A DYNAMIC SUPPLY CHAIN INVENTORY MODEL CONSIDERING SUPPLIER SELECTION, JOINT REPLENISHMENT AND TRANSPORTATION COST

A Thesis in
Industrial Engineering and Operations Research

by
Chenxi Li

© 2015 Chenxi Li

Submitted in Partial Fulfillment
of the Requirements
for the Degree of
Master of Science

December 2015
The thesis of Chenxi Li was reviewed and approved* by the following:

Jose A. Ventura  
Professor of Industrial and Manufacturing Engineering  
Thesis Advisor

Vikash V. Gayah  
Assistant Professor of Civil and Environmental Engineering

David A. Nembhard  
Associate Professor of Industrial and Manufacturing Engineering  
Graduate Program Coordinator

*Signatures are on file in the Graduate School
Abstract

Companies currently face fierce global competition and so they must improve their supply chain efficiency. The development of appropriate decision support tools is important to enable such improvements. The thesis presents a multi-period inventory lot-sizing model for multiple products in a serial supply chain, where raw materials are purchased from multiple suppliers at the first stage, and external demand from vendors occurs at the last stage. Joint replenishment is considered when ordering raw materials. The demand is known and may change from period to period. Stages of this production-distribution serial structure correspond to inventory locations. The first two stages stand for storage areas holding raw materials and finished products in a manufacturing facility; additional intermediate stages represent either manufacturing facilities or centralized warehouses. The last stage is a distribution center that directly serves customer demand. The problem is modeled as a time-expanded transshipment network, which is defined by the nodes and arcs that can be reached by feasible material flows. At first, an integrated approach model aimed at minimizing total cost of the entire supply chain is developed to determine dynamic supplier selection and inventory planning policy at the same time. In addition, a sequential approach model is developed so that inventory planning policy is solved first, and supplier selection strategy is obtained according to that. A comparison between integrated and sequential approaches is shown in the thesis; sensitivity analysis on key factors for the integrated model is presented as well. Results show that total cost is reduced under the integrated approach; major and minor ordering costs, and transportation cost are important factors for supplier selection under the integrated approach.

Keywords:
Serial Supply Chain
Multi-product
Joint Replenishment
Integrated and Sequential Approach


<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Figures ................................................................................................................................. vi</td>
</tr>
<tr>
<td>List of Tables ........................................................................................................................................ vii</td>
</tr>
<tr>
<td>Acknowledgement ................................................................................................................................. ix</td>
</tr>
<tr>
<td>Chapter 1 Introduction and Overview ................................................................................................. 1</td>
</tr>
<tr>
<td>1.1 Introduction ..................................................................................................................................... 1</td>
</tr>
<tr>
<td>1.2 Supply Chain Structure .................................................................................................................. 2</td>
</tr>
<tr>
<td>1.3 Operations Research in Supply Chain Management .......................................................................... 4</td>
</tr>
<tr>
<td>1.4 Inventory Management in Supply Chain .......................................................................................... 6</td>
</tr>
<tr>
<td>1.5 Research Objectives and Contributions ......................................................................................... 7</td>
</tr>
<tr>
<td>1.6 Overview ......................................................................................................................................... 8</td>
</tr>
<tr>
<td>Chapter 2 Literature Review ................................................................................................................ 10</td>
</tr>
<tr>
<td>2.1 Introduction ..................................................................................................................................... 10</td>
</tr>
<tr>
<td>2.2 Supply Chain Inventory Models ..................................................................................................... 10</td>
</tr>
<tr>
<td>2.3 Supply Chain Inventory Models with Ordering Quantity Discount .............................................. 11</td>
</tr>
<tr>
<td>2.4 Supplier Selection Models .............................................................................................................. 13</td>
</tr>
<tr>
<td>2.5 Integrated Supplier Selection Models ............................................................................................. 15</td>
</tr>
<tr>
<td>Chapter 3 Dynamic Inventory Model for a Serial Supply Chain Structure ...................................... 18</td>
</tr>
<tr>
<td>3.1 Introduction ..................................................................................................................................... 18</td>
</tr>
<tr>
<td>3.2 Dynamic Joint Replenishment Problem .......................................................................................... 18</td>
</tr>
<tr>
<td>3.3 Problem Statement and Formulation ............................................................................................... 20</td>
</tr>
<tr>
<td>3.4 Integrated Model Approach ............................................................................................................ 24</td>
</tr>
<tr>
<td>3.5 Sequential Model Approach ............................................................................................................ 31</td>
</tr>
<tr>
<td>Chapter 4 Computational Results .......................................................................................................... 35</td>
</tr>
<tr>
<td>4.1 Introduction ..................................................................................................................................... 35</td>
</tr>
<tr>
<td>4.2 Illustrative Example .......................................................................................................................... 35</td>
</tr>
<tr>
<td>4.3 Solution for the Integrated Model Approach .................................................................................. 39</td>
</tr>
<tr>
<td>4.4 Solution for the Sequential Model Approach .................................................................................. 41</td>
</tr>
<tr>
<td>4.5 Comparison of Sequential Approach vs Integrated Approach .................................................... 45</td>
</tr>
<tr>
<td>4.6 Sensitivity Analysis .......................................................................................................................... 49</td>
</tr>
<tr>
<td>Chapter 5 Conclusions and Future Work .............................................................................................. 57</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 1-1. The Supply Chain Process.................................................................3

Figure 3-1. Dynamic Joint Replenishment Problem Cost Structure..........................20

Figure 3-2. Simplified Dynamic Serial Supply Chain Network.................................21

Figure 3-3. A Static Supply Chain Network with 3 Suppliers, 4 Stages and 5 Time Periods.....24

Figure 3-4. Time-expanded Supply Chain Network for the Sequential Approach.............33

Figure 4-1. Customer Demand.................................................................................36

Figure 4-2. Optimal Solution for the Integrated Approach.........................................41

Figure 4-3. Optimal Solution for the Sequential Approach.......................................44

Figure 4-4: Optimal Allocation to each Supplier for Sequential and Integrated Approaches.....46

Figure 4-5: Inventory at Each Stage for Both Sequential and Integrated Approach.........48
LIST OF TABLES

Table 4-1. Production and Inventory Capacities (expressed in units per month) .................. 37
Table 4-2. Bill of Material Ratio (expressed in units of raw material per unit of final products) ......................................................................................................................... 37
Table 4-3. Equivalence Factors $\omega^m_{raw}$ and $\omega^p_f$ (expressed in units per container) ...... 37
Table 4-4. Description of Raw Material Suppliers .................................................................. 37
Table 4-5. Inventory Holding and Production Cost by Period (expressed in dollars per unit per month) ............................................................................................................ 38
Table 4-6. Major and Minor Fixed Costs during Manufacture Process (expressed in dollars) .... 38
Table 4-7. Initial and Ending Inventory Levels Required at Each Stage (expressed in units) .... 38
Table 4-8. Price Interval for Raw Materials (expressed in dollars) ........................................ 38
Table 4-9. Truck Capacity for Raw Materials and Final Products .......................................... 38
Table 4-10. In-transit Inventory Holding Cost (expressed in dollars) .................................... 38
Table 4-11. Optimal Production and Shipping Strategy for the Integrated Approach (expressed in units) ........................................................................................................... 39
Table 4-12. Optimal Raw Material Procurement Strategy for the Integrated Approach (expressed in units) ........................................................................................................ 40
Table 4-13. Optimal Inventory Levels per Stage for the Integrated Approach (expressed in units) .................................................................................................................. 40
Table 4-14. Optimal Production and Shipping Strategy for the Sequential Approach (expressed in units) ......................................................................................................... 42
Table 4-15. Optimal Raw Material Procurement Strategy for the Sequential Approach (expressed in units) .............................................................................................. 42
Table 4-16. Optimal Inventory Levels per Stage for the Sequential Approach (expressed in units) ..................................................................................................................................................43

Table 4-17. Optimal Solutions for the Integrated and Sequential Approaches .................................................................49

Table 4-18. Sensitivity Analysis of the Optimal Solution, under Different scenarios of Ordering and Holding Costs ..........................................................................................................................51

Table 4-19. Comparison of Supplier Selection under Sensitivity Analysis for Sequential vs Integrated Approach .............................................................................................................................................52

Table 4-20. Sensitivity Analysis of the Optimal Solution, under Different Scenarios of Transportation Cost .................................................................................................................................................53

Table 4-21. Sensitivity Analysis of the Optimal Solution, under Different Scenarios of Minimum Perfect Rate .......................................................................................................................................................54

Table 4-22. Sensitivity Analysis of the Optimal Solution, under Different Price Intervals for Supplier 1 .....................................................................................................................................................55

Table 4-23. Sensitivity Analysis of the Optimal Solution, under Different Discount Quantities.56
ACKNOWLEDGEMENTS

I would like to express gratitude to my advisors, Dr. Jose A. Ventura and Dr. Vikash V. Gayah for the invaluable support they provided to me on the thesis. This thesis would not have been possible without their guidance and encouragement. I feel privileged to have had the chance to work with them.

I am highly grateful for coming to the Department of Industrial and Manufacturing Engineering at The Pennsylvania State University for graduate study. I learned a lot in this field, and had a very good time.

I would also like to thank my parents, who have constantly supported me and encouraged me to pursue my dreams. Thank you, Mom and Dad.
Chapter 1 Introduction and Overview

1.1 Introduction

A supply chain is a network of facilities and distribution options that performs functions of procurement of materials, process of these materials into finished products and the distribution of finished products to customers for manufacturing organizations, it can also focus on resource integration by managing flows on both the demand and supply side for service organizations (Flint, 2010). Although the complexity of a supply chain may vary greatly from industry to industry and firm to firm, it always exists in both manufacturing and service organizations (Ganeshan and Harrison, 2002).

In the current global economy, companies have been forced to develop more efficient processes in order to satisfy customer demand at lower costs and with improved service levels. The continuous growth of international markets has significantly impacted supply chain designs and the execution of their associated strategies. Even small companies have customers and suppliers around the world. This situation can be accentuated by improvement to the part of product design during manufacturing process so that changes can be shown in product design and supply chain practices, which could influence global supply chain behavior and capabilities (Marsillac and Roh, 2014).

In modern supply chains, products can be classified as functional or innovative, resulting in flow dynamics that add additional complexity (Fisher, 1997). Functional products are physical products without any added value that usually have long life cycles and stable demand, but low profit margins. Companies innovate in order to make their products more attractive to customers, which will create competitive advantages. Increased competition shortens the life cycle for innovative products and produces highly dynamic demand patterns. Additionally, customer-focused market orientation trends force supply chains to satisfy demands for products and services, further increasing complexity (Hines, 2014).

Given the growing importance that companies and researchers have placed on supply chain optimization, multiple areas of research have been developed during the last few decades. Extensive literature has been dedicated to inventory, production, transportation and distribution.
Most of the work has been focused on analyzing one of these problems or on optimizing individual components of the supply chain. Thus, under current economic conditions characterized by high competitiveness and growing supply chain complexity, considering all these factors within a theoretical framework while analyzing them for the entire supply chain has become a high priority (Turker and Altuntas, 2014).

Based on a 2003 Accenture study, nearly 90% of the companies said that supply chain management is critical or very important, and 51% said that the importance of supply chain management had increased significantly in the 5 years leading up to the survey. And supply chain management could take up almost 70% of the companies’ operating costs and comprised at least half of all the typical company’s assets. It can be seen that efficient management of the supply chain has become a competitive differentiator for many companies (Ravindran and Warsing, 2013).

1.2 Supply Chain Structure

Previously, organizational structure, has been considered within a single firm or organization (Ghoshal et al., 1961). Marketing, distribution, planning, manufacturing and the purchasing organizations along the supply chain organized individually. They have their own objectives, but the objectives are always conflicting. The result of competition among these organizations is that there is not a single, integrated plan for the organization. Hence supply chain management is a strategy through all the stages including both tactical and strategic planning levels that such an integration can be achieved (Rushton et al., 2014).

In general terms, the supply chain is integrated by two basic processes: the production planning process, the distribution and logistics process (see Figure 1-1).
Bozarth and Handfield (2006) define the supply chain as a network of manufacturers and service providers that work together to convert and move goods from the raw material stage through to the end users. Similarly, according to Chopra and Meindl (2007), a supply chain includes all parties directly or indirectly involved in fulfilling a customer request, including manufacturers, suppliers, transporters, warehouses, distribution centers, retailers, and even customers themselves. Stadtler et al. (2015) define the term supply chain management as the task of integrating organizational units along a supply chain and coordinating material, information and financial flows in order to fulfill customer demands with the aim of improving the competitiveness of a supply chain as a whole.

Lee et al. (1997) introduce a mechanism that supply chain can be classified according to the centralization of their decisions. A single decision maker usually defines optimal policies for a centralized system, which more often exists in companies that manage their own distribution networks. While in decentralized supply chains, multiple decision makers make independent decisions in order to optimize their individual objectives, which is more common in practice. Schmitt et al. (2015) investigate optimal systems in a multi-location system, examine cost and cost variances of the system in a centralized and decentralized way. They find that centralization is optimal due to the risk-pooling effect.

Furthermore, Min and Zhou (2002) define decisions for supply chain management into two broad categories – strategic and operational. Where strategic decisions are made typically over a longer time horizon that closely linked to corporate strategy. On the other hand, operational strategies are short term, and focus on activities over a day-to-day basis. And they
describe four major areas that need decision making in supply chain management: 1) Location, 2) Production, 3) Inventory, and 4) Transportation. Hazen et al. (2014) address supply chain is inundated with data nowadays, more and more organizations adopt and perfect data analysis (data science, predictive analytics and big data) in order to enhance supply chain processes.

Chopra and Meindl (2007) address that the success of a supply chain is closely tied to the design and management of supply chain flows. Multiple decisions need to be made simultaneously regarding the flow of information, products, and money. These decisions are classified as strategic, tactical or operational depending of their frequency and duration. A good supply chain shapes when these decisions cope with each other closely.

### 1.3 Operations Research in Supply Chain Management

Chopra and Meindl (2007) define supply chain management as “the management of flows between and among supply chain stages to maximize supply chain profitability”. And a more complete definition by Simchi-Levi et al. (2003) states, “Supply chain management is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities to the right locations, and the right time in order to minimize system-wide costs while satisfying service level requirements.” One of the applications for operations research in this field is to build optimization models aimed at solving issues among supply chain stages. In the past, types of supply chain models are shown in this field, and most of them work on two conflicting objectives in supply chain management: minimizing supply chain costs and maximizing customer service. Brandenburg et al. (2014) provide a content analysis of 134 carefully identified papers on quantitative, formal models that focus on multi criteria decision making, and make conclusions that numerous possibilities and insights can be gained from expanding types of tools and factors in formal operations research modeling efforts.

Among the issues, one main focus on supply chain behavior is the bullwhip effect. To deal with the bullwhip effect, a variety of causes have been identified by researchers. Warburton (2004) captures the systemic nature of the bullwhip effect by studying the fundamental differential delay equations that describe the evolution of the retailer’s inventory level in the
supply chain, which were initially described by Forrester. Chatfield et al. (2006) present a simulation study to evaluate causes of the bullwhip effect, and found that lead time variability is a significant cause, while information sharing is the most direct way to reduce the bullwhip effect. Besides, making use of information without sharing it is typically worse than completely ignoring the information. Dominguez et al. (2014) address a comparative analysis of the bullwhip effect between a serial supply chain network versus a complex divergent one. Results show great difference between the two networks where customer demand is in violent disturbance.

To enhance supply chain overall behavior, a lot of operations research models have been developed to focus on combining different parts of supply chain stages into an integral system (Ex: supply chain partnerships). Goyal and Gupta (1989) provide a review of price-quantity discount approaches, and a situation of dynamic demand is more realistic within the supply chain. Bhatnagar et al. (1993) provide a review of coordination models by exploring integrated planning across pairs of firm processes. Along the lines, Slats et al. (1995) recognize that operations research models and techniques are well suited to analyze the local performance of logistics sub-chains and processes when coordinated to effectively model the entire conceptual system. Amin and Zhang (2012) address a closed loop supply chain, among which they propose an integrated model by considering supplier selection and order allocation altogether using fuzzy method to evaluate suppliers based on qualitative criteria.

Operations research would hopefully enhance the design, implementation, and generalization of supply chain partnerships across all industries. An efficient approach involves analyzing the entire supply chain, thereby avoiding local sub-optimization produced by the independent analysis of each process. In fact, during the last two decades, companies have recognized that significant cost savings can be achieved by integrating production plans, inventory control, and transportation decisions (Muriel and Simchi-Levi, 2003). For example, during the HP/Compaq merger, it was estimated that the merger would result in a savings of $2.5 billion, of which $1.8 billion was due to supply chain efficiency. Similar condition happens to Walmart, Dell and many global companies (Ravindran and Warsing, 2013).
1.4 Inventory Management in Supply Chain

In supply chain, inventory is used to balance the mismatch between supply and demand. It can be held in raw materials, work in process and final products. Inventory is a major expense that highly impacts supply chain responsiveness (Chopra and Meindl, 2007). Since we could exploit economies of scales in raw material procurement, product manufacturing and transportation/distribution stages, making good inventory decisions could take a significant role in cost reduction for companies (Stadtler, 2015).

An efficient way to reduce costs and increase working capital is to correctly implement inventory management policies. However, inventories in complex supply chain networks can only be efficiently managed with appropriate analytic decision support tools. Bregman (1990) estimates that a typical firm with annual sales of $500 million could save nearly $100,000 in annual inventory holding costs for every 1% reduction in finished goods inventory. Similarly, big retailers like Macy’s, Target and Walgreens benefit a lot from inventory management (Schulz, 2013). Thus, inventory management decisions are important, given their significant impact on working capital and product availability.

“Inventory Management” answers “When” and “How much” to order among the supply chain system. According to this, inventory control policies need to solve the questions related to that. In this context, periodic and continuous reviews are the two most common inventory control policies used to minimize costs while satisfying minimum service levels. Under a periodic review policy, a decision maker places periodic orders every $r$ periods to bring inventories up to a pre-specified level $S$. For this reason, a periodic review policy is also called a $(r,S)$ policy, where the order quantity is a random variable. On the other hand, under a continuous review policy, a decision maker orders up to a fixed quantity $Q$ when inventory falls below predefined reorder point $s$. For this reason, it is also called a $(s,Q)$ policy, where the time between orders is a random variable. The economic order quantity model is the most basic method for determining order quantities in the continuous review system. The economic order quantity model defines the order quantity $Q$, minimizing ordering and holding costs. Once $Q$ is determined, safety stock, reorder points and number of orders per year can also be calculated (Valdebenito, 2010).
In traditional supply chain inventory management, orders are the only information that firms within each stage can exchange. The development of information technology allows us to share demand and inventory information. Cachon and Fisher (2000) compare a traditional information policy that not sharing information versus sharing full information. They found that among the full information model, total cost is saved and lead time is cut as well, but information distortion would cause the bullwhip effect. More researchers address on this topic as well. Lee et al. (2000) address a simple two-level supply chain example, and show the value of information sharing. Yu et al. (2001) illustrate the benefits of supply chain partnership based on information sharing, supply chain members can reap benefits in terms of reductions among inventory levels and cost savings from partnerships with one another. Ganesh (2014) extends the model to a multi-product, multi-level supply chain, and shows the substitution among different products reduces the value of information sharing for all firms. If the firms are more upstream, the degree of substitution and number of substitution would be higher, and demand would be more correlated. In other words, the degree of information sharing is higher, the reduction in supply chain risk would be higher.

Given that the more and more complicated supply chain systems nowadays, companies would constantly review their inventory policies, to make balance between their working capital and supply chain responsiveness, related costs are even higher. So that the development of decision support tools to handle material flows and inventory strategy would be highly important in real world settings. Cruz-Gonzalez (2015) states that integration of sustainability dimensions (economic, social, environmental, ethical and time) into decision-support models would become more sustainable, practical and popular.

1.5 Research Objectives and Contributions

The main objective of this thesis is to develop mathematical programming models that can be used as a decision support tool for integrated supply chain to provide system-wide analysis for the entire chain. Joint replenishment cost is considered when ordering raw materials and through manufacture process. The integrated approach considers the supply chain as a whole, while the sequential approach solves inventory planning problem first, and supplier
selection problem second. Comparison between two approaches would show differences on costs and inventory planning schedules under each. Sensitivity analysis for the integrated model would show key factors that influence supplier selection.

The problem is formulated as a multi-period, multi-product inventory lot-sizing model in a serial supply chain, where raw materials are purchased from multiple suppliers at the first stage, and external demand from vendors occurs at the last stage. The demand is known and may change from period to period. Stages of this production-distribution serial structure correspond to different inventory locations. The first two stages stand for storage areas holding raw materials and finished products in a manufacturing facility, additional intermediate stages represent either manufacturing facilities or centralized warehouses. Last stage is a distribution center that directly serves customer demand. The objective function solving the problem is to minimize purchasing, production, inventory holding, and transportation costs, while completely satisfying raw material and product demand, as well as raw material quality requirement. Even though the problem is formulated as a serial supply chain network structure, the model can be expanded to more general chains.

The model is formulated as a multiple product problem, where replenishment, production, inventory and transportation operations are jointly considered in order to take advantage of potential economies of scale. An all-unit discount strategy is applied to stimulate demand as well as supplier competitiveness when ordering raw materials from suppliers. Full-truck-load is the shipment mode for shipping raw materials and final products. Positive lead times are allowed for all stages in the supply chain.

To overcome disadvantages associated with the increase in problem size, an analytical approach to reduce the size of the respective time-expanded supply chain network is proposed. This method is based on the identification of nodes and arcs in the transshipment network that cannot be used or accessed by any feasible solution due to positive lead times and a finite planning horizon.

1.6 Overview

Remainder of the thesis is organized as follow. Chapter 2 gives literature review for some of the drivers of supply chain optimization, especially when it is an integrated supply chain
system. A general idea and some relevant work in this field are described, innovation of this article is also presented.

Chapter 3 presents a mixed integer linear programming model for the dynamic multi-period inventory problem with supplier selection in a raw material supply chain structure. The proposed model analyzes the multi-product case, where multiple suppliers can all provide different types of raw materials, and it assumes that multiple raw materials may be used to manufacture each final product. A joint replenishment cost structure is considered for raw material procurement and production operations. In this chapter, mathematical model is described in two ways. At first, the model is solved integrally. And secondly, the model is solved sequentially, where a solution for inventory and production schedules is obtained first, and the result is used to obtain an optimal solution for supplier selection at the raw material procurement stage.

Chapter 4 gives an illustrative example based on the parameters and variables in chapter 3. Results are shown when solving the mathematical model integrally and sequentially. Comparison in supplier selection, raw material and final product inventory at each stage, and total cost under the two approaches are described. Results of sensitivity analysis for key factors to the integrated model is presented as well.

Chapter 5 presents the concluding remarks obtained from this thesis research and also new opportunities that have been identified as extension of the work.
Chapter 2 Literature Review

2.1 Introduction

In traditional supply chain management, decisions are usually made under the sequential approach, where every component or stage of the entire supply chain is optimized independently. Thus, the optimal solution of the prior stage is the input for the next decision stage. However, given that the sequential approach will generally lead to local optimal solutions, further improvement in cost savings can be overlooked. In this chapter, a review of some drivers of supply chain optimization is provided, including raw material procurement, inventory management, production scheduling and distribution planning. A general description on supplier selection models including raw material ordering quantity discount is presented as well. Finally, relevant research work on integrated supply chain optimization models is reviewed, where supplier selection, inventory planning and joint replenishment are considered simultaneously.

2.2 Supply Chain Inventory Models

Supply chain inventory systems are commonly implemented by companies when customers are distributed over large geographical areas. In these cases, products pass through multiple stages before they are received by the final customers. Considering the complexity produced by the interactions between different levels, the determination of inventory policies for multi-stage systems is considered to be a difficult problem.

For multi-stage, multi-customer supply chain system, Khouja (2003) addresses a three-stage supply chain model. Three inventory coordination mechanisms, “equal cycle time”, “integer-multiplier at each stage” and “integer powers of two multipliers at each firm” are discussed to solve a cost minimization problem. Result comparison between each method shows which type of coordination mechanisms behaves best under every condition. Raj et al. (2015) propose an integrated production-inventory model considering multiple items and multiple suppliers. They study the impact of different business policies (Ex, demand rate, optimum order size) to the inventory planning strategy at each stage of integrated marketing system.
To deal with a complicated inventory control process, companies categorize inventory items into a few groups, then take similar inventory control policy for each group. Mohammaditabar et al. (2012) present a method to categorize the items and find the best policy simultaneously. The authors use simulated annealing to search for appropriate solutions of inventory at each stage. “ABC Analysis” is a well-established categorization technique based on the “Pareto Principle” for determining which items should get priority in a company’s inventory management. Some other researchers also discuss steps implemented by “ABC Analysis” to analyze the actual case, and prove this technique has important theoretical and practical value in practice.

Multi-criteria optimization models have also been developed to address distribution concerns. Thirumalai (2001) develops such a model for serial cases with three stages that take both deterministic and stochastic demands into account. Rangarajan and Ravindran (2005) present a base period policy for decentralized supply chains. This policy considers that every retailer orders in integer multiples of some base period, which is arbitrarily set by the warehouse. The problem is solved over a finite planning horizon, using Wagner-Whitin’s (1958) model. Scott et al. (2014) design an integrated method using a combined analytic hierarchy process-quality function deployment with chance constrained programming and multiple criteria decision making techniques to deal with supplier selection and inventory planning strategy.

Researchers also conduct related investigations according to uncertainties. Arikan et al. (2014) investigate the impact of transportation lead-time variability on the economic and environmental performance of supply chain inventory system, and conduct a simulation study based on empirical data from an international container shipping supply chain. Kaya et al. (2014) develop a mixed integer linear programming model for a whole system focusing on strategic decisions and regarding uncertain demand in the system. By conducting numerical analysis toward uncertain characters in the system, they find that uncertain of the return amounts of used products effect the inventory strategy of entire supply chain more than uncertainty of demand.

2.3 Supply Chain Inventory Models with Ordering Quantity Discount

Quantity discount policy is decision-making for trade-off prices between suppliers and manufacturers while production is changeable due to demand fluctuations in a real market and it
is used as a mechanism under deterministic or stochastic environment dealing with buyer-vendor coordination. In practice, quantity discounts are frequently offered by transportation costs or raw material ordering costs to stimulate demand for larger and more profitable quantity orders. Therefore, research on quantity discounts becomes an ever-increasing important field in supply chain management.

Corbett (2000) derives the optimal quantity discount policy under asymmetric information and compares it to the situation where the supplier has full information. Munson and Rosenblatt (2001) suggest a mechanism that a company could create an integrated plan to dictate order and production quantities throughout a three-firm channel. They show the benefits of using quantity discounts on both ends of supply chain to decrease costs and incorporating quantity discounts on both ends decrease costs significantly comparing with that only on lower end. Giannoccaro and Pontrandolfo (2004) address that supply chain coordination can be pursued by using supply chain contracts under a decentralized decision-making approach. They proposed an example aimed at coordinating a three-stage supply chain based on revenue sharing mechanism to show supply chain inefficiency is decreased from 12.4% to 3.5% under supply chain contract. Lin and Lin (2014) develop an integrated inventory system involving defective items and quantity discount for optimal pricing and procurement schedules. Under the condition vendors produce larger quantity to reduce set-up cost, and offering all-unit quantity discount. They develop an algorithm to find optimal solutions for the entire chain.

Li and Liu (2006) develop a model to illustrate how to use quantity discount policy to achieve supply chain coordination. They first show that a coordination mechanism for joint decisions has better result than the decentralized decision situation. Secondly, by implementing quantity discount strategy, coordination could be achieved. Adeinat and Ventura (2013) propose a mixed integer nonlinear programming model regarding a supply chain inventory problem to find optimal number of orders for buyers, and decide order quantities to take use of quantity discounts by minimizing total replenishment cost per time. Zhang et al. (2014) address a modified quantity discount function based on order quantity and advance payment. The ranges of advanced payment ratio and quantity discount are derived to show the results for comparison.

Sher and Kim (2014) develop a supply chain coordination mechanism including a dominant manufacturer delivering seasonal products to a group of buyers where manufacturers could offer quantity discount to buyers. They develop a twice-stage ordering and production
system, where first order is placed some time in advance of selling season, and second order is
close to selling period. Demand forecast error is reduced and production time is smoothed out.
Heydari and Norouzinasab (2015) address a discount model to coordinate pricing and ordering
decisions in a two-echelon supply chain. Decentralized decision making on selling price and
order size is investigated at first, joint pricing and ordering decisions are extracted after that.
Finally, a coordination mechanism based on quantity discount is proposed to coordinate both
pricing and ordering decisions simultaneously.

2.4 Supplier Selection Models

In a manufacturing environment, even though raw material orders could be assigned to
only one selected supplier, maintaining a multi-supplier structure could enhance inventiveness
while reducing the sourcing risks. For example, in a serial supply chain network, supplier 1 is
competitive on costs, supplier 2 is competitive on quality of raw materials, while supplier 3 is
more competitive on minimizing the risk. So that manufacturers could select the “right” supplier
according to their qualifications. Hence it would be important to maintain a multiple-supplier
structure while reducing sourcing risks.

Supplier selection problems can be solved by the information gathering from the entire
supply chain. Lee et al. (2001) present a supplier selection and management system utilizing
information obtained from supplier selection and management system, purchasing strategy
system, supplier selection system, and supplier management system applied to an entire real-
world system. And then the effectiveness of the system can be verified by t-test as well as
correlation analysis.

Minner (2003) reviews inventory models with supply options. At first the author
discusses strategic aspects of supplier competition and the role of operational flexibility in global
sourcing. And then inventory models with several suppliers to avoid or reduce the effects of
shortage situations are presented. Ha and Krishnan (2006) outline a hybrid method incorporating
multiple techniques into an evaluation process, which enables a purchaser to do single sourcing
and multiple sourcing by calculating a combined supplier score, which represent both qualitative
and quantitative factors that impact on supply chain management.
Mendoza and Ventura (2010) develop a mathematical model for a serial system. They use a 98% effective power-of-two inventory policy to determine a near optimal order allocation and supplier selection problem. They conclude this policy is advantageous as it is simple to compute and can yield near optimal solutions. Mendoza and Ventura (2012) develop two mixed integer nonlinear programming models to select proper suppliers and determine allocation of order quantities in a multi-stage model while minimizing annual total cost for the entire supply chain. One model allows independent order quantity for each supplier, while the second model restricts order quantity to be of equal size. They investigate on the influence of transportation cost to supplier selection and inventory planning. Mendoza and Ventura (2013) compare full-truck-load transportation mode versus less-than-truck-load transportation mode for the entire supply chain, and conclude that considering transportation cost can not only affect order quantity allocation, but the selection of suppliers.

Fuzzy set theory is also used to solve the supplier selection problem. Amid et al. (2006) address a fuzzy multi-objective linear model to overcome the vagueness of the information in a fuzzy supplier selection problem. In the model, asymmetric fuzzy-decision making technique is applied so that decision maker could assign different weights to various criteria. Chen et al. (2006) present a fuzzy decision-making approach to deal with the supplier selection. First, linguistic values are used to assess ratings and weights for these factors. Then a hierarchy multiple criteria decision-making model based on fuzzy-sets theory is proposed to deal with it. Distance to both solutions are calculated to rank all suppliers mostly. Paul (2015) presents a rule-based fuzzy inference system model considering qualitative and quantitative selection criteria to select the most excellent suppliers. And risk factors are incorporated in the model by developing fuzzy input and output criteria.

Multi-criteria optimization models have also been developed to address supplier selection. Wadhwa and Ravindran (2007) address the supplier selection problem using a multi-level objective model that considers both qualitative and quantitative factors. In this research, goal programming, weighted objective and compromise programming are analyzed. Ravindran and Wadhwa (2009) present an overview of various multi-criteria techniques in the context of the Department of Defense. These authors suggest a two-step approach for the supplier selection problem. First, a multi-criteria optimization model is used to rank and pre-screen suppliers. Second, alternative multi-criteria methods are used to evaluate the set of suppliers selected from
the initial step. Abdollahi et al. (2015) use a combination of multiple criteria decision-making methods, then apply analytical network process to determine the weight of each supplier, and apply data envelopment analysis to rank them.

2.5 Integrated Supplier Selection Methods

Key role for supplier selection problem is a combination of the performance on cost, quality, delivery and service in achieving the objectives of a supply chain. Most of the literatures described above only address some of the key factors, thus integrated supplier selection models are important to have knowledge of overall.

Lee (2005) considers a single-manufacturer single-buyer supply chain serial problem where joint economic lot sizes of manufacturer’s raw material ordering, production and buyer’s ordering are determined to minimize the total cost for the entire supply chain per time unit. Alimardani et al. (2015) extend to a three-echelon, multi-product supply chain including multiple suppliers, multiple manufacturing facilities and multiple retailers. Aiming at achieving an agile network, a cost structure is defined that captures all kinds of operational costs. They use fuzzy-analytical network process framework to evaluate suppliers’ agility level, and then select suitable suppliers. They also develop a hybrid metaheuristic to provide production-inventory planning schedules.

Supplier selection can be treated as a multiple criteria decision-making problem that is affected by conflicting factors. Ghodsypour and Brien (1996) present a multi-criteria problem to decide how much to purchase from each supplier where capacity constraints exists considering both quantitative and qualitative factors. In the model, an analytical hierarchy process and linear programming method is proposed to choose best suppliers and optimum ordering quantity displayed among them to maximize the total value of purchasing. They work on a similar problem by providing a mixed integer non-linear programming model (with an algorithm proposed) to solve multiple sourcing problem in 2001. In that paper, total cost of the entire supply chain is considered, and buyer limitation is taken into account. Hsu et al. (2014) expand this method in electronics industry. By proving 13 criteria and utilizing analytic network process to determine relative weights to evaluate carbon performance among suppliers, the authors
demonstrate how to select the most appropriate suppliers in accordance with carbon management using the VlseKriterijumska Optimizacija I Kompromisno Resenje technique.

Dealing with the situation where input information is not known precisely, Amid et al. (2009) present a fuzzy multi-objective model to simultaneously consider imprecision of information and determine the order quantities to each supplier based on price breaks. A fuzzy weighted additive and mixed integer linear programming model is developed to construct relative decision functions where objectives have different relative importance. They present a max-min fuzzy model in 2011 to handle the vagueness of input data and an analytical hierarchy process is used to determine different weights of criteria. An order of suppliers could be listed with their performance on cost, quality and service.

Kheljani et al. (2009) address on the optimal solution for all members in a supply chain by optimizing the benefits of all the members and alignments of decisions between entities of a supply chain. So that total cost of the supply chain is minimized rather just the buyer’s side. Heydari (2014) proposes a time-based temporary price discount in each replenishment cycle to optimize safety stock globally, and determines an appropriate discount that is acceptable for both parties. The author shows safety stock coordination is profitable and supply chain members can share extra benefits fairly.

Chen (2011) addresses a structured methodology based on the supply chain integration architecture. The strategy is first identified using strengths, weaknesses, opportunities, and threats analysis. Data envelopment analysis is then used to screen potential suppliers, and multi-attribute decision-making method is adapted to rank potential suppliers. Sawik (2014) presents a stochastic mixed integer programming approach to integrated supplier selection and customer order scheduling. The researcher considers single sourcing (one supplier) and dual sourcing strategy (two suppliers) with conditional value-at-risk measure, and gives insights on the choice between the two sourcing schedules.

Ventura et al. (2013) present a multi-period inventory lot-sizing model for a single product in a serial supply chain, they develop a mixed integer nonlinear programming model to determine optimal inventory policy as well as raw material procurement in order to minimize total cost. Two transportation cost approximate functions are provided by them to simplify the model and reduce computational time. Sawik (2014) presents a stochastic mixed integer linear approach to integrated supplier selection and customer order scheduling in the presence of
supply chain disruption risks. The decision maker need to decide which single supplier, or which two different suppliers as the choice to minimize the total cost or to maximize the customer service level. Scott et al. (2014) propose an integrated method and design a decision support system using a combined analytic hierarchy process-quality function deployment and chance constrained optimization algorithm approach to select appropriate suppliers and allocates orders optimally between them.

In this thesis, we develop a mixed integer linear programming model to solve a problem including raw material supplier selection and inventory control strategy for each stage by extending Ventura et al.’s paper (2013) to the multiproduct case. We consider quantity discount for each supplier when ordering raw materials, and expand manufacturing process to two stages. Comparison for the sequential and integrated approaches to solve the problem is shown and sensitivity analysis on key factors is conducted in later chapters.
Chapter 3 Dynamic Inventory Model for a Serial Supply Chain Structure

3.1 Introduction

In this chapter, we aim to solve a dynamic joint replenishment problem in a multi-period inventory lot-sizing situation for multiple products in a serial supply chain. We describe a time-expanded static supply chain network in order to solve the problem under both integrated and sequential approaches. An integrated approach model aimed at minimizing total cost of the entire supply chain is developed to determine dynamic supplier selection and inventory planning policy at the same time. A sequential approach model is also developed that inventory planning and manufacturing policy is solved first, and supplier selection strategy is obtained according to that.

3.2 Dynamic Joint Replenishment Problem

Although the single-product situation is the most commonly used among inventory control theories, Goyal and Satir (1989) mention that when a product is packaged into more than one type of containers after production, cost savings are realized when these products are jointly manufactured and then individually packaged. Hence, each product does not need to be accountable for a major setup cost. Robinson and Lawrence (2004) present an application of the dynamic joint replenishment problem to a chemical company. The firm produces a variety of lubricants that are characterized based on their chemical compositions and package configurations. The manufacturing process considers a major setup cost related to a common operation where raw materials are blended and processed. Even though different products can be produced from a single batch of lubricant, minor setup operations are needed to recalibrate the packaging line and change levels every time products are switched. The authors also mention that the delivery of vaccines from manufacturing facilities to distribution centers is another example of the problem. In this case, given quality and security issues, each replenishment shipment from the manufacturing facility to a distribution center requires a dedicated refrigerated truck (major setup cost). In addition, each product incurs a minor setup cost related to product
labeling, packaging, temperature control, quality inspection, lot-size definition for product tracking and quality control, and forms for the Food and Drug Administration. Abouee-Mehrizi et al. (2015) consider a finite horizon multi-period inventory system where in each period two retailers can replenish their inventory from a supplier or via transshipment from the other retailer. Joint replenishment cost happens among inventory planning and ordering from suppliers. They study a stochastic control problem to determine the optimal joint replenishment and transshipment policies to minimize total expected cost over the season. From the analysis, optimal joint replenishment and transshipment policy partition among each region is obtained.

In the dynamic joint replenishment problem, \( n_M \) raw materials must be replenished for production to satisfy a deterministic dynamic demand over a planning horizon of \( n_T \) time periods. In this context, \( T = \{1, 2, ..., n_T\} \) defines the set of planning periods, and \( M = \{1, 2, ..., n_M\} \) defines the set of raw materials under consideration. Ordering costs are assumed to be comprised of a major ordering cost and several minor ordering costs. A major ordering cost is incurred each time raw materials are jointly replenished. Let \( s^t \) be the respective cost when an order takes place at time period \( t \). On the other hand, a minor ordering cost, denoted by \( s^{m,t} \), is charged each time raw material \( m \) is included in an order placed during period \( t \). Let \( \tilde{M} \subseteq M \) be the set of raw materials that incurred with minor ordering cost. So that \( s^t > s^{m,t} \), for any \( m \in \tilde{M} \). Therefore, joint-replenishment cost will be \( s^t + \sum_{m \in \tilde{M}} s^{m,t} \).

In this context, \( p^{m,t} \) and \( Q^{m,t} \) represent the price and order quantity of raw material \( m \) at time period \( t \). Hence total ordering cost would be \( s^t + \sum_{m \in \tilde{M}} s^{m,t} + p^{m,t} Q^{m,t} \). Figure 3-1 provides a graphical representation of the dynamic joint replenishment problem cost structure for \( n_M \) raw materials during time period \( t \). In this example, major ordering cost is shared among all raw materials in the order, while each individual raw material has respective minor and variable ordering costs associated with it.
3.3 Problem Statement and Formulation

A dynamic lot-sizing model with supplier selection in a serial supply chain structure is presented in this section. Let $J = \{1, 2, ..., n_J\}$ be the set of suppliers, $K = \{1, 2, ..., n_K\}$ be the set of supply chain stages, $T = \{1, 2, ..., n_T\}$ defines the set of planning periods and $M = \{1, 2, ..., n_M\}$ be the set of raw materials. In this context, stages 1 and 2 represent the raw material and finished product warehouses at the manufacturing facility, $K_D = \{2, 3, 4, ..., n_K - 1\}$ is the set of additional manufacturing facilities that hold finished products (including stage 2), and $n_K$ is the last distribution stage which ships products to customers. Figure 3-2 shows a simplified dynamic serial supply chain network. Let $l_{0,j}$ be the number of delivery lead time periods from supplier $j \in J$ to stage 1, and $l_k$ be the number of delivery lead time periods from stage $k \in K_D$ to stage $k + 1$. Finally, let $J_t = \{j \in J: t - l_{0,j} \geq 0\}$ be the subset of suppliers that can provide raw materials at period $t$. Note that at time period $t$ only suppliers with lead times less than $t$ can supply raw materials to stage 1.
A (time-expanded) static supply chain network is defined and represented by the general transshipment network $G_S = (N_S, A_S)$. In this network, the set of nodes $N_S$ and the set of arcs $A_S$ are defined as follows (Ventura et al., 2013):

$$N_S = \{(0, j, t): j \in J, t \in T\} \cup \{(k, t): k \in K, t \in T\},$$

and

$$A_S = \left\{ \left( (0, j, t), (1, t + l_{0,j}) \right): j \in J, t \in \{1, 2, ..., n_T - l_{0,j}\} \right\}$$

$$\cup \left\{ \left( (k, t), (k, t + 1) \right): k \in K, t \in T \setminus \{n_T\} \right\}$$

$$\cup \left\{ \left( (k, t), (k + 1, t + l_k) \right): k \in K \setminus \{n_K\}, t \in \{1, 2, ..., n_T - l_k\} \right\}.$$
Hence, nodes $(0, j, t_1)$ and $(k, t_2)$ will be feasible if and only if $t_1 \in T_{0,j}$ and $t_2 \in T_k$.

Consequently, the reduced network can be defined as follows (Ventura, et al., 2013):

$$G'_s = (N'_s, A'_s),$$

where

$$N'_s = \{(0, j, t): j \in J, t \in T_{0,j}\} \cup \{(k, t): k \in K, t \in T_k\},$$

and

$$A'_s = \\{(0, j, t), (1, t + l_{0,j})\}: j \in J, t \in T_{0,j}\}
\cup \\{(k, t), (k + 1, t + l_k)\}: k \in K\{n_K\}, t \in T_k\}.$$

In the dynamic joint replenishment problem, the demand of product $p$ is assumed to be known for each of the $n_T$ time periods during the planning horizon. Let $d^{p,t}$ represents the demand of product $p$ at time period $t$. The dynamic joint replenishment problem consists of the optimal order quantity $Q_{0,j}^{m,t}$ for raw material $m$ at time period $t$ from supplier $j$ and inventory level $I_k^{p,t}$ for product $p$ at time period $t$ at stage $k$. While $I_k^{p,0}$ represents the initial inventory of product $p$; $I_m^{m,0}$ represents initial inventory of raw material $m$.

An illustration of a time-expanded static supply chain network with three suppliers($n_J = 3$), four stages($n_K = 4$), and a planning horizon of five periods ($n_T = 5$) with the transshipment network $G_s$ as well as the reduced transshipment network $G'_s$ is presented in Figure 3-3, where the reduced transshipment network is included in dashed line. For example, node $(0, 1, 1)$ at supplier level stands for the node at stage 0, supplier 1 at time period 1. And it can provide raw materials to node $(1, 1)$, which stands for stage 1 at time period 1. Node $(1, 1)$ can ship final products to the warehouse, node $(2, 1)$, which stands for stage 2 at time period 1, and then connect to the distribution center stage, node $(3, 1)$ and the vendor stage, node $(4, 2)$. In this supply chain network, two types of raw materials (1 and 2) are required to manufacture two types of finished products (1 and 2). Lead times are positive for supplier 2 and 3, where orders are delivered one time unit (period) after they are submitted. Customer demand for product $p$ at time period $t$ is defined by $d^{p,t}$, $p \in P$ and $t \in T$. Notice that in this illustration, given the delivery lead times from raw material suppliers, only supplier 1 can satisfy the order quantity $Q_{0,1}^{m,t}$, $m \in M$ and $t \in T_{0,j}$ in the same period in which it is requested. Consequently, due to positive delivery lead times, raw materials and products need to be ordered in advance or
maintained in inventory in order to satisfy the demand observed at each time period. In the graphical illustration of the supply chain shown in the next page, flows are represented as two-dimensional vectors, indicating flows for raw materials (1 and 2) or finished products (1 and 2). Stages 1 and 2 represent manufacturing center, where raw materials could be processed into final products. Stage 3 is a centralized warehouse holding finished products for manufacturing center, while stage 4 defines the distribution center that directly serves customer demand to vendors. On-stage and in-transit inventory holding costs are considered in the problem. Both of them are linear cost functions, which means holding costs are directly related to the type and amount of raw materials or final products.

Assuming that trucks are the primary means of transportation. We use all-unit quantity discount policy when ordering raw materials from suppliers, which means average ordering cost is lower when the quantity is large. Let \( q \in Q_{m,j} = \{1,2,\cdots, n_q\} \) denotes price interval ordering raw material \( m \). Raw materials and final products are measured in storage capacity they occupy, and then loaded onto trucks. Full-truck-load is the only way of transportation we consider in this thesis, which means no matter the truck is loaded full or not, charge is the same. Let \( R = \{1,2,\cdots, n_R\} \) be the set of different types of trucks. Different types of trucks are measured in the volume of space they can load.
Figure 3-3: A Static Supply Chain Network with 3 Suppliers, 4 Stages and 5 Time Periods

3.4 Integrated Model Approach

This problem can be solved using operations research techniques, a mixed integer linear programming model is proposed to solve the dynamic inventory problem with supplier selection.
The problem is solved using the integrated approach, where raw material procurement, manufacturing and distribution decisions are considered simultaneously. The objective function for the model minimizes the total variable cost over \( n_T \) planning periods, including raw material purchasing, manufacturing, holding, and transportation costs. Formulation of the mixed integer linear programming model is presented below (P 3.1):

(P 3.1)

**Indices**

\( j \): number of suppliers providing raw materials, \( j \in J = \{1,2,\cdots,n_J\} \).

\( m \): type of raw materials, \( m \in M = \{1,2,\cdots,n_M\} \).

\( p \): type of finished products, \( p \in P = \{1,2,\cdots,n_P\} \).

\( k \): number of supply chain stages, \( k \in K = \{1,2,\cdots,n_K\} \).

\( t \): number of supply chain time periods, \( t \in T = \{1,2,\cdots,n_T\} \).

\( r \): type of trucks, \( r \in R = \{1,2,\cdots,n_R\} \).

\( q \): price interval for ordering material \( m \), \( q \in Q_{m,j} = \{1,2,\cdots,n_Q\} \).

**Parameters**

\( A \): very large number.

\( B \): very large number.

\( w_{rm} \): number of units of raw material \( m \) corresponding to fill one unit of storage capacity, \( m \in M \).

\( w_{rp} \): number of units of final product \( p \) corresponding to fill one unit of storage capacity, \( p \in P \).

\( r^p \): relative production rate for product \( p \) (in units/unit of capacity), \( p \in P \).

\( b_{m,t} \): capacity of supplier \( j \) for raw material \( m \) at time \( t \) (in units/time unit), \( j \in J, m \in M, t \in T_{0,j} \).

\( b^t_1 \): effective production capacity at stage 1 in period \( t \) (in units/time unit), \( t \in T_1 \).
\( b_{k}^{inv,t} \): inventory capacity at stage \( k \) in period \( t \) (in units/time unit), \( k \in K, t \in T_k \). Note that \( k = 1 \) represents the inventory capacity of raw materials, and \( k = 2, \ldots, n_K \) indicates the inventory capacity of final products.

\( b_{m}^{p} \): bill of materials ratio. It is defined as the number of units of raw material \( m \) required to produce one unit of final product \( p \), \( m \in M, p \in P \).

\( a_{m,j}^{m} \): perfect rate of supplier \( j \) for raw material \( m \) (probability that a unit is acceptable), \( j \in J, m \in M \).

\( a^{m} \): minimum acceptable perfect rate for raw material \( m \), \( m \in M \).

\( d_{p}^{t} \): demand for product \( p \) in time period \( t \) (in units/time unit), \( p \in P, t \in T \).

\( h_{1}^{m,t} \): unit holding cost for raw material \( m \) at the manufacturing stage from period \( t \) to period \( t + 1 \) (in \$/unit/time unit), \( m \in M, t \in T_1 \).

\( h_{k}^{p,t} \): unit holding cost for final product \( p \) at stage \( k \) from period \( t \) to period \( t + 1 \) (in \$/unit/time unit), \( p \in P, k \in K_D \cup \{n_K\}, t \in T_k \).

\( u_{1,j}^{m,t} \): holding cost for in-transit inventory of raw material \( m \) shipped from supplier \( j \) (stage 0) to stage 1 for periods \( t \) to \( t + l_{0,j} - 1 \) (in \$/unit/time unit), \( j \in J, m \in M, t \in T_{0,j} \).

\( u_{k+1}^{p,t} \): holding cost for in-transit inventory of final product \( p \) shipped from stage \( k \) to stage \( k+1 \) for periods \( t \) to \( t + l_{k} - 1 \) (in \$/unit/time unit), \( k \in K_D, p \in P, t \in T_k \).

\( p_{0,j,q}^{m,t} \): unit price of raw material \( m \) for supplier \( j \) in period \( t \) at price interval \( q \) (in \$/unit), \( j \in J, m \in M, t \in T_{0,j}, q \in Q_{m,j} \).

\( s_{0,j}^{t} \): major ordering cost for each order submitted to supplier \( j \) in period \( t \) (in \$/order), \( j \in J, t \in T_{0,j} \).

\( s_{0,j}^{m,t} \): minor ordering cost for raw material \( m \) included in the order submitted to supplier \( j \) in period \( t \) (in \$/order), \( j \in J, m \in M, t \in T_{0,j} \).

\( p_{1}^{p,t} \): variable production cost for finished product \( p \) in period \( t \) (in \$/unit), \( p \in P, t \in T_1 \).

\( s_{1}^{t} \): major ordering cost at the manufacturing stage during period \( t \) (in \$/order), \( t \in T_1 \).

\( s_{1}^{p,t} \): minor ordering cost for each finished product \( p \) manufactured at stage 1 in period \( t \) (in \$/order), \( p \in P, t \in T_k \).

\( s_{k}^{t} \): major ordering cost at stage \( k \) during period \( t \) (in \$/order), \( k \in K_D, t \in T_k \).
\(v_{0,j,r}^t\): fixed charge for a full truck load from supplier \(j\) to stage 1 when using truck type \(r\) (in $/truck), 
\(j \in J, t \in T_{0,j}, r \in R\).

\(\hat{v}_{k,r}^t\): fixed charge rate for a full truck load from stage \(k\) to stage \(k + 1\) when using truck type \(r\) (in $/truck), \(k \in K_D, t \in T_k, r \in R\).

\(VC_r\): capacity of truck type \(r\) (in units/truck), \(r \in R\).

\(Q_{j,m}^q\): boundary quantity of raw material \(m\) ordered from supplier \(j\) (in units), \(j \in J, m \in M, q \in Q_{m,j}\).

Continuous Variables

\(Q_{0,j}^{m,t}\): quantity of raw material \(m\) ordered from supplier \(j\) in period \(t\) (in units/order/time unit), \(j \in J, m \in M, t \in T_{0,j}\).

\(Q_{0,j,q}^{m,t}\): quantity of raw material \(m\) ordered from supplier \(j\) at price interval \(q\) in period \(t\) (in units/price interval/order/time unit), \(j \in J, m \in M, t \in T_{0,j}, q \in Q_{m,j}\).

\(I_{1}^{m,t}\): inventory level held at the manufacturing stage from time period \(t\) to time period \(t + 1\) (in units/time unit), \(t \in T_1, m \in M\).

\(I_{k}^{p,t}\): inventory level held at stage \(k\) from period \(t\) to period \(t + 1\) (in units/time unit), \(k \in K, t \in T_k, p \in P\).

\(x_{1}^{p,t}\): production lot size at the manufacturing stage at the beginning of period \(t\) (in units/time unit), \(t \in T_1, p \in P\).

\(R_{k}^{p,t}\): replenishment order quantity from stage \(k\) to stage \(k + 1\) in period \(t\) (in units/time unit), \(t \in T_k, p \in P, k \in K_D\).

Integer Variables

\(L_{0,j,r}^t\): number of full truck loads assigned to transport raw material from supplier \(j\) to stage 1 in period \(t\) using truck type \(r\), \(j \in J, t \in T_{0,j}, r \in R\).

\(L_{k,r}^t\): number of full truck loads assigned to transport finished product from stage \(k\) to stage \(k + 1\) in period \(t\) using truck type \(r\), \(k \in K_D, t \in T_k, r \in R\).
**Binary Variables**

- $\delta_{0,j}^t$: 1 if a replenishment order is submitted to supplier $j$ in period $t$; otherwise, 0; $j \in J, t \in T_{0,j}$.
- $\delta_{0,j}^{m,t}$: 1 if a replenishment order for raw material $m$ is submitted to supplier $j$ in period $t$; otherwise, 0; $j \in J, m \in M, t \in T_{0,j}$.
- $\delta_k^t$: 1 if a replenishment order is placed to stage $k$ in period $t$; otherwise, 0; $k \in K_D \cup \{n_k\}, t \in T_k$.
- $\delta_1^{p,t}$: 1 if a manufacturing order for finished product $p$ is placed to stage 1 in period $t$; otherwise, 0; $p \in P, t \in T_k$.
- $\varphi_{0,j,q}^{m,t}$: 1 if raw material $m$ is ordered from supplier $j$ to stage 1 in period $t$ falls at price interval $q$; otherwise, 0; $j \in J, t \in T_{0,j}, m \in M, q \in Q_{m,j}$.
- $\delta_1^t$: 1 if a replenishment order is placed at manufacturing stage in period $t$; otherwise, 0; $t \in T_1$.

**Objective Function**

Minimize $Z = \sum_{j \in J} \sum_{t \in T_{0,j}} (s_{0,j}^t \delta_{0,j}^t + \sum_{m \in M} (s_{0,j}^{m,t} \delta_{0,j}^{m,t} + \sum_{q \in Q_{m,j}} p_{0,j,q}^{m,t} \varphi_{0,j,q}^{m,t}))$

\[ + \sum_{t \in T_1} \left( s_{1}^t \delta_{1}^t + \sum_{p \in P} (s_{1}^{p,t} \delta_{1}^{p,t} + p_{1}^{p,t} \chi_{1}^{p,t}) \right) + \sum_{k \in K_D} \sum_{t \in T_k} s_k^t \delta_k^t \]

\[ + \sum_{m \in M} \sum_{t \in T_{1}} h_1^{m,t} r_1^{m,t} + \sum_{p \in P} \sum_{k \in K_D \cup \{n_k\}} \sum_{t \in T_k} h_k^{p,t} l_k^{p,t} \]

\[ + \sum_{j \in J} \sum_{m \in M} \sum_{t \in T_{0,s}} u_{1,j}^{m,t} q_{0,j}^{m,t} + \sum_{k \in K_D} \sum_{p \in P} \sum_{t \in T_k} u_k^{p,t} v_k^{p,t} \]

\[ + \sum_{r \in R} \sum_{j \in J} \sum_{t \in T_{0,j}} (v_{0,j,r}^t L_{0,j,r}^t) + \sum_{r \in R} \sum_{k \in K_D} \sum_{t \in T_k} (v_k^t L_k^t). \quad (3.1) \]

**Constraints**

Subject to
\[
\sum_{j \in J} Q_{0,j}^{m,t-1} + l_{1}^{m,t-1} = l_{1}^{m,t} + \sum_{p \in P} b_{m}^{p} x_{1}^{p,t}, \quad m \in M, t \in T_{1},
\]
\[
\sum_{q \in Q_{m,j}} Q_{0,j,q}^{m,t} = Q_{0,j}^{m,t}, \quad m \in M, j \in J, t \in T_{0,j},
\]
\[
x_{1}^{p,t} + l_{2}^{p,t-1} = l_{2}^{p,t} + y_{2}^{p,t}, \quad p \in P, t \in T_{2},
\]
\[
y_{k}^{p,t-1} + l_{k}^{p,t-1} = y_{k}^{p,t} + y_{k}^{p,t}, \quad p \in P, k \in K_{D} \setminus \{2\}, t \in T_{k},
\]
\[
y_{n_{k}}^{p,t-1} + l_{n_{k}}^{p,t-1} = d_{n_{k}}^{p,t} + l_{n_{k}}^{p,t}, \quad p \in P, t \in T_{n_{k}},
\]
\[
\sum_{j \in J} a_{0,j}^{m} x_{0,j}^{m,t-1} \geq a_{m} \sum_{j \in J} Q_{0,j}^{m,t-1}, \quad m \in M, t \in T_{1},
\]
\[
Q_{0,j}^{m,t} \leq l_{0,j}^{m,t} \frac{Q_{0,j}^{m,t}}{s_{0,j}^{m,t}}, \quad m \in M, j \in J, t \in T_{0,j},
\]
\[
\sum_{m \in M} A o_{j}^{m,t} \leq A o_{j}^{m,t}, \quad j \in J, t \in T_{0,j},
\]
\[
x_{1}^{p,t} \leq B o_{1}^{p,t}, \quad p \in P, t \in T_{1},
\]
\[
\sum_{p \in P} x_{1}^{p,t} \leq b_{1}^{t} \delta_{1}^{m}, \quad t \in T_{1},
\]
\[
\sum_{p \in P} y_{k}^{p,t} \leq B o_{k}^{m,t}, \quad k \in K_{D}, t \in T_{k},
\]
\[
\sum_{m \in M} w_{\text{raw}}^{m} l_{1}^{m,t} \leq b_{1}^{t} \text{inv}_{t}, \quad t \in T_{1},
\]
\[
\sum_{p \in P} w_{f}^{p} l_{k}^{p,t} \leq b_{k}^{t} \text{inv}_{t}, \quad k \in K_{D} \cup \{n_{k}\}, t \in T_{k},
\]
\[
\sum_{m \in M} w_{\text{raw}}^{m} Q_{0,j}^{m,t} \leq \sum_{r \in R} V C_{r} l_{0,j,r}^{t}, \quad j \in J, t \in T_{0,j},
\]
\[
\sum_{m \in M} w_{f}^{p} v_{k}^{p,t} \leq \sum_{r \in R} V C_{r} l_{k,r}^{t}, \quad k \in K_{D}, p \in P, t \in T_{k},
\]
\[
\varphi_{0,j,q}^{m,t} \varphi_{j,m}^{q-1} \geq Q_{0,j}^{m,t} \leq \varphi_{0,j,q}^{m,t} Q_{j,m}^{q}, \quad j \in J, q \in Q_{m,j}, m \in M, t \in T_{0,j},
\]
\[
\sum_{q \in Q_{m,j}} \varphi_{0,j,q}^{m,t} \leq 1, \quad j \in J, m \in M, t \in T_{0,j},
\]
\[
Q_{0,j}^{m,t} \geq 0, \quad \delta_{0,j}^{m,t} \in \{0,1\}, \quad j \in J, m \in M, t \in T_{0,j},
\]
\[
Q_{0,j,q}^{m,t} \geq 0, \quad j \in J, m \in M, t \in T_{0,j}, q \in Q_{m,j},
\]

(3.2)
In this formulation, the total cost is minimized while satisfying all time demands. Equation (3.1) represents the total variable cost function which includes purchasing, manufacturing, inventory, and transportation costs for \( n_T \) periods. Notice that no transportation cost is considered between stages 1 and 2, since all flows occur internally in the manufacturing facility. Equation set (3.2) guarantees flow balance between raw materials and finished products at the manufacturing stage. In these equations, in order to correctly calculate the raw material requirement for each unit of finished product, the manufacturing lot size of product \( p \) at the beginning of period \( t \), defined by \( X_{1p,t} \), is multiplied by the respective bill of materials ratio \( b_{m,p} \) (Valdebenito, 2010). Equation set (3.3) defines product balance at stage 2, where production is directly received from the manufacturing stage. Notice that since stages 1 and 2 are assumed to be in the same location, delivery lead times between these stages are assumed to be zero. Equation set (3.4) guarantees product balance for intermediate stages between stage 2 and the final stage, where customer demand is observed. Equation sets (3.5) and (3.6) determine that customer demand must be completely satisfied for each product and time period. Constraint set (3.7) guarantees that the average minimum acceptable perfect rate for each raw material will be satisfied at each period. In this case, given the effect of the delivery lead times, this requirement is imposed at the moment a raw material is received at the manufacturing stage. Constraint set (3.8) guarantees that the supplier capacity for each type of raw material will be satisfied at each period. This constraint set also activates the binary variable \( \delta_{0j}^{m,t} \), associated with each product included in the specific supplier order. Thus, the minor cost associated with each product can be correctly addressed in the objective function. Constraint set (3.9) is defined as a trigger constraint to activate the binary variable \( \delta_{0j}^{t} \), which identifies the time period \( t \) when an order is allocated.
to supplier $j$. Considering that the maximum number of raw materials that can be included in the order is equivalent to the total number of raw materials, the cardinality of set $A$ is used as the upper bound for this constraint. Constraint set (3.10) is used to trigger the binary variable $\delta_{p,t}^j$, which is activated every time an order is received by the manufacturing stage to produce product $p$ during time period $t$. Constraint set (3.11) defines the maximum production capacity throughout each time period. Given that a flexible production line is assumed, then manufacturing capacity is defined as joint capacity, where changeover times between products are assumed to be imperceptible.

Assuming unlimited transportation capacity for final products from the manufacturing stages and distribution centers, Constraint set (3.12) is only used as a trigger for the variable $\delta_{k,t}$, where $B$ represents a very large number. Sets of inequalities (3.13) and (3.14) correspond to the inventory capacity for raw materials and finished products for different stages in the supply chain. In those cases, inventory capacity is counted in equivalent units of raw material and final product. Constraint set (3.15) calculates the total transportation requirement for each raw material supplier and time period. The right-hand sides of the equations classify the transportation volume, which guarantees raw materials ordered from suppliers could be loaded by trucks in stage 1. Constraint set (3.16) calculates the total transportation requirements for each stage and time period. The right-hand side of the equation classifies the transportation volume, which guarantees finished products could be loaded by trucks on stage 2 and stage 3. Constraint sets (3.17) and (3.18) guarantee that raw material order quantity from suppliers will fall in the corresponding price interval. Finally, Constraint sets (3.19) - (3.27) describe the nature of the variables considered in the model.

3.5 Sequential Model Approach

As shown in the literature review, the sequential approach is the most common method used in practice. When taking this type of approach, raw material supply decisions are made independent of the manufacturing and distribution decisions. Under this approach, the inventory planning problem (Scenario 1) is solved first, raw material requirements for each period during the planning horizon (Scenario 2) is generated according to the solution of scenario 1. Based on
this requirement, a second problem is solved, from which the optimal raw material procurement schedule is defined. Thus, the sequential approach solves the problem in two independent scenarios, which are only linked by raw material requirement for each time period.

In the first scenario, the manufacturing and inventory planning problem is analyzed, and optimal solution is obtained under model (P 3.2). This mixed integer linear programming model minimizes the production, inventory, and transportation costs of the supply chain problem. The mathematical model is defined as follows:

(P 3.2)

\[
\begin{align*}
\text{Minimize } Z &= \sum_{t \in T_1} \left( s_1^t \delta_1^t + \sum_{p \in P} \left( s_1^p \delta_1^p + p_1^p \chi_1^p \right) \right) + \sum_{k \in K_D} \sum_{t \in T_k} s_k^t \delta_k^t + \sum_{p \in P} \sum_{k \in K_D} \sum_{t \in T_k} k_k^p l_k^p \\
&\quad + \sum_{k \in K_D} \sum_{p \in P} \sum_{t \in T_k} u_{k+1}^p y_k^p + \sum_{r \in R} \sum_{k \in K_D} \sum_{t \in T_k} (v_r^t I_k^t).
\end{align*}
\]

Subject to equations (3.4) – (3.6), (3.10) – (3.12), (3.14), (3.16), (3.21), (3.23) and (3.27).

In this formulation, equation (3.28) represents the total variable cost function which includes production, on-stage and in-transit inventory holding costs for final products, and transportation costs during \( n_T \) time periods. Raw material procurement schedule is assumed to be irrelevant to the production and distribution decisions. Remaining constraints are defined as before. Correspondingly, a supply chain network is shown as Figure 3-4 below.
Once the first scenario of the sequential approach model has been solved, then optimal raw material procurement schedule is obtained from a second optimization scenario.

Setting $q^{m,t} = \sum_{p \in P} b^p_{m,t} X^{p,t}_1$, $m \in M$ and $t \in T_1$, where $X^{p,t}_1$ is the optimal production level for
product \( p \) at time period \( t \) generated by model (P 3.2), then another mixed integer linear programming model to solve the raw material procurement problem is defined by model (P 3.3) as follows:

\((P \ 3.3)\)

\[
\text{Minimize } Z = \sum_{j \in J} \sum_{t \in T_{0,j}} \left( s_{0,j}^t \delta_{0,j}^t + \sum_{m \in M} \left( s_{0,j}^m \delta_{0,j}^m + \sum_{q \in Q_{m,j}} p_{0,j,q}^m Q_{0,j,q}^m \right) \right) + \sum_{m \in M} \sum_{t \in T_1} h_{1,t}^m l_{1,t}^m \\
+ \sum_{j \in J} \sum_{m \in M} \sum_{t \in T_{0,j}} u_{1,j}^m Q_{0,j}^m + \sum_{r \in R} \sum_{j \in J} \sum_{t \in T_{0,j}} \left( v_{0,j,r}^t l_{0,j,r}^t \right). 
\]  

(3.29)

Subject to

\[
\sum_{j \in J} Q_{0,j}^m l_{0,j}^m + I_{1,t}^m = I_{1,t}^m + q^m, \quad m \in M, t \in T_1. 
\]  

(3.30)

And Equations (3.3), (3.7) – (3.9), (3.12), (3.13), (3.15), (3.17) – (3.20), (3.22), (3.24) and (3.26).

The objective function for this part is represented by equation (3.29), which minimizes raw material procurement costs (ordering, variable and transportation costs), and raw material inventory costs. Equation (3.30) represents raw material balance. Remaining constraints are defined as before. Notice that this problem is solved as a one-stage warehouse problem, where the optimal ordering policy is defined to satisfy a known deterministic demand.
Chapter 4 Computational Results

4.1 Introduction

In this chapter, a four-stage serial supply chain example including manufacturer (2 stages), warehouse and distribution center (similar to Valdebenito, 2010) is presented to illustrate the model in chapter 3. Under this supply chain system, three raw material suppliers can provide two types of raw materials to the manufacturing stage in order to process two types of final products. When ordering raw materials, a quantity discount strategy is taken into account only for supplier 2 and supplier 3.

The problem has been solved using both sequential and integrated approaches. Under the integrated approach, the entire supply chain is analyzed altogether at once. Under the sequential approach, production/distribution decisions are solved first and raw material procurement schedule is obtained according to manufacturing policies. Results for both are discussed in this chapter.

4.2 Illustrative Example

In the supply chain network, the first two stages are assumed to be located at the manufacturing factory. Stage 1 allows storage for raw materials while stage 2 allows storage for final products. Raw materials are processed into final products during stage 1, then sent to storage in stage 2. Stage 3 represents a regional warehouse. The last stage stands for a distribution center near customer locations that can directly distribute deterministic demand of two types of final products to vendors during a five time-period (month) planning horizon. Raw materials and final products are both measured in the volume they occupy.

In order to deliver raw materials as well as final products, we consider full-truck-load as transportation mode. Three types of trucks can load raw materials and final products from one stage to the next. In-transit and on-stage holding costs are considered in the example. Related parameters are shown below.
Figure 4-1 defines customer demand. Table 4-1 describes production capacity and inventory capacities at each stage. Table 4-2 describes bill of material ratio, and Table 4-3 describes equivalent amount of raw materials and final products measured in cubic meters. Table 4-4 defines information of each supplier. Table 4-5 defines production cost as well as inventory holding cost for each stage. Table 4-6 provides information for major and minor ordering costs during manufacture process. Table 4-7 defines initial and ending inventory for raw materials or final products at each stage. Table 4-8 defines price intervals when ordering raw materials. Table 4-9 shows information of different types of trucks shipping raw materials and final products. Table 4-10 provides in-transit inventory holding cost. Minimum acceptable perfect rate for raw materials is assumed to be $\alpha^m = 0.955$ for $m \in M$.

![Customer Demand](image)

**Figure 4-1: Customer Demand**

Relative production rates are assumed to be $r^1 = 2.0$ units/unit of capacity for product 1 and $r^2 = 1.5$ units/unit of capacity for product 2 at one production line. In this example, unit of capacity is defined as hour. Each production line has a total production capacity of 400 hours per month. Total production capacities are assumed to change through the planning horizon as the number of production lines available during time period 1 through time period 5 are 5, 3, 2, 2 and 5. Hence, by multiplying the number of production lines and production line capacity, we
can get production capacity (measured in hours) during each month. Conversely, inventory capacities are assumed to be constant for the four stages during the entire planning horizon. In this case, equivalent units are expressed in cubic meters ($m^3$), assuming that storage capacity is mainly limited by the space occupied by raw materials and final products.

**Table 4-1: Production and Inventory Capacities**

<table>
<thead>
<tr>
<th>Period (month)</th>
<th>Production Capacity (hours/month)</th>
<th>Stage 1 ($m^3$/month)</th>
<th>Stage 2 ($m^3$/month)</th>
<th>Stage 3 ($m^3$/month)</th>
<th>Stage 4 ($m^3$/month)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,000</td>
<td>15,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
</tr>
<tr>
<td>2</td>
<td>1,200</td>
<td>15,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
</tr>
<tr>
<td>3</td>
<td>800</td>
<td>15,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
</tr>
<tr>
<td>4</td>
<td>800</td>
<td>15,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
</tr>
<tr>
<td>5</td>
<td>2,000</td>
<td>15,000</td>
<td>12,000</td>
<td>12,000</td>
<td>12,000</td>
</tr>
</tbody>
</table>

**Table 4-2: Bill of Material Ratio (expressed in units of raw materials per unit of final product)**

<table>
<thead>
<tr>
<th>Product</th>
<th>Raw Material 1</th>
<th>Raw Material 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0</td>
<td>3.0</td>
</tr>
<tr>
<td>2</td>
<td>3.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

**Table 4-3: Equivalence Factors $w_{raw}^m$ and $w_f^p$ (expressed in $m^3$/unit)**

<table>
<thead>
<tr>
<th>Type</th>
<th>Material</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.8</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>2.0</td>
<td>3.0</td>
</tr>
</tbody>
</table>

**Table 4-4: Description of Raw Material Suppliers**

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Lead Time (months)</th>
<th>Capacity Material 1 (units/month)</th>
<th>Capacity Material 2 (units/month)</th>
<th>Quality Rate 1</th>
<th>Quality Rate 2</th>
<th>Major Cost ($/order)</th>
<th>Minor Cost Material 1 ($/order)</th>
<th>Minor Cost Material 2 ($/order)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>9,000</td>
<td>9,000</td>
<td>0.95</td>
<td>0.95</td>
<td>9,000</td>
<td>8,000</td>
<td>9,000</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3,500</td>
<td>3,500</td>
<td>0.96</td>
<td>0.97</td>
<td>5,000</td>
<td>4,000</td>
<td>3,000</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>4,000</td>
<td>4,000</td>
<td>0.97</td>
<td>0.96</td>
<td>4,000</td>
<td>3,000</td>
<td>3,500</td>
</tr>
</tbody>
</table>
Table 4-5: Inventory Holding and Production Costs by Period

<table>
<thead>
<tr>
<th>Period (month)</th>
<th>Stage 1 ($/unit/month)</th>
<th>Stage 2 ($/unit/month)</th>
<th>Stage 3 ($/unit/month)</th>
<th>Stage 4 ($/unit/month)</th>
<th>Production Cost ($/unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m=1 m=2 p=1 p=2 p=1 p=2 p=1 p=2 p=1 p=2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7 6 9 10 10 9 10 12 240 250</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>7 6 9 10 10 9 10 12 250 300</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7 6 9 10 10 9 10 12 290 360</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>7 6 9 10 10 9 10 12 300 360</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>7 6 9 10 10 9 10 12 240 300</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Note: m stands for raw materials, p stands for products)

Table 4-6: Major and Minor Fixed Costs during Manufacture Process

<table>
<thead>
<tr>
<th></th>
<th>Stage 1 ($/order)</th>
<th>Later stages ($/order)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major fixed</td>
<td>5,000</td>
<td>4,000</td>
</tr>
<tr>
<td>Minor product 1</td>
<td>1,500</td>
<td></td>
</tr>
<tr>
<td>Minor product 2</td>
<td>1,250</td>
<td></td>
</tr>
<tr>
<td>Fixed Cost</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4-7: Initial and Ending Inventory Levels Required at Each Stage (expressed in units)

<table>
<thead>
<tr>
<th>Period</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
<th>Stage 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material 1</td>
<td>Material 2</td>
<td>Product 1</td>
<td>Product 2</td>
</tr>
<tr>
<td>Initial</td>
<td>100</td>
<td>100</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>Ending</td>
<td>400</td>
<td>300</td>
<td>500</td>
<td>400</td>
</tr>
</tbody>
</table>

Table 4-8: Price Interval for Ordering Raw Materials (expressed in dollars per unit)

<table>
<thead>
<tr>
<th>Interval</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
<th>Supplier 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material 1</td>
<td>Material 2</td>
<td>Material 1</td>
</tr>
<tr>
<td>Interval 1 (&lt;1000)</td>
<td>9</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Interval 2 (1000-2000)</td>
<td>9</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Interval 3 (&gt;2000)</td>
<td>8</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 4-9: Truck Information for Shipping Raw Materials and Final Products

<table>
<thead>
<tr>
<th>Type</th>
<th>Truck Capacity (m^3/truck)</th>
<th>Fixed charge at stage 0 ($/truck)</th>
<th>Stage 2 ($/truck)</th>
<th>Stage 3 ($/truck)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Supplier 1</td>
<td>Supplier 2</td>
<td>Supplier 1</td>
</tr>
<tr>
<td>Type 1</td>
<td>280</td>
<td>600</td>
<td>550</td>
<td>525</td>
</tr>
<tr>
<td>Type 2</td>
<td>300</td>
<td>625</td>
<td>575</td>
<td>550</td>
</tr>
<tr>
<td>Type 3</td>
<td>320</td>
<td>800</td>
<td>750</td>
<td>725</td>
</tr>
</tbody>
</table>

Table 4-10: In-transit Inventory Holding Cost (expressed in dollars per unit per shipment)

<table>
<thead>
<tr>
<th>Type</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
<th>Supplier 3</th>
<th>Stage 2</th>
<th>Stage 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
<td>12</td>
<td>11</td>
<td>10</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>Type 2</td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>
4.3 Solution for the Integrated Model Approach

The illustrative example has been implemented on SAS 9.4 under the optimization package with “MILP” solver, using the “Branch and Cut” algorithm with solution status “Optimal within Relative Gap”. Running times for solving both problems are always less than 30 seconds. Model (P 3.1) is used to find the optimal strategy for the entire supply chain. The optimal overall strategy for the integrated approach is presented in the tables and figure below.

Under the integrated approach, three tables and one figure will show the production and shipping schedules at each node. Optimal solution is shown in Table 4-11. From the table, more amount of product 2 is manufactured than product 1; production occurs at time periods 1, 2 and 5. Behavior of shipping is measured from stage 3 to stage 4 (warehouse to distribution center). The shipping of final products occurs only at time period 1 and time period 3. More amount of raw material 1 is required than raw material 2, and they are purchased at time periods 1 and 4.

Table 4-11: Optimal Production and Shipping Schedule for the Integrated Approach
(expressed in units per month)

<table>
<thead>
<tr>
<th>Period (month)</th>
<th>Production</th>
<th>Shipping</th>
<th>Raw Material Req.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Product 1</td>
<td>Product 2</td>
<td>Product 1</td>
</tr>
<tr>
<td>1</td>
<td>--</td>
<td>1,857</td>
<td>800</td>
</tr>
<tr>
<td>2</td>
<td>1,205</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>--</td>
<td>--</td>
<td>1,600</td>
</tr>
<tr>
<td>4</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>795</td>
<td>643</td>
<td>--</td>
</tr>
<tr>
<td>Total</td>
<td>2,000</td>
<td>2,500</td>
<td>2,400</td>
</tr>
<tr>
<td>Average</td>
<td>1,000</td>
<td>1,250</td>
<td>1,200</td>
</tr>
</tbody>
</table>

The optimal procurement schedule under the integrated approach is presented in Table 4-12. Under this solution, both raw material 1 and raw material 2 are ordered from supplier 1 and supplier 3. While supplier 3 provides more amount of raw materials than supplier 1.
Table 4-12: Optimal Raw Material Procurement Schedule for the Integrated Approach (expressed in units per month)

<table>
<thead>
<tr>
<th>Period (month)</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
<th>Supplier 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material 1</td>
<td>Material 2</td>
<td>Material 1</td>
<td>Material 2</td>
</tr>
<tr>
<td>1</td>
<td>5,471</td>
<td>3,614</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Total</td>
<td>5,471</td>
<td>3,614</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Average</td>
<td>5,471</td>
<td>3,614</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

Optimal inventory levels for the integrated approach are presented in Table 4-13. On stage 1, inventories for raw materials 1 and 2 are always 0. On stage 2, more amount of product 1 is held in inventory than product 2. On stage 3, only product 2 is held in inventory. On stage 4, inventories for product 1 and product 2 are held at time periods 1, 2 and 4.

Table 4-13: Optimal Inventory Levels per Stage for the Integrated Approach (expressed in units per month)

<table>
<thead>
<tr>
<th>Period (month)</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
<th>Stage 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material 1</td>
<td>Material 2</td>
<td>Product 1</td>
<td>Product 2</td>
</tr>
<tr>
<td>Initial</td>
<td>100</td>
<td>100</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>500</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1,705</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>105</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>105</td>
<td>7</td>
</tr>
<tr>
<td>Ending</td>
<td>400</td>
<td>300</td>
<td>500</td>
<td>400</td>
</tr>
</tbody>
</table>

A graphical representation of the optimal solution for the integrated approach is presented in the following Figure 4-2.
4.4 Solution for the Sequential Model Approach

Under the sequential approach, the optimization problem is divided into two scenarios. In scenario 1, we solve the production/distribution problem to obtain manufacture and inventory
planning schedules. Then we move forward to scenario 2 to obtain the raw material procurement schedule.

Based on this condition, model (P 3.2) is solved first. After the calculation of raw material requirements at each time period, we solve the model (P 3.3) to obtain raw material procurement schedule. Results are collected in the following tables and figure.

In Table 4-14, more amount of product 2 is manufactured than product 1; production occurs at time periods 1 and 5. Behavior of shipping is measured from stage 3 to stage 4. The shipping of final products occurs at time period 1 and time period 3. More amount of raw material 1 is required than raw material 2, and they are purchased during time periods 1 and 4.

Table 4-14: Optimal Production and Shipping Schedule for the Sequential Approach
(expresssed in units per month)

<table>
<thead>
<tr>
<th>Period (month)</th>
<th>Production</th>
<th>Shipping</th>
<th>Raw Material Req.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Product 1</td>
<td>Product 2</td>
<td>Product 1</td>
</tr>
<tr>
<td>1</td>
<td>1,100</td>
<td>2,175</td>
<td>2,360</td>
</tr>
<tr>
<td>2</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>--</td>
<td>--</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>900</td>
<td>325</td>
<td>--</td>
</tr>
<tr>
<td>Total</td>
<td>2,450</td>
<td>2,850</td>
<td>2,400</td>
</tr>
<tr>
<td>Average</td>
<td>612.5</td>
<td>712.5</td>
<td>1,200</td>
</tr>
</tbody>
</table>

The optimal procurement schedule for the sequential approach is presented in Table 4-15. Under this solution, raw material 1 is ordered from supplier 1 and supplier 3, while raw material 2 is ordered from all suppliers.

Table 4-15: Optimal Raw Material Procurement Schedule for the Sequential Approach
(expresssed in units per month)

<table>
<thead>
<tr>
<th>Period (month)</th>
<th>Supplier 1</th>
<th>Supplier 2</th>
<th>Supplier 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material 1</td>
<td>Material 2</td>
<td>Material 1</td>
<td>Material 2</td>
</tr>
<tr>
<td>1</td>
<td>8,625</td>
<td>7,550</td>
<td>--</td>
<td>2,517</td>
</tr>
<tr>
<td>2</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Total</td>
<td>8,625</td>
<td>7,550</td>
<td>--</td>
<td>2,517</td>
</tr>
<tr>
<td>Average</td>
<td>8,625</td>
<td>7,550</td>
<td>--</td>
<td>2,517</td>
</tr>
</tbody>
</table>
Optimal inventory levels for the sequential approach are presented in Table 4-16. On stage 1, two types of raw materials are held from time period 2 to time period 4. On stage 2, only product 2 is held in inventory. On stage 3, more amount of product 2 is held in inventory than product 1. On stage 4, more amount of product 1 is held in inventory than product 2.

Table 4-16: Optimal Inventory Levels per Stage for the Sequential Approach (expressed in units per month)

<table>
<thead>
<tr>
<th>Period (month)</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
<th>Stage 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material 1</td>
<td>Material 2</td>
<td>Product 1</td>
<td>Product 2</td>
</tr>
<tr>
<td>Initial</td>
<td>100</td>
<td>100</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>85</td>
</tr>
<tr>
<td>2</td>
<td>2,875</td>
<td>2,517</td>
<td>0</td>
<td>85</td>
</tr>
<tr>
<td>3</td>
<td>2,875</td>
<td>2,517</td>
<td>0</td>
<td>85</td>
</tr>
<tr>
<td>4</td>
<td>2,875</td>
<td>2,517</td>
<td>0</td>
<td>85</td>
</tr>
<tr>
<td>Ending</td>
<td>400</td>
<td>300</td>
<td>500</td>
<td>400</td>
</tr>
</tbody>
</table>

A graphical representation (Figure 4-3) for the optimal solution of the sequential approach is shown as below:
Figure 4-3: Optimal solution for the Sequential Approach
4.5 Comparison of Sequential Approach vs Integrated Approach

Required CPU times for these two approaches are slightly different. It takes 19 seconds to solve the two sub-problems under the sequential approach, and 25 seconds to solve the problem under the integrated approach. The optimal procurement schedule is significantly impacted by the selected optimization approach. Comparing to the sequential approach, 6.00% of total cost is saved when we apply the integrated approach, which is $120,513.

The percentage of volume allocated to each supplier for each type of raw material is graphically represented in Figure 4-4, where graph (a) and (b) show the percentage of raw materials allocated to each supplier under the sequential approach, and graph (c) and (d) show the raw material allocation percentage under the integrated approach.

Under the integrated approach, 73% of raw material 1 and 67% of raw material 2 are allocated to supplier 1. Under the sequential approach, nearly 50% of product 1 and over 75% of product 2 are assigned at the first time period defined by solving the inventory planning problem. To satisfy the respective raw material requirements, given the positive delivery lead time of suppliers 2 and 3, supplier 1 is the only alternative for the problem. Consequently, the optimal solution for the production/distribution problem forces the selection of relatively expensive raw material supplier among the available alternatives. Thus, when the problem is sequentially solved, it is possible to realize that production/distribution decisions impact the raw material procurement schedule negatively.

Conversely, when the integrated approach is implemented, about 46% of raw material 1 and 32% of raw material 2 are ordered from supplier 1. Remaining raw materials are allocated to supplier 3, and supplier 2 is no longer a choice under this approach. As the problem is solved integrally, supplier inventory no longer plays a critical role in raw material procurement such that some orders of raw material 1 and raw material 2 are shifted from the more expensive supplier 1 to the relatively cheaper supplier 3.
A graphical representation of inventory levels at each time period is presented in Figure 4-5. In this figure, it can be seen that the sequential approach tends to maintain higher inventory levels than the integrated approach in general.

On stage 1, the sequential approach solution has more stable inventory for raw materials 1 and 2 from time period 2 till time period 4, while the integrated approach solution has no
inventory for raw material 1 and raw material 2 during all time periods. On stage 2, the sequential approach solution holds more inventory for product 2, while the integrated approach solution has more inventory on hold for product 1. On stage 3, the integrated approach outperforms the sequential approach with less amount of product 2 on hold throughout each time period. Finally on stage 4, although both of the approaches have inventory for product 1 and product 2, the integrated approach outperforms the sequential approach with less inventory during time period 2 and time period 3. On average, the sequential approach generates higher inventory levels than the integrated approach. This situation can be explained by the larger production lot sizes concentrated within one time period under the sequential approach with respect to the integrated approach.

In general, the integrated approach should generate a superior solution than the sequential approach. For the example we described, a total of 6.00% cost savings are obtained when the integrated approach is used (Table 4-17). Even though the sequential approach takes advantage of the economies of scale produced by larger production and distribution lot sizes, the integrated approach can generate a superior solution by taking the effects of raw material procurement decisions into account. Departing the total costs into variable and fixed costs, the sequential approach generates more competitive alternative (0.09% cost savings) for fixed costs (major and minor ordering costs and shipping cost). And the integrated approach compensates for the extra variable costs (raw material procurement cost, production cost and inventory holding cost) by generating cost savings of 6.84%. Consequently, the integrated approach is able to account for all the factors that finally produces a superior solution to the entire problem.
Figure 4-5: Inventory at each Stage for both Sequential and Integrated Approach
Table 4-17: Optimal Solutions for the Integrated and Sequential Approaches

<table>
<thead>
<tr>
<th>Type</th>
<th>Item</th>
<th>Total Cost ($)</th>
<th>Percentage (a-b)/b * 100%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Sequential (a)</td>
<td>Integrated (b)</td>
</tr>
<tr>
<td>Variable</td>
<td>Raw Material</td>
<td>182,683</td>
<td>169,428</td>
</tr>
<tr>
<td></td>
<td>Production</td>
<td>1,121,250</td>
<td>1,149,200</td>
</tr>
<tr>
<td></td>
<td>Holding</td>
<td>581,504</td>
<td>446,071</td>
</tr>
<tr>
<td></td>
<td>Subtotal</td>
<td>1,885,437</td>
<td>1,764,699</td>
</tr>
<tr>
<td>Fixed</td>
<td>Raw Material</td>
<td>51,500</td>
<td>47,000</td>
</tr>
<tr>
<td></td>
<td>Shipping</td>
<td>160,350</td>
<td>156,075</td>
</tr>
<tr>
<td></td>
<td>Production</td>
<td>31,500</td>
<td>40,500</td>
</tr>
<tr>
<td></td>
<td>Subtotal</td>
<td>243,350</td>
<td>243,575</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>2,128,787</td>
<td>2,008,274</td>
</tr>
</tbody>
</table>

4.6 Sensitivity Analysis

In order to measure the effects of some important factors of raw material purchasing decision on the integrated approach, 7 key parameters are changed to see how procurement behavior is influenced: “Major Ordering Cost”, “Minor Ordering Cost”, “Inventory Holding Cost”, “Minimum Perfect Rate”, “Price Interval”, “Discount Quantity” and “Supplier Transportation Cost”. A total of 28 scenarios are considered, results are shown in the following tables.

In Table 4-18, “Major Ordering Cost”, “Minor Ordering Cost” and “Inventory Holding Cost” are maintained (100%), decreased (0 or 50%), or increased (150%). Holding costs include on-stage and in-transit inventory holding costs, while changes in holding and major ordering costs affect all stages in the supply chain, changes in minor ordering cost only affect the raw material procurement and manufacturing stages.

Considering inverse effect of ordering and inventory holding costs on order quantities, it is important to avoid neutralization of these two parameters. Thus, when holding costs are increased or decreased, major and minor ordering costs are maintained. Similarly, when major and minor ordering costs are changed simultaneously, they should follow the same direction (decrease or increase). Note that the percentages for purchasing raw materials are rounded to the closest second decimal, so that small quantities can be identified.
The original optimal solution for the problem (scenario 1) allocates all the volume to supplier 1 and supplier 3. Under this solution, 46.36% of raw material 1 and 53.64% of raw material 2 are allocated to supplier 1, 32.27% of raw material 1 and 67.73% of raw material 2 are allocated to supplier 3, while supplier 2 is not considered.

When holding cost is individually increased (scenario 2), very little change occurs in supplier selection. When holding cost is decreased individually (scenario 3), volume of raw material 1 allocated to supplier 1 is increased from 46.36% to 61.01%. And the allocated volume of raw material 2 to the same supplier is increased from 32.27% to 42.59%. It can be explained that lowering holding cost makes supplier 1 a more attractive supplier in general as it has relatively higher in-transit inventory holding cost than supplier 2 and supplier 3.

When minor ordering cost is changed (increase 50%, decrease 50%, or decrease to 0) individually (scenario 4 to scenario 6), there is no significant change to the optimal solution of the problem. Minor ordering cost seems to have no significant influence on the optimal solution under the integrated approach.

When major ordering cost is increased individually (scenarios 8 and 9), no significant change occurs in the optimal solution. When major ordering cost is decreased (decrease 50%, or decrease to 0), significant change of raw material allocation happens to raw material 2, but not to raw material 1. Volume of raw material 2 allocated to supplier 2 is increased from 0 to 29.46%, the volume allocated to supplier 3 is decreased from 67.73% to 35.71%, and the volume allocated to supplier 1 is changed slightly, from 32.27% to 34.82%. In these two scenarios, when major ordering cost is reduced, supplier 2, with relatively higher ordering cost, is more attractive than supplier 3.

From scenario 10 to scenario 13, major and minor ordering costs are changed simultaneously in the same direction. In scenario 10, when major and minor ordering costs are increased while inventory holding cost is maintained, no significant change occurs in the optimal solution. In scenario 11, where major and minor ordering costs are decreased simultaneously and holding cost is maintained, significant change happens to raw material 2, but not to raw material 1. Volume of raw material 2 allocated to supplier 2 is increased from 0 to 29.46%, volume allocated to supplier 3 is decreased from 67.73% to 35.21%. Considering we have analyzed the effect of decreasing major ordering cost on the optimal solution of the problem, this result can still be explained that lower ordering cost would help supplier 2 become more attractive as it has
relatively higher ordering costs originally. Similar condition happens to scenario 13 where major and minor ordering costs are decreased and inventory holding cost is increased. In scenario 12, when major and minor ordering costs are increased and inventory holding cost is decreased, significant change happens to the allocation of raw material 1 and raw material 2. 61.03% of raw material 1 and 42.57% of raw material 2 is allocated to supplier 1, 38.97% of raw material 1 and 35.71% of raw material 2 is allocated to supplier 3, while 29.46% of raw material 2 is allocated to supplier 2. It can be explained as cheaper inventory holding cost makes supplier 1 and supplier 2 more attractive than before.

In general, when major ordering cost is decreased, the volume allocated to supplier 2 is increased and the volume allocated to supplier 3 is decreased. Similarly, when inventory holding cost is decreased, the volume allocated to supplier 1 and supplier 2 is increased and the volume allocated to supplier 3 is decreased.

Table 4-18: Sensitivity Analysis of the Optimal Solution, under Different Scenarios of Ordering and Holding Costs

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Setup Cost (%)</th>
<th>Holding (%)</th>
<th>Time (seconds)</th>
<th>Total Cost ($)</th>
<th>Raw Material 1</th>
<th>Raw Material 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Major</td>
<td>Minor</td>
<td></td>
<td></td>
<td>Supplier 1</td>
<td>Supplier 2</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>25</td>
<td>2,008,274</td>
<td>46.36%</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>100</td>
<td>150</td>
<td>15</td>
<td>2,231,000</td>
<td>46.19%</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>100</td>
<td>50</td>
<td>28</td>
<td>1,778,987</td>
<td>61.01%</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>150</td>
<td>100</td>
<td>19</td>
<td>2,026,069</td>
<td>46.41%</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>50</td>
<td>100</td>
<td>18</td>
<td>1,990,530</td>
<td>46.19%</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>0</td>
<td>100</td>
<td>12</td>
<td>1,972,744</td>
<td>46.24%</td>
</tr>
<tr>
<td>7</td>
<td>150</td>
<td>100</td>
<td>100</td>
<td>17</td>
<td>2,026,774</td>
<td>46.36%</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>100</td>
<td>100</td>
<td>11</td>
<td>1,988,214</td>
<td>50.00%</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>100</td>
<td>100</td>
<td>20</td>
<td>1,962,092</td>
<td>50.20%</td>
</tr>
<tr>
<td>10</td>
<td>150</td>
<td>150</td>
<td>100</td>
<td>13</td>
<td>2,004,556</td>
<td>46.19%</td>
</tr>
<tr>
<td>11</td>
<td>50</td>
<td>50</td>
<td>100</td>
<td>14</td>
<td>1,970,414</td>
<td>50.71%</td>
</tr>
<tr>
<td>12</td>
<td>150</td>
<td>50</td>
<td>50</td>
<td>24</td>
<td>1,817,481</td>
<td>61.03%</td>
</tr>
<tr>
<td>13</td>
<td>50</td>
<td>50</td>
<td>150</td>
<td>13</td>
<td>2,191,334</td>
<td>50.00%</td>
</tr>
</tbody>
</table>

(Note: significant changes are shown in grey bars)

We do similar sensitivity analysis toward the sequential approach model, then compare the result with the same scenario under the integrated approach, and obtain Table 4-19 as below.
From Table 4-19, significant differences are shown between sequential and integrated approaches in both running time and supplier selection schedule. We could find that under the sequential approach, from scenario 3 to scenario 11, although some parameters are changed individually, requirement for raw materials remains the same, such that allocation of orders from suppliers would not change. In scenarios 2, 12 and 13, as raw material requirement is changed according to production/distribution policies, supplier selection schedule is changed correspondingly.

In Table 4-20, “Transportation cost” is maintained (100%), decreased (50%), and increased (150%) for each supplier individually. Increase of transportation cost for supplier 1 (scenario 14), increase of transportation cost for supplier 2 (scenario 16) and decrease of
transportation cost for supplier 3 (scenario 19) produce slight change to the supplier selection strategy.

In scenario 15, when transportation cost for supplier 1 is decreased, the volume of raw material 1 allocated to supplier 1 is increased from 46.19% to 55.85%, and the volume allocated to supplier 3 is decreased from 53.81% to 44.15%. For raw material 2, the volume allocated to supplier 1 is increased from 32.27% to 38.93%, the volume allocated to supplier 2 is increased from 0 to 29.46%, and the volume allocated to supplier 3 is decreased from 67.73% to 31.61%. This can be explained as lower transportation cost for supplier 1 would make this supplier more attractive, and differences in production and distribution lot sizes would influence the choice of suppliers conversely, hence supplier 2 becomes a choice to provide raw materials.

In scenario 17, when transportation cost for supplier 2 is decreased, the volume of raw material 1 allocated to supplier 1 is increased from 46.19% to 56.15%, the volume allocated to supplier 2 is increased from 0 to 25.12%, and the volume allocated to supplier 3 is decreased from 53.64% to 18.73%. For raw material 2, the volume allocated to supplier 1 is increased from 32.27% to 39.14%, the volume allocated to supplier 2 is increased from 0 to 60.86%, while the volume allocated to supplier 3 is decreased to 0. Similar results happen to scenario 18 when transportation cost for supplier 3 is increased. The reason is that decreasing transportation cost for supplier 2 or increasing transportation cost for supplier 3 can make supplier 2 more attractive, and changes in production/distribution stages would influence supplier selection as a whole.

In general, if transportation cost for supplier 1 or supplier 2 is decreased, or transportation cost for supplier 3 is increased individually, raw materials allocated to supplier 1 and 2 would increase, while allocation to supplier 3 would decrease.

Table 4-20: Sensitivity Analysis of the Optimal Solution, under Different Scenarios of Transportation Cost

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Supplier 1 cost (%)</th>
<th>Supplier 2 cost (%)</th>
<th>Supplier 3 cost (%)</th>
<th>Time (seconds)</th>
<th>Total Cost ($)</th>
<th>Raw Material 1</th>
<th>Raw Material 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Supplier 1 Supplier 2 Supplier 3</td>
<td>Supplier 1 Supplier 2 Supplier 3</td>
<td>Supplier 1 Supplier 2 Supplier 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>100 100 100 25</td>
<td>2,008,274 46.36% 0.00% 53.64%</td>
<td>32.27% 0.00% 67.73%</td>
<td>14</td>
<td>150 100 100 23</td>
<td>2,026,013 46.19% 0.00% 53.81%</td>
<td>32.14% 0.00% 67.86%</td>
</tr>
<tr>
<td>15</td>
<td>50 100 100 28</td>
<td>1,988,889 55.85% 0.00% 44.15%</td>
<td>38.93% 29.46% 31.61%</td>
<td>16</td>
<td>100 150 100 27</td>
<td>2,008,262 46.36% 0.00% 53.64%</td>
<td>32.27% 0.00% 67.73%</td>
</tr>
<tr>
<td>17</td>
<td>100 50 100 22</td>
<td>1,995,027 56.15% 25.12% 18.73%</td>
<td>39.14% 60.86% 0.00%</td>
<td>18</td>
<td>100 100 150 20</td>
<td>2,016,996 56.15% 25.05% 18.80%</td>
<td>39.14% 60.86% 0.00%</td>
</tr>
<tr>
<td>19</td>
<td>100 100 50 29</td>
<td>1,983,770 46.19% 0.00% 53.81%</td>
<td>32.14% 0.00% 67.86%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Note: significant changes are shown in grey bars)
Minimum perfect rate is maintained (0.955), decreased (0.95), or increased (0.96, 0.965 and 0.97). And the results of supplier selection under each minimum perfect rate are shown from scenario 20 to scenario 23 in Table 4-21.

In scenario 20, when minimum perfect rate is decreased to 0.95, the volume of raw material 1 allocated to supplier 1 is increased from 46.36% to 69.66%, the volume of raw material 2 allocated to supplier 1 is increased from 32.27% to 65.00%, and same amount of decrease happens to supplier 3. This is mainly because the decrease of minimum perfect rate will not make the high-quality raw material provider (supplier 2) more attractive, but low-quality provider (supplier 1) could attract more customers as its large capacity of providing raw materials. In scenario 21, when minimum perfect rate is increased to 0.96, the volume of raw material 1 allocated to supplier 1 is decreased from 46.36% to 33.40%, same amount of increase happens to supplier 3. Volume of raw material 2 allocated to supplier 1 is decreased from 32.27% to 23.16%, the volume allocated to supplier 2 is increased from 0 to 23.30%, while the volume allocated to supplier 3 is decreased from 67.73% to 53.54%. In scenario 22, when minimum perfect rate is increased to 0.965, supplier 1 is no longer a good choice as its low perfect rate for both raw material 1 and raw material 2; supplier 2 can provide 17.42% of raw material 1 and 80.80% of raw material 2, while supplier 3 can provide 82.58% of raw material 1 and 19.20% of raw material 2. In scenario 23, when minimum perfect rate is 0.97, all raw material 1 is allocated to supplier 3, and all raw material 2 is allocated to supplier 2. This is mainly because only supplier 3 gets 0.97 perfect rate for raw material 1 and only supplier 2 gets 0.97 perfect rate for raw material 2.

In general, a change in the minimum perfect rate for one type of raw material for a supplier will lead to a change in the number of customers for it.

Table 4-21: Sensitivity Analysis of the Optimal Solution, under Different Scenarios of Minimum Perfect Rate

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Perfect Rate</th>
<th>Time (seconds)</th>
<th>Total Cost ($)</th>
<th>Material 1</th>
<th>Material 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Supplier 1</td>
<td>Supplier 2</td>
</tr>
<tr>
<td>20</td>
<td>0.95</td>
<td>24</td>
<td>2,003,970</td>
<td>69.66%</td>
<td>0.00%</td>
</tr>
<tr>
<td>1</td>
<td>0.955</td>
<td>25</td>
<td>2,008,274</td>
<td>46.36%</td>
<td>0.00%</td>
</tr>
<tr>
<td>21</td>
<td>0.96</td>
<td>23</td>
<td>2,032,010</td>
<td>33.40%</td>
<td>0.00%</td>
</tr>
<tr>
<td>22</td>
<td>0.965</td>
<td>18</td>
<td>2,106,030</td>
<td>0.00%</td>
<td>17.42%</td>
</tr>
<tr>
<td>23</td>
<td>0.97</td>
<td>12</td>
<td>2,142,148</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>
At initial statement of the example, only supplier 2 and supplier 3 have price discount intervals. Here we want to make supplier 1 having price discount intervals as well to see how it would influence supplier selection decisions. The price discount intervals we set are “9, 7, and 5” ($/unit) for raw material 1 and “8, 6, and 4” ($/unit) for raw material 2. Results under different price discount intervals are shown from scenario 24 to scenario 26 in Table 4-22.

In scenarios 24 and 25, when price discount interval happens to raw material 1 and raw material 2 individually, the volume of raw material 1 allocated to supplier 1 is increased from 46.36% to 55.82%, same amount of decrease happens to supplier 3. For raw material 2, the volume allocated to supplier 1 is increased from 32.27% to 38.91%, the volume allocated to supplier 2 is increased from 0 to 29.46%, and the volume allocated to supplier 3 is decreased from 67.73% to 31.63%.

In scenario 26, when both raw material 1 and raw material 2 have got price discount intervals simultaneously, the volume of raw material 1 allocated to supplier 1 is increased from 46.36% to 61.03%, same amount of decrease happens to supplier 3. For raw material 2, the volume allocated to supplier 1 is increased from 32.27% to 42.57%, the volume allocated to supplier 2 is increased from 0 to 30.70%, and the volume allocated to supplier 3 is decreased from 67.73% to 26.73%.

In general, price discount intervals make supplier 1 a more attractive supplier than before, and changes of production/distribution schedules would impact raw material procurement strategy as well, hence supplier 2 also becomes an alternative. Order of raw material 2 initially from supplier 3 would transfer to supplier 2.

Table 4-22: Sensitivity Analysis of the Optimal Solution under Different Price Intervals for Supplier 1

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Price Interval ($)</th>
<th>Total Cost ($)</th>
<th>Time (seconds)</th>
<th>Raw Material 1</th>
<th>Raw Material 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Material 1</td>
<td>Material 2</td>
<td></td>
<td># of Orders Supplier 1</td>
<td># of Orders Supplier 2</td>
</tr>
<tr>
<td>1</td>
<td>9,9,9</td>
<td>7,7,7</td>
<td>2,008,274</td>
<td>25</td>
<td>1 46.36%</td>
</tr>
<tr>
<td>24</td>
<td>9,9,9</td>
<td>8,6,4</td>
<td>1,997,288</td>
<td>18</td>
<td>1 55.82%</td>
</tr>
<tr>
<td>25</td>
<td>9,7,5</td>
<td>7,7,7</td>
<td>1,983,974</td>
<td>16</td>
<td>1 55.92%</td>
</tr>
<tr>
<td>26</td>
<td>9,7,5</td>
<td>8,6,4</td>
<td>1,970,210</td>
<td>15</td>
<td>1 64.03%</td>
</tr>
</tbody>
</table>

We change the initial discount quantities (measured in units) from (0, 1000, and 2000) to (0, 500, and 1000) and (0, 2000, and 4000) respectively. Results under different discount quantities are shown in scenarios 27 and 28 in Table 4-23 as below.
In scenario 27, where discount quantity is changed to (0, 500 and 1000), very little change is produced to the optimal solution. It seems shrinking discount quantity would not affect supplier selection. In scenario 28, where discount quantity is changed to (0, 2000 and 4000), slight change happens to the optimal solution. The volume of raw material 1 allocated to supplier 1 is decreased from 46.36% to 41.10%, and same amount of increase happens to supplier 3. For raw material 2, the volume allocated to supplier 1 is decreased from 32.27% to 28.57%, and same amount of increase happens to supplier 3. This can be explained that supplier 3, with lower price for larger orders, can attract more customers if discount quantity is increased.

It seems that increasing discount quantity would make supplier 3 become more attractive, while supplier 2 is still not considered into account.

**Table 4-23: Sensitivity Analysis of the Optimal Solution under Different Discount Quantities**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Quantity 1 (units)</th>
<th>Quantity 2 (units)</th>
<th>Quantity 3 (units)</th>
<th>Time (seconds)</th>
<th>Total Cost ($)</th>
<th>Raw Material 1 Supplier 1</th>
<th>Raw Material 1 Supplier 2</th>
<th>Raw Material 1 Supplier 3</th>
<th>Raw Material 2 Supplier 1</th>
<th>Raw Material 2 Supplier 2</th>
<th>Raw Material 2 Supplier 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0~1000</td>
<td>1000~2000</td>
<td>&gt;2000</td>
<td>25</td>
<td>2,008,274</td>
<td>46.36%</td>
<td>0.00%</td>
<td>53.64%</td>
<td>32.27%</td>
<td>0.00%</td>
<td>67.73%</td>
</tr>
<tr>
<td>27</td>
<td>0~500</td>
<td>500~1000</td>
<td>&gt;1000</td>
<td>20</td>
<td>2,008,274</td>
<td>46.36%</td>
<td>0.00%</td>
<td>53.64%</td>
<td>32.27%</td>
<td>0.00%</td>
<td>67.73%</td>
</tr>
<tr>
<td>28</td>
<td>0~2000</td>
<td>2000~4000</td>
<td>&gt;4000</td>
<td>29</td>
<td>2,017,945</td>
<td>41.10%</td>
<td>0.00%</td>
<td>58.90%</td>
<td>28.57%</td>
<td>0.00%</td>
<td>71.43%</td>
</tr>
</tbody>
</table>

(Note: significant change is shown in grey bar)

In general, decreasing inventory holding cost, decreasing major ordering cost, change of transportation cost for suppliers, change of minimum perfect rate, introducing price discount intervals and increasing discount quantities have significant influence on optimal raw material procurement schedule under the integrated approach. Even though it may appear that production and distribution lot sizing decisions are unrelated to raw material procurement schedule, selection of suppliers is affected by the changes in production/distribution stages. It is clear that both decisions are closely related, and the best way is to analyze them integrally.
Chapter 5 Conclusions and Future Work

5.1 Conclusions

This thesis introduces a new mixed integer linear programming formulation for the multiproduct dynamic inventory problem with supplier selection in a serial supply chain structure. Raw material procurement and production and distribution decisions can both be analyzed simultaneously, or sequentially (production/distribution problem first, raw material procurement problem second). The problem has been analyzed in the context of a dynamic joint replenishment problem, where major and minor ordering costs are incurred every time an order is placed at the raw material procurement and manufacturing stages. An all-unit price discount policy is adopted when ordering raw materials, economies of scale associated with larger ordering lot sizes and the logistic and economic advantages by coordinating product replenishment orders are considered in the model as well. We regard inventory holding costs as on-stage and in-transit inventory holding costs, which are both linear functions. Full-truck-load is the only transportation mode between stages.

Two types of raw materials can be manufactured into two types of final products with a deterministic dynamic demand over a planning horizon of five time periods have been considered as part of the illustrative example. Thus, by considering the entire supply chain behavior, it is possible to evaluate the tradeoff between ordering and variable costs under the two approaches, and then select a better strategy.

According to the results described in this paper, although the integrated approach needs relatively longer running time to obtain solutions, the total cost under the integrated approach is decreased with respect to the sequential approach. And the differentiation between the two approaches in total costs is mainly reflected in variable cost. Even if the sequential approach reduces ordering costs by increasing production and distribution lot sizes, significant cost in inventory holding is incurred this way, and raw material purchasing cost is increased as raw material procurement decisions are not considered initially. While in real industry, sequential approach model would be more common and achievable as it requires less information sharing
and can decrease the risk of lacking safety stock. Some companies may choose the integrated approach by making a tradeoff in decreasing total cost for the entire supply chain.

From the results of sensitivity analysis described in the thesis, inventory holding cost and major ordering cost seem to play quite important role in supplier selection. When inventory holding cost and major ordering cost decrease simultaneously, suppliers with cheaper costs cannot attract as many customers as before. Same situation happens when we change supplier transportation cost, decreasing transportation cost will attract more customers for that supplier. Besides, suppliers with price quantity discount have more attractions to manufacturing factories than those without quantity discount. A change in the minimum perfect rate for one type of raw material for a supplier will lead to a change in the number of customers for it. Increasing discount quantities can help the supplier with relatively cheaper price to obtain larger orders.

5.2 Future Work

Further research could consider the effect of variability in some key parameters like supplier lead time, supplier and inventory holding capacity, and transportation costs between stages under both integrated and sequential approaches. When ordering raw materials, besides all-unit discount strategy, piecewise-unit discount strategy can also be considered into the model, hence we could compare purchasing cost as well as the total cost under the two discount strategies. Similar condition happens to transportation modes, we only consider full-truck-load transportation mode in this thesis, while less than full-truck-load can be taken into account as well. Different functions of inventory holding costs can be researched, so that we could obtain a comprehensive idea of which function behaves best under particular situation. Alternative objective of this problem can also transfer from a cost-minimization problem to a profit-maximization problem. And we can broaden the model from a serial distribution network to a non-serial one. Also, in order to correspond to all kinds of situations, we could try to make customer demand stochastic other than deterministic. Linear or quadratic model for demand function could be researched, and robustness of the model can be measured in this way.
References


