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**SUPPLIER SELECTION PROBLEM IN CROSS DOCKING  
COLD CHAIN**

A Thesis in

Industrial Engineering

by

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## ABSTRACT

Cross docking is an integrated warehouse where incoming freights are directly loaded onto outbound trucks. It has been widely used in the retail industry to reduce supply chain costs. Every retailer wants to sell relatively high quality products while reduce the operation costs to the minimum. In recent years, the purchasing function has assumed to take larger proportion of the contribution to the supply chain, and one of the most important purchasing functions is the selection of the suppliers. The supplier selection problem could be solved in two phases. The first phase is to reduce the large amount of candidate suppliers to a relatively manageable small size. The second phase is to form a multiple criteria optimization model to allocate order quantities among the shortlisted suppliers. This thesis examines the strategy of supplier selection in a cross docking cold chain. Buyers usually do not consider from this perspective, but this method may help them to save extra money. In the first phase, Borda Count,  $L_\infty$  Norm, and Analytical Hierarchy Process (AHP) have been used to reduce the original suppliers. An optimization model has been stated in the second phase to determine which supplier to choose from the reduced suppliers selected in the first phase. And the two-phase supplier selection method has been performed in an example. The sensitivity analysis of the example showed that the order allocation among the suppliers would not change when we varied the inbound shipment capacity.

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# 1 Introduction

## 1.1 Supplier Selection Problem

In this highly competitive world, an effective supplier selection process is significant to the success of retailers. In most cases, buyers from a company need to choose among a set of suppliers by using some predetermined criteria such as quality, reliability, technical capability, lead-times, etc. Therefore, two basic and interrelated decisions must be made by a company before building any long-term relationships with suppliers, as presented in the following questions:

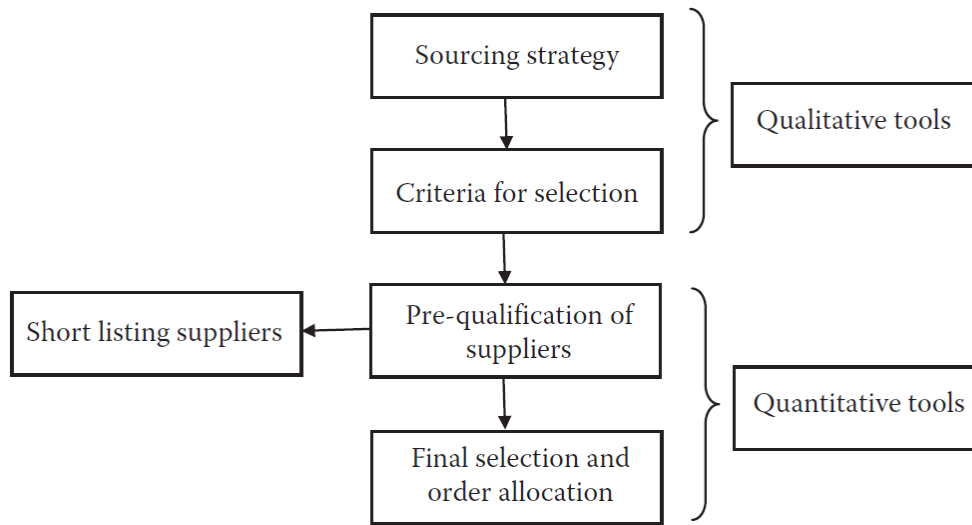
1. Which suppliers to do business with?
2. How much to order from each supplier?

Weber et al. define this pair of decisions as the supplier selection problem (Weber, et al., 1991).

The supplier selection problem is considered to be complicated because most of the criteria for selecting are conflicting. Supplier selection is a multiple criteria optimization problem that requires the decision maker considering the trade-offs among different qualitative and quantitative factors (Ravindran & Warsing, 2012).

The supplier selection methods are explicitly discussed in Dr. Ravindran's book. (Ravindran & Warsing, 2012)

Figure 1 shows the typical decision making procedures for supplier selection problem.



*Figure 1 Supplier Selection Problem (Ravindran & Warsing, 2012)*

### **1.1.1 Sourcing Strategy**

Depending on the type of items being purchased, the sourcing strategy could be either strategic or tactical. When the item being purchased is expensive and critical, and can only be bought from certain suppliers, the sourcing strategy tends to be more strategic; when the item could be easily purchased from several suppliers in the market, the supplier selection is a tactical decision.

### **1.1.2 Criteria for Selection**

Criteria for supplier selection have been studied since the 1960s. Pal et al. thoroughly reviewed present paper regarding to the supplier selection criteria and methods in supply chains and concluded that price, delivery, and quality are considered to be the top three most important criteria for supplier selection (Pal, et al., 2013).

### **1.1.3 Pre-Qualification of Suppliers**

Pre-qualification is defined as the process of reducing a large set of potential suppliers to a smaller manageable number by ranking the suppliers under a pre-defined set of criteria.

(Holt, 1998) The benefits of pre-qualification of suppliers are presented as following:

1. The possibility of rejecting good suppliers at an early stage is reduced.
2. Resource commitment of the buyer toward purchasing process is optimized.
3. With the application of pre-selected criteria, the pre-qualification process is rationalized.

### **1.1.4 Final Selection**

In this step, the purchaser decides which suppliers to do business with and allocates order quantities among the chosen suppliers. As proposed by Ghodsypour & O'Brien, there are two types of basic supplier selection problem (Ghodsypour & O'Brien, 2001):

1. Single Sourcing, which supposes that each one of the suppliers could satisfy the buyer's requirements of demand, quality, delivery, etc.
2. Multiple Sourcing, which considers that there are certain limitations in suppliers' capacity, quality, etc. so that multiple suppliers have to be used.

## **1.2 An Overview of Cross Docking**

Cross docking is a "process of consolidating freight with the same destination (but coming from several origins), with minimal handling and with little or no storage between unloading and loading of the goods" (Belle, et al., 2012). Cross docking has been widely used by many companies such as Wal-Mart (United States), Carrefour (France), Albert Heijn (the Netherlands), and Tesco (United Kingdom). Implementation of proper cross

docking has many advantages compared to traditional warehouses such as reduction or even elimination of merchandise storage, and order-picking. Thus the inventory holding cost and labor cost would be reduced (Galbreth, et al., 2008).

Figure 2 shows a schematic representation of a cross docking terminal (Stephan & Boysen, 2011). At the inbound doors, trucks are unloaded and shipments are registered. The shipments will be checked for completeness and intactness, and be sorted according to their destinations. Then shipments will be moved across the dock to their temporary storage area which has been assigned by the intended destination of the shipments. In the meantime, value adding services would be performed such as labeling while the shipments are waiting for the outbound truck. Lastly, shipments will be loaded onto outbound trucks to leave the terminal for their next destinations.

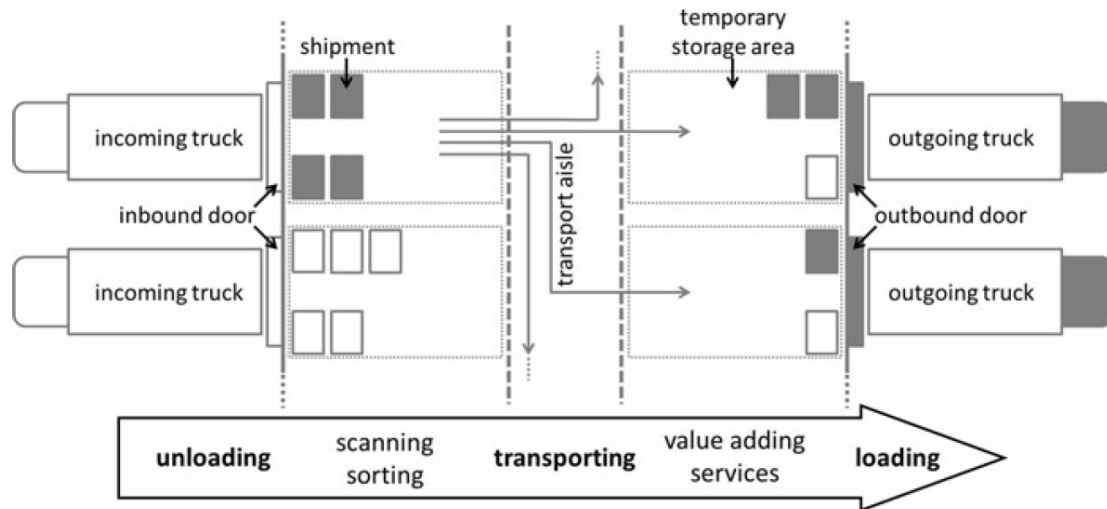


Figure 2 Schematic Representation of a Cross Docking Terminal (Stephan & Boysen, 2011)

### **1.3 An Overview of Cold Chain**

A cold chain protects a large variety of products from deterioration such as food, pharmaceutical and chemical products. It insulates them from degradation, improper exposure to temperature, humidity, light or particular contaminants, and keeps them frozen, chilled and fresh (Bishara, 2006). The typical cold chain infrastructure contains pre-cooling facilities, cold storages, refrigerated carriers, packaging, warehouse, traceability, retailer, and customers. And these facilities are under control of the information management systems (Montanari, 2008). However, Manikas & Terry stated that the efficiency of the food cold chain is not high enough even though automatic machines have been applied to this industry. Large parts of this logistic process is still handled manually such as picking process since it is hard to control the process by machines (Manikas & Terry, 2009).

### **1.4 Objective and Thesis Outlines**

Many retailers have a large amount of suppliers to choose from. This thesis extended a two-phase supplier selection strategy in a cross docking cold chain. The main objective of supplier selection for retailers is to reduce purchase risk, to maximize overall value, and to develop long-term relationship with suppliers in this competitive industrial world. The supplier selection problem, cross docking arrangement and cold supply chain have been studied by many researchers. However, very few studies address the perspective: supplier selection in cross docking cold chain. This thesis aims to fill this gap of researches.

The next chapter provides the literature review on studies of supplier selection problem, cross docking operation and cold chain management. Chapter 3 discussed the two-phase supplier selection method in the cross docking cold chain. Borda Count,  $L_\infty$  Norm, and

AHP have been used in the first phase to reduce the original suppliers. Then an optimization model of supplier selection problem in cross docking cold chain is presented in the second phase, to make final selection of suppliers. Parameters and decision variables used in the optimization model are discussed; assumptions for mathematical formulation of the model are made. The solution to this optimization model is computationally intractable when the problem size grows even modestly. Thereafter, the model is implemented to an example using the modeling tool LINDO and the results are analyzed. The thesis ends with conclusion of the final remarks.

## 2 Literature Review

### 2.1 Supplier Selection Problem

An Analytical Hierarchy Process (AHP)-based model was formulated and implemented to a real case study by Tam & Tummala to examine its feasibility in selecting suppliers for a telecommunication system. The results showed that using the proposed model, the group decision making in selecting vendors, which can satisfy customer demands, was improved. The time consuming pairwise comparison judgements could be avoided by applying the suggested five-point rating system (Tam & Tummala, 2001).

Humphreys et al. developed a framework for integrating environmental factors into the supplier selection process. For example, quality and flexibility are traditional factors that companies would consider for evaluating supplier performance. However, many companies start to consider environmental criteria and measure their suppliers' environmental performance to accommodate the increase of the environmental pressure. A knowledge-based system was constructed and could guide the buyers to select suppliers from an environmental point of view (Humphreys, et al., 2003).

A comparison was made within the weighted sum of the selection number of rank vote by Liu & Hai, after determining the weights in a selected rank. A novel weighting procedure other than AHP's paired comparison for selecting suppliers was presented. A voting analytic hierarchy process was formulated which is simpler than AHP. Even though the presented method is simpler, it does not lose the systematic approach when deriving the weights and scoring the performance of suppliers. The author expected that this method



could be applied to other issues such as policy making, business strategies and performance assessment in the near future (Liu & Hai, 2005).

Shyur & Shih developed a hybrid model for supporting the vendor selection. Firstly, the combination of the multi-criteria decision-making (MCDM) approach and a five-step hybrid process formulated the vendor evaluation problem. Secondly, this modified technique for order performance by similarity to idea solution is used to rank the overall performances of competing products. Lastly, this new ANP approach will yield the relative weights of the multiple evaluation criteria. The effectiveness and feasibility of the model was demonstrated by solving an empirical example (Shyur & Shih, 2006).

Yan & Wei described a procedure of preference adjustments, which was based on a minimax principle, with a finite number of steps to find compromise weights. This paper discussed the problem of the existence of optimal solutions thoroughly. In order to avoid the selection of optimal solutions, the authors defined a set of “very worst preference order”. They also proved that compromise weights could be achieved within a finite number of adjustments on preference orders. A numerical example was presented for illustration. However, this unique method is only appropriate for this special problem described in this paper, and cannot be directly applied to other problems (Yan & Wei, 2002).

Mendoza et al. introduced a three-phase multi-criteria methodology to the supplier selection problem. They initially reduced the number of alternatives for supplier selection by simple linearization and  $L_2$  metric combination, and used AHP to determine the criteria weights and ranking of suppliers. And afterwards, they calculated the efficient allocation

of orders of each potential supplier by applying preemptive goal programming (GP). (Mendoza, et al., 2008)

Velazquez et al. did a study to find the best combination of weighting and scaling methods considering single or multiple decision makers. The experiments were conducted with real decision makers. The weighting methods of rating, ranking (Borda count), and AHP were discussed. The scaling methods of ideal value, linear normalization, and vector normalization using  $L_p$  norm were studied. It was concluded that the best scaling method is influenced by which weighting method has been chosen, and the best combination is scaling by  $L_\infty$  norm and ranking by Borda count. Same results were found for both single and multiple decision makers. (Velazquez, et al., 2010)

## **2.2 Cross Docking**

Lim et al. studied the transshipment through cross docking with inventory and time window constraints. There are two steps to solve this problem. The objective of the first step is to find a flow with minimum cost while meeting all the demand and capacity constraints. The second step forms a new model which can be considered as cross docking because it is aimed to minimize or even to eliminate holdover inventory. Also, the model includes the supplier and customer time windows and takes into account the capacity and holding costs of the cross docking. The objective of this new model is also to minimize the cost (transportation costs and inventory holding costs), while meeting the demand, time window and capacity constraints at the same time. The author showed that when multiple departures and deliveries within a time window were considered, the new model could be reduced to

a flow problem; when certain times of departures and deliveries within a time window were allowed, the problem would be NP-complete in strong sense. (Lim, et al., 2004)

In the work of Ma et al., a new shipment consolidation and transportation problem in cross docking distribution networks was studied, where a single product can be shipped directly or via the cross docking. The trade-offs between transportation costs, inventory and time scheduling requirements were considered. The authors formulated an Integer Program model and proved it to be NP-complete in the strong sense. Moreover, the authors presented a two-stage heuristic framework to solve this problem. In the first stage, a full truckload plan (TL) and an initial less-than-truckload plan (LTL) were constructed. In the second stage, the initial LTL was developed by applying Squeaky Wheel Optimization (SWO) heuristic and a Genetic Algorithm (GA). The computational experiments indicated that the heuristic approaches are more efficient considering runtime and solution quality. (Ma, et al., 2011)

Boysen & Flidner suggested an optimization model which aimed at minimizing the (weighted) number of shipments delayed until the next day. It would solve problems in cross docking with fixed outbound schedules. Postal services and less-than-truck load providers always rely on fixed outbound schedules. The outbound trucks would depart the terminal as scheduled regardless of whether all dedicated products or shipments have been loaded. Since it is assumed in the model that the outbound trucks are fixed, the inbound schedule has to be determined by a short-term truck scheduling at other inbound doors. The authors concluded that the model is NP-hard in the strong sense. (Boysen & Flidner, 2010)

According to Tiwari, an optimization model in a cross docking operation in the presence of multiple items, several suppliers and deterministic demand over a time horizon was stated. This master thesis focused on integrating transportation, inventory decisions and efficient labor management. The author discussed the methodology to choose inbound and outbound schedules cost effectively when the labor utilization or the inventory level was relatively low. The model was implemented with a small pilot study and was used to the real industrial world with the real cross docking operation data provided by a large 3PL provider. (Tiwari, 2003)

### **2.3 Cold Chain**

Giannakourou & Taoukis discussed the vitamin C loss for four green vegetables at the temperature range of freezing storage. These four types of vegetables were exposed to temperature -18.5 °C for 10 days, -22.3 °C for 10 days, -16.1 °C for 20 days, -14.4 °C for 20 days. The results indicated that the type of vegetable determines the deterioration rates of vitamin C. In order to fit the experimental result, an Arrhenius equation was formed and the model was then used to estimate the remaining product shelf life under dynamic temperature conditions. (Giannakourou & Taoukis, 2003)

Koutsoumanis et al. made a survey on time-temperature situations in cold chain for pasteurized milk in Greece. The authors used the survey data to generate a probabilistic model to evaluate the growth of *Listeria Monocytogenes* in this milk cold chain using a Monte Carlo simulation. The model is appropriate because it takes into account the strain variability. And the paper concluded that the domestic storage time significantly influences the concentration of *Listeria Monocytogenes* in the milk. (Koutsoumanis, et al., 2010)

Shi et al. developed a three-stage optimization model of fresh food cold chain with RFID application. In the initial planning model, the objective was to minimize the estimated transportation cost of moving products from farms to packers, from packers to DCs, from DCs to retailers, and to minimize the estimated value loss of food products during packing, transportation and distribution. In the stage-one and stage-two planning model, the decision made in previous planning stage need to be reexamined because the estimated value loss would have been detected with real data using RFID technology. Thus, the objective of stage-one planning and stage-two planning were to minimize the transportation cost of shipping products to the following echelons: the value loss of products, and the penalty costs caused by unmet order quantities for retailers or extra quantities shipped to retailers. The authors concluded that decision making model, product flow visibility and product quality information would help cold supply chain management to better fulfil the customer demand. (Shi, et al., 2010)

#### **2.4 Cross Docking in Cold Chain**

Qiu et al. creatively combined the cross docking logistics and food cold-chain and proposed a new model. The internal workflow in this cross docking distribution center was studied. The advantages of applying cross docking in food cold-chain were presented. Firstly, because the model was consistent with JIT strategy, it could provide in-time protection for its enterprises. Secondly, the inventory space would be saved and cost of inventory, distribution and labor would be reduced. Thirdly, the processes of goods shelves, picking, packing and other operations would be shortened. (Qiu, et al., 2009)

### 3 Methodology

The supplier selection problem could be solved in two phases. The first phase is to reduce the large amount of candidate suppliers to a relatively manageable small size. The second phase is to form a multiple criteria optimization model to allocate order quantities among the shortlisted suppliers. (Ravindran & Warsing, 2012)

#### 3.1 Definitions

##### $L_p$ Norm

$$L_p \text{ norm} = \sum_{j=1}^n [|X_j|^p]^{\frac{1}{p}}, \text{ for } p = 1, 2, \dots, \infty.$$

The most common values of  $p$  are  $p=1, 2$  and  $\infty$ .

$$L_1 \text{ norm} = \sum_{j=1}^n |X_j|$$

$$L_2 \text{ norm} = \left[ \sum_{j=1}^n |X_j|^2 \right]^{\frac{1}{2}}$$

$$L_\infty \text{ norm} = \max[|X_j|]$$

where vector  $X \in R^n$ ,  $j = 1, 2, \dots, n$ .

In this method, scaling is done by dividing the criteria values by their respective  $L_p$  norms.

## **Borda Count**

This method is named after Jean Charles de Borda, an 18<sup>th</sup> century French physicist. The method is defined as follows:

1. The n criteria are ranked 1 when most important, to n when least important. Criterion ranked 1 gets n points, 2 gets n-1 points, and the last place gets 1 point.

2. Weights for the criteria are calculated as follows:

$$\text{Criterion ranked 1} = \frac{n}{s}$$

$$\text{Criterion ranked 2} = \frac{n-1}{s}$$

$$\text{Criterion ranked n} = \frac{1}{s}$$

where s is the sum of all the points  $s = \frac{n(n+1)}{2}$ .

## **AHP**

The AHP was developed by (Saaty, 1980) and has been thoroughly discussed in (Ravindran & Warsing, 2012). It is a multiple criteria decision making method for ranking alternatives.

In the supplier selection process, AHP includes not only quantitative but also qualitative factors, such as financial stability, feeling of trust, etc. There are six steps to implement AHP.

- Step 1: Carry out a pair-wise comparison between criteria using the 1-9 degree of importance scale as shown Table 1.

Degree of Importance	Definition
1	Equal importance
3	Weak importance of one over other
5	Essential or strong importance
7	Demonstrated importance
9	Absolute importance
2,4,6,8	Intermediate values between two adjacent judgments

*Table 1 Degree of Importance Scale in AHP*

The pair-wise comparison matrix for the criteria is given by  $A_{(n \times n)} = [a_{ij}]$ , if there are  $n$  criteria to evaluate.  $a_{ij}$  represents the relative importance of criterion  $i$  with respect to criterion  $j$ .  $a_{ii} = 1$  and  $a_{ji} = \frac{1}{a_{ij}}$ .

Step 2: Compute the normalized weights for the main criteria.  $L_1$  norm is most commonly used. The two step process for calculating the weights is presented as follows:

- Normalize each column of  $A$  matrix using  $L_1$  norm:

$$r_{ij} = \frac{a_{ij}}{\sum_{i=1}^n a_{ij}}$$

- Average the normalized values across each row to get the criteria weights:

$$w_i = \frac{\sum_{j=1}^n r_{ij}}{n}$$



Step 3: Check consistency of the pair-wise comparison matrix using Eigen value theory as follows:

- Compute the vector  $AW$ , where  $A$  is the pair-wise comparison matrix and  $W$  is the weight matrix. Let the vector  $X = (X_1, X_2, X_3, \dots, X_n)$  denote the values of  $AW$ .
- Compute

$$\lambda_{max} = Average\left[\frac{X_1}{W_1}, \frac{X_2}{W_2}, \frac{X_3}{W_3}, \dots, \frac{X_n}{W_n}\right]$$

- Consistency index (CI) is given by

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

The values of RI are listed in Table 2

N	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.11	1.25	1.35	1.4	1.45	1.49

*Table 2 the Value of RI*

Consistency ratio (CR) is defined as  $CR = \frac{CI}{RI}$ . If  $CR < 0.15$ , the pair-wise comparison matrix is consistent.

Step 4: Compute the relative importance of the sub-criteria in the same way as the process done for the main criteria. Step 2 and Step 3 are performed for every pair of sub-criteria with respect to their main criterion. The final weights of the sub-criteria are the product of the weights calculated first in step 4 and its corresponding main criterion weight.

Step 5: Repeat Step 1, 2 and 3 to obtain:

- Pair-wise comparison of alternatives with respect to each criterion using the ratio scale.
- Normalized scores of all alternatives with respect to each criterion. Note that a  $S_{(m \times m)}$  matrix is obtained, and  $S_{ij}$  is noted as normalized score for alternative  $i$  with respect to criterion  $j$  and  $m$  is the number of alternatives.

Step 6: Compute the total score (TS) for each alternative.  $TS_{(m \times 1)} = S_{(m \times n)}W_{(n \times 1)}$ , where  $W$  is the weight vector obtained after step 4. Hence the alternatives will be ranked with the TS.

### **Capacity Utilization**

The percentage of the full capacity being utilized.

### **Fixed Cost**

Fixed cost is a one-time cost that incurred if a supplier is used, irrespective of the number of units bought from that supplier.

### **Less-Than-Truckload (LTL)**

A quantity of freight which is less the required freight for a truckload. The historical definition of LTL is the shipments of freight under 10,000 pounds.

### **Truckload (TL)**

A quantity of freight which could fill a truck. The historical definition of TL is a shipment of freight with 10,000 pounds or more.

## **Global and Local Optima**

A global optimal solution for a specific model is a feasible solution which has an objective value as good as or better than all other feasible solutions. The properties of constraints and objective functions determine whether a globally optimal solution could be obtained. Linear optimization models satisfy these properties.

A local optimal solution for a specific model is a feasible solution which has an objective value as good as or better than all other feasible solutions in the immediate neighborhood. Although no better solution could be found in the immediate neighborhood, a better solution may exist at some distance away. Nonlinear optimization models may have several local optima.

## **Convexity**

A geometric definition of convexity is defined that when a function is convex, for any two points on the function, a straight line connecting this two points lies entirely on or above the function. For minimizing a convex function, a global optimal solution could be found.

## **Integer and Mixed Integer Linear Problems**

Integer programming (IP) models are linear programming models with binary (0-1) decision variables. Mixed integer programming (MIP) models refer to general IP models which include regular integer variables (non 0-1) and continuous variables. An example of MIP model is shown as follows:

Objective:

$$\min \sum_{j \in B} a_j x_j + \sum_{j \in I} b_j x_j + \sum_{j \in C} c_j x_j$$

Constraints:

$$\sum_{j \in B} d_{ij} x_j + \sum_{j \in I} e_{ij} x_j > f_i \quad (i = 1, 2, \dots, m)$$

$$l_j \leq x_j \leq u_j \quad (j \in I)$$

$$x_j \in \{0,1\} \quad (j \in B)$$

$$x_j \in \text{int} \quad (j \in I)$$

$$x_j \in \text{real} \quad (j \in C)$$

where B is the set of 0-1 variables, I is the set of integer variables, and C is the set of continuous variables.  $l_j$  and  $u_j$  are the lower and upper bound values for variable  $x_j$ .

## **3.2 First Phase**

In this thesis, considering the first phase,  $L_\infty$  Norm is used to scale the criteria, Borda count will be used to rank these criteria, the number of initial suppliers will be reduced, and AHP will be conducted to rank the reduced suppliers.

## **3.3 Second Phase**

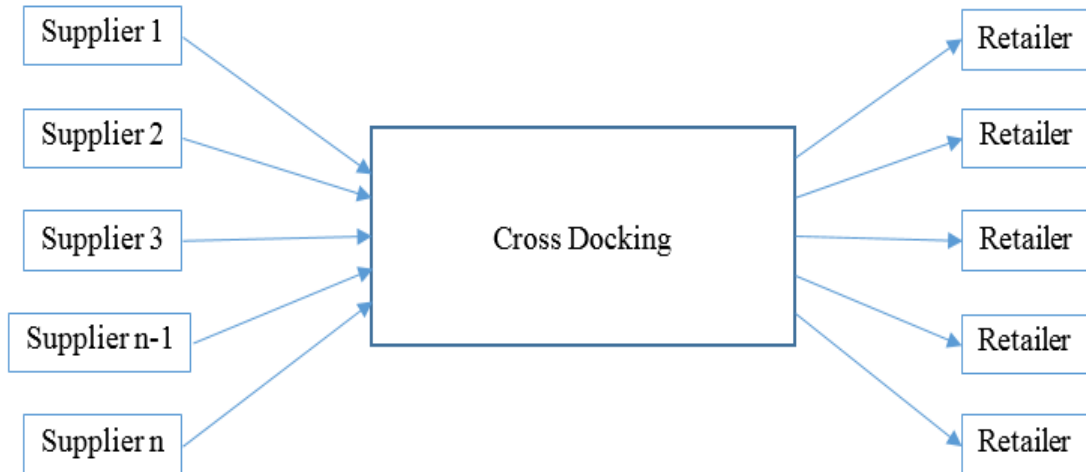
In the second phase, an optimization model for final selection of suppliers and order allocation including cross docking and cold chain constraints will be solved among the shortlisted suppliers determined in the first phase.

### **3.3.1 Model**

A supply chain may be made of several different companies. These companies could range from suppliers to distribution centers to retailers. This thesis investigates a win-win situation for both cross docking operators and the retailers.

The objective of the optimization model is to minimize the major costs in the cold supply chain, namely the fixed and variable costs of the suppliers, the transportation costs, and the value loss costs in cold chain subject to certain constraints. These constraints are based on operating conditions of the cold chain cross docking and the terms of business between the cross docking operator, the suppliers and the retailers.

The operations we aim to model include suppliers, a cross docking facility and retailers. The cross docking facility receives products from suppliers, thereafter, products are unloaded from inbound trucks, consolidated, and loaded onto the outbound trucks.



*Figure 3 The Supply Chain for the Model*

### **3.3.2 Assumptions**

To mathematically formulate the operations of the supply chain shown in Figure 3, we made some assumptions as follows:

1. The demand for product  $i$  is deterministic.
2. Lead time is not considered in this model. There is no lead time of delivery of products to the next echelon.
3. Each supplier is responsible for getting items ready for pick up.
4. No shortage or delay occurs for picking up items.
5. The penalty costs of not being able to fulfill the demand are not included in the objective function. All demands are met.
6. Charges of holding the inventory are ignored.

7. The optimal route for the movement of trucks from cross docking to retailers is pre-determined.
8. Initial inventory is zero.
9. The outbound shipments are consolidated at the cross docking and hence consist of various products received during the horizon.
10. All products shipped from the suppliers have the same initial quality.

### 3.3.3 Indices

Below are the indices used throughout the model.

$I$	Index of product, $i=1,2,\dots,n$
$T$	Index of time horizon, $t=1,2,\dots,T$
$n$	Number of products managed by the cross docking facility in periods $t=1,2,\dots,T$
$J$	Index of potential suppliers for each product, $j=1,2,\dots,J$

### 3.3.4 Parameters

Below are some parameters used in the model.

$K_{i,j}^I$	Inbound cost per container of product $i$ from supplier $j$
$K^O$	Outbound cost per container
$C^O$	Capacity of the outbound trucks
$C^I$	Capacity of the inbound trucks
$C_{i,j}$	Capacity of supplier $j$ for product $i$ in the time horizon $T$
$v_i$	Volume occupied by product $i$

$\lambda_i$	Per period demand of product $i$ at the retailer
$U_{i,j}$	Unit price of product $i$ shipped from supplier $j$
$F_{i,j}$	Fixed costs of using supplier $j$ for product $i$ . It will occur no matter supplier $j$ has been used in which period
$h_{i,j}^I$	Value lost per unit of product $i$ in inbound transportation from supplier $j$
$h_i^{CD}$	Value lost per unit of product $i$ in cross docking
$h_i^O$	Value lost per unit of product $i$ in the outbound transportation
$D_{i,j}$	Defect percentage of product $i$ from supplier $j$

### 3.3.5 Decision Variables

We introduce the decision variables and give their definitions below.

$\alpha_{i,j,t}$	Fraction of the total horizon demand of product $i$ shipped from supplier $j$ to the cross docking in period $t$
------------------	--

**Note:** The total horizon demand of product  $i = \lambda_i T$

$\omega_{i,t}$	Fraction of the total horizon demand of product $i$ shipped from the cross docking to the retailer in period $t$
----------------	--

$n_{i,j,t}^I$	Number of inbound shipments of product $i$ from supplier $j$ in period $t$
---------------	--

$n_t^O$	Number of outbound shipments in period $t$
---------	--

$\delta_{i,j}$	$\begin{cases} 1, & \text{if supplier } j \text{ is used} \\ 0, & \text{otherwise} \end{cases}$
----------------	---



### 3.3.6 Problem Formulation

We present the methodology for formulating the math program in this section.

#### Objective Function

Minimizing the sum of fixed costs of the suppliers, variable costs of the suppliers, inbound transportation costs, outbound transportation costs, values loss costs in cold chain.

Minimize Total Cost =

$$\begin{aligned} & \sum_{i=1}^n \sum_{j=1}^J F_{i,j} \delta_{i,j} + \sum_{i=1}^n \sum_{j=1}^J \sum_{t=1}^T \alpha_{i,j,t} U_{i,j} (\lambda_i T) + \sum_{i=1}^n \sum_{j=1}^J \sum_{t=1}^T n_{i,j,t}^I K_{i,j}^I + \sum_{t=1}^T n_t^O k^O \\ & + \sum_{i=1}^n \sum_{j=1}^J \sum_{t=1}^T \alpha_{i,j,t} D_{i,j} (\lambda_i T) (h_{i,j}^I + h_i^{CD} + h_i^O) \end{aligned}$$

#### Constraints

1. Receive and Ship all Products in Cross Docking:

$$\sum_{j=1}^J \sum_{t=1}^T \alpha_{i,j,t} = 1, \forall i$$

$$\sum_{t=1}^T \omega_{i,t} = 1, \forall i$$

2. Capacity Constraints of the suppliers:

$$\sum_{t=1}^T \alpha_{i,j,t} (\lambda_i T) \leq C_{i,j} \delta_{i,j}, \forall i, j$$

### 3. Inbound Transportation:

$$v_i \times \alpha_{i,j,t} \times (\lambda_i T) \leq n_{i,j,t}^l \times C^l, \forall i, j, t$$

The inbound transportation constraint restricts the shipping capacity not being violated on the inbound side.

$$n_{i,j,t}^l \leq M \delta_{i,j}, \forall i, j$$

where M is a very large real number. This constraint ensures that, when a supplier j not be chosen, the inbound shipment number for that supplier would be zero.

### 4. Outbound Transportation

$$\sum_{i=1}^n v_i \times \omega_{i,t} \times (\lambda_i T) \leq n_t^o \times C^o, \forall t$$

The outbound transportation constraint restricts the shipping capacity not being violated on the outbound side.

### 5. Binary, Integer and Non-negativity Constraints

$$\alpha_{i,j,t} \geq 0, \forall i, j, t$$

$$\omega_{i,t} \geq 0, \forall i, t$$

$$n_{i,j,t}^l \in \text{integer}, \forall i, j, t$$

$$n_t^o \in \text{integer}, \forall t$$

$$\delta_{i,j} \in (0,1), \forall i, j$$

### Final Mixed Integer Linear Optimization Model:

Minimize Total Cost =

$$\begin{aligned} & \sum_{j=1}^J F_{i,j} \delta_{i,j} + \sum_{i=1}^n \sum_{j=1}^J \sum_{t=1}^T \alpha_{i,j,t} U_{i,j}(\lambda_i T) + \sum_{i=1}^n \sum_{j=1}^J \sum_{t=1}^T n_{i,j,t}^l K_{i,j}^l + \sum_{t=1}^T n_t^o k^o \\ & + \sum_{i=1}^n \sum_{j=1}^J \sum_{t=1}^T \alpha_{i,j,t} D_{i,j}(\lambda_i T) (h_{i,j}^l + h_i^{CD} + h_i^o) \end{aligned}$$

Subject to

$$\sum_{j=1}^J \sum_{t=1}^T \alpha_{i,j,t} = 1, \forall i$$

$$\sum_{t=1}^T \omega_{i,t} = 1, \forall i$$

$$\sum_{t=1}^T \alpha_{i,j,t}(\lambda_i T) \leq C_{i,j} \delta_{i,j}, \forall i, j$$

$$v_i \times \alpha_{i,j,t} \times (\lambda_i T) \leq n_{i,j,t}^l \times C^l, \forall i, j, t$$

$$n_{i,j,t}^l \leq M \delta_{i,j}, \forall i, j$$

where M is a very large real number.

$$\sum_{i=1}^n v_i \times \omega_{i,t} \times (\lambda_i T) \leq n_t^o \times C^o, \forall t$$

$$\alpha_{i,j,t} \geq 0, \forall i, j, t$$

$$\omega_{i,t} \geq 0, \forall i, t$$

$$n_{i,j,t}^I \in \text{integer}, \forall i, j, t$$

$$n_t^O \in \text{integer}, \forall t$$

$$\delta_{i,j} \in (0,1), \forall j$$

## 4 Example and Discussions

In this section, a pilot problem with 2 products, 2 suppliers for each product in a time horizon spanning 3 days is presented to illustrate how the supplier selection is carried out using the two-phase methodology proposed in Chapter 3.

### 4.1 First Phase

There are 15 potential suppliers for product 1 which are randomly chosen from top 200 suppliers for Product 1. And there are also 15 potential suppliers for Product 2 which are randomly chosen from top 200 suppliers for Product 2. We firstly reduce the suppliers to 10, 5 for Product 1 and 5 for Product 2 by using  $L_\infty$  Norm and Borda Count. Secondly, ranking these reduced suppliers for each product by AHP, and obtaining top 2 suppliers for each product. Table 3 and Table 4 show the suppliers chosen in the first place for Product 1 and Product 2 respectively. Data used in the First Phase is partially from one of my course project, Goal Programming Approaches of Franchise Selection, other team members in this project are Xuan Li, Maiteng Pornthip, and Vara-Urairat Putthipan.

<b>Supplier No.</b>	<b>Rank in Top 200</b>
1	1
2	3
3	4
4	5
5	12
6	15
7	38
8	39
9	43
10	46
11	48
12	53
13	54
14	67
15	80

*Table 3 Potential Suppliers for Product 1*

<b>Supplier No.</b>	<b>Rank in Top 200</b>
1	17
2	18
3	34
4	47
5	84
6	125
7	136
8	137
9	142
10	145
11	150
12	180
13	186
14	188
15	195

*Table 4 Potential Suppliers for Product 2*

## Screening Process with $L_\infty$ Norm and Borda Count

Fourteen criteria are considered in this stage and are denoted as criterion 1, criterion 2, criterion 3 and etc. Criterion 1 to 8 and criterion 14 need to be minimized, while criterion 9 to criterion 13 need to be maximized.

Using  $L_\infty$  Norm and Borda Count here is trying to reduce the initial suppliers. This makes it much easier for us to do AHP since we will reduce the suppliers to ten, five for product 1 and five for product 2. We use  $L_\infty$  Norm to scale and use Borda Count to rank the suppliers.

- Define the  $L_\infty$  Norm of each criterion and sub-criterion.  $L_\infty$  Norm is calculated as:

$$L_\infty \text{ Norm} = \max[|X_j|]$$

To do the scaling, the criterion values is divided by their respective  $L_\infty$  Norm. Table 5 and Table 6 show the potential supplier criteria data for product 1 and product 2 respectively. Table 7 and Table 8 list the normalized criterion data for each product.

Supplier	Criterion													
	I		O			R		S		P				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	45,000	986,350	4.00%	4.00%	57,600	0	750,000	20	2160	2.73	59	12.20%	4.30%	1
2	45,000	1,264,900	5.00%	5.00%	2,160	1,500,000	750,000	20	448	1.25	62	7.00%	4.60%	3
3	15,000	157,775	8.00%	4.50%	13,286	30,000	195,000	20	112	0.47	40	9.00%	6.70%	4
4	50,000	1,360,600	4.50%	4.00%	7,500	1,500,000	500,000	20	440	1.22	55	3.60%	3.90%	5
5	45,000	1,148,300	5.50%	4.50%	10,000	1,500,000	750,000	25	392	1.30	50	6.80%	0.60%	12
6	25,000	265,825	5.50%	4.00%	15,435	250,000	75,000	10	0	0.72	47	6.40%	5.30%	15
7	50,000	1,208,900	5.00%	5.00%	10,000	1,500,000	750,000	0	240	1.37	32	4.70%	1.30%	38
8	15,000	733,000	4.00%	4.20%	12,500	1,000,000	180,700	20	280	0.88	49	-1.10%	-2.30%	39
9	0	124,823	5.00%	9.00%	1,432	50,000	150,000	10	120	0.68	29	10.60%	7.20%	43
10	50,000	2,021,500	5.00%	4.00%	20,085	6,000,000	2,500,000	10	440	2.96	45	3.80%	1.00%	46
11	0	1,178,556	4.00%	4.00%	20,000	1,000,000	350,000	20	152	1.48	51	3.20%	0.20%	48
12	35,000	1,034,600	5.00%	4.00%	36,000	1,000,000	500,000	20	440	1.06	22	13.30%	1.20%	53
13	10,000	1,437,900	4.00%	5.00%	20,150	1,000,000	300,000	20	560	1.12	53	5.50%	1.20%	54
14	35,000	1,196,500	4.00%	5.50%	0	1,000,000	300,000	20	0	1.21	30	7.50%	4.30%	67
15	25,000	281,200	6.00%	1.50%	10,500	1,500,000	500,000	10	112	0.98	12	19.60%	21.60%	80
L <sub>∞</sub> Norm	50,000	2,021,500	8.00%	9.00%	57,600	6,000,000	2,500,000	25	2,160	3	62	19.60%	21.60%	80

Table 5 Potential Suppliers for Product 2



Supplier	Criterion													
	I		O			R		S		P				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	40,000	294,020	5.90%	5.00%	3,200	250,000	125,000	20	0	0.66	59	5.60%	3.90%	17
2	35,000	482,700	4.50%	4.00%	2,165	500,000	300,000	10	160	1.52	50	7.30%	6.20%	18
3	35,000	396,950	5.00%	6.00%	3,000	750,000	400,000	15	112	0.56	70	11.10%	2.30%	34
4	25,000	618,500	5.50%	4.50%	5,000	1,500,000	500,000	20	0	1.70	54	1.70%	2.00%	47
5	40,000	888,000	6.50%	4.00%	8,160	2,000,000	1,000,000	15	240	1.33	74	8.50%	7.80%	84
6	25,000	2,975,000	10.00%	4.50%	3,400	450,000	250,000	10	240	0.58	12	5.00%	10.50%	125
7	42,000	286,075	6.00%	3.00%	1,980	250,000	100,000	10	24	0.33	20	1.20%	-0.30%	136
8	27,000	194,875	7.00%	1.00%	675	400,000	50,000	20	0	0.36	23	8.00%	13.40%	137
9	35,000	536,300	5.00%	5.00%	2,500	2,500,000	1,000,000	10	240	0.67	7	10.20%	8.30%	142
10	25,000	328,100	6.00%	4.00%	1,089	250,000	80,000	10	0	0.55	23	5.20%	5.20%	145
11	20,000	407,500	5.00%	3.00%	1,200	1,500,000	750,000	10	400	1.34	20	14.30%	3.70%	150
12	25,000	119,400	6.00%	6.00%	5,000	250,000	75,000	10	0	0.29	41	10.30%	2.50%	180
13	25,000	829,000	5.00%	2.00%	1,262	4,500,000	300,000	6	480	2.05	7	15.20%	14.00%	186
14	30,000	143,333	6.00%	1.50%	3,000	400,000	120,000	20	80	0.26	27	28.80%	24.00%	188
15	30,000	452,098	6.00%	4.00%	2,700	1,500,000	600,000	20	0	0.80	21	9.80%	-0.90%	195
L <sub>∞</sub> Norm	42,000	2,975,000	10.00%	6.00%	8,160	4,500,000	1,000,000	20	480	2	74	28.80%	24.00%	195

Table 6 Initial Supplier Data for product 2

Supplier	Criterion													
	I		O			R		S		P				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.90	0.49	0.50	0.44	1.00	0.00	0.30	0.80	1.00	0.92	0.95	0.62	0.20	0.01
2	0.90	0.63	0.63	0.56	0.04	0.25	0.30	0.80	0.21	0.42	1.00	0.36	0.21	0.04
3	0.30	0.08	1.00	0.50	0.23	0.01	0.08	0.80	0.05	0.16	0.65	0.46	0.31	0.05
4	1.00	0.67	0.56	0.44	0.13	0.25	0.20	0.80	0.20	0.41	0.89	0.18	0.18	0.06
5	0.90	0.57	0.69	0.50	0.17	0.25	0.30	1.00	0.18	0.44	0.81	0.35	0.03	0.15
6	0.50	0.13	0.69	0.44	0.27	0.04	0.03	0.40	0.00	0.24	0.76	0.33	0.25	0.19
7	1.00	0.60	0.63	0.56	0.17	0.25	0.30	0.00	0.11	0.46	0.52	0.24	0.06	0.48
8	0.30	0.36	0.50	0.47	0.22	0.17	0.07	0.80	0.13	0.30	0.79	-0.06	-0.11	0.49
9	0.00	0.06	0.63	1.00	0.02	0.01	0.06	0.40	0.06	0.23	0.47	0.54	0.33	0.54
10	1.00	1.00	0.63	0.44	0.35	1.00	1.00	0.40	0.20	1.00	0.73	0.19	0.05	0.58
11	0.00	0.58	0.50	0.44	0.35	0.17	0.14	0.80	0.07	0.50	0.82	0.16	0.01	0.60
12	0.70	0.51	0.63	0.44	0.63	0.17	0.20	0.80	0.20	0.36	0.35	0.68	0.06	0.66
13	0.20	0.71	0.50	0.56	0.35	0.17	0.12	0.80	0.26	0.38	0.85	0.28	0.06	0.68
14	0.70	0.59	0.50	0.61	0.00	0.17	0.12	0.80	0.00	0.41	0.48	0.38	0.20	0.84
15	0.50	0.14	0.75	0.17	0.18	0.25	0.20	0.40	0.05	0.33	0.19	1.00	1.00	1.00

Table 7 Normalized Supplier Data for Product 1

Supplier	Criterion													
	I		O			R		S		P				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.95	0.10	0.59	0.83	0.39	0.06	0.13	1.00	0.00	0.32	0.80	0.19	0.16	0.09
2	0.83	0.16	0.45	0.67	0.27	0.11	0.30	0.50	0.33	0.74	0.68	0.25	0.26	0.09
3	0.83	0.13	0.50	1.00	0.37	0.17	0.40	0.75	0.23	0.28	0.95	0.39	0.10	0.17
4	0.60	0.21	0.55	0.75	0.61	0.33	0.50	1.00	0.00	0.83	0.73	0.06	0.08	0.24
5	0.95	0.30	0.65	0.67	1.00	0.44	1.00	0.75	0.50	0.65	1.00	0.30	0.33	0.43
6	0.60	1.00	1.00	0.75	0.42	0.10	0.25	0.50	0.50	0.28	0.16	0.17	0.44	0.64
7	1.00	0.10	0.60	0.50	0.24	0.06	0.10	0.50	0.05	0.16	0.27	0.04	-0.01	0.70
8	0.64	0.07	0.70	0.17	0.08	0.09	0.05	1.00	0.00	0.17	0.31	0.28	0.56	0.70
9	0.83	0.18	0.50	0.83	0.31	0.56	1.00	0.50	0.50	0.33	0.09	0.35	0.35	0.73
10	0.60	0.11	0.60	0.67	0.13	0.06	0.08	0.50	0.00	0.27	0.31	0.18	0.22	0.74
11	0.48	0.14	0.50	0.50	0.15	0.33	0.75	0.50	0.83	0.66	0.27	0.50	0.15	0.77
12	0.60	0.04	0.60	1.00	0.61	0.06	0.08	0.50	0.00	0.14	0.55	0.36	0.10	0.92
13	0.60	0.28	0.50	0.33	0.15	1.00	0.30	0.30	1.00	1.00	0.09	0.53	0.58	0.95
14	0.71	0.05	0.60	0.25	0.37	0.09	0.12	1.00	0.17	0.12	0.36	1.00	1.00	0.96
15	0.71	0.15	0.60	0.67	0.33	0.33	0.60	1.00	0.00	0.39	0.28	0.34	-0.04	1.00

Table 8 Normalized Supplier Data for Product 2

- Ask decision maker (DM) for preference information between pairs of criteria. In the pairwise comparison matrix,  $p_{ij} = 1$  when  $i$  is preferred to  $j$ , meanwhile,  $p_{ji} = 0$ , and vice versa.  $p_{ij} = p_{ji} = 1$  when  $i$  and  $j$  are equally preferred.  $p_{ii} = 1$  all the time. The preference matrix is shown in Table 9.

Criterion	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	0	1	1	1	1	1	1	1	1	1	1	1	1
4	1	0	0	1	1	1	1	1	1	1	1	1	1	1
5	1	0	0	0	1	0	0	0	0	0	1	0	0	0
6	1	0	0	0	1	1	1	0	1	0	1	0	1	1
7	1	0	0	0	1	0	1	0	1	0	1	0	1	1
8	1	0	0	0	1	1	1	1	1	0	1	0	0	1
9	1	0	0	0	1	0	0	0	1	0	0	0	0	0
10	1	0	0	0	1	1	1	1	1	1	1	1	1	1
11	1	0	0	0	0	0	0	0	1	0	1	0	0	0
12	1	0	0	0	1	1	1	1	1	0	1	1	1	0
13	1	0	0	0	1	0	0	1	1	0	1	0	1	0
14	1	0	0	0	1	0	0	0	1	0	1	1	1	1

Table 9 Preference Matrix of each Criterion

- Rank criteria and get weights using Borda Count, as listed in Table 10.

Criterion	1	2	3	4	5	6	7	8	9	10	11	12	13	14
<b>Borda Count Rank</b>	1	14	13	12	3	8	7	8	3	11	3	9	6	7
<b>Weight</b>	0.01	0.13	0.12	0.11	0.02	0.07	0.06	0.07	0.02	0.10	0.02	0.08	0.05	0.06
	0	3	4	4	9	6	7	6	9	5	9	6	7	7

Table 10 Borda Count Rank and Weight of each Criterion

- Get the score of each supplier. For a supplier, the score is getting from the sum of the product of each normalized criterion and its respective weight. If the criterion needs to be minimized, multiply it by -1 first. Table 11 and Table 12 show the score of suppliers for each product.

<b>Supplier</b>	<b>Score</b>	<b>Rank</b>
1	-0.080	1
15	-0.085	2
6	-0.125	3
9	-0.167	4
3	-0.177	5
2	-0.215	6
7	-0.215	7
4	-0.221	8
11	-0.231	9
12	-0.237	10
8	-0.237	11
5	-0.251	12
14	-0.254	13
13	-0.260	14
10	-0.342	15

*Table 11 Score and Rank of Suppliers for Product 1*

Supplier	Score	Rank
2	-0.099	1
14	-0.111	2
13	-0.115	3
11	-0.159	4
8	-0.173	5
10	-0.207	6
1	-0.214	7
3	-0.219	8
7	-0.227	9
4	-0.238	10
12	-0.259	11
5	-0.266	12
9	-0.292	13
15	-0.319	14
6	-0.377	15

*Table 12 Score and Rank of Suppliers for Product 2*

- Thus we choose Supplier 1, 15, 6, 9, and 3 for Product 1, while Supplier 2, 14, 13, 11, and 8 for Product 2 at this stage.

### **Criterion Weights and Ranking with AHP**

Here we will reduce the 5 suppliers to 2 for each product. The motivation to use AHP to rank suppliers is that this technique allows the selection to involve the assessment not only numerical but also intangible factors. Figure 4 shows the supplier selection criteria in AHP used for this example.

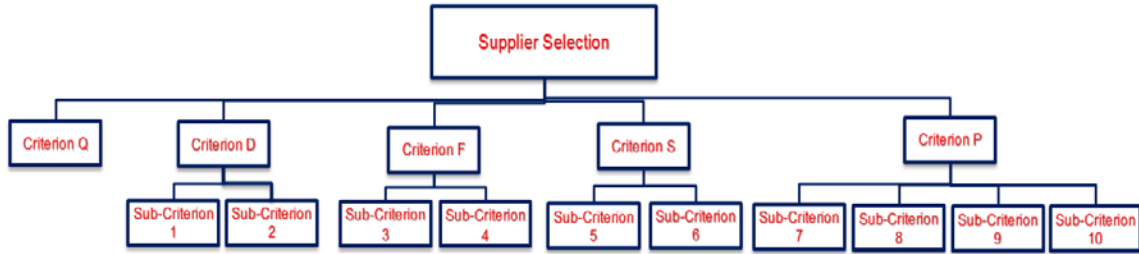


Figure 4 Supplier Selection Criteria

- Do a pairwise comparison of the main criteria using the scale in Table 1. Form the matrix  $A = [a_{ij}]$ , where  $a_{ij}$  represents the relative importance of criterion 'i' with regard to criterion 'j'. Let  $a_{ii} = 1 \forall i$  and  $a_{ji} = \frac{1}{a_{ij}}$ . The results for Product 1 and Product 2 are shown in Table 13 and Table 14 respectively.

	Criterion				
	Q	D	F	S	P
Q	1	0.5	2	2	0.33333
D	2	1	3	3	1
F	0.5	0.33333	1	0.5	0.33333
S	0.5	0.33333	2	1	0.25
P	3	1	3	4	1

Table 13 Pairwise Comparison Matrix for Product 1

	Criterion				
	Q	D	F	S	P
Q	1	2	4	4	3
D	0.5	1	2	2	2
F	0.25	0.5	1	3	2
S	0.25	0.5	0.33333	1	0.5
P	0.33333	0.5	0.5	2	1

Table 14 Pairwise Comparison Matrix for Product 2

- Compute the normalized weights for the main criteria from matrix A by  $L_1$  norm.

Following computation process below, we can get the results displayed in Table 15

and Table 16: Compute  $r_{ij} = \frac{a_{ij}}{\sum_{i=1}^n a_{ij}}$ , then average the  $r_{ij}$  values to get the weights,

$$w_i = \frac{\sum_j r_{ij}}{n}.$$

Normalization						Weights
$A_{norm}$	Q	D	F	S	P	
Q	0.143	0.158	0.182	0.190	0.114	0.157
D	0.286	0.316	0.273	0.286	0.343	0.301
F	0.071	0.105	0.091	0.048	0.114	0.086
S	0.071	0.105	0.182	0.095	0.086	0.108
P	0.429	0.316	0.273	0.381	0.343	0.348

Table 15 Normalized Matrix for Product 1

Normalization						Weights
$A_{norm}$	Q	D	F	S	P	
Q	0.429	0.444	0.511	0.333	0.353	0.414
D	0.214	0.222	0.255	0.167	0.235	0.219
F	0.107	0.111	0.128	0.250	0.235	0.166
S	0.107	0.111	0.043	0.083	0.059	0.081
P	0.143	0.111	0.064	0.167	0.118	0.120

Table 16 Normalized Matrix for Product 2

Pairwise comparison and normalized weights are continuously performed throughout every criteria and sub-criteria. Table 17 shows the weights of each criterion for 2 products. The final weight of a sub-criterion is the product of the weights with the corresponding branch.

Criterion/Sub-criterion Weight	Product 1	Product 2
<b>Q</b>	0.157	0.414
<b>D</b>	0.156	0.153
- Sub-Criterion 1	0.145	0.066
- Sub-Criterion 2	0.029	0.055
<b>F</b>	0.057	0.111
- Sub-Criterion 3	0.027	0.054
- Sub-Criterion 4	0.081	0.027
<b>S</b>	0.158	0.019
- Sub-Criterion 5	0.090	0.054
- Sub-Criterion 6	0.051	0.035
<b>P</b>	0.048	0.013
- Sub-Criterion 7	0.157	0.414
- Sub-Criterion 8	0.156	0.153
- Sub-Criterion 9	0.145	0.066
- Sub-Criterion 10	0.029	0.055

Table 17 Criterion Weights

- Check consistency of the pairwise comparison matrix, using the Consistency Index (CI) and the Consistency Ratio (CR). AHP has a procedure to check the consistency of the DM's response. If the DM is perfectly consistent, then, A (before normalization) has the following property:

$$A \cdot \vec{w} = \begin{bmatrix} 1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & 1 & \cdots & w_2/w_n \\ \vdots & \vdots & \ddots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & 1 \end{bmatrix} \cdot \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = n\vec{w}.$$

If A is perfectly consistent then  $\lambda_{max} = n$ ; where  $\lambda_{max} = Average[A_{1 \cdot} \cdot w/w_1, A_{2 \cdot} \cdot w/w_2, \dots, A_{n \cdot} \cdot w/w_n]$ . To measure the degree of inconsistency, we use the following indicators:

$$CI = \frac{\lambda_{max} - n}{n - 1}; CR = \frac{CI}{RI}$$



where RI is a random index, obtained from Table 2. If  $CR < 0.1$ , the pairwise comparison matrix would be accepted.

Finally, we check the consistency of pairwise comparison matrix for Product 1 and Product 2, and they are all acceptable. Table 18 and Table 19 show the results:

<b>Consistency Check</b>			
<b>AW</b>	<b><math>\lambda</math></b>		
0.811	5.153	<b><math>\lambda_{\max}</math></b>	5.122
1.545	5.141	<b>CI</b>	0.031
0.435	5.062	<b>RI</b>	1.110
0.546	5.057	<b>CR</b>	0.028
1.810	5.200	<b>Acceptance</b>	<b>Y</b>

*Table 18 Consistency Check for Product 1*

<b>Consistency Check</b>			
<b>AW</b>	<b><math>\lambda</math></b>		
2.200	5.314	<b><math>\lambda_{\max}</math></b>	5.192
1.160	5.304	<b>CI</b>	0.048
0.862	5.184	<b>RI</b>	1.110
0.409	5.076	<b>CR</b>	0.043
0.612	5.083	<b>Acceptance</b>	<b>Y</b>

*Table 19 Consistency Check for Product 2*

At this point, we should proceed to rank all the suppliers by comparing the suppliers with regard to each criterion using AHP. The column of Score Matrix (S) is formed by the weights computed for each criterion. The Total Score (TS) of the suppliers is determined by Equation shown below, where w corresponds to the criteria weights, and S represents the Score Matrix. The suppliers are ranked by their TS values, the higher the better.

$$TS = \sum_{i=1}^{11} [S \times w]$$

The results are shown in Table 20 and Table 21:

<b>Supplier</b>	<b>Total Score</b>	<b>Rank</b>
1	0.297	1
15	0.225	2
9	0.189	3
6	0.149	4
3	0.141	5

*Table 20 AHP Rank for Product 1*

<b>Supplier</b>	<b>Total Score</b>	<b>Rank</b>
14	0.299	1
8	0.257	2
2	0.164	3
11	0.142	4
13	0.139	5

*Table 21 AHP Rank for Product 2*

- Therefore, we choose Supplier 1 and Supplier 15 for Product 1, while choosing Supplier 14 and Supplier 8 for Product 2 up to this stage.

## 4.2 Second Phase

For a better illustration, we match the reduced suppliers for Product 1 and Product 2 with Supplier 1 and Supplier 2 for Product 1, and Supplier 1 and Supplier 2 for Product 2 correspondingly, as presented in Table 22.

<b>Product 1</b>	<b>Noted as</b>	<b>Product 2</b>	<b>Noted as</b>
Supplier 1	Supplier 1	Supplier 14	Supplier 1
Supplier 15	Supplier 2	Supplier 8	Supplier 2

*Table 22 Notation of Reduced Suppliers*

The parameter values are chosen as:

$n$  (Number of Products) = 2;

$T$  (Time horizon) = 3 days;

$J$  (Number of suppliers for each product) = 2;

$K_{1,1}^I, K_{1,2}^I, K_{2,1}^I, K_{2,2}^I$  (Cost of an inbound shipment for Product  $i$  from Supplier  $j$ ) = \$150, \$100, \$100, \$200 per shipment for Product 1, 2;

$K^O$  (Cost of an outbound shipment) = \$2000 per outbound shipment;

$C^O$  (Outbound capacity) = 500 units;

$C^I$  (Inbound capacity) = 500 units;

$C_{1,1}, C_{1,2}, C_{2,1}, C_{2,2}$  (The capacity of each supplier for Product  $i$ ) = 50, 70, 40, 50;

$v_1, v_2$  (Volume occupied) = 2, 2 units for products 1, 2 respectively;

$\lambda_1, \lambda_2$  (Daily demands of Product 1, 2) = 60, 60;

$U_{1,1}, U_{1,2}, U_{2,1}, U_{2,2}$  (Unit price of Product 1, 2 from Supplier 1, 2) = \$2, \$3, \$9, \$7;

$F_{1,1}, F_{1,2}, F_{2,1}, F_{2,2}$  (Fixed cost of using Supplier j for Product i) = \$1300, \$1400, \$2100, \$2200;

$h_{1,1}^I, h_{1,2}^I, h_{2,1}^I, h_{2,2}^I$  (Value lost per unit in inbound of Product i from Supplier j) = \$1, \$1, \$2, \$3;

$h_1^{CD}, h_2^{CD}$  (Value lost per unit in cross docking of Product 1, 2) = \$1, \$3;

$h_1^O, h_2^O$  (Value lost per unit of Product 1, 2 in outbound) = \$2, \$2;

$D_{1,1}, D_{1,2}, D_{2,1}, D_{2,2}$  (The defect percentage of Product i from Supplier j) = 2%, 7%, 2%, 8%;

LINDO optimizer is used to analyze this problem.

The summary of the model is given in Table 23:

<b>Variables</b>	<b>Numbers</b>
Total	36
Non-linear	0
Integer	21
<b>Constraints</b>	
Total	35
Non-linear	0

*Table 23 Model Summary*

The total minimum cost is computed as \$11195.60. Supplier 1 and Supplier 2 for Product 1 are both selected. However, only Supplier 2 is selected for Product 2. The number of inbound shipments of Product 1 from Supplier 1 in Time Period 2 is 1 while the number of inbound shipments of Product 1 from Supplier 2 in Time Period 3 is 1. And the number of inbound shipments of Product 2 from Supplier 2 in Time Period 3 is 1. In Period 2 and Period 3, the number of outbound shipments are both 1. The fraction of the total horizon demand of Product 1 shipped from cross docking to the retailer in Period 2 and Period 3 are 38.9% and 61.1% respectively. The fraction of the total horizon demand of Product 2 shipped from cross docking to the retailer in Period 2 is 100%. The value of the rest variables are 0 if not stated above. The order allocation is concluded in Table 24:

Product	Supplier	Time Period	Percent
1	1	1	0
		2	55.56%
		3	0
		Sum	55.56%
	2	1	0
		2	0
		3	44.44%
		Sum	44.44%
2	1	1	0
		2	0
		3	0
		Sum	0
	2	1	0
		2	0
		3	100%
		Sum	100%

*Table 24 Order Allocated to Each Supplier (in Percent)*

For the original supplier notation, the order allocation would be as shown in Table 25:

Product	Supplier	Percent
1	1	55.56%
	15	44.44%
2	14	0
	8	100%

*Table 25 Order Allocated to Original Supplier Notation (in Percent)*

The LINDO formulation and the complete results are attached in Appendix A and Appendix B respectively.

### 4.3 Sensitivity Analysis

Several scenarios were analyzed to see if the order allocation would change. Here we varied the inbound capacity of the containers, which may be a representation of different trailer sizes that are being used by the logistics companies. The other parameters remain the same.

Inbound Capacity	Product 1						Product 2						Total Cost
	Supplier 1			Supplier 15			Supplier 14			Supplier 8			
	T 1	T 2	T 3	T 1	T 2	T 3	T 1	T 2	T 3	T 1	T 2	T 3	
700	0	55.56 %	0	0	0	44.44 %	0	0	0	0	100%	0	\$11,195.6
600	0	55.56 %	0	0	0	44.44 %	0	0	0	0	100%	0	\$11,195.6
500	0	55.56 %	0	0	0	44.44 %	0	0	0	0	0	100%	\$11,195.6
400	0	55.56 %	0	0	0	44.44 %	0	0	0	100 %	0	0	\$11,195.6
300	0	0	55.56 %	44.44 %	0	0	0	0	0	100 %	0	0	\$11,395.6
200	0	0	55.56 %	0	0	44.44 %	0	0	0	0	100%	0	\$11,395.6
100	0	27.78 %	27.78 %	0	44.44 %	0	0	0	0	0	72.22 %	27.78 %	\$12,045.6

Table 26 Allocation for Different Scenarios

In Table 26, we observe that though the order allocation would change in different time periods, the allocation for different suppliers remains unchanged. Therefore, the model is insensitive to the capacity of inbound trucks.

## 5 Conclusions

Supplier selection is an essential part of the purchasing process. However, very few studies addressed the perspective of supplier selection in cross docking cold chain. This thesis formed a two-phase supplier selection problem with cross docking and cold chain constraints. The two-phase methodology presented here allows the retailer to make sound decisions about supplier selection. In particular, first phase screens a large number of potential suppliers to a manageable amount. The AHP in the first phase provides a strategic approach to evaluate alternatives and enables retailers to make selections based on both qualitative and quantitative criteria. A mathematical optimization model has been developed to decide the order allocation in the second phase.

The mathematical model presented in this research have been set up for deterministic retailer demands. The model could be extended for stochastic demands. And other constraints such as quality constraints, lead time constraints and price break constraints could be considered for further study.



## REFERENCES

- Belle, J. V., Valckenaers, P. & Cattrysse, D., 2012. Cross-docking: State of the art. *Omega the International Journal of Management Science*, 40(6), pp. 827-846.
- Bishara, R. H., 2006. *American Pharmaceutical Review*. [Online] Available at: <http://www.sensitech.com/assets/articles/lbsharaapr.pdf>
- Boysen, N. & Fliedner, M., 2010. Cross Dock Scheduling: Classification, Literature Review and Research Agenda. *Omega the International Journal of Management Science*, 38(6), pp. 413-422.
- Buffa, F. & Jackson, W., 1983. A Goal Programming Model for Purchase Planning. *International Journal of Purchasing and Materials Management*, 19(3), p. 27.
- Cook, R. L., 1997. Case-Based Reasoning Systems in Purchasing: Applications and Development. *International Journal of Purchasing and Materials Management*, 33(4), pp. 32-39.
- Ding, H., Benyoucef, L. & Xie, X., 2003. *A simulation-optimization approach using genetic search for supplier selection*. New Orleans, IEEE.
- Dobler, D., Lee, L. & Burt, D. N., 1995. *Purchasing and Supply Management*. 6th edition ed. s.l.:McGraw-Hill Companies.
- Galbreth, M. R., Hill, J. A. & Handley, S., 2008. An Investigation of the Value of Cross-Docking for Supply Chain Management. *Journal of Business Logistics*, 29(1), pp. 225-239.

- Ghodsypour, S. & O'Brien, C., 2001. The total cost of logistics in supplier selection, under conditions of multiple sourcing, multiple criteria and capacity constraint. *International Journal of Production Economics*, 73(1), pp. 15-27.
- Giannakourou, M. & Taoukis, P., 2003. Kinetic modelling of vitamin C loss in frozen green vegetables under variable storage conditions. *Food Chemistry*, 83(1), pp. 33-41.
- Holt, G. D., 1998. Which contractor selection methodology?. *International Journal of Project Management*, 16(3), pp. 153-164.
- Humphreys, P., Wong, Y. & Chan, F., 2003. Integrating environmental criteria into the supplier selection process. *Journal of Materials Processing Technology*, 138(1-3), pp. 349-356.
- Koutsoumanis, K., Pavlis, A., Nychas, G.-J. E. & Xanthiakos, K., 2010. Probabilistic Model for *Listeria monocytogenes* Growth during Distribution, Retail Storage, and Domestic Storage of Pasteurized Milk. *Applied and Environmental Microbiology*, 76(7), pp. 2181-2191.
- Lim, A., Miao, Z., Rodrigues, B. & Xu, Z., 2004. *Transshipment Through Crossdocks with Inventory and Time Windows*. Jeju Island, s.n.
- Liu, F.-H. F. & Hai, H. L., 2005. The voting analytic hierarchy process method for selecting supplier. *International Journal of Production Economics*, 97(3), pp. 308-317.
- Ma, H., Miao, Z., Lim, A. & Rodrigues, B., 2011. Crossdocking Distribution Networks with Setup Cost and Time Window Constraint. *Omega the International Journal of Management Science*, 39(1), pp. 64-72.

- Manikas, L. & Terry, L. A., 2009. A case study assessment of the operational performance of a multiple fresh produce distribution centre in the UK. *British Food Journal*, 111(5), pp. 421-435.
- Mendoza, A., Santiago, E. & Ravindran, A. R., 2008. A Three-Phase Multicriteria Method to the Supplier Selection Problem. *International Journal of Industrial Engineering*, 15(2), pp. 195-210.
- Montanari, R., 2008. Cold chain tracking: a managerial perspective. *Trends in Food Science & Technology*, 19(8), pp. 425-431.
- Mummalaneni, V., Dubas, K. M. & Chao, C.-n., 1996. Chinese purchasing manager's preferences and trade-offs in supplier selection and performance evaluation. *Industrial Marketing Management*, 25(2), pp. 115-124.
- Nydick, R. L. & Hill, R. P., 1992. Using the Analytic Hierarchy Process to Structure the Supplier Selection Procedure. *International Journal of Purchasing and Materials Management*, 28(2), p. 31.
- Pal, O., Gupta, A. K. & Garg, R., 2013. Supplier Selection Criteria and Methods in Supply Chains: A review. *International Journal of Social, Behavioral, Educational, Economic and Management Engineering*, 7(10).
- Qiu, Q., Zhang, Z., Song, X. & Gui, S., 2009. *Application Research of Cross Docking Logistics in Food Cold-Chain Logistics*. Xi'an, IEEE.
- Ravindran, A. R. & Warsing, D. P., 2012. *Supply Chain Engineering: Models and Applications*. s.l.:CRC Press.

- Saaty, T. L., 1980. *The Analytic Hierarchy Process*. New York: McGraw Hill.
- Shi, J., Zhang, J. & Qu, X., 2010. Optimizing Distribution Strategy for Perishable Foods Using RFID and Sensor Technologies. *Journal of Business & Industrial Marketing*, 25(8), pp. 596-606.
- Shyur, H.-J. & Shih, H.-S., 2006. A hybrid MCDM model for strategic vendor selection. *Mathematical and Computer Modelling*, 44(7-8), pp. 749-761.
- Soukup, W. R., 1987. Supplier Selection Strategies. *Journal of Purchasing and Materials Management*, 23(2), pp. 7-13.
- Stephan, K. & Boysen, N., 2011. Cross-docking. *Journal of Management Control*, 22(1), pp. 129-137.
- Stringer, M. & Hall, M., 2007. A generic model of the integrated food supply chain to aid the investigation of food safety breakdowns. *Food Control*, 18(7), pp. 755-765.
- Tam, M. C. & Tummala, V., 2001. An application of the AHP in vendor selection of a telecommunications system. *Omega the International Journal of Management Science*, 29(2), pp. 171-182.
- Tiwari, G., 2003. *Optimization Models for Cross Docking Operations*, State College: s.n.
- Velazquez, M. A., Claudio, D. & Ravindran, A. R., 2010. Experiments in multiple criteria selection problems with multiple decision makers. *International Journal of Operational Research*, 7(4), pp. 413-428.

- Vokurka, R. J., Choobineh, J. & Vadi , L., 1996. A prototype expert system for the evaluation and selection of potential suppliers. *International Journal of Operations & Production Management*, 16(12), pp. 106-127.
- Weber, C. A., 1996. A data envelopment analysis approach to measuring vendor performance. *Supply Chain Management: An International Journal*, 1(1), pp. 28-39.
- Weber, C., Current, J. & Benton, W., 1991. Vendor Selection Criteria and Methods. *European Journal of Operational Research*, pp. 2-18.
- Yan, H. & Wei, Q., 2002. Determining Compromise Weights for Group Decision Making. *The Journal of the Operational Research Society*, 53(6), pp. 680-687.
- Yu, M., Li, X., Pornthip, M. & Putthipan, V.-U., 2014. *Goal Programming Approaches of Franchise Selection*, State College: s.n.
- Zenz, G. J., 1993. *Purchasing and the Management of Materials*. 7 edition ed. s.l.:Wiley.

# Appendix A

## LINDO Formulation of the Model

```
MIN      150 I(1,1,1) + 150 I(1,1,2) + 150 I(1,1,3) + 100 I(1,2,1)
          + 100 I(1,2,2) + 100 I(1,2,3) + 100 I(2,1,1) + 100 I(2,1,2)
          + 100 I(2,1,3) + 200 I(2,2,1) + 200 I(2,2,2) + 200 I(2,2,3) + 2000
O(1)
          + 2000 O(2) + 2000 O(3) + 1300 D(1,1) + 1400 D(1,2) + 2100 D(2,1)
          + 2200 D(2,2) + 374.4 A(1,1,1) + 374.4 A(1,1,2) + 374.4 A(1,1,3)
          + 590.4 A(1,2,1) + 590.4 A(1,2,2) + 590.4 A(1,2,3) + 1645.2
A(2,1,1)
          + 1645.2 A(2,1,2) + 1645.2 A(2,1,3) + 1375.2 A(2,2,1) + 1375.2
A(2,2,2)
          + 1375.2 A(2,2,3)

SUBJECT TO

A(1,1,1)  A(1,1,2) + A(1,1,3) + A(1,2,1) + A(1,2,2) + A(1,2,3) =    1
A(2,1,1)  A(2,1,2) + A(2,1,3) + A(2,2,1) + A(2,2,2) + A(2,2,3) =    1
W(1,1)    W(1,2) + W(1,3) =    1
W(2,1)    W(2,2) + W(2,3) =    1

        6) - 100 D(1,1) + 180 A(1,1,1) + 180 A(1,1,2) + 180 A(1,1,3) <=
0
        7) - 150 D(1,2) + 180 A(1,2,1) + 180 A(1,2,2) + 180 A(1,2,3) <=
0
```

$$8) - 170 D(2,1) + 180 A(2,1,1) + 180 A(2,1,2) + 180 A(2,1,3) \leq 0$$

$$9) - 190 D(2,2) + 180 A(2,2,1) + 180 A(2,2,2) + 180 A(2,2,3) \leq 0$$

$$I(1,1,1) - 50 D(1,1) \leq 0$$

$$I(1,1,2) - 50 D(1,1) \leq 0$$

$$I(1,1,3) - 50 D(1,1) \leq 0$$

$$I(1,2,1) - 50 D(1,2) \leq 0$$

$$I(1,2,2) - 50 D(1,2) \leq 0$$

$$I(1,2,3) - 50 D(1,2) \leq 0$$

$$I(2,1,1) - 50 D(2,1) \leq 0$$

$$I(2,1,2) - 50 D(2,1) \leq 0$$

$$I(2,1,3) - 50 D(2,1) \leq 0$$

$$I(2,2,1) - 50 D(2,2) \leq 0$$

$$I(2,2,2) - 50 D(2,2) \leq 0$$

$$I(2,2,3) - 50 D(2,2) \leq 0$$

$$22) - 500 I(1,1,1) + 360 A(1,1,1) \leq 0$$

$$23) - 500 I(1,1,2) + 360 A(1,1,2) \leq 0$$

$$24) - 500 I(1,1,3) + 360 A(1,1,3) \leq 0$$

$$25) - 500 I(1,2,1) + 360 A(1,2,1) \leq 0$$

$$26) - 500 I(1,2,2) + 360 A(1,2,2) \leq 0$$

27) - 500 I(1,2,3) + 360 A(1,2,3) <= 0  
 28) - 500 I(2,1,1) + 360 A(2,1,1) <= 0  
 29) - 500 I(2,1,2) + 360 A(2,1,2) <= 0  
 30) - 500 I(2,1,3) + 360 A(2,1,3) <= 0  
 31) - 500 I(2,2,1) + 360 A(2,2,1) <= 0  
 32) - 500 I(2,2,2) + 360 A(2,2,2) <= 0  
 33) - 500 I(2,2,3) + 360 A(2,2,3) <= 0  
 34) - 500 O(1) + 360 W(1,1) + 360 W(2,1) <= 0  
 35) - 500 O(2) + 360 W(1,2) + 360 W(2,2) <= 0  
 36) - 500 O(3) + 360 W(1,3) + 360 W(2,3) <= 0

END

GIN I(1,1,1)

GIN I(1,1,2)

GIN I(1,1,3)

GIN I(1,2,1)

GIN I(1,2,2)

GIN I(1,2,3)

GIN I(2,1,1)

GIN I(2,1,2)

GIN I(2,1,3)

GIN I(2,2,1)



GIN	I(2,2,2)	
GIN	I(2,2,3)	
GIN	O(1)	
GIN	O(2)	
GIN	O(3)	
SUB	D(1,1)	1.00000
INTE	D(1,1)	
SUB	D(1,2)	1.00000
INTE	D(1,2)	
SUB	D(2,1)	1.00000
INTE	D(2,1)	
SUB	D(2,2)	1.00000
INTE	D(2,2)	

## Appendix B

### The Solution Reports Obtained from LINDO

#### OBJECTIVE FUNCTION VALUE

1) 11195.60

VARIABLE	VALUE	REDUCED COST
I(1,1,1)	0.000000	150.000000
I(1,1,2)	1.000000	150.000000
I(1,1,3)	0.000000	150.000000
I(1,2,1)	0.000000	100.000000
I(1,2,2)	0.000000	100.000000
I(1,2,3)	1.000000	100.000000
I(2,1,1)	0.000000	100.000000
I(2,1,2)	0.000000	100.000000
I(2,1,3)	0.000000	100.000000
I(2,2,1)	0.000000	200.000000
I(2,2,2)	0.000000	200.000000

I(2,2,3)	1.000000	200.000000
O(1)	0.000000	2000.000000
O(2)	1.000000	2000.000000
O(3)	1.000000	2000.000000
D(1,1)	1.000000	1180.000000
D(1,2)	1.000000	1400.000000
D(2,1)	0.000000	2100.000000
D(2,2)	1.000000	2200.000000
A(1,1,1)	0.000000	590.400024
A(1,1,2)	0.555556	0.000000
A(1,1,3)	0.000000	0.000000
A(1,2,1)	0.000000	0.000024
A(1,2,2)	0.000000	0.000024
A(1,2,3)	0.444444	0.000000
A(2,1,1)	0.000000	1645.199951
A(2,1,2)	0.000000	269.999939
A(2,1,3)	0.000000	269.999939
A(2,2,1)	0.000000	0.000000

A(2,2,2)	0.000000	-0.000049
A(2,2,3)	1.000000	0.000000
W(1,2)	0.388889	0.000000
W(1,3)	0.611111	0.000000
W(2,2)	1.000000	0.000000
W(2,3)	0.000000	0.000000
W(1,1)	0.000000	0.000000
W(2,1)	0.000000	0.000000

ROW SLACK OR SURPLUS DUAL PRICES

A(1,1,1)	0.000000	-590.400024
A(2,1,1)	0.000000	-1375.199951
W(1,1)	0.000000	0.000000
W(2,1)	0.000000	0.000000
6)	0.000000	1.200000
7)	70.000000	0.000000
8)	0.000000	0.000000

9)	10.000000	0.000000
I(1,1,1)	50.000000	0.000000
I(1,1,2)	50.000000	0.000000
I(1,1,3)	50.000000	0.000000
I(1,2,1)	50.000000	0.000000
I(1,2,2)	50.000000	0.000000
I(1,2,3)	50.000000	0.000000
I(2,1,1)	0.000000	0.000000
I(2,1,2)	0.000000	0.000000
I(2,1,3)	0.000000	0.000000
I(2,2,1)	50.000000	0.000000
I(2,2,2)	50.000000	0.000000
I(2,2,3)	50.000000	0.000000
22)	0.000000	0.000000
23)	300.000000	0.000000
24)	0.000000	0.000000
25)	0.000000	0.000000
26)	0.000000	0.000000

27)	340.000000	0.000000
28)	0.000000	0.000000
29)	0.000000	0.000000
30)	0.000000	0.000000
31)	0.000000	0.000000
32)	0.000000	0.000000
33)	140.000000	0.000000
34)	0.000000	0.000000
35)	0.000000	0.000000
36)	280.000000	0.000000

NO. ITERATIONS= 283

BRANCHES= 39 DETERM.= 1.000E 0