_b=win_xformed_acf_sum_b +
dk*autocorr_vals_avg(k)*... 
        (cos(2*pi*frequencies_fuzzy(counter)*(k-1)*delTAU)+... 
         li*sin(2*pi*frequencies_fuzzy(counter)*(k-1)*delTAU));
    end

    % Single-sided PSD using Tukey-Hanning Window
    GxxTukey_b(counter)=2*delTAU*win_xformed_acf_sum_b;
    % GxxTukeyb(counter)(counter2) = abs(GxxTukeyb(counter)(counter2));
end

% Integrate the PSD to get the mean square power in the spectrum
net_mean_sq_gxx_b=sum(GxxTukey_b)/(Nk*delTAU);

% Determine the scaling factor between the FFT with and without the
% Tukey-Hanning window
scaling_factor_b=net_mean_sq_sxx/net_mean_sq_gxx_b;

% Rescale the PSD
GxxTukey_b_scaled=GxxTukey_b.*scaling_factor_b;

% Apply a 3 point smoothing filter to the spectrum
GxxTukey_smooth_b=sgolayfilt(GxxTukey_b_scaled,1,3);
% VII. Plot the PSDs
%========================================================================%close all

% Plot the PSD output by the Tukey Hanning Window
figure;
loglog(frequencies_fuzzy(1:length(frequencies_fuzzy)/2),abs((GxxTukey_avg(1:length(frequencies_fuzzy)/2))));
title('Averaged GxxTukey');

% Plot the PSD output by the Tukey Hanning Window
figure;
loglog(frequencies_fuzzy(1:length(frequencies_fuzzy)/2),abs((GxxTukey_b_scaled(1:length(frequencies_fuzzy)/2))));
title('Averaged GxxTukey');

% Plot the PSD output by the Tukey Hanning Window
figure;
loglog(frequencies_fuzzy(1:length(frequencies_fuzzy)/2),GxxTukey_smooth(1:length(frequencies_fuzzy)/2));
title('Smoothed GxxTukey');

% Plot the PSD output by the FFT function
figure;
loglog(frequencies_fuzzy(1:length(frequencies_fuzzy)/2),abs(real(Sxx_avg(1:length(frequencies_fuzzy)/2))));
title('PSD calculated via FFT');

% Plot both PSDs on the same axis
figure;
loglog(frequencies_fuzzy(1:length(frequencies_fuzzy)/2),GxxTukey_smooth(1:length(frequencies_fuzzy)/2));
fig2fig(3,4);
legend(gca,'Gxx','Sxx');

% Plot the PSD output by the Tukey Hanning Window
figure;
loglog(frequencies_fuzzy(1:length(frequencies_fuzzy)/2),abs(GxxTukey_smooth_b_scaled(1:length(frequencies_fuzzy)/2)));
title('Smoothed GxxTukey - Avg''ed Autocorrelations');

clc;

%========================================================================%
Bibliography

URL https://www.flickr.com/photos/lockheedmartin


URL http://www.journals.cambridge.org/abstract_S0022112071000053


URL http://linkinghub.elsevier.com/retrieve/pii/S0142727X14000976

URL http://www.journals.cambridge.org/abstract_S0022112005005148

URL http://www.journals.cambridge.org/abstract_S0022112004000151


111


URL http://www.lehmanns.de/shop/naturwissenschaften/2398975-9783540661559-schlieren-and-shadowgraph-techniques 31


URL http://link.springer.com/10.1007/s00193-014-0540-5 36

URL http://caltechbook.library.caltech.edu/51/1/multiph.htm 36

URL www.topas-gmbh.de/datein/prospekt/dehs{ }prspe.pdf 36


[77] Nave, R., “Blue Sky,”. URL http://hyperphysics.phy-astr.gsu.edu/hbase/atmos/blusky.html 38


