POST-EARTHQUAKE COLLAPSE PROGNOSTICATION OF STRUCTURAL SYSTEMS USING SPARSE RESPONSE DATA

A Thesis in
Civil Engineering

by

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ABSTRACT

In order to reduce the impact of major earthquakes on communities, a rapid assessment of the state of structural systems for the purpose of re-occupancy of safe buildings is necessary. However, this determination is typically carried out through time-consuming visual inspections. Structural health monitoring is a promising alternative but existing techniques require a dense array of instrumentation on the structure rendering them impractical for many reasons including prohibitive costs of installation, maintenance, and data management and processing. A methodology is proposed in this study that uses sparse response data, for example the accelerations at a few floor levels, to identify the state of nonlinear numerical models that are further used for prognosticating collapse under future seismic hazards. The utility of the proposed methodology is demonstrated using the data from the shake table tests performed on a 4-story scaled model. The methodology is able to successfully predict the observed collapse of the physical structure. Furthermore, it has been observed that the vulnerability to collapse following an earthquake is greatly increased even to lower intensity earthquakes due to the accumulation of residual deformations and stresses in the structure.
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Dedication

I dedicate this thesis to my father.
Chapter 1

Introduction

Background

During a major earthquake, some amount of damage to structures is unavoidable. The current seismic design philosophy permits structures to deform in-elastically. For example, ASCE 7-10 [1] incorporates the response modification coefficient $R$, which allows the design lateral strength to be lower than that required for the structure to remain in the elastic range. This results in the development of residual deformations and stresses leading to damage in the structure. A rapid assessment of the state of such damaged structural systems from a life safety perspective is necessary to reduce the socio-economic impact of major earthquakes on communities.

Currently, visual structural inspections [2-3] are performed to determine the safety of a structure for re-occupancy. Depending on the extent of the damage observed in the structure, colored placards are used to tag the structure as safe, limited-use or unsafe. However, tagging of structures through visual inspections is time consuming, subjective, qualitative, unreliable [4] and often presents hazardous working environments for the inspectors. Structural health monitoring (SHM) is a promising alternative approach for the assessment and prognosis of the safety of the structural systems. SHM involves the installation of sensors on the structure which trigger a notification when a damage threshold is exceeded. However, current SHM technologies rely upon a dense array of sensors on the structures to detect damage which is impractical for many reasons including the prohibitive costs of installation, maintenance, long term reliability of monitoring system, data management and processing. Recognizing these issues associated with obtaining complete response data, efforts have been made to identify linear systems using partial structural
response data [5-8] but no research exists for nonlinear systems. Therefore, a rapid and an automated state assessment procedure for nonlinear structural systems that requires minimal sensory information is needed.

**Research objective**

The goal of the current study is to develop and evaluate a methodology for the parameter estimation of nonlinear structural systems using sparse response data i.e. the response measured at a proper subset of the system’s degrees of freedom (DOFs) to be further used for the purpose of prognostication and reliability analysis. A potential benefit of the proposed methodology is that it facilitates an automated near real-time assessment of the vulnerability to collapse following an earthquake which can be used for better decision-making pertaining to the re-occupancy of safe buildings thereby contributing to a faster recovery of communities.

**Research scope**

The scope of this study is limited to structural systems exhibiting material nonlinearities during a major earthquake. A-prior information pertaining to the characterization of nonlinearity is assumed to be known. Two-dimensional, low-rise moment resisting frames which do not exhibit a soft-story behavior are studied. Side-sway collapse in which lateral displacements of floor levels cause significant P-Δ effects in the lateral force resisting systems is the type of collapse studied in the current research.
Chapter 2

Methodology

The proposed methodology for collapse prognostication using sparse response data is shown in Figure 2-1 and consists of three steps. The first step is to formulate a mathematical model of the physical structure and to obtain its response at those DOFs that will result in a unique identification of the structure. The second step is to estimate the nonlinear material parameters of the model for the purpose of state determination using the sparse response data and nonlinear optimization. The third step is to prognosticate the post-earthquake collapse probability using fragility analyses. A detailed explanation of each of these steps is presented in the following sections.

Figure 2-1. Flowchart of the proposed methodology.
**Step 1: Model formulation and selection of sparse response data**

The methodology for collapse prognostication begins with the model formulation. Based on the first principles of dynamics and from a knowledge of the boundary conditions, geometry and mechanical properties of the physical structure, a numerical model that is characterized by a finite set of parameters \( \{ p \} \) and degrees of freedom (DOFs) is formulated. For the purpose of collapse prognostication in particular, the model has to be numerically stable at the large values of deformations leading to collapse and should incorporate all the significant nonlinearities and modes of deterioration. It should be noted that the process of developing a mathematical model of a physical system is a complex problem and detailed discussion of it is outside the scope of this study.

The parameter estimates \( \{ \hat{p} \} \) of the formulated model are determined by fitting its response with that of the physical structure measured at limited locations. Due to the limited nature of the measured response data, the estimation process becomes ill conditioned leading to non-uniqueness of the parameters. A proper consideration of the location of the limited measured responses is critical to arrive at a unique identification. For example, Udwadia et al. [9] have shown that for a multi-story shear type building modeled as a coupled mass-spring-damper linear system, if the motion of the base and the resulting response of the roof are known, then it leads to multiple estimates of the parameters; all resulting in models having identical responses at the roof. On the other hand, the motion of the base and response of the mass at the first floor level has been shown to be necessary and sufficient to uniquely determine the stiffness and damping parameters for the entire structure. Although no such rigorous mathematical proofs currently exist for nonlinear systems, a heuristic numerical study detailed in Appendix A suggests the base motion along with the first and second floor responses might be sufficient to uniquely estimate the nonlinear material parameters.
Step 2: Parameter and state estimation using sparse response data

The estimated parameter vector \( \{ \hat{p} \} \) of the formulated model is obtained by minimizing a single objective function \( J \), which is defined as the sum of the squares of the errors \( e_{ij} \) between the simulated response history \( Y_{Sij} \) and the measured response history \( Y_{Mij} \) at the location of the limited degrees of freedom expressed as:

\[
J = \sum_{i=1}^{N_s} \sum_{j=1}^{N_t} \left( e_{ij}(p) \right)^2 = \sum_{i=1}^{N_s} \sum_{j=1}^{N_t} \left( Y_{Sij}(p) - Y_{Mij} \right)^2
\]

where \( N_s \) is the number of sensor locations and \( N_t \) is the number of data points in the response.

Equations 1 and 2 lead to the classical nonlinear least squares problem and there exist several approaches in the literature \([10-13]\) to solve this minimization problem. In this study, genetic algorithms (GAs) are employed to minimize the objective function of Equation 1. The advantage of using GAs is that they circumvent the computationally intensive Hessian and Jacobian matrix formulations present in the other approaches. Further, in the case of a discontinuous nonlinear objective function or unobservable states, estimation of the gradients, or the Hessian matrix, is not possible \([14]\). On the other hand, GAs lack a definitive proof that the final solution represents the true global optimum; although certain techniques such as regeneration have been exclusively developed to ensure genetic diversity and thus finding the global optimum. The present methodology utilizes a modified version of a GA developed by Perry et al. \([8]\) that was shown to be capable of parameter estimation for a linear structure with sparse response data. The primary adaptations to the Perry et al. GA include the usage of the full data length, a normalized fitness function and a penalty function which identifies and removes individuals that cause numerical
instabilities along with the omission of the search space reduction method. A flowchart illustrating the adapted Perry et al. GA is presented in Figure 2-2.

Figure 2-2. Flowchart of the genetic algorithm (adapted from Perry et al. [8]).
The best individual in species 1 at the end of the prescribed number of iterations is assumed to be the optimal estimate. A study aimed at evaluating the efficiency of this modified GA for the parameter estimation of shear type frames exhibiting material nonlinearities using only sparse response data at various levels of noise intensities is presented in Appendix A. The damaged state of the model is determined by subjecting the undeformed parameter estimated model to the ground excitation that was used in the estimation process. This damaged state of the model includes the residual deformations and residual stresses that are representative of the conditions of the physical structure due to the ground excitation. The damaged model can be compared with the undamaged model to locate the damage incurred due to the earthquake.

**Step 3: Collapse prognostication using fragility analyses**

Collapse prognostication is carried out by developing collapse fragility functions for the parameter estimated models of Step 2. A fragility function is defined as the conditional probability of exceedance of a limit state of the structure for a given intensity measure \( IM \) of ground excitation expressed as

\[
P(R > LS \mid IM = x) = \Phi \left( \frac{\ln(x/\theta)}{\beta} \right)
\]

where \( P(R > LS \mid IM = x) \) is the conditional probability of the structural response \( R \) exceeding a defined limit state \( LS \) due to a ground motion with \( IM = x \); \( \Phi() \) is the standard normal cumulative distributive function; \( \theta \) and \( \beta \) are the median and logarithmic standard deviations respectively. The limit state \( LS \) in this methodology is taken as the collapse limit state and the intensity measure \( IM \) is taken as the spectral acceleration at 5% damping and at the fundamental period of the structure \( S_a(T_1, 5\%) \) as it incorporates the characteristics of both the
ground motion and the structure. Collapse fragilities functions are determined using the multiple stripes analysis approach [15] with the aid of maximum likelihood method. The benefit of multiple stripes approach over other methods is that it does not require the incremental scaling of ground motions to high $IM$ values to determine the collapse fragilities [15]. The parameters of the fragility function in Equation 3 are estimated using the GA described in Step 2 by maximizing the logarithm of the likelihood function defined in [15] and given by

$$
\left\{ \hat{\theta}, \hat{\beta} \right\} = \arg \max_{\theta, \beta} \sum_{j=1}^{m} \ln \left( \frac{n_j}{z_j} \right) + z_j \ln \Phi \left( \frac{\ln \left( \frac{x_j}{\theta} \right)}{\beta} \right) + (n_j - z_j) \ln \left( 1 - \Phi \left( \frac{\ln \left( \frac{x_j}{\theta} \right)}{\beta} \right) \right)
$$

(4)

where $\left\{ \hat{\theta}, \hat{\beta} \right\}$ are the estimated fragility parameters, $m$ is the number of $IM$ levels, $z_j$ is the number of collapses observed out of the $n_j$ ground motions at $IM$ level $x_j$, and $\Phi()$ is the standard normal cumulative distributive function. With the multiple strip analysis approach, nonlinear response history analyses of the parameter estimated model and the damaged model are performed at discrete values of $IM$ levels. The ground motions at each $IM$ level are selected and scaled so that their mean and variance matches that of a target mean response spectrum, representing the seismic hazard scenario at that $IM$ level. The target mean response spectrum in the current methodology is chosen as the conditional mean spectrum (CMS) proposed by Baker [16] as other commonly used spectra such as the uniform hazard spectrum (UHS) are considered overly conservative. Based on the location of the physical structure and the condition of the surrounding soil, seismic hazard curves derived from probabilistic seismic hazard analysis (PSHA) [17] that relate the annual frequency of exceedance of earthquakes at different spectral accelerations are obtained from [18]. With this information and employing Campbell and Bozorgnia [19] ground motion prediction model, hazard deaggregation is performed using [20] to compute conditional mean spectrum. An algorithm developed by Jayaraman et al. [21] is used to
select and scale ground motions to match the CMS mean and variance. Nonlinear dynamic analyses due to these ground motions are performed on the damaged model and the resulting number of collapses are determined. The fraction of collapses at each IM level are fitted to the lognormal distribution to obtain the collapse fragility functions.
Chapter 3

Application

To illustrate the utility of the proposed methodology and to evaluate the results, data from the nonlinear response of a structural system subjected to ground excitations until collapse is required. So, the publicly available data from shake table tests performed by Lignos et al. [22] at the Network for Earthquake Engineering Simulation (NEES) facility at The State University of New York at Buffalo along with the data from component testing at Stanford University is selected from the NEES Project Warehouse [22] in this study. The Lignos et al. tests consisted of two nominally identical 4-story, 2-bay, 1:8 scaled aluminum test frames, which are subjected to a series of ground motions with increasing intensities until collapse. These test frames are based on a 4-story steel prototype frame that was designed in accordance with the IBC [23], AISC [24] and FEMA-350 [25] provisions.

Overview of experimental setup

The experimental setup of the structural system is shown in Figure 3-1 which consists of two substructures. A test frame which provides lateral resistance and a gravity load simulator which introduces P-Δ moments into the test frame. The substructures are interconnected by axially rigid pinned links at all the floor levels. Rectangular steel plates at every floor level supported by rigid pinned links form the gravity load simulator. The test frame is an aluminum mechanism that is designed to remain elastic throughout the various testing phases. It is held in position by plastic hinge elements consisting of steel coupons. A detail of the steel coupon is
shown in Figure 3-1c). The dimensions of these steel coupons are adjusted accordingly so as to match the scaled rotational characteristics of the prototype frame. Further information regarding the experiment can be found in [26].

![Figure 3-1. Illustration showing the a) Test frame b) Plastic hinge element and c) Steel coupons [26].](image)

**Numerical model development**

As per Step 1 of the methodology, a finite element model of the physical test frame is developed in an Open System for earthquake engineering simulation software platform (OpenSees) [27]. The schematic of the model is shown in Figure 3-2. The test frame is modeled as a 2D plane frame using centerline dimensions and is assumed to be rigidly connected to the
shake table. The P-Δ effects due to the gravity load simulator are modeled using an axially rigid leaning column that is connected to the adjacent frame with axially rigid truss elements. Lateral mass and gravity loads are applied on the leaning columns at each floor level. All nodes in a particular floor level are constrained to have equal horizontal displacements. All beams, columns and other connecting aluminum members are modeled as elastic beam-column elements, whose mechanical properties are based on those of Aluminum 2024 and dimensions based on the section sizes reported in [26]. Four different coupons widths were used in the test frame resulting in four different plastic hinge types.

![Diagram of numerical model showing locations of plastic hinges and frictional hinges.](image)

Figure 3-2. Illustration of the numerical model showing the locations of different plastic hinges and frictional hinges.

Plastic hinges are modeled using zero-length rotational spring elements which follow a bilinear hysteretic response based on the modified Ibarra Krawinkel deterioration model [28-31], which has been shown in [29] to replicate global collapse mechanisms. Figure 3-3 illustrates the modified Ibarra Krawinkel deterioration model showing the backbone curve and the various modes of cyclic deterioration. The parameters required to numerically describe this behavior are obtained from the various component tests performed at the John A. Blume Earthquake Engineering Center at Stanford University [22, 26, 29].
The elastic rotational stiffness $K_e$ and yield moment $M_y$ of the modified Ibarra Krawinkler deterioration model of all the plastic hinges are treated as the unknown material parameters for the current testing conditions which are to be estimated using the sparse response data. The experimental estimates of the strain hardening ratio $a_s$ are very low which is indicative of a perfectly plastic behavior of the plastic hinges in the shake table tests. So its effect on the global response of the frame is considered insignificant and are therefore treated as known constants.

It has been reported by Lignos et al. in [29] that the response of the test frame decayed rapidly during pulse type tests due to friction in the vertical pinned links of the gravity load simulator. Because of the significant contribution of the friction to the overall damping in the system, the energy dissipation through friction was modeled by incorporating frictional hinges on the ends of the leaning column at every floor level as shown in Figure 3-2. They consist of zero-length rotational springs with a rigid perfectly-plastic behavior as shown in Figure 3-4. A moment equal to the yield moment is required to overcome the frictional resistance and allow relative
Details pertaining to the estimation of these yield moments have been presented in Appendix B. In additional to the frictional elements, 3% Rayleigh damping was specified at the first and second modes of vibration to account for all sources of viscous damping in the aluminum frame.

![Diagram](image)

**Figure 3-4. Illustration of the rigid perfectly-plastic behavior of frictional hinges.**

**Selection of sparse response data**

For a successful parameter estimation, a ground excitation that results in inelastic behavior in all the plastic hinges must be selected. It is observed that the application of the design level earthquake (DLE) to the test frame has not resulted in inelastic deformations in all the plastic hinges. Thus, the subsequent higher intensity excitation which is the maximum considered earthquake (MCE) is concatenated to the DLE to obtain a single ground motion that serves as the input to the numerical model for the purpose of parameter estimation. The corresponding responses observed at the first and the second floor levels of the test frame along with the concatenated ground excitation are considered as the sparse response data.
Parameter estimation of the numerical model using sparse response data

Step 2 of the methodology involves parameter and state estimation using the sparse response data. The initial state of the numerical model is taken as the undeformed state as no significant residual deformations were observed in the test frame at the beginning of the DLE test [26]. Parallel implementations of the GA in MATLAB [32] and the numerical model in OpenSees which are capable of simultaneous execution on multiple processors have been exclusively developed to reduce the computational time. Although the GA and the numerical model are compiled in different programming environments, the resulting data exchange gap has been bridged using ASCII files. Table 3-1 summarizes the various GA parameters employed for the parameter estimation.

<table>
<thead>
<tr>
<th>GA parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial population</td>
<td>3 X 35 individuals per species</td>
</tr>
<tr>
<td>Total number of generations</td>
<td>150</td>
</tr>
<tr>
<td>Cross over rate</td>
<td>0.8</td>
</tr>
<tr>
<td>Mutation rate</td>
<td>0.4</td>
</tr>
<tr>
<td>Migration rate</td>
<td>0.05</td>
</tr>
<tr>
<td>Number of regenerations</td>
<td>10</td>
</tr>
<tr>
<td>Number of reintroductions</td>
<td>15</td>
</tr>
</tbody>
</table>

During the optimization routine, it has been observed that certain combinations of parameters lead to dynamically unstable models resulting in numerical non-convergence. This is due to locally formed mechanisms leading to partial collapse of the frame. All such parameter sets do not represent the physical structure and are therefore assigned a fitness value of zero to prevent their passage into subsequent generations. This is computationally achieved by appending a penalty function operator to the GA which searches and removes individuals with zero fitness prior to reproduction. The final parameter estimates for the four different plastic types upon the completion of the optimization routine is summarized in Table 3-2. It can be observed that the parameters estimated by the GA are close to the expected experimental estimates with a
maximum error of 21%. Yield moments are estimated more accurately than the elastic rotational stiffness likely because of the higher sensitivity of global response to the changes in yield moments.

Table 3-2. Summary of GA estimated plastic hinge parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Plastic hinge type</th>
<th>Lower search limit</th>
<th>Upper search limit</th>
<th>GA estimate</th>
<th>Experimental estimate</th>
<th>Percentage error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic rotational stiffness $K_e$ (kip-in/ rad)</td>
<td>1</td>
<td>10</td>
<td>22000</td>
<td>11800</td>
<td>11200</td>
<td>5.36</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>10</td>
<td>26000</td>
<td>11300</td>
<td>13000</td>
<td>-13.1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10</td>
<td>41000</td>
<td>24000</td>
<td>20600</td>
<td>16.5</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>10</td>
<td>50000</td>
<td>22400</td>
<td>25700</td>
<td>-12.8</td>
</tr>
<tr>
<td>Yield moment $M_y$ (kip-in)</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td>15.1</td>
<td>14.2</td>
<td>6.34</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td>40</td>
<td>19.4</td>
<td>19.8</td>
<td>-2.02</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1</td>
<td>50</td>
<td>21.4</td>
<td>27.0</td>
<td>-20.7</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1</td>
<td>70</td>
<td>37.9</td>
<td>34.5</td>
<td>9.86</td>
</tr>
</tbody>
</table>

Table 3-3 shows a comparison of natural periods between the physical test frame and a numerical model formulated using OpenSees with the GA estimated plastic hinge parameters. The natural periods of the numerical model agree well with the test frame values.

Table 3-3. Comparison of natural periods.

<table>
<thead>
<tr>
<th>Vibrational mode</th>
<th>Experimental test frame [33]</th>
<th>Numerical frame formulated with GA estimated parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.470</td>
<td>0.481</td>
</tr>
<tr>
<td>2</td>
<td>0.140</td>
<td>0.145</td>
</tr>
</tbody>
</table>

Figure 3-5 shows the comparison of acceleration histories of the test frame measured in the experiment with that of the numerical model. The responses are in phase and the amplitudes agree well with each other. Not only do the responses of floor levels 1 and 2 agree (which were the only structural responses available for parameter estimation) but also the responses of floor levels 3 and 4 match as well with the test frame response, indicating a successful parameter estimation of all the plastic hinge types. The deformed state is obtained by subjecting a numerical model formulated with the GA estimated plastic hinge parameters to DLE and MCE ground motions.
Figure 3-5. Comparison of the test frame response with the estimated model response.
Collapse prognostication of the test frame

The 4-story prototype structure which serves as the basis for the scaled test frame was designed for a site in Los Angeles assuming soil type D [29]. For this site, seismic hazard curves were generated for the time periods of 1.0s and 2.0s from [18] which were interpolated in the log space to the scaled time period of the parameter estimated model as shown in Figure 3-6.

![Figure 3-6. Interpolated seismic hazard curve for the parameter estimated structure.](image)

Hazard deaggregation from [20] is only available for discrete values of exceedance probabilities. So the numerical values of spectral accelerations are chosen such that their annual frequency of exceedance from the hazard curve in Figure 3-6 corresponds to these discrete exceedance values. For example, spectral acceleration of 0.323g is considered as its annual frequency of exceedance from Figure 3-6 is 0.005017 which corresponds to the exceedance probability of 10% in 21 years, a value for which hazard deaggregation is readily available in [20]. Suites of 20 ground motions
for each spectral acceleration value are obtained using [21] such that their mean and variance matches that of the conditional mean spectrum determined from the hazard deaggregation. All ground motions are also scaled temporally to account for similitude laws. Earlier studies involving collapse assessment [34-36], have considered the maximum inter story drift ratio at the incipient of collapse to range from 7% to 10% for steel frame buildings. For this study, collapse is assumed to have occurred if the inter story drift ratio in any of the stories exceeds 8%.

Fragility analyses were performed on the deformed states of the parameter estimated model resulting from the application of DLE and MCE to study the change in the vulnerability to collapse due to residual deformations and stresses. It was observed for these deformed states that certain ground motions which sway the frame in the same direction as the residual displacements cause collapse but when applied with their direction reversed help stabilize the model by reducing the inter story drift ratio. This is due to the bias generated in the model resulting from the residual deformations. For example, Figure 3-7 shows the variation of inter story drift ratio obtained by applying the 1999 Chi-Chi earthquake to the deformed state of model after DLE. When the ground motion was applied in the positive X direction it causes collapse while the application in the negative X direction results in a non-collapse indicating that the direction of a ground motion is an important consideration for collapse assessment of structures. Due to the lack of a detailed geophysical model of the site surrounding the prototype structure, it is assumed that the ground motions are equally likely to occur in the positive and negative X directions and a collapse due to the ground motion application in either of these directions is considered as collapse.
Table 3-4 summarizes the collapse fragility function parameters for the model at various levels of residual deformations. These parameters were obtained by maximizing Equation 3 using the GA discussed in Chapter 2. The resulting fragility functions along the fraction of the collapses observed in the multiple stripes analysis are shown in Figure 3-8. It can be seen that the fragility functions have shifted significantly towards lower values of spectral accelerations for the models with residual deformations indicating an increase in their vulnerability to collapse even at lower intensities of ground motions under future seismic hazards. Collapse of the physical test frame in the experiment is reported to occur for the deformed model after MCE at a spectral acceleration value of 1.08g which agrees well with the obtained fragility functions of Figure 3-8. Thus the methodology was able to successfully predict the observed collapse of the structure.

<table>
<thead>
<tr>
<th>Model ID</th>
<th>Description of the model</th>
<th>Fragility function parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Undeformed model</td>
<td>Median θ</td>
</tr>
<tr>
<td>1</td>
<td>Undeformed model</td>
<td>1.38</td>
</tr>
<tr>
<td>2</td>
<td>Deformed model after design level earthquake</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>Deformed model after maximum considered earthquake</td>
<td>0.31</td>
</tr>
</tbody>
</table>
Figure 3-8. Illustration showing the collapse fragility functions of the model at various levels of residual deformations and residual stresses.
Chapter 4

Summary and Conclusions

Some amount of damage to structures during a major earthquake is unavoidable. In order to reduce its impact on the community a rapid and an automated approach for the assessment of the damaged state of the structure that uses minimal sensory data is necessary. In this regard, a methodology for the collapse prognostication of structures following an earthquake event that uses the vibrational response of a few floor levels i.e. sparse response data was proposed. The methodology seeks to expedite the decision making regarding re-occupancy of safe buildings thereby contributing to the faster recovery of communities. The methodology utilizes the framework of nonlinear parameter estimation via genetic algorithms and fragility function development from multiple stripes analysis for collapse prognostication. To demonstrate the feasibility of the methodology, data from shake table tests and component tests performed by Lignos et al. [22] on scaled 4-story frame that was subjected to sequential ground motions of increasing intensity until collapse is used. Based on the observations made during the application of the proposed methodology to the data set, the following conclusions are offered:

1. The methodology is shown to rapidly estimate the nonlinear material parameters using sparse response data. The resulting parameter estimates agree well with the experimental estimates with a maximum error of 20%.

2. The methodology is shown to estimate the post-earthquake damaged state using the parameter estimated model and capture the near real-time change in the collapse fragilities.

3. The methodology is able to successfully prognosticate the observed collapse of the physical structure using its sparse response data.
4. The capacity of the structure against collapse decreases significantly with the increase in residual deformations leading to a higher probability of collapse at even moderate intensity future earthquakes.

5. The direction of the ground motion is important for the collapse assessment of structures having residual deformations arising from previous earthquakes or fabrication/construction errors because of the bias generated by the residual deformations.

**Future work**

This research has shown that sparse response data such as the acceleration response histories of a few floor levels might be sufficient to prognosticate the collapse of structural systems. Collapse prognostication is however strongly dependent on the accuracy of the collapse assessment procedure. Rigorous mathematical collapse assessment procedures can be developed and incorporated in the methodology instead of the existing quasi-collapse indicators such as peak inter-story drift ratio limit or peak roof drift ratio limits. While the proposed methodology is applied to the collapse limit state of the structure, other performance limit states which correspond to intermediate levels of damage can be studied. The current work can be extended to higher fidelity 3-dimensional models which better represent the structural systems improving the accuracy of collapse prognostication. A mathematical approach that assists in the selection of the DOFs at which the sparse response data is measured can be developed to replace the existing heuristic approach. The effect of the uncertainty in estimating the parameters on the collapse prognostication of structures can also be studied.
Appendix A

Validation and performance evaluation of modified genetic algorithm

This appendix details the validation and performance evaluation of the modified GA explained in Chapter 2 with respect to the parameter estimation of nonlinear structural systems using sparse response data. A parallel version of the GA which is capable of simultaneous execution on multiple processor cores was implemented in MATLAB [32] to reduce the computational time of the optimization routine. The numerical response of an ideal shear building model as shown in Figure A-1 exhibiting material nonlinearities resulting from a ground excitation was used in the study. In shear building models, masses are lumped at the respective floor levels which act as rigid diaphragms having horizontal translational degrees of freedom.

Figure A-1. Illustration of the 4 story shear building model showing the system’s DOFs and the ground excitation.
Inter story columns connecting various floor levels provide the necessary restoring forces that are developed due to the relative movement of floor levels. The equation of motion for this type of structural system when subjected to a horizontal ground acceleration \( \ddot{u}_g \) is given by

\[ [M]\{\ddot{u}\} + f(\dot{u}, u) = -[M]\{1 \ 1 \ 1\}^T \ddot{u}_g \]  \hspace{1cm} (5)

where \([M]\) is the diagonal mass matrix given by

\[ [M] = \begin{bmatrix}
  m_1 & 0 & 0 & 0 \\
  0 & m_2 & 0 & 0 \\
  0 & 0 & m_3 & 0 \\
  0 & 0 & 0 & m_4 \\
\end{bmatrix} \]  \hspace{1cm} (6)

wherein \(m_i; \ i=1,2,3,4\) is the lumped mass of the \(i\)th floor as summarized in Table A-1; \(\{\ddot{u}\}\) is the relative acceleration of the floor levels with respect to the ground and \(f(\dot{u}, u)\) is the nonlinear restoring force developed in the columns which is assumed to be of the form

\[ f(\dot{u}, u) = [C]\{\dot{u}\} + f_w \]  \hspace{1cm} (7)

where \(\{\dot{u}\}\) is the relative velocity of the floor levels with respect to the ground; \([C]\) is the time invariant Rayleigh damping matrix given by

\[ [C] = a_0[M] + a_1[K_e] \]  \hspace{1cm} (8)

\(a_0\) and \(a_1\) are the Rayleigh damping coefficients given by

\[ a_0 = \frac{2\zeta \omega_1 \omega_2}{(\omega_1 + \omega_2)} \]  \hspace{1cm} (9)

\[ a_1 = \frac{2\zeta}{(\omega_1 + \omega_2)} \]  \hspace{1cm} (10)

where \(\zeta\) is the viscous damping ratio, assumed to be 3% of critical damping for the first two modes of vibration and \(\omega_1, \omega_2\) are the circular natural frequencies of the first and second modes respectively. \(K_e\) is the initial elastic stiffness matrix of the shear building given by
where \( k_i; i=1, 2, 3, 4 \) is the elastic stiffness of the \( i \)th story as given in Table A-1.

\[
\begin{bmatrix}
 k_{e1} + k_{e2} & -k_{e2} & 0 & 0 \\
 -k_{e2} & k_{e2} + k_{e3} & -k_{e3} & 0 \\
 0 & -k_{e3} & k_{e3} + k_{e4} & -k_{e4} \\
 0 & 0 & -k_{e4} & k_{e4}
\end{bmatrix}
\]

(11)

Table A-1 Summary of numerical model parameters.

<table>
<thead>
<tr>
<th>Level</th>
<th>Mass ( m ) (kip-s^2/in)</th>
<th>Story stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Elastic stiffness ( k_e ) (kip/in)</td>
<td>Yield story shear force ( V_y ) (kip)</td>
</tr>
<tr>
<td>Floor 1</td>
<td>100</td>
<td>800</td>
</tr>
<tr>
<td>Floor 2</td>
<td>100</td>
<td>800</td>
</tr>
<tr>
<td>Floor 3</td>
<td>100</td>
<td>500</td>
</tr>
<tr>
<td>Roof</td>
<td>50</td>
<td>500</td>
</tr>
</tbody>
</table>

\( f_{sp} \) is the nonlinear spring force acting at a floor level, given by

\[
f_{sp} = \begin{bmatrix} f_1, f_2, f_3, f_4 \end{bmatrix}^T = \{(V_1 - V_2), (V_2 - V_3), (V_3 - V_4), (V_4)\}^T
\]

(12)

where \( V_i; i=1, 2, 3, 4 \) are the inter story shear forces developed in the columns which are modeled using a bilinear isotropic hardening hysteretic spring whose behavior is shown in Figure A-2. It can be observed that the elastic stiffness \( k_e \), yield story shear force \( V_y \) and the inelastic stiffness \( k_p \) are necessary and sufficient to uniquely characterize the hysteresis behavior. This model is subjected to the unscaled North-South component of the May 18th, 1940 El Centro earthquake to ensure that the nonlinearities at all the floor levels are adequately excited. The nonlinear response of the model is computed using the iterative Newton-Raphson method and Newmark’s average acceleration method.
Figure A-2. Illustration of the bilinear isotropic hardening hysteresis model.

For the purpose of validating the GA in terms of estimating the parameters using sparse response data; elastic stiffness $k_e$, yield story shear force $V_y$ and the inelastic stiffness $k_p$ shown in Figure A-2 of each story are assumed to be the unknown parameters. The problem statement would then be to estimate these quantities from only the acceleration time histories of the ground (system input), first and second floors (system response). Further, in order to evaluate the performance of the GA, parameter estimation is carried out initially using noise free data and subsequently with progressively increasing noise levels. The approach followed by Distenfano and Rath in [37] is used to corrupt the acceleration time histories for its relatively simplistic approach. Accordingly, if $x$ be an uncorrupted signal, then its corresponding corrupted signal $\bar{x}$ is obtained as

$$\bar{x} = x \times (1 + r)$$

(13)

where $r$ is a uniformly distributed random variable in the interval $(-\alpha, +\alpha)$. Three values of $\alpha = 0, \alpha = 0.1$ and $\alpha = 0.2$ are considered as the noise intensities. The search limits for all the parameters are considered to be from half of the true parameter value to twice the true parameter...
value. Various GA parameters used in the identification process are summarized in Table A-2. The parameter estimates thus obtained from the GA for various noise levels are summarized in Table A-3.

Table A-2 Summary of GA parameters for the validation and performance evaluation study.

<table>
<thead>
<tr>
<th>GA parameter</th>
<th>Numerical value</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial population</td>
<td>3 X 35 individuals per species</td>
<td></td>
</tr>
<tr>
<td>Total number of generations</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Cross over rate</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Mutation rate</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>Migration rate</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Number of regenerations</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Number of reintroductions</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

It can be observed from the GA estimated parameters that the determination of elastic stiffness and yield story shear force is excellent while that of the inelastic stiffness is relatively poor. Nevertheless, the GA performs excellently as the parameter estimates are off by only a maximum of 5.1% ($\alpha = 0$). The reason for this discrepancy is that the sensitivity of the limited observed responses is low to the changes in the inelastic stiffness values for the given level of earthquake intensity. With the introduction of noise, this effect is pronounced leading to substantial errors in the inelastic stiffness estimates though the other parameters are still estimated to a reasonably good accuracy with a maximum error of 4% and -10.3% for ($\alpha = 0.1$) and ($\alpha = 0.2$) respectively. Thus the GA is able to successfully estimate the parameters using only the sparse response data and performance of the GA under moderate noise corruption levels, similar to those observed in laboratory data acquisition systems is good.
## Table A-3 Summary of the performance of the GA at various noise levels

<table>
<thead>
<tr>
<th>Stiffness parameter</th>
<th>Level</th>
<th>Minimum search limit</th>
<th>Maximum search limit</th>
<th>True value</th>
<th>GA estimate</th>
<th>Error percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \alpha = 0 )</td>
<td>( \alpha = 0.1 )</td>
</tr>
<tr>
<td>Elastic stiffness ( k_e ) (kip/in)</td>
<td>Story 1</td>
<td>400</td>
<td>1600</td>
<td>800</td>
<td>800.25</td>
<td>803.80</td>
</tr>
<tr>
<td></td>
<td>Story 2</td>
<td>400</td>
<td>1600</td>
<td>800</td>
<td>796.33</td>
<td>793.63</td>
</tr>
<tr>
<td></td>
<td>Story 3</td>
<td>250</td>
<td>1000</td>
<td>500</td>
<td>499.62</td>
<td>497.32</td>
</tr>
<tr>
<td></td>
<td>Story 4</td>
<td>250</td>
<td>1000</td>
<td>500</td>
<td>500.46</td>
<td>490.07</td>
</tr>
<tr>
<td>Yield story shear force ( V_y ) (kip)</td>
<td>Story 1</td>
<td>600</td>
<td>2400</td>
<td>1200</td>
<td>1196.19</td>
<td>1154.64</td>
</tr>
<tr>
<td></td>
<td>Story 2</td>
<td>600</td>
<td>2400</td>
<td>1200</td>
<td>1208.19</td>
<td>1217.77</td>
</tr>
<tr>
<td></td>
<td>Story 3</td>
<td>350</td>
<td>1400</td>
<td>700</td>
<td>683.13</td>
<td>728.08</td>
</tr>
<tr>
<td></td>
<td>Story 4</td>
<td>350</td>
<td>1400</td>
<td>700</td>
<td>689.51</td>
<td>714.54</td>
</tr>
<tr>
<td>Inelastic stiffness ( k_p ) (kip/in)</td>
<td>Story 1</td>
<td>20</td>
<td>80</td>
<td>40</td>
<td>41.58</td>
<td>46.61</td>
</tr>
<tr>
<td></td>
<td>Story 2</td>
<td>20</td>
<td>80</td>
<td>40</td>
<td>41.63</td>
<td>46.51</td>
</tr>
<tr>
<td></td>
<td>Story 3</td>
<td>12</td>
<td>50</td>
<td>25</td>
<td>26.28</td>
<td>30.60</td>
</tr>
<tr>
<td></td>
<td>Story 4</td>
<td>12</td>
<td>50</td>
<td>25</td>
<td>24.73</td>
<td>30.58</td>
</tr>
</tbody>
</table>
Appendix B

Modeling of frictional damping

It has been reported by Lignos et al. in [29] that the response of the test frame decayed rapidly due to considerable friction at the spherical hinges of the gravity links supporting the mass plates. Based on pulse type tests, an equivalent viscous damping of 15% was obtained. Analytically simulating such a high value of viscous damping is not reasonable and so a 3% Rayleigh damping at the first and second modes in conjunction with rotational frictional elements is used in the current study. This appendix details the modeling of these frictional elements which were added to the leaning column of the numerical model at every floor level as shown in Figure 3-2. The frictional elements are modeled as zero-length rotational springs with rigid perfectly plastic behavior as shown in Figure 3-4. It can observed that the yield moment $M_y$ is necessary and sufficient to characterize the behavior. For the purpose of determining the yield moments of the frictional elements, the acceleration time histories of all the floor levels of the test frame from the application of service level earthquake (SLE) is employed. Experimental estimates for the rotational characteristics of the plastic hinge elements as obtained from the component tests [22] and summarized in Table 3-1 were used in the numerical model. The GA described in Chapter 2 is used to estimate the yield moments by minimizing the objective function of Equation 1 and the results obtained through this optimization routine are summarized in Table B-1. It is assumed that the values of these yield moments remain constant for all the subsequent ground motion intensities, namely the design level earthquake (DLE) and maximum considered earthquake (MCE), which shall be used for parameter estimation using sparse response data.
Table B-1. Summary of frictional yield moments ($M_y$) estimated by the GA

<table>
<thead>
<tr>
<th>No.</th>
<th>Location of frictional element</th>
<th>Lower limit (kip-in/rad)</th>
<th>Upper limit (kip-in/rad)</th>
<th>GA estimate (kip-in/rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Top of story 1</td>
<td>0</td>
<td>10</td>
<td>6.3</td>
</tr>
<tr>
<td>2</td>
<td>Bottom of story 2</td>
<td>0</td>
<td>10</td>
<td>4.8</td>
</tr>
<tr>
<td>3</td>
<td>Top of story 2</td>
<td>0</td>
<td>10</td>
<td>1.9</td>
</tr>
<tr>
<td>4</td>
<td>Bottom of story 3</td>
<td>0</td>
<td>10</td>
<td>2.0</td>
</tr>
<tr>
<td>5</td>
<td>Top of story 3</td>
<td>0</td>
<td>10</td>
<td>3.4</td>
</tr>
<tr>
<td>6</td>
<td>Bottom of story 4</td>
<td>0</td>
<td>10</td>
<td>3.4</td>
</tr>
<tr>
<td>7</td>
<td>Top of story 4</td>
<td>0</td>
<td>10</td>
<td>0.9</td>
</tr>
</tbody>
</table>
Appendix C

Effect of sparse response data on the identifiability of model parameters

This appendix details a numerical study on the identifiability of nonlinear parameters based on the amount of available sparse response data. This study is aimed at answering the question of whether more amount of response data shall lead to an asymptotic improvement in the estimation of the model parameters. This study is carried by using the finite element model of the test frame formulated in OpenSees. Details regarding the model formulation can be found in Chapter 3. In this study, the elastic rotational stiffness $K_e$ and yield moment $M_{yp}$ of the plastic hinge elements along with yield moments $M_{yf}$ of the frictional elements are assumed to be the unknown parameters. A ground motion that is obtained by concatenating the maximum considered earthquake with the design level earthquake which adequately excites the nonlinearities in all the plastic hinges is chosen as the input to the system. Different combinations of the resulting responses measured at the floor levels along with the concatenated ground motion are used to estimate the unknown parameters. The modified GA described in Chapter 2 is used for the optimization process. Each optimization problem is run for 75 generations and the resulting parameter estimates are compared.

The estimates of the parameters thus obtained are summarized in Table C-1 and the resulting errors are depicted in Figure C-1. It can be observed that the errors in estimating the elastic rotational stiffness for Type 1, 2 and 4 plastic hinges along with the yield moment estimates for Type 3 and 4 are significantly high when only floor 1 response is known. This could be due to the insensitivity of floor 1 response to variation of these parameters. With further data from the response of floor 2, the errors decrease rapidly for most of the parameters. Type 2 plastic hinges, which are located in the beams at floors 1 and 2 have a significant decrease in the errors potentially due to their higher contribution to floor 1 and 2 responses.
Table C-1. Summary of parameter estimates for different combinations of sparse response data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Hinge type/ID</th>
<th>Experimental estimate</th>
<th>Combining of sparse response data</th>
<th>Ground motion &amp; Floor 1</th>
<th>Ground motion &amp; Floor 1, 2</th>
<th>Ground motion &amp; Floor 1, 2, 3</th>
<th>Ground motion &amp; Floor 1, 2, 3, 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>GA estimate % error</td>
<td>GA estimate % error</td>
<td>GA estimate % error</td>
<td>GA estimate % error</td>
<td>GA estimate % error</td>
</tr>
<tr>
<td>Elastic rotational</td>
<td>1</td>
<td>11200</td>
<td>6720 -40.0</td>
<td>12600 12.5</td>
<td>15000 33.9</td>
<td>8220 -26.6</td>
<td></td>
</tr>
<tr>
<td>stiffness $K_e$ (kip-in/rd)</td>
<td>2</td>
<td>13000</td>
<td>20000 53.8</td>
<td>12600 -3.1</td>
<td>11500 -11.5</td>
<td>12400 -4.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>20600</td>
<td>21400 3.9</td>
<td>19300 -6.3</td>
<td>15400 -25.2</td>
<td>18200 -11.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>25700</td>
<td>39700 54.5</td>
<td>20600 -19.8</td>
<td>21845 -15.0</td>
<td>25100 -2.3</td>
<td></td>
</tr>
<tr>
<td>Yield moment of</td>
<td>1</td>
<td>14.2</td>
<td>15.5 9.3</td>
<td>15.0 5.6</td>
<td>15.1 6.6</td>
<td>14.3 1.0</td>
<td></td>
</tr>
<tr>
<td>plastic hinge $M_{y_p}$</td>
<td>2</td>
<td>19.8</td>
<td>27.6 39.4</td>
<td>18.5 -6.8</td>
<td>17.2 -12.9</td>
<td>17.6 -11.0</td>
<td></td>
</tr>
<tr>
<td>(kip)</td>
<td>3</td>
<td>27.0</td>
<td>15.7 -41.9</td>
<td>35.3 30.7</td>
<td>35.1 29.9</td>
<td>34.5 27.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>34.5</td>
<td>35.9 4.1</td>
<td>37.2 7.9</td>
<td>36.4 5.6</td>
<td>32.6 -5.6</td>
<td></td>
</tr>
<tr>
<td>Yield moment of</td>
<td>1</td>
<td>-</td>
<td>4.2 -</td>
<td>6.2 -</td>
<td>7.8 -</td>
<td>7.8 -</td>
<td></td>
</tr>
<tr>
<td>frictional element $M_{y_f}$ (kip)</td>
<td>2</td>
<td>-</td>
<td>2.7 -</td>
<td>2.4 -</td>
<td>2.1 -</td>
<td>1.8 -</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-</td>
<td>0.2 -</td>
<td>1.0 -</td>
<td>2.1 -</td>
<td>3.6 -</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-</td>
<td>0.8 -</td>
<td>3.1 -</td>
<td>2.6 -</td>
<td>1.7 -</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>-</td>
<td>3.6 -</td>
<td>1.0 -</td>
<td>1.7 -</td>
<td>2.1 -</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>-</td>
<td>3.4 -</td>
<td>2.7 -</td>
<td>1.2 -</td>
<td>2.4 -</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>-</td>
<td>3.6 -</td>
<td>0.9 -</td>
<td>1.4 -</td>
<td>0.3 -</td>
<td></td>
</tr>
</tbody>
</table>
However, by adding floor 3 response in the estimation process, we observe an increase in the errors in the elastic rotational stiffness with the exception of Type 4 hinge. Only marginal fluctuations in the yield moment estimates are observed. Further addition of floor 4 response improves the estimated values for all the parameters but no better than the estimates with only floor 1 and 2 responses. The potential reason could be that increase in the amount of data results in slower convergence of the objective function thereby requiring more generations to find the optimal solution. Further analyses where the GA is run for longer durations is necessary which may prove that the estimation process improves asymptotically with the available sparse response data. Also other combinations of floor responses which are not considered in this study such as floor 1 and 3; floor 1 and 4 can be used to study the identifiability of parameters.

Figure C-1. Variation in the identifiability of a) elastic rotational stiffness b) yield moment of plastic hinge elements for different combinations of sparse structural data.
Appendix D

Modified genetic algorithm source code

This appendix contains the source code of the modified genetic algorithm that was implemented in MATLAB environment which is capable on simultaneous execution on multiple cores.

*initial_population_OpenSees.m*

```matlab
%% editing notes
% initial_population_OpenSees.m
% parallel run with OpenSees trials
% reintroduction strategy is altered
% rouge combinations are removed in fitness operator
% generation average fitness value is calculated and appended to the best solution

%% Files needed to run this file
% crossover_operator.m
% migration_operator.m
% mutation_operator_2OpenSees.m
% mutation_operator_3OpenSees.m
% mutation_operator_4OpenSees.m
% fitness_operator_with_rank_OpenSees.m
% fitness_operator_with_rank_OpenSees_parallel.m
% parsave.m
% reintroduction_operator.m
% selection_operator_OpenSees_based_on_score.m
% selection_operator_OpenSees_based_on_rank.m

%% user input
DOF=4;  % DOF is the number of node outputs requested by the
        % recorders, used to reshape the Acce.out file
variables=13;  % total # of unknown parameters
```
% search limits (minimum values in row1; % maximum values in row2)
global_search_limits=[0 0 0 0 0 0 0 0 0 0 0 0; 22000 0.02 30 26000 0.02 40 41000 0.02 50 50000 0.02 70 0.8];

% enter the DOF at which response is known (row vector please)'
obs_sys_DOF=[1 2];

% selection strategy [enter 'rank' or 'score']
selection_strategy='rank';

%% file name to save (saves as *.mat in the current directory)
% change last line

%% Shift observed response
% It is observed that OpenSees response is shifted by one time step. So to % be computationally efficient, shift the observed response rather than % shifting all OpenSees responses

%% create a matrix A_obs_shifted with column == DOF % each column is the observed response at that DOF

%% rate inputs
individuals_in_each_species=35;
gen_num_max=150; % change regen, reintro accordingly
cross_rate=0.8;
muta_rate=0.2;
gen_between_regen=25;
migr_rate=0.05;
% SSRM_ratio=0.75; % SSRM is evaluated when gen_num==SSRM_ratio X gen_num_max
reintro_gen_num=9;
regen_gen_num=gen_between_regen;

tic

%% generation of random initial population
S1=[];
S2=zeros(individuals_in_each_species,variables);
S3=zeros(individuals_in_each_species,variables);
S4=zeros(individuals_in_each_species,variables);

% populating Species 2
for i=1:individuals_in_each_species
for j=1:variables
    r=rand(1);
    S2(i,j)=global_search_limits(1,j)+r*(global_search_limits(2,j)-
    global_search_limits(1,j));
end
end

% populating Species 3
for i=1:individuals_in_each_species
    for j=1:variables
        r=rand(1);
        S3(i,j)=global_search_limits(1,j)+r*(global_search_limits(2,j)-
        global_search_limits(1,j));
    end
end

% populating Species 4
for i=1:individuals_in_each_species
    for j=1:variables
        r=rand(1);
        S4(i,j)=global_search_limits(1,j)+r*(global_search_limits(2,j)-
        global_search_limits(1,j));
    end
end

%% initial search limits are global search limits
gen_search_limits=global_search_limits; % initialize the limits to user
defined limits

for gen_num=1:gen_num_max
    % cross over
    S2=crossover_operator(S2,cross_rate);
    S3=crossover_operator(S3,cross_rate);
    S4=crossover_operator(S4,cross_rate);

    % migration
    [S2,S3]=migration_operator(S2,S3,migr_rate);
    [S3,S4]=migration_operator(S3,S4,migr_rate);

    % mutation
    S2=mutation_operator_2OpenSees(S2,muta_rate,global_search_limits);
    % let S2 search across the entire user given limits [modification from
    % iGAMAS]
    S3=mutation_operator_3OpenSees(S3,muta_rate,gen_search_limits,gen_num,gen_between_regen);
    % S3 searches only within the generation search
    % limits
    S4=mutation_operator_4OpenSees(S4,muta_rate,gen_search_limits,gen_num,gen_num_max);
    % S4 searches only within the generation search
    % limits
%% fitness with rank [S2 S3 S4 are sorted wrt score && rogue combinations are removed here]
S2=fitness_operator_with_rank_OpenSees_parallel(S2,DOF,A_obs_shifted,obs_sys_DOF);
S3=fitness_operator_with_rank_OpenSees_parallel(S3,DOF,A_obs_shifted,obs_sys_DOF);
S4=fitness_operator_with_rank_OpenSees_parallel(S4,DOF,A_obs_shifted,obs_sys_DOF);

%% artificial selection [store best result of S2,S3,S4 in S1]
% average score is determined here
S_temp=[S2;S3;S4]; % combine into a single matrix
[S_temp_row,S_temp_col]=size(S_temp);
S_temp=sortrows(S_temp,(S_temp_col-1)); % sort this new matrix wrt score
gen_avg_score=mean(S_temp(:,S_temp_col-1)); % determine the average score of all individuals in this generation
S1(gen_num,:)=[S_temp(S_temp_row,:),gen_avg_score]; % last row is the best individual

%% reintroduction [replace less fit individuals in S4 with individuals from S1]
% if S4 has only 5 individuals out of the 15==individuals_in_each_species initially started with then only 5 best out of the S1 & S4 are selected and NOT 15 % strategy #2 (see initial_population_NONLIN1.m for strategy #1)
if mod(gen_num,reintro_gen_num)==0
    S4=reintroduction_operator(S1,S4);
end

%% selection [reproduction] USE EITHER RANK OR SCORE BUT NOT BOTH!
% if true, then
sel_str_logic=strcmp(selection_strategy, 'rank');
% if false, then
sel_str_logic=0
if sel_str_logic==1 % based on rank
    S2=selection_operator_OpenSees_based_on_rank(S2,individuals_in_each_species);
    S3=selection_operator_OpenSees_based_on_rank(S3,individuals_in_each_species);
    S4=selection_operator_OpenSees_based_on_rank(S4,individuals_in_each_species);
else % based on score

S2 = selection_operator_OpenSees_based_on_score(S2, individuals_in_each_species);
S3 = selection_operator_OpenSees_based_on_score(S3, individuals_in_each_species);
S4 = selection_operator_OpenSees_based_on_score(S4, individuals_in_each_species);
end

%% regeneration [of species S2 and S3 only]
if mod(gen_num, regen_gen_num) == 0
    % species 2
    % K2 searches across the entire user given limits. Hence use global search limits [modification from iGAMAS]
    % Use the same search limits for regeneration and mutation.
    % Otherwise k can be negative
    for i = 1: individuals_in_each_species
        for j = 1: variables
            r = rand(1);
            S2(i, j) = global_search_limits(1, j) + r * (global_search_limits(2, j) - global_search_limits(1, j));
        end
    end
    % species 3
    % K3 searches only within the generation search limits
    % Use the same search limits for regeneration and mutation.
    % Otherwise k can be negative
    for i = 1: individuals_in_each_species
        for j = 1: variables
            r = rand(1);
            S3(i, j) = global_search_limits(1, j) + r * (global_search_limits(2, j) - global_search_limits(1, j));
        end
    end
end

save('C:\Users\pck123.COEACCESS\Documents\MATLAB\CE 600 Matlab files\iGAMAS OpenSees parallel\GA_output\parallel_trials.mat');
end % generations complete

toc

$\text{end}$
%% cross over function
function [S_crossover]=crossover_operator(S_crossover_temp,cross_rate)
% tic
[cross_row,cross_col]=size(S_crossover_temp);

%% simple cross over
% randomly selecting individuals for simple cross over
j=0;  % # of individuals selected for crossover
while j==0  % to ensure that atleast two individuals are selected for crossover
    T=[];  % initialise T
    for i=1:cross_row
        r=rand(1);
        if r<=cross_rate
            j=j+1;
            S_simple_crossover(j,:)=S_crossover_temp(i,:);
            T=[T;i];  % matrix which stores the row number selected for cross over
        end
    end
% shuffle the selected individuals
S_simple_crossover=S_simple_crossover(randperm(size(S_simple_crossover, 1)),:);
% to have even number of individuals in matrix
    if mod(j,2)==1
        j=j-1;  % if j==0 it enters while loop so that atleast two individuals are selected for crossover
    end
end

% cross over [SIMPLE]
for i=1:2:j
    r=ceil((cross_col-1)*rand(1));  % cross over location
    if r+1<=ceil(cross_col/2)  % if swapping position is less than half the individual length; swap only left side of swapping position
        for m=1:r  % actual swapping
            swap_temp=S_simple_crossover(i,m);
            S_simple_crossover(i,m)=S_simple_crossover(i+1,m);
            S_simple_crossover(i+1,m)=swap_temp;
        end
    else  % if swapping position is more than half the individual length; swap only right side of swapping position
        for m=r+1:cross_col  % actual swapping
            swap_temp=S_simple_crossover(i,m);
            S_simple_crossover(i,m)=S_simple_crossover(i+1,m);
            S_simple_crossover(i+1,m)=swap_temp;
        end
    end
end
end
end
end

% remap to original matrix
S_crossover_temp(T,:)=S_simple_crossover;

%%% multipoint cross over
% randomly selecting individuals for multi point cross over
j=0;
while j==0
    T=[]; % initialise T as the size in multi cross over could be different than single cross over
    for i=1:cross_row
        r=rand(1);
        if r<=cross_rate
            j=j+1;
            K_multi_crossover(j,:)=S_crossover_temp(i,:);
            T=[T;i]; % matrix which stores the row number selected for cross over
        end
    end
end

% shuffle the selected individuals
K_multi_crossover=K_multi_crossover(randperm(size(K_multi_crossover,1)),:);
    % to have even number of individuals in matrix
    if mod(j,2)==1
        j=j-1;
    end
end

% cross over [MULTI POINT]
for i=1:2:j
    for m=1:cross_col
        r=rand(1);
        if r<=cross_rate
            swap_temp=K_multi_crossover(i,m);
            K_multi_crossover(i,m)=K_multi_crossover(i+1,m);
            K_multi_crossover(i+1,m)=swap_temp;
        end
    end
end

% remap to original matrix
S_crossover_temp(T,:)=K_multi_crossover;

% function return
S_crossover=S_crossover_temp;
% toc
function [SA_migration, SB_migration] = migration_operator(SA_migration_temp, SB_migration_temp, migr_rate)

% tic
%migr_rowA,~] = size(SA_migration_temp);
%migr_rowB,~] = size(SB_migration_temp);

%%%% randomly selecting individuals for migration
%%%% for first matrix
jA = 0;
TA = [];
while jA == 0 % to ensure at least one individual is selected
    for i = 1:migr_rowA
        r = rand(1);
        if r <= migr_rate
            jA = jA + 1;
            SA_random_migration(jA, :) = SA_migration_temp(i, :);
            TA = [TA; i]; % matrix which stores the row number selected
        end
    end
end

% for second matrix
jB = 0;
TB = [];
while jB == 0 % to ensure at least one individual is selected
    for i = 1:migr_rowB
        r = rand(1);
        if r <= migr_rate
            jB = jB + 1;
            SB_random_migration(jB, :) = SB_migration_temp(i, :);
            TB = [TB; i]; % matrix which stores the row number selected
        end
    end
end

%%%% shuffle the selected individuals
SA_random_migration = SA_random_migration(randperm(size(SA_random_migration, 1)));
SB_random_migration = SB_random_migration(randperm(size(SB_random_migration, 1)));

%%%% migrate the first min(jA, jB) rows in each of the matrices
jC = 1:min(jA, jB);
jC = jC';
KC_temp(jC, :) = SB_random_migration(jC, :);
SA_random_migration(jC, :) = KC_temp(jC, :);
SB_random_migration(jC, :) = SA_random_migration(jC, :);
% remap to original matrix
SA_migration_temp(TA,:) = SA_random_migration;
SB_migration_temp(TB,:) = SB_random_migration;

% function return
SA_migration = SA_migration_temp;
SB_migration = SB_migration_temp;
% toc

mutation_operator_2OpenSees.m

%% species 2 mutation operator
function [S2_mutation] = mutation_operator_2OpenSees(S2_mutation_temp, muta_rate, search_limits)
% tic
[muta_row, muta_col] = size(S2_mutation_temp);

% random mutation of each individual element [different from mutation_operator_2.m]
for i = 1:muta_row
    for j = 1:muta_col
        r = rand(1);
        if r <= muta_rate
            %% actual mutation
            S2_mutation_temp(i, j) = search_limits(1, j) + rand(1) * (search_limits(2, j) - search_limits(1, j));
        end
    end
end

% function return
S2_mutation = S2_mutation_temp;
% toc

mutation_operator_3OpenSees.m

%% species 3 mutation operator
function [S3_mutation] = mutation_operator_3OpenSees(S3_mutation_temp, muta_rate, search_limits, generation_number, gen_between_regen)
% tic
[muta_row, muta_col] = size(S3_mutation_temp);

% non uniform cyclic mutation
for i = 1:muta_row
for j=1:muta_col
    r=rand(1);
    if r<=muta_rate
        % actual mutation
        r=rand(1);
        if r<=0.5 % mutate about upper search limit
            S3_mutation_temp(i,j)=S3_mutation_temp(i,j)+(search_limits(2,j)-
                S3_mutation_temp(i,j))*(1-rand(1)^((1-
                    (0.9*(mod(generation_number,gen_between_regen))/gen_between_regen)));
        else % mutate about lower search limit
            S3_mutation_temp(i,j)=S3_mutation_temp(i,j)+(search_limits(1,j)-
                S3_mutation_temp(i,j))*(1-rand(1)^((1-
                    (0.9*(mod(generation_number,gen_between_regen))/gen_between_regen))));
        end
    end
end
end

% function return
S3_mutation=S3_mutation_temp;
% toc
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

mutation_operator_4OpenSees.m

%% species 4 mutation operator
function [S4_mutation]=mutation_operator_4OpenSees(S4_mutation_temp,muta
    _rate,search_limits,generation_number,tot_generations)
    % tic
    [muta_row,muta_col]=size(S4_mutation_temp);
    % non unifrom local mutation
    for i=1:muta_row
        for j=1:muta_col
            r=rand(1);
            if r<=muta_rate
                % actual mutation
                r=rand(1);
                if r<=0.5 % mutate about upper search limit
                    S4_mutation_temp(i,j)=S4_mutation_temp(i,j)+0.5*(search_limits(2,j)-
                        S4_mutation_temp(i,j))*(1-rand(1)^((1-
                            (generation_number/tot_generations))));
                else % mutate about lower search limit
                    S4_mutation_temp(i,j)=S4_mutation_temp(i,j)+0.5*(search_limits(1,j)-
                        S4_mutation_temp(i,j))*(1-rand(1)^((1-
                            (generation_number/tot_generations))));
                end
            end
        end
    end
end

% function return
S4_mutation=S4_mutation_temp;
% toc
function [S_fitness]=fitness_operator_with_rank_OpenSees_parallel(S_fitness_temp,DOF,obs_sys_partial_response_shifted,obs_sys_DOF)
% tic
[S_fit_row,S_fit_col]=size(S_fitness_temp);
S_fitness_temp_temp=zeros(S_fit_row,S_fit_col+1); % preallocation for speed
[obs_row,obs_col]=size(obs_sys_partial_response_shifted); % needs to determined only once and hence outside of the for loop
parfor i=1:S_fit_row
    % identify the worker ID
    t=getCurrentTask();
    worker_ID=t.ID;
    if (worker_ID==1)
        parsave(sprintf('individual_%d.txt',worker_ID),S_fitness_temp(i,:)); % ! OpenSees_1.exe Worker1.tcl
        fileID=fopen('Concentrated_Dynamic_Output_1\Acce.out','r');
        A_temp=fscanf(fileID,'%f');
        A_temp=reshape(A_temp,DOF+1,[]); % DOF is the number of node outputs requested by the recorders
        A_temp=A_temp'; % first column is time
        fclose('all'); % close all open files
    end
    % removing the time column
    index=2:DOF+1; % first column is time, so remove it
    A=A_temp(:,index); % these accelerations are relative w.r.t. ground as a Timeseries is NOT DEFINED in OpenSees recorder
    % Score determination
    calculated_sys_partial_response=A(:,obs_sys_DOF); % assuming partial acceleration of the system is known
    [cal_row,~]=size(calculated_sys_partial_response);
    if cal_row<obs_row % if OpenSees fails to converge or terminate prematurely
end
% toc

% function return
S4_mutation=S4_mutation_temp;
score_perDOF_perstep=0; % assign zero fitness to rouge combinations
else
    response_error_matrix=calculated_sys_partial_response-
    obs_sys_partial_response_shifted;
SSE_perDOF_perstep=sumsqr(response_error_matrix)/(obs_row*obs_col);
    score_perDOF_perstep=1/(10^(-3)+SSE_perDOF_perstep);
end
S_fitness_temp_temp(i,:)=[S_fitness_temp(i,:),score_perDOF_perstep];
end
if (worker_ID==2)
parsave(sprintf('individual_%d.txt',worker_ID),S_fitness_temp(i,:));
    ! OpenSees_2.exe Worker2.tcl
    fileID=fopen('Concentrated_Dynamic_Output_2\Acce.out','r');
    A_temp=fscanf(fileID,'%f');
    A_temp=reshape(A_temp,DOF+1,[]); % DOF is the number of node outputs requested by the recorders
    A_temp=A_temp'; % first column is time
    fclose('all'); % close all open files

    index=2:DOF+1; % first column is time, so remove it
    A=A_temp(:,index); % these accelerations are relative w.r.t. ground as a Timeseries is NOT DEFINED in OpenSees recorder

    % Score determination
    calculated_sys_partial_response=A(:,obs_sys_DOF); % assuming partial acceleration of the system is known
    [cal_row,~]=size(calculated_sys_partial_response);

    if cal_row<obs_row % if OpenSees fails to converge or terminate prematurely
        score_perDOF_perstep=0; % assign zero fitness to rouge combinations
    else
        response_error_matrix=calculated_sys_partial_response-
        obs_sys_partial_response_shifted;
SSE_perDOF_perstep=sumsqr(response_error_matrix)/(obs_row*obs_col);
    score_perDOF_perstep=1/(10^(-3)+SSE_perDOF_perstep);
end
S_fitness_temp_temp(i,:)=[S_fitness_temp(i,:),score_perDOF_perstep];
end
if (worker_ID==3)
parsave(sprintf('individual_%d.txt',worker_ID),S_fitness_temp(i,:));

! OpenSees_3.exe Worker3.tcl
fileID=fopen('Concentrated_Dynamic_Output_3\Acce.out','r');
A_temp=fscanf(fileID,'%f');
A_temp=reshape(A_temp,DOF+1,[]); % DOF is the number of node outputs requested by the recorders
A_temp=A_temp'; % first column is time
fclose('all'); % close all open files

% removing the time column
index=2:DOF+1; % first column is time, so remove it
A=A_temp(:,index); % these acceleartions are relative w.r.t. ground as a Timeseries is NOT DEFINED in OpenSees recorder

% Score determination
calculated_sys_partial_response=A(:,obs_sys_DOF); % assuming partial acceleration of the system is known
[cal_row,~]=size(calculated_sys_partial_response);
if cal_row<obs_row % if OpenSees fails to converge or terminate prematurely
   score_perDOF_perstep=0; % assign zero fitness to rouge combinations
else
   response_error_matrix=calculated_sys_partial_response-obs_sys_partial_response_shifted;
   SSE_perDOF_perstep=sumsqr(response_error_matrix)/(obs_row*obs_col);
   score_perDOF_perstep=1/(10^(-3)+SSE_perDOF_perstep);
end

S_fitness_temp_temp(i,:)=[S_fitness_temp(i,:),score_perDOF_perstep];
end % responses of all individuals are calculated

% remove individuals with zero fitness value
% sorting in increasing order wrt the score column
S_fitness_temp_temp=sortrows(S_fitness_temp_temp,S_fit_col+1);

index_counter=0;
for i=1:S_fit_row
   if S_fitness_temp_temp(i,S_fit_col+1)==0 % if score is zero
      index_counter=index_counter+1;
   end
end

index=(index_counter+1):S_fit_row;
S_nonzero_fitness_individuals=S_fitness_temp_temp(index,:)
%% ranking of individuals based on score

for i=1:S_fit_row-index_counter
    S_nonzero_fitness_individuals(i,S_fit_col+2)=i;
end

S_fitness=S_nonzero_fitness_individuals;
% toc

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

parsave.m

%% function to save variables in parfor loop

function parsave(file_name, variable_to_save)
    save(file_name, 'variable_to_save','-ASCII')
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

reintroduction_operator.m

%% reintroduction operator
function [S4_output]=reintroduction_operator(S1_temp,S4_temp)
%tic

%% remove average fitness column from S1_temp
[S1_row,S1_col]=size(S1_temp);
index=1:(S1_col-1); %last column is avg fitness of the gen
S1_temp_temp=S1_temp(:,index);

[S4_row,S4_col]=size(S4_temp);

%% combine avg fitness removed S1 and S4 matrices into a bigger matrix
S_combination=[S1_temp_temp;S4_temp];

%% sort in increasing order wrt score column
S_combination=sortrows(S_combination,S4_col-1);

%% select last S4_row rows of S_combination matrix
index=S1_row+1:S1_row+S4_row;
S4_output=S_combination(index,:);

%toc
% selection operator based on RANK
function [S_selection] = selection_operator_OpenSees_based_on_rank(S_selection_temp, num_of_selections)
% tic
[S_sel_row, S_sel_col] = size(S_selection_temp);
rank_sum = sum(S_selection_temp(:, S_sel_col)); % sum of ranks [located in S_sel_col column]

% roulette wheel selection
S_roulette_sel = zeros(num_of_selections, S_sel_col); % preallocation for speed
for i = 1:num_of_selections % == # of times selection is carried out == # of individuals in matrix
    r = rank_sum * rand(1); % random number in (0, rank_sum)
    n = ((sqrt(1 + 8 * r)) - 1) / 2; % n(n+1)/2 == r solve n
    n = ceil(n);
    if n > S_sel_row;
        n = S_sel_row;
    end
    S_roulette_sel(i, :) = S_selection_temp(n, :);
end

% remove the score, rank data from the individuals
S_roulette_sel_temp = mat2cell(S_roulette_sel, num_of_selections, [S_sel_col-2, 2]);
S_selection = S_roulette_sel_temp{1};
% toc
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% selection operator based on SCORE [NOT based on rank]
function [S_selection] = selection_operator_OpenSees_based_on_score(S_selection_temp, num_of_selections)
% tic
[S_sel_row, S_sel_col] = size(S_selection_temp);
score_cumsum = cumsum(S_selection_temp(:, S_sel_col-1)); % determining the cumulative sum of scores [located in (S_sel_col-1) column],
score_cumsum == column vector

% roulette wheel selection
S_roulette_sel = zeros(num_of_selections, S_sel_col); % preallocation for speed
for i=1:num_of_selections % == # of times selection is carried out == #
of individuals in matrix
    r=score_cumsum(S_sel_row,1)*rand(1); % random number in (0,last
element of score_cumsum)
    logic_find=find(score_cumsum<=r); % stores the locations (indices)
of elements

    %% if logic_find is null matrix then select the first row of
    S_selection_temp
        empty=isempty(logic_find);
        if empty==1 % i.e. If logic_find is empty
            S_roulette_sel(i,:)=S_selection_temp(1,:);
        else
            S_roulette_sel(i,:)=S_selection_temp(max(logic_find)+1,:);
        end
    end

    %% remove the score,rank data from the individuals
    S_roulette_sel_temp=mat2cell(S_roulette_sel,num_of_selections,[S_sel_co
    ol-2,2]);
    S_selection=S_roulette_sel_temp{1};
    % toc
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
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